PHILOSOPHICAL

TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCXXXVII.

PART I.

LONDON:
PRINTED BY RICHARD AND JOHN E. TAYLOR, RED LION COURT, FLEET STREET.
MDCCCXXXVII.

ADVERTISEMENT.

The Committee appointed by the *Royal Society* to direct the publication of the *Philosophical Transactions*, take this opportunity to acquaint the Public, that it fully appears, as well from the Council-books and Journals of the Society, as from repeated declarations which have been made in several former *Transactions*, that the printing of them was always, from time to time, the single act of the respective Secretaries till the Forty-seventh Volume; the Society, as a Body, never interesting themselves any further in their publication, than by occasionally recommending the revival of them to some of their Secretaries, when, from the particular circumstances of their affairs, the *Transactions* had happened for any length of time to be intermitted. And this seems principally to have been done with a view to satisfy the Public, that their usual meetings were then continued, for the improvement of knowledge, and benefit of mankind, the great ends of their first institution by the Royal Charters, and which they have ever since steadily pursued.

But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed, to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions*; which was accordingly done upon the 26th of March 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings, contained in the several papers so published, which must still rest on the credit or judgement of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon any subject, either of Nature or Art, that comes before them. And therefore the thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

A List of Public Institutions and Individuals, entitled to receive a copy of the Philosophical Transactions of each year, on making application for the same directly or through their respective agents, within five years of the date of publication.

In the British Dominions.

The King's Library.
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The Royal Geographical Society.
The United Service Museum.
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The Society of Antiquaries.
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The Society for the Encouragement of Arts.
The Geological Society.
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The Royal Astronomical Society.
The Royal Asiatic Society.
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The Medical and Chirurgical Society.
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The Observatory at Armagh.
The Observatory at the Cape of Good Hope.
The Observatory at Madras.
The Observatory at St. Helena.
The Observatory at Paramatta.

Denmark.

The Royal Society of Sciences at Copenhagen.
The Royal Observatory at Altona.

France.

The Royal Academy of Sciences at Paris.
The Royal Academy of Sciences at Thoulouse.
The E'cole des Mines at Paris.
The Geographical Society at Paris.
The Entomological Society of France.
The Dépôt de la Marine, Paris.
The Geological Society of France.
The Jardin des Plantes, Paris.

Germany.

The University at Göttingen.
The Cæsarean Academy of Naturalists at Bonn.
The Observatory at Manheim.

Italy.

The Italian Society of Sciences at Modena.
The Royal Academy of Sciences at Turin.

Switzerland.

The Société de Phys. et d'Hist. Nat. at Geneva.

Belgium.

The Royal Academy of Sciences at Brussels.

Netherlands.

The Royal Institute of Amsterdam.

Spain.

The Royal Observatory at Cadiz.

Portugal.

The Royal Academy of Sciences at Lisbon.

Prussia.

The Royal Academy of Sciences at Berlin.

Russia.

The Imperial Academy of Sciences at St. Petersburgh.

Sweden and Norway.

The Royal Academy of Sciences at Stockholm.
The Royal Society of Sciences at Drontheim.

United States.

The American Philosophical Society at Philadelphia.
The New York Philosophical Society.
The American Academy of Sciences at Boston.
The Library of Harvard College.

The *fifty* Foreign Members of the Royal Society.

A List of Public Institutions and Individuals, entitled to receive a copy of the Astronomical Observations made at the Royal Observatory at Greenwich, on making application for the same directly or through their respective agents, within five years of the date of publication.

In the British Dominions.

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The Board of Ordnance.
The British Museum.
The Royal Society.
The Bodleian Library, Oxford.
The Savilian Library, Oxford.
The Library of Trinity College, Cambridge.
The King's Observatory at Richmond.
The Royal Observatory at Greenwich.
The University of Aberdeen.
The University of St. Andrews.
The University of Dublin.
The University of Edinburgh.
The University of Glasgow.
The Observatory at Oxford.
The Observatory at Cambridge.
The Observatory at Dublin.
The Observatory at Armagh.
The Observatory at the Cape of Good Hope.
The Observatory at Paramatta.
The Observatory at Madras.
The Observatory at St. Helena.
The Royal Institution of Great Britain.
The Royal Society, Edinburgh.
The Astronomical Institution, Edinburgh.
The President of the Royal Society.
The Lowndes's Professor of Astronomy, Cambridge.
The Plumian Professor of Astronomy, Cambridge.
Francis Baily, Esq. V.P. and Treas. R.S.
Thomas Henderson, Esq. of Edinburgh.
John William Lubbock, Esq.
Captain W. H. Smyth, R.N. of Bedford.
Sir James South, Observatory, Kensington.
Lieutenant Stratford, R.N.

In Foreign Countries.

The Royal Academy of Sciences at Berlin.
The Royal Academy of Sciences at Paris.
The Imperial Academy of Sciences at St. Petersburgh.
The Royal Academy of Sciences at Stockholm.
The Royal Society of Sciences at Upsal.
The Board of Longitude of France.
The University of Göttingen.
The University of Leyden.
The Academy of Bologna.
The American Academy of Sciences at Boston.
The American Philosophical Society at Philadelphia.
The Library of Harvard College.
The Observatory at Helsingfors.
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The Observatory at Berlin.
The Observatory at Brussels.
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The Observatory at Tubingen.
The Observatory at Wilna.
Professor Bessel, of Königsberg.
Dr. William Olbers, of Bremen.
The Dépôt de la Marine, Paris.
The Bowden College, United States.
The Waterville College, United States.

ROYAL MEDALS.

HIS MAJESTY KING WILLIAM THE FOURTH, in restoring the Foundation of the Royal Medals, graciously commanded a Letter, of which the following is an extract, to be addressed to the Royal Society, through His Royal Highness the DUKE OF SUSSEX, K.G., President:

"Windsor Castle, March 25, 1833.

"It is HIS MAJESTY'S wish,—

"First, That the Two Gold Medals, value of Fifty Guineas each, shall "henceforth be awarded on the day of the Anniversary Meeting of the "Royal Society, on each ensuing year, for the most important discoveries "in any one principal subject or branch of knowledge.

"Secondly, That the subject matter of inquiry shall be previously settled "and propounded by the Council of the Royal Society, three years pre-"ceding the day of such award.

"Thirdly, That Literary Men of all nations shall be invited to afford the "aid of their talents and research: and,

"Fourthly, That for the ensuing three successive years, the said Two "Medals shall be awarded to such important discoveries, or series of in-"vestigations, as shall be sufficiently established, or completed to the "satisfaction of the Council, within the last five years of the days of award, "for the years 1834 and 1835, including the present year, and for which "the Author shall not have previously received an honorary reward.

(Signed) "H. TAYLOR."

The Royal Medals for the year 1833 were awarded to

Sir JOHN FREDERICK WILLIAM HERSCHEL, K.H. F.R.S.,

for his Paper on the Investigation of the Orbits of Revolving Double Stars; and to

Professor AUGUSTE PYRAME DE CANDOLLE, of Geneva, Foreign Member of the Royal Society,

for his Discoveries and Investigations in Vegetable Physiology.

Those for 1834 were awarded to

JOHN WILLIAM LUBBOCK, Esq., V.P. & Treas. R.S.,

for his Papers on the Tides published in the Philosophical Transactions; and to

CHARLES LYELL, Esq., F.R.S.,

for his Work entitled "Principles of Geology."

Those for 1835 were awarded to

MICHAEL FARADAY, D.C.L., F.R.S.,

for his Investigations and Discoveries contained in the Series of Experimental Researches in Electricity, published in the Philosophical Transactions, and more particularly for the Seventh Series, relating to the definite nature of electro-chemical action; and to

Sir WILLIAM ROWAN HAMILTON, Andrews' Professor of Astronomy in the University of Dublin, and Royal Astronomer of Ireland,

for the Papers published by him in the 16th and 17th volumes of the Transactions of the Royal Irish Academy, entitled "Supplement to an Essay on the Theory of "Systems of Rays," and more particularly for those Investigations at the conclusion of the third and last Supplement, which relate to the discovery of Conical Refraction.

The Council propose to give one of the Royal Medals in the year 1836, to the most important unpublished paper in Astronomy, communicated to the Royal Society for insertion in their Transactions, after the present date (May 13th, 1833,) and prior to the month of June in the year 1836.

The Council also propose to give one of the Royal Medals in the year 1836 to the most important unpublished paper in Animal Physiology, communicated to the Royal Society for insertion in their Transactions, after the present date (May 13th, 1833,) and prior to the month of June in the year 1836.

The Council propose to give one of the Royal Medals in the year 1837 to the most important unpublished paper in Physics, communicated to the Royal Society for insertion in their Transactions, after the present date (November 27th, 1834,) and prior to the month of June in that year.

The Council also propose to give one of the Royal Medals in the year 1837 to the author of the best paper, to be entitled "Contributions towards a System of Geological Chronology founded on an examination of fossil remains, and their attendant "phenomena," such paper to be communicated to the Royal Society after the present date (December 1st, 1834,) and prior to the month of June 1837:—but in case no paper is presented to the Society fulfilling the conditions implied by the above Resolution, or possessing sufficient merit, the Council propose to give one of the Royal Medals in the year 1837 to the author of the best paper in Geology and Mineralogy, communicated to the Royal Society for insertion in their Transactions after the present date and prior to the month of June in that year.

The Council propose to give one of the Royal Medals in the year 1838 to the most important unpublished paper on Chemistry, communicated to the Royal Society for insertion in their Transactions, after the present date (November 12th, 1835,) and prior to the month of June 1838.

The Council also propose to give one of the Royal Medals in the year 1838 to the most important unpublished paper in Physics, communicated to the Royal Society for insertion in the Philosophical Transactions, after the present date (November 19, 1835,) and prior to the month of June 1838.

Those for 1836 were awarded to

SIR JOHN FREDERICK WILLIAM HERSCHEL, K.H. F.R.S.,

for his Papers on Nebulæ and Clusters of Stars, published in the Philosophical Transactions for 1833; and to

GEORGE NEWPORT, ESQ.,

for his Series of Investigations on the Anatomy and Physiology of Insects, contained in his two Papers published in the Philosophical Transactions within the last three years.

The Council propose to give one of the Royal Medals in the year 1839 to the most important unpublished Paper in Astronomy, communicated for insertion in their Transactions after the present date, (November 30th, 1836,) and prior to the termination of the Session in June 1839.

The Council also propose to give one of the Royal Medals in the year 1839 to the most important unpublished Paper in Physiology, communicated for insertion in their Transactions after the present date, (November 30th, 1836,) and prior to the termination of the Session in June 1839.

The Council have appointed Mr. H. F. Talbot's Paper, entitled "Further Observations on the Optical Phenomena of Crystals," the Bakerian Lecture for the present year.

ADJUDICATION of the MEDALS of the ROYAL SOCIETY by HIS ROYAL HIGHNESS the PRESIDENT and COUNCIL.

The COPLEY MEDAL, for the year 1833, was not awarded.

A COPLEY MEDAL, for the year 1834, to Professor PLANA, of Turin, for his work, entitled, "Théorie du Mouvement de la Lune."

The RUMFORD MEDAL, for the year 1835, to M. MELLONI, for his discoveries relative to Radiant Heat.

A COPLEY MEDAL, for the year 1835, to WILLIAM SNOW HARRIS, Esq., F.R.S., for his "Experimental Investigations of the Forces of Electricity of High Intensity," contained in his paper published in the Philosophical Transactions for the year 1834.

A COPLEY MEDAL, for the year 1836, to Professor BERZELIUS, for his Systematic Application of the Doctrine of Definite Proportions to the Analysis of Mineral Bodies, as contained in his "Nouveau Systême de Minéralogie," and in other of his works.

A COPLEY MEDAL, for the year 1836, to FRANCIS KIERNAN, Esq., F.R.S., for his Discoveries relating to the Structure of the Liver, as detailed in his paper communicated to the Royal Society, and published in the Philosophical Transactions for 1833.

A COPLEY MEDAL, for the year 1837, to M. BECQUEREL, for his various Memoirs on the subject of Electricity, published in his "Mémoires de l'Académie Royale des Sciences de l'Institut de France," and particularly those on the Production of Crystals of Metallic Sulphurets and Sulphur, by the long-continued action of Electricity of very low tension, and published in the tenth volume of those Mémoires.

Another COPLEY MEDAL, for the year 1837, to JOHN FREDERICK DANIELL, Esq., F.R.S., for his two papers "On Voltaic Combinations," published in the Philosophical Transactions for 1836.

The ROYAL MEDAL, in the department of Physics, for the year 1837, to the Rev. WILLIAM WHEWELL, M.A., F.R.S., for his "Researches connected with the Theory of the Tides," communicated to the Royal Society, and published in its Transactions within the last three years.

The ROYAL MEDAL, in the department of Geology, for the year 1837, was not awarded.

The Council propose to give one of the ROYAL MEDALS in the year 1838 for the most important unpublished paper on Chemistry, &c.

The Council also propose to give one of the ROYAL MEDALS in the year 1838 for the most important unpublished paper on Mathematics, &c.

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APPENDIX.

Meteorological Journal kept at the Apartments of the Royal Society, by order of the President and Council.

ERRATUM.

In announcing the Prize Questions for the Royal Medals to be given in the year 1838, as published in the Philosophical Transactions for 1835, Part II.; 1836, Parts I. and II.; and 1837, Part I.; for "*Physics*" read "*Mathematics.*"

PHILOSOPHICAL TRANSACTIONS.

I. *Researches in the Integral Calculus.—Part* II. *By* H. F. Talbot, *Esq. F.R.S.*

Received October 26,—Read November 17, 1836.

§ 1.

HAVING explained a general method of finding the sums of integrals, I propose to apply it to discover the properties of different transcendents, beginning with those of the simplest nature.

In the first place, therefore, I will show its application to the arcs of the circle and conic sections.

As there will be frequent occasion to make use of cubic equations, I shall suppose their general form to be

$$x^3 - p\,x^2 + q\,x - r = 0.$$

When therefore the letters $p\ q\ r$ occur without explanation, it will be understood that they represent these coefficients.

§ 2. *Application to the Circle.*

Let us take the integral $\int \frac{d\,x}{1 + x^2}$, and suppose nothing to be previously known concerning the properties of the function which it represents.

Let us put, in the first place,

$$1 + x^2 = v\,x \quad \therefore x^2 - v\,x + 1 = 0.$$

The two variables $x\ y$ will be roots of this equation, so that they must satisfy the condition $x\,y = 1$. Also

$$\frac{d\,x}{1 + x^2} + \frac{d\,y}{1 + y^2} = \frac{d\,x}{v\,x} + \frac{d\,y}{v\,y} = \frac{1}{v}\,\mathrm{S}\,\frac{d\,x}{x} = 0,$$

because $\mathrm{S}\,\frac{d\,x}{x} = 0$ in any equation whose last term is constant.

$$\therefore \int \frac{d\,x}{1 + x^2} + \int \frac{d\,y}{1 + y^2} = \text{const.}$$

We thus obtain a characteristic property of the function

$$\int \frac{d\,x}{1+x^2} = f.x,$$

namely, that if $x\,y = 1$,

$$f\,x + f\,y = \text{const.}$$

The truth of which is otherwise evident, for if $x = \tan\theta$, then $y = \text{cotan}\,\theta$

$$f\,x = \theta \text{ and } f\,y = 90° - \theta.$$

$$\therefore f\,x + f\,y = \text{const.}$$

Next let us investigate such a relation between *three* of these integrals that they may have an algebraic sum.

Assume $\frac{1}{1+x^2} = \frac{v+x}{a\,x}$, whence

$$x^3 + v\,x^2 + (1-a)\,x + v = 0,$$

where a is any constant quantity.

The three variables $x\;y\;z$ must be roots of this equation, which however gives only one necessary relation between them, viz.

$$x + y + z = x\,y\,z.$$

We have

$$\frac{a}{1+x^2} = \frac{v}{x} + 1$$

$$\therefore a\,\text{S}\frac{d\,x}{1+x^2} = v\,\text{S}\frac{d\,x}{x} + \text{S}\,d\,x.$$

But $\text{S}\,\frac{d\,x}{x} = \frac{d\,v}{v}$, and $\text{S}\;d\,x = -\,d\,v$,

$$\therefore a\,\text{S}\frac{d\,x}{1+x^2} = d\,v - d\,v = 0,$$

$$\therefore \text{S}\int\frac{d\,x}{1+x^2} = \text{const.}$$

whence we obtain this well-known theorem in trigonometry.

If the sum of three tangents equals their product, the sum of the arcs is constant. The constant = 180°.

Next let us suppose

$$\frac{1}{1+x^2} = \frac{v+x}{a\,x^2}$$

$$\therefore x^3 + (v-a)\,x^2 + x + v = 0.$$

This gives only one necessary relation between the roots, viz.

$$q = x\,y + x\,z + y\,z = 1.$$

For the two other coefficients $(v-a)$ and v, may be made to agree with any two arbitrary quantities. Since we have

$$\frac{a}{1+x^2} = \frac{v}{x^2} + \frac{1}{x}$$

$$\therefore a\,\mathrm{S}\frac{d\,x}{1+x^2} = v\,\mathrm{S}\frac{d\,x}{x^2} + \mathrm{S}\frac{d\,x}{x}.$$

But $\mathrm{S}\,\frac{1}{x} = \frac{q}{r}$, and here

$$q = 1 \text{ and } r = -v$$

$$\therefore \mathrm{S}\frac{1}{x} = -\frac{1}{v},$$

whence

$$\mathrm{S}\frac{d\,x}{x^2} = -\frac{d\,v}{v^2}$$

and

$$v\,\mathrm{S}\frac{d\,x}{x^2} = -\frac{d\,v}{v}.$$

Also

$$\mathrm{S}\frac{d\,x}{x} = \frac{d\,v}{v}.$$

Therefore

$$a\,\mathrm{S}\frac{d\,x}{1+x^2} = -\frac{d\,v}{v} + \frac{d\,v}{v} = 0$$

$$\therefore \mathrm{S}\int\frac{d\,x}{1+x^2} = \text{const.},$$

which furnishes this other well-known theorem, viz. *If three tangents are such that the sum of their products* = 1, *then the sum of the arcs is constant.* The constant in this case = 90°.

The same theorem results from the supposition

$$\frac{1}{1+x^2} = v + a\,x;$$

for this gives

$$x^3 + \frac{v}{a}x^2 + x + \frac{v-1}{a} = 0,$$

and $q = 1$ is the only necessary relation between the roots. Also

$$\mathrm{S}\frac{d\,x}{1+x^2} = v\,\mathrm{S}\,d\,x + a\,\mathrm{S}\,x\,d\,x.$$

But

$$\mathrm{S}\,x^2 = p^2 - 2\,q = \frac{v^2}{a^2} - 2,$$

whence

$$\mathrm{S}\,x\,d\,x = \frac{v\,d\,v}{a^2}.$$

Also

$$\mathrm{S}\,d\,x = -\frac{d\,v}{a}.$$

Therefore

$$\mathrm{S}\frac{d\,x}{1+x^2} = -\frac{v\,d\,v}{a} + \frac{v\,d\,v}{a} = 0. \qquad \textit{Q.E.D.}$$

These theorems, and the analogous ones which exist between any number of tangents, are well known. But when we apply the method to the integral $\int \frac{dx}{\sqrt{1-x^2}}$, we obtain relations between circular arcs which appear to be of a more novel description, and perhaps have not hitherto been noticed. Of which I will proceed to give an example. Let us suppose

$$\frac{1}{\sqrt{1-x^2}} = v\,x + 1,$$

whence

$$x^3 + \frac{2}{v}x^2 + \left(\frac{1}{v^2} - 1\right)x - \frac{2}{v} = 0.$$

In this instance the symmetrical $v = \frac{2}{r}$, and therefore making this substitution, we have

$$x^3 + r\,x^2 + \left(\frac{r^2}{4} - 1\right)x - r = 0.$$

There are therefore two necessary relations between the three roots, viz.

$$p = -r \qquad q = \frac{r^2}{4} - 1.$$

And since

$$\frac{1}{\sqrt{1-x^2}} = v\,x + 1$$

$$\mathrm{S}\,\frac{dx}{\sqrt{1-x^2}} = v\,\mathrm{S}\,x\,dx + \mathrm{S}\,dx.$$

But

$$\mathrm{S}\,x^2 = p^2 - 2\,q = r^2 - \left(\frac{r^2}{2} - 2\right) = \frac{r^2}{2} + 2$$

$$\therefore \mathrm{S}\,x\,dx = \frac{r\,dr}{2}$$

$$v\,\mathrm{S}\,x\,dx = \frac{2}{r}\cdot\frac{r\,dr}{2} = dr.$$

Also

$$\mathrm{S}\,dx = -\,dr$$

$$\therefore \mathrm{S}\,\frac{dx}{\sqrt{1-x^2}} = dr - dr = 0;$$

whence this theorem:

If the sines of three circular arcs are roots of the equation

$$x^3 + r\,x^2 + \left(\frac{r^2}{4} - 1\right)x - r = 0,$$

the sum of the arcs is constant.

I will give a numerical example of this theorem.

The value of r is arbitrary. Suppose it to be

$$= 3 - \sqrt{12} = -\,0{\cdot}4641016.$$

The roots of the equation then have the following values:

$$\begin{aligned} x &= 0{\cdot}5 = \sin 30^\circ &&= \sin\theta \\ y &= 0{\cdot}94565 = \sin 71^\circ\, 1' &&= \sin\theta' \\ z &= -\,0{\cdot}98154 = \sin -(78^\circ\, 59') &&= \sin\theta''; \end{aligned}$$

and the theorem gives the sum of the arcs, or $S\,\theta = \text{const.}$ The word *sum* is used in an algebraic sense, as including the case where one or more of the arcs are to be taken *negatively*, or its definition is

$$S\,\theta = \pm\,\theta \pm \theta' \pm \theta''.$$

The same ambiguity in the signs pervades the whole of this class of formulæ. In the present instance

$$\begin{aligned} S\,\theta &= \theta + \theta' - \theta'' \\ &= 30^\circ + 71^\circ\, 1' + 78^\circ\, 59' = 180^\circ \end{aligned}$$

$\therefore$ the constant is a semicircle.

Ex. 2. Let $r = 0$.

$$\therefore x^3 - x = 0;$$

and the roots are

$$\begin{aligned} x &= 0 &&= \sin 0^\circ \\ y &= 1 &&= \sin 90^\circ \\ z &= -\,1 &&= \sin -90^\circ \end{aligned}$$

$$\therefore \theta = 0^\circ \quad \theta' = 90^\circ \quad \theta'' = -\,90^\circ;$$

and the same formula gives, as before,

$$\theta + \theta' - \theta'' = 180^\circ.$$

A very extensive class of formulæ respecting the arcs of the circle may be obtained in a similar manner, by applying the method more generally. Thus, if we make the supposition

$$[1.] \qquad \frac{1}{\sqrt{1 - x^2}} = a_0 + a_1\, x + \ldots\ldots + a_{n-1}\, x^{n-1},$$

where $a_0, a_1, \ldots\ldots a_{n-1}$ are constants, or any entire rational functions whatever of the variable v, we have an equation of $2\,n$ dimensions, of which x is a root.

If $x = \sin\theta_{{}_{\prime}}$, and the other roots are $\sin\theta_2$, $\sin\theta_3, \ldots\ldots\ldots \sin\theta_{2n}$, then

$$\int \frac{d\,x}{\sqrt{1 - x^2}} = \theta_{{}_{\prime}},$$

and the other integrals $= \theta_2, \theta_3 \ldots \theta_{2n}$. And by a direct process we obtain the final equation

$$S\,\theta, \text{ or } \theta_1, + \theta_2 + \ldots\ldots + \theta_{2n} = f.\,v + \text{const.},$$

$f.\,v$ being an entire rational function of v.

But since it is generally admitted that no combination of circular arcs can be equal to an algebraic quantity, I conclude that we have generally

$$f.\,v = 0.$$

If we consider the generality of the supposition [1.], which admits any number of arbitrary quantities, it certainly appears remarkable that this equation $f.v = 0$ should be always verified.

§. 3. *Application to the Parabola.*

If the tangent at the vertex of a parabola be taken for the axis of abscissæ, and the semiparameter $= 1$, then if x be the abscissa, the equation of the curve will be

$$2\,y = x^2,$$

and the arc, which may be designated as arc x,

$$= \int d\,x\,\sqrt{1+x^2}.$$

The known value of this is

$$[2.] \qquad \text{Arc}\,x = \tfrac{1}{2}\,x\sqrt{1+x^2} + \tfrac{1}{2}\log\left(x+\sqrt{1+x^2}\right).$$

This is a function of x, the properties of which appear to have been hitherto little examined. I will establish two theorems concerning it, which are of considerable simplicity.

Theorem I.—*If three abscissæ are the roots of the equation*

$$x^3 - r\,x^2 + \left(\frac{r^2}{4} + 1\right)x - r = 0,$$

the sum of the arcs equals the sum of the abscissæ.

Since each arc is greater than its corresponding abscissa, it is evident that the word *sum* is to be understood in an algebraic sense, or that at least one of the arcs must be taken negatively.

Theorem II.—*If three abscissæ are the roots of the equation*

$$x^3 - a\,x^2 + \left(\frac{a^2}{4} - a\,b - \frac{a^2\,b^2}{4}\right)x + \frac{a^2\,b}{2} = 0,$$

the sum of the arcs equals the product of the abscissæ.

This theorem is remarkable for its simplicity, when it is considered that it contains two arbitrary quantities, a and b, which, as it appears, may have any values.

Demonstration of Theorem I.

Put $\sqrt{1+x^2} = x^2 + v\,x + 1$: whence

$$[1.] \qquad x^3 + 2\,v\,x^2 + (v^2+1)\,x + 2\,v = 0.$$

Also

$$\text{S}\,d\,x\sqrt{1+x^2} = \text{S}\,x^2\,d\,x + v\,\text{S}\,x\,d\,x + \text{S}\,d\,x$$

$$\therefore \text{S}\int d\,x\sqrt{1+x^2} = \frac{\text{S}\,x^3}{3} + \int v\,\text{S}\,x\,d\,x + \text{S}\,x.$$

The first term $\frac{\text{S}\,x^3}{3} = \frac{p^3}{3} - p\,q + r$ in all equations. Here $p = -2\,v \quad q = v^2 + 1$ $r = -2\,v$.

$$\therefore \frac{\mathrm{S}\, x^3}{3} = \frac{-8\, v^3}{3} + 2\, v\, (v^2 + 1) - 2\, v = -\frac{2\, v^3}{3}.$$

To find the value of the second term $\int v\, \mathrm{S}\, x\, d\, x$, we have

$$\mathrm{S}\, x^2 = p^2 - 2\, q = 4\, v^2 - (2\, v^2 + 2) = 2\, v^2 - 2$$

$$\therefore \mathrm{S}\, x\, d\, x = 2\, v\, d\, v$$

and

$$\int v\, \mathrm{S}\, x\, d\, x = \int 2\, v^2\, d\, v = \frac{2\, v^3}{3}.$$

Therefore these two terms destroy each other. Consequently we have simply

$$\mathrm{S} \int d\, x \sqrt{1 + x^2} = \mathrm{S}\, x + \mathrm{C}.$$

It appears by trial that $\mathrm{C} = 0$, and the equation between the roots [1.] becomes, by writing for v its value $\frac{-r}{2}$,

$$x^3 - r\, x^2 + \left(\frac{r^2}{4} + 1\right) x - r = 0$$

$\therefore$ the sum of three arcs $= \mathrm{S}\, x = r$. *Q.E.D.*

Example.—Let us suppose $r = 4 + 2\sqrt{2}$
$= 6{\cdot}828427$.

The three roots will be

$$x = 1$$
$$y = 4{\cdot}2042580$$
$$z = 1{\cdot}6241690.$$

Calculating the arcs accurately by the formula [2.], we have

$$\text{Arc } x = 1{\cdot}147793$$
$$\text{Arc } y = 10{\cdot}156004$$
$$\text{Arc } z = 2{\cdot}179773$$

In forming the sum we must notice that arc x and arc z are to be accounted negative. Consequently we find *by subtraction*,

$$\begin{aligned} \text{Arc } y &= 10{\cdot}156004 \\ \text{Arc } x + \text{Arc } z &= 3{\cdot}327566 \\ \hline \text{Sum} &= 6{\cdot}828438 \\ r &= 6{\cdot}828427 \\ \hline \text{Error of calculation} &= 0{\cdot}000011 \end{aligned}$$

Thus the calculation verifies the theorem with considerable exactness, and shows that no constant is required to be added to the integral.

Since the sum of these three arcs is algebraic, and that each contains a logarithmic part, the sum of these three logarithms must be $= 0$: for if not, it must be an algebraic quantity, which is considered to be impossible. This is verified by calculation; for

$$2 \text{ arc } x = x\sqrt{1+x^2} + \log(x + \sqrt{1+x^2}).$$

Calling $\log(x+\sqrt{1+x^2}) = f.x$, we have

$$\begin{array}{rlrl} fx = & 0{\cdot}881372 & f.y = & 2{\cdot}143099 \\ fz = & 1{\cdot}261722 & fx + fz = & 2{\cdot}143094 \\ \hline & 2{\cdot}143094 & \text{sum} = & 0{\cdot}000005 \end{array}$$

This sum approaches zero very nearly. The quantities fx, fz are subtractive, being parts of 2 arc x and 2 arc z, which have been already shown to be so.

Demonstration of Theorem II.

Let $v\sqrt{1+x^2} = n\,x^2 + x + v$, where n is a constant,

$$\therefore\ x^3 + \frac{2}{n}\,x^2 + \frac{1 + 2\,v\,n - v^2}{n^2}\,x + \frac{2\,v}{n^2} = 0,$$

and

$$v\,\mathrm{S}\sqrt{1+x^2}\,.\,d\,x = n\,\mathrm{S}\,x^2\,d\,x + \mathrm{S}\,x\,d\,x,$$

the term $v\,\mathrm{S}\,d\,x$ being omitted; because, since $\mathrm{S}\,x = \frac{-2}{n}$ is constant, the factor

$$\mathrm{S}\,d\,x = 0.$$

The formula $\mathrm{S}\,x^3 = p^3 - 3\,p\,q + 3\,r$ gives

$$\frac{\mathrm{S}\,x^3}{3} = \frac{p^3}{3} - p\,q + r,$$

$\therefore$ $\left(\text{observing that } p \text{ is constant and } = -\frac{2}{n}\right)$ $\mathrm{S}\,x^2\,d\,x = \frac{2}{n}\,d\,q + d\,r.$ Therefore the first term, or

$$n\,\mathrm{S}\,x^2\,d\,x = 2\,d\,q + n\,d\,r.$$

The formula $\mathrm{S}\,x^2 = p^2 - 2\,q$ gives the second term, or

$$\mathrm{S}\,x\,d\,x = -\,d\,q$$

$$\therefore\ n\,\mathrm{S}\,x^2\,d\,x + \mathrm{S}\,x\,d\,x = d\,q + n\,d\,r,$$

or

$$v\,\mathrm{S}\sqrt{1+x^2}\,.\,d\,x = d\,q + n\,d\,r.$$

Now we have (omitting constants),

$$q = \frac{2}{n}\,v - \frac{v^2}{n^2}$$

$$\therefore\ d\,q = \frac{2}{n}\,d\,v - \frac{2\,v\,d\,v}{n^2},$$

and

$$n\,d\,r = -\frac{2}{n}\,d\,v$$

$$\therefore\ d\,q + n\,d\,r = -\frac{2\,v\,d\,v}{n^2}.$$

Therefore

$$\mathrm{S}\sqrt{1+x^2}\,.\,d\,x = -\frac{2\,d\,v}{n^2}$$

$$\mathrm{S}\int\sqrt{1+x^2}\,.\,d\,x = -\frac{2\,v}{n^2} = r.$$

Now writing $n = -\frac{2}{a}$, $v = b$, we have the equation in the form given above, viz.

$$x^3 - a\,x^2 + \left(\frac{a^2}{4} - a\,b - \frac{a^2\,b^2}{4}\right)x + \frac{a^2\,b}{2} = 0\,;$$

and therefore the theorem is demonstrated.

Examples.

Ex. 1. Let $a = 2 + \sqrt{2}$, $b = 1$, the roots of the equation are

$$x = 1$$

$$y = \tfrac{1}{2}\,.\,\frac{\sqrt{5}+1}{\sqrt{2}-1}$$

$$-z = \tfrac{1}{2}\,.\,\frac{\sqrt{5}-1}{\sqrt{2}-1},$$

or

$$\begin{array}{rl} x = 1 & \therefore \text{arc}\ x = 1\cdot147793 \\ y = 3\cdot906278 & \therefore \text{arc}\ y = 8\cdot911399 \\ z = -\ 1\cdot492065 & \therefore \text{arc}\ z = 1\cdot935186 \end{array}$$

$$\therefore \mathrm{S}\,x = 3\cdot414213 = 2 + \sqrt{2} = a$$

and

$$x\,y\,z = -\,(3 + 2\,\sqrt{}\,2) = -\,5\cdot828426.$$

Now we have

$$\text{Arc}\ y = 8\cdot911399 = (1.)$$

$$\text{Arc}\ x + \text{Arc}\ z = 3\cdot082979 = (2.)$$

$$\text{Sum (\textit{subtractive})} = -\,5\cdot828420 = (2.) - (1.)$$

$$x\,y\,z = -\,5\cdot828426$$

$$\text{Error} = 0\cdot000006$$

The quantity which we previously called $f\,x = \log\,(x + \sqrt{1+x^2})$ has the following values:

$$f\,x = 0\cdot881372$$

$$f\,y = 2\cdot071728$$

$$f\,z = 1\cdot190354$$

$$\therefore \text{we have}\ f\,x + f\,z = 2\cdot071726$$

$$f\,y = 2\cdot071728$$

$$\text{Error} = 0\cdot000002$$

Thus it is seen that the logarithmic parts destroy each other as in the first theorem.

Ex. 2. Let $x = \frac{3}{4}$ $y = \frac{4}{3}$ be *assumed* for two roots of the equation; in which case we find $a = -\frac{5}{6}$, $b = \frac{42}{5}$, and the third root $z = -\frac{35}{12}$.

Since $x y = 1$ in this example, the theorem gives the sum of three arcs $= x y z = z$, which we propose to verify.

Now the formula

$$2 \text{ arc } x = x \sqrt{1 + x^2} + \log (x + \sqrt{1 + x^2})$$

gives

$$2 \text{ arc } \left(\frac{3}{4}\right) = \frac{15}{16} + \log 2$$

$$2 \text{ arc } \left(\frac{4}{3}\right) = \frac{20}{9} + \log 3$$

$$\text{the sum of which two } = \frac{455}{144} + \log 6$$

$$\text{and } 2 \text{ arc } \left(\frac{35}{12}\right) = \frac{1295}{144} + \log 6.$$

Therefore the sum (*subtractive*)

$$= -\frac{840}{144} = -\frac{70}{12}$$

$$\therefore \text{ arc } x + \text{arc } y - \text{arc } z = -\frac{35}{12}$$

But on the other hand we have

$$z = -\frac{35}{12}.$$

Therefore the sum of the arcs $= z$: which was to be shown.

§ 4. *Analogous Properties of the Circle and Parabola.*

There is a manifest analogy between the *area* of the circle and the *arc* of the parabola, the former being expressed by $\int d x \sqrt{1 - x^2}$, the latter by $\int d x \sqrt{1 + x^2}$, which only differ in the sign. The same analogy is seen in the theorems which may be deduced respecting these integrals. Thus, for instance, the Theorem II., which we have demonstrated in the parabola, may be applied, with a slight modification, to the circle. If we put

$$v \sqrt{1 \pm x^2} = n x^2 + x + v,$$

we find the sum of three integrals of the form

$$\int d x \sqrt{1 \pm x^2} = \pm r,$$

the constant being $= 0$. The upper sign applies to the parabola, the lower to the circle. The demonstration of the latter case is omitted for brevity, being exactly similar to that of the former.

The three variables $x\,y\,z$ are roots of

$$[1.]\qquad x^3 - a\,x^2 + \left(\frac{a^2}{4} - a\,b \mp \frac{a^2\,b^2}{4}\right) x + \frac{a^2\,b}{2} = 0,$$

the upper sign applying to the parabola; a and b being two arbitrary quantities.

To exemplify this theorem in the circle.—Since

$$2\int d\,x\sqrt{1-x^2} = x\sqrt{1-x^2} + \text{arc}\sin x,$$

the theorem gives

$$-\,2\,r = \mathrm{S}\,x\sqrt{1-x^2} + \mathrm{S}\,\text{arc}\sin x.$$

The latter term, being the sum of three circular arcs, cannot form any part of the quantity $-\,2\,r$: therefore we must have this other equation,

$$\mathrm{S}\,\text{arc}\sin x = 0,$$

which we propose to verify.

Ex. 1. Suppose $a = \frac{6}{5}$, $b = \frac{2}{3}$, the equation [1.] becomes

$$x^3 - \frac{6}{5}\,x^2 - \frac{7}{25}\,x + \frac{12}{25} = 0,$$

and its roots are

$$x = \frac{4}{5}$$

$$y = 1$$

$$z = -\,\frac{3}{5}$$

$$\therefore\ \text{arc}\sin x = 53^\circ\ 8' = \theta$$

$$\text{arc}\sin y = 90^\circ = \theta'$$

$$\text{arc}\sin z = -\ 143^\circ\ 8' = \theta''$$

$$\therefore\ \theta + \theta' + \theta'' = 0,$$

in accordance with the theorem.

We may assume two of the arcs arbitrarily, and thence determine the third, so as to satisfy the theorem.

Ex. 2. Thus, let $x = \frac{4}{5}$, $y = \frac{12}{13}$, we find $a = \frac{56}{65}$, $b = \frac{12}{7}$. Here it happens that $x\,y = \frac{a\,b}{2} = \frac{48}{65}$: therefore, dividing the equation

$$x\,y\,z = -\,\frac{a^2\,b}{2}$$

by the equation

$$x\,y = \frac{a\,b}{2},$$

we find the third root

$$z = -\,a.$$

Now these three values satisfy the theorem; for we have

$$\text{arc sin}\,\frac{4}{5} = 53^\circ\ 8' = \theta$$

$$\text{arc sin}\,\frac{12}{13} = 67^\circ\ 23' = \theta'$$

$$\text{arc sin} - \frac{56}{65} = -\ 120^\circ\ 31' = \theta''$$

and

$$\theta + \theta' + \theta'' = 0.$$

§ 5. *Application to the Ellipse.*

In order to obtain a relation between three elliptic integrals, the simplest supposition which we can make appears to be

$$\sqrt{\frac{1 - e^2 x^2}{1 - x^2}} = v\,x + 1,$$

whence

$$x^3 + \frac{2}{v}\,x^2 + \frac{1 - e^2 - v^2}{v^2}\,.\,x - \frac{2}{v} = 0.$$

This determines the value of the symmetrical v to be $= \frac{2}{r}$: and therefore making this substitution we have

$$x^3 + r\,x^2 + \left(\frac{1 - e^2}{4}\,.\,r^2 - 1\right) x - r = 0$$

and

$$\sqrt{\frac{1 - e^2 x^2}{1 - x^2}} = \frac{2}{r}\,x + 1,$$

whence

$$\text{S}\,d\,x\sqrt{\frac{1 - e^2 x^2}{1 - x^2}} = \frac{2}{r}\,\text{S}\,x\,d\,x + \text{S}\,d\,x.$$

But since

$$\text{S}\,x^2 = p^2 - 2\,q = r^2 - \left(\frac{1 - e^2}{2}\,.\,r^2 - 2\right) = \frac{1 + e^2}{2}\,.\,r^2 + 2$$

$$\therefore \text{S}\,x\,d\,x = \frac{1 + e^2}{2}\,r\,d\,r$$

$$\therefore \frac{2}{r}\,\text{S}\,x\,d\,x = (1 + e^2)\,d\,r.$$

Also

$$\text{S}\,d\,x = -\,d\,r$$

$$\therefore \frac{2}{r}\,\text{S}\,x\,d\,x + \text{S}\,d\,x = e^2\,d\,r$$

$$\therefore \text{S}\,d\,x\sqrt{\frac{1 - e^2 x^2}{1 - x^2}} = e^2\,d\,r$$

$$\therefore \text{S}\int d\,x\sqrt{\frac{1 - e^2 x^2}{1 - x^2}} = e^2\,r + \text{C}.$$

Or, if we suppose the radical to have a negative sign,

$$\mathrm{S}\int d\,x\sqrt{\frac{1-e^2\,x^2}{1-x^2}}=\mathrm{C}-e^2\,r\,;$$

whence the following theorem: *If three abscissæ of an ellipse are roots of the equation*

$$x^3+r\,x^2+\left(\frac{1-e^2}{4}\,.\,r^2-1\right)x-r=0,$$

the sum of the arcs $= 2\,\mathrm{Q}-e^2\,r$, *Q being the quadrant of the ellipse.*

Ex. 1. Let $e=0$, or the ellipse be a circle; the theorem then assumes this form: *If three abscissæ of a circle are roots of the equation*

$$x^3+r\,x^2+\left(\frac{r^2}{4}-1\right)x-r=0,$$

the sum of the arcs is a semicircle; the truth of which has been demonstrated previously (vide page 4.).

Fagnani's theorem becomes illusory when $e=0$: it is therefore interesting to observe that the present theorem, on the contrary, has a real application to the circle.

Ex. 2. Let e have any value, and $r=0$; then the roots are

$$\begin{array}{ll} x=0 & \therefore \text{ arc } x=0=(1.) \\ y=1 & \quad\text{arc } y=\mathrm{Q}=(2.) \\ z=-1 & \quad\text{arc } z=-\mathrm{Q}=(3.) \end{array}$$

and the sum, viz.

$$(1.)+(2.)-(3.)=2\,\mathrm{Q},$$

which is therefore the value of the constant.

Ex. 3. When x is not actually $=0$, as in the last example, but has an indefinitely small value $=\omega$, it will be found that the values of y and z differ from 1 and -1 by a quantity of the order of ω^2. But nevertheless the arcs which subtend these abscissæ differ from a quadrant of the ellipse by a quantity of the order of ω. This arises from the direction of the arc at the extremities of the axis being perpendicular to the abscissa, so that its increment is infinitely greater than that of the latter. It will be well to show the truth of the theorem in this case. When $x=\omega$ we have (putting $1-e^2=b^2$)

$$y=1-\frac{b^2\,\omega^2}{8}$$

$$z=-y.$$

For from these values we deduce $y+z=0$, and thence (neglecting quantities of the order ω^3)

$$x+y+z=x=\omega$$

$$x\,y+x\,z+y\,z=y\,z=-y^2=\frac{b^2\,\omega^2}{4}-1$$

$$x\,y\,z=-\omega.$$

So that $x\,y\,z$ are roots of

$$x^3 - \omega x^2 + \left(\frac{1-e^2}{4}\omega^2 - 1\right)x + \omega = 0,$$

which agrees with the given form by putting $r = -\omega$.

We have now to find the sum of the arcs.

The arc subtending the abscissa ($x = \omega$) may be considered as equal to it.

The arc subtending the abscissa y differs from the elliptic quadrant by an arc which may be considered equal to the *ordinate* which corresponds to y. And the same with respect to z.

Let y' be the ordinate corresponding to y. The equation of the curve gives

$$y' = b\sqrt{1-y^2};$$

but since

$$y^2 = 1 - \frac{b^2\omega^2}{4} \therefore \sqrt{1-y^2} = \frac{b\,\omega}{2} \quad \therefore y' = \frac{b^2\omega}{2},$$

and the arc subtending $y = \mathrm{Q} - \frac{b^2\omega}{2}$.

The arc z has the same value. Therefore

$$\operatorname{arc} y + \operatorname{arc} z = 2\,\mathrm{Q} - b^2\omega;$$

adding arc $x = \omega$, we have

$$\text{Sum of arcs} = 2\,\mathrm{Q} + e^2\omega,$$

(or, since $\omega = -r$)

$$= 2\,\mathrm{Q} - e^2 r,$$

in accordance with the theorem.

Ex. 4. Let $1 - e^2 = \frac{1}{3}$.

And also let $r = 9 - 3\sqrt{10} = -0{\cdot}4868331$;

the roots of the equation

$$x^3 + r\,x^2 + \left(\frac{r^2}{12} - 1\right)x - r = 0,$$

are

$$\begin{aligned} x &= 0{\cdot}5 &&= \sin 30^\circ \\ y &= 0{\cdot}98019 &&= \sin 78^\circ\,34' \\ z &= -0{\cdot}99336 &&= \sin 83^\circ\,24' \end{aligned}$$

Entering Legendre's Table IX. with modulus $e = \sqrt{\frac{2}{3}} = \sin 54^\circ\,44'$ and these amplitudes, we find

$$\begin{aligned} \operatorname{arc} x &= 0{\cdot}5081 \\ \operatorname{arc} y &= 1{\cdot}1446 \\ \operatorname{arc} z &= 1{\cdot}1944 \\ \hline \text{Sum} &= 2{\cdot}8471 \end{aligned}$$

On the other hand we have

$$\begin{aligned} 2\,Q = 2 \text{ arc } (90^\circ) &= 2{\cdot}5224 \\ -\,e^2\,r &= 0{\cdot}3246 \\ \hline 2\,Q - e^2\,r &= 2{\cdot}8470 \\ \text{Sum of arcs} &= 2{\cdot}8471 \\ \hline \text{Error} &= 0{\cdot}0001 \end{aligned}$$

I will now indicate two other theorems respecting the sum of *three elliptic arcs.*

I. We may put the integral $\int d\,x \sqrt{\frac{1 - e^2\,x^2}{1 - x^2}}$ in the form

$$\int (1 + e\,x)\,d\,x \sqrt{\frac{1 - e\,x}{(1 + e\,x)\,(1 - x^2)}},$$

and assume $\frac{1 - e\,x}{(1 + e\,x)\,(1 - x^2)}$ to be a symmetrical $= \frac{1}{v}$. This gives

$$x^3 + \frac{1}{e}\,x^2 - (v + 1)\,x + \frac{v - 1}{e} = 0,$$

and the result which I find is, that if three abscissæ are the roots of this equation, the sum of the corresponding arcs $= 2\,e\sqrt{v} + C.$

II. We may put the integral in the form

$$\int \frac{d\,x}{1 + x} \sqrt{\frac{(1 + x)\,(1 - e^2\,x^2)}{1 - x}},$$

and assume $\frac{(1 + x)\,(1 - e^2\,x^2)}{1 - x} = v$, whence

$$x^3 + x^2 - \frac{v + 1}{e^2}\,x + \frac{v - 1}{e^2} = 0.$$

The result which I find is, that if three abscissæ are the roots of this equation, the sum of the arcs $= 2\sqrt{v} + C.$

These theorems respecting the sums of elliptic arcs appear to be some of the simplest which exist; but an unlimited number of theorems of a higher order and more complicated nature are obtainable, the discussion of which would lead too far at present.

Thus if we assume

$$\sqrt{\frac{1 - e^2\,x^2}{1 - x^2}} = a_{n-1}\,x^{n-1} + a_{n-2}\,x^{n-2} + \&c.$$

where the coefficients are constants, or entire rational functions of v, we have an equation of $2\,n$ dimensions, which gives the sum of $2\,n$ elliptic arcs in terms of v.

There is no difficulty, beyond the length of the operation, in deducing these theorems, as they are all obtainable by an uniform method. But it will be of importance to show the relation between them and the previously received doctrines respecting elliptic integrals as established by LEGENDRE and others, the connexion between them not being at first sight very evident.

§ 6. *Application to the Equilateral Hyperbola.*

In order to obtain a relation between three values of the integral $\int \frac{d\,x}{x^3} \sqrt{1 + x^4}$, which expresses the arc of the equilateral hyperbola, we may put

$$\sqrt{1 + x^4} = v\,x + 1,$$

whence

$$x^3 - v^2\,x - 2\,v = 0,$$

we have therefore $2\,v = r$, and making this substitution,

$$x^3 - \frac{r^2}{4}\,x - r = 0.$$

Also

$$\frac{\sqrt{1 + x^4}}{x^2} = \frac{r}{2\,x} + \frac{1}{x^2}$$

$$\therefore \ \mathrm{S}\,\frac{\sqrt{1 + x^4}}{x^2}\,.\,d\,x = \frac{r}{2}\,\mathrm{S}\,\frac{d\,x}{x} + \mathrm{S}\,\frac{d\,x}{x^2}.$$

Now we have

$$\frac{r}{2}\,\mathrm{S}\,\frac{d\,x}{x} = \frac{r}{2}\,.\,\frac{d\,r}{r} = \frac{d\,r}{2},$$

and

$$\mathrm{S}\,\frac{1}{x} = \frac{q}{r} = -\frac{r}{4}$$

$$\therefore \ \mathrm{S}\,\frac{d\,x}{x^2} = \frac{d\,r}{4}$$

$$\therefore \ \mathrm{S}\,\frac{\sqrt{1 + x^4}}{x^2}\,.\,d\,x = \frac{d\,r}{2} + \frac{d\,r}{4} = \frac{3\,d\,r}{4}$$

$$\therefore \ \mathrm{S}\int\frac{\sqrt{1 + x^4}}{x^2}\,.\,d\,x = \frac{3}{4}\,r + \mathrm{C}\,;$$

so that if three abscissæ of the equilateral hyperbola are roots of the equation

$$x^3 - \frac{r^2}{4}\,x - r = 0,$$

the sum of the arcs $= \frac{3}{4}\,r + \mathrm{C}$, which is the theorem which I originally met with concerning the hyperbolic arc*.

It will be seen how very simply and directly we are conducted to it by the present method of investigation. Next let us suppose

$$-\sqrt{1 + x^4} = v\,x^3 + 1,$$

whence

$$x^3 - \frac{x}{v^2} + \frac{2}{v} = 0.$$

Put $v = -\frac{2}{r}$,

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* Philosophica Transactions, 1836, Part I. p. 185.

$$\therefore x^3 - \frac{r^2}{4} x - r = 0,$$

and

$$-\frac{\sqrt{1 + x^4}}{x^2} = -\frac{2\,x}{r} + \frac{1}{x^2}.$$

Therefore

$$-\,\mathrm{S}\,\frac{\sqrt{1 + x^4}}{x^2}\,.\,d\,x = -\frac{2}{r}\,\mathrm{S}\,x\,d\,x + \mathrm{S}\,\frac{d\,x}{x^2},$$

but

$$\mathrm{S}\,\frac{d\,x}{x^2} = \frac{d\,r}{4},$$

as in the last example; and

$$\mathrm{S}\,x^2 = \frac{r^2}{2} \quad \therefore -\frac{2}{r}\,\mathrm{S}\,x\,d\,x = -\,d\,r$$

$$\therefore -\,\mathrm{S}\,\frac{\sqrt{1 + x^4}}{x^2}\,.\,d\,x = \frac{d\,r}{4} - d\,r = -\frac{3}{4}\,d\,r$$

$$\therefore \mathrm{S}\int \frac{\sqrt{1 + x^4}}{x^2}\,.\,d\,x = \frac{3}{4}\,r + \mathrm{C}.$$

This result therefore agrees with the last example, and gives the same theorem, but it supplies a different demonstration of it.

We will now suppose

$$\frac{\sqrt{1 + x^4}}{x^2} = 1 + \frac{a}{x} + \frac{v}{x^2},$$

a being a constant. This gives

$$x^3 + \frac{a^2 + 2\,v}{2\,a}\,x^2 + v\,x + \frac{v^2 - 1}{2\,a} = 0,$$

and I find this result, that if three abscissæ are roots of this equation, which may be written

$$x^3 - p\,x^2 + q\,x - r = 0,$$

then the sum of the arcs

$$= p - \frac{v^2}{r} + \text{const.} = \varphi\,v + \mathrm{C}.$$

This sum is therefore constant if $\varphi\,v$ is so.

Let $v = k$, $v = k'$, be two values of v, which give the same value to $\varphi\,v$, or $p - \frac{v^2}{r}$.

Let the three hyperbolic arcs in the first case be $\alpha\,\alpha'\,\alpha''$, and in the second case $\beta\,\beta'\,\beta''$, then

$$\alpha + \alpha' + \alpha'' = \beta + \beta' + \beta''.$$

All the abscissæ have the same origin at the centre of the curve, therefore the arcs have the same origin, and therefore can be subtracted from one another. Therefore putting $\alpha - \beta = \gamma$, $\alpha' - \beta' = \gamma'$, $\alpha'' - \beta'' = \gamma''$, we have

$$\gamma + \gamma' + \gamma'' = 0.$$

This appears to me to show the possibility of finding three arcs such that (neglecting their signs) the sum of two of them shall be equal to the third (though not superposable in any part). I believe that it has been hitherto held that this equality is impossible in the ellipse and hyperbola, without the addition of *some algebraic quantity*. I should have wished therefore to have added some numerical illustration of such a result, but the length of the calculation has hitherto prevented me from doing so.

II. *Researches towards establishing a Theory of the Dispersion of Light. No. III. By the Rev.* Baden Powell, *M.A. F.R.S., Savilian Professor of Geometry in the University of Oxford.*

Received October 20, 1836—Read January 19, 1837.

Introductory Remarks.

In two former portions of researches on the subject of dispersion *, I have discussed all the observed refractive indices for definite rays, in different media, which had come to my knowledge; consisting of those for ten media determined by **Fraunhofer**, and those in ten other cases by M. **Rudberg**, comparing them with the calculated results of the theory of M. **Cauchy**; and the agreement is sufficiently close. In those papers, and elsewhere, I have remarked the importance of extending the inquiry, especially to media of higher dispersive power; in which cases (as appears from the nature of the formula) the theory would be put to a more precise test.

In the former instances the work of determining the indices was done to my hands, and I could proceed to the theoretical computations with the most perfect confidence in the accuracy of experimental data, furnished from the labours of observers so well known for precision and skill, and obtained, too, before the formula of theory had been deduced.

In any comparison of theory with experiment, it is, in all points of view, far more satisfactory that such comparison should be made with the observations of others rather than those of the theoretical computer himself. In the present instance, however, this desirable condition has not been fulfilled. Though the importance of obtaining a series of indices for the standard rays in different media had been long since pointed out and acknowledged by the most eminent philosophers, yet no observer was found to undertake the task of carrying on the work which **Fraunhofer** and **Rudberg** had so successfully begun. I was thus left to make an attempt myself to supply the deficiency; and my observations were communicated, and printed copies distributed, to the Physical section of the British Association at the Bristol meeting in August 1836†.

From the remarks prefixed to those results, the scientific reader will, I trust, be sufficiently enabled to judge of the nature and degree of accuracy of the observations.

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* Philosophical Transactions, 1835, Part I.; and 1836, Part I.

† This tract now forms one of the series of memoirs published by the Oxford Ashmolean Society.

I conceive, in general, the indices deduced may be relied on as exact, to at least three places of decimals. Some exceptions are noticed in the tract as less certain, viz. the oils of Angelica, Cummin, and Pimento, and the balsam of Peru. Such as they are, however, these results form (as far as I am aware) the only existing data for pursuing the comparison with theory. But I trust they may not be thought insufficient, when we consider that in the present stage of the inquiry the object to be aimed at seems chiefly such a general comparison as may enable us to see whether the main principle of the undulatory explanation of the dispersion is applicable, with a sufficient approach to precision, to encourage us to pursue the theory, or whether it must be abandoned, and some new principle sought.

With respect to media of low dispersive power, little doubt can exist. I have therefore not thought it worth while to go through the calculations for many of this class, but have confined my examination in the present instance chiefly to the higher cases to which my observations have extended.

Method of Calculation.

It may be necessary to premise a notice of the method of calculation adopted in the present communication, as it differs from that employed in my two former papers. That method consisted in finding, in the first instance, by a tentative process, (virtually equivalent to assuming the two extreme indices from observation,) the fundamental arc θ, from which the others were derived on dividing by λ for each ray, so as to fulfill the conditions of the approximate formula

$$\frac{1}{\mu} = \mathrm{H}\left\{\frac{\sin\left(\frac{\theta}{\lambda}\right)}{\left(\frac{\theta}{\lambda}\right)}\right\}.$$

It also appears from the investigations given in several consecutive papers in the London and Edinburgh Journal of Science, &c., that this formula is in fact obtained by supposing the sum of a series of analogous terms collected into one, with a common constant coefficient H. This simplified hypothesis (from the accordances already obtained) is evidently very near the truth for the whole range of media hitherto examined.

Professor Sir W. R. Hamilton afterwards pointed out (besides a direct process for performing this approximate calculation) a method of investigating the exact expression when we do not allow the above assumption as to the coefficients, viz.

$$\left(\frac{1}{\mu}\right)^2 = \mathrm{S}\left\{\mathrm{H}^2\left\{\frac{\sin\left(\frac{\theta}{\lambda}\right)}{\left(\frac{\theta}{\lambda}\right)}\right\}^2\right\}.$$

This is explained at large in the journal just named*; and in a subsequent number†

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* March 1836. † August 1836.

I have stated some further particulars illustrative of the process. It is this method which I shall use in the present investigation, and therefore here merely quote the resulting formula from the last-mentioned paper. It is as follows;

$$\mu_i - \mu_F = a_i(\mu_F - \mu_B) + b_i(\mu_H - 2\mu_F + \mu_B):$$

where the three indices of refraction are assumed from observation for the respective rays B, F, and H, in the particular medium; while μ_i is the index sought, any other of the four remaining of the seven standard rays, corresponding to which a_i and b_i are constants for the ray, independent of the particular medium, and which have been found from the values of λ in the paper last referred to, as follows:

$\log a_C = \bar{1}\cdot95433$ $\log b_C = \bar{2}\cdot65253$

$\log a_D = \bar{1}\cdot80441$ $\log b_D = \bar{1}\cdot06281$

$\log a_E = \bar{1}\cdot49646$ $\log b_E = \bar{1}\cdot03196$

$\log a_G = \bar{1}\cdot74027$* $\log b_G = \bar{1}\cdot62954$*.

The nature of the process of computation is obvious from an inspection of the formula; and the data here stated will suffice for those who may wish to verify the calculations. I proceed to give the results in a tabular form, and then to offer such general conclusions as I think may be safely derived from them.

Comparison of Refractive Indices from CAUCHY's *Theory and from observation.*

1. Nitric Acid.				3. Sulphuric Acid.			
Ray.	Index observed.	Index calculated.	Difference.	Ray.	Index observed.	Index calculated.	Difference.
B	1·3988			B	1·4321		
C	1·3998	1·3998	·0000	C	1·4329	1·4329	·0000
D	1·4026	1·4024	− ·0002	D	1·4351	1·4349	− ·0002
E	1·4062	1·4058	− ·0004	E	1·4380	1·4375	− ·0005
F	1·4092			F	1·4400		
G	1·4155	1·4156	+ ·0001	G	1·4440	1·4448	+ ·0008
H	1·4211			H	1·4468		
2. Muriatic Acid.				4. Oil Angelica.			
B	1·4050			B	1·4836		
C	1·4065	1·4061	− ·0004	C	1·4863	1·4849	− ·0014
D	1·4095	1·4089	− ·0006	D	1·4887	1·4882	− ·0005
E	1·4130	1·4125	− ·0005	E	1·4932	1·4924	− ·0008
F	1·4160			F	1·4963		
G	1·4217	1·4222	+ ·0005	G	1·5049	1·5035	− ·0014
H	1·4265			H	1·5099		

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* The two last numbers include the correction of an error which I detected in those given in the paper referred to.

5. Oil Cummin.

Ray.	Index observed.	Index calculated.	Difference.
B	1·5023		
C	1·5043	1·5039	— ·0004
D	1·5075	1·5079	+ ·0004
E	1·5130	1·5136	+ ·0006
F	1·5196		
G	1·5326	1·5317	— ·0009
H	1·5432		

6. Oil Sassafras.

B	1·5252		
C	1·5267	1·5268	— ·0001
D	1·5312	1·5315	+ ·0003
E	1·5380	1·5376	— ·0004
F	1·5441		
G	1·5568	1·5569	+ ·0001
H	1·5687		

7. Oil Pimento.

B	1·5281		
C	1·5317	1·5300	— ·0017
D	1·5347	1·5348	+ ·0001
E	1·5422	1·5412	— ·0010
F	1·5478		
G	1·5599	1·5605	+ ·0006
H	1·5719		

8. Kreosote.

B	1·5327		
C	1·5341	1·5344	+ ·0003
D	1·5387	1·5392	+ ·0005
E	1·5457	1·5455	— ·0002
F	1·5520		
G	1·5645	1·5646	+ ·0001
H	1·5762		

9. Balsam of Peru.

B	1·5846		
C	1·5869	1·5869	·0000
D	1·5931	1·5935	+ ·0004
E	1·6029	1·6028	— ·0001
F	1·6130		
G	1·6336	1·6336	·0000
H	1·6533		

10. Oil of Anise. Temp. 21°·2.

B	1·5452		
C	1·5473	1·5473	·0000
D	1·5533	1·5533	·0000
E	1·5623	1·5615	— ·0008
F	1·5705		
G	1·5883	1·5887	+ ·0004
H	1·6061		

11. Oil of Anise. General Mean.

Ray.	Index observed.	Index calculated.	Difference.
B	1·5469		
C	1·5489	1·5490	+ ·0001
D	1·5553	1·5551	— ·0002
E	1·5642	1·5635	— ·0007
F	1·5726		
G	1·5904	1·5909	+ ·0005
H	1·6082		

12. Oil of Anise. Temp. 15°·2.

B	1·5486		
C	1·5506	1·5509	+ ·0003
D	1·5574	1·5570	— ·0004
E	1·5661	1·5656	— ·0005
F	1·5747		
G	1·5926	1·5932	+ ·0006
H	1·6103		

13. Sulphuret of Carbon. Temp. 22°.

B	1·6145		
C	1·6176	1·6178	+ ·0002
D	1·6272	1·6269	— ·0003
E	1·6405	1·6392	— ·0013
F	1·6521		
G	1·6763	1·6776	+ ·0013
H	1·7009		

14. Sulphuret of Carbon. Temp. 12°.

B	1·6234		
C	1·6258	1·6268	+ ·0010
D	1·6369	1·6361	— ·0008
E	1·6496	1·6485	— ·0011
F	1·6616		
G	1·6864	1·6870	+ ·0006
H	1·7103		

15. Oil of Cassia.

B	1·5885		
C	1·5918	1·5913	— ·0005
D	1·6017	1·6000	— ·0017
E	1·6155	1·6135	— ·0020
F	1·6295		
G	1·6607	1·6646	+ ·0039
H	1·7002		

General Conclusions.

With regard to some of the first media above compared, the agreement of observation and theory is sufficiently near; but they are low in the scale of dispersion. The cases before mentioned as being somewhat uncertain do not allow of a close comparison. The balsam of Peru (though only from approximate data) and the kreosote, present higher numbers, and the agreement is therefore the most satisfactory. This is also the case with oil of anise seed, though perhaps not to the same degree; but the differences are probably not greater than what may reasonably be allowed.

In all these cases, however, we trace some uniformity in the character of the differences; theory being always in defect for the ray E, and in excess for G. When we proceed to the sulphuret of carbon this becomes more apparent; and the differences increase for the more refrangible rays. And lastly, in oil of cassia the same regular order of deviation is more marked, and the differences much greater, especially in the rays E and G. This regularity of character, as well as increase in the amount of the discrepancy, at once shows that it is at least partly due to some other cause than errors of observation. Yet we must remark, that even in these cases there is a sort of general accordance preserved between observation and theory, the agreement being still accurate to the second place of decimals.

The following considerations, then, suggest themselves:

1. It is to be observed that in all these cases closer accordances might be obtained if we took slightly different values for the assumed indices of B, F, and H; such as would be consistent with the probable errors of observation. In oil of cassia, however, I have found that the errors, even when thus distributed among all the indices, are still too great.

2. The constants a and b are derived solely from the values of λ for each ray, taken from the well-known determinations of FRAUNHOFER from interference. It is doubtless possible that these determinations may be affected by errors. In computing, by the method here used, some of FRAUNHOFER's indices, in which small discrepancies were found, Sir W. R. HAMILTON undertook to investigate what amount of alteration in the values of λ would account for those differences. He communicated his researches in a letter to myself, with the values of λ, and consequently those of a and b, thus altered. I have repeated the calculation for oil of cassia, but find this change quite insufficient to remove the discrepancy; and that, in fact, a much larger alteration than could for a moment be allowed in FRAUNHOFER's data, must be supposed, in order to produce any sensible effect.

3. The entire method of computation here followed, though founded on the exclusion of those approximate suppositions which are allowed in the simpler formula, is yet dependent on the omission of terms in the series for μ beyond the three first*.

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* See London and Edinburgh Journal of Science, &c., March 1836, eq. 9 and 13.

Since in this analysis we know nothing of the values of the coefficients of this series, but proceed solely by the elimination of them, it is still possible that in the neglected terms we may find the means of explaining the observed discrepancy. In a word, it remains an important subject for future research whether a further prosecution of the analysis, either by this method or by that which M. Cauchy has himself recently pointed out, or that of Mr. Kelland, may not lead to more successful results.

For the present I will only remark, by way of recapitulation, that upon the whole I conceive the formula, as already deduced from the undulatory theory, applies sufficiently well to the case of media whose dispersion is as high as that of oil of anise seed. It also represents, with a certain general approximation to the truth, the indices of some more highly dispersive bodies. It is therefore extremely probable that the essential principle of the theory has some real foundation in nature. While, looking at the regularity of the deviation, it seems likely that the formula only requires to receive some further development or extension, in order to make it apply accurately to the higher cases, while it shall still include the simpler form which so well accords with the lower.

October 10, 1836.

III. *On the Optical Phenomena of certain Crystals.* *By* H. F. Talbot, *Esq. F.R.S.*[*]

Received April 20,—Read May 5, 1836.

SOME time ago I had the honour to communicate to the Royal Society an account of my invention of the polarizing microscope[†]. This instrument possesses so great a power of developing the internal structure of transparent bodies, even in their minutest visible particles, that I feel confident the employment of it will lead to many new and interesting results. At present I mean to confine myself to the description of a phenomenon which shows strikingly the beautiful order and regularity with which nature disposes the fabric of some of her minutest visible works.

The object I speak of is a kind of minute crystallization which may be obtained in peculiar circumstances, and I doubt not, in many different ways; but the manner in which it has presented itself to my observation is as follows.

A crystal of borax is placed in a drop of phosphoric acid, somewhat diluted, upon a plate of glass, and then moderately heated until the crystal dissolves in the acid. It is then set aside to crystallize. It is well to prepare a number of these plates at once, varying the relative proportion of the acid and salt, in order that the desired kind of crystallization may be found in one or other of them; for there is a considerable variety in the crystalline forms obtained by this method, some of which indeed are very singular. But when that kind of crystallization takes place which it is more particularly my intention to speak of, the field of view of the microscope is seen covered with minute circular spots, each of which is like a tuft of silk radiating from a centre, and is composed of a close assemblage of delicate acicular crystals forming a star. But besides these, are seen interspersed among them a number of circular transparent bodies, which are evidently modifications of the former, being, in fact, tufts or stars of acicular crystals in such close assemblage as to be in optical contact with each other and to produce the appearance of a single individual. Now let us suppose a group of these circles to be under examination with the polarizing microscope, and when the polarizers[‡] are crossed, we observe the following phenomenon. The field

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[*] This paper appeared in the Philosophical Magazine for October 1836, *after* it had been ordered by the Council to be printed in the second part of the Philosophical Transactions for that year. The circumstance arose entirely from a mistake of the person to whom the superintendence of the printing both of the Transactions and Magazine is entrusted, and neither Mr. Talbot, the Council, nor the Officers of the Royal Society had any cognizance of the error till the paper appeared in the latter publication.

[†] See London and Edinburgh Philosophical Magazine, vol. v. p. 321.

[‡] By this term, for the sake of brevity, I here designate the polarizing and analysing prisms of single-image calcareous spar, or the plates of tourmaline which may be employed in their stead.

of view being dark, the little circles become luminous, and we see upon each of them a well-defined and dark cross, dividing the crystal into four equal parts. All these crosses are placed similarly, and are parallel to each other, and their direction remains unaltered when the crystals are turned round in their own plane by revolving the plate of glass upon which they stand. This beautiful appearance can be seen with a moderate magnifying power. I measured the diameter of some of the larger crystals, which I found to be from $\frac{1}{300}$ to $\frac{1}{600}$ of an inch. But there are many much smaller, and indeed they may be seen decreasing in size, until nothing remains visible of their structure but the four luminous quadrants, appearing like four minute dots of coloured light placed close together.

I proceeded to examine the circles with a high magnifying power, and under favourable circumstances of illumination, and I observed in them a very admirable structure.

Each circle has upon it one or more coloured rings arranged concentrically, but the number as well as the colour of these rings is different in different individuals.

The innermost ring is deeply coloured or black, and incloses a central space of white light, which is traversed by the arms of the cross intersecting in the centre. This part of the cross, which stands within the innermost ring, is beautifully well defined, and perfectly black. The general appearance resembles the figure 98, in Brewster's Optics, which is a representation of the rings seen in uniaxal crystals. It especially resembles it in the circumstance above mentioned, viz. the more defined outline of the part of the cross which is within the innermost ring.

We have hitherto supposed the polarizers to be crossed, but if we place them in a parallel position we shall see a phenomenon complementary to the above. The circle now presents four patches of coloured light, one in each quadrant; and we generally see near the centre four black or obscure spots, which correspond to the arms of the cross in the other position.

Such is an outline of the microscopic appearances presented by these little crystals, which are probably the minutest bodies in which so complicated an optical structure has hitherto been witnessed. I find that the smaller the circles are, the more perfect is their form and the brighter their colours.

These crystals, as I have already observed, probably consist of spicula diverging from a point, but which are in the closest possible contact, and in a state of complete mechanical cohesion. It seems to follow as a consequence from such a structure that their density must increase from their circumference towards their centre. Now it is worthy of remark, that Sir David Brewster has discovered very similar phenomena by polarized light in the crystalline lenses of certain fishes, which are known by direct experiment to increase in density towards the centre. Indeed the figure which he has given of the lens of the codfish in the Philosophical Transactions for 1816 (Plate XII. fig. 1.) is so like the appearance of one of the crystals which I have described, that it might be supposed to have been intended for a representation of it.

Having pointed out this resemblance, I may also mention another class of facts to which I think those I have described possess a considerable analogy. I mean the optical figures which Brewster has discovered in spheres of glass whose density was rendered variable by heating them.

He says* that, "if we take a cold sphere of glass and immerse it in a trough of hot oil, placed in a polarizing apparatus, we shall observe *a black cross with four sectors of polarized light.* If the sphere is turned round it will exhibit in every position the very same figure. If we now suppose the trough to be filled with such spheres they will exhibit the same phenomena in whatever direction the polarized light is transmitted through them, and even if they were in a state of motion. A fluid composed of such spherical particles would exhibit the same polarizing structure in every possible direction, and even if it were in a rapid state of gyration. If the particles possessed the structure that produces circular polarization the fluid would develop the phenomena exhibited by oil of turpentine, &c."

And again†, "The structure of the particles of a circularly polarizing fluid must be exactly the same along every one of its diameters; that is, the structure must be symmetrical round the centre of the particle, or analogous to that which takes place in common polarization when a sphere of glass has its density regularly increasing or regularly diminishing towards its centre."

I have quoted these remarkable passages at length, because it appears to me that what is there advanced merely as a hypothesis, acquires a considerable degree of probability from the facts which I have stated, since I have succeeded in rendering actually visible circular particles of excessive minuteness, in each of which the microscope detects the very structure imagined by Brewster, viz. the black cross and four sectors of light. So that it appears not improbable that the circular-polarizing properties of fluids may be owing to the presence of multitudes of particles similar to these, which they hold in solution.

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* Library of Useful Knowledge, art. "Polarization of Light," p. 51.

† Ibid. p. 45.

IV. *Further Observations on the Optical Phenomena of Crystals.*

By H. F. Talbot, *Esq. F.R.S.*

Received October 26,—Read December 15, 1836.

§ 1.

In my former paper on this subject I have described the remarkable circular mode of crystallization which sometimes occurs when borax crystallizes from a solution in phosphoric acid.

I have stated that when examined by the polarizing microscope, a black cross and four sectors of light are seen upon each crystal; and upon that kind which is most easily and frequently obtained, there are seen in addition one or more rings of vivid colour. Some deviations, however, from this usual form occur occasionally; one of which, being extremely beautiful as a microscopic object, deserves a separate mention. This variety of crystalline circles differs from the one first described in the following particulars.

1. The circles are much larger, attaining the diameter of $\frac{1}{10}$th of an inch; whereas those first observed did not exceed $\frac{1}{200}$dth of an inch in diameter.

2. They are flat, whereas the former ones were convex, and frequently I believe of a spherical form.

3. In consequence of which probably, they are seen to exhibit no coloured rings, but only a cross.

4. The cross is brightly coloured, red, green, blue, &c. upon a white ground (the polarizers being supposed to be parallel to each other). This has a beautiful appearance, especially when several circles seen at once have crosses of different colours.

5. When the polarizers are placed at right angles, the phenomenon complementary to the above is seen. For instance, the circle which presented a red cross upon a white ground now presents a black cross upon a green ground.

6. In an intermediate position of the polarizers, the circle just mentioned presents a red cross alternating with a green one, thus dividing the circle into eight sectors of coloured light.

Other circles present other colours, but they all follow the same analogy, and the crosses upon all the crystals are in a parallel position.

7. These crystals last longer than the former ones. I have found some of these retain their structure for two months; the former kind seldom last in perfection more than a day.

8. It sometimes happens that their circumference is imperfect, and presents a

notched or jagged outline. These have a very beautiful appearance, and have been almost universally compared by those who have looked at them to highly coloured flowers with four petals; the cross upon them being so dark as to have the appearance of being a division between the petals.

All the circles, when viewed by common light, appear transparent, white, and very uniform. If they are composed of acicular crystals diverging from a point, these latter must be exceedingly slender and numerous, and in perfect optical contact, since a high magnifying power does not render them separately visible.

§ 2.

With respect to the chemical nature of these crystals, it appears to me evident that they consist of boracic acid. They are obtained by dissolving borax in phosphoric acid; and it may be inferred that this latter substance unites with the alkali and isolates the boracic acid. In order to see if this supposition were correct, I dissolved boracic acid in alcohol, and I found that a drop of this solution evaporated on a plate of glass frequently yielded an abundant crop of the crystalline spherules. But these are generally exceedingly small, requiring a high power to display in them the cross and four sectors of light, and they speedily grow opake; for which reason they are not so well suited for observation as those prepared by the former method. They establish the fact, however, that this mode of crystallizing is a property of the boracic acid. It is highly improbable that it should be peculiar to that substance, but I have not yet met with it in any other.

§ 3. *Explanation of some of the Optical appearances.*

1. When any doubly refracting crystal is examined with the polarizing microscope, (the polarizers being transverse to each other, and the field of view consequently dark,) if it be turned round in one plane, it is seen to grow four times *luminous* and four times *dark* in the course of one revolution. This I have found to be universally the case with all the substances which I have tried, and it also is in accordance with theory.

2. In the case of an acicular crystal, one of the optical axes always coincided with the axis of figure, or length of the crystal; so that if a crystal of this sort appears unilluminated, all the others that are either parallel to it or perpendicular to it are likewise dark.

3. It results from the above that a circle composed of acicular crystals diverging from a point, must present the appearance of a black cross, and that the crosses on all the circles will be parallel.

4. With respect to the rings of colour, they are a consequence of the variable thickness of the crystalline circle at different distances from its centre. Their being visible, and indeed very conspicuous, upon a body of such small diameter, arises from the very energetic action of boracic acid upon polarized light.

§ 4.

I have remarked that the circular crystallization of boracic acid is frequently entirely superseded by other modes of crystalline formation; which circumstance appears to be chiefly owing to the presence or absence of combined water. Some of these variations deserve to be particularly specified.

1. Instead of circles there often occurs a formation of crystals resembling *two opposite sectors of a circle combined.* This form may be traced in different crystals, from its commencement when the angle of the sector is small, through all degrees of increase, until at length the opposite sectors unite and form a complete circle.

2. The crystals are frequently of a very irregular elongated shape, which does not approximate either to a prismatic or a cylindrical form. This stem, as it may be termed, subdivides itself *at both extremities* into an immense multitude of diverging fibres, giving it the appearance of a bundle of elastic filaments firmly held together in the central part, but with its extremities left at liberty to diverge*.

3. Another variety resembles in the same way irregular stems or branches, which, however, instead of being subdivided, are abruptly truncated at both extremities perpendicularly to the general line of their direction.

4. Sometimes, on the contrary, the ramification is much more developed, and then resembles two plumes united by a common stem.

5. Crystals of regular geometric form. These appear to require the presence of combined water.

Whichever of these formations occurs, it is for the most part seen in all the crystals at once, to the exclusion of any of the other forms.

§ 5.

These crystals generally undergo a spontaneous change in the course of one or two days after they have been formed. Those (No. 4) resembling plumes usually break up and resolve themselves into small rhombs and other geometric forms. The elongated crystals (Nos. 2, 3) undergo a remarkable change. They become traversed with innumerable fissures transverse to their length, and thus break up into thin plates, which either cohere loosely or separate entirely.

§ 6.

All these forms are very pleasing objects for the polarizing microscope. This arises from the very high depolarizing power of boracic acid, which enables its thinnest plates to exhibit colours of great variety and brilliancy, and causes even its dust or smallest particles to appear luminous. The more energetically any substance acts upon polarized light, the closer and more crowded are the bands and lines of

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* This appearance is not very uncommon in the crystallization of other substances, though I believe it has not yet been described. The divergence of the filaments suggests the idea of electrical repulsion as being at least its primary cause.

colour which appear upon its crystals. These isochromatic lines, of which there are frequently many alternations, denote lines of equal thickness in the crystal. In the case of boracic acid, when anhydrous or nearly so, these lines are more crowded than in any other crystal that I have yet examined, insomuch that to exhibit them distinctly is as fine a test of the performance of a microscope as to resolve the more difficult lines on the scales of a butterfly's wing, or any other of the known test-objects. And in many cases the microscope only indicates the existence of a still more delicate structure, which, at least in its present state, it has not power distinctly to exhibit.

§ 7. *On Analytic Crystals.*

I now come to describe a property of crystals which I met with while employed in pursuing the above investigation. This is the power which certain crystals have of analysing polarized light in a manner analogous to the tourmaline; for which reason I shall propose for them the name of *Analytic Crystals.*

If I am not mistaken, this property has been hitherto confined to the tourmaline and a few other natural minerals; and it has not been known that their effects could be imitated, much less surpassed, by crystals artificially made. I trust, therefore, that it will be of some interest to describe a method of procuring such crystals.

In the following experiments it will be understood that the *analysing plate* of the microscope (or the polarizer next the eye) is removed.

1. A good example of this kind of crystal is obtained by dissolving the sulphate of chromium and potash in tartaric acid by the aid of heat. A drop of this solution placed on a plate of glass soon yields by evaporation filmy crystals, which very frequently have the characteristic property of tourmaline: that is to say, that if polarized light is transmitted through them, in one position they suffer it to pass freely, but if they are turned round 90°, they arrest and absorb it entirely.

When the experiment has been successfully conducted, the crystals will not in this position allow the smallest portion of light to pass.

If now we consider the extreme thinness of these crystalline films, it will appear how energetic must be their action upon light: since although white and transparent, they are able to produce an absorption equalling that of the best tourmalines, notwithstanding that the effect of the latter is aided by their natural dark colour.

But if these crystals are analogous to the tourmaline, they must have the power which that substance has of analysing the light that has been transmitted through other crystals.

Accordingly, if we place in the path of the polarized ray a plate of sulphate of lime of a proper thickness, the crystal, which before absorbed the light and appeared black, becomes splendidly coloured with that colour which the sulphate of lime produces, and which a tourmaline would show if it were employed as an analysing plate.

On reversing the polarization of the ray (or turning round the crystal), the complementary tint appears. The same results occur if the crystal is employed in the first place to polarize the light, and tourmaline or calcareous spar is used to analyse it: so that the analogy or rather identity of effect with the tourmaline is complete.

I will now mention some other crystals which possess the analysing property, but not in such a high degree.

2. *Boracic acid.*—If dissolved in boiling water, it yields in cooling irregular crystals which have considerable analytic power. A crystal which in one position is so translucent as to be hardly distinguishable from the water in which it floats, is in the transverse position very strongly defined. It does not become dark all over, but *only in its outline.*

If now we employ it to analyse the tints of sulphate of lime, its *outline* becomes beautifully coloured. Nothing can exceed the delicacy of colouring which a number of these crystals exhibit when viewed together: those which lie in one direction appearing, for instance, *green*; those in a transverse direction *red.* The appearance is very unlike any other optical phenomenon that I know of, in consequence of two colours being seen in strong contrast, and without any intermediate tints; and also from the *outline* only of the objects being coloured, while their interior remains without colour. It is only when the crystals have a fibrous or striated structure that the tint extends over all their surface.

The boracic acid has the same analytic property and precisely the same appearance when it crystallizes from a solution of borax in phosphoric acid. The plumose crystals of it (No. 4. *suprà*) are very delicately coloured with the two opposite tints.

I obtained a very beautiful result by placing a drop of phosphoric acid upon a group of circular crystals. This caused a fresh deposition of boracic acid upon them as nuclei, which assumed the form of very delicate cilia, spreading in all directions as from a centre. These fringed circles showed the analytic property in an admirable manner, exhibiting four quadrants coloured alternately with complementary colours of great vivacity.

3. Another instance which is worthy of mention is the *oxalate af potash and chromium,* a salt whose optical properties have been investigated by Sir DAVID BREWSTER*. If some gum arabic is added to a solution of this salt, and a drop of it put between two plates of glass, it abandons its usual mode of crystallization for another, which resembles a microscopic vegetation composed of minute prisms growing one out of another, and variously arranged in sprigs and branchlets; while in other places it assumes an undulating capillary form, much resembling in miniature the tufts or locks of a species of *conferva* which is seen growing in pools of water or in the sea. Now these objects are possessed of a high analytic power, insomuch that when a plate of sulphate of lime is placed beneath them they assume a colour of great intensity and splendour, which is changed for the complementary tint when the polarization of the incident ray is reversed.

4. *Nitre.*—If nitre and gum arabic are dissolved together in hot water, a drop of

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* Philosophical Transactions for 1835, p. 91.

the solution put on a glass plate yields very good analytic crystals. These have a branched or plumose appearance, and assume beautiful colours in polarized light when a plate of sulphate of lime is placed beneath them. The microscope shows the colour to reside principally in the outline, but to the naked eye the whole film appears coloured. As these films may be obtained of large size, the phenomenon can be well seen by the unassisted eye.

A very interesting experiment, and one which throws much light upon the cause of these appearances, is to transmit a beam of polarized light *very obliquely* through a small prism of nitre immersed in gum, and viewed with the microscope. Its outline then generally exhibits two colours instead of one: for while the edge of the prism which is on that side from whence the ray of light comes is, for instance, of a red colour, the opposite edge will appear green. Reverse the polarization of the light, and these colours are exchanged one for the other. This observation enables us to explain the origin of the phenomenon in a satisfactory manner, and to show why it only occurs in crystals possessing strong double refraction, like nitre, in which the refractive indices of the two rays are materially different.

When a ray of common light is incident upon such a crystal, and therefore divides itself into two rays oppositely polarized, both rays are transmitted through the central parts of the crystal, which are bounded by parallel planes, or by planes approaching to parallelism. But when the bounding planes of the crystal are much inclined to each other, and therefore refract the light in the manner of a prism, the refractive indices of the rays may differ so much, that while one of them passes freely through such a prism, the other cannot pass at all, but suffers total internal reflexion, and is thereby dispersed; just as if the prism had a larger refracting angle with respect to that ray than to the other. Therefore if two oppositely polarized rays are presented successively to such a crystal, as in our experiment, one of them will be transmitted, and the other not. That this is the true explanation appears from this, that when the oblique planes are well formed and clearly defined by the microscope, the colour also is accurately limited by the same boundary: so that while this part analyses the tints of a plate of sulphate of lime, the rest of the crystal is inactive.

It may be inferred by analogy, that the same cause produces the analysing power of striated or fibrous surfaces, and of those in which the striæ are too minute to be discernible (as in No. 1. *suprà,* page 32): for it is not the property of all crystals with striated surfaces to have the analytic power, but only of such as are doubly refractive in a high degree.

I have said that the capillary crystals (No. 3.) possess the analytic property, although their diameter is often evanescent even with a microscope. An important inference may be drawn from this, viz. that a ray of light, *immediately* on entering one of these crystals, subdivides itself into two rays of different refractive indices, or at least that the thickness of crystal which is requisite to produce this effect is insensible to observation.

When nitre is made to crystallize in gum, it often shoots into prismatic crystals, which are very interesting objects, the more so, that they are of a permanent nature, and not liable to spontaneous change. When examined by polarized light, these prisms, in one position of their axis, frequently disappear completely. This arises from the refractive power of the gum being equal to that of one of the two rays in the crystal. Reverse the polarization of the ray, and the crystal appears, as it were, to start into existence, acquiring great strength and blackness of outline, and, not unfrequently, entire opacity. Again, when the sulphate of lime is interposed, this opacity disappears, and the crystal becomes brightly coloured.

Since it is probable that many better methods may be found of obtaining this kind of crystals than have hitherto presented themselves, I have hopes that it will be possible to obtain large and permanent artificial crystals, which may possess the advantages of the tourmaline without the inconvenience resulting from its dark colour.

V. *Observations on the Electro-chemical Influence of long-continued Electric Currents of Low Tension. By* GOLDING BIRD, *F.L.S., F.G.S., Lecturer on Experimental Philosophy at Guy's Hospital, &c. Communicated by* THOMAS BELL, *Esq., F.R.S.*

Received January 30,—Read February 2, 1837.

1. SCARCELY any branch of scientific investigation has yielded more important and interesting results than electro-chemistry, which, as is well known, in the hands of a DAVY, demonstrated the existence of the alkaline metals, and formed a memorable and important epoch in the history of chemical science. Sir HUMPHRY DAVY availed himself of the energetic influence of voltaic currents in a high state of tension, and elicited by means of large batteries. To M. BECQUEREL, however, we are almost entirely indebted for our knowledge of the chemical agency of feeble currents in reducing certain refractory oxides to the metallic state, although it must not be forgotten that Dr. EDMUND DAVY applied the power of weak currents of electricity to the detection of the metallic poisons, by reducing them to the reguline state. BUCHOLZ appears, in 1807, to have succeeded in obtaining crystals of metallic copper by the aid of a simple voltaic circle. But it is to our illustrious countryman Dr. FARADAY that we owe our acquaintance with the interesting circumstance of the power possessed by a current of very low intensity (elicited by a single pair of platinum and zinc plates) in decomposing several saline combinations, as the iodide of potassium, sulphate of soda, &c., and isolating their respective constituents. In offering the following observations, I have not sufficient presumption to suppose them to be possessed of any very important or original value; but being, as they are, the results of carefully repeated experiments, and containing an account of what I believe to be some previously unobserved facts, I deem myself justified in submitting them to the notice of the Royal Society.

2. The facts recently pointed out by M. BECQUEREL of the energetic power exerted by weak electric currents in effecting the reduction not only of the oxides of copper, lead, or tin, but even of glucina, alumina, and silica, are probably very well known to every one. This philosopher obtained these interesting results by means of a single pair of plates, placing the solution of the metallic salt in a glass tube closed at one end by means of a plug of moistened clay, and immersed in a weak solution of common salt: on placing then a compound metallic arc formed of zinc and platinum in the solutions in such a manner that the platinum leg might be immersed in the tube containing the metallic solution, (to which M. BECQUEREL applies the general term of "negative tube,") whilst the zinc dips in the solution of salt, decomposition ensues, and after a lapse of time, varying from a few hours to some weeks, the metal is gene-

rally deposited from its solution on the platinum plate in a more or less crystalline form. M. Becquerel did not attribute the reduction of the metal to the electric current alone, but conceived that three distinct causes, at least, concurred in producing this effect. The decomposition of the water and of the common salt by the electric current set in motion, and the transference of hydrogen and soda through the clay diaphragm to the negative tube, where the alkali unites with the acid holding the metal in solution, causing the deposition of its oxide, which, while in its nascent state, is reduced by the hydrogen, and precipitated in its metallic form on the negative electrode; thus regarding the hydrogen furnished by the decomposition of the water as the actual reducing agent. In some cases, a fourth cause is supposed to be superadded to these, as when a body is used for the negative electrode, for which the metal in solution has a certain degree of affinity; a well-known example of which is found in the reduction of potassium from a solution of potassa when submitted to comparatively weak voltaic action in contact with mercury. Mercury is not the only metal applicable to this purpose, M. Becquerel having frequently used iron with success. He states that the solutions of the pure chlorides of zirconium, glucium, titanium, silicon, &c., refused to yield to the reducing action of weak electric currents, until after the addition of a small quantity of chloride of iron: this the current readily decomposed, precipitating the iron in a crystalline form on the platinum plate, (negative electrode,) which deposit speedily *induces* the commencement of the decomposition of the more refractory salts. This circumstance he attributes to the affinity of the iron for the other metal tending to the formation of an alloy, and expressly states, that when *perfectly pure* the above-mentioned chlorides *did not undergo the slightest decomposition.* I have ascertained, however, that if the electric current is *continuous,* notwithstanding its weak tension, this reduction and decomposition may be effected without the presence of iron, or indeed of any other metal in solution, excepting that which we are attempting to reduce. I must apologize for the above rapid sketch of some of M. Becquerel's researches, the introduction of which into this paper appeared to be necessary for the better understanding the results of my own more limited investigations.

3. On commencing my experiments on the chemical power of electric currents of weak tension, I soon found the want of an apparatus capable of affording an *equal* and *continuous* current of low intensity, which appeared to me to be absolutely necessary for the success of my experiments. In the common electromotor the currents are at first very energetic, but soon cease, or become so feeble as to be scarcely able to traverse a fluid medium unless fresh portions of the exciting fluid is added from time to time. Even Professor Daniell's very ingenious single-pair battery, although most excellent for electro-magnetic purposes, and affording considerable quantities of electricity, yet required the occasional addition of fresh acid and sulphate of copper, which was inconvenient when required to be kept in constant action for some weeks, besides occasioning an irregularity in the intensity of the current. The *quan-*

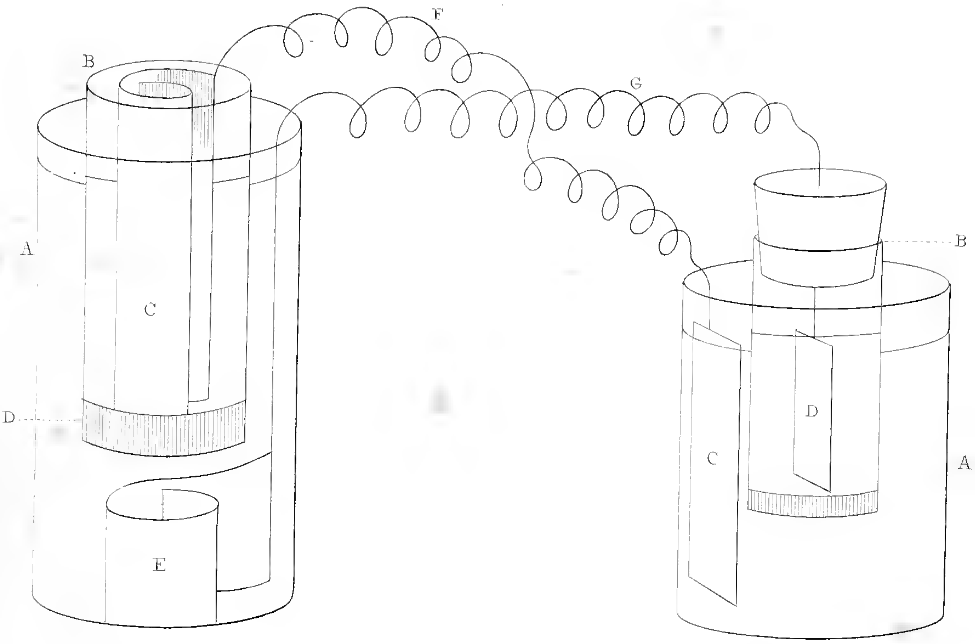

Fig. 1.

Fig. 2.

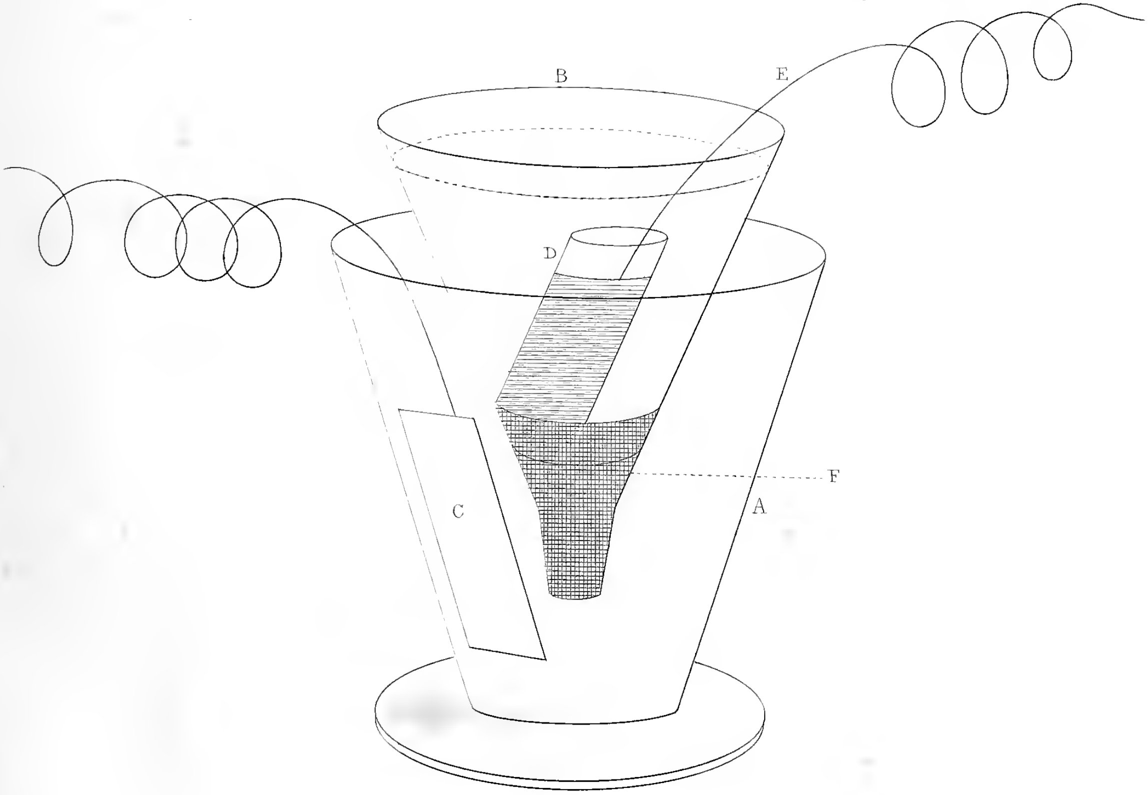

Fig. 3.

Fig. 1. The battery connected with the decomposing apparatus.
2. The decomposing apparatus.
3. A modification of Fig. 2 for obtaining the amalgams of the alcaline metals.
N.B. The dotted line in Fig 3, shews the level of fluid in the funnel above the mercury.

tity of electricity appearing to be by no means so important as a *continuous and equable current*. For similar reasons Mr. **Mullins**'s modification of Professor **Daniell**'s battery was found equally objectionable; added to which the currents evolved are liable to be materially affected by the admixture of the exciting fluids through the membranous partition, which always takes place sooner or later by endosmosmic action.

4. After several experiments I was induced to prefer the following apparatus, (which after all is but a slight modification of Professor **Daniell**'s,) in consequence of its affording a constant and regular current of electricity of very weak tension, continuing for several weeks or even longer without any fresh addition of exciting fluid. A glass cylinder, 1·5 inch in diameter and 4 inches in length, was closed at one end by means of a plug of plaster of Paris 0·7 inch in thickness: this cylinder was fixed by means of corks inside a cylindrical glass vessel about 8 inches deep and 2 inches in diameter. A piece of sheet copper, 6 inches long and 3 inches wide, (having a copper conducting wire soldered to it,) was loosely coiled up, and placed in the small cylinder with the plaster bottom: a piece of sheet zinc of equal size was also loosely coiled up, and placed in the larger external cylinder (being furnished like the copper plate with a conducting wire). The larger cylindrical glass being then nearly filled with weak brine, and the smaller with a saturated solution of sulphate of copper, the two fluids being prevented from mixing by the plaster of Paris diaphragm, the apparatus is complete*; and if care is taken that the fluids in the two cylinders are at the same level, will continue to afford a continuous current of electricity for some weeks, the sulphate of copper being very slowly decomposed. So feeble is the current evolved by an apparatus of this kind, that on connecting the two conducting wires with a common galvanometer, (having but one needle suspended on a pivot,) a deviation to only 10° or 12° took place: with **Nobili**'s galvanometer, with nearly astatic needles, a deviation to 90° immediately ensued, as might be expected from the greater delicacy of the instrument. So small, indeed, is the quantity of electricity evolved by the apparatus I have described, as compared to that evolved by the ordinary electromotor, that I was unable to produce the simplest form of electro-magnetic rotation by its aid. After it has been in action for some weeks, chloride of zinc is found in the external cylinder, and beautiful crystals of metallic copper, frequently mixed with the ruby protoxide (closely resembling the native ruby copper ore) and large crystals of sulphate of soda, are found adhering to the copper plate in the smaller cylinder, especially on that part where it touches the plaster diaphragm.

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* So simple an apparatus scarcely requires an illustration: in the accompanying outline sketch I have, however, figured it, to prevent any error arising from the account in the next not being sufficiently explicit.

Fig 1. represents the battery connected with the apparatus described in paragraph 4.

A. The external cylinder.

B. The smaller one; with D. The plaster of Paris bottom.

C. The coil of copper in the cylinder B, having the conducting wire F soldered to it.

E. The coil of zinc with the wire G soldered to it.

5. If the two copper conducting wires belonging to the little apparatus just described are immersed in water acidulated with sulphuric acid, action soon commences, bubbles of hydrogen appearing at the wire connected with the zinc plate, whilst that connected with the copper plate became tarnished, oxydized, and at last partly dissolved, giving a blueish tint to the fluid; affording an approach to the decomposition of water by a single pair of plates. For the success of this experiment, it is, as might be expected, necessary that the *positive* electrode at least should be formed of a readily oxidizable metal; for when both wires were of platinum, no evidence of decomposing action ensued.

6. If, instead of acidulated water, the wires were immersed into a solution of nitrate or acetate of lead, no *immediate* action ensued, but in about fifteen minutes, or even less, some elegant and delicate feathers of metallic lead, which rapidly increased in size, appeared at the negative electrode. This effect did not occur when *both* conducting wires were of platinum; but when the *negative* electrode only was composed of that metal, the reduction of the lead continued with apparently increased energy. From these experiments, as well as many others of a similar kind which it is unnecessary to detail, it appears fair to presume, that in availing ourselves of the *reducing* agency of feeble currents, or at least of those elicited by a single pair of plates, it is necessary that the positive electrode should be composed of a readily oxidizable metal: thus using a kind of battery of *two* cells, in which the wires forming the electrodes, and the fluid submitted to experiment, form the contents of the second cell.

7. But few metallic solutions yield so readily as those of lead to the reducing agency of weak currents; and where a longer time and continuance of action is required to effect the reduction, the decomposing apparatus of M. Becquerel will be found a useful addition to the little battery (4.), with the substitution of a plug of plaster of Paris for one of clay. This piece of apparatus is, in fact, a counterpart of the battery itself, consisting, like it, of two glass cylinders, one within the other, the smaller one having a bottom or floor of plaster of Paris fixed into it: this smaller tube may be about half an inch wide and three inches in length, and is intended to hold the metallic solution submitted to experiment, the external tube in which it is immersed being filled with a weak solution of common salt *. Into the latter solution a slip of amalgamated zinc, (for the positive electrode,) soldered to the wire coming from the copper plate of the battery (4.), is immersed, whilst for the negative electrode a slip of platina foil, fixed to the wire from the zinc plate of the battery, passes through a

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* Fig. 2. in the sketch represents this apparatus connected with the battery.

A. The larger tube.

B. The smaller one, with the plaster bottom.

C. The electrode of amalgamated zinc connected by the wire F with the copper plate C of the battery (Fig. 1.).

D. The negative electrode of platinum connected by the wire G with the zinc plate of the battery (Fig. 1.).

cork fixed in the mouth of the smaller tube, and dips into the metallic solution it contains.

8. When a solution of the chlorides or nitrates of iron, copper, tin, zinc, bismuth, antimony, lead, or silver, is placed in the smaller tube of the apparatus (7.), and connexion made with the battery* in the manner already described, action is almost instantly apparent, water is decomposed, and torrents of minute bubbles of hydrogen are evolved at the surface of the platinum plate, (negative electrode,) which continues for a short time, sometimes, indeed, lasting for hours; a circumstance depending apparently upon the degree of facility with which the metal under experiment is reduced. Thus with solutions of copper, scarcely a bubble appears, the metal being almost immediately reduced, all the hydrogen being probably employed from the instant of completing the circle, for that purpose: with solutions of lead, tin, or silver, the evolution of hydrogen continues for a short time only, and ceases as soon as the minutest portion of reduced metal appears on the platinum plate; but with solutions of iron and manganese the evolution of gas frequently continues for six, eight, or ten hours, or even longer; the evolution of hydrogen thus seeming to bear something like an inverse ratio to the ease with which metal is reduced. After the hydrogen has ceased to appear at the negative electrode, striæ of the reduced metal, which rapidly increase, are deposited on the surface of the platinum.

9. The metals thus reduced generally, but not invariably possess a perfectly metallic lustre, are always more or less crystalline, and often very beautifully so, affording a considerable contrast to the irregular soft spongy masses obtained from the same solutions by means of large batteries. The crystals of copper obtained by the process just detailed (8.), rival in hardness and malleability the finest specimens of native copper, which they much resemble in appearance. The crystallization of bismuth, lead, and silver by these means, is very beautiful, that of the former metal being lamellar, of a lustre approaching to that of iron, but with the reddish tint peculiar to this metal. Silver may be thus obtained of a snowy and indeed dazzling whiteness, usually under the form of needles.

10. The metallic solutions hitherto mentioned as yielding to the action of the little battery are, as is well known, equally acted on by larger voltaic batteries, consisting of a considerable number of alternations, the metal being reduced in a spongy form, often destitute of a metallic appearance. But there are some metals which are deposited from their solutions as oxides only, when acted on by currents from large batteries, and yet are deposited in a brilliant metallic form if submitted to the action of the currents from the little apparatus already described (4). Of these nickel is an example: a solution of its chloride or sulphate, when placed in the smaller tube of the decomposing apparatus (7.), yielding after some hours a crust of metallic nickel on the negative electrode, often of a silvery lustre on the surface immediately

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* It may here be proper to remark, that by the word battery in the course of the following observations I always allude to the modification of Prof. Daniell's battery described in § 4.

applied to the platinum, that portion of the crust more in contact with the fluid being generally black, and frequently covered with a layer of the hydrated and gelatinous green oxide.

11. Finding that by means of this apparatus I could command a weak but continuous current capable of reducing even the more refractory metallic oxides, I was anxious to ascertain whether the current was sufficiently energetic to cause the reduction of those oxides which (as silica) do not yield to powerful batteries, and which M. Becquerel obtained only alloyed with iron.

12. The solution of silicon used by M. Becquerel was prepared by dissolving gelatinous silicic acid in hydrochloric acid of commerce, which always contains iron; this on being submitted to the action of a single pair of plates deposited an alloy of iron and silicon on the negative electrode. As this solution contains but a very small quantity of silicon, I substituted a solution of fluoride of silicon in alcohol obtained by passing a current of the gaseous fluoride into strong alcohol. On filling the smaller tube of the decomposing apparatus (7.) with this solution, and making the connexion with the battery in the manner already described, bubbles of hydrogen were copiously evolved at the surface of the platinum plate (negative electrode), which continued for eight or ten hours, when the platinum appeared to be tarnished, and in twenty-four hours a copious deposit of silicon had taken place on the platinum, to the surface of which it firmly adhered. Around the reduced silicon, and suspended in the fluid, was a dense gelatinous cloud of silicic acid. On quickly withdrawing the slip of platinum, dipping it in water, and then pressing it between folds of bibulous paper it was dried, and freed from any adhering solution. The silicon was nearly black and granular, under a lens, exhibiting a tendency to a crystalline form. It was not deposited on the platinum in a confused or irregular manner, but in longitudinal striæ, which appeared to follow the direction of certain lines of minute eminences on the surface of the piece of platinum, produced apparently by scouring it with fine sand and a piece of cork before being used for the construction of the negative electrode.

13. The silicon thus procured becomes of a snowy whiteness when ignited in the flame of a spirit lamp, and falls off the platinum in thin flakes, being in fact converted into silicic acid. It is not very easy to oxidate the whole, in consequence of the flakes of the acid forming an incrustation over the subjacent silicon, and protecting it from the oxidating influence of the air even at a red heat. A portion of the silicon *removed* from the platinum did not appear to dissolve in hydrochloric acid; but when the platinum itself with the firmly adhering silicon was immersed in the acid, slow action ensued, bubbles of hydrogen being evolved from the *exposed surface of platinum,* the silicon very slowly disappearing; the solution being probably occasioned by the formation of a simple voltaic circle, the silicon and platinum being the metals, and the acid the exciting fluid. When an aqueous solution of hydrofluosilicic acid is substituted for the fluoride of silicon, the metalloid is reduced, but slower and in

smaller quantity; differences arising in all probability from the smaller quantity of silicon present in the solution.

14. I have frequently had occasion to observe that when an aqueous solution of hydrofluosilicic acid has been submitted to the action of currents of low tension (from the battery already described (4.)) continued during two or three weeks, a considerable deposition of gelatinous silicic acid takes place around the reduced silicon; mixed with which, or precipitated in a zone on the sides of the tube, especially if of small diameter, or even upon the platinum electrode itself, frequently appear minute crystalline grains of sufficient hardness to scratch glass, and appearing translucent under the microscope. These minute crystals I have no hesitation in stating to be crystallized silicic acid, closely resembling its natural form of *quartzose sand*.

15. I next attempted to form potassium with the same apparatus, but failed, as I had anticipated, from the presence of water, which indeed would react on the potassium as soon as reduced. I therefore endeavoured to form its amalgam, well knowing that when dissolved in mercury a very weak electric current is sufficient to prevent the oxidating influence of water upon it; and by using a modification of the decomposing apparatus before described (7.) I succeeded perfectly. In place of the smaller tube containing the metallic solution, I used a small glass funnel*, the beak of which was carefully filled up with plaster of Paris: on this plaster floor I placed a piece of glass tube closed at one end, about 0·5 inch in length and 0·2 inch in diameter, and half filled with pure mercury; this tube was not placed vertically, but inclined so as to form an angle of about 40° with the plaster floor of the funnel, which with its contents was partly immersed in the weak brine contained in the larger cylinder of the decomposing apparatus. The external cylinder communicated as before with the copper plate of the battery, by means of a slip of amalgamated zinc dipping into the brine it contained. The funnel was then nearly filled with a solution of chloride of potassium, and a piece of platinum wire connected with the zinc plate of the battery being twisted into a flat spiral at one end so as to present a larger surface, was immersed in the mercury contained in the little tube submerged in the saline contents of the funnel. The circuit being thus completed, galvanic action soon became apparent, bubbles of hydrogen being evolved from the surface of the mercury (which now formed the negative electrode) in a very curious manner, not in confused and rapid streams, but in large and distinct bubbles, which very slowly appeared, and performed several gyratory movements on the surface of the fluid metal before they were evolved. Not unfrequently a single bubble only was seen, which continued

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* This variety of the apparatus is shown in fig. 3. of the outline sketch.

A. The external vessel containing the salt and water.

B. The funnel containing the alkaline salt required, with its plaster floor F.

C. The positive electrode of amalgamated zinc communicating with the copper plate of the battery (fig. 1.).

D. The little tube containing mercury immersed in the alkaline solution with a platinum wire E, connected with the zinc plate of the battery (fig. 1.) dipping into it.

playing on the surface of the mercury for half an hour, or even longer, before it rose to the surface of the fluid. In about eight or ten hours the mercury had swollen to double its former bulk, and part of it had actually crept* up the platinum wire to the height of 0·3 inch above the level of the other portion, adhering to the wire like so much tenacious mucilage. On dipping a piece of turmeric paper into the contents of the funnel it turned brown, demonstrating the presence of an alkali. The mercury was removed from the little tube as quickly as possible, and poured into distilled water, which acted upon it, causing the evolution of hydrogen gas from its whole surface, and became alkaline from the formation and solution of the oxide of potassium or potass. The film of mercury adhering to the platinum wire remained on it for some days, giving it the appearance of having been amalgamated. This experiment, several times repeated, yielded precisely similar results, from which I think that I am justified in stating that potassa can be reduced by means of the feeble current elicited by a single pair, or as the positive electrode was formed of an oxidizable metal, in the opinion of some, perhaps of two pairs of plates.

16. By submitting in the same apparatus a solution of chloride of sodium to the influence of the battery, analogous results were obtained. An amalgam of sodium being formed, although a much longer time was required, and the result of the experiment, although quite decided, was by no means so distinct as in the case of the reduction of potassium.

17. But of all the saline solutions that I have yet submitted to experiment, none afforded such conclusive and interesting results as those of ammonia. The ammonium being reduced with almost as much ease as copper or tin, when a solution of its chloride (hydrochlorate of ammonia) is submitted to the action of the voltaic current in contact with mercury, in the same manner as chloride of potassium or sodium. The same adhesion and creeping up of the mercury along the wire (15.) is observed, and after a few hours the fluid metal swells to five or six times its former bulk. On removing it quickly and drying it, by allowing it to fall on bibulous paper the amalgam of ammonium is obtained of a buttery consistence, possessing a dull silvery colour, and yielding a peculiar crackling, or (if I may be allowed the expression) an emphysematous sensation to the finger on pressing it: on being immersed in water it very slowly gave off hydrogen, and yielded a solution of ammonia.

18. By far the most satisfactory method of obtaining this amalgam is by using for the negative electrode a piece of platinum wire coiled up at one end, after it has been amalgamated by dipping it into the ammoniacal amalgam obtained by the last described process (17.). A minute quantity of mercury is thus made to adhere to the wire, which being connected with the zinc side of the battery, is dipped into a solution of hydrochlorate of ammonia contained in the smaller tube of the apparatus used in effecting the reduction of silicon (7.). The circuit being completed, a few bubbles

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* This peculiar creeping up of the mercury along the wire does not take place if the little tube holding the fluid metal is placed in a vertical position.

of hydrogen are disengaged from the amalgamated wire, which soon cease, and in an hour or two, a leaden grey spongy mass is observed adhering to the wire, which is sometimes sufficiently bulky to fill the tube, and putting on much of the external appearance of a mass of cellular galena. This mass consists of a spongy amalgam of ammonium, containing a very minute proportion of mercury; it is lighter than the solution in which it is immersed, for on adroitly separating a portion of it, it rises to the surface and rapidly decomposes water, hydrogen being evolved and ammonia formed.

19. It is a very curious and interesting fact, that although this spongy ammoniacal amalgam cannot be kept immersed in water even for a few instants without the formation of ammonia, yet as long as it is connected with the negative electrode of the battery, it may be preserved without change for days and weeks. The instant the connexion with the battery is broken, a mass of this amalgam, as large as a walnut, appears to vanish in a few seconds, torrents of minute bubbles being given off, and a scarcely appreciable quantity of mercury being left on the wire. On again closing the connexion with the battery decomposition recommences, and the amalgam is reproduced.

20. From a review of the results of these experiments, we cannot help being struck with the very energetic power of electric currents of weak tension; currents of sufficient energy to reduce to the metallic state oxides on which currents of higher tension from large batteries are comparatively powerless. This fact, although pointed out by Becquerel and other philosophers, has (as far as I am aware) never been before shown to hold good in the reduction of the alkaline metals. Potassium and ammonium not having, I believe, been previously obtained by the weak current emanating from the chemical action of saline solutions on a single pair of plates; and silicon, although obtained by Becquerel combined with iron, has not been procured before in a pure state by electric currents, at least by those of feeble tension*.

In conclusion I may be permitted to observe, that in applying weak electric currents to the reduction of metallic oxides, it is *absolutely necessary that a continuous current be employed, and that its cessation even for an instant is often fatal to the success of the experiment on hand,* which cessation or suspension, as far as my experiments have gone, the modification I have proposed of Professor Daniell's battery appears to be capable of obviating.

Guy's Hospital,
January 20, 1837.

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* Some other curious circumstances connected with the decomposing electro-chemical power of currents of low tension have fallen under my observation, but have not yet been sufficiently examined to authorize their publication as facts. Some of these I may perhaps at a future period, with the permission of the Society, have the honour of submitting to its notice.

VI. *Inquiries respecting the Constitution of Salts. Of Oxalates, Nitrates, Phosphates, Sulphates, and Chlorides. By* Thomas Graham, *Esq. F.R.S. Edin., Professor of Chemistry in the Andersonian University of Glasgow, Corr. Member of the Royal Academy of Sciences of Berlin, &c. Communicated by* Richard Phillips, *Esq. F.R.S.*

Received June 23,—Read November 24, 1836.

From the results obtained in a former paper upon water as a constituent of sulphates, it seemed likely that a close analogy would generally be found to exist between any hydrated acid and the magnesian salt of that acid. The sulphate of water is constituted like the sulphate of magnesia; and so do I now find the oxalate of water to resemble the oxalate of magnesia, and the nitrate of water to resemble the nitrate of magnesia. Indeed it appears probable that the correspondence between water and the magnesian class of oxides (as we may call the metallic oxides isomorphous with magnesia) extends beyond their character as *bases*,—that in certain subsalts of the magnesian class of oxides we have the metallic oxide replacing the water of crystallization of the neutral salt, or discharging a function which was thought peculiar to water.

In the formation of a double sulphate a certain kind of substitution or displacement was observed, such as the displacement of an atom of water pertaining to the sulphate of magnesia, by an atom of sulphate of potash, to form the double sulphate of magnesia and potash. The same kind of displacement appears to occur likewise in the construction of double oxalates; and the tracing of it enables us to form an idea of the constitution both of the double and of the superoxalates, and to explain their derivation, as in the case of the sulphates.

I. *Of the Oxalates.*

The oxalates promised ample scope for investigation from their number and variety. For we have not only neutral oxalates, double oxalates, and binoxalates, but likewise an unparalleled combination, the quadroxalate of potash, of which the true constitution or proximate composition is a most interesting subject of inquiry.

1. *Oxalate of Water, or Hydrated Oxalic Acid.*

$$\dot{\mathrm{H}}\ \overset{\cdots}{\mathrm{CC}}\ \dot{\mathrm{H}}^{2}.$$

The recent and accurate experiments of Berzelius, Gay-Lussac, and Turner, leave no doubt that the crystals of oxalic acid contain three atoms of water. I find the acid to crystallize with this proportion of water in a variety of circumstances, and believe that it is never deposited from its aqueous solution in any other state. Of these three atoms of water one atom is basic, which is expressed in the formula by placing its symbol *before* that of the acid; while the other two atoms of water are attached to this oxalate of water, and may be termed the *constitutional* water of the oxalate of water. These two atoms of water are found in the oxalate of magnesia, the oxalate of zinc, and the other oxalates of the magnesian class, as well as in the oxalate of water.

It is well known that oxalic acid can likewise exist in combination with no more than one atom of water (its basic water), and is obtained in that state by drying it at a temperature a little above 212° Fahr., or on subliming the hydrated acid by a higher temperature. I have made many experiments in order to discover whether, in the case of the other two atoms of water, one is retained more strongly than the other, or whether an oxalate of water with one additional atom of water, instead of two, could be obtained. The common crystals were dried at various temperatures, both in air and in vacuo, but either none of the water was lost, or the entire two atomic proportions. There is certainly no intermediate hydrate.

2. *Oxalate of Zinc.*

$$\dot{\mathrm{Zn}}\ \overset{\cdots}{\mathrm{CC}}\ \dot{\mathrm{H}}^{2}.$$

In the oxalate of water we observe a contracted solubility, and all the oxalates of the magnesian class of oxides are very sparingly soluble in water. They may be obtained by precipitation, on mixing a solution of oxalate of potash with sulphate of zinc, &c. Cold solutions of the salts were always made use of in our experiments; and the precipitates, which were always granular and more or less distinctly crystalline, were washed with cold water, and dried by exposure to the air for a week or two, without the application of artificial heat.

The oxalate of zinc is admitted to possess two atoms of water, and these I find are retained pretty strongly, as in the case of oxalate of water. It was observed that 24·95 grains of the salt lost only 0·44 grain by three days exposure to 212° Fahr.; but by a few hours at 315° Fahr. the salt lost in all 4·87 grains of water, and appears to have become anhydrous.

3. *Oxalate of Magnesia.*

$$\dot{\mathrm{Mg}}\ \overset{\cdots}{\mathrm{CC}}\ \dot{\mathrm{H}}^{2}.$$

The oxalate of magnesia retains its two atoms of constitutional water very strongly, and it is doubtful whether they can be expelled without decomposing the salt; 13·74 grains of the salt lost only 0·32 grain by an exposure to 212°, protracted for several days; and by two days at 300° Fahr. the whole loss amounted only to 0·47 grain. 22·36 grains of the same salt, ignited, left 5·94 grains of caustic magnesia, or one part of the salt contains 0·2656 magnesia. A salt constituted with two atoms of water should contain 0·2759 magnesia, of which the specimen analysed falls a little short, probably from containing some hygrometric moisture.

The *oxalate of manganese* lost nothing at 212°, and was found by analysis to contain 0·2416 water, which approaches very closely the quantity equivalent to two atoms, namely, 0·2474 water in one hydrated oxalate of manganese.

In regard to several other oxalates of this class, namely, the oxalate of the protoxide of iron, of oxide of nickel, of oxide of cobalt, and of oxide of copper, I believe it is impossible to obtain them in a state of sufficient purity for analysis. They appear to carry down with them portions of the precipitating salts; and they alter manifestly in appearance and composition during the progress of the washing, to which they must be submitted for the purpose of purification. In the case of oxalate of copper, which was examined most particularly, the results were so anomalous that no inference whatever could be drawn from them.

It will appear, however, that a neutral oxalate of copper with two atoms of water can exist but in combination with oxalate of potash, or with oxalate of ammonia, as a double salt.

None of the oxalates of the magnesian class of oxides is more soluble in oxalic acid than in water, and none of them combines with that acid to form a binoxalate. The crystals, which are obtained on mixing together solutions of binoxalate of potash and sulphate of magnesia, and which have been supposed to be a binoxalate of magnesia, are really a mixture of oxalate of magnesia and of quadroxalate of potash. Hence there is no combination of oxalate of magnesia with oxalate of water; which illustrates the fact that bodies of the same class, such as these two oxalates are, have no disposition to enter into union and form a new compound.

4. *Oxalate of Lime.*

$$\dot{\mathrm{C}}\mathrm{a}\,\overset{\cdots}{\mathrm{CC}}\,\dot{\mathrm{H}}^{2}.$$

The oxalate of lime contains two atoms of water, like the oxalate of magnesia, but parts with its water more freely than that salt. Thus 12·06 grains of the hydrated oxalate of lime were found to lose 1·6 grain of water at 212° Fahr. in the course of two days, 1·68 grain in three days, 1·84 grain in six days, and nothing more in nine days. The salt originally consisted of 100 oxalate of lime united to 27·85 water, of which last it has lost 19·53 parts, and retained 8·32 at 212°. It is probable therefore that the constitution of hydrated oxalate of lime is the same as that of hydrated oxalate of magnesia, that oxalate of lime forms only one definite hydrate, containing two atoms of water, but that it parts with the whole of its constitutional water at a moderate temperature.

5. *Oxalate of Barytes.*

$$\dot{\mathrm{Ba}}\,\overset{\cdots}{\mathrm{CC}}\,\dot{\mathrm{H}}.$$

This oxalate differs from all the preceding, and contains only one atom of water. It was formed by digesting an excess of oxalic acid upon carbonate of barytes, and afterwards washing the resulting oxalate with cold water. 20·60 grains of the oxalate, calcined by a low red heat, left 16·45 grains carbonate of barytes, equivalent to 12·77 barytes. Hence it follows that the oxalate consisted of

		Composition of $\dot{\mathrm{Ba}}\,\overset{\cdots}{\mathrm{CC}}\,\dot{\mathrm{H}}$.
Barytes	100	100
Volatile matter	61·32	59·08
	161·32	159·08

Before being washed this oxalate had a sour taste, and the volatile portion of it amounted to 67·01 parts instead of 61·32; but it was evidently the neutral oxalate with a little free oxalic acid. It was not a binoxalate; nor did such a salt present itself on digesting the neutral oxalate in oxalic acid, so that I am constrained to deny the existence of a binoxalate of barytes. Indeed it is scarcely a matter of doubt that no supersalt whatever exists of barytes, strontian, lime, or of the magnesian class of oxides.

6. *Oxalate of Potash.*

$$\dot{\mathrm{K}}\,\overset{\cdots}{\mathrm{CC}}\,\dot{\mathrm{H}}.$$

Oxalate of potash is known to crystallize from solution with one atom of water, and with no other proportion. The crystals speedily become white and opake at 212° Fahr., from the loss of water, but cannot, I believe, be made quite anhydrous at that temperature; at least a portion of the salt, which had been exposed to 212° for four days, still retained water, consisting of 100 oxalate of potash and 3·4 water, which is rather less than a third of the water which the salt originally contained (10·8 parts). The oxalate of potash becomes quite anhydrous when dried at 300°. Of salt so dried 100 parts reabsorbed 10·63 water in a damp atmosphere with the greatest avidity. The oxalate of potash has therefore a certain attraction for a single atom of water, and this is an important feature of the salt.

7. *Binoxalate of Potash.*

$$\dot{\mathrm{K}}\,\overset{\cdots}{\mathrm{CC}} + \dot{\mathrm{H}}\,\overset{\cdots}{\mathrm{CC}}\,\dot{\mathrm{H}}^2.$$

This salt has hitherto been supposed to contain only two, but it certainly contains three atoms of water.

21·37 grains of the salt, calcined by a full red heat, which is necessary for complete decomposition, left 10·14 grains of carbonate of potash. Allowing the potash an equivalent proportion of oxalic acid, the salt must consist by this experiment of

Potash	32·23
Oxalic acid . . .	49·38
Water	18·39
	100·

The water almost coincides with three atoms, which would amount to 18·42 per cent. of the salt.

In the formation of the binoxalate of potash, the constitutional atom of water of the neutral oxalate of potash appears to be displaced by an atom of hydrated oxalic acid; so that the formula of binoxalate of potash represents anhydrous oxalate of potash, followed by oxalate of water with two atoms of water, as given above. The same principle of derivation applies most happily to that anomalous salt, the quadroxalate of potash.

8. *Quadroxalate of Potash.*

Analytic formula . . . $\dot{K}\,(\ddot{C}\dot{C})^4\,\dot{H}^7$.

Rational formula . . . $\dot{K}\,\ddot{C}\dot{C} + \dot{H}\,\ddot{C}\dot{C} + 2\,(\dot{H}\,\ddot{C}\dot{C}\,\dot{H}^2)$.

The formula of the preceding salt is terminated by two atoms of water: let us replace them by two atoms of hydrated oxalic acid, and we have the quadroxalate of potash. We thus derive the quadroxalate from the binoxalate, in the same way that the binoxalate itself is derived from the oxalate.

There can be no doubt, from the accurate analysis of Berzelius, that this salt contains seven atoms of water. He found 100 parts of the quadroxalate of potash to yield by ignition 27·225 carbonate of potash. In an experiment in which 17·3 grains of the salt were ignited by us, there resulted 4·7 carbonate of potash; which is 27·11 carbonate of potash from 100 quadroxalate. Berzelius determined the water directly by igniting the salt with oxide of copper, and found it to amount to 24·8 per cent. of the salt. Calculated from our experiment, the water comes out 25·05 per cent., while the theory of seven atoms of water in the salt requires 24·72 per cent.

10·87 grains of this salt, dried by a nitre-bath, of which the temperature was 240°, lost eventually 1·46 grains; or 100 salt lost 13·43. Four atoms of water amount to 14·12 per cent. of the salt, to which the experimental result approximates sufficiently to prove that this salt parts readily with four of its seven atoms of water. These four atoms of water are evidently the constitutional water of the two atoms of hydrated oxalic acid, which the quadroxalate contains. When the salt is still more strongly heated, oxalic acid itself goes off, partly as a sublimate and partly in a decomposed state.

9. *Oxalates of Ammonia.*

The oxalate and the binoxalate of ammonia agree exactly in composition with the corresponding salts of potash, the hypothetic oxide of ammonium being substituted for potash. It has been supposed that no quadroxalate of ammonia exists; but this is a mistake. Such a salt is formed, on dissolving together equal weights of binoxalate of ammonia and hydrated oxalic acid, and is analogous in form and composition to the quadroxalate of potash.

10. *Oxalate of Soda.*

$$\dot{\mathrm{Na}}\,\overset{\cdot\cdot\cdot}{\mathrm{CC}}.$$

This salt is perhaps the least soluble of the salts of soda, and presents itself as a granular precipitate on saturating carbonate of soda with oxalic acid. Of the oxalate of soda dried in air without the application of heat, 23·44 grains left 18·52 carbonate of soda when strongly ignited, or 100 oxalate yield 79·01 carbonate of soda. Now 100 anhydrous oxalate of soda should yield 79·09 carbonate of soda. Hence the oxalate of soda is correctly stated to be anhydrous. It nevertheless combines with hydrated oxalic acid, and forms a binoxalate. In this compound we have simply the attachment of an atom of the oxalate of water, to the atom of oxalate of soda, without the displacement of an atom of water, as in the formation of the binoxalate of potash. Probably the absence of the atom of water in the oxalate of soda indicates an indifference on the part of this salt to enter into further combination. There is certainly a binoxalate of soda; but this binoxalate cannot support the further attachment to it of two atoms of hydrated oxalic acid, and there is no quadroxalate of soda.

11. *Binoxalate of Soda.*

$$\dot{\mathrm{Na}}\,\overset{\cdot\cdot\cdot}{\mathrm{CC}} + \dot{\mathrm{H}}\,\overset{\cdot\cdot\cdot}{\mathrm{CC}}\,\dot{\mathrm{H}}^2.$$

This salt I find to resemble the binoxalate of potash, in containing three atoms of water. 22·11 grains, strongly ignited, left 8·05 grains fused carbonate of soda; or 100 binoxalate leave 40·67 carbonate of soda, equivalent to 23·84 soda; while a binoxalate with three atoms of water should yield 23·95 per cent. soda, or almost exactly the experimental result. The binoxalate of soda lost little more than 1 per cent. of its weight when kept at 212° Fahr. over sulphuric acid in vacuo. But by a heat approaching 300° Fahr. the salt lost 14·64 per cent. of water, which is a little more than two atomic proportions, namely, 13·78 per cent. Hence this salt retains the whole of its constitutional water at 212°, but loses two atoms of it at a higher temperature, retaining strongly the third atom of water, which is basic.

Double Oxalates.

The number of double oxalates is not so great as is generally supposed. On mixing a solution of binoxalate of potash either with the muriate or the sulphate of magnesia, zinc, &c., the oxalate of magnesia or of zinc precipitates, while the quadroxalate of potash is formed, and remains in solution or crystallizes, being very sparingly soluble, according to circumstances. When binoxalate of potash is digested upon magnesia or upon oxide of zinc, a portion of the oxide is dissolved, but is quickly deposited again as an insoluble oxalate, and no double salt formed. But one member at least of the magnesian class of oxides, namely, oxide of copper, is dissolved by the binoxalates of the alkalies, and forms double salts, which were discovered and carefully examined by M. Vogel of Bayreuth.

12. *Oxalate of Copper and Potash.*

$$\dot{\mathrm{K}}\,\ddot{\mathrm{C}}\dot{\mathrm{C}} + \dot{\mathrm{C}}\mathrm{u}\,\ddot{\mathrm{C}}\dot{\mathrm{C}}\,\dot{\mathrm{H}}^2;$$

and also

$$\dot{\mathrm{K}}\,\ddot{\mathrm{C}}\dot{\mathrm{C}} + \dot{\mathrm{C}}\mathrm{u}\,\ddot{\mathrm{C}}\dot{\mathrm{C}}\,\dot{\mathrm{H}}^2 + \dot{\mathrm{H}}^2.$$

The binoxalate of potash, when considerably diluted, and digested with heat upon the oxide of copper, dissolves it easily, and a double salt of sparing solubility crystallizes, presenting itself generally in two forms, one of which contains two and the other four atoms of water, according to the analyses of M. Vogel, which I have repeated and confirmed so far as the water is concerned. The crystals containing four atoms of water soon become opake by exposure to the air, and lose two atoms of water by efflorescence.

Binoxalate of ammonia likewise dissolves oxide of copper, and does so still more readily than the binoxalate of potash, which may depend upon the circumstance that the resulting double salt of ammonia is considerably more soluble in water than the double salt of potash. The oxalate of copper and ammonia crystallized in plates of a blue colour, and seemed to affect one form only. Of these plates, 9·38 grains were readily decomposed by heat, and left 2·37 grains black oxide of copper, or 25·27 per cent., which is almost exactly the proportion of that of the oxide of copper, which a salt of two atoms water should contain, namely, 25·37 per cent. This salt loses water readily at 212° Fahr.; and of the 11·52 per cent. which it is supposed to have on the theory of its containing two atoms of water, 11·46 per cent. escaped by the exposure of the salt to that temperature. M. Vogel describes two other double oxalates of copper and ammonia; but it is evident that they contain ammonia and not oxide of ammonium; so that they do not come under our consideration at present.

It is to be remarked that the oxalate of copper and potash is represented above by a formula quite analogous to that of binoxalate of potash, oxide of copper being simply substituted for basic water. We have oxalate of potash in both cases, to which there is attached oxalate of copper with two atoms of water in the one formula, and oxalate of water with two atoms of water in the other. It is to be remembered that in the case of the sulphates, the double sulphate of copper and potash was shown to have a similar analogy in constitution to the bisulphate of potash.

Oxalate of Chromium and Potash, of Peroxide of Iron and Potash, of Peroxide of Iron and Soda, &c.

$$\overset{\cdots}{\mathrm{Cr\,Cr}}\,\overset{\cdots}{\mathrm{C}\mathrm{C}}{}^{3}+3\,\dot{\mathrm{K}}\,\overset{\cdots}{\mathrm{C}\mathrm{C}}+\dot{\mathrm{H}}{}^{6}.$$

$$\overset{\cdots}{\mathrm{Fe\,Fe}}\,\overset{\cdots}{\mathrm{C}\mathrm{C}}{}^{3}+3\,\dot{\mathrm{K}}\,\overset{\cdots}{\mathrm{C}\mathrm{C}}+\dot{\mathrm{H}}{}^{6}.$$

$$\overset{\cdots}{\mathrm{Fe\,Fe}}\,\overset{\cdots}{\mathrm{C}\mathrm{C}}{}^{3}+3\,\dot{\mathrm{Na}}\,\overset{\cdots}{\mathrm{C}\mathrm{C}}+\dot{\mathrm{H}}{}^{10}.$$

This group of salts has not hitherto been submitted to analysis, although they occupy the same important position among the oxalates which the alums do among the sulphates.

13. *Oxalate of Chromium and Potash.*

This remarkable salt was first described by Dr. Gregory, and its optical properties have been made the subject of a memoir by Sir David Brewster*. It is easily prepared by the following process, which is Dr. Gregory's, with the proportions slightly altered, from a knowledge of the composition of the salt. One part of bichromate of potash, two parts binoxolate of potash, and two parts hydrated oxalic acid, are dissolved together in hot water. There is a copious evolution of carbonic acid gas, arising from the deoxidation of the chromic acid, and nothing fixed remains except the salt in question; of which a pretty concentrated solution crystallizes upon cooling in prismatic crystals, which are black by reflected light, but of a splendid blue colour by transmitted light, when sufficiently thin to be translucent.

This salt, strongly dried without decomposition, was found to lose 11·67 per cent. of water.

The oxide of chromium cannot be precipitated from it completely by means of an alkaline carbonate, and it is very remarkable that only a small portion of the oxalic acid is thrown down from this salt by chloride of calcium.

To determine the proportion of oxalic acid, the salt was heated in strong sulphuric acid, and the gases allowed to escape through a tube containing chloride of calcium. 15·19 grains of the crystals lost 6·71 grains by this treatment, which loss is the weight of the oxalic acid. Hence this salt contains 44·17 per cent. of oxalic acid.

When this double oxalate is ignited, carbonic oxide escapes, and the residuary salt is a mixture of chromate and carbonate of potash, which is entirely soluble in water, and contains no oxide of chromium. In four experiments the fused residuary salt amounted to 0·5458, 0·5411, 0·5454, and 0·5425 of the weight of the crystals operated upon, while it should be 0·5433, provided this residuary salt is a mixture of two atoms chromate and one atom carbonate of potash, and that the composition of the crystals is as follows:

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* Philosophical Transactions, 1835.

One atom oxide of chromium, $\overset{\cdots}{\mathrm{Cr}}\mathrm{Cr}$ . . .	1003·6	16·28
Three atoms potash, $3\,\dot{\mathrm{K}}$	1769·7	28·70
Six atoms oxalic acid, $6\,\overset{\cdots}{\mathrm{C}}\mathrm{C}$	2717·4	44·07
Six atoms water, $6\,\dot{\mathrm{H}}$	675·	10·95
	6165·7	100·

The results in regard to the water and oxalic acid narrated above, agree completely with this view, and so does the determination of the oxide of chromium. 26·01 grains of the crystals left, when ignited, 14·08 grains of the mixed chromate and carbonate of potash, which were dissolved in water, and being acidulated with acetic acid, the chromic acid was precipitated by acetate of lead, and gave 17·45 grains chromate of lead, equivalent to 4·28 grains oxide of chromium. Hence by this experiment the crystals contain 16·46 per cent. of oxide of chromium, which approaches very nearly to the theoretical result. The fused residuary chromate and carbonate of potash amounted to 0·5425 of the weight of the crystals, which is so near the theoretical result, namely, 0·5433, that we may safely conclude that the quantity of potash in the salt agrees with our theoretical estimate.

This salt is clearly, therefore, a compound of one atom oxalate of chromium, containing three atoms oxygen in the oxide and nine atoms oxygen in the acid, with three atoms oxalate of potash; and the salt has six atoms of water of crystallization. The oxygen in the oxide of chromium being 1, that in the potash is also 1, that in the water 2, and that in the oxalic acid 6.

I made several attempts to crystallize the oxalate of chromium itself, but without success, so that I had no opportunity of studying its constitution in relation to the constitution of the preceding double salt.

14. *Oxalate of Peroxide of Iron and Potash.*

This salt, which has not hitherto been described, is formed by dissolving the hydrated peroxide of iron to saturation in binoxalate of potash. There is no effervescence, but a sap-green solution results, which, when concentrated, deposits the salt in question in tabular crystals, of which the form has no resemblance to that of the corresponding oxalate of chromium and potash, and which are of a beautiful grass-green colour. These crystals are permanent in the air, unless it is very dry, when they lose water by efflorescence and become brown and opake. The solution of the salt is decomposed by ammonia, and the peroxide of iron completely thrown down. The salt, when ignited, leaves peroxide of iron and carbonate of potash. It loses 10·56 per cent. of water at a temperature not exceeding 230° Fahr., but is partially decomposed at 300°. Below, the theoretical composition of this salt is placed in juxtaposition with the results of an analysis.

	Theory.	Experiment.
One atom peroxide of iron . . .	15·93	16·13
Three atoms potash	28·82	29·07
Six atoms oxalic acid	44·25	43·74
Six atoms water	11·00	10·56
	100·	99·50

Hence its composition is the same as that of the preceding salt, iron being substituted for chromium.

15. *Oxalate of Peroxide of Iron and Soda.*

This salt is formed by dissolving the hydrated peroxide of iron in binoxalate of soda. It crystallizes in solid green crystals. It is composed as follows, the water being calculated from the loss on the analysis:

	Theory.	Experiment.
One atom peroxide of iron . . .	16·32	16·56
Three atoms soda	19·57	19·66
Six atoms oxalic acid	45·34	45·51
Ten atoms water	18·77	18·27
	100·	100·

Of the ten atoms water which this salt contains it readily loses six at 212° Fahr., and retains four atoms water at that temperature. It differs, therefore, in composition, and it does so also in form, from either of the preceding double oxalates.

A corresponding oxalate of chromium and soda was produced by a similar process, and crystallized with some difficulty in solid dark crystals, which appeared to have the same form as the preceding soda-salt, and were found, like it, to contain ten atoms of water.

There is also a double oxalate of alumina and potash, which may be made by dissolving hydrated alumina in binoxalate of potash, and crystallizes in white tables of a pearly lustre, which have the same form as the oxalate of iron and potash.

II. *Of Nitrates.*

1. *Hydrated Nitric Acid, the Nitrate of Water.*

$$\dot{H}\,\overset{\cdots}{\ddot{N}}\,\dot{H}^{3}.$$

Nitric acid combines with one atom of water as base, and with three atoms more by a less powerful affinity. The well-defined character of the combination containing four atoms water, which is the acid of specific gravity 1·42, is evinced in its high boiling point and in an appearance of saturation which it exhibits. The true and complete nitrate of water has therefore three atoms of constitutional water attached to it. And in the case of the nitrates of those metallic oxides which correspond with water in their basic character, we find the water of crystallization likewise to be three atoms, or a multiple of three, and no other number.

2. *Nitrate of Copper.*

$$\dot{\mathrm{Cu}}\,\overset{\cdot\,\cdot\,\cdot}{\overset{\cdot\,\cdot}{\mathrm{N}}}\,\dot{\mathrm{H}}^{3};$$

and also

$$\dot{\mathrm{Cu}}\,\overset{\cdot\,\cdot\,\cdot}{\overset{\cdot\,\cdot}{\mathrm{N}}}\,\dot{\mathrm{H}}^{3}+3\,\dot{\mathrm{H}}.$$

There are two nitrates of copper, one of which crystallizes in prisms, and the other in rhomboidal plates of a lighter blue colour than the prisms; the first of which I find to contain three and the other six atoms of water. Both are deliquescent to a certain degree, the salt which contains the large proportion of water being more so than the other.

(1.) Of the dark blue prisms, 28·12 grains carefully calcined left 8·98 grains black oxide, or 31·94 per cent. In a second experiment, 22·9 grains left 7·34 grains oxide, or 32·05 per cent. The salt being neutral in composition, the quantity of nitric acid may be inferred from the oxide of copper, and the difference between their sum and the weight of the salt operated upon is the water. By the first analysis the water amounts to 3·38, and by the second to 3·28 atomic proportions. The excess above three atoms is probably hygrometric moisture, to remove which from this salt we cannot employ the ordinary means. In a third experiment upon a portion of the same salt, which had been dried over sulphuric acid till it began to effloresce, 33·19 grains of nitrate left 11·04 grains oxide, which gives 2·83 atomic proportions of water to the salt, or the result is a little below the three atoms. Hence this nitrate may safely be supposed to possess three atoms of water.

(2.) Of the lighter coloured crystals in plates, 10·60 grains left 2·78 grains oxide of copper when ignited, or 27·36 per cent. Hence the salt is composed of

			With six atoms water.
Nitrate of copper . . .	6·57	100·	100·
Water	4·03	61·33	57·54
	10·6	161·33	157·54

The experimental determination is a little above the theoretical estimate, as might be expected from the deliquescent nature of the salt.

The crystals speedily became opake over sulphuric acid in vacuo, and 10·6 grains lost 2·18 grains water in a night, retaining 1·85 water; which is 28·16 water retained to 100 anhydrous salt, or almost exactly three atoms of water. Hence this salt parts easily with half its water. The other three atoms of water are retained more strongly; for by a second day's exposure over sulphuric acid there was an additional loss of only 0·15 grain water; or the water retained was reduced to 25·87 parts united to 100 anhydrous salt.

3. *Subnitrate of Copper.*

$$\dot{H}\,\overset{\cdot\cdot\cdot\cdot\cdot}{N}\,\dot{C}u^{3}.$$

It is well known that when the nitrate of copper is heated to the temperature of 400° or 500° Fahr., it is decomposed, nitric acid and water being expelled, and a subnitrate remaining, which consists of one atom of nitric acid, one atom of water, and three atoms of oxide of copper. This decomposition I find to take place and be completed at a very moderate temperature, not exceeding 150° Fahr.; and it appears, besides, that none of the three constitutional atoms of water of the nitrate of copper can be expelled without a certain corresponding loss of acid: that on heating the salt in question, nitric acid and water go off together, in the form of nitrate of water with its three atoms water. Thus, three atoms of crystallized nitrate of copper, containing three atoms acid, three atoms oxide, and nine atoms of water, are resolved into two atoms nitrate of water, each containing one atom acid and four water; and one atom of subnitrate of copper, which contains one atom acid, one water, and three oxide of copper.

Experiment.—In a stove of which the temperature never exceeded 150° Fahr., 27·54 grains crystallized nitrate of copper, containing three atoms of water, exposed on a capsule, suffered the following gradual reduction of weight: a loss of 2·59 grains in one day, of 9·62 in six days, of 11·1 in seven days, of 13·35 in eleven days, of 13·47 in twelve days, of 13·58 in sixteen days, of 13·60 in eighteen days, and nothing more afterwards by a heat of 300° Fahr., continued for several hours. Of the crystallized nitrate, 27·54 grains have left 13·94 grains subnitrate; or we have 0·5062 subnitrate from 1 nitrate. By calculation the residuary subnitrate should be 0·5026, with which the experimental result closely corresponds.

Another portion of the same nitrate of copper, dried exactly in the same way, lost 1 per cent. of its weight when afterwards heated to 400° Fahr.; and thereafter, being ignited, was found to consist of

	Experiment.	Theory.
Oxide of copper	100·	100·
Volatile matter	53·19	53·1
	153·19	153·1

I am satisfied that no other subnitrate except the preceding, which contains three atoms of oxide of copper, can be obtained by the decomposition of the neutral nitrate by means of heat. For a quantity of the subnitrate of copper of the first experiment narrated above being gradually exposed in a platinum crucible to a heat above the melting point of lead, by means of a sand-bath, so as actually to reduce a portion of the subsalt in contact with the bottom of the crucible to the state of black oxide, yet the major portion of the subsalt, which still retained its green colour, was found to be little altered in composition. After this extreme heating the subsalt consisted of

Oxide of copper . .	100·
Volatile matter . .	50·6
	150·6

Or the proportion of volatile matter in the subsalt has suffered only a small reduction, namely, from 53·1 to 50·6 parts. This last subnitrate afforded drops of nitric acid with fumes of nitrous acid when heated in a tube, so that the subnitrate of copper retains water even at a temperature above the melting point of lead.

The subnitrate of copper merits a careful consideration; for the subsalts of the magnesian class of oxides, which can be had of a definite composition, are really much fewer in number than is generally supposed. What constitution ought to be assigned to this salt? It will be observed that I have represented it by the singular formula

$$\dot{H}\,\overset{\cdot\,\cdot\,\cdot}{\ddot{N}}\,\dot{C}u^3;$$

implying that the single atom of water which it contains is really the base of the salt, and that the three atoms of oxide of copper are in the place of the constitutional water of this nitrate of water. This opens a new view of the constitution of subsalts. The excess of metallic oxide which they contain may not be basic at all in certain cases like the present, but discharge a function in the constitution of the salt which has hitherto been recognised only as executed by water. For if we find water and oxide of copper strongly resembling each other as bases, why may not the analogy between them extend further, and oxide of copper be capable of discharging the function of constitutional water or water of crystallization in the composition of a salt? Indeed the speculation that all salts whatever are neutral in composition is highly probable. Where the metallic oxide is in excess, as in what are called subsalts, we can attribute another function than that of base to the whole or a portion of the metallic oxide, and thus preserve the salt neutral in composition, or according to its formula. To this subject I shall again recur.

The following observation is particularly favourable to the view which we are taking of the constitution of subnitrate of copper. When the black oxide of copper is drenched with the strongest nitric acid, it is a subnitrate of copper which is formed, although the nitric acid may be in great excess. The black oxide is converted into a green powder, from which the excess of nitric acid should be drained off as well as possible, and the powder will be found to be in great part insoluble in water. The explanation seems to be, that the concentrated nitric acid employed does not contain the constitutional water which the neutral nitrate of copper requires, and accordingly that salt is not formed; but the nitrate of water supplies itself with oxide of copper in the place of its deficient constitutional water; so that the result is a nitrate of water with three atoms of oxide of copper attached. But when nitric acid of a specific gravity not exceeding 1·42 is digested upon the same black oxide of copper, the neutral nitrate of copper only is formed, and no subnitrate.

This view seems likewise to be necessary to account for the great force with which the single atom of water is retained by the subnitrate of copper. The water cannot be expelled without decomposing the salt, notwithstanding the great excess of oxide of copper present.

4. *Nitrate and Subnitrate of Bismuth.*

$$\dot{\text{Bi}}\,\overset{\cdot\,\cdot\,\cdot}{\ddot{\text{N}}}\,\dot{\text{H}}^3$$

$$\dot{\text{H}}\,\overset{\cdot\,\cdot\,\cdot}{\ddot{\text{N}}}\,\dot{\text{Bi}}^3.$$

The neutral nitrate is admitted to contain three atoms of water, like the nitrate of copper, and its constitution appears to be similar.

No portion of the constitutional water of this salt can be expelled without decomposing the salt. Indeed this salt loses acid by exposure to dry air at a temperature not exceeding 80° Fahr. The crystals of the salt are resolved by a heat of 212° into a solid and fluid portion, the first of which is probably the subnitrate, while the last is hydrated nitric acid containing much nitrate of bismuth in solution, and not a supernitrate of bismuth. But the fluid portion fixes so readily upon cooling that the solid product cannot be obtained in a definite state.

Experiment. 28·61 grains of nitrate of bismuth in good crystals, being exposed to a gradual ignition, left 14·16 grains of fused oxide of bismuth. This result accords with the view which is taken above of the composition of this salt:

		Experiment.	Theory.
Oxide of bismuth	14·16	100·	100·
Nitric acid and water . . .	14·45	102·04	102·72
	28·61	202·04	202·72

It appears likewise that three atoms of the hydrated nitrate of bismuth are resolved, when dried at a high temperature, into two atoms hydrated nitrate of water and one atom subnitrate of bismuth, which last is of the same constitution as the subnitrate of copper.

Experiment 1. Dried on the sand-bath at a temperature above the melting point of tin, 28·61 grains of nitrate of bismuth lost 9·29 grains, and retained 5·16 grains of volatile matter, or consisted of

		Experiment.	Subnitrate by theory.
Oxide of bismuth.	14·16	100·	100·
Volatile matter	5·16	36·44	34·24
	19·32	136·44	134·24

Experiment 2. A portion of nitrate dried in a stove at a temperature not exceeding 180° Fahr., till it ceased to lose weight, was thereafter found to consist of

Oxide of bismuth	21·24	100·
Volatile matter	7·57	35·64
	28·81	135·64

It appears from the second of these experiments that the subnitrate of bismuth may be produced at a temperature so low as 180° Fahr., and from the first experiment that the subnitrate may be exposed to a temperature of 500° Fahr. without decomposition.

Several experiments were made to produce another definite subnitrate, containing a greater proportion of oxide of bismuth, by the action of heat upon this subnitrate, but without success. The salt was partially decomposed at various temperatures under redness, but no definite compound resulted. Hence the subnitrate described is probably the only definite subnitrate of bismuth that can exist. The small pearly crystals obtained on throwing the neutral nitrate of bismuth into a moderate quantity of water, are of the same composition as the subnitrate obtained by heat.

5. *Nitrate of Zinc.*

$$\dot{Zn}\,\overset{\cdot\cdot\cdot\cdot\cdot}{N}\,\dot{H}^3 + 3\,\dot{H}.$$

This salt is easily obtained by dissolving zinc in nitric acid. It is very soluble in water, and moderately deliquescent.

Experiment. 29·17 grains of the crystals ignited, left 7·86 grains oxide of zinc. In this experiment we have 0·2694 oxide from one salt, which is very near 0·2713 oxide, the proportion which should be left, supposing the salt to contain six atoms of water. By efflorescence at 212° one part of this salt loses 0·18 water, which is one half of the whole water which the salt is assumed to contain, namely, 0·3639 water. It loses no acid at 212°. Hence this salt is of the same constitution as the nitrate of copper, but is not decomposed at so low a temperature. The proportion of water, however, cannot be reduced below three atoms without a loss of acid, and there appears to be a subnitrate of zinc resembling the subnitrate of copper.

6. *Nitrate of Magnesia.*

$$\dot{Mg}\,\overset{\cdot\cdot\cdot\cdot\cdot}{N}\,\dot{H}^3 + 3\,\dot{H}.$$

Experiment. 27·12 grains of crystals of nitrate of magnesia, when calcined, left 4·3 grains caustic magnesia; a result which indicates 6·17 atomic proportions of water in the salt, or the salt contains six atoms of water.

The nitrate of magnesia stands exposure to a heat which would melt lead without losing acid. At that high temperature the proportion of water is reduced to one atom, which cannot be expelled without loss of acid. The salt remains in a fused state and transparent, and dissolves afterwards completely in water.

Experiment. 18·40 grains of the crystals, containing 7·71 grains water, lost 6·60 grains by a strong sand-bath heat continued till the salt ceased to lose weight. This is a loss of exactly five sixths of the water contained in the salt.

Experiment. 19·76 grains, containing 8·28 water, by similar treatment lost 6·77, which approaches very closely to 6·90 grains, the number representing five atomic proportions of water.

This single atom of water retained by the nitrate of magnesia, is not displaced and expelled upon heating the salt, together with an atomic proportion of nitrate of potash to 600° or 700° Fahr., so that the retention of an atom of water does not indicate a disposition, upon the part of nitrate of magnesia, to form a double salt. It is probable that this peculiar and intimate combination of nitrate of magnesia with one atom water does not exist in the crystals or ordinary hydrate of nitrate of magnesia, but is the result of a new arrangement of the constituents of the salt at a high temperature. There are indications of the existence of a similar nitrate of water.

There does not appear to be a subnitrate of magnesia like the subnitrate of copper.

Supposed Double Nitrates and Supernitrates.

As double nitrates are said to exist, I have repeatedly attempted to form them; but when nitrate of magnesia, nitrate of zinc, or nitrate of copper was mixed with nitrate of potash or with nitrate of ammonia, the salts uniformly separated again in crystallizing. There is no proof of the existence of a single supernitrate.

Most of the nitrates of oxides not belonging to the magnesian class are anhydrous salts, such as the nitrates of potash, soda, barytes, strontian, lead, &c., and do not suggest any new subject matter of inquiry.

III. *Of Phosphates.*

In the present state of our knowledge phosphoric acid is quite peculiar in being capable of combining with bases in three different proportions, forming, besides the usual class of salts containing one atom of acid to one atom of protoxide as base, two other anormal classes of salts, in which two and three atoms of base are united to one atom of acid, namely, the pyrophosphates and the common phosphates. Arsenic acid forms only one class of salts, but that class is anormal, every member of it containing three atoms of base to one atom of acid, like the common phosphates. These anormal classes of phosphates and arseniates, with perhaps the phosphites, are, I believe, the only known salts to which the ordinary idea of a subsalt is truly applicable; or in the formulæ of these salts only, ought more than one atom of any protoxide to appear in a basic relation to one atom of acid. All other reputed subsalts are probably neutral in composition, as I have endeavoured to show in the case of the subnitrate of copper; for to this salt they all bear an analogy in their small solubility and other properties, while they exhibit little resemblance to those classes of phosphates and arseniates which really possess more than one atom of base. The following Table contains the formulæ of the most important phosphates, with a new nomenclature of these salts, which I offer for consideration.

First Class.

Monobasic phosphate of water (metaphosphate of water)	$\dot{\mathrm{H}}\,\overset{\cdot\cdot\cdot\cdot\cdot}{\mathrm{P}}.$
Monobasic phosphate of soda (metaphosphate of soda)	$\dot{\mathrm{Na}}\,\overset{\cdot\cdot\cdot\cdot\cdot}{\mathrm{P}}.$

Second Class.

Bibasic phosphate of water (pyrophosphate of water)	$\dot{\mathrm{H}}^2\,\overset{\cdot\cdot\cdot\cdot\cdot}{\mathrm{P}}.$
Bibasic phosphate of soda and water (bipyrophosphate of soda)	$\dot{\mathrm{Na}}\,\dot{\mathrm{H}}\,\overset{\cdot\cdot\cdot\cdot\cdot}{\mathrm{P}}.$
Bibasic phosphate of soda (pyrophosphate of soda)	$\dot{\mathrm{Na}}^2\,\overset{\cdot\cdot\cdot\cdot\cdot}{\mathrm{P}} + 10\,\dot{\mathrm{H}}.$

Third Class.

Tribasic phosphate of water (common phosphate of water)	$\dot{\mathrm{H}}^3\,\overset{\cdot\cdot\cdot\cdot\cdot}{\mathrm{P}}.$
Tribasic phosphate of water and soda (biphosphate of soda)	$\dot{\mathrm{Na}}\,\dot{\mathrm{H}}^2\,\overset{\cdot\cdot\cdot\cdot\cdot}{\mathrm{P}} + 2\,\dot{\mathrm{H}}.$
Tribasic phosphate of soda and water (phosphate of soda)	$\dot{\mathrm{Na}}^2\,\dot{\mathrm{H}}\,\overset{\cdot\cdot\cdot\cdot\cdot}{\mathrm{P}} + 24\,\dot{\mathrm{H}}.$
Tribasic phosphate of soda (subphosphate of soda)	$\dot{\mathrm{Na}}^3\,\overset{\cdot\cdot\cdot\cdot\cdot}{\mathrm{P}} + 24\,\dot{\mathrm{H}}.$
Tribasic phosphate of soda, ammonia, and water (microcosmic salt)	$\dot{\mathrm{Na}}\,\dot{\mathrm{N}}\mathrm{H}^4\,\dot{\mathrm{H}}\,\overset{\cdot\cdot\cdot\cdot\cdot}{\mathrm{P}} + 8\,\dot{\mathrm{H}}.$
Tribasic phosphate of magnesia and water (phosphate of magnesia)	$\dot{\mathrm{M}}\mathrm{g}^2\,\dot{\mathrm{H}}\,\overset{\cdot\cdot\cdot\cdot\cdot}{\mathrm{P}} + 2\,\dot{\mathrm{H}} + 12\,\dot{\mathrm{H}}.$
Tribasic phosphate of magnesia and ammonia (ammoniaco-magnesian phosphate)	$\dot{\mathrm{M}}\mathrm{g}^2\,\dot{\mathrm{N}}\mathrm{H}^4\,\overset{\cdot\cdot\cdot\cdot\cdot}{\mathrm{P}} + 2\,\dot{\mathrm{H}} + 10\,\dot{\mathrm{H}}.$

It is my object to get rid of the trivial names pyrophosphates, metaphosphates, and common phosphates, which have tended to keep up an erroneous impression that the phosphoric acid is of a different nature in these classes of salts, or is modified in some way unknown. This notion has arisen from the pertinacity with which phosphoric acid continues combined with a constant number of atoms of base, whether it be one, two, or three, although the base itself be repeatedly changed by decomposing the original combination. But this is an occurrence quite analogous to the formation of different sets of sulphurets or of chlorides, when we decompose two or more different oxides of the same metal, such as the oxide and suboxide of mercury, by sulphuretted hydrogen or by muriatic acid. The metal continues in the same relative state of saturation throughout a series of such decompositions; and so does the phosphoric acid, because in both cases the decomposition is effected by an equivalent substitution.

A difficulty occurs in naming two members of the tribasic class, so as to distinguish them from each other, namely, the biphosphate of soda and phosphate of soda, both of which contain soda and water as base. But this difficulty is obviated by placing first in the name that base of which two atoms are present. Thus the biphosphate of soda is "the phosphate of water and soda," and the phosphate of soda is the phosphate of soda and water, both being at the same time characterized as "tribasic."

What I have to add at present in regard to the phosphates relates chiefly to the last three salts, of which formulæ are given in the preceding Table, which belong to classes of tribasic phosphates that were not examined in my former paper upon the phosphates*. But I may premise a few observations, which are more strictly supplementary to the results of that paper.

1. The bibasic phosphate of water (pyrophosphate of water) is possessed of very considerable stability. Both weak and concentrated solutions of this salt have been kept for five or six months without any sensible change or production of the tribasic phosphate of water.

2. It appears to be impossible to crystallize any bibasic phosphate (pyrophosphate) of potash. Such salts can exist in solution, but not in the dry state. The same observation applies to the bibasic phosphates of ammonia, or we have no pyrophosphates of ammonia except in solution. Indeed, the solution of the bibasic phosphate of water and ammonia assumes another atom of basic water when the evaporation is carried far, and crystallizes as the tribasic phosphate of water and ammonia (biphosphate of ammonia).

3. In the case of tribasic phosphates containing potash, I have succeeded in crystallizing the tribasic phosphate of potash, and the tribasic phosphate of water and potash, but not the tribasic phosphate of potash and water, or what would be considered on the old view as the neutral phosphate of potash.

4. Both the bibasic and tribasic phosphates of water may be treated with an excess of caustic potash in solution without the formation of any precipitate or sparingly soluble combination. It is only in the monobasic phosphate of water that a sparingly soluble combination is formed by potash, such as that which is described by Dr. Thomson under the name of diphosphate of potash.

I. *Tribasic Phosphate of Soda, Ammonia, and Water. (Phosphate of Soda and Ammonia: Microcosmic Salt.)*

$$\dot{\mathrm{Na}}\,\dot{\mathrm{N}}\mathrm{H}^4\,\dot{\mathrm{H}}\,\overset{\cdot\cdot\cdot\cdot\cdot}{\mathrm{P}} + 8\,\dot{\mathrm{H}}.$$

I have repeated more than once the analysis of this salt, and obtained the same result as M. Mitscherlich. It appeared to contain 0·5094 of volatile matter: and there may be derived from an atom of this salt one atom of phosphoric acid, of soda and of ammonia respectively, and ten atoms of water. It has hitherto been viewed as a double phosphate or combination of phosphate of soda with phosphate of ammonia; but no reason can be assigned why these particular salts should combine together, and combinations of salts of soda and ammonia are exceedingly unusual.

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* Philosophical Transactions, 1833.

The view expressed above in the formula is much more likely to be true, namely, that this salt is simply a tribasic phosphate, of which the three atoms of base are all different: they are soda, oxide of ammonium, and water; and the salt possesses eight atoms of water of crystallization. By a graduated heat it is possible to expel the water of crystallization of this salt, and likewise the ammonia of its oxide of ammonium; and the water of the last remaining as base, the salt $\dot{Na}\,\dot{H}^2\,\overset{\cdot\cdot\cdot\cdot\cdot}{P}$ is produced.

M. Mitscherlich now admits that there is no tribasic phosphate corresponding with this, but containing potash instead of oxide of ammonium, a conclusion of which I have ascertained the accuracy.

I endeavoured to form a tribasic phosphate to contain two atoms soda and one atom of oxide of ammonium, but such a salt appears to have no existence. For when ammoniacal gas was passed into a strong and hot solution of the common phosphate of soda, a slight deposition of the tribasic phosphate of soda took place, followed by the rhomboidal crystals of the common phosphate unchanged.

It likewise appears that when the bibasic phosphate of soda and the bibasic phosphate of potash (pyrophosphates) are mixed together, no new salt is produced; but the former may be crystallized out, and the latter remains uncrystallizable.

II. *Tribasic Phosphates containing Oxides of the Magnesian Class.*

1. *Tribasic Phosphate of Zinc and Water.* (*Phosphate of Zinc.*)

$$\dot{Zn}^2\,\dot{H}\,\overset{\cdot\cdot\cdot\cdot\cdot}{P}+2\,\dot{H}.$$

This salt, which is nearly insoluble, is obtained in minute silvery plates, by mixing three ounces of sulphate of magnesia with four ounces of phosphate of soda, each dissolved in two pounds of cold water. These crystalline plates consist of

		Theory of $\dot{Zn}\,\dot{H}\,\overset{\cdot\cdot\cdot\cdot\cdot}{P}+2\,\dot{H}$.
Anhydrous salt . . .	100·	100·
Water	19·63	17·77
	119·63	117·77

Dried above the melting point of tin the crystals still retained a glistening appearance, but had lost two thirds of their water; for they now consisted of

		Theory of $\dot{Zn}\,\dot{H}\,\overset{\cdot\cdot\cdot\cdot\cdot}{P}$.
Anhydrous salt . . .	100·	100·
Water	6·08	5·92
	106·08	105·92

The two atoms of water which are expelled in the above experiment are, notwithstanding, pretty strongly attached to the salt, being retained at the boiling point of water. Indeed these two atoms of water are highly constitutional, and are found in all the phosphates of this class.

This phosphate fuses at a red heat, after it becomes anhydrous, but it continues soluble in dilute acids.

2. *Tribasic Arseniate of Magnesia and Water.* (*Arseniate of Magnesia.*)

$$\dot{\mathrm{M}}\mathrm{g}^2\,\dot{\mathrm{H}}\,\overset{\cdot\cdot\cdot\cdot\cdot}{\mathrm{As}} + 2\,\dot{\mathrm{H}} + 12\,\dot{\mathrm{H}}.$$

This salt precipitated on mixing dilute solutions of 500 grains of arseniate of soda and 300 grains of sulphate of magnesia. It consisted of

		Theory of $\dot{\mathrm{M}}\mathrm{g}^2\,\dot{\mathrm{H}}\,\overset{\cdot\cdot\cdot\cdot\cdot}{\mathrm{As}} + 14\,\dot{\mathrm{H}}$.
Anhydrous salt . . .	100·	100·
Water	86·58	86·25
	186·58	186·25

This salt contains in all fifteen atoms of water, of which three are retained and twelve expelled at the boiling point of water. Dried at 212° it consisted of

		Theory of $\dot{\mathrm{M}}\mathrm{g}^2\,\dot{\mathrm{H}}\,\overset{\cdot\cdot\cdot\cdot\cdot}{\mathrm{As}} + 2\,\dot{\mathrm{H}}$.
Anhydrous salt . . .	100·	100·
Water	17·17	17·25
	117·17	117·25

It therefore retains pretty strongly two atoms of water besides its basic atom, resembling the preceding salt in this respect.

This arseniate and the corresponding phosphate are rendered insoluble in dilute acids by the effect of a strong red heat.

3. *Tribasic Phosphate of Magnesia and Water.* (*Phosphate of Magnesia.*)

$$\dot{\mathrm{M}}\mathrm{g}^2\,\dot{\mathrm{H}}\,\overset{\cdot\cdot\cdot\cdot\cdot}{\mathrm{P}} + 2\,\dot{\mathrm{H}} + 12\,\dot{\mathrm{H}}.$$

This salt appears in distinct prismatic crystals in the course of twenty-four hours, upon mixing two ounces of sulphate of magnesia with three ounces phosphate of soda, each dissolved in two pounds of water. Cold water is capable of dissolving about one thousandth part of its weight of these crystals. They have been stated erroneously to be much more soluble. The proportion of water which they contain has hitherto been stated at fourteen atoms instead of fifteen, which is the truth. By analysis the crystals were found to consist of

		Theory of $\dot{\mathrm{M}}\mathrm{g}^2\,\dot{\mathrm{H}}\,\overset{\cdot\cdot\cdot\cdot\cdot}{\mathrm{P}} + 14\,\dot{\mathrm{H}}$.
Anhydrous salt . . .	100·	100·
Water	121·7	119·76
	221·7	219·76

I find that the proportion of water retained by this salt is readily reduced at 212°, from fifteen atoms to seven, by the escape of eight atoms of water. Of the seven atoms retained one is basic, and therefore expelled with difficulty; but from a variety of experiments which I have performed it appears probable (although I have never attained very precise results) that the other six atoms go off in pairs at different temperatures between 212° and 350° Fahr. But even at 410° the quantity of water retained by this salt was sensibly above one atomic proportion. We may with considerable probability represent the consecutive combinations of this salt with water by such a formula as the following:

$$\dot{M}g^2\,\dot{H}\,\overset{\cdot\cdot\cdot}{\ddot{P}}+2\,\dot{H}+2\,\dot{H}+2\,\dot{H}+8\,\dot{H}.$$

Besides the preceding salt there is a tribasic phosphate of magnesia, which is obtained as an insoluble precipitate on mixing tribasic phosphate of soda with sulphate of magnesia. Of this salt the whole three atoms of base are magnesia, as its name implies. Dried at 212° it retains five atoms of water. At a red heat it glows, but it continues soluble in acids even after exposure to a white heat. But I did not succeed in forming the other tribasic salt, containing two atoms of water and one atom of magnesia as bases, which is wanted to complete the series. Such a salt does not appear to exist.

It may be mentioned here in reference to the monobasic phosphate of magnesia (metaphosphate of magnesia), that although this salt does not present itself on mixing the monobasic phosphate of soda with the sulphate of magnesia, yet it is readily precipitated in the form of a soft viscid body, on using the acetate of magnesia instead of the sulphate.

4. *Tribasic Phosphate of Magnesia and Ammonia.* (*Ammoniaco-magnesian Phosphate.*)

$$\dot{M}g^2\,\dot{N}H^4\,\overset{\cdot\cdot\cdot}{\ddot{P}}+2\,\dot{H}+10\,\dot{H}.$$

This salt is the well-known granular precipitate formed on adding a salt of magnesia to any soluble tribasic phosphate with which ammonia or a salt of ammonia has been mixed. I was much interested in ascertaining the true constitution of this salt, and have carefully analysed seven or eight different specimens of it, prepared with and without free ammonia in the liquors. The result is that only one tribasic salt of these constituents exists, although two have often been admitted; while in this compound there exists only one atom of ammonia instead of two, as M. Riffault has supposed. I subjoin the preparation and analysis of one specimen of this salt. 350 grains of crystallized phosphate of soda, 100 grains of chloride of ammonium, and 200 grains of *aqua ammoniæ* were dissolved together in four pounds of cold water, and 200 grains of crystallized sulphate of magnesia were added to that mixture. The precipitation was gradual, and the liquor remained alkaline. The precipitate was slightly washed with cold water, and afterwards dried in the air for ten days, the thermometer being 65° Fahr., without artificial heat. The true proportions of water, which this and many other precipitates affect, have often been mistaken, and definite hydrates not obtained, from using hot solutions in their preparation. Of this precipitate 26·8 grains lost by ignition 14·5 grains, or one part of the precipitate contains 0·541 volatile matter. For the ammonia, the volatile matter from 9·65 grains of the precipitate was sent over quicklime contained in a tube, so as to arrest the water. The loss, or the ammonia, amounted to 0·67 grain, or to 0·0695 of the precipitate. Hence this precipitate consists of

		Theory of $\dot{\mathrm{M}}\mathrm{g}^2\dot{\mathrm{N}}\mathrm{H}^4\overset{\cdot\cdot\cdot\cdot\cdot}{\mathrm{P}} + 2\dot{\mathrm{H}} + 10\dot{\mathrm{H}}$.
Anhydrous salt . . .	45·90	45·85
Ammonia	6·95	6·98
Water	47·15	47·17
	100·	100·

From the manner in which this specimen of the salt was prepared, it should contain the maximum proportion of ammonia of which the salt admits, and yet that proportion is one atom only, and not two, as it was estimated by Riffault. A salt of the same composition was obtained from the same materials, omitting the caustic ammonia. In that case the product was not so abundant, and the mother liquor remained acid from the production of tribasic phosphate of water and soda, which has an acid reaction. When this salt, contained in a little retort, is heated in a very gradual manner to 212° by means of a water-bath, it is possible to distil over ten atomic proportions of the water without any ammonia whatever. Of the three atoms of water which remain, (the whole quantity originally present in the salt being thirteen atoms,) one appears to be combined with the ammonia in the formation of oxide of ammonium, while the other two are the constitutional water of the tribasic phosphate of magnesia and water.

It appears, then, that this salt is not a double phosphate, or combination of two phosphates, but that it is formed from the tribasic phosphate of magnesia and water, by the substitution of oxide of ammonium for the basic water of that salt; and it is a tribasic phosphate of magnesia and oxide of ammonium. The oxygen in the magnesia is double that in the oxide of ammonium.

This salt is the type of a class of tribasic phosphates, in which the magnesia is replaced by the other oxides, which are isomorphous with that base. Two of these salts were discovered and carefully examined by Dr. Otto of Brunswick *.

Dr. Otto's analysis of what we may call the tribasic arseniate of manganese and ammonia corresponds exactly with the analysis given above of the magnesian salt, except that he derives only twelve instead of thirteen atoms of water from his salt. The deficiency in the proportion of water found by him, I attribute to the use which he made of hot water in washing his salt.

His analysis of the tribasic phosphate of the protoxide of iron and ammonia is particularly interesting, as it proves that this salt is precipitated, containing no more than three atoms of water, or exactly of the composition of the magnesian salt dried

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* Journal für Praktische Chemie von Erdman und Schweigger-Seidel, 1834, p. 409.

at 212°, as we have described. The constitution of this salt of iron I would therefore represent by the formula

$$\dot{\mathrm{Fe}}^{2}\,\dot{\mathrm{N}}\mathrm{H}^{4}\,\overset{\cdot\cdot\cdot}{\ddot{\mathrm{P}}}+2\,\dot{\mathrm{H}}.$$

In the same paper Dr. Otto describes another extraordinary phosphate, under the name of paraphosphate of soda, ammonia, and oxide of manganese, which does not belong to any class of phosphates that I have examined, but may possibly be a combination of two bibasic phosphates. Its constituents are $2\,\overset{\cdot\cdot\cdot}{\ddot{\mathrm{P}}}$, $2\,\dot{\mathrm{M}}\mathrm{n}$, $\dot{\mathrm{N}}\mathrm{H}^{4}$ and $6\,\dot{\mathrm{H}}$. It is prepared from bibasic phosphates, and would be said in the old language to contain pyrophosphoric acid.

IV. *Of Sulphates.*

In a former paper upon water as a constituent of sulphates[*], I examined particularly the constitution of hydrated sulphuric acid and of the sulphates of the magnesian class of oxides. All these salts contain one atom of constitutional water, which is displaced in the formation of the double sulphates by an atom of an alkaline sulphate. This view is illustrated by the following formulæ:

Sulphate of water (acid of sp. gr. 1·78)	$\dot{\mathrm{H}}\,\dddot{\mathrm{S}}\,\dot{\mathrm{H}}$
Sulphate of magnesia	$\dot{\mathrm{M}}\mathrm{g}\,\dddot{\mathrm{S}}\,\dot{\mathrm{H}}+6\,\dot{\mathrm{H}}$
Sulphate of magnesia and potash	$\dot{\mathrm{M}}\mathrm{g}\,\dddot{\mathrm{S}}\,(\dot{\mathrm{K}}\,\dddot{\mathrm{S}})+6\,\dot{\mathrm{H}}$
Sulphate of water and potash (bisulphate of potash) .	$\dot{\mathrm{H}}\,\dddot{\mathrm{S}}\,(\dot{\mathrm{K}}\,\dddot{\mathrm{S}})$.

It will be found upon experiment that the salts sulphate of magnesia and sulphate of zinc become anhydrous at much lower temperatures when mixed with sulphate of potash than by themselves, the sulphate of potash displacing the constitutional water of the other salt at a very moderate heat, although the salts are mixed in the state of dry powders.

In that paper the opinion was supported, originally suggested I believe by M. Mitscherlich, that the bisulphate of potash is a double sulphate of water and potash, and therefore really neutral in composition. The only difficulty which stood in the way of generalizing this result, and maintaining that *all* the salts usually considered as bisalts are really neutral in composition, was the composition of the bichromate or red chromate of potash, a salt which unquestionably is anhydrous. Here, it might be said, is a true bisalt. But M. H. Rose has lately published some observations in regard to anhydrous sulphuric acid, which, I think, afford a clue to the discovery of the true constitution of the red chromate of potash. It appears that the vapour of anhydrous sulphuric acid is absorbed by sulphate of potash and by chloride of potassium, without decomposition, and definite compounds formed; which, however, are destroyed by solution in water. Here we appear to have a class of

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[*] Edinburgh Transactions, vol. xiii. p. 297; or London and Edinburgh Philosophical Magazine, 3rd series, vol. vi. pp. 327, 417.

combinations of sulphuric acid *with salts*. Chromic acid, which is isomorphous with sulphuric, forms combinations which I consider as analogous to these. With the neutral or yellow chromate of potash it forms the red chromate of potash, and with chloride of potassium it forms M. Peligot's salt; which differ only from M. Rose's corresponding combinations of sulphuric acid, in being more permanent. The superior stability of these chromic acid combinations unquestionably depends upon the little affinity for water which their acid possesses, while the affinity of sulphuric acid for water is very great. Hence we may suppose that the red chromate of potash is not a direct combination of two atoms of chromic acid with one atom of potash, but a combination of one atom of chromic acid with one atom of yellow chromate of potash; and it may be represented as follows:

$$(\dot{K}\,\dddot{Cr})\,\dddot{Cr}.$$

The red chromate of potash will thus belong to a new order of combinations, differing essentially from proper salts, which contain an oxide as base. This salt, therefore, cannot be adduced as militating against the law that "all salts are neutral in composition"; the only known exceptions to which law are, I believe, afforded by the anormal classes of phosphates, phosphites, and arseniates.

I have devoted much time to the examination of *subsulphates* of the magnesian class of oxides, particularly of the subsulphate of zinc and the subsulphate of copper. These salts were generally formed by the partial precipitation of sulphate of zinc or sulphate of copper by means of caustic potash. They have both a disposition to carry down sulphate of potash, which is never entirely removed from them by washing; while one of them, the subsulphate of zinc, is itself decomposed by washing. When most successfully prepared, they were found to contain four atoms of metallic oxide to one atom of acid, (instead of three atoms oxide, as M. Berzelius supposed,) together with four atoms of water. I have not hitherto been able to form a distinct idea of their constitution, or to decide between different views which may be taken of it. But the force with which water is retained in these subsalts is very remarkable. The subsulphate of copper loses no portion of its four atoms of water at 212°, and I have not been able to reduce the quantity of water retained by this salt so low as one atomic proportion, even at the melting point of lead.

The constitution of the subsulphate of copper appears to be changed when it is made anhydrous by heat. In the progress of the desiccation of the salt, its colour passes from a dull blue to an olive green, and it finally becomes of a chocolate brown, and is then anhydrous. Water poured upon the brown matter comes off of a blue colour, dissolving out a considerable portion of the soluble sulphate of copper. It appears, therefore, that the water originally present in the subsulphate must discharge some important function in its constitution, the subsalt being obviously decomposed when made anhydrous.

The *Alums* form a most important class of the sulphates, but I have never had it in my power to compare their constitution with that of the sulphate of alumina itself, which is not easily obtained in a crystallized state. This salt, however, is described as containing eighteen atoms of water, while the alums have twenty-four. At present I would merely throw out the conjecture, that in the alums we may have simply an alkaline sulphate with the sulphate of alumina attached, that salt carrying along with it its whole water of crystallization, and acquiring six atoms more. The quantity of water in potash alum may be reduced by efflorescence to six atoms in a stove of the temperature of 150° Fahr. Hence potash alum may perhaps be represented as follows:

$$\dot{\text{K}}\dddot{\text{S}} + (\overset{\cdot\,\cdot\,\cdot}{\text{AlAl}}\dddot{\text{S}}^{3} + 6\,\dot{\text{H}} + 18\,\dot{\text{H}}).$$

I have shown by an analysis conducted in very favourable circumstances, that soda-alum contains, like potash-alum, twenty-four, and not twenty-six atoms of water.

V. *Of Chlorides.*

The affinity which the hydracids exhibit for water is weak. Of the lower hydrates of muriatic acid we know nothing, the volatility of the acid putting it out of our power to form and examine such hydrates; but it is likely that they will correspond with the hydrates of the chloride of magnesium, &c., which can be examined.

The law in the case of the chlorides of the magnesian class of metals appears to be, that they have two atoms of water pretty strongly attached to them, and which we may consider as constitutional. Thus chloride of copper crystallizes with two atoms of water, and with no lower proportion; but several chlorides of this class have two or four atoms more, the proportion of water advancing by a multiple of two atoms.

1. *Chloride of Copper.*

$$\text{CuCl}\,\dot{\text{H}}^{2}.$$

The blue prismatic crystals of chloride of copper become brown and lose the greater proportion of their water at a temperature not exceeding the boiling point of water. Fifteen grains of the crystals, exposed to a much higher temperature, lost 3·23 grains of water, leaving 11·77 grains of chloride of copper; and when this quantity of chloride of copper was exposed to the atmosphere, it quickly recovered 3·16 grains of water, and resumed the blue colour of the crystallized salt. I believe this method of reabsorption, in the case of constitutional water, often to give hydrates of which the composition is even more exact than if they had been obtained from solution, owing to the absence of that water, which is often mechanically interposed between the plates of crystals. The hydrated chloride of copper obtained in this way consisted of

			Theory of $\text{CuCl}\,\dot{\text{H}}^{2}$.
Chloride of copper . . .	11·77	100.	100.
Water	3·16	26·85	26·84
	14·93	126·85	126·84.

2. *Chloride of Manganese.*

$$\mathrm{MnCl}\,\dot{\mathrm{H}}^2 + 2\,\dot{\mathrm{H}}.$$

Experiment.—Of the flesh-coloured crystals, 15·53 grains, precipitated by nitrate of silver, gave 22·57 grains of chloride of silver, equivalent to 5·56 chlorine, or to 9·92 chloride of manganese, which leaves 5·61 grains water in the salt, or 36·12 per cent. of water. Now a chloride of manganese with four atoms of water would contain 36·33 per cent. of water.

This salt readily lost half its water when dried at 212° in open air, or when dried over sulphuric acid in the vacuum of an air-pump at the ordinary temperature. But when the exposure of the salt in such circumstances was long protracted, a little of the constitutional water also was lost.

3. *Protochloride of Iron.*

$$\mathrm{Fe\,Cl}\,\dot{\mathrm{H}}^2 + 2\,\dot{\mathrm{H}}.$$

In three experiments made upon different specimens of crystallized protochloride of iron, all newly and very carefully prepared, 13·69 grains chloride of silver were precipitated from 9·72 salt, 17·20 chloride of silver from 12·44 salt, and 15·75 chloride of silver from 11·21 salt. These experiments almost coincide in their results, which are, that 1 part of the salt contains 0·3466, 0·3463, and 0·3461 of chlorine. But such proportions of chlorine are decidedly under the proportion which a neutral salt with four atoms of water should contain, namely, 0·3593 chlorine. Indeed, the quantity of water in the salt is indicated by these experiments to be four and a half atomic proportions almost exactly. By crystallizing from an acid solution Bonsdorff has lately obtained this salt in a state of purity, and containing four atoms of water.

4. *Chloride of Magnesium.*

$$\mathrm{MgCl}\,\dot{\mathrm{H}}^2 + 4\,\dot{\mathrm{H}}.$$

Of the crystals of this salt, which are decidedly deliquescent, 12·65 grains were found to contain 4·29 chlorine; or the salt contains 33·91 per cent. of chlorine, which approaches sufficiently near to the theoretical proportion 34·69 per cent., supposing the salt to contain six atoms of water.

