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OF

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FOR THE YEAR MDCCCLXXVII.

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MDCCCLXXVIII.

CONTENTS.

PART II.

LIST OF ILLUSTRATIONS.

ADJUDICATION of the MEDALS of the ROYAL SOCIETY for the year 1877 by the PRESIDENT and COUNCIL.

The COPLEY MEDAL to Professor JAMES DWIGHT DANA, for his Biological, Geological, and Mineralogical Investigations, carried on through half a century, and for the valuable works in which his conclusions and Discoveries have been published.

A ROYAL MEDAL to Mr. FREDERICK AUGUSTUS ABEL, F.R.S., for his Physico-Chemical Researches on Gun-Cotton and Explosive Agents.

A ROYAL MEDAL to Professor OSWALD HEER, for his numerous Researches and Writings on the Tertiary Plants of Europe, of the North Atlantic, North Asia, and North America, and for his able generalizations respecting their affinities and their geological and climatic relations.

The DAVY MEDAL to ROBERT WILHELM BUNSEN, For. Mem. R.S., and GUSTAV ROBERT KIRCHHOFF, For. Mem. R.S., for their Researches and Discoveries in Spectrum Analysis.

The BAKERIAN LECTURE was delivered by Professor W. C. WILLIAMSON, F.R.S.: it was entitled "On the Organization of the Fossil Plants of the Coal-measures.—Part IX. The latest Researches into the Organization of the Fossil Plants of the British Coal-measures, especially of the *Calamites* and *Lepidodendra*."

The Paper "On the Mechanical Effects and on the Electrical Disturbance consequent on Excitation of the Leaf of *Dionæa muscipula*," by Dr. J. B. SANDERSON, F.R.S., and F. J. M. PAGE, was appointed as the CROONIAN LECTURE.

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CONTENTS

OF VOL. 167.

PART I.

PART II.

ERRATA.

Page 464, line 19. Aji Bārang, *for* Lat. 2° 25′ S. *read* 7° 25′ S.

Page 475, line 2. Declination. At sea—Lat. 17° 15′ S., Long. 292° 45′ E. *should be expunged.*

LIST OF ILLUSTRATIONS.

X. *On Hyperjacobian Surfaces and Curves.*

By William Spottiswoode, *M.A., Treas. R.S.*

Received April 23,—Read May 17, 1877.

§ 1. *Introductory.*

In a paper published in the 'Mathematische Annalen' (vol. iii. p. 459), Brill has discussed the question of curves having three-point contact with a doubly infinite pencil of curves; and in particular he has investigated some of the properties of the curve passing through all the points of contact with the individual curves of the pencil. In the same Journal (vol. x. p. 221) Krey of Kiel has applied a method similar to that of Brill with partial success to the question of curves having four-point contact with a triply infinite pencil. Some formulæ, however, given in my paper "On the Sextactic Points of a Plane Curve" (Phil. Trans. 1865, p. 657) have proved to be directly applicable to both questions. An application of them to Brill's problem will be found in the 'Comptes Rendus' for 1876 (2nd semestre, p. 627), and a solution of Krey's problem in the 'Proceedings of the London Mathematical Society for the same year (vol. viii. p. 29).

The present subject was in the first instance suggested by the foregoing papers; and from one point of view it may be regarded as an attempt to extend the question to the case of surfaces; viz. to determine a curve which shall pass through the points of contact of a given surface U with certain surfaces belonging to a pencil V, and to investigate some of its properties. From a slightly different point of view, however, it may be considered as an extension of two ideas, viz. first, that of the Jacobian surface, or locus of the points whose polar planes with regard to four surfaces meet in a point; and secondly, that of the Jacobian curve, or locus of points whose polar planes with regard to three surfaces have a right line in common. More particularly, commencing with the facts, first, that if a surface of the form $a\phi+b\psi+c\chi$ touch a surface U, the point of contact is a point on the Jacobian, and secondly, that if a surface of the form $a\phi+b\psi$ touch a surface U, the point of contact is a point on the Jacobian curve, I have endeavoured to extend them to higher degrees of contact.

Consider, then, a surface $U=0$ of the degree n, and other surfaces $\phi=0$, $\psi=0$, . . all of the degree m, where m is in general different from n. Let the first differential coefficients of U, ϕ, ψ, . . be thus expressed:

$$\left.\begin{array}{l}(\partial_x,\ \partial_y,\ \partial_z,\ \partial_t)\ U=u\,,\ v\,,\ w\,,\ k\,,\\(\partial_x,\ \partial_y,\ \partial_z,\ \partial_t)\ \phi=a\,,\ b\,,\ c\,,\ d\,,\\(\partial_x,\ \partial_y,\ \partial_z,\ \partial_t)\ \psi=a',\ b',\ c',\ d',\\\qquad\qquad\qquad\quad\ \vdots\quad\ \vdots\quad \vdots\quad \vdots\quad \vdots\end{array}\right\}\ .\ .\ .\ .\ .\ .\ .\ .\quad(1)$$

Then, when the number of surfaces is equal to that of the variables, the determinant

$$\left.\begin{array}{llll} u\,, & v\,, & w\,, & k \\ a\,, & b\,, & c\,, & d \\ a'\,, & b'\,, & c'\,, & d' \\ a''\,, & b''\,, & c''\,, & d'' \end{array}\right\} \quad \cdots\cdots\cdots\cdots \quad (2)$$

is called the Jacobian of U, ϕ, ψ, χ, and is usually designated by the formula

$$J\,(U, \phi, \psi, \chi) \quad \cdots\cdots\cdots\cdots \quad (3)$$

If, however, the number of surfaces be less by unity than that of the variables, the first differential coefficients (1) will form the matrix

$$\left.\begin{array}{llll} u\,, & v\,, & w, & k \\ a\,, & b\,, & c\,, & d \\ a', & b', & c', & d' \end{array}\right\} \quad \cdots\cdots\cdots\cdots \quad (4)$$

from which there may be derived four determinants, whereof, however, two only are independent. Any two of these will represent surfaces, which may be called Jacobian (more strictly Hypojacobian) surfaces; and their intersection, the Jacobian curve of the system. The pair of surfaces, or their curve of intersection, may conveniently be designated by the formula

$$J((U, \phi, \psi)) \quad \cdots\cdots\cdots\cdots \quad (5)$$

Again, if ΔU, $\Delta' U$, . . be any derivatives of U beyond the first, we may, by means of them, form from the system of surfaces U, ϕ, ψ, . . the following matrix :—

$$\left.\begin{array}{lllllll} u\,, & v\,, & w, & k, & \Delta U, & \Delta' U, & .\,. \\ a\,, & b\,, & c\,, & d, & \Delta\phi\,, & \Delta'\phi\,, & .\,. \\ a', & b', & c', & d', & \Delta\psi, & \Delta'\psi\,, & .\,. \\ : & : & : & : & : & : & :\,: \end{array}\right\} \quad \cdots\cdots\cdots \quad (6)$$

in which it is to be understood that the number of surfaces is such that the number of columns exceeds that of the lines by unity. From this matrix two independent determinants may be formed; and the surfaces which they represent may, in conformity with previous nomenclature, be called the *Hyperjacobian surfaces*, and their intersection the *Hyperjacobian curve* of the system. These will be designated by the formula

$$J\,((U, \phi, \psi, .\,.\,)) \quad \cdots\cdots\cdots\cdots \quad (7)$$

The principal properties of the Jacobian surface (2), and of the Jacobian curve (5) are known; and the object of the present paper is to investigate some of the properties of the Hyperjacobian surfaces and curves.

The surfaces and curves (5) are in themselves independent of any particular mode of origination; but they are here considered as arising out of the contact of the surface U with a surface passing through the intersection of ϕ and ψ; or, say, with one or more particular surfaces of the singly infinite or *one-fold pencil* $a\phi+b\psi=V$, where a and b are constants. For the equations expressing the condition of contact between U and V will enable us to eliminate the ratio a : b in two ways, and give rise to the curve $J((U, \phi, \psi))=0$. These equations, combined with $U=0$, will suffice to determine the coordinates of the points of contact; and if the values so determined be substituted in the equation $V=0$, the values of a : b, that is, the particular surfaces of the pencil for which such contact obtains, will be found.

Similarly, the Jacobian (2) may be considered as arising out of the contact of the surface U with the doubly infinite or *two-fold pencil*, $V=a\phi+b\psi+c\chi$.

Again, by selecting suitable derivatives of U, ϕ, ψ, . . for the terms ΔU, $\Delta' U$, . . the Hyperjacobians (6) and (7) may be considered as arising from contact of higher degrees than common (or two-branch) contact between U and some of the surfaces of the pencil $V=a\phi+b\psi+$ . . And we shall in each such case have three equations, viz. $U=0$, and the equations of the curve, which will give the values of the coordinates of the points of contact. These values, substituted in the equation for V, will determine one of the ratios a : b : . ., and thereby a pencil (whose multiplicity is less by unity than that of the given pencil), for which the contact obtains.

The properties here considered are those which appertain to the points, if any, through which all the surfaces pass, or, as they may be termed, the *principal points* of the system; and consist mainly in the nature of the contact of the Hyperjacobian surfaces with the surface U, and the multiplicity of the Hyperjacobian curve at the points in question.

The present investigation extends to the cases of two-branch contact of the given surface with a one-fold and with a two-fold pencil, and of three-branch contact with a four-fold pencil. In the latter case, notice is also made of some properties appertaining to the points, if any, where all the surfaces touch one another, or, as they may be termed, the *secondary points* of the system. In particular, it is shown that, in the case of common, or two-branch contact and a one-fold pencil, the Jacobian curve has a double point at the principal points; while in the case of three-branch contact and a four-fold pencil, the Hyperjacobian curve has a triple point at the same points.

§ 2. *The Jacobian Surfaces and Curve of a one-fold pencil.*

Consider a surface $U=0$ of the degree n, and two other surfaces $\phi=0$, $\psi=0$, each of the degree m, where m is in general different from n; also the one-fold pencil of surfaces

$$V=a\phi+b\psi=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where a and b are constants. If U and V have common, or two-branch contact, we shall have, beside (1), the following conditions, viz.

$$\partial_x V=\theta u, \ \partial_y V=\theta v, \ \partial_z V=\theta w, \ \partial_t V=\theta k, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where θ is indeterminate. If for V there be substituted its value given by (1), these equations will serve to eliminate the ratios a : b : θ in two ways; and by this means we shall obtain two equations in x, y, z, t, which, being independent of a, b, will hold good for any particular surfaces of the pencil U, and will consequently represent a curve passing through all the points of contact of the surface U with the pencil V. The resultants, combined with the equation U=0, will determine the coordinates of all the points of contact; and the particular surfaces of the pencil which actually touch U will be found by substituting successively the values of $x:y:z:t$, so determined, in the equation (1), and thence deducing the corresponding values of the ratio a : b.

The two resultants may be represented by the following formulæ:—

$$\left.\begin{matrix} \mathrm{P},\ \mathrm{Q},\ \mathrm{R},\ \mathrm{S}= & u, & v, & w, & k & =\mathrm{J}((\mathrm{U},\ \phi,\ \psi))=0, \\ & a, & b, & c, & d & \\ & a', & b', & c', & d' & \end{matrix}\right\} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which P, Q, R, S represent the four determinants which can be formed from the matrix (3) by the omission of each of the four columns in succession. Of these, of course, two only are independent; and they represent, as mentioned in the Introduction, the Jacobian curve of the system U, ϕ, ψ. The Jacobian curve presents some peculiarities at the principal points of the system, *i. e.* at the points where U=0, $\phi=0$, $\psi=0$. In order to examine them, it will be convenient to transform the expressions (3) as follows:—

$$\mathrm{Q}z-\mathrm{R}y=-\begin{matrix} zw, & k, & u \\ zc, & d, & a \\ zc', & d', & a' \end{matrix} \quad -\begin{matrix} k, & u, & yv \\ d, & a, & yb \\ d', & a', & yb' \end{matrix}$$

$$=\begin{matrix} yv+zw, & u, & k \\ yb+zc, & a, & d \\ yb'+zc', & a', & d' \end{matrix} \quad = \begin{matrix} u\mathrm{U}, & u, & k, \\ m\phi, & a, & d \\ m\psi, & a', & d' \end{matrix}$$

or, putting $(n-m):m=\mathrm{m}$, the above expression takes the form

$$\mathrm{Q}z-\mathrm{R}y=\mathrm{U}+\mathrm{m}\begin{matrix}\mathrm{U}, & u, & k \\ \phi, & a, & d \\ \psi, & a', & d'\end{matrix}=\mathrm{A}+\mathrm{mUA},\ \text{suppose.}$$

More generally, let

$$\left.\begin{matrix} \mathrm{A}_1,\ \mathrm{B}_1,\ \mathrm{C}_1,\ \mathrm{F}_1,\ \mathrm{G}_1,\ \mathrm{H}_1= & a, & b, & c, & d, \\ & a', & b', & c', & d', \end{matrix}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

i. e. A_1, B_1, . . are the determinants formed from the matrix (4) by omitting the columns two and two in the usual order, viz. 2, 3; 3, 1; 1, 2; 1, 4; 2, 4; 3, 4. Also let

$$\left.\begin{array}{r} A,\ B,\ C,\ F,\ G,\ H=\overset{*}{U},\ u,\ v,\ w,\ k, \\ \phi,\ a,\ b,\ c,\ d, \\ \psi,\ a',\ b',\ c',\ d', \end{array}\right\} \quad \ldots\ldots\ldots \quad (5)$$

i.e. A, B, . . are the determinants formed from the matrix (5), by retaining the first column (on that account marked by the asterisk $*$), and omitting the four others, two and two, in the same order as before. We may then form the following system:—

$$\left.\begin{array}{ll} (Qz-Ry)\ :\ m=A+\mathrm{m}A_1, & (Pt-Sx)\ :\ m=F+mF_1, \\ (Rx-Pz)\ :\ m=B+\mathrm{m}B_1, & (Qt-Sy)\ :\ m=G+mG_1, \\ (Py-Qx)\ :\ m=C+\mathrm{m}C_1, & (Rt-Sz)\ :\ m=HmH_1, \end{array}\right\} \quad \ldots \quad (6)$$

any two of which may be regarded as the equations of the curve in question.

By means of these equations it may easily be shown that at the principal points each of the surfaces (6), or say each of the surfaces

$$\left.\begin{array}{l} x,\ y,\ z,\ t=0 \\ P,\ Q,\ R,\ S \end{array}\right\} \quad \ldots\ldots\ldots\ldots \quad (7)$$

touches, or has two-branch contact with U. For, on differentiating the first equation of (6), we obtain

$$\partial_x(Qz-Ry)\ :\ m=\partial_x A+\mathrm{m}uA_1+mU\partial_x A_1.$$

But, on reference to (5), it is clear that at the points in question $(\partial_x, \partial_y, \partial_z, \partial_t)A=0$, or more generally

$$(\partial_x, \partial_y, \partial_z, \partial_t)(A, B, C, F, G, H)=0. \quad \ldots\ldots\ldots \quad (8)$$

Hence

$$\partial_x(Qz-Ry)\ :\ u=\partial y(Qz-Ry)\ :\ v=\partial_z(Qz-Ry)\ :\ w=\partial_t(Qz-Ry)\ :\ k=m\mathrm{m}A_1\,;$$

in other words, the surface $Qz-Ry$ touches U at the principal points of the system. Still more generally, we may write, for the whole group, the following formula, viz.

$$\left.(\partial_x, \partial_y, \partial_z, \partial_t)\begin{pmatrix} x,\ y,\ z,\ t \\ |P,\ Q,\ R,\ S| \end{pmatrix}\right\} \quad \ldots\ldots\ldots \quad (9)$$

$$=m\mathrm{m}\ (u,\ v,\ w,\ k)(A_1,\ B_1,\ C_1,\ F_1,\ G_1,\ H_1),$$

which expresses the fact that, at the principal points of the system, each of the surfaces (7) has a two-branch contact with U. It remains to show that the same is the case with each of the surfaces P, Q, R, S. Since each of the expressions for P, Q, R, S vanishes at the points under consideration, we have

$$\partial_x(Rx-Pz)=x\partial_x R-z\partial_x P=m\mathrm{m}B_1 u,$$

$$\partial_x(Py-Qx)=y\partial_x P-x\partial_x Q=m\mathrm{m}C_1 u.$$

But on multiplying the first of these equations by z and the second by y, and subtracting, we obtain

$$x(x\partial_x\mathrm{P}+y\partial_x\mathrm{Q}+z\partial_x\mathrm{R})-(x^2+y^2+z^2)\partial_x\mathrm{P}=m\mathrm{m}(z\mathrm{B}_1-y\mathrm{C}_1)u.$$

But, since $x\mathrm{P}+y\mathrm{Q}+z\mathrm{R}=0$, this reduces itself to

$$\partial_x\mathrm{P}:u=m\mathrm{m}(z\mathrm{B}_1-y\mathrm{C}_1):(x^2+y^2+z^2).$$

By a similar process applied to the other equations, and by writing $x^2+y^2+z^2=r^2$, we should obtain

$$\left.\begin{array}{l}\partial_x\mathrm{P}:u=\partial_y\mathrm{P}:v=\partial_z\mathrm{P}:w=\partial_t\mathrm{P}:k=m\mathrm{m}(z\mathrm{B}_1-y\mathrm{C}_1):r^2,\\ \partial_x\mathrm{Q}:u=\partial_y\mathrm{Q}:v=\partial_z\mathrm{Q}:w=\partial_t\mathrm{Q}:k=m\mathrm{m}(x\mathrm{C}_1-z\mathrm{A}_1):r^2,\\ \quad\vdots\qquad\quad\vdots\qquad\quad\vdots\qquad\quad\vdots\qquad\qquad\quad\vdots\end{array}\right\} \quad . \; . \; . \; . \quad (10)$$

Hence each of the surfaces P, Q, R, S has two-branch contact with U at the principal points of the system. In other words, each of the Jacobian surfaces $\mathrm{J}((\mathrm{U}, \phi, \psi))=0$ touches the surface U, and consequently they touch one another at the principal points of the system. At the same points the Jacobian curve therefore has a node, and along each of its two branches the contact of the Jacobian surfaces is three-pointic.

It should here be noticed that if all the surfaces are of the same degree, *i. e.* if $n=m$, then $\mathrm{m}=0$, and consequently

$$(\partial_x, \partial_y, \partial_z, \partial_t)(\mathrm{P}, \mathrm{Q}, \mathrm{R}, \mathrm{S})=0\,; \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (11)$$

so that, in this case, the Hessian of each Jacobian surface vanishes at the principal points; in other words, the principal points are parabolic points on the Jacobians.

§ 3. *The Hyperjacobian Surfaces and Curve of a two-fold Pencil.*

Consider the surface $\mathrm{U}=0$, as before; the three surfaces ϕ, ψ, χ; and the pencil

$$\mathrm{V}=\mathrm{a}\phi+\mathrm{b}\psi+\mathrm{c}\chi=0. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

Now, it is well known that if two surfaces touch one another, the curve of intersection has a double point at the point of contact, and that along each of the branches the contact is three-pointic. The formulæ for determining the directions of these two branches are as follows. Adopting the notation of my memoir, "On the Contact of Surfaces" (Phil. Trans. 1872, p. 259), in which α, β, γ, δ, α', β', γ', δ', are arbitrary constants, and

$$\alpha x+\beta y+\gamma z+\delta t=\varpi,$$

$$\alpha' x+\beta' y+\gamma' z+\delta' t=\varpi',$$

$$\mathrm{A}=\alpha'\varpi-\alpha\varpi',$$

$$\mathrm{B}=\beta'\varpi-\beta\varpi',$$

$$\mathrm{C}=\gamma'\varpi-\gamma\varpi',$$

$$\mathrm{D}=\delta'\varpi-\delta\varpi',$$

$$\left.\begin{array}{l}
\mathrm{H}=\begin{array}{lllll} u_1, & w', & v', & l', & \mathrm{A} \\ w', & v_1, & u', & m', & \mathrm{B} \\ v', & u', & w_1, & n', & \mathrm{C} \\ l', & m', & n', & k_1, & \mathrm{D} \\ \mathrm{A}, & \mathrm{B}, & \mathrm{C}, & \mathrm{D}, & . \end{array} \qquad
\Delta=\begin{array}{llllll} u_1, & w', & v', & l', & \mathrm{A}, & \partial_x \\ w', & v_1, & u', & m', & \mathrm{B}, & \partial_y \\ v', & u', & w_1, & n', & \mathrm{C}, & \partial_z \\ l', & m', & n', & k_1, & \mathrm{D}, & \partial_t \\ \mathrm{A}, & \mathrm{B}, & \mathrm{C}, & \mathrm{D}, & . & . \\ \partial_x, & \partial_y, & \partial_z, & \partial_t, & . & . \end{array} \\
\qquad\qquad \mathrm{n}=1+2(m-1):(n-1);
\end{array}\right\} \quad \cdots \quad (2)$$

then the required formulæ will be

$$\partial_x\mathrm{V}:u=\partial_y\mathrm{V}:v=\partial_z\mathrm{V}:w=\partial_t\mathrm{V}:k=\Delta\mathrm{V}:n\mathrm{H}=\theta, \quad \cdots\cdots \quad (3)$$

the last of which, being a quadratic in $\varpi:\varpi'$, will determine the two directions sought.

If the value of V given by the equation (1) be inserted in (3), we may eliminate the ratios $\mathrm{a}:\mathrm{b}:\mathrm{c}:\theta$ in two different ways. The resultants will be the Hyperjacobian surfaces, and their intersection the Hyperjacobian curve.

The two independent results may be comprised in the formula

$$\left.\mathrm{P, Q, R, S, T}=\begin{array}{lllll} u, & v, & w, & k, & \mathrm{nH}=0. \\ a, & b, & c, & d, & \Delta\phi \\ a', & b', & c', & d', & \Delta\psi \\ a'', & b'', & c'', & d'', & \Delta\chi \end{array}\right\} \quad \cdots\cdots \quad (4)$$

Among these, the expressions P, Q, R, S may be combined in the same manner as the corresponding expressions in § 1; and if we then write

$$\left.\mathrm{A, B, C, F, G, H}=\begin{array}{llllll} \overset{*}{\mathrm{U}}, & u, & v, & w, & k, & \overset{*}{\mathrm{nH}}, \\ \phi, & a, & b, & c, & d, & \Delta\phi, \\ \psi, & a', & b', & c', & d', & \Delta\psi, \\ \chi, & a'', & b'', & c'', & d'', & \Delta\chi, \end{array}\right\} \quad \cdots\cdots \quad (5)$$

$$\left.\mathrm{A}_1, \mathrm{B}_1, \mathrm{C}_1, \mathrm{F}_1, \mathrm{G}_1, \mathrm{H}_1=\begin{array}{lllll} a, & b, & c, & d, & \overset{*}{\Delta\phi}, \\ a', & b', & c', & d', & \Delta\psi, \\ a'', & b'', & c'', & d'', & \Delta\chi, \end{array}\right\} \quad \cdots\cdots \quad (6)$$

we may take instead of the four expressions P, Q, R, S, the six

$$\mathrm{A}+\mathrm{mUA}_1, \ldots \quad \mathrm{F}+\mathrm{mUF}_1, \quad \cdots\cdots \quad (7)$$

and any two of these equated to zero, or any one of them combined with $\mathrm{T}=\mathrm{T}_0+\mathrm{mUT}_1=0$, will serve for the two equations required. From these it is easy to see that, in the same way as in § 1, it may be shown that the Hyperjacobian surfaces touch the surface U at the principal points of the system.

But in this case we may carry the question of the contact of the Hyperjacobians a step further. In fact, bearing in mind that if p, q be any two rational, integral, and homogeneous functions of x, y, z, t, the nature of the operation Δ is such that, if we put

$$\Delta=(\mathfrak{A}, \mathfrak{B}, \mathfrak{C}, \mathfrak{D}, \mathfrak{F}, \mathfrak{G}, \mathfrak{H}, \mathfrak{L}, \mathfrak{M}, \mathfrak{N})(\partial_x, \partial_y, \partial_z, \partial_t)^2, \quad . \quad . \quad . \quad . \quad (8)$$

then

$$\Delta pq=p\Delta q+q\Delta p+2(\mathfrak{A}, \mathfrak{B}, \,.\,.\,)(\partial_x p, \partial_y p, \partial_z p, \partial_t p)(\partial_x q, \partial_y q, \partial_z q, \partial_t q). \quad . \quad . \quad (9)$$

This being so, if we put

$$\left.\begin{aligned}\mathfrak{A}\partial_x+\mathfrak{H}\partial_y+\mathfrak{G}\partial_z+\mathfrak{L}\partial_t&=\mathrm{A}',\\ \mathfrak{H}\partial_x+\mathfrak{B}\partial_y+\mathfrak{F}\partial_z+\mathfrak{M}\partial_t&=\mathrm{B}',\\ \mathfrak{G}\partial_x+\mathfrak{F}\partial_y+\mathfrak{C}\partial_z+\mathfrak{N}\partial_t&=\mathrm{C}',\\ \mathfrak{L}\partial_x+\mathfrak{M}\partial_y+\mathfrak{N}\partial_z+\mathfrak{D}\partial_t&=\mathrm{D}',\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

it follows that

$$\mathrm{A}'\mathrm{U} : x=\mathrm{B}'\mathrm{U} : y=\mathrm{C}'\mathrm{U} : z=\mathrm{D}'\mathrm{U} : t=\mathrm{H}(n-1). \quad . \quad . \quad . \quad . \quad . \quad (11)$$

If, then, we operate with Δ upon the equation $(t : m)\mathrm{T}=\mathrm{T}_0+\mathrm{mUT}_1$, and put τ_1 for the degree of T_1 in x, y, z, t, we shall obtain the following result:—

$$(t : m)\Delta\mathrm{T}+2\mathrm{D}'\mathrm{T} : m=\Delta\mathrm{T}_0+4\mathrm{mHT}+2\mathrm{m}\tau_1\mathrm{HT}_1 : (n-1).$$

Substituting for $\mathrm{D}'\mathrm{T}$ from the equations $\partial_x\mathrm{T} : u=.\,.=m\mathrm{mT}_1 : t$, we find

$$(t : m)\Delta\mathrm{T}=\Delta\mathrm{T}_0+2\mathrm{m}\{-1 : (n-1)+2+\tau_1 : (n-1)\}\mathrm{HT}_1. \quad . \quad . \quad . \quad (12)$$

But if τ represent the degree of T_1 it is easily seen that $\tau+1=\tau_1+n$, or $2\tau_1+2n-4=2(\tau-1)$; so that the coefficient of HT_1 will be $=2+2(\tau-1) : (n-1)$. Again, omitting terms which vanish with $\mathrm{U}, \phi, \psi, \chi$, and, for brevity, writing down only the first line of each determinant, we find

$$\begin{aligned}\Delta\mathrm{T}_0=\ &4\mathrm{H},\ u,\ v,\ w\\ &+2u,\ \mathrm{A}'(u,\ v,\ w)\\ &+2v,\ \mathrm{B}'(u,\ v,\ w)\\ &+2w,\ \mathrm{C}'(u,\ v,\ w)\\ &+2k,\ \mathrm{D}'(u,\ v,\ w),\end{aligned}$$

where the operations $\mathrm{A}', .\,.$ are supposed to affect all the columns which follow them; thus:

$$\begin{aligned}u,\ \mathrm{A}'(u,\ v,\ w)=\ &u,\ \mathrm{A}'u,\ v,\ w\\ &+u,\ u,\ \mathrm{A}'v,\ w\\ &+u,\ u,\ v,\ \mathrm{A}'w.\end{aligned}$$

This being so,

$$\left.\begin{aligned}\Delta\mathrm{T}_0=\ &4\mathrm{H},\ u,\ v,\ w\\ &+2\mathrm{D}'(k,\ u,\ v,\ w)\\ &-8\mathrm{H},\ u,\ v,\ w\\ =&-4\mathrm{H}u,\ v,\ w\\ &+2\mathrm{D}'\mathrm{T}.\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

But, writing it in full, the first term of this

$$\begin{array}{ll} =\mathrm{n}\mathrm{H}+(4-\mathrm{n})\mathrm{H}, & u,\ v,\ w=-\mathrm{T}+(4-\mathrm{n})\mathrm{HT}_1=(4-\mathrm{n})\mathrm{HT}_1, \\ \quad\Delta\phi, & a\ ,\ b\ ,\ c \\ \quad\Delta\psi, & a'\ ,\ b',\ c' \\ \quad\Delta\chi, & a'',\ b'',\ c'', \end{array}$$

since T is supposed to vanish. Hence

$$\left.\begin{aligned} \Delta\mathrm{T}_0&=\{-4+\mathrm{n}+2(n-m):(n-1)\}\mathrm{HT}_1 \\ &=\{-3n+8+2m-2+2n-2m\}\mathrm{HT}_1:(n-1) \\ &=-\mathrm{HT}_1. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (14)$$

So that, substituting in (12), we finally find

$$\left.\begin{aligned} (t:m)\Delta\mathrm{T}&=2\mathrm{m}\{1+(\tau-1):(n-1)\}\mathrm{HT}_1-\mathrm{HT}_1 \\ &=\mathrm{m}\{1+2(\tau-1):(n-1)\}\mathrm{HT}_1+(n-2m)\mathrm{HT}_1:m. \end{aligned}\right\} \quad . \quad . \quad . \quad (15)$$

If the degree of U be double that of ϕ, ψ, . . , *i. e.* if $n=2m$, the last term of this expression will vanish, and m=1; and the equation (15) will be identical with the equation for determining the two branches of three-point contact of T with U at a principal point of the system. But (15) has been formed on the supposition that $\varpi:\varpi'$ satisfies the equation $\Delta\mathrm{V}=\mathrm{n}\theta\mathrm{H}$, viz. the equation for determining the two branches of thre-epoint contact of V with U, at the same point. Hence, in the case where $n=2m$, and at the principal points of the system, the branches of three-point contact of T and U coincide with those of V with U.

It is moreover clear that a similar process may be applied to the functions $\mathrm{Q}z-\mathrm{R}y$, . ., since they are all of the form $\mathrm{A}+\mathrm{mUA}_1$, . ., and that similar results will be obtained. And a transformation similar to that adopted in § 1 will show that the same theorem holds good for the surfaces P, Q, R, S, as for the surface T.

It follows also, as in § 1, that the parabolic points of P, Q, R, S, T coincide with those of U; and also that when $m=n$, the parabolic points of the Hyperjacobian surfaces generally coincide with the principal points of the system.

It is perhaps worth while to calculate the Hessian of the Hyperjacobians at the principal points of the system. And it will be observed that the following calculation applies to all functions which can be expressed in the form $(t:m)\mathrm{P}=\mathrm{P}_0+\mathrm{mUP}_1$. In the first place, forming the Hessian (say H_0) of the left-hand side of this equation, and writing down only the first lines of the determinants, we find

$$\begin{aligned} \mathrm{H}_0(\mathrm{P}t:m)&=(t:m)^4\ \partial_x^2\mathrm{P},\ \partial_x\partial_y\mathrm{P},\ \partial_x\partial_z\mathrm{P},\ \partial_x\partial_t\mathrm{P}+\partial_x\mathrm{P}:t \\ &=(p:t)(t:m)^4\ \partial_x^2\mathrm{P},\ \partial_x\partial_y\mathrm{P},\ \partial_x\partial_z\mathrm{P},\ \partial_x\mathrm{P} \\ &=p(p-1)^{-1}(t:m)^4\ \partial_x^2\mathrm{P},\ \partial_x\partial_y\mathrm{P},\ \partial_x\partial_z\mathrm{P},\ \partial_x\partial_t\mathrm{P} \\ &=p(p-1)^{-1}(t:m)^4\mathrm{H}_0\mathrm{P}. \end{aligned}$$

Again, differentiating the right-hand side of the same equation, we obtain

$$\begin{array}{l}
\partial_x^2(\mathrm{P}t:m)\partial_x^2\mathrm{P}_0 \quad +\mathrm{m}\{u_1\mathrm{P}_1+u\partial_x\mathrm{P}_1+u\partial_x\mathrm{P}_1+\mathrm{U}\partial_x^2\mathrm{P}_1 \quad \} \\
\partial_y\partial_x(\mathrm{P}t:m)\partial_y\partial_x\mathrm{P}_0+\mathrm{m}\{w'\mathrm{P}_1+u\partial_y\mathrm{P}_1+v\partial_x\mathrm{P}_1+\mathrm{U}\partial_y\partial_x\mathrm{P}_1\} \\
\qquad\qquad \vdots \qquad\qquad \vdots \qquad\qquad \vdots
\end{array}$$

Hence

$$\begin{array}{l}
\mathrm{m}^{-4}\mathrm{H}_0(\mathrm{P}t:m) \\
\quad = \begin{array}{lll}
\partial_x^2\mathrm{P}_0:\mathrm{m}+u_1\mathrm{P}_1+u\partial_x\mathrm{P}_1+u\partial_x\mathrm{P}_1, & \partial_x\partial_y\mathrm{P}_0:\mathrm{m}+w'\mathrm{P}_1+u\partial_x\mathrm{P}_1+u\partial_y\mathrm{P}_1, & .\ . \\
\partial_y\partial_x\mathrm{P}_0:\mathrm{m}+w'\mathrm{P}_1+u\partial_y\mathrm{P}_1+v\partial_x\mathrm{P}_1, & \partial_y^2\mathrm{P}_0:\mathrm{m}+v_1\mathrm{P}_1+v\partial_y\mathrm{P}_1+v\partial_y\mathrm{P}_1, & .\ . \\
\vdots & \vdots & \vdots\ \vdots
\end{array}
\end{array}$$

$$= \begin{array}{cccccc}
\partial_x^2\mathrm{P}_0:\mathrm{m}+u_1\mathrm{P}_1, & \partial_x\partial_y\mathrm{P}_0:\mathrm{m}+w'\mathrm{P}_1 & . & . & \partial_x\mathrm{P}_1, & u \\
\partial_y\partial_x\mathrm{P}_0:\mathrm{m}+w'\mathrm{P}_1 & \partial_y^2\mathrm{P}_0:\mathrm{m}+v_1\mathrm{P}_1 & . & . & \partial_y\mathrm{P}_1, & v \\
\partial_z\partial_x\mathrm{P}_0:\mathrm{m}+v'\mathrm{P}_1 & \partial_z\partial_y\mathrm{P}_0:\mathrm{m}+u'\mathrm{P}_1 & . & . & \partial_z\mathrm{P}_1, & w \\
\partial_t\partial_x\mathrm{P}_0:\mathrm{m}+l'\mathrm{P}_1 & \partial_t\partial_y\mathrm{P}_0:\mathrm{m}+m'\mathrm{P}_1 & . & . & \partial_t\mathrm{P}_1, & k \\
\partial_x\mathrm{P}_1 & \partial_y\mathrm{P}_1 & & & . & -1 \\
u & v & . & . & -1, & 1
\end{array}$$

But if p_0, p_1, represent the degrees of P_0, P_1, respectively, we have

$$\begin{array}{l}
x(\text{col. }1)+y(\text{col. }2)+z(\text{col. }3)+t(\text{col. }4)-(n-1)\mathrm{P}_1(\text{col. }6) \\
=(p_0-1)\partial_x\mathrm{P}_0:\mathrm{m}+(n-1)u\mathrm{P}_1-(n-1)u\mathrm{P}_1=(p_0-1)\partial_x\mathrm{P}_0 \\
\quad (p_0-1)\partial_y\mathrm{P}_0:\mathrm{m}+(n-1)v\mathrm{P}_1-(n-1)v\mathrm{P}_1 \quad (p_0-1)\partial_y\mathrm{P}_0 \\
\qquad\quad \vdots \qquad\qquad\qquad \vdots \qquad\qquad\quad \vdots \qquad \vdots \\
\qquad\qquad\qquad (p_1+n-1)\mathrm{P}_1 \qquad\qquad\qquad (p_0+n-1)\mathrm{P}_1 \\
\qquad\qquad\qquad\qquad\quad 0 \qquad\qquad\qquad\qquad\qquad 0
\end{array}$$

But if P be of the same form as T in this section, or of any form having u, v, w, k as its first four columns, then at the principal points $(\partial_x, \partial_y, \partial_z, \partial_t)\mathrm{P}_0=0$; and consequently all the terms of this column will vanish except the fifth, which will $=(p_0+n-1)\mathrm{P}_1$. Further, if we operate in a similar manner upon the lines, viz. if, for line 6, we write

$$x(\text{line }1)+y(\text{line }2)+z(\text{line }3)+t(\text{line }4)-(n-1)\mathrm{P}_1\text{ line }6,$$

the whole expression will $=(p_1+n-1)^2(n-1)^{-2}\times$

$$\begin{array}{lll}
\partial_x^2\mathrm{P}_0:\mathrm{m}+u_1\mathrm{P}_1, & \partial_x\partial_y\mathrm{P}_0:\mathrm{m}+w'\mathrm{P}_1, & .\ . \\
\partial_y\partial_x\mathrm{P}_0:\mathrm{m}+w'\mathrm{P}_1, & \partial_y^2\mathrm{P}_0:\mathrm{m}+v_1\mathrm{P}_1, & .\ . \\
\partial_z\partial_x\mathrm{P}_0:\mathrm{m}+v'\mathrm{P}_1, & \partial_z\partial_y\mathrm{P}_0:\mathrm{m}+u'\mathrm{P}_1, & .\ . \\
\partial_t\partial_x\mathrm{P}_0:\mathrm{m}+l'\mathrm{P}_1, & \partial_t\partial_y\mathrm{P}_0:\mathrm{m}+m'\mathrm{P}_1, & .\ .
\end{array}$$

Similarly, at the points in question, P being of the form indicated above,

$$(\partial_x,\ \partial_y,\ \partial_z)^2\mathrm{P}_0=0,$$

$$\partial_x\partial_t\mathrm{P}_0:\mathrm{m}=\partial_x\mathrm{P}:\mathrm{m}=m\mathrm{P}_1u:t,$$

$$\partial_y\partial_t\mathrm{P}_0:\mathrm{m}=\partial_y\mathrm{P}:\mathrm{m}=m\mathrm{P}_1v:t,$$

$$\partial_z\partial_t\mathrm{P}_0:\mathrm{m}=\partial_z\mathrm{P}:\mathrm{m}=m\mathrm{P}_1w:t,$$

$$\partial_t^2\mathrm{P}_0:\mathrm{m}=\partial_t\mathrm{P}:\mathrm{m}=m\mathrm{P}_1k:t.$$

Hence, finally, the expression sought $=(p_1+u-1)^2(u-1)^{-2}\mathrm{P}_1^4\times$

$$\begin{matrix} u_1 & w' & v' & l'+mu:t, \\ w' & v_1 & u' & m'+mv:t, \\ v' & u' & w_1 & u'+mw:t \\ l'+mu:t & m'+mv:t & n'+mw:t & k_1+mk:t, \end{matrix}$$

$$=(p_1+n-1)^2(n-1)^{-2}(m+n-1)^2\mathrm{P}_1^4t^{-2}\times \begin{matrix} u_1, & w', & v', & u', \\ w', & v_1, & u', & v', \\ v', & u', & w_1, & w', \\ u', & v', & w', & k_1 \end{matrix}$$

$$=(p_1+n-1)^2(n-1)^{-2}(m+n-1)^2\mathrm{P}_1^4\mathrm{H}_0.$$

That is to say, at the principal points the Hessian of P vanishes with that of U.

§ 4. *Hyperjacobian Surfaces and Curve of a four-fold Pencil.*

Consider as before the surface U, and the four-fold pencil

$$\mathrm{V}=\mathrm{a}\phi+\mathrm{b}\psi+\mathrm{c}\chi+\mathrm{d}\omega+\mathrm{e}\varrho=0. \quad \ldots\ldots\ldots\ldots \quad (1)$$

If we now form the conditions for three-branch contact between U and V, we shall be able to eliminate the ratios a : b : . . . in two different ways, and thus deduce as before the Hyperjacobian curve. In order to form the expressions required, it will be convenient to write

$$\left.\begin{aligned} \Delta\ &=\varpi^{12}\Delta_{00}-2\varpi'\varpi\Delta_{01}+\varpi^2\Delta_{11}, \\ \mathrm{H}\ &=\mathrm{I}\varpi^{12}\ \ -2\mathrm{J}\varpi'\varpi\ \ +\mathrm{K}\varpi^2, \\ \Delta\phi &=\mathrm{L}\varpi^{12}\ \ -2\mathrm{M}\varpi'\varpi\ +\mathrm{N}\varpi^2, \\ \Delta\psi &=\mathrm{L}'\varpi^{12}\ \ -2\mathrm{M}'\varpi'\varpi\ +\mathrm{N}'\varpi^2, \\ \vdots\ \ & \quad \vdots \qquad\quad \vdots \qquad\qquad \vdots \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (2)$$

we shall then have for the Hyperjacobian curve the following expression:—

$$\left.\begin{array}{l} \mathrm{P}, \mathrm{Q}, \ldots = \begin{array}{ccccccc} u, & v, & w, & k, & \mathrm{nI}, & \mathrm{nJ}, & \mathrm{nK}, \\ a, & b, & c, & d, & \mathrm{L}, & \mathrm{M}, & \mathrm{N}, \\ a', & b', & c', & d', & \mathrm{L}', & \mathrm{M}', & \mathrm{N}'; \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{array} \end{array}\right\} \quad . \; . \; . \; . \; . \; . \; . \quad (3)$$

and it is not difficult to see that, by a transformation similar to that used in § 1, we may write

$$(t:m)(\mathrm{P}, \mathrm{Q}, \ldots) = \mathrm{P}_0 + \mathrm{mUP}_1, \; \mathrm{Q}_0 + \mathrm{mUQ}_1, \ldots \quad . \; . \; . \; . \; . \; . \quad (4)$$

and consequently that

$$(t:m)(\partial_x, \partial_y, \partial_z, \partial_t)(\mathrm{P}, \mathrm{Q}, \ldots) = \mathrm{m}(u, v, w, k)(\mathrm{P}_1, \mathrm{Q}_1, \ldots); \quad . \; . \; . \; . \quad (5)$$

that is to say, the Hyperjacobian surfaces touch the given surface at the principal points, and that the Hyperjacobian curve has a node at those points.

Again, a transformation similar to that employed in § 3 will give

$$(t:m)\Delta \mathrm{P} = \mathrm{m}\{1+2(m-1):(n-1)\}\mathrm{HP}_1 + (n-2m)\mathrm{HP}_1 : m. \quad . \; . \; . \quad (6)$$

But it is to be observed that a similar process would have led to the relations

$$(t:m)\Delta_{00}\mathrm{P} = \mathrm{m}\{1+2(m-1):(n-1)\}\mathrm{H}_{00}\mathrm{P}_1 + (n-2m)\mathrm{H}_{00}\mathrm{P}_1 : m, \quad . \; . \quad (7)$$

as well as to the corresponding relations with the suffixes 0, 1; 1, 1 respectively. These show that, in the case considered before, viz. where $n=2m$, the Hyperjacobian surfaces have three-branch contact with the given surface, and consequently with one another, at the principal points. At the same points the Hyperjacobian curve will have a triple point.

§ 5. *Nature of the Contact at the Secondary Points of the System.*

We have hitherto considered the degree of the contact of the Hyperjacobian surfaces, and the nature of the points on the Hyperjacobian curve, at the principal points of the system. Suppose that, at some of the principal points, the surfaces U, ϕ, ψ, ... not only meet, but touch one another; and let these points be called the *secondary points* of the system. When this is the case we shall have the relations

$$\left.\begin{array}{l} u : v : w : k \\ =a : b : c : d \\ =a' : b' : c' : d' \\ =\ldots \end{array}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

Suppose now that P, Q have the same values as in § 4, and that P represents a determinant containing the first four columns; say, let

$$\mathrm{P}, \mathrm{Q}, \mathrm{R} = \overset{*}{u}, \overset{*}{v}, \overset{*}{w}, \overset{*}{k}, \mathrm{nI}, \mathrm{nJ}, \mathrm{nK}. \quad . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

It is then, in the first place, clear that in virtue of these relations we may, at the points in question, regard any two of the first four columns of the determinant P as, à une facteur près, equal to one another; and, consequently, any derivative of P in which any two of those first four columns remain unaffected will *ipso facto* vanish.

Hence, for our present purpose, we shall have

$$\mathrm{P}=0,\ (\partial_x, \partial_y, \partial_z, \partial_t)\mathrm{P}=0,\ \Delta\mathrm{P}=0.\ \ .\ .\ .\ .\ .\ .\ .\ .\ \ (3)$$

Also, if we write $(\partial_x\Delta)$ for $(\partial_x\mathrm{A}, \ldots)(\partial_x, \partial_y, \partial_z, \partial_t)^2$, then

$$\partial_x\Delta\mathrm{P}=(\partial_x\Delta)\mathrm{P}+\Delta\partial_x\mathrm{P}.$$

But $(\partial_x\Delta)\mathrm{P}$ vanishes for the same reason as $\Delta\mathrm{P}$; hence operating upon the first derivative of the equation $(t:m)\mathrm{P}=\mathrm{P}_0+\mathrm{mUP}_1$, viz. upon the equation

$$(t:m)\partial_x\mathrm{P}=\partial_x\mathrm{P}_0+\mathrm{m}u\mathrm{P}_1+\mathrm{mU}\partial_x\mathrm{P}_1,$$

and putting p, p_1, p', p_1' for the degrees of P, P_1, $\Delta\mathrm{P}$, $\Delta\mathrm{P}_1$ respectively, we shall obtain

$$\left.\begin{aligned}(t:m)\partial_x\Delta\mathrm{P}=\Delta\partial_x\mathrm{P}_0+\mathrm{m}\{&\mathrm{P}_1\Delta u+u\Delta\mathrm{P}_1+2\mathrm{H}\partial_x\mathrm{P}_1+4\mathrm{H}\partial_x\mathrm{P}_1\\ &+\mathrm{U}\Delta\partial_x\mathrm{P}_1+2(p_1-1)\mathrm{H}\partial_x\mathrm{P}_1;(n-1)\}.\end{aligned}\right\}\ \ .\ .\ .\ \ (4)$$

But since P_0 contains the columns U, u, v, w, it follows that $\partial_x\Delta\mathrm{P}_0$ must contain either the column U, which vanishes, or two of the columns u, v, w, any two of which have been shown to be, à une facteur près, identical. Hence

$$(\Delta\partial_x, \Delta\partial_y, \Delta\partial_z, \Delta\partial_t)\mathrm{P}_0=0.\ \ .\ .\ .\ .\ .\ .\ .\ .\ .\ \ (5)$$

Also, since P_1 contains the three columns u, v, w, it follows that

$$(\partial_x, \partial_y, \partial_z, \partial_t)\mathrm{P}_1=0;\ \ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ \ (6)$$

and we may therefore conclude from (4) that in the present case

$$\partial_x\Delta\mathrm{P}:u=\partial_y\Delta\mathrm{P}:v=\partial_z\Delta\mathrm{P}:w=\partial_t\Delta\mathrm{P}:k=m\mathrm{m}\Delta\mathrm{P}_1:t;\ \ .\ .\ .\ .\ \ (7)$$

that is to say, at the secondary points of the system the Hyperjacobian surfaces have four-branch contact with the given surface, and consequently with one another; and the Hyperjacobian curve has a quadruple point at these points.

It does not, however, appear that the contact between the Hyperjacobian surface and the given surface is more than four-branched. This will be seen from the following process, which, although leading to only a negative result, is perhaps worth placing on record on account of the peculiarity of the algebraical result.

Operating with Δ upon the equation $(t:m)\mathrm{P}=\mathrm{P}_0+\mathrm{mUP}_1$, we obtain

$$\begin{aligned}(t:m)\Delta\mathrm{P}+2\mathrm{D}'\mathrm{P}:m&=\Delta\mathrm{P}_0+4\mathrm{mHP}_1+\mathrm{mU}\Delta\mathrm{P}_1+2\mathrm{m}p_1\mathrm{HP}_1:(n-1)\\ &=\Delta\mathrm{P}_0+2\mathrm{m}\{2+p_1:(n-1)\}\mathrm{HP}_1+\mathrm{mU}\Delta\mathrm{P}_1.\end{aligned}$$

Operating a second time:

$$\begin{aligned}(t:m)\Delta^2\mathrm{P}+2\mathrm{D}'\Delta\mathrm{P}:m+2(\Delta\mathrm{D}')\mathrm{P}:m+2\mathrm{D}'\Delta\mathrm{P}:m+4(\mathrm{A},\ldots)(\partial_x\mathrm{D}',\ldots)(\partial_x\mathrm{P},\ldots):m\\ =\Delta^2\mathrm{P}_0+2\mathrm{m}\{2+p_1:(n-1)\}\{\mathrm{P}_1\Delta\mathrm{H}+\mathrm{H}\Delta\mathrm{P}_1+2(\mathrm{A},\ldots)(\partial_x\mathrm{H},\ldots)(\partial_x\mathrm{P}_1,\ldots)\}\\ +\mathrm{m}\{4\mathrm{H}\Delta\mathrm{P}_1+\mathrm{U}\Delta^2\mathrm{P}_1+2p_1'\mathrm{H}\Delta\mathrm{P}_1:(n-1)\}.\end{aligned}$$

But, in virtue of (3) and (6),

$$(\Delta\mathrm{D})\mathrm{P}=0,\ (\mathrm{A},\ldots)(\partial_x\mathrm{H},\ldots)(\partial_x\mathrm{P}_1,\ldots)=0,\ (\mathrm{A},\ldots)(\partial_x\mathrm{D}',\ldots)(\partial_x\mathrm{P},\ldots)=0;\quad .\quad (8)$$

and consequently

$$(t:m)\Delta^2\mathrm{P}+4\mathrm{m}\mathrm{H}\Delta\mathrm{P}_1:(n-1)=\Delta^2\mathrm{P}_0+2\mathrm{m}\{4+(p_1+p_1'):(n-1)\}\mathrm{H}\Delta\mathrm{P}_1.\quad .\quad (9)$$

Again (writing down only the first lines of the determinants), since

$$\Delta\mathrm{P}_0=-\mathrm{H}\mathrm{P}_1+\mathrm{U},\ \Delta(u,\ldots),\quad .\ .\ .\ .\ .\ .\ .\ .\ .\ .\quad (10)$$

$$\begin{aligned}\Delta^2\mathrm{P}_0=-\mathrm{H}\Delta\mathrm{P}_1-\mathrm{P}_1\Delta\mathrm{H}-2(\mathrm{A},\ldots)(\partial_x\mathrm{H},\ldots)(\partial_x\mathrm{P}_1,\ldots)\\ +4\mathrm{H},\ \Delta(u,\ldots)+\mathrm{U},\ \Delta^2(u,\ldots)+2u,\ \mathrm{A}'\Delta(u,\ldots)\\ +2v,\ \mathrm{B}'\Delta(u,\ldots)\\ +2w,\ \mathrm{C}'\Delta(u,\ldots)\\ +2k,\ \mathrm{D}'\Delta(u,\ldots);\end{aligned}$$

or omitting terms which vanish,

$$\begin{aligned}\Delta^2\mathrm{P}_0=-\mathrm{H}\Delta\mathrm{P}_1+4\mathrm{H},\ \Delta(u,\ldots)-8\mathrm{H},\ \Delta(u,\ldots)+2\mathrm{A}'\{u,\ \Delta(u,\ldots)\}\\ +2\mathrm{B}'\{v,\ \Delta(u,\ldots)\}\\ +2\mathrm{C}'\{w,\ \Delta(u,\ldots)\}\\ +2\mathrm{D}'\{k,\ \Delta(u,\ldots)\}.\end{aligned}$$

But

$$u,\ \Delta(u,\ldots)=\Delta(u,u,\ldots)-\Delta u,\ u,\ldots-2(\mathrm{A},\ldots)(u_1,w',v')(\partial_x,\ldots)(u,\ldots);$$

and of these terms $\Delta(u,u,\ldots)$ vanishes identically, and $\Delta u, u, \ldots$ contains the three columns u, v, w; so that $\mathrm{A}'\{\Delta u, u, \ldots\}$ will contain two of them, and will consequently vanish.

But if we retain only terms which contain not more than two of the columns u, v, w, k, and which, after the operations $\mathrm{A}', \mathrm{B}', \mathrm{C}', \mathrm{D}'$, will consequently contain only one such column, we shall have

$$\begin{aligned}u,\ \Delta(u,\ldots)=2u,\ \mathrm{A}'u,\ w',\ w,\ldots+2u,\ \mathrm{A}'u,\ v,\ v',\ldots\\ +2u,\ \mathrm{B}'u,\ v_1,\ w,\ldots+2u,\ \mathrm{B}'u,\ v,\ u',\ldots\\ +\ldots\qquad\qquad\qquad +\ldots\end{aligned}$$

and consequently

$$\begin{aligned}\mathrm{A}'\{u,\ \Delta(u,\ldots)\}=2u,\ \mathrm{A}'u,\ w',\ \mathrm{A}'w,\ldots+2u,\ \mathrm{A}'u,\ \mathrm{A}'v,\ v',\ldots\\ +2u,\ \mathrm{B}'u,\ v_1,\ \mathrm{A}'w,\ldots+2u,\ \mathrm{B}'u,\ \mathrm{A}'v,\ u',\ldots\\ +\ldots\qquad\qquad\qquad +\ldots\end{aligned}$$

But since col. $w=\lambda$ col. u, it follows that

$$\mathrm{A}' \text{ col. } w = \text{col. } u\,.\,\mathrm{A}'\lambda+\lambda\,\mathrm{A}' \text{ col. } u\,;$$

so that the expression $\mathrm{A}'\{u, \Delta(u, \ldots)\}$ will vanish. The same will obviously be the case with the results of B'_1 and C' on the same expression; and we may, in fact, conclude as follows:—

$$\begin{aligned}
\mathrm{A}'\{u, \Delta(u, \ldots)\}&=0,\\
\mathrm{B}'\{u, \Delta(u, \ldots)\}&=0,\\
\mathrm{C}'\{u, \Delta(u, \ldots)\}&=0,\\
\mathrm{D}'\{u, \Delta(u, \ldots)\}&=\mathrm{D}'\Delta\mathrm{P}.
\end{aligned}$$

Moreover,

$$\begin{aligned}
4\mathrm{H}, \Delta(u, \ldots)&=\{4\mathrm{H}, \Delta(u, v, w), \mathrm{nI}, \mathrm{nJ}, \mathrm{nK}\}\\
&=(4-\mathrm{n})\mathrm{H}\{\Delta(a, b, c), \mathrm{L}, \mathrm{M}, \mathrm{N}\}\\
&=(4-\mathrm{n})\mathrm{H}\Delta\{a, b, c, \mathrm{L}, \mathrm{M}, \mathrm{N}\}\\
&=(4-\mathrm{n})\mathrm{H}\Delta\mathrm{P}_1.
\end{aligned}$$

Hence, finally,

$$\left.\begin{aligned}
\Delta^2\mathrm{P}_0&=\{-1-4+\mathrm{n}+2m\mathrm{m}:(n-1)\}\mathrm{H}\Delta\mathrm{P}_1\\
&=\{-1+(2m-2-3n+3+2n-2m):(n-1)\}\mathrm{H}\Delta\mathrm{P}_1\\
&=-2\mathrm{H}\Delta\mathrm{P}_1.
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (11)$$

Collecting the various terms, we find

$$(t:m)\Delta^2\mathrm{P}=2\{-1+\mathrm{m}[4+(p_1+p'_1-2):(n-1)]\}\mathrm{H}\Delta\mathrm{P}_1.$$

But since

$$\begin{aligned}
p' &=\text{degree of } \Delta\mathrm{P}\,, & \therefore p' &= 2n+p-6,\\
p_1 &=\text{degree of } \mathrm{P}_1\,, & p_1 &= -n+p+1,\\
p'_1 &=\text{degree of } \Delta\mathrm{P}_1, & p'_1 &= 2n+p_1-6,
\end{aligned}$$

consequently

$$\begin{aligned}
&p_1+p'_1=2p-4,\\
&p_1+p'_1-2+4(n-1)=2p'+2,\\
&(4p'+4)(n-m):m(n-1)-2,\\
&\qquad=\{(4p'+4)(n-m)-2m(n-1)\}:m(n-1)\\
&\qquad=\{[4p'+4-(2n-2)](n-m)+2(n-1)(n-2m)\}:m(n-1)\\
&\qquad=\{[1+2(p'-1):(n-1)]+[1+2(p-1):(n-1)]\}\mathrm{m}=2(n-2m):m.
\end{aligned}$$

Hence, in the case considered before, viz. where $n=2m$,

$$(t:m)\Delta^2\mathrm{P}=(\mathrm{n}'+\mathrm{n})\mathrm{H}\Delta\mathrm{P}_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where $\mathrm{n}'=1+2(p'-1):(n-1)$; *i. e.* n' has the same relation to $\Delta\mathrm{P}$ that n has to P. But since n can in no case vanish, it follows that the Hyperjacobian surfaces of a four-fold pencil cannot in general have more than four-branch contact with the given surface at the secondary points of the system.

XI. *On a new Form of Tangential Equation. By* JOHN CASEY, *LL.D., F.R.S., M.R.I.A., Professor of Mathematics in the Catholic University of Ireland.*

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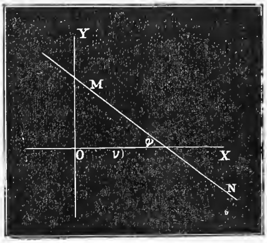
INTRODUCTION.

ART. 1. The tangential equation of a curve is, as is well known, a relation among the coefficients in the equation of a variable line, which being fulfilled, the line must be a tangent to the curve.

Fig. 1.



Let O be the origin, OX, OY the axes; and let a variable line MN in any of its positions make an intercept ν on OX and an angle ϕ with it; then the equation of the line is

$$x+y\cot\phi-\nu=0,$$

and ν and ϕ, the quantities which determine the position of the line, may be called its coordinates. From this it follows that any relation between ν and ϕ, such as

$$\nu=f(\phi), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

will be the tangential equation of a curve which is the envelope of the line.

This form of equation will be the special subject of this paper. Occasionally our investigations will embrace collateral subjects, when their importance will be such as to justify the digression.

It will be seen that our form of equation admits of easy transformation into all the known forms of equation; that it adapts itself with great facility to the various problems of the Integral Calculus relating to curves, such as Rectification, Curvature, Involutes, &c., and gives its results in very simple forms.

In most of the methods of Modern Geometry, such as Pedals, Parallel Curves, Reciprocation, &c., it solves in a very simple manner problems that are very difficult by any other method. I have illustrated it throughout by numerous examples, most of which are of historical interest. Some of the problems discussed are, I believe, now solved for the first time, among which I may mention the rectification of Bicircular Quartics by Elliptic Functions. To this outline of the subject of this paper I may add that the form of equation is suggested by *Kinematics*. For if we differentiate the equation $\nu=f(\phi)$ with respect to a variable t (denoting the time) we get

$$\frac{d\nu}{dt}=f'(\phi)\frac{d\phi}{dt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

Now if we suppose a rigid body to move so that a fixed point in it, say the centre of gravity, describes a right line, then $\frac{d\nu}{dt}$ will be the linear velocity of the centre of gravity, and $\frac{d\phi}{dt}$ will be the angular velocity with which the body revolves round the same point. Then the equation (2) will be the most general equation of the motion of such a body. It gives linear velocity divided by angular velocity as a function of the angle through which the body has rotated. From this it will be seen that some of our results will have a physical as well as a purely mathematical interest. With these remarks we proceed to the subject of the paper.

CHAPTER I.

SECTION I.—*Transformation of Cartesian into Tangential Equations.*

2. *Definition.*—We shall find it convenient to call the line OX, on which the variable line makes the intercept ν, and with which it makes the angle ϕ, the *director* line.

3. If the Cartesian equation of a curve be $U=0$, we can by the usual process find the condition that the line $x+y\cot\phi-\nu=0$ touches it; this condition will be our tangential equation. For this purpose the equation of the line may be written in the form

$$y=(\nu-x)t, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

where t denotes $\tan\phi$; and eliminating y between this and the equation $U=0$, we shall have an equation in x of the form

$$(A_0, A_1, A_2, \ldots A_n)(x-1)^n=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

The discriminant of this will be the tangential equation required. It can be transformed into the usual form of tangential equation by changing ν into $-\frac{\nu}{\lambda}$ and t into $\frac{\lambda}{\mu}$. This is evident by comparing the equations

$$x+y\cot\phi-\nu=0, \quad \lambda x+\mu y+\nu=0.$$

Cor. The usual form of tangential equation can be transformed into our form as follows:—Let

$$\psi(\lambda, \mu, \nu)=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

be the tangential equation, say of the nth degree; divide by λ^n, and change $\frac{\nu}{\lambda}$ into $-\nu$, and $\frac{\mu}{\lambda}$ into $\cot\phi$.

4. The coefficients in equation (4) are deserving of notice. Equated to zero, they are the tangential equations of curves which possess interesting geometrical relations to the curve U. For the purpose of examining their properties, let the curve U be given by the equation

$$\begin{aligned}(a_0, a_1, a_2 \ldots a_n \between x, y)^n + n(b_1\, b_2 \ldots b_n \between x, y)^{n-1}& \\ + \frac{n \,.\, n-1}{\underline{|2}}(c_2, c_3 \ldots c_n \between x, y)^{n-3} + \&\text{c.} = 0\,; \quad \ldots \ldots \ldots &\end{aligned} \tag{6}$$

then substituting in this the value of y from equation (3), and equating the result with equation (4), we get the following system of identities:—

$$\left.\begin{aligned}
A_0 &\equiv (a_0,\ a_1,\ a_2 \ldots a_n \between 1,\ -t)^n = 0, \\
A_1 &\equiv \nu t(a_1,\ a_2 \ldots a_n \between 1,\ -t)^{n-1} \\
&\quad + (b_1,\ b_2, \ldots b_n \between 1,\ -t)^{n-1} = 0, \\
A_2 &\equiv \nu^2 t^2 (a_2,\ a_3, \ldots a_n \between 1,\ -t)^{n-2} \\
&\quad + 2\nu t(b_2,\ b_3 \ldots b_n \between 1,\ -t)^{n-2} \\
&\quad + (c_2,\ c_3, \ldots c_n \between 1,\ -t)^{n-2} = 0, \\
A_3 &\equiv \nu^3 t^3 (a_3,\ a_4 \ldots a_n \between 1,\ -t)^{n-3} \\
&\quad + 3\nu^2 t^2 (b_3,\ b_4 \ldots b_n \between 1,\ -t)^{n-3} \\
&\quad + 3\nu t(c_3,\ c_4 \ldots c_n \between 1,\ -t)^{n-3} \\
&\qquad (d_3,\ d_4 \ldots d_n \between 1,\ -t)^{n-3} = 0, \\
&\qquad \&\text{c.} \qquad \&\text{c.} \qquad \&\text{c.}
\end{aligned}\right\} \quad \ldots \ldots \ldots \tag{7}$$

5. The system of identities (7) are remarkable for their symmetry, the equation $A_0 = 0$ being independent of all but the coefficients of the highest powers of x and y, A_1 of all the homogeneous terms lower than the $(n-1)$th in x and y, &c. Transformed into the usual form of tangential coordinates, they become

$$\left.\begin{aligned}
A_0 &\equiv (a_0,\ a_1,\ a_2 \ldots a_n \between \mu,\ -\lambda)^n = 0, \\
A_1 &\equiv \nu(a_1,\ a_2,\ a_3 \ldots a_n \between \mu, -\lambda)^{n-1} \\
&\quad - \mu(b_1,\ b_2,\ b_3 \ldots b_n \between \mu,\ -\lambda)^{n-1} = 0, \\
A_2 &\equiv \nu^2 (a_2,\ a_3 \ldots a_n \between \mu,\ -\lambda)^{n-2} \\
&\quad - 2\mu\nu(b_2, b_3 \ldots b_n \between \mu,\ -\lambda)^{n-2} \\
&\quad + \mu^2 (c_2,\ c_3 \ldots b_n \between \mu,\ -\lambda)^{n-2} = 0, \\
A_3 &\equiv \nu^3 (a_3, a_4 \ldots a_n \between \mu,\ -\lambda)^{n-3} \\
&\quad - 3\mu\nu^2 (b_3,\ b_4 \ldots b_n \between \mu,\ -\lambda)^{n-3} \\
&\quad + 3\mu^2\nu(c_3, c_4 \ldots c_n \between \mu,\ -\lambda)^{n-3} \\
&\quad - \mu^3 (d_3,\ d_4 \ldots d_n \between \mu,\ -\lambda)^{n-3} = 0.
\end{aligned}\right\} \quad \ldots \ldots \ldots \tag{8}$$

6. We shall now examine the geometrical interpretation of the equations (8), first, for the sake of illustration, in special cases, and then we shall give the general results.

We may remark in passing that all the contravariants of curves can be expressed in terms of these tangential curves; for instance, if U be a cubic, the envelope of the line which cuts it in three points, whose distances are in arithmetical progression, is the curve

$$A_0^2 A_3 + 2A_1^3 - 3A_0 A_1 A_2 = 0 ; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and if U be a quartic, the envelope of the line which it cuts harmonically is the determinant

$$\begin{vmatrix} A_0, & A_1, & A_2 \\ A_1, & A_2, & A_3 \\ A_2, & A_3, & A_4 \end{vmatrix} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

7. Let the curve $U=0$ be a conic, then the equation (4) becomes

$$(A_0, A_1, A_2 \between x, 1)^2 = 0.$$

Now if $A_1=0$, it is evident the line $y=(\nu-x)t$ will cut the curve in two points, which are equally distant from the axis of y; but when $n=2$, A_1 becomes

$$\nu(a_1\mu - a_2\lambda) - \mu(b_1\mu - b_2\lambda) = 0 ; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

that is, a conic section. Hence we have the following theorems, the second of which is the projection of the first, and follows from the equation in λ, μ, ν, as the first does from the corresponding one in ν and t:—

1st. *If a variable line intersect a conic section, and if the locus of its middle point be a right line, its envelope is a conic section.*

2nd. *If a variable line be cut harmonically by a conic section and a pair of lines, its envelope is a conic section touching the pair of lines.*

8. Let U be the cubic

$$(a_0, a_1, a_2, a_3 \between x, y)^3 + 3(b_1, b_2, b_3 \between x, y)^2 + 3(c_2, c_3 \between x, y) + d_3 = 0, \quad . \quad . \quad (12)$$

and the curve A_1 will be

$$\nu t(a_1, a_2, a_3 \between 1-t)^2 + (b_1, b_2, b_3 \between 1-t)^2 = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

This equation is the condition that the locus of the mean centre of the points where the line $x + y \cot \phi - \nu$ meets the curve is the axis of y; and since the axis of y may be any line, we have the following theorem:—*If a variable line intersect a cubic in such a manner that the locus of the mean centre of the points where it meets the cubic is a right line, its envelope is a curve of the third class.*

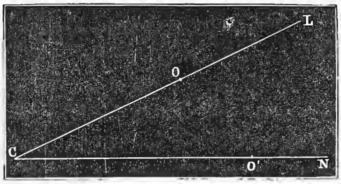
9. The equation (13), expressed in the usual notation of tangential coordinates, is

$$\nu(a_1, a_2, a_3 \between \mu - \lambda)^2 - \mu(b_1, b_2, b_3 \between \mu - \lambda)^2 = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

This is the analytical statement of the theorem we get by projecting that of the last article; and since in projection the line at infinity becomes a finite line, it may be expressed as follows:—*Being given a curve of the third degree*, $U=0$, *and two fixed lines* L *and* N; *then if* O, O′ *be two variable points on* L *and* N *respectively, such that the polar line of* O *with respect to* U *passes through* O′, *the envelope of the line* OO′ *is a curve of the third class.*

10. Since the highest power of ν contained in equation (14) is the first, the tangential cubic A_1 which it represents has one double tangent, namely the line joining the points λ and μ, which we may call the line $(\lambda\mu)$. Similarly the line $(\mu\nu)$ is a single tangent. The same thing can be shown geometrically, as follows:—Let the lines L and N intersect in C, then C is the point whose equation is $\mu=0$. Now since, the polar line of O passes through O′, then the polar conic of O′ passes through O; but this conic intersects the line L in two points, and the line joining O′ to each of them is a tangent to A_1. Hence from any point of the line CN can in general two tangents be drawn to A_1; and we shall see immediately that CN itself is a tangent. This agrees with the fact of the curve being of the third class. Let the polar conic of C intersect CL in the points Ω, Ω', then the lines $C\Omega$, $C\Omega'$ are tangents to A_1; in other words, CL is a double tangent, and it is plain that Ω, Ω' are its points of contact. Again, let the polar line of C intersect CN in H, then H is a point of contact, so that CN is a tangent.

Fig. 2.



11. Since the point O′ moves on CN, its polar conic will pass through four fixed points, namely, the four poles of CN with respect to U. Hence any line will be cut in involution by the polar conics of the points O′; and we have the following theorem:—*If from any three points in* CN *three pairs of tangents be drawn to* A_1, *these will meet its double tangent in six points in involution, and the two points of contact of the double tangent belong to the involution.*

12. We find the limiting points of the involution as follows:—Let the pole conic of the line CL with respect to U intersect CN in the points Σ, Σ'; then since the pole-conic is the locus of points whose polar conics touch CL, the polar conics of the points Σ, Σ' will touch CL. Let the points where they touch it be denoted by Δ, Δ', then Δ, Δ' will be the double points of the involution. *Or thus, the double points will be the points of contact of the two conics, which can be drawn through the four poles of* CN *to touch* CL.

13. From the last article, it is plain that each of the lines $\Sigma\Delta$, $\Sigma'\Delta'$ is a pair of coincident tangents to the curve A_1; and since CN is itself a tangent we see that from each of the points Σ, Σ' can be drawn only two tangents to A_1; but the curve is of the third class, therefore it must pass through Σ and Σ'. Hence we have the following theorem:—

The curve A_1 *intersects the line* CN *in the points where the pole-conic of* CL *meets it, and it touches* CN *in the point whose polar conic passes through* C.

14. The polar line of the point C with respect to U will cut CN at its point of contact with A_1. The same polar line will be a tangent to the pole-conic of CL, and will be the polar of the point C with respect to the polar conic of C. *Hence it will with* C *divide harmonically the segment of* CL *included between the points of contact with* A_1.

15. We can get the equation of the line of which A_1 is the envelope as follows:—Since $y=(\nu-x)t$ we have $\nu t=(y+xt)$; and substituting in equation (13) we get

$$\alpha_3 xt^3-(2a_2x-a_3y-b_3)t^2+(a_1x-2a_2y-2b_2)t+a_1y+b, \quad . \quad . \quad . \quad . \quad (15)$$

which is the required line, and the discriminant with respect to t will be Cartesian equation of A_1. This discriminant is

$$\left.\begin{aligned} &27a_3^2x^2(a_1y+b_1)^2+4a_3x(a_1x-2a_2y-2b_2)^3 \\ &\quad -4(a_1y+b_1)(2a_2x-a_3y-b_3)^3 \\ &\quad -(2a_2x-a_3y-b_3)^2(a_1x-2a_2y-2b_2)^2 \\ &\quad +18a_3x(2a_2x-a_3y-b_3)(a_1x-2a_2y-2b_2)(a_1y+b_1)=0. \end{aligned}\right\} \quad . \quad . \quad . \quad (16)$$

This equation is of the fourth degree, as it ought, since the curve has a double tangent.

16. If we denote the equation (15) by T, and since a cusp is a point at which three consecutive tangents intersect, the conditions that there shall be a cusp are that

$$T=0, \quad \frac{dT}{dt}=0, \quad \frac{d^2T}{dt^2}=0\,;$$

and eliminating x and y from these equations, we get the following determinant:—

$$\begin{vmatrix} a_3t-2a_2\,, & a_3\,, & b_3\,, \\ a_1-2a_2t, & a_3t-2a_2\,, & b_3t-2b^2\,, \\ a_1t\,, & a_1-2a_2t, & b_1-2b_2t, \end{vmatrix}=0 \quad . \quad . \quad . \quad . \quad . \quad (17)$$

This determinant is a cubic in t, showing that there are three cusps. The values of t, got from this equation, if substituted in equation (15), will give us the three cuspidal tangents.

17. If we denote the singularities by the following notation—

Class ν,
Double tangents τ,
Cusps $\varkappa$,

Degree μ,
Double points δ,
Points of inflection ι,

we have the singularities of the curve A_1 as follows:—

$$\left.\begin{aligned} \nu=3, \quad \mu=4, \quad \tau=1, \\ \varkappa=3, \quad \iota=0, \quad \delta=0, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Observation.—The curve A_2 for a cubic has properties similar to A_1. They differ only in that the lines CL and CN are interchanged, for CL is a single and CN a double tangent to A_2.

18. If U be the general curve of the nth degree, $A_1=0$ gives the following theorem:—

Given a curve of the nth *degree, and two lines* CL *and* CN, *then if* O, O′ *be two movable points on these lines, such that the polar line of* O *with respect to* U *may pass through* O′, *the envelope of* OO′ *will be a curve of the* nth *class, to which the line* CL *will be a multiple tangent of the order* $(n-1)$.

19. If in the equation for A_1 given in art. 4 we substitute for ν its value $y+xt$, as in art. (15), we shall find the equation of OO′ in the form

$$at^n+nbt^{n-1}+\frac{n.n-1}{\underline{|2}}ct^{n-2}+\&c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (19)$$

Hence (see SALMON'S 'Higher Curves,' second edition, p. 66) we have

$$\left.\begin{array}{lll}\nu=n, & \mu=2(n-1), & \varkappa=3(n-2),\\ \delta=(n-2)(n-3), & \tau=\frac{1}{2}(n-1)(n-2), & \iota=0.\end{array}\right\} \quad . \quad . \quad . \qquad (20)$$

All this will also follow from the propositions of the following articles, of which this and the preceding are special cases.

20. We will now examine the general case $A_m=0$.

The equation $A_m=0$ gives us the following theorem:—If $U=0$ *be a curve of the* nth *degree, and* CL, CN *two given lines, then if* O, O′ *be two points taken on these lines, such that the* mth *polar of* O *with respect to* U *passes through* O′, *then the envelope of* OO′ *is the curve of the* nth *class* $A_m=0$.

21. The curve A_m touches the line CL in $(n-m)$ points and CN in m points.

Demonstration.—Since the mth polar of O passes through O′, the $(n-m)$th polar of O′ passes through O. Hence we have two ways of generating the curve. Now let the point O′ move along CN until it becomes consecutive to C, and it is evident that the $(n-m)$ points in which its $(n-m)$th polar intersects CL will be points of contact of CL with A_m. In like manner the m points in which the mth polar of a point consecutive to C on the line CL intersects the line CN will be points of contact. Hence the proposition is proved.

Cor. The number of double tangents which A_m has $=\frac{n^2-n-2mn+2m^2}{2}$.

For the line CL is equivalent to

$$\frac{(n-m)(n-m-1)}{2} \text{ double tangents,}$$

and the line CN to

$$\frac{m(m-1)}{2} \text{ double tangents;}$$

∴ we have

$$2\tau=n^2-n-2mn+2m^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (21)$$

22. The curve A_m is of the degree $2m(n-m)$.

Demonstration.—If O′ be any point on the line CN, then since the $(n-m)$th polar of O′ cuts CL in $(n-m)$ points, the lines drawn from O′ to these points will make up $(n-m)$ tangents, and the line CL itself counts for m tangents. Hence the n tangents which can be drawn from O′ are accounted for. Now if the point O′ itself be on the curve A_m, only $(n-1)$ tangents can be drawn from it, and two of the points in which the line CL is intersected by the polar curve of the $(n-m)$th degree must coincide, that is the polar curve must touch CL. Hence we have to find the points on CN whose polar curves of the $(n-m)$th degree, with respect to U, will touch CL. In order to find the number of solutions of this problem, we will use trilinear coordinates. Let (a, b, c) (a', b', c') be the coordinates of two fixed points on CN, then the coordinates of any variable point on it are $a+ka'$, $b+kb'$, $c+kc'$, and the polar curve of this point of the $(n-m)$th degree, with respect to U, is

$$\left\{(a+ka')\frac{d}{dx}+(b+kb')\frac{d}{dy}+(c+kc')\frac{d}{dz}\right\}^m \mathrm{U}=0. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (22)$$

Now this equation contains the variables in the degree $n-m$, and its coefficients contain k in the mth degree. Hence the condition that it will touch any given line will contain k in the degree $2m(n-m-1)$; and this is the number of points in which the curve A_m intersects the line CN, but it touches CN in m points; ∴ the total number of points in which the curve meets CN is $2m(n-m)$.

Hence the proposition is proved.

23. The following are the singularities for the curve A_m:—

$$\nu=n, \qquad \mu=2m(n-m), \qquad \iota=0.$$

$$\left.\begin{aligned} 2\tau&=n^2-n-2mn+2m^2,\\ \delta&=2m^2(n-m)^2-10m(n-m)+4n,\\ \kappa&=6mn-6m^2-3n.\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (23)$$

Cor. $\mu+2\tau=n^2-n$, and is therefore the same for the curves A_1, A_2, &c.; that is, it is independent of m.

Cor. 2. The curves A_m, A_{n-m} have the same singularities.

Examples.

(1) Find the tangential equation of the cuspidal cubic $ay^2=x^3$.

Eliminating y between this and the equation $y=(\nu-x)t$, we get

$$x^3-at^2x^2+2a\nu t^2x-a\nu^2t^2=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (24)$$

The discriminant of this is

$$\nu=\frac{4a}{27}t^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (25)$$

which is the required tangential equation.

In the usual notation this is

$$4a\lambda^3+27\mu^2\nu=0. \qquad (26)$$

The equation (24) shows that the sum of the x's of the points where any line cuts $ay^2=x^3$ is proportional to the square of the tangent of the angle which the line makes with the axis of x, and the sum of their reciprocals is proportional to the reciprocal of the intercept which the same line makes on the same axis.

Cor. It is evident that similar theorems hold for the curve $ay^{n-1}=x^n$.

(2) Let the curve be $x^3+y^3-3axy=0$.

The tangential equations are

$$\nu^4-(6a^2\cot\phi)\nu^2-4a^3(1+\cot^3\phi)\nu+3a^4\cot^2\phi=0, \qquad (27)$$

$$\nu^4-6a^2\lambda\mu\nu^2+4a^3(\lambda^3+\mu^3)\nu+3a^4\lambda^2\mu^2=0. \qquad (28)$$

(3) Find the tangential equations of the cissoid.

They are

$$(2a-\nu)^3=27a^2\nu\cot^2\phi, \qquad (29)$$

$$(2a\lambda+\nu)^3+27a^2\mu^2\nu=0. \qquad (30)$$

(4) Find the tangential equation A_1 for a cubic in its canonical form—that is, referred to its three chords of inflection as axes. This question is solved by supposing the coefficients in the equation (12) to vanish, except a_0, a_3, b_2, d_3; then equation (14) becomes the conic

$$a_3\nu\lambda+2b_2\mu^2=0, \qquad (31)$$

and the curve A_1 for the Hessian of the cubic is

$$3(a_0b_3d_3)^{\frac{1}{3}}b_2^2\nu\lambda=(a_0b_3d_3+2b_2^3)\mu^2, \qquad (32)$$

a curve which has double contact with the former.

(5) Find the equations of the curves A_1, A_2, A_3 for the trinodal quartic

$$(a,\ b,\ c,\ f,\ g,\ h)\!\!\between\!\!(x^{-1},\ y^{-1},\ z^{-1})^2.$$

$$A_1\equiv\mu^2\lambda(g\lambda-f\mu+c\nu)=0, \qquad (33)$$

$$A_3\equiv\mu^2\nu(a\lambda-h\mu+g\nu)=0, \qquad (34)$$

$$A_2\equiv(a,\ b,\ c,\ -f,\ 2g,\ -h)\!\!\between\!\!(\lambda,\ \mu,\ \nu)^2=0. \qquad (35)$$

(6) The points where the curve A_m intersects the line CN may be found as follows:—If a variable point moves along the line CL the envelope of its polar curve of the mth degree with respect to U will be a curve of the degree $2m(n-m-1)$ which will cut CN in the required points. Similarly the points where it cuts CL may be found.

SECTION II.—*Transformation of Polar into Tangential Equations.*

24. The polar equation of a curve being given, to find its tangential equation.

Let the polar equation be $\varrho=F(\theta)$, then

$$\tan\psi=\frac{F(\theta)}{F'(\theta)}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Also we have $\nu\sin\phi=\varrho\sin\psi$, that is, we have

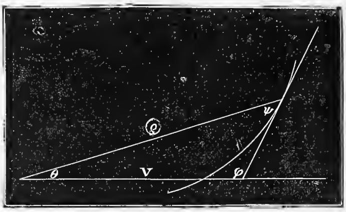
$$\nu\sin\phi=F(\theta)\sin\psi \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and

$$\theta+\phi+\psi=\pi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Then eliminating θ and ψ between equations (1), (2), (3). The result will be the tangential equation.

Fig. 3.



Ex. Let the polar equation be

$$\varrho^m=a^m\sin m\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

We find, by taking logarithmic differentials,

$$\tan\psi=\tan m\theta;$$

$$\therefore\ \psi=m\theta,$$

$$\text{and}\ \ \nu^m\sin^m\phi=\varrho^m\sin^m\psi=a^m\sin^{m+1}\psi.$$

Hence the tangential equation is

$$\nu\sin\phi=a\left\{\sin\frac{m(\pi-\phi)}{m+1}\right\}^{\frac{m+1}{m}},$$

or, putting ϕ in place of $\pi-\phi$,

$$\nu\sin\phi=a\left\{\sin\frac{m\phi}{m+1}\right\}^{\frac{m+1}{m}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

25. The family of curves represented by equation (36) includes several important species. The following Table contains the principal, with their corresponding tangential equations.

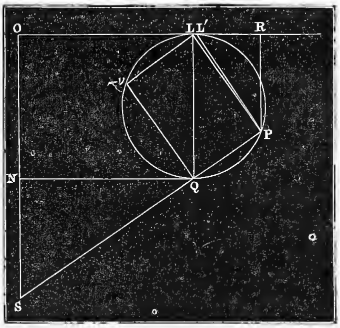
Value of m.	Name of curve.	Tangential equation of curve.	
2	Lemniscate....	$\nu\sin\phi=a\left(\sin\frac{2\phi}{3}\right)^{\frac{3}{2}}$	(38)
-2	Equilateral hyp.	$\nu\sin\phi=a(\sin 2\phi)^{\frac{1}{2}}$	(39)
		or $\nu=a\sqrt{2\cot\phi}$	(40)
$-\frac{1}{2}$	Parabola	$\nu=-a\,\mathrm{cosec}^2\phi$	(41)
		The parabola has another form of tangential equation, namely, $\nu=a\tan\phi$............	(42)
		The *director* line in this form of equation is the tangent at the vertex. In the other forms it is the axis.	
$\frac{1}{2}$	Cardioide	$\nu=a\sin^3\left(\frac{\phi}{3}\right)$	(43)
1	Circle	$\nu\sin\phi=a\sin^2\frac{1}{2}\phi$	(44)
		or $\nu=\frac{a}{2}\tan\frac{1}{2}\phi$	(45)

CHAPTER II.

SECTION I.—*Transformation of Tangential into Cartesian Equations.*

26. The tangential equations $\nu = f(\phi)$ of a curve being given, to find its Cartesian equations.

Fig. 4.



Consider two consecutive positions of the line $x + y\cot\phi - \nu = 0$, such as LP, L′P intersecting in P; then P is the point of contact of LP with its envelope, and the diameter of the circle described about the infinitesimal triangle LL′P is evidently equal $\frac{\text{LL}'}{d\phi} = \frac{d\nu}{d\phi} = f'(\phi)$;

$\therefore$ LQ$= f'(\phi)$. Hence PR$= f'(\phi)\sin^2\phi$ and

$$\text{OR} = \text{OL} + \text{LR} = f\phi + f'(\phi)\sin\phi\cos\phi\,;$$

$\therefore$ the Cartesian coordinates of the point P are

$$x = f(\phi) + f'(\phi)\sin\phi\cos\phi \quad . \quad . \quad (46)$$

$$y = \quad -f'(\phi)\sin^2\phi \quad . \quad . \quad . \quad . \quad (47)$$

These values can be got also by the analytical method of finding envelopes. For differentiating the equation $x + y\cot\phi - f\phi = 0$ with respect to ϕ, we get $y = -f'\phi\sin^2\phi$; and substituting this in the equation $x + y\cot\phi - f\phi$, we get $x = f(\phi) + f'(\phi)\sin\phi\cos\phi$.

27. From the results of the last article we get

$$\text{the subtangent} = f'(\phi)\sin\phi\cos\phi\,. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

$$\text{subnormal } = f'(\phi)\sin^2\phi\tan\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

28. If the movable line $x + y\cot\phi - \nu = 0$ be a double tangent, it is evident that for the same values of ν and ϕ we must have two different values for LP, one value corresponding to each point of contact. Hence, since LP$= f'(\phi)\sin\phi$, we must have two different values for $f'(\phi)$. This will happen when $f'(\phi)$ is given in the form of a fraction whose numerator and denominator each vanish. Thus, suppose the equation of the curve to be given in the form

$$\text{F}(\nu, \phi) = 0\,;$$

then we have

$$\frac{d\nu}{d\phi} = -\frac{d\text{F}}{d\phi} \div \frac{d\text{F}}{d\nu}\,;$$

and therefore the conditions for a double tangent are

$$\frac{d\text{F}}{d\phi} = 0, \ \frac{d\text{F}}{d\nu} = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

which correspond with the conditions in Cartesian coordinates for a double point.

29. Professor CAYLEY considers a "curve as described (see SALMON'S 'Higher Curves,' second edition, p. 33) by a point which moves along a line at the same time that the line revolves round the point. There is, then, this peculiarity at a point of inflection, the line first becomes stationary and then reverses the sense of its motion." From this it follows that the line $x+y\cot\phi-\nu$ will cut off a maximum or minimum intercept on the *director* line when it passes through a point of inflection, and also it will make in the same case with the same line a maximum or minimum angle. Hence when

$$x+y\cot\phi-\nu$$

is an inflectional tangent,

$$f(\phi)=\text{maximum or minimum}$$

and

$$\phi=\text{maximum or minimum.}$$

Examples.

(1) If a line of constant length slide along two rectangular lines, to find its envelope. In this case we have evidently

$$\nu=a\cos\phi\,;\ \therefore\ f(\phi)=a\cos\phi.$$

Hence from equations (46), (47) we get

$$\left.\begin{array}{l} x=a\cos^3\phi, \qquad y=a\sin^3\phi\,; \\ \therefore\ x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}} \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

(2) If from any point in an ellipse perpendiculars be let fall on the axes, find the envelope of the line joining their feet. In this case $f(\phi)=\frac{a^2}{\sqrt{a^2+b^2\tan^2\phi}}$, and the required equation is

$$\left(\frac{x}{a}\right)^{\frac{2}{3}}+\left(\frac{y}{b}\right)^{\frac{2}{3}}=1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

(3) Let $\nu=k\tan^m\phi$; then if we put $\frac{(m+1)^{m+1}}{m^m}k=a$, we get the Cartesian equation

$$x^{m+1}=ay^m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

(4) If $\nu=c\left\{1+(\cot\phi)^{\frac{2}{m}}\right\}^{\frac{m}{2}}$, the Cartesian equation is

$$x^{\frac{2}{2-m}}+y^{\frac{2}{2-m}}=c^{\frac{2}{2-m}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

Compare equation (51).

Cor. If in this example we substitute $-2n$ for m, we get

$$x^{\frac{1}{n+1}}+y^{\frac{1}{n+1}}=c^{\frac{1}{n+1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

as the Cartesian equation of the curve

$$\nu=c\left\{1+(\tan\varphi)^{\frac{1}{n}}\right\}^{-n}.$$

(5) Let $\nu=\frac{c^2}{(a^2+b^2\tan^2\varphi)^{\frac{1}{2}}}$, where $c^2=a^2-b^2$. This curve is the evolute of the ellipse. The Cartesian equation is

$$(ax)^{\frac{2}{3}}+(by)^{\frac{2}{3}}=c^{\frac{2}{3}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

(6) To find a curve in which the subnormal is constant, let the constant be $2a$; then from equation (49) we have

$$\left.\begin{aligned} f'(\varphi)\sin^2\varphi\tan\varphi=2a\,; \\ \therefore f(\varphi)=-a\,\mathrm{cosec}^2\varphi, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

which is the common parabola (see art. 25).

Cor. In like manner the curve in which the subtangent is constant is

$$\nu=a\log\tan\varphi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

or

$$\frac{\mu}{\lambda}=e^{\frac{a\lambda}{\nu}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

(7) If in fig. 4 (art. 26) PL be produced to meet OS in T, required to find the curve in which PL : LT in a given ratio, say n : 1. Here we have evidently

$$\frac{f'(\varphi)\sin\varphi\cos\varphi}{f(\varphi)}=n,$$

$$\therefore \log f(\varphi)=\mathrm{C}+n\log\tan\varphi,$$

$$\therefore f(\varphi)=k\tan^n\varphi, \qquad \text{if } k=e^{\mathrm{C}}\,;$$

$\therefore$ the required curve is

$$\nu=k\tan^n\varphi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

or

$$\mu^n\nu+k\lambda^{n+1}=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

SECTION II.—*Transformation of the Tangential into the Intrinsic Equation.*

30. If we differentiate the value of x given in art. 26, we get

$$\frac{dx}{d\varphi}=2f'(\varphi)\cos^2\varphi+f''(\varphi)\sin\varphi\cos\varphi\,;$$

but

$$\frac{dx}{d\varphi}=\frac{dx}{ds}\times\frac{ds}{d\varphi}=\cos\varphi\,\frac{ds}{d\varphi},$$

$$\therefore \frac{ds}{d\varphi}=2f'(\varphi)\cos\varphi+f''(\varphi)\sin\varphi, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62)$$

$$\therefore s=f'(\varphi)\sin\varphi+\int f'(\varphi)\cos\varphi\,d\varphi.$$

Hence if $\nu = f(\phi)$ be the tangential equation of a curve, its intrinsic equation is

$$s = f'(\phi)\sin\phi + \int f'(\phi)\cos\phi\, d\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

The result in equation (62) may be written in a form which in practice we shall find more useful. Thus

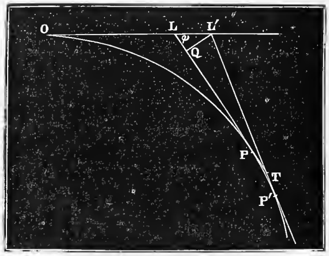
$$\frac{ds}{d\phi} = \frac{\frac{d}{d\phi}(f'(\phi)\sin^2\phi)}{\sin\phi}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

31. Equation (63) may be established geometrically as follows:—Let LP, L′P′ be two consecutive positions of the movable line, P, P′ their points of contact with the envelope, and T their point of intersection. Let L′Q be a perpendicular on LP. Now PP′ is an element of the curve, and denoting it by ds, we have

$$\begin{aligned} ds &= \mathrm{P'T} + \mathrm{TP} = \mathrm{P'L'} - \mathrm{QP} = \mathrm{P'L'} - \mathrm{PL} + \mathrm{LQ} \\ &= d(\mathrm{LP}) + \mathrm{LL'}\cos\phi = d(\mathrm{PL}) + d\nu\cos\phi\,; \\ \therefore\ s &= \mathrm{PL} + \int d\nu\cos\phi \\ &= f'(\phi)\sin\phi + \int f'(\phi)\cos\phi\, d\phi. \end{aligned}$$

Which is the same result as before.

Fig. 5.



Cor.

$$s = \mathrm{PL} + \int \mathrm{PQ}\, d\phi \text{ (see fig. art. 26)}\,; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

$$\therefore\ \frac{ds}{d\phi} = \mathrm{PQ} + \frac{d}{d\phi}(\mathrm{PL}). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

From this it will be seen that the triangle LPQ is an important one in this theory.

Observation.—The geometrical method of proof shows that this theorem holds even when the director line OL (see def. art. 2) is any plane curve; and we shall further on have to make use of this generalization.

32. Before giving examples of the process of this section we will give the following integral reduction.

To reduce $\int \frac{z\,dz}{\sqrt{1+z^3}}$ to the normal form of elliptic integrals.

Let $z = \sqrt{3}\cot^2\frac{1}{2}\theta - 1$, and $\sqrt{1 - \frac{2+\sqrt{3}}{4}\sin^2\theta} = \Delta(\theta)$. Then after some easy reductions, we get

$$\int \frac{z\,dz}{\sqrt{1+z^3}} = \frac{2}{(\sqrt{3}-1)\sqrt[4]{3}} \int \frac{d\theta}{\Delta\theta} - 2\sqrt[4]{3} \int \frac{(1+\cos\theta)\,d\theta}{\sin^2\theta\,\Delta(\theta)}.$$

Now

$$\int \frac{\cos\theta\, d\theta}{\sin^2\theta\,.\,\Delta(\theta)} = -\frac{\Delta\theta}{\sin\theta},$$

and

$$\int \frac{d\phi}{\sin^2\theta\,.\,\Delta\theta} = \mathrm{F}(k,\ \theta) - \mathrm{E}(k,\ \theta) - \cot\theta\,.\,\Delta\theta,$$

where

$$k=\frac{\sqrt{3}+1}{2\sqrt{2}}.$$ (See Durège, 'Theorie der elliptischen Functionen.')

Hence

$$\int\frac{zdz}{\sqrt{1+z^3}}=\frac{1-3^{\frac{1}{2}}}{3^{\frac{1}{4}}}\,\mathrm{F}(k,\,\theta)+2\,.\,3^{\frac{1}{4}}\mathrm{E}(k,\,\theta)+2\,.\,3^{\frac{1}{4}}\cot\tfrac{1}{2}\theta\,.\,\Delta(\theta). \quad . \quad (67)$$

Examples.

(1) Let the tangential equation be $\nu=k\tan^n\theta$, to find the intrinsic equation. Here we have $f(\phi)=k\tan^n\phi$;

$$\therefore\ f'(\phi)\sin^2\phi=nk\tan^{n+1}\phi.$$

Hence from equation (64) $\dfrac{ds}{d\phi}=n(n+1)k\tan^{n-1}\phi\sec^3\phi$;

$$\therefore\ s=n(n+1)k\int\tan^{n-1}\phi\sec^3\phi\,d\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

We can get a formula of reduction for this integral as follows:—Put $\mathrm{P}=\tan^{n-2}\phi\sec^3\phi$, then by differentiation and reduction,

$$\frac{d\mathrm{P}}{d\phi}=(n+1)\tan^{n-1}(\phi)\sec^3\phi+(n-2)\tan^{n-3}\phi\sec^2\phi;$$

$$\therefore\ \int\tan^{n-1}(\phi)\sec^3\phi\,d\phi=\frac{\tan^{n-2}\phi\,.\,\sec^3\phi}{n+1}-\frac{n-2}{n+1}\int\tan^{n-3}\phi\sec^3\phi\,d\phi, \quad . \quad (69)$$

which is the required formula; and the integral will ultimately depend on known forms.

(2) Let the tangential equation be that of the evolute of an ellipse,

$$\nu=\frac{c^2}{\sqrt{a^2+b^2\tan^2\phi}}.$$

We have

$$f'(\phi)\sin^2\phi=\frac{-b^2c^2\tan^3\phi}{(a^2+b^2\tan^2\phi)^{\frac{3}{2}}}.$$

Hence, from equation (64),

$$\frac{ds}{d\phi}=-\frac{3b^2}{a}\cdot\frac{\sin\phi\cos\phi}{\Delta^5(\phi)},$$

where

$$\Delta(\phi)=\sqrt{1-e^2\sin^2\phi};$$

$$\therefore\ s=\frac{b^2}{a}\cdot\frac{1}{\Delta^3(\phi)}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

and this is the intrinsic equation of the evolute of an ellipse.

(3) To find a curve in which the radius of curvature bears a constant ratio to the normal, the given condition is expressed by the equation

$$\frac{f'(\phi)\sin^2\phi}{\cos\phi}=\frac{\frac{1}{a}\frac{d}{d\phi}(f'(\phi)\sin^2\phi)}{\sin\phi};$$

$$\therefore\ \frac{\frac{d}{d\phi}(f'(\phi)\sin^2\phi)}{f'(\phi)\sin^2\phi}=\frac{a\sin\phi}{\cos\phi},$$

$$\therefore\ f'(\phi)\sin^2\phi=\frac{e^c}{(\cos\phi)^a},$$

$$\therefore\ f(\phi)=e^c\int\frac{d\phi}{\sin^2\phi\,(\cos\phi)^a},$$

$$\therefore\ \nu=e^c\int\frac{d\phi}{\sin^2\phi\,.\,(\cos\phi)^a}. \qquad (71)$$

If a be any even integer the integration on the right-hand side can be performed. See WILLIAMSON'S 'Integral Calculus.'

(4) To find a curve whose tangential equation is the same as its intrinsic equation.

Here we have $f'(\phi)\sin\phi+\int f'(\phi)\cos\phi\,d\phi=f(\phi)$,

$$\text{or } \frac{d}{d\phi}(f'(\phi)\sin^2\phi)=f'(\phi)\sin\phi;$$

$$\therefore\ f'(\phi)\sin^2\phi=\mathrm{C}\tan\tfrac{1}{2}\phi,$$

$$\therefore\ f(\phi)=a\{\tan^2\tfrac{1}{2}\phi+\log\tan^2\tfrac{1}{2}\phi)\}, \qquad (72)$$

where a stands for $\frac{\mathrm{C}}{4}$.

(5) If the tangential equation of a curve be $\nu=f(\phi)$, and the intrinsic equation $s=f'(\phi)$, find the curve.

We have $f'(\phi)\sin\phi+\int f'(\phi)\cos\phi\,d\phi=f'(\phi)$;

$$\therefore\ \frac{f''(\phi)}{f'(\phi)}=\frac{2\cos\phi}{1-\sin\phi}.$$

Hence $f'(\phi)=\frac{\mathrm{C}_1}{(1-\sin\phi)^2}$;

$$\therefore f(\phi)=\mathrm{C}_2+\mathrm{C}_1\left\{\frac{\cot\left(\frac{\pi}{4}-\frac{\phi}{2}\right)}{2}+\frac{\cot\left(\frac{\pi}{4}-\frac{\phi}{2}\right)^3}{6}\right\} \qquad (73)$$

(6) To find the intrinsic equation of the curve

$$\nu^{\frac{1}{3}}=1+(\cot\phi)^{\frac{1}{3}}.$$

This is the curve whose ordinary tangential equation is

$$\lambda^{\frac{1}{3}}+\mu^{\frac{1}{3}}+\nu^{\frac{1}{3}}=0,$$

or the curve whose trilinear equation is

$$\alpha^{-\frac{1}{2}}+\beta^{-\frac{1}{2}}+\gamma^{-\frac{1}{2}}=0.$$

We have $f'(\phi)\sin^2\phi=-(\tan^{\frac{1}{3}}\phi+1)^2$;

$$\therefore\ \frac{d}{d\phi}(f'(\phi)\sin^2\phi)=-\tfrac{2}{3}(\cot^{\frac{1}{3}}\phi+\cot^{\frac{2}{3}}\phi)\sec^2\phi;$$

∴ from equation (64) we have

$$s=-\tfrac{2}{3}\int\cot^{\frac{1}{3}}\phi\sec^2\phi\operatorname{cosec}\phi\,d\phi-\tfrac{2}{3}\int\cot^{\frac{2}{3}}\phi\sec^2\phi\operatorname{cosec}\phi\,d\phi. \quad . \quad . \quad . \quad (74)$$

We reduce the first of these integrals to the normal form of elliptic integrals as follows:—

Let $z=\cot^{\frac{2}{3}}\phi$, and we find

$$\int\cot^{\frac{1}{3}}\phi\sec^2\phi\operatorname{cosec}\phi\,d\phi=-\tfrac{3}{2}\int\frac{dz}{z^2\sqrt{1+z^3}}-\tfrac{3}{2}\int\frac{zdz}{\sqrt{1+z^3}}$$

$$=\frac{\tfrac{3}{2}\sqrt{1+z^3}}{z}-\tfrac{9}{4}\int\frac{zdz}{\sqrt{1+z^3}}$$

$$=\tfrac{3}{2}\cdot\frac{1}{\sin^{\frac{1}{3}}\phi\cos^{\frac{2}{3}}\phi}-\tfrac{9}{4}\int\frac{zdz}{\sqrt{1+z^3}}$$

$$=\tfrac{3}{2}\cdot\frac{1}{\sin^{\frac{1}{3}}\phi\cos^{\frac{2}{3}}\phi}-\frac{3^{\frac{3}{4}}}{2}\cot\tfrac{1}{2}\theta\,.\,\Delta\theta+\frac{3^{\frac{1}{4}}(3^{\frac{1}{2}}-1)}{2}\,\mathrm{F}(k,\theta)-\frac{3^{\frac{3}{4}}}{2}\,\mathrm{E}(k,\theta), \quad . \quad . \quad . \quad (75)$$

where θ and ϕ are connected by the equation

$$\cos\theta=\frac{\cos^{\frac{2}{3}}\phi-(\sqrt{3}-1)\sin^{\frac{2}{3}}\phi}{\cos^{\frac{2}{3}}\phi+(\sqrt{3}+1)\sin^{\frac{2}{3}}\phi}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$

The second integral in equation (74) may be derived from the first by changing the sign and putting $\left(\frac{\pi}{2}-\phi\right)$ for ϕ. Hence we have at once

$$\int\cot^{\frac{2}{3}}\phi\sec^2\phi\operatorname{cosec}\phi\,d\phi$$

$$=-\tfrac{3}{2}\frac{1}{\sin^{\frac{2}{3}}\phi\cos^{\frac{1}{3}}\phi}+\frac{3^{\frac{3}{4}}}{2}\cot\tfrac{1}{2}\theta'\,.\,\Delta\theta-\frac{3^{\frac{1}{4}}(3^{\frac{1}{2}}-1)}{2}\,\mathrm{F}(k,\theta')+\frac{3^{\frac{3}{4}}}{2}\,\mathrm{E}(k,\theta'), \quad . \quad . \quad (77)$$

where θ' is given by the equation

$$\cos\theta'=\frac{\sin^{\frac{2}{3}}\phi-(\sqrt{3}-1)\cos^{\frac{2}{3}}\phi}{\sin^{\frac{2}{3}}\phi+(\sqrt{3}+1)\cos^{\frac{2}{3}}\phi}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (78)$$

and substituting from equations (75) (77) in (74), we get the required intrinsic equation

$$\left.\begin{aligned}s=\frac{\cos^{\frac{1}{3}}\phi-\sin^{\frac{1}{3}}\phi}{\sin^{\frac{2}{3}}\phi\cos^{\frac{2}{3}}\phi}&+3^{\frac{5}{4}}\{\cot\tfrac{1}{2}\theta\,.\,\Delta(\theta)-\cot\tfrac{1}{2}\theta'\,.\,\Delta\theta'\}\\&-\frac{3^{\frac{3}{4}}(3^{\frac{1}{2}}-1)}{2}\{\mathrm{F}(k,\theta)-\mathrm{F}(k,\theta')\}\\&+3^{\frac{5}{4}}\{\mathrm{E}(k,\theta)-\mathrm{E}(k,\theta')\},\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (79)$$

where $k=\dfrac{\sqrt{3}+1}{2\sqrt{2}}$, and θ, θ' are given by the equations (76), (78).

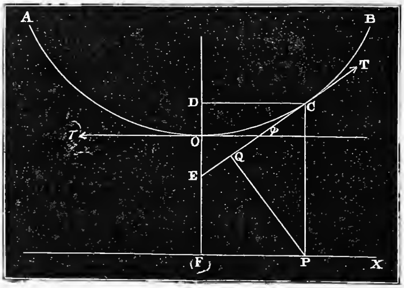
SECTION III.—*Transformation of the Intrinsic into the Tangential Equation.*

33. We shall have much use to make of the intrinsic equation of the catenary in this

and in subsequent sections; for this reason, and also on account of its extremely elementary character, we give here an investigation of the leading properties of that curve.

Let O be the lowest point of a uniform string AOB, suspended at the points A and B, and let the tension at O be denoted by τ, and at any other point C by T. Then if we consider the equilibrium of the portion O C we find that the forces acting on it are τ, T, and its own weight W; and these are parallel respectively to the sides of the triangle C D E. Hence, by the property of the triangle of forces,

Fig. 6.



$$\frac{\mathrm{W}}{\tau}=\tan \mathrm{E\,C\,D}=\tan \phi,$$

where ϕ is the angle which the tangent at C makes with the tangent at O. Now if s be the length of O C and c the length of a portion whose weight is equal to τ, we have, since the string is uniform,

$$\frac{s}{c}=\frac{\mathrm{W}}{\tau};$$

$$\therefore s=c\tan\phi. \qquad (80)$$

34. The equation $s=c\tan\phi$, which we have just obtained, is the intrinsic equation of the catenary; we get the Cartesian equation from it as follows:—Make O F$=c$, and draw FX parallel to C D. Then we shall take these lines as axes. Now let the coordinates of the point C be denoted by x and y, and we have

$$\frac{dy}{ds}=\sin\phi\text{; but } ds=c\sec^2\phi\, d\phi\text{, equation (80),}$$

$$\therefore y=c\sec\phi.$$

Again, we have

$$\frac{dx}{ds}=\cos\phi,$$

$$\therefore dx=c\sec\phi\, d\phi,$$

and

$$x=c\log(\sec\phi+\tan\phi),$$

$$\therefore e^{\frac{x}{c}}=e^{\frac{x}{c}}=2\sec\phi.$$

Hence

$$y=\frac{c}{2}(e^{\frac{x}{c}}+e^{-\frac{x}{c}})\,; \qquad (81)$$

and this is the Cartesian equation of the catenary.

35. From the value

$$x=c\log(\sec\phi+\tan\phi)$$

we get

$$e^{\frac{x}{c}}-e^{-\frac{x}{c}}=2\tan\phi.$$

Hence, from equation,

$$s=\frac{c}{2}(e^{\frac{x}{c}}-e^{-\frac{x}{c}}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (82)$$

36. If from the point P we let fall the perpendicular PQ on the tangent at C, we have evidently

$$\mathrm{PQ}=y\cos\phi\,;\ \text{but } y=c\sec\phi,$$

$$\therefore \mathrm{PQ}=c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

Again we have

$$\frac{\mathrm{CQ}}{\mathrm{PQ}}=\tan\phi,$$

$$\therefore \mathrm{CQ}=s. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (84)$$

Hence the locus of the point Q is the involute of the catenary.

37. From the diagram we have

$$\frac{\mathrm{T}}{\mathrm{W}}=\operatorname{cosec}\phi$$

and

$$\frac{\mathrm{CP}}{\mathrm{CQ}}=\operatorname{cosec}\phi=\frac{y}{s}\,;$$

and since s is the length of a portion of the string whose weight is W, y is the length of a portion whose weight is T.

38. The Intrinsic Equation of a curve being given, to find its Tangential Equation.

This problem is the converse of the one solved in art. 30, Section II.

Let $s=\mathrm{F}(\phi)$ be the given intrinsic equation,

$$\therefore \frac{ds}{d\phi}=\mathrm{F}'(\phi).$$

Hence from equation (64) we have

$$\frac{d}{d\phi}(f'(\phi)\sin^2\phi)=\mathrm{F}'(\phi)\sin\phi\,;$$

$$\therefore f(\phi)=\int\operatorname{cosec}^2\phi\{\int\mathrm{F}'(\phi)\sin\phi d\phi\}d\phi.$$

Hence the required tangential equation is

$$\nu=\int\operatorname{cosec}^2\phi\{\int\mathrm{F}'(\phi)\sin\phi d\phi\}d\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (85)$$

Examples.

(1) Find the tangential equation of the catenary.

Here $\mathrm{F}(\phi)=c\tan\phi$. See equation (80).

Hence

$$\int F'(\phi) \sin \phi d\phi = c \sec \phi,$$

$$\therefore \nu = c\left\{\log \tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right) - \operatorname{cosec} \phi\right\} + c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (86)$$

(2) Find the tangential equation of the involute of the catenary; that is, the tangential equation of the locus of the point Q (see fig. art. 33). The intrinsic equation of the involute of the catenary is

$$s = c\int \tan \phi \, d\phi.$$

Hence we have

$$F(\phi) = c\int \tan \phi d\phi,$$

$$\therefore \int F'(\phi) \sin \phi d\phi = c\{\log(\sec \phi + \tan \phi) - \sin \phi\}.$$

Hence

$$\nu = c\int \frac{\log(\sec\phi+\tan\phi)}{\sin^2\phi} - c\int \frac{d\phi}{\sin\phi};$$

and integrating the first integral by parts, we find it equal

$$-\cot \phi \,.\, \log\{\sec \phi + \tan \phi\} + \int \frac{d\phi}{\sin \phi}.$$

Hence

$$\nu = C - c \cot \phi \,.\, \log(\sec \phi + \tan \phi),$$

where C is the constant of integration, which is evidently equal to c; therefore the required tangential equation is

$$\nu = c\left\{1 - \frac{\log(\sec\phi+\tan\phi)}{\tan\phi}\right\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (87)$$

(3) Let the intrinsic equation be $s = a \sin n\phi$. Then we find

$$\nu = \frac{2na}{n^2-1} \sin^2 \frac{n\phi}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (88)$$

This formula fails when $n=1$; but in that case we have $F(\phi) = a \sin \phi$, and we find

$$\nu = \frac{a\phi}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (89)$$

(4) Find the equation of a curve, being given

$$\nu = ns.$$

Here we have, if

$$\nu = f(\phi), \quad s = nf(\phi);$$

$$\therefore \frac{d}{d\phi}\left(f'(\phi) \sin^2 \phi\right) = nf'(\phi) \sin \phi. \quad \text{See equation (64).}$$

$$\therefore f'(\phi) \sin^2 \phi = C\left(\tan \tfrac{1}{2}\phi\right)^n,$$

$$\therefore f(\phi) = \frac{C}{2}\left\{\frac{(\tan\frac{1}{2}\phi)^{n-1}}{n-1} + \frac{(\tan\frac{1}{2}\phi)^{n+1}}{n+1}\right\},$$

or, since C is an arbitrary constant,

$$f\phi = C\left\{\frac{(\tan\frac{1}{2}\phi)^{n-1}}{n-1} + \frac{(\tan\frac{1}{2}\phi)^{n+1}}{n+1}\right\}. \qquad (90)$$

(5) It is required to find the equation of two curves A and B, which are so related that the Tangential Equation of A is the same as the Intrinsic of B, and the Tangential of B the same as the Intrinsic of A.

Let $\nu = F(\phi)$ be the curve A,
$\nu = f(\phi)$,, ,, B.

Then by the first condition we have

$$F'(\phi)\sin\phi + \int F'(\phi)\cos\phi d\phi$$
$$= f'(\phi)\sin 2\phi + f''(\phi)\sin^2\phi + 2\int f'(\phi)\cos^2\phi d\phi + \int f''(\phi)\sin\phi\cos\phi d\phi,$$

and by the second condition

$$f(\phi) = F'(\phi)\sin\phi + \int F'(\phi)\cos\phi d\phi\,;$$

$$\therefore f(\phi) = f'(\phi)\sin 2\phi + f''(\phi)\sin^2\phi + 2\int f'(\phi)\cos^2\phi + \int f''(\phi)\sin\phi\cos\phi d\phi.$$

And by differentiating and some easy reduction we get

$$5f''(\phi)\sin 2\phi + 6f'(\phi)\cos 2\phi - f'''(\phi)\cos 2\phi + f'''(\phi) = 0,$$

or

$$\frac{d}{d\phi}\{3f'(\phi)\sin 2\phi - f''(\phi)\cos 2\phi + f''(\phi)\} = 0.$$

Hence

$$3f'(\phi)\sin 2\phi + f''(\phi)\{1 - \cos 2\phi\} = 4C_1,$$

the multiple 4 being put to the arbitrary constant in order to avoid fractions;

$$\therefore 3f'(\phi)\cos\phi + f''(\phi)\sin\phi = \frac{2C_1}{\sin\phi}. \qquad (\alpha)$$

This may be written

$$\left.\begin{aligned} &\frac{d}{d\phi}(f'(\phi)\sin^3\phi) = 2C_1\sin\phi, \\ &\therefore f'(\phi)\sin^3\phi = -2C_1\cos\phi + 2C_2, \end{aligned}\right\} \qquad (\beta)$$

$2C_2$ being an arbitrary constant;

$$\therefore f(\phi) = \frac{C_1 - C_2\cos\phi}{\sin^2\phi} + C_2\log\tan\tfrac{1}{2}\phi + C_3.$$

This is the tangential equation of the curve B.

To find the equation of A we have, from equation (β),

$$f'(\phi)\cos\phi = -\frac{2C_1\cos^2\phi}{\sin^3\phi} + \frac{2C_2\cos\phi}{\sin^3\phi},$$

and subtracting this from equation (α) we get

$$2f'(\phi)\cos\phi+f''(\phi)\sin\phi=\frac{2C_1}{\sin^3\phi}-\frac{2C_2\cos\phi}{\sin^3\phi};$$

$$\therefore \frac{dS}{d\phi}=\frac{2C_1}{\sin^3\phi}-\frac{2C_2\cos\phi}{\sin^3\phi},$$

$$\therefore S=\frac{C_1\cos\phi-C_2}{\sin^2\phi}+C_1\log\tan\tfrac{1}{2}\phi+C_4.$$

This is the intrinsic equation of B, and therefore the tangential equations of A and B respectively are

$$\nu=\frac{C_1\cos\phi-C_2}{\sin^2\phi}+C_1\log\tan\tfrac{1}{2}\phi+C_4 \quad \ldots\ldots\ldots \quad (91)$$

$$\nu=\frac{C_1-C_2\cos\phi}{\sin^2\phi}+C_2\log\tan\tfrac{1}{2}\phi+C_3 \quad \ldots\ldots\ldots \quad (92)$$

CHAPTER III.

SECTION I.—*Evolutes.*

39. If the tangential equation of a curve be

$$\nu=f(\phi),$$

we have proved, in art. 30,

$$\frac{ds}{d\phi}=\frac{\frac{d}{d\phi}(f'(\phi)\sin^2\phi)}{\sin\phi}.$$

Hence if ρ denote the radius of curvature, we have

$$\rho\sin\phi=\frac{d}{d\phi}(f'(\phi)\sin^2\phi);$$

$\therefore$ if $\nu=f(\phi)$ be the tangential equation of a curve, the intrinsic equation of its evolute is

$$s\sin\phi=\frac{d}{d\phi}(f'(\phi)\sin^2\phi) \quad \ldots\ldots\ldots\ldots \quad (93)$$

40. If our movable line had been given by the equation

$$y=x\tan\phi+f(\phi),$$

we get in the usual manner

$$x=-f'(\phi)\cos^2\phi,\quad y=f(\phi)-f'(\phi)\sin\phi\cos\phi.$$

Hence

$$\frac{dx}{d\phi}=\cos\phi(2f'(\phi)\sin\phi-f''(\phi)\cos\phi),$$

$$\frac{dy}{d\phi}=\sin\phi(2f'(\phi)\sin\phi-f''(\phi)\cos\phi);$$

$$\therefore \frac{ds}{d\phi} = 2f'(\phi)\sin\phi - f''(\phi)\cos\phi\,;$$

and, as in the last art., we find

$$s\cos\phi = -\frac{d}{d\phi}(f'(\phi)\cos^2\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (94)$$

41. The Tangential Equation of a Curve being given, to find the Tangential Equation of its Evolute. This problem is solved by articles 38 and 39.

For if $\nu = f(\phi)$ be the tangential equation, the intrinsic equation of the evolute is

$$s = 2f'(\phi)\cos\phi + f''(\phi)\sin\phi. \quad \text{(Art. 39.)}$$

Let this be denoted by $F(\phi)$, and, by art. 38, the tangential equation is

$$\nu = \int \operatorname{cosec}^2\phi \{\int F'(\phi)\sin\phi d\phi\} d\phi.$$

Now we have, from the value of $F(\phi)$,

$$F'(\phi)\sin\phi = 3f''(\phi)\sin\phi\cos\phi + f'''(\phi)\sin^2\phi - 2f'(\phi)\sin^2\phi\,;$$

and integrating by parts we easily get

$$\int F'(\phi)\sin\phi d\phi = f''(\phi)\sin^2\phi + f'(\phi)\sin\phi\cos\phi - f(\phi).$$

Multiplying by $\operatorname{cosec}^2\phi$, and integrating again, we get

$$f'(\phi) + f(\phi)\cot\phi.$$

Hence the tangential equation required,

$$\nu = f'(\phi) + f(\phi)\cot\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (95)$$

42. The foregoing result may be obtained very simply from geometrical considerations as follows. In fig. 4 (art. 26) the line PS is a tangent to the evolute, and the angle $OSP = \phi$; then we have

$$\begin{aligned} OS &= ON + NS \\ &= LQ + OL\cot\phi \\ &= f'(\phi) + f(\phi)\cot\phi. \end{aligned}$$

Hence if OS be taken as the directing-line, the tangential equation of the envelope of SP is $\nu = f'(\phi) + f(\phi)\cot\phi$; but the envelope of SP is the evolute, and therefore we have the same result as before.

43. The right-hand side of equation (95) may be written

$$\frac{\frac{d}{d\phi}(f(\phi)\sin\phi)}{\sin\phi}.$$

Hence if ν_1, ν_2, ν_3, &c. represents for the successive evolutes what we have denoted by ν for the curve itself, we have

$$\nu_1 \sin\phi = \frac{d}{d\phi}(\nu \sin\phi);$$

similarly

$$\nu_2 \sin\phi = \frac{d}{d\phi}(\nu_1 \sin\phi),$$

$$\therefore\ \nu_2 \sin\phi = \frac{d^2}{d\phi^2}(\nu \sin\phi).$$

Hence in general

$$\nu_n \sin\phi = \frac{d^n}{d\phi^n}(\nu \sin\phi). \qquad \qquad (96)$$

44. Since

$$\nu = f(\phi), \quad \nu \sin\phi = f(\phi)\sin\phi;$$

and denoting this by $\pi(\phi)$ and the corresponding functions for the evolutes by $\pi_1(\phi)$, $\pi_2(\phi)$, &c., we have, from equation (96),

$$\pi_n(\phi) = \frac{d^n}{d\phi^n}(\pi(\phi)) \ . \qquad \qquad (97)$$

45. In art. 26 we have found the coordinates of a point on the curve $\nu = f(\phi)$:—

$$x = f(\phi) + f'(\phi)\sin\phi\cos\phi; \ y = -f'(\phi)\sin^2\phi.$$

These assume, if we substitute from art. 44 for $f(\phi)$ the value $\frac{\pi(\phi)}{\sin\phi}$, the symmetrical form

$$x = \pi(\phi)\sin\phi + \pi'(\phi)\cos\phi, \qquad \qquad (98)$$

$$y = \pi(\phi)\cos\phi - \pi'(\phi)\sin\phi; \qquad \qquad (99)$$

and hence, from art. 44, if we denote by x_n, y_n the coordinates of a point on the nth evolute,

$$x_n = \left\{\sin\phi\left(\frac{d}{d\phi}\right)^n + \cos\phi\left(\frac{d}{d\phi}\right)^{n+1}\right\}\pi(\phi), \qquad \qquad (100)$$

$$y_n = \left\{\cos\phi\left(\frac{d}{d\phi}\right)^n - \sin\phi\left(\frac{d}{d\phi}\right)^{n+1}\right\}\pi(\phi). \qquad \qquad (101)$$

46. By using Leibnitz's theorem, we find, from equation (98),

$$\frac{d^n x}{d\phi^n} = \sin\phi\frac{d^n\pi}{d\phi} + n\cos\phi\frac{d^{n-1}\pi}{d\phi^{n-1}} - \frac{n\,.\,n-1}{\underline{|2}}\sin\phi\frac{d^{n-2}\pi}{d\phi^{n-2}} - \frac{n\,.\,n-1\,.\,n-2}{\underline{|3}}\cos\phi\frac{d^{n-3}\pi}{d\phi^{n-3}} + \&c.$$

$$+\cos\phi\frac{d^{n+1}\pi}{d\phi^{n+1}} - n\sin\phi\frac{d^n\pi}{d\phi^n} - \frac{n\,.\,n-1}{\underline{|2}}\cos\phi\frac{d^{n-1}\pi}{d\phi^{n-1}} + \frac{n\,.\,n-1\,.\,n-2}{\underline{|3}}\sin\phi\frac{d^{n-1}\pi}{d\phi^{n-2}} - \&c.$$

Hence, by equations (100), (101), we get

$$\frac{d^n x}{d\phi^n} = x_n + ny_{n-1} - \frac{n\,.\,n-1}{\underline{|2}}x_{n-2} - \frac{n\,.\,n-1\,.\,n-2}{\underline{|3}}y_{n-3} + \&c. \qquad \qquad (102)$$

Similarly, from equation (99), we get

$$\frac{d^ny}{d\phi^n}=y_n-ux_{n-1}-\frac{n\,.\,n-1}{\lfloor 2}\,y_{n-2}+\frac{n\,.\,n-1\,.\,n-2}{\lfloor 3}\,x_{n-3}+\&c. \quad . \quad . \quad . \qquad (103)$$

47. The Intrinsic Equation of a Curve being given, we can find the Tangential Equation of the Evolute thus :—

Let $s=f(\phi)$ be the given intrinsic equation, then the intrinsic equation of the evolute is

$$s=f'(\phi).$$

(see WHEWELL, "On the Intrinsic Equation of Curves," Phil. Trans. vol. viii. p. 659); and therefore, by art. 38, equation (85),

$$\nu=\int \operatorname{cosec}^2\phi\{\textstyle\int f''(\phi)\sin\phi\, d\phi\}d\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (104)$$

Cor. The tangential equation of the second evolute is

$$\nu=\int \operatorname{cosec}^2\phi\{\textstyle\int f'''(\phi)\sin\phi\, d\phi\}d\phi, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (105)$$

and, in general, of the *n*th evolute

$$\nu=\int \operatorname{cosec}^2\phi\{\textstyle\int f^{(n+1)}(\phi)\sin\phi\, d\phi\}d\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (106)$$

Examples.

(1) Find the tangential equation of the evolute of the catenary.

Here we have $f(\phi)=c\tan\phi$;

$$\therefore \textstyle\int f''(\phi)\sin\phi\, d\phi=c\{\sec\phi\tan\phi-\log(\sec\phi+\tan\phi)\},$$

$$\therefore \int \operatorname{cosec}^2\phi\{\textstyle\int f''(\phi)\sin\phi\, d\phi\}d\phi$$

$$=c\{\sec\phi+\cot\phi\,.\,\log(\sec\phi+\tan\phi)\},. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (107)$$

which is the required equation.

The following three examples are illustrations of art. 39.

(2) To find the intrinsic equation of the evolute of the curve $\nu=(1+\cot^{\frac{1}{3}}\phi)^3$:—

$$f(\phi)=(1+\cot^{\frac{1}{3}}\phi)^3\,;$$

$$\therefore\; s\,.\,\sin\phi=-\tfrac{2}{3}\{\cot^{\frac{1}{3}}\phi+\cot^{\frac{2}{3}}\phi\}\sec^2\phi$$

(see example 6, Section II., Chapter II.),

$$\therefore\; s=-\tfrac{2}{3}\left\{\frac{1}{\sin^{\frac{4}{3}}\phi\cos^{\frac{5}{3}}\phi}+\frac{1}{\sin^{\frac{5}{3}}\phi\cos^{\frac{4}{3}}\phi}\right\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (108)$$

(3) If the curve be the lemniscate,

$$f\phi=a\left(\sin\frac{2\phi}{3}\right)^{\frac{3}{2}}\operatorname{cosec}\phi \text{ (see art. 25),}$$

and

$$\therefore\ s=\frac{a}{3\sqrt{\sin\frac{2\phi}{3}}}\ \text{(see art. 39)}. \qquad (109)$$

is the intrinsic equation of the evolute.

(4) Let the given curve be the equilateral hyperbola, we have

$$f(\phi)=a\sqrt{2\cot\phi}\,;$$

$$\therefore\ \text{the evolute is } s=a(\operatorname{cosec}2\phi)^{\frac{3}{2}}. \qquad (110)$$

The next five examples are illustrations of art. 43.

(5) Let the curve be $\nu=e^{\sin\phi}$, its evolute will be

$$\nu=e^{\sin\phi}(\cot\phi+\cos\phi). \qquad (111)$$

(6) The tangential equations of the successive evolutes of the curve $\nu=a\cos\phi$ are

$$\nu_1=\frac{a\cos2\phi}{\sin\phi},$$

$$\nu_2=-4a\cos\phi=-4\nu\,;$$

and in general

$$\nu_{2m}=\pm4^m\nu, \qquad (112)$$

$$\nu_{2m+1}=\pm4^m\nu_1, \qquad (113)$$

where the sign + or − is to be used according as m is even or odd.

(7) Find the evolute of the logarithmic curve.

The Cartesian equation of the curve is $y=e^{cx}$,

and the tangential is $\nu=a\log\tan\phi$;

and therefore the tangential equation of its evolute is

$$\nu=a\cot\phi\{\log\tan\phi+\sec^2\phi\}. \qquad (114)$$

(8) Let the curve be the polar one, $\rho^m=a^m\sin m\phi$.

The tangential equation is

$$\nu=a\left\{\sin\frac{m\phi}{m+1}\right\}^{\frac{m+1}{m}}\operatorname{cosec}\phi,$$

and the evolute is

$$\nu_1=\nu\cot\left(\frac{m\phi}{m+1}\right). \qquad (115)$$

This result could be easily obtained geometrically.

(9) The tangential equation of the evolute of the curve

$$\nu=k\tan^n\phi$$

is

$$\nu_1=\nu\{(n+1)\cot\phi+n\tan\phi\}. \qquad (116)$$

Hence the tangential equation of the evolute of the common parabola is

$$\nu_1=\nu(2\cot\phi+\tan\phi). \qquad (117)$$

SECTION II.—*Involutes.*

48. From the equations in art. 43 for the successive evolutes of a curve, we can conversely infer the equations of the successive involutes: thus, let the tangential equation of a curve be

$$\nu = f(\phi),$$

the tangential equations of the successive involutes are

$$\nu_{-1} \sin \phi = \int \nu \sin \phi \, d\phi,$$

or, as it may be written,

$$\nu_{-1} \sin \phi = \int_{d\phi} \nu \sin \phi,$$

$$\nu_{-2} \sin \phi = \iint_{d\phi} \nu \sin \phi,$$

$$\nu_{-3} \sin \phi = \iiint_{d\phi} \nu \sin \phi\,;$$

and in general, for the nth involute,

$$\nu_{-n} \sin \phi = \iiint_{d\phi}^{\dots(n)} \nu \sin \phi.. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (118)$$

Mathematicians have recognized it as legitimate to interpret the symbol of differentiation with a negative index, as denoting integration; therefore we may write the equation (118) as follows:—

$$\nu_{-n} \sin \phi = \left(\frac{d}{d\phi}\right)^{-n} (f(\phi) \sin \phi). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (119)$$

Hence the equation (96) includes the formulæ both for evolutes and involutes, according as n is regarded as positive or negative.

By an extension of the notation of art. 44, the last equation may be written

$$\pi_{-n}(\phi) = \left(\frac{d}{d\phi}\right)^{-n} (\pi(\phi)). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (120)$$

49. If x_{-1}, y_{-1} denote the coordinates of a point on the first involute, x_{-2}, y_{-2} those of a point on the second involute, &c., we have

$$x_{-1} = \cos \phi (\pi(\phi)) + \sin \phi \int_{d0} \pi(\phi), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (121)$$

$$y_{-1} = -\sin \phi (\pi(\phi)) + \cos \phi \int_{d\phi} \pi(\phi)\,; \quad . \quad . \quad . \quad . \quad . \quad . \quad (122)$$

and, in general,

$$x_{-n} = \left\{ \sin \phi \left(\frac{d}{d\phi}\right)^{-n} + \cos \phi \left(\frac{d}{d\phi}\right)^{-(n-1)} \right\} \pi(\phi), \quad . \quad . \quad . \quad . \quad . \quad (123)$$

$$y_{-n} = \left\{ \cos \phi \left(\frac{d}{d\phi}\right)^{-n} - \sin \phi \left(\frac{d}{d\phi}\right)^{-(n-1)} \right\} \pi(\phi). \quad . \quad . \quad . \quad . \quad . \quad (124)$$

50. The Tangential Equation of a Curve being given, to find the Intrinsic Equation of its Involute.

This problem is solved by articles 30 and 48. Thus, if $\nu = \mathrm{F}(\phi)$ be the tangential equation of the involute,

$$\frac{ds}{d\phi}=\frac{\frac{d}{d\phi}(\mathrm{F}'(\phi)\sin^2\phi)}{\sin\phi}\quad\text{(see equation (64));}$$

but by article 48 we have

$$\mathrm{F}(\phi)=\frac{\int f(\phi)\sin\phi d\phi}{\sin\phi};$$

$$\therefore\frac{ds}{d\phi}=f'(\phi)\sin\phi+f(\phi)\cos\phi+\int f(\phi)\sin\phi\,d\phi:$$

that is,

$$\frac{ds}{d\phi}=\frac{d}{d\phi}(f(\phi)\sin\phi)+\int f(\phi)\sin\phi\,d\phi;\quad\quad(125)$$

$$\therefore s=f(\phi)\sin\phi+\iint(f(\phi)\sin\phi\,d\phi)\,d\phi,\quad\quad(126)$$

or, as it may be written,

$$s=\left\{1+\left(\frac{d}{d\phi}\right)^{-2}\right\}(f(\phi)\sin\phi).\quad\quad(127)$$

Hence we have the following theorem:—

If $\nu=f(\phi)$ be the tagential equation of a curve, the intrinsic equation of its involute is

$$s=\left\{1+\left(\frac{d}{d\phi}\right)^{-2}\right\}(f(\phi)\sin\phi).$$

Cor. 1. Since s is the length of the involute, $\frac{ds}{d\phi}$ is the length of the given curve.

Hence from equation (125) we have the following theorem:—If $\nu=f(\phi)$ be the tangential equation of a curve, the length of the curve is given by the equation

$$s=\left\{\frac{d}{d\phi}+\left(\frac{d}{d\phi}\right)^{-1}\right\}f(\phi)\sin\phi.\quad\quad(128)$$

Cor. 2. The equation (127) is equivalent to the following:—

$$s=\int f'(\phi)\sin\phi\,d\phi+\iint(f'(\phi)\cos\phi\,d\phi)\,d\phi;\quad\quad(129)$$

for we have proved, art. 30, that if $\nu=f(\phi)$ be the tangential equation, the intrinsic equation is

$$s=f(\phi)\sin\phi+\int f'(\phi)\cos\phi\,d\phi,$$

and we get the intrinsic equation of the involute from this by integration.

51. From the intrinsic equation to find the tangential of the involute.

Let $s=f(\phi)$ be the given equation, then the intrinsic equation of the involute is

$$s=\int f(\phi)d\phi.$$

Hence from equation (85), art. 38, the tangential equation of the involute is

$$\nu=\int\operatorname{cosec}^2\phi\{f(\phi)\sin\phi\,d\phi\}d\phi.\quad\quad(130)$$

Observation.—Under each of the heads Evolute and Involute it will be observed we have solved three problems, which may be stated briefly as follows:—

Given	To find
Tangential equation of a curve,	Tangential equation of its evolute, involute.
Tangential ,, ,,	Intrinsic ,, ,, ,,
Intrinsic ,, ,,	Tangential ,, ,, ,,

We have omitted the problems given the intrinsic equation of a curve to find the intrinsic equation of its evolute and involute, because these had been previously solved by WHEWELL (see 'Cambridge Philosophical Transactions,' already cited).

Examples.

Examples 1–3 are illustrations of art. 48, 4 and 5 of art. 50, and 6–8 of art. 51.

(1) Let $\nu = k\tan^n\phi$ be the equation, it is required to find the involute.

From equation (118), art. 48, we have

$$\nu_{-1}\sin\phi = k\int\tan^n\phi\sin\phi\,d\phi.$$

We can get a formula of reduction for this integral as follows:—

$$\text{Put }\ P = \tan^{n-1}(\phi)\sin\phi\,;$$

$$\therefore\ \frac{dP}{d\phi} = n\tan^{n-2}\phi\sin\phi + (n-1)\tan^n\phi\sin\phi,$$

$$\therefore\ \int\tan^n\phi\sin\phi\,d\phi = \frac{\tan^{n-1}(\phi)\sin\phi}{n-1} - \frac{n}{n-1}\int\tan^{n-2}\phi\sin\phi\,d\phi, \quad . \quad . \quad (131)$$

which is the required formula.

Cor. If $\nu = n\tan^{n-2}\phi + (n-1)\tan^n\phi$ be the equation of a curve, the equation of its involute is

$$\nu_{-1} = \tan^{n-1}\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (132)$$

Compare equation (116).

(2) Find the involute of the curve

$$\nu = a\log\tan\phi$$

(that is, of the logarithmic curve), we have

$$\nu_{-1}\sin\phi = a\int\sin\phi\,(\log\tan\phi)\,d\phi$$

$$= C - a\left\{\cos\phi\,.\,\log\tan\phi + \log\tan\frac{\phi}{2}\right\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (133)$$

Cor. $$\nu_{-1}\sin\phi + \nu\cos\phi = C - a\log\tan\frac{\phi}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (134)$$

(3) Let the curve be

$$\nu = (1 + \cot^{\frac{1}{3}}\phi)^3,$$

then the tangential equation of its involute is

$$\nu_{-1}\sin\phi=\int\sin\phi\,d\phi+3\int\cos^{\frac{1}{3}}\phi\,\sin^{\frac{2}{3}}\phi\,d\phi$$
$$+3\int\cos^{\frac{2}{3}}\phi\,\sin^{\frac{1}{3}}\phi\,d\phi+\int\cos\phi\,\sin\phi\,d\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (135)$$

Of these four integrals, the first and fourth are elementary, and the third is derived from the second by putting $\left(\frac{\pi}{2}-\phi\right)$ in place of ϕ and changing signs. Hence the question will be solved if we integrate

$$\int\cos^{\frac{1}{3}}\phi\,\sin^{\frac{2}{3}}d\phi.$$

To reduce this to elliptic integrals, let $z^3=\cot^2\phi$, and we easily find

$$\int\cos^{\frac{1}{3}}\phi\,\sin^{\frac{2}{3}}\phi=\frac{\sin^{\frac{5}{3}}\phi}{\cos^{\frac{2}{3}}\phi}+\int\frac{dz}{z^2\sqrt{1+z^3}}.$$

Now

$$\frac{d}{dz}\left\{\frac{\sqrt{1+z^3}}{z}\right\}=\frac{1}{2}\cdot\frac{z}{\sqrt{1+z^3}}-\frac{1}{z^2\sqrt{1+z^3}}.$$

Hence

$$\int\cos^{\frac{1}{3}}\phi\,\sin^{\frac{2}{3}}\phi\,d\phi=-\frac{\cos^{\frac{4}{3}}\phi}{\sin^{\frac{1}{3}}\phi}+\frac{1}{2}\int\frac{2dz}{\sqrt{1+z^3}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (136)$$

and the question is completely solved. (See art. 32.)

(4) Let $\nu=a\cos\phi$ be the tangential equation of a curve, then (see art. 50) the intrinsic equation of its involute is

$$s=\left\{1+\left(\frac{d}{d\phi}\right)^{-1}\right\}f(\phi)\sin\phi$$
$$=\frac{3a\sin 2\phi}{8}+\frac{a\phi}{4}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (137)$$

(5) Let $\nu=a\cos\ \phi$; then we find for the involute

$$s=\frac{2a\sin 3\phi}{9}+\frac{a\phi}{3}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (138)$$

(6) Let the intrinsic equation of a curve be

$$s=a\cos^3\phi;$$

then (see art. 51) the tangential equation of its involute is

$$\nu=-\frac{a}{4}\int\frac{\cos^4\phi\,d\phi}{\sin^2\phi}$$
$$=\frac{a}{16}(4\cot\phi+2\phi+\sin 2\phi). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (139)$$

Similarly, if $s=a\cos^5\phi$,

$$\nu=\frac{a}{96}\{16\cot\phi+30\,\phi+9\sin 2\phi-4\sin^3\phi\cos\phi\}. \quad . \quad . \quad . \quad (140)$$

is its involute.

(7) Let $s=c\tan^2\phi$, then $f\phi=c\tan^2\phi$;

$$\therefore\ \int f(\phi)\sin\phi\,d\phi=c(\cos\phi+\sec\phi),$$

$$\therefore\ \nu=c\int(\cos\phi+\sec\phi)\operatorname{cosec}^2\phi\,d\phi$$

$$=c\{\log(\sec\phi+\tan\phi)-2\operatorname{cosec}\phi\}. \qquad (141)$$

(8) Find the tangential equation of the involute of a circle.

The intrinsic equation of the circle is

$$s=a\phi;$$

$$\therefore\ \int f(\phi)\sin\phi\,d\phi=a(\sin\phi-\phi\cos\phi),$$

$$\therefore\ \int\operatorname{cosec}^2\phi\{f(\phi)\sin\phi\,d\phi\}d\phi=\frac{a\phi}{\sin\phi};$$

∴ the tangential equation of the involute of the circle is

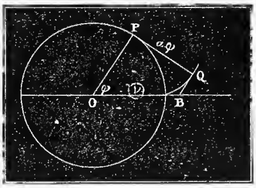
$$\nu=\frac{a\phi}{\sin\phi}. \qquad (142)$$

We can verify the foregoing result geometrically as follows:—For in the annexed diagram, which represents a circle and its involute, we have $PQ=a\phi$, and $OB=\nu$; and since QB is parallel to PO, we have at once

$$OB\sin\phi=PQ;\ \text{that is,}\ \nu\sin\phi=a\phi,$$

which proves the proposition.

Fig. 7.



CHAPTER IV.

SECTION I.—*Positive Pedals.*

52. If we make the perpendicular to our *director* line the initial line, it is evident that the polar equation of the first positive pedal of the curve

$$\nu=f(\phi)$$

$$\text{is}\ \rho=f(\phi)\sin\phi. \qquad (143)$$

Hence the tangential equation of any curve is at once transformed into the polar equation of its first positive pedal by changing ν into ρ, and multiplying the function on the right-hand side by $\sin\phi$.

Thus the tangential equation of the parabola is

$$\nu=a\tan\phi\ \text{(see art. 25)};$$

hence its first positive pedal is

$$\varrho=\frac{a\sin^2\phi}{\cos\phi}, \qquad (144)$$

which equation represents, as is well known, the cissoid.

Again, the pedal of the logarithmic curve is

$$\varrho=a\sin\phi\log\tan\phi, \qquad (145)$$

and of the ellipse

$$\varrho=\sqrt{a^2\sin^2\phi+b^2\cos^2\phi}. \qquad (146)$$

This curve is a bicircular quartic.

53. The tangential equation of the evolute of the curve $\nu=f(\phi)$ is

$$\nu=f(\phi)\cot\phi+f'(\phi).$$

Hence the polar equation of the first positive pedal of the evolute is

$$\varrho=f(\phi)\cos\phi+f'(\phi)\sin\phi. \qquad (147)$$

54. The foregoing result can be shown geometrically as follows (see fig. art. 26):—The perpendicular OT on PQ is the radius vector of the pedal of the evolute; but

$$\begin{aligned}\text{OT}&=\text{OL}\cos\phi+\text{LP}\\&=f(\phi)\cos\phi+f'(\phi)\sin\phi.\end{aligned}$$

Cor. 1. The equation (147) may be written

$$\varrho=\frac{d}{d\phi}(f(\phi)\sin\phi). \qquad (148)$$

This also appears from art. 43; and from the same article we see that the first positive pedal of the nth evolute is

$$\varrho_n=\left(\frac{d}{d\phi}\right)^n(f(\phi)\sin\phi). \qquad (149)$$

Cor. 2. If in the last equation n be taken as negative, we have the first positive pedal of the nth involute.

Cor. 3. If ϱ_1 and ϱ_{-1} denote the radii vectores of the first positive pedals of the evolute and involute of $\nu=f(\phi)$, we have

$$\begin{aligned}\varrho_1\ &=f'(\phi)\sin\phi+f(\phi)\cos\phi,\\ \varrho_{-1}&=-f(\phi)\cos\phi+\int f'(\phi)\cos\phi\,d\phi;\\ \therefore\ \varrho_1+\varrho_{-1}&=f'(\phi)\sin\phi+\int f'(\phi)\cos\phi\,d\phi,\\ \therefore\ \varrho_1+\varrho_{-1}&=s. \qquad (150)\end{aligned}$$

Hence we have the following theorem:—

The length of a curve is equal to the sum of the radii vectores of the first positive pedals of its evolute and involute.

Cor. 4. If ϱ denote the radius vector of the first positive pedal of a curve,

$$s=\frac{d\varrho}{d\phi}+\int\varrho\, d\phi. \quad \ldots \ldots \quad (151)$$

SECTION II.—*Negative Pedals.*

55. We have seen in art. 52 that the polar equation of the first positive pedal of a curve is obtained from its tangential equation by changing ν into ϱ, and multiplying the function on the right-hand side by $\sin\phi$. Hence, conversely, we have the following theorem:—If $\varrho=\mathrm{F}(\phi)$ be the polar equation of a curve, the tangential equation of its first negative pedal is

$$\nu=\frac{\mathrm{F}(\phi)}{\sin\phi}. \quad \ldots \ldots \quad (152)$$

Thus the polar equation of a parabola is

$$\varrho=4a\tan\phi\sec\phi;$$

∴ the tangential equation of its first negative pedal is

$$\nu=4a\sec^2\phi, \quad \ldots \ldots \quad (153)$$

or, in Cartesian coordinates,

$$(x-4a)^3=27a\,y^2, \quad \ldots \ldots \quad (154)$$

showing that it is in the semicubical parabola.

56. The equation of the line whose envelope is the negative pedal is

$$x\sin\phi+y\cos\phi-\mathrm{F}(\phi)=0.$$

Hence the points where this line meets its envelope are given by the equations

$$x=\mathrm{F}(\phi)\sin\phi+\mathrm{F}'(\phi)\cos\phi, \quad \ldots \ldots \quad (155)$$

$$y=\mathrm{F}(\phi)\cos\phi-\mathrm{F}'(\phi)\sin\phi; \quad \ldots \ldots \quad (156)$$

and by eliminating ϕ between these equations, we get the equation of the pedal.

Cor.

$$\left.\begin{aligned} x^2+y^2&=(\mathrm{F}(\phi))^2+(\mathrm{F}'(\phi))^2 \\ \text{or} \quad x^2+y^2&=\varrho^2+\left(\frac{d\varrho}{d\phi}\right)^2 \end{aligned}\right\} \quad \ldots \ldots \quad (157)$$

Hence the distance from the extremity of ϱ to where the perpendicular to it meets its envelope is $\left(\frac{d\varrho}{d\phi}\right)$.

Examples.

(1) Find the first negative pedal of the cardioide.

The polar equation of this curve is, taking the perpendicular to the cuspidal tangent as the initial line,

$$\varrho=a(1+\sin\phi);$$

$$\therefore F(\phi)=a(1+\sin\phi);$$

$$\therefore x=2a+a\sin\phi,$$

$$y=a\cos\phi,$$

$$\therefore (x-2a)^2+y^2=a^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (158)$$

Therefore the pedal is a circle.

(2) Find the negative pedal of $\varrho=k\tan^2\phi\sec\phi$.

$$\therefore \frac{x}{k}=2\tan\phi+4\tan^3\phi,$$

$$\frac{y}{k}=-\tan^2\phi-3\tan^4\phi.$$

Then if we put

$$N=\{\sqrt{k-12y}-k^2\},$$

the result of eliminating ϕ will be

$$\frac{k^4x^2}{2}=\frac{N}{27}\{N+3k^2\}^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (159)$$

By differentiating the values of x and y given in equations (155), (156), then squaring and adding &c., we get the length of the first negative pedal,

$$s=F'(\phi)+\int F(\phi)d\phi, \quad . \quad . \quad . \quad . \quad . \quad . \quad (160)$$

an equation which agrees with equation (151), but expressed in a different notation.

57. If in art. 39 we substitute $\left(\frac{F\phi}{\sin\phi}\right)$ for $f(\phi)$, we get, from equation (93),

$$s=\left\{1+\left(\frac{d}{d\phi}\right)^2\right\}F(\phi).$$

Hence we have the following theorem:—

If $\varrho=F(\phi)$ *be the polar equation of a curve, the intrinsic equation of the evolute of its first negative pedal is*

$$s=\left\{1+\left(\frac{d}{d\phi}\right)^2\right\}F(\phi). \quad . \quad . \quad . \quad . \quad . \quad . \quad (161)$$

In like manner, from art. 50, *the intrinsic equation of the involute of the first negative pedal is*

$$s=\left\{1+\left(\frac{d}{d\phi}\right)^{-2}\right\}F(\phi). \quad . \quad . \quad . \quad . \quad . \quad . \quad (162)$$

Cor. *The intrinsic equation of the* n*th evolute is*

$$s=\left\{\left(\frac{d}{d\phi}\right)^{n-1}+\left(\frac{d}{d\phi}\right)^{n+1}\right\}F(\phi), \quad . \quad . \quad . \quad . \quad . \quad (163)$$

and of the n*th involute is*

$$s=\left\{\left(\frac{d}{d\phi}\right)^{-(n-1)}+\left(\frac{d}{d\phi}\right)^{-(n+1)}\right\}F(\phi). \quad . \quad . \quad . \quad . \quad (164)$$

Examples.

Find the intrinsic equation of the first negative pedal of an ellipse.

The polar equation of the ellipse,

$$\varrho=\frac{b}{\Delta(\phi)},\ \text{where } \Delta(\phi)=\sqrt{1-e^2\sin^2\phi}.$$

Hence by art. 56, equation (160), the equation of its first negative pedal is

$$s=\frac{be^2\sin\phi\cos\phi}{\Delta^3\phi}+b\int\frac{d\phi}{\Delta\phi},$$

or

$$s=b\left\{\frac{e\sin\phi\cos\phi}{\Delta^3\phi}+\mathrm{F}(e,\phi)\right\}, \quad (165)$$

and the intrinsic equation of the evolute of the pedal is

$$s=\frac{b}{\Delta(\phi)}\left\{1+\frac{e^2\cos 2\phi}{\Delta^2\phi}+\frac{b}{4}\left(\frac{e^2\sin^2\phi}{\Delta^2\phi}\right)^2\right\}. \quad (166)$$

58. The converse of the problem solved in art. 56 is, being given the intrinsic equation of a curve to find the polar equation of its first positive pedal.

Let $s=f(\phi)$ be the given intrinsic equation, then we have, from equation (151),

$$\frac{d\varrho}{d\phi}+\int\varrho\,d\phi=f(\phi);$$

$$\therefore\ \frac{d^2\varrho}{d^2\phi}+\varrho=f'(\phi),$$

$$\therefore\ \varrho=\left\{1+\left(\frac{d}{d\phi}\right)^2\right\}^{-1}f'(\phi). \quad (167)$$

Cor. 1. The polar equation of the positive pedal of the evolute is

$$\varrho=\left\{1+\left(\frac{d}{d\phi}\right)^{-2}\right\}^{-1}f(\phi), \quad (168)$$

and of the involute

$$\varrho=\left\{1+\left(\frac{d}{d\phi}\right)^2\right\}^{-1}f'(\phi). \quad (169)$$

Cor. 2. The equation (167) may be written

$$\varrho=\sin\phi\int\cos\phi\,f'(\phi)d\phi-\cos\phi\int\sin\phi\,f'(\phi)d\phi$$
$$+\mathrm{C}_1\cos\phi+\mathrm{C}_2\sin\phi. \quad (170)$$

See BOOLE'S 'Differential Equations,' where the reader will find illustrations of the cases in which the symbol $\left(1+\left(\frac{d}{d\phi}\right)^2\right)^{-1}$ on the right-hand side of equation (167) may be usefully expanded in ascending powers of $\left(\frac{d}{d\phi}\right)^2$, and thus the integration on the right-hand side rendered unnecessary, for the process then will be performed by differentiation.