5. *Chloride of Calcium.*

$$\mathrm{CaCl}\,\dot{\mathrm{H}}^2 + 4\,\dot{\mathrm{H}}.$$

The crystals of this deliquescent salt, dried in vacuo till they began to effloresce, were found to contain six atoms of water, the proportion usually allotted to them; but it is remarkable, that, continued in vacuo over sulphuric acid for ten days during the heat of summer, the crystals became opake and of a talky lustre, without being disintegrated, and their proportion of water was reduced to two atoms.

6. *Double Chloride of Copper and Ammonium.*

$$NH^4 Cl + Cu Cl \dot{H}^2.$$

Hydrated chloride of copper dissolved with chloride of ammonium, in the proportion of eleven of the first to seven of the last, readily affords a double salt, in which we appear to have an atom of chloride of ammonium with an atom of the hydrated chloride of copper attached. This double salt is less soluble than the chloride of copper itself, and retains more strongly the two constitutional atoms of water of that salt; illustrating in both of these points what appear to be two very general occurrences: namely, 1st, the reduced solubility of double salts; and, 2nd, the closer attachment which constitutional water exhibits for a salt when that salt itself enters into combination.

	Analysis.	Theory of $NH^4Cl + CuCl\dot{H}^2$.
Chlorine	51·03	51·08
Copper	23·35	22·83
Ammonium (NH^4) .	13·20	13·10
Water	12·09	12·99
	100·67	100·

The water cannot be entirely expelled without risking the sublimation of chloride of ammonium, and hence the quantity of water obtained is under the truth. The copper is above the truth, from having been precipitated by caustic potash in the state of oxide, which last when so obtained always retains a little potash.

There is a corresponding chloride of copper and potassium, but I did not succeed in forming analogous double salts with chloride of magnesium or with any other chloride of the class in the place of the chloride of copper.

The chlorides have probably their analogues in the cyanides, but with the single cyanides of iron, copper, &c., we are less acquainted. It is worthy of remark, however, that the disposition of the protocyanide of iron and of the cyanide of copper to combine with two atoms of cyanide of potassium may depend upon the cyanides of iron and of copper possessing two atoms of constitutional water, (like the corresponding chlorides,) which are displaced by two atoms of the alkaline cyanide in the formation of the double cyanides. In "ferrocyanic acid" we have the protocyanide of iron combined with two atoms of hydrocyanic acid, in the place of the same two atoms of water.

VII. *Researches on the Tides.—Seventh Series. On the Diurnal Inequality of the Height of the Tide, especially at Plymouth and at Singapore; and on the Mean Level of the Sea. By the Rev.* W. Whewell, *M.A. F.R.S., Fellow of Trinity College, Cambridge.*

Received March 7,—Read March 9, 1837.

THE Inequality of the Tides which is the subject of the present paper, though theoretically very curious, and practically very important, has hitherto been hardly noticed, and its laws have never been generally stated. By means of the materials which I have had in my hands, I have not only been able to obtain a rule agreeing with the observations to an extraordinary degree of precision, but I have found and analysed a case in which this inequality assumes a very remarkable form, so as materially to disguise the general circumstances of the tides, and to explain other cases in which the usual features are entirely obliterated.

The inequality of which I speak is the Diurnal Inequality, by which the tide of the morning and evening of the same day differ. The difference is often very considerable, especially in the height of the water; and naval officers have often found the preservation or destruction of a ship to be caused by this difference, without being aware that it was subject to steady rules, and was capable of being predicted. The small number of places for which I have been able to procure the proper observations will not permit me at present to state the circumstances of the inequality as they occur all over the surface of the ocean; but I am, by fortunate circumstances, able to trace its laws in some very remarkable instances, situated in very widely separate regions of the globe.

Sect. I. *Diurnal Inequality at Plymouth.*

I will first treat of the diurnal inequality as it appears at Plymouth, at which port good tide observations are regularly made under the direction of Mr. Alexander Lumsdale and Mr. William Walker, the Master Attendant and Assistant Master Attendant of the Dock-yard.

It has long been known that both at Plymouth and at other places there is commonly a difference in the morning and evening tide of the same day. It is stated by Colepress in 1668*, that at that port "the diurnal tides from about the latter end of March till the latter end of September are about a foot higher in the evening than in the morning; that is, every tide which happens after twelve in the day before twelve at

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* Philosophical Transactions, vol. iii. p. 633.

night, and *vice versâ* the rest of the year." But we shall soon see that this way of expressing the fact, by speaking of morning and evening tides, is quite inaccurate.

It is easily seen that the theory of the tides, which supposes them to be produced by the ocean assuming its form of equilibrium under the influence of the moon's attraction, would give a diurnal difference of the tides: for if the moon have 20° north declination, the tide spheroid will have one pole in latitude 20° north, and the other in 20° south latitude; and as the earth revolves, a place in 50° north latitude will have the tide which belongs to these two poles alternately: and as it is 30° from one pole and 70° from the other, the two tides will be very unequal.

Now it has been found, with regard to all the other inequalities of the tides, that they follow the *laws* of the tides of the equilibrium-theory, although the *constant elements* (the *magnitudes* and *epochs*) can be determined only by observation. Finding that the diurnal inequality was very clearly marked in the Plymouth observations, I did not hesitate to attempt to trace its laws, by assuming this kind of correspondence with the equilibrium-theory. The result confirmed the assumption in the most striking manner, as I shall show.

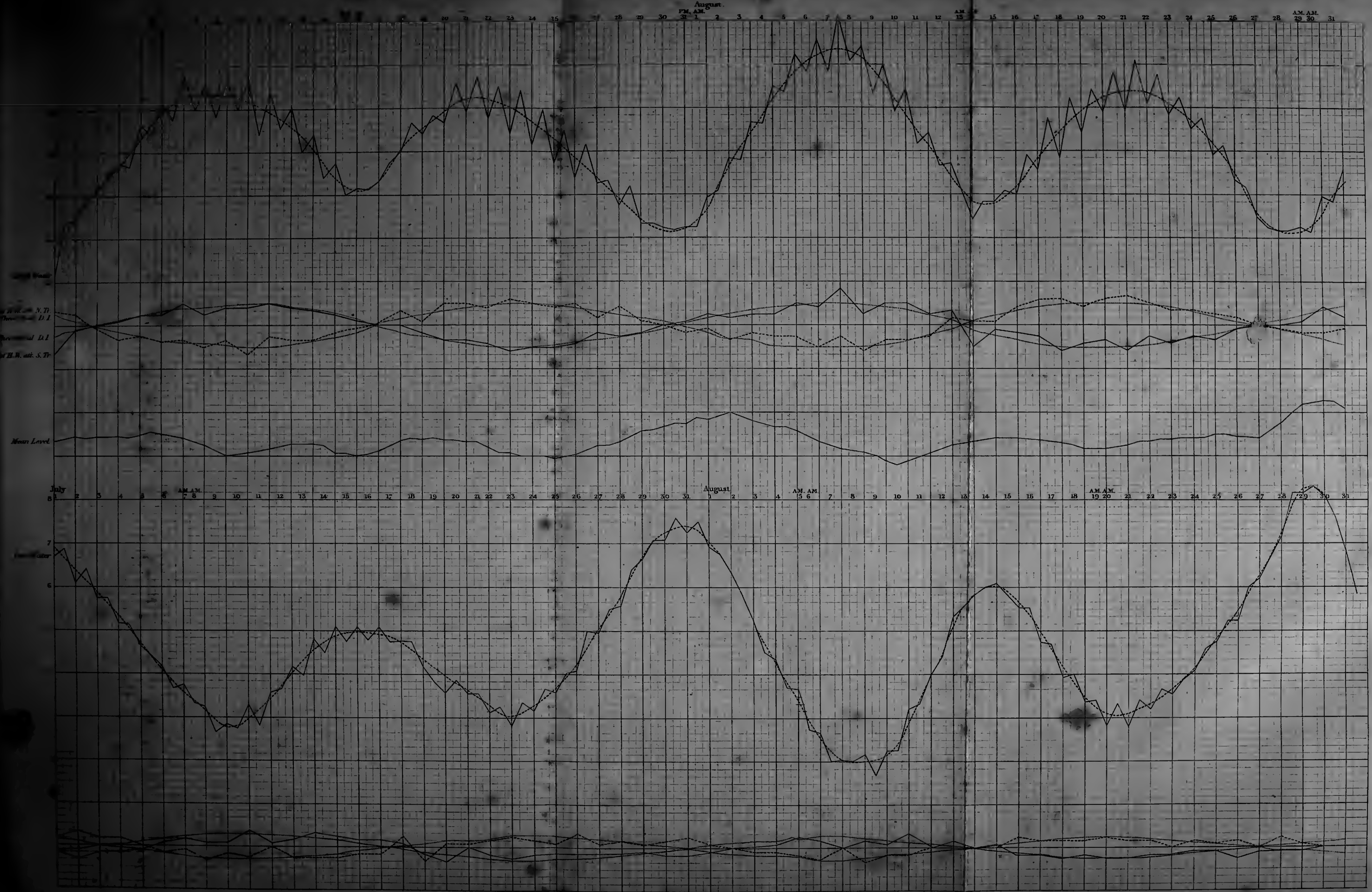
According to the equilibrium-theory, the tide which belongs to a *south* transit of the moon should be the *greater* (of the two on the same day) when the moon's declination is *north*; when the moon crosses the equator, the difference of the two tides vanishes; when she has *south* declination, the tide which belongs to her *south* transit is the *smaller*. The contrary (as to greater and smaller) will be true of the tide which belongs to the *north* or *inferior* transit.

We cannot know, except by observation, to what transit of the moon any tide *belongs*; but it is manifest that if we begin with any tide, the tides must belong alternately to south and north transits, and therefore the above alternation of greater and smaller tides, as the moon has north or south declination, must come into view.

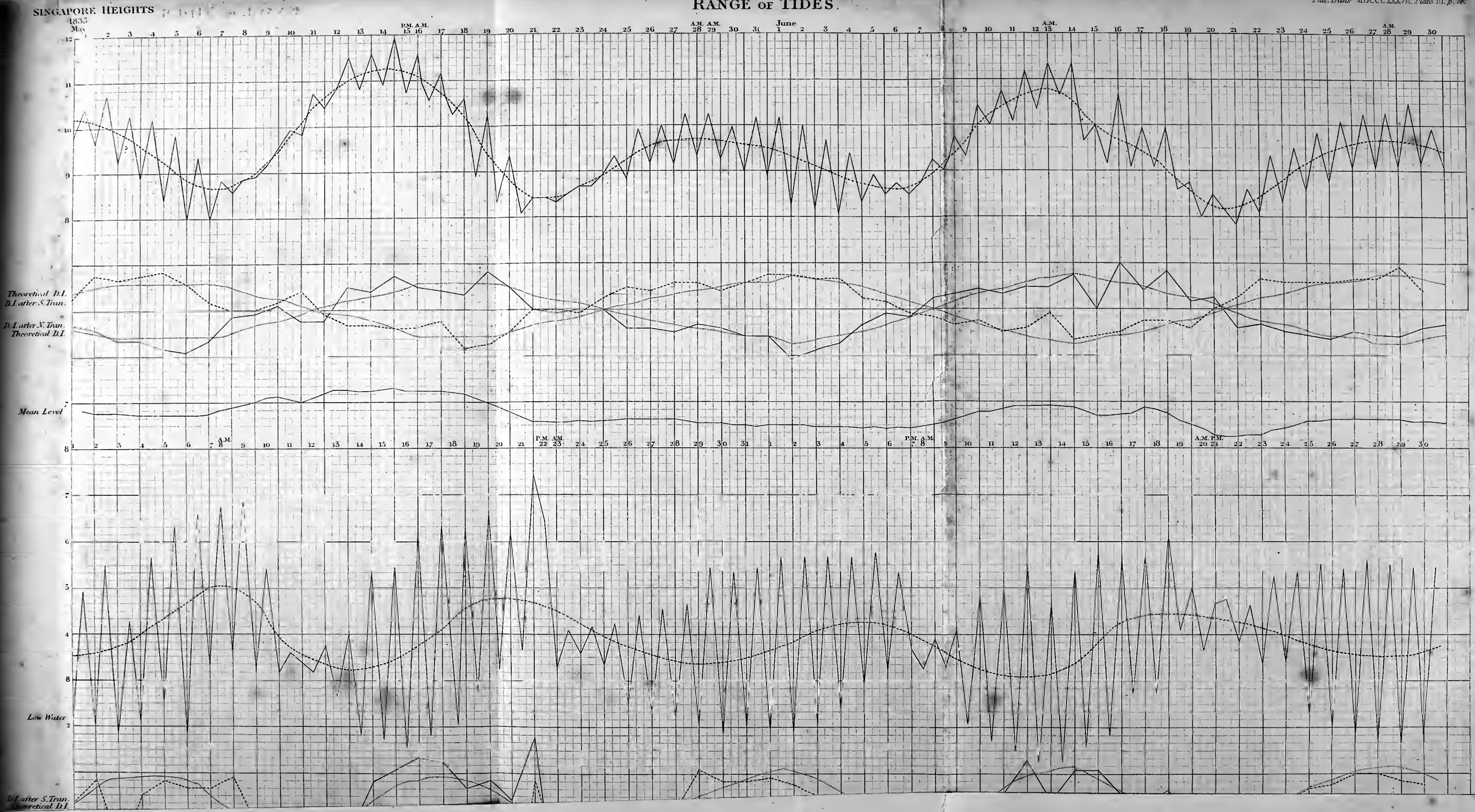
Accordingly I set off the observed heights of high water at Plymouth as ordinates of a curve, as seen in Plates II. and III. The zigzag form of the lines, appearing, vanishing, and reappearing, about once a fortnight, with great steadiness, showed that the diurnal inequality really existed in the observations. A line was drawn *by the eye*, cutting off from these zigzags equal portions above and below, and this was taken as the mean high water cleared of the diurnal inequality. The *excesses* or *defects* of the observed height and this mean height were then set off from another axis; those which belonged to the high water next following a south transit forming one curve, and the alternate tides, which follow a north transit, forming another curve. These curves were both of them found to have their ordinates alternately positive and negative at intervals, corresponding to the change of the moon's declination from north to south, and the contrary.

But though the order and cycles of these changes were the same for the observed diurnal inequality and for the lunar declination, the epochs of the changes were not the same. According to theory, as we have said, the diurnal inequality ought to

RANGE OF TIDES



RANGE OF TIDES.



vanish when the moon is in the equator. But it appeared that in fact the diurnal inequality did not vanish till about four days after that period.

By taking the moon's declination four days anterior to the day of observation, and reducing it to a proper scale, it was found that the amount of the diurnal inequality could be represented with great accuracy, as may be seen in the Plates, which are specimens of a comparison of this kind made for the whole of the years 1833 and 1834.

It is to be observed, however, that the calculation of the diurnal inequality from the declination was made by means of a coefficient which was somewhat different in different months. Thus the usual multiplier of the declination for the diurnal inequality of high water at Plymouth is $\frac{1}{4}$; that is, 4° of lunar declination produce a difference of height of 1 inch: but in some cases the coefficient is $\frac{1}{3}$ or more; in others it is $\frac{1}{6}$. These differences appear to arise in part from the height of the tide itself; for the inequality is theoretically proportional to the whole lunar tide; partly to the effect of the sun, according to different seasons of the year. Yet there appears to be still some other unexplained cause of the variation of this multiplier; for there are differences in its value which cannot be referred to the causes just mentioned, and which operate, during a lunation or a semilunation, too uniformly to be accidental. We may take the coefficient at Plymouth to be $\frac{1}{4}$, which is the value on which the curves represented in Plate II. are constructed. I am persuaded that no one accustomed to the comparison of theoretical formulæ with observation can look at those curves without being persuaded that the formula exhibits the true law of nature.

As has been said, the declination of an anterior period has been taken. The period employed was the *fifth* lunar transit preceding the tide. Thus the diurnal inequality of January 6, 1834, is determined by the declination on January 2, at 5^h 44^m A.M., the time of the moon's transit. The assumption of this period is confirmed by the general agreement of the results.

From what has been said, the inaccuracy of the statements of this inequality, as an excess of the evening tide at particular seasons, and of the morning tide at other seasons, will readily appear; for the high water at Plymouth is, on the average, five hours after the moon's transit. Suppose the moon to move in the ecliptic, which is her average path: when the sun's right ascension is five hours, (that is, about June 7,) the tide which follows the moon's transit will follow the sun's transit also, as soon as the moon is north of the equator; that is, if the diurnal inequality were regulated by the moon's place on the same day, the afternoon tide would be greatest; and so it would continue till the moon was seven hours after the sun, at which period the tide would be twelve hours after the sun, and the tide following the moon would become the morning tide. But at the same time the moon would pass to the south of the equator; and therefore the tide following the moon would be the smaller. Therefore in this situation the evening tide would be the greater during the whole lunation.

But suppose the sun's right ascension to be eight hours, (July 21,) then, when the moon begins to have north declination, the tide which follows her, and which is therefore the greatest, is three hours before the sun, and is a morning tide. When the moon's right ascension becomes three hours, (that is, after about one eighth of a lunation,) the tide following the moon, which is still the greater, (because the declination is still north,) becomes the evening tide. The evening tide continues the greater till the moon's right ascension becomes twelve hours, when she passes to the south of the equator, and the tide following the moon, which is nine hours after the sun, and still the evening tide, becomes the smaller; and this continues till the moon is seven hours from the sun, or in fifteen hours right ascension, at which period the tide which follows her becomes the morning tide, and the evening tide is again the greater. Thus in this position the morning tide is greater during six hours of the moon's motion in right ascension (from the sun), and the evening tide is greater during the remaining eighteen hours; that is, the evening tide is the greater during three fourths of the lunation.

We might in the same manner trace the changes which take place in other positions of the sun; but this is unnecessary. The effect of the inequality may be calculated by the tables which are added at the end of this paper.

The height of low water at Plymouth is also affected by a diurnal inequality. It follows the same law as the inequality of high water; its epoch is the same; and its multiplier for May, June, July, August, 1834, is $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{9}$, respectively.

Sect. II. *Diurnal Inequality at Singapore.*

By the Hydrographical Office of the Admiralty I was furnished with about a year's observations of the tides of Singapore, from August 1834 to August 1835, made by Mr. W. Scott, the Master Attendant at that port, in pursuance of directions given by the Directors of the East India Company.

These observations, from the very curious nature of the results to which they lead, I consider as more remarkable and valuable than any series of equal extent which has fallen under my notice.

On laying down the heights of high water, it appeared that the early part of the series was very irregular, obviously from the imperfection of the observations; but beginning with January 1835, the curve was tolerably regular; and during the greater part of the subsequent time, the inequalities (which the observers could not know) were so clearly marked, and so steady in their course, that it was impossible to doubt the goodness of the observations.

I proceeded to examine these in the manner already described for the Plymouth observations, and found a diurnal inequality nearly agreeing in law and in amount with that at Plymouth; the only difference being, that instead of four days it was here found necessary to take the lunar declination *a day and a half* preceding the tide, or, more exactly, at the *interpolated* or north lunar transit which intervened between the second and third south transit preceding the tide.

The amount of the inequality is nearly the same as at Plymouth, or rather greater, being, in the most regular parts of the series, one inch of height for every three degrees of declination.

In these parts of the series (May, June, July, 1835,) the coincidence of the formula with observation is as close as at Plymouth. In other months (March, April, and August,) there are discrepancies; but we cannot consider these as throwing any doubt on the general correctness of the formulæ, when we see how well it represents the observed diurnal inequality of low water, which is much more marked than that of high water.

The diurnal inequality of low water at Singapore is of a magnitude which it would have been impossible to anticipate. It makes a difference in many cases of not less than six feet between the height of the morning and evening tide; the whole rise of the mean tide being only seven feet at spring tides, and the difference of mean spring and neap tides not more than two feet.

This enormous diurnal inequality conforms, with deviations which are slight considering its magnitude, to the same formula which we have already stated, the epoch being the same as that for high water; that is, thirty-six lunar hours anterior to the last transit. The multiplier is different in different months, varying from $\frac{3}{4}$ to 1; so that each degree of the moon's declination produces an effect of nearly an inch in the height of low water, or two inches in the difference of two successive low waters.

Sect. III. *On the Diurnal Inequality at some other places, and on the General Laws of its Progress.*

I have not found any register of tide observations which exhibits the diurnal inequality so clearly and regularly as Plymouth and Singapore, although I have tried many series observed in different parts of the world. It may however be detected in many, perhaps in most, places. The comparison of the circumstances of this inequality in different places is curious and interesting, and especially the change which the *epoch* undergoes; that is, the anterior period at which the moon's declination corresponds to the amount and direction of the inequality.

Bristol.—Mr. Bunt, who has bestowed very great labour upon the analysis of tide observations made at Bristol, has, among other inquiries, endeavoured to determine the diurnal inequality at that port. The results are not very regular, but they lead him to the conclusion that the inequality vanishes at nearly the distance of five days' motion of the moon from her nodes; that is, the epoch is *five days.* The amount of the inequality is five or six inches each way, at the greatest.

Liverpool.—The diurnal inequality of the heights at Liverpool has been detected by Mr. Bywater from the observations, and introduced by him into his tide tables. I have already remarked in these Researches*, that the epoch of the diurnal inequality at this port is about *six days and a quarter*; but I do not conceive the determina-

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* Fifth Series. Philosophical Transactions, 1836, p. 133.

tion to be very exact, since the inequality has been tabulated by means of the calendar months, and thus has been referred to the moon's mean motion in the ecliptic instead of being referred to her actual motion in her own orbit. The greatest effect is about *half a foot* in excess and in defect.

Leith.—Tide observations have been made at Leith Harbour for several years. I have examined these for the diurnal inequality, but it does not appear with any great steadiness and regularity. Still its existence is very obvious; and as the determination of its epoch is a curious point, I attempted it in the following manner:

Leith Tides, 1835.

Periods of Max. Diurn. Ineq.	Tides after S. Transit.	Middle of Max.	Inequality vanishes.	Moon's dec. vanishes.	Difference. Days.
Feb. 15 to 25.	less	Feb. 20.	Feb. 27.	N. Feb. 15.	12
March 1 to 13.	greater	March 7.	March 13.	S. March 1.	12
March 15 to 25.	less	March 20.	March 27.	N. March 15.	12
March 28 to April 8.	greater	April 3.	April 11.	S. March 29.	13
April 15 to 22.	less	April 19.	April 26.	N. April 11.	13
April 29 to May 9.	greater (*small*)	May 4.	May 9.	S. April 24.	15
May 11 to 18.	less	May 15.	May 22.	N. May 9.	13
May 22 to June 7.	greater (*small*)	May 30.	June 6.	S. May 21.	[16]
June 9 to 14.	less	June 12.	June 17.	N. June 5.	12
June 15 to 30.	irregular	June 23.	June 29.	S. June 18.	[11]
July 1 to 8.	less (*small*)	July 5.	July 10.	N. July 2.	[8]
July 12 to 19.	greater (*irreg.*)	July 16.	July 20.	S. July 15.	[5]
July 21 to 27.	less (*small*)	July 24.	August 3.	N. July 30.	[5]
August 10 to 17.	greater	August 14.	August 22.	S. August 11.	11
Aug. 26 to Sept. 4.	less	August 31.	Sept. 6.	N. August 26.	10
September 8 to 18.	greater	Sept. 13.	Sept. 19.	S. Sept. 8.	11
September 20 to 30.	irregular	Sept. 25.	Sept. 30.	N. Sept. 22.	[8]
October 1 to 11.	greater	October 6.		S. October 5.	

Among all the irregularities of the Leith tides, it is easily seen from the curves, when they are laid down, that there is a diurnal inequality, in consequence of which the *tide following the south transit* of the moon becomes alternately the greater and the smaller, as the moon's declination changes from south to north, and the reverse. The times when this inequality is large can be picked out more decidedly than the times when it vanishes, and I therefore determined the epoch by means of the greatest inequality, supposing the times when it vanishes to be midway between two successive maxima, as may be seen in the preceding Table.

Rejecting those cases in which the inequality is very small or altogether irregular, it appears that the inequality vanishes *twelve days* after the moon's inclination vanishes. This is certainly a very extraordinary result; for it is difficult to conceive how the effect of the moon's action can require so much time to manifest itself. Yet there can hardly be any doubt of the fact; for it is verified in 11 semilunations out of 17, and is inconsistent with none; the variations in the interval being not greater than might be expected, supposing the law to be true. It may be observed, that by these variations the inequality is in some cases thrown back more than a whole semilunation. Thus the inequality which prevails before April 26, and vanishes about that day, is not produced by the series of declinations which vanish on April 24, but by the series which vanish on April 11. To suppose the reverse would be impossible; for that would make it necessary to suppose that the inequality vanishes on Feb. 27, in consequence of the declination vanishing two days *later*, or March 1; that is, that the effect precedes the cause.

In the system of tide observations made on the coasts of Europe and America in June 1835, of the results of which an account was given in the Sixth Series of these Researches*, it appeared that the diurnal inequality on the east coast of Scotland was, during that semilunation, irregular, passing over a tide in the middle of the series. This and other anomalies in the diurnal inequality, as it appears on the coasts of the German Ocean, appear to show that the waters in that region are affected by the mixture of more than one tide. In the most material point, however, the observations of June 1835 confirm the results of our present inquiry; namely, in showing that the diurnal inequality travels more slowly than the other inequalities. On the east coast of America, the changes of this inequality appear to be contemporaneous with the corresponding changes of the moon's declination, and the epoch is *zero*. On the coasts of Spain, Portugal, and France, it is successively *two* and *three days*. And this is quite consistent with the fact that this epoch is *four* days on the coast of Cornwall and Devonshire, *five* days at Bristol, *six* at Liverpool, and *twelve* at Leith. That the diurnal inequality should thus creep from place to place on successive days is difficult to explain; but the laws of fluid motion are so little known, that we cannot collect from hydraulical views any good reason for doubting this curious fact. The fact is certainly not easily reconciled with our conception of the

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* Philosophical Transactions, 1836, Part II. p. 304.

tides of remote places, as produced successively by the motion of the same "tide-wave;" but it is already established beyond doubt, by the observations made on the two sides of the Atlantic in 1835, that tides which were supposed to be brought by the same tide-wave differ materially in their circumstances. As I have already stated*, "On the 9th, 10th, and 11th of June 1835, when the diurnal inequality was great in America it was nothing in the West of Europe; and on the 18th and 19th, when this inequality had vanished in America, it was great in Europe." Are we to doubt whether the tide-wave which brings high water to America and to Europe at the same moment be the same wave? A sound hydrodynamical view of all the circumstances must enable us to decide; but for this purpose more observations are needed, especially observations on the coast of America, where the diurnal inequality is great, and where, on several accounts, a knowledge of its laws would be interesting to us.

Another remarkable circumstance in the progress of the diurnal inequality is, that it appears much more distinctly and steadily at some places than at others which are near them: nor does it seem easy to assign any rule which it follows in this respect. It is very marked and almost universal on the coast of the United States, and was conspicuous in the observations of June 1835 on the coasts of Spain and Portugal, the west coast of France, and parts of the west coast of Ireland. Yet at places interjacent among those at which it was thus displayed it could not be detected; nor did the circumstances easily allow of my ascribing this to any defect of exactness in the observations. In like manner it is large on the east coast of New Holland, as we know from Cook's account of his getting his ship off a reef by means of it; and the north and south coasts of Australia appear to exhibit the extreme case of it, as we shall see. We might therefore suppose that it affects the whole of the Indian Ocean: yet at Keeling Island, in the centre of that ocean, it does not decidedly show itself. Such, at least, is the result of observations made by Captain Fitz Roy, from April 2 to April 8, 1836, with which I have been furnished by his kindness.

Sect. IV. *On extreme Cases of great Diurnal Inequality.*

If we consider the motion of the surface of the water in cases where, as at Singapore, the diurnal inequality is very great, we shall see that this motion is very different from the alternate equal ascent and descent which would occur if there were no such inequality. In order to exhibit this peculiarity, I have represented this motion in Plate IV. for Plymouth and for Singapore, as observed in the months of May and June. It will be seen that at Plymouth the curve of the motion of the surface oscillating upwards and downwards by nearly equal distances; the main feature of inequality is the difference of spring and neap tides, although the diurnal inequality is very clearly visible. But at Singapore the alternate oscillations make no approach to equality; at some parts of the series the alternate tides seem to be on the point

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* Philosophical Transactions, 1836, Part II. p. 302.

RANGE of TIDES.

Phil. Trans. MDCCCXXXVII. Plate IV. p. 75

Engraved by J. & C. Walker

of disappearing; and the progress of this alternation affects the tides as much as the independent alternation of springs and neaps.

It is easy to conceive the diurnal inequality carried a little further than it is at Singapore; so that at a certain stage of it the alternate tides would vanish. This is equivalent to supposing the highest low water and the lowest high water to have the same height.

There are statements of navigators respecting various places at which there is "only one tide in twenty-four hours." From what has been said it appears that this may happen during a part of each semilunation by the effect of the inequality now under consideration, but that it cannot in this way be constantly the case.

I am fortunately enabled to throw some light on this subject by the kindness of Captain Fitz Roy. King George's Sound on the south coast of New Holland is one of the places to which these *Single-day Tides* have been ascribed*. In March 1836 Captain Fitz Roy, aware of the interest of this position in respect to tide phenomena, caused observations to be made every half hour for some days, and for a portion of the time, every quarter of an hour. The result was that on March the 7th and 8th there were two very unequal tides, and that on the 9th and 10th there was only one tide; but a recession and return in the high water, which had been barely perceptible on the 11th, became more and more marked on the 12th, 13th, and 14th, so as again to give two tides each day. Thus at this place it appears to be only at one particular period of the semilunation that we have a single-day tide, agreeably to our general view. I insert the curve of the motion of the surface at King George's Sound in Plate IV.

The single-day tides of Tonquin† were referred by Newton to the interference of two tides, which arrive by different channels. The great diurnal inequality of Singapore, which is in the same seas, appears to be clearly due to the effect of the moon's declination; and the establishment of this point, and the circumstances ascertained to occur in the reputed single-day tide of King George's Sound, throw some doubt on the explanation just referred to, which cannot be removed till the tides of those seas have been more fully observed.

Sect. V. *On the Mean Height of the Sea.*

The question of the fluctuations of the mean height of the sea is not especially connected with the diurnal inequality. But as the curves which I had to draw in the course of this investigation give me the means of exhibiting very clearly these fluctuations, I will here say a word on the subject.

In Plate II. a line is drawn representing the *mean height*; that is, the height midway between low water and high water each day. It is obtained by taking the mean of the two curves of high and low water by which the diurnal inequality is cut off. The same is done for Singapore in Plate III.

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* Flinders, vol. i. p. 71. King, vol. ii. p. 380.

† See Philosophical Transactions, vol. xiv. p. 162.

It appears that in all these cases the mean height of the sea is very nearly constant. This is most remarkable at Singapore, where, though the successive low waters often differ by six feet, the mean level only oscillates through a few inches. At Plymouth the mean level is not quite so steady. The fact is, that at that port the low water varies more by the difference of springs and neaps than the high water does; and hence the mean level slightly follows the low water, and is lowest at spring tides, and highest at neap tides, or perhaps more exactly a day or two later.

"The level of the sea at low water," a phrase sometimes used by surveyors, is altogether erroneous, and may lead to material error. From the instances just quoted (and indeed from the nature of the case) it is certain that the mean height of the sea is far more nearly constant than low or high water, under whatever assumed conditions. A *level surface* drawn from any point (that is a surface of *stagnant* water) would probably be nearly parallel to the points of mean water at different places. This becomes more manifest when we consider that at places near each other the tide often differs greatly in amount. At St. David's Head in Pembrokeshire the range of the tides is near thirty feet; on the opposite coast of Ireland it is only two or three: if the sea were level at low water the difference of the mean heights on the two sides of the Channel (which is only about fifty miles) would be fourteen feet. Such an average elevation of one side of a narrow sea above the other is quite inconsistent with the laws of fluids.

I cannot conclude this paper without again pointing out that a great number of curious facts in fluid motion are established by these Tide Researches, of which it may be hoped the theory of hydrodynamics will one day be able to render a reason. Why is it that at places near each other the range of the oscillations of the sea from low to high water is so different? Why is it that the sun affects the low water at Plymouth more than the high water, and that the moon's declination at Singapore affects the low water four times as much as the high water, while at Plymouth it affects it less? Above all, why is it that while the effect of the sun, and of the moon's declination and parallax, in the monthly course of the tides, produces the effect due to the equilibrium of the forces in one or two days, the moon's declination does not produce its effect upon the diurnal oscillation till after three, four, five, and six days; and in some cases probably not till the moon is exerting forces which tend absolutely to reverse the effect?

TABLE of the DIURNAL INEQUALITY of the Height of High Water AT PLYMOUTH.
To be used with the moon's declination *four days anterior*.

For *N*. decl., *add* to the tide following moon's S. transit, *subtract* from the tide following moon's N. transit.
For *S*. decl., *subtract* from the tide following moon's S. transit, *add* to the tide following moon's N. transit.

Moon's Declination	0° to 4°	5° to 9°	10° to 14°	15° to 18°	19° to 21°	22° to 24°	25° to 26°	27° to 28°	29°	30°
Diurnal Inequality	0^{in}	1^{in}	2^{in}	3^{in}	4^{in}	5^{in}	6^{in}	7^{in}	8^{in}	9^{in}

Postscript.

I will take the liberty of mentioning the only way in which it appears to me mechanically possible to conceive the slow propagation of the diurnal inequality which I have described in Sect. III.

If we suppose equal semidiurnal tides to be propagated along the length of a wide canal; and if we suppose, in addition to these, a *transverse* oscillation of the water to take place in the direction of the width of the canal, the time of this oscillation (from maximum to maximum) being a whole tide day; we shall have successive tides alternately greater and less by a diurnal inequality. And we may easily suppose this transverse oscillation to be propagated *gradually and slowly along* the canal, by the contact of the particles of the water. In this manner we may represent phenomena following laws like those above described.

But it may be further observed, that we may conceive the semidiurnal tide, as well as the diurnal inequality, to be propagated along the canal by means of transverse oscillations, the time of this oscillation being half a lunar day; and the rate of propagation of this undulation may easily be supposed to be different from that of the diurnal oscillation. In this way we may conceive the possibility of the different inequalities of the tides being propagated from place to place at different rates, and thus having different epochs, as from the recent researches on the subject contained in the Philosophical Transactions they appear to have.

Moreover, it is by no means necessary, in order to make this explanation applicable, that the transverse undulations should be perpendicular to the direction in which the tide is propagated: they may be oblique to it at any angle, and the result will still be the same.

It appears possible, also, that such a supposition may be modified, so as to explain other phenomena of the tides; for instance, the smallness of the tides in the central parts of wide seas.

But the application of such a supposition to the actual phenomena of the ocean, and the determination of those tracts of sea which must, on this view of the case, be looked upon as tide-canals, would be a matter of no small difficulty, even if our materials were sufficient for the purpose, and would probably be impossible without more knowledge of the tides on the shores of the great oceans than has yet been published.

Trinity College, Cambridge,
May 5, 1837.

VIII. *On the Structure of the Brain in Marsupial Animals. By* RICHARD OWEN, *Esq. F.R.S. Hunterian Professor of Anatomy to the Royal College of Surgeons.*

Received October 31, 1836,—Read January 26, 1837.

THE brain in Mammalia is essentially characterized by the complexity and magnitude of the apparatus by which its different masses are brought into communication with one another. With respect to size, the cerebral hemispheres are in many species proportionally inferior to those of Birds; and in most Insectivorous and Rodent Mammalia they present an equally smooth and uniform external surface; but notwithstanding the absence of convolutions and diminished size of the cerebral hemispheres in such Mammalia, a large apparatus of medullary fibres is present, which connect together either the opposite hemispheres, or the distant parts of the same hemisphere; and this apparatus, or great commissure, is superadded to the anterior, posterior, and soft commissures, which, with the exception of a very slight rudiment of the fornix, are alone developed in birds for the purpose of uniting the opposite hemispheres. In the higher Mammalia, in which the cerebral hemispheres acquire superior size and increased extent of surface by means of convolutions, the superadded commissural apparatus presents a corresponding development and a highly complicated structure; its several parts being distinguished as the corpus callosum, fornix, and their intercommunicating laminæ, termed the septum lucidum. The fornix, by means of its posterior crura and the intermediate medullary tract termed the lyra, brings the hippocampi majores into communication with each other, and with the posterior folds of the corpus callosum*; by means of its anterior crura it establishes a communication between the hippocampi and the optic thalami; and by means of the septum lucidum its connexion with the corpus callosum is continued to the anterior fold of that body†.

In the Human brain the fornix, though of complex structure and developed as a very distinct part, is of small size as compared with the corpus callosum; while the delicate

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* "The fasciculi from the fornix form in part the covering of the hippocampus, and in part its loose fold, the tænia hippocampi."—REIL in MAYO's Anatomical Commentaries, p. 116.

"L'envelope medullaire de la corne d'Ammon se continue avec la partie posterieure du corps calleux, et en partie aussi avec le pilier posterieur de la voute: c'est dans ce dernier que va se jeter le corps frangé tout entier."—MECKEL, *Anatomie Descript.* tom. ii. p. 679.

† "Ainsi la voute represente une chaine très complexe qui unit les deux hémisphères l'un avec l'autre sur plusieurs points, et qui, de plus, établit une communication entre la partie anterieure et la partie posterieure de chaque hémisphère."—MECKEL, *Anatomie Descript.* tom. ii. p. 658.

laminæ of the septum lucidum by which the fornix is connected with the corpus callosum, present an extent of surface corresponding to the degree to which the corpus callosum and fornix recede vertically from one another as they advance from behind forwards. In tracing the modifications of these different parts through the mammiferous series, the disproportion of the fornix to the corpus callosum is found to decrease as the parts, to the connexion of which they are subservient, alter in their relative size. For as the superincumbent masses of the cerebral hemispheres diminish in the placental Mammalia, the corpus callosum is proportionally restricted in its development; while the hippocampi and their free processes, called the tæniæ hippocampi, maintaining a remarkable uniformity in their absolute size, the fornix also continues large, and undergoes modifications of form which more distinctly manifest its relation as a commissure to the hippocampi than its structure in the human brain would indicate. Thus in the brain of the Sheep the tæniæ hippocampi, instead of being lost in the posterior crura of the fornix, are continued along its lateral margins, augmenting its breadth: they converge and unite above the anterior crura of the fornix, which here appear as small subordinate appendages sent off into the optic thalami below, from the union of the tæniæ above; the tæniæ are then again separated and are continued downwards and forwards into the anterior lobes of the hemispheres, bringing these parts into communication with the hippocampi behind, whilst the point of union of the opposite tæniæ becomes continuous with the anterior fold of the corpus callosum above.

As the corpus callosum and fornix recede vertically from one another in a less degree in most Mammalia than in Man, the two laminæ of the septum lucidum are consequently of less extent, but are proportionally stronger; they are formed not merely by the epithelium of the lateral ventricles, but by fibrous laminæ extending from the anterior and upper surface of the fornix to the opposite surface of the corpus callosum. In the simple and depressed forms of brain, such as the Rodentia present, the fornix, or hippocampal commissure, and the corpus callosum, or hemispheric commissure, are in contact, so that their uniting medium cannot with propriety be termed the septum lucidum.

The corpus callosum is the principal bond of union between the opposite hemispheres, extending horizontally above the ventricles, its middle fibres passing transversely, while those of its extremities, which are more or less bent beneath its body, radiate, and all intermix, in apposition with the ascending and diverging fibres of the peduncles of the cerebral hemispheres. It has hitherto been considered as the great characteristic of the brain in the Mammalia, and, taking the human brain as the term of comparison, to be developed in the ratio of the magnitude of the cerebral hemispheres.

In the placental Mammalia this is a pretty accurate expression of the relations of the corpus callosum; and as the posterior lobes of the hemispheres are the first to disappear in the descending comparison, so the corpus callosum diminishes in longitudinal extent from behind forwards, and thus the corpora quadrigemina, pineal gland, and posterior part of the optic thalami are successively brought into view on divaricating the cerebral hemispheres in the different Mammalia which exhibit this progressive degradation of the great commissure.

The researches of TIEDEMANN, as is well known, have shown that the anterior part, which is the most constant in the mammiferous series, is that from which the development of the corpus callosum commences in the human brain.

The aim of the present paper is not, however, to trace step by step the various modifications of the commissural apparatus of the hemispheres through the mammiferous class, but is limited to the description of a remarkable modification in that apparatus in the brains of the marsupial animals, to the detection of which I was led by observing that the commissural system presented the essential difference between the brains of the oviparous and mammiferous Vertebrata, and by associating the greater perfection of the brain, resulting from the development of the great commissure with the placental mode of development in the true Mammalia.

The connexion subsisting between placentation and high cerebral organization may be one of simple coincidence, yet it is certain that of all the great organic systems, the cerebral or sentient organ is that which alone offers a marked improvement of gradational structure in the animals developed by a placenta.

An attentive study of the manners of different Marsupiata in confinement, and an inspection of the exterior forms of the brain in some of the species, induced me to allude in a former paper to an inferiority of intelligence and a low development of the cerebral organ as being the circumstances in the habits and structure of these singular animals which were most constantly associated with the peculiarities of their generative economy*. I have since derived the most satisfactory confirmation of this coincidence from repeated dissections of the brains of Marsupiata belonging to different genera; and although unable to explain how a brief intra-uterine existence and the absence of a placental connexion between the mother and fœtus can operate (if it be really effective) in arresting the development of the brain, yet it is a coincidence which has been so little suspected, and is so interesting in various points of view, that I believe the evidence of it will be acceptable both to the physiologist and the naturalist.

In order to obtain satisfactory proof of the difference in the structure of the brain in the marsupial and placental quadruped, I dissected and compared together, step by step, the brains of a Wombat and Beaver. These animals, as is well known, are of nearly similar bulk, and manifest so many mutual affinities in their structure, that they have been, and still are, by some naturalists, classed in the same order of Mammalia. The Wombat is, in fact, in all its exterior characters, save the marsupial pouch, a Rodent; and in its internal anatomy, especially its digestive organs, more nearly resembles the Beaver than do many of the true rodent animals. The

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* Philosophical Transactions, 1834, p. 358.

brain of the Beaver was also preferred for this comparison of internal organization, because on an outward inspection it would be pronounced to be the less highly organized of the two; the hemispheres in the Wombat presenting a few convolutions (Plate V. fig. 3.), whilst in the Beaver they are perfectly smooth (Plate V. fig. 1.).

In the Beaver, however, the cerebrum is extended further backward, though still leaving the cerebellum quite uncovered; while in the Wombat a portion of the optic lobes (corpora quadrigemina) is also exposed.

On divaricating the hemispheres of the brain in the Beaver, we bring into view, about three lines below the surface, the corpus callosum; and on removing the cerebral substance to a level with this body, its fibres are observed to diverge into the substance of each hemisphere, in the usual manner, some bending upwards, but a greater proportion arching downwards, and embracing the cerebral nuclei; the anterior fibres radiating into the anterior, the posterior fibres into the posterior extremities of the hemispheres. (Plate VI. fig. 3.)

The portions of the brain which are removed in thus tracing the extent of the corpus callosum, bring into view the corpora bigemina and the pineal gland; but the optic thalami are concealed by the great commissure above described.

On separating the hemispheres of the brain of the Wombat, not only the bigeminal bodies and pineal gland, but the optic thalami are immediately brought into view, and instead of a broad corpus callosum, we perceive, situated deeply at the bottom of the hemispheric fissure, a small commissural medullary band, *m*, (Plate VII. fig. 4.) passing in an arched form over the anterior part of the thalami, and extending beneath the overlapping internal or mesial surfaces of the hemispheres, which thus appear, as in the Bird, to be wholly disunited.

On gently raising the hemispheres from above the commissure, and pressing them outwards with the handle of a scalpel, the instrument passes into the fissure upon which the hippocampus is folded; and on continuing the pressure the hippocampus is torn through, and the lateral ventricle is exposed. The mesial wall of the hemisphere is continued from the superior and internal border of the hippocampus, and is composed in the Wombat, as in the Bird, of a thin lamina of medullary substance analogous to the septum lucidum. In the Kangaroo, the mesial parietes of the lateral ventricles are stronger, being about two lines in thickness.

The posterior transverse fibres of the commissure are continued outwards and backwards, beneath the more longitudinal fibres, which overlap them as they pass from the tæniæ hippocampi forwards to the anterior cerebral lobes. All the fibres of the commissure pass along the floor of the lateral ventricles into the substance of the hippocampi majores, which are of proportionally very large size. (See Plate VI. and VII. fig. 4, *n*.)

Thus the commissure which is brought into view on divaricating the cerebral hemispheres in the Wombat is seen to be partly the bond of union of the two hippocampi majores in the transverse direction, and partly of the hippocampus and anterior lobe of the same hemisphere in the longitudinal direction. It also fulfils the other function of the fornix by sending down from the inferior surface two small nerve-like processes, which extend vertically, behind the anterior commissure, through the substance of the optic thalami, near their mesial surfaces, to the corpus albicans, at the base of the brain.

The superior view of the connexions of the hippocampal commissure of the Wombat is given at Plate VI. fig. 4.

Returning to the Beaver's brain, we raise the posterior thickened margin of the corpus callosum, and at the middle of its inferior surface we find it closely connected with the centre of a commissural band of fibres, arching over the anterior part of the optic thalami, and passing outwards and backwards along the floor of the lateral ventricles into the substance of the hippocampi, which are as largely developed as in the Wombat. The anterior part of the corpus callosum is bent downwards, and is attached along the middle line of its inferior surface by a uniting medium of medullary substance, representing the septum lucidum, to the hippocampal commissure or fornix. The tæniæ hippocampi, which form the lateral parts of this commissure, extend forwards, as in the Wombat, into the anterior lobes.

The corpus callosum being removed, and the commissural fibres of the hippocampi being left behind (as shown on the left side at Plate VI. fig. 5.), the view of the Beaver's brain now corresponds with that obtained in the previous dissection of the brain of the Wombat; which we regard, therefore, as wanting the corpus callosum, septum lucidum, and consequently the fifth ventricle. The artery of the plexus choroides, in both the Beaver and Wombat, enters the lateral ventricle, where the hippocampus commences at the base of the hemisphere, and the plexus is continued along the under surface of the tænia hippocampi, and passes beneath the fornix, through the usual foramen, to communicate with its fellow in the third ventricle, immediately behind the anterior crura of the fornix, which are sent down in the Beaver, as in the Wombat, from the centre of the inferior surface of the hippocampal commissure.

If we expose the lateral ventricle by removing its outer parietes in a marsupial and placental quadruped, as shown in Plate VII. figg. 4 and 5, in the Kangaroo and Ass, the hippocampus major (*n*), the tænia hippocampi (*o*), the plexus choroides (*p*), and the foramen Monroianum (*γ*) are brought into view. If a style be thrust transversely through the internal wall of the ventricle, immediately above the hippocampus, in the placental quadruped, it perforates the septum lucidum (*q*), and enters the opposite ventricle below the corpus callosum. If the same be done in the marsupial brain, the style passes into the opposite ventricle, but is immediately brought into view from above by divaricating the hemispheres, and is seen lying above the commissure of the hippocampi.

This commissure may nevertheless be regarded as representing, besides the fornix, he rudimental commencement of the corpus callosum; but this determination does not invalidate the fact that the great commissure which unites the supraventricular masses of the hemispheres in the Beaver and all other placentally developed Mammalia, and which exists in addition to the hippocampal commissure, is wanting in the brain of the Wombat: and as the same deficiency exists in the brain of the Great and Bush Kangaroos, the Vulpine Phalanger, the Ursine and Mauge's Dasyures, and the Virginian Opossum, it is most probably the characteristic of the marsupial division of Mammalia.

In the modification of the commissural apparatus above described, the Marsupialia present a structure of brain which is intermediate to that of the placental Mammalia and Birds, in which class the great commissure is wholly wanting, and the hemispheres, though comparatively larger than in many of the Mammalia, are brought into communication only by means of the anterior, posterior, and soft commissures, and a slight trace of the fornix or hippocampal commissure.

Of the other peculiarities of the marsupial brain, the relatively large size of the anterior commissure is most worthy of notice; its development is correspondent with the large size of the cerebral ganglion, which forms the chief origin of the olfactory nerve, and some of the anterior fibres arch forwards, and are directly continued into those nerves.

In the position, superficial transverse fissure, and solidity of the bigeminal bodies, the marsupial brain adheres to the mammiferous type, as also in the exterior transverse fibres of the commissure of the cerebellum, forming the pons Varolii, the presence of which is in relation with the development of the lateral lobes of the cerebellum.

Other minor points of difference between the brains of the Marsupiata themselves will be explained in the description of the figures.

Meanwhile their agreement in so important a modification of the cerebral organ as the absence of a corpus callosum and septum lucidum, affords additional and strong grounds for regarding the Marsupiata as a distinct and peculiar group of Mammalia; and when to this modification of cerebral structure are added the traces of the oviparous type of structure presented in the circulating and absorbent systems, together with the peculiarities of the osseous and generative apparatus, we may with reason suspect that distribution of the Marsupiata to be artificial and founded on an imperfect knowledge of their mutual affinities which, from a modification of the teeth and extremities alone, would separate and disperse the species amongst corresponding groups of the placental Mammalia.

Cuvier has observed that the marsupial group of quadrupeds embraces forms which typify different orders of the ordinary Mammalia*; and M. De Blainville regards them as forming, with the Monotremata, a division apart from the placental Mammalia. The metropolis of this subclass is the continent of Australasia, where the different carnivorous, insectivorous, omnivorous, and herbivorous genera act

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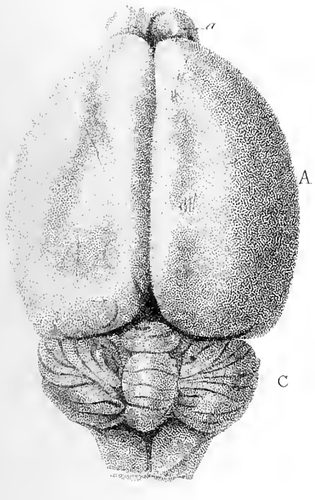
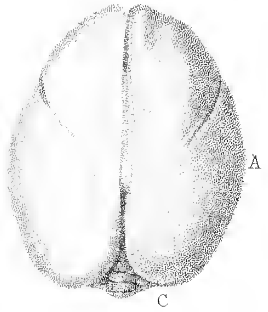
* "Les Marsupiaux—nous paraissent devoir former un ordre à part, tant ils offrent de singularités dans leur économie, et surtout parceque l'on observe en quelque sorte la representation de trois ordres bien différents."—Règne Anim. i. p. 172.

Fig. 1.



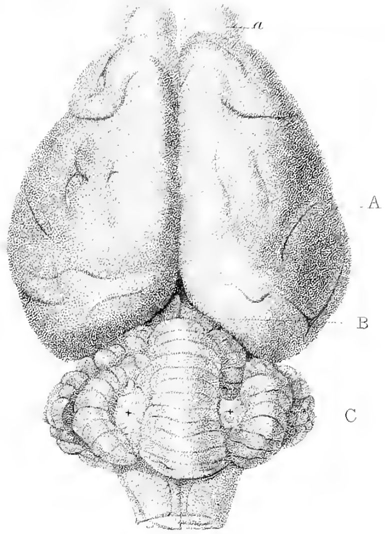
Beaver

2



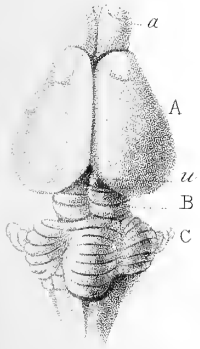
Midas rufimanus

3



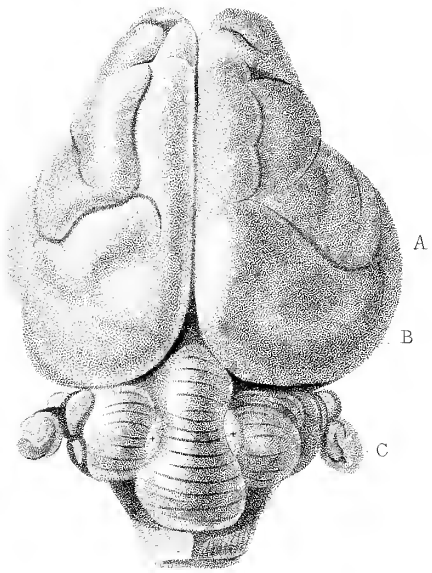
Wombat

6



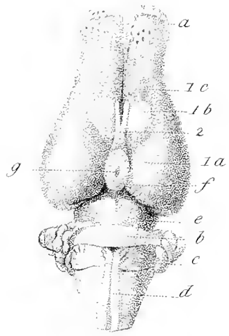
Opoſsum

4



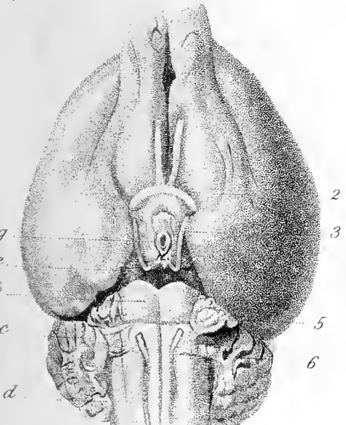
Kangaroo

9



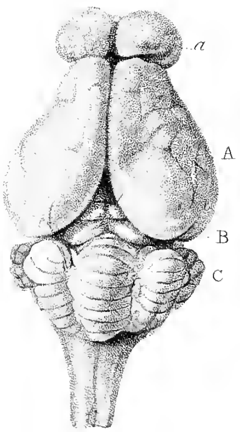
Opoſsum

7



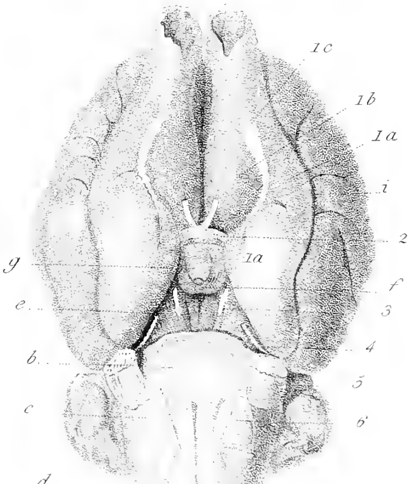
Beaver

5



Ursine Dasyure

8



Wombat

Owen, del!

corresponding parts to those performed by the placental Mammalia on a larger theatre, in which the avoidance of more numerous and powerful enemies, or the capture of more varied and subtle prey, demands the manifestation of more courage, the practice of more address, and the possession of more resources than appear to be called for by the exigencies of the Marsupiata in their more limited sphere.

Description of the PLATES.

PLATE V.

External form.

Fig 1. The upper surface of the brain of a Beaver (*Castor Fiber*, L.).
2. The upper surface of the brain of a Monkey (*Midas rufimanus*, Geoff.).
3. The upper surface of the brain of a Wombat (*Phascolomys Wombatus*, Bl.).
4. The upper surface of the brain of a Kangaroo (*Macropus major*, Shaw.).
5. The upper surface of the brain of an Ursine Dasyure (*Dasyurus ursinus*, Geoff.).
6. The upper surface of the brain of a Virginian Opossum (*Didelphys Virginiana*, Shaw).
7. The base of the brain of a Beaver.
8. The base of the brain of a Wombat.
9. The base of the brain of a Virginian Opossum.

From these figures it will be seen that the convolution of the surface of the hemispheres of the brain does not take place in proportion as the hemispheres themselves are developed in superficial extent. They are fewer, for example, in the *Midas*, in which the hemispheres extend, as in most of the Quadrumana, over the greater part of the cerebellum, than in the Kangaroo or Wombat, where the cerebellum is left quite exposed. The brains of two species of herbivorous and two of carnivorous Marsupials are figured in this plate, to show indications of superior development which distinguish the brain of the herbivora, in the greater proportional development of the cerebrum, its convoluted surface, and the smaller proportional size of the olfactory tubercles. In all the species, but especially the carnivorous Marsupials, the greater relative size of the vermiform process is deserving of notice, as indicating the approach to the oviparous type of cerebral structure: it is associated with a corresponding diminution of the pons Varolii, as is strikingly shown in fig. 9.

PLATE VI.

Fig. 1. Side view of the brain of the Kangaroo.
2. Side view of the brain of the Virginian Opossum.

Structure.

Fig. 3. Brain of the Beaver, with the substance of the hemispheres removed to the level of the corpus callosum.

4. Brain of the Wombat, with the substance of the hemispheres removed to the level of the hippocampal commissure, except on the right side, where part of the thin internal wall of the lateral ventricle is left.

5. Brain of the Beaver, with the left cerebral hemisphere cut down to a level with the commissure of the hippocampus, and the lateral ventricle exposed. The corpus callosum has been vertically divided, and the left half removed, together with the hemisphere: the right hemisphere is entire.

6. A similar dissection of the brain of the Kangaroo, with the right hemisphere entire, and turned aside, showing the absence of the hemispheric commissure, corresponding to the corpus callosum of the Beaver.

The small size of the corpus striatum, *r*, as compared with the Wombat and Beaver, is shown in this view. The posterior bigeminal bodies are the broadest, the anterior the longest, in this animal as in the Beaver and Wombat.

PLATE VII.

Fig. 1. A vertical bisection of the brain of the Opossum (*Didelphys Virginiana*, Shaw), showing the large proportional size of the anterior commissure, *y*.

2. A vertical bisection of the brain of a Goose.

3. A lateral section of the left hemisphere, showing the lateral ventricle and hippocampus major in the Opossum.

The roof of the lateral ventricle is raised, showing it to be formed by fibres arching over the hippocampus, and continued from the inner margin of that part into those which radiate from the corpus striatum externally.

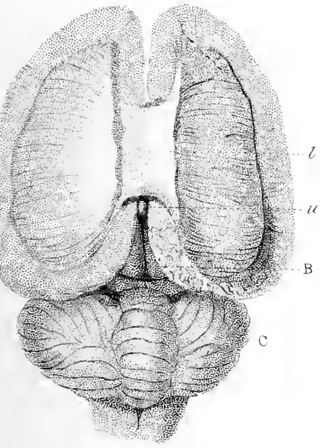
4. A similar dissection of the brain of the Kangaroo.

In this species the roof of the ventricle is proportionally thicker than in the carnivorous Opossum. Besides the diverging fibres of the crus cerebri, and those which pass in an arched form from the inner border of the hippocampus, over that body to the corpus striatum, there are others which form a thin layer, and pass into the tænia hippocampi, closely embracing the hippocampus: some of these are shown at *x x*.

The general disposition of the hemispheric fibres is such, that, supposing them contractile, they would draw the superficies of the hemisphere towards the crus cerebri, as to a fixed point, and compress the bodies projecting into the ventricles.

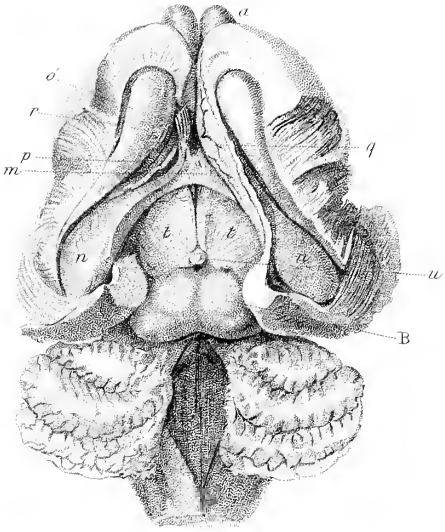
5. A similar dissection of the brain of the Ass. The dotted line shows the extent of the corpus callosum.

Fig. 3.



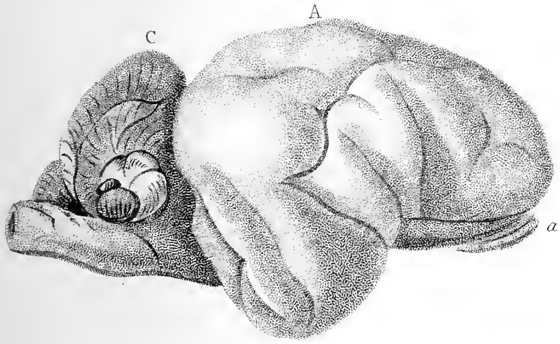
Beaver. N°11.

Fig. 4.



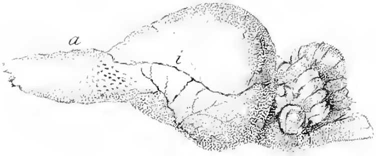
Wombat. N°12.

Fig. 1.



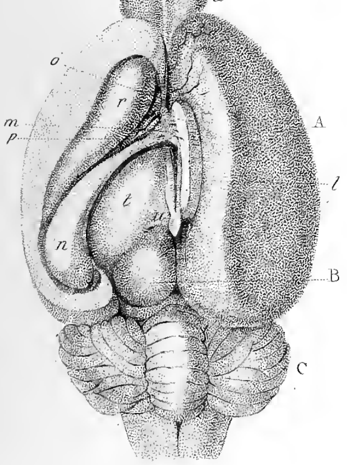
Kangaroo.

Fig. 2.



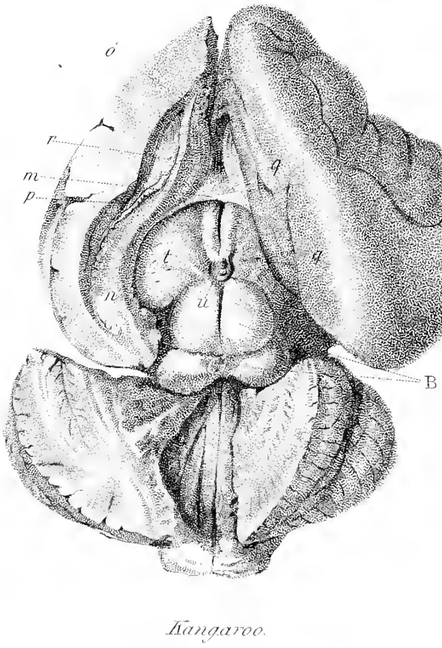
Opoſsum.

Fig. 5.



Beaver.

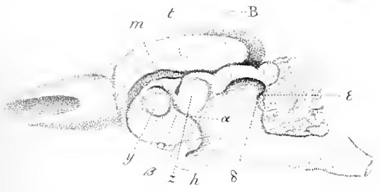
Fig. 6.



Kangaroo.

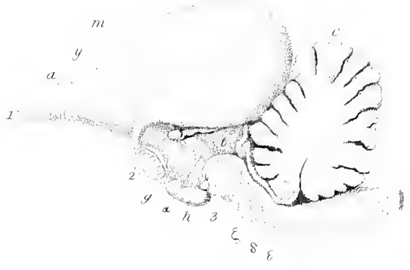
hard Owen, del.t

Fig. 1.



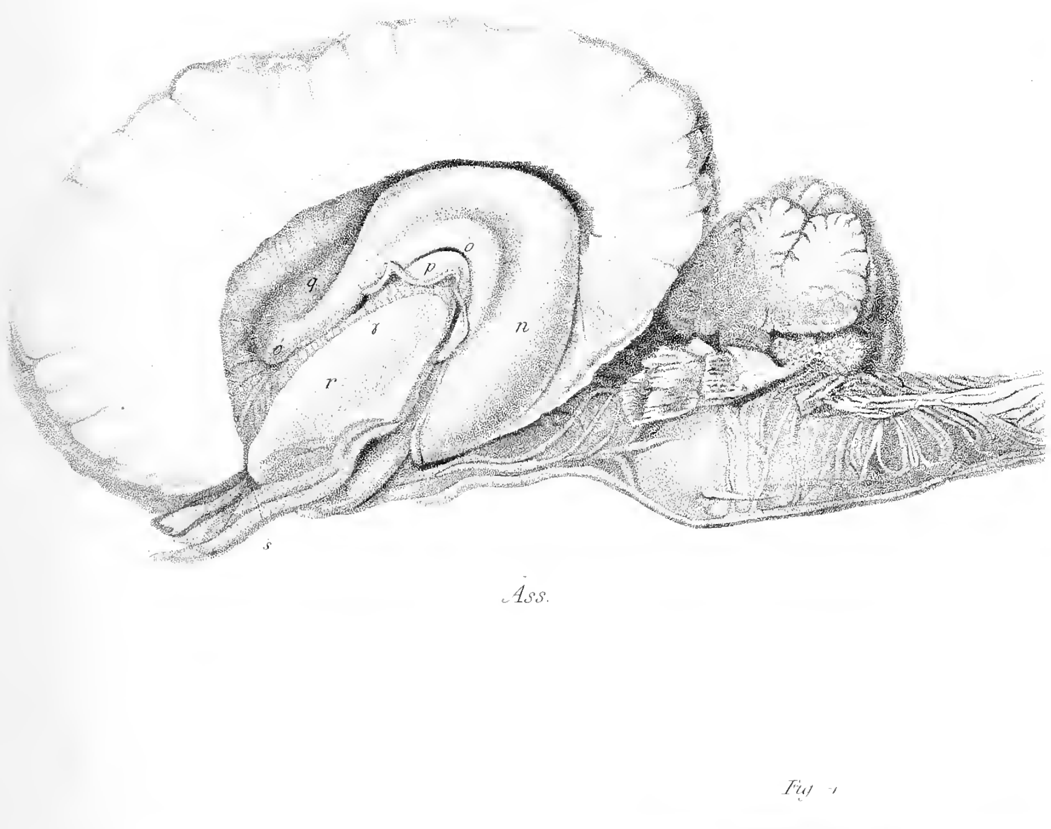
Opossum

Fig. 2



Goose.

Fig. 5



Ass.

Fig. 3.

Opoſsum

Fig. 4

Kangaroo.

del^{t}

The same letters indicate the same parts in each figure.

A. Cerebral hemispheres.
B. Optic lobes, or corpora quadrigemina.
C. Cerebellum.
* Place between the vermiform process and lateral lobes, where the medullary matter of the cerebellum is superficial.
a. Olfactory lobes or ganglions.
b. Pons Varolii, or cerebellic commissure.
c. Corpus trapezoideum.
d. Corpora pyramidalia.
e. Crura cerebri.
f. Corpus albicans.
g. Infundibulum.
h. Pituitary gland.
1 *a*. Natiform protuberance, giving off the external root of the olfactory nerve.
1 *b*. Pyriform protuberance, forming the origin of the internal root of the olfactory nerve.
1 *c*. Medullary root of the olfactory nerve emerging from a longitudinal fissure in the natiform protuberance.
i. Fissure dividing the external root of olfactory nerve from the superincumbent hemisphere.
2. Chiasma of the optic nerves.
3. Third pair of nerves.
4. Fourth pair of nerves.
5. Fifth pair of nerves.
6. Sixth pair of nerves.
k. Corpus trapezoideum.
l. Corpus callosum, or commissure of the hemispheres.
m. Fornix, or commissure of the hippocampi.
n. Hippocampus major.
o. Tænia hippocampi.
o'. Anterior fibres of the tænia hippocampi continued into the anterior lobes of the hemispheres.
p. Plexus choroides.
q. Septum lucidum, or internal wall of the lateral ventricle (Plate VII. fig. 5.).
r. Corpus striatum.
s. Continuation of the lateral ventricle into the olfactory nerve.
t. Optic thalami.
u. Pineal gland.
x. Part of a thin stratum of medullary fibres arching over the hippocampus major, and continued into the internal wall of the ventricle.

y. The anterior commissure.
z. Soft commissure.
m. Hippocampal commissure.
α. The third ventricle.
β. The iter ad infundibulum.
γ. The foramen Monroianum.
δ. The iter ad quartum ventriculum.
ε. The valvula Vieussenii.
ζ. The fold of the valve corresponding to the posterior commissure.

IX. *On the Tides.* *By* John William Lubbock, *Esq., V.P.R.S.*

Received March 15,—Read March 16, 1837.

In my last paper on the Tides I endeavoured to point out the remarkable agreement which obtains in some respects between Bernoulli's theory and results obtained from observations at the London Docks. Since that time my attention has been directed to the following points:

1. To ascertain, from a discussion of the Liverpool observations with reference to a previous transit [*], whether they present the same kind of agreement with theory as those of London. (See Tables I. to XII.)

2. To ascertain whether, by taking into account a greater number of observations, the results given in my last paper remain sensibly unaltered. (See Tables XV. to XXVIII.)

3. To ascertain whether the *establishment* of the port varies sensibly in different years, and whether the removal of the old Bridge has occasioned any difference at London. (See Tables XIV. and XXX.)

Numerous tables have been computed for me, in order to elucidate these points, by Mr. Jones and Mr. Russell, having been enabled to procure their valuable assistance in these laborious investigations by means of a further sum of money placed at my disposal for the purpose by the British Association for the Advancement of Science, to which distinguished body I take this opportunity of offering my grateful acknowledgements.

The succeeding transits of the moon being denoted by the letters A, B, C, D, E, F; and F being the time of the moon's transit which immediately precedes the time of high water at London, my last discussion of the London Dock observations was instituted with reference to transit B: the present discussion of the observations made at Liverpool between the 1st of January 1774 and the 31st of December 1792, by Mr. Hutchinson, has been instituted with reference to transit A[†], or that which precedes the time of high water at Liverpool by about $2^{d}\ 0^{h}\ 20^{m}{\cdot}9$. This paper contains, in fact, two sets of tables precisely similar; the one set deduced from 13,391 observations of high water made at Liverpool by Mr. Hutchinson, and the other set deduced from 24,592 observations of high water at the Wapping entrance of the

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[*] The former discussion by Mr. Dessiou, given in the Philosophical Transactions for 1835, was made with reference to the transit immediately preceding.

[†] I had intended the transit B to be used as the argument: the mistake was not perceived until the work was too far advanced to make it worth while to have recommenced.

London Docks made under the direction of the late Mr. Peirce; the latter tables differing from those given in the Philosophical Transactions for 1836, Part I., chiefly by being founded upon nearly double the number of observations. The high waters at Liverpool considered in this paper occur about 48 hours after the transit of the moon to which they are referred; the tides at London which are considered occur about 50 hours after the transit to which they are referred in this discussion, so that in fact all the *intervals* of the Liverpool Tables given in this paper ought to be increased by 36 hours, and all the *intervals* in the London Tables by 48 hours. The tide which makes high water at Liverpool arrives at the same instant somewhere on the north-east coast of Scotland, and at London about fifteen hours later. This is proved by the epoch of the semimenstrual inequality.

I find by interpolation from Table II. the interval for the moon's transit A

	d	h	m
At 3 o'clock . . .	1	23	40·5
and at 9 o'clock . . .	2	1	2·0

The difference is $1^{h}\,21^{m}{\cdot}5$, which converted into space

$$= 20^{\circ}\,22' \qquad \log\tan 20^{\circ}\,22' = 9{\cdot}56965 = \log (A). \text{ (See p. 117.)}$$

If we take the difference between the greatest and least heights $= 5{\cdot}52$ from Table III.,

$$(E) = \frac{5{\cdot}52}{2(A)} = 7{\cdot}4353 \text{ for Liverpool,} \quad \log (E) = 0{\cdot}87130.$$

If we take the greatest height $= 17{\cdot}66$ from Table III.,

$$17{\cdot}66 = D + \{1 + (A)\}\,(E) = D + \{1{\cdot}3712\}\,(E);$$

and hence definitively for Liverpool in the year 1786,

$$\log (A) = 9{\cdot}56965, \quad \log (E) = 0{\cdot}87130, \quad D = 7{\cdot}46,$$

D being reckoned from the datum in the East Wall of the Canning Dock.

And I find in the same manner for London in the year 1820

$$\log (A) = 9{\cdot}58418, \quad \log (E) = 0{\cdot}64690, \quad D = 16{\cdot}69,$$

D being reckoned from the sill of the London Dock gates at the Wapping entrance.

I conceive that the best if not the only method of investigating alterations in the height of the land above the water in any given locality where the water is influenced by the tides, will be to examine carefully whether any alteration has taken place in the values of the constants D and (E) for that place, the height of high water being of course always reckoned from some fixed mark in the land.

The semimenstrual[*] inequality of the interval at Liverpool presents the same remarkable agreement with observation which has been noticed before, while the form or law of the semimenstrual inequality of the height is also the same as that indi-

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[*] The semimenstrual inequality is an inequality of high water or of the semidiurnal wave.

cated by the observations; but in order to render the agreement complete, it would be necessary to change the epoch by half an hour*. This remarkable circumstance also obtains in the London correction, as may be seen by reference to the plate which accompanies my last paper.

The results contained in the Tables here given are laid down in diagrams, without which they could not be so readily understood; but as they are similar in nature to those contained in my last paper, they do not require an extended description.

The calendar month inequality at Liverpool, considered as resulting implicitly from the corrections due to changes in the declinations of the luminaries and in the sun's parallax, agrees generally with the equilibrium theory, and with the results deduced from the London observations given in my last paper. The diagrams in Plate I. show that the spring equinoctial tides are greater than the neap equinoctial tides, and that the neap solstitial tides are greater than the spring solstitial tides, confirming what is stated by LAPLACE in the *Exposition du Système du Monde*, 5e ed., p. 83, and by NEWTON: "In quadraturis autem solstitialibus majores ciebunt æstus quam in quadraturis æquinoctialibus, eo quod Lunæ jam in æquatore constitutæ effectus maxime superat effectum Solis. Incidunt igitur æstus maximi in syzygias et minimi in quadraturas luminarium, circa tempora æquinoctii utriusque. Et æstum maximum in syzygiis comitatur semper minimus in quadraturis, ut experientiâ compertum est." LAPLACE says, "Elles [les marées] augmentent et diminuent avec le diamétre et le parallaxe lunaire, *mais dans un plus grand rapport*;" but the diagrams in Plate II. appear to confirm the truth of this passage only at neap tides.

It is desirable to establish the laws which regulate the diurnal inequality in the height of high water in different parts of the globe; at present the data are very insufficient. Mr. WHEWELL remarks, "that it would be easy to enumerate many actual cases in which the safety or loss of a ship has been determined by this inequality." Mr. WHEWELL was the first specially to notice, in his examination of the results of the tide observations made on the coasts of Europe and America in June 1835, contained in the Philosophical Transactions for 1836, the changes which this inequality presents in passing from one place to another.

This inequality depends chiefly upon the sign and amount of the moon's declination. The observations at London and Liverpool indicate no difference between tides corresponding to upper and lower transits, or between those corresponding to A.M. transits and transits P.M. six months afterwards; hence in endeavouring to determine the *diurnal inequality* at London and Liverpool, I have confounded in Tables XII. and XIII. the results corresponding to upper and lower transits, and those corresponding to A.M. transits and transits P.M. six months afterwards. I have also added to these those which ought, according to the preceding remarks, to differ only

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* Or, adopting BERNOULLI's views in other respects, the epoch of the correction for the height is not the same as that for the interval.

in sign, and I have taken the mean of the whole for the result, as in the following example.

	h m		ft.		ft.
Moon's transit A.—Liverpool, Jan.	0 30	A.M.	−·63	substituting	−·56
	. . .	P.M.	+·57	mean of all	+·56
July	. . .	A.M.	+·60	with proper	+·56
	. . .	P.M.	−·42	sign	−·56

4)2·22

·56

In the comparison of the heights in Plates I. II. and III. the London corrections have been multiplied by 1·7, that being the ratio of the quantities (E) for London and Liverpool, agreeably to the remark made in my last paper, p. 223. As the London discussion contained in my last paper was instituted with reference to transit B, and this discussion of the Liverpool observations has been made with reference to transit A, and as the tides which correspond to P.M. transits B correspond to A.M. transits A about twenty-five minutes less, in comparing our London and Liverpool results in all the Plates it was necessary to change the epoch, or to place the London corrections more to the left by half an hour, and to substitute in Plate III. for the London results corresponding to transits P.M. those corresponding to transits A.M. The diurnal inequality therefore, as it is laid down in Plate III. for London and Liverpool, has reference to the same tide or semidiurnal wave, making high water at London about fifteen hours later than at Liverpool.

I have already remarked that the laws to which the wave producing the semidiurnal inequality is subject, agree remarkably with Bernoulli's theory. The equilibrium theory also implies the existence of another wave producing a *diurnal inequality*. $2\psi - 2\varphi$ and 2ψ are the arguments of the *semidiurnal* inequality, $\psi - \varphi$ and ψ of the *diurnal inequality*. If we suppose the diurnal inequality-wave to move with a different velocity from the other, the diurnal inequality in the height may still be represented by the expression

$$d h = B \{A \sin 2\delta \cos(\psi - \varphi) + \sin 2\delta' \cos\psi\},$$

and may be calculated by means of Tables X. and XI., δ being the sun's declination, and δ' that of the moon, but the constants which accompany ψ and φ will be different from those which accompany 2ψ and 2φ; and if we consider the constants to be included in the quantities ψ and φ, at high water, $\cos\psi$ may no longer be nearly equal to ± 1 in the last expression, but it will nearly equal $\pm$ some other constant, supposing the angle ψ still to increase by 180° in passing from one high water to the next; and the diurnal inequality, if the smaller term due to the sun's declination be neglected, may be represented approximately by

$$d\psi = \frac{G \tan\delta'}{1 + A\cos 2\varphi} \qquad d h = C \sin 2\delta' \text{ (for a given transit A.M. or P.M.),}$$

C and G being constant for any given place. Probably the amount depends also upon the moon's parallax, and then the expression for d h will be

$$C \times \frac{P'^3}{(57')^3} \sin 2\,\delta' \text{ (for a given transit A.M. or P.M.).}$$

But this expression will not afford results agreeing with those which I have obtained from the observations at Liverpool if the declination of the moon be employed belonging to the time of the transit A; and it is necessary to employ the moon's declination at some time previous; that is, several days before the high water under consideration. This is not at variance with what is stated in the *Exposition*, except that, although LAPLACE considers the two waves separately*, he has not, I think, referred distinctly to the change in the epoch for different places, or to the difference between the epoch of the original diurnal and semidiurnal waves, which produce the derived tides observed on our coasts. If, however, the diurnal inequality-wave travels more slowly than the semidiurnal inequality-wave, the epoch also will be different, and thus it may depend upon the moon's declination several days earlier.

If this view be correct, the diurnal inequality of high water has a maximum (geographically) at those places on the coast at which the diurnal inequality-wave and the semidiurnal inequality-wave arrive simultaneously, and there will be places intermediate at which the diurnal inequality of high water is imperceptible, but where the diurnal inequality of low water is a maximum. This theory agrees with observation in giving no difference in the diurnal inequality for upper or lower transits.

The diurnal inequality in the interval at Liverpool is inappreciable; the diurnal inequality in the height has been laid down in Plate III. from the approximate expression

$$\mathrm{d}\,h = B\,\{A \sin 2\,\delta \cos(\psi - \varphi) + \sin 2\,\delta' \cos\psi\} \quad \text{(for transits A.)}$$

The moon's declination δ' was taken from Table X., where it is given for the time of the moon's transit A, but the curve evidently requires to be shifted more to the right; it is difficult to decide exactly how much. In the observations of the height at Liverpool in May 1836 (see Plate IV.) the diurnal inequality vanishes on the 15th, the moon having crossed the equator on the 11th. If we consider that the theory curve requires to be shifted to the right about two hours, this would amount to referring the diurnal inequality at Liverpool to the moon's declination about four days previously.

Mr. RUSSELL has extended the discussion of the London observations given in my last paper by employing those made between 1802 and 1807, and those between 1827 and 1835, omitted before, so that we have now obtained tables similar to those contained in my last paper from the concurrence of no less than 24,592 observations.

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* J'ai determiné la grandeur de ce flux et l'heure de son *maximum* dans le port de Brest. J'ai trouvé un cinquième de mètre [7·4 inches] à fort peu près pour sa grandeur; et un dixième de jour environ, pour le temps dont il precède à Brest, l'heure du *maximum* de la marée semidiurne. (Exposition du Système du Monde, 5e ed., p. 286.)

In consequence of this additional number of observations some of the jumps or irregularities which the former tables presented have been removed*, but the differences are in general less than I anticipated.

It is evident from the diagrams in Plate III. that a diurnal inequality in the interval at London is distinct although small. The value of the constant C is different from that which obtains for Liverpool. It is evident from Plate III. that the diurnal inequality in passing from Liverpool to London becomes reversed, that is to say, if a and b denote two successive heights of high water at Liverpool, and a', b' successive heights at London caused by the same tides,

$$\text{if } a > b \text{ then generally } \quad a' < b'.$$

The character of the diurnal inequality is generally manifest in the observations of a single month, as may be seen by those which are laid down in Plate IV. When the change is remarked which takes place in the diurnal inequality in passing from Plymouth to Portsmouth, it will not excite surprise that this inequality should be so different for places more distant from each other, as for London and Liverpool.

The calculations or predictions of the time of high water at any given place have long been made to depend upon what is called the *establishment of the port*, or a certain quantity presumed to be constant and independent of the distances and declinations of the luminaries, but which may be influenced by local circumstances. It seemed to me desirable to ascertain carefully how much this quantity has fluctuated during the time the observations were made at Liverpool by Mr. HUTCHINSON, which we have employed, and since the observations at the London Docks were instituted. Tables XIV. and XXX., which give these fluctuations, have been computed by Mr. RUSSELL. The changes of the Liverpool establishment, and the fluctuations of the average height of high water at Liverpool are given in Table XIV., and are exhibited in fig. 1. Plate V.: which shows the time and height of high water from 1802 to 1835 at the London Docks on the full and change of the moon; the moon's parallax being 57′, and the declinations of the luminaries 15°, i. e. *the establishment* and the fluctuations in the average height of high water during the same interval. All the intervals and heights have been carefully reduced to horizontal parallax 57′ and declination 15°.

The changes of the London establishment, and the fluctuations of the average height, are given in Table XXX., and are exhibited in fig. 2. Plate V. These fluctuations in the *interval* and in the *height* present an insuperable obstacle to extreme accuracy in tide predictions, unless they can be explained.

"In 1832 none of the lower portions of old London Bridge, (with the exception of two piers,) which prevented the natural flow of the tidal waters, were removed; and in the second year (1833) almost the whole of that structure was cleared away as

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* See, for instance, the calendar month correction for the interval in January, and the correction for the height corresponding to H. P. 56′.

regarded the masonry and starlings, although the section of the river was far from completed, many portions still remaining one or two feet above low-water mark, and which were finally removed in the year 1834*."

The time of high water appears now to be nearly as late as in 1804; in 1821 it was about ten minutes sooner.

I am much indebted to Mr. **Yates** for notice of a very ancient tide table which exists in a MS. in the British Museum. It is in the Codex Cottonianus, Julius DVII., which appears to have been written in the 13th century, and to have belonged to St. Albans Abbey. It contains calendar and other astronomical or geographical matters, some of which are the productions of **John Wallingford**, who died Abbot of St. Albans A.D. 1213. At p. 45 b. is a table on one leaf, showing the time of high water at London Bridge, "flod at london brigge", thus:

Ætas Lunæ.	h	m
1	3	48
2	4	36
3	5	24
4	6	12
....	....	....
....	....	....
28	1	24
29	2	12
30	3	0

N.B. The numbers increase by a constant difference of forty-eight minutes. The first column gives the moon's age in days.

Hence it would appear that high water at London on full and change was at that epoch $3^h 48^m$, or more than an hour later than at present. The time of high water at London on full and change is given in Mr. **Riddle**'s *Navigation* and in other works $2^h 45^m$: **Flamsteed** made it 3^h†.

Note.—*On the Fluctuations of the Height of High Water due to changes in the Atmospheric Pressure.*

Read June 15.

M. **Daussy** having ascertained that at Brest the ocean rises when the barometer is depressed, I verified the existence of the same fact at Liverpool and London, and I found that at Liverpool when the barometer falls ·91 inch the tide rises 10·1 inches. As the *range* of the barometer is 3 inches‡, the correction which arises from change in the atmospheric pressure is by no means inconsiderable, its *range* being at Liverpool about 33 inches. At London I have found that when the barometer falls ·9 inch

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[*] **Rennie**, Report on Hydraulics, p. 512.

[†] See Philosophical Transactions, vol. xii. p. 12.

[‡] Between the tropics the fluctuations of the barometer do not much exceed one fourth of an inch, while beyond this space they reach to 3 inches. **Daniell**'s Meteorological Essays, p. 108.

the tide rises 6·3 inches, and hence the *range* of the correction here is about 21 inches. Hence it is evident that in many inquiries relative to the tides, and particularly when observations are employed throughout only a limited period, the *correction* due to the atmospheric pressure may require to be attended to. Here, however, a question arises of some interest; does the surface of the ocean rise in narrow seas *simultaneously* with the depression of the barometer, or otherwise? In order to acquire some information upon this point, I requested Mr. Russell to calculate carefully from our Tables the height of high water at Liverpool and London for May and June 1836, and to compare the calculations with the observations, which is done in the accompanying Table, and the *errors*, together with the height of the barometer at Liverpool and London, are exhibited in fig. 3. Plate V.

Table showing the difference between the Height of High Water as calculated, and the Heights derived from observations at the London and Liverpool Docks.

1836. May.	Liverpool.	London.	1836. May.	Liverpool.	London.	1836. June.	Liverpool.	London.	1836. June.	Liverpool.	London.
	O – C.	O – C.		O – C.	O – C.		O – C.	O – C.		O – C.	O – C.
	inches.	inches.		inches.	inches.		inches.	inches.		inches.	inches.
1	− 1	− 1	17		+ 1	1.		+ 6	17.	+ 11	+ 9
	− 7	+ 5		− 8	0		− 3	+ 4		+ 12	+ 9
2	− 11	+ 13	18	− 6	− 1	2.	0	+ 6	18.	+ 14	+ 5
		+ 18		− 4	+ 1		+ 6	+ 6		+ 14	+ 5
3	− 15	+ 11	19	− 3	+ 1	3.	+ 11	+ 1	19.	+ 14	+ 7
	− 11	+ 8		− 1	− 1		+ 14	+ 1		+ 13	+ 10
4	− 6	+ 6	20	+ 1	− 1	4.	+ 17	+ 4	20.	+ 12	+ 8
	− 2	+ 5		+ 2	+ 1		+ 18	+ 4		+ 12	+ 5
5	− 1	+ 2	21	+ 3	+ 4	5.	+ 14	+ 8	21.	+ 11	+ 5
	− 4	+ 5		+ 3	+ 5		+ 12			+ 10	+ 4
6	− 5	+ 7	22	+ 4	+ 2	6.	+ 12	+ 17	22.	+ 12	0
	− 3	+ 5		+ 1	+ 1		+ 5	+ 6		+ 14	0
7	− 4	+ 6	23	0	+ 5	7.	+ 12	+ 6	23.	+ 16	+ 1
	− 2	+ 4		− 3	+ 7		+ 9	+ 8		+ 15	+ 2
8	− 2	+ 4	24	− 1	+ 6	8.	+ 6	+ 7	24.	+ 11	+ 2
	0	+ 3		+ 1	0		+ 9	+ 6		+ 10	+ 1
9	+ 1	+ 4	25	+ 1	0	9.	+ 9	+ 6	25.	+ 9	0
	0	+ 2		− 3	+ 1		+ 9	+ 4		+ 4	+ 1
10	0	+ 1	26	− 2	+ 5	10.	+ 9	+ 3	26.	0	+ 3
	− 1	− 3		− 2	+ 1		+ 5	+ 5		− 2	+ 2
11	0	− 5	27	− 2	− 1	11.	+ 3		27.	− 1	
	0	− 5		− 2	− 1		+ 5	+ 6		+ 2	+ 1
12	0		28	− 0		12.	+ 5	+ 4	28.	+ 3	0
	− 3	+ 1		− 2	0		+ 3	+ 2		+ 2	+ 2
13	− 5	0	29	− 3	0	13.	+ 1	+ 3	29.	0	+ 3
	− 8	− 2		− 3	+ 1		+ 1	+ 4		0	+ 3
14	− 9	− 2	30	− 2	0	14.	+ 4	+ 5	30.		+ 3
	− 9	− 1		− 3	+ 1		+ 5	+ 7		− 1	+ 2
15	− 9	− 1	31	− 0	+ 3	15.	+ 7	+ 9			
	− 7	− 1		− 3	+ 2			+ 11			
16	− 7	0				16.	+ 11	+ 10			
	− 7	+ 1					+ 11	+ 9			

The above differences, O — C, are not the differences between Calculation and *actual* observation, but between Calculation and what it is presumed observation would be if freed of diurnal inequality by drawing an intermediate curve between those given in Plate IV.

Results deduced from Observations made at

LIVERPOOL.

Table I. (*a*.)

Showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the Liverpool Docks, corresponding to the Apparent Solar Time of the Moon's Transit A* in each month of the year, from 13,391 observations made at the Liverpool Docks, between the 1st of January 1774 and the 31st of December 1792.

January.

Number of Observations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.
	h m	h m	ft. in.	°	′
91	0 30·1	12 10·1	17 10·1	18·6	57·6
94	1 30·4	11 57·1	17 3·7	14·9	57·4
96	2 29·7	11 45·5	16 4·1	9·9	57·4
104	3 29·7	11 38·8	15 0·8	5·6	57·0
100	4 29·9	11 42·7	13 4·8	4·9	56·7
104	5 30·0	12 3·5	12 6·8	8·5	56·8
96	6 30·1	12 36·2	12 6·8	13·5	56·7
92	7 27·6	13 2·6	13 9·1	17·9	56·9
92	8 29·2	13 7·3	15 3·8	20·6	57·1
86	9 30·2	12 59·4	16 2·2	22·7	57·5
89	10 30·2	12 44·7	17 3·5	22·6	57·6
86	11 30·7	12 28·7	17 9·0	21·9	57·7

Sun's Declination S. 21°.

February.

Number of Observations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.
	h m	h m	ft. in.	°	′
88	0 31·4	12 12·9	18 3·6	9·9	57·3
91	1 30·7	11 57·7	17 7·1	5·2	57·4
90	2 29·9	11 47·1	16 4·9	4·8	57·2
91	3 29·1	11 38·2	14 8·7	8·3	57·1
91	4 29·3	11 42·3	12 11·5	13·4	56·9
86	5 29·5	12 3·6	11 9·5	18·4	56·9
85	6 29·0	12 40·6	11 11·2	20·9	56·7
83	7 29·5	13 5·2	13 7·0	22·3	57·0
81	8 29·0	13 8·7	15 0·5	22·9	57·0
83	9 29·8	12 59·4	16 6·3	21·7	57·2
81	10 30·4	12 43·9	17 7·9	18·9	57·5
87	11 30·3	12 28·2	18 2·8	14·9	57·3

Sun's Declination S. 13°.

March.

Number of Observations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.
102	0 28·8	12 13·1	18 4·3	4·5	57·7
101	1 29·1	11 58·9	17 5·0	8·0	57·7
100	2 29·9	11 45·5	15 11·9	12·8	57·3
92	3 30·0	11 35·9	14 1·1	17·5	56·9
91	4 29·3	11 37·7	12 3·5	20·7	56·8
89	5 29·7	12 0·2	11 2·5	22·3	56·7
89	6 31·2	12 42·2	11 8·8	22·8	56·2
84	7 30·1	13 6·2	13 3·6	21·7	56·8
94	8 29·7	13 8·8	15 0·7	19·7	56·8
98	9 30·2	12 58·4	16 8·2	15·2	57·2
98	10 30·9	12 44·1	17 10·3	10·6	57·4
92	11 30·4	12 28·4	18 6·4	6·0	57·7

Sun's Declination S. 2°.

April.

Number of Observations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.
94	0 29·0	12 13·1	17 6·8	12·4	57·5
93	1 30·0	11 57·7	16 9·6	16·8	57·6
87	2 30·7	11 43·1	15 4·8	20·6	57·5
86	3 30·6	11 34·1	13 10·1	22·2	57·2
88	4 30·7	11 37·7	12 3·3	23·0	57·0
85	5 30·7	12 2·2	11 5·8	22·2	57·0
91	6 30·3	12 39·6	11 10·9	19·8	56·7
91	7 30·5	13 1·6	13 8·3	15·8	57·1
94	8 29·4	13 4·5	15 5·1	11·1	57·1
96	9 28·4	12 56·9	16 10·1	6·2	57·2
103	10 29·0	12 43·5	17 8·3	4·4	57·2
94	11 29·4	12 29·1	18 0·5	7·4	57·3

Sun's Declination N. 10°.

May.

Number of Observations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.
87	0 28·6	12 12·8	16 9·0	20·3	57·3
89	1 29·8	11 57·1	16 2·3	22·1	57·4
83	2 30·6	11 43·1	15 3·3	22·8	57·5
87	3 29·3	11 35·2	14 0·7	22·1	57·3
95	4 29·4	11 38·9	12 11·7	20·0	57·1
96	5 29·7	12 3·2	12 4·1	15·1	57·1
101	6 28·9	12 35·7	12 10·8	11·9	57·1
106	7 29·5	12 57·1	14 1·6	6·6	57·2
103	8 30·0	13 1·3	15 4·4	4·5	57·0
101	9 30·5	12 55·5	16 5·7	7·2	57·3
98	10 31·3	12 43·0	16 11·6	11·9	57·2
89	11 29·9	12 29·3	17 3·5	16·9	57·4

Sun's Declination N. 19°.

June.

Number of Observations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.
82	0 28·7	12 11·1	16 9·1	23·1	57·3
85	1 28·9	11 56·8	16 3·3	22·1	57·3
87	2 29·4	11 44·2	15 5·9	19·5	57·2
92	3 28·6	11 38·7	14 5·5	15·9	57·1
99	4 30·0	11 44·5	13 8·0	11·5	57·1
96	5 29·7	12 4·6	12 11·2	6·4	57·0
108	6 30·2	12 35·4	13 3·0	4·8	57·0
97	7 30·9	12 57·2	14 0·5	7·5	57·2
96	8 30·1	13 2·6	15 2·5	13·4	57·3
91	9 30·4	12 57·2	16 2·0	17·0	57·4
85	10 29·8	12 45·1	16 8·0	20·4	57·6
84	11 29·5	12 29·4	16 9·4	22·2	57·3

Sun's Declination N. 23°.

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* The succeeding transits of the moon are denoted by the letters A, B, C, D, E, F; F being the transit immediately preceding the time of high water at London.

Table I. (*a*.) (Continued.)

July.						August.					
Number of Observations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.	Number of Observations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.
	h m	h m	ft. in.	°	′		h m	h m	ft. in.	°	′
93	0 30·3	12 13·3	17 2·5	19·4	57·4	97	0 29·6	12 14·3	18 0·5	10·7	57·7
90	1 30·1	11 58·5	16 10·1	16·1	57·3	103	1 28·3	12 0·0	17 4·1	6·1	57·6
101	2 29·8	11 45·4	16 1·1	11·0	57·1	105	2 28·0	11 47·0	16 5·0	4·5	57·3
101	3 30·8	11 41·8	14 9·0	6·4	56·9	100	3 28·4	11 40·1	14 9·5	7·0	56·9
100	4 29·7	11 45·3	13 8·2	4·7	56·8	100	4 28·7	11 41·3	13 1·1	12·8	56·8
102	5 29·0	12 5·2	12 8·5	7·5	56·8	93	5 28·8	12 2·4	11 9·4	17·5	56·7
99	6 29·1	12 37·0	12 7·9	12·7	56·9	93	6 29·0	12 39·7	12 0·4	20·6	56·7
97	7 29·7	13 0·8	13 10·8	17·2	57·1	84	7 29·1	13 4·9	13 4·7	22·2	56·9
92	8 30·6	13 6·2	14 11·1	20·7	57·1	90	8 29·3	13 9·7	14 9·0	22·9	56·9
85	9 29·5	13 0·2	16 1·6	22·2	57·6	88	9 30·4	13 0·2	16 3·3	21·6	57·1
88	10 29·1	12 44·3	16 10·1	22·9	57·6	92	10 30·6	12 44·8	17 2·8	19·4	57·4
89	11 30·0	12 29·2	17 3·3	21·9	57·6	92	11 30·3	12 28·5	18 0·6	15·5	57·7
Sun's Declination N. 21°.						Sun's Declination N. 13°.					

September.						October.					
98	0 30·5	12 13·5	18 4·9	4·5	57·6	95	0 28·7	12 12·9	18 0·0	11·3	57·3
98	1 30·9	11 58·7	17 6·5	7·0	57·5	93	1 28·3	11 57·0	17 3·1	16·3	57·5
96	2 30·4	11 45·6	16 2·5	12·4	57·5	92	2 28·7	11 41·3	15 8·0	20·2	57·3
93	3 30·0	11 36·6	14 4·9	16·8	57·1	91	3 28·2	11 35·0	14 3·2	22·0	57·4
88	4 29·6	11 37·9	12 6·7	19·9	56·9	97	4 29·8	11 34·9	12 7·4	23·0	57·1
82	5 28·4	11 59·3	11 6·0	22·1	56·8	89	5 31·6	12 0·4	11 8·1	21·9	57·2
92	6 29·2	12 39·5	12 1·6	23·3	56·7	88	6 29·7	12 37·7	12 1·9	20·2	56·9
83	7 30·4	13 4·9	13 6·9	22·1	57·0	100	7 29·6	13 3·3	13 10·6	17·0	57·0
88	8 29·7	13 8·2	15 2·4	20·0	56·9	96	8 30·7	13 5·7	15 6·8	12·4	56·9
91	9 29·9	12 58·7	16 9·2	16·0	57·2	100	9 30·0	12 57·3	17 0·6	6·9	57·1
94	10 29·7	12 44·0	17 11·2	11·2	57·4	98	10 29·3	12 43·5	17 8·9	4·5	57·3
98	11 30·2	12 28·5	18 3·8	6·5	57·5	100	11 28·7	12 28·8	18 6·8	6·6	57·3
Sun's Declination N. 3°.						Sun's Declination S. 9°.					

November.						December.					
86	0 31·1	12 11·9	17 5·2	19·7	57·4	85	0 29·6	12 12·0	17 1·2	23·1	57·3
82	1 30·6	11 56·0	16 6·8	22·2	57·2	87	1 29·8	11 55·2	16 5·8	22·1	57·3
83	2 29·9	11 41·7	15 6·4	22·9	57·2	86	2 29·1	11 42·7	15 7·6	19·3	57·1
85	3 29·6	11 34·1	14 2·8	22·2	57·1	97	3 28·7	11 38·3	14 9·2	15·8	57·1
94	4 31·3	11 37·2	12 10·7	20·2	57·0	101	4 30·3	11 41·6	13 6·0	11·0	56·9
89	5 31·0	12 2·4	12 5·5	16·4	57·1	104	5 30·1	12 3·7	12 10·3	6·6	56·9
100	6 30·2	12 37·5	12 11·5	12·3	56·9	106	6 30·2	12 35·8	13 2·0	4·9	56·9
98	7 30·0	12 57·7	14 2·9	7·3	57·0	103	7 30·0	12 57·8	14 3·4	7·1	57·1
98	8 28·7	13 2·4	15 9·6	4·5	57·1	101	8 29·5	13 2·3	15 7·7	12·3	57·3
99	9 28·4	12 54·4	16 11·7	6·2	57·3	90	9 29·1	12 57·0	16 7·7	16·3	57·5
94	10 28·3	12 43·9	17 8·0	11·6	57·4	93	10 29·1	12 45·1	17 3·3	20·1	57·5
94	11 29·9	12 28·8	17 10·1	16·5	57·4	85	11 30·3	12 28·2	17 6·3	22·2	57·6
Sun's Declination S. 19°.						Sun's Declination S. 23°.					

Table II. (*b*.) (Interpolated from Table I.)

Showing the Interval between the Apparent Solar Time of the Moon's Transit A, and the Time of High Water at the Liverpool Docks for each month in the year.

Apparent Solar Time of Moon's Transit A.	January.	February.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Mean.
h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
0 30	12 9·9	12 13·2	12 12·6	12 12·6	12 12·3	12 10·6	12 13·3	12 14·0	12 13·4	12 12·4	12 12·1	12 11·9	12 12·3
1 30	11 56·8	11 57·5	11 58·1	11 57·2	11 56·1	11 56·2	11 58·2	11 59·1	11 58·4	11 56·1	11 55·9	11 54·9	11 57·0
2 30	11 44·8	11 46·8	11 45·0	11 42·4	11 43·4	11 43·8	11 45·2	11 46·1	11 44·9	11 40·6	11 41·4	11 42·3	11 43·9
3 30	11 38·8	11 38·0	11 36·1	11 33·7	11 34·5	11 38·4	11 42·0	11 40·2	11 36·4	11 34·0	11 33·9	11 38·0	11 37·0
4 30	11 43·5	11 42·7	11 38·3	11 37·7	11 38·8	11 44·3	11 45·9	11 42·1	11 38·3	11 34·7	11 36·9	11 41·8	11 40·4
5 30	12 3·7	12 4·0	12 0·8	12 1·9	12 3·2	12 4·8	12 5·9	12 3·4	12 0·3	11 59·4	12 1·8	12 3·8	12 2·8
6 30	12 35·8	12 40·7	12 40·6	12 39·0	12 36·4	12 35·3	12 37·4	12 39·8	12 39·5	12 37·7	12 37·8	12 35·6	12 37·9
7 30	13 2·8	13 5·3	13 5·7	13 1·7	12 57·7	12 57·5	13 1·1	13 4·8	13 4·8	13 3·4	12 57·7	12 58·0	13 1·7
8 30	13 7·5	13 8·7	13 8·3	13 4·7	13 1·3	13 3·3	13 6·4	13 9·5	13 8·0	13 5·5	13 2·7	13 3·0	13 5·7
9 30	13 0·2	12 59·7	12 58·7	12 56·9	12 56·1	12 57·9	13 1·1	13 0·5	12 59·0	12 57·5	12 53·6	12 57·6	12 58·2
10 30	12 45·2	12 44·5	12 44·7	12 43·5	12 43·5	12 45·6	12 44·7	12 45·3	12 44·3	12 43·6	12 43·9	12 45·4	12 44·5
11 30	12 29·1	12 28·4	12 28·7	12 29·0	12 29·4	12 29·4	12 29·4	12 28·8	12 28·8	12 28·5	12 28·9	12 28·5	12 29·0

Moon's Hor. Par. 57′.

TABLE III. (*c.*) (Interpolated from Table I.)

Showing the Height of High Water at the Liverpool Docks, corresponding to the Apparent Solar Time of the Moon's Transit A, in each month of the year.

Apparent Solar Time of Moon's Transit A.	January.	February.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Mean.
h m	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
0 30	17·59	18·18	18·06	17·35	16·61	16·62	17·04	17·75	18·16	17·86	17·27	16·97	17·46
1 30	17·15	17·43	17·12	16·55	16·02	16·13	16·71	17·07	17·33	17·02	16·50	16·35	16·78
2 30	16·17	16·33	15·87	15·21	15·33	15·40	16·05	16·26	16·02	15·53	15·45	15·57	15·77
3 30	15·06	14·66	14·13	13·76	13·93	14·38	14·81	14·79	14·37	14·06	14·18	14·70	14·40
4 30	13·51	12·98	12·35	12·30	12·93	13·63	13·75	13·14	12·59	12·58	12·92	13·55	13·02
5 30	12·65	11·82	11·33	11·49	12·30	12·93	12·78	11·89	11·56	11·62	12·43	12·90	12·14
6 30	12·69	12·06	12·03	12·03	12·87	13·25	12·71	12·16	12·26	12·20	13·00	13·21	12·54
7 30	13·85	13·58	13·38	13·64	14·06	13·94	13·87	13·45	13·58	13·88	14·24	14·24	13·81
8 30	15·30	15·07	15·15	15·40	15·37	15·09	14·87	14·81	15·26	15·59	15·79	15·53	15·27
9 30	15·98	16·45	16·60	16·79	16·35	16·01	15·90	16·23	16·69	17·01	16·89	16·46	16·45
10 30	17·04	17·43	17·68	17·62	16·87	16·42	16·60	17·05	17·76	17·63	17·52	17·07	17·22
11 30	17·46	18·10	18·34	17·91	17·12	16·65	17·03	17·76	18·11	18·45	17·67	17·28	17·66

Moon's Hor. Par. 57′.

TABLE IV. (*d.*)

Showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water at the Liverpool Docks, corresponding to the Apparent Solar Time of the Moon's Transit A, for every minute of her Horizontal Parallax.

Hor. Par. 54′.					Hor. Par. 55′.				
Number of Observations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Number of Observations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
	h m	h m	ft. in.	°		h m	h m	ft. in.	°
201	0 29·8	12 12·3	16 4·0	14·5	159	0 29·4	12 13·2	16 7·8	14·8
191	1 29·9	11 53·9	15 7·7	15·0	169	1 29·6	11 56·1	15 11·2	14·8
191	2 29·1	11 39·6	14 8·4	15·3	175	2 30·3	11 39·7	14 10·9	15·0
167	3 28·6	11 30·2	13 4·2	15·1	193	3 28·3	11 31·5	13 7·6	14·8
150	4 27·7	11 32·2	11 11·5	15·5	236	4 30·4	11 34·1	12 3·0	15·8
108	5 29·0	11 58·0	10 10·4	15·3	242	5 30·2	12 0·2	11 3·0	14·8
120	6 30·1	12 41·6	11 4·0	16·3	266	6 29·1	12 40·7	11 6·9	15·3
133	7 30·8	13 8·5	12 8·6	14·8	214	7 28·5	13 6·7	12 10·9	15·3
174	8 29·3	13 12·7	14 2·3	15·8	208	8 29·1	13 13·7	14 7·3	15·5
179	9 29·3	13 4·9	15 5·1	14·8	186	9 29·8	13 4·6	15 9·7	15·1
196	10 29·9	12 46·4	16 2·9	15·1	169	10 29·4	12 46·4	16 6·4	14·4
190	11 29·6	12 29·9	16 6·9	14·8	168	11 29·6	12 29·4	16 9·1	14·7
Sun's Declination 15°.					Sun's Declination 15°.				

Hor. Par. 56′.					Hor. Par. 57′.				
116	0 30·1	12 12·7	16 11·4	14·4	105	0 32·0	12 12·4	17 3·3	14·1
118	1 30·8	11 56·2	16 2·8	14·6	103	1 31·2	11 57·2	16 9·3	14·1
119	2 28·8	11 37·4	15 6·2	13·9	106	2 30·6	11 43·9	15 11·3	13·6
147	3 30·5	11 35·2	14 0·0	14·6	118	3 30·3	11 39·5	14 5·2	14·4
141	4 31·4	11 38·9	12 6·8	14·6	128	4 29·4	11 41·9	13 0·4	14·3
143	5 29·1	12 0·6	11 7·1	15·1	144	5 29·5	12 2·2	12 1·6	14·9
153	6 29·5	12 38·4	12 0·4	14·8	136	6 31·9	12 38·4	12 6·6	14·9
143	7 29·5	13 3·6	13 4·2	14·7	136	7 29·5	13 1·0	13 9·8	14·5
139	8 29·7	13 7·3	14 9·5	14·5	113	8 29·0	13 5·9	15 3·0	14·4
125	9 30·0	12 59·2	16 0·8	14·5	102	9 27·5	12 55·5	16 3·4	14·0
124	10 30·0	12 44·2	16 10·0	14·6	99	10 29·2	12 43·0	17 1·9	14·1
107	11 30·9	12 29·3	17 2·7	14·6	109	11 30·4	12 29·0	17 6·3	14·3
Sun's Declination 15°.					Sun's Declination 15°.				

TABLE IV. (*d.*) (Continued.)

Hor. Par. 58′.					Hor. Par. 59′.				
Number of Observations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Number of Observations.	Apparent Solar Time of Moon's Transit A.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
	h m	h m	ft. in.	°		h m	h m	ft. in.	°
92	0 27·4	12 13·7	17 6·2	13·5	94	0 29·1	12 13·3	18 4·9	14·0
101	1 28·6	11 58·0	17 2·2	13·8	98	1 31·7	11 58·4	17 8·2	14·1
101	2 30·0	11 46·1	16 0·1	14·0	114	2 30·9	11 47·3	16 6·5	14·6
126	3 30·3	11 38·9	14 9·6	14·4	133	3 30·2	11 41·0	15 2·2	14·2
133	4 31·7	11 43·5	13 4·6	14·5	204	4 31·7	11 46·1	13 10·9	15·0
154	5 31·9	12 5·5	12 8·4	14·5	285	5 31·0	12 7·1	13 2·4	15·4
147	6 29·0	12 36·5	13 0·2	14·9	301	6 28·1	12 34·9	13 6·6	15·0
144	7 30·0	12 58·1	14 2·9	14·3	196	7 27·9	12 55·7	14 6·9	15·3
114	8 28·8	13 2·9	15 8·0	13·7	143	8 28·5	13 2·8	16 0·8	14·7
114	9 30·9	12 56·6	16 8·9	13·9	113	9 26·7	12 54·2	17 2·5	14·3
104	10 29·9	12 43·3	17 6·2	13·4	97	10 28·3	12 44·0	18 0·8	14·4
88	11 28·7	12 28·6	17 8·9	14·1	98	11 29·5	12 29·0	18 6·1	13·8
Sun's Declination 15°.					Sun's Declination 15°.				
Hor. Par. 60′.					Hor. Par. 61′.				
113	0 31·6	12 13·3	18 8·9	14·8	191	0 29·7	12 13·2	19 3·0	14·7
112	1 30·3	11 59·8	18 1·8	14·4	213	1 27·6	12 1·2	18 6·1	15·1
161	2 31·3	11 49·3	16 11·4	14·2	140	2 26·5	11 50·3	17 5·1	15·6
219	3 30·0	11 43·7	15 7·7	15·5	16	3 12·4	11 46·1	16 1·1	18·6
150	4 25·7	11 47·2	14 4·7	16·1					
33	5 15·3	11 58·6	13 8·4	15·2					
30	6 43·6	12 41·4	13 9·6	15·9					
150	7 33·9	12 55·6	15 0·9	15·4					
212	8 29·7	12 58·9	16 5·7	15·3	19	8 48·9	12 57·9	16 9·1	19·8
162	9 27·4	12 53·8	17 9·3	14·6	127	9 35·6	12 52·6	18 0·8	15·1
123	10 29·6	12 42·2	18 7·1	14·4	201	10 31·4	12 40·3	18 10·9	14·9
106	11 30·2	12 27·0	19 0·0	14·1	195	11 28·7	12 27·8	19 5·1	15·1
Sun's Declination 15°.					Sun's Declination 15°.				

TABLE V. (*e.*)

Interpolated from Table IV., and reduced to Moon's Declination 15°.

Apparent Solar Time of Moon's Transit A.	H. P. 54′.		H. P. 55′.		H. P. 56′.		H. P. 57′.	
	Interval.	Height of Tide.	Interval.	Height of Tide.	Interval.	Height of Tide.	Interval.	Height of Tide.
h m	h m	feet.	h m	feet.	h m	feet.	h m	feet.
0 30	12 12·1	16·31	12 13·0	16·64	12 12·6	16·92	12 12·8	17·23
1 30	11 53·9	15·64	11 56·0	15·91	11 56·3	16·22	11 57·3	16·74
2 30	11 39·5	14·70	11 39·6	14·93	11 36·9	15·42	11 43·6	15·86
3 30	11 30·2	13·32	11 31·4	13·58	11 35·0	13·99	11 39·3	14·40
4 30	11 33·0	11·94	11 34·4	12·32	11 38·4	12·58	11 41·7	12·98
5 30	11 58·6	10·88	12 0·1	11·24	12 1·1	11·59	12 2·5	12·12
6 30	12 41·4	11·42	12 40·3	11·61	12 38·7	12·03	12 37·4	12·53
7 30	13 8·4	12·69	13 6·8	12·96	13 3·8	13·35	13 1·3	13·80
8 30	13 12·3	14·27	13 13·4	14·66	13 7·5	14·77	13 6·1	15·24
9 30	13 4·9	15·43	13 4·6	15·82	12 59·4	16·04	12 55·3	16·27
10 30	12 46·4	16·25	12 46·4	16·52	12 44·3	16·81	12 43·0	17·11
11 30	12 29·8	16·57	11 29·3	16·75	12 29·5	17·21	12 29·2	17·49
	H. P. 58′.		H. P. 59′.		H. P. 60′.		H. P. 61′.	
0 30	12 12·9	17·41	12 13·0	18·34	12 13·7	18·74	12 13·1	19·16
1 30	11 57·5	17·08	11 58·6	17·64	11 59·8	18·12	12 0·6	18·49
2 30	11 45·8	15·95	11 47·4	16·54	11 49·3	16·93	11 49·9	17·40
3 30	11 38·7	14·77	11 40·7	15·16	11 44·0	15·67		
4 30	11 43·0	13·39	11 45·8	13·95	11 45·8	14·37		
5 30	12 4·4	12·69	12 6·7	13·24				
6 30	12 37·0	13·03	12 35·9	13·58				
7 30	12 58·4	14·20	12 55·9	14·65	12 54·6	15·02		
8 30	13 3·3	15·62	13 3·0	16·10	12 58·7	16·51		
9 30	12 57·1	16·65	12 53·8	17·24	12 53·4	17·81	12 53·6	17·96
10 30	12 43·6	17·42	12 43·7	18·06	12 42·2	18·57	12 40·6	18·89
11 30	12 28·3	17·69	12 29·0	18·43	12 27·2	18·94	12 27·4	19·45

Table VI. (*f.*)

Showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the Liverpool Docks, corresponding to the Apparent Solar Time of the Moon's Upper and Lower Transit A, P.M. and A.M.

January.									
Upper Transits A, P.M.					Lower (Interpolated) Transits A, A.M.				
Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
h m	′	h m	ft. in.	°	h m	′	h m	ft. in.	°
0 30·1	57.6	12 11·3	18 7·6	S. 18·4	0 28·7	57·8	12 12·6	17 2·7	S. 19·2
1 30·4	58·0	11 56·3	18 5·3	S. 14.7	1 28·4	57·0	11 57·9	16 7·8	S. 14·7
2 28·7	57·2	11 45·7	17 0·3	S. 10·0	2 29·9	57·8	11 45·1	15 10·9	S. 9·9
3 29·9	56·9	11 38·3	15 9·7	S. 4·2	3 29·4	57·0	11 39·8	14 4·7	S. 3·9
4 29·4	56·9	11 42·9	14 3·1	N. 2·1	4 29·6	56·6	11 41·9	12 8·2	N. 2·2
5 30·6	56·5	12 3·8	12 10·1	N. 7·9	5 29·8	56·8	12 3·2	12 1·0	N. 7·6
6 33·5	56·7	12 39·0	12 10·7	N. 13·8	6 27·3	56·5	12 35·7	12 2·9	N. 13·5
7 28·9	56·8	13 1·9	13 9·5	N. 17·8	7 27·0	56·7	13 2·0	13 7·4	N. 17·9
8 29·3	56·9	13 8·5	15 4·1	N. 20·3	8 28·7	56·7	13 6·1	15 3·7	N. 20·6
9 31·2	57·2	12 59·9	16 1·6	N. 23·1	9 24·2	57·6	12 59·8	16 5·1	N. 22·5
10 34·2	57·2	12 45·4	16 11·2	N. 22·5	10 25·6	57·3	12 46·5	17 4·7	N. 22·2
11 32·1	57·8	12 29·2	17 4·6	N. 21·5	11 30·7	57·5	12 30·0	17 7·1	N. 22·2
Upper Transits A, A.M.					Lower (Interpolated) Transits A, P.M.				
0 29·1	57·4	12 13·5	17 2·5	N. 18·5	0 32·1	57·4	12 11·2	18 2·1	N. 18·2
1 33·2	56·9	11 57·0	16 8·7	N. 14·9	1 29·4	57·6	11 57·3	17 8·5	N. 15·2
2 30·7	57·3	11 46·1	15 8·8	N. 10·1	2 29·5	57·2	11 45·5	16 6·4	N. 8·6
3 29·0	57·0	11 39·7	14 7·1	N. 3·7	3 30·5	57·1	11 37·3	15 4·5	N. 4·0
4 30·8	56·6	11 43·1	12 11·5	S. 2·5	4 29·8	56·9	11 42·8	13 10·9	N. 0·5
5 29·8	57·2	12 5·0	12 6·8	S. 7·8	5 29·6	56·8	12 3·0	12 7·9	S. 8·9
6 28·9	56·9	12 34·8	12 8·3	S. 13·4	6 31·0	56·7	12 35·6	12 5·4	S. 13·1
7 33·1	56·9	13 2·3	14 0·7	S. 18·2	7 23·3	57·2	13 0·4	13 8·5	S. 17·7
8 35·1	57·4	13 5·9	15 10·2	S. 20·9	8 24·8	57·2	13 8·5	14 10·5	S. 20·5
9 32·1	57·8	12 58·7	16 4·7	S. 22·4	9 31·9	57·3	12 58·9	15 10·1	S. 22·8
10 26·6	57·8	12 45·1	17 9·5	S. 22·7	10 34·1	57·9	12 42·8	17 1·5	S. 22·9
11 26·1	57·5	12 28·9	18 1·0	S. 22·1	11 34·2	58·2	12 26·9	17 10·6	S. 21·5
February.									
Upper Transits A, P.M.					Lower (Interpolated) Transits A, A.M.				
0 32·1	56·7	12 12·1	18 4·1	S. 9·9	0 30·9	57·9	12 12·1	17 11·2	S. 10·2
1 31·5	58·2	11 58·0	18 4·6	S. 3·8	1 30·8	57·1	11 57·9	16 9·8	S. 3·4
2 30·2	57·2	11 44·6	16 10·7	N. 2·3	2 28·9	57·3	11 46·3	15 7·8	N. 0·6
3 30·8	56·9	11 37·0	15 0·7	N. 8·6	3 27·7	57·3	11 36·2	14 2·8	N. 8·7
4 30·1	56·9	11 42·5	13 8·0	N. 13·4	4 28·8	56·8	11 41·5	12 8·8	N. 13·4
5 28·8	56·5	12 0·6	11 11·9	N. 18·1	5 29·8	56·6	12 3·9	11 9·0	N. 18·6
6 29·9	56·6	12 39·8	11 10·5	N. 21·3	6 29·9	56·7	12 40·0	12 1·2	N. 21·0
7 32·2	56·7	13 4·1	13 3·6	N. 22·6	7 26·7	56·7	13 3·3	13 9·8	N. 21·9
8 34·1	57·2	13 5·6	15 1·2	N. 22·8	8 25·5	56·7	13 8·2	14 11·8	N. 23·2
9 32·8	57·1	12 57·7	16 6·2	N. 21·2	9 28·1	57·0	12 58·8	16 9·6	N. 21·6
10 31·4	57·4	12 42·8	17 8·9	N. 18·9	10 28·8	57·6	12 40·9	17 7·7	N. 18·8
11 29·4	56·9	12 26·2	17 10·8	N. 15·1	11 27·8	57·2	12 27·9	18 7·2	N. 15·2
Upper Transits A, A.M.					Lower (Interpolated) Transits A, P.M.				
0 28·9	57·9	12 15·0	18 5·1	N. 9·8	0 33·3	57·1	12 12·6	18 5·9	N. 9·6
1 29·7	57·0	11 57·9	17 4·5	N. 3·3	1 31·0	57·8	11 56·8	18 0·9	N. 3·8
2 31·7	57·5	11 47·9	16 3·7	S. 2·3	2 28·7	56·9	11 49·6	16 8·3	S. 2·9
3 30·1	57·3	11 40·5	14 6·4	S. 7·5	3 27·5	57·1	11 39·6	15 0·4	S. 7·3
4 29·9	56·8	11 43·3	12 6·5	S. 13·4	4 28·7	57·3	11 41·5	13 0·9	S. 13·9
5 32·2	57·1	12 7·1	11 8·7	S. 18·9	5 27·4	57·4	12 3·4	11 7·8	S. 18·1
6 28·3	56·9	12 41·7	12 0·2	S. 20·8	6 27·9	56·7	12 40·9	11 8·9	S. 20·7
7 28·2	57·4	13 7·6	13 10·7	S. 22·3	7 30·3	57·3	13 6·1	13 4·4	S. 22·4
8 30·0	57·2	13 10·7	15 4·4	S. 23·2	8 26·9	57·2	13 10·2	14 8·6	S. 22·6
9 32·6	56·9	13 0·0	16 7·7	S. 21·8	9 25·6	57·6	13 1·2	16 1·3	S. 22·3
10 33·2	57·7	12 45·3	18 2·9	S. 18·2	10 28·6	57·2	12 46·2	17 0·9	S. 19·6
11 30·7	57·9	12 28·5	18 9·4	S. 15·3	11 32·8	57·1	12 29·9	17 9·7	S. 14·1

TABLE VI. (*f*.) (Continued.)

Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
March.									
Upper Transits A, p.m.					Lower (Interpolated) Transits A, a.m.				
h m	′	h m	ft. in.	°	h m	′	h m	ft. in.	°
0 30·7	57·8	12 12·3	18 10·4	N. 1·2	0 25·5	57·3	12 14·1	17 7·5	N. 1·2
1 31·0	57·2	11 58·3	17 5·1	N. 8·7	1 26·1	58·0	11 59·5	17 1·9	N. 6·7
2 29·9	57·6	11 46·0	16 4·6	N. 12·6	2 28·2	57·0	11 44·4	15 6·1	N. 12·9
3 29·1	56·6	11 35·4	14 2·3	N. 17·4	3 29·9	56·9	11 36·4	13 9·3	N. 18·0
4 30·1	56·8	11 35·7	12 2·8	N. 20·9	4 28·6	56·7	11 41·0	12 1·9	N. 20·5
5 28·6	56·7	11 56·6	10 10·7	N. 22·8	5 29·3	56·6	12 1·6	11 1·7	N. 21·9
6 31·8	56·5	12 42·5	11 6·8	N. 22·6	6 31·0	56·4	12 42·8	11 8·2	N. 22·6
7 29·4	56·6	13 5·9	13 1·4	N. 21·4	7 32·2	56·6	13 6·9	13 4·6	N. 21·9
8 28·5	56·6	13 9·2	14 10·1	N. 20·0	8 28·9	56·9	13 8·4	15 3·0	N. 19·4
9 31·1	56·9	12 57·8	16 6·2	N. 15·1	9 30·8	57·4	12 57·9	16 10·8	N. 15·4
10 29·5	57·6	12 43·5	17 10·8	N. 10·3	10 35·1	57·0	12 43·2	17 11·1	N. 10·2
11 29·4	57·2	12 29·9	18 1·0	N. 4·8	11 31·3	57·8	12 27·5	18 9·8	N. 4·8
Upper Transits A, a.m.					Lower (Interpolated) Transits A, p.m.				
0 31·7	58·1	12 12·2	18 3·9	S. 1·4	0 27·6	57·7	12 13·7	18 7·1	S. 0·6
1 28·9	57·8	11 58·7	17 3·9	S. 7·1	1 30·1	57·5	11 59·2	17 8·6	S. 8·3
2 31·9	57·3	11 45·2	15 11·4	S. 13·1	2 29·2	57·6	11 46·9	16 3·7	S. 12·7
3 35·0	57·3	11 34·6	14 1·1	S. 18·2	3 26·7	57·1	11 35·9	14 3·3	S. 17·4
4 30·1	56·9	11 37·2	12 5·8	S. 20·8	4 28·6	56·9	11 36·7	12 3·5	S. 20·8
5 30·5	56·7	12 2·1	11 6·8	S. 22·3	5 30·4	56·9	12 0·8	11 3·2	S. 22·2
6 32·4	56·8	12 44·4	12 1·9	S. 23·2	6 29·7	56·7	12 39·2	11 6·5	S. 22·9
7 31·2	57·1	13 5·2	13 8·7	S. 21·6	7 27·6	56·9	13 6·9	12 11·5	S. 21·9
8 32·5	56·8	13 7·9	15 4·8	S. 19·1	8 29·1	56·8	13 8·5	14 9·0	S. 20·1
9 27·6	57·6	12 58·9	17 2·2	S. 15·6	9 30·6	57·1	12 59·2	16 2·6	S. 14·9
10 29·0	57·3	12 42·7	18 2·0	S. 11·2	10 29·7	57·9	12 43·8	17 5·1	S. 10·3
11 30·9	58·0	12 27·4	19 1·1	S. 5·7	11 30·5	57·8	12 28·2	18 0·9	S. 5·7
April.									
Upper Transits A, p.m.					Lower (Interpolated) Transits A, a.m.				
0 29·5	57·6	12 14·3	17 9·7	N. 13·6	0 27·7	57·2	12 12·7	17 1·4	N. 11·9
1 30·1	57·0	11 58·3	16 7·2	N. 16·6	1 31·6	57·7	11 56·3	16 7·9	N. 17·0
2 32·1	57·5	11 43.2	15 4·3	N. 20·7	2 30·3	57·2	11 42·7	15 2·8	N. 20·2
3 33·1	57·1	11 33·6	13 7·5	N. 22·6	3 27·6	57·0	11 33·4	13 9·4	N. 22·0
4 34·0	56·8	11 38·8	11 10·0	N. 22·7	4 28·6	56·6	11 36·7	12 3·2	N. 22·9
5 34·7	56·6	12 5·3	11 1·4	N. 22·0	5 28·1	57·0	12 0·9	11 6·3	N. 22·4
6 30·9	56·7	12 40·3	11 9·5	N. 20·1	6 30·3	56·7	12 39·5	12 0·9	N. 19·0
7 32·6	56·8	13 2·8	13 9·4	N. 15·3	7 31·9	57·0	13 2·2	13 9·1	N. 16·1
8 31·9	57·0	13 5·3	15 6·7	N. 10·7	8 30·7	57·0	13 3·6	15 7·1	N. 11·7
9 26·6	57·3	12 58·4	16 11·6	N. 5·7	9 29·8	57·3	12 56·0	17 0·2	N. 5·1
10 27·5	57·1	12 42·9	17 8·6	S. 0·8	10 32·6	57·4	12 42·7	17 10·0	S. 0·5
11 26·6	57·9	12 29·6	18 3·6	S. 6·8	11 32·6	57·3	12 27·1	18 1·1	S. 7·4
Upper Transits A, a.m.					Lower (Interpolated) Transits A, p.m.				
0 26·9	57·3	12 13·7	17 7·3	S. 11·5	0 33·0	58·0	12 11·3	17 10·3	S. 13·0
1 29·2	58·0	11 58·3	17 1·4	S. 17·4	1 28·8	58·0	11 58·0	16 10·5	S. 16·4
2 28·4	57·9	11 43·5	15 10·9	S. 20·6	2 31·4	57·6	11 43·2	15 1·7	S. 20·8
3 30·7	57·3	11 34·3	14 2·8	S. 21·9	3 31·1	57·3	11 35·3	13 8·5	S. 22·4
4 31·6	57·3	11 37·5	12 10·2	S. 23·4	4 28·8	57·3	11 37·7	12 2·2	S. 22·9
5 31·8	57·2	12 1·6	12 0·6	S. 22·1	5 27·8	57·2	12 1·0	11 2·6	S. 22·3
6 32·5	56·7	12 41·3	12 3·1	S. 19·6	6 27·3	56·8	12 37·5	11 6·0	S. 20·5
7 31·4	57·2	12 59·8	13 11·9	S. 15·7	7 25·4	57·3	13 1·3	13 2·3	S. 16·1
8 27·2	57·2	13 3·9	15 7·6	S. 11·9	8 28·4	57·2	13 5·0	15 0·7	S. 10·8
9 27·1	56·7	12 57·0	16 10·7	S. 6·0	9 30·4	57·4	12 56·4	16 5·1	S. 5·0
10 26·4	57·5	12 44·1	18 0·1	N. 0·6	10 29·4	57·0	12 44·4	17 3·5	N. 0·5
11 28·3	56·9	12 30·2	18 0·1	N. 6·7	11 29·7	57·6	12 29·5	17 9·5	N. 7·3

TABLE VI. (*f.*) (Continued.)

May.

Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
Upper Transits A, P.M.					Lower (Interpolated) Transits A, A.M.				
h m	′	h m	ft. in.	°	h m	′	h m	ft. in.	°
0 27·2	57·1	12 13·4	16 5·9	N. 20·9	0 29·9	57·4	12 12·9	17 0·3	N. 20·2
1 30·4	57·1	11 57·6	15 9·9	N. 22·3	1 31·1	57·1	11 56·0	16 3·2	N. 21·8
2 31·4	57·5	11 43·2	14 10·7	N. 22·8	2 32·3	57·2	11 41·5	15 4·7	N. 23·3
3 29·4	57·0	11 35·7	13 7·9	N. 21·7	3 29·1	57·1	11 35·3	14 2·4	N. 21·9
4 26·3	57·0	11 39·2	12 8·0	N. 20·6	4 30·7	56·9	11 38·1	13 3·3	N. 19·6
5 28·9	56·9	12 2·8	12 2·2	N. 15·9	5 28·9	56·9	12 1·9	12 7·6	N. 16·2
6 29·2	56·9	12 36·3	12 11·7	N. 11·5	6 27·7	57·0	12 33·6	13 1·3	N. 12·5
7 26·9	57·2	12 57·5	14 2·2	N. 5·4	7 29·9	57·1	12 57·6	14 3·1	N. 5·9
8 28·9	56·8	13 2·5	15 6·2	S. 0·8	8 29·6	57·4	13 0·8	15 5·8	N. 0·4
9 29·9	57·8	12 56·0	16 8·6	S. 6·9	9 29·5	57·2	12 55·0	16 4·6	S. 6·8
10 31·1	57·1	12 43·9	17 4·0	S. 11·9	10 31·6	57·7	12 42·9	17 1·1	S. 12·2
11 32·5	57·2	12 28·2	17 8·7	S. 17·5	11 26·8	57·1	12 30·9	17 4·8	S. 16·4
Upper Transits A, A.M.					Lower (Interpolated) Transits A, P.M.				
0 29·3	57·6	12 12·2	17 3·9	S. 19·9	0 27·9	57·2	12 12·7	16 8·4	S. 20·3
1 27·3	57·5	11 56·6	16 8·1	S. 21·9	1 30·3	57·8	11 58·5	15 11·7	S. 22·5
2 28·8	57·7	11 43·1	15 9·6	S. 22·8	2 30·4	57·7	11 44·4	14 11·8	S. 22·6
3 29·7	57·5	11 33·7	14 7·7	S. 22·6	3 28·8	57·7	11 36·2	13 8·5	S. 22·2
4 31·0	57·3	11 38·9	13 7·5	S. 19·6	4 29·2	57·2	11 39·2	12 3·5	S. 20·3
5 30·4	57·4	12 3·4	12 10·7	S. 16·3	5 30·6	57·1	12 4·9	11 9·4	S. 15·4
6 26·9	56·9	12 34·4	13 2·6	S. 12·2	6 31·5	57·4	12 38·1	12 4·4	S. 11·4
7 28·1	57·0	12 55·9	14 3·0	S. 5·9	7 33·6	57·4	12 57·4	13 9·7	S. 6·3
8 29·6	57·1	13 0·7	15 5·8	N. 0·5	8 31·9	56·9	13 1·7	15 0·0	N. 0·9
9 32·5	56·9	12 54·8	16 4·3	N. 6·9	9 29·9	57·2	12 56·5	16 6·2	N. 6·4
10 34·1	57·1	12 40·3	16 7·0	N. 12·0	10 28·7	56·9	12 44·4	16 10·0	N. 11·8
11 30·1	57·0	12 29·7	17 0·5	N. 16·6	11 29·9	56·9	12 28·6	16 11·9	N. 17·0

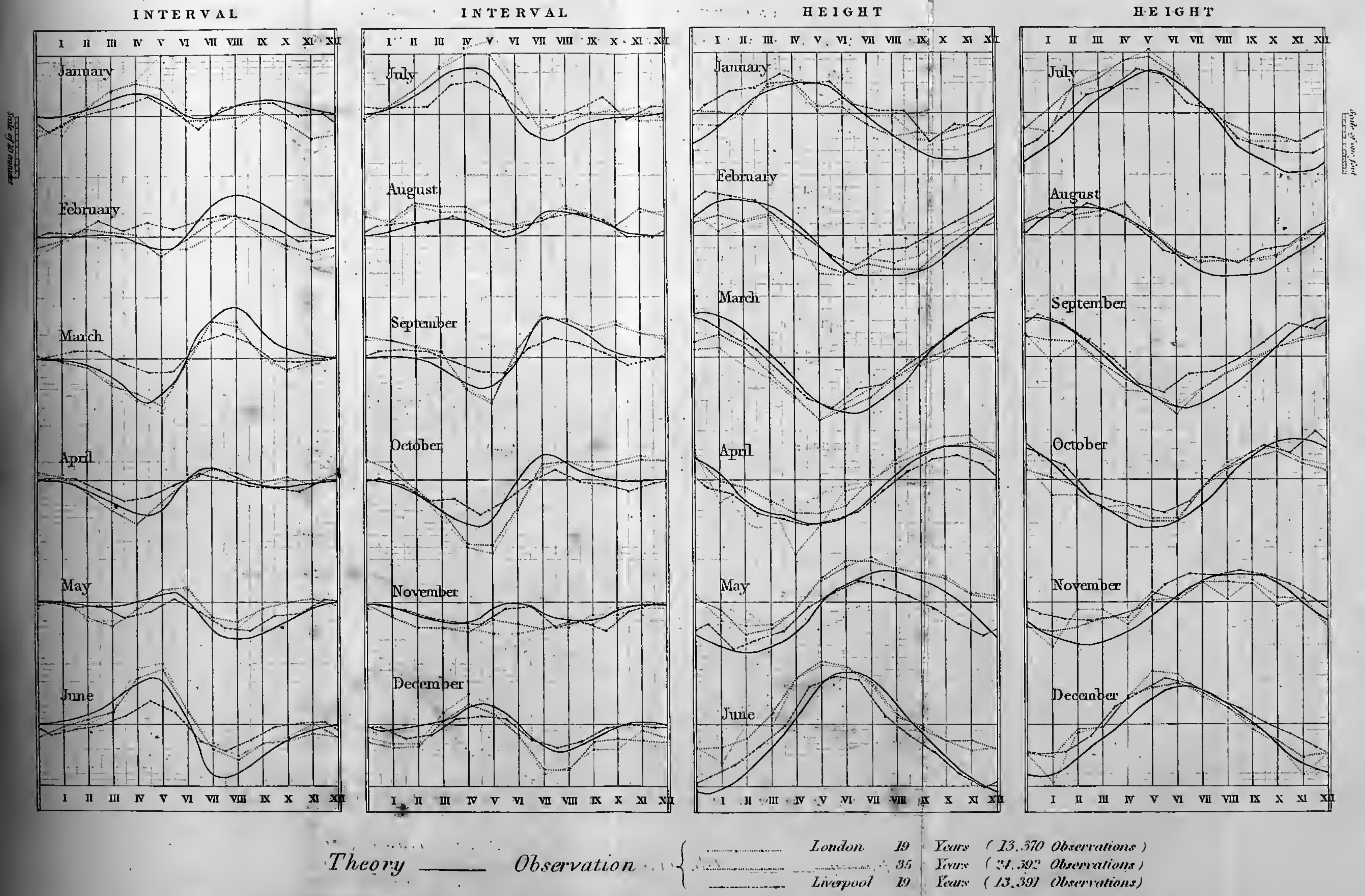
June.

Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
Upper Transits A, P.M.					Lower (Interpolated) Transits A, A.M.				
0 30·8	57·4	12 12·1	16 3·7	N. 23·0	0 28·9	57·1	12 11·5	16 11·5	N. 23·1
1 26·1	57·2	11 57·7	15 10·9	N. 22·1	1 32·6	57·1	11 55·1	16 5·7	N. 21·6
2 29·4	56·8	11 43·4	14 10·9	N. 19·9	2 32·3	57·2	11 43·2	15 9·2	N. 18·8
3 31·7	56·8	11 37·6	13 10·9	N. 15·2	3 27·3	56·9	11 37·5	14 9·5	N. 16·4
4 30·9	57·1	11 45·0	13 2·3	N. 10·9	4 30·7	56·9	11 43·8	13 9·2	N. 11·4
5 23·4	56·7	12 5·0	12 6·9	N. 4·9	5 32·2	57·3	12 3·0	13 3·1	N. 4·9
6 29·5	57·0	12 37·5	13 1·8	S. 0·9	6 28·0	56·9	12 32·6	13 2·5	S. 0·8
7 31·8	57·2	12 57·8	14 1·7	S. 7·5	7 29·4	57·3	12 56·9	13 11·8	S. 7·2
8 33·3	57·4	13 2·2	15 6·9	S. 12·4	8 26·4	57·6	13 1·9	15 0·6	S. 11·9
9 34·9	57·7	12 56·9	16 7·8	S. 18·0	9 26·0	57·6	12 56·8	15 10·7	S. 16·4
10 28·6	57·9	12 45·3	17 3·0	S. 20·3	10 29·4	57·6	12 44·1	16 4·8	S. 20·5
11 26·6	57·4	12 30·5	17 4·3	S. 22·3	11 31·9	57·6	12 28·6	16 5·7	S. 22·4
Upper Transits A, A.M.					Lower (Interpolated) Transits A, P.M.				
0 25·3	57·3	12 10·9	17 3·0	S. 23·2	0 30·3	57·9	12 9·7	16 5·1	S. 22·9
1 27·3	57·8	11 57·0	16 10·7	S. 22·2	1 29·3	57·6	11 57·6	15 9·2	S. 22·5
2 27·4	57·6	11 44·1	16 4·6	S. 19·5	2 28·5	57·4	11 44·8	15 0·5	S. 19·7
3 25·6	57·3	11 37·8	15 4·4	S. 16·7	3 31·9	57·3	11 38·8	13 11·3	S. 15·7
4 26·3	57·2	11 44·3	14 5·6	S. 10·8	4 32·7	57·4	11 45·3	13 0·4	S. 10·9
5 27·5	57·1	12 4·2	13 7·9	S. 5·8	5 29·9	56·9	12 6·0	12 4·1	S. 5·2
6 31·4	57·2	12 35·9	13 10·3	N. 1·1	6 31·7	56·9	12 35·6	12 8·8	N. 1·0
7 33·9	56·9	12 57·5	14 2·4	N. 7·0	7 28·5	57·3	12 56·4	13 9·7	N. 6·9
8 30·2	56·9	13 4·1	15 2·0	N. 12·3	8 29·5	57·0	13 2·3	15 0·0	N. 12·4
9 28·9	57·0	12 58·4	15 11·8	N. 16·4	9 32·7	57·4	12 55·9	16 4·4	N. 17·2
10 31·0	57·0	12 45·9	16 3·8	N. 20·6	10 30·1	57·9	12 44·9	16 9·9	N. 20·4
11 32·7	57·1	12 29·2	16 5·7	N. 22·1	11 26·8	56·9	12 29·4	16 9·9	N. 21·9

TABLE VI. (*f.*) (Continued.)

Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
July.									
Upper Transits A, P.M.					Lower (Interpolated) Transits A, A.M.				
h m	′	h m	ft. in.	°	h m	′	h m	ft. in.	°
0 34·5	56·9	12 13·2	16 10·2	N. 18·9	0 28·6	57·4	12 12·8	17 9·9	N. 19·7
1 34·5	57·2	11 58·2	16 5·5	N. 15·6	1 29·5	56·9	11 56·7	17 3·5	N. 16·5
2 31·3	57·0	11 44·0	15 6·7	N. 10·8	2 29·6	56·8	11 42·8	16 5·8	N. 10·9
3 30·4	56·6	11 41·4	14 4·8	N. 4·9	3 29·0	57·0	11 40·8	15 1·9	N. 5·0
4 30·8	56·9	11 45·3	13 4·5	S. 0·9	4 28·4	56·7	11 44·3	13 11·9	S. 0·7
5 28·8	56·7	12 5·5	12 6·1	S. 7·6	5 30·5	57·0	12 5·3	13 0·6	S. 7·6
6 28·8	56·9	12 37·8	12 9·0	S. 13·2	6 30·9	57·2	12 35·1	12 10·1	S. 12·1
7 25·7	57·3	12 59·5	13 11·8	S. 17·4	7 31·9	57·2	13 0·7	13 7·4	S. 17·2
8 25·1	57·2	13 4·9	15 3·3	S. 20·0	8 34·1	57·2	13 6·8	14 7·7	S. 21·3
9 26·7	57·9	13 0·6	16 5·2	S. 22·6	9 31·0	57·7	12 59·3	15 9·6	S. 22·1
10 30·9	57·8	12 44·5	17 5·2	S. 22·9	10 26·2	57·9	12 46·7	16 3·5	S. 23·2
11 31·8	57·9	12 28·0	17 10·1	S. 21·5	11 26·9	58·1	12 30·8	16 11·3	S. 22·3
Upper Transits A, A.M.					Lower (Interpolated) Transits A, P.M.				
0 25·7	57·8	12 14·5	17 11·8	S. 19·7	0 31·5	57·4	12 12·9	16 5·4	S. 19·3
1 25·4	57·2	11 59·2	17 4·2	S. 16·8	1 31·4	57·8	12 0·4	16 2·2	S. 15·6
2 29·2	57·4	11 48·4	16 8·4	S. 11·0	2 29·3	57·2	11 45·7	15 5·6	S. 11·4
3 31·2	57·3	11 39·5	15 6·4	S. 4·9	3 32·4	56·9	11 44·7	14 2·3	S. 4·6
4 29·0	56·6	11 44·7	14 0·8	N. 1·7	4 31·2	57·2	11 47·5	13 1·0	N. 1·8
5 29·4	56·8	12 5·6	13 2·0	N. 6·2	5 27·4	56·7	12 4·4	12 1·6	N. 6·8
6 26·9	56·8	12 35·9	12 9·4	N. 12·7	6 29·6	56·7	12 38·8	12 3·7	N. 12·9
7 27·6	56·9	13 0·1	13 7·4	N. 17·0	7 33·1	57·1	13 3·1	13 9·3	N. 17·1
8 29·5	57·1	13 6·7	14 8·2	N. 20·7	8 34·1	56·8	13 6·6	15 0·6	N. 21·0
9 28·3	57·3	13 2·6	15 9·7	N. 21·9	9 32·6	57·3	12 57·9	16 3·9	N. 22·2
10 28·9	57·6	12 45·9	16 5·7	N. 22·6	10 30·2	57·2	12 40·3	17 1·1	N. 22·9
11 31·6	57·2	12 28·6	16 8·9	N. 22·1	11 29·5	57·2	12 29·6	17 8·4	N. 21·6
August.									
Upper Transits A, P.M.					Lower (Interpolated) Transits A, A.M.				
0 31·7	57·5	12 14·1	17 6·7	N. 10·8	0 29·4	57·3	12 13·8	18 2·2	N. 10·6
1 31·4	57·3	11 59·7	16 10·6	N. 4·4	1 24·9	57·7	12 1·0	18 0·0	N. 5·6
2 28·2	57·3	11 46·1	16 2·6	S. 1·1	2 26·6	57·1	11 47·8	16 7·6	S. 0·7
3 26·2	56·7	11 41·4	14 7·7	S. 6·3	3 31·1	57·1	11 39·3	14 11·6	S. 6·4
4 27·4	56·9	11 42·5	13 1·8	S. 13·4	4 31·5	56·8	11 41·9	13 2·9	S. 12·1
5 24·5	56·7	12 1·0	12 0·1	S. 17·0	5 31·2	56·9	12 2·0	11 9·5	S. 18·2
6 24·9	56·7	12 37·3	12 2·7	S. 20·5	6 31·6	56·9	12 41·8	11 11·3	S. 20·7
7 26·3	57·2	13 4·3	13 9·4	S. 22·4	7 31·3	57·3	13 4·2	13 2·1	S. 22·5
8 28·8	56·9	13 9·3	15 2·8	S. 22·9	8 30·6	56·9	13 10·7	14 3·2	S. 22·8
9 29·0	57·6	13 1·6	16 11·6	S. 21·8	9 32·8	57·6	12 58·9	15 10·0	S. 22·1
10 26·6	57·8	12 46·8	18 0·9	S. 19·8	10 34·5	57·4	12 44·7	16 8·3	S. 18·9
11 26·2	57·8	12 28·0	18 8·6	S. 15·7	11 35·3	58·2	12 28·2	17 5·5	S. 14·9
Upper Transits A, A.M.					Lower (Interpolated) Transits A, P.M.				
0 30·7	57·8	12 13·6	18 9·3	S. 10·5	0 25·8	58·1	12 15·7	17 5·6	S. 10·9
1 29·3	58·0	11 59·4	18 4·0	S. 5·3	1 27·2	57·3	12 0·1	16 7·9	S. 5·2
2 28·6	56·9	11 46·7	16 9·0	N. 0·9	2 28·9	58·1	11 47·5	15 10·9	N. 1·3
3 28·9	57·1	11 40·2	15 3·7	N. 6·7	3 27·6	56·8	11 39·4	14 4·0	N. 6·4
4 26·9	56·9	11 41·2	13 4·1	N. 12·5	4 28·9	56·9	11 39·6	12 8·1	N. 13·4
5 28·4	56·5	12 2·8	11 9·3	N. 18·1	5 30·8	56·8	12 3·6	11 7·2	N. 16·7
6 28·6	56·5	12 41·2	11 11·7	N. 20·4	6 31·4	56·8	12 38·4	11 11·6	N. 20·8
7 24·9	56·6	13 4·9	12 11·9	N. 21·8	7 33·6	56·6	13 6·2	13 6·9	N. 22·4
8 26·3	56·8	13 9·2	14 4·5	N. 22·9	8 32·3	56·9	13 9·3	15 2·1	N. 23·2
9 27·4	56·9	13 1·0	15 9·2	N. 21·9	9 32·2	56·7	12 59·5	16 6·2	N. 20·9
10 29·6	56·9	12 42·9	16 8·6	N. 19·5	10 31·0	57·4	12 45·0	17 8·5	N. 19·3
11 30·7	57·8	12 29·1	17 7·4	N. 15·3	11 29·7	57·3	12 28·9	18 3·4	N. 15·9

Diagram showing a comparison between the Calendar Month Inequality in the Interval and in the Height of high water as deduced from Bernoullis' Theory and from Observations at the London and Liverpool Docks.—See Tables IX and XXIII. p. 118 and 192.



In this comparison of the London and Liverpool results: the London corrections have been shifted to the left half an hour agreeably to the remark p.100 and the London height corrections have been multiplied by 1·7. The abscissa represents the apparent solar time of moon's transit A.

TABLE VI. (*f.*) (Continued.)

Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
September.									
Upper Transits A, P.M.					Lower (Interpolated) Transits A, A.M.				
h m	′	h m	ft. in.	°	h m	′	h m	ft. in.	°
0 30·0	57·7	12 14·3	18 2·1	S. 0·9	0 31·9	57·3	12 12·2	18 6·6	S. 0·9
1 29·8	57·5	11 58·7	17 4·2	S. 6·9	1 34·7	57·8	11 53·6	17 9·4	S. 7·1
2 30·5	57·7	11 46·4	16 0·2	S. 12·7	2 31·4	57·4	11 45·2	16 3·6	S. 11·9
3 26·8	56·9	11 36·9	14 6·8	S. 16·8	3 30·6	57·3	11 36·3	14 5·2	S. 16·9
4 25·9	57·0	11 38·1	12 10·7	S. 19·5	4 32·4	56·9	11 37·2	12 3·8	S. 20·7
5 23·9	57·1	11 56·9	11 11·1	S. 22·3	5 34·4	56·9	12 0·8	11 4·0	S. 22·1
6 26·7	56·8	12 34·1	12 5·0	S. 23·1	6 32·7	57·0	12 42·1	12 3·5	S. 23·1
7 28·4	57·3	13 3·4	13 11·9	S. 22·1	7 32·6	57·3	13 5·2	13 4·5	S. 22·2
8 25·6	56·9	13 8·3	15 7·9	S. 20·1	8 33·9	57·0	13 8·2	14 11·8	S. 19·9
9 26·3	57·8	12 58·5	17 6·1	S. 15·9	9 33·7	57·4	12 57·7	16 8·1	S. 15·4
10 29·2	57·2	12 44·9	18 5·8	S. 11·2	10 29·5	57·8	12 41·5	17 5·1	S. 11·1
11 29·8	57·8	12 25·4	19 2·9	S. 5·2	11 29·6	57·4	12 29·5	17 10·1	S. 6·4
Upper Transits A, A.M.					Lower (Interpolated) Transits A, P.M.				
0 28·0	57·5	12 13·9	18 11·9	S. 0·3	0 31·9	57·8	12 14·1	17 9·9	N. 1·6
1 28·0	57·3	11 59·7	18 1·0	N. 6·4	1 31·4	57·3	12 2·5	16 11·9	N. 6·6
2 27·5	57·7	11 45·0	16 8·8	N. 12·0	2 32·3	57·7	11 45·3	15 9·8	N. 12·7
3 29·5	56·8	11 35·9	14 5·9	N. 16·8	3 32·5	57·1	11 37·1	14 1·9	N. 16·6
4 28·5	56·8	11 38·8	12 8·0	N. 19·1	4 31·8	56·8	11 37·7	12 4·1	N. 20·2
5 25·7	56·4	11 59·2	11 4·1	N. 21·7	5 29·3	56·9	12 0·3	10 5·4	N. 22·4
6 27·9	56·7	12 42·3	11 8·8	N. 23·7	6 29·9	56·5	12 40·3	12 0·4	N. 23·1
7 27·6	56·8	13 5·4	13 3·4	N. 22·0	7 32·5	56·6	13 5·6	13 8·2	N. 22·0
8 26·8	56·8	13 8·4	14 8·1	N. 20·4	8 32·5	56·8	13 7·8	15 6·1	N. 19·6
9 29·1	56·8	12 59·3	16 1·8	N. 15·9	9 30·3	56·9	12 59·7	16 9·9	N. 16·6
10 28·3	57·6	12 45·6	17 7·0	N. 11·2	10 31·3	56·9	12 43·8	18 0·7	N. 11·0
11 30·0	56·9	12 30·4	17 8·2	N. 5·9	11 31·9	57·9	12 28·8	18 9·9	N. 5·1
October.									
Upper Transits A, P.M.					Lower (Interpolated) Transits A, A.M.				
0 26·7	56·9	12 13·9	17 4·9	S. 11·2	0 30·8	57·6	12 12·2	18 5·2	S. 11·6
1 26·5	57·7	11 58·5	17 4·2	S. 16·4	1 28·2	57·5	11 56·4	17 4·5	S. 15·8
2 24·1	57·3	11 44·1	16 1·1	S. 19·8	2 30·9	57·4	11 43·1	15 6·3	S. 20·3
3 22·6	57·6	11 36·3	14 9·5	S. 21·9	3 31·3	57·7	11 35·5	14 1·5	S. 22·3
4 24·8	57·0	11 34·8	13 0·9	S. 23·2	4 33·1	57·5	11 34·4	12 2·9	S. 22·6
5 27·8	57·3	11 59·0	12 3·2	S. 22·1	5 32·2	57·2	12 1·4	11 3·0	S. 22·1
6 27·8	57·0	12 35·2	12 7·7	S. 20·1	6 30·7	56·9	12 38·5	11 9·8	S. 20·6
7 27·3	57·3	12 59·8	14 3·6	S. 16·9	7 33·5	57·1	13 3·9	13 5·4	S. 16·9
8 30·3	56·9	13 6·1	15 11·8	S. 12·8	8 33·4	57·1	13 4·9	15 0·6	S. 12·4
9 29·3	57·2	12 58·8	17 5·7	S. 5·6	9 29·9	57·3	12 56·8	16 5·7	S. 6·5
10 25·7	57·6	12 43·7	18 7·6	S. 1·0	10 28·8	56·9	12 44·5	17 6·7	S. 0·4
11 26·9	56·8	12 29·1	19 0·2	N. 5·6	11 30·2	57·8	12 27·6	18 2·9	N. 5·9
Upper Transits A, A.M.					Lower (Interpolated) Transits A, P.M.				
0 28·4	57·7	12 12·1	18 8·4	N. 10·9	0 29·4	56·9	12 13·2	17 8·1	N. 11·1
1 27·0	57·3	11 57·4	17 4·6	N. 16·4	1 31·1	57·3	11 56·2	16 10·9	N. 16·6
2 28·9	57·1	11 40·9	15 6·2	N. 20·5	2 30·9	57·4	11 36·6	15 6·6	N. 20·2
3 27·4	57·1	11 34·5	13 10·8	N. 21·5	3 31·5	57·4	11 33·7	14 3·3	N. 22·2
4 26·9	57·2	11 34·3	12 5·9	N. 23·1	4 34·1	56·9	11 35·9	12 7·5	N. 23·2
5 28·9	56·9	11 56·1	11 1·6	N. 22·1	5 37·5	56·8	12 5·6	11 11·4	N. 21·5
6 25·5	56·6	12 35·9	11 10·5	N. 20·4	6 34·8	56·8	12 41·2	12 3·9	N. 19·9
7 27·4	56·7	13 3·4	13 6·3	N. 17·1	7 30·3	56·9	13 2·8	14 5·4	N. 17·2
8 29·0	56·9	13 5·3	15 3·2	N. 12·2	8 30·5	56·7	13 6·1	15 9·7	N. 12·2
9 28·9	56·9	12 57·2	16 10·6	N. 6·9	9 31·7	57·2	12 56·5	17 4·6	N. 6·0
10 29·7	57·2	12 43·3	17 4·6	S. 0·5	10 32·7	57·3	12 42·3	17 6·2	S. 0·7
11 26·4	57·7	12 29·7	18 5·2	S. 4·9	11 30·9	57·0	12 28·8	18 5·7	S. 6·5

Table VI. (*f.*) (Continued.)

Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Apparent Solar Time of Moon's Transit A.	Moon's Parallax.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.
November.									
Upper Transits A, P.M.					Lower (Interpolated) Transits A, A.M.				
h m	′	h m	ft. in.	°	h m	′	h m	ft. in.	°
0 31·5	57·4	12 11·3	17 9·8	S. 19·9	0 29·5	57·7	12 12·4	17 5·1	S. 19·8
1 28·9	57·1	11 56·1	16 11·5	S. 21·7	1 32·0	57·6	11 55·8	16 5·2	S. 22·2
2 28·4	57·2	11 41·8	16 0·9	S. 23·1	2 29·6	57·5	11 40·9	15 3·3	S. 22·9
3 29·8	57·3	11 33·8	14 9·8	S. 22·0	3 28·3	57·3	11 33·7	13 9·9	S. 22·6
4 28·4	57·3	11 38·7	14 0·7	S. 20·1	4 30·9	57·2	11 38·4	12 5·2	S. 20·3
5 27·3	57·3	11 59·5	13 1·9	S. 16·5	5 36·9	57·1	12 6·3	11 11·6	S. 16·2
6 29·4	56·9	12 36·2	13 4·6	S. 12·0	6 32·0	57·2	12 37·8	12 2·7	S. 12·5
7 29·4	57·2	12 56·5	14 7·0	S. 6·6	7 29·5	57·1	12 57·0	13 9·4	S. 6·6
8 27·3	57·3	13 1·0	16 0·1	S. 0·9	8 30·5	57·0	13 2·7	15 3·9	S. 0·8
9 29·8	57·1	12 50·8	17 1·3	N. 5·5	9 27·8	57·4	12 55·3	16 4·1	N. 4·5
10 30·7	57·5	12 43·1	17 9·5	N. 11·5	10 27·2	57·3	12 44·1	17 2·3	N. 11·3
11 30·4	57·3	12 28·5	17 8·6	N. 17·0	11 30·6	57·3	12 28·8	17 7·6	N. 16·3
Upper Transits A, A.M.					Lower (Interpolated) Transits A, P.M.				
0 35·6	57·1	12 11·2	17 2·5	N. 19·8	0 27·8	57·3	12 12·7	17 3·1	N. 19·3
1 33·2	57·4	11 55·7	16 4·9	N. 22·0	1 28·5	57·0	11 56·5	16 6·1	N. 22·7
2 29·4	57·0	11 42·4	15 0·3	N. 22·9	2 32·2	57·1	11 41·7	15 8·1	N. 22·9
3 25·5	56·9	11 34·0	13 9·7	N. 22·2	3 34·2	56·8	11 34·7	14 5·3	N. 21·9
4 30·1	56·6	11 36·7	12 5·0	N. 20·5	4 36·4	56·9	11 34·3	12 11·0	N. 19·6
5 31·5	57·0	12 4·2	12 0·0	N. 16·1	5 28·2	56·8	11 59·3	12 8·7	N. 16·7
6 28·4	57·1	12 38·2	12 10·7	N. 12·4	6 31·2	56·7	12 38·0	13 1·8	N. 12·5
7 29·2	56·7	12 58·8	14 1·2	N. 7·2	7 32·3	57·3	12 58·7	14 7·7	N. 5·8
8 28·5	57·0	13 2·5	15 10·1	N. 0·8	8 28·3	56·9	13 3·5	16 0·8	S. 0·2
9 28·6	57·5	12 55·6	17 1·9	S. 4·8	9 27·2	57·1	12 56·6	17 2·0	S. 4·7
10 30·0	57·0	12 44·3	17 9·4	S. 12·2	10 25·2	57·8	12 44·1	18 0·2	S. 11·2
11 32·9	57·9	12 28·4	18 2·5	S. 16·6	11 26·5	57·2	12 29·5	17 10·3	S. 16·2
December.									
Upper Transits A, P.M.					Lower (Interpolated) Transits A, A.M.				
0 32·8	57·6	12 10·1	17 8·7	S. 23·2	0 25·9	57·3	12 12·8	16 11·3	S. 23·2
1 29·2	57·6	11 55·8	17 3·6	S. 22·2	1 31·4	57·3	11 54·6	16 2·4	S. 21·9
2 28·4	57·2	11 43·0	16 5·6	S. 19·7	2 33·4	57·4	11 42·6	15 3·1	S. 19·1
3 28·5	56·9	11 35·2	15 9·0	S. 15·1	3 27·4	57·7	11 42·9	14 3·7	S. 16·2
4 31·8	57·3	11 41·0	14 5·6	S. 11·4	4 27·0	56·8	11 41·0	12 11·9	S. 11·6
5 31·8	56·9	12 3·8	13 8·1	S. 4·8	5 29·9	57·5	12 5·2	12 10·3	S. 5·5
6 31·6	56·8	12 36·6	13 9·3	N. 1·1	6 28·3	57·1	12 34·4	12 8·3	N. 0·3
7 30·8	57·3	12 58·1	14 7·5	N. 6·7	7 27·7	57·0	12 57·1	14 1·0	N. 5·8
8 29·0	57·0	13 3·5	15 9·8	N. 12·4	8 30·2	57·3	13 1·5	15 5·9	N. 12·2
9 29·2	57·4	12 57·2	16 9·3	N. 16·7	9 29·5	57·6	12 55·9	16 5·7	N. 15·5
10 26·8	57·5	12 46·8	17 1·1	N. 19·9	10 30·6	57·1	12 44·1	17 0·0	N. 20·8
11 27·5	57·6	12 29·8	17 4·1	N. 22·0	11 34·1	57·2	12 26·6	17 6·0	N. 21·9
Upper Transits A, A.M.					Lower (Interpolated) Transits A, P.M.				
0 27·0	57·2	12 14·0	16 8·2	N. 23·2	0 33·9	57·1	12 10·8	17 1·7	N. 22·7
1 29·4	56·9	11 55·8	15 10·6	N. 22·5	1 29·2	57·3	11 54·8	16 7·8	N. 21·8
2 29·3	56·8	11 42·9	14 11·2	N. 19·0	2 25·2	56·9	11 42·3	15 11·5	N. 19·6
3 29·3	56·9	11 42·5	14 0·2	N. 15·8	3 29·5	56·4	11 34·6	14 9·4	N. 16·1
4 30·6	56·6	11 42·5	12 8·2	N. 10·6	4 32·2	57·0	11 42·1	14 0·5	N. 10·5
5 30·0	56·7	12 4·2	12 3·5	N. 6·2	5 28·7	56·6	12 2·5	12 7·7	N. 4·9
6 31·4	57·0	12 36·3	12 11·7	S. 1·2	6 29·4	56·8	12 35·6	13 1·4	S. 0·5
7 30·3	56·7	12 58·2	14 1·7	S. 6·1	7 32·0	57·3	12 57·6	14 4·3	S. 5·6
8 28·8	57·7	13 1·9	15 8·3	S. 12·3	8 30·0	57·3	13 2·4	15 6·8	S. 12·1
9 24·7	57·5	12 58·6	16 11·5	S. 15·8	9 32·6	57·5	12 56·2	16 5·0	S. 17·3
10 27·5	57·6	12 45·2	17 6·9	S. 19·7	10 32·2	58·1	12 44·1	17 4·7	S. 20·1
11 34·4	57·7	12 26·5	17 10·3	S. 22·9	11 23·7	57·7	12 30·7	17 4·5	S. 21·8

Table VII. (*g*.)

Showing the Diurnal Inequality at Liverpool, or the Difference in the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Interval in Table II., and the Difference between the Height of High Water and the Height in Table III.

Diurnal Inequality.									
Apparent Solar Time of Moon's Transit A.	January.			February.			March.		
	Interval.	Height.	Moon's Declination.	Interval.	Height.	Moon's Declination.	Interval.	Height.	Moon's Declination.
h m	m	feet.	°	m	feet.	°	m	feet.	°
P.M. 0 30	+ 1·4	+ ·57	S. 18·3	− 0·2	+ ·29	S. 9·8	− 0·1	+ ·42	N. 0·9
1 30	− 0·8	+ ·60	S. 15·0	− 0·5	+ ·50	S. 3·8	− 1·6	+ ·25	N. 8·5
2 30	+ 0·4	+ ·50	S. 9·3	+ 0·2	+ ·45	N. 2·7	+ 0·3	+ ·25	N. 12·7
3 30	− 1·1	+ ·55	S. 5·0	+ 0·2	+ ·37	N. 8·0	+ 0·5	+ ·13	N. 17·4
4 30	− 0·3	+ ·60	N. 1·3	− 0·8	+ ·30	N. 13·7	− 1·7	− ·06	N. 20·8
5 30	− 1·3	+ ·27	N. 8·4	− 1·0	+ ·03	N. 18·1	− 1·7	− ·23	N. 22·5
6 30	− 0·1	+ ·08	N. 13·5	0·0	− ·13	N. 21·0	− 0·7	− ·10	N. 22·8
7 30	− 1·1	+ ·02	N. 18·9	− 0·5	− ·28	N. 22·5	+ 1·9	− ·24	N. 21·7
8 30	+ 1·3	− ·16	N. 20·4	− 0·4	− ·27	N. 22·7	+ 1·4	− ·22	N. 20·0
9 30	0·0	− ·14	N. 22·9	+ 0·2	− ·27	N. 21·7	+ 0·1	− ·26	N. 15·0
10 30	+ 0·3	− ·32	N. 22·7	+ 1·5	− ·22	N. 19·3	− 0·4	− ·35	N. 10·3
11 30	+ 0·1	− ·27	N. 21·5	+ 0·1	− ·33	N. 14·6	+ 0·6	− ·35	N. 5·1
A.M. 0 30	0·0	− ·63	N. 18·8	+ 0·1	− ·38	N. 10·0	− 0·1	− ·35	S. 1·3
1 30	+ 0·9	− ·45	N. 14·9	+ 0·4	− ·36	N. 3·4	− 0·2	− ·29	S. 6·9
2 30	− 0·2	− ·57	N. 10·0	− 0·2	− ·49	S. 1·5	− 0·4	− ·21	S. 13·4
3 30	+ 1·0	− ·56	N. 3·8	− 0·5	− ·43	S. 8·1	− 0·6	− ·18	S. 18·1
4 30	0·0	− ·53	S. 2·3	+ 0·3	− ·28	S. 13·4	+ 1·5	+ ·03	S. 20·7
5 30	+ 0·3	− ·32	S. 7·7	+ 1·1	+ ·01	S. 18·8	+ 1·5	+ ·17	S. 22·1
6 30	+ 0·1	− ·07	S. 13·5	+ 0·5	+ ·10	S. 20·9	+ 1·5	+ ·02	S. 22·9
7 30	− 0·9	− ·03	S. 18·0	+ 0·7	+ ·09	S. 22·2	+ 0·1	+ ·22	S. 21·8
8 30	− 1·4	− ·19	S. 20·8	+ 1·4	+ ·43	S. 23·2	− 0·4	+ ·20	S. 19·3
9 30	+ 0·4	− ·17	S. 22·4	− 0·2	+ ·27	S. 21·7	+ 0·3	+ ·08	S. 15·5
10 30	+ 0·2	− ·37	S. 22·4	− 0·6	+ ·25	S. 18·5	− 1·1	+ ·26	S. 10·7
11 30	+ 0·2	− ·18	S. 22·2	− 0·2	+ ·25	S. 15·3	− 0·7	+ ·20	S. 5·3
	Sun's Declination 21° S.			Sun's Declination 13° S.			Sun's Declination 2° S.		

	April.			May.			June.		
P.M. 0 30	+ 0·6	− ·15	N. 13·3	0·0	− ·31	N. 20·6	+ 0·3	− ·54	N. 23·0
1 30	+ 0·4	− ·05	N. 16·5	+ 1·8	− ·19	N. 22·4	+ 0·2	− ·50	N. 22·3
2 30	+ 0·2	− ·13	N. 20·7	− 0·4	− ·43	N. 22·7	+ 0·6	− ·44	N. 19·8
3 30	+ 0·1	− ·15	N. 22·5	+ 0·6	− ·43	N. 22·0	+ 0·6	− ·42	N. 15·5
4 30	0·0	− ·29	N. 22·8	+ 0·6	− ·55	N. 18·5	− 0·1	− ·54	N. 11·0
5 30	+ 0·6	− ·27	N. 22·1	+ 0·7	− ·30	N. 15·7	+ 0·7	− ·40	N. 5·0
6 30	− 0·3	− ·31	N. 20·3	+ 0·8	− ·33	N. 11·4	+ 1·9	− ·32	S. 1·0
7 30	+ 0·5	− ·10	N. 15·6	+ 0·5	− ·18	N. 5·8	+ 0·4	− ·09	S. 7·2
8 30	+ 0·6	− ·17	N. 10·8	+ 0·3	− ·03	S. 0·9	− 0·6	+ ·07	S. 12·4
9 30	+ 0·8	− ·20	N. 5·4	+ 1·1	+ ·03	S. 6·6	+ 1·9	+ ·26	S. 17·6
10 30	− 0·2	− ·14	S. 0·7	+ 0·7	+ ·26	S. 11·9	+ 0·2	+ ·26	S. 20·4
11 30	+ 0·2	− ·15	S. 7·0	− 0·7	+ ·18	S. 17·3	− 0·1	+ ·38	S. 22·1
A.M. 0 30	− 0·1	− ·12	S. 11·7	+ 0·1	+ ·55	S. 19·5	− 0·3	+ ·34	S. 23·2
1 30	− 0·6	+ ·01	S. 17·2	− 0·3	+ ·31	S. 21·9	− 0·4	+ ·47	S. 21·9
2 30	+ 0·7	+ ·06	S. 20·4	− 1·8	+ ·27	S. 23·0	− 0·7	+ ·51	S. 19·2
3 30	− 2·1	− ·12	S. 21·9	− 0·7	+ ·35	S. 22·3	− 1·1	+ ·58	S. 16·6
4 30	− 0·6	+ ·26	S. 23·2	− 0·7	+ ·50	S. 19·6	0·0	+ ·43	S. 11·1
5 30	− 1·0	+ ·14	S. 22·3	− 0·5	+ ·38	S. 16·3	− 1·1	+ ·47	S. 5·4
6 30	+ 1·7	+ ·26	S. 19·3	− 1·0	+ ·32	S. 12·4	− 0·9	+ ·30	N. 0·9
7 30	− 0·7	+ ·16	S. 15·9	− 0·5	+ ·18	S. 5·7	− 0·6	+ ·07	N. 7·1
8 30	− 0·6	+ ·21	S. 11·8	+ 0·2	·00	N. 0·5	+ 0·5	− ·05	N. 12·1
9 30	+ 0·1	+ ·16	N. 5·6	− 1·0	·00	N. 6·9	+ 0·1	− ·15	N. 16·4
10 30	+ 0·4	+ ·19	N. 0·6	− 0·8	− ·03	N. 12·1	− 0·5	− ·10	N. 20·6
11 30	− 0·2	+ ·10	N. 7·0	− 3·4	− ·12	N. 18·7	+ 0·2	− ·35	N. 22·3
	Sun's Declination 10° N.			Sun's Declination 19° N.			Sun's Declination 23° N.		

TABLE VII. (*g*.) (Continued.)

Diurnal Inequality.

Apparent Solar Time of Moon's Transit A.	July.			August.			September.		
	Interval.	Height.	Moon's Declination.	Interval.	Height.	Moon's Declination.	Interval.	Height.	Moon's Declination.
h m	m	feet.	°	m	feet.	°	m	feet.	°
P.M. 0 30	+ 0·5	− ·42	N. 19·1	+ 0·4	− ·57	N. 10·9	+ 0·3	− ·49	S. 1·3
1 30	+ 1·3	− ·56	N. 15·6	+ 0·1	− ·53	N. 4·9	+ 1·7	− ·31	S. 6·8
2 30	− 0·5	− ·58	N. 11·1	− 0·9	− ·52	S. 1·2	+ 0·2	− ·35	S. 12·7
3 30	+ 1·7	− ·40	N. 4·8	+ 0·9	− ·26	S. 6·4	+ 0·6	− ·03	S. 16·5
4 30	− 1·8	− ·50	S. 1·4	− 0·4	− ·24	S. 13·4	+ 0·3	+ ·05	S. 19·8
5 30	+ 1·4	− ·36	S. 7·2	+ 0·3	− ·04	S. 18·9	0·0	+ ·09	S. 22·4
6 30	− 4·7	− ·17	S. 13·0	− 1·3	+ ·08	S. 20·7	− 2·7	+ ·15	S. 23·1
7 30	+ 0·8	− ·08	S. 17·3	+ 0·2	+ ·27	S. 22·4	− 0·6	+ ·28	S. 22·0
8 30	− 0·7	+ ·32	S. 20·5	− 0·2	+ ·45	S. 23·0	− 0·1	+ ·39	S. 19·9
9 30	− 1·3	+ ·33	S. 22·4	+ 0·1	+ ·49	S. 21·3	+ 0·3	+ ·40	S. 16·3
10 30	− 1·6	+ ·43	S. 20·7	+ 0·9	+ ·61	S. 19·6	+ 0·2	+ ·48	S. 11·2
11 30	− 0·9	+ ·53	S. 21·5	− 0·6	+ ·53	S. 15·8	− 1·3	+ ·58	S. 5·1
A.M. 0 30	− 0·6	+ ·60	S. 19·7	− 0·5	+ ·49	S. 10·6	− 0·4	+ ·44	S. 0·6
1 30	− 1·1	+ ·51	S. 16·6	− 0·4	+ ·69	S. 5·5	− 1·9	+ ·37	N. 6·8
2 30	+ 0·3	+ ·50	S. 11·0	+ 0·7	+ ·38	N. 0·8	− 0·9	+ ·25	N. 11·9
3 30	− 2·7	+ ·37	S. 4·9	− 0·7	+ ·28	N. 6·6	− 0·4	+ ·10	N. 16·8
4 30	− 0·3	+ ·37	N. 1·3	+ 0·1	+ ·21	N. 12·3	− 0·2	− ·06	N. 17·6
5 30	+ 0·7	+ ·37	N. 6·9	− 0·4	+ ·01	N. 18·1	+ 0·1	− ·10	N. 21·9
6 30	− 1·4	+ ·12	N. 12·4	+ 1·1	− ·12	N. 20·6	+ 2·4	− ·22	N. 23·4
7 30	− 0·7	− ·24	N. 17·1	− 0·1	− ·28	N. 24·6	+ 0·7	− ·29	N. 22·1
8 30	+ 0·7	− ·34	N. 21·0	+ 0·5	− ·40	N. 22·9	+ 0·1	− ·40	N. 20·1
9 30	− 0·9	− ·28	N. 21·9	0·0	− ·55	N. 22·0	− 0·2	− ·30	N. 15·5
10 30	+ 0·3	− ·48	N. 22·9	− 0·7	− ·46	N. 19·2	− 0·4	− ·54	N. 11·1
11 30	− 0·7	− ·44	N. 22·2	− 0·7	− ·61	N. 15·1	+ 1·1	− ·61	N. 6·2
	Sun's Declination 21° N.			Sun's Declination 13° N.			Sun's Declination 3° N.		

	October.			November.			December.		
P.M. 0 30	+ 0·6	− ·34	S. 11·1	− 0·1	+ ·11	S. 19·6	− 0·6	+ ·34	S. 22·9
1 30	+ 0·9	− ·15	S. 16·5	+ 0·1	+ ·19	S. 22·3	− 0·2	+ ·45	S. 22·0
2 30	− 1·3	+ ·09	S. 20·0	+ 0·1	+ ·36	S. 23·0	− 2·4	+ ·58	S. 19·6
3 30	− 0·3	+ ·20	S. 22·1	− 0·5	+ ·51	S. 21·9	− 2·4	+ ·62	S. 15·7
4 30	+ 0·7	+ ·20	S. 23·2	− 1·0	+ ·59	S. 19·9	− 1·2	+ ·69	S. 10·8
5 30	+ 1·4	+ ·53	S. 21·8	− 1·3	+ ·46	S. 16·6	− 0·6	+ ·35	S. 4·9
6 30	− 0·3	+ ·31	S. 20·0	− 0·6	+ ·35	S. 12·3	0·0	+ ·29	N. 0·8
7 30	− 2·0	+ ·50	S. 17·0	− 1·9	+ ·29	S. 6·2	+ 0·9	+ ·05	N. 6·2
8 30	0·0	+ ·38	S. 12·5	− 0·1	+ ·27	S. 0·6	+ 0·4	− ·09	N. 12·4
9 30	+ 0·5	+ ·33	S. 5·8	− 0·1	+ ·24	N. 5·1	− 0·4	− ·09	N. 17·0
10 30	− 0·5	+ ·35	S. 0·9	− 0·3	+ ·17	N. 11·4	+ 0·6	− ·12	N. 20·0
11 30	+ 0·2	+ ·35	N. 6·0	− 0·2	+ ·00	N. 16·6	+ 0·8	− ·19	N. 21·9
A.M. 0 30	− 0·4	+ ·54	N. 11·3	+ 0·5	− ·10	N. 19·8	+ 0·3	− ·29	N. 23·2
1 30	− 0·2	+ ·16	N. 16·1	− 0·1	− ·26	N. 22·1	+ 0·3	− ·34	N. 22·2
2 30	+ 0·9	− ·12	N. 20·4	+ 1·1	− ·11	N. 22·9	+ 0·4	− ·49	N. 19·0
3 30	+ 0·1	− ·23	N. 21·9	− 0·3	− ·38	N. 22·4	+ 3·9	− ·70	N. 16·0
4 30	− 1·2	− ·38	N. 22·9	+ 0·9	− ·54	N. 20·4	+ 3·4	− ·63	N. 11·2
5 30	− 1·0	− ·43	N. 22·1	+ 1·4	− ·45	N. 16·2	+ 0·4	− ·36	N. 5·9
6 30	+ 0·3	− ·26	N. 20·5	+ 0·8	− ·46	N. 12·5	0·0	− ·40	S. 0·8
7 30	− 2·7	− ·54	N. 17·0	− 0·1	− ·43	N. 6·9	− 0·4	− ·07	S. 5·9
8 30	− 0·4	− ·45	N. 12·3	− 0·1	− ·20	N. 0·8	− 0·3	− ·15	S. 12·3
9 30	− 0·3	− ·39	N. 6·7	+ 2·4	− ·27	S. 4·7	+ 0·2	+ ·07	S. 15·7
10 30	+ 0·3	− ·18	S. 0·9	+ 0·2	− ·09	S. 11·7	− 0·4	+ ·08	S. 20·3
11 30	− 0·1	− ·45	S. 5·5	+ 0·2	− ·02	S. 16·5	− 0·6	+ ·17	S. 22·4
	Sun's Declination 9° S.			Sun's Declination 19° S.			Sun's Declination 23° S.		

TABLE VIII. (*h.*)

Showing a Comparison between the Semimenstrual Correction at Liverpool in the Interval and in the Height, as deduced from theory and from the results of observation contained in Tables II. and III.

Moon's Hor. Par. 57′, and Decl. 15°.

Apparent Solar Time of Moon's Transit A.	Interval. ψ + constant.		Height. h.	
	Theory.	Observation.	Theory.	Observation.
h m	h m	h m	feet.	feet.
0 0	12 21·2		17·67	
0 30	12 13·2	12 12·3	17·51	17·46
1 0	12 5·3		17·37	
1 30	11 58·7	11 57·0	17·00	16·78
2 0	11 51·0		16·58	
2 30	11 45·0	11 43·9	16·58	15·77
3 0	11 40·2		15·38	
3 30	11 38·2	11 37·0	14·70	14·40
4 0	11 38·2		13·97	
4 30	11 41·2	11 40·4	13·30	13·02
5 0	11 50·3		12·69	
5 30	12 2·8	12 2·8	12·29	12·14
6 0	12 21·2		12·15	
6 30	12 36·5	12 37·9	12·27	12·54
7 0	12 51·9		12·65	
7 30	13 1·5	13 1·7	13·25	13·81
8 0	13 4·4		13·92	
8 30	13 4·2	13 5·7	14·72	15·27
9 0	13 2·0		15·39	
9 30	12 58·2	12 58·2	16·00	16·45
10 0	12 51·6		16·59	
10 30	12 44·5	12 44·5	17·02	17·22
11 0	12 37·0		17·36	
11 30	12 29·0	12 29·0	17·52	17·66

$\alpha' =$ Moon's Right Ascension. $\alpha =$ Sun's Right Ascension.

$\mu =$ Sidereal Time.

$$\mu - \alpha' = \psi \qquad \alpha - \alpha' = \phi \qquad \mu - \alpha = \psi - \phi.$$

$h =$ Height of High Water.

The columns headed "Theory" have been calculated from the expressions

$$\tan 2\,\psi = \frac{(A) \sin 2\,\phi}{1 + (A) \cos 2\,\phi}$$

$$h = D + (E)\,\{(A) \cos (2\,\psi - 2\,\phi) + \cos 2\,\psi\}$$

$$\log (A) = 9{\cdot}56965 \qquad \log (E) = 0{\cdot}871{\cdot}30 \qquad D = 7{\cdot}46$$

TABLE IX. (*i.*)

Showing the Calendar-month Inequality in the Interval and in the Height of High Water, as deduced from BERNOULLI's theory and from the results of observation contained in Tables II. and III. See Plate I.