Cor. 3. The equation (169) may be written

$$\varrho=\sin\phi\int\cos\phi\, f(\phi)d\phi-\cos\phi\int\sin\phi\, f(\phi)d\phi$$
$$+C_1\cos\phi+C_2\sin\phi. \quad \ldots \ldots \quad (171)$$

59. Since $(1+x^{-1})^{-1}=x(1+x)^{-1}$, if we put for x the symbol $\left(\frac{d}{d\phi}\right)^2$, we get

$$\left\{1+\left(\frac{d}{d\phi}\right)^{-2}\right\}^{-1}=\left(\frac{d}{d\phi}\right)^2\left\{1+\left(\frac{d}{d\phi}\right)^2\right\}^{-1}. \quad \ldots \ldots \quad (172)$$

∴ if the right-hand side of equation (169) be differentiated twice with respect to ϕ, we get the right-hand side of equation (168). Hence equation (168) may be written

$$\varrho=\cos\phi\int\sin\phi\, f(\phi)d\phi-\sin\phi\int\cos\phi\, f(\phi)d\phi$$
$$+f(\phi)+C_1\cos\phi+C_2\sin\phi. \quad \ldots \ldots \quad (173)$$

60. From equation (160) art. 56 we have at once the following theorems.

If we have three polar curves given by the equations

$$\varrho=F(\phi), \quad \varrho=F_1(\phi), \quad \varrho=mF(\phi)+nF_1(\phi);$$

then, 1°, if the corresponding lengths of their negative pedals be denoted by

$$s, \quad s_1, \quad S,$$

we shall have

$$S=ms+ns_1. \quad \ldots \ldots \quad (174)$$

2°. If the corresponding lengths of the nth evolutes of their first negative be

$$\sigma, \quad \sigma_1, \quad \text{and } \Sigma,$$

then

$$\Sigma=m\sigma+n\sigma_1. \quad \ldots \ldots \quad (175)$$

61. To find the curve whose length bears a constant ratio to the radius vector of its first positive pedal. The given condition is expressed by the equation

$$kf(\phi)\sin\phi=f'(\phi)\sin\phi+\int f'(\phi)\cos\phi\, d\phi;$$

$$\therefore\ k(f'(\phi)\sin\phi+f(\phi)\cos\phi)=2f'(\phi)\cos\phi+f''(\phi)\sin\phi.$$

Hence

$$f(\phi)=\frac{e^{m\phi}}{\sin\phi}, \quad \ldots \ldots \quad (176)$$

where $m=k+\frac{1}{k}$;

$$\therefore\ \nu=\frac{e^{m\phi}}{\sin\phi}.$$

This curve is the equiangular spiral; and we infer from the form of its equation that its reciprocal with respect to a circle whose radius is k is another equiangular spiral whose equation in polar coordinates is

$$\varrho=k^2e^{-m\phi}. \qquad (177)$$

Cor. The positive and negative pedals of equiangular spirals are also equiangular spirals, and so is the inverse. So that every geometrical transformation of this curve is another curve of the same species.

SECTION III.—*Reciprocal Curves.*

62. We have seen that the polar equation of the first positive pedal of the curve

$$\nu=f(\phi)$$

is

$$\varrho=f(\phi)\sin\phi\,;$$

and the reciprocal of a curve being the inverse of its first positive pedal, then the polar equation of the reciprocal of $\nu=f(\phi)$ is

$$\frac{k^2}{\varrho}=f(\phi)\sin\phi. \qquad (178)$$

Thus the reciprocal of the parabola is

$$\frac{k^2}{\varrho}=a\tan\phi\sin\phi, \qquad (179)$$

or, in Cartesian coordinates,

$$y^2=\frac{k^2}{a}x, \qquad (180)$$

which is another parabola, as it ought, since the centre of reciprocation is a point on the curve.

63. Since the value of ϱ derived from art. 178 is

$$\varrho=\frac{k^2}{f(\phi)\sin\phi},$$

we infer, from art. 55, that the equation of the first negative pedal of the curve is

$$\nu=\frac{k^2}{f(\phi)\sin^2\phi}.$$

Hence we have the following theorem:—*If* $\nu=f(\phi)$ *be the tangential equation of a curve the reciprocal of its first positive pedal or the first negative pedal of its reciprocal is*

$$\nu=\frac{k^2}{f(\phi)\sin^2\phi} \qquad (181)$$

64. If the intrinsic equation be given, say $s=\mathrm{F}(\phi)$, then we have, from equation (167), the polar equation of its reciprocal,

$$\frac{k^2}{\varrho}=\left\{1+\left(\frac{d}{d\phi}\right)^2\right\}^{-1}\mathrm{F}'(\phi) \qquad (182)$$

Cor. The polar equation of the reciprocal of the nth evolute is

$$\frac{k^2}{\rho}=\left\{1+\left(\frac{d}{d\phi}\right)^2\right\}^{-1}\mathrm{F}^{(n+1)}(\phi) \quad \ldots\ldots\ldots \quad (183)$$

65. If in equation (182) we put $\varrho=\psi(\phi)$, we find

$$\mathrm{F}'(\phi)=\left\{1+\left(\frac{d}{d\phi}\right)^2\right\}\frac{k^2}{\psi(\phi)};$$

$$\therefore\ \mathrm{F}(\phi)=k^2\int\frac{d\phi}{\psi(\phi)}-k^2\frac{\psi'\phi}{(\psi\phi)^2}, \quad \ldots\ldots\ldots \quad (184)$$

an equation which gives the intrinsic equation of a curve in terms of the polar equation of its reciprocal.

66. If in equation (178) we put $\varrho=\psi(\phi)$, we get

$$f(\phi)=\frac{k^2}{\psi(\phi)\sin\phi}.$$

Hence if $\varrho=\psi(\phi)$ be the polar equation of a curve,

$$\nu=\frac{k^2}{\psi(\phi)\sin\phi} \quad \ldots\ldots\ldots\ldots \quad (185)$$

is the tangential equation of its reciprocal.

Obs.—The problems we have solved in this section may be briefly stated thus:—

Given	To find
Tangential equation of a curve,	Polar equation of its reciprocal.
Polar ,, ,,	Tangential equation of its reciprocal.
Intrinsic ,, ,,	Polar ,, ,,
Polar ,, ,,	Intrinsic ,, ,,

Examples.

(1) Let it be required to find the reciprocal of the catenary.

The intrinsic equation is

$$s=c\tan\phi;$$

$\therefore\ \mathrm{F}'(\phi)=c\sec^2\phi$, and, substituting in equation (182),

$$\frac{k^2}{\varrho}=c\left\{1+\left(\frac{d}{d\phi}\right)^2\right\}^{-1}\sec^2\phi.$$

Hence, from equation (170), we have

$$\frac{k^2}{\varrho}=c\sin\phi\int\sec\phi\,d\phi=c\cos\phi\int\sec\phi\tan\phi\,d\phi+\mathrm{C}_1\cos\phi+\mathrm{C}_2\sin\phi;$$

then performing the integrations, and determining the constants by the condition that

ϱ must be infinite when $\phi=0$, we have $C_1=c$ and $C_2=0$, and the required equation is

$$\frac{k^2}{\varrho}=c\sin\phi\log(\sec\phi+\tan\phi)-2c\sin^2\tfrac{1}{2}\phi\,. \qquad (186)$$

(2) Find the reciprocal of the curve

$$\varrho^m=a^m\sin m\phi.$$

$$\therefore\ \psi(\phi)=a(\sin m\phi)^{\frac{1}{m}};$$

$\therefore$ the tangential equation of its reciprocal is

$$\nu=\frac{k^2}{a\sin\phi\,(\sin m\phi)^{\frac{1}{m}}} \qquad (187)$$

(3) Find the reciprocal of the cycloid.

The intrinsic equation is

$$s=4a\cos\phi.$$

Hence, from equations (182) and (170),

$$\frac{k^2}{\varrho}=2a\phi\sin\phi-a\cos\phi+C_1\cos\phi+C_2\sin\phi.$$

Now it is evident that ϱ must be infinite when ϕ vanishes, and that $\frac{k^2}{\varrho}$ must be equal to $a\pi$ when $\phi=\frac{\pi}{2}$. Hence $C_1=a$, $C_2=0$, and therefore the required reciprocal curve is

$$\frac{k^2}{\varrho}=2a\phi\sin\phi \qquad (188)$$

(4) The reciprocal of the logarithmic curve

$$\nu=a\log\tan\phi$$

is

$$\frac{k^2}{\varrho}=a\sin\phi\log\tan\phi,$$

or, in Cartesian coordinates,

$$x=ye^{\frac{ay}{k^2}} \qquad (189)$$

(5) Find the reciprocal of the curve

$$\nu=(1+\cot^{\frac{1}{3}}\phi)^3.$$

Here we have, from equation (178),

$$\frac{k^2}{\varrho}=(1+\cot^{\frac{1}{3}}\phi)^3\sin\phi,$$

$$k^2=(x^{\frac{1}{3}}+y^{\frac{1}{3}})^3;$$

$$\therefore\ x^{\frac{1}{3}}+y^{\frac{1}{3}}=k^{\frac{2}{3}} \qquad (190)$$

CHAPTER V.

SECTION I.—*The Cycloid.*

67. There is one curve which, though we have very seldom mentioned hitherto in our memoir, was the one which led to the discovery of its methods. This curve is the cycloid; and the reason it has not been more frequently used in our illustrations is that we consider its importance demands a chapter to itself. The novelty of the methods and of most of the results is our apology for devoting so much space to its investigation.

68. In the figure, art. 26, it is evident that the point Q is the centre of instantaneous rotation for the line LP, because the motion of the points L and P are respectively at right angles to the lines LQ and PQ respectively, and since the coordinates of the point Q are OL and LQ. Hence the locus of the centres of instantaneous rotation of the line LP, whose position is given at any time by the quantities ν and ϕ, where $\nu=f(\phi)$, is the curve obtained by eliminating ϕ between the equations

$$\left\{\begin{array}{l} x=f(\phi), \\ y=f'(\phi). \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (191)$$

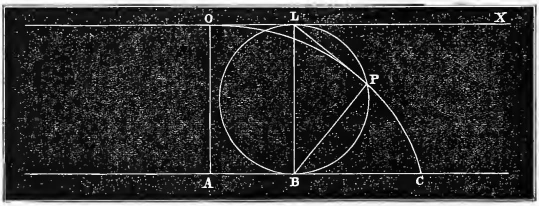
69. In the same fig., if LV, VQ be at right angles to LP, PQ, and since $LQ=f'(\phi)$, the values of LV, VQ will be $f'(\phi)\cos\phi$, $f'(\phi)\sin\phi$; and therefore the motion of the line LP will be given by supposing a curve whose equation is the system

$$\left\{\begin{array}{l} x=f'(\phi)\cos\phi, \\ y=f'(\phi)\sin\phi \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (192)$$

to roll on the curve whose equation is the system (191), and the line LP will be the axis of y with respect to this rolling curve.

70. Let $f(\phi)=2a\phi$, then $\nu=2a\phi$; let O be the origin, $OL=\nu$, the angle $XLP=\phi$; then if P be the point of contact of LP with its envelope we have, by art. 26, the diameter of the circle touching OX at L and passing through the point $P=f'(\phi)=2a$. Hence if we erect LB at right angles to OX, and PB to LP, the diameter LB of the circle LPB will be constant and equal to $2a$, and the arc LP of the same circle will be equal $2a\phi$; $\therefore$ the arc LP=the line OL=the line AB. Hence if we make $AC=\pi a$, the arc PB

Fig. 8.



will be equal to the line BC; and ∴ the point P may be considered as fixed in the circle LPB, and the locus of P will be the curve described by a fixed point in the circle LPB rolling on the line AC. In other words the locus of P is a cycloid.

71. Since $f(\phi)=2a\phi$, the equations (191) denote a right line, and the equations (192) the circle $x^2+y^2=4a^2$. Hence the cycloid $\nu=2a\phi$ is the envelope of a fixed diameter of the circle $x^2+y^2=4a^2$, which rolls along the line $y=-2a$. Therefore we have two methods of generating the same cycloid, either as a locus or an envelope.

72. The coordinates of the point P are, from equations (46), (47), the system

$$\text{I.}\quad \begin{cases} x=a(2\phi+\sin 2\phi), & \qquad (193) \\ y=-2a\sin^2\phi. & \qquad (194) \end{cases}$$

From equation (62) we have the intrinsic equation

$$\text{II.}\quad s=4a\sin\phi, \qquad (195)$$

and from (61)

$$\text{III.}\quad \rho=4a\cos\phi. \qquad (196)$$

If we differentiate the equation $\nu=2a\phi$ we have the differential equation of the cycloid

$$\text{IV.}\quad \frac{d\nu}{d\phi}=2a=\text{constant}. \qquad (197)$$

73. From equation (93), art. 39, the intrinsic equation of the evolute is

$$s=4a\cos\phi=4a\sin\left(\frac{\pi}{2}-\phi\right). \qquad (198)$$

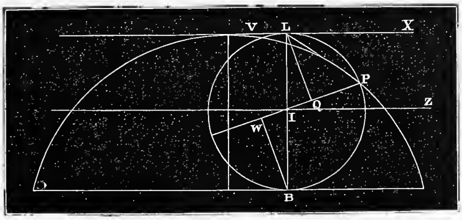
Hence the evolute is another cycloid.

We can show the same thing geometrically; for we have seen that the arc PB = the line BC. Hence denoting BC by ν, and the angle PBC by θ, we have $\nu=2a\theta$, and therefore the envelope of PB is a cycloid.

Cor. If the line PB be produced to R, making BR=BP, then R is the centre of curvature.

74. From L let fall the perpendicular LQ on the diameter VP of the revolving circle,

Fig. 9.



then it is evident that the angle XLQ=2XLP=2ϕ; and denoting this angle by ψ, we have

$$\nu=a\psi,$$

or

$$\frac{d\nu}{a\psi}=a.$$

Hence the envelope of LQ is a cycloid, and it is evident that Q is the point of contact. This is the cycloid that would be described by a fixed point in the circumference of the circle, whose diameter is the line IL, rolling on the line Z.

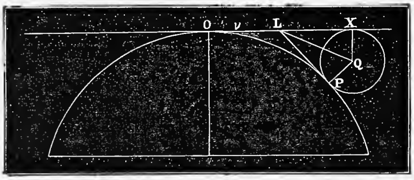
Cor. From B let fall the perpendicular BW on the diameter VP of the rolling circle, then we have IW=IQ; and therefore the locus of the point W is the envelope of VP, and it is the evolute of the cycloid described by Q.

75. The circle whose centre is P, and which touches the line LX, also touches the locus of Q. Hence we have the following theorem:—

If a variable circle has its centre on a given cycloid, and if it touches the tangent at its vertex, its envelope is another cycloid.

76. *If a variable circle touch a given cycloid, and also touch the tangent at the vertex, the locus of its centre is a cycloid.*

Fig. 10.



Or we may give a direct proof of this last theorem: let the angle $XLQ=\frac{1}{2}XLP=\frac{1}{2}\phi$ be denoted by θ; now we have

$$\nu=2a\phi=4a\theta;$$

hence the envelope of LQ is a cycloid. Again, $LP=LQ\cos\theta$, but $LP=2a\sin 2\theta$, $\therefore$ $LQ=4a\sin\theta$; and therefore Q is the point of contact of LQ with its envelope, and the proposition is proved.

77. If LP, L′P′, L″P″ be three fixed tangents to a variable cycloid, we have

$$\nu=2a\phi,\quad \nu'=2a\phi',\quad \nu''=2a\phi''.$$

Hence

$$\frac{\nu'-\nu}{\nu''-\nu'}=\frac{\phi'-\phi}{\phi''-\phi'}=\text{constant}.$$

Hence the tangent at the vertex of the cycloid is divided in a given anharmonic ratio by the three given tangents and the line at infinity. Hence we have the following theorem:—

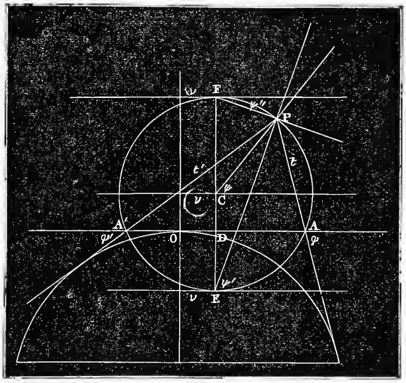
Being given three fixed tangents to a variable cycloid, the envelope of the tangent at the vertex is a parabola.

78. If four fixed tangents to a cycloid be given, the tangent at the vertex is a common tangent to two parabolas. Now being given two parabolas they have, in addition to the

common tangent at infinity, three finite common tangents. Hence we have the following theorem :—

Four lines being given, three cycloids can be described to touch them.

Fig. 11.



79. If two variable tangents ($t\,t'$) to a cycloid intersect at a constant angle, and a circle be described about the triangle formed by $t\,t'$ and the tangent at the vertex of the cycloid, then (1°) *the envelope of the diameter of this circle passing through the points* $t\,t'$ *is a cycloid*; (2°) *the envelopes of the chords passing through the same point, and through the highest and lowest points of the circle, are cycloids.*

Let the tangents $t\,t'$ intersect in P, and let C be the centre of the circle APA′; then since the angle APA′ is constant, $\phi+\phi'$ is constant, $\therefore$ $AA'=2a(\phi+\phi')$ is constant. Hence the base and the vertical angle of the triangle APA′ is constant; $\therefore$ the diameter of the circumscribing circle is constant, and it is evident that the loci of the points E, C, F are right lines parallel to AA′.

Again, since D is the middle point of AA′,

$$OD=\tfrac{1}{2}(OA-OA')=a(\phi-\phi')=a(\widehat{FCP}).$$

Hence

$$\nu=a\left(\frac{\pi}{2}-\phi\right),$$

$$\therefore\ \frac{d\nu}{d\phi}=-a=\text{constant};$$

therefore (see art 72, equation (197)) the envelope of CP is a cycloid.

(2) Since $OD=a(\widehat{FCP})=2a(\widehat{FEP})$, we have, for finding the envelope of EP,

$$\nu=2a\left(\frac{\pi}{2}-\psi'\right);$$

therefore it is a cycloid.

(3) The angle $FEP=\psi''$, $\therefore$ $\nu=2a\psi''$, and the envelope of FP is a cycloid.

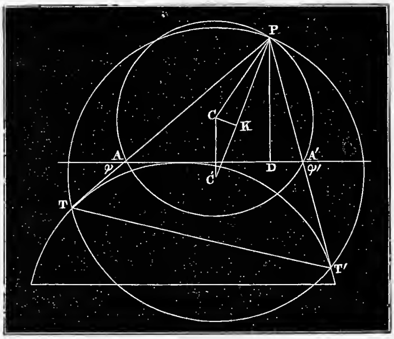
80. *If* C, C′ *be the centres of the circles* APA′ *and* TPT′, *then* CC′ *is perpendicular to the tangent at the vertex of the cycloid, and equal to the radius of its generating circle.*

Demonstration.—Since $AT=2a\sin\phi$, and $A'T'=2a\sin\phi'$, we have

$$\frac{AT}{A'T'}=\frac{\sin\phi}{\sin\phi'}=\frac{PA'}{PA};$$

$\therefore$ PA . AT=PA′ . A′T′. Hence the radical axis of the two circles is parallel to AA′, and therefore CC′ is perpendicular to AA′.

Fig. 12.



Again, the radical axis of the two circles passes through P; hence, by a known property of coaxal circles, the rectangle

$$PA . AT=2CC' . PD;$$

that is,

$$PA . AT=2CC' . PA . \sin\phi,$$

$$\therefore AT=2CC' \sin\phi;$$

but

$$AT=2a \sin\phi,$$

$$\therefore CC'=a. \qquad \text{Q. E. D.}$$

81. If the angle TPT′ be constant, *the locus of* C′, *the centre of the circle described about the triangle formed by the two tangents and the chord of contact, is a right line.* This is evident, since CC′=a and perpendicular to AA′, and the locus of C is a right line. (See art. 79.)

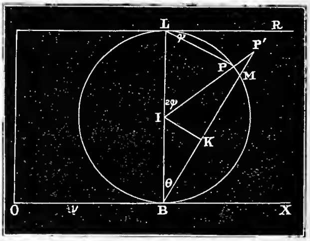
Section II.—*Intern and Extern Cycloids.*

82. *Definition.*—When the extremity of the revolving radius of the generating circle describes a cycloid, a fixed point in the radius describes a curve, which, according as the point is *inside* or *outside* the circle, I shall call the *intern* or *extern* cycloid.

These curves are usually called the *prolate* and the *curtate* cycloid; but the names I have adopted are more suggestive.

83. To find the intrinsic equation of an extern cycloid.

Fig. 13.



Let BPL be the generating circle of the cycloid, P the point which describes it, and P′ the point which describes the extern cycloid; then denoting IP, IP′ by a, b, and the angles as in the diagram, we have, since $\nu=2a\phi$, the coordinates of the point P′ given by the equations

$$\left.\begin{aligned} x&=2a\phi+b\sin 2\phi, \\ y&=\ a\ \ +b\cos 2\phi. \end{aligned}\right\} \quad . \quad . \quad . \quad (199)$$

If we differentiate these equations with respect to ϕ, then square and add, we get

$$\left(\frac{ds}{d\phi}\right)^2=4(a+b)^2-16\,ab\sin^2\phi;$$

$$\therefore s=2(a+b)\,\mathrm{E}(c', \phi), \quad \ldots\ldots\ldots \quad (200)$$

where

$$c'^2=\frac{4ab}{(a+b)^2}. \quad \ldots\ldots\ldots \quad (201)$$

Hence the arc of an extern cycloid is equal in length to an elliptic arc.

If we make $c'=\frac{2\sqrt{c}}{1+c}$, we find from equation (201) $c=\frac{a}{b}$; but from the triangle IBP′ we have

$$a : b :: \sin(2\phi-\theta) : \sin\theta.$$

Hence

$$\sin(2\phi-\theta)=c\sin\theta.$$

Hence we can apply LANDEN'S formula of transformation to the function on the right of equation (200), and we get (see CAYLEY'S 'Elliptic Functions,' p. 329)

$$2(a+b)\,\mathrm{E}(c', \phi)=2b\mathrm{E}(c, \theta)+\frac{a^2-b^2}{b}\mathrm{F}(c, \theta)+2a\sin\theta;$$

$$\therefore s=2b\mathrm{E}(c, \theta)+\frac{a^2-b^2}{b}\mathrm{F}(c, \theta)+2a\sin\theta \quad \ldots\ldots\ldots \quad (202)$$

Now since B is the centre of instantaneous rotation, the locus of the point P′ will be at right angles to P′B; that is, the tangent at P′ will be perpendicular to P′B, and the tangent at the highest point will be perpendicular to IB; hence the angle between these tangents will be equal to θ, and therefore the transcendental equation (202) is the intrinsic equation of the extern cycloid.

84. If the point P′ be inside the circle, that is, if the curve be an intern cycloid, the formula (202) will still hold, and be the intrinsic equation of the curve; but the modulus c of the functions E and F will be greater than unity. A simple transformation of that formula will give one in which c is changed into its reciprocal. If we interchange the quantities a and b in equation (200), the value of s remains unaltered; hence when the point P′ is inside, and b less than a, if we wish that the modulus of the functions E and F should be less than unity, instead of formula (202) we shall have the following:—

$$s=2a\mathrm{E}(c, \theta)+\frac{b^2-a^2}{a}\mathrm{F}(c, \theta)+2b\sin\theta. \quad \ldots\ldots\ldots \quad (203)$$

In this formula $c=\frac{b}{a}$, and the angle θ is the angle IP′B.

85. If we differentiate the equation (202) we get, after some slight reduction,

$$\frac{ds}{d\theta}=\frac{b(c\cos\theta+\Delta(\theta))^2}{\Delta(\theta)}.$$

Hence if ϱ denote the radius of curvature of an extern cycloid, we have

$$\varrho=\frac{b(c\cos\theta+\Delta(\theta))^2}{\Delta(\theta)} \quad \ldots\ldots\ldots \quad (204)$$

In the figure, art. 82, we have evidently

$$\mathrm{P'K}=b(\Delta(\theta)),\ \ \mathrm{P'B}=b(c\cos\theta+\Delta(\theta));$$

$$\therefore\ \varrho=\frac{\mathrm{P'B}^2}{\mathrm{P'K}}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (205)$$

Hence we have the following elegant construction for the centre of curvature of an intern or extern cycloid:—

From the centre I *of the generating circle let fall the perpendicular* IK *on the normal to the curve, and then the third proportional* P′Q *to* P′K *and* P′B *will be the radius of curvature.*

86. Since

$$\mathrm{P'Q}=\frac{b(c\cos\theta+\Delta(\theta))^2}{\Delta\theta}$$

and

$$\mathrm{P'B}=b(c\cos\theta+\Delta(\theta)),$$

we have

$$\mathrm{BQ}=\frac{a\cos\theta\,(c\cos\theta+\Delta'\theta))}{\Delta(\theta)}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (206)$$

Again, if the line BP′, that is, the normal to the curve, meet the polar of the point P′, with respect to the generating circle, in the point N, then the line BN is divided harmonically, and we have

$$\frac{1}{\mathrm{BP'}}+\frac{1}{\mathrm{BN}}=\frac{2}{\mathrm{BM}}$$

or

$$\frac{1}{b(c\cos\theta+\Delta(\theta))}+\frac{1}{\mathrm{BN}}=\frac{1}{bc\cos\theta};$$

$$\therefore\ \mathrm{BN}=\frac{a\cos\theta\,(c\cos\theta+\Delta(\theta))}{\Delta(\theta)},\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (207)$$

$$\therefore\ \mathrm{BQ}=\mathrm{BN}.$$

Hence we have the following theorem:—*The portion of the normal to an intern or extern cycloid at any point* P′ *of the curve included between the polar of the point* P′ *with respect to the centre of the generating circle and the corresponding centre of curvature is bisected by the centre of instantaneous rotation.*

87. By art. 30, equation (64), if $\nu=f(\theta)$ be the tangential equation of a curve,

$$\frac{ds}{d\theta}=\frac{\frac{d}{d\theta}(f'(\theta)\sin^2\theta)}{\sin\theta};$$

but by art. 85,

$$\frac{ds}{d\theta}=\frac{b\,(c\cos\theta+\Delta\,(\theta))^2}{\Delta\theta}.$$

Hence

$$(\theta)\sin^2\theta=a\sin^2\theta-b\cos\theta\,.\,\Delta(\theta)+b\,;$$

$$\therefore f(\theta)=a\sin^{-1}\{\sin\theta(c\cos\theta+\Delta\theta)\}+\frac{b(\Delta(\theta)-\cos\theta)}{\sin\theta}.$$

Hence the tangential equation of an intern or extern cycloid is

$$\nu=a\sin^{-1}\{\sin\theta(c\cos\theta+\Delta(\theta))\}+b\left(\frac{\Delta\theta-\cos\theta}{\sin\theta}\right). \quad . \quad . \quad . \quad . \quad . \quad (208)$$

88. In the triangle IBP′, art. 83, we have

$$\sin 2\phi : \sin\theta :: \mathrm{BP}' : b,$$

$$\therefore \sin 2\phi=\sin\theta(c\cos\theta+\Delta(\theta));$$

and from the equation of the cycloid described by the point P we have

$$\nu=2a\phi;$$

∴ eliminating ϕ, we have for the envelope of the line BP′

$$\nu=a\sin^{-1}\{\sin\theta(c\cos\theta+\Delta(\theta))\}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (209)$$

and this is the tangential equation of the evolute of an intern or extern cycloid.

89. The tangential equation (208) can be expressed very simply as follows. For if we take the conic

$$\frac{x^2}{b^2-a^2}+\frac{y^2}{b^2}=1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (210)$$

we easily find its tangential equation to be

$$\nu=\frac{b(\Delta(\theta)-\cos\theta)}{\sin\theta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (211)$$

Hence we have the following theorem:—

If $\nu=\mathrm{F}(\theta)$ be the tangential equation of the evolute (see equation (209)), and $\nu=\mathrm{G}(\theta)$ the tangential equation of the ellipse (210), then the tangential equation of the intern or extern cycloid is

$$\nu=\mathrm{F}(\theta)+\mathrm{G}(\theta). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (212)$$

Cor. 1. The intrinsic equation of the evolute of an intern or extern cycloid is

$$s=\frac{b(c\cos\theta+\Delta(\theta))^2}{\Delta(\theta)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (213)$$

Cor. 2. If σ, σ', σ'' denote the lengths of an extern or intern cycloid, its evolute, and its auxiliary conic (see equation (210)), taken on the three curves from points whose tangents are parallel to other three points whose tangents also are parallel, then

$$\sigma=\sigma'+\sigma''. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (214)$$

Cor. 3. If ρ, ρ', ρ'' be the radii of curvature of the same three curves at points whose tangents are parallel,

$$\rho=\rho'+\rho''. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (215)$$

Cor. 4. ρ will be infinite when either ρ' or ρ'' is infinite; but ρ'' will be infinite when the auxiliary conic is a hyperbola and the point of contact at infinity. Now if b is less than a, we have the following theorem:—

An intern cycloid has two points of inflection, the tangents at which are parallel to the asymptotes of the auxiliary conic.

SECTION III.

90. *If two tangents to a cycloid intersect at a constant angle, the locus of their point of intersection is an extern cycloid.*

Demonstration.—Since the angle APA′ (see fig. art. 79) between the tangents $t\,t'$ is constant, $\phi+\phi'$ is constant, $\therefore$ $AA'=2a(\phi+\phi')$ is constant; but the diameter of the circle about the triangle APA′

$$=\frac{AA'}{\sin P}=\frac{2a(\phi+\phi')}{\sin(\phi+\phi')};$$

$$\therefore CP=\frac{a(\phi+\phi')}{\sin(\phi+\phi')}.$$

Hence CP is constant. Again we have (see art. 79)

$$\frac{dv}{d\psi}=-a=\text{constant}.$$

Hence if CP were equal to a, the locus of P would be a cycloid; but since $\phi+\phi'$ is always greater than $\sin(\phi+\phi')$, CP is greater than a, and therefore the locus of P is an extern cycloid.

Lemma. *If two tangents,* PT, PT′, *to any given curve be inclined at a constant angle, the circle described about the triangle formed by the two tangents and the chord of contact touches the locus of* P.

Demonstration.—Let P′ be a consecutive point on the locus, then the tangents from P′ touch the curve in the points T, T′. Hence, since the angle TPT′=TP′T′, the quadrilateral is inscribed in a circle, and the line joining the consecutive points is a tangent to the circle. Hence the proposition is proved.

91. *If two tangents to a given cycloid make a given angle, the centre of the circle described about the triangle formed by the two tangents and the chord of contact is the centre of instantaneous rotation for the extern cycloid, which is the locus of the intersection of the tangents.*

Demonstration.—Since the angle TPT′ is given, the locus of P is an extern cycloid, and therefore, by the preceding lemma (see fig. art. 80), C′P is normal to the locus of P.

Again, since P is a point in the revolving radius of a circle whose centre is C and radius a, and we have proved $CC'=a$, the circle rolls on the locus of C′. Hence the proposition is proved.

92. If the angle TPT′ be constant, the radius of the circle TPT′ is a mean propor-

tional between the radius of the circle APA′ and half the chord of curvature at P, passing through the centre of APA′.

Demonstration.—Let the radii of the circles be r, r', and the angle CPC′ be ω; then, by art. 85, we have $\mathrm{PK}:\mathrm{PC}'::\mathrm{PC}':\varrho$. But $\mathrm{PK}=r\cos\omega$, $\therefore r\cos\omega : r' :: r' : \varrho$;

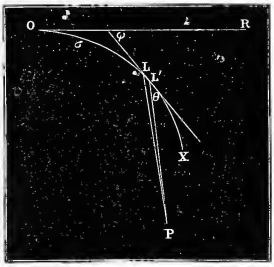
$$\therefore r'^2=r\times\varrho\cos\omega. \qquad \text{Q. E. D.}$$

CHAPTER VI.

SECTION I.—*Epicycloids.*

93. The form of tangential equation employed in the previous portion of this memoir may be usefully generalized as follows:—Thus, instead of taking a directing line OX (see art. 1) and a variable line LP, making an angle ϕ with OX at the distance ν from the origin, let us take a directing curve OX, and a variable line LP, making an angle θ with the curve at L, and denoting the arc OL by σ; then any relation between σ and θ, such as $\sigma=f(\theta)$, may be called the tangential equation of the curve which the line LP envelopes.

Fig. 14.



Let us take a consecutive position, L′P of LP, then P is the point where LP touches its envelope, and $\mathrm{LL}'=d\sigma$. Let the intrinsic equation of the directing curve be $\sigma=f_1(\phi)$, then the angle LPL′ is easily seen to be $d\phi+d\theta$; and if δ denote the diameter of the circle described about the infinitesimal triangle LPL′, we have

$$\frac{1}{\delta}=\frac{d\phi+d\theta}{d\sigma}.$$

Hence if ϱ denotes the radius of curvature of the directing curve at L, we have

$$\frac{1}{\delta}=\frac{1}{\varrho}+\frac{1}{f'(\theta)};$$

$$\therefore \delta=\frac{\varrho f'(\theta)}{\varrho+f'(\theta)}.$$

Hence

$$\mathrm{LP}=\frac{\varrho f'(\theta)\sin\theta}{\varrho+f'(\theta)}.$$

If s denotes the length of the curve which is the envelope of LP from some fixed point in it up to P, then (see art. 31)

$$ds=\mathrm{LL}'\cos\theta+d(\mathrm{LP})$$

$$=d\sigma\cos\theta+d(\mathrm{LP}).$$

Hence

$$s=\mathrm{LP}+\int\cos\theta d\sigma;$$

$$\therefore\ s=\frac{\varrho f'(\theta)\sin\theta}{\varrho+f'(\theta)}+\int\cos\theta d\sigma\,;$$

that is,

$$s=\frac{f'(\theta)f'_1(\varphi)\sin\theta}{f'(\theta)+f'_1(\varphi)}+\int\cos\theta f'(\theta)d\theta \quad . \quad . \quad . \quad . \quad . \quad (216)$$

If we denote the angle which LP makes with OR by ψ, we have evidently

$$d\psi=d\varphi+d\theta, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (217)$$

and we have also

$$f(\theta)=f_1(\varphi). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (218)$$

Hence eliminating θ and φ between the three last equations, there will be a resulting equation between s and ψ, say

$$s=\mathrm{F}(\psi), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (219)$$

and this will be the intrinsic equation of the envelope of LP.

94. Let the directing curve be the catenary, and let the functional symbols f, f_1 be the same; then, since $\sigma=f(\theta)=f_1(\varphi)$, we have $\theta=\varphi$; $\therefore\ \psi=2\theta$.

Now we have, from the intrinsic equation of the catenary,

$$f'_1(\varphi)=c\sec^2\varphi\,;\ \therefore f'(\theta)=c\sec^2\theta.$$

Hence, making these substitutions in equation (216), and putting $\theta=\varphi=\frac{\psi}{2}$, we have the required intrinsic equation

$$s=c\left\{\tfrac{1}{2}\sec\frac{\psi}{2}\tan\frac{\psi}{2}+\log\left(\sec\frac{\psi}{2}+\tan\frac{\psi}{2}\right)\right\}. \quad . \quad . \quad . \quad . \quad . \quad (220)$$

95. Let the directing curve be the cycloid, and let, as before, f, f_1 be the same, then we get the intrinsic equation

$$s=2a(1-\cos\psi), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (221)$$

a curve which we shall find to be a parallel to the cycloid.

96. The most interesting application of our general equation is where the directing curve is a circle, and the relation between σ and θ is linear; that is,

$$\sigma=\delta_{-1}(\theta),$$

where δ_{-1} is a constant. It will be seen that in this simple case the envelope belongs to the class of curves known as epicycloids and hypocycloids, and it will belong to one or the other according as δ_{-1} is positive or negative, or, what comes to the same thing, according as θ is positive or negative.

97. Let $\sigma=\delta_{-1}\theta$, and let the radius of the directing circle ALC be ϱ, then, from art. 93,

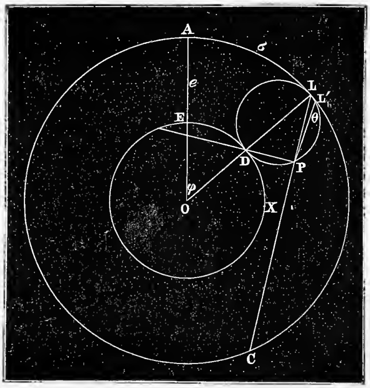
$$\frac{1}{\delta}=\frac{1}{\varrho}+\frac{1}{\delta_{-1}}\,; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (222)$$

$$\therefore\ \delta_{-1}(\varrho-\delta)=\delta\varrho.$$

but
$$\rho\phi = \delta_{-1}\theta,$$
$$\therefore (\rho-\delta)\phi = \delta\theta;$$

that is, since $LD=\delta$, the arc $ED=$ arc LP; and making the arc $EX=$ semicircle LPD, we

Fig. 15.



have the arc $XD=$ arc DP. Hence the locus of P is an epicycloid described by the rolling of the circle DPL on the circle EDX.

98. *Def.* We shall call the circle DPL the *generating* circle of the epicycloid, and the circle EDX, on which it rolls, the *base.*

It is evident that the motion of the circle DPL with respect to the director circle ALC is that of pure sliding, and its motion with respect to EDX is that of pure rolling.

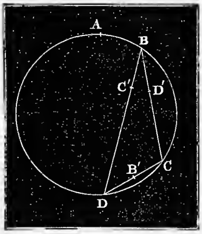
99. Since L is the centre of similitude of the circles DPL and ALC, we have
$$\frac{LP}{PC} = \frac{\delta}{2\rho-\delta};$$
but, from equation, (222)
$$\frac{\delta}{2\rho-\delta} = \frac{\delta_{-1}}{2\rho+\delta_{-1}},$$
$$\therefore \frac{LP}{PC} = \frac{\delta_{-1}}{2\rho+\delta_{-1}}.$$

Again, since
$$AL = \delta_{-1}\theta,$$
and arc
$$LC = 2\rho\theta,$$
we have
$$\frac{\text{arc } AL}{\text{arc } AC} = \frac{\delta_{-1}}{2\rho+\delta_{-1}}.$$

Hence
$$LP : PC :: \text{arc } AL : \text{arc } AC \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (223)$$

Hence, if a variable arc, AC, *has one extremity,* A, *fixed, and be divided in a given* *ratio in the point* L, *the envelope of the chord* LC *is an epicycloid, which it touches in a point* P, *determined by the proportion* LP : PC : : *arc* AL : *arc* AC.

Fig. 16.



100. If A be a fixed point, and B, C, D variable points, and if the ratios be given arc AB : arc BC : arc CD, then, from the last article, the envelope of each side of the triangle BCD is an epicycloid touching the circle in the point A. Hence we have the following theorem:—*If a variable polygon be inscribed in a circle, and if the envelopes of all the sides but one be epicycloids which have a common point of contact with the circle, then the envelope of the remaining side is another epicycloid, having the same point of contact with the circle.*

Cor. If the points of contact of the sides of the triangle BCD, with their respective envelopes, be B′, C′, D′, the three lines BB′, CC′, DD′, are concurrent. This is evident from art. 99.

101. In the figure (art. 97), since the arc XD=arc PD, then denoting XD by σ' and the angle which PD makes with the circle XDE by θ', we have

$$\sigma'=\delta\theta'; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (224)$$

therefore the envelope of PD is an epicycloid, whose directing circle is the circle XDE.

Hence, the evolute of an epicycloid is another epicycloid, and the director circle of one is the base of the other.

102. If δ_1 denote the diameter of the generating circle of the evolute, we have, as in art. 97,

$$\frac{1}{\delta_1}=\frac{1}{\rho-\delta}+\frac{1}{\delta}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (225)$$

But δ_1 and δ denote respectively the diameters of the generating circles of an epicycloid and its involute. *Hence, the difference between the reciprocals of the diameters of the generating circles of an epicycloid and its involute equals reciprocal of radius of directing circle of the epicycloid.*

Cor. *In the equation $\sigma=\delta_{-1}\theta$, the constant δ_{-1} is the diameter of the generating circle of the involute.*

This follows from the present article combined with equation (222). It was on this account that the negative suffix was put to δ.

103. In the figure (art. 97), if PD meet its envelope in P′, then P′ is the centre, and PP′ the radius of curvature at P; but PD$=\delta\cos\theta$, and P′D$=\delta_1\cos\theta$,

$$\therefore \text{PD} : \text{P}'\text{D} :: \delta : \delta_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (226)$$

That is, *the base of an epicycloid divides its radii of curvature in the constant ratio of the diameters of the generating circles of the epicycloid and its evolute.*

104. Let P″ (see fig., art. 97) be the point where LC meets the epicycloid, which is

the involute of the locus of P, then from the last article we have

$$\frac{P''L}{PL}=\frac{\delta_{-1}}{\delta}=\frac{\varrho}{\varrho-\delta} \text{ from art. 97};$$

$$\because \frac{P''L-LP}{P''P}=\frac{\delta}{2\varrho-\delta}=\frac{LP}{PC} \text{ from art. 99.}$$

Hence the points P″, P are harmonic conjugates to the points L and C.

Cor. 1. *Every radius of curvature of an epicycloid is divided harmonically by the base of the epicycloid.*

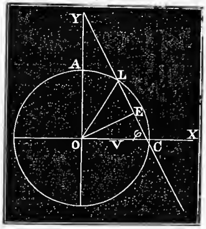
Cor. 2. *If in the figure* (art. 100) B″, C″, D″ *be the points on the involute of the three epicycloids which touch the sides of the triangle* BCD, *corresponding to the points of contact of these epicycloids, then the points* B″, C″, D″ *are collinear.*

105. From art. 99 we see that the arc AL : arc AC : : $\delta_{-1} : 2\varrho+\delta_{-1}$. Hence, letting fall the perpendicular OE, and denoting the angle AOE by ϕ, the angle LOE will be $m\phi$; if $m=\frac{\varrho}{\varrho+\delta_{-1}}$, and if we denote the radius of the circle by a, we have $OE=a\cos m\phi$. Hence the equation of the tangent to the epicycloid is

$$x\sin\phi+y\cos\phi=a\cos m\phi \quad . \quad . \quad . \quad . \quad (227)$$

For examples of the case in which the envelope of this line is an algebraic curve, see Salmon's 'Higher Curves,' p. 270.

Fig. 17.



106. The equation (227) may be written in the form $x+y\cot\phi=a\cos m\phi\operatorname{cosec}\phi$. Hence if the line OX be taken as the director line, the tangential equation of the epicycloid is

$$\nu=a\cos m\phi\,.\operatorname{cosec}\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (228)$$

107. In order to find the intrinsic equation we have $f(\phi)=a\cos m\phi\,.\operatorname{cosec}\phi$.

Hence from equation (64), art. 30, we find

$$\frac{ds}{d\phi}=a(1-m^2)\cos m\phi;$$

$$\therefore\ s=\frac{a(1-m^2)}{m}\sin m\phi, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (229)$$

which is the required intrinsic equation.

The same result may be obtained from art. 93, equation (216).

Cor. If we substitute for m its value we find, from art. 97, equation (222) combined with (229), that

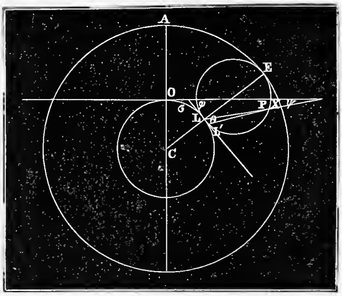
$$s=(\delta+\delta_{-1})\sin\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (230)$$

Hence putting $\theta=\frac{\pi}{2}$ and doubling, we have the whole length of the epicycloid from cusp to cusp=twice the sum of the diameters of the generating circles of the curve and its involute.

SECTION II.—*The Hypocycloid.*

108. Having discussed at considerable length the properties of the epicycloid, we shall treat very briefly those of the hypocycloid. In fact, analytically, the latter curve differs from the former only in the sign of a parameter; hence the properties of one curve are with slight modifications true of the other. The most interesting are those which are found by considering the curves in combination.

Fig. 18.



109. In the equation $\sigma=\delta_{-1}\theta$ of art. 96 let θ denote the angle which LP makes externally with the tangent to the director circle (which comes to the same thing as to consider θ negative. Now if θ change its sign, since we must regard σ as positive, δ_{-1} must change sign; in other words δ_{-1} has changed direction).

Also let ψ denote the angle which LP makes with the tangent to the director circle at the origin; then we have $\psi=\theta-\phi$;

$$\therefore\ \frac{d\psi}{d\sigma}=\frac{d\theta}{d\sigma}-\frac{d\phi}{d\sigma}.$$

Hence, if ${}^{\backprime}\delta$ denote the diameter of the circle LPE described about the infinitesimal triangle LL′P, we have

$$\frac{1}{{}^{\backprime}\delta}=\frac{1}{\delta_{-1}}-\frac{1}{\varrho}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (231)$$

and we find, as in art. 97,

$$(\varrho+{}^{\backprime}\delta)\phi={}^{\backprime}\delta\theta;$$

that is, the arc AE=arc LP; and making the arc AX=semicircle LPE, X will be a fixed point, and we shall have the arc EP=arc EX. Hence the locus of P is the hypocycloid generated by the rolling of the circle EPL on the circle AEX.

110. Since $\frac{1}{{}^{\backprime}\delta}=\frac{1}{\delta_{-1}}-\frac{1}{\varrho}$, see equation (231),

and $\frac{1}{\delta}=\frac{1}{\delta_{-1}}+\frac{1}{\varrho}$, see equation (222),

$$\therefore\ \frac{1}{\delta}+\frac{1}{{}^{\backprime}\delta}=\frac{1}{\delta_{-1}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (232)$$

Hence if an epicycloid and hypocycloid have the same director circle, and if the generating circles of their involutes be equal to one another, the diameter of the generating circles of their involutes is a harmonic mean between the diameters of the generating circles of the curves themselves.

111. From equation (231) we get

$$\varrho+{}^{\backprime}\delta=\frac{\varrho^2}{\varrho-\delta_{-1}},$$

and from equation (222)

$$\varrho-\delta=\frac{\varrho^2}{\varrho+\delta_{-1}}.$$

Hence $\varrho+\delta$, ϱ, $\varrho-\delta$ are in harmonical progression. *Hence, with the same hypothesis as in the last article, the radius of the common director circle is a harmonic mean between the radii of their bases.*

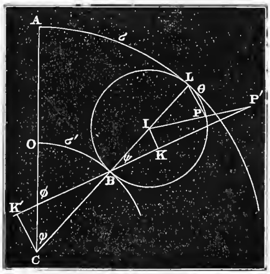
112. Several propositions proved for the cycloid may with scarcely any modification of the demonstration be extended to epi- and hypocycloids. Thus:—1°. *If from the point where the generating circle of an epi- or hypocycloid touches the base a perpendicular be let fall on the revolving radius, the envelope of the perpendicular is an epi- or hypocycloid.* 2°. *If the perpendicular be let fall from the point where the generating circle touches the director circle, the envelope is an epi- or hypocycloid.* 3°. *The envelope of the revolving radius is an epi- or hypocycloid.* 4°. *If two tangents*, PT, PT′, *to an epi- or hypocycloid meeting the director circle in the points* A, A′ *make a constant angle, the locus of the centre of the circle about the triangle* APA′ *is a circle.* 5°. *The envelope of the diameter of this circle which passes through* P *is an epi- or hypocycloid.* 6°. *The envelopes of the chords passing through* P *and through the highest or lowest points are epi- or hypocycloids.*

Section III.—*Extern Epicycloids.*

113. In the same manner as we have called the curve described by a fixed point in the revolving radius of the generating circle of a cycloid an in- or extern cycloid, we shall call the curve described by a fixed point in the plane of the generating circle of an epicycloid an in- or extern epicycloid according as the point is inside or outside the circumference of the circle. Similarly we shall have an in- or extern hypocycloid; so that the curve embraces four distinct species; but as they differ only in the magnitude or sign of a parameter, their properties are virtually the same; hence we shall discuss only the extern hypocycloid.

114. Let P′ be the point in the radius IP; then, since B is the centre of instantaneous rotation, BP′ will be a normal to the curve, and P′Z perpendicular to BP′ will be a tangent. The curve will have points of inflection. This follows at once from a beautiful theorem of Professor Ball's, Astronomer Royal of Ireland:—"That if a plane figure is moving in a plane according to any law, there is always a circle of points rigidly connected with it, such that three consecutive positions of each point are in a right line"*. (See 'Proceedings of the Royal Irish Academy,' December 11, 1871.) Another proof will be given in the course of our investigations.

Fig. 19



Let the equation of the curve described by the point P be

$$\sigma=n\theta;$$

---

* This circle is called the "circle of inflections." The theorem in the text was originally given by Savary in his 'Leçons des Machines.'—November 1877.

then if the diameter of the generating circle be $2a$, we have, from art. 97,

$$n=\frac{2a\varrho}{\varrho-2a}.$$

Now let the angle IBP′ be denoted by ψ and IP′ by b, and we have

$$\sin 2\theta : \sin\psi :: \mathrm{BP}' : b.$$

But

$$\mathrm{BP}'=\mathrm{BK}+\mathrm{KP}'=b(c\cos\psi+\Delta\psi),\ \text{if}\ c=\frac{a}{b};$$

hence

$$2\theta=\sin^{-1}\{\sin\psi(c\cos\psi+\Delta\psi)\}.$$

Again, we have

$$\sigma : \sigma' :: \varrho : \varrho-2a;$$

$$\therefore\ \sigma'=2a\theta,$$

$$\therefore\ \sigma'=a\sin^{-1}\{\sin\psi(c\cos\psi+\Delta\psi)\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (233)$$

And this is the equation of the evolute of the extern epicycloid.

115. From the diagram we have $\Phi=\phi+\psi$;

$$\therefore\ \frac{d\Phi}{d\sigma'}=\frac{d\phi}{d\sigma'}+\frac{d\psi}{d\sigma'}.$$

Now let a consecutive position of the line P′B intersect P′B in the point Q, and the arc OB in the point B′; then if D denote the diameter of the circle described about the infinitesimal triangle BB′Q, we have

$$\frac{d\Phi}{d\sigma'}=\frac{1}{\mathrm{D}},$$

and from equation (233) we get

$$\frac{d\psi}{d\sigma'}=\frac{\Delta(\psi)}{a(c\cos\psi+\Delta(\psi))};$$

$$\therefore\ \frac{1}{\mathrm{D}}=\frac{1}{\varrho-2a}+\frac{\Delta(\psi)}{a(c\cos\psi+\Delta(\psi))}.$$

Hence

$$\frac{1}{\mathrm{D}\cos\psi}=\frac{1}{(\varrho-2a)\cos\psi}+\frac{\Delta(\psi)}{a\cos\psi(c\cos\psi+\Delta\psi)}. \quad . \quad . \quad . \quad . \quad . \quad (234)$$

If the polar of the point P′ with respect to the generating circle meet BP′ in N, we have, see art. 86, equation (207),

$$\mathrm{BN}=\frac{a\cos\psi(c\cos\psi+\Delta\psi)}{\Delta(\psi)}.$$

Hence, from equation (234), if CK′ be perpendicular to BP′, we have

$$\frac{1}{\mathrm{BQ}}=\frac{1}{\mathrm{BK}'}+\frac{1}{\mathrm{BN}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (235)$$

Therefore BQ is half the harmonic mean between BK′ and BN; and this gives a geometric construction for the centre of curvature at the point P′.

116. From equation (235) we get

$$\mathrm{BQ}=\frac{\mathrm{BK'}\,.\,\mathrm{BN}}{\mathrm{K'N}}.$$

Now if we make BQ′=BN, the point Q′ would be the centre of curvature of an extern cycloid, see art. 86. Hence by subtraction

$$\mathrm{QQ'}=\frac{\mathrm{BN}^2}{\mathrm{K'N}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (236)$$

That is, *the distance between the centres of curvature of an extern cycloid and epicycloid is a third proportional to the lines* K′N *and* BN. This vanishes, as it ought, when K′ is at infinity.

117. From equation (234) we get the value of D cos ψ, that is, of BQ; thus

$$\mathrm{BQ}=\frac{a(\varrho-2a)\cos\psi\,(\varrho\cos\psi+\Delta(\psi))}{ac\cos\psi-(\varrho-a)\,\Delta(\psi)}$$

and

$$\mathrm{BP'}=b(\varrho\cos\psi+\Delta\psi).$$

Hence, remembering that $a=bc$, we get P′Q; that is, the radius of curvature at P′

$$=\frac{b\,(\varrho-a)\,\{c\cos\psi+\Delta\psi\}^2}{ac\cos\psi+(\varrho-a)\,\Delta\psi}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (237)$$

118. The following geometrical expression for the radius is remarkable for its simplicity and symmetry.

From equation (235),

$$\mathrm{BQ}=\frac{\mathrm{BK'}\,.\,\mathrm{BN}}{\mathrm{K'N}},$$

and from art. 86,

$$\frac{1}{\mathrm{BP'}}=\frac{1}{\mathrm{BK}}-\frac{1}{\mathrm{BN}};$$

$$\therefore\ \mathrm{BP'}=\frac{\mathrm{BK}\,.\,\mathrm{BN}}{\mathrm{KN}},$$

$$\therefore\ \mathrm{P'Q}=\mathrm{BN}\left\{\frac{\mathrm{BK}}{\mathrm{KN}}+\frac{\mathrm{BK'}}{\mathrm{K'N}}\right\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (238)$$

Cor. 1. The anharmonic ratio of the four points,

$$\mathrm{N, K, B, K'}=\mathrm{BP'}:\mathrm{BQ}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (239)$$

Cor. 2.

$$\frac{1}{\mathrm{BP'}}+\frac{1}{\mathrm{BQ}}=\frac{1}{\mathrm{BK}}+\frac{1}{\mathrm{BK'}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (240)$$

119. If P′ be a point of inflection, the radius of curvature at P′ will be infinite, and

therefore the denominator of the fraction on the right of equation (237) will vanish; and therefore

$$ac\cos\psi=(\varrho-a)(-\Delta\psi). \qquad (241)$$

Hence $\Delta(\psi)$ must be negative. But $\Delta(\psi)=\mathrm{P'K}\div b$, $\therefore$ P′K is negative; or the point P′ must lie between B and K, and therefore IP′ is less than IB; that is, the curve must be an intern epicycloid; and, from what we have proved, we see that the angle IP′B must be obtuse.

Again, we have $c\cos\psi=\frac{\mathrm{BK}}{b}$ and $\Delta\psi=\frac{\mathrm{P'K}}{b}$; and therefore the following conditions for determining a point of inflection:—

1st. The angle IP′B must be obtuse.

2nd. BK : KP′ :: $\varrho-a : a$ from equation (241); that is, BK : KP′ :: CI : IB. Hence the triangle IP′B can be constructed; and it follows that there are two points of inflection in each revolution of the generating circle.

120. Since

$$\sigma'=2a\theta \text{ (see art. 114)},$$

and also

$$\sigma'=(\varrho-2a)\phi,$$

we have

$$\phi=\frac{a}{\varrho-2a}\sin^{-1}\{\sin\psi(c\cos\psi+\Delta\psi)\};$$

but

$$\varphi=\psi+\phi,$$

$$\therefore \frac{d\varphi}{d\psi}=\frac{ac\cos\psi+(\varrho-a)\Delta\psi}{(\varrho-2a)\Delta(\psi)}. \qquad (242)$$

Again

$$\frac{ds}{d\varphi}=\text{radius of curvature};$$

$$\therefore \frac{ds}{d\varphi}=\frac{b(\varrho-a)\{c\cos\psi+\Delta(\psi)\}^2}{ac\cos\psi+(\varrho-a)\Delta\psi};$$

$$\therefore \frac{ds}{d\psi}=\frac{b(\varrho-a)\{c\cos\psi+\Delta(\psi)\}^2}{(\varrho-2a)\Delta(\psi)}. \qquad (243)$$

Now, if s' denote the corresponding arc of the extern cycloid, we have, from art. 85,

$$\frac{ds'}{d\psi}=\frac{b(\cos\psi+\Delta(\psi))^2}{\Delta(\psi)};$$

$$\therefore ds=\left(\frac{\varrho-a}{\varrho-2a}\right)ds' \qquad (244)$$

This formula, which connects the extern epicycloid with the extern cycloid, may be also obtained as follows, for we have evidently

$$x=(\varrho-a)\sin\phi+b\sin\left(\frac{\varrho-a}{a}\right)\phi,$$

$$y=(\varrho-a)\cos\phi+b\cos\left(\frac{\varrho-a}{a}\right)\phi;$$

$$\therefore\ \frac{ds}{d\phi}=\frac{\varrho-a}{a}(a+b)\Delta(c'\theta),\ c'=\frac{2\sqrt{ab}}{a+b}.$$

But

$$\frac{d\phi}{d\theta}=\frac{2a}{\varrho-2a},$$

$$\therefore\ \frac{ds}{d\theta}=\frac{\varrho-a}{\varrho-2a}2(a+b)\Delta(c'\theta),$$

$$\therefore\ s=\frac{\varrho-a}{\varrho-2a}+2(a+b)\mathrm{E}(c'\theta); \qquad (245)$$

but

$$s'=2(a+b)\mathrm{E}(c',\theta)\ \text{(see art. 83)},$$

$$\therefore\ s=\left(\frac{\varrho-a}{\varrho-2a}\right)s'. \qquad (246)$$

The same result as before.

CHAPTER VII.

SECTION I.—*Parallel Curves.*

121. The intercept which a parallel at the distance k from the movable line $x+y\cot\phi-\nu=0$ at either side makes on the directing line is $\nu\pm k\operatorname{cosec}\phi$, the choice of sign depending on the position of the parallel with respect to the origin. Hence we have the following theorem:—

If $\nu=f(\phi)$ be the tangential equation of a curve, the tangential equation of a parallel curve at the distance k is

$$\nu=f(\phi)\pm k\operatorname{cosec}\phi. \qquad (247)$$

Thus the parallel to the parabola is

$$\nu=a\tan\phi\pm k\operatorname{cosec}\phi, \qquad (248)$$

and the parallel to the cissoid

$$(2a-\nu)^3=27a^2\nu\cot^2\phi$$

is

$$\{(2a-\nu)\sin\phi\pm k\}^3=27a^2(\nu\sin\phi\pm k)\cos^2\phi. \qquad (249)$$

122. By the method of art. 26 we get the coordinates of a point on the parallel curve to be

$$x=f(\phi)+f'(\phi)\sin\phi\cos\phi\pm k\sin\phi, \qquad (250)$$

$$y=-f'(\phi)\sin^2\phi\pm k\cos\phi; \qquad (251)$$

and eliminating ϕ between these equations, we have the Cartesian equation of the parallel curve.

123. Since the ordinary tangential equation of a curve is the envelope of the line $\lambda x+\mu y+\nu=0$, the tangential equation of the parallel curve is the envelope of $\lambda x+\mu y+\nu\pm k\sqrt{\lambda^2+\mu^2}=0$. Hence we have the following theorem:—

If the tangential equation of a curve is

$$F(\lambda, \mu, \nu)=0,$$

the tangential equation of the parallel curve is

$$F(\lambda, \mu, \nu\pm k\sqrt{\lambda^2+\mu^2})=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (252)$$

124. If the result of the last article be expanded by TAYLOR's theorem, it may be written in the form

$$P\pm Qk\sqrt{\lambda^2-\mu^2}=0;$$

or, cleared of radicals,

$$P^2-Q^2k^2(\lambda^2+\mu^2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (253)$$

Hence the class of the parallel curve is twice the class of the original, and is independent of the sign of k. This shows the figures got by taking k plus and minus are both included in the equation of the parallel curve.

125. As in art. 30 we get for the intrinsic equation of the parallel curve*

$$\frac{ds}{d\phi}=2f'(\phi)\cos\phi+f''(\phi)\sin\phi+k,$$

$$\therefore s=f'(\phi)\sin\phi+\int f'(\phi)\cos\phi\,d\phi+k. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (254)$$

Cor. From the value of $\frac{ds}{d\phi}$ we see that the radius of curvature of the parallel curve differs from the radius of curvature of the original curve by the quantity k, as is otherwise evident.

Examples.

(1) Find the intrinsic equation of the parallel to the curve

$$\nu=\int\frac{d\theta}{\Delta(\theta)},$$

the function on the right being the elliptic integral of the first species.

---

* The following is an elegant focal property of parallel curves:—*Every single focus of the original curve is a double focus of the parallel curve.*

Demonstration. Let a tangent from a point I meet the original curve in two consecutive points P, P′; then if P, P′ be the centres of two circles, each of which passes through I, the line IP will be a normal to each, and therefore a normal to any curve of which these circles are generators. Now let the point I be one of the circular points at infinity; and since the parallel to any curve is the envelope of a circle of constant radius whose centre moves along the given curve, the line IP will be a normal to the parallel curve at I, and therefore a tangent at I; hence if two tangents be drawn to the original curve from the circular points at infinity, these tangents will touch the parallel curve at the circular points. Hence the theorem is proved.—*November* 1877.

Here we have

$$f'(\theta)=\frac{1}{\Delta(\theta)};$$

$$\therefore s=\frac{\sin\theta}{\Delta\theta}+\int\frac{\cos\theta d\theta}{\Delta\theta}+k\theta$$

$$=\frac{\sin\theta}{\Delta\theta}+\frac{1}{c}\sin^{-1}(c\sin\theta)+k\theta. \quad . \; . \; . \; . \; . \; . \; . \; . \qquad (255)$$

(2) Find the intrinsic equation of the parallel to the curve

$$\nu=\int\Delta(\theta)d\theta,$$

or

$$\nu=\mathrm{E}(c\theta).$$

In this case $f'(\theta)=\Delta(\theta)$; and we find, as before,

$$s=\frac{3\sin\theta\,.\,\Delta(\theta)}{2}+\frac{1}{2c}\sin^{-1}(c\sin\theta)+k\theta. \quad . \; . \; . \; . \; . \; . \; . \; . \qquad (256)$$

(3) The Cartesian equation of the curve parallel to the parabola is, by the equation of art. 122, the result of eliminating ϕ between the equations

$$x=2a\tan\phi+k\sin\phi,$$
$$y=-a\tan^2\phi-k\cos\phi.$$

This problem may be solved more easily by finding the envelope of the line

$$x+y\cot\phi-a\tan\phi-k\operatorname{cosec}\phi,$$

or, what is the same thing, the line

$$x\sin 2\phi+(a+y)\cos 2\phi-2k\cos\phi+(y-a-2k)=0.$$

Writing this in the form

$$\mathrm{A}\sin 2\phi+\mathrm{B}\cos 2\phi+\mathrm{C}\cos\phi+\mathrm{D}=0,$$

we get, by a known method, the required envelope to be

$$\{432(\mathrm{A}^2+\mathrm{B}^2)\mathrm{D}+9(\mathrm{D}-6\mathrm{B}^2)\mathrm{C}-2\mathrm{D}^3\}^2=4\{12(\mathrm{A}^2+\mathrm{B}^2)+3\mathrm{C}^2+\mathrm{D}^2\}^3. \quad . \qquad (257)$$

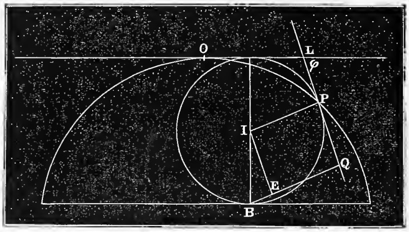
Cor. The characteristics of this curve are

$$\left.\begin{matrix}\mu=4, & \nu=6, & \varkappa=6\\ \delta=4, & \tau=3, & \iota=0\end{matrix}\right\}. \quad . \; . \; . \; . \; . \; . \; . \; . \qquad (258)$$

We shall find the reciprocal of this curve in a future article.

(4) *The envelope of a fixed tangent to the generating circle of a cycloid is a parallel to another cycloid whose generating circle has a diameter equal to half the diameter of the former.*

Fig. 20.



Let PQ be the tangent at the point P fixed in the revolving circle; then, since B is the centre of instantaneous rotation, Q, the foot of the perpendicular from B on PQ, will be the point of contact of PQ with its envelope. From I let fall the perpendicular IE on BQ; then it is easy to see that the locus of the point E will be a cycloid whose generating circle will have IB for diameter, and EB will be the normal at E to the cycloid. Now, since EQ=IP =radius of generating circle, we see that the locus of Q is a parallel to the cycloid.

The same result may be shown thus:—It is evident that $OL=a\phi+a\tan\frac{1}{2}\phi$; the equation of the curve which is the envelope of LP is

$$\nu=a\phi+a\tan\tfrac{1}{2}\phi,$$

and the intrinsic equation of this is

$$s=a\phi+2a\sin\phi, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (259)$$

which is a parallel to a cycloid.

Cor. 1. *The envelope of any line in rigid connexion with the generating circle of a cycloid is a parallel to another cycloid.*

Cor. 2. *In like manner the envelope of a fixed tangent* to, *or of any line in rigid connexion* with, *the generating circle of an epicycloid is a parallel to another epicycloid.*

126. From the equation (254) we infer that if $s=F(\phi)$ be the intrinsic equation of a curve, the equation of the parallel to it is

$$s=F(\phi)\pm k(\phi);$$

and hence (see art. 64, equation (182)) the polar equation of the reciprocal of the parallel curve is

$$\frac{a^2}{\varrho}=\left\{1+\left(\frac{d}{d\phi}\right)^2\right\}^{-1}\{F'(\phi)\pm k\}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (260)$$

or

$$\left.\begin{aligned}\frac{a^2}{\varrho}=\sin\phi\int\cos\phi(F'\phi\pm k)\,d\phi-\cos\phi\int\sin\phi(F'(\phi)\pm k)\,d\phi\\ +C_1\cos\phi+C_2\sin\phi.\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (261)$$

In this equation we have used a^2 as the numerator to ϱ on the left side of the equation instead of k^2 of recent articles in order to avoid confusion of notation.

127. The reciprocal of the parallel curve is found at once from the tangential equation. For it is evident that the polar equation of the reciprocal of the parallel to the curve

$$\nu=f(\varphi)$$

is

$$\frac{k^2}{\varrho}=f(\varphi)\sin\varphi\pm r, \quad \ldots\ldots\ldots \quad (262)$$

where r is the distance between the curve and its parallel, and k is the radius of the circle of reciprocation.

Examples.

(1) To find the reciprocal of the parallel to a parabola.

We have

$$f(\varphi)=a\tan\varphi,$$

$$\therefore\ \frac{k^2}{\varrho}=\frac{a\sin^2\varphi}{\cos\varphi}\pm r$$

is the required equation. This, expressed in Cartesian coordinates, is

$$(k^2x-ay^2)^2=r^2x^2(x^2+y^2). \quad \ldots\ldots\ldots \quad (263)$$

This curve has three double points, namely, the origin and the points where the conic k^2x-ay^2 meets the line at infinity.

Again, the curve is evidently the envelope of the conic

$$x^2+y^2+2\mu(k^2x-ay^2)+\mu^2r^2x^2. \quad \ldots\ldots\ldots \quad (264)$$

The discriminant of this conic is

$$(1-2a\mu)\mu^2k^2.$$

This shows that there are two values of μ, for which the conic breaks up into a pair of lines; hence the curve has four double tangents. Therefore the characteristics of the curve are

$$\mu=4, \qquad \delta=3, \qquad \tau=4,$$
$$\nu=6, \qquad \iota=6, \qquad \varkappa=0.$$

(2) Find the reciprocal of the parallel to the curve

$$\varrho^m=a^m\sin m\theta.$$

From equation (37), art. 24 we have at once the polar equation of the reciprocal

$$\frac{k^2}{\varrho}=a\left\{\sin\frac{m\varphi}{m+1}\right\}^{\frac{m+1}{m}}\pm r. \quad \ldots\ldots\ldots \quad (265)$$

(3) Find the intrinsic equation of the lemniscate.

The tangential equation is (see art. 25)

$$\nu=a\left(\sin\frac{2\varphi}{3}\right)^{\frac{3}{2}}\operatorname{cosec}\varphi,$$

and, by the method of art. 30, we find the intrinsic equation to be

$$s=\frac{a}{3}\int\frac{d\phi}{\sqrt{\sin\frac{2\phi}{3}}}. \quad \ldots \ldots \ldots \quad (266)$$

If we put $\sqrt{\sin\frac{2\phi}{3}}=\cos\theta$, this equation becomes

$$s=\frac{a}{\sqrt{2}}\int\frac{d\theta}{\sqrt{1-\frac{1}{2}\sin^2\theta}},$$

or

$$s=\frac{a}{\sqrt{2}}\mathrm{F}\left(\frac{1}{\sqrt{2}}.\theta\right). \quad \ldots \ldots \ldots \quad (267)$$

(4) Find the reciprocal of the parallel to an ellipse.

Here

$$f(\phi)=\sqrt{a^2+b^2\cot^2\phi},$$

$$\therefore \frac{k^2}{\rho}=\sqrt{a^2\sin^2\phi+b^2\cos^2\phi}\pm r$$

is the required equation.

This curve, in Cartesian coordinates, is

$$4k^4r^2\varrho^2=(a^2x^2+b^2y^2-k^4-r^2\varrho^2)^2. \quad \ldots \ldots \ldots \quad (268)$$

This curve can by linear transformation be changed into a bicircular quartic. For writing the equation in full by putting x^2+y^2 in place of ϱ^2, and then changing y into $\left(\frac{a^2-r^2}{b^2-r^2}\right)^{\frac{1}{2}}y$, we get

$$(x^2+y^2)^2+\frac{k^4r^2}{a^2-r^2}\left\{\frac{r^2-2(a^2-r^2)(b^2-r^2)}{(a^2-r^2)(b^2-r^2)}\right\}(x^2+y^2)$$

$$+\frac{k^4}{(a^2-r^2)^2}\left\{k^4-\frac{4r^2}{b^2-r^2}(b^2x^2+a^2y^2)\right\}=0. \quad \ldots \ldots \quad (269)$$

128. To find the reciprocal of a bicircular quartic, with respect to one of its circles of inversion.

The following method of generating these curves is given in my memoir on "Bicircular Quartics" (see Transactions of the Royal Irish Academy, vol. xxiv. p. 460):—

Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ be a conic F, called the focal conic, $(x-f)^2+(y-g)^2-r^2=0$ a circle J; then if from the centre O of the circle we let fall a perpendicular OT on any tangent to F, and take two points, P, P′, in opposite directions from T on OT, such that

$$\mathrm{OT}^2-\mathrm{TP}^2=\mathrm{OT}^2-\mathrm{TP}'^2=r^2,$$

the locus of the points P, P′ is a bicircular quartic. Now, denoting OT by p, and OP by ϱ, we get from this construction

$$2p\varrho=r^2+\varrho^2,$$

or

$$2\{\sqrt{a^2\cos^2\alpha+b^2\sin^2\alpha}-(f\cos\alpha+g\sin\alpha)\}\varrho=r^2+\varrho^2. \quad \ldots \quad (270)$$

Again, since OP . OP$'=r^2$, the points P, P$'$ are inverse points with respect to the circle J, and the perpendicular through P to the line OP will be the polar of P$'$; therefore the envelope of this perpendicular will be the reciprocal of the bicircular quartic. Now, let ϕ be the angle which the perpendicular makes with the axis of x, or the *directing* line, and ν=intercept, then we have $\phi=90-\alpha$, and $\varrho=\nu\sin\phi$.

Therefore the tangential equation of the reciprocal of the bicircular quartic is

$$\nu^2\sin^2\phi+r^2=2\{\sqrt{a^2\sin^2\phi+b^2\cos^2\phi}-(f\sin\phi+g\cos\phi)\}\nu\sin\phi. \quad . \quad . \quad (271)$$

Cor. The equation (271) is also the first negative pedal of a bicircular quartic.

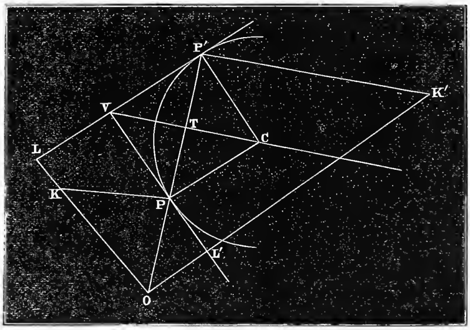
129. If we divide the equation (271) by $\sin^2\phi$, and then change ν into $-\frac{\nu}{\lambda}$ and $\cot\phi$ into $\frac{\mu}{\lambda}$, we get, after a slight reduction,

$$\{r^2(\lambda^2+\mu^2)+\nu^2-2f\nu\lambda-2g\lambda\mu\}^2=4(a^2\lambda^2+b^2\mu^2)\nu^2, \quad . \quad . \quad . \quad . \quad (272)$$

which is in the ordinary form of tangential equations.

130. From the equation (271) it is plain that to each value of ϕ there are two values of ν. This is otherwise evident; for erecting the perpendicular PK and P$'$K$'$ to OP and OP$'$, these perpendiculars will make intercepts on the director line, which will be the required values of ν. Let C be the centre of the generating circle, then C will be a point on the focal conic, and CP, CP$'$ will be normals to the quartic, and PV, P$'$V will be tangents. Now if K$'$ be the point of contact of P$'$K$'$ with its envelope, then the angle P$'$K$'$O=OP$'$V, and therefore OK$'$ is parallel to CP, and OK to CP$'$. Hence, drawing from the point O two parallels to the normals at P, P$'$, they will meet PK, P$'$K$'$ in the points of contact of these lines with their envelopes, and they will intersect the tangents PV, P$'$V perpendicularly in the points L, L$'$.

Fig. 21.



Cor. 1. The locus of the points L, L$'$ is evidently the first positive pedal of the bicircular quartic.

Cor. 2. The first positive pedal of a bicircular quartic is the inverse of its first negative pedal; for evidently

$$\text{OL}' . \text{OK}'=\text{OP} . \text{OP}'=r^2.$$

Cor. 3. OK′−OK=diameter of generating circle of the bicircular quartic; for, denoting the angle K′OP′=KOP by ψ, we have

$$\text{OK}'\cos\psi-\text{OK}\cos\psi=\text{OP}'-\text{OP}=2\text{CP}\cos\psi\,;$$

∴ OK′−OK=2CP=diameter of generating circle.

Section II.—*Rectification of Bicircular Quartics.*

131. If through the point O (see diagram, art. 130) we draw a consecutive line OQQ′, then the perpendiculars to this line at the points Q, Q′ will pass through the points K, K′, and we have evidently

$$\frac{\text{P}'\text{Q}'-\text{PQ}}{d\varphi}=\text{OK}'-\text{OK}.$$

Hence, denoting the elements P′Q′, PQ of the quartic by ds' and ds, we have

$$\frac{ds'-ds}{d\varphi}=2\varrho \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (273)$$

if ϱ denotes the radius CP of the generating circle (see *Cor.* 3, art. 130).

132. Mr. W. Roberts showed, in Liouville's Journal, vol. xv. p. 194, "Sur les arcs des Lignes Aplanétiques," that the difference between two arcs of a Cartesian oval is expressed by an arc of an ellipse; and Professor Genocchi showed some time afterwards, in Tortolini's 'Annali,' that the arc of a Cartesian oval is the sum of three elliptic arcs. We propose in this section to extend these theorems to bicirculars in general. We will show that Genocchi's theorem is an immediate inference from Roberts's, and that each is only a particular case of a more general theorem which holds for all bicirculars, and which can be expressed in terms of the radii of the generating circles of these curves.

In order that we may not have to be referring to other writings, we shall investigate briefly the leading properties of these curves, referring for a fuller discussion to the author's memoir on Bicirculars.

133. In art. 128, equation (270), we have the polar equation of a bicircular quartic. This, expressed in Cartesian coordinates, is

$$4(a^2x^2+b^2y^2)=(x^2+y^2+2fx+2gy+r^2)^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (274)$$

This equation is the envelope of the conic

$$\text{S}+\mu\text{C}+\mu^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (275)$$

where S represents the expression $a^2x^2+b^2y^2$, and C the circle $x^2+y^2+2fx+2gy+r^2=0$. Now the discriminant of the equation (275) is

$$\frac{\mu^2f^2}{a^2+\mu}+\frac{\mu^2g^2}{a^2+\mu}=\mu r^2\mu^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (276)$$

a biquadratic equation showing that there are four pairs of double tangents.

If the four values of μ be denoted by μ_1, μ_2, μ_3, μ_4, we have the equations of the four pairs of double tangents to the bicircular (see my memoir "On Bicirculars," art. 47). These pairs of lines are

$$\text{S}+\mu_1\text{C}+\mu^2,\ \ \text{S}+\mu_2\text{C}+\mu_2^2,\ \ \&\text{c.};$$

and, from the same article, the double points of these pairs of lines are the four centres of inversion of the quartic. Since one value of μ is obviously $=0$ in the foregoing equation, we see that the pair of double tangents drawn from the centre of the circle of inversion $J\,((x-f)^2+(y-g)^2=r^2)$ will, when that centre is taken as origin, be $a^2x^2+b^2y^2=0$. Hence, if the other centres be taken respectively as origin, the equations of the other pairs of double tangents will be

$$(a^2+\mu_2)x^2+(b^2+\mu_2)y^2=0, \quad \ldots \ldots \quad (277)$$

$$(a^2+\mu_3)x^2+(b^2+\mu_3)y^2=0, \quad \ldots \ldots \quad (278)$$

$$(a^2+\mu_4)x^2+(b^2+\mu_4)y^2=0. \quad \ldots \ldots \quad (279)$$

Now since the pair of lines $a^2x^2+b^2y^2=0$ are the asymptotes of the reciprocal of the conic $\frac{x^2}{a^2}+\frac{y^2}{b^2}-1=0$, we infer that the pairs of lines (277), (278), (279) are the asymptotes of the reciprocals of the other focal conics. Hence we have the following system as the equations of these conics:—

$$\frac{x^2}{a^2+\mu_2}+\frac{y^2}{b^2+\mu_2}=1, \quad \ldots \ldots \quad (280)$$

$$\frac{x^2}{a^2+\mu_3}+\frac{y^2}{b^2+\mu_3}=1, \quad \ldots \ldots \quad (281)$$

$$\frac{x^2}{a^2+\mu_4}+\frac{y^2}{b^2+\mu_4}=1. \quad \ldots \ldots \quad (282)$$

Hence the four focal conics of a bicircular quartic are confocal.

134. Since the equation (276) may be written in the form

$$\frac{f^2}{a^2+\mu}+\frac{g^2}{b^2+\mu}=1+\frac{x^2}{\mu},$$

and this is the discriminant of $\mu F+J$ (where J and F have the values in art. 128; see SALMON'S 'Conics,' p. 324), we infer that the same values of μ which will make $\mu F+J$ a pair of lines will make $S+\mu C+\mu^2$ a pair of lines; the two pairs of lines will have the same double point, their equations referred to that point as origin being

$$\frac{x^2(a^2+\mu)}{a^2}+\frac{y^2(b^2+\mu)}{b^2}=0, \quad \ldots \ldots \quad (283)$$

$$x^2(a^2+\mu)+y^2(b^2+\mu)=0. \quad \ldots \ldots \quad (284)$$

Hence we have the following theorem:—If F and J be a corresponding focal conic and circle of inversion of a bicircular quartic, and if μ_1, μ_2, μ_3 be the roots of the cubic which is the discriminant of $\mu F+J$, then if F be given in its canonical form $\frac{x^2}{a^2}+\frac{y^2}{b^2}-1=0$, the equations of the other three focal conics are got from this by changing a^2, b^2 respectively into $a^2+\mu_1$, $b^2+\mu_1$; $a^2+\mu_2$, $b^2-\mu_2$; and $a^2+\mu_3$, $b^2+\mu_3$.

135. When $S+\mu C+\mu^2$ represents a pair of lines, the coordinates of the double point are, by the usual process,

$$\frac{-\mu f}{a^2+\mu},\quad \frac{-\mu g}{b^2+\mu}$$

if referred to the centre of J as origin, and

$$\frac{a^2 f}{a^2+\mu},\quad \frac{b^2 g}{b^2+\mu}$$

if referred to the centre of F as origin. Hence we have the following theorem:—

If $F=\frac{x^2}{a^2}+\frac{y^2}{b^2}-1=0$ and $J=(x-f)^2+(y-g)^2-r^2=0$ be a corresponding focal conic and circle of inversion of a bicircular quartic, and if μ_1, μ_2, μ_3 be the three roots of the equation which is the discriminant of $\mu F+J$, then the coordinates of the centres of the three other circles of inversion are:—

$$\frac{a^2 f}{a^2+\mu_1},\quad \frac{b^2 g}{b^2+\mu_1}, \quad \ldots \ldots \quad (285)$$

$$\frac{a^2 f}{a^2+\mu_2},\quad \frac{b^2 g}{b^2+\mu_2}, \quad \ldots \ldots \quad (286)$$

$$\frac{a^2 f}{a^2+\mu_3},\quad \frac{b^2 g}{b^2+\mu_3}. \quad \ldots \ldots \quad (287)$$

136. Being given $F\equiv\frac{x^2}{a^2}+\frac{y^2}{b^2}-1=0$, and $J\equiv(x-f)^2+(y-g)^2-r^2=0$, the equation of the quartic is

$$4(a^2x^2+b^2y^2)-(x^2+y^2+2fx+2gy+r^2)^2=0. \quad \ldots \ldots \quad (288)$$

Again, being given

$$F'\equiv\frac{x^2}{a^2+\mu_1}+\frac{y^2}{b^2+\mu_1}-1=0,$$

$$J'\equiv\left(x-\frac{a^2 f}{a^2+\mu_1}\right)^2+\left(y-\frac{b^2 g}{b^2+\mu_1}\right)^2-r'^2=0,$$

the equation of the same quartic is

$$4\{(a^2+\mu_1)x^2+(b^2+\mu_1)y^2\}-\left\{x^2+y^2+\frac{2a^2 f}{a^2+\mu_1}x+\frac{2b^2 g}{b^2+\mu_1}y+r'^2\right\}^2. \quad \ldots \quad (289)$$

In order to compare the equations (288) and (289), which represent the same curve, they must be referred to the same origin; we will therefore transform (288) to the same origin as (289), and we get

$$4\left\{a^2\left(x+\frac{\mu_1 f}{a^2+\mu_1}\right)^2+b^2\left(y+\frac{\mu_1 g}{b^2+\mu_1}\right)^2\right\}-\left\{\left(x+\frac{\mu_1 f}{a^2+\mu_1}\right)^2+\left(y+\frac{\mu_1 g}{b^2+\mu_1}\right)^2\right.$$

$$\left.+2f\left(x+\frac{\mu_1 f}{a^2+\mu_1}\right)+2g\left(y+\frac{\mu_1 g}{b^2+\mu_1}\right)+r'^2\right\}^2=0. \quad \ldots \quad (290)$$

Since the equations (289) and (290) represent the same curve, their absolute terms must be the same. Now if the absolute term in the latter equation be reduced by means of the relation $\frac{f^2}{a^2+\mu_1}+\frac{g^2}{b^2+\mu_1}=1+\frac{r^2}{\mu_1}$, it becomes

$$\left\{\frac{\mu_1{}^2 f^2}{(a^2+\mu_1)^2}+\frac{\mu_1{}^2 g^2}{(b^2+\mu_1)^2}-r^2\right\}^2,$$

and the absolute term in equation (289) is r'^4; hence we get

$$r^2+r'^2=\left(\frac{\mu_1 f}{a^2+\mu_1}\right)^2+\left(\frac{\mu_1 g}{b^2+\mu_1}\right)^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (291)$$

That is, the sum of the squares of the radii of J and J′ equals square of the distance between their centres. Hence J and J′ cut each other orthogonally.

137. The propositions established in articles 133–136 are those which we shall require for the present investigation. They are proved in the memoir already cited, but by another method. It is useful to recapitulate them here:—

(1) A bicircular quartic is the envelope of a variable circle whose centre moves on a given conic F, called the focal conic, and which cuts a given circle J orthogonally.

(2) The circle J is a circle of inversion of the quartic.

(3) There are four circles of inversion and four focal conics.

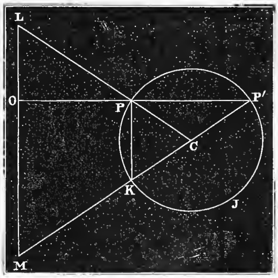
(4) The four focal conics are confocal.

(5) The four circles of inversion are mutually orthogonal.

(6) The centres of the circles of inversion are such that any three will form a self-conjugate triangle with respect to the circle which has the fourth for centre; in other words, the four centres form the angular points and the point of intersection of perpendiculars of a plane triangle.

138. The proposition proved in art. 131 is our fundamental one for the rectification of bicirculars; it will be seen that it is a generalization of Mr. Roberts's theorem already referred to. On account of its importance we will here give an elementary proof of it, but under a slightly different enunciation.

Fig. 22.



If OPP′ be any line cutting a circle J in the points P, P′, then if two circles passing through O touch J in the points P, P′ respectively, the difference between their diameters is equal to the diameter of J.

Demonstration.—Let C be the centre of J. Join CP, CP′, and produce them to meet the line LOM drawn perpendicular to OP. Join PK. Now, evidently, PL=KM;

$$\therefore\ P'M-PL=P'K=\text{diameter of J.}$$

Hence the proposition is proved.

Cor. If the point O be within the circle J, we shall have the sum of the diameters equal the diameter of J.

139. If we denote the diameter of J by 2ϱ, and if a line OQQ′ (see last diagram) infinitely near OP make an angle $d\theta$ with OP, then $\frac{P'Q'}{d\theta}=P'M$ and $\frac{PQ}{d\theta}=PL$.

Hence by art. 138 we have

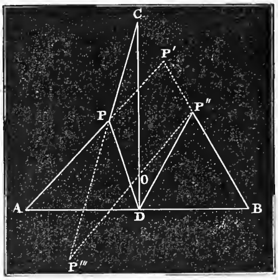
$$P'Q'-PQ=2\varrho d\theta. \qquad (292)$$

Cor. If the point O be inside J we have

$$P'Q'+PQ=2\varrho d\theta. \qquad (293)$$

140. If four circles be mutually orthogonal, and if any figure be inverted with respect to each of the four circles in succession, the fourth inversion will coincide with the original figure *.

Fig. 23.



Demonstration.—It will plainly be sufficient to prove the proposition for a single point, for the general proposition will then follow.

Since the four circles are mutually orthogonal, their four centres will form the angular points and the intersection of the perpendiculars of a plane triangle. Let them be the points A, B, C, O; CO produced will intersect AB perpendicularly in D, and the squares of the radii of the four circles will be equal to the four rectangles

$$AB\,.\,AD, \quad BA\,.\,BD, \quad -CO\,.\,OD, \quad CD\,.\,OD,$$

one of the circles being imaginary, namely the one the square of whose radius is $-CO\,.\,OD$. Now let P be the point we operate on, and let P′ be its inverse with respect to the circle A, and P″ the inverse of P′ with respect to the circle B. Join P″O and CP meeting in P‴. Now since P′ is the inverse of P with respect to the circle A, the square of whose radius is AB . AD, we have the rectangle $AB\,.\,AD=AP\,.\,AP'$. The triangle ADP is similar to the triangle AP′B, therefore the angle ADP=angle AP′B; in like manner, the angle BDP″=angle AP′B, therefore the triangles ADP and BDP″ are equiangular, and the rectangle AD . DB=rectangle PD . DP″. Again, because O is the intersection of the perpendiculars of the triangle ABC, the rectangle AD . DB= CD . OD; hence CD . OD=PD . DP″, and the angles CDP and P″DO being the com-

---

* An important extension of this theorem can be got by combining it with the following proposition, which is proved in art. 95 of my memoir on "Cyclides and Sphero-Quartics":—If a sphero-quartic be projected on one of the planes of circular section of any quadric passing through it by lines parallel to the greatest or least axis of the quadric, the projection will be a bicircular quartic whose centres of inversion will be the projection of the centres of inversion of the sphero-quartic. The extension is as follows. *There exists in sphero-quartics a series of inscribed quadrilaterals* ABCD, *whose sides* AB, BC, CD, DA, *taken in order, pass through the vertices of the four cones of the sphero-quartic.*

plements of equal angles are equal: therefore the triangles P″DO and CDP are equiangular, and the angle OP″D=PCD; hence the four points C, P″, D, P‴ are concyclic, and therefore the point P‴ is the inverse of P″ with respect to the imaginary circle the square of whose radius is −CO . OD, and whose centre is the point O.

Again, the angle ODP=P″DO=OP‴C; therefore the points O, D, P‴, P are concyclic, and P is the inverse of P‴ with respect to the circle whose centre is C, and the square of whose radius is the rectangle CD . OD. Hence the proposition is proved.

Cor. 1. If the point C be at infinity, the point O will coincide with D, and the point P will be the reflection of P‴ with respect to the line AB.

Cor. 2. If the points A, B, C, O be the centres of inversion of a bicircular quartic, and if the point P be on the curve, the points P′, P″, P‴ will also be on the curve.

141. Let the radii of the generating circles of the bicircular quartic which touch it at the four pairs of points (PP′), (P′P″), (P″P‴), (P‴P) be denoted by ϱ, ϱ', ϱ'', ϱ''' respectively. Let the angle which the line APP′ makes with any fixed line in the plane, say the axis of x, be denoted by θ, and the angles which the lines BP′P″, OP″P‴, CPP‴ make with the same line by θ', θ'', θ'''.

Now if the points P, P′, P″, P‴ describe infinitesimal arcs, we have (see art. 139), denoting these arcs by ds, ds', &c.,

$$\begin{aligned} ds' - ds &= 2\varrho\, d\theta, \\ ds' - ds'' &= 2\varrho'\, d\theta', \\ ds'' + ds''' &= 2\varrho''\, d\theta'', \\ ds''' - ds &= 2\varrho'''\, d\theta'''. \end{aligned}$$

Hence

$$ds' = \varrho d\theta + \varrho' d\theta' + \varrho'' d\theta'' + \varrho''' d\theta''' ; \quad \ldots \ldots \quad (294)$$

$$\therefore \quad s' = \int \varrho d\theta + \int \varrho' d\theta' + \int \varrho'' d\theta'' + \int \varrho''' d''' . \quad \ldots \ldots \quad (295)$$

Hence the arc of a bicircular quartic is the sum of four similar integrals. We shall find that each of them is expressed in terms of elliptic integrals. This theorem is our generalization of ROBERTS'S and GENOCCHI'S theorems*.

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* The following proof of the theorem art. 138 will apply equally to sphero-quartics, and will lead to an important extension of the theorem of this article:—

Let CV, C′V′ be two consecutive tangents to the focal conic F of the bicircular quartic (or, in the case of a sphero-quartic, to the focal sphero-conics), and OPP′, OQQ′ two perpendiculars to CV, C′V′ (see fig. art. 130). If CV, C′V′ intersect the generating circle in the points R, R′, it is evident, from geometrical considerations, that

$$RR' = \tfrac{1}{2}(P'Q' - PQ).$$

But $RR' = \varrho d\theta$ for bicircular quartics and $= \sin \varrho d\theta$ for sphero-quartics; hence, remembering the theorem in the footnote to art. 140, we have, for sphero-quartics,

$$s' = \int \sin \varrho d\theta + \int \sin \varrho' d\theta' + \int \sin \varrho'' d\theta'' + \int \sin \varrho''' d\theta''',$$

and the rectification of sphero-quartics is reduced to elliptic integrals.—*November* 1877.

142. If the bicircular becomes a Cartesian oval, the point C will be at infinity, and we shall have (see cor. 1, art. 140),

$$ds'''-ds=0.$$

Hence

$$\varrho'''d\theta'''=0,$$

and

$$ds'=\varrho d\theta+\varrho' d\theta'+\varrho'' d\theta'\,; \quad \dots\dots\dots\dots \quad (296)$$

$$\therefore\ s'=\int\varrho d\theta+\int\varrho' d\theta'+\int\varrho'' d\theta''. \quad \dots\dots\dots\dots \quad (297)$$

This is GENOCCHI's theorem.

Cor. By integrating the equation

$$ds'-ds=2\varrho d\theta$$

we get

$$s'-\ s=2\int\varrho d\theta, \quad \dots\dots\dots\dots \quad (298)$$

which is ROBERTS's theorem.

143. To reduce the integral $\int\varrho d\theta$ to the normal form of elliptic integrals.

Let the focal conic of the bicircular be $F\equiv\frac{x^2}{a^2}+\frac{y^2}{b^2}-1=0$, and the corresponding circle of inversion $J\equiv(x-f)^2+(y-g)^2-k^2=0$. The equation of a tangent to the conic is

$$x\cos\theta+y\sin\theta=\sqrt{a^2\cos^2\theta+b^2\sin^2\theta},$$

or say

$$x\cos\theta+y\sin\theta=p\,;$$

therefore if x' y' be the point of contact, we have

$$\frac{x'}{a^2}=\frac{\cos\theta}{p},$$

$$\frac{y'}{b^2}=\frac{\sin\theta}{p}.$$

Now if ϕ be the eccentric angle, $x'=a\cos\phi$, $y'=b\sin\phi$;

$$\therefore\ \frac{\cos\phi}{a}=\frac{\cos\theta}{p},$$

$$\frac{\sin\phi}{b}=\frac{\sin\theta}{p}\,;$$

$$\therefore\ \tan\theta=\frac{b}{a}\tan\phi.$$

Hence

$$d\theta=\frac{abd\phi}{a^2\sin^2\phi+b^2\cos^2\phi} \quad \dots\dots\dots\dots \quad (299)$$

Again, since ϱ is the radius of the generating circle whose centre is the point $(x'\ y')$ on the focal conic F, and which cuts the circle J orthogonally, we have

$$\varrho=\sqrt{(a\cos\phi-f)^2+(b\sin\phi-g)^2-k^2}\,; \quad . \quad . \quad . \quad . \quad . \quad . \qquad (300)$$

$$\therefore\ \varrho d\theta=\frac{\varrho^2 d\theta}{\varrho}=\frac{ab\{(a\cos\phi-f)^2+(b\sin\phi-g)^2-k^2\}d\phi}{\varrho\{a^2\sin^2\phi+b^2\cos^2\phi\}}$$

$$=-\frac{abd\phi}{\varrho}+\frac{ab\{a^2+b^2+f^2+g^2-k^2-2af\cos\phi-2bg\sin\phi\}d\phi}{\varrho\{a^2\sin^2\phi+b^2\cos^2\phi\}}.$$

Put $\sin\phi=\frac{2z}{1+z^2}$; then $\cos\phi=\frac{1-z^2}{1+z^2}$, and $d\phi=\frac{2dz}{1+z^2}$. Making these substitutions, we get

$$\varrho d\theta=-\frac{2abdz}{\sqrt{\mathrm{Z}}}+\frac{2ab\{(a^2+b^2+f^2+g^2-k^2)(1+z^2)^2-2af(1-z^4)-4bgz(1+z^2)\}dz}{\{4a^2z^2+b^2(1-z^2)\}\sqrt{\mathrm{Z}}}, \quad . \qquad (301)$$

where Z stands for the quartic

$$\{a(1-z^2)-f(1+z^2)\}^2+\{2bz-g(1+z^2)\}^2-k^2(1+z^2)^2\ . \quad . \quad . \quad . \qquad (302)$$

144. In order to reduce still further the expression (301), we must decompose

$$2ab\,\frac{\{(a^2+b^2+f^2+g^2-k^2)(1+z^2)^2-2af(1-z^4)-4bgz(1+z^2)\}}{4a^2z^2+b^2(1-z^2)^2}$$

into simpler fractions, or say the fraction Φ into simpler fractions.

Let us, for shortness, put t^2 for the expression $a^2+b^2+f^2+g^2-k^2$. Making this substitution, and denoting the eccentricity of the conic $\frac{x^2}{a^2}+\frac{y^2}{b^2}-1$ by e, we find, by dividing &c.,

$$\Phi=\frac{2a}{b}\,(t^2+2af)-\frac{2ab\{4b^3gz^3+4a^2\{e^2t^2+af(1+e^2)\}z^2+4b^3gz+b^2t^2+4ab^2f\}}{\{b^2z^2+a^2(1+e)^2\}\{b^2z^2+a^2(1-e)^2\}}.$$

Then decomposing the fraction still remaining, and substituting, we get, after some reduction,

$$\int\varrho d\theta=\frac{2a}{b}\{(a+f)^2+g^2-k^2\}\int\frac{dz}{\sqrt{\mathrm{Z}}}$$

$$-\frac{4b^2g}{a}\int\frac{zdz}{\{(1+e)z^2+(1-e)\}\sqrt{\mathrm{Z}}}-\frac{4b^2g}{a}\int\frac{zdz}{\{(1-e)z^2+(1+e)\}\sqrt{\mathrm{Z}}}$$

$$-\frac{b\{8afe+(1-e)(1+3e)t^2\}}{2ae(1+e)}\int\frac{dz}{\{(1+e)z^2+(1-e)\}\sqrt{\mathrm{Z}}}$$

$$-\frac{b\{8afe-(1+e)(1-3e)t^2\}}{2ae(1-e)}\int\frac{dz}{\{(1-e)z^2+(1-e)\}\sqrt{\mathrm{Z}}} \quad . \quad . \quad . \quad . \qquad (303)$$

Since Z is a quartic function of the variable, each of these integrals belongs to the domain of the elliptic integrals. (See CAYLEY's 'Elliptic Functions.')

145. If the bicircular quartic be a Cartesian oval, the focal conic F is a circle; and taking the line joining the centres of J and F as the axis of x, we may write their equations in the forms

$$F \equiv x^2+y^2-a^2=0, \quad J \equiv (x-f)^2+y^2-k^2=0,$$

and we get

$$\int \rho \, d\theta = \int \sqrt{a^2+f^2-k^2-2af\cos\theta} \, d\theta \, ; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (304)$$

and this represents an arc of an ellipse. Hence ROBERTS's and GENOCCHI's theorems are proved.

XII. *On the Bicircular Quartic.—Addition to Professor* CASEY'S *Memoir "On a new Form of Tangential Equation." By* A. CAYLEY, *LL.D., F.R.S., Sadlerian Professor of Pure Mathematics in the University of Cambridge.*

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PROFESSOR CASEY communicated to me the MS. of the foregoing Memoir, and he has permitted me to make to it the present Addition, containing further developments on the theory of the bicircular quartic.

Starting from his theory of the fourfold generation of the curve, Prof. CASEY shows that there exist series of inscribed quadrilaterals ABCD whereof the sides AB, BC, CD, DA pass through the centres of the four circles of inversion respectively; or (as it is convenient to express it) the pairs of points (A, B), (B, C), (C, D), (D, A) belong to the four modes of generation respectively, and may be regarded as depending upon certain parameters (his θ, θ', θ'', θ''', or say) ω_1, ω_2, ω_3, ω respectively, any three of these being in fact functions of the fourth. Considering a given quadrilateral ABCD, and giving to it an infinitesimal variation, we have four infinitesimal arcs AA′, BB′, CC′, DD′; these are differential expressions, AA′ and BB′ of the form $M_1 d\omega_1$, BB′ and CC′ of the form $M_2 d\omega_2$, CC′ and DD′ of the form $M_3 d\omega_3$, DD′ and AA′ of the form $M d\omega$; or, what is the same thing, AA′ is expressible in the two forms $M d\omega$ and $M_1 d\omega_1$, BB′ in the two forms $M_1 d\omega$ and $M_2 d\omega_2$, &c., the identity of the two expressions for the same arc of course depending on the relation between the two parameters. But any such monomial expression $M d\omega$ of an arc AA′ would be of a complicated form, not obviously reducible to elliptic functions; CASEY does not obtain these monomial expressions at all, but he finds geometrically monomial expressions for the differences and sum BB′−AA′, CC′−BB′, DD′+CC′, DD′−AA′ (they cannot be all of them differences), and thence a quadrinomial expression $AA'=N_1 d\omega_1+N_2 d\omega_2+N_3 d\omega_3+N d\omega$ (his $ds'=\varrho d\theta+\varrho' d\theta'+\varrho'' d\theta''+\varrho''' d\theta'''$); and that without any explicit consideration of the relations which connect the parameters.

I propose to complete the analytical theory by establishing the monomial equations $AA'=M d\omega=M_1 d\omega_1$, &c., and the relations between the parameters ω, ω_1, ω_2, ω_3 which belong to an inscribed quadrilateral ABCD, so as to show what the process really is by which we pass from the monomial form to a quadrinomial form

$$AA' \text{ (or } dS)=N d\omega+N_1 d\omega_1+N_2 d\omega_2+N_3 d\omega_3,$$

wherein each term is separately expressible as the differential of an elliptic integral; and to further develop the theory of the transformation to elliptic integrals: we require to establish for these purposes the fundamental formulæ in the theory of the bicircular quartic.

I remark that in the various formulæ f, g, θ, θ_1, θ_2, θ_3 are constants which enter only in the combinations $f+\theta$, $f-g$, $\theta_1-\theta$, $\theta_2-\theta$, $\theta_3-\theta$, that X, Y are taken as current coordinates, and these letters, or the same letters with suffixes, are taken as coordinates of a point or points on the bicircular quartic; the letters (x, y), (x_1, y_1), (x_2, y_2), (x_3, y_3) are used throughout as variable parameters, viz. we have

$$\begin{aligned}
(f+\theta\ )\,x^2+(g+\theta\ )\,y^2&=1,\\
(f+\theta_1)\,x_1^2+(g+\theta\ )\,y_1^2&=1,\\
(f+\theta_2)\,x_2^2+(g+\theta_2)\,y_2^2&=1,\\
(f+\theta_3)\,x_3^2+(g+\theta_3)\,y_3^2&=1\,;
\end{aligned}$$

so that $x, y=\frac{\cos\omega}{\sqrt{f+\theta}}, \frac{\sin\omega}{\sqrt{g+\theta}}$, are functions of a single parameter ω, and similarly (x_1, y_1), (x_2, y_2), (x_3, y_3) are functions of the parameters ω_1, ω_2, ω_3 respectively; we sometimes use these or similar expressions of (x, y) &c. as trigonometrical functions of a single parameter, but more frequently retain the pair of quantities, considered as connected by an equation as above, and so as equivalent to a single variable parameter.

Formulæ for the fourfold generation of the Bicircular Quartic.—Art. Nos. 1 to 5.

1. We have four systems of a dirigent conic and circle of inversion, each giving rise to the same bicircular quartic: viz. the bicircular quartic is the envelope of a generating circle, having its centre on a dirigent conic, and cutting at right angles the corresponding circle of inversion; or, what is the same thing, it is the locus of the extremities of a chord of the generating circle, which chord passes through the centre of the circle of inversion, and cuts at right angles the tangent (at the centre of the generating circle) to the dirigent conic; the two extremities of the chord are thus inverse points in regard to the circle of inversion. The four systems are represented by letters without suffixes, or with the suffixes 1, 2, 3 respectively, and we say that the system, or mode of generation, is 0, 1, 2, or 3 accordingly.

2. The dirigent conics are confocal, and their squared semiaxes may therefore be represented by $f+\theta$, $g+\theta$, $f+\theta_1$, $g+\theta_1$, $f+\theta_2$, $g+\theta_2$, $f+\theta_3$, $g+\theta_3$ (which are in fact functions of the five quantities $f+\theta$, $f-g$, $\theta_1-\theta$, $\theta_2-\theta$, $\theta_3-\theta$); and we can in terms of these data express the equations as well of the dirigent conics as of the circles of inversion; viz. taking X, Y as current coordinates, the equations are

$$\begin{aligned}
&\frac{X^2}{f+\theta}+\frac{Y^2}{g+\theta}=1,\ (X-\alpha\ )^2+(Y-\beta\ )^2-\gamma^2=0,\ \text{or}\ X^2+Y^2-2\alpha X-2\beta Y+k=0,\\
&\frac{X^2}{f+\theta_1}+\frac{Y^2}{g+\theta_1}=1,\ (X-\alpha_1)^2+(Y-\beta_1)^2-\gamma_1^2=0,\ \text{or}\ X^2+Y^2-2\alpha_1X-2\beta_1Y+k_1=0,\\
&\frac{X^2}{f+\theta_2}+\frac{Y^2}{g+\theta_2}=1,\ (X-\alpha_2)^2+(Y-\beta_2)^2-\gamma_2^2=0,\ \text{or}\ X^2+Y^2-2\alpha_2X-2\beta_2Y+k_2=0,\\
&\frac{X^2}{f+\theta_3}+\frac{Y^2}{g+\theta_3}=1,\ (X-\alpha_3)^2+(Y-\beta_3)^2-\gamma_3^2=0,\ \text{or}\ X^2+Y^2-2\alpha_3X-2\beta_3Y+k_3=0,
\end{aligned}$$

where

$$\sqrt{\frac{f+\theta\,.\,f+\theta_1\,.\,f+\theta_2\,.\,f+\theta_3}{f-g}}=(f+\theta)\alpha=(f+\theta_1)\alpha_1=(f+\theta_2)\alpha_2=(f+\theta_3)\alpha_3,$$

$$\sqrt{\frac{g+\theta\,.\,g+\theta_1\,.\,g+\theta_2\,.\,g+\theta_3}{g-f}}=(g+\theta)\beta=(g+\theta_1)\beta_1=(g+\theta_2)\beta_2=(g+\theta_3)\beta_3.$$

$$\begin{aligned}
f+\theta\ .\ g+\theta\ .\ \gamma^2 &= \theta-\theta_1\,.\,\theta-\theta_2\,.\,\theta-\theta_3,\\
f+\theta_1\,.\,g+\theta_1\,.\,\gamma_1^2 &= \theta_1-\theta\ .\ \theta_1-\theta_2\,.\,\theta_1-\theta_3,\\
f+\theta_2\,.\,g+\theta_2\,.\,\gamma_2^2 &= \theta_2-\theta\ .\ \theta_2-\theta_1\,.\,\theta_2-\theta_3,\\
f+\theta_3\,.\,g+\theta_3\,.\,\gamma_3^2 &= \theta_3-\theta\ .\ \theta_3-\theta_1\,.\,\theta_3-\theta_2.
\end{aligned}$$

$$f+g+\theta+\theta_1+\theta_2+\theta_3=k+2\theta=k_1+2\theta_1=k_2+2\theta_2=k_3+2\theta_3.$$

3. The geometrical relations between the dirigent conics and circles of inversion are all deducible from the foregoing formulæ; in particular the conics are confocal, and as such intersect each two of them at right angles; the circles intersect each two of them at right angles. Considering a dirigent conic and the corresponding circle of inversion, the centres of the remaining three circles are conjugate points in regard as well to the first-mentioned conic, as to the first-mentioned circle; or, what is the same thing, they are the centres of the quadrangle formed by the intersections of the conic and circle.

4. The centre of the conics and the centres of the four circles lie on a rectangular hyperbola, having its asymptotes parallel to the axes of the conics. Given the centres of three of the circles (this determines the centre of the fourth circle) and also the centre of the conic, these four points determine a rectangular hyperbola (which passes also through the centre of the fourth circle); and the axes of the conics are then the lines through the centre, parallel to the asymptotes of the hyperbola.

5. The equation of the bicircular quartic may be expressed in the four forms

$$\begin{aligned}
&(\mathrm{X}^2+\mathrm{Y}^2-k\ )^2-4[(f+\theta\ )(\mathrm{X}-\alpha\ )^2+(g+\theta\ )(\mathrm{Y}-\beta\ )^2]=0,\\
&(\mathrm{X}^2+\mathrm{Y}^2-k_1)^2-4[(f+\theta_1)(\mathrm{X}-\alpha_1)^2+(g+\theta_1)(\mathrm{Y}-\beta_1)^2]=0,\\
&(\mathrm{X}^2+\mathrm{Y}^2-k_2)^2-4[(f+\theta^2)(\mathrm{X}-\alpha_2)^2+(g+\theta_2)(\mathrm{Y}-\beta_2)^2]=0,\\
&(\mathrm{X}^2+\mathrm{Y}^2-k_3)^2-4[(f+\theta_3)(\mathrm{X}-\alpha_3)^2+(g+\theta_3)(\mathrm{Y}-\beta_3)^2]=0,
\end{aligned}$$

the equivalence of which is easily verified by means of the foregoing relations.

Determination as to Reality.—Art. Nos. 6 and 7.

6. To fix the ideas suppose that $f-g$ is positive; then in order that the centres of the four circles of inversion may be real we must have $f+\theta\,.\,f+\theta_1\,.\,f+\theta_2\,.\,f+\theta_3$ positive, but $g+\theta\,.\,g+\theta_1\,.\,g+\theta_2\,.\,g+\theta_3$ negative; and this will be the case if $f+\theta$, $f+\theta_1$, $f+\theta_2$, $f+\theta_3$ are all positive, but $g+\theta$, $g+\theta_1$, $g+\theta_2$, $g+\theta_3$ one of them negative, and the other three

positive. In reference to a figure which I constructed I found it convenient to take θ_3, θ_1, θ_0, θ_2 to be in order of increasing magnitude: this being so we have $f+\theta_3$ positive, $g+\theta_3$ negative; and the other like quantities $f+\theta_1$, $f+\theta_0$, $f+\theta_2$, $g+\theta_1$, $g+\theta_0$, $g+\theta_2$ all positive: we then have γ_3^2 and γ_1^2 each positive, γ_0^2 negative, γ_2^2 positive: viz. the conics and circles are

Hyperbola H_3 corresponding to real circle C_3,
Ellipse E_1 ,, real circle C_1,
,, E_0 ,, imaginary circle C_0,
(viz. the radius is a pure imaginary)
,, E_2 ,, real circle C_2,

and where the confocal ellipses E_1, E_0, E_2 are in order of increasing magnitude. The centre C_0 is here a point within the triangle formed by the remaining three centres C_1, C_2, C_3. It will be convenient to adopt throughout the foregoing determination as to reality.

7. It may be remarked that a circle of a pure imaginary radius γ, $=i\lambda$, where λ is real, may be indicated by means of the concentric circle radius λ, which is the concentric orthotomic circle; and that a circle which cuts at right angles the original circle cuts diametrally (that is, at the extremities of a diameter) the substituted circle radius λ: we have thus a real construction in relation to a circle of inversion of pure imaginary radius.

Investigation of dS.—Art. Nos. 8 to 17.

8. The coördinates of a point on the dirigent conic $\frac{X^2}{f+\theta}+\frac{Y^2}{g+\theta}=1$ may be taken to be $(f+\theta)x$, $(g+\theta)y$: and we hence prove as follows the fundamental theorem for the generation of the bicircular quartic. Consider the generating circle, centre $(f+\theta)x$, $(g+\theta)y$, which cuts at right angles the circle of inversion $(X-\alpha)^2+(Y-\beta)^2=\gamma^2$. If for a moment the radius is called δ, then the equation of the generating circle is

$$(X-\overline{f+\theta}x)^2+(Y-\overline{g+\theta}y)^2=\delta^2\,;$$

the condition for the intersection at right angles is

$$(\alpha-\overline{f+\theta}x)^2+(\beta-\overline{g+\theta}y)^2=\gamma^2+\delta^2,$$

and hence eliminating δ^2, the equation of the generating circle is

$$X^2+Y^2-k-2(X-\alpha)(f+\theta)x-2(Y-\beta)(g+\theta)y=0\,;$$

and considering herein x, y as variable parameters connected by the foregoing equation $(f+\theta)x^2+(g+\theta)y^2=1$, we have as the envelope of this circle the required bicircular quartic.

9. It is convenient to write $R=\frac{1}{2}(X^2+Y^2-k)$; the equation then is

$$R-(X-\alpha)(f+\theta)x-(Y-\beta)(g+\theta)y=0\,;$$

the derived equation is

$$(X-\alpha)(f+\theta)dx+(Y-\beta)(g+\theta)dy=0\,;$$

and from these two equations, together with the equation in (x, y) and its derivative, we find $X-\alpha=Rx$, $Y-\beta=Ry$; from these last equations, and the equations $R=\frac{1}{2}(X^2+Y^2-k)$, $(f+\theta)x^2+(g+\theta)y^2=1$, eliminating x, y, R, we have

$$(f+\theta)(X-\alpha)^2+(g+\theta)(Y-\beta)^2=R^2,$$

that is

$$(X^2+Y^2-k)^2-4[(f+\theta)(X-\alpha)^2+(g+\theta)(Y-\beta)^2]=0,$$

the required equation of the bicircular quartic.

10. We have thus $X-\alpha=Rx$, $Y-\beta=Ry$, as the equations which serve to determine the bicircular quartic: if from these equations, together with $R=\frac{1}{2}(X^2+Y^2-k)$, we eliminate X, Y, we have R expressed as a function of x, y; and thence also X, Y expressed in terms of x, y; that is in effect the coordinates X, Y of a point of the bicircular quartic expressed as functions of a single variable parameter. The process gives $2R+k=(\alpha+Rx)^2+(\beta+Ry)^2$, viz. this is

$$R^2(x^2+y^2)-2(1-\alpha x-\beta y)R+\gamma^2=0,$$

or putting for shortness

$$\Omega=(1-\alpha x-\beta y)^2-\gamma^2(x^2+y^2),$$

this is

$$R=\frac{1-\alpha x-\beta y+\sqrt{\Omega}}{x^2+y^2},$$

or say the two values are

$$R=\frac{1-\alpha x-\beta y+\sqrt{\Omega}}{x^2+y^2},\quad R'=\frac{1-\alpha x-\beta y-\sqrt{\Omega}}{x^2+y^2}\,;$$

to preserve the generality it is proper to consider $\sqrt{\Omega}$ as denoting a determinate value (the positive or the negative one, as the case may be) of the radical.

11. Considering the root R′ we have $X=\alpha+R'x$, $Y=\beta+R'y$, and from these equations we obtain

$$dX=R'dx+xdR',$$
$$dY=R'dy+ydR'\,;$$

but from the equation for R′ we have

$$[R'(x^2+y^2)-(1-\alpha x-\beta y)]dR'+R'^2(xdx+ydy)+R'(\alpha dx+\beta dy)=0,$$

that is

$$-\sqrt{\Omega}\,dR'+R'(Xdx+Ydy)=0,$$

whence

$$dX=R'dx+\frac{R'x}{\sqrt{\Omega}}(Xdx+Ydy),$$

$$dY=R'dy+\frac{R'y}{\sqrt{\Omega}}(Xdx+Ydy).$$

12. The differentials dx, dy can be expressed in terms of a single differential $d\omega$, viz. writing

$$x=\frac{\cos\omega}{\sqrt{f+\theta}},\quad y=\frac{\sin\omega}{\sqrt{g+\theta}},\ \text{and}$$

$$\Theta=(f+\theta)(g+\theta),$$

then we have

$$dx=-\frac{g+\theta}{\sqrt{\Theta}}\,y\,d\omega,\quad dy=\frac{f+\theta}{\sqrt{\Theta}}\,x\,d\omega.$$

It is to be observed that when the dirigent conic is an ellipse, ω is a real angle, and Θ is positive (whence also $\sqrt{\Theta}$ is real and positive), but when the dirigent conic is a hyperbola, ω is imaginary, and Θ is negative; we have, however, in either case

$$dx^2+dy^2=\frac{(f+\theta)^2x^2+(g+\theta)^2y^2}{\Theta}\,d\omega^2,$$

and we may therefore write

$$\frac{d\omega}{\sqrt{\Theta}}=\frac{ds}{\sqrt{(f+\theta)^2x^2+(g+\theta)^2y^2}},$$

where $\sqrt{(f+\theta)^2x^2+(g+\theta)^2y^2}$ is positive; ds is the increment of arc on the conic $(f+\theta)x^2+(g+\theta)y^2=1$, this arc being measured in a determinate sense, and therefore ds being positive or negative as the case may be: $\frac{d\omega}{\sqrt{\Theta}}$ has thus a real positive or negative value, even when ω is imaginary, and it is convenient to retain it in the formulæ.

13. It may further be noticed that if υ denote the inclination to the axis of x of the tangent to the dirigent conic at the point $\sqrt{f+\theta}\cos\omega$, $\sqrt{g+\theta}\sin\omega$ (υ is CASEY's θ), then

$$x=\frac{\cos\upsilon}{\sqrt{\mathrm{U}}},\quad y=\frac{\sin\upsilon}{\sqrt{\mathrm{U}}},\ \text{where } \mathrm{U}=(f+\theta)\cos^2\upsilon+(g+\theta)\sin^2\upsilon,$$

viz. we have

$$\frac{\cos\omega}{\sqrt{f+\theta}}=\frac{\cos\upsilon}{\mathrm{U}},\quad \frac{\sin\omega}{\sqrt{g+\theta}}=\frac{\sin\upsilon}{\mathrm{U}},$$

giving, as is easily verified, $\frac{d\omega}{\sqrt{\Theta}}=\frac{d\upsilon}{\mathrm{U}}$; we have therefore

$$\frac{d\omega}{(x^2+y^2)\sqrt{\Theta}}=\frac{d\upsilon}{\upsilon(x^2+y^2)},\ =d\upsilon,$$

or

$$\frac{d\omega}{\sqrt{\Theta}}=(x^2+y^2)d\upsilon,$$

which is another interpretation of $\frac{d\omega}{\sqrt{\Theta}}$.

14. Substituting for dx, dy their values, the formulæ become

$$d\mathrm{X}=\frac{\mathrm{R}'}{\sqrt{\Theta}}\left\{-(g+\theta)y+\frac{x}{\sqrt{\Omega}}(-(g+\theta)y\mathrm{X}+(f+\theta)x\mathrm{Y})\right\}d\omega,$$

$$d\mathrm{Y}=\frac{\mathrm{R}'}{\sqrt{\Theta}}\left\{\ (f+\theta)x+\frac{y}{\sqrt{\Omega}}(-(g+\theta)y\mathrm{X}+(f+\theta)x\mathrm{Y})\right\}d\omega.$$

We have

$$xX+yY=\alpha x+\beta y+(x^2+y^2)R'$$
$$=1-\sqrt{\Omega}\,;$$

that is

$$1=\frac{1-xX-yY}{\sqrt{\Omega}}\,;$$

and consequently the foregoing expressions of dX, dY become

$$dX=\frac{R'd\omega}{\sqrt{\Theta}\,\sqrt{\Omega}}\{(g+\theta)y(xX+yY-1)+x(-(g+\theta)yX+(f+\theta)xY)\}$$

$$=\frac{R'd\omega}{\sqrt{\Theta}\,\sqrt{\Omega}}\{(\overline{g+\theta}\,y^2+\overline{f+\theta}\,x^2)Y-(g+\theta)y\},$$

$$dY=\frac{R'd\omega}{\sqrt{\Theta}\,\sqrt{\Omega}}\{(f+\theta)x(1-xX-yY)+y(-(g+\theta)yX+(f+\theta)xY)\}$$

$$=\frac{R'd\omega}{\sqrt{\Theta}\,\sqrt{\Omega}}\{(f+\theta)x-((f+\theta)x^2+(g+\theta)y^2)X\},$$

or finally

$$dX=\frac{R'd\omega}{\sqrt{\Theta}\,\sqrt{\Omega}}\{Y-(g+\theta)y\},\quad =\frac{R'd\omega}{\sqrt{\Theta}\,\sqrt{\Omega}}\{R'y+\beta-(g+\theta)y\},$$

$$dY=\frac{-R'd\omega}{\sqrt{\Theta}\,\sqrt{\Omega}}\{X-(f+\theta)x\},\quad =\frac{-R'd\omega}{\sqrt{\Theta}\,\sqrt{\Omega}}\{R'x+\alpha-(f+\theta)x\}.$$

15. We have

$$(R'x+\alpha-\overline{f+\theta}\,x)^2+(R'y+\beta-\overline{g+\theta}\,y)^2$$
$$=R'^2(x^2+y^2)-2R'(1-\alpha x-\beta y)$$
$$+(\alpha-\overline{f+\theta}\,x)^2+(\beta-\overline{g+\theta}\,y)^2\,;$$

viz. this is

$$=(\alpha-\overline{f+\theta}\,x)^2+(\beta-\overline{g+\theta}\,y)^2-\gamma^2,$$

$=\delta^2$, the radius of the generating circle.

Hence if dS, $=\sqrt{dX^2+dY^2}$, be the element of arc of the bicircular quartic, this element being taken to be positive, we have

$$dS=\frac{\varepsilon'\,R'\,\delta\,d\omega}{\sqrt{\Omega}\,\sqrt{\Theta}},$$

where ε' denotes a determinate sign, + or −, as the case may be.

16. I stop to consider the geometrical interpretation; introducing dv, the formula may be written

$$dS=\frac{\varepsilon' . R'(x^2+y^2)\delta\, dv}{\sqrt{\Omega}},$$

we have $(x^2+y^2)R'=1-\alpha x-\beta y-\sqrt{\Omega}$, or

$$\frac{(x^2+y^2)R'}{\sqrt{\Omega}}=\frac{1-\alpha x-\beta y}{\sqrt{\Omega}}-1\,;$$

$\frac{1-\alpha x-\beta y}{\sqrt{x^2+y^2}}$ is the perpendicular from the centre of the circle of inversion upon the tangent to the dirigent conic, and $\frac{\sqrt{\Omega}}{\sqrt{x^2+y^2}}$ is the half-chord which this perpendicular forms with the generating circle. Hence $\frac{1-\alpha x-\beta y}{\sqrt{\Omega}}-1=$ (perpendicular — half-chord) ÷ half-chord, the numerator being in fact the distance of the element dS (or point X, Y) from the centre of inversion: the formula thus is

$$dS=\pm\frac{\varrho\,.\,\delta}{\frac{1}{2}c}\,dv,$$

where δ is the radius of the generating circle, ϱ the distance of the element from the centre of the circle of inversion, and c the chord which this distance forms with the generating circle. If we consider the two points on the generating circle, and write dS' for the element at the other point, then we have $(dS\pm dS')=\pm\frac{(\varrho-\varrho')\delta\, dv}{\frac{1}{2}c}$, $=2\delta\, dv$ (which is CASEY'S formula $ds'-ds=2\varrho\, d\phi$ (273)).

17. The foregoing forms of dX, dY are those which give most directly the required value of dS, but I had previously obtained them in a different form. Writing

$$\Delta=\beta\alpha-\alpha y+(f-g)xy,$$

then

$$x\Delta=\beta x^2-\alpha xy+[(f+\theta)x^2-(g+\theta)y^2]\,;$$

or since

$$(f+\theta)x^2=1-(g+\theta)y^2,$$

this is

$$x\Delta=\beta x^2-\alpha xy+[1-(g+\theta)(x^2+y^2)],\ =y(1-\alpha x-\beta y)+(x^2+y^2)(\beta-(g+\theta)y),$$
$$=(x^2+y^2)\{yR'+\beta-(g+\theta)y\}+y\sqrt{\Omega}\,;$$

that is

$$x\Delta-y\sqrt{\Omega}=(x^2+y^2)\{yR'+\beta-(g+\theta)y\},$$

and similarly

$$-y\Delta-x\sqrt{\Omega}=(x^2+y^2)\{xR'+\alpha-(f+\theta)x\}.$$

We have therefore

$$dX=\frac{R'd\omega}{(x^2+y^2)\sqrt{\Theta}\sqrt{\Omega}}(x\Delta-y\sqrt{\Omega}),$$

$$dY=\frac{R'd\omega}{(x^2+y^2)\sqrt{\Theta}\sqrt{\Omega}}(y\Delta+x\sqrt{\Omega}),$$

and thence a value of dS which, compared with the former value, gives

$$\Omega+\Delta^2=(x^2+y^2)\delta^2,$$

an equation which may be verified directly.

Formulæ for the Inscribed Quadrilateral.—Art. Nos. 18 to 22.

18. We consider on the curve four points, A, B, C, D, forming a quadrilateral, ABCD. The coordinates are taken to be (X, Y), (X_1, Y_1), (X_2, Y_2), (X_3, Y_3) respectively. It is assumed that (A, B), (B, C), (C, D), (D, A) belong to the generations 1, 2, 3, 0, and depend on the parameters (x_1, y_1), (x_2, y_2), (x_3, y_3), (x, y) respectively.

We write

$$\begin{aligned}\Omega\ &=(1-\alpha\,x\ -\beta\,y\ )^2-\gamma^2(x^2+y^2),\\ \Omega_1&=(1-\alpha_1x_1-\beta_1y_1)^2-\gamma_1^2(x_1^2+y_1^2),\\ \Omega_2&=(1-\alpha_2x_2-\beta_2y_2)^2-\gamma_2^2(x_2^2+y_2^2),\\ \Omega_3&=(1-\alpha_3x_3-\beta_3y_3)^2-\gamma_3^2(x_3^2+y_3^2),\end{aligned}$$

and then, $\sqrt{\Omega}$ denoting as above a determinate value, positive or negative as the case may be, of the radical, and similarly $\sqrt{\Omega_1}$, $\sqrt{\Omega_2}$, $\sqrt{\Omega_3}$ denoting determinate values of these radicals respectively, each radical having its own sign at pleasure, we further write

$$\begin{aligned}(x^2+y^2)R'\ &=1-\alpha\,x\ -\beta\,y\ -\sqrt{\Omega}\ , & (x_1^2+y_1^2)R_1&=1-\alpha_1x_1-\beta_1y_1+\sqrt{\Omega_1},\\ (x_1^2+y_1^2)R_1'&=1-\alpha_1x_1-\beta_1y_1-\sqrt{\Omega_1}, & (x_2^2+y_2^2)R_2&=1-\alpha_2x_2-\beta_2y_2+\sqrt{\Omega_2},\\ (x_2^2+y_2^2)R_2'&=1-\alpha_2x_2-\beta_2y_2-\sqrt{\Omega_2}, & (x_3^2+y_3^2)R_3&=1-\alpha_3x_3-\beta_3y_3+\sqrt{\Omega_3},\\ (x_3^2+y_3^2)R_3'&=1-\alpha_3x_3-\beta_3y_3-\sqrt{\Omega_3}, & (x^2+y^2)R\ &=1-\alpha\,x\ -\beta\,y\ +\sqrt{\Omega}\,;\end{aligned}$$

and this being so, we must have

$$\begin{aligned}X\ &=\alpha\ +R'x\ =\alpha_1+R_1x_1, & Y\ &=\beta\ +R'y\ =\beta_1+R_1y_1, & R'\ &=\tfrac{1}{2}(X^2+Y^2-k\ ), & R_1&=\tfrac{1}{2}(X^2+Y^2-k_1),\\ X_1&=\alpha_1+R_1'x_1=\alpha_2+R_2x_2, & Y_1&=\beta_1+R_1'y_1=\beta_2+R_2y_2, & R_1'&=\tfrac{1}{2}(X_1^2+Y_1^2-k_1), & R_2&=\tfrac{1}{2}(X_1^2+Y_1^2-k_2),\\ X_2&=\alpha_2+R_2'x_2=\alpha_3+R_3x_3, & Y_2&=\beta_2+R_2'y_2=\beta_3+R_3y_3, & R_2'&=\tfrac{1}{2}(X_2^2+Y_2^2-k_2), & R_3&=\tfrac{1}{2}(X_2^2+Y_2^2-k_3),\\ X_3&=\alpha_3+R_3'x_3=\alpha\ +R\,x\ , & Y_3&=\beta_3+R_3'y_3=\beta\ +R\,y\ , & R_3'&=\tfrac{1}{2}(X_3^2+Y_3^2-k_3), & R\ &=\tfrac{1}{2}(X_3^2+Y_3^2-k\ );\end{aligned}$$

and then from the values of X, Y, R′, R, we have

$$\begin{aligned}\alpha-\alpha_1+\mathrm{R}'x-\mathrm{R}_1x_1&=0,\\ \beta-\beta_1+\mathrm{R}'y-\mathrm{R}_1y_1&=0,\\ (\theta-\theta_1)+\mathrm{R}'-\mathrm{R}_1&=0,\end{aligned}$$

giving

$$(\beta-\beta_1)(x-x_1)-(\alpha-\alpha_1)(y-y_1)+(\theta-\theta_1)(xy_1-x_1y)=0\,;$$

and similarly

$$\begin{aligned}(\beta_1-\beta_2)(x_1-x_2)-(\alpha_1-\alpha_2)(y_1-y_2)+(\theta_1-\theta_2)(x_1y_2-x_2y_1)&=0,\\ (\beta_2-\beta_3)(x_2-x_3)-(\alpha_2-\alpha_3)(y_2-y_3)+(\theta_2-\theta_3)(x_2y_3-x_3y_2)&=0,\\ (\beta_3-\beta)(x_3-x)-(\alpha_3-\alpha)(y_3-y)+(\theta_3-\theta)(x_3y-xy_3)&=0,\end{aligned}$$

which are the relations connecting the parameters (x, y), (x_1, y_1), (x_2, y_2), (x_3, y_3) of the quadrilateral.

19. We have thus apparently four equations for the determination of four quantities, or the number of quadrilaterals would be finite; but if from the first and second equations we eliminate (x_1, y_1), or from the third and fourth equations we eliminate (x_3, y_3), we find in each case the same relation between (x, y), (x_2, y_2), viz. this is found to be

$$\Omega\Omega_2=(1-\alpha x_2-\beta y_2)^2(1-\alpha_2x-\beta_2y)^2\,;$$

and we have thus the singly infinite series of quadrilaterals. We have, of course, between (x_1, y_1), (x_3, y_3) the like relation,

$$\Omega_1\Omega_3=(1-\alpha_1x_3-\beta_1y_3)^2(1-\alpha_3x_1-\beta_3y_1)^2.$$

20. The relation between (x, y), (x_1, y_1) may be expressed also in the two forms:

$$1-\alpha(x+x_1)-\beta(y+y_1)+(f+\theta_1)xx_1+(g+\theta_1)yy_1+\frac{x^2+y^2}{xy_1-x_1y}(\overline{\alpha-\alpha_1}\,y_1-\overline{\beta-\beta_1}\,x_1)=0,$$

$$1-\alpha_1(x+x_1)-\beta_1(y+y_1)+(f+\theta)xx_1+(g+\theta)yy_1+\frac{x_1^2+y_1^2}{x_1y-xy_1}(\overline{\alpha_1-\alpha}\,y-\overline{\beta_1-\beta}\,x)=0.$$

In fact, the first of these equations is

$$\begin{aligned}\{1+(f+\theta_1)xx_1+(g+\theta_1)yy_1\}(xy_1-x_1y)\ &-\{\alpha(x+x_1)+\beta(y+y_1)\}(xy_1-x_1y)\\ &+\{(\alpha-\alpha_1)y_1-(\beta-\beta_1)x_1\}(x^2+y^2)=0,\end{aligned}$$

which, by virtue of the original form of relation, is

$$\begin{aligned}-\{1+(f+\theta_1)xx_1+(g+\theta_1)yy_1\}&\frac{(\beta-\beta_1)(x-x_1)-(\alpha-\alpha_1)(y-y_1)}{\theta-\theta_1}\\ -\{\alpha(x+x_1)+\beta(y+y_1)\}(xy_1-x_1y)&+\{(\alpha-\alpha_1)y_1-(\beta-\beta_1)x_1\}(x^2+y^2)=0\,;\end{aligned}$$

or, in the first term, writing

$$-\frac{\beta-\beta_1}{\theta-\theta_1}=\frac{\beta}{g+\theta_1},\ \frac{\alpha-\alpha_1}{\theta-\theta_1}=\frac{-\alpha}{f+\theta_1},$$

and in the third term

$$\alpha-\alpha_1=-\frac{(\theta-\theta_1)\alpha}{f+\theta_1},\quad -(\beta-\beta_1)=\frac{(\theta-\theta_1)\beta}{g+\theta_1},$$

this is

$$(1+(f+\theta_1)xx_1+(g+\theta_1)yy_1)\left(\frac{\beta(x-x_1)}{g+\theta_1}-\frac{\alpha(y-y_1)}{f+}\right)$$

$$-\{\alpha(x+x_1)+\beta(y+y_1)\}(xy_1-x_1y)-\left(\frac{\alpha(\theta-\theta_1)}{f+\theta_1}y_1-\frac{\beta(\theta-\theta_1)}{g+\theta_1}x_1\right)(x^2+y^2)=0\,;$$

and in this equation the coefficients of α and of β are separately $=0$: in fact the coefficient of β is

$$\frac{x-x_1}{g+\theta_1}+\frac{f+\theta_1}{g+\theta_1}xx_1(x-x_1)+(x-x_1)yy_1-(y+y_1)(xy_1-x_1y)+\frac{\theta-\theta_1}{g+\theta_1}x_1(x^2+y^2)$$

$$=\frac{x}{g+\theta_1}\{1-(f+\theta_1)x_1^2-(g+\theta_1)y_1^2\}-\frac{x_1}{g+\theta_1}\{1-(f+\theta)x^2-(g+\theta)y^2\}=0\,;$$

and similarly the coefficient of α is $=0$.

And in like manner the second equation may be verified.

21. The two equations are:

$$1-\alpha x-\beta y-(x^2+y^2)\mathrm{R}'=\alpha x_1+\beta y_1-(f+\theta_1)xx_1-(g+\theta_1)yy_1,$$
$$1-\alpha_1x_1-\beta_1y_1-(x_1^2+y_1^2)\mathrm{R}_1=\alpha_1x+\beta_1y-(f+\theta\,)xx_1-(g+\theta\,)yy_1\,;$$

or, substituting for R′, R$_1$ their values, these are

$$\sqrt{\Omega}=\alpha\,x_1+\beta\,y_1-(f+\theta_1)x\,x_1-(g+\theta_1)y\,y_1,\quad \sqrt{\Omega_1}=-\alpha_1x-\beta_1y+(f+\theta\,)x\,x_1+(g+\theta\,)y\,y_1,$$

and similarly

$$\sqrt{\Omega_1}=\alpha_1x_2+\beta_1y_2-(f+\theta_2)x_1x_2-(g+\theta_2)y_1y_2,\quad \sqrt{\Omega_2}=-\alpha_2x_1-\beta_2y_1+(f+\theta_1)x_1x_2+(g+\theta_1)y_1y_2,$$
$$\sqrt{\Omega_2}=\alpha_2x_3+\beta_2y_3-(f+\theta_3)x_2x_3-(g+\theta_3)y_2y_3,\quad \sqrt{\Omega_3}=-\alpha_3x_2-\beta_3y_2+(f+\theta_2)x_2x_3+(g+\theta_2)y_2y_3,$$
$$\sqrt{\Omega_3}=\alpha_3x+\beta_3y-(f+\theta\,)x_3x-(g+\theta\,)y_3y\,,\quad \sqrt{\Omega}=-\alpha\,x_3-\beta_2y_3+(f+\theta_3)x_3x+(g+\theta_3)y_3y\,.$$

Differentiating the equation

$$(\beta-\beta_1)(x-x_1)-(\alpha-\alpha_1)(y-y_1)+(\theta-\theta_1)(xy_1-x_1y)=0,$$

we have

$$[(\beta-\beta_1)+(\theta-\theta_1)y_1]dx-[(\alpha-\alpha_1)+(\theta-\theta_1)x_1]dy$$
$$-[(\beta-\beta_1)+(\theta-\theta_1)y\,]dx_1+[(\alpha-\alpha_1)+(\theta-\theta_1)x\,]dy_1=0\,;$$

and writing herein

$$dx=-\frac{(g+\theta)}{\sqrt{\Theta}}y\,d\omega,\quad dx_1=\frac{-(g+\theta_1)}{\sqrt{\Theta_1}}y_1d\omega_1,$$

$$dy=\frac{f+\theta}{\sqrt{\Theta}}x\,d\omega,\quad dy_1=\frac{(f+\theta_1)}{\sqrt{\Theta_1}}x_1d\omega_1,$$

we find

$$-\frac{d\omega}{\sqrt{\Theta}}\{(g+\theta)(\beta-\beta_1)y+(f+\theta)(\alpha-\alpha_1)x+(\theta-\theta_1)((f+\theta)xx_1+(g+\theta)yy_1)\}$$

$$+\frac{d\omega_1}{\sqrt{\Theta_1}}\{(g+\theta_1)(\beta-\beta_1)y_1+(f+\theta_1)(\alpha-\alpha_1)x_1+(\theta-\theta_1)((f+\theta_1)xx_1+(g+\theta_1)yy_1)\}=0\,;$$

viz., dividing by $\theta-\theta_1$, this becomes

$$-\sqrt{\Omega_1}\frac{d\omega}{\sqrt{\Theta}}-\sqrt{\Omega}\frac{d\omega_1}{\sqrt{\Theta_1}}=0,\text{ that is }\frac{d\omega}{\sqrt{\Theta}\sqrt{\Omega}}+\frac{d\omega_1}{\sqrt{\Theta_1}\sqrt{\Omega_1}}=0\,;$$

or, completing the system, we have

$$\frac{d\omega}{\sqrt{\Theta}\sqrt{\Omega}}=\frac{-d\omega_1}{\sqrt{\Theta_1}\sqrt{\Omega_1}}=\frac{d\omega_2}{\sqrt{\Theta_2}\sqrt{\Omega_2}}=\frac{-d\omega_3}{\sqrt{\Theta_3}\sqrt{\Omega_3}},$$

which are the differential relations between the parameters $\omega, \omega_1, \omega_2, \omega_3$, or (x, y), (x_1, y_1), (x_2, y_2), (x_3, y_3).

22. From the equations $X=\alpha+R'x$, $Y=\beta+R'y$, we found

$$dX=\frac{R'd\omega}{\sqrt{\Omega}\sqrt{\Theta}}\{Y-(g+\theta)y\},$$

$$dY=\frac{R'd\omega}{\sqrt{\Omega}\sqrt{\Theta}}\{X-(f+\theta)x\}\,;$$

the new values, $X=\alpha_1+R_1x_1$, $Y=\beta_1+R_1y_1$ give in like manner

$$dX=-\frac{R_1d\omega_1}{\sqrt{\Omega_1}\sqrt{\Theta_1}}\{Y-(g+\theta_1)y_1\},$$

$$dY=-\frac{R_1d\omega_1}{\sqrt{\Omega_1}\sqrt{\Theta_1}}\{X-(f+\theta_1)x_1\}\,;$$

and in virtue of the relation just found between $d\omega$ and $d\omega_1$ these two sets of values will agree together if only

$$R'\{Y-(g+\theta)y\}=R_1\{Y-(g+\theta_1)y_1\},$$
$$R'\{X-(f+\theta)x\}=R_1\{X-(f+\theta_1)x_1\}.$$

These are easily verified: the first is

$$R'Y-(g+\theta)(Y-\beta)=(R'-\theta+\theta_1)Y-(g+\theta_1(Y-\beta_1),$$

viz. this is $(g+\theta)\beta-(g+\theta_1)\beta_1=0$, which is right; and similarly the second equation gives $(f+\theta)\alpha-(f+\theta_1)\alpha_1=0$, which is right.

From the first values of dX, dY we have, as above,

$$dS=\frac{\epsilon' R'\delta\, d\omega}{\sqrt{\Omega}\sqrt{\Theta}},$$

and the second values give in like manner

$$dS=\frac{\varepsilon_1 R_1 \delta_1\, d\omega_1}{\sqrt{\Omega_1}\,\sqrt{\Theta_1}},$$

where ε_1 is $=\pm 1$. It will be observed that we have in effect, by means of the relation $(\beta-\beta_1)(x-x_1)-(\alpha-\alpha_1)(y-y_1)+(\theta-\theta_1)(xy_1-x_1y)=0$, proved the identity of the two values of dS.

Considering the quadrilateral ABCD, and giving it an infinitesimal variation, so as to change it into A'B'C'D', then dS is the element of arc AA'; and writing in like manner dS_1, dS_2, dS_3 for the elements of arc BB', CC', DD', we have, of course, a like pair of values for each of the elements dS_1, dS_2, dS_3.

Formulæ for the elements of Arc dS, dS_1, dS_2, dS_3.—Art. Nos. 23 to 27.

23. The formulæ are

$$\begin{aligned}
dS &=\varepsilon' R'\delta\frac{d\omega}{\sqrt{\Omega}\,\sqrt{\Theta}} &&=\varepsilon_1 R_1\delta_1\frac{d\omega_1}{\sqrt{\Omega_1}\,\sqrt{\Theta_1}},\\
dS_1 &=\varepsilon_1' R_1'\delta_1\frac{d\omega_1}{\sqrt{\Omega_1}\,\sqrt{\Theta_1}} &&=\varepsilon_2 R_2\delta_2\frac{d\omega_2}{\sqrt{\Omega_2}\,\sqrt{\Theta_2}},\\
dS_2 &=\varepsilon_2' R_2'\delta_2\frac{d\omega_2}{\sqrt{\Omega_2}\,\sqrt{\Theta_2}} &&=\varepsilon_3 R_3\delta_3\frac{d\omega_3}{\sqrt{\Omega_3}\,\sqrt{\Theta_3}},\\
dS_3 &=\varepsilon_3' R_3'\delta_3\frac{d\omega_3}{\sqrt{\Omega_3}\,\sqrt{\Theta_3}} &&=\varepsilon\, R\,\delta\,\frac{d\omega}{\sqrt{\Omega}\,\sqrt{\Theta}},
\end{aligned}$$

where the ε's each denote ± 1. Supposing as above that γ^2 is negative, but that γ_1^2, γ_2^2, γ_3^2 are positive; then R', R have opposite signs: but R_1', R_1 have the same sign, as have also R_2' and R_2, and R_3' and R_3. We may take δ, δ_1, δ_2, and δ_3 as each of them positive: the signs of $\frac{d\omega}{\sqrt{\Omega}\,\sqrt{\Theta}}$, $\frac{d\omega_1}{\sqrt{\Omega_1}\,\sqrt{\Theta_1}}$, $\frac{d\omega_2}{\sqrt{\Omega_2}\,\sqrt{\Theta_2}}$, $\frac{d\omega_3}{\sqrt{\Omega_3}\,\sqrt{\Theta_3}}$ are $+$, $-$, $+$, $-$, or $-$, $+$, $-$, $+$: hence to make dS, dS_1, dS_2, dS_3 all positive,

$$\begin{array}{l}
\varepsilon'\ ,\quad \varepsilon_1'\ ,\ \varepsilon_2'\ ,\quad \varepsilon_3'\ ,\quad \varepsilon_1\ ,\ \varepsilon_2\ ,\quad \varepsilon_3\ ,\ \varepsilon,\\
\text{must have signs of}\\
R',\ -R_1',\ R_2',\ -R_3',\ -R_1,\ R_2,\ -R_3,\ R\,;
\end{array}$$

or else the reverse signs: hence in either case $\varepsilon'=-\varepsilon$, $\varepsilon_1'=\varepsilon_1$, $\varepsilon_2'=\varepsilon_2$, $\varepsilon_3'=\varepsilon_3$; or the equations are

$$\begin{aligned}
dS &=-\varepsilon R'\delta\frac{d\omega}{\sqrt{\Omega}\,\sqrt{\Theta}} &&=\varepsilon_1 R_1\delta_1\frac{d\omega_1}{\sqrt{\Omega_1}\,\sqrt{\Theta_1}},\\
dS_1 &=\quad \varepsilon_1 R_1'\delta_1\frac{d\omega_1}{\sqrt{\Omega_1}\,\sqrt{\Theta_1}} &&=\varepsilon_2 R_2\delta_2\frac{d\omega_2}{\sqrt{\Omega_2}\,\sqrt{\Theta_2}},\\
dS_2 &=\quad \varepsilon_3 R_2'\delta_2\frac{d\omega_2}{\sqrt{\Omega_2}\,\sqrt{\Theta_2}} &&=\varepsilon_3 R_3\delta_3\frac{d\omega_3}{\sqrt{\Omega_3}\,\sqrt{\Theta_3}},\\
dS_3 &=\quad \varepsilon_3 R_3'\delta_3\frac{d\omega_3}{\sqrt{\Omega_3}\,\sqrt{\Theta_3}} &&=\varepsilon\, R\,\delta\,\frac{d\omega}{\sqrt{\Omega}\,\sqrt{\Theta}}.
\end{aligned}$$

24. But we have $R'-R=\dfrac{-2\sqrt{\Omega}}{x^2+y^2}$ &c.; and hence, putting for shortness

$$\frac{\delta}{(x^2+y^2)\sqrt{\Theta}},\ \frac{\delta_1}{(x_1^{\,2}+y_1^{\,2})\sqrt{\Theta_1}},\ \frac{\delta_2}{(x_2^{\,2}+y_2^{\,2})\sqrt{\Theta_2}},\ \frac{\delta_3}{(x_3^{\,2}+y_3^{\,2})\sqrt{\Theta_3}}=\mathrm{P},\ \mathrm{P}_1,\ \mathrm{P}_2,\ \mathrm{P}_3,$$

$$\begin{aligned}
d\mathrm{S}+d\mathrm{S}_3&=+2\varepsilon\,\mathrm{P}\,d\omega,\\
d\mathrm{S}_1-d\mathrm{S}&=-2\varepsilon_1\mathrm{P}_1d\omega_1,\\
d\mathrm{S}_2-d\mathrm{S}_1&=-2\varepsilon_2\mathrm{P}_2d\omega_2,\\
d\mathrm{S}_3-d\mathrm{S}&=-2\varepsilon_3\mathrm{P}_3d\omega_3,
\end{aligned}$$

and consequently

$$\begin{aligned}
d\mathrm{S}&=\varepsilon\mathrm{P}d\omega+\varepsilon_1\mathrm{P}_1d\omega_1+\varepsilon_2\mathrm{P}_2d\omega_2+\varepsilon_3\mathrm{P}_3d\omega_3,\\
d\mathrm{S}_1&=\varepsilon\mathrm{P}d\omega-\varepsilon_1\mathrm{P}_1d\omega_1+\varepsilon_2\mathrm{P}_2d\omega_2+\varepsilon_3\mathrm{P}_3d\omega_3,\\
d\mathrm{S}_2&=\varepsilon\mathrm{P}d\omega-\varepsilon_1\mathrm{P}_1d\omega_1-\varepsilon_2\mathrm{P}_2d\omega_2+\varepsilon_3\mathrm{P}_3d\omega_3,\\
d\mathrm{S}_3&=\varepsilon\mathrm{P}d\omega-\varepsilon_1\mathrm{P}_1d\omega_1-\varepsilon_2\mathrm{P}_2d\omega_2-\varepsilon_3\mathrm{P}_3d\omega_3,
\end{aligned}$$

which are the required formulæ for the elements of arc.

25. The determination of the signs has been made by means of the particular figure; but it is easy to see that the pairs of terms could not for instance be $d\mathrm{S}-d\mathrm{S}_3$, $d\mathrm{S}_1-d\mathrm{S}$, $d\mathrm{S}_2-d\mathrm{S}_1$, $d\mathrm{S}_3-d\mathrm{S}$, or any other pairs such that it would be possible to eliminate $d\mathrm{S}$, $d\mathrm{S}_1$, $d\mathrm{S}_2$, $d\mathrm{S}_3$, and thus obtain an equation such as

$$\varepsilon\mathrm{P}d\omega+\varepsilon_1\mathrm{P}_1d\omega_1+\varepsilon_2\mathrm{P}_2d\omega_2+\varepsilon_3\mathrm{P}_3d\omega_3=0\,;$$

this would, by virtue of the relations between $d\omega$, $d\omega_1$, $d\omega_2$, $d\omega_3$, become

$$\varepsilon\frac{\delta\sqrt{\Omega}}{x^2+y^2}-\varepsilon_1\frac{\delta_1\sqrt{\Omega_1}}{x_1^{\,2}+y_1^{\,2}}+\varepsilon_2\frac{\delta_2\sqrt{\Omega_2}}{x_2^{\,2}+y_2^{\,2}}-\varepsilon_3\frac{\delta_3\sqrt{\Omega_3}}{x_3^{\,2}+y_3^{\,2}}=0,$$

an equation not deducible from the relations which connect ω, ω_1, ω_2, ω_3, and which therefore cannot be satisfied by the variable quadrilateral.

26. The differentials of the formulæ are, it will be observed, of the form $\mathrm{P}d\omega$

$$=\frac{\delta\,d\omega}{(x^2+y^2)\sqrt{\Theta}},$$

where $\sqrt{\Theta}$, $=\sqrt{f+\theta\,.\,g+\theta}$ is a mere constant, $x,\ y=\dfrac{\cos\omega}{\sqrt{f+\theta}},\ \dfrac{\sin\omega}{\sqrt{g+\theta}}$, and

$$\delta^2=\big((f+\theta)x-\alpha\big)^2+\big((g+\theta)y-\beta\big)-\gamma^2,$$

viz. the form is

$$\frac{\sqrt{(\cos\omega\,\sqrt{f+\theta}-\alpha)^2+(\sin\omega\,\sqrt{g+\theta}-\beta)^2-\gamma^2}}{\sqrt{\Theta}\,.\left(\dfrac{\cos^2\omega}{f+\theta}+\dfrac{\sin^2\omega}{g+\theta}\right)}\,d\omega,$$

which is, in fact, the same as CASEY's form in ϕ (equation (300), his ϕ being $=90^\circ-\omega$).

Writing as before υ in place of his θ, the differential expression becomes simply $=\delta\,d\upsilon$:

but δ^2 expressed as a function of υ is an irrational function $\mathrm{M}+\mathrm{N}\sqrt{\mathrm{U}}$, and δ would be the root of such a function; so that if the form originally obtained had been this form $\delta\, d\upsilon$, it would have been necessary to transform it into the first-mentioned form $\frac{\delta\, d\omega}{(x^2+y^2)\sqrt{\Theta}}$, in which δ is expressed as a function of (x, y), that is of ω.

27. The system of course is

$$\begin{aligned}
d\mathrm{S}\ &=\varepsilon\delta d\upsilon+\varepsilon_1\delta_1 d\upsilon_1+\varepsilon_2\delta_2 d\upsilon_2+\varepsilon_3\delta_3 d\upsilon_3,\\
d\mathrm{S}_1&=\varepsilon\delta d\upsilon-\varepsilon_1\delta_1 d\upsilon_1+\varepsilon_2\delta_2 d\upsilon_2+\varepsilon_3\delta_3 d\upsilon_3,\\
d\mathrm{S}_2&=\varepsilon\delta d\upsilon-\varepsilon_1\delta_1 d\upsilon_1-\varepsilon_2\delta_2 d\upsilon_2+\varepsilon_3\delta_3 d\upsilon_3,\\
d\mathrm{S}_3&=\varepsilon\delta d\upsilon-\varepsilon_1\delta_1 d\upsilon_1-\varepsilon_2\delta_2 d\upsilon_2-\varepsilon_3\delta_3 d\upsilon_3,
\end{aligned}$$

where $d\upsilon=\frac{d\omega}{(x^2+y^2)\sqrt{\Theta}}$ &c.; and this is the most convenient way of writing it.

Reference to Figure.—Art. No. 28.

28. I constructed a bicircular quartic consisting of an exterior and interior oval with the following numerical data: $(f+\theta_3=48,\ f+\theta_1=56,\ f+\theta_0=60,\ f+\theta_2=80;\ g+\theta_3=-6,\ g+\theta_1=2, g+\theta_0=6, g+\theta_2=26)$,—not very convenient ones, inasmuch as the exterior oval came out too large. The annexed figure shows 0, 1, 2, 3, the centres of the circles of inversion, the interior oval, and a portion of the exterior oval, also the origin and axes; it will be seen that the centres 0, 2 lie inside the interior oval, the centres 1, 3 outside the exterior oval: I add further the values

$$\begin{array}{llll}
\sqrt{f+\theta_3}=6{\cdot}93, & \sqrt{-(g+\theta_3)}=2{\cdot}45, & \alpha_3=10{\cdot}18, & \beta_3=-\ {\cdot}98,\\
\sqrt{f+\theta_1}=7{\cdot}48, & \sqrt{g+\theta_1}\quad\ =1{\cdot}41, & \alpha_1=\ 8{\cdot}73, & \beta_1=+2{\cdot}94,\\
\sqrt{f+\theta_0}=7{\cdot}75, & \sqrt{g+\theta_0}\quad\ =2{\cdot}45, & \alpha_0=\ 8{\cdot}15, & \beta_0=+\ {\cdot}98,\\
\sqrt{f+\theta_2}=8{\cdot}94, & \sqrt{g+\theta_2}\quad\ =5{\cdot}09, & \alpha_2=\ 6{\cdot}10, & \beta_2=+\ {\cdot}23.
\end{array}$$

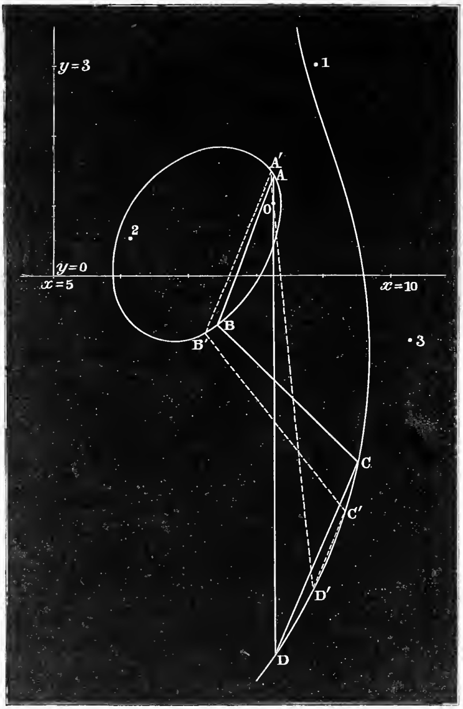
We thus see how there exists a series of quadrilaterals ABCD, where A, B are situate on the interior oval, C, D on the exterior oval. Considering the sides as drawn in the senses A to B, B to C, C to D, D to A, and representing the inclinations measured from the positive infinity on the axis of x in the sense x to y, by υ_1, υ_2, υ_3, υ respectively, then in passing to the consecutive quadrilateral A′B′C′D′, we have υ_1 and υ_2 decreasing, υ_3 and υ increasing, that is, $d\upsilon_1$ and $d\upsilon_2$ negative, $d\upsilon_3$ and $d\upsilon$ positive; then reckoning the elements AA′, BB′, CC′, DD′, that is $d\mathrm{S}_1$, $d\mathrm{S}_2$, $d\mathrm{S}_3$, $d\mathrm{S}$, as each of them positive, we have

$$\begin{aligned}
d\mathrm{S}_2-d\mathrm{S}_1&=-2\delta_1 d\upsilon_1,\\
d\mathrm{S}_3-d\mathrm{S}_2&=-2\delta_2 d\upsilon_2,\\
d\mathrm{S}\ -d\mathrm{S}_3&=+2\delta_3 d\upsilon_3,\\
d\mathrm{S}_1+d\mathrm{S}\ &=+2\delta\, d\upsilon,
\end{aligned}$$

and thence

$$\begin{aligned}
d\mathrm{S} &= \delta dv - \delta_1 dv_1 - \delta_2 dv_2 + \delta_3 dv_3,\\
d\mathrm{S}_1 &= \delta dv + \delta_1 dv_1 + \delta_2 dv_2 - \delta_3 dv_3,\\
d\mathrm{S}_2 &= \delta dv - \delta_1 dv_1 + \delta_2 dv_2 - \delta_3 dv_3,\\
d\mathrm{S}_3 &= \delta dv - \delta_1 dv_4 - \delta_2 dv_2 - \delta_3 dv_3,
\end{aligned}$$

which are the correct signs in regard to the particular figure.



Reduction of $\int \frac{\delta\, d\omega}{(x^2+y^2)\sqrt{\Theta}}$ *to Elliptic Integrals.*—Art. No. 29.

29. The expression in question is

$$\int d\omega \,.\, \frac{\sqrt{(\cos\omega\sqrt{f+\theta}-\alpha)^2+(\sin\omega\sqrt{g+\theta}-\beta)^2-\gamma^2}}{\left\{\frac{\cos^2\omega}{f+\theta}+\frac{\sin^2\omega}{g+\theta}\right\}\sqrt{\Theta}},$$

where $\sqrt{\Theta}$ is a mere constant; and we may apply it to the Gaussian transformation,

$$\cos\omega=\frac{a+a'\cos\mathrm{T}+a''\sin\mathrm{T}}{c+c'\cos\mathrm{T}+c''\sin\mathrm{T}},$$

$$\sin\omega=\frac{b+b'\cos\mathrm{T}+b''\sin\mathrm{T}}{c+c'\cos\mathrm{T}+c''\sin\mathrm{T}},$$

where the coefficients $a, b, c, a', b', c', a'', b'', c''$ are such that identically

$$\cos^2\omega+\sin^2\omega-1=\frac{1}{(c+c'\cos\mathrm{T}+c''\sin\mathrm{T})^2}\{\cos^2\mathrm{T}+\sin^2\mathrm{T}-1\}$$

$$(\cos\omega\sqrt{f+\theta}-\alpha)^2+(\sin\omega\sqrt{g+\theta}-\beta)^2-\gamma^2,\text{ that is}$$

$$\cos^2\omega(f+\theta)+\sin^2\omega(g+\theta)-2\alpha\sqrt{f+\theta}\cos\omega-2\beta\sqrt{g+\theta}\sin\omega+k$$

$$=\frac{1}{(c+c'\cos\mathrm{T}+c''\sin\mathrm{T})}(\mathrm{G}_1-\mathrm{G}_2\cos^2\mathrm{T}-\mathrm{G}_3\sin^2\mathrm{T}).$$

30. It is found that G_1, G_2, G_3 are the roots of a cubic equation

$$(\mathrm{G}+\theta-\theta_1)(\mathrm{G}+\theta-\theta_2)(\mathrm{G}+\theta-\theta_3),$$

which being so, we may assume $\mathrm{G}_1=\theta_1-\theta$, $\mathrm{G}_2=\theta_2-\theta$, $\mathrm{G}_3=\theta_3-\theta$, or the second condition in fact is

$$(f+\theta)\cos^2\omega+(g+\theta)\sin^2\omega-2\alpha\sqrt{f+\theta}\cos\omega-2\beta\sqrt{g+\theta}\sin\omega+k$$

$$=\frac{1}{(c+c'\cos\mathrm{T}+c''\sin\mathrm{T})^2}\{\theta_1-\theta-(\theta_2-\theta)\cos^2\mathrm{T}-(\theta_3-\theta)\sin^2\mathrm{T}\};$$

and this being so, we find without difficulty the values

$$\begin{array}{lll}
a^2=\dfrac{g+\theta_1\cdot f+\theta_2\cdot f+\theta_3}{f-g\cdot\theta_1-\theta_2\cdot\theta_1-\theta_3}, & b^2=\dfrac{f+\theta_1\cdot g+\theta_2\cdot g+\theta_3}{g-f\cdot\theta_1-\theta_2\cdot\theta_1-\theta_3}, & c^2=\dfrac{f+\theta_1\cdot g+\theta_1}{\theta_1-\theta_2\cdot\theta_1-\theta_3}, \\[2ex]
a'^2=-\dfrac{g+\theta_2\cdot f+\theta_1\cdot f+\theta_3}{f-g\cdot\theta_2-\theta_1\cdot\theta_2-\theta_3}, & b'^2=-\dfrac{f+\theta_2\cdot g+\theta_1\cdot g+\theta_3}{g-f\cdot\theta_2-\theta_1\cdot\theta_2-\theta_3}, & c'^2=-\dfrac{f+\theta_2\cdot g+\theta_2}{\theta_2-\theta_1\cdot\theta_2-\theta_3}, \\[2ex]
a''^2=-\dfrac{g+\theta_3\cdot f+\theta_1\cdot f+\theta_2}{f-g\cdot\theta_3-\theta_1\cdot\theta_3-\theta_2}, & b''^2=-\dfrac{f+\theta_3\cdot g+\theta_1\cdot g+\theta_2}{g-f\cdot\theta_3-\theta_1\cdot\theta_3-\theta_2}, & c''^2=-\dfrac{f+\theta_3\cdot g+\theta_3}{\theta_3-\theta_1\cdot\theta_3-\theta_2},
\end{array}$$

(to make these positive the order of ascending magnitude must, however, be not as heretofore θ_3, θ_1, θ_2, but θ_3, θ_2, θ_1, viz. we must have $f+\theta_1$, $f+\theta_2$, $f+\theta_3$, $g+\theta_1$, $g+\theta_2$, $-(g+\theta_3)$, $\theta_1-\theta_3$, $\theta_1-\theta_2$, $\theta_2-\theta_3$ all positive).

31. The above are the values of the squares of the coefficients; we must have definite relations between the signs of the products aa', bb', ab, &c., viz. we may have

$$a'a''=\frac{f+\theta_1}{f-g\,.\,\theta_2-\theta_3}\sqrt{\frac{\Theta_2\Theta_3}{\theta_3-\theta_1\,.\,\theta_1-\theta_2}},\quad a''a=\frac{f+\theta_2}{f-g\,.\,\theta_3-\theta_1}\sqrt{\frac{-\Theta_3\Theta_1}{\theta_1-\theta_2\,.\,\theta_2-\theta_3}},\quad aa'=\frac{f+\theta_2}{f-g\,.\,\theta_1-\theta_2}\sqrt{\frac{-\Theta_1\Theta_2}{\theta_2-\theta_3\,.\,\theta_3-\theta_1}},$$

$$b'b''=\frac{g+\theta_1}{g-f\,.\,\theta_2-\theta_3}\sqrt{\quad ,,\quad},\quad b''b=\frac{g+\theta_2}{g-f\,.\,\theta_3-\theta_1}\sqrt{\quad ,,\quad},\quad bb'=\frac{g+\theta_2}{g-f\,.\,\theta_1-\theta_2}\sqrt{\quad ,,\quad},$$

$$c'c''=\frac{1}{\theta_2-\theta_3}\sqrt{\quad ,,\quad},\quad c''c=\frac{1}{\theta_3-\theta_1}\sqrt{\quad ,,\quad},\quad cc'=\frac{1}{\theta_1-\theta_2}\sqrt{\quad ,,\quad},$$

and further

$$ab=\frac{1}{f-g\,.\,\theta_3-\theta_1\,.\,\theta_1-\theta_2}\sqrt{-\Theta_1\Theta_2\Theta_3},\quad bc=-\frac{f+\theta_1}{\theta_3-\theta_1\,.\,\theta_1-\theta_2}\sqrt{\frac{g+\theta_1\,.\,g+\theta_2\,.\,g+\theta_3}{g-f}},\quad ca=-\frac{g+\theta_1}{\theta_3-\theta_1\,.\,\theta_1-\theta_2}\sqrt{\frac{f+\theta_1\,.\,f+\theta_2\,.\,f+\theta_3}{f-g}},$$

$$a'b'=\frac{-1}{f-g\,.\,\theta_1-\theta_2\,.\,\theta_2-\theta_3}\sqrt{\quad ,,\quad},\quad b'c'=\frac{f+\theta_2}{\theta_1-\theta_2\,.\,\theta_2-\theta_3}\sqrt{\quad ,,\quad},\quad c'a'=\frac{g+\theta_2}{\theta_3-\theta_1\,.\,\theta_1-\theta_2}\sqrt{\quad ,,\quad},$$

$$a''b''=\frac{-1}{f-g\,.\,\theta_2-\theta_3\,.\,\theta_3-\theta_1}\sqrt{\quad ,,\quad},\quad b''c''=\frac{f+\theta_3}{\theta_3-\theta_3\,.\,\theta_3-\theta_1}\sqrt{\quad ,,\quad},\quad c'a''=\frac{g+\theta_3}{\theta_2-\theta_3\,.\,\theta_3-\theta_1}\sqrt{\quad ,,\quad},$$

and also

$$b'c''+b''c'=\frac{2g+\theta_2+\theta_3}{\theta_2-\theta_3}\sqrt{\frac{g+\theta_1\,.\,f+\theta_2\,.\,f+\theta_3}{g-f\,.\,\theta_3-\theta_1\,.\,\theta_1-\theta_2}},\quad c'a''+c''a'=\frac{2f+\theta_2+\theta_3}{\theta_2-\theta_3}\sqrt{\frac{f+\theta_1\,.\,g+\theta_2\,.\,g+\theta_3}{f-g\,.\,\theta_3-\theta_1\,.\,\theta_1-\theta_2}},$$

$$b''c+bc''=\frac{2g+\theta_3+\theta_1}{\theta_3-\theta_1}\sqrt{-\frac{g+\theta_2\,.\,f+\theta_3\,.\,f+\theta_1}{g-f\,.\,\theta_1-\theta_2\,.\,\theta_2-\theta_3}},\quad c''a+ca''=\frac{2f+\theta_3+\theta_1}{\theta_3-\theta_1}\sqrt{-\frac{f+\theta_2\,.\,g+\theta_3\,.\,g+\theta_1}{f-g\,.\,\theta_1-\theta_2\,.\,\theta_2-\theta_3}},$$

$$bc'+b'c=\frac{2g+\theta_1+\theta_2}{\theta_1-\theta_2}\sqrt{-\frac{g+\theta_3\,.\,f+\theta_1\,.\,f+\theta_2}{g-f\,.\,\theta_2-\theta_3\,.\,\theta_3-\theta_1}},\quad ca'+c'a=\frac{2f+\theta_1+\theta_2}{\theta_1-\theta_2}\sqrt{-\frac{f+\theta_3\,.\,g+\theta_1\,.\,g+\theta_2}{f-g\,.\,\theta_2-\theta_3\,.\,\theta_3-\theta_1}}.$$

32. These values, in fact, satisfy the several relations which exist between the nine coefficients, viz. the original expressions of $\cos\omega$, $\sin\omega$, in terms of $\cos\mathrm{T}$, $\sin\mathrm{T}$ give conversely expressions of $\cos\mathrm{T}$, $\sin\mathrm{T}$ in terms of $\cos\omega$, $\sin\omega$, the two sets being

$$\cos\omega=\frac{a+a'\cos\mathrm{T}+a''\sin\mathrm{T}}{c+c'\cos\mathrm{T}+c''\sin\mathrm{T}},\qquad \cos\mathrm{T}=-\frac{a'\cos\omega+b'\sin\omega-c'}{a\cos\omega+b\sin\omega-c},$$

$$\sin\omega=\frac{b+b'\cos\mathrm{T}+b''\sin\mathrm{T}}{c+c'\cos\mathrm{T}+c''\sin\mathrm{T}},\qquad \sin\mathrm{T}=-\frac{a''\cos\omega+b''\sin\omega-c''}{a\cos\omega+b\sin\omega-c},$$

and we have then the relations

$$\cos^2\omega+\sin^2\omega-1=\frac{1}{(c+c'\cos\mathrm{T}+c''\sin\mathrm{T})^2}(\cos^2\mathrm{T}+\sin^2\mathrm{T}-1),$$

$$\cos^2\mathrm{T}+\sin^2\mathrm{T}-1=\frac{1}{(a\cos\omega+b\sin\omega-c)^2}(\cos^2\omega+\sin^2\omega-1),$$

$$(\theta+f)\cos^2\omega+(\theta+g)\sin^2\omega-2\alpha\sqrt{\theta+f}\cos\omega-2\beta\sqrt{\theta+g}\sin\omega+k$$

$$=\frac{1}{(c+c'\cos\mathrm{T}+c''\sin\mathrm{T})^2}\{(\theta_1-\theta)-(\theta_2-\theta)\cos^2\mathrm{T}-(\theta_3-\theta)\sin^2\mathrm{T}\},$$

$$(\theta_1-\theta)-(\theta_2-\theta)\cos^2\mathrm{T}-(\theta_3-\theta)\sin^2\mathrm{T}$$

$$=\frac{1}{(a\cos\omega+b\sin\omega-c)^2}\{(\theta+f)\cos^2\omega+(\theta+g)\sin^2\omega-2\alpha\sqrt{\theta+f}\cos\omega-2\beta\sqrt{\theta+g}\sin\omega+k\},$$

giving the four sets each of six equations

$$\begin{array}{ll}
a^2+b^2-c^2=-1, & a'a''+b'b''-c'c''=0,\\
a'^2+b'^2-c'^2=+1, & a''a+b''b-c''c=0,\\
a''^2+b''^2-c''^2=+1, & aa'+bb'-cc'=0,
\end{array}$$

$$\begin{array}{ll}
-a^2+a'^2+a''^2=+1, & -bc+b'c'+b''c''=0,\\
-b^2+b'^2+b''^2=+1, & -ca+c'a'+c''a''=0,\\
-c^2+c'^2+c''^2=-1, & -ab+a'b'+a''b''=0,
\end{array}$$

$$\begin{array}{llllll}
(\theta+f)a^2 & +(\theta+g)b^2 & -2\alpha\sqrt{\theta+f}\;ac & -2\beta\sqrt{\theta+g}\;bc & +kc^2 & =\theta_1-\theta,\\
(\theta+f)a'^2 & +(\theta+g)b'^2 & -2\alpha\sqrt{\theta+f}\;a'c' & -2\beta\sqrt{\theta+g}\;b'c' & +kc'^2 & =-\theta_2+\theta,\\
(\theta+f)a''^2 & +(\theta+g)b''^2 & -2\alpha\sqrt{\theta+f}\;a''c'' & -2\beta\sqrt{\theta+g}\;b''c'' & +kc''^2 & =-\theta_3+\theta,
\end{array}$$

$$(\theta+f)a'a''+(\theta+g)b'b''-\alpha\sqrt{\theta+f}(a'c''+a''c')-\beta\sqrt{\theta+g}(b'c''+b''c')+kc'c''=0,$$

$$(\theta+f)a''a+(\theta+g)b''b-\alpha\sqrt{\theta+f}(a''c+ac'')-\beta\sqrt{\theta+g}(b''c+bc'')+kc''c=0,$$

$$(\theta+f)aa'+(\theta+g)bb'-\alpha\sqrt{\theta+f}(ac'+a'c)-\beta\sqrt{\theta+g}(bc'+b'c)+kcc'=0,$$

$$\begin{array}{ll}
(\theta_1-\theta)a^2-(\theta_2-\theta)a'^2-(\theta_3-\theta)a''^2=\theta+f, & \text{or say } (\theta_1+f)a^2-(\theta_2+f)a'^2-(\theta_3+f)a''^2=0,\\
(\theta_1-\theta)b^2-(\theta_2-\theta)b'^2-(\theta_3-\theta)b''^2=\theta+g, & \quad ,, \quad (\theta_1+g)b^2-(\theta_2+g)b'^2-(\theta_3+g)b''^2=0,\\
(\theta_1-\theta)c^2-(\theta_2-\theta)c'^2-(\theta_3-\theta)c''^2=k, & \quad ,, \quad \theta_1c^2-\theta_2c'^2-\theta_3c''^2=k+\theta,
\end{array}$$

$$-(\theta_1-\theta)bc+(\theta_2-\theta)b'c'+(\theta_3-\theta)b''c''=-\beta\sqrt{\theta+g},$$

$$-(\theta_1-\theta)ca+(\theta_2-\theta)c'a'+(\theta_3-\theta)c''a''=-\alpha\sqrt{\theta+f},$$

$$(\theta_1-\theta)ab-(\theta_2-\theta)a'b'-(\theta_3-\theta)a''b''=0;$$

all which formulæ are in fact satisfied by the foregoing values of the expressions a^2, b^2, a'^2, &c.

33. We then have

$$d\omega = \frac{d\mathrm{T}}{c+c'\cos\mathrm{T}+c''\sin\mathrm{T}};$$

and the radical which multiplies $d\omega$ being

$$=\frac{1}{c+c'\cos\mathrm{T}+c''\sin\mathrm{T}}\sqrt{\theta_1-\theta_2\cos^2\mathrm{T}-\theta_3\sin^2\mathrm{T}},$$

the differential becomes

$$=\frac{d\mathrm{T}\sqrt{\theta_1-\theta_2\cos^2\mathrm{T}-\theta_3\sin^2\mathrm{T}}}{\left(\frac{\cos^2\omega}{f+\theta}+\frac{\sin^2\omega}{g+\theta}\right)(c+c'\cos\mathrm{T}+c''\sin\mathrm{T})^2\sqrt{\Theta}},$$

that is

$$=\frac{d\mathrm{T}\sqrt{\theta_1-\theta_2\cos^2\mathrm{T}-\theta_3\sin^2\mathrm{T}}}{\left\{\frac{1}{f+\theta}(a+a'\cos\mathrm{T}+a''\sin\mathrm{T})^2+\frac{1}{g+\theta}(b+b'\cos\mathrm{T}+b''\sin\mathrm{T})^2\right\}\sqrt{\Theta}}.$$

The denominator could, of course, be reduced to the form $(*\between 1, \cos\mathrm{T}, \sin\mathrm{T})^2$; but the actual form seems preferable, inasmuch as it puts in evidence the linear factors

$$\frac{1}{\sqrt{f+\theta}}(a+a'\cos\mathrm{T}+a''\sin\mathrm{T})\pm\frac{i}{\sqrt{g+\theta}}(b+b'\cos\mathrm{T}+b''\sin\mathrm{T}),$$

and there seems to be no advantage in further reducing the integral.

XIII. *Contributions to Terrestrial Magnetism.*—No. XV.

By General Sir Edward Sabine, *R.A., K.C.B., F.R.S.*

Received June 14,—Read June 15, 1876.

[Plates 17-19.]

The paper now offered to the Society forms the XVth and last of a series of papers printed in the Philosophical Transactions, entitled "Contributions to Terrestrial Magnetism." The whole Fifteen Numbers are related to each other as "Contributions to the Magnetic Survey of the Globe." Four of them (viz. XI., XIII., XIV., and the present paper) contain the complete statement of this Survey in the double form of "Catalogue" or "Tables" and of "Magnetic Maps;" of these maps there are twelve, one for each of the three magnetic elements in each of the four papers. The present paper consists (as did its last predecessor, No. XIV.) of four zones, each 10° in breadth :—

Zone 1, comprehending from the equator to 10° S.
Zone 2, " " lat. 10° S. " 20° S.
Zone 3, " " lat. 20° S. " 30° S.
Zone 4, " " lat. 30° S. " 40° S.

In the Tables the observations are entered in each zone in the succession of their longitudes, beginning with the meridian of Greenwich. The statements in the introduction to No. XIII. regarding the different magnetic elements apply to the present paper, as they did also to the preceding paper (No. XIV.).

The question of correction for secular change next presents itself. Happily the greater part of the observations were made within, or very near to, the "mean epoch," viz. 1840–1845. Sea observations were not generally corrected for differences of epoch in the previous papers; but in the present paper such corrections have been introduced for observations within the range of places (land stations) where the rate of secular change has been sufficiently established.

Nos. XI. & XIII., published earlier, comprise the northern and southern portions of the globe, from either pole to lat. 40°. These are the regions which have long been recognized as offering to the magnetician at once the most arduous and the most important field of research. In the middle or equatorial portions of the globe, comprised in the last and present papers, the magnetic relations are simpler, and the laying down of the lines representing them derives much aid from the adjacent portions of the North and South Polar Maps. Therefore, both as regards observations and treatment, less abundant as well as less exact evidence may, it may be hoped, suffice.

SOUTH EQUATORIAL ZONE I.—EQUATOR TO 10° S.

Authorities.

Belcher (Sir Edward)	Sabine in Phil. Trans. 1841, and MSS. in the Magnetic Office.
Bonite	Voyage de la Bonite (Paris, 1842).
Novara (Austrian Frigate)	Reise um die Erde (Wien, 1862–65).
Vénus (La)	Voyage de la Vénus (Paris, 1841).
FitzRoy	Voyage of the 'Beagle' (1849).
H.M.S. 'Fly'	MSS. in the Magnetic Office, Kew Observatory.
Boteler	L. S. Kämtz, MSS. in the Magnetic Office, Kew Observatory.
Owen	
Laplace	
Prussian Ships	
Bougainville	
Duperrey	
D'Urville	
Boussingault	
Foster	
Home	
Jehenne	
Rumker	
Elliot	Magnetic Survey of the Indian Archipelago, Phil. Trans. 1851.
Stanley	Contributions to Terrestrial Magnetism (Sabine in Phil. Trans.).
Blosseville	Schlagintweit's Scientific Mission to India and High Asia (Leipzig, 1861).
Dayman	Contributions to Terrestrial Magnetism (Sabine in Phil. Trans.).
Sulivan	
Dunlop	
Erman	Reise um die Erde (Berlin, 1831).
Friesach	Memoirs of the Imperial Academy of Sciences, Vienna, vols. xxix. to xliv.
Lütke	Mem. by Lenz in the Sci. Mem. of the Academy of St. Petersburg, 1838; and L. S. Kämtz, MS.
Lefroy	Contributions to Terrestrial Magnetism, 1846; and U. S. Coast Survey.
Kellett	MSS. in the Magnetic Office, received from Admiral Kellett.
Harkness	Smithsonian Contributions, vol. xviii.
Du Petit Thouars	Sabine in Phil. Trans. 1849.
Ross	MSS. in the Magnetic Office, communicated by Admiral Sir J. C. Ross.
Pasley	MSS. in the Magnetic Office.
Bérard	MSS. in the Magnetic Office, received from Admiral Duperrey.
Allen	MSS. in the Magnetic Office, received from the Author.
Rokeby	MSS. in the Magnetic Office.
Rattlesnake	H.M.S. 'Rattlesnake,' MSS. in the Hydrographic Office.
Denham	H.M.S. 'Herald,' MSS. in the Hydrographic Office.
H.M.S. 'Hecate'	MSS. in the Hydrographic Office.
Richards	MSS. in the Hydrographic Office.

SOUTH EQUATORIAL ZONE I.—Equator to 10° S.

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observers.
				Observed.	Correction to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′		°	° ′		°				
Annc ona	1 28	5 38	1829	21 01 w.	......	21·0 w.	......	......	......	......	......	......	Boteler.
Cape pez	0 36	8 40	1826	19 48 w.	......	19·8 w.	......	......	......	......	......	......	Owen.
Magu a	3 23	10 35	1826	21 24 w.	......	21·4 w.	......	......	......	......	......	......	Owen.
Loan	4 40	11 42	1826	20 36 w.	......	20·6 w.	......	......	......	......	......	......	Owen.
St. P de Loanda	8 46	13 09	1825	22 00 w.	......	22·0 w.	......	......	......	......	......	......	Owen.
Cong iver	6 05	13 12	1826	21 42 w.	......	21·7 w.	......	......	......	......	......	......	Owen.
Assu ion Island	9 46	46 28	1824	12 54 w.	......	12·9 w.	......	......	......	......	......	......	Owen.
Provi ice Island	9 14	51 05	1824	8 54 w.	......	8·9 w.	......	......	......	......	......	......	Owen.
eych es	4 36	55 31	1825	6 30 w.	......	6·5 w. } 5·3° w.	......	......	......	......	......	......	Owen.
			1830	7 31 w.	......	7·5 w.	......	......	......	......	......	......	Laplace.
			1842	2 01 w.	......	2·0 w.	32 03 s.	......	32·1 s.	8·01	......	8·01	Belcher.
t sea observations)	9 39	81 52	1837	0 25 w.	......	0·4 w.	......	......	......	......	......	......	Bonite.
t sea observations)	7 37	82 52	1857	0 19 w.	......	0·3 w.	......	......	......	......	......	......	Novara.
t sea observations)	0 39	82 52	1857	0 29 E.	......	0·5 E.	......	......	......	......	......	......	Novara.
t sea observations)	7 06	83 33	1837	0 48 E.	......	0·8 E.	......	......	......	......	......	......	Bonite.
t sea observations)	1 35	83 40	1857	0 16 E.	......	0·3 E.	......	......	......	......	......	......	Novara.
sea observations)	2 32	84 05	1857	0 50 E.	......	0·8 E.	......	......	......	......	......	......	Novara.
alem agan	0 12	100 10	1847	1 37 E.	......	1·6 E.	16 47 s.	......	16·8 s.	......	......	......	Elliot.
esan	0 08	100 12	1847	1 47 E.	......	1·8 E.	16 33 s.	......	16·5 s.	......	......	......	Elliot.
onjo	0 01	100 14	1847	1 36 E.	......	1·6 E.	16 39 s.	......	16·7 s.	......	......	......	Elliot.
enir o	0 13	100 14	1847	1 32 E.	......	1·5 E.	17 01 s.	......	17·0 s.	......	......	......	Elliot.
ort d Kock	0 13	100 27	1847	1 09 E.	......	1·1 E.	17 00 s.	......	17·0 s.	......	......	......	Elliot.
adar	0 59	100 31	1847	1 24 E.	......	1·4 E.	18 32 s.	......	18·5 s.	8·40	......	8·40	Elliot.
adar Panjang	0 22	100 43	1847	1 34 E.	......	1·6 E.	17 48 s.	......	17·8 s.	......	......	......	Elliot.
lok	0 47	100 56	1847	1 39 E.	......	1·7 E.	17 51 s.	......	17·9 s.	......	......	......	Elliot.
ort V de Capellen	0 28	101 03	1847	1 28 E.	......	1·5 E.	17 12 s.	......	17·2 s.	......	......	......	Elliot.
yacc oo	0 13	101 05	1847	1 30 E.	......	1·5 E.	16 38 s.	......	16·6 s.	......	......	......	Elliot.
a P āng	0 28	101 08	1847	1 22 E.	......	1·4 E.	17 11 s.	......	17·2 s.	......	......	......	Elliot.
onjc	0 42	101 20	1847	1 22 E.	......	1·4 E.	17 50 s.	......	17·8 s.	......	......	......	Elliot.
sea observations)	8 49	101 33	1851	0 24 w.	......	0·4 w.	......	......	......	......	......	......	Kellett.
sea observations)	7 58	102 19	1826	0 22 w.	......	0·4 w.	......	......	......	......	......	......	Prussian ships.
ncoo	3 54	102 29	1847	1 05 E.	......	1·1 E.	23 54 s.	......	23·9 s.	8·66	......	8·66	Elliot.
sea observations)	7 30	102 44	1851	0 17 w.	......	0·3 w.	......	......	......	......	......	......	Kellett.
ngin	0 12	104 37	1846	1 19 E.	......	1·3 E.	......	......	......	......	......	......	Elliot.
sea (observations)	9 21	105 17	1829	1 13 w.	......	1·2 w.	......	......	......	......	......	......	Lütke.
mpo	5 26	105 20	1847	1 13 E.	......	1·2 E.	26 16 s.	......	26·3 s.	8·83	......	8·83	Elliot.
ering	6 22	105 47	1846	0 51 E.	......	0·9 E.	27 34 s.	......	27·6 s.	8·90	......	8·90	Elliot.
ebili	6 47	105 49	1846	0 21 E.	......	0·3 E.	28 41 s.	......	28·7 s.	8·83	......	8·83	Elliot.
amb gan	6 31	105 55	1846	0 59 E.	......	1·0 E.	28 09 s.	......	28·1 s.	8·91	......	8 91	Elliot.
jeer	6 03	106 00	1842	1 07 E.	......	1·1 E. } 1·0 E.	26 38 s.	......	26·6 s. } 26·6° s.	......	......	......	Belcher.
			1846	0 58 E.	......	1·0 E.	26 32 s.	......	26·5 s.	8·82	......	8·82	Elliot.
sea (observations)	2 08	106 00	1824	0 36 w.	......	0·6 w.	......	......	......	......	......	......	Prussian ships.
sea (observations)	0 18	106 01	1841	......	......	......	13 50 s.	......	13·8 s.	......	......	......	Stanley.
nong ādap	6 28	106 06	1846	0 53 E.	......	0·9 E.	27 32 s.	......	27·5 s.	8·96	......	8·96	Elliot.
elang han	6 54	106 07	1846	0 14 E.	......	0·2 E.	28 24 s.	......	28·4 s.	8·84	......	8·84	Elliot.
oron Goonong	6 11	106 10	1846	0 40 E.	......	0·7 E.	27 33 s.	......	27·5 s.	8·92	......	8·92	Elliot.
sea (bservations)	4 34	106 13	1845	1 45 E.	......	1·7 E.	......	......	......	......	......	......	H.M.S. 'Fly.'
ram	6 07	106 15	1846	0 34 E.	......	0·6 E.	27 14 s.	......	27·2 s.	8·83	......	8·83	Elliot.
ngang han	7 31	106 19	1846	0 10 E.	......	0·2 E.	29 44 s.	......	29·7 s.	9·11	......	9·11	Elliot.
sea (bservations)	1 25	106 25	1858	1 00 E.	......	1·0 E.	......	......	......	......	......	......	Novara.
ebrān	6 57	106 26	1846	0 35 E.	......	0·6 E.	28 31 s.	......	28·5 s.	9·01	......	9·01	Elliot.
Iotoe	7 11	106 27	1846	0 28 E.	......	0·5 E.	28 54 s.	......	28·9 s.	9·02	......	9·02	Elliot.
sea (bservations)	4 41	106 34	1842	......	......	......	22 47 s.	......	22·8 s.	......	......	......	Belcher.
n Co er's Bay	7 05	106 37	1846	0 32 E.	......	0·5 E.	29 22 s.	......	29·4 s.	9·03	......	9·03	Elliot.
oaro ukasso	7 28	106 38	1846	0 13 E.	......	0·2 E.	30 08 s.	......	30·1 s.	9·04	......	9·04	Elliot.
yper and	6 02	106 41	1828	0 31 E.	......	0·5 E.	25 33 s.	......	25·5 s.	......	......	......	Blosseville.
rang ugga	6 58	106 48	1846	1 13 E.	......	1·2 E.	28 24 s.	......	28·4 s.	9·02	......	9·02	Elliot.
sea (servations)	4 57	106 54	1858	0 26 E.	......	0·4 E.	......	......	......	......	......	......	Novara.
sea (servations)	1 32	106 57	1842	......	......	......	16 40 s.	......	16·7 s.	......	......	......	Belcher.
avia	6 10	106 58	1828	0 31 E.	......	0·5 E. } 0·8 E.	25 56 s.	......	25·9 s. } 26·9 s.	......	......	......	Blosseville.
			1846	0 47 E.	......	0·8 E.	27 05 s.	......	27·1 s.	8·87	......	8·87	Elliot.
			1858	1 05 E.	......	1·1 E.	27 35 s.	......	27·6 s.	......	......	......	Novara.

SOUTH EQUATORIAL ZONE I.—Equator to 10° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.				Inclination.				Force in British units.			Obser rs.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.		Ob-served.	Cor. to Epoch 1842·5.	Corrected.		Ob-served.	Cor. to Epoch. 1842·5.	Corrected.	
	° ′	° ′		° ′		°		° ′		°					
At sea (2 observations)	2 21	106 59	1858	1 42 E.	......	1·7 E.		......	......	......		......	......	......	Novara.
Tegu	6 43	106 59	1846	0 12 E.	......	0·2 E.		28 45 s.	......	28·7 s.		......	......	......	Elliot.
Pangerango	6 51	106 59	1846	......	......	......		29 46 s.	......	29·8 s.		......	......	......	Elliot.
Bejong Petair	7 14	107 02	1846	0 16 E.	......	0·3 E.		29 37 s.	......	29·6 s.		9·11	......	9·11	Elliot.
Sidang Bārang	7 30	107 10	1846	0 05 E.	......	0·1 E.		30 15 s.	......	30·3 s.		9·01	......	9·01	Elliot.
Chunjūr	6 50	107 10	1846	1 35 E.	......	1·6 E.		28 26 s.	......	28·4 s.		8·97	......	8·97	Elliot.
Bandong	6 56	107 41	1846	0 26 E.	......	0·4 E.		28 34 s.	......	28·6 s.		9·04	......	9·04	Elliot.
Permangpek	7 39	107 45	1846	0 20 E.	......	0·3 E.		30 15 s.	......	30·3 s.		9·06	......	9·06	Elliot.
Garoet	7 14	107 55	1846	0 25 E.	......	0·4 E.		29 02 s.	......	29·0 s.		9·06	......	9·06	Elliot.
Samadang	6 51	108 05	1847	0 30 E.	......	0·5 E.		28 00 s.	......	28·0 s.		9·00	......	9·00	Elliot.
Kandang Aur	6 24	108 05	1847	0 18 E.	......	0·3 E.		......	......	......		......	......	......	Elliot.
Cherūgnūktok	7 38	108 10	1847	0 18 E.	......	0·3 E.		30 11 s.	......	30·2 s.		9·13	......	9·13	Elliot.
Chāwee	7 10	108 23	1847	0 33 E.	......	0·5 E.		28 42 s.	......	28·7 s.		9·07	......	9·07	Elliot.
Indramāyu	6 20	108 26	1847	0 41 E.	......	0·7 E.		27 31 s.	......	27·5 s.		8·96	......	8·96	Elliot.
Cheribon	6 44	108 42	1847	0 32 E.	......	0·5 E.		27 52 s.	......	27·9 s.		......	......	......	Elliot.
Banjeer	7 23	108 42	1847	0 28 E.	......	0·5 E.		29 10 s.	......	29·2 s.		......	......	......	Elliot.
Kalipoochen	7 39	108 53	1847	0 58 E.	......	1·0 E.		29 54 s.	......	29·9 s.		9·12	......	9·12	Elliot.
Chilāchap	7 44	108 57	1847	0 37 E.	......	0·6 E.		29 46 s.	......	29·8 s.		9·12	......	9·12	Elliot.
Aji Bārang	2 25	109 04	1847	0 55 E.	......	0·9 E.		27 22 s.	......	27·4 s.		......	......	......	Elliot.
Tegal	6 52	109 16	1847	0 38 E.	......	0·6 E.		28 05 s.	......	28·1 s.		9·01	......	9·01	Elliot.
Kārang Bolong	7 46	109 27	1847	0 32 E.	......	0·5 E.		29 56 s.	......	29·9 s.		9·16	......	9·16	Elliot.
Pantiānak	0 01	109 30	1846	1 31 E.	......	1·5 E.		12 45 s.	......	12·7 s.		8·33	......	8·33	Elliot.
At sea (2 observations)	3 24	109 45	1841	......	......	......		19 40 s.	......	19·7 s.		......	......	......	Stanley
Succadāna	1 16	109 57	1846	1 23 E.	......	1·4 E.		17 02 s.	......	17·0 s.		8·46	......	8·46	Elliot.
Munoori	7 35	110 04	1847	0 18 E.	......	0·3 E.		29 21 s.	......	29·3 s.		9·13	......	9·13	Elliot.
Pulo Kumpal	2 44	110 07	1840	0 39 E.	......	0·7 E.		19 49 s.	......	19·8 s.		8·74	......	8·74	Belcher
Ambarawa	7 16	110 29	1847	0 33 E.	......	0·5 E.		29 28 s.	......	29·5 s.		9·15	......	9·15	Elliot.
Samarang	7 00	110 31	1847	0 24 E.	......	0·4 E.		27 05 s.	......	27·1 s.		8·92	......	8·92	Elliot.
Balembang	7 21	110 37	1847	......	......	......		29 02 s.	......	29·0 s.		......	......	......	Elliot.
Japara	6 36	110 38	1847	0 25 E.	......	0·4 E.		27 30 s.	......	27·5 s.		8·98	......	8·98	Elliot.
Solo	7 35	110 54	1847	0 36 E.	......	0·6 E.		29 13 s.	......	29·2 s.		9·12	......	9·12	Elliot.
Patchitan	8 13	111 06	1847	0 20 E.	......	0·3 E.		30 36 s.	......	30·6 s.		9·16	......	9·16	Elliot.
Nyjāwee	7 24	111 29	1847	0 29 E.	......	0·5 E.		29 00 s.	......	29·0 s.		9·19	......	9·19	Elliot.
Kedeeri	7 48	112 00	1847	0 28 E.	......	0·5 E.		29 52 s.	......	29·9 s.		9·12	......	9·12	Elliot.
Bankāwa, Solo River	7 00	112 21	1847	0 29 E.	......	0·5 E.		27 47 s.	......	27·8 s.		9·07	......	9·07	Elliot.
Soorabāya	7 16	112 45	1844	0 58 E.	......	1·0 E.	1·1 E.	28 48 s.	......	28·8 s.	28·8 s.	8·81	......	9·02	H.M.S Fly.'
			1847	0 52 E.	......	0·9 E.		28 53 s.	......	28·9 s.		9·22	......		Elliot.
			1860	1 20 E.	......	1·3 E.		......	......	......		......	......	......	Denha
At sea (4 observations)	7 18	112 59	1860	1 15 E.	......	1·2 E.		......	......	......		......	......	......	Denha
Bezooke	7 43	113 43	1847	0 30 E.	......	0·5 E.		27 08 s.	......	27·1 s.		9·00	......	9·00	Elliot.
Sūmenap	7 00	113 51	1847	0 44 E.	......	0·7 E.		27 46 s.	......	27·8 s.		9·10	......	9·10	Elliot.
At sea (10 observations)	7 20	114 19	1845	0 46 E.	......	0·8 E.		......	......	......		......	......	......	H.M.S Fly.'
Solombo Island	5 35	114 23	1840	1 24 E.	......	1·4 E.		24 16 s.	......	24·3 s.		8·99	......	8·99	Belche
At sea (4 observations)	8 22	114 41	1860	0 44 E.	......	0·7 E.		......	......	......		......	......	......	Denha.
Kangelang Island	6 48	115 00	1825	0 00	......	0·0		......	......	......		......	......	......	Bouga ville.
Pulo Kuneeang	6 52	115 17	1847	0 32 E.	......	0·5 E.		27 26 s.	......	27·4 s.		9·09	......	9·09	Elliot.
At sea (2 observations)	9 14	115 39	1860	1 24 E.	......	1·4 E.		......	......	......		......	......	......	Denha.
At sea (3 observations)	9 28	117 29	1860	1 00 E.	......	1·0 E.		......	......	......		......	......	......	Denha.
Macassar	5 08	119 23	1840	0 29 E.	......	0·5 E.		23 42 s.	......	23·7 s.		8·98	......	8·98	Belche
At sea	6 11	121 59	1824	1 00 E.	......	1·0 E.		24 02 s.	......	24·0 s.		......	......	......	Dupery.
At sea	8 25	124 19	1827	0 16 E.	......	0·3 E.		......	......	......		......	......	......	D'Urv e.
On shore (2 observns.)	8 22	124 52	1840	......	......	......		30 06 s.	......	30·1 s.		......	......	......	Stanle
At sea (6 observations)	9 55	125 45	1848	......	......	......		31 00 s.	......	31·0 s.		......	......	......	Rattle ake.
At sea (6 observations)	9 31	126 55	1848	......	......	......		30 35 s.	......	30·6 s.		......	......	......	Rattle ake.
Bouro Island	3 23	127 06	1848	1 06 E.	......	1·1 E.		20 23 s.	......	20·4 s.		8·84	......	8·84	Belch
Cayeli	3 22	127 21	1823	0 32 E.	......	0·5 E.		20 08 s.	......	20·1 s.		......	......	......	Dupery.
At sea (6 observations)	9 12	127 38	1848	......	......	......		30 10 s.	......	30·2 s.		9·40	......	9·40	Rattle ake.
Amboyna	3 42	128 10	1823	0 28 E.	......	0·5 E.	0·9 E.	20 32 s.	......	20·5 s.	20 9 s.	......	......	......	Dupery.
			1840	1 14 E.	......	1·2 E.		21 10 s.	......	21·2 s.		8 94	......	8·94	Belch
At sea (6 observations)	9 48	128 33	1843	1 19 E.	......	1·3 E.		......	......	......		......	......	......	H.M.S Fly.'
At sea (6 observations)	9 20	128 52	1848	......	......	......		30 27 s.	......	30·5 s.		9·26	......	9·26	Rattle ake.
On shore (2 observns.)	4 07	129 07	1841	......	......	......		22 34 s.	......	22·6 s.		......	......	......	Stanle

SOUTH EQUATORIAL ZONE I.—Equator to 10° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observers.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′		°	° ′		°				
a (2 observations)	1 56	129 21	1827	2 30 E.	......	2·5 E.	......	......	......	......	......	......	D'Urville.
....	0 02	130 42	1823	1 02 E.	......	1·0 E.	13 31 S.	......	13·5 S.	8·36	......	8·36	Duperrey.
terdam Island ...	0 20	132 08	1840	1 24 E.	......	1·4 E.	15 09 S.	......	15·1 S.	8·50	......	8·50	Belcher.
hore (2 observns.)	5 39	133 35	1840	......	......	...... °	25 27 S.	......	25·5 S.	......	......	......	Stanley.
ri Island	0 52	133 59	1824	1 36 E.	......	1·6 E. } 1·8 E.	......	......	......	......	......	......	Duperrey.
			1827	2 06 E.	......	2·1 E.	......	......	......	......	......	......	D'Urville.
a (6 observations)	1 16	136 12	1827	2 50 E.	......	2·8 E.	......	......	......	......	......	......	D'Urville.
Rock..............	1 57	136 21	1840	3 00 E.	......	3·0 E.	18 16 S.	......	18·3 S.	8·70	......	8·70	Belcher.
Island	1 50	136 41	1840	4 09 E.	......	4·1 E.	17 28 S.	......	17·5 S.	8·65	......	8·65	Belcher.
a......................	0 20	138 19	1823	2 00 E.	......	2·0 E.	12 41 S.	......	12·7 S.	......	......	......	Duperrey.
a......................	1 37	140 12	1823	2 10 E.	......	2·2 E.	16 08 S.	......	16·1 S.	......	......	......	Duperrey.
a (7 observations)	3 08	142 47	1827	3 16 E.	......	3·3 E.	......	......	......	......	......	......	D'Urville.
nnia Island	3 19	143 29	1840	4 55 E.	......	4·9 E.	22 02 S.	......	22·0 S.	8·65	......	8·65	Belcher.
(11 observations)	9 04	143 40	1845	3 55 E.	......	3·9 E.	......	......	......	......	......	......	H.M.S. 'Fly.'
ley Island.........	9 35	143 45	1845	......	......	......	32 06 S.	......	32·1 S.	9·16	......	9·16	H.M.S. 'Fly.'
(11 observations)	9 33	143 51	1845	3 44 E.	......	3·7 E.	......	......	......	......	......	......	H.M.S. 'Fly.'
a......................	3 05	144 04	1823	5 12 E.	......	5·2 E.	17 57 S.	......	17·9 S.	......	......	......	Duperrey.
ble Cay............	9 08	144 15	1849	4 23 E.	......	4·4 E.	......	......	......	......	......	......	Dayman.
(10 observations)	8 10	144 50	1845	5 06 E.	......	5·1 E.	......	......	......	......	......	......	H.M.S. 'Fly.'
ar Bay	9 18	146 52	1849	5 20 E.	......	5·3 E.	......	......	......	......	......	......	Dayman.
a (6 observations)	6 06	148 35	1827	5 40 E.	......	5·7 E.	......	......	......	......	......	......	D'Urville.
a......................	3 28	150 55	1823	5 00 E.	......	5·0 E.	17 28 S.	......	17·5 S.	......	......	......	Duperrey.
a......................	4 42	152 40	1827	6 40 E.	......	6·7 E.	......	......	......	......	......	......	D'Urville.
Ireland	4 41	152 44	1840	7 13 E.	......	7·2 E.	20 49 S.	......	20·8 S.	8·80	......	8·80	Belcher.
Praslin	4 50	152 48	1823	6 48 E.	......	6·8 E.	20 40 S.	......	20·7 S.	......	......	......	Duperrey.
a Island	5 05	154 29	1823	7 20 E.	......	7·3 E.	......	......	......	......	......	......	Duperrey.
....	5 17	156 00	1823	6 36 E.	......	6·6 E.	20 08 S.	......	20·1 S.	......	......	......	Duperrey.
(5 observations)	2 38	160 39	1858	8 24 E.	......	8·4 E.	......	......	.. ...	......	......	......	Novara.
(2 observations)	5 59	160 44	1858	7 33 E.	......	7·5 E.	......	......	......	......	......	......	Novara.
(2 observations)	8 12	161 09	1858	7 48 E.	......	7·8 E.	......	......	......	......	......	......	Novara.
(8 observations)	8 26	161 48	1858	9 24 E.	......	9·4 E.	......	......	......	......	......	......	Novara.
....	0 40	174 19	1824	7 45 E.	......	7·7 E.	8 04 S.	......	8·1 S.	......	......	......	Duperrey.
(2 observations)	1 44	175 07	1824	7 45 E.	......	7·7 E.	3 25 S.	......	3·4 S.	......	......	......	Duperrey.
....	2 57	175 15	1824	7 45 E.	......	7·7 E.	6 28 S.	......	6·5 S.	......	......	......	Duperrey.
....	6 23	176 02	1824	8 05 E.	......	8·1 E.	12 25 S.	......	12·4 S.	......	......	......	Duperrey.
....	7 31	176 45	1824	8 30 E.	......	8·5 E.	15 11 S.	......	15·2 S.	......	......	......	Duperrey.
....	8 45	177 24	1824	10 32 E.	......	10·5 E.	16 34 S.	......	16·6 S.	......	......	......	Duperrey.
(7 observations)	7 55	200 19	1863	6 17 E.	......	6·3 E.	......	......	......	......	......	......	Richards.
(4 observations)	3 10	201 12	1863	5 06 E.	......	5·1 E.	......	......	......	......	......	......	Richards.
(3 observations)	0 46	201 50	1863	6 12 E.	......	6·2 E.	......	......	......	......	......	......	Richards.
(10 observations)	8 10	215 17	1848	5 06 E.	......	5·1 E.	......	......	......	......	......	......	H.M.S. 'Fly.'
....	9 36	215 35	1830	4 59 E.	......	5·0 E.	......	......	......	......	......	......	Erman.
....	9 22	215 43	1830	......	......	......	15 19 S.	......	15·3 S.	8·12	......	8·12	Erman.
....	8 37	216 06	1830	4 32 E.	......	4·5 E.	......	......	......	......	......	......	Erman.
(11 observations)	9 32	216 09	1845	3 40 E.	......	3·7 E.	......	......	......	......	......	......	H.M.S. 'Fly.'
(11 observations)	9 04	216 22	1845	3 55 E.	......	3·9 E.	......	......	......	......	......	......	H.M.S. 'Fly.'
(2 observations)	7 56	216 32	1830	......	......	......	12 11 S.	......	12·2 S.	7·63	......	7·63	Erman.
(3 observations)	7 03	217 00	1830	4 41 E.	......	4·7 E.	10 07 S.	......	10·1 S.	7·73	......	7·73	Erman.
(2 observations)	5 43	217 45	1830	......	......	......	7 50 S.	......	7·7 S.	7·66	......	7·66	Erman.
....	5 14	218 25	1830	4 24 E.	......	4·4 E.	......	......	......	......	......	......	Erman.
(3 observations)	4 01	218 45	1830	4 08 E.	......	4·1 E.	4 03 S.	......	4·1 S.	7·55	......	7·55	Erman.
(2 observations)	2 46	219 38	1830	......	......	......	1 08 S.	......	1·1 S.	7·42	......	7·42	Erman.
....	2 53	219 45	1830	4 17 E.	......	4·3 E.	......	......	......	......	......	......	Erman.
....	1 52	220 17	1830	4 26 E.	......	4·4 E.	......	......	......	......	......	......	Erman.
's Island.........	8 56	220 20	1840	6 16 E.	......	6·3 E.	14 06 S.	......	14·1 S.	7·93	......	7·93	Belcher.
(4 observations)	2 00	220 45	1830	......	......	......	0 01 S.	......	0·0	7·50	......	7·50	Erman.
ristine	9 56	220 50	1838	......	......	......	18 15 S.	......	18·3 S.	......	......	......	Vénus (La).
....	1 59	220 50	1830	......	......	......	......	......	......	7·47	......	7·47	Erman.
....	2 14	221 06	1830	4 12 E.	......	4·2 E.	......	......	......	......	......	......	Erman.
a......................	9 44	221 20	1859	4 55 E.	......	4·9 E.	18 45 S.	......	18·7 S.	8·20	......	8·20	Friesach.
(4 observations)	1 38	221 58	1830	......	......	......	0 35 N.	......	0·6 N.	7·44	......	7·44	Erman.
....	1 38	222 06	1830	4 12 E.	......	4·2 E.	......	......	......	......	......	......	Erman.

SOUTH EQUATORIAL ZONE I.—Equator to 10° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Obser rs.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	
	° '	° '		° '		°	° '		°				
At sea....................	1 28	222 48	1830	4 16 E.	......	4·3 E.	......	......	......	......	......	......	Erman.
At sea (2 observations)	1 29	223 09	1830	......	......	......	0 50 N.	......	0·8 N.	7·51	......	7·51	Erman.
At sea....................	1 40	224 04	1830	4 34 E.	......	4·6 E.	......	......	......	......	......	......	Erman.
At sea (4 observatious)	1 51	224 24	1830	......	......	......	0 15 N.	......	0·3 N.	7·34	......	7·34	Erman.
At sea (3 observations)	1 52	225 31	1830	3 43 E.	......	3·7 E.	0 11 S.	......	0·2 S.	7·29	......	7·29	Erman.
At sea (3 observations)	1 53	226 26	1830	3 38 E.	......	3·6 E.	0 06 N.	......	0·1 N.	7·48	......	7·48	Erman.
At sea (2 observations)	1 53	226 59	1830	......	......	......	0 11 S.	......	0·2 S.	......	......	......	Erman.
At sea (2 observations)	1 49	227 03	1830	......	......	......	0 09 N.	......	0·1 N.	7·62	......	7·62	Erman.
At sea (2 observations)	1 19	227 35	1830	......	......	......	1 19 N.	......	1·3 N.	7·69	......	7·69	Erman.
At sea....................	1 29	227 37	1830	3 19 E.	......	3·3 E.	......	......	......	......	......	......	Erman.
At sea....................	0 53	228 02	1830	4 29 E.	......	4·5 E.	......	......	......	......	......	......	Erman.
At sea (3 observations)	0 41	228 16	1830	......	......	......	2 22 N.	......	2·4 N.	7·32	......	7·32	Erman.
At sea....................	0 10	231 26	1830	4 44 E.	......	4·7 E.	......	......	......	......	......	......	Erman.
At sea (3 observations)	0 05	231 26	1830	......	......	......	3 11 N.	......	3·2 N.	7·39	......	7·39	Erman.
At sea................ ...	0 12	231 57	1830	......	......	......	3 09 N.	......	3·1 N.	7·44	......	7·44	Erman.
At sea..........	0 56	233 17	1827	4 53 E.	......	4·9 E.	2 11 N.	......	2·2 N.	......	......	......	Lütke.
At sea....................	1 10	234 31	1827	5 09 E.	......	5·1 E.	1 34 N.	......	1·6 N.	......	......	......	Lütke.
At sea....................	1 15	235 30	1827	4 19 E.	......	4·3 E.	2 14 N.	......	2·2 N.	......	......	......	Lütke.
At sea	2 02	236 04	1827	3 59 E.	......	4·0 E.	0 36 N.	......	0 6 N.	......	......	......	Lütke.
At sea....................	2 29	236 26	1827	4 00 E.	......	4·0 E.	0 28 S.	......	0·5 S.	......	......	......	Lütke.
At sea (5 observations)	1 12	238 01	1834	4 07 E.	......	4·1 E.	......	......	......	......	......	......	Prussian hips.
At sea....................	4 20	238 13	1827	4 24 E.	......	4·4 E.	3 54 S.	......	3·9 S.	......	......	......	Lütke.
At sea (3 observations)	0 04	238 30	1857	4 10 E.	......	4·2 E.	......	......	......	......	......	......	Richard
At sea....................	6 01	240 08	1827	4 19 E.	......	4·3 E.	6 54 S.	......	6·9 S.	......	......	......	Lütke.
At sea (4 observations)	1 37	240 39	1857	5 29 E.	......	5·5 E.	......	......	......	......	......	......	Richard
At sea (6 observations)	3 17	242 42	1857	6 00 E.	......	6·0 E.	......	......	......	......	......	......	Richard
At sea (8 observations)	3 00	242 43	1830	4 35 E.	......	4·6 E.	......	......	......	......	......	......	Prussiar hips.
At sea....................	9 38	243 25	1827	5 45 E.	......	5·7 E.	15 04 S.	......	15 1 S.	......	......	......	Lütke.
At sea (3 observations)	4 19	244 38	1857	6 08 E.	......	6·1 E.	......	......	......	......	......	......	Richard
At sea (3 observations)	8 16	246 04	1835	6 06 E.	......	6·1 E.	......	......	......	......	......	......	FitzRoy
At sea (3 observations)	6 27	246 35	1835	6 15 E.	......	6·3 E.	......	......	......	......	......	......	Prussiar hips.
At sea (3 observations)	5 20	246 48	1857	6 02 E.	......	6·0 E.	......	......	......	......	......	......	Richard
At sea (12 observations)	7 11	250 04	1857	6 39 E.	......	6 6 E.	......	......	......	......	......	......	Richard
At sea (3 observations)	6 16	253 08	1835	7 13 E.	......	7·2 E.	......	......	......	......	......	......	FitzRoy
At sea (5 observatious)	9 04	253 14	1857	7 02 E.	......	7·0 E.	......	......	.. ...	......	......	......	Richard
At sea (7 observations)	6 50	254 41	1827	7 29 E.	......	7·5 E.	......	......	......	......	......	......	Prussiar hips.
At sea (2 observations)	4 06	256 35	1835	7 15 E.	......	7·3 E.	......	......	......	......	......	......	FitzRoy
At sea (2 observations)	9 50	260 56	1828	8 49 E.	......	8·8 F.	......	......	......	......	......	......	Prussiar hips.
At sea (2 observations)	0 31	261 17	1835	8 40 E.	......	8·7 E.	......	......	......	......	......	......	FitzRoy
Galapagos Island. Post-office Bay, Charles Island	1 15	269 28	1838	......	......	......	8 14 N.	......	8·2 N.	......	......	......	Vénus (ı).
			1846	8 50 E.	......	8·8 E.	......	......	......	......	......	......	Kellett.
	0 15	269 29	1835	9 30 E.	......	9·5 E.	9 29 N.	......	9·5 N.	......	......	......	FitzRoy
Chatham Isl....	0 50	270 23	1835	9 35 E.	......	9·6 E.	8 41 N.	......	8·7 N.	......	......	......	FitzRoy
At sea....................	1 00	272 18	1836	8 20 E.	......	8·3 E.	......	......	......	......	......	......	Bonite.
At sea....................	7 32	274 54	1823	10 47 E.	......	10·8 E.	3 51 S.	......	3·9 S.	......	......	......	Duperr
At sea....................	9 55	274 54	1833	10 18 E.	......	10·3 E.	......	......	......	......	......	......	Prussia hips.
At sea (2 observations)	3 50	274 58	1835	9 25 E.	......	9·4 E.	......	......	......	......	......	......	FitzRoy
At sea....................	6 23	275 17	1823	10 48 E.	......	10·8 E.	0 51 S.	......	0·9 S.	......	......	......	Duperr
At sea....................	1 25	275 40	1836	7 37 E.	......	7·6 E.	......	......	......	......	......	......	Bonite.
At sea (2 observations)	7 17	278 34	1823	8 23 E.	......	8·4 E.	0 56 N.	......	0·9 N.	......	......	......	Duperr
Payta	5 06	278 54	1823	8 56 E.	......	8·9 E. } 9·0 E.	3 56 N.	......	3·9 N. } 4·5 N.	......	......	7·39	Duperr
			1832	......	......	......	4 40 N.	......	4·7 N.	......	......		Boussin ult.
			1835	9 00 E.	......	9·0 E.	......	......	......	......	......		FitzRoy
			1836	9 00 E.	......	9·0 E.	4 25 N.	......	4·4 N.	......	......		Bonite.
			1838	......	......	......	4 32 N.	......	4·5 N.	......	......		La Vén
			1866	8 53 E.	......	8·9 E.	4 58 N.	......	5·0 N.	7·39	......		Harkne
At sea....................	8 23	279 10	1823	7 42 E.	......	7·7 E.	1 41 S.	......	1·7 S.	......	......	.....	Duperr
At sea (2 observations)	3 51	279 20	1827	9 51 E.	......	9·9 E.	......	......	......	......	......	......	Prussia ships.
Tumbez River	3 31	279 30	1835	8 50 E.	......	8·8 E.	......	......	......	......	......	......	FitzRoy
At sea....................	8 54	279 32	1823	7 42 E.	......	7·7 E.	2 19 S.	......	2·3 S.	......	......	......	Duperr
At sea....................	3 18	279 32	1836	8 55 E.	......	8·9 E.	......	......	......	......	......	......	Bonite.

SOUTH EQUATORIAL ZONE I.—Equator to 10° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.				Inclination.				Force in British units.			Observers.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.		Ob-served.	Cor. to Epoch 1842 5.	Corrected.		Ob-served.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	°	° ′	° ′	°	°				
Pur	2 47	280 05	1835	9 00 E.	......	9·0 E.	8·8 E.	......	......	......	9·4 N.	......	......	......	FitzRoy.
			1836	8 27 E.	......	8·5 E.		9 46 N.	......	9·8 N.		......	......	......	Bonite.
			1838	8 56 E.	......	8·9 E.		9 01 N.	......	9·0 N.		......	......	......	Belcher.
Gua quil	2 13	280 06	1832	......	......	......		12 15 N.	......	12·3 N.		......	......	......	Boussingault.
			1835	8 30 E.	......	8·5 E.		......	......	... ..		......	......	......	FitzRoy.
Pac ayo Point (5 st ons)...........	7 23	280 12	1835	9 25 E.	......	9·4 E.		......	......	......		......	......	......	FitzRoy.
At s (3 observations)	8 59	280 29	1836	9 03 E.	......	9·1 E.		......	......	......		......	......	......	Bonite.
Qui	0 14	281 16	1831	......	......	......		16 32 N.	......	16·5 N.		......	......	......	Boussingault.
Riol aba	1 42	281 16	1831	......	......	......		12 35 N.	......	12·6 N.		......	......	......	Boussingault.
am co Bay (4 sta- ti)	9 28	281 32	1835	9 30 E.	......	9·5 E.		......	......	......		......	......	......	FitzRoy.
ara	1 26	311 30	1830	1 14 E.	1 00 W.	0·2 E.		23 27 N.	......	23·5 N.	23·8 N.	......	......	......	Foster.
			1838	......	......	......		24 08 N.	......	24·1 N.		......	......	......	Home.
ar am..................	2 32	315 39	1822	......	......	......	0·3 W.	23 06 N.	......	23·1 N.	23·3 N.	......	......	......	Sabine.
			1828	0 31 W.	1 10 W.	1·7 W.		23 21 N.	......	23·3 N.		......	......	......	Foster.
			1832	2 00 E.	0 50 W.	1·2 E.		......	......	......		......	......	......	Owen.
			1835	......	......	......		23 31 N.	......	23·5 N.		......	......	......	Home.
ear	3 44	321 29	1865	8 29 W.	2 06 E.	6·4 W.		21 23 N.	......	21·4 N.		6·99	......	6·99	Harkness.
t se 3 observations)	2 39	321 36	1842	6 19 W.	......	6·3 W.		......	......	......		......	......	......	Jehenne.
t se	0 27	324 44	1839	......	......	......		27 32 N.	......	27·5 N.		7·62	......	7·62	Sulivan.
t s	2 54	325 03	1839	......	......	......		22 32 N.	......	22·5 N.		7·30	......	7·30	Sulivan.
err ıbuco	8 04	325 09	1836	5 54 W.	0 47 W.	6·7 W.	7·5 W.	13 13 N.	−0 13	13·0 N.	13·0 N.	......	......	6·64	FitzRoy.
			1839	......	......	......		13 09 N.	−0 7	13·0 N.		6·74	......		Sulivan.
			1865	11 00 W.	2 40 E.	8·3 W.		12 08 N.	+0 45	12·9 N.		6·54	......		Harkness.
t se 2 observations)	8 03	325 10	1836	5 10 W.	0 45 W.	5·9 W.		......	......	......		......	......	......	FitzRoy.
se 2 observations)	6 23	325 20	1839	......	......	......		15 57 N.	......	15·9 N.		7·03	......	7·03	Sulivan.
se 4 observations)	7 39	325·52	1829	3 58 W.	1 31 W.	5·5 W.		......	......	......		......	......	......	Rumker.
se	7 28	326 48	1837	9 45 W.	0 38 W.	10·4 W.		......	......	......		......	......	......	Du Petit Thouars.
se 7 observations)	2 16	327 02	1829	5 25 W.	1 34 W.	7·0 W.		......	......	......		......	......	......	Rumker.
se	6 20	327 20	1839	......	......	......		15 57 N.	......	15·9 N.		7·16	......	7·16	Sulivan.
se	6 24	327 24	1840	......	......	......		......	......	......		6·72	......	6·72	Ross.
sea	3 51	327 26	1837	9 30 W.	0 38 W.	10·1 W.		......	......	......		......	......	.....	Du Petit Thouars.
sea	7 16	327 27	1840	8 57 W.	0 17 W.	9·2 W.		......	......	......		......	......	......	Ross.
sea	7 50	327 28	1840	......	......	......		12 08 N.	......	12·1 N.		6·65	......	6·65	Ross.
rna o de Noronha	3 50	327 35	1832	7 00 W.	1 13 W.	8·2 W.		......	......	......		......	......	......	FitzRoy.
sea 2 observations)	2 14	327 39	1837	9 45 W.	0 38 W.	10·4 W.		......	......	......		......	......	......	Du Petit Thouars.
sea	4 49	327 43	1840	......	......	......		17 49 N.	......	17·8 N.		6·90	......	6·90	Ross.
sea	2 36	327 52	1837	10 10 W.	0 38 W.	10·8 W.		......	......	......		......	......	......	Du Petit Thouars.
sea	9 21	327 58	1840	......	......	......		9 36 N.	......	9·6 N.		6·52	......	6·52	Ross.
sea 3 observations)	3 54	328 00	1832	7 53 W.	1 13 W.	9·1 W.		......	......	......		......	......	......	FitzRoy.
sea	9 23	328 03	1838	......	......	......		6 41 N.	......	6·7 N.		......	......	......	Stanley.
sea	9 41	328 05	1830	8 39 W.	1 27 W.	10·1 W.		9 36 N.	......	9·6 N.		6·49	......	6 49	Erman.
sea	9 47	328 07	1840	8 39 W.	0 17 W.	8·9 W.		......	......	......		......	......	......	Ross.
sea observations)	7 57	328 31	1830	9 01 W.	1 27 W.	10 5 W.		......	......	......		......	......	......	Erman.
sea	3 18	328 31	1840	......	......	......		18 44 N.	......	18·7 N.		7·02	......	7·02	Ross.
sea	3 02	328 41	1840	10 15 W.	0 17 W.	10·5 W.		......	......	......		......	......	......	Ross.
sea	2 35	328 50	1838	......	......	......		21 55 N.	......	21·9 N.		7·25	......	7·25	Sulivan.
sea observations)	6 21	328 57	1830	8 53 W.	1 27 W.	10·3 W.		......	......	... ..		......	......	......	Erman.
sea	5 19	329 04	1830	......	......	......		17 42 N.	......	17·7 N.		6·82	......	6·82	Erman.
sea observations)	9 03	329 05	1846	9 15 W.	0 25 E.	8·8 W.		......	......	......		......	......	......	Sulivan.
sea	3 51	329 06	1830	8 58 W.	1 27 W.	10·4 W.		20 29 N.	......	20·5 N.		6·92	......	6·92	Erman.
sea	5 02	329 06	1842	9 27 W.	......	9·5 W.		......	......	......		......	......	......	Jehenne.
sea	6 46	329 07	1846	9 46 W.	0 24 E.	10·4 W.		......	......	......		......	......	.. ...	Sulivan.
sea.	3 18	329 11	1846	9 50 W.	0 24 E.	9·4 W.		......	......	......		......	......	......	Sulivan.
sea.	5 10	329 16	1846	9 46 W.	0 24 E.	9·4 W.		......	......	......		......	......	......	Sulivan.
sea.	1 37	329 17	1840	......	......	......		23 04 N.	......	23·1 N.		7·19	......	7·19	Ross.
sea.	1 04	329 20	1837	12 20 W.	0 38 W.	13·0 W.		......	......	......		......	......	......	Du Petit Thouars.
sea.	1 53	329 24	1830	......	......	......		23 27 N.	......	23·5 N.		7·37	......	7·37	Erman.
sea.	2 56	329 31	1830	9 37 W.	1 27 W.	11·1 W.		......	......	......		......	......	......	Erman.
sea observations)	7 30	329 33	1826	8 43 W.	1 55 W.	10·6 W.		......	......	......		......	......	......	Lutke.
sea.	2 50	329 34	1846	10 18 W.	0 24 E.	9·9 W.		......	......	......		......	......	......	Sulivar .

SOUTH EQUATORIAL ZONE I.—Equator to 10° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Obser s.
				Observed.	Correction to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	° ′		°				
At sea……………………	1 21	329 39	1830	9 19 w.	1 27 w.	10·8 w.	……	……	……	……	……	……	Erman.
At sea (2 observations)	0 36	329 43	1830	9 51 w.	1 27 w.	11·3 w.	……	……	……	……	……	……	Erman.
At sea (3 observations)	0 19	329 48	1832	8 26 w.	1 13 w.	9·7 w.	……	……	……	……	……	……	FitzRoy.
At sea……………………	0 28	330 02	1840	……	……	……	24 28 N.	……	24·5 N.	7·19	……	7·19	Ross.
At sea (2 observations)	8 06	330 02	1840	11 01 w.	0 17 w.	11·3 w.	……	……	……	……	……	……	Pasley.
At sea……………………	0 46	330 03	1846	10 54 w.	0 24 E.	10·5 w.	……	……	……	……	……	……	Sulivan.
At sea……………………	0 07	330 08	1840	10 35 w.	0 17 w.	10·9 w.	……	……	……	……	……	……	Ross.
At sea……………………	4 15	330 40	1848	……	……	……	15 19 N.	……	15·3 N.	……	……	……	Stanley.
At sea… ……………	7 47	330 48	1847	13 41 w.	0 31 E.	13·2 w.	……	……	……	……	……	……	Stanley.
At sea (3 observations)	7 41	330 53	1847	……	……	……	10 26 N.	……	10·4 N.	6·76	……	6·76	Rattlesna.
At sea……………………	5 59	331 08	1842	13 04 w.	……	13·1 w.	……	……	……	……	……	……	Jehenne.
At sea (2 observations)	8 10	331 16	1842	12 00 w.	……	12·0 w.	……	……	……	……	……	……	Bérard.
At sea (2 observations)	6 53	331 23	1830	9 30 w.	1 27 w.	10·9 w.	……	……	……	……	……	……	Prussian ips.
At sea (2 observations)	9 25	331 32	1842	11 14 w.	……	11·2 w.	……	……	……	……	……	……	H.M.S. 'y.'
At sea……………………	1 00	332 08	1838	……	……	……	20 13 N.	……	20·2 N.	……	……	……	Stanley.
At sea (3 observations)	4 58	332 12	1847	……	……	……	14 58 N.	……	15·0 N.	6·89	……	6·89	Rattlesna.
At sea……………………	8 56	332 50	1843	10 12 w.	……	10·2 w.	7 24 N.	……	7·4 N.	……	……	……	Ross.
At sea……………………	7 01	332 56	1843	11 52 w.	……	11·9 w.	11 01 N.	……	11·0 N.	……	……	……	Ross.
At sea (2 observations)	3 46	333 00	1847	14 30 w.	0 31 E.	14·0 w.	……	……	……	……	……	……	Stanley.
At sea……………………	5 26	333 06	1843	11 10 w.	……	11·2 w.	13 25 N.	……	13·4 N.	……	……	……	Ross.
At sea……………………	3 47	333 13	1843	11 36 w.	……	11·6 w.	16 15 N.	……	16·3 N.	……	……	……	Ross.
At sea (2 observations)	7 52	333 26	1842	13 46 w.	……	13 8 w.	……	……	……	……	……	……	Jehenne.
At sea……………………	6 06	333 27	1842	……	……	……	13 48 N.	……	13·8 N.	6·74	……	6·74	H.M.S. 'ly.'
At sea……………………	1 53	333 39	1843	12 48 w.	……	12·8 w.	19 59 N.	……	20·0 N.	……	……	……	Ross.
At sea (3 observations)	2 25	333 53	1847	……	……	……	17 58 N.	……	18·0 N.	6·94	……	6·94	Rattlesna.
At sea……………………	0 19	334 00	1847	12 51 w.	0 31 E.	12·3 w.	22 34 N.	……	22·6 N.	……	……	……	Ross.
At sea (3 observations)	2 11	334 23	1826	11 24 w.	1 55 w.	13·3 w.	……	……	……	……	……	……	Lütke.
At sea (2 observations)	7 00	334 34	1836	12 38 w.	0 45 w.	13·4 w.	……	……	……	……	……	……	Bonite.
At sea (3 observations)	2 48	334 43	1842	14 27 w.	……	14·5 w.	……	……	……	……	……	……	H.M.S. 'ly.'
At sea (3 observations)	1 50	334 56	1842	15 15 w.	……	15·3 w.	……	……	……	……	……	……	Bérard.
At sea (3 observations)	6 34	335 13	1850	……	……	……	11 20 N.	……	11·3 N.	6·93	……	6·93	Rattlesn e.
At sea (3 observations)	4 01	335 20	1850	……	……	……	15 21 N.	……	15·3 N.	7·22	……	7·22	Rattlesn e.
At sea (8 observations)	2 43	335 34	1830	13 52 w.	1 27 w.	15·3 w.	……	……	……	……	……	……	Prussiar hips.
At sea (3 observations)	0 28	335 42	1847	……	……	……	20 10 N.	……	20·2 N.	7·04	……	7·04	Rattlesn e.
At sea (3 observations)	9 21	335 44	1850	……	……	……	6 36 N.	……	6·6 N.	6·90	……	6·90	Rattlesn e.
At sea……………………	7 48	335 46	1842	14 29 w.	……	14·5 w.	……	……	……	……	……	……	Jehenne
At sea……………………	0 19	335 50	1847	15 05 w.	0 34 E.	14·5 w.	……	……	……	……	……	……	Stanley.
At sea (6 observations)	0 34	336 00	1850	……	……	……	20 40 N.	……	20·7 N.	7·50	……	7·50	Rattlesn e.
At sea……………………	6 20	336 05	1822	11 30 w.	2 33 w.	14·1 w.	11 07 N.	……	11·1 N.	……	……	……	Duperre
At sea (2 observations)	3 42	336 23	1822	12 00 w.	2 33 w.	14·5 w.	16 44 N.	……	16·7 N.	……	……	……	Duperre
At sea (3 observations)	8 01	336 33	1826	11 30 w.	2 03 w.	13·5 w.	……	……	……	……	……	……	D'Urvil
At sea (2 observations)	0 57	336 52	1822	13 12 w.	2 33 w.	15·7 w.	19 09 N.	……	19·1 N.	……	……	……	Duperr
At sea (2 observations)	2 35	337 44	1839	15 23 w.	0 24 w.	15·8 w.	……	……	……	……	……	……	Du Peti Thouars.
At sea……………………	9 45	337 53	1831	……	……	……	3 09 N.	……	3·1 N.	……	……	……	Dunlop.
At sea (6 observations)	0 56	338 00	1846	……	……	……	18 33 N.	……	18·5 N.	7·12	……	7·12	H.M.S. Fly.'
At sea (4 observations)	2 00	338 40	1829	15 24 w.	1 40 w.	17·1 w.	……	……	……	……	……	……	Lütke.
At sea (2 observations)	2 26	338 43	1836	14 33 w.	0 48 w.	15·3 w.	……	……	……	……	……	……	Bonite.
At sea……………… …	3 18	338 45	1839	15 58 w.	0 24 w.	16·4 w.	……	……	……	……	……	……	Du Pet Thouars.
At sea……………………	1 05	338 52	1842	……	……	……	……	……	……	7·43	……	7·43	Lefroy.
At sea (2 observations)	2 31	339 02	1826	13 10 w.	2 03 w.	15·2 w.	……	……	……	……	……	……	D'Urvi
At sea……………………	8 10	339 50	1831	……	……	……	5 12 N.	……	5·2 N.	……	……	……	Dunlo
At sea (6 observations)	2 50	340 10	1851	18 07 w.	1 20 E.	16·8 w.	……	……	……	……	……	……	Kellett.
At sea (3 observations)	3 10	340 14	1846	……	……	……	11 35 N.	……	11·6 N.	6·80	……	6·80	H.M.S. Fly.'
At sea (2 observations)	1 48	340 39	1846	17 32 w.	0 26 E.	17·1 w.	……	……	……	……	……	……	Bérard
At sea……………………	6 02	341 02	1831	……	……	……	8 24 N.	……	8·4 N.	……	……	……	Dunlo
At sea……………………	3 00	341 04	1842	……	……	……	……	……	……	7·01	……	7·01	Lefroy.
At sea……………………	4 29	341 16	1839	16 33 w.	0 27 w.	17·0 w.	……	……	……	……	……	……	Du Pet Thouars.
At sea (3 observations)	2 47	341 20	1846	18 02 w.	0 28 E.	17·6 w.	……	……	……	……	……	……	Bérard
At sea……………………	4 03	341 37	1831	……	……	……	11 28 N.	……	11·5 N.	……	……	……	Dunlo
At sea (2 observations)	9 49	341 45	1836	15 57 w.	0 51 w.	16 8 w.	……	……	……	……	……	……	FitzRo
At sea (4 observations)	8 42	342 02	1846	18 08 w.	0 28 E.	17·7 w.	……	……	……	……	……	……	H.M.S Fly.'
At sea…………… … …	3 58	342 04	1842	……	……	……	……	……	……	6·79	……	6·79	Lefroy.

SOUTH EQUATORIAL ZONE I.—Equator to 10° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.				Inclination.				Force in British units.			Observers.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.		Ob-served.	Cor. to Epoch 1842·5.	Corrected.		Ob-served.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°		° ′	° ′	°					
At s (3 observations)	5 06	342 08	1846	......	......	......		6 45 N.	......	6·8 N.		6·62	......	6·62	H.M.S. 'Fly.'
At s	1 52	342 31	1831	......	......	......		14 40 N.	......	14·7 N.		......	......	......	Dunlop.
At s	4 41	342 49	1842	......	......	......		......	......	......		6·68	......	6·68	Lefroy.
At s (2 observations)	7 58	343 10	1824	15 45 W.	2 32 W.	18·3 W.		......	......	......		......	......	......	Prussian ships.
At s (2 observations)	6 34	343 20	1839	19 15 W.	0 27 W.	19·7 W.		......	......	......		......	......	......	Prussian ships.
At s (2 observations)	5 42	343 28	1846	19 14 W.	0 27 E.	18·8 W.		......	......	......		......	......	......	Bérard.
At s (6 observations)	6 23	343 32	1846	......	......	......		3 04 N.	......	3·1 N.		6·48	......	6·48	H.M.S. 'Fly.'
At s	9 17	343 44	1843	19 09 W.	......	19·1 W.		0 30 N.	......	0·5 N.		......	......	......	Ross.
t s (4 observations)	6 17	344 04	1851	20 01 W.	1 20 E.	18·7 W.		......	......	......		......	......	......	Kellett.
t s 3 observations)	7 04	344 06	1846	......	......	......		1 54 N.	......	1·9 N.		6·43	......	6·43	H.M.S. 'Fly.'
t s	0 53	344 36	1831	......	......	......		17 22 N.	......	17·4 N.		......	......	......	Dunlop.
t se 3 observations)	1 50	344 40	1837	17 45 W.	0 44 W.	18·5 W.		......	......	......		......	......	......	Bonite.
t se 2 observations)	8 02	344 48	1832	17 21 W.	2 44 W.	20·1 W.		......	......	......		......	......	......	Prussian ships.
t se 2 observations)	6 55	344 57	1839	18 39 W.	0 27 W.	19·1 W.		......	......	......		......	......	......	Du Petit Thouars.
t se 6 observations)	8 03	345 08	1846	......	......	......		0 40 S.	......	0·7 S.		6·36	......	6·36	H.M.S. 'Fly.'
se	7 59	345 25	1843	......	......	......		0 19 N.	......	0·3 S.		......	......	......	Ross.
se 6 observations)	8 35	345 32	1846	......	......	......		1 50 S.	......	1·8 S.		6·29	......	6·29	H.M.S. 'Fly.'
ce on *	7 54	345 36	1830	20 10 W.	1 12 W.	21·4 W.		......	......	......		......	......	......	Foster.
			1834	......	......	......		1 57 N.	−2 3	0·1 S.	° 0·2 S.	......	......	......	Allen.
			1836	17 36 W.	0 36 W.	18·2 W.	° 18·6 W.	1 39 N.	−1 34	0·1 N.		......	......	......	FitzRoy.
			1839	18 31 W.	0 18 W.	18·8 W.		0 06 N.	−0 51	0·7 S.		......	......	......	Du Petit Thouars.
			1842	......	......	......		0 08 S.	......	0·1 S.		......	......	......	Allen.
			1842	19 16 W.	......	......		......	......	......		6·61	......	6·61	Belcher.
			1846	19 16 W.	0 24 E.	18·9 W.		......	......	......		......	......	......	Bérard.
			1861	21 45 W.	......	......		......	......	......		......	......	......	Denham.
			1863	21 38 W.	......	......		4 47 S.	......	......		6·08	......	6·08	H.M.S. 'Hecate.'
			1864	......	......	......		5 57 S.	......	......		6·33	......	6·33	Rokeby.
se 4 observations)	7 29	345 37	1834	18 57 W.	1 16 W,	20·2 W.		......	......	......		......	......	......	Prussian ships.
sea observations)	7 45	345 43	1829	18 19 W.	2 01 W.	20·3 W.		......	......	......		......	......	......	Lütke.
sea observations)	7 04	345 44	1837	18 41 W.	0 48 W.	19·5 W.		......	......	......		......	......	......	Bonite.
sea observations)	8 42	345 44	1846	19 21 W.	0 31 E.	18·8 W.		......	......	......		......	......	......	H.M.S. 'Fly.'
sea	8 07	345 50	1843	......	......	......		0 17 N.	......	0·3 N.		......	......	......	Ross.
sea observations)	8 08	345 55	1851	21 00 W.	1 30 E.	19·5 W.		......	......	......		......	......	......	Kellett.
sea observations)	8 04	346 03	1846	19 21 W.	0 31 E.	18·8 W.		......	......	......		......	......	......	Bérard.
sea	6 11	346 05	1842	......	......	......		......	......	......		6·59	......	6·59	Lefroy.
sea	9 38	346 22	1842	......	......	......		......	......	......		6·18	......	6·18	Lefroy.
sea observations)	8 55	346 31	1836	17 40 W.	0 58 W.	18·6 W.		......	......	......		......	......	......	FitzRoy.
sea observations)	9 59	346 44	1846	......	......	......		4 48 S.	......	4·8 S.		6·30	......	6·30	H.M.S. 'Fly.'
sea observations)	9 03	347 02	1839	19 16 W.	0 31 W.	19·8 W.		......	......	......		......	......	......	Du Petit Thouars.
sea	9 30	347 47	1846	19 17 W.	0 31 E.	18·8 W.		......	......	......		......	......	......	Bérard.

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* The effects of local disturbance on land being so great, the mean Declination is assumed from results of observations made on board ships in the anchorage.

SOUTH EQUATORIAL ZONE II.—LATITUDE 10° TO 20° S.

Authorities.

FitzRoy	Voyage of the 'Beagle,' 1849.
Belcher	Sabine in Phil. Trans. 1840, 1842, and 1843.
Sulivan	
Wickham	
Dunlop	
Crozier	
Erebus	
Terror	
Pagoda	
Ross	Sabine in Phil. Trans. 1842 and 1849; and St.-Helena Observations, vol. i.
Jehenne	
Pasley	
Du Petit Thouars	
Stanley	
Owen	L. S. Kämtz, MSS.
Laplace	
Duperrey	
Prussian Ships	
Lütke	
Dumont d'Urville	
Richardson	
Kotzebue	
Irland	
Bruce	
Lartigue	
Rumker	
Beechey	
Smythe	St.-Helena Observations, vol. ii.
Bonite	Voyage (Paris, 1842).
La Vénus	Voyage autour du Monde (Paris, 1841).
Bérard	MSS. in Mag. Office, received from Admiral Duperrey.
Novara (Austrian Frigate)	Reise um die Erde (Wien, 1862–65).
Erman	Reise um die Erde (Berlin, 1841).
Livingstone (Charles)	MSS. in Mag. Office.
Elliot	Phil. Trans. (1851), Art. xii.
Harkness	Smithsonian Contrib. vol. xviii.
Friesach	Mem. Imp. Acad. of Sciences, Vienna, vols. xxix. to xliv.
Rattlesnake	MSS. in the Hydrographic Office.
Richards	MSS. in the Hydrographic Office.
Kellett	MSS. in the Hydrographic Office.
Denham	MSS. in the Hydrographic Office.
'Fly'	MSS. in the Hydrographic Office.
'Hecate'	MSS. in the Hydrographic Office.

SOUTH EQUATORIAL ZONE II.—Lat. 10° to 20° S.

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observers.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	° ′		°				
t se 3 observations)	19 43	0 44	1846	……	……	……	30 57	……	31·0 s.	6·23	……	6·23	H.M.S. ‘Fly.’
ape egro	15 47	11 46	1825	23 00 w.	1 10 w.	24·2 w.	……	……	……	……	……	……	Owen.
ape rio	18 23	11 57	1825	23 18 w.	1 10 w.	24·5 w.	……	……	……	……	……	……	Owen.
eng la	12 34	13 14	1825	22 00 w.	1 10 w.	23·2 w.	……	……	……	……	……	……	Owen.
ett	16 09	33 28	1858	……	……	…… } 15·1 w.	48 10	……	48·2 s. } 48·2 s.	7·83	……	} 7·83	Livingstone.
			1859	15 07 w.	……	15·1 w.	……	……	……	……	……		Livingstone.
ak mois Island	16 02	35 01	1859	14 56 w.	……	14·9 w.	48 41	……	48·7 s.	7·86	……	7·86	Livingstone.
xpe ion Island	18 25	35 51	1858	14 57 w.	……	15·0 w.	51 10	……	51·2 s.	7·74	……	7·74	Livingstone.
uil ane	18 01	36 56	1825	21 00 w.	……	21·0 w.	……	……	……	……	……	……	Owen.
ian Nova	17 15	42 30	1825	18 00 w.	……	18·0 w.	……	……	……	……	……	……	Owen.
offi sland	17 29	43 42	1825	18 00 w.	……	18·0 w.	……	……	……	……	……	……	Owen.
urr Island	18 41	43 53	1825	17 48 w.	……	17·8 w.	……	……	……	……	……	……	Owen.
mc	12 10	44 20	1825	12 30 w.	……	12·5 w.	……	……	……	……	……	……	Owen.
mb oo-ka	15 43	46 15	1825	15 00 w.	……	15·0 w.	……	……	……	……	……	……	Owen.
aja o Bay	15 14	47 00	1842	12 10 w.	……	12·2 w.	48 19	……	48·3 s.	8·46	……	8·46	Belcher.
ori Islands	11 35	47 19	1825	13 00 w.	……	13·0 w.	……	……	……	……	……	……	Owen.
ssa ava	13 28	48 10	1825	12 30 w.	……	12·5 w.	……	……	……	……	……	……	Owen.
mo o	19 55	48 47	1825	12 00 w.	……	12·0 w.	……	……	……	……	……	……	Owen.
ma e	18 10	49 31	1825	13 00 w.	……	13·0 w.	……	……	……	……	……	……	Owen.
ego uarez	12 15	49 34	1825	11 00 w.	……	11·0 w.	……	……	……	……	……	……	Owen.
M 's Island	17 00	49 49	1825	14 00 w.	……	14·0 w.	……	……	……	……	……	……	Owen.
Go	15 14	50 25	1825	13 00 w.	……	13·0 w.	……	……	……	……	……	……	Owen.
ady sland	15 54	54 21	1830	11 00 w.	……	11·0 w.	……	……	……	……	……	……	Laplace.
sea	19 54	57 55	1845	9 27 w.	……	9·4 w.	……	……	……	……	……	……	Pagoda.
rgo s-Garajos	16 22	59 44	1824	9 54 w.	……	9·9 w.	……	……	……	……	……	……	Owen.
sea observations)	19 02	61 13	1836	8 35 w.	……	8·6 w.	……	……	……	……	……	……	FitzRoy.
sea observations)	18 39	62 57	1836	7 51 w.	……	7·8 w.	……	……	……	……	……	……	FitzRoy.
sea	19 50	65 32	1837	8 16 w.	……	8·3 w.	……	……	……	……	……	……	Bonite.
sea	19 04	68 10	1837	5 45 w.	……	5·7 w.	……	……	……	……	……	……	Bonite.
sea	18 27	69 29	1837	5 54 w.	……	5·9 w.	……	……	……	……	……	……	Bonite.
sea	18 06	70 39	1837	5 02 w.	……	5·0 w.	……	……	……	……	……	……	Bonite.
sea observations)	17 31	70 39	1836	4 45 w.	……	4·7 w.	……	……	……	……	……	……	FitzRoy.
sea observations)	18 48	71 24	1846	……	……	……	50 24	……	50·4 s.	9·18	……	9·18	H.M.S. ‘Fly.’
sea.	17 15	71 55	1837	4 36 w.	……	4·6 w.	……	……	……	……	……	……	Bonite.
sea observations)	16 57	73 01	1836	3 40 w.	……	3·7 w.	……	……	……	……	……	……	FitzRoy.
sea.	15 54	73 51	1837	4 09 w.	……	4·1 w.	……	……	……	……	……	……	Bonite.
sea observations)	18 00	74 52	1846	……	……	……	48 40	……	48·7 s.	9·16	……	9·16	H.M.S. ‘Fly.’
sea observations)	19 11	75 47	1834	4 46 w.	……	4·8 w.	……	……	……	……	……	……	Prussian ships.
sea.	13 44	76 33	1837	3 36 w.	……	3·6 w.	……	……	……	……	……	……	Bonite.
ea (observations)	17 55	77 49	1846	……	……	……	48 52	……	48·9 s.	9·16	……	9·16	H.M.S. ‘Fly.’
ea (observations)	17 45	79 26	1846	……	……	……	48 31	……	48·5 s.	9·13	……	9·13	H.M.S. ‘Fly.’
ea.	10 12	81 02	1837	0 42 w.	……	0·7 w.	……	……	……	……	……	……	Bonite.
ea.	17 30	81 27	1846	……	……	……	47 27	……	47·5 s.	9·19	……	9·19	H.M.S. ‘Fly.’
ea (observations)	19 57	82 56	1851	7 42 w.	……	7·7 w.	……	……	……	……	……	……	Kellett.
ea.	18 33	84 03	1824	0 37 w.	……	0·6 w.	48 23	……	48·4 s.	……	……	……	Duperrey.
ea (observations)	13 59	85 10	1857	3 24 w.	……	3·4 w.	……	……	……	……	……	……	Novara.
sea (observations)	17 06	85 18	1857	4 22 w.	……	4·4 w.	……	……	……	……	……	……	Novara.
sea (observations)	17 23	85 20	1846	……	……	……	48 30	……	48·5 s.	9·32	……	9·32	H.M.S. ‘Fly.’
sea (observations)	16 56	85 24	1833	2 40 w.	……	2·7 w.	……	……	……	……	……	……	Prussian ships.
sea (observations)	18 06	87 30	1851	6 09 w.	……	6·1 w.	……	……	……	……	……	……	Kellett.
sea..	17 35	89 15	1846	……	……	……	48 30	……	48·5 s.	9·43	……	9·43	H.M.S. ‘Fly.’
ea (observations)	13 08	89 23	1836	0 38 w.	……	0·6 w.	……	……	……	……	……	……	FitzRoy.
ea (observations)	16 20	90 16	1851	5 28 w.	……	5·5 w.	……	……	……	……	……	……	Kellett.
ea (observations)	13 58	91 12	1827	2 21 w.	……	2·3 w.	……	……	……	……	……	……	Prussian ships.
ea (observations)	14 27	93 08	1851	4 30 w.	……	4·5 w.	……	……	……	……	……	……	Kellett.
ea (observations)	18 02	93 28	1846	……	……	……	48 44	……	48·7 s.	……	……	……	H.M.S. ‘Fly.’
ea..	12 21	94 04	1836	0 04 w.	……	0·1 w.	……	……	……	……	……	……	FitzRoy.
ea (1 bservations)	18 41	94 58	1860	5 35 w.	……	5·6 w.	……	……	……	……	……	……	Denham.
ea (bservations)	13 06	95 45	1851	3 40 w.	……	3·7 w.	……	……	……	……	……	……	Kellett.
ea..	19 33	96 18	1846	……	……	……	48 43	……	48·7 s.	9·88	……	9·88	H.M.S. ‘Fly.’
ling lands	12 06	96 50	1836	1 12 w.	……	1·2 w. } 1·2 w.	……	……	…… } 39·3 s.	……	……	} 9·40	FitzRoy.
			1848	1 11 w.	……	1·2 w.	39 19	……	39·3 s.	9·40	……		Elliot.

SOUTH EQUATORIAL ZONE II.—Lat. 10° to 20° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observers.
				Observed.	Correction to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	° ′		°				
At sea (3 observations)	17 35	97 24	1829	3 01 W.	......	3·0 W.	......	......	......	......	......	......	Lütke.
At sea (2 observations)	11 52	97 42	1851	2 35 W.	......	2·6 W.	......	......	......	......	......	......	Kellett.
At sea (7 observations)	10 40	99 12	1851	2 01 W.	......	2·0 W.	......	......	......	......	......	......	Kellett.
At sea (9 observations)	15 45	100 40	1860	3 40 W.	......	3·7 W.	......	......	......	......	......	......	Denham.
At sea (20 observations)	14 58	103 02	1860	1 43 W.	......	1·7 W.	......	......	......	......	......	......	Denham.
At sea (3 observations)	15 37	103 17	1829	2 33 W.	......	2·5 W.	......	......	......	......	......	......	Lütke.
At sea (6 observations)	18 55	103 47	1848	......	......	......	46 43	......	46·7 S.	10·15	......	10·15	Rattlesnake.
At sea (2 observations)	13 50	105 08	1829	2 27 W.	......	2·5 W.	......	......	......	......	......	......	Lütke.
At sea (3 observations)	17 49	105 25	1848	......	......	......	44 52	......	44·9 S.	10·16	......	10·16	Rattlesnake.
At sea (12 observations)	14 20	107 50	1860	0 30 W.	......	0·5 W.	......	......	......	......	......	......	Denham.
At sea (3 observations)	14 47	111 00	1848	......	......	......	39 46	......	39·8 S.	9·93	......	9·93	Rattlesnake.
At sea (3 observations)	13 46	113 23	1848	......	......	......	38 14	......	38·2 S.	......	......	......	Rattlesnake.
At sea (3 observations)	13 36	115 04	1848	......	......	......	37 52	......	37·9 S.	......	......	......	Rattlesnake.
At sea (6 observations)	18 19	115 05	1827	0 59 W.	......	1·0 W.	......	......	......	......	......	......	Dumont d'Urville.
At sea (8 observations)	12 41	116 09	1860	0 26 E.	......	0·4 E.	......	......	......	......	......	......	Denham.
At sea (3 observations)	13 16	117 56	1848	......	......	......	36 53	......	36·9 S.	9·78	......	9·78	Rattlesnake.
At sea (3 observations)	10 21	119 11	1860	1 19 E.	......	1·3 E.	......	......	......	......	......	......	Denham.
At sea (3 observations)	12 23	119 33	1848	......	......	......	35 27	......	35·5 S.	9·70	......	9·70	Rattlesnake.
At sea (5 observations)	16 35	120 44	1827	0 21 W.	......	0·4 W.	......	......	......	......	......	......	Dumont d'Urville.
At sea (6 observations)	11 52	121 05	1848	......	......	......	34 44	......	34·7 S.	9·66	......	9·66	Rattlesnake.
At sea (9 observations)	13 02	121 09	1827	0 28 W.	......	0·5 W.	......	......	......	......	......	......	Dumont d'Urville.
Point Swan	16 21	123 03	1838	......	......	......	43 07	......	43·1 S.	9·92	......	9·92	Wickham.
Port Usborne	16 39	123 34	1838	......	......	......	43 26	......	43·4 S.	10·04	......	10·04	Wickham.
Timor, Koepang Fort	10 10	123 36	1863	1 24 E.	......	1·4 E.	32 58	......	33·0 S.	9·58	......	9·58	H.M.S. 'Hecate.'
At sea (3 observations)	10 33	124 01	1848	......	......	......	32 27	......	32·5 S.	9·55	......	9·55	Rattlesnake.
Port George IV.	15 20	124 40	1838	......	......	......	41 29	......	41·5 S.	10·00	......	10·00	Wickham.
At sea (6 observations)	10 06	124 57	1848	......	......	......	31 24	......	31·4 S.	9·47	......	9·47	Rattlesnake.
At sea (4 observations)	11 25	125 59	1860	1 07 E.	......	1·1 E.	......	......	......	......	......	......	Denham.
At sea (3 observations)	11 13	128 57	1860	2 21 E.	......	2·4 E.	......	......	......	......	......	......	Denham.
At sea (8 observations)	10 53	131 45	1848	......	......	......	33 12	......	33·2 S.	9·38	......	9·38	Rattlesnake.
Port Essington, on shore	11 23	132 12	1845	2 10 E.	......	2·2 E.	35 21	......	35·3 S.	9·49	......	9·49	H.M.S. 'Fly.'
Port Essington, on shore (3 obsrvtns.)	11 23	132 12	1848	......	......	......	35 15	......	35·3 S.	9·68	......	9·68	Rattlesnake.
Port Essington, on board (6 obsrvtns.)	11 14	132 12	1848	......	......	......	33 48	......	33·8 S.	9·44	......	9·44	Rattlesnake.
Hammond Island	10 32	142 12	1860	4 44 E.	......	4·7 E.	......	......	......	......	......	......	Denham.
Evans Bay, C. York ...	10 44	142 31	1848	......	......	...... } 6·4 E.	33 11	......	33·2 S. } 33·0 S.	9·41	......	} 9·41	Rattlesnake.
			1863	6 25 E.	......	6·4 E.	32 45	......	32·8 S.	......	......		H.M.S. 'Hecate.'
Mount Adolphus	10 39	142 40	1843	4 00 E.	......	4·0 E.	33 48	......	33·8 S.	9·43	......	9·43	H.M.S. 'Fly.'
Sir C. Hardy's Islet ...	11 56	143 39	1843	3 12 E.	......	3·2 E.	35 31	......	35·5 S.	9·68	......	9·68	H.M.S. 'Fly.'
Claremont Isld., No. 5	13 39	143 45	1848	......	......	......	38 12	......	38·2 S.	......	......	......	Rattlesnake.
R. Sir Ch. Hardy......	11 56	143 47	1831	5 30 E.	......	5·5 E.	......	......	......	......	......	......	Richardson.
Raine Island (3 obs.)...	11 36	144 02	1860	5 21 E.	......	5·3 E.	......	......	......	......	......	......	Denham.
Raine Island (1 obs.)...	11 36	144 02	1844	4 00 E.	......	4·0 E.	35 01	......	35·0 S.	9·60	......	9·60	H.M.S. 'Fly.'
Lizard Island, Torres Straits	14 40	145 28	1843	6 50 E.	......	6·8 E. } 6·8 E.	39 30	......	39·5 S. } 39·5 S.	10·04	......	} 10·08	H.M.S. 'Fly.'
			1848	......	......	......	39 32	......	39·5 S.	10·12	......		Rattlesnake.
Rockingham Bay	18 15	146 05	1843	6 53 E.	......	6·9 E. } 6·7 E.	45 01	......	45·0 S. } 44·8 S.	10·57	......	} 10·76	H.M.S. 'Fly.'
			1863	6 30 E.	......	6·5 E.	44 37	......	44·6 S.	10·96	......		H.M.S. 'Hecate.'
Round Islet, Rockingham Bay	17 55	146 10	1848	......	......	......	44 13	......	44·2 S.	10·66	......	10·66	Rattlesnake.
Cape Upstart	19 43	147 50	1843	6 58 E.	......	7·0 E.	47 15	......	47·3 S.	10·67	......	10·67	H.M.S. 'Fly.'
Willis Island (2 obs.)...	16 13	150 02	1860	7 11 E.	......	7·2 E.	......	......	......	......	......	......	Denham.
Barnard I. No. 2 (3 obs.)	17 40	151 13	1848	......	......	......	44 09	......	44·1 S.	......	......	......	Rattlesnake.
Alert Reef (2 obs.) ...	17 07	152 07	1860	8 03 E.	......	8·1 E.	......	......	......	......	......	......	Denham.
Louisiade Archi. (2 obs.)	11 18	152 51	1849	......	......	......	32 42	......	32·7 S.	......	......	......	Rattlesnake.
Sand Cay (3 obsrvs.) ...	17 24	155 53	1860	8 30 E.	......	8·5 E.	......	......	......	......	......	......	Denham.
Chesterfield Group (6 observations) ...	19 53	158 21	1859	9 22 E.	......	9·4 E.	......	......	......	......	......	......	Denham.
At sea (3 observations)	10 17	162 40	1858	9 11 E.	0 16 E.	9·4 E.	......	......	......	......	......	......	Novara.
At sea (6 observations)	11 59	169 24	1828	10 02 E.	0 14 W.	9·8 E.	......	......	......	......	......	......	Dumont d'Urville.
Tanna Island...........	19 32	169 29	1840	11 37 E.	0 03 W.	11·6 E.	39 53	......	39·9 S.	10·40	......	10·40	Belcher.

SOUTH EQUATORIAL ZONE II.—Lat. 10° to 20° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.				Inclination.				Force in British units.			Observers.
				Observed.	Correction to Epoch 1842·5.	Corrected.		Observed.	Cor. to Epoch 1842·5.	Corrected.		Observed.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	°	° ′		°	°				
t se 13 observations)	19 49	175 33	1855	11 34 E.	......	11·6 E.		......	......	......		......	......	......	Denham.
t s	16 48	176 14	1823	8 00 E.	0 19 W.	7·7 E.		31 17	......	31·3 S.		......	......	......	Duperrey.
t se 10 observations)	18 24	177 02	1826	10 49 E.	0 16 W.	10·5 E.		......	......	... ..		......	......	......	Dumont d'Urville.
Beng Island...........	18 20	178 10	1840	10 21 E.	0 02 W.	10·3 E.		36 30	......	36·5 S.		11·34	......	11·34	Belcher.
Nuk u Island	18 10	178 31	1840	10 25 E.	0 02 W.	10·4 E.		36 09	......	36·1 S.		9·78	......	9·78	Belcher.
iw 2 observations)	17 57	178 36	1855	9 38 E.	......	9·6 E.		......	......	......		......	......	......	Denham.
evu	17 41	178 50	1855	10 07 E.	......	10·1 E.		......	......	......		......	......	......	Denham.
fair 4 observations)	17 53	179 20	1855	8 57 E.	......	8·9 E.		......	......	......		......	......	......	Denham.
t se 5 observations)	17 48	179 23	1855	9 25 E.	......	9·4 E.		......	......	......		......	......	......	Denham.
ova eef (2 obs.) ...	18 39	179 34	1855	10 41 E.	......	10·7 E.		......	......	......		......	......	......	Denham.
oto 3 observations)	18 55	179 50	1855	11 00 E.	......	11·0 E.		......	......	......		......	......	......	Denham.
latu (3 observtns.)	19 09	180 10	1855	11 02 E.	......	11·0 E.		......	......	......		......	......	......	Denham.
t se 8 observations)	16 00	185 12	1863	8 44 E.	......	8·7 E.		......	......	......		......	......	......	Richards.
avs sland	18 39	186 00	1837	......	......	......	9·5 E.	35 25	.....	35·4 S.	35·3 S.	......	......	9·65	Anonymous.
			1840	9 34 E.	0 02 W.	9·5 E.		35 07	......	35·1 S.		9·65	......		Belcher.
poli sland	14 00	188 00	1824	6 58 E.	0 18 W.	6·7 E.		......	......	......		......	......	......	Kotzebue.
pia ., Upolu Island	13 50	188 14	1863	9 16 E.	... ..	9·3 E.		27 32	......	27·5 S.		9·08	......	9·08	H.M.S. 'Hecate.'
se	11 56	188 34	1824	10 47 E.	0 18 W.	10·5 E.		24 12	......	24·2 S.		......	......	......	Duperrey.
se 5 observations)	13 01	189 53	1863	7 54 E.	......	7·9 E.		......	......	......		......	......	......	Richards.
tu	14 16	189 23	1837	......	......	......		26 04	......	26·1 S.		......	......	......	Anonymous.
se 2 observations)	11 22	192 52	1863	7 34 E.	......	7·6 E.		......	......	......		......	......	......	Richards.
se 4 observations)	20 48	195 47	1835	9 28 E.	0 07 W.	9·3 E.		......	......	......		......	......	......	FitzRoy.
se 1 observations)	10 25	197 58	1863	7 07 E.	......	7·1 E.		......	......	......		......	......	......	Richards.
se 4 observations)	18 38	201 14	1835	9 09 E.	0 03 W.	9·1 E.		......	......	......		......	......	......	FitzRoy.
se 2 observations)	18 21	203 07	1835	7 43 E.	0 03 W.	7·7 E.		......	......	......		......	......	......	FitzRoy.
llin hausen Island	15 48	205 30	1824	6 00 E.	0 09 W.	5·8 E.		......	......	......		......	......	......	Kotzebue.
sea 4 observations)	17 26	207 21	1835	7 37 E.	0 03 W.	7·6 E.		......	......	......		......	......	......	FitzRoy.
anp Island	16 26	207 48	1823	7 10 E.	0 09 W.	7·0 E.		......	......	......		......	......	......	Duperrey.
iat	16 42	208 37	1837	......	......	......		28 44	......	28·7 S.		......	......	......	Anonymous.
sea 3 observations)	17 20	208 41	1835	7 57 E.	0 03 W.	7·9 E.		......	......	......		......	......	......	FitzRoy.
sea	18 24	209 32	1830	6 22 E.	0 06 W.	6·3 E.		......	......	......		......	......	......	Erman.
sea	19 06	209 52	1830	......	......	......		31 59	......	32·0 S.		......	......	......	Erman.
sea	19 28	209 52	1830	7 25 E.	......	7·4 E.		......	......	......		9·44	......	9·44	Erman.
sea	17 25	210 13	1830	......	......	......		29 16	......	29·3 S.		......	......	......	Erman.
sea 2 observations)	18 23	210 13	1859	7 10 E.	......	7·2 E.		......	......	......		......	......	......	Novara.
sea	16 27	210 26	1830	......	......	......		27 05	......	27·1 S.		8·62	......	8·62	Erman.
pie	17 32	210 27	1837	......	......	......	6·8 E.	29 07	......	29·1 S.	28·6 S.	......	......	8·48	Anonymous.
			1838	6 11 E.	......	6·2 E.		27 07	......	27·1 S.		......	......		La Vénus.
			1859	6 54 E.	......	6·9 E.		28 43	......	28·7 S.		8·40	......		Friesach.
			1859	7 27 E.	......	7·4 E.		29 21	......	29·3 S.		8·57	......		Novara.
ahei	17 29	210 30	1823	6 40 E.	......	6·7 E.	7·1 E.	30 04	......	30·1 S.	30·0 S.	......	......	8·77	Duperrey.
			1824	6 50 E.	......	6·8 E.		29 03	......	29·0 S.		......	......		Kotzebue.
			1826	7 33 E.	......	7·5 E.		......	......	......		......	......		Beechey.
			1830	......	......	......		30 26	......	30·4 S.		8·85	......		Erman.
			1831	7 30 E.	......	7·5 E.		......	......	......		......	......		Irland.
			1835	7 34 E.	......	7·6 E.		30 14	......	30·2 S.		......	......		FitzRoy.
			1837	7 10 E.	......	7·2 E.		......	......	......		......	......		Bruce.
			1840	6 30 E.	......	6·5 E.		30 18	......	30·3 S.		8·68	......		Belcher.
ses observations)	15 55	210 31	1859	6 51 E.	......	6·8 E.		......	......	......		......	......	......	Novara.
sea observations)	15 57	210 41	1830	7 02 E.	......	7·0 E.		......	......	......		......	......	......	Erman.
sea observations)	14 46	211 41	1830	6 35 E.	......	6·6 E.		......	......	......		......	......	......	Erman.
sea observations)	14 43	212 06	1830	6 10 E.	......	6·2 E.		24 23	......	24·4 S.		8·19	......	8·19	Erman.
sea observations)	14 25	212 12	1859	6 29 E.	......	6·5 E.		......	......	......		......	......	......	Novara.
sea observations)	14 45	212 33	1830	6 19 E.	......	6·3 E.		......	......	......		......	......	......	Erman.
sea observations)	15 04	213 33	1830	6 58 E.	......	7·0 E.		24 55	......	24·9 S.		8·46	......	8·46	Erman.
sea observations)	14 33	214 02	1830	6 20 E.	......	6·3 E.		......	......	......		......	......	......	Erman.
sea observations)	13 27	214 08	1830	5 46 E.	......	5·8 E.		......	......	......		......	......	......	Erman.
sea observations)	13 53	214 19	1830	......	......	......		22 57	......	23·0 S.		8·33	......	8·33	Erman.
sea observations)	13 02	214 21	1830	......	......	......		21 19	......	21·3 S.		8·45	......	8·45	Erman.
sea observations)	11 58	214 30	1830	......	......	......		19 23	......	19·4 S.		8·27	......	8·27	Erman.
sea observations)	13 37	214 31	1859	5 58 E.	......	6·0 E.		......	......	......		......	......	......	Novara.
sea observations)	11 10	214 45	1830	5 09 E.	......	5·1 E.		......	......	......		......	......	......	Erman.

SOUTH EQUATORIAL ZONE II.—Lat. 10° to 20° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observe
				Observed.	Correction to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′		°	° ′		°				
At sea (2 observations)	10 48	214 55	1830	......	......	......	17 48	......	17·8 s.	8·16	......	8·16	Erman.
At sea (7 observations)	14 26	216 30	1859	7 12 E.	......	7·2 E.	......	......	......	......	......	......	Novara.
At sea (7 observations)	13 49	217 27	1859	6 21 E.	......	6·3 E.	......	......	......	......	......	......	Novara.
At sea (2 observations)	12 57	217 53	1859	6 26 E.	......	6·4 E.	......	......	......	......	......	......	Novara.
Bow Island	18 05	219 07	1840	6 43 E.	......	6·7 E.	......	......	......	......	......	......	Belcher.
Moller Island	17 43	219 18	1823	6 32 E.	......	6·5 E.	......	......	.....	......	......	......	Duperrey.
At sea (2 observations)	14 00	219 30	1859	6 05 E.	......	6·1 E.	......	......	......	......	......	......	Novara.
Predpriatir Island ...	15 58	219 54	1824	5 00 E.	......	5·0 E.	......	......	......	......	......	......	Kotzebue.
At sea (2 observations)	15 16	220 07	1859	6 46 E.	......	6·8 E.	......	......	......	......	......	......	Novara.
Lanciers Island........	18 30	220 52	1825	7 03 E.	......	7·0 E.	......	......	......	......	......	......	Beechey.
Narcisse Island........	17 21	221 30	1823	5 22 E.	......	5·4 E.	......	......	......	......	......	......	Duperrey.
At sea (2 observations)	15 38	222 15	1859	6 20 E.	......	6·3 E.	......	......	......	......	......	......	Novara.
Clermont Tonnere Isl.	18 28	223 34	1823	4 51 E.	......	4·8 E.	......	......	......	......	......	......	Duperrey.
At sea (5 observations)	16 58	225 30	1859	6 44 E.	......	6·7 E.	......	......	......	......	......	......	Novara.
At sea (2 observations)	19 22	227 20	1859	7 07 E.	......	7·1 E.	......	......	......	......	......	......	Novara.
At sea....................	13 11	227 49	1835	6 20 E.	......	6·3 E.	......	......	......	......	......	......	FitzRoy.
At sea (2 observations)	12 00	231 58	1835	5 35 E.	......	5·6 E.	......	......	......	......	......	......	FitzRoy.
At sea (3 observations)	11 30	234 55	1835	5 14 E.	......	5·2 E.	......	......	......	......	......	......	FitzRoy.
At sea (3 observations)	10 03	240 12	1835	5 16 E.	......	5·3 E.	......	......	......	......	......	......	FitzRoy.
At sea (4 observations)	11 47	248 09	1827	5 54 E.	......	5·9 E.	......	......	......	......	......	......	Lütke.
At sea....................	13 09	251 20	1827	8 05 E.	......	8·1 E.	20 36	......	20·6 s.	7·89	......	7·89	Lütke.
At sea (5 observations)	11 23	257 05	1857	8 32 E.	......	8·5 E.	......	......	......	......	......	......	Richards.
At sea (5 observations)	17 17	260 34	1827	9 01 E.	......	9·0 E.	......	......	......	......	......	......	Lütke.
At sea (5 observations)	13 30	260 43	1857	9 49 E.	......	9·8 E.	......	......	......	......	......	......	Richards.
At sea....................	17 23	261 08	1827	10 23 E.	......	10·4 E.	......	......	......	......	......	......	Prussian ps.
At sea (2 observations)	15 02	263 21	1857	11 02 E.	......	11·0 E.	......	......	......	......	......	......	Richards.
At sea (4 observations)	12 21	265 05	1825	10 03 E.	......	10·0 E.	......	......	......	......	......	......	Prussian ps.
At sea (5 observations)	17 27	265 31	1857	11 46 E.	......	11·8 E.	......	......	......	......	......	......	Richards.
At sea (17 observations)	19 07	268 07	1857	12 56 E.	......	12·9 E.	......	......	......	......	......	......	Richards.
At sea (3 observations)	10 46	276 60	1830	10 33 E.	......	10·5 E.	......	......	......	......	......	......	Prussian ps.
At sea....................	10 52	281 37	1836	9 02 E.	......	9·0 E.	......	......	......	......	......	......	Bonite.
At sea (6 observations)	11 43	281 44	1832	10 31 E.	......	10·5 E.	......	......	......	......	......	......	Prussian ips.
Huacho Pt. (3 stations)	11 12	282 24	1835	9 54 E.	......	9·9 E.	......	......	......	......	......	......	FitzRoy.
At sea....................	15 43	282 38	1836	11 30 E.	......	11·5 E.	......	......	......	......	......	......	Bonite.
Callao....................	12 03	282 53	1823	9 30 E.	......	9·5 E.	8 33	......	8·5 s.	......	......		Duperrey
			1827	10 40 E.	......	10·7 E.	......	......	......	......	......		Prussian ips.
			1835	10 18 E.	......	10·3 E.	7 03	......	7·0 s.	......	......		FitzRoy.
			1836	10 23 E.	......	10·4 E. } 10·3 E.	......	......	...... } 7·0 s.	......	......	7·32	Bonite.
			1838	10 44 E.	......	10·7 E.	6 14	......	6·2 s.	7·59	......		Belcher.
			1838	10 17 E.	......	10·3 E.	6 49	......	6·8 s.	......	......		La Vénus
			1866	10 30 E.	......	10·5 E.	6 28	......	6·5 s.	7·05	......		Harkness
At sea....................	13 00	283 05	1823	8 02 E.	......	8·0 E.	8 26	......	8·4 s.	......	......	......	Duperrey
Lima	12 04	283 07	1858	10 40 E.	......	10·7 E.	7 10	......	7·2 s.	7·07	......	7·07	Friesach.
At sea....................	14 06	283 14	1823	9 33 E.	......	9·5 E.	9 55	......	9·9 s.	......	......	......	Duperrey
At sea....................	16 52	283 15	1823	9 16 E.	.....	9·3 E.	14 50	......	14·8 s.	......	......	......	Duperrey
At sea....................	19 43	283 19	1823	9 47 E.	......	9·8 E.	20 11	......	20·2 s.	......	......	......	Duperrey
Sangallan Island	13 50	283 29	1823	9 33 E.	......	9·5 E.	......	......	.....	......	......	......	Duperrey
Pisco (2 stations)	14 00	283 46	1835	10 15 E.	......	10·2 E.	......	......	......	......	......	......	FitzRoy.
At sea....................	17 53	284 50	1836	11 10 E.	......	11·2 E.	......	......	......	......	......	......	Bonite.
San Juan (3 stations) .	15 43	285 22	1835	10 50 E.	......	10·8 E.	......	......	......	......	......	......	FitzRoy.
At sea....................	19 42	286 44	1836	10 25 E.	......	10·4 E.	......	......	......	......	......	......	Bonite.
At sea (6 observations)	16 52	286 51	1829	10 38 E.	......	10·6 E.	......	......	......	......	......	......	Prussian ips.
Puno, Peru	15 50	287 18	1860	10 44 E.	......	10·7 E.	7 58	......	8·0 s.	6·77	......	6·77	Friesach
Jacua, Peru	18 01	287 28	1859	11 08 E.	......	11·1 E.	......	......	......	......	......	......	Friesach.
Mellends	17 02	287 53	1823	10 00 E.	......	10·0 E.	......	......	......	......	......	......	Lartique
Islay (4 stations)	17 05	288 14	1835	10 58 E.	......	11·0 E.	......	......	......	......	......	......	FitzRoy.
Arequipa	16 24	288 23	1835	12 27 E.	......	12·4 E.	10 00	......	10·0 s.	6·75	......	6·75	Friesach
Ylo........................	17 36	288 35	1823	10 15 E.	......	10·2 E.	......	......	......	......	......	......	Lartigue
La Paz, Bolivia.........	16 30	288 48	1860	10 30 E.	......	10·5 E.	8 38	......	8·6 s.	6·71	......	6·71	Friesach
Arica	18 28	289 35	1827	10 45 E.	......	10·7 E.	......	......	......	......	......		Prussian ips.
			1835	11 00 E.	......	11·0 E. } 10·9 E.	......	......	...... } 13·7 s.	......	......	6·85	FitzRoy
			1858	10 53 E.	......	10·9 E.	13 40	......	13·7 s.	6·85	......		Friesach

SOUTH EQUATORIAL ZONE II.—Lat. 10° to 20° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observers.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	° ′		°				
Mea f (4 stations)...	19 12	289 40	1835	11 30 E.	......	11·5 E.	......	......	......	......	......	......	FitzRoy.
At s	17 15	292 45	1837	4 36 w.	......	4·6 w.	......	......	......	......	......	......	Bonite.
At s	18 25	321 05	1843	3 10 w.	......	3·3 w.	......	......	......	......	......	......	Pasley.
At s 4 observations)	17 19	321 11	1832	1 18 w.	0 59 w.	2·3 w.	......	......	......	......	......	......	FitzRoy.
Abr os Island	17 58	321 18	1832	2 00 w.	0 59 w.	3·0 w.	......	......	......	......	......	......	FitzRoy.
Bahi	12 59	321 30	1832	4 18 w.	1 16 w.	5·6 w. } 4·7 w.	4 56	......	4·9 N. } 5·0 N.	......	......	} 6·33	FitzRoy.
			1837	......	......	......	5 35	......	5·6 N.	......	......		Wickham.
			1839	......	......	......	5 01	......	5·0 N.	6·43	......		Sulivan.
			1842	3 30 w.	......	3·5 w.	......	......	......	......	......		Bérard.
			1865	7 57 w.	3 00 E.	5·0 w.	4 24	......	4·4 N.	6·23	......		Harkness.
t se 5 observations)	14 48	321 30	1832	2 19 w.	1 19 w.	3·6 w.	......	......	......	......	......	......	FitzRoy.
t se 4 observations)	13 23	321 32	1832	1 52 w.	1 18 w.	3·2 w.	......	......	......	......	......	......	FitzRoy.
t se 4 observations)	17 36	321 32	1832	1 56 w.	1 19 w.	3·2 w.	......	......	......	......	......	......	FitzRoy.
t se 4 observations)	18 01	321 39	1832	2 19 w.	1 19 w.	3·6 w.	......	......	......	......	......	......	FitzRoy.
t se 3 observations)	13 12	321 43	1842	4 50 w.	......	4·8 w.	......	......	......	......	......	......	Bérard.
t se 2 observations)	15 24	322 05	1842	3 45 w.	......	3·7 w.	......	......	......	......	......	......	Bérard.
se 5 observations)	12 46	322 18	1836	3 15 w.	0 52 w.	4·1 w.	......	......	......	......	......	......	FitzRoy.
se	13 36	322 20	1839	......	......	......	4 22	......	4·4 N.	6·61	......	6·61	Sulivan.
se 3 observations)	15 43	322 46	1832	3 17 w.	1 18 w.	4·6 w.	......	......	......	......	......	......	FitzRoy.
se	18 35	323 00	1838	......	......	......	......	......	......	6·61	......	6·61	Sulivan.
se	17 47	323 24	1837	7 20 w.	0 31 w.	7·8 w.	......	......	......	......	......	......	Du Petit Thouars.
se	15 10	323 26	1839	......	......	......	1 37	......	1·6 N.	6·50	......	6·50	Sulivan.
se 2 observations)	18 50	323 26	1842	4 08 w.	......	4·1 w.	......	......	......	......	......	......	Bérard.
se 4 observations)	17 13	323 29	1829	0 26 w.	1 40 w.	2·1 w.	......	......	......	......	......	......	Rumker.
se 2 observations)	12 49	323 37	1836	4 18 w.	0 49 w.	5·1 w.	......	......	......	......	......	......	FitzRoy.
sea	16 00	323 42	1838	......	......	......	1 25	......	1·4 s.	6·61	......	6·61	Sulivan.
sea	17 41	323 43	1847	7 00 w.	0 34 w.	6·6 w.	......	......	......	......	......	......	Stanley.
sea observations)	17 35	323 47	1847	......	......	......	4 17	......	4·3 s.	6·33	......	6·33	Rattlesnake.
sea	15 44	323 58	1839	......	......	......	0 22	......	0·4 s.	6·47	......	6·47	Sulivan.
sea observations)	16 28	324 04	1837	7 28 w.	0 42 w.	8·2 w.	......	......	......	......	......	......	Du Petit Thouars.
sea observations)	18 10	324 04	1832	3 02 w.	1 18 w.	4·3 w.	......	......	......	......	......	......	FitzRoy.
sea	18 48	324 16	1836	3 41 w.	0 42 w.	4·4 w.	......	......	......	......	......	......	Bonite.
sea observations)	12 01	324 17	1842	8 00 w.	......	8·0 w.	......	......	......	......	......	......	Bérard.
sea	17 52	324 36	1839	......	......	......	3 37	......	3·6 s.	6·40	......	6·40	Sulivan.
sea	13 50	324 41	1838	......	......	......	1 50	......	1·8 N.	6·60	......	6·60	Sulivan.
sea observations)	12 39	324 42	1829	3 23 w.	1 34 w.	5·0 w.	......	......	......	......	......	......	Rumker.
sea.	15 10	324 45	1837	7 35 w.	0 38 w.	8·2 w.	......	......	......	......	......	......	Du Petit Thouars.
sea.	19 45	324 50	1830	3 25 w.	1 27 w.	4·9 w.	......	......	......	......	......	......	Erman.
sea observations)	17 48	324 52	1826	3 11 w.	1 55 w.	5·1 w.	......	......	......	......	......	......	Lütke.
sea.	18 59	324 52	1830	4 39 w.	1 27 w.	6·1 w.	7 25	......	7·4 s.	5·86	......	5·86	Erman.
sea.	19 36	324 56	1830	4 06 w.	1 27 w.	5·5 w.	7 40	......	7·7 s.	5·82	......	5·82	Erman.
sea observations)	18 14	325 01	1829	1 34 w.	1 30 w.	3·1 w.	......	......	......	......	......	......	Prussian ships.
sea.	19 56	325 05	1839	......	......	......	7 07	......	7·1 s.	6·30	......	6·30	Sulivan.
sea.	18 30	325 09	1830	4 28 w.	1 45 w.	6·2 w.	......	......	......	......	......	......	Erman.
sea observations)	18 00	325 18	1830	4 40 w.	1 45 w.	6·4 w.	......	......	......	......	......	... ..	Prussian ships.
sea observations)	14 52	325 25	1847	8 13 w.	0 24 E.	7·8 w.	0 35	......	0·6 s.	6·37	......	6·37	Rattlesnake.
sea.	16 48	325 41	1836	4 56 w.	0 45 w.	5·7 w.	......	......	......	......	......	......	Bonite.
sea.	10 08	325 42	1838	......	......	......	10 50	... ..	10·8 N.	6·82	......	6·82	Sulivan.
sea.	17 32	325 48	1830	4 31 w.	1 27 w.	6·0 w.	4 34	......	4·6 s.	5·73	......	5·73	Erman.
sea.	11 55	325 50	1837	8 12 w.	0 38 w.	8·8 w.	......	......	......	......	......	......	Du Petit Thouars.
sea.	16 45	326 02	1830	5 15 w.	1 27 w.	6·7 w.	......	......	......	......	......	......	Erman.
sea (bservations)	16 07	326 26	1830	......	......	......	2 00	......	2·0 s.	5·89	......	5·89	Erman.
sea.	14 59	326 40	1830	6 50 w.	1 27 w.	8·3 w.	0 27	......	0·4 N.	6·12	......	6·12	Erman.
sea.	15 12	326 51	1836	6 45 w.	0 45 w.	7·5 w.	......	......	......	......	......	......	Bonite.
sea (bservations)	13 43	327 00	1823	4 55 w.	2 16 w.	7·2 w.	......	......	......	......	......	......	Prussian ships.
sea (bservations)	12 23	327 01	1826	5 24 w.	1 55 w.	7·3 w.	......	......	......	......	......	......	Lütke.
sea..	14 24	327 01	1830	6 08 w.	1 27 w.	7·6 w.	1 30	......	1·5 N.	6·26	......	6·26	Erman.
sea..	13 22	327 15	1830	6 43 w.	1 27 w.	8·2 w.	3 20	......	3·3 N.	5·93	......	5·93	Erman.
sea..	12 38	327 36	1830	7 16 w.	1 27 w.	8·7 w.	......	......	......	......	......	......	Erman.
sea (bservations)	12 38	327 36	1847	9 56 w.	0 31 E.	9·4 w.	......	......	......	......	......	......	Stanley.
sea..	11 44	327 39	1830	7 25 w.	1 27 w.	8·9 w.	......	......	......	......	......	......	Erman.
sea..	11 31	327 41	1830	......	......	......	6 30	......	6·5 N.	......	......	......	Erman.

SOUTH EQUATORIAL ZONE II.—Lat. 10° to 20° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observs.
				Observed.	Correction to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	° ′		°				
At sea (3 observations)	12 40	327 43	1847	......	......	......	3 06	......	3·1 N.	6·37	......	6·37	Rattlesna
At sea......................	10 49	327 45	1842	10 45 W.	......	10·7 W.	......	......	......	......	......	......	Bérard.
At sea......................	10 58	327 52	1830	7 11 W.	1 27 W.	8·6 W.	......	......	......	......	......	......	Erman.
At sea......................	12 48	327 58	1847	9 01 W.	0 31 E.	8·5 W.	......	......	......	......	......	......	Stanley.
At sea (4 observations)	13 09	327 59	1828	5 25 W.	1 42 W.	7·1 W.	......	......	......	......	......	......	Prussian ps.
At sea (2 observations)	16 08	328 13	1833	4 54 W.	1 15 W.	6·1 W.	......	......	......	......	......	......	Prussian ps.
At sea......................	11 03	328 22	1839	......	......	......	......	......	......	6·38	......	6·38	Erebus.
At sea......................	13 15	328 34	1836	8 06 W.	0 45 W.	8·8 W.	......	......	......	......	......	......	Bonite.
At sea......................	12 32	328 57	1839	......	......	......	......	......	......	6·29	......	6·29	Erebus.
At sea......................	12 28	329 00	1839	8 25 W.	0 24 W.	8·8 W.	......	......	......	......	......	......	Ross.
At sea......................	18 11	329 06	1846	9 11 W.	0 24 E.	8·8 W.	......	......	......	......	......	......	Sulivan.
At sea......................	12 29	329 08	1846	9 13 W.	0 24 E.	8·8 W.	......	......	......	......	......	......	Sulivan.
At sea (2 observations)	12 37	329 10	1833	7 30 W.	1 15 W.	8·7 W.	......	......	......	......	......	......	Prussian ips.
At sea......................	15 35	329 16	1846	8 31 W.	0 24 E.	8·1 W.	......	......	......	......	......	......	Sulivan.
At sea......................	17 20	329 16	1846	9 01 W.	0 24 E.	8·6 W.	......	......	......	......	......	......	Sulivan.
At sea......................	14 17	329 19	1846	9 00 W.	0 24 E.	8·6 W.	......	......	......	......	......	......	Sulivan.
At sea......................	10 15	329 25	1847	12 28 W.	0 31 E.	12·0 W.	......	......	......	......	......	......	Stanley.
At sea (2 observations)	13 42	329 26	1839	8 29 W.	0 24 W.	8·9 W.	......	......	......	......	......	......	Ross.
At sea......................	14 00	329 28	1839	......	......	......	......	......	......	6·16	......	6·16	Erebus.
At sea......................	14 55	329 53	1839	8 33 W.	0 24 W.	9·0 W.	......	......	......	......	......	......	Ross.
At sea......................	19 02	329 58	1846	8 58 W.	0 21 E.	8·6 W.	......	......	......	......	......	......	Sulivan.
At sea......................	15 04	330 06	1839	......	......	......	......	......	......	6·16	......	6·16	Erebus.
At sea (3 observations)	14 00	330 17	1842	......	......	......	0 12	......	0·2 N.	6·50	......	6·50	H.M.S. 'y.'
At sea (3 observations)	15 45	330 20	1842	......	......	......	3 13	......	3·2 S.	6·41	......	6·41	H.M.S. 'y.'
At sea......................	11 46	330 27	1836	8 00 W.	0 45 W.	8·7 W.	......	......	......	......	......	......	Bonite.
At sea......................	16 52	330 27	1839	......	......	......	......	......	......	6·12	......	6·12	Erebus.
At sea......................	16 57	330 30	1839	9 09 W.	0 24 W.	9·5 W.	......	......	......	......	......	......	Ross.
At sea (3 observations)	18 10	330 33	1842	......	......	......	6 58	......	7·0 S.	6·30	......	6·30	H.M.S. 'y.'
At sea......................	18 02	330 36	1839	9 28 W.	0 24 W.	9·9 W.	......	......	......	......	......	......	Erebus.
At sea......................	19 07	330 42	1839	9 48 W.	0 24 W.	10·2 W.	......	......	......	......	......	......	Ross.
At sea......................	19 01	330 45	1839	......	......	......	......	......	......	6·10	......	6·10	Erebus.
At sea (3 observations)	11 30	330 48	1842	......	......	......	4 40	......	4·7 N.	6·51	......	6·51	H.M.S. 'y.'
At sea (2 observations)	12 08	330 55	1836	8 43 W.	0 45 W.	8·5 W.	......	......	......	......	......	......	FitzRoy.
At sea......................	12 08	331 05	1836	8 40 W.	0 45 W.	9·4 W.	......	......	......	......	......	......	FitzRoy.
At sea......................	10 05	331 48	1836	9 24 W.	0 45 W.	10·1 W.	......	......	......	......	......	......	Bonite.
At sea (3 observations)	17 52	332 07	1826	7 18 W.	1 39 W.	9·0 W.	......	......	......	......	......	......	Dumont Urville
At sea......................	19 47	332 45	1831	......	......	......	10 13	......	10·2 S.	6·44	......	6·44	Dunlop.
At sea......................	18 09	332 48	1831	......	......	......	7 10	......	7·2 S.	6·29	......	6·29	Dunlop.
At sea (2 observations)	11 47	333 06	1859	14 28 W.	1 55 E.	12·5 W.	......	......	......	......	......	......	Novara.
At sea......................	16 12	333 10	1831	......	......	......	4 02	......	4·0 S.	6·59	......	6·59	Dunlop.
At sea......................	19 22	333 37	1859	13 52 W.	1 55 E.	12·0 W.	......	......	......	......	......	......	Novara.
At sea......................	14 31	334 30	1831	......	......	......	1 05	......	1·1 S.	6·52	......	6·52	Dunlop.
At sea (2 observations)	18 44	335 34	1850	......	......	......	9 29	......	9·5 S.	6·54	......	6·54	Rattlesn
At sea (2 observations)	15 59	336 00	1850	......	......	......	4 58	......	5·0 S.	6·45	......	6·45	Rattlesn
At sea......................	12 51	336 10	1831	......	......	......	0 35	......	0·6 N.	6·59	......	6·59	Dunlop.
At sea (2 observations)	14 20	336 14	1850	......	......	......	1 34	......	1·6 S.	6·60	......	6·60	Rattlesn
At sea......................	11 25	336 37	1836	13 06 W.	0 52 W.	14·0 W.	......	......	......	......	......	......	FitzRoy.
At sea......................	10 12	337 42	1831	......	......	......	1 38	......	1·6 N.	6·52	......	6·52	Dunlop.
At sea......................	10 06	340 27	1842	15 23 W.	......	15·4 W.	......	......	......	......	......	......	Jehenne.
At sea......................	10 07	341 02	1836	15 57 W.	0 52 W.	16·8 W.	......	......	......	......	......	......	FitzRoy.
At sea......................	11 10	342 37	1842	16 00 W.	......	16·0 W.	......	......	......	......	......	......	Jehenne.
At sea (4 observations)	11 38	343 46	1842	17 10 W.	......	17·2 W.	......	......	......	......	......	......	Jehenne.
At sea......................	11 59	344 09	1846	20 58 W.	0 28 E.	20·5 W.	......	......	......	......	......	......	Bérard.
At sea......................	12 09	345 01	1842	17 36 W.	......	17·6 W.	......	......	......	......	......	......	Jehenne.
At sea......................	19 20	345 44	1840	18 44 W.	0 12 W.	18·9 W.	......	......	......	......	......	......	Ross.
At sea......................	19 12	345 45	1840	......	......	......	......	......	......	6·11	......	6·11	Terror.
At sea......................	18 53	345 46	1840	......	......	......	......	......	......	6·12	......	6·12	Erebus.
At sea......................	18 43	346 00	1840	17 46 W.	0 13 W.	18·0 W.	......	......	......	......	......	......	Ross.
At sea......................	17 44	346 10	1840	......	......	......	......	......	......	6·12	......	6·12	Terror.
At sea......................	17 30	346 15	1840	......	......	......	......	......	......	6·14	......	6·14	Erebus.
At sea......................	17 37	346 23	1840	19 26 W.	0 13 W.	19·6 W.	......	......	......	......	......	......	Ross.
At sea......................	17 23	346 29	1840	......	......	......	......	......	......	6·11	......	6·11	Terror.

SOUTH EQUATORIAL ZONE II.—Lat. 10° to 20° S. (continued).

tations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observers.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	° ′		°				
t sea ………………	17 11	346 40	1840	……	……	……	……	……	……	6·12	……	……	Erebus.
t sea ………………	17 08	346 43	1840	20 12 w.	0 13 w.	20·4 w.	……	……	……	……	……	……	Ross.
t sea ………………	13 07	347 00	1842	19 41 w.	……	19·7 w.	……	……	……	……	……	……	Jehenne.
t sea ………………	16 43	347 02	1840	……	……	……	……	……	……	6·09	……	……	Terror.
t sea ………………	16 41	347 07	1840	19 32 w.	0 15 w.	19·8 w.	……	……	……	……	……	……	Ross.
t sea ………………	16 30	347 17	1840	……	……	……	……	……	……	6·11	……	……	Erebus.
t sea ………………	16 22	347 25	1840	18 18 w.	0 15 w.	18·5 w.	……	……	……	……	……	……	Ross.
t sea ………………	15 25	347 50	1840	……	……	……	……	……	……	6·12	……	……	Terror.
t sea ………………	10 34	347 54	1837	18 05 w.	0 44 w.	18·8 w.	……	……	……	……	……	……	Bonite.
t sea ………………	15 22	347 58	1840	20 52 w.	0 17 w.	21·1 w.	……	……	……	……	……	……	Ross.
t sea ………………	15 30	347 58	1840	……	……	……	……	……	……	6·15	……	……	Terror.
t sea ………………	15 15	348 01	1840	……	……	……	……	……	……	6·15	……	……	Erebus.
sea ………………	15 24	348 06	1840	20 40 w.	0 17 w.	21·0 w.	……	……	……	……	……	……	Ross.
sea ………………	15 20	348 07	1840	……	……	……	……	……	……	6·09	……	……	Erebus.
sea ………………	15 49	348 09	1840	……	……	……	……	……	……	6·11	……	……	Erebus.
sea ………………	15 41	348 09	1840	……	……	……	……	……	……	6·34	……	……	Terror.
sea ………………	15 31	348 15	1840	20 16 w.	0 17 w.	20·5 w.	……	……	……	……	……	……	Ross.
sea ………………	15 44	348 15	1840	19 10 w.	0 17 w.	19·4 w.	……	……	……	……	……	……	Ross.
sea ………………	15 37	348 27	1840	19 11 w.	0 17 w.	19·5 w.	……	……	……	……	……	……	Ross.
sea ………………	15 37	348 32	1840	……	……	……	……	……	……	6·12	……	……	Terror.
sea ………………	15 40	348 35	1840	18 56 w.	0 17 w.	19·2 w.	……	……	……	……	……	……	Ross.
sea ………………	15 30	348 51	1840	……	……	……	……	……	……	6·07	……	……	Erebus.
sea observations)	11 12	348 54	1851	21 52 w.	1 30 E.	20 4 w.	……	……	……	……	……	……	Kellett.
sea observations)	12 13	349 16	1832	19 14 w.	1 24 w.	20 6 w.	……	……	……	……	……	……	Prussian ships.
sea observations)	11 57	348 56	1846	……	……	……	11 43 s.	……	11·7 s.	6·26	……	……	H.M.S. 'Fly.
sea. ………………	14 45	349 22	1840	……	……	……	……	……	……	6·13	……	……	Terror.
sea. ………………	10 47	349 31	1825	18 40 w.	2 20 w.	21·0 w.	3 04 s.	……	3·1 s.	……	……	……	Duperrey.
sea. ………………	11 22	349 32	1839	20 14 w.	0 28 w.	20·7 w.	……	……	……	……	……	……	Du Petit Thouars.
sea. ………………	14 33	349 37	1840	……	……	……	……	……	……	6·11	……	……	Erebus.
sea. ………………	12 25	349 42	1837	18 40 w.	0 44 w.	19·4 w.	……	……	……	……	……	……	Bonite.
sea. ………………	14 22	349 54	1840	19 31 w.	0 17 w.	19·8 w.	……	……	……	……	……	……	Ross.
sea observations)	12 53	350 08	1829	20 39 w.	1 34 w.	22·2 w.	……	……	……	……	……	……	Lütke.
sea. ………………	14 28	350 19	1840	……	……	……	……	……	……	6·21	……	……	Terror.
sea. ………………	13 44	350 20	1840	……	……	……	……	……	……	6·20	……	……	Terror.
sea. ………………	14 26	350 21	1840	……	……	……	……	……	……	6·19	……	……	Terror.
sea.. ………………	14 54	350 25	1840	……	……	……	……	……	……	6·27	……	……	Terror.
sea.. ………………	14 53	350 26	1840	……	……	……	……	……	……	6·11	……	……	Terror.
sea.. ………………	14 08	350 28	1840	……	……	……	……	……	……	6·26	……	……	Terror.
sea.. ………………	14 11	350 28	1840	21 24 w.	0 15 w.	21·6 w.	……	……	……	……	……	……	Ross.
sea.. ………………	14 38	350 30	1840	21 33 w.	0 15 w.	21·8 w.	……	……	……	……	……	……	Ross.
sea.. ………………	14 51	350 31	1840	……	……	……	……	……	……	6·11	……	……	Erebus.
sea.. ………………	13 37	350 33	1840	……	……	……	……	……	……	6·16	……	……	Erebus.
sea.. ………………	14 19	350 33	1840	……	……	……	……	……	……	6·09	……	……	Erebus.
sea.. ………………	13 02	350 53	1836	19 56 w.	0 39 w.	20·6 w.	……	……	……	……	……	……	FitzRoy.
sea (bservations)	13 32	351 00	1846	……	……	……	16 14 s.	……	16·2 s.	6·17	……	……	H.M.S. 'Fly.'
sea.. ………………	12 40	351 05	1839	21 16 w.	0 21 w.	21·6 w.	……	……	……	……	……	……	Du Petit Thouars.
sea.. ………………	13 06	351 11	1825	18 45 w.	1 45 w.	20·5 w.	8 47 s.	……	8 8 s.	……	……	……	Duperrey.
sea (bservations)	14 14	351 19	1840	21 13 w.	0 15 w.	21·5 w.	……	……	……	……	……	……	Ross.
sea.. ………………	14 08	351 31	1840	……	……	……	……	……	……	6·18	……	……	Erebus.
sea.. ………………	14 08	351 32	1840	21 32 w.	0 15 w.	21·8 w.	……	……	……	……	……	……	Ross.
sea.. ………………	14 55	351 52	1840	……	……	……	……	……	……	6·21	……	……	Terror.
sea.. ………………	14 19	351 53	1840	……	……	……	……	……	……	6·26	……	……	Terror.
sea.. ………………	14 36	351 53	1840	21 08 w.	0 15 w.	21·4 w.	……	……	……	……	……	……	Ross.
sea.. ………………	15 03	351 54	1840	……	……	……	……	……	……	6·18	……	……	Erebus.
sea.. ………………	14 26	351 57	1840	……	……	……	……	……	……	6·18	……	……	Erebus.
sea.. ………………	15 04	351 57	1840	21 10 w.	0 15 w.	21·4 w.	……	……	……	……	……	……	Ross.
sea (bservations)	13 34	351 59	1839	21 47 w.	0 21 w.	22·1 w.	……	……	……	……	……	……	Du Petit Thouars.
sea.. ………………	13 36	352 00	1840	……	……	……	……	……	……	6·25	……	……	Terror.
sea.. ………………	14 11	352 00	1840	21 06 w.	0 15 w.	21·3 w.	……	……	……	……	……	……	Ross.
sea.. ………………	13 43	352 01	1840	……	……	……	……	……	……	6·16	……	……	Erebus.
sea.. ………………	13 34	352 02	1840	22 09 w.	0 15 w.	22·4 w.	……	……	……	……	……	……	Ross.
sea.. ………………	15 14	352 03	1840	……	……	……	……	……	……	6·22	……	……	Terror.