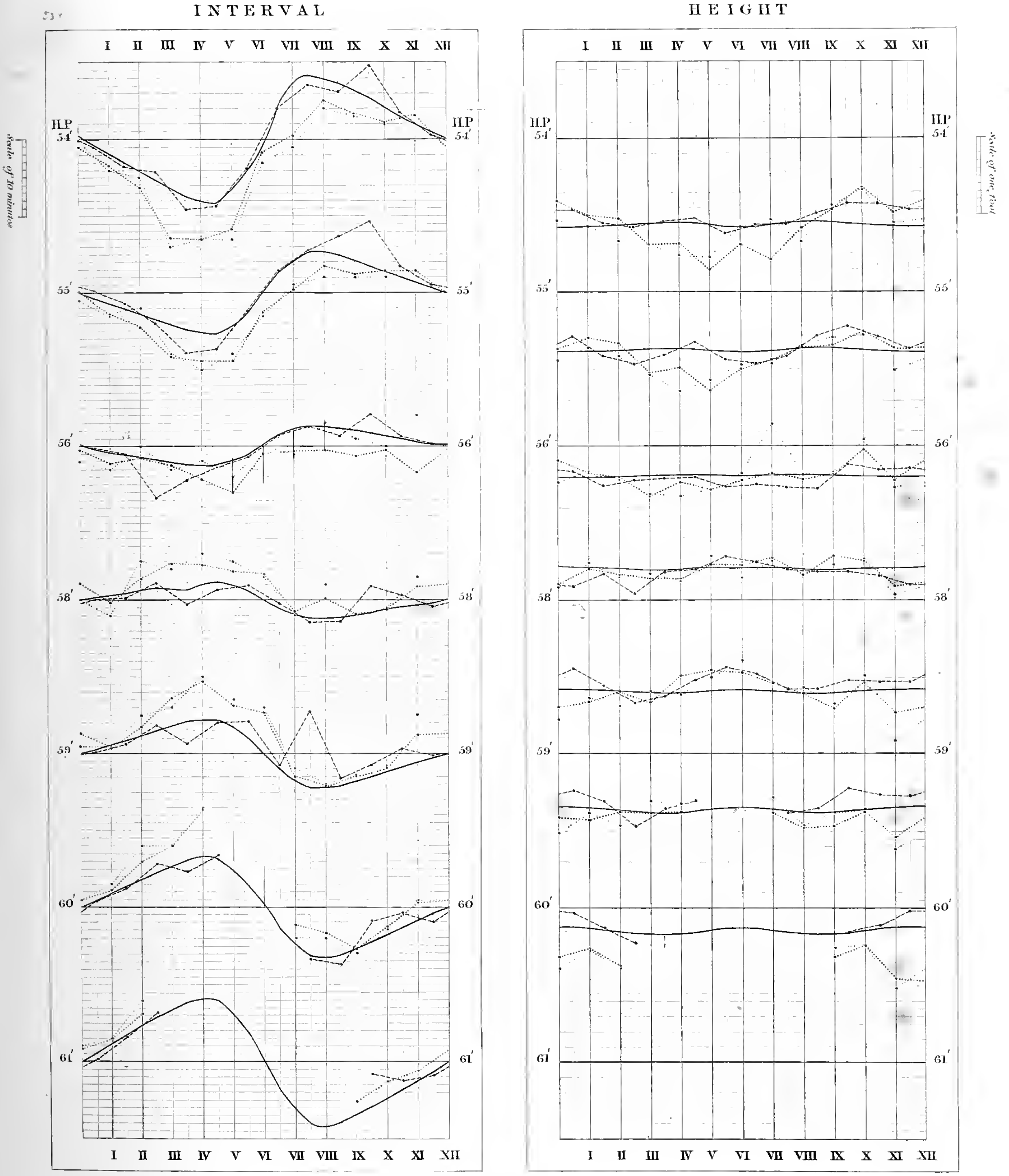
Apparent Solar Time of Moon's Transit A.	January.					February.					March.					Apparent Solar Time of Moon's Transit. A.
	d ψ		d h		Moon's Declination.	d ψ		d h		Moon's Declination.	d ψ		d h		Moon's Declination.	
	Theory.	Observation.	Theory.	Observation.		Theory.	Observation.	Theory.	Observation.		Theory.	Observation.	Theory.	Observation.		
h m	m	m	feet.	feet.	°	m	m	feet.	feet.	°	m	m	feet.	feet.	°	h m
0 30	−0·2	−2·4	−·39	+·13	19	0·0	+0·9	+·44	+·72	10	−0·1	+0·3	+·74	+·60	5	0 30
1 30	+0·5	−0·2	−·05	+·37	15	+0·2	+0·5	+·60	+·65	5	−0·6	+1·1	+·59	+·34	8	1 30
2 30	+1·9	+0·9	+·26	+·42	10	+0·4	+2·9	+·55	+·58	5	−1·9	+1·1	+·26	+·12	13	2 30
3 30	+3·3	+1·8	+·44	+·66	6	0·0	+1·0	+·40	+·26	8	−4·5	−0·9	−·12	−·27	17	3 30
4 30	+3·4	+3·1	+·50	+·49	5	−1·7	+2·3	+·06	−·04	13	−7·2	−2·1	−·59	−·67	21	4 30
5 30	+1·3	+0·9	+·44	+·51	8	−1·9	+1·2	−·36	−·32	18	−3·9	−2·0	−·82	−·81	22	5 30
6 30	−0·5	−2·1	+·12	+·15	14	+2·8	+2·8	−·64	−·48	21	+4·2	+2·7	−·91	−·51	23	6 30
7 30	+0·7	+1·1	−·19	+·04	18	+6·3	+3·6	−·65	−·23	22	+7·9	+4·0	−·68	−·43	22	7 30
8 30	+2·1	+1·8	−·49	+·03	21	+6·4	+3·0	−·65	−·20	23	+6·1	+2·6	−·37	−·12	20	8 30
9 30	+2·4	+2·0	−·71	−·47	23	+4·2	+1·5	−·51	·00	22	+2·5	−0·5	+·13	+·15	15	9 30
10 30	+1·3	−0·2	−·75	−·18	23	+1·7	−0·9	−·20	+·21	19	+0·9	−0·7	+·45	+·46	11	10 30
11 30	+0·5	+0·1	−·67	−·20	22	+0·3	−0·6	+·14	+·44	15	+0·1	−0·3	+·72	+·68	6	11 30
	Sun's Decl. S. 21°, and Par. 8″·94.					Sun's Decl. S. 13°, and Par. 8″·90.					Sun's Decl. S. 2°, and Pa . 8″·84.					

	April.					May.					June.					
0 30	0·0	+0·3	+·25	−·11	12	+0·1	0·0	−·63	−·35	20	+0·3	−1·7	−1·11	− ·84	23	0 30
1 30	−0·8	+0·2	−·10	−·23	17	−0·2	−0·9	−·78	−·76	22	+1·0	−0·8	− ·92	− ·65	22	1 30
2 30	−2·8	−1·5	−·46	−·54	21	−0·7	−0·5	−·80	−·44	23	+2·5	−0·1	− ·58	− ·35	20	2 30
3 30	−4·6	−3·3	−·57	−·63	22	−0·4	−2·5	−·62	−·47	22	+5·5	+1·4	− ·13	− ·02	16	3 30
4 30	−5·8	−2·7	−·71	−·72	23	+0·7	−1·6	−·30	−·09	20	+7·7	+3·9	+ ·40	+ ·61	11	4 30
5 30	−2·4	−0·9	−·67	−·65	22	+1·6	+0·4	+·18	+·16	15	+4·1	+2·0	+ ·81	+ ·79	6	5 30
6 30	+1·7	+1·1	−·48	−·51	20	−2·1	−1·5	+·38	+·33	12	−4·1	−2·6	+ ·83	+ ·71	5	6 30
7 30	+1·5	0·0	−·10	−·17	16	−5·7	−4·0	+·51	+·25	7	−8·5	−4·2	+ ·54	+ ·13	8	7 30
8 30	−0·5	−1·0	+·24	+·13	11	−5·8	−4·4	+·42	+·10	5	−6·7	−2·4	+ ·07	− ·18	13	8 30
9 30	−1·2	−1·3	+·48	+·34	6	−4·0	−2·1	+·29	−·10	7	−3·6	−0·3	− ·33	− ·44	17	9 30
10 30	−0·8	−1·9	+·55	+·40	4	−1·9	−1·9	+·02	−·35	12	−1·5	+0·2	− ·73	− ·80	20	10 30
11 30	−0·2	0·0	+·48	+·25	7	−0·3	+0·4	−·37	−·54	17	−0·3	+0·4	−1·00	−1.01	22	11 30
	Sun's Decl. N. 10°, and Par. 8″·76.					Sun's Decl. N. 19°, and Par. 8″·70.					Sun's Decl. N. 23°, and Par. 8″·66.					

	July.					August.					September.					
0 30	+0·4	+1·0	−·65	−·42	19	+0·3	+1·7	+·21	+·29	11	+0·1	+1·1	+·65	+·70	4	0 30
1 30	+1·9	+1·2	−·25	−·07	16	+1·4	+2·1	+·42	+·29	6	+0·1	+1·4	+·53	+·55	7	1 30
2 30	+4·5	+1·3	+·07	+·30	11	+2·3	+2·2	+·45	+·51	5	−0·6	+1·0	+·26	+·27	12	2 30
3 30	+7·3	+5·0	+·40	+·41	6	+2·9	+3·2	+·39	+·39	7	−3·0	−0·6	−·14	−·03	17	3 30
4 30	+7·7	+5·5	+·62	+·73	5	+2·1	+1·7	+·14	+·12	13	−5·0	−2·1	−·46	−·43	20	4 30
5 30	+3·4	+3·1	+·66	+·64	8	−0·2	+0·6	−·12	−·25	17	−3·0	−2·5	−·73	−·58	22	5 30
6 30	−2·7	−0·5	+·40	+·17	13	+1·3	+1·9	−·48	−·38	21	+3·3	+1·6	−·82	−·28	23	6 30
7 30	−4·1	−0·6	·00	+·06	17	+4·1	+3·1	−·67	−·36	23	+6·3	+3·1	−·64	−·23	22	7 30
8 30	−1·9	+0·7	−·53	−·40	21	+3·6	+3·8	−·67	−·46	23	+4·6	+2·3	−·39	−·01	20	8 30
9 30	−0·8	+2·9	−·76	−·55	22	+2·2	+2·3	−·61	−·22	22	+1·8	+0·8	−·01	+·24	16	9 30
10 30	−0·3	−0·7	−·97	−·62	23	+0·5	−0·1	−·36	−·17	19	+0·3	−1·1	+·36	+·54	11	10 30
11 30	−0·1	+1·2	−·93	−·63	22	0·0	−0·2	−·12	+·10	16	−0·1	−0·2	+·61	+·45	6	11 30
	Sun's Decl. N. 21°, and Par. 8″·66.					Sun's Decl. N. 13°, and Par. 8″·70.					Sun's Decl. N. 3°, and Par. 8″·76.					

	October.					November.					December.					
0 30	−0·2	+0·1	+·42	+·40	11	−0·3	0·4	−·45	−·19	20	−1·0	−0·4	−·85	−·49	23	0 30
1 30	−1·3	−0·9	+·07	+·24	16	−1·4	−1·1	−·62	−·28	22	−0·5	−2·1	−·70	−·43	22	1 30
2 30	−3·6	−3·3	−·30	−·22	20	−2·7	−2·5	−·70	−·30	23	+0·1	−1·6	−·36	−·18	19	2 30
3 30	−6·3	−3·0	−·55	−·34	22	−3·2	−3·1	−·58	−·22	22	+1·5	+1·0	−·09	+·30	16	3 30
4 30	−7·6	−5·7	−·76	−·44	23	−2·4	−3·5	−·38	−·10	20	+3·4	+1·4	+·28	+·53	11	4 30
5 30	−3·3	−3·4	−·76	−·52	22	−0·1	−1·0	−·06	+·29	16	+1·9	+1·0	+·55	+·76	7	5 30
6 30	+2·6	−0·2	−·57	−·34	20	−0·6	−0·6	+·22	+·46	12	−2·0	−2·3	+·61	+·67	5	6 30
7 30	+3·8	+1·7	−·23	+·07	17	−2·6	−4·0	+·43	+·43	7	−4·3	−3·7	+·45	+·43	7	7 30
8 30	+1·5	−0·2	+·21	+·32	12	−3·0	−3·0	+·46	+·52	5	−3·1	−2·7	+·17	+·26	12	8 30
9 30	+0·2	−0·7	+·51	+·56	7	−2·2	−4·6	+·43	+·44	6	−1·2	−0·6	−·12	+·01	16	9 30
10 30	−0·1	−1·8	+·65	+·41	4	−0·7	−1·5	+·18	+·30	12	+0·1	−0·0	−·51	−·15	20	10 30
11 30	0·0	−0·5	+·60	+·79	7	−0·3	−0·1	−·19	+·01	17	+0·3	−0·5	−·74	−·38	22	11 30
	Sun's Decl. S. 9°, and Par. 8″·84.					Sun's Decl. S. 19°, and Par. 8″·90.					Sun's Decl. S. 23°, and Par. 8″·94.					

Diagram showing a comparison between the Moon's Parallax inequality in the Interval and in the Height of high water as deduced from Bernoulli's Theory and from Observations at the London and Liverpool Docks. See Tables X and XXIV. p. 119. and p. 133.



Theory ——— *Observation* { ·— ·— ·— *London 19 Years* ··········· *35 Years* –·–·–·–· *Liverpool*

In this comparison of the London and Liverpool results the London corrections have been shifted to the left half an hour agreeably to the remark in p. 100 and the London height corrections have been multiplied by 1.7 The abscissa represents the apparent solar time of moon's transit A.

TABLE X. (*j.*)

Showing the Moon's Parallax Inequality in the Interval and in the Height of High Water, as deduced from BERNOULLI's theory and from the results of observation contained in Table V. See Plate II.

Apparent Solar Time of Moon's Transit A.	H. P. 54′.				H. P. 55′.				Apparent Solar Time of Moon's Transit A.
	d ψ		d *h*		d ψ		d *h*		
	Theory.	Observation.	Theory.	Observation.	Theory.	Observation.	Theory.	Observation.	
h m	m	m	feet.	feet.	m	m	feet.	feet.	h m
0 30	− 1·0	− 0·7	−1·16	−0·92	− 0·7	+ 0·2	− ·79	− ·59	0 30
1 30	− 3·0	− 3·4	−1·14	−1·10	− 2·0	− 1·3	− ·77	− ·83	1 30
2 30	− 5·3	− 4·1	−1·11	−1·16	− 3·4	− 4·0	− ·75	− ·93	2 30
3 30	− 7·4	− 9·1	−1·09	−1·08	− 4·9	− 7·9	− ·74	− ·82	3 30
4 30	− 8·3	− 8·7	−1·10	−1·04	− 5·3	− 7·3	− ·75	− ·66	4 30
5 30	− 4·0	− 3·9	−1·15	−1·24	− 2·6	− 2·4	− ·78	− ·88	5 30
6 30	+ 4·0	+ 4·0	−1·15	−1·11	+ 2·6	+ 2·9	− ·78	− ·92	6 30
7 30	+ 8·3	+ 7·1	−1·10	−1·11	+ 5·3	+ 5·5	− ·75	− ·84	7 30
8 30	+ 7·4	+ 6·2	−1·09	−0·97	+ 4·9	+ 7·3	− ·74	− ·58	8 30
9 30	+ 5·3	+ 9·6	−1·11	−0·84	+ 3·4	+ 9·3	− ·75	− ·45	9 30
10 30	+ 3·0	+ 3·4	−1·14	−0·86	+ 2·0	+ 3·4	− ·77	− ·59	10 30
11 30	+ 1·0	+ 0·6	−1·16	−0·92	+ 0·7	+ 0·1	− ·79	− ·74	11 30
	H. P. 56′.				H. P. 57′.				
0 30	− 0·3	− 0·2	− ·40	− ·31	0·0	0·0	0·0	0·0	0 30
1 30	− 1·0	− 1·0	− ·40	− ·52	0·0	0·0	0·0	0·0	1 30
2 30	− 1·7	− 6·7	− ·39	− ·44	0·0	0·0	0·0	0·0	2 30
3 30	− 2·3	− 4·3	− ·38	− ·41	0·0	0·0	0·0	0·0	3 30
4 30	− 2·5	− 2·7	− ·38	− ·40	0·0	0·0	0·0	0·0	4 30
5 30	− 1·2	− 1·4	− ·39	− ·53	0·0	0·0	0·0	0·0	5 30
6 30	+ 1·2	+ 1·3	− ·39	− ·50	0·0	0·0	0·0	0·0	6 30
7 30	+ 2·5	+ 2·5	− ·38	− ·44	0·0	0·0	0·0	0·0	7 30
8 30	+ 2·3	+ 1·4	− ·38	− ·46	0·0	0·0	0·0	0·0	8 30
9 30	+ 1·7	+ 4·1	− ·39	− ·23	0·0	0·0	0·0	0·0	9 30
10 30	+ 1·0	+ 1·3	− ·40	− ·30	0·0	0·0	0·0	0·0	10 30
11 30	+ 0·3	+ 0·3	− ·40	− ·28	0·0	0·0	0·0	0·0	11 30
	H. P. 58′.				H. P. 59′.				
0 30	+ 0·3	+ 0·1	+ ·42	+ ·18	+ 0·6	+ 0·2	+ ·85	+1·11	0 30
1 30	+ 0·9	+ 0·2	+ ·41	+ ·34	+ 1·8	+ 1·3	+ ·84	+0·90	1 30
2 30	+ 1·6	+ 2·2	+ ·40	+ ·09	+ 3·0	+ 3·8	+ ·81	+0·68	2 30
3 30	+ 2·2	− 0·6	+ ·39	+ ·37	+ 4·2	+ 1·4	+ ·80	+0·76	3 30
4 30	+ 2·3	+ 1·3	+ ·41	+ ·40	+ 4·4	+ 4·1	+ ·81	+0·97	4 30
5 30	+ 1·1	+ 1·9	+ ·42	+ ·57	+ 2·0	+ 4·2	+ .84	+1·12	5 30
6 30	− 1·1	− 0·4	+ ·42	+ ·50	− 2·0	− 1·5	+ ·84	+1·05	6 30
7 30	− 2·3	− 2·9	+ ·41	+ ·40	− 4·4	− 5·4	+ ·81	+0·85	7 30
8 30	− 2·2	− 2·8	+ ·39	+ ·38	− 4·2	− 3·1	+ ·80	+0·86	8 30
9 30	− 1·6	+ 1·8	+ ·40	+ ·38	− 3·0	− 1·5	+ ·81	+0·97	9 30
10 30	− 0·9	+ 0·6	+ ·41	+ ·31	− 1·8	+ 0·7	+ ·84	+0·95	10 30
11 30	− 0·3	− 0·9	+ ·42	+ ·20	− 0·6	− 0·2	+ ·85	+0·94	11 30
	H. P. 60′.				H. P. 61′.				
0 30	+ 0·9	+ 0·9	+1·30	+1·51	+ 1·2	+ 0·3	+1·76	+1·93	0 30
1 30	+ 2·7	+ 2·5	+1·28	+1·38	+ 3·5	+ 3·3	+1·73	+1·75	1 30
2 30	+ 4·5	+ 5·7	+1·24	+1·07	+ 5·9	+ 6·3	+1·68	+1·54	2 30
3 30	+ 6·1	+ 4·7	+1·22	+1·27	+ 7·9		+1·66		3 30
4 30	+ 6·2	+ 6·9	+1·24	+1·39	+ 8·0		+1·69		4 30
5 30	+ 2·9		+1·28		+ 3·7		+1·74		5 30
6 30	− 2·9		+1·28		− 3·7		+1·74		6 30
7 30	− 6·2	− 6·7	+1·24	+1·22	− 8·0		+1·69		7 30
8 30	− 6·1	− 7·4	+1·22	+1·27	− 7·9		+1·66		8 30
9 30	− 4·5	− 1·9	+1·24	+1·54	− 5·9	− 1·7	+1·68	+1·69	9 30
10 30	− 2·7	− 0·8	+1·28	+1·46	− 3·5	− 2·4	+1·73	+1·78	10 30
11 30	− 0·9	− 2·0	+1·30	+1·45	− 1·2	− 1·8	+1·76	+1·96	11 30

Table XI. (*k*.)

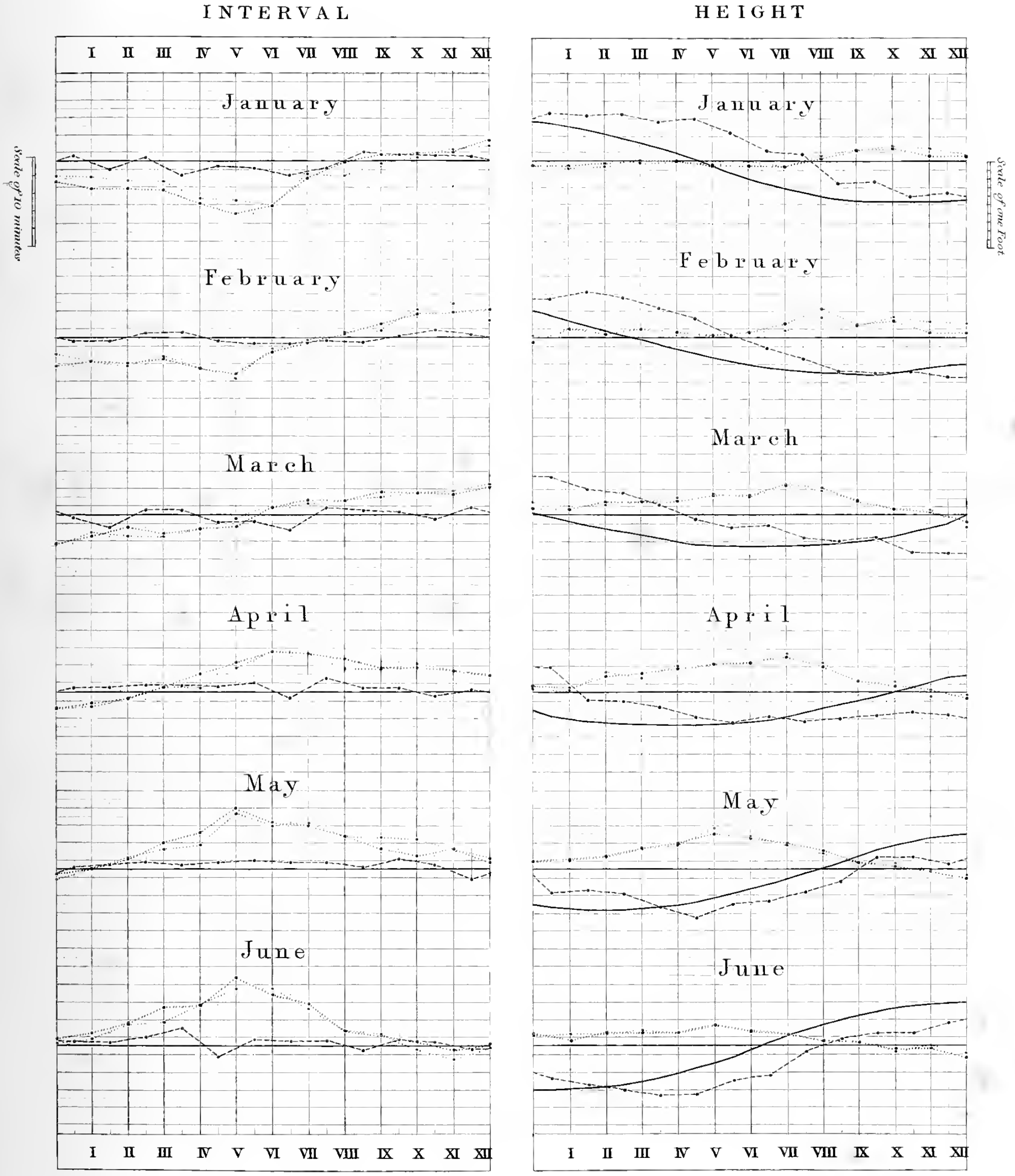
Showing the Diurnal Inequality in the Interval and in the Height in the first six months of the year, for the Moon's Transit A, P.M. See Plate III.

Apparent Solar Time of Moon's Transit A.	January.			February.			March.			Apparent Solar Time of Moon's Transit A.
	d ψ.	d *h*.	Moon's Declination.	d ψ.	d *h*.	Moon's Declination.	d ψ.	d *h*.	Moon's Declination.	
	Observation.	Observation.		Observation.	Observation.		Observation.	Observation.		
P.M. h m	m	feet.	°	m	feet.	°	m	feet.	°	h m
0 30	+ 0·6	+ ·56	S. 19	− 0·3	+ ·43	S. 10	− 0·2	+ ·43	N. 1	0 30
1 30	− 1·0	+ ·53	S. 15	− 0·4	+ ·52	S. 4	− 1·4	+ ·30	N. 7	1 30
2 30	+ 0·4	+ ·54	S. 10	+ 0·5	+ ·46	N. 2	+ 0·5	+ ·27	N. 13	2 30
3 30	− 1·6	+ ·47	S. 5	+ 0·6	+ ·34	N. 7	+ 0·5	+ ·11	N. 17	3 30
4 30	− 0·6	+ ·50	N. 2	− 0·4	+ ·21	N. 13	− 0·9	− ·05	N. 20	4 30
5 30	− 0·9	+ ·33	N. 8	− 0·7	+ ·02	N. 18	− 0·8	− ·15	N. 22	5 30
6 30	− 1·6	+ ·11	N. 13	− 0·7	− ·11	N. 21	− 1·8	− ·12	N. 23	6 30
7 30	− 0·9	+ ·09	N. 18	− 0·4	− ·23	N. 23	+ 0·8	− ·26	N. 22	7 30
8 30	+ 1·0	− ·25	N. 21	− 0·6	− ·39	N. 23	+ 0·5	− ·30	N. 20	8 30
9 30	+ 0·7	− ·23	N. 22	+ 0·1	− ·40	N. 22	+ 0·2	− ·26	N. 16	9 30
10 30	+ 0·6	− ·40	N. 22	+ 0·9	− ·39	N. 19	− 0·5	− ·41	N. 11	10 30
11 30	+ 0·5	− ·36	N. 22	+ 0·4	− ·43	N. 15	+ 0·9	− ·43	N. 5	11 30
	July.			August.			September.			
	April.			May.			June.			
0 30	+ 0·4	+ ·29	N. 12	+ 0·2	− ·27	N. 20	+ 0·4	− ·38	N. 23	0 30
1 30	+ 0·5	− ·09	N. 17	+ 0·6	− ·24	N. 22	+ 0·3	− ·44	N. 22	1 30
2 30	+ 0·8	− ·10	N. 20	+ 0·9	− ·29	N. 23	+ 1·0	− ·50	N. 19	2 30
3 30	+ 0·7	− ·18	N. 22	+ 0·5	− ·42	N. 22	+ 2·0	− ·58	N. 16	3 30
4 30	+ 0·6	− ·28	N. 23	+ 0·8	− ·55	N. 20	− 1·2	− ·57	N. 11	4 30
5 30	+ 1·0	− ·34	N. 22	+ 1·0	− ·40	N. 16	+ 0·7	− ·40	N. 5	5 30
6 30	− 0·7	− ·29	N. 20	+ 0·8	− ·37	N. 12	+ 0·5	− ·33	S. 1	6 30
7 30	+ 1·5	− ·33	N. 16	+ 0·8	− ·27	N. 6	+ 0·6	− ·07	S. 7	7 30
8 30	+ 0·4	− ·30	N. 12	+ 0·2	− ·13	S. 1	− 0·5	+ ·09	S. 12	8 30
9 30	+ 0·4	− ·27	N. 6	+ 1·2	+ ·14	S. 6	+ 0·7	+ ·14	S. 17	9 30
10 30	− 0·4	− ·22	S. 1	+ 0·5	+ ·14	S. 12	+ 0·4	+ ·14	S. 29	10 30
11 30	+ 0·2	− ·26	S. 6	− 1·1	+ ·08	S. 17	− 0·4	+ ·26	S. 22	11 30
	October.			November.			December.			

The tide depending on the Moon's Transit A.M. for the last six months has the same inequality and the same signs as the above; and in the first six months A.M. and the last six months P.M. the same values obtain, but with a contrary sign.

The quantities in the columns headed "Observation" have been obtained by taking the mean of January and July, February and August, &c. A.M. and P.M., as explained in p. 100. The corresponding moon's declination has been obtained in a similar manner.

Diagram showing the Diurnal inequality in the Interval and in the Height of high water as deduced from Observations at the London and Liverpool Docks_ See Tables XI. & XXV. p. 120 and p. 134



Theory ———— *Observation*

London 35 *Years* ....
London 19 *Years* . . . . . .
Liverpool — · — · — · —

In this comparison of the London and Liverpool results the London corrections have been shifted to the left half an hour agreeably to the remark in p. 100 and the signs reversed. The abscissa represents the apparent solar time of moons transit A.P.M.

Diagram showing the Establishment of the Port of Liverpool. — See Table XIV. p. 121.

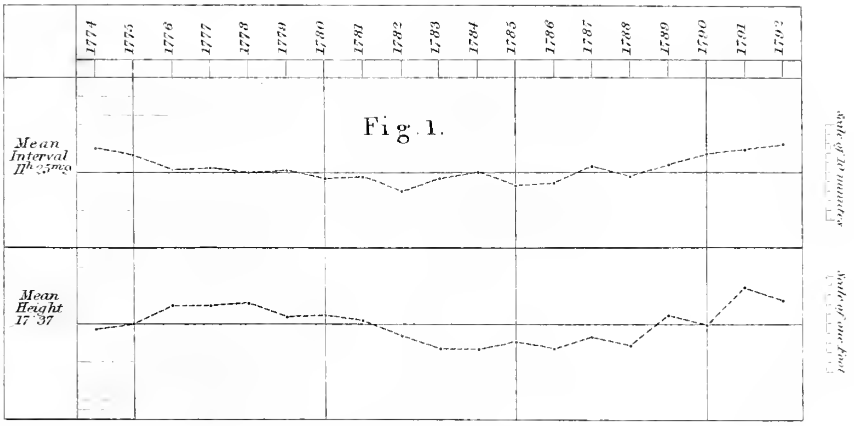


Fig. 1.

Diagram showing the Establishment of the Port of London. — See Table XXX. p. 136.

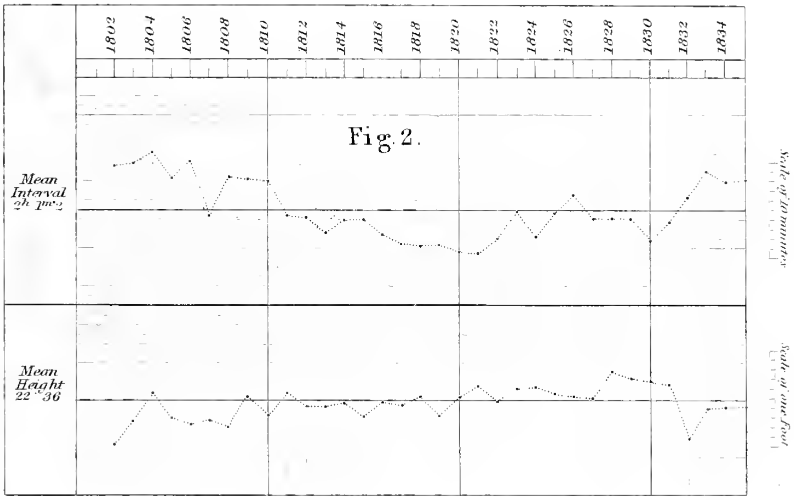


Fig. 2.

Diagram showing the errors of calculated Heights of Highwater for May & June and the corresponding Heights of the Barometer at Liverpool & London. — See. p. 104.

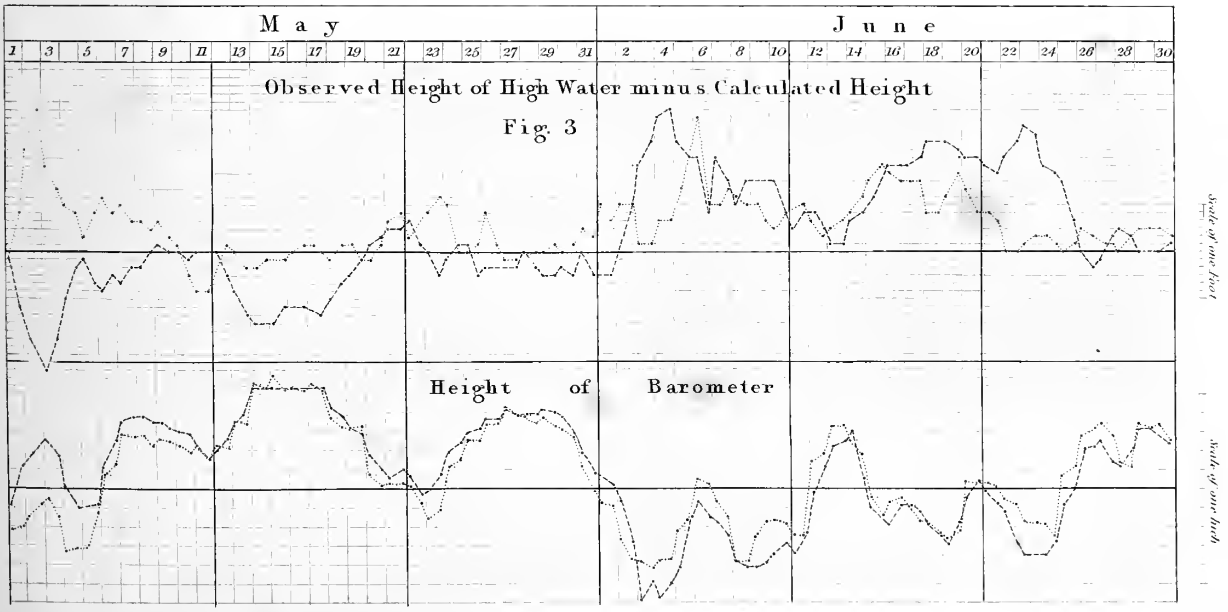


Fig. 3

Liverpool ————————

London ……………………

TABLE XII. (*l.*)

Showing the Interval and Height of High Water at the Liverpool Docks, with the Sun's and Moon's Declinations, and the Moon's Horizontal Parallax (for the mean of all the Moon's Transits A occurring between 0^h and 1^h) for every year from 1774 to 1792.

Year.	Number of Observations.	Moon's Transit A.	Interval.	Height.	Moon's Declination.	Moon's Hor. Par.	Sun's Declination.
		h m	h m	ft. in.	°	′	°
1774	58	0 27·5	12 15·4	18 0·9	10·9	57·6	14·0
1775	57	0 29·3	12 14·3	17 10·7	11·6	57·4	14·0
1776	59	0 30·8	12 12·3	17 11·7	12·7	57·3	14·5
1777	59	0 32·2	12 12·1	17 10·1	14·0	57·3	14·6
1778	52	0 30·6	12 12·1	17 10·0	15·1	57·4	14·6
1779	58	0 29·3	12 12·7	17 8·9	15·8	57·6	14·4
1780	62	0 29·3	12 11·9	17 6·7	16·8	57·3	14·4
1781	59	0 29·5	12 11·9	17 6·0	16·9	57·3	14·2
1782	53	0 29·6	12 10·5	17 4·4	17·2	57·4	13·6
1783	57	0 30·3	12 11·6	17 4·7	16·8	57·7	13·7
1784	59	0 29·4	12 12·5	17 2·0	17·3	57·3	14·1
1785	55	0 31·3	12 11·4	17 4·1	16·5	57·4	14·3
1786	62	0 29·6	12 11·3	17 3·1	16·1	57·3	14·4
1787	62	0 28·3	12 13·1	17 7·0	15·1	57·6	14·1
1788	55	0 30·0	12 11·9	17 5·8	14·0	57·4	14·7
1789	54	0 28·1	12 13·7	17 11·1	12·4	57·4	14·0
1790	56	0 28·2	12 14·8	17 9·8	11·9	57·3	14·4
1791	55	0 29·6	12 14·8	18 3·8	11·7	57·6	14·9
1792	62	0 29·3	12 15·4	18 1·0	11·2	57·3	14·4

TABLE XIII. (*m.*)

Interpolated from Table XII. by reducing each quantity to Moon's Transit A (0^h 30^m), and correcting the quantities for deviation from mean Declinations and Parallax.

Moon's Transit A = 0^h 30^m.

Year.	Interval.	Height.
	h m	feet.
1774	12 14·7	17·40
1775	12 14·1	17·46
1776	12 12·5	17·65
1777	12 12·7	17·65
1778	12 12·3	17·67
1779	12 12·5	17·53
1780	12 11·7	17·55
1781	12 11·8	17·49
1782	12 10·4	17·34
1783	12 11·7	17·20
1784	12 12·3	17·19
1785	12 11·0	17·27
1786	12 11·2	17·20
1787	12 12·9	17·32
1788	12 11·9	17·24
1789	12 13·2	17·55
1790	12 14·3	17·45
1791	12 14·7	17·84
1792	12 15·2	17·69

TABLE XIV. (*n.*)

Showing the *Establishment* of the Port of Liverpool obtained from Table XIII. by altering the argument from Transit A to Transit D, and reducing it to 0^h 0^m from 0^h 30^m. Moon's Hor. Par. 57′, and Decl. 15°.

Moon's Transit D = 0^h 0^m.

Year.	Interval.*	Height.
	h m	feet.
1774	11 28·3	17·31
1775	11 27·7	17·37
1776	11 26·1	17·56
1777	11 26·3	17·56
1778	11 25·9	17·58
1779	11 26·1	17·44
1780	11 25·3	17·46
1781	11 25·4	17·40
1782	11 24·0	17·25
1783	11 25·3	17·11
1784	11 25·9	17·10
1785	11 24·6	17·18
1786	11 24·8	17·11
1787	11 26·5	17·23
1788	11 25·5	17·15
1789	11 26·8	17·46
1790	11 27·9	17·36
1791	11 28·3	17·75
1792	11 28·8	17·60

* i. e. *Establishment.*

Results deduced from Observations made at

LONDON.

Table XV. (*a.*)

Showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the London Docks, corresponding to the Apparent Solar Time of the Moon's Transit B in each month of the year, from 24,592 observations made at the London Docks, between the 1st of September 1801 and the 31st of August 1836.

January.						February.					
Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.	Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.
	h m	h m	ft. in.	°	′		h m	h m	ft. in.	°	′
167	0 30·1	3 3·9	22 9·1	19	57·1	165	0 30·2	3 5·5	22 10·8	10	57·3
166	1 29·7	2 49·7	22 6·3	15	57·3	169	1 28·3	2 50·9	22 8·2	6	57·2
186	2 29·1	2 38·1	21 11·4	10	57·1	172	2 28·4	2 38·5	22 1·4	5	57·2
188	3 29·6	2 30·5	21 4·8	6	56·9	175	3 29·1	2 27·1	21 3·8	8	56·9
189	4 29·9	2 28·9	20 6·0	5	56·9	166	4 29·9	2 22·9	20 0·9	14	56·7
194	5 30·1	2 41·7	19 7·3	9	56·9	152	5 29·6	2 32·5	19 2·2	18	56·6
179	6 29·3	3 9·4	19 7·0	13	56·9	151	6 29·7	3 9·6	19 1·5	21	56·7
176	7 29·3	3 40·7	20 1·8	18	57·0	144	7 29·0	3 43·8	19 11·2	22	56·8
169	8 29·9	3 52·8	20 11·9	21	57·2	142	8 28·7	3 56·9	20 9·4	23	56·9
154	9 29·2	3 50·7	21 8·9	22	57·2	149	9 28·6	3 51·1	21 6·7	21	57·1
162	10 29·1	3 37·8	22 3·2	23	57·2	157	10 30·0	3 37·2	22 4·1	19	57·2
158	11 29·1	3 19·9	22 7·4	21	57·2	157	11 30·8	3 20·6	22 9·5	15	57·3
Sun's Declination 21°.						Sun's Declination 13°.					
March.						April.					
178	0 29·3	3 7·8	22 10·8	5	57·2	170	0 29·0	3 9·0	23 0·0	12	57·3
185	1 29·9	2 50·5	22 8·2	8	57·1	165	1 28·9	2 51·8	22 5·8	17	57·2
176	2 29·6	2 35·1	22 0·0	13	56·9	161	2 29·0	2 34·7	21 10·5	20	57·0
170	3 28·7	2 21·6	20 11·8	17	56·8	154	3 29·2	2 21·0	20 11·2	22	56·8
167	4 28·8	2 16·7	19 9·9	20	56·7	155	4 29·1	2 15·4	19 10·4	23	56·7
166	5 30·0	2 28·4	18 10·8	22	56·7	157	5 29·0	2 31·4	19 1·6	22	56·5
163	6 29·6	3 9·5	19 0·4	23	56·8	165	6 28·8	3 9·6	19 2·9	20	56·7
163	7 28·9	3 47·4	19 10·0	22	56·9	174	7 29·2	3 43·5	20 1·9	16	56·7
177	8 29·0	3 58·3	20 11·8	19	57·2	175	8 29·2	3 54·5	21 3·3	11	57·0
175	9 29·6	3 50·5	21 10·7	16	57·3	186	9 30·0	3 49·8	22 2·1	6	57·2
180	10 30·3	3 36·7	22 6·6	11	57·4	175	10 29·1	3 39·5	22 9·4	5	57·3
187	11 30·5	3 23·7	22 11·2	6	57·5	179	11 28·9	3 23·0	23 1·4	7	57·4
Sun's Declination 3°.						Sun's Declination 10°.					
May.						June.					
166	0 30·0	3 6·9	22 10·3	20	57·6	157	0 28·2	3 5·8	22 6·8	23	57·3
161	1 29·8	2 51·6	22 5·6	22	57·5	156	1 29·3	2 51·2	22 3·8	22	57·4
165	2 30·0	2 34·5	21 8·1	23	57·2	163	2 29·5	2 38·1	21 10·6	19	57·3
163	3 30·1	2 22·5	20 10·4	22	56·9	172	3 30·0	2 28·7	21 2·9	16	57·0
170	4 29·7	2 21·8	20 1·3	20	56·8	176	4 30·0	2 30·6	20 7·2	11	56·8
171	5 29·4	2 37·5	19 8·2	16	56·6	186	5 29·6	2 45·0	20 0·9	7	56·7
183	6 29·4	3 12·0	19 9·7	12	56·5	184	6 29·8	3 12·0	19 11·9	5	56·6
192	7 29·7	3 39·3	20 6·7	7	56·7	179	7 29·3	3 37·5	20 7·2	7	56·8
188	8 29·8	3 50·6	21 5·0	5	56·8	179	8 29·4	3 47·7	21 2·5	12	56·9
183	9 30·0	3 49·3	22 2·0	7	57·0	161	9 29·0	3 47·6	21 10·0	17	57·0
176	10 29·2	3 39·0	22 8·0	12	57·2	161	10 29·3	3 36·4	22 3·4	20	57·1
175	11 28·9	3 24·4	22 9·9	17	57·3	149	11 28·8	3 22·4	22 6·6	22	57·2
Sun's Declination 19°.						Sun's Declination 23°.					

TABLE XV. (*a*.) (Continued.)

July.

Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.
	h m	h m	ft. in.	°	′
159	0 30·0	3 6·6	22 7·4	19	57·2
167	1 28·4	2 52·7	22 6·3	16	57·1
186	2 29·7	2 40·7	22 2·1	11	57·1
186	3 29·7	2 34·7	21 6·0	6	57·1
198	4 29·3	2 33·4	20 8·7	5	56·9
191	5 29·3	2 46·6	20 1·0	8	56·9
185	6 29·0	3 14·4	19 10·4	13	56·8
175	7 29·2	3 39·2	20 4·0	17	56·9
165	8 29·0	3 52·3	21 0·4	20	56·9
162	9 29·2	3 50·6	21 8·5	22	57·1
154	10 29·6	3 39·5	22 2·8	23	57·1
164	11 30·5	3 22·3	22 5·6	22	57·2

Sun's Declination 21°.

August.

Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.
	h m	h m	ft. in.	°	′
179	0 30·6	3 10·0	22 10·3	11	57·0
178	1 29·7	2 53·4	22 7·9	6	56·9
181	2 28·8	2 42·0	22 1·6	5	57·0
184	3 29·0	2 31·2	21 4·4	8	56·9
182	4 29·4	2 27·5	20 5·4	13	56·7
174	5 29·0	2 38·5	19 6·4	17	56·7
175	6 29·2	3 11·7	19 4·4	20	56·9
168	7 30·2	3 43·8	20 0·2	22	57·0
162	8 30·0	3 57·7	20 11·2	23	57·1
165	9 29·9	3 53·7	21 8·4	22	57·2
165	10 30·0	3 40·6	22 3·6	19	57·3
171	11 29·8	3 28·2	22 8·5	16	57·3

Sun's Declination 14°.

September.

Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.
179	0 29·4	3 10·9	22 11·9	4	57·4
182	1 29·7	2 54·0	22 7·4	7	57·1
173	2 29·8	2 38·3	22 0·9	12	57·0
163	3 29·0	2 27·3	21 0·9	17	56·9
164	4 29·3	2 17·5	19 11·4	20	56·7
157	5 30·0	2 28·8	19 1·7	22	56·7
155	6 29·9	3 11·6	18 11·5	23	56·7
160	7 29·1	3 47·7	19 10·8	22	56·8
164	8 28·4	3 59·7	20 11·5	20	57·0
174	9 29·5	3 54·5	21 10·5	16	57·2
176	10 30·1	3 44·1	22 6·7	12	57·3
177	11 29·8	3 27·7	22 11·1	6	57·4

Sun's Declination 4°.

October.

Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.
175	0 30·0	3 10·6	22 11·1	12	57·5
173	1 29·7	2 53·0	22 6·4	16	57·4
166	2 29·8	2 34·9	21 10·3	20	57·0
164	3 28·9	2 21·3	20 10·5	22	56·9
157	4 28·9	2 12·5	19 10·8	23	56·8
164	5 29·2	2 24·8	19 0·8	22	56·6
170	6 30·6	3 5·6	19 1·0	20	56·6
167	7 30·6	3 44·6	20 1·2	17	56·6
180	8 29·7	3 56·5	21 2·5	12	56·9
187	9 28·7	3 52·3	22 1·0	7	57·1
190	10 29·0	3 41·4	22 8·9	5	57·4
189	11 30·1	3 26·8	22 11·3	7	57·5

Sun's Declination 9°.

November.

Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.
162	0 28·9	3 7·3	22 6·8	20	57·4
158	1 28·9	2 49·2	22 3·2	22	57·4
152	2 29·2	2 33·2	21 9·4	23	57·3
163	3 29·3	2 23·4	21 0·6	22	57·2
161	4 29·4	2 18·8	20 0·6	20	56·8
169	5 28·7	2 32·2	19 6·6	17	56·6
185	6 30·0	3 7·0	19 6·0	12	56·7
177	7 30·2	3 37·4	20 4·8	7	56·8
182	8 29·9	3 50·6	21 3·6	5	56·8
174	9 29·4	3 46·8	22 0·9	7	57·1
176	10 29·6	3 37·9	22 5·0	11	57·2
159	11 29·3	3 23·6	22 9·5	16	57·3

Sun's Declination 18°.

December.

Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Mean of Moon's Declination.	Mean Horizontal Parallax.
157	0 29·4	3 4·6	22 6·2	22	57·3
159	1 29·6	2 47·6	22 2·8	22	57·3
165	2 30·0	2 33·4	21 11·1	20	57·2
175	3 30·4	2 27·9	21 1·7	16	57·1
185	4 29·7	2 26·7	20 5·6	11	56·9
197	5 29·8	2 38·9	19 10·6	7	56·9
181	6 29·2	3 8·2	19 10·2	5	56·7
193	7 28·4	3 34·3	20 5·2	8	56·7
184	8 29·2	3 46·4	21 3·3	12	56·9
167	9 29·3	3 47·6	21 10·7	17	57·1
160	10 28·6	3 36·9	22 3·5	20	57·2
163	11 28·5	3 21·5	22 5·1	22	57·2

Sun's Declination 23°.

TABLE XVI. (*b*.) (Interpolated from Table XV.)

Showing the Interval between the Apparent Solar Time of the Moon's Transit B, and the Time of High Water at the London Docks for each month in the year.

Apparent Solar Time of Moon's Transit B.	January.	February.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Mean.
h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
0 30	3 3·8	3 5·3	3 7·5	3 8·5	3 6·6	3 5·1	3 6·5	3 10·2	3 10·5	3 10·3	3 6·8	3 4·2	3 7·1
1 30	2 49·3	2 50·3	2 50·4	2 51·4	2 51·1	2 50·6	2 52·2	2 53·4	2 53·8	2 52·5	2 48·5	2 47·2	2 50·9
2 30	2 37·7	2 37·8	2 35·2	2 34·5	2 34·1	2 37·4	2 40·6	2 41·8	2 38·3	2 34·9	2 32·5	2 33·0	2 36·5
3 30	2 30·9	2 27·5	2 22·3	2 21·7	2 22·9	2 28·7	2 34·3	2 31·6	2 27·7	2 21·7	2 22·7	2 27·5	2 26·7
4 30	2 29·3	2 24·2	2 18·2	2 16·9	2 22·8	2 31·5	2 33·9	2 28·9	2 18·9	2 13·6	2 19·8	2 27·2	2 23·8
5 30	2 42·1	2 34·4	2 29·7	2 34·0	2 39·5	2 46·5	2 47·4	2 40·3	2 30·1	2 26·9	2 34·6	2 39·4	2 37·1
6 30	3 10·0	3 10·3	3 10·0	3 10·7	3 13·1	3 12·8	3 15·2	3 12·3	3 12·2	3 6·0	3 7·5	3 9·1	3 10·8
7 30	3 40·7	3 43·9	3 47·5	3 43·4	3 39·2	3 37·5	3 39·3	3 43·8	3 47·8	3 44·1	3 37·3	3 34·4	3 41·7
8 30	3 53·1	3 56·8	3 58·6	3 54·5	3 50·3	3 47·6	3 52·2	3 57·8	3 59·6	3 56·4	3 50·3	3 46·3	3 53·6
9 30	3 50·9	3 50·9	3 50·8	3 50·1	3 49·3	3 47·4	3 50·6	3 54·0	3 54·7	3 52·2	3 46·8	3 47·6	3 50·4
10 30	3 37·8	3 37·4	3 37·1	3 39·5	3 39·0	3 36·3	3 39·5	3 40·8	3 44·3	3 41·5	3 38·0	3 36·8	3 39·0
11 30	3 19·7	3 20·8	3 23·8	3 22·7	3 24·1	3 22·1	3 22·4	3 28·1	3 27·6	3 26·8	3 23·4	3 21·1	3 23·6

TABLE XVII. (*c.*) (Interpolated from Table XV.)

Showing the Height of High Water at the London Docks, corresponding to the Apparent Solar Time of the Moon's Transit B, in each month of the year.

Apparent Solar Time of Moon's Transit B.	January.	February.	March.	April.	May.	June.	July.	August.	Sept.	October.	Nov.	Dec.	Mean.
h m	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
0 30	22·74	22·84	22·86	22·94	22·75	22·50	22·58	22·86	22·91	22·82	22·49	22·46	22·73
1 30	22·45	22·62	22·66	22·43	22·37	22·23	22·48	22·68	22·60	22·45	22·17	22·15	22·44
2 30	21·91	22·05	22·01	21·85	21·63	21·80	22·16	22·11	22·07	21·86	21·69	21·88	21·92
3 30	21·42	21·33	21·00	20·96	20·90	21·24	21·46	21·37	21·08	20·87	20·98	21·12	21·14
4 30	20·53	20·16	19·87	19·93	20·16	20·66	20·74	20·52	20·02	19·94	20·10	20·49	20·22
5 30	19·64	19·30	18·99	19·27	19·80	20·16	20·10	19·61	19·23	19·18	19·66	19·91	19·55
6 30	19·61	19·20	19·07	19·31	19·94	20·09	19·92	19·40	19·03	19·17	19·57	19·93	19·52
7 30	20·17	19·99	19·87	20·25	20·64	20·65	20·37	20·02	19·96	20·18	20·44	20·55	20·26
8 30	20·95	20·83	20·97	21·29	21·46	21·24	21·08	20·91	21·00	21·24	21·34	21·31	21·14
9 30	21·72	21·57	21·84	22·13	22·17	21·85	21·71	21·66	21·84	22·09	22·06	21·89	21·89
10 30	22·24	22·30	22·47	22·73	22·64	22·27	22·22	22·24	22·50	22·67	22·38	22·27	22·42
11 30	22·59	22·74	22·84	23·05	22·77	22·55	22·44	22·66	22·85	22·85	22·74	22·40	22·71

In reducing the above Tables from Table XV., the quantities have been corrected for the deviation from a mean Horizontal Parallax (57′).

TABLE XVIII. (*d.*)

Showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water at the London Docks, corresponding to the Apparent Solar Time of the Moon's Transit B, for every minute of her Horizontal Parallax.

Hor. Par. 54′.						Hor. Par. 55′.					
Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Sun's Declination.	Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Sun's Declination.
	h m	h m	ft. in.	°	°		h m	h m	ft. in.	°	°
409	0 29·2	3 6·2	22 2·7	14·5	14·3	280	0 30·3	3 6·5	22 3·4	14·4	14·5
417	1 29·3	2 48·3	21 10·2	14·5	14·3	289	1 29·2	2 48·9	22 1·4	14·4	14·9
383	2 29·0	2 30·7	21 4·0	14·9	14·7	322	2 29·9	3 32·3	21 6·3	14·0	14·5
375	3 28·9	2 14·5	20 5·8	14·7	15·0	343	3 29·2	2 19·1	20 7·8	14·8	15·0
331	4 29·2	2 10·7	19 6·9	14·9	14·9	385	4 30·0	2 15·0	19 8·0	14·8	15·0
293	5 28·9	2 25·4	18 8·4	15·5	14·9	412	5 30·1	2 29·2	18 11·3	14·9	15·2
298	6 30·0	3 7·3	18 8·7	15·0	15·3	412	6 28·9	3 6·0	18 11·1	15·0	15·2
334	7 30·6	3 42·9	19 4·2	15·1	15·5	385	7 29·3	3 42·6	19 8·6	14·9	14·7
364	8 29·6	3 57·5	20 5·7	14·8	14·9	347	8 29·6	3 55·7	20 8·9	14·4	15·1
402	9 29·3	3 54·3	21 3·8	14·7	14·8	293	9 29·1	3 53·5	21 5·3	14·6	14·3
406	10 29·3	3 41·5	21 11·0	14·3	14·4	294	10 30·1	3 42·1	21 11·9	14·5	14·3
410	11 29·8	3 24·8	22 2·4	14·1	14·2	290	11 29·8	3 24·6	22 3·9	14·2	14·3
Hor. Par. 56′.						Hor. Par. 57′.					
219	0 29·2	3 5·9	22 7·0	14·3	14·2	166	0 30·0	3 6·2	22 8·4	14·5	14·3
203	1 29·5	2 49·5	22 3·0	14·2	14·3	182	1 30·1	2 51·5	22 4·8	14·9	14·4
228	2 29·0	2 35·3	21 8·5	14·6	15·1	201	2 30·2	2 36·6	21 10·8	14·9	14·6
234	3 30·2	2 24·9	20 10·4	14·8	14·9	215	3 29·9	2 27·2	21 2·9	14·7	14·3
259	4 29·4	2 19·7	20 0·5	14·9	15·1	233	4 29·9	2 24·0	20 3·6	15·0	15·2
276	5 29·4	2 32·9	19 4·2	14·7	15·2	256	5 29·2	2 37·2	19 8·3	15·5	15·0
271	6 29·8	3 8·0	19 3·3	14·8	15·4	254	6 29·4	3 8·8	19 6·3	15·0	15·3
253	7 28·8	3 41·4	20 0·3	14·7	14·9	240	7 28·6	3 42·0	20 2·7	15·1	15·2
254	8 28·7	3 56·3	20 10·5	15·1	14·5	210	8 29·1	3 52·8	21 1·5	15·0	14·5
226	9 29·0	3 52·3	21 7·6	14·7	14·6	202	9 29·0	3 51·3	21 10·2	14·8	14·5
217	10 28·5	3 40·1	22 2·9	14·5	14·5	187	10 27·7	3 39·6	22 3·0	14·7	14·9
206	11 29·0	3 25·3	22 5·8	14·6	14·7	182	11 30·0	3 21·7	22 9·0	14·0	14·0

TABLE XVIII. (*d.*) (Continued.)

Hor. Par. 58′.						Hor. Par. 59′.					
Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Sun's Declination.	Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Sun's Declination.
	h m	h m	ft. in.	°	°		h m	h m	ft. in.	°	°
158	0 31·1	3 8·0	22 9·9	14·6	14·5	180	0 28·2	3 9·4	23 0·8	14·2	14·3
173	1 29·1	2 51·5	22 7·9	14·5	14·2	182	1 30·0	2 52·4	22 10·0	14·1	14·4
197	2 29·7	2 39·7	22 1·3	14·7	14·9	204	2 30·4	2 40·1	22 4·5	14·7	14·8
208	3 29·6	2 32·0	21 4·8	15·0	14·4	282	3 31·0	2 34·3	21 7·8	14·8	14·4
268	4 28·9	2 28·3	20 5·9	14·8	15·1	404	4 32·3	2 33·6	20 10·0	14·9	14·6
288	5 29·9	2 41·7	19 11·0	14·9	14·8	533	5 30·0	2 44·1	20 3·5	15·0	15·1
290	6 28·5	3 11·7	19 8·8	15·4	14·9	518	6 29·6	3 14·2	20 1·5	15·1	15·0
254	7 28·1	3 40·2	20 6·2	14·6	15·0	381	7 27·6	3 38·6	20 8·6	14·7	15·0
220	8 27·9	3 53·0	21 3·2	15·3	14·7	258	8 27·8	3 49·3	21 6·9	14·2	14·7
184	9 28·3	3 49·4	22 1·7	14·9	14·8	216	9 28·6	3 48·4	22 2·5	14·6	14·2
171	10 30·0	3 38·1	22 6·9	14·7	14·5	176	10 29·7	3 37·5	22 10·0	14·2	14·2
174	11 29·4	3 23·7	22 10·0	14·7	14·7	164	11 28·0	3 24·6	23 0·4	14·4	14·4
Hor. Par. 60′.						Hor. Par. 61′.					
200	0 29·3	3 7·4	23 4·2	14·9	14·8	390	0 29·7	3 8·0	23 5·8	14·3	14·5
223	1 30·5	2 53·7	23 0·7	14·4	14·3	301	1 28·4	2 55·0	23 3·4	14·6	14·5
327	2 31·5	2 42·4	22 7·2	14·3	14·2	172	2 22·8	2 44·2	22 9·0	15·1	15·5
380	3 28·1	2 35·7	21 11·9	14·5	15·0	10	3 13·7	3 14·2	21 9·3	16·6	15·7
180	4 22·7	2 35·4	21 2·0	15·2	15·6						
12	5 7·6	2 50·2	20 4·5	17·2	16·2						
28	6 42·7	3 25·4	20 2·9	16·4	14·2						
207	7 35·0	3 41·0	21 0·1	15·1	15·5						
388	8 31·1	3 49·0	21 9·0	15·5	15·9	18	8 47·2	3 52·4	21 10·2	16·7	17·9
309	9 26·8	3 46·3	22 5·7	14·0	13·8	195	9 36·4	3 44·7	22 8·8	15·3	15·5
229	10 28·2	3 37·0	22 11·6	14·4	14·2	342	10 31·2	3 36·2	23 2·0	14·3	14·6
203	11 28·7	3 22·7	23 3·4	14·0	14·0	386	11 30·1	3 22·8	23 4·7	14·3	14·3

TABLE XIX. (*e.*) (Interpolated from Table XVIII.)

Apparent Solar Time of Moon's Transit B.	H. P. 54′.		H. P. 55′.		H. P. 56′.		H. P. 57′.	
	Interval.	Height of Tide.	Interval.	Height of Tide.	Interval.	Height of Tide.	Interval.	Height of Tide.
h m	h m	feet.	h m	feet.	h m	feet.	h m	feet.
0 30	3 6·1	22·22	3 6·4	22·26	3 5·7	22·55	3 6·2	22·68
1 30	2 48·2	21·83	2 48·7	22·06	2 49·4	22·21	2 51·6	22·40
2 30	2 30·5	21·31	2 32·2	21·52	2 35·0	21·67	2 36·7	21·90
3 30	2 14·5	20·44	2 19·1	20·62	2 24·9	20·86	2 27·4	21·23
4 30	2 10·9	19·52	2 15·0	19·74	2 19·6	20·03	2 23·9	20·30
5 30	2 26·2	18·69	2 29·0	18·94	2 32·9	19·34	2 37·9	19·66
6 30	3 7·3	18·72	3 6·5	18·94	3 8·1	19·26	3 9·1	19·51
7 30	3 42·8	19·34	3 42·7	19·73	3 41·7	20·03	3 42·3	20·23
8 30	3 57·5	20·47	3 56·0	20·73	3 56·0	20·90	3 52·5	21·14
9 30	3 54·2	21·32	3 53·4	21·44	3 52·1	21·64	3 51·0	21·84
10 30	3 41·3	21·90	3 42·1	21·98	3 39·7	22·24	3 39·2	22·27
11 30	3 24·8	22·16	3 24·6	22·29	3 25·0	22·46	3 21·7	22·71
	H. P. 58′.		H. P. 59′.		H. P. 60′.		H. P. 61′.	
0 30	3 8·3	22·80	3 8·9	23·03	3 7·2	23·35	3 7·9	23·47
1 30	2 51·3	22·63	2 52·4	22·79	2 53·8	23·05	2 54·6	23·24
2 30	2 39·6	22·09	2 40·2	22·37	2 42·7	22·60	2 42·8	22·61
3 30	2 32·2	21·39	2 34·5	21·62	2 35·4	21·92		
4 30	2 28·4	20·46	2 33·3	20·88	2 36·7	21·00		
5 30	2 41·7	19·92	2 44·1	20·29				
6 30	3 12·5	19·77	3 14·4	20·12				
7 30	3 40·8	20·54	3 39·3	20·76	3 40·0	20·92		
8 30	3 52·6	21·33	3 49·4	21·60	3 49·2	21·74		
9 30	3 49·1	22·17	3 48·1	22·18	3 45·6	22·45	3 45·9	22·61
10 30	3 38·0	22·56	3 37·4	22·80	3 36·6	22·97	3 36·6	23·13
11 30	3 23·5	22·82	3 24·1	23·02	3 22·4	23·24	3 22·8	23·34

In forming the above Table, the quantities have been corrected for deviations from mean Declinations.

Table XX. (*f.*)

Showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the London Docks, corresponding to the Apparent Solar Time of the Moon's Transit B, A.M. and P.M.

A.M.						P.M.					
Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Horizontal Parallax.	Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Horizontal Parallax.
January.											
	h m	h m	ft. in.	°	′		h m	h m	ft. in.	°	′
84	0 30·8	3 3·0	22 8·3	N. 19·0	57.1	83	0 29·3	3 5·0	22 10·2	S. 19·1	57·2
82	1 30·4	2 48·0	22 5·5	N. 15·3	57·4	84	1 29·0	2 51·3	22 7·2	S. 15·1	57·1
94	2 29·3	2 35·5	21 11·2	N. 10·0	57·1	92	2 28·8	2 40·8	21 11·7	S. 9·9	57·2
91	3 29·2	2 28·2	21 5·3	N. 4·1	56·9	97	3 29·9	2 32·7	21 4·7	S. 4·0	56·9
97	4 29·6	2 24·2	20 5·9	S. 1·7	56·8	92	4 30·3	2 33·6	20 6·1	N. 2·1	56·9
95	5 29·2	2 35·5	19 6·3	S. 8·2	56·9	89	5 31·0	2 47·7	19 8·1	N. 8·2	56·8
91	6 28·9	3 3·9	19 5·8	S. 13·1	56·9	88	6 29·8	3 15·0	19 8·2	N. 13·5	56·9
89	7 30·1	3 38·0	20 0·9	S. 17·7	57·0	87	7 28·5	3 43·4	20 2·7	N. 17·5	57·0
83	8 30·4	3 51·2	21 0·1	S. 20·8	57·2	86	8 29·5	3 54·3	20 11·7	N. 20·7	57·1
73	9 28·1	3 50·7	21 10·8	S. 22·4	57·2	81	9 30·2	3 50·7	21 7·1	N. 22·5	57·2
83	10 28·0	3 38·0	22 4·9	S. 22·5	57·1	79	10 30·3	3 37·5	22 1·4	N. 22·9	57·4
79	11 28·3	3 19·2	22 8·0	S. 21·2	57·2	79	11 30·0	3 20·5	22 6·8	N. 21·1	57·2
Sun's Declination S. 21°.											
February.											
86	0 29·2	3 3·5	22 9·2	N. 10·0	57·2	79	0 31·2	3 7·8	23 0·4	S. 9·8	57·3
81	1 27·5	2 48·6	22 10·0	N. 4·3	57·2	88	1 29·1	2 52·9	22 6·3	S. 4·0	57·2
91	2 28·0	2 36·0	22 1·5	S. 1·8	57·1	81	2 28·9	3 41·1	22 1·3	N. 2·2	57·2
86	3 28·9	2 27·1	21 4·6	S. 7·8	57·0	89	3 29·3	2 27·1	21 3·2	N. 8·0	56·9
84	4 30·1	2 19·8	20 1·1	S. 13·6	56·6	82	4 29·7	2 26·4	20 0·7	N. 13·8	56·8
77	5 30·0	2 28·2	19 1·3	S. 18·1	56·5	75	5 29·0	2 37·1	19 3·1	N. 18·1	56·6
73	6 29·6	3 6·9	19 1·6	S. 20·6	56·7	78	6 29·9	3 12·0	19 1·3	N. 20·6	56·7
74	7 28·5	3 41·3	20 0·2	S. 22·4	56·7	70	7 29·7	3 46·2	19 10·1	N. 22·5	56·9
73	8 29·4	3 54·4	20 11·9	S. 22·5	56·8	69	8 28·0	3 59·5	20 6·9	N. 22·8	56·9
71	9 28·5	3 50·9	21 8·6	S. 21·4	57·1	78	9 28·7	3 51·2	21 5·0	N. 21·4	57·1
80	10 29·9	3 37·8	22 6·6	S. 18·5	57·3	77	10 30·1	3 36·6	22 1·5	N. 18·5	57·2
81	11 31·6	3 22·1	22 9·8	S. 15·0	57·3	76	11 30·0	3 19·0	22 9·2	N. 14·7	57·3
Sun's Declination S. 13°.											
March.											
91	0 29·1	3 5·7	22 11·6	S. 1·1	57·0	87	0 29·5	3 9·4	22 9·9	N. 1·7	57·3
89	1 28·7	2 50·6	22 9·0	S. 6·9	57·1	96	1 31·0	2 50·5	22 7·5	N. 7·4	57·1
94	2 28·8	2 34·8	22 1·4	S. 12·6	56·9	82	2 30·5	2 35·6	21 10·3	N. 13·0	56·9
84	3 29·1	2 20·9	21 1·8	S. 17·3	56·8	86	3 28·4	2 22·2	20 9·8	N. 17·3	56·8
83	4 29·1	2 15·9	20 0·5	S. 20·2	56·7	84	4 28·4	2 17·6	19 7·3	N. 20·4	56·6
84	5 30·0	2 27·1	19 1·6	S. 21·9	56·7	82	5 30·0	2 29·9	18 7·9	N. 22·3	56·7
84	6 30·2	3 9·0	19 2·6	S. 22·8	56·8	79	6 28·9	3 10·2	18 10·5	N. 22·7	56·8
80	7 29·7	3 46·9	20 2·3	S. 21·8	56·9	83	7 28·1	3 47·9	19 6·0	N. 21·7	56·9
86	8 28·7	3 57·8	21 3·0	S. 19·2	57·2	91	8 29·3	3 58·7	20 8·8	N. 19·0	57·2
89	9 28·9	3 51·9	22 0·4	S. 15·7	57·3	86	9 30·3	3 49·0	21 9·0	N. 15·3	57·3
89	10 30·5	3 38·2	22 7·8	S. 10·6	57·6	91	10 30·2	3 35·2	22 5·4	N. 10·4	57·3
96	11 30·5	3 24·4	22 11·9	S. 4·7	57·4	91	11 30·3	3 23·0	22 10·5	N. 4·7	57·6
Sun's Declination S. 2°.											
April.											
84	0 29·0	3 8·2	23 0·8	S. 12·2	57·3	86	0 29·2	3 9·6	22 11·3	N. 12·4	57·3
83	1 29·0	2 51·7	22 6·9	S. 16·4	57·3	82	1 28·8	2 52·1	22 4·6	N. 17·0	57·2
79	2 28·9	2 35·0	22 0·6	S. 20·0	57·0	82	2 29·0	2 34·6	21 8·5	N. 19·9	57·0
78	3 29·0	2 22·6	21 1·4	S. 21·8	56·8	76	3 29·1	2 19·8	20 9·0	N. 21·9	56·9
79	4 30·2	2 17·9	20 1·8	S. 22·6	56·7	76	4 28·0	2 12·6	19 6·8	N. 22·6	56·7
79	5 30·8	2 34·7	19 5·1	S. 21·9	56·6	78	5 27·2	2 28·0	18 10·0	N. 21·8	56·5
78	6 29·1	3 11·6	19 7·3	S. 20·0	56·7	87	6 28·4	3 7·9	18 10·7	N. 19·7	56·7
89	7 28·9	3 44·6	20 6·0	S. 15·7	56·7	85	7 29·6	3 42·1	19 9·7	N. 15·9	56·7
87	8 29·3	3 56·4	21 6·3	S. 11·5	57·0	88	8 29·1	3 52·6	21 0·3	N. 11·0	56·9
95	9 29·9	3 50·4	22 4·0	S. 5·4	57·2	91	9 30·0	3 49·3	22 0·2	N. 5·3	57·1
83	10 28·2	3 40·6	22 10·5	N. 0·6	57·4	92	10 29·9	3 38·6	27 8·4	S. 0·8	57·3
92	11 28·5	3 23·9	23 1·3	N. 6·3	57·4	87	11 29·3	3 22·1	23 1·4	S. 6·9	57·4
Sun's Declination N. 19°.											

TABLE XX. (*f.*) (Continued.)

A.M.						P.M.					
Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Horizontal Parallax.	Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Horizontal Parallax.
May.											
	h m	h m	ft. in.	°	′		h m	h m	ft. in.	°	′
81	0 29·3	3 7·5	22 11·3	S. 19·7	57·7	85	0 30·7	3 6·4	22 9·3	N. 19·9	57·5
82	1 28·7	2 51·7	22 7·4	S. 21·9	57·5	79	1 31·0	2 51·5	22 3·8	N. 21·7	57·5
83	2 29·4	2 37·2	21 10·3	S. 22·5	57·2	82	2 30·6	2 31·8	21 5·9	N. 22·8	57·2
80	3 29·4	2 26·6	21 2·1	S. 21·8	56·9	83	3 30·9	2 18·4	20 7·6	N. 21·8	56·9
86	4 29·2	2 24·8	20 4·4	S. 19·9	56·8	84	4 30·3	2 18·6	19 10·1	N. 19·7	56·9
87	5 29·4	2 43·5	20 0·2	S. 16·3	56·7	84	5 29·3	2 31·4	19 4·2	N. 16·4	56·6
90	6 29·7	3 16·3	20 1·8	S. 11·7	56·5	93	6 29·2	3 7·7	19 5·8	N. 11·4	56·5
95	7 29·0	3 42·5	20 9·8	S. 5·9	56·7	97	7 30·5	3 36·1	20 3·5	N. 5·7	56·7
94	8 28·7	3 52·9	21 7·0	N. 0·0	56·8	94	8 30·9	3 48·4	21 3·1	S. 0·2	56·8
94	9 30·2	3 50·8	22 3·3	N. 6·1	57·0	89	9 29·8	3 47·7	22 0·7	S. 6·4	57·0
91	10 30·4	3 39·2	22 8·2	N. 12·0	57·3	85	10 28·0	3 38·7	22 7·7	S. 11·6	57·2
85	11 30·1	3 25·5	22 9·8	N. 16·9	57·4	90	11 27·6	3 23·6	22 9·9	S. 16·6	57·3
Sun's Declination N. 19°.											
June.											
77	0 26·3	3 7·7	22 8·3	S. 23·0	57·3	80	0 30·1	3 3·8	22 5·3	N. 22·7	57·3
79	1 27·4	2 54·3	22 5·3	S. 21·9	57·5	77	1 31·2	2 48·0	22 2·2	N. 21·8	57·2
83	2 28·4	2 41·7	22 0·3	S. 19·2	57·3	80	2 30·7	2 34·3	21 9·0	N. 19·3	57·3
86	3 29·3	2 33·6	21 4·5	S. 16·3	56·9	86	3 30·7	2 23·9	21 1·4	N. 15·8	57·1
90	4 30·2	2 35·9	20 9·4	S. 11·6	56·9	86	4 29·7	2 25·2	20 4·8	N. 11·0	56·8
94	5 30·4	2 52·3	20 1·9	S. 5·9	56·6	92	5 28·8	2 37·5	19 11·8	N. 5·9	56·8
90	6 30·4	3 18·2	20 2·2	N. 0·7	56·7	94	6 29·2	3 6·1	19 9·7	S. 0·5	56·5
91	7 29·5	3 41·3	20 8·5	N. 6·3	56·7	88	7 29·1	3 33·7	20 5·8	S. 6·8	56·9
87	8 29·1	3 47·9	21 3·0	N. 12·1	56·9	92	8 29·7	3 47·5	21 2·1	S. 11·9	56·9
82	9 28·4	3 47·4	21 10·0	N. 16·5	57·0	79	9 29·7	3 47·7	21 10·0	S. 17·2	57·0
81	10 28·9	3 36·1	22 3·0	N. 20·0	57·1	80	10 29·7	3 36·7	22 3·9	S. 19·9	57·1
76	11 29·2	3 21·6	22 6·4	N. 22·1	57·4	73	11 28·3	3 23·3	22 6·7	S. 21·9	57·1
Sun's Declination N. 23°.											
July.											
82	0 30·0	3 9·9	22 7·6	S. 19·1	57·1	77	0 30·2	3 2·9	22 7·2	N. 19·3	57·3
80	1 28·3	2 57·8	22 7·0	S. 15·9	57·3	87	1 28·5	2 48·0	22 5·8	N. 15·4	57·0
97	2 30·3	2 44·3	22 2·4	S. 11·0	57·1	89	2 28·9	2 36·8	22 1·8	N. 11·0	57·1
90	3 30·3	2 39·5	21 6·1	S. 4·8	57·1	96	3 29·1	2 30·2	21 5·9	N. 4·9	57·1
98	4 28·8	2 38·6	20 8·8	N. 0·9	56·9	100	4 29·8	2 28·3	20 8·7	S. 1·4	56·9
96	5 28·3	2 52·3	20 1·4	N. 7·0	56·9	95	5 30·4	2 40·8	20 0·7	S. 7·1	56·9
92	6 28·3	3 18·9	19 10·6	N. 12·6	56·8	93	6 29·7	3 9·9	19 10·3	S. 12·6	56·8
88	7 28·1	3 40·3	20 4·2	N. 16·4	56·8	87	7 30·0	3 38·4	20 3·8	S. 17·1	57·0
82	8 28·2	3 51·5	20 11·7	N. 20·1	56·9	83	8 29·7	3 53·1	21 1·3	S. 20·1	56·9
80	9 28·2	3 49·2	21 7·7	N. 22·3	57·1	82	9 30·1	3 52·0	21 9·3	S. 22·3	57·1
79	10 29·2	3 37·6	22 1·5	N. 23·1	57·1	75	10 30·1	3 41·4	22 4·2	S. 22·8	57·1
83	11 31·1	3 19·4	22 4·6	N. 21·4	57·1	81	11 29·9	3 25·2	22 6·6	S. 21·6	57·3
Sun's Declination N. 21°.											
August.											
92	0 30·1	3 13·9	22 10·0	S. 10·9	57·0	87	0 31·1	3 6·0	22 10·7	N. 10·4	57·0
87	1 29·1	2 57·1	22 7·3	S. 5·2	56·9	91	1 30·3	2 49·9	22 8·6	N. 4·8	56·9
91	2 28·2	2 45·3	22 0·5	N. 0·8	57·0	90	2 29·4	2 38·6	22 2·8	S. 1·2	57·0
92	3 27·9	2 36·1	21 3·0	N. 7·1	56·8	92	3 30·2	2 26·2	21 5·8	S. 7·4	56·9
93	4 29·2	2 31·7	20 4·0	N. 12·4	56·8	89	4 29·6	2 23·3	20 6·9	S. 12·6	56·7
88	5 29·2	2 43·1	19 5·7	N. 17·2	56·9	86	5 28·9	2 33·8	19 7·2	S. 17·4	56·5
87	6 29·6	3 13·0	19 3·2	N. 20·2	56·9	88	6 28·7	3 10·5	19 5·6	S. 20·3	56·9
86	7 31·4	3 42·6	19 9·7	N. 22·2	57·0	82	7 29·0	3 44·7	20 2·8	S. 22·3	57·0
81	8 31·9	3 54·6	20 9·0	N. 22·3	57·1	81	8 28·3	4 1·1	21 1·6	S. 22·7	57·1
82	9 31·5	3 50·2	21 6·6	N. 21·7	57·3	83	9 28·3	3 57·2	21 10·2	S. 21·7	57·2
80	10 31·4	3 35·2	22 1·7	N. 19·4	57·3	85	10 28·8	3 45·7	22 5·4	S. 19·3	57·3
87	11 30·5	3 24·0	22 7·3	N. 15·8	57·2	84	11 29·1	3 32·5	22 9·8	S. 15·7	57·3
Sun's Declination N. 14°.											

TABLE XX. (*f*.) (Continued.)

A.M.						P.M.					
Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Horizontal Parallax.	Number of Observations.	Apparent Solar Time of Moon's Transit B.	Interval between the Moon's Transit and the Time of high water.	Height of Tide.	Moon's Declination.	Horizontal Parallax.
September.											
	h m	h m	ft. in.	°	′		h m	h m	ft. in.	°	′
83	0 30·0	3 15·7	22 11·0	S. 0·3	57·4	96	0 28·8	3 6·7	23 0·8	N. 0·1	57·4
93	1 29·0	2 58·7	22 6·3	N. 6·6	57·1	89	1 30·5	2 49·1	22 8·5	S. 6·6	57·1
86	2 29·4	2 40·7	21 11·1	N. 11·9	57·1	87	2 30·2	2 36·0	22 2·7	S. 12·3	56·9
83	3 29·0	2 31·0	20 11·2	N. 16·5	56·9	80	3 29·1	2 23·5	21 2·7	S. 16·8	56·9
82	4 30·0	2 19·8	19 9·2	N. 20·1	56·7	82	4 28·7	2 15·2	20 1·6	S. 20·1	56·7
81	5 31·7	2 30·2	18 10·7	N. 22·1	56·6	76	5 28·3	2 27·0	19 4·8	S. 22·0	56·7
76	6 30·9	3 10·0	18 7·8	N. 22·8	56·7	79	6 27·8	3 14·4	19 3·1	S. 23·0	56·7
78	7 29·5	3 43·1	19 6·4	N. 22·1	56·9	82	7 28·7	3 52·1	20 2·9	S. 22·0	56·7
82	8 28·4	3 55·8	20 7·2	N. 19·7	57·1	82	8 28·4	4 3·6	21 3·8	S. 19·9	56·9
88	9 29·9	3 50·5	21 8·6	N. 16·4	57·2	86	9 29·1	3 58·6	22 0·5	S. 16·1	57·2
89	10 30·6	3 41·0	22 5·7	N. 11·0	57·3	87	10 29·5	3 47·5	22 7·6	S. 11·9	57·3
82	11 29·3	3 23·7	22 10·7	N. 6·1	57·3	95	11 30·2	3 31·1	22 11·4	S. 5·4	57·5
Sun's Declination N. 3°.											
October.											
86	0 29·8	3 14·1	22 10·1	N. 11·5	57·5	89	0 30·2	3 7·5	23 0·1	S. 11·5	57·5
89	1 29·9	2 55·0	22 6·4	N. 15·6	57·4	84	1 29·5	2 51·0	22 6·4	S. 16·3	57·4
84	2 30·9	2 36·2	21 7·5	N. 19·8	56·9	82	2 28·5	2 33·6	22 1·2	S. 19·2	57·1
80	3 30·0	2 21·8	20 7·5	N. 22·0	56·9	84	3 27·8	2 20·9	21 1·3	S. 21·8	56·9
77	4 29·3	2 11·2	19 7·6	N. 22·7	56·8	80	4 28·4	2 13·8	20 1·8	S. 22·7	56·8
81	5 29·0	2 20·4	18 9·0	N. 22·2	56·7	83	5 29·3	2 29·1	19 4·4	S. 22·0	56·6
87	6 30·9	2 58·0	18 9·3	N. 19·6	56·7	83	6 30·4	3 13·3	19 4·8	S. 20·0	56·6
83	7 31·6	3 37·8	19 8·1	N. 16·9	56·6	84	7 29·7	3 51·3	20 6·3	S. 16·6	56·6
92	8 30·3	3 51·4	20 10·6	N. 12·6	56·8	88	8 29·1	4 1·9	21 6·7	S. 12·4	56·9
90	9 28·8	3 47·8	21 11·3	N. 7·0	57·0	97	9 28·5	3 56·5	22 2·5	S. 7·0	57·1
98	10 28·8	3 36·6	22 8·2	N. 0·9	57·4	92	10 29·2	3 46·5	22 9·9	S. 0·3	57·4
95	11 29·7	3 23·4	22 11·1	S. 5·6	57·6	94	11 30·3	3 30·6	22 11·6	N. 5·7	57·3
Sun's Declination S. 9°.											
November.											
81	0 30·5	3 8·5	22 5·2	N. 19·8	57·5	81	0 27·2	3 6·2	22 8·4	S. 19·2	57·4
80	1 31·5	2 48·7	22 1·7	N. 21·9	57·4	78	1 26·3	2 49·7	22 4·7	S. 21·7	57·4
75	2 31·8	2 32·6	21 7·3	N. 22·6	57·3	77	2 26·7	2 33·7	21 11·5	S. 22·5	57·3
81	3 30·9	2 21·4	20 9·1	N. 21·8	57·2	82	3 27·7	2 25·4	21 4·0	S. 22·1	57·2
79	4 30·5	2 14·4	19 8·1	N. 20·4	56·9	82	4 28·4	1 23·1	20 4·9	S. 19·8	56·8
85	5 29·5	2 24·7	19 1·0	N. 17·0	56·6	84	5 27·9	2 39·8	20 0·2	S. 17·0	56·6
91	6 30·2	3 0·4	19 1·6	N. 12·1	56·7	94	6 29·8	3 13·3	19 10·3	S. 12·1	56·7
87	7 29·7	3 30·6	20 0·7	N. 6·4	56·8	90	7 30·6	3 44·1	20 8·5	S. 6·2	56·8
94	8 29·7	3 45·8	21 0·7	N. 0·5	56·8	88	8 30·1	3 55·9	21 6·8	S. 0·1	56·8
85	9 29·6	3 43·7	22 0·1	S. 5·6	57·1	89	9 29·3	3 49·7	22 1·6	N. 5·8	57·1
89	10 29·0	3 35·6	22 4·4	S. 11·2	57·2	87	10 30·2	3 40·3	22 5·6	N. 11·6	57·2
78	11 28·3	3 20·6	22 9·7	S. 16·2	57·4	81	11 30·2	3 26·6	22 9·3	N. 16·1	57·3
Sun's Declination S. 18°.											
December.											
81	0 28·6	3 4·9	22 4·7	N. 22·4	57·4	76	0 30·2	3 4·4	22 7·9	S. 22·4	57·2
78	1 29·6	2 47·3	22 1·9	N. 22·2	57·3	81	1 29·6	2 48·0	22 3·7	S. 21·8	57·3
82	2 29·2	2 32·2	21 9·3	N. 19·6	57·3	83	2 30·7	2 34·6	22 0·8	S. 19·6	57·1
89	3 29·5	2 24·3	21 0·1	N. 16·0	57·1	86	3 31·4	2 31·6	21 3·4	S. 16·0	57·1
91	4 29·2	2 22·7	20 4·5	N. 11·5	57·0	94	4 30·2	2 30·6	20 6·8	S. 10·9	56·9
101	5 29·9	2 30·9	19 6·7	N. 5·5	56·8	96	5 29·6	2 47·3	20 2·7	S. 5·3	56·9
89	6 29·7	3 2·0	19 7·7	S. 0·2	56·7	92	6 28·7	3 14·3	20 0·7	N. 0·7	56·7
97	7 28·4	3 28·4	20 3·7	S. 6·6	56·7	96	7 28·3	3 40·4	20 6·8	N. 6·7	56·7
89	8 28·5	3 43·0	21 2·6	S. 12·1	57·0	95	8 29·8	3 49·6	21 4·1	N. 12·3	56·9
87	9 28·7	3 45·5	21 9·7	S. 16·4	57·1	80	9 29·9	3 49·9	21 11·8	N. 16·9	57·1
82	10 29·9	3 35·4	22 4·5	S. 19·7	57·1	78	10 27·3	3 38·4	22 2·5	N. 19·7	57·2
81	11 30·0	3 21·2	22 5·3	S. 22·3	57·3	82	11 27·0	3 21·7	22 4·9	N. 22·1	57·1
Sun's Declination S. 23°.											

In the above Table the Lower Transits have been incorporated with the upper, the declinations are those corresponding to the Upper Transits.

TABLE XXI. (*g.*)

Showing the Diurnal Inequality at London, or the Difference in the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Interval in Table XVI., and the Difference between the Height of High Water and the Height in Table XVII.

Diurnal Inequality.									
Apparent Solar Time of Moon's Transit B.	January.			February.			March.		
	Interval.	Height.	Moon's Declination.	Interval.	Height.	Moon's Declination.	Interval.	Height.	Moon's Declination.
h m	m	feet.	°	m	feet.	°	m	feet.	°
P.M. 0 30	+ 0·9	+ ·07	S. 19·1	+ 2·6	+ ·14	S. 9·8	+ 1·6	− ·09	N. 1·7
1 30	+ 1·7	+ ·12	S. 15·1	+ 2·2	− ·15	S. 4·0	+ 0·2	− ·05	N. 7·4
2 30	+ 2·5	·00	S. 9·9	+ 2·7	·00	N. 2·2	+ 0·7	− ·12	N. 13·0
3 30	+ 2·2	·00	S. 4·0	0·0	− ·05	N. 8·0	+ 0·5	− ·20	N. 17·3
4 30	+ 4·6	+ ·02	N. 2·1	+ 3·1	− ·04	N. 13·8	+ 1·5	− ·19	N. 20·4
5 30	+ 5·9	+ ·12	N. 8·2	+ 4·9	+ ·07	N. 18·1	+ 1·5	− ·24	N. 22·3
6 30	+ 5·3	+ ·09	N. 13·5	+ 2·1	− ·02	N. 20·6	+ 1·0	− ·15	N. 22·7
7 30	+ 3·0	+ ·08	N. 17·5	+ 2·3	− ·12	N. 22·5	+ 0·7	− ·31	N. 21·7
8 30	+ 1·3	+ ·02	N. 20·7	+ 2·5	− ·19	N. 22·8	+ 0·4	− ·26	N. 19·0
9 30	+ 0·1	− ·17	N. 22·5	+ 0·2	− ·14	N. 21·4	− 1·3	− ·16	N. 15·3
10 30	+ 0·1	− ·20	N. 22·9	− 0·6	− ·22	N. 18·5	− 1·7	− ·08	N. 10·4
11 30	+ 0·8	− ·05	N. 21·1	− 1·8	− ·02	N. 14·7	− 0·5	− ·07	N. 4·7
A.M. 0 30	− 0·7	− ·07	N. 19·0	− 2·1	− ·11	N. 10·0	− 2·0	+ ·11	S. 1·1
1 30	− 1·6	− ·06	N. 15·3	− 2·5	+ ·14	N. 4·3	− 0·2	+ ·05	S. 6·9
2 30	− 2·5	− ·01	N. 10·0	− 2·4	+ ·01	S. 1·8	− 0·4	+ ·10	S. 12·6
3 30	− 2·3	+ ·03	N. 4·1	− 0·4	+ ·03	S. 7·8	− 0·7	+ ·18	S. 17·3
4 30	− 4·2	+ ·01	S. 1·7	− 2·6	+ ·04	S. 13·6	− 0·8	+ ·23	S. 20·2
5 30	− 5·8	− ·09	S. 8·2	− 4·1	− ·04	S. 18·1	− 1·3	+ ·23	S. 21·9
6 30	− 5·3	− ·10	S. 13·1	− 2·7	·00	S. 20·6	− 0·8	+ ·19	S. 22·8
7 30	− 2·7	− ·10	S. 17·7	− 2·5	+ ·13	S. 22·4	− 0·6	+ ·35	S. 21·8
8 30	− 1·6	− ·01	S. 20·8	− 2·7	+ ·21	S. 22·5	− 0·5	+ ·27	S. 19·2
9 30	− 0·2	+ ·18	S. 22·4	− 0·2	+ ·16	S. 21·4	+ 1·3	+ ·15	S. 15·7
10 30	− 0·1	+ ·18	S. 22·5	+ 0·6	+ ·19	S. 18·5	+ 1·7	+ ·05	S. 10·6
11 30	− 1·0	+ ·06	S. 21·2	+ 1·7	+ ·02	S. 15·0	+ 0·8	+ ·05	S. 4·7

	April.			May.			June.		
P.M. 0 30	+ 0·7	− ·06	N. 12·4	− 0·3	− ·08	N. 19·9	− 1·5	− ·12	N. 22·7
1 30	+ 0·3	− ·11	N. 17·0	+ 0·1	− ·14	N. 21·7	− 2·5	− ·07	N. 21·8
2 30	− 0·1	− ·16	N. 19·9	− 2·6	− ·16	N. 22·8	− 3·6	− ·11	N. 19·3
3 30	− 1·5	− ·20	N. 21·9	− 4·1	− ·22	N. 21·8	− 5·2	− ·13	N. 15·8
4 30	− 2·6	− ·32	N. 22·6	− 3·9	− ·25	N. 19·7	− 5·4	− ·20	N. 11·0
5 30	− 2·5	− ·32	N. 21·8	− 6·1	− ·34	N. 16·4	− 7·6	− ·13	N. 5·9
6 30	− 1·5	− ·35	N. 19·7	− 4·2	− ·33	N. 11·4	− 5·5	− ·08	S. 0·5
7 30	− 1·4	− ·36	N. 15·9	− 3·4	− ·29	N. 5·7	− 3·7	− ·13	S. 6·8
8 30	− 2·0	− ·23	N. 11·0	− 2·2	− ·18	S. 0·2	− 0·2	− ·04	S. 11·9
9 30	− 0·7	− ·13	N. 5·3	− 1·6	− ·11	S. 6·4	+ 0·2	− ·01	S. 17·2
10 30	− 0·7	− ·09	S. 0·8	− 0·6	− ·01	S. 11·6	+ 0·4	+ ·03	S. 19·9
11 30	− 0·8	·00	S. 6·9	− 1·1	+ ·01	S. 16·6	+ 0·8	+ ·02	S. 21·9
A.M. 0 30	− 0·8	+ ·07	S. 12·2	+ 0·3	+ ·06	S. 19·7	+ 1·4	+ ·11	S. 23·0
1 30	− 0·2	+ ·06	S. 16·4	− 0·2	+ ·13	S. 21·9	+ 2·8	+ ·07	S. 21·9
2 30	+ 0·3	+ ·18	S. 20·0	+ 2·6	+ ·18	S. 22·5	+ 3·4	+ ·13	S. 19·2
3 30	+ 1·6	+ ·18	S. 21·8	+ 4·1	+ ·29	S. 21·8	+ 4·5	+ ·14	S. 16·3
4 30	+ 2·3	+ ·30	S. 22·6	+ 3·1	+ ·25	S. 19·9	+ 4·8	+ ·15	S. 11·6
5 30	+ 2·0	+ ·28	S. 21·9	+ 5·6	+ ·31	S. 16·3	+ 7·3	+ ·12	S. 5·9
6 30	+ 1·9	+ ·37	S. 20·0	+ 4·1	+ ·33	S. 11·7	+ 5·7	+ ·16	N. 0·7
7 30	+ 1·2	+ ·34	S. 15·7	+ 3·3	+ ·27	S. 5·9	+ 3·7	+ ·14	N. 6·3
8 30	+ 1·9	+ ·25	S. 11·5	+ 2·3	+ ·19	0·0	+ 0·2	+ ·05	N. 12·1
9 30	+ 0·6	+ ·16	S. 5·4	+ 1·5	+ ·10	N. 6·1	− 0·3	+ ·01	N. 16·5
10 30	+ 1·0	+ ·08	N. 0·6	+ 0·5	− ·03	N. 12·0	− 0·4	− ·03	N. 20·0
11 30	+ 0·8	·00	N. 6·3	+ 1·4	− ·02	N. 16·9	− 0·7	− ·02	N. 22·1

TABLE XXI. (*g*.) (Continued.)

Diurnal Inequality.

Apparent Solar Time of Moon's Transit B.	July.			August.			September.		
	Interval.	Height.	Moon's Declination.	Interval.	Height.	Moon's Declination.	Interval.	Height.	Moon's Declination.
h m	m	feet.	°	m	feet.	°	m	feet.	°
P.M. 0 30	− 3·8	− ·04	N. 19·3	− 3·9	+ ·03	N. 10·4	− 4·3	+ ·08	N. 0·1
1 30	− 4·6	− ·02	N. 15·4	− 3·3	+ ·06	N. 4·8	− 4·7	+ ·10	S. 6·6
2 30	− 4·0	− ·03	N. 11·0	− 3·3	+ ·11	S. 1·2	− 2·1	+ ·17	S. 12·3
3 30	− 4·5	− ·02	N. 4·9	− 5·0	+ ·15	S. 7·4	− 3·8	+ ·15	S. 16·8
4 30	− 5·2	+ ·01	S. 1·4	− 4·2	+ ·12	S. 12·6	− 2·1	+ ·16	S. 20·1
5 30	− 6·4	− ·01	S. 7·1	− 3·8	+ ·13	S. 17·4	− 0·9	+ ·24	S. 22·0
6 30	− 4·9	− ·02	S. 12·6	− 1·0	+ ·10	S. 20·3	+ 3·3	+ ·31	S. 23·0
7 30	− 0·9	− ·05	S. 17·1	+ 1·1	+ ·23	S. 22·3	+ 4·4	+ ·38	S. 22·0
8 30	+ 0·8	+ ·06	S. 20·1	+ 3·3	+ ·24	S. 22·7	+ 3·8	+ ·38	S. 19·9
9 30	+ 1·5	+ ·04	S. 22·3	+ 3·2	+ ·18	S. 21·7	+ 4·0	+ ·18	S. 16·1
10 30	+ 2·0	+ ·11	S. 22·8	+ 4·8	+ ·17	S. 19·3	+ 3·3	+ ·07	S. 11·9
11 30	+ 2·8	+ ·06	S. 21·6	+ 4·2	+ ·11	S. 15·7	+ 3·5	+ ·01	S. 5·4
A.M. 0 30	+ 3·3	+ ·03	S. 19·1	+ 3·7	− ·03	S. 10·9	+ 5·0	− ·07	S. 0·3
1 30	+ 4·9	+ ·01	S. 15·9	+ 3·6	− ·06	S. 5·2	+ 4·6	− ·11	N. 6·6
2 30	+ 3·8	+ ·05	S. 11·0	+ 3·2	− ·11	N. 0·8	+ 2·1	− ·18	N. 11·9
3 30	+ 4·8	+ ·03	S. 4·8	+ 5·1	− ·12	N. 7·1	+ 3·7	− ·14	N. 16·5
4 30	+ 5·3	·00	N. 0·9	+ 3·9	− ·15	N. 12·4	+ 2·2	− ·17	N. 20·1
5 30	+ 6·2	+ ·03	N. 7·0	+ 3·6	− ·12	N. 17·2	+ 0·9	− ·20	N. 22·1
6 30	+ 4·9	+ ·02	N. 12·6	+ 1·1	− ·11	N. 20·2	− 2·2	− ·32	N. 22·8
7 30	+ 1·3	+ ·06	N. 16·4	− 1·5	− ·25	N. 22·2	− 4·7	− ·40	N. 22·1
8 30	− 0·9	− ·05	N. 20·1	− 3·0	− ·23	N. 22·3	− 3·8	− ·38	N. 19·7
9 30	− 1·6	− ·05	N. 22·3	− 3·1	− ·20	N. 21·7	− 3·9	− ·16	N. 16·4
10 30	− 2·0	− ·11	N. 23·1	− 5·1	− ·18	N. 19·4	− 3·0	− ·09	N. 11·0
11 30	− 2·7	− ·08	N. 21·4	− 4·0	− ·08	N. 15·8	− 4·1	− ·01	N. 6·1

	October.			November.			December.		
P.M. 0 30	− 3·1	+ ·09	S. 11·5	− 1·5	+ ·12	S. 19·2	+ 0·2	+ ·16	S. 22·4
1 30	− 2·0	·00	S. 16·3	− 0·1	+ ·08	S. 21·7	+ 0·4	+ ·09	S. 21·3
2 30	− 1·7	+ ·19	S. 19·2	− 0·9	+ ·13	S. 22·5	+ 1·5	+ ·19	S. 19·6
3 30	− 0·5	+ ·21	S. 21·8	+ 1·9	+ ·24	S. 22·1	+ 3·7	+ ·16	S. 16·0
4 30	+ 1·4	+ ·24	S. 22·7	+ 4·5	+ ·34	S. 19·8	+ 3·8	+ ·11	S. 10·9
5 30	+ 4·2	+ ·30	S. 22·0	+ 8·0	+ ·46	S. 17·0	+ 8·5	+ ·34	S. 5·3
6 30	+ 7·8	+ ·33	S. 20·0	+ 6·4	+ ·36	S. 12·1	+ 6·3	+ ·21	N. 0·7
7 30	+ 6·9	+ ·44	S. 16·6	+ 6·6	+ ·30	S. 6·2	+ 6·1	+ ·12	N. 6·7
8 30	+ 5·4	+ ·36	S. 12·4	+ 5·3	+ ·27	S. 0·1	+ 3·2	+ ·06	N. 12·3
9 30	+ 4·1	+ ·13	S. 7·0	+ 2·9	+ ·06	N. 5·8	+ 2·4	+ ·07	N. 16·9
10 30	+ 5·1	+ ·08	S. 0·3	+ 2·5	+ ·05	N. 11·6	+ 1·2	− ·06	N. 19·7
11 30	+ 3·9	+ ·07	N. 5·7	+ 3·3	− ·02	N. 16·1	− 0·2	+ ·01	N. 22·1
A.M. 0 30	+ 3·5	− ·08	N. 11·5	+ 1·5	− ·16	N. 19·8	+ 0·1	− ·16	N. 22·4
1 30	+ 2·1	·00	N. 15·6	+ 0·2	− ·09	N. 21·9	− 0·3	− ·07	N. 22·2
2 30	+ 1·6	− ·20	N. 19·8	− 0·2	− ·11	N. 22·6	− 1·5	− ·20	N. 19·6
3 30	+ 0·5	− ·21	N. 22·0	− 2·0	− ·25	N. 21·8	− 3·6	− ·13	N. 16·0
4 30	− 1·4	− ·26	N. 22·7	− 5·1	− ·39	N. 20·4	− 4·3	− ·14	N. 11·5
5 30	− 3·7	− ·35	N. 22·2	− 7·9	− ·46	N. 17·0	− 7·6	− ·29	N. 5·5
6 30	− 8·0	− ·34	N. 19·6	− 6·7	− ·37	N. 12·1	− 6·4	− ·22	S. 0·2
7 30	− 7·0	− ·45	N. 16·9	− 6·7	− ·33	N. 6·4	− 5·9	− ·14	S. 6·6
8 30	− 5·3	− ·33	N. 12·6	− 4·8	− ·23	N. 0·5	− 3·4	− ·06	S. 12·1
9 30	− 4·6	− ·13	N. 7·0	− 3·1	− ·06	S. 5·6	− 2·2	− ·07	S. 16·4
10 30	− 4·9	− ·05	N. 0·9	− 2·4	− ·04	S. 11·2	− 1·3	+ ·08	S. 19·7
11 30	− 3·4	− ·03	S. 5·6	− 3·3	+ ·01	S. 16·2	+ 0·1	− ·01	S. 22·3

TABLE XXII. (*h.*)

Showing a Comparison between the Semimenstrual Inequality at London in the Interval and in the Height, as deduced from theory and from the results of observation contained in Tables XVI. and XVII.

Moon's Hor. Par. 57′, and Decl. 15°.

Apparent Solar Time of Moon's Transit B.	Interval. ψ + a constant.		Height. *h.*	
	Theory.	Observation.	Theory.	Observation.
h m	h m	h m	feet.	feet.
0 0	3 15·3		22·76	
0 30	3 7·1	3 7·1	22·77	22·72
1 0	2 58·8		22·70	
1 30	2 51·3	2 50·9	22·58	22·44
2 0	2 43·1		22·35	
2 30	2 36·8	2 36·5	22·09	21·92
3 0	2 30·8		21·73	
3 30	2 26·8	2 26·7	21·35	21·14
4 0	2 24·8		20·90	
4 30	2 23·3	2 24·0	20·47	20·23
5 0	2 29·8		20·10	
5 30	2 37·8	2 37·5	19·75	19·57
6 0	2 52·8		19·58	
6 30	3 10·8	3 10·8	19·47	19·55
7 0	3 25·8		19·64	
7 30	3 42·8	3 41·5	19·85	20·26
8 0	3 48·8		20·25	
8 30	3 53·8	3 53·4	20·63	21·15
9 0	3 51·8		21·10	
9 30	3 49·8	3 50·4	21·50	21·89
10 0	3 44·8		21·89	
10 30	3 38·8	3 39·0	22·22	22·42
11 0	3 30·8		22·47	
11 30	3 23·8	3 23·6	22·66	22·70

The above Inequalities from theory are the same as for the preceding London Discussion*, excepting that the constant applied to ψ is now $3^h\ 8^m{\cdot}4$, formerly it was $3^h\ 6^m{\cdot}6$. They have been calculated from the expressions (See p. 117.)

$$\tan 2\psi = \frac{(A)\sin 2\phi}{1 + (A)\cos 2\phi}$$

$$h = D + (E)\{(A)\cos(2\psi - 2\phi) + \cos 2\psi\}$$

$$\log(A) = 9{\cdot}56284 \qquad \log(E) = 0{\cdot}63749 \qquad D = 16{\cdot}79$$

The columns headed "Observation" have been deduced from the quantities headed "Mean" in Tables II. and III., by applying to them proper corrections for the deviations from declination 15°.

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* Philosophical Transactions, 1836.

TABLE XXIII. (*i.*)

Showing the Calendar-month Inequality in the Interval and in the Height of High Water, as deduced from BERNOULLI's theory and from the results of observation contained in Tables XVI. and XVII. See Plate I.

Apparent Solar Time of Moon's Transit B.	January.					February.					March.					Apparent Solar Time of Moon's Transit. B.
	d ψ		d *h*		Moon's Declination.	d ψ		d *h*		Moon's Declination.	d ψ		d *h*		Moon's Declination.	
	Theory.	Observation.	Theory.	Observation.		Theory.	Observation.	Theory.	Observation.		Theory.	Observation.	Theory.	Observation.		
h m	m	m	feet.	feet.	°	m	m.	feet.	feet.	°	m	m	feet.	feet.	°	h m
0 30	0·0	−3·3	−·49	+·02	19	0·0	−1·8	+·08	+·12	10	0·0	+0·4	+·32	+·14	5	0 30
1 30	+0·3	−1·6	−·36	+·01	16	+0·1	−0·6	+·16	+·18	6	−0·4	−0·5	+·25	+·22	8	1 30
2 30	+1·2	+1·2	−·13	−·01	11	+0·2	+1·3	+·16	+·13	5	−1·7	−1·3	+·10	+·09	13	2 30
3 30	+2·8	+4·2	+·03	+·28	6	0·0	+0·8	+·10	+·19	8	−4·1	−4·4	−·11	−·14	17	3 30
4 30	+3·5	+5·3	+·10	+·30	5	−2·2	+0·2	−·09	−·17	14	−7·2	−5·8	−·36	−·36	22	4 30
5 30	+2·5	+4·6	+·08	+·07	9	−3·6	−3·1	−·28	−·27	18	−7·8	−7·8	−·52	−·58	23	5 30
6 30	0·0	−0·8	+·01	+·06	13	0·0	−0·5	−·45	−·35	21	0·0	−0·8	−·58	−·48	23	6 30
7 30	+0·9	−0·8	−·24	−·09	18	+7·0	+2·4	−·54	−·27	23	+7·8	+6·0	−·49	−·39	22	7 30
8 30	+3·0	−0·3	−·45	−·20	21	+6·9	+3·4	−·50	−·32	22	+6·5	+5·2	−·26	−·18	19	8 30
9 30	+3·3	+0·5	−·57	−·17	23	+5·2	+0·5	−·45	−·32	22	+3·7	+0·4	−·06	−·05	16	9 30
10 30	+2·1	−1·2	−·62	−·18	23	+2·4	−1·6	−·29	−·12	19	+1·7	−1·9	+·16	+·05	11	10 30
11 30	+0·1	−3·9	−·60	−·11	21	+0·7	−2·8	−·07	+·04	14	+0·3	+0·2	+·29	+·14	6	11 30
	Sun's Decl. 21°, and Par. 8″·94.					Sun's Decl. 13°, and Par. 8″·90.					Sun's Decl. 3°, and Par. 8″·84.					
	April.					May.					June.					
0 30	0·0	+1·4	+·22	+·22	13	0·0	−0·5	−·17	+·03	20	0·0	−2·0	−·34	−·22	23	0 30
1 30	−0·5	+0·5	+·04	−·01	17	−0·2	+0·2	−·28	−·07	22	+0·7	−0·3	−·29	−·21	22	1 30
2 30	−2·0	−2·0	−·12	−·07	20	−0·7	−2·4	−·30	−·29	23	+2·2	+0·9	−·14	−·12	20	2 30
3 30	−4·1	−5·0	−·24	−·18	22	−0·6	−3·8	−·22	−·24	22	+5·1	+2·0	+·11	+·10	16	3 30
4 30	−6·4	−7·1	−·31	−·30	23	+0·2	−1·2	−·07	−·07	20	+7·8	+7·5	+·36	+·43	11	4 30
5 30	−5·3	−3·5	−·32	−·30	22	+1·8	+2·0	+·10	+·23	17	+8·5	+9·0	+·55	+·59	7	5 30
6 30	0·0	−0·1	−·22	−·24	20	0·0	+2·3	+·34	+·39	12	0·0	+2·0	+·67	+·54	5	6 30
7 30	+1·9	+1·9	−·02	−·01	16	−5·4	−2·3	+·42	+·38	7	−8·2	−4·0	+·55	+·39	8	7 30
8 30	−0·1	+1·1	+·20	+·14	11	−6·4	−3·1	+·41	+·31	5	−8·2	−5·8	+·34	+·09	12	8 30
9 30	−1·2	−0·3	+·34	+·24	6	−4·7	−1·1	+·34	+·28	7	−4·7	−3·0	+·06	−·04	17	9 30
10 30	−0·6	+0·5	+·38	+·31	5	−2·4	0·0	+·20	+·22	12	−2·2	−2·7	−·14	−·15	20	10 30
11 30	−0·3	−0·9	+·37	+·35	7	−0·7	+0·5	−·02	+·07	17	−0·6	−1·5	−·29	−·15	22	11 30
	Sun's Decl. 10°, and Par. 8″·76.					Sun's Decl. 19°, and Par. 8″·70.					Sun's Decl. 23°, and Par. 8″·66.					
	July.					August.					September.					
0 30	0·0	−0·6	−·14	−·14	20	0·0	+3·1	+·30	+·14	11	0·0	+3·4	+·46	+·19	5	0 30
1 30	+1·3	+1·3	+·05	+·04	16	+0·9	+2·5	+·41	+·24	7	0·0	+2·9	+·39	+·16	8	1 30
2 30	+2·8	+4·1	+·26	+·24	11	+2·2	+5·3	+·44	+·19	4	−0·9	+1·8	+·26	+·15	12	2 30
3 30	+6·8	+7·6	+·41	+·32	6	+2·9	+4·9	+·35	+·23	8	−2·7	+1·0	+·02	−·06	17	3 30
4 30	+8·7	+9·9	+·48	+·51	5	+2·2	+4·9	+·22	+·29	13	−5·4	−5·1	−·18	−·21	20	4 30
5 30	+7·1	+9·9	+·50	+·53	8	−0·3	+2·8	·00	+·04	18	−6·4	−7·4	−·36	−·34	22	5 30
6 30	0·0	+4·4	+·42	+·37	13	0·0	+1·5	−·16	−·15	21	0·0	+1·4	−·45	−·52	23	6 30
7 30	−3·7	−2·2	+·20	+·11	17	+3·0	+2·3	−·21	−·24	22	+6·4	+6·3	−·36	−·30	22	7 30
8 30	−2·2	−1·2	−·01	−·07	20	+3·7	+4·4	−·24	−·24	23	+4·7	+6·2	−·18	−·15	20	8 30
9 30	−0·7	+0·2	−·22	−·18	23	+2·3	+3·6	−·19	−·23	22	+2·3	+4·3	+·02	−·05	17	9 30
10 30	−0·5	+0·5	−·27	−·20	23	+0·4	+1·8	−·03	−·18	19	+0·9	+5·3	+·26	+·08	12	10 30
11 30	−0·5	−1·2	−·25	−·26	22	−0·2	+4·5	+·10	−·04	16	−0·1	+4·0	+·41	+·15	7	11 30
	Sun's Decl. 21°, and Par. 8″·66.					Sun's Decl. 14°, and Par. 8·″70.					Sun's Decl. 4°, and Par. 8″·76.					
	October.					November.					December.					
0 30	0·0	+ 3·2	+·13	+·10	12	0·0	−0·3	−·42	−·23	20	0·0	−2·9	−·77	−·26	23	0 30
1 30	−0·9	+ 1·6	−·05	+·01	16	−1·0	−2·4	−·53	−·27	22	−0·5	−3·7	−·70	−·29	22	1 30
2 30	−2·8	− 1·6	−·24	−·06	20	−2·3	−4·0	−·56	−·23	23	−0·2	−3·5	−·53	−·04	20	2 30
3 30	−5·3	− 5·0	−·37	−·27	22	−3·2	−4·0	−·47	−·16	22	+1·3	+0·8	−·27	−·02	16	3 30
4 30	−8·2	−10·4	−·45	−·29	23	−3·4	−4·8	−·34	−·13	20	+2·6	+3·2	−·01	+·26	11	4 30
5 30	−6·7	−10·6	−·46	−·39	22	−1·2	−2·9	−·22	+·09	18	+4·1	+1·9	+·15	+·34	7	5 30
6 30	0·0	− 4·8	−·37	−·38	20	0·0	−3·3	+·05	+·02	12	0·0	−1·7	+·25	+·38	5	6 30
7 30	+3·3	+ 2·6	−·21	−·08	17	−2·4	−4·2	+·14	+·18	7	−3·8	−7·1	+·13	+·29	8	7 30
8 30	+1·7	+ 3·0	+·04	+·09	12	−2·8	−3·1	+·15	+·19	5	−3·0	−7·1	−·04	+·16	12	8 30
9 30	0·0	+ 1·8	+·19	+·20	7	−2·1	−3·6	+·09	+·17	7	+0·9	−2·8	−·32	·00	17	9 30
10 30	+0·2	+ 2·5	+·26	+·25	5	−0·8	−1·0	·00	−·04	12	+0·2	−2·2	−·53	−·15	20	10 30
11 30	+0·1	+ 3·2	+·24	+·15	7	−0·2	−0·2	−·23	+·04	16	+0·6	−2·5	−·70	−·30	23	11 30
	Sun's Decl. 9°, and Par. 8″·84.					Sun's Decl. 18°, and Par. 8″·90.					Sun's Decl. 23°, and Par. 8″·94.					

TABLE XXIV. (*j.*)

Showing the Moon's Parallax Inequality in the Interval and in the Height of High Water, as deduced from BERNOULLI's theory and from the results of observation contained in Table XIX. See Plate II.

Apparent Solar Time of Moon's Transit B.	H. P. 54′.				H. P. 55′.				Apparent Solar Time of Moon's Transit B.
	d ψ		d *h*		d ψ		d *h*		
	Theory.	Observation.	Theory.	Observation.	Theory.	Observation.	Theory.	Observation.	
h m	m	m	feet.	feet.	m	m	feet.	feet.	h m
0 30	0·0	− 0·1	− ·66	− ·46	0·0	+ 0·2	− ·45	− ·42	0 30
1 30	− 2·0	− 3·4	− ·66	− ·57	− 1·3	− 2·9	− ·45	− ·34	1 30
2 30	− 4·2	− 6·2	− ·64	− ·59	− 2·7	− 4·5	− ·44	− ·38	2 30
3 30	− 6·5	−12·9	− ·62	− ·79	− 4·2	− 8·3	− ·42	− ·61	3 30
4 30	− 8·7	−13·0	− ·61	− ·78	− 5·6	− 8·9	− ·42	− ·56	4 30
5 30	− 8·4	−11·7	− ·64	− ·97	− 5·2	− 8·9	− ·44	− ·72	5 30
6 30	0·0	− 1·8	− ·66	− ·79	0·0	− 2·6	− ·45	− ·57	6 30
7 30	+ 8·4	+ 0·5	− ·64	− ·89	+ 5·2	+ 0·4	− ·44	− ·50	7 30
8 30	+ 8·7	+ 5·0	− ·61	− ·67	+ 5·6	+ 3·5	− ·42	− ·41	8 30
9 30	+ 6·5	+ 3·2	− ·62	− ·52	+ 4·2	+ 2·4	− ·42	− ·40	9 30
10 30	+ 4·2	+ 2·1	− ·64	− ·37	+ 2·7	+ 2·9	− ·44	− ·29	10 30
11 30	+ 2·0	+ 3·1	− ·66	− ·55	+ 1·3	+ 2·9	− ·45	− ·42	11 30
	H. P. 56′.				H. P. 57′.				
0 30	0·0	− 0·5	− ·23	− ·10	0·0	0·0	·00	·00	0 30
1 30	− 0·6	− 2·2	− ·23	− ·19	0·0	0·0	·00	·00	1 30
2 30	− 1·3	− 1·7	− ·23	− ·23	0·0	0·0	·00	·00	2 30
3 30	− 2·0	− 2·5	− ·21	− ·37	0·0	0·0	·00	·00	3 30
4 30	− 2·7	− 4·3	− ·21	− ·27	0·0	0·0	·00	·00	4 30
5 30	− 2·5	− 6·0	− ·22	− ·32	0·0	0·0	·00	·00	5 30
6 30	0·0	− 1·0	− ·23	− ·25	0·0	0·0	·00	·00	6 30
7 30	+ 2·5	− 0·6	− ·22	− ·20	0·0	0·0	·00	·00	7 30
8 30	+ 2·7	+ 0·5	− ·21	− ·24	0·0	0·0	·00	·00	8 30
9 30	+ 2·0	+ 1·1	− ·21	− ·20	0·0	0·0	·00	·00	9 30
10 30	+ 1·3	+ 0·5	− ·23	− ·03	0·0	0·0	·00	·00	10 30
11 30	+ 0·6	+ 3·3	− ·23	− ·25	0·0	0·0	·00	·00	11 30
	H. P. 58′.				H. P. 59′.				
0 30	0·0	+ 2·1	+ ·24	+ ·12	0·0	+ 2·7	+ ·49	+ ·35	0 30
1 30	+ 0·6	− 0·3	+ ·24	+ ·23	+ 1·2	+ 0·8	+ ·48	+ ·39	1 30
2 30	+ 1·3	+ 2·9	+ ·23	+ ·19	+ 2·5	+ 3·5	+ ·47	+ ·47	2 30
3 30	+ 2·0	+ 4·8	+ ·22	+ ·16	+ 3·8	+ 7·1	+ ·45	+ ·39	3 30
4 30	+ 2·5	+ 4·5	+ ·22	+ ·16	+ 4·8	+ 9·4	+ ·45	+ ·58	4 30
5 30	+ 2·0	+ 3·8	+ ·23	+ ·26	+ 4·2	+ 6·2	+ ·46	+ ·63	5 30
6 30	0·0	+ 3·4	+ ·24	+ ·26	0·0	+ 5·3	+ ·49	+ ·61	6 30
7 30	− 2·0	− 1·5	+ ·23	+ ·31	− 4·2	− 3·0	+ ·46	+ ·53	7 30
8 30	− 2·5	+ 0·1	+ ·22	+ ·19	− 4·8	− 3·1	+ ·45	+ ·46	8 30
9 30	− 2·0	− 1·9	+ ·22	+ ·33	− 3·8	− 2·9	+ ·45	+ ·34	9 30
10 30	− 1·3	− 1·2	+ ·23	+ ·29	− 2·5	1·8	+ ·47	+ ·53	10 30
11 30	− 0·6	+ 1·8	+ ·24	+ ·11	− 1·2	+ 2·4	+ ·48	+ ·31	11 30
	H. P. 60′.				H. P. 61′.				
0 30	0·0	+ 1·0	+ ·75	+ ·67	0·0	+ 1·7	+1·01	+ ·79	0 30
1 30	+ 1·8	+ 2·2	+ ·73	+ ·65	+ 2·3	+ 3·0	+0·99	+ ·84	1 30
2 30	+ 3·6	+ 6·0	+ ·72	+ ·70	+ 4·7	+ 6·1	+0·97	+ ·71	2 30
3 30	+ 5·5	+ 8·0	+ ·70	+ ·69	+ 7·1		+0·95		3 30
4 30	+ 6·9	+12·8	+ ·69	+ ·70	+ 8·9		+0·94		4 30
5 30	+ 6·1		+ ·71		+ 7·6		+0·97		5 30
6 30	0·0		+ ·75		0·0		+1·01		6 30
7 30	− 6·1	− 2·3	+ ·71	+ ·69	− 7·6		+0·97		7 30
8 30	− 6·9	− 3·3	+ ·69	+ ·60	− 8·9		+0·94		8 30
9 30	− 5·5	− 5·4	+ ·70	+ ·61	− 7·1	− 5·1	+0·95	+ ·77	9 30
10 30	− 3·6	− 2·6	+ ·72	+ ·70	− 4·7	− 2·6	+0·97	+ ·86	10 30
11 30	− 1·8	+ 0·7	+ ·73	+ ·53	− 2·3	− 1·1	+0·99	+ ·63	11 30

Table XXV. (*k*.)

Showing the Diurnal Inequality in the Interval and in the Height of High Water for the first six months of the year, for the Moon's Transit B, p.m. See Plate III.

Apparent Solar Time of Moon's Transit B.	January.		February.		March.		Apparent Solar Time of Moon's Transit B.
	d ψ.	d h.	d ψ.	d h.	d ψ.	d h.	
	Observation.	Observation.	Observation.	Observation.	Observation.	Observation.	
P.M.							
h m	m	feet.	m	feet.	m	feet.	h m
0 30	+ 2·3	+ ·05	+ 3·2	+ ·05	+ 3·2	− ·09	0 30
1 30	+ 3·2	+ ·05	+ 2·8	− ·10	+ 2·5	− ·08	1 30
2 30	+ 3·2	+ ·02	+ 3·0	− ·05	+ 1·4	− ·15	2 30
3 30	+ 3·3	·00	+ 2·5	− ·10	+ 2·1	− ·16	3 30
4 30	+ 4·9	·00	+ 3·5	− ·08	+ 1·6	− ·17	4 30
5 30	+ 6·0	+ ·05	+ 4·1	− ·03	+ 1·2	− ·22	5 30
6 30	+ 5·1	+ ·05	+ 1·6	− ·06	− 0·9	− ·23	6 30
7 30	+ 2·0	+ ·06	+ 0·6	− ·17	− 1·8	− ·36	7 30
8 30	+ 0·2	− ·02	− 0·4	− ·22	− 1·7	− ·32	8 30
9 30	− 0·7	− ·11	− 1·5	− ·16	− 2·7	− ·17	9 30
10 30	− 0·9	− ·15	− 2·7	− ·20	− 2·5	− ·08	10 30
11 30	− 1·0	− ·07	− 2·9	− ·06	− 2·2	− ·04	11 30
	April.		May.		June.		
0 30	+ 1·9	− ·08	+ 0·6	− ·10	− 0·8	− ·14	0 30
1 30	+ 1·2	− ·05	0·0	− ·11	− 1·4	− ·08	1 30
2 30	+ 0·8	− ·18	− 1·1	− ·15	− 2·6	− ·15	2 30
3 30	− 0·5	− ·20	− 3·0	− ·23	− 4·3	− ·15	3 30
4 30	− 2·0	− ·28	− 4·2	− ·30	− 4·6	− ·15	4 30
5 30	− 3·3	− ·31	− 7·0	− ·40	− 7·8	− ·23	5 30
6 30	− 4·7	− ·34	− 5·3	− ·35	− 5·9	− ·17	6 30
7 30	− 4·2	− ·40	− 5·0	− ·30	− 4·9	− ·12	7 30
8 30	− 2·7	− ·30	− 3·8	− ·21	− 1·7	− ·05	8 30
9 30	− 3·4	− ·13	− 2·2	− ·09	− 1·1	− ·04	9 30
10 30	− 2·9	− ·08	− 1·5	− ·03	− 0·4	+ ·04	10 30
11 30	− 2·3	− ·03	− 2·2	+ ·01	+ 0·5	+ ·02	11 30

The tide depending on the Moon's Transit a.m. for the last six months has the same inequality and the same signs as the above; and in the first six months a.m. and the last six months p.m. the same values obtain, but with a contrary sign.

The quantities in the columns headed "Observation" have been obtained by taking the mean of January and July, February and August, &c., a.m. and p.m., as explained in p. 100.

TABLE XXVI.

Showing that part of the Diurnal Inequality in the Height of High Water depending on the Moon, calculated from the expression $d\,h = B \sin 2\,\delta'$, assuming for Parallax 57′, $B = 0{\cdot}5$ feet.

$d\,h$.

Moon's Declination.	Moon's Horizontal Parallax.				
	54′.	55′.	57′.	59′.	61′.
3	·04	·05	·05	·06	·06
6	·09	·09	·10	·11	·12
9	·13	·14	·15	·17	·18
12	·17	·18	·20	·22	·24
15	·21	·22	·25	·27	·30
18	·25	·26	·29	·32	·35
21	·28	·30	·33	·37	·40
24	·31	·33	·37	·40	·44
27	·34	·36	·40	·44	·48
30	·37	·39	·43	·47	·51

For Moon's Upper Transit the correction in the above Table has the same sign as the Declination. For the Lower Transit it has a contrary sign.

This Table was originally calculated for an hourly variation of φ, and for a Mean Parallax from the expression $d\,h = B \sin 2\,\delta \cos \psi$; but it being found that as long as the factor B was less than unity, $\cos \psi$ might always be considered equal to 1, it was thought preferable to Tabulate it in the above form, which admits of B varying as the cube of the Moon's Parallax.

TABLE XXVII.

The following Table contains the part of the Diurnal Inequality in the Height depending upon the Sun's Declination, calculated from the expression

$(A)\ B \sin 2\,\delta \cos \varphi \qquad \log (A) = 9{\cdot}56965.$

φ.	Sun's Declination.								φ.
	3°.	6°.	9°.	12°.	15°.	18°.	21°.	24°.	
0	+ ·02	+ ·04	+ ·06	+ ·08	+ ·10	+ ·11	+ ·13	+ ·14	360
15	+ ·02	+ ·04	+ ·06	+ ·08	+ ·09	+ ·11	+ ·13	+ ·14	345
30	+ ·02	+ ·04	+ ·05	+ ·07	+ ·09	+ ·10	+ ·12	+ ·13	330
45	+ ·02	+ ·03	+ ·05	+ ·06	+ ·08	+ ·09	+ ·11	+ ·12	315
60	+ ·01	+ ·03	+ ·04	+ ·05	+ ·06	+ ·07	+ ·08	+ ·09	300
75	+ ·01	+ ·02	+ ·02	+ ·03	+ ·04	+ ·04	+ ·05	+ ·06	285
90	·00	·00	·00	·00	·00	·00	·00	·00	270
105	− ·01	− ·02	− ·02	− ·03	− ·04	− ·04	− ·05	− ·06	255
120	− ·01	− ·03	− ·04	− ·05	− ·06	− ·07	− ·08	− ·09	240
135	− ·02	− ·03	− ·05	− ·06	− ·08	− ·09	− ·11	− ·12	225
150	− ·02	− ·04	− ·05	− ·07	− ·09	− ·10	− ·12	− ·13	210
165	− ·02	− ·04	− ·06	− ·08	− ·09	− ·11	− ·13	− ·14	195
180	− ·02	− ·04	− ·06	− ·08	− ·10	− ·11	− ·13	− ·14	180

Table XXVIII. (*l.*)

Showing the Interval and Height of High Water at the London Docks, with the Sun's and Moon's Declinations, and the Moon's Horizontal Parallax, (for the Mean of all the Moon's Transits B occurring between 0^h and 1^h) for every year from 1802 to 1835.

Year.	Number of Observations.	Moon's Transit B.	Interval.	Height of Tide.	Moon's Declination.	Moon's Hor. Par.	Sun's Declination.
		h m	h m	ft. in.	°	′	°
1802	58	0 29·4	3 12·2	22 3·8	17·0	57·5	14·1
1803	59	0 29·1	3 12·4	22 6·4	17·1	57·4	14·3
1804	56	0 29·2	3 13·8	22 10·7	15·7	57·6	13·8
1805	54	0 27·9	3 11·2	22 7·0	15·8	57·3	14·8
1806	57	0 30·3	3 12·4	22 7·2	14·4	57·5	14·5
1807	57	0 31·3	3 6·7	22 7·7	13·5	57·3	14·6
1808	55	0 32·5	3 10·1	22 7·2	12·7	57·3	14·1
1809	59	0 29·2	3 10·6	22 11·0	12·0	57·2	14·7
1810	54	0 29·3	3 10·7	22 10·2	11·6	57·7	15·1
1811	60	0 29·8	3 7·2	23 0·4	11·1	57·3	14·3
1812	53	0 27·8	3 6·7	23 1·1	11·3	57·4	14·0
1813	59	0 27·7	3 5·3	22 9·9	12·8	57·1	14·4
1814	59	0 29·0	3 6·3	22 10·5	13·5	57·5	14·4
1815	59	0 29·5	3 5·9	22 7·0	15·0	57·1	14·5
1816	57	0 27·9	3 5·1	22 9·2	15·2	57·2	13·8
1817	57	0 29·2	3 3·4	22 7·8	16·6	57·1	14·3
1818	57	0 28·4	3 3·5	22 9·4	17·5	57·4	14·5
1819	55	0 28·7	3 3·6	22 6·7	17·0	57·3	14·1
1820	58	0 30·2	3 2·7	22 8·7	17·3	57·3	14·2
1821	58	0 31·1	3 2·1	22 9·5	17·5	57·0	14·1
1822	59	0 30·7	3 3·6	22 8·1	17·2	57·2	14·4
1823	57	0 29·7	3 7·1	22 10·5	15·7	57·2	14·1
1824	61	0 29·6	3 4·1	22 11·3	15·3	57·3	14·1
1825	63	0 30·4	3 7·0	22 10·7	14·2	57·3	14·9
1826	59	0 28·0	3 9·5	22 11·2	13·0	57·3	14·5
1827	53	0 29·6	3 6·1	22 11·3	12·4	57·4	14·9
1828	61	0 31·5	3 5·5	23 2·0	11·5	57·1	14·6
1829	56	0 29·9	3 6·0	23 2·0	11·3	57·3	15·1
1830	57	0 31·9	3 2·2	23 1·1	11·6	57·2	14·7
1831	57	0 31·3	3 5·1	23 0·5	12·1	57·1	14·3
1832	55	0 30·0	3 8·5	22 5·0	13·5	57·2	15·3
1833	60	0 29·1	3 11·6	22 9·3	13·5	57·4	14·4
1834	57	0 27·8	3 11·0	22 8·6	15·5	57·4	14·9
1835	53	0 28·0	3 11·1	22 8·3	15·5	57·5	13·7

Table XXIX. (*m.*)

Interpolated from Table XXVIII. by reducing each quantity to Moon's Transit (B) $0^h 30^m$, and correcting for deviation from Mean Declinations and Parallax.

Year.	Interval.	Height.
	h m	ft.
1802	3 11·8	22·27
1803	3 12·0	22·51
1804	3 13·3	22·79
1805	3 10·4	22·54
1806	3 12·2	22·48
1807	3 6·8	22·52
1808	3 10·6	22·44
1809	3 10·3	22·76
1810	3 10·1	22·58
1811	3 6·7	22·80
1812	3 5·9	22·85
1813	3 4·6	22·67
1814	3 5·7	22·70
1815	3 5·7	22·56
1816	3 4·4	22·71
1817	3 3·1	22·68
1818	3 2·9	22·77
1819	3 3·0	22·57
1820	3 2·6	22·75
1821	3 2·4	22·88
1822	3 3·7	22·72
1823	3 7·0	22·85
1824	3 3·8	22·88
1825	3 6·9	22·80
1826	3 8·8	22·77
1827	3 5·8	22·75
1828	3 5·8	23·03
1829	3 5·8	22·97
1830	3 2·6	22·93
1831	3 5·5	22·90
1832	3 8·4	22·32
1833	3 11·2	22·63
1834	3 10·1	22·65
1835	3 10·3	22·66

Table XXX. (*n.*)

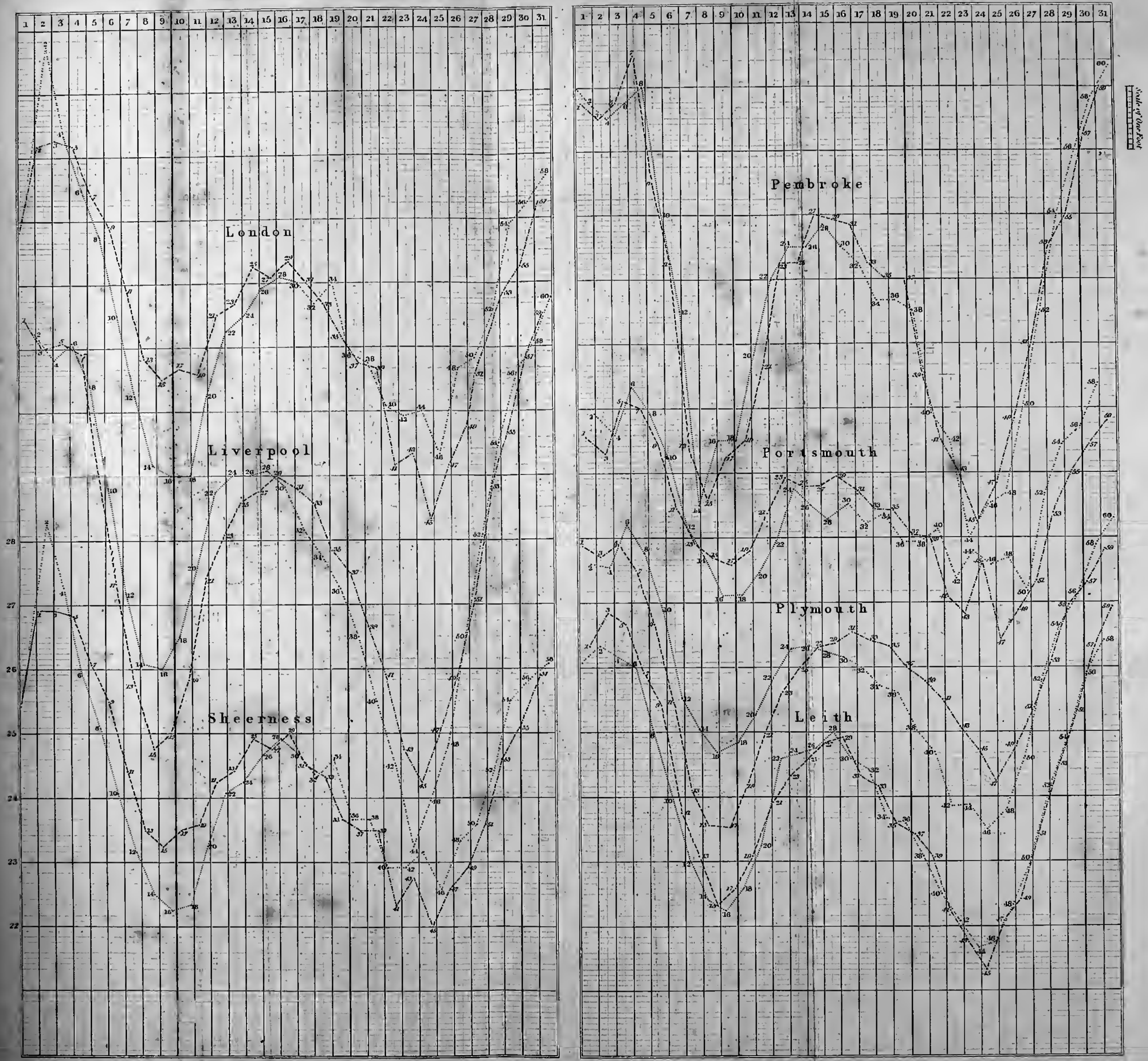
Showing the *Establishment* of the Port of London since 1802, obtained from Table XXIX. by altering the argument from Transit B to Transit F, and reducing it to $0^h 0^m$ from $0^h 30^m$. Moon's Hor. Par. 57′, and Decl. 15°.

Moon's Transit F = $0^h 0^m$.

Year.	Interval*.	Height.	Year.	Interval.	Height.
	h m	ft.		h m	ft.
1802	2 5·9	21·90	1819	1 57·1	22·20
1803	2 6·1	22·14	1820	1 56·7	22·38
1804	2 7·4	22·42	1821	1 56·5	22·51
1805	2 4·5	22·17	1822	1 57·8	22·35
1806	2 6·3	22·11	1823	2 1·1	22·48
1807	2 0·9	22·15	1824	1 57·9	22·51
1808	2 4·7	22·07	1825	2 1·0	22·43
1809	2 4·4	22·39	1826	2 2·9	22·40
1810	2 4·2	22·21	1827	1 59·9	22·38
1811	2 0·8	22·43	1828	1 59·9	22·66
1812	2 0·0	22·48	1829	1 59·9	22·60
1813	1 58·7	22·30	1830	1 56·7	22·56
1814	1 59·8	22·33	1831	1 59·6	22·53
1815	1 59·8	22·19	1832	2 2·5	21·95
1816	1 58·5	22·34	1833	2 5·3	22·26
1817	1 57·2	22·31	1834	2 4·2	22·28
1818	1 57·0	22·40	1835	2 4·4	22·29

* i. e. *Establishment.*

Heights of High Water in May 1836 — See Table XXXI.



All the Heights of Tides depending upon { Upper Transits A. A. M. are thus connected ----- } and are marked
A. P. M ———— ----- } by odd numbers
Lower Transits A. P. M ———— ...... } and are marked
A. A. M ———— ...... } by even numbers

The Moon's declination is north from May 12th to May 26th The Heights caused by the same Tide are marked with the

Table XXXI.

Observations of High Water in May 1836.

Date.	Plymouth.		Portsmouth.		London Docks.		Pembroke.		Liverpool. Salthouse Dock.		Leith.	
	Time.	Height.	Time.	Height.	Time.	Height.	Time.	Height.	Time.	Height.	Time.	Height.
1836.	h m	ft. in.	h m	ft. in.	h m	ft. in.	h m	ft. in.	h m	ft. in.	h m	ft. in.
May 1. A.M.	5 12	17 10½	11 28	18 6·9	1 40	22 10	6 20	22 9	11 6	18 5	2 12	16 1
P.M.	5 45	17 7¼	11 48	18 11·8	2 10	23 8	6 40	22 9	11 25	18 3	2 24	16 4
2. A.M.	6 5	17 8½	……	……	2 20	24 2	7 0	22 6	11 50	18 0	2 46	16 4
P.M.	6 22	17 6½	0 5	18 3·9	2 45	25 9	7 15	22 6	……	……	3 4	16 10
3. A.M.	6 44	17 10¾	0 35	18 7·2	2 55	24 3	7 35	22 9	0 5	17 10	3 23	16 2
P.M.	7 6	18 2¼	0 55	19 1·5	3 25	24 4	8 0	22 9	0 30	18 1	3 54	16 8
4. A.M.	7 28	17 6	1 15	19 4·9	3 40	24 2	8 0	23 6	0 50	18 1	4 18	16 0
P.M.	7 54	17 9¼	1 35	19 0·2	4 0	23 6	8 25	23 0	1 12	17 11	4 43	15 11
5. A.M.	8 13	16 8	1 58	18 11·4	4 25	23 5	8 50	21 6	1 26	17 5	5 8	15 0
P.M.	8 22	16 10½	2 30	18 5·2	4 50	22 9	9 10	21 0	2 0	16 0	5 35	15 5
6. A.M.	9 1	15 5½	2 50	18 3·2	5 15	22 11	9 30	20 3	2 20	15 10	6 5	14 0
P.M.	9 32	15 6¼	3 25	17 5·5	5 35	21 6	10 10	19 6	2 50	14 4	6 27	13 9
7. A.M.	9 56	14 1¼	3 45	17 2·0	6 10	21 11	10 45	17 5	3 15	14 2	7 0	13 0
P.M.	10 17	15 1½	4 30	16 11·0	6 20	20 3	11 10	16 6	3 49	12 9	7 40	13 1
8. A.M.	11 2	13 6¾	5 15	16 8·6	7 15	20 10	11 55	16 7	4 29	13 1	8 18	12 5
P.M.	11 37	14 8¼	5 45	16 8·7	7 35	19 2	……	……	5 6	11 9	9 6	12 4
9. A.M.	……	……	6 15	16 1·2	8 35	20 6	12 25	17 6	5 50	13 0	9 30	12 3
P.M.	0 32	13 6¼	7 0	16 7·0	9 20	19 0	12 55	17 3	6 40	12 0	10 18	12 7
10. A.M.	12 55	14 10	7 20	16 1·3	10 5	20 8	1 10	17 6	7 16	13 6	10 34	12 7
P.M.	1 47	14 2¼	8 5	16 9·6	10 30	19 0	2 30	17 6	7 50	12 10	11 28	13 1
11. A.M.	2 19	15 3¼	8 43	16 5·7	11 15	20 7	2 55	18 10	8 25	14 7	11 36	13 3
P.M.	2 58	14 11¾	9 8	17 4·7	11 30	20 3	3 45	18 8	8 55	14 5	……	……
12. A.M.	3 15	15 10	9 42	16 11·5	……	……	4 5	20 0	9 14	15 9	0 10	13 11
P.M.	3 40	15 7½	10 5	17 11·0	12 10	21 6	4 30	20 3	9 42	15 1	0 22	14 7
13. A.M.	3 53	16 3½	10 20	17 9·0	12 25	21 3	4 50	20 6	9 57	16 0	0 58	14 4
P.M.	4 23	16 0	10 46	17 9·5	1 0	21 8	5 10	20 3	10 16	15 7	1 2	14 8
14. A.M.	4 43	16 4	10 55	17 6·0	1 10	21 6	5 30	20 6	10 34	16 0	1 32	14 7
P.M.	5 2	16 3¾	11 15	17 9·8	1 25	22 3	5 30	21 0	10 48	15 9	1 38	14 9
15. A.M.	5 19	16 3	11 32	17 3·5	1 45	21 11	5 45	20 10	11 10	16 1	1 55	14 10
P.M.	5 28	16 4½	11 51	17 11·2	2 10	22 1	6 10	20 11	11 24	16 0	2 12	15 0
16. A.M.	5 47	16 2	……	……	2 20	22 1	6 35	20 6	11 45	15 10	2 29	14 11
P.M.	6 4	16 6¼	0 14	17 6·2	2 45	22 4	6 45	20 10	11 58	15 10	2 57	14 7
17. A.M.	6 26	16 0	0 24	17 8·7	3 5	22 0	7 15	20 3	……	……	3 13	14 4
P.M.	6 35	16 5	0 39	17 2·8	3 15	22 0	7 35	20 3	0 16	15 2	3 32	14 5
18. A.M.	6 47	15 8¾	0 58	17 5·3	3 30	21 8	7 50	19 8	0 30	15 6	3 50	14 2
P.M.	7 4	16 4	1 20	17 4·5	3 50	21 8	8 0	20 0	0 46	14 9	4 10	13 8
19. A.M.	7 31	15 7¼	1 35	17 5·0	4 10	22 0	8 25	19 8	1 4	14 10	4 20	13 7
P.M.	7 36	16 0	1 50	16 11·3	4 20	21 2	8 35	20 0	1 22	14 3	4 38	13 7
20. A.M.	7 53	15 0¼	2 0	17 0·5	4 35	20 11	8 45	19 6	1 35	14 6	4 58	13 5
P.M.	8 1	15 9½	2 20	16 11·3	4 45	20 9	9 0	18 6	2 0	13 6	5 26	13 1
21. A.M.	8 36	14 8¼	2 37	17 0·0	5 10	20 9	9 30	17 11	2 15	13 8	5 41	13 1
P.M.	8 45	15 6	3 0	17 1·6	5 15	20 8	9 45	17 6	2 40	12 6	6 8	12 6
22. A.M.	9 6	13 10¼	3 14	16 1·0	5 55	20 1	10 10	17 6	3 0	12 11	6 37	12 3
P.M.	9 29	15 1	3 30	16 4·5	6 0	19 2	10 30	17 0	3 19	11 6	7 5	12 1
23. A.M.	9 53	13 10½	3 56	15 9·8	6 35	19 11	11 0	16 0	3 50	11 9	7 36	11 9
P.M.	10 36	14 8¾	4 27	16 9·8	7 0	19 4	11 25	16 3	4 15	10 3	8 14	11 8
24. A.M.	11 2	13 6	5 18	16 7·5	7 35	20 0	……	……	4 55	11 3	8 36	11 4
P.M.	11 48	14 3	5 43	16 7·0	7 55	18 4	12 15	16 6	5 40	11 0	9 13	11 9
25. A.M.	……	……	6 18	15 5·1	8 55	19 4	12 35	16 10	6 20	12 0	9 53	12 1
P.M.	0 34	13 9¾	6 50	16 8·2	9 20	19 2	1 30	16 8	7 0	11 10	10 16	12 4
26. A.M.	1 8	14 9¼	7 40	15 10·5	10 10	20 8	1 55	17 10	7 20	12 10	10 46	12 5
P.M.	1 44	14 7¼	8 12	16 2·0	10 45	19 9	2 30	18 0	7 58	13 6	11 13	13 0
27. A.M.	2 16	15 4½	8 38	16 4·0	11 20	20 10	3 10	19 0	8 28	14 1	11 33	13 5
P.M.	2 49	15 9½	9 5	17 8·0	11 35	20 7	3 40	19 6	8 46	15 1	……	……
28. A.M.	3 6	16 1¼	9 21	17 4·6	……	……	3 50	20 6	9 11	15 10	0 5	14 2
P.M.	3 37	16 7¾	9 56	18 5·1	12 5	21 7	4 20	21 0	9 35	16 6	0 27	14 6
29. A.M.	4 0	16 11½	10 12	18 0·2	12 20	21 10	4 45	21 0	9 55	16 8	0 52	14 11
P.M.	4 27	17 1¼	10 37	18 8·5	12 50	22 11	4 45	22 0	10 16	17 7	1 17	15 4
30. A.M.	4 47	17 3¾	11 4	18 5·2	1 10	22 3	5 10	22 3	10 40	17 10	1 34	15 11
P.M.	5 26	17 10¾	11 23	19 4·6	1 35	23 3	5 45	22 10	11 0	18 1	2 8	16 4
31. A.M.	5 42	17 10	11 42	18 9·5	1 50	23 3	6 10	23 0	11 23	18 6	2 24	16 5
P.M.	6 5	18 3½	……	……	2 20	23 8	6 40	23 4	11 45	18 9	2 58	16 11

Index to the Tables.

Results deduced from Observations made at LIVERPOOL.

These observations were made at Liverpool by Mr. HUTCHINSON, Dockmaster at that place: they are now in the possession of the Liverpool Lyceum, and they were granted with the greatest kindness and liberality to the author for the purposes of this inquiry by the Committee of that Institution.

The *intervals* in Tables I. II. IV. V. and VI. should be increased by 36 hours to give the real *interval* between the moon's transit A and the time of high water.

I have concluded that Mr. HUTCHINSON's observations are given in *apparent solar* time.

Table I. (*a*.), showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the Liverpool Docks, corresponding to the Apparent Solar Time of the Moon's Transit A in each month of the year, from 13,391 observations made at the Liverpool Docks, between the 1st of January 1774 and the 31st of December 1792.

Table II. (*b*.) (Interpolated from Table I.), showing the Interval between the Apparent Solar Time of the Moon's Transit A, and the Time of High Water at the Liverpool Docks for each month in the year.

Table III. (*c*.) (Interpolated from Table I.), showing the Height of High Water at the Liverpool Docks, corresponding to the Apparent Solar Time of the Moon's Transit A, in each month of the year.

Table IV. (*d*.), showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water at the Liverpool Docks, corresponding to the Apparent Solar Time of the Moon's Transit A, for every minute of her Horizontal Parallax.

Table V. (*e*.), Interpolated from Table IV., and reduced to Moon's Declination 15°.

Table VI. (*f*.), showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the Liverpool Docks, corresponding to the Apparent Solar Time of the Moon's Upper and Lower Transit A, A.M. and P.M.

Table VII. (*g*.), showing the Diurnal Inequality at Liverpool, or the Difference in the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Interval in Table II., and the Difference between the Height of High Water and the Height in Table III.

Table VIII. (*h*.), showing a Comparison between the Semimenstrual Correction at Liverpool in the Interval and in the Height, as deduced from theory and from the results of observation contained in Tables II. and III.

Table IX. (*i*.), showing the Calendar-month Inequality in the Interval and in the Height of High Water, as deduced from BERNOULLI's theory and from the results of observation contained in Tables II. and III. See Plate I.

Table X. (*j*.), showing the Moon's Parallax Inequality in the Interval and in the

Height of High Water, as deduced from Bernoulli's theory and from the results of observation contained in Table V. See Plate II.

Table XI. (*k.*), showing the Diurnal Inequality in the Interval and in the Height for the first six months of the year for the Moon's Transit A, P.M. See Plate III.

Table XII. (*l.*), showing the Interval and Height of High Water at the Liverpool Docks, with the Sun's and Moon's Declinations, and the Moon's Horizontal Parallax (for the mean of all the Moon's Transits A occurring between 0^h and 1^h) for every year from 1774 to 1792.

Table XIII. (*m.*), interpolated from Table XII. by reducing each quantity to Moon's Transit A ($0^h\ 30^m$), and correcting the quantities for deviation from mean Declinations and Parallax.

Table XIV. (*n.*), showing the *Establishment* of the Port of Liverpool obtained from Table XIII. by altering the argument from Transit A to Transit D, and reducing it to $0^h\ 0^m$ from $0^h\ 30^m$. Moon's Hor. Par. 57′, and Decl. 15°.

Results deduced from Observations made at London.

These observations were made at the London Docks under the direction of the late Mr. Peirse, and they are now in the possession of the Royal Society.

The intervals in Tables XV. XVI. XVIII. XIX. and XX. must be increased by 48 hours to give the real interval between the moon's transit B and the time of high water.

I have concluded that these observations are given in *mean solar* time.

Table XV. (*a.*), showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the London Docks, corresponding to the Apparent Solar Time of the Moon's Transit B in each month of the year, from 24,592 observations made at the London Docks, between the 1st of September 1801 and the 31st of August 1836.

Table XVI. (*b.*) (Interpolated from Table XV.), showing the Interval between the Apparent Solar Time of the Moon's Transit B, and the Time of High Water at the London Docks for each month in the year.

Table XVII. (*c.*) (Interpolated from Table XV.), showing the Height of High Water at the London Docks, corresponding to the Apparent Solar Time of the Moon's Transit B, in each month of the year.

Table XVIII. (*d.*), showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water at the London Docks, corresponding to the Apparent Solar Time of the Moon's Transit B, for every minute of her Horizontal Parallax.

Table XIX. (*e.*), Interpolated from Table XVIII.

Table XX. (*f.*), showing the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Height of High Water at the London Docks, corresponding to the Apparent Solar Time of the Moon's Transit B, A.M. and P.M.

Table XXI. (*g*.), showing the Diurnal Inequality at London, or the Difference in the Interval between the Apparent Solar Time of the Moon's Transit and the Time of High Water, and the Interval in Table XVI., and the Difference between the Height of High Water and the Height in Table XVII.

Table XXII. (*h*.), showing a Comparison between the Semimenstrual Inequality at London in the Interval and in the Height, as deduced from theory and from the results of observation contained in Tables XVI. and XVII.

Table XXIII. (*i*.), showing the Calendar-month Inequality in the Interval and in the Height of High Water, as deduced from Bernoulli's theory and from the results of observation contained in Tables XVI. and XVII. See Plate I.

Table XXIV. (*j*.), showing the Moon's Parallax Inequality in the Interval and in the Height of High Water, as deduced from Bernoulli's theory and from the results of observation contained in Table XIX. See Plate II.

Table XXV. (*k*.), showing the Diurnal Inequality in the Interval and in the Height of High Water for the first six months of the year, for the Moon's Transit B, P.M. See Plate III.

As the London discussion contained in this paper has been made with reference to transit B, and the discussion of the Liverpool observations has been made with reference to transit A, it was necessary to pay attention to this circumstance in the comparisons on the Plates. This has been done for the present roughly, by placing the London corrections more to the left by half an hour. The interval corrections would strictly require, in extreme cases, a slight alteration, which may be obtained from Tables XXIII. XXV. and XXVII., given in a former paper*.

Table XXVI., showing that part of the Diurnal Inequality in the Height of High Water depending on the Moon, calculated from the expression $\mathrm{d}\,h = B \sin 2\,\delta'$, assuming for Parallax $57'$, $B = 0{\cdot}5$ feet.

Table XXVII. contains the part of the Diurnal Inequality in the Height depending upon the Sun's Declination, calculated from the expression

$$(A)\ B \sin 2\,\delta \cos \varphi \qquad \log (A) = 9{\cdot}56965.$$

Table XXVIII. (*l*.), showing the Interval and Height of High Water at the London Docks with the Sun's and Moon's Declinations, and the Moon's Horizontal Parallax, for the Mean of all the Moon's Transits B occurring between 0^{h} and 1^{h} for every year from 1802 to 1835.

Table XXIX. (*m*.), interpolated from Table XXVIII. by reducing each quantity to Moon's Transit (B) $0^{\mathrm{h}}\ 30^{\mathrm{m}}$, and correcting for deviation from Mean Declinations and Parallax.

Table XXX. (*n*.), showing the *Establishment* of the Port of London since 1802, obtained from Table XXIX. by altering the argument from Transit B to Transit F, and reducing it to $0^{\mathrm{h}}\ 0^{\mathrm{m}}$ from $0^{\mathrm{h}}\ 30^{\mathrm{m}}$. Moon's Hor. Par. $57'$, and Decl. 15°.

Table XXXI. Observations of High Water in May 1836. See Plate IV.

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* Philosophical Transactions, 1836, p. 255.

X. *Further Observations on Voltaic Combinations. In a Letter addressed to* MICHAEL FARADAY, *Esq. D.C.L. F.R.S., Fullerian Prof. Chem. Royal Institution, Corr. Memb. Royal & Imp. Acadd. of Sciences, Paris, Petersburgh, &c. By* J. FREDERIC DANIELL, *F.R.S., Prof. Chem. in King's College, London, &c.*

Received March 30,—Read April 6, 1837.

MY DEAR FARADAY,

I HAD intended, ere this, to have addressed you upon the subject of the *measure of affinity* which the *constant battery* may be made to supply, as indicated by the concluding experiment of my last letter; but my attention has been diverted, and the whole of my leisure occupied by what I found to be a necessary preliminary investigation of the effects of changes of temperature upon the voltaic action. In the course of my experiments upon this principal subject, I have also been led to observe some curious disturbances and diversions of the battery current, from secondary combinations; and I now submit the results of the whole inquiry to your judgment, not without a hope that you may consider them of sufficient interest and importance to be communicated to the Royal Society.

You may perhaps recollect that the standard charge, which I finally adopted in the use of the constant battery, was a mixture of eight parts of water with one of oil of vitriol on the side of the zinc, and a saturated solution of sulphate of copper in contact with the copper; and that the average amount of its work, as measured by the voltameter, was 11 cubic inches of mixed gases per five minutes. It occurred to me that the resistance to the current might again be reduced by dissolving the salt in the standard acid instead of water; and upon making the experiment I found the action increased from 11 cubic inches to 13 cubic inches, at which rate it steadily maintained itself; the following being the result of one series of observations.

Time.		Interval.	Voltameter.
h	m	′	Cubic inches.
11	0		
11	5	5	13
11	10	5	26
11	15	5	39
11	20	5	52

Upon one occasion I prepared the charge by adding one part of oil of vitriol to eight parts of the saturated solution of the sulphate, and poured it into the cells whilst of the high temperature produced by the disengagement of heat during the mixture, which was about 110°; and the following series of experiments will show the great increase of action which followed this accession of heat, and its rapid decline with the temperature. The observations were made at intervals of two minutes; but I have added the calculated rate per five minutes to facilitate comparison with former experiments.

Time. h m	Interval. ′	Voltameter. Cubic inches.	5 min. rate. Cubic inches.
12 52			
12 54	2	8·8	22·
12 59			
12 61	2	8·	20·
1 3			
1 5	2	7·3	18·2
1 7			
1 9	2	7·	17·5
1 11			
1 13	2	6·5	16·2
1 14			
1 16	2	6·	15·
1 17			
1 19	2	5·7	14·2
1 21	2	4·8	12·

Wishing to follow up this indication of the influence of temperature, I caused a tub to be made which would just receive the ten cells of the battery standing upon small blocks of wood, between which there was room for the syphon tubes to pass. In this situation it was fresh charged with the acid solution; and henceforth I carefully noted the temperature, which in this instance was 68°. Its action was steady for three quarters of an hour at 13·8 cubic inches per five minutes. Two fluid ounces of fresh standard acid were then poured into the inner cells, and the tub was filled with hot water of the temperature of 130° to nearly the top of the cells; the action was now found to be 20 cubic inches per five minutes. The temperature of the exterior solution of sulphate was 106°, and of the interior acid 100°. The experiment was terminated by the bursting of all the membranes which had been exposed for five weeks to the acid solution.

Having remounted the battery, I proceeded to ascertain the effect of different charges in connection with this decided influence of temperature; and, in the first place, having poured a plain aqueous solution of sulphate of copper into the exterior cells, I filled the interior with pure water, making use at the same time of fresh amalgamated zinc rods. At the commencement of the experiment the temperature was 74°; and there was no appreciable decomposition in the voltameter. The tub was then filled with water at 120°. When the temperature of the interior cell had reached 100° a slow action commenced, which became steady at 0·6 cubic inches per five minutes. The voltameter itself being now also heated to 115°, the following results were obtained:

Time. h m	Interval.	Voltameter. Cubic inches.	5 min. rate. Cubic inches.
11 21			
11 26	5	0·8	0·8
11 31	10	1·6	0·8
11 51	30	5·0	0·8

At the expiration of the experiment the rods were found to be only slightly tarnished, and no copper had penetrated to the interior cells.

The next charge which I made trial of was a solution of muriate of ammonia, in the proportion of two pounds of the salt to a gallon of water for the interior cells, and aqueous solution of sulphate of copper for the exterior. The results were as follow:

	per 5 min. Cubic inches.
At 74° out of water	5·5
At 94 in water	8·8
At 124 in water	12·5

Upon this occasion I first observed that a portion of the current was discharged by the water in which the battery was immersed.

The battery plunged in water of the atmospheric temperature of 74° had been working steadily for twenty minutes at the rate of 3·1 cubic inches per five minutes: when the water was drawn off from the tub the rate immediately rose, and was maintained at 4·2 cubic inches per five minutes.

That a discharge may take place from the copper of one cell to the copper of the next, when the regular circuit is interrupted between the two, I had many opportunities of observing in the powerful currents with which I had been experimenting; for I have frequently seen it pass in the form of a spark when the cells had been too much approximated in the air; and when in water it was indicated by the frothing between the two from the disengagement of gas. In such a case there is no doubt that one of the zinc rods is thrown out of action, and the copper of that cell merely acts as an electrode to the antecedent zinc. I shall hereafter point out to you how readily a portion of a current may be diverted from its principal course to by-paths, if I may so express myself, which may be open to it.

The decomposition of the secondary electrolytes, and the course of the ions in this combination, are worthy of some remark. In several instances when the battery had been in action for a considerable period, the zinc rods were found thickly studded with beautiful, large, transparent crystals of sulphate of zinc. The solution of this salt was abundantly precipitated by nitrate of baryta and by potassa, the precipitate in the latter case being redissolved by an excess of the alkali. It was not in the slightest degree affected by nitrate of silver; proving that no muriate or chloride existed in it. There were no indications of free ammonia in the exterior cell: the precipitated copper however did not exhibit the beautiful bright pink hue which it ordinarily presents, but was of a dull, greyish, earthy appearance, resembling that of copper over which ammoniacal gas has been passed at a red heat, and probably contained some combined nitrogen. I had not, however, time to examine a product which is worthy of further investigation.

I tried one more experiment with a view to complete the inquiry into the probable advantage of changing the battery charge. I placed a saturated solution of common salt in contact with the zinc, and filled the exterior division of the cells with a saturated aqueous solution of sulphate of copper. With a temperature of 70° the following series of observations was made:

Time.		Interval.	Voltameter.	5 min. rate.
h	m	′	Cubic inches.	Cubic inches.
11	11			
11	15	4	3·5	4·3
11	19	8	5·9	3·
11	23	12	8·0	2·6
11	27	16	9·9	2·3
11	31	20	11·6	2·1
11	34			
11	54	20	4·	1·0

A fluid ounce of muriatic acid was added to each interior cell, and the action was only brought up to 2·7 cubic inches per five minutes. It thus appeared that the substitution of solutions of the muriates for dilute sulphuric acid, was in every way disadvantageous; and it was moreover found that when the circuit was broken the copper became seriously injured by their action and the formation of a submuriate of that metal.

Wishing now to extend the inquiry into the influence of high temperatures upon the voltaic current, and finding that the membranous tubes would not be able to resist the action of the acid under these circumstances, I endeavoured to find some substitute for the partition of the cells which would not be liable to injury from this cause. After several trials I found that porous earthenware, of the same texture as that of which wine coolers are commonly made, would answer my purpose sufficiently; and the arrangement which I ultimately adopted simplifies the construction of the battery to such a degree as to render it advantageous even under circumstances of ordinary temperature.

The interior cells consist of tubes of this earthenware, closed at the lower end, of the diameter of 1½ inches, and of the same height as the copper cells. The bottoms of the latter are fitted with sockets, in which the tubes are placed, and which confine them in their proper positions; the perforated copper plates, or colanders, for the reception of the solid sulphate of copper, pass over their upper ends. The tubes can be easily removed and instantly replaced; and the facility of emptying and refilling them renders the addition of syphon tubes unnecessary, except in very particular circumstances. Previously to their use they require to be thoroughly soaked in dilute acid; and I have not found them liable to injury, provided the sulphate of zinc be not allowed to crystallize within them; in which case they become disintegrated from the expansive force of crystallization. When the battery is out of action they may always be removed and emptied, and preserved in a state for immediate use by immersing them in very dilute acid. Liquid conduction is not carried on quite so perfectly through their substance as through the membranes, on account of the less perfect communication between the liquids on their opposite sides; but when the acid sulphate of copper is used the amount of action at ordinary temperatures is from 7 to 8 cubic inches per five minutes, which is quite sufficient for all ordinary purposes, and is moreover perfectly constant and steady.

Having thus prepared the battery, I caused a circular steam vessel to be made of tin plate, round which the cells could be placed upon blocks of wood, and closed in with a cover, in which there was a socket which could at pleasure be connected with the steam pipe of a boiler. Two other sockets were also conveniently placed, which were stopped with corks, through which the electrodes of the battery could pass, when the proper connections were made. I intended to have experimented with a series of ten cells, but owing to a mistake only nine could be conveniently arranged in the steamer.

The general result of numerous experiments was, that, the working rate of the battery having been ascertained at the ordinary atmospheric temperature, when steam was admitted it immediately began to rise; and at the full temperature of 212° was more than doubled, provided no *secondary action* interfered with it. The following series will be sufficient to illustrate the progress of the current: the temperatures were taken by a thermometer immersed in the steam.

Time.	Interval.	Voltameter.	5 min. rate.	Temperature.
h m	'	Cubic inches.	Cubic inches.	°
10 0				
10 5	5	7·5	7·5	58
10 10	10	15·	7·5	58
		Steam admitted	. . . . .	110
10 25				
10 30	5	9·	9·	170
10 35	10	20·5	11·5	195
10 40	15	37·	16·5	200
10 41				
10 46	5	19·	19·	205
10 51	10	39·	20·	206

Wishing to ascertain whether any portion of this increased effect were owing to a simple increase of conducting power in the electrolyte, or whether it were wholly dependent upon the electro-motive force, or increased energy of affinity, I set the battery in action with a voltameter included in the circuit, the body of which was immersed in water, which could be conveniently raised to the boiling temperature by means of a lamp. The observations were commenced at the temperature of 58°.

Time.	Interval.	Voltameter.	5 min. rate.
h m	′	Cubic inches.	Cubic inches.
11 25			
11 30	5	6·5	6·5
11 35	10	13·	6·5
Voltameter heated to 130°.			
11 46			
11 51	5	6·5	6·5
Voltameter heated rapidly to 212°.			
11 56	10	14·	7·5
12 1	15	21·5	7·5
12 6	20	29·	7·5

The battery itself was then heated by steam to 135°.

Time.	Interval.	Voltameter.	5 min. rate.
h m	′	Cubic inches.	Cubic inches.
2 10			
2 15	5	13	13

A cell was then included in the circuit charged with cold standard acid, and without any sulphate of copper.

Time.	Interval.	Voltameter.	5 min. rate.
h m	′	Cubic inches.	Cubic inches.
2 20			
2 25	5	8	8
Cold cell removed.			
2 30	10	20	12
Cold cell restored with acid sulphate of copper.			
2 35	15	29·5	9·5

The temperature of the battery was then increased to 160°.

Time.	Interval.	Voltameter.	5 min. rate.
h m	′	Cubic inches.	Cubic inches.
2 38			
2 43	5	11	11
2 47	4	20·5	14

A small portion only of the increased effect would thus appear to depend upon the simple conducting power, which alone could have any influence in the voltameter with platinum electrodes; whilst the increased energy of the electro-motive force was checked by including in the circuit a cell whose powers had not been exalted by the increased temperature which had been communicated to the others.

At this period of my investigation some unexpected and highly interesting phenomena occurred, which turned out to be of a very complicated nature, and the unravelling of which cost me much time and labour. After careful consideration of the subject, I have come to the conclusion, that the clearest mode of giving you an account of these will be to adhere nearly to the order in which I followed them out.

In heating the battery in the steamer, it frequently happened (and indeed in the very first series of experiments which I made with it) that when the thermometer had nearly reached the boiling point, and the action of the battery was at its maximum, a sudden cessation would take place; the decomposition of water in the voltameter, which was proceeding at the rate of 18 or 20 cubic inches per five minutes, would stop as suddenly as by the lifting of one of the connecting wires; and this suspension of power would continue for hours, provided the high temperature were maintained. Upon turning the steam off, and quickly cooling the steamer, the action would return as suddenly as it had ceased, though generally not to the full amount; falling mostly from about 20 cubic inches to 14 or 15. Upon turning the steam on, it would again stop, and again be renewed by cooling. Upon closely examining the voltameter upon these occasions, it was found that the current was not wholly stopped, but that there was a small residual action amounting to $\frac{1}{4}$ cubic inch per five minutes.

These experiments were often repeated with the same general results; and yet there were times when every care had been taken not to vary any of the circumstances of the arrangement, when they could not be reproduced.

In seeking for the cause of these phenomena, there were two which naturally suggested themselves as probable: the first was the unequal action of heat upon the different elements of the battery, exciting thermo-electric currents; and the second the possible excitement of opposing currents from the metallic steam case.

In heating and cooling the battery by the means which I have described, it is obvious that the temperature of its different parts would be unequally affected; and, in fact, I found by the thermometer that a difference of ten or twenty degrees would occasionally exist between the liquid in contact with the zinc and that in contact with the copper: these differences I attempted to increase and modify in numberless experiments. On one occasion I charged the battery with a cold acid solution of sulphate of copper on the outside, and poured boiling standard acid into the interior tube: the inequality did not long exist, but its action was steady at $8\frac{1}{2}$ cubic inches per five minutes. I heated the connecting wires to different degrees, and ultimately placed spirit-lamps under them, to maintain them at a low red heat; but the working rate of the battery, and its steadiness as measured by the voltameter, were not sensibly affected thereby. In short, I convinced myself, that though heat was obviously connected with the phenomena, heat alone was not their exciting cause.

With the view of ascertaining whether the residual current, after the sudden cessation of the greater action, were in the same direction as the original battery current, I included a galvanometer of a coarse construction, and consisting of a single large needle, in the circuit with the voltameter. I found that the needle was strongly deflected to the E., and that the deflection increased as the action rose with the rising temperature. Upon the sudden cessation of the action in the voltameter, the needle was suddenly released from its coercing force, and swang violently backwards and forwards W. and E. till it finally rested a few degrees to the W.; proving that the residual current which passed through the voltameter was in the opposite direction to the original current, and was the excess of a counter force which had power enough to arrest its course.

The cover of the steamer was now removed, and it was found that the bottom was covered with a considerable quantity of condensed water, which was slightly acid, from the leakage of one or two of the cells.

Upon taking out some of the connecting wires between the cells, some action still remained both upon the needle and voltameter, which was not wholly stopped till four wires were removed; proving the existence of a current in some circuit which was different from the main circuit of the battery.

I now tested the direction of the current in each separate cell of the battery, by leading extra wires from the zinc and copper to a second galvanometer, and found all in the normal direction (causing a permanent deflection of the needle 50° E.) except one, which presented a contrary current, and deflected the needle 30° W. Upon another occasion I found the current in six separate cells E., and three in the opposite direction, or W.; and once again one cell was found to be neutral, two reverse, and six direct. I proceeded to investigate with great care a phenomenon which promised at first to afford the explanation of the stoppage of the battery current. Let fig. 1. (Plate VIII.) represent the section of the voltaic combination: *a b c d* will be the steam-vessel, and the nine cells, with their connecting wires, will be represented by the nine smaller circles 1, 2, 3, 4, 5, 6, 7, 8, 9. In the experiment which I am about to describe, the electrodes *e* and *f* were unconnected, but all the short connecting wires were in their places. Each cell was tested by the galvanometer by means of extra wires, as at *g*. The cells 1, 2, 3, 4, 5, 6, 7, separately deflected the needle in the normal direction, or from 30° to 40° E. No. 8. deflected it in the opposite direction, or 40° W. The connexion between 8 and 9 was broken, and the same cell deflected the needle 35° E. The connection was then broken between 8 and 7, and the deflection was 30° E.; and when both wires were replaced, the needle returned to 40° W.

The whole circuit was then completed by the connection of 1 and 9 by a short wire, when, notwithstanding a path was open for the circulation of the battery current, the deflection caused by the single cell No. 8. increased to 55° W., which was contrary to the direction of the main circulation. While the circuit was thus complete, the other cells were again tested, with the following result:—

Fig 1.

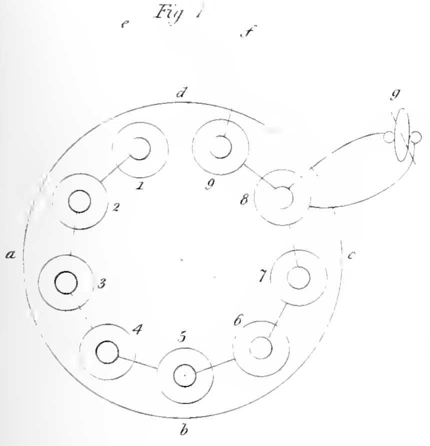


Fig 2.

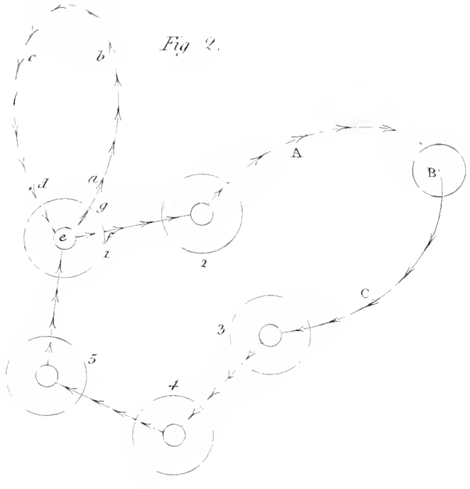


Fig 3

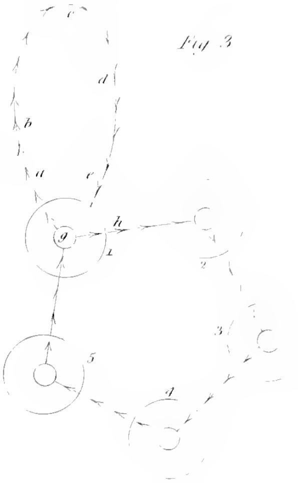


Fig. 5.

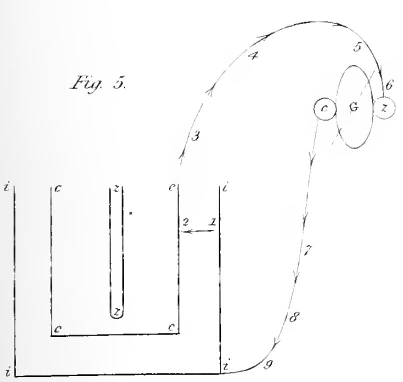


Fig 6.

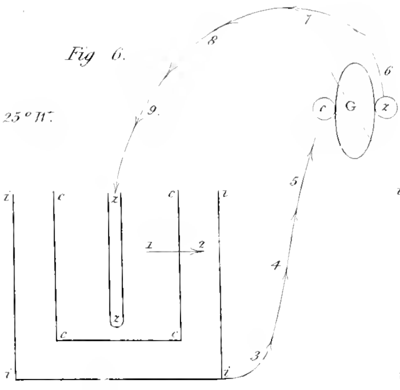


Fig 7.

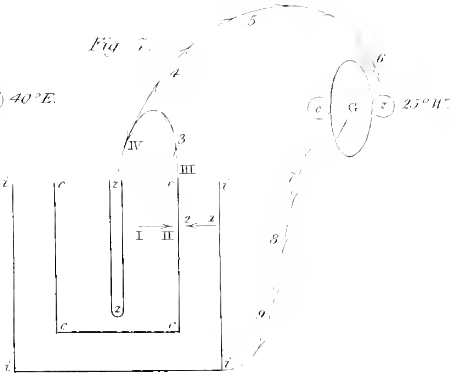


Fig. 4.

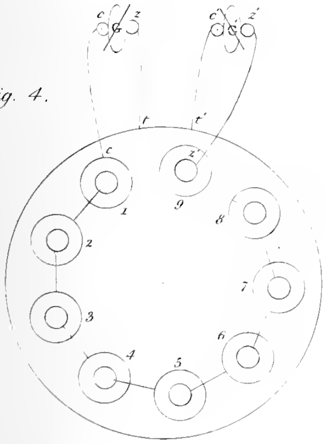


Fig. 8.

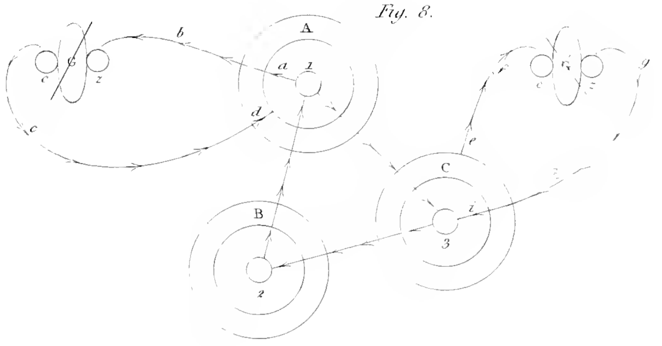


Fig 9.

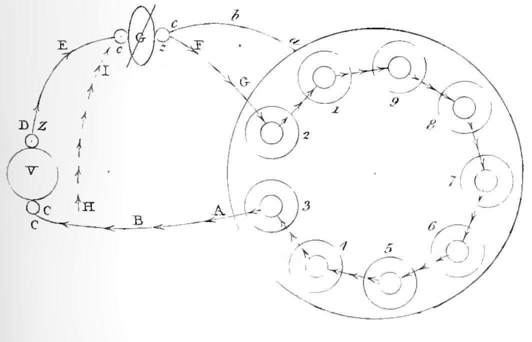
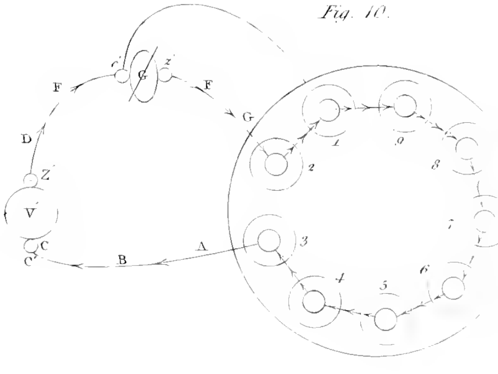


Fig. 10.



No. 8.—55° W.
7.—45 E.
6.—35 W.
5.—10 E.

With No. 4. the needle oscillated from one side of the coil to the other in the most extraordinary manner, first striking with considerable force against the pin on one side and then upon the other. Sometimes it seemed to hesitate between the two, and then to recede in one direction and advance in the opposite by sudden starts and jumps. These oscillations lasted for more than an hour, during which the experiment was continued, with equal force. The other cells were found,

No. 3.—10° W.
2.—20 E.

There was a strong spark upon breaking the connections of all these secondary currents, but they would not pass through the voltameter.

These experiments were frequently repeated with the same general results; sometimes a cell in one position indicating a reverse current, sometimes one in another, the same cell occasionally passing to the normal direction, and at times oscillating violently between the two.

To determine whether the metal steam-vessel had any influence upon these currents, five of the cells were removed to a table, and connected in series with a galvanometer, the needle of which was permanently deflected 90° E. When, in addition to the galvanometer connection, the two extreme cells were connected by a similar wire to those between the other cells, the needle was still deflected 15° E.: so that notwithstanding a shorter path was open to the battery current, and that through a conductor of considerably greater substance than the wires of the galvanometer, a portion still passed by the longer path. Under these circumstances, No. 1. cell was tested by a separate galvanometer, which was deflected 40° W. When the extra short connection of the battery was broken, the deflection from No. 1. fell to 30° W.; and when this cell was totally unconnected with the others, its current was in the normal direction, or 50° E. When the short connections were all restored, it again returned to 40° W. I must here observe, that the galvanometers, not being of the same construction, were not used as accurate measures of the force, but only to indicate the direction of the currents, and occasionally to show that the force was increasing or declining. The other cells were tested in the same way, with the following results:—

No. 2. Battery connected by short wire . . .	30°	E.
Short connection broken . . .	55	E.
No. 3. Battery connected	30	W.
but changed to E.		
Unconnected	55	E.

No. 4. Battery connected . . . . 20 E.
Unconnected . . . . 50 E.
Again connected . . . . 40 W.
Again unconnected . . . . . 50 E.
Again connected . . . . 40 W.
changed to 30° E.
and swang violently E. & W.
No. 5. Battery connected . . . . 35 E.
Unconnected . . . . 55 E.
Connected . oscillated E. & W. 30

Hence it appears that this variable current may be produced from the single cells of the battery, under ordinary circumstances, when the whole series is connected by short wires.

I was still desirous of ascertaining whether these currents would be produced in the simplest possible arrangement, viz. when the elements consisted of amalgamated zinc, copper, and dilute sulphuric acid. For this purpose the five cells were thoroughly cleaned, and charged entirely with standard acid, without sulphate of copper. They were connected in series with a voltameter, and each cell was separately tested by the galvanometer, with the following results:

Time.	Interval.	Voltameter.	5 min. rate.	Galvanometer.
1 56				
61	5	$1\frac{1}{4}$	$1\frac{3}{4}$	Cell 1.—45° E.
2 6	10	3	$1\frac{1}{4}$	Cell 2.—45 E.
11	15	4	1	Cell 3.—40 E.
16	20	$4\frac{3}{4}$	$0\frac{3}{4}$	Cell 4.—40 E.
21	25	$5\frac{1}{2}$	$0\frac{3}{4}$	Cell 5.—40 E.

The voltameter was removed, and the circuit completed by a short wire: the separate cells were again tested, with the following result:—

Cell 1.—25° E.
Cell 2.—20 W.
Cell 3.—20 W.
Cell 4.—25 W.
Cell 5.—25 E.

When the cells 2, 3, and 4, whose secondary currents were all W., or in the direction opposite to that of the main current, were all included in the secondary circuit, the deflection of the needle was 30° W.; but the current produced no decomposition in the voltameter when included with them. The deoxidation of the oxide of copper by the hydrogen was thus proved not to have been the exciting cause of the secondary currents.

I was now desirous of making a comparison of all these effects with the similar phenomena of the battery with its usual charge. The results of the latter are shown in the following Tables:

Time.	Interval.	Voltameter.	5 min. rate.	Galvanometer.
h m				
3 7				
3 12	5	5	5	Cell 1.—42½° E.
3 17	10	10	5	Cell 2.—45 E.
3 22	15	15	5	Cell 3.—45 E.
3 27	20	19½	4½	Cell 4.—47½ E.
3 32	25	24¼	4¾	Cell 5.—47½ E.

Circuit completed by short wire.

Cell 1.—25° W.
Cell 2.—30 W.
Cell 3.—30 W.
Cell 4.—30 E.
Cell 5.—30 E.

The assisting action of the sulphate of copper was thus found to have increased the decomposing power of the battery current from 1·1 cubic inch to 5 cubic inches, and the force of the secondary current, as measured by the deviation of the galvanometer, from 40° to 47°.

From these experiments we find that when the course of the main battery current is obstructed by causing it to pass through the long wire of a galvanometer, or through the electrolyte of a voltameter, the course of the secondary current from each separate cell is always normal, or in the same direction; but that when the battery current is allowed to flow with the least possible resistance, as by completing the main circuit by a short wire, the secondary current of the separate cells becomes opposite. Hence it might be inferred that the resistance might be so adjusted as that the secondary current should altogether disappear, or vary between the two directions. To ascertain the effect of different degrees of resistance, the following experiments were made.

A galvanometer was included in the main circuit of the battery, formed of a wire one twentieth of an inch in diameter and thirty-two feet in length: this we will designate as No. 1. An extra connection was also made of the circuit by means of a wire of the same thickness, fifty-three feet four inches long. The diameter of the wire forming the battery connections was one seventh of an inch. A secondary circuit was also formed from a single cell through another galvanometer, which we will call No. 2.

The deviation of No. 1. was 70° E.
No. 2. — 40 E.

and both the needles were perfectly steady.

When the extra connection was reduced to half, or twenty-six feet eight inches,

The deviation of No. 1. was 60° E.
No. 2. — 35 E.

and No. 2. wavered a little.

Extra connection again halved, or thirteen feet four inches,

The deviation of No. 1. was 40° E.
No. 2. — 30 E.

and No. 2. began to oscillate.

Extra connection once more halved, or six feet eight inches,

The deviation of No. 1. was 28° E.

and No. 2. oscillated from E. and W. very strongly.