SOUTH EQUATORIAL ZONE II.—Lat. 10° to 20° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observers.
				Observed.	Correction to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′		°	° ′		°				
At sea	15 23	352 06	1840	......	......	......	......	......	......	6·13	......	......	Erebus.
At sea (2 observations)	13 42	352 12	1836	19 52 w.	0 39 w.	20·5 w.	......	......	......	......	......	......	Prussian ships.
At sea	15 26	352 20	1840	21 39 w.	0 15 w.	21·9 w.	......	......	......	......	......	......	Ross.
At sea	15 27	352 30	1840	21 27 w.	0 15 w.	21·7 w.	......	......	......	......	......	......	Ross.
At sea (4 observations)	14 31	352 31	1851	23 16 w.	1 10 E.	22·1 w.	......	......	......	......	......	......	Kellett.
At sea	15 11	352 32	1840	......	......	......	......	......	......	6·18	......	......	Terror.
At sea	15 13	352 33	1840	......	......	......	......	......	......	6·28	......	......	Terror.
At sea	15 17	352 35	1840	......	......	......	......	......	......	6·14	......	......	Erebus.
At sea	14 30	352 40	1836	19 50 w.	0 39 w.	19·5 w.	......	......	......	......	......	......	FitzRoy.
At sea	18 53	352 45	1840	......	......	......	......	......	......	6·19	......	......	Erebus.
At sea	14 28	352 52	1839	22 18 w.	0 21 w.	22·6 w.	......	......	......	......	......	......	Du Petit Thouars.
At sea	18 33	352 52	1840	......	......	......	......	......	......	6·16	......	......	Terror.
At sea	18 29	352 55	1840	23 11 w.	0 15 w.	23·4 w.	......	......	......	......	......	......	Ross.
At sea (3 observations)	15 00	352 58	1846	......	......	......	19 53 s.	......	19·9 s.	6·10	......	......	H.M.S. 'Fly.'
At sea (8 observations)	16 14	353 00	1840	21 52 w.	0 15 w.	22·1 w.	......	......	......	......	......	......	Ross.
At sea	15 19	353 06	1840	......	......	......	......	......	......	6·26	......	......	Terror.
At sea	15 20	353 07	1840	......	......	......	......	......	......	6·21	......	......	Terror.
At sea	15 19	353 13	1840	......	......	......	......	......	......	6·14	......	......	Erebus.
At sea	17 38	353 19	1840	21 37 w.	0 15 w.	21·9 w.	......	......	......	......	......	......	Ross.
At sea	15 15	353 21	1840	21 29 w.	0 15 w.	21·7 w.	......	......	......	......	......	......	Ross.
At sea	17 26	353 28	1840	......	......	......	......	......	......	6·16	......	......	Terror.
At sea	17 10	353 33	1840	20 54 w.	0 15 w.	21·1 w.	......	......	......	......	......	......	Ross.
At sea	17 14	353 33	1840	......	......	......	......	......	......	6·15	......	......	Terror.
At sea	15 00	353 36	1840	......	......	......	......	......	......	6·12	......	......	Terror.
At sea	15 02	353 36	1840	......	......	......	......	......	......	6·17	......	......	Terror.
At sea	15 07	353 44	1840	......	......	......	......	......	......	6·21	......	......	Erebus.
At sea	15 10	354 05	1840	......	......	......	......	......	......	6·26	......	......	Terror.
At sea	15 21	354 07	1840	23 27 w.	0 15 w.	23·7 w.	......	......	......	......	......	......	Ross.
At sea	15 05	354 08	1840	......	......	......	......	......	......	6·18	......	......	Erebus.
St. Helena Observatory, Longwood*	15 55	354 17	1840	22 53 w.	......	23·2 w.	......	......	......	6·08	......	......	Ross.
			1840	......	......		21 15 s.	......	21·6 s.	......	......	......	Observatory.
			1841	22 58 w.	......		21 26 s.	......		......	......	......	Observatory.
			1842	23 05 w.	......		21 25 s.	......		......	......	......	Observatory.
			1843	23 14 w.	......		21 45 s.	......		......	......	......	Observatory.
			1844	23 20 w.	......		21 56 s.	......		......	......	......	Observatory.
			1845	23 27 w.	......		21 55 s.	......		......	......	......	Observatory.
			1846	......	......	......	22 14 s.	......	......	......	......	......	Observatory.
			1847	......	......	......	22 37 s.	......	......	......	......	......	Observatory.
			1848	......	......	......	22 49 s.	......	......	......	......	......	Observatory.
Sister's Walk, St. Helena	15 55	354 18	1840	......	......	......	......	......	......	6·28	......	......	Crozie
			1840	......	......	......	18 21 s.	......	......	6·28	......	......	Ross.
			1842	......	......	......	17 01 s.	......	......	......	......	......	Belche
			1846	......	......	......	19 23 s.	......	......	6·29	......	......	Smyth
Anchorage, St. Helena	15 55	354 17	1836	18 00 w.	......	......	......	......	......	......	......	......	FitzRc
			1839	22 17 w.	......	......	17 55 s.	......	......	......	......	......	Du Pe Thouars
			1846	23 11 w.	......	......	......	......	......	......	......	......	Bérarc
At sea	15 40	354 19	1840	......	......	......	......	......	......	6·13	......	......	Terror
At sea	15 57	354 26	1836	19 43 w.	0 32 w.	20·2 w.	......	......	......	......	......	......	FitzRc
At sea (3 observations)	16 08	354 39	1846	......	......	......	22 05 s.	......	22·1 s.	6·16	......	......	H.M.S Fly.'
At sea	16 27	355 24	1842	24 03 w.	......	24·0 w.	......	......	......	......	......	......	Jehen
At sea (3 observations)	19 32	355 51	1846	22 46 w.	0 17 E.	22·5 w.	......	......	......	......	......	......	Bérarc
At sea (5 observations)	18 01	356 10	1846	23 38 w.	0 17 E.	23·3 w.	......	......	......	......	......	......	Bérarc
At sea (2 observations)	17 40	356 20	1842	24 39 w.	......	24·6 w.	......	......	......	......	......	......	Jehen
At sea (3 observations)	17 11	356 16	1846	......	......	......	24 06 s.	......	24·1 s.	6·17	......	......	H.M.S Fly.'
At sea (3 observations)	17 33	356 21	1834	20 17 w.	0 42 w.	21·0 w.	......	......	......	......	......	......	Prussi ships.
At sea (8 observations)	17 12	356 22	1851	24 53 w.	1 05 E.	23·8 w.	......	......	......	......	......	......	Kellet
At sea (3 observations)	17 13	356 24	1846	23 50 w.	0 18 E.	23·5 w.	......	......	......	......	......	......	Bérar
At sea	17 04	356 32	1837	22 28 w.	0 27 w.	22·9 w.	......	......	......	......	......	......	Bonite
At sea (3 observations)	18 19	357 08	1829	23 09 w.	1 08 w.	24·3 w.	......	......	......	......	......	......	Lütke
At sea	18 52	357 15	1842	25 15 w.	......	25·3 w.	......	......	......	......	......	......	Jehen
At sea (3 observations)	18 01	357 30	1829	20 36 w.	1 08 w.	21·7 w.	......	......	......	......	......	......	Pruss ships.
At sea (3 observations)	18 20	358 14	1846	......	......	......	27 02 s.	......	27·0 s.	6·28	......	......	H.M. Fly.'

* The observations at St. Helena and the adjacent anchorage being so much affected by local attraction, the only data entered in the Map are those obtained at the Observatory, the site of which (Longwood) was carefully decided on as the least objectionable place.

SOUTH EQUATORIAL ZONE III.—LATITUDE 20° TO 30° S.

Authorities.

Bonite	Voyage de la Bonite (Paris, 1842).
FitzRoy	Voyage of the 'Beagle,' 1849.
Du Petit Thouars	Sabine in Philosophical Transactions, 1849.
Moore	Contributions to Terrestrial Magnetism (Sabine in Phil. Trans. 1840).
Clerk	
Dayman	
Sulivan	
Dunlop	
Pagoda	
Erebus	
Terror	
La Vénus	Voyage de la Vénus (Paris, 1841).
Bérard	MSS. received from Admiral Duperrey.
Blosseville	Schlagintweit, Sci. Mission to India and High Asia (Leipzig and London, 1861).
Novara (Austrian Frigate)	Reise um die Erde (Wien, 1862–65).
Denham	MSS. in the Magnetic Office, Kew Observatory, received from the Hydrographic Office.
Belcher	MS. in the Mag. Office, and Contrib. to Terr. Mag. (Sabine in Phil. Trans. 1840).
Erman	Reise um die Erde (Berlin, 1841).
Prussian Ships	L. S. Kämtz, MSS. in the Magnetic Office, Kew Observatory.
Jehenne	
Duperrey	
Owen	
Burchell	
Dumont d'Urville	
Young	
King	
Laplace	
Rumker	
Foster	
Biscoe	
Lütke	
Friesach	Mem. Imp. Acad. of Sciences, Vienna, vols. xxix. to xliv.
Beechey	Contributions to Terrestrial Magnetism (Sabine in Phil. Trans. 1840).
Helmreicher	Sabine in Phil. Trans. 1849, and L. S. Kämtz, MS.
Harkness	Smithsonian Contrib. vol. xviii.
Stanley	Sabine in Phil. Trans. 1849.
Pasley	Sabine in Phil. Trans. 1849.
Kellett	MSS. in the Magnetic Office, Kew Observatory, received from Admiral Kellett.
Ross	Sabine in Phil. Trans. 1849.
Rattlesnake	MSS. in the Hydrographic Office.
H.M.S. 'Fly'	MSS. in the Hydrographic Office.
H.M.S. 'Hecate'	MSS. in the Hydrographic Office.
H.M.S. 'Nassau'	MSS. in the Hydrographic Office.
Richards	H.M.S. 'Plumper;' MSS. in the Hydrographic Office.

SOUTH EQUATORIAL ZONE III.—Lat. 20° to 30° S.

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observers.
				Observed.	Correction to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	° ′		°				
At sea (4 observations)	21 16	0 28	1851	25 18 w.	0 50 E.	24·5 w.	......	......	......	......	......	......	Kellett.
At sea (3 observations)	21 50	0 59	1829	23 10 w.	1 12 w.	24·4 w.	......	......	......	......	......	......	Prussian ships.
At sea (3 observations)	22 25	2 07	1829	23 57 w.	1 12 w.	25·1 w.	......	......	......	......	......	......	Lütke.
At sea....................	20 51	2 38	1839	25 27 w.	0 14 w.	25·7 w.	......	......	......	......	......	......	Du Petit Thouars.
At sea (4 observations)	24 10	2 58	1851	26 06 w.	0 40 E.	25·4 w.	......	......	......	......	......	......	Kellett.
At sea (3 observations)	21 30	3 00	1846	......	......	......	34 39 s.	......	34·6 s.	6·34	......	6·34	H.M.S. ‘Fly.’
At sea (4 observations)	23 04	4 36	1829	24 08 w.	1 12 w.	25·3 w.	......	......	......	......	......	......	Prussian ships.
At sea (2 observations)	22 37	4 51	1836	24 11 w.	0 27 w.	24·6 w.	......	......	......	......	......	......	FitzRoy.
At sea (6 observations)	26 03	5 30	1851	26 50 w.	0 40 E.	26·2 w.	......	......	......	......	......	......	Kellett.
At sea (3 observations)	23 35	5 35	1846	......	......	......	38 53 s.	......	38·9 s.	6·56	......	6·56	H.M.S. ‘Fly.’
At sea (3 observations)	26 55	6 03	1829	25 45 w.	1 12 w.	27·0 w.	......	......	......	......	......	......	Lütke.
At sea....................	25 54	7 28	1839	26 28 w.	0 27 w.	26·9 w.	......	......	......	......	......	......	Du Petit Thouars.
At sea (3 observations)	25 09	7 30	1846	......	......	......	42 16 s.	......	42·3 s.	6·58	......	6·58	H.M.S. ‘Fly.’
At sea....................	26 35	7 33	1839	26 28 w.	0 27 w.	26·9 w.	......	......	......	......	......	......	Du Petit Thouars.
At sea....................	27 35	8 05	1842	28 00 w.	0 02 w.	28·0 w.	......	......	......	......	......	......	Jehenne.
At sea (2 observations)	27 49	8 19	1832	27 03 w.	0 42 w.	27·7 w.	......	......	......	......	......	......	Prussian ships.
At sea....................	27 23	8 32	1839	27 08 w.	0 14 w.	27·4 w.	......	......	......	......	......	......	Du Petit Thouars.
At sea (4 observations)	27 21	8 38	1839	26 44 w.	0 14 w.	27·0 w.	......	......	......	......	......	......	Du Petit Thouars.
At sea....................	29 02	9 18	1824	25 30 w.	1 14 w.	26·7 w.	41 17 s.	......	41·3 s.	......	......	......	Duperrey.
At sea (3 observations)	29 05	9 47	1851	27 43 w.	0 20 E.	27·4 w.	......	......	......	......	......	......	Kellett.
At sea....................	27 48	9 55	1837	26 06 w.	0 22 w.	26·5 w.	......	......	......	......	......	......	Bonite.
At sea (3 observations)	26 52	9 56	1846	......	......	......	45 07 s.	......	45·1 s.	6·83	......	6·83	H.M.S. ‘Fly.’
At sea....................	24 31	10 34	1837	25 11 w.	0 22 w.	25·5 w.	......	......	......	......	......	......	Bonite.
At sea....................	28 26	10 53	1837	23 50 w.	0 22 w.	24·2 w.	......	......	......	......	......	......	Bonite.
At sea....................	29 33	10 58	1839	26 51 w.	0 07 w.	27·0 w.	......	......	......	......	......	......	Du Petit Thouars.
At sea (3 observations)	28 24	12 17	1846	......	......	......	46 56 s.	......	46·9 s.	7·05	......	7·05	H.M.S. ‘Fly.’
At sea....................	29 45	13 08	1837	28 15 w.	0 11 w.	28·4 w.	......	......	......	......	......	......	Bonite.
Cape Voltas	28 44	16 26	1825	29 00 w.	0 35 w.	29·6 w.	......	......	......	......	......	......	Owen.
Litakou	27 06	24 35	1824	27 07 w.	......	27·1 w.	......	......	......	......	......	......	Burchell.
At sea (2 observations)	27 59	34 43	1836	21 16 w.	......	21·3 w.	......	......	......	......	......	......	FitzRoy.
At sea (6 observations)	28 57	37 45	1845	......	......	......	59 14 s.	......	59·2 s.	......	......	......	Moore.
At sea (7 observations)	28 57	37 49	1845	23 37 w.	......	23·6 w.	......	......	......	......	......	......	Pagoda.
At sea (4 observations)	28 57	37 52	1845	......	......	......	59 08 s.	......	59·1 s.	8·20	......	8·20	Clerk.
At sea (5 observations)	28 57	37 55	1845	......	......	......	......	......	......	8·32	......	8·32	Moore.
At sea (2 observations)	29 05	38 38	1834	23 37 w.	......	23·6 w.	......	......	......	......	......	......	Prussian ships.
At sea (3 observations)	28 48	38 50	1846	......	......	......	60 38 s.	......	60·6 s.	8·54	......	8·54	H.M.S. ‘Fly.’
At sea....................	29 21	39 06	1845	......	......	......	60 01 s.	......	60·0 s.	8·10	......	8·10	Dayman.
Bassa da India	21 29	39 35	1825	21 54 w.	......	21·9 w.	......	......	......	......	......	......	Owen.
At sea (5 observations)	28 36	40 14	1845	22 34 w.	......	22·6 w.	......	......	......	......	......	......	Pagoda.
At sea (12 observations)	29 32	40 14	1861	24 23 w.	......	24·4 w.	......	......	......	......	......	......	Denham.
Europa Island	22 22	40 19	1825	21 00 w.	......	21·0 w.	......	......	......	......	......	......	Owen.
At sea (4 observations)	28 35	40 24	1845	......	......	......	58 54 s.	......	58·9 s.	8·45	......	8·45	Clerk.
At sea (3 observations)	27 28	40 44	1836	21 08 w.	......	21·1 w.	......	......	......	......	......	......	FitzRoy.
At sea (6 observations)	29 09	41 11	1846	......	......	......	58 18 s.	......	58·3 s.	8·47	......	8·47	H.M.S. ‘Fly.’
At sea (4 observations)	28 44	42 01	1845	......	......	......	59 01 s.	......	59·0 s.	8·37	......	8·37	Clerk.
At sea (4 observations)	28 49	42 07	1845	21 57 w.	......	21·9 w.	......	......	......	......	......	......	Pagoda.
At sea....................	28 15	42 18	1845	......	......	......	59 37 s.	......	59·6 s.	8·14	......	8·14	Dayman.
At sea (3 observations)	29 17	42 21	1851	24 31 w.	......	24·5 w.	......	......	......	......	......	......	Kellett.
At sea....................	28 02	42 40	1845	......	......	......	58 10 s.	......	58·2 s.	......	......	......	Moore.
At sea (4 observations)	28 24	43 00	1845	......	......	......	58 52 s.	......	58·9 s.	8·43	......	8·43	Clerk.
At sea (3 observations)	28 19	43 07	1845	21 19 w.	......	21·3 w.	......	......	......	......	......	......	Pagoda.
St. Augustin Bay	23 35	43 50	1825	20 48 w.	......	20·8 w.	......	......	......	......	......	......	Owen.
At sea....................	29 34	43 53	1837	24 26 w.	......	24·4 w.	......	......	......	......	......	......	Bonite.
Mourondoca	20 18	44 17	1825	18 42 w.	......	18·7 w.	......	......	......	......	......	......	Owen.
At sea (14 observations)	27 26	45 09	1861	20 31 w.	......	20·5 w.	......	......	......	......	......	......	Denham.
At sea (2 observations)	28 34	45 33	1837	22 21 w.	......	22·3 w.	......	......	......	......	......	......	Bonite.
At sea....................	29 21	45 34	1838	21 02 w.	......	21·0 w.	......	......	......	......	......	......	Prussian ships.
At sea....................	26 54	45 47	1845	......	......	......	58 41 s.	......	58·7 s.	......	......	......	Dayman.
At sea (4 observations)	27 14	45 50	1845	......	......	......	58 32 s.	......	58·5 s.	8·68	......	8·68	Moore.
At sea (5 observations)	27 12	46 02	1845	20 25 w.	......	20·4 w.	......	......	......	......	......	......	Pagoda.
At sea (4 observations)	27 12	46 09	1845	......	......	......	58 44 s.	......	58·7 s.	8·46	......	8·46	Clerk.
At sea....................	25 42	46 42	1836	16 16 w.	......	16·3 w.	......	......	......	......	......	......	FitzRoy.

SOUTH EQUATORIAL ZONE III.—Lat. 20° to 30° S. (continued).

ations.	Lat. S.	Long. E.	Date.	Declination.				Inclination.				Force in British units.			Observers.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.		Ob-served.	Cor. to Epoch 1842·5.	Corrected.		Ob-served.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′		°		° ′		°					
t sea	27 33	47 17	1837	18 52 w.	......	18·9 w.		......	......	......		......	......	......	Bonite.
Iana atoo.........	24 17	47 20	1827	17 00 w.	......	17·0 w.		......	......	......		......	......	......	Owen.
t sea observations)	26 49	48 14	1824	19 43 w.	......	19·7 w.		......	......	......		......	......	......	Prussian ships.
t sea observations)	26 26	48 20	1845	......	......	......		58 46 s.	......	58·8 s.		8·72	......	8·72	Moore.
t sea	29 37	48 44	1829	24 01 w.	......	24·0 w.		......	......	......		......	......	......	Lütke.
t sea	25 42	49 06	1845	......	......	......		58 43 s.	......	58·7 s.		8·35	......	8·35	Dayman.
t sea observations)	26 25	49 12	1845	......	......	......		58 36 s.	......	58·6 s.		8·69	......	8·69	Clerk.
t sea observations)	26 30	49 20	1845	16 23 w.	......	16·4 w.		......	......	......		......	......	......	Pagoda.
t sea observations)	25 47	49 40	1845	15 09 w.	......	15·1 w.		......	......	......		......	......	......	Pagoda.
t sea observations)	25 39	49 47	1861	16 49 w.	......	16·8 w.		......	......	......		......	......	......	Denham.
t sea observations)	26 52	50 24	1829	20 10 w.	......	20·2 w.		......	......	......		......	......	......	Prussian ships.
t sea	25 13	51 29	1845	......	......	......		58 34 s.	......	58·6 s.		8·53	......	8·53	Dayman.
t sea observations)	23 44	51 48	1845	14 22 w.	......	14·4 w.		......	......	......		......	......	......	Pagoda.
t sea	25 01	51 58	1837	16 45 w.	......	16·7 w.		......	......	......		......	......	......	Bonite.
sea observations)	26 14	52 30	1851	18 24 w.	......	18·4 w.		......	......	......		......	......	......	Kellett.
sea observations)	21 54	53 00	1845	13 44 w.	......	13·7 w.		......	......	......		......	......	......	Pagoda.
sea	24 28	53 21	1837	16 12 w.	......	16·2 w.		......	......	......		......	......	......	Bonite.
sea observations)	21 50	53 25	1845	......	......	......		54 51 s.	......	54·8 s.		8·70	......	8·70	Moore.
sea	23 17	53 31	1837	15 47 w.	......	15·8 w.		......	......	......		......	......	......	Bonite.
sea observations)	21 44	53 34	1845	......	......	......		54 38 s.	......	54·6 s.		8·83	......	8·83	Clerk.
sea	24 05	54 20	1837	15 15 w.	......	15·2 w.		......	......	......		......	......	......	Bonite.
sea observations)	29 55	54 35	1846	......	......	......		61 59 s.	......	62·0 s.		9·40	......	9·40	H.M.S. ‘Fly.’
sea	21 36	54 36	1837	13 57 w.	......	13·9 w.		......	......	......		......	......	......	Bonite.
sea observations)	26 30	54 52	1829	15 21 w.	......	15·3 w.		......	......	......		......	......	......	Lütke.
e Bc bon............	20 51	55 30	1824	13 46 w.	......	13·8 w.	° 14·3 w.	......	......	......	55·4 s.	......	......	......	Duperrey.
			1827	14 58 w.	......	15·0 w.		......	......	......		......	......	......	Blosseville.
			1837	14 10 w.	......	14·2 w.		55 33 s.	... ..	55·5 s.		......	......	......	Bonite.
			1839	......	......	......		55 12 s.	......	55·2 s.		......	......	......	La Vénus.
sea observations)	20 50	55 32	1845	11 15 w.	......	11·2 w.		......	......	......		......	......	......	Pagoda.
sea.	20 50	55 36	1837	11 04 w.	......	11·1 w.		......	......	......		......	......	......	Bonite.
sea.	20 59	56 09	1837	12 23 w.	......	12·4 w.		......	......	......		......	......	......	Bonite.
sea(observations)	23 38	56 11	1861	14 06 w.	......	14·1 w.		......	......	......		......	......	......	Denham.
sea (observations)	24 08	56 54	1847	......	......	......		56 14 s.	......	56·2 s.		9·21	......	9·21	Rattlesnake.
sea (observations)	29 22	56 58	1847	......	......	......		60 44 s.	......	60·7 s.		9·30	......	9·30	Rattlesnake.
sea.	24 45	57 03	1845	......	......	......		59 13 s.	......	59·2 s.		......	......	......	Dayman.
sea (observations)	28 30	57 15	1846	......	......	......		60 08 s.	......	60·1 s.		8·95	......	8 95	H.M.S. ‘Fly.’
t Lc s.............	20 09	57 31	1824	13 46 w.	......	13·8 w.	11·6 w.	53 47 s.	......	53·8 s.	53·9 s.	......	......	8·65	Duperrey.
			1836	11 18 w.	......	11·3 w.		54 01 s.	......	54·0 s.		......	......		FitzRoy.
			1845	......	......	......		54 14 s.	......	54·2 s.		8·66	......		Clerk and Moore.
			1845	......	......	......		53 38 s.	......	53·6 s.		8·65	......		Moore.
			1845	9 44 w.	......	9·7 w.		......	......	......		......	......		Pagoda.
			1847	......	......	......		53 49 s.	......	53·8 s.		8·84	......		Rattlesnake.
sea (bservations)	21 18	57 49	1847	......	......	......		54 00 s.	......	54·0 s.		9·18	......	9·18	Rattlesnake.
sea (bservations)	27 05	57 52	1847	......	......	......		58 52 s.	......	58·9 s.		9·32	......	9·32	Rattlesnake.
sea (bservations)	26 00	58 35	1847	......	......	......		58 01 s.	......	58·0 s.		9·36	......	9 36	Rattlesnake.
sea..	24 36	58 37	1845	......	......	......		58 17 s.	......	58·3 s.		8·83	......	8·83	Dayman.
sea..	21 01	58 43	1837	9 25 w.	......	9·4 w.		......	......	......		......	......	......	Bonite.
sea (bservations)	20 31	59 42	1845	......	......	......		53 59 s.	......	54·0 s.		8·77	......	8·77	Clerk.
sea (bservations)	20 30	59 42	1845	9 44 w.	......	9·7 w.		......	......	......		......	......	......	Pagoda.
sea..	24 43	59 46	1845	......	......	.....		57 59 s.	......	58·0 s.		8·87	......	8·87	Dayman.
sea (bservations)	26 52	60 11	1846	......	......	......		59 11 s.	......	59·2 s.		9·25	......	9·25	H.M.S. ‘Fly.’
sea (bservations)	26 23	60 31	1847	......	......	......		58 44 s.	......	58·7 s.		9·41	......	9·41	Rattlesnake.
sea (bservations)	25 35	60 54	1851	14 41 w.	......	14·7 w.		......	......	......		......	......	......	Kellett.
sea (bservations)	27 33	61 00	1847	......	......	......		59 06 s.	......	59·1 s.		9·44	......	9·44	Rattlesnake.
sea..	24 50	61 11	1845	......	......	......		58 15 s.	......	58·3 s.		8·97	......	8·97	Dayman.
sea (bservations)	23 07	62 00	1824	12 41 w.	......	12·7 w.		......	......	......		......	......	......	Prussian ships.
sea (bservations)	23 50	62 18	1832	11 44 w.	......	11·7 w.									
sea (bservations)	25 42	62 25	1846	......	......	......		58 22 s.	......	58·4 s.		9·49	......	9·49	H.M.S. ‘Fly.’
sea (bservations)	22 21	62 26	1861	10 40 w.	......	10·7 w.		......	......	......		......	......	......	Denham.
sea..	24 23	62 54	1845	......	......	......		57 57 s.	......	57·9 s.		8·82	......	8·82	Dayman.
sea (bservations)	20 40	62 58	1845	......	......	......		53 53 s.	......	53·9 s.		8·79	......	8·79	Moore.
sea (bservations)	20 39	63 01	1845	8 27 w.	......	8·4 w.		......	......	......		......	......	......	Pagoda.
sea (bservations)	28 02	63 15	1847	......	......	......		59 36 s.	......	59·6 s.		......	......	......	Rattlesnake.

SOUTH EQUATORIAL ZONE III.—Lat. 20° to 30° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observe
				Ob-served.	Correction to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch. 1842·5.	Corrected.	
	° ′	° ′		° ′		°	° ′		°				
At sea (4 observations)	24 57	64 17	1851	13 55 w.	......	13·9 w.	......	......	......	......	......	......	Kellett.
At sea (3 observations)	25 26	64 25	1838	11 25 w.	......	11·4 w.	......	......	......	......	......	......	Prussian sl s.
At sea....................	20 02	64 59	1837	7 51 w.	......	7·8 w.	......	......	......	......	......	......	Bonite.
At sea....................	23 52	64 59	1845	......	......	......	56 52 s.	......	56·9 s.	8·96	......	8·96	Dayman.
At sea (2 observations)	22 34	65 05	1834	8 36 w.	......	8·6 w.	......	......	......	......	......	......	Prussian sbs.
At sea (3 observations)	23 22	65 25	1846	......	......	......	55 50 s.	......	55·8 s.	9·54	......	9·54	H.M.S. 'F'
At sea (4 observations)	21 02	66 02	1845	......	......	......	54 03 s.	......	54·1 s.	8 85	......	8·85	Clerk.
At sea (5 observations)	21 01	66 10	1845	......	......	......	......	......	......	8·83	......	8·83	Moore.
At sea (3 observations)	21 01	66 24	1845	7 46 w.	......	7·8 w.	......	......	......	......	......	......	Pagoda.
At sea (3 observations)	23 39	66 25	1829	9 06 w.	......	9·1 w.	......	......	......	......	......	......	Lütke.
At sea (7 observations)	21 01	66 50	1845	......	......	......	53 49 s.	......	53·8 s.	......	......	......	Moore.
At sea (8 observations)	21 12	67 29	1845	......	......	......	53 39 s.	......	53·6 s.	9·00	......	9·00	Clerk.
At sea (2 observations)	21 12	67 29	1845	7 37 w.	......	7·6 w.	......	......	......	......	......	......	Pagoda.
At sea (5 observations)	21 12	67 29	1845	......	......	......	53 59 s.	......	54·0 s.	8·92	......	8 92	Moore.
At sea (3 observations)	27 57	67 30	1847	......	......	......	59 22 s.	......	59·4 s.	9·72	......	9·72	Rattlesnak
At sea (2 observations)	23 33	67 34	1851	12 05 w.	......	12·1 w.	......	......	......	......	......	......	Kellett.
At sea (7 observations)	23 00	67 53	1861	8 58 w.	......	9·0 w.	......	......	......	......	......	......	Denham.
At sea (8 observations)	21 11	67 54	1845	......	......	......	53 02 s.	......	53·0 s.	9·01	......	9·01	Clerk.
At sea (3 observations)	21 11	67 54	1845	6 34 w.	......	6·6 w.	......	......	......	......	......	......	Pagoda.
At sea (5 observations)	21 11	67 54	1845	......	......	......	53 46 s.	......	53·8 s.	9·04	......	9·04	Moore.
At sea....................	21 08	68 04	1845	......	......	......	53 01 s.	......	53·0 s.	......	......	......	Moore.
At sea (5 observations)	20 58	68 12	1845	8 00 w.	......	8·0 w.	......	......	......	......	......	......	Pagoda.
At sea (4 observations)	21 06	68 12	1845	......	......	......	53 10 s.	......	53·2 s.	8·92	......	8·92	Clerk.
At sea (3 observations)	21 00	68 33	1846	......	......	......	53 40 s.	......	53·7 s.	9·09	......	9·09	H.M.S. 'F'
At sea (12 observations)	20 34	69 24	1845	......	......	......	53 01 s.	......	53·0 s.	9·06	......	9·06	Moore.
At sea (2 observations)	20 34	69 37	1845	6 07 w.	......	6·1 w.	......	......	......	......	......	......	Pagoda.
At sea (5 observations)	20 26	69 48	1845	......	......	......	52 19 s.	......	52·3 s.	......	......	......	Moore.
At sea....................	22 41	69 54	1845	......	......	......	55 54 s.	......	55·9 s.	9·10	......	9·10	Dayman.
At sea (3 observations)	20 26	70 36	1845	......	......	......	......	......	......	9·03	......	9·03	Moore.
At sea (4 observations)	20 28	70 46	1845	......	......	......	52 51 s.	......	52·8 s.	8·98	......	8·98	Clerk.
At sea (2 observations)	20 27	70 49	1845	6 35 w.	......	6·6 w.	......	......	......	......	......	......	Pagoda.
At sea (4 observations)	22 30	71 07	1851	10 10 w.	......	10·2 w.	......	......	......	......	......	......	Kellett.
At sea....................	22 33	72 20	1851	......	......	......	55 04 s.	......	55·1 s.	9·07	......	9·07	Dayman.
At sea (16 observations)	23 35	72 32	1861	8 50 w.	......	8·8 w.	......	......	......	......	......	......	Denham.
At sea (2 observations)	25 01	73 18	1840	10 22 w.	......	10·4 w.	......	......	......	......	......	......	Prussian ps.
At sea....................	20 45	73 20	1845	5 54 w.	......	5·9 w.	......	......	......	......	......	......	Pagoda.
At sea....................	22 28	74 18	1845	......	......	......	54 41 s.	......	54·7 s.	9·11	......	9·11	Dayman.
At sea (4 observations)	21 52	75 13	1829	6 57 w.	......	6·9 w.	......	......	......	......	......	......	Lütke.
At sea (4 observations)	22 07	75 54	1851	9 06 w.	......	9·1 w.	......	......	......	......	......	......	Kellett.
At sea....................	22 38	76 10	1845	......	......	......	54 45 s.	......	54·7 s.	9·26	......	9·26	Dayman.
At sea (8 observations)	20 29	76 22	1845	......	......	......	52 20 s.	......	52·3 s.	9·16	......	9·16	Clerk.
At sea (3 observations)	20 28	76 23	1845	6 01 w.	......	6·0 w.	......	......	......	......	......	......	Pagoda.
At sea (12 observations)	20 39	77 43	1845	......	......	......	51 59 s.	......	52·0 s.	9·27	......	9·27	Clerk.
At sea (12 observations)	20 39	77 43	1845	......	......	......	51 58 s.	......	52·0 s.	9·24	......	9·24	Moore.
At sea (3 observations)	20 39	77 45	1845	5 22 w.	......	5·4 w.	......	......	......	......	......	......	Pagoda.
At sea....................	22 35	78 08	1845	......	......	......	54 26 s.	......	54·4 s.	9·27	......	9·27	Dayman
At sea (24 observations)	20 44	78 31	1845	......	......	......	52 00 s.	......	52·0 s.	9·25	......	9·25	Moore.
At sea (19 observations)	20 44	78 31	1845	......	......	......	52 03 s.	......	52·1 s.	9·28	......	9·28	Clerk.
At sea (21 observations)	20 44	78 34	1845	5 29 w.	......	5·5 w.	......	......	......	......	......	......	Pagoda.
At sea (12 observations)	24 36	79 03	1861	9 25 w.	......	9·4 w.	......	......	......	......	......	......	Denham
At sea (5 observations)	20 36	79 10	1845	......	......	......	51 46 s.	......	51·8 s.	9·09	......	9·09	Moore.
At sea (7 observations)	20 36	79 20	1845	4 43 w.	......	4·7 w.	......	......	......	......	......	......	Pagoda.
At sea (8 observations)	20 36	79 22	1845	......	......	......	51 50 s.	......	51·8 s.	9·34	......	9·34	Clerk.
At sea (3 observations)	21 09	79 27	1851	8 47 w.	......	8·8 w.	......	......	......	......	......	......	Kellett.
At sea....................	22 34	80 10	1845	......	......	......	54 23 s.	......	54·4 s.	9·45	......	9·45	Dayman
At sea (3 observations)	20 25	82 00	1845	......	......	......	51 05 s.	......	51·1 s.	9·35	......	9·35	Moore.
At sea (8 observations)	20 26	82 22	1845	......	......	......	51 39 s.	......	51·6 s.	9·42	......	9·42	Clerk.
At sea (5 observations)	20 25	82 32	1845	5 08 w.	......	5·1 w.	......	......	......	......	......	......	Pagoda.
At sea....................	22 08	84 17	1845	......	......	......	53 52 s.	......	53·9 s.	9·37	......	9·37	Dayman
At sea (5 observations)	20 31	84 41	1829	4 30 w.	......	4·5 w.	......	......	......	......	......	......	Lütke.
At sea (3 observations)	23 34	84 43	1857	7 47 w.	......	7·8 w.	......	......	......	......	......	......	Novara.
At sea (2 observations)	29 04	84 49	1857	10 01 w.	......	10·0 w.	......	......	......	......	......	......	Novara.

SOUTH EQUATORIAL ZONE III.—Lat. 20° to 30° S. (continued).

ations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observers.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′		°	° ′		°				
t sea · observations)	20 37	85 02	1845	......	......	......	51 21 s.	......	51·4 s.	9·46	......	9·46	Moore.
t sea · observations)	20 38	85 26	1845	......	......	......	51 14 s.	......	51·2 s.	9·48	......	9·48	Clerk.
t sea · observations)	20 37	85 32	1845	5 20 w.	......	5·3 w.	......	......	......	......	......	......	Pagoda.
t sea	22 11	86 30	1845	......	......	......	53 20 s.	......	53·3 s.	9·55	......	9·55	Dayman.
t sea . observations)	22 56	86 58	1860	7 19 w.	......	7·3 w.	......	......	......	......	......	......	Denham.
t sea observations)	20 38	87 50	1845	......	......	......	51 33 s.	......	51·5 s.	9·73	......	9 73	Moore.
t sea observations)	20 46	87 59	1845	......	......	......	50 57 s.	......	50·9 s.	9·70	......	9·70	Clerk.
t sea observations)	20 46	88 06	1845	4 45 w.	......	4·7 w.	......	......	......	......	......	......	Pagoda.
t sea observations)	21 44	89 38	1845	......	......	......	51 45 s.	......	51·7 s.	9·85	......	9·85	Moore.
t sea observations)	21 53	89 42	1845	4 23 w.	......	4·4 w.	......	......	......	......	......	......	Pagoda.
t sea observations)	21 50	89 44	1845	......	......	......	52 17 s.	......	52·3 s.	9·94	......	9·94	Clerk.
t sea	22 17	89 57	1845	.....	......	......	53 11 s.	......	53·2 s.	9·65	......	9·65	Dayman.
t sea observations)	22 46	90 40	1845	5 56 w.	......	5·9 w.	......	......	......	......	......	......	Pagoda.
t sea observations)	22 47	91 00	1845	......	......	......	52 49 s.	......	52·8 s.	9·92	......	9·92	Clerk.
t sea	22 19	91 16	1845	......	......	......	53 11 s.	......	53·2 s.	9·57	......	9·57	Dayman.
sea observations)	24 02	92 07	1845	......	......	......	52 44 s.	......	52·7 s.	10·24	......	10·24	Moore.
sea	24 05	92 11	1845	6 34 w.	......	6·6 w.	......	......	......	......	......	......	Pagoda.
sea	22 54	93 48	1845	......	......	......	53 26 s.	......	53·4 s.	10·02	......	10·02	Dayman.
sea observations)	24 17	93 50	1845	......	......	......	54 07 s.	......	54·1 s.	10·13	......	10·13	Moore.
sea observations)	24 17	94 06	1845	5 31 w.	......	6·5 w.	......	......	......	......	......	......	Pagoda.
t sea	23 11	95 40	1845	......	......	......	53 44 s.	......	53·7 s.	10·00	......	10·00	Dayman.
t sea observations)	23 56	95 46	1845	6 10 w.	......	6·2 w.	......	......	......	......	......	......	Pagoda.
t sea observations)	23 50	95 56	1845	......	......	......	54 26 s.	......	54·4 s.	10·32	......	10·32	Moore.
t sea observations)	24 00	96 06	1845	......	......	......	54 16 s.	......	54·3 s.	10·23	......	10·23	Clerk.
t sea observations)	28 48	96 33	1848	......	......	......	58 51 s.	......	58·8 s.	11·02	......	11·02	Rattlesnake.
sea observations)	20 33	97 11	1846	......	......	......	50 14 s.	......	50·2 s.	9·95	......	9·95	H.M.S. ‘Fly.’
sea observations)	24 01	97 25	1845	......	......	......	54 18 s,	......	54·3 s.	10·35	......	10·35	Moore.
sea observations)	24 01	97 30	1845	......	......	......	54 03 s.	......	54·1 s.	10·33	......	10·33	Clerk.
sea observations)	24 01	97 34	1845	7 08 w.	......	7·1 w.	......	......	......	......	......	......	Pagoda.
sea observations)	24 51	97 34	1848	......	......	......	54 51 s.	......	54·8 s.	......	......	......	Rattlesnake.
sea observations)	21 12	98 47	1846	......	......	......	49 55 s.	......	49·9 s.	10·04	......	10·04	H.M.S. ‘Fly.’
sea observations)	23 58	99 06	1845	......	......	......	53 46 s.	......	53·8 s.	10·35	......	10·35	Moore.
sea.	20 11	99 13	1829	3 02 w.	......	3·0 w.	......	......	......	......	......	......	Lütke.
sea (bservations)	23 58	99 21	1845	5 14 w.	......	5·2 w.	......	......	......	......	......	......	Pagoda.
sea.	27 45	99 23	1827	5 30 w.	......	5·5 w.	......	......	......	......	......	......	D'Urville.
sea (bservations)	24 00	99 23	1845	......	......	......	54 28 s.	......	54·5 s.	10·24	......	10·24	Clerk.
sea.	24 00	99 33	1845	......	......	......	54 20 s.	......	54·3 s.	10·54	......	10·54	Dayman.
sea (bservations)	21 36	100 30	1846	......	......	......	51 50 s.	......	51·8 s.	10·19	......	10·19	H.M.S. ‘Fly.’
sea.	24 50	101 31	1845	......	......	......	54 44 s.	......	54·7 s.	10·60	......	10·60	Dayman.
sea (bservations)	24 07	102 28	1845	5 32 w.	......	5·5 w.	......	......	......	......	......	......	Pagoda.
sea (bservations)	23 02	102 50	1846	......	......	......	52 44 s.	......	52·7 s.	10·34	......	10·34	H.M.S. ‘Fly.’
sea.	25 52	102 58	1845	......	......	......	55 50 s.	......	55 8 s.	10·65	......	10·65	Dayman.
sea.	23 22	103 18	1827	3 00 w.	......	3·0 w.	......	......	......	......	......	......	D'Urville.
sea (bservations)	24 45	103 48	1846	......	......	......	55 34 s.	......	55·6 s.	10·71	......	10·71	H.M.S. ‘Fly.’
sea.	26 44	104 26	1845	......	......	......	56 54 s.	......	56·9 s.	10·98	......	10·98	Dayman.
sea (bservations)	25 46	104 55	1845	......	......	......	55 05 s.	......	55·1 s.	10·84	......	10·84	Moore.
sea.	28 04	105 06	1845	......	......	......	58 33 s.	......	58·5 s.	11·19	......	11·19	Dayman.
sea (bservations)	26 00	105 11	1845	......	......	......	55 09 s.	......	55·2 s.	11·01	......	11·01	Clerk.
sea (bservations)	22 04	105 13	1836	3 40 w.	......	3·7 w.	......	......	......	......	......	......	FitzRoy.
sea (bservations)	26 15	105 15	1846	......	......	......	56 47 s.	......	56·8 s.	10·79	......	10·79	H.M.S. ‘Fly.’
sea (bservations)	26 10	105 16	1845	5 30 w.	......	5·5 w.	......	......	......	......	......	......	Pagoda.
sea.	29 40	105 28	1845	......	......	......	59 37 s.	......	59·6 s.	11·52	......	11·52	Dayman.
sea (bservations)	27 35	106 32	1845	......	......	......	57 26 s.	......	57·4 s.	11·07	......	11·07	Moore.
sea (bservations)	27 41	106 35	1845	6 33 w.	......	6·5 w.	......	......	......	......	......	......	Pagoda.
sea (bservations)	27 47	106 36	1845	......	......	......	57 17 s.	......	57·3 s.	11·16	......	11·16	Clerk.
sea (bservations)	29 16	106 49	1845	......	......	......	59 30 s.	......	59·5 s.	11·23	......	11·23	Moore.
sea (bservations)	29 20	106 55	1845	6 30 w.	......	6·5 w.	......	......	......	......	......	......	Pagoda.
sea (bservations)	29 20	106 55	1845	......	......	......	59 19 s.	......	59·3 s.	11·64	......	11·64	Clerk.
sea (bservations)	27 30	108 28	1846	......	......	......	58 49 s.	......	58·8 s.	11·22	......	11·22	H.M.S. ‘Fly.’
sea (bservations)	27 28	108 49	1836	5 09 w.	......	5·1 w.	......	......	......	......	......	......	FitzRoy.
sea (bservations)	29 05	111 45	1846	......	......	......	57 38 s.	......	57·6 s.	11·66	......	11·66	H.M.S. ‘Fly.’

SOUTH EQUATORIAL ZONE III.—Lat. 20° to 30° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observe
				Observed.	Correction to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′		°	° ′		°				
Dirk Hartog Islands (3 observations)...	25 52	113 04	1858	2 57 W.	......	3·0 W.	......	......	......	......	......	......	Denham.
Shark's Bay (23 observations).........	25 59	113 28	1858	2 42 W.	......	2·7 W.	......	......	......	......	......	......	Denham.
New Cape	25 04	113 30	1858	3 02 W.	......	3·0 W.	......	......	......	......	......	......	Denham.
Port Molle	20 20	148 55	1844	......	......	......	47 19 S.	......	47·3 S.	10·55	......	10·55	H.M.S. 'Fl
No. 2 Percy Island ...	21 40	150 17	1847	......	......	......	49 02 S.	......	49·0 S.	......	......	......	Rattlesnake
			1859	8 04 E.	......	8·1 E.	......	......	......	......	......	......	Denham.
Port Bowen	22 31	150 49	1843	8 50 E.	......	8·8 E.	50 16 S.	......	50·3 S.	10·96	......	10·96	H.M.S. 'Fl
Keppel Island	22 09	150 56	1847	......	......	......	50 48 S.	......	50·8 S.	......	......	......	Rattlesnake
Facing Island, Port Curtis...............	23 51	151 04	1847	......	......	......	51 30 S.	......	51·5 S.	......	......	......	Rattlesnake
Sandy Cape	24 43	153 15	1843·5	7 50 E.	......	7·8 E.	52 24 S.	......	52·4 S.	11·14	......	11·14	H.M.S. 'F
Moreton Island (5 observations)......	27 06	153 25	1849	......	......	......	55 18 S.	......	55·3 S.	11·72	......	11·72	Rattlesnake
			1858	9 43 E.	......	9·7 E. } 9·5 E.	......	......	......	......	......	......	Denham.
			1863	9 19 E.	......	9·3 E.	......	......	......	......	......	......	H.M.S. 'H[illegible]te.'
Frederick Reef (4 observations)......	21 02	154 25	1859	8 44 E.	......	8·7 E.	......	......	......	......	......	......	Denham.
Cato Bank (2 observations)	23 15	155 34	1858	9 21 E.	......	9·3 E.	......	......	......	......	......	......	Denham.
Wreck Reef (4 observations).........	22 10	155 39	1858	9 43 E.	......	9·7 E.	......	......	......	......	......	......	Denham.
Kenn Reef (3 observations)	21 16	155 49	1859	9 00 E.	......	9·0 E.	......	......	......	......	......	......	Denham.
At sea (3 observations)	27 43	156 33	1858	9 52 E.	.....	9·9 E.	......	......	......	......	......	......	Novara.
At sea (2 observations)	25 17	157 28	1858	8 59 E.	......	9·0 E.	......	......	......	......	......	......	Novara.
At sea (2 observations)	22 33	157 49	1858	9 24 E.	......	9·4 E.	......	......	......	......	......	......	Novara.
Observatory Islet (4 observations)......	21 24	158 53	1858	9 20 E.	......	9·3 E.	......	......	......	......	......	......	Denham.
Bellona Shoal (2 observations).........	21 47	159 35	1858	9 32 E.	......	9·5 E.	......	......	......	......	......	......	Denham.
At sea (7 observations)	25 11	160 17	1855	10 54 E.	......	10·9 E.	......	......	......	......	......	......	Denham.
At sea (6 observations)	21 03	164 56	1827	11 14 E.	......	11·2 E.	......	......	......	......	......	......	D'Urville.
At sea (18 observations)	29 08	167 20	1855	12 38 E.	......	12·6 E.	......	......	......	......	......	......	Denham.
Norfolk Island (5 observations)......	29 12	167 27	1855	12 34 E.	......	12·6 E.	......	......	......	......	......	......	Denham.
At sea (6 observations)	29 07	167 53	1855	12 55 E.	......	12·9 E.	......	......	......	......	......	......	Denham.
At sea (2 observations)	28 24	168 33	1828	11 30 E.	......	11·5 E.	......	......	......	......	......	......	D'Urville
At sea (5 observations)	22 07	168 33	1855	11 01 E.	......	11·0 E.	......	......	......	......	......	......	Denham.
At sea (6 observations)	29 26	171 14	1855	12 58 E.	......	13·0 E.	......	......	......	......	......	......	Denham.
At sea (3 observations)	28 00	172 21	1863	10 34 E.	......	10·6 E.	......	......	......	......	......	......	Richards.
At sea (6 observations)	25 16	173 03	1863	11 29 E.	......	11·5 E.	......	......	......	......	......	......	Richards.
Conway Reef (4 observations).........	21 45	174 38	1855	10 34 E.	......	10·6 E.	......	......	......	......	......	......	Denham.
At sea (8 observations)	24 26	174 54	1855	12 57 E.	......	12·9 E.	......	......	......	......	......	......	Denham.
At sea.....................	29 04	176 24	1824	10 00 E.	......	10·0 E.	......	......	......	......	......	......	Duperrey
At sea (5 observations)	23 03	176 56	1863	10 40 E.	......	10·7 E.	......	......	......	......	......	......	Richards
At sea (2 observations)	29 41	179 08	1835	13 20 E.	......	13·3 E.	......	......	......	......	......	......	FitzRoy.
At sea (2 observations)	23 24	180 05	1854	8 36 E.	......	8·6 E.	......	......	......	......	......	......	Denham.
At sea (2 observations)	23 35	180 08	1854	10 00 E.	......	10·0 E.	......	......	......	......	......	......	Denham.
At sea (3 observations)	23 09	180 21	1854	10 05 E.	......	10·1 E.	......	......	......	......	......	......	Denham.
At sea (2 observations)	22 57	180 30	1854	10 35 E.	......	10·6 E.	......	......	......	......	......	......	Denham.
At sea (2 observations)	23 18	180 46	1854	10 43 E.	......	10·7 E.	......	......	......	......	......	......	Denham.
Minerva Reefs (North and South)	23 45	181 11	1854	10 47 E.	......	10·8 E.	......	......	......	......	......	......	Denham.
At sea (4 observations)	27 06	181 25	1835	11 27 E.	......	11·4 E.	......	......	......	......	......	......	FitzRoy.
At sea (2 observations)	21 57	181 35	1854	9 18 E.	......	9·3 E.	......	......	......	......	......	......	Denham
At sea (4 observations)	28 28	181 54	1854	10 44 E.	......	10·7 E.	......	......	......	......	......	......	Denham.
Sunday Island	29 15	182 00	1854	11 19 E.	......	11·3 E.	......	......	......	......	......	......	Denham
At sea (2 observations)	21 30	182 15	1863	10 20 E.	......	10·3 E.	......	......	......	......	......	......	Richards
At sea (4 observations)	29 06	182 30	1863	10 26 E.	......	10·4 E.	......	......	......	......	......	......	Denham
At sea (2 observations)	25 33	182 53	1863	9 58 E.	......	10·0 E.	......	......	......	......	......	......	Denham

SOUTH EQUATORIAL ZONE III.—Lat. 20° to 30° S. (continued).

tations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observers.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	° ′		°				
At sea 3 observations)	25 50	182 55	1863	11 42 E.	......	11·7 E.	......	......	......	......	......	......	Denham.
At sea observations)	26 08	182 57	1863	10 39 E.	......	10·6 E.	......	......	......	......	......	......	Denham.
At sea observations)	28 39	183 02	1863	10 54 E.	......	10·9 E.	......	......	......	......	......	......	Denham.
At sea observations)	28 19	183 19	1863	11 45 E.	......	11·7 E.	......	......	......	......	......	......	Denham.
At sea observations)	28 29	183 29	1863	11 12 E.	......	11·2 E.	......	......	......	......	......	......	Denham.
At sea observations)	25 12	183 45	1863	10 33 E.	......	10·5 E.	......	......	......	......	......	......	Denham.
At sea observations)	26 32	183 57	1863	10 04 E.	......	10·1 E.	......	......	......	......	......	......	Denham.
At sea observations)	26 00	184 29	1863	9 48 E.	......	9·8 E.	......	......	......	......	......	......	Denham.
At sea observations)	24 34	184 59	1863	10 31 E.	......	10·5 E.	......	......	......	......	......	......	Denham.
At sea observations)	20 53	185 12	1827	10 48 E.	......	10·8 E.	......	......	......	......	......	......	Dumont d'Urville.
t sea observations)	24 36	185 18	1854	9 17 E.	......	9·3 E.	......	......	......	......	......	......	Denham.
t sea observations)	22 41	190 59	1835	10 58 E.	......	11·0 E.	......	......	......	......	......	......	FitzRoy.
t sea observations)	20 48	195 47	1835	9 28 E.	......	9·5 E.	......	......	......	......	......	......	FitzRoy.
arot a Island......	21 12	200 14	1840	8 34 E.	0 03 E.	8·6 E.	36 09 s.	......	36·2 s.	9·96	......	9·96	Belcher.
t sea observations)	27 25	205 59	1859	8 48 E.	9 17 W.	8·5 E.	......	......	......	......	......	......	Novara.
t sea	22 17	209 33	1830	......	......	......	36 33 s.	......	36·5 s.	9·00	......	9·00	Erman.
t sea	23 17	209 45	1830	7 19 E.	0 12 E.	7·5 E.	......	......	......	......	......	......	Erman.
sea observations)	22 58	209 47	1859	8 56 E.	0 17 W.	8·6 E.	......	......	......	......	......	......	Novara.
sea	20 11	209 51	1830	7 25 E.	0 12 E.	7·6 E.	......	......	......	......	......	......	Erman.
sea	27 07	209 55	1830	8 55 E.	0 12 E.	9·1 E.	......	......	......	......	......	......	Erman.
t sea observations)	26 51	210 02	1830	8 24 E.	0 12 E.	8·6 E.	43 06 s.	......	43·1 s.	10·18	......	10·18	Erman.
t sea	27 43	210 04	1830	......	......	......	44 03 s.	......	44·0 s.	10·00	......	10·00	Erman.
t sea observations)	25 03	210 07	1830	8 03 E.	0 12 E.	8·2 E.	40 20 s.	......	40·3 s.	9·57	......	9·57	Erman.
t sea observations)	21 42	210 09	1859	7 24 E.	0 17 W.	7·1 E.	......	......	......	......	......	......	Novara.
sea	26 01	210 27	1830	7 21 E.	......	7·3 E.	......	......	......	......	......	......	Erman.
sea	28 26	210 57	1830	8 16 E.	......	8·3 E.	......	......	......	......	......	......	Erman.
sea	29 00	211 08	1830	7 57 E.	......	7·9 E.	......	......	......	......	......	......	Erman,
sea	29 06	212 45	1830	7 54 E.	......	7·9 E.	......	......	......	......	......	......	Erman.
sea observations)	28 56	213 24	1830	......	......	......	45 25 s.	......	45·4 s.	9·77	......	9 77	Erman.
sea observations)	29 21	213 31	1830	8 40 E.	......	8·7 E.	......	......	......	......	......	......	Erman.
sea..	28 37	213 42	1830	9 31 E.	......	9·5 E.	......	......	......	......	......	......	Erman.
cairn sland.........	25 02	226 52	1825	6 00 E.	......	6·0 E.	......	......	......	......	......	......	Beechey.
sea observations)	23 57	227 10	1859	7 39 E.	......	7·6 E.	......	......	......	......	......	......	Novara.
sea observations)	27 32	229 08	1859	8 17 E.	......	8·3 E.	......	......	......	......	......	......	Novara.
sea..	21 51	268 05	1827	10 45 E.	......	10·7 E.	32 06 s.	......	32·1 s.	8·13	......	8·13	Lütke.
sea observations)	22 26	269 12	1827	11 26 E.	......	11·4 E.	......	......	......	......	......	......	Lütke.
sea observations)	20 43	270 46	1857	13 04 E.	......	13·1 E.	......	......	......	......	......	......	Richards.
sea observations)	22 15	273 23	1857	13 55 E.	......	13·9 E.	......	......	......	......	......	......	Richards.
sea..	24 26	276 45	1857	15 10 E.	......	15·2 E.	......	......	......	......	......	......	Richards.
sea observations)	27 48	277 17	1827	13 17 E.	......	13·3 E.	......	......	......	......	......	......	Lütke.
sea..	29 38	278 56	1827	12 47 E.	......	12·8 E.	40 01 s.	......	40 0 s.	8·96	......	8·96	Lütke.
sea bservations)	26 25	279 43	1857	16 33 E.	......	16·5 E.	......	......	......	......	......	......	Richards.
sea bservations)	28 27	280 47	1850	14 47 E.	......	14·8 E.	......	......	......	......	......	......	Young.
sea bservations)	22 34	281 59	1830	11 16 E.	......	11·3 E.	......	......	......	......	......	......	Prussian ships.
sea..	28 32	283 22	1857	14 58 E.	......	15·0 E.	......	......	......	......	......	......	Richards.
sea bservations)	27 02	286 32	1827	14 05 E.	......	14·1 E.	......	......	......	......	......	......	Prussian ships.
sea...............	28 01	286 33	1836	12 52 E.	......	12·9 E.	......	......	......	......	......	......	Bonite.
sea...............	25 19	286 52	1836	13 00 E.	......	13·0 E.	......	......	......	......	......	......	Bonite.
sea...............	20 28	287 36	1836	11 08 E.	......	11·1 E.	......	......	......	......	......	......	Bonite.
quimb	29 59	288 34	1828	14 24 E.	......	14·4 E. } 14·3 E.	......	......	......	......	......	......	Beechey.
			1868	14 17 E.	......	14·3 E.	29 55 s.	......	29·9 s.	7·31	......	7·31	H.M.S. 'Nassau.'
an of stations ...	29 56	288 37	1835	14 21 E.	......	14·3 E.	......	......	......	......	......	......	FitzRoy.
asco stations) ...	28 41	288 40	1835	13 33 E.	......	13 5 E.	......	......	......	......	......	......	FitzRoy.
sea...............	23 33	288 52	1836	11 30 E.	......	11·5 E.	......	......	......	......	......	......	Bonite.
an of stations ...	27 23	288 58	1835	13 31 E.	......	13·5 E.	......	......	......	......	......	......	FitzRoy.
iapo	27 13	289 02	1827	13 25 E.	......	13·4 E.	......	......	......	......	......	......	Prussian ships.
ata tations) ...	25 46	289 18	1835	13 25 E.	......	13·4 E.	......	......	......	......	......	......	FitzRoy.
ija (2 ations) ...	23 00	289 30	1835	12 39 E.	......	12·6 E.	......	......	......	......	......	......	FitzRoy.
r R. L (2 stations)	21 44	289 46	1835	12 03 E.	......	12·1 E.	......	......	......	......	......	......	FitzRoy.
Para	29 00	300 56	1846	10 40 E.	......	10·7 E.	......	......	......	......	......	......	Sulivan.
riente Fort.........	27 27	301 16	1846	9 24 E.	......	9·4 E.	......	......	......	......	......	......	Sulivan.
ncion Paraguay...	25 16	302 32	1860	8 29 E.	......	8·5 E.	18 04 s.	......	18·1 s.	6·51	......	6·51	Friesach.

SOUTH EQUATORIAL ZONE III.—Lat. 20° to 30° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.				Inclination.				Force in British units.			Observers.
				Observed.	Correction to Epoch 1842·5.	Corrected.		Observed.	Cor. to Epoch 1842·5.	Corrected.		Observed.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	°	° ′		°	°				
St. Catherine...........	27 26	311 27	1827	6 30 E.	1 31 w.	5·0 E.	5·1 E.	......	......	......	20·9 s.	......	......	......	King.
			1832	6 30 E.	1 01 w.	5·5 E.		......	......	......		......	......	......	FitzRoy.
			1836	......	......	......		21 40 s.	......	21·7 s.		......	......	......	Beechey.
			1838	......	......	......		21 07 s.	......	21·1 s.		......	......	......	Sulivan.
			1860	3 00 E.	1 45 E.	4·7 E.		19 47 s.	......	19·8 s.		6·32	......	6·32	Friesach.
At sea.....................	29 53	311 48	1832	6 27 E.	0 58 w.	5·5 E.		......	......	......		......	......	......	FitzRoy.
At sea.....................	27 56	312 42	1837	4 30 E.	0 38 w.	3·9 E.		......	......	......		......	......	......	Du Petit Thouars.
At sea.....................	26 35	312 55	1838	......	......	......		21 17 s.	......	21·3 s.		6·43	......	6·43	Sulivan.
At sea.....................	29 03	313 06	1836	5 14 E.	0 36 w.	4·6 E.		......	......	......		......	......	......	Bonite.
At sea.....................	29 10	313 35	1827	6 24 E.	1 25 w.	5·0 E.		24 27 s.	......	24·5 s.		7·18	......	7·18	Lütke.
At sea.....................	29 53	313 43	1830	......	......	......		25 33 s.	......	25·6 s.		6·60	......	6·60	Erman.
At sea.....................	28 20	314 10	1836	3 10 E.	0 35 w.	2·6 E.		......	......	......		......	......	......	Bonite.
At sea (3 observations)	27 20	314 12	1827	3 59 E.	1 25 w.	2·6 E.		......	......	......		......	......	......	Lütke.
At sea.....................	26 09	314 18	1837	2 44 E.	0 30 w.	2·2 E.		......	......	......		......	......	......	Du Petit Thouars.
At sea.....................	28 40	314 35	1830	4 14 E.	1 08 w.	3·1 E.		......	......	......		......	......	......	Erman.
At sea (4 observations)	27 04	314 35	1832	4 29 E.	0 58 w.	3·4 E.		......	......	......		......	......	......	FitzRoy.
At sea.....................	27 58	314 36	1830	......	......	......		22 01 s.	......	22·0 s.		6·39	......	6·39	Erman.
At sea.....................	27 50	315 10	1830	4 10 E.	1 09 w.	3·0 E.		......	......	......		......	......	......	Erman.
At sea.....................	25 50	315 12	1830	3 33 E.	1 09 w.	2·4 E.		......	......	......		......	......	......	Erman.
At sea.....................	26 38	315 44	1830	3 33 E.	1 09 w.	2·4 E.		......	......	......		......	......	......	Erman.
At sea.....................	26 22	315 46	1830	......	......	......		19 46 s.	......	19·8 s.		6·27	......	6·27	Erman.
At sea.....................	29 47	315 50	1839	......	......	......		24 00 s.	......	24·0 s.		6·60	......	6·60	Sulivan.
At sea.....................	26 49	315 54	1836	3 06 E.	0 35 w.	2·5 E.		......	......	......		......	......	......	Bonite.
At sea.....................	24 35	315 55	1838	......	......	......		17 55 s.	......	17·9 s.		6·51	......	6·51	Sulivan.
At sea (3 observations)	27 37	315 58	1831	2 55 E.	1 03 w.	1·9 E.		......	......	......		......	......	......	Prussian sl
At sea.....................	24 12	316 23	1837	2 04 E.	0 30 w.	1·6 E.		......	......	......		......	......	......	Du Petit Thouars.
At sea (2 observations)	24 11	316 29	1830	1 12 E.	1 09 w.	0·0		......	......	......		......	......	......	Erman.
At sea (2 observations)	24 18	316 32	1830	......	......	......		15 55 s.	......	15·9 s.		6·12	......	6·12	Erman.
At sea.....................	23 30	316 40	1837	2 00 E.	0 30 w.	1·5 E.		......	......	......		......	......	......	Du Petit Thouars.
At sea.....................	25 02	316 41	1830	2 40 E.	1 09 w.	1·5 E.		......	......	......		......	......	......	Erman.
Tijuĉo, near Rio	22 58	316 49	1860	1 24 w.	1 53 w.	3·3 w.		11 55 s.	......	11·9 s.		6·11	......	6·11	Friesach.
Rio de Janeiro	22 55	316 51	1825	3 11 E.	1 53 w.	1·3 E.	0·8 E.	......	......	......	13·2 s.	......	......	6·39	Beechey.
			1826	2 37 E.	1 47 w.	0·8 E.		14 04 s.	......	14·1 s.		......	......		King.
			1827	3 00 E.	1 41 w.	1·3 E.		14 35 s.	......	14·6 s.		6·88	......		Lütke.
			1830	2 09 E.	1 21 w.	0·8 E.		13 35 s.	......	13·6 s.		6·72	......		Erman.
			1832	2 00 E.	1 08 w.	0·9 E.		......	......	......		......	......		Laplace.
			1832	2 00 E.	1 08 w.	0·9 E.		13 37 s.	......	13·6 s.		......	......		FitzRoy.
			1836	......	......	......		12 54 s.	......	12·9 s.		......	......		Beechey.
			1837	0 51 E.	0 36 w.	0·3 E.		13 19 s.	......	13·3 s.		......	......		Du Petit Thouars.
			1838–9	......	......	......		13 00 s.	......	13·0 s.		......	......		Sulivan.
			1838	......	......	......		13 01 s.	......	13·0 s.		6·40	......		Sulivan.
			1838	......	......	......		13 56 s. 13 37 s.	....	13·9 s. 13·6 s.		6·41	....		Sulivan.
			1839	......	......	......		13 05 s.	......	13·1 s.		6·40	......		Sulivan.
			1839	......	......	......		13 10 s.	......	13·2 s.		6·39	......		Sulivan.
			1845	0 13 E.	0 16 E.	0·5 E.		......	......	......		......	......		Helmreich.
			1847	......	......	......		12 17 s.	......	12·3 s.		6·38	......		Rattlesna
			1857	0 45 w.	1 37 E.	0·9 E.		11 48 s.	......	11·8 s.		6·25	......		Novara.
			1866	2 49 w.	2 36 E.	0·2 w.		11 47 s.	......	11·8 s.		6·08	......		Harkness.
			1867	......	......	......		......	......	......		6·03	......		H.M.S. 'ssau.'
At sea (2 observations)	24 20	316 51	1830	2 54 E.	1 39 w.	1·3 E.		......	......	......		......	......	......	Erman.
At sea (2 observations)	23 35	317 01	1832	1 48 E.	1 03 w.	0·7 E.		......	......	......		......	......	......	FitzRoy.
At sea.....................	23 16	317 04	1830	2 59 E.	1 35 w.	1·4 E.		......	......	......		......	......	......	Erman.
At sea.....................	23 09	317 05	1839	......	......	......		13 55 s.	......	13·9 s.		6·37	......	6·37	Sulivan.
At sea.....................	24 00	317 09	1847	0 47 E.	0 27 E.	1·2 E.		......	......	......		......	......	......	Stanley.
At sea (2 observations)	23 09	317 17	1830	1 39 E.	1 35 w.	0·1 E.		......	......	......		......	......	......	Erman.
At sea.....................	24 38	317 19	1844	1 00 E.	0 09 E.	1·1 E.		......	......	......		......	......	......	Pasley.
At sea.....................	24 40	317 35	1839	......	......	......		16 57 s.	......	16·9 s.		6·34	......	6·34	Sulivan.
At sea.....................	25 23	317 36	1836	1 09 E.	0 45 w.	0·4 E.		......	......	......		......	......	......	Bonite.
At sea.....................	23 12	317 41	1830	1 39 E.	1 27 w.	0·2 E.		......	......	......		......	......	......	Erman.
At sea (2 observations)	23 11	318 02	1829	3 12 E.	1 34 w.	1·6 E.		......	......	......		.....	......	......	Rumker.
Cape Frio	23 00	318 03	1828	1 07 E.	1 42 w.	0·6 w.	0·9 w.	......	......	......		......	......	......	Foster.
			1831	0 12 E.	1 20 w.	1·1 w.		......	......	......		......	......	......	Kellett.
At sea.....................	24 07	318 07	1836	0 30 E.	0 44 w.	0·2 w.		......	......	......		......	......	......	Bonite.

SOUTH EQUATORIAL ZONE III.—Lat. 20° to 30° S. (continued).

ations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British Units.			Observers.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	° ′		°				
At sea	23 05	318 09	1836	2 30 E.	0 44 W.	1·8 E.	......	......	......	......	......	......	Bonite.
At sea	29 17	318 13	1843	0 00	......	0·0	......	......	......	......	......	......	Pasley.
At sea	23 30	318 13	1830	1 03 E.	1 15 W.	0·2 W.	......	......	......	......	......	......	Erman.
At sea observations)	28 14	318 25	1844	0 00	0 10 W.	0·2 W.	......	......	......	......	......	......	Pasley.
At sea	24 56	318 37	1836	0 23 E.	0 39 W.	0·3 W.	......	......	......	......	......	......	Bonite.
At sea	27 11	318 38	1844	0 00	......	0·0	......	......	......	......	......	......	Pasley.
At sea	23 49	318 45	1830	0 28 E.	1 15 W.	0·8 W.	......	......	......	......	......	......	Erman.
At sea	23 51	318 51	1830	......	......	......	16 27 S.	......	16·5 S.	......	......	......	Erman.
At sea observations)	22 53	319 03	1832	0 15 E.	1 03 W.	0·8 W.	......	......	......	......	......	......	FitzRoy.
t sea	22 53	319 18	1837	2 12 W.	0 33 W.	2·7 W.	......	......	......	......	......	......	Du Petit Thouars.
t sea	25 59	319 19	1847	1 33 W.	0 30 E.	1·0 W.	......	......	......	......	......	......	Stanley.
t sea observations)	25 57	319 21	1847	......	......	......	17 43 S.	......	17·7 S.	6·54	......	6·54	Rattlesnake.
t sea observations)	24 15	319 36	1830	0 50 W.	1 15 W.	2·1 W.	16 35 S.	......	16·6 S.	6·45	......	6·45	Erman.
t sea	23 11	319 49	1826	3 18 E.	1 27 W.	2·8 W.	......	......	......	......	......	......	Prussian ships.
t sea	24 50	320 31	1830	......	......	......	17 27 S.	......	17·5 S.	6·56	......	6·56	Erman.
t sea	20 50	320 32	1843	3 00 W.	......	3·0 W.	......	......	......	......	......	......	Pasley.
t sea	22 07	320 39	1847	3 01 W.	0 27 E.	2·6 W.	......	......	......	......	......	......	Stanley.
t sea observations)	22 09	320 42	1847	......	......	......	10 54 S.	......	10·9 S.	6·37	......	6·37	Rattlesnake.
t sea observations)	25 54	321 00	1829	3 24 E.	1 21 W.	2·0 W.	......	......	......	......	......	......	Rumker.
t sea	24 43	321 12	1830	1 15 W.	1 15 W.	2·5 W.	......	......	......	......	......	......	Erman.
t sea observations)	22 08	321 35	1826	0 09 E.	1 39 W.	1·5 W.	......	......	......	......	......	......	Prussian ships.
t sea observations)	27 26	321 49	1847	......	......	......	20 31 S.	......	20·5 S.	......	......	......	Rattlesnake.
t sea	23 27	321 50	1839	......	......	......	12 52 S.	......	12·9 S.	6·17	......	6·17	Sulivan.
t sea observations)	21 21	321 58	1833	2 20 W.	0 57 W.	3·3 W.	......	......	......	......	......	......	Prussian ships.
t sea	20 12	322 06	1847	5 05 W.	0 27 E.	4·6 W.	......	......	......	......	......	......	Stanley.
t sea observations)	20 05	322 08	1847	......	......	......	8 15 S.	......	8·3 S.	6·26	......	6·26	Rattlesnake.
t sea.	27 52	322 42	1847	2 20 W.	0 27 E.	1·9 W.	......	......	......	......	......	......	Stanley.
t sea.	24 58	322 46	1830	1 21 W.	1 15 W.	2·6 W.	......	......	......	......	......	......	Erman.
sea observations)	27 23	322 48	1857	3 54 W.	1 27 E.	2·4 W.	......	......	......	......	......	......	Novara.
sea.	23 05	323 39	1839	......	......	......	13 00 S.	......	13·0 S.	6·09	......	6·09	Sulivan.
sea observations)	28 23	323 47	1847	......	......	......	22 36 S.	......	22·6 S.	6·46	......	6·46	Rattlesnake.
sea observations)	24 50	323 48	1830	2 29 W.	1 15 W.	3·7 W.	......	......	......	......	......	......	Erman.
sea.	24 53	324 26	1830	......	......	......	18 35 S.	......	18·6 S.	6·36	......	6·36	Erman.
sea.	20 56	324 42	1830	3 45 W.	1 15 W.	5·0 W.	9 47 S.	......	9·8 S.	6·03	......	6·03	Erman.
sea.	20 10	324 52	1830	3 19 W.	1 15 W.	4·6 W.	7 56 S.	......	7·9 S.	5·44	......	5·44	Erman.
sea observations)	22 29	324 52	1830	3 33 W.	1 15 W.	4·8 W.	......	......	......	......	......	......	Erman.
sea..	22 39	324 58	1842	3 30 W.	......	3·5 W.	......	......	......	......	......	......	Bérard.
sea observations)	24 58	325 03	1830	2 49 W.	1 15 W.	4·1 W.	......	......	......	......	......	......	Erman.
sea..	20 29	325 07	1826	2 00 W.	1 39 W.	3·6 W.	......	......	......	......	......	......	Lütke.
sea observations)	24 03	325 09	1830	3 17 W.	1 15 W.	4·5 W.	......	......	......	......	......	......	Erman.
sea..	24 44	325 10	1830	3 05 W.	1 15 W.	4·3 W.	......	......	......	......	......	......	Erman.
sea observations)	24 16	325 12	1830	......	......	......	16 07 S.	......	16·1 S.	6 05	......	6·05	Erman.
sea..	23 21	325 15	1830	3 12 W.	1 15 W.	4·4 W.	......	......	......	......	......	......	Erman.
sea..	22 21	325 34	1839	......	......	......	12 37 S.	......	12·6 S.	6 00	......	6·00	Sulivan.
sea..	21 40	325 55	1839	......	......	......	10 55 S.	......	10·9 S.	6·21	......	6·21	Sulivan.
sea observations)	28 56	327 03	1857	6 36 W.	1 27 E.	5·1 W.	......	......	......	......	......	......	Novara.
sea observations)	26 02	328 18	1842	4 30 W.	......	4·5 W.	......	......	......	......	......	......	Bérard.
sea..	21 11	329 31	1822	3 20 W.	2 00 W.	5·3 W.	12 42 S.	......	12·7 S.	......	......	......	Duperrey.
sea observations)	28 36	329 42	1850	......	......	......	20 55 S.	......	20·9 S.	6·54	......	6·54	Rattlesnake.
sea..	25 38	329 54	1826	4 55 W.	1 39 W.	6 6 W.	......	......	......	......	......	......	D'Urville.
sea..	22 01	330 22	1826	6 10 W.	1 39 W.	7·8 W.	......	......	......	......	......	......	D'Urville.
sea..	20 30	330 37	1839	8 01 W.	0 21 W.	8·4 W.	......	......	......	......	......	......	Ross.
sea observations)	27 28	330 38	1842	5 40 W.	......	5·7 W.	......	......	......	......	......	......	Bérard.
land Trinidad ...	20 31	330 38	1839	......	......	......	......	......	......	6 09	......	6·09	Erebus.
sea observations)	20 15	330 44	1842	......	......	......	12 03 S.	......	12·1 S.	6·23	......	6·23	H.M.S. 'Fly.'
sea..	21 30	330 46	1839	9 43 W.	0 21 W.	10·1 W.	......	......	......	......	......	......	Ross.
sea..	21 31	330 47	1839	......	......	......	......	......	......	6·12	......	6·12	Erebus.
sea..	21 47	330 50	1839	......	......	......	......	......	......	6·13	......	6·13	Erebus.
sea..	22 40	330 52	1839	9 01 W.	0 21 W.	9·4 W.	......	......	......	......	......	......	Ross.
sea observations)	23 14	330 55	1839	......	......	......	......	......	......	6·17	......	6·17	Erebus.
sea..	23 19	330 56	1839	7 45 W.	0 21 W.	8·1 W.	......	......	......	......	......	......	Ross.
sea..	20 01	331 00	1830	6 04 W.	1 15 W.	7·3 W.	......	......	......	......	......	......	Biscoe.

SOUTH EQUATORIAL ZONE III.—Lat. 20° to 30° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British Units.			Observer
				Observed.	Correction to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	° ′		°				
At sea……………………	24 16	331 45	1839	……	……	……	……	……	……	6·21	……	6·21	Erebus.
At sea……………………	24 28	331 57	1839	……	……	……	……	……	……	6·20	……	6·20	Erebus.
At sea……………………	24 42	332 05	1839	7 08 w.	0 21 w.	7·5 w.	……	……	……	……	……	……	Ross.
At sea (3 observations)	25 50	332 20	1842	……	……	……	20 37 s.	……	20·6 s.	6·28	……	6·28	H.M.S. 'Fly
At sea (2 observations)	25 40	332 46	1839	……	……	……	……	……	……	6·20	……	6·20	Erebus.
At sea……………………	21 08	333 04	1831	……	……	……	12 12 s.	……	12·2 s.	6·37	……	6·37	Dunlop.
At sea (2 observations)	26 12	333 22	1839	……	……	……	……	……	……	6·24	……	6·24	Erebus.
At sea……………………	26 58	333 24	1839	9 24 w.	0 21 w.	9·7 w.	……	……	……	……	……	……	Ross.
At sea……………………	26 37	333 26	1839	8 02 w.	0 21 w.	8·4 w.	……	……	……	……	……	……	Ross.
At sea……………………	26 49	333 30	1839	……	……	……	……	……	……	6·29	……	6·29	Erebus.
At sea……………………	26 52	333 30	1839	……	……	……	……	……	……	6·28	……	6·38	Erebus.
At sea……………………	28 16	333 48	1859	12 45 w.	1 39 E.	11·1 w.	……	……	……	……	……	……	Novara.
At sea (3 observations)	26 52	334 00	1850	……	……	……	20 40 s.	……	20·7 s.	6·51	……	6·51	Rattlesnake
At sea (3 observations)	27 30	334 00	1842	……	……	……	24 16 s.	……	24·3 s.	6·30	……	6·30	H.M.S. 'Fly
At sea (2 observations)	27 04	334 13	1839	……	……	……	……	……	……	6·26	……	6·26	Erebus.
At sea……………………	27 04	334 14	1839	10 04 w.	0 21 w.	10·4 w.	……	……	……	……	……	……	Ross.
At sea (3 observations)	29 05	334 33	1842	……	……	……	25 16 s.	……	25·3 s.	6·41	……	6·41	H.M.S. 'Fl
At sea……………………	23 06	334 39	1831	……	……	……	15 12 s.	……	15·2 s.	6·29	……	6·29	Dunlop.
At sea (4 observations)	26 30	334 39	1859	13 33 w.	1 22 E.	12·2 w.	……	……	……	……	……	……	Novara.
At sea (3 observations)	21 04	334 46	1850	……	……	……	12 32 s.	……	12·5 s.	6·42	……	6·42	Rattlesnake
At sea (4 observations)	26 02	334 46	1859	13 19 w.	1 22 E.	12·0 w.	……	……	……	……	……	……	Novara.
At sea (4 observations)	25 11	334 46	1859	13 23 w.	1 22 E.	12·0 w.	……	……	……	……	……	……	Novara.
At sea (3 observations)	23 10	334 51	1850	……	……	……	15 19 s.	……	15·3 s.	6·27	……	6·27	Rattlesnake
At sea (2 observations)	27 44	335 06	1839	……	……	……	……	……	……	6·33	……	6·33	Erebus.
At sea……………………	27 40	335 06	1839	9 37 w.	0 17 w.	9·9 w.	……	……	……	……	……	……	Ross.
At sea (2 observations)	26 06	335 19	1839	……	……	……	……	……	……	6·24	……	6·24	Erebus.
At sea……………………	25 30	335 20	1839	10 35 w.	0 17 w.	10·9 w.	……	……	……	……	……	……	Ross.
At sea……………………	26 55	335 21	1839	9 48 w.	0 17 w.	10·1 w.	……	……	……	……	……	……	Ross.
At sea……………………	26 53	335 26	1839	……	……	……	……	……	……	6·29	……	6·29	Erebus.
At sea……………………	25 21	335 28	1839	……	……	……	……	……	……	6·19	……	6·19	Erebus.
At sea……………………	25 45	335 43	1839	10 22 w.	0 17 w.	10·6 w.	……	……	……	……	……	……	Ross.
At sea (3 observations)	26 20	335 43	1850	……	……	……	21 06 s.	……	21·1 s.	6·56	……	6·56	Rattlesnak
At sea (6 observations)	25 32	335 53	1850	……	……	……	19 09 s.	……	19·2 s.	6·56	……	6·56	Rattlesnak
At sea……………………	26 12	336 12	1839	……	……	……	……	……	……	6·21	……	6·21	Erebus.
At sea……………………	29 30	336 40	1831	……	……	……	26 02 s.	……	26·0 s.	6·44	……	6·44	Dunlop.
At sea……………………	25 02	336 50	1831	……	……	……	17 45 s.	……	17·7 s.	6·22	……	6·22	Dunlop.
At sea……………………	26 51	337 11	1839	11 10 w.	0 17 w.	11·4 w.	……	……	……	……	……	……	Ross.
At sea……………………	29 06	337 17	1831	……	……	……	26 20 s.	……	26·3 s.	6·44	……	6·44	Dunlop.
At sea (2 observations)	27 05	337 25	1839	……	……	……	……	……	……	6·24	……	6·24	Erebus.
At sea……………………	27 51	337 51	1831	……	……	……	24 57 s.	……	24·9 s.	6·14	……	6·14	Dunlop.
At sea……………………	27 44	338 36	1839	12 00 w.	0 17 w.	12·3 w.	……	……	……	……	……	……	Ross.
At sea (2 observations)	27 44	338 37	1839	……	……	……	……	……	……	6·24	……	6·24	Erebus.
At sea……………………	26 12	339 12	1831	……	……	……	19 50 s.	……	19·8 s.	6·37	……	6·37	Dunlop.
At sea……………………	27 02	339 15	1831	……	……	……	23 46 s.	……	23·8 s.	6·07	……	6·07	Dunlop.
At sea……………………	28 17	339 59	1840	……	……	……	……	……	……	6·08	……	6·08	Terror.
At sea……………………	28 15	340 08	1840	13 23 w.	0 12 w.	13·6 w.	……	……	……	……	……	……	Ross.
At sea……………………	28 19	340 10	1840	……	……	……	……	……	……	6·20	……	6·20	Erebus.
At sea……………………	25 56	340 34	1831	……	……	……	22 01 s.	……	22·0 s.	6·14	……	6·14	Dunlop.
At sea……………………	28 03	341 41	1840	……	……	……	……	……	……	6·05	……	6·05	Terror.
At sea……………………	28 01	341 45	1840	……	……	……	……	……	……	6·20	……	6·20	Erebus.
At sea……………………	27 55	341 50	1840	13 13 w.	0 12 w.	13·4 w.	……	……	……	……	……	……	Ross.
At sea……………………	27 48	341 53	1840	……	……	……	……	……	……	6·15	……	6·15	Terror.
At sea……………………	27 33	342 27	1840	13 41 w.	0 12 w.	13·9 w.	……	……	……	……	……	……	Ross.
At sea……………………	27 26	342 28	1840	……	……	……	……	……	……	6·13	……	6·13	Terror.
At sea……………………	27 26	342 29	1840	……	……	……	……	……	……	6·22	……	6·22	Erebus.
At sea……………………	25 50	342 55	1840	……	……	……	……	……	……	6·20	……	6·20	Terror.
At sea……………………	25 48	342 55	1840	14 11 w.	0 12 w.	14·4 w.	……	……	……	……	……	……	Ross.
At sea (2 observations)	25 34	342 58	1840	……	……	……	……	……	……	6·17	……	6·17	Erebus.
At sea……………………	26 50	342 58	1840	……	……	……	……	……	……	6·18	……	6·18	Terror.
At sea……………………	26 46	342 58	1840	……	……	……	……	……	……	6·18	……	6·18	Erebus.
At sea……………………	24 28	343 00	1840	……	……	……	……	……	……	6·11	……	6·11	Terror.
At sea……………………	24 39	343 02	1840	14 26 w.	……	14·4 w.	……	……	……	……	……	……	Ross.

SOUTH EQUATORIAL ZONE III.—Lat. 20° to 30° S. (continued).

tions.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observers.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′		°							
t sea bservations)	24 10	343 05	1840	......	......	......	......	......	......	6·08	......	6·08	Erebus.
t sea.	22 56	343 30	1840	......	......	......	......	......	......	6·12	......	6·12	Terror.
t sea.	22 54	343 36	1840	14 59 w.	......	15·0 w.	......	......	......	......	......	......	Ross.
t sea (bservations)	22 41	343 42	1840	......	......	......	......	......	......	6·08	......	6·08	Erebus.
t sea.	21 43	344 13	1840	......	......	......	......	......	......	6·08	......	6·08	Terror.
t sea (bservations)	21 30	344 17	1840	......	......	......	......	......	......	6·07	......	6·07	Erebus.
t sea.	20 31	345 05	1840	......	......	......	......	......	......	6·00	......	6·00	Terror.
t sea.	20 20	345 07	1840	17 50 w.	......	17·8 w.	......	......	......	......	......	......	Ross.
sea (bservations)	20 20	345 12	1840	......	......	.. ...	......	......	......	6·07	......	6·07	Erebus.
sea.	29 56	345 53	1846	17 48 w.	......	17·8 w.	......	......	......	......	......	......	Bérard.
sea.	27 00	346 33	1840	17 53 w.	......	17·9 w.	......	......	......	......	......	......	Ross.
sea (bservations)	27 03	346 33	1840	......	......	......	......	......	......	6·32	......	6·32	Erebus.
sea.	26 51	346 37	1840	......	......	......	......	......	......	6·21	......	6·21	Terror.
sea.	27 54	346 42	1840	......	......	......	......	......	......	6·26	......	6·26	Terror.
sea (bservations)	27 56	346 43	1840	......	......	......	......	......	......	6·25	......	6·25	Erebus.
sea..	27 53	346 43	1840	18 30 w.	......	18·5 w.	......	......	......	......	......	......	Ross.
sea (bservations)	26 08	347 03	1840	......	......	......	......	......	......	6·19	......	6·19	Erebus.
sea..	26 03	247 07	1840	......	......	......	......	......	......	6·16	......	6·16	Terror.
sea..	26 10	347 18	1840	17 51 w.	......	17·8 w.	......	......	......	......	......	......	Ross.
sea..	25 23	347 49	1840	19 55 w.	......	19·9 w.	......	......	......	......	......	......	Ross.
sea..	25 38	347 41	1840	......	......	......	......	......	......	6·21	......	6·21	Erebus.
sea..	25 18	347 54	1840	......	......	......	......	......	......	6·15	......	6·15	Terror.
sea (bservations)	25 20	347 54	1840	......	......	......	......	......	......	6·14	......	6·14	Erebus.
sea (bservations)	24 26	348 12	1840	......	......	......	......	......	......	6·14	......	6·14	Erebus.
sea..	28 48	348 14	1840	18 19 w.	......	18·3 w.	......	......	......	......	......	......	Ross.
sea..	28 47	348 15	1840	......	......	......	......	......	......	6·32	......	6·32	Terror.
sea (bservations)	24 35	348 34	1840	......	......	......	......	......	......	6·11	......	6·11	Terror.
sea (bservations)	24 33	348 39	1840	......	......	......	......	......	......	6·12	......	6·12	Erebus.
sea...	24 41	348 39	1840	20 21 w.	......	20·3 w.	......	......	......	......	......	......	Ross.
sea (servations)	29 03	348 46	1840	......	......	......	......	......	......	6·26	......	6·26	Erebus.
sea...	28 05	349 15	1846	19 56 w.	......	19·9 w.	......	......	......	......	......	......	Bérard.
sea (servations)	23 47	350 29	1840	......	......	......	......	......	......	6·12	......	6·12	Erebus.
sea (servations)	23 37	350 35	1840	......	......	......	......	......	......	6·10	......	6·10	Terror.
sea...	29 56	350 42	1840	......	......	......	......	......	......	6·27	......	6·27	Terror.
sea...	29 58	350 52	1840	19 55 w.	......	19·9 w.	......	......	......	......	......	......	Ross.
sea...	23 32	351 04	1840	21 39 w.	......	21·6 w.	......	......	......	......	......	......	Ross.
sea (servations)	23 35	351 05	1840	......	......	......	......	......	......	6·14	......	6·14	Erebus.
sea...	22 00	351 19	1840	22 17 w.	......	22·3 w.	......	......	......	......	......	......	Ross.
sea (servations)	22 21	351 22	1840	......	......	......	......	......	......	6·12	......	6 12	Erebus.
sea (servations)	22 14	351 25	1840	......	......	......	......	......	......	6·19	......	6·19	Terror.
sea (servations)	22 02	351 27	1840	......	......	......	......	......	......	6·15	......	6·15	Erebus.
sea (servations)	22 00	351 30	1840	......	......	......	......	......	......	6·20	......	6·20	Terror.
sea (servations)	20 51	351 54	1840	......	......	......	......	......	......	6·17	......	6·17	Erebus.
sea...	20 15	352 04	1840	23 12 w.	......	23·2 w.	......	......	......	......	......	......	Ross.
sea...	20 22	352 06	1840	......	......	......	......	......	......	6·14	......	6·14	Terror.
sea...	24 50	353 32	1846	21 09 w.	......	21·1 w.	......	......	......	......	......	......	Bérard.

SOUTH EQUATORIAL ZONE IV.—LATITUDE 30° TO 40° S.

Authorities.

Bérard	MS. received from Admiral Duperrey.
Bonite	Voyage de la Bonite (Paris, 1842).
Novara	(Austrian Frigate) Reise um die Erde (Wien, 1862–65).
Vénus (La)	Voyage de la Vénus (Paris, 1841).

Authorities (continued).

Erman	Reise um die Erde (Berlin, 1841).
Dunlop	Contributions to Terrestrial Magnetism (Sabine in Philosophical Transactions, 1840 and 1849).
Smith	
Moore	
Clerk	
Pagoda	
Erebus	
Terror	
Bethune	
Wickham	
Dayman	
Sulivan	
Stanley	
Ross	
Crozier	
Pasley	
Du Petit Thouars	Sabine in Phil. Trans. 1849.
D'Urville	L. S. Kämtz, MSS. in the Magnetic Office, Kew Observatory.
Lütke	
Prussian Ships	
Jehenne	
Owen	
Bellamy	
Stirling	
Duperrey	
Bougainville	
Herd	
Cécille	
Laplace	
King	
Morrell	
Kotzebue	
FitzRoy	Voyage of the 'Beagle' (1849).
Rattlesnake	H.M.S. 'Rattlesnake,' MSS. in the Hydrographic Office.
Belcher	MSS. in the Magnetic Office, and Sabine in Phil. Trans. 1840.
Anonymous	MSS. in the Magnetic Office, received from Hydrographic Office.
D'Orbigny	L. S. Kämtz, MSS.
Biscoe	L. S. Kämtz, MSS.
Rumker	L. S. Kämtz, MSS.
Denham	MSS. in Magnetic Office, received from Hydrographic Office.
Beechey	MSS. in Magnetic Office, received from Admiral Sir E. Belcher.
Harkness	Smithsonian Contributions, vol. xviii.
H.M.S. 'Fly'	MSS. in the Hydrographic Office.
H.M.S. 'Nassau'	MSS. in the Hydrographic Office.
Macrae (U.S. Navy)	Astronomical Expedition from Santiago de Chilé to Buenos Ayres.
Stokes	H.M.S. 'Acheron,' MSS. in the Hydrographic Office.
Drury	H.M.S. 'Pandora,' MSS. in the Hydrographic Office.
Kerr	H.M.S. 'Pandora,' MSS. in the Hydrographic Office.
Kellett	H.M.S. 'Herald,' MSS. in the Hydrographic Office.
Richards	H.M.S. 'Hecate,' MSS. in the Hydrographic Office.

SOUTH EQUATORIAL ZONE IV.—Lat. 30° to 40° S.

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observers.
				Observed.	Correction to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	° ′		°				
t se ……	38 44	0 16	1831	……	……	……	43 44 s.	……	43·7 s.	6·82	……	6·82	Dunlop.
t se ……	38 52	1 01	1847	23 41 w.	0 15 E.	23·4 w.	……	……	……	……	……	……	Stanley.
t se ……	32 00	1 48	1840	24 07 w.	0 06 w.	24·2 w.	……	……	……	……	……	……	Ross.
t se ……	31 57	2 02	1840	……	……	……	……	……	……	6·64	……	6·64	Terror.
t se ……	32 01	2 17	1840	……	……	……	……	……	……	6·63	……	6·63	Ross.
t se ……	39 03	3 16	1831	……	……	……	44 48 s.	……	44·8 s.	6·82	……	6·82	Dunlop.
t se 3 observations)	38 28	3 37	1847	……	……	……	48 06 s.	……	48·1 s.	7·27	……	7·27	Rattlesnake.
t se ……	32 39	4 18	1840	……	……	……	……	……	……	6·85	……	6·85	Ross.
t se ……	38 00	4 20	1844	……	……	……	52 00 s.	……	52·0 s.	……	……	……	Smith.
t se ……	32 35	4 20	1840	……	……	……	……	……	……	6·67	……	6·67	Terror.
t se ……	32 41	4 24	1840	24 49 w.	0 06 w.	25·9 w.	……	……	……	……	……	……	Ross.
t se 3 observations)	35 23	4 25	1857	26 19 w.	0 44 E.	25·6 w.	……	……	……	……	……	……	Novara.
t se ……	38 19	4 37	1847	24 30 w.	0 14 E.	24·3 w.	……	……	……	……	……	……	Stanley.
t se ……	39 45	4 40	1831	……	……	……	46 24 s.	……	46·4 s.	6·89	……	6·89	Dunlop.
t se ……	33 02	5 40	1840	……	……	……	……	……	……	6·89	……	6·89	Terror.
se ……	33 09	5 48	1840	……	……	……	……	……	……	6·84	……	6·84	Ross.
se ……	33 14	6 03	1840	26 48 w.	0 04 w.	26·9 w.	……	……	……	……	……	……	Ross.
se ……	39 10	6 20	1831	……	……	……	46 55 s.	……	46·9 s.	6·97	……	6·97	Dunlop.
se 2 observations)	35 58	7 17	1826	24 40 w.	0 33 w.	25·2 w.	……	……	……	……	……	……	D'Urville.
se 2 observations)	33 25	7 30	1840	……	……	……	……	……	……	6·91	……	6·91	Ross.
se ……	33 20	7 32	1840	……	……	……	……	……	……	6·88	……	6·88	Terror.
se 3 observations)	37 42	7 37	1847	……	……	……	49 33 s.	……	49·5 s.	7·35	……	7·35	Rattlesnake.
t se ……	38 28	7 45	1844	……	……	……	53 03 s.	……	53·0 s.	……	……	……	Smith.
t se ……	33 29	7 48	1840	27 13 w.	0 04 w.	27·3 w.	……	……	……	……	……	……	Ross.
t se ……	33 10	9 02	1840	……	……	……	……	……	……	6·91	……	6·91	Terror.
se ……	33 21	9 04	1840	……	……	……	……	……	……	6·93	……	6·93	Ross.
se ……	33 27	9 06	1840	28 27 w.	0 04 w.	28·5 w.	……	……	……	……	……	……	Ross.
se ……	37 10	9 28	1847	27 12 w.	0 10 E.	27·0 w.	……	……	……	……	……	……	Stanley.
se ……	33 01	9 52	1840	28 21 w.	0 04 w.	28·4 w.	……	……	……	……	……	……	Ross.
sea ……	33 03	9 55	1840	……	……	……	……	……	……	7·04	……	7·04	Terror.
sea ……	38 11	10 07	1842	26 00 w.	……	26·0 w.	……	……	……	……	……	……	Bérard.
sea ……	33 08	10 11	1840	……	……	……	……	……	……	6·97	……	6·97	Ross.
sea ……	39 00	10 12	1831	……	……	……	48 31 s.	……	48·5 s.	6·97	……	6 97	Dunlop.
sea observations)	36 48	10 19	1847	……	……	……	50 27 s.	……	50·4 s.	7·43	……	7·43	Rattlesnake.
sea ……	33 14	10 37	1840	29 22 w.	0 05 w.	29·5 w.	……	……	……	……	……	……	Ross.
sea ……	30 02	11 37	1839	26 21 w.	0 07 w.	26·5 w.	……	……	……	……	……	……	Du Petit Thouars.
sea ……	33 08	11 43	1840	……	……	……	……	……	……	7·15	……	7·15	Terror.
sea observations)	36 41	11 48	1847	……	……	……	50 50 s.	……	50·8 s.	7·48	……	7·48	Rattlesnake.
sea ……	33 11	11 57	1840	……	……	……	……	……	……	6·99	……	6·99	Ross.
sea ……	39 06	12 00	1844	……	……	……	55 42 s.	……	55·7 s.	……	……	……	Smith.
sea ……	36 40	12 05	1847	27 50 w.	……	27·8 w.	……	……	……	……	……	……	Stanley.
sea ……	31 25	13 21	1839	27 19 w.	……	27·3 w.	……	……	……	……	……	……	Du Petit Thouars.
sea observations)	35 24	13 23	1845	……	……	……	51 19 s.	……	51·3 s.	6·99	……	6·99	Moore.
sea observations)	37 25	13 24	1845	……	……	……	51 44 s.	……	51·7 s.	7·23	……	7·23	Clerk.
sea observations)	35 10	13 25	1845	25 40 w.	……	25·7 w.	……	……	……	……	……	……	Pagoda.
sea observations)	35 12	13 28	1845	……	……	……	51 35 s.	……	51·6 s.	7·12	……	7·12	Clerk.
sea ……	33 00	13 36	1840	28 44 w.	……	28·7 w.	……	……	……	……	……	……	Ross.
sea observations)	34 14	13 44	1857	29 31 w.	0 15 E.	29·3 w.	……	……	……	……	……	……	Novara.
sea ……	32 56	13 48	1840	……	……	……	……	……	……	7·16	……	7·16	Terror.
sea ……	36 20	13 48	1847	28 27 w.	……	28·5 w.	……	……	……	……	……	……	Stanley.
sea ……	35 17	14 00	1845	27 15 w.	……	27·3 w.	51 16 s.	……	51·3 s.	6·92	……	6·92	Moore and Clerk.
sea. ……	32 57	14 00	1840	……	……	……	……	……	……	7·09	……	7·09	Ross.
sea. ……	39 07	14 00	1831	……	……	……	49 55 s.	……	49·9 s.	7·19	……	7·19	Dunlop.
sea observations)	36 24	14 02	1847	……	……	……	52 12 s.	……	52·2 s.	7·60	……	7·60	Rattlesnake.
sea. ……	30 40	14 08	1837	28 35 w.	0 05 w.	28·7 w.	……	……	……	……	……	……	Bonite.
sea observations)	32 59	14 18	1829	27 40 w.	0 13 w.	27·9 w.	……	……	……	……	……	……	Lütke.
sea. ……	32 53	14 21	1840	29 36 w.	……	29·6 w.	……	……	……	……	……	……	Ross.
sea. ……	38 43	14 25	1845	25 09 w.	……	25·1 w.	……	……	……	……	……	……	Pagoda.
sea observations)	38 42	14 27	1845	……	……	……	53 31 s.	……	53·5 s.	7·33	……	7·33	Moore.
sea observations)	38 37	14 27	1845	……	……	……	52 39 s.	……	52·6 s.	7·55	……	7·55	Clerk.
sea observations)	39 18	14 28	1845	28 20 w.	……	28·3 w.	……	……	……	……	……	……	Pagoda.
sea observations)	33 14	14 29	1836	28 53 w.	……	28·9 w.	……	……	……	……	……	……	Prussian ships.

SOUTH EQUATORIAL ZONE IV.—Lat. 30° to 40° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observers.
				Observed.	Correction to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	° ′	′	°				
At sea (7 observations)	39 10	14 38	1845	......	......	......	54 12 s.	......	54·2 s.	7·22	......	7·22	Moor
At sea (4 observations)	39 10	14 41	1845	......	......	......	54 14 s.	......	54·2 s.	7·41	......	7·41	Clerk
At sea (4 observations)	35 26	15 08	1845	28 39 w.	......	28·7 w.	......	......	......	......	......	......	Pago.
At sea (6 observations)	35 29	15 09	1845	......	......	......	51 27 s.	......	51·5 s.	7·25	......	7·25	Moor
At sea	32 14	15 20	1840	......	......	......	......	......	......	7·27	......	7·27	Terr
At sea	32 33	15 24	1840	......	......	......	......	......	......	7·15	......	7·15	Ross.
At sea	32 57	15 27	1842	31 16 w.	......	31·3 w.	......	......	......	......	......	......	Jeher e.
At sea	33 21	15 38	1837	31 22 w.	......	31·4 w.	......	......	......	......	......	......	Bonit
At sea	39 42	15 44	1844	......	......	......	57 06 s.	......	57·1 s.	......	......	......	Smitl
At sea	32 23	15 52	1840	29 23 w.	......	29·4 w.	......	......	......	......	......	......	Ross.
At sea	33 45	15 51	1837	28 43 w.	......	28·7 w.	......	......	......	......	......	......	Bonit
At sea	32 03	15 52	1837	27 16 w.	......	27·3 w.	......	......	......	......	......	......	Bonit
At sea	33 28	15 57	1839	26 37 w.	......	26·6 w.	......	......	......	......	......	......	Du P it Thoua
At sea	39 11	15 59	1843	28 22 w.	......	28·4 w.	......	......	......	......	......	......	Ross.
At sea	39 52	16 04	1843	26 38 w.	......	26·6 w.	......	......	......	......	......	......	Crozi.
At sea (3 observations)	33 33	16 15	1846	......	......	......	52 39 s.	......	52·6 s.	7·24	......	7·24	H.M. 'Fly.'
At sea	35 59	16 22	1843	27 50 w.	......	27·8 w.	......	......	......	......	......	......	Crozi.
At sea	32 44	16 27	1840	......	......	......	......	......	......	7·44	......	7·44	Terr
At sea (6 observations)	30 39	16 28	1846	......	......	......	51 03 s.	......	51·1 s.	7·29	......	7·29	H.M. 'Fly.'
At sea	37 44	16 36	1840	......	......	......	......	......	......	7·37	......	7·37	Terr
At sea	32 45	16 37	1840	......	......	......	......	......	......	7·12	......	7·12	Ross.
At sea	38 26	16 39	1843	29 24 w.	......	29·4 w.	......	......	......	......	......	......	Ross.
At sea	33 01	16 41	1840	......	......	......	......	......	......	7·31	......	7·31	Terr
At sea	35 42	16 44	1843	27 11 w.	......	27·2 w.	......	......	......	......	......	......	Crozi.
At sea	33 03	16 46	1840	......	......	......	......	......	......	7·27	......	7·27	Ross.
At sea	33 13	16 46	1840	......	......	......	......	......	......	7·52	......	7·52	Terr
At sea	32 16	16 52	1840	......	......	......	......	......	......	7·31	......	7·31	Terr
At sea	32 49	16 53	1840	29 29 w.	......	29·5 w.	......	......	......	......	......	......	Ross.
At sea (2 observations)	33 17	16 55	1840	......	......	......	......	......	......	7·41	......	7·41	Ross.
At sea	32 33	16 55	1840	29 25 w.	......	29·4 w.	......	......	......	......	......	......	Ross.
At sea (3 observations)	34 45	16 57	1846	......	......	......	53 11 s.	......	53·2 s.	7·47	......	7·47	H.M. 'Fly.'
At sea	32 17	17 00	1840	......	......	......	......	......	......	7·22	......	7·22	Ross.
At sea	38 47	17 00	1840	......	......	......	......	......	......	7·49	......	7·49	Terr
At sea (4 observations)	33 56	17 04	1851	29 20 w.	......	29·3 w.	......	......	......	......	......	......	Kelle.
At sea	33 21	17 07	1840	29 34 w.	......	29·6 w.	......	......	......	......	......	......	Ross.
At sea	32 59	17 08	1840	29 46 w.	......	29·8 w.	......	......	......	......	......	......	Ross.
At sea	39 22	17 24	1831	......	......	......	50 59 s.	......	51·0 s.	7·34	......	7·34	Dunl.
At sea	37 16	17 24	1840	......	......	......	......	......	......	7·41	......	7·41	Terr
At sea	38 58	17 26	1840	......	......	......	......	......	......	7·64	......	7·64	Terr
At sea	34 16	17 34	1840	......	......	......	......	......	......	7·31	......	7·31	Terr
At sea (4 observations)	34 42	17 36	1845	29 51 w.	......	29·9 w.	......	......	......	......	......	......	Pago.
At sea	32 31	17 41	1840	......	......	......	......	......	......	7·11	......	7·11	Ross.
At sea	32 31	17 45	1840	......	......	......	......	......	......	7·15	......	7·15	Terr
At sea (7 observations)	34 46	17 46	1845	......	......	......	53 34 s.	......	53·6 s.	7·38	......	7·38	Moor
At sea (3 observations)	34 44	17 50	1840	......	......	......	52 56 s.	......	52·9 s.	7·35	......	7·35	Clerk
At sea	34 37	17 51	1840	30 10 w.	......	30·2 w.	......	......	......	......	......	......	Ross
At sea	34 20	17 57	1840	......	......	......	......	......	......	7·36	......	7·36	Ross.
At sea	34 38	17 58	1839	27 45 w.	......	27·7 w.	......	......	......	......	......	......	Du P it Thou
At sea	34 18	18 03	1840	29 33 w.	......	29·5 w.	......	......	......	......	......	......	Ross
At sea	33 52	18 04	1840	......	......	......	......	......	......	7·42	......	7·42	Terr
At sea (3 observations)	34 55	18 09	1857	30 58 w.	......	31·0 w.	......	......	......	......	......	......	Nova.
At sea	33 56	18 10	1840	......	......	......	......	......	......	7·45	......	7·45	Ross.
At sea (3 observations)	36 10	18 22	1842	......	......	......	57 29 s.	......	57·5 s.	7·77	......	7·77	H.M 'Fly.'
At sea	36 52	18 25	1840	......	......	......	......	......	......	7·46	......	7·46	Terr
Simon's Bay	34 11	18 26	1825	28 12 w.	0 26 w.	28·6 w.	......	......	......	......	......		Owe
			1829	28 00 w.	0 20 w.	28·3 w.	......	......	......	......	......		Bella y.
			1836	28 30 w.	0 10 w.	28·7 w.	52 35 s.	+34	53·1 s.	7·57	......		Fitzl y.
			1837	......	......	...... } 28°·9 w.	52 26 s.	+29	52·9 s.	......	......	7·46	Beth e.
			1837	......	......	......	52 54 s.	+29	53·4 s. } 53·3 s.	......	......		Wic m.
			1839	29 09 w.	0 05 w.	29·2 w.	53 22 s.	+18	53·7 s.	......	......		Du P it Thoua
			1840	29 04 w.	0 04 w.	29·1 w.	......	......	......	......	......		Ross
			1840	29 40 w.	0 04 w.	29·7 w.	53 03 s.	+13	53·3 s.	7·35	......		Ross nd Crozie

SOUTH EQUATORIAL ZONE IV.—Lat. 30° to 40° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observers.
				Observed.	Correction to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	° ′	° ′	°				
			1841	29 07 w.	0 02 w.	29·1 w.	53 09 s.	+ 8	53·3 s.	......	......		Observatory.
			1842	29 06 w.	......	29·1 w.	53 14 s.	......	53·2 s.	......	......		Observatory.
			1843	29 04 w.	......	29·1 w.	53 19 s.	— 2	53·3 s.	......	......		Observatory.
			1844	29 06 w.	0 02 E.	29·1 w.	53 28 s.	— 7	53·3 s.	......	......		Observatory.
			1844	......	......	......	53 31 s.	— 7	53·4 s.	7·49	......		Observatory.
			1844	......	......	......	53 50 s.	— 7	53·7 s.	7·53	......		Observatory.
			1845	......	......	......	53 37 s.	— 13	53·4 s.	......	......		Observatory.
f Good Hope rvatory	33 56	18 29	1845	......	......	...... } 29·1 w.	53 34 s.	— 13	53·3 s. } 53·3 s.	......	......	} 7·53	Observatory.
			1845	......	......	......	53 28 s.	— 13	53·3 s.	7·58	......		Observatory.
			1845	29 07 w.	0 04 E.	29·1 w.	53 28 s.	— 13	53·3 s.	......	......		Observatory.
			1846	29 09 w.	0 05 E.	29·1 w.	53 33 s.	— 18	53·3 s.	......	......		Observatory.
			1847	29 12 w.	0 07 E.	29·1 w.	53 41 s.	— 24	53·3 s.	......	......		Observatory.
			1848	29 14 w.	0 08 E.	29·1 w.	53 47 s.	— 29	53·3 s.	......	......		Observatory.
			1849	29 16 w.	0 10 E.	29·1 w.	53 52 s.	— 34	53·3 s.	......	......		Observatory.
			1850	29 18 w.	0 11 E.	29·1 w.	53 58 s.	— 40	53·3 s.	......	......		Observatory.
			1851	......	......	......	54 02 s.	— 45	53·3 s.	......	......		Observatory.
			1842	29 10 w.	......	29·2 w.	53 12 s.	......	53·2 s.	7·43	......		Belcher.
servatory	33 56	18 29	1845	......	......	...... } 29·2 w.	53 30 s.	— 13	53·3 s. } 53·3 s.	7·50	......	} 7·55	Moore.
			1857	29 34 w.	0 22 E.	29·2 w.	54 36 s.	—1 15	53·4 s.	7·71	......		Novara.
2 observations)	35 14	18 27	1840	......	......	......	54 32 s.	......	54·5 s.	7·37	......	7·37	Ross.
.6observations)	34 12	18 27	1845	29 15 w.	......	29·3 w.	......	......	......	......	......	......	Pagoda.
....	36 11	18 35	1840	30 05 w.	......	30·1 w.	......	......	......	......	......	......	Ross.
3 observations)	37 25	18 35	1857	29 21 w.	......	29·3 w.	......	......	......	......	......	......	Novara.
....	34 18	18 40	1839	29 38 w.	......	29·6 w.	......	......	......	......	......	......	Du Petit Thouars.
....	35 52	18 41	1840	30 08 w.	......	30·1 w.	......	......	......	......	......	......	Ross.
....	35 48	18 47	1840	......	......	......	54 18 s.	......	54·3 s.	7·48	......	7·48	Ross.
2 observations)	36 00	19 00	1840	......	......	......	55 24 s.	......	55·4 s.	......	......	......	Ross.
2 observations)	34 20	19 00	1851	30 18 w.	......	30·3 w.	......	......	......	......	......	......	Kellett.
3 observations)	35 46	19 02	1837	31 09 w.	......	31·1 w.	......	......	......	......	......	......	Bonite.
....	35 47	19 05	1832	28 54 w.	0 15 w.	29·1 w.	......	......	......	......	......	......	Prussian ships.
....	36 04	19 19	1840	......	......	......	......	......	......	7·40	......	7·40	Erebus.
observations)	34 57	19 31	1846	......	......	......	53 23 s.	......	53·4 s.	7·65	......	7·65	H.M.S. ‘Fly.’
observations)	34 55	19 33	1845	28 57 w.	......	28·9 w.	......	......	......	......	......	......	Pagoda.
....	34 48	19 33	1845	......	......	......	54 50 s.	......	54·8 s.	7·37	......	7·37	Dayman.
....	39 06	20 00	1831	......	......	......	52 03 s.	......	52·1 s.	7·27	......	7·27	Dunlop.
....	36 16	20 04	1840	......	......	......	......	......	......	7·46	......	7·46	Terror.
observations)	35 08	20 24	1845	28 47 w.	......	28·8 w.	......	......	......	......	......	......	Pagoda.
observations)	37 33	20 25	1826	29 20 w.	0 22 w.	29·7 w.	......	......	......	......	......	......	D'Urville.
....	39 55	20 35	1840	31 09 w.	......	31·1 w.	......	......	......	......	......	......	Ross.
observations)	36 16	20 42	1840	30 15 w.	......	30 3 w.	......	......	......	......	......	......	Ross.
....	36 11	20 42	1840	......	......	......	......	......	......	7·47	......	7·47	Erebus.
.. ..	35 06	20 46	1845	27 56 w.	......	27·9 w.	......	......	......	.... .	......	......	Pagoda.
observations)	35 07	20 46	1845	......	......	......	55 08 s.	......	55·1 s.	7·59	......	7 59	Clerk.
observations)	36 07	20 55	1840	......	......	......	55 50 s.	......	55·8 s.	......	......	......	Ross.
observations)	36 25	21 15	1840	30 40 w.	......	30·7 w.	......	......	......	......	......	......	Ross.
observations)	36 29	21 16	1840	......	......	......	55 38 s.	......	55·6 s.	......	......	......	Ross.
observations)	38 16	21 20	1840	32 10 w.	......	32·2 w.	......	......	......	......	......	......	Ross.
....	36 35	21 20	1840	......	......	......	......	......	......	7·57	......	7·57	Erebus.
observations)	37 19	21 26	1840	30 51 w.	......	30·9 w.	......	......	......	......	......	......	Ross.
....	38 13	21 30	1840	......	......	......	......	......	......	7·83	......	7·83	Erebus.
observations)	37 20	21 37	1840	......	......	......	......	......	......	7·92	......	7·92	Erebus.
observations)	37 19	21 37	1840	......	......	......	56 03 s.	......	56·1 s.	......	......	......	Ross.
observations)	35 40	21 37	1845	29 16 w.	......	29·3 w.	......	......	......	......	......	......	Pagoda.
observations)	35 10	21 37	1846	......	......	......	55 28 s.	......	55·5 s.	7·80	......	7·80	H.M.S. ‘Fly.’
observations)	35 40	21 40	1845	......	......	......	55 31 s.	......	55·5 s.	7·74	......	7·74	Moore.
observations)	35 36	21 40	1845	......	......	......	55 18 s.	......	55·3 s.	7·68	......	7·68	Clerk.
....	34 29	21 44	1845	......	......	......	55 36 s.	......	55·6 s.	......	......	......	Dayman.
observations)	34 37	21 50	1851	30 48 w.	......	30·8 w.	......	......	......	......	......	......	Kellett.
observations)	38 11	22 00	1840	......	......	......	55 35 s.	......	55·6 s.	......	......	......	Ross.
bservations)	35 31	22 09	1829	28 40 w.	0 18 w.	29·0 w.	......	......	......	......	......	......	Lütke.
....	34 04	22 45	1845	......	......	......	56 43 s.	......	56·7 s.	......	......	......	Dayman.
bservations)	34 49	22 52	1836	27 41 w.	0 09 w.	27·8 w.	......	......	......	......	......	......	FitzRoy.
....	39 00	23 00	1831	......	......	......	54 01 s.	......	54·0 s.	7·57	......	7·57	Dunlop.

SOUTH EQUATORIAL ZONE IV.—Lat. 30° to 40° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Obs vers.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′		°	° ′		°				
At sea…………………	39 33	23 05	1844	……	……	……	59 32 s.	……	59·5 s.	……	……	……	Smith.
At sea (3 observations)	37 08	23 11	1846	……	……	……	56 37 s.	……	56·6 s.	8·00	……	8·00	H.M.S. Fly.'
At sea (4 observations)	35 46	23 35	1845	……	……	……	56 08 s.	……	56·1 s.	7·74	……	7·74	Clerk.
At sea (5 observations)	35 39	23 35	1845	29 26 w.	……	29·4 w.	……	……	……	……	……	……	Pagoda
At sea…………………	34 40	24 16	1845	……	……	……	56 32 s.	……	56·5 s.	7·52	……	7·52	Dayma
At sea (2 observations)	35 59	24 24	1829	28 01 w.	0 15 w.	28·3 w.	……	……	……	……	……	……	Prussia ships.
At sea (3 observations)	38 07	24 43	1857	30 25 w.	0 17 E.	30·1 w.	……	……	……	……	……	……	Novara
At sea…………………	38 04	25 00	1831	……	……	……	55 10 s.	……	55·2 s.	7·94	……	7·94	Dunlo
At sea…………………	34 36	25 23	1845	……	……	……	56 35 s.	……	56·6 s.	7·65	……	7·65	Dayma
At sea…………………	34 35	25 31	1845	……	……	……	56 45 s.	……	56·7 s.	……	……	……	Dayma
At sea…………………	35 12	25 40	1837	26 54 w.	……	26·9 w.	……	……	……	……	……	……	Bonite
At sea (3 observations)	36 50	25 48	1846	……	……	……	56 08 s.	……	56·1 s.	8·22	……	8·22	H.M.S Fly.'
At sea (3 observations)	37 58	26 22	1842	……	……	……	59 01 s.	……	59·0 s.	8·33	……	8·33	H.M.S Fly.'
At sea (3 observations)	34 27	26 23	1861	29 25 w.	……	29·4 w.	……	……	……	……	……	……	Denha
At sea…………………	34 44	26 50	1845	28 41 w.	……	28·7 w.	……	……	……	……	……	……	Pagoda
At sea…………………	39 33	26 52	1844	……	……	……	61 47 s.	……	61·8 s.	7·58	……	7·58	Smith.
At sea…………………	34 02	26 53	1845	……	……	……	57 23 s.	……	57·4 s.	7·58	……	7·58	Dayma
At sea (6 observations)	34 18	27 03	1851	30 40 w.	……	30·7 w.	……	……	……	……	……	……	Kellet
At sea (4 observations)	34 31	27 04	1845	……	……	……	57 06 s.	……	57·1 s.	7·93	……	7·93	Clerk.
At sea (3 observations)	36 44	27 40	1847	……	……	……	57 19 s.	……	57·3 s.	8·29	……	8 29	Rattle ake.
At sea (2 observations)	34 17	27 53	1834	28 38 w.	0 08 w.	28·8 w.	……	……	……	……	……	……	Prussi ships.
At sea (6 observations)	36 11	27 54	1846	……	……	……	57 19 s.	……	57·3 s.	8·11	……	8·11	H.M.S Fly.'
At sea…………………	39 50	28 36	1844	……	……	……	62 08 s.	……	62·1 s.	……	……	……	Smith
At sea (2 observations)	34 04	28 55	1829	30 10 w.	0 11 w.	30·3 w.	……	……	……	……	……	……	Lütke.
At sea (3 observations)	38 05	29 30	1847	……	……	……	58 31 s.	……	58·5 s.	8·45	……	8·45	Rattle ake.
At sea…………………	32 17	29 34	1845	……	……	……	58 16 s.	……	58·3 s.	7·69	……	7·69	Daym .
At sea (4 observations)	33 01	29 36	1845	……	……	……	57 34 s.	……	57·6 s.	7·84	……	7·84	Clerk.
At sea (4 observations)	32 54	29 49	1845	28 44 w.	……	28·7 w.	……	……	……	……	……	……	Pagod
At sea…………………	39 16	30 27	1831	……	……	……	58 11 s.	……	58·2 s.	8·24	……	8·24	Dunlo
At sea (5 observations)	32 54	30 44	1851	29 23 w.	……	29·4 w.	……	……	……	……	……	……	Kellet
At sea (3 observations)	34 42	30 54	1846	……	……	……	59 39 s.	……	59·6 s.	8·53	……	8·53	H.M.S 'Fly.'
At sea (5 observations)	31 06	31 26	1845	……	……	……	57 24 s.	……	57·4 s.	7·96	……	7·96	Moore
At sea (9 observations)	31 09	31 31	1845	26 46 w.	……	26·8 w.	……	……	……	……	……	……	Pagod
At sea (4 observations)	31 06	31 34	1845	……	……	……	57 28 s.	……	57·5 s.	7·95	……	7·95	Clerk.
At sea…………………	33 52	31 42	1837	29 21 w.	……	29·3 w.	……	……	……	……	……	……	Bonit
At sea…………………	31 19	32 21	1845	……	……	……	58 32 s.	……	58·5 s.	7·84	……	7·84	Daym .
At sea (3 observations)	38 09	32 36	1847	……	……	……	59 39 s.	……	59·7 s.	8·50	……	8·50	Rattl ake.
At sea……………… …	30 35	33 13	1845	25 09 w.	……	25·1 w.	……	……	……	……	……	……	Pagod
At sea (4 observations)	30 33	33 19	1845	……	……	……	57 19 s.	……	57·3 s.	8·13	……	8·13	Clerk
At sea…………………	39 26	33 30	1831	……	……	……	59 05 s.	……	59·1 s.	8·39	……	8·39	Dunlo
At sea (3 observations)	32 27	33 36	1846	……	……	……	59 07 s.	……	59·1 s.	8·14	……	8·14	H.M. 'Fly.'
At sea (2 observations)	30 27	33 41	1845	……	……	……	56 37 s.	……	56·6 s.	8·28	……	8·28	Moor
At sea (6 observations)	30 30	33 42	1845	27 28 w.	……	27·5 w.	……	……	……	……	……	……	Pagod
At sea (6 observations)	31 25	33 52	1861	27 24 w.	……	27·4 w.	……	……	……	……	……	……	Denh .
At sea (4 observations)	32 09	34 25	1829	27 59 w.	0 13 w.	28·2 w.	……	……	……	……	……	……	Lütke
At sea…………………	33 26	34 40	1837	22 35 w.	0 05 w.	22·7 w.	……	……	……	……	……	……	Bonit
At sea (6 observations)	31 27	34 59	1851	27 09 w.	……	27·2 w.	……	……	……	……	……	……	Kelle
At sea (8 observations)	32 18	35 17	1837	26 25 w.	0 05 w.	26·5 w.	……	……	……	……	……	……	Pruss n ships.
At sea…………………	39 16	35 37	1831	……	……	……	60 01 s.	……	60·0 s.	8 24	……	8·24	Dunl .
At sea (2 observations)	30 18	35 55	1845	26 29 w.	……	26·5 w.	……	……	……	……	……	……	Pago .
At sea (3 observations)	30 05	36 06	1846	……	……	……	59 06 s.	……	59·1 s.	8·14	……	8·14	H.M. 'Fly.'
At sea…………………	30 10	36 17	1845	……	……	……	59 08 s.	……	59·1 s.	8·06	……	8·06	Dayn .
At sea…………………	37 30	37 27	1831	……	……	……	58 11 s.	……	58·2 s.	8·09	……	8·09	Dunl .
At sea (2 observations)	30 31	38 06	1851	26 10 w.	……	26·2 w.	……	……	……	……	……	……	Kelle
At sea…………………	31 31	38 57	1837	26 05 w.	0 05 w.	26 2 w.	……	……	……	……	……	……	Bonit
At sea…………………	35 20	39 07	1831	……	……	.. ..	58 33 s.	……	58·5 s.	8·32	……	8·32	Dunl .
At sea…………………	35 10	40 50	1831	……	……	……	59 54 s.	……	59·9 s.	8·54	……	8·54	Dunl .
At sea (2 observations)	30 31	41 46	1839	23 50 w.	……	23·8 w.	……	……	……	……	……	……	Pruss n ships.
At sea (3 observations)	30 36	42 23	1846	……	……	……	58 22 s.	……	58·4 s.	8·74	……	8·74	H.M. 'Fly.'
At sea (2 observations)	30 58	42 44	1829	26 21 w.	0 13 w.	26·6 w.	……	……	……	……	……	……	Lütk
At sea…………………	35 54	43 16	1831	……	……	……	61 08 s.	……	61·1 s.	8·92	……	8·92	Dunl .
At sea (3 observations)	37 56	44 08	1826	30 30 w.	0 16 w.	30·8 w.	……	……	……	……	……	……	D'Ur le.

SOUTH EQUATORIAL ZONE IV.—Lat. 30° to 40° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observers.
				Observed.	Correction to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	° ′		°				
(3 observations)	38 16	45 23	1847	......	......	......	63 20 s.	......	63·3 s.	9·10	......	9·10	Rattlesnake.
(3 observations)	35 29	48 00	1842	......	......	......	63 49 s.	......	63 8 s.	9·18	......	9·18	H.M.S. ‘Fly.’
....	35 01	48 06	1831	......	......	......	60 27 s.	......	60·5 s.	8·84	......	8·84	Dunlop.
(3 observations)	30 21	48 15	1846	......	......	......	60 35 s.	......	60·6 s.	......	......	......	H.M.S. ‘Fly.’
(6 observations)	30 22	51 24	1846	......	......	......	62 05 s.	......	62·1 s.	8·99	......	8·99	H.M.S. ‘Fly.’
....	36 18	52 48	1831	......	......	......	62 46 s.	......	62·8 s.	9·52	......	9·52	Dunlop.
(3 observations)	36 56	53 00	1847	......	......	......	64 10 s.	......	64 2 s.	9·45	......	9·45	Rattlesnake.
(3 observations	34 33	54 10	1847	......	......	......	64 04 s.	......	64 1 s.	9·61	......	9·61	Rattlesnake.
3 observations)	36 50	55 16	1842	......	......	......	64 17 s.	......	64·3 s.	9·41	......	9·41	H.M.S. ‘Fly.’
....	37 07	55 20	1831	......	......	......	63 13 s.	......	63·2 s.	9·52	......	9·52	Dunlop.
....	37 40	59 15	1831	......	......	......	63 54 s.	......	63·9 s.	9·67	......	9·67	Dunlop.
....	39 34	60 55	1844	......	......	......	67 27 s.	......	67 5 s.	......	......	......	Smith.
....	37 46	63 38	1831	......	......	......	63 51 s.	......	63 9 s.	9·59	......	9·59	Dunlop.
3 observations)	38 00	64 16	1826	24 38 w.	0 16 w.	24·9 w.	......	......	......	......	......	......	D’Urville.
....	39 00	65 44	1844	......	......	......	67 19 s.	......	67·3 s.	......	......	......	Smith.
....	37 50	66 04	1831	......	......	......	64 10 s.	......	64·2 s.	9·82	......	9·82	Dunlop.
3 observations)	31 50	67 56	1847	......	......	......	65 39 s.	......	65·7 s.	10·12	......	10·12	Rattlesnake.
....	38 31	68 45	1844	......	......	......	66 45 s.	......	66·7 s.	......	......	......	Smith.
....	38 15	69 31	1831	......	......	......	65 04 s.	......	65·1 s.	10·34	......	10·34	Dunlop.
3 observations)	33 39	70 05	1847	......	......	......	66 30 s.	......	66·5 s.	10·23	......	10·23	Rattlesnake.
....	38 22	70 10	1844	......	......	......	67 06 s.	......	67·1 s.	......	......	......	Smith.
3 observations)	35 23	71 55	1847	......	......	......	67 31 s.	......	67·5 s.	10·35	......	10·35	Rattlesnake.
....	38 08	73 35	1844	......	......	......	66 45 s.	......	66·7 s.	......	......	......	Smith.
....	39 31	74 00	1831	......	......	......	65 02 s.	......	65·0 s.	10·27	......	10·27	Dunlop.
3 observations)	36 04	74 02	1847	......	......	......	67 36 s.	......	67·6 s.	10·74	......	10·74	Rattlesnake.
....	38 10	75 22	1844	......	......	......	66 42 s.	......	66·7 s.	......	......	......	Smith.
....	38 25	76 44	1844	......	......	......	66 54 s.	......	66·9 s.	......	......	......	Smith.
d off St. Paul..	......	......	1842	......	......	......	66 58 s.	......	67·0 s.	11·19	......	11·19	H.M.S. ‘Fly.’
....	38 43	77 31	1857	23 25 w.	0 15 E.	23·2 w.	66 56 s.	......	66·9 s	11·33	......	11·33	Novara.
....	38 48	77 50	1844	......	......	......	67 14 s.	......	67·2 s.	......	......	......	Smith.
observations)	38 29	77 50	1857	21 47 w.	0 15 E.	21·5 w.	......	......	......	......	......	......	Novara.
....	39 04	79 45	1844	......	......	......	67 17 s.	......	67·3 s.	......	......	......	Smith.
observations)	35 30	80 01	1847	......	......	......	67 20 s.	......	67·3 s.	11·00	......	11·00	Rattlesnake.
....	39 27	81 00	1831	......	......	......	66 07 s.	......	66·1 s.	......	......	......	Dunlop.
observations)	34 40	82 43	1857	18 12 w.	0 15 E.	17·9 w.	......	......	......	......	......	......	Novara.
....	39 58	84 00	1844	......	......	......	67 43 s.	......	67·7 s.	......	......	......	Smith.
observations)	35 20	84 12	1847	......	......	......	67 19 s.	......	67·3 s.	11·15	......	11·15	Rattlesnake.
observations)	31 50	84 34	1857	13 58 w.	0 15 E.	13·7 w.	......	......	......	......	......	......	Novara.
observations)	36 45	85 27	1826	18 40 w.	0 16 w.	18·9 w.	......	......	......	......	......	......	D’Urville.
observations)	35 34	87 32	1847	......	......	......	67 37 s.	......	67·6 s.	11·26	......	11·26	Rattlesnake.
observations)	39 00	88 50	1842	......	......	......	67 46 s.	......	67·8 s.	11·18	......	11·18	H.M.S. ‘Fly.’
observations)	35 50	91 02	1847	......	......	......	67 45 s.	......	67·7 s.	11·52	......	11·52	Rattlesnake.
observations)	35 57	92 43	1826	16 12 w.	0 08 w.	16·3 w.	......	......	......	......	......	......	D’Urville.
observations)	36 15	94 42	1847	......	......	......	67 55 s.	......	67·9 s.	11·83	......	11·83	Rattlesnake.
observations)	39 13	95 30	1842	......	......	......	67 49 s.	......	67·8 s.	11·82	......	11·82	H.M.S. ‘Fly.’
observations)	30 59	97 01	1848	......	......	......	60 23 s.	......	60·4 s.	......	......	......	Rattlesnake.
observations)	39 43	98 10	1842	......	......	......	68 33 s.	......	68·6 s.	12·12	......	12·12	H.M.S. ‘Fly.’
....	39 52	99 22	1844	......	......	......	69 00 s.	......	69·0 s.	......	......	......	Smith.
....	30 05	99 51	1827	7 02 w.	0 07 w.	7·1 w.	......	......	......	......	......	......	D’Urville.
observations)	37 24	101 05	1847	......	......	......	68 50 s.	......	68 8 s.	12·46	......	12·46	Rattlesnake.
....	39 54	102 00	1844	......	......	......	69 00 s.	......	69·0 s.	......	......	......	Smith.
observations)	37 57	103 09	1847	......	......	......	69 34 s.	......	69·6 s.	12 47	......	12·47	Rattlesnake.
observations)	35 15	104 44	1848	......	......	......	64 26 s.	......	64·4 s.	......	......	......	Rattlesnake.
observations)	38 43	105 44	1842	......	......	......	69 30 s.	......	69·5 s.	12·28	......	12·28	H.M.S. ‘Fly.’
observations)	38 35	106 38	1847	......	......	......	79 14 s.	......	70·2 s.	12·66	......	12 66	Rattlesnake.
observations)	35 46	107 09	1826	12 50 w.	......	12·8 w.	......	......	......	......	......	......	D’Urvillo.
....	33 37	108 24	1845	......	......	......	62 18 s.	......	62·3 s.	12·01	......	12·01	Dayman.
observations)	30 25	109 07	1845	7 20 w.	......	7·3 w.	......	......	......	......	......	......	Pagoda.
observations)	38 57	110 20	1847	......	......	......	70 28 s.	......	70·5 s.	12 82	......	12·82	Rattlesnake.
....	33 47	111 04	1845	......	......	......	63 23 s.	......	63·4 s.	12·06	......	12·06	Dayman.
observations)	32 24	111 26	1845	......	......	......	62 22 s.	......	62·4 s.	11·79	......	11·79	Moore.
observations)	32 32	111 36	1845	......	......	......	62 14 s.	......	62·2 s.	12·09	......	12·09	Clerk.

SOUTH EQUATORIAL ZONE IV.—Lat. 30° to 40° S. (continued).

Stations.	Lat. S	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Obs vers.
				Observed.	Correction to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch. 1842·5.	Corrected.	
	° ′	° ′		° ′		°	° ′		°				
At sea (5 observations)	32 42	111 43	1845	6 36 w.	......	6·6 w.	......	......	......	......	......	......	Pagoda
At sea......................	34 58	112 59	1845	......	......	......	65 16 s.	......	65·3 s.	12·63	......	12 63	Dayma
At sea (4 observations)	34 16	113 01	1845	......	......	......	64 44 s.	......	64·7 s.	12·30	......	12·30	Clerk.
At sea (4 observations)	34 18	113 12	1845	6 37 w.	......	6·6 w.	......	......	......	......	......	......	Pagoda
At sea (3 observations)	34 17	113 50	1846	......	......	......	65 39 s.	......	65·6 s.	12·62	......	12·62	H.M.S.Fly.'
At sea......................	35 46	114 14	1845	......	......	......	65 48 s.	......	65·8 s.	......	......	......	Dayma
At sea (3 observations)	39 34	114 20	1847	......	......	......	71 05 s.	......	71·1 s.	13·08	......	13·08	Rattles ke.
At sea......................	35 30	114 35	1845	......	......	......	64 54 s.	......	64·9 s.	12·53	......	12·53	Moore.
At sea (3 observations)	35 09	114 41	1826	9 28 w.	......	9·5 w.	......	......	......	......	......	. ....	D'Urvi .
At sea (3 observations)	35 33	114 42	1845	5 41 w.	......	5·7 w.	......	......	......	......	......	......	Pagoda
At sea (4 observations)	35 36	114 44	1845	......	......	......	65 28 s.	......	65·5 s.	12·65	......	12·65	Clerk.
At sea......................	37 00	115 10	1845	......	......	......	66 35 s.	......	66·6 s.	......	......	......	Dayma
At sea (3 observations)	38 04	115 20	1842	......	......	......	67 35 s.	......	67·6 s.	13·17	......	13·17	H.M.S Fly.'
At sea (3 observations)	31 55	115 22	1846	......	......	......	63 26 s.	......	63·4 s.	12·22	......	......	H.M.S Fly.'
Rottnest Island.........	31 58	115 24	1829	5 20 w.	......	5·3 w.	......	......	......	......	......	......	Stirling
At sea......................	36 24	115 33	1845	......	......	......	66 44 s.	......	66·7 s.	......	......	......	Dayma
At sea......................	35 42	115 40	1845	5 59 w.	......	6·0 w.	......	......	......	......	......	......	Pagoda
Swan River	32 03	115 41	1827	4 17 w.	......	4·3 w.	......	......	......	......	......	......	Stirling
			1837	......	......	......	62 24 s.	......	62·4 s.	12·39	......	12·39	Wickh .
Fremantle	32 03	115 47	1843	4 30 w.	......	4·5 w.	62 44 s.	......	62·7 s.	12·19	......	12·19	H.M.S Fly.'
			1846	......	......	......	63 06 s.	......	63·1 s.	12·52	.. ...	12·52	H.M.S Fly.'
At sea (3 observations)	35 20	115 48	1836	7 22 w.	......	7·4 w.	......	......	......	......	......	......	FitzRo
At sea (5 observations)	38 40	116 15	1845	......	......	......	68 04 s.	......	68·1 s.	12 90	......	12·90	Moore
At sea (4 observations)	38 52	116 15	1845	......	......	......	68 49 s.	......	68·8 s.	13·02	......	13·02	Clerk.
At sea (4 observations)	38 40	116 17	1845	6 56 w.	......	6·9 w.	......	......	......	......	......	......	Pagoda
At sea (3 observations)	36 51	116 36	1845	4 31 w.	......	4·5 w.	......	......	......	......	......	......	Pagoda
At sea......................	36 06	116 42	1845	......	......	......	66 27 s.	......	66·5 s.	12·85	......	12·85	Dayma
At sea (8 observations)	36 10	116 43	1845	4 52 w.	......	4·9 w.	......	......	......	......	......	......	Pagoda
At sea (4 observations)	36 11	116 48	1845	......	......	......	66 00 s.	......	66·0 s.	12·51	......	12·51	Moore
At sea (4 observations)	36 12	116 50	1845	......	......	......	65 28 s.	......	65·5 s.	12 54	......	12·54	Clerk.
At sea (4 observations)	37 03	116 57	1845	......	......	......	66 46 s.	......	66·8 s.	12·70	......	12·70	Clerk.
At sea (6 observations)	37 00	116 57	1845	......	......	......	66 21 s.	......	66·3 s.	12·57	......	12·57	Moore
At sea......................	35 28	117 04	1845	6 55 w.	......	6·9 w.	......	......	......	......	......	......	Pagoda
At sea (4 observations)	35 18	117 07	1845	......	......	......	65 48 s.	......	65·8 s.	12·75	......	12·75	Clerk.
At sea (9 observations)	35 21	117 11	1846	......	......	......	66 03 s.	......	66·1 s.	12·51	......	12·51	H.M.S Fly.'
At sea (4 observations)	35 07	117 28	1845	......	......	......	65 24 s.	......	65·4 s.	12·69	......	12·69	Moore
At sea......................	36 58	117 38	1845	......	......	......	67 19 s.	......	67·3 s.	......	......	......	Dayma
At sea (5 observations)	35 12	117 41	1845	6 20 w.	......	6·3 w.	......	......	......	......	......	......	Pagod
At sea......................	35 22	117 46	1845	......	......	......	65 53 s.	......	65·9 s.	12·90	......	12·90	Dayma
At sea (3 observations)	39 54	117 50	1847	......	......	......	71 58 s.	......	72·0 s.	13·34	......	13·34	Rattle ake.
At sea (2 observations)	35 06	117 55	1845	......	......	......	65 59 s.	......	66·0 s.	......	......	......	Moore
King George's Sound..	35 02	117 56	1826	5 35 w.	......	5·6 w. } 5°·6 w.	......	......	...... } 65°·0 s.	......	......	} 12·65	D'Urv e.
			1836	5 36 w.	......	5·6 w.	64 41 s.	......	64·7 s.	12·80	......		FitzR
			1845	......	......	......	65 11 s.	......	65·2 s.	12·65	......		Clerk.
			1845	......	......	......	64 51 s.	......	64·9 s.	12·61	......		Clerk.
			1845	......	......	......	64 55 s.	......	64·9 s.	12·65	......		Moore
			1845	......	......	......	64 52 s.	......	64·9 s.	12·53	......		Moore
			1845	......	......	......	65 07 s.	......	65·1 s.	......	......		Moore
			1845	......	......	......	65 02 s.	......	65·0 s.	12·65	......		Moore
			1845	5 33 w.	......	5·5 w.	......	......	......	......	......		Pagod
			1858	5 42 w.	......	5·7 w.	......	......	......	......	......		Denha .
At sea......................	35 10	118 06	1845	5 57 w.	......	5·9 w.	......	......	......	......	......	......	Pagod
At sea......................	36 42	118 35	1845	......	......	......	67 03 s.	......	67·1 s.	......	......	......	Daym .
At sea (2 observations)	36 00	118 40	1836	5 45 w.	......	5·7 w.	......	......	......	......	.. ...	......	FitzR
At sea......................	36 28	118 57	1845	......	......	......	67 08 s.	......	67·1 s.	12·86	......	12·86	Daym .
At sea (3 observations)	34 55	119 26	1846	......	......	......	65 22 s.	......	65·4 s.	12·33	......	12·33	H.M.S Fly.'
At sea (3 observations)	36 02	120 33	1846	......	......	......	66 03 s.	......	66·1 s.	12·52	......	12·52	H.M.S Fly.'
At sea......................	37 13	120 40	1845	......	......	......	68 06 s.	......	68·1 s.	......	......	......	Daym .
At sea......................	37 14	121 58	1845	......	......	......	67 07 s.	......	67·1 s.	......	......	......	Daym .
At sea......................	37 52	122 29	1845	......	......	......	67 39 s.	......	67·7 s.	......	......	......	Daym .
At sea......................	38 21	122 46	1845	......	......	......	67 27 s.	......	67·5 s.	13·31	......	13·31	Daym .
At sea (4 observations)	38 28	123 25	1836	3 19 w.	......	3·3 w.	......	......	......	......	......	......	FitzR

SOUTH EQUATORIAL ZONE IV.—Lat. 30° to 40° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.				Inclination.				Force in British units.			Observers.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.		Ob-served.	Cor. to Epoch 1842·5.	Corrected.		Ob-served.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′		°		° ′		°					
se 3 observations)	35 30	123 28	1846	......	......	......		66 24 s.	......	66·4 s.		12·68	......	12·68	H.M.S. 'Fly.'
se	38 00	123 38	1844	......	......	......		68 21 s.	......	68·3 s.		13·48	......	13·48	Dayman.
se	39 25	124 04	1844	......	......	......		69 40 s.	......	69·7 s.		13·43	......	13·43	Dayman.
se 2 observations)	37 06	124 11	1826	3 35 W.	......	3·6 W.		......	......	......		......	......	.. ...	D'Urville.
se	37 14	124 58	1845	......	......	......		......	......	......		13·27	......	13·27	Dayman.
se 3 observations)	34 55	125 10	1846	......	......	......		65 40 s.	......	65·7 s.		12·86	......	12·86	H.M.S. 'Fly.'
se 3 observations)	35 25	127 33	1846	......	......	......		67 41 s.	......	67·7 s.		12·91	......	12·91	H.M.S. 'Fly.'
se 6 observations)	35 12	130 02	1846	......	......	......		66 42 s.	......	66·7 s.		12·75	......	12·75	H.M.S. 'Fly.'
se 4 observations)	38 39	134 53	1826	0 37 W.	......	0·6 W.		......	......	......		......	......	......	D'Urville.
se 3 observations)	35 45	135 33	1846	......	......	......		66 47 s.	......	66·8 s.		12·68	......	12·68	H.M.S. 'Fly.'
se 3 observations)	35 40	137 04	1846	......	......	......		66 26 s.	......	66·4 s.		12·62	......	12·62	H M.S. 'Fly.'
se 3 observations)	34 58	138 33	1846	......	......	......		66 21 s.	......	66·3 s.		12·60	......	12·60	H.M.S. 'Fly.'
en	34 58	138 33	1846	6 16 E.	......	6·3 E.		65 28 s.	......	65·5 s.		12·89	......	12·89	H.M.S. 'Fly.'
se 3 observations)	36 15	138 37	1846	......	......	......		65 58 s.	......	66·0 s.		12·50	......	12·50	H.M.S. 'Fly.'
se 3 observations)	37 48	139 30	1846	......	......	......		66 46 s.	.. ...	66·8 s.		12·99	......	12·99	H.M.S. 'Fly.'
se 3 observations)	38 42	141 36	1846	......	......	......		67 16 s.	......	67·3 s.		12·98	......	12·98	H.M.S. 'Fly.'
rt ilip	37 52	144 53	1846	......	......	......		66 57 s.	......	66·9 s.	67·1 s.	13·16	......	13·17	H.M.S. 'Fly.'
			1848	9 09 E.	.. ...	9·1 E.		67 15 s.	......	67·3 s.		13·19	......		Rattlesnake.
se 3 observations)	37 29	146 41	1826	8 31 E.	......	8·5 E.		......	......	......		......	......	......	D'Urville.
se observations)	37 54	150 20	1841	10 38 E.	......	10·6 E.		......	......	......		......	......	......	Terror.
se observations)	37 48	150 21	1841	11 01 E.	......	11·0 E.		......	......	.. ...		......	......	......	Erebus.
se observations)	38 17	150 22	1841	......	......	......		66 57 s.	......	66·9 s.		13·37	......	13·37	Terror.
se	37 50	150 22	1841	......	......	......		66 36 s.	......	66·6 s.		......	......	......	Erebus.
se 3 observations)	37 36	150 58	1841	......	......	......		......	......	......		13·15	......	13·15	Erebus.
se observations)	33 56	151 00	1841	11 18 E.	......	11·3 E.		......	......	......		......	......	......	Terror.
de sland, Port ach n	33 51	151 17	1824	8 56 E.	......	8·9 E.	9·6 E.	62 18 s.	......	62·3 s.	62·7 s.	......	......	12·67	Duperrey.
			1825	8 03 E.	......	8·1 E.		......	......	......		......	......		Bougainville.
			1826	9 17 E.	......	9·3 E.		......	......	......		......	......		D'Urville.
			1831	......	......	......		62 53 s.	......	62·9 s.		......	......		Bethune.
			1831	......	......	......		62 51 s.	......	62·9 s.		12·48	......		Dunlop.
			1836	10 24 E.	......	10·4 E.		62 49 s.	......	62·8 s.		12·62	......		FitzRoy.
			1837	......	......	......		62 48 s.	......	62·8 s.		......	......		Anonymous.
			1838	......	......	......		62 51 s.	......	62·9 s.		12·60	......		Wickham.
			1841	......	......	......		62 59 s.	......	63 0 s.		12·73	......		Terror.
			1841	......	......	......		62 48 s.	......	62·8 s.		12 63	......		Erebus.
			1841	......	......	......		62 52 s.	......	62·9 s.		12·88	......		Terror.
			1841	......	......	....		62 42 s.	......	62·7 s.		12·78	......		Erebus.
			1841	9 51 E.	......	9·9 E		......	......	......		......	......		Erebus.
			1842	......	......	......		62 37 s.	......	62·6 s.		......	......		H.M.S. 'Fly.'
			1844	9 25 E.	......	9·4 E.		62 45 s.	......	62·8 s.		......	......		H.M.S. 'Fly.'
			1845	......	......	......		62 31 s.	......	62·5 s.		......	......		H.M.S. 'Fly.'
			1848	10 05 E.	......	10·1 E.		......	......	......		......	......		Rattlesnake.
			1849	10 09 E.	......	10·1 E.		62 44 s.	......	62·7 s.		12·59	......		Rattlesnake.
			1852	......	......	......		62 44 s.	......	62·7 s.		......	......		Kerr.
			1858	10 00 E.	......	10 0 E.		62 41 s.	......	62·7 s.		12·70	......		Novara.
sea observations)	34 06	151 19	1841	......	......	......		62 58 s.	......	63·0 s.		......	......	......	Terror.
sea(observations)	33 51	151 20	1841	......	......	......		62 47 s.	......	62·8 s.		......	......	......	Erebus.
sea observations)	35 14	151 29	1841	......	......	......		......	......	......		13·02	......	13·02	Terror.
sea.	34 26	151 30	1826	9 09 E.	......	9·1 E.		......	......	......		......	......	......	D'Urville.
sea observations)	37 28	151 30	1841	......	......	......		66 22 s.	......	66·4 s.		13·07	......	13·07	Terror.
sea(observations)	37 10	151 32	1841	11 32 E.	......	11·5 E.		......	......	......		......	......	......	Terror.
sea observations)	37 21	151 33	1841	......	......	......		66 01 s.	......	66·0 s.		......	......	......	Erebus.
sea(observations)	37 14	151 34	1841	9 31 E.	......	9·5 E.		......	......	......		......	......	......	Erebus.
sea observations)	34 56	151 35	1841	......	......	......		......	......	......		12·89	......	12 89	Erebus.
sea observations)	36 21	151 39	1841	......	......	......		66 11 s.	......	66·2 s.		......	......	......	Terror.
sea observations)	36 01	151 48	1841	......	......	......		65 04 s.	......	65·1 s.		......	......	......	Erebus.
rt St hens	32 40	152 02	1842	9 45 E.	......	9·8 E.		61 24 s.	......	61·4 s.		12·30	......	12·30	H.M.S. 'Fly.'
			1844	......	......	......		61 29 s.	......	61·5 s.		12·30	......	12·30	H.M.S. 'Fly.'
sea.	31 37	153 28	1858	8 58 E.	......	9·0 E.		.. ...	......	......		......	......	......	Novara.
sea observations)	33 58	153 35	1841	......	......	......		62 30 s.	......	62·5 s.		......	......	......	Terror.
sea observations)	33 58	153 36	1841	......	......	......		......	......	......		12·76	......	12·76	Terror.
sea observations)	34 09	153 42	1850	......	......	......		61 36 s.	......	61·6 s.		12·41	......	12·41	Rattlesnake.
sea observations)	33 54	153 50	1841	10 07 E.	......	10·1 E.		......	......	......		......	......	......	Erebus.