The extra connection was then made by only four inches of the same wire:

The deviation of No. 1. was 15° E.
No. 2. — 30 W.

and the needle was only slightly unsteady.

The extra connection was then entirely removed, and

The deviation of No. 1. was 80° E.
No. 2. — 45 E.

Now with regard to the main battery current, when the extra connection was wholly removed, the whole passed through No. 1. galvanometer with a certain resistance, and was measured by its deflection 80°: when the long wire was added, a portion was diverted into the new channel, and was measured by the decline of the needle to 70°. As the length of the extra wire was shortened, the resistance of this passage decreased, and more and more of the current was diverted from the galvanometer till the deflection of the needle only amounted to 15°, and nearly the whole passed through the extra wire.

The effect of these varying resistances upon the secondary current I think I can explain with the help of the annexed diagram. Let the circles 1, 2, 3, 4, 5, (fig. 2.) represent sections of the five battery cells, and the lines between 3 and 4, 4 and 5, 5 and 1, and 1 and 2, with the arrow heads, the short wires between the zinc of each cell and the copper of the next, with the direction of that conventional current which is supposed to flow from one to the other. Let A B C represent the long wire of a galvanometer, or the electrodes of a voltameter through which the circuit is completed between 2 and 3, and by which the current is resisted. When a secondary connection is formed by the wire *a b c d*, between the zinc *e* and the copper *g* of the cell No. 1, a portion of the main current, which tends to pass through the electrolyte to the copper at *f*, being obstructed in this direction, passes to *g*, and completes its circuit through the wire *a b c d*, and the *diverted current* obviously will flow in the same direction as the main current, or from the copper through the wire to the zinc.

But if, instead of a resisting communication, the primary circuit be completed between 2 and 3 by a short wire, as between the other cells, and as represented at fig. 3., and a secondary circuit be formed as before with the cell No. 1. by the wire *a b c d e*, a portion of the main current flowing from the copper of No. 5. to the zinc of No. 1, instead of passing through the electrolyte from *g* to *h*, finds a passage through the wire *a b c d e* to *h*, and consequently will appear to flow in an opposite direction to the primary current or from the zinc to the copper. The resistance of the circuits may be so adjusted that the current may sometimes take one course and sometimes the opposite, and produce those oscillations of the needle from E. to W. which have been just described.

The breaking of the secondary circuit did not affect the galvanometer of the primary current, but the breaking of the primary circuit always turned the secondary current into the normal direction, and increased the deviation of its needle; it reduced it, in fact, to the condition of the direct current from the single cell.

It is however obvious that these *diverted* currents of the complete circuit would not of themselves be sufficient to account for the stoppage of the main battery current of which we are in search; nor has it yet been shown how they were produced when the main circuit was broken, as represented at fig. 1. In search of the explanation of these phenomena I turned my attention to the influence of the metallic steam-vessel upon the voltaic arrangement.

When the battery was connected in the usual way with a galvanometer, and the needle was deflected 80° E., if the zinc electrode were lifted and made to touch the tin case in any part, it would remain deflected in the same direction 30°. If, on the other hand, the main circuit were broken at the copper electrode, and it were brought into contact with the tin, the deflection would be to the same amount in the same direction. The difference of that connection is shown at fig. 4, where *t z* represents the zinc electrode in connection with the tin, and passing to the zinc cup of the galvanometer G and the copper electrode C C in its usual position. On the other hand, $z' z'$ represents the zinc electrode in its usual communication with the galvanometer, and the copper electrode $c' t'$ in connection with the tin. The most striking result of this experiment is, that notwithstanding the connection of the tin with the galvanometer is reversed, the current is in the same direction; or that the current, which we must conceive to flow from the tin in one connection, must flow to the tin in the other. To simplify the conditions of the experiment, I repeated it with a single cell. At the atmospheric temperature of 52°, when the circuit was completed in the usual way through the galvanometer, the deflection was 57° E.; but no deflection was obtained by making connection with the tin. When steam was admitted, the ordinary battery current increased to 65°, and then contact with the hot tin produced a deflection of 20° E. by either electrode.

Four cells were now placed in the steam case upon plates of glass, and the action of the battery with the usual connections being first tried at the temperature of 54°, was found as follows:

Galvanometer alone	70° E.
Galvanometer with voltameter . . .	50° E.

Voltameter alone	5 cubic inches per five minutes.
Voltameter with galvanometer	4 cubic inches per five minutes.

At this temperature there was no current from the tin by either connection.

When the steam was turned on, the battery current increased rapidly to 85° E. by the galvanometer, and there was a current from the tin connections notwithstanding the glass insulation. This *extra current* was also found to exist during the flow of the main battery current; and when the two were measured by separate galvanometers,

The battery current was . . .	85° E.
And the extra current	30 E.

It again made no difference whether the tin took the place of the zinc or the copper in the arrangement; the deflection was always in the same direction.

The breaking of neither current affected the other.

To ascertain whether heat alone, independent of the steam, was the cause of this extra current, a tin plate was placed upon the hot sand of a sand-bath, and the battery cells transferred to glass plates upon it. At first no extra current was detected; but as the temperature rose to 150° and 200°, the second galvanometer was affected to the amount of 50° or 60°; and this whether the battery circuit was complete or not. A deflection was even occasioned by making the contact with any part of the iron stove, however distant, with which the tin plate was in contact.

These experiments were repeated and varied numberless times, but not with uniform results: sometimes the extra current had sufficient intensity to pass through a voltameter, producing slow decomposition of the water; but most frequently, however great the deflection of the needle, it would not pass through this obstacle. At other times, in apparently similar circumstances, the extra current could not be detected at all. Whenever produced, however, it was always observed to flow from the tin to the battery, whether the connection with the latter were made with the zinc or the copper.

I now placed one of the battery cells upon a piece of wood in an iron case made to receive it, of the same height, but having a space all round it of about an inch. When the primary circuit was completed by means of a galvanometer, the deflection was 60° E.; but there was no action upon a second galvanometer included in a secondary circuit between the iron and the zinc or the iron, and the copper of the cell. A little dilute sulphuric acid was then poured into the iron case, and immediately a strong extra current was produced. Under these circumstances

The primary current was	60° E.
The extra current copper and iron connection . .	20 E.
Zinc and iron	40 E.

The analysis of the phenomena of these extra currents was most satisfactorily performed in the following manner:—in figg. 5, 6, and 7, let *i i i i* represent the section of the iron case, *c c c c* the section of the copper cell, and *z z* the zinc rod: let G also represent the situation of the secondary galvanometer with its different connections with the circuit. In fig. 5. the connection with the iron is made with the copper cup of the galvanometer, and the zinc cup is connected with the copper of the cell; and we see at once that a current is established, which, setting out from the iron, passes through the electrolyte to the copper, and completes its circuit through the galvanometer in the direction 1, 2, 3, 4, 5, 6, 7, 8, 9. In fig. 6. the connection with the iron remaining the same, the battery cell is connected by its zinc rod with the zinc cup of the galvanometer, and we have a powerful reversed current, which we must suppose to set out from the zinc, and to pass through a portion of the electrolyte to the copper, and from the copper through another portion of electrolyte to the iron, and to complete its course in the direction 1, 2, 3, 4, 5, 6, 7, 8, 9. In fig. 7. the main battery circuit is likewise completed, and the primary current will flow in the direction I., II., III., IV.; while the extra circuit, although apparently connected, as in fig. 6, with the zinc, is in fact connected with the copper, as in fig. 5, by means of the main battery connection, and will convey the extra current from the iron through the electrolyte to the copper, and from the copper through the galvanometer back again to the iron in the direction 1, 2, 3, 4, 5, 6, 7, 8, 9.

It will be seen that the two currents coincide in their direction in that part of their circuits which is common to both, viz. III., IV., and 3, 4.

There is no difficulty at all in understanding how an extra current is established from the iron to the copper in addition to the main current from the zinc to the copper; but I was for a long time puzzled to make out how an extra current could pass from the zinc through the electrolyte to the copper, and from the copper through the electrolyte to the iron: it seemed to me that the interposed copper must act as a retarding plate, upon the opposite surfaces of which hydrogen and oxygen must be evolved; and that the intensity of a single circle could not be sufficient to force this passage. You will, I dare say, remember suggesting an experiment which led to the explanation of the difficulty.

Some dilute sulphuric acid was poured into a basin, and a platinum crucible containing some solution of sulphate of copper was placed in it. An amalgamated zinc rod wrapped in filtering paper moistened with dilute acid, to prevent it from precipitating the copper by its local action, was held in the crucible. A plate of iron was also immersed in the acid in the basin; contact with the platinum being carefully avoided. A metallic communication was then made by means of a wire with the zinc and the iron, and we had thus the exact circumstances of the battery cell in the iron case, except that platinum was substituted for copper. No current, however, was thus formed, and no copper was precipitated upon the platinum from the solution of sulphate. A piece of copper-plate was now placed under the platinum crucible, and in contact with it: the current immediately passed, and copper was precipitated upon the interior surface of the crucible from the sulphate. The current of the single circle could not pass by the retarding plate of platina, when oxygen must have been evolved on one side and hydrogen on the other; but when the oxygen was absorbed by the copper, and the hydrogen by the oxide of copper, these concurring affinities enabled the current affinity to make good its circuit.

To vary the experiment with regard to the metallic part of the combination, three of the battery cells were placed upon zinc plates with interposed flannel moistened with dilute sulphuric acid; the copper was thus placed between two generating plates of the same metal. When the three were connected together in regular series with a galvanometer, the deflection of the needle was 90° E. When a secondary connection was made from the zinc rod of each cell in succession through another galvanometer with the zinc plate on which it stood, the deflection occasioned by the extra current was 20° W. In this case it must have flowed from the zinc through the electrolyte in the flannel to the copper; from the copper through the electrolyte in the cell to the zinc rod; and from the zinc rod through the wire back to the zinc plate. When the main circuit was broken the extra current changed its direction, and occasioned a deflection of the needle 20° E. Upon restoring the primary current the extra current again returned to its original direction, and so invariably upon many successive repetitions of the experiment.

When the extra connection was made between the copper of the cell and the zinc plate, the deflection of the second galvanometer was always 40° W., or opposite to the main current, and was not disturbed by any interruption of the latter.

Being now satisfied that these extra currents were hydro-electric, and dependent upon the action of a liquid upon the metal which was brought into association with the regular voltaic combination, I examined more carefully the circumstances of the arrangement in which I had supposed that I had insulated the cells, and cut them off from any such influence by placing them upon thick glass plates. I now ascertained that the establishment of the extra current was owing to a thin film of moisture formed upon the glass, either by the condensation of steam, or slight leakage from the cells. At ordinary temperatures no action was thus excited, but when the temperature of the combination was sufficiently exalted very energetic currents were sometimes developed by a quantity of moisture, which might well have escaped ordinary observation. When great care was taken to make the glass plates perfectly bright and dry, the extra current was never produced.

It was now also clear that not only could independent extra currents be established, but that different portions of the main battery current could be diverted into this secondary path, and thus the occasional decomposition of water in the voltameter by the extra current could be accounted for.

There is one more relation of the *battery current*, the *diverted current* and the *extra current*, which it may be worth while to point out when they are all three established at the same moment. Let 1, 2, 3 (fig. 8) represent the section of three battery cells, all standing upon blocks of wood in iron cases, the sections of which are represented by A, B, C, the bottoms of the cases being covered with dilute acid. The main connections of the battery are made, and the principal current flows from 1 to 3, from 3 to 2, from 2 to 1. A diverted current may be led off from 1, and may pass in the direction *a b c d* through the galvanometer G, or sometimes in the opposite direction. At the same time an extra current may be established from C, the iron case of the cell 3, through the galvanometer G′, in the direction *e f g h i*.

The making or breaking of this extra current had no effect upon the diverted current *a b c d*, but the two were always in opposite directions. When the first moved the needle to the E. the second deflected its needle W. When the diverted current was W. the breaking the main current always turned it in the normal direction E, and at the same moment the needle of the extra current changed to the W. Upon restoring the main current both needles returned to their former position.

When the battery current, instead of being allowed to flow through short connections, was led through a separate galvanometer, each of the other currents passing also to separate galvanometers, the diverted current varied in the different cells from 45° to 20°, but was always in the normal direction, and the extra current was opposite. When the short connection was added to the battery, as well as the long one through the galvanometer, the latter fell from 90° E. to about 20° E., and the diverted current oscillated rapidly E. and W., and the needle of the extra current changed with it in the opposite directions W. and E.

I could now have no doubt that the explanation of the sudden stoppage and reversal of the battery current, of which I was in search, was to be found either in this diversion which I have described, or from the opposition of extra currents exalted in their power by heat, or possibly from some combination of the two. I therefore returned to the original combination of the battery in the steamer.

I soon ascertained that the extra current could be produced by a connection from any part of the tin case to any of the cells of the battery standing upon wooden blocks, and that its energy was increased both by acidulating the condensed water and by heat. I found also that by leading this current through the same galvanometer and voltameter as the battery current, that it interfered with it in different degrees, even to its stoppage and reversal. I must not attempt to give you the details of the numerous series of observations which I have made upon the subject, but will content myself with stating as concisely as possible the results of the last combination, which have proved always constant.

At figg. 9 and 10 I have represented, as before, a section of the arrangement: the course of the main current is marked by the arrow heads, and is conducted by the electrodes through the voltameter V and the galvanometer G. The battery was first charged in the usual way, and the cells were placed in the steamer upon blocks of damp wood standing in a little acidulated water. The observation commenced at the temperature of 52°, and the following are the tabulated results.

Time.		Interval.	Voltameter.	5 min. rate.	Galvanometer.
h	m	′	Cubic inches.	Cubic inches.	°
10	58				
11	3	5	6	6	60 E.
11	5				
11	10	5	12	6	60 E.

A connection was now made between the tin and the zinc cup of the galvanometer, as in fig. 9, by *a b c*, and the action was decreased.

Time.	Interval.	Voltameter.	5 min. rate.	Galvanometer.
h m	′	Cubic inches.	Cubic inches.	°
11 11				
11 16	5	17	5	55 E.
11 21	5	21¼	4¼	

The cover of the steamer was next put on, and the connections made as before by the electrodes passing through the cocks.

Time.	Interval.	Voltameter.	5 min. rate.	Galvanometer.
h m	′	Cubic inches.	Cubic inches.	°
11 40				
11 45	5	6·5	6·5	60 E.

Connection was again made from the tin (cover) to the zinc cup.

Time.	Interval.	Voltameter.	5 min. rate.	Galvanometer.
h m	′	Cubic inches.	Cubic inches.	°
11 50	5	11·5	5·	55 E.
11 52	The steam was turned on.			
11 57	5	19·	7·5	65 E.
12 2	5	27·	8·	67·5 E.
12 7	5	36·	9·	70 E.
12 9				
12 14	5	11·	11·	77 E.
12 19	5	21·5	10·5	

The tin was again connected as before.

Time.	Interval.	Voltameter.	5 min. rate.	Galvanometer.
h m	′	Cubic inches.	Cubic inches.	°
12 24	5	25·	3·5	40 E.
12 29	5	stopped.		0

Now we may observe that in this arrangement the extra current, which we have already found completing its circuit, according to circumstances, either to the zinc or copper of the battery, has a path open to it by the wire F G to the zinc of No. 2, or through the galvanometer and voltameter by the wire B A to the copper of No. 3, the latter being in opposition to the battery current. That in this series of experiments it tended to pass in the latter direction, is proved by the gradual retardation and ultimate neutralization of the latter. When the resistance to the main current was diminished by throwing the voltameter V out of the circuit by a wire passing from H to I, it again passed in the regular course, turning the needle of the galvanometer to the E.

When the secondary connection was made with the tin by the copper cup C of the galvanometer, instead of the zinc cup, the battery current through the voltameter was again stopped, but the needle of the galvanometer turned 80° E., indicating a powerful current through it in the normal direction.

These experiments were frequently repeated with the same results.

In attempting to account for all the variable phenomena of these extra and diverted currents, it must not be overlooked that it may be possible for the whole battery, or for a certain number of the series of the battery, to force itself a passage through the electrolyte on each side of the copper to the tin, and thus to discharge itself by what would appear to be a reversed current through the secondary communication. This would be determined by the amount of different resistances in the paths which might be open to it. In this way we can account for the different degrees of power in the extra current, and for its being able to pass through the voltameter at times and not at others.

The only difficulty in now accounting for the original stoppage of the battery current which I observed, consists in my not having been aware that there was any metallic communication between the tin and any of the battery connections. I am now however convinced that, notwithstanding precautions were taken to avoid this, contact must have taken place. Indeed the distance between the connecting wires and the rim of the tin cover was but small, and the jarring occasioned by placing the cover in its place may easily have occasioned sufficient disturbance in the arrangement. This supposition also sufficiently explains why the stoppage could not invariably be produced when desired.

I think that I do not deceive myself in believing that the preceding observations may not be without interest and importance to those who are actively engaged in advancing by experiment our knowledge of one of the most wonderful and widely-diffused agencies with which matter has been endowed. If they should answer no higher purpose, they may very probably prevent the application of much labour and thought in the explanation of phenomena of a very striking but perplexing nature, which are very likely to be observed by those who are working in this field of inquiry, and of which in my own case the preceding pages are a very brief abstract. At the same time they afford an exemplification of the advantages of the *constant battery*; for both the diverted currents and the effects of temperature would have been masked and lost in the variable results of the common voltaic combinations.

The effects of heat upon single voltaic circuits have been ably investigated both by M. Marianini[*] and by Mr. Rogers[†]; but although both these gentlemen purposed to extend their observations to compound circuits, or the battery, they have probably been prevented by the cause which I have indicated. It is now, however, apparent that in the exact measures of different effects which an invariable current of electricity will enable experimentalists to undertake, the variations of atmospheric temperature even must not be neglected.

I remain, my dear Faraday,
Yours very faithfully,
J. F. Daniell.

King's College,
March, 1837.

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[*] Annales de Chimie, tom. xxxiii. p. 132.

[†] Silliman's Journal, January, 1835.

Postscript.

I have just completed a *constant battery* of large dimensions, the effects of which exceed my most sanguine expectations, and open new views of the possible application of the extraordinary powers of the voltaic current to economical purposes. It consists of only ten copper cells 20 inches high, $3\frac{1}{2}$ inches diameter, as in the first battery. The interior partitions are formed by merely tying the open ends of the oxen's gullets to the rings of the colanders for supporting the sulphate of copper, and which are made deeper than before, and suspending them in the cells, to the bottoms of which they reach. These membranous bags contain each rather more than a quart of the dilute acid. The zinc rods are of the diameter of $\frac{5}{8}$th of an inch, well amalgamated, and the connections are made as before described. At the temperature of 67° this battery produces, in the voltameters which I have all along employed in these researches, 12 cubic inches of the mixed gases per minute, or 720 cubic inches per hour. Its powers of ignition are very great; and while it will maintain 6 inches of platinum wire $\frac{1}{100}$th of an inch diameter red hot, it will still decompose water at the rate of 14 cubic inches per five minutes. The permanence of this result is very striking.

When the battery is not in use the rods are taken out and wiped, and the membranous bags carefully lifted out of the cells, emptied of their acid, filled with water, and suspended from a frame placed for their reception. By this treatment I do not find that they are liable to any change of texture or deterioration; and I have now membranes which have been in use for several months and are quite perfect. If the acid be perfectly washed out of them they may even be dried with impunity; but it is better to preserve them in a moist state, as when dry they are liable to crack. The acid solution of sulphate of copper remains in the cells without injury, and in ten minutes the battery, when required, may be brought into action. There is no reason to think that the limits of efficiency have yet been nearly attained, and the gullets could easily be connected together so as to obtain bags of any required length. I scarcely, however, think that in simplicity and cheapness of construction the battery can be further improved.

J. F. D.

King's College,
15*th June,* 1837.

XI. *Analysis of the Roots of Equations. By the Rev.* R. Murphy, *M.A., Fellow of Caius College, Honorary Member of various Philosophical Societies. Communicated by* J. W. Lubbock, *Esq. F.R.S.*

Received April 6,—Read April 27, 1837.

1. THE object of this memoir is to show how the constituent parts of the roots of algebraical equations may be determined, by considering the conditions under which they vanish, and conversely to show the signification of each such constituent part.

2. In equations of degrees higher than the second the same constituent part of the root is found in several places governed by the same radical sign, but affected with the different corresponding roots of unity as multipliers.

3. The root of every equation, of which the coefficients are rational, contains a rational part, for the sum of the roots could not otherwise be rational.

This rational part, as such, is insusceptible of change in the different roots of the same equation, consequently its value is the coefficient of the second term (with a changed sign) divided by the number of roots, or index of the first term.

4. The supposed evanescence of any of the other constituent parts implies that a relation exists between the roots; if such a relation be expressed by equating a function of the roots to zero, that constituent part will be the product of all such functions, and a numerical factor.

5. The joint evanescence of various constituent parts implies the co-existence of various relations between the roots, and thus an interpretation may be given to each of the constituent parts, riveting the expression of the root in the memory, and beautifully converting the solution of a problem into a condensed enunciation of various theorems.

For simplicity these principles are first applied to equations of lower degrees.

6. Let us take for example the general quadratic equation

$$x^2 + a\,x + b,$$

the two roots of which are represented by x_1, x_2 in the formulæ,

$$x_1 = -\frac{a}{2} + \sqrt{\alpha}$$

$$x_2 = -\frac{a}{2} - \sqrt{\alpha}.$$

To find α suppose $\alpha = 0$, a relation is then established between the quantities x_1 and x_2, viz.

$$x_1 - x_2 = 0,$$

$x_1 - x_2$ vanishing with α is a factor of it; and since the roots must be symmetrically involved in α, the other factor is $x_2 - x_1$, whence

$$\alpha = k\,(x_1 - x_2)^2,$$

where k is simply a number.

Put for $-\frac{a}{2}$ its expression in terms of the roots, and we thus have

$$x_1 = \frac{x_1 + x_2}{2} + \frac{x_1 - x_2}{2} \,.\, \sqrt{4\,k},$$

$$x_2 = \frac{x_1 + x_2}{2} - \frac{x_1 - x_2}{2} \sqrt{4\,k},$$

whence

$$x_1 - x_2 = (x_1 - x_2) \sqrt{4\,k}, \text{ or } k = \frac{1}{4};$$

then

$$\alpha = \frac{1}{4}\,(x_1 - x_2)^2,$$

which is a symmetrical function, and therefore easily expressed by the coefficients.

7. From this it follows that $\alpha = 0$ is the condition that the equation may have two equal roots; but if the proposed quadratic be represented by $\varphi = 0$, and its derived equation by $\varphi' = 0$, which is the same as $2\,x + a = 0$, the condition for two equal roots is obtained by eliminating x between these equations, which by the theory of elimination gives as the sought condition

$$\varphi'(x_1) \,.\, \varphi'(x_2) = 0\,;$$

we have thus

$$\alpha = k'\,\varphi'(x_1) \,.\, \varphi'(x_2),$$

k' being numerical.

Now $\varphi(x) = (x - x_1)\,(x - x_2)$, therefore $\varphi'(x) = (x - x_1) + (x - x_2)$, whence

$$\varphi'(x_1) = x_1 - x_2, \quad \varphi'(x_2) = x_2 - x_1,$$

consequently $k' = -\frac{1}{4}$; and converting the sum and product of x_1, x_2 into the coefficients, we obtain

$$x_1 = -\frac{a}{2} + \sqrt{\left(\frac{a^2}{4} - b\right)} \qquad x_2 = -\frac{a}{2} - \sqrt{\left(\frac{a^2}{4} - b\right)}.$$

8. The constituent parts in the roots which have been the objects of investigation were $-\frac{a}{2}$ and α, and with respect to their evanescence we have the following theorem.

The vanishing of that part of the root of a quadratic which is under the radical sign implies the existence of two equal roots.

The vanishing of the other part which is unaffected by that sign signifies that the roots are equal, but with contrary signs.

By the aid of this theorem we shall be able to find two of the three constituent parts of the roots of a cubic equation.

9. Let us now extend the same views to equations of the third degree, and let $x_1\, x_2\, x_3$ be the three roots of the cubic $x^3 + a\,x^2 + b\,x + c = 0$. Put

$$x_1 = -\frac{a}{3} + \sqrt[3]{\alpha} + \sqrt[3]{\beta}$$

$$x_2 = -\frac{a}{3} + \theta\sqrt[3]{\alpha} + \theta^2\sqrt[3]{\beta}$$

$$x_3 = -\frac{a}{3} + \theta^2\sqrt[3]{\alpha} + \theta\sqrt[3]{\beta}.$$

The numbers θ, θ^2 are the imaginary cube roots of unity, and we may observe that the formulæ for x_3, x_1 differ only from that for x_2 in having θ^2, θ^3 respectively instead of θ.

10. The quantities α, β, which are obviously similarly involved, are the roots of a quadratic, and of the forms

$$\alpha = \alpha' + \sqrt{\alpha''}$$
$$\beta = \alpha' - \sqrt{\alpha''};$$

the three quantities $-\frac{a}{3}$, α' and α'' are the constituent parts of the roots of the cubic in the sense in which those words have been used; the first is the same as $\frac{x_1 + x_2 + x_3}{3}$, and the other two can be found from the conditions of their evanescence as follows.

11. Suppose $\alpha'' = 0$, the theorem of art. 8. gives us $\alpha = \beta$, whence we find $x_2 = x_3$; now $x_2 - x_3$ vanishing with α'' is a factor of it; and the other symmetrical factors are $x_3 - x_2$, $x_1 - x_3$, $x_3 - x_1$, $x_1 - x_2$, $x_2 - x_1$, k being a number, we must therefore have

$$\alpha'' = k\,(x_1 - x_2)^2\,(x_1 - x_3)^2\,(x_2 - x_3)^2.$$

12. To find α' in like manner, suppose $\alpha' = 0$ the theorem of art. 8 before referred to, in this case makes $\alpha = -\beta$, and the three roots of the cubic accordingly are changed to the following:

$$x_1 = -\frac{a}{3}$$

$$x_2 = -\frac{a}{3} + (\theta - \theta^2)\sqrt[3]{\alpha}$$

$$x_3 = -\frac{a}{3} - (\theta - \theta^2)\sqrt[3]{\alpha};$$

whence we readily find that $2\,x_1 = x_2 + x_3$, therefore $2\,x_1 - x_2 - x_3$ is a factor of α', and the other symmetrical factors are $2\,x_2 - x_1 - x_3$, and $2\,x_3 - x_1 - x_2$; hence if k' be a numerical factor

$$\alpha' = k'\,(2\,x_1 - x_2 - x_3)\,(2\,x_2 - x_1 - x_3)\,(2\,x_3 - x_1 - x^2);$$

these symmetrical functions can be expressed by the given coefficients of the equation.

13. The constants k, k' may be easily found in various ways, perhaps the simplest means is to suppose $\alpha' = 0$ to find k, and $\alpha'' = 0$ to find k'.

If we put $x_3 = 0$ and $x_1 = 2\,x_1$, we have $\alpha' = 0$, and $\alpha'' = 4\,k\,x_1^6$, hence $\alpha = 2\,x_1^3 \sqrt{k}$, $\beta = -\,2\,x_1^3 \sqrt{k}$, and substituting in the formula for x_2, we have (since $a = -\,3\,x_1$)

$$2\,x_1 = x_1 + x_1\,\theta \sqrt[3]{2 \sqrt{k}} - x_1\,\theta^2 \sqrt[3]{2 \sqrt{k}}$$

or

$$1 = \theta\,(1 - \theta) \sqrt[3]{2 \sqrt{k}},$$

therefore

$$2 \sqrt{k} = \frac{1}{(-\theta)^3} = \frac{1}{3\,(\theta^2 - \theta)}$$

and

$$4\,k = \frac{1}{9\,(\theta - 2 + \theta^2)},$$

but $1 + \theta + \theta^2 = 0$; therefore

$$k = \frac{-\,1}{2^2 \,.\, 3^3}.$$

14. In like manner to find k' suppose $x_2 = x_3 = 0$, which makes $\alpha'' = 0$, and $\alpha' + 2\,k' \,.\, x_1^3$, and $\alpha = \beta = \alpha'$, therefore

$$x_1 = \frac{x_1}{3} + 2\,x_1 \sqrt[3]{2\,k'}$$

or

$$\frac{1}{3} = \sqrt[3]{2\,k'},$$

whence

$$k' = \frac{1}{2 \,.\, 3^3}$$

15. The preceding analysis furnishes the following formula,

$$\begin{aligned} 3\,x_1 = (x_1 + x_2 + x_3) &+ \sqrt[3]{\Big\{\tfrac{1}{2}\,(2\,x_1 - x_2 - x_3)\,(2\,x_2 - x_1 - x_3)\,(2\,x_3 - x_1 - x_2)} \\ &\qquad \overline{+ \tfrac{3}{2} \sqrt{-\,3\,(x_1 - x_2)^2\,(x_1 - x_3)^2\,(x_2 - x_3)^2}\Big\}} \\ &+ \sqrt[3]{\Big\{\tfrac{1}{2}\,(2\,x_1 - x_2 - x_3)\,(2\,x_2 - x_1 - x_3)\,(2\,x_3 - x_1 - x_2)} \\ &\qquad \overline{- \tfrac{3}{2} \sqrt{-\,3\,(x_1 - x_2)^2\,(x_1 - x_3)^2\,(x_2 - x_3)^2}\Big\}} \end{aligned}$$

the corresponding formulæ for $3\,x_2$, $3\,x_3$ being obtained by writing θ and θ^2 before the cubic radical signs.

In consequence of the negative multiplier $-\,3$ under the sign of square root it is visible that this formula is not arithmetically applicable when the three roots are real and unequal, which is usually termed the irreducible case.

16. The cubic surds in the formula above given are actually extractible, which verifies the solution.

To this end let $2x_1 - x_2 - x_3 = \mathrm{A}$, and $x_2 - x_3 = \mathrm{B}$, then

$$2x_2 - x_1 - x_3 = \frac{1}{2}(3\,\mathrm{B} - \mathrm{A})$$

$$2x_3 - x_1 - x_2 = \frac{1}{2}\,.\,(-3\,\mathrm{B} - \mathrm{A})$$

therefore

$$\frac{1}{2}(2x_1 - x_2 - x_3)(2x_2 - x_1 - x_3)(2x_3 - x_1 - x_2) = \frac{1}{8}(\mathrm{A}^3 - 9\,\mathrm{A}\,\mathrm{B}^2).$$

Again

$$x_1 - x_2 = \frac{1}{2}(\mathrm{A} - \mathrm{B})$$

$$x_1 - x_3 = \frac{1}{2}(\mathrm{A} + \mathrm{B})$$

therefore

$$\frac{3\sqrt{-3}}{2}\,.\,(x_1 - x_2)(x_1 - x_3)(x_2 - x_3) = \frac{3\sqrt{-3}}{8}(\mathrm{A}^2\,\mathrm{B} - \mathrm{B}^3)$$

the total quantity under the first cubic surd thus becomes

$$\frac{1}{8}\{\mathrm{A}^3 + 3\,\mathrm{A}^2\,.\,(\mathrm{B}\sqrt{-3}) + 3\,\mathrm{A}\,.\,(\mathrm{B}\sqrt{-3})^2 + (\mathrm{B}\sqrt{-3})^3\},$$

the cube root of which is $\frac{1}{2}\{\mathrm{A} + \mathrm{B}\sqrt{-3}\}$, and the actual root of the second surd is similarly $\frac{1}{2}\{\mathrm{A} - \mathrm{B}\sqrt{-3}\}$. But

$$\frac{1}{2}(\mathrm{A} + \mathrm{B}\sqrt{-3}) = x_1 - x_2\,.\,\frac{1 - \sqrt{-3}}{2} - x_3\,.\,\frac{1 + \sqrt{-3}}{2}$$

$$= x_1 + x^2\,\theta^2 + x^3\,\theta.$$

And

$$\frac{1}{2}(\mathrm{A} - \mathrm{B}\sqrt{-3}) = x_1 + x_2\,\theta + x_3\,\theta\,;$$

the formulæ of this solution become then those of Vaudermonde, viz.

$$3x_1 = (x_1 + x_2 + x_3) + (x_1 + x_2\,\theta^2 + x_3\,\theta) + (x_1 + x_2\,\theta + x_3\,\theta^2)$$

$$3x_2 = (x_1 + x_2 + x_3) + (x_1\,\theta + x_2 + x_3\,\theta^2) + (x_1\,\theta^2 + x_2 + x_3\,\theta)$$

$$3x_3 = (x_1 + x_2 + x_3) + (x_1\,\theta^2 + x_2\,\theta + x_3) + (x_1\,\theta + x_2\,\theta^2 + x_3),$$

which contain a complete verification.

17. The two constituent parts α' α'' of the roots of a cubic equation have been resolved into factors by observing the relations established between the roots by their evanescence; another mode exists for forming the same quantities by elimination between the proposed equation and its first and second derived equations.

Let the given equation $x^3 + a\,x^2 + b\,x + c = 0$ be represented by $\varphi(x) = 0$, and the first derived, viz. $3x^2 + 2\,a\,x + b = 0$ by $\varphi'(x) = 0$, the second derived $6x + 2a = 0$ by $\varphi''(x) = 0$.

18. When $\alpha'' = 0$ we have seen that the equation $\varphi(x) = 0$ has two equal roots. But when $\varphi(x) = 0$ has equal roots, is expressed by making the result of eliminating x between $\varphi(x) = 0$ and $\varphi'(x) = 0$ to vanish, and by the theory of elimination this result is $\varphi'(x_1) \cdot \varphi'(x_2) \cdot \varphi'(x_3)$; hence we have ($h$ being a number),

$$\alpha'' = h\,\varphi'(x_1) \cdot \varphi'(x_2) \cdot \varphi'(x_3).$$

19. Suppose now the joint evanescence of α' and α'', the equation has then three equal roots $x_1 = x_2 = x_3$, the system of equations $\alpha' = 0$ $\alpha'' = 0$, the first of three dimensions relative to the roots, the second of six, are therefore the two conditions necessary for the existence of three equal roots.

The results of the elimination of x between $\varphi(x) = 0$ $\varphi''(x) = 0$ and $\varphi(x) = 0$ $\varphi'(x) = 0$, give also the two conditions for three equal roots of the same dimensions as the above, with which this system is identical, we have thus, h' being numerical,

$$\alpha' = h'\,\varphi''(x_1) \cdot \varphi''(x_2) \cdot \varphi''(x_3).$$

20. Since

$$\varphi(x) = (x - x_1)(x - x_2)(x - x_3)$$
$$\varphi'(x) = (x - x_2)(x - x_3) + (x - x_1)(x - x_3) + (x - x_3)(x - x_2)$$
$$\varphi''(x) = 2(x - x_3) + 2(x - x_2) + 2(x - x_1);$$

therefore

$$\varphi'(x_1) \cdot \varphi'(x_2) \cdot \varphi'(x_3) = -(x_1 - x_2)^2 (x_1 - x_3)^2 (x_2 - x_3)^2$$
$$\varphi''(x_1) \cdot \varphi''(x_2) \cdot \varphi''(x_3) = 8(2x_1 - x_2 - x_3)(2x_2 - x_1 - x_3)(2x_3 - x_1 - x_2);$$

the values of α' α'' are therefore conformable to those before found, and

$$h = -k = \frac{1}{2^2 \cdot 3^3} \qquad h' = \frac{k'}{8} = \frac{1}{2^4 \cdot 3^3}.$$

21. In the elimination of a quantity between two equations into which that quantity enters rationally, it is in general indifferent which of the two equations is selected that its roots may be substituted for such quantity, for the dimensions of the product is the same whether we substitute the n roots of an equation of n dimensions in one of m, or the m roots of the latter in the former of n dimensions, and take the products; these products are not only of the same dimensions but imply the coexistence of the same system of equations, and can only differ from each other by numerical multipliers, when the numerical coefficient in the one differs from that of the other.

In the present instance, if ξ_1 ξ_2 be the roots of the equation $\varphi' = 0$, and X that of $\varphi'' = 0$, the values of α' α'' may also be expressed in the following form,

$$\alpha'' = \mathrm{H}\,\varphi(\xi_1) \cdot \varphi(\xi_2)$$
$$\alpha'' = \mathrm{K} \cdot \varphi(\mathrm{X})$$

the factors H, K being numbers.

22. Now if we observe that by the equation $\varphi'(\xi_1) = 0$,

$$\xi_1{}^2 = -\frac{2a}{3} \cdot \xi_1 - \frac{b}{3}$$

and therefore

$$\xi_1^3 = -\frac{2a}{3}.\xi_1^2 - \frac{b}{3}.\xi_1 = \left(\frac{4a^2}{9} - \frac{b}{3}\right).\xi_1 + \frac{2ab}{9}$$

it will follow that $\varphi(\xi_1)$ or $\xi_1^3 + a\xi_1^2 + b\xi_1 + c$, is the same as $A\xi_1 + B$, putting

$$A = -\left(\frac{2a^2}{9} - \frac{2b}{3}\right)$$

$$B = c - \frac{ab}{9}.$$

Hence

$$\varphi(\xi_1).\varphi(\xi_2) = (A\xi_1 + B)(A\xi_2 + B);$$

and since $\xi_1\xi_2 = \frac{b}{3}$ and $\xi_1 + \xi_2 = -\frac{2a}{3}$, therefore

$$\alpha'' = H\left(A^2.\frac{b}{3} - \frac{2a}{3}.AB + B^2\right)$$

$$= H\left(c^2 - \frac{2ab}{3}.c + \frac{4a^3}{27}.c + \frac{4b^3}{27} - \frac{a^2b^2}{27}\right).$$

Again since $X = -\frac{a}{3}$ therefore

$$\alpha' = K\left(c - \frac{ab}{3} + \frac{2a^3}{27}\right).$$

Suppose a and b to vanish, then $\sqrt{\alpha'} = c\sqrt{H}$, $\alpha'' = K.c$; therefore

$$x_1 = \sqrt[3]{c}\{K + \sqrt{H}\} + \sqrt[3]{c}\{K - \sqrt{H}\} = \sqrt[3]{-c}: \text{ hence}$$

$$k + \sqrt{H} = -1\ K - \sqrt{H} = 0;$$

therefore

$$K = -\frac{1}{2}, H = \frac{1}{4};$$

thus the transformation is affected from symmetrical functions to given coefficients.

23. Expressions for α', α'' having been found by various methods, we have also the following properties with respect to their evanescence,

$$x_1 = -\frac{a}{3} + \sqrt[3]{(\alpha' + \sqrt{\alpha''})} + \sqrt[3]{(\alpha' - \sqrt{\alpha''})}.$$

When the quantity (α'') under the quadratic and cubic radicals vanishes, the proposed equation has two equal roots.

When the quantity (α') under the cubic but not under the quadratic surd vanishes, two of the three differences of the roots are equal, or one root is one half the sum of the other two.

When both these quantities (α', α'') vanish conjointly, the given equation has three equal roots.

Conversely when the proposed has two equal roots the equation of condition is $\alpha'' = 0$, when it has three equal roots the two equations of condition are $\alpha' = 0$, $\alpha'' = 0$.

When the quantity $\left(-\frac{a}{3}\right)$ in the expression for the root which is unaffected by surds vanishes, one root is equal to the sum of the other two with changed signs.

When $-\frac{a}{3}$ and α'' vanish conjointly, two of the roots are each one half of the third with a changed sign.

When $-\frac{a}{3}$ and α' vanish jointly, the equation has two roots equal, but with contrary signs, and the third root is zero.

When $-\frac{a}{3}$, α', α'' all vanish simultaneously, the three roots are equal to each other and to zero.

24. *Biquadratic Equations.*—Let x_1, x_2, x_3, x_4 be the four roots of the equation

$$x^4 + a\,x^3 + b\,x^2 + c\,x + d,$$

the number 4 being composite allows the subdivision of the sums of the roots taken two and two into three pairs, which have rational sums, thus

$$x_1 + x_2 + \frac{a}{2} = -\left(x_3 + x_4 + \frac{a}{2}\right), \left(x_1 + x_3 + \frac{a}{2}\right) = -\left(x_2 + x_4 + \frac{a}{2}\right),$$

$$x_1 + x_4 + \frac{a}{2} = -\left(x_2 + x_3 + \frac{a}{2}\right),$$

therefore the equation, of which the roots are

$$x_1 + x_2 + \frac{a}{2},\ x_1 + x_3 + \frac{a}{2},\ x_1 + x_4 + \frac{a}{2},\ -\left(x_3 + x_4 + \frac{a}{2}\right), \text{ \&c.}$$

is one of six dimensions, but without terms involving the odd powers of the unknown quantity, and therefore these quantities are the square roots of the roots of a cubic equation. Put therefore

$$x_1 + x_2 + \frac{a}{2} = 2\sqrt{\alpha}$$

$$x_1 + x_3 + \frac{a}{2} = 2\sqrt{\beta}$$

$$x_1 + x_4 + \frac{a}{2} = 2\sqrt{\gamma},$$

therefore

$$2\,x_1 + (x_1 + x_2 + x_3 + x_4) + \frac{3\,a}{2} = 2\,(\sqrt{\alpha} + \sqrt{\beta} + \sqrt{\gamma}),$$

or

$$x_1 = -\frac{a}{4} + \sqrt{\alpha} + \sqrt{\beta} + \sqrt{\gamma},$$

the quantities α, β, γ being the roots of a cubic equation, are of the form

$$\alpha = \alpha' + \sqrt[3]{\beta'} + \sqrt[3]{\beta''}$$

$$\beta = \alpha' + \theta\sqrt[3]{\beta'} + \theta^2\sqrt[3]{\beta''}$$

$$\gamma = \alpha' + \theta^2\sqrt[3]{\beta'} + \theta\sqrt[3]{\beta''},$$

when θ, θ^2 are the imaginary cube roots of unity, and β', β'' being the roots of a quadratic are of the forms

$$\beta' = \alpha'' + \sqrt{\alpha'''}$$
$$\beta'' = \alpha'' - \sqrt{\alpha'''}.$$

The three other roots of the biquadratic are

$$x_2 = 2\sqrt{\alpha} - \left(x_1 + \frac{a}{2}\right) = -\frac{a}{4} + \sqrt{\alpha} - \sqrt{\beta} - \sqrt{\gamma}$$
$$x_3 = 2\sqrt{\beta} - \left(x_1 + \frac{a}{2}\right) = -\frac{a}{4} - \sqrt{\alpha} + \sqrt{\beta} - \sqrt{\gamma}$$
$$x_4 = 2\sqrt{\gamma} - \left(x_1 + \frac{a}{2}\right) = -\frac{a}{4} - \sqrt{\alpha} - \sqrt{\beta} + \sqrt{\gamma}.$$

The constituent or essentially different parts of the roots are $\frac{a}{4}$, α', α'', α''', which we proceed to analyse by the conditions of their evanescence.

25. Suppose $\alpha''' = 0$, then by art. 23 two of the roots of the cubic are equal, or $\beta = \gamma$, from whence we have $x_3 = x_4$, therefore $x_3 - x_4$ is a factor of α''', and forming all the other symmetrical factors, we have

$$\alpha''' = k\,(x_1 - x_2)^2\,(x_1 - x_3)^2\,(x_1 - x_4)^2\,(x_2 - x_3)^2\,(x_2 - x_4)^2\,(x_3 - x_4)^2,$$

k being a numerical multiplier.

26. Next suppose $\alpha'' = 0$, then by the properties of the roots of the cubic already demonstrated we have $2\,\alpha = \beta + \gamma$, or $\alpha - \beta = \gamma - \alpha$, whence

$$(\sqrt{\alpha} + \sqrt{\beta})\,(\sqrt{\alpha} - \sqrt{\beta}) = (\sqrt{\gamma} + \sqrt{\alpha})\,(\sqrt{\gamma} - \sqrt{\alpha}),$$

therefore

$$(x_1 - x_4)\,(x_2 - x_3) = (x_1 - x_3)\,(x_4 - x_2)\,;$$

one factor of α'' is found thus to be

$$(x_1 - x_4)\,(x_2 - x_3) + (x_1 - x_3)\,(x_2 - x_4).$$

The two remaining symmetrical factors are

$$(x_1 - x_2)\,(x_3 - x_4) + (x_1 - x_4)\,(x_3 - x_2)$$
$$(x_1 - x_3)\,(x_4 - x_2) + (x_1 - x_2)\,(x_4 - x_3)\,;$$

and α'' is the product of all three multiplied by a numerical factor k'.

27. Again, suppose $\alpha' = 0$, then by the article above referred to $\alpha + \beta + \gamma = 0$; but

$$\beta + \gamma = (x_1 - x_2)^2 + (x_3 - x_4)^2$$
$$\alpha + \gamma = (x_1 - x_3)^2 + (x_2 - x_4)^2$$
$$\alpha + \beta = (x_1 - x_4)^2 + (x_2 - x_3)^2,$$

the sum of which being *per se* symmetrical, shows that α' has no other but a numerical factor; therefore

$$\alpha' = k''\,\{(x_1 - x_2)^2 + (x_1 - x_3)^2 + (x_1 - x_4)^2 + (x_2 - x_3)^2 + (x_2 - x_4)^2 + (x_3 - x_4)^2\}.$$

28. The numbers k, k', k'' may be found by the values already given for the roots of a cubic, by which we have

$$\alpha = \alpha' + \sqrt[3]{(\alpha'' + \sqrt{\alpha'''})} + \sqrt[3]{(\alpha'' - \sqrt{\alpha'''})}$$

and

$$\alpha' = \frac{\alpha + \beta + \gamma}{3}$$

therefore

$$\mathrm{K}'' = \frac{1}{2 \,.\, 3}$$

$$\alpha'' = \frac{1}{2 \,.\, 3^3} \,.\, (2\,\alpha - \beta - \gamma)\,(2\,\beta - \alpha - \gamma)\,(2\,\gamma - \alpha - \beta)$$

and the factors into which k' is multiplied are the same as

$$\frac{1}{2^3} \,.\, (2\,\alpha - \beta - \gamma) \,.\, (2\,\beta - \alpha - \gamma) \,.\, (2\,\gamma - \alpha - \beta)$$

therefore

$$k' = \frac{1}{2^4\,3^3}.$$

Lastly

$$\alpha''' = -\frac{1}{2^2 \,.\, 3^3} \,.\, (\alpha - \beta)^2\,(\alpha - \gamma)^2\,(\beta - \gamma)^2.$$

But since

$$\alpha + \gamma = (x_1 - x_3)^2 + (x_2 - x_4)^2$$

and

$$\beta + \gamma = (x_1 - x_4)^2 + (x_2 - x_3)^2$$

therefore

$$\alpha - \beta = 2\,(x_1\,x_4 - x_1\,x_3 + x_2\,x_3 - x_2\,x_4)$$
$$= 2\,(x_1 - x_2)\,(x_4 - x_3).$$

Similarly $\alpha - \gamma$ and $\beta - \gamma$ are expressed, and comparing the expression for α''' thus arising with that found before, we have

$$-\,2^2 \,.\, 3^3\,\alpha''' = 2^6 \,.\, \frac{\alpha'''}{k} \text{ or } k = -\frac{2^4}{3^3}.$$

29. Let us next seek the same quantities α', α'', α''', by the theory of elimination.

When $\alpha''' = 0$, the proposed equation which it will be convenient to express by $\varphi\,(x) = 0$ has two equal roots, the condition for which is also obtained by eliminating x between $\varphi\,(x) = 0$ and the first derived $\varphi'\,(x) = 0$, the function of the coefficients arising from this elimination is of the same dimensions, and expresses the same condition as the constituent quantity α''', and therefore only differs from it by a numerical multiplier.

This quantity in a symmetrical form relative to the roots is therefore

$$\alpha''' = h \,.\, \varphi'\,(x_1) \,.\, \varphi'\,(x_2) \,.\, \varphi'\,(x_3) \,.\, \varphi'\,(x_4)$$

and since

$$\varphi'\,(x_1) = (x_1 - x_2)\,(x_1 - x_3)\,(x_1 - x_4)$$

we have the same result as by the former method and $h = k$.

30. When α'' vanishes jointly with α''', then since $\alpha = \beta = \gamma$ we have also

$x_2 = x_3 = x_4$, therefore the equation $\phi(x) = 0$ has three equal roots, and since $\alpha''' = 0$ denotes the existence of two equal roots, therefore $\alpha'' = 0$ is the additional condition for a third, or the system of equations $\alpha'' = 0$, $\alpha''' = 0$, are equivalent to the system $\phi(x) = 0$, $\phi'(x) = 0$, $\phi''(x) = 0$.

Now $\frac{\phi''(x)}{2} = 6x^2 + 3ax + b = z + b$ for abridgment, we shall next form an equation in z indicative of two equal roots, and eliminating z by the equation $z + b = 0$ we shall obtain α''.

When $x_1 = x_2$, then
$$\begin{aligned} 6x_1^2 + 3ax_1 &= 6x_1^2 - 3x_1(2x_1 + x_3 + x_4) \\ &= -3(x_1x_3 + x_1x_4) \\ &= -3(x_1x_3 + x_2x_4) \end{aligned}$$
the equation
$$z + 3(x_1x_3 + x_2x_4) = 0$$
expresses that $x_1 = x_2$, and forming a cubic equation by taking all the symmetrical simple equations, a condition in z for the existence of any pair of equal roots is obtained, viz.
$$\text{F}(z) = \{z + 3(x_1x_3 + x_2x_4)\} \cdot \{z + 3(x_1x_2 + x_3x_4)\} \cdot \{z + 3(x_1x_4 + x_2x_3)\} = 0$$
or
$$\text{F}(z) = z^3 + 3bz^2 + 9(ac - 4d)z + 27\{d(a^2 - 4b) + e^2\} = 0;$$
and if z be eliminated between this equation, and $z + b = 0$, the result multiplied by a constant will be α'' or
$$\alpha'' = h'\,\text{F}(-b).$$

Now one factor of
$$\begin{aligned} \text{F}(-b) &= 2(x_1x_3 + x_2x_4) - (x_1x_2 + x_1x_4 + x_2x_3 + x_3x_4) \\ &= (x_1 - x_2)(x_3 - x_4) + (x_1 - x_4)(x_3 - x_2); \end{aligned}$$
the other two being symmetrical with it gives the same value of α'' as before, and $h' = k'$.

31. If $\alpha' = 0$, $\alpha'' = 0$, $\alpha''' = 0$ simultaneously, then $x_1 = x_2 = x_3 = x_4$. Now α' is a function of two dimensions, as is that arising by eliminating x between $\phi''(x) = 0$, $\phi'''(x) = 0$, and gives the same condition, but $\phi'''(x) = 6(4x + a)$, therefore $\phi''\left(\frac{-a}{4}\right)$ is the result of this elimination, therefore $\alpha' = h''\phi''\left(\frac{-a}{4}\right)$. Now
$$\phi''(x) = 2(6x^2 + 3ax + b)$$
therefore
$$\phi''\left(\frac{-a}{4}\right) = 2b - \frac{3a^2}{4},$$
and our former value of α' is
$$k''\{\Sigma 3x_1^2 - \Sigma 2x_1x_2\} = k''(3a^2 - 8b)$$
hence
$$h'' = -4k''.$$

32. Collecting the results of the last three articles they give us

$$\alpha' = -\frac{2}{3} \cdot \varphi''\left(\frac{-a}{4}\right)$$

$$\alpha'' = \frac{1}{2^4 \,.\, 3^3} \cdot \mathrm{F}\,(-b)$$

$$\alpha''' = -\frac{2^4}{3^3} \cdot \varphi'(x_1) \cdot \varphi'(x_2) \cdot \varphi'(x_3) \cdot \varphi'(x_4),$$

all of which may be expressed easily in terms of the coefficients.

33. *Theorems deduced.*—The root of the biquadratic $x^4 + a\,x^3 + b\,x^2 + c\,x + d = 0$ being expressed by

$$\begin{aligned} x_1 = -\frac{a}{4} &+ \sqrt{\{\alpha' + \sqrt[3]{(\alpha'' + \sqrt{\alpha'''})} + \sqrt[3]{(\alpha'' - \sqrt{\alpha'''})}\}} \\ &+ \sqrt{\{\alpha' + \theta\sqrt[3]{(\alpha'' + \sqrt{\alpha'''})} + \theta^2\sqrt[3]{(\alpha'' - \sqrt{\alpha'''})}\}} \\ &+ \sqrt{\{\alpha' + \theta^2\sqrt[3]{(\alpha'' + \sqrt{\alpha'''})} + \theta\sqrt[3]{(\alpha'' - \sqrt{\alpha'''})}\}}, \end{aligned}$$

where θ, θ^2 are the imaginary cube roots of unity, then the condition $\alpha''' = 0$ denotes the existence of two equal roots in the proposed equation.

The condition $\alpha'' = 0$ denotes the following relation of the roots

$$(x_1 - x_4)(x_2 - x_3) + (x_1 - x_3)(x_2 - x_4) = 0.$$

The system of coexisting conditions $\alpha'' = 0$, $\alpha''' = 0$ are necessary and sufficient for the existence of three equal roots.

The condition $\alpha' = 0$ denotes the following relation of the roots,

$$\Sigma\,(x_1 - x_2)^2 = 0.$$

The simultaneous system of conditions $\alpha' = 0$, $\alpha'' = 0$, $\alpha''' = 0$ essentially and sufficiently express the coexistence of four equal roots.

The rational part of the root $-\frac{a}{4}$ only vanishes with the sum of the roots.

34. We now proceed to determine the constituent parts of the roots of equations of the fifth degree by the conditions of their evanescence.

θ, θ^2 represent the imaginary cube roots of unity.

ω, ω^2, ω^3, ω^4 the imaginary fifth roots of unity.

x_1, x_2, x_3, x_4, x_5 are the five roots of the proposed equation of the fifth degree, viz.

$$x^5 + a\,x^4 + b\,x^3 + c\,x^2 + d\,x + e = 0.$$

$$x_1 = -\frac{a}{5} + \sqrt[5]{\alpha} + \sqrt[5]{\beta} + \sqrt[5]{\gamma} + \sqrt[5]{\delta}$$

$$x_2 = -\frac{a}{5} + \omega\sqrt[5]{\alpha} + \omega^2\sqrt[5]{\beta} + \omega^3\sqrt[5]{\gamma} + \omega^4\sqrt[5]{\delta}$$

$$x_3 = -\frac{a}{5} + \omega^2\sqrt[5]{\alpha} + \omega^4\sqrt[5]{\beta} + \omega\sqrt[5]{\gamma} + \omega^3\sqrt[5]{\delta}$$

$$x_4 = -\frac{a}{5} + \omega^3 \sqrt[5]{\alpha} + \omega \sqrt[5]{\beta} + \omega^4 \sqrt[5]{\gamma} + \omega^2 \sqrt[5]{\delta}$$

$$x_5 = -\frac{a}{5} + \omega^4 \sqrt[5]{\alpha} + \omega^3 \sqrt[5]{\beta} + \omega^2 \sqrt[5]{\gamma} + \omega \sqrt[5]{\delta};$$

the formulæ for x_3, x_4, x_5, x_1 are derived from the formula for x_2, by writing successively ω^2, ω^3, ω^4, ω^5 instead of ω.

$$\alpha = \alpha' + \sqrt{\beta'} + \sqrt{\gamma'} + \sqrt{\delta'}$$
$$\beta = \alpha' + \sqrt{\beta'} - \sqrt{\gamma'} - \sqrt{\delta'}$$
$$\gamma = \alpha' - \sqrt{\beta'} + \sqrt{\gamma'} - \sqrt{\delta'}$$
$$\delta = \alpha' - \sqrt{\beta'} - \sqrt{\gamma'} + \sqrt{\delta'},$$

such being the forms of the roots of a biquadratic.

Again, the expressions for β', γ', δ' as roots of a cubic, are

$$\beta' = \alpha'' + \sqrt[3]{\beta''} + \sqrt[3]{\gamma''}$$
$$\gamma' = \alpha'' + \theta \sqrt[3]{\beta''} + \theta^2 \sqrt[3]{\gamma''}$$
$$\delta' = \alpha'' + \theta^2 \sqrt[3]{\beta''} + \theta \sqrt[3]{\gamma''}.$$

Lastly, β'', γ'' as roots of a quadratic, are expressed by the following formulæ:

$$\beta'' = \alpha''' + \sqrt{\alpha^{\text{IV}}}$$
$$\gamma'' = \alpha''' - \sqrt{\alpha^{\text{IV}}}.$$

The quantities $-\frac{a}{5}$, α', α'', α''', α^{IV} are the constituent or essentially distinct parts of the roots x_1, x_2, x_3, x_4, x_5, and the analysis of their formation is to be sought by observing all the conditions under which each may vanish.

35. If for $-\frac{a}{5}$ we put $\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$, it is obvious that the system of five equations for the roots is equivalent to one of only four, viz.

$$5 \sqrt[5]{\alpha} = x_1 + \omega^4 x_2 + \omega^3 x_3 + \omega^2 x_4 + \omega x_5$$
$$5 \sqrt[5]{\beta} = x_1 + \omega^3 x_2 + \omega x_3 + \omega^4 x_4 + \omega^2 x_5$$
$$5 \sqrt[5]{\gamma} = x_1 + \omega^2 x_2 + \omega^4 x_3 + \omega x_4 + \omega^3 x_5$$
$$5 \sqrt[5]{\delta} = x_1 + \omega x_2 + \omega^2 x_3 + \omega^3 x_4 + \omega^4 x_5$$

the right hand members of which equations differ from each other only by the particular fifth root of unity in use, which considered as ω in the first, will be ω^2, ω^3, ω^4 respectively in the second, third, and fourth.

36. Suppose $\alpha^{\text{IV}} = 0$, then by art. 33 two roots of the biquadratic must be equal.

First let $\gamma = \delta$, which can only happen under the five following relations,

$$\sqrt[5]{\gamma} = \sqrt[5]{\delta},\ \sqrt[5]{\gamma} = \omega \sqrt[5]{\delta},\ \sqrt[5]{\gamma} = \omega^2 \sqrt[5]{\delta},\ \sqrt[5]{\gamma} = \omega^3 \sqrt[5]{\delta},\ \sqrt[5]{\gamma} = \omega^4 \sqrt[5]{\delta},$$

which furnish five factors, linear functions of the differences of the roots, and since these differences may be taken either way, we have also from the same equations five other factors equal to the former and with contrary signs.

In this manner ten factors of α^{IV} may be found by equating any pair of the four quantities α, β, γ, δ, and the number of pairs being six, the whole number of factors of α^{IV} is sixty, these factors are very easily formed, and here we present the first thirty factors, the remaining thirty being formed merely by changing the signs of these, or, which is the same, inverting the order of the differences of the roots in each factor.

37. For greater clearness we shall subdivide these thirty factors into five groups, from the first of which x_1 is excluded, from the second x_2, and so on; each subdivision contains six factors, four of which are of one form, and two of a different form; they are as follow:

$$\left.\begin{array}{rl}
(x_2 - x_4) + \omega\,(x_3 - x_4) + \omega^2\,(x_3 - x_5) & \ldots\ldots\ (1.)\\
(x_3 - x_2) + \omega\,(x_5 - x_2) + \omega^2\,(x_5 - x_4) & \ldots\ldots\ (2.)\\
(x_4 - x_5) + \omega\,(x_2 - x_5) + \omega^2\,(x_2 - x_3) & \ldots\ldots\ (3.)\\
(x_5 - x_3) + \omega\,(x_4 - x_3) + \omega^2\,(x_4 - x_2) & \ldots\ldots\ (4.)\\
(x_2 - x_5) + (\omega^2 + \omega^3)\,(x_3 - x_4) & \ldots\ldots\ (5.)\\
(x_3 - x_4) + (\omega^2 + \omega^3)\,(x_5 - x_2) & \ldots\ldots\ (6.)
\end{array}\right\}(A.)$$

$$\left.\begin{array}{rl}
(x_1 - x_4) + \omega\,(x_5 - x_4) + \omega^2\,(x_5 - x_3) & \ldots\ldots\ (1.)\\
(x_3 - x_5) + \omega\,(x_4 - x_5) + \omega^2\,(x_4 - x_1) & \ldots\ldots\ (2.)\\
(x_4 - x_3) + \omega\,(x_1 - x_3) + \omega^2\,(x_1 - x_5) & \ldots\ldots\ (3.)\\
(x_5 - x_1) + \omega\,(x_3 - x_1) + \omega^2\,(x_3 - x_4) & \ldots\ldots\ (4.)\\
(x_1 - x_3) + (\omega^2 + \omega^3)\,(x_5 - x_4) & \ldots\ldots\ (5.)\\
(x_5 - x_4) + (\omega^2 + \omega^3)\,(x_3 - x_1) & \ldots\ldots\ (6.)
\end{array}\right\}(B.)$$

$$\left.\begin{array}{rl}
(x_1 - x_2) + \omega\,(x_4 - x_2) + \omega^2\,(x_4 - x_5) & \ldots\ldots\ (1.)\\
(x_2 - x_5) + \omega\,(x_1 - x_5) + \omega^2\,(x_1 - x_4) & \ldots\ldots\ (2.)\\
(x_4 - x_1) + \omega\,(x_5 - x_1) + \omega^2\,(x_5 - x_2) & \ldots\ldots\ (3.)\\
(x_5 - x_4) + \omega\,(x_2 - x_4) + \omega^2\,(x_2 - x_1) & \ldots\ldots\ (4.)\\
(x_1 - x_5) + (\omega^2 + \omega^3)\,(x_4 - x_2) & \ldots\ldots\ (5.)\\
(x_4 - x_2) + (\omega^2 + \omega^3)\,(x_5 - x_1) & \ldots\ldots\ (6.)
\end{array}\right\}(C.)$$

$$\left.\begin{array}{rl}
(x_1 - x_5) + \omega\,(x_3 - x_5) + \omega^2\,(x_3 - x_2) & \ldots\ldots\ (1.)\\
(x_2 - x_3) + \omega\,(x_5 - x_3) + \omega^2\,(x_5 - x_1) & \ldots\ldots\ (2.)\\
(x_3 - x_1) + \omega\,(x_2 - x_1) + \omega^2\,(x_2 - x_5) & \ldots\ldots\ (3.)\\
(x_5 - x_2) + \omega\,(x_1 - x_2) + \omega^2\,(x_1 - x_3) & \ldots\ldots\ (4.)\\
(x_1 - x_2) + (\omega^2 + \omega^3)\,(x_3 - x_5) & \ldots\ldots\ (5.)\\
(x_3 - x_5) + (\omega^2 + \omega^3)\,(x_2 - x_1) & \ldots\ldots\ (6.)
\end{array}\right\}(D.)$$

$$\left.\begin{array}{rr}
(x_1 - x_3) + \omega\,(x_2 - x_3) + \omega^2\,(x_2 - x_4) \quad . \; . \; . \; . \; . \; . & (1.) \\
(x_2 - x_1) + \omega\,(x_4 - x_1) + \omega^2\,(x_4 - x_3) \quad . \; . \; . \; . \; . \; . & (2.) \\
(x_3 - x_4) + \omega\,(x_1 - x_4) + \omega^2\,(x_1 - x_2) \quad . \; . \; . \; . \; . \; . & (3.) \\
(x_4 - x_2) + \omega\,(x_3 - x_2) + \omega^2\,(x_3 - x_1) \quad . \; . \; . \; . \; . \; . & (4.) \\
(x_1 - x_4) + (\omega^2 + \omega^3)\,(x_2 - x_3) \quad . \; . \; . \; . \; . \; . & (5.) \\
(x_2 - x_3) + (\omega^2 + \omega^3)\,(x_4 - x_1) \quad . \; . \; . \; . \; . \; . & (6.)
\end{array}\right\}(\text{E.})$$

The other thirty factors of α^{IV} differ from these only in having ω^4 instead of ω, thus changing the sign of (1.), (A.), to obtain the first of these factors, and writing it in an inverted order it gives

$$\omega^2\,\{(x_5 - x_3) + \omega^4\,(x_4 - x_3) + \omega^3\,(x_4 - x_2)\}$$

differing from (4.), (A.) in having ω^4 for ω, for the numerical factor ω^2 may be rejected since $(\omega^2)^{30} = 1$.

The quantity α^{IV} is the product of these sixty factors and a numerical constant k.

38. Formation of the factors of α'''.

The factors in the group A of the preceding article denote for abridgment by their number placed as a subindex to A, and so for all the others, thus by B_3 is meant the third factor in the group B.

The quantity α''' is composed of the three factors of ten dimensions each. Every such factor is the sum of two parts, each decomposable into ten simple factors.

In the first pair of these simple factors x_1 does not enter, in the second pair x_2 is excluded, and so on.

These three compound factors are found as follows: First,

$$A_1 = \frac{5}{\omega\,(\omega - 1)}\,(\sqrt[5]{\gamma} - \sqrt[5]{\delta}) \qquad A_2 = -\frac{5}{\omega\,(\omega - 1)}\,(\sqrt[5]{\beta} - \sqrt[5]{\delta})$$

$$B_2 = \frac{5}{\omega^3\,(\omega - 1)}\,(\sqrt[5]{\gamma} - \omega\sqrt[5]{\delta}) \qquad B_3 = -\frac{5}{\omega^4\,(\omega - 1)}\,(\sqrt[5]{\beta} - \omega^2\sqrt[5]{\delta})$$

$$C_3 = \frac{5}{(\omega - 1)}\,(\sqrt[5]{\gamma} - \omega^2\sqrt[5]{\delta}) \qquad C_4 = -\frac{5}{\omega^2\,(\omega - 1)}\,(\sqrt[5]{\beta} - \omega^4\sqrt[5]{\delta})$$

$$D_4 = \frac{5}{\omega^2\,(\omega - 1)}\,(\sqrt[5]{\gamma} - \omega^3\sqrt[5]{\delta}) \qquad D_1 = -\frac{5}{(\omega - 1)}\,(\sqrt[5]{\beta} - \omega\sqrt[5]{\delta})$$

$$E_1 = \frac{5}{\omega^4\,(\omega - 1)}\,(\sqrt[5]{\gamma} - \omega^4\sqrt[5]{\delta}) \qquad E_2 = -\frac{5}{\omega^3\,(\omega - 1)}\,(\sqrt[5]{\beta} - \omega^3\sqrt[5]{\delta})$$

$$A_3 = \frac{5}{\omega\,(\omega - 1)}\,(\sqrt[5]{\alpha} - \sqrt[5]{\gamma}) \qquad A_4 = -\frac{5}{\omega\,(\omega - 1)}\,(\sqrt[5]{\alpha} - \sqrt[5]{\beta})$$

$$B_4 = \frac{5}{(\omega - 1)}\,(\sqrt[5]{\alpha} - \omega^2\sqrt[5]{\gamma}) \qquad B_1 = -\frac{5}{(\omega - 1)}\,(\sqrt[5]{\alpha} - \omega\sqrt[5]{\beta})$$

$$C_1 = \frac{5}{\omega^4\,(\omega - 1)}\,(\sqrt[5]{\alpha} - \omega^4\sqrt[5]{\gamma}) \qquad C_2 = -\frac{5}{\omega^4\,(\omega - 1)}(\sqrt[5]{\alpha} - \omega^2\sqrt[5]{\beta})$$

$$D_2 = \frac{5}{\omega^3\,(\omega - 1)}\,(\sqrt[5]{\alpha} - \omega\sqrt[5]{\gamma}) \qquad D_3 = -\frac{5}{\omega^3\,(\omega - 1)}\,(\sqrt[5]{\alpha} - \omega^3\sqrt[5]{\beta})$$

$$E_3 = \frac{5}{\omega^2(\omega - 1)}(\sqrt[5]{\alpha} - \omega^3 \sqrt[5]{\gamma}) \qquad E_4 = -\frac{5}{\omega^2(\omega - 1)}(\sqrt[5]{\alpha} - \omega^4 \sqrt[5]{\beta})$$

$$A_5 = \frac{5}{\omega^2(\omega - 1)}(\sqrt[5]{\beta} - \sqrt[5]{\gamma}) \qquad A_6 = \frac{5}{\omega^2(\omega - 1)}(\sqrt[5]{\alpha} - \sqrt[5]{\delta})$$

$$B_5 = -\frac{5}{\omega - 1}(\sqrt[5]{\beta} - \omega \sqrt[5]{\gamma}) \qquad B_6 = -\frac{5}{\omega(\omega - 1)}(\sqrt[5]{\alpha} - \omega^3 \sqrt[5]{\delta})$$

$$C_5 = \frac{5}{\omega^3(\omega - 1)}(\sqrt[5]{\beta} - \omega^2 \sqrt[5]{\gamma}) \qquad C_6 = -\frac{5}{\omega - 1}(\sqrt[5]{\alpha} - \omega \sqrt[5]{\delta})$$

$$D_5 = -\frac{5}{\omega(\omega - 1)}(\sqrt[5]{\beta} - \omega^3 \sqrt[5]{\gamma}) \qquad D_6 = \frac{5}{\omega^4(\omega - 1)}(\sqrt[5]{\alpha} - \omega^4 \sqrt[5]{\delta})$$

$$E_5 = \frac{5}{\omega^4(\omega - 1)}(\sqrt[5]{\beta} - \omega^4 \sqrt[5]{\gamma}) \qquad E_6 = \frac{5}{\omega^3(\omega - 1)}(\sqrt[5]{\alpha} - \omega^2 \sqrt[5]{\delta})$$

Therefore

$$\left.\begin{aligned} \gamma - \delta &= h \,.\, A_1 B_2 C_3 D_4 E_1 \\ \beta - \delta &= -h \,.\, A_2 B_3 C_4 D_1 E_2 \\ \alpha - \gamma &= h \,.\, A_3 B_4 C_1 D_2 E_3 \\ \alpha - \beta &= -h \,.\, A_4 B_1 C_2 D_3 E_4 \\ \beta - \gamma &= h \,.\, A_5 B_5 C_5 D_5 E_5 \\ \alpha - \delta &= h . A_6 . B_6 . C_6 . D_6 . E_6 \end{aligned}\right\}, h = \left(\frac{\omega - 1}{5}\right)^5.$$

39. Now the conditions necessary for the evanescence of α''' are by art. 33.

$$(\alpha - \delta)(\beta - \gamma)' + (\alpha - \gamma)(\beta - \delta) = 0$$
$$(\alpha - \gamma)(\delta - \beta) + (\alpha - \beta)(\delta - \gamma) = 0$$
$$(\alpha - \beta)(\gamma - \delta) + (\alpha - \delta)(\gamma - \beta) = 0.$$

Substitute the values of $\alpha - \delta$, $\beta - \gamma$, &c. above found, and including h^2 in the numerical multiplier k of the whole, we shall have

$$\begin{aligned} \alpha''' = k' &(A_5 A_6 . B_5 B_6 . C_5 C_6 . D_5 D_6 . E_5 E_6 - A_2 A_3 . B_3 B_4 . C_4 C_1 . D_1 D_2 . E_2 E_3) \\ \times &(A_5 A_6 . B_5 B_6 . C_5 C_6 . D_5 D_6 . E_5 E_6 + A_4 A_1 . B_1 B_2 . C_2 C_3 . D_3 D_4 . E_4 E_1) \\ \times &(A_2 A_3 . B_3 B_4 . C_4 C_1 . D_1 D_2 . E_2 E_3 + A_4 A_1 . B_1 B_2 . C_2 C_3 . D_3 D_4 . E_4 E_1). \end{aligned}$$

40. The condition that α'' may vanish is $\Sigma(\alpha - \beta)^2 = 0$ by art. 33. Hence

$$\begin{aligned} \alpha'' = k'' (&A_1^2 B_2^2 C_3^2 D_4^2 E_1^2 + A_2^2 B_3^2 C_4^2 D_1^2 E_2^2 + A_5^2 B_5^2 C_5^2 D_5^2 E_5^2 \\ &+ A_3^2 B_4^2 C_1^2 D_2^2 E_3^2 + A_4^2 B_1^2 C_2^2 D_3^2 E_4^2 + A_6^2 B_6^2 C_6^2 D_6^2 E_6^2), \end{aligned}$$

k'' being a numerical quantity.

41. The terms which compose α' are indicated below by the indices which enter them placed between brackets.

$$[5] = 4\,\Sigma\, x_1^5$$
$$[4, 1] = -5\,\Sigma\, x_1^4 x_2$$
$$[3, 2] = -10\,\Sigma\, x_1^3 x_2^2$$

$$\begin{aligned}
[3, 1, 1] = & - 20\, x_1^3 (x_2 x_3 + x_2 x_4 + x_3 x_5 + x_4 x_5 - 4\, x_2 x_5 - 4\, x_3 x_4) \\
& - 20\, x_2^3 (x_1 x_4 + x_1 x_5 + x_3 x_4 + x_3 x_5 - 4\, x_1 x_3 - 4\, x_4 x_5) \\
& - 20\, x_3^3 (x_1 x_2 + x_1 x_4 + x_2 x_5 + x_4 x_5 - 4\, x_1 x_5 - 4\, x_2 x_4) \\
& - 20\, x_4^3 (x_1 x_3 + x_1 x_5 + x_2 x_3 + x_2 x_5 - 4\, x_1 x_2 - 4\, x_3 x_5) \\
& - 20\, x_5^3 (x_1 x_2 + x_1 x_3 + x_2 x_4 + x_3 x_4 - 4\, x_1 x_4 - 4\, x_2 x_3)
\end{aligned}$$

$$\begin{aligned}
[2, 2, 1] = & - 30\, \{x_1^2 x_2^2 (x_3 + x_5 - 4\, x_4) + x_1^2 x_3^2 (x_4 + x_5 - 4\, x_2) \\
& + x_1^2 x_4^2 (x_2 + x_3 - 4\, x_5) + x_1^2 x_5^2 (x_2 + x_4 - 4\, x_3) \\
& + x_2^2 x_3^2 (x_1 + x_4 - 4\, x_5) + x_2^2 x_4^2 (x_1 + x_5 - 4\, x_3) \\
& + x_2^2 x_5^2 (x_3 + x_4 - 4\, x_1) + x_3^2 x_4^2 (x_2 + x_5 - 4\, x_1) \\
& + x_3^2 x_5^2 (x_1 + x_2 - 4\, x_4) + x_4^2 x_5^2 (x_1 + x_3 - 4\, x_2)\}
\end{aligned}$$

$$[2, 1, 1, 1] = - 60\, \Sigma\, x_1^2 x_2 x_3 x_4$$

$$[1, 1, 1, 1, 1] = 480\, x_1 x_2 x_3 x_4 x_5.$$

The quantity α' is the sum of all these multiplied by a constant k'''.

42. It remains to give the value of the constants k, k', k'', k''', which may be easily found by comparison of the above values with the constituent parts of the roots of a biquadratic; they are as follows:

$$k = - \left(\frac{\omega - 1}{5}\right)^{60} \cdot \frac{2^4}{3^3}$$

$$k' = \left(\frac{\omega - 1}{5}\right)^{30} \cdot \frac{1}{2^4 \cdot 3^3}$$

$$k'' = \left(\frac{\omega - 1}{5}\right)^{10} \cdot \frac{1}{2 \cdot 3}$$

$$k''' = \left(\frac{1}{5}\right)^{5} \cdot \frac{1}{4};$$

and the term uninfluenced by surds is $\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$.

43. All the constituent parts except the last-mentioned vanish when all the roots are equal, but for their separate evanescence the condition for the equality of roots is insufficient, the requisite conditions are easily seen from the factors of such parts already given; they are of two classes, arising from the different relations of the couples $\{(\omega, \omega^2), (\omega, \omega^3), (\omega^3, \omega^4), (\omega^3, \omega^4)\}$, and of the two couples $\{(\omega^2, \omega^3), (\omega, \omega^4)\}$, each member of which contains the same imaginary part with the other, which is not the case with the former couples.

44. I cannot at present, from the pressure of other engagements, pursue these investigations further; if any additional light to the analyst is furnished by the preceding imperfect reflections on a subject so often treated, the author's object will in a great measure be attained; they at least tend to show the imperfection of our

knowledge with respect to the conditions resulting from elimination, where more than two equations are concerned, and they exhibit in the higher powers relations between pairs of roots which have not been as yet expressed, even by the differential calculus.

London, April 1, 1837.

PHILOSOPHICAL TRANSACTIONS.

XII. *First Memoir on the Theory of Analytical Operations.* *By the Rev.* R. MURPHY, *M.A. Fellow of Caius College, Honorary Member of various Philosophical Societies. Communicated by* J. W. LUBBOCK, *Esq. F.R.S.*

Received December 8,—Read December 22, 1836.

§ 1.

1. THE elements of which every distinct analytical process is composed are three, namely, first the *Subject,* that is, the symbol on which a certain notified operation is to be performed; secondly, the *Operation* itself, represented by its own symbol; thirdly, the *Result,* which may be connected with the former two by the algebraic sign of equality.

Thus let a be the *subject* representing, we may suppose, some quantity, b the symbol for multiplication by b, and c the result or product; for greater distinctness let the subject be inclosed in square brackets, the analytical process in this case is $[a]\ b = c$.

Again, let x^n be the subject, ψ a symbol of operation denoting that x must be changed into $x + h$, and $(x + h)^n$ will evidently be the result, or

$$[x^n]\ \psi = (x + h)^n.$$

Again, let a^x be the subject, Δ a symbol of operation, which indicates that we are to subtract the subject itself from that which it becomes when x is changed into $x + h$, which is usually called taking the finite difference, then the result is

$$a^{x+h} - a^x,$$

or

$$[a^x]\ \Delta = [a^x]\ (a^h - 1).$$

Lastly, let d_x denote the operation of taking the finite difference, and after dividing it by h, then putting $h = 0$, which is the same as finding the differential coefficient of the subject, which we may suppose represented by u, then

$$[u]\ d_x = \frac{d\,u}{d\,x}.$$

2. The operations written as above are *monomial*, consisting of only one term; and *polynomial* operations give the sums or the differences of the results of the respective monomials of which they are formed, according as these monomials are affected by the signs $+$ or $-$.

Thus if 1 as an operation be understood as the multiplying of the subject by unity which leaves it unaltered, and the symbols ψ, Δ have the same signification as in art. 1, then

$$[u]\,(\psi - 1) = [u]\,\Delta$$
$$[u]\,(\Delta + 1) = [u]\,\psi,$$

where the subject u is any quantity whatever.

When general relations, such as these, between different symbols exist independently of the particular value of the subject, we may abstract the consideration of the latter, and the sign $=$ between symbols of operation being understood to indicate that they are universally equivalent, the symbols used in art. 1 would have the following relations independent of the subject.

$$\Delta = \psi - 1, \quad \psi = \Delta + 1, \quad \psi - \Delta = 1,$$
$$d_x = \frac{\Delta}{h} = \frac{\psi - 1}{h},$$

when h is put $= 0$.

3. A compound operation consists of a series of simple defined operations, monomial or polynomial, the subject of each individual in this series after the first being the result of all the preceding operations.

Thus $[x^2]\,a\,\psi\,\Delta\,d_x$ denotes that first x^2 must be multiplied by a, which gives $a\,x^2$; then the operation ψ, which denotes the putting $x + h$ for x, gives $a\,(x + h)^2$; next the operation Δ will give $a\,(x + 2\,h)^2 - a\,(x + h)^2$, or $a\,(2\,h\,x + 3\,h^2)$; and lastly, the symbol of taking the differential coefficient relative to x gives $2\,h\,a$: the final or complete result is therefore

$$[x^2]\,a\,\psi\,\Delta\,d_x = 2\,h\,a.$$

When all the symbols in a compound operation are exactly the same, then for abridgment the whole operation is represented by writing an index to the right of the symbol for the simple operation over it, this index expressing the number of times the simple operation is repeated. Thus

$$[x^2]\,\psi^3 = [x^2]\,\psi\,\psi\,\psi = [(x + h)^2]\,\psi\,\psi = [(x + 2\,h)^2]\,\psi = (x + 3\,h)^2.$$

But when the simple operations are different, they must be written consecutively in the order in which they are to be performed, unless that order of arrangement by the mutual relations of the operations should be indifferent. Thus

$$[x^n]\,x\,\psi = [x^{n+1}]\,\psi = (x + h)^{n+1}$$
$$[x^n]\,\psi\,x = [(x + h)^n]\,x = x\,(x + h)^n.$$

But

$$[x^n]\, a\, \psi = [a\, x^n]\, \psi = a\,(x+h)^n$$
$$[x^n]\, \psi\, a = [(x+h)^n]\, a = a\,(x+h)^n.$$

in the latter case the order of the operations is indifferent, because the operation ψ does not act on the multiplier a, and for the contrary reason the order of $x\, \psi$ is *fixed* in the first case.

Operations are therefore relatively *fixed* or *free*; in the first case a change in the order in which they are to be performed would affect the result, in the second case it would not do so.

In a compound operation any part of the symbols may be taken conjointly with the subject in the square brackets, their result being the subject for the compound operation of the remaining symbols. Thus

$$[x^2]\, a\, \psi\, \Delta\, d_x = [a\, x^2]\, \psi\, \Delta\, d_x.$$

§ 2.

4. Linear operations in analysis are those of which the action on any subject is made up by the several actions on the parts, connected by the sign + or −, of which the subject is composed.

Let p denote the operation of multiplying by a quantity p, then

$$[a+b]\, p = [a]\, p + [b]\, p;$$

this operation is therefore linear.

Let ψ denote the operation of changing x into $x + h$, then if $f(x)$, $\varphi(x)$ be any functions of x, we have

$$[f(x) + \varphi(x)]\, \psi = f(x+h) + \varphi(x+h) = [f(x)]\, \psi + [\varphi(x)]\, \psi,$$

which shows that ψ is also a linear operation.

Let $X + \xi$ represent the subject acted on, and θ, θ' any linear operations, then

$$\begin{aligned}[X+\xi]\,(\theta+\theta') &= [X+\xi]\,\theta + [X+\xi]\,\theta' \\ &= [X]\,\theta + [\xi]\,\theta + [X]\,\theta' + [\xi]\,\theta' \\ &= [X]\,(\theta+\theta') + [\xi]\,(\theta+\theta');\end{aligned}$$

hence polynomial operations of which the parts are linear possess themselves the same character.

Thus Δ the operation of Finite Differences is linear, because $\Delta = \psi - 1$, the operation ψ of changing x into $x + h$ and the multiplying by unity being both linear.

Also

$$\begin{aligned}[X+\xi]\,\theta\,\theta' &= [X\,\theta + \xi\,\theta]\,\theta' \\ &= [X]\,\theta\,\theta' + [\xi]\,\theta\,\theta',\end{aligned}$$

which shows that the compounds of linear operations are also linear.

The operation of taking the differential coefficient is therefore linear, for the operation Δ of finite differences and that of dividing by h being both of that character, the compound $\Delta . \frac{1}{h}$ is generally linear, and must remain so in the limiting state when h vanishes. Hence every function of a linear operation is itself of the same class of operations.

5. The composition of polynomial linear operations is effected in the same manner as algebraic multiplication; with, however, this peculiarity—the order of the compound operations, when not relatively free, must be strictly preserved.

Thus let θ, θ' ι, ι' represent linear operations: then

$$[u]\,(\theta + \theta')\,(\iota + \iota') = [u]\,(\theta\,\iota + \theta'\,\iota + \theta\,\iota' + \theta'\,\iota');$$

for $\iota\,\iota'$ being linear will act on each of the parts $[u]\,\theta$, $[u]\,\theta'$, which form their subject.

Again, when θ, θ' are relatively fixed, $[u]\,(\theta + \theta')^2$ will not, as in algebraic involution, be identical with $[u]\,(\theta^2 + 2\,\theta\,\theta' + \theta'^2)$, its correct value being $[u]\,(\theta^2 + \theta'\,\theta + \theta\,\theta' + \theta'^2)$; which, however, is the same as the former expression, when θ, θ' are relatively free; for then $\theta'\,\theta = \theta\,\theta'$.

Similarly

$$(\theta + \theta')^3 = (\theta^3 + \theta'\,\theta^2 + \theta\,\theta'\,\theta + \theta'^2\,\theta) + (\theta^2\,\theta' + \theta'\,\theta\,\theta' + \theta\,\theta'^2 + \theta'^3);$$

or putting $\theta^{(2)}\,\theta'$ for the sum of the compound operations in which θ twice enters, and θ' and $\theta'^{(2)}$ for the sum of those containing θ once and θ' twice, this may also be written

$$(\theta + \theta')^3 = \theta^3 + \theta^{(2)}\,\theta' + \theta\,\theta'^{(2)} + \theta'^3;$$

and employing a similar notation, we shall have, when n is a positive integer,

$$(\theta + \theta')^n = \theta^{(n)} + \theta^{(n-1)}\,\theta' + \theta^{(n-2)}\,\theta'^{(2)} + \ldots\ldots \theta^{(2)}\,\theta'^{(n-2)} + \theta\,\theta'^{(n-1)} + \theta'^{(n)}.$$

The term $\theta^{(n-1)}\,\theta'$ in this formula is the sum of n terms, formed by placing θ' at the beginning, at the end, and in all the $n - 2$ intermediate positions of the expression $\theta\,\theta\,\theta \ldots\ldots (n - 1)$ times. Similarly, by the known theory of permutations, $\theta^{(n-2)}\,\theta'^{(2)}$ is the sum of $\frac{n\,(n-1)}{1\,.\,2}$ terms, &c.

Hence when θ, θ' are relatively free, we have

$$(\theta + \theta')^n = \theta^n + n\,\theta^{n-1}\,\theta' + \frac{n\,(n-1)}{1\,.\,2}\,.\,\theta^{n-2}\,\theta'^2 + \ldots . n\,\theta\,\theta'^{n-1} + \theta'^n.$$

6. The following theorems are immediately deducible from the general expansion of the preceding article:

Since $\Delta = \psi - 1$, therefore

$$\Delta^n = (\psi - 1)^n = \psi^n - n\,\psi^{n-1} + \frac{n\,(n-1)}{1\,.\,2}\,.\,\psi^{n-2} - \frac{n\,(n-1)\,(n-2)}{1\,.\,2\,.\,3}\,.\,\psi^{n-3} + \ldots . + (-1)^n;$$

or, if we introduce the subject $f\,(x)$, and observe that $[f\,(x)]\,\psi^n = f\,(x + n\,h)$, then

$$\Delta^n f\,(x) = f\,(x + n\,h) - n f\,\{x + (n-1)\,.\,h\} + \frac{n\,(n-1)}{1\,.\,2}\,.\,f\,\{x + (n-2)\,h\} -, \&c. \quad \text{(I.)}$$

Again, $\psi = \Delta + 1$, therefore

$$\psi^n = (\Delta + 1)^n = 1 + n\Delta + \frac{n(n-1)}{1.2} . \Delta^2 + \frac{n(n-1)(n-2)}{1.2.3} . \Delta^3 \ldots\ldots + \Delta^n \ldots;$$

or, introducing the subject $f(x)$,

$$f(x+nh) = f(x) + n\,\Delta f(x) + \frac{n(n-1)}{1.2} . \Delta^2 f(x) + \frac{n(n-1)(n-2)}{1.2.3} . \Delta^3 f(x) + \ldots\ldots \quad \text{(II.)}$$

Again, $1 = \psi - \Delta$, therefore

$$1 = (\psi - \Delta)^n = \psi^n - n\psi^{n-1}\Delta + \frac{n(n-1)}{1.2} . \psi^{n-2}\Delta^2 - \frac{n(n-1)(n-2)}{1.2.3} . \psi^{n-3}\Delta^3 \ldots\ldots (-1)^n \Delta^n,$$

or

$$\left.\begin{aligned} & f(x) = f(x+nh) - n\,\Delta f\{x + (n-1).h\} + \frac{n(n-1)}{1.2} . \Delta^2 \\ & f\{x + (n-2).h\} - \frac{n(n-1)(n-2)}{1.2.3}\,\Delta^3 f\{x + (n-3)h\}, \&c. \end{aligned}\right\} \quad \ldots\ldots \quad \text{(III.)}$$

In the expansion (II.) put $n h = k$, or $n = \frac{k}{h}$: therefore

$$f(x+k) = f(x) + k\frac{\Delta}{h} f(x) + \frac{k.(k-h)}{1.2} . \left(\frac{\Delta}{h}\right)^2 f(x) + \frac{k.(k-h)(k-2h)}{1.2.3} . \left(\frac{\Delta}{h}\right)^3 f(x), \&c.$$

Now suppose n to increase infinitely, k remaining constant, the quantity h, which is the increment of x, diminishes infinitely, and the operation $\frac{\Delta}{h}$ in the limiting state when h vanishes, becomes d_x. Hence

$$f(x+k) = f(x) + k\,d_x f(x) + \frac{k^2}{1.2}\,d_x^{\,2} f(x) + \frac{k^3}{1.2.3}\,d_x^{\,3} f(x) +, \&c. \quad . \quad \text{(IV.)}$$

The expansions (II.) (IV.) are TAYLOR's theorems for the development of functions by means of their finite differences and their differential coefficients respectively.

Again, if h be written for k in the expansion (IV.), and the subject be omitted, it becomes

$$\psi = 1 + h\,d_x + \frac{h^2 d_x^{\,2}}{1.2} + \frac{h^3 d_x^{\,3}}{1.2.3} +, \&c. \quad \ldots\ldots\ldots \quad \text{(V.)}$$

and

$$\Delta = h\,d_x + \frac{h^2 d_x^{\,2}}{1.2} + \frac{h^3 d_x^{\,3}}{1.2.3}, \&c.$$

§ 3.

7. The expansion given for the operation ψ, of changing x into $x + h$, possesses remarkable properties, which we propose to develop in the present section, from the importance of the theorem of TAYLOR, which it expresses.

Representing, as usual, by ψ the operation of changing x into $x + h$, and by ψ' that of changing x into $x + h'$, the quantities h, h' being independent of x, and, lastly, denoting by $\psi_{\prime}$ the operation which changes x into $x + h + h'$, we have obviously the identity

$$\psi\,\psi' = \psi_{\prime};$$

and putting for these symbols their expansions found in the preceding article, we get

$$\left\{1 + h\,d_x + \frac{h^2\,d_x^2}{1\,.\,2} + \frac{h^3\,d_x^3}{1.2.3} +, \&c.\right\}\left\{1 + h'\,d_x + \frac{h'^2\,d_x^2}{1\,.\,2} + \frac{h'^3\,d_x^3}{1\,.\,2\,.\,3} +, \&c.\right\}$$

$$= 1 + (h + h')\,d_x + \frac{(h+h')^2\,d_x^2}{1\,.\,2} + \frac{(h+h')^3\,d_x^3}{1\,.\,2\,.\,3} +, \&c.\,;$$

a relation which may be verified by actually compounding the two polynomials of the first member.

Now in this act of verification the operations $h\,d_x$ $h'\,d_x$ have only such properties as are common to any two linear operations which are relatively free: hence if θ, θ' represent any such operations, we have generally

$$\left\{1 + \theta + \frac{\theta^2}{1\,.\,2} + \frac{\theta^3}{1\,.\,2\,.\,3} +, \&c.\right\}\,.\,\left\{1 + \theta' + \frac{\theta'^2}{1\,.\,2} + \frac{\theta'^3}{1\,.\,2\,.\,3} +, \&c.\right\}$$

$$= 1 + (\theta + \theta') + \frac{(\theta+\theta')^2}{1\,.\,2} + \frac{(\theta+\theta')}{1\,.\,2\,.\,3}:$$

and it is easy to extend an identity exactly similar to any number of operations which are all relatively free; for in introducing a new polynomial, $1 + \theta'' + \frac{\theta''^2}{1.2} + \frac{\theta''^3}{1.2.3} +, \&c.$, we have only to regard $\theta + \theta'$ as itself a free linear operation, and therefore the result would be

$$1 + (\theta + \theta' + \theta'') + \frac{(\theta+\theta'+\theta'')^2}{1\,.\,2} + \frac{(\theta+\theta'+\theta'')^3}{1\,.\,2\,.\,3} +, \&c.$$

8. If the subject be a function of two variables, x and y, then using ψ_x to denote that x must be changed into $x + h$, and ψ_y that y must become $y + k$, these operations are relatively free, it being of no consequence which operation is first performed; therefore the operations Δ_x, Δ_y of taking the corresponding finite differences are also free; from whence, lastly, the differentiations relative to x and y, represented by d_x, d_y, must be of the same character.

Now since

$$\psi_x = 1 + h\,d_x + \frac{h^2\,d_x^2}{1\,.\,2} + \frac{h^3\,d_x^3}{1\,.\,2\,.\,3} +, \&c.$$

and

$$\psi_y = 1 + k\,d_y + \frac{k^2\,d_y^2}{1\,.\,2} + \frac{k^3\,d_y^3}{1\,.\,2\,.\,3} +, \&c.$$

therefore, by the general identity of (7.), we have

$$\psi_x\,\psi_y = 1 + (h\,d_x + k\,d_y) + \frac{(h\,d_x + k\,d_y)^2}{1\,.\,2} + \frac{(h\,d_x + k\,d_y)^3}{1\,.\,2\,.\,3} +, \&c.$$

And now introducing the subject $f(x, y)$, and expanding the terms in the right member of this identity by the formula given in the preceding section, it will become, in the common notation,

$$f(x+h, y+k) = f(x,y) + \frac{df(x,y)}{dx} \cdot h + \frac{d^2 f(x,y)}{dx^2} \cdot \frac{h^2}{1.2} + \frac{d^3 f(x,y)}{dx^3} \cdot \frac{h^3}{1.2.3} +, \&c.$$

$$+ \frac{d.f(x,y)}{dy} \cdot k + \frac{d^2 f(x,y)}{dx\, dy} \cdot h\,k + \frac{d^3 f(x,y)}{dx^2\, dy} \cdot \frac{h^2 k}{1.2} +, \&c.$$

$$+ \frac{d^2 f(x,y)}{dy^2} \cdot \frac{k^2}{1.2} + \frac{d^3 f(x,y)}{dx\, dy^2} \cdot \frac{h\,k^2}{1.2} +, \&c.$$

$$+ \frac{d^3 f(x,y)}{dy^3} \cdot \frac{k^3}{1.2.3} +, \&c.$$

If a third variable z becomes $z + l$ by a third operation ψ'', then the actual composition of the equivalent polynomial $1 + l\,d_z + \frac{l^2 d_z^2}{1.2} + \frac{l^3 d_z^3}{1.2.3} +$, &c. gives

$$\psi\,\psi'\,\psi'' = 1 + (h\,d_x + k\,d_y + l\,d_z) + \frac{1}{1.2}(h\,d_x + k\,d_y + l\,d_z)^2 + \frac{1}{1.2.3}(h d_x + k d_y + l d_z)^3 +, \&c.;$$

and so this method may be extended, whatever be the number of variables.

9. Let θ denote any linear operation, and make

$$\Theta = 1 + \theta + \frac{\theta^2}{1.2} + \frac{\theta^3}{1.2.3} +, \&c.,$$

then Θ is itself a linear operation; and if in (7.) we put $\theta = \theta' = \theta'' =$, &c., and suppose the number of these operations to be n, we have by that article

$$\Theta^n = 1 + n\,\theta + \frac{n^2\,\theta^2}{1.2} + \frac{n^3\,\theta^3}{1.2.3} +, \&c.$$

Again, suppose

$$\phi = 1 + \frac{\theta}{m} + \frac{\theta^2}{1.2\,m^2} + \frac{\theta^3}{1.2.3\,m^3} +, \&c.,$$

m being any positive integer, we have by the same principles

$$\phi^m = 1 + \theta + \frac{\theta^2}{1.2} + \frac{\theta^3}{1.2.3} +, \&c. = \Theta$$

$$\phi^n = 1 + \frac{n}{m} \cdot \theta + \frac{n^2}{m^2} \cdot \frac{\theta^2}{1.2} + \frac{n^3}{m^3} \frac{\theta^3}{1.2.3} +, \&c.$$

Hence if ϕ denote an operation which, repeated m times, gives Θ, and in which we shall employ the notation $\Theta^{\frac{1}{m}}$, then ϕ^n denotes the same as $\Theta^{\frac{n}{m}}$, the latter being the operation which, repeated m times, is the same as Θ^n. With this meaning understood, it follows that

$$\Theta^{\frac{n}{m}} = 1 + \frac{n}{m} \cdot \theta + \frac{n^2}{m^2} \cdot \frac{\theta^2}{1.2} + \frac{n^3}{m^3} \cdot \frac{\theta^3}{1.2.3} +, \&c.$$

Lastly, if we put

$$\Omega = 1 - n\,\theta + \frac{n^2\,\theta^2}{1.2} - \frac{n^3\,\theta^3}{1.2.3} +, \&c.,$$

compound this by the formula of (7.) with the operation

$$\phi^n = 1 + n\,\theta + \frac{n^2\,\theta^2}{1.2} + \frac{n^3\,\theta^3}{1.2.3} +, \&c.,$$

we have

$$\Omega\,\phi^n = 1 + (n - n)\,.\,\theta + \frac{(n-n)^2\theta^2}{1\,.\,2} + \&c. = 1\,;$$

or if we introduce any subject $f(x)$; Ω, ϕ^n being necessarily free,

$$[f(x)]\,\Omega\,\phi^n = [f(x)]\,\phi^n\,\Omega = f(x).$$

Hence Ω, ϕ^n are mutually *inverse* operations, the action of the one on the result of the other restoring the original subject. Ω may be represented by ϕ^{-n}, attaching to this symbol the meaning here assigned.

If n represent any quantity, positive negative, integer, or fractional, understanding the conventional notation by the definitions laid down, it follows, that if

$$\Theta = 1 + \theta + \frac{\theta^2}{1\,.\,2} + \frac{\theta^3}{1\,.\,2\,.\,3} +, \&c.,$$

then shall

$$\Theta^n = 1 + n\,\theta + \frac{n^2\,\theta^2}{1\,.\,2} + \frac{n^3\,\theta^3}{1\,.\,2\,.\,3} +, \&c.$$

10. Suppose θ to be simply the operation of multiplying by unity, then

$$\Theta = 1 + 1 + \frac{1}{1.2} + \frac{1}{1.2} +, \&c.\,;$$

and putting as usual ϵ for the sum of this series, Θ represents the operation of multiplying by ϵ; put therefore in the formulæ of art. 9. 1 for θ, and ϵ for Θ, and then

$$\epsilon^n = 1 + n + \frac{n^2}{1.2} + \frac{n^3}{1\,.\,2\,.\,3} +, \&c.$$

The properties of this series when any way involved are common, as has been seen, to those in a series where θ, any linear operation, is put for n, and therefore we may write the purely symbolical identity

$$\epsilon^\theta = 1 + \theta + \frac{\theta^2}{1.2} + \frac{\theta^3}{1\,.\,2\,.\,3} +, \&c.,$$

where θ may be an *imaginary* multiplier, or any species of linear operation.

Thus if $\theta = h\,d_x$ and ψ_x denote the operation of changing x into $x + h$, we have

$$\psi_x = \epsilon^{h\,d_x}.$$

Similarly

$$\psi_y = \epsilon^{k\,d_y}$$

$$\psi_x\,\psi_y = \epsilon^{(h\,d_x + k\,d_y)},$$

all of which are proved by the formulæ of art. 8.

11. Having seen in the course of the investigations of this section the signification of the indices of operations when fractional negative or even purely symbolical of linear operations, it is easy to prove by similar steps that in all cases where θ, θ' are relatively free,

$$(\theta + \theta')^n = \theta^n + n\,\theta^{n-1}\,\theta' + \frac{n\,(n-1)}{1\,.\,2}\,.\,\theta^{n-2}\,\theta'^2 +, \&c.\,;$$

for since

$$(\theta + \theta')^n (\theta + \theta')^m = (\theta + \theta')^{n+m},$$

it follows that the composition of the polynomials

$$\left\{\theta^n + n\,\theta^{n-1}\theta' + \frac{n(n-1)}{1\,.\,2}\,.\,\theta^{n-2}\theta'^2 +, \&c.\right\}.\left\{\theta^m + m\,\theta^{m-1}\theta' + \frac{m(m-1)}{1\,.\,2}\,.\,\theta^{m-2}\theta'^2 +, \&c.\right\}$$

$$= \theta^{n+m} + (n+m)\,\theta^{n+m-1}\theta' + \frac{(n+m)\,(n+m-1)}{1\,.\,2}\,.\,\theta^{n+m-2}\theta'^2, \&c.;$$

and since nothing in the actual verification of this identity depends on their being integers, for which case the expansion has been proved, the identity holds generally, and therefore if $m = n$, and we take p, such polynomials, we have

$$\left\{\theta^n + n\theta^{n-1}\theta' + \frac{n(n-1)}{1\,.\,2}\,.\,\theta^{n-2}\theta'^2 + \,.\,.\right\}^p = \theta^{np} + np\,\theta^{np-1}\theta' + \frac{(np)(np-1)}{1\,.\,2}\,.\,\theta^{np-2}\theta'^2$$

$$\left\{\theta^n + n\,\theta^{n-1}\theta' + \,.\,.\,.\,.\,.\right\}^q \qquad = \theta^{nq} + nq\,\theta^{nq-1}\theta' + \&c.$$

Put $n = \frac{q}{p}$,

$$\therefore \left\{\theta^{\frac{q}{p}} + \frac{q}{p}\,.\,\theta^{\frac{q}{p}-1}\theta' + \,.\,.\,.\,.\,.\right\}^p = \theta^q + q\,\theta^{q-1}\theta' + \,.\,.\,.\,. = (\theta + \theta')^q,$$

or

$$(\theta + \theta')^{\frac{q}{p}} = \theta^{\frac{q}{p}} + \frac{q}{p}\,.\,\theta^{\frac{q}{p}-1}\theta' + \frac{\frac{q}{p}\left(\frac{q}{p} - 1\right)}{1\,.\,2}\,.\,\theta^{\left(\frac{q}{p}-2\right)}\theta'^2 + \&c.$$

Again, since

$$\left\{\theta^n + n\theta^{n-1}\theta' + \frac{n(n-1)}{1\,.\,2}\theta^{n-2}\theta'^2 + \&c.\right\}.\left\{\theta^{-n} - n\theta^{-n-1}\theta' + \frac{n(n+1)}{1\,.\,2}\theta^{-n-2}\theta'^2 - \&c.\right\} = 1,$$

we have

$$(\theta + \theta')^{-n} = \theta^{-n} - n\,\theta^{-n-1}\theta' + \&c.$$

These formulæ applied to quantities or *fixed* operations, suffice after the usual methods for calculating their finite differences, differential coefficients, &c.

§ 4.

12. Suppose θ to represent any operation which performed on a subject $[u]$ gives y as the result, then the inverse operation is denoted by θ^{-1}, and is such that when $[y]$ is made the subject u becomes the result.

Thus b denoting the operation of multiplying by a quantity b, we have

$$[a]\,b = c \quad \therefore [c]\,b^{-1} = a.$$

Again, ψ denoting the operation of changing x into $x + h$, we have

$$[f(x)]\,\psi = \varphi(x), \qquad \text{where } \varphi(x) = f(x + h)$$

$$\therefore [\varphi(x)]\,\psi^{-1} = f(x), \qquad \text{where } f(x) = \varphi(x - h)$$

and in general if

$$[u]\,\theta = y \qquad \text{then } [y]\,\theta^{-1} = u, \qquad \text{and therefore } [y]\,\theta^{-1}\,\theta = y,$$

the compound operation $\theta^{-1}\,\theta$ or θ^0 being equivalent to no operation.

To invert a compound operation we must invert the *order* as well as the nature of the component operations, which rule must be strictly observed when the latter are relatively fixed. For let

$$[x]\,\theta = y \qquad [y]\,\theta' = z \qquad \text{and therefore } [x]\,\theta\,\theta' = z.$$

$$\text{Then} \quad [z]\,\theta'^{-1} = y \qquad [y]\,\theta^{-1} = x \qquad \text{and} \ldots\ldots [z]\,\theta'^{-1}\,\theta^{-1} = x,$$

which proof is applicable, whatever be the number of the component operations.

If all the component operations be alike, we have then but to change the sign of the index to obtain the inverse, thus if

$$[u]\,\theta^n = y, \qquad \text{then } [y]\,\theta^{-n} = u,$$

as in the last section.

13. The consideration of inverse operations leads to the introduction of the *appendage*, which when the operations are linear must be annexed to the result to give it the most general value of which it is susceptible, for the inverses of such operations are themselves linear; thus if θ be a linear operation,

$$[X + \xi]\,\theta = [X]\,\theta + [\xi]\,\theta.$$

Put $[X]\,\theta = X_1$, $[\xi]\,\theta = \xi_1$,

$$\text{hence } [X_1 + \xi_1]\,\theta^{-1} = X + \xi$$

$$= [X_1]\,\theta^{-1} + [\xi_1]\,\theta^{-1},$$

which shows the linearity of θ^{-1}.

Now suppose the nature of θ to be such that $[P]\,\theta = 0$, the subject P being thus in some way connected with the nature of the operation θ, then if we suppose

$$[X]\,\theta = y, \qquad \text{we have also } [X + P]\,\theta = y, \qquad \text{hence } [y]\,\theta^{-1} = X + P,$$

this being the same as writing $y + 0$ for y, since $[0]\,\theta^{-1} = P$.

The appendage therefore in a linear operation is the result of its action on zero; P will express a *form*, but its magnitude must be susceptible of an infinity of values, that is, it contains arbitrary constants which enter as multipliers, for if a be such a constant, we have in general $[X]\,a\,\theta = [X]\,\theta\,a$; and supposing $X = 0$, we have $[0\,.\,a]\,\theta = [0]\,\theta\,a$: therefore whatever particular value may be assigned to $[0]\,\theta$, a more comprehensive value is attained by its arbitrary multiplication by a. A multiplier is the most general form in which the operation represented by a can enter when X is a function of but one variable; but it admits of other forms more extended in cases of several variables, as may easily be perceived: thus $[fy]$ representing a function independent of x, then $[fy]\,\psi_x = f(y)$, ψ_x denoting the operation of changing x into $x + h$: hence $[f(y)]\,\Delta_x = 0$. Then if X be any function of x, and ξ be any particular value of $[X]\,\Delta_x^{-1}$, we shall have more generally $[X]\,\Delta_x^{-1} = \xi + f(y)$, which includes the former; since, the form of $f(y)$ being arbitrary, we have ξ alone, amongst the infinite number of values of $\xi + f(y)$.

In compound operations, the appendage obtained by the first simple operation becomes a new subject for the succeeding operations, each of which may in like manner introduce a new appendage.

14. The operation ψ_x, taken directly or inversely, is incapable of introducing any appendage: for suppose $[0]\,\psi_x^{-1} = \varphi(x)$, then $[\varphi(x)]\,\psi_x = 0$, or $\varphi(x+h) = 0$, which identity being general, we may put x instead of $x+h$, which gives $\varphi(x) = 0$; from which it also follows that $[0]\,\psi_x^{-n} = 0$.

To find the appendage introduced by d_x^{-1}, suppose in the same way $[0]\,d_x^{-1} = \varphi(x)$, hence $[\varphi(x)]\,d_x = 0$, therefore $[\varphi(x)]\,d_x^2 = 0$ $[\varphi(x)]\,d_x^3 = 0$, &c.; and since

$$\varphi(x+h) = \varphi(x) + h\,\frac{d\,\varphi(x)}{d\,x} + \frac{h^2}{1\,.\,2}\cdot\frac{d^2\,\varphi(x)}{d\,x^2} + \&c.$$

hence we have $\varphi(x+h) = \varphi(x)$, and h being arbitrary, we may put it $= -x$, therefore $\varphi(x) = \varphi(0)$, that is, $\varphi(x)$ is constant relative to x; if therefore C be any arbitrary quantity independent of x, we have $[0]\,d_x^{-1} = \mathrm{C}$.

Again,

$$[0]\,d_x^{-2} = [0]\,d_x^{-1}\,d_x^{-1} = [\mathrm{C}]\,d_x^{-1} + [0]\,d_x^{-1},$$

as above stated; but since $[\mathrm{C}_x]\,d_x = \mathrm{C}$, and $[0]\,d_x^{-1} = \mathrm{C}'$, any constant therefore $[0]\,d_x^{-2} = \mathrm{C}\,x + \mathrm{C}'$, and in general

$$[0]\,d_x^{-n} = \mathrm{A}_1\,x^{n-1} + \mathrm{A}_2\,x^{n-2} + \mathrm{A}_3\,x^{n-3} + \ldots . + \mathrm{A}_n,$$

where $\mathrm{A}_1\ \mathrm{A}_2 \ldots . \mathrm{A}_n$ denote constant multipliers.

Lastly, let $[0]\,\Delta_x^{-1} = \varphi(x)$, or $[\varphi(x)]\,\Delta_x = 0$, therefore $\varphi(x+h) = \varphi(x)$, and by Taylor's theorem (dividing by $\varphi(x)$),

$$0 = \frac{\varphi'(x)}{\varphi(x)}\,h + \frac{\varphi''(x)}{\varphi(x)}\cdot\frac{h^2}{1\,.\,2} + \&c.$$

where $\varphi'(x)$ $\varphi''(x)$ &c. are the differential coefficients of $\varphi(x)$; this identity being independent of x, the latter quantity must disappear from the series: put therefore

$$\frac{\varphi'(x)}{\varphi(x)} = \frac{n\sqrt{-1}}{h}, \; n \text{ being independent of } x; \text{ hence}$$

$$\varphi''(x) = \frac{n\sqrt{-1}}{h}\,\varphi'(x) = -\frac{n^2}{h^2}\cdot\varphi(x) \quad \varphi'''(x) = -\frac{n^3\sqrt{-1}}{h^3}\,\varphi(x), \;\&c.$$

therefore

$$1 = \sqrt{-1}\left\{n - \frac{n^3}{1\,.\,2\,.\,3} + \frac{n^5}{1\,.\,2\,.\,3\,.\,4\,.\,5} - \&c.\right\} + \left\{1 - \frac{n^2}{1\,.\,2} + \frac{n^4}{1\,.\,2\,.\,3\,.\,4} - \&c.\right\} \quad (a.)$$

At present suppose n the least real value which satisfies this equation, then

$$\varphi(x) = \mathrm{C}\,\varepsilon^{\frac{n\,x\sqrt{-1}}{h}},$$

also since $\varphi(x) = \varphi(x + h)$ change x into $x + h$ $\therefore$ $\varphi(x + h) = \varphi(x + 2h)$ and generally $\varphi(x) = \varphi(x \pm m h)$, m being an integer; and since $\varphi(x) = \mathrm{C}\,\varepsilon^{\frac{mnn\sqrt{-1}}{mh}}$ satisfies this equation, it follows that $\pm n \pm 2n \pm 3n$, &c. satisfy the equation (*a.*); the complete appendage will therefore be

$$\mathrm{A}_1\,\varepsilon^{\frac{nx\sqrt{-1}}{h}} + \mathrm{A}_2\,\varepsilon^{\frac{2nx\sqrt{-1}}{h}} + \ldots\ldots + \mathrm{B}_1\,\varepsilon^{\frac{-nx\sqrt{-1}}{h}} + \mathrm{B}_2\,\varepsilon^{\frac{-2nx\sqrt{-1}}{h}} + \ldots\ldots$$

the number of constants being infinite.

§ 5.

15. When the simple operations which compose a compound one are relatively free their places are transmutable, but when fixed a mutation of places will require an alteration in the operations themselves.

Let ψ_x denote the changing of x into $x + h$ as before, and let θ_x be any operation affecting x, or, which is the same, fixed relative to ψ_x, then considering θ_x as a subject, put $[\theta_x]\,\psi_x = \theta'_x$, and if the compound operation $[u]\,\theta_x\,\psi_x$ be proposed, its value by transmutation is $[u]\,\psi_x\,\theta'_x$, for in the first compound operation ψ_x affects all the preceding symbols as forming its subject.

Again, let $[u]\,\theta_x\,\Delta_x = y$ be proposed for transmutation, we have

$$\begin{aligned} y &= [u]\,\theta_x\,(\psi_x - 1) \\ &= [u]\,(\psi_x\,\theta'_x - \theta_x), \end{aligned}$$

and putting $\Delta_x + 1$ for ψ_x, and $\overline{\theta_x\,\Delta_x}$ for the finite difference of θ_x considered as *one* operation, we have

$$y = [u]\,(\Delta_x\,\theta'_x + \overline{\theta_x\,\Delta_x}) = [u]\,\theta_x\,\Delta_x.$$

Lastly, divide this identity by h, and then put $h = 0$. When $\frac{\Delta_x}{h}$ becomes d_x, and θ'_x becomes θ_x, we get for the transmutation of $\theta_x\,d_x$,

$$[u]\,\theta_x\,d_x = [u]\,(d_x\,\theta_x + \overline{\theta_x\,d_x}).$$

These formulæ of transmutation separated from the subject are respectively

$$\begin{gathered} \theta_x\,\psi_x = \psi_x\,\overline{\theta_x\,\psi_x} \\ \theta_x\,\Delta_x = \Delta_x\,\overline{\theta_x\,\psi_x} + \overline{\theta_x\,\Delta_x} \\ \theta_x\,d_x = d_x\,\theta_x + \overline{\theta_x\,d_x}. \end{gathered}$$

When θ_x is constant or not dependent on x, then

$$\overline{\theta_x\,\psi_x} = \theta_x \quad \overline{\theta_x\,\Delta_x} = 0 \quad \overline{\theta_x\,d_x} = 0,$$

and these formulæ will then express merely that θ_x, ψ_x, &c. are relatively free.

For example, let θ_x simply denote a multiplier, then $\overline{\theta_x\,\psi}$ will be θ_{x+h}, also a multiplier, $\overline{\theta_x\,\Delta_x} = \theta_{x+h} - \theta_x$, which may be represented by ${}^{\backprime}\theta_x$ and $\overline{\theta_x\,d_x}$ = the limit of $\frac{\theta_{x+h} - \theta_x}{h}$, when h vanishes, or the differential coefficient of θ_x, which may be denoted by θ'_x, then the general formula becomes

$$[u]\,\theta_x\,\psi_x = [u]\,\psi_x\,\theta_{x+h}$$

$$[u]\,\theta_x\,\Delta_x = [u]\,(\Delta_x\,\theta_{x+h} + {}^{\backprime}\theta_x)$$

$$[u]\,\theta_x\,d_x = [u]\,(d_x\,\theta_x + \theta'_x).$$

Again, let the subject be $f(x, y)$, and suppose now θ_x to be the operation of changing y into $y + \varphi(x)$, then $\overline{\theta_x\,\psi}$ or θ_{x+h} is the operation of changing y into $y + \varphi(x + h)$, $\overline{\theta_x\,\Delta_x}$ or ${}^{\backprime}\theta_x$ is the operation which changes $f(x, y)$ into

$$f\{x, y + \varphi(x + h)\} - f\{x, y + \varphi(x)\} = f\{x, y + \varphi(x)\}$$
$$+ \frac{d f\{x, y + \varphi(x)\}}{d y}\,\varphi'(x)\,.\,h + \&c. - f\{x, y + \varphi(x)\},$$

and therefore $\overline{\theta_x\,d_x}$ will give $\dfrac{d f\{x, y + \varphi x\}}{d y}$ and is equivalent to $\theta_x\,d_y\,\varphi'(x)$; in this case we should have

$$[f(x, y)]\,\theta_x\,d_x = [f(x, y)]\,(d_x\,\theta_x + \theta_x\,d_y\,\varphi'(x)),$$

which result may be also deduced by putting for θ_x its equivalent symbol $\varepsilon^{d_y\,\varphi(x)}$, which gives

$$\overline{\theta_x\,d_x} = \varepsilon^{d_y\,\varphi(x)}\,.\,d_y\,\varphi'(x) = \theta_x\,d_y\,\varphi'(x),$$

where $\varphi'(x)$ is written for $\dfrac{d\,\varphi(x)}{d x}$.

This example shows how operations may themselves be the subjects of other operations.

16. We now proceed to consider the transformed values of $\theta_x\,\psi^n_x$, $\theta_x\,\Delta^n_x$, $\theta_x\,d^n_x$, when n is any positive integer. First,

$$\theta_x\,\psi_x = \psi_x\,\overline{\theta_x\,\psi_x} = \varphi_x;$$

suppose therefore

$$\theta_x\,\psi_x^2 = \varphi_x\,\psi_x = \psi_x\,\overline{\varphi_x\,\psi_x}.$$

Now $\overline{\varphi_x\,\psi_x}$ regards φ_x solely as the subject of the operation ψ_x, and

$$\overline{\theta_x\,\psi_x}\,\psi_x = \psi_x\,\overline{\theta_x\,\psi_x\,\psi_x}$$

by the first formula; therefore

$$\psi_x\,\overline{\varphi_x\,\psi_x} = \psi^2_x\,\overline{\theta_x\,\psi^2_x}.$$

and in general if we suppose

$$\theta_x\,\psi_x^{n-1} = \psi_x^{n-1}\,\overline{\theta_x\,\psi_x^{n-1}} = \varphi_x,$$

then

$$\theta_x\,\psi_x^n = \varphi_x\,\psi_x = \psi_x\,\overline{\varphi_x\,\psi_x};$$

but

$$\overline{\theta_x\,\psi_x^{n-1}}\,\psi_x = \psi_x\,\overline{\theta_x\,\psi_x^n}$$

by the first formula; therefore

$$\theta_x\,\psi_x^n = \psi_x^{n-1}\,\psi_x\,\overline{\theta_x\,\psi_x^n} = \psi_x^n\,\overline{\theta_x\,\psi_x^n}.$$

This general formula may be more readily deduced by considering that $\psi_x{}^n$ is the operation of changing x into $x + n\,h$, and consequently $\psi_x{}^n$ may at once be substituted in the first formula for ψ_x.

Secondly,

$$\theta_x\,\Delta_x = \Delta_x\,\overline{\theta_x\,\psi_x} + \overline{\theta_x\,\Delta_x},$$

therefore

$$\theta_x\,\Delta_x{}^2 = \Delta_x\,\overline{\theta_x\,\psi_x}\,\Delta_x + \overline{\theta_x\,\Delta_x}\,\Delta_x,$$

but

$$\overline{\theta_x\,\psi_x}\,\Delta_x = \Delta_x\,.\,\overline{\theta_x\,\psi_x{}^2} + \overline{\theta_x\,\psi_x\,\Delta_x},$$

writing $\overline{\theta_x\,\psi_x}$ for θ_x; and similarly

$$\overline{\theta_x\,\Delta_x}\,\Delta_x = \Delta_x\,.\,\overline{\theta_x\,\psi_x\,\Delta_x} + \overline{\theta_x\,\Delta_x{}^2},$$

whence

$$\theta_x\,\Delta_x{}^2 = \Delta_x{}^2\,.\,\overline{\theta_x\,\psi_x{}^2} + 2\,\Delta_x\,.\,\overline{\theta_x\,\psi_x\,\Delta_x} + \overline{\theta_x\,\Delta_x{}^2}.$$

generally suppose

$$\theta_x\,\Delta_x{}^{n-1} = \Delta_x{}^{n-1}\,.\,\overline{\theta_x\,\psi_x{}^{n-1}} + (n-1)\,\Delta_x{}^{n-2}\,.\,\overline{\theta_x\,\psi_x{}^{n-2}\,\Delta_x}$$
$$+ \frac{(n-1)\,(n-2)}{1\,.\,2}\,.\,\Delta_x{}^{n-3}\,.\,\overline{\theta_x\,\psi_x{}^{n-3}\,\Delta_x{}^2}.$$

Now if we write $\overline{\theta_x\,\psi_x{}^m}$ for θ_x in the fundamental formula, we have

$$\overline{\theta_x\,\psi_x{}^m}\,\Delta_x = \Delta_x\,.\,\overline{\theta_x\,\psi_x{}^m} + \overline{\theta_x\,\psi_x{}^{m-1}\,\Delta_x}$$

each term when we put for m, $n-1$, $n-2$, &c. successively, will thus be divided into two, which being placed in two distinct lines will give

$$\theta_x\Delta_x{}^n = \Delta_x{}^n.\overline{\theta_x\psi_x{}^n} + (n-1)\,\Delta_x{}^{n-1}\,.\,\overline{\theta_x\psi_x{}^{n-1}\,\Delta_x} + \frac{(n-1)\,(n-2)}{1.\,2}\,.\,\Delta_x{}^{n-2}.\overline{\theta_x\psi_x{}^{n-2}\,\Delta_x{}^2} + \&c.$$

$$+ \Delta_x{}^{n-1}\,.\,\overline{\theta_x\psi_x{}^{n-1}\,\Delta_x} + \frac{(n-1).2}{1\,.\,2}\,.\,\Delta_x{}^{n-2}\,.\,\overline{\theta_x\,\psi_x{}^{n-2}\Delta_x{}^2} + \&c.$$

$$= \Delta_x{}^n\overline{\theta_x\psi_x{}^n} + n\Delta_x{}^{n-1}\,.\,\overline{\theta_x\psi_x{}^{n-1}\Delta_x} + \frac{n\,(n-1)}{1\,.\,2}\,.\,\Delta_x{}^{n-2}\,.\,\overline{\theta_x\,\psi_x{}^{n-2}\,\Delta_x{}^2} + \&c.$$

which is the general formula sought for.

Divide now by h^n and observe that $\frac{\Delta_x}{h}$ becomes d_x and ψ_x becomes unity as a multiplier when $h = 0$, hence the third general formula

$$\theta_x\,d_x{}^n = d_x{}^n\,\theta_x + n\,d_x{}^{n-1}\,.\,\overline{\theta_x\,d_x} + \frac{n\,(n-1)}{1\,.\,2}\,.\,d_x{}^{n-2}\,.\,\overline{\theta_x\,d_x{}^2} + \&c.$$

which when θ_x represents quantity is the theorem commonly called LEIBNITZ's.

17. We next proceed to investigate the formulæ for negative indices. First since $\psi_x{}^{-1}$ denotes simply the changing x into $x - h$, we may write $\psi_x{}^{-1}$ for ψ_x in the first formula.

Therefore

$$\theta_x\,\psi_x{}^{-1} = \psi_x{}^{-1}\,.\,\overline{\theta_x\,\psi_x{}^{-1}}$$

more generally

$$\theta_x\,\psi_x^{-n} = \psi_x^{-n} \,.\, \overline{\theta_x\,\psi_x^{-n}}.$$

Secondly, since

$$\theta_x\,\Delta_x = \Delta_x\,\overline{\theta_x\,\psi_x} + \overline{\theta_x\,\Delta_x}$$

therefore

$$\Delta_x^{-1}\,\theta_x\,\Delta_x = \overline{\theta_x\,\psi_x} + \Delta_x^{-1}\,\overline{\theta_x\,\Delta_x}.$$

Put now $\overline{\theta_x\,\psi_x^{-1}}$ for θ_x, and therefore θ_x for $\overline{\theta_x\,\psi_x}$; hence

$$\Delta_x^{-1}\,\overline{\theta_x\,\psi_x^{-1}}\,\Delta_x = \theta_x + \Delta_x^{-1} \,.\, \overline{\theta_x\,\psi_x^{-1}}\,\Delta_x$$

therefore

$$\theta_x\,\Delta_x^{-1} = \Delta_x^{-1} \,.\, \overline{\theta_x\,\psi_x^{-1}} - \Delta_x^{-1} \,.\, \overline{\theta_x\,\psi_x^{-1} \,.\, \Delta_x} \,.\, \Delta_x^{-1}.$$

Put $\overline{\theta_x\,\psi_x^{-1}}$ for θ_x, thence we have

$$\theta_x\,\Delta_x^{-1} = \Delta_x^{-1} \,.\, \overline{\theta_x\,\psi_x^{-1}} - \Delta_x^{-2} \,.\, \overline{\theta_x\,\psi_x^{-2}\,\Delta_x} \,. + \Delta_x^{-2} \,.\, \overline{\theta_x\,\psi_x^{-2}\,\Delta_x^2} \,.\, \Delta_x^{-1},$$

or we continue this process indefinitely

$$\theta_x\,\Delta_x^{-1} = \Delta_x^{-1} \,.\, \overline{\theta_x\,\psi_x^{-1}} - \Delta_x^{-2} \,.\, \overline{\theta_x\,\psi_x^{-2}\,\Delta_x} + \Delta_x^{-3} \,.\, \overline{\theta_x\,\psi_x^{-3}\,\Delta_x^2} - \&c.$$

which is the same as the general formula for $\theta_x\,\Delta_x^n$ when $n = -1$.

Again

$$\theta_x\,\Delta_x^{-2} = \Delta_x^{-1} \,.\, \overline{\theta_x\,\psi_x^{-1}} \,.\, \Delta_x^{-1} - \Delta_x^{-1} \,.\, \overline{\theta_x\,\psi_x^{-1}\,\Delta_x} \,.\, \Delta_x^{-2},$$

but

$$\Delta_x^{-1}\,\overline{\theta_x\,\psi_x^{-1}} \,.\, \Delta_x^{-1} = \Delta_x^{-2} \,.\, \overline{\theta_x\,\psi_x^{-2}} - \Delta_x^{-2} \,.\, \overline{\theta_x\,\psi_x^{-2}\,\Delta_x} \,.\, \Delta_x^{-1},$$

and

$$\Delta_x^{-1} \,.\, \overline{\theta_x\,\psi_x^{-1}\,\Delta_x} \,.\, \Delta_x^{-1} = \Delta_x^{-2} \,.\, \overline{\theta_x\,\psi_x^{-2}\,\Delta_x} - \Delta_x^{-2} \,.\, \overline{\theta_x\,\psi_x^{-2}\,\Delta_x^2} \,.\, \Delta_x^{-1}.$$

Hence

$$\theta_x\,\Delta_x^{-2} = \Delta_x^{-2} \,.\, \overline{\theta_x\,\psi_x^{-2}} - 2\,\Delta_x^{-2} \,.\, \overline{\theta_x\,\psi_x^{-2}\,\Delta_x} \,.\, \Delta_x^{-1} + \Delta_x^2 \,.\, \overline{\theta_x\,\psi_x^{-2}\,\Delta_x^2} \,.\, \Delta_x^{-2},$$

and in a similar manner it is easy to prove generally in a terminating series

$$\theta_x\,\Delta_x^{-n} = \Delta_x^{-n} \,.\, \overline{\theta_x\,\psi_x^{-n}} - n\,\Delta_x^{-n} \,.\, \overline{\theta_x\,\psi_x^{-n}\,\Delta_x} \,.\, \Delta_x^{-1} + \frac{n\,(n-1)}{1\,.\,2} \,.\, \overline{\theta_x\,\psi_x^{-n}\,\Delta_x^2} \,.\, \Delta_x^{-2} - \&c.$$

or in an infinite series,

$$\theta_x\Delta_x^{-n} = \Delta_x^{-n} \,.\, \overline{\theta_x\psi_x^{-n}} - n\,\Delta_x^{-(n+1)} \,.\, \overline{\theta_x\psi_x^{-(n+1)}} \,.\, \Delta_x + \frac{n\,(n+1)}{1\,.\,2}\Delta_x^{-(n+2)}\overline{\theta_x\psi_x^{-(n+2)}}\Delta_x^2 - \&c.$$

Thirdly, divide Δ_x by h, and then put $h = 0$, whence

$$\begin{aligned}\theta_x\,d_x^{-1} &= d_x^{-1} \,.\, \theta_x - d_x^{-1} \,.\, \overline{\theta_x\,d_x}\,d_x^{-1} \\ &= d_x^{-1} \,.\, \theta_x - d_x^{-2} \,.\, \overline{\theta_x\,d_x} + d_x^{-3} \,.\, \overline{\theta_x\,d_x^2} - \&c.\end{aligned}$$

$$\begin{aligned}\theta_x\,d_x^{-n} &= d_x^{-n} \,.\, \theta_x - n\,d_x^{-n} \,.\, \overline{\theta_x\,d_x} \,.\, d_x^{-1} + \frac{n\,(n-1)}{1\,.\,2} \,.\, d_x^{-n} \,.\, \overline{\theta_x\,d_x^2} \,.\, d_x^{-2} - \&c. \\ &= d_x^{-n} \,.\, \theta_x - n\,d_x^{-(n+1)}\,\overline{\theta_x\,d_x} + \frac{n\,(n+1)}{1\,.\,2} \,.\, \theta_x^{-(n+2)} \,.\, \overline{\theta_x\,d_x^2} - \&c.\end{aligned}$$

which formulæ admit of most extensive applications, whether θ_x be regarded as a quantity or a fixed operation.

§ 6.

18. Before proceeding further in the search of the fundamental formulæ for the transformation of operations, we shall exemplify the theory which precedes by inverting binomial operations and applying the results to some simple cases.

Let θ, θ' denote two linear operations relatively fixed or free, and let us seek the value of $(\theta - \theta')^{-1}$.

Put $(\theta - \theta')^{-1} = \theta^{-1} + \eta_1$; the latter being the difference of two linear operations must itself be linear.

Hence

$$1 = (\theta^{-1} + \eta_1)(\theta - \theta') = 1 - \theta^{-1}\theta' + \eta_1(\theta - \theta')$$

therefore

$$\eta_1(\theta - \theta') = \theta^{-1}\theta'.$$

Similarly put

$$\eta_1 = \theta^{-1}\theta'\theta^{-1} + \eta_2$$

which gives

$$\eta_1(\theta - \theta') = \theta^{-1}\theta' - (\theta^{-1}\theta')^2 + \eta_2(\theta - \theta')$$

whence

$$\eta_2(\theta - \theta') = (\theta^{-1}\theta')^2$$

so again put

$$\eta_2 = (\theta^{-1}\theta')^2\theta^{-1} + \eta_3$$

$$\eta_3 = (\theta^{-1}\theta')^3\theta^{-1} + \eta_4$$

$$\&c. = \&c.$$

We thus obtain

$$(\theta - \theta')^{-1} = \theta^{-1} + (\theta^{-1}\theta')\theta^{-1} + (\theta^{-1}\theta')^2\theta^{-1} + \ldots\ldots + (\theta^{-1}\theta')^{n-1}\theta^{-1} + \eta_n$$

where η_n represents the compound operation $(\theta^{-1}\theta')^n(\theta - \theta')^{-1}$.

The same formula continued to infinity would be obtained by first putting $\theta^{-1}(1 - \theta^{-1}\theta')^{-1}$ for $(\theta - \theta')^{-1}$; and since the operations represented respectively by 1 and $\theta^{-1}\theta'$ are relatively free, we should have by art. 11.

$$(1 - \theta^{-1}\theta')^{-1} = 1 + \theta^{-1}\theta' + (\theta^{-1}\theta')^2 + \&c. \text{ ad infin.}$$

When θ, θ' are relatively free the theorem becomes

$$(\theta - \theta')^{-1} = \theta^{-1} + \theta^{-2}\theta' + \theta^{-3}\theta'^2 + \theta^{-4}\theta'^3 + \ldots.\theta^{-n}\theta'^{n-1} + \theta^{-n}\theta'^n(\theta - \theta')^{-1}.$$

19. For a first example suppose Δ_x to denote the finite difference, on the supposition that by the operation ψ_x the quantity x is changed into $x + h$.

Then Δ_x^{-1} is the finite integral, and in the usual notation of analysts is denoted by Σ_x, we have therefore

$$[f(x)]\Sigma_x = [f(x)](\psi_x - 1)^{-1} = [f(x)]\{\psi_x^{-1} + \psi_x^{-2} + \psi_x^{-3} + \ldots\psi_x^{-(n-1)} + \psi_x^{-n}\Sigma_x\}$$
$$= f(x - h) + f(x - 2h) + f(x - 3h) + \ldots$$
$$+ f\{x(n - 1)h\} + \Sigma_x f(x - nh);$$

where it may be remarked that if $\frac{x}{h}$ be an integer, the final terms of the series would be $\ldots f(2h) + f(h) + f(0)$, at any of which, if we suppose the series to stop, its finite difference would be obviously $f(x)$.

For the next example suppose the subject to be $f(x+y)$, and that by the operation ψ_x, x receives an increment h, and y the *same* increment by the operation ψ_y, then it is obvious that $[f(x+y)]\ (\psi_x - \psi_y) = 0$, therefore $[f(x+y)]\ (\Delta_y - \Delta_x) = 0$; hence $f(x+y)$ must be included in the general value of $[0]\ (\Delta_y - \Delta_x)^{-1}$.

Now

$$[0]\ (\Delta_y - \Delta_x)^{-1} = [0]\ \{\Delta_y^{-1} + \Delta_y^{-2} \,.\, \Delta_x + \Delta_y^{-3}\, \Delta_x^{2} + \Delta_y^{-4}\, \Delta_x^{3} + \&\text{c.}\}$$

and also $[0]\ \Delta_y^{-1} = \varphi(x)$ an arbitrary function of x.

$[0]\ \Delta_y^{-2} = \varphi(x) \,.\, \frac{y}{h}$, omitting the appendage, which being a function of x would not vanish with y, and which in the succeeding terms, if included, would only generate another series similar to that now formed from $\varphi(x)$, and consequently in the present case not add to its generality.

Again,

$$[0]\ \Delta_y^{-3} = \varphi(x) \,.\, \frac{y(y-h)}{1\,.\,2\,.\,h^2} \text{ for } [y\,.\,(y-h)]\ \Delta_y = (y+h)\,y - y\,(y-h) = 2\,h\,y.$$

Similarly

$$[0]\ \Delta_y^{-4} = \varphi(x) \,.\, \frac{y(y-h)(y-2h)}{1\,.\,2\,.\,3\,.\,h^3},\ \&\text{c.}$$

Hence

$$[0]\ (\Delta_y - \Delta_x)^{-1} = \varphi(x) + \frac{y}{h} \,.\, \Delta\,\varphi(x) + \frac{y\,.\,(y-h)}{1\,.\,2\,.\,h^2} \,.\, \Delta^2\,\varphi(x)$$
$$+ \frac{y\,(y-h)\,(y-2h)}{1\,.\,2\,.\,3\,.\,h^3} \,.\, \Delta^3\,\varphi(x) + \&\text{c.};$$

and since $f(x+y)$ is included in this general expression, the particular form to be assigned to the arbitrary function $\varphi(x)$ is known by making $y = 0$, which gives $\varphi(x) = f(x)$; in this formula the ordinary notation has been employed.

Suppose, for instance, $f(x) = a^x$ and $h = 1$, then $\Delta^n f(x) = a^x\,(a-1)^n$; therefore

$$a^{x+y} = a^x + y\,.\,a^x\,.\,(a-1) + \frac{y\,(y-1)}{1\,.\,2}\, a^x\,(a-1)^2 + \frac{y\,(y-1)\,(y-2)}{1\,.\,2\,.\,3} \,.\, a^x\,(a-1)^3,\ \&\text{c.}$$

or putting $a = 1 + b$, and expunging a^x from both sides,

$$(1+b)^y = 1 + y\,.\,b + \frac{y(y-1)}{1\,.\,2} \,.\, b^2 + \frac{y\,(y-1)\,(y-2)}{1\,.\,2\,.\,3} \,.\, b^3,\ \&\text{c.}$$

which is the binomial theorem, whatever may be the value of y.

For the next example let us take the same subject, $f(x+y)$, and let d_x, d_y denote the differential coefficients relative to x and y; then since

$$[f(x+y)]_{\frac{\Delta_y - \Delta_x}{h}} = 0$$

put $h = 0$ and $\frac{\Delta_x}{h}$ becomes d_x, $\frac{\Delta_y}{h}$ being similarly d_y; therefore

$$[f(x+y)]\,(d_y - d_x) = 0;$$

hence $f(x+y)$ must be included in the general value of $[0]\,(d_y - d_x)^{-1}$.

But

$$[0]\,(d_y - d_x)^{-1} = [0]\,(d_y^{-1} + d_y^{-2}\,d_x + d_y^{-3}\,d_x^{\,2} + d_y^{-4}\,d_x^{\,3} +, \&c.)$$

Now

$$[0]\,d_y^{-1} = \varphi(x)$$

an arbitrary function of x; therefore

$$[0]\,d_y^{-2} = \varphi(x)\,.\,y$$

omitting the appendage for the reason abovementioned; also

$$[0]\,d_y^{-3} = \varphi(x)\,.\,\frac{y^2}{1\,.\,2}$$

since $[y^2]\;d_y = 2\,y$; and

$$[0]\,d_y^{-4} = \varphi(x)\,.\,\frac{y^3}{1\,.\,2\,.\,3}$$

substituting we have

$$[0]\,(d_y - d_x)^{-1} = \varphi(x) + y\,\frac{d\,\phi\,x}{d\,x} + \frac{y^2}{1\,.\,2}\,.\,\frac{d^2\,\varphi(x)}{d\,x^2} + \frac{y^3}{1\,.\,2\,.\,3}\,.\,\frac{d^3\,\varphi(x)}{d\,x^3} +, \&c.$$

employing the common notation; the form of $\varphi(x)$ is determined by making $y = 0$, which gives $\varphi(x) = f(x)$; therefore

$$f(x+y) = f(x) + y f'(x) + \frac{y^2}{1\,.\,2}\,.\,f''(x) +, \&c.$$

where $f'(x)$ $f''(x)$, &c. are the successive differential coefficients of $f(x)$; this is TAYLOR's expansion.

If we put $f(x) = a^x$ and for the limit of $\frac{a^h - 1}{h}$ write log (a), we get from this

$$a^y = 1 + y\,\mathrm{l.}\,.(a) + \frac{y^2}{1\,.\,2}\,.\,(\mathrm{l.}\,.\,a)^2 +, \&c.$$

These examples suffice to show the mode and use of the inversion of binomial operations.

§ 7.

20. To return to the general theory, suppose θ, ι, $\varkappa$ to represent three operations connected by the equation $\theta\,\iota = \iota\,\varkappa$, where the subject is omitted, the identity being supposed general; the symbol ι represents an operation which may be said to be *intermediate* to those designed by θ, $\varkappa$.

If either of the extreme operations θ, $\varkappa$ be given, and the intermediate ι be also given, the other extreme may be readily found for

$$\theta = \iota\,\varkappa\,\iota^{-1} \text{ and } \varkappa = \iota^{-1}\,\theta\,\iota.$$

A remarkable property of intermediate operations is that they are also intermediate between any operations which are the same functions of the extremes.

For let

$$\theta\,\iota = \iota\,\varkappa$$

then performing the operation $\varkappa$

$$\theta\,\iota\,\varkappa = \iota\,\varkappa^2$$

put now for $\iota\,\varkappa$ its equivalent operation $\theta\,\iota$, and we have

$$\theta^2\,\iota = \iota\,\varkappa^2.$$

Similarly if we suppose

$$\theta^{n-1}\,\iota = \iota\,\varkappa^{n-1}$$

then

$$\theta^{n-1}\,\iota\,\varkappa = \iota\,\varkappa^{n}$$

but

$$\iota\,\varkappa = \theta\,\iota;$$

therefore

$$\theta^{n}\,\iota = \iota\,\varkappa^{n}.$$

Again, suppose the subject in the last equation to be one on which the operation θ^{-n} has been performed, then that equation becomes

$$\iota = \theta^{-n}\,\iota\,\varkappa^{n},$$

or

$$\theta^{-n}\,\iota = \iota\,\varkappa^{-n}.$$

Again, suppose K an operation satisfying the equation

$$\theta^{\frac{n}{m}}\,\iota = \iota\,\mathrm{K}.$$

We have by the parts of the proof already given in this article,

$$\theta^{n}\,\iota = \iota\,\mathrm{K}^{m} = \iota\,\varkappa^{n},$$

or

$$\mathrm{K}^{m} = \varkappa^{n},\ \mathrm{K} = \varkappa^{\frac{n}{m}};$$

hence

$$\theta^{\frac{n}{m}}\,\iota = \iota\,\varkappa^{\frac{n}{m}}.$$

From these premised equations it follows, that if $f(\theta)\,f(\varkappa)$ represent the aggregates of any similar *powers* of the operations θ, $\varkappa$, with the same coefficients, we must have generally

$$f(\theta)\,.\,\iota = \iota f(\varkappa).$$

By this theorem, if $f(\theta)$ be known, $f(\varkappa)$ can be found, supposing that we know ι the operation intermediate to θ, $\varkappa$.

21. We shall now apply this theorem to cases where θ, ι are given, and therefore $\varkappa$ known, as above shown.

Let one extreme θ represent the operation of differentiating relatively to x, and the intermediate ι that of multiplying by ε^{ax}, then we have

$$d_x\,\varepsilon^{ax} = \varepsilon^{ax}\,\varkappa$$

therefore

$$\varepsilon^{-ax}\, d_x\, \varepsilon^{ax} = \varkappa\,;$$

but by § 5.

$$\theta_x\, d_x = d_x\, \theta_x + \overline{\theta_x\, d_x},$$

therefore

$$\varepsilon^{-ax}\, d_x = d_x\, \varepsilon^{-ax} - a\, \varepsilon^{-ax} = (d_x - a)\, \varepsilon^{-ax}.$$

Hence

$$d_x - a = \varkappa \text{ the extreme required.}$$

Now if $\theta\,\iota = \iota\,\varkappa$, it has been shown that $f(\theta)\,.\,\iota = \iota f(\varkappa)$.

In this case therefore

$$f(d_x)\,.\,\varepsilon^{ax} = \varepsilon^{ax} f(d_x - a)$$

To find the intermediate operation between $d_x + b$ and $d_x + c$; b and c not containing x.

Put $f(d_x) = d_x + b$ in the last identity, and $b - a = c$ we then have

$$(d_x + b)\,.\,\varepsilon^{(b-c)x} = \varepsilon^{(b-c).x}\,(d_x + c)$$
$$f(d_x + b)\,.\,\varepsilon^{(b-c).x} = \varepsilon^{(b-c)x} f(d_x + c).$$

22. Suppose the intermediate operation ι to denote $\varepsilon^{{}^{\backprime}\mathrm{P}}$ considered as a multiplier, ${}^{\backprime}\mathrm{P}$ being a function of x, of which the differential coefficient is P, and θ to represent d_x as before, it is required to find $\varkappa$. Since

$$d_x\, \varepsilon^{{}^{\backprime}\mathrm{P}} = \varepsilon^{{}^{\backprime}\mathrm{P}}\,.\,\varkappa\,;$$

therefore

$$\varkappa = \varepsilon^{-{}^{\backprime}\mathrm{P}}\,.\,d_x\,.\,\varepsilon^{{}^{\backprime}\mathrm{P}}.$$

But

$$\varepsilon^{-{}^{\backprime}\mathrm{P}}\, d_x = d_x\, \varepsilon^{-{}^{\backprime}\mathrm{P}} + \overline{\varepsilon^{-{}^{\backprime}\mathrm{P}}}\, d_x = d_x\, \varepsilon^{-{}^{\backprime}\mathrm{P}} - \mathrm{P}\,\varepsilon^{-{}^{\backprime}\mathrm{P}} = (d_x - \mathrm{P})\,\varepsilon^{-{}^{\backprime}\mathrm{P}}.$$

Hence

$$\varkappa = d_x - \mathrm{P}.$$

Corollary;

$$f(d_x)\,.\,\varepsilon^{{}^{\backprime}\mathrm{P}} = \varepsilon^{{}^{\backprime}\mathrm{P}} f(d_x - \mathrm{P}).$$

And if ${}^{\backprime}\mathrm{Q}$ be a function, of which Q is the differential coefficient, we have in like manner

$$f(d_x)\,.\,\varepsilon^{{}^{\backprime}\mathrm{Q}} = \varepsilon^{{}^{\backprime}\mathrm{Q}} f(d_x - \mathrm{Q})\,;$$

hence

$$\varepsilon^{{}^{\backprime}\mathrm{P}} f(d_x - \mathrm{P})\,.\,\varepsilon^{-{}^{\backprime}\mathrm{P}} = \varepsilon^{{}^{\backprime}\mathrm{Q}} f(d_x - \mathrm{Q})\,\varepsilon^{-{}^{\backprime}\mathrm{Q}},$$

or

$$f(d_x - \mathrm{P})\,.\,\varepsilon^{{}^{\backprime}\mathrm{Q} - {}^{\backprime}\mathrm{P}} = \varepsilon^{{}^{\backprime}\mathrm{Q} - {}^{\backprime}\mathrm{P}} f(d_x - \mathrm{Q}),$$

that is, $\varepsilon^{{}^{\backprime}(\mathrm{Q}-\mathrm{P})}$ is the intermediate to the operations $d_x - \mathrm{P}$, $d_x - \mathrm{Q}$.

23. Let ι now signify the operation of changing y into $y + {}^{\backprime}\varphi(x)$, θ being, as before, the operation of differentiating relative to x, and the subject being a function of x and y, and ${}^{\backprime}\varphi(x)$ the quantity, of which $\varphi(x)$ is the differential coefficient.

Here $$\varkappa = \iota^{-1}\, d_x\, \iota.$$

Now ι^{-1} is the operation which changes y into $y - {}^{\backprime}\varphi(x)$, and therefore, as in § 5, $\iota^{-1}\, d_x = d_x\, \iota^{-1} - \varphi(x)\, \iota^{-1}\, d_y$; therefore

$$\varkappa = d_x - \varphi(x)\,.\,d_y.$$

Corollary;

$$f(d_x)\,\iota = \iota f(d_x - \phi(x)\,d_y);$$

and as in the last article, if ι' be the operation of changing y into $y + \int_x \{\phi(x) - \mathrm{F}(x)\}$. Then

$$f(d_x - \mathrm{F}(x)\,d_y)\,\iota' = \iota' f(d_x - \phi(x)\,d_y).$$

The example last given is capable of being extended to any number of variables in the subject of the operations; if that subject contain x, y, z, u, &c., and ι denote the operation of changing y into $y + \phi(x)$, z into $z + \varpi(x)$, u into $u + \Omega(x)$, &c., where $\phi(x)\ \varpi(x)\ \Omega(x)$, &c. represent any functions of x, then will ι be the intermediate operation, of which d_x and $d_x - \phi'(x)\,.\,d_y - \varpi'(x)\,d_z - \Omega'(x)\,.\,d_u -$, &c. form the extremes, the accents being used to represent the differential coefficients of the functions over which they are placed.

24. Let one of the extremes, θ, be now supposed to represent the operation Δ_x of finite differences, on the supposition that h is the increment of x.

Let ι represent a multiplier P_x, constant or variable with x. Then

$$\varkappa = \mathrm{P}_x^{-1}\,\Delta_x\,\mathrm{P}_x.$$

But by § 5,

$$\mathrm{P}_x^{-1}\,\Delta_x = \Delta_x\,.\,\mathrm{P}^{-1}_{x+h} + \overline{\mathrm{P}_x^{-1}\,\Delta_x};$$

thence

$$\varkappa = \Delta_x\,.\,\frac{\mathrm{P}_x}{\mathrm{P}_{x+h}} + .\,\overline{\mathrm{P}_x^{-1}\,\Delta_x}\,.\,\mathrm{P}_x;$$

and then

$$f(\Delta_x)\,.\,\mathrm{P}_x = \mathrm{P}_x f\left(\Delta_x\,.\,\frac{\mathrm{P}_x}{\mathrm{P}_{x+h}} + \overline{\mathrm{P}_x^{-1}\,\Delta_x}\,.\,\mathrm{P}_x\right).$$

Thus, if

$$\mathrm{P}_x = a^{-\frac{x}{h}}\ \therefore\ \mathrm{P}_{x+h} = a^{-1}\,.\,a^{-\frac{x}{h}}, \qquad \overline{\mathrm{P}_x^{-1}\Delta_x} = (a-1)\,.\,a^{\frac{x}{h}},$$

we obtain

$$f(\Delta_x)\,.\,a^{-\frac{x}{h}} = a^{-\frac{x}{h}} f(a\,\Delta_x + a - 1).$$

Let ψ_x denote the operation of changing x into $x + h$, then $\psi_x = \Delta_x + 1$, and the general formula of this article becomes

$$f(\Delta_x)\,\mathrm{P}_x = \mathrm{P}_x f(\psi_x\,.\frac{\mathrm{P}_x}{\mathrm{P}_{x+h}} - 1).$$

Again, if the subject be a function of x and y, let ι represent the operation of changing y into $y + \phi(x)$ and ι' that which would change y into $y - \Delta_x\{\phi(x)\}$, where $\Delta_x\,\phi(x)$ according to the common notation stands for $\phi(x+h) - \phi(x)$, then by a similar procedure we shall obtain the identity

$$f(\Delta_x)\,\iota = \iota f(\psi_x\,\iota' - 1).$$

25. In the identity

$$f(\Delta_x)\,.\,a^{-\frac{x}{h}} = a^{-\frac{x}{h}} f a\left(\Delta_x + \frac{a-1}{a}\right) \text{ put } \frac{a-1}{a} = -b\ \therefore\ a = \frac{1}{1+b}$$

hence

$$f(\Delta_x)\,(1+b)^{\frac{x}{h}} = (1+b)^{\frac{x}{h}} f(\Delta_x - b).$$

Now by the nature of an identity both sides in this expression would be alike, if expanded according to the powers of b, the coefficients of like powers must therefore be equal, and in consequence the identity must remain if b instead of being a multiplier represent any operation which is free, relative to Δ_x, thus if $b = \Delta_y$, $1 + b = \psi_y$, and therefore

$$f(\Delta_x)\,.\,\psi_y^{\frac{x}{h}} = \psi_y^{\frac{x}{h}} f(\Delta_x - \Delta_y)$$

As a particular example of this, suppose the subject to be $\varphi(y)$ independent of x, and $f(\Delta_x)$ merely to stand for Δ_x, then

$$[\varphi(y)]\,(\Delta_x) = 0,$$

the general identity therefore becomes

$$[\varphi(y)]\,\psi_y^{\frac{x}{h}}\,(\Delta_x - \Delta_y) = 0,$$

or

$$\left[\varphi\left(y + \frac{k\,x}{h}\right)\right](\Delta_x - \Delta_y) = 0,$$

k being the increment of y; now

$$\left[\varphi\left(y + \frac{k\,x}{h}\right)\right]\psi_x = \varphi\left(y + k\,.\,\frac{x+h}{h}\right) = \varphi\left(y + k + \frac{k\,x}{h}\right) = \left[\varphi\left(y + \frac{k\,x}{h}\right)\right]\psi_y$$

which verifies the result deduced from the general identity.

26. Thus when the intermediate operation and one extreme are given the other extreme may generally be found, but it seems more difficult to discover the intermediate operation when both extremes are known; here follow some examples of the latter process.

Let θ represent any linear operation, d_x that of taking the differential coefficient of the subject relative to x, and ι the required intermediate.

Then

$$\begin{aligned}\theta\,\iota &= \iota\,d_x \\ &= d_x\,\iota + \overline{\iota\,d_x}\end{aligned}$$

perform on both sides the inverse operation ι^{-1}.

Hence

$$\theta = d_x + \overline{\iota\,d_x}\,\iota^{-1}$$

or

$$\overline{\iota\,d_x}\,.\,\iota^{-1} = \theta - d_x.$$

Suppose

$$\iota = \varepsilon^{f(\theta - d_x)} = 1 + f(\theta - d_x) + \left\{\frac{f(\theta - d_x)}{1\,.\,2}\right\}^2 \&c.$$

then by § 3,

$$\iota^{-1} = \varepsilon^{-f(\theta - d_x)} = 1 - f(\theta - d_x) + \frac{f(\theta - d_x)^2}{1.2} - \&c.,$$

and

$$\overline{\iota\, d_x} = \varepsilon^{f(\theta - d_x)} . \overline{f(\theta - d_x)\, d_x}$$

also

$$(\theta - d_x)\, \iota = (\theta - d_x)\, \varepsilon^{f(\theta - d_x)}.$$

Now $\theta - d_x$ is free relative to its own functions, hence

$$\theta - d_x = \overline{f(\theta - d_x)\, d_x}\,;$$

therefore

$$f(\theta - d_x) = \overline{(\theta - d_x) . d_x^{-1}}.$$

from whence ι is known.

For a second example, suppose ψ the operation of changing x into $x + h$, Δ that of taking the finite difference on the same hypothesis, θ any linear operation, then ι is required to be such that

$$(\psi - \theta)\, \iota = \iota\, \Delta\, \theta.$$

By art. 15,

$$\begin{aligned} \iota\, \Delta &= \Delta\, \overline{\iota\, \psi} + \overline{\iota\, \Delta} \\ &= \psi\, \overline{\iota\, \psi} - \overline{\iota\, \psi} + \overline{\iota\, \Delta} \\ &= (\psi - \iota\, .\, \overline{\iota\, \psi}^{-1})\, \overline{\iota\, \psi}. \end{aligned}$$

Substituting we obtain

$$(\psi - \theta)\, \iota = (\psi - \iota\, .\, \overline{\iota\, \psi}^{-1})\, \overline{\iota\, \psi}\, \theta.$$

To satisfy this identity, suppose

$$\iota\, .\, \overline{\iota\, \psi}^{-1} = \theta\,;$$

now $\overline{\iota\, \psi}$ is free relative to ι, hence

$$\overline{\iota\, \psi}^{-1}\, \iota = \theta,$$

prefix to each side the operation $\overline{\iota\, \psi}$, which only alters the subject, which is perfectly general; therefore

$$\iota = \overline{\iota\, \psi}\, .\, \theta\,;$$

hence the preceding supposition fully satisfies; therefore we have this theorem, if ι be determined such that

$$\iota\, .\, \overline{\iota\, \psi}^{-1} = \theta,$$

then shall

$$(\psi - \theta)\, \iota = \iota\, \Delta\, \theta,$$

and

$$f(\psi - \theta)\, \iota = \iota\, f(\Delta\, \theta).$$

As a particular case, suppose θ to be a multiplier P_x, then ι will be another multiplier v_x, such that

$$v_x\, .\, (v_{x+h})^{-1} = P_x,$$

or

$$(\log v_x)\,\Delta_x = -\log \mathrm{P}_x,$$

from whence v_x or ι is determinable.

§ 8.

27. In this section we shall give some examples of the use of the formulæ investigated in the last section. In general

$$d_x\,\varepsilon^{ax} = \varepsilon^{ax}\,(d_x - a),$$

therefore

$$d_x - a = \varepsilon^{-ax}\,d_x\,\varepsilon^{ax}.$$

Put for a in this formula the terms of the series $0, h, 2h, \ldots\ (n-1)\,.\,h$, and compound all the binomials which result from these substitutions, hence

$$d_x(d_x-h)(d_x-2h)\ldots\ldots(d_x-(n-1)h) = d_x.\varepsilon^{-hx}d_x\,\varepsilon^{hx}\,.\,\varepsilon^{-2hx}d_x\,\varepsilon^{2hx}\ldots\ldots\varepsilon^{-(n-1)hx}d_x\,\varepsilon^{(n-1)hx}$$
$$= d_x\,\varepsilon^{-hx}\,d_x\,\varepsilon^{-hx}\,d_x\,\varepsilon^{-hx}\ldots\ldots d_x\,\varepsilon^{-hx}\,d_x\,\varepsilon^{(n-1)hx}.$$

Now

$$d_x\,\varepsilon^{-hx} = h\,d_y, \text{ putting } y = \varepsilon^{-hx},$$

therefore

$$d_x\,(d_x-h)\,(d_x-2h)\ldots(d_x-(n-1)\,h) = d_y^n\,.\,h^n\,\varepsilon^{nhx}.$$

Example. The expansions of $(a+b)^n$, viz.

$$a^n + n\,a^{n-1}\,b + \frac{n\,(n-1)}{1\,.\,2}\,.\,a^{n-2}\,b^2 +, \&c.$$

and

$$1 + n\,\mathrm{l}.\,.\,(a+b) + \left\{\frac{n\,.\,\mathrm{l}.\,(a+b)}{1\,.\,2}\right\}^2 +, \&c.$$

being identical when n is a quantity, ought to remain so when n is a linear operation, to verify which, suppose $n = d_x$, now it has been shown that $[f(x)]\,\varepsilon^{h\,d_x} = f(x+h)$; hence $[f(x)]\,(a+b)^{d_x} = f\{x + \mathrm{l}.\,.\,(a+b)\}$.

But

$$(a+b)^{d_x} = a^{d_x} + d_x\,a^{d_x-1}\,b + \frac{d_x(d_x-1)}{1\,.\,2}\,.\,a^{d_x-2}\,b^2 + \frac{d_x(d_x-1)(d_x-2)}{1\,.\,2\,.\,3}\,.\,a^{d_x-3}\,b^3 +, \&c.$$

$$= a^{d_x}\left\{1 + d_x\,.\,\frac{b}{a} + \frac{d_x(d_x-1)}{1\,.\,2}\,.\left(\frac{b}{a}\right)^2 + \frac{d_x(d_x-1)(d_x-2)}{1\,.\,2\,.\,3}\,.\left(\frac{b}{a}\right)^3 +, \&c.\right\}$$

But by this article

$$d_x\,(d_x-1)\,(d_x-2)\ldots(d_x-n+1) = d_y^n\,.\,y^n, \text{ if } y = \varepsilon^x;$$

therefore

$$(a+b)^{d_x} = a^{d_x}\left\{1 + d_y\,.\,\frac{by}{a} + \frac{d_y^2}{1\,.\,2}\,.\left(\frac{by}{a}\right)^2 + \frac{d_y^3}{1\,.\,2\,.\,3}\,.\left(\frac{by}{a}\right)^3 +, \&c.\right\},$$

and now introducing the subject $f(x)$, we get

$$f\{x + \mathrm{l}.\,.\,(a+b)\} = f(x + \mathrm{l}.\,(a)) + \frac{by}{a}\,.\,\frac{d\,.f(x+\mathrm{l}.\,a)}{dy} + \frac{b^2y^2}{a^2}\,.\,\frac{d^2\,.f(x+\mathrm{l}.\,(a))}{1\,.\,2\,dy^2} +, \&c.$$

which series, if we substitute for x its value $\mathrm{l}.\,.\,y$, and put $f(\mathrm{l}.\,.\,y) = \varphi\,(y)$, becomes

$$\varphi(ay + by) = \varphi(ay) + \frac{by}{a} \cdot \frac{d\varphi(ay)}{dy} + \frac{b^2 y^2}{1.2} \cdot \frac{d^2 . \phi(ay)}{a^2 dy^2} +, \&c.$$

which being also deducible from TAYLOR's theorem gives the required verification.

28. Let $v_1\, v_2\, v_3 \ldots\ldots v_n$ represent functions of x as multipliers, or other operations fixed relative to d_x, it is required to find the polynomial arranged according to the powers (usually called orders) of d_x, which shall be equivalent to the compound operation represented by $v_1\, d_x\, v_2\, d_x\, v_3\, d_x \ldots\ldots v_n\, d_x$.

It is easy to see that this polynomial cannot contain the powers of the differential coefficients of any of the multipliers, that is such as $\left(\frac{d v_1}{d x}\right)^2$, &c.; moreover, if we suppose $v_1 = \varepsilon^{\alpha_1 x}$, then $\frac{d^n v_1}{d x^n} = \alpha_1^{\,n} . \varepsilon^{\alpha_1 x}$; thus the order of the differential coefficients of v_1 in the required result will be the same as the power of the multiplier $\alpha_1^{\,n}$ when we substitute $\varepsilon^{\alpha_1 x}$ for v_1; and the same method will apply to discover how $v_2\, v_3$, &c. are involved; and when $\alpha_1\, \alpha_2$, &c. do not enter as multipliers we thence know that $v_1\, v_2$ &c. are themselves multipliers, and not their differential coefficients. This being considered, the question is reduced to find the value of the compound operation $\varepsilon^{\alpha_1 x}\, d_x\, \varepsilon^{\alpha_2 x}\, d_x\, \varepsilon^{\alpha_3 x}\, d_x \ldots\ldots \varepsilon^{\alpha_n x}\, d_x$ arranged according to the decreasing indices of d_x, and then where we have $\alpha_1^{\,m}\, \varepsilon^{\alpha_1 x}$ if we put $\frac{d^m v_1}{d x^m}$, and when we have $\alpha_2^{\,p}\, \varepsilon^{\alpha_2 x}$ put $\frac{d^p v_2}{d x_p}$, &c., we shall obtain more readily the result of the question proposed.

Now by the last section

$$\varepsilon^{\alpha_1 x}\, d_x = (d_x + \alpha_1) . \varepsilon^{\alpha_1 x};$$

therefore

$$\varepsilon^{\alpha_1 x}\, d_x\, \varepsilon^{\alpha_2 x}\, d_x = (d_x + \alpha_1) . \varepsilon^{(\alpha_1 + \alpha_2) x}\, d_x = (d_x + \alpha_1)(d_x + \alpha_1 + \alpha_2)\, \varepsilon^{(\alpha_1 + \alpha_2) x}.$$

Similarly

$$\varepsilon^{\alpha_1 x}\, d_x\, \varepsilon^{\alpha_2 x}\, d_x\, \varepsilon^{\alpha_3 x}\, d_x = (d_x + \alpha_1)(d_x + \alpha_1 + \alpha_2)(d_x + \alpha_1 + \alpha_2 + \alpha_3)\, \varepsilon^{(\alpha_1 + \alpha_2 + \alpha_3) x};$$

and generally

$$\varepsilon^{\alpha_1 x} d_x \varepsilon^{\alpha_2 x} d_x \varepsilon^{\alpha_3 x} d_x \ldots \varepsilon^{\alpha_n x} d_x = (d_x + \alpha_1)(d_x + \alpha_1 + \alpha_2) \ldots (d_x + \alpha_1 + \alpha_2 + \ldots + \alpha_n)\, \varepsilon^{(\alpha_1 + \alpha_2 + \ldots \alpha_n) x}.$$

If therefore we expand according to the decreasing indices of d_x, the compound operation,

$$(d_x + \alpha_1)(d_x + \alpha_1 + \alpha_2) \ldots (d_x + \alpha_1 + \alpha_2 + \ldots + \alpha_n) . \varepsilon^{\alpha_1 x} . \varepsilon^{\alpha_2 x} . \varepsilon^{\alpha_3 x} \ldots \varepsilon^{\alpha_n x},$$

we shall then have only to put v_1 for $\varepsilon^{\alpha_1 x}$, v_2 for $\varepsilon^{\alpha_2 x}$, &c.

$$\frac{d v_1}{d x} \text{ for } \alpha_1\, \varepsilon^{\alpha_1 x}, \quad \frac{d v_2}{d x} \text{ for } \alpha_2\, \varepsilon^{\alpha_2 x}, \&c.$$

To effect the composition above indicated, let us seek the product (arranged according to the powers of x) of

$$(x + \alpha_1)(x + \alpha_1 + \alpha_2)(x + \alpha_1 + \alpha_2 + \alpha_3) \ldots (x + \alpha_1 + \alpha_2 + \alpha_3 + \ldots + \alpha_n).$$

Represent this product by

$$x^n + A_1 x^{n-1} + A_2 x^{n-2} + \ldots + A_m x^{n-m} + \ldots + A_{n-1} x + A_n,$$

the general coefficient A_m being the sums of products, each of which contain m factors.

For A_1 it is easily seen that its value is

$$n\,\alpha_1 + (n-1)\,\alpha_2 + (n-2)\,\alpha_3 + \ldots 2\,\alpha_{n-1} + \alpha_n.$$

Again, A_2 consists of products such as $\alpha_1\,\alpha_2$, $\alpha_1\,\alpha_3$, $\alpha_2\,\alpha_3$, &c., and pure powers, as α_1^2, α_2^2, &c.; the general form of the first class of terms is $\alpha_p\,\alpha_q$, and we now proceed to find its coefficient, or the number of times this combination occurs, which number may be denoted by $(\alpha_p\,\alpha_q)$, and supposing p less than q, no factor preceding $x + \alpha_1 + \alpha_2 + \ldots \alpha_p$ will be concerned in forming the combination in question, and in the factor itself and the succeeding ones the terms preceding α_p may also be neglected.

The factors commencing from the above, arranged horizontally, will form this diagram.

$$\begin{array}{l}
x + \alpha_1 + \ldots + \alpha_p \\
x + \alpha_1 + \ldots + \alpha_p + \alpha_{p+1} \\
x + \alpha_1 + \ldots \alpha_p + \alpha_{p+1} + \alpha_{p+2} \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
x + \alpha_1 + \ldots \alpha_p + \alpha_{p+1} + \alpha_{p+2} + \ldots + \alpha_q \,.\, * \\
x + \alpha_1 + \ldots \alpha_p + \ldots\ldots\ldots\ldots\ldots\ldots + \alpha_q + \alpha_{q+1} \\
x + \alpha_1 + \ldots \alpha_p + \ldots\ldots\ldots\ldots\ldots\ldots\ldots \alpha_q + \alpha_{q+1} + \alpha_{q+2}, \\
\quad \text{\&c.} \qquad\qquad\qquad\qquad\qquad\qquad \text{\&c.}
\end{array}$$

Now if α_{q+1} were placed where the asterisk stands, the combination of α_p with α_q and α_{q+1} would be alike, $\therefore (\alpha_p\,\alpha_q) - (\alpha_p\,\alpha_{q+1}) =$ the number of combinations of one term at the asterisk with the terms in the vertical column of α_p, except *that* α_p which is the same horizontal line with the asterisk; it is therefore the number of terms minus one in that column which (since $p-1$ factors precede the first above written) will be $n-p$.

Therefore Δ denoting the finite difference, when q increases by unity we have

$$\Delta\,(\alpha_p\,\alpha_q) = -(n-p);$$

therefore

$$(\alpha_p\,\alpha_q) = (n-p)\,(c-q),\ c \text{ being independent of } q.$$

Suppose $q=n$, $(\alpha_p\,\alpha_n)$ will be the number of terms minus one in the column of α_p, since α_n enters only once; that is $(\alpha_p\,\alpha_n) = n-p$, therefore $c-n=1$, or $c=n+1$ which gives

$$(\alpha_p\,\alpha_q) = (n-p)\,(n-q+1).$$

As for the coefficients of the powers as α_p^2, denoting such by a similar notation (α_p^2), they will not be affected by the supposition that

$$\alpha_1 = 0\ \alpha_2 = 0 \ldots \alpha_{p-1} = 0\ \alpha_{p+1} = 0 \ldots \alpha_n = 0,$$

they are therefore the same as in $(1+\alpha_p)^{n-p+1}$ that is $\frac{(n-p)(n-p+1)}{1.2}$ which is half of the formula obtained by putting $q=p$.

Hence

$$A_2 = \frac{n.(n-1)}{1.2}.\alpha_1^2 + \frac{(n-1)(n-2)}{1.2}.\alpha_2^2 + \frac{(n-2)(n-3)}{1.2}.\alpha_3^2 + \ldots (n-1)(n-1)\alpha_1\alpha_2$$
$$+ (n-1)(n-2)\alpha_1\alpha_3 + (n-1)(n-3)\alpha_1\alpha_4 + \ldots$$
$$+ (n-2)(n-2)\alpha_2\alpha_3 + (n-2)(n-3)\alpha_2\alpha_4 + \ldots$$
$$+ (n-3)(n-3)\alpha_3\alpha_4 + \ldots$$

In like manner we may classify the terms of which A_3 is composed into terms of the forms $\alpha_p\,\alpha_q\,\alpha_r$, $\alpha_p^2\,\alpha_q$, α_p^3 respectively, p, q, r being arranged according to magnitude; their coefficients may be represented as before by the same letters in brackets.

Every combination of $\alpha_p\,\alpha_q$ may be combined with α_r, except such as are formed from the α_p and α_q which are in the same horizontal line with it, if these are erased the number n is reduced to $n-1$, and the combinations of $\alpha_p\,\alpha_q$ are then by what has been already shown only $(n-p-1)(n-q)$ in number, therefore the excess of the number of the combinations of α_r with $\alpha_p\,\alpha_q$ above that of α_{r+1} is $(n-p-1)(n-q)$ or taking the finite difference in reference to r

$$\Delta(\alpha_p\,\alpha_q\,\alpha_r) = -(n-p-1)(n-q);$$

thence

$$(\alpha_p\,\alpha_q\,\alpha_r) = (n-p-1)(n-q)(c-r),$$

and putting $r=n$ we find as before $c=n+1$; therefore

$$(\alpha_p\,\alpha_q\,\alpha_r) = (n-p-1)(n-q)(n-r+1)$$

and generally if $s>r$, $r>q$, $q>p$, &c., then by the same process

$$(\alpha_s\,\alpha_r\,\alpha_q\,\alpha_p\ldots.) = (n-s+1)(n-r)(n-q-1)(n-p-2)\ldots..$$

Again, if we erase the α_p which is in the same horizontal line with α_r the number of combinations of the remaining terms α_p (in number $n-p$) are $\frac{(n-p)(n-p-1)}{1.2}$, and since the number of terms in the vertical line where α_q stands is $n-q+1$, it follows that

$$(\alpha_p^2\,\alpha_q) = \frac{(n-p)(n-p-1)}{1.2}.(n-q+1),$$

and generally

$$\left(\overset{s'}{\underset{s}{\alpha}}\;\overset{r'}{\underset{r}{\alpha}}\;\overset{q'}{\underset{q}{\alpha}}\ldots\right)$$
$$= \frac{(n-s+1)(n-s)\ldots(s'\text{ times})}{1.2\ldots s'}.\frac{(n-r)(n-r-1)\ldots(r'\text{ times})}{1.2\ldots r'}.\frac{(n-q-1)(n-q-2)\ldots q'\text{ times}}{1.2\ldots q'}\ldots$$

Lastly $(\alpha_p{}^3)$ is the same as if all the terms $\alpha_1\,\alpha_2$ &c. were zero, except α_p and is therefore

$$\frac{(n-p+1)(n-p)(n-p-1)}{1\,.\,2\,.\,3}.$$

More generally

$$(\alpha_p{}^{p'}) = \frac{(n-p+1)(n-p)\ldots(p' \text{ times})}{1\,.\,2\ldots p'}.$$

We have thus investigated the coefficients of every combination which enter the whole product, and then if only $\frac{d^{p'}v_p}{d\,x^{p'}}$ be substituted for any general symbol $(\alpha_p^{p'})$, the required development is completely obtained.

It may be remarked that the coefficients of the combinations of consecutive terms are pure powers, thus

$$(\alpha_1\,\alpha_2) = (n-1)^2 \quad \alpha_2\,\alpha_3 = (n-2)^2, \text{\&c.} \quad \alpha_1\,\alpha_2\,\alpha_3 = (n-2)^3.$$

29. By the preceding investigation we have obtained the following general formula, in which the subject is any function of x:

$$\begin{aligned}\frac{v_1\,d_x\,v_2\,d_x\,v_3\,d_x\ldots v_n\,d_x}{v_1\,.\,v_2\,.\,v_3\ldots v_n} = {} & d_x{}^n + d_x{}^{n-1}\left\{\frac{n\,d\,v_1}{v_1\,d\,x} + (n-1)\frac{d\,v_2}{v_2\,d\,x} + (n-2)\frac{d\,v_3}{v_3\,d\,x} + \ldots\frac{d\,v_n}{v_n\,d\,x}\right\} \\ & + d_x{}^{n-2}\left\{(n-1)(n-1)\frac{d\,v_1}{v_1\,d\,x}\cdot\frac{d\,v_2}{v_2\,d\,x}\right. \\ & \quad + (n-1)(n-2)\frac{d\,v_1}{v_1\,d\,x}\frac{d\,v_3}{v_3\,d\,x} + (n-1)(n-3)\frac{d\,v_1}{v_1\,d\,x}\frac{d\,v_4}{v_4\,d\,x} + \ldots \\ & + (n-2)(n-2)\frac{d\,v_2}{v_2\,d\,x}\cdot\frac{d\,v_3}{v_3\,d\,x} + (n-2)(n-3)\frac{d\,v_2}{v_2\,d\,x}\cdot\frac{d\,v_4}{v_4\,d\,x} + \ldots \\ & + (n-3)(n-3)\frac{d\,v_3}{v_3\,d\,x}\cdot\frac{d\,v_4}{v_4\,d\,x} + \ldots \\ & + \ldots\ldots\ldots\ldots\ldots\ldots \\ & + \frac{n(n-1)}{2}\cdot\frac{d^2\,v_1}{v_1\,d\,x^2} + \frac{(n-1)(n-2)}{2}\cdot\frac{d^2\,v_2}{v_2\,d\,x^2} \\ & \quad \left. + \frac{(n-2)(n-3)}{2}\cdot\frac{d^2\,v_3}{v_3\,d\,x^2} + \ldots\right\} \\ & + d_x{}^{n-3}\left\{(n-2)(n-2)(n-2)\,.\,\frac{d\,v_1}{v_1\,d\,x}\cdot\frac{d\,v_2}{v_2\,d\,x}\frac{d\,v_3}{v_3\,d\,x} +, \text{\&c.}\right\} \\ & +, \text{\&c.}\end{aligned}$$

Put $v_1 = v_2 = v_3 = \ldots = v_n = v$ hence,

$$\begin{aligned}(v\,d_x)^n = {} & d_x{}^n\,v^n + \frac{n(n+1)}{2}\,d_x{}^{n-1}\,v^{n-1}\,.\,\frac{d\,v}{d\,x} \\ & + d_x{}^{n-2}\left\{\frac{(n-1)\,n\,(n+1)(3\,n-2)}{2\,.\,3\,.\,4}\,.\,v^{n-2}\,\frac{d\,v^2}{d\,x^2} + \frac{(n-1)\,.\,n\,(n+1)}{2\,.\,3}\,.\,v^{n-1}\,\frac{d^2\,v}{d\,x^2}\right\}\end{aligned}$$

and similarly if we put in the general formula $v_1 = 1$, and write v_1 for v_2, v_2 for v_3, &c. and then multiply by v_n, finally making all equal to v, we obtain

$$(d_x v)^n = d_x^{\,n} v^n + \frac{(n-1).n}{2} d_x^{\,n-1} v^{n-1} \frac{dv}{dx}$$

$$+ d_x^{\,n-2} \left\{ \frac{(n-2)(n-1)n(3n-5)}{2.3.4} v^{n-2} \frac{dv^2}{dx^2} + \frac{(n-2).(n-1)n}{2.3} v^{n-1} \frac{d^2 v}{dx^2} \right\} +, \&c.$$

Put now $\frac{1}{v}$ for v in this formula, whence

$$(d_x v^{-1})^n = d_x^{\,n} v^{-n} - \frac{(n-1).n}{2} . d_x^{\,n-1} v^{-n-1} \frac{dv}{dx}$$

$$+ d_x^{\,n-2} \left\{ \frac{(n-2)(n-1)n(n+1)}{2.4} . v^{-n-2} \frac{dv^2}{dx^3} - \frac{(n-2)(n-1)n}{2.3} . v^{-n-1} \frac{d^2 v}{dx^2} \right\} +, \&c.$$

30. Change of the independent variable.

When u is a function of y, and y of x, then it is easily shown that $\frac{du}{dy} = \frac{du}{dx} . \left(\frac{dy}{dx}\right)^{-1}$, or $d_y = d_x . \left(\frac{dy}{dx}\right)^{-1}$ omitting the subject u; hence by substitution in the preceding general formula we have

$$(d_y)^n = d_x^{\,n} . \left(\frac{dy}{dx}\right)^{-n} - \frac{(n-1).n}{2} . d_x^{\,n-1} . \left(\frac{dy}{dx}\right)^{-n-1} . \left(\frac{d^3 y}{dx^2}\right)$$

$$+ d_x^{\,n-2} \left\{ \frac{(n-2)(n-1).n(n+1)}{2.4} . \left(\frac{dy}{dx}\right)^{-n-2} . \left(\frac{d^2 y}{dx^2}\right)^2 - \frac{(n-2).(n-1).n}{2.3} . \left(\frac{dy}{dx}\right)^{-n-1} . \frac{d^3 y}{dx^3} \right\}$$

$+, \&c.$

Thus, for example, if $n = 3$,

$$\frac{d^3 u}{dy^3} = \frac{d^3 u}{dx^3} . \left(\frac{dy}{dx}\right)^{-3} - 3 . \frac{d^2 u}{dx^2} . \left(\frac{dy}{dx}\right)^{-4} . \frac{d^2 y}{dx^2} + \frac{du}{dx} \left\{ 3 \left(\frac{dy}{dx}\right)^{-5} \left(\frac{d^2 y}{dx^2}\right)^2 - \left(\frac{dy}{dx}\right)^{-4} . \frac{d^3 y}{dx^3} \right\};$$

and

$$\frac{d^3 x}{dy^3} = 3 \left(\frac{dy}{dx}\right)^{-5} . \left(\frac{d^2 y}{dx^3}\right)^2 - \left(\frac{dy}{dx}\right)^{-4} . \frac{d^3 y}{dx^3}.$$

As I am not aware that any formula has heretofore been given for the *general* change of the independent variable, I shall here add a reinvestigation of the same subject on simple principles.

When x is changed into $x + h$, suppose y to become $y + k$, and u to be changed into $u + l$.

Now $u + l$ may be expressed by TAYLOR's theorem in two ways,

$$u + l = u + \frac{du}{dx} . h + \frac{d^2 u}{dx^2} . \frac{h^2}{1.2} + \frac{d^3 u}{dx^3} . \frac{h^3}{1.2.3} +, \&c.$$

$$= u + \frac{du}{dy} . k + \frac{d^2 u}{dy^2} . \frac{k^2}{1.2} + \frac{d^3 u}{dy^3} . \frac{k^3}{1.2.3} +, \&c.$$

Hence $\frac{d^n u}{dy^n} . \frac{1}{1.2 \ldots n}$ is the coefficient of k^n in l, that is, in the expression

$$l = \frac{du}{dx} . h + \frac{d^2 u}{dx^2} . \frac{h^2}{1.2} + \frac{d^3 u}{dx^3} . \frac{h^3}{1.2.3} + \ldots + \frac{d^n u}{dx^n} . \frac{h^n}{1.2 \ldots n} +, \&c.$$

and since it is visible that h may be expanded in the form $\mathrm{A}\,k + \mathrm{B}\,k^2 +$, &c., it will be unnecessary to consider in l any term after $\frac{d^n u}{dx^n} \cdot \frac{h^n}{1.2\ldots n}$.

Hence $\frac{d^n u}{dy^n}$ is the coefficient of k^n, when for h we substitute its value in terms of k in the polynomial.

$$\mathrm{H} = \frac{d^n u}{dx^n}.h^n + n.\frac{d^{n-1} u}{dx^{n-1}}.h^{n-1} + n\,(n-1)\frac{d^{n-2} u}{dx^{n-2}}.h^{n-2} + \ldots 2.3.4\ldots n\frac{du}{dx}.h.$$

Again, k is given in terms of h by TAYLOR's theorem.

$$k = \frac{dy}{dx}.h + \frac{d^2 y}{dx^2}\cdot\frac{h^2}{1.2} + \frac{d^3 y}{dx^3}\cdot\frac{h^3}{1.2.3} +, \&c.$$

Put for abridgement

$$\mathrm{Y} = \frac{d^2 y}{dx^2}\cdot\left(\frac{dy}{dx}\right)^{-1}\cdot\frac{h}{2} + \frac{d^3 y}{dx^3}\cdot\left(\frac{dy}{dx}\right)^{-1}\cdot\frac{h^2}{2.3} + \frac{d^4 y}{dx^4}\cdot\left(\frac{dy}{dx}\right)^{-1}\cdot\frac{h^3}{2.3.4} +, \&c.$$

The equation for determining h in terms of k becomes then

$$h - \left\{k\left(\frac{dy}{dx}\right)^{-1} - h\,\mathrm{Y}\right\} = 0.$$

In a memoir on the Resolution of Algebraic Equations, published in the fourth volume of the Transactions of the Cambridge Philosophical Society, I have proved the following rule.

If $\varphi(x) = 0$ be an equation which contains only entire positive powers of x, and $f(x)$ any other function of the same kind, of which the differential coefficient or derived function is $f'(x)$, then the value of $f(x)$ will be found by taking the coefficient of $\frac{1}{x}$ in the expression $-f'(x)\,\mathrm{l}.\frac{\varphi(x)}{x}$.