SOUTH EQUATORIAL ZONE IV.—Lat. 30° to 40° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Ob rvers.
				Observed.	Correction to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	° ′		°				
At sea (7 observations)	36 20	153 58	1826	9 05 E.	......	9·1 E.	......	......	......	......	......	......	D'Urv le.
At sea (5 observations)	33 52	154 07	1841	......	......	......	62 47 s.	......	62·8 s.	12·69	......	12·69	Erebu
At sea (4 observations)	33 56	156 38	1841	......	......	......	61 46 s.	......	61·8 s.	12·58	......	12 58	Terro
At sea (5 observations)	33 51	157 18	1841	......	......	......	62 07 s.	......	62·1 s.	12·49	......	12·49	Erebu
At sea (3 observations)	32 32	157 23	1858	11 07 E.	0 16 w.	10·9 E.	......	......	......	......	......	......	Novar
At sea (5 observations)	33 12	158 10	1854	8 30 E.	0 12 w.	8·3 E.	......	......	......	......	......	......	Denha .
At sea (2 observations)	33 09	159 18	1863	11 47 E.	......	11·8 E.	......	......	......	......	......	......	Richa s.
At sea (4 observations)	33 31	160 20	1841	......	......	......	61 04 s.	......	61·1 s.	12·52	......	12·52	Terro
At sea (5 observations)	33 27	160 43	1841	...	......	......	61 30 s.	......	61·5 s.	12·40	......	12·40	Erebu
At sea (5 observations)	31 11	161 04	1855	12 04 E.	10 13 w.	11·9 E.	......	......	......	......	......	......	Denha .
At sea (4 observations)	33 00	161 24	1854	10 49 E.	0 12 w.	10·6 E.	......	......	......	......	......	......	Denha .
At sea (2 observations)	32 22	161 47	1858	11 16 E.	0 16 w.	11·0 E.	......	......	......	......	......	......	Novar
At sea (8 observations)	33 33	162 01	1841	12 02 E.	......	12 0 E.	......	......	......	......	......	......	Erebu
At sea (3 observations)	32 41	162 04	1863	12 10 E.	......	12·2 E.	......	......	......	......	......	......	Richa s.
At sea (10 observations)	33 35	162 47	1841	14 26 E.	......	14·4 E.	......	......	......	......	......	......	Terro
At sea (5 observations)	33 38	163 42	1841	......	......	......	60 48 s.	......	60·8 s.	12·30	......	12·30	Erebu
At sea (4 observations)	33 42	163 50	1841	......	......	......	......	......	......	12·42	......	12·42	Terro
At sea (4 observations)	33 42	164 05	1841	......	......	......	60 52 s.	......	60·9 s.	......	......	......	Terro
At sea (2 observations)	34 35	164 32	1836	13 27 E.	0 05 E.	13·5 E.	......	......	......	......	......	......	FitzR .
At sea (3 observations)	31 33	165 04	1863	12 34 E.	......	12·6 E.	......	......	......	......	......	......	Richa s.
At sea (3 observations)	31 26	165 19	1850	......	......	......	56 11 s.	......	56·2 s.	......	......	......	Rattl ake.
At sea (2 observations)	33 41	166 23	1841	......	......	......	......	......	......	12·18	......	12·18	Erebu
At sea (8 observations)	33 41	166 26	1841	13 34 E.	......	13·6 E.	......	......	......	......	......	......	Erebu
At sea (3 observations)	33 36	166 27	1858	12 39 E.	0 16 w.	12·4 E.	......	......	......	......	......	......	Nova .
At sea (5 observations)	33 38	166 28	1841	......	......	......	60 07 s.	......	60·1 s.	......	......	......	Erebu
At sea (16 observations)	33 48	166 29	1841	13 40 E.	......	13·7 E.	......	......	......	......	......	......	Terro
At sea (10 observations)	33 44	166 37	1841	......	......	......	59 55 s.	......	59·9 s.	......	......	......	Terro
At sea (4 observations)	33 45	166 38	1841	......	......	......	......	......	......	12·19	......	12·19	Terro
At sea (3 observations)	33 17	166 41	1854	11 30 E.	0 12 w.	11·3 E.	......	......	......	......	......	......	Denh .
At sea (7 observations)	31 05	167 22	1863	12 21 E.	......	12·3 E.	......	......	......	......	......	......	Richa s.
At sea (7 observations)	33 32	167 35	1841	13 27 E.	......	13·5 E.	......	......	......	......	......	......	Erebu
At sea (5 observations)	33 34	167 37	1841	......	......	......	......	......	......	11·99	......	11·99	Terro
At sea (9 observations)	33 33	167 38	1841	......	......	......	59 58 s.	......	60·0 s.	......	......	......	Terro
At sea (5 observations)	33 22	167 40	1841	......	......	......	59 39 s.	......	59·7 s.	12·12	......	12·12	Erebu
At sea (3 observations)	34 09	167 48	1850	......	......	......	59 47 s.	......	59·8 s.	11·88	......	11·88	Rattl ake.
At sea (12 observations)	33 37	168 04	1841	15 02 E.	......	15·0 E.	......	......	......	......	......	......	Terro
At sea (3 observations)	33 00	168 22	1850	......	......	.... .	57 16 s.	......	57·3 s.	......	......	......	Rattl ake.
At sea (6 observations)	31 28	168 25	1850	......	......	......	55 37 s.	......	55·6 s.	11·35	......	11 35	Rattl ake.
At sea (2 observations)	33 25	169 15	1854	12 28 E.	0 12 w.	12·3 E.	......	......	......	......	......	......	Denh .
At sea (4 observations)	32 58	169 20	1841	......	......	......	......	......	......	12·04	......	12 04	Terro
At sea (9 observations)	33 00	169 20	1841	......	......	......	58 43 s.	......	58·7 s.	......	......	......	Terro
At sea (5 observations)	32 58	169 20	1841	......	......	......	59 04 s.	......	59·1 s.	12·04	......	12·04	Erebu
At sea (11 observations)	33 42	169 44	1841	12 54 E.	......	12·9 E.	......	......	......	......	......	......	Erebu
At sea (4 observations)	32 12	170 27	1841	......	......	......	58 33 s.	......	58·5 s.	......	......	......	Erebu
At sea (3 observations)	33 21	170 53	1850	......	......	......	57 21 s.	......	57·3 s.	11·60	......	11·60	Rattl ake.
At sea (6 observations)	32 12	170 54	1841	......	......	......	......	......	......	11·91	......	11·91	Terro
At sea (11 observations)	32 11	171 01	1841	......	......	......	57 28 s.	......	57·5 s.	......	......	......	Terro
At sea (8 observations)	33 57	171 58	1841	......	......	......	58 24 s.	......	58·4 s.	......	......	......	Erebu
At sea (5 observations)	33 55	171 59	1841	......	......	......	58 24 s.	......	58·4 s.	......	......	......	Terro
At sea (6 observations)	33 32	171 59	1841	......	......	......	......	......	......	11·86	......	11·86	Ereb
At sea (6 observations)	33 57	172 03	1841	......	......	......	......	......	......	12·00	......	12·00	Terro
At sea (7 observations)	33 58	172 06	1841	......	......	......	58 14 s.	......	58·2 s.	......	......	......	Terro
Three Kings Islands..	34 06	172 10	1852	14 55 E.	......	14·9 E.	......	......	......	......	......	......	Drur
At sea (2 observations)	33 51	172 32	1854	14 02 E.	0 12 w.	13·8 E.	......	......	......	......	......	......	Denh n.
At sea (9 observations)	34 15	172 33	1841	13 45 E.	......	13·7 E.	......	......	......	......	......	......	Terro
At sea (5 observations)	34 15	172 50	1841	......	......	......	58 48 s.	......	58·8 s.	11·97	......	......	Terro
Parengarenga	34 31	173 02	1852	13 30 E.	......	13·5 E.	......	......	......	......	......	......	Drur
At sea (3 observations)	34 22	173 05	1850	......	......	......	57 32 s.	......	57·5 s.	11·56	......	11·56	Rattl ake.
At sea (4 observations)	34 35	173 21	1827	13 34 E.	0 15 E.	13·8 E.	......	......	......	......	......	......	D'Ur lle.
Hokianga	35 32	173 23	1851	......	......	......	59 46 s.	......	59·8 s.	12·10	......	12·10	Kerr.
Cape Kara Kara	34 47	173 25	1852	13 15 E.	......	13·2 E.	......	......	......	......	......	......	Drur
At sea (2 observations)	34 32	173 28	1836	13 35 E.	0 06 E.	13·7 E.	......	......	.....	......	......	......	FitzR .

SOUTH EQUATORIAL ZONE IV.—Lat. 30° to 40° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.				Inclination.				Force in British units.			Observers.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.		Ob-served.	Cor. to Epoch 1842·5.	Corrected.		Ob-served.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°		° ′		°					
1 observations)	34 31	173 28	1841	13 56 E.	......	13·9 E.		......	......	......		......	......	......	Erebus.
za	35 31	173 32	1825	13 23 E.	0 17 E.	13·7 E.		......	......	......		......	......	......	Herd.
6 observations	34 29	173 36	1841	......	......	......		58 26 s.	......	58·4 s.		......	......	......	Erebus.
7 observations	34 24	173 43	1841	......	......	......		59 00 s.	......	59·0 s.		12·13	......	12·13	Terror.
5 observations)	34 32	173 47	1841	13 42 E.	......	13·7 E.		......	......	......		......	......	......	Terror.
oa	35 02	173 47	1853	......	......	......		58 27 s.	......	58·4 s.		......	......	......	Kerr.
			1854	14 10 E.	......	14·2 E.		......	......	......		......	......	......	Drury.
......	33 22	173 48	1858	12 26 E.	0 16 w.	12·2 E.		......	......	......		......	......	......	Novara.
2 observations)	34 18	173 57	1854	13 57 E.	0 12 w.	13·7 E.		......	......	......		......	......	......	Denham.
slands	35 16	174 00	1824	13 22 E.	0 18 E.	13·7 E.	13·7 E.	59 35 s	......	59·6 s.	59·4 s.	......	......	12 01	Duperrey.
			1827	13 08 E.	0 15 E.	13·4 E.		......	......	......		......	......		D'Urville.
			1835	14 00 E.	0 07 E.	14·1 E.		59 32 s.	......	59·5 s.		......	......		FitzRoy.
			1838	13 30 E.	0 04 E.	13·6 E.		......	......	......		......	......		Cécille.
			1838	13 33 E.	0 04 E.	13·6 E.		58 58 s.	......	59·0 s.		......	......		La Vénus.
			1841	13 36 E.	......	13·6 E.		......	......	......		......	......		Erebus.
			1841	......	......	......		59 36 s.	......	59 6 s.		12·05	......		Terror.
			1841	......	......	......		59 25 s.	......	59·4 s.		12·06	......		Terror.
			1841	......	......	......		59 31 s.	......	59·5 s.		11·94	......		Erebus.
			1849	14 20 E.	0 07 w.	14·2 E.		......	......	......		......	......		Stokes.
			1850	13 27 E.	0 08 w.	13·3		59 33 s.	......	59·5 s.		11·97	......		Rattlesnake.
nouth	39 04	174 05	1849	14 45 E.	......	14·7 E.		......	......	......		......	......	......	Stokes.
......	36 20	174 11	1852	......	......	......		60 43 s.	......	60·7 s.		12·10	......	12·10	Kerr.
ıru Harbour..	35 24	174 22	1849	14 25 E.	......	14·4 E.		......	......	......		......	......	......	Stokes.
observations)	35 16	174 23	1841	......	......	......		59 28 s.	......	59·5 s.		......	......	......	Erebus.
Harbour ...	35 51	174 32	1849	14 20 E.	......	14·3 E.		......	......	......		......	......	......	Stokes.
observations)	35 15	174 39	1841	......	......	......		......	......	......		12·07	......	12·07	Erebus.
Harbour ...	36 50	174 49	1841	13 53 E.	......	13·9 E.	14·0 E.	61 10 s.	... ..	61·2 s.	61·0 s.	12·33	......	12·41	Novara.
			1848	14 11 E.	......	14·2 E.		......	......	......		......	......		Stokes.
			1851	......	......	......		60 43 s.	......	60·7 s.		12·50	......		Kerr.
observations)	33 55	174 54	1854	14 03 E.	0 12 w.	13·9 E.		......	......	......		......	......	......	Denham.
Island......	36 49	175 11	1853	......	......	......		59 56 s.	......	59·9 s.		......	......	......	Kerr.
rier Island...	36 09	175 21	1848	13 33 E.	......	13·5 E.		......	......	......		......	......	......	Stokes.
observations)	35 53	175 26	1858	12 50 E.	0 16 w.	12·6 E.		......	......	......		......	......	......	Novara.
observations)	32 01	175 32	1835	14 15 E.	0 07 w.	14·1 E.		......	......	......		......	......	......	FitzRoy.
Bay......	36 49	175 45	1852	......	......	......		61 19 s.	......	61·3 s.		......	......	......	Kerr.
Harbour ...	37 36	176 11	1853	15 00 E.	......	15·0 E.		......	......	......		......	......	......	Drury.
bservations)	36 05	176 17	1841	......	......	......		59 20 s.	......	59·3 s.		......	......	......	Terror.
bservations)	37 11	176 29	1827	14 11 E.	0 15 E.	14·4 E.		......	......	......		......	......	......	D'Urville.
......	39 29	176 55	1855	16 00 E.	......	16·0 E.		......	......	......		......	......	......	Drury.
......	37 58	177 18	1855	14 46 E.	......	14·8 E.		......	......	......		......	......	......	Drury.
a......	35 38	177 22	1859	13 12 E.	0 17 w.	12 9 E.		......	......	......		......	......	......	Novara.
bservations)	36 20	177 27	1841	......	......	......		......	......	......		12·11	......	12·11	Terror.
bservations)	36 27	177 34	1841	......	......	......		59 54 s.	......	59·9 s.		12·08	......	12·08	Erebus.
bservations)	36 39	177 58	1841	14 24 E.	......	14·4 E.		......	......	......		......	......	......	Erebus.
ay	38 45	177 59	1855	14 49 E.	......	14·8 E.		......	......	......		......	......	......	Drury.
away	37 31	178 00	1855	14 47 E.	......	14·8 E.		......	......	......		......	......	......	Drury.
bservations)	31 31	178 05	1854	13 18 E.	0 12 w.	13·1 E.		......	......	......		......	......	......	Denham.
......	37 32	178 21	1853	......	......	......		59 41 s.	......	59·7 s.		......	......	......	Kerr.
			1855	14 47 E.	......	14·8 E.		......	......	......		......	......	......	Drury.
bservations)	30 42	178 50	1854	12 12 E.	0 12 w.	12·0 E.		......	......	......		......	......	......	Denham.
bservations)	38 17	179 31	1841	......	......	......		......	......	......		12·09	......	12·09	Erebus.
bservations)	38 03	179 32	1841	14 55 E.	......	14·9 E.		......	......	......		......	......	......	Terror.
bservations)	38 14	179 47	1841	......	......	......		......	......	......		12·24	......	12·24	Terror.
bservations)	38 02	179 51	1841	14 44 E.	......	14·7 E.		......	......	......		......	......	......	Erebus.
bservations)	38 17	179 51	1841	......	......	......		60 34 s.	......	60·6 s.		......	......	......	Erebus.
bservations)	38 16	179 58	1841	......	......	......		60 37 s.	......	60·6 s.		......	......	......	Terror.
bservations)	34 54	181 14	1850	......	......	......		56 18 s.	......	56·3 s.		11·32	......	11·32	Rattlesnake.
a......	30 29	181 20	1827	12 00 E.	0 15 E.	12·3 E.		......	......	......		......	......	......	D'Urville.
a......	33 41	181 22	1859	12 36 E.	0 17 w.	12·3 E.		......	......	......		......	......	......	Novara.
bservations)	38 55	182 05	1841	......	......	......		......	......	......		12·29	......	12·29	Terror.
bservations)	38 54	182 17	1841	......	......	......		61 21 s.	......	61·3 s.		......	......	......	Terror.
bservations)	39 08	182 30	1841	......	......	......		61 34 s.	......	61·6 s.		......	......	......	Erebus.

SOUTH EQUATORIAL ZONE IV.—Lat. 30° to 40° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Obs ers.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	
	° '	° '		° '	° '	°	° '		°				
At sea (4 observations)	39 10	182 35	1841	……	……	……	……	……	……	12·10	……	12·10	Erebus.
At sea(11 observations)	39 29	182 42	1841	16 55 E.	……	16·9 E.	……	……	……	……	……	……	Terror.
At sea(13 observations)	39 10	182 43	1841	14 43 E.	……	14·7 E.	……	……	……	……	……	……	Erebus.
At sea (8 observations)	39 22	182 57	1841	……	……	……	……	……	……	12·27	……	12·27	Terror.
At sea (16 observations)	39 21	182 57	1841	……	……	……	61 15 s.	……	61·3 s.	……	……	……	Terror.
At sea (3 observations)	35 29	183 46	1850	……	……	……	56 18 s.	……	56·3 s.	11·20	……	11·20	Rattles ke.
At sea (3 observations)	36 25	185 27	1850	……	……	……	56 46 s.	……	56·8 s.	11·39	……	11·39	Rattles ke.
At sea (3 observations)	36 44	187 20	1850	……	……	……	57 19 s.	……	57·3 s.	11·38	……	11·38	Rattles ke.
At sea (2 observations)	35 00	188 32	1859	11 40 E.	0 17 W.	11·4 E.	……	……	……	……	……	……	Novara.
At sea (3 observations)	37 40	190 54	1850	……	……	……	57 20 s.	……	57·3 s.	11·23	……	11·23	Rattles ke.
At sea (3 observations)	38 29	193 54	1850	……	……	……	57 43 s.	……	57·7 s.	11·49	……	11·49	Rattles ke.
At sea (3 observations)	35 29	195 03	1859	10 58 E.	0 17 W.	10·7 E.	……	……	……	……	……	……	Novara.
At sea (3 observations)	39 21	197 37	1850	……	……	……	58 24 s.	……	58·4 s.	11·32	……	11·32	Rattles ke.
At sea (3 observations)	34 59	197 47	1859	11 02 E.	0 17 W.	10·7 E.	……	……	……	……	……	……	Novara.
At sea (3 observations)	32 14	202 35	1859	9 59 E.	0 17 W.	9·7 E.	……	……	……	……	……	……	Novara.
At sea……………………	30 24	213 18	1830	8 24 E.	0 12 E.	8·6 E.	47 21 s.	……	47·3 s.	10·44	……	10·44	Erman.
At sea……………………	32 21	214 32	1830	……	……	……	49 35 s.	……	49·6 s.	10·57	……	10·57	Erman.
At sea……………………	32 41	215 02	1830	8 29 E.	0 12 E.	8·7 E.	……	……	……	……	……	……	Erman.
At sea……………………	34 35	216 48	1830	7 44 E.	0 12 E.	7·9 E.	51 14 s.	……	51·2 s.	10·34	……	10·34	Erman.
At sea……………………	37 08	218 13	1830	7 34 E.	0 12 E.	7·8 E.	……	……	……	……	……	……	Erman.
At sea……………………	37 39	218 18	1830	……	……	……	54 16 s.	……	54·3 s.	11·18	……	11·18	Erman.
At sea……………………	35 02	218 42	1830	7 30 E.	0 12 E.	7·7 E.	52 26 s.	……	52·4 s.	10·44	……	10·44	Erman.
At sea……………………	36 17	220 05	1830	……	……	……	52 18 s.	……	52·3 s.	10·62	……	10·62	Erman.
At sea (2 observations)	34 19	220 14	1830	6 24 E.	0 12 E.	6·6 E.	……	……	……	……	……	……	Erman.
At sea……………………	35 21	220 21	1830	8 15 E.	0 12 E.	8·5 E.	……	……	……	……	……	……	Erman.
At sea……………………	34 28	220 33	1830	……	……	……	50 34 s.	……	50·6 s.	10·82	……	10·82	Erman.
At sea……………………	30 40	236 28	1859	9 34 E.	0 17 W.	9·3 E.	……	……	……	……	……	……	Novara
At sea……………………	31 51	241 34	1859	10 42 E.	0 17 W.	10·4 E.	……	……	……	……	……	……	Novara
At sea (2 observations)	36 17	271 00	1827	12 58 E.	0 15 E.	13·2 E.	……	……	……	……	……	……	Prussia ships.
Mas Afuera Island …	33 45	279 06	1832	16 00 E.	0 10 E.	16·2 E.	……	……	……	……	……	……	Laplac
At sea……………………	38 01	280 11	1836	15 33 E.	0 06 E.	15·7 E.	……	……	……	……	……	……	Bonite.
Juan Fernandez ……	33 38	281 07	1830	17 13 E.	……	17·2 E.	44 50 s.	……	44·8 s.	……	……	……	King.
At sea……………………	35 11	281 35	1859	17 51 E.	……	17·9 E.	……	……	……	……	……	……	Novara
At sea……………………	38 08	282 48	1834	17 27 E.	……	17·5 E.	……	……	……	……	……	……	FitzRo
At sea……………………	36 15	283 20	1836	18 24 E.	……	18·4 E.	……	……	……	……	……	……	Bonite.
At sea (9 observations)	33 02	283 59	1827	16 34 E.	……	16·6 E.	……	……	……	……	……	……	Lütke.
At sea……………………	33 16	284 41	1859	17 07 E.	……	17·1 E.	……	……	……	……	……	……	Novara
At sea……………………	34 28	284 51	1836	16 05 E.	……	16·1 E.	……	……	……	……	……	……	Bonite.
At sea (3 observations)	32 17	285 09	1859	15 09 E.	……	15·1 E.	……	……	……	……	……	……	Novara
At sea……………………	34 52	285 09	1836	17 31 E.	……	17·5 E.	……	……	……	……	……	……	Bonite.
At sea (4 observations)	37 22	285 17	1828	16 27 E.	……	16·5 E.	……	……	……	……	……	……	Prussia ships.
At sea (3 observations)	36 37	285 32	1827	16 38 E.	……	16·6 E.	……	……	……	……	……	……	Lütke.
At sea (4 observations)	32 34	285 56	1859	15 11 E.	……	15·2 E.	……	……	……	……	……	……	Novara
Mocha Island ………	38 19	286 00	1823	17 22 E.	……	17·4 E.	……	……	……	……	……	……	Morrel
Mean of 3 stations …	37 40	286 15	1835	17 10 E.	……	17·2 E.	……	……	……	……	……	……	FitzRo
At sea (3 observations)	33 12	286 24	1827	15 50 E.	……	15·8 E.	……	……	……	……	……	……	Prussia ships.
Valdivia (2 stations)…	39 51	286 37	1835	18 18 E.	……	18·3 E.	46 46 s.	……	46 8 s.	……	……	……	FitzRo
Concepcion …………	36 42	286 50	1823	16 16 E.	……	16·3 E. } 16·5 E.	44 42 s.	……	44·7 s. } 44·7 s.	9·60	……	9·60	Duperr
			1824	15 00 E.	……	15·0 E.	……	……	……				Kotzebu
			1825	16 49 E.	……	16·8 E.	……	……	……				Beeche
			1827	17 02 E.	……	17·0 E.	45 33 s.	……	45·5 s.				Lütke.
			1829	16 47 E.	……	16·8 E.	45 10 s.	……	45·2 s.				King.
			1835	16 48 E.	……	16·8 E.	43 15 s.	……	43·3 s.				FitzRo
Near Maule (2 stations)	36 01	287 10	1835	16 36 E.	……	16·6 E.	……	……	……	……	……	……	FitzRo
At sea (2 observations)	38 18	287 30	1835	17 38 E.	……	17·6 E.	……	……	……	……	……	……	FitzRo
At sea……………………	33 07	287 48	1836	14 19 E.	……	14·3 E.	……	……	……	……	……	……	Bonite.
At sea……………………	33 04	287 51	1834	17 16 E.	……	17·3 E.	……	……	……	……	……	……	FitzRo
At sea……………………	35 11	288 15	1835	16 34 E.	……	16·6 E.	……	……	……	……	……	……	FitzRo

SOUTH EQUATORIAL ZONE IV.—Lat. 30° to 40° S. (continued).

ations.	Lat. S.	Long. E.	Date.	Declination.				Inclination.				Force in British units.			Observers.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.		Ob-served.	Cor. to Epoch 1842·5.	Corrected.		Ob-served.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°		° ′		°					
			1823	15 41 E.	......	15·7 E.		......	......	......		......	......	......	Morrell.
			1825	15 52 E.	......	15·9 E.		......	......	......		......	......	......	Beechey.
			1827	14 26 E.	......	14·4 E.		39 56 s.	......	39·9 s.		......	......	......	Lütke.
			1830	15 18 E.	......	15·3 E.		40 11 s.	......	40·2 s.		......	......	......	King.
			1831	15 00 E.	......	15·0 E.		......	......	......		......	......	......	Laplace.
			1835	15 18 E.	......	15·3 E.		38 03 s.	......	38·1 s.		......	......	......	FitzRoy.
lpar o............	33 02	288 19	1836	......	......	......	15·4 E.	37 05 s.	......	37·1 s.	37·6 s.	......	......	7·77	Beechey.
			1837	15 35 E.	......	15·6 E.		38 20 s.	......	38·3 s.		......	......	......	La Vénus.
			1838	.....	......	......		38 43 s.	......	38·7 s.		......	......	......	La Vénus.
			1838	15 36 E.	......	15·6 E.		38 12 s.	......	38·2 s.		......	......	......	La Vénus.
			1859	15 40 E.	......	15·7 E.		35 40 s.	......	35·7 s.		7·94	......	......	Novara.
			1866	15 51 E.	......	15·9 E.		35 23 s.	......	35·4 s.		7·76	......	......	Harkness.
			1868	......	......	......		34 23 s.	......	34·4 s.		7·62	......	......	H.M.S. 'Nassau.'
pud ıy (2 stations)	32 19	288 27	1835	15 18 E.	......	15·3 E.		......	......	......		......	......	......	FitzRoy.
aco Montenegro.	32 46	289 20	1852	16 22 E.	......	16·4 E.		35 10 s.	......	35·2 s.		7·94	......	7·94	Macrae.
ıtiag	33 26	289 22	1852	16 27 E.	......	16·5 E.		35 37 s.	......	35·6 s.		7·94	......	7·94	Macrae.
ıuch e la Cumbro	32 49	289 50	1852	......	......	......		34 52 s.	......	34·9 s.		......	......	......	Macrae.
salla	32 34	290 33	1852	15 06 E.	......	15·1 E.		......	......	......		7·73	......	7·73	Macrae.
ndo	32 51	291 03	1852	15 05 E.	......	15·1 E.		34 23 s.	......	34·4 s.		7·76	. ....	7·76	Macrae.
roc o	33 26	292 18	1852	14 56 E.	......	14·9 E.		34 26 s.	......	34·4 s.		7·73	......	7·73	Macrae.
Lu le la Punta .	33 17	293 33	1852	14 40 E.	......	14·7 E.		33 25 s.	......	33·4 s.		7·63	......	7·63	Macrae.
a d . Concepcion	33 07	295 37	1852	13 28 E.	......	13·5 E.		32 38 s.	......	32·6 s.		7·50	......	7·50	Macrae.
e tr Station	32 58	297 28	1852	13 00 E.	......	13·0 E.		31 50 s.	......	31·8 s.		7·40	......	7·40	Macrae.
orad River (4 atio	39 47	297 55	1835	15 40 E.	0 28 w.	15·2 E.		......	......	......		......	......	......	FitzRoy.
nco y	38 57	298 01	1828	13 39 E.	0 56 w.	12·7 E.	13·5 E.	......	......	......		......	......	......	D'Orbigny.
			1833	15 00 E.	0 36 w.	14·4 E.		43 12 s.	......	43·2 s.		......	......	......	FitzRoy.
rmo Mount (5 atio	39 01	298 03	1832	15 08 E.	0 40 w.	14·5 E.		......	......	......		......	......	......	FitzRoy.
sea (bservations)	39 00	298 21	1833	15 10 E.	0 36 w.	14·6 E.		... ..	......	......		......	......	......	FitzRoy.
Rosa	32 56	299 28	1852	12 01 E.	......	12·0 E.		30 57 s.	......	30·9 s.		7·27	......	7·27	Macrae.
ada Santa Fé...	31 41	299 34	1846	13 14 E.	0 16 E.	13·5 E.		......	......	......		......	......	......	Sulivan.
			1829	13 10 E.	0 52 w.	12·3 E.		......	......	......		......	......	......	D'Orbigny.
nos res	34 36	301 38	1833	11 40 E.	0 36 w.	11·1 E.	11·7 E.	......	......	......		......	......	......	FitzRoy.
			1852	11 45 E.	......	11·8 E.		32 11 s.	......	32·2 s.		7·32	......	7·32	Macrae.
gua iver.........	33 41	301 53	1845	12 06 E.	0 10 E.	12·3 E.		......	......	......		......	......	......	Sulivan.
ly I nd...........	32 20	301 55	1845	11 14 E.	0 10 E.	11·4 E.		......	......	......		......	......	......	Sulivan.
mia	34 28	302 11	1844	11 36 E.	0 06 E.	11·7 E.		......	......	......		......		......	Sulivan.
e C ientes (4 ation	38 00	302 21	1832	13 50 E.	0 42 w.	13·1 E.		......	......	......		......	......	......	FitzRoy.
ea...	34 42	302 28	1832	11 33 E.	0 42 w.	10 9 E.		......	......	......		......	......	......	FitzRoy.
e San ntonio (4 tion	35 48	302 53	1832	12 37 E.	0 42 w.	11·9 E.		......	......	......		......	......	......	FitzRoy.
ea (7 servations)	34 51	303 10	1832	12 13 E.	0 42 w.	11·5 E.		......	......	......		......	......	......	FitzRoy.
ea (3 servations)	35 41	303 12	1832	11 49 E.	0 42 w.	11·1 E.		......	......	......		......	......	......	FitzRoy.
ea...	38 50	303 20	1838	......	......	......		40 15 s.	......	40·3 s.		8·09	......	8·09	Sulivan.
ea (3 servations)	36 57	303 25	1832	12 36 E.	0 40 w.	11·9 E.		......	......	......		......	......	......	FitzRoy.
ea (3 servations)	34 50	303 35	1833	11 46 E.	0 36 w.	11·2 E.		......	......	......		......	......	......	FitzRoy.
ea (3 servations)	34 57	303 42	1832	12 28 E.	0 40 w.	11·8 E.		......	......	......		......	......	......	FitzRoy.
			1827	12 07 E.	1 00 w.	11·1 E.		36 28 s.	......	36·5 s.		......	......	......	King.
			1829	11 43 E.	0 52 w.	10·9 E.		......	......	......		......	......	......	D'Orbigny.
			1830	11 42 E.	0 48 w.	10·9 E.		......	......	......		......	......	......	Duperrey.
			1833	12 40 E.	0 36 w.	12·1 E.		34 51 s.	......	34·9 s.		......	......	......	FitzRoy.
			1836	10 35 E.	0 24 w.	10·2 E.		34 50 s.	......	34·8 s.		......	......	......	Bonite.
te Vi o............	34 53	303 47	1838	......	......	......	11·0 E.	34 03 s.	......	34·1 s.	33·5 s.	7·52	......	7·20	Sulivan.
			1843	10 42 E.	......	10·7 E.		......	......	......		......	......	......	Sulivan.
			1844	10 53 E.	0 06 E.	11·0 E.		......	......	......		......	......	......	Sulivan.
			1852	10 13 E.	0 40 E.	10 9 E.		32 07 s.	......	32·1 s.		7·27	......	......	Macrae.
			1866	9 21 E.	1 36 E.	10·9 E.		31 06 s.	......	31·1 s.		7·05	......	......	Harkness.
			1867	......	......	......		31 02 s.	......	31·0 s.		6·95	......	......	H.M.S. 'Nassau.'
iti	34 57	305 03	1829	13 48 E.	0 52 w.	12·9 E.	12·4 E.	35 06 s.	......	35·1 s.		......	......	......	King.
			1833	12 28 E.	0 36 w.	11·9 E.		......	......	......		......	......	......	FitzRoy.
de la ata.........	35 14	305 13	1836	10 57 E.	0 24 w.	10·5 E.		......	......	......		......	......	......	Bonite.
ea.....	38 37	305 45	1837	10 34 E.	0 20 w.	10·2 E.		......	......	......		......	......	......	Du Petit Thouars.
ea.....	35 52	306 15	1836	8 40 E.	0 24 w.	8·3 E.		......	......	......		......	......	......	Bonite.

SOUTH EQUATORIAL ZONE IV.—Lat. 30° to 40° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British Units.			Observers.
				Observed.	Correction to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	Observed.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	° ′		°				
At sea....................	34 05	306 48	1836	8 25 E.	0 24 W.	8·0 E.	......	......	......	......	......	......	Boni
At sea....................	35 01	306 51	1836	10 14 E.	0 24 W.	9·8 E.	......	......	......	......	......	......	Boni
At sea....................	34 57	307 30	1836	9 10 E.	0 24 W.	8·8 E.	......	......	......	......	......	......	Boni
At sea....................	34 09	307 57	1832	10 27 E.	0 40 W.	9·8 E.	......	......	......	......	......	......	Fitzly.
At sea....................	37 52	308 22	1830	12 00 E.	0 48 W.	11·2 E.	......	......	......	......	......	......	Bisec
At sea (4 observations)	37 48	308 31	1827	12 12 E.	1 00 W.	11·2 E.	......	......	......	......	......	......	Lütk
At sea....................	39 49	308 55	1830	11 44 E.	0 48 W.	10·9 E.	40 19 S.	......	40·3 S.	7·36	......	7·36	Erm.
At sea (2 observations)	33 26	309 01	1837	6 33 E.	0 20 W.	6·2 E.	......	......	......	......	......	......	Du it Thou
At sea....................	33 16	309 26	1836	8 18 E.	0 24 W.	7·9 E.	......	......	......	......	......	......	Boni.
At sea....................	38 29	309 31	1830	11 19 E.	0 48 W.	10·5 E.	......	......	......	......	......	......	Erm.
At sea....................	33 42	309 39	1836	6 56 E.	0 24 W.	6·5 E.	......	......	......	......	......	......	Boni.
At sea....................	38 17	309 40	1830	......	......	......	37 53 S.	......	37·9 S.	7·36	......	7·36	Erm.
At sea (2 observations)	37 55	309 54	1830	9 44 E.	0 48 W.	8·9 E.	......	......	......	......	......	......	Erm.
At sea....................	37 09	309 55	1830	......	......	......	36 42 S.	......	36 7 S.	7·31	......	7·31	Erm.
At sea....................	36 54	310 00	1830	9 34 E.	0 48 W.	8·8 E.	......	......	......	......	......	......	Erm.
At sea....................	35 46	310 35	1830	8 21 E.	0 48 W.	7·5 E.	34 21 S.	......	34·3 S.	6 79	......	6·79	Erm.
At sea....................	31 44	310 40	1838	......	......	......	28 22 S.	......	28·4 S.	7·12	......	7·12	Suliv.
At sea....................	34 49	310 55	1830	8 08 E.	0 56 W.	7·2 E.	......	......	......	......	......	......	Erm.
At sea (3 observations)	30 18	311 33	1832	6 55 E.	0 45 W.	6·2 E.	......	......	......	......	......	......	Fitzy.
At sea (3 observations)	32 02	311 37	1827	8 26 E.	1 07 W.	7·3 E.	......	......	......	......	......	......	Lütk
At sea....................	30 56	311 43	1836	4 44 E.	0 27 W.	4·3 E.	......	......	......	......	......	......	Boni.
At sea....................	36 46	312 10	1830	11 00 E.	0 56 W.	10·1 E.	......	......	......	......	......	......	Bisc.
At sea....................	33 18	312 11	1830	7 18 E.	0 56 W.	6·4 E.	......	......	......	......	......	......	Erm.
At sea....................	33 04	312 15	1830	......	......	......	30 03 S.	......	30·1 S.	6·97	......	6·97	Erm.
At sea....................	32 18	312 43	1830	7 17 E.	0 56 W.	6·3 E.	......	......	......	......	......	......	Erm.
At sea (7 observations)	33 12	312 58	1831	6 46 E.	0 52 W.	5·9 E.	......	......	......	......	......	......	Prusan ship
At sea....................	30 51	313 22	1830	5 15 E.	0 56 W.	4·3 E.	......	......	......	......	......	......	Erm.
At sea (3 observations)	37 41	314 19	1850	......	......	......	31 53 S.	......	31·9 S.	7·46	......	7·46	Rattsnake.
At sea (2 observations)	37 32	315 29	1826	9 50 E.	1 06 W.	8·7 E.	......	......	......	......	......	......	Prusan ship
At sea (3 observations)	36 01	315 34	1850	......	......	......	30 11 S.	......	30·2 S.	7·04	......	7·04	Rattsnake.
At sea (2 observations)	32 18	315 47	1830	6 15 E.	0 56 W.	5·3 E.	......	......	......	......	......	......	Bise.
At sea (3 observations)	35 19	316 03	1850	......	......	......	29 04 S.	......	29·1 S.	6·99	......	6·99	Rattsnake.
At sea....................	38 48	316 08	1839	......	......	......	36 15 S.	......	36·3 S.	7·35	......	7·35	Sulin.
At sea....................	30 23	316 15	1846	4 15 E.	0 24 E.	4·7 E.	......	......	......	......	......	......	Sulin.
At sea....................	36 10	317 05	1839	......	......	......	34 30 S.	......	34·5 S.	7·07	......	7·07	Sulin.
At sea....................	30 12	318 18	1843	0 00	......	0 0	......	......	......	......	......	......	Pasl.
At sea....................	31 55	318 30	1844	0 00	0 06 E.	0·1 E.	......	......	......	......	......	......	Pasl.
At sea....................	30 06	318 45	1846	2 10 E.	0 18 E.	2·5 E.	......	......	......	......	......	......	Sulin.
At sea (3 observations)	35 20	320 21	1850	......	......	......	28 49 S.	......	28·8 S.	6·97	......	6·97	Rattsnake.
At sea....................	31 26	320 34	1844	1 20 W.	0 06 E.	1·2 W.	......	......	......	......	......	......	Pasl.
At sea....................	30 22	321 00	1844	1 00 W.	0 06 E.	0·9 W.	......	......	......	......	......	......	Pasl.
At sea....................	31 20	322 06	1844	1 17 W.	0 06 E.	1·2 W.	......	......	......	......	......	......	Pasl.
At sea (6 observations)	31 31	322 41	1850	......	......	......	27 40 S.	......	27·7 S.	6·93	......	6·93	Rattsnake.
At sea....................	39 10	322 47	1859	2 10 W.	1 14 E.	0·9 W.	......	......	......	......	......	......	Nova.
At sea (3 observations)	33 10	323 06	1847	......	......	......	28 00 S.	......	28·0 S.	6·72	......	6·72	Rattsnake.
At sea....................	33 13	323 09	1847	2 48 W.	0 20 E.	2·5 W.	......	......	......	......	......	......	Stary.
At sea....................	30 50	323 14	1847	2 24 W.	0 20 E.	2·1 W.	......	......	......	......	......	......	Stany.
At sea (3 observations)	30 44	323 16	1847	......	......	......	25 02 S.	......	25·0 S.	6·57	......	6·57	Rattsnake
At sea (2 observations)	33 34	323 46	1829	1 38 E.	1 05 W.	0·5 E.	......	......	......	......	......	......	Runer.
At sea (3 observations)	38 49	324 03	1829	4 14 E.	1 05 W.	3·1 E.	......	......	......	......	......	......	Runer.
At sea....................	35 07	324 14	1847	2 49 W.	0 24 E.	2·4 W.	......	......	......	......	......	......	Stary.
At sea (3 observations)	35 13	324 23	1847	......	......	......	31 27 S.	......	31·5 S.	6·87	......	6·87	Ratsnake.
At sea (3 observations)	32 12	325 52	1850	......	......	......	24 50 S.	......	24·8 S.	6·89	......	6·89	Ratsnake.
At sea (3 observations)	36 38	326 48	1847	......	......	......	33 55 S.	......	33·9 S.	6·80	......	6·80	Ratsnake.
At sea....................	36 37	326 57	1847	2 37 W.	0 24 E.	2·2 W.	......	......	......	......	......	......	Stary.
At sea (3 observations)	30 18	327 30	1850	......	......	......	22 23 S.	......	22·4 S.	6·67	......	6·67	Ratsnake.
At sea....................	37 24	328 42	1847	4 09 W.	0 24 E.	3·7 W.	......	......	......	......	......	... ..	Stary.
At sea (3 observations)	37 22	328 55	1847	......	......	......	34 30 S.	......	34·5 S.	......	......	......	Ratsnake.
At sea (3 observations)	36 51	332 00	1847	......	......	......	34 52 S.	......	34·9 S.	6·86	......	6·86	Ratsnake.

SOUTH EQUATORIAL ZONE IV.—Lat. 30° to 40° S. (continued).

ations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observers.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	° ′		°				
……………	36 50	332 01	1847	6 32 w.	0 24 E.	6·1 w.	……	……	……	……	……	……	Stanley.
……………	36 32	333 25	1847	7 51 w.	0 24 E.	7·5 w.	……	……	……	……	……	……	Stanley.
observations)	30 00	335 07	1857	10 51 w.	1 15 E.	9·6 w.	……	……	……	……	……	……	Novara.
observations)	36 33	335 40	1847	……	……	……	34 51 s.	……	34·9 s.	6·83	……	6·83	Rattlesnake.
observations)	38 34	336 10	1846	7 40 w.	0 20 E.	7·3 w.	……	……	……	……	……	……	Bérard.
observations)	30 13	336 27	1826	7 34 w.	1 22 w.	8·9 w.	……	……	……	……	……	……	D'Urville.
……………	36 31	336 28	1847	10 00 w.	0 23 E.	9·6 w.	……	……	……	……	……	……	Stanley.
observations)	31 33	337 04	1842	12 40 w.	……	12·7 w.	……	……	……	……	……	……	Bérard.
……………	30 18	337 05	1831	……	……	……	27 11 s.	……	27·2 s.	6·44	……	6·44	Dunlop.
……………	35 18	337 22	1846	11 03 w.	0 12 E.	10·9 w.	……	……	……	……	……	……	Bérard.
……………	30 47	337 26	1831	……	……	……	28 05 s.	……	28·1 s.	6·52	……	6·52	Dunlop.
……………	32 01	338 08	1831	……	……	……	30 13 s.	……	30·2 s.	6·59	……	6·59	Dunlop.
observations)	32 30	338 30	1842	……	……	……	31 45 s.	……	31·8 s	6·47	……	6·47	H.M.S. 'Fly.'
observations)	36 08	338 40	1847	……	……	……	35 45 s.	……	35·7 s.	6·79	……	6·79	Rattlesnake.
……………	35 31	340 26	1847	13 07 w.	0 18 E.	12·8 w.	……	……	……	……	……	……	Stanley.
……………	33 39	341 00	1831	……	……	……	32 46 s.	……	32·8 s.	6·67	……	6·67	Dunlop.
bservations)	32 50	341 03	1846	13 10 w.	0 12 E.	13·0 w.	……	……	……	……	……	……	Bérard.
bservations)	36 16	341 07	1847	……	……	……	35 54 s.	……	35·9 s.	6·74	……	6·74	Rattlesnake.
……………	37 01	341 09	1847	11 13 w.	0 18 E.	10 9 w.	……	……	……	……	……	……	Stanley.
……………	36 52	341 31	1847	11 58 w.	0 18 E.	11·7 w.	……	……	……	……	……	……	Stanley.
bservations)	30 03	341 51	1826	12 20 w.	1 22 w.	13·7 w.	……	……	……	……	……	……	D'Urville.
……………	38 02	343 00	1847	13 07 w.	0 27 E.	12 7 w.	……	……	……	……	……	……	Stanley.
bservations)	38 08	343 31	1847	……	……	……	39 32 s.	……	39·5 s.	6·90	……	6·90	Rattlesnake.
……………	35 31	344 15	1831	……	……	……	35 44 s.	……	35·7 s.	6·59	……	6·59	Dunlop.
……………	30 41	344 37	1846	17 16 w.	0 21 E.	16·9 w.	……	……	……	……	……	……	Bérard.
……………	36 36	347 24	1831	……	……	……	38 02 s.	……	38·0 s.	6·59	……	6·59	Dunlop.
……………	31 36	347 32	1857	19 40 w.	1 27 E.	18·2 w.	……	……	……	……	……	……	Novara.
……………	37 57	349 19	1847	14 30 w.	0 27 E.	14·1 w.	……	……	……	……	……	……	Stanley.
bservations)	34 32	350 20	1842	16 00 w.	……	16·0 w.	……	……	……	……	……	……	Bérard.
……………	37 12	350 24	1831	……	……	……	39 16 s.	……	39 3 s.	6·74	……	6·74	Dunlop.
bservations)	30 15	351 37	1840	……	……	……	……	……	……	6·37	……	6 37	Ross.
bservations)	37 29	352 15	1847	……	……	……	41 49 s.	……	41·8 s.	6·89	……	6·89	Rattlesnake.
……………	37 27	352 40	1847	18 40 w.	0 27 E.	18·2 w.	……	……	……	……	……	……	Stanley.
……………	30 58	353 26	1840	21 48 w.	0 15 w.	22·1 w.	……	……	……	……	……	……	Ross.
……………	37 37	353 36	1831	……	……	……	40 50 s.	……	40·8 s.	6·82	……	6·82	Dunlop.
……………	31 00	353 40	1840	……	……	……	……	……	……	6·39	……	6·39	Terror.
bservations)	31 08	353 56	1840	……	……	……	……	……	……	6·35	……	6·35	Ross.
……………	36 05	355 10	1847	20 02 w.	0 27 E.	19·6 w.	……	……	……	……	……	……	Stanley.
bservations)	35 27	355 21	1847	……	……	……	42 19 s.	……	42·3 s.	6·83	……	6·83	Rattlesnake.
……………	31 32	355 37	1840	21 54 w.	0 15 w.	22·1 w.	……	……	……	……	……	……	Ross.
……………	34 45	355 39	1847	21 43 w.	0 27 E.	21·3 w.	……	……	……	……	……	……	Stanley.
……………	37 40	355 51	1831	……	……	……	41 51 s.	……	41·9 s.	6·89	……	6·89	Dunlop.
……………	34 00	356 02	1857	23 40 w.	1 13 E.	22 5 w.	……	……	……	……	……	……	Novara.
……………	31 40	356 30	1840	……	……	……	……	……	……	6·39	……	6·39	Terror.
……………	31 46	356 38	1840	……	……	……	……	……	……	6·53	……	6·53	Ross.
……………	35 23	356 39	1847	23 08 w.	0 23 E.	22·7 w.	……	……	……	……	……	……	Stanley.
(3 servations)	35 24	356 45	1847	……	……	……	43 33 s.	……	43·5 s.	6·93	……	6·93	Rattlesnake.
……………	35 30	357 00	1847	22 41 w.	0 18 E.	22·4 w.	……	……	……	……	……	……	Stanley.
……………	35 42	357 08	1842	21 00 w.	……	21·0 w.	……	……	……	……	……	……	Bérard.
……………	31 13	358 26	1840	……	……	……	……	……	……	6·42	……	6·42	Terror.
……………	36 58	358 28	1847	22 16 w.	0 23 E.	21·9 w.	……	……	……	……	……	……	Stanley.
(3 servations)	36 51	358 28	1847	……	……	……	44 21 s.	……	44·3 s.	7·02	……	7·02	Rattlesnake.
……………	31 13	358 38	1840	……	……	……	……	……	……	6·53	……	6·53	Ross.
……………	31 01	359 26	1840	23 29 w.	0 10 w.	23·7 w.	……	……	……	……	……	……	Ross.
(2 servations)	31 08	359 26	1840	……	……	……	……	……	……	6·64	……	6·64	Ross.
(3 servations)	38 15	359 26	1847	……	……	……	45 46 s.	……	45·8 s.	7·16	……	7·16	Rattlesnake.
……………	31 07	359 28	1840	……	……	……	……	……	……	6 57	……	6·57	Terror.
……………	31 13	359 31	1840	23 02 w.	0 10 w.	23·2 w.	……	……	……	……	……	……	Ross.
……………	30 30	359 36	1840	23 08 w.	0 10 w.	23·3 w.	……	……	……	……	……	……	Ross.

SOUTH EQUATORIAL ZONE IV.—Lat. 30° to 40° S. (continued).

Stations.	Lat. S.	Long. E.	Date.	Declination.			Inclination.			Force in British units.			Observers.
				Ob-served.	Correction to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	Ob-served.	Cor. to Epoch 1842·5.	Corrected.	
	° ′	° ′		° ′	° ′	°	° ′		°				
At sea......................	38 23	359 37	1847	22 28 w.	0 18 E.	22·2 w.	......	......	......	......	......	......	Stanle
At sea......................	31 28	359 38	1840	23 41 w.	0 10 w.	23·9 w.	......	......	......	......	......	......	Ross.
At sea (3 observations)	33 25	359 40	1842	......	......	......	43 53 s.	......	43·9 s.	6·63	......	6·63	H.M.S Fly.'
At sea......................	31 19	359 40	1840	23 08 w.	0 10 w.	23·3 w.	......	......	......	......	......	......	Ross.
At sea......................	30 37	359 48	1840	23 23 w.	0 10 w.	23·5 w.	......	......	......	......	......	......	Ross.
At sea......................	31 15	359 48	1840	......	......	......	......	......	......	6·56	......	6·56	Terror
At sea (4 observations)	30 20	359 50	1840	......	......	......	......	......	......	6·57	......	6·57	Ross.
At sea......................	30 10	359 53	1840	......	......	......	......	......	......	6·39	......	6·39	Terror
At sea (2 observations)	31 19	359 53	1840	......	......	......	......	......	......	6·56	......	6·56	Ross.
At sea......................	30 12	359 54	1840	......	......	......	......	......	......	6·56	......	6·56	Terror

In the following Tables I have placed in comparison with each other the values of the magnetic Elements at every fifth degree of latitude between the Equator and 40° S., and at every tenth degree of longitude between 0° and 360°, as shown (1) in the Table published by MM. Gauss and Weber, in the 'Atlas des Erdmagnetismus' (Leipsic, 1840), and (2) in the Tables and Maps of the present paper. For the values of the Magnetic Force, which in the Atlas of MM. Gauss and Weber are expressed in the Arbitrary Scale, of which the fundamental value is 1·372, or (as written by M. Gauss) 1372=the Force in London in 1836, I have substituted the Absolute Values corresponding to 10·28 as the Absolute Force in London at the same Epoch, in the scale which was originally adopted in conformity with the Report of the Committee of Physics of the Royal Society, 1840, page 21. In all the three Elements there are some blanks in the columns derived from the data in the present paper, owing to observations being either wanting or insufficient in those localities.

Declination.

Latitudes.	Gauss.	Sabine.	Gauss.	Sabine.	Gauss.	Sabine.	Gauss.	Sabine.	Gauss.	Sabine.	Gauss.	Sabine.	Latitudes.
	Long. 0° E.		Long. 10° E.		Long. 20° E.		Long. 30° E.		Long. 40° E.		Long. 50° E.		
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	°
00 s.	21 49 w.	......	23 04 w.	19 32 w.	22 14 w.	16 31 w.	19 14 w.	13 39 w.	14 26 w.	10 54 w.	8 45 w.	7 50 w.	00 s.
05 s.	21 50 w.	......	23 37 w.	21 15 w.	23 20 w.	18 57 w.	20 47 w.	15 47 w.	16 13 w.	12 32 w.	10 31 w.	8 50 w.	05 s.
10 s.	21 51 w.	......	24 11 w.	22 37 w.	24 30 w.	20 56 w.	22 28 w.	18 14 w.	18 15 w.	14 21 w.	12 39 w.	10 12 w.	10 s.
15 s.	21 49 w.	......	24 41 w.	24 04 w.	25 37 w.	22 28 w.	24 13 w.	20 34 w.	20 30 w.	17 03 w.	15 11 w.	12 43 w.	15 s.
20 s.	21 39 w.	......	25 03 w.	25 36 w.	26 38 w.	24 11 w.	25 56 w.	22 22 w.	22 53 w.	19 43 w.	18 04 w.	15 12 w.	20 s.
25 s.	21 16 w.	......	25 09 w.	26 47 w.	27 22 w.	25 48 w.	27 27 w.	24 18 w.	25 13 w.	22 00 w.	21 07 w.	18 14 w.	25 s.
30 s.	20 38 w.	......	24 56 w.	27 16 w.	27 45 w.	27 25 w.	28 37 w.	26 29 w.	27 19 w.	24 14 w.	24 10 w.	21 08 w.	30 s.
35 s.	19 42 w.	......	24 21 w.	......	27 42 w.	29 12 w.	29 20 w.	29 08 w.	29 02 w.	27 01 w.	26 59 w.	24 04 w.	35 s.
40 s.	18 34 w.	......	23 28 w.	26 00 w.	27 17 w.	30 00 w.	29 39 w.	31 07 w.	30 20 w.	30 00 w.	29 27 w.	26 50 w.	40 s.
	Long. 60° E.		Long. 70° E.		Long. 80° E.		Long. 90° E.		Long. 100° E.		Long. 110° E.		
00 s.	3 29 w.	4 42 w.	0 17 E.	1 44 w.	2 11 E.	......	2 30 E.	......	1 54 E.	......	1 12 E.	......	00 s.
05 s.	5 06 w.	5 23 w.	1 08 w.	2 33 w.	0 58 E.	0 21 w.	1 30 E.	......	1 12 E.	......	0 50 E.	......	05 s.
10 s.	7 09 w.	6 30 w.	3 00 w.	3 37 w.	0 41 w.	1 34 w.	0 08 E.	0 41 w.	0 11 E.	0 31 w.	0 15 E.	0 10 w.	10 s.
15 s.	9 43 w.	8 05 w.	5 26 w.	4 42 w.	2 52 w.	3 05 w.	1 41 w.	2 19 w.	1 12 w.	2 00 w.	0 39 w.	1 17 w.	15 s.
20 s.	12 48 w.	10 08 w.	8 27 w.	7 00 w.	5 37 w.	4 43 w.	4 00 w.	4 05 w.	2 59 w.	3 44 w.	1 52 w.	2 31 w.	20 s.
25 s.	16 18 w.	13 51 w.	12 02 w.	9 49 w.	8 56 w.	7 50 w.	6 51 w.	6 40 w.	5 14 w.	5 52 w.	3 28 w.	4 05 w.	25 s.
30 s.	20 03 w.	17 45 w.	16 03 w.	14 32 w.	12 47 w.	12 06 w.	10 14 w.	10 10 w.	7 58 w.	8 53 w.	5 31 w.	6 08 w.	30 s.
35 s.	23 48 w.	21 20 w.	20 18 w.	19 38 w.	17 02 w.	17 29 w.	14 07 w.	15 35 w.	11 15 w.	12 52 w.	8 04 w.	9 03 w.	35 s.
40 s.	27 21 w.	24 24 w.	24 36 w.	22 56 w.	21 27 w.	21 22 w.	18 30 w.	20 00 w.	15 08 w.	16 20 w.	11 15 w.	11 36 w.	40 s.
	Long. 120° E.		Long. 130° E.		Long. 140° E.		Long. 150° E.		Long. 160° E.		Long. 170° E.		
00 s.	1 04 E.	1 09 E.	1 51 E.	......	3 37 E.	......	5 58 E.	......	8 16 E.	......	9 49 E.	......	00 s.
05 s.	1 04 E.	......	2 13 E.	1 46 E.	4 13 E.	3 44 E.	6 37 E.	......	8 47 E.	......	10 04 E.	......	05 s.
10 s.	0 54 E.	0 11 E.	2 25 E.	1 00 E.	4 39 E.	3 45 E.	7 08 E.	5 51 E.	9 11 E.	7 27 E.	10 15 E.	8 31 E.	10 s.
15 s.	0 30 E.	0 38 w.	2 25 E.	1 08 E.	4 55 E.	4 10 E.	7 29 E.	6 40 E.	9 28 E.	8 31 E.	10 22 E.	9 26 E.	15 s.
20 s.	0 11 w.	1 33 w.	2 11 E.	0 35 E.	4 59 E.	4 38 E.	7 40 E.	7 37 E.	9 38 E.	9 20 E.	10 28 E.	10 22 E.	20 s.
25 s.	1 11 w.	2 31 w.	1 41 E.	0 11 E.	4 51 E.	7 46 E.	7 43 E.	......	9 44 E.	10 13 E.	10 34 E.	......	25 s.
30 s.	2 33 w.	3 28 w.	0 54 E.	0 07 w.	4 29 E.	8 36 E.	7 38 E.	8 36 E.	9 48 E.	......	10 44 E.	......	30 s.
35 s.	4 22 w.	4 30 w.	0 14 w.	0 13 w.	3 54 E.	5 47 E.	7 26 E.	9 30 E.	9 59 E.	......	11 00 E.	......	35 s.
40 s.	6 44 w.	5 55 w.	1 48 w.	0 18 w.	3 03 E.	5 49 E.	7 07 E.	10 31 E.	9 59 E.	13 17 E.	11 28 E.	14 52 E.	40 s.
	Long. 180° E.		Long. 190° E.		Long. 200° E.		Long. 210° E.		Long. 220° E.		Long. 230° E.		
00 s.	10 15 E.	......	9 36 E.	8 00 E.	8 22 E.	......	7 10 E.	......	6 32 E.	......	6 39 E.	......	00 s.
05 s.	10 10 E.	8 29 E.	9 12 E.	8 00 E.	7 44 E.	6 47 E.	6 26 E.	5 33 E.	5 49 E.	4 14 E.	6 05 E.	3 57 E.	05 s.
10 s.	10 05 E.	8 58 E.	8 54 E.	8 08 E.	7 22 E.	6 55 E.	5 57 E.	5 52 E.	5 23 E.	5 05 E.	5 46 E.	4 53 E.	10 s.
15 s.	10 02 E.	9 29 E.	8 45 E.	8 26 E.	7 06 E.	7 20 E.	5 46 E.	6 29 E.	5 15 E.	5 58 E.	5 45 E.	5 52 E.	15 s.
20 s.	10 04 E.	10 00 E.	8 47 E.	9 00 E.	7 11 E.	7 56 E.	5 54 E.	7 16 E.	5 28 E.	6 53 E.	6 05 E.	6 51 E.	20 s.
25 s.	10 13 E.	......	9 01 E.	......	7 32 E.	8 43 E.	6 24 E.	8 04 E.	6 05 E.	7 47 E.	6 48 E.	7 50 E.	25 s.
30 s.	10 30 E.	......	9 29 E.	......	8 13 E.	9 25 E.	7 16 E.	8 43 E.	7 07 E.	8 33 E.	7 58 E.	8 34 E.	30 s.
35 s.	11 01 E.	......	10 15 E.	......	9 16 E.	10 08 E.	8 35 E.	9 20 E.	8 38 E.	9 20 E.	9 36 E.	9 16 E.	35 s.
40 s.	11 48 E.	......	11 24 E.	......	10 45 E.	......	10 23 E.	......	10 40 E.	......	11 47 E.	......	40 s.
	Long. 240° E.		Long. 250° E.		Long. 260° E.		Long. 270° E.		Long. 280° E.		Long. 290° E.		
00 s.	7 21 E.	4 46 E.	8 14 E.	6 21 E.	8 52 E.	7 53 E.	8 53 E.	......	8 00 E.	......	6 08 E.	6 19 E.	00 s.
05 s.	7 01 E.	5 05 E.	8 12 E.	6 38 E.	9 07 E.	8 07 E.	9 23 E.	......	8 43 E.	......	6 59 E.	7 17 E.	05 s.
10 s.	6 55 E.	5 54 E.	8 21 E.	7 10 E.	9 32 E.	8 44 E.	10 01 E.	10 07 E.	9 31 E.	10 14 E.	7 53 E.	8 15 E.	10 s.
15 s.	7 05 E.	6 46 E.	8 44 E.	7 53 E.	10 08 E.	9 28 E.	10 48 E.	11 05 E.	10 26 E.	11 17 E.	8 53 E.	9 25 E.	15 s.
20 s.	7 33 E.	7 37 E.	9 23 E.	8 40 E.	10 58 E.	10 18 E.	11 46 E.	12 01 E.	11 29 E.	12 22 E.	9 59 E.	10 50 E.	20 s.
25 s.	8 23 E.	8 23 E.	10 21 E.	9 28 E.	12 03 E.	11 27 E.	12 56 E.	12 57 E.	12 41 E.	13 28 E.	11 12 E.	12 20 E.	25 s.
30 s.	9 38 E.	9 05 E.	11 40 E.	10 26 E.	13 24 E.	12 36 E.	14 19 E.	13 55 E.	14 03 E.	14 34 E.	12 32 E.	13 52 E.	30 s.
35 s.	11 20 E.	9 50 E.	13 22 E.	11 50 E.	15 04 E.	13 53 E.	15 54 E.	15 00 E.	15 34 E.	......	13 58 E.	......	35 s.
40 s.	13 33 E.	11 21 E.	15 31 E.	13 20 E.	17 04 E.	15 14 E.	17 44 E.	......	17 14 E.	......	15 30 E.	......	40 s.
	Long. 300°.		Long. 310°.		Long. 320°.		Long. 330°.		Long. 340°.		Long. 350°.		
00 s.	3 14 E.	3 44 E.	0 33 w.	0 34 E.	5 02 w.	4 53 w.	9 54 w.	10 43 w.	14 42 w.	15 39 w.	18 52 w.	19 26 w.	00 s.
05 s.	4 11 E.	4 53 E.	0 27 E.	1 26 E.	4 02 w.	3 57 w.	8 58 w.	10 00 w.	13 57 w.	15 42 w.	18 26 w.	......	05 s.
10 s.	5 10 E.	5 50 E.	1 27 E.	2 11 E.	3 03 w.	3 10 w.	8 05 w.	9 20 w.	13 14 w.	15 23 w.	18 01 w.	......	10 s.
15 s.	6 12 E.	6 47 E.	2 29 E.	3 02 E.	2 03 w.	2 34 w.	7 10 w.	8 42 w.	12 29 w.	14 51 w.	17 33 w.	......	15 s.
20 s.	7 18 E.	7 53 E.	3 35 E.	4 00 E.	1 00 w.	1 57 w.	6 12 w.	8 05 w.	11 41 w.	14 07 w.	17 00 w.	20 35 w.	20 s.
25 s.	8 31 E.	9 09 E.	4 45 E.	5 13 E.	0 07 E.	1 07 w.	5 09 w.	7 18 w.	10 45 w.	13 22 w.	16 17 w.	19 47 w.	25 s.
30 s.	9 49 E.	10 38 E.	6 01 E.	6 28 E.	1 21 E.	0 00	3 58 w.	6 20 w.	9 40 w.	12 27 w.	15 22 w.	18 43 w.	30 s.
35 s.	11 10 E.	12 15 E.	7 20 E.	8 05 E.	2 38 E.	1 34 E.	2 43 w.	5 05 w.	8 27 w.	11 12 w.	14 15 w.	17 16 w.	35 s.
40 s.	12 36 E.	13 43 E.	8 42 E.	10 00 E.	3 57 E.	3 25 E.	1 24 w.	3 20 w.	7 09 w.	9 32 w.	12 59 w.	15 44 w.	40 s.

Inclination.

Latitudes.	Gauss.	Sabine.	Gauss.	Sabine.	Gauss.	Sabine.	Gauss.	Sabine.	Gauss.	Sabine.	Gauss.	Sabine.	Latitudes.
	Long. 0° E.		Long. 10° E.		Long. 20° E.		Long. 30° E.		Long. 40° E.		Long. 50° E.		
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	°
00 s.	7 36 N.	......	2 38 s.	......	12 58 s.	......	21 44 s.	......	27 42 s.	......	30 24 s.	......	00 s.
05 s.	1 18 s.	8 31 s.	11 27 s.	......	21 30 s.	......	29 58 s.	......	35 45 s.	......	38 29 s.	......	05 s.
10 s.	9 57 s.	16 11 s.	19 36 s.	......	29 04 s.	......	36 59 s.	......	42 28 s.	......	45 11 s.	41 44 s.	10 s.
15 s.	18 03 s.	23 40 s.	26 53 s.	......	35 30 s.	......	42 45 s.	......	47 53 s.	......	50 33 s.	47 50 s.	15 s.
20 s.	25 21 s.	30 18 s.	33 10 s.	37 29 s.	40 49 s.	......	47 19 s.	......	52 03 s.	51 47 s.	54 41 s.	52 23 s.	20 s.
25 s.	31 40 s.	34 56 s.	38 24 s.	41 46 s.	45 02 s.	48 04 s.	50 48 s.	52 22 s.	55 07 s.	54 42 s.	57 42 s.	56 13 s.	25 s.
30 s.	37 03 s.	39 31 s.	42 42 s.	45 31 s.	48 21 s.	51 10 s.	53 22 s.	54 50 s.	57 17 s.	57 34 s.	59 50 s.	60 06 s.	30 s.
35 s.	41 33 s.	......	46 11 s.	49 37 s.	50 55 s.	53 49 s.	55 14 s.	57 32 s.	58 44 s.	60 41 s.	61 12 s.	......	35 s.
40 s.	45 19 s.	......	49 03 s.	52 19 s.	52 57 s.	56 19 s.	56 37 s.	60 18 s.	59 46 s.	......	62 11 s.	......	40 s.
	Long. 60° E.		Long. 70° E.		Long. 80° E.		Long. 90° E.		Long. 100° E.		Long. 110° E.		
00 s.	30 04 s.	......	27 29 s.	......	23 47 s.	......	20 09 s.	......	17 27 s.	......	16 03 s.	......	00 s.
05 s.	38 22 s.	32 22 s.	36 08 s.	30 35 s.	32 50 s.	......	29 33 s.	......	27 04 s.	24 14 s.	25 41 s.	22 18 s.	05 s.
10 s.	45 18 s.	40 34 s.	43 27 s.	38 59 s.	40 37 s.	37 28 s.	37 46 s.	35 28 s.	35 34 s.	33 19 s.	34 16 s.	32 02 s.	10 s.
15 s.	50 54 s.	46 47 s.	49 28 s.	45 25 s.	47 09 s.	44 18 s.	44 46 s.	42 54 s.	42 54 s.	41 22 s.	41 44 s.	40 00 s.	15 s.
20 s.	55 17 s.	52 11 s.	54 18 s.	51 11 s.	52 31 s.	50 22 s.	50 38 s.	49 19 s.	49 09 s.	47 59 s.	48 11 s.	46 44 s.	20 s.
25 s.	58 33 s.	57 00 s.	58 05 s.	56 07 s.	56 52 s.	55 19 s.	55 31 s.	54 35 s.	54 27 s.	53 50 s.	53 43 s.	53 13 s.	25 s.
30 s.	60 56 s.	61 47 s.	60 58 s.	61 07 s.	60 21 s.	60 14 s.	59 35 s.	59 41 s.	58 58 s.	59 17 s.	58 32 s.	59 07 s.	30 s.
35 s.	62 37 s.	......	63 09 s.	......	63 10 s.	......	63 00 s.	......	62 51 s.	......	62 45 s.	......	35 s.
40 s.	63 51 s.	......	64 52 s.	......	65 30 s.	......	65 52 s.	......	66 13 s.	......	66 29 s.	......	40 s.
	Long. 120° E.		Long. 130° E.		Long. 140° E.		Long. 150° E.		Long. 160° E.		Long. 170° E.		
00 s.	15 41 s.	13 34 s.	15 45 s.	13 15 s.	15 27 s.	13 25 s.	14 07 s.	......	11 25 s.	......	7 35 s.	......	00 s.
05 s.	25 10 s.	21 54 s.	24 56 s.	22 16 s.	24 18 s.	22 15 s.	22 37 s.	21 18 s.	19 44 s.	18 16 s.	15 53 s.	13 07 s.	05 s.
10 s.	33 39 s.	31 32 s.	33 10 s.	31 44 s.	32 18 s.	31 43 s.	30 22 s.	31 00 s.	27 26 s.	27 25 s.	23 44 s.	22 19 s.	10 s.
15 s.	41 04 s.	39 25 s.	40 27 s.	39 22 s.	39 19 s.	39 19 s.	37 21 s.	38 57 s.	34 30 s.	35 44 s.	31 03 s.	31 44 s.	15 s.
20 s.	47 33 s.	46 39 s.	46 49 s.	46 40 s.	45 37 s.	46 47 s.	43 39 s.	46 06 s.	40 57 s.	43 45 s.	37 49 s.	40 00 s.	20 s.
25 s.	53 09 s.	51 59 s.	52 26 s.	53 08 s.	51 14 s.	53 01 s.	49 22 s.	52 31 s.	46 52 s.	51 11 s.	44 05 s.	48 03 s.	25 s.
30 s.	58 07 s.	59 06 s.	57 29 s.	58 57 s.	56 21 s.	58 37 s.	54 36 s.	58 05 s.	52 21 s.	56 44 s.	49 54 s.	54 13 s.	30 s.
35 s.	62 33 s.	......	62 03 s.	......	61 03 s.	......	59 28 s.	......	57 28 s.	......	55 19 s.	60 20 s.	35 s.
40 s.	66 34 s.	......	66 15 s.	......	65 25 s.	......	64 03 s.	......	62 18 s.	......	60 24 s.	......	40 s.
	Long. 180° E.		Long. 190° E.		Long. 200° E.		Long. 210° E.		Long. 220° E.		Long. 230° E.		
00 s.	3 21 s.	......	0 22 N.	......	2 55 N.	......	4 14 N.	......	4 45 N.	......	5 12 N.	......	00 s.
05 s.	11 48 s.	6 40 s.	8 20 s.	......	6 08 s.	......	5 15 s.	6 17 s.	5 08 s.	5 45 s.	5 04 s.	5 13 s.	05 s.
10 s.	19 56 s.	16 45 s.	16 51 s.	19 40 s.	15 02 s.	18 37 s.	14 29 s.	16 17 s.	14 44 s.	15 27 s.	15 02 s.	14 41 s.	10 s.
15 s.	27 38 s.	26 43 s.	24 58 s.	27 20 s.	23 30 s.	26 27 s.	23 15 s.	25 17 s.	23 42 s.	24 30 s.	24 09 s.	......	15 s.
20 s.	34 50 s.	35 35 s.	32 33 s.	34 46 s.	31 22 s.	35 42 s.	31 16 s.	32 56 s.	31 50 s.	32 00 s.	32 24 s.	......	20 s.
25 s.	41 29 s.	43 46 s.	39 32 s.	42 18 s.	38 34 s.	41 04 s.	38 33 s.	40 00 s.	39 06 s.	38 50 s.	39 38 s.	......	25 s.
30 s.	47 38 s.	51 05 s.	45 58 s.	49 17 s.	45 08 s.	47 36 s.	45 06 s.	46 04 s.	45 32 s.	44 57 s.	45 58 s.	......	30 s.
35 s.	53 21 s.	57 25 s.	51 53 s.	55 11 s.	51 07 s.	53 42 s.	51 00 s.	......	51 15 s.	51 20 s.	51 30 s.	......	35 s.
40 s.	58 40 s.	......	57 22 s.	......	56 36 s.	......	56 21 s.	......	56 23 s.	......	56 23 s.	......	40 s.
	Long. 240° E.		Long. 250° E.		Long. 260° E.		Long. 270° E.		Long. 280° E.		Long. 290° E.		
00 s.	6 15 N.	......	8 15 N.	......	11 15 N.	......	14 59 N.	......	19 00 N.	......	22 46 N.	......	00 s.
05 s.	4 19 s.	4 40 s.	2 29 s.	3 47 s.	0 31 N.	1 42 s.	4 26 N.	1 03 N.	8 45 N.	9 10 N.	12 56 N.	10 54 N.	05 s.
10 s.	14 31 s.	14 19 s.	12 54 s.	13 28 s.	10 01 s.	11 00 s.	6 08 s.	7 55 s.	1 41 s.	3 53 s.	2 43 N.	2 05 N.	10 s.
15 s.	23 55 s.	......	22 33 s.	......	19 54 s.	21 47 s.	16 11 s.	17 29 s.	11 50 s.	12 50 s.	7 25 s.	6 49 s.	15 s.
20 s.	32 19 s.	......	31 09 s.	......	28 47 s.	......	25 21 s.	26 55 s.	21 15 s.	21 43 s.	17 01 s.	15 42 s.	20 s.
25 s.	39 36 s.	......	38 36 s.	......	36 28 s.	......	33 22 s.	......	29 37 s.	30 00 s.	25 42 s.	23 43 s.	25 s.
30 s.	45 54 s.	......	44 58 s.	......	43 03 s.	......	40 15 s.	......	36 52 s.	38 40 s.	33 18 s.	31 01 s.	30 s.
35 s.	51 19 s.	......	50 23 s.	......	48 37 s.	......	46 04 s.	......	43 01 s.	......	39 48 s.	40 17 s.	35 s.
40 s.	56 01 s.	......	55 02 s.	......	53 20 s.	......	50 59 s.	......	48 13 s.	......	45 20 s.	......	40 s.
	Long. 300° E.		Long. 310° E.		Long. 320° E.		Long. 330° E.		Long. 340° E.		Long. 350° E.		
00 s.	25 49 N.	......	27 41 N.	......	28 00 N.	......	26 25 N.	......	22 35 N.	......	16 16 N.	9 47 N.	00 s.
05 s.	16 22 N.	15 19 N.	18 36 N.	18 19 N.	19 10 N.	19 16 N.	17 42 N.	13 09 N.	13 51 N.	11 04 N.	7 27 N.	2 02 N.	05 s.
10 s.	6 27 N.	7 25 N.	8 57 N.	10 41 N.	9 46 N.	11 34 N.	8 29 N.	8 36 N.	4 46 N.	2 26 N.	1 30 s.	5 41 s.	10 s.
15 s.	3 35 s.	1 07 s.	0 53 s.	1 37 N.	0 08 N.	2 43 N.	0 56 s.	0 37 s.	4 23 s.	6 26 s.	10 13 s.	13 41 s.	15 s.
20 s.	13 12 s.	10 18 s.	10 31 s.	7 10 s.	9 21 s.	6 36 s.	10 09 s.	9 41 s.	13 10 s.	14 22 s.	18 23 s.	21 22 s.	20 s.
25 s.	22 08 s.	19 20 s.	19 29 s.	16 29 s.	18 14 s.	15 47 s.	18 45 s.	17 42 s.	21 16 s.	21 36 s.	25 42 s.	27 52 s.	25 s.
30 s.	30 01 s.	26 21 s.	27 31 s.	24 01 s.	26 14 s.	23 15 s.	26 29 s.	24 16 s.	28 28 s.	28 00 s.	32 06 s.	33 05 s.	30 s.
35 s.	36 50 s.	34 38 s.	34 31 s.	31 19 s.	33 14 s.	29 48 s.	33 14 s.	30 53 s.	34 42 s.	33 46 s.	37 34 s.	38 11 s.	35 s.
40 s.	42 40 s.	......	40 32 s.	......	39 15 s.	......	39 03 s.	......	40 02 s.	......	42 12 s.	......	40 s.

Force in British Units.

Latitudes.	Gauss.	Sabine.	Gauss.	Sabine.	Gauss.	Sabine.	Gauss.	Sabine.	Gauss.	Sabine.	Gauss.	Sabine.	Latitudes.
	Long. 0° E.		Long. 10° E.		Long. 20° E.		Long. 30° E.		Long. 40° E.		Long. 50° E.		
°													°
00 s,	6·55	……	6·47	……	6·56	……	6·79	……	7·07	……	7·32	……	00 s.
05 s.	6·32	……	6·31	……	6·54	……	6·85	……	7·19	……	7·48	……	05 s.
10 s.	6·18	……	6·28	……	6·57	……	6·94	……	7·33	……	7·66	8·0	10 s.
15 s.	6·12	……	6·31	……	6·63	……	7·05	……	7·47	……	7·84	8·2	15 s.
20 s.	6·15	……	6·37	……	6·73	……	7·16	……	7·61	8·0	8 02	8·4	20 s.
25 s.	6·25	……	6·49	……	6 86	7·2	7·29	7·7	7·76	8·2	8·21	8·7	25 s.
30 s.	6·43	……	6·67	……	7·03	7·3	7·47	7·8	7·95	8·4	8 44	8·9	30 s.
35 s.	6 70	……	6·92	7·0	7·27	7·6	7·70	8·1	8·19	8·7	8·72	9·3	35 s.
40 s.	7·06	……	7·27	7·5	7·60	7·9	8·02	8·5	8·52	9·0	9·08	9·6	40 s.
	Long. 60° E.		Long. 70° E.		Long. 80° E.		Long. 90° E.		Long. 100° E.		Long. 110° E.		
00 s.	7·51	……	7·62	……	7·72	……	7·80	……	7·89	……	7·96	……	00 s.
05 s.	7 69	8·1	7·83	……	7·93	……	8·04	……	8·16	……	8·26	……	05 s.
10 s.	7·91	8·3	8·08	8·6	8·24	……	8·39	……	8 56	9·2	8·71	9·3	10 s.
15 s.	8·13	8·5	8·38	8·8	8·60	9·0	8·82	9·3	9·06	9·5	9·27	9·7	15 s.
20 s.	8·37	8·8	8 69	9·0	9·00	9·5	9·31	9·8	9·63	10·0	9·91	10·3	20 s.
25 s.	8 63	9·0	9·03	9·5	9·44	9·9	9·85	10·4	10·25	10·7	10·61	11·0	25 s.
30 s.	8·92	9·4	9·41	9·9	9·91	10·5	10·41	11·0	10·91	11·4	11·34	11·7	30 s.
35 s.	9·26	9·8	9·83	10·4	10·42	11·0	11·01	11·6	11·58	12·0	12·09	12·4	35 s.
40 s.	9·68	10·2	10·31	10·8	10·97	11·5	11·63	12·1	12·26	12·5	12·83	13·6	40 s.
	Long. 120° E.		Long. 130° E.		Long. 140° E.		Long. 150° E.		Long. 160° E.		Long. 170° E.		
00 s.	7·97	……	7·90	……	7·76	……	7·55	……	7·33	……	7·15	……	00 s.
05 s.	8 31	8·9	8·28	……	8·15	9·1	7·94	9·0	7·69	……	7·46	……	05 s.
10 s.	8 81	9·4	8·81	9·5	8·68	9 6	8·49	9·5	8·21	……	7·94	……	10 s.
15 s.	9·42	9·9	9·46	10·0	9·37	10·1	9·16	10·1	8·88	9·9	8·57	……	15 s.
20 s.	10·13	10·5	10·21	10·7	10·15	10·7	9·96	10·7	9·66	10·5	9·34	10·2	20 s.
25 s.	10·88	11·3	11·02	11·4	11·00	11·4	10 83	11·4	10·55	11·1	10·22	10·8	25 s.
30 s.	11·68	12·0	11·87	12·1	11·90	12·2	11·76	12·1	11·51	11·8	11·18	11·4	30 s.
35 s.	12·49	12·7	12·73	12·9	12·81	12·8	12 72	12·7	12·49	12 4	12·19	12·0	35 s.
40 s.	13·28	……	13·57	……	13·70	……	13·66	……	13·48	12·9	13·20	……	40 s.
	Long. 180° E.		Long. 190° E.		Long. 200° E.		Long. 210° E.		Long. 220° E.		Long. 230° E.		
00 s.	7·04	……	7·05	……	7·15	……	7·32	……	7·53	……	7·74	……	00 s.
05 s.	7·30	……	7·23	……	7·27	……	7·38	……	7·54	……	7·70	……	05 s.
10 s.	7·72	9·0	7·59	……	7·57	……	7·63	8·2	7·74	8·0	7·86	……	10 s.
15 s.	8·30	9·5	8·13	9·2	8·05	……	8·05	8·7	8·12	8·5	8 19	……	15 s.
20 s.	9·04	9·9	8·81	9·6	8·69	9·3	8·64	9·1	8·65	8·9	8·66	……	20 s.
25 s.	9·90	10·5	9·63	10·1	9·46	9·9	9·36	9 7	9·31	9·5	9 26	……	25 s.
30 s.	10·85	11·0	10·56	10·7	10·33	10·3	10·18	10 2	10·06	10·0	9 94	……	30 s.
35 s.	11·86	11·6	11·54	11·3	11·28	10·9	11·07	10·8	10·88	10·6	10·69	……	35 s.
40 s.	12·88	……	12·56	……	12·26	……	11·99	……	11·73	……	11·47	……	40 s.
	Long. 240° E.		Long. 250° E.		Long. 260° E.		Long. 270° E.		Long. 280° E.		Long. 290° E.		
00 s.	7·92	……	8·06	……	8·15	……	8·19	……	8·19	……	8·13	……	00 s.
05 s.	7·83	……	7·91	……	7·93	……	7·89	……	7·81	……	7·70	……	05 s.
10 s.	7·94	……	7·95	……	7·89	……	7·77	……	7·61	……	7·42	……	10 s.
15 s.	8·21	……	8·16	……	8·03	……	7·82	……	7·58	……	7·31	6·9	15 s.
20 s.	8·63	……	8·52	……	8·31	……	8 03	……	7·70	……	7 37	6·8	20 s.
25 s.	9·16	……	8·98	……	8·71	……	8·36	……	7·97	7·7	7·56	7 0	25 s.
30 s.	9·78	……	9·53	……	9·20	……	8·79	……	8·33	8·1	7·87	7·5	30 s.
35 s.	10·45	……	10·14	……	9·75	……	9·29	……	8·79	……	8·29	8·1	35 s.
40 s.	11·16	……	10·79	……	10·35	……	9·85	……	9·32	……	8·78	……	40 s.
	Long. 300° E.		Long. 310° E.		Long. 320° E.		Long. 330° E.		Long. 340° E.		Long. 350° E.		
00 s.	8·03	……	7·87	……	7·65	……	7·38	……	7·07	……	6·77	……	00 s.
05 s.	7·56	……	7·38	……	7·17	6·9	6·92	6·9	6 67	6·7	6·44	……	05 s.
10 s.	6·48	6·9	7·02	6·8	6·80	6·6	6·58	6·5	6·37	6·4	6·21	……	10 s.
15 s.	7·06	6·7	6·81	6·6	6·57	6·4	6·36	6·2	6·18	……	6·09	……	15 s.
20 s.	7·04	6·6	6·75	6·5	6·49	6·3	6·27	……	6·12	……	6·07	……	20 s.
25 s.	7·18	6·8	6·83	6·6	6·54	6 4	6·32	……	6·18	……	6·15	6·2	25 s.
30 s.	7·44	7 0	7·05	6·8	6·73	6·6	6·49	6·4	6·35	……	6·33	……	30 s.
35 s.	7·81	7·5	7·39	……	7 04	6·8	6·78	6·7	6 63	……	6·60	……	35 s.
40 s.	8·28	……	7·83	……	7·46	……	7·19	……	7·02	……	6 98	……	40 s.

In the preceding Numbers of these Contributions the great assistance received for many years past from the Hydrographic Office of the Admiralty has been thankfully acknowledged. In tracing now the last lines of the present number, which completes the series, these acknowledgments must be expressed anew, and with redoubled earnestness; for without the cordial and truly invaluable assistance of Captain EVANS, and of Staff-Commander E. W. CREAK of his Department, its preparation, under the weight of broken health and increasing weakness, would have been impossible.

ERRATA IN CONTRIBUTION No. XV.

Page 464, line 19. Aji Bārang, *for* Lat. 2° 25′ S. *read* 7° 25′ S.

Page 475, line 2. Declination. At sea—Lat. 17° 15′ S., Long. 292° 45′ E. *should be expunged.*

XIV. *On Friction between Surfaces moving at Low Speeds.* *By* Fleeming Jenkin, *F.R.SS.L. & E., Professor of Engineering in the University of Edinburgh, and* J. A. Ewing.

Received March 8,—Read April 19, 1877.

[Plate 20.]

The following paper contains an account of an investigation of the Friction between surfaces in motion under circumstances such as do not appear to have been examined before.

The general subject of Friction has received the attention of many writers, beginning with Amontons in 1699. In 1799 Coulomb began to investigate it, and in 1781 he communicated to the Academy of Sciences a paper containing the result of his experiments, entitled "Mémoire sur la théorie des machines simples," which is published in vol. x. of the 'Savans Étrangers.' Coulomb pointed out the necessity of distinguishing between the friction which resists the relative movement of surfaces already in motion, or what is now called kinetic friction, and the friction which tends to prevent surfaces at rest from being set in motion, or what is now called static friction. He found that with two dry metallic surfaces there was no difference between the static and kinetic friction. In the other cases which he examined there was a more or less considerable difference, the static being always greater than the kinetic. He found also that the static friction depended on the length of time during which the surfaces were at rest; a prolongation of the time of rest had the effect of increasing the friction, and the rate of this increase varied much in different cases. When the intensity of pressure was great the static friction reached its highest value in a shorter time than when the pressure was small. Coulomb also examined the influence of velocity on the kinetic friction; but his means of observing this seem to have been somewhat rough. He was, however, able to point out that generally, although subject to several exceptions, friction is independent of the extent of surface in contact, directly proportional to the pressure, and independent of the velocity.

In 1784 Vince laid before the Royal Society the results of some experiments, which, although not very conclusive, agreed in the main with those of Coulomb.

The Philosophical Transactions for 1829 contain a paper on friction by G. Rennie, in which an interesting account is given of the early history of the subject. Rennie's experiments with hard surfaces such as those of metal or wood confirmed, as far as they went, the conclusions of Coulomb.

One part of RENNIE's work is of special interest on account of the light which has been thrown upon it by the comparatively recent progress of scientific theory. He observed that between ice and ice and between ice and steel (the temperature of the atmosphere being about 28° Fahr.) the coefficient of friction *diminished* largely as the pressure increased. The friction was far from regular; but the diminution of the coefficient under increased pressure was very marked, and amounted in one case to a change from 0·125 to 0·018. This result, which seems to be in complete accordance with J. THOMSON's discovery of the effect of pressure in lowering the melting-point of ice (Trans. R. S. E. 1849), does not appear ever to have had the attention directed to it that it deserves.

Our most definite knowledge of the phenomena of friction is due to MORIN, who executed from 1830 to 1834 an elaborate series of experiments, the results of which were communicated to the Paris Academy of Sciences, and published in four memoirs. MORIN introduced the system of automatically registering the motion of a body along a horizontal plane surface under the action of certain forces, one of which was the friction to be measured, and he applied this method with great care to the determination of the friction between a large number of different substances. He not only confirmed the laws enunciated by COULOMB, but showed that all the numerous exceptions which COULOMB had mentioned conformed to the general laws when tested by the new and more accurate methods. The results of MORIN have been accepted as conclusive, and the work of subsequent experimenters has been practical rather than directly scientific in its object.

MORIN agreed with COULOMB in distinguishing between static and kinetic friction, and although he did not observe that the time of rest affected the result to nearly so great an extent as COULOMB had affirmed, he found that the static value was usually greater than, but sometimes sensibly equal to, the kinetic. He also noticed that in many cases a slight shock was enough to destroy the distinction between the two.

It occurred to us that instead of there being an abrupt change from the static to the kinetic value of friction at the instant in which motion begins, there might possibly be continuity between the two kinds, and hence that in those cases in which the static coefficient considerably exceeded the kinetic, the latter would be affected by changes of the velocity when the velocity was very small, in such a way as to increase as the velocity diminished. The experiments of MORIN showed that this change in the kinetic value, if it took place at all, must have been confined to very low velocities, so low that his method of observation did not enable him to detect it. The question of whether the friction is affected by changes of velocity under a velocity of, say, 0·01 foot per second, is left by the researches of MORIN an entirely open one; for the length of time which elapsed in his experiments between the instant at which motion began or ended and that at which the velocity was only 0·01 foot per second must have been far too short to allow any definite measurement to be made during it of the rate of acceleration of the moving body.

By means of an apparatus which differs essentially from any previously employed, we have been able to make definite measurements of the friction between surfaces whose relative velocity varied from about 0·01 foot per second down to about 0·0002 foot per second, and have found that in those cases in which the static coefficient largely exceeds the kinetic, the kinetic friction gradually increases as the velocity diminishes between those limits, so that in all probability there is continuity between the two kinds.

In designing an apparatus for the purpose of carrying out this inquiry the following requirements suggested themselves:—That the velocity of the rubbing surfaces (besides being exceedingly low) should change very gradually, so that the acceleration might be capable of measurement at velocities differing only slightly from one another, and differing as little as possible from rest. That the change should be from motion to rest rather than from rest to motion, so as to avoid any jerk at the instant of passage from one to the other state. That no force whatever except the friction to be measured should take part in the action; especially that the means adopted of registering the motion should be such as to cause no retardation. That the surfaces in contact should not change progressively during the motion, but should be periodically restored several times during the motion, so that any change observed to take place in the friction might not be due to a specific change in the surfaces: this last condition might perhaps be more generally expressed by saying that the whole apparatus should periodically return to exactly the same configuration. It was also desirable that a very small change in the velocity should cause a complete change of the rubbing surfaces.

The present inquiry being entirely limited to the question of what influence velocity has on friction at very low speeds, we did not consider it necessary to provide means of measuring the friction at speeds exceeding about 0·01 foot per second, and that is the highest velocity to which our determinations extend. Further, in order to make the apparatus as well conditioned as possible for this particular purpose, it was not arranged so that the intensity of pressure on the rubbing surfaces could be either measured or varied.

The apparatus with which the experiments were conducted, and which was designed with a view to the fulfilment of the above conditions, is shown in fig. 1, Plate 20. A is a disk of cast iron turned on both sides as well as round the circumference, so as to be exactly cylindrical, and weighing 86·2 lbs. Its diameter is nearly 2 feet and its thickness $\frac{3}{4}$ inch. The disk is supported by means of a steel spindle (*a*), the ends of which, resting on the bearings *b b*, are equal cylinders with a diameter of only about 0·1 inch. The small ends of the spindle are shown more clearly in fig. 2, which gives a section of the apparatus in a vertical plane through the axis of the disk, and also in figs. 3 & 4, which give a full-size side elevation and vertical section through the centre of one bearing. The bearings (*b b*) which carry the ends of the axle (*a*) were made successively of the various materials whose friction against the steel axle it was desired to determine. These pieces are fixed by screws to strong iron uprights forming part of the frame B. The bearings consist of rectangular notches, the lower surfaces of both being in the same horizontal plane and each pair of corresponding sides in the same vertical plane.

The breadth of the notch is considerably greater than the diameter of the axle. Thus when the disk is caused to revolve in the direction of the arrow in fig. 1 the axle rolls along the bottom of the notch until it reaches the vertical side on the right hand, and then any subsequent motion can only occur by the sliding of the circumference of the axle upon the bottom and side of each bearing. To prevent the disk from sliding laterally in the event of the bearings ceasing to be exactly level, steel end-plates (*c c*) are provided which can be adjusted by means of screws so as just to touch the ends of the spindle without sensible pressure, and these are slightly rounded at the ends so that their points of contact with the end-plates may lie as nearly as possible in the geometrical axis of the axle. The end-plate is omitted for the sake of clearness in fig. 1, but it is shown in fig. 3, displaced, however, from its proper vertical position in order to allow the end of the axle and the bearing to be fully seen. There was no difficulty in adjusting the end-plates so that their pressure on the axle should cause an infinitesimally small retardation of the motion of the disk, in comparison with that due to the friction of the axle upon the bottom and side of its bearings. Owing to the great weight of the disk this friction was so considerable that the resistance of the air could safely be neglected so long as the velocity of rotation of the disk did not greatly exceed the greatest velocity which occurred in our experiments. When, therefore, the disk was caused to revolve by the temporary application of any force, and was then left to itself, it gradually came to rest in virtue of one cause only—the friction of the axle upon its bearings. By observing the rate of (negative) acceleration of the disk throughout its motion the value of the friction could be determined for all velocities of the rubbing surfaces from the greatest or initial velocity down to the lowest velocity at which the acceleration could be measured.

In order to determine the rate of retardation of the disk at all times throughout its revolution it was necessary to devise some means of recording with great exactness the angular distances moved through by it during successive short intervals of time. This was effected by recording the linear distances moved through by its circumference during successive semiperiods of oscillation of a short pendulum. To obtain a permanent record of these spaces without introducing any new source of retardation whatever, such as would be introduced if a pencil or brush were caused to press either continuously or intermittently against any part of the moving disk, we adopted the method of recording which Sir WILLIAM THOMSON invented for the purpose of registering the arrival of electrical impulses through long submarine cables, and which has found practical application in his siphon recorder. The recording apparatus is shown on the right-hand side in fig. 1. A pendulum (C) is supported on a horizontal knife-edge (*d*) attached to a fixed stand (D), and is capable of oscillating in a plane perpendicular to that of the disk, its position of rest being directly opposite the middle of the circumference of the disk. The knife-edge (*d*) enters a hollow cylinder on the top of the pendulum-rod, and this hollow cylinder is of greater internal diameter at the centre than at the ends, so that the knife-edge bears against it only at the ends. The pendulum is therefore free to oscillate about the line joining these two bearing points. Its period can be altered by shifting up or down the bob *e*. At *f* a small cradle is soldered to the pendulum-rod, and carries a fine glass siphon (*g*), the shorter end of which dips into a box (*h*) containing ink (aniline blue dissolved in water). The longer end (*i*) of the siphon is bent in the manner shown in fig. 1, and stands at a distance of rather less than one tenth of an inch from a strip of paper (E E) two and a half inches broad which is fastened round the circumference of the disk, and projects about three quarters of an inch on each side of it. The lateral stiffness of the paper makes it assume a cylindrical form when placed round the disk, and the ends are fastened together so as to make the cylinder complete. The breadth of the paper is somewhat greater than the maximum amplitude of oscillation of the point *i* of the siphon fixed to the pendulum. In order to make the ink run through the siphon and be deposited on the strip of paper, the ink is continually maintained in a state of electrification. This is effected as follows:—The ink-box (*h*) stands on an insulating rod of vulcanite (*k*), and has fixed to it a small horizontal brass plate (*m*). At a short distance above *m* is the point of a brass rod (*l*) which slides up and down in a V-groove cut in the side of *n*, another rod of vulcanite forming the bracket to which the knife-edge of the pendulum is secured. The rod *l* is pressed into the notch on *n* by the spring *o*. To the top of the rod *l* a wire is fastened which leads to an inductive electrical machine. The machine which we employed was identical in construction with the "mouse-mill" which is used to electrify the ink in the siphon recorder. (See Sir W. THOMSON's 'Electrostatics and Magnetism,' and 'Journal of the Society of Telegraph Engineers,' vol. v. 1877.) The plate *m* becomes electrified by aërial convection from the point of the rod *l*. The rate of electrification of the ink may be varied by raising or lowering the rod *l*.

When the siphon becomes electrified to a certain extent, the attraction so developed between its point *i* and the paper strip is sufficient to cause the long limb of the siphon to bend until the point *i*, or rather the particle of ink projecting from it, just touches the paper. When this takes place a very small drop of ink is deposited on the paper, and the siphon, ink-holder, and plate *m* are instantaneously diselectrified. The point *i* then recedes from the paper, drawn back by the elasticity of the long limb, until the electrification (continuously communicated by the rod *l*) is again sufficient to cause an advance of *i* towards the paper, when another drop of ink is deposited, and so on. The point of the siphon is by these means kept in a state of rapid vibration towards and from the paper, every advance being accompanied by the deposit of a particle of ink. The rate of electrification is adjusted (by moving the rod *l* up or down) so that the time taken to recharge the siphon, inkholder, and plate corresponds to the period of vibration due to the elasticity of the glass. If the electrification be too rapid the point *i* will be checked in its recession from the paper before the completion of its semiperiod of free vibration. If, on the other hand, the electrification be too slow, the impulse given by the new accumulation of electricity following each discharge will not come soon enough. The adjustment of the rate of electrification is a matter requiring some attention, and it is sometimes difficult to see whether the rate is too great or too small, since both errors have much the same effect in preventing the siphon from vibrating properly. The deposit of ink must be made in a succession of particles by the rapid vibration of the siphon in the manner described, and not as a continuous "brush" or "glow."

In our experiments the length of the longer limb of the siphon was about $3\frac{1}{2}$ inches, and its period of vibration when under the influence of the electrification was about 0·028 second; in other words the particles of ink were deposited on the paper at intervals of 0·028 second of time. The distance between successive spots was in some cases as great as $\frac{3}{4}$ of an inch. The thickness and length of the siphon were both much greater than those of the siphons used in the siphon recorder, and the period of vibration was consequently much greater also. We at first attempted to use siphons of the same size as those used in the recorder, but owing to the much greater speed of the paper in our apparatus, as well as the necessity of keeping the siphon-point at a greater distance from the paper, so that any irregularity in the paper band might pass without scraping against the point, we found it to be necessary to use much longer and coarser tubes.

When the pendulum is at rest, and the disk is caused to revolve, if the rod *l* be continuously electrified the vibrations of the siphon deposit a series of particles of ink forming a dotted straight line in the centre of the paper strip. If when the disk is revolving the pendulum is made to oscillate, a curved line will be traced out by the drops of ink, crossing the central straight line at intervals depending on the velocity of revolution of the disk. The distances between the successive points of intersection of the curve with the central line are the distances described by the circumference of the disk in equal intervals of time, these intervals being equal to the semiperiod of oscillation of the pendulum.

The experiments were conducted as follows:—A pair of bearings (*b b*) were selected of the material whose friction against steel was to be measured. Rectangular notches were cut in them, and they were secured in their places on the uprights as shown in figs. 1 & 2, and exactly levelled. The disk with its spindle was then set on them and the end-plates (*c c*) adjusted. A strip of strong paper (E E) 6·3 feet long and 2·5 inches broad was stretched round the periphery and its ends secured by gum, one end overlapping the other for an inch or two. The paper band was continuous except for this junction. The pendulum was then caused to oscillate, and the electrical machine in connexion with *l* was put in action. When the siphon was vibrating rapidly and the ink was being freely deposited on the paper, the disk was set revolving in the direction of the arrow (fig. 1) by means of an impulse given by hand during only a short part of a revolution. It was then left to itself, and usually performed from three to eight complete revolutions before coming to rest. The motion of the point of the siphon relatively to the paper on the disk was registered in the form of an undulating curve of dots crossing the central position at intervals which gradually diminished as the velocity of the disk became less. Fig. 5 shows a short portion of the paper strip on which the dotted curve formed by the deposited drops of ink has been completed by a line drawn through the dots. The portion marked 8 was the first to be traced by the siphon; then 7, which was traced one complete revolution later, and so on down to 1, which was the final portion of the curve traced just as the disk was coming to rest. P is the point at which the siphon first crossed the central position after the motion of the disk had ceased.

When an unguent was used both bearings were well supplied with it just before the disk was set revolving; and when necessary the supply was kept up during the revolution. The total time during which the motion of the disk lasted never exceeded one minute.

The pendulum was next allowed to come to rest, and while the disk was slowly turned by hand, the central line was traced out by the siphon. The strip of paper was then cut across at one place and removed from the disk.

The distances from P (fig. 5) to the successive points at which the curve crossed the central line were measured by means of a rule graduated to six-hundredths of a foot, and the differences between the successive values were found. These differences, which may be called Δs, are the distances described by a point in the circumference of the disk during a half beat, Δt, of the pendulum. $\frac{\Delta s}{\Delta t}$ is the mean velocity during the time Δt, and this mean is the actual velocity at the middle of the time Δt, provided that the acceleration is uniform during that time. Now even if the force due to friction were to change very considerably with changes in the velocity, this force, and therefore the acceleration, would be sensibly constant during the short interval Δt, during which the velocity undergoes exceedingly little change. Hence we are completely justified in taking the successive values of $\frac{\Delta s}{\Delta t}$ as accurately representing the velocities at times differing by Δt; and since Δt is constant, the successive values of Δs are proportional to these velocities.

A curve was next drawn, as O A B in fig. 6, in which the ordinates were the successive values of Δs, and the abscissæ differed by the constant quantity Δt. This curve expressed the velocity as a function of the time, and would be a straight line when the acceleration was uniform. When the acceleration was greater at low than at high velocities, the curve would be convex upwards, as shown in the figure, and the acceleration at any point, such as A, would be proportional to the tangent of the inclination of the tangent at that point, or $\frac{\delta y}{\delta x}$.

This means of finding the value of the acceleration at any point, in which the method of tangents was only once used, was obviously much more accurate than if the first curve plotted had been simply one connecting the distances moved through by the disk with the times, and the method of tangents had been applied to that curve in order to enable a second curve to be drawn connecting the velocities with the times.

A single numerical example will suffice to explain the method of calculation by which, in each experiment, the acceleration due to friction was deduced from the graphic record. The particular case chosen is that of steel rubbing against bearings of beech, without any unguent. The curve is a comparatively short one, extending over only three complete revolutions of the disk. Column I. Table I. gives the distance (measured in six-hundredths of a foot) from a point which corresponds in this example to the point P in fig. 5, to the successive points in which the curve crosses the central line. Column II. gives the differences between these successive values, or what we have called Δs—that is, the distance moved through by a point in the circumference of the disk in times equal to the half-period of the pendulum.

TABLE I.

I. s in $\frac{\text{feet}}{600}$.	II. Δs in $\frac{\text{feet}}{600}$.	I. s in $\frac{\text{feet}}{600}$.	II. Δs in $\frac{\text{feet}}{600}$.	I. s in $\frac{\text{feet}}{600}$.	II. Δs in $\frac{\text{feet}}{600}$.
8	8	1533	163	5692	315
24	16	1705	172	6021	329
49	25	1887	182	6357	336
85	36	2077	190	6703	346
128	43	2275	198	7054	351
183	55	2483	208	7419	365
244	61	2701	218	7787	368
317	73	2925	224	8173	386
397	80	3161	236	8561	388
487	90	3402	241	8964	403
586	99	3659	257	9368	404
694	108	3921	262	9788	420
809	115	4195	274	10211	423
937	128	4475	280	10648	437
1071	134	4768	293	11093	445
1217	146	5063	295	11542	449
1370	153	5377	314		

s for one revolution $= 3718 \frac{\text{feet}}{600}$.

When the curve is drawn corresponding (in this example) to that shown in fig. 6, the irregularities in the successive values of Δs, which are due to the fact that the central line has not been exactly central, disappear, and the curve turns out to be exactly straight. Hence between the limits of velocity to which this experiment extends the friction is perfectly constant. The tangent of the inclination of the line is measured and found to be ·01515, δy (see fig. 6) being expressed in feet, and δx in terms of the unit Δt. To find $\frac{d^2s}{dt^2}$, the acceleration in the direction of motion of a point in the circumference of the disk in feet and seconds, we must divide this quantity by $(\Delta t)^2$ in seconds. Throughout all the experiments Δt was 0·3571 second. Hence for this example

$\frac{d^2s}{dt^2}=0\cdot1188$ foot per second. If ω be the angular velocity of the disk, the angular acceleration $=\frac{d\omega}{dt}=\frac{1}{\mathrm{R}}\frac{d^2s}{dt^2}$, where R is the radius, or 0·9857 foot. The moment, M′, of the couple due to friction, measured in absolute kinetic units, is $\mathrm{I}\frac{d\omega}{dt}$, where I is the moment of inertia of the disk about its axis. Since the mass of the disk is 86·2 lbs., and its radius of gyration 0·697 foot, I=41·9. Hence, in the above example, M′=5·05. To reduce this to M, the moment of the couple due to friction where the force is expressed in terms of the gravitation unit, we must divide by 32·2; hence M=0·157.

The value of this couple having been obtained, the coefficient of friction, μ, expressing the ratio of the force due to friction to the normal pressure, remains to be found. During the revolution of the disk the axle presses against one side and the bottom of each bearing in the manner shown in fig. 7. If P_v be the pressure against the bottom, P_h the pressure against the side, and W the weight of the disk, we have

$$\mathrm{P}_h=\mu\mathrm{P}_v$$

and

$$\mathrm{P}_v=\mathrm{W}-\mu\mathrm{P}_h=\mathrm{W}-\mu^2\mathrm{P}_v.$$

Hence

$$\mathrm{P}_v=\frac{\mathrm{W}}{1+\mu^2}\text{ and }\mathrm{P}_h=\frac{\mu\mathrm{W}}{1+\mu^2}.$$

The couple due to friction (M) is $\mu\mathrm{P}_vr+\mu\mathrm{P}_hr$, where r is the radius of the axle.

We have therefore a quadratic equation for determining μ:

$$\mu^2(\mathrm{W}r-\mathrm{M})+\mu\mathrm{W}r=\mathrm{M};$$

or, to put it in a form better suited for arithmetical work,

$$\mu^2+\mu\frac{\mathrm{W}r}{\mathrm{W}r-\mathrm{M}}=\frac{\mathrm{W}r}{\mathrm{W}r-\mathrm{M}}-1.$$

This equation has only one positive root.

It has here been assumed that μ is the same for both the places where sliding occurs. Even in cases where this assumption is not quite warrantable (as when the bearings are made of wood, and the motion is at the bottom in a plane perpendicular to the fibres, and at the side in the direction of the fibres), the amount of error so introduced into the determination of μ will be exceedingly small; for, since P_v is much greater than P_h, the value of μ, as above determined, is very approximately that corresponding to the bottom surface.

Substituting for W and r their numerical values, viz. W=86·2 lbs. and $r=\cdot004135$ foot, we have

$$\mu^2+\mu\frac{0\cdot3564}{0\cdot3564-\mathrm{M}}=\frac{0\cdot3564}{0\cdot3564-\mathrm{M}}-1.$$

In the example cited M=0·157; whence

$$\mu=0\cdot366.$$

The greatest and least values of the relative velocity of the sliding surfaces in this experiment are determined as follows:—The greatest value of Δs (Table I.) is 449 six-hundredths of a foot, and this distance was passed through by a point in the circumference of the disk in Δt, or 0·3571 second. The ratio of the radius of the disk to that of the axle is 238·4. Hence the greatest velocity of sliding between the circumference of the axle and the bearings is 0·0088 foot per second. The lowest limit of velocity of sliding down to which the observation can be said to extend depends, of course, upon how near to velocity $=0$ in the curve shown in fig. 6 the tangent can be drawn by which the acceleration is determined. In this case any change in the inclination of the curve could certainly be observed down to as low a velocity as that corresponding to $\Delta s=10$. This ordinate corresponds to a velocity of sliding of 0·0002 foot per second. Hence the above determination of μ extends from a velocity of 0·0088 foot per second, as the higher limit, down to 0·0002 foot per second, as the lower limit, and between these two limits the value of μ remains perfectly constant.

This limit of 0·0002, or *one five-thousandth of a foot per second*, as the least relative velocity of the sliding surfaces for which the determination of the coefficient of friction is definite, is approximately the same for all the experiments that follow. In every case the determination of μ is definite for as low a velocity as this. In some cases it is definite for even lower velocities. The higher limit of velocity to which the experiments extend varies in different cases, and is stated in each.

This exceedingly low limit of velocity could only be secured by making the diameter of the axle very small. As a result of this, the state of balance of the disk was rarely perfect. Although the disk was turned with the greatest care, so as to be truly cylindrical, it proved, when placed in its bearings, to be slightly heavier on one side than on the other. This irregularity was easily removed by applying a small counterpoise near the periphery; but we found that the state of balance so produced was not permanent. This was, no doubt, due to an almost infinitesimal bending of the axle from time to time. Although this irregularity could not be observed by the eye when the disk was revolving, it became apparent when the curve corresponding to fig. 6 was drawn, its effect on the curve being to introduce an undulation whose period was that of one revolution of the disk. Of course, if the friction was uniform throughout the movement, tangents drawn to the curve at corresponding phases in the successive undulations should be parallel. In this way the irregularity due to a want of balance in the disk was eliminated, and the friction determined throughout the whole curve. The example which has been given above is one of the comparatively rare cases in which the disk was for the time being in a state of sensibly perfect balance.

The cases examined were as follows:—Steel on steel, steel on brass, steel on polished agate, steel on greenheart, and steel on beech; in each case under the three different conditions, dry, oiled, and wet with water. The oil made use of was the fine oil employed by watchmakers. In order that no trace of unguent might be present when dry or wet surfaces were being examined, the oil was removed from the metallic surfaces by washing them with solution of caustic potash; while in the case of wood the friction was first determined when the surfaces were dry; then they were wetted with water, and the friction again measured; and, lastly, when the water had dried off, oil was applied. In every case the same specimens were used in all three determinations. When the bearings were of wood the fibres were vertical, and the coefficient of friction was approximately that corresponding to motion in a plane normal to the fibres. The following is a summary of the results:—

I. *Steel on Steel. Dry.*

1. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0057 foot per second . . $\mu=0\cdot337$
2. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0089 foot per second . . $\mu=0\cdot350$
3. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0086 foot per second . . $\mu=0\cdot365$

Mean . . . . $\mu=0\cdot351$

In this series, although the friction remained sensibly uniform during the time each experiment lasted, there was a progressive increase of the coefficient, probably due either to a tearing of the surface or to chemical action. The time that elapsed between the successive experiments was great relatively to the time taken up by any one of them, and a good deal of motion took place between the surfaces in each interval. We have observed several cases in which there is a similar progressive increase, but none in which there is a progressive decrease in the value of μ.

II. *Steel on Steel. Oiled.*

In this case the friction appeared to be somewhat *less* at the lower than at the higher velocities. This result is quite anomalous; we have observed no other instance in which the same thing occurs. The change, however, in the value of μ is not great, and it is possible that this result may be due to error of observation; we therefore state it with caution, although the different observations show a remarkably close agreement amongst themselves.

1. For velocity 0·0002 foot per second . . . . $\mu=0\cdot119$
   For velocity 0·0046 foot per second . . . . $\mu=0\cdot130$

Here 0·0046 foot per second is the greatest velocity observed.

2. For velocity 0·0002 foot per second . . . . $\mu=0\cdot116$
   For velocity 0·0060 foot per second . . . . $\mu=0\cdot130$

From this to the greatest velocity, 0·0065 foot per second, μ remains sensibly constant.

3. For velocity 0·0002 foot per second . . . . $\mu=0{\cdot}119$
For velocity 0·0043 foot per second . . . . $\mu=0{\cdot}130$

Here 0·0043 is the greatest velocity observed.

4. For velocity 0·0002 foot per second . . . . $\mu=0{\cdot}116$
For velocity 0·0040 foot per second . . . . $\mu=0{\cdot}127$

From this to the greatest velocity observed, 0·0059 foot per second, μ remains sensibly constant.

Mean value of μ at velocity 0·0002 foot per second . . 0·118
" " " about 0·0050 foot per second . . 0·129

It is to be observed in connexion with these values, and, indeed, in connexion with our experiments with unguents generally, that, owing to the very great intensity of pressure on the small bearing-surfaces of the axle, the unguent must have been to a great extent forced out, so as to leave the surfaces in the state described by MORIN as "unctuous."

III. *Steel on Steel. Wet with water.*

1. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0058 foot per second . . $\mu=0{\cdot}178$
2. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0052 foot per second . . $\mu=0{\cdot}205$
3. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0054 foot per second . . $\mu=0{\cdot}241$

Mean . . . . $\mu=0{\cdot}208$

The remarks made in Case I. apply here. On comparing these values with those for dry steel on steel, we see that the presence of water here diminishes the friction.

IV. *Steel on Brass. Dry.*

1. Friction very irregular, but apparently unaffected by velocity. From velocity 0·0002 foot per second to greatest velocity observed, 0·0059 foot per second . . $\mu=0{\cdot}180$
2. Very irregular. From velocity 0·0002 to 0·0053 . . . $\mu=0{\cdot}202$
3. Irregular. From velocity 0·0002 to 0·0056. . . . . $\mu=0{\cdot}202$

Mean . . . . $\mu=0{\cdot}195$

Although the friction was very irregular here, it seemed to be quite independent of the velocity. The coefficient is strikingly less than in the case of dry steel surfaces.

V. *Steel on Brass.* *Oiled.*

1. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0044 foot per second . $\mu=0{\cdot}146$
2. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0064 foot per second . $\mu=0{\cdot}146$

Mean . . . . $\mu=0{\cdot}146$

Here the coefficient appears to be entirely unaffected by change of velocity.

VI. *Steel on Brass.* *Wet with water.*

1. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0048 foot per second . $\mu=0{\cdot}106$
2. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0045 foot per second . $\mu=0{\cdot}102$
3. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0041 foot per second . $\mu=0{\cdot}107$

Mean . . . . $\mu=0{\cdot}105$

The coefficient here is remarkably low, indicating, when compared with the preceding, that, at least when the intensity of pressure is very great, water is a better unguent than oil for surfaces of steel and brass.

VII. *Steel on Polished Agate.* *Dry.*

1. Friction not very regular, but apparently independent of velocity. From velocity 0·0002 foot per second to greatest velocity observed, 0·0064 foot per second . . $\mu=0{\cdot}168$
2. Ditto. From velocity 0·0002 foot per second to greatest velocity observed, 0·0053 foot per second . . . . . . $\mu=0{\cdot}191$
3. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0065 foot per second . $\mu=0{\cdot}240$

Mean . . . . $\mu=0{\cdot}200$

The remarks made in Case I. apply here.

VIII. *Steel on Polished Agate. Oiled.*

1. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0054 foot per second . $\mu=0\cdot106$
2. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0041 foot per second . $\mu=0\cdot107$
3. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0054 foot per second . $\mu=0\cdot107$

Mean . . . . $\mu=0\cdot107$

IX. *Steel on Polished Agate. Wet with water.*

The determinations here are not perfectly definite, but appear to agree in showing that the friction increases slightly at very low velocities.

Taking the mean of three observations, we have

For velocity 0·0002 foot per second . . . . $\mu=0\cdot166$
For velocity 0·0060 foot per second . . . . $\mu=0\cdot146$

The greatest velocity observed was 0·0069 foot per second, and it is doubtful whether μ was not still sensibly diminishing at that velocity.

X. *Steel on Greenheart. Dry.*

1. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0058 foot per second . . $\mu=0\cdot212$
2. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0047 foot per second . . $\mu=0\cdot213$
3. Friction uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0064 foot per second . . $\mu=0\cdot221$

Mean . . . . $\mu=0\cdot215$

XI. *Steel on Greenheart. Oiled.*

In this case there was a very marked increase of friction as the velocity diminished. From a velocity of 0·0002 to about 0·005 foot per second, the friction gradually diminished, and at the latter velocity it appeared to have become nearly constant.

1. For velocity 0·0002 foot per second . . . . $\mu=0\cdot123$
   For velocity 0·005 foot per second . . . . $\mu=0\cdot094$
   Greatest velocity observed 0·0064 foot per second.
2. For velocity 0·0002 foot per second . . . . $\mu=0\cdot126$
   For velocity 0·004 foot per second . . . . $\mu=0\cdot098$
   Greatest velocity observed 0·0042 foot per second.
3. For velocity 0·0002 foot per second . . . . $\mu=0\cdot124$
   For velocity 0·005 foot per second . . . . $\mu=0\cdot097$
   Greatest velocity observed 0·0050 foot per second.

Mean value of μ for velocity of 0·0002 foot per second . . . 0·124
Mean value of μ for higher limit of velocity . . . . 0·096

This change in the value of μ seems to be far greater than experimental errors can account for. The three (independent) observations given above agree closely both as regards the initial and final values of μ.

The specimens of greenheart which were used in the above experiments were new, that is, their surfaces were freshly cut at the time these experiments were made. They were then laid aside without having the oil which had been put on them removed, and after an interval of six months the observation was repeated, fresh oil being applied. Two determinations were made, giving results which agreed closely in all respects. These results were

For velocity 0·0002 foot per second . . . . μ=0·077
For velocity 0·004 foot per second . . . . μ=0·062
Greatest velocity observed 0·0055 foot per second.

Here it is noticeable that although the prolonged exposure to oil had altered the bearing-surface of the wood so as greatly to diminish the coefficient of friction, the effect of change of velocity was the same as before. From velocity 0·004 foot per second upwards the coefficient underwent little or no appreciable change.

XII. *Steel on Greenheart. Wet with water.*

In this case there was an equally marked increase of friction at low speeds.

1. For velocity 0·0002 foot per second . . . . μ=0·290
   For velocity 0·005 foot per second . . . . μ=0·232
   Greatest velocity observed 0·0066 foot per second.
2. For velocity 0·0002 foot per second . . . . μ=0·292
   For velocity 0·005 foot per second . . . . μ=0·243
   Greatest velocity observed 0·0056 foot per second.
3. For velocity 0·0002 foot per second . . . . μ=0·290
   For velocity 0·006 foot per second . . . . μ=0·237

Mean value of μ for velocity 0·0002 foot per second . . . 0·291
Mean value of μ for higher limit of velocity . . . . . 0·237

The diminution of μ as the velocity increases amounts to about twenty per cent. of its value at the lower limit. This proportion is almost identical with that observed in the case of steel rubbing on oiled greenheart.

In both of the foregoing cases (XI. & XII.) in which this marked change took place in the value of μ, the experiments were at first made under the following conditions:—The oil (or water) was liberally supplied to the bearings just before the disk was caused to revolve, and the supply was not renewed during the revolution of the disk. It was suspected that the increase of μ might be due to the absorption of the unguent by the wood during the time that the disk was revolving. To test whether this was the case, the experiments were in both cases repeated, the supply of unguent being kept up continuously during the revolution of the disk. Precisely the same change in μ was observed under these circumstances as under the circumstances in which the experiments were at first made.

As a further confirmation of the result already stated for Case XII., the following determinations of μ were made with a new pair of greenheart bearings, but with the same steel axle.

4. For velocity 0·0002 foot per second . . . . $\mu=0·253$
For velocity 0·007 foot per second . . . . $\mu=0·181$
Greatest velocity observed 0·007 foot per second.

5. For velocity 0·0002 foot per second . . . . $\mu=0·262$
For velocity 0·0056 foot per second . . . . $\mu=0·203$
Greatest velocity observed 0·0056 foot per second.

6. For velocity 0·0002 foot per second . . . . $\mu=0·262$
For velocity 0·008 foot per second . . . . $\mu=0·207$
Greatest velocity observed 0·0106 foot per second.

Mean value of μ for velocity 0·0002 foot per second . . . . 0·259
Mean value of μ for higher limit of velocity . . . . 0·197

Here the change is equally marked; the diminution is rather more than twenty per cent. of the higher value. It will be observed that although in each set of determinations the separate observations agree well, the values obtained from the second set are considerably lower than those obtained from the first, no doubt on account of some specific difference in the bearing-surfaces. Taking a general mean from the two sets of observations, we have

Value of μ for velocity 0·0002 foot per second . . . . . . 0·275
Value of μ for higher limit of velocity . . . . 0·217

Under this set of conditions μ appears to undergo little change when the velocity exceeds about 0·006 foot per second.

XIII. *Steel on Beech. Dry.*

1. Friction perfectly uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0088 foot per second . . . . $\mu=0·366$
2. Friction perfectly uniform from velocity 0·0002 foot per second to greatest velocity observed, 0·0089 foot per second . . . . $\mu=0·367$

Mean . . . . $=0·3665$

This result differs from that of Case X. only in the magnitude of the coefficient.

XIV. *Steel on Beech. Oiled.*

The results obtained in the corresponding experiment with greenheart appear again here.

1. For velocity 0·0002 foot per second . . . . μ=0·126
   For velocity 0·005 foot per second . . . . μ=0·100
   Greatest velocity observed 0·0057 foot per second.
2. For velocity 0·0002 foot per second . . . . μ=0·126
   For velocity 0·005 foot per second . . . . μ=0·101
   Greatest velocity observed 0·0055 foot per second.
3. For velocity 0·0002 foot per second . . . . μ=0·126
   For velocity 0·0037 foot per second . . . . μ=0·110
   Greatest velocity observed 0·0037 foot per second.

In the last experiment it appears that the higher limit of the velocity was not high enough to enable μ to assume its minimum value. We therefore reject it in estimating the means, which are as follows:—

Mean value of μ for velocity 0·0002 foot per second . . . . 0·126
Mean value of μ for higher limit of velocity . . . . 0·101

The change in the value of μ occurs here below a velocity of 0·005 foot per second. The total change is almost exactly twenty per cent. of the higher value.

XV. *Steel on Beech. Wet with water.*

Here again we have an equally unmistakable increase of the coefficient of friction as the speed diminishes.

1. For velocity 0·0002 foot per second . . . . μ=0·344
   For velocity 0·007 foot per second . . . . μ=0·254
   Greatest velocity observed 0·007 foot per second.
2. For velocity 0·0002 foot per second . . . . μ=0·353
   For velocity 0·008 foot per second . . . . μ=0·290
   Greatest velocity observed 0·008 foot per second.
3. For velocity 0·0002 foot per second . . . . μ=0·370
   For velocity 0·0074 foot per second . . . . μ=0·292
   Greatest velocity observed 0·0074 foot per second.

The curves for these observations connecting μ with the velocity indicate pretty clearly that the greatest velocity observed was insufficient to bring μ to its least value. Taking, however, the means of the observations as they stand, we have

Mean value of μ for velocity 0·0002 foot per second . . . . 0·356
Mean value of μ for highest velocity observed . . . . 0·279

A comparison of all the above results shows that, omitting the doubtful cases of oiled steel (II.) and wet agate (IX.), the coefficient of friction remained entirely unaffected by changes in the velocity except where the bearings were made of wood, and that even in these circumstances no change of the coefficient could be detected so long as the surfaces were dry; but when oil or water was present the coefficient of friction increased in a very marked manner as the velocity diminished, this increase of the coefficient of friction taking place under a limit of velocity of about 0·01 foot per second. It is to be observed that the cases in which this increase of the coefficient at low speeds occur are precisely those in which COULOMB and MORIN have found that there is a very marked difference between the static and kinetic values of the coefficient of friction. In the case of dry metal surfaces both these writers are agreed that no difference can be detected between the friction of rest and that of motion; and we find that in such cases no change takes place in the value of μ as the velocity varies. MORIN also found that the friction between unctuous metallic surfaces was the same or nearly the same for rest as for motion, a result which agrees with the absence of almost any change of the coefficient of friction in our investigation of Cases II. & V. The friction between wet metallic surfaces is not spoken of by COULOMB, and received comparatively little attention from MORIN, who does not say whether he found any difference between its static and kinetic values. We have not seen any account of experiments on the friction between metals and agate. As to the remaining sets of conditions, MORIN says that he found no sensible difference between the static and kinetic values of the friction of dry metals on wood (Mémoire i. pp. 104, 106). In fact, out of all the cases which we have examined, the only ones in which there is known to be a marked difference between the friction of rest and that of motion are those in which steel slides on oiled or wetted surfaces of wood, and in these cases, and these only, we have detected a very considerable increase in the coefficient of friction at very low speeds.

Although the varying want of balance of the disk, which has been already referred to, was never so great as to throw any doubt on the fact that the rate of acceleration did change in these particular cases (and we are unable to see any cause for the change of the rate of acceleration except a change in the coefficient of friction), still it was considerable enough to make the determination of the precise way in which the coefficient changed with changes in the velocity a matter of the greatest difficulty. We have been able to draw only very roughly curves connecting the coefficient μ with the relative velocity of the sliding surfaces. These all agree in showing that $\frac{d\mu}{dv}$, or the rate of change of the coefficient of friction relatively to change of velocity, becomes greater as the velocity becomes less. This being so it is perfectly possible that at velocities below 0·0002 foot per second, which is the lowest limit to which our observation may be taken as extending, a considerable further increase of the coefficient may take place before motion entirely ceases.

It is in fact highly probable that in those cases in which the static coefficient of friction is greater than the kinetic (that is, the coefficient which is observed when the surfaces are moving at a moderate speed), the latter gradually increases when the velocity becomes extremely small, so as to pass without discontinuity into the former. The experiments of COULOMB show that the friction between surfaces at rest is itself not constant, but increases as the time of rest is prolonged. It seems doubtful whether this result is due to a real change in the static coefficient or not; but if it is we may suppose that not only does the coefficient of friction increase continuously as the state of the surfaces changes from motion to rest, but continues for a time to increase after the latter state has been reached. To prove this with certainty is probably impossible, both because of the difficulty of observing the rate of retardation of moving surfaces down to the point at which motion wholly ceases, and because of the impossibility of measuring the static coefficient between surfaces which have not been in contact for a finite length of time; but the results of the experiments which have been stated above seem to give a strong colour of probability to the hypothesis.

The friction of steel on greenheart wetted with water (Case XII.) has special interest. MORIN has endeavoured to explain the excess of the friction of rest over that of motion, by supposing that the unguent present is more or less expelled when the surfaces are at rest. This view may perhaps in some cases be correct, but it is certainly not always tenable. With wetted steel and greenheart surfaces the static coefficient is very much greater than the kinetic, although the presence of water has an effect the reverse of that of an unguent (compare X. & XII.). It seems impossible that the expulsion of what has the effect of increasing the friction should give rise to a further increase.

We found that our apparatus was not suited for determining the value of the friction between surfaces at rest. Attempts were made to use it for this purpose, but the results were not satisfactory. The static coefficients showed a fair general agreement with those given above; the most marked divergences appeared in the cases where the kinetic values increased at low speeds. In these the static values of the coefficients were considerably greater than even the greatest kinetic values.

In making practical deductions from experiments on friction, it is to be observed that the phenomena are so dependent on apparently insignificant variations of the conditions of the observation that it is hazardous to state, for the guidance of the engineer, results which are obtained under conditions greatly different from those met with in practice. We therefore only draw attention to two points, both of which seem to be novel:—(1) The excellence of greenheart, whether dry or oiled, as a material for bearings; and (2) the very small friction between steel and brass when the surfaces are wetted with water.

We regret that the laboriousness of the measurements and calculations which have had to be made in connexion with each observation has prevented the number of cases examined from being more numerous than they are. For the same reason, the experiments, of which the above is an account, although for the most part made in the summer of 1876, have only now been prepared for publication.

Table II. gives a synopsis of the results. In Column IV. the value of μ is given, corresponding to the lowest velocity to which the observations extended. Columns V. and VI. show the change in the value of μ wherever any was observed, Column V. giving the changed value of μ, and Column VI. the velocity at which the change appeared to be sensibly complete. Columns V. and VI. are left blank wherever no change was detected in the value of μ from the least velocity up to the greatest velocity observed, which is given in Column VII.

TABLE II.

I. Number.	II. Surfaces.	III. Condition.	IV. μ at velocity 0·0002 ft. per sec.	V. μ changes to	VI. At velocity (ft. per sec.).	VII. Greatest velocity to which observation extends (ft. per sec.).	VIII. Remarks.
I.	Steel on Steel.	Dry.	0·351	......	......	0·0089	
II.	,, ,,	Oiled.	0·118	(?) 0·129	0·005	0·0065	Condition of surfaces rapidly changing, so as to increase μ.
III.	,, ,,	Wet with water.	0·208	......	......	0·0058	
IV.	Steel on Brass.	Dry.	0·195	......	......	0·0059	Very irregular.
V.	,, ,,	Oiled.	0·146	......	......	0·0064	
VI.	,, ,,	Wet with water.	0·105	......	......	0·0048	
VII.	Steel on Agate.	Dry.	0·200	......	......	0·0064	Condition of surfaces rapidly changing, so as to increase μ.
VIII.	,, ,,	Oiled.	0·107	......	......	0·0054	
IX.	,, ,,	Wet with water.	0·166	(?) 0·146	0·006	0·0069	
X.	Steel on Greenheart.	Dry.	0·215	......	......	0·0064	
XI.	,, ,,	Oiled.	0·124	0·096	0·005	0·0064	
	,, ,,	,,	0·077	0·062	0·004	0·0055	After prolonged exposure to oil.
XII.	,, ,,	Wet with water.	0·275	0·217	0·006	0·0106	
XIII.	Steel on Beech.	Dry.	0·366	......	......	0·0089	
XIV.	,, ,,	Oiled.	0·126	0·101	0·005	0·0057	Change in μ still going on at greatest velocity observed.
XV.	,, ,,	Wet with water.	0·356	0·279	0·008	0·0080	

XV. *On the Structure and Development of the Skull in the Urodelous Amphibia.*—Part I.

By William Kitchen Parker, *F.R.S.*

Received November 9,—Read November 23, 1876.

[Plates 21–29.]

Introductory Remarks.

Before submitting to the Royal Society a *third* paper on the Skull of the Batrachia, it would seem to be better for me to bring forward the results of my work on one of the simpler kinds of Amphibia, namely the Axolotl, a type belonging to the "Urodela."

Through the kindness of friends* I have been enabled to follow the Axolotl through many stages, and several other types have been worked out both in their larval and adult stages; some of these fill up lacunæ in the series of Axolotls.

That type, however, is here given in *nine* larval stages, besides the *Amblystoma* stage into which certain individuals pass.

Ten or twelve stages are not at all more than are needed for following up and catching every important modification in the skull of a Vertebrate no higher than a Salamander —a type which, practically, stands between the "Pisces Dipnoi" and the true Reptiles. Moreover this leaves a large fore margin to the embryologist proper, whose most important work dovetails into mine where mine begins.

To the morphologist the group of the Urodela is of great interest, first in its relations to the Batrachia, the Cæcilians, and the extinct Labyrinthodonts, and then to the more generalized Fishes below and the Reptiles above.

And not only with the Reptiles, for the more reptilian types of Birds, such as the Dinornis, the Emu, and the Cassowary, have skulls that are easily interpreted by one who is familiar with the Salamandrian skull.

But perhaps the most instructive comparisons are those which may be made of the skull (chondrocranium) of the tailed Amphibia with that of the Elasmobranch Fishes; and, besides my own researches into the development of the skull in *Raia* and *Scyllium* (now being published), I gladly avail myself of the labours of Professors Gegenbaur and Huxley in that and related groups†.

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* In these researches I have been most liberally supplied with materials by the undernamed scientific friends, namely—Messrs. Alex. Agassiz, Flower, Günther, Mivart, Murie, and, above all, Tegetmeier; my specimen of *Proteus anguinus* belonged to the late Henry Christy, Esq., F.R.S., and was given me by his brother, Edmund Christy, Esq.

† The work of the former of these excellent authors here referred to is his 'Untersuchungen zur verglei-

I am also greatly indebted to Professor HUXLEY for his paper on *Menobranchus* (Proc. Zool. Soc. 1874, pp. 184–204, plates 29–31), and his article on the *Amphibia* in the last (ninth) edition of the 'Encyclopædia Britannica.'

But those unspeakably important labours that dovetail into the front margin of my own work are now being carried on by a native worker of the highest ability: I refer to FRANCIS M. BALFOUR, Esq., M.A.*

The reader is referred to the following memoirs, namely:—FRIEDRICH and GEGENBAUR, "Der Schädel des Axolotl (*Siredon pisciformis*)," in the 'Berichte der königlichen zootomischen Anstalt zu Würzburg,' 1849; Dr. ROBERT WIEDERSHEIM on *Salamandrina* [*Seironota*] *perspicillata* (Würzburg, 1875), and his exquisite work, 'Das Kopfskelet der Urodelen' (Leipzig, 1877).

In working out the skull of these forms, then, I have had much more light and help than in beginning the Batrachian skull, besides personal experience considerably enlarged, and new means of research that I have learned from younger labourers†.

In the present paper my endeavour has been to bring into view all the morphological changes or metamorphoses undergone by a tailed Amphibian. But as the Axolotl (*Siredon*), in its occasional metamorphosis into a caducibranchiate Salamandrian, does not even in that give a full measure of the changes undergone by this group, I have added another type, namely *Seironota*.

Moreover the larva of *Seironota* is very instructive, and fills up a lacuna left among my earlier stages of the Axolotl.

Then, as the Perennibranchs are comparable to the larvæ of the Caducibranchs, I have given here the structure of the skull in the lowest type of the former group, namely, that of *Proteus anguinus*—a type in which the form of the Fish is very thinly veiled, and whose relationship to both the Dipnoi and other generalized fishes is evident and unmistakable.

It is in the Amphibia, both tailed and tailless, that the axial elements of the skull are combined with olfactory and auditory capsules that are beginning to show metamorphic modifications, which by plain and evident steps lead us onward and upward, as by a guiding hand, towards the almost untranslatable complexity seen in these organs in the higher types.

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chenden Anatomie der Wirbelthiere' (Leipzig, 1872). Professor HUXLEY's work in this department, here especially referred to, is a paper on the skull of *Ceratodus forsteri* (Proc. Zool. Soc. 1876, pp. 24–59).

* Besides Mr. BALFOUR's part in FOSTER and BALFOUR's 'Elements of Embryology' (London: Macmillan, 1874), I would refer the reader to his following invaluable memoir, 'A Preliminary Account of the Development of the Elasmobranch Fishes' (London: J. C. Adlard, 1874), reprinted from the 'Quarterly Journal of Microscopical Science,' Oct. 1874. Further papers by Mr. BALFOUR on this subject will be found in 'The Journal of Anatomy and Physiology,' vol. x. pp. 517–570 and 672–688. Also by the same "A Comparison of the Early Stages in the Development of Vertebrates," in 'Studies from the Physiological Laboratory in the University of Cambridge,' pp. 1–20 (Cambridge, 1876).

† It is due to my son, Mr. T. J. Parker, to state that in modes of colouring objects, and in making fine sections, his help has been of great value to me.

These things relate to extant and familiar forms; but types like the Urodela, that possess external branchiæ (all for a time, and many persistently) and that never have internal branchiæ, must, it appears to me, be related to groups of extinct forms of low vermiform kinds of Fish, far below any known brain-bearing fishes now existent.

Indeed to me the interest attached to every stage, stage by stage, in these metamorphosing *Ichthyopsida* has been greatly enhanced by the supposition that each of these grades shows me a pattern according to which family after family of the finny tribes was developed—modified, undoubtedly, in these forms by exoskeletal structures that gave the members of each group a perfect fitness for their own conditions of life.

I feel that I need only suggest this view of the matter for it to be accepted by the thinking zoologist and anatomist; the contemplation of such forms as I figured in my last paper, namely, the larvæ of *Dactylethra*, and the larvæ here figured of *Siredon*, unconsciously set the imagination to work in an attempt to pass the great gulf at present fixed between the brainless *Amphioxus* and the lowest brain-bearing Fishes extant, such as the Hag-fish and the Lamprey.

That, however, is not the primary purpose of the work of which the present paper forms a part; the goal aimed at is the interpretation of the skull as part of the Vertebrate skeleton. This, in detail, means to ascertain what relation the cranium bears to the vertebral column, the visceral arches to the ribs, and the subcutaneous cartilages to those superadded arches of the body that sustain the limbs.

The morphological space traversed by a Salamander or Newt in passing from the first fixation of the cartilaginous beams and bars that form its first chondroskeleton, and especially its first chondrocranium, to its adult state is immense; the first grade shows the framework of a fish lower by far than any save the Lancelet, and the last lands us among forms that lie on the border of the true Reptilia.

Here, amongst these types, must the morphologist search for the *alphabet*, and, as the mind learns to arrange the facts, synthetically construct the *grammar*, of his science.

To a great extent the writer has studied the relations of both the nervous and muscular systems as correlated with the skeletal; here, however, he needs much friendly help; that help has already been given in some degree, but most of the work has yet to be done.

On the Skull of the Axolotl.—First Stage. Unhatched Embryos, one fourth of an inch long.

The larva of *Siredon* at this stage is entirely (or almost entirely) composed of cells; but its development is, embryologically, far advanced, for in the head, at least, the greater number of the organs can be made out.

Indeed in the specimen figured, which has been preserved for three or four years in weak chromic-acid solution, very much that is highly instructive could be seen from the exterior; views of these are here given from the side, from above, and from below (Plate 21. figs. 1–3).

A sectional view (fig. 4) throws much further light upon this stage; and altogether we see the condition of the neural axis, the skeletal axis (notochord), the paired sense-capsules, the developing visceral arches and branchiæ, and the opening visceral clefts.

The section (fig. 4) not being exactly in the mid line, only exposes the pharyngeal or branchial region of the alimentary canal, which, however, was developed at a much earlier period; the oral passage is still incomplete.

My work here yokes on to that of the embryologist; the stage which immediately precedes this, as well as earlier stages, can be studied in GÖTTE's magnificent work on *Bombinator igneus*, for the development of the Amphibian embryo, but still more instructively in one of Mr. BALFOUR's papers.

In the paper here referred to ('Studies from the Physiological Laboratory,' pp. 1–20, pl. 1: Cambridge, 1876) the embryo of the Elasmobranch is compared with that of the Amphibian.

The third stage of the Amphibian embryo (fig. 3 C, p. 455) is that which comes in directly before my first morphological stage; this and the other two (A and B) are given by Mr. BALFOUR as "diagrammatic longitudinal sections of *Bombinator igneus*, reproduced with modifications from GÖTTE."

This variously shaded woodcut shows what part the three main layers of the blastoderm take in the formation of the embryo. In the earliest stage (3 A) there was a segmentation cavity, but no alimentary cavity. In the second (3 B) the alimentary cavity is formed, but is closed at both ends, and the segmentation cavity is disappearing. In the third (3 C) the latter has disappeared, and the alimentary canal is open behind; the neural canal opens into it close to this hinder opening, which is the "anus of RUSCONI;" it does not correspond with the proper or permanent anal opening*.

In the third embryological stage the embryo is longer than the diameter of the yolk, which has become elongated and flattened, and is enclosed in a layer of epiblast cells, the future epidermis. Both the head end and the tail end of the embryo, which is like an inverted boat, project beyond the yolk-mass, and, indeed, this mass is now enclosed by a layer of mesoblastic cells inside those of the epiblast. The cephalic and caudal ends of the embryo are curved downwards; and there is a layer of mesoblastic cells both above and below the neural axis, which is swollen where the cerebral vesicles will soon appear. There is no mouth, but at the part below the cephalic end of the embryo the mesoblastic layer is deficient. At this part the closed alimentary cavity is very large; it is roofed throughout by hypoblast, which becomes developed into epithelium.

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* Of the "blastopore" or anus of RUSCONI, Mr. BALFOUR says:—"This is the primitive opening by which the alimentary canal communicates with the exterior, or, in other words, the opening of the alimentary involution. It is a distinctly marked structure in *Amphioxus* and the Batrachians, and is also found in a less well-marked form in the Selachians; in Birds no trace of it is any longer seen. In all those Vertebrates in which it is present, it closes up, and does not become the anus of the adult. The final anus nevertheless corresponds very closely in position with the anus of RUSCONI." ('A Comparison of the Early Stages in the Development of Vertebrates,' pp. 18, 19.)

This must suffice as an introduction to my first morphological stage; but the papers just quoted should be studied before the details here to be given are gone into.

My drawings only reach to the middle of the yolk-mass, a large quantity of which has not yet been used up.

The head is bent down at a right angle by the "mesocephalic flexure." The hind brain (C^3) is long and lanceolate, its direction is straight. The mid brain (C^2) is large and rounded; it is *frontal* in position, forming the actual end of the embryo. The fore brain (C^1) is egg-shaped; it is half the size of the mid brain, and its direction is downwards, with its front margin considerably behind the front of the mid brain. The upper half of the huge hind brain is occupied by the vesicle of the fourth ventricle, and the cranial roof over this part is very thin, showing the cavity in the external views (figs. 1 & 2). These vesicles and their contents are epiblastic products; between them and the epidermis (also epiblastic) there is a stratum of mesoblast which will become differentiated into "cutis vera," cranium, brain-membranes, and vessels.

Rudiments of the chief organs of sense are now visible: the foremost pair are the nasal sacs; they are now circular elevations of the skin, with a shallow, saucer-like depression in the middle; each rimmed rudiment is placed on the hinder part of the lateral depression between the hind and fore brain. Correlated with the budding of the "optic vesicle," right and left, from the fore brain is an elevation of the outer structures; a mass of cells in shape like a kidney, with its notch looking downwards and backwards, is placed in the lower part of the valley which runs between the hind and mid brain. This rough shetch as it were of the outer part of the eyeball shows that the lens is only partially developed.

A very similar mass of cells, but rounder, is seen on the side of the hind brain; at its middle this rounded mass has an imperfect rounded opening above; this is the ear-sac (*au.*), and the opening is the primary involution or "aqueduct."

From the outside, also, we can see the rudimentary, facio-branchial arches, with the commencements, between them, of the visceral clefts, and in front of them, below, that azygous, transverse dehiscence of the facial floor which becomes the "deuterostomatous" mouth.

The mandibular and hyoid arches are marked out externally (figs. 1 & 3, *mn.*, *hy.*); but the arches that follow—the branchials—are largely hidden by the opercular curtain which grows backwards from the hind edge of the hyoid arch (*op.*).

In the side view (fig. 1) the upper part of the mandibular arch is seen as an arched thickening of the facial wall, below and behind the eye, and having a backward bend. In the recess between the thickenings for the eye and the nose there is a small elevation; this is all that is seen at present of the "maxillary rudiment." In the lower view and in the section a band of skin is seen on the front margin of the oral region, *behind* the fore brain; this is the "naso-frontal process"; it becomes the upper lip, but acquires no labial cartilages; ultimately it will have growing into it the fore ends of the trabeculæ in the middle, and laterally their out-turned cornua; these do not exist at present.

Behind the hyoid arch, and partly hidden under its opercular fold, there are, on each side, three dermal vegetations; the first of these is lowest in position, and the last is highest. These are the rudimentary "external branchiæ;" they are decurved, claw-shaped buds, the terminal claw of which is ensheathed in a thick base, which itself gives off two smaller buds externally: thus we have here the first *three* of the branchial filaments.

If we look at the ventral surface of the throat (fig. 3, *cl.* 3–5) we can see three gaping spaces where the skin has undergone dehiscence, and these correspond to the fossæ seen on the inner side of the pharynx (fig. 4, *cl.* 2–5); these clefts are lined on the inside with hypoblast, and on the outside with epiblast, these two laminæ meeting here at these spaces, as in the early "blastopore." The first cleft (figs. 1 & 4, *cl.* 1) is a very small, roundish, thin space, which, as far as I can see, never becomes quite open either in Urodeles or Anura. This cleft is the first to appear in the Shark according to Mr. BALFOUR (Develop. of Elasmobranch Fishes, p. 539), who shows it in his stage "G." "The alimentary canal in the region of the head exhibits on each side a slight bulging outwards, the rudiment of the first visceral cleft. This is represented in the figure (plate 24. fig. G) by two lines (fig. 1, *v.c.*). The visceral clefts at this stage [a stage corresponding to the one next before my first] consist of a pair of simple diverticula from the alimentary canal, and there is no communication between the throat and the exterior."

The sectional view of the Axolotl embryo shows this diverticulum with three or four behind it; it is a temporary structure in the Urodele, but becomes the tympano-Eustachian cavity in the Batrachia, and is persistently open outside in the Elasmobranchs as the "spiracle;" the "annulus" of the Frog is the counterpart of the spiracular cartilage of an Elasmobranch, which is generally a single "visceral ray," corresponding to the numerous rays on the branchial arches*.

The hyoid and mandibular arches are nearly of a size, and they both project forwards where the two halves meet below; the latter projects most, and in front of this "chin" there is a transverse fossa, somewhat in the form, as to outline, of an hourglass. The section (fig. 4, *m*) shows that dehiscence of the ventral wall has taken place to some degree, and the naso-frontal process forms a "rest" for the mentum. If the lips are parting, the mouth-cavity is still considerably occluded by a mass of cells, that, in this

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* In GEGENBAUR's work on the skull of the Elasmobranchs, the "spiracular cartilage" is shown to be single as a rule; in *Scymnus* (plate 11. fig. 1, *kr.*) there are *two*, and in *Centrophorus calceus* (plate 12. fig. 1, *kr.*) there are *three*. Now my own drawings show *four* short external branchial filaments in the spiracle of an embryo of *Scyllium canicula* (which was nearly an inch in length); there can be therefore no difficulty about the homology of the ray or rays growing from that arch as tending to form an operculum to these temporary gills.

In the Batrachia, where the first cleft is much more developed than in the Urodela, although it does not open externally, this mandibular branchial ray, or opercular cartilage, reappears, becoming first crescentic and then circular, and serves for an "annulus tympanicus" in these tribes, whose external auditory apparatus becomes on a sudden, as it were, so highly developed.

bent condition of the head, lie below the apex of the notochord, behind the pituitary floor, and above the cleaving lips*.

The notochord (Plate 21. fig. 4, *nc.*) reaches the pituitary body by its sheath; it is turned downward, and this deflection takes place at both ends (see GÖTTE's and BALFOUR's plates). At this stage all obscurity as to the anterior termination of the notochord has vanished, and the solid chromic-acid preparation, whose section is here figured, gave precisely the same results as similar sections of early embryos of the Salmon (see Phil. Trans. 1873, plate iii. figs. 3 & 5, *nc.*). In both these types the watery tissue of the notochord ceases close behind the pituitary body; but its dense sheath passes forwards and upwards beyond the hinder margin of the pituitary body and infundibulum. This Axolotl embryo is at an earlier stage than those of the Salmon, and its notochordal sheath wedges in between the hind and mid brain.

This empty sheath is suddenly folded, as it were, upon itself, and the lower or returning layer develops a cup-shaped process at right angles with the notochord, which forms a well-fitted "rest" for the globular pituitary body (*py.*). This scooped plate, in turn, rests against a loose mass of mesoblastic cells, which are evidently parental to the stroma which afterwards ossifies to form the parasphenoid.

Thus, notwithstanding that the cephalic end of the notochord is, like the caudal end, turned downwards, it seems to seek to grow through the neural region to the upper wall of the head nearly parallel with an ideal line passing through the pituitary and pineal rudiments (*pn.*, *py.*).

The apex of the notochord first straightens, and then acquires more or less of an upward curve. I see this in the embryo of *Scyllium* (11 lines in length), and it is shown in my figured sections (the third, fourth, and fifth) of the embryo Salmon's head. But this is shown very remarkably in the hot-blooded types, as the Fowl and the Pig ("Fowl's Skull," plate lxxxi. fig. 3, *nc.*; and "Pig's Skull," plate xxviii. fig. 6, *nc.*).

I am careful to notice the direction taken by this axial and most fundamental part of the skeleton; around its cranial end cluster most of the difficulties that beset the labourer in this field†.

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* These cells evidently correspond to those described by Mr. BALFOUR as obscuring the view of the notochord in his stage "G" of the Selachian embryo; and as I must now describe the notochord of the Axolotl, it will be profitable to give his observations on that of the Shark. "Another structure which became developed in even a younger embryo than 'C' is now for the first time visible in the living embryo. This is the notochord: it extends from almost the extreme posterior end of the embryo. It lies between the ventral wall of the spinal canal and the dorsal wall of the intestine; and round its posterior end these two walls become continuous with each other (plate 24, G). Anteriorly the termination of the notochord cannot be seen; it can only be traced into a mass of mesoblast at the base of the brain, which there separates the epiblast from the hypoblast." This cell-mass is seen coloured rusty red in the figure of the embryo of *Bombinator igneus* ('Early Stages in the Development of Vertebrates,' plate 1. fig. C 3), and is black in the woodcut in the paper on the Elasmobranchs (p. 545, figs. B, C). With these papers and descriptions before him the reader will much better be able to follow my description and to understand the figure (4).

† The ventral ends of the non-chondrified visceral bars are coloured (fig. 4) to assist the eye; they are becoming quite distinct and solid: below the hinder of these, and in front of the unused yolk-cells, the rudimentary heart (*h*) is seen; it is partly subdivided by a constriction.

The latest observations that I have seen on the origin of the notochord are those made by Mr. BALFOUR (Dev. of Elasm., Journ. of Anat. and Phys. vol. x. p. 682); these quite corroborate his former assertions, namely, that it is developed from the "hypoblast." I must refer the reader to his paper for the proofs of this, and for the views of other anatomists, and proceed to describe the morphology of the Axolotl's skull in the soecnd stage.

Skull of Axolotl.—Second Stage. Embryos unhatched, and measuring from 4 to 4½ lines in length.

In embryos one fourth longer than the last (Plate 21. figs. 5–7) a great change has taken place; the mesocephalic flexure is fast vanishing, and the mouth, now cleft through, is nearer the fore part of the head.

The hind brain (C^3) is now more perfectly roofed in, and is relatively much smaller; the mid brain (C^2) projects but little over the fore brain (C^1).

The mid brain is now symmetrical on its upper surface, being equally divided into two crescentic elevations, placed back to back; these developments in the roof of the mid brain are the optic lobes*.

The azygous rudiment of the cerebral hemispheres now lying near the fore front of the head, the part attached to the hinder margin of the investment of this vesicle is of necessity brought near to the front: this part is the naso-frontal process (*f.n.p.*), and is the rudiment of the upper lip and of the parts within and behind it.

On each side of this naso-frontal selvedge the olfactory rims (*ol.*) are more marked out, but they still retain their place below the head as in Selachians.

The eyeballs (*e*) are almost formed, a fissure existing where there was an open "hilus" before; the distinctly globular ear-sacs (*au.*) still show the primitive opening or involution.

The fore part of the lower lip, and its contained mandibular rudiments, is projected forwards, and being broad makes the mouth to be a transverse cleft, as in the Selachians: but there is a considerable lateral extension of the opening; this is overhung by large labial growths from the so-called "maxillary rudiments;" they resemble the "flews" of a hound.

The elevations caused by the enclosed mandibular and hyoid bars, which did stoop forward above, now lean backward; this direction increases more and more, and then

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* BALFOUR, 'Development of Elasmobranch Fishes,' p. 563. At p. 560 the round fore brain is spoken of as the "impaired rudiment of the cerebral hemispheres;" from these the "optic vesicles," the essential part of the eyeballs, arise. In a note to p. 560, Mr. BALFOUR gives another account of the origin of the "thalamencephalon" to that in Prof. HUXLEY's 'Elements,' where (p. 131) this part, "the vesicle of the third ventricle," is said to arise from the posterior part of the fore brain. The note runs as follows:—"The part of the brain which I have here called mid brain, and which unquestionably corresponds to the part called mid brain in the embryos of higher Vertebrates, becomes in the adult what MIKLUCHO-MACLAY and GEGENBAUR called the vesicle of the third ventricle or thalamencephalon. I shall always speak of it as the mid brain."

lessens; the Urodela, however, go through less of this change of direction than the Batrachia*.

The first two postoral bars, with their enfolding skin, help to make the head very large relatively; and the opercular vallance (*op.*) is now very large, and conceals the lesser or branchial arches to a great degree.

Each of these gill-arches has one gill (one on each side), and these single gills sprout out from the arch higher and higher up.

The opercular fold receives two lesser plaits laterally; for the first gill-arch partly overlaps the second with its gill, and this in turn overlaps the third arch with its gill; the fourth arch is barren.

These three-knobbed, sprouting gills now most resemble the hand of the Aye-Aye (*Cheiromys*), save that they only bear three fingers. This tridigitate condition is not long kept; for the appearance of new buds will result in the production of an elegant branchial feather, of great length; but no new suckers will grow out from the outer edge of the arch, and none appear further in.

The embryos, at this stage, are becoming lively, and soon acquire considerable consistence of their skeletal structure, with their fairly differentiated muscular segments: chondrification has begun.

But the youngest embryo in which it has been possible to dissect and display the nascent skeletal structures was half a line longer than the one whose portrait I have given and described (Plate 21. figs. 5–7). This larger embryo was 4½ lines long (Plate 22. figs. 1, 2). Professor HUXLEY's illustration (Proc. Zool. Soc. 1874, plate xxxi. fig. 1) is from the study of Axolotls one line longer (5½), and corresponds with my next (*third*) stage†.

The results given in the two figures (Plate 22. figs. 1, 2) were obtained from an embryo that had been hardened in absolute alcohol. It was scalped, washed, cleared of all loose tissue, especially brain-cells, stained with carmine, and mounted in glycerine.

The free mandible is given in both figures, but the other arches only in fig. 2, which was drawn as an upper view also, from an object made by slicing away the whole of the lower face. These arches must be conceived of as existing in the object shown in fig. 1, but hidden beneath the basicranial structures.

This is the simplest and most fundamental vertebrate skull I have been able as yet to demonstrate and depict; nor have I seen any thing figured by other labourers that displays so many of the unopened buds, so to speak, that afterwards expand and grow into the complex cranium and face.