Applying this rule to the case before us we find that H is the coefficient of $\frac{1}{h}$ in the formula

$$-\frac{d\mathrm{H}}{dh}.\mathrm{l}.\left\{1 - \left(\frac{k}{h}\cdot\left(\frac{dy}{dx}\right)^{-1} - \mathrm{Y}\right)\right\},$$

and consequently $\frac{d^n y}{dx^n}$ is the coefficient of $\frac{k^n}{h}$ in the same formula.

The first $(n-1)$ terms in the expansion of the logarithm do not contain k^n and may therefore be neglected, instead then of

$$-\mathrm{l}.\left\{1 - \left(\frac{k}{h}\cdot\left(\frac{dy}{dx}\right)^{-1} - \mathrm{Y}\right)\right\}$$

we may use the series

$$\frac{1}{n}\left\{\frac{k}{h}\cdot\left(\frac{dy}{dx}\right)^{-n} - \mathrm{Y}\right\}^n + \frac{1}{n+1}\left\{\frac{k}{h}\cdot\left(\frac{dy}{dx}\right)^{-1} - \mathrm{Y}\right\}^{n+1} +, \&c. = \mathrm{S},$$

and the value of

$$\frac{d\mathrm{H}}{dh}\text{ is } n\,\frac{d^n u}{dx^n}.h^{n-1} + n\,(n-1)\,\frac{d^{n-1}.u}{dx^{n-1}}.h^{n-2}$$

$$+n(n-1)(n-2)\cdot\frac{d^{n-2}u}{dx^{n-2}}h^{n-3}+\ldots 1.2\ldots n\frac{du}{dx}$$

in the product of both which series the coefficient of $\frac{k^n}{h}$ being sought will give the required value of $\frac{d^n u}{dy^n}$ and this is manifestly of the form

$$A_1\cdot\frac{d^n u}{dx^n}+A_2\frac{d^{n-1}u}{dx^{n-1}}+A_3\cdot\frac{d^{n-2}u}{dx^{n-2}}+\ldots+A_n\cdot\frac{du}{dx}$$

A_1 is the coefficient of $\frac{k^n}{h}$ in $n\,h^{n-1}$ S, or of $\frac{k^n}{h^n}$ in n S

A_2 .... of $\frac{k^n}{h}$ in $n\,(n-1)\,h^{n-2}$ S, or of $\frac{k^n}{h^{n-1}}$ in $n\,(n-1)$. S, &c.

Now if we observe that Y contains h as a factor it follows that the coefficient of k^n in the series S is of the form

$$\frac{\alpha_1}{h^n}+\frac{\alpha_2}{h^{n-1}}+\frac{\alpha_3}{h^{n-2}}+\text{\&c.}$$

and consequently

$A_1=n\alpha_1\quad A_2=n(n-1)\alpha_2\quad A_3=n(n-1)(n-2)\alpha_3\ldots.A_n=n(n-1)(n-2)\ldots.1.\alpha_n.$

Therefore

$$\frac{d^n u}{dy^n}=n\,\alpha_1\frac{d^n u}{dx^n}+n\,(n-1)\,\alpha_2.\frac{d^{n-1}u}{dx^{n-1}}+n\,(n-1)\,(n-2)\,\alpha_3\frac{d^{n-2}u}{dx^{n-2}}+,\text{\&c.}$$

By taking in fact the coefficient of k^n in S and multiplying it by h^n we find that the product, viz. $\alpha_1+\alpha_2 h+\alpha_3 h^2+\ldots.\alpha_n h^{n-1}+$, &c. is equivalent to

$$\frac{1}{n}\cdot\left(\frac{dy}{dx}\right)^{-n}-.\left(\frac{dy}{dx}\right)^{-n}.Y+\frac{n+1}{2}\cdot\left(\frac{dy}{dx}\right)^{-n}.Y^2-\frac{(n+1)(n+2)}{2.3}\cdot\left(\frac{dy}{dx}\right)^{-n}.Y^3\ldots$$

$$=\frac{1}{n}\cdot\left(\frac{dy}{dx}\right)^{-n}-h\left(\frac{dy}{dx}\right)^{-n-1}\cdot\left\{\frac{d^2y}{dx^2}\cdot\frac{1}{2}+\frac{d^3y}{dx^3}\cdot\frac{h}{2.3}+,\text{\&c.}\right\}$$

$$+\frac{n+1}{2}h^2\left(\frac{dy}{dx}\right)^{-n-2}\left\{\frac{d^2y}{dx^2}\cdot\frac{1}{2}+\frac{d^3y}{dx^3}\cdot\frac{h}{2.3}\text{\&c.}\right\}^2$$

$$-\frac{(n+1)(n+2)}{2.3}.h^3.\left(\frac{dy}{dx}\right)^{-n-3}\left\{\frac{d^2y}{dx^2}\cdot\frac{1}{2}+\frac{d^3y}{dx^3}\cdot\frac{h}{2.3},\text{\&c.}\right\}^3$$

$$+\text{\&c.}$$

Hence

$$\alpha_1=\frac{1}{n}\cdot\left(\frac{dy}{dx}\right)^{-n}$$

$$\alpha_2=-\left(\frac{dy}{dx}\right)^{-n-1}\cdot\frac{d^2y}{dx^2}\cdot\frac{1}{2}$$

$$\alpha_3=\frac{n+1}{2}\cdot\left(\frac{dy}{dx}\right)^{-n-2}\cdot\left(\frac{d^2y}{2\,dx^3}\right)^2-\left(\frac{dy}{dx}\right)^{-n-1}\cdot\frac{d^3y}{2.3\,dx^3}$$

$$\alpha_4 = -\frac{(n+1)(n+2)}{2\,.\,3}\,.\left(\frac{dy}{dx}\right)^{-n-3}.\left(\frac{d^2 y}{2\,dx^2}\right)^3$$

$$+\frac{n+1}{2}\,.\left(\frac{dy}{dx}\right)^{-n-2}.\,2\,.\left(\frac{d^2y}{2dx^2}\right)\left(\frac{d^3y}{2\,.\,3\,dx^3}\right)-\left(\frac{dy}{dx}\right)^{-n-1}.\frac{d^4y}{dx^4}.\frac{1}{2\,.\,3\,.\,4}.$$

In general let

$$\left(\frac{d^2y}{dx^2}.\frac{1}{2}+\frac{d^3y}{dx^3}.\frac{h}{2\,.\,3}+\frac{d^4y}{dx^4}.\frac{h^2}{2.3.4}+,\&c.\right)^m = y_{m,0}+y_{m,1}\,.\,h+y_{m,2}\,.\,h^2\,\&c.$$

Then

$$(-1)^{m-1}\,\alpha_m = \frac{(n+1)(n+2)\ldots(n+m-2)}{2\,.\,3\ldots(m-1)}\,.\left(\frac{dy}{dx}\right)^{-n-m+1}.\,y_{m-1,0}$$

$$-\frac{(n+1)(n+2)\ldots(n+m-3)}{2\,.\,3\ldots(m-2)}\,.\left(\frac{dy}{dx}\right)^{-n-m+2}.\,y_{m-2,1}$$

$$+\frac{(n+1)(n+2)\ldots(n+m-4)}{2\,.\,3\ldots(m-3)}\,.\left(\frac{dy}{dx}\right)^{-n-m+3}.\,y_{m-3,2}$$

. . . .

$$\mp\left(\frac{dy}{dx}\right)^{-n-1}.\,y_{1,m-2},$$

m being > 1.

Corollary. Put $u = x$, then

$$\frac{d^n x}{dy^n} = n\,(n-1)\,(n-2)\ldots 1\,\alpha_n.,$$

where

$$(-1)^{n-1}\,.\,\alpha_n = \frac{(n+1)\,(n+2)\ldots(2n-2)}{2\,.\,3\ldots n-1}\,.\left(\frac{dy}{dx}\right)^{-2n+1}.\,y_{n-1,0}$$

$$-\frac{(n+1)\,(n+2)\ldots(2n-3)}{2\,.\,3\ldots(n-2)}\,.\left(\frac{dy}{dx}\right)^{-2n+2}.\,y_{n-2,1},\ \&c.$$

$$\therefore (-1)^{n-1}\,.\,\frac{d^n x}{dy^n} = \Big\{n\,(n+1)\ldots(2n-2)\,.\,\frac{dy}{dx}\,.\,y_{n-1,0}$$

$$-(n-1)\,.\,n\,(n+1)\ldots(2n-3)\,.\left(\frac{dy}{dx}\right)^2.\,y_{n-2,1}$$

$$+(n-2)\,(n-1)\ldots(2n-4)\left(\frac{dy}{dx}\right)^3.\,y_{n-3,2},\ \&c.\Big\}\,.\left(\frac{dy}{dx}\right)^{-2n}.$$

Thus

$$\frac{dx}{dy} = \left\{\frac{dy}{dx}\,.\,y_{0,0}\right\}.\left(\frac{dy}{dx}\right)^{-2} = \left(\frac{dy}{dx}\right)^{-1}$$

$$\frac{d^2x}{dy^2} = \left\{2\frac{dy}{dx}\,.\,y_{1,0}\right\}.\left(\frac{dy}{dx}\right)^{-4} = \left(\frac{dy}{dx}\right)^{-3}.\,\frac{d^2y}{dx^2}$$

$$\frac{d^3x}{dy^3} = \left\{3\,.\,4\,.\,\frac{dy}{dx}\,.\,y_{2,0}-2\,.\,3\,.\left(\frac{dy}{dx}\right)^2.\,y_{1,1}\right\}\left(\frac{dy}{dx}\right)^{-6}$$

$$=3\,.\left(\frac{dy}{dx}\right)^{-5}.\left(\frac{d^2y}{dx^2}\right)^2-\left(\frac{dy}{dx}\right)^{-4}.\,\frac{d^3y}{dx^3},\ \&c.\ \&c.$$

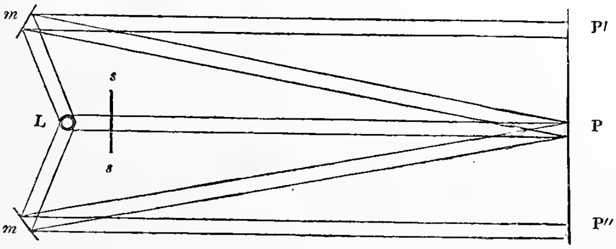
XIII. *On the Adaptation of different Modes of Illuminating Lighthouses; as depending on their Situations and the Object contemplated in their Erection. By* William Henry Barlow, *Esq. In a Letter addressed to* Peter Barlow, *Esq., F.R.S. &c., and communicated by Him.*

Received April 27,—Read May 4, 1837.

Constantinople, March 14th, 1837.

Having made several experiments with the **Drummond** light, and other means of illuminating light-houses, undertaken at the request of the Turkish Government, with a view to placing lights at the entrance of the Bosphorus from the Black Sea, I have been led to observe some facts regarding the illuminating powers of the lights themselves, and the increase obtained by the use of reflectors and lenses, which, I trust, may not be found uninteresting.

On the increase of illuminating power obtained by Lenses and Reflectors.



Let L in the annexed figure represent a lamp; *m, m,* two reflectors, which may be so adjusted as to throw the reflected images either in parallel lines on a screen at P′ and P″, or at such an inclination as to unite with that of the light itself at the centre point P. Let also *s s* represent a screen of such imperfect transparency as to absorb the same quantity of light in transmission as the mirrors *m, m* absorb by reflection; then in the first case the three images P′, P, P″ will have equal surfaces and intensities*, and the illuminating power will be three times that of the central lamp; and when by a different adjustment of the mirrors the three images are blended in one, then the surface will be equal to that of the central image, but the intensity three times greater, so that in either case the illuminating power will be proportional to the number of mirrors, or to the surface of those mirrors. If, therefore, we conceive the whole space between *m, m* to be filled with mirrors, to reflect the light in parallel lines on the screen P′, P, P″, it is clear that the illuminating power of the

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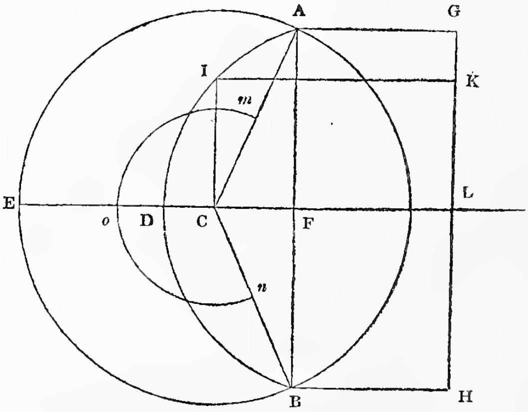
* We reject here the difference in the length of the trajectory of the direct and reflected light.

lens will be expressed by the number of times the surface of the central image is contained in the whole surface of the screen P′, P″; and this is true whether we consider the several images to be thrown in parallel lines, or condensed in a focus, or dispersed over a larger surface, for as the illuminated surface is contracted, the intensity is increased, and as it is extended, the intensity is diminished in the same proportion, so that under all circumstances the product of surface and intensity will be a constant quantity. Hence the illuminating power (abstracting from absorption) will be increased by the reflector in the ratio of the surface of the lights to the surface of the end or section of the reflector. Or in other words, the area of the end of the reflector divided by the area of the light, will be a numerical measure of the illuminating power.

This result is obtained by supposing the reflector to be composed of a number of small plane reflectors, each throwing the light in parallel lines, and each image therefore as having the same intensity as the direct light (screened as above) when viewed from the same distance; but with a continuous curve surface, such as a parabolic reflector, we must consider the divergency of the emanating ray at the point where it falls on the reflector, which will vary inversely as the square of the distance of that point from the centre of the light, or directly as the square of the sine of half the angle which the light subtends from that point, and therefore as the versed sine of half the same angle; and the sum of all these must be compared with the area of the reflector, that is of its section or end, which varies also as the versed sine of half the angle which its extreme edge subtends at the light.

In order, therefore, to compute the increase of illuminating power due to a parabolic reflector, according to this principle, we must find a *mean focal distance*, that is, a distance (from which to estimate the constant angle subtended by the light) that shall be equivalent to the several variable distances.



Let A D B be a parabolic reflector and C its focus, then will D C be the minimum and A C the maximum focal distance. Now if the light at C emanated from a point, all the rays intercepted by the surface A D B would be projected forward in parallel lines and cover the plane surface G H = A B at whatever distance it might be placed from the reflector, and the light at G, K, L would be that due to the distances A C, I C, D C respectively: if then a segment of a sphere *m o n* be described intercepting the same number of rays as A D B, and whose surface is equal to the area of A B or H G, we shall have the same quantity of light equally distributed over the same surface; hence the radius of the segment *m o n* will be the mean focal distance with which all the light may be conceived to leave the reflector.

Describe the circle A E B; then, because A D B is a parabola, and A E B a circle described about it with the radius C A, and because $\mathrm{C\,A} = \mathrm{D\,F} + \mathrm{D\,C}$, the height D F of the parabola $= \frac{1}{2}$ the height E F of the segment A E B.

But $\mathrm{E\,F} = \mathrm{E\,C} + \mathrm{C\,F} = \mathrm{A\,C} + \mathrm{C\,F}$, therefore $\mathrm{D\,F} = \frac{\mathrm{A\,C} + \mathrm{C\,F}}{2}$,

and D C the minimum focal distance $= \mathrm{D\,F} - \mathrm{C\,F} = \frac{\mathrm{A\,C} - \mathrm{C\,F}}{2}$. Let $\mathrm{A\,C} = r$, $\mathrm{C\,F} = h$, then $r =$ maximum focal distance, and $\frac{r-h}{2} =$ minimum focal distance, $(2\,r \times 3{\cdot}1416)\,(r+h) =$ surface of segment A F B; and $4\,(r^2 - h^2)\,.7854 =$ area of A B or G H.

Let $x =$ radius of segment *m o n*: now the surface of the segment A E B is to the surface of the segment *m o n* as r^2 to x^2, and the area of the end A B is equal to the surface of the segment *m o n*, therefore

$$(2\,r \times 3{\cdot}1461)\,(r+h) : 4\,(r^2 - h^2)\,{\cdot}7854 :: r^2 : x^2$$

or

$$2\,r\,(r+h)\,x^2 = (r^2 - h^2)\,r^2$$

whence

$$x^2 = \frac{(r^2 - h^2)}{2\,r\,(r+h)}\,r^2 = \left(\frac{r-h}{2}\right)r$$

or

$$x = \sqrt{r\left(\frac{r-h}{2}\right)}$$

But $r =$ maximum focal distance and $\frac{r-h}{2}$ minimum focal distance. Therefore x, the mean focal distance, is a mean proportional between the maximum and minimum focal distances. Let therefore A represent the angle subtended by the reflector from the centre of the light, and $a =$ the angle subtended by the light from the reflector at the mean focal distance, then

$$\frac{\text{versed sine } \frac{1}{2}\,\mathrm{A}}{\text{versed sine } \frac{1}{2}\,a}.$$

will be the amount of illuminating power obtained by the reflector, that of the lamp being 1.

This result differs in its numerical value very little from the former, viz. the area of the reflector divided by the area of the light. Thus, for example, let a reflector whose maximum focal distance is twelve inches, and minimum three inches, be illuminated with a standard Argand lamp, the diameter of the *flame* of which is one inch, and its altitude $1\frac{3}{4}$ inch. Here the depth of reflector is 9 inches and the area of its end $4\,(12^2 - 6^2)\cdot7854 = 339\cdot28$ inches. And by the first rule $\frac{339\cdot28}{1\cdot75} = 193\cdot8$ is the amount of power obtained.

By the second rule we have the angle subtended by the reflector equal 240°; mean focal distance $= \sqrt{12 \times 3} = 6$ inches. The angle subtended by the flame of an Argand lamp, which is in the form of a cylinder, will be greater in the vertical direction than in the horizontal; in order therefore that we may be able to measure the surface of the segment by its versed sine, we will assume that the light is in the form of a sphere whose apparent surface and intensity is equal to that of the lamp, and therefore equal to it in illuminating power.

Now the angle subtended by a sphere whose apparent surface is 1·75 at a distance of 6 inches is 14° 18′; therefore by the second rule $\frac{\text{vers. } 120^\circ}{\text{vers. } 7^\circ\, 9'} = 192\cdot9$ amount of illuminating power obtained.

Let us now suppose Drummond's lime ball to be placed in the focus to find its illuminating effect. Here the section of the ball, the diameter being $\frac{3}{8}$ths of an inch, is ·110445, and on the first principle

$$\frac{339\cdot28}{\cdot110445} = 3079 \text{ amount of power,}$$

and by the second

$$\frac{\text{vers. } 120^\circ}{\text{vers. } 1^\circ\, 47'\, 27''} = 3071 \text{ amount of power}[*].$$

And as it is known that the illuminating power of the lime ball when $\frac{3}{8}$ths of an inch in diameter is equal to 16·6 Argand lamps, it follows that a reflector of the above dimensions will give a light equal to $3079 \times 16\cdot6 = 51112$ Argand lamps, or 264 such reflectors illuminated with Argand lamps; which agrees with Drummond's observations[†].

These rules are equally applicable to lenses, the same effect being produced in them by refraction as in the reflectors by reflection, except the difference between the light absorbed and transmitted.

It is, however, almost impossible here to determine the mean focal distance very exactly, the lens being built in pieces; and its form being square increases the difficulty; still if we take the mean between the distance of the focal point from the

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[*] It may not be seen immediately why these rules do not give precisely the same numerical results, but it will be found that if the angle of divergence be very great, the position of the reflector will at the extreme edge have a considerable obliquity to the line of direction in which it acts and its apparent surface, and consequently its illuminating powers will be reduced. The difference, however, is very small when the mean divergence is under 20°.

[†] Phil. Trans. 1830, p. 390.

centre and extreme angle of the lens, in the middle of the thickness of the glass, we obtain a tolerably close approximation.

Also, the lens being square, and eight of them forming the circle or system of lenses, $2\left(\frac{\sin 22^\circ\, 30'}{8}\right)$ will be the expression for the light intercepted.

For example, let it be required to find the increase of illuminating power obtained by the French lens with its lamp, as used by Drummond in his experiments, the lens being 30 inches square, and the lamp having an intensity equal to 4, and illuminating power equal to 10·4 Argand lamps.

Here the surface of the flame will be 4·55 inches; therefore by the first rule $\frac{30^2}{4{\cdot}55} = 198$ increase of power. Again, the mean focal distance being about 39 inches, a sphere whose apparent surface is 4·55 inches will subtend 3° 31′; hence by the second rule

$$\frac{\frac{1}{4}\sin 22^\circ\, 30'}{\text{vers. } 1^\circ\, 45'\, 30''} = 200.$$

These examples being sufficient for the purpose of illustration, we may now state the conclusion which is derived from the above investigation; namely, that all reflectors and lenses of the same diameter have the same illuminating power when illuminated with the same lamp, and that decreasing the focal distance, and intercepting more rays, does not increase the illuminating power, but simply the divergence, and consequently the surface or space over which it acts.

On the Comparison of Lenses and Reflectors in reference to their Perfection as Optical Instruments.

The results obtained by the above rules, as to the actual increase of illuminating power produced by the use of reflectors and lenses, will of course be considerably greater than would be found in practice, no account being taken of the absorption, obstruction, or undue dispersion of the light; still, however, by comparing their computed powers with those obtained by experiment, we shall be enabled to ascertain their merits as optical instruments.

The French lens with its lamp was found by experiment to be equal to 9·1 reflectors 21 inches diameter, illuminated with Argand lamps[*].

Now by computation (the lens being 30 inches square) and the intensity of its lamp 4, give $30^2 \times 4 = 3600$ for its illuminating power.

And a reflector 21 inches diameter, with a lamp whose intensity is 1, gives 346 illuminating power.

Therefore the illuminating power of the lens ought to be equal to $\frac{3600}{346} = 10{\cdot}4$ reflectors.

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[*] Philosophical Transactions, 1830, p. 383.

But we can hardly expect the lens to be so perfect an instrument as the reflector, not only from the obstruction of light caused by the zones, but from its being composed of separate pieces of glass, each of which has its own focus, which foci will coincide more or less according to the accuracy with which the instrument is made; and it is doubtless from the want of mathematical exactness in the adjustment and curvature of the pieces of which it is composed, that a small surface of light, such as the lime ball, is observed to produce but little effect when placed in its focus, and that the observed divergence of the lens is greater than the computed.

Taking the diameter of the lamp at 3 inches, the maximum horizontal divergence would be 5°, whereas in a revolution of 8 minutes the observed duration of the light was 7 seconds, making the observed angle of divergence 5° 15′.

In the reflector the duration was 25 seconds, which gives the observed divergence 18° 45′, while the computed divergence is 19° 10′, the computed in this instrument exceeding the observed.

Yet as eight lenses may be applied to one light, as well as some additional apparatus which increases their illuminating power, it possesses an advantage in point of economy when applied to a lighthouse; for with a lamp, consuming the oil of fifteen or sixteen ARGAND burners, we are enabled to illuminate eight times 5° 15′ or 42° of the horizon, the full power of the light being (with the additional apparatus) equal to 10·4 reflectors 21 inches diameter; while a reflector frame, such as that at Beechy Head, consumes the oil of thirty burners to illuminate three times 18° 45′ or 56° 15′ of the horizon, the full power of the light being equal to ten reflectors; whence it appears that the lenses illuminate three fourths of the number of degrees in the horizon which the reflectors do with the consumption of one half of the oil.

But although the sum of the angles of maximum divergence of the eight sides of a system of lenses is equal to three fourths of the sum of the angles of maximum divergence of the three sides of a reflector frame in the horizontal direction, yet the vertical divergence of the reflectors is far greater than that of the lenses.

The following are the maximum horizontal and vertical divergences of the reflector and lens.

	Max. Hor. Div.	Max. Vert. Div.
Reflector	18° 45′	31° 54′
French lens	5 15	3 0

The computed vertical divergence is less than this, but the observed horizontal divergence being found more than the computed, the 3° above stated would probably be found sufficiently correct in practice.

Hence it appears that the advantage gained by the use of lenses over that of reflectors, is not dependent upon their greater perfection as optical instruments, but from their using the light more economically, by reason of their having less divergence both horizontally and vertically, and illuminating much less space in the horizon.

On Divergences.

The actual quantity of divergence necessary in any case is a question of considerable importance. As regards that in the horizontal direction, we have only to consider the practical question of the proportion of light to darkness in a revolution of the light frame. Thus, for example, three faces, each illuminating 16° of the horizon, and revolving in 8 minutes, would produce the same effect as six faces, each illuminating 8°, and revolving in 16 minutes. The decision will then be made by reference to the divergence in the vertical direction, the value of which will depend upon the situation of the lighthouse; for if the vertical divergence be too small, a ship might under peculiar circumstances be so near the lighthouse as not to catch the light at all, being, in fact, under the lowest rays; whereas should the situation of the lighthouse be such that a vessel never can approach within two or three miles of it, it is useless to throw away light upon a part where it can never be wanted; bearing in mind however that the refraction which now and then takes place might, in the event of the divergence being too small, prevent a vessel on the horizon from seeing the light, even were she at a proper distance to be able to discern it in the natural state of the atmosphere: some little excess therefore of the practical above the computed angle is requisite.

The following Table shows the vertical divergence required in lighthouses from 100 to 500 feet above the level of the sea, the distance at which the light is first required to be seen being from half to four miles from the lighthouse.

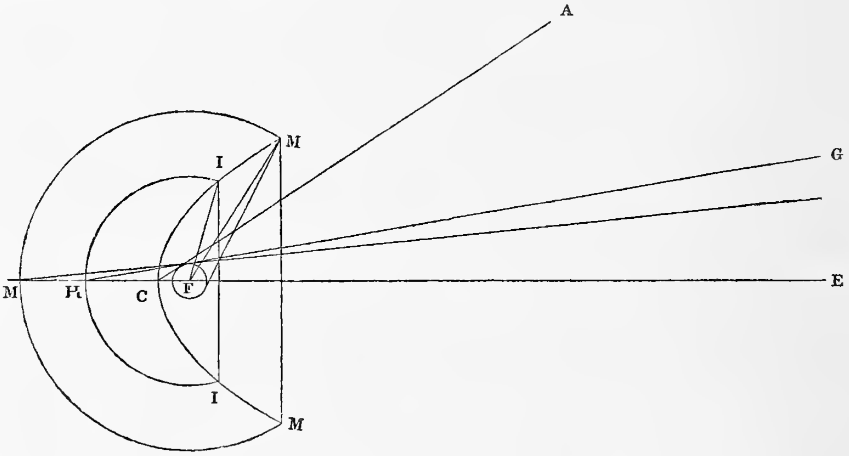
Height of Lighthouse in feet.	Distance at which the light is first required to be seen.				
	½ Mile.	1 Mile.	2 Miles.	3 Miles.	4 Miles.
100	4° 20′	2° 12′	1° 7′	0° 46′	0° 36′
200	8 40	4 22	2 12	1 30	1 10
300	12 58	6 32	3 18	2 14	1 42
400	17 14	8 42	4 22	2 56	2 14
500	20 28	10 50	5 28	3 40	2 46

The axis of the instrument being supposed to be horizontal, the vertical divergence given in the table is twice the angle formed between the horizontal line and a line drawn from the light to the sea at the distance stated at the head of each column. Thus supposing a lighthouse 300 feet above the level of the sea, and that a vessel could not approach within four miles of it, we should only require 1° 42′ vertical divergence; but should a light of that height be so situated that a vessel might pass within one mile of it, we should require 6° 32′. Under which latter circumstances it is clear that the French lens having only 3° could not be made use of.

Hitherto, however, we have only spoken of the maximum divergences, without considering the minimum; but this is far too important a part of the subject to be overlooked, as it is only within the range or angle of minimum divergence that we can see the whole light of the reflector.

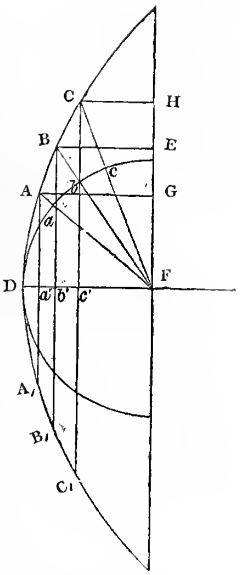
As regards the vertical divergences, if the maximum be sufficient for the situation of the lighthouse, the minimum will in most cases be enough to admit of a vessel approaching so near that some diminution in the power of the light will not be felt; and for this reason it is unnecessary to enter very minutely into this part of the question; we shall proceed therefore at once to notice the action of reflectors and lenses in reference to their horizontal divergences.

If we examine the effect of a reflector as it revolves on its frame, when seen from a distance of several miles (that is to say at such a distance that lines drawn from either side of the reflector to the eye may be considered as parallel) it will be found that it first begins to give its assistance when the eye is in the line A C, which is the extreme of the angle of maximum divergence, or when the angle formed between the line of the axis C E and a line drawn from the eye to the reflector is equal to half the angle subtended by the light in its focus at the minimum focal distance C F, and the reflected light will first appear at C.



As the reflector continues its revolution the eye is brought into the line G H; G H E being the angle formed between a line drawn from the eye to the reflector and the line of its axis, and F H will be the distance at which the light would subtend that angle: now here the reflected light will have extended itself from C along the surface of the reflector to that part where its distance from the focal point is equal to H F. With H F describe the arc I H I, then I C I is the part of the reflector acting at that time, and the area of the circle of which I I is the diameter multiplied into the intensity of the light will be the illuminating power at that time.

In the same manner the illuminating power will continue to increase until the angle formed between the axis of the reflector and a line drawn from it to the eye is equal to half the angle subtended by the light in the focus, at the maximum focal distance F M, when it will have attained its full power, at which it will continue until, by the revolution of the reflector, the same angle is formed on the other side of the axis, that is to say, the light will have its full power only during the range of the angle of minimum divergence, when it will begin to diminish; the reflected light receding from the edges of the reflector, and ceasing at the apex C.



But in a parabolic reflector the area of the circles A a' $A_{,}$, B b' $B_{,}$, C c' $C_{,}$, &c. are to each other as $\overline{Aa'}^2$, $\overline{Bb'}^2$, $\overline{Cc'}^2$, &c., or as D a', Db', D c', &c., or as their equals A a, B b, C c. Let D F A, &c. be any variable angle $= \theta$, the corresponding absciss x, and make D F $= p$ one fourth of the parameter. Then

$$(p + x) \cos \theta = p - x,$$

whence

$$x = \frac{1 - \cos \theta}{1 + \cos \theta} p.$$

And as the angle θ depends upon the angle formed between the axis of the reflector, and a line drawn from it to the eye, we are enabled to ascertain the ratio in which the illuminating power increases and diminishes, when the reflector makes a revolution in a given time.

The following Table shows the illuminating power of the reflector and lens at the end of every second of time during the passage of the light. The reflectors with Argand lamps being supposed to be placed on three sides, and revolving in 8 minutes, that with the lime ball and the lens having eight sides, and revolving in 21 minutes 20 seconds, making the time from the appearance to the reappearance of the light in each 160 seconds. The illuminating powers are expressed in Argand lamps as placed in reflectors 21 inches diameter.

Seconds of Time.	3 Sides.	3 Sides.	8 Sides.	8 Sides.
	Single Reflector with Argand Lamp.	Ten Reflectors with Argand Lamp.	Single Reflector with Lime Ball.	French Lens.
1	·0285	·285	7·72	?
2	·0617	·617	16·51	?
3	·1019	1·019	27·33	10·4
4	·1501	1·501	40·66	10·4
5	·2133	2·133	57·48	10·4
6	·2937	2·937	79·35	10·4
7	·4019	4·019	108·98	10·4
8	·5555	5·555	151·35	10·4
9	·7905	7·905	216·98	10·4
10	1·0000	10·000	264·00	10·4
11	1·0000	10·000	264·00	10·4
12	1·0000	10·000	264·00	10·4
13	1·0000	10·000	264·00	10·4
14	1·0000	10·000	264·00	10·4
15	1·0000	10·000	264·00	10·4
16	1·0000	10·000	264·00	10·4
17	·7905	7·905	216·98	10·4
18	·5555	5·555	151·35	?
19	·4019	4·019	108·98	?
20	·2937	2·937	79·35	Dark.
21	·2133	2·133	57·48	——
22	·1501	1·501	40·66	——
23	·1019	1·019	27·33	——
24	·0617	·617	16·51	——
25	·0285	·285	7·72	——
26	Dark.	Dark.	Dark.	——

From the above Table it will be seen, that taking the full power of a reflector in connection with the duration due to its maximum divergence, gives a very inadequate idea of the quality of the instrument; and that from the high ratios in which the illuminating power increases and diminishes immediately before and after the brightest period, the duration due to the minimum divergence is all we can depend on, if the distance and state of the weather be such as to require the full power of the instrument; and as it is under circumstances like these that our beacon lights are most called for to give their aid to the benighted mariner, the duration of the brightest period becomes one of the most essential qualities to be attended to.

In comparing the lenses with the reflectors in this respect, we find (by referring to the above Table) that when the revolution is made so that the time from the appearance to the re-appearance of the light is equal in each, the duration of the brightest period is as 14 to 6, that is to say, the number of degrees of the horizon illuminated with the brightest light by the eight sides of a system of lenses, is to the number illuminated with the brightest light by the three sides of a reflector frame as 14 is to 6. Consequently we should require seven sides to our reflector frame to be equal in this respect to the lenses; and as each side must have ten reflectors to be equal in power to the lens with its additional apparatus, we should require to consume the oil of seventy ARGAND lamps to produce the same effect with the present construction of our reflectors, as that which is obtained in the lenses with the consumption of fifteen or sixteen ARGAND lamps.

Before, however, we give an opinion as to whether the lens or the reflectors is the best instrument for our lighthouses, or before we examine whether any improvement can be made in them, we must first consider the situation and purposes of the lighthouses to which they are to be applied.

On the Situations and Purposes of Lighthouses, and the Application of Lenses and Reflectors to the Duties required in them.

Lighthouses may be divided into three classes, viz.

First. Beacon or warning lights, being those which are placed to warn a vessel, and to which she can never be nearer than three or four miles.

Secondly. Guiding or leading lights, being those which are placed to guide a vessel, and to which she may approach very closely.

Thirdly. Lights which have both these duties to perform, being those which are placed to warn a vessel from a danger at a considerable distance in one direction, while they may be approached with safety in another.

In the first we require great illuminating power and a long duration of the brightest period, with a small angle of vertical divergence. In the second less illuminating power is required, but a large angle of vertical divergence, the duration of the extreme brightness not being of so much importance. In the third we require great illuminating power, a long duration of the brightest period, and a large angle of vertical divergence.

The comparative qualities of the French system of lenses, and a first class light with reflectors, such as that at Beechy Head, as to their application to the above duties, will be seen in the following statement.

	First Class light with ten Reflectors on a side 21 in. diameter.	French Lens with its additional apparatus.
Oil consumed in $3\frac{1}{4}$ hours	4 qts. 1 pt.	2 qts. $\frac{1}{2}$ pt.
Illuminating power without the instruments expressed in Argand lamps	30	10·4
Illuminating power with the instruments expressed in reflectors with Argand lamps	10	10·4
Maximum horizontal divergence per side	18° 45′	5° 15′
Minimum horizontal divergence per side	4° 46′	4° 4′
Number of sides	3	8
Degrees of horizon illuminated	56° 15′	42° 0′
Degrees of horizon with the full power	14° 18′	32° 32′
Maximum vertical divergence	31° 54′	3° 0′

From the above it will be seen that for a beacon light, when not more than 3° of vertical divergence is required, the lens is far the cheapest and best instrument, on account of its greater power and longer duration of the brightest period, although the illuminating power without the instrument is obtained at a greater consumption of oil than in the Argand burners; and that the reflectors as at present constructed are best adapted for leading or guiding lights, where a very large angle of vertical divergence is required. But in all those situations where more than 3° of vertical divergence are wanted, the lens is inapplicable, and where less than 30° are required there would be a useless waste of light in the reflectors. Now this is undoubtedly by far the most numerous class of lighthouses, we shall therefore examine with what advantage an alteration may be effected in the vertical divergences of the above instruments; but as many circumstances, such as the accuracy with which the instrument is made and fixed, atmospheric refraction, and a little variation in the height of the flame of the lamp would all tend to render variable the range of the angle of vertical divergence, we shall not attempt to employ very great exactness in this respect.

Considering these circumstances, the natural vertical divergence of the lens is the least that ought to be employed in any situation; and as the height of the flame which oil is capable of supporting to burn with advantage is nearly the same whatever may be the dimension of the lamp, the only way in which this divergence can be increased is by shortening the focal distance, to do which we must either reduce the area of each lens or the number of sides; the one would produce a loss of illuminating power, and in the others the figure of the instrument would become more distorted, and the greater obliquity with which the rays would strike and leave the lens would also be a cause of loss of light. Gas might perhaps be applied here with advantage, for by having five or six concentric burners the same intensity of light might be obtained, while the greater height of the flame would give a greater vertical diverging angle.

But in the reflector where the vertical divergence admits of so much reduction, we are enabled to increase the focal distance, and consequently to employ a large reflector, by which means a considerable saving would be effected in the consumption of oil, or using the same oil, we might obtain a much greater illuminating power. If, for example, we employed a reflector 42 inches diameter, with six inches focal distance, the illuminating power depending on the area of the end would be equal to four of the usual reflectors, but the divergences would all be reduced one half, we should consequently require six sides instead of three to illuminate the same number of degrees in the horizon, so that by reducing the vertical divergence to 15° we obtain the same illuminating power with half the consumption of oil. Instead, however, of employing twice as many reflectors to produce the same durations, we might double the diameters of the lamps and use the same number as at present.

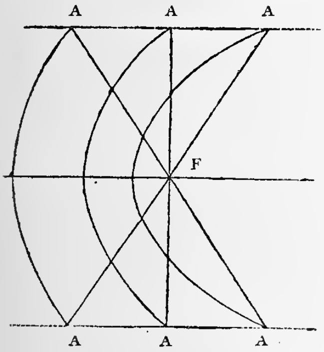
Before proceeding further with this part of the subject, it will be necessary to say a few words regarding the consumption of oil to produce a given illuminating power in lamps of different sizes and constructions. In the French lamp the consumption of oil to produce the same illuminating power as the Argand burners is nearly as $1\frac{1}{2}$ to 1, but here there are four concentric wicks, the largest of which is not more than three inches in diameter, and there would in consequence be a great intensity of heat, which would cause a considerable portion of the oil to pass off in vapour; but by increasing the diameter of a single wick, the intensity of heat would not be increased, we may therefore consider that the consumption would be exactly as the illuminating power. I am not aware of any experiments having been made to ascertain the consumption in lamps of two and three wicks, and it would perhaps be found to vary a little with their dimensions, but as we find the illuminating power to be obtained at 1 to 1 in the Argand lamp which has one wick, and as $1\frac{1}{2}$ to 1 in the French lamp which has four, it will be sufficiently near for our purpose to assume that the consumption of oil to produce a given illuminating power will be as follows:

In a lamp with one wick as 1 to 1.
In a lamp with two concentric wicks as $1\frac{1}{4}$ to 1.
In a lamp with three concentric wicks as $1\frac{1}{3}$ to 1.
In a lamp with four concentric wicks as $1\frac{1}{2}$ to 1.

And as we find that a lamp with one wick has an intensity of 1, and that the French lamp which has four wicks has an intensity of 4, it appears that the intensity increases directly as the number of wicks.

Calculating then as above, we should evidently obtain the illuminating power of ten reflectors at the cheapest rate, by employing a reflector the area of whose end should be equal to ten, with a lamp having only one wick; but as this would lead to such unwieldy instruments, it would doubtless be preferable to diminish the size of the apparatus by increasing the intensity of the lamps, and if it be within practicable limits to make reflectors four feet in diameter, and seven inches focal distance, we might by employing three such reflectors with lamps 2¼ inches in diameter, having two concentric wicks, produce the same illuminating power and the same durations as the Beechy Head lighthouse, the vertical divergence being 14°, and the consumption equal to about seventeen or eighteen ARGAND burners.

Still, however, there would be only 14° 14′ of the horizon illuminated with the brightest light, but an improvement may be made in this respect by altering the form of the reflector, for as all reflectors of the same diameter have the same illuminating power when illuminated with the same lamp, that which has the greatest angle of minimum divergence, or the longest duration of the bright period, will be that in which the distance A F is the shortest, that is to say when the focus is in the plane of the end of the reflector, and it is not improbable that having by this means less reflecting surface there would be less absorption and a better light. But in increasing the duration of the brightest period we diminish the angles of maximum divergence both horizontally and vertically. The divergences of a reflector four feet diameter and 7 inches focal distance with a lamp 2¼ inches diameter would be,



Vert. Div.	Max. Hor. Div.	Min. Hor. Div.
15° 22′	18° 34′	4° 42′

whereas if the focus were in the plane of its end they would be

Vert. Div.	Max. Hor. Div.	Min. Hor. Div.
8° 22′	10° 46′	5° 22′

But notwithstanding the great reduction in the angles of maximum divergence, I have no hesitation in considering this to be the best form of reflector when the vertical divergence required by the situation of the lighthouse will admit of its application, as it gives the longest duration to the bright period; for if the distance and state of the weather be such that it becomes a question whether the light be seen or not, the spread of weaker light will of course be invisible, and if the light be seen distinctly this weak light is not required, the regular appearance and disappearance of the light at known intervals being all that is necessary; such light, however, ought not to be used for purposes of distinction, as its duration is subject to great variations from distance or the state of the weather.

What number of degrees in the horizon it is necessary to illuminate, and more particularly what number it is necessary to illuminate with the brightest light, is a question upon which no determination appears to have been arrived at, but there can be no doubt in applying a given illuminating power (say that of ten reflectors 21 inches diameter,) to a lighthouse, that having attained a sufficient vertical divergence for its situation, the rest of the light cannot be better employed than in increasing as much as possible the length of the period during which the full power of the instrument is acting, or the number of degrees in the horizon illuminated with the full power of the instrument, however much the spread of weaker light may be diminished by so doing.

From the foregoing it would appear that the best mode of applying lenses and reflectors to lighthouses when illuminated with oil is as follows.

When less than 3° of vertical divergence is required.

The French lens is to be preferred with its additional apparatus by which 32° 82′ of the horizon is illuminated with a light equal to 10·4 reflectors 21 inches diameter, the sum of the angles of maximum horizontal divergence being 42° 0′, and the consumption of oil equal to fifteen or sixteen ARGAND burners.

For more than 3° and less than 8° vertical divergence.

Reflectors four feet diameter should be employed, if practicable, with the focus in the plane of the end. Four such reflectors with lamps $2\frac{1}{4}$ inches diameter having two concentric wicks would illuminate 21° 28′ of the horizon, with a light equal in power to 10·4 reflectors 21 inches diameter, the sum of the angles of maximum horizontal divergence being 43° 4′, and the probable consumption equal to twenty-three ARGAND burners.

Or three such reflectors with lamps $2\frac{1}{2}$ inches diameter having three concentric wicks would illuminate 17° 54′ of the horizon, with a light equal in power to fifteen reflectors 21 inches diameter, the sum of the angles of maximum horizontal divergence being 35° 54′, and the probable consumption equal to thirty ARGAND burners.

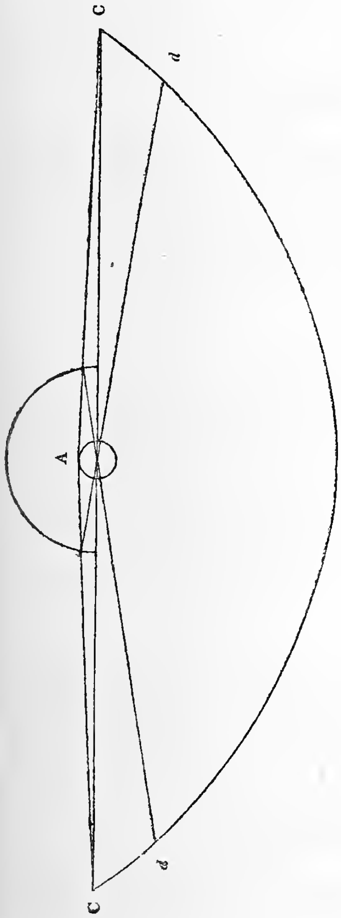
For more than 8° and less than 15° vertical divergence.

Reflectors four feet in diameter may be employed with the focal distance so arranged as to give the vertical divergence required.

And for more than 15° vertical divergence it will perhaps be found better to use smaller reflectors and more in number. This last, however, is a case which will rarely occur in lights where great illuminating power is required.

It is proper, however, to observe generally, that the lens is liable to one practical objection, viz. that depending on one light only, any accident whereby it becomes extinguished leads to total darkness, which is not so likely to happen in a system of reflectors which has several lights to depend on.

P.S. Since writing the above, it has occurred to me that a considerable increase of illuminating power would be obtained in a reflector whose focus was in the plane of its end by the application of a spherical reflector (as in the annexed figure) to intercept the rays which would otherwise be lost, and thus to return them through the light itself, and thereby increase its intensity.



The spherical reflector ought to be made about one fifth of the diameter of the parabolic reflector, and a little less than a semisphere, so as just to be without the lines A C. Here it will be seen that the light upon the part *d d* will be doubled through all its divergences, minus the absorption of light in the spherical reflector, and the area of light obstructed by it. If the spherical reflector were made of glass silvered (in which I find from experiment, that the absorption of light is about two fifths,) the increase of illuminating power obtained by this means in the parabolic reflector would be about one third or one fourth.

The effect of a coloured light might also probably be produced by a similar segment of coloured glass between the lamp and reflector.

XIV. *Researches on the Tides.—Eighth Series. On the Progress of the Diurnal Inequality Wave along the Coasts of Europe. By the Rev.* W. Whewell, *M.A. F.R.S., Fellow of Trinity College, Cambridge.*

Received June 14,—Read June 15, 1837.

In the Seventh Series of these Researches I have pointed out the laws which the diurnal inequality of the height of high water follows, and which I believe had never before been collected from the facts of observation, or indeed stated at all. I have also shown that these laws are modified so as to exhibit very remarkable differences at different places, and to give rise to some difficulty in conceiving the mechanical propagation of the tide-wave. I suggested what appeared to me a possible solution of the difficulty; but as this suggestion was founded upon the facts of a few places, and as other modes of propagation might perhaps also be conceived and adapted to the same facts, the subject remained incomplete.

I resolved therefore to attempt to trace the progress of the wave which brings the diurnal inequality, on some of the coasts on which simultaneous observations were made at my request in June 1835, and the present memoir will give some account of the conclusions to which I have been led by this investigation.

The diurnal inequality of the height of high and low water may be conceived to arise from an oscillating wave, of which the maximum height comes to each place once in twenty-four (lunar) hours; the minimum height arriving, of course, at the intermediate twelve hours. If the time of the maximum height of this wave arriving at any port coincides every day with the time of high water, the alternate high water, being at twelve hours' interval, will be affected alternately with the greatest and least heights of the diurnal wave; and the intermediate low waters will coincide with the mean height of this wave, and will not be affected at all. In this case there will be a decided diurnal inequality in the height of the high water, but no diurnal inequality in the height of low water. In like manner if the time of the maximum height of the diurnal wave coincide with the time of low water, the height of low water will be marked with a diurnal inequality, while the height of high water will exhibit no such feature. But if the diurnal wave arrive every day at a time intermediate between high and low water, it will elevate both the high water and the low water which are nearest to it, and will depress both the high and the low water which happen in the other half of the day. Hence both the high waters taken separately, and the low waters taken separately will be marked by a diurnal inequality; and this inequality will be greater for high water or for low water, according as the time of the maximum of the diurnal wave is nearer to the time of high or of low water.

Hence by taking the diurnal inequality of high and of low water at any place, and by combining these effects, we may determine the time of the arrival of the diurnal portion of the tide, and also its magnitude; and may thus separate this tide wave from the semidiurnal wave which brings *every* tide. And the time and magnitude of the diurnal wave being thus determined at a series of places along any coast, we trace its progress nearly in the same manner as we do that of the tide itself.

This is what I have done in the Tables subjoined at the end of this memoir. The heights of high water, for example, observed in June 1835 (from the 9th to the 28th) were laid down as ordinates, and a line was drawn connecting them. This line, when the diurnal inequality was manifest, was a zigzag line, such as is represented for Plymouth and for Singapore, in the figures to the Seventh Series of these Researches, and for several places in America and Europe in the Sixth Series. The line of *mean heights* was then drawn, cutting off all the diurnal inequalities. The same was done for low water; and the diurnal inequalities of the high and low waters, thus cut off, were tabulated in order. In general they were, of course, alternately two additive and two subtractive sums. These sums were laid down as ordinates at certain intervals, which intervals represent half tides (six lunar hours); and the curve drawn through the extremities of these ordinates is the diurnal wave according to its changes from day to day at the same place. The assemblage of the circumstances of such waves at different places gives the progress of the wave along the coast.

The forms of the curves thus representing the diurnal wave being determined for a sufficient number of places, it is easy to see what relations among these forms would indicate the different modes of propagation of the diurnal inequality which may be supposed. In all cases this inequality, depending as it does upon the moon's declination north and south, would increase from nothing to a maximum, and decrease to nothing again in about a fortnight; after which the inequality becomes negative, increases to a negative maximum, and decreases to nothing again in another fortnight, and so on. The epochs at which the inequality vanishes *correspond* to the times when the moon crosses the equator, but occur *after* those times at intervals varying from a few hours to four or five days, and perhaps more. It appeared to me, from the cases which I considered in my last memoir, that the epoch gradually increases as we proceed along the coast in the direction of the progress of the semidiurnal tide wave; and that this increase of epoch goes on much more rapidly than the increase of epoch for the inequalities due to the moon's parallax and declination; so that the diurnal inequality is propagated much more slowly than the other inequalities, and employs, for example, two days or more to make its way from the coasts of Spain to those of England; or, as I have before expressed it, the diurnal inequality *creeps* along the coast from day to day. Another mode in which we might explain different modifications of the diurnal inequality which the observations at

DIURNAL WAVE

different places exhibit is this: we may suppose that the diurnal wave has the same epoch as the semidiurnal wave, but that the former wave travels with *a different velocity* from the latter. The consequence of this would be that the diurnal inequality would at one port be thrown entirely upon the high water, at a place at some distance, where the diurnal wave had gained (or lost) six (lunar) hours upon the semidiurnal wave, the diurnal inequality would fall entirely upon the low water, and would not appear in the high water at all; and at intermediate places it would affect both high and low waters. If neither of these cases appear to agree with the facts, there appears to be no supposition remaining but that the diurnal wave travels irregularly, so as to affect only or principally sometimes high water, sometimes low water, sometimes both, with no regular progression. And in this case it may be conceived that the diurnal wave at some places vanishes or becomes very small, as I have shown in the Sixth Series of these Researches that the semidiurnal wave does, even in the near neighbourhood of places where it is of considerable magnitude.

The form of the curve which represents the diurnal wave at a series of ports would be modified in the following manner on these different suppositions. If the epoch of this wave changes more rapidly than that of the other inequalities, the sinuous curve which represents the diurnal wave, will have its zero ordinates, and its maximum ordinates, gradually transferred from one half day to a succeeding one, and so on, as we proceed in the direction of its propagation. Each of the sinuous *swells*, corresponding to the successive tides, may remain in the same place, but the *assemblage* of them, corresponding to a semimenstrual series of north or of south lunar declinations, will glide forwards by an alteration of the values of the maximum ordinates in these diurnal swells. On the other hand, if the epoch were the same at different places, and the velocity of the diurnal wave different from that of the semidiurnal wave, *each* diurnal swell will slide on, separating itself more and more from the corresponding high (or low) water, but undergoing no progressive change in its magnitude.

The form of the diurnal wave curve was thus determined for several series of places, and I will state the conclusions to which these series respectively led*.

First Series.—Ferrol, Port Magee (west end of Valentia Island), Doonkeghan (Mayo), Sligo, Ballynass (in the north-west of Ireland), Scrabsters (near Thurso), Buckie, Uzon (near Montrose), North Berwick (Frith of Forth), Berwick-upon-Tweed, and Clay Hole (Lincolnshire).

This series begins on the west coast of Spain, and proceeding by the west coast of Ireland to the north of Scotland, turns round the north-east point of Scotland, and goes on along the east coast of Britain.

It appears, in the first place, by the inspection of these curves, that there is no such slow propagation of the diurnal inequality as I had supposed. The inequality vanishes at all these places about the 10th and 22nd of June, the moon's declination having

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* The curves for a series of places on the coasts of the British Channel are given in Plate XIV.

vanished on the 6th and on the 18th. Thus the epoch is the same, or nearly the same, at all these places, namely, about four days, which is the value I had already assigned to it from several years' observations at Plymouth. It appears to be half a day, or perhaps a day greater than this on the east coast of Britain, but on that coast the tide has been from half a day to a day longer in arriving; so that we have here nothing to favour the opinion that the diurnal inequality is transferred at a different rate from the other inequalities of the tides, and the suggestions contained in my last memoir respecting the laws and causes of the supposed peculiar movement of the diurnal inequality must be rejected. They were founded principally on observations made at Leith, in which the diurnal inequality was very imperfectly exhibited; the rejection of them is founded on observations made at sixty-five places, taken in order along the coasts of England, Ireland, and Scotland; for I have examined the diurnal inequality at many places on those coasts, besides those for which I have drawn the curves of the first series, and I find a general agreement in the features of contiguous places.

The slow propagation of the diurnal inequality from day to day being thus rejected, we have next to consider the motion of the diurnal wave for each day, by means of our curves. It will be observed that in each curve the alternate strong ordinate lines belong to high waters, and the intermediate lines to low waters. The maxima of the *swells* of the curves, above and below the axis, are the summits of elevation and depression of the diurnal wave, and by the position of these summits with regard to high and low water, we see whether the diurnal wave arrives before or after the semidiurnal wave, and by how much. And as we know at each place how long the semidiurnal wave arrives after the moon's transit, we thus can refer the diurnal wave to the moon's transit.

In doing this we must make a distinction between the superior and inferior transit, which does not affect the semidiurnal waves. The diurnal wave *belonging* to the superior transit will (by theory) increase the tide when the moon's declination is north, and diminish the tide when the declination is south. Hence if we consider our diurnal wave as belonging to the moon's superior transit, since from June 6th to June 18th, 1835, the moon's declination is south, we must take the lower summits of the curve from June 10th to June 22nd; and the mode of proceeding is obvious.

At Ferrol, for example, it will be seen that the lower summits of the diurnal wave occur in general about two hours before the high water; and this is the high water which follows a superior transit; for on June 16th, for instance, it is the morning tide which occurs at $6^h\,42^m$, the moon's transit occurring June 16th $4^h\,53^m$ A.M. Now the tide at Ferrol, taking the average interval (the "corrected establishment" of my former Researches), follows the moon's transit at an interval of 2 hours and thirty minutes. Therefore the diurnal wave at Ferrol follows the moon's superior transit at an interval of thirty minutes.

The following is the result of the investigation in this series of places.

Comparing the diurnal wave, which brings the diurnal inequality of high and of low water, with the semidiurnal wave, which brings every tide, we find that

At Ferrol, the diurnal wave is about $2\frac{3}{4}$ hours earlier.
At Port Magee, about 4 hours earlier.
At Doonkeghan, about 2 hours earlier.
At Sligo, about $1\frac{1}{2}$ hour earlier.
At Ballynass, about 1 hour *later*.
At Scrabsters, about 4 hours earlier.
At Buckie, about 4 hours earlier.
At Uzon, about 5 hours earlier.
At North Berwick, about 3 hours earlier.
At Berwick-upon-Tweed, about 4 hours earlier.
At Clay Hole, about 2 hours earlier.

These quantities are unavoidably somewhat vague; for the place of the summit of the wave, as determined by four points of the curve, is necessarily liable to uncertainty, arising from its form not being known; besides which it is affected by accidental causes. And it may be seen by the diagrams that the distance of the summit from high or low water often differs considerably on different days. The curve at Ballynass, where the diurnal wave differs most from the general average, is very irregular. The above quantities, therefore, do not afford us any clear evidence of a *progressive* separation of the diurnal from the semidiurnal wave. And the variations which take place in the diurnal inequality at different places, may be referred to a partial acceleration or retardation of the diurnal wave. Thus on the east coast of Scotland (at Uzon, near Montrose), the diurnal wave shoots on before the semidiurnal, so as to arrive five hours sooner than that; consequently it nearly coincides with low water, and the diurnal inequality of low water is great, while that of high water almost vanishes. But at North Berwick, in the Frith of Forth, this displacement of the diurnal wave is almost corrected, the diurnal inequality affecting high and low waters almost equally.

We appear to be led by this course of investigations to the conclusion, that the differences of diurnal inequality at different places are governed by local circumstances, and do not form a progressive series. We need the less be surprised at this, having already seen (in the Sixth Series of these Researches,) that the amount of the rise of the tide differs very much even within small distances along the coast, or across the sea, and follows no progressive course of increase or decrease. And we may hence explain the cases, many of which occur, in which places having no diurnal inequality are interposed in a line of coast along which the inequality prevails: for example, at Baltimore, near the south-west point of Ireland, the diurnal inequality is not perceptible, either at high or low water, in the observations of June 1835, (which were carefully made,) although it is very conspicuous both on the west and on the south coast of the island.

Second Series.—I now proceed to consider another series of places taken on the coasts of the British Channel, namely, Ferrol, Brest, Cherbourg, Havre, on the continental coast, and Penzance, Plymouth, Bridport, Lulworth and Portsmouth on our own shores.

As before, comparing the diurnal with the semidiurnal wave, we find that

At Ferrol, the diurnal wave is $2\frac{3}{4}$ hours earlier.
At Brest, $3\frac{1}{2}$ hours earlier.
At Penzance, $2\frac{1}{4}$ hours earlier.
At Plymouth, $2\frac{1}{2}$ hours earlier.
At Cherbourg, 4 hours earlier.
At Havre, 3 hours earlier.

So far the two waves appear to go on nearly with the same velocity; but the Isle of Wight appears to produce a disturbance.

For proceeding onwards, we find that

At Bridport, the diurnal wave coincides with the semidiurnal.
At Lulworth, the diurnal wave is five hours *later*.
At Portsmouth, it is $4\frac{1}{2}$ hours *later*.

It appears, therefore, that at this point, where St. Alban's Head and the Isle of Wight interpose themselves in its course, the diurnal wave receives a check which almost reverses its position, and makes the inequalities very different at places before and after that point. Nor does this assertion rest upon any arbitrary mode of combining the facts, but it appears in the facts themselves. For instance, if we compare the high waters at Plymouth, and at Lepe, near Southampton, we shall find that their relation is contrary, the morning high waters being lower than the mean, and the evening high waters higher than the mean, from June 14 to 18 at Plymouth, and the same being the case at Lepe; although the morning tide of one place is identical with the evening tide of the other.

We may observe, that we have here a new proof of that, of which the recent examination of the tides has supplied many proofs, that we can by no means reason on the supposition that the waters of the ocean approximate to a level surface, or that an elevation at one place is necessarily shared by the surrounding seas. We may also observe, that the part of the Channel where the diurnal wave is thus held back, had already been found to be marked by peculiar tidal features; the cotidal lines turning round the promontories above mentioned as a kind of hinges, in consequence of the slow progress of the tide-wave near the shore. It is probable that the peculiarities which we thus discover to coexist in the motion of the diurnal and of the semidiurnal wave would be found to be connected, if we could analyse the hydrodynamical laws of the ocean.

The motion of the diurnal wave being thus irregular, we can account for the variety of values of the diurnal inequality at different places. We can also conceive this wave to move in a manner even more irregular than we have yet described; and some of the facts do appear to indicate this further irregularity. For example, the diurnal inequality appears sometimes, for several days, to leap from its proper tides to the alternate tides, without vanishing in the transition, as in rule it does. Now this irregular movement may be accounted for by supposing that the diurnal wave, which usually completes its oscillation and returns in twenty-four (lunar) hours, does sometimes occupy a longer or shorter time in the oscillation. If, for instance, this wave, after arriving four hours before the *superior* high water of one day, arrive again twenty hours afterwards and therefore eight hours before the superior tide of the next day, it will be only four hours after the *inferior* high water of the second day; and therefore the diurnal inequality will pass from the superior to the inferior tide by a leap. There appear to be several cases implying a change of this kind, but the subject is so complex and so laborious that I shall not now pursue it.

I venture to observe, that what has been recently done in the prosecution of our knowledge of the tides, proves the claims which it has for the same kind of attention and support which is given to other branches of astronomy. The immense labour of following out any one portion of this subject may be judged of, from a very slight attention to any part of the researches of Mr. LUBBOCK and myself upon it. In the present memoir I have selected the best-conditioned and most carefully made observations out of the general mass of those made in June 1835. I have had the curves of high and low water drawn for seventy-one such places. From among these, I have taken the nineteen places mentioned in this memoir, and have caused the diurnal wave curves to be tabulated and drawn, of which a series is represented in the diagram. These calculations and diagrams have been performed by Mr. D. Ross of the Admiralty, without whose services, placed at my disposal by the First Lord of the Admiralty, it would have been impossible for me to proceed. And even with all the assistance which can thus be given, the superintendence of the subject of the tides alone might fully employ one man of science, with great advantage to the progress of our knowledge; the subject being now so far opened that it is pretty clear in what manner research may be profitably pursued.

There is another reason why tide investigations should be made a national work in civilised maritime states. The peculiarities of the tides in each country are such as to make each shore a study by itself, and our best generalizations will be collected from results obtained in separate parts and combined. I have given less of my labour to the coasts of the United States of North America than might have been due to the interest of the tides of that part of the Atlantic, because I was obliged to limit my labour in some direction, and I hoped, and do hope, that the subject will be taken up by the government of that country as well as our own.

Suffolk Street,
June 15, 1837.

List of Places examined for the Progress of the Diurnal Wave.

Spain.

Ferrol.

Coast of France.

Brest.
St. Servan.
Cherbourg.
Havre.
D'Ouessant.

Scotland toward Thames.

4 *a*[*] Scrabsters.
6 *a* Buckie.
6 *a'* Cullen.
6 *e* Fraserburgh.
7 *e* Cove Bay.
8 *c* Uzon.
10 *a* North Berwick.
11 *a* Berwick-upon-Tweed.
14 *h* Clay Hole.
18 *f* Kessingland.

Thames to Land's End.

259 Kingsdown.
26 *e* No. 3 Tower, near Dovor.
26 *k* Fort Sutherland.
29 *l* Chichester Harbour.
30 *l* Langstone Harbour.
30 *d* Portsmouth.
30 *g* Hamble River.
31 *a* Cowes Harbour.
31 *b* Ryde.
31 *d* Sandown.
31 *g* Freshwater.
31 *i* Newtown.
32 *b* Lepe.
32 *g* Christchurch.
34 *a* Lulworth.
34 *c* Weymouth.
35 *c* Bridport Harbour.
35 *e* Lyme Cobb.
36 *a'* Weston.
36 *d* Dawlish.
37 *d* Torcross.
37 *f* Salcombe.
37 *h* Challabro.
38 *c* Bovisand.
Plymouth.
38 *f* Port Winkle.
39 *c* Polruan.
40 *d* Coverack.
41 *b* Penzance.

Coast of Ireland.

49 Arthurstown.
53 *e* Baltimore.
54 White Horse Station.
55 *b* Ballinskelligs.
55 *c* Port Magee.
56 *b* Dingle.
57 *a* Castle Gregory.
58 *a* Kilrush.
59 *c* Ballyonughan.
59 *g* Littermore.
61 *c* Innisbuffin.
62 Elly Bay.
63 *a* Doonkeghan.
63 *b* Port Tulin.
63 *f* Kilcummin.
64 *dd* Sligo.
65 *b* Trybane.
65 *e* Port Nov.
66 Guidore.
66 *c* Port Ballynass.
68 *f* Torr Head.
70 *f* Ballywater.
71 *c* St. John's Point.
72 *e* Dunany Point.
73 *c* Balbriggan.
73 *m* Howth.

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[*] These numbers and letters refer to the Coast Guard Stations as arranged in the Sixth Series of these Researches.

Tables of the Effect of the Diurnal Inequality on Low and High Water in June 1837.

[From these Tables curves were constructed, by means of which the conclusions contained in the preceding memoir respecting the diurnal wave were established.]

	Ferrol.				Ferrol.				Ferrol.		
	Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.
1835.		h m		1835.		h m		1835.		h m	
June 9 A.M.	H.	53	0	June 16 A.M.	L.	35	− 5	June 23 A.M.	H.	56	− 1
	L.	6 57	− 2		H.	6 49	− 7		L.	7 17	− 1
P.M.	H.	1 19	0	P.M.	L.	42	+ 5	P.M.	H.	1 13	+ 2
	L.	7 29	+ 2		H.	7 17	+10		L.	7 17	+ 2
10 A.M.	H.	1 40	+ 3	17 A.M.	L.	1 22	− 1	24 A.M.	H.	1 40	− 4
	L.	7 50	− 1		H.	7 43	− 7		L.	7 33	− 1
P.M.	H.	2 7	0	P.M.	L.	1 52	− 5	P.M.	H.	1 50	+ 2
	L.	8 12	− 2		H.	8 15	+ 8		L.	7 50	+ 1
11 A.M.	H.	2 33	− 3	18 A.M.	L.	2 22	− 1	25 A.M.	H.	2 13	− 1
	L.	8 38	+ 1		H.	8 50	− 6		L.	8 18	0
P.M.	H.	2 48	+ 3	P.M.	L.	3 2	+ 3	P.M.	H.	2 29	+ 2
	L.	9 12	− 2		H.	9 8	+ 6		L.	8 42	0
12 A.M.	H.	3 28	− 4	19 A.M.	L.	3 37	− 2	26 A.M.	H.	2 49	− 5
	L.	9 26	+ 4		H.	10 7	− 5		L.	8 40	0
P.M.	H.	3 47	+ 5	P.M.	L.	3 50	+ 3	P.M.	H.	3 4	+ 5
	L.	9 56	− 5		H.	10 23	+ 4		L.	9 10	0
13 A.M.	H.	4 11	− 5	20 A.M.	L.	4 33	−12	27 A.M.	H.	3 30	− 3
	L.	10 10	+ 4		H.	11 6	− 3		L.	9 9	0
P.M.	H.	4 17	+ 7	P.M.	L.	5 1	+ 1	P.M.	H.	3 46	+ 5
	L.	10 56	− 5		H.	11 16	+ 3		L.	9 24	− 1
14 A.M.	H.	5 3	− 6	21 A.M.	L.	5 30	− 2	28 A.M.	H.	4 14	− 6
	L.	10 58	+ 4		H.	11 58	− 1		L.	9 49	+ 1
P.M.	H.	5 19	+ 7	P.M.	L.	5 52	+ 1	P.M.	H.	4 14	+ 8
	L.	11 42	− 5	22 A.M.	H.	1	− 1		L.	10 30	− 1
15 A.M.	H.	5 58	− 7		L.	6 5	− 1				
	L.	11 52	+ 4	P.M.	H.	32	+ 1				
P.M.	H.	6 19	+ 9		L.	6 32	+ 1				

	Brest.				Brest.				Brest.		
June 9 A.M.	H.	1 49	− 1	June 16 A.M.	L.	1 40	− 7	June 23 A.M.	H.	1 49	+ 2
	L.	8 10	+ 1		H.	7 45	− 5		L.	8 2	0
P.M.	H.	2 17	+ 1	P.M.	L.	2 5	+ 4	P.M.	H.	2 7	0
	L.	8 38	− 1		H.	8 9	+ 6		L.	8 24	− 2
10 A.M.	H.	2 44	− 1	17 A.M.	L.	2 31	− 3	24 A.M.	H.	2 27	0
	L.	9 2	+ 1		H.	8 36	− 4		L.	8 45	+ 6
P.M.	H.	3 15	0	P.M.	L.	3 1	+ 3	P.M.	H.	2 50	+ 6
	L.	9 30	− 1		H.	9 6	+ 7		L.	9 7	0
11 A.M.	H.	3 38	+ 1	18 A.M.	L.	3 29	− 2	25 A.M.	H.	3 7	− 1
	L.	9 57	+ 2		H.	9 31	− 4		L.	9 22	0
P.M.	H.	3 59	+ 4	P.M.	L.	3 53	+ 2	P.M.	H.	3 24	+ 1
	L.	10 19	− 3		H.	10 4	+ 5		L.	9 40	− 2
12 A.M.	H.	4 25	− 2	19 A.M.	L.	4 32	− 3	26 A.M.	H.	3 42	− 3
	L.	10 45	+ 3		H.	10 36	− 3		L.	10 1	+ 1
P.M.	H.	4 51	+ 4	P.M.	L.	5 0	+ 1	P.M.	H.	4 4	+10
	L.	11 9	− 4		H.	11 8	+ 3		L.	10 23	0
13 A.M.	H.	5 12	− 4	20 A.M.	L.	5 36	− 1	27 A.M.	H.	4 25	− 2
	L.	11 33	+ 5		H.	11 40	− 2		L.	10 40	0
P.M.	H.	5 38	+ 5	P.M.	L.	6 4	+ 2	P.M.	H.	4 37	+ 2
	L.	11 58	− 4	21 A.M.	H.	2	+ 3		L.	10 55	0
14 A.M.	H.	6 2	− 4		L.	6 27	− 2	28 A.M.	H.	4 56	− 5
P.M.	L.	23	+ 5	P.M.	H.	0 32	− 2		L.	11 9	+ 1
	H.	6 28	+ 6		L.	6 56	+ 1	P.M.	H.	5 12	+ 4
15 A.M.	L.	49	− 4	22 A.M.	H.	59	0		L.	11 30	− 1
	H.	6 53	− 4		L.	7 18	− 1				
P.M.	L.	1 11	+ 3	P.M.	H.	1 28	0				
	H.	7 17	+ 6		L.	7 47	+ 1				

TABLES, &c. (Continued.)

	Penzance.				Penzance.				Penzance.		
	Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.
1835.		h m		1835.		h m		1835.		h m	
June 9 A.M.	H.	2 50	− 3	June 16 A.M.	L.	2 55	− 3	June 23 A.M.	H.	2 55	− 2
	L.	9 30	0		H.	8 30	− 7		L.	9 25	− 1
P.M.	H.	3 10	+ 2	P.M.	L.	3 25	+ 3	P.M.	H.	3 20	+ 2
	L.	9 50	+ 1		H.	9 0	+ 7		L.	9 45	0
10 A.M.	H.	3 30	− 1	17 A.M.	L.	3 50	− 3	24 A.M.	H.	3 35	− 1
	L.	10 15	0		H.	9 25	− 6		L.	10 0	+ 2
P.M.	H.	3 55	+ 2	P.M.	L.	4 5	+ 3	P.M.	H.	3 50	+ 2
	L.	10 40	0		H.	10 0	+ 9		L.	10 15	0
11 A.M.	H.	4 20	− 1	18 A.M.	L.	4 30	− 3	25 A.M.	H.	4 15	−11
	L.	11 5	0		H.	10 25	− 8		L.	10 45	− 4
P.M.	H.	4 50	+ 2	P.M.	L.	5 5	+ 2	P.M.	H.	4 35	− 4
	L.	11 40	− 1		H.	11 10	+ 6		L.	10 55	− 1
12 A.M.	H.	5 20	− 4	19 A.M.	L.	5 50	− 2	26 A.M.	H.	4 50	− 1
	L.	11 55	+ 1		H.	11 30	− 4		L.	11 10	+ 4
P.M.	H.	5 40	+ 3	P.M.	L.	6 20	+ 2	P.M.	H.	5 0	+ 5
13 A.M.	L.	30	− 2		H.	11 55	+ 4		L.	11 30	0
	H.	6 5	− 4	20 A.M.	L.	6 45	− 1	27 A.M.	H.	5 15	− 5
P.M.	L.	50	+ 3	P.M.	H.	35	− 5		L.	11 40	− 1
	H.	6 25	+ 4		L.	7 15	+ 2	P.M.	H.	5 25	+ 2
14 A.M.	L.	1 10	− 2	21 A.M.	H.	55	+ 4		L.	12 0	− 1
	H.	6 45	− 5		L.	7 35	− 2	28 A.M.	H.	5 45	− 2
P.M.	L.	1 35	+ 3	P.M.	H.	1 10	− 3	P.M.	L.	20	+ 3
	H.	7 15	+ 6		L.	8 5	+ 1		H.	6 10	+ 5
15 A.M.	L.	2 5	− 4	22 A.M.	H.	1 55	− 1				
	H.	7 35	− 7		L.	8 25	− 2				
P.M.	L.	2 25	+ 4	P.M.	H.	2 30	+ 2				
	H.	8 0	+ 6		L.	9 0	+ 3				

	Plymouth.				Plymouth.				Plymouth.		
June 9 A.M.	H.	3 55	0	June 16 A.M.	L.	3 11	− 6	June 23 A.M.	H.	3 32	− 2
	L.	10 3	0		H.	9 20	− 7		L.	9 46	− 1
P.M.	H.	4 13	+ 1	P.M.	L.	3 34	+ 3	P.M.	H.	4 0	0
	L.	10 20	0		H.	9 43	+ 7		L.	10 5	− 1
10 A.M.	H.	4 33	− 1	17 A.M.	L.	3 43	− 3	24 A.M.	H.	4 15	− 1
	L.	11 0	0		H.	10 16	− 6		L.	10 13	+ 5
P.M.	H.	5 2	+ 1	P.M.	L.	4 20	+ 3	P.M.	H.	4 23	+ 3
	L.	11 43	0		H.	10 34	+ 8		L.	10 40	0
11 A.M.	H.	5 54	− 2	18 A.M.	L.	4 34	− 3	25 A.M.	H.	4 50	0
P.M.	L.	1	+ 1		H.	10 58	− 7		L.	11 4	− 6
	H.	6 8	+ 3	P.M.	L.	5 8	+ 2	P.M.	H.	5 18	− 2
12 A.M.	L.	22	− 1		H.	11 27	+ 3		L.	11 25	− 2
	H.	6 33	− 5	19 A.M.	L.	5 36	− 2	26 A.M.	H.	5 45	− 9
P.M.	L.	39	+ 2	P.M.	H.	8	− 3		L.	11 47	+ 5
	H.	6 43	+ 3		L.	6 13	+ 1	P.M.	H.	5 54	+ 8
13 A.M.	L.	1 5	− 5	20 A.M.	H.	31	+ 2	27 A.M.	L.	25	+ 1
	H.	7 16	− 3		L.	6 40	− 1		H.	6 15	− 3
P.M.	L.	1 27	+ 4	P.M.	H.	1 5	− 1	P.M.	L.	42	− 3
	H.	7 24	+ 4		L.	7 21	+ 3		H.	6 42	+ 4
14 A.M.	L.	1 47	− 3	21 A.M.	H.	1 28	+ 4	28 A.M.	L.	55	0
	H.	8 7	− 5		L.	7 52	− 2		H.	7 15	− 4
P.M.	L.	2 13	+ 4	P.M.	H.	2 7	− 4	P.M.	L.	1 21	+ 4
	H.	8 24	+ 5		L.	8 17	+ 2		H.	7 25	+ 8
15 A.M.	L.	2 36	− 5	22 A.M.	H.	2 28	− 2				
	H.	8 47	− 6		L.	8 41	− 2				
P.M.	L.	2 54	+ 5	P.M.	H.	3 4	+ 3				
	H.	8 58	+ 7		L.	9 21	+ 4				

Tables, &c. (Continued.)

	Bridport.				Bridport.				Bridport.		
	Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.
1835.		h m		1835.		h m		1835.		h m	
June 9 A.M.	H.	4 15	0	June 16 A.M.	L.	4 0	0	June 23 A.M.	H.	4 0	− 3
	L.	10 0	+ 1		H.	10 15	− 4		L.	9 30	− 1
P.M.	H.	4 30	0	P.M.	L.	4 0	0	P.M.	H.	4 13	0
	L.	10 30	− 1		H.	10 0	+ 5		L.	10 0	+ 2
10 A.M.	H.	5 0	0	17 A.M.	L.	4 15	0	24 A.M.	H.	5 0	0
	L.	10 45	0		H.	10 45	− 3		L.	10 30	0
P.M.	H.	5 30	0	P.M.	L.	4 30	0	P.M.	H.	5 15	+ 3
	L.	11 0	0		H.	11 0	+ 7		L.	11 0	− 3
11 A.M.	H.	5 45	0	18 A.M.	L.	5 0	0	25 A.M.	H.	5 15	− 3
	L.	11 30	0		H.	11 45	− 2		L.	11 15	+ 3
P.M.	H.	6 30	+ 1	P.M.	L.	5 15	+ 1	P.M.	H.	5 45	+ 1
12 A.M.	L.	30	+ 1		H.	11 45	− 6		L.	11 45	− 4
	H.	7 0	− 3	19 A.M.	L.	5 45	− 2	26 A.M.	H.	6 0	0
P.M.	L.	45	− 1	P.M.	H.	15	+ 2		L.	12 0	+ 4
	H.	7 20	+ 2		L.	6 15	+ 1	P.M.	H.	6 15	+ 7
13 A.M.	L.	1 15	0	20 A.M.	H.	45	+ 1	27 A.M.	L.	30	− 1
	H.	8 15	− 1		L.	6 30	− 2		H.	6 30	− 8
P.M.	L.	1 45	+ 9	P.M.	H.	1 15	− 9	P.M.	L.	1 0	− 2
	H.	8 30	− 2		L.	7 0	+ 3		H.	7 0	+ 4
14 A.M.	L.	2 0	0	21 A.M.	H.	1 45	+ 2	28 A.M.	L.	1 15	0
	H.	8 40	+ 2		L.	7 15	− 1		H.	7 30	− 5
P.M.	L.	2 15	− 2	P.M.	H.	2 30	− 1	P.M.	L.	1 30	+ 3
	H.	9 0	+ 1		L.	8 0	+ 1		H.	8 0	+ 7
15 A.M.	L.	2 30	+ 1	22 A.M.	H.	3 0	0				
	H.	9 30	− 4		L.	8 45	− 2				
P.M.	L.	3 0	0	P.M.	H.	3 30	+ 2				
	H.	9 45	+ 4		L.	9 15	+ 1				

	Cherbourg.				Cherbourg.				Cherbourg.		
June 9 A.M.	H.	6 23	− 2	June 16 A.M.	L.	6 8	− 7	June 23 A.M.	L.	13	0
P.M.	L.	49	0		H.	11 52	− 5		H.	5 58	− 1
	H.	6 41	+ 2	P.M.	L.	6 30	+ 9	P.M.	L.	39	+ 1
10 A.M.	L.	1 20	0	17 A.M.	H.	12	+ 5		H.	6 18	+ 1
	H.	7 1	− 1		L.	7 1	− 8	24 A.M.	L.	52	− 2
P.M.	L.	1 40	0	P.M.	H.	38	− 3		H.	6 43	− 1
	H.	7 21	+ 2		L.	7 20	+ 9	P.M.	L.	1 25	+ 8
11 A.M.	L.	2 9	− 1	18 A.M.	H.	59	+ 4		H.	6 40	+ 5
	H.	7 52	− 2		L.	7 47	− 8	25 A.M.	L.	1 44	− 3
P.M.	L.	2 34	+ 2	P.M.	H.	1 40	− 3		H.	7 24	− 2
	H.	8 17	+ 2		L.	8 18	+ 6	P.M.	L.	1 49	+ 2
12 A.M.	L.	2 57	− 3	19 A.M.	H.	2 6	+ 2		H.	7 21	+ 2
	H.	8 43	− 5		L.	9 1	− 3	26 A.M.	L.	2 29	− 3
P.M.	L.	3 24	+ 4	P.M.	H.	2 46	+ 1		H.	7 49	− 5
	H.	9 43	+ 7		L.	9 42	+ 3	P.M.	L.	2 25	+ 8
13 A.M.	L.	3 52	− 5	20 A.M.	H.	3 19	− 1		H.	8 0	+12
	H.	9 26	− 4		L.	9 51	− 3	27 A.M.	L.	2 40	− 1
P.M.	L.	4 14	+ 7	P.M.	H.	3 51	0		H.	8 32	− 3
	H.	9 47	+ 4		L.	10 34	+ 5	P.M.	L.	3 1	+ 1
14 A.M.	L.	4 31	− 4	21 A.M.	H.	4 5	+ 1		H.	8 35	+ 3
	H.	10 21	− 4		L.	10 50	− 4	28 A.M.	L.	3 31	− 4
P.M.	L.	4 58	+ 7	P.M.	H.	4 38	0		H.	8 58	− 6
	H.	10 34	+ 4		L.	11 17	0	P.M.	L.	3 44	+ 7
15 A.M.	L.	5 17	− 6	22 A.M.	H.	5 7	− 1		H.	9 11	+ 5
	H.	11 3	− 4		L.	11 38	0				
P.M.	L.	5 40	+ 8	P.M.	H.	5 30	+ 2				
	H.	11 21	+ 4								

TABLES, &c. (Continued.)

	Havre.				Havre.				Havre.		
	Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.
1835.		h m		1835.		h m		1835.		h m	
June 9 A.M.	L.	3 0	0	June 16 A.M.	H.	1 15	+ 4	June 23 A.M.	L.	2 32	− 1
	H.	7 55	− 2		L.	8 45	− 5		H.	8 1	− 1
P.M.	L.	3 26	0	P.M.	H.	1 55	− 4	P.M.	L.	2 55	+ 1
	H.	8 22	+ 2		L.	8 53	+ 2		H.	8 32	+ 2
10 A.M.	L.	3 49	0	17 A.M.	H.	2 11	+ 3	24 A.M.	L.	3 25	− 4
	H.	8 45	− 3		L.	9 19	− 2		H.	8 51	− 5
P.M.	L.	4 15	0	P.M.	H.	3 4	− 1	P.M.	L.	3 35	+10
	H.	9 16	+ 3		L.	9 53	+ 3		H.	9 14	+ 4
11 A.M.	L.	4 54	− 2	18 A.M.	H.	3 13	+ 2	25 A.M.	L.	3 54	− 2
	H.	9 39	− 2		L.	10 10	− 4		H.	9 43	− 1
P.M.	L.	5 8	+ 2	P.M.	H.	3 53	− 1	P.M.	L.	4 18	+ 3
	H.	10 8	+ 3		L.	10 44	+ 6		H.	10 11	+ 2
12 A.M.	L.	5 38	− 3	19 A.M.	H.	4 12	0	26 A.M.	L.	4 51	− 4
	H.	10 21	− 4		L.	11 16	− 1		H.	9 53	− 4
P.M.	L.	5 59	+ 2	P.M.	H.	4 51	+ 1	P.M.	L.	5 2	+ 3
	H.	10 43	+ 3		L.	11 41	+ 1		H.	10 36	+13
13 A.M.	L.	6 33	− 3	20 A.M.	H.	5 22	− 2	27 A.M.	L.	5 21	0
	H.	11 17	− 2	P.M.	L.	8	− 1		H.	10 25	− 2
P.M.	L.	6 49	+ 5		H.	6 12	+ 2	P.M.	L.	5 41	+ 1
	H.	11 41	+ 3	21 A.M.	L.	46	+ 6		H.	10 35	+ 2
14 A.M.	L.	7 10	− 6		H.	6 15	− 2	28 A.M.	L.	6 4	− 5
P.M.	H.	7	− 4	P.M.	L.	1 19	− 4		H.	11 1	− 6
	L.	7 27	+ 5		H.	6 41	+ 1	P.M.	L.	6 23	+ 7
15 A.M.	H.	21	+ 3	22 A.M.	L.	1 44	− 1		H.	11 13	+ 4
	L.	7 47	− 7		H.	7 19	− 2				
P.M.	H.	1 0	− 4	P.M.	L.	2 1	+ 1				
	L.	8 15	+ 8		H.	7 39	+ 2				

	Lulworth.				Lulworth.				Lulworth.		
June 9 A.M.	L.	45	+ 4	June 16 A.M.	L.	6 20	−15	June 23 A.M.	H.	4 40	0
	H.	5 38	− 6		H.	11 8	+ 2	P.M.	L.	30	+ 1
P.M.	L.	1 10	− 1	P.M.	L.	6 52	+12		H.	5 20	0
	H.	6 2	+ 3		H.	11 29	− 3	24 A.M.	L.	55	− 2
10 A.M.	L.	1 45	− 2	17 A.M.	L.	7 20	−14		H.	5 45	0
	H.	6 36	− 5		H.	11 59	+ 4	P.M.	L.	1 10	+ 5
P.M.	L.	2 10	+ 4	P.M.	L.	7 45	+13		H.	5 59	− 2
	H.	6 46	+ 2	18 A.M.	H.	32	− 4	25 A.M.	L.	1 40	− 5
11 A.M.	L.	2 40	− 4		L.	8 10	−12		H.	6 30	0
	H.	7 25	0	P.M.	H.	50	+ 4	P.M.	L.	2 1	+ 4
P.M.	L.	3 7	+ 4		L.	8 30	+ 9		H.	6 59	0
	H.	7 58	+ 1	19 A.M.	H.	1 10	− 2	26 A.M.	L.	2 38	− 5
12 A.M.	L.	3 34	− 5		L.	8 52	− 4		H.	7 38	+ 6
	H.	8 20	− 1	P.M.	H.	1 40	+ 6	P.M.	L.	2 58	+ 5
P.M.	L.	3 49	+ 8		L.	9 10	+ 3		H.	7 59	+ 8
	H.	8 39	0	20 A.M.	H.	2 12	− 8	27 A.M.	L.	3 15	− 6
13 A.M.	L.	4 12	−11		L.	9 35	− 2		H.	8 17	− 2
	H.	8 52	+ 2	P.M.	H.	2 38	+ 3	P.M.	L.	3 40	+11
P.M.	L.	4 36	+ 5		L.	10 12	+ 9		H.	8 25	+ 1
	H.	9 18	0	21 A.M.	H.	2 59	− 3	28 A.M.	L.	3 55	− 9
14 A.M.	L.	5 5	− 4		L.	10 52	− 3		H.	8 40	− 1
	H.	9 55	− 6	P.M.	H.	3 35	+ 2	P.M.	L.	4 15	+ 8
P.M.	L.	5 30	+ 8		L.	11 10	+ 2		H.	8 50	+ 2
	H.	10 25	− 2	22 A.M.	H.	4 5	− 2				
15 A.M.	L.	5 53	−12		L.	11 40	0				
	H.	10 39	+ 4	P.M.	H.	4 25	+ 4				
P.M.	L.	6 5	+13		L.	11 55	− 1				
	H.	10 50	− 3								

Tables, &c. (Continued.)

	Portsmouth.				Portsmouth.				Portsmouth.		
	Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.
1835.		h m		1835.		h m		1835.		h m	
June 9 A.M.	L.	2 20	+ 4	June 16 A.M.	H.	3 10	− 2	June 23 A.M.	L.	2 15	− 1
	H.	9 55	− 2		L.	8 10	− 6		H.	9 37	0
P.M.	L.	2 55	0	P.M.	H.	3 35	+ 2	P.M.	L.	2 37	+ 2
	H.	10 15	+ 3		L.	8 35	+ 6		H.	9 57	0
10 A.M.	L.	3 15	− 2	17 A.M.	H.	4 0	− 4	24 A.M.	L.	2 57	− 3
	H.	10 45	− 3		L.	9 0	− 6		H.	10 17	+ 6
P.M.	L.	3 45	+ 2	P.M.	H.	4 35	+ 9	P.M.	L.	3 17	+ 2
	H.	11 10	+ 2		L.	9 35	+ 6		H.	10 39	− 6
11 A.M.	L.	4 11	− 1	18 A.M.	H.	5 0	− 6	25 A.M.	L.	3 39	0
	H.	11 32	0		L.	10 0	− 8		H.	10 55	− 5
P.M.	L.	4 33	+ 1	P.M.	H.	5 35	+ 4	P.M.	L.	3 55	− 3
12 A.M.	H.	11	+ 1		L.	10 38	0		H.	11 10	+ 3
	L.	5 11	− 2	19 A.M.	H.	6 2	− 3	26 A.M.	L.	4 10	− 7
P.M.	H.	34	− 2		L.	11 2	+ 1		H.	11 35	+ 1
	L.	5 34	+ 1	P.M.	H.	6 40	+ 5	P.M.	L.	4 36	+10
13 A.M.	H.	48	+ 2		L.	11 40	− 1		H.	11 57	0
	L.	5 50	0	20 A.M.	H.	7 10	− 9	27 A.M.	L.	5 2	+ 5
P.M.	H.	1 10	+ 3	P.M.	L.	10	− 1	P.M.	H.	21	− 1
	L.	6 10	− 2		H.	7 45	+ 5		L.	5 21	− 2
14 A.M.	H.	1 36	+ 1	21 A.M.	L.	45	+ 1	28 A.M.	H.	41	+ 2
	L.	6 37	0		H.	7 55	− 4		L.	5 41	− 6
P.M.	H.	2 0	0	P.M.	L.	0 55	− 1	P.M.	H.	1 0	− 2
	L.	7 0	+ 4		H.	8 22	+ 1		L.	6 0	+ 3
15 A.M.	H.	2 22	− 1	22 A.M.	L.	1 23	− 2				
	L.	7 22	− 6		H.	8 47	− 1				
P.M.	H.	2 45	+ 3	P.M.	L.	1 47	+ 1				
	L.	7 45	+ 6		H.	9 15	0				

	Port Magee.				Port Magee.				Port Magee.		
June 9 A.M.	H.	1 50	− 3	June 16 A.M.	L.	1 40	− 3	June 23 A.M.	H.	2 20	− 2
	L.	8 0	− 2		H.	8 15	− 4		L.	8 43	− 1
P.M.	H.	2 30	+ 2	P.M.	L.	2 30	+ 1	P.M.	H.	2 34	+ 1
	L.	8 40	+ 2		H.	8 30	+ 4		L.	8 53	+12
10 A.M.	H.	2 52	− 2	17 A.M.	L.	2 43	− 2	24 A.M.	H.	3 10	+ 1
	L.	9 7	− 1		H.	9 10	− 3		L.	9 23	+10
P.M.	H.	3 18	+ 2	P.M.	L.	3 18	+ 9	P.M.	H.	3 40	+ 4
	L.	9 32	0		H.	9 30	+ 1		L.	9 55	+ 3
11 A.M.	H.	3 47	− 1	18 A.M.	L.	3 40	− 1	25 A.M.	H.	4 0	− 2
	L.	9 57	+ 1		H.	10 0	− 1		L.	10 10	− 1
P.M.	H.	4 9	+ 2	P.M.	L.	4 15	+ 1	P.M.	H.	4 0	+ 1
	L.	10 20	− 1		H.	10 20	+ 1		L.	10 15	+ 5
12 A.M.	H.	4 30	− 2	19 A.M.	L.	4 35	− 2	26 A.M.	H.	4 18	− 5
	L.	10 41	+ 3		H.	11 5	− 2		L.	10 30	+ 3
P.M.	H.	4 54	+ 3	P.M.	L.	5 20	+11	P.M.	H.	4 24	+ 3
	L.	11 9	− 5		H.	11 20	+ 3		L.	10 30	− 2
13 A.M.	H.	5 19	− 3	20 A.M.	L.	5 30	+ 5	27 A.M.	H.	4 43	− 3
	L.	11 30	+ 6		H.	12 0	− 2		L.	10 58	+ 1
P.M.	H.	5 44	+ 4	P.M.	L.	6 20	0	P.M.	H.	5 17	+ 5
	L.	12 0	− 5	21 A.M.	H.	40	0		L.	11 25	− 1
14 A.M.	H.	6 10	− 2		L.	6 45	− 4	28 A.M.	H.	5 30	− 3
P.M.	L.	17	+ 6	P.M.	H.	1 10	0		L.	11 47	+ 3
	H.	6 25	+ 2		L.	7 15	+ 1	P.M.	H.	5 45	+ 2
15 A.M.	L.	33	− 5	22 A.M.	H.	1 35	0				
	H.	6 53	− 2		L.	7 45	− 1				
P.M.	L.	1 15	+ 8	P.M.	H.	1 45	+ 1				
	H.	7 10	+ 3		L.	7 50	+ 1				

TABLES, &c. (Continued.)

	Doonkeghan.				Doonkeghan.				Doonkeghan.		
	Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.
1835.		h m		1835.		h m		1835.		h m	
June 9 A.M.	H.	3 24	− 1	June 16 A.M.	L.	3 0	− 6	June 23 A.M.	H.	3 30	− 2
	L.	9 34	− 5		H.	9 20	−12		L.	9 42	− 4
P.M.	H.	3 49	+ 1	P.M.	L.	3 25	+ 9	P.M.	H.	3 45	+ 1
	L.	9 58	+ 7		H.	10 4	+14		L.	10 2	+ 4
10 A.M.	H.	4 2	− 1	17 A.M.	L.	4 20	− 7	24 A.M.	H.	4 10	− 1
	L.	10 2	− 2		H.	10 30	− 6		L.	10 15	0
P.M.	H.	4 36	+ 3	P.M.	L.	4 15	+ 6	P.M.	H.	4 27	+ 4
	L.	10 26	+ 2		H.	10 50	+ 9		L.	10 37	0
11 A.M.	H.	4 55	− 4	18 A.M.	L.	5 11	− 5	25 A.M.	H.	4 38	−11
	L.	11 1	− 3		H.	11 40	−12		L.	10 45	− 2
P.M.	H.	5 29	+ 5	P.M.	L.	5 54	+ 5	P.M.	H.	5 0	+ 1
	L.	11 29	+ 1	19 A.M.	H.	10	+ 8		L.	11 9	+ 1
12 A.M.	H.	5 50	− 6		L.	6 17	− 6	26 A.M.	H.	5 17	− 2
P.M.	L.	11 43	+ 2	P M.	H.	28	− 7		L.	11 5	+ 3
	H.	6 3	+ 9		L.	6 30	+ 9	P.M.	H.	5 27	+ 6
13 A.M.	L.	15	− 3	20 A.M.	H.	35	+ 9		L.	11 40	− 3
	H.	6 26	− 8		L.	7 15	− 6	27 A.M.	H.	5 51	− 9
P.M.	L.	18	+ 4	P.M.	H.	1 50	− 6		L.	11 40	+ 1
	H.	6 46	+ 9		L.	8 3	+ 5	P.M.	H.	5 52	+ 9
14 A.M.	L.	1 14	− 6	21 A.M.	H.	2 5	+ 3	28 A.M.	L.	20	− 2
	H.	7 40	− 9		L.	8 10	− 6		H.	6 20	− 8
P.M.	L.	1 11	+ 5	P.M.	H.	2 20	− 3	P.M.	L.	20	+ 9
	H.	7 40	+12		L.	8 35	+ 5		H.	6 27	+ 7
15 A.M.	L.	1 57	− 6	22 A.M.	H.	2 45	+ 3				
	H.	8 16	−12		L.	9 0	+ 6				
P.M.	L.	2 12	+ 6	P.M.	H.	3 30	+ 1				
	H.	8 32	+13		L.	9 20	+ 6				

	Sligo.				Sligo.				Sligo.		
June 9 A.M.	H.	3 45	+ 1	June 16 A.M.	L.	3 20	− 6	June 23 A.M.	H.	3 55	0
	L.	10 0	− 6		H.	9 40	−15		L.	10 10	− 6
P.M.	H.	4 20	− 1	P.M.	L.	3 25	+ 4	P.M.	H.	4 20	0
	L.	10 30	+ 6		H.	9 50	+15		L.	10 30	+ 4
10 A.M.	H.	4 40	0	17 A.M.	L.	4 10	− 3	24 A.M.	H.	4 35	+ 6
	L.	10 45	− 3		H.	9 55	−15		L.	10 40	0
P.M.	H.	5 15	+ 2	P.M.	L.	4 20	+ 2	P.M.	H.	4 50	+ 9
	L.	11 25	+ 4		H.	10 40	+10		L.	11 0	0
11 A.M.	H.	5 40	− 2	18 A.M.	L.	5 0	− 2	25 A.M.	H.	5 10	− 4
	L.	11 40	0		H.	11 30	−10		L.	11 20	0
P.M.	H.	6 0	+ 4	P.M.	L.	5 50	+ 5	P.M.	H.	5 30	+ 3
12 A.M.	L.	10	0	19 A.M.	H.	10	+13		L.	11 45	0
	H.	6 20	− 5		L.	6 30	− 5	26 A.M.	H.	6 0	− 2
P.M.	L.	30	0	P.M.	H.	1 0	−11	P.M.	L.	10	0
	H.	6 45	+ 7		L.	7 10	+ 8		H.	6 20	+ 7
13 A.M.	L.	1 0	0	20 A.M.	H.	1 30	+13	27 A.M.	L.	30	0
	H.	7 15	− 8		L.	7 50	− 4		H.	6 35	− 9
P.M.	L.	1 15	+ 3	P.M.	H.	2 0	−11	P.M.	L.	35	0
	H.	7 30	+ 9		L.	8 0	+ 2		H.	6 45	+19
14 A.M.	L.	1 40	− 6	21 A.M.	H.	2 15	+ 9	28 A.M.	L.	1 0	0
	H.	8 0	−10		L.	8 25	− 5		H.	7 10	− 8
P.M.	L.	2 0	+ 3	P.M.	H.	2 35	− 6	P.M.	L.	1 15	0
	H.	8 15	+11		L.	8 50	+ 1		H.	7 30	+ 7
15 A.M.	L.	2 15	− 3	22 A.M.	H.	3 10	+ 9				
	H.	8 45	−13		L.	9 25	− 1				
P.M.	L.	2 30	+ 3	P.M.	H.	3 30	0				
	H.	9 0	+14		L.	9 40	+ 4				

Tables, &c. (Continued.)

	Port Ballynass.				Port Ballynass.				Port Ballynass.		
	Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.
1835.		h m		1835.		h m		1835.		h m	
June 9 A.M.	H.	4 30	− 1	June 16 A.M.	L.	3 54	0	June 23 A.M.	H.	4 12	0
	L.	11 10	− 2		H.	10 0	−12		L.	10 18	0
P.M.	H.	4 45	+ 1	P.M.	L.	4 30	0	P.M.	H.	4 25	0
	L.	11 15	+ 2		H.	10 30	+15		L.	10 30	+ 1
10 A.M.	H.	5 0	− 3	17 A.M.	L.	4 43	+ 1	24 A.M.	H.	4 37	+ 3
	L.	11 55	− 1		H.	10 58	− 5		L.	10 45	− 1
P.M.	H.	5 40	+ 2	P.M.	L.	4 50	− 2	P.M.	H.	4 50	− 1
11 A.M.	L.	10	+ 1		H.	11 25	+ 3		L.	11 2	+ 3
	H.	6 5	0	18 A.M.	L.	5 30	0	25 A.M.	H.	5 20	+ 2
P.M.	L.	56	0		H.	11 50	− 5		L.	11 30	− 4
	H.	6 20	+ 1	P.M.	L.	6 2	0	P.M.	H.	5 35	− 5
12 A.M.	L.	54	0	19 A.M.	H.	30	+10		L.	11 55	0
	H.	6 45	− 2		L.	6 20	0	26 A.M.	H.	6 10	+15
P.M.	L.	1 20	0	P.M.	H.	56	− 4	P.M.	L.	13	− 1
	H.	6 57	+ 4		L.	6 50	0		H.	6 20	+ 1
13 A.M.	L.	1 42	+ 1	20 A.M.	H.	1 18	+ 7	27 A.M.	L.	30	+ 3
	H.	7 35	− 6		L.	6 35	+ 1		H.	6 45	− 1
P.M.	L.	2 1	− 1	P.M.	H.	1 55	− 5	P.M.	L.	54	0
	H.	7 50	+ 7		L.	7 40	− 2		H.	6 40	+ 3
14 A.M.	L.	2 25	− 1	21 A.M.	H.	2 15	+ 4	28 A.M.	L.	1 10	0
	H.	8 10	− 4		L.	8 0	+ 1		H.	7 20	− 4
P.M.	L.	2 35	0	P.M.	H.	2 45	− 3	P.M.	L.	1 18	0
	H.	8 34	+ 5		L.	8 34	− 1		H.	7 25	+ 5
15 A.M.	L.	3 0	0	22 A.M.	H.	3 5	+ 3				
	H.	9 2	− 4		L.	9 25	+ 2				
P.M.	L.	3 1	− 1	P.M.	H.	3 36	− 1				
	H.	9 25	+ 7		L.	9 40	0				

	Scrabsters.				Scrabsters.				Scrabsters.		
June 9 A.M.	L.	20	+10	June 16 A.M.	H.	15	+ 9	June 23 A.M.	L.	2	+ 1
	H.	6 45	− 0		L.	6 45	− 9		H.	6 35	+ 1
P.M.	L.	45	− 6	P.M.	H.	1 0	− 6	P.M.	L.	40	− 2
	H.	7 0	0		L.	7 0	+14		H.	6 50	− 2
10 A.M.	L.	1 10	+ 5	17 A.M.	H.	1 20	+10	24 A.M.	L.	1 15	+ 1
	H.	7 30	0		L.	7 40	−15		H.	7 25	+ 1
P.M.	L.	1 45	− 4	P.M.	H.	1 50	−13	P.M.	L.	1 40	0
	H.	8 0	0		L.	7 50	+ 4		H.	7 45	+ 6
11 A.M.	L.	2 0	+ 8	18 A.M.	H.	2 10	+ 9	25 A.M.	L.	2 0	0
	H.	8 15	− 1		L.	8 40	− 5		H.	8 0	− 3
P.M.	L.	2 25	− 3	P.M.	H.	2 50	− 4	P.M.	L.	2 15	0
	H.	8 40	+ 1		L.	9 0	+ 6		H.	8 25	+ 1
12 A.M.	L.	2 55	− 6	19 A.M.	H.	3 20	+ 4	26 A.M.	L.	2 35	− 1
	H.	9 20	− 2		L.	9 20	−10		H.	8 40	− 3
P.M.	L.	3 10	+ 3	P.M.	H.	3 45	− 5	P.M.	L.	2 50	+ 4
	H.	9 45	+ 5		L.	9 45	+11		H.	9 0	+ 7
13 A.M.	L.	3 45	− 3	20 A.M.	H.	4 10	+ 8	27 A.M.	L.	3 0	− 2
	H.	10 10	− 2		L.	10 30	− 7		H.	9 15	− 4
P.M.	L.	4 15	+ 3	P.M.	H.	4 45	− 6	P.M.	L.	3 15	− 1
	H.	10 30	+ 1		L.	10 50	+ 3		H.	9 30	+ 3
14 A.M.	L.	4 40	− 4	21 A.M.	H.	5 10	0	28 A.M.	L.	3 38	− 5
	H.	11 0	− 5		L.	11 20	− 5		H.	9 45	− 4
P.M.	L.	5 10	+ 9	P.M.	H.	5 35	+ 2	P.M.	L.	3 55	+ 4
	H.	11 20	+ 9		L.	11 45	+ 9		H.	10 3	+ 6
15 A.M.	L.	5 35	−11	22 A.M.	H.	5 50	0				
	H.	11 50	− 9		L.	11 55	− 1				
P.M.	L.	6 0	+ 9	P.M.	H.	6 15	0				

TABLES, &c. (Continued.)

	Buckie.				Buckie.				Buckie.		
	Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.
1835.		h m		1835.		h m		1835.		h m	
June 9 A.M.	L.	4 10	+3	June 16 A.M.	H.	3 30	+8	June 23 A.M.	L.	4 0	+5
	H.	10 30	0		L.	10 0	−7		H.	10 0	−2
P.M.	L.	4 30	−3	P.M.	H.	4 5	−5	P.M.	L.	4 20	−1
	H.	11 0	+1		L.	10 15	+17		H.	10 20	−2
10 A.M.	L.	5 0	+4	17 A.M.	H.	4 0	+9	24 A.M.	L.	4 50	−1
	H.	11 15	0		L.	11 10	−10		H.	10 35	0
P.M.	L.	5 20	−2	P.M.	H.	5 10	−11	P.M.	L.	5 0	+2
	H.	11 45	0		L.	11 20	+10		H.	11 0	0
11 A.M.	L.	6 0	0	18 A.M.	H.	5 25	+1	25 A.M.	L.	5 30	+2
P.M.	H.	5	0	P.M.	L.	10	−6		H.	11 10	−1
	L.	6 25	0		H.	6 10	−1	P.M.	L.	5 45	0
12 A.M.	H.	20	+1	19 A.M.	L.	25	+12		H.	11 40	+4
	L.	6 40	−3		H.	6 15	+1	26 A.M.	L.	6 10	−2
P.M.	H.	40	−1	P.M.	L.	1 0	−9	P.M.	H.	10	−2
	L.	7 0	+5		H.	7 15	−5		L.	6 20	+2
13 A.M.	H.	1 5	0	20 A.M.	L.	1 30	+6	27 A.M.	H.	30	+3
	L.	7 20	−5		H.	7 35	+7		L.	6 40	0
P.M.	H.	1 30	+1	P.M.	L.	2 0	−7	P.M.	H.	35	−3
	L.	7 40	+9		H.	8 0	−6		L.	7 0	0
14 A.M.	H.	1 50	+2	21 A.M.	L.	2 20	+3	28 A.M.	H.	1 10	+3
	L.	8 20	−9		H.	8 15	0		L.	7 20	−6
P.M.	H.	2 20	−4	P.M.	L.	2 45	−5	P.M.	H.	1 40	−2
	L.	8 35	+8		H.	8 35	+1		L.	7 40	+4
15 A.M.	H.	2 35	+4	22 A.M.	L.	3 10	+5				
	L.	9 5	−10		H.	9 0	−1				
P.M.	H.	3 15	−5	P.M.	L.	3 20	+4				
	L.	9 15	+10		H.	9 45	+4				

	North Berwick.				North Berwick.				North Berwick.		
June 9 A.M.	H.	23	0	June 16 A.M.	H.	6 16	+3	June 23 A.M.	H.	26	+4
	L.	6 36	+7	P.M.	L.	23	−10		L.	6 40	+4
P.M.	H.	48	+1		H.	6 45	−5	P.M.	H.	47	−3
	L.	7 0	−4	17 A.M.	L.	53	+16		L.	6 54	−4
10 A.M.	H.	1 11	0		H.	7 10	+9	24 A.M.	H.	1 12	0
	L.	7 21	+2	P.M.	L.	1 22	−11		L.	7 15	−3
P.M.	H.	1 40	+1		H.	7 46	−6	P.M.	H.	1 29	0
	L.	7 43	−2	18 A.M.	L.	1 54	+8		L.	7 33	0
11 A.M.	H.	2 11	−1		H.	8 10	0	25 A.M.	H.	1 50	0
	L.	8 15	+1	P.M.	L.	2 24	−10		L.	7 54	+1
P.M.	H.	2 34	+2		H.	8 45	+6	P.M.	H.	2 0	+1
	L.	8 37	0	19 A.M.	L.	2 58	+15		L.	8 5	+1
12 A.M.	H.	3 0	−3		H.	8 58	+3	26 A.M.	H.	2 19	0
	L.	9 4	−3	P.M.	L.	3 24	−6		L.	8 24	−1
P.M.	H.	3 24	+2		H.	9 44	−5	P.M.	H.	2 41	−1
	L.	9 32	+4	20 A.M.	L.	3 50	+6		L.	8 44	0
13 A.M.	H.	3 46	−2		H.	10 16	+4	27 A.M.	H.	3 4	+1
	L.	9 56	−6	P.M.	L.	4 22	−5		L.	9 5	−3
P.M.	H.	4 12	+3		H.	10 40	−2	P.M.	H.	3 20	0
	L.	10 16	+8	21 A.M.	L.	5 0	+6		L.	9 17	+5
14 A.M.	H.	4 34	0		H.	11 10	−3	28 A.M.	H.	3 36	0
	L.	10 41	−10	P.M.	L.	5 16	−9		L.	9 38	−9
P.M.	H.	4 57	0		H.	11 32	+3	P.M.	H.	3 58	0
	L.	11 4	+10	22 A.M.	L.	5 37	+5		L.	10 0	+4
15 A.M.	H.	5 24	0		H.	11 57	−3				
	L.	11 32	−9	P.M.	L.	6 6	−2				
P.M.	H.	5 50	−2								
	L.	11 56	+10								

Tables, &c. (Continued.)

	Berwick-upon-Tweed.				Berwick-upon-Tweed.				Berwick-upon-Tweed.		
	Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.
1835.		h m		1835.		h m		1835.		h m	
June 9 A.M.	H.	27	− 3	June 16 A.M.	H.	6 25	+ 5	June 23 A.M.	H.	56	+ 5
	L.	6 37	+ 7	P.M.	L.	35	−10		L.	7 0	+ 4
P.M.	H.	49	+ 2		H.	7 3	− 4	P.M.	H.	1 4	− 4
	L.	7 5	− 5	17 A.M.	L.	1 8	+14		L.	7 4	− 3
10 A.M.	H.	1 25	0		H.	7 33	+ 8	24 A.M.	H.	1 30	+ 2
	L.	7 32	+ 3	P.M.	L.	1 36	−11		L.	7 40	0
P.M.	H.	1 44	0		H.	8 10	− 6	P.M.	H.	1 54	− 1
	L.	8 4	− 3	18 A.M.	L.	2 10	+11		L.	8 0	+ 1
11 A.M.	H.	2 14	0		H.	8 30	0	25 A.M.	H.	2 17	+ 1
	L.	8 26	+ 1	P.M.	L.	2 35	− 8		L.	8 20	+ 1
P.M.	H.	2 30	0		H.	9 5	+ 4	P.M.	H.	2 23	− 1
	L.	8 45	− 1	19 A.M.	L.	2 37	+ 2		L.	8 30	+ 2
12 A.M.	H.	2 47	− 1		H.	9 15	+ 7	26 A.M.	H.	2 29	+ 2
	L.	9 15	− 2	P.M.	L.	3 4	− 3		L.	8 55	− 3
P.M.	H.	3 15	+ 1		H.	9 40	− 4	P.M.	H.	2 45	− 3
	L.	9 30	+ 3	20 A.M.	L.	4 10	+ 6		L.	9 5	+ 2
13 A.M.	H.	3 30	− 1		H.	10 7	+ 4	27 A.M.	H.	3 8	+ 3
	L.	10 7	− 5	P.M.	L.	4 35	− 6		L.	9 20	− 3
P.M.	H.	4 20	+ 2		H.	10 45	− 3	P.M.	H.	3 28	− 2
	L.	10 28	+ 9	21 A.M.	L.	4 48	+ 5		L.	9 35	+ 5
14 A.M.	H.	4 50	0		H.	11 15	− 1	28 A.M.	H.	4 0	0
	L.	10 45	− 7	P.M.	L.	5 5	−10		L.	9 46	− 7
P.M.	H.	5 19	0		H.	11 55	+ 5	P.M.	H.	4 20	0
	L.	11 2	+ 6	22 A.M.	L.	5 47	+ 6		L.	10 5	+ 6
15 A.M.	H.	5 33	+ 2	P.M.	H.	25	− 3				
	L.	11 32	− 7		L.	6 20	− 3				
P.M.	H.	6 5	− 4								
	L.	12 0	+12								

	Clay Hole.				Clay Hole.				Clay Hole.		
June 9 A.M.	H.	4 50	− 4	June 16 A.M.	L.	4 13	+ 8	June 23 A.M.	H.	4 25	+ 6
	L.	10 55	+ 7		H.	10 10	+ 8		L.	10 57	+10
P.M.	H.	5 0	+ 1	P.M.	L.	5 5	− 8	P.M.	H.	4 50	− 3
	L.	12 0	− 5		H.	11 0	−15		L.	10 50	− 1
10 A.M.	H.	5 33	0	17 A.M.	L.	5 0	+ 7	24 A.M.	H.	5 5	0
P.M.	L.	8	+ 6		H.	10 50	+18		L.	11 35	+ 1
	H.	5 53	+ 1	P.M.	L.	6 5	− 5	P.M.	H.	5 25	0
11 A.M.	L.	48	− 3	18 A.M.	H.	5	−16		L.	11 45	− 2
	H.	6 18	− 1		L	5 40	+ 9	25 A.M.	H.	5 55	− 3
P.M.	L.	55	+ 2	P.M.	H.	1	+ 3	P.M.	L.	1	− 1
	H.	6 46	+ 1		L.	6 35	−10		H.	6 10	+14
12 A.M.	L.	1 33	− 2	19 A.M.	H.	1 5	− 4	26 A.M.	L.	35	+15
	H.	7 3	0		L.	6 40	+27		H.	6 20	− 2
P.M.	L.	1 50	0	P.M.	H.	50	+ 7	P.M.	L.	43	− 8
	H.	7 40	− 1		L.	7 35	− 4		H.	6 45	− 1
13 A.M.	L.	2 20	0	20 A.M.	H.	2 15	−20	27 A.M.	L.	56	+16
	H.	7 53	+ 1		L.	7 45	+ 4		H.	6 59	+ 4
P.M.	L.	2 45	0	P.M.	H.	2 20	+ 4	P.M.	L.	1 35	+ 2
	H.	8 23	0		L.	8 48	− 6		H.	7 35	− 2
14 A.M.	L.	3 0	+ 6	21 A.M.	H.	2 55	− 4	28 A.M.	L.	1 35	− 4
	H.	8 45	+ 1		L.	8 55	+ 6		H.	7 55	0
P.M.	L.	3 35	−10	P.M.	H.	3 2	− 2	P.M.	L.	2 13	− 8
	H.	9 23	− 4		L.	9 37	−11		H.	8 13	0
15 A.M.	L.	3 30	+ 8	22 A.M.	H.	3 47	+ 5				
	H.	9 30	+ 6		L.	10 7	+ 6				
P.M.	L.	4 22	− 7	P.M.	H.	4 7	− 8				
	H.	10 15	− 9		L.	10 17	− 3				

Tables, &c. (Continued.)

	Uzon.				Uzon.				Uzon.		
	Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.		Tide.	Time.	Diurnal Inequal.
1835.		h m		1835.		h m		1835.		h m	
June 9 A.M.	H.	3	0	June 16 A.M.	H.	5 45	0	June 23 A.M.	L.	5 59	+ 3
	L.	6 5	+ 5		L.	11 48	− 4	P.M.	H.	22	+ 6
P.M.	H.	17	0	P.M.	H.	6 32	− 7		L.	6 35	− 2
	L.	6 26	− 5	17 A.M.	L.	6	+13	24 A.M.	H.	33	− 1
10 A.M.	H.	45	0		H.	6 38	+ 9		L.	6 47	0
	L.	6 52	+ 3	P.M.	L.	57	−12	P.M.	H.	43	− 2
P.M.	H.	1 6	+ 1		H.	7 35	− 8		L.	7 6	+ 2
	L.	7 16	− 1	18 A.M.	L.	1 12	+ 8	25 A.M.	H.	1 15	+ 1
11 A.M.	H.	1 25	− 1		H.	7 47	+ 1		L.	7 26	− 2
	L.	7 39	+ 1	P.M.	L.	1 46	− 8	P.M.	H.	1 25	− 1
P.M.	H.	1 49	+ 2		H.	8 40	+ 5		L.	7 44	+ 2
	L.	8 9	+ 1	19 A.M.	L.	2 24	+16	26 A.M.	H.	1 46	0
12 A.M.	H.	2 15	− 2		H.	8 43	+ 5		L.	8 5	− 2
	L.	8 31	− 3	P.M.	L.	3 10	− 7	P.M.	H.	2 10	− 2
P.M.	H.	2 44	+ 1		H.	9 25	− 5		L.	8 14	+ 3
	L.	8 52	+ 4	20 A.M.	L.	3 29	+ 7	27 A.M.	H.	2 26	+ 2
13 A.M.	H.	3 3	− 1		H.	9 27	+ 2		L.	8 33	− 4
	L.	9 22	− 6	P.M.	L.	3 55	− 6	P.M.	H.	2 45	− 1
P.M.	H.	3 36	+ 1		H.	10 40	− 2		L.	8 47	+ 4
	L.	9 46	+ 9	21 A.M.	L.	4 23	+ 5	28 A.M.	H.	2 57	0
14 A.M.	H.	3 55	− 2		H.	10 25	0		L.	9 14	− 7
	L.	10 11	−10	P.M.	L.	4 49	− 9	P.M.	H.	3 31	0
P.M.	H.	4 37	− 3		H.	11 42	+ 5		L.	9 21	+ 9
	L.	10 38	+ 9	22 A.M.	L.	5 18	+ 6				
15 A.M.	H.	4 48	− 3		H.	11 30	− 4				
	L.	10 55	−11	P.M.	L.	5 40	− 2				
P.M.	H.	5 30	+ 6		H.	11 50	+ 2				
	L.	11 24	+ 8								

XV. *On the Connexion between the Phenomena of the Absorption of Light, and the Colours of thin Plates. By Sir* David Brewster, *K.H. LL.D. F.R.S.*

Received May 9,—Read May 11, 1837.

Since the phenomena of the absorption of light by coloured media began to be studied with attention, various philosophers have regarded them as inexplicable by the theory of the colours of thin plates, and have consequently regarded Sir Isaac Newton's theory of the colours of natural bodies as either defective in generality, or altogether unfounded. Mr. Delaval[*] was the first person who brought an extensive series of experiments to bear upon this subject. Dr. Thomas Young[†] considered it "impossible to suppose the production of natural colours perfectly identical with those of thin plates," unless the refractive density of the particles of colouring bodies was at least twenty or thirty times as great as that of glass or water, which he considered as "difficult to believe with respect to any of their arrangements constituting the diversities of material substances." Sir John Herschel has expressed a still more decided opinion upon this subject. He regards "the speculations of Newton on the colours of natural bodies" as only "a premature generalization," and "limited to a comparatively narrow range; while the phenomena of absorption, to which he considers the great majority of natural colours as referable, have always appeared to him to constitute a branch of photology *sui generis*[‡]."

The general opinion advanced by these three philosophers I have long entertained[§]; and with the view of supporting them I have analysed a great variety of colours which are exhibited by the juices of plants. In a paper "On the Colours of Natural Bodies[||]," I have shown that the *green* colour of plants, the most prevalent of all the colours of natural bodies, in place of being a *green of the third order*, as Newton and his commentators assert, is a colour of no order whatever, and having in its composition no relation at all to the colours of thin plates.

In arriving at these conclusions, however, and drawing a distinct line between the phenomena of absorption and those of thin plates, two classes of facts are compared under very different circumstances. In the one case philosophers have studied *in cumulo* the result of the successive actions of an infinite number of the colorific particles upon the intromitted light, whereas in the other case they have observed only the

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[*] Manchester Memoirs, vol. ii. p. 131.

[†] Ed. Nat. Phil. vol. i. p. 469, 481. and vol. ii. p. 638.

[‡] London and Edinburgh Philosophical Magazine, December 1833, vol. iii. p. 401. See also his Treatise on Light, Encyc. Metrop. p. 580, 581.

[§] Life of Newton, chap. vii.

[||] Edinburgh Transactions, vol. xii.

colour of a single particle, whose thickness is equal to that of the films of air, water, glass and mica submitted to experiment. The impracticability of combining a number of such films, and studying their united action upon light, was doubtless the reason which prevented natural philosophers from bringing the two series of facts under the same conditions. Sir Isaac Newton, indeed, had spoken so confidently of the result of such a combination, as to discourage any attempts to effect it; and it is a singular fact that his successors have never called in question his bold though ingenious assumption. "If a thinned or plated body," says he, "which being of an even thickness, appears all over of an uniform colour, shall be slit into threads or broken into fragments of the same thickness with the plate, I see no reason why every thread or fragment should not keep its colour, and by consequence why a heap of those threads or fragments should not constitute a mass or powder of the same colour which the plate exhibited before it was broken. And the parts of all natural bodies being like so many fragments of a plate, must on the same grounds exhibit the same colours."

This remarkable opinion I have often been desirous to submit to the test of direct experiment, in the conviction that the result would be different from what is here stated; but I have been baffled in every attempt to make such an experiment; and had not accidental circumstances placed in my hands two substances, in which thin plates were combined nearly in the very manner which I wished, and which I believe had never before been submitted to examination, the problem might have remained long without a solution.

The first of these substances to which my attention was called, is the remarkable nacreous body which Mr. Horner has described in the last volume of the Transactions, and whose singular optical properties I have explained in a letter which accompanies his paper. This substance consists of laminæ of considerable transparency, separated by extremely thin films, which exhibit in the most brilliant manner the colours of thin plates.

In order to compare the effect produced by a number of such films with that of a single film, we must either analyse the light reflected and transmitted by a single film by means of a fine prism placed in front of a telescope, or examine the prismatic spectrum produced by such an apparatus when it is reflected or transmitted by the film in question. When we thus examine the reflected tints of the three first orders of colours, we find them to consist of that part of the spectrum which gives the predominating colour of the tint mixed with the rays on each side of it. The reflected *green* of the *third* order, for example, consists of the green part of the spectrum, bounded on one side with some blue, and on the other side with some yellow rays, all the rest of the spectrum being wanting, having passed, as it were, into the transmitted beam. In analysing, therefore, the transmitted beam, its spectrum is found to consist only of the violet and blue, and the orange and red spaces, a dark band corresponding to the reflected spectrum separating it into two parts. In the higher orders of colours the reflected spectrum consists of two or more portions separated by perfectly dark bands, while the transmitted light exhibits analogous bands, which are much less dark in consequence of the tint being diluted with a portion of white light. The coloured bands of the reflected spectrum occupy the same place among the fixed lines of the spectrum as the dark bands of the transmitted one; and if the two spectra were superposed they would form a perfect spectrum, whose rays when united would form white light. Hence the reflected and the transmitted tints are complementary to each other.

When this analysis is made with a highly magnified spectrum, the numerous lines of which are distinctly seen, it forms one of the most splendid experiments in optics. The spectrum is crossed throughout its whole extent with alternate dark and coloured bands, increasing in number and diminishing in magnitude with the thickness of the plate by which the tint is produced.

If we use a thin film of mica, of such a thickness as polarizes the *white* of the first order, the transmitted spectrum will be crossed by upwards of three hundred dark and three hundred luminous bands, thirty-four of each being included between the lines C and D of Fraunhofer, a space less than one tenth of the whole spectrum.

When we use polarized light, and interpose a doubly refracting plate, and subsequently analyse the transmitted beam, the spectrum is crossed with an analogous series of bands, which are still more splendid and more perfect than those given by a singly refracting film. The bands in the complementary spectra are equally and perfectly dark; and when the tints are pure as in calcareous spar, the colours are nearly identical with those of thin plates. Through the natural faces of a rhomb of calcareous spar about one sixth of an inch thick, I observed in the space C D above mentioned hundreds of the most minute lines almost as sharp and black as those in the solar spectrum.

In the phenomena of periodical colours which we have now described, there are three peculiarities which demand our attention. 1. The dark lines change their place by inclining the plate which produces them. 2. Two or more lines never coalesce into one, and one line of the series is never seen without all the rest being equally visible. 3. The colours of the luminous bands in the complementary spectra are the same as those of the original spectrum when the thin plate is perfectly colourless. In the case of polarized tints this similarity is not general.

In order to obtain a correct idea of the phenomena of absorption, I shall describe those which are exhibited by a solid, a fluid, and a gaseous body,—by the common smalt blue glass, by the green sap of vegetables, and by nitrous acid gas.

Dr. Young has described the smalt blue glass as dividing the spectrum "into *seven* distinct portions." I have given in the Edinburgh Transactions* rude coloured drawings of the effect it produces on the spectrum, and Sir John Herschel† has represented its action in a different manner. Excepting in the single circumstance of the spectrum being divided into bands, there appears no analogy whatever between this

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[*] Vol. ix. p. 439. Plate XXVII.

[†] Ibid. p. 449. Plate XXVIII.

phenomenon and those of thin plates. The bands diminish in number as the thickness of the plate increases, and their colour suffers no other change by inclining the plate but that which arises from the small increase of thickness which the ray traverses. There is one remarkable point of difference between the two classes of phenomena which requires to be specially attended to. The *colours of some of the luminous bands are not the same as those of the spectrum,* and therefore the glass has removed certain colours while it has left others of exactly the same refrangibility. The *green,* for example, is changed into *yellow* by the removal of blue rays, and in certain glasses a band, *almost white,* is produced. The colours thus removed are said to be absorbed; and by an extensive series of experiments with such absorbing substances I have been able to insulate white light in the spectrum, which no prism can decompose, and to establish the existence of three equal and superposed spectra of red yellow and blue light.

Analogous phenomena are exhibited in an alcoholic solution of the colouring matter of the green leaves of vegetables. The spectrum which it forms consists of *six* luminous bands separated by *five* dark ones*, and the phenomena have the same character as those of the blue glass.

When the spectrum is viewed through nitrous acid gas the phenomena are still more remarkable. While the gas exerts a general absorbent action over the violet extremity of the spectrum, it attacks it when in a diluted state in definite lines as sharp and distinct as those in the solar spectrum; and what is still more important, it acts upon the same parts of light as the cause which produces the fixed lines in the sun's spectrum. In other respects the character of its action is similar to that of the blue glass and the green sap of plants.

In thus comparing the phenomena of absorption with those of thin plates, we find no connecting link but that of giving a divided or a mutilated spectrum; and even this common fact has not the same character in both. In coloured media the bands of light and darkness have no fixed relation, as in periodical colours; and the light removed from the dark portions, as well as the tints from some of the coloured spaces, have wholly disappeared, in place of being found in the reflected beam.

I have already mentioned that by the aid of two substances I have been able to study this subject under a new aspect, and that the nacreous substance described by Mr. Horner was the one which first exhibited to me the connexion between absorption and periodical action.

This substance when it contains no thin plates acts generally in absorbing the violet and blue end of the spectrum; but when it includes within it, or has on its surface thin films which act like thin plates, it exercises an additional action upon the spectrum. In some cases when the thickness of the plate is small, it produces bands perfectly identical with those of thin plates, but in other cases the bands are

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* A full account of this experiment, and a coloured drawing of the divided spectrum, will be found in the Edinburgh Transactions, vol. xii.

exactly similar to those of coloured media. In one specimen I obtained a dark and distinct band in the orange space at D, with another faint band in the red. These bands were parallel to the fixed line D at a vertical incidence, but by inclining the plate the bands moved towards the green space, and became inclined to the line D. In a recent specimen I obtained the darkest band in the green space, with other lesser bands of unequal size and breadth in the other spaces, all of which moved along the spectrum, while new ones advanced from the red extremity during the inclination of the plate. In a third specimen the phenomena were still more varied, and what was a new feature in the results, the *colour* of the tints was changed exactly as in the phenomena of absorption. It is very obvious that these results are not produced by the same action which causes the orange colour of the substance, for this action could not vary by the inclination excepting in producing a greater absorption of the more refrangible rays; but in order to place this beyond a doubt, I detached a film which had none of the colours of thin plates, and which, as I expected, produced none of the bands above described. In these experiments the nacreous plate was placed in Canada Balsam to remove the imperfect smoothness of its surface, but the phenomena were essentially the same with plates surrounded by air. I now divided the first of the plates above mentioned into two, and having viewed the spectrum through both, I found the principal black band considerably widened, as happens with absorbent media.

When the light reflected from the nacreous plates is examined in a similar manner, the division of the spectrum into bands is extremely brilliant and beautiful, and the phenomena the same; but owing to the light having entered the substance to different depths before it was reflected, the spectrum is by no means complementary to the one seen by transmission.

Satisfactory as these experiments are, I was still desirous of obtaining similar results with perfectly transparent plates, but after failing in every attempt to combine them, I thought of trying the iridescent films of decomposed glass*. This idea succeeded beyond my most sanguine expectations. I obtained combinations of films which gave me by transmitted light the most rich and splendid colours, surpassing anything that I had previously seen either among the colours of nature or of art. I obtained the deepest and richest blues shading off into the palest, and the finest reds and yellows, with all those intermediate and mixed tints which are seen only in the vegetable kingdom. The reflected tints had quite a different character. They possessed all the brilliancy of metallic reflexion, like the colours in the Diamond Beetle and other insects, and the tints varying within a considerable range were disposed in straight lines and bands, as if the film had formed part of a regularly organized body†.

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* For a very fine collection of these films I have been indebted to the kindness of Mrs. BUCKLAND, the Marquis of NORTHAMPTON, and Mr. CHILDREN.

† The surface of these films is beautifully mammillated, the parts that are curves on one side being concave on the other.

The reflected tints of course vary with the obliquity of the incident light; and at great incidences the transmitted ones, however splendid and varied, all become pale yellow. When these combinations of glass films are immersed in a balsam or an oil, their colours whether transmitted or reflected all disappear, excepting a pale yellow light like that which is transmitted at great incidences. These facts prove, beyond a doubt, that the transmitted colours, though wholly unlike to those of thin plates, are yet produced by the same cause and are residuary, and generally complementary to the hue of the reflected tints.

The analysis of these colours by the prism affords a series of most beautiful and instructive phenomena, and it is only by coloured drawings that any adequate idea of them can be conveyed. All the phenomena of coloured media, with bands of various breadths and various intensities of illumination, are exhibited in great perfection, so as to identify completely *in this feature* the two classes of facts. But what is still more striking, the colours of the bands are changed, and we thus find that the characteristic phenomenon of absorption is produced by the action of thin plates. To such a degree indeed is the change of tint carried, that I have insulated a white band in the orange part of the spectrum.

Notwithstanding this identification of absorption and periodical action in their primary features, there are two points of difference which separate widely the two classes of phenomena. The first of these is, that the bands and tints of absorbing media are not changed by obliquity, and the second, that the reflected tints are not visible in such media. Sir Isaac Newton endeavoured to remove the first of these difficulties by supposing that the particles of bodies on which their colours depended have an enormous refractive power; and M. Biot* has endeavoured to meet it more effectually by introducing two new suppositions, viz. that the particles are capable of transmitting light *only through their centre of gravity*, and that the lateral transmissions may be prevented or turned aside by the inflecting forces which act at a distance on the luminous molecules which approach them.

These explanations of the uniformity of the tints at all incidences have been rendered necessary, not perhaps by the real difficulties of the case, but in consequence of Sir Isaac Newton and his followers taking it for granted that the colours of natural bodies were pure tints of a particular order. Hence it becomes a necessary assumption in the theory that the particles had sizes corresponding to these pure tints, and that the light which composed them should not pass through different thicknesses of these particles. As I have demonstrated, however, in a paper already referred to, that the tint which Newton reckoned one of the third order, has no connexion whatever with that or with any other order, and that all other tints of absorbent media are in the same predicament, we are not only free from the difficulty which embarrassed Newton; but it is actually necessary to have recourse to particles of an ordinary refractive power, and having such forms and occupying such positions as will

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* Traité de Physique, tom. iv. p. 126.

permit lateral transmissions and thus produce compound tints, such as we actually observe in natural bodies, and as we have shown to be produced by thin plates.

Now if we suppose the colouring particles to be spherical, or to have the form of plates or cubes, or other solids disseminated through the fluid or solid bodies which they colour, the tints would be permanent and compound as we find them in nature.

The second point of difference to which I have referred, namely the absolute disappearance of the reflected tints in several coloured solids, fluids, and gases, is one of great magnitude. Newton has evaded this difficulty in his theory; but from the manner in which he gets rid of the intromitted light in black bodies, it is obvious that he would ascribe the disappearance of the reflected tints to their being "variously reflected to and fro until they happened to be stifled and lost."

As I shall have occasion to discuss this subject experimentally in a paper on the permanent colours of natural bodies, I shall only state at present that I have succeeded by particular methods in rendering reflected tints visible in many coloured fluids and glasses, but I cannot consider them as equivalent to the reflexions of thin plates.

I have endeavoured to corroborate the views contained in the preceding pages by a series of collateral experiments on the periodical colours of polarized light. When we divide the spectrum into bands by doubly refracting plates, the phenomena are beautiful beyond all description. If we dissect or subdivide the luminous bands in the spectrum, as seen by one analysing prism, by means of successive plates and prisms, the result is very remarkable; and if the doubly refracting plates are inclined to each other or to the incident beam, the black bands will also be inclined to each other, and the luminous spaces have the form of a triangle either complete or truncated at its apex. By using plates of the same or of various substances*, and placing their axes in different azimuths to the plane of primitive polarization, we obtain extremely singular spectra, in which the bands approximate to those of absorbing media.

But there is another result of this class of experiments to which I would especially call the attention of philosophers. The colours of the bands thus produced have no

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* I have constructed apparatuses of this kind made out of composite crystals of calcareous spar, including one and more thin plates of its own substance. The beautiful and apparently capricious tints which such crystals exhibit when properly cut into prisms, or when prisms are applied to their surface, are nothing more than the luminous bands of the spectrum subdivided by one or more dissections. I have now before me such a crystal, in which a prism cemented externally brings out the spectrum, which would otherwise have suffered total internal reflexion. A virtual prism forming part of the rhomb polarizes the incident light, an included hemitrope plate affords the polarized tints, and a second virtual prism analyses the light which the plate transmits. In some parts of the rhomb there are plates of different thickness, by which the luminous bands are beautifully subdivided. In this manner by the slight aid of an applied prism we are furnished with a complicated optical apparatus. Such a combination, which it is easy to make artificially by inclosing thin doubly refracting plates between prisms of calcareous spar, affords an ocular explanation of those beautiful forms of the system of polarized rings which are produced in composite crystals of calcareous spar. These subdivided bands, indeed, are portions of that system seen obliquely by prismatic refraction.

resemblance to those of the original spectrum, so that the spectrum has actually been analysed by dissection. This effect is so decided, that even by a single subdivision of a banded spectrum I have succeeded in insulating a band nearly white, and of course incapable of being decomposed by the prism.

Hence we deduce from the phenomena of thin plates, and polarized tints, the existence of a new property of light, in virtue of which the reflecting force selects, as it were, out of differently coloured rays of the same refrangibility rays of a particular colour, allowing the others to pass into the transmitted beam; or to use the language of the undulatory theory, the colour produced by the interference of homogeneous pencils reflected from the first and second surfaces of thin plates, is different from the colour produced by the interference of the transmitted light with that which has suffered two internal reflexions within the plate. If, for example, we use the greenish yellow light of the spectrum between the lines D and E, the system of reflected rings will be more yellow than the transmitted rings towards E, and more *green* than the same rings towards D, a result which in so far as the transmitted tints are concerned, is seen in the colours of smalt blue glass.

Here then we have a principle not provided for in either of the theories of light to which the phenomena of absorption, produced by nacrite, by decomposed films of glass and by polarizing plates are distinctly referable. Here also we have the probable cause of certain remarkable phenomena of dichroism in doubly refracting bodies, in which rays of the same refrangibility but of different colours pass into the ordinary and extraordinary pencils.

Allerly by Melrose,
May 5th, 1837.

XVI. *On the Development and Extinction of regular doubly refracting Structures in the Crystalline Lenses of Animals after Death. By Sir* DAVID BREWSTER, *K.H. LL.D. F.R.S. &c. &c.*

Received May 10,—Read 1st June, 1837.

SINCE the year 1816, when I communicated to the Royal Society an account of the doubly refracting structures which exist in the crystalline lenses of fishes and other animals, I have examined a great variety of recent lenses, with the view of ascertaining the origin of these structures, the order of their succession in different lenses, and the purpose which they answered in the animal œconomy. Although I had found that in the lenses of the cod, the salmon, the haddock, the frog-fish, the skate, and several other fishes there were three structures, the innermost of which had negative double refraction, the next *positive*, and the outermost negative double refraction, yet in the lenses of animals the greatest discrepancies presented themselves. In every case, however, excepting one, I have found the central structure in all quadrupeds* to be positive, while it is always negative in fishes when there are three structures, but this positive structure sometimes existed alone, with faint traces of a negative structure; sometimes it was followed by another *positive* structure, separated from the first by a black neutral circle, in which the double refraction disappeared. Sometimes these two positive structures were succeeded by an external negative structure. Sometimes the central and external positive structures were separated by a negative structure, and at other times the lens exhibited *four* structures, a negative and a positive one alternating. As these discrepancies appeared in the lenses of animals of the same species, I conceived that they were owing to differences of age or sex, or to some change in the health of the animal. I was therefore led to make new observations in reference to these probabilities, and to observe the phenomena with additional attention when the structure differed from that which was most common. In these observations I sometimes noticed in the dark or neutral line, which separated two positive structures, something like a trace of an intervening structure, which was either about to disappear, or about to be developed. This conjecture was confirmed by observations on the lenses of a cow eleven years old.

The lenses after being carefully taken out, were freed from the adhering portions of the vitreous humour by the gentle application of blotting paper, so as not to disturb their internal structure. The lenses were elliptical. Their longest diameter was 0·774 inch, their shortest diameter 0·747 inch, and their thickness 0·513 of an inch. The first lens which I exposed to polarized light was in the highest perfection, and the symmetry of the optical figure unusually beautiful. I have represented it in

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* Excepting the hare. See Phil. Trans. 1836, p. 37.

Plate XV. fig. 1., in which only two structures, or two series of positive sectors, are visible*. The lens was now a day old, and there seemed to be a faint light within the two black rings, especially in the outer one, which was either the remains of an old, or the germ of a new structure. If this were the case, then the anomalous combination of two positive structures would be converted into a combination of four structures, in which a negative and a positive one alternated.

On the following day I prepared the other lens with the same care, and found my conjecture completely verified. In the middle black ring, which was distinctly brownish in the first lens, the negative structure had evidently commenced at one part, and the colour of the whole ring was a brighter brown than in the first lens. In the outer black ring another negative structure had also appeared, and had advanced considerably upon the positive structure. These phenomena I have represented in fig. 2., where the four structures are distinctly seen, the second being a faint blue of the first order. On the third day the two new structures had become more prominent. The structure No. 2, now a pale white of the first order, was completely developed, having encroached upon and almost obliterated the third structure. The structure No. 4, which was not in existence on the first day, had now the maximum tint, namely a bright white of the first order. On the fourth day the structures No. 2 and 4, which at first were not in existence, are now the structures with the maximum tints, and No. 3, which had the maximum tint, is now almost obliterated, a little faint brown light remaining in one of the quadrants.

On the fifth day the four sectors of the inner structure No. 1, have almost disappeared. No. 3 has disappeared entirely, and No. 2, which is almost the only polarizing structure, exhibits a more intense white of the first order than appeared in any part of the lens. The ring No. 2 divides the radius of the lens equally.

On the sixth day the structure No. 2 was still bright and uniform, but the polarized light had disappeared from every other part of the lens.

On the seventh day the lens, which was always placed in water, burst its capsule, and there was no longer any trace of distinct polarizing structures.

My next observations were made on the lenses of a cow nearly twenty years old. The following were the dimensions of the eye and the lenses.

	Inch.
Diameter of eyeball	1·66
Chord of the cornea (largest)	1·30
Chord of the cornea (shortest)	1·02
Longest diameter of lens	0·827
Shortest diameter of lens	0·793
Thickness of lens	0·50

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* Upon referring to my earlier observations, I find that in both the lenses of an ox there was only one structure which was a positive one, and which had not yet divided itself into two structures, as in that of the cow under consideration. There was the appearance of a black space near the margin of the lens, but the polarized light both within and without that black ring was positive.

In the lens of another ox, and of a bull, I found the *positive* structure separated into two positive structures by a distinct black ring, while an external negative structure was clearly developed.

Fig. 1.

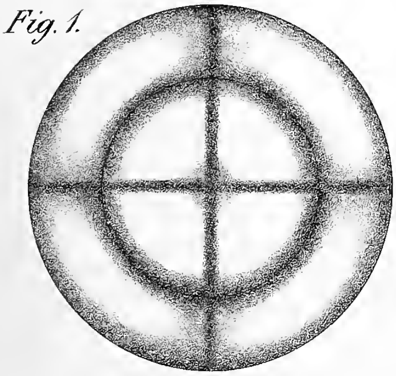


Fig. 2.

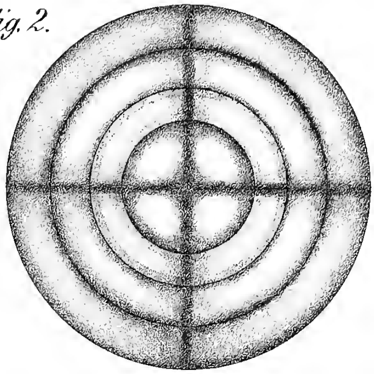


Fig. 3.

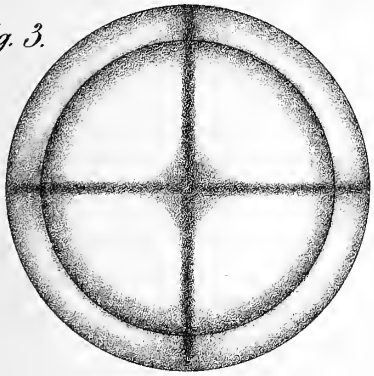


Fig. 4.

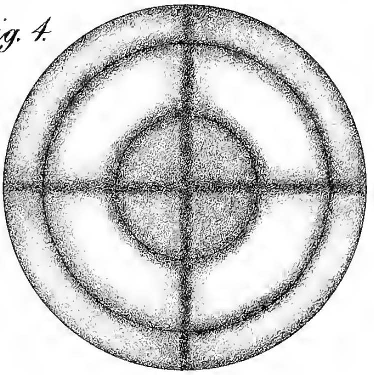


Fig. 5.

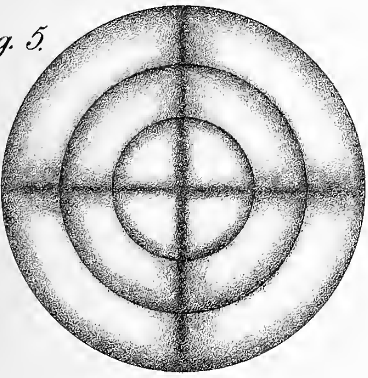


Fig. 6.

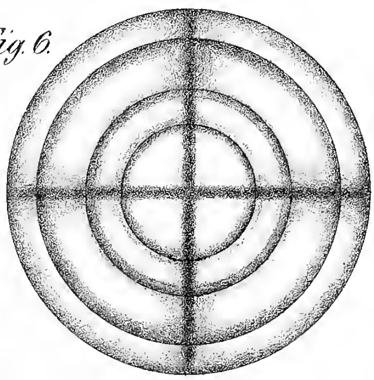


Fig. 7.

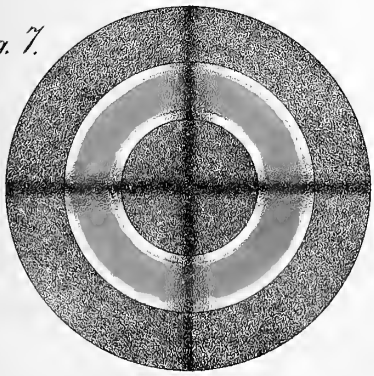
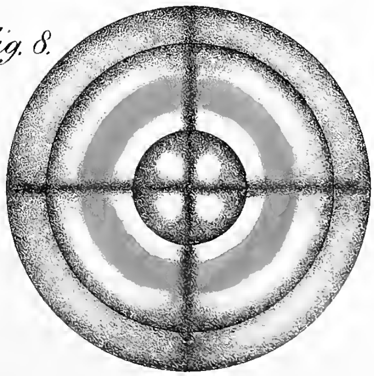


Fig. 8.



J. Basire, lith.

Both the lenses of the cow exhibited when taken out of the eye four beautiful structures, in which the positive and negative structures alternate. The first and fourth were very faint, being the palest white of the first order. The third was also faint, but the second was both bright and large, and its tint was a *brilliant yellow* of the first order. After lying four days in water the lenses swelled so much that their dimensions were as follows:

	Inch.
Diameter of one lens	0·807
Diameter of the other	0·793
Thickness of the first	0·647
Thickness of the second	0·620

The lenses were still transparent, and the tint of the structure No. 2 had risen to an *orange red* of the first order.

Having experienced great difficulty in the course of the preceding experiments in preserving the capsule of the lens transparent for several days, I made trial of various fluids, but found distilled water more suited to my purpose than any other. I therefore began a regular course of observations on the crystalline lens of the sheep when placed in distilled water, which have afforded me very satisfactory results.

The lens of a sheep a year and a quarter old, when newly taken out of the eye, exhibited in the distinctest manner only one structure, with slight traces of an external one. This structure was *positive*, and occupied almost the whole of the lens, as shown in fig. 3. The traces of an external structure, when carefully examined, showed it to be negative. On the following day this lens burst in the direction of the three septa.

In the lenses of another sheep I found two structures like the preceding, but with this difference, that the external negative structure was more developed, as in fig. 4. On the following day this negative structure had extended itself inwards, but in consequence of an accident the lenses burst their capsule.

In the lenses of another Cheviot sheep, where the external negative structure had just begun to appear, the wide positive structure shown in fig. 3 had just begun to separate itself by a dark neutral line, which was seen only in one of its four sectors, and which divided that sector into two.

In another Cheviot sheep the principal positive structure had distinctly divided itself into two positive structures, separated by a dark neutral ring, as shown in fig. 5. The same appearance was shown in the other lens; and I have found it a very common structure in the lenses of sheep at that age when they are killed for the table.

When this division of the principal structure takes place the central one is at first faint, and the other a bright white of the first order, as in fig. 4. It becomes, however, brighter and brighter till it nearly rivals the other in the intensity of its polarized tint, as in fig. 5, when another change begins to show itself.

This change, similar to that which I have described in the lens of the cow, arises from the absorption of distilled water by the capsule of the lens. It first shows itself by the appearance of a brown tint in the dark neutral ring which separated the two positive structures. In the middle of the brownish black ring a trace of faint blueish light appears, generally in one of the sectors only, but gradually extends itself into a blue ring, which has negative double refraction and which is separated by distinctly formed black rings from the two positive structures, between which it lies. This state of the polarizing structure is shown in fig. 6, which is nearly the same as in the lens of the cow.

The structure No. 1, beginning at the centre, was pretty bright, but No. 3 was much more so, and No. 4 very faint, though perfectly distinct.

On the second day the blue ring No. 2 was much enlarged, and had encroached greatly on the brightest structure No. 3, having reduced it both in breadth and intensity. No. 4 has also extended itself at the expense of No. 3.

On the third day the new structure No. 2 had become the brightest of all. No. 4 had increased also, whilst No. 1 had become smaller and fainter, and No. 3 was wholly obliterated.

In another pair of lenses one of them burst at this stage of the development of the polarizing structures, while in the other the effect was singularly fine. No. 3 was wholly, and No. 1 nearly obliterated; while the two new structures, which had no existence at first, were the only ones that remained. The new negative structure No. 2 consisted of four beautiful blue sectors of polarized light; but in consequence of the great absorption of distilled water, and the consequent distension of the lens, it soon burst.

I have already remarked that only one case has occurred in the course of my experiments in which the central structure of the lenses of quadrupeds was negative, as in fishes. In this case, however, the centre of the lens had its structure affected by some change in the condition of the fibres at their union in the three septa, which were not only distinctly seen, but had the polarizing structure clearly related to them. The polarized light filled up each of the three angles of 120° which lay between the three septa, and the intensity of the light was a maximum close to the three septa. Hence it is evident that the central *negative* structure was the result of an induration of the lens related to the septa, and had obliterated the *positive* structure which would otherwise have existed there.

In examining the lenses of the *horse* I have observed the progressive development of its three structures as the animal advanced in age, and the extinction of all of them but one when the age of the animal was great.

In both the lenses of a young horse three years old I found only one positive structure.

In both the lenses of a horse whose age was unknown, I observed three structures beautifully developed. The central ones, which were extremely distinct and more beautiful in form and more intensely luminous than in any other quadruped which I had examined, were *positive*, the next structure *negative*, and the external one *positive*.

In the lens of another horse, whose age was also unknown to me, the remains of three structures were visible; but the two positive ones, namely, the central and external structures, had just disappeared, but were not encroached upon by the intermediate negative one. They were therefore black when seen by polarized light, as shown in fig. 7, while the remaining *negative* one was of the most brilliant yellow colour.

In the lenses of a third horse, probably of an intermediate age, I found a structure intermediate between that of the two preceding ones. The following were the dimensions of its lenses.

	First Lens. inch.	Second Lens. inch.
Longest diameter	0·827	0·820
Shortest diameter	0·793	0·793
Thickness	Not measured	0·500

The first lens having been carefully prepared and immersed in distilled water, exhibited the beautiful optical figure which is but imperfectly represented in fig. 8. The central sectors were *positive*, but faintly illuminated. The wide and brilliant yellow and white structure was *negative*, and the external structure, which had just begun to appear, was *positive*.

On the second day the black mass round the central sectors had enlarged itself, and become very black, having the form of a square lozenge. The *yellow* ring has risen in its tint to a *brilliant pink* yellow at the edges, the white ring within it having increased in width, and the white ring without it having diminished.

On the third day the diameter of the lens had increased to 0·86 in all directions, and its thickness from 0·50 to 0·717 of an inch. The coloured ring has not changed greatly.

On the fourth day the bright pink of the negative structure has risen to a bright blue, the pink and yellow being seen at its margin; and the external positive structure seems to be now conjoined with the blue negative structure, in consequence, no doubt, of the extension of the latter to the margin of the lens. The thickness of the lens was now upwards of 0·86, and the capsule came off, in consequence of which two of the blue sectors have become of a pale pink colour. The instant the capsule came off the lens shrunk in all its dimensions nearly the *tenth* of an inch.

The *second* lens on the *third* day gave exactly the optical figure shown in fig. 8, having been newly placed in distilled water; but the external ring seems to be slightly *negative*, like the yellow one. Its appearance was greyish and indistinct.

On the *fourth* day the yellow ring had risen to a *pale pink* of the first order, and the outer ring was *negative*, as on the preceding day.

On the *fifth* day the *pink ring* had increased in intensity, and the other structures remained the same as before.

On the *sixth* day the *pink* had risen to a very *bright blue*. The diameter of the lens was now 0·867 of an inch, and its thickness 0·733, being an increase of 0·233 of an inch in thickness.

On the *seventh* day the capsule burst, and upon removing it and the soft pulp which formed about *one tenth* of an inch of the outer margin of the lens, the pink ring, with the white band both within and without it, and the black mass at the centre of the rectangular cross, were as distinct as ever. Hence it is manifest that the rise of the tint from *yellow* was not the effect of any expansive pressure produced by the swelling of the lens and the reaction of the capsule.

The descent of the tint from *bright blue* to pink was no doubt owing to the polarizing action of the extended capsule being withdrawn. In order to prove this I took the capsule, which is a tough and elastic membrane, and having stretched it, I found that it polarized, just before it tore, a white of the first order. Now the value of this tint is nearly equal to the difference between the values of the *pink* and *blue* of the second order of colours.

The preceding results throw much light on the physiology of the crystalline lens; and I shall have occasion, in a separate paper, to point out the conclusions to which they lead respecting the cause and cure of cataract.

Allerly by Melrose,
May 6*th*, 1837.

XVII. *On the Temperature of Insects, and its connexion with the Functions of Respiration and Circulation in this Class of Invertebrated Animals. By* George Newport, *Esq., Member of the Royal College of Surgeons, and of the Entomological Society of London. Communicated by* P. M. Roget, *M.D. Sec. R.S.*

Received June 5,—Read June 15, 1837.

EVERY naturalist is aware that many species of insects, particularly of hymenopterous insects, which live in society, maintain a degree of heat in their dwellings considerably above that of the external atmosphere, but no one, I believe, has hitherto demonstrated the interesting facts that every individual insect when in a state of activity maintains a separate temperature of body considerably above that of the surrounding atmosphere, or medium in which it is living, and that the amount of temperature varies in different species of insects, and in the different states of those species. Previously, therefore, to considering the connection which subsists between the evolution of animal heat and the functions of respiration and circulation in insects, I shall endeavour to prove that every species maintains a distinct temperature of body, the amount of which differs in the different states of the insect.

I was first led to the particular consideration of the subject of temperature in insects by some observations on the temperature of wild bees in their natural haunts, which were made by myself at Richborough, near Sandwich in Kent, in the autumn of 1832, at the suggestion of Dr. Marshall Hall, for the purpose,—similar to that of my observations on respiration, as noticed on a former occasion[*],—of ascertaining what relation, if any, subsists between the natural heat of these insects in their hybernating condition and the irritability of their muscular fibre. The results of these observations on the temperature of Bees are shown on Table III., Nos. 1 to 14, and together with many other facts connected with the physiology of insects were communicated to Dr. Hall a short time afterwards[†]. These observations were

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[*] Philosophical Transactions, Part II. 1836, p. 551.

[†] In submitting these observations on the Temperature of Insects to the consideration of the Royal Society, I have felt myself imperatively called upon to make the above remark, in explanation of the nature of my supposed obligations to Dr. Marshall Hall, with regard to this and other subjects connected with the Physiology of Insects, in consequence of certain misrepresentations which were made on a recent occasion respecting my communications with that gentleman; and I beg further to state, that many of the views here advanced respecting the temperature of insects, and also most of the subjoined Tables, particularly those on the temperature of the Hive Bee, from the commencement of my observations to the month of May 1836, were communicated by myself to Dr. Marshall Hall, at his own particular request, in the beginning of July 1836, in the presence of my intelligent friend, and late pupil, Mr. John Osborn, who assisted me in making the observations, and unto whom I am indebted for much valuable assistance during my investigations.

made in the usual manner, by placing a considerable number of insects of the same species together, and then introducing the thermometer among them. But it was a few days previously to making these observations that I first noticed the interesting fact, that each individual insect maintains its own temperature, which is perceptible externally by the thermometer, and that the amount of this varies in the different conditions of the same insect. The observation was first made on the larva of *Sphinx Atropos*, Linn., and on that of *Pygæra bucephala*, Steph., as will presently be shown.

During the time I have been engaged in preparing the present communication I have become acquainted, through the kindness of Dr. Forbes of Chichester, with the recently published views of Dr. Berthold, of Gottingen, who has made a series of observations on the temperature of cold-blooded animals*, and among them several on insects, somewhat similar to those which I now have the honour of submitting to the Society. But excellent as are the views of that gentleman, he does not appear to have paid sufficient attention to the conditions of activity or rest in the insects at the time of making his experiments, and consequently has omitted to observe the important fact of the existence of a distinct temperature of body in individual insects†, and also those circumstances which augment or lessen its amount, and has estimated the temperature by placing many individuals together, which, as will presently be seen, is open to several objections. Dr. Berthold has, however, anticipated me in the expression of one opinion, unto which we have mutually been led by our observations, viz. that at all events the higher classes of invertebrated animals ought not to be considered as *cold-blooded*, since it is found that under certain conditions they have a temperature of body higher than that of the surrounding medium. Hausmann‡ made an observation as long ago as the year 1803, which ought to have led to a proper understanding of the nature of the temperature of insects. He placed a perfect specimen of *Sphinx Convulvuli*, Linn. in a small glass phial when the temperature of the atmosphere was 17° Reaum. (70°·25 Fahr.), together with a small thermometer, and at the expiration of half an hour the temperature of the phial was 19° Reaum. (74°·75 Fahr.), but soon afterwards he found that the temperature of the phial had sunk again to the previous standard 17° Reaum. He then repeated the observation with six specimens of *Carabus hortensis*, Linn. with similar results. From what will subsequently be shown respecting the temperature of *Carabi*, which do not develop so large a quantity of heat, it is very probable, as suggested by Dr. Berthold §, that the results obtained by Hausmann arose from the bottle which contained the insects being touched by the hand of the operator. Dr. Berthold has observed this in his experiments, and I have constantly remarked the same thing myself when proper care was not

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* New Experiments on the Temperature of Cold-Blooded Animals, by A. H. Berthold, M.D., Gottingen, 1835.

† *Ibid.* p. 36. Experiment 59.

‡ De Animalium exsanguinum Respiratione. Götting. 1803. p. 68.

§ Neue Versuche, &c., p. 11.

taken to guard against its occurrence. **Rengger**[*] observed a distinct temperature in *Melolonthæ* when many of them were collected together in an earthen vessel, but could not detect a distinct temperature in water-insects, or in Caterpillars. **Juch**[†] likewise made observations on the temperature of the bee-hive, the ant-hill, and on the common Blister-flies. In a vessel containing a large quantity of the latter insects, the *Lyttæ,* he found the thermometer rise several degrees above the temperature of the atmosphere. Dr. **Davy**, according to **Berthold**[‡], in making observations on several species of insects, *Scarabœus pilularis, Lampyris, Blatta, Gryllus,* and *Apis,* found only a slight difference, except in the *Gryllus,* in which the difference amounted to five or six degrees, while in the Scorpion and Centipede he found a temperature lower than that of the atmosphere. Dr. **Burmeister**, in his Manual, recently translated by Mr. **Shuckard**, has spoken of the temperature of insects, but only of insects in society, and has referred to the observations of **Juch**, **Reaumur**, &c., and although he believes in the existence of individual temperature in insects, has given no observation of his own to prove the fact, while Dr. **Berthold**, in the work just noticed, (experiment 59,) made on a single insect, could not detect it, nor could he do so in every species when the observation was made on a number of individuals collected together. It is evident, therefore, that although the existence of individual temperature is inferred from experiments on insects collected together, it yet remains to be proved that every individual insect in a state of activity invariably maintains a certain amount of temperature, which is readily appreciable by the instruments we are enabled to employ.

Before detailing the results of my observations it is necessary to explain the manner in which the observations themselves have been made, and to point out those circumstances which seem to have been overlooked by other inquirers in their experiments on the temperature of insects. It is only by a careful attention to those circumstances that we are enabled to detect the existence of temperature in single insects, and to understand the causes of its variations at different periods.

The thermometers employed by me on every occasion are of the smallest possible calibre, with cylindrical bulbs about half an inch in length, and scarcely larger than crow-quills, and are similar to those employed by Professor **Daniel** for the purpose of ascertaining the dew point. They were made by Mr. **Newman** of Regent Street, and are graduated from zero, or from a few degrees below freezing to about 110° or 120°. Whenever great delicacy of observation is required, in order to observe the varying temperature of an insect during a state of partial rest, it is necessary to use the same instrument for ascertaining the temperature of the atmosphere as for that of the insect, otherwise a great difficulty will arise, from the well known circumstance that two thermometers, be they ever so delicately constructed, and carefully compared

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[*] Physiologische Untersuchungen über die thierische Haushaltung der Insecten. Tübingen, 1817, p. 39.

[†] Ideen zu einer Zoochemie, Bd. 1. 1800, p. 92.

[‡] Neue Versuche, &c. p. 12, 13.

with each other, will seldom if ever both indicate precisely the same amount of temperature in exactly the same space of time. The mode of taking the temperature is either by allowing the insect to remain with the soft ventral surface of its abdomen pressing against the bulb of the thermometer when in a state of rest, or by pressing the thermometer firmly against its body when in a state of excitement, the insect being held during the time between a pair of forceps covered with woollen, in order that the contact of the fingers of the operator may not interfere with the correctness of the observation by unnaturally increasing the temperature of the insect. It is also further necessary to guard the hand with a glove, or non-conducting substance, to prevent the thermometer itself from becoming affected by it during the experiment. Much caution also is necessary when the same thermometer is employed to ascertain the temperature both of the atmosphere and of the excited insect, to guard against one very material source of error. It is necessary *first* to ascertain the temperature of the atmosphere, and *then* that of the insect, because if this be not attended to, and the experiment be made by taking the temperature of the insect *before* observing that of the atmosphere, the moisture on the bulb of the instrument occasioned by the condensation of the *cutaneous perspiration* from the body of the animal will occasion during its drying or evaporation, while taking the temperature of the atmosphere, an indication of a lower amount of atmospheric temperature than what really exists, and consequently the apparent difference between the temperature of the insect, previously taken, and that of the atmosphere, will be much too great, and thereby appear to indicate a higher temperature than what the body of the insect really possesses. When the temperature is taken during a state of rest, the thermometer is placed beneath, and as completely covered by the abdomen of the insect as possible, while a second thermometer, which has been very carefully compared with the first, is placed on the same level with and at a short distance from it to indicate the temperature of the atmosphere. When the temperature of active volant insects is to be taken, it is preferable to inclose them singly in a small phial, introducing them with the forceps as before, and being particularly careful not to touch the phial with the fingers. The degree of activity or quiescence of the insect must always be particularly noticed, and also the number of inspirations. By attending to these facts we acquire a knowledge of the amount of respiration compared with the quantity of heat evolved, as indicated by the thermometer. The temperature of the insect taken on the exterior of the body is always a little lower than that of the interior; but the difference is not so great as might at first be imagined, so that I have generally preferred taking the exterior temperature, because the observations are then less complicated by unnatural causes. The interior temperature is seldom if ever more than a degree and a half, or at most two degrees above the exterior, and often not even half a degree, when the insect is in a state of perfect rest. Perhaps it may be urged as an objection, that when the bulb of the thermometer is applied to the exterior of the body, it can seldom be so completely covered as to indicate the whole amount of heat developed. But this objection, although at first plausible, must be considered valid only when the observations are made very quickly. But even were the objection substantiated it would be of but little consequence, because it is only the relative amount of heat developed by one insect as compared with that of another, when the observations on both are conducted in a similar manner, which is ultimately sought for, it being almost impossible to ascertain the exact amount evolved by any single insect. It may also be urged as an objection to this mode of taking the temperature of insects in a state of excitement, that when an insect is respiring very rapidly, the friction of the segments of its body against the bulb of the thermometer may evolve a certain amount of heat independent of the natural heat of the insect, and thereby indicate a higher temperature in the insect than that which really exists. In order to meet this objection, I made a number of trials with my thermometers, by using, as nearly as could be ascertained, about the same amount of attrition against the bulb of the instruments as that which is exerted by the segments of the excited insect during its laboured respiration and efforts to escape, and found that so small a quantity of heat is evolved that it is not in the slightest degree indicated on the scale of the thermometer. Hence I have not in general found it necessary to take the temperature of the interior of the body, although I have done so in a few instances, because there are also other circumstances which interfere with the correctness of the observation. The first of these is the large size of the instrument employed compared with that of the body of the insect into which it is inserted, and the consequent necessary loss of a certain amount of caloric, which becomes latent in the thermometer, before there is any indication of increased temperature on the scale, and because also of the unavoidable escape of a large amount of caloric into the surrounding atmosphere, and because still further it is only at the very instant after the introduction of the thermometer into the body of the insect that the real perceptible amount of temperature is indicated, while the insect under observation is every moment losing the power of generating and of maintaining its temperature, owing to the injury that has been inflicted upon it. These objections do not occur when the observations are made on the exterior of the insect, which from its being uninjured, continues to possess its power of generating heat unaffected by those circumstances which tend very materially to interfere with or destroy it, while a sufficient length of time is afforded for the production of its full amount of heat after a certain quantity has become latent in the thermometer, before the observation of the amount is taken.

These are the principal circumstances to be attended to in ascertaining the temperature of insects, and which have directed me in my observations.

I. *Temperature of the different States of Insects.*

1. *The Larva.*

The temperature of the larva is always lower than that of the perfect insect of the same species, provided both individuals be in a similar state of activity relative to their usual condition. This circumstance must never be neglected when making comparative observations on the different states of the same insect. Thus the larva of the more perfect hymenopterous insects, the common Humble Bees, *Bombi*, *Anthophoræ*, *Euceræ*, &c., which in all their stages have a temperature higher than perhaps any other insects, in their active larva state vary from about 2° to 4° Fahr. above the temperature of the surrounding medium, while the same individuals in their perfect state, when moderately active, have a temperature of from 3° to 8° or 10° Fahr. higher than that medium; but when the same insect is very greatly excited the amount of difference is raised to a much greater extent. There is a similar difference between the temperature of the larva of the common Flesh Fly, *Musca vomitoria*, Linn. and that of its perfect insect, only that the amount is not so great as in the hymenopterous insects. In the Musca the amount of temperature in the larva state seldom exceeds 1°·5, and in the perfect perhaps not more than 2°·5, above that of the surrounding medium. It is probable that this estimate of the difference between the larva and perfect state of dipterous insects may be rather too little, owing to the difficulty of making observations on these insects individually, their small size rendering precision in the experiment almost impossible. But the fact is sufficiently clear that they have not so high a temperature as hymenopterous insects. The same difficulty does not exist in making observations on large insects, particularly on the large soft-bodied larvæ of the Sphinges, and accordingly it is found that in these lepidopterous insects we are better enabled to ascertain the maximum amount of heat evolved by the larva, and the difference which exists between its powers of generating heat and that of its perfect insect. This difference is greater in lepidopterous insects than in dipterous, and approaches nearer to the hymenopterous. It was in the larvæ of lepidopterous insects that I first observed the existence, and the varying amount of temperature in individual insects. These observations were commenced in September 1832. At 2½ p.m. September 14, the temperature of the atmosphere being 62°·5 Fahr., the bulb of a thermometer was applied to the under surface of the body of a full-grown larva of *Sphinx Atropos*, Linn., which had discontinued feeding preparatory to undergoing its transformation. The insect then weighed 365½ grains. Previously to the observation it had been for a considerable time in a state of violent excitement, and was moving about with great rapidity. Its temperature, as indicated by the thermometer, was then 70° Fahr., or 7°·5 higher than that of the atmosphere. This, however, was much higher than its real temperature, which is probably not more than 3°, and was occasioned, as I subsequently had reason to believe, by holding the insect in my hand while making the observation. At 12½ midnight, atmosphere 60°·5, the larva perfectly at rest had a temperature of only 61° Fahr.; and at 7 o'clock on the following morning, September 15, having remained perfectly quiet and apparently asleep since the last observation, the temperature of the atmosphere continuing at 60°·5 Fahr., when the bulb of the thermometer was gently pressed against its side without disturbing it, and allowed to remain there for a quarter of an hour, the mercury was not perceptibly affected, the temperature of the larva, now in a complete state of rest, being exactly that of the surrounding atmosphere. Observations in every respect similar to these were also made at the same time on the larva of the Bull-headed Moth, *Pygæra bucephala,* Steph. At midnight the temperature of the atmosphere, as before stated, being 60°·5, the thermometer was applied to the under surface of a larva that had been lying perfectly at rest for several hours, and although it now became slightly aroused its temperature was only 61° Fahr. At 7 on the morning of the 15th, the larva still perfectly quiet, and the thermometer placed in contact with it, and, as with the *Sphinx Atropos,* allowed to remain for a quarter of an hour, there was no indication of any increase of temperature, the temperature of the insect being exactly that of the atmosphere; but a few hours afterwards, when the thermometer was again applied to the same insect, which had become slightly active, the mercury rose to 60°·5, the temperature of the atmosphere being then 60° Fahr. At 6½ on the morning of the 17th the observations on this species were repeated. The temperature of the atmosphere was then 62° Fahr.; and when the bulb of the thermometer was applied to a full-grown larva, which had been remaining several hours at rest, the mercury rose very nearly to 63° Fahr. The observation was then repeated on several other individuals of the same species, which had been lying at rest, and with precisely similar results. The bulb of the thermometer was then placed in a box which was filled with these larvæ, and being completely covered with them was suffered to remain for ten minutes, during which time they were in a state of great activity, and the mercury rose to 63°·3 Fahr., a difference of 1°·3 Fahr. Subsequent observations on the temperature of other species of lepidopterous insects confirmed these observations; and it was remarkable that the amount of temperature in the larvæ of different tribes of this order is pretty nearly the same. On the 26th of June 1834 I examined the full-grown larva of *Pavonia minor,* which like the preceding species had been at rest for several hours, and found that the temperature of the atmosphere being 68°, the temperature of the insect was only 68°·3. The insect then became a little excited, and the mercury rose to 68°·7; and when still further excited to 68°·9, and ultimately to 69°·3, being a difference of 1°·3 above that of the atmosphere, thus proving that the temperature of an insect increases immediately it becomes active, and that the increase is in proportion to the degree of activity, and probably also to the quantity of respiration of the insect. From these facts it is sufficiently clear that individual insects possess a temperature of body above that of the surrounding medium, and that the amount is not constant in the same insect, but varies according to certain conditions of the insect. These views were still further confirmed and extended by observations on the *Sphinx ligustri, S. populi, S. ocellata*, Linn., and *Cerura vinula*, Steph. The first and last of these insects, from their large size and frequency of occurrence, afford us the means of ascertaining all the facts connected with the temperature of larvæ, and are those on which most of my subsequent observations have been made. It is at about the fifth or sixth day after the larva of *Sphinx ligustri* has assumed its last skin, that it evolves the greatest quantity of heat. It then feeds most voraciously, and usually weighs about 80 grains. Its greatest temperature is then 1°·3 above the temperature of the atmosphere. I have seldom or ever found it higher, while on the eighth or ninth day it seldom exceeds nine tenths, and a little while before its change into the pupa state perhaps not more than five tenths. Its quantity of respiration at that time is diminished, and its temperature is reduced by copious cutaneous perspiration, which becomes very apparent when the insect is much excited. The difference which exists in the maximum amount of heat generated by the larvæ of different species of the same class of insects, appears to have some reference to the habits of those species. The greatest amount, so far as I have yet ascertained, excepting only the *Sphinx Atropos* before noticed, appears to be generated by the larva of the Puss Moth, *Cerura vinula*, Steph., which usually lives on the boughs of trees, and subsequently undergoes its changes on the trunk or limbs of the tree a few feet from the ground, has a higher temperature of body, and a quicker circulation of its fluids than the larva of the Sphinx, which undergoes its changes in the earth. The larva of the *Cerura* in its most active condition sometimes has a temperature of 1°·8, or nearly half a degree higher than the Sphinx; but I have not observed the same difference between the temperatures of the perfect insects of these species, both of which constantly reside in the open air. The amount of difference between the perfect insect and larva in these species, like that of the hymenopterous insects, is very great. A perfectly healthy specimen of *Sphinx ligustri* in its perfect condition after violent exertion, has sometimes a temperature of nearly 8° above that of its larva. The usual difference is about 5°, and the same is the case with the *Cerura*.

When the *internal* temperature of a larva of the Sphinx or *Cerura* is taken, it is found to vary from ·5 of a degree to 1° above that of the external. But all observations on the internal temperature of larvæ, more particularly of soft-bodied larvæ, are necessarily uncertain, on account of the reasons before stated. Still it is sometimes desirable to ascertain its amount, particularly when the specimens have been kept in a steady medium. When the internal temperature of the larva of *Anthophora retusa*, Steph. is taken with the necessary care, it is found to be nearly or quite a degree above that of the exterior; but the difficulty in making correct observations on these larvæ is exceedingly great, owing to the rapidity with which they part with their natural heat when exposed to a varying medium. Hence when the observations are attempted to be made, even with regard to external temperature, in the natural haunts of these insects, they seldom afford very satisfactory results. In order therefore to ascertain the real temperature of these larvæ I collected a number of separate nidi, each of which inclosed a larva, and placed them for a few days in a room, the temperature of which varied but very slightly. Each larva was then submitted to observation immediately it was removed from its cell. The temperature of the room in which the nidi were kept was 57° Fahr. The first specimen examined had been lying partly exposed for a short time, and the larva perhaps had thereby had its temperature diminished. When the bulb of the thermometer was inserted into its abdomen the mercury rose only to 57°·8, while its external temperature was scarcely above that of the atmosphere. The second specimen had been better preserved from exposure, but the mercury rose again only to 57°·8. In a third, and apparently very healthy specimen, it rose to 58°. In a fourth, in every respect healthy, to 60° for about a moment, but rapidly sunk again to a little more than 59°; in a fifth it rose also to 60°; in a sixth to 59°·5; and in a seventh and eighth to 60°. On another occasion, when the medium in which the larvæ were kept was 57°·3, the temperature of the under surface of a larva was 60°, but when the bulb of a thermometer was carefully passed into its abdomen the mercury rose to 61° Fahr. In the larvæ of *Musca vomitoria,* Linn., treated in a similar manner, the temperature of the atmosphere being then 56°·8, the mercury rose to 57°·8, but was maintained at that height only for a few seconds, owing to causes before noticed.

I have not yet had an opportunity of examining the larvæ of coleopterous insects, which judging from their similarity to those of the hymenopterous and dipterous classes, it is fair to infer evolve a similar amount of heat. Neither have I been able to examine the orthopterous and hemipterous larvæ, which, from their approaching very near to the condition of the perfect insect, probably differ but little in their production of heat and the quantity of respiration.

2. *The Pupa.*

The pupa state being in all insects which undergo a complete metamorphosis a condition of absolute rest, the temperature of the individual is in general lower than at any previous or subsequent period of its existence, and is only equal to, or at most but very little above that of the surrounding medium. But in those insects which do not undergo a complete metamorphosis, the temperature probably is intermediate between that of the larva and perfect condition. In those species the individuals continue active during their whole life. These exceptions include most of the hemipterous, orthopterous, and a few coleopterous insects, and cannot properly be included under the designation of pupa, the term being here intended to apply strictly to the lepidopterous, dipterous, hymenopterous, and a few coleopterous insects.

The only periods during which the temperature of a pupa is higher than that of the surrounding medium, are, first at the period of, or within a short time after its change from the larva state, while it is still active, and respiring very freely, and before it has completely subsided into a state of rest. At that time, when the whole of its energies are called into activity in effecting its transformation, the temperature of the pupa may be considerably higher than that of the surrounding medium. Thus I have found it in the Sphinx, immediately after changing, equal to that of the active larva. When the temperature of its cell in the earth was 68°·3, the temperature of the newly-changed pupa within it was 69°·5, a difference of 1°·2; but within a single hour afterwards, while the body of the pupa was yet soft, the difference was scarcely more than three tenths of a degree. So likewise when a pupa is very much disturbed for the purpose of experiment, its temperature becomes considerably increased. Also when the medium in which the pupa is living is suddenly diminished, or when the pupa is removed from a warmer to a colder medium; and lastly, when the pupa, aroused by the stimulus of gradually increasing external temperature, begins again to respire freely, during a short time before it is developed into the perfect insect. In each of these cases its temperature may be more or less high, according, in the first place, to the rapidity with which the temperature of the surrounding medium has been diminished, and in the second according to its quantity of respiration in a given time. The increased temperature of a lepidopterous pupa arising, as it appears to do, with increased respiration, is coincident with the power which the insect gradually acquires before it is able to fissure its prison-house and liberate itself from the puparium; while the hymenopterous insect, which lives in society, and remains during its nymph or pupa state inclosed in an almost impervious cocoon, has its temperature artificially increased by the incubation of insects already developed.

It is very shortly after an insect has entered the pupa state that its respiration is diminished, and its temperature sinks down very nearly to that of the surrounding medium. At 8 A.M., November 10, two pupæ of *Sphinx ligustri*, which had remained during several weeks with other specimens entirely undisturbed, were carefully removed with the forceps into glass-stoppered phials, the temperature of which was exactly that of the room in which the pupa had previously been kept. They were examined during three succeeding days, the temperature of the atmosphere being also very carefully noted. The temperature of the phials varied a little, but there was not the slightest difference between the temperature of the atmosphere of the phials and of their respective pupæ, even when the thermometer was allowed to remain in contact with the pupæ for several minutes. The variations in the temperature of the phials are shown in the following Table.

TABLE I. Temperature of Pupæ.

Period of observation.	Atmosphere.	Phials.	Diff.	Remarks.
Nov. 10, 1834. A.M. 8	53°·4	No. 1. 53·4		
		No. 2. 53·4		
P.M. 1½	54·5	No. 1. 54·7	·2	Pupa had been a little excited.
		No. 2. 54·5		
11 P.M. 1	51·5	No. 1. 51·6	·1	Atmospheric temperature sinking.
		No. 2. 51·6	·1	
12 A.M. 9	51·9	No. 1. 51·8		Atmospheric temperature rising.
		No. 2. 51·9		
13 A.M. 9½	50·9	No. 1. 51	·1	Atmospheric temperature sinking.
		No. 2. 51·1	·2	

From these observations it is seen that when the pupa was disturbed there was a slight evolution of heat; the amount of this was greatest when the temperature of the atmosphere was subsiding. But as it appears reasonable to infer, *à priori*, that the internal temperature of the pupa may be higher than that of the surrounding atmosphere, although the thermometer be not perceptibly affected when applied to the thick exterior of the puparium, another specimen of the same insect was subjected to examination. This specimen had been lying for several weeks on the surface of the ground, in the shade, exposed to all the variations of the atmosphere. During that period the temperature of the air had seldom been more than a few degrees above freezing, while on the three nights immediately preceding the making of this observation, on the morning of the 23rd of March, the temperature of the atmosphere had ranged from 2° to 4° FAHR. below 32° FAHR. On the night of the 22nd it was from 3° to 4° below that standard. Under these circumstances there appeared to be a favourable opportunity of ascertaining the real internal temperature of the pupa. Accordingly at $7\frac{1}{2}$ A.M., atmosphere perfectly calm, and its temperature 32°·6 FAHR., and gradually but very slowly rising, an incision was made quickly with a pair of scissors through the posterior part of the pupa, which was held for the moment between a pair of forceps that had previously been cooled down to the temperature of the atmosphere. The fluids of the insect instantly gushed out, and the entire cylindrical bulb of a small thermometer was immediately passed into the body of the pupa. It was the same thermometer which only a moment before had been used to ascertain the temperature of the atmosphere. The mercury in the scale immediately sunk to 32°·3 FAHR., or three tenths below that of the atmosphere, and it was maintained at that standard for fifteen minutes, while the temperature of the atmosphere was still slowly rising. At the expiration of that time the pupa was slightly compressed with the forceps, and its temperature rose slowly to 32°·7, that of the atmosphere being 32°·8. In this observation there was not the objection of part of the bulb of the thermometer being exposed, nor of evaporation taking place from the surface of the wetted bulb. Hence it is fair to conclude that the internal temperature of a pupa, perfectly at rest, is scarcely above that of the surrounding medium, when the temperature of that medium is stationary. We have still further evidence that this is really the case, when instead of a single specimen a considerable number of pupæ are employed. When the bulb of a thermometer was completely covered with the pupæ of the Flesh Fly, *Musca vomitoria*, LINN., the temperature of the atmosphere being 56°·5 FAHR., the mercury was not in the slightest affected, but continued exactly at the same standard. But when the more delicate pupæ, or nymphs, are employed, as those of Bees, the temperature of a number of them which have been somewhat disturbed is generally a little above that of the surrounding medium; and this is also the case when a single specimen is employed, if its temperature be taken during the summer, when the nymph is active and preparing to pass into the perfect state, as shown in Table III., No. 39. But this difference very soon becomes reduced by the greater rapidity with which insects in the condition of nymphs part with their natural heat than even the larva; and this apparently is the reason why most hymenopterous insects select those situations for their young which are found to be the worst conductors of heat. This evidently is why the *Anthophora* incloses its larvæ in cells constructed in the vertical sections of banks of earth which are exposed to the morning sun, and why the Hive and Humble Bees crowd over those cells which are about to produce the perfect insect, when the inclosed nymphs are most in need of increased temperature to invigorate them for the change they are about to undergo.