Only the inferior arches are chondrified; the corpuscles of the cartilage are

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* In that group the mandibular pier has at first its direction downwards and backwards, as in the *first stage* of the Axolotl; it soon becomes nearly *horizontal*, and then, during metamorphosis, swings backwards again, regaining, in the adult, the direction it had during the existence of the mesocephalic flexure.

† They were bred by me, and we both worked from the same source. The smallest I supplied my friend with were nearly half an inch long.

large, placed crosswise, and the bars are so small and thin that each cell can be counted.

The only arch that is subdivided is the mandibular, the pier or upper part of which is still granular, indifferent tissue; so also are the trabeculæ (Plate 22. fig. 1, *q*, *tr.*).

There is no cartilage in any of the sense-capsules; the whole cranial cavity is enclosed by a "membrano-cranium," the first morphological stage of this important chamber.

I was not able to work out the cranial nerves; but safe landmarks were found in solid masses of nerve-cells, from which were developing the Gasserian ganglion of the fifth, and, not distinct from it to my view, the geniculate ganglion (5, 7); whilst behind and mesiad of the auditory sac (*au.*) there is the large ganglion of the glossopharyngeal and vagus nerves.

As this specimen was in advance of that whose outward appearance has been just described, the head had become almost straight again; hence the three cerebral regions are now in almost one plane.

Yet the fore brain (C^1) is in front of the mouth (*m.*); the mid brain (C^2) behind both mouth and eyes (*e.*). The floor of the hind brain has all been removed to show the subjacent structures.

The most important of these is the notochord (*nc.*), the dip downward of whose apex is not shown in the figure, nor the ascending process of its sheath (see Plate 21. fig. 4, *nc.*).

Of this massive notochord two thirds belong to the skull and one third to the neck. The two first pairs of muscular segments belong also to the head, but the occipital arch is mere membrane. The investing mass or parachordal cartilages, right and left, will be best seen in my fourth stage; but this notochord is embraced by the trabeculæ, and full half of the cranial notochord will be enveloped by cartilage which is the hinder half of the trabeculæ.

These small, non-cartilaginous trabeculæ are attached, like the horns of an ox, to the fore end of the notochord; they are rounded in front and flat behind. The space whose sides are enclosed by these little rods is not the mere counterpart of the proper "pituitary space;" it is the outline of the vesicle of the third ventricle (or thalamencephalon) which is here marked out. The infundibulum has been cut away from the pituitary body (*py.*), which takes up but little of the room embraced by the trabeculæ. Hence we see that the sudden arrest at this part of the axial structures has arisen from something more than the suspension of the pituitary body; the vesicular mid brain has been a still more important factor.

The morphological meaning of the modified, stunted, non-segmentary axis, and especially of the cartilaginous rods that grow forwards in front of the notochord, is still an unsolved mystery*.

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* The anterior part of the developed trabeculæ for several years seemed to me to be visceral or *pleural*; the base of the septum nasi, the cornua trabeculæ, and the prenasal cartilage, together, appeared to be the foundation of the "intermaxillary arch." (See "Ostrich's Skull," Phil. Trans. 1866, p. 122.)

All that can be done here is to show what parts grow from, and what parts are attached to, these rods. The Axolotl appears to me to be an excellent subject for this purpose.

The fore ends of these bars do not reach quite to the front of the vesicle they embrace, and a space equal to their length divides them in front. Looked at in this view, they do not suggest the idea of serial relationship to the mandibles. A side view (fig. 3, beginning of next stage) lends itself better to such a theory of their nature*.

At present there is nothing that can be satisfactorily determined as to *preoral* visceral arches; but the postoral arches present no difficulty. The first of these, the mandibular, is very large relatively; it is the principal element that goes to form the gates and bars of the face and mouth. The pier of this inverted arch (fig. 1, *q.*) is still mere granular indifferent tissue; it is an incurved, succulent leaf, with its rounded apex or "pedicle" directed upwards and inwards, and the front of its base resting upon its free inferior segment, the cartilaginous Meckelian rod (*mk.*). This stage represents a highly modified vertebrate skull, if the apex of the mandibular pier is the next following attaching point of a "visceral arch" to the apex of the trabecular bar. Now it lies a little outside the fore part of that bar; it grows outwards and downwards, and its free bar (*mk.*) sweeps across the floor of the face, almost at a right angle to the trabeculæ.

The Meckelian rods do not meet by a space equal to a fourth of their length. They are stout, sigmoid cartilages, and fit obliquely to the interior angle of their expanded pier (*q*). Of course they lie in a lower plane than the trabeculæ; this is not so evident as it should be in the figure. A mass of cells (fig. 2, *sp.*) in the inside of each rod is ready to become the "splenial" teeth and bone.

But this free part of the mandible is seen in the next figure (2) to be the first of a series of six pairs of rods. Of these the second pair is a free arch, whose "pier" never develops in this creature, as in many of the Urodela. The last two never segment off a part answering to the lower element of the mandible. This must be made plain afterwards, and the difference between these types (in the development of their pleural arches) and the Selachians fully explained.

The sickle-shaped second pair of rods are the hyoid arch (*hy.*). They are connected at the mid line by indifferent tissue, as are all the rods; no basal piece is chondrified as yet, and the hyoid does not acquire one for itself.

The rest of the bars (*br.* 1–4) are feebler and narrower than the hyoid. They are

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* After Professor HUXLEY had satisfied himself that the whole of the two trabecular rods were pleural, and I had found them in the frog distinct from the investing mass, and also, during the cephalic flexure, dipping so as to be almost parallel with the primary mandibular rods, the question appeared to be settled.

For the last two years or more my doubts have been growing as to the truth of this view; for the trabeculæ do too much work in skull-building to be mere visceral bars. I find also that GÖTTE and BALFOUR doubt or deny their pleural nature. I am inclined to put their "cornua" into that category, and to regard them as undivided representatives of the primary moieties of the vertebral "centra," and as, naturally, developing an undivided neural wall, posterior and anterior, sphenoidal, and ethmoidal.

sigmoid in shape, lessen rapidly in the series, and are so delicate that in some parts they are only composed of a single row of cartilage-cells, arranged crosswise.

The slight shading at the apices of the first three branchial arches shows where the fingered gills arise.

The nasal involution (fig. 1, *ol.*) is becoming differentiated into layers; the eyeball (*e.*) is almost perfect, and the ear-ball (*au.*) shows through its membranous walls seven or eight crystals, the otoconial particles (*ot.*). Behind the ear-sac and ganglion is a gland, evidently the "thymus" (*tm.g.*).

Third Stage. Axolotl "fry" 5 to $5\frac{1}{2}$ lines long.

The process of development takes place rapidly in larvæ that have become free, although they do not increase very fast in size—not above a line in the first week after hatching.

Nevertheless they soon take on the form of the adult perennibranchiate individuals, although the head at first is relatively very large.

The specimen whose skull I now have to describe was only the twenty-fourth of an inch longer than the last; yet its cranial bend was much lessened, and the short lower jaws had grown so as to give the "underhung" form to the face seen in larval *Pipæ* and *Dactylethræ* (Plate 22. fig. 3, *mk.*).

The relationship of the Urodele to the Batrachian is shown in many ways; but the *time* at which any particular morphological stage is attained in either is very different; processes that in one case take weeks, and even months, in the other may be gone through in as many days.

But this is the case to a lesser degree within the margin of each of these groups, so that particular types break the fall from the one to the other.

Here is an instance of the slowness of growth and change of one part and the quickness in another:—In larvæ of the Common Toad (Phil. Trans. 1876) the trabeculæ are as much developed in the first stage as they are in the Axolotl in individuals two thirds of an inch in length, and intermediate between my third and fourth stages (Plate 23. figs. 1, 2, and Plate 22. figs. 4, 5).

But the mandible, its pier, and the relation to it of the hyoid cornu—these are as far forwards in the beginning of my third stage in the Axolotl (Plate 22. fig. 3, *mk.*, *chy.*) as they were in Toads whose metamorphosis was almost complete, the tail having disappeared (Phil. Trans. 1876, part 2, plate 55. fig. 6).

A sphenoidal neural crest has now grown up from the middle of each trabecula; but the hinder end is flat, embracing the notochord, and the fore end rounded (Plate 23. figs. 1, 2, *nc.*, *tr.*).

The mandibular pier inclines very little forward at present. After growing forwards in succeeding stages, it retires somewhat, but in this type never recovers the vertical position. In tadpoles of the Common Frog and Toad at this stage this bar is almost horizontal, but it was vertical in the first stage ("Frog's Skull," plate 3. figs. 2 & 3).

The apex of the pier is thick, its upper part (seen in the figure) is the "ascending process"; the lower spur or "pedicle" is not seen in this view. But from the apex the elbowed part is seen embracing the outer face of the auditory capsule (*au.*). This part is the rudiment of the otic process. It is now, as to this process, quite like a newly metamorphosed Frog or Toad (see "Skull of Batrachia," part 2, plate 55. fig. 6).

The notch on the hinder edge of the quadrate cartilage (mandibular pier) exactly corresponds with the concavity for the condyle of the hyoid cornu of a tadpole; but its position under the ear is attained very late by the Batrachia. In their larva this point is beneath the eye ("Frog's Skull," plate 5. fig. 1, *hy.*).

In an extremely short time the mandible has doubled its relative length (figs. 1 & 3, *mk.*), and from terete it has become flat and steep; it is turned down in front, and the mass of elevated cells seen in fig. 2 have developed into a "splenial" bone and teeth (*sp.*). Moreover the two rami meet at the chin.

The segmentation of the other five rods is now evident, and no other subdivision will appear. The Urodeles divide these arches more than the Batrachia, but far less than fishes (Elasmobranchs especially). There is no segment in any of these answering to the apex of the quadrate or mandibular pier, which in the Skate is distinct, and may be a serial homologue of the "pharyngo-branchials" of fishes; but in some Batrachia, such a segment appears, *sooner or later*, in the hyoid region.

But in this, and in nearly every kind of Urodele, a joint appears near the lower end of the hyoid cornu, the "hypo-hyal." It exists in the Ray (*Raia clavata*), but not in the Dogfish (*Scyllium canicula*), and is present in Osseous Fishes, where it receives two osseous centres. In these, and in the Sturgeon, it is segmented off (M. Micr. Journ. June 1873, plate 20. fig. 1, *h.hy.*), and not indicated merely by a separate bony deposit.

The rest of the hyoid is a flattish bar, gently arcuate, and having now its apex filling in the notch of the quadrate. This apex is the serial homologue of the "angular process" of Meckel's cartilage (*mk.*). There is no basihyal in these types; but the hypohyal becomes attached to the fore end of the first basibranchial, a part not yet chondrified perfectly.

But the next two arches (1st and 2nd branchials) segment themselves higher up. They have no lower segment or hypo-branchial, so constant in Elasmobranchs, Teleostei, &c., nor does the apex of the arch become detached as a pharyngo-branchial, as in fishes. The longer, upper segment is here the "epibranchial," and the lower, shorter piece the "cerato-branchial"*.

The 3rd and the 4th (or empty arch, *e.br.* 4) are undivided. These are very small rods.

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* As we were working at the same types, Professor Huxley and I agreed that this was the proper nomenclature of these segments, which are so named in his invaluable memoir on *Menobranchus* (P. Z. S., Mar. 17, 1874).

The auditory sac (Plate 22. fig. 3, *au.*) shows two masses of otoconial crystals, and its floor is undergoing chondrification.

If the *primordial* trabeculæ have to be eliminated from the pleural category, there is nothing of the kind in front of the mouth at present. But as a long series of more and more imperfect vertebral segments are developed in various types beyond the *secondary* anal opening, so it is no wild search to be looking for rudiments in front of the *secondary* mouth. These will be discussed in the succeeding stages.

In larvæ less than half an inch in length ($5\frac{1}{2}$ lines) a still further advance is to be found (Plate 23); but there is no new element*.

The cephalic part of the notochord (Plate 23. figs. 1 & 2, *nc.*) is still large, and full half of it in front is invested by trabecular cartilage. These rods have grown much backward and upward, and but little forward. They are now composed of solid cartilage. They only embrace the sides of the thalamencephalon, and bound it in behind, not quite meeting, being kept apart by the apex of the notochord.

Behind these expansions there is no cartilage, the whole occipital ring being still membranous, and the two muscular segments (*ms.*) being visible from above when the hind brain is removed.

The hind brain is now walled in at the sides by the great trabecular crest (figs. 1, 5, & 6, *tr.*), the height of which is equal to the breadth of the broadest part of the bar whence it arises. The temporal muscle (fig. 6) arises from this crest, and is inserted into the coronoid region of MECKEL's cartilage.

There is no distinction between alisphenoid and orbito-sphenoid either in Amphibians or Selachians; and there must be some meaning shut up in the fact that the greatest similitude to vertebral segments is seen in the most specialized types of skull. The trabeculæ are oval on section in front (fig. 4, *tr.*), and end opposite the eyeballs on the side, and the junction of the mid and fore brain within. At present, then, the ethmoidal and nasal regions and the lateral "horns" are absent. Only the common, paired rudiments of the posterior and anterior sphenoids are present. There is no "anterior clinoid cartilage," and the "posterior clinoid wall" does not rise behind the small pituitary body (*py.*).

Attached to the under face of the fore half of the free trabecular rod is an oval, bony plate with bristling teeth. This is the palatine bone; and in front of it is a similar toothed plate, the vomer (*p.pg.*, *v.*).

Near the frontal edge of the face, below, a very small pair of bony spicules are set transversely, each bearing two recurved teeth. These are the premaxillaries (*px.*). Between these and the vomer, but further outwards, the nasal sacs (*ol.*) are seen; they are becoming perforated.

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* These minute skulls have received most careful attention, in preparing them for both lateral and bird's-eye views: this has been done by staining some of them with carmine, and by *imbedding* others for sections, and the colouring of such sections with picro-carmine, which gives a rose-pink colour to the cartilage, and makes the bony plates scarlet and the epidermis brown. This latter was my son's work.

The nerve-cells that are becoming the ganglia of the 5th and 7th nerves in front, and the ganglia of the 9th and 10th nerves behind (5, 7, 9, 10, figs. 1 & 2, and 5, fig. 8) are large and conspicuous masses.

The auditory sacs are now chondrified to a great extent below and outside. Prof. HUXLEY's figure, *op. cit.* plate xxxi. fig. 1, *au.*, shows them as membranous at this stage; but the stained preparations correct this error. A large oval tract above is membranous, and a lesser tract in front of this is granular; the rest is thin cartilage. The base (fig. 2, *au.*) is well chondrified, and the fenestral cleft has not yet appeared. The sections (Plate 21. figs. 8–10) show the structure and condition of these capsules. In fig. 8 the anterior and horizontal canals are shown well in section (*a.s.c.*, *h.s.c.*). Below and in front of the capsule the ganglionic mass of the 5th and 7th nerves is shown (Plate 23. fig. 8), and behind the capsule the ganglion of the 9th and 10th (9, 10, Plate 21. fig. 10).

The large mandibular arch is now well developed, and its pier shows three out of four of its spurs or processes. The pterygoid outgrowth is much later.

As no amount of controversy has sufficed to make Professor HUXLEY's observations on the mandibular pier quite agree with mine, I shall show what I have seen, stage by stage, gladly naming the regions with his well-chosen terms.

The cartilage of the "pier" closely embraces the front of the ear-capsule; the flattened hinder part, which thins out and is afterwards clamped by the squamosal, is the "otic process" (*st.p.*).

The true apex of the mandible is roughly bifurcate, and the upper knob, which has no counterpart in the Frog, is the "ascending process" (*a.p.*).

The lower knob, which corresponds to the band which in the Frog coalesces with the elbow of the trabecula ("Frog's Skull," plate 5. figs. 1–4, *m.pg.*; see it also *free* in the embryo Salmon, "Salmon's Skull," plate 2. figs. 3 & 7, *mt.pg.*), is the pedicle (*pd.*). It lies *below* the orbito-nasal nerve and the ascending process above it.

Neither of these processes is yet modified by coalescence with the trabecula. The pedicle *never does* coalesce (as in the adult Frog, where it is the only inner process); but the "ascending process" does unite, by cartilage, to the alisphenoidal wall.

Even now the ascending process has come closer to the trabecula than the pedicle; the attachment of the pedicle is now (and always in the Urodeles) to the inner face of the ear-capsule in front. This relationship is secondary in the Frog, whose pedicle wastes above, and then expands further down into a large facet ("Frog's Skull," plate 8, *m.pg.*, and plate 9. figs. 2 & 7, *m.pg.*), whose attachment is to a like cartilaginous surface on the ear-capsule; this surface is a "plaster" derived from the basal plate.

The free mandibular bar (*mk.*) continues to elongate, and it is losing the deflection at the chin. A dentary plate (*d*) is now added to the outside of the rod, besides the dentigerous "splenial."

The series of sections (Plate 23. figs. 3–8 and Plate 21. figs. 8–10) will help to illustrate the present condition of both face and skull.

The *first* (Plate 23. fig. 3) is through the fore brain (C^1) and nasal sacs (*ol.*). Two pairs of bony plates are cut through the vomers (*v.*) inside the nasal capsules and the dentaries (*d.*) near the chin.

The *second* (Plate 23. fig. 4) was made through the front of the mid brain (C^2) and eyeballs. There is no cartilage in the upper lips and cheek, but the terete ends of the trabeculæ (*tr.*) are severed. These have the palatine toothed plate attached.

Below, Meckel's cartilages are similarly protected on their inner face by the splenial (*sp.*).

The next section (fig. 5), half of which is shown, is a little behind the last, and shows the orbito-sphenoidal crest of the trabecula (*tr.*).

The *fourth* section (fig. 6) is through the back of the eye and mid brain (*e.*, C^2). In its sensible thickness both the position of the pituitary body (*py.*) and of the apex of the notochord (*nc.*) are shown.

Here the alisphenoidal crest of the trabecula is at its highest. Just in front of the mandibular pier the temporal muscle is seen to pass from this crest to the crested coronoid region of Meckel's *cartilage* (wrongly lettered *q*). The transverse floor of the mouth shows *seven* cartilaginous rods in section. Those near the mouth-angle are the hyoid cornua (*c.hy.*), and the cluster in the middle are the first cerato-branchials outside (*c.br.*^{1}); the second cerato-branchials above and within (*c.br.*^{2}), and the second basibranchial below, at the mid line (*b.br.*^{2}). These rods are seen in a full-sized Perennibranch in Plate 25. fig. 5: that figure will explain the position of these sections.

In the *fifth* section (fig. 7), a little further back, the notochord (*nc.*) is cut through, and we see the descending part of the alisphenoidal crest of the trabecula (*tr.*). Here is seen the great height of the ascending process of the mandibular pier (*a.p.*) and the stunted pedicle. Below these the orbito-nasal nerve is seen cut through.

The pier (*q*) then descends, thick and solid, to the articular region. The free mandible escapes this section, and the flattened upper part of the hyoid (*c.hy.*) is severed.

The second basibranchial (*b.br.*^{2}) is now flattening towards its hind part, and the first two branchials (*c.br.*^{1}, *c.br.*^{2}) are cut through close to their epihyal region.

The *sixth* section (*half*) is through the front of the ear-capsule and hind brain (fig. 8, *au.*, C^3). The trabeculæ and Gasserian ganglion are shown (*tr.*, 5), the otic process of the "pier" (*ot.p.*), the hyoid cornu (*c.hy.*), and an oblique slice is seen of the first epibranchial (*e.br.*^{1}).

The *seventh* (Plate 21. fig. 8) is through the middle of the hind brain and auditory capsule (C^3). Then we have a section of the apices of the trabecular plates at the first third of the notochord (*nc.*, *tr.*). The outer and lower part of the ear-capsule is chondrified, and the anterior and horizontal canals are severed (*a.s.c.*, *h.s.c.*).

An apparently meaningless cavity, lanceolate in section, is seen in this and the next figure, the lower part of which reaches the membranous *inner roof* of the labyrinth. This is evidently the primary involution or "aqueduct."

The mouth-floor shows sections of the same parts as in the last.

The *eighth* section (Plate 21. fig. 9), without the floor, is a front view of a section with some thickness, showing the back of the ear-sac cavity and the thick part of the notochord, behind the trabeculæ (*nc.*, *m.s.*). Here there is only muscle and connective tissue on each side of the notochord. The hyoid (*c.hy.*) has been severed near its apex.

The *ninth* section (Plate 21. fig. 10) is a back view of another section taken between the ear-sacs (*au.*), and showing the ganglia of the 9th and 10th nerves (9, 10).

The front of this section belongs to the back of the eighth. Here the actual apices of the hyoid cornua (*c.hy.*) are caught, but the branchials are not figured. These hind sections explain the two main figures (Plate 23. figs. 1, 2).

Fourth Stage. Young Larvæ $\frac{3}{4}$ of an inch long.

In this stage the "fry" were nearly twice the length of the last, so that ample time had elapsed for very important structural modifications to appear.

In this stage the pituitary body (Plate 22. figs. 4, 5, *py.*) is halfway between the frontal wall and the occipital condyles; and the cephalic notochord is as long as the prepituitary part of the skull. The front half of the chorda is a long cone with an obtuse apex; the hind half is cylindrical, slightly inclined to the hour-glass shape. There are two equal *parachordal* tracts of cartilage; those which embrace the fore part of the notochord are the slabs of *early* cartilage that form the postpituitary region of the trabeculæ (*tr.*); the *late*, newly-developed cartilages are the moieties of the investing mass, the hinder or Huxleyan "parachordals." These latter are sickle-shaped tracts, whose ends curve outwards away from the notochord. The fore part is sharp, and wedges in between the trabecular plate and the ear-capsule. The hind part is knobbed; it is separated from the ear-sac by the ganglion of the 9th and 10th nerves, and is the rudiment of the occipital condyles. The membranous tract separating these two pairs of cartilage is the primordial boundary-line between the basioccipital and basisphenoidal regions. It is represented in the Mammal, afterwards, when ossification has set in, by the "spheno-occipital synchondrosis."

This primordial landmark, then, separates the occipital from the posterior sphenoidal regions. Its development is very late in the Amphibia. The two tracts appeared to be quite contemporaneous in their chondrification in the Salmon ("Salmon's Skull," plates 1 & 2). They were separate, even in the fifth stage, in that type (plate 4. figs. 2 & 3, *tr.*, *iv.*) in "fry" the second week after hatching; but the ends of the trabeculæ, which overlie the hinder tract (*iv.*), had lost the retiring notochord. In the early stages (plates 1 & 2) they scarcely embraced it at all.

In the Fowl ("Fowl's Skull," plates 81 & 82, *lg.*, *iv.*) the fore end of the investing mass is turned outwards and is truncate, the trabecular apices retire outwards from the notochordal apex, which also retires backwards at the same time.

I do not, then, find these two pairs of basicranial cartilages distinct now for the first time. What is new and suggestive is the immense size of the parachordal region of the trabeculæ, and its earlier birth than that of the hinder plates. Some cause or causes, unknown, have given rise to the anachronism of the vertebræ directly behind the occiput and the trabeculæ on the one hand, and the great occipital ring on the other.

The common sphenoidal wall is chondrified fully a fortnight (perhaps three weeks, even) before the lateral occipital wall. I strongly suspect that the auditory capsules oppress the growth of the occipital ring (or segment), making it to be later in its growth than other parts*.

Whilst the hind brain has only the protection of the implanted ear-capsules, the mid brain has a high wall, the common sphenoidal crest of the trabeculæ; this part is somewhat incurved, but does not form a "tegmen cranii."

The fore brain has merely membranous walls, but it has a cartilaginous floor in front; for the round trabecular rods have become longer and twice as near together in front, and besides this they are conjugated in front by a transverse plate of new cartilage.

This plate has three regions; the middle is the "internasal plate" (*i.n.c*), and the outer pair are the trabecular cornua (*c.tr.*).

These appear first, and lacking a stage, in the Axolotl; to show this I refer to my last Plate (Plate 29. figs. 1, 2), which, happily, shows separate trabeculæ, running to the frontal wall, each giving out a pedate "cornu:" the internasal plate is not yet developed.

These three regions (Plate 22. figs. 4, 5, *i.n.c.*, *c.tr.*) have equal, rounded, front margins; the cornua make a falcate curve; and on their outer edge the nasal sacs (*ol.*) partly rest; the internal nostril (*i.n.*) is seen in this concavity. The primordial pituitary region is now a large oval fontanelle, elegantly regular in form; and this egg-shaped space is about equal in area to the narrower but longer space behind, which is floored on the right and left by cartilage. The roofs of the nasal sacs (*ol.*) are membranous as yet; of the chondrification of the sclerotic I take no note, as it does not graft itself on the basicranial stems; but the ear-capsules seem as closely related to the basilar cartilages as carpels to an axis.

The simple egg-like auditory sac, only partially chondrified in the last stage, has now a complete shell of hyaline cartilage, which has grown big with various swellings that are modelled on the membranous labyrinth, whose three "canals" are now well seen, both in opaque and in transparent preparations of the skull (Plate 22. fig. 4, *a.s.c.*, *h.s.c.*, *p.s.c.*).

Below (fig. 5), the vestibule, with its contained otoliths (*ot.*), swells the general surface; and here the cartilage which was perfectly distributed over the membranous contents has undergone dehiscence.

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* The consideration of *morphological anachronism* is being constantly forced upon me by the study of the Amphibia; this is so great in the common Batrachia, in certain parts, notably the upper parts of the hyoid arch, that Professor HUXLEY himself has not found it easy to harmonize their homologies.

It is as though the cartilaginous coat had been filled to bursting, and giving way, a crescentic cleft, with its convex edge outwards, was formed, a little mesiad of the under surface of the horizontal canal. This space beneath the canal and outside the cleft is the first marking off of the "tegmen tympani."

At the back of this morphological rupture, the fast-increasing otolith is seen, partly floored by cartilage and partly by membrane; the cleft itself is the "fenestra ovalis," opened for the first time in the Vertebrata in this group of the Urodeles.

The thin ragged inner edge of this natural rent, whose lips have been torn by growth and expansion, like the sundering of the leafy rays in the Fan-palm, contains the substance out of which a new morphological element, the stapes, will soon be formed—an individuated leafy bud.

Nor is the mandibular pier of less interest than the metamorphosing ear-capsule; it is "laid out and full of meaning," serving as a key to the modification of this suspensory pier right through the air-breathing *Ovipara*.

This large cartilage, besides grafting its cartilaginous substance on to the alisphenoidal crest of the trabecula, has closely clasped the outer fore edge of the ear-capsule (fig. 4, *a.p.*, *ot.p.*, *au.*). This pedate process ends inwards as a small bud, which, however, grows no further in that direction, but stops against the ampulla of the anterior canal (*a.s.c.*).

The lower part of the apex has grown no nearer to the skull, but it has swollen into an evident condyle in front of the auditory sac (fig. 5, *pd.*).

Thus, the "pedicle" in this case is ready to hinge itself upon the prootic region, below, as it does *after segmentation*, when the apex has been absorbed, in the Frog.

At this stage the otic process exactly corresponds with what is seen in newly curtailed Frogs and Toads: in *Pseudis paradoxa* a like state of things is well seen in individuals with the tail reduced to one third its full *larval* length*.

The lower part or quadrate region is as narrow as the contiguous part of Meckel's cartilage; and this latter meets its fellow at the chin, forming a large, bent bow, united by the raised points of its equal horns.

About four pairs of bony plates have been added to the skull and face, besides the azygous pieces; there are now, in all, nine pairs and two median bones. Some of these subcutaneous bony deposits are brought into more direct union with the chondrocranium than others, but all are drawn to it as by a morphological affinity.

The first of the median bones to be noticed is a deposit which ensheaths the apex of the notochord, like a rudimentary centrum; a narrow bridge of cartilage divides this little bony *cephalostyle* from the pituitary body (*py.*). Under the whole basicranial fontanelle, and also extending back beneath the fore half of the notochord and the trabecular apices, is the thin submucous parasphenoid (*pa.s.*); it is lanceolate in shape.

A film of bone, spatulate above and stalked below, binds down the "heel" of the otic process, and serves as a splint both to the outer face of the auditory sac and the

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* This triple fixation of the mandibular stem, grafted, socketed, and embracing, is in strong contrast with that of the Osseous Fishes, where the simple pedicle is let down to a distance from the skull.

outer edge of the mandibular suspensorium; this is the squamosal (*sq.*), whose name, amongst Fishes, is "præopercular."

On the roof, supplementary to the general deficiency of the cartilaginous "tegmen," there are four nearly equal, thin shells of subcutaneous bone; these are the frontals and parietals (*f.*, *p.*); in this their early deficiency they still leave a lozenge-shaped fontanelle above.

The next five pairs of bones were present in the last (third) stage; the foremost of these, margining the face in front, are the premaxillaries (*px.*); these bones do not yet meet at the mid line, but they have sent upwards a long styloid "nasal process."

The vomers (*v.*) have changed but little; but the palatines are now "pterygo-palatines" (*p.pg.*), for the dentigerous plate in the ethmoid region has sent backwards and outwards a long, sigmoid, ragged, fenestrate plate, without teeth; this is the pterygoid process of the bone; it just reaches the quadrate.

This answers to the "palato-pterygoid" bone of the lower Urodeles, as *Proteus* (see Plate 28. *p.pg.*) and *Menobranchus* (HUXLEY, *op. cit.*); and it is also seen in certain Fishes whose relationship to the Amphibia is most evident and sure, namely the "Dipnoi." In the Batrachia, as in the Sturgeon, there is, from the first, a palatine bone distinct from the pterygoid; and in Osseous Fishes a third primary centre, the mesopterygoid.

The dentary and splenial bones on the mandible (*d.*, *sp.*) have increased in size; and the rod is now ensheathed on its inner side, above, by a rudimentary "articulare."

At this stage we can profitably compare the main nerves with those described by Professor HUXLEY in *Menobranchus* (Proc. Zool. Soc. 1874, p. 191); one description might serve for both.

The olfactory nerves pass to the nasal sacs beneath the nasal processes of the premaxillaries; they are not yet built into a solid wall of cartilage.

The optic nerves pass through the sphenoidal crest of the trabeculæ, and they serve as a landmark to show where the orbital wing begins and where the alisphenoid ends; they are well displayed in the large specimens (Plate 27. figs. 1, 2).

Leaving out of consideration the small 3rd, 4th, and 6th nerves as of minor importance in this research, we find the trigeminal full of interest in this survey.

As in *Menobranchus*, "the orbito-nasal (ophthalmic) division of the trigeminal nerve [fig. 5^1] passes beneath [the ascending process (*a.p.*)], which therefore, morphologically speaking, ascends higher than the eye, inasmuch as the orbito-nasal nerve, as it passes forwards, runs above the optic nerve [Plate 27. figs. 1, 2, 5 ?].

"The orbito-nasal nerve actually leaves the skull by a considerable foramen, common to it and the other divisions of the fifth [fig. 5^2, 5^3], which lies between the trabecula internally and below, the prootic [region] externally and behind, and the parietal bone above. And this foramen is undivided; but, as the ascending process of the suspensorium passes between the orbito-nasal nerve on its inner and anterior side, and the second and third divisions of the fifth on its outer and posterior side, it looks as if the process in question divided the foramen of exit of the trigeminal nerve into two parts.

"The ganglia of the trigeminal and of the seventh nerves are situated, close together, above the trabecula, where it passes into the floor of the auditory capsule—the Gasserian ganglion [Plate 22. fig. 4] lying in front of the anterior wall of the capsule, while the ganglion of the seventh, which is very closely connected with the auditory nerve, is placed rather on the ventral side of the anterior end of the capsule [Plate 22. fig. 5, 7)].

"The posterior division of the seventh nerve (which answers to what is commonly called the facial nerve, and may be termed the *hyomandibular* division of the seventh, runs directly in front of the auditory capsule, and beneath the otic process of the suspensorium [fig. 5, *ot.p.*, 7^2]. The anterior division (palatine or Vidian division) of the seventh [7^1, fig. 5], on the contrary, passes directly forwards, close to the pedicle of the suspensorium, parallel with the orbito-nasal, and below but external to it" (HUXLEY, *op. cit.* pp. 191, 192). The ganglion of the glossopharyngeal and vagus nerve (9, 10) is seen wedged in between the posterior internal face of the auditory capsule and the investing mass (*iv.*). The auditory or eighth nerve cannot be seen in these views; it enters the inner face of the capsule not far behind the Gasserian ganglion.

Before leaving this *4th stage*, I would remark upon the importance of the new *pterygoid* wing that has grown backwards from the dentigerous, primary palatine. In this now enlarged tract we have one bony centre reaching from the quadrate to the vomer. I have already remarked upon the presence of this bone in the "Dipnoi"—a group where we see the dermal plates appearing as *deep layers*, and becoming especially correlated to certain territories of the chondrocranium. We have a similar relation of a few subcutaneous or submucous bones to the cartilaginous endocranium in the low Urodeles and in the larvæ of the higher types.

It is, however, extremely remarkable that in many of the "Sauropsida" one bony bar runs from the quadrate to the vomer—in Snakes and Lizards to the greatest extent, the elongating face necessitating the intercalation of the whole (Snakes) or part (Lizards) of the palatine between the end of this palato-pterygoid bar and the vomer of that side.

In *Anguis fragilis* the palatine intervenes very little, and, contrary to what is general in Reptiles, the fore half of the palato-pterygoid bone is segmented from the hind half to form a "mesopterygoid;" we shall see this state of things higher up.

In *Hatteria* (GÜNTHER, Phil. Trans. 1867, part 2, plate 1, p. 5) there is a long palato-pterygoid bone, undivided, and reaching the whole distance from the quadrate to the vomer of that side. In the Ostrich (*Struthio camelus*) (see my paper on the skull of the Ostrich tribe) the pointed end of the pterygoid scarcely reaches the huge vomer (a double coalesced bone); but in the other types (*Rhea*, *Dromæus*, *Casuarius*, *Tinamus*) these bones meet.

But in the Carinate birds generally (and I have examined and figured these parts in a large number of types) the rule is for the pterygoid bones to grow forwards to the hind part of the vomer (or vomers), and afterwards, as in *Anguis fragilis*, for the pointed anterior part of each to become segmented off as a mesopterygoid. This segment loosed from the pterygoid then coalesces with the upper edge of the inner part of the main palatine bone (" Skull of Ægithognathous Birds," Trans. Zool. Soc. 1875, plates 54–62)*.

Fifth Stage. Young Axolotls $1\frac{1}{4}$ *inch long.*

In this stage (Plate 22. figs. 6, 7) the chondrocranium shows several points in advance of what was seen in the last. The two pairs of basilar cartilages have united with each other and with the auditory capsules, and, leaving out the postmandibular arches, all but the *new parts* are welded together; so that we have now a cartilaginous skull exactly comparable to that of the Elasmobranch (excepting, of course, the lack of the tessellated calcifications).

The occipital condyles (fig. 7, *oc.c.*) are now fashioned, and in front of them the basilar cartilage has sent a lip right and left beneath the notochord (*nc.*). These basilar lips are distinct, and so also are the crests that have grown upwards from the basilar plate to form the *ex*- and *supra*-occipital regions of cartilage (figs. 6, 7, *e.o.*, *s.o.*).

The notochord is not only belted below, it is also capped with the increased and increasing bony *cephalostyle*; a broad selvedge of cartilage also passes between this rudimentary cranial " centrum " and the pituitary body; it is a flat postclinoid conjugation of the trabecular *parachordal* tracts.

The prechordal part of the trabeculæ now exists as a high wall from the auditory to the nasal sacs; and this wall, convex without and concave within, is turned over a little into the roof, and to a greater degree into the floor. But the roof is open from end to end, and the floor is a gaping space for the foremost three fifths of its extent.

But the vicarious exoskeleton keeps up with the requirements of this openwork of cartilage.

The conjugational " internasal plate " (*i.n.c.*) is thickening, especially at the middle, and its hinder margin has become convex; it has retained its relative extent antero-posteriorly; but the lateral leafy growths, the cornua trabeculæ (*c.tr.*), are now large flabelliform outgrowths, having an emargination between them which forms three fourths of a circle. The anterior conjugation of the trabeculæ (internasal plate) may well be the foremost growth of the curiously generalized basilar and neural parts of the skeletal axis; yet, dying out here in the frontal wall, they have sent out a pair of *pleural rudiments.*

But behind the internasal band there is a pair of " pleural rudiments " which are not mere outgrowths. They are separate elements; these are the antorbital cartilages or ethmopalatines (*e.pa.*).

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* We shall soon come to a modification, by segmentation and displacement, of this primary pterygo-palatine bone that will tax our knowledge of these parts and the interpretation of them in a large series of Vertebrate types.

These cartilages are slightly arcuate, and have a forward turn; they have rounded ends, and are broadest above: they are, as far as I have seen, universal in the Urodeles.

Here, again, the Elasmobranchs come in to our help. I have studied their growth in Rays and Dogfish: GEGENBAUR shows, in his invaluable memoir ('Untersuchungen zur vergleichenden Anatomie der Wirbelthiere,' part 3, plates 1–17), that in those types the antorbital is sometimes continuous with the skull and sometimes distinct*.

In Birds also we find an antorbital cartilage, which early coalesces with the antorbital portion of the nasal sac at its lower angle; it chondrifies quite separately from the trabecular (basicranial) bar. In *Corythaix* and *Musophaga* it might easily be taken, in its osseous state, for an "os transversum;" but it runs across the face too far forwards, starting from the ethmoidal and not from the presphenoidal region. In the Passerinæ there is a second pair of *transverse* cartilages, but these are in the middle part of the palate; of these I shall speak in the next stage.

As for the coalescence of the little antorbital bar with the nasal capsule, this does take place even in the Urodela, namely, in *Salmandra maculosa* and *Triton cristatus.*

Another pair of cartilages are to be seen over the nasal sac; these are the foremost of the "paraneural" elements of Professor HUXLEY (*op. cit.* p. 198), who, in discussion with me, spoke of them as upgrowths from the trabeculæ. They soon coalesce with these bars by their lower edge; but I have satisfied myself of their primary distinctness, not only in *Siredon*, but also in *Notophthalmus viridescens* in its larval state.

These crescentic shells of cartilage have their concave edge looking forwards and outwards, and their convex edge looking backwards and inwards (Plate 22. figs. 6, 7, *na.*).

We have all the *essential* elements here, in this stage of the Axolotl, for the morphological development of the nasal labyrinth; superadded parts may come in, as the bony plates which become correlated to or grafted upon the labyrinth; and in many cases there are supplementary cartilages from the "labial" category, but these do not appear in the Urodeles. In the consolidated chondrocranium of the Selachians there is a notable azygous cartilage which grows forwards between the cornua trabeculæ; this is not seen here, but turns up in metamorphosed Salamanders (*S. maculosa*); the bar referred to is large in the embryo bird (the "prenasal rostrum").

Besides general increase of size there is this to be remarked now, in the mandibular pier, that the otic process (fig. 6, *ot.p.*) and the pedicle (fig. 7, *pd.*) are much more solid and massive; the ascending process (*a.p.*) is now of great breadth, especially at its line of union with the skull wall. The articular concavity (fig. 7, *q.*) is now complete.

The chondrocranium is adding fresh bony plates *in the same order* in which they

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* This "lower antorbital" or ethmo-palatine is very common in the Carinate Birds, and I have lately described it in them in papers recently published in the Transactions of the Linnean and Zoological Societies; in these types it is called the "os uncinatum" (Magnus). It is very distinct as a hooked process, both in the "Carinatæ" and in the Emu and Cassowary (see "Ostrich Skull," plate x. fig. 18, *a.i.t.*, and plate xiv. figs. 1 & 7, *a.i.t.*, pp. 151 & 158).

appear in an ascending survey of the Urodeles; the new pair are the maxillaries (*mx.*). These are small dentigerous styles applied to the outer side of the trabecular cornua and nasal roof-plates: in *Menobranchus* there is *sometimes* one of these bones (HUXLEY, *op. cit.* p. 190); in *Proteus* they are absent (Plate 28).

The premaxillaries, frontals, parietals, and squamosals are fast growing into their typical size and shape (figs. 6 & 7, *px.*, *f.*, *p.*, *sq.*), and the parasphenoid (fig. 7, *pa.s.*) is now notched in front and subalate behind; it reaches to the basilar belt of cartilage (*iv.*).

But the pterygo-palatines (*p.pg.*) are most modified; the dentigerous part is now a small territory compared to the long, flat pterygoid wing, which is very narrow in front and gently widens to its oblique end beneath the suspensorium.

This narrow neck between the toothed and toothless regions of the bone is becoming ready for dislocation in the next stage. The only bone that can be as yet called *endoskeletal* is the notochordal "cap."

The free postoral arches need not take our attention for some time to come.

Sixth Stage. Young Axolotls 2¼ inches long.

Every moderate increase of size in these "fry" of the Axolotl is attended with some important and instructive morphological change.

If the metamorphosis of this type was studied for its own sake, irrelatively to the structure and development of the skull in other Vertebrata, it would be fraught with great interest.

But the details of each stage are full of instruction; and when these are compared and commented on, stage by stage, we seem to be acquiring the very grammar of this difficult language, so as to be in a position to decipher these most ancient hieroglyphics.

This stage is perhaps the one which presents us with the greatest number of suggestive conditions in its changed and still changing elements, and nearly every thing one sees here tends to send the mind hither and thither, throughout the length and breadth of the kingdom of the Vertebrata.

The occipital condyles are still more elegantly finished than in the last stage, and the basicranial lips have now united beneath the notochord (Plate 24. fig. 2, *iv.*). The edges of the ascending part of the arch are united now as much, relatively, as they will be (fig. 2, *s.o.*). The trabecular half of the notochord (figs. 2 & 4, *nc.*) is an alate *centrum*, whose jagged bony edges are growing into the substance of the symmetrical cartilages, right and left. This is a "præ-basioccipital" bone; it is not followed by a posterior joint like itself, the rest of the notochord and its investing mass remaining soft. In one important respect this type lies on the level of the "Dipnoi," for it has a pair of small exoccipital bones (figs. 2 & 4, *e.o.*); these are formed as rings round the 9th and 10th nerves (9, 10)*.

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* These bones are very small in *Ceratodus forsteri* (see HUXLEY, Proc. Zool. Soc. Jan. 4, 1876, p. 38) as compared with those of *Lepidosiren*; in the cranium proper these are the only bones that can be called *intrinsic* in those types and in this *larva*.

If we compare this with the foregoing stages it will be seen that the great basal fontanelle has become twice as long (in proportion to its breadth) as it was at first. Its high walls, inbent both above and below, are still unossified; the cross wall (internasal plate) has become a thick mass of cartilage, through the sides of which the olfactory nerves pass obliquely.

The lateral parts now show their morphological meaning clearly; large hollow, cartilaginous crescents cover the inner and posterior regions of the nasal sacs, and the flabelliform trabecular cornua floor the nostrils and form a curved model on which the premaxillaries and maxillaries are grown (Plate 24. figs. 1–3, *px.*, *mx.*, *na.*, *c.tr.*).

In the notch formed by their posterior margin, the internal nostrils appear surrounded by a frame of fibrous tissue and mucous membrane. The primary independence of the curved flap of cartilage behind the simple nasal roof is here well shown (Plate 24. figs. 1, 3, *e.pa.*); it is neither part of the nasal roof nor a mere outgrowth of the trabecular wall: it may *coalesce* with both, as in *Triton cristatus.*

The forward curve of the antorbital is caused by this tendency; it follows the curve of the nasal roof, drawn to it as a tendril is drawn to some stout stem. For in the Selachians it generally runs backwards (GEGENBAUR, *op. cit.*), and it does in *Proteus* (Plate 28); but in the Batrachia it grows both ways, and is like the letter T, the stem running inward and upward.

But the part which I have lately called "ethmo-, pre-, and post-palatine" in *Bufo vulgaris* (Phil. Trans. 1876, part 2, plate 54. figs. 3, 4) in its posterior crus is most probably continuous with another facial cartilage, represented in *Siredon* by a distinct piece or element (Plate 24. figs. 1–3, *pt.pa.*).

This element, which may be called the "postpalatine," is new to me in these low types, this species and this stage having alone yielded it; it is a short flat bar, rounded at both ends, and lying obliquely upon the narrowest part of the pterygo-palatine bone, a little behind the antorbital, and having a similar outward and forward direction. This cartilage is attached by fibrous tissue to the fore part of the pterygoid outgrowth of the suspensorium—the "symplectic" process of the mandibular pier.

A transpalatine cartilage has long been familiar to me in the palate of Passerine Birds (see Monthly Micr. Journ. 1873, plates viii.-x. "Turdidæ," plates xxxiv.–xxxix. "Corvidæ," and plates ii.–xi. "Paridæ;" Trans. Zool. Soc. vol. ix. part 5, Dec. 1875, plate lv. *Corvus* and *Ruticilla*; and Trans. Linn. Soc. ser. 2, Zool. vol. i. plate xxi *Linota* and *Troglodytes*).

This ornithic palatine element, like its counterpart in *Siredon*, belongs to the anterior sphenoidal region, and therefore comes next in front of the mandibular arch, whose suspensory relation is to the posterior sphenoid.

In the Bird this cartilage soon becomes ossified, endosteally, and then rapidly coalesces with a bone (the main palatine) which runs from the front of the pterygoid to the side of the prenasal rostrum. Along *these regions*, then, in *Siredon* it soon coalesces with the *chondropterygoid*, and then, having become the apex of that process, degenerates into fibrous tissue*.

In the fifth stage (Plate 22. figs. 6, 7) the suspensorium gave forth nothing from its front margin, but now there is a long tongue of cartilage (Plate 24. figs. 1, 2, *pg.*), the *chondropterygoid*. This process in its development in the Urodela is curiously like the modifications and stages of the symplectic process of the hyomandibular in Fishes (see GEGENBAUR, 'Selachians,' p. 175, figs. A, B, C, D, E). I have missed that stage in *Siredon* in which the pterygoid was budding, answering to fig. B in M. GEGENBAUR'S woodcut, but I have seen and drawn this stage in larvæ of *Spelerpes* and *Triton*.

The larvæ of Batrachians throw no light upon the interpretation of the pterygopalatine arcade; for in them it is at first, and indeed for many days, merely a conjugating band between the ethmoidal region of the trabecula and the distal part of the suspensorium†.

So that, equally belonging to both bars, in the larval state, it has to become developed into its adult condition before it teaches any thing. In the Toad (see my last paper) it becomes very instructive, and shows, at least, the independence of the ethmopalatine.

The huge mandibular pier, which has sucked the life out of the pier of the hyoid arch, now holds to the skull by the broad band of cartilage that has become completely welded to the skull-wall. But below this "ascending process" the "pedicle" (*pd.*) has grown into a ball-shaped bud, which is made to sink into a socket of cartilage just where the trabecula is fused with the ear-capsule; it rests on a bed of fibrous tissue, for there is here (contrary to what we see in the Frog) no joint-cavity (figs. 2, 4).

The otic process (*ot.p.*) is a blunted triangular mass of cartilage, strapped by a fibrous ligament to the ear-sac, and held down by the lathy, splintery squamosal (or preopercular).

The ear-capsule at this stage corroborates the older views of the writer, namely, that the stapes is cut out (or segmented off) from the preformed cartilage of the ear-capsule in the Urodeles. In the last two stages (Plate 22. figs. 4, 5, 6, 7) the inner and anterior part of the fenestral cleft was fringed by ragged cartilage; it is now (Plate 24. figs. 2 & 4, *st. f.o.*) ragged no longer.

There is now a half-severed flap of cartilage, like a *stonecrop*-leaf, which serves as a *stapedial* lid to the vestibular fenestra.

The study of the development of these parts in various Urodeles shows how it is that the pedicle in its stunted condition (*primary* in the Urodeles, but *secondary* in the Anura) articulates with the prootic region of the ear-capsule.

The young of *Triton cristatus* shows clearly that as soon as the parachordal cartilage

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* At present I am not aware of any thing but a *membrane-bone* in this part of the palate in Snakes, Lizards, and Crocodiles—the "os transversum," or osseous transpalatine. The early embryos of Serpents have failed me here; but the development of the skull in the other Reptiles has only very partially been worked out by me.

† Professor HUXLEY will bear me witness that I am not the only morphologist whose mind has oscillated (not once or twice) as to the *independence*, or the *secondary nature*, of the pterygo-palatine arcade.

has become fused with the flat part of the trabecula, the common basal plate so formed plasters, as it were, the antero-internal face of the ear-capsule. In that type the prootic appears early; but when largely spread over the sac, this bone has, covering its fore part, a mass of cartilage derived from the basal plate. The socket for the pedicle is sunk in the lower part of this investing cartilage, and, above, it appears as an outstanding wing over the suspensorium and between the ascending and otic processes*. The seventh nerve (7^2, Plate 24. figs. 2 & 4) escapes from the skull directly beneath this embracing cartilage; the tract in front of that nerve belongs to it, and not to the auditory sac.

In this stage the prootic has not yet made its appearance; nor is it seen in much larger specimens.

The osteocranial elements are just becoming perfect; the squamosals especially (*sq.*) are broadening, and show now that peculiarity of their form in the Axolotl's skull, namely, the fingered splinters at their upper or auditory end.

But the most important thing to be noticed is the segmentation of the small dentigerous palatine from the large pterygoid wing which it had acquired in the fourth stage (Plate 22. figs. 4, 5, *p.pg.*).

Both these newly disparted bones are blunt-pointed where they overlap, having been parted asunder obliquely.

I have long been familiar with the segmentation of the bony bar which in the embryo of *Carinate* birds runs from the quadrate to the vomer. *There*, however, it is the mesopterygoid which is cut off from the front of the pterygoid, to be added to the palatine by ankylosis. *Here* the primary palatine acquires a pterygoid and then moults it again, and the two bones diverge (in the highest or Salamandroid stage) from each other; this will be explained in the *tenth stage*.

The parasphenoid (Plate 24. figs. 2 & 4, *pa.s.*) is now a very elegant and delicate lamina of bone, almost oblong; it has a rounded emargination in front, has basitemporal angular processes, and is rounded behind, where it only ends with the median part of the investing mass; so that it partly underlies the internasal plate in front, and behind reaches the foramen magnum. The rough bony sheath (*o.nc.*) of the fore end of the notochord lies on this bone, and is early more or less united with it. It is seen that both the maxillary and premaxillary form an eave to the adze-shaped trabecular cornu; the former, however, is related also to the nasal capsule and to the outside of the ethmoidal region (figs. 1–3).

Seventh Stage. Young Axolotls $3\frac{1}{4}$ *inches long.*

This young Axolotl was 1 inch longer than the last; it shows several things worth notice, and amongst them a curious want of symmetry, right and left, the left auditory

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* Prof. Huxley mentions this process in the Siren's skull (Art. "Amphibia," Encycl. Brit. p. 758). It is the "sphenotic" process of the chondrocranium, and is common in Selachian, Ganoid, and Teleostean Fishes. Wiedersheim figures it: see his last work, plate i. figs. 11, 12, *V.F.*

sac being larger than the right, and thrusting the suspensorium further forward (Plate 25. figs. 1, 2, *au.q.*).

In this specimen the notochord showed more on the lower than on the upper side, and the sheath of bone was now unapparent, save perhaps as a slight groove on the parasphenoid; in all the Urodeles this sheath is very evanescent.

The exoccipitals (*eo.*) had doubled in relative size, but I could not detect the prootic centres; this chondrocranium is therefore still the equivalent of that of the "Dipnoi."

A greater approach to the Selachian chondrocranium is evident in the growth inwards of more cartilage in the floor of the skull, making the fontanelle less and less. This is principally in front and at the sides, although, behind, there is more cartilage; this, however, is a very temporary increment; it soon wastes again there. The internasal plate has grown all round, and is very thick and solid at the middle; the nasal roofs are now continuous with it and with the very broad cornua trabeculæ (*c.tr.*). Also we can see a greater distinctness between the fore end of the skull-wall and the precranial growths. Thickening of the cartilage, generally, has caused a greater obscuration of the canals of the ear-capsule as seen from without (fig. 1); below, the *stapes* has become free, and is now a large tongue of cartilage, with its broad end foremost and turned a little inwards. Behind, it does not yet accurately fit to the irregular fenestra ovalis. A bony (intrinsic) centre has appeared in the quadrate region (*q.*); it is spatulate, with the "handle" upwards.

The large size of the left ear-capsule throws that suspensorium further forwards than the other; its ascending process is confluent with the cranial wall; the pedicle is still a knob, and the otic process a large blunt spur below, with a pedate process above.

Now, with the superadded postpalatine cartilages, the chondropterygoid nearly reaches the antorbital (*pg.*, *a.o.*); it is now, relatively, nearly as large as in the Selachian, whose mandibular suspensorium is a swinging quadrate, there being no pedicle or any ascending process, but only a huge chondropterygoid, growing anteromesially.

This bar, in *Siredon*, bears no teeth, and the mouth is armed and surrounded by exoskeletal toothed bones, that have no existence in the Selachian.

The vomers (*v.*) are now longer and more arcuate, and the little, toothed palatine bone is a small distance from the huge "osteopterygoid."

Eighth Stage. Fully grown perennibranchiate Axolotls 8½ inches long.

This is a very important stage; it is, indeed, the adult of this species, as a rule; exceptional individuals undergo great metamorphic changes, but mostly this type grows to this large size and retains its gills.

The higher kinds (Caducibranchs) are generally small; and so is the Salamandrian form (*Amblystoma*) which exceptional young Axolotls change into.

As compared with the skull of an *Amblystoma*, this is low and larval; yet it has gained several new things since the last stage, and has become greatly modified in shape.

This chondrosteous skull is a very exact intermedium between the chondrocranium of a Selachian and the osteocranium of a Reptile. Several pairs of bony centres now exist that are intimately blended with, grafted upon, and by ingrowth destructive of, large tracts of cartilage.

But still larger tracts are soft, and only one pair of bones occurs on the inferior arches that have any right to be called endoskeletal, namely the "articular" pieces (Plate 25. fig. 5, and Plate 26. fig. 5, *ar.*).

On the whole this skull agrees well with that of any rather large larva of a Caducibranchiate Urodele.

True to the Amphibian type, there is neither a floor-bone nor a keystone to the occipital arch; the tendency once shown to vertebral segmentation has now vanished, and the notochord, deprived of its bony sheath, has now become a mere thread in the basal cartilage.

On each side of the fontanelle, in front, there is a tract of bony masonry (Plate 26. figs. 3, 4); but for the most part the parachordal and trabecular regions run to the frontal wall as one solid mass of cartilage.

As compared with former stages, the fontanelle is less; but that is due to the huge plate and "horns" in front, for the basal plate (*b.o.*) runs no further forward than the occipital roof; these two territories were extremely unequal (see last stage) (Plate 25. figs. 1 & 2). The upper fontanelle is much larger than the lower, the sides of the latter having grown into the floor considerably; especially in front is the floor cartilaginous, and the two representatives of the many olfactory foramina of a Mammal are conspicuous with their nerve (1) from above.

The exoccipital bones (*e.o.*) are wider apart below than above; but in both cases there is a clear tract of cartilage between them; the cartilage of the condyles (*oc.c.*) looks downwards. Above, these bones run but little in front of a transverse line that might be drawn across the top of the foramen magnum (*f.m.*); but on each side the auditory capsules are largely hardened by them. Also below, shunning the median basal plate, the exoccipitals grow out into a large "opisthotic" shell, which floors most of the vestibular bulb (fig. 4). A little more specialization, and this plate would have become an independent opisthotic centre.

Nearly all the posterior canal and its ampulla is invested by the upper part of the exoccipital bone; therefore that part is an *epiotic region* of the bone.

But the horizontal canal, which burrows the outer edge of the capsule, has its own, evidently independent, periotic centre; this irregular shell of bone, which forms a rough tegmen tympani, is the "pterotic."

The prootic is perhaps the most curious bone of all: it begins as a film on the fore face of the capsule, where the anterior canal and its ampulla is enclosed. During growth it finds its way down to the floor of the front part of the capsule between it and the *plaster of newer cartilage* which was derived from the basal plate, and which persists as a soft socket for the "pedicle" of the suspensorium (*pd.*). But the prootic (which in the Batrachia often vicariously ossifies the alisphenoidal region) here forms a stony coping to the soft hinder half of the skull-wall (fig. 3, *pro.*); it alters but little of the soft wall beneath it, and ends as a spike near the bony ethmo-sphenoidal tract (*e.sp.*). Beneath (fig. 4) there is no such strength added to the foundation of the wall, although the floor, like the roof, has an extraneous source of strength, in the latter derived from the parietals and frontals, and in the former from the parasphenoid (figs. 1 & 2, *pa.s.*, *f.*, *p.*). The stapes (*st.*) will be described with the suspensorium. The ethmo-sphenoid bones (*sp.e.*) are manifestly the lateral rudiments of the Batrachian girdle-bone, without the upper azygous piece (the superethmoid), and left unfinished in their growth. The part formed by the original fore ends of the trabeculæ is here very solid, and is well covered with bone-deposit, which ends abruptly both before and behind, above; but in front on the lower face the bones form sharp wedges, piercing the huge flat underface of the internasal plate (fig. 4). Below, also, they are their own width apart; above, they meet in front of the lower fontanelle and the fore brain, and the olfactory crura rest upon these conjugating processes (1, fig. 3).

At the margin of the upper fontanelle these bones are wide apart; and here the cartilage rises into a crescentic mound, whose concave edge looks backwards on each side; and, in front, the cartilage is bevelled.

In front, the internasal tract ends so as to form a semielliptical notch between the large, flat, trabecular horns (*i.n.c.*, *c.tr.*); on these the reniform nasal capsules are mounted, and with them they are coalesced. The united crura are largest where they have attached to them the small antorbital cartilages (*a.o.*), and the underface of this ethmonasal tract is large, flat, and roughly pentagonal; for below, the "horns," where they become free, are bevelled (fig. 4); they diverge gently, and end in front with a sinuous margin. Above (fig. 3), the naso-trabecular building is very elegant; for these most simple nasal roof-cartilages (first "paraneurals") rest in front on the middle of the cornua, and behind overlap the ethmoidal region; the outer nostril (*e.n.*) is finished behind by a fibrous valve, and not by a cartilaginous labial, as in the Frog.

The inner nostril (*i.n.*) is much further outwards, being external to both the vomer and the dentigerous osseo palatine.

The suspensorium of the mandible is a part of the chondrocranium, and a very large part too, and may be fitly described here.

The quadrato-pterygoid cartilage is now a morphological counterpart of that of the Selachians, with this great difference, namely, that it has three processes attaching it to the cranium that are mostly represented by membrane in them.

These processes, the pedicle and the ascending and otic processes, are, two of them, attached by a strong fibrous web to the basisphenoidal and auditory regions; the ascending process keeps its confluence with the side wall of the skull. That process (figs. 3, 5, *a.p.*) is a thick rod, and it is separated by a shallow groove from the shorter but still stouter pedicle (*pd.*); through the angle of the fork run the Vidian and orbito-nasal nerves. The postero-external or otic process (*ot.p.*), as also the main part of the suspensorium, is convex above and hollowed out below. A large bony spatula now runs upwards and backward from the articular condyle (*q.*) to the top of the front half of the otic process, that is its handle: the broad, lower part takes up much of the cartilage; but higher up it is only a hard wall, with cartilage before and behind it. The otic process ends as a lobe behind, and the lower part of the suspensorium is subalate also.

The condyle is well scooped; its upper edge overlaps the lower. The chondropterygoid is a huge tongue, whose base extends from the hinge, externally, to the pedicle within; it is narrow and terete in front, where it has gained the small postpalatine element, and the blunt point just reaches another cartilage, the antorbital (*a.o.*).

The small osseo palatine (*pa.*) is hatchet-shaped; its blade is on the inside, and its handle behind; this part just touches the "osteo pterygoid" (*pg.*), a large, triangular plate of bone, serving, like the one in front, as a splint to the underface of the quadrato-pterygoid arcade.

The sinuous sides of the bone (figs. 2 & 4) only reveal a little of the cartilage below, which overlaps the bone within and behind; the axes of the two bony plates are coincident.

The suspensorium is tied to the cranium at a fourth place; the stapes (figs. 2, 4, & 5, *st.*) is unusually solid and projecting, its outstanding process looking a little forward. From that process a ligament arises which spreads into a fan-like fascia, which is inserted along the under and outer edges of the suspensorium from the lobe of the otic process to the lobe of the quadrate.

This fascia helps to form the rudimentary tegmen tympani; it lies some height above the portio dura as it passes to the hyoid cornu and the mandible*.

The mandible (Plate 25. figs. 4, 5, and Plate 26. fig. 5) is a large cartilaginous arch, with three bony plates wrapped over it. The dentary (*d.*) is almost entirely external; it reaches nearly to the angle; the splenial (*sp.*) not half so large, but, like the dentary, dentigerous, lies entirely on the inner side; it occupies most of the anterior three fifths in extent. The articulare (*ar.*) forms a trough in which the thicker proximal part of the bar lies; it is principally internal, and reaches nearly to the chin; the thick longitudinal convex condyle rises out of the bone, and appears above it. The quadrate half

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* Professor Huxley pointed out this anomaly to me, showing me that this ligament cannot correspond to the "suspensorio-stapedial" ligament of *Menobranchus* (*op. cit.* p. 192). Until quite lately this enigma has remained unsolved; I had found more or less ossified cartilages attached, either directly or by ligament, to the stapes in certain North-American Newts (*Desmognathus fuscus*, *Spelerpes salmonea*, and *S. rubra*); but until I dissected the Menopome they remained uninterpreted.

In the latter type it is impossible to mistake the stem of the stapes (partly ossified where it joins that plate) for any thing else than a "spiracular cartilage," a structure so common in the Selachians (see Gegenbaur). The spiracular cartilage is an offshoot from the top of the mandibular pier (=the hyoidean and branchial rays of the Selachians), and in the Frog is, during the larval stage, a part of the suspensorium; in that type it becomes separated, and is specialized into the cartilaginous "annulus tympanicus." Professor Huxley, in his paper on *Ceratodus* (P. Z. S. 1876, p. 42), rightly compares the spiracular cartilage of *Cestracion* to the "otic process" of the Frog; it does correspond to the permanent process in the metamorphosed Frog (see "Frog's Skull," plates 5–9, where the small mandibular ray, afterwards spiracular cartilage, then tympanic ring, is lettered *s.h.m.*). Wiedersheim ('Das Kopfskelet der Urodelen,' plate 2. fig. 24) represents the facial nerve as passing *over* the stem of the stapes (*op.*); this is an error. I learn from him that the same part occurs in *Cryptobranchus*, *Ellipsoglossa*, and *Ranodon*.

embraces the condyle of the mandible; externally, it seems to lie in it, the hinder part of the articular condyle being somewhat hollowed for the outer lip of the quadrate. The other five arches, notwithstanding their size and solidity, are entirely cartilaginous (Plate 25. fig. 5, and Plate 26. fig. 5); they are for the most part gently arched and oval in section.

In degree of transverse segmentation they are intermediate and between those of the Selachian, where the hyo-branchial series has most superimposed pieces, and those of the larval Batrachian, which has fewest.

In the latter, indeed, the true (inner) branchial arches are distal rudiments of the cerato-branchial pieces, attached to hypobranchial plates; in Selachians (GEGENBAUR, *op. cit.* plates 14–19) there is great subdivision of the bars and permanent separateness.

In these latter types there is so much regularity of these numerous segments, where the hyo-branchial cage is most highly specialized and has its most massive development, that it can be used as an exemplar and rule both for comparison and for nomenclature.

In the Osseous Fishes (see "Salmon's Skull," plate 88. fig. 9) these parts are still more specialized by bony deposit, pharyngeal teeth, &c.; but the segments easily take the same names. The small distal hypohyal (*h.hy.*) of *Siredon* has no representative in its branchial arches, and in the hyoid arch the upper piece or hyo-mandibular is entirely suppressed.

The cerato-hyal (with the small distal segment) corresponds to the free mandible, to which it is attached by the mandibulo-hyal ligament (*m.h.l.*); the hyoid cornu is also strapped to the suspensorium by the hyo-suspensorial ligament (*h s.l.*).

The next two arches have a suspensorial piece more than twice the size of the lower or free cornu; the next two have no lower segment. In the two large front arches even there is therefore no hypobranchial below nor pharyngo-branchial above. The latter part is represented by a sort of hammer-head to the two middle bars, the first and fourth bowing towards this transverse process (Plate 26. fig. 5, *e.br.*). The two cerato-branchials (*c.br.* 1, 2) are stout, straight styles, the first articulating below to the sides of the basal piece behind, and the next to its end. The three first epibranchials have a long, single, feather-like gill (*e.br.*) attached outside the upper third; these are about the length of the bar from which they arise. The first bar has a snag near where the gill is attached. Tooth-like processes interdigitate between the arch and form the usual gill-colander in this group. The first basibranchial (*b.br.* 1) is a long, oval, thick, high cartilage, rounded in front, and having the hyoid arch attached to it by ligaments. It serves as a conjugational piece for the hyoid and the first two branchial arches. The second basibranchial (*b.br.* 2) has no arch attached to it; it is compressed in front and flat behind, where it becomes spatulate. It is twice as long as the first, and it is the homologue of the so-called "uro-hyal" of the bird*.

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* It should be borne in mind that the suspensorium of the mandible, like the hyo-mandibular (with its symplectic peg), answers to the pharyngo- and epibranchials together. The mandible, the hyoid cornu, and the cerato-branchials also correspond.

Whilst the gills keep their full size and function, this second basal piece keeps its larval condition. Its transformation is a correlate of gill-wasting; when these wither the most remarkable transformations are set up.

Certain investing bones which were appearing in the last stage are now well seen. The prootic and pterotic have been developed *in* the otic capsule, and now the nasals and "osteo-ectoethmoids" (*e.eth.*) have appeared. The nasals (Plate 26. fig. 1, *n.*) are thin shells of bone that form the inner side of the valvular outer nostril (*e.n.*); they are subtriangular, and are wedged in between the nasal processes of the intermaxillaries and the next bone. This bone, the outer ethmoid (*e.eth.*), is here a mere splint; in Osseous Fishes it forms the ossifying plate of the great prefrontal cartilage ("Salmon's Skull," plate 5. fig. 7, *l.e.*). This bone is twice the size of the nasal, is much like the shell of a Mussel or a *Cythere*; it articulates with the ascending plate of the maxillary (*mx.*), and, like the nasal, overlaps the frontal (*f.*). The investing ethmoid and the nasal apply themselves, without ingrafting, to the cartilaginous nasal roof (*na.*). The large semioval frontals (*f.*) creep beneath these bones and the nasal processes of the premaxillaries, and in their turn overlie the parietals (*p.*), which reach the ethmoidal plates. The two great roof-bones on each side are nearly of a size; the parietals have a temporal process and fossa; a notch divides them behind, where they nearly cover the superoccipital plate.

The growing premaxillaries and maxillaries (*px.*, *mx.*) now form a very round fore face; their outline altogether forms half a neat ellipse.

The squamosals are still, like preoperculars, stout and nail-like below, where they bind on the quadrate; they are digitate, with many splintery divisions above, where they thatch the auditory eave.

The vomers and parasphenoid (*v.*, *pa.s.*) are still quite larval; the former are mere arcuate, obliquely placed plates, bearing a rasp of teeth, and the latter is a splintery plank, somewhat scooped above, where it floors the great basicranial fontanelle: almost oblong, it has some little side-growth into the ethmoidal and basitemporal regions.

I have the more carefully gone into the details of this stage because of its morphological interest, as being one good illustration of a perennibranch. When once its very low and primordial gills wither, then a new life crops up in every part of the creature, transforming it both without and within. It is good and profitable for comparison with what is seen in *Proteus*, *Menobranchus*, and *Siren*, but has also many things that are the likeness of what is found in the Selachians and the Dipnoi. It has, indeed, more likeness to them than to its neighbours, the Batrachia.

The next stage takes rank with those Urodeles, such as the great Japanese *Cryptobranchus* and the Menopome, in which the branchiæ almost disappear. These form an intermediate group of great value, which may be called the *Cryptobranchiata*.

Ninth Stage. Large Axolotl ($8\frac{1}{4}$ *inches long*), *with gills reduced to* $\frac{1}{4}$ *inch in length.*

This instructive stage I got from Mr. Tegetmeir's vivarium. It was about the size of the last, and, metamorphosing late, probably would never have gone beyond the cryptobranch condition.

On the whole, the description just given will serve for this; but there are several points of difference that must be noticed.

The bony deposits and plates are denser, and there is altogether a nearer approach to the Salamandrian type of skull. The hyo-branchial cartilages are as large as in the last, but the second basibranchial has acquired a bony centre (Plate 27. figs. 1 & 5).

The exoccipitals, prootics, pterotics, and spheno-ethmoids (Plate 24. fig. 5, and Plate 27. figs. 1 & 2, *eo.*, *pro.*, *pto*, *sp.e.*) are unaltered. The stapes is now a very solid cone of bone, and its fan-shaped ligament is seen to pass *above* the portio dura to be inserted into the edge of the suspensorium (Plate 24. fig. 5, and Plate 27. fig. 1, *st.* 7). Thus we have here what may may be called a *spiracular fascia*, the counterpart of the spiracular cartilage and bone of the Menopome and others.

There is still a large tract of cartilage in the cranial wall, through the fore part of which the optic nerve (2) passes. The bony tract (*sp.e.*) only reaches to the front margin of the great basal fontanelle, which is margined by a huge tract of internasal cartilage (*i.n.c.*), the outer angles of which have the olfactory crura lying across them.

A great change has taken place in the antorbital cartilage (Plate 27. fig. 3, *e.pa.*). Instead of being a tongue-like flap, projecting outwards and looking forwards, it is now a flat, oval wedge, with its narrow end in front, and closely applied to the articular face of the projection of the ethmoid to which it belongs.

This is a rare condition in the Urodela, for this cartilage generally retains its relative size and coalesces with the back of the nasal roof. In the next stage we shall find the palatine bone assuming a rare condition also, being directed transversely outward, instead of gaining a longitudinal position (Plate 24. fig. 6, *pa.*).

But at present the palatine bones (Plate 24. fig. 5, *pa.*) are more nearly parallel with the axis of the skull than they were. One thing is noteworthy, namely, that there is a definite space between them and the pterygoid bones.

The vomers, premaxillaries, maxillaries, and squamosals all retain their larval condition (Plate 24. fig. 5, and Plate 27. figs. 1 & 2, *v.*, *px.*, *mx.*, *sq.*); but the parasphenoid (*pa.s.*) has become more elegant in form, and is nearer to the state it assumes in the Salamandrian stage (Plate 24. figs. 5 & 6); its ethmoidal region is more outspread, and the basitemporal angles are more definite.

Professor HUXLEY has always contended with the writer for a continuity of both the pedicle and ascending process with the base and side wall of the skull in the Axolotl. This I could never see in the pedicle. I have shown that the ascending process is confluent with the cartilaginous skull-wall; but the pedicle is very short for some time, and never coalesces with the skull. Its relation is to the antero-inferior part of the auditory mass, where that is plastered over with secondary cartilage.

A section through the top of the suspensorium at this stage (Plate 27. fig. 4) shows that the pedicle is quite distinct from the auditory capsule (*pro.*, *a.s.c.*). The section was made *behind* the confluence of the ascending process with the alisphenoidal wall; consequently it also is free in this section.

This section also shows how the squamosal (*sq.*) binds on the outer face of the suspensorium, and the pterygoid bone (*pg.*) on its under face. The substance of the suspensorium is being converted into the quadrate bone (*q.*).

One of the most interesting correlations of structure is that of the second basibranchial with the rudimentary larynx in this group. In the perfect Axolotls (Plate 25. fig. 5) this part is a slightly gapped, broad-edged chisel; it is now a delicate fork (Plate 27. fig. 5, *b.br.* 2). More than the hinder half is ossified—all but the tips of the prongs, and these have the larynx lying behind and between them. They are related to that organ like the single spatulate "urohyal" of a bird*.

This is only a step, however, to complete separation of the second basibranchial from its old attachments (Plate 25. fig. 6, and Plate 27. fig. 7, *b.br.* 2).

These modifications, which have taken place in a full-sized Axolotl, whose gills were only partially absorbed, are but faint indications of what the innate metamorphic energy is capable of performing, when the changes take place timely, so as to convert the Axolotl into a kind of Salamander.

Tenth Stage. Skull of an adult Amblystoma opacum, 3 *inches* 10 *lines long.*

This is a North-American species, closely related to, if not identical with, the form into which the Axolotl changes when its metamorphosis is complete.

The whole appearance of the skull is changed; it is altogether neater, more solid, and its narrowed, more elongate form is much more like that of a Frog.

In fact it has undergone changes quite similar to those which the skull of a metamorphosing Frog undergoes in passing from the short-tailed condition to that of the adult.

Nevertheless the adult skull of the Caducibranchiate Urodeles and that of the Batrachia are very different; the former types belong to a lower and more lateral *fork* than the latter, but both are terminal or culminating. There is no inosculation. Indeed, whilst the Urodeles may be considered as an upgrowth from the Dipnoi, the Batrachia must needs have had ichthyic forefathers, in whom the characters of a Lamprey and a Chimæra were combined ("Frog's Skull," pp. 193, 194).

The process of ossification in the occipito-auditory region has been very intense, and has obliterated all the sutural landmarks.

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* In one of our discussions Prof. Huxley controverted my description of the formation of the "thyro-hyals" of the Frog (see "Frog's Skull," p. 172), where they are described as primarily symmetrical and double, because his observations on the Newt showed that its thyro-hyal is formed of a basal azygous piece. They are double, however (as in Mammals), in the Batrachia. That instance, of inferring from the Urodele what would be found in the Batrachian, was parallelled by me when I inferred from the larval Salamander that the stapes of the Frog would be developed by segmentation off of a part of ready-made cartilage taken from the ear-capsule.

But a broad tract of cartilage still divides the two sides of the occipital ring, both above and below (Plate 24. fig. 6, and Plate 27. fig. 6, *s.o.*, *b.o.*).

The ear-masses now stretch outwards as in the Frog and Lizard, and their divergent growths are extended into drooping arms by the addition of the transverse suspensoria, now forming even more than a right angle with the axis of the skull.

The elegant form of the canals of the labyrinth can be well seen in the solid bone that has been fashioned as a mould over them (Plate 27. figs. 6 & 7, *pro.*, *e.o.*). They are, however, partly hidden by the parietals and squamosals (*p.*, *sq.*). The bulging, bony floor of the vestibule forms a widely crescentic bulla, and in the notch the fenestra ovalis contains a small lenticular stapes, the centre only of which is ossified. The spiracular ligament fastens the stapes to the back of the top of the suspensorium (Plate 24. fig. 6, *st.*, *st.l.*).

Further forwards there is an unossified alisphenoidal tract between the fifth and optic nerves (Plate 24. fig. 6, and Plate 27. fig. 7. 5, 2), but the lateral walls are ossified from the optic foramen up to a short distance behind the inner nostrils (*i.n.*). The rest of the endoskeleton is unossified; the cornua trabeculæ lessen considerably after that the vomers have fairly taken their form and pattern; the internasal cartilage (*i.n.c.*) is seen below, with its rounded notch in front of a mimetic notch on the fore margin of the parasphenoid (*pa.s.*). Outside (Plate 27. fig. 7, *e.n.*), the external nostrils are seen to be surrounded by the nasal roof-cartilage.

The roof of the skull is now exquisitely imbricated—all the dense, polished, well-shaped bones overlapping from before backward.

The premaxillary margin, arcuate instead of transverse, passes into the lateral sweep of the long maxillaries (compare Plate 27. figs. 2 & 6, *px.*, *mx.*). The palatal portion of these foremost bones is but little developed, only showing a headland for the close-set teeth. The nasal processes of the premaxillaries (Plate 27. fig. 6, *px.*) are thus visible below (Plate 24. fig. 6). They are well approximated, leaving a chink where the median passage was, and they are large, flat splints, and prefigure, very accurately, those of a bird. The maxillaries (*mx.*) have a narrow palatine edge within the teeth. They have a good face-plate (Plate 27. fig. 7), on which rest the nasal and ecto-ethmoid bones (*n.*, *e.eth.*). As in the Frog, a small rhinal ossification, the septo-maxillary (*s.mx.*), rests upon that part of the maxillary which is articulated to the dentary angle of the premaxillary. The zygomatic process now rivals in height and length that of a high type, such as a Lizard or a Bird; and, exceptionally in this type, it is surmounted at its end by a small seed-like bone, manifestly the jugal (*j.*). There is no quadrato-jugal as in the Frog; but the Urodeles are not of the same stock as the Batrachia. In their ascent, however, they often run almost parallel with them. The nasals (*n.*) are thin shells of bone, with a concave margin towards the nostril, a straight one towards the nasal processes of the premaxillaries, and a ragged, wedge-like hinder margin, which overlaps the frontals, as their outer edge does the ecto-ethmoids (*e.eth.*). These latter bones are now much larger than in the *Siredon* form; they rest upon, and are somewhat overlapped by, the face-plate of the maxillaries. Each bone is twice the size of the nasal. Bending inwards from its fore, broad, overlapped part, it helps to give an orbital margin to the skull in front, and then becomes attenuated where it ends in the middle of the superorbital region, just where the parietal runs under the frontal*.

This very *Varanian* skull has no superorbital or lacrymal on the outside of the dense, large ecto-ethmoid. The frontals and parietals (*f.*, *p.*) are nearly equal in size; they are dense and smooth. The former dip a little towards the median suture, and the latter rise into a very gentle parietal crest. The frontals run under the nose-roofs nearly to the outer nostril; behind they end at a transverse line over the optic foramina.

The parietals (*p.*) form the coping of the skull-wall all along, reaching so far forwards as to underlie the ecto-ethmoidal tiles. A little deficiency appears between the ends of the frontals, precisely where, by more perfect ossification, the Lizard finishes its small pinhole-shaped "parietal fontanelle."

The correlated growth of these now dense and finished parietals to the hind skull and ear-capsules has so completely modelled them over these parts, that it would seem as though they had been applied in a soft state, and then worked on to the subjacent structures.

Although mainly roof-bones, their *parietal* portion is greater than in the Monitor; as in that Lizard there is a supratemporal crest and fossa; and in like manner the edge of the bone is notched gently to fit over the swelling on the prootic caused by the arch and ampulla of the anterior canal.

The junction of the anterior and posterior canals (Plate 27. fig. 6) is just roofed over by the parietals at their posterior angle. They then run transversely across, with dentate edges, overlapping the front part of the occipital roof.

If we would compare the parasphenoid of *Amblystoma* (Plate 24. fig. 6, *pa.s.*) with that of *Monitor*, it should be for contrast. In the latter it is a most delicate style; in the Urodele it is an almost complete floor for the large, flat, barge-shaped skull.

It is roundly notched in front, extended outwards towards the antorbital region, subcarinate in the pituitary region, alate beneath the foramen ovale and Gasserian ganglion, and narrows rapidly towards its transverse hinder end.

The vomers that did but form the back of the narrow arcuate rasp (Plate 24. fig. 5, *v.*) now (fig. 6, *v.*) are leaves of bone, as large as the leafy trabeculæ of the *Siredon*. The row of teeth is arched in the opposite direction, namely, backwards, and occupies only the hind edge of the bone, where it overlaps the parasphenoid.

Each vomer is now roundly notched at its outer margin for the internal nostril.

The oblique outer margin in front fits by a jagged edge against the rudimentary palatal plate of the maxillary.

These vomers are very Batrachian, but they are still larger than in the Frogs and Toads. The relation of the ragged outer edge to the narrow ingrowth of the maxillary

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* If the reader will compare this description and the figures with the skull of a *Monitor* and of a young bird, he will see how nearly this Amphibian skull approaches them.

is quite similar to what are seen in the long-faced Monitor, whose vomers are long, and only moderately broad, planks.

We saw that in the branchial stage the teeth were distributed in two concentric semicircular rows. Those of the premaxillary had the vomerine rows within and behind them, and the maxillary series were similarly imitated by those of the bony palatine (Plate 24. fig. 5). Now all is changed (fig. 6); for not only are the vomerine rasps turned the other way, but the little palatine, with its row, has its gently convex edge looking forwards, and *vice versâ*, the opposite to the row on the vomer, and nearly at right angles to the maxillary series; this type is a "lechriodont."

This little bone, which has turned outwards, like a railway signal, is steep; it helps to surround the inner nostril, and ties the vomer to the maxillary.

This is not a rare condition for the palatine bone of a Urodele; it occurs also in the Batrachia; for instance, in *Ceratophrys*, where it is attached to the under surface of the ethmo-palatine ectostosis.

But as a simple bony plate it may be compared with the palatine of the Monitor Lizard, which has but little longitudinal extent, and, growing directly outwards, forms a second or anterior "os transversum," tying the vomer to the maxillary, as in *Amblystoma*. That process of the pterygoid bone which fastens on to the palatine (nearly reaching the vomer), in the Monitor, is absent in *Amblystoma*; but the apex of its pterygoid runs outwards towards the jugal exactly as in that Lizard. Thus the palatine and pterygoid plates, which were in the larval state one bone, are now far apart, and their axes, which were coincident, are now at a right angle with each other.

Moreover, in the fourth stage, the pterygoid was seen to have arisen as a mere process of the palatine; it is now twenty times the size of the old stock from which it detached itself.

The pterygoid is, like the palatine (or "ethmo-palatine"), composed of a cartilage and a bone; for the palatine dentigerous plate is the bony counterpart (or companion) of the antorbital cartilage; and the pterygoid bone, by using up the substance of the "chondropterygoid," has enucleated, as it were, a distinct epipterygoid cartilage (Plate 27. fig. 7, *e.pg.*)*.

The pterygoid (Plate 24. fig. 6, *pg.*) now binds more completely under the pedicle, which it hides, and the quadrate, or lower part of the suspensorium. The quadrate bony centre has ossified the greater part of the suspensorium, and even the narrowed ascending process is bony (Plate 27. fig. 7).

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* This unossified rod is partly the fixed postpalatine cartilage of a former stage, and partly the anterior end of the tongue of cartilage which grew from the front of the suspensorium.

In the Chelonians and Lizards a similar rod becomes *enucleated* from the indifferent tissue covering the bony pterygoid; this becomes semicartilaginous, and is then invested with a bony sheath. It keeps its half-prostrate posture in the Chelonia, but in Lizards uplifts itself, gradually, to an almost erect position; this "columella" has its base, then, resting on the bony pterygoid, and its capital leaning against the anterior part of the prootic above (*Monitor*), or reaching, *by its upper epiphysis*, to the lower edge of the parietal (*Lœmanctus*).

The direction of the squamosal (*sq.*) shows that the angle formed by the suspensorium with the axis of the skull is more than a right angle; this bone is now a very dense strong plate, knobbed and transversely ridged above and below, spiked in front, and lobate behind.

The squamosal of the Amphibia represents the preopercular and the great "supra-temporal" lying over and above it in *Clarias capensis*, both in one piece. But in *Polypterus* (TRAQUAIR, *op. cit.* plate 6. fig. 7, *y*) we have the exact counterpart of the Amphibian squamosal. In the figure referred to there are four "post-spiracular" plates lettered *z* (p. 180); one of these appears on the right side only in *Amblystoma* (Plate 27. fig. 6, *s.t.*). I have seen this bone in no other of the group; but it is as valuable a remnant as the spiracular cartilage of *Menopoma*, *Spelerpes*, and *Desmognathus*.

The bone which Professor HUXLEY calls "angulare" now can claim the title I have all along given to it, namely, "articulare" (Plate 27. fig. 7, and Plate 25. fig. 7, *ar.*). It is not now even a deep trough, merely, for the articular part of the mandible, but has ensheathed the cartilage, converting much of it into bone, and rises high, and turns in considerably, in the coronoid region.

The dentary (*d.*) nearly reaches the angle; it has much less affinity for cartilage than the articulare; it is gently angular in the middle, and roundly inturned towards the mentum. The splenial (*sp.*), with its fine saw-like row of teeth, is slender, and occupies most of the fore half of the inner face of the mandible.

The two rami (Plate 25. fig. 6, *d.ar.*) together form half a long and elegant ellipse; their apposed distal ends are enlarged, and tied strongly together by ligamentous substance.

The shrinking and then the absorption of the gills has been attended with very curious changes of the other arches (compare Plate 25. figs. 5 & 6).

The hyoid cornu (*c.hy.*) was oval in section, therefore thick; it is now a flat tape; it is still attached to the mandible by the mandibulo-hyoid, and to the suspensorium by the hyo-suspensorial ligament (*m.h.l.*, *h.s.l.*).

The hypo-hyal (*h.hy.*) was only a gently attenuating fore part of the main rod, just separated by a tract of fibrous tissue; now it is a very slender terete rod turned backwards from the cerato-hyal by a sharp angle, and it has coalesced with the front corner of the first basibranchial (*b.br.* 1).

The first epibranchial is still a stout cartilage, but the distal piece, or first cerato-branchial (*c.br.* 1), has escaped from it, and thus the upper piece has travelled down to the basal piece.

Moreover, the distal piece is now a mere hypo-branchial as to size, and, like the hypo-hyal, it has turned up, in the manner of a railway signal, becoming nearly parallel with the piece in front.

The second cerato-branchial and both the next arches are absorbed, and the second epibranchial (*e.br.* 2) articulates with the narrow end of the first basibranchial (*b.br.* 1).

That element is flatter in its front part, which is ossified, and longer in its hind part.

The second basibranchial (Plate 27. fig. 7, and Plate 25. fig. 6, *b.br.* 2) has lost its shaft, and only the bony forks, with soft extremities, remain, as the V-shaped thyro-hyal, now part of the laryngeal apparatus.

It is easy to harmonize this hyo-branchial structure with the so-called hyoid of a Bird. The symmetrical cartilages that form the skeleton of its tongue are hypo-hyals that unite and partly ossify; the so-called basihyal of the Bird answers to the first basibranchial of the Newt, and the hinder piece is the second basibranchial. The long cornua of the Bird are the moieties of the first branchial arch—epi- and cerato-branchials ("Fowl's Skull," plate 87. fig. 11).

I think that the stages of this single type will be considered very instructive by all whose pleasure it is to work out the morphology of the skull; to me it seems to be a lamp giving light to all around.

For want of space I am only able to give the lesser part of my researches into the structure and development of the Urodelous skull; but to add to the supposed value of the account of the type above given, I am anxious to describe the *lowest* kind of adult skull, that of *Proteus anguinus*, and that of one of the *highest*, namely, *Seironata* (*Salamandrina*) *perspicillata.*

The larva of the latter has yielded some important conditions of the ground-plan of the skull, and these will be also described.

Skull of the adult Proteus anguinus.

The simple skull of this the lowest known Perennibranchiate Urodele is in a general way comparable to that of small larvæ of *Siredon*; but whilst in some things it is comparable even to the second stage, just described, in others it has to be compared with the largest gill-bearing Axolotls.

It is, however, in some respects unique, and in others only comparable to the most generalized and ichthyic of the group; the difference between its skull and that of the Caducibranch next to be described (compare Plates 28 & 29) is immense.

The half-ossified chondrocranium (Plate 28. figs. 1, 4, 5), simple as it looks, is not in a *primitive* condition; certain tracts of cartilage have been absorbed so as to give it an appearance of bilateral separateness which it did not possess at first, and, on the other hand, bony deposit has in some places bridged over tracts that were always scant of cartilage.

The fewness of the investing bones is very instructive,—just a few more than we find on the chondrocranium of the Lepidosiren; and these are those that appear first in the larvæ of the higher kinds of Urodeles.

Yet this skull-building would have been but half complete if these "shingles" had not been superadded; for here there is no such free growth of cartilage as is seen in those types (the Selachians) whose dermal bones retain their independence, and show no affinity for the skull.

But few as these subcutaneous hard tracts are, they are brought completely under the influence of the chondrocranium, which, itself also, has acquired the power of converting considerable tracts of its own substance into a hardish kind of bone.

This skull, comparable to that of an Eel or a Snake, has its lateral halves only conjugated slightly before and behind; thus the main part is devoid of any intrinsic floor; the basal fontanelle is very large indeed.

If the hind pair of sense-capsules (*pro.*, *ep.*) were not fused with the skull proper, it would indeed be a feeble framework—just a pair of feebly out-bent bars slightly soldered together at either end, and having the facial rods very loosely swung to each side-piece, the main pair, only, confluent at one small point (*pd.*)*.

The superoccipital ring was imperfect in its cartilaginous condition (Plate 28. fig. 4, *s.o.*); but the bony substance has formed a very narrow keystone. Below, the shrunk and retired notochord lies on a narrow ossified bridge of cartilage, and thus the ring is complete. The condyles (*oc.c.*) are large, non-pedunculate, and look inwards; the large notch between them betokens the presence of an odontoid process on the first vertebra, not the homologue of the mammalian spike, but an aborted intercalary vertebra. The occipital floor is inseparable behind from the parasphenoid, which it thickens postero-laterally, the bony matter running thence into the base of the ear-capsules on their inner side. By the analogy of *Menobranchus* I should suppose that the epiotics were at first distinct; they are not now. The ear-capsules are like egg-shaped fruits, and are very large, relatively; unlike those of most of their order, they show scarcely any impress of the arched tubes and swollen bags within. The whole capsule, like the Diatomaceous *Isthmia*, is composed of two hard cases united by a zonular intermedium; *this* is composed of unchanged cartilage; *they* are the prootic and epiotic bones (*pro.*, *ep.*) (the latter includes the opisthotic), and their appearance is as if they were ready to dehisce transversely, like the pyxidium of the Pimpernel (*Anagallis*). These thin divisions are subequal, the prootic being the larger. Infero-laterally there is a long fenestra ovalis, and its adapted stapes (*st.*) is a long oval shell of bone, a little soft at its narrower front end. The epiotic is roughened with bony granules where it projects backwards as the hindermost part of the skull; in front the capsule is lowest, and the granules mark the uplifted epiotic apicular cap.

Instead of the prootic applying a long wall-plate to the side of the skull, as in *Siredon*, it sends forwards a most minute cup, and into this the end of the trabecula fits. That was not the *end*, but the *elbow* of the bar, whose notochordal outspread portion has been absorbed. As in the Snake, the trabeculæ persist as filiform cartilages; they are

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* Professor HUXLEY's paper on the kindred form, *Menobranchus lateralis* (P. Z. S., March 17, 1874) will be incessantly referred to in this description; whilst writing it I have dissected the skull of one $10\frac{1}{2}$ inches long, and probably somewhat younger than his specimen. In mine there is a superoccipital band $\frac{1}{20}$ of an inch across, and an ossified bridge of cartilage below the notochord, just in front of the foramen magnum. The huge epiotics were thoroughly distinct, which is a remarkable character, as they are generally only a tract of the exoccipital. The absence of bony matter in the sphenethmoid region is very remarkable; the want of cartilage, across, behind the pituitary body, is due to absorption of the flat parachordal apices, or hind end of the trabeculæ; it is not a primitive condition.

thicker towards each end than in the middle, and only become bony in the ethmoidal region (*tr.*, *sp.e.*). Thus, in this respect, these bars correspond with their earliest condition in the other types; in a larval *Siredon* less than half an inch in length they have a high crest; and in *Menobranchus*, where they do not ossify, they are high, or crested.

They are almost absolutely parallel from their prootic cup to their ethmoidal shaft-bone; the appearance of inbending, at the middle, is due to attenuation of the bars. A little behind the inner nostrils (*i.n.*) the trabeculæ gently turn inwards, and this inflected part is two fifths the length of the straight or interorbital part.

But the great lower fontanelle is extended forwards to nearly the middle of the inflected tracts; there the rods are ossified; the rest is soft, and to the frontal wall are almost entirely confluent.

This confluent part is a flat internasal cartilage (*i.n.c.*); it is wedge-shaped. Nearly the anterior third is notched, thus forming two short straight horns, the trabecular cornua (*c.tr.*).

Proximally, on the inflected part, there is on each side a free horn of cartilage; it is gently arcuate, looking outwards and a little backwards; this is the antorbital or ethmo-palatine element (*e.pa.*). The internal nostril (*i.n.*) lies in the obtuse angle in front of this bar, and outside the ossified tract of the trabeculæ; its outer margin is a strong fibrous band or fascia. Inside this opening the ethmoidal bony tract of the trabecula is burrowed by the olfactory nerve (1).

As in *Menobranchus*, there is no cartilaginous nasal roof; these types, therefore, correspond to very young Axolotls, and are below *Siren* and *Menopoma*, the former possessing a small distinct nasal roof-cartilage, and the latter a very perfect cartilaginous capsule, with which the large antorbital cartilage coalesces*.

The next cartilaginous element to be considered is the suspensorium, whose free segment is the mandible or, rather, its pith, the articulo-Meckelian rod.

The direction of this arch and its pier is almost directly forward, as in the newly hatched Axolotl (Plate 23. figs. 1, 2, and Plate 22. fig. 3) and as in Batrachian larvæ.

Also, as in Batrachian larvæ, the suspensorium has only one junctional process at its apex, the pedicle (*pd.*), which is fused with the trabecula below where it is ensheathed by the prootic (fig. 7, *tr.*, *pd.*)†.

The "tuberculum" of this bar, the otic process (*ot.p.*), is a remarkably slender, *free*, digitiform outgrowth; it passes obliquely over the front and outer face of the ear-capsule, the portio dura (7^2) emerging beneath its middle.

The only sign of a pterygoid process is in the rather suddenly convex outline of the flat fore margin of the upper, unossified part of the suspensorium. The lower half is a very solid but compressed mass, ossified save at the scooped articular face; its inner side has the styloid end of the osteo-pterygoid process attached to it (*q.*, *pp.g.*).

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* WIEDERSHEIM (*op. cit.* plate 1. fig. 1, N.K.) figures the skull of *Menobranchus* with a fenestrate nasal cartilage.

† In the abstract of my second paper on the Batrachian Skull (Proc. Roy. Soc. No. 165, 1875, p. 141) it is said that the ascending process only is present; this is a mistake.

The free arches will be described after the bony plates of the skull.

The upper surface of the skull is imbricated with four pairs of long, splintery, bony plates, and the lower surface is faced with two pairs and an odd one; these bones are the counterparts of the earliest bones in the larvæ of higher kinds.

Above and in front, the premaxillaries (*px.*) are seen to be almost entirely composed of the nasal process, the dentary part being only one third the length of these roofing spars, whose hind half covers the apices of the long frontals. These latter bones (*f.*) are wedges more than half the length of the skull; their broad interocular part is gently convex above.

The parietals (*p.*) are of the same length, their fore half wedging in under the frontals; they are still more convex than the frontals, for they form much of both the side walls, as well as the roof of the skull (see fig. 8, *f.*, *p.*, *pa.s.*, *tr.*). Near the ear-capsules they send out the normal angular process; they rise very gently towards the mid line, and their outer superauditory margin is almost straight. Behind, they form an arched emargination, parallel with, and slightly in front of, the foramen magnum above. Both sides, behind, and the hind margin also, are raised and thickened at the edge. Running parallel with and a little outside their outer margin, behind, there runs a pair of bones, the squamosals (*sq.*); the fore half of these plates is free of the cranium, and runs along the postero-external edge of the suspensorium, with the "habit" of a preopercular.

The supratemporal and the preopercular halves of the squamosal are both lanceolate, but the upper half is nearly twice as broad as the antero-inferior. The two halves are united by a slender middle portion, and are gently bent on each other, the arched margin being in front. A little below the middle the concave margin gives off a slender process, at an acute angle, half the length of the lower part; it runs backwards, and is bent also a little outwards, and passing over the seventh nerve (7^2) is attached to the fore edge of the stapes (*st.*).

This may be called the "spiracular process" of the squamosal; it exists in rudiment in *Menopoma*, and is above its spiracular cartilage; in *Menobranchus* it is almost as large as in *Proteus* (Huxley, *l. c.*).

The down-bent part has copied the curve of the suspensorium exactly; the elegant, lanceolate supratemporal part protects the broad cartilaginous zone of the ear-capsule, and runs halfway along the epiotic.

The parasphenoid (*pa.s.*) runs but little short of either end of the skull; in front it is wedged between the internasal cartilage and the vomers (*i.n.c.*, *v.*), and behind it has gradually broadened until it forms a huge floor, whose slightly extended basitemporal angles protect the otic zone, nearly to the fenestra ovalis, on each side; and then with a narrowing margin, five times notched, it underlies and coalesces with the feebly developed occipital floor (*eo.*). The trabecular rods can just be seen outside the edges of the parasphenoid (figs. 3 & 8, *pa.s.*, *tr.*).

The vomers (*v.*) are dentigerous, they have a single outer row of teeth; wedging in behind the feeble, facial, dentigerous parts of the premaxillaries, they run back to the antorbital cartilage, and are one third the length of this skull. The inner nostril (*i.n.*) is outside their hinder part, and they there join the pterygo-palatines (*p.pg.*).

The lanceolate vomers diverge considerably behind, and are slightly out-turned; they form the bony margin of the face in front, as there are neither maxillaries, nor nasals, nor ectoethmoids in this type.

The next pair are of the same length as the vomers; these bones, the pterygo-palatines (*p.pg.*), are more developed here than in *Siren lacertina*, where there is no pterygoid process, the plate being arrested at the same stage as in newly hatched Axolotls; in *Proteus* they correspond with the *4th stage* of the Axolotls (Plate 22. figs. 4, 5, *p.pg.*), in which the short dentigerous bony plate has sent a ragged process of bone backwards into the pterygoid region. In *Proteus* the bone is shaped like a "battledoor," but the handle is pointed; a few teeth run along the middle of the broad part; the pointed end binds inside the suspensorium.

The mandible, as in *Siren*, is very strong and steep; its front three fifths is surmounted by a phalanx of high flattened teeth; its hinder part rises high as a rounded boss of cartilage, which lies in the deep trough of the strong articulare (*ar.*), whose angle is tuberculate, like the epiotic.

The steep strong dentary (*d.*) reaches nearly to this angle; the articulare runs far forwards; but the inner face of the mandible shows much of Meckel's cartilage, unprotected. Here, unlike other Urodeles, there is no "splenial," but there is a rough, clubbed bar of bone in front, formed by ossification of the distal part of the cartilage; and although not distinct from the dentary, it corresponds with the "mento-Meckelian" bone of the wide-chinned Frog.

This absence of the splenial, and ossification of the cartilage in front, is a second point in which this low type agrees with the Batrachia*; the other was its having no "ascending process" above the pedicle.

The hyoid arch also is most anomalous, and but for the ossification of the lower piece, would correspond rather with a Shark than with any known Urodele.

Certain kinds, namely, *Siren* and *Menopoma*, have an upper cartilage (hyo-mandibular); but in them it is scarcely one fourth the size of that of *Proteus*, and is partly confluent with the suspensorium.

In the two former types the upper piece is about as much developed as in *Ceratodus* (HUXLEY, *l. c.* p. 35, fig. 6, *H.M.*). In *Proteus* it is larger than in *Cestracion* (*ibid.* p. 42, fig. 8, *H.M.*) and many times the size of that of *Notidanus* (*ibid.* p. 44, fig. 9).

The hyo-mandibular of *Proteus* (*h.m.*) is a large, short ray, thick and solid above, but scooped on its narrowed, lower part, to form an oblique concavity for the condyloid head of the cerato-hyal. The axis of the whole hyoid is parallel with that of the whole mandibular apparatus; the extended top of the hyo-mandibular is attached to the lower

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* WIEDERSHEIM (*op. cit.* plate 2. fig. 16) gives a splenial to the mandible of *Proteus*.

edge of a large fascia which runs along from the upper part of the suspensorium, behind and beneath the stapes (*st.*) and is attached to the infero-lateral face of the epiotic (*ep.*).

The cerato-hyal is a stout rod, not so thick as the upper piece, and it is invested with a strong ectosteal sheath, all but its extremities; the lower end is bent upwards, and has no hypo-hyal segment, another anomaly in this type; for that is a very constant segment in the Urodeles.

Antero-superiorly, the cerato-hyal is attached by a broad hyo-suspensorial ligament to the quadrate, and by a narrow mandibulo-hyoid ligament to the angle of the articulare (*h.s.l.*, *m.h.l.*).

Proteus belongs to a small minority of the Urodeles that have no *fourth* branchial arch; *Menobranchus* also has only three, and certain lechriodont Caducibranchs, namely *Spelerpes*, have only three, as I find in the larvæ of *S. rubra* and *S. salmonea*.

In *Menobranchus* only the second basibranchial is ossified of all the postmandibular structures; in *Proteus* only one small segment in the middle of the second branchial arch, besides the hyo-mandibular, remains soft.

The first branchial arch (*br.* 1) is very stout; it is normal in having a very long epibranchial and a very short cerato-branchial (*e.br.* 1, *c.br.* 1); its pharyngo-branchial, like that of the second and third, is represented by its unossified apex.

The next, much smaller bar is similar, but it has a short, thick, unossified segment wedged in between the upper and lower pieces; to this the third branchial is attached, and has no lower piece unless the intercalary segment belongs to it. This inwedged piece does not occur in *Menobranchus*, nor in the Urodela generally; in that type the second cerato-branchial is very small.

The well-ossified basibranchials are of the same length; but the foremost is the thicker by far; its fore end is strongly tied to the distal parts of the hyoid (*b.br.* 1, *c.hy.*); the hinder piece is not metamorphosed in relation to the larynx.

This most instructive type thus yields a skull well worthy of being placed between the Dipnoi and the higher Urodeles. Of *sixteen* species of its Order, worked out by me, it is the most abnormal.

On the Skull of Seironota perspicillata.—*First Stage. Larvæ ½ an inch long.*

This Caducibranch is one of the smallest I have worked out; the adult is only 2 inches long. It is a native of South Europe.

The larva, besides coming in well as an intermediate stage between my third and fourth stages of *Siredon* (Plates 22 & 23), has also some important characters of its own; characters not so clearly shown in the fry of the Axolotl.

The adult, also, carries the *Salamandrian* modification of an Amphibian skull to the highest level, the *finish* of the cranial building being very perfect, and analogous to what is seen in the skull of the higher Batrachia, and even in the Reptiles.

The larval skull (Plate 29. figs. 1, 2) shows what, a year or two since, Professor Huxley considered to be a thing not known, namely, the perfectly distinct chondrification of the parachordals (*iv.*) behind, from the trabeculæ (*tr.*) further forward.

In the third stage of *Siredon* (Plate 23. figs. 1, 2, *tr.*) the trabeculæ were *parachordally related* to the notochord, taking in full half of its cranial part; but in front they still only reached to the vomers, not up, even, to the nasal sacs.

In the next stage (Plate 22. figs. 4, 5) the parachordals are well developed and *quite distinct*; the trabeculæ, also, have grown forward into the frontal wall; not, however, by the direct elongation of the primary rods, which have now grown only to the middle of the nasal sacs; but a large flat transverse "internasal plate," ending in lunate horns, finishes the skull in front.

There is evidently some difference in the method in which the highly modified *axial skeleton* ends, even among the Caducibranchiate Amphibia; for in *Seironota* (Plate 29. figs. 1 & 2, *tr.*, *c.tr.*) the paired rods grow directly forward, only gently lessening in size, right into the frontal wall; there they send out a small *facial lobe*, the trabecular cornu.

At this stage the internasal plate has no existence; it is formed afterwards by a commissural growth bridging over the space between the two rods.

Already the hind part of each trabecula has coalesced with its counterpart of the other side; so that, now, the point of the notochord reaches only halfway to the pituitary space; this state of things corresponds with that of the fifth stage of the Axolotl (Plate 22. figs. 6, 7). The hind part of the trabeculæ is flat; each, being thin, bends outward and then turns forward, thickening at the side, and gently lessening in width.

The arcuation of each bar is not great; and the two are some distance apart, even close to the frontal wall. The hinder half of each has arisen into a sphenoidal crest, which curves inwards, above, fitting itself to the dura mater. There is just a perceptible groove between the rod and its cornu in front; for the process is thick and is hooked outward and backward.

Instead of the proper parachordals, coming up close to the notochordal part of the trabeculæ, as in the fourth stage of *Siredon* (Plate 22. figs. 4, 5), there is an ample space running athwart the floor of the skull obliquely between them, entirely composed of membrane. These spaces are two thirds the size of the parachordals or moieties of the investing mass (*iv.*).

Each moiety is concentric in shape, turning its concavity to the ear-capsule; but the opposite side is scooped, for there is a rudiment now of the ascending part—the occipital wall.

Below (fig. 2), the halves of the basal plate run somewhat under the notochord, and closely embrace it above (fig. 1); thus the axial rod can be fully seen only on this aspect. This rod is composed of two parts, that are more sharply defined than in any other species I have examined as yet.

The foremost part of the notochord is a high, blunt-topped cone; the second is roughly hourglass-shaped, that is to say, it is exactly like the notochordal axis of the succeeding vertebræ, whose arches are formed of a pair of cartilages, manifestly the *serial homologues* of the parachordals that form the occipital ring.

The faintness of this attempt to form *cranial vertebræ* is displayed in a peculiar manner; for the conical segment is enclosed in an ectosteal bony sheath, exactly like that on the first following vertebræ of the spine; this is what may be called a *cephalostyle.*

But the neural laminæ of the cephalostyle are formed by the trabeculæ.

The second occipital segment, so well defined, and so perfect in shape, has *merely calcareous grains* dispersed over its surface, and exactly resembling the semicrystalline points that gather and cluster round ossifying cartilage-cells.

Although a bony cephalostyle occurs very constantly in the larvæ of Caducibranchs, this is yet the most perfect I have seen; in *Salamandra maculosa* it seems to be a tubular process of the parasphenoid, and in all it is *transient*, only continuing for some stages as a groove with lateral ridges on the upper surface of the parasphenoid.

It is as well to say at once that here, in these types, the first segment belongs to the postpituitary region, therefore to the hinder half of the basisphenoid, and the second only to the basioccipital*.

In this species the first *vertebra* is a large joint, as large as its successors; but in several Caducibranchs there is a structure in this part, so curious and instructive, that I must mention it here; although the copious illustrations of these types, prepared by me, must wait for publication at some future period.

The Common Newt (*Triton cristatus*), *Spelerpes salmonea*, *S. rubra*, and *Notophthalmus viridescens* have shown me this structure most clearly in their larval state.

There is a deep notch between the occipital condyles, which, like those of the large vertebra to which they are articulated (its "pro-zygapophyses"), are pedunculated.

Where the notochord lies between the occipital condyles, there the parachordal cartilage is deficient, for a time; but a small oval *posterior parachordal* arises on each side. As the peduncles of the anterior articular facets of the first vertebra are very long, there is a considerable tract of the notochord left uncovered, in front by the parachordals, and behind by that vertebra.

That tract acquires its own bony sheath; the small, intercalary parachordals become separately ossified, like the exoccipitals, and then coalesce with the anterior end of the long notochordal style.

We thus get what at first sight appears to be a mere "odontoid process," but which is a true vertebra, having all the essential elements thereof †.

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* I am now satisfied that in the Vertebrata, generally, the hind part of the trabeculæ, which *lies upon* the front part of the parachordals, is the source from whence the posterior clinoid wall grows.

† See Professor Huxley's article "Amphibia," Encycl. Brit. vol. ix. p. 752. After saying that a similar process is seen in the Rays, he goes on to say, "The first spinal nerve which has the distribution of the hypoglossal of the higher *Vertebrata* passes out of the spinal canal either between the first and second vertebræ, or through the foramen in the arch of the first, in the *Amphibia*, which have no proper suboccipital nerve. This is a very curious circumstance, and requires elucidation by the study of development."

I cannot help thinking that both the intercalary vertebra, just described, and also the one through or behind which the hypoglossal or suboccipital nerve passes, are both undifferentiated from the occipital arch in the

In the presence of such curious facts as the skulls of larval Caducibranchs disclose, the mind naturally seeks to know what have been the factors in that marvellous modification of the axis seen in the skull of a Vertebrate animal. I shall reconsider these things in my "Summary."

The auditory capsules of the larval *Seironota* (Plate 29. figs. 1, 2) are thoroughly chondrified, and well show the form of the enclosed membranous labyrinth, with its three canals above (fig. 1, *a.s.c.*, *h.s.c.*, *p.s.c.*), and the crescentic slit, becoming the fenestra ovalis (fig. 2, *f.o.*), below.

Mesiad of this oblique *rent* in the capsule the ragged and imperfect cartilage has not yet formed itself into a stapes.

The suspensorium has all its three upper processes formed; and the ascending (fig. 1, *a.p.*) is grafting its apex on to the alisphenoidal crest. The pedicle (fig. 2, *pd.*) is blunt and rounded; a considerable space separates it from the trabecular convexity.

The otic process (*ot.p.*) is equally well seen in both aspects; but above (fig. 1) it sends forwards and inwards a pedate lobe that cleaves close to the ear-sac against the anterior ampulla, its normal terminus.

The lower surface is gently convex; but the upper is hollowed, relatively, to the ascent of the otic and ascending processes. The direction of the clubbed quadrate region is forwards, and so far outwards, that a line, parallel with the axis of the skull, which should pass through the otic process behind, would cut the inner face of the condyle in front.

Both the articular faces are somewhat convex and then sloping, the movement of the articular end of the MECKEL's cartilage (*mk.*) on the quadrate being loose and free, like a Cow's jaw.

The two cartilaginous mandibles make the face somewhat *underhung*; together they form half an ellipse, and become small, by degrees, to the chin, where they are united by fibrous tissue.

In this early state we miss the pterygoid process of the suspensorium, and the ethmo-palatine visceral rudiment.

Bony laminæ are fast appearing over this simple chondrocranium. Above (fig. 1, *f.*, *p.*, *sq.*, *px.*, *d.*, *sp.*) we see these films, that already have taken on the outline and form of the frontals, parietals, squamosals, premaxillaries, dentaries, and the dentigerous splenials.

The articulars, nasals, ethmoids (outer and inner), the exoccipitals and the prootics, none of these have appeared.

Beneath (fig. 2, *pa.s.*), the large parasphenoid is flooring the unfinished skull with its open rafters (*tr.*); and in the olfactory region there is a triangular tract of teeth on

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higher Vertebrata. I have long ago shown that the notochord of the early chick is *submoniliform* ("Fowl's Skull," plate 82. fig. 3); and it is not an unscientific use of the imagination to suppose that the Sauropsida and the Mammalia have a series of three or four, or even more vertebræ suppressed in the region of the cranial notochord.

each side, adhering to the trabecula, its point in front, and its notched base behind (fig. 2, *v.*, *pa.*). This little crop of teeth is being attached to an undergrowth of bone-cells; the antero-external part is vomerine, and the postero-internal part palatine; they arise, however, as one single upgrowth of pointed, recurved papillæ, that rapidly become denticles.

In such a skull the main nerves are easily seen (figs. 1 & 2; 5, 7, 9, 10); their size, relations, and distribution are precisely like those of *Menobranchus* (HUXLEY, *op. cit.*) and those I have just described in *Siredon.*

The postmandibular visceral arches are conveniently studied as a separate object in this *micro-morphological* work.

The upper, or hyo-mandibular element never appears in this species; the lower cornu (fig. 3, *c.hy.*, *h.hy.*) is a flattish sigmoid bar, which is blunt-pointed after it has become broad above. Below it is more terete, and a short distal segment, the hypohyal, is evident; each hypohyal is attached by a ligament to the fore end of the first keystone piece of the branchial apparatus.

There are four branchial arches, the last gill-less, and these are conjugated by two azygous pieces (fig. 3, *br.* 1–4, *b.br.*).

These bars rapidly lessen backwards, and the hinder two are unsegmented; the segments in the front two are a short ceratobranchial (*c.br.*) and a long epibranchial (*e.br.*); the last two have no ceratobranchial part or region, and are carried on the third. Both the ceratobranchials articulate with the first basibranchial, which is twice as large as the second or free hind piece (*b.br.* 1, *b.br.* 2).

Last Stage. The Skull of the adult Seironota perspicillata.

The changes through which this larval skull has passed must be conceived of as essentially like what I have described in going through the various stages of *Siredon* to that of *Amblystoma.*

If not in this, yet in several others, I have traced every important change.

The general form of the adult skull is oval as to outline and very flat, and is no larger than a "Ladybird"; it is yet a very finished structure, and represents, very fairly, the culmination of the Caducibranchiate type of skull. Each bone, like the composite cranium, has its own perfectness.

The three capsular regions are almost of equal length (Plate 29. figs. 4, 5, 6); the *optic* is modified, in the Urodele, mainly as a recess; the *nasal* roofs are cartilaginous and almost entirely hidden by outer bones; but the *auditory* sacs are the largest, are densely ossified, and freely exposed.

The occipital arch is strong and largely confluent with the ear-masses; above, its two bony halves meet and unite by an irregular, short, superoccipital suture, which is somewhat overlapped by the parietals (fig. 4, *e.o.*, *p.*).

Below (fig. 5) there is a wider space between the two half-rings; and in this synchondrosis there is no visible trace of the notochord and its evanescent vertebral segments.

The suboval, convexo-concave condyles (*oc.c.*) look mainly downwards and a little inwards; they have a very short neck.

More of the ossified endoskeleton is hidden by the single parasphenoid below than by the paired parietals above.

One large and some lesser teeth of a suture can be seen between the prootic (*pro.*) and the epiotic overgrowth of the exoccipitals. I say *overgrowth*, because in these Caducibranchs I find no separate epiotic or opisthotic, both these regions gathering bone from the edge of the exoccipital.

The prootic ossification (*pro.*) is very large; it occupies all but the hinder fourth of the capsule; and it is remarkable that the only part unossified by it (except that which belongs to the exoccipital) is on its anterior face (figs. 4 & 5), from the front of the anterior ampulla to the facet for the pedicle. The stapes (*st.*) is a little bony lozenge.

The soft tract is due to an ensheathing of the capsule by the basal plate; thus the pedicle articulates with that, and not with the proper capsule.

The chondrocranium is like a little boat with the bottom out, but replaced by an extraneous plank, the parasphenoid (*pa.s.*). The sides of this boat-shaped skull are hardened by ossification, the cartilage ultimately yielding to true bone: this bony tract reaches from the prootic and foramen ovale to near the internal nostril (*i.n.*); it is deficient above, from the optic foramen (2) to the ear-capsule.

I know of no Urodele in which an overlying "supraethmoidal" bone conjugates into one "girdle-bone" these two lateral tracts or "sphenethmoids," although *Siren* has two such superficial plates, and the Menopome one.

The rest of the chondrocranium is cartilaginous; it consists in the Urodele of the nasal roofs, coalescent with the internasal plate, or the remainder of the trabecular cornua in front of the unossified wall of the skull or of the sphenoethmoidal tracts.

Behind the internal nares (*i.n.*) there is a thick lip of cartilage, the antorbital or ethmo-palatine (*e.pa.*); it is confluent in the adult with the nasal capsule. In the larva (figs. 1 & 2) it had not appeared.

Part of the nasal roof can be seen in the outer nostril (*e.n.*)*. Up to the ethmoidal region the skull is very bony; in some, as *Salamandra maculosa*, there is a band of cartilage, across the floor, some distance in front of the parachordal region, which is ossified by the prootics; in *Triton cristatus* this band vanishes in the adult.

In Osseous Fishes this part is large, and is ossified by the prootic, forming the *prootic bridge* ("Salmon's Skull," plates 7 & 8); it is formed by the *trabeculæ*; and thus there is in many types a perfectly clear distinction between those bars and the investing mass throughout life.

The roof of the skull in the adult *Seironota* is a strong piece of masonry.

The parietals (figs. 4 & 6, *p.*) are about the size of the frontals (*f.*); they are

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* In a future communication I shall show the condition of the chondrocranium in the adult *Triton* and *Salamander*, after the outer bones have been removed.

perfectly distinct from each other and from the frontals. Each bone has a raised line, or low crest, outside its middle, the shelving part outside the ridge being part of the temporal fossa.

The coronal suture between the parietals and frontals is sinuous, and projects forwards in the middle. The abrupt coronal junction of the parietals and frontals is a great advance upon the perennibranchiate type of skull, where the parietals run under the frontals up to the prefrontals.

The frontals are quite distinct from each other; they become deficient at the middle, in front, but are very large behind.

Each bone has a median elevation, like that on the parietals; but outside the ridge the bone is grooved and multiperforate; outside this groove there is a strong and well-formed superorbital ridge, which makes a good crescent with the bone in front.

The parietals (fig. 6, *p.*) run into the wall of the cranial cavity, and are not mere roof-bones; the frontals (*f.*) do this still more perfectly; they have a good orbital plate.

Part of the roof and the front part of the orbital edge is formed by the *external* prefrontals. These (*e.eth.*) are strong bony wedges, thrust in between the frontals and nasals (*n.*); they reach the open fontanelle, inwards.

The nasals (figs. 4, 6, *n.*) are irregularly lobate shells of bone; they cover the nasal capsules, which are here as wide apart as in the Selachians; they are fairly fixed on between the prefrontal behind, the maxillaries externally, and the premaxillaries in front.

The latter bones (*px.*) close in the cranio-facial box; in front they remain separate, have very short nasal processes, between which and their body there is a foramen; and they have a large, well-developed palatine plate (fig. 5). Each bone projects so as to leave an emargination in the front of the face; together, they largely help to form the elegant semielliptical *upper jaw*, which is dentigerous up to the commencement of the zygoma.

Each bone, externally, forms the antero-inferior third of the outer narial opening (fig. 6, *e.n.*), to which it gives a thickened rim; this opening has a triradiate series of sutures, the two hinder of which separate the maxillary from the premaxillary below and the nasal above, the other is between the nasal and premaxillary.

The maxillary now (figs. 4–6, *mx.*) is a large, well-grown bone; it has a high, outer facial plate, a considerable palatine plate (fig. 5), and its dentary edge runs for half the extent of the bone, which ends behind in a large, arcuate, zygomatic process.

In this, perhaps the smallest of adult Vertebrate skulls, I can discern no septo-maxillary, although it is very common in the Caducibranchs.

The next bones to be described are the vomers (fig. 5, *v.*); but I must first remark upon the large opening (*m.n.c.*) at the mid line between these bones and the palatine plates of the premaxillaries.

In the Salmon ("Salmon's Skull," plates 7 & 8, *m.n.c*) there is a median nasal canal; and this evidently *Petromyzine* structure is very constant in the Caducibranchs; it is surrounded by bone, and appears in *Spelerpes* as a *well* walled round by the nasal process of the azygous premaxillary.

In *Seironota* (figs. 4 & 5, *m.n.c.*) it is behind the palatal plates of the premaxillaries and between the great vomerine blades (*v.*). Above, it is seen in the middle of the precranial fontanelle. We see that this skull is much like that of a Shark, as to the position of the fontanelle and the separation of the nasal roofs.

As in the larva, so in the adult, the vomers and the palatines are continuous; but, for the same time during the latter part of larval life, they were distinct. The curious behaviour of the dentigerous palatine, after it has once become independent of the vomer, may be traced in all the higher Urodeles.

We have seen that in *Siredon* this bony tooth-bearing plate sent backwards and outwards an edentulous process, and that that untoothed part became the larger bone by far, and was segmented from the part from which it sprung.

The same thing has taken place here; but *Seironota* agrees with most Caducibranchs in having its dentigerous palatine become confluent with the end of the vomer just where it has retained a few teeth, and not turned outwards, as in *Amblystoma*.

All the fore part of each vomer is now a large toothless blade of bone, forming, with its fellow, much of this very strong, *hard palate*, and elegantly notched at its side for the inner nostril.

The sutures on this palate, like the bony plates, are of great extent; the edges of each vomer are denticulated; but on the whole the vomer is just more than right-angled, where it fits between the premaxillary and maxillary palatine plates.

Behind the middle nasal *canal* the left vomer binds strongly on the right; they both then pass insensibly into the long, divaricating, dentigerous palatine.

Thus we have, from the ethmoidal region to the ear-capsule, a bony tract whose outer outline is concave or crescentic; behind the middle there is nothing but a steep ridge of bone bearing teeth.

These bars reach to the *basitemporal angle* of the parasphenoid (*pa.s.*); and if we look at the larval skull (figs. 1 & 2) we shall see that these tooth-tracts run along the trabecular line even then.

Whilst giving off the bony pterygoid plate the palatines turned outwards, and then loosing themselves from their new segment, a separate osseous "stolon," they gradually went back to their old position.

In some Caducibranchs the hinder part of this long rod becomes segmented off, also, forming a postpalatine bone.

The edentulous separated piece, or bony pterygoid (figs. 5 & 6, *pg.*), applies itself as an ectosteal plate to a process of cartilage which grows forwards from the suspensorium.

These diverse parts meet and unite, and now in the adult the bony plate has metamorphosed the cartilaginous process; that process was the quadrato-pterygoid, the homologue of the main part of the "upper jaw" of a Selachian (that bar ends behind in the quadrate).

In the Selachians, however, the process is huge and persistent; in Urodeles it is at best a thin wedge, and becomes largely ossified.

The quadrate (*q.*) is well ossified, and reaches below to the transverse kidney-shaped condyle, which forms the base of the vertical suspensorium.

This *pier* is clamped on the inside by the pterygoid bone (*pg.*), and outside by the squamosal (*sq.*), a strong, triradiate bone, which, like that of the Frog, grows forwards in front of the ear-capsule.

In the Frog it is *free* in front; here it applies its split fore end to the postfrontal process, and thus forms a temporal bridge; below, it resembles the preoperculum of a Fish.

The lower jaw (Plate 29. figs. 6, 7) is strong and gently arched; the dentary (*d.*) nearly reaches to the angle, outside; the dentigerous splenial (*sp.*) runs back two thirds the length of the jaw; and the articular cartilage is well embraced by the large articulare (*ar.*).

The hyo-branchial series has undergone a curious transformation (see figs. 3 & 8).

The ceratohyals (fig. 8, *c.hy.*) are but little altered, except that they are pointed below and have lost their hypohyal segment.

That piece has coalesced with its fellow, and also with the fore end of the first basibranchial (*h.hy.*, *b.br.* 1), giving it an alate and emarginate appearance.

That bar has become very large, crested above, flat below, and ossified for three fourths its extent, that is to say, up to the attachment of the ventral ends of the first and second branchials.

These two bars (*e.br.*) have lost their smaller (distal) piece, the ceratobranchial; the first is a thick rod, and is attached to the bone, behind; the second is small, is attached to the sides of the cartilaginous end of the basal piece, and is partly confluent with the large bar in front.

The second basibranchial has entirely disappeared; I have found the same state of things in an *old* Newt (*Triton cristatus*); in some species there is merely left, here, a moss-like growth of cartilage in front of a similar growth of laryngeal origin.

This loss of independence of the second branchial brings this *hyoid apparatus* very near to that of the Bird, whose so-called "basihyal" is, in truth, the homologue of the first basibranchial of the Ichthyopsida, and its "urohyal" of the second basibranchial: this piece, in the Bird, is often dilated at its end where it lies beneath the larynx; the Cryptobranch *Siredon* (Plate 27. fig. 5, *b.br.* 2) foreshadows, accurately, this avian structure.

Of course the so-called paired thyro-hyals or "cornua majora" of the Bird correspond to the first branchial arch of an Ichthyopsidan; each is composed of an epi- and a ceratobranchial piece.

If the figures of Caducibranchiate skulls given in this paper be compared with figures of the skulls of the various species of *Dinornis*[*], it will at once be seen how clear a prophecy we get in this low group of that generalized bird's skull.

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[*] See the invaluable series of papers on these Birds (with their excellent illustrations) by Professor Owen, in the Transactions of the Zoological Society.

SUMMARY.

As the present paper is but a fraction of the work already done on this particular plot of ground, I must refer, in making general remarks upon the skull in this group, to illustrations only available to the writer, and not to the reader.

I also want to show the likeness and the unlikeness of the Urodelous type of skull to that of the Batrachia; but the work given before, on the latter, will be profitable for that purpose; I refer to my two papers on the Skull of the Batrachia (Phil. Trans. 1871 & 1876).

But these two examples of cranial structure—the Urodelous and the Batrachian—are well fitted for comparison with any skulls that are; for the place of the Amphibia in Nature is in the midst of the Vertebrate tribes; they stand, as it were, at the parting of the way, and you can, in leaving them, go back, at once, to the Fishes, or forwards, immediately, to the Reptilian, Avian, or Mammalian groups.

For in these low creatures the morphological force, like a spirit of change, is rife; and although you begin, in their beginning, with the lowest kind of Fish you can conceive of, yet you end, in their ending, with a creature whose endowments, by metamorphosis, enable it to tread upon the heel of the very noblest forms.

If any of the Vertebrata may be said to be *generalized*, these may, their relations are so radiating and complex; yet they become specialized in structure in many ways, anticipating a great deal of what occurs in groups far above them.

If we compare the Amphibia with any culminating group of a Class, such as the Teleostei among the Fish, the Lacertilia among the Reptiles, or the Carinatæ among the Birds, we shall be struck with the marvellous uniformity of structure in these, and the constant variation of the Amphibia—as though the morphological leaven were, in them, still in full ferment.

To come to particulars: *Rana pipiens*, the Bullfrog, differs in its skull far more from that of *Rana temporaria*, the common kind, than can be seen in the whole Teleostean group, if we except the Siluroids and the Murænoids.

There is no such difference in the skull of any Carinate bird as is seen between the skulls of *Bufo vulgaris* and *Bufo agua*.

To say nothing of the want of uniformity among the low, *quasi-larval* Perennibranchiate Urodeles, there is more difference between the skulls of the various *genera* of the Caducibranchs than can be seen in those of the *families* of the Lacertilia.

The difference between the Urodelous and Batrachian types of skull is of great importance, but difficult to express because of the great variability—a variability in fundamentals and essentials, and not in slight things.

First Stage.—In embryos still coiled up in the jelly there are several differences to be noted.

In the Axolotl there is no appearance of the transverse band which runs across the frontal wall in the Frog*.

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* In the following comparisons the reader is referred to my papers on the Batrachian skull (Parts I. & II.) and to the illustrations in the present paper. The *stages* spoken of now are the same as I have described in these three papers, and I wish not to confuse the text in this part by incessant reference to plates and figures.

In both the notochord is thick and blunt at the apex, and it has a downward curve, but not equal to what Mr. Balfour shows in Selachians at the same stage.

There is no true cartilage at present; but solid rods are formed of dense granular tissue, in which a separation of the cells that will form the perichondrium can be seen as distinct from the pith, within.

These rods in the Axolotl are the visceral arches; in the Frog these with the trabeculæ in front of them.

For in the Axolotl the trabeculæ are very indistinct tracts of the mesoblast beneath the membranous cranium; but in the Frog they are quite as distinct as the visceral rods, and are, indeed, the first pair of the series of rods, and the largest as well.

This parallelism of the trabeculæ with the postoral rods is due to the bend of the head upon itself, whereby the floor of the fore and mid brain becomes vertically placed.

The *clefts* between the arches are already visible as slits, in the Axolotl, all but the first, which never becomes truly open.

In the Frog, also, it scarcely opens on the outside, although it is a deep sulcus within; but in these embryos the second and following clefts are still mere grooves.

In the Axolotl the external branchiæ on the third, fourth, and fifth bar are large and trifoliate; in the Frog there is merely one small rounded papilla on the face of the third and fourth bar (first and second branchial arches).

In the Axolotl the fossa which is opening to form the mouth is large and transverse; in the Frog it is very small and 4-angled, two angles looking outward and the other two fore and aft.

Second Stage.—In embryos that are straightening, and getting free from their glairy envelope, the divarication of the two types goes on increasing.

In both the head is recovering from its bend; the clefts are distinct, and the mouth open. In the Axolotl the three pair of external gills are very long, but only bifid; in the Frog they are shorter, and the two that are visible are palmate with about eight digitiform processes.

The mouth of the Axolotl is now a transverse slit with outturned angles; in the Frog it has passed from the lozenge to a square with rounded angles; and whilst the lips of the Urodele are thin, those of the Batrachian are thick and solid, and contain the rudiments of four cartilages.

In both kinds the first *cleft* is imperfectly open, externally; in both cartilage has begun to form in the facial rods; but whilst in the Frog the trabeculæ still appear to be the first of the series, in the Axolotl they are not yet chondrified, whilst the visceral arches are, all but the suspensorium of the mandible.

That suspensorium in the Axolotl is as long and thrice the breadth of the small trabeculæ, which grow like very minute horns, whose base is attached to the sides of the apex of the immense notochord.

But Meckel's cartilage is already a strong cartilage, sigmoid and transversely placed, just bending forwards to the chin; the suspensorium passes outwards and downwards at right angles to the trabecula, and has its apex near the fore end of that tract.

So that, in the Axolotl, the trabeculæ are wholly incomparable in position to the halves of the first visceral arch, instead of seeming, as in the embryo Frog, to be direct predecessors or "serial homologues."

In the Axolotl the mandible (Meckel's cartilage) is chondrified first, before its suspensory part, and before the trabeculæ; whilst in the Frog this rod appears as a small bud, detaching itself from the upper or main part of the arch.

In both the hind ends of the trabeculæ touch the apex of the notochord; but in the Axolotl they lie along it much more, and that part is flattened out more.

Externally the Axolotl has developed a large free operculum from the hyoid region and arch, and this is complete below, as well as at the sides, although of less extent.

In the Frog this part is very small and only covers the proximal part of the first external gill.

Third Stage.—Embryos that have quite recovered from the cephalic flexure, and in which the trabeculæ and visceral arches are well chondrified, show a great difference in their habit of growth—as great a difference as we should find between the development of the embryo of a Bird on the one hand, and of a Mammal on the other.

In the Axolotl the external gills go on growing, and, developing new papillæ, become pinnate; in the Frog they are at their height, and ready to decline, the *internal* gills absorbing them.

In the Axolotl the mouth is widely gaping, the lower jaw underhung, and the form and relative size is like that of the adult Frog; there are no labial cartilages.

In the Frog, at this stage, the mouth is small, round, suctorial, and has horny jaws, and two pairs of labial cartilages in the closely fitting lips; there are no bones or dentigerous patches that acquire bone.

In the mouth and palate of the Axolotl there are, already, *five* pairs of bony, dentigerous plates, namely, the premaxillaries, vomers, palatines, dentaries, and splenials; here, this early appearance of persistent osseous elements is a correlate of the suppression of the "labial cartilages."

In the Tadpole of the Frog, which is a sort of *temporary* Lamprey, the bones spoken of do not appear for two or three weeks to come, when the larva is beginning to abort its *second series* of gills and to acquire lungs, and is therefore just passing out of the larval state.

The Axolotl, however, does not, *in most individuals*, ever cease to be a larva; *now* it is half an inch in length, but becomes eight inches long or more.

The chondrocranium is now composed of a number of cartilaginous bars, and these bars are developing processes and crests.

In both kinds there are two pairs of muscular segments below the hind brain, a pair on each side of the notochord; the *third* pair are cervical; in neither kind are there any "parachordal cartilages" behind the investing hind end of the trabeculæ, and in both kinds these bars are distinct from each other.

In neither is there any roof or floor cartilage, save below, in the trabecular rods themselves; the nasal region is not chondrified; the auditory capsules are cartilaginous below and at their sides, but membranous above.

For the rest the divergence is extreme; in the Axolotl almost half each trabecula is *parachordal*, and the prochordal part does not reach to the nasal sacs, and only encloses the sides of the great pituitary fontanelle; each trabecula is sending up an *alisphenoidal crest.*

In the Frog or Toad the trabeculæ have only a small *parachordal* tract; they are long, grow beyond the pituitary fontanelle as a pair of diverging, but short cornua, but do not quite meet in front of the subcerebral fontanelle.

In the Axolotl the suspensorium is a solid cartilage, growing forwards and outwards; it has three upper processes, all distinct as yet from the trabecula; these are the pedicle and its *ascending* fork, and the otic process; there is no quadrato-pterygoid process for some weeks to come.

The suspensorium is about two fifths the length of the free mandible, which meets its fellow in front of and under the upper part of the face.

In the Batrachian the suspensorium is of the same size, or nearly, as the trabecula; its apex or pedicle is a simple band of cartilage, which has already fused with the elbow of the trabecula behind, and its distal part is continuous with the ethmoidal region in front.

There is no ascending and, for some weeks to come, no otic process; the bar is narrow like the trabecula, runs parallel with it to the nasal region, and the two are, as we have seen, twice conjugated, like the filaments of *Zygæna.*

Instead of being much longer than its pier, the mandible is one fourth the length only; and like the "horns" of the trabeculæ, these two bars are quite in the front of the head; they cross under the fore face, like folded arms, and are evidently, for a long while, non-functional, the labials being, as in the Lamprey, the working jaws.

In both there is no *epihyal* segment, or pier, to the hyoid arch; when it does appear in the Batrachian, after two months or more, it comes with new credentials, and on another mission, than as a support to the arch of the tongue: in the Axolotl, and most of the Urodeles, it is entirely suppressed.

But the *cerato-hyal* or free hyoid cornu is large in both; in *position* and *relation* that of the Axolotl agrees *now* with that of an almost transformed Tadpole, being under the ear-sacs and attached to the back of a short suspensorium, and having a tape-like form.

In the Frog and Toad it is a short, massive, obliquely 4-sided and 4-angled plate, whose attachment to the back of the suspensorium is *in front of the eye.*

In the Axolotl the four *flat* branchial arches are all distinct, and the two foremost are composed of two pieces; there are two basibranchials, and the last arch has no gills. In the Frog and its congeners the four gill-arches are all functional, and are all conjoined together above and below; this series of *pouched* cartilages is conjugated by a single basibranchial.

In the Urodeles they are formed on the same plan as in Selachians, Ganoids, and

Teleostei, but are deficient in segments; in the Batrachian they are manifestly merely a modification of the Marsipobranchiate type, and the *inner* rods are arrested*.

Fourth and two or three following Stages.—Some minor stages may be considered together now, so as to bring the Axolotl up to its Cryptobranchiate stage, and the Frog to the time when its second series of gills are losing their functions; we can then compare the skull of the adult in each type.

In the Axolotl the hind part of the cranial notochord acquires a new pair of cartilages to invest it—the parachordals. I am not certain of their perfect distinctness in the Batrachian; their nasal roof is not so distinct from the trabeculæ in front as in the Urodeles.

Very soon in both cases there is a continuous basal cartilage from the occipital condyles to the frontal wall; for in both cases the trabeculæ become continuous with the basal cartilages behind, grow together between the nasal sacs in front, and send out two free cornua.

In the Axolotl neither a roof nor a floor of cartilage is ever formed in the middle or interorbital region of the skull; in the Tadpole a floor soon forms between the divaricated trabeculæ.

In the Axolotl a slight ethmoidal roof is formed in front and an occipital roof behind; but besides these, which are better developed in the young Frog, there is also in it a band of cartilage (tegmen) over the posterior sphenoidal region.

Whilst the Frog continues *larval* the palato-quadrate band remains as a short, joining tract, and the pedicle remains continuous, behind, with the trabecula.

In the Axolotl after two or three weeks an "ethmo-palatal" cartilage grows from the trabeculæ, behind the nasal sacs, and a process of cartilage grows forwards from the front of the suspensorium; this free cartilage and that process, together, represent the *palato-quadrate* band of the Tadpole.

Of course as long as that band stops in its arrested stage the quadrate hinge runs up to the sides of the nasal region, and the mandibles are transversely placed; this is marvellously unlike the state of things in an Axolotl larva, large or small.

The auditory capsules are now well chondrified in both types, and a crescentic slit, whose convex edge is outwards, appears beneath the capsule.

In the Axolotl the thin and ragged edge of the cartilage mesiad of the slit forms itself into a leaf of cartilage which becomes free, and shapes itself into the elliptical stapes.

In the Frog and Toad the soft tissue filling the chink chondrifies separately as the stapes.

In the Axolotl the ascending fork of the pedicle coalesces with the alisphenoidal crest of the trabecula, the lower process becomes bulbous, but remains free; the otic process is pedate above and embraces the capsule.

In the Tadpole the pedicle, without any ascending fork, is fixed, and the "elbow" of the suspensorium is attached to the capsule by a band of cartilage which is pedate behind; this is the *primary*, transitory, otic process—a "spiracular ray," as in the Sharks.

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* In the Shark there are two sets of arches, *outer* and *inner*; the outer are arrested and the inner developed; the contrary takes place in the Tadpole.

The suspensorium of the Tadpole is of a huge size, and in one kind, that of *Pseudis paradoxa*, reaches by its quadrate condyle to the front of the face. A little behind the trabecular band it grows upwards as a large leaf of cartilage that overarches the temporal muscle and "trigeminal" branches, and in the Toad, as in the Lamprey, coalesces with the ethmoid.

I have not yet found a trace of this structure in any Urodele.

In the Axolotl the hyoid cornu alters very little from what it was at first; in the Tadpole it retains its massiveness, breadth, and shortness until the gills begin to shrink; then it soon becomes a long, narrow tape, loosens itself from the suspensorium, and attaches itself to the auditory capsule.

To the end of the tailed and branchial stage the gill-arches are persistent in the Tadpole, but become bands; in the Axolotl they are massive bars that only alter if the *Siredon* turns into an *Amblystoma.*

Up to that time also the pterygoid process of the suspensorium alters but little except that it grows larger and adds a separate piece to its apex, a segment comparable to the autogenous pharyngobranchials of a Skate.

But the osseous plates that apply themselves to the cartilage are very different in their origin in the two types.

In the Axolotl the little dentigerous palatine sends a toothless process backwards and outwards to the suspensorium; this then separates, the toothed piece turns outwards in *Amblystoma*, and inwards and backwards under the skull in Caducibranchs, generally.

In the Batrachian a thin independent toothless bone applies itself to the under surface of the ethmopalatine bar, as its ectosteal plate; and another larger plate applies itself to the inner face of the suspensorium, to its pterygoid process, and in Toads to the pedicle; in Frogs there is a separate mesopterygoid applied to that process, and which in them becomes large and detached from the trabecula.

In *Bufo agua*, under the huge ethmo-palatine ectostosis, a counterpart of the little transverse palatine of *Amblystoma* appears.

Whilst these things are taking place in the Frog the palato-quadrate band keeps lengthening, the gape widening, and consequently the quadrate gets further and further backwards, until at last the suspensorium forms an obtuse angle with the basis cranii. By the time the tail has disappeared in the Frog the primary otic process has become a free trifoliate "spiracular cartilage;" this becomes the cartilaginous "annulus tympanicus:" it is always, at any stage, *above* the portio dura nerve.

In certain Urodeles, namely *Menopoma*, *Spelerpes*, *Desmognathus*, the two latter being Caducibranchs, this cartilage grows to the stapes, and generally fits its narrow posterior end into a cupped process of the stapedial bony centre; in some it is independently ossified and free.

These two specializations of that peculiar *Selachian* cartilage are of great interest, suggesting the possibility of many curious transformations of ichthyic elements in the higher Classes.

These are some of the most important of the modifications in the morphology of the skull of a Urodele as compared with that of a Batrachian; there are several more, but these must suffice for this present summary.

Last Stage. Skull of adult Urodeles and Batrachians.

A reference to the figures in my papers on the Batrachian skull, and to those of *Amblystoma* and *Seironota* in the present communication, will serve to show at once the sharp distinctness between the two types, and yet their general resemblance.

One great difficulty in comparing them arises from the fact that several of the Urodeles are perennibranchiate, and therefore permanently quasi-*larval*; there are no such Batrachians, and one of that group, namely *Pipa*, is nearly abranchiate.

Looking at the skulls of adult (Caducibranchiate) Urodeles and Batrachians from the surface we see the following differences:—

The roof-bones remain distinct in the Urodele; in the Batrachian the frontal and parietal of the same side coalesce.

In the Urodele the nasal is supplemented by *an external* prefrontal bone, and is therefore much smaller.

The premaxillaries are distinct in the lower and about half the higher Urodeles; in the rest there is an azygous bone with two long nasal processes.

These bones are constantly distinct in Batrachia, and the nasal processes are short.

The maxillary grows further backwards along the face in Batrachia than in the Urodeles; in *one* of these, namely *Amblystoma*, there is a small jugal; in none a quadratojugal, a constant bone in the Batrachia, binding the cheek to the quadrate.

There is only one " temporal bone " in both kinds, with the exception, again, of *Amblystoma*; and in the Urodeles this does not so often grow into the postorbital region.

Beneath, the parasphenoid is much smaller and more specialized in the Batrachia than in the Urodeles; it begins much earlier in the latter, in the Batrachia it develops into a dagger with a large basitemporal *guard.*

The vomers are double in both, save in *Dactylethra*; the palatine base is only transverse (as in the Batrachians) in some "Lechriodonts"; in most Urodeles of the higher kinds it lies under the parasphenoid, and is ankylosed to the much enlarged vomer in front; these are called " Mecodonts."

In both we have the double occipital articulation and the absence of basal and keystone pieces in the occiput. Yet in some larval Caducibranchiate Urodeles *two basal vertebræ* are partly developed and then disappear.

In most species of both kinds the prootic is large, and the rest of the ear-capsule is ossified by an overgrowth from the exoccipital.

Except in *Dactylethra* the two wall bones (sphenethmoids) are fused into a girdle by the help of superethmoidal bone; in the Urodeles the later bony tracts run back past the optic foramen, but never, or seldom (for example, in *Siren lacertina*), unite at the mid line.

In a few Urodeles, namely, *Siren lacertina* and *Salamandra maculosa*, there is a short tongue-shaped prenasal in the adult; and these also show a free prorhinal, or lobe of the cornu trabeculæ, on each side.

These paired processes are very constant in the Batrachia, but in them the prenasal is generally a mere bud.

In the Batrachia the nasal capsules, with the conjoined trabecular (subnasal) laminæ, are more developed than in the other group; they also have a larger "appendix alæ nasi," formed of the outer three fourths of the upper labial.

The suspensorium develops a *quadrate bone* in Urodeles, save in *Siren.* This is absent in Batrachia generally, but present in *Dactylethra* and *Bufo agua.*

The "pedicle" is single in *Proteus*, double in the rest of the group. In that type, in *Menobranchus*, *Siren*, and *Menopoma*, it grows up to the trabecula, as in the Batrachia.

In most Urodeles, however, the lower fork, or *pedicle proper*, is attached to a plaster of investing cartilage on the face of the ear-capsules by a mass of fibrous tissue.

In the Frog (*Rana temporaria*) the attached part of the simple pedicle is absorbed, and the lower, swollen part forms a condyle, and there is a joint-cavity, instead of a fibrous bed, where it hinges on the capsule.

In the Toad (*Bufo vulgaris*) the apex of the pedicle merely coalesces with the front of the capsule, and is not absorbed.

In the Urodeles, with the doubtful exception of *Proteus**, there is a dentigerous splenial, besides the dentary and articulare. In the Batrachia there are only the two latter bones.

In the Urodeles the little mento-Meckelian bone, which characterizes the Frog's mandible, only imperfectly appears in *Proteus.*

The epivisceral element of the second arch occurs as a *large* hyo-mandibular in *Proteus*, and as a *smaller* segment in *Siren* and *Menopoma*; but as a rule it is suppressed, and the stapes is merely connected to the suspensorium by a ligament, the "suspensorio-stapedial."

The cerato-visceral is large, but it does not ascend to the ear-capsule. It is connected to the suspensorium by the hyo-suspensorial, and to the angle of the mandible by the mandibulo-hyoid ligament.

Distally it sometimes gives off its hypohyal piece to coalesce with the first basibranchial at its fore end. In some kinds this distal piece becomes very slender and retrally directed. In the Menopome it is subdivided into three pieces; in Osseous Fishes it has two bony centres.

In the Frog the hyoid arch becomes very slender, coalesces with the basihyal, and, above, ascends to attach itself to the auditory sac. This, however, is only the cerato-hyal element; it has no hypohyal piece; and this is a common thing, namely, for two hypovisceral elements in one type to be represented by a basivisceral in another.

In *Bufo vulgaris* the top of this piece (the stylohyal region) coalesces with the ear-

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* Wiedersheim figures it in his new work on the Urodeles (plate 2. fig. 16).

capsule as in Mammals. In *Dactylethra* it is suspended by a ligament; in *Bufo agua* it is largely, and in *Pipa* wholly, absorbed.

The epihyal piece is mostly small in the Batrachia; it is developed *early* in *Pipa* and *very late* in *Bufo*, *Rana*, and *Dactylethra*.

This small *hyo-mandibular* element becomes the "columella auris." In Toads it has two bony centres and an orbicular symplectic end. In Frogs it becomes two distinct pieces, and has a spatulate symplectic end (the extrastapedial).

In the Urodeles the two last branchials are absorbed; also the second arch becomes simple. Sometimes the first ceratobranchial is lost. The first basibranchial becomes very large, and the second ossifies behind, and is absorbed in front; this bony remnant becomes a transverse, azygous thyro-hyal; it sometimes is a mere rudiment, and at others is totally absorbed in old age.

In the Batrachia two rudiments of arches remain on the hind part of the side of the hyo-branchial lingual plate.

Their common hypobranchial plates grow further backwards, become terete, ossify, and form symmetrical *thyro-hyals*.

These are some of the more important morphological differences between the two groups. I may mention a *histological* difference, namely, that the *Selachian* incrustation of the cartilage seen in the Batrachia (not in *tesseræ*, but in large, irregular patches) does not occur in the Urodeles.

At the risk of prolixity, I have thus compared these two groups as to their cranial morphology. Much of the work already done has not had time to see the light; and much more will be done, if possible, as no other Vertebrata run through so large and instructive a series of metamorphoses, and no other types have such extensive relationships.

I may remark that the Urodeles seem to approximate most to the "Sauropsida," and the Batrachia to the Mammalia. The *largest* species are invariably the most generalized; and some of the larger Batrachia, such as *Rana pipiens* and *Bufo agua*, have many things in them that remind the observer of the "Labyrinthodonts."

The two *aglossal* Toads, lately described, so extremely unlike in most respects, and yet agreeing in being tongueless, suggest most extensive *lacunæ* in the group. The discrepancies between the larger "Perennibranchs" do the same.

The Amphibia are not studied, however, for their own sakes; but it is sought to gain more and more insight into the meaning of the skull in the Vertebrata generally, and not of the skull only, but of the whole skeleton.

That framework, also, is considered in relation to all the other structures, eminently the nervous system; and thus we seek to have a clear view of a vertebrated creature throughout its adult complexity of structure, and also of all the stages through which it passes.

Of late, the invaluable researches of my friend Mr. BALFOUR have come in to open up more and more the secrets of embryological growth. His memoirs, and the later morphological work done by Professor HUXLEY, have been giving new life to my own slow research.

I venture to offer now a few remarks on the growth and the architecture of the skull of vertebrated animals generally.

Concluding Remarks on the Formation of the Skull.

The paired mesoblastic plates, that on each side of the azygous notochord support the neural axis, may be considered to be the *first foundation* of the skeleton.

Whether the notochord be developed from the underlying hypoblast, or be a median tract of mesoblast, does not affect the argument.

A vertebrated animal may be imagined which should be equally modified, or as little modified, at either end. It would be *acephalous.*

The least specialization in its sensory nerves, and in the part of the nervous axis from which they spring, by which simple sensibility should develop into the power of appreciating the sapid or odorous qualities of surrounding bodies, or, still further, give to the creature some feeling of sound or light,—such an exaltation of the sensory endowment in the fore end of the animal would be tantamount to the specialization of the *head*, as distinct from the rest of the body.

We can conceive of this taking place with but little change in a vermiform animal. The endoskeleton may be supposed to remain throughout life as a pair of mesoblastic tracts (enclosing the notochord) more solid than the rest of the tissues formed from that embryonic layer. The neural axis would be a simple tube, ending in points, *amphioxine*; and the hypoblastic tube, or digestive canal, might be as simple as the rest of the creature.

Leaving the actual *Amphioxus* out of the question, what we see take place in the embryos of the Vertebrata generally is a rapid departure from the *actual*, and a wide divergence from any *supposed*, simplicity of form.

To get light upon the almost hopeless question of the morphological divergence of the head from the trunk we must, in imagination, remove all the factors of cephalic specialization, and thus, in idea, reduce the head into the mere fore end of the animal, unenlarged, and not transformed, as we actually see it.

What are those factors? If they can be set in array, and their work and influence truly appreciated, we shall be able the better to read the interpretation of this hard chapter in morphology.

The development of nerves of special sense is a correlate of two things:—first, a large regional increase in bulk of the nervous axis; and, secondly, the budding-out of it on each side of special sense-capsules.

But as the mesoblastic beams on which the nervous axis is laid must grow with the growth of that axis, and be modified with its every modification, we have here a great factor in this skull-building.