3. *The Imago, or Perfect State.*

When an insect has assumed its last or perfect condition it has a higher temperature of body than at any other period of its life, and when in a state of activity is not so much influenced by sudden changes of atmospheric temperature as in its earlier states of existence as larva or pupa; and it has also a greater power of generating as well as of maintaining its temperature. But it is not until some time after an insect has assumed its perfect form that it is able to support its full temperature. This period is longer or shorter, according to the habits of the species. When a lepidopterous insect leave its puparium with its whole body soft and delicate, and its wings undeveloped and hanging uselessly like little buds from the sides of its thorax, it so rapidly parts with its temperature that it appears to have a lower degree of heat than at the time when it was about to pass from the larva to the pupa state, and it immediately seeks a retired situation, where it may suspend itself vertically at rest, and complete the development of what are now to become its most important organs of locomotion. In effecting this development it is well known that the insect first begins to breathe very deeply, and it continues to do so for a considerable time. The inspired air passes from the large air-sacs in the abdomen of the insect into the base of the wings, with which the air-sacs have a direct communication*; and while the ramified tracheæ in the wings are becoming elongated and distended, and the wings in consequence developed, the temperature of the insect again begins to increase. But it is not until the wings have become firm and fitted for flight that the insect is enabled to generate its full amount of temperature. Thus in the Puss Moth, *Cerura vinula*, STEPH. half an hour after coming from the pupa the temperature of the insect was only ·2 of a degree above that of the atmosphere; at an hour afterwards ·3; at an hour and a half ·6. During this period the insect was only in a moderate state of activity. But at two hours and a half, and when a little more active, its temperature amounted to one degree and two tenths; and on the following day, when perfectly strong and excited as during rapid flight, it amounted to nearly 7° above that of the atmosphere (Table V. Nos. 25 to 35.).

This is exactly the same with the *Sphinx ligustri*, LINN. An individual which had

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* Mr. GOADBY, Medical Gazette, April 2, 1836.

only left the pupa state about an hour and a quarter had a temperature of only ·4 of a degree above the atmosphere; but at the expiration of two hours and a quarter, when it had become strong and had just taken its first flight, it had a temperature of 5°·2 (Table V. No. 7.); while another specimen, which had been longer exerting itself in rapid flight, had a temperature of 9° above that of the atmosphere (Table V. No. 12.). Now these very species in their larva state, as we have before seen, have not more than 1°·3 and 1°·8 above that of the atmosphere. The circumstances connected with the power of generating heat are nearly the same in the development of hymenopterous as in lepidopterous insects, the only difference being that those hymenopterous insects which live in society have their heat augmented artificially before leaving the cocoon or pupa case. But when the young bee comes forth it parts with its temperature most rapidly, unless it be immediately protected by warmth afforded to it by the bodies of other individuals. But when the same insect a few hours afterwards has become fully able to perform all the duties of its existence, it sometimes has a temperature of perhaps 20° Fahr. above that of the surrounding medium, while the temperature of its larva is scarcely more than 3° or 4° Fahr.

During the whole of my observations I have not met with a single instance in which I was unable to detect a certain amount of external temperature in perfect insects in a state of activity, and it may therefore be regarded as proved that the whole class develop a certain amount of external heat. This uniformity of results, however, has not been observed in the experiments by Dr. Berthold*, before alluded to, and I can only attribute the discrepancy which exists between his observations and my own, to the circumstance of his omitting to attend particularly to the degree of activity or rest in the insects on which he experimented. I am the more inclined to attribute it to this omission, because in his 58th experiment, page 36, he says that in "twenty chamber flies there was no development of heat or external temperature," the observation being made in a steady atmospheric temperature of 17 Reaumur (70°·25 Fahr.). In my own observations upon insects of this order, as in an experiment with about the same number of specimens of *Musca vomitoria*, Linn. in their perfect state, the atmosphere being about 52° Fahr., the insects in a state of activity evolved from 1° to 1°·9 Fahr. of external heat, while in the same individuals in a state of partial rest the amount of heat did not exceed ·6 of a degree. Again, in Dr. Berthold's 59th experiment, which evidently was made in order to ascertain whether single insects evolve any appreciable heat, the bulb of a thermometer was passed into the body of a "single chaffer," through an opening under the wing-covers, and examined half-hourly for about two or three hours, but no heat was detected. In several experiments made by myself in a similar manner to this by Dr. Berthold, particularly on the *Melolontha vulgaris*, Steph. (Table VI. Nos. 45 to 52), the amount of heat developed varied from 2° to 9° above that of the atmosphere, and was always in proportion to the activity of the insect.

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* Neue Versuche, &c., p. 36.

These facts are sufficient to prove that insects have a high temperature of body, and that it is higher in their perfect than in their larva or infant condition. They also beautifully accord with the facts ascertained, and the views deduced from them by Dr. Edwards, respecting the difference between the temperature of the young mammiferous animals and their perfect adults.

II. *Temperature of Insects as influenced by various conditions.*

Abstinence, Inactivity, Sleep, Hybernation and inordinate Excitement.

1. *Abstinence.*

Having shown the difference between the temperature of the larva and perfect insect in a state of activity, we come next to the consideration of certain conditions under which the temperature both of the perfect insect and of the larva will sometimes subside, almost to that of the surrounding medium. When an insect, whether it be in its earlier or later condition, has been long deprived of food, its power of generating and of maintaining its natural heat is diminished. But this diminution of power does not keep pace with the length of time it has been fasting, but is only in an inverse degree. In the larva of *Sphinx ligustri*, Table XII., and in *Acrida viridissima,* Table VI. Nos. 9 to 16, the amount of heat is much below the usual quantity evolved when the insect is not deprived of food, and in a state of activity. When the proper quantity of food is again supplied to these insects, their respiration is restored to its original condition, and they again evolve a full amount of heat. When a larva that has been deprived of food, or has been fed sparingly, is preparing for transformation, its natural temperature is reduced to within two or three tenths of a degree of that of the surrounding medium. This was the case with the larva of *Cerura vinula,* Steph. (Table X. No. 30, B.), which although actively employed spinning its cocoon, had, at one time, a temperature of only two tenths of a degree above that of the atmosphere; while the other specimen, No. 1. A, which had been supplied with its full amount of food of proper quality, had a temperature under similar circumstances, and almost at the same hour, of ·7 tenths above that of the atmosphere. In another larva of *Sphinx ligustri,* which having been inadequately supplied with food soon after it had assumed its last skin, and thereby retarded three or four days beyond the usual period before it began to prepare for transformation, the temperature of its body, while in the state of the greatest muscular excitement in attempting to rupture and cast off its exuviæ, was only ·3 tenths of a degree above that of the atmosphere.

2. *Inactivity.*

Another source of diminished temperature in insects is *inactivity.* In this condition, as in a state of abstinence, the quantity of respiration is also diminished. When an insect becomes quiet, after having continued for some time in a state of moderate activity, its temperature gradually subsides, and continues to be diminished in proportion to the length of time it remains inactive, until it has approached very near to the temperature of the atmosphere. Thus many of those insects which have a comparatively high temperature when in a state of active exertion in the early part of the summer, have their temperature greatly reduced when they become inactive at the end of autumn; and when an insect passes from a state of inactivity into that of natural sleep, its temperature subsides even during summer, very nearly to that of the surrounding medium. This was the case with the larvæ of *Sphinx Atropos* and *Bombyx bucephala,* as shown in the observations on larvæ.

3. *Sleep.*

All insects enjoy a periodical state of repose, or natural sleep. They are endowed with this privilege of life for the renovation of their voluntary energies in common with other animals. It is at this period that the involuntary functions of the body, which, together with the voluntary, are exercised to their utmost amount during the willing activity of the individual, begin steadily to subside, in order to restore the equilibrium which ought to exist between the healthy capability of the organs employed and the amount of energy expended. Respiration, circulation, digestion, and the evolution of animal heat are all diminished, until a fresh amount of voluntary power is again generated, and the animal is aroused to the enjoyment of it either by its superabundance, or through the agency of external stimuli. It is no small amount of this privilege that is enjoyed by insects. I have witnessed sleeping in almost every order of insects, and am satisfied that they enjoy as great a proportion of rest as any other animals. Many insects will remain in a state of rest during ten, twelve, or twenty hours at a time, even in their seasons of activity, influenced as they are by external stimuli. Every one is aware that the common May Chaffer, *Melolontha vulgaris,* will often continue sleeping on the leaves of the lime tree throughout the whole of a fine summer's day, and not become active until near sunset. The case is the same with nearly the whole tribe of Sphinges and Moths, while many Butterflies which are active during sunshine, will often remain for two or three days, when the weather is gloomy, affixed to the very same spot. The common Honey Bee, *Apis mellifica,* LINN., notwithstanding the bustle and activity of the hive, enjoys its share of repose as well as other insects, even amidst the apparent commotion of its own dwelling. HUBER observed that his bees often inserted their heads and part of their bodies into the empty combs, and remained there for a considerable time. They were then quietly sleeping in the cells. At other times they appear to sleep for short intervals on the surface of the combs. I have seen them towards the latter end of summer sleeping in the cells in great numbers for many hours together. It is there also where many of them pass a portion of their winter, doubtless in a state of hybernation, or most profound sleep; and it is an interesting fact, that this inactivity of the inhabitants of the hive during winter, is accompanied by a diminution of heat in their dwelling, as I shall presently have an opportunity of proving. The common Humble Bee, *Bombus terrestris*, even in the month of April, will continue in a state of rest* approaching to the condition of hybernation for at least twenty hours, while its temperature becomes diminished in proportion to the diminution of its quantity of respiration, which also is diminished in proportion to the length of time it remains in a state of rest. This is always the case with insects when the temperature of the surrounding atmosphere is stationary. But if the temperature of the atmosphere is gradually increasing when an insect is passing into a state of repose, the temperature of the insect will continue to rise also, accompanying that of the atmosphere, but not so rapidly as it would have done were the insect in a state of activity, so that the temperature of the air and of the insect will at length arrive at exactly the same level; and if, when this is the case, the temperature of the atmosphere continues rising, that of the insect will also accompany it for a certain time; but if the increase of atmospheric temperature be very rapid, the temperature of the insect will at length be found to be one or two tenths of a degree below that of the atmosphere. When this has happened the insect generally becomes slightly aroused, fetches one or two deep inspirations, and its temperature very quickly rises to that of the atmosphere, while the insect relapses again into its previous slumber. On the other hand, if the temperature of the atmosphere be gradually diminishing, that of the insect will also continue to be diminished, but will remain for a longer period higher than that of the atmosphere when the atmosphere is rising, or is remaining stationary, since the insect during sleep can neither acquire nor part with its heat so rapidly as the atmosphere around it. But if the temperature of the atmosphere continues to subside rapidly, the temperature of the insect during the whole period of its most profound sleep may continue considerably higher than that of the surrounding medium. These facts may be readily demonstrated by careful observations on the smooth-bodied larvæ of Lepidoptera, the best of which for this purpose are the larvæ of the Sphinges, in which besides the varying amount of temperature, the correspondent rate of pulsation may also be observed with great accuracy. The larva of *Sphinx ligustri* upon which the observations detailed in Table No. II. were made, had arrived at the seventh day of its age after assuming its last skin, or at about the thirtieth day after coming from the egg, and consequently was nearly full grown, and beginning to feed rather less voraciously than on the two preceding days. At the time my observations were commenced it had been lying at rest about an hour, having fed plentifully in the morning. The whole period of observation, throughout which it was sleeping almost uninterruptedly, was about nine hours. During this period the thermometer was allowed to remain entirely undisturbed on a table in close contact with the ventral surface of the insect, while a second thermometer, with which the one employed to take the temperature of the insect had been carefully compared, was used to take the temperature of the atmosphere, which throughout the obser-

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* Philosophical Transactions, 1836, Part II., p. 555, Table I., No. 27.

vations was perfectly calm. Thrice during this period of repose the insect became slightly aroused, and each time, as shown on the Table, the number of its pulsations and its temperature slightly increased, but subsided again as the insect relapsed into its previous condition. Once also it was disturbed by the passing of excrement, immediately after which there was a slight increase of its temperature, and of the pulsation of its dorsal vessel, and the insect continued awake for a few minutes, but having relapsed into its former sleep its temperature and pulsation again subsided.

TABLE II.

Exhibiting the diminished temperature of body during sleep, and also a coincident diminution in the rate of pulsation of the dorsal vessel in different conditions of the insect during the last three days of the larva state of the *Sphinx Ligustri*, LINN.

No.	Species.	Period of Observation.	Atmosphere.	Insect.	Difference.	Pulsation.	Weight.	Fæces.	Loss.	Age.	Remarks.
		1834. ′ ″	°	°	°		grs.	grs.	grs.		
1	Sphinx ligustri, larva...	Aug. 29. A.M. 11 15	66·8	67·3	·5	27	....	......	......	7th day.	After last change of skin, sleeping.
2	Sphinx ligustri, larva...	A.M. 11 30	67·6	68·0	·4	27	....	......	......	....	Sleeping.
3	Sphinx ligustri, larva...	A.M. 11 40	67·8	68·1	·3	28	....	......	......	....	Sleeping.
4	Sphinx ligustri, larva...	A.M. 11 45	68·0	68·2	·2	29	....	......	......	....	Sleeping.
5	Sphinx ligustri, larva...	A.M. 11 50	68·4	68·6	·2	30	....	......	......	....	Sleeping.
6	Sphinx ligustri, larva...	A.M. 12 0	68·5	68·6	·1	30	....	......	......	....	Sleeping.
7	Sphinx ligustri, larva...	A.M. 12 8	68·8	68·9	·1	31	....	......	......	....	Sleeping.
8	Sphinx ligustri, larva...	A.M. 12 15	69·1	69·1	·0	32	....	......	......	....	Sleeping.
9	Sphinx ligustri, larva...	A.M. 12 23	69·2	69·2	·0	31	....	......	......	....	Sleeping.
10	Sphinx ligustri, larva...	A.M. 12 30	69·3	69·4	·1	33	....	......	......	....	Sleeping, but slightly aroused.
11	Sphinx ligustri, larva...	P.M. 12 38	69·5	69·5	·0	31	....	......	......	....	Sleeping.
12	Sphinx ligustri, larva...	P.M. 12 45	69·6	69·6	·0	32	....	......	......	....	Sleeping.
13	Sphinx ligustri, larva...	P.M. 12 50	69·7	69·7	·0	32	....	......	......	....	Sleeping.
14	Sphinx ligustri, larva...	P.M. 1 0	69·7	69·7	·0	32	....	......	......	....	Sleeping.
15	Sphinx ligustri, larva...	P.M. 1 8	69·8	69·8	·0	32	....	......	......	....	Sleeping.
16	Sphinx ligustri, larva...	P.M. 1 15	69·9	69·9	·0	32	....	......	......	....	Sleeping.
17	Sphinx ligustri, larva...	P.M. 2 0	69·7	69·8	·1	31	....	......	......	....	Sleeping.
18	Sphinx ligustri, larva...	P.M. 2 45	69·6	69·7	·1	31	....	......	......	....	Sleeping.
19	Sphinx ligustri, larva...	P.M. 3 15	69·2	69·6	·4	30	....	......	......	....	Sleeping.
20	Sphinx ligustri, larva...	P.M. 3 30	69·3	69·6	·3	30	....	......	......	....	Sleeping.
21	Sphinx ligustri, larva...	P.M. 4 0	69·3	69·8	·5	30	... .	......	......	....	Sleeping.
22	Sphinx ligustri, larva...	P.M. 4 15	69·4	69·9	·5	30	....	......	......	....	Sleeping.
23	Sphinx ligustri, larva...	P.M. 5 0	69·4	69·8	·4	29	....	6·1	......	....	Sleeping.
24	Sphinx ligustri, larva...	P.M. 5 15	69·4	69·9	·5	31	....	......	......	....	Arousing, changing colour for transformation.
25	Sphinx ligustri, larva...	P.M. 5 30	69·4	69·9	·5	30	....	......	......	....	Sleeping.
26	Sphinx ligustri, larva...	P.M. 6 0	69·1	69·7	·6	29	....	......	......	....	Sleeping.
27	Sphinx ligustri, larva...	P.M. 6 30	68·8	69·3	·5	29	....	......	......	....	Sleeping.
28	Sphinx ligustri, larva...	P.M. 7 0	68·7	69·6	·9	36	141·4	......	......	....	Aroused and active.
29	Sphinx ligustri, larva...	Aug. 30. A.M. 6 0	65·0	65·4	·4	25	....	......	......	8th day.	Sleeping.
30	Sphinx ligustri, larva...	A.M. 7 0	65·5	65·8	·3	25	136·6	3·5	......	....	Sleeping.
31	Sphinx ligustri, larva ..	A.M. 8 0	66·0	66·3	·3	24	....	......	......	....	Sleeping, much discoloured.
32	Sphinx ligustri, larva...	A.M. 9 0	67·4	......	......	26	....	......	......	....	Aroused, very active.
33	Sphinx ligustri, larva...	Aug. 31. A.M. 11 0	66·6	67·0	·4	18	110·4	......	......	9th day.	Just entered the earth, very active.
34	Sphinx ligustri, pupa...	Sept. 4. A.M. 10 0	66·1	66·4	·3	12	79·4	Skin. 3·8	27·2	13th day.	Pupa within one hour after changing, has been much disturbed.

4. *Hybernation.*

From a state of profound sleep we pass to that of hybernation, which, as shown in the hybernating Mammalia*, appears to be almost identical with the natural repose of all animals. In insects, however, hybernation seems to differ from natural rest in some of its exciting causes. Thus there are reasons for believing that this disposition to pass into a profound sleep, bears some relation to the changes which take

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* Dr. M. HALL, Philosophical Transactions, 1832, Part I.

place at certain periods in the capacity of the respiratory organs, which seem to become oppressed, and their full expansion prevented by the remarkable accumulations of fat which always exist in the bodies of insects before passing into the true hybernating condition. Thus before the larva assumes the condition of pupa it feeds most voraciously, and an immense quantity of fat is collected within it, and if it has been properly supplied with food, it acquires its utmost size and weight many hours before it changes to a pupa. During the interval which elapses between its full development as a larva and its change into the pupa state it is often much less active, and has the appearance of an animal suffering from repletion: it ceases to eat, it is more sluggish in its movements, often sleeps a great deal, and perspires copiously; its average temperature is lower than it had been a day or two previously, and its quantity of respiration is also diminished. These appear to be conditions which induce the phenomena of its transformation, because I have repeatedly found that if a larva be deprived of its proper quantity of food, its change into the pupa state does not take place so early, but is retarded for two or three days. On the other hand if the insect be supplied to repletion, its change will be slightly hastened. Thus if several specimens of the larva of the Sphinx be hatched at about the same time but supplied with different kinds of food, those which are fed upon one kind of plant will often arrive at maturity and undergo their changes before those which are fed upon another. In these cases it is inferred that a plethoric condition, which is supposed always to precede the change to the pupa state, occasioned by the accumulated fat within the body compressing the respiratory organs, and thereby preventing the full aeration of the circulatory fluids, is induced in the one instance earlier than in the other, owing to the more nutritious quality of the food supplied to the insect during the first few days after it has left the egg. There is also another strong reason for believing that this condition of body is closely connected with the phænomena of transformation, in the circumstance that, although for many hours immediately preceding the change, the quantity of respiration, relatively to the size of the insect, becomes diminished, yet within one hour of the actual period of rupturing and throwing off its skin, the insect makes several very powerful and laboured inspirations; and it is then probably that those tracheæ which seem to have become compressed and diminished in calibre during the plethoric state, begin again to be distended, previously to their subsequent development into the large respiratory sacs of the perfect insect. This enlargement of the sacs is slowly progressive during the earlier, but most rapidly so during the latter period of the pupa state, while particularly in the Sphinx, it is almost suspended in the middle, or intervening period of this state, the period when the insect is in its most complete state of hybernation. The enlargement, as suggested on a former occasion*, seems to keep pace with the gradually diminishing size of the alimentary canal, and with the absorption of the accumulated fat, and since it is well known that a higher or lower degree of atmospheric temperature will either accelerate or retard the completion of these changes in the pupa, it may not be unreason-

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* Philosophical Transactions, 1836, Part II. p. 534.

able to infer that the subsequent arousing of the insect from this hybernating condition arises, in addition to the stimulus of increased temperature in the surrounding medium, partly also from the stimulus of a more perfect aeration of its fluids, through means of the greater quantity of air which necessarily enters its enlarging respiratory organs. These opinions are supported by the facts that some insects pass into the pupa state at two different periods of the year, and that their subsequent development into the perfect state depends upon the period at which they enter into the pupa. Thus the common Cabbage Butterflies, *Papilio brassicæ*, Linn., and *P. Napi*, Linn., when changed from larvæ to pupæ in the middle of summer, become perfect insects within a fortnight; but when the change into the pupa state takes place at the end of summer, the perfect insects are not developed until the following spring, unless, as shown long ago by Reaumur, they are placed in a warm atmosphere, when they may at any time be developed within a few days, even in the months of December and January. Besides these facts, and a variety of others which are equally well known, every one is aware that the hybernation of many insects occurs at comparatively high degrees of temperature. The facts connected with the presumed plethoric condition of insects before hybernating are equally referable to those perfect insects which pass the winter months in hybernacula as to larvæ which are about to pass into the pupa state, since it is found that they always have a much larger accumulation of fat in the autumn than at other seasons of the year. This is the case in the bodies of *Vanessa Atalanta*, Steph., *V. Io*, Steph., *V. urticæ*, and in the Cabbage Butterflies just noticed; and it is well known to the cottager that when the flowers have not yielded an abundance of honey in the latter part of the summer, the bees in his hives will have less chance of existing through the winter than when the production of honey has been plentiful. This latter circumstance may, perhaps, be said to arise from a deficiency in the quantity of honey stored up by the bees, but I have strong reasons for believing that it arises chiefly from the bees being in a worse bodily condition, and having but a small quantity of nutriment stored up within their own systems, which alone enables them to pass some portion of the winter in a state of repose. If the female of the common Humble Bee, *Bombus terrestris*, Steph., which sleeps through the winter and appears early in the following spring, be examined about the end of September, its abdomen is found to be supplied with large bags of fat. At that period the insect is less active, and evolves a smaller quantity of heat than in the spring when there is a much lower temperature of the atmosphere. And if at that period the insect be deprived of food it will continue to live, very much longer than it would have lived, under similar circumstances, and exactly at the same temperature of the atmosphere in the month of April. About the end of September I confined two large females, *Bombus terrestris* and *B. lapidarius*, Steph., in the same box without food, and placed them in my sitting room, the temperature of which was seldom lower than 60° Fahr. and often 65°, during the whole time of their confinement. When first confined they were both very active. *B. terrestris* died on the 27th of October, and *B. lapidarius* on the 5th of November, having each of them been confined about a month or five weeks. Now the very same species when confined in the early part of the spring and summer, at a temperature of at least 10° lower than the present, would have perished within forty-eight hours. Hence it is not diminished temperature alone that induces a state of hybernation. Now during the confinement of these individuals I examined other specimens of the same species, and found the abdomen in each of them well filled with fat, while the respiratory organs appeared to be diminished in calibre, and somewhat compressed by its accumulation. This was particularly the case with one specimen which I examined, and the circumstance became the more interesting to me from a knowledge of the fact that both the amount of respiration and the quantity of heat evolved by the insect are at this period diminished. But without going further with the causes of hybernation of insects, and which do not directly belong to this subject, it may be inquired how it happens that if the sleep of the hybernating insect be induced by a plethoric condition of body, that there are certain species, as, for instance, the *Anthophora retusa*, Steph., which assume the perfect form and begin to hybernate during the summer, even at the end of August, but do not leave their abodes until April or May in the following spring, although the morning sun shines brightly on their dwellings, and sometimes raises the exterior surface of the bank in which they are deposited to a temperature of 80° Fahr. or upwards? Unto this it may be replied that the bodies of those insects, having so recently changed from the larva to the perfect state, are still provided with a full supply of nourishment; that the soil in which they are nidificating has not its temperature increased to a sufficient depth to arouse them into activity, and that even if its temperature be sufficiently increased for a day or two, it does not continue at the same standard, but gradually declines with the approach of autumn; while on the other hand, on the approach of spring the mean temperature of the atmosphere is daily augmented, and the insect becomes aroused from its long slumbers by the steadily increasing warmth of its dwelling; its respiration is then excited, its fluids circulate more quickly, and the nutriment stored up within its body when it entered its sleeping condition having become exhausted, it is soon stimulated by the calls of hunger*, which the more perfect aeration of its fluids and the activity of all its functions induce within it; it makes a powerful effort to escape from its prison house, and pioneers its way through the soil to a new life, a life of activity,—directed in its proper course by the less consolidated state of the earth, in the passage to its abode, with which, many months before, the careful parent bee had securely closed the entrance, to protect her delicate offspring from the intrusion of enemies. I have seen this insect at the moment of its first leaving its abode. It always takes several very deep and powerful inspirations before it first takes wing, and its temperature is then scarcely more than a degree or two above that of the nidus it has just left. The comparative amount of the temperature of this insect in its different states during the period of hybernation, as compared with the temperature of the soil in which it is living and with its temperature in the perfect and active period, is very interesting, and will best be shown in the accompanying Table.

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* Dr. M. Hall on Hybernation, Philosophical Transactions, 1832, Part I. p. 22.

Table III.

Showing the temperature of the Wild Bee, *Anthophora retusa*, Steph., in its larva, nymph, and perfect state, as compared with the temperature of the atmosphere and of its nidus during the period of hybernation, and also with that of the same insect in a state of activity.

No. of Obs.	Period of Observation.	Wind.	No. of Specimens.	Duration of Experiment.	Atmosphere.		Soil or Nidus.					Remarks.	
					Sun.	Shade.	Empty.	With Honey.	Larva.	Nymph.	Perfect.		
				m	°	°	°	°	°	°	°		
1	Sept. 17, 1832. P.M. 3		14	10		64	65·7				65·7	These observations were made in the open air at the natural haunts of the insects, when the temperature of the atmosphere was gradually subsiding, and at a season of the year when the perfect insect, which had never yet been exposed to the open atmosphere, was in a state of hybernation.	
2	P.M. 3½		24	5		64·5	65·7			64·5			
3	P.M. 4		24	5		64				64			
4	P.M. 4½		14	5		63·5			65·5				
5	P.M. 4½		14	5		63·5			63·5				
6	P.M. 4½		14	5		63			63				
7	P.M. 4¾		14	5		62					62·3		
8	P.M. 5		8	5		60	63·5		62				
9	P.M. 5¼					60	63		62·3	60·7			
10	P.M. 6½					59·5			62·3	60·5			
11	P.M. 6½					58·5			63				
12	21 P.M. 3		30	5		64	66·3		68·7				
13	P.M. 3½		30	25		63·7			67·3				
14	P.M. 3¾					63·5	66			67	66		
15	April 6, 1834. A.M. 12½	Brisk E.			60	54	56		59·5		59	Nidi with larva and perfect insect 1½ inch deep in soil, *dry sand*	Richborough bank, all these cells one inch and a half from the surface; *soil, dry sand.*
16	A.M. 12½				60 to 81	54	55·7		59·5		59·7		
17	A.M. 12½				60 to 81	54	55·7		59·5		58·5	Bank, but a little exposed to sun	
18	June 10, 1834. A.M. 10	Brisk S.E.			67·7 to 73	67·5		84·4			84	Nidus open, without larva	
19	A.M. 10					67·5		80				Nidus closed, with larva	
20	A.M. 10					67·5		79·8				Nidus closed, with larva	
21	A.M. 10					67·5		78				Nidus closed, with larva	
22	A.M. 10					67·5		78				Nidus closed, with larva	
23	A.M. 10					67·5		77·7				Nidus closed, with larva	
24	A.M. 10					67·5		82·5			84	Nidus open, perfect female very active	
25	June 17, 1834. A.M. 10	Brisk S.W.			63	62·5	63·8	63·8				Nidus open	
26	A.M. 10					62·5		69				Nidus closed	
27	A.M. 10					62·5		69				Nidus closed	
28	A.M. 10					62·5		69				Nidus closed	
29	A.M. 10					62·5		68·5				Nidus closed	
30	A.M. 10					62·5		67·4				Nidus closed 2½ inches deep	Richborough bank, exposed to sun; *soil, dry clay.*
31	A.M. 10					62·5		67				Nidus closed	
32	A.M. 10					62·5		66·4				Nidus closed, 3 inches deep	
33	A.M. 11				72	66·4		75·7				Nidus closed, 1¼ inch deep	
34	A.M. 11				72	66·4		74·3				Nidus closed, 1¾ inch deep	
35	A.M. 11				72	66·4		78·7			80·7	Nidus open, 2 inches deep, female just returned from flight in cell	
36	A.M. 11				72	66·4		68				Nidus closed, 3 inches deep	
37	June 19, 1833. A.M. 8½		1		84·5	62·5	71	71	73·5		82	Larva in its cell, female just returned from flight, (near Canterbury.)	
38	June 20, 1833. P.M. 1		1			66	67	68	68·5		72·7	Female in her cell, inactive	Richborough.
39	P.M. 1		30	5		66	67	68	71				
40	Aug. 21, 1834. P.M. 1		1			70·5	76·2	74		74·7		Very few nymphs have yet become perfect	

From this Table it is seen that in the autumn, while the larva of the Anthophora continues active in its cell, its temperature is higher than that of either nymph or perfect insect, while the nymph, which has in reality a lower temperature than either the larva or perfect insect, being at that time in a state of activity, or degree of excitement inferior to that of the larva, and superior to that of the perfect insect, has a temperature in its cell intermediate between that of these two conditions. It was evident to me while making these observations that these apparently contradictory facts arose only from the circumstance of the perfect insect being then in a state of far more complete hybernation than the nymph, which, as well as the larva, was less able to maintain its temperature when raised to a certain amount than the perfect insect. But when the season of hybernation is over, and the swarthy female bee is roving abroad in the sunshine of the months of May and June, she has a temperature, as shown at Nos. 18, 24, 35, 37, and 38, very far above her temperature in the states of larva and nymph, or than what is possessed by her only a short time before she quits her cell in the months of March and April, when her temperature is scarcely higher than that of the larva, as shown in Nos. 15, 16, and 17. But if the perfect bee be taken from her cell cither at the end of March or at the commencement of her hybernation in September, her temperature of body after a few inspirations will be raised to two or three degrees above that of the atmosphere, but if undisturbed the insect always endeavours to sink again into a state of repose, and the temperature of her body becomes that of the surrounding medium. The soil in which the hybernacula of these insects are formed being of sand or clay, which are bad conductors of heat, always continues of a more uniform temperature than the open atmosphere, and is less subject to variations through the alternating and often suddenly changed temperatures of day and night, so that the insects are neither exposed on the one hand to the chilling hoar frosts of midnight, nor to the scorching sun of noon, which even in April, as shown on the Table, Nos. 16 and 17, may raise the thermometer to 81° Fahr. on the surface of the bank, while the insects in their nidi at only $1\frac{1}{2}$ inch or 2 inches deep are preserved in an almost uniform temperature of 56° Fahr.; and when the perfect insects have left their dwellings and are again filling the bank with cells and storing them with ova and with honey-paste for the support of the future young, the temperature of the same cells may be raised to 80° or upwards, a temperature which perhaps is then necessary for hatching the ova, and rearing the larvæ in their earliest condition.

5. *Inordinate Excitement.*

The great rapidity with which, as we have just seen, the temperature of an insect is raised from being almost on a level with that of the surrounding medium to several degrees above it, would naturally lead us to conclude that a much larger amount of heat is in reality generated than what is indicated by the thermometer, and that since the heat evolved within the body of the insect becomes perceptible through means of the thermometer so very rapidly, it is fair to suppose that the insect parts with it with nearly equal facility, and that a very large proportion evolved passes off to the surrounding atmosphere or medium in which the insect is inclosed, and that when such medium is of given small extent its temperature becomes raised as well as that of the insect, and is appreciable by the thermometer. This is in reality the case; and Dr. Burmeister* has already imagined it to be so, but he does not appear to have made any observations of his own in order to prove it, but refers to the observations before noticed by Hausmann†. I remarked the fact during my earlier observations on the temperature of insects in 1834, when endeavouring to ascertain the actual amount of temperature in the common Humble Bee in a state of rest and in a state of great excitement, and when endeavouring also to ascertain whether the amount of temperature in a single insect is equal to that of an indefinite number of individuals. I had long suspected that this could not be the case, and that, for instance, the temperature of a hive of bees in winter, stated by Huber to be equal to 80° Fahr., could not be equal to that of a single individual at the same period. Previous observations had induced me to believe that the temperature of a single insect is only a few degrees above that of the medium in which it is living, and that the actual heat of the insect is increased in proportion to the amount of its respiration; that when an insect is at rest its temperature is comparatively low, and that it becomes greatly increased during violent activity; and further, that a number of individuals confined in a given space can raise the temperature of that space to a great amount. With these views I inclosed a single female of *Bombus terrestris* in a glass-stoppered phial of three cubic inches capacity, having first noted the temperature of the atmosphere within the phial, and of that of the external atmosphere immediately around it, both of which stood at 66°·9 Fahr. The bee was allowed to remain about five minutes in the phial in a state of great activity, and its temperature was then taken by pressing the bulb of a thermometer against its abdomen. The mercury rose to 73°·4 Fahr., or 6°·5 above the temperature of the atmosphere, while the temperature of the atmosphere of the phial was raised to 68°·2 Fahr., or 2°·3 above that of its original temperature. Three other individuals of the same species were then added, and the whole four continued in a state of excitement until the mercury rose to 74°·5. It was thus proved that a single individual when excited raises the temperature of the surrounding medium, and that several individuals collectively will increase the temperature of that medium beyond what it could possibly be increased by only one.

In the next experiment, the atmosphere being 69°·4 Fahr., five individuals of the same species were confined in the same sized phial as the one just employed, and after remaining in a state of great excitement raised the temperature of the phial to 72°·5, a difference of 3°·1, while the temperature of the five excited bees was 76°·3. In another experiment, when a single bee was allowed to remain at rest with the thermo-

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[*] Manual of Entomology, p. 403. Translated by W. E. Shuckard, Esq. M.E.S. 1836.

[†] De Anim. Ex. Respirat. p. 68.

meter pressed against its abdomen until it had become perfectly quiet, the mercury rose only to about one degree above that of the surrounding medium. These experiments appeared to indicate that the quantity of heat evolved is in the ratio of the degree and activity of respiration.

On the 9th of June 1834 three female specimens of *Bombus terrestris, B. lapidarius,* and *B. muscorum,* all of which had been captured about three hours previously, were submitted to experiment, great caution being taken to prevent anything from interfering with the correctness of the observations. The temperature of the atmosphere and of the phials employed on this occasion was 68° Fahr., and the time occupied in each observation was five minutes. *Bombus terrestris* raised the temperature of the phial to 72° Fahr., and maintained it at that height during the whole of the experiment, while the temperature of its own body was 77° Fahr. That of *B. lapidarius* at the end of the observations was 71°·5, and of *B. muscorum* 72°·2 Fahr. In the first of these observations the temperature of *B. terrestris* was gradually raised from the temperature of rest, or only two or three degrees above that of the atmosphere, 68° Fahr., to 77°. During the whole five minutes the insect continued in violent motion, and maintained the temperature of the stoppered phial at 72°, or 4° above the temperature of the phial at the commencement of the observation, while that of the insect itself was raised to 9°, or 5°·5 above that of the medium around it, which it had itself raised 4°.

At that time I imagined that this great amount of temperature, nearly 10° Fahr., was very nearly or quite the maximum amount of temperature that a single insect can generate, since a little more exertion, or longer continuance of excitement, would have made the insect perspire copiously. The occurrence of this phenomenon in insects, as in vertebrated animals, must be looked upon as the natural cooling process, and beyond which the temperature of the animal cannot be raised in a state of health. The second specimen, *B. lapidarius,* was feeble, and only in a moderate state of activity, and consequently did not raise the temperature of its body above the usual standard. The third specimen, *B. muscorum,* was very much excited, and its temperature rose to 72°·2, or 4° above that of the atmosphere. On the 9th of July 1834, atmosphere 69°·8 Fahr., I placed a single specimen of *B. Jonella* immediately after it was captured in the stoppered phial employed in the previous experiments. The phial was closed, and the insect continued in a highly excited state for six or eight minutes. When it had become quiet a thermometer was very carefully introduced to the bottom of the phial without touching the insect, and the mercury rose to, and was maintained at 74°·7, or 5°·8 above that of the atmosphere and of the phial at the commencement of the observations. The insect then became excited, and the thermometer was held near enough to touch the tips of its wings. The temperature of the air in the phial immediately sunk to 72°·5, being a diminution of 2°·2. This observation was several times repeated with the same results, so that while confirming the previous conclusion respecting the evolution of heat, it shows also another interesting fact, viz. that the vibration of the wings tends to cool the body of the insect during flight, and moderate its temperature. But the power of radiating from its body into the surrounding atmosphere is not confined to the insect in its perfect state only, but exists also in the larva, as I have had opportunities of observing in the larvæ of the Sphinges, Puss Moth, &c. From these observations it is clear that a very large proportion of the heat evolved by insects in all their states passes off into the surrounding medium, and that the amount of heat evolved is in proportion to the degree of excitement and consequent quantity of respiration.

III. *Temperature of different Tribes of Insects.*

Having found that every insect maintains its own temperature of body, and that the amount of this temperature differs in the different states of each insect, it yet remains to be seen which are the families that generate the greatest amount, and what relation that amount in the different families bears to the habits and localities of the species. Our previous observations lead us to anticipate the fact that the volant insects in their perfect state have the highest temperature, while on pursuing the inquiry it is found that those species which have the lowest temperature are constantly located on the earth. Among the volant insects, those hymenopterous and lepidopterous species have the highest temperature which pass nearly the whole of their active condition on the wing in the open atmosphere, either busily engaged in the face of day despoiling the blossoms of their honied treasures, or flitting wantonly from flower to flower and breathing the largest amount of atmospheric influence. Of these it may be almost superfluous to remark, the Hive Bee and its long train of near and distant affinities, and the elegant and sportive Butterflies have the highest. Next to these probably are their predatory enemies the Hornets and Wasps, and others of the same order; and lastly, a tribe of insects which have always attracted attention, and in general are located on the ground, but sometimes enjoy the volant condition, —the Ants, the temperature of whose dwellings has been found to be considerably above that of the atmosphere: according to JUCH the temperature of an ant-hill was 17° REAUM. (70°·25 FAHR.), while that of the atmosphere was 10° REAUM. (54°·5 FAHR.). Next below the diurnal insects are the crepuscular, the highest of which are the Sphinges and Moths, and almost equal with these are the *Melolonthæ*. But the following experiments with the different tribes, while they still further illustrate the causes of the variability of temperature in insects, will also show the relative amount of heat evolved by different species.

Melolontha vulgaris, STEPH.

May 20, 1835, 7 A.M.—Having captured many individuals of this species of Chaffer Beetle on the preceding evening, I now found them perfectly quiet. The temperature of the external atmosphere was 60° FAHR., and that of the interior of the box in which they had been confined during the night was 61°·3, while on carefully introducing the bulb of the thermometer among the beetles, without disturbing them, the mercury rose to 61°·5 F. I then took a single beetle which had been remaining quiet, and having secured him with the forceps, opened the abdomen quickly with a pair of scissors, and introduced the bulb of a fine thermometer. The mercury immediately rose to 63°·3 Fahr., a difference of 2° above the temperature of the box, and 3°·3 above that of the atmosphere, and it was maintained at that height more than ten minutes, after which it sunk two or three tenths of a degree, as the energies of the insect became impaired. Half an hour after the above observations the temperature of the box had risen to 63° Fahr., and the insects were in motion; and when the bulb of the thermometer was merely allowed to rest upon the backs of several specimens, the mercury rose immediately to 65°·3 Fahr. When the beetles were again examined on the 23rd of May, at 7 a.m., they were perfectly quiet, having fasted since the last observation, being now a space of eighty-two hours since they were captured and had taken food. Atmosphere 60°·5 Fahr., of the box with the beetles 61°·3, thermometer introduced carefully among the beetles 61°·5, but when introduced as above into the body of a single beetle it rose to 63°·3 Fahr. One hour after this, at 8 a.m., atmosphere 64°, the temperature of the box was 66°, and the temperature of the interior of the body of a quiet beetle was 69°·2. At 8½ a.m., atmosphere 64°·5, thermometer applied to the exterior of the body of a female beetle that had been respiring very rapidly and preparing for flight, the mercury rose to 69°·3, and continued to rise in proportion to the degree of respiration of the insect. At 8¾ a.m. the insect just employed was placed on its back for half an hour, during which time it was respiring very rapidly, and endeavouring to escape, and its temperature had risen at the expiration of this period to 74°·5, while that of the atmosphere was 65°·5, a difference of 9°, so that although this insect had now been entirely without food for nearly eighty-four hours, its long abstinence had very little diminished its power of generating heat. A male specimen was then placed under almost precisely similar circumstances, and its temperature rose to 74°. At 6 p.m., atmosphere 64°·1, the same female specimen which had been employed in the morning, but which subsequently had been lying at rest for several hours, and was still reposing, had a temperature of 66°·3, a difference of only 2°·2, while the same male specimen that had been employed in the morning and had since been at rest, but was now respiring again very freely, and attempting to escape, had a temperature of 69°·1, a difference of 5° above that of the atmosphere, thus fairly leading to the inference that the amount of temperature is in proportion to the quantity of respiration.

At 7 p.m. May 24.—The temperature of a female specimen which had been at rest since the morning in its natural haunts clinging to the leaves of a lime tree was very carefully taken without disturbing it, by applying the thermometer to its abdomen, and was found to be only 62°·6, or one tenth of a degree only above that of the atmosphere; so that, like the temperature of the hybernating Mammalia, it had sunk down during its rest almost to a level with that of the surrounding atmosphere.

Melolontha solstitialis, STEPH.

June 26, 1834, A.M.—The specimens employed on the present occasion were captured on the evening of the 25th, the temperature of the atmosphere being 70°·5; a single specimen, which had been lying for some time at rest, had a temperature only of 70°·8. Five specimens which had previously been very active, and were now perspiring profusely, raised the temperature of a phial, whose cubic bulk was about two inches, from 70°·5 to 71°·4. Nine insects in a similar-sized phial raised the temperature in four minutes from 70°·5 to 72°·2, and a few minutes afterwards to 73°·2. During this time the insects were in a state of the greatest excitement. The bulb of the thermometer was not brought into contact with the bodies of the insects. When the thermometer was placed among the beetles, and in contact with their bodies, the mercury rose to 74°·5, a difference of at least 4° above the original temperature of the bottle; but this was far from being the full amount of the heat of these insects. During these observations I found that a large amount of heat generated by the insects confined in the phial becomes latent, and also that much caloric is radiated from the exterior of the phial, which becomes heated by the beetles and warm air within, as is proved by the fact that when the thermometer is held very close to the side of the phial without touching it, the mercury is considerably affected, and when the bulb of the thermometer is held in contact with the phial the mercury ascends the scale. In the present experiment it rose more than a degree when the bulb of the thermometer touched the side of the phial. It must not be forgotten that besides this difficulty in our observations on the temperature of insects, there is another which prevents us from knowing the exact amount of heat generated by the insect under examination. It is seen in these observations on the *Melolonthæ*, as before shown in the *Bombi*, that a large amount of the heat generated by the body of an insect quickly passes off into the surrounding medium. But if the excited state of the insect be excessive, and the consequent evolution of heat greatly exceed its usual amount, nature has resorted to another expedient for cooling down the animal body, through means of a profuse perspiration, which is carried on in insects perhaps to a greater extent than in other animals. Thence the amount of heat believed to be generated under certain conditions is only comparative; but when, as in experiments made on many specimens collected together, a profuse perspiration breaks out among the insects, the amount of temperature indicated by the thermometer introduced among them is much lower than the real amount that has been produced. This was the case in the present instance: the specimens were in a state of profuse perspiration, besides which they had fasted about eighteen hours. These facts were further illustrated by a subsequent experiment, in which eighteen specimens were employed in the same sized phial; they were crowded together, and allowed to remain about a quarter of an hour in a state of great activity, until they became gradually weakened, were bathed with perspiration, and were becoming quiet and asphyxiated with the carbonic acid gas produced during their confined respiration. The temperature of the atmosphere of the phial at the commencement of the observation was 71°·3, at the termination 73°·2, a difference of only 1°·9.

Lucanus cervus, Linn.

July 9, 1834, 9 A.M.—The temperature of the atmosphere being 67°, that of a male specimen of this insect, the great Stag Beetle, which had been fasting about two days, was very carefully taken while the insect was lying at rest, by placing the bulb of the thermometer for several minutes against the surface of its abdomen. The mercury rose to 67°·3, a difference of only ·3 of a degree of external temperature. At 9¼ A.M. I inclosed the insect in a stoppered phial of about three cubic inches capacity. The temperature of the atmosphere and of the phial was 66°·9. The insect remained perfectly at rest for a quarter of an hour, at the expiration of which the atmosphere of the phial was 67°·1. At the expiration of half an hour it was 67°·2, and the external temperature of the insect itself was 67°·4. During this period the insect had remained perfectly quiet, but at the expiration of an hour it began to find itself uneasy, and became slightly active, probably from the presence of carbonic acid gas in the phial, which had been generated during respiration. At 10½ the atmosphere of the phial was raised to 68°·5, or 1°·5 higher than at the commencement of the observation. The temperature of the atmosphere was now 66°·6. The insect was then removed from the phial, and the bulb of a delicate thermometer passed beneath its elytra, and the mercury rose to 68°·2. The insect was then placed on its back upon a smooth table, which occasioned it to exert itself greatly in order to recover its proper position. The bulb of the thermometer was applied as before, and the mercury rose to 69°·2, or 2°·6 above that of the atmosphere. At 4 P.M., atmosphere 71°, temperature of the insect beneath the elytra as before, was 71°·5.

Coccinella septempunctata, Linn.

It is almost impossible to ascertain with any precision the temperature of these interesting little insects, the Lady Cows, but I have sufficient reason for believing that it is very considerable, and corresponds with the views which ought to place them in the class of volant diurnal insects of high temperature. Had a larger number of specimens been employed, I have no doubt that the amount of heat evolved would have corresponded with the very high degree of respiration which they are found to possess. July, 9 A.M., atmosphere 67°·1, eight specimens were confined in a cubic inch phial, the temperature of which was 68°·2 , and when four of them were clinging to the bulb of the thermometer the mercury immediately rose, and was maintained at 68°·5, and after a short interval, when the insects had been moderately active, the thermometer stood at 69°, a difference of nearly one degree.

Meloe proscarabæus and *M. violaceus*, Linn.

These insects (the Oil Beetles), like their congeners the Blister Beetles (*Lyttæ*), have a temperature corresponding to their natural habits. The temperature of a number of *Lyttæ vesicatoriæ* was found by Juch to be several degrees above that of the atmosphere. This, to a certain extent, is the case with the Meloes, which love to bask in the heat of the sun, and respire a large quantity of atmospheric air. On the 1st of May I examined the temperature of a female *Meloe proscarabæus* soon after it was captured, and found its temperature amounted to very nearly 3° above that of the atmosphere when the insect was a little excited; but half an hour afterwards, when the insect had become more calm, it had subsided to 1°·5. I have in general found that the temperature of a single Meloe varies from one to two degrees above that of the atmosphere when not excited, and it seldom sinks down to the temperature of the atmosphere, because during the season in which the perfect Meloe is found it is almost always active. But when the newly developed Meloe first leaves its nidus in the earth in the beginning of March or end of February, I have seldom been able to detect more than one, or at most two tenths of a degree, in those of one species which I have had opportunities of examining, *Meloe cicatricosus*; and the same is the case with the nymph of the same species found in the month of August.

Gryllus viridissimus, Linn.

All the Grylli or locust tribes have comparatively a high temperature, and exist but a short time when the atmosphere around them becomes vitiated. This accords with their usual habits. We find them in the most sunny places, basking in the hottest rays, or chirping among the bushes at some distance from the ground. Hence we should conclude, *à priori*, that they have a high temperature. In a female specimen of *G. viridissimus*, captured on the 14th of July, when confined for a short time, atmosphere 73°·7, the temperature of the air of the phial had risen to 74°·7, and that of the insect at rest to 75°·4, but when excited 75°·8, a difference of 2°·1. When the insect had been confined in a phial about an hour it respired at the rate of 37 irregular and forcible contractions per minute. It was then becoming affected by the carbonic acid in the phial, the atmosphere of which had been raised to 74°·9, while the insect was perfectly at rest. In a subsequent experiment the temperature of the insect was 76°, that of the atmosphere continuing at 73°·7. When the observations were repeated at 7 o'clock on the morning of the 15th, atmosphere 63°·3, the temperature of the phial was soon raised to 67°·4, while that of the insect not excited was 68°, a difference of 4°·7. At 11 a.m., atmosphere 71°·6, insect 73°·6, phial 72°·7, and on the morning of the 16th at 7, the insect having fasted for thirty-six hours, atmosphere 69°·1, phial 70°, insect 70°·5, but when excited 70°·8, thus proving that a great diminution of its power of generating heat had taken place during its abstinence.

Staphylinus olens, and *S. erythropterus,* LINN.

Both these species of Rove Beetles have a comparatively low temperature, and it is often difficult even to detect the existence of distinct temperature in these insects, unless the individuals have become considerably excited. I have never yet examined *S. Olens* in the autumn before it retires to its hybernaculum, but in a specimen found in April, the temperature of the atmosphere being 60°·2, that of the insect was 61°·2. I could seldom find it rise higher, and it was often difficult to detect its existence at all. In *S. Erythropterus* I have seldom found the temperature higher than about ·5 above the atmosphere. It must thus be seen that there is a marked difference between the power which these insects possess of generating heat, and those which are more constantly in the open air; and when we examine the *Carabi* and *Tenebriones,* this difference of power is still more remarkable.

Carabus monilis, C. violaceus, and *C. nemoralis,* LINN.

June 18, 1834.—A specimen of the Ground Beetle, *Carabus monilis,* without being touched with the fingers, was carefully placed in a stoppered phial, the temperature of which, as well as of the atmosphere, was 67°·4. When the bulb of the thermometer was pressed against the under surface of the insect the mercury was not perceptibly affected, nor was there any change in the temperature of the closed phial during five minutes, all which time the insect was in a state of great excitement. This observation being made precisely as in the cases with the hymenopterous insects, the temperature of the *Carabus,* consequently, is exceedingly low. It ought to be remarked, however, that this insect had fasted during eighteen hours, and of course could not be expected to generate so great an amount of heat as the recently fed specimens. A second specimen, which had been recently captured, was then placed with the first in the same phial, and within a few minutes the atmosphere of the phial was raised to 67°·6, or ·2 of a degree above the previous temperature. A specimen of *Carabus violaceus* was then added to the number, and the three insects continued in a state of great excitement for several minutes, when the inside of the phial was found to be 67°·7, or ·3 of a degree above its original standard; but only a very slight additional effect was produced on the thermometer when applied to the body of the insects.

April 11, 1836, 3½ P.M.—I examined a single female specimen of *Carabus nemoralis* which had recently been captured. The insect was lying quiet when I made the first observation, by applying the thermometer to the under surface of its abdomen. The temperature of the atmosphere was then 61°·6, and that of the insect 61°·8. The insect then became active, and at the expiration of half an hour was 62°·8, that of the atmosphere having risen to 62°·5, while in ten minutes after this observation, the atmosphere being 63°, that of the insect was 63°·4. The difference, therefore, in this specimen in a state of great excitement, was only ·4 of a degree, while, as we have before seen, the difference in a Hymenopterous or Lepidopterous insect, in the perfect state, and under precisely similar circumstances, would have amounted to at least eight or ten degrees. From these observations it is evident that the natural heat of the *Carabi* is exceedingly low, and that their external temperature is scarcely more than ·2 or ·3 of a degree above that of the medium in which they are living; and although the respiration of these insects is higher than might at first be supposed from the small amount of their external temperature, yet they have the power of bearing the privation of oxygen for a very long time, and also of supporting the presence of some noxious gases; while they often reside in the coldest, dampest, and most unaerated situations. It was a specimen of this species that I once kept for several hours in hydrogen, and at the end of the observation found that it had expired a considerable quantity of carbonic acid gas during its confinement.

Blaps Mortisaga, Linn.

June 26, 1834.—The temperature of this species (which is truly a nocturnal one,) appears to be lower even than that of the *Carabus*. I placed two specimens in a phial, the temperature of which, and of the surrounding atmosphere, was 71°; but the thermometer was raised only ·1 of a degree, even after the insects had been for a considerable time in a state of activity. Two more specimens were then added, and the four insects were in a state of great activity for five minutes, when the temperature of the phial was only 71°·1, that of the insects themselves 71°·4, a difference of only ·4 of a degree above the medium in which they were confined. Thus the amount of power of developing heat in the *Blaps*, as in the *Carabus*, corresponds with the capability of supporting existence in a noxious medium, and also with its power of sustaining life during long abstinence. The *Blaps* will live for several minutes in a mixture of the most noxious gases, carbonic and even nitrous acid gas. I have confined one of this species in nitrous acid gas for three minutes, and it recovered in a quarter of an hour after being again exposed to the atmosphere. Another specimen was confined in nitrous acid gas for fifteen or sixteen minutes, and although it did not give any indications of recovering after being again exposed to the atmosphere for more than an hour, yet on my beginning to dissect the specimen, and after I had removed the whole under surface of the abdomen it began to recover, and in less than four minutes was so completely restored as to be able to walk about with nearly its usual speed. I have also confined other specimens in hydrogen for several hours, during which time they evolved a considerable quantity of carbonic acid gas, and did not appear to be at all inconvenienced by the medium in which they were placed. The low amount of heat in the species corresponds also with its power of going without food. One of this species is stated to have lived three years in confinement without food, and I have myself kept several individuals of this species about nine months fasting; it must be remarked, however, that this was during the winter months, from the latter part of autumn to the following spring, and may derive some explanation from what is now known with regard to the condition of insects during the season of hybernation. Yet I have also kept this insect nearly three months without food during the summer, the season of activity, but it has generally died at the expiration of that period.

These observations on different species will sufficiently show the great difference which exists between volant and creeping insects, in the power which they possess of generating heat, while, comparing all the physiological conditions of the species with each other, they seem to point to the source or cause of the development of heat. Thus the amount of heat is found to approach very nearly in volant Coleoptera to the amount in Hymenoptera. In both these tribes of insects the organs of respiration are of large extent, and the quantity and activity of respiration in both are great, while the quantity of heat developed appears to be in proportion to the quantity of respiration. Further, these observations lead to the conclusion that some of the volant Coleoptera (*Melolonthæ*) have a higher temperature, even in a quiescent state, than some of the terrestrial Coleoptera in a state of moderate activity, while the amount is increased in a much greater degree in volant insects in a state of activity, than in those Coleoptera which live entirely on the ground. It also appears that the temperature of *Crepuscular* insects, *Melolonthæ*, *Sphinges*, &c. is lower than that of the diurnal Hymenoptera, and this we might naturally expect would be the case, *Crepuscular* insects having, compared with their size, a lower degree of respiration than Hymenopterous insects, nearly all of which are diurnal species, and bear the privation of atmospheric air with greater difficulty than any other tribes.

Table IV.

A Table exhibiting the Temperature of Insects of different Species under various circumstances, and in their different states, compared with the Temperature of the Atmosphere at the time of making the observation.

Division 1. Volant Insects. (*a.*) Diurnal Species.

No. of Exp.	Order.	Species and state.	Period of observation.	No. of Specimens.	Atmosphere.	Insect.	Difference.	Remarks.
					°	°	°	
1	Hymenoptera, 1834.	Bombus terrestris, perfect.	June 7. A.M.	1	66·9	73·4	6·5	In each of these observations, which were all made within two or three hours of the insects being captured, the individuals were in a state of great excitement, excepting only *Bombus lapidarius*, which is a species that appears to be less readily excited than the others.
2	Hymenoptera, 1834.	Bombus terrestris, perfect.	A.M.	5	66·9	76·2	9·3	
3	Hymenoptera, 1834.	Bombus terrestris, perfect.	A.M.	1	66·9	73·4	6·5	
4	Hymenoptera, 1834.	Bombus terrestris, perfect.	A.M.	5	69·4	76·2	6·8	
5	Hymenoptera, 1834.	Bombus terrestris, perfect.	9. A.M. 12	1	68	77·5	9·5	
6	Hymenoptera, 1834.	Bombus lapidarius, perfect.	A.M. 12	1	68	71·5	3·5	
7	Hymenoptera, 1834.	Bombus muscorum, perfect.	A.M. 12	1	68	72·2	4·2	
8	Hymenoptera, 1834.	Bombus Jonella, perfect.	P.M. 6	1	68·9	74·7	5·8	
9	Hymenoptera, 1834.	Bombus Jonella, perfect.	P.M. 6	1	68·9	74·7	5·8	
10	Hymenoptera, 1834.	Bombus Jonella, perfect.	P.M. 6	1	69	75·4	6·4	
11	Hymenoptera, 1834.	Bombus Jonella, perfect.	P.M. 8½	1	68·2	71·3	3·1	
12	Hymenoptera, 1834.	Bombus terrestris, perfect.	29. P.M. 5	1	59	67·5	8·5	After great excitement.
13	Hymenoptera, 1834.	Bombus terrestris, perfect.	July 10. A.M. 12	......	70·5	77	6·5	Temperature of a nest of this species containing about thirty individuals and brood comb. The nest was contained in a box about seven inches square, and closed at night with a lid. Insects excited, but not in contact with the thermometer.
14	Hymenoptera, 1834.	Bombus terrestris, perfect.	P.M. 1½	......	70	80·2	10·2	
15	Hymenoptera, 1834.	Bombus terrestris, perfect.	P.M. 2½	......	70·5	80·4	9·9	
16	Hymenoptera, 1834.	Bombus terrestris, perfect.	Midnight 12½	......	68·5	80·3	11·8	
17	Hymenoptera, 1834.	Bombus terrestris, perfect.	July 11. A.M. 6	......	67	77·3	10·3	
18	Hymenoptera, 1834.	Bombus terrestris, perfect.	Midnight 12	......	67·5	78	10·5	
19	Hymenoptera, 1834.	Bombus terrestris, perfect.	July 12. A.M. 7	......	68·7	76·5	7·8	
20	Hymenoptera, 1834.	Bombus terrestris, perfect.	13. A.M. 8	1	71·8	84·1	12·3	Nurse Bees moderately excited. Nursing on a single cell, which contained a nymph that was developed from it about eight hours afterwards; during this incubating the Nurse Bee respired at the rate of 120 per minute.
21	Hymenoptera, 1834.	Bombus terrestris, perfect.	A.M. 8½	4	72·5	89·2	16·7	
22	Hymenoptera, 1834.	Bombus terrestris, perfect.	A.M. 8¾	7	72·5	90·2	17·7	
23	Hymenoptera, 1834.	Bombus terrestris, perfect.	A.M. 9	7	72·5	92·3	19·8	
24	Hymenoptera, 1834.	Bombus terrestris, perfect.	A.M. 9	1	72·5	91·5	19	
25	Hymenoptera, 1834.	Bombus terrestris, perfect.	A.M. 9½	7	72·7	91	18·3	
26	Hymenoptera, 1834.	Bombus terrestris, perfect.	A.M. 12	7	70·2	92·5	22·3	
27	Hymenoptera, 1834.	Bombus terrestris, perfect.	P.M. 1	7	72·2	85	12·8	
28	Hymenoptera, 1834.	Bombus terrestris, perfect.	P.M. 2½	4	72·5	94·1	21·6	
29	Hymenoptera, 1834.	Bombus terrestris, perfect.	P.M. 4	1	73·5	92	19·5	These observations show the great power which the Nurse Bees have of producing heat at will during the period of developing the nymphs. This evolution of heat is never produced when the insect is remaining perfectly quiet, but always occurs when the individual is much excited and respiring very rapidly. In those cases in the Table where a small degree of heat is indicated the insect was comparatively but little excited.
30	Hymenoptera, 1834.	Bombus terrestris, perfect.	P.M. 10	1	69	73·5	4·5	
31	Hymenoptera, 1834.	Bombus terrestris, perfect.	Midnight 12	1	68	83·2	15·2	
32	Hymenoptera, 1834.	Bombus terrestris, perfect.	14. P.M. 1½	1	69·5	89·4	19·9	
33	Hymenoptera, 1834.	Bombus terrestris, perfect.	P.M. 2	1	69·5	92·2	22·7	
34	Hymenoptera, 1834.	Bombus terrestris, perfect.	P.M. 3	1	69·5	91	21·5	
35	Hymenoptera, 1834.	Bombus terrestris, perfect.	P.M. 5	4	73·4	94·2	20·8	
36	Hymenoptera, 1834.	Bombus terrestris, perfect.	P.M. 11	1	68	83	15	
37	Hymenoptera, 1834.	Bombus terrestris, perfect.	15. A.M. 8½	1	68·2	88·2	20	
38	Hymenoptera, 1834.	Bombus terrestris, perfect.	A.M. 9½	1	71	91	20	
39	Hymenoptera, 1834.	Bombus terrestris, perfect.	A.M. 10½	1	72	93·2	21·2	
40	Hymenoptera, 1834.	Bombus terrestris, perfect.	P.M. 10	1	72·2	91·9	18·7	
41	Hymenoptera, 1834.	Bombus terrestris, perfect.	P.M. 11	1	71·6	85	13·4	

TABLE V.—A Table exhibiting the Temperature of Insects of different Species under various circumstances, and in their different states, compared with the Temperature of the Atmosphere.

Division 1. VOLANT INSECTS. (*b.*) Crepuscular Species.

No. of Exp.	Species.	Period of observation.	Bulk in cubic in.	Weight in grs.	Atmosphere.	Insect.	Diff.	Respirations.	Pulse.	Remarks.
1	Sphinx ligustri, A. (perfect male)	May 25. P.M. 2	....	34·2	64°·6	65°	°·4	....	....	At rest, but not sleeping.
2	Sphinx ligustri, A. (perfect male)	P.M. 2¼	....	34	64·6	65·2	·6	....	....	At rest for a quarter of an hour.
3	Sphinx ligustri, A. (perfect male)	P.M. 2½	....	....	64·4	64·7	·3	....	....	Still at rest.
4	Sphinx ligustri, A. (perfect male)	P.M. 2¾	....	34	64·3	65	·7	....	....	A little disturbed.
5	Sphinx ligustri, A. (perfect male)	P.M. 3	....	34	64·2	65·4	1·2	....	....	Has been aroused, but is again at rest.
6	Sphinx ligustri, A. (perfect male)	P.M. 3	....	....	64·2	69	4·8	....	....	Violently vibrating its wings as in rapid flight.
7	Sphinx ligustri, A. (perfect male)	P.M. 3¼	....	....	64·2	69·4	5·2	....	....	Just taken its first flight since leaving the pupa state.
8	Sphinx ligustri, A. (perfect male)	P.M. 3½	....	....	64·2	65·2	1	....	....	Has been sleeping since last observation.
9	Sphinx ligustri, A. (perfect male)	P.M. 3¾	....	....	64·3	64·7	·4	....	....	Still sleeping.
10	Sphinx ligustri, A. (perfect male)	P.M. 5¼	....	....	63·3	63·5	·2	....	....	Still sleeping.
11	Sphinx ligustri, A. (perfect male)	P.M. 6	....	....	63·4	63·7	·3	....	....	Still sleeping.
12	Sphinx ligustri, B. (perfect male)	31. A.M. 11	....	....	70	79	9	....	127	Has been in rapid flight around my room.
13	Sphinx ligustri, B. (perfect male)	June 1. A.M. 12½	....	....	68	70	2	....	60	Active, but not excited.
14	Sphinx ligustri, B. (perfect male)	P.M. 1	....	....	68	72·6	4·6	....	151	Violently excited, as in rapid flight.
15	Sphinx ligustri, B. (perfect male)	P.M. 1′ 6″	....	....	68	72·3	4·3	....	110	At rest since the last observation.
16	Sphinx ligustri, B. (perfect male)	P.M. 1½	....	....	68	70·6	2·6	....	50	Perfectly quiet.
17	Sphinx ligustri, B. (perfect male)	P.M. 1¾	....	....	68	69·8	1·8	....	45	Still at rest.
18	Sphinx ligustri, B. (perfect male)	P.M. 1¾	....	....	67·8	69·4	1·6	....	41	Still at rest.
19	Sphinx ligustri, B. (perfect male)	P.M. 2¼	....	....	67·6	69·6	2·0	....	53	A little aroused.
20	Sphinx ligustri, B. (perfect male)	P.M. 2¾	....	....	68·8	70	1·2	....	41	At rest.
21	Sphinx ligustri, B. (perfect male)	P.M. 3	....	....	69·5	75	5·5	42	139	Greatly excited in flight for a long time.
22	Sphinx ligustri, B. (perfect male)	P.M. 3½	....	....	68·8	69·3	·5	18	49	Has been at rest since last observation.
23	Sphinx ligustri, B. (perfect male)	P.M. 4¼	·18	22·4	68·8	69	·2	15	42	Perfectly quiet.
24	Sphinx ligustri, C. (perfect female)	2. P.M. 6	....	39	67·5	73	5·5	....	....	Excited, three hours after coming from the pupa state.
25	Cerura vinula, D. (perfect)	Apr. 22. P.M. 4	....	24·6	63·7	63·9	·2	....	....	Half an hour after coming from the pupa state.
26	Cerura vinula, D. (perfect)	P.M. 4½	....	....	63·5	63·8	·3	....	....	A little active.
27	Cerura vinula, D. (perfect)	P.M. 5½	....	22·6	61·4	62	·6	....	....	Slightly disturbed.
28	Cerura vinula, D. (perfect)	P.M. 6½	....	22·6	61·2	62·4	1·2	....	....	A little more excited.
29	Cerura vinula, D. (perfect)	23. A.M. 7½	....	....	59·1	60·1	1	....	....	Quiet for several hours, but is now moving.
30	Cerura vinula, D. (perfect)	A.M. 7¾	....	....	59·1	61·6	2·5	....	....	Is becoming excited.
31	Cerura vinula, D. (perfect)	A.M. 8	....	....	59	62·7	3·7	....	....	Considerably excited.
32	Cerura vinula, D. (perfect)	A.M. 10	....	....	58	60·2	2·2	....	....	Has been quiet for two hours, is now excited.
33	Cerura vinula, D. (perfect)	P.M. 2¼	....	....	65·3	66·4	1·1	....	....	Quiet for some hours.
34	Cerura vinula, D. (perfect)	P.M. 2′ 20″	....	....	65·3	70·3	5	....	....	Very much excited.
35	Cerura vinula, D. (perfect)	P.M. 2½	....	....	65·3	72·9	6·6	....	....	Violently excited as in rapid flight.
36	Cerura vinula, E.	P.M. 2½	....	....	65·3	65·6	·3	....	....	Sleeping many hours in the same box with No. 29 (D).
37	Cerura vinula, D.	P.M. 3¼	....	....	64·5	65	·5	....	....	Has been resting with its abdomen on the thermometer since last observation.
38	Cerura vinula, D.	P.M. 5	....	....	61	62·4	1·4	....	....	Has been disturbed, but is now quiet.
39	Cerura vinula, D.	P.M. 5¼	....	....	60·4	61·9	1·5	....	....	Quite at rest, temperature of the atmosphere sinking.
40	Cerura vinula, D.	P.M. 5½	....	....	60·3	61·9	1·6	....	....	Slightly disturbed.
41	Cerura vinula, D.	P.M. 5¾	....	....	60·3	61·8	1·5	....	....	A little disturbed.
42	Cerura vinula, D.	P.M. 6	....	....	60·3	61·8	1·5	....	....	A little disturbed.
43	Cerura vinula, D.	P.M. 6¼	....	19·9	60·1	61·6	1·5	....	....	At rest.
44	Cerura vinula, D.	P.M. 6½	....	19·9	59·4	65·3	5·9	....	....	Violently excited as in rapid flight.

TABLE VI.

A Table exhibiting the Temperature of Insects of different Species under various circumstances compared with the Temperature of the Atmosphere.

Division 1. VOLANT INSECTS. (*b.*) Crepuscular Species.

No. of Exp.	Order.	Species.	Period of Observation.	No. of Specimens.	Bulk in cubic ins.	Weight in grains.	Atmosphere.	Insect.	Difference.	Remarks.
			1835.				°	°	°	
45	Coleoptera.	Melolontha vulgaris, perf.	May 23 A.M. 7	1	......	......	61·3	63·3	2·	Male, quiet; internal temperature of body.
46	Coleoptera.	Melolontha vulgaris, perf.	A.M. 7½	6	......	......	61·3	61·5	·2	All the specimens perfectly quiet.
47	Coleoptera.	Melolontha vulgaris, perf.	A.M. 8	1	......	......	66	69·2	3·2	Quiet; internal temperature of body.
48	Coleoptera.	Melolontha vulgaris, perf.	No. 1. A.M. 8¼	1	......	......	64·5	69·3	4·8	Respiring quick, preparing for flight.
49	Coleoptera.	Melolontha vulgaris, perf.	1. A.M. 8¾	1	......	......	65·5	74·5	9·	Female. Respiration violent and long continued.
50	Coleoptera.	Melolontha vulgaris, perf.	No. 2. A.M. 9	1	......	......	65·5	74	8·5	Male. Under similar circumstances.
51	Coleoptera.	Melolontha vulgaris, perf.	No. 1. P.M. 6	1	......	......	64·1	66·3	2·2	Female; has been long at rest.
52	Coleoptera.	Melolontha vulgaris, perf.	No. 2. P.M. 6	1	......	......	64·1	69·1	5·	Male; respiring rapidly and trying to escape.
53	Coleoptera.	Melolontha vulgaris, perf.	No. 3. P.M. 7	1	......	......	62·5	62·6	·1	Female just taken from her natural haunts, in the open air in a state of perfect rest.
54	Coleoptera.	Mel. solstitialis, perf.......	June 27 A.M.	1	......	......	70·5	70·9	·4	Quiet.
55	Coleoptera.	Mel. solstitialis, perf.......	A.M.	5	......	......	70·5	71·9	1·4	Very active and perspiring profusely.
56	Coleoptera.	Mel. solstitialis, perf.......	A.M.	9	......	......	70·5	72·3	1·8	Very active.
57	Coleoptera.	Mel. solstitialis, perf.......	A.M.	9	......	......	71·3	74·5	3·2	Very much excited.
58	Coleoptera.	Mel. solstitialis, perf.......	A.M.	18	......	......	71·3	73·6	2·3	Quiet, becoming asphyxiated.
59	Coleoptera.	Coccinella 7-punctata......	July 9 A.M.	8	......	......	68·2	68·5	·3	The insect had been moderately active.
60	Coleoptera.	Coccinella 7-punctata......	A.M.	8	......	......	68·2	69	·8	Very active; had raised the atmosphere of the phial to 68°·5.
61	Coleoptera.	Lucanus cervus	A.M.	1	......	......	67	67·3	·3	Insect had been lying perfectly quiet.
62	Coleoptera.	Lucanus cervus	A.M. 9¼	1	......	......	66·9	67·4	·5	Perfectly quiet, but raised the temperature of the phial in ½ an hour to 67°·2.
63	Coleoptera.	Lucanus cervus	A.M. 10½	1	......	......	66·6	68·6	2·	A little active; temperature of the phial at 1¼ raised to 68°·5.
64	Coleoptera.	Lucanus cervus	A.M. 10¾	1	......	......	66·6	69·2	2·6	After great exertion.
65	Coleoptera.	Lucanus cervus	P.M. 4	1	......	......	71	71·5	·5	Insect has been lying quiet.
Division 2. TERRESTRIAL INSECTS. (*a.*) Diurnal Species.										
1	Coleoptera.	Proscarabæus violaceus...	April 11 P.M. 3½	1	·04*	11	62·4	63	·6	A very small female, somewhat excited.
2	Coleoptera.	Proscarabæus violaceus...	P.M. 3¾	1	·04	11	64·3	65	·7	Still excited; has been fasting.
3	Coleoptera.	Proscarabæus violaceus...	22 P.M. 6	1	·04	10·8	61·2	62·3	1·1	Active; has been feeding in warmer atmosphere.
4	Coleoptera.	Proscarabæus violaceus...	P.M. 6¼	1	·04	10·8	60·4	61·7	1·3	Has been active.
5	Coleoptera.	Proscarabæus violaceus...	P.M. 6½	1	·04	10·8	60	61·2	1·2	A little excited.
6	Coleoptera.	Proscarabæus violaceus...	23 P.M. 7	1	·05	12·7	58·2	59·3	1·1	A little excited.
7	Coleoptera.	Proscarabæus vulgaris ...	May 1 P.M. 3	1	·09	21·5	60·2	62·9	2·7	Just after being captured; excited.
8	Coleoptera.	Proscarabæus vulgaris ...	P.M. 3½	1	·09	21·5	63·1	64·6	1·5	Is now more quiet.
9	Orthoptera.	Acrida viridissima	July 14 P.M. 3½	1	......	......	74·7	75·4	·7	Female quiet, fasting for 2 days.
10	Orthoptera.	Acrida viridissima	P.M. 3½	1	......	......	74·7	75·8	1·1	A little excited.
11	Orthoptera.	Acrida viridissima	P.M. 3½	1	......	......	73	74·1	1·1	Insect confined one hour; quiet, but respiring irregularly and forcibly at 37 per minute: during this violent respiration at rest.
12	Orthoptera.	Acrida viridissima	P.M. 3½	1	......	......	74·9	76	1·1	
13	Orthoptera.	Acrida viridissima	15 A.M. 7	1	......	......	67·4	68	·6	Insect quiet.
14	Orthoptera.	Acrida viridissima	A.M. 11	1	......	......	72·7	73·6	·9	Insect a little active.
15	Orthoptera.	Acrida viridissima	16 A.M. 7	1	......	......	70	70·5	·5	A little excited; has fasted for the last 48 hours.
16	Orthoptera.	Acrida viridissima	A.M. 7	1	......	......	70	70·8	·8	Very much excited;
17	Coleoptera.	Staphylinus olens	April 23 P.M. 7	1	......	3·8	60·2	61·2	1·	Active.
18	Coleoptera.	S. erythropterus	18 P.M. ½	1	......	......	64·5	65·1	·6	Male specimen; active.
19	Coleoptera.	S. erythropterus	P.M. ½	1	......	......	65	65·6	·6	Male specimen; active.
(*b.*) Crepuscular Species.										
20	Coleoptera.	Carabus nemoralis	April 11 P.M. 3¼	1	·05	12·5	61·6	61·8	·2	Female; quiet.
21	Coleoptera.	Carabus nemoralis	P.M. 3¼	1	·05	12·5	62·5	62·8	·3	A little active.
22	Coleoptera.	Carabus nemoralis	P.M. 3¼	1	·05	12·5	63	63·4	·4	Very much excited.
23	Coleoptera.	Carabus monilis............	June 18 A.M.	1	......	......	67·4	67·4	......	Insect was excited, but did not evolve perceptible heat; had fasted for 18 hours.
24	Coleoptera.	Carabus monilis............	A.M.	2	......	......	67·4	67·6	·2	Active.
25	Coleoptera.	Carabus violaceus	A.M.	3	......	......	67·4	67·7	·3	In a state of great excitement.
26	Coleoptera.	Blaps Mortisaga............	26 A.M.	2	......	......	71	71·1	·1	Very active.
27	Coleoptera.	Blaps Mortisaga............	A.M.	4	......	......	71	71·3	·3	Still more active.

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* In my Paper on the Respiration of Insects in the Philosophical Transactions, Part II. 1836, p. 552, Table I. the cubic bulk of *Carabus cancellatus*, Nos. 33 and 34, and of *Meloe violaceus*, No. 36, has been erroneously printed 0·4 instead of 0·04.

IV. *Temperature of Insects which live in Society.*

We pass now to those insects which live in society, all of which belong to that great division the Hymenoptera, which have been shown to possess the highest temperature and greatest amount of respiration. Naturalists hitherto have examined only two genera of this great division with reference to the subject of temperature of these insects in their dwellings. These are the *Apis Mellifica*, or common Honey Bee, and the society of Ants; and the existence of a higher temperature than that of the atmosphere in the other families has only been inferred. Those species unto which I have devoted particular attention are the *Bombus terrestris* and *Apis Mellifica*.

Bombus terrestris.—1. *Temperature of Nests under observation.*

During the summer of 1830, having obtained a colony of this species, with the original parent bee, from the neighbourhood of Richborough, near Sandwich, (which locality had before that time afforded me opportunity of observing the habits of other species of this interesting family of insects,) I removed it from its locality in the earth to my own residence, the distance of a mile, and placed it in a small insect breeding cage for the purpose of more closely watching the economy of this species. The bees at first were somewhat irritable, and of course were kept in close confinement, and were fed with moistened sugar; but within a day or two they became quite accustomed to their new residence, and I had ample opportunity of watching the economy of the nest. On the third day they were placed on a table in my sitting-room near the window, which remained open, and also the door of the cage, that the bees might go abroad and return at pleasure, which they did with as much regularity after the first day or two as if the nest had been placed in its proper locality in the earth. I had thus most ample opportunity of watching their habits. The nest consisted of from forty to fifty individuals, and it gave me great pleasure in being able to confirm many of the statements made respecting these insects by Huber. During the time the bees were in my possession, a period of nearly three weeks, I observed upon introducing a thermometer among them, that the temperature of the nest varied at different times, and was considerably higher when they were in a state of excitement; but the circumstance did not then attract my particular attention. In the summer of 1834, while engaged with the observations before detailed, I determined to repeat the observation which I then remembered having made in 1830; and accordingly on the 10th of July 1834, having taken a nest of *Bombus terrestris* with brood comb, it was placed on a table near the window of my apartment, in a small box about eight inches square, and four deep, covered with green gauze, and after the first day's confinement the bees were allowed to go and return as on the former occasion. Soon after commencing my observation, I was interested in observing that the bees were at first greatly affected and agitated by the slightest noise, such as the removal of a chair, or one's footsteps about the room, or the passing of carriages along the road, which was at least thirty feet distant from the window of the apartment; but they were not in the slightest degree affected by persons talking loudly in the room, while a gentle tap with one's finger on the table put them immediately into a state of the greatest agitation. Hence during the observations it was necessary to be cautious, and not disturb the bees when wishing to take the temperature of the nest. The bees, however, in the course of a day or two became accustomed to their situation, and were not disturbed by slight noises or vibrations; and I was then enabled to take their temperature under all circumstances. The observations were commenced at 12 A.M., July 10, about two hours after the bees were placed in the box. The temperature of the atmosphere was then 70°·5 Fahr., that of the box and nest 73°; but when they became excited it soon rose to 77° but gradually subsided again to 73° as the bees became quiet. The thermometer was introduced very carefully under the gauze covering, and was not allowed to touch the bodies of the bees in this and the subsequent observations. At 1½ P.M., the insects having remained at rest for more than a quarter of an hour, atmosphere 70°, the thermometer, introduced as before, rose to 75°, and in a few minutes afterwards, when the bees had become much excited, to 80°·2, a difference of 10°·2 between the temperature of the atmosphere and that of the box; and when the body of a bee touched the bulb of the thermometer, even but for an instant, the mercury immediately rose at least a degree on the scale. At 2½, atmosphere 70°·5, bees quiet, atmosphere of the box 76°; but when they became much excited it rose in four minutes to 80°·4. At 12½ midnight, atmosphere 68°·5, interior of the box was 73°, the bees having been quiet during the previous nine hours; but when they became greatly excited it rose to 80°·3, a difference of 11°·8. At 6 o'clock on the following morning, July 11, atmosphere 67°, the interior of the box was 71°, but when the bees became much excited it rose to 77°·3. At 12½ midnight, atmosphere 67°·5, box with bees at rest 73°, when agitated 78°. At 7 A.M., July 13, the box in which the bees were confined had remained closed during the night, which had been perfectly calm and still, and at the time of making the present observation there was not a breath of wind stirring; indeed the air was suffocatingly calm, and its temperature 68°·7; when the thermometer was carefully introduced under the lid of the box the mercury rose to 72°, which was the temperature of the interior of the box around the nest, but when the thermometer was placed in the nest itself the temperature stood at 76°·5.

2. *Nest of Bombus in its natural haunts.*

Having proceeded thus far with my observations on the temperature of the nest, removed from its proper locality in the earth for the purpose of experiment, it became a matter of interest to endeavour to ascertain its temperature while undisturbed in its natural haunts. Having at length discovered the nest of a species of Bombus nearly allied to *Bombus terrestris* situated in a shaded chalk bank near the ground, and about eight inches from the surface, at 10 A.M.,—the temperature of the atmosphere in the shade four feet from the ground being 68°·7, while that of the exterior of the chalk bank in which the nest was situated, and near the entrance to it was 66°,—I very carefully introduced a small thermometer without disturbing the inmates, and found that the temperature of the interior of the nest was 83°, but in a few minutes it rose to 85°; it was thus evident that the temperature of the nest upon which I had made the preceding observations was at about its average temperature in its natural haunts.

3. *Nurse Bees.—Voluntary Power of generating Heat.*

The above experiments on the nest of *Bombus terrestris* thus confirmed the results of my observations made a short time before on individual insects with regard to the rapid transmission of heat from the body of the animal when in a state of excitement, and also in a less degree when in a state of rest; but during the time I was engaged upon them they also afforded me a new and totally unexpected phenomenon, and one which is not a little interesting and important as regards its connection with the origin of animal heat;—it was the capability which these insects possess during the act of incubation on the cells which contain nymphs, of increasing their own temperature many degrees above that of the surrounding medium, of in fact a voluntary power of generating heat through means of respiration. Huber has stated that there are certain individuals in the nests of the Humble Bees, and among the bees in a hive, which at a particular season of the year are employed to impart warmth from their bodies to the young bees in the combs by brooding over them, and these he called Nurse Bees. It gives me great pleasure in being able to bear testimony to the correctness of his statement, particularly with regard to those in the nest of the Humble Bee, which I had ample opportunity of observing. These individuals are chiefly the young female bees, and at the period of the hatching of nymphs they seem to be occupied almost solely in increasing the heat of the nest and communicating warmth to the nymphs in the cells by crowding upon them and clinging to them very closely, during which time they respire very rapidly, and evidently are much excited. These bees begin to crowd upon the cells of the nymphs about ten or twelve hours before the nymph makes its appearance as a perfect bee. The incubation during this period is very assiduously persevered in by the Nurse Bee, who scarcely leaves the cell for a single minute; when one bee has left another in general takes its place: previously to this period the incubation on the cell is performed only occasionally, but becomes more constantly attended to the nearer the hour of development. The manner in which the bee performs its office is by fixing itself upon the cell of the nymph, and beginning at first to respire very gradually; in a short time its respiration becomes more and more frequent, until it sometimes respires at the rate of 120 or 130 in a minute. I have seen a bee upon the combs perseveringly continue to respire at this rate for eight or ten hours, at the expiration of which time its body has become of a very high temperature, and on attentive observation the insect is often found in a state of great perspiration; when this is the case the bee generally discontinues her office for a time, and another individual will sometimes take her place. Very frequently the Nurse Bee respires with much less rapidity, and remains many hours on the cells. The very high temperature unto which the insects are able to raise their own bodies, and the cells upon which they are incubating at this period, will be best shown by detailing the continued observations on the nest.

At 8 A.M., July 13, when the temperature of the atmosphere was 71°·8, and the temperature of the interior of the box around the nest 72°·5, I inserted the bulb of a fine thermometer very carefully between the abdomen of several bees and the cells upon which they were incubating, and which contained nymphs, and found the body of a single nursing bee was 84°·1, while the exterior of some cells that contained nymphs, but which were not covered, was 76·5. At 8½ the temperature of the outside of the waxen cover, or top of the nest, was 77°·7, and that of the atmosphere 72°·5, while the interior of the nest, where the bulb of the thermometer was introduced among four bees which were nursing upon the cells, was 89°·2. At 8¾, atmosphere as before, when the thermometer was introduced among seven nursing bees at the same spot, three of which were large females, and the others males, which also assist in the process of incubating, the mercury of the thermometer rose to 90°·2 Fahr. At 9 A.M., atmosphere still 72°·5, the temperature of the same bees still incubating was 92°·3, and of others incubating in another part of the same nest 91°·5; at 9½, atmosphere 72°·7, that of the bees still nursing was 91°. At 12 A.M. the observations were resumed: in the interval between the last observation and the present time there had been a gentle shower with light wind, and the atmosphere had sunk to 70°·2; the temperature of the Nurse Bees on the cell was now 92°·5. The thermometer was raised to this height within about ten minutes, and was maintained at that standard as long as the bulb of the instrument was allowed to remain in contact with the bodies of the insects, while the temperature of some of the adjoining cells beneath the same cover, but which were not covered by the bees, was maintained at only 80°·2. Within a quarter of an hour after these observations were made three large female bees were hatched from the cells upon which the seven bees had been incubating; the temperature of the atmosphere was then 72°·2, while that of the Nurse Bees, which had now desisted from incubating, and consequently were respiring less rapidly, had sunk to 85°. It was thus evident that the greatest amount of heat is generated by the Nurse Bees just before the young bees are liberated from the combs, at which period they require the greatest amount of invigorating heat. It is at this period also, as before noticed, that the young bee is most susceptible of diminished temperature; it is then exceedingly sleek, soft, and covered with moisture; perspires profusely, and is highly sensitive of the slightest current of air. It crowds eagerly among the combs and among the other bees, and everywhere where there is the greatest warmth. In the course of a few hours it becomes a little stronger, and is less sensitive, and better able to bear a diminished temperature. It then moves about with less circumspection, and its wings, which at first are soft and weak, and bent upon its trunk, become plain and straight. When the young bee first leaves its cell it is entirely of a whitish or pale grey colour, but within half an hour the black markings on the thorax become very distinct, although they retain a tinge of grey colour for a much longer period; the yellow bands on the body and thorax are at first quite white, and it is not until an hour or two has elapsed that the principal yellow band on the thorax begins at length to gain colour, while it is several hours before the yellow bands acquire their full shade or degree of colour. During all this time the bee continues in an enfeebled state and takes no part in the business of the nest, but seeks for itself the warmest place among the combs, and it is not until sometime after it has acquired its proper degree of colour that it becomes active like the other bees, and is able to maintain its own proper temperature. It is thus evident, that the same principle which has been shown by Dr. Edwards to prevail with regard to the young of some of the mammiferous animals, that they are unable for a certain period after birth to generate and maintain within themselves a proper amount of temperature, but require to be cherished by external warmth, regulates also the development of the individuals of this family of Hymenopterous insects, from their pupa or nymph to their perfect state, and further tends to prove to us how universal and simple are the great laws which regulate the continuance of animal life. It is a curious fact that these bees do not incubate on the cells which contain only larvæ, the temperature of the atmosphere of the nest being sufficiently high for them in that condition; consequently the larvæ at an advanced period do not require so high a temperature before changing into nymphs as that which has just been shown to be required by the nymphs before coming forth as perfect insects. This will be shown in some observations made on larvæ in the nest now under examination, at the same time with those just described, and also with others which were made on nymphs. The temperature of the atmosphere being 76°, some of the cells which were open and contained larvæ were exposed in the nest, and the Nurse Bees therefore covered them lightly with dried grass, of which the nest of this species of Bombus is usually composed; but when the temperature of the atmosphere a few hours afterwards had risen to 73°·5, most of the dried grass with which these cells had been covered was removed, and the larvæ were more exposed; the temperature of these cells and the larvæ being 77°·4, while that of the cell of a nymph, with the Nurse Bee upon it, in another part of the nest was 92°, and subsequently when four large females were nursing around it was 94°·1, the temperature of the atmosphere being still 72°·5.

When there are no longer any nymphs which are soon to be developed into perfect insects the necessity for generating a larger amount of heat is diminished, and the Nurse Bees remain in a state of quietude; the temperature of the nest is then much lower than when young bees are about to be produced. This was the case on the 14th of July; the atmosphere was then 69°, while that of the nest was in no part higher than 72°·5; and even when the bulb of the thermometer was in contact with the bodies of several of the bees, the mercury scarcely rose to 73°·5, while at 12 o'clock on the preceding night, when the atmosphere was 68°, and several young bees were soon to come forth, the temperature of the box was 70°·5, and that of some bees very moderately excited in the act of nursing 83°·2. It is not only at the moment when the young bee is about to come forth that the Nurse Bees produce a larger amount of heat; they keep up the heat to a considerable amount for some time after the young bee is developed. At 1½ P.M., July 14, the bees were again incubating, the atmosphere 69°·5; the cells immediately beneath the cover of the nest were 89°·4. At 2 P.M., atmosphere 69°·5, the same cells were 92°·2, at which time most of the bees were crowding around this part of the comb, from which at 6 P.M. several young ones came forth. At 3 P.M., atmosphere 69°·5, the temperature of a single bee nursing on these cells was 91°. At 5 P.M., atmosphere 73°·4, atmosphere of the box was 75°·3, and that of four bees nursing 94°·2; while at 11 P.M., five hours after the young bees had been developed from this part of the comb and when no bees were present, the temperature at the very same spot was only 68°, exactly that of the open atmosphere; but in another part of the nest where the bees were again nursing it stood at 83°. It was in this way that the nurse bees constantly raised their own temperature and that of the cells upon which they were incubating whenever new bees were to be produced. In order to prove that this great amount of heat resulted directly from the temperature of the nursing bee, I placed the bulb of a thermometer on the back of a single individual that was nursing on the upper surface of a comb that was exposed to the temperature of the atmosphere, 71°·6, when it rose to and was maintained exposed as it was at 85°, while the temperature of the cell immediately after the bee had quitted it was 75°·3, and it was maintained at that temperature several minutes. In other observations I found that on one occasion, when the atmosphere was 72°·5, a single female bee while nursing upon a single cell, from which a perfect insect was developed about eight hours afterwards, had a temperature of 92°·3: the bulb of the thermometer in this instance was placed upon the cell immediately beneath the abdomen of the bee, which was respiring at the rate of 120 per minute. In another observation, when the temperature of the atmosphere was still the same, 72°·5, a single bee while nursing had a temperature of 94°·5, but a little while afterwards when the atmosphere was 72°·7 it had subsided to 91°.

These facts distinctly prove that bees have a voluntary power of evolving heat, while it seems only fair to conclude, on comparing the facts, that the quantity of heat produced in a given time and space, has relation to the number of respirations performed by the individual; and from the quantity of atmospheric air consumed, and of carbonic acid gas evolved, that animal heat is greatly and perhaps almost entirely dependent upon the chemical changes which take place in the air respired.

Temperature of the Hive Bee, Apis mellifica, LINN., *during the Winter.*

The many curious facts connected with the production of heat in the Humble Bee and other insects, naturally disposed me to wish to extend my inquiries to the ascertainment of that of the inhabitants of the hive, and fortunately circumstances enabled me to carry my wishes into execution, and commence my observations in the summer of 1835. They were continued almost uninterruptedly until the spring of the present year. I had long doubted the statements of naturalists that the Hive Bee does not hybernate, but maintains a very high temperature in its dwelling throughout the whole winter. This statement is so at variance with everything that is known with regard to the habits of insects in this country, especially those of the same class, the Humble Bees, that were it really the case it could not fail to be looked upon as quite anomalous in the economy of British insects. SWAMMERDAM, REAUMUR, and HUBER were all of opinion that the Hive Bee does not at all enter into a state of hybernation, but continues active during the winter. HUBER states expressly*, that so far from bees becoming torpid in winter, the temperature of a populous hive ranges from 86° to 88° FAHR. when the thermometer in the open air is several degrees below freezing. But these authors have been deceived with regard to the real fact. The Hive Bee certainly does not become *torpid*, but if entirely undisturbed it passes into that condition in which its temperature of body and quantity of respiration are very greatly diminished;—a state of deep sleep in the combs, but a sleep which, so far from being continued at a very low atmospheric temperature, then becomes broken, and is only continued at a moderate temperature. It is true that when the hive is disturbed in the winter, and it becomes so very readily, its temperature is soon raised to a great height. There can be no doubt but that this was the case in the observations made by the authors just noticed. They must necessarily have disturbed the bees when they introduced the thermometer to take the temperature of the hive, since, as I am about to prove, there are periods during the winter when the temperature of the hive is so greatly reduced, and the bees are so inactive, that the temperature is scarcely above that of the open atmosphere; and when the temperature of the air is increased rapidly, that of the hive is even below it for a short period, just as we saw in the observations on the temperature of larvæ during sleep; but if at that very period the hive become disturbed, its temperature is raised in the course of a few minutes by the excitement of the bees to a very great amount above that of the atmosphere, as shown in Table XVI. Nos. 204, 205, so that we may fairly conclude that HUBER and the other naturalists were deceived in their observations by arousing the bees while introducing the thermometer.

The observations detailed in the accompanying Tables on the hive were commenced in October, when only a very few bees venture abroad, and were continued with but few intermissions to the end of September in the following year, when the bees are becoming inactive, and the temperature of the hive is very much reduced. All my observations on the Hive Bee were confirmatory of the conclusions deduced from observations on other insects, and proved that this useful and interesting little species does not form an exception to the general rule.

From previous observations on the temperature of insects I had found that the

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* New Observations on the Natural History of Bees, by F. HUBER. (Translation.) Third Edition. Edinburgh, 1821, p. 224.

amount of heat developed in a given time was in proportion to the quantity and activity of respiration, and that the temperature of each species of insect can only be increased to a certain extent above the temperature of the medium in which it is living, and that when it has arrived at that point, whatever it happens to be, a copious cutaneous transpiration takes place; and if the temperature be still increased, the body of the insect becomes bathed in perspiration, and its temperature is immediately begun to be reduced. Now the degree unto which the temperature of insects may be increased above that of the medium in which they are living, varies in the different species as well as in the different genera of insects; each species has a certain standard of its own, beyond which its increase of temperature cannot be carried. In some insects, as in the Hive Bee, this may perhaps amount to from fifteen to twenty degrees, while in others it perhaps scarcely exceeds one or two degrees above the temperature of the surrounding medium. Besides this, it has been found that insects have a power of generating heat when confined in a given space, and that this power is in proportion to the activity of respiration. I have had numerous proofs of this fact in my observations on the varying temperature of the hive.

My experiments on the hive were conducted in the following manner: a common straw hive was placed with its entrance hole in the direction of another wooden hive, which was standing beside it in a bee-house, which was so constructed that the whole of the back part of the house could be removed or closed at pleasure. The proper entrance for the bees at the front of the bee-house was directly into the wooden hive, from the side of which there was a little covered communication with the entrance hole of the straw hive, to serve as a passage for the bees and a connection between the wooden and straw hive. The object of this was to prevent any sudden effect upon the temperature of the hive by changes which might occur in the temperature of the air without. The interior of the straw hive was thus subjected as little as possible to the variations in the open atmosphere, since the bees were obliged to pass through the empty wooden hive to its entrance hole before they could reach the open air. In order to make the experiment with the greatest accuracy, it was necessary that the bees should never be disturbed while making an observation, and therefore a small crow-quill sized thermometer, with a long free bulb, was passed through a hole just large enough to admit it in the top of the straw hive, about eight inches from the centre, and retained there during the whole of my subsequent observations without being removed or touched. The bees at first seemed a little inconvenienced by its presence, but within two or three days they became accustomed to it, and, as I had reason to believe, removed the comb and wax from around it, so that the bulb of the instrument was remaining about an inch within the free space of the hive, and the observations were then made at intervals, and with the greatest accuracy. The temperature of the atmosphere was taken with a thermometer of similar size and construction to the one used for the hive, and the two had been carefully compared before the first was passed into the hive. It was thus only necessary to notice from time to time the rise and fall of each thermometer, and to note the difference between them, the temperature of the air being of course taken in the immediate vicinity of the bee-house. By this course of observation it was found that the temperature of the hive, when the bees are in a state of repose, varies with that of the atmosphere, but that the change within the hive is never so rapid as in the atmosphere, unless the bees have been disturbed. When the temperature of the atmosphere has risen very suddenly, I have found it exceed that of the hive by one or two degrees, as in Table XVI. No. 173, provided the bees continue in a state of entire rest; but if, on the contrary, the temperature of the atmosphere be suddenly diminished, that of the hive will subside also, but with much less rapidity. These facts are shown in the observations, Table XIV. Nos. 85 and 86, and also in all the observations on the tables which were made after one o'clock at noon on each day during the winter. Sometimes the thermometers became exactly equal to each other, as in No. 124. On the other hand, when the bees are in a state of activity and respiring quickly, the hive is even then affected in the winter months by great changes in the temperature of the external atmosphere, particularly if these changes occur late in the autumn or in the beginning of the winter season. But a change in the temperature of the atmosphere in summer does not so readily affect the temperature of the hive, because in summer, when the general temperature of the atmosphere ranges from 45° Fahr. upwards, the bees are always in a state of activity, and are not themselves so readily affected by sudden atmospheric changes of temperature; while in winter, when the temperature of the season ranges from 45° Fahr. downwards, the bees are very soon affected by diminished heat, and become disposed to pass into a state of hybernation, in which state, as we have before shown, scarcely any respiration takes place, and the temperature of the little animals sinks down, or very nearly so, to the temperature of the medium in which they are placed; and if there be a direct and free communication between that medium and the external atmosphere, even down to that also. The amount of temperature in the individual bee I have been led to believe, as before stated, is in general from 10° to 15° Fahr. above the temperature of the medium in which it is living, when in a state of moderate excitement, but it seems liable to be still further increased at certain periods, as in the hive a short time before swarming, and when clustering together on the alighting board of the hive a short time before the colony departs. In some observations made on the 5th and 27th of June, when the temperature of the atmosphere ranged only from 56° to 58° Fahr., the temperature of the hive was 96° and 98°, being at least 40° above that of the atmosphere. Now the occurrence of this amazingly high temperature at these periods is readily explained by what we have learned of the habits of bees in incubating on the combs, and voluntarily increasing their heat, by means of respiration, before the new bees come forth, that being the season in which the population of the hive is perhaps doubled within a very few days. A similar explanation is also afforded to us, i. e. the excitement of the insects, and consequent greatly increased quantity and activity of their respiration,—of the surprising amount of temperature that may suddenly be developed in the hive, even in the midst of winter, when the bees are disturbed, as in the observations 190, 193, 195, 205, 214, 221, and many others on these tables. I have found that be the insects ever so quietly at rest, and even passing into a state of hybernating sleep, and although the temperature of the atmosphere be very much reduced, as in the observations just noticed, and also in Nos. 52, 134, 137, and 139, yet by exciting and arousing them, by gently tapping and shaking the hive, the bees are immediately put into a state of great agitation, and in less than ten or fifteen minutes the mercury will be raised on the scale of the thermometer upwards of 30° Fahr. above the temperature of the hive immediately preceding the experiment, when the bees were quiet, although the temperature of the atmosphere may scarcely exceed 35° Fahr., and although the temperature of the hive itself had previously been not more than 6° above that of the atmosphere. But this is not the greatest difference I have observed between the temperature of the excited hive and that of the atmosphere. It may appear surprising that any part of a well-peopled hive should at any time have a temperature lower than that of freezing, 32° Fahr., yet I have occasionally found this to be the case both during the last winter, 1836–37, and once in the preceding of 1835–36. In the latter instance it occurred but once, as indicated by the thermometer. This was in the hive upon which I have made the whole of my series of observations, and the hive at the time was well populated. It happened on the morning of January 2, 1836, at a quarter past seven, just before sunrise, when there was a clear intense frost, and the thermometer stood at 17°·5 Fahr. The bees were perfectly quiet, and the thermometer which had been untouched since its first introduction into the hive stood at 30°, or only 12°·5 above that of the atmosphere. The bees were then aroused in the usual manner by tapping the exterior of the hive, and in sixteen minutes the mercury of the thermometer had risen to 70° Fahr., but I was unable to excite the hive sufficiently to increase the temperature beyond this standard. This was 52° Fahr. above that of the external atmosphere, and 40° Fahr. above the previous temperature of the hive at that spot; but this was only the apparent, and not the real temperature of the hive, and resulted from the great accumulation of excited bees in the immediate vicinity of the bulb of the thermometer, within the hive, because a second thermometer having been introduced at a corresponding part of the top of the hive, at about five inches' distance from the first, indicated a temperature in that part of only 45° Fahr. These observations were sufficient to prove the incorrectness of attempting to ascertain the temperature of a hive of bees by occasionally introducing a thermometer among them and taking the temperature of the bees when excited by its presence. This circumstance was not lost sight of in my subsequent observations. At 12 a.m. on the same day the temperature of the atmosphere had risen to 30°·7 Fahr., while that of the hive, as indicated by the first thermometer, had subsided to 46° Fahr., and the bees within had become perfectly quiet. On the 5th of January at 1 p.m., the temperature of the atmosphere having risen to 50° Fahr., that of the hive stood only at 55° Fahr., while the bees aroused by the suddenly increased temperature of the atmosphere were becoming active; and when the hive was again excited by tapping it for a few minutes, the thermometer rose to 82°·2, a difference of 32° above that of the atmosphere, and 27° above that of the previous temperature of the hive, after which the temperature of the hive was maintained at 78° during several hours, while the bees continued in a state of activity, the temperature of the atmosphere being then congenial to their habits, and equal to the average temperature of the month of April, when the hive is again becoming active. But these are not the greatest amounts of temperature observed in the hive on its becoming excited during winter. In a second straw hive, which was exposed like the usual cottage hives to the open air, I found the interior temperature, at 10 a.m., on the 2nd of February, after the hive had been disturbed by tapping on its exterior, raised to 102° Fahr., the temperature of the atmosphere being 34°·5, a difference of 67°·5, while the first hive, which had not been disturbed, was then 48°·5, a difference of only 14° Fahr. between it and the surrounding atmosphere.

Although the hive be very much disturbed and its temperature be greatly increased by exciting the bees during the middle of winter, it will soon become quiet, and its temperature be reduced again to within ten or twelve degrees of the temperature of the atmosphere within ten hours, as in the observations No. 205 and following, made on the 2nd of January.

When the temperature of the hive has been increased suddenly, during the earlier or latter part of the winter, which we have just seen is the case when the hive is disturbed, the sudden increase of heat in their dwelling becomes intolerable to the little inhabitants, and they immediately endeavour to reduce it by ventilation, provided the temperature of the external atmosphere be not too low to endanger them, by exposing themselves at the entrance of the hive. When the temperature of the atmosphere is at or near 40° Fahr., at the time when the hive is disturbed the heat soon becomes oppressive, and although the degree of excitement within the hive be very great, its temperature is quickly moderated by the assiduity of the bees. I have often been amused by observing them, after the hive has been disturbed for a short time, although but a few minutes before there was not a single bee on the alighting board, come hastily to the entrance of the hive, and having arranged themselves within three fourths of an inch of the doorway, begin to fan with their wings most laboriously, to occasion a current of cool air through the interior of the hive. This act is the more assiduously performed, when, as in the hive under observation, there is not a free communication between the interior of the hive and the open atmosphere. On one occasion, No. 138, when the temperature of the hive had been raised to about 70° Fahr., the external atmosphere being scarcely more than 40° Fahr., the bees at midday maintained the temperature of the hive steadily at 57° by this mode of ventilating, the hive still continuing excited.

Although the bee can bear the transition from a hot to a cool atmosphere without inconvenience during the spring when the temperature of the atmosphere is only 45° Fahr., yet it cannot bear a sudden transition from hot to cold in the winter, even when the temperature of the atmosphere is at 40° Fahr. I had a striking proof of this while making the above observation, No. 138, at 11 a.m., on the 14th of November. The hive had been for a considerable time in a state of excitement, and its apparent temperature was raised to nearly 70°, while a great many bees were ventilating at the entrance, and others flew abroad into the open air while the sun was shining, but they very soon returned to the hive again. Shortly after this I found one individual lying within the entrance of the wooden hive apparently dead. On exposing it for a few minutes to the sun it began to revive, and was completely recovered, and able to fly again to the entrance of the hive, in six minutes. A thermometer placed close to the torpid bee in the sun rose only to 53°·5 Fahr. It was thus shown that the bee cannot bear a sudden transition in winter from a high to a low temperature, yet it will be seen by the Tables at Nos. 116 and 133, that the bees were active when the temperature of the hive was not higher than 43°, that of the atmosphere being 35° Fahr., so that it is not until the medium in which the bees are residing is below 40°, that the insects begin to pass into a state of repose.

From a gradually increased temperature through the months of March and April, the hive acquires its maximum amount of temperature in the months of May and June, the periods of the greatest activity, and when the largest proportion of young bees is produced. We are now aware of the circumstances connected with the great amount of temperature in the hive at this season, and of the power which the bees themselves possess of increasing it at pleasure, or as the necessity for imparting it to the young may demand. These facts will explain a circumstance connected with the temperature of the hive, which without a previous knowledge of them might have been of difficult solution. It is the circumstance before alluded to of one part of the hive being of a higher temperature than another. This is the case in the hive even when the bees are not in a state of excitement. I had been led to the observation of this fact during the winter when making experiments on the bees in a state of excitement. Being anxious to know whether this was also the case in the spring and summer, I introduced another thermometer through the top of the straw hive, at the same distance from the centre, but on the side opposite to the one previously inserted. This was on the evening of the 12th of May, when the temperature of the atmosphere was 58° Fahr. The instrument on passing through the top of the hive was plunged into a cell of honey, and the mercury rose to 78° Fahr., which of course indicated the real temperature at that time of the honey and interior of the hive. The mercury in the first or original thermometer was very quickly raised to 90° Fahr. in consequence of the excitement of the bees within the hive, but shortly afterwards sunk to 84°. During this time the temperature of the opposite side of the hive, as indicated by the newly introduced thermometer, rose to and remained at 79° Fahr. Here then we have a clear proof that the sudden increase of temperature when a thermometer is passed into the hive arises from the bees flocking around it, and it is also a proof that the natural temperature of these insects in a state of excitement may be raised to 20° Fahr. above that of the medium in which they are living, as shown in the observations on the Humble Bees. But this variation in the amount of temperature in different parts of the hive does not very much affect our means of judging of the average amount of the temperature of the hive at different periods when the thermometer remains entirely undisturbed, because it is found that when the temperature of the air is examined at about the same hour of the day, on two or more successive days, and all other circumstances being nearly the same, there will be but little variation in the average amount of temperature; so that we find the temperature of the hive, at the period of swarming, amounts to about 96° Fahr., while in the month of August it is seldom more than 80° Fahr., or perhaps 86°, even in the middle of the day, when the temperature of the atmosphere is often more than 78° Fahr. The cause of this difference between the amount of heat in the hive at this period and in the time of swarming is readily explained by reference to the facts connected with the production of heat. Less heat is in reality produced from the same volume of air consumed at the high temperature of 78° Fahr. than when the atmosphere is not more than 66° Fahr., as is often the case at the period of swarming, while in reality a far less volume of air is consumed in August than in May, because the bees are not in the same state of excitement. These facts readily account for the diminished temperature of the hive in the month of August, when the temperature of the atmosphere is in general higher than when the bees are most active.

During the period of swarming in 1836 I availed myself of the opportunity afforded me by the annular eclipse of the sun on the afternoon of the 15th of May, of watching the effect of diminished light and atmospheric temperature on the temperature of my hives, and the activity of their inhabitants, and found, as shown in the accompanying Table, that in proportion to the diminution of light the hives became quiet, and the temperature of the hives decreased until after the eclipse had passed its maximum, when as the light began again to increase, the activity of the hives became restored, and with it a considerable increase of heat.

Table VII.

Showing the variation in the Temperature of two Bee-hives compared with the Temperature of the atmosphere, as observed at Chichester, in Sussex, during the Annular Eclipse of the Sun on the afternoon of May 15, 1836.

No. of Exp.	Period of Observation.	Wind.	Weather, &c.	Atmosphere.	Hive No. 1.	Difference.	Hive No. 2.	Difference.	Remarks.
1	1836. ′ ″ May 15 A.M. 9	E.	Light wind, fine	67·7°	87°	18·3°	90·8°	23·1°	The bees have been clustering on the alighting board of the hive No. 2 for the last two days. The hive was now raised an inch to prevent swarming. In No. 1 there are no indications of swarming.
2	A.M. 10	E.	Light wind, fine	69·3	88·5	19·2	91·2	21·9	Hives calm, but not many bees abroad at work.
3	A.M. 12	E.	Calm, fine	71·5	93·6	22·1	90·7	19·2	Drones beginning to come abroad, no bees clustering.
4	P.M. 1½	N.	Light wind, fine	70	92·3	22·3	92	22	○ Abundance of bees around the hives, loud humming.
5	P.M. 2	N.N.W.	Light wind, fine	69·5	93·8	24·3	92·6	23·1	◔ *Eclipse* has commenced, many drones abroad, bees greatly excited, flying around the hives.
6	P.M. 2¼	N.	Light wind, fine	69·3	92·3	23	93·3	24	◔ Sunlight sensibly diminished, bees flocking home, very few go abroad.
7	P.M. 2½	N.	Calm, fine	67·5	93	25·5	93	25·5	◔ Light still diminishing, scarcely a bee goes abroad.
8	P.M. 2¾	N.	Light wind, fine	63·5	91·5	28	92·5	29	◑ Bees flocking home very rapidly, a few drones still abroad.
9	P.M. 3	N.	Light wind, fine	62	91·4	29·4	92·5	30·5	◕ Light greatly diminished. *Geotrupes stercorarius* on the wing.
10	P.M. 3¼	N.	Light wind, fine	59	91·3	32·3	91·7	32·7	● Light more obscured, hives quiet as in the evening, not a bee goes abroad; cocks crowing, town in the distance hazy, cool light wind, sky very clear.
11	P.M. 3 20	N.	Light wind, fine	57·5	87·5	30	90·8	33·3	◕ Eclipse past its maximum, two bees have just come home again.
12	P.M. 3½	N.	Light wind, fine	58	87·2	29·2	91·4	33·4	◕ Light sensibly increased, bees at the entrance of the hives and going abroad.
13	P.M. 3¾	N.	Less wind, fine..	57	85·5	28·5	90·7	33·7	◕ Light still increasing, a few bees going abroad.
14	P.M. 3 50	N.	Less wind, fine..	57·5	85·7	28·2	90·9	33·4	◑ Light much increased, bees still going abroad.
15	P.M. 4	N.	Less wind, fine..	57·8	87·1	29·3	91·4	33·6	◔ Great increase of light; one bee has again returned with pollen.
16	P.M. 4¼	N.	Light wind, fine	58·5	87·5	29	90·5	32	◔ Buzzing and activity in the hives increasing, bees departing.
17	P.M. 4 20	N.	Light wind, fine	58·7	86·7	28	89·8	31·1	◔ Eclipse nearly terminated. But few bees abroad from No. 1.
18	P.M. 4½	N.	Calm, fine	59·5	87·5	28	89·9	30·4	○ Bees abroad from No. 2; eclipse terminated.
19	P.M. 5	N.	Calm, fine	61·5	86·5	25	90·3	28·8	Bees abroad from both hives; sky clear, very fine.

5. *Quantity of Free Heat in the Hive.*

Having endeavoured to ascertain the quantity of heat radiated from the bodies of single insects, and also from one species of bee in society, I was desirous of gaining some information respecting the quantity of free heat developed within the hive. The information derived from the thermometer inserted at the top of the hive was not sufficiently satisfactory, owing, as before stated, to the bulb being very frequently in contact with the bodies of the bees. I therefore made the following trial, both with the view of preventing the hive from swarming and of ascertaining the amount of heat radiated from the bodies of the bees. So late as the middle of June, the bees in the hive No. 1 had not swarmed, but appeared at that time as if about to do so. I therefore elevated the straw hive upon a wooden one, of about thirteen inches square, with a hole in the top of it about eight inches in diameter, which allowed of a very free communication with the straw hive. In the back of this wooden hive was a window for observing what occurred within, and the bees were obliged to pass and repass through this wooden hive into the straw one above it. The hive being thus enlarged the bees did not swarm, but extended their combs from above downwards, and filled about one fourth of the interior with them. When they had become perfectly reconciled to their enlarged dwelling, a second thermometer, similar to the one introduced through the top of the straw hive, was passed through the side of the box, about three inches from the top, so that it might not touch the combs, from which it was distant about three or four inches, while its bulb extended about an inch into the interior of the box or wooden hive, and the mercury in the scale indicated from time to time the amount of free heat developed, uninfluenced by contact with the bodies of the bees. The original thermometer still indicated the apparent temperature at the top of the hive among the combs as before. The observations were begun upon the temperature of the wooden, or sub-hive, in the middle of July, when the bees had become more quiet than in the time of swarming, and when the internal temperature of the hive is diminishing. It was soon evident that the quantity of free heat developed under these circumstances in the lower part of the hive, where there were no bees congregating, was very considerable, and was often equal to, or even greater than that of the apparent heat of the top of the straw hive, where the bees were in a state of great activity. Sometimes the quantity of free heat at the bottom of the hive amounted to 12°·8 above that of the external atmosphere, when its temperature was 67°·2 Fahr., and when the temperature at the top of the hive was only 13°·1 above, even at 3 o'clock in the afternoon, at which time, in the month of July, the hive is generally hottest, from the numbers of bees which then return from the fields. Sometimes in the evening, when the temperature of the atmosphere is almost always sinking, the free heat in the lower part of the hive has amounted to 16°·8 above that of the external atmosphere at a temperature of 64°, while at the top of the hive the difference has been only 15°·7. In these cases the quantity of free heat developed must very far have exceeded the amount indicated by the thermometer, since the constant ventilation at the entrance of the hive admitted the cool air, and expelled the warm. In all the observations thus made care was taken to notice through the window at the back of the hive that there were no bees in contact with the bulb of the thermometer. This I had ample opportunities of doing, and found that when a bee alighted, even but for a moment, upon the bulb of the thermometer, the mercury rose in the scale at least one degree, and immediately subsided again when the bee had departed. This is a further proof that the temperature of a single bee in a state of activity is greatly above that of the medium in which it is living. But it may be urged, perhaps, that this proves very little, and that the rising of the thermometer may occur from the circumstance that the bee which came into contact with the bulb had passed suddenly from the top, and heated part of the hive, to the lower and cooler, and that the transition of the insect from one part of the hive to the other was too sudden to have allowed of its being cooled down to the temperature of the lower medium before it touched the thermometer. That this was not the case is proved by the circumstance that the same thing occurred both when the temperature of the medium in the upper hive was hotter, and also when it was cooler than that of the lower, and also when both were of exactly the same temperature. When the temperature of the external atmosphere is very high, as at 75° or upwards, the temperature of the interior of the hive, except at the period of swarming, is seldom more than a few degrees above it, either at the top or in the free space at the bottom of the hive. The bees then are generally very inactive, the heat becomes oppressive to them, and they leave the hive in great numbers.

6. *Mean Temperature during Summer and Winter.*

We have seen that the natural temperature of the hive during the winter is very much lower than during the summer, and that instead of the hive possessing a temperature of 86° Fahr., as stated by Huber and other naturalists, it occasionally has a temperature even below 32° in very low states of the atmosphere, while its mean, or average amount in the months of January and December, when it appears to have the lowest temperature, may not exceed 45°. It is, however, regulated by the temperature of the external atmosphere, being in a very mild season higher, and in a very severe season lower than its usual mean. Without very much digressing from the subject of the present paper, I cannot help remarking that a knowledge of these facts may lead us to a practical application of them, in the preservation and culture of the valuable insect which is the subject of these remarks, the Honey-bee. It tends to confirm our opinion of the utility and prudence of the practice which is adopted by some cultivators, of placing their beehives during the winter in vaults, or other subterranean recesses, where they may remain in quietude, and in an almost uniform temperature, unaffected by the changes of the varying season.

From the accompanying tables of the mean temperature of the hive, throughout nearly the whole year, it is seen that the mean temperature in the different days and months constantly maintains in every hive a certain relative amount of difference above the temperature of the atmosphere, and that although occasionally interfered with by casual circumstances, it is gradually increased from its minimum, in the month of January, when probably it is not more than 6° or 7° above the atmosphere, to its maximum, in May and June, when it amounts to from 25° to 26° or 27° Fahr., after which it again declines through the months of July, August, and September, until in the months of October and November it amounts to no more than 8° or 9°, and the bees are again passing into a state of inactivity. The mean difference of the first half of the year from February to the end of May, or up to the period of swarming, greatly exceeds that of the second half, from June to the end of November; in the first half of the year the difference varies from 17° to 21° Fahr., while in the second half it is only from 10° to 8° Fahr. It will also be seen from one of the accompanying tables that the mean hourly difference of temperature is almost uniform at the same hour and day of the same month in different years, even when the observations are made in different states and temperatures of the atmosphere.

V. *Temperature of Insects as connected with the other Functions of Life.*

On reviewing all the circumstances connected with the temperature of insects, we cannot fail to observe the remarkable coincidence between the amount of heat produced, and the quantity of respiration in these animals, under all the circumstances of their existence. We have seen that whether sleeping or waking,—whether inactive or in a state of great excitement,—the quantity of heat evolved by an insect is always in proportion to the quantity and activity of its respiration. But there are other circumstances which also claim our attention. When the temperature is increased, the circulation of the fluids of the insect are also much accelerated, and there is a greater amount of gaseous expenditure from the surface of the body. On the other hand it is observed, that when the process of digestion is suspended, not only is there a less expenditure of gaseous and fæcal matter from the surface of the body and from the alimentary canal, but the power and velocity of the circulation, the quantity of heat, and the activity of the respiration of the insect are diminished. These circumstances are readily demonstrated by experiments on insects, and lead us to inquire what relation subsists between the great functions of life, and the production, and variations, of temperature in these "little miniatures of creation," and whether the temperature of their bodies depends mainly upon one or more of these functions, or upon the agency of that inexplicable source of all the voluntary energies of the animal,—the nervous system.

1. *Respiration.*

The circumstances which affect the respiration of insects have been particularly considered on a former occasion*. It was then seen that the contractions of the segments of the body in insects correspond with the acts of respiration in other animals, and that these are greatest during a state of activity, and less frequent during a state of repose. It is exceedingly difficult to determine the number of these respiratory motions, per minute, in the larva state, even of the large Lepidopterous insects, and to ascertain what relation they bear to the temperature, quantity of respiration, and rate of pulsation of the dorsal vessel; but from a great number of observations on the larva of the Sphinx in its fifth or last period, I am inclined to think that they are not so frequent as in the perfect insect. It has been suggested by some naturalists that since the progressive movements of the larva are mainly performed by means of the longitudinal contractions of the body, that these are concerned in the function of respiration, and this appears highly probable from the circumstances which take place when a larva is submerged in spirits of wine or other fluid for the purpose of destroying it. At first it does not appear to be incommoded by contact with the spirit, but as soon as it attempts to inspire it is immediately affected, and the four posterior segments contract, and the whole body becomes shortened, as in the act of forcible expiration,

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* Philosophical Transactions, 1836, Part II. p. 547. *et seq.*

while strings of air bubbles issue from the spiracles, particularly from the posterior ones; an interval of a few moments succeeds, and then another contraction follows, and more air-bubbles issue forth; and this alternate contraction of the segments, and expiration of bubbles, takes place until the insect is completely asphyxiated, while its body becomes contracted both in length and diameter. From these circumstances it seems highly probable that the contraction of the longitudinal muscles of the body of the larva, during its progressive motions, are connected with the *expiratory* act of respiration of the insect, just as similar parts in the body and thorax of the perfect individual of the species are connected with the respiratory functions during the motions of flight. In every condition of the insect the number of respirations is in accordance with the activity of the animal, and with the quantity of air it deteriorates in a given time, and they are also in accordance with the amount of heat developed. Thus in the pupa state I have not observed more than three inspirations per minute, and these only when the pupa has been disturbed; and the number of these corresponds with the small amount of respiration, and the low power of generating heat in this condition. In the perfect insect of the same species, the Sphinx, when in a state of excitement after great exertion, Table V. No. 21, I have counted forty-two, but at the expiration of an hour and a quarter, No. 23, when the insect had become quiet, there has been only fifteen inspirations per minute. In the Hive Bee and Humble Bees, the number of respirations has amounted to from one hundred and ten to one hundred and twenty, when in a state of excitement, but when very moderately active, to no more than forty. The same, and even greater difference, is found in the Wild Bee, *Anthophora retusa*, Steph., in which, in a state of violent excitement, the number of respirations once amounted to two hundred and forty in a minute*; while in the very same insect when first removed from its hybernaculum in the autumn, or in the spring of the year, and when it has a temperature only a little above that of the medium in which it has been living, it has scarcely more than two or three respirations in the same space of time. In the common Green Grasshopper, when moderately excited, Table VI. Nos. 11, 12, and after it had fasted during several hours, there were about thirty-seven or thirty-eight. In all these cases the number corresponds with the amount of respiration or quantity of air deteriorated.

2. *Velocity of the Circulation.*

But there is not merely an accordance between the activity of the insect, its quantity of respiration, and amount of heat developed, but also between these and the general rate of pulsation, or the circulation of the blood in its body. This therefore demands our particular attention.

When an insect is remaining perfectly at rest, its rate of pulsation, like its respiration and temperature, is greatly diminished. We are enabled to observe the pulsation of the heart, or dorsal vessel, both in the larva and perfect state of many insects,

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* Philosophical Transactions, 1836, Part II. p. 550.

but in none better than in the large Moths and Sphinges. When an insect has remained for some time in a state of repose, both the power and rate of pulsation are greatly diminished, but are again increased immediately the insect awakes. The manner in which the pulsation takes place, as seen through the delicate skin of the larva of the Puss Moth, *Cerura vinula*, Steph., appears to be as follows: at the moment the insect begins to awake there is a slight extension of the posterior segments of its body, followed immediately by a slight contraction of the same parts; and almost immediately afterwards there is an increased motion in the posterior part of the dorsal vessel, in the twelfth or penultimate segment, where the vessel is broadest, and as shown by Carus and Wagner, receives a current of blood which flows into it on either side. The contraction, or ventricular action of the vessel, commences first in this segment, and is gradually continued onwards through the chambers of the vessel in the preceding segments by a series of successive impulses, from behind forwards, communicated in succession by the valves in each chamber*, but which in the Cerura and Sphinx are not observed through the skin of the insect. These contractions force along the blood through the chambers of the heart in the ninth, eighth, seventh, sixth, and fifth segments with intermitted or pulsatory motion, so that while the middle and anterior chambers are contracting the posterior is again filling. The auricular, expanding, or receiving action of the vessel begins also in the twelfth segment, where, indeed, the greatest amount of blood seems to be received from the body, although it is also received by the other valves in the different segments. Immediately the posterior valve has impelled the blood onward to the next one, it begins again to expand. If the action of the vessel be carefully examined, the expansion and contractions of the chambers in the different segments in gradual succession from behind forwards, at every impulse, may be readily observed. Each pulsation of the vessel is, I think, divisible into three periods: first, the *auricular*, or filling, which is rather the longest; second, the *ventricular*; and thirdly, the *period of rest*, which is immediately subsequent to the ventricular, but is of rather shorter duration. From these causes the true arterial motion of the fluids through the thorax of the insect is later by one whole contraction of the vessel than in the posterior segment or division of the organ; and it is also evident, on watching the motions of the vessel, that the period of rest is longer in the anterior or aortal portion of the vessel, which passes through the thorax, than in the posterior or true dorso-abdominal. It has been shown in other parts of this paper that after the insect has arrived at its full size as a larva there is a gradual diminution in its quantity of respiration and temperature; and it is interesting to observe that this is coincident with a similar diminution both in its actual weight and in the pulsation of its dorsal vessel, and that the diminution continues until after the insect has changed to its pupa state, as shown in the accompanying Table.

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* See Bowerbank on Circulation of Insects, Entomological Mag. vol. i. p. 240.

Table VIII.

A Table exhibiting the Temperature, Pulsation, Weight, &c. of the Larva of *Sphinx ligustri* during its last or adult period, and their gradual and coincident diminution after the ninth and tenth days of that period.

No. of Exp.	Species.	Period of Observation.	Atmosphere.	Insect.	Difference.	Pulsation.	Weight in Grains.	Fæces in Grains.	Increase in Grains.	Age.	Remarks.
		1834. h ′	°	°	°						
1	Sphinx ligustri (larva) ...	July 30. A.M. 8	70	......	......	...	....	......	......	....	Just entered its fifth and last skin.
2	Sphinx ligustri (larva) ...	P.M. 4	74·6	75·5	·9	...	15·1	......	......	8 hours	Quiet; has voided no fæces for 10½ hours.
3	Sphinx ligustri (larva) ...	31. P.M. 4	......	......	......	...	19·9	6	4·8	2 days	
4	Sphinx ligustri (larva) ...	Aug. 1. P.M. 4	......	......	......	...	27·4	......	7·5	3 days	Has voided no fæces.
5	Sphinx ligustri (larva) ...	2. P.M. 7	73·8	·2	·4	...	41·3	6·5	13·9	4 days	Sleeping.
6	Sphinx ligustri (larva) ...	3. P.M. 4 15	72·4	73·4	1·	50	56·6	8·6	15·3	5 days	Quiet.
7	Sphinx ligustri (larva) ...	4. P.M. 5	71·9	72·9	1·	50	69·1	19	13·5	6 days	Quiet, but not feeding.
8	Sphinx ligustri (larva) ...	P.M. 5 45	72·5	73·8	1·3	56	71·5	......	......	....	Active and feeding.
9	Sphinx ligustri (larva) ...	5. A.M. 9	71·3	72·6	1·3	51	77·5	11·4	6	7 days	Aroused and beginning to feed.
10	Sphinx ligustri (larva) ...	P.M. 8 30	69·9	71·1	1·2	51	85	10·8	7·5	....	Just aroused.
11	Sphinx ligustri (larva) ...	6. A.M. 7 30	71·1	72·3	1·2	50	90·5	16·9	5·5	8 days	Aroused; beginning to feed.
12	Sphinx ligustri (larva) ...	P.M. 5 30	70	71·2	1·2	47	93	12·5	3·5	....	Feeding.
13	Sphinx ligustri (larva) ...	7. A.M. 6	68·3	68·7	·4	36	98·8	14	5·8	9 days	Sleeping.
14	Sphinx ligustri (larva) ...	A.M. 6 15	68·4	69·3	·9	42	98·8	......	......	....	Aroused and beginning to feed.
15	Sphinx ligustri (larva) ...	P.M. 4 30	69·2	70·3	1·1	43	100·1	12·6	1·3	....	Quiet; feeding.
16	Sphinx ligustri (larva) ...	8. P.M. 3 30	72	72·9	·9	42	92·1	23	......	10 days	Very active; discoloured; refuses food.
17	Sphinx ligustri (larva) ...	P.M. 5 30	71·3	72·1	·8	40	91·9	......	......	....	Active; more discoloured; pulse laborious.
18	Sphinx ligustri (larva) ...	P.M. 6 30	71·5	72·3	·8	40	91·7	......	......	....	Very active; no food eaten.
19	Sphinx ligustri (larva) ...	P.M. 7 30	70·4	71·4	1·	40	91·5	......	......	....	Much excited; fasting; no fæces passed.
20	Sphinx ligustri (larva) ...	P.M. 10 30	68·3	69·1	·8	37	90	·9	......	....	Active; more discoloured; voided soft discoloured fæces.
21	Sphinx ligustri (larva) ...	9. A.M. 7 30	67·4	67·8	·4	24	88·7	......	......	11 days	Has slept during several hours.
22	Sphinx ligustri (larva) ...	A.M. 7 45	68·5	69·1	·6	28	88·6	......	......	....	Awaking; temperature of air rising rapidly.
23	Sphinx ligustri (larva) ...	P.M. 11 30	68·3	68·8	·5	29	80·3	......	......	....	In incessant action; about to enter the earth.

This difference in the velocity of the circulation at certain periods is an important circumstance as connected with the present subject,—the relation of the velocity of circulation to the temperature and respiration of the insect. For the purpose of ascertaining the rate of pulsation at different periods of the larva state with precision, I selected a healthy specimen of *Sphinx ligustri*, and commenced my observations upon it exactly seventy hours after it had left the ovum. At the moment of leaving the ovum it weighed only one eightieth part of a grain, but I was accidentally prevented from watching the rate of pulsation at that time. This individual was kept apart from other specimens from the moment it escaped from the egg until it changed into the pupa state. During this time, its weight, fæcal expenditure, rate of increase from the making of one observation to the making of another, were all carefully noted, as well as the velocity of the circulation at different periods of its growth. Unfortunately, however, I was then without my thermometers, which prevented me from observing the temperature of the insect, and thereby completing the examinations. From these observations it appeared that the rate of pulsation is greatest during the first and second periods of the larva state, or before it has entered its third skin, and when its weight is no more at most than two thirds of a grain. From not knowing the temperature of the atmosphere at the period of making the observations on this insect in its second skin, I am doubtful whether the rate of pulsation be not in reality greatest during the earlier life of the larva, before it has thrown off its first skin, because this was really the case in all the observations, if we except only two which were made on the afternoon of the same day, when the larva was at about the age of two hundred and seventeen hours. These observations being excepted, it will be seen from Table IX. that the rate of pulsation is gradually diminished from the earliest period of the larva state until the insect has changed into a pupa,—that while the rate of pulsation within a few hours after the insect has left the egg varies from seventy-five to ninety, and in its second skin, or at an average age of about two hundred and forty hours, it is but very little lower, it becomes in its third reduced to an average of seventy-five, in its fourth to less than sixty, in the middle period of its fifth to a maximum of fifty-five, and the latter period of the same to scarcely more than thirty-two pulsations per minute. These are interesting facts as connected with the power which the insect possesses of generating heat. It is, as before stated, at about the middle period of its fifth state or condition as a larva, when it is feeding most voraciously, that the insect is able to generate the greatest amount of heat.

Although it will be seen from the additional facts about to be stated that both during sleeping and activity, when most vigorous as a larva, as also when passing into the enfeebled condition of a pupa, there is a coincident and correspondent activity or diminution in the rate of pulsation with the increase of motion, respiration, or digestion; yet the primary source of the development of heat is not dependent upon the velocity or rapidity of the circulation, since the period in which there is the greatest rapidity of circulation is that in which the larva is least able to generate and maintain its greatest amount of temperature. Another circumstance which tends greatly to prove that the amount of heat does not necessarily depend upon the rapidity of the circulation is the different rates of pulsation when the insect is placed in different temperatures, or when in different states of health in the same temperature. In the first case the rate of pulsation may be very considerably increased, while the amount of temperature remains nearly, or perhaps exactly the same. In the latter instance the temperature may continue exactly the same, but the rate of pulsation be diminished. Thus in two specimens of *Sphinx ligustri* which were both of the same age, and in similar conditions of activity, feeding in the same atmospheric temperature, when the observations were made upon both at the same time, the temperature of the insects was exactly the same, ·9 above that of the atmosphere, but the rate of pulsation in one specimen, which was perfectly healthy, was forty-one beats per minute; while in the other, which was unhealthy, it was only thirty-eight.

TABLE IX.

A Table showing the rate of Pulsation of the Dorsal Vessel at different periods of the Larva and Pupa state of the *Sphinx ligustri*, LINN.

No. of Exp.	Species.	Period of observation.	Pulsation.	Weight in grains.	Fæces in grains.	Increase.	Loss.	Age in days or hours.	Remarks.
		1835.							
1	Sphinx ligustri (larva)...	July 14 P.M. 2	......	$\frac{1}{80}$	......	......	......	1 day	Larva has just burst from the egg.
2	Sphinx ligustri (larva)...	15		......	......	......	......		
3	Sphinx ligustri (larva)...	16 P.M. 5	......	$\frac{1}{35}$	......	......	......	3 days, or 51	After leaving the egg.
4	Sphinx ligustri (larva)...	17 A.M. 12	85	$\frac{1}{25}$	......	......	......	4 days, or 70	
5	Sphinx ligustri (larva)...	18 P.M. 2	85	$\frac{1}{20}$	......	......	......	5 days, or 96	Quiet, but not sleeping.
6	Sphinx ligustri (larva)...	19 P.M. 2	90	$\frac{1}{15}$	......	......	......	6 days, or 120	A little excited.
7	Sphinx ligustri (larva)...	P.M. 4	75	$\frac{1}{15}$	......	......	......	 122	Has been perfectly at rest for an hour.
8	Sphinx ligustri (larva)...	20 P.M. 2	80	$\frac{1}{14}$	......	......	......	7 days, or 144	Sleeping.
9	Sphinx ligustri (larva)...	21 P.M. 2½	32	$\frac{1}{8}$	·04	......	......	8 days, or 168½	Sleeping, preparing for change.
10	Sphinx ligustri (larva)...	22 A.M. 12	......	......	......	......	......	9 days, or 190	Has just assumed its *second* skin.
11	Sphinx ligustri (larva)...	P.M. 3½	80	$\frac{1}{10}$	......	......	·02	.... 193½	Sleeping, but has not yet eaten.
12	Sphinx ligustri (larva)...	23 A.M. 7½	73	......	......	......	......	10 days, or 209½	Quiet, but not sleeping.
13	Sphinx ligustri (larva)...	P.M. 1	108	$\frac{1}{4}$	·05	......	......	 215	Sleeping.
14	Sphinx ligustri (larva)...	P.M. 3½	103	......	......	......	......	.... 217½	Quiet, but not sleeping.
15	Sphinx ligustri (larva)...	24 P.M. 3½	85	$\frac{2}{3}$	......	......	......	11 days, or 241½	Sleeping.
16	Sphinx ligustri (larva)...	25 A.M. 9½	87	......	......	......	......	12 days, or 259½	Sleeping, and preparing for change.
17	Sphinx ligustri (larva)...	26 A.M. 8	......	......	......	......	......	13 days, or 282	Has just assumed its *third* skin.
18	Sphinx ligustri (larva)...	A.M. 12	79	$\frac{9}{10}$	·25	......	......	 286	Sleeping.
19	Sphinx ligustri (larva)...	27	......	......	......	......	......	14 days	
20	Sphinx ligustri (larva)...	28 A.M. 7½	70	1·9	......	1·0	......	15 days, or 329½	Feeding, atmospheric temperature reduced.
21	Sphinx ligustri (larva)...	P.M. 1	50	2·3	......	·4	......	 335	Sleeping.
22	Sphinx ligustri (larva)...	29 A.M. 7½	50	......	......	......	......	16 days, or 353½	Quiet, but not sleeping.
23	Sphinx ligustri (larva)...	A.M. 10	36	3·5	·7	1·2	......	 356	Sleeping, preparing for change.
24	Sphinx ligustri (larva)...	30 P.M. 1	......	......	......	......	......	17 days, or 383	Has just assumed its *fourth* skin.
25	Sphinx ligustri (larva)...	P.M. 7½	44	3·5	·2	......	......	.... 389½	Sleeping, has fed a little.
26	Sphinx ligustri (larva)...	31 P.M. 6	56	5·9	1·4	2·4	......	18 days, or 412	Sleeping.
27	Sphinx ligustri (larva)...	Aug. 1 P.M. 3½	53	9·4	2·	3·5	......	19 days, or 433½	Sleeping.
28	Sphinx ligustri (larva)...	P.M. 9½	48	10·5	·5	1·1	......	.... 439½	Sleeping.
29	Sphinx ligustri (larva)...	2 A.M. 11¼	37	13·4	1·6	2·9	......	20 days, or 453¼	Sleeping.
30	Sphinx ligustri (larva)...	P.M. 3½	47	14·7	·7	1·3	......	.... 457½	Sleeping.
31	Sphinx ligustri (larva)...	P.M. 6½	52	15·1	·6	·4	......	.... 460½	Quiet, but not sleeping.
32	Sphinx ligustri (larva)...	3 A.M. 7	33	18·3	2·4	3·2	......	21 days, or 473	Sleeping.
33	Sphinx ligustri (larva)...	P.M. 1¾	43	19·7	1·1	1·4	......	.... 479¾	Sleeping.
34	Sphinx ligustri (larva)...	4 A.M. 12	29	......	......	......	......	22 days, or 502	Has been sleeping 12 hours for changing.
35	Sphinx ligustri (larva)...	5 P.M. 5	......	......	......	......	......	23 days, or 531	Has just assumed its *fifth* skin.
36	Sphinx ligustri (larva)...	P.M. 10½	34	17·1	·7	0	2·6	.... 536½	Sleeping.
37	Sphinx ligustri (larva)...	6 A.M. 12	39	19·7	1·6	2·6	......	24 days, or 550	Sleeping.
38	Sphinx ligustri (larva)...	7 A.M. 7½	38	27·7	5·	8·	......	25 days, or 569½	Sleeping, pulse irregular.
39	Sphinx ligustri (larva)...	P.M. 7¾	37	33·3	3·1	5·6	......	.... 581¾	Quiet.
40	Sphinx ligustri (larva)...	8 A.M. 8¾	41	40·2	3·8	6·9	......	26 days, or 594¾	Quiet, pulse full and quick.
41	Sphinx ligustri (larva)...	P.M. 1½	47	42·6	2·7	2·4	......	.... 599½	Feeding.
42	Sphinx ligustri (larva)...	P.M. 10	39	49·1	4·2	6·5	......	 620	Sleeping.
43	Sphinx ligustri (larva)...	9 A.M. 6½	28	54	5·	4·9	......	27 days, or 628½	Perfectly at rest, and sleeping.
44	Sphinx ligustri (larva)...	A.M. 11½	36	58·1	2·75	4·1	......	.... 633½	At rest.
45	Sphinx ligustri (larva)...	A.M. 12	55	59·1	0	1	......	 634	Feeding.
46	Sphinx ligustri (larva)...	P.M. 5½	43	63·6	5·6	4·5	......	.... 639½	Sleeping.
47	Sphinx ligustri (larva)...	10 A.M. 7	29	72·7	8·3	9·1	......	28 days, or 653	Sleeping.
48	Sphinx ligustri (larva)...	P.M. 1½	53	83·5	4·9	10·8	......	.... 659½	Sleeping.
49	Sphinx ligustri (larva)...	P.M. 7	46	86·7	8·9	3·2	......	 665	Sleeping.
50	Sphinx ligustri (larva)...	11 A.M. 7	29	90·2	13	3·5	......	29 days, or 677	Sleeping.
51	Sphinx ligustri (larva)...	P.M. 2½	50	102	8·7	11·8	......	.... 684½	Sleeping.
52	Sphinx ligustri (larva)...	P.M. 7½	45	106·6	9·6	4·6	......	.... 689½	Feeding.
53	Sphinx ligustri (larva)...	12 A.M. 7½	44	118·2	19·3	11·6	......	30 days, or 701½	Feeding.
54	Sphinx ligustri (larva)...	A.M. 12	52	117·4	18·7	0	·8	 706	Feeding.
55	Sphinx ligustri (larva)...	P.M. 1¼	52	116·7	2·2	0	·7	.... 707¼	Has been feeding during the last hour.
56	Sphinx ligustri (larva)...	P.M. 9½	47	123	13	6·3	......	.... 715½	Quiet.
57	Sphinx ligustri (larva)...	13 A.M. 6½	33	114·4	0	0	8·6	31 days, or 724½	Has escaped unfed during the night.
58	Sphinx ligustri (larva)...	P.M. 4	34	123·3	10·4	8·9	......	 734	Quiet.
59	Sphinx ligustri (larva)...	14 A.M. 7½	28	124·7	17·2	1·4	......	32 days, or 749½	Sleeping.
60	Sphinx ligustri (larva)...	P.M. 1¾	36	118·2	4·7	0	6·5	.... 755¾	Active, and discoloured for change.
61	Sphinx ligustri (larva)...	15 A.M. 9	34	100·1	0	0	18·1	33 days, or 775	Very active, preparing for change.
62	Sphinx ligustri (larva)...	A.M. 12½	31	97·2	0	0	2·9	.... 778½	Restless, discoloured.
63	Sphinx ligustri (larva)...	P.M. 5	26	......	......	......	......	 783	Just entered the earth for changing.
64	Sphinx ligustri (pupa)...	20 A.M. 10	22	71·1	Skin. 3·2	......	28·1	38 days, or 903	Pupa still soft, has very recently changed.

Thus also in the larva of the Puss Moth, Table X. A. No. 7 to 27. Although the temperature of the atmosphere was gradually raised through twelve successive hours from 69°·5 Fahr. at 5½ a.m.,—when the larva, which had been sleeping through several hours, and had a temperature of only °·5 above that of the atmosphere, and its pulse was beating at the rate of forty-seven per minute,—to 80°·4 Fahr. at 5½ p.m. the insect then had a temperature of only °·8, while its pulse was beating at the rate of eighty-eight per minute. Again, at 7 on the following morning, atmosphere 75°·2 Fahr., the temperature of the insect at rest was only °·9; at the expiration of one hour and a half it had not been increased, and the insect was still at rest, but the pulse had risen to sixty-eight, while at 9 a.m., when the insect was aroused and feeding, its amount of temperature was still the same, but the number of its pulsations then amounted to seventy-two. At 7 o'clock on the following morning, when the insect was active and preparing for transformation, its temperature being °·7, its pulsations were at the rate of sixty per minute; but half an hour afterwards, when the temperature of the insect was °·9, the number of pulsations was not increased; and at the expiration of an hour, when the temperature had again sunk to °·7, the pulse had also subsided to fifty-four. This very insect, A. No. 1, which immediately after it was captured had been placed in a box in my coat-pocket, and after remaining there for some time, excited by immoderate warmth, had a temperature of 13°·5 Fahr. above that of the atmosphere, which was then 68° Fahr., while the pulse of the insect was ninety-nine per minute. But one hour afterwards, when its temperature had sunk to 2°·3, the pulsations were only sixty-four. At the expiration of another quarter of an hour they had risen again to seventy-two, while the temperature of the insect had sunk to 1°·6 Fahr. Thus then, although in general we cannot fail to observe the almost constant uniformity or correspondence between the number of pulsations and the temperature of the insect, as in Nos. 6, 14 and 17, it is evident that the amount of temperature does not necessarily depend upon the rate or mere velocity of pulsation.

On examining the Table now referred to it will be seen that there is a remarkable difference in the rate of pulsation, as well as in the temperature of the larva of the Puss Moth and of the *Sphinx ligustri* of the same age, and at about the same temperature of the atmosphere as on Tables VIII. and X., from which it is seen that neither the temperature of body nor the rate of pulsation is so great in the Sphinx as in the Puss Moth, while in both is observed the general coincidence of the rate of pulsation with the amount of temperature. In both the Tables VIII. and IX. it is seen that when the larva is about to change into the pupa state the pulsations are reduced from thirty-two to twenty-eight, and even to twenty-six; and when the change into the pupa state is completed, the rate of pulsation is not more, in some instances, than twelve beats per minute. When the insect is in its most complete state of hybernation the circulation in the pupa is reduced to its lowest condition, and there is perhaps an almost entire absence of pulsation, although I have reason to believe that the fluids still circulate even when there is no development of external heat.

TABLE X.

Showing that the Temperature is greater and the Pulsation more frequent in the larvæ of those insects which undergo their metamorphoses in the open air, as the Puss Moth (*Cerura vinula*), than in those which undergo their changes in the earth, as the *Sphinx ligustri*, and others.

No. of Exp.	Species.	Period of observation.	Atmosphere.	Insect.	Difference.	Pulsation.	Age.	Remarks.
1	Cerura vinula (larva), A.	1834. h m July 16 A.M. 9	68°	81·5°	13·5°	99	7th day.	Just captured, and confined in my box in my pocket, perspiring copiously.
2	Cerura vinula (larva), A.	A.M. 10	70·5	72·8	2·3	64	....	Insect active, but more calm; pulse full, sinking.
3	Cerura vinula (larva), A.	A.M. 10 15	72·7	74·3	1·6	72	....	Very active, in constant motion, pulse small.
4	Cerura vinula (larva), A.	A.M. 10 30	......	......	......	66	....	Has rested a few minutes, asleep.
5	Cerura vinula (larva), A.	A.M. 11	73·5	......	......	64	....	Has been sleeping half an hour.
6	Cerura vinula (larva), A.	A.M. 11 15	73·5	74·8	1·3	71	....	Aroused and excited.
7	Cerura vinula (larva), A.	17 A.M. 5 30	69·5	70	·5	47	8th day.	Has been sleeping during several hours.
8	Cerura vinula (larva), A.	A.M. 7	71·4	72·3	·9	64	....	Moderately active.
9	Cerura vinula (larva), A.	A.M. 7 30	72	72·9	·9	57	....	At rest.
10	Cerura vinula (larva), A.	A.M. 7 45	72·3	72·9	·6	55	....	Sleeping.
11	Cerura vinula (larva), A.	A.M. 8	72·5	73·2	·7	56	....	Still sleeping.
12	Cerura vinula (larva), A.	A.M. 9	72·2	73·2	1·0	68	....	Active, and feeding.
13	Cerura vinula (larva), A.	A.M. 9 15	72·2	73·3	1·1	59	....	Resting.
14	Cerura vinula (larva), A.	A.M. 9 30	73·1	74·2	1·1	70	....	Feeding.
15	Cerura vinula (larva), A.	A.M. 9 45	73·2	74·4	1·2	68	....	Still feeding.
16	Cerura vinula (larva), A.	A.M. 10 15	73·2	74·3	1·1	67	....	Active, but not feeding.
17	Cerura vinula (larva), A.	A.M. 11	74·4	75·7	1·3	72	....	Very active.
18	Cerura vinula (larva), A.	A.M. 12 45	78·5	80	1·5	77	....	Very active.
19	Cerura vinula (larva), A.	P.M. 1 15	78·5	80·2	1·7	78	....	Still very active.
20	Cerura vinula (larva), A.	P.M. 4 45	80·5	81·9	1·4	88	....	Moderately active.
21	Cerura vinula (larva), A.	P.M. 5 30	80·4	81·2	·8	88	....	Less active.
22	Cerura vinula (larva), A.	18 A.M. 7	75·2	76·1	·9	66	9th day.	Sleeping, or quiet.
23	Cerura vinula (larva), A.	A.M. 8 30	75·4	76·3	·9	68	....	Quiet.
24	Cerura vinula (larva), A.	A.M. 9	76·1	77	·9	72	....	Aroused and feeding.
25	Cerura vinula (larva), A.	19 A.M. 7	70·7	71·4	·7	60	10th day.	Changing colour for transformation.
26	Cerura vinula (larva), A.	A.M. 7 30	70·9	71·8	·9	60	....	More discoloured.
27	Cerura vinula (larva), A.	A.M. 8	70·7	71	0·3	54	....	Preparing to spin its cocoon.
28	Cerura vinula (larva), B.	16 A.M. 10 30	71·8	72·3	·5	49	....	After feeding 36 hours, just fed, sleeping.
29	Cerura vinula (larva), B.	A.M. 10 45	71·8	72·5	·7	......	....	A little active.
30	Cerura vinula (larva), B.	17 A.M. 5 30	68·5	68·7	·2	......	....	Is spinning its cocoon for transformation.
31	Cerura vinula (larva), B.	P.M. 1 15	78·5	78·9	·4	50	....	Still spinning its cocoon.
32	Cerura vinula (larva), B.	P.M. 4 45	78·9	79·2	·3	46	....	Still spinning.
33	Cerura vinula (larva), B.	19 A.M. 7 30	70·9	71·3	·4	31	....	Has been retarded from changing.

But it is not only at the period of change into the pupa state that the pulsation is greatly reduced, the same thing takes place immediately before each change of skin in the larva, as shown on Table IX. Nos. 9, 23, 34, and 63. At those periods the temperature and respiration are also reduced, and the insect ceases to eat; but soon after the change of skin has taken place the respiration and temperature are again increased; but the average rate of pulsation is never so great as before the previous change of skin, and it continues to be diminished at each succeeding change.

The following observations made on larvæ of *Sphinx ligustri* of the same age, at different periods after entering their fifth or last skin, and when the pulse in each was regular and full, will further illustrate the general accordance which exists between the rate of pulsation and amount of temperature when the pulsation has not been accelerated by inordinate activity or other causes.

Table XI.

Period of observation.	No.	Age of the Insects.	Atmosphere.	Insect.	Difference.	Pulse.
July 31, 1834.	1	*Three* days in last skin, feeding	71°·2	72°·3	1°·1	54
	2	*Three* days in last skin, resting	71·2	72·2	1	49
	3	*Five* days in last skin, feeding	71·2	72·2	1	49
	4	*Five* days in last skin, feeding	71·6	72·6	1	50
	5	Seven days in last skin; has been long sleeping	71·2	71·6	·4	29
	6	Seven days in last skin; aroused and active	71·6	72·4	·8	38

The same general accordance which exists in the larva between the quantity of respiration, amount of heat developed, and number of pulsations, exists also in the perfect insect. In order to observe the number of pulsations in the perfect insect it is necessary to denude the dorsal surface of the abdomen of its thick covering of scales, and when this has been done completely the pulsation of the vessel is readily observed. In a male specimen of *Sphinx ligustri* which had been exerting itself in active flight for several minutes around my sitting-room, I found the number of pulsations was 127 per minute, while the insect then had a temperature of 9° Fahr. above that of the atmosphere, which was 70° Fahr. On the following day, after it had been exerting itself in a similar manner for a much longer space of time, the temperature of the atmosphere being 69°·5, the number of its pulsations was then 139, and its number of respirations forty-two per minute, but its amount of heat was only 5°·5 Fahr. When it had remained at rest about half an hour its temperature was only °·5, while the number of its respirations was eighteen, and of its pulsations forty-nine; and at the expiration of three quarters of an hour, when it was perfectly quiet and apparently asleep, its temperature was only °·2, its number of respirations fifteen, and its pulse forty-two. In these instances the accordance between the number of respirations and pulsations, and the temperature of the insect was nearly uniform, but in some of the other observations the same uniformity between the amount of heat developed and the number of pulsations is not so strictly observed. Thus in No. 12, Table V., the temperature of the insect after violent exertion was 9° Fahr., the number of pulsations 127, while in No. 14 the temperature was only 4°·6, but the pulsations amounted to 151; and in No. 15 the temperature was 4°·3, but the pulsations only 110.

It is thus evident that in the perfect insect, as in the larva, there are sometimes similar irregularities in the rate, or velocity of pulsation, and which irregularities when compared with each other do not appear to have relation to the quantity of heat developed, while the general, or what appears to be the average rate of pulsation, is in almost uniform accordance with the amount of heat and number of respirations. But these apparent discrepancies may, perhaps, be explained by the circumstance, that when the pulsations are excessive in number they are small, rapid, and intermittent, like the pulsation in certain excited states in the human body, and this is the case in every instance of excessive pulsation, both in the larva and perfect insect; while in those instances in which there is a near accordance between the rate of pulsation, amount of heat developed, and number of respirations, the pulsatory motions are full, regular, and without intermissions, so that the relative quantity of blood which is steadily submitted to the influence of the air in the respiratory organs is perhaps greater in the latter than in the former instances. This circumstance may also account perhaps for the smaller amount of heat generated by the larva in its earlier than in its latter condition, although the number of its pulsations is more than double in the earlier than in the latter period. In the full grown larva the pulsations are steady and full, with much power, but in the earlier state of the larva they are small, rapid, and intermitting. From these circumstances we may fairly infer that the quantity of heat developed is more dependent upon the quantity of respiration than upon the velocity of the circulation.

3. *Digestion.*

The influence which the process of digestion exercises over the production of heat is very considerable. We have before seen that in the larva the greatest amount of heat is produced after the insect has fed, or while it is feeding and becoming much excited. It is at these periods that it deteriorates the greatest quantity of air, which quantity is then necessarily required during its respiration in assimilating the new matter which has just been taken into its circulation through means of the digestive process. In the perfect insect the circumstances are exactly the same, its temperature is greatest after it has fed, and is then exerting itself, and at that time it respires the greatest quantity of air. On the other hand, when the insect is fasting, the quantity of heat evolved by it, even during great exertion, is much diminished, while the quantity of air consumed is smaller than the quantity consumed under similar excitement after it has taken food.

4. *Gaseous, or Cutaneous Expenditure of the Body.*

The cutaneous expenditure of the body is closely connected, both with the digestive process and with the regulation of the temperature of the insect. It is seen in the observations on *Melolontha solstitialis* and other species, that the amount of gaseous expenditure is exceeding great, and that after the temperature of the insect has been raised to a certain amount, a profuse perspiration breaks out, which is the natural cooling process of the body. The pulse also is considerably affected by it, as shown in the larva of the Puss Moth, which had been subjected to high temperature, and which soon became bathed in perspiration, Table X. No. 1 and 2. The exact correspondence which exists between the quantity of gaseous, or cutaneous expenditure, acceleration or subsidence of the pulse, increase or decrease of weight, and quantity of respiration in every period of the larva, pupa, and perfect state, is very remarkable. The quantity appears to be at its maximum in the very active perfect insect, and is greater than in the larva, or in the pupa, in which it is at its minimum when the pupa has the smallest amount of respiration; but in all cases it is least during the state of most complete inactivity. In the common Hive Bee in a state of activity the amount is prodigious, and very soon becomes evident, if the bee be confined in a very small glass phial, closely stoppered, and kept in a state of excitement. The perspiration from the insect is then condensed upon the interior of the phial, and if several bees be confined together, the bodies of the little insects themselves become bathed with perspiration. In the summer of 1832, I endeavoured to ascertain the quantity of gaseous expenditure in the larvæ of Lepidoptera compared with the weight, quantity of food eaten, increase, and fæcal expenditure of the insect, in a given time, and it was then found that the quantity of gaseous is equal to, or even greater than the quantity of fæcal expenditure, even in these animals in which the latter is so enormous. The first subject of my observations was my old favourite, the larva of *Sphinx ligustri.* The specimen on which my observations were commenced had been confined fasting about twelve hours, when it weighed 79·8 grains, having at the commencement of the twelve hours weighed 83·3 grains. During this period of fasting it had passed two masses of fæces, which weighed only 1·7 grain, consequently it had expended by the skin and respiratory organs 1·8 grain, an excess of one tenth of a grain in the gaseous expenditure. It was then supplied with fresh food, of which it ate 2·8 grains, and weighed 82·1 grains at the expiration of the first hour; had passed no fæces, but had expended ·5 of a grain from the skin and respiratory organs. It was then made to fast for an hour, and afterwards weighed again to ascertain whether there was any difference in the quantity of gaseous expenditure during abstinence. It had discharged one mass of fæces weighing ·9 of a grain, and itself weighed 80·8 grains, so that during the hour of fasting only ·4 had passed off in the gaseous form instead of ·5 as in the previous hour of taking food. At this time, while the insect was lying at rest, the dorsal vessel pulsated at the rate of thirty-six beats per minute. The insect was then allowed to feed for another hour and weighed again; at the expiration of that time it had passed no fæces, had eaten 3·4 grains of food, and weighed 83·6 grains. Thus one whole grain had now been expended in the gaseous form. It then fasted for three hours, but during that time it passed only one mass of fæces, which weighed 1·2 grains, and itself weighed 81·6, so that it had now lost only ·8 in the gaseous form during three hours' fasting. It was thus evident that the greatest amount of gaseous expenditure occurs during the period of taking food, and that the quantity of gaseous expenditure decreases in proportion to the length of time the insect is kept fasting, and also that less gaseous expenditure takes place when there is the greatest amount of fæcal. When the insect had been fed for another hour, and had eaten 2·7 grains of food, it weighed 83·9, but had passed no fæces, consequently it had now expended ·4 of a grain in the gaseous form. It was thus evident that the quantity which passes off in the gaseous form during a certain length of time when the animal is taking food varies considerably, and sometimes amounts to one whole grain per hour, while at other times it is only about ·4 of a grain. These observations were continued through two successive days, with similar results. Thus after the insect had been fasting for twelve hours, during which time its amount of gaseous expenditure had been very trifling, the very first time it was weighed after feeding for one hour it had expended ·5 of a grain; but when it was kept fasting, the very next hour its expenditure was only ·4 of a grain. Similar experiments were also made at the same time upon the larvæ of the Puss Moth, *Cerura vinula*, Steph., and *Sphinx Elpenor,* Linn., with precisely the same results relative to the quantity of gaseous expenditure. In the observations on the *Sphinx ligustri*, it will be seen by the Table XII. that the heat developed during fasting is much less than during the period of taking food.

Table XII.—A Table* exhibiting the quantity of food eaten, with the rate of increase of weight, and the gaseous and fæcal expenditure, and their effect on the Temperature of a Larva of *Sphinx ligustri*.

No. of Exp.	Period of Observation.	Feeding.	Fasting.	Temp. of Atmos.	Insect.	Difference.	Weight of larva in grains.	Weight of food eaten in grains.	Increase.	Gaseous expenditure.	Fæcal expenditure.	Remarks.
	1832.			°	°	°	grs.					
1	Aug. 18 P.M. to 19 A.M. 10	......	Twelve hours	......	......	......	79·8	......	......	......	......	
2	A.M. 11	One hour	......	......	......	......	82·1	2·8	2·3	·5	......	
3	A.M. 12	......	One hour	......	......	......	80·8	......	......	·4	·9	
4	P.M. 1	One hour	......	......	......	......	83·6	3·4	2·8	·6	......	
5	P.M. 4	......	Three hours	......	......	......	81·6	......	......	·8	1·2	
6	P.M. 5	One hour	......	......	......	......	83·9	2·7	2·3	·4	......	
7	P.M. 6	One hour	......	......	......	......	85·1	4·5	1·2	2	1·3	
8	P.M. 7	One hour	......	......	......	......	85·6	3·2	·5	2·7	......	
9	P.M. 8	One hour	......	......	......	......	85	1·5	......	·6	1·5	
10	P.M. 9	One hour	......	......	......	......	86·5	2·1	1·5	·6	......	
11	Aug. 19 P.M. 9 to 20 A.M. 6	Nine hours	......	......	......	......	88	16·5	1·5	7	8	
12	A.M. 7	......	One hour	......	......	......	87·6	......	......	·4	......	
13	A.M. 8	......	One hour	......	**	**	85·6	......	......	·4	1·6	
14	A.M. 9	......	One hour	65	65·5	·5	85·55	......	......	·05	......	
15	A.M. 10	......	One hour	65·5	65·9	·4	85·5	......	......	·05	......	
16	A.M. 11	......	One hour	66·4	70	·6	85·4	......	......	·1	......	
17	A.M. 12	......	One hour	67	67·7	·7	85·2	......	......	·2	......	Active.
18	P.M. 1	......	One hour	68	68·7	·7	85	......	......	·2	......	
19	P.M. 2	......	One hour	69	69·4	·4	83·6	......	......	·05	1·35	At rest.
20	P.M. 3	......	One hour	69·5	69·7	·2	83·55	......	......	·05	......	At rest.
21	P.M. 4	One hour	......	69·5	70·4	·9	86·1	3·6	2·55	1·05	......	Feeding.
22	P.M. 5	One hour	......	70·1	71·1	1	88·85	3·6	2·75	·85	......	Very active.
23	P.M. 6	One hour	......	69·5	70·4	·9	89·15	2·4	·3	1	1·1	Feeding.
24	P.M. 7	One hour	......	69	70·1	1·1	90	1·8	·85	·95	......	Active.
25	Aug. 20 P.M. 7 to 21 A.M. 7	Twelve hours	......	68·5	69·4	·9	92·6	25·5	2·6	11·6	11·3	Sleeping.
26	A.M. 8	One hour	......	68·7	69·9	1·2	93·9	1·95	1·3	·65	......	Active.
27	A.M. 9	One hour	......	69	70	1	94·65	4·4	·75	2·1	1·55	Active.
	Total increase in 47 hours			......	......	......	14·85	79·95	......	35·3	29·8	

A Table exhibiting the gradually decreasing amount of Weight and Gaseous Expenditure in proportion to the length of time of fasting in a Larva of *Sphinx Elpenor*.

28	Aug. 20 A.M. 10	......	One hour	65·5	......	......	65·3	......	......	......	......	
29	A.M. 11	......	One hour	66·4	......	......	65·1	......	......	·2	......	Insect in constant motion.
30	A.M. 12	......	One hour	67	......	......	64·9	......	......	·2	......	
31	P.M. 1	......	One hour	68	......	......	64·8	......	......	·1	......	At rest.
32	P.M. 2	......	One hour	69	......	......	64·7	......	......	·1	......	Sleeping.
33	P.M. 3	......	One hour	69·5	......	......	64·65	......	......	·05	......	Sleeping.
34	P.M. 4	......	One hour	69·5	......	......	64·5	......	......	·15	......	A little aroused.
35	P.M. 5	......	One hour	70·1	......	......	64·4	......	......	·1	......	
36	P.M. 6	......	One hour	69·5	......	......	64·2	......	......	·2	......	Very active.
37	P.M. 7	......	One hour	69	......	......	64	......	......	·2	......	
38	A.M. 7	......	Twelve hours	68·5	......	......	62·15	......	......	1	·85	
39	A.M. 8	......	One hour	68·7	......	......	62	......	......	·15	......	Very active.
40	A.M. 9	One hour	......	69	......	......	63·65	3·45	1·65	1·8	......	
	Total decrease in 23 hours			......	......	......	1·65	......	......	4·25	·85	

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* These Tables on the quantity of food eaten, loss and increase of weight, gaseous and fæcal expenditure, and temperature of the atmosphere at the time of making the observations, were made, as noticed below, in August 1832; but the two columns which indicate the temperature of the insect** were not made at that period, but have been added subsequently, having been made in the summer of 1834 upon the larva of the Sphinx under circumstances similar to those of August 1832. Indeed from the precautions necessary to be attended to while taking the temperature of the insect, as noticed in the beginning of the present paper, it will be seen that it is impossible to make the whole of the observations here detailed upon the same individual at the same time, the excitement produced in the insect while handling it in order to ascertain its weight unavoidably interfering with the correctness of the observations on its temperature. Two specimens therefore of the same weight and age must always be employed.

Table XII. (Continued.)

A Table of the Weight and Rate of Increase and Decrease, with the Fæcal and Gaseous expenditure of a Larva of *Cerura vinula*.

No.	Period of Observation.	Feeding.	Fasting.	Temp. of Atmos.	Insect.	Difference.	Weight of larva in grains.	Weight of food eaten in grains.	Increase.	Gaseous expenditure.	Fæcal expenditure.	Remarks.
				°			°	grs.				
41	1832. Aug. 20 A.M. 9¼	One hour	......	65	......	......	76·5	......	......	......	......	This larva was fed throughout the whole of the observations upon stale food.
42	A.M. 10¼	One hour	......	65·5	......	......	78·1	3·65	1·6	1·55	·5	
43	A.M. 11¼	One hour	......	66·4	......	......	77·2	2·4	......	1·45	1·55	
44	A.M. 12¼	One hour	......	67	......	......	77·6	2·55	·4	1·2	·95	
45	P.M. 1¼	One hour	......	68	......	......	76·6	1·6	......	1·25	1·35	
46	P.M. 2¼	One hour	......	69·5	......	......	76·3	2·5	......	1·45	1·35	
47	P.M. 3¼	One hour	......	69·5	......	......	74·95	2·1	......	1·05	2·1	
48	P.M. 4¼	One hour	......	70	......	......	75·9	4·4	·95	2·1	1·35	
49	P.M. 5¼	One hour	......	70·2	......	......	74·4	1·75	......	1·1	2·15	
50	P.M. 6¼	One hour	......	69·5	......	......	75·05	2·7	·65	1·25	·8	
51	P.M. 7¼	......	......	69	......	......	75·7	2·1	·65	·95	·5	
52	A.M. 7¼	......	Twelve hours	68·5	......	......	69·75	......	......	2·55	4·6	
53	A.M. 8¼	One hour	One hour	68·7	......	......	69·65	......	......	·1	......	
54	A.M. 9¼	One hour	......	69	......	......	71·1	3·7	1·45	1·75	·5	
		Decrease in Weight in 26 hours 5·4						Food eaten 29·45		17·75	11·70	

From this Table we deduce the following facts:—First that the expenditure which takes place from the cutaneous surface of the insect and from its respiratory organs is greater than its whole amount of fæcal expenditure, is more regular and continued, and decreases in proportion to the length of time which the insect remains fasting, but never entirely ceases. It is greatest while the insect is in motion and least when it is lying entirely at rest. Thus in the observations on *Sphinx Elpenor*, Linn., which was fasting during nearly the whole of the period of observation, twenty-two hours, the insect lost only ·85 of a grain of fæcal expenditure, but 2·45 of grains by the respiratory and cutaneous surfaces, and of this expenditure, when the insect was lying at rest, only ·05 of a grain per hour, but when in violent motion the loss amounted to ·15 per hour. This difference of quantity is readily accounted for by the quicker circulation of the fluids in the active state of the insect, when its respiration is greater, and consequently a greater amount of heat is generated, and requires to be regulated by the transpiration from the surface of the body. This Table also indicates the fact that the whole process of digestion may be completed in the larva of the Sphinx in about two hours and a half, and that the average quantity of fæcal expenditure in the latter period of a moderate sized larva is about one grain per hour.

But the connection or correspondence between the quantity of respiration, temperature and gaseous expenditure in a given time, is beautifully illustrated in what occurs in the pupa state. On the 3rd of April, 1836, I weighed several pupæ of *Sphinx ligustri*, and found that one of them which on the 20th of the preceding August, immediately after it had changed to a pupa, weighed 71·1 grains, had not expended, during the long interval of nearly eight months, or two hundred and twenty-eight days inclusive, more than 3·7 grains in weight, the whole of which must have passed off from the respiratory and cutaneous surfaces. This was the identical specimen which I had watched from the egg, and whose rate of pulsation is noticed on Table IX. At the time of entering into the pupa state in August, it weighed, as above stated, 71·1 grains. At the present time it weighed 67·4 grains. This diminution was during the period of hybernation, and is in beautiful accordance with the greatly diminished quantity of respiration during this state, respiration being reduced to its minimum in this condition of the insect, as shown in my previous observations. On the 24th of May, fifty-one days after the first weighing, the perfect insect was developed from this pupa, and then weighed only thirty-six grains, and when weighed again on the following day, only thirty-four grains, Table V. A, being an amazing diminution of nearly one half of the whole weight of the pupa in the short space of fifty-three days. Now it will be remembered that, as shown in the Tables on the Respiration[*] of the pupa of *Sphinx ligustri* in the month of April, that the quantity of respiration at that period is gradually increasing, and is in proportion to the degree of animation in the insect; and the degree of animation is proportioned to the quantity of stimuli, external temperature, &c., so that, as shown by Reaumur in the pupæ of the common Cabbage Butterfly, if the pupæ be kept in a very low temperature, as in that of an ice-house, development into the perfect state is greatly retarded; and as now shown, respiration, owing to the absence of a proper amount of external stimulus, being reduced to its minimum, the circulation of blood is almost suspended, the development of heat scarcely, if at all perceptible, and the expenditure of solid matter from the body of the insect in a gaseous form is so insignificant that the powers of life are in no way injured by retarded development, and the insect revives in its full vigour whenever the natural stimuli of life are sufficiently increased. At the moment of weighing the above pupa in April, I weighed several others which had entered the pupa state about the same time. One of them at the expiration of fifty-three days, on the 26th of May, had lost thirteen grains, another eight grains, a third nine grains, and a fourth ten grains, and the respiration of these had increased in the ratio of their loss of weight.

There may, perhaps, be some difficulty in ascertaining with certainty the chemical constituents of this gaseous expenditure from the body of the insect in its different stages, since a large proportion appears to be aqueous vapour, but I am satisfied that sometimes there is also a quantity of carbonic acid. However, I could not discover the carbonic acid in a quantity of vapour expelled from the bee hive and condensed during the night, but I very readily detected it in the pupa, in my earlier observations on the respiration of insects, in April 1829. A pupa of *Sphinx ligustri*, after being carefully washed to prevent the adhesion of air to the surface of its body, was placed for a few hours in a glass stoppered phial, completely filled with perfectly clear lime-water, and at the expiration of two or three hours, I had the satisfaction of detecting carbonate of lime deposited both within the entrance of the spiracles and also in the minute punctures which are distributed over the whole body of the pupa; a certain

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[*] Philosophical Transactions, 1836, Part II. p. 552, Table I. No. 3 to 10.

proof this, both that the pupa was transpiring through the pupa case, and also that the transpired matter contained carbonic acid.

Conclusion.

The very great length unto which this paper has already been extended, necessarily prevents me from entering so fully into all the circumstances connected with the evolution of heat in insects as the great importance attached to this interesting subject demands; I shall, therefore, review the contents of this paper, and other circumstances connected with the production of animal heat, with as much conciseness as possible.

On comparing the whole of the facts we have just examined, we cannot fail to observe the very close relation which subsists between the amount of heat developed, and the quantity of respiration. We have seen in the larva, the pupa, and the perfect insect, that when the respiration is accelerated the temperature is also increased, and that when respiration is diminished the temperature subsides. When the insect is sleeping, its respiration gradually becomes slower, and its temperature continues to lessen until the insect is aroused, when immediately after the first respirations it is again increased. When the insect falls into a state of hybernation, and its respiration is suspended, its evolution of heat becomes so likewise. When the insect is most active, and respiring most voluminously, its amount of temperature is at its maximum, and is very great, and corresponds with the quantity of respiration, and, as in the Bee, an immense quantity of heat passes off into the surrounding medium. When the insect wishes to impart heat to its young it can do so at pleasure, and can voluntarily increase its own temperature. It does this by accelerating its respiration. At those times, as shown in the comparative observations, the insect evolves in one hour, in this state of activity and excitement, at least twenty times the amount of heat, and consumes nearly twenty times the quantity of air, which it consumes at the same temperature when in a state of repose. In insects which live in society the temperature of their dwellings is increased in proportion to the activity of the inmates, and consequent amount of their respiration. In the hive it is steadily increased until the time of swarming. In the winter when the bees are quiet, and their respiration is exceedingly low, and when not a bee is observed ventilating at the entrance, the temperature of the hive may be raised in a few minutes, very many degrees, by disturbing the inmates, and thereby increasing their respiration, until such an amount of heat is evolved, and so much air is deteriorated, as to become oppressive and noxious to the bees, many of whom, although the open atmosphere be too cold for them to venture abroad, will come to the entrance of the hive and begin as laboriously to ventilate the interior, by vibrating their wings, as in the midst of summer. The quantity of free heat is always greatest in the hive when the bees are most active, and least when they are most quiet. With regard to the habits and anatomical structure of insects, the amount of heat is by far greatest in volant insects; these always have the largest respiratory organs, and breathe the greatest quantity of air. In the terrestrial insects the amount of heat is greatest in those which have the largest respiratory organs, and breathe the greatest quantity of air, whatever be the condition of their nervous system. In the larva state the respiratory organs are smaller than in the perfect insect, compared with the size of the body, and the larva, we have seen, has the lowest temperature. But in these comparisons we must observe that the activity of respiration is equal in the individuals which are compared. Thus although the respiratory organs are larger in the pupa than in the larva, the physiological condition of the insect is lower, its respiration is inactive. These facts, it will be seen, are all in strict accordance with each other, and point to the chemical changes in the air during respiration as the immediate source of animal heat. But it may be matter of inquiry how it is that the heat evolved within the body of the insect, during respiration, becomes evident so rapidly. This, it may be urged, tends to show that it results from the influence of the nervous system. But when we remember that in insects the circulatory vessels are in close and most extensive communication with the respiratory organs over the whole body of the individual, and that, unlike the vessels in those vertebrated warm-blooded animals which have extensive respiration, they are neither strictly venous nor arterial, but probably intermediate between the two, may it not arise from only a very small amount of heat evolved at each respiration becoming latent, while nearly the whole becomes free, and is liberated as quickly as produced, and that this is the occasion of the temperature of the insect being so quickly raised during its respiration, and so rapidly diminished as the acts of respiration become less frequent? That, in other words, in insects the capacity of the blood for caloric is but very little increased during respiration? With these facts in consideration, and looking at the analogical condition of insects, and with Professor Grant[*] and Mr. Owen[†], comparing the vast extent of their respiratory organs, distributed over the whole body, with a like extensive respiration in birds, and finding that, like birds, insects have also a greater activity of respiration, and a higher temperature of body than any other class in the division of animals unto which they respectively belong, we can hardly withhold our assent to the opinions which have long been advocated by many of our best physiologists, that animal heat is the direct result of the chemical changes which take place in the air respired. But it may be urged that activity of respiration is coincident with increased rapidity of circulation, and hence that the latter may, perhaps, precede the former, and be in reality the source of heat. Unto this it may be replied that the larva in its earlier state has a more rapid circulation, but develops less heat than in its latter. In many of the observations on the Tables it is shown that the pulse may be rapid with a low amount of heat. It is shown in the larva, when arousing, that the pulse is not increased until

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[*] Lectures on Comparative Anatomy.—*Lancet*, 1833–34.

[†] Cyclopædia of Anatomy and Physiology, vol. i. p. 341.

after the first respirations*, when the heat is becoming apparent. With regard to the digestive process, we have seen that when the animal is taking food it has the greatest amount of gaseous expenditure from its body, and that the greatest amount of heat, when in a state of quietude, is then generated. But a greater quantity of air is then consumed, in assimilating the new matter which has been taken into the system, and the quantity of heat is still further increased if the animal becomes active, and this is regulated by the increased expenditure from the surface of the body. Lastly, we have seen that in the more perfect volant insects, the Bees, Sphinges, &c., there is the largest amount of heat produced, and the greatest quantity of air consumed, but the nervous system is also largely developed, and hence it may fairly be supposed to have much influence in the development of heat. But on the other hand we find many insects, as the *Melöe* and its congeners, which produce a large amount of heat, in which the nervous system is comparatively small, while these insects have large respiratory organs, and a large amount of respiration. In the *Staphylinus* the nervous system is exceedingly large, compared with the size of the body, but the respiratory organs are by no means small, while the amount of heat is very moderate. In the *Carabus* the nervous system is also large, as are likewise the organs of respiration, but the amount of heat and activity of respiration are low, and the same is the case in the *Blaps*, in which the nervous system is rather small. If the development of heat depends upon the nervous system, or the number of ganglia, the *Leech*, which has twenty-two ganglia, ought to generate more heat than the larvæ of lepidopterous insects, which have but ten or twelve, and the larva ought to generate as much as the perfect insect. In the larvæ of the Bee, the Hornet, Ichneumon, and Tenthredo, which generate so large an amount of heat, the nervous system is exceedingly small; and if, as some suppose, heat is the result of muscular contraction, surely it ought to be most developed where there is the greatest amount of muscular contractility; it ought to be generated more in the *Leech* than in other articulated animals, and in those *Vertebrata* which are peculiarly noted as cold-blooded. These facts con-

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* This is in perfect accordance with the condition of the circulation in the human body during sleep, and at the moment of waking, as noticed by BLUMENBACH, and as I myself once had an opportunity of observing in a female patient who was suffering from severe fracture of the skull, for which she had been trephined; subsequently to which, a large portion of the bone (the right parietal) became affected with necrosis and was removed by operation, and the patient afterwards gradually recovered. At least one-third of the whole parietal bone had been removed, and a large surface of the dura mater being thus exposed, the activity of the circulation in the brain was readily observed. I thought this a fair opportunity, as the patient was recovering, for observing the state of the circulation during sleep, and at the moment of waking. The patient was sleeping soundly at the time of the observation, and while she remained entirely undisturbed, the pulsations in the arteries of the dura mater were at the rate of ninety-four beats per minute, and were perfectly synchronous with the pulsations at the wrists; but immediately she began to inspire deeply at the moment of waking, the pulsations became much accelerated. At the instant of waking, the patient fetched a full and deep inspiration, and in less than a minute and a half after this, the patient being perfectly awake, the pulsations amounted to 104 beats per minute, thus making a difference of about 600 beats per hour in the rate of pulsation when sleeping and immediately after waking.

sidered, and connected with that very remarkable one, the voluntary power of producing heat possessed by the Bee, must lead us to conclude, that although, doubtless, the whole of the functions of the body are more or less remotely concerned in the production of heat, yet that the immediate source of its evolution seems to be chemical changes effected during respiration, and that the nervous system is only secondarily concerned.

Appendix.

Since the preceding paper on the Temperature of Insects was submitted to the Royal Society, circumstances have enabled me to ascertain a few additional facts respecting the temperature of some other species which I had not heretofore any opportunity of examining, and these the Council have kindly permitted me to subjoin to my paper.

I am not aware that the temperature of the nest of the common wasp has ever before been examined, and it is therefore pleasing to find that all the circumstances connected with the evolution of heat in the nest of this species are in perfect accordance with the observations made on the neighbouring families of hive and humble bees.

On the 11th of August, during the past summer (1837), I dug away the soil from the top of a nest of *Vespa vulgaris* which was situated in a bank of earth at the depth of about seven inches from the surface. The nest was nine inches in diameter, so that the colony was by no means a small one. The temperature of the atmosphere, when the covering of the nest was removed, at 4½ P.M. was 70° Fahr. When the thermometer was passed through the top of the nest the mercury rose immediately to 80°. In about ten or fifteen minutes afterwards, when the colony had become disturbed, and the thermometer was passed a little deeper into the nest, the mercury rose to 95°. This distinctly proves that the evolution of heat in the wasps' nest is greatly increased, as in the beehive, when the insects have become excited. At 6½ P.M. the temperature of the atmosphere was 65° Fahr., and the wasps having now become more quiet, the temperature of the nest, which had remained with its upper surface exposed since the last observation, was only 90° Fahr.; but an hour afterwards, when the temperature of the atmosphere had sunk to 63°, that of the nest had risen to 91°, the thermometer having remained undisturbed in the nest since the last observation. This increase of temperature was readily explained by a great number of the excited insects, which had been flying around the spot, having now returned to the nest. Thus the circumstances connected with the evolution of heat in the nests of the predaceous and in the melliferous Hymenoptera are precisely similar; and they are similar also in another interesting family of this order—the ants. It is elsewhere noticed* that Juch found the temperature of an ant-hill about 15° Fahr. above that of the atmosphere. My own observations are in accordance with this statement. On the 27th of July 1837 I examined the temperature of the nest of *Formica herculanea*, Linn. The temperature of the atmosphere in the shade, at 11 A.M., was 76° Fahr., but when the thermometer was exposed on the ground to the full rays of the sun the mercury rose to 95° Fahr. The nest was rather a small one, and at the time of commencing the observations was completely undisturbed. When the thermometer was first passed into it, to the depth of five inches, the temperature was maintained steadily at 84° Fahr.; but within six or eight minutes afterwards, when the insects had become excited by the presence of the thermometer, and were running about in every direction in a state of the greatest agitation, the temperature of the nest rose to 93° Fahr., and in a few minutes after this, when the insects were still more excited, to 95°·5, and a little nearer the surface, where the commotion was greatest, to 98°·6 Fahr. During these observations the ant-hill was carefully shaded from the rays of the sun, in order to avoid all source of error. When the ant-hill was again exposed to the sun, and the thermometer placed upon its surface, the mercury rose to 108° Fahr. This was a temperature much too great

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[*] Page 283.

for the insects to bear, since nearly the whole of them immediately retired beneath the covering of the nest, and there was scarcely a single ant to be seen. On the 2nd of September I repeated my observations on the same ant-hill. On this occasion the day was very gloomy, with steady light rain, and the temperature of the atmosphere at 11 o'clock A.M. was only 54°. The temperature of the ant-hill varied but little in its different parts but it was now greatest near the surface. At a depth of one inch it was 65°, at two inches 66°, below which it gradually diminished. At this time I also examined another nest of the same species, but which was about twice the size of the first. The atmosphere being, as before stated, 54°, the mean temperature of this nest, when the insects were a little excited, was 74°.

During the summer and autumn of the present year I have repeated my observations on the temperature of the bee-hive, and have found but little variation in its average amount at similar periods in the two years. I have also examined the nests of *Bombus lapidarius,* and *Bombus sylvarum,* and in both have found that the ordinary temperature, which is about 10° or 15° above that of the atmosphere, is considerably increased during the period of incubation, exactly the same as in the nest of *Bombus terrestris.*

On the following day after examining the nest of the wasp, I examined the temperature and pulsation of the larva of the same species. The specimens examined had been removed from the nest on the previous evening, but had not been removed from their cells. The results are given on the accompanying table. I examined also the larva of the hornet, *Vespa Crabro,* LINN., which was still contained in its cell, but had been some days removed from the nest. In this instance the temperature of the larva was found to be about 2°·5 FAHR. above that of the atmosphere, but its rate of pulsation was only thirty-two beats per minute. I should have attributed this low rate of pulsation to the specimen having been so long removed from the nest, had not the rate of pulsation in this larva been examined by my friend Mr. ORSBORN a few days before, and almost immediately after the specimen was obtained from its nest, and found at that time not to exceed thirty-three or thirty-four beats per minute. These facts therefore are in accordance with the observations on the larva of *Anthophora retusa* and *Bombus terrestris,* and also accord with other observations on the larvæ of that very destructive tenthredo or saw fly *Athalia centifolia,* KLUG; which has been so obnoxious to the agriculturist by destroying his crops of turnips during the last three summers.

London, November 7th, 1837.

TABLE.—TEMPERATURE OF LARVÆ.

No. of Exp.	Name of Species.	Period of observation.	No. of Specimens.	Atmosphere.	Soil.	Insect.	Difference.	Pulsation.	Remarks.
1	Vespa Crabro (larva)	July	1	70°·	....	72°·5	2°·5	32	Full grown; has fasted three or four days.
2	Vespa vulgaris (larva)	Aug. 12	1	72·7	....	75·8	3·1	56	Nearly full grown; very active.
3	Vespa vulgaris (larva)	12	1	72·7	....	74·	1·3	52	Full grown.
4	Vespa vulgaris (larva)	12	1	72·7	....	75·2	2·5	52	Full grown.
5	Athalia (larvæ)................	Sept. 6	50	64·5	....	66·5	2·0	....	Larva nearly full grown; very active.
6	Athalia (larvæ)................	6	....	64·7	....	66·	1·3	....	
7	Athalia (larvæ)................	6	....	66·3	....	66·8	0·5	....	Larva inactive.
8	Athalia (larvæ)................	6	200	65·3	....	67·3	2·	....	Full grown; active.
9	Athalia (larvæ)................	6	50	65·3	....	67·3	2·	....	
10	Melolontha vulgaris (larva)	Oct. 7	1	61·5	59·7	60·2	0·5	....	No. 1. Full grown larvæ; the temperature taken while lying in their cells and compared with the temperature of the soil.
11	Melolontha vulgaris (larva)	7	1	61·5	59·7	60·3	0·6	....	No. 2.
12	Melolontha vulgaris (larva)	8	1	64·7	64·6	64·7	0·1	....	No. 1.
13	Melolontha vulgaris (larva)	8	1	64·7	64·6	64·8	0·2	....	No. 2.
14	Melolontha vulgaris (larva)	12	1	63·5	63·5	63·7	0·2	....	No. 1.
15	Melolontha vulgaris (larva)	12	1	63·5	63·5	63·7	0·2	....	No. 2.

TABLE XIII.

Showing the difference between the Temperature of the Atmosphere and that of the Bee-hive No. 1, through many succeeding days, both when undisturbed and when excited.

No. of Exp.	Period of observation.	Weather.	Wind.	Atmosphere.	Hive No. 1.	Difference.	Excited.	Difference.	Remarks.
	1835.			°		°	°	°	
1	Oct. 23 A.M. 7	Fine, hoar frost	....	36	50	14	......	......	Hive had remained undisturbed through the night.
2	A.M. 10	Sunshine	....	49·5	55	5·5	67·5	18	Bees readily excited.
3	P.M. 2	Sunshine	....	53	62·5	9·5	80	27	Bees active, loud humming in the hive.
4	P.M. 5½	Twilight	....	51	66	15	......	......	Hive slightly disturbed; cool evening.
5	24 A.M. 10½	Fine, brisk wind......	W.	53·4	58·3	4·9	59	5·6	A few bees abroad, some return with *yellow pollen*.
6	P.M. 3½	Wind and rain	W.S.	52	61·8	9·8	......	......	Not a single bee abroad; faint humming in the hive.
7	P.M. 5½	Windy, cloudy	S.W.S.	53	60	7	......	......	Hive perfectly quiet.
8	25 A.M. 7¼	Frost, calm, fine......	E.	40·5	52	11·5	......	......	Bees beginning to hum, and becoming active.
9	A.M. 10½	Dull, foggy, calm ...	W.S.	53	55·3	2·3	71	18	Hive quiet; a few bees abroad returning with *pollen*.
10	P.M. 2	High wind and rain	S.	56	58	2	......	......	Hive quiet; tempestuous rain.
11	26 A.M. 8	High wind, fine......	W.	48	53·8	5·8	......	......	Quiet; wind and rain tempestuous during the night.
12	A.M. 8½	High wind, fine......	W.	50	54	4	......	......	Quiet.
13	A.M. 11	High wind, sunny ...	W.	52·5	59	6·5	......	......	Hive quiet, but few bees abroad.
14	P.M. 2	Strong wind, showery	W.S.W.	52·9	57	4·1	......	......	Faint humming in the hive; few bees abroad.
15	P.M. 5¾	Calm, cloudy	W.S.	44	54	10	......	......	Twilight, hive quiet.
16	27 A.M. 6½	Fair, light clouds ...	W.N.	40	49·5	9·5	50·5	10·5	Much rain in the night; bees irritable.
17	A.M. 8	Fair, light wind	W.N.	41	49·4	8·4	......	......	Perfectly quiet.
18	A.M. 10	Light wind, sunny...	W.	50·5	51·9	1·4	......	......	Many bees at the entrance of the hive; irritable.
19	P.M. 2	Light wind, fine......	W.N.	50·8	58·7	7·9	73	22·2	Few bees abroad; soon excited.
20	P.M. 5½	Heavy clouds	W.N.	44	52	8	......	......	Twilight, slight humming in the hive.
21	28 A.M. 6¾	Sharp hoar frost	W.N.	28·5	45	16·5	......	......	Slightly disturbed; humming; morning calm.
22	A.M. 7½	Fair	W.N.	30	52·5	22·5	......	......	Quiet.
23	A.M. 10½	Calm, fine	W.	49·3	63·7	14·4	......	......	Few bees abroad; slight humming.
24	P.M. 2	Calm, dull	W.	48·3	57	8·7	......	......	Slight humming.
25	P.M. 5	Calm, foggy	W.	44	56	12	......	......	Quiet; calm damp foggy evening.
26	29 A.M. 7	Hard rain, wind......	S.W.	51	52·5	1·5	......	......	Hive quiet; hard rain and wind during the night.
27	P.M. 2½	Dull, damp............	S.W.	57	67	10	......	......	Many bees abroad with *orange yellow pollen*; irritable.
28	P.M. 5	Calm evening.........	W.	51	61·5	10·5	......	......	Perfectly quiet.
29	30 A.M. 6½	Fair, misty	E.N.	38	49·5	11·5	......	......	A few sounds heard in the hive.
30	A.M. 10½	Fair, cold wind	E.	51·5	54	2·5	76	24·5	Many bees enter with *pollen*; irritable.
31	A.M. 12	Fair, signs of rain ...	E.S.	51	65	14	......	......	Bees abroad in numbers flocking home with *pollen*.
32	P.M. 2½	Hard rain	E.S.	50·5	60	9·5	......	......	Bees quiet; began to rain about 1 P.M.
33	P.M. 5	Steady rain	E.S.	49·5	58	8·5	......	......	Quiet; light wind.
34	31 A.M. 7	Misty	S.	52	56	4	......	......	Quiet; heavy continued rain during the night.
35	A.M. 10½	Sunshine	S.	53	59	6	......	......	Fine morning; many bees abroad.
36	A.M. 12	Sunny, light clouds...	W.S.	57·6	67·5	9·9	......	......	Many bees abroad; a few with *pollen*.
37	P.M. 2	Fair	W.S.	58	67	9	83·5	25·5	Many bees abroad.
38	P.M. 5	Calm, light clouds ...	W.N.	50·6	76·5	25·9	......	......	Twilight, dull evening; slight humming.
39	Nov. 1 A.M. 6¾	Fine.....................	N.E.	39	56	17	......	......	Light clouds; sun just risen; hive quiet.
40	A.M. 8	Fine morning.........	N.E.	42	57·5	15·5	......	......	Quiet.
41	A.M. 11	Very fine..............	N.E.	50·5	58·4	7·9	78	27·5	Bees at entrance of the hive; very little *pollen* collected.
42	P.M. 2	Very fine..............	N.E.	51	65·4	14·4	80	29	Loaded bees numerous; quantity of *pollen* scanty.
43	P.M. 5	Fair, calm	W.	43·5	68	24·5	......	......	Quiet.
44	2 A.M. 7	Fair, light clouds ...	E.	39·5	51	11·5	......	......	Perfectly quiet.
45	P.M. 1	Fair......................	S.W.	55·5	58·3	2·8	......	......	Hive quiet; many bees abroad.
46	P.M. 6	Cloudy, rain	S.W.W.	49·5	55·2	5·7	......	......	Dark evening; hive quiet; beginning to rain.
47	3 A.M. 7½	Misty rain	S.E.	45	52·4	7·4	......	......	Quiet; light steady rain through the night.
48	A.M. 10½	Light misty rain......	S.E.	47·5	52·4	4·9	......	......	Quiet; no bees abroad.
49	P.M. 3	Brisk wind	S.E.	47	57·7	10·7	......	......	Quiet; heavy clouds; signs of rain.
50	P.M. 5	Wind...	S.E.	46	53	7	......	......	Hive quiet; no bees abroad to day.
51	4 A.M. 7	Fair, with clouds......	S.E.	43	49·7	6·7	......	......	Quiet; heavy clouds; cold wind.
52	A.M. 10	Fair......................	E.	43·5	49·6	6·1	70	26·5	Quiet, but soon excited; brisk sharp wind.
53	P.M. 2	Cloudy, wind	E.	43·2	51·5	8·3	......	......	Brisk wind; quiet.
54	P.M. 5	Rain with hail	E.	42	50·5	8·5	......	......	Quiet; brisk wind.
55	5 A.M. 7½	Heavy clouds	E.	43	49	6	......	......	Quiet; but little rain during the night.
56	A.M. 10	Steady rain	E.	45·7	49·7	4	......	......	Quiet; heavy clouds.
57	P.M. 2½	Misty rain	E.	44·6	49·8	5·2	......	......	Slight noise in the hive; light wind.
58	P.M. 5	Light rain	E.N.	43	49·8	6·8	......	......	Quiet.
59	6 A.M. 7¾	Hoar frost	N.E.	32·7	44·1	11·4	......	......	Rather misty; slight sound in the hive.
60	A.M. 9	Very calm	N.E.	37·5	44·6	7·1	......	......	Hive quiet.
61	A.M. 10	Calm, sunshine	N.E.	41	45·3	4·3	......	......	Hazy sunshine.
62	A.M. 11	Calm, sunshine	N.E.	45	47	2	......	......	Hive quiet.

TABLE XIII. (Continued.)

No. of Exp.	Period of observation.	Weather.	Wind.	Atmosphere.	Hive No. 1.	Difference.	Excited.	Difference.	Remarks.
	1835.			°	°	°	°	°	
63	Nov. 6 P.M. 1½	Calm, sunshine	E.	47·7	53·7	6			Loud humming in the hive; a few bees abroad.
64	P.M. 2½	Calm, sunshine	N.E.	45·5	51·7	6·2			A few bees return with a little *pollen.*
65	P.M. 3	Calm, fair	N.E.	44·9	50·6	5·7			A few bees still abroad.
66	P.M. 3¼	Calm, sunshine	N.E.	45·3	49·5	4·2			A few bees abroad; dew begins to condense on the grass in the shade.
67	P.M. 3½	Calm, fair	N.E.	44·7	49·1	4·4			Slight humming.
68	P.M. 3¾	Calm, fair	N.E.	44·5	48·9	4·4			Slight humming.
69	P.M. 4	Calm, fair	N.E.	43·4	49	5·6			Sky cloudless, but slightly hazy.
70	P.M. 4¼	Calm, fair	N.E.	41·5	48·1	6·6			Quiet.
71	P.M. 4½	Calm, light clouds	E.	41·5	53·9	12·4			Light hazy clouds.
72	P.M. 4¾	Light wind	E.S.E.	40·5	50·4	9·9			Wind shifting.
73	P.M. 5	Cloudy	S.E.	41·2	49	7·8			Thermometer varying.
74	P.M. 5¼	More cloudy	S.E.	41·7	50	8·3			Signs of rain in the horizon.
75	7 A.M. 7	Very cloudy	N.N.W.	41·5	46	4·5			Hive quiet; a little rain last night.
76	A.M. 7¼	Light clouds	N.N.W.	41·4	46·6	5·2			Hive a little excited without evident cause.
77	A.M. 7½	Light clouds	N.N.W.	41·7	46·6	4·9			Hive quiet; atmosphere clearer.
78	A.M. 7¾	Sun peeping	N.	42·2	46·5	4·3			Quiet.
79	A.M. 8	More cloudy	N.	43·1	48	4·9			Slight humming; clouds thickening from the east.
80	A.M. 8¼	Signs of rain	N.	43·5	47	3·5			Slight humming.
81	A.M. 8½	Clouds breaking	N.E.	44·7	47·4	2·7			Slight humming; a few drops of rain have fallen.
82	A.M. 8¾	Fairer	N.E.	45·6	47·5	1·9			Slight humming.
83	A.M. 9	Fair	N.E.	46·9	47·6	·7			Hive quiet; clouds dispersing.
84	A.M. 9¼	Light wind and clouds	E.N.E.	47·7	48	·3			Slight humming.
85	A.M. 9½	Fair	E.N.E.	48·7	48·3				Clouds dispersing; hive ·4 of degree lower than the temperature of atmosphere.
86	A.M. 9¾	Fair	E.N.E.	48·7	48·7				A few bees have been abroad.
87	A.M. 10	Fair	E.N.	49			73·5	24·5	When excited temperature of hive rose in 10 minutes to 73°·5.
88	A.M. 11½	Calm, dull	E.N.	50·6	64·9	14·3			Many bees abroad; two have returned with *pollen.*
89	P.M. 1	Calm, dull	E.	52·6	70	17·4	78·3	25·7	Bees irritable; many abroad; a few with *pollen.*
90	P.M. 3	Calm, fine	E.S.	51·4	75	23·6			Many bees still abroad.
91	P.M. 5	Steady rain	E.S.	49	70	21			Hive quiet.
92	8 A.M. 7	Light frost, calm	W.	35	53	18			Quiet; much rain fell last night.
93	A.M. 9	Fine, calm	W.	40·6	52·3	11·7			Quiet; cold, fine; a few bees dead on the alighting board.
94	A.M. 12½	Brisk wind	N.W.	48	60·3	12·3			Sunshine; many bees abroad.
95	P.M. 2	Wind, sunshine	N.W.	50	59·3	9·3			Bees abroad; no *pollen* collected.
96	P.M. 5	Calm, clear	N.W.N.	42·2	57	14·8			Hive quiet; calm evening.
97	9 A.M. 7½	Dull, light wind	N.E.	40·6	49	8·4			Quiet; cold dull morning; no dew on the grass.
98	A.M. 9	Cold wind	N.E.	41·7	49	7·3			Quiet; cloudy.
99	A.M. 10	Cold wind	N.E.	42	49·4	7·4			Quiet; cold brisk wind.
100	A.M. 11	Cold, sunny	N.	42·4	50·2	7·8			No bees abroad.
101	A.M. 12	Cold, sunny	N.	43·6	49·7	6·1			Quiet.
102	P.M. 1	Cold wind	N.E.N.	42·4	49·9	7·5			Quiet.
103	P.M. 2½	Cold wind	N.E.	41·9	50	8·1			Quiet; cold misty rain.
104	P.M. 3	Wind and rain	N.E.	39·3	49·5	10·2			Quiet; light driving rain.
105	P.M. 5	No rain	N.E.	38	48	10			Quiet; cold wind with driving clouds.
106	10 A.M.	Cold brisk wind	N.E.						A severely cold day; wind bitingly keen.
107	11 A.M. 7½	Hazy, cold	N.E.N.	33	43·6	10·6			Quiet; cold windy morning.
108	A.M. 9	Hazy, cold	N.E.N.	35	43·	8			Quiet; a little snow has just fallen.
109	A.M. 10	Sun peeping	N.E.N.	38·5	43·9	5·4			Slight humming; a little snow falling.
110	A.M. 11	Fine rain	N.E.N.	40·4	44·4	4			Quiet; light rain.
111	A.M. 12	Light rain	N.	41	45	4			Quiet; sunny with rain.
112	P.M. 1	Dull, hazy	N.	41·8	45·2	3·4			Calm; dull.
113	P.M. 2	Hazy	N.N.W.	41·6	45·2	3·6			A shrill humming at intervals of a single bee is heard in the hive.
114	P.M. 4	Hazy	N.N.W.	40	44·7	4·7			Hive quiet.
115	P.M. 4¾	Hazy	N.N.W.	39·6	44·5	4·9			Quiet; heavy clouds.
116	12 A.M. 7½	Fine, light wind	N.	35	43·4	8·4			Cold dry wind; hive quiet.
117	A.M. 9	Fine, calm	N.	40	43·3	3·3			Quiet; but excited by the slightest noise.
118	A.M. 10	Fine, calm	N.	43	44·2	1·2			Quiet; bright sunshine.
119	A.M. 11	Fine, calm	N.E.N.	45·6	45·4				Quiet; hive ·2 of degree lower than temperature of the atmosphere.
120	A.M. 12	Fine	N.E.N.	46·6	46·3				Hive quiet; ·3 of a degree the temperature of atmosphere.
121	P.M. 1	Sunny, clouds	N.E.N.	48	48·1	·1			Slight humming; a few bees abroad.
122	P.M. 2	Heavy clouds	N.E.	45·3	47·9	2·6			Hive quiet.
123	P.M. 3	Heavy clouds	N.E.	42·6	47	4·4			Quiet; very heavy clouds passing.
124	P.M. 4	Heavy clouds	N.E.	41·2	41·2				Quiet; signs of rain; rainbow.
125	13 A.M. 7½	Light rain	N.E.	36	43·7	7·7			Quiet; hard rain this morning.

TABLE XIII. (Continued.)

No. of Exp.	Period of observation.	Weather.	Wind.	Atmosphere.	Hive No. 1.	Difference.	Excited.	Difference.	Remarks.
	1835.			°	°	°	°	°	
126	Nov. 13 A.M. 9	Light rain	N.E.	37·3	43·7	6·4	......	......	Quiet; cold brisk wind.
127	A.M. 10	Light continued rain......	N.E.	37·6	43·9	6·3	......	......	Quiet; less wind; gloomy morning.
128	A.M. 11	Light rain	N.E.	39·6	44·1	4·5	......	......	Quiet; very light wind; sky clearing.
129	A.M. 12½	Sunny	N.E.	39·8	44·9	5·1	......	......	Clouds passing.
130	P.M. 2	Calm	N.E.	41·6	45·8	4·2	......	......	Sunshine; bees attacked by the sparrows.
131	P.M. 3	Rain	E.	39·6	45·8	6·2	......	......	Quiet; heavy clouds, with rain; sky very gloomy.
132	P.M. 4	Rain	E.	38	45·2	7·2	......	......	Quiet; heavy clouds.
133	14 A.M. 7½	Calm, cloudy	N.W.N.	35	43·2	8·2	......	......	Hive a little disturbed.
134	A.M. 8	Calm, fair	N.W.N.	35·4	43·2	7·8	67·3	31·9	When the hive was excited temp. rose in 14^{m} to 67′·3.
135	A.M. 9	Calm, fair	N.W.N.	35·4	47	11·6	......	......	Hive nearly quiet.
136	A.M. 10	Calm, fair	N.W.N.	39·4	46·3	6·9	......	......	Slight humming.
137	A.M. 11	Calm, fair, cold	N.E.N.	40·4	46·9	6·5	69·5	29·1	Raised in 11^{m} to 69′·5; bees appear, but return directly to the hive; air too cold.
138	A.M. 12	Calm, fine	E.N.	42·5	48·5	6	57	14·5	Great excitement; temperature maintained at 57′; bees ventilating, and going abroad.
139	P.M. 2½	Calm, fine	E.N.	42·2	57	14·8	65	22·8	A few bees still abroad; hive still excited.
140	P.M. 4	Calm, fine	E.N.	38	66	28	......	......	Slight humming; very fine evening.
141	P.M. 4½	Calm, fine evening.........	N.W.	36	56	20	......	......	Slight humming.
142	15 A.M. 8	Calm, fair	N.E.	41	49	8	......	......	Hive quiet; atmosphere rather hazy.
143	A.M. 9	Heavy clouds	N.E.	42·6	50	7·4	......	......	Humming; clouds passing.
144	A.M. 10	Sunny	N.E.	45	50·2	5·2	......	......	Clouds passing.
145	P.M. 1	Bleak wind..................	N.E.	47	54	7	......	......	Quiet; sky dark, cloudy.
146	P.M. 2	Cloudy, cold	N.E.	45·4	53·6	8·2	......	......	Faint humming; wind bleak.
147	P.M. 4½	Heavy clouds	N.	43·5	51	7·5	......	......	Faint humming; signs of rain.
148	16 A.M. 7½	Dull, cloudy	W.	36·8	46	9·2	......	......	Faint humming.
149	A.M. 8½	Fair, light wind	W.	37·6	46·2	8·6	......	......	Hive quiet.
150	A.M. 9½	Fairer	W.	40·3	46·4	6·1	......	......	Faint humming.
151	A.M. 10½	Dull sky	W.	40·9	46·7	5·8	......	......	Hive quiet.
152	A.M. 11½	Fair	W.N.	44	47·1	3·1	......	......	Humming.
153	A.M. 12½	Fair	W.N.	45·1	48	2·9	......	......	Humming.
154	P.M. 1½	Fair	W.N.	44·7	48·1	3·4	......	......	Quiet, excepting that the humming of a single bee is sometimes heard.
155	P.M. 2½	Fair	W.N.	44	47·8	3·8	......	......	Quiet.
156	P.M. 3½	Fair	N.	40·8	47·2	6·4	......	......	Quiet.
157	P.M. 4½	Calm, clear..................	N.	37·6	46·7	9·1	......	......	Hive quiet; calm clear evening.
158	17 A.M. 7	Misty, calm..................	W.	39	45	6	......	......	Quiet; dull misty morning.
159	A.M. 7½	Misty, calm..................	W.	39·5	45	5·5	......	......	Brisk humming without evident cause.
160	A.M. 8	Misty, calm..................	W.	40	45·3	5·3	......	......	Hive more quiet.
161	A.M. 8½	Misty, calm..................	W.N.	40·6	45·4	4·8	......	......	Quiet.
162	A.M. 9	Misty, calm..................	N.W.	41·4	45·3	3·9	......	......	Humming of a single bee.
163	A.M. 9½	Misty, light wind	N.W.	43·6	45·5	1·9	......	......	Quiet.
164	A.M. 10	Light misty rain...........	N.W.W.	44·6	46·1	1·5	......	......	Quiet.
165	A.M. 10½	Light misty rain...........	W.N.	45·2	46·8	1·6	......	......	Quiet; no bees abroad.
166	A.M. 11½	Light misty rain...........	W.	47·4	47·4	......	......	......	Quiet; heavy clouds.
167	A.M. 12	Light misty rain...........	W.	48·2	48·1	......	......	......	Quiet; hive °·1 of degree below the atmosphere.
168	A.M. 12½	Light rain	W.	48·4	48·1	......	......	......	Quiet; °·3 of degree below.
169	P.M. 1½	Clouds breaking...........	W.	48·6	48·3	......	......	......	Quiet; °·3 below; wind increasing; no rain.
170	P.M. 4	No rain	W.	47·6	48·5	·9	......	......	Quiet; cloudy.
171	18 A.M. 8	Very dull	W.	47·8	48·2	·4	......	......	Hive quiet; signs of rain.
172	A.M. 9	Brisk wind	W.	49·2	48·9	......	......	......	Hive quiet; °·3 below atmosphere; cloudy.
173	A.M. 10	Brisk wind	W.	51·4	49·7	......	......	......	Quiet; 1°·7 below the atmosphere; sunshine.
174	A.M. 11	Brisk wind	W.	52	50·9	......	......	......	Quiet; 1°·1 below; dull, cloudy.
175	A.M. 12	Brisk wind	W.S.	52·6	51	......	......	......	Quiet; 1°·4 below; sunny, with clouds.
176	P.M. 2½	Brisk wind	W.S.	51·9	51·6	......	......	......	Very quiet; °·3 below; a few bees abroad.
177	30 P.M. 3	Light rain	S.	54·3	60	5·7	80	25·7	Humming; bees undisturbed for the last 12 days.
178	Dec. 2 P.M. 1	Fine, calm	W.S.	51	75	24	......	......	Many bees abroad; excited without evident cause.
179	P.M. 2½	Fair, sunny..................	W.S.	49	72·5	23·5	......	......	Bees ventilating; still excited.
180	P.M. 3	Fair, sunny..................	S.W.	48	73·5	25·5	79·8	31·8	Very irritable.
181	P.M. 3¾	Fair, calm	S.W.	46·5	73	26·5	......	......	Very irritable.
182	P.M. 4	Fair, calm	S.W.	46	......	......	78	32	Clear; moonlight.
183	3 A.M. 8	Windy, cold	S.	49·2	71·5	22·3	......	......	Air damp and cold; bees abroad and very busy.
184	A.M. 9	Brisk wind, cloudy.........	S.	50	61·5	11·5	......	......	Hive quiet; heavy clouds.
185	A.M. 12½	Brisk wind, cloudy.........	S.	51	60·7	9·7	......	......	Hive quiet.
186	P.M. 4¼	Brisk wind, cloudy.........	S.S.E.	49·2	60·2	11	......	......	Hive quiet.
187	4 A.M. 10	Fair, calm	W.	44·2	50·8	6·6	......	......	Light clouds and rain; hive quiet.
188	5 A.M. 8	Fine, clear	W.	34	43·2	9·2	......	......	Hive quiet.
189	P.M. 4	Fine, calm	S.W.	44	47	3	......	......	Hive quiet.
190	12 A.M. 8	Fine morning, light wind	N.	23	39	16	72	49	Hoar frost; has frozen hard during the last 36 hours; bees active, although entirely undisturbed during the last six days; raised the therm. in 11^{m} to 49° above the temperature of the atmosphere.

TABLE XIII. (Continued.)

No. of Exp.	Period of Observation.	Weather.	Wind.	Atmosphere.	Hive No. 1.	Difference.	Hive No. 2.	Difference.	Excited.	Difference.	Remarks.
191	1835. Dec. 12 P.M. 1	Dull, cloudy	W.	38°	47°·8	9°·8	......°	......°	74°·2	36°·2	Bees more quiet but readily excited.
192	P.M. 4	Dull, cloudy	W.	36	56	20	......	......	......	......	Bees quiet.
193	13 P.M. 2	Fine, calm	N.W.	41·2	45·2	4	......	......	75·2	34	Slightly active; soil hard frozen in the shade.
194	14 A.M. 11½	Cloudy	S.W.	42·6	45·6	3	......	......	63·5	20·9	
195	23 P.M. 4	Clear frost	N.E.	27	38	11	......	......	72·3	45·3	Bees excited by the slightest noise, although entirely undisturbed for the last nine days, during four of which the temperature of atmosphere has been below 32°, sometimes so low as 24° FAHR. Bees raised the thermometer to 72°·3 in ten minutes.
196	24 A.M. 12	Calm, misty	N.E.	31	42·7	11·7	......	......	......	......	Bees quiet; soil still hard frozen.
197	P.M. 4	Calm, hazy	N.E.	29·5	42	12·5	......	......	......	......	
198	25 A.M. 10½	Calm, fine	N.E.	23	38·4	15·4	......	......	......	......	Bees quiet, but excitable; intense hoary frost.
199	27 A.M. 8	Cloudy, calm	W.	29	39·5	10·5	......	......	......	......	Frost during the night; temperature rising; bees quiet.
200	P.M. 4	Cloudy, thawing	W.	37	45·4	8·4	......	......	......	......	Bees quiet; a gentle thaw.
201	28 A.M. 8	Cloudy	W.S.	42·7	45	2·3	......	......	......	......	Bees quiet.
202	29 P.M. 1	Fine	W.	42·8	46·2	3·4	......	......	......	......	Bees quiet.
203	30 A.M. 8	Light cold rain	N.W.	40	44·9	4·9	......	......	......	......	Bees quiet.
204	1836. Jan. 1 P.M. 2	High wind, sleet	N.E.	31·5	44·1	12·6	......	......	72	40·5	Quiet, but excitable; frost, with wind and sleet all day.
205	2 A.M. 7¼	Clear, intense frost	E.	17·5	30	12·5	......	......	70	52·5	Day-break; starlight; hive excited; temperature raised in 16 minutes, and maintained for several minutes at 70° FAHR., but at 5 inches distant from this part of hive, temperature only 45°, thus giving a temperature of 25° for the bodies of the bees.
206	A.M. 7¾	Sun just risen	E.	16·5	63	46·5	......	......	......	......	
207	A.M. 8¼	Light wind	E.	16·5	59	42·5	......	......	......	......	Very fine.
208	A.M. 8½	Light wind	E.	17·5	59	41·5	......	......	......	......	Hive more quiet; very fine.
209	A.M. 9¼	Light wind	E.	18·5	49	30·5	......	......	......	......	Only very faint sounds in the hive.
210	A.M. 12¼	Light clouds	E.S.	30·7	46	15·3	......	......	......	......	Hive quiet; wind shifting; temperature rising rapidly.
211	P.M. 1½	Rather cloudy	S.E.	32·3	49	16·7	......	......	......	......	Bees irritable; wind shifting.
212	P.M. 2⅓	Cloudy	S.E.	31·2	45	13·8	......	......	......	......	Hive quiet.
213	3 A.M. 10	Light clouds	W.	37	43·5	6·5	......	......	......	......	Hive quiet; frost broke suddenly.
214	5 P.M. 1	Sunny, fair	S.W.	50	55	5	......	......	82·2	32·2	Bees undisturbed for three days; excited temperature continued at 70° for several hours; many bees going abroad.
215	13 A.M. 8	Hoar frost	W.	28·5	45	16·5	......	......	......	......	
216	28 A.M. 8	Fair	W.	43·5	59·5	16	......	......	......	......	
217	February 19 A.M. 9	Fine day	N.W.	35	47·5	12·5	51	16	......	......	On the 15th inst., a very fine day, I saw many bees enter the hive with *orange, brown* and *grey pollen.*
218	A.M. 11	Fine day	N.W.	39·2	48·2	9	52·3	13·1	......	......	
219	P.M. 2	Fine day	N.	48·5	50·2	1·7	55·3	6·8	......	......	Fine day.
220	20 A.M. 8	Fine, calm	N.E.	24	44	20	45	21	......	......	Hive quiet; hard frost all night.
221	A.M. 10	Fine	N.E.	34·5	48·5	14	91	56·5	102	67·5	Hive No. 2. very active; light clouds.
222	A.M. 11	Light clouds	E.	39·5	48·1	8·6	93	53·5	......	......	Calm cold morning.
223	P.M. 2½	Very fine	E.	41·8	58·5	6·7	84·4	42·6	......	......	Bees go abroad but return quickly; air too cold.

Table XIV., showing the Difference between the Temperature of the Atmosphere and that of the same Hive at half-hourly observations, made at precisely the same periods in succeeding years, 1836 and 1837.

		1836.					1837.				
No. of Exp.	Period of observation.	Wind, &c.	Atmosphere.	Hive No. 1.	Difference.	Remarks.	Wind, &c.	Atmosphere.	Hive No. 1.	Difference.	Remarks.
	1836.		°	°	°			°	°	°	
1	March 22 A.M. 7	S.W.S. misty, calm...	47	61	14	Dull misty morning.	E. by S. calm, frost	28·6	46·3	17·7	Snow on the ground 2 inches deep; has continued to fall since 12 P.M.; very faint sounds in hive; thick, cloudy.
2	A.M. 7½	S.W.S. misty	47	62	15	Dull; hive quiet; no bees abroad.	E.S. light	29·1	46·4	17·3	Faint sounds; clouds breaking; snow falling.
3	A.M. 8	S.W.S. light wind ...	48	62·2	14·2	No rain; no bees abroad; mist dispersing	S.E.E. calm	30·4	46·4	16	Hive quiet; clouds breaking; sun peeping.
4	A.M. 8½	....	......	......	......	....	S.E.E. calm	33	46·8	13·8	Hive quiet; sun peeping.
5	A.M. 9	....	......	......	......	....	S.E.E. fine, calm	34·1	47·5	13·4	Sunny; hive quiet; birds singing.
6	A.M. 9½	S. verydull, light wind	49·2	62·6	13·4	Has been a light shower	S.E.S. light........	35·5	48·6	13·1	Hive quiet; dull, cloudy.
7	A.M. 10	S. dull, moist cold air	48·3	64	15·7	Hive quiet; no bees abroad	S.E. light	33·9	49	15·1	Very dull; cloudy; snowing very fast.
8	A.M. 10½	S. fairer	48·5	64·1	15·6	Hive quiet; horizon clearing	S.E.E. brisk	32·4	48·9	16·5	Hive perfectly quiet; thick fall of snow.
9	A.M. 11	S. very dull..........	49·6	62·8	13·2	A few bees have just gone abroad	S.E. light	34·3	48·8	14·5	Hive quiet; snow still falling.
10	A.M. 11½	....	......	......	......	....	S.E.S. light........	34·6	49	14·4	Snow still falling.
11	A.M. 12	S. fair	48·3	63·6	15·3	Hive quiet; no bees abroad............	S. light	37	49·3	12·3	Sky fair; a little snow; fair.
12	A.M. 12½	S. rain, calm	......	......	......	Hard steady rain; hive quiet	S. light	37·8	49·9	12·1	Hive quiet; snow falling.
13	P.M. 1	S. rain, calm	48·2	63·9	15·7	Hive quiet; steady rain	S. light	38·6	50·6	12	Fine; a little snow falling.
14	P.M. 1½	S. rain, calm	49·8	64	14·2	Steady rain; hive quiet	S. light	36·6	50·4	13·8	Brisk wind; hive quiet.
15	P.M. 2	S.W.S. rain, calm ...	49·8	63·8	14	A few bees abroad; irritable	S. light	37·5	50·3	12·8	A few bees moving; none abroad; sunny.
16	P.M. 2½	....	......	......	......	....	S. fine	37·6	50·6	13	Hive quiet; sunny; light wind.
17	P.M. 3	S.W.S. rain, calm ...	48·1	64	15·9	A few bees abroad; steady rain	S. by E. fair	37·3	50·6	13·3	Hive quiet; no bees abroad; sunny.
18	P.M. 3½	S.W.S. fair	48·7	63·9	15·2	No rain; bees abroad, but no *pollen* collected	S. by E. light......	36·8	50·9	14·1	Faint sounds in the hive; fair.
19	P.M. 4	S.W.S. fair	49·3	63·3	14	Many bees abroad; irritable	S. light	35·7	50·1	14·4	Hive quiet; fair; cold.
20	P.M. 4½	....	......	......	......	....	S. fair, light	35·7	50	14·3	Fleecy clouds; hive quiet.
21	P.M. 5	S.W.S. dull...........	49·2	63·1	13·9	No rain; a few bees abroad............	S. fair, light	34·7	49·6	14·9	Sunny; light.
22	P.M. 5½	....	......	......	......	....	S. calm	33·7	49·3	15·6	Hive quiet; calm.
23	P.M. 6	S.W.S. very dull......	48·3	62·5	14·2	Hive quiet; no rain	S. by W. calm......	32·5	49	16·5	Dull evening.
24	P.M. 6½	S.W.S. dull...........	48	62·5	14·5	No rain; evening hazy.................	S. by W. calm......	31·5	48	16·5	Dull; cold; hive perfectly quiet.
25	23 A.M. 7	S.S.W. calm, cloudy	45·5	60	14·5	Hive quiet; light rain	S. by W. light......	32	45	13	Hive quiet; cloudy; thick snow last night.
26	A.M. 7½	S.S.W. hazy	45·7	60·7	15	Hive quiet; misty rain................	S. by W. brisk ...	32·6	45·4	12·8	Hive quiet; cloudy.
27	A.M. 8	S.S.W. fair	47·2	60·3	13·1	Light wind; sun peeping..............	S. by W. brisk ...	33·3	46·9	13·6	Very faint humming; very cloudy.
28	A.M. 8½	S.S.W. fair	47·3	61	13·7	Light rain and wind; clouds dispersing	S.E. by S. brisk ...	33·9	48·4	14·5	Faint humming; slightly disturbed.
29	A.M. 9	S.S.W. fair	49·3	61·6	12·3	Hive quiet; a few bees come abroad ...	E. by S. brisk......	34	49·2	15·2	Very faint humming; cloudy.
30	A.M. 9½	S.S.W. fair	51	61	10	More wind; more bees abroad	E. by S. brisk......	34·7	50·9	16·2	
31	A.M. 10	S.S.W. brisk	51	63	12	Bees active; *scarlet* and *orange pollen* ...	E. brisk	35·5	52·6	17·1	Hive quiet; fair.
32	A.M. 10½	S.S.W. brisk	50·6	64	13·4	Many bees with *pollen*	E. by S. brisk......	36·9	53·6	16·7	Hive aroused; sunshine.
33	A.M. 11	S.S.W. brisk	49·7	62	12·3	Bees active............................	E. by S. brisk......	36·8	54	17·2	
34	A.M. 11½	S.S.W. brisk, cloudy	50·2	63	12·8	Quantity of *pollen* collected scanty	E. brisk	36·6	55·2	18·6	Slight humming.
35	A.M. 12	S.S.W. brisk	49·9	63·9	14	Hive quiet; bees abroad	E. brisk	36·5	55·5	19	Hive quiet; fair; cold.
36	A.M. 12½	S.S.W. fair	49·7	64·6	14·9	Very light rain; few bees abroad	E. brisk	37	55·9	18·9	Slight humming; cloudy.
37	P.M. 1	S.S.W. rain, calm ...	50	64·7	14·7	Steady rain............................	E. brisk	38·3	56·5	18·2	Slight humming; cloudy; snow has fallen.
38	P.M. 2	S.S.W. rain...........	47·3	64·7	17·4	Continued hard rain....................	E. brisk	37·5	56	18·5	Hive a little excited; sunny, with clouds.
39	P.M. 5	S. brisk	45	61·2	16·2	Hard continued rain since 2 o'clock ...	E. calm	32	56·2	24·2	Fine afternoon; calm; hive quiet; frost.
40	24 A.M. 7	W. fair, light wind...	37	57	20	Sunny; heavy dew on grass..............					
41	A.M. 7½	W. fair, light wind...	41	58	17	Hive quiet; sunshine; cold wind					
42	A.M. 8	W. brisk	42·5	59	16·5	No bees yet abroad					
43	A.M. 11	W. brisk, sunshine...	50	59·2	9·2	Bees active; *orange* and *yellow pollen* ...					
44	P.M. 1	W. brisk	50	62	12	Many bees with *pollen*					
45	P.M. 1½	W. cold, brisk wind	49·6	62·1	12·5	*Pollen* still collected					
46	P.M. 4	W. cold	45	61	16	Hive quiet; no bees abroad.............					
74	P.M. 5	W. cold	44·5	60·3	15·8	Hive quiet, and signs of rain					

Table XV.

Showing the Temperature of the Atmosphere and of the Bee Hives No. 1 and No. 2 every fifteen minutes during thirteen successive hours on the 2nd April, 1837, and of Hive No. 1 on the same day in April, 1836.

1837.								1836.				
No. of Exp.	Period of observation.	Wind.	Atmosphere.	Hive No. 1.	Difference.	Hive No. 2.	Difference.	Wind.	Atmosphere.	Hive No. 1.	Difference.	Remarks on the Hives and Weather, 1837.
	1837.		°	°	°	°	°		°	°	°	
1	April 2. A.M. 6	Calm N.W.N.	26·2	53·5	27·3	52·8	26·6	......	......	......	......	Hard frost; very fine; slight humming in the hive.
2	A.M. 6¼	Calm N.W.N.	26·2	53·5	27·3	52·2	26	......	......	......	......	Slight humming; light fleecy sky.
3	A.M. 6½	Calm N.W.N.	27·6	54·4	26·8	54	26·4	High N.W.......	35	59	24	Slight humming; stratified clouds in the west.
4	A.M. 6¾	Calm N.W.N.	29·3	54·4	25·1	54·8	25·5	High N.W.......	36	61	25	Slight humming in both hives.
5	A.M. 7	Light N.W.N.	33	54·5	21·5	57·6	24·6	High N.W.......	36·8	59	22·2	Very fine; clouds fleecy.
6	A.M. 7¼	Light N.W.N.	33·5	54·6	21·1	59	25·5	......	......	......	......	Very fine.
7	A.M. 7½	Light N.N.W.	35·7	56·3	20·6	60·3	24·6	High N.W.......	38·5	58·9	20·4	Both hives quiet; very fine morning.
8	A.M. 7¾	Light N.N.W.	36·9	57·6	20·7	60·8	23·9	......	......	......	......	Quiet; stratified clouds in the west; fine.
9	A.M. 8	Light N.N.W.	38·5	56·7	18·2	60·7	22·2	High N.W.......	39·5	61·1	21·6	Hives still quiet.
10	A.M. 8¼	Light N.N.W.	42	57·6	15·6	64	22	......	......	......	......	Very fine; hives quiet.
11	A.M. 8½	Light N.N.W.	43·2	59·5	16·3	63·8	20·6	High N.W.......	41	60·5	19·5	Not a bee has yet gone abroad.
12	A.M. 8¾	Light N.N.W.	44·3	59·6	15·3	63·2	18·9	......	......	......	......	Male of *Anthophora retusa* abroad.
13	A.M. 9	Light N.N.W.	45·7	60·7	15	63·3	17·6	High N.W.......	42	60·3	18·3	Very fine; fleecy clouds in the west.
14	A.M. 9¼	Light N.	47·2	61·7	14·5	64·3	17·1	......	......	......	......	Not a bee has yet left the hives.
15	A.M. 9½	Light N.	47·6	63·7	16·1	67·3	19·7	......	......	......	......	First bee left the hive at 9h 35″.
16	A.M. 9¾	Light N.	49·3	64·7	15·4	67	17·7	......	......	......	......	Sky fleecy; two or three more bees abroad.
17	A.M. 10	Light N.	50·4	65·4	15	69·1	18·7	High N.W.......	44	61·3	17	First bee just returned with *pollen*.
18	A.M. 10¼	Light N.	49·5	66·5	17	75·6	26·1	......	......	......	......	Three more bees have returned with *pollen*.
19	A.M. 10½	Light N.	50·5	69·2	18·7	76·1	25·6	High N.W.......	44·5	61·4	16·9	Many bees abroad; sky fleecy; very fine.
20	A.M. 10¾	Light N.	49·5	70	20·5	77	27·5	......	......	......	......	Many bees returning with pollen.
21	A.M. 11	Light N.	48·3	73·9	25·6	78·3	30	......	......	......	......	Much *yellow pollen* collected, some *orange*.
22	A.M. 11¼	Light N.	47	72·7	25·7	80	33	......	......	......	......	Fine bright clouds; bees irritable.
23	A.M. 11½	Light N. by E.	45·6	72·6	27	79	33·4	......	......	......	......	Very fine; *pollen* collected.
24	A.M. 11¾	Light N. by E.	45	74	29	78·2	33·2	......	......	......	......	Very fine; hives active.
25	A.M. 12	Light N.	46	73·8	27·8	78·6	32·6	......	......	......	......	Seven loaded bees return per minute.
26	A.M. 12¼	Light N. by W.	47·5	73·6	26·1	79·3	31·8	......	......	......	......	Wind shifting; much *pollen collected*.
27	Noon 12½	Light N.N.W.	45·2	74·4	29·2	79	33·8	High N.W.W.	48	64·7	16·7	Fourteen loaded bees return per minute.
28	P.M. 12¾	Light N.N.W.	45·6	75·3	29·7	78·8	33·2	......	......	......	......	Sky fleecy; few bees going abroad; many returning.
29	P.M. 1	Light N.N.W.	49·2	75·3	26·1	82·7	33·5	......	......	......	......	Ten loaded bees per minute.
30	P.M. 1¼	Brisk N.N.W.	48·9	74·6	25·7	81	32·1	......	......	......	......	Bees fighting at entrance of the hive.
31	P.M. 1½	Light W.......	48·2	74·6	26·4	81	32·8	High N.W.W.	48	62·8	14·8	Fair; bees flocking home hastily; few go abroad.
32	P.M. 1¾	Light W. by S.	47·7	75·9	28·2	78	30·3	......	......	......	......	Rather overcast; very few bees abroad.
33	P.M. 2	Brisk W.S. ...	47·7	75·3	27·6	74	26·3	......	......	......	......	Overcast; scarcely a bee goes abroad.
34	P.M. 2¼	Brisk W.S. ...	47·5	74·4	26·9	75	27·5	......	......	......	......	Dull; cold wind.
35	P.M. 2½	Brisk W.S. ...	46·2	74·9	28·7	72	25·8	High N.W.W.	46·8	64	17·2	Dull; not a bee abroad.
36	P.M. 2¾	Brisk W.S. ...	45·5	73·6	28·1	70	24·5	......	......	......	......	Dull; slight humming in the hive.
37	P.M. 3	Brisk W.S. ...	45·3	74	28·7	68·2	22·9	High N.W.W.	44	65·2	21·2	Very dull; signs of rain.
38	P.M. 3¼	Brisk W.S. ...	45·5	73·5	28	67·3	21·8	......	......	......	......	Very dull.
39	P.M. 3½	Brisk W.S. ...	44·5	73·5	29	67	22·5	High N.W.W.	42·6	62·5	19·9	Very dull.
40	P.M. 3¾	Light S.W. ...	43·5	72	28·5	65	21·5	......	......	......	......	Very dull; hives quiet.
41	P.M. 4	Light S.	45·3	71·5	26·2	63·3	18	High N.W.W.	40·5	62	21·5	Fair.
42	P.M. 4¼	Light S.	43·1	72	28·9	64·3	21·2	......	......	......	......	Fair.
43	P.M. 4½	Light S.	42·5	71	28·5	64	21·5	......	......	......	......	Sunny, with clouds.
44	P.M. 4¾	Light S.	42·5	71	28·5	66	23·5	......	......	......	......	Dull; moist cool wind.
45	P.M. 5	Light S.W. ...	42·6	71	28·4	67	24·4	Brisk N.W.W.	41	60·3	19·3	Dull; hazy.
46	P.M. 5¼	Light W.......	41·8	69·2	27·4	64	22·2	......	......	......	......	Dull; slight humming in the hive.
47	P.M. 5½	Light W.......	41·6	69·	27·4	63	21·4	......	......	......	......	Very dull.
48	P.M. 5¾	Light W.......	41·5	68·9	27·4	62·5	21	......	......	......	......	Dull.
49	P.M. 6	Light W.......	40·3	67·2	26·9	62·	21·7	Brisk N.W. ...	41	61·1	20·1	Dull; hives quiet.
50	P.M. 6¼	Light W.......	40·5	67·1	26·6	62·8	22·3	......	......	......	......	Dull.
51	P.M. 6½	Light W.S....	40·2	67·2	27	62	21·8	......	......	......	......	Dull.
52	P.M. 6¾	Light N.W....	40	68	28	63	23	......	......	......	......	Dull.
53	P.M. 7	Light N.	39	67·6	28·6	62·5	23·5	Brisk N.W. ...	40·3	61	20·7	Very much overcast; hives quiet.
54	P.M. 10	Calm	32	......	......	......	......	......	......	......	......	Very cloudy.

TABLE XVI.

Mean daily Temperature of the Atmosphere and Bee Hive No 1. as deduced from observations made at about the hours of seven, nine, and twelve in the morning, and two and five in the afternoon, from October 23, 1835, to November 18, 1835; and of the Hives No. 1 and 2 from February 19, to September 30, 1836.

No. of Exp.	Period of observation.	Prevailing.	Wind.	Weather.	Atmosphere.	Hive No. 1.	Difference.	Hive No. 2.	Difference.	Remarks.
	1835.				°	°	°	°	°	
1	Oct. 23	....	....	Fine	47·37	58·37	11	......	......	Bees active.
2	24	W. by S.	Light	Cloudy	52·8	60·03	7·23	......	......	*Pollen* collected in the morning.
3	25	S.	High	Hard rain......	47·5	55·1	7·6	......	......	*Pollen* collected in the morning.
4	26	W. by S.	High	Showery	49·48	55·56	6·08	......	......	Scarcely a bee abroad.
5	27	W. by N.	Light	Light clouds ...	45·26	52·3	7·04	......	......	A few bees abroad.
6	28	W.	Calm	Fine	40·2	54·84	14·64	......	......	A few bees abroad.
7	29	S.W.	Calm	Hazy	54·33	60·33	6	......	......	*Pollen* collected; many bees abroad.
8	30	E. by S.	Light	Haze and rain	48·1	57·3	9·2	......	......	Much *pollen* collected in the morning.
9	31	S.W.	Shifting	Fair	54·24	65·20	10·96	......	......	*Pollen* scanty; many bees abroad.
	Mean temperature in October..................				48·80	57·67	8·87			
10	Nov. 1	N.E.	Light	Fine	45·2	61·06	15·86	......	......	*Pollen scarce*; many bees abroad.
11	2	S.W.	Light	Fair	48·16	54·87	6·71	......	......	Hive quiet; but a few bees abroad.
12	3	S.E.	Brisk	Misty rain	46·37	53·87	7·50	......	......	No bees abroad.
13	4	E.	Brisk	Showery	42·67	50·32	7·65	......	......	Hive quiet; no bees abroad.
14	5	E.	Light	Steady rain......	44·15	49·57	5·42	......	......	Hive quiet.
15	6	N.E.	Calm	Fine	40·38	47·28	6·90	......	......	A few bees abroad; a little *pollen* collected.
16	7	N.E.	Shifting	Fair	47·88	61·5	13·62	......	......	A little *pollen* collected; bees disturbed.
17	8	N.W.	Brisk	Fine	43·16	56·38	13·22	......	......	Many bees abroad.
18	9	N.E.	Brisk	Cloudy	41·16	49·14	7·98	......	......	Cold bleak wind; hive quiet.
19	11	N.E.N.	Calm	Hazy rain	38·03	44·26	6·23	......	......	Hive quiet; a little sleet this morning.
20	12	N.E.N.	Light	Light clouds ...	41·62	44·42	2·80	......	......	A few bees abroad, but return quickly.
21	13	N.E.	Light	Rain & clouds	38·34	44·66	6·32	......	......	No bees abroad.
22	14	N.E.N.	Shifting	Calm, fair	38·22	50·34	12·12	......	......	Bees greatly disturbed.
23	15	N.E.	Bleak	Cloudy	43·9	55·52	11·62	......	......	Hive nearly quiet.
24	16	W. by N.	Light	Fair	40·76	46·98	6·22	......	......	Hive quiet.
25	17	W.	Calm	Misty rain	44·96	47·04	2·08	......	......	Clouds and light wind.
26	18	W. by S.	Brisk	Cloudy	50·37	49·92	......	......	......	A few bees go abroad,
	Mean temperature in November..................				43·35	51·0	7·65			
	1836.									
27	Feb. 19	N.W.	Light	Fine	40·9	48·63	7·73	53·03	12·13	*Pollen* was collected on the 15th inst.
28	20	N.E.E.	Calm	Fine	35·7	50·2	14·5	74·13	38·43	Bees in No. 2 much disturbed.
29	21	N.W.	Light	Fine	38·3	51·1	12·8	60·26	21·96	Many bees go abroad, but soon return.
30	22	W.	Light	Fine	43·07	50	6·93	58·47	15·40	*Pollen* again collected.
31	23	W. by S.	Light	Light clouds ...	43	50·1	7·1	54·86	11·86	*Pollen* collected; bees fighting.
32	24	S.W.	Light	Showery	42·6	50·7	8·1	56·16	13·56	*Pollen* collected; bees of No. 1. fighting; have *diarrhœa*.
33	25	W.	Brisk	Fine	42·3	54·4	12·1	60·7	18·4	*Pollen* collected.
34	26	N.E.	Light	Rain and sleet	38·9	49·47	10·57	54·25	15·35	Hives quiet; rain, sleet, snow, and wind.
35	27	W.	Light	Cloudy	39·	48·22	9·22	53·98	14·98	Hives quiet.
36	28	W.	Light	Fair	39·82	48·37	8·55	54·72	14·90	Hives less quiet; no bees abroad.
37	29	W.	Light	Fine	40·26	49·96	9·70	55·15	14·89	*Pollen* collected, bees fighting, have *diarrhœa*.
	Mean temperature in February				40·35	50·10	9·75	57·79	17·44	
38	March 1	S.S.E.	Brisk	Rain, cloudy...	45·66	54·8	9·14	61·5	15·84	Hives quiet; bees disposed to come abroad; *diarrhœa*.
39	2	W.	Brisk	Fine	45·4	52·4	7	58·73	13·33	Many bees abroad fighting.
40	3	W.	Light	Rain	46·08	55·8	9·72	59·18	13·10	Bees abroad; only one returned with *pollen*.
41	4	W.	Light	Fine	47·2	58·48	11·28	58·74	11·54	*Pollen* collected; many bees abroad.
42	5	S.W.	High	Hard rain	46	53·54	7·54	61·92	15·92	Scarcely a bee abroad to-day.
43	6	S.S.E.	High	Continued rain	47·6	55·12	7·52	57·67	10·07	Hives quiet.
44	7	S.S E.	Light	Fair	44·6	53·04	8·44	55·2	11·60	Bees abroad, fighting, have *diarrhœa*.
45	8	N.N.E.	Light	Cloudy	40·03	50·2	10·17	55·	14·97	A little *pollen* collected.
46	9	S. by E.	Shifting	Cloudy	37·93	47·6	9·67	51·83	13·90	No bees abroad.
47	12	S.W.	High	Rain	47·6	55·7	8·1	......	......	Weather tempestuous; a few bees abroad.
48	13	W.	Brisk	Fair	47·48	57·26	9·78	......	......	Many bees with *pollen*.
49	14	W.	High	Rain	49·3	58·4	9·1	......	......	No bees abroad.
50	15	W.	Tremendous	Hard rain	50	57·1	7·1	......	......	Scarcely a bee abroad.
51	16	N.W.	Light	Fine	43·17	51·97	8·80	......	......	*Pollen* scanty; a few bees abroad.
52	17	W. by S.	High	Cloudy	49·5	55·3	5·8	......	......	No bees abroad.
53	18	S.S.W.	Light	Fine	53·34	65·5	12·16	......	......	*Pollen* in abundance; bees fighting.
54	19	E.S.E.	Light	Fine	54·64	71·6	16·96	......	......	*Pollen* collected in abundance; fighting.
55	20	W.	Light	Hazy	49·36	65·46	16·10	......	......	*Pollen* scarce, only a few bees abroad.
56	21	S.W.	Light	Misty rain	48·14	63·04	14·90	......	......	Very few bees abroad.

Table XVI. (Continued.)

No. of Exp.	Period of observation.	Prevailing.	Wind.	Weather.	Atmosphere.	Hive No. 1.	Difference.	Hive No. 2.	Difference.	Remarks.
	1836.				°	°	°	°	°	
57	Mar. 22	S.W.S.	Var. light...	Clouds and rain	48·7	62·82	14·12	......	......	Scarcely a bee abroad; no *pollen* collected.
58	23	S.S.W.	Light	Clouds, rain ...	47·4	62·28	14·88	......	......	*Pollen* collected in the morning.
59	24	W.	Brisk	Very fine.........	44·72	59·52	14·80	......	......	Bees active; much *orange* and *yellow pollen*.
60	25	S.W.	Brisk	Fine light clouds	51·3	67·6	16·3	......	......	*Pollen* collected; bees very active.
61	26	W.N.W.	High	Stormy	42·64	60·64	18	......	......	No bees abroad; rain and hail.
62	27	S.S.E.	Brisk	Fine..............	46·3	61·84	15·54	......	......	*Pollen*, *orange* and *yellow*, collected.
63	28	S.W.	Var. high...	Rain	43·8	62·6	18·8	......	......	No bees abroad; hive quiet.
64	29	W.	High	Heavy clouds ...	46·16	60·8	14·64	......	......	But few bees abroad; no *pollen*.
65	30	S.W.	Var. high...	Hard rain	49·8	63·8	14	......	......	Hive quiet; weather tempestuous.
66	31	W.	High	Fair	47·2	54·9	7·7	......	......	*Scarlet* and *orange pollen* collected; many bees abroad.
		Mean temperature in March.........			46·93	58·59	11·66	57·75	10·82	Mean temperature of No. 2, in the first nine days of March.
67	April 1	N.E.E.	High	Rain and snow	41·5	59·5	18	......	......	No bees abroad; hard continued rain with snow.
68	2	N.W.W.	High	Cloudy	42·9	61·66	18·76	......	......	A few bees abroad with *pollen*.
69	3	N.W.	High	Stormy	48·75	63·05	14·30	......	......	Many bees abroad perished with the wind.
70	4	N.N.W.	Light	Very fine.........	45·9	72·3	26·4	......	......	*Orange* and *yellow pollen* in abundance.
71	5	S.S.W.	Light	Fine..............	48·66	71·44	22·78	97·3	48·64	*Pollen* in abundance; Hive No. 2 much disturbed.
72	6	S.E.	Var. light...	Fair	45·9	71·2	25·3	67·3	21·4	But few bees abroad; cloudy, fair.
73	7	S.W.	Brisk	Sunny	51·76	72·03	20·27	68·06	16·30	*Pollen*, *scarlet*, *orange*, and *yellow* in abundance.
74	8	S.W.S.	High	Cloudy	47·5	68·2	20·7	65·27	17·77	No *pollen* collected; few bees abroad.
75	9	S.W.	Light	Fair	49	69·54	20·54	67·16	18·16	*Pollen* in abundance, *scarlet*, *white*, *yellow*, *brown*.
76	10	S.E.	Var. light...	Showery	50·56	73·44	22·88	71·42	20·86	*Pollen* abundant, *orange*, *yellow*, *scarlet*, *grey*, *white*, *brown*.
77	11	N. by W.	Light	Light clouds ...	51·46	72·68	21·22	72·42	20·96	Pollen abundant, *orange*, *yellow*, *scarlet*, *grey*, *white*.
78	12	S.W.	Brisk	Fair	48·42	69·75	21·33	69·87	21·45	Few bees abroad; signs of rain.
79	13	S.W.	Brisk	Fair	52·06	72·84	20·78	72·54	20·48	Bees very busy; *pollen orange*, *scarlet*, *yellow*, *grey*.
80	14	S.	Var. light...	Dull..............	51·45	73·5	22·05	73·75	22·30	Bees very busy; damp atmosphere.
81	15	S.E.	Light	Fair	57·76	76·72	18.96	81·6	23·84	Signs of rain; bees still very active.
82	16	S.E.	Light	Fine..............	54·42	76·26	21·84	85·86	31·44	*Pollen* in abundance, *scarlet*, *orange*, *grey*, *yellow*, *brown*.
83	17	S.W.S.	Var. calm...	Light rain	47·26	74·66	27·40	76·98	29·72	A few bees abroad.
84	18	W.	Light	Light rain	49·06	78·78	29·72	77·34	28·28	A few bees abroad, no *pollen collected*.
85	19	W.	Calm	Fine..............	50·72	74·77	24·05	79·62	28·90	*Pollen* in abundance, *yellow*, *white*, *grey*, *orange*, *scarlet*.
86	20	W.	Brisk	Fair	53·3	75·9	22·6	85·55	32·25	Bees very active; working.
87	21	W.	Light	Fine..............	51·2	73·46	22·26	86·7	35·5	Many bees abroad; working.
88	22	S.W.	High	Very fine.........	54·88	76·74	21·86	86·8	31·92	*Pollen* in abundance, chiefly dirty *white* and *orange*.
89	23	S.W.	Brisk	Rain and fair ...	51·98	75·26	23·28	85·52	33·54	But few bees abroad; showery.
90	24	S.	Calm	Steady rain	48·6	75	26·4	79·5	30·9	Very few bees abroad.
91	25	N.E.	Light	Fine..............	54·42	77·2	22·78	81·5	27·08	Bees very active; working.
92	26	N.	Light	Fine..............	55·22	81·65	26·43	82·2	26·98	Many bees abroad; *pollen abundant*.
93	27	N.E.	Var. brisk...	Fair	48·5	78·75	30·25	79·62	31·12	Few bees abroad.
94	28	N.	Light	Cloudy	47·25	74·4	26·15	80·9	32·65	Few bees abroad.
95	29	N.	High	Fair	48·36	75·83	26·47	80·93	31·57	Pollen nearly all *deep orange*; a fall of snow this morning.
96	30	E. by N.	Var. light...	Fine..............	49·26	75·62	26·36	78	28·74	Thick ice this morning; abundance of *pollen*, chiefly *deep orange*.
		Mean temperature in April			49·93	73·07	23·14	78·22	28·29	
97	May 1	N.	High	Fair	55·3	79·65	24·35	81·65	26·35	Many bees abroad; *pollen* abundant.
98	2	E.	Tremendous.	Hard rain	47·1	74·16	27·06	76·53	29·43	Snow this morning; rain all day.
99	3	N.E.N.	Very high	Fine..............	53·25	77·7	24·45	79·27	26·02	Abundance of *pollen*, *orange* and *scarlet*.
100	4	N.E.	Brisk	Fine..............	55·67	80·5	24·83	83·57	27·90	Abundance of *pollen*, *orange* and *white*.
101	5	E.	Light	Showery	50·36	84·54	34·18	83·5	33·14	Many bees abroad.
102	6	E.	Light	Very fine.........	56·67	84·12	27·45	87·87	31·20	Great commotion in the hives; young bees hatching.
103	7	E.	Light	Very fine.........	59·74	85·26	25·52	91·42	31·68	*Abundance* of *pollen*, *scarlet*, *yellow*, *brown*, *grey*, *orange*.
104	8	E. by S.	Brisk	Very fine.........	57·54	83·5	25·96	91·7	34·16	Many bees abroad; working.
105	9	N.E.	Light	Very fine.........	57·84	82	24·16	92·28	34·34	Bees very active.
106	10	E.	Brisk	Fine..............	57·92	83·16	25·24	89·66	31·74	Drone bees have just appeared.
107	11	W.	Light	Very fine.........	57·78	81·92	24·14	89·22	31·44	Hoar frost last night; *pollen* collected.
108	12	W.	Light	Very fine.........	51·8	87·16	35·36	91·66	39·86	Many drones abroad; bees active.
109	13	W.	Light	Very fine.........	66·98	87·06	20·08	96·17	29·19	Drones abroad; bees beginning to lay out for swarming.

TABLE XVI. (Continued.)

No. of Exp.	Period of observation.	Prevailing.	Wind.	Weather.	Atmosphere.	Hive No. 1.	Difference.	Hive No. 2.	Difference.	Remarks.
	1836.									
110	May 14	N. by E.	Calm ...	Very fine......	68°·36	87°·36	19°	92°·82	24°·46	Bees hanging out; loud humming in the hives.
111	15	N.	Light ...	Very fine......	66·74	89·48	22·74	90·94	24·20	Hive No. 2 lifted to prevent swarming. *Sun eclipsed* at 2 P.M.
112	16	S.	Var. light	Fine...........	67·52	88·8	21·28	96·06	28·54	Bees hanging out, much excited; hive replaced.
113	17	E.	Light ...	Very fine......	65·4	89·28	23·88	93·98	28·58	Bees hanging out; drones abroad.
114	18	S.E.	Light ...	Very fine......	71·06	93·9	22·84	95·5	24·44	Bees hanging out.
115	19	E.	Light ...	Light clouds	67·74	92·8	25·06	93·22	25·48	No bees hanging out, except a few in the morning.
116	20	S.E.	Light ...	Very fine......	67·54	94·04	26·50	94·36	26·82	Bees again hanging out.
117	21	E.	Light ...	Dull day......	61·17	95·37	34·20	93·7	32·53	Bees still hanging out.
118	22	E.	Brisk ...	Fine...........	61·77	92·83	31·06	92·83	31·06	No bees hanging out; very few abroad.
119	23	E.	Brisk ...	Dull day......	52·75	88·41	35·66	88	35·25	No bees hanging out; very few abroad.
120	24	N.E.	Light ...	Fair...........	59·75	88·45	28·70	90·2	30·45	Many bees abroad, but not hanging out.
121	25	N.E.	Light ...	Fair...........	57·08	86·7	29·62	91·8	34·72	Many bees abroad, but not hanging out.
122	27	E.	Brisk ...	Fine...........	63·03	92·43	29·40	93	29·97	*Hive No. 2 swarmed suddenly* at 2½ P.M.
Temperature of No. 2 up to period of swarming.................								90·06	29·89	
123	28	E.	Light ...	Fair...........	50·04	90·28	40·24	86·24	36·20	A dead drone and *queen nymph* thrown out from the swarmed Hive No. 2.
124	29	N.E.	Light ...	Very fine......	63·5	89·75	26·25	91·75	28·25	Very few bees at entrance of Hive No. 2.
125	30	N.E.	Light ...	Fine...........	65·12	90·1	24·98	90·75	25·63	*Three queen nymphs* ejected from Hive No. 2.
126	31	N.E.	Light ...	Fine...........	68·8	91·72	22·92	92·4	24·6	Abundance of *pollen* brought home.
Mean temperature in May					60·17	87·08	26·91	90·06	29·89	
Total mean temperature of four months before swarming					49·34	67·21	17·87	70·95	21·61	
127	June 1	N.E.	Light ...	Showery	60·37	87·7	27·33	90·1	29·73	Bees of both hives very busy.
128	2	N.E.	Shifting	Rain	56·02	84·37	28·35	86·4	30·38	Bees abroad in the morning.
129	3	S.S.W.	High ...	Rain	51·7	87·25	35·55	88·7	37	Bees very active.
130	4	S.S.W.	High ...	Fair...........	60·5	86·16	25·66	85·16	24·66	Few bees abroad.
131	5	S.W.W.	Brisk ...	Dull..	61·8	86	24·2	88	26·2	Signs of rain; few bees abroad.
132	6	S.W.	High ...	Fine...........	63·37	90	26·63	89·45	26·08	Many bees abroad.
133	7	S.	Brisk ...	Fine...........	58·4	89·15	30·75	89·8	31·4	Many bees abroad.
134	8	S.	Brisk ...	Rain	61·85	90·5	28·65	89	27·15	Bees of No. 1 working in the side box.
135	9	S.	Light ...	Dull...........	62·5	91·5	29	....	......	Working in side box; many abroad.
136	11	S.W.	High ...	Fair...........	65·75	90·7	24·95	85·95	20·20	Bees very busy; (many bees at entrance of Hive No. 1.)
137	12	S.W.	High ...	Very fine......	64·6	88·37	23·77	86·95	22·35	Symptoms of swarming again in Hive No. 2.
138	13	S.W.	Light ...	Very fine......	70·6	94·83	24·23	93	22·4	Bees much agitated at entrance of No. 2.
139	14	N.E.	Var. light	Very fine......	71·2	92·24	21·04	90·16	18·96	Loud sounds in both hives; *pollen* abundant.
140	15	E.	Light ...	Fair...........	73·64	92·14	18·50	91·94	18·30	Evening showery, and *bees hanging out again* from No. 2.
141	16	W.	Light ...	Fine...........	65·12	88·84	23·72	88·32	23·20	Evening stormy, with thunder; bees hanging out.
142	17	E.	Light ...	Fine...........	66·22	90·38	24·16	89·54	23·32	Evening stormy; bees hanging out.
143	18	S.W.	Brisk ...	Very fine......	66·04	87·17	21·13	82·66	16·62	*Second swarm* left Hive No. 2 at 7 A.M.
144	19	W.	Light ...	Fine...........	64·4	86·43	22·03	83·56	19·16	Showery; Hive No. 2 very thin.
145	20	W.	Light ...	Very fine......	65·82	89·37	23·55	84·75	18·93	Temperature of Hive 2 reduced.
Mean temperature in June.................					63·67	*89·11	25·44	†87·96	24·29	
146	July 20	W.	Light ...	Fair...........	58·32	78·65	20·33	....	......	Has rained hard for thirty-six hours; bees active.
147	21	W.	Light ...	Showery	62	78	16	....	......	Bees very active; *pollen black*.
148	22	W.	Light ...	Showery	60·98	80·33	20·35	....	......	Many bees abroad; no drones have yet been killed.
149	23	S.W.	Var. light	Fair...........	63·96	84·14	20·18	....	......	Bees in No. 1 disturbed.
150	24	S.W.	Light ...	Fair...........	63·92	79·47	15·55	....	......	Killing drones in the *first swarm* from No. 2.
151	25	W.	Light ...	Light clouds	67	78·15	11·15	....	......	Bees very active.
152	26	W.	Light ...	Very fine......	68·15	80·15	12	....	......	Many bees abroad.
153	27	W.	Brisk ...	Dull...........	63	79	16	....	......	Massacre of drones in the swarm continues.
154	28	E. by S.	Light ...	Fine...........	65·8	80·1	14·3	....	......	Bees very active.
Mean temperature in July					63·68	79·77	16·09			
155	Aug. 2	W.	Light ...	Very fine......	65·47	75·1	9·63	....	......	Attacking the drones in *second* swarm from No. 2.
156	4	S.	Light ...	Fair...........	67·73	81·2	13·47	....	......	Bees very active; light rain and clouds.
157	5	S.	Light ...	Rain	66·1	81·79	15·69	....	......	Bees very active; light rain.
158	7	S.E.	Light ...	Very fine......	70·83	81·03	10·20	....	......	Heavy dew last night.
159	8	S.E.	Light ...	Fine...........	69·16	79·36	10·20	....	......	Bees disturbed.
160	12	S.E.E.	Light ...	Fine...........	71·75	81·15	9·40	....	......	Massacre of drones continues.

* Mean temperature in June of No. 1, the unswarmed hive.

† Mean temperature of Hive 2, in June, after *twice swarming*.

TABLE XVI. (Continued.)

No. of Exp.	Period of observation.	Prevailing.	Wind.	Weather.	Atmosphere.	Hive No. 1.	Difference.	Hive No. 2.	Difference.	Remarks.
	1836.				°	°	°	°	°	
161	Aug. 13	E.	Light ...	Very fine......	76·4	81·1	4·7	......	......	Bees abroad in great numbers.
162	14	S. by E.	Light ...	Very fine......	77·45	82·9	5·45	......	......	Abundance of *pollen, white, orange, brown, green,* and *grey*.
163	15	W.	Light ...	Fair...........	71·64	84·24	12·60	......	......	Still killing the drones in the young swarms.
164	16	S.E.E.	Light ...	Very fine......	70·06	83·58	13·52	......	......	Beginning to kill the drones in Hive No. 1.
165	17	S.W.	Brisk ...	Very fine......	71·57	80·92	9·35	......	......	*Pollen* in abundance; killing drones.
166	18	W.	Brisk ...	Dull...........	63·98	78·74	14·76	74·32	10·34	Loud sounds in the hive; few bees abroad.
167	19	N.W.	Light ...	Fine...........	69·12	78·52	9·40	83·75	14·63	Bees very active.
168	20	N.W.	High ...	Hard rain ...	59·05	73·31	14·26	74·62	15·57	Three nymphs expelled from No. 2.
169	21	N.W.	Brisk ...	Fine...........	64·9	75·1	10·2	80·93	16·03	Three nymphs ejected from the first swarm from No. 2.
170	22	S.W.	High ...	Fair...........	66·62	74·92	8·30	76·37	9·75	A drone turned out from swarm.
171	23	S.E.	Light ...	Rain	63·38	73·26	9·88	75·28	11·90	Bees very active.
172	24	N.E.	High ...	Fair...........	60·64	74·88	14·24	75·28	15·64	Few bees abroad; drones have all perished.
173	25	S.	Light ...	Fine...........	63·22	74·32	11·10	74·87	11·65	Few bees abroad.
174	26	S.W.	High ...	Dull...........	68·56	75·23	6·67	74·63	6·07	Few bees abroad.
175	27	S.W.	High ...	Fine...........	65·16	73·14	7·98	76·26	11·10	Few bees abroad.
176	28	S.W.	Light ...	Showery	65·15	76·57	11·42	78	12·85	Abundance of bees abroad with *pollen.*
177	29	N.	Calm ...	Very fine......	67·3	74·82	7·52	78·5	11·2	Many bees abroad with *orange pollen.*
178	30	W.	Light ...	Light clouds	67·3	74.82	7·52	78·5	11·2	Bees very active.
179	31	S.S.W.	Light ...	Very fine......	68·7	74·86	6·16	75·3	6·6	Abundance of *pollen, yellow, orange, grey.*
		Mean temperature in August......			67·64	77·79	10·15	76·9	10·82	
180	Sept. 1	S.W.	Brisk ...	Fine...........	66·14	76·64	10·50	73·22	7·08	Much *pollen* collected, *orange, grey, yellow, and brown.*
181	2	W.	Brisk ...	Light clouds	59·58	71·08	11·50	72·96	13·38	*Pollen* less abundant; showery.
182	3	S.S.W.	Light ...	Fine...........	63·02	69·02	6	72·45	9·43	*Pollen* collected from the mignionette.
183	4	S.	Brisk ...	Showery	67·05	72	4·95	73·4	6·35	Not many bees abroad; showery.
184	5	W.	Brisk ...	Light clouds	59·18	69·89	10·71	70	10·82	Fine, but windy.
185	6	W.	High ...	Rain	56·03	64·5	7·47	67·86	11·83	A cold rainy day, with wind.
186	7	N.	Light ...	Very fine......	57·07	66·3	9·23	69·3	12·23	Many bees abroad, working.
187	8	N.	Light ...	Fine...........	60·92	67·87	6·95	69·77	8·85	A little showery, with light wind.
188	9	S.W.	Light ...	Showery	57·23	67·6	10·37	65·66	8·43	Hard rain all the morning.
189	10	W.	Brisk ...	Stormy	56·02	64·46	8·44	66·66	10·64	Bees abroad at noon; weather rough.
190	11	N.W.	Brisk ...	Fine...........	56·86	61·02	4·16	67·54	10·68	Cold wind, but fine; few bees abroad.
191	12	N.	High ...	Fine...........	57·84	64·46	6·62	66·62	8·78	Not many bees abroad.
192	13	N.	High ...	Fair...........	58·66	64·8	6·14	68·08	9·42	Signs of rain; hives quiet.
193	14	N.N.E.	Light ...	Fair...........	57·47	64·82	7·35	68·1	10·63	Signs of rain.
194	15	E.	Brisk ...	Fine...........	58·75	63·45	4·70	67·82	9·07	Bees abroad.
195	16	N.E.	Brisk ...	Fine...........	57·5	66	8·5	68·8	11·3	Weather unsettled; showery.
196	17	N.W.	Light ...	Fair...........	60·67	68·03	7·36	70·1	10·43	A few bees abroad.
197	19	N.	Light ...	Fair...........	63·87	69·20	5·33	70·38	6·51	Weather unsettled.
198	21	N.	Calm ...	Dull...........	57·4	61	3·6	65·7	8·3	Bees still abroad.
199	22	S.	Light ...	Dull...........	55·73	58·26	2·53	62·02	6·29	Hives quiet; no bees abroad.
200	23	W.	High ...	Hard rain ...	60·83	66·43	5·60	67·76	6·93	Hives quiet; hard rain to-day.
201	24	W.	Brisk ...	Very fine......	66·18	68·64	2·46	72·96	6·78	Great commotion at the entrance of the hives.
202	25	S.W.	Light ...	Fine...........	64·65	69·45	4·80	70·7	6·05	Fair; many bees abroad.
203	26	S.	Brisk ...	Dull...........	63·46	69·78	6·32	70·94	7·48	A few bees abroad.
204	27	S.W.	Light ...	Dull...........	66·06	69·1	3·04	71·4	4·34	No bees abroad.
205	28	S.W.W.	High ...	Showery	61·3	64·93	3·63	65·53	4·23	Fair, windy; a few bees abroad.
206	29	S.W.	High ...	Rain	53·1	55·7	2·6	57·03	3·93	Hives quiet; rain and wind all day.
207	30	N.W.	Light ...	Fine...........	58·76	67·83	9·07	69·2	10·44	Bees abroad again.
Mean temperature in September					60·04	66·5	6·46	66·68	6·64	
Total mean temperature of four months after swarming					63·75	78·29	14·54	77·18	13·43	
Total mean temperature of the periods before and after swarming................................					56·54	72·75	16·21	74·06	17·52	

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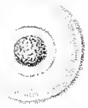

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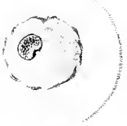

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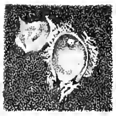

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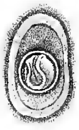

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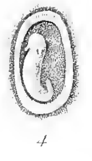

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XVIII. *On the first Changes in the Ova of the Mammifera in consequence of Impregnation, and on the Mode of Origin of the Chorion. By* Thomas Wharton Jones, *Esq. Communicated by* Richard Owen, *Esq. F.R.S.*

Received March 16,—Read April 27, 1837.

Part I.—*On the Changes in the Envelopes.*

HAVING previously described the structure of the ovum of mammiferous animals, as it exists in the ovary before impregnation*, I now proceed to relate some facts respecting the changes which it undergoes in consequence of that act.

My observations in reference to this point are the following.

Observation 1.—On Wednesday the 16th, and Thursday the 17th September, 1835, I examined the internal organs of generation of a Rabbit, which had been impregnated on the afternoon of the Saturday preceding, and which was killed on the afternoon of Tuesday. The ovaries of both sides presented corpora lutea.

In the Fallopian tube of the right side, near where it enters the horn of the uterus, I found six ova. In the same place on the left side there were only two. They differed very remarkably from the ova as they exist in the ovaries before impregnation, inasmuch as the former presented, in addition to the component parts of the ovum of the ovary, a thick gelatinous matter surrounding it, similar to what is observed in the ovum of the Frog. The addition of this gelatinous envelope made the diameter of the whole body about $\frac{1}{70}$th of an inch. Plate XVI. fig. 1. represents one of these ova magnified 40 diameters, and fig. 2. the ovum of the Frog when recently laid, magnified 2 diameters.

I could not detect the germinal vesicle in the ova in question. The granulary matter of the yelk was coherent. The application of weak vinegar to the ova rendered the yelk transparent. Dilute nitric acid made the superadded gelatinous envelope contract, but by the addition of more water it gradually expanded again.

The question which this observation suggests is, "Where do the ova acquire the additional gelatinous envelope; in the Fallopian tubes or in the ovaries?" The two following observations give the answer, "In the ovaries."

Observation 2.—March 6, 1236. Examined a female Rabbit to-day, 41 hours and 40 minutes after impregnation. There were *no ova* in the horns of the uterus, nor in the Fallopian tubes.

The right ovary presented on its surface a very large and prominent Graafian ve-

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* See Lond. and Edin. Phil. Mag. vol. vii. p. 209.

sicle, quite transparent, except at its most projecting point, where there was a spot of blood. I perceived nothing peculiar in the ovum contained in this vesicle. I did not detect a germinal vesicle in it.

Besides this large and prominent Graafian vesicle there were on the surface of the right ovary other five prominent vesicles filled with coagulated blood. At the most projecting point of each of these there was a small whitish mammillary elevation, within which was contained the ovum, surrounded by a transparent gelatinous substance, the same as that described in the preceding observation; only it is to be remarked, that in the Fallopian tubes this gelatinous looking substance had swelled out and acquired a greater diameter than it presented in the ovary. I did not detect a germinal vesicle in the ova forming the subject of this observation.

In the left ovary I found only one vesicle, containing the coagulated blood and the ovum surrounded by the gelatinous looking envelope.

Observation 3.—A Rabbit 48 hours after impregnation presented appearances much the same as the above.

Is any trace of the gelatinous looking envelope of the ovum to be observed before impregnation? In the ova of the Rabbit, &c., before impregnation, the proligerous disc, in which the ovum is imbedded, is observed to be composed of a gelatinous substance interspersed with grains, but as yet there appears no distinctly circumscribed envelope*.

The gelatinous looking envelope of the ovum I have just described must not be confounded with the vitellary membrane of the ovum, which was fully considered in my former paper. The former appears to be analogous to the cortical membrane surrounding the ovum of the *Ornithorhynchus paradoxus*, &c. while still in the ovary, described by Mr. Owen†. That it, and not the vitellary membrane, as I formerly imagined, forms the chorion, will be made evident by the following observations.

I would, however, premise some remarks on the ova of the batrachian reptiles, in order to place in a more striking point of view the circumstances I am about to relate in regard to the ova of the mammifera.

Fig. 2. exhibits the ovum of the Frog magnified 2 diameters. It is composed of a yelk, black on its surface, and whitish inside. The yelk is surrounded by a vitellary membrane, thicker than that of the bird's egg, but thinner in proportion than that of the ova of the mammifera. Outside the vitellary membrane is a gelatinous envelope, which is added in the oviduct, the two preceding parts being formed in the ovary. When the ova are laid the gelatinous envelope rapidly absorbs water, and swells out to great thickness.

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* Dr. Karl Krause of Göttingen, however, in a late Number of Müller's Archiv., speaks as if the gelatinous substance really formed a well defined envelope. From his observations on the ovum *before* impregnation he has been led to form much the same opinion regarding the origin of the chorion as is recorded in this memoir.

† Philosophical Transactions, 1834, p. 561.

The ovum of the Newt differs from that of the Frog, inasmuch as the gelatinous-like matter which surrounds the yelk and its membrane is of an oval form, and is somewhat hardened on the surface, so as to form a kind of shell, inside which is a fluid substance, in which the yelk and its membrane can freely revolve and glide from one end to the other. The vitellary membrane is thinner in the Newt than in the Frog. Fig. 3. is the ovum of a Newt, in which development has commenced; magnified rather more than twice.

But what I wish particularly to insist on, in regard to the ova of the batrachian reptiles, and especially of the Newt, is, that when the embryo of the latter has attained a certain size, but still at an early period, the vitellary membrane gives way, and then the embryo is only contained within the cavity of the substance, which is added to the ovum in the oviduct, fig. 4.

In the case of the Frog the vitellary membrane does not give way, until about the time the Tadpole is ready to burst all its envelopes. With the development of the embryo the cavity circumscribed by the vitellary membrane increases to as much as one fifth of an inch in diameter, and always retains its spherical form. There is a limpid fluid in the interior of the vitellary membrane, which seems to serve the purpose of an amniotic fluid, fig. 5.

Observation 4.—March 18 and 19, 1836. Examined a female Rabbit seven days after impregnation. The right ovary presented four corpora lutea, the left ovary two. I found only one ovum in each horn of the uterus; they were about $\frac{1}{50}$th of an inch in diameter*. Fig. 6, magnified 40 diameters.

No vitellary membrane was to be seen. The gelatinous-looking envelope constituted the only covering of the yelk, which now formed a vesicular blastoderma. The cavity of the gelatinous-looking envelope was much larger than the vesicular blastoderma. The inner surface of the gelatinous coat presented what I supposed to be fragments of the vitellary membrane adhering to it.

In both ova the vesicular blastoderma was irregular on one side, that on which I supposed the embryo was about to be developed. It was beginning to present the separation into layers, and had the same peculiar friable globular structure as the blastoderma of the hen's egg.

Observation 5.—This observation, which refers to the human ovum, agrees with that just related in regard to the ovum of the Rabbit.

In the spring of 1836 I examined a small human ovum sent to me to Cork, where I then was, from Glasgow, by Dr. Mackenzie. In his letter to me, dated November 29, 1835, he describes it thus: "A very small human ovum. It came along with the entire decidua from a patient of mine. It lay in the middle of one of the parietes of

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* The reason I found but two ova is, perhaps, that from their great transparency they may have escaped notice. The gelatinous coat was so transparent that I could with difficulty see the outline of it under the glass when it was observed by transmitted light. The vesicular blastoderma being opaque was the only circumstance that enabled me to detect the ova at all.

the decidua, rather near its upper edge, and was about the size of a marrowfat pea, before being put into spirits. The decidua covering it, towards the hydroperionic cavity, was thin and semitransparent, but the opposite portion of the decidual nidamentum was thick, and marked with foramina, as if from vessels which had penetrated and adhered to it. Having opened the nidamentum and taken out the ovum, I observed what will immediately strike you, that one side of it was bald and the other shaggy with the villi of the chorion. The bald part lay towards the hydroperionic cavity. A small puncture was made through the chorion, and perhaps through the amnion, by which some fluid escaped: nothing more was attempted. The Fallopian portions of the decidua measured nearly half an inch, and were both entire."

In a subsequent letter Dr. MACKENZIE says, in reference to the age of this ovum, "The ovum in question I consider as three or four weeks old. The lady had missed one menstrual period, and thought herself four weeks gone."

On laying open the ovum, by carefully cutting and reversing the bald side of the chorion, the following appearances (delineated, natural size, in fig. 7.) presented themselves. The whole cavity of the chorion was filled with a fine gelatinous cellular tissue, imbedded in which, towards one extremity of the ovum, was a small round body.

It was evidently the vesicular blastoderma; on being taken out and examined under the microscope it presented the same friable globular structure found in the vesicular blastoderma of the Rabbit in the preceding observation. There was no vitellary membrane to be seen.

From observation 4. it may be inferred, that in the progress of the development of the ovum of the Rabbit the vitellary membrane gives way, as in the ova of the Newt and indeed of many of the oviparous animals; that the gelatinous coat acquired by the ovum in the ovary, and more especially circumscribed and defined after impregnation, constitutes the only covering of the vesicular blastoderma after the giving way of the vitellary membrane; that this gelatinous-looking coat forms the chorion, which in the rodents at a further stage of development presents itself under the form of a thin and transparent membrane, very like the vitellary membrane of the bird's egg, situated immediately outside the non-vascular and reflected layer of the umbilical vesicle.

The conclusions to be drawn regarding the human ovum from observation 5. are the same as the above. The human ovum as regards the vesicular blastoderma was in much the same stage as the ova of the Rabbit seven days after impregnation; the vitellary membrane had disappeared, or been resolved into the gelatinous cellular tissue filling the interior of the chorion; and the embryo had not yet appeared though the vesicular blastoderma was undergoing the preparatory changes. As regards the chorion, the human ovum was more developed than that of the Rabbit, but it is to be remarked that even in an after stage of development the same difference in structure continues to prevail.

APPENDIX.

I think it right to mention that in the Rabbit which formed the subject of observation 1, I observed the following other points:

Having cut off a piece from the ovarian extremity of the Fallopian tube of the right side, I put it into a glass capsule, and having laid it open, examined its contents with the microscope; I observed among the numerous shreds of the lining mucous membrane a small body, transparent, and of a very peculiar shape. Having succeeded in transferring it from the capsule to a flat plate of glass, and having removed the shreds of membrane, I was enabled to examine it with a stronger power, and see better its very extraordinary form and structure, which are well represented in fig. 8. I had not a micrometer at the time to measure it, but I think it was about $\frac{1}{70}$th of an inch in diameter at its globular extremity. The calculation was made by comparing it with an ovum from the ovary. It revolved through the water when the latter was put in motion, and in doing so the part *a* was forced to turn sometimes to the one side and sometimes to the other.

In the next piece of the Fallopian tube of the same side which I examined, I found a transparent body not quite round, but prominent on one side, and close by the prominent point there was a small oval vesicular projection, fig. 9. There was an appearance of circular lines on it which touched each other at the prominent point; three of the lines were particularly evident, and the prominent point had a brilliant appearance under the microscope.

In the next piece of Fallopian tube examined I found a body, fig. 10, which on the whole resembled the preceding, but as I might say not so far developed.

Could the three bodies described have been blighted ova? They were all about $\frac{1}{70}$th of an inch in diameter, and therefore corresponding in size to the real ova, already described as being found in the same Rabbit.

PART II.—*On the Changes in the Vitellus.*

What I have to communicate in this second part of my memoir is of a much less definite character than that which is given in the first part. From the nature of the subject it in many cases necessarily consists of inferences rather than observed facts. It relates chiefly to the ova of the batrachian reptiles, and is added here merely for the purpose of throwing some light on the changes which take place in the yelk of the ova of the mammifera, previously to the commencement of the evolution of the embryo.

In approaching this subject the first question which presents itself is: "When does the germinal vesicle of the ova of the mammifera disappear, before or after impregnation?" It is known that in birds and reptiles the germinal vesicle disappears before impregnation. In the ova of the Frog, contained in the oviduct, and also in the more advanced of those contained in the ovary, no trace of the germinal vesicle is to be observed. The black blastoderma surrounds the whole yelk, with the exception of a small spot* on the opposite side to that where the primitive streak appears. In the furthest advanced ova contained in the ovary of the Newt, the blastoderma was formed, and I think I perceived the place where the germinal vesicle had been. As to the ova of the mammifera, I have found many in which there was no germinal vesicle, and which certainly had not been impregnated. It is to be remarked that in such ova the vitelline grains were for the most part coherent and formed the vesicular blastoderma.

It being determined that the disappearance of the germinal vesicle is prior to impregnation and not dependent on it, the next question which arises is "how does the germinal vesicle disappear?" My observations on the ova of the water Newt are the only ones I have which bear upon this question. From what I have observed in them I think the mode of disappearance is the following:

The vesicle, at first imbedded in the substance of the yelk, approaches more and more the surface of it, until it comes to lie immediately underneath the vitellary membrane, in the manner represented in fig. 12. The coat of the vesicle having now become very soft and weak gives way, and the contained fluid is effused on the surrounding surface of the yelk. The coat of the vesicle being of extreme tenuity cannot be seen after it has given way. The small depression in which the vesicle was situate now forms the cicatricula, fig. 13.

I think that the fluid contained in the germinal vesicle being effused gives a degree of consistence to the matter composing the surface of the yelk, and thus promotes the formation of the blastoderma.

If then the germinal vesicle is not dependent on impregnation, it may be asked, what is the first change which takes place in the ova in consequence of impregnation? Of all ova the ova of the Frog are those in which such change can be most directly observed. In them *the breaking up of the surface of the yelk into crystalline forms*, described by Prévost, and Dumas, is the first change I have seen.

March 17th, 1835. I examined to-day the spawn taken from a Frog yesterday, part of which was impregnated and part not; that which was impregnated presented the appearance delineated in fig. 11. The unimpregnated ova presented no change. The surface of the yelk becomes every day still more broken up, the crystalline forms becoming smaller and smaller, until the surface of the black blastoderma appears under a magnifying glass like shagreen. The blastoderma, consisting of an aggregation of clear globules, different from those of the rest of the yelk, is now fully

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* This small spot of the ova of the frog which is white, (from the exposure of the white yelk,) always turns to the most depending side. The germinal point is thus always uppermost. I turned a mass of spawn upside down; the white spot was exhibited by all, but in a short time the white spot had turned downwards and the germinal surface again became uppermost. In this case, does the vitellus alone revolve, or does the vitellus and its membrane turn round together in the gelatinous substance surrounding the ovum? It appeared to me that the latter was the way in which the revolution took place.

formed; it has extended itself so as to close in the white spot. Evolution then proceeds.

The change which takes place in the yelk of the bird's egg appears to be limited to the neighbourhood of the cicatricula. In the ovum of the mammifera, there being little more than a blastoderma to be formed, the whole of the vitelline grains undergo a change, and are resolved into a vesicular blastoderma, presenting the same peculiar friable and globular texture as the blastoderma of the egg of the Newt, Frog, Bird, &c. The matter contained in the cavity of the yelk of the bird's egg seems to be a substance of the same nature as the blastoderma, and to serve for the extension of it. The blastoderma of the bird's egg being once formed by the effusion of the fluid of the vesicle of Purkinge, and animated by fecundation, probably has the power to assimilate the matter in the cavity of the yelk to its own substance, without the assistance of a fluid such as that of the vesicle of Purkinge, which was first required to promote its formation. There is no central cavity in the ova of the Frog and Newt, because the blastoderma is formed at once all round the ovum.

Description of the PLATE.

PLATE XVI.

Fig. 1. An ovum found in the Fallopian tube of a Rabbit the third day after impregnation; magnified forty diameters.

Fig. 2. The ovum of the Frog when recently laid; magnified two diameters.

Fig. 3. The ovum of a water Newt in which development has commenced; magnified rather more than twice.

Fig. 4. A diagram showing the embryo of the Newt after the vitellary membrane has given way, contained only within the cavity of the substance which is added to the ovum in the oviduct.

Fig. 5. A diagram showing the embryo of the Frog still surrounded by the vitellary membrane as well as the gelatinous substance which is added to the ovum in the oviduct.

Fig. 6. An ovum found in the horn of the uterus of a Rabbit seven days after impregnation; magnified forty diameters.

Fig. 7. A human ovum aborted at the third or fourth week; natural size.

Figs. 8, 9, 10. Bodies found in the right Fallopian tube of the Rabbit which forms the subject of observation 1; magnified about fourteen diameters.

Fig. 11. This exhibits the breaking up into crystalline forms, observed on the surface of the Frog's ovum after impregnation; magnified about six diameters.

Fig. 12, 13. Diagrams illustrating the mode of disappearance of the germinal vesicle.

XIX. *Sequel to an Essay on the Constitution of the Atmosphere, published in the Philosophical Transactions for* 1826; *with some Account of the Sulphurets of Lime. By* JOHN DALTON, *D.C.L., F.R.S., &c.*

Received June 9,—Read June 15, 1837.

IN an essay of mine on the constitution of the atmosphere, which was printed in the Transactions for 1826, I signified my intention of following it with a sequel of experiments to ascertain if possible which of the two views therein developed was most countenanced by facts. I now proceed to give an account of such investigations relating to this subject as have engaged my attention during a long period of years.

It may be needful to premise certain facts which are, I believe, universally admitted as indisputable; namely, that the atmosphere consists principally of two elastic fluids, azote and oxygen, either mixed by some mechanical law, or otherwise combined by a chemical principle in proportion nearly as four parts of the former to one of the latter in volume; that the two elastic fluids may be obtained separately in a state of purity; that when thus obtained they may be mixed in all possible proportions; and that the aggregate volumes in such cases are just equal to the sum of the two volumes of the ingredients: also, that any body which has a chemical affinity for either of them so as to combine with it in a separate state, will also combine with it in the mixed state.

It is also pretty generally admitted that oxygen and azote are capable of chemical combinations in five or more definite proportions, namely,

2 vol. of azote with 1 vol. of oxygen—forming 2 vol. of nitrous oxide.
1 vol. of azote with 1 vol. of oxygen—forming 2 vol. of nitrous gas.
1 vol. of azote with $1\frac{1}{2}$ vol. of oxygen—forming $1\frac{1}{2}$ vol. of hyponitrous acid.
1 vol. of azote with 2 vol. of oxygen—forming 2 vol. of nitrous acid vapour.
1 vol. of azote with $2\frac{1}{2}$ vol. of oxygen—forming $2\frac{1}{2}$ vol. of nitric acid.

There does not appear to be a doubt of the reality of five combinations, but all chemists are not agreed as to the proportions of the volumes being precisely as above specified, chiefly because no general law has been found to obtain in such gaseous compounds.

These compounds are never formed nor decomposed without manifest chemical agency; they all contain oxygen, but no portion of it can be abstracted from any one of them without some chemical operation; whereas nitrous gas will immediately seize the oxygen from any of the aforementioned mixtures, the same as if it was alone, whatever may be the proportions. Atmospheric air itself, or any artificial mixture of the two gases in the same proportion as common air, is equally affected by nitrous gas and by every other agent.

Waving at present any consideration as to the nature and properties of the above chemical compounds, I shall now proceed to state the means by which the proportions of oxygen and azote in mixtures of these two gases may best be determined. Having been engaged in this investigation occasionally for more than forty years, I may be entitled to give my opinion on this important subject in practical chemistry.

Various methods of analysing common air have been discovered in the last fifty years. I have principally directed my attention to three, namely, (1.) by the use of Volta's eudiometer and hydrogen, or (2.) by nitrous gas, or (3.) by quadrisulphuret of lime, to abstract the oxygen from the azote.

First Method, by Volta's *Eudiometer.*

Mr. Cavendish was one of the first to investigate the changes produced by firing mixtures of hydrogen and common airs in various proportions. (Vid. Philos. Trans., 1784.) The following Table will exhibit a lasting monument of his skill in effecting such an investigation. Many have attempted since to improve the methods of analysis, and have brought out results widely differing from those to be derived from his table; but it is now universally allowed that his results are nearer approximations to the truth than most of those we have seen since.

His method was to take 100 measures of common air and mix them with various proportions of hydrogen, beginning with upwards of 100, and gradually descending till about 20; then, firing each mixture by an electric spark, he marked the diminution of the mixture each time as under.

The following results are extracted from Mr. Cavendish's Table, except the last column, "Amendment," which I have attached, for reasons assigned below.

Exp.	Common Air.		Inflammable Air.		Diminution on firing.	Amendment.
1.	100	measures mixed with	124·1	gave	68·6	66·3
2.	100	———	105·5	——	64·2	65·8
3.	100	———	70·6	——	64·7	64·9
4.	100	———	42·3	——	61·2	60·6
5.	100	———	33·1	——	47·6	47·4
6.	100	———	20·6	——	29·4	29·5

In the first three experiments no oxygen was found in the residuary gas; in the fourth a trace of oxygen was found; and in the fifth and sixth, considerable quantities of oxygen were found in the residues.

It is obvious that Mr. Cavendish began intentionally with an overdose of hydrogen, probably expecting the diminution to be a constant quantity till the hydrogen became deficient, and then of course the diminution must be lessened; this was not the case exactly; but the reason is easily discovered, and it proves the accuracy of the observations.

Hydrogen gas is rarely obtained quite pure: it frequently holds two or three per cent. of common air, detached from the water through which it bubbles and by other means; this air increases as more water enters the hydrogen bottle, till sometimes it amounts to ten per cent. at the last, as every one knows who has had a due share of experience. Now as Mr. Cavendish does not mention the purity of his hydrogen, we must try it by the means now generally known, as the reported results will guide us in the investigation.

On looking at the column headed "diminution on firing" it is easy to see there is a discrepancy in the first three experiments in that column; if the hydrogen used contained any oxygen the diminution on firing ought to have continually decreased, whereas it was greater in the third than in the second experiment. This it must be allowed is a proof of inaccuracy in one or both of the experiments; but it is no greater error than usually occurs if we trust to a single experiment with any gaseous mixture. The average of two or three experiments on mixtures of the same proportions should be taken. The fourth experiment clearly shows that the hydrogen contained oxygen as well as azote; for a diminution of 61·2 would denote the union of 20·4 oxygen with 40·8 hydrogen; hence there must have been 1·5 common air in the hydrogen. I have formed the column "amendment" by assuming the hydrogen in all the experiments to contain $4\frac{1}{2}$ per cent. common air. If we combine the results of the third and fourth experiments, either by assuming Mr. Cavendish's diminution or that of the amendment, we shall obtain a very good approximation to the quantity of oxygen in atmospheric air, the former experiment giving too great diminution by reason of the excess of hydrogen and that containing some oxygen, and the latter giving too little diminution for want of the requisite quantity of hydrogen; the former will give 20·98 per cent. oxygen, and the latter 20·92 per cent. oxygen in atmospheric air. If any doubt should remain as to Mr. Cavendish's hydrogen containing oxygen, it is removed by the consideration that his first experiment would indicate 22·9 oxygen per cent. in air, which cannot be allowed; and his last experiment that 8·8 oxygen must have combined with 20·6 hydrogen instead of 17·6, which is equally inadmissible.

Since the period 1784 it has been found by various chemists that in mixtures of oxygen and hydrogen, as well as in other similar ones, the electric spark does not always cause an explosion, and when it does a complete combination does not always take place, but that in the residue sometimes portions of both the ingredients may be found. The limitations and restrictions are now pretty generally known; and with regard to the mixtures of common air and hydrogen, I published a letter in the 10th volume of the Annals of Philosophy, (New Series) page 304, in which I showed the limitations found by my own experience to be as under:

Common air and hydrogen in which the oxygen is only $\frac{1}{15}$th, or from six to seven per cent. of the whole mixture, do not explode.

Common air and hydrogen in which the oxygen is only $\frac{1}{14}$th, or seven per cent., explode imperfectly, leaving both oxygen and hydrogen.

Common air and hydrogen in which the oxygen is from $\frac{1}{13}$th to $\frac{1}{6{\cdot}86}$th, or from eight to fourteen or fifteen per cent., fire leaving hydrogen and azote only.

Common air and hydrogen in which the hydrogen is $\frac{1}{3{\cdot}38}$th to $\frac{1}{7}$th, or from fourteen to thirty per cent., fire and leave oxygen and azote only.

Common air and hydrogen in which the hydrogen is $\frac{1}{8}$th to $\frac{1}{14}$th, or from eight to twelve per cent., fire imperfectly, and leave oxygen, hydrogen, and azote.

Common air and hydrogen in which the hydrogen is $\frac{1}{15}$th or less than seven per cent., do not explode.

It should be observed that when one of the gases is so far deficient as not to allow of an explosion by a single spark, the effect may be obtained by a current of sparks for a longer or shorter period, accompanied by the requisite diminution of volume. In such instances where the effect is produced only by a current of sparks it may be proper here to suggest the reason. When mixtures explode perfectly but feebly, we see the flame, lighted by the spark, to run down the eudiometer till it reaches the water; when they explode still more feebly, the flame runs perhaps half way down the tube and is extinguished before it reaches the water. There scarcely can be a doubt that the extinction must be occasioned by the cooling effect of the eudiometer and of the intermixture of the mass of air which has to be heated by the feeble flame. Another spark in its passage will re-alight the flame, to suffer a quicker extinction, and so on till at length the combustion is complete. This reason will also explain the excessively slow combustion of azote by the electric spark, as ascertained by Mr. Cavendish, and as I have found by repeated experience. Query, might not this experiment succeed better by heating the eudiometer?

From what we have stated it must be obvious that in order to secure the complete abstraction of either oxygen or hydrogen from mixtures by Volta's eudiometer, we should avoid too near an approach to the limitations we have pointed out; or if that cannot be, we should carefully examine the residue for both gases. The best test for very small portions of oxygen is undoubtedly nitrous gas; for somewhat larger portions of oxygen or hydrogen, additions of those gases might be made so as to bring the mixtures into proportions capable of being exploded.

Second Method, by Nitrous Gas.

The nitrous gas eudiometer is of singular utility on many occasions. No other can exceed it in accuracy when mixtures contain very little, as one or two per cent. of oxygen; or on the other hand when nearly the whole of the gas is oxygen. But when the mixture of gases contains from twenty to eighty per cent. of oxygen, as in the case of common air, it is not the best when great exactness is required. The reason is well known; when oxygen and nitrous gas combine, the combination is not like that of oxygen and hydrogen, in uniform proportion. We may take one third of the diminution for oxygen, when mixed over water; but this can be considered only as a first approximation. One hundred parts of oxygen may combine with 130 or 360 parts, or any intermediate quantity of nitrous gas, according to circumstances. When only 1 or 2 per cent. of oxygen are expected I put in 5 or 10 per cent. of nitrous gas, and take one third of the diminution for oxygen. When the oxygen (freed from carbonic acid) is judged to be 90 or more per cent. pure, I put 100 parts of nitrous gas of known purity (say 98 ±) to 100 of the oxygen, and mark the diminution; I next put in 40 nitrous and mark the diminution, and so on, till there is manifestly a slight portion of nitrous left; then this is to be removed by a small portion of oxygen; finally, knowing the quantity of azote which was in the nitrous gas, the rest must have been introduced by the oxygen.

In this way I find a perfect agreement, whether the nitrous test or the hydrogen is used; but with common air the residue is so enlarged with azote as to render the measuring of it not so accurate.

Third Method, by Quadrisulphuret of Lime.

Quadrisulphuret of lime is an excellent test for oxygen, and may be applied to common air or to other mixtures of which oxygen is a part, up to the purest oxygen. As this and other similar compounds seem to me destined to act an important part in chemical operations, it may not be improper here to give some account of their origin and their constitution, as far as actual experiments have demonstrated.

The alkalies and the alkaline earths that are soluble in water have been long known to combine with sulphur, both in the dry and humid way. In the last century they went by the name of *hepar sulphuris*, or liver of sulphur, from their colour.

Scheele was the first to use the quadrisulphuret of lime to abstract oxygen from atmospheric air. Lavoisier also made use of the same article; but it was to De Marti of Spain we owe the most successful attempt with the quadrisulphuret of lime to abstract the oxygen from atmospheric air. His memoir, printed in 1795, and reprinted in the Journal de Physique, vol. lii., 1801, may still be read with interest. All the *hepars*, when dissolved in water, have usually gone by the harsh name of *hydroguretted sulphurets* in our English works of chemistry since the commencement of the present century.

In 1798 Berthollet published an essay on the nature and combinations of sulphuretted hydrogen, with reference to the part it acts in the sulphurets. Proust afterwards controverted some of Berthollet's opinions in the 59th volume of the Journal de Physique, 1804. Gay-Lussac, in the 78th volume of the Annales de Chimie, 1811, gives some important results on the mutual action of metallic oxides and alkaline hydrosulphurets; he finds amongst other results that no *sulphates* are formed, that water is formed, that sulphites or sulphuretted sulphites, and often metallic sulphurets are formed; and that consequently it is not possible to obtain the simple metallic bases of hydrosulphurets by means of hydrosulphurets of their oxides;

and that when a sulphuret is dissolved in water, no sulphate is ever formed, as is commonly imagined, but sulphites and sulphuretted sulphites. Some proofs are afterwards given[*]. VAUQUELIN, in the 6th volume of the Annales de Chimie et de Physique, 1817, presents us with a laboured series of experiments on the alkaline sulphurets, the chief object of which is to ascertain the state of the alkali in the sulphuret, whether it is that of a metal or of an oxide. After many experiments on the sulphurets of potash, soda, and lime in the dry way, and one on sulphuret of lime in the humid way, the author sums up, and notwithstanding his leaning to the opinion that the alkalies exist in sulphurets in the state of *metals*, he is obliged at last to acknowledge "that it is probable, *but not yet demonstrated*, that in all the sulphurets formed by means of the alkaline oxides by a red heat, these last lose their oxygen, and are united to sulphur in the metallic state, as is the case with the other metals." GAY-LUSSAC, in the sequel of the same volume, page 322, in a memoir, animadverts on the before-cited paragraph; and allowing that sulphuric acid is formed when a sulphuret of potash made by a red heat is dissolved in water, he contends, according to a suggestion of BERTHOLLET, that the acid is formed in the instant of solution from the reciprocal action of the sulphuret and the water, rather than from the oxygen of the potash and sulphur. This opinion is countenanced by several combinations of a similar nature, which he has adduced, and which are worth the attention of chemists.

Without adverting at present to my own experiments, I may observe that Sir JOHN HERSCHEL, in an essay in the first volume of the Edinburgh Philosophical Journal, 1819, was the first writer who published an atomic view of the class of salts called sulphuretted sulphites, or hyposulphites, that accorded with what I had long entertained and demonstrated by reiterated and decisive experiments[†]. In the above-mentioned essay he showed clearly that the hyposulphurous acid is composed of two atoms of sulphur and two of oxygen, which united to one atom of base, as potash or lime, compose an atom of a hyposulphite. The formation of those of lime, potash, soda, barytes, and some metallic oxides is more particularly explained. A saturated solution of hyposulphite of lime at 50° he found to be 1·30 specific gravity[‡].

In the 14th volume of the Annales de Chimie et de Physique, GAY-LUSSAC has given the principal results of HERSCHEL's essays on the hyposulphurous acid with some judicious remarks, but he leaves the subject as one requiring further investigation.

In 1822 BERZELIUS published a memoir on the alkaline sulphurets. The results of his experiments seemed to him confirmatory of the previous notion of VAUQUELIN. Those experiments were on the sulphurets of potash and lime made in the dry way; he made only one on lime, which agreed very well with the theory; but this very

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[*] See also vol. lxxxv. p. 199.

[†] See New System of Chemical Philosophy, vol. ii. Preface, and p. 105.

[‡] Dr. THOMSON, in a paper on the compounds of chromium in the Transactions of the Royal Society for 1826, disputes the accuracy of this constitution of hyposulphurous acid. I have never had any doubt concerning it since 1815.

delicate experiment was not enough to establish so important a law of combination, and I do not find that any one besides has obtained the same result[*].

Though I am not prepared to deny that sulphurets of potassium and calcium can be obtained by the process of Berzelius, I am quite satisfied that sulphurets of potash and lime, &c. may be easily procured in the dry way: of that of lime I have had numberless instances. As the compounds of sulphur and the alkaline earths have been very little subjected to investigation by chemists in general, we find great vacancy in the accounts given of them by the modern compilers of chemical books. For this reason I shall introduce here a few of the results I have obtained in a long series of experiments on this branch of chemical inquiry.

Sulphuret of Lime, in the dry way.

In 1806 I formed, for the first time, the protosulphuret of lime by heating 50 grains of fallen lime with 50 sulphur in a covered crucible not quite air-tight, so that the escape and combustion of the excess of sulphur might be allowed; when raised to a red heat an addition was made to the weight of the lime; by repeating the dose of the sulphur and heating, a further addition was made to the weight; but repeating the operation a third time seldom made any further addition. The weight of the compound was 65 grains; it was a white powder with a tinge of yellow, not caustic, but bitter to the taste.

In 1809 I examined this powder more minutely, and found it was best made by mixing equal weights of pure hydrate of lime and flowers of sulphur, putting the mixture into a covered crucible and heating it slowly to red; when the escape of the sulphur fumes ceases, cool the contents, and again mix them with the same weight of sulphur as in the first operation, and again heat it as above; at last it will be found that 32 parts of hydrate of lime = 24 lime have combined with 14 of sulphur, or one atom to one[†]. In the work referred to I have stated that pounded lime and sulphur scarcely form any union by this process, and carbonate of lime and sulphur still less. An ingenious pupil of mine, Mr. William Barnett Watson of Bolton, has succeeded in uniting lime and sulphur by heat; instead of taking pounded lime, which has a harsh gritty feel, he takes hydrate of lime, and expels the water by a red heat continued till 32 parts of hydrate are reduced to 24; this is a fine soft powder; when 24 parts of this pure and finely divided lime freed from water are well mixed with 24 parts of sulphur and heated red in a covered crucible, a partial combination takes place, and an increase of weight to the lime; this operation is to be repeated till the additional weight becomes 14 grains, after which no further addition can be effected. Mr. Watson found it require several repetitions. I have since found it may be effected by two or three only. This sulphuret is not used in eudiometry.

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[*] Annals of Philosophy, 1822.

[†] See New System of Chemical Philosophy, vol. ii. pages 99 and 102.

Quadrisulphuret of Lime, in the humid way.

When sulphur and hydrate of lime in almost any proportions are boiled together in water, quadrisulphuret of lime is formed and dissolved in the water; the solution is of a deep yellow colour, and has a very bitter taste. I have not seen in any author the proportion that ought to be used, nor the quantity and specific gravity of the liquid solutions. These are subjects which have engaged my attention. If lime is in excess, the liquid consists of *lime water* holding in solution quadrisulphuret of lime. If sulphur is in excess, the liquid consists of *water* holding in solution quadrisulphuret of lime. I have long known that the economical proportions to be used are 32 parts of dry hydrate of lime by weight with 56 of sulphur, that is, one atom of lime with four atoms of sulphur. If more lime than that above be used, it will be found prevalent in the residue; if more sulphur, then the redundant sulphur will be found in the residue. A few ounces of the mixed ingredients may be gently boiled in an iron pan for an hour or more, stirring the liquor occasionally, and covering the pan with a lid to prevent the too free admission of atmospheric air. Or, in order to prevent the action of oxygen on the liquid, a flask may be substituted for the pan; the materials may be put into the flask nearly filled with water, and the flask loosely corked may be immersed in a pan of boiling water so as to be almost covered by the water. The liquor to be preserved should be kept in green glass bottles nearly full, and having ground stoppers. After the boiled liquor has cooled and the sediment subsided, the clear liquor may be decanted; if it be strong or deep coloured the sediment may be washed with a little water, and another quantity of the liquor obtained of inferior strength. The sediment may be dried if necessary, and subjected to analysis, as I have mostly done. The quantity and specific gravity of the clear liquors should then be ascertained.

The first quadrisulphuret of lime I made was in 1804; it was very weak, since it only absorbed one fourth of its bulk of oxygen gas; the next that was made took its bulk of oxygen. The next, made in 1806, took $2\frac{1}{2}$ times its bulk of oxygen. In these no account was taken of quantities or residues of lime and sulphur. After this I saw the necessity of investigating, (1.) the quantities of lime and sulphur mixed; (2.) the quantity and specific gravity of the liquid obtained; and (3.) the quantity and proportion of the materials left in the residue, in order that the rationale of the changes effected might be explained. From 1806 to the present time (1837) I have made no quadrisulphuret of lime without attending to all those particulars. In this period I have made it 23 times, six of which were in flasks, and the rest in iron pans covered as mentioned above; the difference of the two methods I found to be very little; it consisted chiefly in traces of sulphuret of iron being found in the residues when pans were used.

A few trials of the various liquids obtained soon furnished me with a formula for ascertaining the quantities of sulphur and lime in a liquid of given specific gravity;

namely, multiply the three leading decimals in the specific gravity of the liquid by 13, and the product will give the aggregate weight in grains of sulphur and lime in 1000 water grain measures of the liquid; of this aggregate $\frac{9}{13}$th will be sulphur, and $\frac{4}{13}$th lime.

With regard to the residue after boiling and its analysis, it is obvious the residue must consist chiefly of sulphur and lime, which for want of due continuance of the ebullition have escaped combination; and there may be some impurities in the sulphur, or the hydrate of lime may not be free from carbonate, &c.; but when the residue is comparatively small no material disturbance of proportions in the quadrisulphuret can take place. If the residue be chiefly sulphur, its quantity may be approximated by ignition; but if lime is in excess, it may be estimated by the quantity of muriatic acid required to saturate it.

The following Table exhibits a selection of the principal varieties in the proportions of ingredients and products obtained so as to illustrate the foregoing statements.

Table of Proportions in Quadrisulphuret of Lime.

	Quantities of hydrate of lime and sulphur mixed.	Proportions of lime and sulphur.	Quantity of liquor obtained in water grain measures, and quantities of lime and sulphur in it.	Measures of oxygen required to saturate 100 liquid.	Quantity of residue when dried.
	Hydrate. Sulphur.	Lime. Sulph.			
1	120 = 90 lime + 210	4 : $9\frac{1}{3}$	3100 of 1·056 containing 70 lime + 156 sulph.	900	56 = 16 lime + 40 sulph.
2	50 = $37\frac{1}{2}$ lime + 50*	4 : $5\frac{1}{3}$	2200 of 1·0240 containing 21 lime + 47 sulph.	400	20 = 12 lime + 4 sulph. + loss.
3	150 = $112\frac{1}{2}$ lime + 200	4 : 7+	1450 of 1·146 containing 85 lime + 190 sulph.	2350	†20 = 7 lime + 13 sulph.
4	96 = 72 lime + 168 sulph.‡	4 : $9\frac{1}{3}$	2800 of 1·056 containing 63 lime + 141 sulph.	900 §	34 = 9 lime + 25 sulph.
5	35 = 26 lime + 140 sulph.	4 : 21·6	1600 of 1·037 containing 23·7 lime + 53·3 sulph.	600 §	83 all sulph.

On the Quantity of Oxygen in the Atmosphere.

Since the commencement of the present century it has been ascertained beyond dispute that the chief constituents of the atmosphere, oxygen gas and azotic gas, are in the same proportion in all countries and at all times, except when influenced by local circumstances; namely, 21 per cent. of volume of oxygen, and 79 per cent. of azote, neglecting fractions: other elements are found in the atmosphere, but they are comparatively insignificant in quantity, namely aqueous vapour, carbonic acid, &c. The experiments have generally been made on air collected at the surface of the earth; and it may be remembered that I have endeavoured to prove in various essays that the diffusion of gases one amongst another as well as in *vacuo*, is owing to the repulsive powers peculiar to the particles of each particular gas, otherwise we

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* Boiled in a flask loosely corked.

† Lost some of the ingredients by boiling over; hence a deficiency.

‡ Boiled in a flask with great care.

§ The oxygen was determined by especial care in these two cases.

should never have the feeble efforts of carbonic acid and aqueous vapour diffusing those elements against the immense pressure of the atmosphere. The principle I contend for has, I believe, obtained general assent; but I apprehend few have been aware of the consequences. If we suppose a carbonic acid atmosphere of 15 inches of mercury pressure and a hydrogen atmosphere of the same pressure, together constituting a mixture of the two amounting to 30 inches of pressure, were to surround the earth, I think no one would hazard a conjecture that these two would be found in equal proportions at every elevation in the atmosphere; yet a similar supposition seems prevalent with regard to our present atmosphere of oxygen and azote. It has been an object of investigation with me for many years to find how the fact stands in this respect; that is, whether the oxygen is more abundant relatively in the lower strata of the atmosphere than in the higher, as it ought to be in a stagnant column; or whether the constant agitation of the atmosphere and the predominant mechanical power of the azotic part of it do not prevent that equilibrium which a stagnant mixture of aerial fluids of different specific gravities would effect. From the experiments about to be related, I have reason to believe that the higher regions of the atmosphere are somewhat less abundant in the proportion of oxygen than the lower, though the reverse might be expected from the enormous consumption of oxygen by daily processes on the surface of the earth, when we know of no proportionate consumption of azote. It appears, however, that the disproportion of the two elements at different elevations is by no means so great as theory requires; and therefore we must conclude the unceasing agitation of the atmosphere by currents and counter-currents is sufficient to maintain an almost uniform mixture at the different elevations to which we have access.

The subject is one involving an important principle. I have kept it continually in view for the last forty years, and have made innumerable experiments with a view to its elucidation. As the value of such experiments depends much upon a thorough acquaintance with the nature of the operations and the several sources of error to which they are liable, it may be needful to point out certain particulars, which, as long experience has taught me, require attention in order to secure a due approximation to accuracy. I allude more particularly to the use of VOLTA's eudiometer as applied to determine the proportions and quantities of oxygen and hydrogen gases.

1. Hydrogen gas procured over water is sure to contain some common air, whether the water has been previously boiled or not; it arises out of the water and may amount to 1 or 2 per cent.; the same observation applies to oxygen gas; the proportion of oxygen and azote is usually that in common air nearly. When a phial of hydrogen gas, by long keeping or by accident, has acquired a portion of common air, and then stood some weeks after, the oxygen seems to diminish, either by slow combustion or by absorption in the water, and so leaves the azote and oxygen in another proportion to that of common air. Before using such hydrogen the oxygen in it should be tested by nitrous gas, and the percentage of hydrogen by oxygen gas.

It is best not to rely too much upon hydrogen taken from a bottle half filled with water.

2. Oxygen gas, and others, will show carbonic acid by sending them up through a narrow eudiometer tube filled with lime-water, provided the acid gas amounts to $\frac{1}{2}$ per cent. of the original; but it does not show any carbonic acid in this way in atmospheric air, though the acid is always present to the amount perhaps of $\frac{1}{1000}$th part. The proportion of pure oxygen in any sample containing from 90 to 100 per cent. of that gas, may be found either by hydrogen gas or nitrous gas; and if great accuracy is required, I recommend testing it both ways, as has already been mentioned under the head nitrous gas.

3. The gradual deterioration of oxygen, hydrogen, nitrous gas, common air, &c., when by use the phial becames $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$ filled with trough water, is a circumstance by no means to be overlooked. The entrance of water that has been sometime stagnant in the cistern, though preserved carefully from any material impurities, always affects the remaining air, though the phial be well corked and immersed in a cup of water. The cause is obvious to those acquainted with the laws that regulate the absorption of gases by water. The common air in the water (the quantity of which varies much as to the oxygen part) is continually either making its escape into the incumbent air of the phial, or this last air is entering the water, so that the degree of purity is continually changing in a small degree. This renders it necessary to test the actual state of this gas after it has been some time in the phial, before we recommence the use of it. A phial of air may be pure at first, and only 90 per cent. at its conclusion. I have known samples of common air kept in bottles at first containing 21 per cent. of oxygen, and after some months a small residue was found to contain only 19 per cent.

4. It may not be improper here to relate some unpublished results which I formerly obtained when experimenting on subjects here discussed. In my memoranda for 1816, I find that I took water well boiled (supposed $\frac{1}{4}$ of an hour or more) and then poured it gently into a Florence flask, filling it up into the narrowest part of the neck, and left it so, exposed to the atmosphere for three days without any agitation. At the end of this, 2700 grains of water imbibed 49 grain measures of atmospheric air by agitation, which is about $\frac{4}{5}$ of a full share; hence $\frac{1}{5}$ of a full share must have been, both the air that was left in after boiling, and that acquired from the atmosphere in three days by absorption from the small exposed surface.

Water boiled in a kettle for three or four minutes, then suddenly cooled and transferred without agitation into a bottle containing 2700 grains, and then agitated with atmospheric air, imbibed 32 measures, which are about half a charge; whence it may be inferred that water boiled for three or four minutes loses about half of its air.

I boiled a kettle full of water for a quarter of an hour; let it stand a day or two to cool, then transferred it carefully by a siphon into a cylindric jar of 8 inches diameter and 10 inches deep; afterwards drew off daily by a siphon 2700 grain measures from the middle or near the bottom of the jar, and charged it with air to the full by agitation. The bottle of water imbibed

The first day . . .	16 measures.	
The second day . .	15 measures.	
The third day . .	12 measures.	
The fourth day . .	10 measures.	
The fifth day . . .	10 measures.	
The sixth day . .	9 measures.	
The seventh day . .	4 measures.	The water taken near the surface.
The eighth day . .	7 measures.	These portions taken up consisted nearly one half of oxygen.
The ninth day . .	9 measures.	
The tenth day . .	7 measures.	
The fifteenth day . .	2 or 3 measures.	

From these experiments it would appear that by boiling water briskly for three or four minutes, about half of the atmospheric air previously in the water escapes along with the steam. But it requires much longer boiling and keeping the atmospheric air as much as possible from the surface of the water to get the rest of the air expelled. It is never all expelled by boiling, except in the construction of a good water hammer. Any one air not chemically combined with water is easily and effectually expelled from it by repeatedly agitating the water with another kind of air.

It also appears that water deprived of its atmospheric air, if kept at rest, acquires the air again slowly, and more so if the surface exposed is small. But if violent agitation of the water, so as to mix the atmospheric air and it intimately together, be used, the full impregnation is effected in one or two minutes, as I have elsewhere shown.

Trough waters being mentioned above (3.) it may be well to explain some of the circumstances affecting it. The waters I use for the chemical trough is *rain-water*; it is preferable to pump water by its freedom from carbonic acid and earthy salts; it is slightly coloured at first when drawn from the cistern, but it soon becomes clarified by standing: my trough contains about nine gallons when in work. I take great care to put nothing in it which can materially affect its purity; small portions of lime water and of some iron and other salts are the chief impurities which are admitted; no sulphurets or hydrosulphurets are allowed to enter, and very little of either acids or alkalies. I examine the state of the water occasionally; lately, after it had been more than half a year in the trough, though not very frequently used, I had the curiosity to examine its state before the trough was emptied. The water was neutral by the colour test; it contained about 50 grains of saline matter in the gallon; it was transparent, but slightly milky; prussiate of potash gave sensible blue; oxalate of ammonia, muriate of barytes, and carbonate of soda produced a white precipitate. The taste was like that of earthy pump water. It had its full share of azotic gas, but rather less than half of its share of oxygen gas; that is, it had about 4 or 5 cubic inches of azote in the gallon, and only 1 cubic inch of oxygen.

In the following train of experiments on the oxygen in the atmosphere I have mostly used from 50 to 70 measures of hydrogen for 100 air, unless otherwise mentioned. Possibly this may not be thought the best proportion for securing the complete abstraction of the oxygen. The limits are, 100 air with 42 of hydrogen for the minimum, and 100 air with 170 hydrogen for the maximum. In the former case the hydrogen is barely sufficient for the oxygen; in the latter case the oxygen is barely enough to admit of a complete combustion, being only $\frac{1}{13}$th of the mixture. Perhaps the best proportion would be 100 air to 100 hydrogen to ensure complete combustion, because it is about the mean of the two extremes; but it must be considered that if the hydrogen should contain even a very small portion of oxygen, the whole of it in 100 measures would be included in the atmospheric oxygen, so that in practice it would probably be safest to use a mean between 40 and 100 of hydrogen. I have mostly endeavoured to keep between 50 and 70 of hydrogen for 100 air.

Experiments on the Quantity of Oxygen in Atmospheric Air.

Air from the Summit of Helvellyn*, July 14, 1824.

A phial, containing about half a pint, was filled with water at a clear rivulet on the ascent; this was emptied at the summit and well corked; the cork was drawn at the foot of the mountain in a trough of clear running water, when a quantity of water was found to enter corresponding to the increased pressure of the atmosphere. The phial was then corked and inverted in a cup of water, and the air analysed a week afterwards.

Average of four experiments on this air with hydrogen, about 50 to 100 air, gave	20·70 oxygen per cent.
Average of four experiments of the common air taken in Manchester at the time of the analysis, and with same phial of hydrogen and same proportion, gave	20·88 oxygen per cent.
Average of seven experiments on Helvellyn air made a day afterwards, gave	20·58 oxygen per cent.
Average of seven experiments on air from an open place in the town next day with same hydrogen, gave	21·1 oxygen per cent.
Average of eight experiments on the country air three miles from Manchester, July 29, with same phial of hydrogen, which now manifested a very slight trace of oxygen, gave	21 oxygen per cent.

1824, November 23.—Barometer 28 inches, very low. Apprehending that this

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* This mountain, situate at the head of Ullswater, separates Cumberland from Westmoreland; its height above the sea, which lies to the S.W., and from which it is distant about 20 miles, is upwards of 3000 feet; it is surrounded by other mountains, mostly of less elevation.

circumstance, attended by rain and a high wind S.E., might have some influence on the proportions of the atmosphere, I made the following experiments.

Average of six experiments gave 20·75 oxygen per cent.

When the remainder of this air had been kept five months in the bottle, it then yielded on an average of three experiments 20·67 oxygen per cent.

1825, January 8.—Barometer 30·94, extremely high, after a week of calm weather. Filled a bottle with air from the town.

Average of four experiments with two parts air and one hydrogen gave 21·12 oxygen per cent.

The remainder of this air, kept till August same year, gave 21·1 oxygen per cent.

June 8.—Average of four experiments from air in the town gave 20·97 oxygen per cent.; barometer 29·90.

June 10.—Air from a field near the town, barometer being 30·30, thermometer 70°, wind S.W.; sunny and sultry. Two parts of the air with one of pure hydrogen being mixed, the average of six experiments gave 20·58 oxygen per cent.

June 14.—Mixed some pure azotic gas with oxygen gas, which was marked 90 per cent. pure, in such proportions as to make a mixture of 21 per cent. oxygen. On trial with hydrogen the mixture gave, first experiment 21 + oxygen per cent.; the second experiment 20·9 oxygen per cent.

November 3.—Air in the town, barometer 28·76, thermometer 46°, rainy, with S.W. wind. Average of ten experiments gave 20·6 oxygen per cent.

Air from the Summit of Snowdon, 3570 feet above the sea, taken by John Blackwall, Esq., May 14, 1826, at 7 P.M.; wind N.E. light, barometer 26·20, thermometer 42°.

May 28.—Average of ten experiments gave 20·65 per cent. oxygen.

Country air three miles from Manchester, analysed the same day, average of six experiments gave 20·8 per cent. oxygen.

Again, Snowdon air in six experiments gave 20·66 oxygen per cent.; but the bottle being now half full of water, I did not examine the rest.

Another bottle of air was taken at the summit on another occasion, May 18, by the same gentleman; wind S.W., light.

May 25.—Analysed; average of six experiments gave 20·59 oxygen per cent.

Country air near Manchester at same time gave average 20·7 per cent.

A second bottle of air from Snowdon, taken at the same time, May 18, gave on an average of four experiments 20·9 oxygen per cent.

Air from the town at the same time, on an average of five experiments, gave 21·04 oxygen per cent.

1826, July, Air from the Summit of Helvellyn.

Average of ten experiments gave 20·63 oxygen per cent.

Average of the town air found at same time was 20·73 oxygen per cent.

Air taken in an Aerial Voyage over Cheshire.

Mr. Grafton was so good as to procure me a bottle of air taken in an aerial voyage over Cheshire with Mr. Green, June 26, 1827; height 9600 feet above the sea*. The air was transferred into two phials.

First Phial.

June 27.—Average of seven experiments of balloon air gave 20·7 oxygen per cent.
Average of seven experiments on town air gave . 20·83
July 2.—Average of eight experiments of balloon air gave 20·2†
Average of eight experiments on town air gave . 20·8

The second phial of balloon air was carefully preserved, the phial being filled and having a ground stopper. It was analysed.

1828, May 28.—Average of three experiments balloon air gave 20·70 oxygen per cent.
Average of three experiments town air gave . 20·80
Aug. 5.—Average of thirteen experiments, being the whole of the balloon air, gave . . . . 20·52
Average of thirteen experiments on town air gave . . . . 20·92

On the last-mentioned day I received a bottle of air from the summit of Snowdon through the care and attention of my friend and pupil Mr. John Hall. It was corked and well sealed with wax; when opened under water a due portion of that fluid entered.

The average of the first two experiments gave 20·44 oxygen per cent.

The rest of the air after these two experiments was divided into two portions, and entered into two phials for examination. These were analysed a week or two afterwards.

Average of five experiments with first phial gave 20·25 oxygen per cent.

Average of four experiments, which emptied the first phial, gave 19·98 oxygen per cent.

Average of seven experiments of second phial gave 20·3 oxygen per cent.; and a considerable portion was left.

Average of the town air was during these experiments nearly 21 oxygen per cent.

I am not aware of any cause why this air was so much inferior in oxygen to that on former occasions.

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* Height found as under:
Capacity of bottle . . . . 10·47 ounces.
On drawing the cork under water there entered . . 2·77 ounces.
Left . . . 7·7 ounces of air.
Also height of barometer and thermometer below given.

† The whole air in the first phial was spent in these fifteen experiments. The deterioration of the air in the first phial, by being kept half full of trough water for five days, is remarkable.

1831, July 4.—Helvellyn air brought down from the summit by me; wind S.W., with rain and fog.

1. July 21.—Mixed two ounce measures of this air with one of hydrogen, so as to make six separate and successive explosions; the hydrogen had $\frac{4}{10}$ths of a grain measure per cent. of oxygen, and this is allowed for in the corrected results. These results on the average gave 20·57 oxygen per cent.; the highest was 20·68, and the lowest was 20·43.

The residues of the six explosions were collected, and found to have 5 per cent. of hydrogen and 1 in 120 of oxygen.

2. Mixed *equal volumes* of this Helvellyn air and the same bottle of hydrogen used above, and fired the mixture in successive portions. The average of six experiments gave 20·8 per cent. of oxygen. No oxygen was found in the residue.

By comparing the results of 1 and 2, it would seem that more oxygen is reduced from common air by firing equal volumes of common air and hydrogen than by firing one volume of common air with half a volume of hydrogen.

August 23.—Mixed 100 measures of town air and 120 of new pure hydrogen; this fired gave 21·5 oxygen per cent.; there was no oxygen in the residue. This would seem to point out $\frac{1}{240}$th of oxygen in the hydrogen, yet nitrous gas scarcely manifested so much.

1832, July 26.—Mr. Green, jun., and Mr. John Taylor of the Manchester gas works, ascended in a balloon from Manchester after 6 P.M., a fine, clear, calm evening, barometer being 30 inches, thermometer 65°; the balloon took a south direction, and landed in Cheshire about fourteen miles off. Mr. Taylor took a bottle of air when at the highest elevation, when the barometer stood at 16·8 inches, thermometer 55°; whence the altitude must have been about 15,000 feet.

Capacity of the bottle = 2406 grains of water.

On opening it under water in temp. 64° there entered 884 grains of water.

The air was soon after its reception on the 27th transferred into two small phials for examination.

The first phial was mixed with 60 per cent. of hydrogen, and fired in five portions; it yielded 20·59 oxygen per cent.

The second phial, mixed in like proportion, gave 20·65 oxygen per cent.

Air from the town the next day, fired with the same phial of hydrogen as the preceding, gave 20·95 on the average of five experiments.

Air from Switzerland, &c.

In the autumn of 1835 I was favoured with three samples of air taken in elevated situations in Switzerland by my friend W. D. Crewdson, jun. Esq., of Kendal. Each of these was taken in a two ounce phial by pouring out the contained water and corking the phial immediately, leaving only a drop or two of water within. The cork was then well closed with sealing-wax. No. 1 was taken on the *Mer de Glace*, August 21, estimated at the height of 6000 feet above the sea; the second on the pass of the *Simplon*, August 29, at the height of 6174 feet above the sea; and the third on the *Wengern Alp* on the 15th September, at the height of 6230 feet. These airs were analysed in October with the following results.

Mer de Glace.—	Average of four first experiments	20·2	oxygen per cent.
	Average of four last experiments	19·4	oxygen per cent.
Simplon.—	Average of four first experiments	19·98	oxygen per cent.
	Average of four last experiments	19·53	oxygen per cent.
Wengern Alp.—	Average of four first experiments	20·45	oxygen per cent.
	Average of four last experiments	20·11	oxygen per cent.

It may not be amiss to subjoin a few experiments on air in close chambers, where a number of people have been congregated for two hours, the air being taken at the moment of breaking up.

1802, March 6.—Got a 20-ounce phial filled at the close of a congregation of 500 people assembled for two hours with 50 candles burning; the air completely neutralized 150 grains of lime water, but took very little more; this accords nearly with 1 per cent. of carbonic acid gas. The oxygen was not examined.

1824, November 28.—Examined the air at the close of an ordinary congregation, perhaps 200 people, retained for two hours.

Average of five experiments gave the oxygen 20·42 per cent.

1826, March 16.—Examined the air from a crowded congregation after two hours' confinement, but some doors open.

Average of four experiments gave the oxygen 20·23 per cent.

There was a very slight appearance of carbonic acid each time a charge was passed up through lime water, a phenomenon never observed in ordinary atmospheric air.

The general conclusions, it seems to me, to be drawn from these experiments are, that the proportion of oxygen to azote in the atmosphere on the surface of the earth is not precisely the same at all places and times; and that in elevated regions the proportion of oxygen to azote is somewhat less than at the surface of the earth, but not nearly so much so as the theory of mixed gases would require; and that the reason for this last must be found in the incessant agitation in the atmosphere from winds and other causes.

June 6, 1837.

XX. *On the Hereditary Instinctive Propensities of Animals. By* Thomas Andrew Knight, *Esq. F.R.S. President of the Horticultural Society, &c. &c.*

Received May 15,—Read May 25, 1837.

In a communication which I had the honour many years ago to address to this Society upon the Economy of Bees, I gave an opinion that families of those insects, in common with those of every species of domesticated animal, are to a greater or less extent governed by a power which I have there called "an instinctive hereditary propensity;" that is, by an irresistible propensity to do that which their predecessors of the same family have been taught or constrained to do, through many successive generations. In that communication I stated that a young Terrier, whose parents had been much employed in destroying Polecats, and a young Springing Spaniel, whose ancestry through many generations had been employed in finding Woodcocks, were reared together as companions, the Terrier not having been permitted to see a Polecat, or any other animal of similar character, and the Spaniel having been prevented seeing a Woodcock, or other kind of game; and that the Terrier evinced, as soon as it perceived the *scent* of the Polecat, very violent anger; and as soon as it *saw* the Polecat attacked it with the same degree of fury as its parents would have done. The young Spaniel, on the contrary, looked on with indifference; but it pursued the first Woodcock which it ever saw with joy and exultation, of which its companion, the Terrier, did not in any degree partake.

I had at that period made a great many analogous experiments, and I have subsequently made a considerable number, chiefly upon one variety of dog, namely, that which is generally used in search of Woodcocks, and is usually called the Springing Spaniel. These experiments were commenced nearly sixty years ago, and occupied a good deal of my attention during more than twenty years, and to a less extent nearly to the present time; and as it does not appear to me probable that any person is now likely to investigate this subject as laboriously, or through so long a period, I have been induced to believe that the facts which I am prepared to communicate may be thought to deserve to be recorded in the Transactions of this Society.

At the period in which my experiments commenced, well-bred and well-taught Springing Spaniels were abundant, and I readily obtained possession of as many as I wanted. I had at first no other object in view than that of obtaining dogs of great excellence; but within a very short time some facts came under my observation which very strongly arrested my attention. In several instances young and wholly inexperienced dogs appeared very nearly as expert in finding Woodcocks as their experienced parents. The woods in which I was accustomed to shoot did not contain Pheasants, nor much game of any other kind, and I therefore resolved never to shoot at anything except Woodcocks, conceiving that by so doing the hereditary propensities above-mentioned would become more obvious and decided in the young and untaught animals; and I had the satisfaction, in more than one instance, to see some of those find as many Woodcocks, and give tongue as correctly, as the best of my older dogs.

Woodcocks are driven in frosty weather, as is well known, to seek their food in springs and rills of unfrozen water, and I found that my old dogs knew about as well as I did the degree of frost which would drive the Woodcocks to such places; and this knowledge proved very troublesome to me, for I could not sufficiently restrain them. I therefore left the old experienced dogs at home, and took only the wholly inexperienced young dogs; but to my astonishment, some of these, in several instances, confined themselves as closely to the unfrozen grounds as their parents would have done. When I first observed this I suspected that Woodcocks might have been upon the unfrozen ground during the preceding night, but I could not discover (as I think I should have done had this been the case) any traces of their having been there; and as I could not do so, I was led to conclude that the young dogs were guided by feelings and propensities similar to those of their parents.

The subjects of my observation in these cases were all the offspring of well-instructed parents, of five or six years old, or more; and I thought it not improbable that instinctive hereditary propensities might be stronger in these than in the offspring of very young and inexperienced parents. Experience proved this opinion to be well-founded, and led me to believe that these propensities might be made to cease to exist, and others be given; and that the same breed of dogs which displayed so strongly an hereditary disposition to hunt after Woodcocks, might be made ultimately to display a similar propensity to hunt after Trufles; and it may, I think, be reasonably doubted whether any dog having the habits and propensities of the Springing Spaniel would ever have been known, if the art of shooting birds on wing had not been acquired.

I possessed one young Spaniel, of which the male parent, apparently a well-bred Springing Spaniel, had been taught to do a great number of very extraordinary tricks (some of which I previously thought it impossible that a dog could be made to learn), and of which the female parent was a well-taught Springing Spaniel; and the puppy had been taught, before it came into my possession, a part of the accomplishments of its male parent. This animal possessed a very singular degree of acuteness and cunning, and in some cases appeared to be guided by something more nearly allied to reason than I have ever witnessed in any of the inferior animals. In one instance I had walked out with my gun and a servant, without any dog, and having seen a Woodcock, I sent for the dog above-mentioned, which the servant brought to me. A month afterwards I sent my servant for it again, under similar circumstances, when it acted as if it had inferred that the track by which the servant had come from me would lead it to me. It left my servant within twenty yards of my house, and was with me in a very few minutes, though the distance which it had to run exceeded a mile. I repeated this experiment at different times, and after considerable intervals, and uniformly with the same results, the dog always coming to me without the servant. I could mention several other instances, nearly as singular, of the sagacity of this animal, which I imagined to have derived its extraordinary powers in some degree from the highly cultivated intellect of its male parent.

I have witnessed, within the period above mentioned, of nearly sixty years, a very great change in the habits of the Woodcock. In the first part of that time, when it had recently arrived in the autumn, it was very tame; it usually chuckled when disturbed, and took only a very short flight. It is now, and has been during many years, comparatively a very wild bird, which generally rises in silence, and takes a comparatively long flight, excited, I conceive, by increased hereditary fear of man.

I procured a puppy of a breed of Setters, which had, through many generations, been employed in setting Partridges for the flight net only, and of whose exploits I had heard many very extraordinary accounts. I employed it as a pointer in shooting Partridges; and for finding coveys of those birds in the open field, I never saw its equal, or in its manner of setting them; but it would never set its game amongst brakes or hedge-rows. Whenever it found a bird in such a situation, it invariably sat down, in the same attitude, and alternately looked into the bush and at me, seeming to think that setting Partridges in such situations was not a part of its duty.

It is well known that very young Pointers, of slow and indolent breeds, will point Partridges without any previous instruction or practice. I took one of those to a spot where I had just seen a covey of small Partridges alight in August, and amongst them I threw a piece of bread to induce the dog to move from my heels, which it had very little disposition to do at any time, except in search of something to eat. On getting amongst the partridges and perceiving the scent of them, its eyes became suddenly fixed, and its muscles rigid, and it stood trembling with anxiety during some minutes. I then caused the birds to take wing, at sight of which, it exhibited strong symptoms of fear, and none of pleasure. A young springing Spaniel, under the same circumstances, would have displayed much joy and exultation, and I do not doubt but that the young Pointer would have done so too, if none of its ancestry had ever been beaten for springing Partridges improperly.

The most extraordinary instance of the power of instinctive hereditary propensity, which I have ever witnessed, came under my observation in the case of a young dog of a variety usually called Retrievers. The proper office of these dogs is that of finding and recovering wounded game, but they are often employed for more extensive purposes, and are found to possess very great sagacity. I obtained a very young puppy* of this family, which was said to be exceedingly well bred, and had

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* It was only one month old when it came into the author's possession.

been brought to me from a distant county. I had walked up the side of the river which passes by my house in search of Wild Ducks, when the dog above mentioned followed me unobserved, and contrary to my wishes, for it was too young for service, not being then quite ten months old. It had not received any other instruction than that of being taught to bring any floating body off a pond, and I do not think that it had ever done this more than three or four times. It walked very quietly behind my gamekeeper upon the opposite side of the river, and it looked on with apparent indifference whilst I killed a couple of Mallards and a Widgeon, but it leaped into the river instantly upon the gamekeeper pointing out the birds to it, and it brought them on shore, and to the feet of the gamekeeper, just as well as the best instructed old dog could have done. I subsequently shot a Snipe, which fell into the middle of a large nearly stagnant pool of water, which was partially frozen over. I called the dog from the other side of the river and caused it to see the Snipe, which could not be done without difficulty: but as soon as it saw it, it swam to it, brought it to me, laid it down at my feet, and again swam through the river to my gamekeeper. I never saw a dog of any age acquit itself so well, yet it was most certainly wholly untaught. I state the circumstances with reluctance, and not without hesitation, because I doubt whether I could myself believe them to be well founded upon any other evidence than that of my own senses: the statement is nevertheless most perfectly correct.

I could add an account of a great many more experiments and observations which were made with other varieties of dogs and upon other species of animals, but as all the facts which I have noticed are confirmations of the truth of the conclusions which I have drawn from those above stated, I shall state the result of one other experiment only, and that solely because it tends to establish a fact which appears to me to be of a good deal of importance.

I stated in a communication to this Society many years ago, "upon the Comparative Influence of the Male and of the Female Parent upon the Offspring of some Species of Animals," that in cases where nature intended the offspring to accompany its parent in flight at an early age, the influence of the parent of one sex upon the form of the offspring differed very widely from that of the other parent, and that when the female parents were of small size and of a small breed, and of permanent habits, and the male of a large size and large breed, and of permanent habits, the length of the legs of the fœtus were given by those of the family of the female parent. I imported some Norwegian Pony Mares with the intention of obtaining cross-bred animals between them and the London Dray Horse; having satisfied myself that the experiment might be made without danger or injury to the smaller animal. The bodies and shoulders of the cross-bred animals which I have obtained are excessively deep, comparatively with the length of their legs, which remains unchanged, except that the joints, being greatly larger, on account of the greatly increased strength of the legs, and being of the same form, necessarily occupy a little more space. The strength of these animals appears to be very great; I believe that they will prove capable of drawing, particularly up-hill, as heavy weights as the London Dray Horses, provided that they be made to draw from a proper level; and I am quite confident that they will prove capable of bearing much more long-continued labour and living upon much less food.

The hereditary propensities of the offspring of the Norwegian Ponies, whether full or half bred, are very singular. Their ancestry have been in the habit of obeying the *voice of their riders* and not the bridle, and the horse-breakers complain, and certainly with very good reason, that it is impossible to give them what is called a mouth; they are nevertheless exceedingly docile, and more than ordinarily obedient where they understand the commands of their master. They appear also to be as incapable of understanding the use of hedges as they are of bridles, for they will walk deliberately, and much at their ease, through a strong hedge; and I therefore conclude that the Norwegian horses are not in the habit of being restrained by hedges similar to those of England.

The male and female parent appear to possess similar powers of transferring to their offspring their hereditary feelings and propensities, except in cases where mule offsprings are produced. In such cases, I think that I have witnessed a decided prevalence of the power of the male parent. The organization of the Mule, which is obtained by cross-breeding between the Horse and the Ass, is well known to be regulated to a much greater extent by the male than by the female parent; and its disposition is, I have some reason to believe, to a very great extent, given by its male parent. I have noticed this in the Mule which is the offspring of a female Ass. I have seen a few only of these animals, but those which I have seen presented the expression of countenance of the Horse, and were perfect horses in temper, and perfectly without the sullenness and obstinacy of the more common Mule. The results of such violations of the ordinary laws of nature appear to be very various in different species of animals, and I should not here have introduced the subject, but that the characters of mules have in many instances misled the judgement of physiologists in their estimates of the comparative influence in ordinary cases of the male and the female upon the offspring.

Whenever I have obtained cross-bred animals by propagating from families of dogs of different permanent habits, the hereditary propensities of the offspring have been very irregular, sometimes those of the male, and at other times those of the female parent being prevalent; and in one instance I saw a very young dog, a mixture of the Springing Spaniel and Setter, which dropped upon crossing the track of a Partridge, as its male parent would have done, and sprang the bird in silence; but the same dog having within a couple of hours afterwards found a Woodcock gave tongue very freely, and just as its female parent would have done. Such cross-bred animals are, however, usually worthless, and the experiments and observations which I have made upon them have not been very numerous or interesting.

XXI. *On the Elementary Structure of the Muscular Fibre of Animal and Organic Life.*

By Frederic C. Skey, *Esq. F.R.S. Assistant Surgeon to St. Bartholomew's Hospital.*

Received June 8,—Read June 15, 1837.

THE volume of the Transactions of the Royal Society for the year 1817 contains a paper which formed the subject of the Croonian Lecture for that year by Sir Everard Home, in which he endeavours to prove the identity of the muscular filaments with the globules of the blood.

To the above paper is appended a plate which exhibits an ultimate muscular filament composed of a string of globules, marked by lateral indentations corresponding to each globule.

It was inferred by Sir Everard Home, from the experiments of Mr. Bauer, which appear to have furnished the material for the paper, that an ultimate muscular filament consists of a string of globules of the blood, estimated by Capt. Kater at the diameter of about the $\frac{1}{5000}$th part of an inch.

This opinion of the composition of muscular fibre, which has been, according to Dutrochet, confirmed by the authorities of France, viz. Beclard, Edwards, Prevost, Dumas, and himself, was first opposed by Messrs. Hodgkin and Lister, whose researches on the subject were published in the year 1832, in an Appendix to the translation by the former gentleman of Dr. Edwards's work, De l'Influence des Agens Physiques sur la Vie.

These authorities were the first to deny the existence of a globular structure, and to assert the uninterrupted continuity of the component parts of the fibre. They proceed to point out a most important distinction "between the minute structures of the muscles of voluntary motion and those of organic life." The former, they assert, "are characterized by innumerable very minute, but clear and fine parallel lines or striæ, which cross the fibre transversely." These they conceive to be the distinguishing feature of true muscle. I shall have occasion again to refer to the valuable though brief observations of Messrs. Hodgkin and Lister on the subject of the muscular fibre of organic life.

Having had the opportunity afforded me, by the kindness of my friend Mr. Henry Goadby, of testing the truth of the opinions of these eminent physiologists by the aid of his admirable microscope, I beg to lay before the Society the results of some inquiries confirmatory of these opinions, and to add some new facts which I hope may be not uninteresting to physiological inquirers.

The microscope which I have employed is an achromatic instrument, possessing a magnifying power of about 600 diameters, by which the objects are exhibited with remarkable distinctness. In order, however, to accomplish this with the best effect, I have invariably submitted the object to a careful preparation under a smaller microscope, used by Mr. Goadby in his able dissections of the anatomy of insects, on the field of which a minute portion of perfectly recent muscle is placed, which has been detached from the mass by means of a lancet, or a fine pair of curved scissars. This, placed on a slip of glass and immersed in water, is observed to consist of many small fasciculi, which may be separated from each other by two pairs of delicate forceps. Of these fasciculi one may be retained, and laid out on the glass with as little violence done to its natural structure as possible.

The connecting medium of the whole of the fasciculi, is a finely reticulated cellular tissue, the tenacity of which is *great* in proportion as the muscle is *fresh*; but it may be at all times divided with scissars without injury either to the form, or to the arrangement of the layer of fibres to be submitted to examination with the larger microscope.

The residue reduced by dissection to a nearly diaphanous state will consist of a single layer of ultimate muscular *fibres*; and of these the object thus prepared may contain from twenty to thirty placed parallel to each other, occupying for the most part the same plane, and straight in direction.

In obedience to the recommendation of Proschaska, and indeed of most physiologists who have made muscular fibre the subject of minute examination, I have subjected the objects I have employed, to boiling and maceration. The result of which has been an increased conviction of the superiority of the perfectly recent fibre. The effect of boiling is that of softening to a considerable degree the cellular tissue, which breaks down readily under the instrument employed, and consequently, the easy separation of the fibres from each other. This I conceive to be a positive objection, inasmuch as a large quantity of cellular tissue retains its connection to the individual fibres, from which, in consequence of its unnatural softness, it cannot be disengaged. Each fibre therefore presents a woolly appearance, and is comparatively indistinct; whereas by the aid of the dissecting microscope, with a little careful manipulation, all the superfluous cellular tissue may be removed from the recent fibre that has not been subjected to this process, in consequence of its unimpaired tenuity of texture, and the fibre exhibits in a very striking degree its natural characters.

Nor could I concur in the recommendation of Proschaska: "Tum lacertus inter duos digitos teritur leniter ac premitur donec mollis et pulposus quasi evadat lacertus." On the contrary, I believe the less violence of any kind employed the better. By a coarse manipulation the fibres may be rendered zigzag or serpentine, but their natural direction I believe to be straight.

Each fibre is connected with its fellow by cellular membrane still finer than that which connects the smaller fasciculi, and so transparent when recent, as not to impair the distinct view of the fibre itself when clearly in focus. If this cellular connection be lacerated, the fibres are drawn asunder and become distorted.

The muscular fibres of animal life, possess a very varying diameter, but their average size, and that which largely predominates, may be stated at about $\frac{1}{400}$th of an inch; but they may be found of all magnitudes, from the $\frac{1}{250}$th to the $\frac{1}{700}$th of an inch. If the object be separated from the mass with a pair of scissars, the extremities of many of the fibres will be compressed and closed, but they retain their natural diameters up to the extremity, if separated with a lancet or knife; this therefore is preferable.

When the object is clearly in focus circular striæ are exposed, crossing the fibre along its whole length (Plate XVII. fig. 1. *a*.). These were known to and delineated by Leuwenhoek, Muys, Proschaska, Fontana, and others. By the former eminent physiologist they were delineated but coarsely, and as existing at irregular intervals from each other; and judging from the plate by Proschaska, they were but imperfectly known to him; yet he has devoted a large portion of one entire chapter to their description. Fontana's work, however, "Sur les Poisons," contains a beautiful representation of the transverse striæ, which both for correctness and for effect cannot well be surpassed. They are seen more or less distinctly in the fibre of animal life in all the examples I have examined, but most distinctly in that of the Ox, the Hog, the Ichneumon Fly, and the *Blatta Americana* (Cockroach). In the two latter they form prominent and elevated bands, resembling in their magnified form the rings of the human trachea (fig. 2. *a b*.). Straus states that the muscular fibre of the *Melolontha vulgaris* (Cockchafer) is similarly serrated.

The transverse striæ are placed closely together, but varying much in thickness and in number, a portion of the length of the fibre equal to its diameter containing from 16 to 25. They sometimes appear uninterrupted in their course across the fibre, and occasionally exhibit the appearance of shorter interrupted lines, which, surrounding it, present the aspect of a cylinder of a polygonal form (fig. 1. *b*.). In the plate of Fontana the striæ are represented of each variety in the same object.

I believe this appearance, which is both frequent and regular, to arise from violence to the fibre, and to be neither natural to its structure nor dependent on optical deception. This arrangement has given rise to the opinion by Proschaska, that the muscular fibres were polyhedral cylinders. It must be observed, however, that this is not an uniform appearance, but that the transverse striæ are more generally arranged in continuous and uninterrupted circular lines around each ultimate fibre. I have remarked that if great care be taken in the preparation of the object while under the dissecting microscope, this broken arrangement is rarely visible; and considering the improbability of the co-existence in the *living fibre* of both the series described, as inconsistent with the simplicity of nature, and the impossibility of converting by any manipulation the *in*terrupted into the continuous and *un*interrupted striæ, I cannot doubt but that this apparently angular arrangement is due to so many artificial depressions of a mutilated *fibre*.

The regularity of the appearance I conceive to be produced by the connection which subsists between the circular striæ and the longitudinal filaments beneath them, the latter being connected together in bands around the tube of the fibre, each band containing about eight or ten filaments, and the appearance of an angular arrangement of the striæ is produced by the partial separation of these portions of the fibre from each other. The uniformity of this separation, of which each fibre is susceptible, appears to warrant its subdivision into these bands, which I propose to name "*Fibrillæ*," these again being subdivided into "*filaments*."

If a fibre be partly unravelled, this irregular and interrupted appearance of the striæ will be rendered still more apparent (fig. 3.).

Of the anatomists whose names I have mentioned, Proschaska has given the most minute description of the transverse striæ, and yet judging from the plates attached to his work, "De Carne Musculari," he must either have seen them with inperfect microscopic powers, or the delineations by the artist have done injustice to his descriptions.

It is, perhaps, somewhat remarkable that the striæ are not seen with equal distinctness, in all the muscular fibre of animal life. When distinct, they present themselves in the form of well-defined rings, the extremities of which may be distinctly traced, encircling the fibre equidistant from each other, uniform in diameter, and apparently elevated from its surface into ridges, leaving depressions between them; and when a fibre is sufficiently bent to render its convex edge somewhat tense, they very apparently stand out from the plane of the fibres, forming circular ridges around it, and presenting the appearance of a fine serrated edge. When very large they occasionally form distinct bifurcations or loops, but pursue their course with the utmost regularity.

Although the striæ exhibit the character above described, of elevated rings crossing the fibre, they present in different examples some variety in appearance. For the most part the dark lines are narrower than the light which alternate with them. Sometimes the dark appear elevated, the light or colourless striæ forming the depressions. At other times this appearance is reversed, and the elevated striæ appear to be formed by the *intervals between the darker lines.*

It is not easy to determine the question by tracing these to the margins of the fibre, because the entire fibre is not in focus at the same time, and the slightest movement of the field of the microscope distracts the eye from the point of observation. After adopting various modes of inquiry which led to no satisfactory conclusion, accident ultimately convinced me that the opinion I at first entertained was erroneous. I obtained a fibre, torn in the longitudinal direction, in which it was evident that the lines of separation corresponded uniformly with the dark striæ, the light, although distorted from their straight direction, remaining unbroken, and pursuing a distinctly continuous course across the fibre (See Plate XIX. fig. 5.).

I infer, therefore, that the light are the *elevated* striæ, and the dark intervening lines, the depressions. After five years immersion in spirit I find them as distinct and well defined as in the recent fibre, and I am informed by Mr. Owen that they remain perfectly unaltered in muscular fibre that has been immersed in spirit since the period of Mr. Hunter.

With respect to their use Proschaska says, "Nil aliud sunt quam profundiora vestigia a vasis, nervis et filis cellulosis, fibram circumdantibus, et ejus vaginam perreptantibus impressa," and Fontana adopts this opinion of Proschaska, while the plates of the two authors bear but a very remote resemblance to each other.

I conceive the arrangement of the transverse striæ to be much too uniform to warrant the explanation of Proschaska and Fontana, for they are not grooves but positive elevations on the fibre. Nor are they invariably found on the muscular fibre of animal life, of which according to the views of Proschaska and Fontana they ought to be the invariable attendants, and with one exception never on that of organic life; besides which, as I shall afterwards endeavour to prove, they are three or four times smaller than the globules of the blood themselves, and consequently cannot be destined to the transmission of blood vessels. They appear to hold some relation rather to the integrity of the fibre.

In the Pharynx the size of the fibres varies from the $\frac{1}{700}$th to the $\frac{1}{300}$th of an inch in diameter, and here is exhibited the greatest variety in the circular striæ. They are invariably large as the fibre is small, while the broader fibres, exceeding greatly the average diameter of $\frac{1}{400}$ of an inch, exhibit the most delicate pencilling and as minute as the eye can detect. I have once observed them varying in size on the same fibre (Plate XVII. fig. 4.).

Is it probable, therefore, that they are destined to the purpose of conveying vessels or nerves, or that they are mere cellular threads? Throughout the general system of animal life, and except in the Pharynx, the circular striæ are most prominent in the large and well-formed fibre, the completeness and integrity of which is its most characteristic feature.

If a portion of muscle, which has degenerated by disease and consequent inaction, be submitted to observation, it will exhibit the outline of the fibres *without any trace* of the striæ or longitudinal filaments; little, indeed, remains beyond the mere form of the fibre. I have examined the gastrocnemeus and soleus muscle of a person for many years bedridden, in which these muscles were wasted to a whitish mass, little exceeding in diameter that of their own tendons.

The striæ appear to bind together the united strands of the fibre, retaining them in position around the cylinder; they are the woof to the warp of the longitudinal filaments, but instead of being interlaced with them they form circles around, and attached to the most prominent part of the longitudinal filaments to which they are intimately united.

The Filaments or Longitudinal Striæ.

I have retained the name of *fibre* to that division of a fasciculus, which though extremely minute, is apparent to ordinary vision.

But each fibre is a compound structure, and is surrounded externally by the circular striæ I have above described. A fibre may be reduced to its apparent elements by a successful manipulation, which will exhibit its ultimate structure, composed of a series of longitudinal lines or *filaments*, placed parallel and in close apposition to each other, around the axis of the tube of the fibre. These are the ultimate known filaments of muscular texture, and of which each fibre of the diameter of $\frac{1}{400}$th of an inch contains from 90 to 100.

They may occasionally be separated from each other, forming a sort of tasselled or brush-like extremity of the fibre they compose. Their diameter I conceive to be about the $\frac{1}{16000}$th part of an inch (Plate XVII. fig. 3. *c c*.).

I have examined these filaments with great care, and with a magnifying power, nearly 200 times greater than that employed by Sir E. Home and Mr. Bauer, and I am compelled to differ from these gentlemen in favour of the opinion, first promulgated by Messrs. Hodgkin and Lister, that they are *un*interrupted threads or cylinders, and neither composed of the globules of the blood, nor possessing even a globular arrangement.

I have carefully compared a filament magnified by 600 diameters with the plate by Sir E. Home in the Transactions of the Society, and I find that neither the human filament nor that of any animal in which I have observed it, is nearly so large nor so distinct as that represented in the above plate. Yet Mr. Bauer's magnifying power was 200 diameters less than that of Mr. Goadby's which I employed. Beclard, M. Edwards, Prevost, Dutrochet, and Dr. Grant, have adopted this view first promulgated by Sir E. Home. Fontana, who has delineated the fibre of muscle so accurately, and who applied a single lens of $\frac{1}{90}$th of an inch focus, asserts them to be cylinders, *hollow* or *solid*, and only occasionally presenting a globular appearance.

It should be particularly observed that the circular striæ which surround each fibre are closely adherent to the most projecting surface of each longitudinal *filament*. These latter, when detached into separate shreds, occasionally exhibit on their surface, the *marks* or *indentations corresponding* to the *distance* between the *circular striæ* on the whole fibre (fig. 3. *d d*.), and I think the filament will present the more or less distinct appearance of a globular structure in proportion to the distinctness of the circular striæ.

In the Haddock and the Cod, the *fibres* of which are very large, and in which the circular striæ are of extreme beauty and delicacy, the ultimate filaments present no appearance of a globular arrangement, but are distinctly continuous and uniform throughout their whole length.

Probably the best test to which they can be submitted is that of placing the globules of the blood and some muscular filaments, under the field of the microscope at the same time. When subjected to this mode of inquiry, the filaments will be observed to be excessively minute, and the globules of the blood may be seen floating between and behind the different *fibres*, in the apparent breadth of about twelve to a single fibre, and from three to four times larger than the reputed globules of Sir E. Home and Mr. Bauer (Plate XVII. fig. 5.).

I have counted on making a successful division of a fibre about 100 filaments, the number mentioned by Leuwenhoek, somewhat less than the half of which were in focus at the same time, those of the opposite side being brought into view, by a new adjustment of the microscope. A single globule suspended behind a separated fibre, would correspond to the breadth of about three filaments.

Now the estimate of the diameter of a globule of blood by Dr. Wollaston and Captain Kater is the $\frac{1}{5000}$th part of an inch, from which the above calculation does not materially differ. A more recent admeasurement by M. Edwards* gives them a diameter of $\frac{1}{325}$th of a line. A single muscular fibre has a diameter of $\frac{1}{400}$th of an inch. A single globule of blood, which is about the twelfth part of the breadth of a fibre, $\frac{1}{4800}$th of an inch. If each fibre contain 100 filaments, something less than the transverse breadth of the fibre, or forty, or allowing for the receding margins multiplied by 400, is 16000, which is the breadth of a single filament. A globule of blood to the diameter of a filament is therefore as 4800 to 16000. If this calculation make any approach towards truth, the filaments cannot be composed of the globules of the blood, and they are not identical.

I believe the appearance of globules, of which the filaments are asserted to be composed, is due to the delicate indentations of the transverse striæ upon them, for the distinctness of the globular appearance is always proportionate to that of the transverse striæ. I was, therefore, very desirous of examining the appearance in the fibre of an animal characterized by delicacy of the striæ, and I found that in the Cod and the Haddock, in which they are most minute, the filaments being disencumbered of their connection to the cross bands or striæ, pursue their course floating and twisted in all directions, without a trace of a globular appearance or mark of any kind, cylindrical, and of uniform thickness throughout.

Glutinous interior of the fibre.

The interior of each fibre appears to contain a glutinous semitransparent substance, covering thickly the inner surface of the longitudinal filaments. It is very soluble in water, and when the end of a fibre is broken up, exhibiting its filamentous structure, no trace of this substance is seen, but it is apparent on the internal surface of each fibre when the tube is exposed. It is this glutinous coating to the interior of the tube, that conceals from view in a degree the long filaments of the opposite surface, when that part of the fibre is brought into focus.

Tube of the fibre.

The divided extremity of each fibre presents the appearance of a jagged circle terminating an apparently hollow tube. For the most part these extremities are con-

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* Encyclopedia of Anatomy and Physiology.

tracted, and are occasionally elongated even to a point. Frequently, however, the fibre retains its natural diameter up to its termination in the jagged circle (Plate XVII. fig. 1. *c.*). This part of the fibre will occasionally exhibit an orifice, such as would appear by the foreshortening of a tube cut obliquely. When a section of the fibre is made in the vertical direction, this appearance is not observed.

It is difficult to obtain a distinct view of the tubular end of a fibre by any careful preparation of the object, but *without* such preparation several fibres in the same object, may exhibit its tubular character. If such a fibre be brought into focus at its extremity, the circular striæ and longitudinal filaments will be exposed, extending to the *near* margin, and if the depth of the fibre be then *penetrated* by the microscope, the longitudinal filaments of the opposite side will be *first* exposed, and secondly, the circular striæ, but neither of these will be distinct, being obscured by the glutinous lining of the interior of the tube. A careful adjustment will thus detect the aperture of the tube of the fibre, which appears in the form of a hollow cylinder, perfectly translucent in its centre, but less so at its sides from their vertical direction to the plane on which they rest. The latter present when in focus the dark outline of the fibre, extending along its length.

If a single fibre be divided in the *longitudinal* direction its cavity may be exposed along a considerable length, the filaments composing the fibre with their investing striæ of the opposite side of the cylinder may then be seen when the near side is out of focus (Plate XVIII. fig. 1. *a.*).

As the tubular character of muscular fibre is not always distinctly apparent, I would add the following arguments in favour of this view of their composition.

1st. A fibre is frequently elongated to a point, up to the extreme external surface of which, the circular striæ are apparent. If the fibre be a solid cylinder, what becomes of the central substance? for it is evidently the external surface that is so attenuated, indicated by the presence of the circular striæ.

2nd. When a fibre is entirely separated into its filaments forming a fringe-like extremity, that surface of the fibre nearest the eye, forms all that portion of the fringe which is distinctly in focus. If the focus be then changed the fringe of the *opposite* side is brought into view, but there is no middle fringe to complete what would *then* be, a solid tassel (Plate XVII. fig. 3.).

3rd. If a few fibres be placed on glass and dried, little remains apparent, beyond the black outline of each fibre, their central portions become obliterated, and consequently the fibre is transparent. If the margin of the fibres are rendered dark by their perpendicularity to the plane below them, *à fortiori*, the *middle* portion of the fibre ought to exhibit the same phenomenon, for it is higher from the surface and consequently thicker.

4th. The separation of a few or more filaments from the body of a fibre, never exhibits a second layer of filaments beneath them. This view of a central filament might reasonably be expected if each fibre were composed of a solid cylinder; and it would be interesting to ascertain the relation which subsists between the central filaments, supposing the fibre to possess them, and the transverse striæ.

I have never seen any appearance like that of filaments projecting from the interior of the fibre at its extremity; for although the exhibition of the tubular character may be rare, involving as it does many conditions, yet it is not unreasonable to imagine, that if the fibre were solid, the extremities of the central filaments would be occasionally as apparent, as those which are arranged on its external surface.

5th. Analogy to other structures would enable us in some degree to comprehend the utility of the circular striæ, supposing them to surround a tube which they probably compress in certain states of its action.

Are the *filaments* like the fibres which they compose *tubular*?

Up to a late period of my inquiries into this subject, I had only the ground of analogy to support the opinion of the tubular character of the filaments, but being engaged in examining the muscular coat of the trachea of a Horse, I was not a little gratified to observe the very apparently tubular composition of these threads, one of which, indeed, placed at right angles to the plane below it, exhibited its cavity to some distance within (Plate XVIII. fig. 2.). Indeed the filaments presented very much the aspect of miniature fibres, in which I could almost fancy I saw some traces of still minuter threads. This though speculative is, I think, not very improbable; but of the tubular nature of these delicate threads, I have no doubt; they were distinctly perceptible to many observers.

From the above I deduce,

That the human muscular fibres of animal life possess an average diameter of $\frac{1}{400}$th of an inch.

That they are surrounded by circular striæ varying in thickness and in number.

That the striæ are actual ridges or elevations on the fibre, leaving depressions between them, considerably smaller than the globules of the blood.

That each fibre is divisible into bands or fibrillæ, which, composed of many ultimate filaments are arranged in parallel longitudinal lines around the axis of the fibre, and that the partial separation of these fibrillæ produces the occasional broken or interrupted appearance of the circular striæ.

That each band or fibrilla is subdivided into filaments, of which every fibre of $\frac{1}{400}$th of an inch diameter contains about 100.

That the muscular filaments possess a diameter of about the third part of a globule of the blood, or $\frac{1}{16000}$th of an inch, and that they are tubular, and that these filaments are arranged longitudinally around the tube of the fibre, which finally contains a soluble gluten.

The human fibre of animal life pervades the whole of the external muscles, and all internal muscles connected to any form of tendinous matter. This will include those of the tongue, palate, larynx, and pharynx, with some portion of the œsophagus prolonged from it, and constituting an exception to this rule; the muscles of the orbit and ear, diaphragm, intercostals, levator and sphincter ani. The muscles of the tympanum incased in bone, composed so largely of tendinous matter, and apparently beyond the reach of voluntary power, must however be classed among the muscles of animal life. Yet they are so intermixed with tendon, that had I not rendered myself familiar with the structure and appearance of tendinous fibre, which possesses a remote resemblance to the muscular fibre of organic life, I should have erroneously concluded that they belonged to that class. They possess, however, all the characters incidental to the fibre of animal life.

Organic Life.

The microscopic view of the muscle of organic or involuntary life exhibits a structure essentially different from that of the fibre of the external muscles.

The difference was first made known by Messrs. **Hodgkin** and **Lister**, who state that "the minute fibrillæ which enter into the composition of the fasciculi of fibres of which this tissue is made up, instead of presenting the transverse striæ, are perfectly smooth, and appear to be continued to a considerable length, of nearly uniform width." They describe the fibre as nearly straight and parallel, occasionally interlacing and dividing among themselves.

In the muscular fibre of organic life there are no distinct and separable fibres, no transverse striæ, with one exception, and no appearance of the larger tubes.

This tissue appears to consist of a series of irregularly disposed lines of various thickness, taking for the most part a longitudinal direction, and forming a kind of untraceable net-work difficult of delineation. Although there exist no single fibres connected by cellular tissue with others around it, yet there is no difficulty in observing the direction of the muscle; for the lines take one course, frequently, however, bending to one side and uniting with others around: but the aggregate, though far from straight, pursue one general longitudinal direction (Plate XVIII. fig. 3.). The cut margin of the object exhibits no projecting fibres, which in the process of preparation have started out from their connection with others, or which have evaded the straight division with the knife; but the whole edge is smooth and uniform. The drawing is taken from the fibre of the small intestine (jejunum).

The muscle of organic life appears to possess a smaller proportion of cellular tissue than that of the voluntary muscles. None is required for the connection of fibres, for in reality there are *no fibres* in the muscle of organic life, which rather consists of filaments interwoven with each other to form the general structure, than arranged in parallel lines around the cylinder of each separate fibre, as observed in the muscular fibre of animal life.

I could imagine it might be artificially imitated by subjecting a thin layer of these latter fibres to a degree of pressure which would destroy the integrity of each fibre, and yet preserve the general direction of its filaments. It may be readily distinguished from tendinous fibre, in which the filaments are uniform in size, pursuing individually one unvarying line, each filament being parallel to those around it. This great regularity in arrangement renders tendinous fibre a microscopic object of singular beauty and delicacy, when it has not been subjected to a coarse manipulation (Plate XVIII. fig. 4. *b.*).

To the general description of the muscular fibre of organic life, the heart forms an important exception (Plate XVIII. fig. 5.). It appears to possess a somewhat compound character of texture. There is a nearer approach to the fibres of animal life, each fibre being more distinct than those of any other internal viscus, and possessing a very delicate pencilling of *transverse striæ,* as observed by Hodgkin and Lister. The fibres are only about one third of the magnitude of the animal fibre of the same subject; they are interwoven with each other, and being more separable than the general fibre of the other organic viscera, project at the cut extremities, where their diameter is very apparent. The net-work which they form is composed of the entire fibre, and not, as in organic life in general, by the filaments of each.

The examination of the pharynx, composed of the fibre of *animal* life, and that of the œsophagus of *organic* life, exhibited some views of considerable interest. This continuous line of tube commences in animal, and ends in organic fibre. I was desirous of ascertaining the nature of the junction, whether by a gradual blending of one description of fibre into the other, or by an admixture of the two.

The constrictor superior, the first agent of deglutition, exhibits the perfect fibre of animal life. The striæ are of ordinary size, of about 24 to the diameter of the fibre.

Those of the constrictor medius exhibit no peculiarity, except that they are strongly marked and distinct; but the cellular tissue is dense, possessing the character of *that* connecting the texture of organic life. The same observations will apply to the constrictor inferior, in which the density of the cellular tissue is yet perhaps greater.

The structure of the first 2 inches and half or 3 inches of the œsophagus is that of animal life, but surrounded with striæ varying much in number and in breadth. The size of the fibres themselves, likewise varies considerably, and may be found from that of the 700th to the 300th of an inch diameter (Plate XIX. fig. 1.). I have generally observed that the *smaller* fibres possessed the *larger* striæ. These frequently appeared to bifurcate in their course around the tube, and at the edge distinctly projected from the surface, forming the serrated appearance I have previously described. The larger the fibre the more delicate are the striæ, which become less and less apparent on the larger fibres, as they descend on the œsophagus. Still the smaller fibre with large striæ may be found as far as the fibre of animal life itself exists, and this junction of the two takes place at about 3 inches from the lower border of the constrictor inferior, where both structures are associated in the same object. One half inch below, and the fibre of animal life ceases entirely, and it is at this precise point that the œsophagus enters the cavity of the chest.

Perhaps the most interesting, as well as the most instructive object exhibiting the muscular structure of organic life, is that of the *arterial system,* the composition of which has presented material of the deepest interest to all physiologists of the last and the present century.

If a portion of the middle coat of an artery, whether of the pulmonary or aortic system, be submitted for examination, it is impossible to distinguish it from the muscular texture of the stomach, intestinal canal, or bladder. It exhibits the perfect composition of the organic muscular texture of these parts (Plate XIX. fig. 2.).

It would be impracticable to determine, with so large a microscopic power as that which this subject demands, the relative proportions of muscular fibre in the larger, compared with that in the smaller arterial tubes; but I have observed that the muscular texture of the smaller vessels, as the internal mammary and the smaller branches of the iliacs, is paler and of a more delicate fabric, but their *relative* proportions could only be appreciated by a different mode of inquiry.

I need hardly state, perhaps, that the fibres are placed circularly around the vessels, and that the muscular, forms the thickest of the coats of these tubes.

I can discover *no resemblance* between the structure of the middle coat of an artery and that of the elastic ligamentous tissues of the body.

If the drawing of the former be compared with that of the muscular fibre of organic life in general, I think it will be found so closely to correspond as to appear almost identical. Possibly the arterial tissue is more delicate, but both apparent composition and arrangement are the same.

I observe, however, no comparison between the arterial tissue and that of the elastic ligamentous structures. These latter are composed of large and distinct filaments placed in a parallel direction, and connected by dense cellular tissue. Each filament possesses its characteristic property of elasticity, and when separated at one extremity from the mass it curls backwards on itself.

The entire structure is likewise more transparent than the arterial tissue, and is much more simple in its arrangement (Plate XIX. fig. 4.).

I have been unable to detect anything approaching to the character of muscular fibre in the structure of the venous system in general. I have observed it, however, in the hepatic veins of the Seal; and it doubtless exists in all animals subject to an arrest of the venous circulation around the heart.

There yet remains a structure in the economy which presents an interest little inferior to that of the arterial system, I mean the iris.

The tenacity of this membrane is greater than that of any other structure I have examined, so much so, as to render it exceedingly difficult of preparation under the dissecting microscope.

When exhibited with the larger power it presents so much the character of the muscular fibre of organic life, that I feel almost inclined to associate it with that system.

As regards the *arrangement,* I have *less doubt* than I have of the *chemical composition* of the iris, which does not possess the semitransparent character of *fibrine.*

Yet there exist some important distinctions, which require considerably more extensive observation than I have hitherto been able to make; and I am anxious not to commit myself by the expression of an opinion hastily formed as to its composition, on which my limited inquiries have hitherto fallen far short of the difficulties of the subject.

It is difficult to explain the experiments of Sir E. Home as regards the muscular texture of the stomach which he employed, and which is of the pure structure of organic life. I have examined each part, and I have been unable to obtain the least trace of animal fibre. The ultimate muscular filaments *may* be seen in the texture of organic, but with by no means the distinctness of animal life, in consequence of its reticulated structure which renders them difficult of separation from the bulk of the fibre.

The muscle of organic life pervades the greater part of the œsophagus, the stomach including that of the *ruminants* and the alimentary canal, the trachea and bronchial tubes, the *uterus*, the urinary bladder, the arterial system, and possibly the iris.

The diameter of all muscular fibres holds a relation to age, being in the human fœtus, as well as in the young of all the animals in which I have observed it, about one third the diameter of that of mature age (Plate XIX. fig. 3.).

On comparing the muscular fibre of animal and organic, or voluntary and involuntary life, it does not appear surprising that there should exist the remarkable variety of structure which I have described. Although both systems are embraced under the general denomination of *muscle*, and possessing the characteristic property of irritability, yet their functions in the economy are so distinct, and the power required by each is so unequal, that we might almost have conceived the existence of an important difference of structure.

In the muscle of *animal* life we find the fibres with their subordinate filaments pursuing a direct course between the attachments of the whole muscle, or deviating from it merely for the purpose of a convenient adhesion to its common tendon. Hence the advantage obtained by a united and cooperating force, by which the whole component fibres of the muscle are called into action at the same time. The fibres possess no independent influence, but all cooperate to one obvious end, that of approximating the extremities of the muscle, and act with a force which, considering the nature of their general adaptation, may well be deemed enormous.

But the power of the muscle of organic life is limited. We find it spread over extensive tubular surfaces of membrane, and contributing to the involuntary functions of internal life, by a slow and gradually extending contraction. It has no antagonist but the contents of the tube it surrounds, its influence on which extends along the surface of the muscle, as the contents descend within the tube.

By means of its matted structure it serves the purpose of a nearly complete investment to the canal it surrounds, while its connecting and reticulated composition enables it at once to transfer the contents of the tube within the influence of the portion prolonged from it, and to communicate the stimulus necessary to their removal.

To this function the heart again forms a striking exception. Its contractions are impetuous, and throughout each division of the organ simultaneous. In the heart therefore we find the modified but *separate* fibre of animal life, with all the physical characters indicating great contractile power, demanded for the important function it is known to possess.

Charterhouse Square,
January 10, 1837.

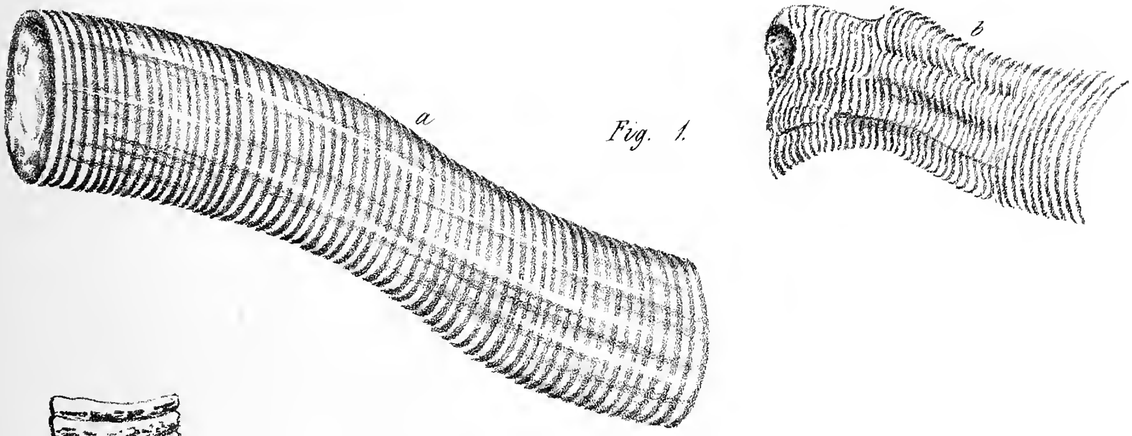
Explanation of the PLATES.

PLATE XVII.

Fig. 1. *a.* An unbroken muscular fibre of animal life, with continuous striæ, magnified about 600 times, linear measurement.
c. Its tubular extremity.
b. A similar fibre broken into fibrillæ, exhibiting the interrupted striæ, and presenting a polygonal appearance.
Fig. 2. *a b.* Muscular fibres of animal life, from the Cockchafer.
Fig. 3. A muscular fibre separated at its extremity into its component filaments.
a, a, a. Striæ continuous across the unbroken fibre.
b, b. The fibre broken into fibrillæ, forming the interrupted striæ.
c, c. Muscular filaments forming a tasselled extremity to the fibre.
d, d. Filaments retaining slight marking of the striæ.
Fig. 4. Circular striæ varying in size on the same fibre.
Fig. 5. *a, a.* Filaments; globules of blood floating behind them, showing their relative diameters.
Fig. 6. Globules magnified 600 times.