Moreover, the sense-capsules have to lie in close proximity with their nerve-supply;

they become large and complex; and of the *three* principal pairs the foremost and hindmost form secondary graftings and intimate blendings with the axial skeleton. The middle pair, also the *eyes*, have the surrounding parts cunningly built over and around them; sockets are sunk in the skull for their reception, albeit they are free themselves.

A temporary change in the direction of the axial nervous mass in front, its *mesocephalic flexure*, whereby the straight embryo is formed into a crozier-like body, this, of necessity, is modifying the growth of the axial skeleton as long as the head is thus bent.

Now the beautiful researches of Mr. BALFOUR show that the notochord becomes (in the Selachians) shaped like a sheep-hook* during the period of embryonic growth; and my own researches (" On the Skull of the Shark and Skate," Trans. Zool. Soc. vol. x. plate 35) show that the part of the mesoblastic plates just in front of this bend grow more rapidly than the part behind, from which they have been, as it were, *dislocated*.

Time has now to be considered in the morphology of the skull; and parts that start first and grow quickest generally overshadow the later and slow-growing parts.

We thus get a morphological *anachronism*—some elements of the skeletal structure standing still and waiting, apparently suppressed, until the proper nick of time occurs in the age and growth of the animal.

But for the modifications undergone by the cephalic structures, the vesicular condition of the neural axis, the development of the organs of special sense, &c., the two plates that run along the sides of the notochord might have been chondrified at one and the same time from end to end of the animal, a little slowness being allowed for the extreme ends.

This would not have been materially affected by the *somatomic* subdivision of the tracts; they might have been separated into moieties for each vertebra, or, obliterating the earlier divisions, chondrification may be imagined as running on (as it does in the neck of Selachians) along parallel tracts continuously.

But in the existing Vertebrata chondrification does not take place in the basal mesoblastic tracts of the head at one and the same time; and it would be well if we knew what causes this anachronism.

This is extremely difficult to account for. It is not equally seen in all types; and there are remarkable variations within the limits of an Order, or even Family.

On the whole, the trabeculæ, or interocular tracts, chondrify first; then the inter-auditory, or parachordals; and, lastly, the internasal, or fore ends of the trabeculæ.

The development of *true* (arrested) vertebræ in the head is possible as far as the *three* essential elements go, namely, the notochord and its pair of investing mesoblastic tracts.

As a fact, such segments, even below the hind brain, are imperfect and transient; under the other two vesicles they are impossible, and in the early embryo only slight traces of somatomic division can be seen, even in the region of the hind brain.

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* Journ. of Anat. and Phys. vol. x. plate 24.

We now come to two causes of modification of the basal plates that are very important, namely, *divarication* of the paired tracts, and *obstruction* of the unpaired axis; the latter is the most important of the two.

The rapid growth of the brain-vesicles, as they turn over the bent skeletal axis, causes the floor of the membranous primordial cranium to bulge, and this bulging affects the hardening bands that are ready to begin to build the "chondrocranium."

The middle tracts (trabeculæ), although starting from the sides of the apex of the notochord, curve round the infero-lateral regions of the brain-sac, and in an early state are merely like two horns, gently curved outwards from their origin, and far apart at their tips (Plate 22. fig. 1, *tr.*).

At their beginning, however, these outbent bars have the notochord between them, so that the posterior trabecular region has in it all the elements that go to form vertebræ.

Now, however, we come to *obstruction.* The posterior part of the oval space, marked out by the divaricating trabeculæ, is occupied by a structure that effectually stops all further forward growth of the notochord.

A process from the postero-inferior region of the fore brain, the infundibulum, here joins on to the *pituitary body*—a tear-shaped sac that passes downwards and backwards from its cerebral attachment, and rests upon the oral mucous membrane.

Here is a perfect barrier; and although the sheath of the notochord may grow upwards between the hind and mid brain, and downwards below the pituitary body, as is seen in the *Axolotl* and *Salmon*, this axial growth is stopped along the basal mid line.

In the *Selachians* it pushes itself against the barrier, and becomes crumpled and beaded; it then smooths out its creases, and presses its fore end against the intrusive body, so as to flatten its point; but it is there arrested.

Therefore it is evident that we can no longer speak of vertebral division of the skull from this point, and this is no further forwards than the "posterior clinoid" region.

From thence we have the highly modified mesoblastic tracts running forwards to the frontal wall, without their proper bond, and subject to conditions that cannot be found in any region of the axis of the body.

I now come to consider another great factor in the specialization of a skull, as such, namely, the sense-capsules, the "paraneurals" of Professor HUXLEY.

Behind the ear-capsules the basal plates are free to grow upwards, and to arch over the neural axis. Between them the basal plate is, as it were, cut away and bevelled; and no upgrowth is possible, as these sacs are built into the sides of the cranium and form much of its side walls.

Moreover, the basal tracts become confluent with these capsules; and afterwards, when ossification sets in, it often works blindly, not keeping to the primary morphological landmarks.

Then, while the neural arch or roof, that should grow up everywhere from the basal plates, is made imperfect by the implantation of these capsules, the arches that should grow ventrad, like the costal arches, miss their point of attachment, and their nervous supply has to find its way round the obstructive masses.

The middle pair of capsules are free; they do not blend themselves with the chondrocranium, but they affect its growth by causing it to harmonize with their size, form, and mobility.

In flat, broad skulls this is the less seen; but in high, compressed skulls the two parallel bands suddenly unite in front of the pituitary body, and grow up into a high wall between the two capsules that mutually become approximated.

In these cases the brain-cavity is lifted up high above this partition, and any *walls* it may acquire grow as wings from the top of this interorbital crest.

Further forwards we have a similar state of things, namely, in the nasal region, these foremost paraneural capsules often approximating very closely.

Here the crest is continuous with that between the eyeballs; but the roof of the nasal labyrinth, right and left, coalesces with the ascending trabecular crest. The capsules here again are blended with the axis.

Only in the lower (ichthyic) types do the basal plates grow up over the brain mass, except behind; so that we have the chondrocranium as a basin or a trough, and not as a finished and roofed structure. This is a great specialization of a neural-arch structure.

The closing in in front of the neural cavity is not necessarily directly over the end of the skeletal axis, and the nasal septum is continued in front of the cranial cavity.

The *ventral* arches of the head are for several reasons very much unlike *costal* arches. They are only perfect behind the mouth, and very independent in their growth.

Costal arches spring, normally, from a vertebral centrum, as directly, indeed, as neural arches do; but the cephalic arches may or may not be attached, even secondarily, to the bands that answer to the distinct vertebral centra.

Where the costal arches grow, there the descending or ventral laminæ are divided into a "splanchnopleure" and a "somatopleure," the pleuro-peritoneal space being between the two, and these arches are formed in the outer layer.

But this division, because of the closing in of that space, does not exist below the head; the old term "visceral arches" still may be used for the bars. They are, to all intents and purposes, *pleural* arches, and they lie close, or near, the outer wall of the throat.

When the visceral arches are attached to the skull-base it is often a secondary attachment, and they, most of them, have a habit of growing over the top of the pharynx, *beneath* the axis.

The intervening clefts, the mode of subdivision of the bars, the structures that are attached to them, and the bony plates that round the mouth are applied to them as *investing* bones; in all these things the *visceral* arches are diverse from the *costal.*

Their *generic* term may be "pleural," but that should not suggest the idea of a true rib or costal arch.

Perchance these two species of arches are but *secular* specializations of one and the same type of ventral arch; but that is a thing hidden from us now.

I can only find two pairs of rudiments of visceral arches in front of the mouth—the ethmo-palatines, which are often separate elements, and the trabecular cornua (with the intervening "rostrum"); but these latter are exogenous processes from the paired basal plates growing into the frontal wall of the embryo.

Description of Plates 21–29.

Plate.	Fig.	Stage.		Number of diameters magnified.
21	1	1st	Unhatched embryo of *Siredon*, $\frac{1}{4}$ inch long. Side view of fore part	26
,,	2	,,	The same. Upper view	26
,,	3	,,	The same. Under view	26
,,	4	,,	The same in section (vertical)	26
,,	5	2nd	Unhatched embryo of *Siredon*, $\frac{1}{3}$ inch long. Side view of fore part	20
,,	6	,,	The same. Upper view	20
,,	7	,,	The same. Under view	20
,,	8	3rd	Hatched embryo of Axolotl, $5\frac{1}{2}$ lines long. Transverse section (7th) of skull through middle of auditory region	20
,,	9	,,	A similar section (8th) further back	20
,,	10	,,	Another section (9th). Occipital region	20
22	1	2nd	Skull of unhatched embryo of *Siredon*, $4\frac{1}{2}$ lines long. Upper view	25
,,	2	,,	Visceral arches of same. Upper view	25
,,	3	3rd	Skull of hatched embryo of Axolotl (*Siredon*), 5 lines long. Side view	20
,,	4	4th	Skull of young Axolotl, $\frac{3}{4}$ inch long. Upper view .	15
,,	5	,,	The same. Lower view	15
,,	6	5th	Skull of young Axolotl, $1\frac{1}{4}$ inch long. Upper view .	10
,,	7	,,	The same. Lower view	10

Plate.	Fig.	Stage.		Number of diameters magnified.
23	1	3rd	Skull of young (hatched) Axolotl, $5\frac{1}{2}$ lines long. Upper view	20
,,	2	,,	The same. Lower view	20
,,	3	,,	Transversely vertical section (1st) of same skull through nasal sacs	20
,,	4	,,	Same; section (2nd) through front of eyeballs . .	20
,,	5	,,	Same; section (3rd) through middle of eyeballs .	20
,,	6	,,	Same; section (4th) through hind part of eyeballs	20
,,	7	,,	Same; section (5th) through suspensorium . . .	20
,,	8	,,	Same; section (6th) through part of ear-capsule .	20
24	1	6th	Skull of young Axolotl, $2\frac{1}{4}$ inches long. Upper view	8
,,	2	,,	The same. Lower view	8
,,	3	,,	Part of upper view of same	16
,,	4	,,	Part of lower view of same	16
,,	5	9th	Skull of cryptobranchiate Axolotl, $8\frac{1}{4}$ inches long. Lower view	$3\frac{1}{2}$
,,	6	10th	Skull of *Amblystoma opacum*, 3 inches 10 lines long. Lower view	5
25	1	7th	Skull of young Axolotl, $3\frac{1}{4}$ inches long. Upper view	5
,,	2	,,	The same. Lower view	5
,,	3	8th	Skull of large Axolotl, full-gilled, $8\frac{1}{2}$ inches long. End view	$3\frac{1}{2}$
,,	4	,,	Lower jaw of same. Inner view	$3\frac{1}{2}$
,,	5	,,	Visceral arches of same. Lower view	$3\frac{1}{2}$
,,	6	10th	Visceral arches of *Amblystoma opacum*. Lower view	5
,,	7	,,	Lower jaw of same. Inner view	5
26	1	8th	Skull of large Axolotl, $8\frac{1}{2}$ inches long. Upper view.	$3\frac{1}{2}$
,,	2	,,	The same. Lower view	$3\frac{1}{2}$
,,	3	,,	The same, with investing bones removed. Upper view	$3\frac{1}{2}$
,,	4	,,	The same. Lower view	$3\frac{1}{2}$
,,	5	,,	Skull and visceral arches of same. Side view . .	$3\frac{1}{2}$
27	1	9th	Skull and visceral arches of cryptobranchiate Axolotl, $8\frac{1}{4}$ inches long. Side view	$3\frac{1}{2}$

Plate.	Fig.	Stage.		Number of diameters magnified.
27	2	9th	The same skull. Upper view	$3\frac{1}{2}$
,,	3	,,	Part of lower view of same	7
,,	4	,,	Section of suspensorium of same	7
,,	5	,,	Median part of visceral arches of same. Upper view	$3\frac{1}{2}$
,,	6	10th	Skull of *Amblystoma opacum.* Upper view . . .	5
,,	7	,,	The same. Side view	5
28	1	Adult	Skull of *Proteus anguinus.* Side view	$5\frac{1}{2}$
,,	2	,,	The same. Upper view	$5\frac{1}{2}$
,,	3	,,	The same. Lower view	$5\frac{1}{2}$
,,	4	,,	The same, with investing bones removed. Upper view	$5\frac{1}{2}$
,,	5	,,	The same. Side view*	8
,,	6	,,	Lower jaw of same. Inner view	$5\frac{1}{2}$
,,	7	,,	Part of skull showing union of pedicle with trabecula	12
,,	8	,,	Section through the same skull. Orbital region .	$5\frac{1}{2}$
29	1	1st	Skull of larval *Seironota perspicillata*, $\frac{1}{2}$ inch long. Upper view	24
,,	2	,,	The same. Lower view	24
,,	3	,,	Visceral arches of same. Upper view	24
,,	4	Adult	Skull of adult of same species, 2 inches long. Upper view	12
,,	5	,,	The same. Lower view	12
,,	6	,,	The same. Side view	12
,,	7	,,	Lower jaw of same. Inner view	12
,,	8	,,	Visceral arches of same. Upper view	12

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* In the Plate this is lettered "× 11," by mistake.

XVI. *Residual Charge of the Leyden Jar.—Dielectric Properties of different Glasses.*

By J. HOPKINSON, *M.A., D.Sc.* *Communicated by Professor Sir* WILLIAM THOMSON, *F.R.S.*

Received November 30, 1876,—Read January 18, 1877.

I. BEFORE proceeding to comparative experiments on different glasses, it appeared desirable to verify experimentally the two following propositions:—

(*a*) If two jars be made of the same glass but of different thicknesses, if they be charged to the same potential for equal times, discharged for equal times and then insulated, the residual charge will after equal times have the same potential in each. In experiments in which potentials and not quantities of electricity are measured the thickness of the jar may be chosen arbitrarily, nor need any inconvenience be feared from irregularities of thickness.

(*b*) Residual charge is proportional to exciting charge.

These propositions may be included in one law—that superposition of simultaneous forces is applicable to the phenomena of residual charge.

To verify (*a*) two flasks were prepared of the glass afterwards referred to as No. 1. One was estimated to be about 1 millim., the other 6 or 7 millims. thick. These were cleansed and insulated in the usual way by filling with strong sulphuric acid without soiling the neck of the flask. They were placed in the same basin of water, which was electrically connected with the outside of the quadrant electrometer. The interiors of the flasks were respectively connected with the two quadrants; they were also connected together by a wire which could at any instant be removed. One DANIELL'S element gave a deflection of 69 scale-divisions. The two flasks were charged together with 48 elements for some minutes, and it was observed that the equal charge of the two quadrants did not deflect the needle. The flasks were discharged for 15 to 20 seconds and insulated, still connected. The connecting wire was then removed, and the subsequent movement of the image observed. If left undisturbed a maximum of about 20 divisions of the scale was attained. But usually the deflection in from 20 to 30 seconds reaching 10 divisions, the thick flask was discharged, and the image was driven from the scale, showing that at that time the potential of either flask was represented by more than 500 scale-divisions, and hence that the difference between them was less than 2 per cent. of either of them. When the charge was negative the error was in favour of the thin flask. This is in complete accord with anomalous results subsequently obtained with the same glass. Correcting for this peculiarity of the glass we may conclude that the law is verified within the limits of these experiments.

The second proposition was confirmed with two different glasses; but the results in one case are not quite accordant, possibly owing to variations of temperature, or to slight unremoved effects of previous chargings; but the irregularities indicate no continuous deviation from the law. In these and all the subsequent experiments the flasks were blown as thin as possible in the body, but with thick necks, the neck being thick that the capacity of any zone might be small.

Flask of optical soft crown, No. 5. The electrometer reads 28½ for one DANIELL'S element. The charging in each case lasted some hours, the discharge 30 seconds. The flask was then insulated and remained insulated; the residual charge was read off from time to time. Column I. gives the time in minutes from insulation; II., III., IV., V., the readings at those times, the exciting electromotive force being respectively that of 48, 48, 24, and 12 elements of the battery.

I.	II.	III.	IV.	V.
1	90	92	46	....
5	218	225	103	51
15	....	344	160	79
30	....	423	197	99
60	478	462	226	114
120	492	....	233	120

Flask of blue glass, No. 2. The reading of the electrometer for one element was 69 divisions. The charge in each case lasted 10 minutes, the discharge 30 seconds; the flask was then insulated. Column I. gives the time from insulation in minutes; II., III., IV., V., the potentials at those times when the batteries which had been employed were respectively of 48, 12, 3, and 1 DANIELL'S elements.

I.	II.	III.	IV.	V.
½	414	102	26½	9
Maximum potential.	472	117½	30¼	10
1½	456	114	29½	10
2½	385	96	24¾	8¼
4½	256	65	16½	6
9½	120	....	8½	3

The agreement in this case, all the experiments being made on the same day, is fairly satisfactory.

II. The following method of treating the question of residual charge was suggested to the author by Professor CLERK MAXWELL; it is essentially similar to that used by BOLTZMANN for the after-effects of mechanical strain ("Zur Theorie der elastischen Nachwirkung," aus dem lxx. Bande der Sitz. der k. Akad. der Wissensch. zu Wien, II. Abth. Oct. Heft, Jahrg. 1874).

Let L be the couple tending to twist a wire or fibre about its axis, θ_t the whole angle of torsion at time t; then L at time t depends upon θ_t, but not wholly on θ_t, for the torsion to which the wire has been submitted at all times previous to t will slightly affect the value of L. Assume only that the effects of the torsion at all previous times can be superposed. The effect of a torsion $\theta_{t-\omega}$ at a time ω before the time considered, acting for a short time $d\omega$, will continually diminish as ω increases; it may be expressed by $-\theta_{t-\omega}f(\omega)d\omega$, where $f(\omega)$ is a function of ω, which diminishes as ω increases. Adding all the effects of the torsion at all times, we have

$$\mathrm{L}=a\theta_t-\int_0^\infty \theta_{t-\omega}f(\omega)d\omega.$$

In the case of a glass fibre BOLTZMANN finds that $f(\omega)=\frac{\mathrm{A}}{\omega}$, where A is constant for moderate value of ω, but decreases when ω is very great.

The after-effects of electromotive force on a dielectric are very similar; to strain corresponds electric displacement, to stress electromotive force. Let x_t be the potential at time t as measured by the electrometer, and y_t the surface-integral of electric displacement divided by the instantaneous capacity of the jar; then, assuming only the law of superposition already proved to be true for simultaneous forces, we may write

$$x_t=y_t-\int_0^\infty y_{t-\omega}\phi(\omega)d\omega, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $\phi(\omega)$ is a function decreasing as ω increases. This formula is precisely analogous to that of BOLTZMANN; but in the case of a glass jar the capacity of which is too small to give continuous currents, it is not easy to measure y_t; hence it is necessary to make x_t the independent variable. From the linearity of the equation (1) as regards $x_t\,y_t$ and the value of $y_{t-\omega}$ for each value of ω, and from the linearity of the equation expressing $x_{t-\omega}$ for each value of ω, it follows that

$$y_t=x_t+\int_0^\infty x_{t-\omega}\,\psi(\omega)d\omega, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where $\psi(\omega)$ decreases as ω increases.

The statement of equations (1) and (2) could be expressed in the language of action at a distance and electrical polarization of the glass, y_t being replaced by the polarization as measured by the potential of the charge which would be liberated if the polarization were suddenly reduced to zero, the jar being insulated. It should be noted that the view of this subject adopted by the author in the previous paper* can be included in equation (2) by assuming that $\psi(\omega)$ is the sum of a series of exponentials.

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* *Vide* Phil. Trans. vol. clxvi. pt. 2.

If $\psi(\omega)$ is determined for all values of ω, the properties of the glass, as regards conduction and residual charge, are completely expressed.

Suppose that in equation (2) $x_t=0$ till $t=0$, and that after that time $x_t=\mathrm{X}$ a constant,

$$y_t=\mathrm{X}\left(1+\int_0^t \psi(\omega)d\omega\right),$$

$$\frac{dy}{dt}=\mathrm{X}\psi(t);$$

now when t is very great, $\frac{dy}{dt}$ is the steady flow of electricity through the glass divided by the capacity. Hence

$$\psi(\infty)=\mathrm{B}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

B is the reciprocal of the specific resistance multiplied by 4π and divided by the electrostatic capacity of the substance.

We have no practicable method of determining y_t; but we may proceed thus:—During insulation y_t is constant; we have then

$$x_t=\mathrm{A}-\int_0^t x_{t-\omega}\,\psi(\omega)d\omega; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

x_t and $x_{t-\omega}$ alone can be measured; (4) is, then, the equation by aid of which $\psi(\omega)$ must be determined.

(α) Let x_t be maintained constant$=$X from time 0 to time t, then insulate; differentiating (4),

$$\left.\begin{aligned}\frac{dx_t}{dt}&=-\mathrm{X}\psi t-\int_0^t \frac{dx_{t-\omega}}{dt}\,\psi\omega d\omega\\&=-\mathrm{X}\psi t\\&=-\mathrm{BX}\text{ when } t \text{ is very great.}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

To find B, charge for a long time to a constant potential, insulate and instantly observe the rate of decrease of the potential.

(β) Let the flask be charged for a very short time τ and then be insulated; at the instant of insulation we have $\frac{dx_t}{dt}=-\mathrm{X}\psi(\tau)$. Hence an approximation may be made to an inferior limit of $\psi(0)$.

(γ) Let x_t be constant $=$X for a long time from $t=-\mathrm{T}$ to $t=0$; discharge and, after a further time t, insulate:—

$$\left.\begin{aligned}x_{t+\tau}&=\mathrm{A}-\mathrm{X}\int_{t+\tau}^{\mathrm{T}+t+\tau}\psi(\omega)d\omega-\int_0^\tau x_{t-\omega+\tau}\,\psi(\omega)d\omega,\\\frac{dx_t}{dt}&=\mathrm{X}(\psi(t)-\mathrm{B})\text{ when } \tau \text{ vanishes.}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

To find $\psi(t)$ in terms of t charge for a very long time, discharge and from time to time insulate and determine $\frac{dx_t}{dt}$.

(δ) Let the charging last during a shorter time τ', then discharge and insulate from time to time as in (γ):—

$$\frac{dx_t}{dt}=\mathrm{X}\{\psi(t)-\psi(\tau'+t)\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

(ε) Charge during time τ', and reverse the charge for time τ'' before discharging:—

$$\frac{dx_t}{dt}=\mathrm{X}\{\psi(t)-2\psi(\tau''+t)+\psi(\tau'+\tau''+t)\}. \quad . \quad . \quad . \quad . \quad (8)$$

III. *Glass* No. 1.—This glass is a compound of silica, soda, and lime. In a damp atmosphere it "sweats," the surface showing a crystalline deposit easily wiped off. For a soda glass it is very white. Density 2·46.

When the flask was mounted, connected with the electrometer, the image from which was deflected 70 divisions by one Daniell's element, and insulated, it was found to steadily develop a negative charge, amounting to 11 scale-divisions in 10 seconds, and increasing to a maximum of 25 divisions. The cause of this the author cannot explain. Two other flasks of the same glass behaved in a similar manner—in one case, with the thin flask of § I., the charge rising to 40 divisions, with the thick flask to only 15 divisions. No sensible effect of the same kind was noticed with any other glass. The effect does not appear to be due to the connecting wires (for these were repeatedly removed and replaced by fresh ones), nor to difference between the acid within and that outside the flask, as this also was changed.

Experiment α.—The flask was charged to 500 divisions for half an hour, insulated, and the potential observed after 5, 10, 15, 20 seconds. The mean of several experiments gave for these times 372, 275, 216, 170: hence the loss in 5 seconds is about 25 per cent.; and from this we may readily deduce $\frac{dx}{dt}$, since the percentage of loss is not materially different in the second interval of 5 seconds, B=3·4, the minute being unit of time.

Experiment β.—An attempt was made to estimate $\psi(0)$. The charging lasted one second. In two seconds from insulation the charge fell from 500 to about 330, which gives $\psi(0)$ certainly greater than 10·2. This can, of course, only be regarded as the roughest approximation.

Experiment γ.—The flask was charged positively for about 19 hours with 48 elements, the electromotive force of which is represented by about 3360 scale-divisions. It was then discharged, and at intervals insulated for 10 seconds, and the residual charge developed in that time observed. Column I. gives the time in minutes from first discharge to the middle of each 10-second period; II. the charge developed in ten seconds; III. the estimated value of $\psi(t)-\mathrm{B}$, obtained by correcting for the negative charge which it was found this flask took in 10 seconds, and dividing by 3360.

These results are certainly much below the true values, for the image moved over the scale much more rapidly in the first than in the second five seconds; but their ratios are probably fair approximations.

I.	II.	III.	I.	II.	III.
½	190	0·36	15	17	0·050
1	106	0·21	20	14	0·045
2	57	0·12	30	11	0·040
3	42	0·094	40	7	0·032
4	36	0·084	50	5	0·029
5	30	0·074	60	3	0·025
7	26	0·066	90	0	0·020
10	22	0·060	180	−5	0·011

Experiment δ.—This experiment was tried both with a positive and negative charge The charge lasted 90 minutes. The readings were made as in γ.

I. gives the time in minutes;
II. the readings when the charge was positive;
III. when the charge was negative;
IV. the mean of II. and III.;
V. the value calculated from γ.

I.	II.	III.	IV.	V.
½	180	190	185	190
1	93	120	106	106
2	45	75	60	57
3	31	68	49	42
4	...	53	...	36
5	22	47	34	30
7	16	43	29	26

The same experiments were made, but with time of charging only 5 minutes.

Columns II. and III. give the means in each case of two separate observations, made on different occasions.

I.	II.	III.	IV.	V.
½	150	170	160	162
1	80	92½	86¼	79
2	22½	41½	32	31
3	10½	29	19¾	18
4	5	23	14	13
5	0	20½	10¼	8
7	−4	18½	7	6

Glass No. 2.—This glass is of a deep blue colour; it is composed of silica, soda, and lime, the quantity of soda being less than in No. 1, but of lime greater. The colour is due to a small quantity of oxide of cobalt. The temperature throughout ranged from 62° F. to 64° F.

Experiment α.—The flask was charged for several minutes, and then insulated. The intensity of the charge before insulation, and at intervals of 5 seconds after, was observed, the whole experiment being repeated three times. The mean is given.

Time	0.	5.	10.	15.	20.	30.	40.
Reading . . .	497	465	433·6	405	379	342	311

$$B = 0{\cdot}77.$$

Experiment β.—The charging lasted 2 seconds. The flask was then insulated, and its charge measured at intervals of 5 seconds. The mean of two fairly accordant observations is given.

Time	0.	5.	10.	15.	20.
Reading . . .	490	390	325	282½	249

Hence $\psi(0) > 2{\cdot}4$, probably much greater.

Experiment γ.—The flask was charged with 48 elements for 8 hours in the first experiment, and subsequently for 3 hours 25 minutes for a second experiment, the effect of the previous charging being still considerable when the charging began. After discharge the flask was from time to time insulated for 20 *seconds*, and the residual charge developed in that time was observed.

I. gives the time from discharge to the middle of the periods of insulation;

II. and III. the observations in the two experiments;

IV. the results corrected by a curve from II. and III.;

V. the values of $\psi(t) - \mathrm{B}$, again not corrected for the rapid decrease in $\frac{dx}{dt}$ after each insulation.

It may be remarked that the image in this case moved in 10 seconds about $\frac{3}{5}$ of what it attained in 20 seconds.

I.	II.	III.	IV.	V.
½	470	468	469	0·42
1	300	325	310	0·28
2	178	183	180	0·16
3	134	134	133	0·12
4	105	107	106	0·094
5	89	91	90	0·080
7	68	69	68	0·061
10	52	54	53	0·047
15	39	41	40	0·036
20	32	36	34	0·030
30	...	29	28	0·025
60	...	20	20	0·018
90	16	...	16	0·014
600	...	6	6	0·005

Experiment δ.—The charging with 48 elements lasted 5 minutes. The experiment was tried twice with positive and negative charges respectively. II. and III. give the readings, whilst IV. gives the value calculated from the curve of γ.

I.	II.	III.	IV.
½	...	385	385
1	212	228	232
2	...	110	112
3	66	67	72
4	47	47	50
5	34½	33½	37
10	11½	10	13
15	5	4½	6

Experiment ε.—The flask was for many hours charged negatively, then positively for 5 minutes, and observations of residual charge were made as before.

Column III. are the values calculated from γ by the formula $\frac{dx}{dt}=\psi(t)-2\psi(\mathrm{T}+t)$.

I.	II.	III.
½	−310	−301
1	−168	−154
2	− 48	− 44
4	+ 8	+ 6
5	+ 17	+ 16
10	+ 28	+ 27
15	+ 27	+ 28
20	+ 27	+ 27

Glass No. 3.—Common window-glass, composed of silica, soda, and lime, the quantity of lime being greater than in No. 2. This glass does not "sweat" in a moist atmosphere. The temperature was 68° F.

Experiment α.—The flask was charged to 425 divisions for about 3½ hours, and was then insulated. After ½ minute the charge was 210; 1 minute, 138; 2 minutes, 74; 3 minutes, 50. Hence B is certainly greater than unity, and lies intermediate between the values for glasses 1 and 2.

Experiment γ gives the *observed values* of $\psi(t)-\mathrm{B}$ throughout a little less than in No. 2. As this flask was not very well blown further experiments were not made.

If the values of $\psi(t)-\mathrm{B}$ could be accurately obtained for these three glasses, they would certainly differ less from each other than they appear to do.

Glass No. 4.—*Optical hard crown.* Density 2·48. Composed of silica, potash, and lime. The composition may be regarded as corresponding to a glass intermediate between 1 and 3, with the soda replaced by potash.

The experiments α and β were made by the following modified method:—The whole battery of 48 elements was used, one pole being connected with the case of the electrometer and the exterior of the flask, the other with the interior of the flask by a cup of mercury and also with one electrode of the electrometer. The other electrode was permanently connected with the interior of the flask. It was ascertained that the image remained at zero whether both quadrants were charged equally or both discharged. The potential of the 48 elements was measured by 6 elements at a time; the extremes were 432 and 437, and the total 3475 scale-divisions. Where the charge of each quadrant is considerable and of the same sign, it cannot be assumed that the deflection for a given difference is the same as if the charges were small, or of equal and opposite sign; in fact, if the potentials of the quadrant and the jar of the electrometer are of the same sign, the sensibility of the instrument will be diminished (*vide* Maxwell's 'Electricity and Magnetism,' vol. i. p. 273). On this account the results for $\psi(t)$ given below should be increased by about $\frac{1}{15}$ part of their value. The experiment consisted in insulating the flask from the battery, and observing the difference of potential between the flask and the battery after a suitable interval.

The flask was charged and instantly insulated at 8.25 P.M. The image traversed 164 divisions in 10 seconds. The flask was again connected with the battery, and insulated from time to time.

I. gives the middle of the period of insulation, measured from 8.25; II. the division traversed; III. the duration of insulation; IV. the value of $\psi(t)$.

I.	II.	III.	IV.
5 seconds.	164	10 seconds.	0·28
1 minute.	26	20 ,,	0·022
2 minutes.	14	20 ,,	0·012
3 ,,	11	20 ,,	0·0094
5 ,,	8	20 ,,	0·0069
10 ,,	34	2 minutes.	0·0049
15 ,,	28	2 ,,	0·0040
20 ,,	22	2 ,,	0·0031
30 ,,	36½	4 ,,	0·0026
60 ,,	25	4 ,,	0·0018
15 hours.	11	6 ,,	0·0005

Glass No. 5.—Optical soft crown. Density 2·55. Composed of silica and potash, with lead and lime in small quantity.

Experiments α and β.—68 divisions of the electrometer-scale equal one Daniell's element.

The flask was charged for 5 seconds, insulated, and the loss in the subsequent 10 seconds observed. The result may be regarded as giving an approximation to $\psi(\frac{1}{6})$. The mean of two experiments gives a fall from 471 to 452½, or $\psi(\frac{1}{6})=0·23$.

Charging for 45 seconds, and observing the loss during 30 seconds, gave $\psi(1)=0·06$.

The flask was connected with the battery continuously, and only insulated at intervals, and connected with the electrometer for a short time to determine the rate of loss. The following values are thence deduced:—

t	5.	10.	30.	60.	120.	180.	300.
$\psi(t)$ . . .	0·025	0·017	0·012	0·009	0·007+	0·007−	0·006

$\psi(\infty)$ probably does not differ much from 0·005 or 0·004.

Experiment γ.—The flask was charged for three days with 48 elements, equal to 3260 divisions, or thereabouts, then discharged.

I. gives the time from first discharge to the middle of the period of insulation;

II. the scale-divisions traversed;

III. the times of insulation in minutes;

IV. the value of $\frac{dx_t}{dt}$;

V. $\psi t - B$.

I.	II.	III.	IV.	V.
½	53	⅙	318	0·098
1	62	⅓	186	0·057
2	64	½	128	0·039
3	62	⅔	93	0·029
5	70	1	70	0·021
10	92	2	46	0·014
15	71½	2	35·75	0·011
20	63	2	31·5	0·0097
30	48½	2	24·25	0·0074
60	109	8	13·6	0·0042
90	89	8	11·12	0·0034
125	69½	8	8·7	0·0027
180	54	8	6·75	0·0021

The results thus obtained agree fairly with those obtained by Experiment β; the differences may be attributed to errors of observation.

Experiment δ.—The charging lasted 5 minutes. The experiment was performed twice, with positive and negative charges respectively.

I. gives the time from first discharge;

II. the period of insulation;

III. and IV. the divisions traversed in that time;

V. their mean;

VI. the value of $\frac{dx_t}{dt}$ thence obtained;

VII. the value of $\frac{dx_t}{dt}$ calculated from the last experiment.

I.	II.	III.	IV.	V.	VI.	VII.
$\frac{1}{2}$	$\frac{1}{6}$	37	...	37	222	252
1	$\frac{1}{3}$	42	43	42$\frac{1}{2}$	127·5	124
2	$\frac{1}{2}$	32	32$\frac{1}{2}$	32$\frac{1}{4}$	64·5	72
5	1	23	22$\frac{1}{2}$	22$\frac{3}{4}$	22·75	24
15	4	23	...	23	5·75	4·25

The differences between VI. and VII. are somewhat large; they may perhaps be in part attributed to the fact that $\frac{dx_t}{dt}$ is deduced from observations on a quantity not uniformly increasing, on the assumption that the increase is uniform, and to the inequality of the times of insulation.

Glass No. 6.—A flint glass containing less lead than No. 7.

Experiments α and β.—66 divisions of the scale equal to one DANIELL'S element.

The flask was continuously connected with the battery, and only insulated for brief periods, to determine the rate of loss, the following values are thence deduced:—

t	1.	5.	15.	120.	240.
$\psi(t)$. . . .	0·013	0·007	0·004	0·0016	0·001

Experiment γ.—The flask was charged for 13 hours with 48 elements, then discharged. The columns are the same as in glass No. 5.

I.	II.	III.	IV.	V.
1	21	$\frac{1}{3}$	63	0·02
5	37$\frac{1}{2}$	2	18·75	0·006
15	48	6	8·0	0·0026
75	60	24	2·5	0·0008

There is a considerable discrepancy between the values of $\psi(1)$ from α and $\psi(1)-B$ from γ; the former may be in error, as it was deduced from the time of traversing 3 divisions only.

Glass No. 7.—Optical "light flint." Density 3·2. Composed of silica, potash, and lead. Almost colourless. The surface neither "sweats" nor tarnishes in the slightest degree. This glass at ordinary temperatures is sensibly a perfect insulator.

A flask was mounted in the usual way on July 15th; it was charged with 48 elements for some hours, the potential being 240 scale-divisions as measured through the "induction-plate" of the electrometer. The charging-wire was then withdrawn. On July 23rd the wire was again introduced and connected with the induction-plate; a charge of 183 scale-divisions still remained, although the temperature of the room was as high as 72° F. The flask was again put away till Aug. 9th, when the charge was found to be 178. On September 14th it was 163. Lastly on October 14th it had fallen to 140.

As might be expected from the last experiment, the residual charge in this glass is small. The flask was charged for nine hours with 48 elements; it was discharged, and after 4 minutes insulated; in 2 minutes the residual charge had only attained $11\frac{1}{2}$ divisions, giving $\psi(5)=0{\cdot}0017$. It was again insulated after 44 minutes; in 12 minutes the charge was $10\frac{1}{2}$, giving $\psi(50)=0{\cdot}00026$.

Since the loss by conduction is so small, the flask may be strongly charged by an electrophorus instead of with the battery. If it is left insulated for a considerable time, and then discharged, and the return charge observed, it may be assumed that the exciting charge has been sensibly constant during the latter portion of the period of insulation.

The flask was strongly charged and remained insulated for 3 hours 40 minutes; it was then discharged, and from time to time was temporarily insulated to ascertain the rate of return of charge.

At $\frac{1}{2}$ minute	250 divisions in	$\frac{1}{6}$ minute	=1500	per minute.		
5 minutes	247	,,	1 ,,	= 247	,,	,,
10 ,,	285	,,	2 minutes	= $142\frac{1}{2}$	,,	,,
15 ,,	304	,,	3 ,,	= 101	,,	,,
30 ,,	326	,,	6 ,,	= 54	,,	,,

It was immediately charged again, insulated for 70 minutes, and then the observations repeated.

At $\frac{1}{2}$ minute	120 divisions in	$\frac{1}{6}$ minute	=720	per minute.		
1 ,,	135	,,	$\frac{1}{3}$,,	=405	,,	,,
2 minutes	125	,,	$\frac{1}{2}$,,	=250	,,	,,
5 ,,	121	,,	1 ,,	=121	,,	,,
10 ,,	142	,,	2 minutes	= 71	,,	,,
15 ,,	106	,,	2 ,,	= 53	,,	,,

The ratios of the numbers in the two experiments agree fairly.

Glass No. 8.—"Dense flint." Density 3·66. Composed of silica, lead, and potash, the proportion of lead being greater than in No. 7.

Experiment α.—The flask was charged for three hours to 500 divisions, and then insulated:—

After 1 minute	from insulation		$499\frac{3}{4}$
,, 5 minutes	,,	,,	499
,, 30 ,,	,,	,,	495

hence $\psi(180)=0{\cdot}0004$.

Experiment β.—The flask was charged for 5 seconds, insulated, and the potential read off at intervals of $\frac{1}{2}$ minutes. The results are the mean of two observations:—

Reading . .	497	$479\frac{1}{2}$	$475\frac{1}{2}$	474	473	$472\frac{1}{2}$	$472\frac{1}{4}$
Time . . .	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3

from this it may be inferred that $\psi(0)$ is considerably greater than $0{\cdot}07$. An experiment on residual charge gives $\psi(1)-B=0{\cdot}017$.

Glass No. 9.—Extra dense flint. Density 3·88. Colour slightly yellow. The proportion of lead is somewhat greater than in No. 8. The surface tarnishes slowly if exposed unprotected to the air.

The flask was charged for 10 seconds to 500, and was then insulated.

After 1 minute the reading was 499
,, 3 minutes ,, ,, ,, $497\frac{3}{4}$
,, 5 ,, ,, ,, ,, 495
,, 30 ,, ,, ,, ,, $486\frac{1}{2}$
,, 60 ,, ,, ,, ,, 479

The flask was charged with 48 elements for $1\frac{1}{2}$ hour, and the residual charge observed,

$$\psi(2)-B=0{\cdot}003.$$

An attempt was made to obtain a knowledge of the form of the function $\psi(t)$ in the same manner as for No. 7. The flask was charged from the electrophorus, and allowed to stand insulated for 22 hours; it was then discharged and temporarily insulated at intervals.

At $\frac{1}{2}$ minute traversed 130 divisions in $\frac{1}{6}$ minute =780 per minute.
,, 1 ,, ,, 160 ,, ,, $\frac{1}{3}$,, =480 ,,
,, 2 minutes ,, 145 ,, ,, $\frac{1}{2}$,, =290 ,,
,, 5 ,, ,, 152 ,, ,, 1 ,, =152 ,,
,, 10 ,, ,, 189 ,, ,, 2 minutes = $94\frac{1}{2}$,,
,, 15 ,, ,, 217 ,, ,, 3 ,, = 72 ,,
,, 30 ,, ,, 275 ,, ,, 6 ,, = 46 ,,
,, 60 ,, ,, 360 ,, ,, 12 ,, = 30 ,,
,, 120 ,, ,, 437 ,, ,, 24 ,, = 18 ,,

It will be remarked that in this case $\psi(t)-B$ deviates further from the reciprocal of the time than in the case of No. 7.

Glass No. 10.—Opal glass. This glass is white and opaque. It is essentially a flint. The reason for examining it was to ascertain if its opacity had any striking effect on its electrical properties.

Experiment α.—The flask was charged to 462 divisions for five hours; on insulation the loss was found to be 4 to 5 divisions in an hour; hence $B=0{\cdot}00016$.

Experiment β.—Charged to 462 for 10 seconds; a loss of 2 in 3 minutes was observed on insulation.

Experiment γ.—The flask was charged with 48 elements, each equal to 67 divisions of the scale, for 5 hours, and was then discharged.

At 1	minute,	$4\frac{1}{2}$	divisions	in	$\frac{1}{3}$	minute.
„ $2\frac{1}{2}$	minutes,	6	„	„	1	„
„ 5	„	6	„	„	2	„

or

$$\psi(1)-B=0{\cdot}004$$
$$\psi(2\tfrac{1}{2})-B=0{\cdot}002$$
$$\psi(5)-B=0{\cdot}001.$$

The residual charge is smaller than in any other glass observed.

A few of the results of the preceding experiments are collected in the following Table for the purpose of ready comparison.

I. The greatest value of ψt observed.
II. „ least „ „ „
III. $\psi(1)\ -B$ as obtained by experiment γ.
IV. $\psi(5)\ -B$ „ „ „
V. $\psi(60)-B$ „ „ „

Glass.	I.	II.	III.	IV.	V.
1.	10·2	3·4	0·21	0·073	0·025
2.	2·45	0·76	0·28	0·08	0·018
3.	......	1·0	......	0·05	0·01
4.	0·28	0·0005	0·0215	0·0064	0·0013
5.	0·23	0·006	0·057	0·021	0·0042
6.	0·013	0·001	0·02	0·006	1·0008*
7.	......	0·00002	......	0·0017	0·00026†
8.	0·07	0·0004	0·017	......	......
9.	0·002	......	0·003‡	......	......
10.	0·0014	0·00016	0·004	0·001	......

From this Table two classes can at once be selected as having well-marked characters. The soda-lime glasses, although the composition and colour vary widely, agree in

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* $\psi(75)-B$. † $\psi(50)-B$. ‡ $\psi(20)-B$.

possessing small insulating power, but exhibit very great return charge. The values of the function $\psi(t)-\mathrm{B}$ for the three glasses agree almost within the limits of these roughly approximate experiments.

At the opposite extreme are the flints or potash-lead glasses, which have great specific resistance. The experiment does not prove that No. 7 conducts electricity at all; for it is not certain that the very slight loss of charge may not be due to conduction over the surface of the glass; but it is certainly not less than 100,000 times as resistant as No. 1. The flints also have very similar values of $\psi(t)-\mathrm{B}$, much smaller than the soda-lime glasses.

IV. It is known that glass at a moderately high temperature conducts electricity electrolytically. The following experiment shows that with the more conductive glasses electrolytic conduction occurs at the ordinary temperature of the air.

A flask of blue glass, No. 2, was very carefully insulated with strong sulphuric acid within the flask, and was placed in a vessel of caustic potash. Platinum wires dipping in the two liquids communicated with the quadrants of the electrometer. On insulation the acid developed a positive charge as follows:—

In	½	minute	15	divisions	of the	scale.
,,	1	,,	22½	,,	,,	,,
,,	2	minutes	33½	,,	,,	,,
,,	5	,,	47	,,	,,	,,
,,	10	,,	55	,,	,,	,,
,,	15	,,	57	,,	,,	,,

one DANIELL'S element giving 68 divisions of the scale.

The experiment was repeated after the flask had stood some days with the two liquids connected by a platinum wire; the potential developed much more slowly, and in 50 minutes was stationary at 38½ divisions.

Summary.—These experiments are subject to many causes of error. Deducing $\frac{dx_t}{dt}$ from an observation of dx_t in a period of many seconds or even minutes gives values of $\psi(t)-\mathrm{B}$ necessarily too low, in some cases very much too low. No attempt was made to keep the glass at a constant temperature; the temperature of the room was occasionally noted, but is not given here, as no conclusion is based upon it. The experiments were performed irregularly at such times as other circumstances permitted. It will be observed that the discords of the experiments of verification are considerable, but they are irregular. It may, perhaps, be assumed that they are within the limits of error, and we may infer that the fundamental hypothesis is verified, viz. that the effects on a dielectric of past and present electromotive forces are superposable. OHM'S law asserts the principle of superposition in bodies in which conduction is not complicated by residual charge. Conduction and residual charge may be treated as parts of the same phenomenon, viz. an after-effect, as regards electric displacement, of electromotive force. The experiments appear to show, though very roughly, that the principle of OHM'S law is applicable to the *whole* phenomenon of conduction through glass.

V. *Effect of Temperature.*

The purpose of the previous experiments being to examine generally the applicability of the formulæ and to compare the values of $\psi(t)$ for different glasses of known composition, no account was taken of temperature, and no attempt made to maintain it constant, although it is well known that changes of temperature greatly affect both conduction and polarization in glass*. It appeared, however, desirable to compare the same glass at different temperatures in the same manner as different glasses at the same temperature.

The flask, carefully filled with sulphuric acid as before, was placed in an earthenware jar containing sulphuric acid, which was in its turn placed in a double cylindrical shell of copper, with oil or water between the cylinders. The jar was covered by two disks of wood, through holes in the centre of which the neck of the flask projected. A cap of sealing-wax, carrying a small cup of mercury for making electrical connexions with the interior, closed the flask. A thermometer dipped into the acid outside the flask for reading the temperature of the glass, whilst a second thermometer was inserted between the cylinders in the oil or water to help the observer in regulating the temperature by means of a spirit-lamp. In the two experiments below freezing-point the earthenware jar was removed from the oil-bath and placed in a freezing-mixture of hydrochloric acid and sulphate of soda. In all cases the temperature was maintained approximately constant for some time before observing. It will be remarked that, as the acid was not stirred, the temperature-readings are subject to a greater probable error than that due to the thermometer itself. But as the changes of temperature of the acid were always very slow, the error thus introduced cannot seriously affect the results. All temperatures are Centigrade. The actual readings are given, and also the temperature, roughly corrected when necessary, for the exposed portion of the stem of the thermometer. The times in these and in most of the previous experiments were taken by ear from a dead-beat seconds clock, the eye being fixed on the image and the scale. In the intervals between the short insulations to determine $\frac{dx}{dt}$, the flask was either connected with the battery or discharged. In all cases the registered time of observation is taken at the middle of the period of insulation; thus, in the experiment at $39\frac{1}{2}$° below, insulation was made one second before the minute, and the reading one second after. Two glasses were examined, Nos. 2 and 7, selected as extreme cases. The whole of the observations made are given, excepting three manifestly in error, although

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* *Vide* Mr. PERRY, Proceedings of the Royal Society, 1875, p. 468; Prof. CLERK MAXWELL, "Electricity and Magnetism," Art. 271.

only a portion are used. The values of $\psi(5)$ and $\psi(10)$, for glasses 2 and 7 respectively, are taken as sensibly equal to B, and are calculated on the assumption that during the short time of insulation the rate of loss at any instant is proportional to the then charge. The values of $\psi(1)-B$ and $\psi(5)-B$ are deduced as though $\frac{dx}{dt}$ were constant during the time of insulation, and are therefore considerably below the truth in all cases. It will be observed that the battery was not quite constant; but the value of 48 elements may be taken as 3160 scale-divisions without serious error.

Glass No. 2.—Temperature 53°. It was roughly estimated that on insulation $\frac{1}{4}$ of the charge was lost within 1 second. Notwithstanding this high conductivity, the residual charge was capable of rising to more than 400 scale-divisions when the flask had been charged with 48 elements and then discharged for a few seconds. This differentiates the polarization in even highly conductive glass from the electrochemical polarization in a voltameter, in a single element of which no electromotive force can give rise to a return force greater than that due to the energy of combination of the constituents of the electrolyte. Subsequently, considerable residual charges were obtained with the same glass up to 150°; at 180° the residual charge was so rapidly lost that it was hardly sensible.

Temperature $39\frac{1}{2}$°.

	h. m.	
Time	6 10.	Charged with 7 elements.
	6 11.	From 462 to 350 in 2 seconds.
	6 12.	,, 463 to 360 ,, ,,
	6 17.	,, 464 to 350 ,, ,,
	6 19.	,, 464 to 350 ,, ,,

$$\left.\begin{array}{r}B=10{\cdot}0\\ \text{Log } B=\ 1{\cdot}0\end{array}\right\}\text{ at } 39\tfrac{1}{2}^{\circ}.$$

At	6 20.	Charged with 48 elements.
		Temperature 41°.
	6 40.	Discharge.
	6 41.	50 in 4 seconds.
	6 42.	28 ,, ,,
	6 43.	18 ,, ,,
		Temperature 41°.

$$\psi(1)-B=0{\cdot}24 \text{ at } 41^{\circ}.$$

h m
Temperature $33\frac{1}{2}^\circ$.
Time 7 50. Charged with 7 elements.
7 51. 462 to 340 in 4 seconds.
7 52. 463 to 340 ,, ,,
7 55. 465 to 343 ,, ,,
Temperature $33\frac{1}{4}^\circ$.

$$\left.\begin{aligned} B&=5{\cdot}4 \\ \text{Log } B&=0{\cdot}73 \end{aligned}\right\} \text{ at } 33\tfrac{3}{8}^\circ.$$

7 56. Charged with 48 elements.
Temperature 35°.
8 30. Discharge.
8 31. 115 in 10 seconds.
8 32. 67 ,, ,,
8 33. 46 ,, ,,
8 35. 29 ,, ,,

$$\left.\begin{aligned} \psi(1)-B&=0{\cdot}22 \\ \psi(5)-B&=0{\cdot}055 \end{aligned}\right\} \text{ at } 35^\circ.$$

Temperature $27\frac{1}{2}^\circ$.
10 2. Charged with 7 elements.
10 3. 459 to 340 in 5 seconds.
10 4. 460 to 360 ,, ,,
10 7. 461 to 368 ,, ,,
Temperature 27°.

$$\left.\begin{aligned} B&=3{\cdot}2 \\ \text{Log } B&=0{\cdot}50 \end{aligned}\right\} \text{ at } 27\tfrac{1}{4}^\circ.$$

10 8. Charged with 48 elements.
Temperature 28°.
10 41. Discharge.
10 42. 140 in 10 seconds.
10 43. 77 ,, ,,
10 44. 53 ,, ,,
10 46. 34 ,, ,,

$$\left.\begin{aligned} \psi(1)-B&=0{\cdot}26 \\ \psi(5)-B&=0{\cdot}064 \end{aligned}\right\} \text{ at } 28^\circ.$$

h m
Temperature 26°.
Time 8 45. Charged with 7 elements.
8 46. 452 to 350 in 6 seconds.
8 47. 453 to 350 ,, ,,
8 50. 455 to 368 ,, ,,
Temperature $25\frac{1}{2}$°.

$$\left.\begin{array}{r}B=2\cdot5\\ \text{Log }B=0\cdot40\end{array}\right\}\text{ at }25\tfrac{3}{4}°.$$

Temperature $24\frac{1}{4}$°.
9 11. Charged with 7 elements.
9 12. 458 to 345 in 8 seconds.
9 13. 458 to 351 ,, ,,
9 16. 457 to 355 ,, ,,
Temperature 24°.

$$\left.\begin{array}{r}B=2\cdot2\\ \text{Log }B=0\cdot34\end{array}\right\}\text{ at }24\tfrac{1}{8}°.$$

Temperature $22\frac{1}{2}$°.
9 38. Charged with 7 elements.
9 39. 455 to 338 in 10 seconds.
9 40. 456 to 340 ,, ,,
9 43. 457 to 352 ,, ,,
Temperature $22\frac{1}{4}$°.

$$\left.\begin{array}{r}B=1\cdot85\\ \text{Log }B=0\cdot27\end{array}\right\}\text{ at }22\tfrac{3}{8}°.$$

9 45. Charged with 48 elements.
Temperature $20\frac{1}{2}$°.
10 15. Discharged.
10 16. 150 in 10 seconds.
10 17. 81 ,, ,,
10 18. 55 ,, ,,
10 20. 33 ,, ,,

$$\left.\begin{array}{r}\psi(1)-B=0\cdot28\\ \psi(5)-B=0\cdot062\end{array}\right\}\text{ at }20\tfrac{1}{2}°.$$

h m
Temperature $7\frac{1}{4}^{\circ}$.
Time 4 40. Charged with 7 elements.
4 41. 466 to 385 in 20 seconds.
4 42. 465 to 397 „ „
4 45. 466 to 411 „ „

$$\left.\begin{array}{r} B=0{\cdot}45 \\ \text{Log } B=\bar{1}{\cdot}65 \end{array}\right\} \text{at } 7\tfrac{1}{4}^{\circ}.$$

4 46. Charged with 48 elements.
Temperature $7\frac{1}{4}^{\circ}$.
5 15. Discharge.
5 16. 250 in 20 seconds.
5 17. 160 „ „
5 18. 110 „ „
5 20. 66 „ „

$$\left.\begin{array}{r} \psi(1)-B=0{\cdot}24 \\ \psi(5)-B=0{\cdot}062 \end{array}\right\} \text{at } 7\tfrac{1}{4}^{\circ}.$$

Temperature -3°, after standing 30 minutes in the freezing-mixture.
7 19. Charged with 7 elements.
7 20. 457 to 417 in 20 seconds.
7 21. 458 to 427 „ „
7 24. 459 to 438 „ „
7 29. 461 to 442 „ „
Temperature -3°.

$$\left.\begin{array}{r} B=0{\cdot}17 \\ \text{Log } B=\bar{1}{\cdot}22 \end{array}\right\} \text{at } -3^{\circ}.$$

7 30. Charged with 48 elements.
Temperature $-1\frac{1}{4}^{\circ}$.
8 3. Discharged.
8 4. 180 in 20 seconds.
8 5. 115 „ „
8 6. 83 „ „
8 8. 56 „ „
Temperature -1°.

$$\left.\begin{array}{r} \psi(1)-B=0{\cdot}17 \\ \psi(5)-B=0{\cdot}053 \end{array}\right\} \text{at } -1\tfrac{1}{8}^{\circ}.$$

h m
Temperature -5°, in a fresh freezing-mixture.
Time 8 48. Charged with 7 elements.
8 49. 463 to 432 in 20 seconds.
8 50. 464 to 438 „ „
8 53. 465 to 447 „ „

$$\left.\begin{array}{r}B=0\cdot14\\ \text{Log } B=\bar{1}\cdot15\end{array}\right\}\text{ at } -5^\circ.$$

8 55. Charged with 48 elements.
Temperature -3°.
9 25. Discharged.
9 26. 176 in 20 seconds.
9 27. 108 „ „
9 28. 80 „ „
9 30. 53 „ „

$$\left.\begin{array}{r}\psi(1)-B=0\cdot17\\ \psi(5)-B=0\cdot050\end{array}\right\}\text{ at } -3^\circ.$$

As in Mr. Perry's experiments the results agree closely with the formula

$$\text{Log } B=a+b\theta,$$

where θ is the temperature, and in this case $a=\bar{1}\cdot35$ and $b=0\cdot0415$. The following Table gives the observed and calculated values:—

Temp. °	B observed.	B from formulæ.
39½	10	9·8
33⅜	5·4	5·4
27¼	3·2	3·0
25¾	2·5	2·6
24¼	2·2	2·2
22⅜	1·85	1·9
7⅛	0·45	0·46
−3	0·17	0·17
−5	0·14	0·14

The residual charge results do not show so great a degree of regularity, probably because the direct deduction of $\frac{dx}{dt}$ as equal to $\frac{\delta x}{\delta t}$ gives a greater error than the method used for obtaining B. This much is quite certain, that the value of $\psi(1)-B$ and $\psi(5)-B$ is rapidly increasing up to 7°. It appears probable that at higher temperatures these do not increase so rapidly if at all; but this is by no means certain, as although shorter times of insulation were used, the values at higher temperatures are notwithstanding more reduced by conduction than at the lower.

Glass No. 7.—Temperature 119°.

h m
Time 6 21. Charged with 7 elements.
6 22. 463 to 390 in 20 seconds.
6 23. 464 to 399 ,, ,,
6 26. 465 to 412 ,, ,,
6 31. 465 to 419 ,, ,,
Temperature 119°.

$$\left.\begin{array}{r} B=0{\cdot}370\\ \text{Log } B=\bar{1}{\cdot}568\\ \psi(1)=0{\cdot}608 \end{array}\right\}\text{ at } 120\tfrac{1}{4}^\circ.$$

6 32. Charged with 48 elements.
Temperature 122°.
7 5. Discharged.
7 6. 226 in 20 seconds.
7 7. 141 ,, ,,
7 8. 104 ,, ,,
7 10. 65 ,, ,,

$$\left.\begin{array}{r} \psi(1)-B=0{\cdot}215\\ \psi(5)-B=0{\cdot}062 \end{array}\right\}\text{ at } 123\tfrac{1}{4}^\circ.$$

Temperature 107°.
7 51. Charged with 7 elements.
7 53. 466 to 437? in 20 seconds.
7 56. 466 to 429 ,, ,,
8 1. 466 to 447 ,, ,,
Temperature 107°.

$$\left.\begin{array}{r} B=0{\cdot}148\\ \text{Log } B=\bar{1}{\cdot}169 \end{array}\right\}\text{ at } 108^\circ.$$

8 2. Charged with 48 elements.
Temperature 107°.
8 36. Discharged.
8 37. 162 in 20 seconds.
8 38. 100 ,, ,,
8 39. 76 ,, ,,
8 41. 51 ,, ,,

$$\left.\begin{array}{r} \psi(1)-B=0{\cdot}155\\ \psi(5)-B=0{\cdot}05 \end{array}\right\}\text{ at } 108^\circ.$$

h m
Time 9 25. Charged with 48 elements.
Temperature 98°.
10 1. Discharged.
10 2. 110 in 20 seconds.
10 3. 74 ,, ,,
10 4. 56 ,, ,,
10 6. 39 ,, ,,
Temperature 97¾°.

$$\left.\begin{array}{l}\psi(1)-B=0{\cdot}11\\ \psi(5)-B=0{\cdot}037\end{array}\right\}\text{ at }98\tfrac{3}{4}^{\circ}.$$

Temperature 172½°.
7 25. Charged with 7 elements.
7 26. 461 to 270 in 3 seconds.
7 27. 462 to 272 ,, ,,
7 30. 463 to 277 ,, ,,
7 35. 465 to 281 ,, ,,
Temperature 172°.

$$\left.\begin{array}{r}B=11{\cdot}9\\ \log B=\ 1{\cdot}076\end{array}\right\}\text{ at }175\tfrac{1}{2}^{\circ}.$$

7 36. Charged with 48 elements.
Temperature 172°.
7 50. Discharged.
7 51. 100 in 5 seconds.
7 52. 50 ,, ,,
7 53. 28 ,, ,,
7 55. 9 ,, ,,
Temperature 171½°.

$$\left.\begin{array}{l}\psi(1)-B=0{\cdot}38\\ \psi(5)-B=0{\cdot}034\end{array}\right\}\text{ at }175^{\circ}.$$

9 0. Charged with 48 elements.
Temperature 150°.
9 30. Discharged.
9 31. 122 in 5 seconds.
9 32. 125 in 10 seconds.
9 33. 96 ,, ,,
9 35. 64 ,, ,,

$$\left.\begin{array}{l}\psi(1)-B=0{\cdot}46\\ \psi(5)-B=0{\cdot}12\end{array}\right\}\text{ at }152\tfrac{1}{4}^{\circ}.$$

h m
Temperature 162°.
Time 10 13. Charged with 7 elements.
10 14. 461 to 330 in 3 seconds.
10 15. 462 to 340 „ „
10 18. 463 to 346 „ „
10 23. 463 to 353 „ „
Temperature 161°.

$$\left.\begin{array}{r} B=6\cdot42 \\ \text{Log } B=0\cdot807 \\ \psi(1)=7\cdot88 \end{array}\right\}\text{ at } 164^\circ.$$

10 24. Charged with 48 elements.
Temperature 165°.
10 58. Discharged.
10 59. 125 in 5 seconds.
11 0. 74 „ „
11 1. 50 „ „
11 3. 26 „ „

$$\left.\begin{array}{r} \psi(1)-B=0\cdot47 \\ \psi(5)-B=0\cdot098 \end{array}\right\}\text{ at } 167\tfrac{1}{2}^\circ.$$

Temperature 143°.
4 48. Charged with 7 elements.
4 49. 469 to 400 in 4 seconds.
4 50. 469 to 403 „ „
4 53. 470 to 410 „ „
4 58. 470 to 420 „ „
Temperature 143½°.

$$\left.\begin{array}{r} B=1\cdot99 \\ \text{Log } B=0\cdot300 \\ \psi(1)=2\cdot82 \end{array}\right\}\text{ at } 145\tfrac{1}{4}^\circ.$$

5 0. Charged with 48 elements.
Temperature 143°.
5 23. Discharged.
5 24. 190 in 10 seconds.
5 25. 115 „ „
5 26. 88 „ „
5 28. 55 „ „

$$\left.\begin{array}{r} \psi(1)-B=0\cdot36 \\ \psi(5)-B=0\cdot105 \end{array}\right\}\text{ at } 144\tfrac{3}{4}^\circ.$$

h m
Temperature 127°.
Time 7 8. Charged with 7 elements.
7 9. 465 to 412 in 10 seconds.
7 10. 466 to 416 ,, ,,
7 13. 467 to 427 ,, ,,
7 18. 468 to 428 ,, ,,

$$\left.\begin{array}{r}B=0{\cdot}63\\ \operatorname{Log}B=\bar{1}{\cdot}80\\ \psi(1)=0{\cdot}86\end{array}\right\}\text{at }128\tfrac{1}{2}^{\circ}.$$

7 20. Charged with 48 elements.
Temperature 126°.
7 58. Discharged.
7 59. 135 in 10 seconds.
8 0. 86 ,, ,,
8 1. 73 ,, ,,
8 3. 47 ,, ,,

$$\left.\begin{array}{r}\psi(1)-B=0{\cdot}26\\ \psi(5)-B=0{\cdot}09\end{array}\right\}\text{at }127\tfrac{1}{2}^{\circ}.$$

Temperature 79°.
9 30. Charged with 7 elements.
9 35. 468 to 448 in 2 minutes.
9 40. 468 to 450 ,, ,,
Temperature 79°.

$$\left.\begin{array}{r}B=0{\cdot}023\\ \operatorname{Log}B=\bar{2}{\cdot}36\end{array}\right\}\text{at }79\tfrac{1}{2}^{\circ}.$$

5 15. Charged with 48 elements.
Temperature 66°.
5 45. Discharged.
5 46. 55 in 40 seconds.
5 47. 31 ,, ,,
5 48. 22 ,, ,,
5 50. 14 ,, ,,
Temperature 64½°.

$$\left.\begin{array}{r}\psi(1)-B=0{\cdot}026\\ \psi(5)-B=0{\cdot}007\end{array}\right\}\text{at }65\tfrac{1}{2}^{\circ}.$$

h m
Temperature 94°.
Time 6 35. Charged with 7 elements.
6 37. 457 to 422 in 1 minute.
6 40. 458 to 432 ,, ,,
6 47. 458 to 433 ,, ,,
Temperature $94\frac{1}{2}$°.

$$\left.\begin{array}{r} B=0\cdot066 \\ \text{Log } B=\bar{2}\cdot82 \end{array}\right\}\text{ at }95^\circ.$$

Temperature $153\frac{1}{2}$°.
8 0. Charged with 7 elements.
8 1. 461 to 340 in 4 seconds.
8 2. 461 to 350 ,, ,,
8 3. 462 to 352 ,, ,,
8 5. 463 to 358 ,, ,,
8 10. 463 to 362 ,, ,,
Temperature $153\frac{1}{2}$°.

$$\left.\begin{array}{r} B=4\cdot36 \\ \text{Log } B=0\cdot64 \\ \psi(1)=5\cdot39 \end{array}\right\}\text{ at }155\tfrac{3}{4}^\circ.$$

Temperature 66°.
10 31. Charged with 7 elements.
10 41. $964\frac{1}{2}$ to 945 in 4 minutes.
Temperature 67°.

$$\left.\begin{array}{r} B=0\cdot0126 \\ \text{Log } B=2\cdot101 \end{array}\right\}\text{ at }66\tfrac{3}{4}^\circ.$$

With this glass the results do not agree so closely with the exponential formula as with glass No. 2. This is perhaps not surprising when it is considered that the temperatures differ more from that of the room, and, consequently, that errors due to unequal heating of the acid, and to exposure of the stem of the thermometer, will be greater.

The observed values of B, and those calculated from the formula $\log B=\bar{4}\cdot17+0\cdot0283\theta$, are given in the following Table :—

θ.	Observed.	Calculated.
175½°	12	14
164	6·4	6·4
155¾	4·4	3·8
145⅛	2·0	1·9
128½	0·63	0·63
120¼	0·37	0·37
108	0·15	0·17
95	0·066	0·073
79½	0·023	0·026
66	0·013	0·011

The values obtained for $\psi(1)$ and B do not in general give a value of $\psi(1)-\mathrm{B}$, which agrees very closely with that obtained by residual charge. This is not astonishing, for $\psi(1)$ and B are both subject to a considerable probable error, and do not differ greatly from each other. On the other hand, at high temperatures, the values of $\psi(1)-\mathrm{B}$ and $\psi(5)-\mathrm{B}$, obtained by residual charge, are undoubtedly much too low. It is interesting to remark, that whereas the values of $\psi(1)-\mathrm{B}$ and $\psi(5)-\mathrm{B}$ from residual charge do not increase with temperature above 160°, the values of $\psi(1)-\mathrm{B}$ obtained by difference show a continually accelerated increase. The observed values of $\psi(1)-\mathrm{B}$ and $\psi(5)-\mathrm{B}$ are collected in the following Table. The values above 140°, if admitted at all, must be regarded as subject to an enormous probable error.

Temperature.	$\psi(1)-\mathrm{B}$.	$\psi(5)-\mathrm{B}$.
175°	0·38	0·034
167½	0·47	0·098
152¼	0·46	0·12
144¾	0·36	0·105
127½	0·26	0·09
123¼	0·215	0·062
108	0·155	0·05
98¾	0·11	0·037
65¼	0·026	0·007

It should be mentioned that the temperature experiments were not made on the same flask as flask No. 7 of the previous experiments, but on a flask of the same composition.

XVII. *On the Physiological Action of the Bark of* Erythrophleum Guinense, *generally called Casca, Cassa, or Sassy Bark. By* T. LAUDER BRUNTON, *M.D., F.R.S., and* WALTER PYE, *Esq.*

Received and read June 15, 1876.

SEVERAL months ago we received from Mr. MONTEIRO a piece of Casca bark, which he had obtained with great difficulty from a native at Bembe during his residence in Angola. This bark is used by the natives as an ordeal, persons suspected of theft, witchcraft, or other crimes being made to drink an infusion of it. If it causes vomiting only, the person is acquitted; but if it causes purging, he is considered to be guilty, and is either allowed to die of the poison or at once killed. Among some tribes a practice prevails of making the accused, after drinking the infusion, walk in a stooping posture under half a dozen low arches made by bending switches and sticking both ends into the ground. Should he fall down in passing under any of the arches, he is at once considered guilty, and, without waiting for a purgative effect to be produced, he is at once put to death.

All the natives agree in their description of the effect produced on a person poisoned by this bark. His limbs are first affected, and he loses all power over them, falls to the ground, and dies quickly, without much apparent suffering.

The same bark, or one having precisely similar effects, is used as an ordeal in Sierra Leone, under the name of "red-water bark," and in Ashantee, under the name of "doom." In both these places the test of vomiting or purging only seems to be employed, and not that of stumbling, as described by Mr. MONTEIRO; but, according to C. A. SANTOS, the missionaries describe the bark as causing vomiting, glazing of the eyes, and loss of the power of contracting the muscles throughout the body; so that, when the poison has fairly commenced its action, the sufferer is incapable of standing or walking, and the head rolls heavily about the breast and shoulders.

Appearance of the Bark.—The pieces given to us by Mr. MONTEIRO were from 8–12 inches long, about 4 inches broad, and $\frac{3}{8}$ of an inch thick, dark brownish red in colour, and deeply grooved externally. Their appearance agreed exactly with the description given by C. A. SANTOS, in the 'American Journal of Pharmacy,' April 1849, p. 96, of the bark which he terms Saucy bark, or Gidu.

Chemical Reactions.—When treated with alcohol it yields a dark brownish-red tincture, and boiling water gives an infusion of a similar colour, which deposits a pale brownish-red precipitate on cooling; but at the same time the supernatant fluid remains turbid from suspended particles, which do not subside, and which are not removed by filtration. It becomes clear when heated, but the turbidity returns on cooling. The tincture, when evaporated, leaves a resinous-looking extract, and when mixed with water gives a pale brownish-red precipitate. The watery solution of either the alcoholic or aqueous extract becomes much darker in colour after exposure to air.

An aqueous solution gives a brownish-black precipitate with ferric perchloride, indicating the presence of some form of tannin. It also gives a precipitate with tincture of galls or tannic acid, mercuric chloride, stannic chloride, gold chloride, silver nitrate, and lead acetate, either neutral or basic. If the precipitate produced by neutral lead acetate be removed by filtration, the addition of basic lead acetate to the filtrate causes very little further turbidity. Platinum chloride causes little or no precipitate.

These reactions closely agree with those given by SANTOS and PROCTER.

SANTOS states that, by passing the dilute tincture through animal charcoal, washing, drying, and boiling in absolute alcohol, he obtained a crystalline substance which was poisonous; but PROCTER failed to obtain a poisonous substance, although he got crystals which were non-poisonous.

From the small quantity of bark at our disposal, we have not attempted to isolate any active principle, as we feared our supply would not be more than sufficient for the investigation of its physiological action.

Professor LIEBREICH, however, has lately succeeded in separating a crystalline substance, which is exceedingly poisonous.

General Action.—Its action on the lower animals has been investigated by SANTOS and LIEBREICH. The former found that a decoction of the bark caused alternate dilatation and contraction of the pupils, appearance of delirium, violent retching, vomiting, symptoms of tetanus, and, finally, death. The crystalline principle obtained from the bark by LIEBREICH caused vomiting and sudden death, without previous loss of consciousness. Death is attributed by him to paralysis of the heart.

In our own experiments on dogs and cats we have observed vomiting, weakness, and death during a convulsion from the effects of the drug, whether introduced into the stomach or injected under the skin, into the peritoneal cavity, or into the veins.

GENERAL ACTION ON MAMMALS.

Action on Cats.

Experiment I.—February 23.

Effects of a very Large Dose.

Four and a half cubic centimetres of a saturated alcoholic solution of Casca[*] were injected into the abdominal cavity of a half-grown kitten weighing 746 grammes. At three minutes after the injection it began to walk stiffly, and a peculiar jerk occurred in the hind legs each time they were lifted. Respirations 60. At 5′ after injection it

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[*] This solution was prepared by extracting the bark with alcohol and evaporating to dryness. The solid extract thus obtained was dissolved in warm alcohol in such quantity that on cooling a deposit occurred. The solution was then filtered and the filtrate employed for experiments.

seemed giddy, and rolled over on trying to walk. Fæces were passed. Respirations 80, gasping. At 10′ it was lying on its side with its mouth wide open; respirations 120. At 10′ 30″ the respirations were 160, gasping. At 11′ it made feeble and unsuccessful attempts to vomit; the respiration became intermittent; there was an oscillating movement of the eyeballs, and the pupils were widely dilated. At 12′ the respiration became slower, as well as irregular. At 16′ it was 60, irregular and laboured. At 21′ the respiration appeared to be entirely diaphragmatic. At 24′ there were strong, ineffectual attempts at vomiting, followed by sudden stoppage of respiration, and death in a condition of emprosthotonos. The pupils at that time were widely dilated.

On opening the thorax immediately after death, the ventricles were found firmly contracted; but they recommenced to pulsate, and continued to do so for a few minutes spontaneously. They responded by a single contraction to irritation for 3½ hours after death. By this time the heart looked quite dry and glazed, and rigor mortis was well marked in the muscles. The lungs were pale.

Experiment II.—February 22.

Effects of a Moderate Dose.

Three cub. centims. of the same alcoholic solution were injected into the abdominal cavity of a cat weighing 2238 grammes. At 25 minutes afterwards the animal vomited; and this was repeated at 35′ and 41′. At 41′ the cat seemed weak; respirations laboured, 60 per minute; pulse 100, regular. At 55′ respirations 80, shallow, irregular. At 75′ respiration deeper and more laboured. At 85′ the gait was staggering, and the limbs were moved with a jerk at the end of each step; respirations 40, more regular; pulse 100, regular. Four hours after injection the animal was sleepy; when roused it walked feebly; there was no further vomiting. Its condition remained unaltered as long as it was observed; and it died between eight and twenty hours after the injection.

On post mortem examination rigor mortis was well marked. The abdomen contained some yellow serum. The stomach contained no solid food, but about two ounces of a greyish turbid alkaline fluid.

Experiment III.—April 28.

Seven cub. centims. of a similar solution to that used in Experiments I. and II. were injected beneath the skin of a moderate-sized, well-nourished cat. In fifteen minutes the animal vomited for the first time, and this vomiting was repeated four times within the next hour and twenty minutes. During the rest of the day it remained quiet, without further vomiting, and with no paralysis or disturbance of muscular movements. It appeared to be quite comfortable.

On the following day it remained sitting in one position, and refused its food and milk. There was no vomiting, and no urination or defæcation. When disturbed, it would immediately return to its former position in a mechanical manner.

On the following day it passed a very small quantity of fæces; it was not observed to urinate, and, as before, it neither ate nor drank any thing. It vomited once, very slightly.

From this time forwards, for a fortnight after the administration of the poison, the cat remained in this condition, neither eating nor drinking, although tempted to do so with milk and meat; and even when a live mouse was placed before it, it merely pricked up its ears, and looked eagerly at it, but did not touch it, nor did it pass urine or fæces once for the last eleven days.

It sat always in one position unless disturbed, and though it got steadily weaker, did not lose flesh in the way an animal starved would have done. The temperature on May 2nd was 38° C.

Five days after the poison was given a subcutaneous abscess formed over the right scapula and ribs. No other lesions were ascertained during life. The abscess did not form at the seat of puncture.

It died, apparently from exhaustion, fourteen days after the poison was administered.

Post mortem Examination.

Rigor mortis well marked.

Subcutaneous tissue contained a fair amount of fat.

There was a large, sloughy, subcutaneous abscess in the situation mentioned above, and another localized collection of pus a little higher up in the skin of the neck. No other superficial abscesses were found.

The muscles were pale and rather dry. There was general congestion throughout the body of the larger venous trunks, but apparently not of the smaller ones.

Abdomen.—The omentum contained rather a large quantity of fat.

The stomach was quite empty, pale and contracted.

The small intestine contained a small quantity of bile-stained mucus; it was otherwise empty.

The large intestine contained bile-stained mucus, and in its lower half a considerable quantity of fæces, also bile-stained. The mucous membrane appeared perfectly healthy.

The bladder contained only a few drops of high-coloured urine, but had not contracted at all firmly. It had the appearance of a bladder which has been dried when inflated, and the air then let out.

The kidneys were pale, although the renal vein was much distended.

The vagina and uterus contained a large quantity of a greenish smeary fluid, which, under the microscope, was seen to be muco-pus. The mucous membrane had, here and there, patches of injection on it (*vide infrà*); near the orifice of the vagina the secretion had quite the character of ordinary pus, but no abscess existed there.

The diaphragm was pale, flabby, and very transparent (*vide infrà*).

Thorax.—The lungs were congested, but otherwise natural.

The heart was very pale and flabby; all the cavities contained moderate quantities of blood.

Microscopical Examination.

Kidneys: epithelium not degenerated.

Heart: muscle-fibres very granular; in many places hardly a trace of transverse striation could be seen.

Voluntary muscles (rectus abdominis) also granular (well marked, but not quite so much as the heart).

Bladder: muscular coat not degenerated.

Intestines: muscular coat not degenerated.

Remarks on Experiments I.–III.

Experiments I. to III. show the effects of the poison on cats when administered in three decreasing doses.

It will be seen that, during life, the most prominent symptoms of a rapidly poisonous dose were in their order of constancy:—1, vomiting; 2, respiratory difficulty; 3, abnormal muscular movements.

After death the condition of the heart and great vessels and of the lungs are most noteworthy.

Vomiting.—This was a constant symptom in all the cats we experimented on, unless they were placed under special circumstances. The vomit consisted, first, of whatever food might be contained in the stomach, and then of a white frothy mucus. On no occasion did it ever have the appearance of intestinal (fæcal) vomiting.

In Experiment I. no actual vomiting occurred. In this case the stomach was found to be empty of food after death; and the absence of the symptoms in this case was doubtless due to the rapid paralysis caused by the very large dose administered.

Respiratory Difficulty.—This is a constant symptom, except when very small doses are administered.

On reference to Experiment I. it will be seen that at one time the respirations rose to 160 per minute. This, however, was exceptional. From 40 to 60 respiratory movements per minute is probably the average rate after administration of a moderately poisonous dose.

Abnormal Muscular Movements.—These are of two kinds; thus, immediately after the administration of any dose, large or small, there is very generally produced a peculiar twitching of the muscles of the limbs, especially of the hind legs. This is especially seen when the animal is walking.

When large, rapidly poisonous doses are administered, symptoms of general muscular paralysis and loss of coordination are developed *pari passu* with the dyspnœa and frequency of the vomiting. The animal rolls and staggers as it walks; its head falls on the ground; and, finally, it falls over on its side and is unable to stand. Death always occurs a very few minutes after the development of these last phenomena.

Appearance of the Heart post mortem.—As a rule, post mortem examination of the heart shows a moderately firm contraction of the ventricles, with a somewhat distended condition of the auricles. The ventricles, however, were never found to be completely emptied of blood, and on one or two occasions the heart was found to be moderately distended, the left ventricle containing well arterialized blood.

On several occasions, but here also with one or two exceptions, a remarkable vitality of the auricles was noticed (Experiment I.).

The post mortem appearances of the heart and their physiological value will be noticed more particularly in the section which treats especially of the action of that organ.

The lungs were, in all cases in which they were noticed, found to be pale, except in Experiment III. (see Experiment XXXI.).

One of the most noteworthy phenomena is the action of a small dose upon a cat in causing an utter refusal to take either food or drink, and that, notwithstanding this total abstinence from nourishment, the animal should live such a long time, should show considerable muscular power (being able to jump from the floor upon a chair up to the day before its death), and should have still retained so much of its subcutaneous and omental fat. Another point to be noted is the occurrence of subcutaneous abscesses, none of which were near the point where the poison had been injected.

This long continuance of life and retention of strength seem to us to indicate that the processes of tissue change had been retarded by the poison; and the granular condition of the striated muscles appears to indicate a diminution especially in the processes of oxidation.

Action on Dogs.

Experiment IV.—May 9.

The effect of the poison on dogs was investigated in the same manner as it had already been on cats in Experiments I.–III. It will be seen that the results do not differ in any noteworthy point from those previously obtained.

Six cub. centims. of the solution were injected beneath the skin of a dog weighing 8 lbs. It vomited for the first time twenty minutes afterwards, and this vomiting continued at gradually increasing intervals of from 15 minutes to three quarters of an hour for the next four hours. At the end of that time the animal was very restless, and continued to be so while it was observed. On the following day its gait became staggering; and, finally, it lay flat on its belly, and died about thirty hours after the drug was administered.

GENERAL ACTION ON BIRDS.

Birds are affected very readily by the poison, and the symptoms produced in them are similar to those observed in mammals. This will be seen by the results of the following experiment.

Experiment V.

A full-grown pigeon had nearly 1 cub. centim. of the solution injected beneath the wing. In ten minutes a quivering motion of the wings was noticed; in a quarter of an hour its feathers were puffed out, its gait was staggering; twenty-four minutes after injection it began to vomit. This was repeated four minutes afterwards more violently, and several times subsequently. Forty minutes after the injection it was unable to stand; and from that time to its death, one hour and thirty-five minutes after injection, it lay flat on the table, occasionally attempting to vomit unsuccessfully. For the last forty minutes its respirations were hardly visible, but it moved when roused. Violent expiratory movements came on just before the respiration finally ceased.

GENERAL ACTION ON FISHES AND FROGS.

In fishes and frogs there is but slight susceptibility to the poison, and the effects produced by it are similar in the two classes. The most obvious general symptoms are muscular paralysis and cessation of respiration, preceded by spasmodic movements. It will be seen later, however (Experiment XVIII.), that the particular action of the drug on the heart is well shown in frogs.

Experiment VI.—February 21.

One third of a cub. centim. of the solution was injected beneath the skin of a medium-sized frog. In two minutes slight tonic contraction of the limbs was observed; in four minutes it was jumping rather actively, but fell over on its back; in twelve minutes the respiratory movements had become almost imperceptible; and from this time the reflex movements of the limbs on irritation gradually got weaker and weaker, and finally ceased thirty-three minutes after the injection.

After death, the ventricle was found firmly contracted, the auricles and venous trunks engorged.

Experiment VII.—February 21.

Experiment VI. was repeated with double the dose. The frog was slightly larger than the one first used; but reflex action ceased within a few seconds of the same time after injection. The general effects were almost the same as in Experiment VI., save that five minutes before reflex movements ceased there were four spasmodic inspirations.

Experiment VIII.—April 18.

The effect on fishes was tried. First, 55 cub. centims. of a $\frac{1}{300}$ watery solution of casca were added to 3 litres of water in which a gold-fish weighing 3 ounces was swimming. At the end of three hours no effect was produced on the fish. 1·3 cub. centim. of the alcoholic solution were then injected into the side of the fish, a little in front of the tail. In five minutes it began to roll from side to side; the respirations were catching. For the next ten minutes it lay chiefly on its side, occasionally swimming about actively.

At the end of twenty-five minutes from the time of injection it appeared to have nearly recovered itself, and 1 cub. centim. more of the alcoholic solution was injected.

In three minutes from this time it lay completely over on its side, having spasmodic twitchings of its fins; in five minutes the respirations again became rapid and gasping; in ten minutes the reflex movements were very weak, but respiration and reflex action did not entirely cease before thirty minutes after the second injection.

GENERAL ACTION ON INVERTEBRATA.

The following experiments (Nos. IX. and X.) show that the drug exerts very little, if any, poisonous action on the Invertebrata.

Experiment IX.

A leech was placed in a watery solution of casca nearly as strong as could be made with cold water. At the end of two hours and a half it seemed but little affected, but was found dead on the following morning.

Experiment X.

Nearly 5 cub. centims. of a concentrated alcoholic solution of casca were injected beneath the back of a common snail. The animal showed no signs of poisoning, and on the following morning appeared to be uninjured.

ACTION ON INFUSORIA.

Experiment XI.

In Experiment XI. we investigated its action on Infusoria by placing a drop of tank-water containing some infusorians under the microscope, and adding a drop of a $\frac{1}{300}$ solution of the watery extract of casca. At the end of two hours no alteration in the movements of the animalcules was observed; and it may therefore be concluded that the drug exerted little or no action on them.

The action of the drug on germination and oxidation processes, and on different ferments and ferment organisms, was investigated in Experiments XII.–XX.

On germination the casca infusion was found to exert no effect.

Experiment XII.

A few mustard seeds were placed on flannel in two saucers, and kept moist in a warm place, the one with an infusion of casca, the other with water. The seeds began to germinate at the same time in each, and no difference was observed in the growth of the shoots for three days afterwards.

Effect on the Development of Bacteria.

Experiment XIII.

This experiment, which was repeated on two other occasions, shows that a weak solution of the alcoholic extract possesses the power of hindering the formation of *Bacteria*, a property not shown by the watery extract, as is shown in Experiment XIV. This difference in the properties of the two extracts does not show itself in the general action of the drug on animals; but the power of the alcoholic extract to prevent the development of *Bacteria*, while it is without action on them after their development, is interesting, as substantiating the results of BUCHHOLZ's experiments on this subject with other drugs*.

Three pieces of fresh muscular tissue were placed in bottles on March 22nd. The first contained a watery solution of the alcoholic extract of casca, the second a $\frac{1}{200}$ solution of sulphate of quinia, and the third distilled water. On March the 29th the bottles were opened; and while the bottle containing water was very offensive, and the water was crowded with *Bacteria*, neither the quinine nor casca solutions contained any *Bacteria* at all.

The bottle containing the casca solution was again examined on May 14th, and was found, as before, quite free from *Bacteria*. Long before this a thick crust of *Penicillium* had formed on its surface.

Experiment XIV.

A piece of fresh cat's liver was placed in a solution of casca of the same strength as that used in the preceding experiment; but the watery extract was used instead of the alcoholic. At the end of two days the liquid was found to be crowded with *Bacteria*.

This experiment was afterwards repeated with muscular and other tissues with the same result.

Effect on the Life of Bacteria.

The effect of the drug on the life of *Bacteria*, when developed, was tried in Experiment No. XV. For this purpose an infusion of hay was made, and found to contain many rod-shaped *Bacteria*. To a drop of this infusion a drop of a $\frac{1}{20}$ solution of both the alcoholic and the watery extracts of casca was added at different times, and the movements of the Bacterians carefully watched under the microscope. They did not, however, seem in any way affected by the addition.

For the sake of comparison a $\frac{1}{100}$ solution of quinia sulphate was added to the hay-infusion. The Bacterian movements were found to be instantly stopped.

Effect on Red and White Blood-corpuscles.

In Experiment XVI. the action on the red and colourless blood-corpuscles of the newt was investigated. The effects produced by the addition of dilute solution of casca to the blood were cessation of amœboid movements and rounding of the white corpuscles, with an irregular shrinking of the nucleus, and general crenation of the red ones. These effects were probably due to the action of the tannic acid contained in the extract.

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* Archiv f. exper. Pathologie u. Pharmakologie, iv. p. 1.

Effect on Ciliary Motion.

The drug appears to have no action on ciliary motion; for when (Experiment XVII.) two preparations of ciliated epithelium were made, the one being placed in ·75 per cent. salt solution, and the other in a $\frac{1}{300}$ solution of casca extract, it was found on microscopic examination that the movements of the cilia ceased in about the same time in both specimens.

Effect on Processes of Oxidation.

It, however, does appear to exercise an inhibitory action on oxidation processes generally. This point was investigated in the following manner:—

Experiment XVIII.—April 20.

Four thin slices of potato were placed in two saucers, and were just covered, the one with distilled water, the other with a $\frac{1}{300}$ watery solution of casca. When a drop or two of the tincture of guaiacum was added, either to the liquid or to the potato slices, the bluing produced was much fainter in the case of the saucer containing the casca than in that containing distilled water. The results of the experiment on organized and unorganized ferments were negative, neither the development of the yeast-plant (Experiment XIX.) nor the digestion of fibrin by pepsin (Experiment XX.) appearing to be in the least degree hindered by the addition of the drug.

ACTION ON THE DIGESTIVE SYSTEM.

One of the most prominent symptoms of poisoning by casca is the violent vomiting which it produces; and, as has already been noticed, its occasional purgative action is used as a test of innocence or guilt. The emetic or purgative action is supposed by some to depend on the administration of a pure infusion, or of one containing the drugs in suspension, and innocence or guilt are thus supposed to be practically decided by the priests, who have it in their power to administer either one or other to the accused.

In order to test this, an infusion was given to one cat, B (Experiment XXI.), and an infusion containing a quantity of powder to another. The latter, however, contrary to expectation, recovered, whereas the former died. The experiment, however, was vitiated by the fact that the infusion was made from the finely pulverized bark, the only kind we had at our disposal at the time, and consequently contained a quantity of it in suspension, which would not have been the case if the infusion had been made from a coarsely pounded bark.

In order to ascertain whether the vomiting and purging were due to the local action of the drug on the stomach and intestines, or to its action on the nervous system after its absorption into the blood, a comparison was made between the effects of the poison when introduced into the stomach and when injected under the skin. Our experiments show that whereas vomiting was invariably produced by the casca, in whatever manner introduced into the system, purging only occurred when the poison was given by the mouth, and was never observed after subcutaneous injection. The purgative action is therefore due to the local action of the drug on the intestines.

The following is a brief account of two experiments we made in investigating the action of the drug when injected into the stomach.

Experiment XXI. A.—May 10.

10 cub. centims. of an infusion of the watery extract of the bark, with the dregs which were deposited when the infusion cooled, were injected into the stomach of a large cat. It appeared quite well for forty minutes, and then vomited. Within the next two hours and a half it vomited five times. A little more than five hours after the exhibition of the drug it passed some solid fæces with great forcing, and from that time recovered.

Experiment XXI. B.—May 10.

10½ cub. centims. of a cold aqueous infusion of the pounded bark, containing numerous fine particles in suspension, were injected into the stomach of a small ill-nourished cat. Vomiting came on thirty-five minutes afterwards, and free purging an hour and a half after the injection. During the rest of that day and on the next it was very feeble, but showed no special symptoms, and it died quietly on the morning of May 12.

Comparative Action of the Alcoholic and Watery Extracts.

The action of the alcoholic and watery extracts of casca, when administered subcutaneously to cats in large doses, is almost identical; and their activity as poisons appears to be about equal, the watery extract, if any thing, being rather the more powerful.

Experiment XXII.—April 28.

Two equal quantities of the alcoholic and watery extracts were dissolved in equal volumes of alcohol and water respectively. The quantities were 3 grammes of the extracts and 4 cub. centims. of the fluids. These solutions were injected beneath the skin of two cats of the same size. In the case of the alcoholic extract vomiting came on fifteen minutes after injection, with the other symptoms of poisoning by the drug (*i. e.* respiratory difficulty and staggering gait). The vomiting was repeated violently, and the animal died one hour and fifteen minutes after injection, death being preceded by general convulsions.

In the case of the watery extract vomiting did not come on for thirty-five minutes, but death occurred, with symptoms similar to those of the former case, in one hour after the injection.

In order to ascertain whether the vomiting was due to the action of the drug upon the sensory nerves in the stomach itself, after it had been conveyed to that organ by the circulation, or to its action upon the nervous centre in the medulla oblongata regulating the movements of vomiting, the vagi were cut, and the chief sensory nerves of the stomach thus divided, before administering the poison. By this procedure the retching and vomiting were either completely prevented or very greatly diminished, the dyspnœa rendering it rather difficult to decide in some cases whether some convulsive movements were due to it or were movements of retching. The vomiting is therefore chiefly, and in all probability entirely, due to the action of the drug on the sensory nerves of the stomach itself, as the retching, if indeed really present, might be due to irritation conveyed to the medulla through the splanchnics after the vagi had been divided.

Experiment XXIII.—March 6.

General Symptoms after Injection, both Vagi having been previously divided.

A cat weighing 3 lb. was chloroformed, and the vagi divided in the neck. In 25 minutes after the operation it had recovered from the effects of the anæsthetic. Its respirations were 18 per minute. Three cub. centims. of a concentrated alcoholic solution of alcoholic extract of casca were injected subcutaneously. Five minutes afterwards the cat had fallen over on its side. The respirations were still regular, 16 per minute. During the next hour, with one temporary disturbance, the animal remained quiet, still breathing quietly and slowly, with no symptoms of sickness and no dyspnœa. It remained on its side the whole time, except when roused. It then staggered a few steps, and again lay down. One hour after the first injection $2\frac{1}{2}$ cub. centims. more were injected. For the next quarter of an hour the animal continued to breathe easily, but appeared weaker. At the end of that time there were some very slight convulsive movements, and then respiration ceased. On beginning artificial respiration one or two gasping inspirations occurred, and then entirely ceased one hour and fifteen minutes after the first injection. On post mortem examination the heart's cavities were found distended. They did not contract on irritation or puncture. The lungs were bright scarlet, and contained a moderate amount of blood. The liver and kidneys were congested ; the stomach was pale ; the brain was normal.

Experiment XXIV.—April 28.

This experiment was in most points an exact repetition of Experiment XXIII., but the results were even more striking. A well-nourished cat was chloroformed, and both vagi were divided in the neck. When it had recovered from the chloroform a solution of ·3 grm. of the alcoholic extract in 4 cub. centims. of alcohol was injected subcutaneously. None of the ordinary symptoms of poisoning by the drug were produced. There was no dyspnœa and no vomiting, except at one time, an hour and fifteen minutes after the injection, when the animal either coughed or vomited up a small quantity of frothy mucus. When seen the next morning it was to all appearance well, and was killed, to prevent suffering being caused by the secondary effects of section of the vagi, which were found to be completely divided.

Experiment XXV.—May 17.

This experiment was similar to Nos. XXIII. and XXIV. As before, no vomiting was produced by injection of the drug after section of the vagi, but death occurred one hour and ten minutes after the injection, in consequence of dyspnœa occasioned by the section of these nerves.

Action of Casca on Respiration.

Powdered casca, when inhaled, acts as a violent sternutatory. All the men employed by us in grinding or pounding the bark suffered severely from the violent and irresistible fits of sneezing which attacked them; and in one instance these were accompanied by great faintness and tendency to syncope.

When injected into the circulation casca greatly accelerates the respirations (Experiments I., II., XXXIV.).

This acceleration appears to be due to stimulation of the pulmonary branches of the vagus, and not to any action of the drug upon the respiratory centre, as no acceleration is noticed when the vagi are divided before the injection of the casca (Experiments XXIII. and XXIV.).

Action on the Intestines.

Experiment XXVI.

In order to ascertain whether the intestinal secretion was increased by casca, a cat was chloroformed, the abdomen opened, and three loops of small intestine ligatured. Into the middle loop 2 cub. centims. of a concentrated solution of the watery extract of casca were injected, and 2 cub. centims. of water into the other two. The cat vomited about an hour afterwards. At the end of about 5 hours, the animal was killed and the body examined. The upper and middle loops were both dry, and the mucous membrane was normal in appearance, except slight congestion at the place of ligature between the upper and middle loop. The lower loop contained several cubic centims. of turbid greyish fluid.

The intestinal secretion is thus seen not to be increased by the drug.

Action of Casca on Circulation.

Experiment XXVII.

Preliminary Experiments on Frog's Heart.

A watery solution of the alcoholic extract and a standard salt solution were prepared; the hearts of two frogs of about the same size were then removed, and placed for a minute or two in ·75 per cent. salt solution. When they had recovered from the shock of removal, and were beating regularly, one was placed in the casca solution, the other in the salt one. At the commencement of the experiment the heart, A, placed in salt solution was beating at the rate of 6 per 10 seconds; the heart, B, in casca at the rate of 75. Both hearts became weaker and their pulsations slower; at the end of 50 minutes the heart in casca stopped entirely, that in salt solution pulsated feebly at the rate of 3 per 10 seconds. In 15 seconds more it stopped.

In Experiment XXVIII. we repeated the foregoing experiment with a much stronger casca solution. The hearts at the commencement of the experiment were beating at the rate of 4 per 10 seconds. At the end of 30 minutes the heart, B, in casca, which had previously got very slow and weak, stopped, while the standard heart, A, was still beating strongly and regularly at the rate of 4 per 10 seconds.

Experiment XXIX.—March 7.

The heart of a frog whose cerebrum had been destroyed was exposed. Pulse 72 before injection.

h m
11 40. Injected 2 cub. centims. alcoholic solution beneath skin of back.
11 50. Pulse 60; regular.
12 2. Pulse 48.
12 10. Pulse 60. Clonic convulsions.
12 15. Ventricle stopped in systole, firmly contracted in its greater part, with a pouch-like dilatation of a small portion. Auricles still contracting.
12 27. Ventricle firmly contracted. Still partial contraction of auricles. Respiration and slight convulsive movements continue.
12 50. Removed from frog-board. The ventricle is not so firmly contracted. Still respiring and occasionally convulsed.
1 0 P.M. Ventricle relaxed. Slight movements of it have recommenced. 1·4 cubic centim. alcoholic solution again injected.
1 7. Ventricle again firmly contracted. No respiration. Still slight reflex movement.
1 30. Died with heart in same condition.

Experiment XXX.—May 10.

The heart of a frog was exposed, and a little extract of casca placed on it. Pulse 34 per minute.

It had no apparent action.

A watery solution of casca was then poured into the thorax. The pulse became slower, =24 per minute.

The ventricle then expanded irregularly; the diastole at the base being later than that at the apex.

Then the distention became imperfect, the ventricle seeming wrinkled.

The heart then stopped in systole, having two pin-point dilated pouches on it.

These experiments show that a very weak solution of casca applied to a frog's heart, when removed from the body, slows its pulsations, while, after the application of a stronger solution, the pulsations become slow, then the systolic contraction ceases to take place instantaneously over the whole surface of the ventricle; lastly the heart stands still in systole.

When the heart of a frog is exposed, but not removed from the body, and a solution of casca is injected beneath the skin, the heart's action is slowed, and is eventually stopped in systole; previous to its stopping, however, pouch-like dilatations are formed; in this respect the action of casca is similar to that of digitalis and other cardiac poisons.

Experiment XXXI.—April 27.

A cat was chloroformed; a cannula placed in the left jugular vein and one in the trachea. Artificial respiration was commenced, and the thorax was opened The heart was beating regularly, but it was difficult to count the pulsations. They were counted by one person as 90, by another as 180.

10 cub. centims. of a saturated watery solution of watery extract of casca were injected into the jugular vein. No apparent effect was produced.

7 cub. centims. more were injected in the course of a few minutes. Within about a minute of the last injection the ventricle no longer contracted as a whole, but became pouched, the upper half seeming to overlap the under half so as to produce a transverse fold.

A few seconds afterwards, the lungs, which had hitherto been rosy, became white, and almost immediately the motions of all cavities of the heart completely ceased. On irritation of the ventricle no movement occurred.

Both vagi were divided, but without effect on the heart.

It was noticed that the rosy colour of the lungs returned, although the heart did not again beat. No pulsations were noticed in the pulmonary vessels. The heart was perfectly firm, and seemed to be in systole; but on tying a ligature round the base so as to include the large vessels, it contracted to about one third of its former size.

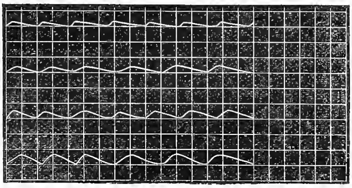
The action of casca on the mammalian heart is seen from this experiment to be similar to its action on the heart of the frog.

Experiment XXXII.

Action on Blood-pressure in the Frog.

The cerebrum of a large frog was destroyed. A cannula was then inserted into the left aorta, and was connected with a small kymograph, the pulse-wave and oscillation being recorded on a revolving drum.

The appended curves give the oscillations of pressure in the aorta, and show that under the influence of casca the blood-pressure in the aorta rises to twice its normal height during systole, although it falls to zero during diastole.



Normal pulse (10 millims.) ....

Pulse three minutes after injection of 1 cub. centim. of a $\frac{1}{300}$ solution of casca into flank ....

Pulse one minute after a second injection of 1 cub. centim.

Pulse three minutes later ....

Experiment XXXIII.—March 31.

Action on the heart and blood-pressure of a large dose of casca. (For action on secretion of urine also, *vide infrà.*)

A bitch, weight 26½ lb., was chloroformed.

A cannula was placed in the trachea.

„ „ left femoral vein.

„ „ right ureter.

„ „ left carotid artery.

The operation was very long, and during it the intestines became much congested.

Time.		Respiration.	Amount of Urine secreted per 10 minutes.	Blood-pressure.	Oscillations.	Pulse.
h. m. s.			minims.			
1 54 57	Normal condition before experiments	....	5	120	10	20
	The rate of secretion of urine was estimated at two successive periods of 10 minutes each.					
2 7 40	4 cub. centims. casca solution $\frac{1}{300}$ injected into femoral vein.					
0 8 10 0 10 30	Showing the ordinary effect of the injection of a small quantity	....	....	140	10	20
0 17 40	Urine increased in quantity, alkaline, turbid, not albuminous	....	7	140	10	20
2 27 40	Urine still further increased	....	24	145	10	20
	Is clear; not albuminous.					
2 34 30 2 34 40	5 cub. centims. more injected.					
2 35 40	Showing effect of further injection	....	....	160	15	20
2 47 40	Urine secreted, clear as before	....	17	160	15	20
	Clot formed. Lost about 15 cub. centims. of blood.					
2 57 40	Urine as above	....	17	140	25	20
	With the increased dose the pulse-oscillations have begun to be affected as well as the blood-pressure.					
3 0 10	10 cub. centims. more injected.					
3 0 30	Showing the immediate effect of a large dose. The systole and diastole are nearly equal; there is a very powerful action of the heart and increased blood-pressure	....	....	165	40 90	10
3 2 40	Showing the later effects of a large dose. The oscillations of the pulse are less, while the blood-pressure is greatly increased. The secretion of urine is much diminished	....	3	200	15 20	20
3 7 40	Secretion of urine stopped	....	....	180	10 20	16
3 17 40	Urine has again commenced to be secreted, the blood-pressure having fallen somewhat	....	5	150	30	15
3 27 40	Urine secretion again increased	....	13	110	50	12½
3 37 40		....	9	115	30	16
3 37 50	Further injection of 15 cub. centims.					
3 38 10	The blood-pressure is now much affected by the respiration after the injection of a large dose	5	....	150 { 160 140	4	30
3 40 0	This is still more marked	5	....	170 { 190 150	2	40
3 42 0	The blood-pressure gradually fell, and the pulse was too feeble to write on the kymograph, while there was still a rise of blood-pressure synchronously with the respirations, which stopped at 3 h. 43 min.					

The thorax was then opened, and the heart found beating; respiratory movements recommenced on opening the thorax.

The bladder was found greatly distended; there had been micturition during the experiment, which was probably only overflow.

Remarks.

We append a diagram (p. 652) showing the coincident variations of blood-pressure and secretion of urine, which will be more fully commented on later. (*Vide* effect on urinary secretion.)

So far as the phenomena of correlation are concerned, this experiment shows that while a small dose of casca slows the pulse, an additional one greatly quickens it. This action of casca closely resembles the effect of digitalis, which first slows the pulse by stimulating the vagus-roots, and then quickens it by paralyzing the ends of the vagus in the heart. It therefore seemed probable that the cardiac ends of the vagus would be found to be paralyzed by large doses of casca.

It was possible that the primary slowing of the heart's action might be due to stimulation of the inhibitory apparatus in the heart itself, and not to the action of the drug on the vagus-roots. Two questions, therefore, were to be settled:—

1st. Is the primary slowing of the pulse due to stimulation of the vagus-roots, or to stimulation of the inhibitory apparatus in the heart?

2nd. Is the secondary acceleration of the pulse due to paralysis of the ends of the vagus in the heart?

To answer these questions the following experiment was performed.

Experiment XXXIV.—March 21.

Action on the Heart and Arterial Pressure of a small dose of Casca.

A dog weighing 8 lb. was chloroformed, and kept under chloroform during the experiment.

A cannula was inserted into the right carotid and into the right femoral vein.

Time.			Blood-pressure.	Pulse in 10 seconds.	Oscillations.	Respiration.
m.	s.		millims.		millims.	
		Condition before injection	110	$18\frac{1}{2}$	10	
0	55	7 cub. centims. $\frac{1}{300}$ watery solution injected into femoral vein	125	$16\frac{1}{2}$	10	
1	0		130	16	10	
1	10					
1	20	Rise of blood-pressure. Commenced prolongation of diastole	155	14	10	
1	45	Systole fairly sharp, diastole very long	140	5	40	
		Commencing fall of blood-pressure.				
2	5	Great fall of blood-pressure and lengthening of diastole.	65	$\frac{1}{3}$	25	
2	20	Diastole extends over 15 seconds.				
		Systole extends over $\frac{1}{2}$ second; does not vary with respiration.				

TABLE (continued).

Time.		Blood-pressure.	Pulse in 10 seconds.	Oscillations.	Respiration.
m. s.		millims.		millims.	
2 40 3 0	Blood-pressure again lower. Diastole further prolonged Diastole 20 seconds. Systole ⅓ second. Diastolic curve slightly affected by respiration.	45	½	30	5
3 40 3 50	Blood-pressure recovering. Diastole shorter Diastole 5 seconds. Systole ½ second. Respirations affect diastolic curve to extent of 3 millims.	50	2	25	9
4 0 4 10	Blood-pressure rising Systole and diastole nearly equal. Both affected by respiration to extent of 4 millims.	100	4	35	10
4 20 4 30	Blood-pressure nearly at the height of commencement of experiments. Systole sharp	130	5	35	
4 40 4 50	Blood-pressure higher. Diastole again prolonged Diastole 4 seconds. Systole ½ second. Respirations affect curves to extent of 10 millims.	140	2½	60	10
5 20 5 30	There was again a fall of blood-pressure with great prolongation of diastole similar to that which occurred at 2.5 to 2.20, but less marked	115	1	50	9
6 0 6 10	Do., more marked	80	1	30	4
6 20 7 30	Gradual fall of blood-pressure. Cessation of heart's action.				

Post mortem (immediately after death).—The heart contained blood, and contracted on puncture. The auricles contracted for three minutes after death.

This experiment shows that the action of a small dose of casca is to raise the blood-pressure and slow the heart at first. Next, when the heart becomes very slow, the pressure falls, and finally the heart ceases to beat and death takes place.

The cardiac pulsations remained slow from the time of the injection of the casca up till death; and although they at one time rose from 1 pulsation in 30 seconds up to 5 pulsations in 10 seconds, they never came at all near to the normal, which in this animal was 18½ pulsations in 10 seconds.

The very slow pulse here indicates that the vagus is probably stimulated by the casca; and the continuance of the blood-pressure at the height of 65 millims., during a cardiac diastole, lasting for 30 seconds, shows unmistakably that the arterioles are strongly contracted by the drug.

ACTION ON VAGUS.

Maximum Irritation.

Experiment XXXV.—March 4.

A cat, weight 4 lb., was chloroformed, and a cannula was placed in the left carotid artery and in the left femoral vein.

Time.		Blood-pressure.	Oscillations.	Pulse in 10 sec.
m. s.		millims.	millims.	
1 0	Normal curve taken 1 minute after connexion of cannula. Oscillation at top of respiratory curve =9, at bottom 2	80	2 9	23
2 15	R. vagus ligatured and cut.			
2 18	L. vagus do. do.			
	Condition after section of vagi	100 110	2	40
4 40	L. vagus irritated. Coil 5°*	60	18	15
	R. vagus irritated. do.	55	12	12
8 10	Condition subsequent to irritation	110	1	42
	It was thus ascertained that irritation of the strength of coil 5° was almost sufficient to stop the heart's action when applied to the peripheral end of either vagus previous to injection of casca.			
8 10 9 12	3½ cub. centims. of casca solution injected gradually into vein.			
10 0	Condition after injection	120	2	40
11 0	A clot formed.			
12 0	Condition after clot was removed	115	4	43
	(Slight improvement of pulse.)			
12 35	Peripheral end of L. vagus irritated. Coil 5°	90	8	35
13 0	Peripheral end of R. vagus irritated. Coil 5°	95	8	35
	Condition between irritations	120	2	35
14 27	Left vagus. Coil 0°	88	5	27
	Right vagus. do.	110	0	0
	Complete stoppage of heart's action without fall of blood-pressure.			
15 20	Experiment repeated with R. vagus. Coil 0°	85	5	26
17 10 17 44	Fresh injection of 4 cub. centims.			
18 30	Condition after injection	115	2	40
19 5	Right vagus. Coil 0°. Commencement of irritation	100	3	30
	Latter part ,,	118	2	36
19 30	Left vagus. do. Commencement ,,	104	4	30
	Latter part ,,	120	2	35
20 0	Left vagus. do. Commencement ,,	121	2	35
	Latter part ,,	150	2	35
20 30	Condition after irritation	120	2	40
22 0 22 25	A third injection of 2 cub. centims. was given.			
24 25	And the central ends of the vagi exposed for irritation.			
24 30	Condition before irritation	120	2	40
25 12	Central end of left vagus irritated. Coil 0°	130	1 ?	30
	(Blood-pressure increased; pulse nearly extinguished.)			
25 50	Condition after irritation	125	2	34
26 30	Right vagus. Coil 0°	130	1	34
	(Effect as before.)			
27 0	Condition after irritation	120	2	36
28 16	The sciatic nerve was exposed and irritated with coil 5°	135	2	37
28 45	Condition after irritation	120	2	36

This experiment shows that when both vagi are divided the injection of a small dose of casca no longer slows the heart; and therefore the slowing usually observed after

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[*] The mark ° here signifies distance in centimetres between primary and secondary coils in DU BOIS REYMOND's induction-apparatus.

its administration is due to stimulation of the vagus-roots and not to stimulation of the inhibitory apparatus in the heart itself.

It also shows that a large dose completely paralyzes the ends of the vagus in the heart, so that a strong interrupted current applied to the trunk of the nerve produces no slowing of the cardiac pulsations.

Action of a small dose on the excitability of the Vagus.

As it has been stated that digitalis in small doses increases the excitability of the ends of the vagus in the heart, so that a slight irritation applied to the trunk of the nerve will cause slowing or stoppage of the heart after the administration of the drug, although previously it had no effect, it seemed advisable to ascertain whether or not a similar action was possessed by casca. The following experiment was therefore tried:—

Minimum Irritation of Vagus (peripheral end).

Experiment XXXVI.—April 6.

A cat, weight 6 lb., was chloroformed, and kept under chloroform the whole time of the experiment.

A cannula was inserted into the right carotid artery and into the right femoral vein.

Both vagi were then cut, and the peripheral end of the right vagus attached to VON BASCH's electrodes.

Operation lasted half an hour; the cat at first very feeble, afterwards recovered.

Time.		Blood-pressure.	Oscillations.	Pulse.
m. s.		millims.	millims.	
1 50	Condition previous to irritation, both vagi being cut	120	1–2	40
2 0	Right vagus irritated. Coil 30°. No effect	120 125	1–2	42
3 20	Right vagus irritated. Coil 25°. This irritation was sufficient to slow the heart and lower blood-pressure	100	5	20
5 0	Condition previous to injection	155	2	44
	As the cat was stronger, the normal minimum irritability was again tried, and coil 25° was again found to be the weakest which produced any effect.			
6 0 6 30	1½ cub. centim. casca solution, as in Experiment XXXIV., injected into femoral vein.			
7 0	Condition after injection	175	2	42
7 30	Vagus irritated. Coil 30°. No effect	175	2	40
8 0	Vagus irritated. Coil 25°. No effect	180	2	42

From this experiment it appears that the excitability of the peripheral terminations of the vagus-nerve is not increased by casca.

Action on the Vagus-roots.

The slowness of the pulse which quickly follows the injection of casca, and which we have already shown to be due to stimulation of the vagus-roots, might be caused either (*a*) by stimulation of the central end of the vagus by increased blood-pressure in the nerve-centres, or (*b*) stimulation by the direct action of the drug itself; (*c*) it was also possible that without actually irritating the vagus-roots the casca might increase their sensibility to other stimuli, reflex or otherwise.

Effect on minimum excitability of the Vagus-roots.

Experiment XXXVII.—March 30.

A cat, weight 4 lb., was chloroformed.
A cannula was inserted into the trachea.
,, ,, left carotid.
,, ,, left saphena vein.
The right vagus nerve was cut, and its central end placed in a Von Basch's electrode.
The left vagus remained intact. A $\frac{1}{300}$ solution of the watery extract was used.

Time.		Respirations in 10 sec.	Blood-pressure.	Oscillations.	Pulse in 10 sec.
m. s.			millims.	millims.	
0	Condition before experiment	4	105	1–2	38
1 0 2 30	The normal excitability of the central end of the right vagus was then tested; it was found that coil 10° produced slight slowing of the pulse and fall of blood-pressure, while the respirations became slower and deeper. This was the slightest irritation which produced any effect	2	100	3	30
3 30	Condition after irritation	2	110	1–2	34
4 0	·5 cub. centim. injected.				
4 30	Condition after injection	3	140	1	34
	Rise of blood-pressure. No alteration of pulse. Respirations quickened and respiratory oscillations increased.				
5 0	Central end of right vagus irritated. Coil 15°	3½	140	1	34
	No effect.				
6 0	Central end of right vagus irritated. Coil 10°	2¼	135	1	38
	Same effect as before injection.				
7 0	Second injection of 1½ cub. centim.				
8 0	Clot formed and removed.				
9 0	Condition before irritation	3	120	2	43
9 30	Irritation with coil 10°	0	120	2	40
	The thorax remained in a state of permanent inspiration during irritation, while the effect on the heart and blood-pressure was *nil*.				
10 0 10 5	Irritation with coil 6°	0	115	2	40
	No effect on heart. Respiration as before.				
10 8	Condition immediately following irritation	3	60	28	12
	Slowing of pulse. Great fall of blood-pressure. Great oscillation. Systole and diastole of same length, with no pause between them.				
15 0 15 30	Irritation with coil 8° As before there was no effect on the blood-pressure or pulse, and there was permanent inspiration during the irritation.	0	120	2	40
15 35	Condition immediately after irritation. See remarks on after effect of coil 6°	4	30	15	16 irreg.
16 0	Gradual cessation of after effect	2	100	7	22
16 30	After effect ceased	0½	120	2	35
17 0	Coil 8°. Irritation repeated with same results.				

In this experiment, as well as in several others, the blood-pressure rose without being accompanied by a slowing of the pulse, and this indicates that the latter is not dependent on the former.

The excitability of the vagus-roots to reflex stimuli does not seem to be increased by casca, as a stimulus of the same strength applied to the central end of one vagus had a similar effect before and after the injection of the drug. We would call attention, however, to the very extraordinary effect which succeeded the application of a stronger stimulus, an effect which seems all the more extraordinary from occurring after the stimulus had ceased, and not during its application.

Irritation of the vagus-roots by the carbonic acid accumulated in the blood during the tetanic inspiration, which lasted during the irritation, at once suggests itself as a cause of the slow pulse which followed the irritation; but the fact that the pulse was not affected when the distance of the coil was 10 centimetres, although the thorax was tetanically expanded, seems to indicate that the slowing which followed the stronger irritation from a secondary coil at 8 or 6 centimetres distance from the primary was due to reflex action, which the first irritation had been too weak to produce.

From Experiment XXXV. it will be seen that after the administration of a large dose of casca, irritation of the vagi, instead of producing slowing or stoppage of the heart's action increased the frequency of its pulsations. The acceleration was equally great after irritation of the left, as after irritation of the right vagus. This shows that the accelerator-fibres in the vagus are not paralyzed by casca, and also that accelerator-fibres, though usually, according to BOEHM, contained only in the right vagus, may occasionally be present in the left.

The effect of irritating the other accelerating nerves of the heart contained in the rami cardiaci or in the sympathetic cord was not examined.

ACTION ON CARDIAC GANGLIA, EFFECT ON PULSE, ETC.

Experiment XXXVIII.—June 10.

A moderate-sized cat was chloroformed, and cannulæ were placed in the carotid artery and jugular vein.

A solution of 3 cub. centims. of saturated alcoholic tincture added to 50 cub. centims. of water was used for injection into the vein.

Time.		Blood-pressure.	Oscillations.	Pulse in 10 sec.
m. s.		millims.	millims.	
0 5	Condition before injection	160	25	23
1 20	Injected 1 cub. centim. casca solution.			
1 30	Condition shortly after injection	180	50	14
	Rise of blood-pressure, slowing of pulse.			
2 0	Later, quickening of pulse	165	7	26
3 0	Same effect, more marked	175	5	36
4 0	Commencing slowing of pulse	170	6	30
4 30	,, ,, ,,	140	8	30
5 0	,, ,, ,,	150	14	24
5 5	Further injection of 1 cub. centim. casca.			
5 40	Secondary slowing of pulse with *fall* of blood-pressure.	115	25	16

From this experiment it is seen that after the primary slowing due to stimulation of the vagus-roots and the quickening due to paralysis of the vagus ends in the heart, a second slowing occurs.

This second slowing might be due either to stimulation of the inhibitory apparatus in the heart or to weakening of the cardiac motor ganglia.

The latter seems improbable, from the fact that each systole during this slow period instead of being weak is exceedingly strong, the pulsation in an artery being felt very powerfully when the finger is laid upon it, and the rise of pressure during it being very great, as shown by the oscillation of the mercurial column of the manometer.

In order to ascertain more exactly whether the inhibitory cardiac ganglia were stimulated or not the following experiment was tried.

Experiment XXXIX.

A dog was chloroformed and 7 cub. centims. of a concentrated watery solution of casca were injected into the jugular vein.

The pulse at the time of injection was 37 in ten seconds. In ten seconds after the injection it sank to 20. After the injection of 3 cub. centims. more the pulse rose to 37. After a further injection of 26 cub. centims. more in divided doses it again sank to 16.

This number was, however, uncertain, as the tracing was a very imperfect one.

The injection of 1 cub. centim. of liquor atropiæ, B. P., did not seem to alter the number of the pulse, but the injection of ½ a cub. centim. more seemed to cause it again to become quick.

This seems to indicate that the slowing is due to an action of the casca on the inhibitory ganglia. The imperfection of the tracing renders the result somewhat uncertain; but want of time prohibited us from repeating the experiment, although we greatly desired to do so.

Experiments XL., XLI.—March 15.

The effects on the capillaries of the frog's web were microscopically observed in Experiment XL. after an injection of casca under the skin of the back, in Experiment XLI. when locally applied to the web.

The capillaries were observed with oc. 2 obj. 4 of Hartnack. In the first case, *i. e.* after the drug had been injected, the results were purely negative. In the second, after application of a drop of strong casca solution to the web, the results were also indecisive. In the capillaries and also in the larger trunks the current was at first slowed, and in some permanent stasis occurred. When slowing only was produced, the partial arrest was followed by reaction, which did not exceed the original rapidity of the circulation. No dilatation or contraction of the vessels was seen to accompany the original slowing.

Although the results of experiments on the frog's web gave no definite information regarding the contraction of the arterioles under the influence of casca, yet no reasonable doubt can be entertained that in mammals they do contract; for this is the only possible explanation of the exceedingly slow fall of the blood-pressure during the intervals between the beats of the heart when these have become slow, either from the action of the drug or from irritation of the vagus-trunk.

In order to ascertain whether this contraction was due to the action of the drug upon the vasomotor centre in the medulla oblongata or in the vessels themselves, the vasomotor centre in the medulla was separated from its connexion with the vessels by division of the spinal cord in the neck previous to the injection of casca.

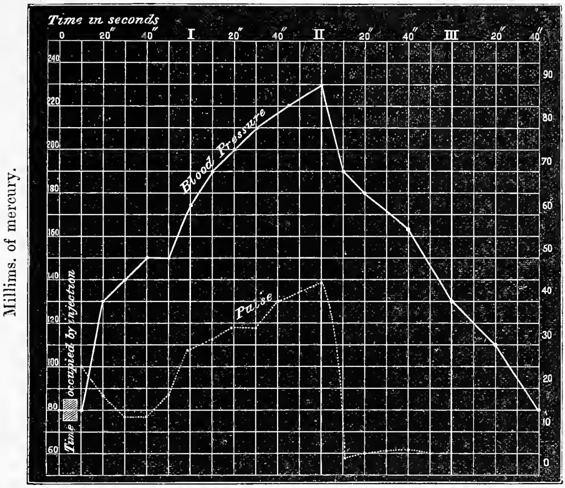
EFFECT ON BLOOD-PRESSURE AFTER DIVISION OF CORD.

Experiment XLII.—May 17.

A large strong cat, weighing 7½ lb., was chloroformed. A cannula was inserted into the left carotid, and another into the jugular vein. The spinal cord was then divided opposite the second cervical vertebra, and artificial respiration kept up.

Time.		Blood-pressure.	Pulse in 10 sec.	Oscillations.
m. s.		millims.		millims.
0 0	Condition after section of cord and previous to injection of casca	80	25	5
0 3 0 7	Injected 1 cub. centim. watery solution of casca.			
0 10		80	25	5
0 20		130	18	12
0 30		140	13	20
0 40		150	13	25
0 50		150	18	12
1 0		175	28	5
1 10		190	32	4
1 20		200	34	3
1 30		210	34	3
1 40		220	41	4
2 0		230	44	3
2 10	Sudden alteration in character of pulse, which becomes slow. The curve flat-topped, and both systole and diastole showing numerous secondary oscillations. Blood-pressure falling	190	3½	40
2 20		180	5	30
2 40		165	6	25
3 0	Blood-pressure rapidly falling	130	6	15
3 20	Both pulse and oscillations are very irregular	100	5	?
3 40		80		
3 50		25		
4 0		10		
	There was no more pulsation after this; but the blood-pressure took 1′ 30″ to fall quite to zero. On opening the thorax the heart was found moderately contracted; electrical stimulation of the phrenic nerve caused contraction of the diaphragm.			

The result of this experiment will be seen all the more clearly by the following diagram, in which they have been graphically represented.



The rise of pressure in this experiment was greater than in any other in which the cord had not been divided. This seemed to us so extraordinary that we thought at first that the cord had not been properly divided; but a careful dissection made immediately after death showed us that the division was complete. A year or two ago this result would have been regarded as a proof that the drug acts on the vessels themselves; but recent researches having shown that much more importance must be attributed to vaso-motor centres in the cord and in the periphery than was previously done, we cannot say whether the drug acts on these centres or on the walls of the vessels themselves. The non-contraction of the vessels of the frog's web would indicate that the action of the drug is rather on nervous centres in the cord or neighbourhood of the vessels than on the vascular walls.

In order to exclude all centres except those in the periphery, Experiment XLIII. was performed.

Experiment XLIII.

The sympathetic cord was divided on the right side of the neck of a rabbit and the animal allowed to come out of the chloroform anæsthesia. The ear of the right side was deeply injected while the left ear was very moderately filled with blood. A dose of casca was then administered. The vessels of both ears became pale, those of the right ear equally so with those of the other.

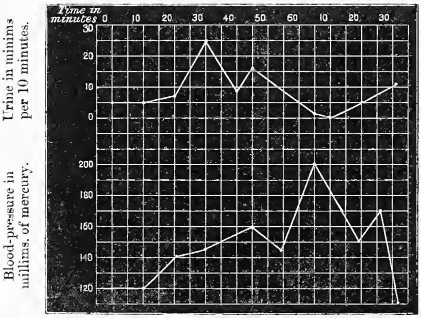
VESSELS AFFECTED BY THE DRUG.

The vessels by which the blood-pressure in the body is chiefly regulated are those of the intestines, those of the skin and muscles being very much less under the influence of the vasomotor centre in the medulla. As casca acts on the vessels without this centre, however, it was natural to suppose that other vessels than those of the intestines might be affected; and this the curves show to be the case. During the stoppage of the heart (Experiment XLII.) for half a minute the pressure fell only slightly. Now Ludwig and Hafiz found that when contraction of the abdominal vessels was produced by irritation of the vasomotor centre in the medulla oblongata, complete stoppage of the heart was followed by a rapid fall in the blood-pressure, the blood finding its way out of the arterial system into the veins through the vessels of the muscles. The slow fall after the administration of casca shows that the vessels of the muscles must be contracted as well as those of the skin and intestines.

Action on Secretion of Urine.

A detailed account of our experiment on the action of the drug on the urinary secretion, together with the accompanying effects on the circulatory system, will be found in pages 642 and 643. Underneath is a diagram showing the close dependence of the rate of secretion of urine upon the blood-pressure.

Experiment XXXIII.—March 31.



Remarks.

The results of this experiment, as regards the vascular phenomena of blood-pressure, pulse, &c., have been already fully noticed.

The action of the drug on the secretion of urine is seen to be very marked and characteristic, and the dependence of the secretion on the blood-pressure is well shown in the accompanying chart.

Thus the average rate of urine secreted before administration of casca being 5 minims in 10 minutes, an increase of 50 millims. in the blood-pressure caused by the drug brought the rate of secretion of urine up to 25 minims. When the action of the drug was further pushed there was first decrease and then total suppression of urine, the blood-pressure at the time of suppression being 200 millims. of mercury.

Subsequently as the blood-pressure fell the secretion of urine recommenced.

The physiological explanation of these successive phenomena appears to be that the primary increase of blood-pressure produces arterial fluxion to the kidney; but that if the action of the drug is pushed, the renal vessels become contracted so as to prevent the blood reaching the kidneys, notwithstanding the high pressure in the arterial system. It is worth notice that the urine collected after the secretion had recommenced did not contain albumen.

In this respect the result of this experiment differs from those obtained by Mr. Power and one of us in our experiments on the action of digitalis; it coincides, however, with those experiments in its general results.

Action on the Pupil and Lacrymal Gland.

In order to see if the drug exerted any local action on either of these organs, we (Experiment XLV.) placed some drops of strong watery solution of casca in the eye of a cat, but with purely negative results.

Action on Muscle.

Effect on Structure of Muscular Tissue.

The effect on fresh muscular tissue of immersion in casca solution was carefully watched with an oc. 3 obj. 7 of Hartnack's microscope without any change in the structure being discovered (Experiment XLVI.). We then (Experiment XLVII., March 22nd) examined the "naked-eye" and microscopical changes produced in muscular tissue by prolonged immersion in a watery solution of the alcoholic extract, the effect of which solution in preventing the development of *Bacteria* has already been detailed (p. 635).

A. The solution in which the muscular tissue had been placed presented, in addition to the absence of *Bacteria*, few noteworthy points; it preserved its original slightly resinous smell, and deposited a fine light-brown sediment, which, under the microscope, appeared as a granular structureless detritus.

B. The muscular tissue to the naked eye appeared hardly altered in consistence: the fibrous sheath was firm; there was no smell. Under obj. 7 Hartnack the fibres were seen to be very granular, in part only preserving their transverse striation; the general appearance closely resembled ordinary fatty degeneration. Some of the fibres were then soaked in ether for twenty-four hours; on examination after this the granulations had in great part disappeared. Many of the fibres appeared to consist merely of collapsed tubes of sarcolemma; where they were not collapsed they showed plain transverse striæ.

Six weeks later the muscle was again examined; it having remained in the same casca solution all the while, it was now reduced to the condition of a rather tough gelatinous pulp; the sheath of the muscle retained its strength. Under the microscope there was seen a mixture of granular and fibrous material, with a large quantity of oil-globules and flat crystals, and when treated with ether these were completely removed.

Effect on the Lifting-power of Muscle.

Experiment XLVIII.

The lifting-power of a frog's gastrocnemius which had been placed in a $\frac{1}{300}$ solution of casca was compared with that of a similar preparation placed in salt solution, by attaching the one muscle as quickly as possible after the other to an apparatus for estimating their lifting-power connected with a revolving drum. The irritations were made with electrodes connected with a Leclanché's battery.

The results of our first experiments appeared to show that the drug possesses a stimulating action on the lifting-power of muscle; but on repeating the experiment this result was not confirmed—the conclusion we drew from the whole series of experiments being that muscles which had been immersed in casca and salt solution respectively possessed nearly the same lifting-power.

Effect on Muscle-curves.

Experiments XLIX., L.

Two frogs were injected with casca, and when they seemed dead, nerve-muscle preparations were made of the gastrocnemii. In Experiment XLIX. the preparation was attached to a Fick's pendulum myograph and a tracing taken. In Experiment L. the nerve muscle was made to trace on a revolving cylinder: the curves obtained in these experiments are evidently normal.

In Experiment LI. the action on the sensibility to electrical stimuli of muscle and motor nerves was tried, also with completely negative results, by making two nerve-muscle preparations of a frog's gastrocnemii, and immersing one in casca solution, the other in salt solution. The sensibility of the two preparations was then tested by various strengths of a Du Bois Reymond's coil connected with a Leclanché's battery. The two muscles responded quite similarly.

Remarks on the Action of Casca on Muscle.

1. When applied to fresh muscular fibre no change is observed in its histological details.

2. In addition to the absence of the development of *Bacteria* which is noticed when muscular tissue is placed in a watery solution of the alcoholic extract, and which has already been remarked upon, the structural changes which the muscular tissue undergoes appear to consist in a fatty metamorphosis, which at first simulates very closely that of ordinary fatty degeneration, while the later appearances resemble those of the more complete fatty changes which go on after the death of a tissue, large oil-globules and abundant crystals of the fatty acids being everywhere found.

3. It does not diminish the lifting-power of muscle in a nerve-muscle preparation, nor

when the irritation is applied to the muscle itself, and it probably does not increase it; for although apparently positive results were attained on the first occasion when the lifting-power was experimented on, these results were not borne out by further experiments.

4. The muscle-curve given by a nerve-muscle preparation taken from a frog poisoned by a large dose of casca appears to be quite a normal one.

5. It also exerts no action on the sensibility of muscle to electrical stimulation if this sensibility be tried quantitatively by estimating the weakest interrupted current which will produce a contraction.

From all these results, then, it may be concluded that while the drug produces a peculiar and characteristic change on muscular tissue immersed in it for some days, it is not a muscle-paralyzer.

ACTION ON MOTOR NERVES.

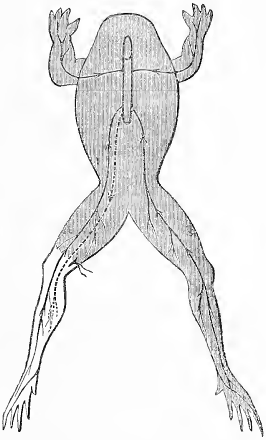
If casca had any paralyzing action on the ends of motor nerves similar to that of curare it would be found that after immersion in a solution of the drug the muscle would respond to electrical stimuli directly applied to it, but not to those applied to the nerve. In Experiment LII., however, the nerve also is seen to preserve its irritability, and therefore we may conclude that casca has no action on motor nerves.

ACTION ON SENSORY NERVES.

Effect on Reflex Excitability.

Experiment LII.—March 22.

The cerebrum of a living frog of medium size was destroyed. The circulation through the left leg was cut off by ligaturing the arterial trunks above the knee (*vide* fig.). The sciatic nerve was left uninjured. $\frac{1}{4}$ cub. centim. of the alcoholic extract was then injected beneath the skin of the back.



The reflex excitability was then tested at intervals of 5 minutes by irritating points above and below the ligature. No difference in the sensibility could be detected.

As the frog was very little affected by the injection, another $\frac{3}{4}$ cub. centim. was injected.

The frog became insensible to reflex irritation in five and twenty minutes; during this time the irritability was tested every five minutes as before; the rates of increase of the insensibility appeared to be equal above and below the ligatures.

In this experiment the poison was applied to the terminations of the sensory nerves above the ligature, but not to those below it. Had it possessed any marked power of diminishing the sensibility of these nerves a stimulus applied above the ligature would have had less effect than one applied below it; but this was not the case.

The poison therefore seems to have no action on sensory nerves, at any rate none of a paralyzing character.

Action on Reflex.

Experiment LIII.

In this experiment the action on reflex was tested by applying a very dilute solution of sulphuric acid to the leg of a frog with its cerebrum destroyed and suspended by its head. Its normal irritability was then tested. The tips of the toes only were immersed in the acid. Contraction was immediate and lasted three minutes (right leg). A $\frac{1}{4}$ cub. centim. alcoholic solution was then injected; the animal immediately hung more flaccidly. Five minutes afterwards, on immersing the tip of the toes of the right leg, slight contraction occurred after 63 seconds. A quarter of an hour later, on immersing half the leg, contraction was immediate and lasted 5 seconds; in ten minutes more a similar immersion produced contraction after 10 seconds, and five minutes afterwards after 15 seconds. Fifty minutes after the injection of the drug the acid solution produced no reflex movements, and only slight ones were excited by pinching. Reflex ceased last in the eyelids one hour and twenty minutes after the casca had been injected.

In Experiment LIII. the reflex excitability disappeared very much more quickly than it usually does.

This might be due to the action of the drug on the spinal cord itself, or to the cessation of circulation caused by the action of the drug on the heart.

In order to decide this the following experiment was made.

Experiment LIV.

The heart of a frog was exposed and casca administered. As soon as the heart had ceased to beat the heart of a second frog was ligatured at the root of the aorta so as completely to arrest the circulation. At first both frogs were able to jump readily; but gradually their movements became more sluggish, and after a jump their legs trailed out behind them and were only slowly drawn up to the body. They became less and less sensitive to pinching, and insensibility and loss of motor power occurred simultaneously in both.

The diminished power of movement and diminished reflex action observed in the frog after the administration of casca is therefore due to the arrest of the circulation caused by it, not to any action of the drug upon the nervous system.

ANTAGONISM BETWEEN CASCA AND ATROPIA AND CHLORAL HYDRATE.

The remarkable result of Experiment XXIV., in which a dose of casca, usually fatal, produced no effect in an animal with divided vagi, seemed to render it probable that such a drug as atropia, which paralyzes the ends of the vagus in the heart, might have an antagonistic action. On trying it, however, it was found that the vomiting caused by the casca was even more violent than usual; and therefore a combination of atropia with chloral hydrate was employed, the chloral being given to lessen the irritability of the vomiting centre in the medulla.

The results were not satisfactory, as will be seen from the two following experiments.

Experiment LV.—May 1, 1876.

About 11.40. Injected 4 cub. centims. of liquor atropiæ under skin of flank of cat A.

12^h. Injected 4 cub. centims. of a saturated alcoholic solution of alcoholic extract of casca under skin of flank of cats A and B.

	A.	B.
h. m. 12 30	....	Sick and vomits. During the intervals between the fits of vomiting seems well.
12 40	Crouching, trembling, and seems about to be sick. Licks its lips. Hind eyelid much drawn up.	
12 52	Very sick; vomiting. Seems more uneasy than B.	Vomiting. Disinclined to move. When disturbed and made to walk its hind legs give a shake as if to shake off something sticking to the feet every time they are drawn up.
1 0	Very sick.	Very sick. Brings up fluid, which appears to be digested meat. Respirations 18 per minute.
1 15	Gives loud squalling cries when retching.	Seems weaker.
1 30	Seems easier; not retching.	Twitch or rather shake of hind legs is very marked.
2 35	Walking about.	Seems unable to move.
2 40	....	Violent retching; crying. Convulsive extension of legs and emprosthotonos. Then two or three sighing respirations; a pause; one or two respirations at intervals; then death.
4 0	Has been vomiting at intervals. Has a violent fit of sickness and dies in the same way as B.	
	Post mortem.	
	Stomach contains a quantity of food. Heart moderately contracted. Ventricles continue to make slight pulsations, auricles not. Lungs somewhat congested. No congestion of interior of stomach.	Stomach empty, not congested. Heart moderately contracted. Lungs normal.

Experiment LVI.

Cat A. Large.

h. m.
12 3. Injected 30 minims liquor atropiæ subcutaneously.
12 20. Injected 5 cub. centims. saturated watery solution and suspension of casca.
12 30. Vomited.
12 45. Vomiting has been repeated 3 or 4 times. Is lying on its side and cannot stand. R. 144. Alæ nasi working.
12 52. Eyelids much drawn up. Respiration irregular.
12 55. Gasps; seems to try but to be unable to vomit. Emprosthotonic spasms.
12 58. Muscular twitchings. Slow sighing respirations. Death.

Cat A lived 38 minutes after injection of casca.

Cat B. Medium.

h. m.
12 45. Injected 15 grs. of chloral hydrate and 15 minims of liquor atropiæ with 5 cub. centims. of casca.
12 58. Mewing.
12 59. Vomited twice.
1 9. Vomited again.
1 14. Loud violent retching, but no vomiting.
1 15. Involuntary extrusion of fæces and urine. Died.

Cat B lived 30 minutes.

Cat C. Large and strong.

h. m.
12 40. Injected 5 cub. centims. of the same casca solution.
12 49. Very restless.
12 52. Vomited for first time. After this it vomited frequently, but remained restless. T. 37°·1.
1 30. Died rather suddenly, with violent gasping and emprosthotonic spasm.

Cat C lived 50 minutes.

In this experiment the cat which had received the casca alone lived longer than the others.

XVIII. *Further Observations on the Locomotor System of Medusæ.*

By George J. Romanes, *M.A., F.L.S., &c.* *Communicated by Prof.* Huxley, *Sec. R.S.*

Received November 21, 1876,—Read January 11, 1877.

[Plates 30 & 31.]

In now submitting the second instalment of my observations on the locomotor system of Medusæ to the consideration of this Society, I should like to preface the communication with a few words to explain why several experiments which I had intended to perform this season have been unavoidably postponed. It is well known to naturalists that, from causes which are not very well understood, the numbers of Medusæ in the same localities are subject, in different years, to considerable variations. Now on the part of the coast where my work has throughout been carried on—viz. the Cromarty Firth, on the east of Scotland—the Medusæ have been this year as scarce as last year they were abundant. Probably the cause of this scarcity in the land-locked position occupied by the Cromarty Firth is in part to be attributed to the unusual prevalence of westerly winds which has this year been observable in that locality. But whatever the cause, the fact of this scarcity having existed has compelled me this year to restrict my observations, almost exclusively, to the genera *Sarsia*, *Tiaropsis*, and *Aurelia*. By way of introduction it is only necessary further to state that, for the sake of facilitating reference, I shall endeavour to construct the present communication on as nearly as possible the same general plan as the last one.

I. FUNDAMENTAL OBSERVATIONS.

§ 1. *Effects of excising the entire margins of Nectocalyces.*—Under this heading I have very little to add to the statements contained in my former paper. Many of the experiments which I have this year performed have necessitated, in a vast number of instances, the removal of the extreme periphery of nectocalyces; and in no one instance have I found an occurrence of even the slightest deviation from the general rule previously enunciated—the rule, namely, that "excision of the extreme margin of a nectocalyx causes immediate, total, and permanent paralysis of the entire organ"*.

§ 2. *Effects of excising the lithocysts from Gonocalyces* (*Umbrellas*).—It will be remem-

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* I may here mention, what perhaps I ought to have mentioned in my former paper, viz. that, in the case of the species *Tiaropsis diademata*, it frequently happens that a single quadrant of the thus mutilated nectocalyx bends slightly inwards at long intervals and without rhythm. I may also mention that I have this year performed the fundamental experiment on three additional species of the genus *Thaumantias*, viz. *T. purpureus* (Forbes), *T. crucifera* (Romanes), and *T. helicobostrica* (Romanes).

bered that excision of the eight marginal bodies of a covered-eyed Medusa is attended with the same degree of paralyzing effect as is the excision of the entire margin. It will also be remembered that this degree of paralyzing effect is not, generally speaking, so complete as it is in the case of the naked-eyed Medusæ; but that, after the operation, occasional contractions at long intervals are usually given, and that sometimes, though very rarely, these after contractions assume a rhythmical character more or less resembling the contractions of the unmutilated animal. All the details concerning the behaviour of mutilated gonocalyces in this and in other respects were carefully recorded in my former communication. Afterwards, however, I had occasion to add a Postscript to that communication, in consequence of my attention having been drawn by Dr. LÜTKEN to a paper by Dr. EIMER, of the existence of which I was previously unaware. In this Postscript I had mainly to deal with certain discrepancies between Dr. EIMER'S account of the behaviour of mutilated gonocalyces and that which I had given in my former communication. Now, as this want of complete accordance between Dr. EIMER'S statements and those which I have published tends to cast doubt upon the accuracy of the latter, it becomes necessary for me in the present communication briefly to recur to the points wherein Dr. EIMER'S results differ from my own. Of course it is needless to say that I have this year given my careful attention to these points, by repeating a great number of times the experiments which have reference to them. The following statements, therefore, may be considered final, so far as I am concerned.

The first point of difference to which I must allude is that which is explained in my Postscript thus:—"It will, of course, have been observed that Dr. EIMER'S view as to the exact seat of spontaneity in *Aurelia aurita* does not coincide with mine. He is careful to state that the ganglionic function is distributed all round what he terms the 'contractile zone,' *i. e.* the crescent-shaped interruption of the margin in which the lithocyst, together with its gelatinous hood, is situated (see Plate 32). On the other hand, I have stated it as my opinion that the lithocyst is alone the locomotor centre; and notwithstanding the account which Dr. EIMER gives of the experiments by which he sought to localize that centre, I still adhere to this opinion. Dr. EIMER'S experiments in this connexion were twofold:—First, that of progressively lessening the amount of contractile tissue left adhering to an excised segment of *Aurelia aurita*; and, second, that of excising the lithocyst without injuring the 'contractile zone.' Of these two methods Dr. EIMER appears to lay most stress upon the first one; for he merely mentions the second method in a short footnote towards the end of his paper, and there states that he only tried it in a few cases. Yet to me it seems that for the object in view the second method is much more trustworthy than the first. I am well aware of the fact pointed out by Dr. EIMER, that upon each contraction of a given segment of *Aurelia aurita* the two arms of the so-called 'contractile zone' approximate each other, and that this gives rise to the appearance of spontaneous action on their part. I think, however, that this appearance is deceptive, being caused only by the absence of resistance at the interrupted part of the margin to the pressure exerted by the contraction of the immediately surrounding tissues. At any rate, so long as this possible explanation has not been thoroughly excluded by experiments conducted on the converse method of removing the lithocysts from between the arms of the contractile zone, so long, it seems to me, must the method we are considering be valueless. The question, then, must be decided by the converse method just alluded to, and by it alone. Now I have made experiments according to this method, and, so far as I remember, in every case, when sufficient care was taken to remove all the lithocysts, the contractile zone entirely ceased its contractions. And not only so, but by removing, with the aid of a well-pointed scissors, the little sac of crystals composing the central part of the lithocyst, without injuring the curious wing-like appendages by which this sac is partly surrounded*, and, conversely, by removing in other specimens these wing-like appendages alone, without injuring the little sac of crystals—by these experiments I was able to satisfy myself that the whole spontaneity of the lithocyst appeared to be exclusively lodged in (or about) the minute sac of crystals referred to."

To these statements I have only to add that scores of experiments on *Aurelia aurita* have this year invariably confirmed those of last year. In no one instance have I been able to detect any physiological evidence of the presence of ganglionic matter in the "contractile zones." In most instances, of course, occasional contractions were given after removal of the lithocysts alone; but I was never able to observe that the spontaneity in these instances proceeded more exclusively, or more frequently, from the contractile zones than from any other part of the general contractile tissue. For after allowing the animal to recover from the shock consequent upon removal of its lithocysts alone, and then observing the degree of spontaneity it manifested, I was in no case able to perceive that by now removing all the contractile zones, this degree of spontaneity was in the smallest degree diminished†.

This allusion to the spontaneity manifested by *Aurelia aurita* after its lithocysts have been removed, leads to the next point of difference between Dr. EIMER and myself. For the sake of conciseness, I shall again quote from my Postscript:—"I cannot quite assent to the description which Dr. EIMER gives of the contractions which sometimes supervene in the umbrella of *Aurelia aurita* when all the lithocysts have been removed. He describes them as 'several *irregular*, *inefficient*, and *feeble* contractions of a local nature which rarely last any considerable time.' This is no doubt partly true of some cases, but it is not true of all. I have frequently seen these after-contractions as rhythmical (though this is rare), as effectual, and as powerful as those which had been previously supplied by the single remaining lithocyst. Moreover, these contractions may usually be seen to emanate from some very localized portion of tissue, and from thence to radiate over the whole substance of the umbrella, just as the contractile waves which

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* The "wing-like appendages" here referred to are shown by injection to be minute diverticula of the nutritive-tube system.

† [In this connexion I may observe that Mr. SCHÄFER has shown the "nerve-epithelium" to be exclusively confined to the region of the lithocysts.—1878.]

emanated from a single remaining lithocyst had previously done. On now cutting out this localized portion of tissue, the umbrella usually becomes again paralyzed, while the portion of tissue which previously animated it may be seen to continue its contractions after the manner of excised lithocysts. I did not pay sufficient attention to the number of hours after excision of the lithocysts during which these secondary movements continued, to admit of my speaking with confidence on this point; but, so far as I can recollect my numerous experiments with this species, it is certainly not correct to say that these contractions 'generally ceased after a few moments.' My impression is that they *usually* last for several days. I agree with Dr. EIMER, however, that, as a general rule, the secondary movements in *Aurelia aurita* are not so persistent as the primary ones; and also that in this species, under some circumstances*, insensibility to stimulation rapidly supervenes upon loss of spontaneity. This, however, is far from being the case with other species—for instance, *Cyanœa capillata* will continue to respond even to slight stimulation *two or three days* after it has been completely paralyzed by the removal of its lithocysts, and this even though it be kept in a small jar without change of water. Again, if the lithocysts be left *in situ* and the animal be kept in a confined body of water, irritability will continue for days after all the usual indications of spontaneity have disappeared. Nevertheless, with slight modifications, I have confirmed Dr. EIMER'S experiment of placing different specimens of *Aurelia aurita* under similar conditions, removing the lithocysts of some and not of others, and observing that those individuals which had been operated upon died sooner than those which had not."

This year I have given careful attention to these secondary contractions manifested by the mutilated swimming-organs of *Aurelia aurita*; and as the fact of their occurrence will afterwards be found of importance in relation to some peculiar points in the physiology of this animal, I shall here detail all the observations regarding the subject which I have been able to make. In the first place, the above-quoted statements with reference to *Aurelia aurita* have been this year, with one exception, fully confirmed by numerous experiments. The one exception refers to my statement as to having "frequently seen these after-contractions as rhythmical (though this is rare), as effectual, and as powerful as those which had been previously supplied by the single remaining lithocyst." The word "rare" here, as applied to the rhythmical nature of these after-contractions, is not sufficiently strong. If I had said "very" or "exceedingly rare," the statement would have been more accurate. For although, so far as I can remember, the statement as it stands would apply without correction to *Cyanœa capillata*†, I have not this year found it to do so in the case of *Aurelia aurita*. The after-contractions of this species are nearly always, as Dr. EIMER describes them, "irregular;" that is to say, they nearly always occur at uncertain intervals. These intervals are

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* That is, in particular, too high a temperature and want of aëration.

† I must trust to memory here, because I was not able this season to obtain a single specimen of this species.

usually very long, though different individuals manifest considerable difference in this respect, in some cases a great many more contractions occurring in a given time than in other cases. It is not unfrequently observable that these after-contractions occur in pairs, and more rarely in triplets; and when this is the case the interval of diastole corresponds with that which occurs in ordinary swimming.

So much for the rhythmical nature of these after-contractions of *Aurelia aurita.* With regard to their other qualities, I have nothing further to add. I am still unable to say that they appear to me either more "inefficient" or more "feeble" than the contractions which proceed from a single remaining lithocyst of average potency, particularly if the comparison be made between the former and the latter after a pause of a minute or more has in each case intervened between the contractions observed and the previous contraction (see III. (A) (*a*)). I have also been unable to perceive that the secondary contractions are any more of a "local" nature than are the primary ones. In almost every instance which I have this year observed, the description just quoted from my Postscript was strictly applicable; and those comparatively few instances in which the secondary contractions were local, were not much, if at all, more numerous than those in which primary contractions proceeding from a single remaining lithocyst were local. Again, my experience has this year been quite at variance with that of Dr. EIMER as regards the time after removal of the lithocysts during which the secondary movements persist. Numerous experiments have shown that it certainly is not correct to say that these contractions "generally cease after a few moments," or even "after several hours." Indeed so far have I found it from being true that they "rarely last any considerable time," that I am now able fully to confirm my original "impression," and to state positively that these after-contractions of *Aurelia aurita* "*usually* last for several days." For in some dozens of experiments I have found no exception to the rule that, if an ordinarily fair chance be given to the mutilated animals by keeping them in a basin-shaped vessel at a temperature not greatly above that of the sea and changing the water once a day, these secondary contractions persist for one day at the least, generally for two or three days, and sometimes longer. In most cases they continue after decomposition of some parts of the tissues has set in, and even after such decomposition has proceeded to the extent of giving the latter a certain ragged appearance, which is due to portions disintegrating into shreds. These remarks apply to many cases in which, not the contractile zones merely, but the entire margin had been removed.

Lastly, with regard to the greater vitality of unmutilated covered-eyed Medusæ with their lithocysts *in situ* than those which have had these bodies removed, I have found my previous statement substantially correct. There are wonderful individual variations, however, in this respect; for sometimes the irritability of the paralyzed specimens will remain after that of unmutilated specimens which are exposed to exactly the same conditions has been destroyed by decomposition. Comparisons, therefore, in this respect can only be fairly made between detached segments of the same individual; and in this case it may be taken as a rule that the segments in which the lithocyst is allowed to remain preserve their irritability longer than the segments from which the lithocysts have been removed. The greatest amount of endurance that I have met with in these experiments was in the case of an individual cut into eight segments, in such a way as to leave one lithocyst in the middle of each arc. Four of the segments were then deprived of their lithocysts, and the paralyzed and unparalyzed segments were left in the same basin at a temperature of about 45° F. The water was changed once in every twenty-four hours, and all the eight segments were perfectly healthy at the end of the sixth day. Next night, however, the temperature rose, and in the morning of the seventh day all the segments were dead.