PLATE XVIII.

Fig. 1. *a.* A tube cut open longitudinally, magnified 400 times, linear.
b. General arrangement of fibres, magnified 200 times, linear.
Fig. 2. Muscular fibre of organic life from the trachea of a Horse, showing the tubular character of the filaments.
Fig. 3. Muscular fibre of organic life (Jejunum).
Fig. 4. *a* and *b.* Tendon; tendon of Pectoralis major muscle.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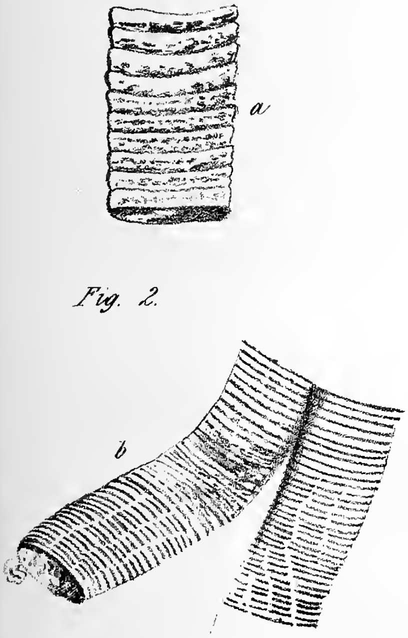
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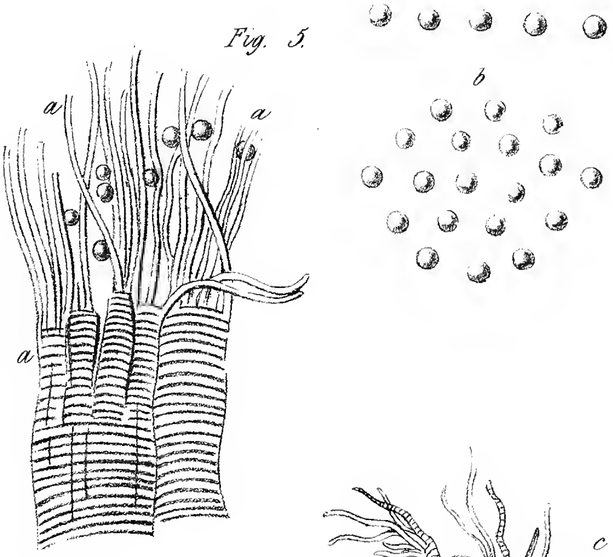
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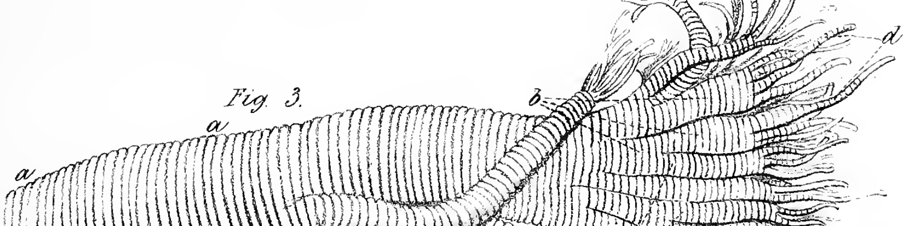
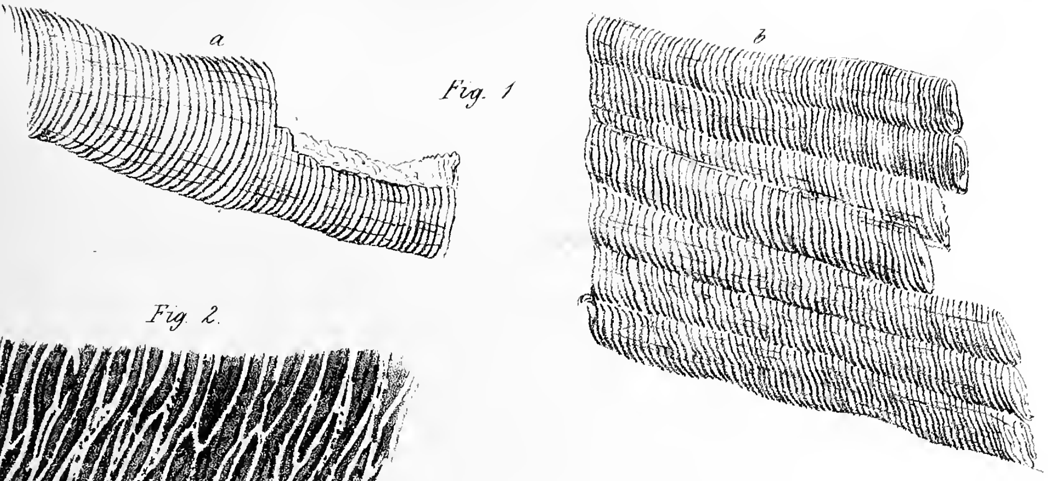


Fig. 3.


Fig. 4.

J. Basire, lith.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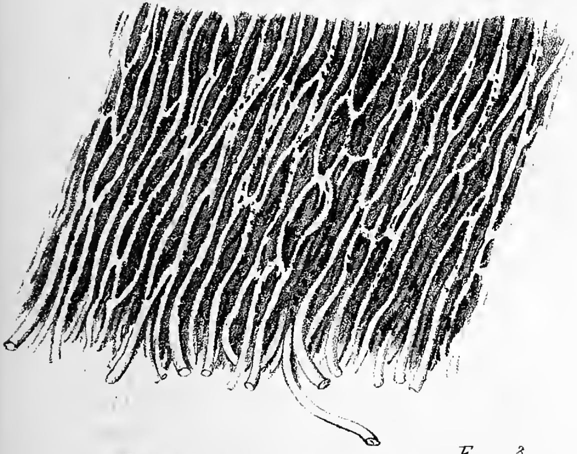

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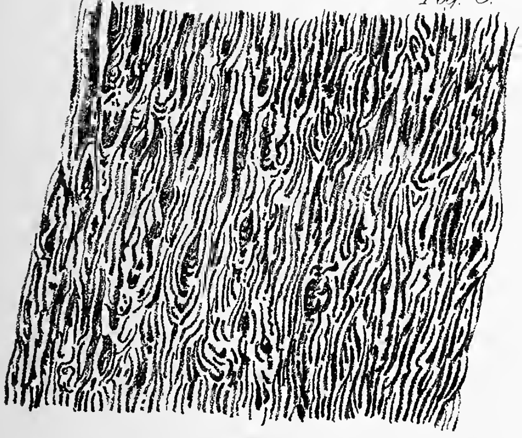

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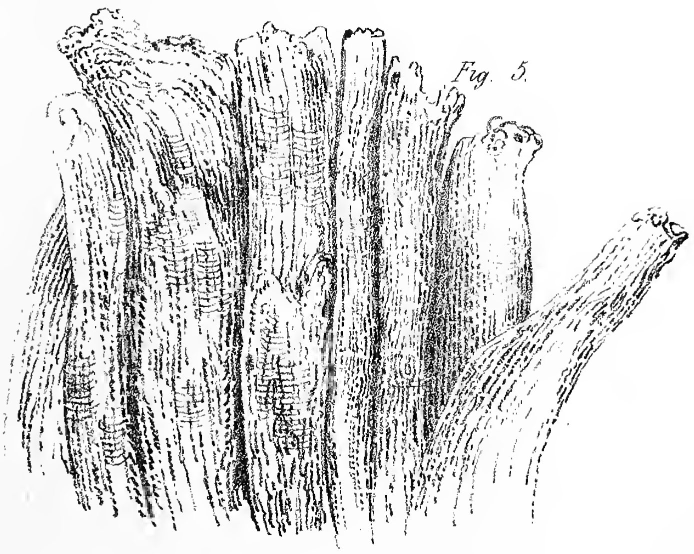

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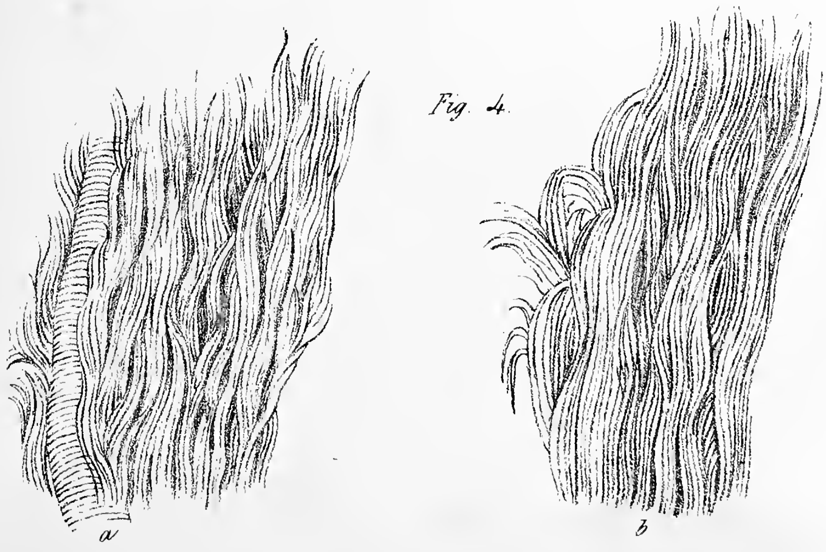

J Basire. lith.

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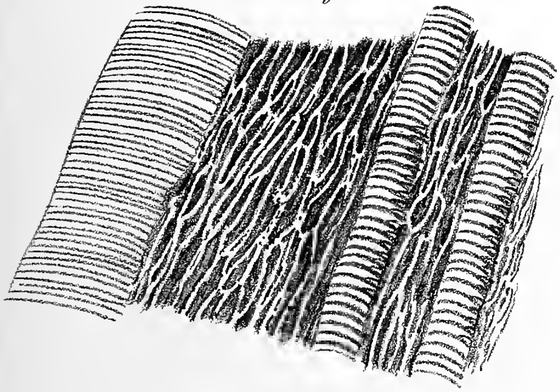

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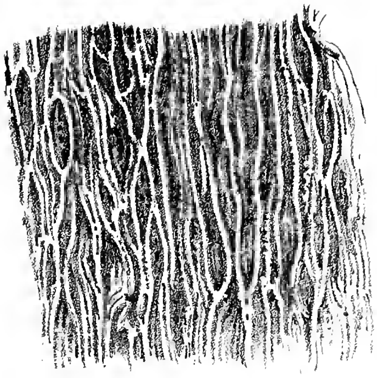

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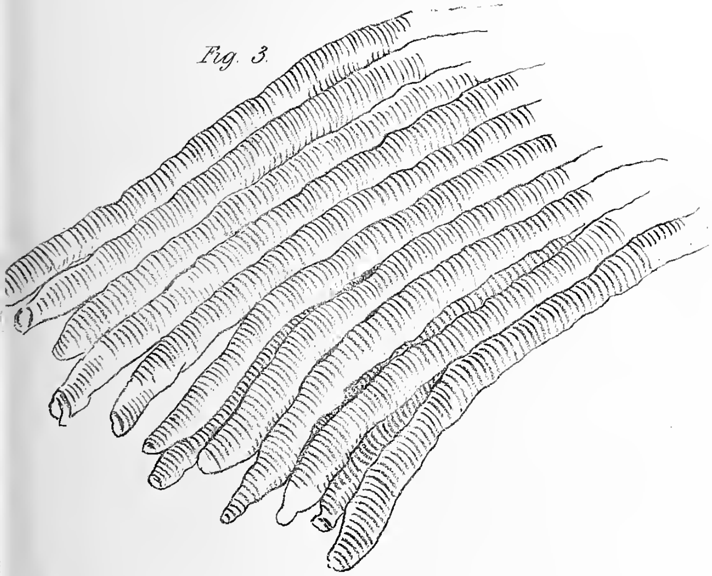

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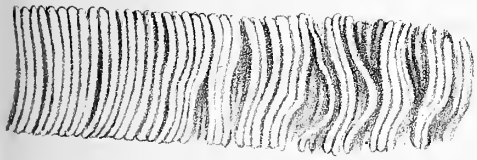

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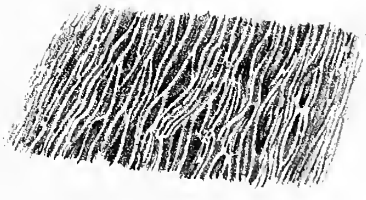

J. Basire, lith.

Fig. 4. *b*. Tendon of Tensor tympani muscle, with a single muscular fibre.
Fig. 5. Heart (human), composed of distinct fibres, with a few circular striæ.

PLATE XIX.

Fig. 1. Muscular fibre taken from the human œsophagus, about three inches below the pharynx, showing the two structures of animal and organic life combined.
Fig. 2. Middle coat of an artery.
Fig. 3. Fœtal fibre of animal life, magnified 300 times, linear.
Fig. 4. Ligamentum nuchæ of the Sheep.
Fig. 5. A fibre of animal life torn longitudinally, exhibiting the separations of the dark striæ, the serrated margins being due to the elevations of the light striæ.
Fig. 6. Middle coat of an artery.

XXII. *Observations on the Minute Structure of some of the higher forms of Polypi, with views of a more Natural Arrangement of the Class. By* ARTHUR FARRE, *M.B. Lecturer on Comparative Anatomy at St. Bartholomew's Hospital. Communicated by* RICHARD OWEN, *Esq. F.R.S.*

Received May 11,—Read June 8th and 15th, 1837.

TO attempt the reformation of any class in the animal kingdom,—the numerous individuals of which are widely spread over the surface of the globe, many therefore difficult of access, and others, though easily obtained, yet extremely perishable, and for the most part so minute, as to require for their examination the utmost penetration of the microscope and unwearying perseverance in the observer—is a task of no little difficulty in the accomplishment, and one that may fairly entitle him who enters upon it to expect to meet with indulgence.

It is probably owing to these retarding circumstances that the class Polypi, as now generally understood, presents such a heterogeneous accumulation of widely differing structures as is perhaps to be found in few similar portions of the animal kingdom: and it is only by a strict investigation of the *intimate structure* of the various forms of animals that have been so indiscriminately heaped together, that any permanent arrangement that shall indicate their true and natural affinities may be hoped for.

The slightest glance at the history of the revolutions which the ideas of naturalists have undergone, with reference to this class since it first became known, will establish the truth of this position, and show the importance of attending to the entire organization of the animal, as far as it can be known, in any attempt at classific arrangement.

It is not wonderful, indeed, that a class of animals to which the name Zoophytes has been so long and universally applied, a name sufficiently expressive of the dubious position which they were supposed to hold in the kingdom of nature, should by the earlier naturalists have been referred entirely to the vegetable or even to the mineral kingdom; and accordingly we find that in the seventeenth century many of these were described as minerals by BOCCONE and GUISON; and by CESALPIN, BAUHIN, LOBEL, TOURNEFORT and RAY as vegetables; the great quantity of earthy materials, produced by many forms of Zoophytes, leading to the former supposition and giving rise to many theories, as to the growth of stones, &c.; whilst the more obvious external characters and habits would, under deficient means of observation, readily favour the latter. And this supposed alliance with the vegetable kingdom seemed to be still further strengthened when, in the commencement of the following century, the animals of some species of coral were described by MARSIGLI as flowers.

This circumstance perhaps more than any other tended to confirm botanists in claiming these bodies for the vegetable kingdom, notwithstanding that it was maintained by chemists that their structure exhibited more of an animal than a vegetable nature, and that even so early as the sixteenth century the animals of several had been distinctly described as such by IMPERATI.

The discoveries and opinions, however, of this observer, who appears to have been the first to ascertain the animal nature of these Zoophytes, as well as the observations of RUMPHIUS made upon many of the living corals in the Archipelago, seem to have been entirely neglected and forgotten; nor does it appear that the botanical theory was disturbed until a similar discovery to that of IMPERATI was communicated to the Acad. des Scien. in 1727 by REAUMUR, founded upon the observations of PEYSSONELL, who maintained that the supposed flowers of MARSIGLI were in fact aggregate animals analogous to Actinia, which latter animal was then, perhaps, the only one of the class to which a vegetable nature was not generally ascribed.

This communication seems to have directed the attention of naturalists more immediately to the subject, and the subsequent discoveries of TREMBLEY of the naked Polypi, in 1740, and the investigations of BERNARD de JUSSIEU, GUETTARD, LÆFLING and DONATI, were greatly instrumental in pointing out the true nature of Zoophytes. But by none was the investigation pursued to so great an extent as by the indefatigable ELLIS, whose systematic work was the first of the kind that appeared upon this subject. In maintaining the entire animality of Zoophytes ELLIS was strongly opposed by LINNÆUS, BASTER, and PALLAS, who still holding an opinion midway between the two that divided naturalists, maintained that they were of a mixed nature, partly animal and partly vegetable.

With LINNÆUS, however, and his contemporaries this view of the subject ceased, and subsequent investigations have completely exposed the fallacy, both of the vegetable and vegeto-animal theories. But the work of ELLIS, as well as that of PALLAS on the same subject, can be considered as but little more than a classification of the more solid, or least perishable, and least important parts, (the part called Polypary by REAUMUR), without reference to the structure of the individual animals, which was then little understood, and was generally supposed to partake in all these cases of the simple nature of Hydra, and they were therefore so called by LINNÆUS.

This mode of classification, by no means likely to lead to a natural arrangement of the subject, was from the same cause adopted in the more recent systems of LAMARK and LAMOUROUX, where the characters of the axis or polypary are again taken as the basis of arrangement; though a considerable advance is made in founding secondary divisions on the structure and form of that part of it, which is immediately inhabited by the individual animals, commonly called the cell. Still, however, from a deficiency of knowledge the most important parts are disregarded, and animals frequently united even in the same *genus* which have not a *classical* relationship.

But Renier and Savigny had already shown that the animals of Botryllus and Alcyonium, Linn., were not, as had been generally supposed, Polypes, but possessed a structure similar to that of Ascidia; whilst the descriptions and figures given by various writers of some of the cortical Polypes showed that these were closely allied to Actinia.

But though some of the larger forms had been thus more accurately investigated, the hundreds of minute species that remained must necessarily have escaped observation, until the more general use of the microscope and the great improvements lately made in that instrument, opened up a wide and almost entirely new field of discovery, which the inefficient instruments of previous investigators had only just enabled them to enter upon.

By this means the currents observed by Spallanzani to be produced by some of these animals, and attributed by him to the action of the arms, were shown by Steinbuch*, by Fleming† in Valkeria, and by Grant‡ in Flustra, to be due to the vibration of cilia, by which the sides of the tentacula were fringed; and to the last-mentioned naturalist we are also indebted for many important observations on the ciliated reproductive gemmules of this family, on the form and growth of the cells, and on the digestive cavity.

It was shortly after discovered by Milne Edwards and Audouin that some of these compound polypes possessed an *anal* as well as an *oral* opening to the alimentary canal; a discovery which Edwards communicated to the Acad. des Sciences in 1828 §, and proposed thereupon to found a division of the class Polypes into different families, according to the forms of the alimentary canal. In this class, however, he also includes Sponges.

A similar discovery was also made about the same time by Ehrenberg, independently of that of Edwards, and was taken by him as the basis of his classification of Polypes ||, dividing these animals into two principal groups, Anthozoa and Bryozoa, according as the alimentary canal has one or two external openings; a division which he has since (1835) modified by separating the Sertulariæ and other hydriform Polypes, which form a third group denominated by him Dimorphæa.

This type of structure, observed shortly after (in 1830) by Thompson¶ in Ireland,

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* Analecten Neuer Beobachtungen und Untersuchungen für die Naturkunde 1802, p. 89, quoted by Dr. Sharpey, Cycl. Anat. art. Cilia, p. 609.

† Mem. of Wern. Soc. Fol. Part V. p. 488.

‡ Edinburgh New Philosophical Journal, vol. iii. 1827.

§ Résumé des Recherches sur les Animaux sans Vertèbres, faites aux îles Chaussay, par MM. Audouin et Milne Edwards. Annales des Sciences Naturelles, t. 15. Sept. 1828; and Recherches Anatomiques, Physiologiques et Zoologiques sur les Eschares, par M. H. M. Edwards. Ib. t. 16. Juillet 1836.

|| Symbolæ Physicæ.

¶ Zoological Researches and Illustrations, Memoir V. Cork, 1830.

apparently without a knowledge of the discoveries of EDWARDS and EHRENBERG, was described by him as indicating a new form of animal, to which he applied the name Polyzoa, to distinguish them from such of the compound animals as partake of the nature of Hydra, and he proposed to elevate them to the class of tunicated Mollusca.

The existence of this type in Flustra has also been since demonstrated by Mr. LISTER, in his paper on "Tubular and Cellular Polypi," communicated to the Royal Society in 1834; which contains also much new and valuable information relative to the economy of the more simple or hydriform Polypes.

The descriptions and illustrations of these last-mentioned observers are in various degrees confirmatory of each other, and are sufficient to indicate in a general way the characters of this more recently discovered form of animal; but the uses of their various organs are often confused and misunderstood, and their minute structure certainly not investigated with that degree of accuracy which it deserves, and which the present state of science demands.

It is with the view of supplying these deficiencies that I am induced to lay before the Society the result of my own observations upon this very interesting portion of the animal kingdom, conceiving that they have been prosecuted to an extent that has not hitherto been effected.

My attention was first particularly directed to the subject in the year 1835, during a short visit to the Isle of Sheppy, for the purpose of exploring the various animal productions, so abundant on that portion of our coast. During this visit the type of structure here referred to came under my notice, and the results of my investigations upon it were then so entirely new to me, that I was induced to repeat these visits at intervals; and upon the specimens thus procured, and also upon similar supplies obtained from the same place, which I have repeatedly received from my friend Mr. BOWERBANK, I have been enabled to continue the investigation beyond the limits that a mere temporary visit to the coast would have enabled me to do.

During the early part of these investigations I was but little acquainted with the observations that had already been made by others upon the subject. But having since been necessarily led to consult these, I find some of my own investigations in various degrees confirmed. Those points therefore that are not new I have either wholly omitted, or touched upon only to the extent that would be necessary to render the subject intelligible.

The facts that I have thought the most interesting and important to be stated are embodied in the descriptions of the various species that furnish the subject matter of the present memoir. Two of these species I believe to be entirely new, and I have ventured to name them accordingly.

A few particulars with regard to the method that I have pursued may not be without their use, though each specimen will frequently require a different manipulation.

A number of glass troughs being at hand (which with the aid of a little cement may be easily constructed of every variety of depth), these should be filled with sea water, and a specimen of the species to be examined placed in each, care being taken to adapt the depth (from side to side) of the trough to the thickness of the specimen, it being very desirable that no more water should intervene between the latter and the sides of the glass than is just sufficient for the purpose.

If the specimens are tolerably recent it will seldom fail that in a short time the animals, which always contract the instant that they are disturbed, will begin to expand themselves, in which state many may be observed by the naked eye, and a very cursory glance under the instrument will then show which are the best adapted for observation.

For this purpose it is necessary that a clear reflected light should be transmitted through the object, care being taken to avoid all artificial light, which is totally inadequate to supply that delicate and perfect definition requisite for the examination of objects so extremely minute as those which form the subject of the present essay. This method, which if rigidly pursued, greatly limits the time during which the investigation may be continued, is yet the only one that can be safely trusted to; and for subjects of this nature I have therefore long ceased to use any other than a clear *daylight**.

The figures which are added in illustration were drawn from the specimens by being previously outlined by the aid of a camera lucida attached to a reserve eyepiece, to allow of its being instantly substituted whenever a favourable specimen should present. By this means a faithful record of appearances is preserved that cannot be equalled, and indeed hardly obtained by other means, although it is scarcely in the power of a drawing to convey an adequate idea of the exquisite beauty of the living objects.

PLATE XX. and XXI.

Bowerbankia densa†, Mihi.

Fig. 1. Found commonly on *Flustra foliacea*, thickly aggregated in masses of half of an inch to one inch diameter.

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* The very perfect instrument, in the possession of my brother, with which I have been enabled to make these observations, was constructed by Mr. Ross, of London, to whom the greatest credit is due for the perfection to which he has brought his glasses.

† This I believe to be either entirely new, or to have been confounded with the *Grape coralline* of Ellis, (Corall. pl. xv. f. 25. c. C. D.) the *Valkeria uva* of Fleming (Brit. Anim., p. 551. gen. lxx. 197.), &c. Whatever be the animal meant by Ellis, it certainly differs materially from the present species, which I cannot refer to any described genus with which I am acquainted. Believing it to be new I have named it after my friend Mr. Bowerbank, whose zeal displayed on this as on many other occasions where the study of natural history may be promoted, was mainly instrumental in inducing me to follow up these investigations, on account of the many supplies that I received from him, and I gladly therefore take the opportunity of acknowledging and recording the obligation that I am under to him.

Fig. 2. The animal when fully expanded is about one twelfth of an inch in length. In its retracted state it is completely inclosed in a delicate horny cell, sufficiently transparent to admit of the whole structure of the contained animal being seen through its parietes. The cells are connected together by a cylindrical creeping stem, upon which they are thickly set, and sessile, ascending from its sides and upper surface.

Fig. 3. *a.* The animal when completely expanded is seen to possess ten arms of about one third the length of the whole body, each arm being thickly ciliated on either side, and armed at the back by about a dozen fine hair-like processes, which project at nearly right angles from the tentacula, remaining motionless, while the cilia are in constant and active vibration.

The tentacula are united together at their base to form a circle, in the centre of which is the mouth, and from which descends the œsophagus (fig. 3. *a* 1), bulging a little at its commencement, and then contracting and passing down nearly straight to its termination. The parietes of the œsophagus, especially at the upper part, which may be more correctly denominated the pharynx, are thickly studded with minute oval spots, arranged closely in contact with each other (Plate XXI. fig. 8.). The whole organ appears to be highly irritable, and contracts vigorously when food is introduced into it.

At the termination of the œsophagus is a distinct cardiac orifice (fig. 3. *a* 2.) that opens into a small globular cavity (*a* 3.) of singular construction, which appears to perform the office of a gizzard. The parietes of this organ are thicker than in any other part of the alimentary canal. They contain two dark round bodies placed opposite to each other, from each of which dark lines are seen radiating. In the space between these two dark bodies may be seen a number of squamiform spots arranged closely in contact, and presenting a beautifully regular tessellated appearance. This appearance was at first supposed to be owing to the crossing of the radiating lines at this point (fig. 4.); but a more accurate examination convinced me that they were distinct bodies lining a part of the interior of the cavity, and probably performing the office of gastric teeth: they have a remarkably definite outline when viewed under favourable circumstances, and by tearing open the gizzard, or bursting it by pressure, may be separated from its inner surface (Plate XXI. fig. 7.). The two dark bodies appear to be the points at which the radiating lines are concentrated. When the organ is in a state of rest and viewed laterally, these are seen projecting into the cavity, and giving it an hour-glass form (fig. 5.); but when it contracts these bodies become elongated, and their inner surfaces are closely applied to each other, and the cavity is obliterated (fig. 6.). The alterations in the form of this organ appear to be entirely due to these apparently muscular bodies. They are conspicuous even in animals not yet arrived at maturity (fig. 3. *c* 1.).

This organ, which I shall call the gizzard, opens downward into the true digestive stomach (fig. 3. *a* 4.), an oblong cavity terminating below in a blunt extremity, and from which it is separated only by the contraction of the parietes. The entire walls of the stomach are thickly studded with spots of a rich brown colour. These appear to be hepatic follicles, and to prepare a fluid that tinges the whole organ, as well as its contents, of a similar hue.

From the upper part of the stomach, and by the side of the entrance from the gizzard, arises the intestine (*a* 6.), by a distinct pyloric orifice (*a* 5.) that is surrounded by vibrating cilia. The intestine passes up straight and narrow by the side of the œsophagus, from which it is entirely separate and free, and terminates by a distinct anal orifice (*a* 7.) in the delicate parietes of the body, close to the outer side of the tentacular ring. The parietes of the intestine are marked with pale spots, something like those of the pharynx, and the whole tube, like the rest of the alimentary canal, possesses a high contractile power. Thus the alimentary canal consists of pharynx or œsophagus, gizzard, stomach and intestine, with subsidiary secreting follicles, and distinct oral, cardiac, pyloric, and anal orifices. The whole floats freely in a visceral cavity, the boundaries of which are formed by the delicate transparent parietes of the animal; the space between the alimentary canal and the parietes being occupied by a clear fluid, and by the muscles which act upon the animal.

That the animal possesses distinct membranous parietes separate from the walls of the alimentary canal, and independent of the cell which it inhabits, does not at first sight strike the observer, but the slightest attention to points hereafter to be mentioned will place this matter beyond a doubt.

The transparent horny cell which closely embraces the body of the animal is nearly unyielding in its lower two thirds, but terminates above by a flexible portion, which serves to protect the upper part of the body when the whole is expanded, in which state it is of the same diameter as the rest of the cell; but when the animal retracts is folded up and drawn in after it, and completely closes the mouth of the cell.

The flexible part consists of two portions, the lower half being a simple continuation of the rest of the cell; the upper consisting of a row of delicate bristle-shaped processes or setæ, which are arranged parallel with each other round the top of the cell, and are prevented from separating beyond a certain distance by a membrane of excessive tenuity, which surrounds and connects the whole. This mode of termination of the cell is one of constant occurrence, as will be described in other species, and is evidently a provision for allowing of the freest possible motion of the upper part of the body in its expanded state, to which it affords at the same time support and protection.

The mechanism by which the acts of *protrusion* and *retraction* are effected is somewhat complicated, and these acts are usually performed with such rapidity, especially that of retraction, that it was only by perseveringly watching the animals for several hours together, and sketching down each step of the process, whenever I could catch more than a momentary glance of one of them in any intermediate position, that I was at length led to a satisfactory knowledge of the precise mode of performing these operations.

For the process of *retraction* two distinct sets of muscles are provided, the one acting upon the *animal*, and the other upon the flexible part of the *cell*.

The muscles for the retraction of the *animal* are contained in the visceral cavity, and consist of two bundles of delicate thread-like chords (fig. 3. *a* 8 and 9.); the one set (*a* 8.) arising from the bottom of the cell to be inserted about the base of the stomach; the other (*a* 9.) also arising from near the bottom of the cell, though generally at the opposite side from the former, and passing up free by the side of the pharynx to be inserted around the line of junction of this organ with the base of the tentacula. The muscles provided for the retraction of the *operculum*, or flexible portion of the cell, have their origin from the inner surface and near the top of the stiff part, and are inserted into the flexible portion, on which they act. (fig. 3. *b* 2 and 3.) They are most distinctly seen when the flexible operculum is completely drawn in, at which time the latter is folded up, so as to occupy the axis of the upper part of the cell, and to it the muscles are seen extending inwards from the opposite sides of the cell from which they have their origin. They consist of six flattened bundles of fibres having a triradiate arrangement. The *upper three* sets (*b* 2.) act upon the flexible part of the cell and are inserted into it. The *lower three* (*b* 3.) are smaller, and are for the purpose of retracting the bundle of setæ by which it is crowned.

It is at this point that the best opportunity is afforded for investigating the structure of this form of muscle. It would appear as if muscular fibre were here reduced to its simplest condition. The filaments are totally disconnected, and are arranged the one above the other in a single series. They pass straight and parallel from their origin to their insertion, and have a uniform diameter throughout their course, except that each filament generally presents a small knot upon its centre, which is most apparent when it is in a state of contraction, at which time the whole filament also is obviously thicker than when relaxed. The filaments have a watery transparency and smooth surface, and under the highest powers of the microscope present neither an appearance of cross markings nor of a linear arrangement of globules.

These muscles, though apparently attached to the inner walls of the cell, must yet have the membranous parietes of the body interposed between their insertions and these walls, if as I suppose the cell is *completely* lined by the integument. In the lower part the integument is only occasionally seen separate from the walls of the cell, but above it may be easily discerned in the expanded animal passing up to be inserted around the tentacular ring, and thus distinctly bounding this part of the body which is always free within the expanded operculum. It is probable, therefore, that the retractors of the *operculum* as well as those of the *body* are within the *visceral cavity*, and that the relation of the origins of both, with regard to the integument and cell is similar to that which exists in the attachment of the muscles, with reference to the mantle and shell of bivalve Mollusca. This is easily understood if we suppose that the integument in this case, as I have ascertained it to be in another species, is attached to the operculum on a line with the *base* of the setæ, which is the highest point upon which the opercular retractors act, and from which it is there carried up free to the tentacular ring.

This being the *mechanism* by which the retraction of the animal within its cell is effected, I proceed to explain the *mode of its operation.*

The tentacula from being expanded in the form of an inverted cone are brought together into a straight line and immediately begin to descend. Their descent is effected by the contraction of the muscle which passes from the base of the cell to the tentacular ring, (fig. 3. *a* 9.) whilst at the same time the stomach is drawn down by *its* retractor. (3. *a* 8.) The whole body, however, does not descend in a mass, but must be folded up in a somewhat complicated manner, in order that the cell may completely inclose it. For this purpose the œsophagus, surmounted by the tentacula, descends first, whilst the integument of the upper part of the body begins to be *inverted* at the point where it has its insertion around the tentacular ring. As the descent of the tentacula proceeds, the inversion of the integument continues, forming a close sheath around them, (Plate XXI. fig. 12. *c*) until the extremities of the arms have descended to a level with the top of the unyielding portion of the cell. The animal is now completely drawn in, the stomach brought close to the bottom of the cell, and the œsophagus bent in the form of a letter S; the tentacula generally lying straight in the axis of the cell encased in their tegumentary sheath, and so separated from the fluid in the general visceral cavity; the centre of which they have the appearance of occupying, whilst they are in effect external to it. The *animal* being thus retracted, the next step of the process is to draw in the *upper part of the cell* after it. This process, however, always commences before the retraction of the body is completed, and by the time that the end of the arms are on a level with the base of the *setæ*. (fig. 11.) These latter bodies are then immediately brought together in a bundle, and begin to descend apparently by the action of the lower of the two sets of opercular retractors already described. Their descent, like that of the tentacles, takes place exactly in the axis of the upper part of the cell, and is accompanied by an inversion around them of its flexible portion, similar to that of the integument of the body around the tentacula during *their* descent (fig. 10.). Whilst the lower set of muscles are drawing down the *setæ*, the upper set complete the retraction of the *flexible part*, and the whole operculum is thus packed closely in the upper part of the cell, the end of which now presents a triangular indentation, corresponding with the triangular arrangement of the opercular retractors (fig. 3. *b*, and fig. 9.).

In this position of the animal it is impossible to define the whole course of the integument, but when the tentacles are drawn unusually low, (fig. 9.) that portion of it which forms their sheath may be readily seen passing up to the base of the setæ, around which it appears to have an attachment, and to be then continued up the sides of the inverted operculum to the angle at the top of the cell, whence it probably again descends to line the sides of the cell.

Thus the whole process of *retraction* may be easily accounted for, and the office of each set of muscles satisfactorily explained; but the *protrusion* of the animal is effected by a totally different mechanism.

Of course the different *stages* of protrusion occur in the inverse order of those of retraction. The bundle of setæ (fig. 10. *a*) first makes its appearance rising out of the apex of the cell, and followed by the flexible portion (fig. 11. *b*) on which it is set. The tentacula next pass up between the setæ and thrust them asunder, while the integument of the animal is seen gradually rolling outwards from around the tentacles. (fig. 12. *c*.) These latter continue to emerge and the integument to be everted from around them, until the base of the tentacles has risen above the top of the expanded setæ, when the act of protrusion is completed, the tentacula separate and expand, and the cilia commence vibrating*.

During repeated observations of the various steps of the process, I had in vain searched for a set of muscles having an antagonist power to the former and lifting the animal out of its cell. But I could discover no structure of this kind; and indeed it is not easy to imagine how such a mechanism could act, since the upper flexible portion of the sheath could afford no fixed point of attachment for elevating muscles, whilst from the want of rigidity in the body, and the manner in which it is folded up in its cell, no muscles arising from a lower point could effect its expansion.

After examining several species for an explanation of this phænomenon, I at length obtained a clue to it from one more favourable for examination than the rest. (Plate XXIV. fig. 3. *a*.)

In this species the body is capable of protruding for some distance beyond the mouth of the cell, in which state its delicate membranous walls may be readily traced downwards on one side to nearly the bottom of the cell, (Plate XXIV. fig. 3. *a* 3.) from the inner surface of which they are capable of being separated in about one third of its circumference and from top to bottom of the cell, but remaining apparently in immediate connexion with the other two thirds.

This separation of a portion of the parietes of the body from the inner surface of the cell, I found invariably to accompany the protrusion of the animal; and on examining further, I discovered upon this part of the body two rows of delicate, short, transverse filaments, arranged at a little distance from, and parallel to, each other. (*a b* and *d* 3.) These fibres were distinctly seen to contract whenever the protrusion of the animal took place, and to become relaxed again upon its retiring into its cell; the walls of the latter being so pellucid that the minutest alteration in the form of these muscles was readily seen. When contracted to their utmost, each filament was reduced to just half its original length, at the same time that its thickness was doubled, and the little knot upon its centre appeared also somewhat thickened. (Plate XXIV. fig. 4 and 5.) During their contraction, the unattached part of the pa-

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* Figs. 9, 10, 11, 12, represent the different stages of protrusion and retraction in the order in which they occur.

rietes upon which they were arranged was seen to recede from the inner surface of the cell, and to be drawn into longitudinal lines, especially at their points of origin and insertion. When the animal retired they returned to their former dimensions. These parietal muscles, which in structure exactly resemble the retractors, I have observed in every species of ciliated polype that has since come under my notice, and have ascertained its existence in Bowerbankia. (Plate XXI. fig. 13.)

These transverse filaments then, acting together from top to bottom of the space upon which they are arranged, must necessarily tend by their contraction to diminish considerably the *diameter* of the visceral cavity, and will therefore exercise a pressure upon the fluid which it contains. The effect of this will be to elongate the body in the direction in which it is most free to move, and it might be supposed that this would satisfactorily account for the act of protrusion; but it must be remembered that the contents of the cavity are folded up in a complicated manner, and the cell closed by its operculum, the whole of which parts have to be unfolded in regular order before the act of expansion is completed. I doubt, therefore, whether this simple apparatus could accomplish this act unassisted; but I believe it to be materially aided by the cooperation of the alimentary canal, which undoubtedly has the power of straightening itself from the sigmoid flexure into which it is thrown when the animal is retracted. I am led to think this from having frequently observed the great extent of motion which the upper part, especially of the alimentary canal, is capable of exercising, independently of any action of the muscles attached to it; and from having also noticed occasionally that, during the rising of the tentacula, the unfolding of the integument from around them seemed rather to follow as the consequence of their advance, than as being the means of effecting it, which cannot well be explained, if we suppose the fluid of the body to be driven upwards by the contraction of the parietal muscles with sufficient force of itself to expand the upper part of the body, and so to carry up the alimentary canal and to thrust out the arms. And this appears the more probable when we observe that even in the simple hydriform polypes, the advance and receding of the animal in its cell is entirely effected by the action of the parietes of the body, which are analogous to the alimentary canal in the present case; the hydriform polypes possessing no distinct muscles to assist in these operations.

To return then to Bowerbankia. Let us see how far these considerations will apply to the explanation of the phænomenon in question. The animal being retracted, with the stomach resting upon the bottom of the cell, begins to erect itself by straightening the alimentary canal; and the tentacula must be the parts first to rise. Before, however, these can protrude from the cell it is necessary that the flexible operculum which closes the mouth of it should be unfolded. As there does not appear to be any separate apparatus for this purpose, and as I have never observed it to occur independently of the motions of the animal, it may be presumed that this is effected by the pressure from below when the animal endeavours to rise.

Thus the tentacles rising first would press up the little bundle of setæ (Plate XXI. fig. 10. *a.*), that lies immediately above them. The pressure continuing, the flexible part of the sheath (fig. 11. *b.*) would be next unfolded, and the whole would then be expanded by the passage of the several parts in succession through them. (fig. 12.)

It is now that the parietal muscles come chiefly into play, and by keeping the teguments tense during the alteration in the position and form of the body, prevent any collapsing of its parietes, which might entangle the operation of any of its parts, and which, for the want of this provision, would be likely to ensue from the pressure of the surrounding fluid when the animal rises from its cell; especially as there does not appear to be that ready communication between the interspace then left in the cell and the surrounding element, by which the water might flow in to supply the vacuum left by the change in the form of the lower part of the body, during the protrusion of the animal. This circumstance I had occasion to prove by noticing the forcible indentation of the stiff horny cell itself, by the pressure of the surrounding fluid, which in some instances followed this act, as is represented in *Lagenella repens* (Plate XXIV. fig. 2. *a.*). Further, by the contraction of these muscles the body may be so much elongated as to carry the base of the arms to some distance above the margin of the cell, by which the freedom of their action is considerably increased, the stomach, being then lifted from the bottom of the cell, hanging suspended in the visceral cavity. It would appear then that the act of *protrusion* is effected by the combined operation of the parietal muscles and of the alimentary canal, which in fact forms the principal part of the substance of the animal, the parietes being purely membranous, and having little else to do than to retain the fluid in which the viscera float.

It is interesting to compare these parietal muscles with similar parts in animals of another class. Having been frequently struck with the close analogy which the general characters of the animal under consideration presents with those of the class *Rotifera*, especially in the character of the *retractor muscles*, I was led to compare the *parietal muscles* also with the parts which, in *Hydatina senta*, for example, are usually considered and represented as the dorsal vessel with its lateral branches. (For it must be understood that the parietal muscles of which I am speaking have no resemblance to the circular fibres that surround the bodies of vermiform animals, and are intimately blended with their integument, but have a totally different character, being simple short filaments, occupying a very small portion only of the circumference of the body, and being apparently connected with the parietes only by their extreme points of attachment). Having procured therefore some specimens of Hydatina, I was not much surprised to find that the parts in the two animals were identical; the transverse lines of Hydatina being obviously *parietal muscles*, which whenever the body becomes elongated may be observed by their contraction to draw that side to which they are attached into longitudinal folds, and to be again elongated whenever the body is shortened by the contraction of the longitudinal muscles, to which the former are evidently the antagonists. In this case the alimentary canal being straight and not folded on itself, and the body being unshackled by a dense covering, the parietal muscles alone are adequate to effect its expansion.

Upon a review then of this description of the organization of Bowerbankia, it must be admitted that the mechanical functions are executed with a degree of perfection, which in a being so exceedingly minute cannot fail to excite our surprise and admiration: not less interesting either is it to observe the more vital operations of this highly organized species.

The little animal, when in full vigour, is seen projecting from its cell with the arms extended and the cilia in full operation, the upper part of the body being frequently turned from side to side over the edge of the cell, the extremity of which, from its peculiar flexibility, moves along with it. The particles, carried to the mouth in the vortex produced by the action of the cilia, after remaining a little while in the pharynx, are swallowed by a vigorous contraction of its parietes, and carried rapidly down the œsophagus and through the cardia to the gizzard, which expands to receive them. Here they are submitted to a sort of crushing operation, the parietes of the organ contracting firmly upon them, and the two dark bodies being brought into apposition. Their residence, however, in this cavity is only momentary, and they are immediately propelled into the true stomach below, where they become mixed up with its contents, which during digestion are always of a dark rich brown colour, being tinged by the secretion of its parietal follicles.

The food appears to be retained for a considerable time in the stomach, and may be frequently seen to be regurgitated into the gizzard, whence, after having been again submitted to its operations, it is returned to the stomach. Here it is rolled about by the contraction of its parietes, and at its upper part is frequently submitted to a rotating motion. This rotation of particles is chiefly near the pyloric orifice, and a mass may be frequently seen projecting through the pylorus into the intestine, and rotating rapidly in the direction of the axis of the orifice. In an animal having a similar form of pylorus to this, but in which the parts were more transparent, I could distinctly see the cilia by which this rotation is effected surrounding the orifice.

The granular matter, after rotating for some time at the pylorus (a provision for preventing its too rapid escape from the stomach), passes into the intestine, where it accumulates in little pellets, that distend the parietes of the tube; and it is possible that it may be here still further acted upon by these parietes, which have a spotted appearance.

By the contraction of the intestine the little pellets of excrementitious matter are carried rapidly upwards to the anal orifice, which is seen to open, and the little pellet to be tilted over its edge, when it is immediately whirled away from the sight in the current produced by the ciliated tentacles, and the orifice of the tube again contracts.

With regard to the mode in which the animals are united together, I could not discover that any connection existed between them beyond that which results from their cells being placed upon a common stock. In almost every case, however, I could see a filament (possibly a tube) passing down from the base of the stomach into the short neck that connects the cell with the main stem, but beyond this I could not trace it. It appeared to be distinct from the retractor muscles, by which it was surrounded. The stem itself appears to be nearly homogeneous throughout, and no motion of particles was ever observed in its interior. Each cell is capable of a slight degree of flexion upon the main stem, but the means by which that is effected are not obvious.

The only mode of *reproduction* that I had the opportunity of witnessing was that by a process of gemmation from the common stock, or creeping stem, from which the young animals in various stages of growth were seen sprouting (Plate XX. fig. 2.).

The smallest gemmæ appeared to be homogeneous in texture, forming little nodules on the parent stem. Those further advanced were seen to present something like a boundary line, indicating the thickness of the parietes of the future cell. Within this, in others, was a little dark mass, which in larger ones presented a rough outline of the form of the complete animal. Those about half grown had all the parts distinctly traced out; the retractor muscles completely formed; the tentacles short and clumsy; the walls of the alimentary canal thick, and its cavity clearly defined, as well as the dark spots in the gizzard (fig. 3. *c* 1.). These were commonly seen in all stages of growth up to full maturity, grouped together on the same stem without any order, the stem terminating in a blunt growing end.

As I shall have occasion to notice in another part of this paper the second mode of reproduction in this class, namely, that by locomotive ciliated gemmules, but which I did not observe in the present species, I defer for the present the consideration of this subject. I cannot, however, avoid referring here to an appearance which I have commonly observed, that seems to be in some way connected with it. It is that of one or more rounded or oval bodies, of a brown colour, lying apparently loose within the visceral cavity, and near the bottom of the cell (Plate XXI. fig. 14 and 15.). From their dark colour they are generally very conspicuous, especially as they remain in the cells long after the animal has perished and disappeared from them. From this circumstance it might be imagined, that they resulted from decomposition, were they not also frequently seen in the living animal. Moreover they have a definite form and size, and when removed from the cell and carefully examined are found to consist of a delicate transparent membrane, inclosing a brown granular matter to which their colour is due. (fig. 16.) It is further remarkable that they are often seen as large in animals not even half matured as in the adults (compare figs. 14 and 15.). Besides these I have sometimes noticed other bodies more nearly spherical, and of a milk-white colour, which when pressed, broke up into minute granules. I have observed as many as three of the white and two of the brown bodies in the same animal, (fig. 14.) but the former are seldom seen. The brown bodies, however, are so extremely common, that I have seldom had occasion to examine any species of this class without detecting them. I have not, however, been able to ascertain their use.

They do not appear to answer any purpose in the economy of the animal; and from their persistence after its death, they would seem to have the power of resisting those forces which cause the decomposition of the other parts. It is most probable that they are connected with the process of reproduction, but whether they may be viewed as ovaries or as immature ova, and what relation the white and brown bodies have to each other, are considerations that might be reserved for further opportunities of elucidation. I have never detected any motion in the granules of which they are composed.

PLATE XXII.

Vesicularia spinosa, THOMPSON, Zool. Research. Mem. V. pl. iii.
Syn. *Corallina confervoides*, ELLIS, Cor. pl. xi. N. 17. b. B. C. D.
Sertularia sericea, PALL., El. Zooph. p. 114. No. 65.
Sertularia spinosa, LINN., GMEL. p. 3855. No. 23.
Valkeria spinosa, FLEM. Brit. Anim. p. 551. Gen. lxx. 198.

This species forms part of the subject of Mr. THOMPSON's memoir, whose name for it I have adopted. In this memoir, which contains but a very rough sketch of the animal, he calls the part that I have described as a gizzard the stomach, which latter organ he mistook for an ovarium.

It might be difficult to select two species differing more in external characters than do this and the previous one, yet in the structure of the *animal* hardly any essential points of distinction can be observed: it is one of the smallest and most delicate species on our coast. The main stem, which is zig-zag, (fig. 1.) sends from each angle two branches that divide dichotomously to their extremities, which in *growing* branches are rounded, but in others sharp and spiny; a circumstance that did not escape the notice of ELLIS. The vesicles are so minute that they can with difficulty be detected by the naked eye. The animal has but *eight arms*, (fig. 3.) which are short and stiff, and during expansion remain nearly motionless in the usual funnel-like form, but may be occasionally seen separated so far from each other as to stand out at nearly right angles from the body: they are ciliated, but not armed with spines. The alimentary canal presents the same character as that already described in Bowerbankia; but allowing for the much smaller size of the animal, is proportionately shorter and stouter.

The muscular apparatus consists of the gastric and tentacular retractors, the former (fig. 8. *a.*) arising from the bottom, the latter, (8. *b.*) a little above from the side of the cell; two rows of parietal muscles (fig. 8. *c.*), and *two* sets of operculum retractors (fig. 8. *d.*), in which respect it differs from the former species, where there are *three* double sets; and the triangular indentation at the upper part of the closed cell is consequently wanting. The cell is also much broader in proportion to its length, having a more oval form. The operculum is finished by a row of setæ. The connecting stem has a remarkably definite character (fig. 2.). Between every dichotomous division is a joint bearing three cells, which, when they drop off leave a circular foramen. The cells are strictly unilateral upon the stem, and diminish in size very regularly towards the growing extremity; the last few being mere buds from its surface. Within the stem may be seen a number of transparent, circular bodies of a tolerably uniform size, that appear to be attached to the inner surface. They very much resemble in appearance the granules which may be seen through the cell attached to the membranous parietes of the animal itself. These latter are smaller than those in the stem, and more scattered. I have not ascertained the use of either.

Ellis was, I believe, the first to notice in this species what appears to be a direct medium of communication between the animals themselves. It consists of a thread of a darker substance than the rest of the stem, running within its upper surface immediately below the base of the cells. Ellis states* that the slightest movements of the animals were communicated to this substance, an observation that I have not been able to confirm; but my specimens were not very lively. The point is one of interest and worthy of further investigation.

PLATE XXIII.

Valkeria cuscuta, Flem., Brit. Anim. p. 550. gen. lxx. 196.
Syn. *Corallina cuscutæ forma,* Ellis, Corall. N. 26. pl. xiv. c. C.
Sertularia cuscuta, Linn., Gmel. p. 3852. No. 18. and Muller, Zool. Dan. 3, p. 62. t. 117. f. 1—3.
Cuscutaria cuscuta, Blainv. Dict. des Scien. Nat. p. 461.
Vesicularia cuscuta, Thomp. Zool. Research. N. IV. pl. ii. 1.

This is the most minute species that I have met with. Its creeping stems are found closely adherent to the filamentous ramifications of a species of Ceramium, on which it grows parasitic, with much of the habit of Dodder, whence its trivial name (fig. 1. and 2.).

It has been placed in the same genus with the species last described, but differs from it, and also from Bowerbankia, in the entire absence of the manducatory organ; a difference which it is of great importance to observe with reference to a natural arrangement of the class†.

The ciliated tentacles are eight in number, slender, and often widely spread, exhibiting a good view of the oral aperture (fig. 4. *a.*).

The alimentary canal is here of some length, and acquires a considerable sigmoid flexure when the animal is at rest (fig. 5.). The intestine near its termination possesses a decided rectal enlargement, which is very distinct even when empty (fig. 5. *e.*).

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* Essai sur l'Hist. Nat. des Corall. p. 36.

† I have for this reason removed it from Vesicularia, and adopted the name previously given to it by Dr. Fleming.

There appears to be but one set of parietal muscles in this instance. The retractors of the body resemble those of Bowerbankia, as do also those of the operculum in their division into an upper and a lower set, but there appear to be only two instead of three of each, as in Bowerbankia (fig. 4. *b.*).

The granules which adhere to the parietes are very distinct in this species, and remain attached to the inner walls of the cell, after the rest of the animal has disappeared (fig. 4. *c.*), together with the brown bodies which I have conjectured to be ovaries or ova. I have not generally observed more than one of these bodies in each animal of this species.

A very remarkable agitation of particles, which was frequently observed in the visceral cavity, and very closely resembled the irregular vibration of cilia, was found, by the aid of a very high power, to be caused by a multitude of minute *cercariæ* (fig. 5.) swimming about with the greatest activity in the fluid with which that cavity is filled. When this cavity was laid open by a needle they escaped and swam away by the serpentine motion of their bodies. They consisted simply of a long slender filament, with a rounded extremity, by which they occasionally fixed themselves (fig. 5. *g.*). Similar parasites were not unfrequently observed in other species.

The cell is terminated by the usual row of setæ (fig. 5. *f.* and fig. 6.). From the quantity of earthy material combined with its horny texture, it is rendered so opake as to present great difficulty in the examination of the contained parts. This character, which pervades the stem also, renders these parts exceedingly tough and strong; and notwithstanding its extreme fineness the stem will bear the exertion of a considerable degree of force without breaking. When these parts are pressed by the dissecting needle they yield a grating sound.

The arrangement of the cells with regard to the stem is intermediate in regularity between that of the two former species. They are generally gathered in clusters which surround the stem (fig. 3. *a.*); the cluster nearest to the growing extremity having its cells gradually diminishing in size, and more obviously springing from opposite sides of the base of support, the long spiny end of which often projects in a remarkable manner beyond the extremity of the branch upon which it creeps. The stem is often seen divided into joints at irregular distances, and the cells are sometimes set on short branches springing from them (fig. 3. *b.*).

PLATE XXIV.

Lagenella repens, Mihi.

Fig. 1. and 2. Parasitic, with a creeping stem, on *Sertularia* and on *Halodactylus diaphanus*. Not very common.

This species has twelve ciliated arms (fig. 3. *a.*), not spiny. The alimentary canal is short and stout, and whilst the animal is expanded remains high up in the body. During retraction the stomach is never brought down to the bottom of the cell, but remains suspended from the upper part of it by the intestine, which appears to have some attachment at this point. The upper part of the tube, however, is generally brought down lower than the stomach, in order that the tentacles may be completely drawn in (fig. 3. *c*.). By this suspension of the stomach from the upper part of the cell a fixed point is obtained, from which the retracted flexed portion of the tube may erect itself with the same effect as if the stomach were in contact with the bottom of the cell. This is a point which it would be important to observe in generic distinctions; but here, as with many other points in this species, my observations were not carried to the extent that they have been in others, as this was one of the specimens with which my investigations were commenced, and I have never since had an opportunity of confirming them. This is the more to be regretted, as from the complete isolation of the cells, and the extreme transparency of their parietes, a clearer view of their contents is obtained than in any species that I have subsequently met with.

The spots upon the pharynx, and their absence in the triangular ciliated space, were remarkably distinct (fig. 3. *a* 1.), as was also the difference between the dark brown colour of the hepatic follicles in the *replenished* stomachs, and their pale and almost inconspicuous character in the *empty* ones. (Compare *a* and *b*. fig. 3.) The position of the cardia was not ascertained. When the body was turned so that the pylorus was presented to view, and this happened to be empty, a row of cilia were distinctly seen surrounding it. The vibration of these cilia, as well as of others which were observed in the stomach, appeared to be entirely under the control of the animal. Their action was frequently observed to be suddenly suspended, when the rotation of particles ceased also, and when it recommenced the motion of particles was renewed. This rotation was often so rapid at the pylorus that I should think from one to two hundred revolutions must have been performed in the minute. When very small animalcules were introduced into the bulging pharynx, several convulsive efforts were sometimes made before they could be swallowed; during these the animalcules not unfrequently escaped again by the mouth, but were intercepted by one of the tentacula being bent forward and striking the animalcule as it rose with a sharp blow that drove it back again into the pharynx. The animalcules did not immediately perish in the stomach, but continued their motions for some time after being introduced into it.

The gastric and tentacular retractors are particularly distinct, and have the usual origins and insertions (fig. 3. *d* 1 and 2.). The double row of parietal muscles (fig. 3. *d* 3.) have been already described in the notes on the species first quoted. There appears to be but one set of retractors of the operculum (fig. 3. *d* 4.), which is generally drawn towards the side of the cell from which they arise, leaving a slight indentation in the top of the latter when retracted (fig. 3. *c*.). The granules on the parietes are less numerous than in most instances, but very conspicuous.

The cells have an oblong form, and are connected to their narrow creeping stem by a short peduncle. The opercular portion terminates in a notched margin, and is very short (fig. 3. *a* 4.). (It is possible that this notched margin may be formed by the extremities of short and broad setæ, but this was not determined.) The cells spring from the sides and upper surface of the stem, and turn upwards as in *Bowerbankia*. They are set at some distance apart.

The gemmæ exhibit the same process of growth as in other cases, and are scattered irregularly amongst the larger cells (fig. 3.).

PLATE XXV. and XXVI.

Halodactylus diaphanus, Mihi*.

Syn. *Alcyonium seu Fucus nodosus et spongiosus*, Ellis, Cor., p. 102. pl. xxxii. fig. d. D.
Alcyonium gelatinosum, Linn., Gmel., p. 3814. No. 11. Lamx. Polyp. flex., p. 350. No. 495. Mull. Zool. Dan. iv. p. 30. t. cxlvii. f. 1—4. Flem. Brit. Anim. p. 517. gen. xl. 86.
Alcyonidium diaphanum, Lamx. Gen. Thalass. p. 71. t. 7. f. 4. Hooker, Flora Scotica, part II. p. 75. London Encyc. of Plants, 1829, p. 928.
Ulva diaphana, Hudson, Flor. Angl., vol. iii. p. 570. Sowerb. Engl. Bot. t. 263.

Extremely common on the Sheppy coast, especially after a gale, when it is cast up in immense quantities, and is found attached to loose stones and shells, in the form of soft, flexible, finger-like processes of very irregular figure, being rounded and smooth upon the surface, or flattened, nodulated and branched, sometimes attaining the length of two or three feet, but generally about six inches long. The animals are, however, so small that such specimens must contain many millions of them. When a portion of this is placed in a trough of sea-water, the little animals are seen quickly to emerge in such numbers as to cover its surface with a coating as it were of the finest down; and they are so closely set that there seems to be hardly room for their several operations (fig. 1.). In this state it is scarcely possible to make any observations upon them, but when a few only are projecting they become from their extreme delicacy and transparency peculiarly favourable subjects for examination. On this account, and also from the length of time that they may be kept in vigour, I have been enabled to prosecute the inquiry further than other specimens enabled me to do.

Plate XXVI. fig. 7. The tentacula are sixteen in number, (occasionally fifteen)† fully two-thirds the length of the body of the animal, and extremely slender and

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* ἅλς et δάκτυλος. The confusion and doubt which have so long pervaded the very ill-defined genera *Alcyonium* and *Alcyonidium* appear likely to be dispelled only by beginning *de novo*, and adopting a new name in conjunction with characters sufficiently definite to preclude all probability of further error. I have therefore renamed the present species as indicative of a new genus, with which other species will probably be found, upon a minute examination of their intimate structure, to possess congeneric affinities.

† Fleming, Lamouroux and others appear to be in error in stating them to be twelve.

flexible. When expanded they are frequently seen to roll up closely upon themselves, (fig. 18.) even down to their base, the revolution taking place either inwardly or outwardly, and in one or more arms at the same time. Their full expansion affords a more perfect campanulate form than is usually met with in this class, each of the arms having a slight curve outwards towards its extremity, which gives to the whole a very elegant appearance. It is remarkable that in some specimens the arms are much shorter on one side of the body than on the other. In many positions this is not very striking, as it might be attributed to an appearance of fore-shortening of that side which happens to be turned towards the observer, (fig. 17.) but when viewed laterally this character is very obvious (fig. 16.). The arms when viewed with an amplifying power of 200 linear are seen to be tubular throughout, (fig. 19. *a.*), and to have an aperture at each extremity. The aperture at the apex is extremely small, and in a lateral view sometimes appears like a slight notch at the extremity of the arm. The apertures at the base are seen more plainly, and are situated in the centre of the tentacular ring, one corresponding with the base of each arm (fig. 8.). I have also sometimes observed what would seem to be a fine canal, running round in the substance of the ring, and apparently uniting the tentacular canals (fig. 10.). It would be exceedingly interesting to ascertain with what parts these tentacular canals communicate. As the tentacles appear to be respiratory as well as prehensile organs, it is most probable that the canals by which they are permeated are for the purpose of allowing a circulation of fluid through them; but from the minuteness of the parts and the agitation of the surrounding medium by the rapid action of their cilia, it would be a matter of great nicety to detect such currents though they should exist.

The action of the tentacular cilia appears entirely under the control of the animal, and they are sometimes seen completely at rest. If, however, a portion of one of the arms be cut off, the action of the cilia continues as vigorous as before, and the isolated part is carried about in the field of the microscope. When the animal is dead, these cilia are seen to be longer and considerably more numerous than they appeared to be when in action (fig. 19. *a.*). As the parts gradually perish the cilia disappear, and the surface of the arms becomes covered with a granular matter and the part shrinks to a mere thread (fig. 19. *b* and *c.*).

The ring upon which the tentacles are set is well marked, and terminates on its inner circumference by a sharp edge projecting to form the mouth of the pharynx. It is probable that this tentacular ring never contracts except to bring the base of the tentacles together. But whilst it remains fixed so as not to alter the arrangement of the arms, the closing of the mouth of the pharynx is effected by the constriction of the parts immediately below it, which there appear as if they had been bound round with a ligature (fig. 8. *d.*).

The pharynx (figs. 7, 8 and 9.) is very short in this species, and its parietes are covered by the peculiar spots already noticed, except in the triangular space at the upper part (fig. 9. *c.*) which is entirely free from them, and where a vibration of cilia may be occasionally detected. On the opposite side, generally, of the pharynx, may be seen a line (figs. 8 and 9. *a.*) running down to the cardia, and if this part be observed for a short time it will be seen that the pharynx is repeatedly distended, as by a sudden act of insufflation, and that whenever this distention takes place the part bounded by the line becomes expanded but quickly returns to its former position. It is probable that there is here a muscular apparatus by which these sudden expansions of the pharynx are effected. When at rest this part has much the appearance of a tube, which is marked by rough cross lines, and wants the spotted character of the surrounding parts (fig. 9.).

This sudden expansion of the parietes of the pharynx, which takes place at irregular intervals, appears to be connected with the act of *respiration* as well as of *nutrition*; for not only are the particles of food thus admitted more freely into the sac, but the water is more readily renewed and brought more effectually into contact with its inner surface.

With a view of determining to what extent the flow of water into the pharynx took place, some particles of carmine were diffused through the fluid in which the animals were placed. As soon as the cilia commenced vibrating, the particles of carmine were put into rapid motion, (fig. 7.) being carried in a stream down the inner surface of each arm, the greater part passing out again between their bases; while of the remainder part turned upwards again and issued from the centre of the expanded arms, (a course best seen when the arms were turned forward) and a few were carried through the mouth into the pharynx, where they were submitted to a rotatory motion by the action of the cilia lining its upper part; and after remaining there a short time some were swallowed, while the rest escaped at the mouth, and their place was supplied by others. From this experiment it did not appear that the flow of fluid into the pharynx was so free as might be expected, seeing that the mouth is almost constantly open; for except during the act of expansion, by which the sac is suddenly filled with water, the parietes are so nearly in contact as to obliterate a large portion of its cavity: at every expansion, however, the greater part of the water must be renewed.

The *size* of the particles which the animal swallows appears to be regulated conjointly by the mouth and tentacula. The aperture within the tentacular ring, which forms the mouth of the pharynx, is not capable of distention like the mouth of Hydra; for if this part were to be engaged in swallowing large prey, the whole tentacular apparatus would be thrown into disorder, and the regular flow of fluid to the pharynx interrupted. Whilst, therefore, the diameter of the mouth prevents the admission of the larger particles, the size of the smaller ones will be regulated by the spaces between the tentacula, which, like a sieve, of a degree of fineness proportioned to the number of the arms and the consequent width of their intermediate spaces, would allow all the finest particles to drain away, and retain in their area only those of an intermediate size. These readily flowing into the pharynx become subject to a selection of a less mechanical nature, in accordance with which some are swallowed and others rejected. In the act of swallowing the mouth is closed by the constriction below the tentacular ring, and the sides of the pharynx being brought into apposition, principally by the action of the part already alluded to, the particles are forced through the cardia, which projects into the pharynx with a nipple-like prominence (fig. 8. *b.*).

As all below this point appears to be concerned more or less in the office of digestion, the stomach may be considered as commencing here, though in form it might rather resemble an œsophagus. The hepatic follicles, however, reach nearly as high as the cardia (fig. 7. *a.*). If then the cardia be taken as the line of demarkation between the pharynx or œsophagus and the stomach, without regard to its variable position in different species, then it will be found that in some the pharynx is short and the stomach very long, as in the present instance, and in *Valkeria cuscuta*; whereas in *Bowerbankia*, where the cardia is low down, the reverse obtains, and the pharynx is of great length, while the stomach is comparatively short.

Those who would follow out the analogy which undoubtedly exists between these animals and those of the class *Tunicata*, and would compare the *pharynx* in the present case to the *respiratory sac* in *Ascidia*, might contend that the *upper* aperture of the pharynx being analogous to the entrance to the respiratory sac in *Ascidia*, then the *lower* aperture should be called the mouth, as being placed at the bottom of the sac. As, however, the pharynx is here *certainly* an organ for the reception and deglutition of food, and only *probably* concerned in respiration; it would be more consistent to use the names as I have applied them; the upper and lower apertures being respectively *mouth* and *cardia*, while the intermediate space may be designated from its probably double function, *respiratory pharynx*. The distinction, if contended for, would be at best but one of names and could not improve the analogy.

The stomach is not furnished with a gizzard in this species. The intestine forms a considerable elbow at its origin, and is short and wide, terminating not as in other cases near the tentacular ring, but about midway up the body, at a point opposite the base of the setæ (fig. 7. *a.*).

A very singular organ (figs. 16, 17 and 18. *b.*) was frequently observed consisting of a little flask-shaped body situated between the base of two of the arms, and attached to the tentacular ring by a short peduncle. The cavity in its interior is lined with cilia which vibrate downwards towards the outer, and upwards towards the inner side; it has an arrow neck and a wide mouth, around which a row of delicate cilia are constantly playing. No flow of fluids could ever be detected through it, nor did the use of carmine assist in showing with what parts the cavity in its interior might communicate. From the circumstance that it is more frequently absent than present, it cannot be an organ of vital importance to the animal: and it is too intimately blended with the sides of the tentacula and too constant in its position to be regarded as a parasite. Does it indicate a difference of sex?

The peculiar fleshy character which caused the name *gelatinosum* to be applied to this species arises from the mode in which the cells are united together. Their arrangement is best seen by making a thin *transverse* slice of the main substance and examining it with a low power (Plate XXV. fig. 3.). The cells are then found to be arranged parallel with one another and having their sides united together so as to form a compact ring, of which the bases constitute the inner and the apices the outer circumference. The centre of the cylinder, of which this is a section, is occupied by a light cellular tissue and a clear fluid, probably water. In such a section similar brown bodies to those already described are seen in great numbers, and not confined to the cells, but dispersed through the whole substance. The arrangement of the cells being thus shown, a more accurate view of their structure is obtained by examining a thin section made *parallel with the surface* (fig. 4.). In this view their ends only are seen having an hexagonal form, from their pressure upon each other, but in each compartment the animal may be discerned with all its parts. Sometimes, however, the cells, instead of being arranged parallel to one another, lie so obliquely, that their sides instead of their apices form the outer surface; an arrangement which bears a close resemblance to that of the cells of *Flustra* (fig. 5.). But in order to witness the different stages of protrusion and retraction a portion of the mass must be viewed edgewise (Plate XXV. fig. 2. and Plate XXVI. fig. 7.). In which position, although the lower part of the cell and animal is generally concealed from view, all the most interesting parts may be observed as they rise in succession above the surface. The stages of protrusion and retraction occur in the same order and with nearly the same phenomena as in *Bowerbankia*. (See the series from fig. 11 to 16. Plate XXVI.) The arms, however, instead of rising straight are often seen bent upon themselves, a provision that appears to be necessary, on account of the great length of some of them, in order that they may be completely inclosed in the cell.

The upper portion of the cell, from its superior transparency and flexibility, appears to contain little, if any, earthy matter. The setæ (fig. 7. *a* and *d*.) are very stout and short, broad at their base, and few in number. The body projects to an unusual distance beyond the mouth of the cell (fig. 7. *a*.), and its delicate parietes may be seen separate from the whole circumference of the cell, except where they are attached to the edge of the operculum at the point whence the setæ arise.

Cercariæ were seen in the bodies of these animals which did not differ in any respect from those of *Valkeria*, and occupied a similar position. On one occasion these were observed drifting rapidly to the upper part of the visceral cavity, and shortly after issued from the centre of the tentacula; but as the animal had in the mean time half withdrawn itself, I lost the opportunity of tracing their course. It would appear from this that there is some external communication with the *cavity* of the body.

The process of reproduction by gemmation (in this case by the growth of young animals and cells amongst the mature ones) may be seen in every specimen (Plate XXV. fig. 4.). The smaller cells are triangular, and the animal forms a mere spot in its centre. As they grow they thrust aside the surrounding cells, and the number of their sides increases until they acquire the irregular hexagonal form of the adult. In the oblique position of the cells (in which they look like a new growth encrusting the old mass like a *Flustra*) the young cells are less angular, and arranged more regularly at the spreading edge (fig. 5.).

This species afforded an opportunity of examining also the reproductive *gemmules*. These are readily seen in spring as minute whitish points just below the surface of the mass (fig. 3. *a a*.). Sometimes they are of a darker colour, and exceedingly numerous, appearing to occupy almost the whole substance. If one of these points be carefully turned out with a needle it is found to consist of a transparent sac (fig. 20. *a*.), in which are contained generally from four to six of the gemmules, which, as soon as the sac is torn, escape and swim about with the greatest activity, affording a most interesting subject for microscopic investigation.

When viewed with a power of forty, linear measure, they are seen to be of an oval or rounded form (fig. 20. *b* and *c*.), convex above, and nearly plane below, and fringed at the margin with a single row of cilia, which appear to vibrate in succession round the whole circumference. Under an amplification of 120 they assume a different aspect (fig. 21. and 22.), and their minute structure is clearly discerned. Viewed as opake objects, both the body and cilia have a silvery whiteness, but by transmitted light the former appears of a dark brown, and the cilia of a golden yellow colour. Upon the most convex part of the body, which is not generally in the centre, but leaning to one side, are set from three to five prominent transparent bosses surrounded by a circle; and other circles are seen extending to the base of the body, the extreme margin of which is bounded by a row of prominent tubercles. These marginal tubercles are from thirty to forty in number, and from the circumstance of the cilia arising from them, it is probable that they are for the purpose of governing their motions, and therefore analogous to the muscular lobes of *Hydatina senta*. No structure, however, could be detected in these, nor in any other part of the body beyond a mere granular parenchyma.

Fig. 22. Under this power the whole character of the ciliary motion is changed, and it is seen that what before appeared to be a single cilium is in fact a wave of cilia, and that their motion, instead of being in the direction of the circumference of the disc, is at right angles to this. The ciliary phenomena are the most readily observed when the gemmule is nearly at rest, or has become languid; it then lies either with the convex or the plane side uppermost, and with the cilia, which are of great length, doubled in the middle upon themselves (fig. 21.), so that their extremities are brought back nearly to touch the margin of the disc from which they arise. The whole fringe of cilia is then suddenly unfolded, and after waving up and down with a fanning motion they are either again folded up towards the under surface of the body, or they commence their peculiar action. As the cilia have the appearance of moving in waves round the disc (fig. 22.), each wave may be thus analysed. From a

dozen to twenty cilia are concerned in the production of each apparent wave, the highest point of which is formed by a cilium extended to its full length, and the lowest point between every two waves by one folded down completely upon itself, the intervening space being completed by others in every degree of extension, so as to present something of the outline of a cone. (And it is remarkable that one of these corresponds very nearly in breadth with one of the supposed muscular lobes.) As, however, the persistence of each cilium in any one of these positions is only of the shortest possible duration, and each takes up in regular succession the action of the adjoining one, so that cilium, which by being completely folded up formed the lowest point between any two waves, now in its turn by its complete extension forms the highest point of a wave; and thus while the cilia are alternately bending and unbending themselves, each in regular succession after the other, the *waves* only travel onward, whilst the *cilia* never change their position in this direction, having in fact no lateral motion. When the waves travel very rapidly they appear smooth on one side and fringed on the other (fig. 23.). The whole of the ciliary motions are so evidently under the entire control of the animal as to leave not the slightest doubt in the mind of the observer as to this point. The whole fringe of cilia may be instantly set in motion, and as instantaneously stopped, and their action regulated to every degree of rapidity. Sometimes one or two only of the waves are seen continuing their action whilst the remainder are at rest, or isolated cilia may be observed slowly bending and unbending themselves, or projecting entirely at rest (fig. 21.). The body is generally somewhat pointed towards one extremity of the oval, and at this part may be observed a bundle of cilia, longer than the rest, and moving very rapidly (fig. 21. *a*.). Their vibrations were in several instances counted very evenly at 230 times in the minute, continuing in action whilst all the others were folded up. These may be respiratory, whilst the others are chiefly locomotive. There can be little doubt that this explanation of the action of the cilia in the gemmules is applicable also to those of the tentacula of the adult animal, and not only in this species but throughout the class generally; for I have already observed that the tentacular cilia are infinitely more numerous when at rest than they *appear* to be in action: and I have also noticed, when their motions become languid, that here also they vibrate, not in the direction of the plane of the arms, but at right angles to it, and with the same hook-like form as in the gemmules. In this way the apparent travelling of the cilia up one side of the arm and down the other, as the eye is seduced to follow the waves which they seem to produce, is at once explained.

It would be impossible to explain the variety of motions which the gemmules are capable of executing, were it not obvious how complete is their control over the action of the cilia, which are their sole locomotive organs. They generally swim with the convex part forwards, and with the greatest rapidity. Sometimes they simply rotate upon their axis, or they tumble over and over; or selecting a fixed point they whirl round it in rapid circles, carrying every loose particle after them. Others creep along the bottom of the watch-glass upon one end and with a waddling gait; but generally after a few hours all motion ceases, and they are found to have attached themselves to the surface of the glass. At the expiration of forty-eight hours the rudiments of a cell were observed extending beyond the margin of the body (fig. 24.); but at this stage the animals invariably perished, and during repeated observations I had no opportunity of witnessing their further metamorphosis. At this stage the cilia had disappeared, and the muscular lobes were no longer apparent. None of these gemmules were spontaneously evolved, and their death appeared to be owing to their premature extraction. The parenchyma of the gemmules has a *contractile* power, somewhat like that of Hydra, but less in degree, by which the form of the body is occasionally altered. If a portion of the margin with the cilia attached be torn off, the cilia continue to vibrate, as when a portion of one of the tentacles has been so isolated.

PLATE XXVII. fig. 1—5.

Membranipora pilosa, Blainv.

Syn. *Eschara Millepora*, Ellis, Corall. pl. xxxi. f. a. A.
Flustra pilosa, Linn., Gmel., p. 3827. No. 3. Blainv., Dict. des Scien. Nat., Art. Zooph., p. 415. Flem., Brit. Anim., p. 537. Gen. lvi. 147.
Flustra dentata, Linn., Gmel., p. 3828. No. 11. Ellis, Corall. pl. xxix. d. Blainv., Dict. des Sc. Nat., p. 414.

This evidently belongs to the genus *Membranipora* of Blainville*, though not included in it by him. It unites *Flustra pilosa* and *dentata*; the only difference between which is in the length of the anterior spine of the cell, a character which varies in every degree even in the same specimen.

Fig. 1. The *animal* in many respects very closely resembles *Halodactylus diaphanus*, but its form is far less elegant. The arms are twelve (rarely eleven) in number, ciliated and furnished with long spines. They are very long in proportion to the body, but thick and rather clumsy, and during expansion are frequently curled inwards at their extremities.

The base of the tentacles appears to be surrounded by a delicate band, which is placed on their outer side as if for the purpose of bringing them together, and immediately within which they unite to form the tentacular ring (fig. 2.). The appearance of a circumferential vessel in the substance of the ring, and the tentacular canals were observed here as in *Halodactylus*.

The pharynx also in every respect confirmed the observations upon that species, especially in its mode of expansion, and in the position of the dark line, the triangular space, &c. The flask-shaped body was here also occasionally observed, but without affording any additional information. It was much larger in proportion to the length

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* Dict. des Sc. Nat. Art. Zooph., p. 411.

of the arms, and was sometimes seen to be much distended and to alter its form occasionally (fig. 3 and 4.).

The cardia is placed about midway between the mouth and the base of the stomach, the whole canal being very short. The intestine terminates in the membranous parietes at a little distance below the ring (fig. 1. *a.*). The separation of the parietes from the cell when the animal emerges is very distinct.

The lateral aperture in the cell is filled by the flexible membranous portion, which does not terminate here by setæ, but has a plain margin (fig. 1. *b.*), forming a close ring round the protruding animal. The cells are met with either isolated or aggregated; in the latter case the growth of the young cells is seen in advance of the older ones, and they appear to spring from the upper and back part of the cells immediately below them (fig. 1. *c.*). The cells are often seen connected by cylindrical stems which do not appear to belong to them.

PLATE XXVII. fig. 6—9.

Notamia loriculata, Flem., Brit. Anim., p. 541. Gen. lx. 158.

Syn. *Corallina cellifera mollis ramosissima*, Ellis, Corall., p. 55. pl. xxi. n. 7. b. B.
Sertularia loriculata, Linn., Gmel., p. 3858. No. 31.
Cellularia loriculata, Pall., Zooph., p. 64. No. 22.
Crisia loriculata, Lamx., Polyp. flex., p. 140. No. 250.
Gemicellaria loriculata, Blainv., Dict. des Sc. Nat., Art. Zooph., p. 425.
Loricaria Europæa, Lamx., Expos. méthodique des genres de l'ordre des Polypiers, p. 7.

Fig. 6 and 7. This is a ramified species very common on the Sheppy coast. The arms are ten in number, ciliated and flexible (fig. 9.). The alimentary canal presents the usual details (fig. 8. *a.*) The pylorus is very distinct, and there is a considerable rectal enlargement. The pharynx is spotted. The gizzard wanting.

Fig. 7. The branches, which are given off generally form opposite points of the main stem, are formed like it of a succession of cells, placed back to back in pairs, the last two or three pairs gradually diminishing in size, with a corresponding degree of development of the contained animals; the terminal pair is generally very small, and apparently homogeneous in texture, and without a trace of its future animal inhabitant (fig. 8. *c.*).

Fig. 8 and 9. From this position of the cells the animals cannot emerge from their extremities, but protrude laterally by the oval aperture in the upper part and side of the cell. This is closed, as in *Membranipora*, by a more flexible portion than that which forms the rest of the cell, (which is only a modified form of the flexible operculum in the foregoing species,) leaving a horse-shoe aperture for the passage of the tentacula and upper part of the body (fig. 9. *a.*). Here the process of gemmation occurs in very regular order, the smaller cells as in *Membranipora*, growing from the upper and inner sides of the larger ones immediately below them.

It is evident that in the construction of the cell, *Membranipora* and *Notamia* are closely allied, and notwithstanding the encrusting character of the former species, and the ramified habit of the latter, it would be easy in imagination to convert the one into the other by supposing two lines of growing cells, such as are often seen in *M. pilosa*, to be attached back to back to each other: from these similar branches arising the encrusting species would be converted into the ramifying one. And the further passage of this to the arrangement in *Flustra* is accomplished simply by the union of a parallel series of such branches.

Such then are the principal facts that have offered themselves to my notice during the investigation of the above described species. They afford evidence of the existence of a very decided type of structure, and one which presents a remarkable uniformity of character, notwithstanding that it was observed in genera differing considerably in less important particulars. To what extent, however, this prevails, and how far it may be modified in other genera, are considerations which must be reserved for a more extended inquiry to determine. Among the points requiring further elucidation, one of the most important is the condition of the nervous system. No trace of either nerves or ganglia could be detected; yet the attributes of a nervous system were so clearly exhibited as to leave no doubt but that this must exist, and probably in some degree of perfection. Not only was the delicacy of their sense of touch very strongly marked, but the operations also consequent upon the enjoyment of such a sense were sometimes singularly striking. This is seen in the instant retiring of the animal on the slightest alarm, and the caution which it sometimes shows before emerging again from its cell*; in the obvious selection of its food; and in the pertinacity with which it refuses to expose itself to water that has become in the least degree deteriorated.

The respiratory system, again, being so intimately connected with the digestive apparatus, it becomes difficult to determine, from witnessing the *combined* operations of the parts, in what degree they contribute to the performance of each function *respectively*. The peculiar action by which the pharynx becomes so frequently distended, and the constancy of the currents produced by the tentacular cilia, (apparently far beyond what would be necessary to afford a *sufficient* supply of food,)

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* On several occasions I observed in *Halodactylus* one or more of the tentacles protruded and turned over the side of the cell before the animal ventured out again, after having been alarmed by the sudden contact of some vibriones that abounded in the water used, as if to ascertain the presence or absence of the intruder (plate xxvi. f. 7. b.); a position of the arms which is also frequently assumed in this species in the act of retiring. So delicate indeed is the sense of touch, that the creeping of a very small animalcule over the top of one of the closed cells was followed instantly by a shrinking of the soft parts beneath.

together with the circumstance that the particles so brought within its reach seem frequently rather a source of annoyance than of advantage to the animal, are points which would encourage the belief that these parts have a considerable share in the process of respiration; but until the existence of a circulatory system and its course shall have been determined, the consideration of this as a respiratory apparatus must yet remain conjectural, though supported by very strong analogies. On the other hand, its use in ensuring a supply of food to the animal cannot be questioned.

The structure and growth of the cells, and their connecting medium, offer many interesting points for consideration. In the ramified and creeping species the cells are connected by a cylindrical stem, which appears nearly homogeneous throughout, and does not present that obvious distinction between hard and soft parts that is observed in the stems of *Sertularia*. If, however, this stem be cut across, especially when decomposition has commenced, a granular matter flows out, leaving the delicate corneous sheath nearly empty. This corneous case of the stem is easily seen to be continuous with the cells that arise from it, but the internal substance cannot be distinctly traced to the animals as in *Sertularia*. It is probable, however, that it passes gradually into the parietes of the body by which the cells are lined. With the facts before us of the progressive growth of the stem, and the production from it of buds or gemmæ, which gradually develop into mature animals, no doubt can be entertained either as to the vitality of this part, or of the direct communication between it and the *young* animals, at least up to the period at which they begin to emerge from their cells and to seek nourishment for themselves. Nor is it reasonable to suppose that this communication ever after ceases, for then it would be impossible to account for the nutrition of the growing parts, and the combined operations by which the regularity of growth of the whole is maintained, as exemplified in the ramified species by the proportionate thickening of the stem to the number of branches which it has to bear, and in the definite forms which each species assumes.

But it might be questioned whether the whole of the stock is a living part, or only the soft interior; while the more dense exterior, together with the cell, might be regarded as a mere exudation from it. From the various phænomena, however, that occur during the growth of these parts, and from the manner in which they are blended in their early state, I am disposed to consider both the one and the other as organized and influenced by one common vitality.

The *two* processes of reproduction here observed offer many points of contrast. That by *gemmæ*, or buds from the common stock, appears to be uninfluenced by season; the young animals, from the earliest period in which form can be traced in them, resemble in some measure the parent; and their subsequent growth is but a development of that form, and at no period are they separated from the parts that produce them. The process by locomotive ciliated *gemmules* is limited to certain seasons, generally spring; these bear no resemblance to the parent; they appear to be the more immediate produce of the individuals, than of the community; and they separate from the parent at an early period, and must undergo metamorphosis before arriving at maturity. This process of reproduction is entitled to be called *gemmuliparous* as contradistinguished from the *gemmiparous* mode.

The gemmiparous mode is precisely similar to that which takes place in the free Hydra. The resemblance is nearest when there is no connecting stem, as in *Membranipora*, *Cellaria*, &c. Here the gemmæ sprout apparently only from the cells, but doubtless also in connection with the parts of the body by which they are lined. When a stem is present the gemmæ do not arise from the cells, but always from this, which is but an extension of the reproductive surface. In either case the buds are at first homogeneous throughout, and their separation into cell, parietes, and alimentary canal, is a subsequent process of growth.

With regard to the mode by ciliated gemmules, it would be important to ascertain the origin and exact condition of these previous to their separation; and also to determine whether they have any relation with the brown bodies so frequently observed in the visceral cavity. These latter I found sometimes, after being kept several days, converted into mere cysts full of living animalcules, which however bore no resemblance to the mature gemmules.

Quitting, then, at this stage of the inquiry the further consideration of this type of structure, it will follow next in order to show what position it will hold with reference to other portions of the animal kingdom; and for this purpose it will be necessary to consider the relative value of its different characters.

In the absence of a knowledge of the condition of the nervous system, the characters, at once the most obvious and important, are derived from the apparatus for entrapping and digesting the prey; and the structure of the tentacula, and the form of the alimentary canal, will of themselves be sufficient to constitute the distinguishing features of this type, connected too as the former appear to be with the very important process of respiration. Indeed the combination of ciliated tentacula with a free alimentary canal, having two external openings, appears so uniform, that the presence of the one being determined, the structure of the other, and indeed more or less of the entire animal, may be fairly inferred.

With this view of the subject I propose for this class the name *Ciliobrachiata*, a name which, by seizing one of the most prominent features, will serve at once to distinguish those animals, to which it is applicable from all inferior types.

The Ciliobrachiata, therefore, will comprehend the fourth family of Polypes of MILNE EDWARDS, the Bryozoa of EHRENBERG, and the Polyzoa of THOMPSON; and in applying a new name to a group of animals previously, but imperfectly, indicated by others, I do so with a wish to stamp upon it those distinguishing features which it has been my object in the present essay to point out, and in preference to adopting others which, as expressive of characters common to it with inferior types, might only tend to carry on the errors that have given rise to so much confusion with regard to the subject.

With a view to a subdivision of this class, after a more natural method than has hitherto been followed in arranging the various forms of Polypes, it would be desirable to regard the varieties presented by the alimentary canal, which from the conspicuous position of the parts are the more easily determined. Thus the presence or absence of the manducatory organ are points of much importance; and the position of the anus may be also worthy of consideration. The structure of the *alimentary canal*, however, will probably be found to present but few essential points of difference, and it may be necessary soon to revert to the form of the *cell* as a secondary means of distinction. The position of the aperture and the character of the operculum then becomes of consequence, especially in its mode of termination, whether by separate spines or by a notched or smooth margin. But if it be found that where the aperture of the cell is *lateral* the manducatory organ is sometimes present, and in other cases absent, as I have shown it to be when the aperture is *terminal*, then the character of the cell must yield in importance to that of the alimentary canal, and the animals be arranged accordingly. With the lateral position of the aperture the operculum is generally simple, and the cells have seldom a distinct connecting medium. But where the aperture is terminal the operculum is more complicated, and the cells are generally united by a ramified or creeping stem; but the passage from the one form of arrangement to the other is shown in *Halodactylus*, in which the cells, though usually placed perpendicularly with their sides in contact, and the aperture terminal, are yet sometimes placed so obliquely as to resemble in arrangement an encrusting species, having the aperture directed laterally. The structure of the cell, however, is not in this case affected by its accidental position.

It will then be of consequence to determine the degree of importance to be attached to those characters which have been erroneously considered primary, namely, those that are derived from the common mass, or polypary. These, however, are generally the most superficial and least important; since a very slight alteration in the arrangement of similar parts will give a very different character to the whole, as exemplified in the readiness with which an encrusting species might be converted into a ramified, and that again into a foliaceous one. But the mode of growth of the stem might occasionally afford useful characters for generic distinctions; thus the definite mode of growth in *Vesicularia spinosa* is contrasted with the irregular arrangement of the cells in *Bowerbankia* and *Lagenella*; whilst trivial characters are readily found in the number of the arms, and similar points of inferior importance.

In natural affinities *Ciliobrachiata* is evidently allied both to *Tunicata* and *Rotifera*. In *Tunicata* the tentacles are reduced to mere rudiments at the entrance to the respiratory sac, and the cilia are distributed over the surface of this cavity, which is in proportion magnified, and is analogous to the pharynx of *Ciliobrachiata*. The more immediate entrance to the alimentary canal, thence called mouth, being situated at the bottom of this sac, corresponds with the part that I have called *cardia*; and the analogies between the remaining course of the alimentary canal, position of the ovary, nature of the external covering, and other points of resemblance between the two classes, are easily traced.

But between *Ciliobrachiata* and *Rotifera* the affinities are still nearer. Taking *Hydatina* as the representative of the latter class, the cilia (which, however, differ in form) are placed on short lobes instead of arms. The pharynx is very short, and leads at once to the manducatory organ, which guards the entrance to the stomach, as in *Bowerbankia*. The muscular apparatus for altering the form of the body is identical in the two classes, and in the general character of the body and position of the contained parts there is a very close resemblance. They vary, however, in their mode of reproduction, position of the anus, and other points.

The Ciliobrachiate polypes being thus separated from the rest of their associates by characters well defined and easily recognized, there yet remain two other types, which may be represented by *Hydra* and *Actinia*, the *Hydriform* and the *Actiniform*, or *Zoanthiform* polypes.

Of these two, the Hydriform polypes, whilst they are the furthest removed from *Ciliobrachiata* in degree of organization, are nevertheless those which have been most frequently confounded with them. For it would appear that in the lower type the superficial characters of the higher are sketched, as it were, in outline; so that whilst they are found to differ materially in intimate structure, there yet remains a sufficient resemblance in external configuration to have caused them to be confounded together. The Hydriform polypes may be recognized by the granular structure of the body, by the entire absence of a stomach distinct from the parietes, by the single external opening to the cavity, and the absence of cilia from the tentacula.

The granular parenchyma of the body having a contractile power in every part, the alterations in its form and dimensions are effected without the necessity for a distinct muscular apparatus. No folding of the body takes place when the animals withdraw into their cells, where they are still left more or less exposed from the absence of a distinct operculum. The food is received at once into the main cavity of the body which constitutes the stomach, there to be acted upon by the granular parietes; and whilst the egesta escape by the same orifice by which they were taken in, as in *Hydra*, the nutrient particles have been traced to the tentacula in the free animals; and in the compound ones as flowing in a stream through the tubular fleshy medium which communicates with the stomach, and by which all the animals are united.

As in the higher type the tentacular cilia appear to be concerned both in nutrition and respiration, so their absence in the present case must be viewed with reference to both these points. With regard to the greater choice of food afforded by the action of cilia so placed, this appears to be less necessary where the body is of such a structure as to be capable of accommodating itself to prey of a greater variety of size; whilst, for purposes of respiration, the exposure of the entire surface of the body to the water, which has free access to the cell, may be sufficient, without the necessity for that constant renewal of it that a more complicated organization appears to require.

The absence of the ciliary character of the arms, in the present case, appears to be as uniform as its presence in the former; and as so much of the economy of the animal turns upon this single point, one more characteristic could not perhaps be selected. I propose, therefore, to unite all those animals, which, partaking of the nature of *Hydra*, present this character, into a second class, to which the name *Nudibrachiata* might be applied.

This class will comprehend the second family of polypes of Edwards, and the *Dimorphœæ* of Ehrenberg. The more interesting forms of it have been so well illustrated by the very interesting descriptions and figures of Mr. Lister, in the paper already referred to, that I have no occasion to add any here.

The points in which *Nudibrachiata* approach nearest to *Ciliobrachiata* are (with the exception of the mere contour of the body), the general habit and mode of growth, and the process of reproduction. The former character I have shown to be extremely superficial, and one that should be considered as among the least important in a natural arrangement. It presents but little essential difference in the two classes. In the structure, however, of the more solid parts there appears to be a deficiency of the earthy material in the lower class, which in the higher is blended in greater or less quantity with the horny matter. The gemmiparous and gemmuliparous mode of reproduction appears to be similar in the two classes.

The *Nudibrachiata* will probably continue to hold that position in the animal kingdom which has been usually assigned to the entire class *Polypi*.

The class *Polypi* being thus deprived of two of its principal divisions, which, whilst they resemble each other so much in superficial character, as to require the aid of the microscope to distinguish them, in their intimate structure hold the two extreme positions; the third division only remains. But this is by far the most extensive, and the animals are seldom or never so small but that their characters may be readily discerned by the unassisted eye, while many attain a considerable size.

This division corresponds with the third family of Polypes of Edwards, and forms the *Anthozoa* of Ehrenberg, (deprived of the *Hydræ* and *Sertulariæ*,) as last constituted by him. In this state I shall leave it, merely adding a few remarks on its natural affinities which necessarily arise out of the consideration of the two former classes, and in order to complete the view of the subject.

The *Anthozoa* will comprehend the corticiferous polypes, together with the free and associated *Actiniæ*, and indeed all those forms to which the familiar term "Animal Flower" has been most frequently applied.

The body is here distinctly membranous, and the stomach forms a separate pouch suspended in its centre. The stomach has but *one* external orifice, which serves for mouth and anus; but posteriorly it communicates with the main cavity of the body. This, in *Actinia*, is divided perpendicularly by septa, passing from the stomach to the sides of the body; and with the chambers thus formed the short tubular arms that are set round the mouth communicate. In these tubular processes a constant circulation of fluids may be observed passing up from the chamber of the body and returning to it again. They appear to answer the double purpose of prehensile tentacula and of respiratory tubes. They are not ciliated externally*, and in form have little resemblance to the tentacles of the other two classes. In this respect, however, they vary very much, being, in some forms of *Actinia*, arranged simply in one or more circles round the mouth; in others, elevated upon semilunar lobes; whilst, in other instances, as in *Xenia*, *Gergonia*, &c., these lobes may be supposed to be drawn out into the conical or cylindrical arms, having a dentated margin; in which cases the whole arm does not correspond with a single arm of *Actinia*, but each of the dentiform processes upon its sides. The character of distinct ovaria producing ciliated gemmules appears to be very prevalent through this class.

The *Anthozoa*, then, are distinguished from *Nudibrachiata* chiefly by the separation of the stomach from the parietes of the body, which has a membranous character; and from *Ciliobrachiata* by the single external opening, and the absence of cilia from the surface of the arms. They appear to hold a place immediately below *Acalepha* and *Echinodermata*, the transition between these three classes being exceedingly gradual.

Thus, with *Asterias*, the affinities are easily traced. The single external opening to the membranous stomach is found equally in *Actinia* and *Asterias*; but while in the former this organ communicates posteriorly with the main cavity of the body, in the latter it is closed in this position, and the immediate communication cut off. Again, in the conical arms of the corticiferous polypes, with their fringe of tubular processes, may be traced the analogue of the rays and respiratory tubes of *Asterias*; both are distended by the fluids which circulate through them, probably for respiratory purposes. Moreover, in the position and form of the ovary the closest resemblance exists; and when to these points is added the stem of the crinoid animals, the affinities between the two classes are rendered still more striking.

Again, between *Anthozoa* and *Acalepha* analogous points of resemblance might be traced; and here again the transition appears so gradual that it might be difficult to determine where the one ends and the other begins.

Thus, then, it appears that under the commonly received name of *Polypi* there exist *three* distinct types of structure, which must be referred to the same number of separate classes, possessing but few points in common, and those generally of the most superficial kind, but which have nevertheless induced naturalists, from the want of a sufficient degree of attention to their intimate structure, to group together, in accordance with such superficial resemblance, animals that have no title to be classically associated.

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* It might be objected that as the arms of *Anthozoa* are not ciliated, at least externally, the term "*Nudibrachiata*" is equally applicable to this class. The *Anthozoa*, however, could never be confounded with either of the other two classes, to the *mutual* distinction of which the names that I have applied to them have reference.

Ciliobrachiata

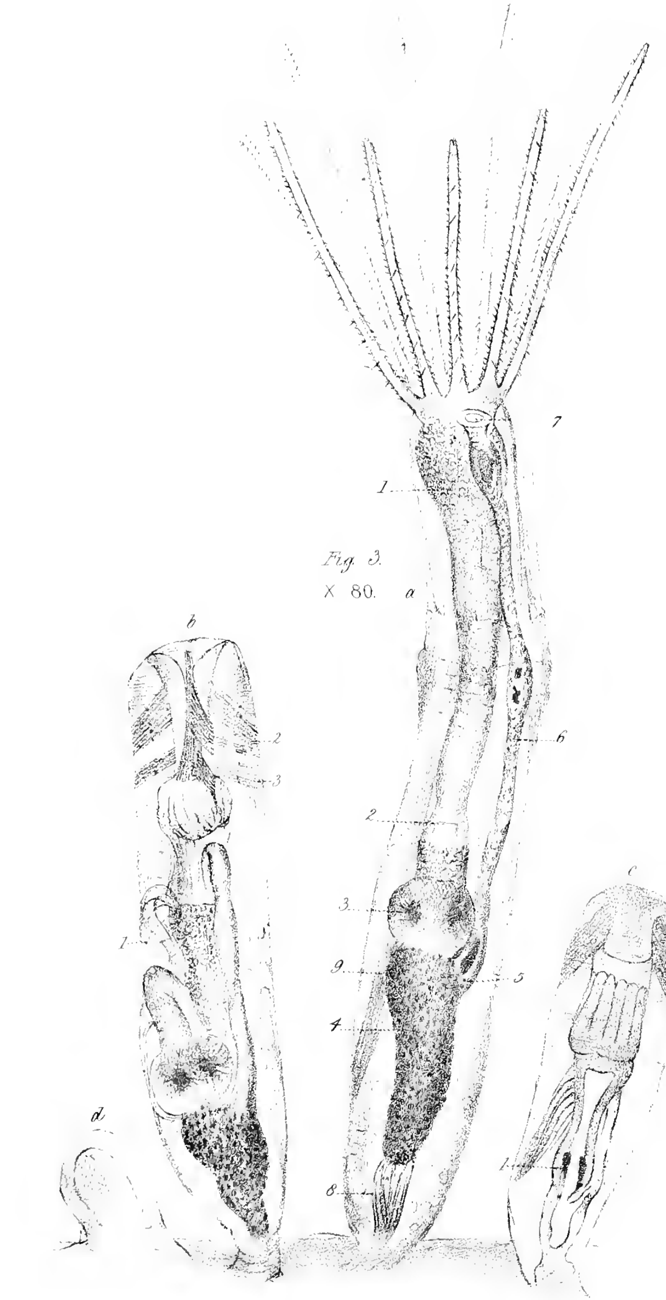


Fig. 3.
× 80.

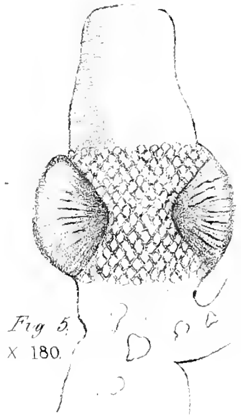


Fig. 5.
× 180.

Fig. 4.
× 80.

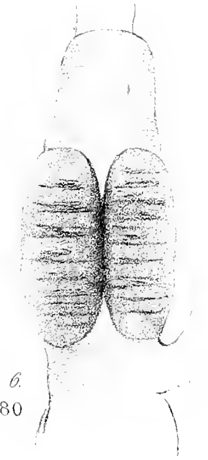


Fig. 6.
× 180.

Fig. 1.

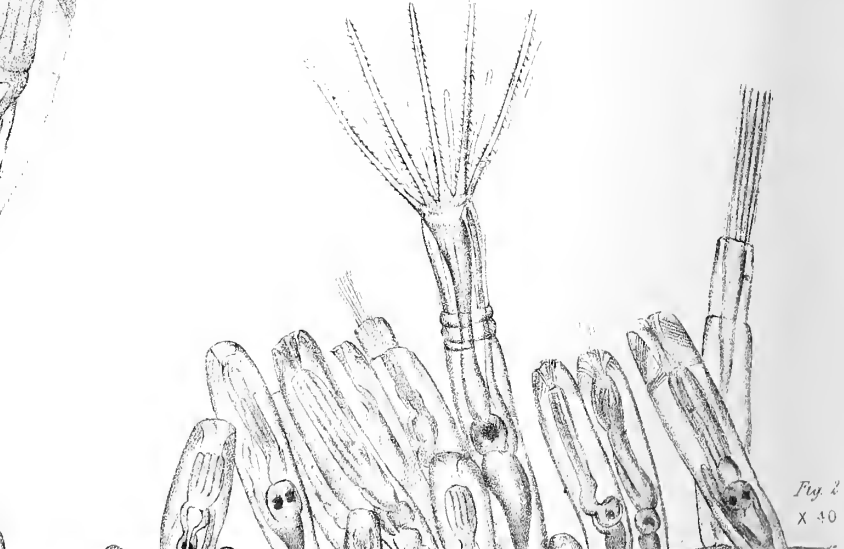


Fig. 2
× 40

Parasitic on Flustra foliacea

Bowerbankia densa

Arthur Farre del.

J. Basire

Fig. 15. X 100

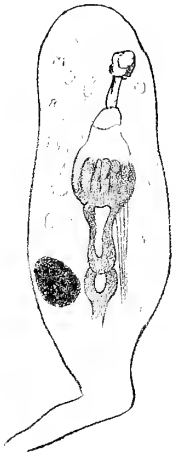


Fig. 14. X 100.

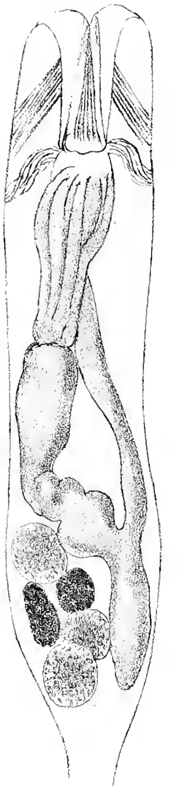
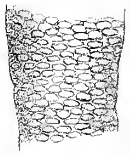


Fig. 8.



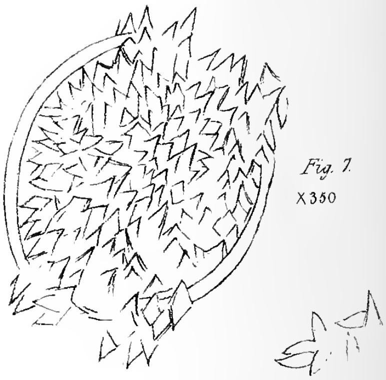


Fig. 7. X350

Fig. 16.

Fig. 12.

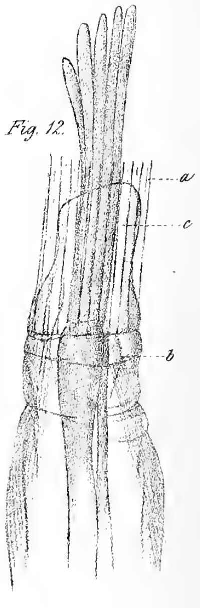


Fig. 13.

Fig. 11.

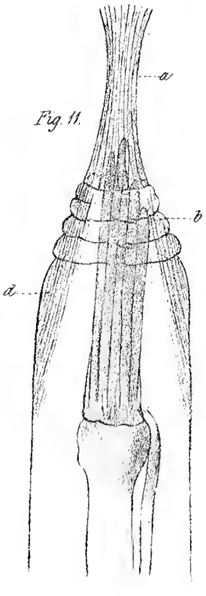


Fig. 10.

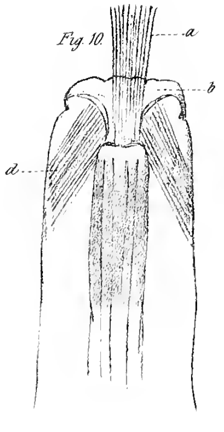
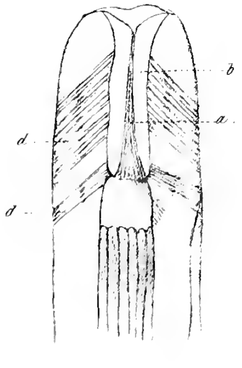


Fig. 9.



Bowerbankia densa.

Ciliobrachiata

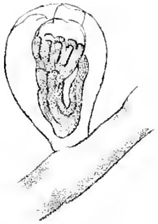


Fig. 6.
X 100.

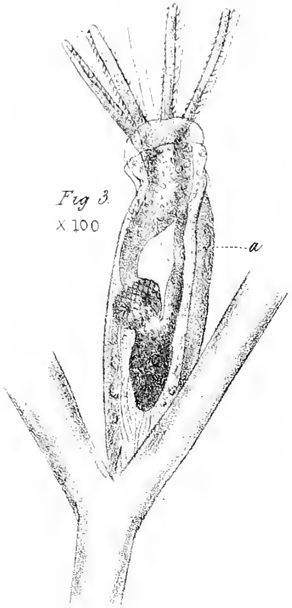


Fig 3.
X 100

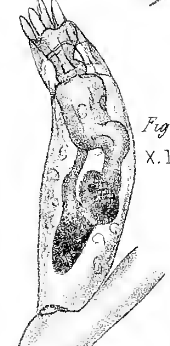


Fig. 4.
X. 100

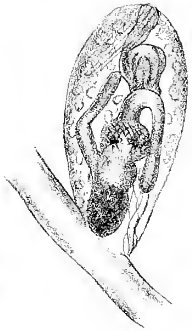


Fig. 5.
X 100

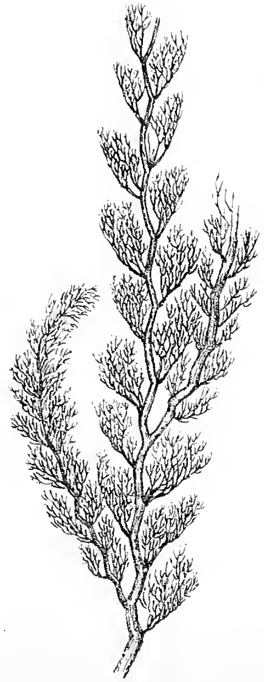


Fig. 1.
X 3

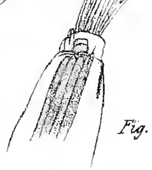


Fig. 7.

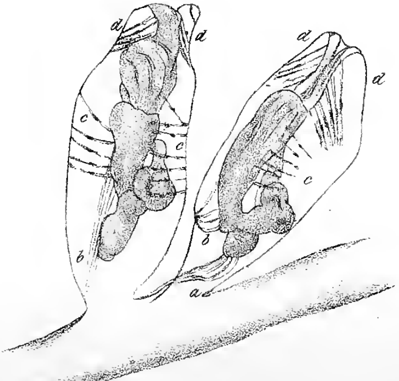


Fig. 8.
X 100

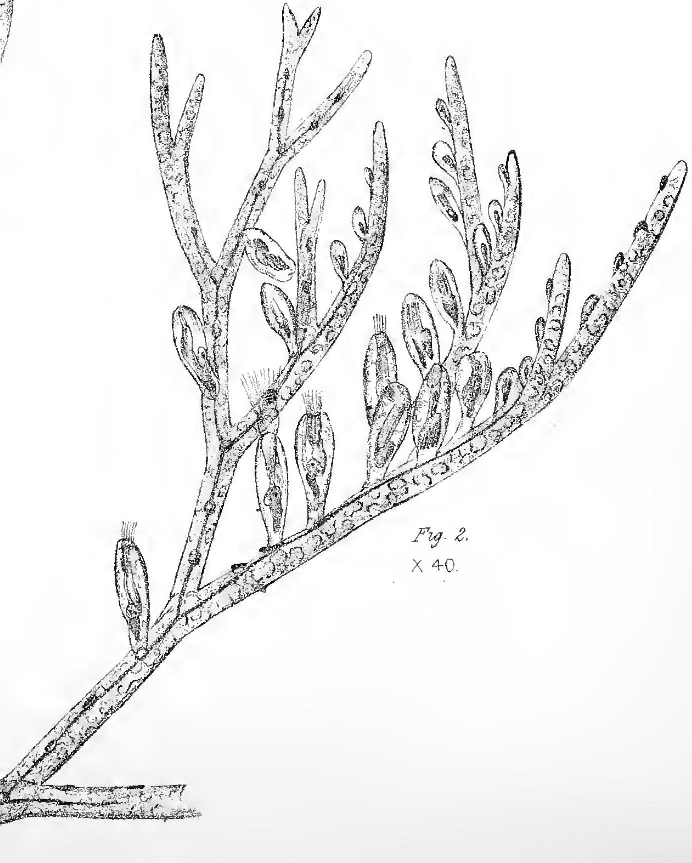


Fig. 2.
X 40.

Arthur Farre, del.

J. Basire, lith.

Vesicularia spinosa. Thomp.

Ciliobrachiata.

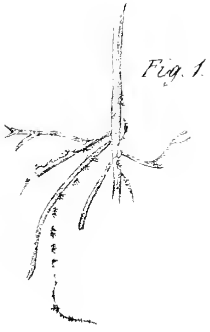


Fig. 1.

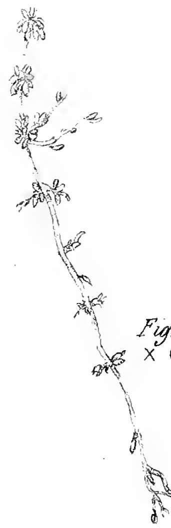


Fig. 2.
× 6.

Fig. 6.

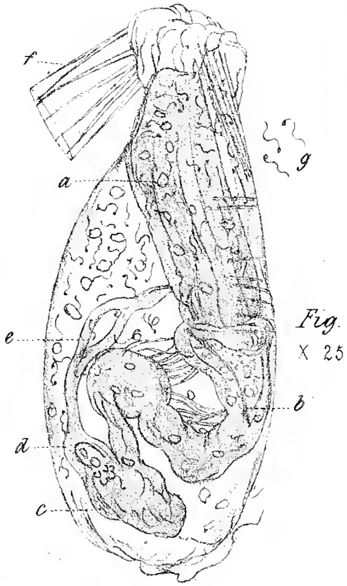


Fig. 5.
× 250.

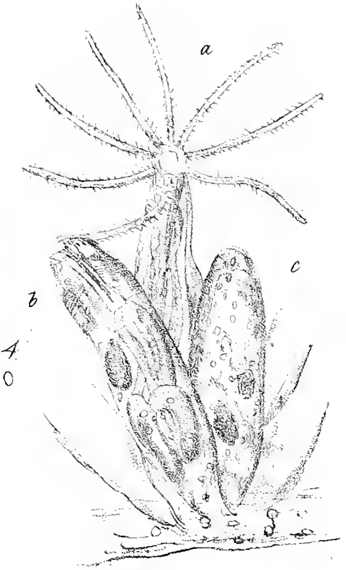


Fig. 4.
× 100

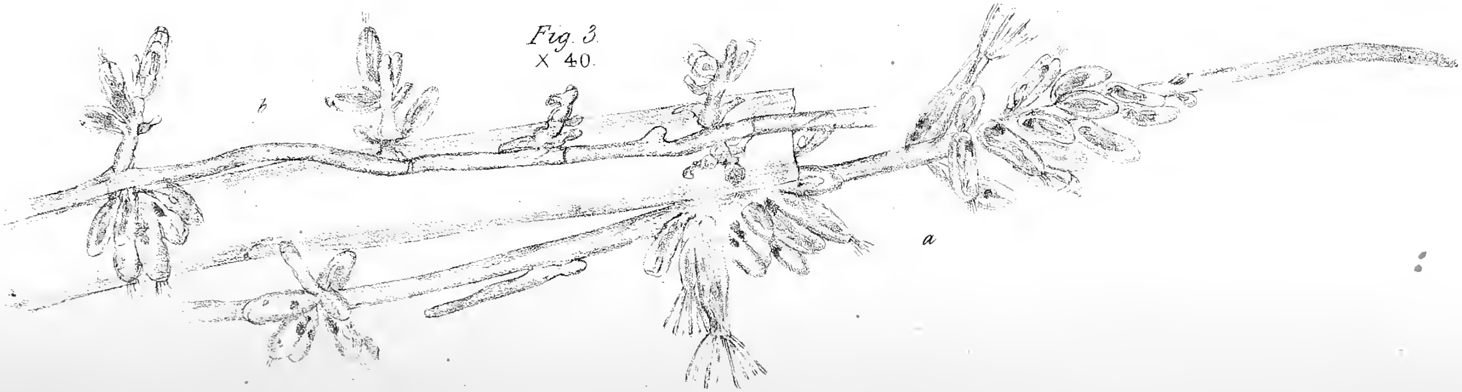


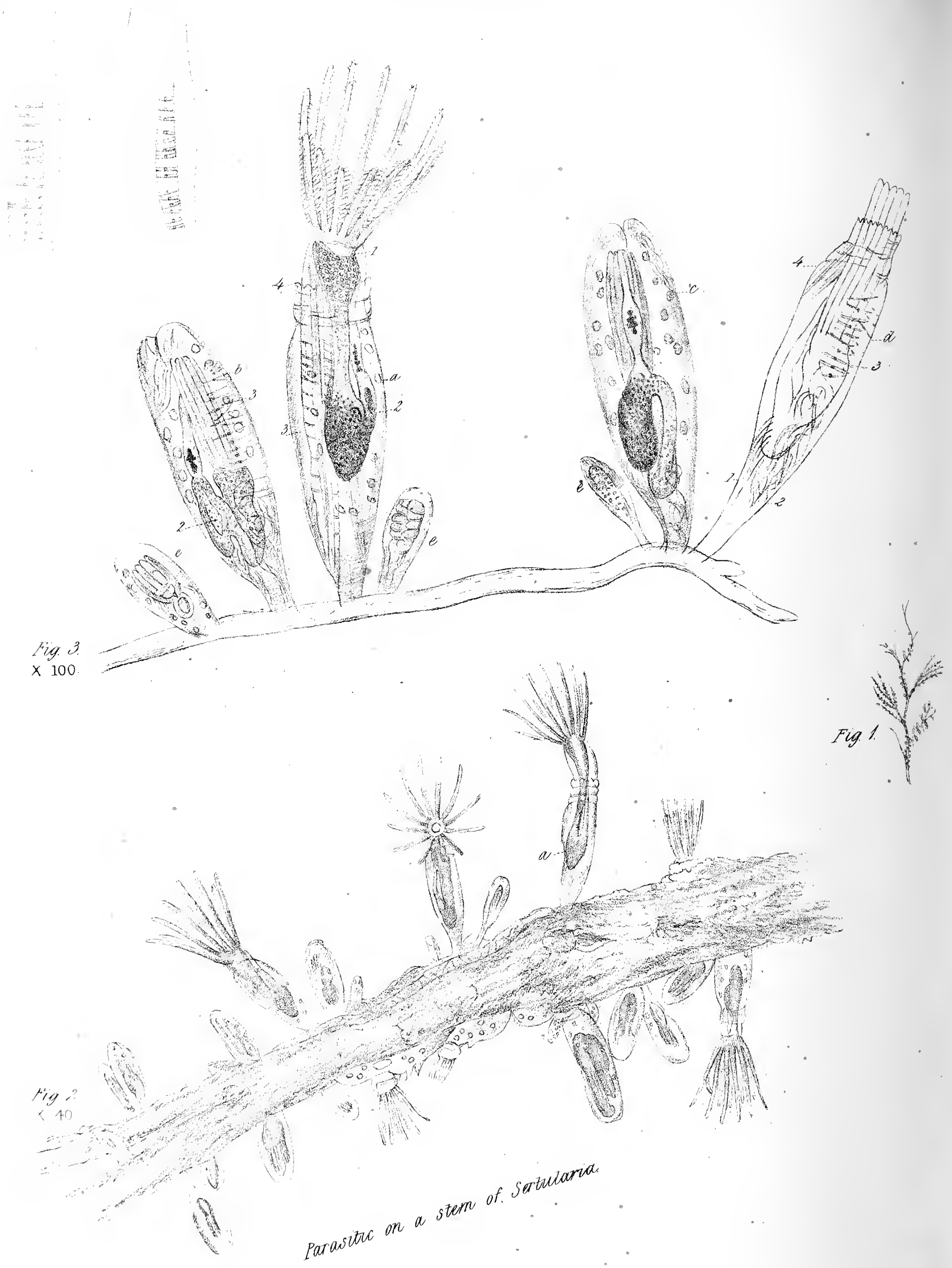
Fig. 3.
× 40.

Valkeria cuscuta. Flem.

Arthur Farre del.

J. Basire lith.

Ciliobrachiata



Lagenella repens.

Arthur Farre del.

J. Basir

Ciliobrachiata.

Fig. 2

X 40

Fig. 1.

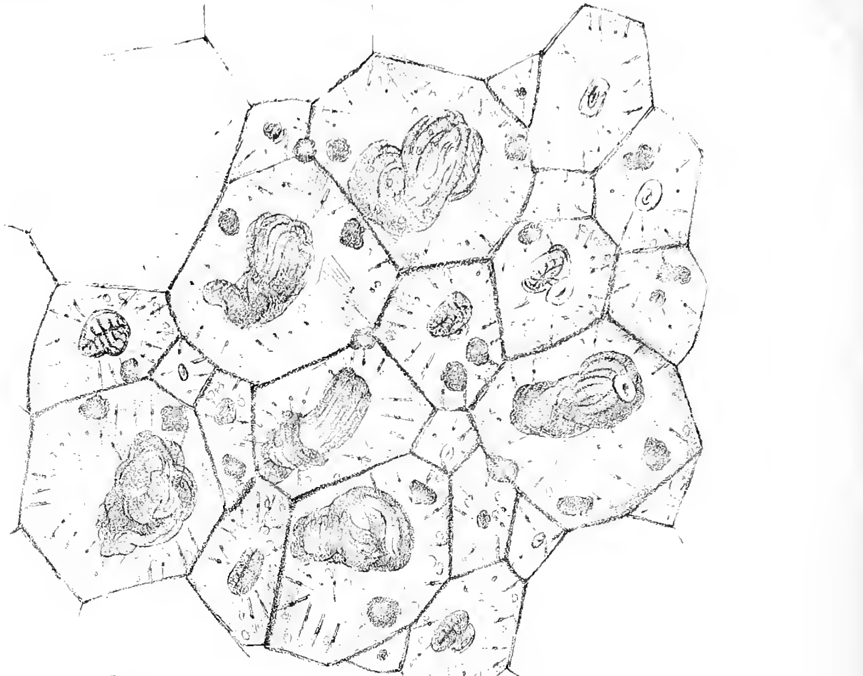


Fig. 4.

X 80

Fig. 3

X 40

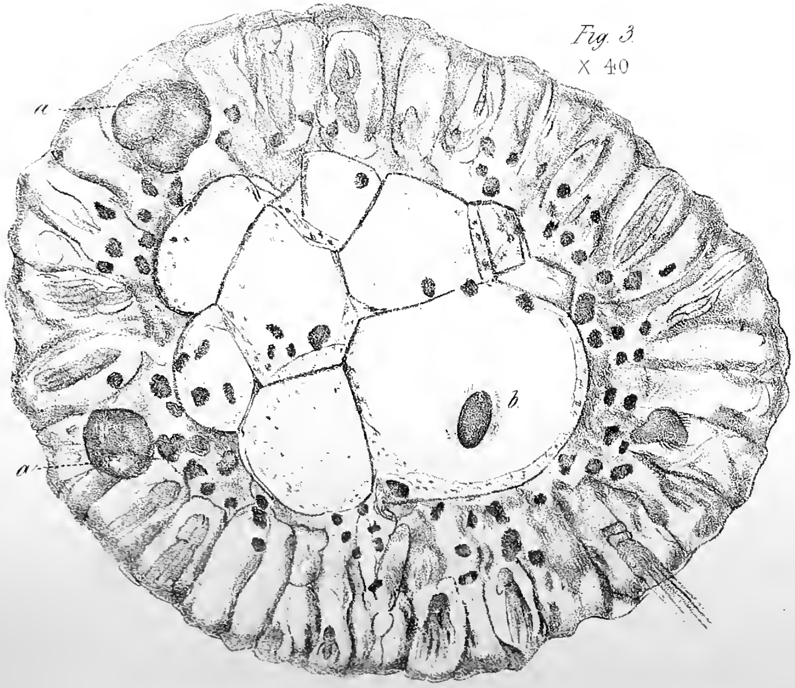


Fig. 6.

X 100

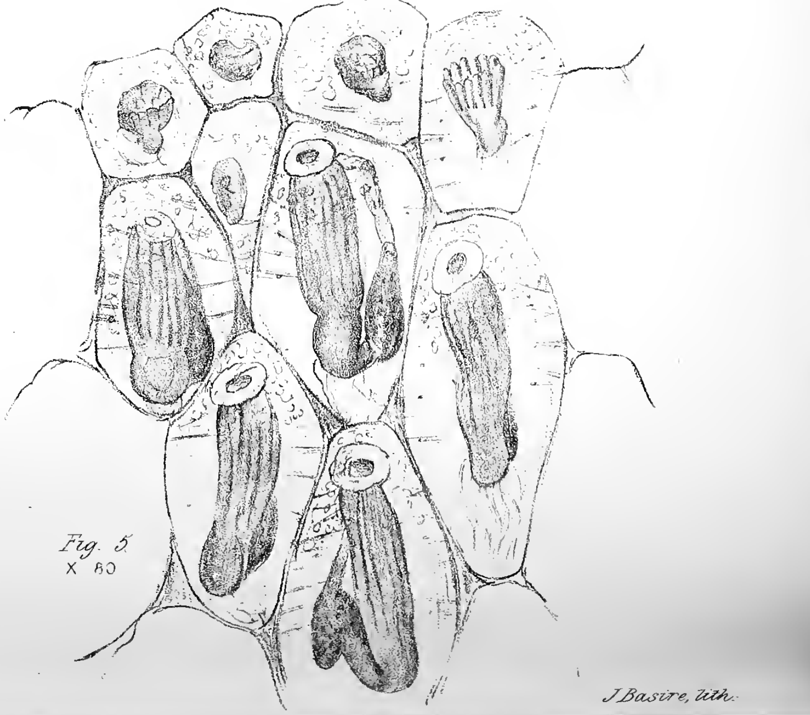


Fig. 5.

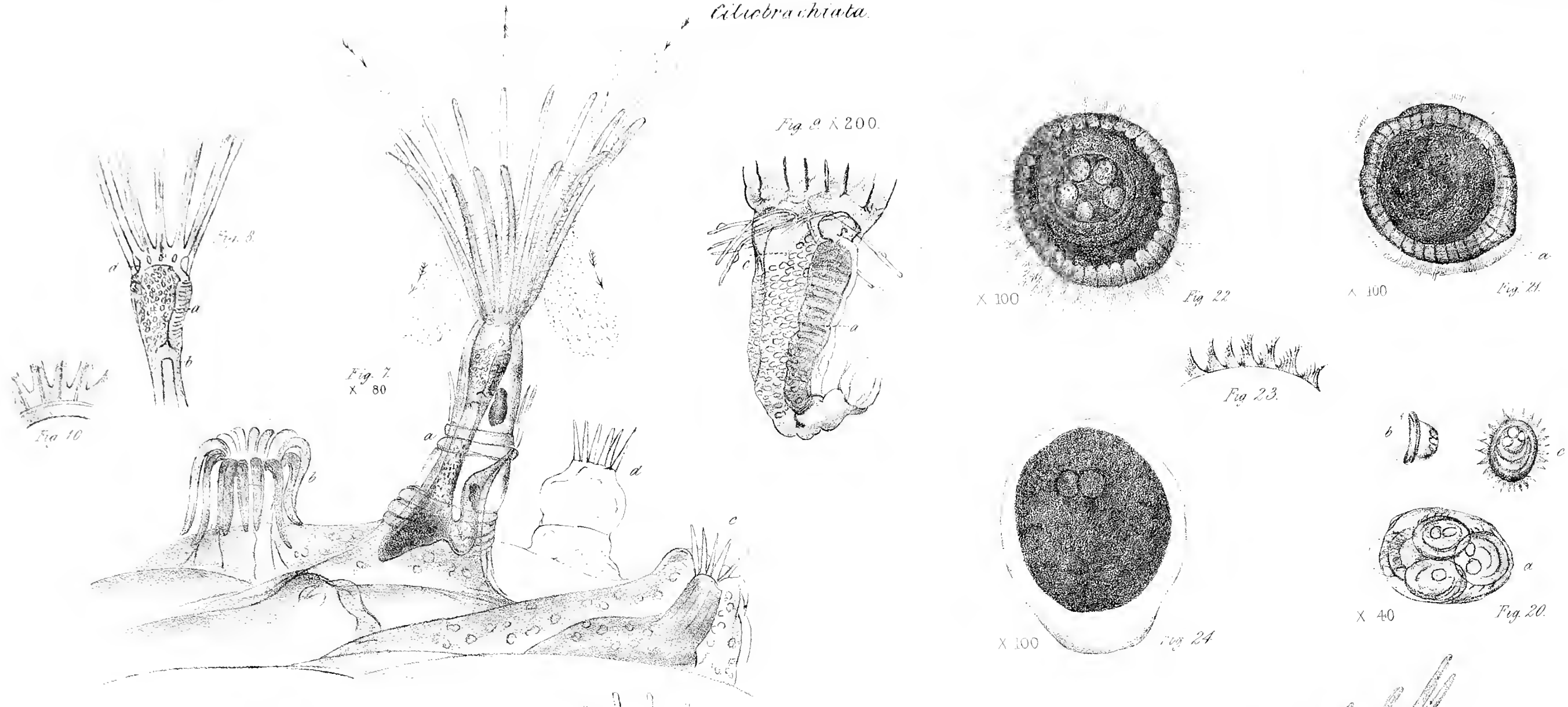
X 80

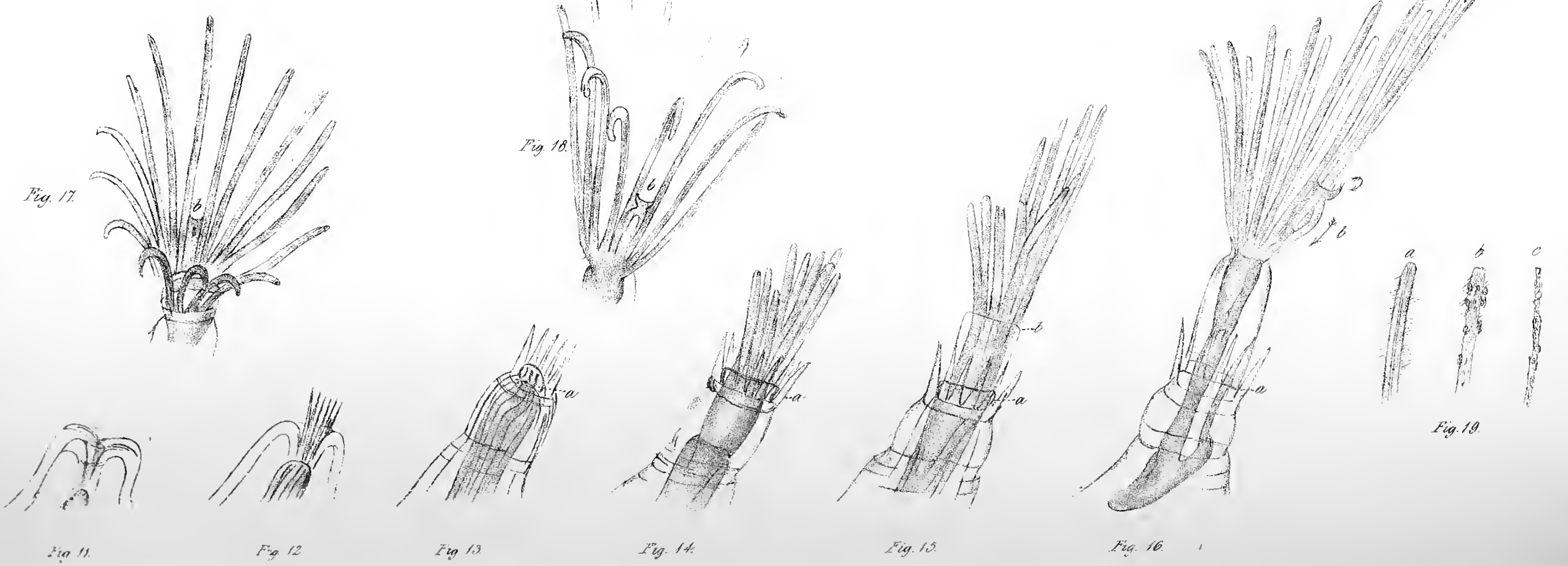
Halodactylus diaphanus.

Arthur Farre del.

J. Basire, lith.

Ciliobrachiata.





Halodactylus diaphanus.

Arthur Farre, del.
J. Basire, lith.

Phil. Trans. MDCCCXXXVII. Plate XXVII. p. 421.

Ciliobrachiata.


Fig. 6.

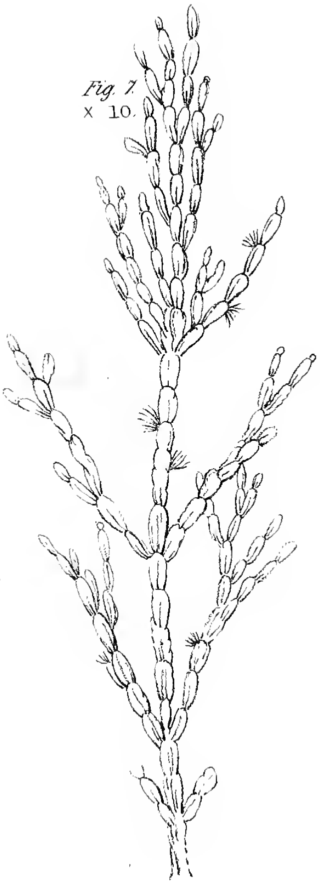


Fig. 7. × 10.

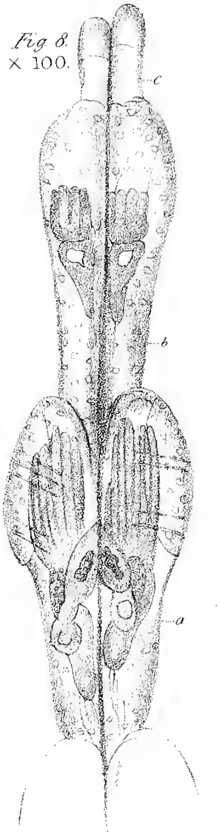


Fig. 8. × 100.

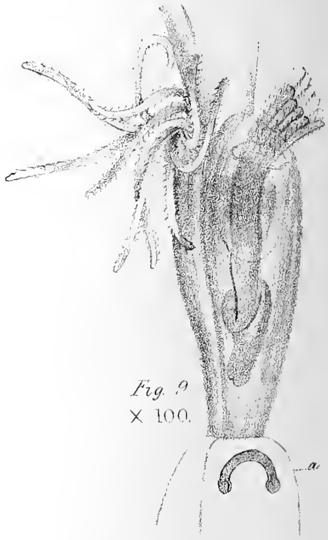


Fig. 9. × 100.

Notamia loriculata. Flem.

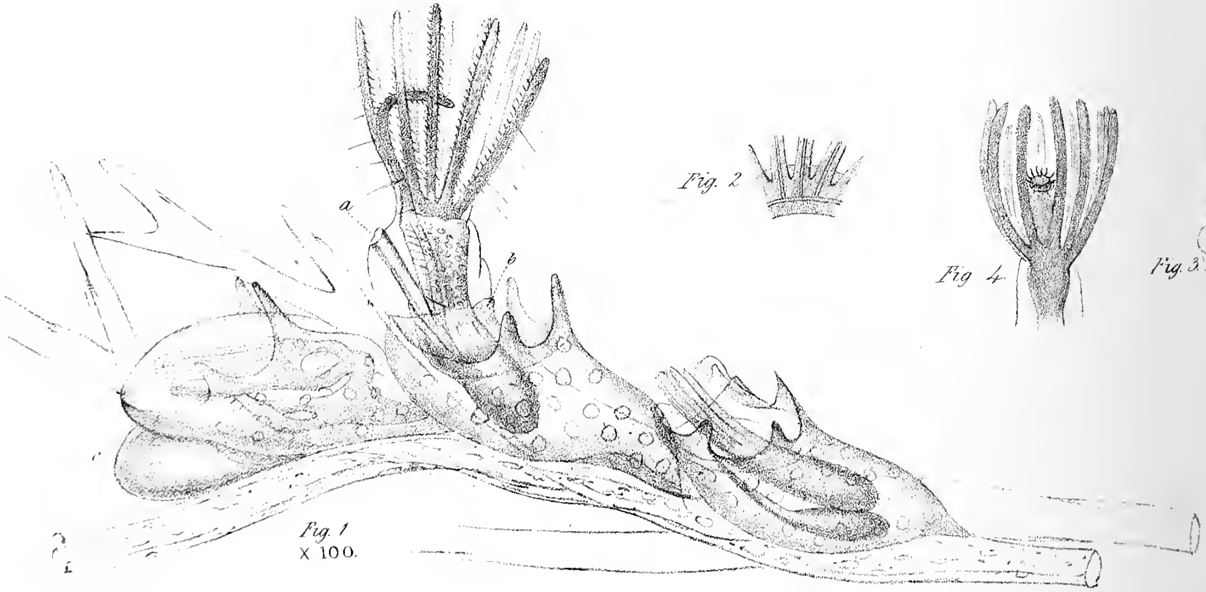


Fig. 1. × 100.


Fig. 2.

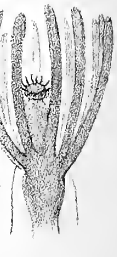


Fig. 4.


Fig. 3.

Arthur Farre del.

J. Basire lith.

Membranipora pilosa. Blainv.

Description of the PLATES*.

The linear enlargement of each figure is expressed by the numbers with the sign × prefixed.

CILIOBRACHIATA.

PLATE XX. and XXI. fig. 1 to 16. p. 391.

Bowerbankia densa.

Fig. 1. *a.* A specimen of the natural size, with the creeping stems closely set upon a piece of *Flustra foliacea.*

b. A portion of the same separated.

Fig. 2. × 40. The same as fig. 1. *b.* The animals are seen in various positions, and in all stages of growth.

Fig. 3. × 80. *a.* One of the animals fully expanded. 1. Pharynx. 2. Cardia. 3. Manducatory organ, or gizzard. 4. Stomach, its parietes studded with the hepatic follicles. 5. Pylorus. 6. Intestine, containing pellets of feculent matter. 7. Anus. The gastric (8) and tentacular (9) retractors are seen within the cavity of the body. The flexible portion of the cell, or the operculum, is seen expanded and surrounding the upper part of the body.

b. A similar animal completely retracted. The stomach drawn to the bottom of the cell. The upper portion of the alimentary canal flexed. The tentacula somewhat distorted by the pressure of the operculum. Their retractor filaments relaxed, 1. The upper part of the cell is occupied by the operculum folded up in its axis, and from it the upper and lower sets of opercular retractors are seen radiating, and in their contracted state. 2 and 3. These filaments are about $\frac{1}{6,500}$ inch diameter in this state.

c. An immature animal. The tentacula and alimentary canal rudely formed; the cavity in the latter very distinct. The tentacular and opercular retractors also shown. 1. The gizzard.

d. One of the gemmæ in its earliest state. The cavity just defined, but no animal distinguishable.

Fig. 4. × 80. Portion of the alimentary canal, showing very distinctly the cardia. Gizzard with the dark points, radiating lines and teeth. Hepatic follicles and pylorus.

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* In the representation of specimens of the *natural size,* I have in most cases selected such small portions as may be just sufficient to afford a recognition of the species, to avoid encumbering the plates with points which are of minor importance, and which, moreover, would be unsuited to pages not devoted to zoological subjects. This remark is particularly applicable to the figure of *Halodactylus,* Pl. XXV. 1., which conveys no idea of the size to which the aggregate masses sometimes attain.

Fig. 5. × 180. The gizzard relaxed, with the dark bodies or constrictor muscles seen on opposite sides, and projecting into its cavity. The inner surface lined with teeth.

Fig. 6. × 180. The same in a state of contraction. The dark bodies elongated and brought into apposition, obliterating the cavity. (Both these figures as seen when the interior of the organ is in focus.)

Fig. 7. The gizzard torn open, and the teeth displayed.

Fig. 8. Portion of the pharynx, with the markings upon its surface.

Fig. 9 to 11. A series to show the mode in which the operculum and upper part of the body is unfolded. The same animal is represented in four different stages.

Fig. 9. *First stage.* The top of the cell completely closed. The setæ, folded up in the centre, 9 *a*, with the flexible portion of the cell *b* inverted and closely surrounding them. The muscles contracted, *d d*.

Fig. 10. *Second stage.* The bundle of setæ *a* rising from the centre of the cell, being forced upward by the pressure of the tentacula. The flexible portion *b* rolling from around the setæ, and the muscles *d* put upon the stretch.

Fig. 11. *Third stage.* The flexible portion *b* completely everted. The setæ *a* still lying together. The tentacles just appearing between them.

Fig. 12. *Fourth stage.* The tentacula appearing above the margin of the operculum. The *integument of the body*, which forms the tentacular sheath half everted, *c*. The operculum completely expanded. The letters refer to the same parts in all. The *last stage* is seen in fig. 3. *a*. where the eversion of the integument is complete and the tentacula separated. (These stages are taken arbitrarily, the process being continuous.)

Fig. 13. The parietal muscles which assist in the act of protrusion. The knot is seen in the centre of each.

Fig. 14. Showing the position of the brown and white bodies. (Query, ova.)

Fig. 15. The same in a young animal.

Fig. 16. *a*. Two of these separated from the body.

b. More highly magnified, showing the external membranous sac with contained granular matter.

Plate XXII. p. 401.

Vesicularia spinosa.

Fig. 1. × 3. A portion of the stem with the branches springing from its angles.

Fig. 2. × 40. Small portion of a growing branch. The cells strictly unilateral, and

showing a regular gradation in growth. The apertures are seen left by the falling off of the cells, and below them the apparent line of connexion. The stem occupied by granules.

Fig. 3. × 100. The animal expanded (not very perfectly). The arms and alimentary canal short compared with the former species. The gizzard and other parts the same. The parietes seen separated from the cell at *a*.

Fig. 4. A similar animal in the act of protruding.

Fig. 5. Another completely retracted. The parietal granules seen in each of these figures.

Fig. 6. A young animal.

Fig. 7. Upper half of a cell, showing the setæ.

Fig. 8. Two animals, showing the various muscles.

a. Gastric retractors.
b. *b*. Tentacular retractors.
c. *c*. *c*. Parietal muscle.
d. *d*. *d*. *d*. Opercular retractors.

PLATE XXIII. p. 402.

Valkeria cuscuta.

Fig. 1. Natural size. Growing on *Ceramium*, like Dodder.

Fig. 2. × 6. A similar portion separated.

Fig. 3. × 40. Two portions showing the arrangement of the cells. At *a* in groups surrounding the stem, and sessile. At *b* raised upon short branches from each joint. On a portion of *Ceramium*.

Fig. 4. × 100. Group of animals.

a. Expanded
b. Retracted.

This figure shows also the opercular retractors, the brown body and the parietal granules. These two latter are also seen in the cell *c*, which in other respects is empty.

Fig. 5. × 250. Single animal magnified to show the cercariæ in the cavity of the body.

g. Separate cercariæ.
a. Tentacula.
b. Line in the pharynx.
c. Stomach.
d. Pylorus.
e. Rectal enlargement of intestine.
f. Operculum protruding.

Parietal granules and muscles distinct. The *gizzard wanting* in this and all the following species.

Fig. 6. Empty cell showing the operculum terminating in setæ.

Plate XXIV. p. 403.

Lagenella repens.

Fig. 1. Natural size, parasitic on *Sertularia.*

Fig. 2. × 40. Portion of the same. The animals in various positions and stages of growth, connected by a creeping stem, and mixed with *Membranipora pilosa.*

a. Side of the cell indented by the pressure of the surrounding fluid when the animal protruded.

Fig. 3. × 100. *a.* Animal protruded. 1. Triangular space in the pharynx. 2. Stomach *during digestion.* The hepatic follicles *large and very distinct.* The food tinged with their secretion. The pylorus well marked. 3. Parietes separated from the sides of the cell by the action of the parietal muscle. 4. Notched margin of operculum.

b. and *c.* Two animals retracted. The operculum drawn towards one side by the single retractor. The stomach suspended by the intestine from the top of the cell; at *b* 2. the stomach is seen *pale and empty,* and the hepatic follicles *barely visible.* *b* 3. Parietal muscles.

This figure (*b.*) exhibits the appearance *invariably* presented by all the animals of this class when they have remained for a day or two without emerging from their cells, and consequently without a fresh supply of food; emptiness of the stomach, being in every case accompanied by a pale and nearly transparent state of its parietes, and a reduction of the follicles to the finest points; while the full stomachs appear as represented at *a.* These characters are so striking when pervading an entire specimen, as at a glance to furnish the observer with a ready test of the purity of the water employed, and of the degree of vigour of the little animals under examination.

d. Figure showing the muscles. 1. Gastric. 2. Tentacular and 4. Opercular retractor. 3. Two sets of parietal muscles.

e. e. e. Gemmæ in various stages.

Fig. 4. Parietal muscles in a state of relaxation.

Fig. 5. The same in a state of contraction. Their diameter doubled.

PLATE XXV. and XXVI. p. 405.

Halodactylus diaphanus.

Fig. 1. A small specimen of the natural size. The animals seen protruding from its surface.

Fig. 2. × 40. The appearance presented by a *lateral view* of the *surface*. The animals in various stages of protrusion. Their diameter doubled.

Fig. 3. × 40. Thin *transverse* section of fig. 1. The centre occupied by a cellular tissue and water. The circumference formed by the cells in close apposition. The brown bodies scattered through the substance.

a. a. Position of the gemmules, enclosed in the sac.

b. One of the gemmules escaped during the section into the central tissue.

Fig. 4. × 80. Thin slice from the surface, with a *terminal view* of the cells, showing their mode of arrangement, and various stages of growth.

Fig. 5. × 80. A similar slice from a specimen which afforded a *lateral* view of the cells, as in *Flustra*. The muscles and granules distinct in both figures.

Fig. 6. × 80. A young animal extracted, rudely formed. The cavities in the alimentary canal and arms are very distinct.

Fig. 7. × 80. Group of animals. (*Lateral* view of the *surface* as in fig. 2.)

a. Fully expanded. The arrows mark the direction of the particles of carmine, in the currents produced by the ciliated arms.

b. The arms turned over the margin of the cell as if to feel, preparatory to the entire protrusion of the animal.

c. Animal nearly retracted.

d. Empty cell.

Fig. 8. Pharynx, with portion of tentacula.

a. Pharyngeal line.

b. Cardia.

d. Constricted portion of pharynx; above are seen the tubes in the arms and the apertures at their base.

Fig. 9. The pharynx, with squamiform spots, triangular space, *c.* and dark line, *a.*

Fig. 10. Appearance of a circumferential vessel in the tentacular ring.

Fig. 11 to 16. A series illustrating the act of protrusion at different stages; at *a.* figs. 14, 15 and 16 is a circular band within the setæ, apparently for the purpose of bringing them together, seen contracted at *a* 13. *b* 15. Integument half inverted, forming tentacular sheath.

Fig. 16. *b.* Flask-shaped body. Lateral view. The arrow denotes the course of the ciliary motion. This specimen shows the oblique termination of the tentacula.

Fig. 17. *b.* Posterior view of flask-shaped body.

Fig. 18. *b.* Anterior ditto. (The cilia are not represented in these latter figures; they are seen in fig. 7.)

Fig. 19. *a.* Portion of recent arm. The cilia very numerous.

b. c. The same when perishing.

Fig. 20. × 40. *a.* Sac containing gemmules.

b. Single gemmule in the act of swimming, with the cilia curved downward.

c. The same viewed from above. The *waves* of cilia appearing as single cilia.

Fig. 21. × 100. A gemmule seen from the under surface. The greater part of the cilia folded up. The muscular lobes shown.

a. The cilia that have a distinct motion from the rest.

Fig. 22. The same from above. The cilia as when slowly acting round the margin in waves. The muscular lobes more distinct.

Fig. 23. The appearance of the cilia when in rapid action.

Fig. 24. The same after forty-eight hours, when it has become fixed and the formation of the cell commenced.

PLATE XXVII. fig. 1 to 5. p. 412.

Membranipora pilosa.

Fig. 1. Three animals in different positions. A fourth growing at *e*.

a. Anus.

b. Margin of operculum. These were parasitic on the filaments surrounding the stem of *Vesicularia spinosa.*

Fig. 2. Circumferential and tentacular canals.

Figs. 3 and 4. Flask-shaped body.

Fig. 6 to 9. p. 413.

Notamia loriculata.

Fig. 6. A branch of the natural size.

Fig. 7. × 10. Portion of ditto.

Fig. 8. × 100. *a.* Animal fully formed. Pylorus and rectal enlargement very distinct, the latter distended by feculent matter.

b. Young animals.

c. Buds yet homogeneous in texture.

Fig. 9. Animals in different stages of protrusion.

a. Horse-shoe aperture in membranous operculum.

XXIII. *On the Ipoh or Upas Poison used by the Jacoons and other Aboriginal Tribes of the Malay Peninsula. By Lieut.* T. J. Newbold, *A.D.C. to Brigadier-General* Wilson, *C.B. Communicated by* P. M. Roget, *M.D. Sec. R.S.*

Received January 26,—Read June 15, 1837.

To tip the slender arrows propelled from the *Súmpitan*, or blow-pipe, the aborigines of the Malay Peninsula make use of three preparations of the *Ipoh* or *Upas* poison, distinguished by the names *Króhi*, *Ténnik* or *Kennik*, and *Malláye*.

The *Króhi* is extracted from the root and bark of the *Ipoh* tree, the roots of the *Túba* and *Kópah*; red arsenic and the juice of limes.

The *Ténnik* is made in the same manner as the *Króhi*, leaving out the *Kópoh* root.

The *Malláye* poison, which is accounted the most potent of the three, is prepared from the roots of the *Túba*, the *Peráchi*, the *Kópah* and the *Chéy*, and from that of the shrub *Malláye*; hence its name.

The process of concocting these preparations is as follows: the roots are carefully selected and cut at a particular age of the moon; I believe about the full. The woody fibre is thrown away, and nothing but the succulent bark used. This is put into a *quáli* (a sort of pipkin made of earth), with as much soft water as will cover the moss, and kneaded well together. This done, more water is added, and the whole is submitted to a slow heat over a charcoal fire until half the water has evaporated. The decoction is next strained through a cotton cloth, again submitted to slow ebullition until it attains the consistence of syrup. The red arsenic (*Warangan*) rubbed down in the juice of the sour lime, the *Límou Assam* of the Malays, is then added and the mixture poured into small bamboos, which are carefully closed up ready for use. Some of the tribes add a little opium, spices and saffron, some the juice of the *Lanchar* and the bones of the *Súnggat* fish burnt to ashes.

A number of juggling incantations are performed and spells gibbered over the seething caldron by the *Poyangs*, (a class of men supposed by this superstitious race to be in league with the powers of darkness,) by whom the fancied moment of the projection of the poisoning principle is as anxiously watched for as that of the philosopher's stone, or the elixir vitæ by the alchymists and philosophers of more enlightened races of men.

When recently prepared the *Ipoh* poisons are all of a dark liver brown colour, of the consistence of syrup, and emit a strong narcotic odour. The deleterious principle appears to be volatile, as the efficacy of the poison diminishes by keeping.

The arrows are very slight slips of wood, scarcely the thickness of a crow-quill, and generally about eight inches long, tapering to a fine point; this is coated with the poison, which is allowed to inspissate thereon for the space of an inch or so. They then cut the arrow slightly all round at the part where the coat of poison ends, consequently it almost invariably snaps off on piercing the flesh of the victim, leaving the envenomed point rankling in the wound. At the other end of the arrow is a cone of light pith-like wood, which is fitted to the tube of the *Súmpitan,* and assists materially in the propulsion and direction of the arrow.

From experiments I caused some of the aborigines to make with these poisoned weapons on living animals in my presence, I am enabled to offer the Society the following results, showing the efficacy of the *Kennik* preparation. A squirrel died in 12 minutes; young dogs in from 37 to 40 minutes; a fowl in two hours: one lingered 7½ hours. Three arrows tipped with the *Malláye* preparation, it is affirmed, would kill a man in less than an hour, and a tiger in less than three hours.

According to the aborigines the only remedy against the poison is the recent juice of the *Lemmah-kapíting,* rubbed round and into the wound, and afterwards over the limb into which the puncture has been made. The arrow seldom penetrates farther than an inch, snapping off as mentioned above.

The following are the symptoms evinced by a strong healthy pup, struck in the right hip; penetration of the arrow about one fourth of an inch only. Six minutes after being wounded it demonstrated signs of uneasiness, yawned and moaned. In 10½ minutes it grew sick; vomited the contents of the stomach; continued vomiting at intervals, bringing up small quantities of a white frothy-looking fluid. In 16 minutes the muscles of the chest and diaphragm were powerfully excited; slight convulsive twitchings in the legs. In 20 minutes it fell on its side, foamed much at the mouth; again got on its legs, and struggled violently as if to get loose. In 23 minutes it was still foaming at the mouth, and had an involuntary alvine evacuation; it then again fell down after painful retching, made ineffectual attempts to vomit, and continued in this state, the efforts to relieve the stomach and chest gradually becoming weaker, till at 37 minutes after the insertion of the poison it died strongly convulsed.

On dissection by Mr. Maurice, the surgeon of the 23rd Regiment, M.N.I., a frothy saliva-like fluid was discovered in the stomach; the gall-bladder distended with bile; the intestines unusually pale. In the cavity of the thorax on each side were found about four drachms of a serous fluid. The brain and spinal chord, I regret to say, were not examined.

By reason of the complicated nature of its preparation, it would be difficult to decide from the above train of symptoms whether the *Upas* poison should or should not be classed, as it has been by some writers, among the narcotico-acrid vegetable poisons.

Quere, whether the *Lemmah-kapíting,* a shrub said by natives to be the only antidote against it, bears any botanical affinity to the *Feuillea cordifolia,* ascertained by Monsieur Drapiez to be a most powerful antidote agains vegetable poison. The native names of the plants I have mentioned will, it is hoped, afford botanists visiting the Straits of Malacca, or the islands of the Indian Archipelago, some clue to a more scientific investigation, both of the plants of which the poison is composed, and of its antidote, the *Lemmah-kapíting*. With regard to the *Ipoh* tree of the Malay peninsula, from the description of it given to me by the natives, I much question its identity with the *Anchar* or *Upas* tree of the Javan forests, described by Dr. HORSFIELD, and the *Arbor toxicaria* of RHUMPHIUS.

It may be superfluous to add, that in the wildest tales related to me by the aborigines regarding the deadly qualities of this poison, there is nothing to corroborate or give rise to the extravagant fictions with which FOERSCH so easily amused the credulity of half Europe.

Bellary, Madras Presidency,
August 7, 1836.

XXIV. *Description of a new Barometer, recently fixed up in the Apartments of the Royal Society; with Remarks on the mode hitherto pursued at various periods, and an account of that which is now adopted, for correcting the observed height of the mercury in the Society's Barometers. By* FRANCIS BAILY, *Esq. Vice-President and Treasurer R.S.*

Received October 25,—Read November 16, 1837.

THE Barometer here alluded to may in some measure be considered as two separate and independent barometers, inasmuch as it is formed of two distinct tubes dipping into one and the same cistern of mercury. One of these tubes is made of *flint* glass, and the other of *crown* glass, with a view to ascertain whether, at the end of any given period, the one may have had any greater chemical effect on the mercury than the other, and thus affected the results. A brass rod, to which the scale is attached, passes through the framework, between the two tubes, and is thus common to both: one end of which is furnished with a fine agate point, which, by means of a rack and pinion moving the whole rod, may be brought just to touch the surface of the mercury in the cistern, the slightest contact with which is immediately discernible*; and the other end of which bears the scale of inches, on which I have set off with great accuracy, from the standard scale of the Royal Astronomical Society, the distance of 30 inches from the above-mentioned agate point. Above and below this mark of 30 inches, the usual scale of inches, tenths, &c. is engraved; and there is a separate vernier for each tube. A piece of thin brass projects from the zero point of each vernier, across its contiguous tube, which, when the height of the mercury is read off, is brought down so that the lower edge of it forms a tangent to the column of mercury, in the usual manner. A small thermometer, the bulb of which dips into the mercury in the cistern, is inserted at the bottom: and an eye-piece is also there fixed, so that the agate point can be viewed with more distinctness and accuracy. The whole instrument is made to turn round in azimuth, in order to verify the perpendicularity of the tubes and the scale.

It is evident that there are many advantages attending this mode of construction, which are not to be found in the barometers as usually formed for general use in this country. The absolute heights are more correctly and more satisfactorily determined; and the permanency of true action is more effectually noticed and secured. For, every part is under the inspection and control of the observer; and any derangement or

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* The motion of this rack-work is much too slow, and might be greatly improved if made more rapid.

imperfection in either of the tubes is immediately detected on comparison with the other. And, considering the care that has been taken in filling the tubes, it may justly be considered as a *Standard Barometer*. The specific gravity of the mercury was determined by Dr. PROUT to be 13·581; the thermometer being at 62°, and the barometer at 30 inches*.

The second part of the present volume of the *Philosophical Transactions* will contain the first register of the observations that have been made with this instrument. The *daily* observations are recorded just as they are read off from the scale, without the application of any correction whatever. This will be found, on due consideration, and after the details which I shall presently state, to be the most simple, and by far the safest plan of registering them; whatever mode may be afterwards adopted of reducing and *discussing* them. At the end of each month the uncorrected *mean* is deduced; which mean, however, will also be given *corrected* agreeably to the usual formulæ, to which I shall now proceed to advert.

The *observed* height of the mercury in a barometer requires several corrections (differing according to the construction of such barometer) in order to determine its *absolute* height, or that point when it may be considered strictly comparable with another barometer, either of the same or of a different construction: and, for effecting this end, certain conditions are previously understood, and universally assented to. Thus, the temperature of the mercury is always supposed to be at the freezing point of water, or 32° FAHRENHEIT: the scale, by which the height is measured, if liable to expansion by heat, is always reduced to the standard temperature, which in this country is 62° FAHRENHEIT: the tube must be corrected for its capillary attraction: and lastly, proper allowance should be made, in certain cases, for the elevation of the place of observation above the mean level of the sea. I shall speak of each of these in their order†. With these corrections duly made, the absolute heights of two barometers might be considered comparable with each other, although separated by the whole diameter of the globe: and with barometers, formed of tubes of a considerable diameter, and having a well adjusted scale, this is probably the case. Yet as, even in the best barometers, there are still certain sources of discordance, some of which, although slight, cannot be altogether avoided notwithstanding our utmost care, such as differences in the specific gravity of the mercury, or in setting off the measure of the scale, or an uncertainty in the height of the station above the mean level of the sea, and, in the more usual ones, others of a more formidable and *variable* nature, depending on circumstances not yet sufficiently accounted for, it is always the most

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* Dr. PROUT has been good enough to inform me that, in taking the specific gravity of mercury in the common mode, it is necessary, in order to expel the whole of the adhering air, to heat repeatedly the mercury in the vessel to nearly the boiling point, and in this state to expose it under the exhausted receiver of an air-pump. This precaution was taken in the present instance.

† In those barometers where the tube dips into a *measured* cistern (similar to that which was constructed for this Society by Mr. DANIEL, to which I shall presently allude) there is another correction requisite, which depends on the relative capacity of the tube and the cistern: but this does not apply to the present barometer

satisfactory method to compare them together, if possible, on the same spot, more especially where great accuracy is required*.

The correction for the temperature of the mercury is by far the most important, since it is in most cases more than ten times the amount of the correction for the expansion of the scale. The correction, for both these sources of discordance and error, may be reduced to one general expression by the following well-known formula: viz.

$$-h \times \frac{m(t-32)-s(t-62)}{1+m(t-32)}$$

where h denotes the observed height, as read off from the scale, which represents English standard inches when at the temperature of 62° FAHR., m the expansion of mercury in volume, and s the expansion of the scale in length, for 1° FAHR.: t denoting the temperature of the mercury and the scale, which are supposed to be the same, and to be ascertained by the thermometer that dips into the cistern of mercury; the slight difference which may exist in the temperature of the scale making no perceptible difference in the results.

According to the accurate experiments of MM. DULONG and PETIT, it appears that mercury expands in volume $\frac{1}{9990}$ (= ·000100100) for each degree of FAHRENHEIT'S thermometer: and, with respect to the linear expansion of brass (of which the present scale is made) we may assume it to be ·000010434 for each degree of FAHRENHEIT. Consequently the above formula becomes

$$-h \times \frac{\cdot 0001001\,(t-32)-\cdot 000010434\,(t-62)}{1+\cdot 0001001\,(t-32)}$$

which, by proper reduction, becomes

$$-h \times \frac{\cdot 000089566\,t-\cdot 002553092}{\cdot 0001001\,t+\cdot 9967968}$$

This expression may be easily formed into a table of double entry, which would be very convenient for correcting the observed heights of the barometer. And it is agreeably to this formula that Professor SCHUMACHER has constructed the tables which are printed in the first volume of his *Astronomische Hülfstafeln*, showing the correction for every difference of half an inch in the height of the mercury, from 27½ to 31 inches; and for every degree of FAHRENHEIT from 6° to 88°, to four places of decimals. These tables, having been afterwards slightly corrected, were (together with some others) printed on a separate sheet, and distributed with No. 114 of his *Astronomische Nachrichten*. They have been recently much enlarged by the distinguished author; and

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* In one of my barometers, the tube of which is about a quarter of an inch in diameter, the mercury has generally stood about a quarter of an inch lower than that of a standard barometer placed by its side, after every correction made for capillarity and temperature, and after a careful examination of the scale. I satisfied myself that there was no *air* in the tube; having had it re-filled with mercury for the express purpose of determining that point, and having also placed it by the side of other excellent standards, and always with the same results. This anomaly, I have since been informed, is by no means rare, and shows the necessity of direct comparison of such barometers with standard ones. Mr. NEWMAN however conceiving that the imperfection arose from *vapour*, has remedied it by drying and wiping out the tube and filling it again with *heated* mercury.

having been extended to every tenth of an inch in the height of the mercury, and to every fifth part of a degree of its temperature, are now printed in his *Jahrbuch* for 1837. It is by these latter tables that the *monthly means*, in the Meteorological Register, are now corrected for temperature.

As I am not aware that any tables of this kind have been printed in England, I shall (with the approbation and consent of the author) give, on this page, some of the values here mentioned: namely, for every half inch in the height of the mercury from 28·0 to 30·5 inches, and for every degree of its temperature from 30° to 90°, which will be found very useful and convenient for the correction of such barometers as are furnished with a continuous brass scale*.

Corrections for a Mercurial Barometer with a continuous Brass scale: all *subtractive.*

Ther.	Barometer.						Ther.	Barometer.					
	in. 28·0	in. 28·5	in. 29·0	in. 29·5	in. 30·0	in. 30·5		in. 28·0	in. 28·5	in. 29·0	in. 29·5	in. 30·0	in. 30·5
30°	·004	·004	·004	·004	·004	·004	60°	·079	·080	·082	·083	·084	·086
31	·006	·006	·006	·007	·007	·007	61	·081	·083	·084	·086	·087	·089
32	·009	·009	·009	·009	·009	·009	62	·084	·085	·087	·088	·090	·091
33	·011	·011	·012	·012	·012	·012	63	·086	·088	·089	·091	·092	·094
34	·014	·014	·014	·014	·015	·015	64	·089	·090	·092	·094	·095	·097
35	·016	·017	·017	·017	·017	·018	65	·091	·093	·095	·096	·098	·100
36	·019	·019	·019	·020	·020	·020	66	·094	·095	·097	·099	·100	·102
37	·021	.022	·022	·022	·023	·023	67	·096	·098	·100	·101	·103	·105
38	·024	·024	·025	·025	·025	·026	68	·099	·101	·102	·104	·106	·108
39	·026	·027	·027	·028	·028	·029	69	·101	·103	·105	·107	·108	·110
40	·029	·029	·030	·030	·031	·031	70	·104	·106	·107	·109	·111	·113
41	·031	·032	·032	·033	·034	·034	71	·106	·108	·110	·112	·114	·116
42	·034	·034	·035	·036	·036	·037	72	·109	·111	·113	·115	·116	·118
43	·036	·037	·038	·038	·039	·040	73	·111	·113	·115	·117	·119	·121
44	·039	·039	·040	·041	·042	·042	74	·114	·116	·118	·120	·122	·124
45	·041	·042	·043	·044	·044	·045	75	·116	·118	·120	·122	·124	·127
46	·044	·045	·045	·046	·047	·048	76	·119	·121	·123	·125	·127	·129
47	·046	·047	·048	·049	·050	·050	77	·121	·123	·125	·128	·130	·132
48	·049	·050	·051	·051	·052	·053	78	·124	·126	·128	·130	·132	·135
49	·051	·052	·053	·054	·055	·056	79	·126	·128	·131	·133	·135	·137
50	·054	·055	·056	·057	·058	·059	80	·129	·131	·133	·135	·138	·140
51	·056	·057	·058	·059	·060	·061	81	·131	·133	·136	·138	·140	·142
52	·059	·060	·061	·062	·063	·064	82	·134	·136	·138	·141	·143	·146
53	·061	·062	·064	·065	·066	·067	83	·136	·138	·141	·143	·146	·148
54	·064	·065	·066	·067	·068	·070	84	·139	·141	·143	·146	·148	·151
55	·066	·067	·069	·070	·071	·072	85	·141	·144	·146	·149	·151	·154
56	·069	·070	·071	·073	·074	·075	86	·144	·146	·149	·151	·154	·156
57	·071	·073	·074	·075	·076	·078	87	·146	·149	·151	·154	·156	·159
58	·074	·075	·076	·078	·079	·080	88	·148	·151	·154	·156	·159	·162
59	·076	·078	·079	·080	·082	·083	89	·151	·154	·156	·159	·162	·164
60	·079	·080	·082	·083	·084	·086	90	·153	·156	·159	·162	·164	·166

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* By a continuous brass scale, I mean one that extends the *whole length* of the tube: and it should be specially borne in mind that the tables, here alluded to, apply *only* to barometers of that construction. For barometers of the ordinary construction, other tables, computed also by Professor SCHUMACHER, will be mentioned in the sequel. See the note in page 437.

The correction for the capillarity of the tube is very slight, and might indeed be safely neglected: but it has been considered proper that every source of anomaly, however small, should be pointed out and scrupulously allowed for. The diameter of the tube of flint glass is ·594 inch, and of the tube of crown glass ·658 inch. The correction for these, agreeably to the formula of LAPLACE, would be respectively + ·0048 and + ·0033: but, in cases where the mercury has been well boiled in the tubes, the correction, as found by the formula, should be somewhat diminished. If we strike off the last figure in each case, we probably shall not be far from the truth: and I have therefore proposed that the correction to be applied should be + ·004 to the flint glass, and + ·003 to the crown glass.

These are all the corrections that, in the case of the present barometer, require to be applied in order to ascertain the absolute height at the place where it is now fixed. The correction for the height of a barometer above the mean level of the sea, is never applied except on especial occasions, and for some definite and express object. The formula for such correction, whenever it may be wanted, is as follows*:

$$d = + \frac{f . h}{24337{\cdot}55 + 58{\cdot}20\, t}$$

where d denotes the addition (in parts of an inch) to the height of the mercury in the barometer, when elevated f feet above the mean level of the sea, in order to show the height at which the mercury would stand, provided the barometer were placed at that level. So that, assuming the height of the station of the present barometer to be 97 feet above the mean level of the sea (and on this subject I shall have some further remarks to make in the sequel), the above expression would become

$$d = + \frac{h}{250{\cdot}90 + {\cdot}60\, t}$$

Whence, if the reading of the barometer, at the place where it is now fixed, were exactly 30 inches, and the temperature 60°, we should have

$$d = + \frac{30}{286{\cdot}9} = + {\cdot}1045$$

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* This formula is easily deduced from that which I have given in my *Astronomical Tables and Formulæ*, page 111, for "computing the difference in the height of two places by means of the barometer." For, there we have

$$f = a . b . c . \log \frac{h'}{h}$$

all known quantities except h'. But $\log\frac{h'}{h}$ is equal to $\log h' - \log h$: and if we make $h' = h + d$ (where d is the required difference in the height of the mercury) we have $\log h' = \log h + \mathrm{M} . \frac{d}{h}$. The formula therefore becomes

$$f = a . b . c . \mathrm{M}\frac{d}{h}$$

whence we obtain

$$d = \frac{f . h}{a . b . c . \mathrm{M}}$$

which is the formula in the text.

Or, in other words, the height of the mercury in the barometer would in such case be 30·1045 inches, if placed at the mean level of the sea, instead of being in the apartments of the Society: and so in the proportion of ·0011 inch for every foot below its present position. But, as I have before remarked, this correction is wholly omitted in the Meteorological Journal.

I have been particular in giving these explanations as to the precise mode in which the corrections should (and are now directed to) be made, since it appears that great irregularity, as well as some inattention, error, or confusion has hitherto occurred on this subject, which ought not to have existed; and the Meteorological Journal of this Society has lost much of its utility, confidence, and importance in consequence thereof.

Prior to the year 1823, the registers of the barometer do not indicate whether the observations are corrected or not: nor can I obtain any satisfactory information on this point. So that a person now referring to them can consider them only as approximate values. The barometer then in use is still in existence.

In January 1823 the registers commence (as I presume*) with the new barometer which had been constructed in the preceding year under the able direction of Mr. Daniel, now Professor of Chemistry at King's College. A description of this barometer is given by him in his *Meteorological Essays and Observations*, page 353. The daily observations are, in the register, said to be *corrected*; but no formula or rule is given, of the mode in which the corrections have been made: and if the observations have been corrected by the small table engraved on the face of the barometer (which is the same as that given by Mr. Daniel in page 372 of his *Essays*), the result will in most cases, for the reasons which I shall presently mention, be slightly erroneous; but more so as the temperature varies from the freezing point. So that although, during the winter months, the results will not be far from the truth, yet in the summer they will not exhibit the correct values†. For, that table has been calculated "from the expansion of mercury and mean dilatation of glass:" it having been originally intended (as I have understood) that the divisions of the scale should have been cut on the glass tube. But this plan having been abandoned, and recourse had to the ordinary mode of construction, it is evident that the expansion of the glass tube does not affect the observed height of the column of mercury sustained by the atmosphere. The only effect which the expansion of the glass can have on the reading of the vernier, will be caused by an alteration in the relative capacity of the tube and the cistern; but this would be so extremely small, on all ordinary occasions, as to be

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* There is nothing stated in the register by which we can judge whether the *old* barometer, or Mr. Daniel's, was at that time used for the daily observations; except that the height of the cistern of the barometer is then stated to be 19 feet higher than before: which was the position in which I find that Mr. Daniel's barometer was placed, as I shall presently explain more fully.

† Taking the thermometer at 70°, and the barometer at 30 inches, the true correction would be ·114; but, according to the table attached to the barometer, it is only ·098: being a difference of ·016 inch.

wholly imperceptible; or at all events now inappreciable, since we are not informed at what temperature the relative measures were ascertained. The true formula for the correction of the expansion of the mercury alone is

$$-h \times \frac{m(t-32)}{1+m(t-32)}$$

where m denotes, as in page 433, the *absolute* expansion of mercury for 1° Fahr. (= ·0001001), and not the *apparent* expansion (= ·0000857339) as assumed in the table above mentioned*.

Besides this correction, there is another, which is peculiar to Mr. Daniel's mode of constructing this barometer, and which is called the correction for the *capacity* of the cistern. As the height of the mercury in the cistern is constantly varying with the variation in the height of the mercury in the tube, it is necessary that the relative capacity, or contents, of the volume of the cistern and the tube should be determined; as also some fixed point on the scale, as the zero of comparison. This has been done with great care by Mr. Daniel; and the capacity of the cistern has been determined to be exactly $\frac{1}{100}$th part of the capacity of the tube, and the neutral point fixed at 30·576 inches†. So that the correction for capacity is

$$+\frac{30{\cdot}576-h}{100}$$

The diameter of the tube is ·530 inch: the correction for capillary attraction is therefore, by Laplace's formula, + ·006; and this is the value that is engraved on the front of the barometer case.

The whole of the corrections therefore for Mr. Daniel's barometer will be as follows‡:

$$-h \times \frac{{\cdot}0001001\,(t-32)}{1+{\cdot}0001001\,(t-32)} + \frac{30{\cdot}576-h}{100} + {\cdot}006$$

There is a short brass scale, of about 4 or 5 inches, on which the divisions are cut: but the expansion of this would, in no possible case, cause an error of more than an unit in the third place of decimals: and as it is screwed to the wooden frame, which is

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* The *absolute* expansion of a liquid is that which is independent of the form, or expansion, of the vessel that contains it: the *apparent* expansion is obtained by deducting 3 times the linear expansion of the containing vessel. Thus, the *absolute* expansion of mercury being ·0001001, and the linear expansion of glass being ·0000047887, we have ·0001001 — ·0000143661 = ·0000857339 for the *apparent* expansion of the mercury. See my Paper on this subject in the *Memoirs of the Astron. Soc.* vol. i. page 383.

† Fifty inches, measured in the upper part of the tube before it was sealed, raised the float in the cistern exactly half an inch.

‡ Amongst the tables, separately printed and distributed with No. 114 of the *Astron. Nach.* by Professor Schumacher (as already mentioned in page 433), there is one showing the value of that part of the expression in the text which is denoted by $-h \times \frac{{\cdot}0001001\,(t-32)}{1+{\cdot}0001001\,(t-32)}$, for every $\frac{1}{2}$ inch from $27\frac{1}{2}$ to 31 inches; and for every degree of Fahrenheit from 6° to 88°. And this is the table that should be used for barometers of the ordinary construction, not furnished with a brass scale extending the *whole length* of the tube. But I am not aware that any such table has been published in this country.

liable to expand and contract with different degrees of moisture, independent of the temperature, no correction for this purpose can be depended upon. This is a great imperfection in the mode of constructing and fixing the scale of a barometer intended for very accurate purposes. The specific gravity of the mercury was ascertained by Mr. Faraday to be 13·624: the thermometer being at 40°, but the height of the barometer not given.

I have already stated that prior to the year 1823, the registers do not indicate whether the observations have been corrected, or not; but that, commencing with January 1823, they profess to give the *corrected* heights of the readings of the barometer, unexplained however as to the mode of correction. This continued till March 20, 1826, when a temporary suspension of the observations took place. From April 6, 1826, down to the end of the year 1836, we are again left in doubt whether the *daily* observations are corrected, or not. But the inference is that they were not corrected; since we find a correction applied to the *monthly* means, for temperature and capillarity. I have ascertained, however, on inquiry, that the daily observations have in all cases been *partially* corrected: that is, the correction for the capacity of the cistern has been applied daily. Why this correction alone, on each day, should have been considered requisite, I have not been able to ascertain; and as it is nowhere mentioned in the meteorological journal, it may perhaps have sometimes led to error. But leaving this part of the subject, I shall now proceed to notice the loose manner in which the remaining correction (for temperature) has been from time to time applied to the monthly means.

From April 6, 1826, to the end of that year, the temperature has been taken from the *external* thermometer, instead of the thermometer which dips into the cistern of mercury. Consequently all the reduced values of the readings are too great. By the *external* thermometer, I mean the thermometer which is placed *outside of the building*, and consequently gives the temperature of the open air.

In the year 1827 this error appears to have been discovered and discontinued; but another of a different nature was at the same time introduced. For, from that epoch to the end of the year 1836, all the corrections are made under the assumption that the height of the mercury in the barometer was exactly 30 inches: when it is well known that the correction will vary according to the variation in the height. In fact there does not appear, at any time, to have been any regular and uniform system of reduction adopted.

Now this state of confusion and uncertainty ought not to exist in a meteorological journal emanating from this Society, more especially as the true values are as easily attainable as the approximate ones. And although, in a general point of view, the minute differences caused by such errors may be unimportant, yet as appeals are frequently made to the barometer of this Society, as a standard, by persons engaged in important researches, the most scrupulous accuracy ought to be adopted and pursued, and the fullest explanation placed on record. And notwithstanding the details which

I have here given may create some doubt respecting the accuracy of the past, yet I am persuaded that the system now pursued will inspire more confidence for the future. It is on this account that I have entered thus at large on the subject: trusting that what I have here stated will not only tend to preserve for the future a more correct and uniform system, but also justify the Council in directing that the register should henceforth contain the daily observations *uncorrected*, and thus prevent the possibility of any similar confusion and mistakes hereafter.

I shall now say a few words respecting the height of the barometer above the mean level of the sea; a subject of much interest to many persons engaged in various pursuits, but which appears, from the notes attached, at different periods, to the meteorological journal of this Society, to be involved in some confusion and uncertainty. Thus, prior to the year 1823, the cistern of the barometer is said to be 81 feet above the level of low-water spring tides at Somerset House; but without any information how this was connected with the sea. From 1823 to 1825, both inclusive, it is said to be 100 feet above the same level. And from 1826 to 1836, both inclusive, the above indication is omitted, and the height is said to be 83 feet 2½ inches above a *fixed mark* on Waterloo Bridge; or "above the mean level of the sea (presumed about) 95 feet." The discordance between the 81 feet and the 100 feet is easily accounted for by the fact that the old barometer, prior to 1823, was fixed up in the Council-room of the Society, or the contiguous ante-room: but when Mr. Daniel's barometer was finished, at the end of the year 1822, it was fixed up in the closet adjoining the library, on the floor which is immediately *over* the Council-room: the assumed difference in the elevation of the two floors (namely, 19 feet) having since been ascertained to be correct.

With respect to the new reference of altitude, namely, the fixed *mark* at Waterloo Bridge, much doubt has frequently been expressed about its existence, since no person had been able to discover it. The fact is that there is no *mark*, in the common acceptation of the term; but the intended reference is nevertheless more conspicuous, more durable, and more convenient than any mark that could have been inscribed by hands. This standard mark, or level, was fixed on by Mr. Bevan in the year 1827, at the request of the Council of this Society: and the same gentleman also ascertained the difference of level between that mark and the floor of the council room. As his Report on the occasion has never yet been made public, and will throw the best light on the subject, as well as be interesting to many persons, I shall here subjoin his letter to the Council, detailing the whole circumstances of the case.

"Gentlemen,—Pursuant to the order I had the honour to receive at the close of "your last session, I have selected a permanent and definite point of reference, or *bench-"mark*, for heights at Waterloo Bridge; and have determined the difference of level "between this point and the floor of the Council room, in the Apartments of the "Society at Somerset House.

"The bench-mark, I have adopted, is the *surface* of the granite pedestal at the

" base of the columns, at the north abutment of the bridge, and on the eastern side; " which is about five feet above the lowest platform, or landing, at the stairs.

" I have ascertained, by levelling from this spot, or bench-mark, to the floor of the " Council room, in which the barometer was kept in June 1826, that the floor in the " centre of the doorway between the two rooms is 62·41 feet above the said bench-" mark. The mercury, in the basin of the barometer, I found above the floor 2·84 " feet; making the rise from the bench-mark to the mercury 65·25 feet.

" I am, Gentlemen, your obedient humble Servant,
" B. Bevan."

Upon what authority it was presumed that the present position of the cistern of the barometer is ninety-five feet above the mean level of the sea, (or, in other words, that the above-mentioned station at Waterloo Bridge is 11 feet 9½ inches above that level) I have not been able to ascertain; since Captain Lloyd's levelling of the river Thames, from Sheerness upwards, as detailed in the *Philosophical Transactions* for 1831, terminated at London Bridge. He says, page 190, "I *concluded* my levellings at a " standard mark sunk in the large plinth of the landing place (near the wall) of the " stairs on the north-east side of the New London Bridge. This standard was 2·3967 " [feet] below the north standard mark at Sheerness." Now, as the north standard mark at Sheerness was ascertained by Captain Lloyd to be 13·1511 feet above the mean level of the sea, we consequently have the surface of the above-mentioned plinth at London Bridge equal to 10·7544 (or 10¾) feet above the mean level of the sea. It therefore only remained to ascertain the difference of level between the surface of this plinth, and the surface of the plinth at Waterloo Bridge.

But a doubt for a long time remained as to the position of the mark at London Bridge, since (as in the case of that at Waterloo Bridge) it had escaped the search of all those who attempted to discover it. It was at length found by Dr. Fitton, who in a note to his paper "On the Strata below the Chalk," inserted in vol. iv. (second series) of the *Transactions of the Geological Society*, page 370, gives the following accurate and circumstantial description of its position. "The mark here referred to is " a flat piece of brass, let into a cavity in one of the two large flags, or slabs of gra-" nite, which form the landing place at the bottom of the second flight of steps, de-" scending from the footway on the north-east side of the bridge. The upper flight " consists of 29 steps; the second (at the foot of which is the mark) of 26. The " lowest flight is more or less commonly covered by the water. The cavity, in which " the mark is lodged, is about 3 inches square, with rounded angles; and is two feet " from the eastern wall, or side of the bridge, and two feet eight inches from the " southern side of the stone. The surface of the brass is about half an inch beneath " that of the stone, which is itself a few inches below the level of the water at high " spring tides."

The propriety of such a position for a *standard* mark may be much questioned, since we know, from what has recently taken place at Blackfriars Bridge, that the steps of a common landing place, abutting on the river, are liable to *settle*; and in course of time to be altogether removed, for the purpose of repairs. It therefore became desirable, on more accounts than one, to connect together the two marks at London and Waterloo Bridges by direct levelling. This has recently been effected, at the request of the Council of this Society, by the direction and under the superintendence of Sir JOHN RENNIE, who readily undertook the determination of this point. In his letter on this subject, dated October 18, 1837, he says, “After repeated trials “ (the greatest variation of which did not exceed $\frac{2}{10}$ of an inch) I find that the difference “ is 3 feet 1·65 of an inch: that is, the mark on Waterloo Bridge is 3 feet 1·65 of an “ inch *above* that on the New London Bridge fixed by Captain LLOYD.”

The height of the cistern of the present barometer above the floor is 1·75 foot: therefore adding all these several quantities together, namely,

$$\begin{array}{r} 19{\cdot}000 \\ 62{\cdot}410 \\ 10{\cdot}754 \\ 3{\cdot}138 \\ 1{\cdot}750 \\ \hline 97{\cdot}052 \end{array}$$

we have, in round numbers, 97 feet for the height of the mercury in the cistern of this barometer above the mean level of the sea.

One word more before I close this paper, as to the propriety of the position of the several meteorological instruments of this Society; on which, comments have occasionally been made. With respect to the *barometer*, I am not aware that any objection can be offered; and as to the *hygrometer*, the observations have been found, by recent trials, not to differ materially from some expressly made in another position, at King's College, which was considered to be more favourable for such experiments. It therefore only remains to speak of the external *thermometer* and of the *rain-gauge*; of which all that can be said on the subject would be merely a repetition of what was justly said sixty years ago by Mr. CAVENDISH on a similar occasion (Philosophical Transactions, 1776), namely, “that, on the whole, the situation is not alto- “ gether such as could be wished, but is the *best* the house affords.”

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TO THE

PHILOSOPHICAL TRANSACTIONS

FOR THE YEAR 1837.

LONDON:
PRINTED BY RICHARD AND JOHN E. TAYLOR,
RED LION COURT, FLEET STREET.