I have dwelt thus at length upon the differences between the results yielded by Dr. EIMER's experiments and those yielded by my own, because, although these differences refer to points that were of minor importance at the time when Dr. EIMER published his paper—a fact which, doubtless, led this observer, so well skilled in dealing with the great and peculiar difficulties that attend such an investigation of these and allied organisms, to neglect repeating these subordinate experiments a sufficient number of times,—still it will shortly be seen that, at the stage which the inquiry has now reached, it is of great importance to be in possession of accurate and precise data with regard to the points which we have just been considering*.

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* [I here withdraw from the proof a footnote mentioning the fact that I had obtained histological evidence of the presence of nerve-cells and fibres in *Aurelia aurita*. I withdraw this note because it is now rendered superfluous by the much more extended and detailed histological researches of my friend Mr. SCHÄFER. Perhaps, however, it is desirable to state that my own histological work was merely of a cursory nature, sufficient only to show that there was here a splendid field for further cultivation. In accordance with my original plan, however, I deferred all systematic inquiry concerning the histological part of the subject till the more important of the physiological experiments should have been completed—not because I underrated the importance of a thorough re-investigation of the histology of the Medusæ, but, on the contrary, because I deemed such a re-investigation to be of so much importance that I hoped best to give it the attention which it deserved by postponing this difficult part of the inquiry until I should be able to devote to it my undivided attention. But as the physiology of the subject continued to open up more and more, I felt it was undesirable, either, on the one hand, to suspend this part of the inquiry, or, on the other hand, any longer to defer a careful development of the histological part. Accordingly I represented the case to Mr. SCHÄFER, who with great kindness consented to spend his leisure time in cooperating with me. The highly interesting character of the results which he has obtained show how desirable it was to render the histology of the Medusæ a subject of elaborate scrutiny; but as he has communicated these results to the Royal Society, it is unnecessary that I should here restate them. I may observe, however, that the elements which he describes were not unfamiliar to me, though I hesitated to pronounce with certainty upon their nervous character. But, lest the fibres which Mr. SCHÄFER describes as nervous should eventually prove to be the anatomical structures that are concerned in performing the physiological functions which I described in my former paper, I adopted in that paper a neutral term whereby to designate the nervous tracts whose presence my experiments, and also those of Dr. EIMER, had revealed. This neutral term was "lines of discharge," a term which is applicable alike to partly or to fully differentiated nerves. And forasmuch as I still deem it premature to conclude that definite histological elements are concerned in the performance of these physiological functions, I think that while treating of the latter it still remains desirable to designate their structural correlatives by some such neutral term as "lines of discharge," rather than by any term of a more precise signification.

I may also take this opportunity of observing that, almost simultaneously with the publication of Mr.

II. RHYTHM.

§ 1. *Relation between Size and Rhythm.*—It will be convenient here to introduce all the observations that I have been able to make with regard to the natural rhythm of the Medusæ. As Dr. EIMER has also made some observations in this connexion, before proceeding with the fresh points having relation to this subject, I shall consider those points to which allusion was made in my Postscript.

In *Aurelia aurita*, as Dr. EIMER has noticed, the rate of the rhythm has a tendency to bear an inverse proportion to the size of the individual. Size, however, is far from being the only factor in determining the differences between the rate of the rhythm of different specimens—the individual variations in this respect being very great even among specimens of the same size. What the other factors in question may be, however, I am unable to suggest.

§ 2. *Relations between the periods of Activity and the periods of Repose.*—Dr. EIMER affirms that the duration of the natural pauses, which in *Aurelia* sometimes alternate with bouts of swimming, bears a direct proportion to the number and strength of the contractions that occurred in the previous bout of swimming. In my Postscript I observed that *Sarsia* are much better adapted than *Aurelia* for determining whether any such precise relation obtains; for, in the first place, the strength of the contractions is more uniform, and, in the next place, the alternation of pauses with bouts of swimming is of a more decided character in *Sarsia* than in *healthy* specimens of *Aurelia*. I further observed that in *Sarsia* I was quite sure no such precise relation did obtain, although in a very general way it was true, as might be expected, that unusually prolonged bouts of swimming were sometimes followed by pauses of unusual duration. At that time I had not taken the trouble actually to count the pulsations and to time the intervals of rest, and so could not say any thing more upon the subject. This year, however, I have made precise observations with regard to this matter, and I find that they fully confirm my previous statements. As all the observations are very much the same, I shall only quote two of them:—

SCHÄFER's work, two other monographs have appeared on the histology of the nervous tissues of the Medusæ. One of these monographs is by Messrs. HERTWIG, and the other by Prof. EIMER. In my next paper I shall give an abstract of both, and shall therefore not burden this Postscript by endeavouring to do so at present. But I should like to take this the earliest opportunity of rectifying an injustice which, as I now learn from Prof. EIMER's last paper, I previously did him in my first paper. In my Postscript to that paper I stated that in our independent observations concerning the effect of excising lithocysts I had a right to claim priority, both as regards observation and publication. Prof. EIMER, however, now explains that, in consequence of his first paper having been bound in the Würtzburg Verhandlungen without its proper titlepage, the reference to the date of his work which occurs in the paper itself ("d. J.") refers, not to the year on the back of the volume, but to the year preceding. Therefore, although I was right in saying that I had anticipated Dr. EIMER in making the "fundamental observation," I was wrong in supposing that I had also anticipated him in publishing this observation. Dr. EIMER has thus a full right to claim priority as regards this and all his other researches concerning the nervous system of Medusæ.—1878.]

Sarsia.		*Sarsia* (another specimen).	
Number of pulsations.	Seconds of rest.	Number of pulsations.	Seconds of rest.
54	90	40	60
20	15	29	90
9	92	32	132
51	40	33	92
38	60	18	59
1	43	8	63
63	45	15	35
1	14	2	85
60	15	11	63
6	50	30	33
38	50	17	81
22	32	19	67
25	12	3	65
56	55	19	36
65	20	41	123
42	15	80	23
35	40	61	150
76	43	45	145
		40	120
		10	97
		14	35

These observations may be taken as samples of others which it would be unnecessary to quote, as it will be seen from the above that there is no precise relation between the number of the pulsations and the duration of the pauses. Nevertheless, that there is a *general* relation may be seen from some cases in which unusually prolonged pauses occur. The following instance will serve to show this:—

Sarsia (another specimen).

Number of pulsations.	Seconds of rest.
38	30
22	35
49	40
30	45
46	20
2	15
24	380
112	20
45	185
894	30
6	45
4	140
2	185
30	240
200	60

In this case the relation between the long pause of 380 seconds and the subsequent prolonged swimming bout of 112 pulsations, is obvious. Also, as the latter was then followed by a short pause of 20 seconds and another comparatively short bout of 45 pulsations, the refreshing influence of the previous 380 seconds of rest may be supposed to have been not quite neutralized by the exhausting effect of the foregoing 112 pulsations. At any rate, looking to the general nature of the previous proportions (viz. in their sum $\frac{185}{211}$), it is certain that $\frac{380}{112}$ leaves a large preponderance in favour of nutrition, which preponderance is not much modified by adding the next succeeding proportion thus, $\frac{380+20}{112+45}=\frac{400}{157}$. Consequently, the organism may fairly be supposed to have entered upon the next prolonged period of rest (viz. 185 seconds) with a large balance of reserve power; so that when to this large balance there was added the further accumulation due to the further rest of 185 seconds, we are not surprised to find the next succeeding swimming-bout comprising the enormous number of 894 pulsations. But this great expenditure of energy seems to have been somewhat in excess of the energy previously accumulated by the prolonged rest; for this unusual expenditure seems next to have entailed an unusually prolonged period of exhaustion. At any rate it is plainly observable that the next succeeding proportions are greatly in favour of repose; for it is not until 360 seconds have elapsed, with only 12 pulsations in the interval, that energy enough has been accumulated to cause a moderate bout of 30 pulsations. But next another long and sustained pause of 240 seconds supervenes, and, the animal being now fully refreshed with a large surplus of accumulated energy, the next succeeding swimming-bout comprises 200 pulsations. Lastly there succeeded 60 seconds of rest, and here the observation terminated*.

§ 3. *Effects of Artificial Segmentation on the Rhythm.*—We have next to consider Dr. EIMER's observations concerning the effects on the rhythm of *Aurelia* which result on cutting the animal into segments; and here, again, I much regret to say that I cannot wholly agree with this author. It will be remembered that he says he found evidence of a very remarkable fact, viz. that by first counting the natural rhythm of an unmutilated *Aurelia*, and then dividing the animal into two halves, one of these halves into two quarters, and one of these quarters into two eighths, the sum of the contractions performed by these four segments in a given time was equal to the number which had previously been performed in a similar time by the unmutilated animal. And not only so, but the number of contractions which each segment contributed to this sum was a number that stood in direct proportion to the size of the segment; so that the half contracted half as many times, the quarter a quarter as many times, and the eighth parts one eighth part the number of times that the unmutilated *Aurelia* had previously contracted in a period of equal duration. I am glad to observe that Dr. EIMER does not regard this rule otherwise than as liable to frequent exceptions; for, as already observed, I cannot say that my experiments have tended to confirm it. As stated in my

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* If the reader takes the trouble to ascertain the average proportion between the number of pulsations and the seconds of rest in the first observations as far down as the first long pause, viz., as above stated, $\frac{185}{211}$, and if he then balances the succeeding income and expenditure of energy over all the rest of the observations, he will find the net result to accord very precisely with the proportion he previously obtained. But, as already stated, any such precision as this is certainly the exception rather than the rule.

It may here be stated that after the 60 seconds of rest above recorded, the animal began another swimming-bout. It was then immediately bisected, and the subsequent observations are detailed in the next footnote.

previous paper, there is a general tendency for the smaller segments of an *Aurelia* divided in this way to contract less frequently than the larger segments; and more careful observation this year has not resulted in establishing any more precise a relationship.

It would be tedious and unnecessary to quote my observations in this connexion; but as these observations brought out very clearly a fact which I had previously suspected, I may detail one experiment to illustrate this point. The fact in question is, that the *potency of the lithocysts* in any given segment of a divided *Aurelia* has more to do with the frequency of its pulsations than has the size of the segment. As mentioned in my other paper, one or more lithocysts may often be observed to be permanently prepotent over the others; and I may here observe that the segmentation experiments just described have shown the converse to be true, viz. that one or more lithocysts are often permanently feebler than the others. Well, if a specimen of *Aurelia* exhibiting decided prepotency in one or more of its lithocysts be watched for a considerable length of time, so as to be sure that the prepotency is not of a merely temporary character, and if the animal be then divided into segments in such a way that the prepotent lithocysts shall occupy the smaller segments, it may be observed, provided time be left for the tissues to recover shock, that the segments containing the prepotent lithocysts, notwithstanding their smaller size, contract more frequently than do the larger segments. Conversely, if the larger segments happen to contain feeble lithocysts, their contractions will be but few. I have, indeed, seen cases in which the lithocysts appeared to be quite functionless, so far as the origination of stimuli was concerned.

The following observations were made on a healthy specimen of *Aurelia* having all its lithocysts in good condition, but prepotency being well marked in the case of one of them, and also, though in a lesser degree, in the case of another. I divided the animal so as to leave one of these two prepotent lithocysts in each of the eighth-part segments, and the next most powerful lithocyst in the quadrant segment. In the following description I shall call the two eighth-part segments A and B, the former letter designating the segment containing the most powerful lithocyst. The *Aurelia* before being divided manifested for several hours a very regular and sustained rhythm of 32 per minute. After its division the various segments contracted at the following rates in one minute intervals:—

Time after operation.	Segment ½.	Segment ¼.	Segment ⅛ A.	Segment ⅛ B.
½ hour.	20	25	27	15
1 ,,	20	25	27	15
2 hours.	29	25	27	16
4 ,,	19	16	27	12

Next morning the water which contained the segments was somewhat foul, and this, as is always the case, gave rise to abnormally long pauses. This effect was much more marked in the case of some of the segments than in that of others. I therefore observed the segments over five minutes intervals, instead of over one minute intervals as on the previous day. The following is a sample of several such observations, all yielding the same general result.

Segment $\frac{1}{2}$.		Segment $\frac{1}{4}$.	Segment $\frac{1}{8}$ A.	Segment $\frac{1}{8}$ B.
Number of pulsations.	Seconds of rest.			
12	120	No motion during the hour of observation.	Continued persistently to contract with a nearly perfect rhythm of 78 in the 5 minutes during the hour of observation.	Rhythm tolerably perfect at 78 in the 5 minutes; but this was occasionally interrupted by long pauses of 4 or 5 minutes duration.
3	10			
2	20			
44	130			
12	20			
73	5 minutes.			
Average rate $14\frac{4}{5}$ per minute.		No motion.	Continuous rhythm at the rate of $15\frac{3}{5}$ per minute.	Interrupted rhythm at the rate of $15\frac{3}{5}$ per minute.

I now transferred all the segments to fresh sea-water, with the following results:—

Rhythm during first quarter of an hour immediately after transference, in five minutes' intervals.

Time.	Segment $\frac{1}{2}$.	Segment $\frac{1}{4}$.	Segment $\frac{1}{8}$ A.	Segment $\frac{1}{8}$ B.
First 5 minutes	139 (irregular).	0	83 (regular).	20 (irregular).
Second 5 minutes ..	0	0	68 ,,	75 (regular).
Third 5 minutes	100 (regular).	39 (irregular).	70 ,,	69 ,,

Rhythm 2 hours after transference (5 minutes' intervals).

Segment $\frac{1}{2}$.	Segment $\frac{1}{4}$.	Segment $\frac{1}{8}$ A.	Segment $\frac{1}{8}$ B.
82 (regular).	77 (regular).	70 (regular).	62 (regular).

Rhythm next day (5 minutes' intervals).

Segment $\frac{1}{2}$.	Segment $\frac{1}{4}$.	Segment $\frac{1}{8}$ A.	Segment $\frac{1}{8}$ B.
68	55	17	Dead.

Next day all the segments were dead except the largest one, in which a single lithocyst still continued to discharge at the rate of 24 in five minutes.

Now, with regard to these Tables it is to be observed that during the first day the prepotent lithocyst in the eighth-part Segment A maintained an undoubted supremacy over all the others, and that the same is true of the comparatively potent lithocysts in the quadrant. (This is not the case with Segment B; probably the degree of prepotency of the lithocyst in this case was not sufficient to counteract the antagonistic influence of the small size of the segment.) But next day the supremacy of the small Segment A was not so marked; for although its rhythm was more *regular* in the stale water than was that of the largest segment, its actual number of contractions in a given time was just about equal to that of the largest segment. Again, after transference to fresh sea-water, the balance began to fall on the side of the larger segments; for even the quadrant, which in the stale water had ceased its motions altogether, now held a middle position between that of the half-segment and the prepotent eighth-part segment. On the next day, again, the balance fell decidedly in favour of the larger segments, and the weaker eighth-part segment died. Lastly, next day all the smaller segments were dead.

Hence the principal facts to be gathered from these observations are, that as time goes on the rhythm of all the segments progressively decreases, and that the decrease is more marked in the case of the smaller than in that of the larger segments. This lesser endurance of the smaller segments also finds its expression in their earlier death. Now as these smaller segments started with a greater proportional amount of ganglionic power than the larger segments, their lesser amount of endurance can only, I think, be explained by supposing that the process of starvation proceeds at a rate inversely proportional to the size of the segment—a supposition which is rendered probable if we reflect that the smaller the segment the greater is the proportional area of severed nutrient-tubes*. And in this connexion it is interesting to observe that, although the endurance of the smaller segments was less than that of the larger as regards the deprivation of nutriment, it was greater than that of the larger segments as regards the deprivation of oxygen. This is shown by the greater regularity of the rhythm mani-

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* It may be thought that the greater area of general tissue-mass in the larger segments than in the smaller, and not the lesser proportional area of tube-section, is the cause of the larger segments living longer than the smaller ones. I am led, however, to reject this hypothesis, because in *Sarsia*, where segmentation entails a comparatively small amount of tube-section, there is no constant rule as to the larger segments showing more endurance than the smaller ones—the converse case, in fact, being of nearly as frequent occurrence. I can only account for this fact by supposing that the endurance of the segments of *Sarsia* is determined by the degree in which the three or four minute open tube-ends become accidentally blocked. This supposition is the only one I can think of to account for the astonishing contrasts as to endurance that are presented by different segments of the same individual, and, I may add, of different individuals when deprived of their margins and afterwards submitted to the same conditions. For instance, a number of equally vigorous specimens had their margins removed, and were then suspended in a glass cage attached to a buoy in the sea. Four days afterwards some of the specimens were putrid, while others were as fresh as they were when first operated on. Again, as an instance of the experiments in segmentation of *Sarsia*, I may quote an experiment in which a score of specimens were divided in all sorts of ways, such as leaving the polypite attached to one half, or three marginal bodies in one portion and the remaining marginal body in the other portion, &c., &c. Yet, although it was very exceptional to find the two portions presenting an equal degree of endurance, no uniform results pointing to the cause of the variations could be obtained. In most cases, however, the energy, as distinguished from the endurance, of the larger segments was conspicuously greater than that of the smaller. But it is curious that in many cases the effects of *shock* appeared to be more marked in the larger than in the smaller segments—the latter, for some time after the operation, contracting much more frequently than the former. To show both these effects, one

fested by the smaller than by the larger segments in the stale water; and the fact is presumably to be accounted for by the consideration that the ganglia in the smaller segments were more potent than those in the larger.

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experiment may be quoted. A specimen of *Sarsia* was divided into two parts, of which one was a quadrant. Immediately after the operation the results were as follows:—

Portion ¼.		Portion ¾.	
Number of pulsations.	Minutes of rest.	Number of pulsations.	Minutes of rest.
20	0	0	5
4	4	10	2
15	5	46	1
6	3	23	2
		49	1
		900	1
		117	1
45	12	1145	13

To show the difference between the *endurance* of two halves of a bisected specimen of *Sarsia*, I may quote one experiment which was performed on the same specimen as the one mentioned in the text to show the general relationship between the duration of the pauses and that of swimming-bouts (see last footnote).

Immediately after Bisection.

½ A.		½ B.	
Number of pulsations.	Seconds of rest.	Number of pulsations.	Seconds of rest.
56	10	82	180
150	150	51	20
68	335	14	60
130	30	13	50
46	45	46	45
2	10	38	65
99	66	18	45
103	360	23	60
12	4	35	130
		105	70
Pauses now become longer, and swimming-bouts shorter.			

Twenty-four hours after the Operation.

½ A.		½ B.	
Number of pulsations.	Seconds of rest.	Number of pulsations.	Seconds of rest.
2	363	50	20
12	362	81	25
4	666	37	101
25	300	2400	60

With regard, therefore, to the original point under consideration, I conclude that although the size of the segments is doubtless one factor in determining the relative frequency of contraction, there are at least two other factors quite as important, viz. the relative potency of the lithocysts, and the length of time that elapses between performing

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But although in the case of *Sarsia* the lesser endurance of the smaller segment than of the larger cannot be regarded as a general rule, it may be so regarded, as already stated, in the case of *Aurelia*. The following experiment exemplifies this particular rule even more prettily than does the one quoted in the text, from the fact that the segments survived the operation for a greater number of days.

An *Aurelia* having a regular and well-sustained rhythm of 20 per minute was divided as already described in the text. In five minutes' intervals on successive days the average rates of the four segments were as follows:—

Four hours after the Operation.			
Seg. ½.	Seg. ¼.	Seg. ⅛ A.	Seg. ⅛ B.
100	100	85	90
Next Day.			
88	90	64	58
Next Day.			
86	82	62	57
Next Day.			
59	45	24	20
Next Day.			
50	49	20	10
Next Day.			
43	33	18	4
Next Day.			
39	32	19	Dead.
Next Day.			
33	7	Dead.	0
Next Day.			
28	Dead.	0	0

Next day the temperature unfortunately rose sufficiently to cause the death of the single surviving segment, which otherwise would probably have lived for one or two days longer.

the operation and observing the rhythm. Hence it is that in my experience I have found but very few examples of Dr. EIMER's rule.

§ 4. *Effects of other forms of Mutilation on the Rhythm.*—The next point I have to dwell upon is one of some interest. If the polypite of *Aurelia*, or of any other covered-eyed Medusa, be suddenly cut off at its base, the swimming-motions of the umbrella immediately become accelerated*. This acceleration, however, only lasts for a few minutes, when it gradually begins to decline, the rate of the rhythm becoming slower and slower, until, finally, it comes to rest at a rate considerably less than was previously manifested by the unmutilated animal. If a circular piece be now cut out from the centre of the umbrella, the rhythm of the latter again becomes temporarily quickened, but, as before, gradual slowing next supervenes. This slowing, however, proceeds further than in the last case; so that the rate at which the rhythm next becomes stationary is even less than before. If, now, another circular ring be cut from the central part of the umbrella,—*i. e.* if the previously open ring into which this organ had been reduced by the former operation be somewhat narrowed from within,—the same effects on the rhythm are again observable; and so on with every repetition of the operation, the rate of the rhythm always being quickened in the first instance, but then gradually slowing down to a point somewhat below the rate it manifested before the previous operation. It will here suffice to quote one experiment among many I have made in this connexion:—

An *Aurelia* manifested a regular and sustained rhythm of	26
Immediately after removal of polypite, rhythm rose to	36
Rate then gradually fell for ¼ of an hour, and became stationary at . .	20
Circular incision just including ovaries caused rhythm to rise to . . .	26
After gradual fall during ¼ of an hour, rhythm became stationary at . .	17
Another circular incision carried round midway between the former one and the margin caused rhythm to rise to	24
Rate again gradually declined, and in ¼ of an hour was	12
Another circular incision was carried round as close to the margin as was compatible with leaving the physiological continuity of all the lithocysts intact. Rhythm rose to	14
Within a few minutes it fell to	6

Excepting the cases where the effects of shock are apparent, some such series of phenomena as those just recorded are always sure to ensue when a covered-eyed Medusa is mutilated in the way described†. But this kind of mutilation, besides producing such marked effects on the *rate* of the rhythm, also produces an effect in impairing the *regularity* of the rhythm. In some specimens the latter effect is more marked than it

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* In some cases this effect is obliterated by that of shock, the swimming-motions ceasing altogether for a time, or becoming more feeble.

† I have not tried any experiments with the naked-eyed Medusæ in this connexion.

is in others. The following series of observations will serve to render a good idea of this effect:—

An *Aurelia* manifested a regular and sustained rhythm of 36. Immediately after the removal of the polypite the rate of rhythm in successive minutes was as follows:—40, 39, 37, 35, 32, 30, 29, 26, 24, 18, 14 (40 seconds' pause), 16, 15, 14, 15, 16 (40 seconds' pause), 22, 20, 19, 15, 16, 17, 14, 13, 13, 15, 16, 16, 17, 18, 14, 12, 13, 11, 12, 9, 15, 16, 14, 12, 9, &c., the rhythm now continuing very irregular. An hour after the operation the following were the number of contractions given in one-minute intervals, the observations being taken at intervals of ten minutes:—15, 15, 12, 22, 14, &c.

In this experiment, therefore, as soon as the acceleration-stage had been passed, viz. about a quarter of an hour after the operation, a great disturbance was observable in regularity of the rhythm; for before the removal of the polypite the Medusa had been swimming for hours with perfect regularity.

It is an important point in connexion with these observations to ascertain whether the remarkable effects of mutilation on the rhythm are transitory or permanent; but as yet I have not been able to determine this point with certainty. So far as my observations have gone, they all tend to prove that the slowing-effect of the mutilation is permanent; but I do not like as yet to say positively that it is so, because even unmutilated specimens of *Aurelia* sometimes slow their rhythm from day to day when kept in confinement. I may here state, for the benefit of those who may repeat these experiments, that when an *Aurelia* is converted into an open ring, the various parts of the ring fall together, especially if it be a narrow one. This unnatural state of things, in consequence of the irritation arising from the different parts of the ring rubbing on one another, causes the remnant of the animal to quicken its swimming-motions. Nor is this source of fallacy obviated by keeping the open ring distended by means of some cylindrical body of appropriate diameter; for the presence of this foreign body acts more or less as a constant stimulus. The best plan, therefore, to adopt is to make the circular incisions only through the thickness of the very tenuous nervo-muscular sheet, leaving the gelatinous tissue everywhere intact. The natural shape of the animal is thus not in the least affected, while the constituent parts of the nervo-muscular sheet are physiologically separated from one another as effectually as if the one were altogether removed from the other.

Before concluding my description of these experiments, it may perhaps be as well to mention one other, which was designed to meet a possible objection to the inferences which, as I shall presently explain, these experiments seem to sustain. It occurred to me, as a remote possibility, that the slowing and irregularity of the rhythm, which are observable about a quarter of an hour after the operations described, might be due to the deprivation of adequate nourishment suffered by the ganglia, in consequence of the escape of nutrient matter from the cut ends of the nutrient tubes. Accordingly, instead of cutting off the polypite, I tried the effect of momentarily immersing it in hot water, and found, as I had expected, that the subsequent disturbances of the rhythm were precisely similar to those which result from removal of the polypite.

Now, to draw any inferences from such meagre facts as the above would be hazardous, unless we recognize that in so doing our inferences are not trustworthy. But, with this recognition, I think there will be no harm in briefly stating the deductions to which the facts, such as they are, would seem to point.

Physiologists are undecided as to the extent in which many apparently automatic actions may not really be actions of a reflex nature. Given, for instance, any ganglio-muscular tissue which is rhythmically contracting, and how are we to know whether the action of the ganglia is truly automatic, or sustained from time to time by stimuli proceeding from other parts of the organism? In most cases experiments cannot be conducted with reference to this question, but in the case of the Medusæ they may be so; and it was with the view of throwing light on this question that the experiments just described were made. Now in these experiments the fact is sufficiently obvious that mutilations of any part of the organism modify the rhythm of the marginal ganglia most profoundly. That this modification does not proceed from shock, would seem to be indicated by the facts that the first effect of the mutilation is to *quicken* the rhythm, that there is a sort of general proportion to be observed between the amount of tissue abstracted and the degree of slowing of the rhythm produced, and that the slowing effects continue for so long a time. All these facts seem to show that we have here something other than mere shock to deal with. A strong suspicion, therefore, arises that the cause of the slowing of the rhythm which results from removing the polypite, or a part of the general contractile tissue of the bell, consists in the destruction of some influence of an afferent character which had previously emanated from the parts of the organism which have been removed; and that the normal rhythm before the operation was partly due to a continuous reception, on the part of the ganglia, of this afferent or stimulating influence. In support of this view are the facts that the first effect of such an operation as we are considering is greatly to accelerate the rhythm, and that this acceleration then gradually declines through a period of about a quarter of an hour. These facts tend to support this view, because, if it is correct, they are what we might anticipate. If the polypite, for instance, while *in situ*, is continually supplying a gentle stimulus to the marginal ganglia, when it is suddenly cut off, the nerve-tracts through which this stimulating influence had previously been conveyed must be cut through; and as it is well known how irritable nerve-fibres are at their points of section, it is to be expected that the irritation caused by cutting these nerve-tracts, and probably also the action of the sea-water on their cut extremities, would cause them to stimulate the ganglia more powerfully than they did before their mutilation. And here I may state that on several occasions, with vigorous specimens, I have observed a sudden removal of the polypite to be followed, not merely with a quickening of the rhythm on the part of the bell, but with a violent and long-sustained spasm. Again, as regards the other fact before us, it is obvious that as soon as the cut extremities of the nerves begin to die down, and so gradually to lose their irritability, the effect on the rhythm would be just what we observe it to be, viz. a gradual slowing till the rate falls considerably below that which was exhibited by the unmutilated animal. And even the *irregularity* which is at this stage so frequently observable is, I think, what we should expect to find if this view as to the essentially reflex character of the natural rhythm is the true one.

If this view is the true one, the question next arises as to the nature of the process which goes on in the excitable tissues, and which afterwards acts as a stimulus on the ganglionic tissues. This question, however, I am quite unable to answer. Whether the process is one of nutrition, of oxygenation, of chemical changes exerted by the sea-water, or a process of any other kind, further experiments may perhaps be able to show; but meanwhile I have no suggestion to offer.

§ 5. *Effects of lessening the amount of Tissue adhering to a single Ganglion**.—The above experiments led me to try the effects of cutting out a single lithocyst of *Aurelia*, and, after the rhythm of the detached segment had become regular, progressively paring down the contractile tissues around the ganglion. I found that this process had no very marked effect on the rhythm until the paring reached within an inch or two of the ganglion. Then, however, the effect began to show itself, and with every successive paring it became more marked. This effect consisted in slowing the rate of the rhythm, but more especially in giving rise to prolonged pauses. Indeed, if only a very little contractile tissue was left adhering to the ganglion, the pauses often became immensely prolonged; so that one might almost suppose the ganglion to have entirely ceased discharging. But if a stimulus of any kind were then applied, the rhythmic discharges at once recommenced. They then generally continued for some little time at a slower rate than that which they had manifested before they were affected by the paring down of the contractile tissue. To illustrate these effects, it will suffice to quote one experiment from my notes:—"In an eighth-part segment of *Aurelia*, measuring 5 inches by 4, the rhythm for several hours was quite regular at 18 per minute. A band of tissue measuring half an inch across was now removed from each side of the segment. Rate immediately rose to 22, and then gradually fell again to 18. Another similar band was now removed. This operation was followed by a pause of several minutes' duration, after which some irregular contractions occurred; but eventually these gave place to a regular rhythm of 16 per minute. The tissue-cone was now truncated by cutting off an inch of its apex. The rate of rhythm rose to 18, and eventually became regular at 13. On now again truncating the cone, by removing from its upper part a band of tissue half an inch wide, the rate again rose slightly, but then became stationary at 10. Lastly, a band of tissue half an inch wide was removed from both the sides of the truncated cone, thus reducing the contractile tissue surrounding the lithocyst to a small piece measuring $\frac{5}{8}$ of an inch long by $\frac{1}{2}$ an inch broad. This final operation was followed by prolonged periods of

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* In view of more recent observations on the histology of lithocysts, I substitute in this paper the term "ganglion" for the term "locomotor centre," which in my previous paper I everywhere used to denote these bodies.

quiescence, which were broken by occasional single contractions and occasional bouts of partly rhythmical swimming-motions. These bouts could at any time be pretty certainly evoked by a stimulus, and the rate of their imperfect rhythm was about 10 per minute."

§ 6. *Effects of Temperature on Rhythm.* (*a*).—The effects of temperature on the rhythm of Medusæ are very decided. For instance, a specimen of *Sarsia* which in successive minutes gave the following numbers of pulsations—16, 26, 0, 0, 26, gave 60 pulsations during the next minute, while a spirit-lamp was held under the water in which the Medusa was swimming. If hot water be added to that in which *Sarsia* are contained until the whole is about milk-warm, their swimming-motions become frantic. If the same experiment be performed after the margins of the *Sarsia* have been removed, the paralyzed bells remain quite passive, while the severed margins exhibit the frantic motions just alluded to.

In the case of *Aurelia aurita* the characteristic effects of temperature on rhythm may be better studied than in that of *Sarsia*, from the fact that the natural motions are more rhythmical and sustained in the former than in the latter genus. I have therefore, in this connexion, made more observations on *Aurelia* than on *Sarsia*. The following, quoted from my notes, may be taken at a typical experiment:—

"A small and active specimen of *Aurelia* contracted with the greatest regularity 33 times per minute in water kept at 34°; but on transference to water kept at 49°, the contractions always became irregular, in respect (*a*) of not having a perfectly constant rhythm, and (*b*) of exhibiting frequent pauses, which was never the case in colder water. The rate of rhythm in the warmer water varied from 37 to 49; and, as in these observations no allowance was made for the occurrence of the pauses, the actual rate of rhythm during the swimming-motions was about 60 per minute. The following are some sample observations in the case of this specimen:—

Rate of Rhythm in successive Minutes.

Temperature of water (Fahr.).	Number of pulsations.	Seconds of rest.
49°	41	5
"	49	4
Transferred to 34	33	0
" "	33	0
" "	33	0
" "	33	0
Transferred to 49	45	4
" "	39	10
" "	37	15
Transferred to 34	20	0
" "	30	0
" "	33	0
" "	33	0
" "	33	0
" "	33	0

"This rate continued quite regularly for a quarter of an hour, when the observation terminated."

It might naturally be supposed that when the alterations of temperature between 34° and 49° produce such marked effects on the rhythm, still greater alterations would be attended with still greater effects. Such, however, is not the case. Water at 70° or 80°, for instance, has the effect of permanently *diminishing* the rate of the rhythm, after having temporarily raised it for a few seconds. The following note will serve to convey a just estimation of these facts:—

"An *Aurelia*, whose rhythm in water at 40° was very regular at 18 per minute, was suddenly transferred to water at 80°. In the immediately succeeding minutes the rhythm was 22, 20, 14. The latter rate continued for nearly half an hour, when the observation terminated."

The effect of very warm water, therefore, is to slow the rhythm, as well, I may add, as to enfeeble the vigour of the contractions. The case of Medusæ thus differs, in the former respect, from that of the heart; and I think the reason of the difference is to be found in the following considerations. Even slight elevations of temperature are quickly fatal to the Medusæ, so it becomes presumable that considerable elevations act very destructively on the nervo-muscular tissues of these animals. This destructive effect of high temperatures may therefore very probably counteract the stimulating effect which such temperatures would otherwise exert on the natural rhythm; and hence a point would somewhere be reached at which the destructive effect would so far overcome the stimulating effect as to slow the rhythm instead of quicken it. That this is probably the true, as it certainly is the only explanation to be rendered, will, I think, be conceded when I further state that if an *Aurelia* be left for some little time in water at 80° and then again transferred to water at 30° or 40°, its original rate of rhythm at the latter temperature does not again return, but the rhythm remains permanently slowed. And, in favour of the explanation just offered, it may be further pointed out that the first effects of sudden immersion in heated water is to *quicken* the rhythm—it not being for a few seconds, or for even a minute or two after the immersion, that the rhythm becomes slowed. Lastly, the slowing takes place gradually; and this is what we should expect if, as is probable, the destructive effect takes somewhat more time to become fully developed than does the stimulating effect.

(*b*) *Cold.*—Before leaving the subject of temperature in relation to rhythm, I must say a few words on the effects of cold. The following may be regarded as typical experiments:—

"An *Aurelia* presenting a regular rhythm of 20 per minute in water at 45° was placed in water at 19°. Soon after the transference the rhythm began to slow, and the strength of the contractions to diminish. Both these phenomena rapidly became more and more pronounced, till the rhythm fell to 10 per minute (still quite regular), and the contractions ceased to penetrate the muscular tissue further than an inch or so from the marginal ganglia. Shortly after this stage pauses became frequent; but mechanical or other irritation always originated a fresh swimming-bout. Next only one very feeble contraction was given at long and irregular intervals—a contraction so feeble that it was restricted to the immediate vicinity of the lithocyst in which it originated. Soon after this stage irritability towards all kinds of stimuli entirely ceased, including even strong spirit dropped on the under surface of the animal when taken momentarily out of the water. All these stages thus described were passed through rapidly, the whole series occupying rather less than five minutes. On now leaving the specimen for ten minutes and then restoring it to its original water at 45°, all the above-mentioned stages were passed through in reverse order. The first faint marginal contraction occurred four minutes after restoration to the warmer water. This contraction was confined to the immediate vicinity of the prepotent lithocyst, and all subsequent contractions continued to be so for the next three minutes. Rhythm very slow. Contractions now began to penetrate round the margin, and in eight minutes from the restoration had gone all the way round, the rate of their rhythm meanwhile increasing. In two minutes more all the umbrella was contracting at the rate of 15 per minute."

In another specimen, subjected to the same conditions, the rate of recovery was even more rapid, occupying only two minutes altogether; but in every case the process of recovery is a gradual one, and differs only in the time it occupies in passing through the various stages.

(c) *Effects of freezing Medusæ.*—In conclusion I will describe some rather interesting experiments that consisted in freezing some specimens of *Aurelia* into a solid block of ice. Of course, as sea-water had to be employed, the cold required was very considerable; but I succeeded in turning out the Medusæ encased on all sides in a continuous block of frozen sea-water. By now immersing this block in warm water, I was able to release the contained specimens, which then presented a very extraordinary appearance. The thick and massive gelatinous bell of a Medusa is, as every one knows, chiefly composed of sea-water, which everywhere enters very intimately into the structure of the tissue. Now all this sea-water was, of course, frozen *in situ*; so that the animals were everywhere and in all directions pierced through by an innumerable multitude of ice crystals, which formed a very beautiful meshwork pervading the whole substance of their transparent tissues.

These experiments were made in order to ascertain whether the Medusæ, after having been thus completely frozen, would survive on being again thawed out; and, if so, whether the freezing process would exert any permanent influence on the rate of their rhythm. Now in all the cases the Medusæ after having been thawed out presented a ragged appearance, which was due to the disintegrating effect exerted by the ice crystals while forming in the tissues; yet, notwithstanding this mechanical injury superimposed on the physiological effects of such extreme cold, all the Medusæ recovered on being restored to sea-water of the normal temperature. The time occupied by the process of recovery varied in different individuals from a few minutes to half an hour or more; and it was observable that those specimens which recovered soonest had the rate of their rhythm least affected by the freezing. In no case, however, that I observed did the rate of the rhythm after the freezing return fully to that which had been manifested before the freezing.

§ 7. *Effects of certain gases on the Rhythm.* (*a*) *Oxygen.*—I will now conclude my remarks on rhythm by very briefly describing the effects of certain gases. Oxygen forced under pressure into sea-water containing *Sarsia* has the effect of greatly accelerating the rate of their rhythm. The following observation on a single specimen will serve to render this apparent.

Number of pulsations given by *Sarsia* in successive five minutes' intervals.

In ordinary sea-water	472, 527, 470
In oxygenated sea-water	800
In ordinary sea-water	268, 350, 430

It will be seen from this observation that the acceleration of the rhythm due to the oxygenation was most marked. Indeed the pulsations followed one another so rapidly that it was no easy matter to count them. It must also be stated that while the animal was under the influence of oxygen, the duration of the natural pauses between the swimming-bouts was greatly curtailed—the swimming-motions, in fact, being almost quite continuous throughout the five minutes that the Medusa was exposed to such influence. Lastly, it will be observed from the above Table that the unnatural amount of activity displayed by the organism while in the oxygenated water entailed on it a considerable degree of exhaustion, as shown by the fact that even a quarter of an hour after its restoration to normal water its original degree of energy had not quite returned.

(*b*) *Carbonic acid.*—As might be expected this gas has the opposite effects to those of oxygen. It is therefore needless to say more about this agent, except that if administered in large doses it destroys both spontaneity and irritability. Nevertheless, if its action is not allowed to last too long, the Medusæ will fully recover on being again restored to normal sea-water.

(*c*) *Nitrous oxide.*—This gas at first accelerates the motions of *Sarsia*, but eventually retards them. I omitted, however, to push the experiment to the stage of complete anæsthesia, which would doubtless have supervened had the pressure of the gas been sufficiently great.

(*d*) *Deficient aëration.*—It may now be stated that the Medusæ are exceedingly sensitive to such slight carbonization of the water in which they are contained as results from their being confined in a limited body of it for a few hours. The rhythm becomes slowed and the contractions feeble, while the pauses between the swimming-bouts become more frequent and prolonged. If the water is not changed, all these symptoms become more marked, and in addition the rhythm becomes very irregular. Eventually the swimming-motions entirely cease; but almost immediately after the animals are restored to normal sea-water, they recover themselves completely, the rate and regularity of their rhythm being then quite natural. The suddenness with which this return to the normal state of things is effected cannot but strike the observer as very remarkable; and, I may observe, it takes place with equal suddenness at whatever stage in the above-described process of asphyxiation the transference to normal sea-water is accomplished [*].

III. STIMULATION.

Under this heading I have a great number of fresh results to communicate; and as these chiefly have reference to electrical stimulation, I think it will be convenient to modify the order observed in my last paper, and to consider the other modes of stimulation first.

§ 1. *Mechanical stimulation.*—The only remark I have to make in this connexion is, that when the paralyzed swimming-organ of *Aurelia aurita* is stimulated by a single mechanical irritation, it often responds with two, and more rarely with three contractions, which are separated from one another by an interval of about the same length as the normal diastole of the unmutilated animal.

§ 2. *Chemical stimulation.*—Dilute spirit, or other irritant, when dropped on the paralyzed swimming-organ of *Aurelia aurita,* often gives rise to a whole series of rhythmical pulsations, the systoles and diastoles following one another at about the same rate as is observable in the normal swimming-motions of the unmutilated animal.

§ 3. *Thermal stimulation.*—It is somewhat difficult, in the case of paralyzed swimming-organs, to prove the occurrence of a contraction in response to thermal stimulation, from the fact that while these tissues are not nearly so sensitive to this mode of excitation as might be anticipated, they are, as observed in my former paper, extraordinarily sensitive to mechanical excitation. It therefore becomes difficult to administer the appropriate thermal stimulus without at the same time causing a sufficient mechanical disturbance to render it doubtful to which of the stimuli the response is due. This may be done, however, by allowing a few drops of heated sea-water to run over the excitable surface while it is exposed to the air. In this and in other ways I have satisfied myself that the paralyzed tissues of swimming-organs respond to sudden elevations of temperature.

§ 4. *Luminous stimulation.*—It is interesting to note that, in the case of some of the naked-eyed Medusæ, the action of light as a stimulus is most marked and unfailing. In the case of *Sarsia,* for instance, a flash of light let fall upon a living specimen almost invariably causes it to respond with one or more contractions. If the animal is vigorous and swimming freely in water, the effect of a momentary flash thrown upon it during one of the natural pauses is immediately to originate a bout of swimming. But if the animal is non-vigorous, or if it is removed from the water and spread flat upon an object-glass, it usually gives only one contraction in response to

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[*] Dr. EIMER has also observed the facts mentioned in this paragraph (*d*).

every flash. There can thus be no doubt that a sudden transition from darkness to light acts upon *Sarsia* as a stimulus, and this even though the transition be but of momentary duration. The question therefore arises as to whether the stimulus consists in the presence of light, or in the occurrence of the sudden transition from darkness to light and from light to darkness. To answer this question I tried the converse experiment of placing a vigorous specimen in sunlight, waiting till the middle of one of the quiescent stages in the swimming-motions had come on, and then suddenly darkening. In no case, however, under these circumstances, did I obtain any response; so that I cannot doubt it is the light *per se*, and not the sudden nature of the transition from darkness to light, which in the former experiment acted as the stimulus. Indeed the effect of the converse experiment just described is rather that of inhibiting contractions; for, if the sunlight be suddenly shut off during the occurrence of a swimming-bout, it frequently happens that the quiescent stage immediately sets in. Again, in a general way, it is observable that *Sarsia* are more active in the light than they are in the dark, the comparative duration of the quiescent stages being less in the former than in the latter case. Light thus appears to act towards these animals as a constant stimulus. Nevertheless, when the flashing method of experimentation is employed, it is observable that the stimulating effect of the flashes progressively declines with their repetition—supposing, of course, that the intensity of the light employed remains constant, and that the rate at which the flashes are thrown in is not so slow as to cause the ganglia to forget, as it were, the occurrence of the previous flash before its successor is thrown in. It is difficult to measure with any exactness the limit of time during which the effect of the previous flash remains, but it appears to be about a quarter of a minute; for if the flashes are thrown in with a greater frequency than this, the progressive decline in their efficiency as stimulants first becomes observable. In all cases the response of *Sarsia* to luminous stimulation appears to be quite as rapid as is their response to any other mode of stimulation, although, as I have not in the case of this genus been able to measure the period of latent stimulation, this statement rests only on the estimate formed by the eye. It is further to be observed that when the efficacy of luminous stimulation is being diminished—either in consequence of frequent repetition at short intervals, or in consequence of the slow dying of the animal on an object-glass—the period of latency is very much prolonged. In this case the first effect of the flash is to cause retraction of the tentacles, to be followed immediately afterwards by a contraction of the bell. (This is also the case in other modes of stimulation of *feeble* specimens.) Lastly, it may be stated that when the marginal bodies of *Sarsia* are removed, the swimming-bell, although still able to contract spontaneously, no longer responds to luminous stimulation of any kind or degree. But if only one body be left *in situ*, or if the severed margin alone be experimented upon, the same unfailing response may be obtained to luminous stimulation as that which is obtained from the entire animal.

Although I have tested many of the Medusæ, I have obtained indications of response to luminous stimulation only in the case of one other species of the naked-eyed division.

This is a species which I have called *Tiaropsis polydiademata*, and the response which it gives to luminous stimulation is even more decided than is that which is given by *Sarsia*. In the opening paragraphs of my former paper, I described a peculiar spasmodic movement that is performed by *Staurophora laciniata* when subjected to marginal stimulation of any kind—a movement quite distinct from the ordinary locomotor contractions. A precisely similar movement is exhibited under similar circumstances by *Tiaropsis polydiademata*, and it is very interesting to observe that a sudden increase of light causes this peculiar spasm in this species quite as invariably and quite as decidedly as does any other kind of stimulation. Now in one remarkable particular this response to luminous stimulation on the part of *Tiaropsis polydiademata* differs from that given by *Sarsia tubulosa*; and the difference consists in the fact that while with *Sarsia* the period of latency is, so far as the eye can judge, as instantaneous in the case of response to luminous stimulation as it is in the case of response to any other kind of stimulation, such is far from being true with *Tiaropsis polydiademata*. The period of latency in the last-named species is, so far as the eye can judge, quite as instantaneous as it is in the case of *Sarsia*, when the stimulus employed is other than luminous; but in response to light the characteristic spasm does not take place till slightly more than a second has elapsed after the first occurrence of the stimulus. As this extraordinary difference in the latent period exhibited by the same animal towards different kinds of stimuli appeared to me a matter of considerable interest, I was led to reflect upon the probable cause of the difference. It occurred to me that the only respect in which luminous stimulation of the Medusæ differed from all the other modes of stimulation I had employed consisted in this—that, as proved by my previous experiments on *Sarsia*, which I repeated on *Tiaropsis*, luminous stimulation directly affected the central nervous tissues alone, while all the other modes of stimulation directly affected the contractile tissues. Now, as in *Tiaropsis polydiademata* luminous stimulation differed from all the other modes of stimulation in giving rise to an immensely longer period of latency, I seemed here to have an index of the difference between the rapidity of response to stimuli by the contractile and by the ganglionic tissues respectively. The next question, therefore, which presented itself was as to whether the enormous length of time occupied by the process of stimulation in the ganglia was due to any necessity on the part of the latter to accumulate the stimulating influence prior to originating a discharge, or to an immensely lengthened period of latent stimulation manifested by the ganglia under the influence of light. This is an interesting question, because if such a lengthened period of latent stimulation occurs in this case, it would stand in curious antithesis to the very short period of latent stimulation manifested by the contractile tissues of the same animal under other modes of irritation. To test these alternative hypotheses, I employed the very simple method of first allowing a continuous flood of light to fall suddenly on the Medusid, and then noting the time at which the responsive spasm first began. This time, as already stated, was slightly more than one second. I next allowed the animal to remain for a few minutes in the dark to recover shock, and, lastly, proceeded to throw in single flashes of light of measured duration. I found that unless the flash of light was of slightly more than one second in its duration, no response was given. That is to say, the minimal duration of a flash required to produce a responsive spasm was just the same as the time during which a continuous flood of light required to operate in order to produce a similar spasm. From this, therefore, I conclude that the enormously long period of latent excitation in response to luminous stimuli was not, properly speaking, a period of latent excitation at all, but that it represented the time during which a certain summation of stimulating influence was taking place in the ganglia, which required somewhat more than a second to accumulate, and which then caused the ganglia to originate an abnormally powerful discharge.

With regard to luminous stimulation it is only necessary further to observe that responses were given equally well to direct sunlight, diffused daylight, and to light reflected from a mirror inclined at the polarizing angle. It must also be stated that responses are given to any of the luminous rays of the spectrum when these are employed separately, but that neither the non-luminous rays beyond the red, nor those beyond the violet, appear to exert the smallest degree of stimulating influence.

§ 5. *Electrical stimulation.* (A) *Latent periods and characters of the subsequent contractions.* (*a*) *Method.*—Under this heading I propose to treat at the same time of the period of latent stimulation and the manner in which this, as well as the responsive contractions, are modified by certain conditions. As *Aurelia aurita* is the only species on which I have experimented in this connexion, my remarks under this heading will be confined to it alone. The method by which I determined the latent period in the case of this species was as follows. A basin containing the Medusid was filled to its brim with sea-water, and placed close beside a cylinder which, while it lay in a horizontal position, could be rotated at a known rate. The *Aurelia** was placed with its concave aspect uppermost, and an inch or two below the surface of the water. The animal was held firmly in this position by means of a pair of compasses thrust through it and forced into a piece of wood which was sunk at the bottom of the basin. The legs of the compasses were provided with india-rubber sliders; so that by placing these under the Medusa, the latter might be kept at any elevation in the water which might be desired. The polypite and lithocysts were now removed, and also a segment of the disk. A light straw was then forced through the gelatinous substance of the umbrella in a radial direction, and close to the gap caused by the missing segment. The other, or free, end of this straw was firmly joined to a capillary glass rod, which was suitably bent to avoid contact with the rim of the basin and also to write on the smoked cylinder. If the straw was not itself sufficient to support the weight of the capillary rod, a small cross-piece of cork might easily be tied to it, so as to add to the floatation-power. A part of the excitable tissue was now raised

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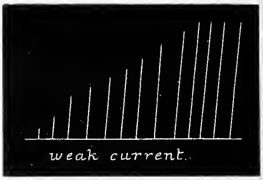
* It may here be stated that, in all the experiments on stimulation subsequently to be detailed, there is no difference to be observed between the behaviour of an entire swimming-organ deprived of its ganglia and that of a portion of any size which may be separated from it.

above the surface of the water by means of a disk of cork placed beneath it, and on the part of the tissue thus raised there were placed a pair of platinum electrodes. These electrodes proceeded from an electro-magnetic apparatus, which was arranged in such a way, that every time the current in it was opened or closed it gave an induction-shock and moved a lever at the same instant of time. This lever was therefore placed upon the cylinder immediately above the capillary glass writer which proceeded from the Medusa, care being taken to place the two writers in the same line parallel to the axis of the cylinder. Such being the arrangement, the cylinder was rotated, and thus two parallel lines were made upon it by the two writers. If the current was now closed, an induction-shock was thrown into the tissue at the same instant that the electro-magnet writer recorded the fact by altering its position on the cylinder. Again, as soon as the paralyzed Medusa responded to the induction-shock, the radii of the vacant segment were drawn apart, and in this way a curve was obtained by the other writer on the rotating cylinder. Now, by afterwards dropping a perpendicular line from the point at which the electro-magnet writer changed its position to the parallel line made by the other writer, and then measuring the distance between the point of contact and the point on the last-mentioned line at which the curve began, the period of latent stimulation was determined. A glance at figs. 2 & 3 will render this description clear to any one who is not already acquainted with the method, when it is stated that the upper line is a record of the movements of the electro-magnet writer, and the lower line that of the movements of the other writer. It will be observed that the point *a* in the upper line marks the point at which the induction-shock was thrown in; so that by first producing the perpendicular till it meets the lower line at *b*, and then measuring the distance between the point *b* and the point *c*, at which the curve in the lower line first begins, the latent period (*bc*) is determined—the time occupied by the rotation of the cylinder from *b* to *c* being known.

(*b*) *Summation of Stimuli.*—In this way I have been able to ascertain the period of latent stimulation in *Aurelia aurita* with accuracy. It must be stated at the outset, however, that this period is subject to great variations under certain varying conditions; so that we can only arrive at a just estimation of it by understanding the nature of the modifying causes. To take the simplest cause first, suppose that the paralyzed *Aurelia* has been left quiet for several minutes in sea-water at 45°, and that it is then stimulated by means of a single induction-shock. The responsive contraction will be comparatively feeble with a very long period of latency, viz. $\frac{5}{8}$ of a second. If another shock of the same intensity be thrown in as soon as the tissue has relaxed, a somewhat stronger contraction with a somewhat shorter latent period will be given. If the process is again repeated, the response will be still more powerful, with a still shorter period of latency; and so on for perhaps eight or ten stages, when the maximum force of contraction of which the tissue is capable will have been attained, while the period of latency will have been reduced to its minimum. This period is $\frac{3}{8}$ of a second, or, in some cases, slightly less.

Now we have here a very remarkable series of phenomena; and as it is a series which never fails to occur under the conditions named, I append tracings to give a better idea of the very marked and striking character of the results. The first tracing (fig. 1) is a record of the successive increments of the responses to successive induction-shocks of the same intensity thrown in at three seconds' intervals—the cylinder being stationary during each response, and rotated a short distance with the hand during each interval of repose.

Fig. 1.



The second tracing (figs. 2 & 3) is a record of the difference between the lengths of the latent period, and also between the strengths of the contraction, in the case, 1st, of the first of such a series of responses (fig. 2), and, 2nd, of the last of such a series (fig. 3). From these tracings it will be manifest, without further comment, how surprising is the effect of a series of stimuli, first in *arousing* the tissue, as it were, to increased *activity*, and second in developing a state of *expectancy*.

Fig. 2.

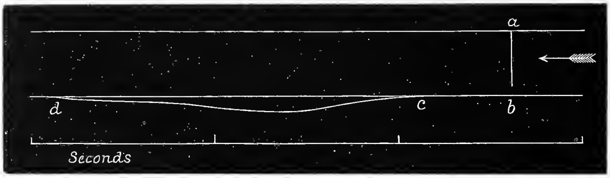
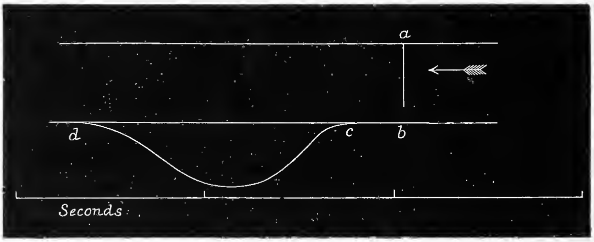


Fig. 3.



Those who are acquainted with Dr. Bowditch's investigation "concerning the irritability of the muscular fibres of the heart"*, cannot fail to be struck by the great simi-

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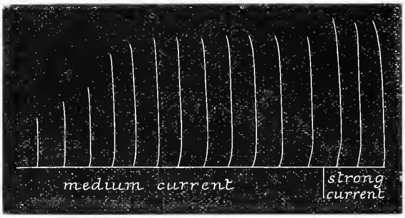
* Ludwig, 'Arbeiten aus der physiologischen Anstalt zu Leipzig,' 1871, pp. 139–176. [It is further to be observed that a similar summation of stimuli has lately been observed to occur, in the case of reflex action, in the frog (Dr. Stirling, 'Journal of Anat. and Physiol.' Jan. 1876) and in the excitable leaf of *Dionæa* (Dr. Burdon Sanderson and Mr. Page, Proc. Roy. Soc. No. 177).—1877.]

larity of the first of the above tracings with those published in his paper. It will be remembered that he experimented on the physiologically severed apex of the heart, and therefore on a tissue which, in the absence of any active ganglionic element, resembles the paralyzed umbrella of *Aurelia.* His tracings were obtained in exactly the same way as my own, so that the fact of tissues separated from one another in the animal scale so widely as are the muscle-fibres of the heart and those of the Medusæ, nevertheless behaving towards stimulation in so peculiar and yet so similar a manner, is to my mind a fact of great interest. Dr. Bowditch does not appear to have determined the effect of what he conveniently terms the staircase action (from the appearance of the tracings to a staircase) upon the period of latent stimulation; but from the fact that in other respects the case of the heart and that of the Medusæ are so similar, there can be little doubt that in the former, as in the latter, the latent period will be found to be greatly influenced by a series of stimuli.

But although the case of the heart and the Medusæ are so wonderfully parallel in the particulars we are considering, there are one or two points of difference between them which must here be noted. In the case of *Aurelia*, after a staircase has been built up by means of a series of stimuli, if a pause of ten seconds be allowed to elapse and the stimulation be then again commenced, I find that the first response given is not quite of maximum intensity, but corresponds with perhaps the second or third step from the top of the previously completed staircase. Again, if a pause of fifteen seconds be allowed to elapse, the first step of the next staircase corresponds with the third or fourth step from the top of the standard one. If, again, a pause of half a minute be allowed to elapse, the first step of the next staircase will correspond in height only with the second or third step from the bottom of the first staircase. Lastly, if a whole minute be allowed to elapse between the maximum effect of one series of stimulations and the first stimulus of another series, it is observable that the tissue has, as it were, completely forgotten the occurrence of the previous series, so that the next staircase has to begin anew from the first step. Now Dr. Bowditch has found, in the case of the heart, that an interval of *five minutes* must be allowed to intervene between two series of stimuli before the effect of the first on the second is thus totally abolished; so that, returning to the metaphor first employed, we may say that the memory of the cardiac tissue is about five times as long as that of the medusoid tissue. But in the case of *exhausted* medusoid tissue the difference may be even greater than this; for in this case I have observed all memory to fade in the course of half a minute. There is one other point of difference between the heart and the Medusæ analogous to the one first stated. From Dr. Bowditch's tracings it appears that the maximum staircase effect is produced on the heart-apex when the successive shocks are thrown in at six seconds' intervals; whereas, in the case of the Medusæ, I found that the more rapidly the shocks are thrown in, the more marked is the beneficial influence of each contraction on its successor,—that is to say, up to the time when the interval between the successive shocks is not so short as to give rise to incipient tetanus by summation of contractions (when, of course, the staircase effect is lost in that of tetanus), progressive shortening of the intervals between successive stimuli has the effect of progressively diminishing the number of steps in a staircase, and this by increasing the difference between the heights of the successive steps. Again, if a staircase has been built up by shocks thrown in at seconds' intervals, and the rate of stimulation be now suddenly changed to six seconds' intervals, the maximum level of the tracing that was obtained by the more rapidly succeeding shocks may be observed slightly to sink. This greater tolerance of the medusoid than of the cardiac tissue towards rapidity in the succession of stimuli is doubtless connected with the same differences in their constitution that leads to what I have called the shorter memory of the one than of the other. In this connexion it may also be stated that a staircase has more steps in it if caused by a weak than if caused by a strong current (compare figs. 1 & 4), and that if the strength of the current be suddenly increased after the maximum level of a staircase has been attained by a feeble current, this level admits of being slightly raised (see fig. 4).

Fig. 4.



Lastly, the cases of the heart and of *Aurelia* appear not quite to coincide in one other particular; for I find that in *Aurelia* the staircase action is so pronounced, that a stimulus which at the bottom of a staircase is of less than minimal intensity, is able, at the top of a staircase, to give rise to a contraction of very nearly maximum intensity. That is to say, by employing an induction stimulus of slightly less than minimal intensity in relation to the original irritability of the tissue, no response is given to the first two or three shocks of a series; but at the third or fourth shock a slight response is given, and from that point onward the staircase is built up as usual. This was the case in the experiment of which fig. 1 is a record, no response having been given to the first two shocks. Now, as Dr. Bowditch does not state that a staircase may thus be built up by means of stimuli which at the beginning of the process are of less than minimal intensity, I infer that in the case of the heart this peculiarity is not observable.

With regard to this interesting staircase action, two questions naturally present themselves. In the first place, we are anxious to know whether the arousing effect which is so conspicuous in a staircase series is due to the occurrence of the previous *stimulations*, or to that of the previous *contractions*; and, in the next place, we should like to know whether, during the *natural* rhythm of the tissue, each contraction exerts a beneficial influence on its successor, analogous to that which occurs in the case of contractions that are due to *artificial* stimuli. Now both these questions admit of being answered by experiments on the Medusæ much more satisfactorily than they do by experiments on the heart—if, indeed, it is possible to answer them in the latter case at all. To answer the first of these questions, therefore, I built up a staircase in the ordinary way, and then suddenly transferred the electrodes to the opposite side of the disk from that on which they rested while constructing the staircase. On now throwing in another shock at this part of the contractile tissue so remote from the part previously irritated, the response resembled in all respects the one previously given, *i. e.* it was a maximum response. Similarly, if the electrodes were transferred in the way just described, not after the maximum effect had been attained, but at any point during the process of constructing a staircase, the response given to the next shock was of an intensity to make it rank as the next step in the staircase. Hence, shifting the position of the electrodes in no wise modifies the peculiar effect we are considering; and this fact conclusively proves that the effect is a general one pervading the whole mass of the contractile tissue, and not confined to the locality which is the immediate seat of stimulation. Nevertheless this fact does not tend to prove that the staircase-effect depends on the process of contraction as distinguished from the process of stimulation, because the wave of the former process must always precede that of the latter. But, on the other hand, in this connexion it is of the first importance to remember a fact recently stated, viz. that a current which at the beginning of a series of stimulations is of slightly less than minimal intensity, presently becomes of minimal, and eventually of much more than minimal intensity—a staircase being thus built up of which the first observable step (or contraction) only occurs in response to the second, third, or even fourth shock of the series. This fact, of course, clearly tends to show, or rather conclusively proves, that the staircase effect, at any rate at its commencement, depends on the process of stimulation as distinguished from that of contraction; for it is obvious that the latter process cannot play any part in thus constructing what we may term the invisible steps of a staircase.

To answer the second of the above questions, I placed an *Aurelia* with its concave surface uppermost, and removed seven of its lithocysts; I then observed the spontaneous discharge of the remaining one, and found it to be conspicuous enough—that after the occurrence of one of the natural pauses (if this were of sufficient duration) the first contraction was feeble, the next stronger, the next still stronger, and so on, till the maximum was attained. This natural staircase action admits of being very prettily shown in another way. If a tolerably large *Aurelia* is cut into a spiral strip of small width and great length, and if all the lithocysts are removed except one at one end of the strip, it may be observed that, after the occurrence of a natural pause, the first discharge only penetrates perhaps about a quarter of the length of the strip, the next discharge penetrates a little further, the next further, and so on, till finally the contractile waves pass from end to end. On now removing the ganglion, waiting a few minutes, and then stimulating with successive induction-shocks, the same progressive penetration is observable as that which previously took place with the ganglionic stimulation. Lastly, the identity of natural and artificial staircase action may be placed beyond all doubt by an experiment in which the effects of induction-shocks and of ganglionic discharges are combined. To accomplish this, all the lithocysts save one are removed, and a staircase is then built up in the ordinary way by successive induction-shocks. It will now occasionally happen that the ganglion originates a discharge during the process of constructing the staircase; and when this does happen, it may invariably be observed that the contraction due to the natural stimulus is just of the proper intensity to take its place in the staircase which is being built up by the artificial stimuli, and this at whatever point the natural contraction happens to come in*.

Of course, in all these experiments with natural discharges, care must be taken to choose a lithocyst of a potency adapted to the object in view. This condition, before it is satisfied, often requires a number of trials to be made. The experiment with the spiral strip may prove particularly tedious in this respect, as it is difficult to obtain such a strip presenting the proper degree of resistance in relation to the terminal ganglion †.

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* I may here observe that if an *Aurelia with all its lithocysts removed* happens, during the construction of a staircase by stimuli, to give a spontaneous contraction, this also ranks as the next step in the staircase, just in the same way as does a spontaneous contraction proceeding from a single remaining lithocyst. This proves that "secondary contractions" are not really, as Dr. EIMER describes them, more "feeble and inefficient" than primary ones; and that the only reason why they usually appear to be so is because they usually occur singly with a long interval between them, so that each secondary contraction usually corresponds with the first step of a staircase. But when, as sometimes happens, two or three secondary contractions follow one another at the rate observable in ordinary swimming, the second contraction is stronger than the first, and the third than the second, and so on, just as in the construction of a staircase by a single remaining lithocyst after the occurrence of a natural pause of adequate duration. It is almost needless to state that, as this effect of the summation of stimuli had not been observed at the time when Dr. EIMER published his paper, he is not to be accused of any inaccuracy in his observations because he described the "secondary contractions" as "feeble and inefficient."

† [As the present communication was originally accepted by the Royal Society, there here followed a lengthy subsection on "Artificial Rhythm." A condensed epitome of the main facts which were detailed in that subsection may be found in the already published abstract of this paper in the Proceedings of the Royal Society. But as the new theory of ganglionic action which it is the main object of this passage to disclose appears to me a theory of sufficient importance to demand its final publication in as complete a form as possible, I have sought and obtained the permission of the Royal Society to withdraw this subsection from the proof. And, as a further reason for postponing publication of the passage on "Artificial Rhythm," I may state that, as this paper is of undue length, it seemed to me desirable, if possible, to relegate a portion of it to my next paper; and I find that this subsection is the most suitable part temporarily to suppress, because the facts of which it treats form in themselves a connected body distinct from all the other facts which are detailed in the rest of this communication; so that while their omission from the present paper does not entail any further modification of the latter, they will admit of being introduced *en masse* in my next paper. When, therefore, I shall have completed the further experiments which in this connexion I have devised, I shall hope to communicate to the Royal Society, in a single connected series, all the facts and inferences which bear upon this subject.—1878.]

(*c*) *Temperature.*—Returning now to the period of latent stimulation in *Aurelia aurita,* we have seen how profoundly this period is modified by the staircase effect we have just been considering. We have next to contemplate the influence of temperature on the latent period and on the character of the contractions. But before stating my results, I may observe that in all my experiments in this connexion I changed the temperature of the Medusæ by drawing off the water in which they floated with a siphon, while at the same time I substituted water of a different temperature from that which I thus abstracted. In this way, without modifying any of the other conditions to which the animals were exposed, I was able to observe the effects of changing the temperature alone.

With regard to the effect of temperature on the latent period of stimulation, the following Table, setting forth the results of one among several experiments, explains itself.

Period of latent stimulation of the deganglionated tissues of *Aurelia aurita* as affected by temperature.

Temperature of water (Fahr.).	Period of latent stimulation.
70°	$\frac{1}{5}$ second
50°	$\frac{1}{3}$ second
35°	$\frac{2}{5}$ second
20°	$\frac{1}{2}$ second

In the case of each observation several shocks were administered before the latent period was taken, in order to decrease this period to its minimum by the staircase action. When this is not done, the latent period at 20° may be as long as $1\frac{1}{5}$ second; but soon after this irritability disappears.

The extraordinary sluggishness of the latent period at very low temperatures is fully equalled by the no less extraordinary sluggishness of the contraction.

I may as well state here that in water at all temperatures, within the limits where responses to stimuli are given at all, the staircase action admits of being equally well produced. But in order to procure the maximum effect for any given temperature, the rate at which the successive stimuli are thrown in must be quicker in warm than in cold water; for in cold water the latent period and the subsequent contraction are both so prolonged, that summation of the contractions is produced by employing such intervals between the successive shocks as yield the maximum staircase effect in warmer water.

(B) *Tetanus.*—The allusion just made to summation of contractions leads us next to the subject of tetanus. As stated in my previous paper, some of the Medusæ, when subjected to faradaic stimulation, respond, not by a single prolonged tetanus, but by a flurried series of rapidly alternating contractions and relaxations. It must now be added that these remarks apply especially to *Sarsia*; for I find that in the case of *Aurelia* tolerably strong faradization does cause a more or less well-pronounced tetanus. The continuity of the spasm is, indeed, often interrupted by momentary and partial relaxations. These interruptions are the more frequent the weaker the current; so that at a certain strength of the latter, the tetanus is of a wild and tumultuous nature; but with strong currents the spasm is tolerably uniform. That in all cases the tetanus is due to summation of contractions, may be very prettily shown by the following experiment. An *Aurelia* is cut into a spiral strip, and all its lithocysts are removed; single induction-shocks are then thrown in with a key at one end of the strip—every shock, of course, giving rise to a contractile wave. If these shocks are thrown in at a somewhat fast rate, two contractile waves may be made at the same time to course along the spiral strip one behind the other; but if the shocks are thrown in at a still faster rate, so as to diminish the distance between any two successive waves, a point soon arrives at which every wave overtakes its predecessor; and if several waves be thus made to coalesce, the whole strip becomes thrown into a state of persistent contraction.

In this way sustained tetanus, or single contractile waves, or any intermediate phase, may be instantly produced at pleasure. In such experiments, moreover, it is interesting to observe that, no matter how long the strip may be, whatever disturbances are set up at one end are faithfully transmitted to the other. For instance, if an *Aurelia* be cut into the longest possible strip with a remnant of the disk left attached at one end—as represented by Plate 33 in my previous paper—then all the peculiar time relations between successive contractions which are intentionally caused by the experimenter at one end of the strip, are afterwards accurately reproduced at the other end of the strip by the remainder of the disk. Now as this fact is observable however complex these time relations may be, and however rapidly the successive stimuli are thrown in, I think it is a point of some interest that these complicated relations among rapidly succeeding stimuli do not become blended during their passage along the thirty or forty inches of contractile tissue. The fact, of course, shows that the rate of transmission is so identical in the case of all the stimuli originated, that the sum of the effects of any series of stimuli is delivered at the distal end of the strip with all its constituent parts as distinct from one another as they were at starting from the proximal end of the strip.

IV. SECTION.

In this division I shall treat of all the evidence I have been able to collect regarding the distribution of nerves and the physiological character of the contractile tissues in the various species of Medusæ which have this year fallen within my observation. By the word "nerves" here I mean certain physiologically differentiated tracts of tissue which either stimulation or section prove to perform the function of conveying impressions to a distance; and by "physiological character of the contractile tissues" I mean the character of these tissues in respect of the degree in which the nervous element shows itself to be physiologically differentiated from the muscular element.

§ 1. *Distribution of Nerves in* Sarsia. (A) *Reflex Action.*—The occurrence of reflex

Fig. 5.

Sarsia, nat. size.

action in *Sarsia* is of a very marked and unmistakable character. I may begin by stating that when any part of the internal surface of the bell is irritated, the polypite responds; but as there is no evidence of ganglia occurring in the polypite, this cannot properly be regarded as a case of reflex action. But, now, the converse of the above statement is likewise true, viz. that when any part of the polypite is irritated, the bell responds; and it is in this that the unequivocal evidence of reflex action consists. Of course the objection at once arises, if response by the polypite to irritation of the bell is explained by the simple supposition of there being a continuous or common excitability pervading these organs, why may not the converse case be explained by the same supposition? But this objection is met by a fact that makes an essential difference between the two cases, viz. that while the sympathy of the polypite with the bell is not in the least impaired by removing the marginal ganglia of the latter, the sympathy of the bell with the polypite is by this operation entirely destroyed.

We have thus very excellent demonstration of the occurrence of reflex action in the Medusæ; and as this is such an admirable instance of it, I shall describe it a little more in detail. Suppose the experimenter to have chosen a vigorous specimen of *Sarsia* with its polypite well retracted, and suppose him to wait for the occurrence of one of the natural pauses in the swimming-motions; if he then suddenly seizes the end of the polypite with a pair of forceps, taking care not to touch any part of the bell, the polypite is instantly retracted even further than it was before, and immediately afterwards the tentacles are drawn up while the bell begins to make violent efforts to escape. But if the experimenter chooses a specimen having its polypite somewhat relaxed, and, after seizing the free end of this organ, follows it up with the forceps during its retreat, he may observe, if he has been dexterous enough not to allow the slightest pull to occur at the point of suspension of the polypite, that the bell does not respond to the stimulus however severe this may have been. By repeating this experiment a number of times, it becomes evident that the reflex action occurs, not between the marginal ganglia and every part of the polypite, but between the marginal ganglia and the point of the bell from which the polypite is suspended. In other words, the marginal ganglia do not respond to any amount of disturbance that may be set up in the polypite itself, provided the contraction of this organ does not cause it to pull upon the point of its attachment with the bell. Now the high degree of sensitiveness shown by the ganglia to the smallest degree of such traction is quite as remarkable as is their lack of sensitiveness to disturbances going on in the polypite*. Consequently

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* When a specimen of *Sarsia* is taken out of the water, laid on an object-glass, slit open along one side, and spread out flat with its inner surface uppermost, and then has its polypite turned back as represented in the figure on p. 697, it may be observed that the bell often responds to irritations of the polypite, even though care be taken to release the latter from the forceps before it is allowed to pull on its point of suspension. This may possibly be due to the fact of the polypite meeting with some friction on the glass during its process of retraction, which friction must cause it to pull very slightly on its point of suspension. At any rate, if it is due to nervous connexions between the substance of the polypite and the marginal ganglia, the conducting-power, as estimated by the response of the distant tissue, is much less in the direction we are now considering than it is in the converse

we have in this a very conclusive proof of the truly reflex character of the action we are considering; for after the removal of the marginal ganglia, the mutilated nectocalyx, although it remains most keenly sensitive to the gentlest stimulation applied to its own irritable tissues, will allow itself to be dragged through the water as rapidly as possible by means of the polypite without giving a single contraction. But, in this experiment, if the smallest atom of marginal tissue be left *in situ*, the ganglion cell or cells contained in that atom will suffice to preserve the reflex action. In this case, however, the responses are neither so ready nor so sure as they are when a larger amount of ganglionic matter is concerned in the process.

(B) *Nervous connexions between the Tentacles of* Sarsia.—When one of the four tentacles of *Sarsia* is very gently irritated, it alone contracts. If the irritation be slightly stronger, all the four tentacles, and likewise the polypite, contract. If one of the four tentacles be irritated still more strongly, the bell responds with one or more locomotor contractions. If in the latter case the stimulus be not too strong, or, better still, if the specimen operated on be in a non-vigorous or in a partly anæsthesiated state, it may be observed that a short interval elapses between the response of the tentacles and that of the bell. Lastly, the polypite is much more sensitive to a stimulus applied to a tentacle, or to one of the marginal bodies, than it is to a stimulus applied at any other part of the nectocalyx*.

These facts clearly point to the inference that nervous connexions unite the tentacles with one another and also with the polypite—or, perhaps more precisely, that each marginal body acts as a coordinating centre between nerves proceeding from it in four directions, viz. to the attached tentacle, to the margin on either side, and to the polypite. This, it will be observed, is the distribution which HAECKEL describes as occurring in *Geryonia*, and SCHULTZ as occurring in *Sarsia*†. It is further the distribution to which my explorations by stimulus of last year would certainly point. But, in order to test the matter still more thoroughly, I tried the effects of section in destroying the

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direction, which is to be immediately considered in the text (B). And as it seems probable that in such a simple case as this the same nerves would serve to convey impressions in both directions, perhaps the most judicious view to take of the difference between the degree of sensitiveness displayed by the polypite when a tentacle is injured, and that displayed by the tentacles when the polypite is injured, is to suppose that in the former case a feeble ganglionic discharge is added to the stimulus, which discharge would of course be absent in the latter case.

* These facts were partly ascertained by the method of experimentation described in the last footnote. It must here be added that in conducting such experiments the greatest care must be taken not to agitate the drops of water in which the animal is contained. The disturbance caused by capillarity on introducing the needle-point into these few drops of water is sufficient to cause the tentacles and polypite to contract, even though they be a long way off. I therefore used two object-glasses separated by a small interval, so as to break the continuity of the water between the point of irritation and that of the tissue whose physiological connexions with such point I wished to ascertain.

† 'Ueber den Bau von *Syncoryne Sarsia* und der zugehörigen Medusa *Sarsia tubulosa*,' by Dr. F. E. SCHULTZ (Leipzig, 1873). My attention has been directed by my friend Professor LANKESTER to this admirable little monograph on *Sarsia*. The histological elements to which the author ascribes a nervous function are quite familiar to me, and I think that at any rate some of his views with regard to them are probably correct.

physiological relations which I have just described. These effects, in the case of the tentacles, were sufficiently precise. A minute radial cut introduced between each pair of adjacent marginal bodies—there being thus four such cuts in all—as a rule completely destroyed the physiological connexion between the tentacles; or if, as in some cases, such connexions were not completely destroyed by this operation, they were at least conspicuously impaired. If only three marginal cuts were introduced, the sympathy between those two adjacent tentacles between which no cut was made continued unimpaired, while the sympathy between them and the other tentacles was either destroyed or greatly impaired. In all cases where the sympathy between tentacles was not wholly destroyed, but merely impaired, the impairment showed itself in this way. Whereas before the introduction of the radial cuts the slightest nip of one tentacle caused an instantaneous response on the part of all the tentacles, after the operation such a stimulus applied to one tentacle would perhaps cause no effect at all on the other tentacles, though on gently *pulling* one of the tentacles the others would retract at the same time as the bell, in response to this severe stimulus, would give a locomotor contraction. And as, before mutilation, the tentacles may be observed to respond to such a stimulus an exceedingly short time *before* the bell, I conclude that, after mutilation, the *time* required for the stimulus to pass round the margin from one tentacle to the others is increased. Hence the cases in which the sympathy between the tentacles is not wholly destroyed by the four minute radial cuts are, I think, to be regarded as cases in which those quadrants of the margin which have been physiologically separated from beneath nevertheless continued united to each other from above. And this junction I conceive to be effected by means of nerve-loops which are composed of smaller fibres than those of the margin, and which may be supposed to join the artificial quadrants of the margin by traversing the muscular tissue of the bell in all directions above the level that is reached by the short radial cuts*.

(C) *Nervous connexion between the Tentacles and the Polypite of* Sarsia.—Having obtained such definite results in the case of the tentacles, I expected to do the same in the case of the polypite. Accordingly I began by severing one of the nutrient tubes across its diameter, and then nipping the tentacle at the end of that tube. The polypite responded as before. Knowing from my previous experiments that the stimulus escaped round the margin of the nectocalyx, I thought it probable that the reason why the polypite now responded was because the stimulus found its way up the three unsevered tubes†. I therefore divided all the four nutrient tubes; but the polypite still continued to respond to a stimulus applied to any of the tentacles. Next, in the same specimen, I made two radial cuts, one on each side of a marginal body, and then irritated the tentacle attached to that body; the polypite contracted as before. Lastly, I treated the other three

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* In the foregoing and subsequent descriptions, by "short radial cuts" I mean cuts which are only just long enough to sever the tissues of the extreme margin.

† For the sake of brevity I speak of the nutrient tubes as themselves the excitable tracts, although anatomically, no doubt, these tracts are distinct from the nutrient tubes.

bodies in the same way, and the polypite still contracted when I irritated any one of the four tentacles.

Now, as these experiments were repeated on a number of specimens—the radial tubes being in some cases divided at the base and in others at the apex of the cone—and as the experiments always yielded the same result, I cannot doubt that the nervous connexions between the margin and the polypite are in no wise restricted to the course of the nutrient tubes.

(D) *Nervous connexion between the Nectocalyx and the Polypite of* Sarsia.—The polypite of *Sarsia* often shows itself more sensitive than does the nectocalyx to stimuli applied to the latter. Moreover, as already stated, the polypite of *Sarsia* is much more sensitive to stimuli applied to a tentacle or to a marginal body of the nectocalyx than to stimuli applied at any other part of that organ. As regards the general surface of the irritable tissue of the nectocalyx, the polypite is more sensitive to stimuli applied at some parts than it is to stimuli applied at other parts; but in different specimens there is no constancy as to the position occupied by these excitable tracts. I think, however, that, as a general rule, stimulation of the parts of the bell nearest to the insertion of the polypite causes the latter to respond more readily than does stimulation of the parts more distant. Over all parts of the bell, however, during such explorations, sudden variations in the responses of the polypite are observable according as the electrodes are shifted a millimetre or less one way or the other; in some cases, indeed, even a few hair-breadths will make all the difference between an active response and no response. Similarly with mechanical stimulation.

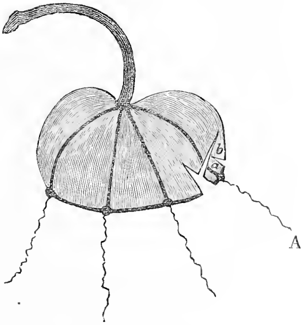
From these explorations, therefore, it would seem that the polypite of *Sarsia* is in very intimate communication with the nectocalyx, and especially with the marginal bodies, by means of a nervous plexus pervading the excitable tissues of the latter, but that the principal nervous tracts by which this communication is effected are not constant as to their position in different individuals.

(E) *Character of the Excitable Tissues of* Sarsia.—It will be remembered that in my former paper I described several experiments which were designed to test the amount of section that the excitable tissues of the Medusæ would tolerate without suffering loss of their "physiological continuity." This term, as previously used by Dr. FOSTER, implies such a condition of excitable tissue that its various parts are in physiological connexion with one another, whether by means of contractile elements (protoplasm, muscle) or conductile elements ("lines of discharge," nerves). Therefore, in my last paper, I employed this term to designate such a condition on the part of contractile tissues as admits of an uninterrupted passage along their substance of what I called "contractile waves." But having in the present paper an important distinction to draw, I must discard the *generic* term "*physiological* continuity," and substitute for it two *specific* terms, viz. "*contractional* continuity" and "*excitational* continuity." By "contractional continuity" I shall wish to be understood such a condition of contractile tissue as admits of the uninterrupted passage of contractile waves; while by "excitational continuity" I shall wish to be understood such a condition of the contractile tissues as admits of one part responding to stimuli applied at another part, *whether or not contractile waves are able to pass along the intervening parts*[*].

With this distinction clearly understood, I shall now proceed to describe some experiments which were devised in order to test the amount of section that the excitable tissues of *Sarsia* would endure without suffering loss of their excitational continuity. The experiments detailed in the foregoing subsections were really experiments of this kind; but such experiments derive a special interest when conducted on the general contractile-sheet of the Medusæ. They do so because, as we have already seen, there is here decided indication of a nervous plexus; and it will be remembered that some of the most interesting questions with which my last paper was concerned, and which it left unanswered, were, first as to the presence of such a nerve-plexus, and next as to the extent in which, if present, it was differentiated from the muscular element.

Now, as stated in my former paper, the contractile tissues of *Sarsia* will endure very severe forms of radial and spiral section without suffering loss of their contractional continuity; but I find that, as a rule, their tolerance is not nearly so great as regards the maintenance of their excitational continuity. To save space I will only give one average case. The figure (fig. 7) represents a specimen of *Sarsia* spread flat on an object-glass after having been slit open along one side. The experiment consisted in determining the

Fig. 6.



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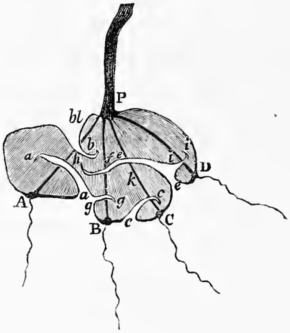
[*] I choose these terms, because they are the only ones I can think of to express the meaning I wish to convey. There is evidently a broad distinction—and a very important one for the purposes of this paper—between the passage of a contractile wave from the point of stimulation A to the point of destination B, and the passage of an unseen molecular wave from A to B, the existence of which is only rendered apparent by the response at B to the stimulus applied at A. And as it is further evident that this distinction has reference to the most fundamental quality wherein the function of nerve is distinguished from that of muscle, viz. the power of setting up responsive contractions at a distance from the seat of irritation, it will be understood that by the term "excitational continuity" I intend to denote the first indications we can perceive in the animal kingdom of the distinguishing function of nerve-tissue. The terms which I have employed are not to be found in our standard dictionaries; but their introduction is justified by the analogy of such words as "emotional," "sensational," &c.

severity of section which the nervous connexions between the nectocalyx and the polypite would endure without being wholly destroyed; and, as these connexions are always most intimate between the polypite and the marginal bodies or the tentacles, the points chosen for irritation were the latter. If the reader will endeavour to imagine all the curved sections represented in fig. 7 to be absent, as they are in the left-hand portion of fig. 6, he will understand that on irritating the tentacle marked A, the polypite P immediately responded. On now introducing the incision marked *a a*, the polypite still continued its response to stimuli applied at A; but on adding the cut marked *b b,* such responses ceased. The connexions, however, between C and P were of course still intact, and they were not destroyed by introducing the cut *c c*; but they were destroyed on adding the cut *e e*. Nevertheless, on now irritating the divided nutrient tube in any point of its division *k*, the polypite still responded; accordingly the cut *e e* was continued to *f*, and this had the effect of destroying the connexions between *k* and P. Excitational continuity still subsisted between B and P; therefore the cut *g g* was introduced: responses still being given, the cut *e e f* was produced to *h*, when the connexions between P and B were destroyed. Lastly, on irritating D, P responded; but, on introducing the cut *i i*, responses ceased.

From this instance it will be seen that, even in the same individual, there are considerable differences in the degree of tolerance manifested towards section by different homologous parts; but a number of experiments of this kind have satisfied me that in *Sarsia* the loss of excitational continuity between the polypite and bell generally supervenes much earlier than does that of contractional continuity between different parts of the bell. The preceding sketch, fig. 6, represents the most extreme instance of tolerance as regards excitational continuity that I have observed in the case of *Sarsia.* In this instance the polypite continued to respond to stimuli applied at the tentacle marked A after the line of junction-tissue marked *a b* had been reduced to the thickness of one fortieth of an inch.

Fig. 7.



(F) *Stimulation of the Convex Surface of the Nectocalyx of* Sarsia.—I have this year tried the effects of stimulating the convex, or external, surface of *Sarsia*, in consequence of its having been suggested to me that, as this is the surface which is naturally the most liable to come in contact with foreign bodies during the natural swimming-motions of the animal, it is to be expected that this part of the animal should receive a nerve-supply from the marginal ganglia. But although this is what we might reasonably expect to be the case, as a matter of fact it is not so. Even strong faradaic stimulation applied to any part of the external surface of the nectocalyx fails to produce the slightest response, provided the current employed is not so strong as to cause escape into the internal surface. That the responses are due to such an escape of the current when over-strong currents are used, may be proved by using weaker currents and progressively pushing the needle-point terminals further and further into the gelatinous substance of the nectocalyx. According to the strength of the current employed, a point will sooner or later be reached at which the escape finds its way to the internal surface, and thereupon causes the bell to contract.

In these experiments it is desirable to use electrical or chemical stimulation, because, when the animal is suspended in water, it is impossible to stimulate the external surface of the bell mechanically without causing the whole animal to move slightly through the water, and to any such disturbance the marginal ganglia are wonderfully sensitive. If, however, it is desired to give mechanical stimuli a fair trial in this connexion, the experiment may be conducted by placing a vigorous specimen of *Sarsia* on a grooved object-glass, when it may be observed that on touching the apex of the bell with a needle no response is given, unless the touch is strong enough to cause the whole animal to slide along the glass. Similarly, if one side of the bell be touched, no response is given until the pressure exerted is sufficiently great to force the subjacent *internal* surface against that of the opposite side.

I have here chosen *Sarsia* for special description, because it is the most active of the Medusæ which have come under my observation. In the case of the covered-eyed Medusæ, no such precautions as those which I have just described are necessary; for the massive nature of their swimming-organs gives them a considerable amount of inertia as they float in the water, and pretty severe mechanical stimulation may be applied to their external surfaces without causing any displacement of the animal.

These experiments, then, conclusively prove that, in the case of all the Medusæ I have examined, irritability of the swimming-organ is exclusively confined to the thin layer of contractile tissue which everywhere constitutes the internal surface of that organ.

§ 2. *Distribution of Nerves in* Tiaropsis indicans. (A) *Reflex action* (?).—We have seen that in *Sarsia* reflex action obtains between the polypite and the bell; we shall now see that in *Tiaropsis indicans* something resembling reflex action obtains between the bell and the polypite. The last-named species is a new one which I have described elsewhere*, and I have called it "*indicans*" from a highly interesting and important peculiarity of function that is manifested by its polypite. The Medusa in question

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* "New Species, Varieties, and Monstrous Forms of Medusæ," Journ. Linn. Soc., Zool. vol. xii. p. 524.

measures about $1\frac{1}{2}$ inch in diameter, and is provided with a polypite of unusual proportional size, its length being about $\frac{5}{8}$ of an inch and its thickness being also considerable. Now if any part of the nectocalyx be irritated, the following series of phenomena takes place. Shortly after the application of the stimulus, the large polypite suddenly contracts, the appearance presented being that of an exceedingly rapid crouching movement. The crouching attitude in which this movement terminates continues for one or two seconds, after which the organ begins gradually to resume its former dimensions. Concurrently with these movements on the part of the polypite, the portion of the nectocalyx which was irritated has been gradually bending inwards towards the polypite, and by the time that the latter has again become fully extended, the portion of the nectocalyx in question has bended inwards as far as it is able. The polypite now begins to deflect itself towards the bent-in portion of the nectocalyx; and this deflection continuing with a somewhat rapid motion, the extremity of the polypite is eventually brought, with unerring precision, to meet the in-bent portion of the nectocalyx. I here introduce a drawing of twice life-size to render a better idea of this *pointing* action by the polypite to a seat of irritation located in the bell (see Plate 30. fig. 1). It must further be stated that in the unmutilated animal such action is quite invariable, the tapered extremity of the polypite never failing to be placed on the exact spot in the nectocalyx where the stimulation is being, or had previously been, applied. Moreover, if the experimenter irritates one point of the nectocalyx, with a needle or a fine pair of forceps for instance, and while the polypite is applied to that point he irritates another point, then the polypite will leave the first point and move over to the second. In this way the polypite may be made to indicate successively any number of points of irritation; and it is interesting to observe that when, after such a series of irritations, the animal is left to itself, the polypite will subsequently continue for a considerable time to visit first one and then another of the points which have been irritated. In such cases it usually dwells longest and most frequently on those points which have been irritated most severely.

I think the object of these movements is probably that of stinging the offending body by means of the urticating cells with which the extremity of the polypite is armed. But, be the object what it may, the fact of these movements occurring is a highly important one in connexion with our study of the distribution of nerves in Medusæ; and the first point to be made out with regard to these movements is clearly as to whether or not they are of a truly reflex character. Accordingly I first tried cutting off the margin and then irritating the muscular tissue of the bell; the movements in question were performed exactly as before. These movements, therefore, supply an additional instance of the rule laid down in my former paper, viz.: "Every Medusa, when its centres of spontaneity have been removed, responds to a single stimulation by once performing that action which it would have performed in response to that stimulation had its centres of spontaneity still been intact." Now the fact of this rule holding good in this case opens up an interesting subject for our contemplation. It will be observed that, so far as the movements in question are themselves concerned, they are undistinguishable from an ordinary reflex act—such, for instance, as we have observed to occur in *Sarsia*. But we are accustomed to hear that for the occurrence of a truly reflex act, the presence of ganglionic or central nerve-matter is requisite. Yet in the case before us we have exceedingly precise action of an apparently reflex kind, which is nevertheless quite independent of the only ganglia that can be shown to occur in the organism. In view of this fact, therefore, I was led to think it probable that the reflex centres I was in search of might be seated in the polypite. Accordingly I cut off the polypite, and tried stimulating its own substance directly. I found, however, that no matter how small a portion of this organ I used, and no matter from what part of the organ I cut it, this portion would do its best to bend over to the side which I irritated. Similarly, no matter how short a stump of the polypite I left in connexion with the nectocalyx, on irritating any part of the latter, the stump of the polypite would deflect itself towards that part of the bell, although of course, from its short length, it was unable to reach it. Hence there can be no doubt that every portion of the polypite—down, at least, to the size which is compatible with conducting these experiments—is independently endowed with the capacity of very precisely localizing a point of irritation which is seated either in its own substance or in that of the bell*.

We have here, then, a curious fact, and one which it will be well to bear in mind during our subsequent endeavours to frame some sort of a conception regarding the nature of these primitive nervous tissues. The localizing function which is so very efficiently performed by the polypite of this Medusa, and which, if any thing resembling it occurred in the higher animals, would certainly have definite ganglionic centres for its structural correlative, is here shared equally by every part of the exceedingly tenuous contractile tissue that forms the outer surface of the organ. I am not aware that such a diffusion of ganglionic function has as yet been actually proved to occur in the animal kingdom; but I can scarcely doubt that future investigation will show such a state of things to be of common occurrence among the lower members of that kingdom†.

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* It must here be added that the same remark may be made with reference to the nectocalyx; for however small a part of this organ is excised, it will bend inwards on being irritated. This case, however, is not nearly so remarkable as is that of the polypite, because, in the first place, there is no choice of *direction* to be exercised, and, in the next place, this bending inward of the nectocalyx upon irritation occurs more or less in the case of all the species of this genus, and is no doubt identical with the spasmodic movement exhibited under similar circumstances by *Staurophora laciniata*, to which attention was directed in my last paper, and which will form the subject of the next section of the present one. The only respect in which, so far as I can see, the inflexion of the seat of irritation in *Tiaropsis indicans* is distinguishable from the spasm of some other species of *Tiaropsis* and of *Staurophora* consists in its being of a local instead of a general character.

† The only case I know which rests on direct observation, and which is at all parallel to the one above described, is the case of the tentacles of *Drosera*. Mr. DARWIN found, when he cut off the apical gland of one of these tentacles together with a small portion of the apex, that the tentacle thus mutilated would no longer respond to stimuli applied directly to itself. Thus far the case differs from that of the polypite of *Tiaropsis indicans*, and in respect of localization of coordinating function resembles that of ganglionic action. But Mr. DARWIN

(B) *Nervous connexions between the Nectocalyx and Polypite of* Tiaropsis indicans.—(*a*) In the last subsection we have virtually seen that if we have regard to *function* alone the localizing action of the polypite of *Tiaropsis indicans* may properly be termed a reflex action, but that if we have regard to the *structure* or mechanism by which this action is performed we cannot properly so term it. We have virtually seen this, because we have seen that at least one element essential to the mechanism whereby reflex action is effected—the element, namely, of a definite localized ganglion—is here absent. The next question, therefore, which arises is, as to whether the other elements which are usually considered essential to the performance of reflex action are likewise absent—the elements, namely, of afferent and efferent nerves. Now I felt so persuaded of the difficulty of answering this question by means of histological research, that hitherto I have postponed the latter, and, giving priority to the method of section, I obtained the most definite results with regard to function which it is possible for this or for any other method to yield.

(*b*) Bearing in mind that in an unmutilated *Tiaropsis indicans* the polypite invariably localizes with the utmost precision any minute point of irritation situated in the bell, the significance of the following fact is unmistakable, viz. that when a cut is introduced between the base of the polypite and the point of irritation in the bell, the localizing power of the former as regards that point in the latter is wholly destroyed. For instance, if such a cut as that represented at *a* (see Plate 30. fig. 1) be made in the nectocalyx of this Medusa, the polypite will no longer be able to localize the seat of a stimulus applied below that cut, as, for instance, at *b*. Now, having tried this experiment a number of times and having always obtained the same result, I conclude that the nervous connexions between the bell and the polypite which render possible the localizing action of the latter, are connexions the functions of which are intensely specialized, and the distribution of which is radial.

(*c*) So far, then, we have highly satisfactory evidence of tissue tracts performing the function of afferent nerves. But another point of great interest arises. Although in the experiment just described the polypite is no longer able to *localize* the seat of stimulation in the bell, it nevertheless continues able to perceive, so to speak, that

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also found that such a "headless tentacle" continued to be influenced by stimuli applied to the glands of neighbouring tentacles—the headless one in that case bending over in whatever direction it was needful for it to bend in order to approach the seat of stimulation. This shows that the analogue of ganglionic function must here be situated in at least more than one part of a tentacle; and I think it is not improbable that, if trials were expressly made, this function would be found to be diffused throughout the whole tentacle.

Of course in here using the term "analogue of ganglionic function," I do not commit myself to supposing that any thing resembling ganglionic *structure* occurs in any part of *Drosera*. On the contrary, I have here alluded to the case of *Drosera* just because it is a case in which we cannot expect to meet with any such structure, and therefore a case confirmatory of the views to which, I think, my own investigations point—viz. that before the functions of ganglion are localized in any characteristic histological structure, they occur in a more or less diffused manner in primitive contractile tissues. But as yet I have not had time to give the histology of *Tiaropsis indicans* the attention which it deserves.

stimulation is being applied in the bell *somewhere*; for every time any portion of tissue below the cut *a* is irritated, the polypite actively dodges about from one part of the bell to another, applying its extremity now to this place and now to that one, as if seeking in vain for the offending body. If the stimulation is persistent, the polypite will every now and then pause for a few seconds, as if trying to decide from which direction the stimulation is proceeding, and will then suddenly move over and apply its extremity, perhaps, to the point that is opposite to the one which it is endeavouring to find. It will then suddenly leave this point and try another, and then another and another, and so on, as long as the stimulation is continued. Moreover, it is important to observe that there are *gradations* between the ability of the polypite to localize correctly and its inability to localize at all—these gradations being determined by the circumferential distance from the end of the cut at which the stimulus is applied, and also by the radial distance between the cut and the point of stimulation. For instance, in figure 1 suppose a cut AB, $\frac{1}{4}$ of an inch long, to be made pretty close to the margin and concentric with it, then a stimulus applied at the point *c*, just below the middle point of A B, would have the effect of making the polypite move about to various parts of the bell, without being able in the least degree to localize the seat of irritation. But if the stimulus be applied at *d*, the polypite will probably be so far able to localize the seat of irritation as to confine its movements, in its search for the offending body, to perhaps the *quadrant* of the bell in which the stimulation is being applied. If the stimulation be now applied at *e*, the localization on the part of the polypite will be still more accurate; and if applied at *f* (that is, *almost* beneath one end of the cut AB) the polypite may succeed in localizing quite correctly.

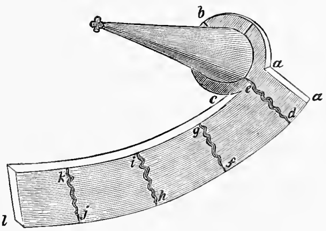
These facts may also be well brought out by another mode of section, which is described in my former paper as "cutting round a greater or less extent of the marginal tissue, leaving one end of the resulting strip free, and the other end attached *in situ*." If this form of section be practised on *Tiaropsis indicans*, as represented at *g*, *k* in the figure, it may always be observed that irritation of a distant point in the marginal strip, such as *g* or *h*, causes the polypite to move in various directions, without any special reference to that part of the bell which the irritated point of the marginal strip would occupy if *in situ*. But if the stimulation be applied only one or two millims. from the point of attachment of the marginal strip, as at *i*, the polypite will confine its localizing motions to perhaps the proper quadrant of the bell; and if the stimulus be applied still nearer to the attachment of the severed strip, as at *j*, the localizing motions of the polypite may become quite accurate.

Again, with regard to *radial* distance, if the cut AB in the figure were situated higher up in the bell, as at A′B′, and the arc, *c*, *d*, *e*, *f*, of the margin irritated as before, the polypite would be able to localize better than if, as before, the radial distance between AB and *c*, *d*, *e*, *f* were less. The greater this radial distance, the better would be the localizing power of the polypite; so that, for instance, if the cut A′B′ were situated nearly at the base of the polypite, the latter organ might be able to localize correctly a stimulus applied, not only as before at *f*, but also at *e* or *d*. In such comparative experiments, however, it is to be understood that the higher up in the bell a cut is placed, the shorter it must be; for a fair comparison requires that the two ends of the cut shall always touch the same two radii of the nectocalyx. Still, if the cut is only a very short one (say 1 or 2 millims. long), this consideration need not practically be taken into account; for such a cut, if situated just above the margin, as represented at *a*, will have the effect of destroying the localizing power of the polypite as regards the corresponding arc of the margin; but if situated high up in the bell, even though its length be still the same, it will not have this effect.

From all this, then, we have seen that the connexions which render possible the *accurate* localizing functions of the polypite are almost, though not quite, exclusively radial. We have also seen that between accurate localization and mere random movements on the part of the polypite there are numerous gradations—the degree of decline from one to the other depending on the topographical relations between the point of stimulation and the end of the section (the section being of the form represented by AB in the figure). These relations, as we have seen, are the more favourable to correct localization, (*a*) the greater the radial distance between the point of stimulation and the end of the section; and (*b*) the less the circumferential distance between the point of stimulation and the radius let fall from the end of the section. But we have seen that the limits, as regards severity of section, within which these gradations of localizing ability occur are exceedingly restricted—a cut of only a few millims. in length, even though situated at the greatest radial distance possible, being sufficient to destroy all localizing power of the polypite as regards the middle point of the corresponding arc of the margin, and a stimulus applied only a few millims. from the attached end of a severed marginal strip entirely failing to cause localizing action of the polypite. Lastly, we have seen that even after all localizing action of the polypite has been completely destroyed by section of the kinds described, this organ nevertheless continues actively, though ineffectually, to search for the seat of irritation.

(*d*) The last-mentioned fact shows that after excitational continuity of a higher order has been destroyed, excitational continuity of a lower order nevertheless persists. Or, to state the case in other words, the fact in question shows that after severance of the almost exclusively radial connexions between the bell and the polypite by which the perfect or impaired localizing function of the latter is rendered possible, other connexions between these organs remain which are not in any wise radial. I therefore next tested the degree in which these non-radial connexions might be cut without causing destruction of that excitational continuity of a lower order which it is their function to maintain. It will here suffice to record one mode of section which has yielded important results. A glance at the accompanying figure (8) will show the manner in which the Medusa is prepared. The margin having been removed (in order to prevent possible conduction by the marginal nerve-fibre), a single deep radial cut (*a a*) is first made, and then a circumferential cut (*a*, *b*, *c*) is carried nearly all the way round the base of the polypite. In this way the nectocalyx, deprived of its margin, is converted into a continuous band of tissue, one of the ends of which supports the polypite. Now it is obvious that this mode of section must be very trying to nervous connexions of any kind subsisting between the bell and the polypite. Nevertheless, in many cases, irritating any part of the band *a l* has the effect of causing the polypite to perform the active random motions previously described. In such cases, however, it is observable that the further away from the polypite the stimulus is applied, the less active is the response of this organ. In very many instances, indeed, the polypite altogether fails to respond to stimuli applied at more than a certain distance from itself. For example, referring to fig 8, the polypite might actively respond to irritation of any point in the division *d*, *e*, *f*, *g*, while to irritation of any point in the division *f*, *g*, *h*, *i* its responses would be weaker, and to irritation of any point in *h*, *i*, *j*, *k* they would be very uncertain or altogether absent. Hence in this form of section we have reached about the limit of tolerance of which the non-radial connexions between the bell and polypite are capable*.

Fig. 8.



(*e*) Another interesting fact brought out by this form of section is, that the tracts occupied by the four radial tubes are tracts of comparatively high irritability as regards the polypite; for the certainty and vigour with which the polypite responds to a stimulus applied at one of the severed radial tubes *f g*, *h i*, or *j k*, contrast strongly with the uncertainty and feebleness with which it often responds to stimuli applied between any of these tubes. Indeed it frequently happens that a specimen which will not respond at all to a stimulus applied *between* the radial tubes *f g*, and *h i*, will respond certainly and well to a stimulus applied much further from the polypite, but *in the course of* the radial tube *g k*.

(*f*) And this leads us to another point of interest. In such a form of section, when

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* This may also be well shown by the spiral form of section. Suppose fig. 2, Plate 30 to represent in a diagrammatic form a specimen of *Tiaropsis indicans* seen in projection from above. A spiral cut ABC is carried round as far as the point C, so as very nearly, though not quite, to overlap the beginning of the cut A. The polypite P may now actively respond to stimuli applied anywhere in the area A, B, C, D (although the response would probably be more active the nearer the stimulation to D). But if the cut be now carried to D, so as just to overlap A, all response on the part of P will cease to stimuli applied anywhere over the area ABC.

any part of the mutilated nectocalyx is irritated, the polypite shows a very marked tendency to touch some point in the tissue-mass *a a d e* (fig. 8) by which it still remains in connexion with the bell, and through which, therefore, the stimulus must pass in order to reach the polypite. And it is observable that this tendency is particularly well marked if the section has been planned as represented in fig. 8—*i. e.* in such a way as to leave the tissue-tract *a a d e* pervaded by a nutrient tube *d e*, this tube being thus left intact. When this is done the polypite most usually points to the uninjured nutrient-tube *d e* every time any part of the tissue-band *a l* is irritated*.

(*g*) Let us now very briefly consider the inferences to which these results would seem to point. The fact that the localizing power of the polypite is completely destroyed as regards all parts of the bell lying beyond an incision in the latter conclusively proves, as already stated, that all parts of the bell are pervaded by radial lines of differentiated tissue, which have at least for one of their functions the conveying of impressions to the polypite†. The fact in question also proves that the particular effect which is produced on the polypite by stimulating any one of these lines cannot be so produced by stimulating any of the other lines. Or, to state the same thing in the words previously used, we have conclusive proof that, so far as their *function* is concerned, these radial lines of differentiated tissue are undistinguishable from what we elsewhere know as afferent nerves. But although these tracts of differentiated tissue thus far resemble afferent nerves in their function, we soon see that in one important particular they differ widely from such nerves; for we have seen that, after they have been divided, stimulation of their peripheral parts still continues to be transmitted to their central parts, as shown by the non-localizing movements of the polypite. Of course this transmission cannot take place through the divided tissue-tracts themselves; and hence the only hypothesis we can frame to account for the fact of its occurrence is that which would suppose these tissue-tracts, or afferent lines, to be capable of vicarious action. Such vicarious action would probably be effected by means of intercommunicating lines, the directions of which would probably be various. In this way we arrive at the hypothesis of the whole contractile sheet being pervaded by an intimate plexus of functionally differentiated tissue, the constituent elements of which are capable of vicarious action in a high degree (see severity of section in fig. 8). And this hypothesis, besides being recommended by the consideration of its being the only feasible one that is open, is confirmed by the fact that the stimuli which it supposes to escape from a severed phalanx of radial lines, and thus

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* It may here be stated that the greater sensitiveness of the nutrient tubes is also made apparent by the responses of the nectocalyx; for to a nip or prick of the general contractile sheet the response by the nectocalyx usually consists, as before stated, in a local bending in of the part irritated; whereas in response to similar irritation of a radial tube, the response is usually a general bending in of the whole nectocalyx—*i. e.* a general spasm.

I may here observe that cutting through all the nutrient tubes at their insertion in the polypite has no effect in preventing the localizing action of the latter as regards other parts of the bell.

† We must not, however, conclude that these lines are radial *structurally*; the evidence only proves that they are so *functionally*.

to reach the polypite after being diffused through many or all of the other radial lines (such stimuli thus converging from many directions), are responded to when they reach the polypite, not by any decided localizing action on the part of the latter, but, as the hypothesis would lead us to expect, by the tentative and apparently random motions which are actually observed. Moreover, we must not neglect to notice that these tentative or random movements resemble in every way the localizing movements, save only in their want of precision. Again, this hypothesis is rendered yet more probable by the occurrence of those *gradations* in the localizing power of the polypite which we have seen to be so well marked under certain conditions. The occurrence of such gradations under the conditions I have named is what the plexus theory would lead us to expect, because the closer beneath a section that a stimulus is applied, the greater will be the lateral spread of the stimulus through the plexus before it reaches the polypite. Similarly, the further the circumferential distance from the nearest end of such a section that the stimulus is applied, the greater will be its lateral spread before reaching the polypite. Lastly, the present hypothesis would further lead us to anticipate the fact that when *Tiaropsis indicans* is prepared as represented in fig. 8, the polypite refers a stimulus applied anywhere in the mutilated nectocalyx to the band of tissue by which it is still left in connexion with that organ; for it is evident that, according to the hypothesis, the radial lines occupying such a band are the only ones whose irritation the polypite is able to perceive; and hence it is to be expected that it should tend to refer to these particular lines a source of irritation occurring anywhere in the mutilated bell*.

It is not quite so clear why, in the last-mentioned experiment, the polypite should tend to refer a seat of irritation to the unsevered nutrient tube, rather than to the tracts

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* It may be objected that the fact of diffused excitability persisting after destruction of the localizing function does not prove that the same tissue-elements are concerned in the two cases. For instance, in fig. 1, Plate 30, before introducing the cut *a*, a stimulus applied at *b* would, as before stated, cause localizing motion of the polypite. Let us call the tissue-tracts by which this localizing motion is effected $x\,x$, and let us call all the other tracts belonging to the same system, but occurring in other parts of the bell, $x'\,x'$. Now the hypothesis above explained supposes that after the cut *a* has been introduced, the random motions of the polypite which ensue on stimulating *b* are due to an escape of the stimulus at *a* from $x\,x$ to $x'\,x'$, *i. e.* all over the bell. But the objector may ask, Why not suppose that the stimulus is conveyed to the polypite by some other tissue-element altogether, *e. g.* by $z\,z$, and hence that the function of the radial elements $x\,x$ is exclusively that of enabling the polypite to localize correctly? Against this objection, of course, nothing can be urged, except that it merely suggests a possibility, and this a very improbable one. If such additional tissue-tracts are present, they must resemble those treated of in the text in at least one distinctive feature, viz. in having the property of conveying impressions to a distance. And forasmuch as the only objection to the above hypothesis consists in its supposing the distinguishing property of nerve to be blended with the property of vicarious action in a high degree, I cannot perceive any advantage in again supposing these properties blended in another tissue-element. On the contrary, any such additional supposition appears to me quite unnecessary, and, if accepted, would certainly serve but to render the explanation offered in the text more complicated than it is. Moreover the fact already alluded to in the text—viz. that tentative movements differ in no respect from the accurately localizing movements, except in their want of precision—this fact, I think, strongly tends to show that the same kind of tissue-elements is concerned in the production of all these movements.

of unsevered tissue on either side of that tube. The fact of its doing so, however, certainly seems to be connected with the fact of the radial tubes being tracts of high excitability. Now the latter fact, it seems to me, can only be accounted for in one of two ways. Either (*a*) we must suppose the course of a radial tube to coincide with that of a chain of ganglionic elements, or else (*b*) we must suppose it to coincide with something resembling a nerve-trunk—the latter being composed of aggregations of the afferent lines which we have already supposed to constitute the rudimentary nerve-plexus. According to the first of these suppositions, the comparatively high excitability of the radial tubes, as well as the pointing of the polypite to the unsevered tube in a specimen prepared as represented in fig. 8, would both be explained by the further supposition that when such a ganglion chain is stimulated—either directly or by conduction of the stimulus from a distance—the ganglionic elements discharge their influence, and so add to the strength of the original stimulus. According to the second of the above suppositions (viz. *b*), the fact of the high excitability of the nutrient tubes would be explained by the consideration that a stimulus applied to the supposed nerve-trunk would *directly* affect a greater *number* of the plexus-elements than would a stimulus applied to any other part of the bell. The additional fact to be met, viz. the pointing of the polypite to a single unsevered tube, is not quite so easy to explain on the present supposition; for if the assumed nerve-trunk is identical in function with a true nerve-trunk, the afferent elements collected in it ought to communicate to the polypite the impression of having had their *distal* terminations irritated; and therefore the fact of a number of such elements being collected into a single trunk ought not to cause the polypite to refer a distant seat of irritation to that trunk rather than to any of the parts from which the plexus-elements may emanate. Concerning this difficulty, however, I may observe that we seem to have in it one of those cases in which it would be very unsafe to argue with any confidence from the highly integrated nervous systems with which we are best acquainted to the primitive nervous systems with which we are now concerned. And although it would occupy too much space to enter into a discussion of this subject, I may further observe that I think it is not at all improbable that the polypite of *Tiaropsis indicans* should, in the absence of more definite information, refer a distant seat of injury to that tract of collected afferent elements through which it actually receives the strongest stimulation.

The first of these two suppositions (viz. *a*) will be found to derive some little support when we come to consider a curious class of facts to be dealt with in the next section. The second of these suppositions (viz. *b*) is supported by the histological researches of Professor HAECKEL on *Geryonia*, a genus which closely resembles *Tiaropsis indicans* in form. This observer describes nerve-filaments following everywhere the course of the radial tubes, and receiving a number of minute filaments from either side. Moreover, the supposition we are now considering is further analogically supported by SCHULTZE's researches as to structure, and my own as to function, of the radial nerve-tracts in *Sarsia*.

On the whole, then, with regard to *Tiaropsis indicans*, I conclude in favour of something resembling a plexus of functionally-differentiated tissue-lines pervading the whole inner surface of the nectocalyx. A number of these tissue-lines I conceive to pursue, in a functional sense, a radial course; and, therefore, when intact, to enable the polypite correctly to localize a seat of irritation. Others of these lines I conceive to pursue various courses, and in so doing to connect the radial lines with one another. All the lines of such a plexus I conceive to be capable, in a high degree, of vicarious action; and I am inclined to suppose that a large proportional number of such lines are aggregated in the courses of the four radial tubes. But in now taking leave of the interesting series of facts which have led to these inferences, I should like to observe that, while I know the former cannot be modified by future investigation, I feel the latter may thus be shown imperfect or untrue. It has not seemed to me, however, that this is a sufficient reason to avoid speculation at the stage which my inquiries have now reached. There can be no doubt that the facts concerning *Tiaropsis indicans* are of a highly suggestive character; and I think it will shortly be found desirable thus to have paved the way towards arriving at some sort of conception with regard to analogous, though far more astonishing, facts which are presently to be adduced concerning *Aurelia aurita.*

§ 3. *Character of the Nervo-muscular Tissue of* Staurophora laciniata.—(*a*) The few individuals of this species which I was this year able to procure I utilized in the study of those peculiar spasmodic movements to which allusion was made in my former paper. These movements, it will be remembered, never occur spontaneously, but only in response to stimulation of certain parts of the organism, under which condition they never fail to occur. It will further be remembered that these spasmodic movements consist of a sudden and most violent contraction of the entire muscle-sheet, the effect of which is to draw together all the gelatinous walls of the nectocalyx in a far more powerful manner than occurs during ordinary swimming. In consequence of this spasmodic action being so strong, the nectocalyx undergoes a change in form of a very marked and distinctive character. The corners of the four radial tubes, being occupied by comparatively resisting tissue, are not so much affected by the spasm as are other parts of the bell; and they therefore constitute a sort of framework upon which the rest of the bell contracts—the whole bell thus assuming the form of an almost perfect square, with each side presenting a slight concavity inwards. These spasmodic movements, however, are quite unmistakable, even when they occur in a very minute portion of detached tissue; for however large or small the portion may be, when in a spasm it folds upon itself with the characteristically strong and persistent contraction. I say "*persistent* contraction" because it will be remembered that a spasmodic contraction, "besides being of unusual strength, is also of unusual duration." That is to say, while an ordinary systolic movement only lasts a short time, a spasm lasts from six to ten seconds or more, and this whether it occurs in a large or in a small piece of tissue. Again, it will also be remembered that the diastolic movements differ very much in the case of an ordinary locomotor contraction and in that of a spasm; for while in the former case the process of relaxation is rapid even to suddenness, in the latter it is exceedingly prolonged and gradual, occupying some four or five seconds in its execution, and, from its slow but continuous nature, presenting a graceful appearance. Lastly, the difference between the two kinds of contraction is shown by the fact that, while a spasm is gradually passing off, the ordinary rhythmical contractions may often be seen to be superimposed on it—both kinds of contraction being thus present in the same tissue at the same time.

(*b*) I have thus given a somewhat fuller description of these movements than that contained in my former paper, because, as the whole of the present section is to be occupied with their consideration, it seems desirable to render a complete account of them at the outset. Now the point with which we shall be especially concerned on the present occasion is one which has already been alluded to incidentally, viz. that in the case of *Staurophora laciniata* it is only stimulation of *certain parts* of the organism which has the effect of throwing it into a spasm. These parts are the margin (including the tentacles) and the courses of the four radial tubes (including the polypite, which in this species is spread over the radial tubes). This limitation, however, is not invariable; for I have often seen individuals of this species respond with a spasm to irritation of the general contractile tissue. Nevertheless, such response to such stimulation in the case of this species is exceptional—the usual response to muscular irritation being an ordinary locomotor contraction, which forms a marked contrast to the tonic spasm that *invariably* ensues upon stimulation of the margin, and *almost* invariably upon the stimulation of a radial tube. Still it must here be carefully noted, that in at least three other species belonging to another genus, viz. *Tiaropsis diademata*, *T. polydiademata*, and *T. oligoplocama*, spasmodic movements in no way distinguishable from those of *Staurophora laciniata* invariably follow upon stimulation of *any part of the organism*, or even of a minute portion of the contractile tissue when excised. Hence the usual failure of *Staurophora laciniata* to respond spasmodically to irritation of its contractile tissues is probably due to their being somewhat less irritable than those of the other species named.

(*c*) For my investigation of these spasmodic movements I chose *Staurophora laciniata* because, from its large size, it admitted of my performing various modes of section which would be impracticable in the case of the other naked-eyed Medusæ. Moreover, as the principal point to be inquired into had reference to the fact that these movements were most readily elicited by stimulating the marginal or radial tubes, this species seemed particularly well suited to the purposes of my inquiry; for, as we have just seen, the degree of its irritability is such as to cause the animal to respond spasmodically to the slightest irritation of the marginal or radial tubes, while even to violent irritation of the contractile tissues the only response is a locomotor contraction. The first point I endeavoured to settle was whether the faculty of originating a tonic spasm in answer to a stimulus was absolutely restricted, in the case of *Staurophora laciniata*, to the marginal and radial tubes, or such faculty was likewise shared by a small band of the contractile tissue in the immediate neighbourhood of these tracts.

To settle this point I tried gently pinching the margin with a fine pair of forceps: in answer to every nip I obtained a general spasm. I then tried nipping the general contractile tissue a millimetre or less from its line of junction with the marginal tissue: I obtained a general spasm. I next applied similar irritation two millimetres from the margin, and obtained no contraction of any kind on the part of the bell as a whole; but in the immediate neighbourhood of the irritation (*i. e.* about a quarter of an inch on either side of it) I obtained a *local* spasm. On now similarly irritating another point of the general contractile tissue, about another millimetre inwards from the margin, a still slighter local spasm was the result; and on repeating the experiment another millimetre or two from the margin no response of any kind was given. The same experiments were then conducted on one of the radial tubes, and they yielded the same results, with the exception that a general spasm was not of so certain occurrence in answer to nipping such a tube as it was in answer to nipping the margin. From these experiments, then, we see that there is no sharp line of demarcation between the tracts of tissue that are able to originate a spasm and those that are not. Nevertheless, I think the probability is that in these experiments the general and local spasms which resulted on irritating a point of the contractile tissue very near to one of these tracts, were really due to a slight sympathetic action on the part of the marginal or radial tube tissues which were so near to the actual point of irritation; for it must be remembered that such a local spasm could not have been obtained by irritating any other part of the general muscle-sheet of *Staurophora laciniata.*

(*d*) The next question I undertook to answer was the amount of section which the excitable tissues of *Staurophora laciniata* would endure without losing their power of conducting the spasmodic contraction from one of their parts to another. This was a very interesting question to settle, because, it may be remembered, *Staurophora laciniata*, like all the other species of discophorous naked-eyed Medusæ, differs from *Aurelia* &c. in that the ordinary contractile waves are very easily blocked by section*. It therefore became very interesting to ascertain whether or not the spasmodic wave admitted of being blocked as easily. First, then, as regards the margin. In my former paper I stated that if this be all cut off in a continuous strip with the exception of one end left attached *in situ*, irritation of any part of the almost severed strip will cause a responsive spasm of the bell, so soon as the wave of stimulation has time to reach the latter. This year, therefore, I continued this form of section into the contractile tissues themselves, carrying the incision round and round the bell in the form of a spiral, as

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* As stated in the Postscript of my former paper, "there appear to be important differences between the discophorous naked-eyed Medusæ and the true *Discophora* in this respect; for in all the species of the former which I have as yet observed, the area of paralysis in the nectocalyx corresponds much more precisely with the line of ganglionic tissue which has been removed from its margin, than it does in the case of the true *Discophora*." I may here explain that this is not, of course, intended to mean that in no case is the spread of a contractile wave observable in the tissues of such naked-eyed Medusæ; but merely that such spread is usually far less extensive than it is in the case of the covered-eyed Medusæ.

represented in the case of *Aurelia* by Plate 33 of my previous paper. In this way I converted the whole Medusa into a ribbon-shaped piece of tissue*; and on now stimulating the marginal tissue at one end of the ribbon, a portion of the latter would go into a spasm. The object of this experiment was to ascertain how far into the ribbon-shaped tissue the spasmodic wave would penetrate. As I had expected, different specimens manifested considerable differences in this respect; but in all cases the degree of penetration was astonishingly great. For it was the exception to find cases in which the spasmodic wave failed to penetrate from end to end of a spiral strip caused by a section that had been carried twice round the nectocalyx; and this is very astonishing when we remember that the ordinary contractile waves, whether originated by stimulation of the contractile tissues, or arising spontaneously from the point of attachment of the marginal strip, usually failed to penetrate further than a quarter of the way round. But still more astonishing is the fact that these spasmodic waves will continue to penetrate such a spiral strip even after the latter has been submitted to a system of interdigitating cuts of a very severe description. Here, again, of course, considerable individual differences as to tolerance of section occur; but the drawing in Plate 30 (fig. 3) will convey some idea of the surprising amount of tolerance that is sometimes shown. This figure explains itself, it having been drawn to measurement from a specimen in which the spasmodic waves, started by stimulation of any point in the severed margin *a b*, would penetrate the whole of the strip *c d e*, while in the same specimen the ordinary contractile waves became blocked at *d*.

(*e*) Now we have here to deal with a class of facts which physiologists will recognize as of a perfectly novel character. Why it should be that the very tenuous tracts of tissue which I have named should have the property of responding even to a feeble stimulus by issuing an impulse of a kind which throws the contractile tissues into a spasm; why it should be that a spasm, when so originated, should manifest a power of penetration to which the normal contractions of the tissues in which it occurs bear so small a proportion; why it is that the contractile tissues should be so deficient in the power of originating a spasm, even in response to the strongest stimulation applied to themselves—these and other questions at once suggest themselves as questions of the highest interest: at present, however, I am wholly unable to answer them. To assume that it is the ganglionic element in the margin, and possibly also in the radial tubes, which responds to direct stimulation by discharging a peculiar impulse which has the remarkable effect in question—to assume this is not to explain the facts. For the sake of rendering the matter quite clear, let us employ a somewhat far-fetched but convenient metaphor. We may compare the general contractile tissues of this Medusa to a mass of gun-cotton, which responds to ignition (direct stimulation) by burning with a quiet

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* It may be stated that while conducting this mode of section on *Staurophora laciniata*, the animal responds to each cut of the contractile tissues with a locomotor contraction (or it may not respond at all); but every time the section crosses one of the radial tubes, the whole bell in front of the section and the whole strip behind it immediately go into a spasm.

flame, but to detonation (marginal stimulation) with an explosion. In the tissue, as in the cotton, every fibre appears to be endowed with the capacity of liberating energy in either of two very different ways; and whenever one part of the mass is made, by the appropriate stimulus, to liberate its energy in one of these two ways, all other parts of the mass do the same—and this no matter how far through the mass the liberating process may have to extend. Now, employing this metaphor, what we have to explain is why this capacity of dual response should exist in contractile tissues, and why certain slender lines of differentiated tissue should have the capacity, when themselves directly stimulated, of determining the explosive response. And to me it seems that neither of these questions is answered by supposing that it is the ganglionic element which, when itself directly stimulated, acts as a detonator. That this may be the case I do not dispute; for it is certainly, to say the least, a curious coincidence that such a marked differentiation of function as the facts imply should everywhere accompany and be restricted to the ganglionic tracts*; but what I desire to insist upon is this—that even if the ganglionic element could be proved to be the detonating element, the facts before us would not thus be accounted for. Doubtless it would be an interesting thing to know that a ganglion-cell may be able to originate two very different kinds of impulse, according as it liberates its energy spontaneously or in answer to direct stimulation†; but this knowledge would merely serve to transfer the questions which now apply to the marginal and polypite tissues in general to the ganglionic tissue in particular. Again, the supposition of the ganglia acting as detonators when themselves directly stimulated, would in no wise tend to explain why it is that the *contractile* tissues are capable of two such very different kinds of *response*. By detonating ordinary cotton, we can never obtain an explosion; and, similarly, unless the general contractile tissues of these Medusæ were in some way prepared to respond in a different manner to the different kinds of marginal or ganglionic discharge, we should never observe a spasm.

(*f*) Another solution of the difficulty may possibly present itself as worthy of consideration. Bearing in mind the facts already adduced with regard to the high degree

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* As the polypite of *Staurophora laciniata* is spread out over the radial tubes, it is not improbable that ganglionic elements may here occur.

† It may perhaps be thought that this is too strong a statement. Considering that, in the various species of *Tiaropsis*, spasmodic motions result when any part of the irritable tissues are stimulated, it may appear that a spasm differs from an ordinary contraction only in the degree of its intensity; and therefore that, in *Staurophora*, direct stimulation of the ganglia is followed by a spasm only because a greater amount of ordinary ganglionic influence is thus caused to be liberated than would be the case in a merely spontaneous discharge. I think, however, that if this were the explanation there ought not to be so immense a contrast as there is between the facility with which a spasm may be caused by stimulation of the margin and of the contractile tissue respectively. The slightest nip of the margin of *Staurophora laciniata*, for instance, is sufficient to cause a spasm, whereas even crushing the contractile tissues with a large pair of dissecting-forceps will probably fail to cause any thing other than an ordinary contraction. Nevertheless, pricking the margin with a fine needle usually has the effect of causing only a locomotor contraction.

of sensitiveness manifested by the radial tubes of *Sarsia*, *Tiaropsis indicans*, &c., it may be suggested that if we imagine a vast number of nerve-tracts coursing from all parts of the bell towards the ganglionic tracts, we may imagine that on irritating any of these nerve-tracts near their central or ganglionic ends (nerve-trunks) a greater effect might be thus produced on the contractile tissues than could be produced by irritating the latter directly. In view of this possibility I tried making a V-shaped incision from the margin inwards, the angle of the V being situated about halfway between the centre and the circumference of the bell. The two sides of the V, however, did not quite meet at this angle, the effect of the section being thus almost, though not quite, to sever a wedge-shaped piece of tissue from the rest of the nectocalyx. Now on irritating the marginal tissue composing the thick end of this wedge, the latter went into a spasm, and, communicating the spasmodic wave through the very narrow tract of tissue at its thin end, it threw the whole nectocalyx into a spasm. This, of course, was what I expected, and from the mode of section it followed that the whole of the influence which caused the nectocalyx to go into a spasm must have passed through the narrow piece of tissue by which the wedge-shaped portion had been left in connexion with the rest of the nectocalyx. Yet, on waiting till the spasm passed off and then crushing with the forceps this narrow connecting piece of tissue, no response at all was given by the nectocalyx. This experiment, therefore, clearly shows that the nerve-trunk hypothesis will not account for the facts; because, as the hypothesis would require us to suppose that the entire nerve-supply by which the spasm-causing influence was conveyed to the nectocalyx passed through the narrow connecting piece of tissue, crushing this tissue ought, according to the hypothesis, to have produced a spasm; for at whatever point in the course of a nerve a stimulus is applied, the effect on the contractile tissue supplied by that nerve is the same[*].

(*g*) There is one other fact worth stating with regard to this wedge-shaped form of section. Immediately after making the cuts which nearly but not quite sever the wedge-shaped mass, it is often observable that the *shock* suffered by the narrow connecting piece of tissue prevents it from conducting the spasmodic influence to the nectocalyx, the wedge itself alone going into a spasm. After a variable time, however, perhaps as long as a quarter of an hour, the conducting-power of the connecting-tissue is restored. But to produce this effect of shock, the connecting piece of tissue must be exceedingly narrow.

(*h*) In conclusion, I may state that anæsthetics have the effect of blocking the spasmodic wave in any portion of tissue that is submitted to their influence. It is always observable, however, that this effect is not produced till after spontaneity has been fully suspended, and even muscular irritability destroyed as regards direct stimulation. Up to this stage, the certainty and vigour of the spasm consequent on marginal irri-

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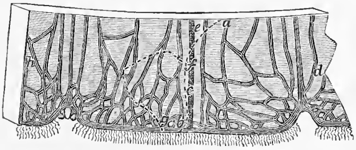
[*] The "avalanche" doctrine as to the summation of influence in a nerve appears to have become obsolete; but even if true it might be neglected in the above considerations—the supposed nerve-length being so small and the contrast between the effects of central and peripheral stimulation being so great.

tation are not perceptibly impaired; but soon after this stage the intensity of the spasm begins to become less, and later still it assumes a *local* character. It is important, also, to notice that at this stage the effect of marginal stimulation is very often that of producing a *general locomotor* contraction, and sometimes a series of two or three such. During recovery in normal sea-water all these phases recur in reverse order.

§ 4. *Character of the Contractile Tissues of* Aurelia aurita. (A) *Contractional Continuity.*—(*a*) This year I have been able to collect some additional facts relating to this subject; and of these facts the first that I shall deal with is an important one, namely, that exploration of the contractile tissues by graduated stimuli affords direct proof that these tissues are in a high degree functionally heterogenous. That is to say, if the swimming-bell of *Aurelia* be paralyzed by removal of its lithocysts, and if its irritable surface be then explored by induction-shocks of graduated intensity, differences in the excitability of different parts may thus be rendered clearly apparent. These differences are observable in all degrees, from being scarcely perceptible to being so pronounced that no strength of stimulation will cause any, except a very local, response. Moreover it is of importance for us to note that if the stimuli be administered by means of needle-point electrodes, it may almost always be observed that an area of low excitability does not graduate into an adjacent area of high excitability by successive stages, but that the one area is separated from the other by an exceedingly well-defined line of demarcation.

(*b*) Having observed these lines of demarcation between areas of different degrees of excitability to be so well defined, I was led to try cutting a spiral strip until the contractile waves became blocked, and then exploring the line of blockage by stimulus. In all cases I found that if I stimulated a fraction of a millim. on the spiral strip side of this line, the spiral strip responded, while the uncut portion of the bell remained passive; and, conversely, that if I stimulated the other side of this line, the uncut portion of the bell responded, while the spiral strip remained passive. To make the fact in question quite clear I may refer to fig. 9, which represents a portion of a broad strip wherein the contractile waves became blocked at the curved line *a b*. On stimulating the tissue at any point as close as possible on one side of this line, as at *c*, a contractile wave would start from that point, and course all the way along the strip in the direction *c d*, but in that direction only; while if the stimulus were applied at any point on the other side of the line *a b*, as at *e*, the contractile wave thus originated

Fig. 9.



would course in the opposite direction $e\,h$, and in that direction only. On stimulating the line $a\,b$ itself, the result might either be no response at all, a contractile wave running in one direction, a contractile wave running in the other direction, or contractile waves running in both directions. These various results are doubtless to be explained by the various degrees in which the current escaped from the slender line $a\,b$.

(*c*) Exploration by stimulus, then, not only shows that different tissue-tracts differ in their degree of irritability; but also that they further differ in the degree of their permeability to the stimulating influence. In the cases just cited, viz. those of complete blocking, we perceive a total absence of this permeability; but it must now be stated that, just as in the cases previously cited of differential irritability, so in those of differential permeability there are all the degrees of difference observable. And, in the one set of cases as in the other, the lines of demarcation between adjacent areas are exceedingly well defined. For example, returning to our former sketch, the area enclosed by the line $b\,f$ and the angular line $b\,g$ was an area the minimal stimulation of any part of which was followed by a local contraction of the area $g\,b\,f$, while minimal stimulation of any point outside of the lines $g\,b\,f$ was followed by a general contraction of the entire strip situated outside of these lines. And of course there are other cases in which even the strongest stimulation of such an insular area as $g\,b\,f$ would fail to elicit any thing further than a local contraction of that area.

(*d*) From all this, then, we see that exploration by graduated stimuli reveals, first, differences in the degrees of excitability of closely contiguous tissue-areas, and, second, differences in the degrees of permeability to stimuli on the part of such areas—there being thus tissue-areas which admit of being more or less physiologically isolated from the rest of the tissue-mass by section. And the fact that in all these cases the lines of demarcation between the differentiated tissue-tracts are so sharply defined constitutes, I think, an additional support to the hypothesis concerning the presence of nerves, or "lines of discharge"*.

(B) *Excitational Continuity.*—(*a*) Closely related to this hypothesis are also the

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* It has been suggested to me that contractile waves are merely muscle-waves, and that their blockage in contractile strips is due to shock. This, of course, would be a delightfully simple solution of the difficulties besetting this subject, and it is naturally the first one that occurs to an experimenter. But further observation shows that the tissues of Medusæ cannot be made to suffer shock unless the tissue-tracts affected by the section are exceedingly narrow, while blocking of contractile waves may take place in strips of considerable width. Moreover, the above results with regard to exploration by stimulus prove that even the *unmutilated* contractile tissues are far from being functionally homogenous. These results also negative the supposition which may possibly occur to some physiologists, viz. that blocking of the contractile waves in a spiral strip is due to a resistance that progressively varies with the length of the strip.

A more plausible explanation would be that the line of blocking is determined by an accidental strain to which that line has been previously subjected. That this is sometimes the case there can be no doubt, as the tissue bears optical indications of having been strained at the line of blocking. But that this cannot be the explanation in all cases is proved by the occurrence of those numerous gradations in different parts of the same tissue-mass, both with regard to excitability of tissue-areas and intimacy of physiological connexions.

facts to be detailed in this subsection. For if it can be proved that the contractile tissues of *Aurelia* are pervaded by tissue-tracts which display the essentially nervous function of establishing what I have termed excitational continuity between different parts, then I think we may be more prepared to believe that the passage of contractile waves depends on the presence of tissue-tracts presenting a nervous character. For the evidence being already in favour of the hypothesis that the passage of contractile waves depends on the presence of certain differentiated elements whose function is presumably nervous, such evidence would be further strengthened if it could be shown that in the very same tissue there occur other differentiated elements whose function is demonstrably nervous. Now that such elements as the last mentioned do occur in all parts of the excitable tissues of *Aurelia aurita* is a fact concerning which there can be no question.

(*b*) If any part of the polypite of *Aurelia* be stimulated, the natural rhythm becomes accelerated and the systoles more powerful. Again, if any part of the irritable surface of the swimming-bell be stimulated, not only do the swimming-motions become more rapid and powerful, but the lobes of the polypite writhe about in answer to the stimulation. These complementary facts clearly prove the occurrence of reflex action between the polypite and the swimming-bell reciprocally. Similarly, if seven lithocysts be removed, and the remaining one be of a somewhat weakly character, so as not to interfere with the observation by its excessive spontaneity, it may often be observed that if a portion of the irritable surface of the swimming-bell be submitted to a stimulus of somewhat less than minimal intensity as regards the starting of a contractile wave from the point of stimulation, a contractile wave will nevertheless start from the single remaining lithocyst so soon as time has been allowed for the stimulus to travel from the point of its application to that of the lithocyst. The greater the distance between these two points, the less certain is the experiment to succeed; but in many specimens it may succeed every time, even though the stimulus be applied at the opposite side of the swimming-bell from that in which the lithocyst is placed. In such cases, however, it is very important to observe that stimulation of certain tracts of tissue is followed by this reflex response, while stimulation of other tracts, even though these be situated much nearer to the lithocyst, is not so followed. Such excitable tracts are by no means constant as to their position in different individuals; but the fact of their occurrence tends to show that the reflex action we are contemplating is effected, not by means of an equally diffused excitability on the part of the whole nervo-muscular sheet, but by means of functionally differentiated tissue-tracts, which, during the experiment, act as afferent nerves.

(*c*) Next I must draw attention to a fact which will presently be proved of the highest importance in relation to the subject we are considering. It may be remembered, from Plate 32 in my former paper, that the entire margin of *Aurelia aurita* is fringed by a number of exceedingly delicate tentacles. These tentacles are highly retractile; and the point to which I have now to direct attention is this, that when any portion of the irritable surface of *Aurelia* is stimulated *too gently to start a contractile wave*, it may nevertheless be sometimes observed that those tentacles which are nearest to the seat of stimulation respond with a sudden contraction, then those next adjacent to them on either side do the same, and so on—there being thus started in the margin two impulses which travel with equal rapidity in opposite directions, and the passage of which is marked by the successive and intensely sudden retraction of the numberless tentacles by which the margin is fringed. This most beautiful expression of the passage of a wave of stimulation does not occur in all, or even in most, specimens of *Aurelia*. It may best be seen in specimens that are perfectly fresh and vigorous—*i. e.*, in general, such as are deeply coloured. Moreover, just as in the case of reflex action, so in this case, the wave of stimulation may be started from some tracts more certainly than from others, although there is no constancy as to the position of these tracts in different individuals. As showing the identity of the wave of stimulation in this case and in that of reflex action, I may mention the following fact. When a single lithocyst is left *in situ* and a point in the nervo-muscular sheet at a distance from the lithocyst is gently irritated, if the passage of the stimulus happens to be marked by the above-described occurrence of what we may term a tentacular wave, it is always observable that the lithocyst never originates its reflex response until after the tentacular wave has reached it, and that it then invariably does so when the requisite period of latent stimulation has elapsed, viz. about half a second after the arrival of the tentacular wave. This experiment may be rendered particularly fascinating if the *Aurelia* has been previously cut into a broad strip, in such a way as to leave the single remaining lithocyst at one end; for on now irritating the other end of this strip, the tentacular wave may be observed to run continuously in the same direction all the way along the margin, and then, after it reaches the terminal ganglion and the period of ganglionic latency has elapsed, the contractile wave which it has been the means of starting from the ganglionic end of the strip courses all the way along the latter in the opposite direction to that which the tentacular wave had previously pursued.

In connexion with these tentacular waves I may further state that it does not signify how much of the tentacular rim is removed from the organism; for, however small a number of tentacles are left adhering to the margin, they exhibit the same action as when the whole series is intact.

(*d*) Now the occurrence of these tentacular waves is invaluable for the purpose of the experiments next to be described. These experiments consist in submitting the swimming-bell of *Aurelia* to various forms of section, with the view of ascertaining the extent to which the nervo-muscular tissues may be thus mutilated without suffering loss of their *excitational* continuity. The importance of any facts established by these experiments will doubtless be appreciated, in view of the theoretical standing of the analogous facts which we have so recently been considering with regard to *contractional* continuity. Of course the value of tentacular waves in these experiments consists in the circumstance that they never occur except in response to stimulation, and that when they do occur they are of so unmistakable a character as to leave not the smallest room for supposing them due to any cause other than the passage of a stimulus.

The result of a number of experiments performed in this connexion is to show that the severity of section which causes blocking of the tentacular waves varies in different individual cases. Very frequently, however, the tolerance of tentacular waves towards section is quite as remarkable as is that of contractile waves. Or, adopting our previous terminology, section proves that in *Aurelia aurita* excitational continuity is as difficult to destroy as is contractional continuity.

To economize space, I shall not describe any of the intermediate degrees of tolerance, but shall pass at once to the most extreme instance which I have met with. The figure (see Plate 31) is one quarter life-size, and was carefully drawn to measurement*. It represents a specimen of *Aurelia aurita*, which, after having had its lithocysts removed, was cut in the same way as already described in the case of *Tiaropsis indicans* (fig. 8). The whole band-shaped length of tissue into which the swimming-bell was thus reduced was then submitted to the tremendously severe form of section which is represented in the figure. Yet on gently stimulating either end of the band-shaped tissue-mass *a b*, a tentacular wave would start from the point of stimulation, and, as represented in the figure, course all the way along the margin from end to end of the band-shaped mass†.

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* *i. e.* the proportional length of the cuts was so drawn. When the animal is so severely mutilated, the swimming-bell floats out in various directions; so that in the Plate the general shape of the bell is to be regarded as diagrammatic.

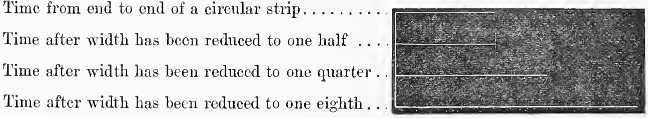
† [In my MS., as it was originally sent to the Royal Society, there here followed some pages showing the confirmation which the above facts supplied to my view that the contractile tissues of *Aurelia* are pervaded by a nerve-plexus, the constituent fibres of which are capable of vicarious action in almost any degree. But as I have now, through Mr. SCHÄFER's collateral work, obtained microscopical demonstration of the presence of such a plexus, it becomes unnecessary to adduce the reasoning from function to structure which these pages contained, and I have therefore struck them out. It seems desirable, however, to quote from these omitted pages the following paragraph, which serves to state the present standing of the question as to whether the waves of contraction in *Aurelia* depend for their passage on the muscular elements alone, or likewise upon the nervous elements—a question which, it will be observed, is not decided even by the histological demonstration of a nerve-plexus:—

"But in addition to these general considerations we must remember, more particularly, that in *Aurelia aurita* we have already obtained evidence of more or less distinct conductile tracts—this evidence being even last year sufficiently strong to render the plexus theory on the whole the most probable one that could be devised. This year that evidence has been further confirmed by explorations by stimuli; so that the only obstacle in the way of our accepting the plexus theory to account for the passage of contractile waves is the enormous amount of section which the contractile tissues will endure without these waves becoming blocked. Now the only reason why this fact is an obstacle in the way of our accepting the plexus theory is because, upon that theory, this fact would require us to suppose the conductile elements to be capable of vicarious action to an almost incredible extent. Hence it was that last year the issue with respect to the passage of contractile waves lay between

§ 5. *Rate of Transmission of Stimuli in* Aurelia aurita. (A) *Contractile Waves.*—(*a*) The rate at which contractile waves traverse spiral strips of *Aurelia* is variable. It is largely determined by the length and width of the strip, so that the best form of strip to use for the purpose of ascertaining the maximum rate is one which I shall call the circular strip. A circular strip is obtained by first cutting out the central bodies, *i. e.* polypite and ovaries, and then, with a single radial cut, converting the animal from the form of an open ring to that of a continuous band. I distinguish this by the name "circular" band or strip, because the two ends tend to preserve their original relative positions, so giving the strip more or less of a circular form. Such a strip has the advantage of presenting all the contractile tissue of the swimming-bell in one continuous band of the greatest possible width, and is therefore the form of strip that yields the maximum rate at which contractile waves are able to pass. The reason why the maximum rate should be the one sought for is because this is the rate which must most nearly approximate the natural rate of contractile waves in the unmutilated animal. This rate, at the temperature of the sea and with vigorous specimens, I find to be 18 inches per second.

(*b*) In a circular strip the rate of the waves is uniform over the whole extent of the strip; so that the time of their transit from one point to another varies directly as the length of the strip. But on now narrowing such a strip, although the rate is thus slowed, the relation between the narrowing and the slowing is not nearly so precise as to admit of our saying that the rate varies inversely as the width. The following figure will serve to show the proportional extent to which the passage of contractile waves is retarded by narrowing the area through which they pass:—

Fig. 10.



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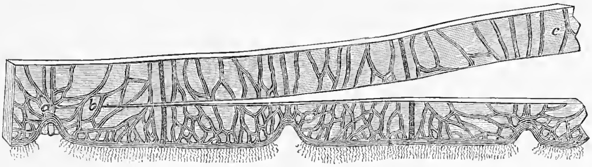
regarding them as muscle-waves, notwithstanding the difficulties connected with their sudden blocking in spiral strips &c., or regarding their passage as due to nervous elements, notwithstanding the difficulties connected with the supposition as to vicarious action which this view necessarily involved. But this year it has been shown that these same tissues manifest a function essentially nervous, and that this function is as difficult to destroy by section as is that on which the passage of contractile waves depends. And forasmuch as this function cannot be regarded as muscular, we are in this case compelled either to adopt the hypothesis as to vicarious action of nerve-fibres, or to abandon the whole subject as inexplicable. Thus it is evident that these later results affect the previous ones to this extent, that they remove any advantage we should otherwise gain on the side of simplicity by regarding the contractile waves as mere muscle-waves; for, even if we do so regard them, we must still face the old difficulty in another form"—KLEINENBERG's hypothesis as to the possible blending of the functions of nerve and muscle in the same tissue-elements having been shown, in a previous part of the omitted portion of this paper, to be here untenable.—September 1877.]

In such experiments it generally happens, as here represented, that reducing the width of a circular strip by one half produces no effect, or only a slight effect, on the rate, while further narrowing to the degree mentioned produces a conspicuous effect. I may also state that when, as occasionally happens, the immediate effect of narrowing a circular strip to one half is to temporarily block the contractile waves, when the latter again force their passage, their rate is slower than it was before. It seems as if the more pervious tissue-tracts having been destroyed by the section, the less pervious ones, though still able to convey the contractile wave, are not able to convey it so rapidly as were the more pervious tracts.

(*c*) In order to ascertain whether certain zones of the circular contractile sheet in all individuals habitually convey more of the contractile influence than do other zones, I tried a number of experiments in the following form of section. Having made a circular strip, I removed the lithocysts and then cut the strip as represented in fig. 11. On now stimulating the end *a*, the resulting contractile wave would bifurcate at *b*, and then pass on as two separate waves through the zones *b c*, *b d*. Now, as these two waves were started at the same instant of time, they ran, as it were, a race in the two zones and in this way the eye could judge with perfect ease which wave occupied the shortest time in reaching its destination. This experiment could be varied by again bisecting each of these two zones, thus making four zones in all, and four waves to run in each race. A number of experiments of this kind showed me that there is no constancy in the relative conductivity of the same zones in different individuals. In some instances the waves occupy less time in passing through the zone *b c* than in passing through the zone *b d*; in other instances the time in the two zones is equal; and, lastly, the converse of the first-mentioned case is of equally frequent occurrence. Very often the waves become blocked in *b c* while they continue to pass in *b d*, and *vice versâ*. Now all these various cases are what we might expect to occur, in view of the variable points at which contractile waves become blocked in spiral strips &c.; for if the contractile tissues are not functionally homogenous, and if the relatively pervious conductile tracts are not constant as to their position in different individuals, the results I have just described are the only ones that could be yielded by the experiments in question. Considering, however, that in these experiments the central zones are not so long as the

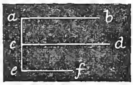
Fig. 11.



peripheral zones, I think it may fairly be said that the conductile power of the latter is greater than that of the former; for otherwise the above experiments ought to yield a large majority of races won by the waves that course through the central zones; and this is not the case. Indeed it is surprising how often the race is, as it were, neck and neck, thus showing that the relative conductivity of all the zones is precisely adjusted to their relative lengths; and forasmuch as in the unmutilated animal this adjustment must clearly serve the purpose of securing to the contractile waves a passage of uniform rate over the whole radius of the umbrella, I doubt not that if it were possible to perform the racecourse section without interrupting any of the lines of conductile tissue, neck and neck races would be of invariable occurrence. It may be added that all the above results are observable when the discharges of a single remaining lithocyst are substituted for artificial stimuli at the undivided end of a strip.

(*d*) Interdigitating cuts, as might be expected, prolong the time of contractile waves in their passage through the tissue in which the cuts are interposed. For example, in a spiral strip measuring 26 inches in length, the time required for the passage of a contractile wave from one end to the other is represented by the line *a b* in the annexed woodcut. But after twenty interdigitating cuts had been interposed, ten on each side of the strip, the time increased to *c d*—the line *e f* representing one second. And more severe forms of section are, of course, attended with a still more retarding influence.

Fig. 12.

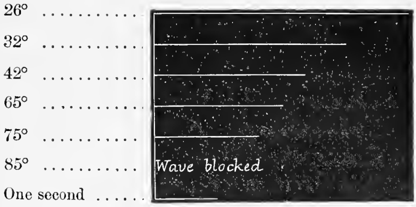


(*e*) The effects of temperature on the rate of contractile waves are very striking. For instance, in a rather narrow strip measuring 28 inches long and 1½ inch wide, the following variations in rate occurred:—

Temperature of water.	Time occupied in passage of contractile waves.
26°	4 seconds.
32°	3 seconds.
42°	2⅖ seconds.
65°	2 seconds.
75°	1⅗ second.
85°	Blocked.

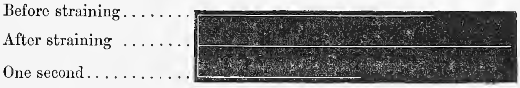
Or, adopting again the graphic method of statement, these variations may be represented as follows:—

Fig. 13.



(*f*) Submitting a contractile strip to slight strains has also the effect of retarding the rate of the waves while they pass through the portions of the strip which have been submitted to strain. The method of straining which I adopted was to pass my finger below the strip, and then, by raising my hand, to bring a portion of the strip slightly above the level of the water. The irritable, or contractile, surface was kept uppermost, and therefore suffered a gentle strain; for the weight of the tissue on either side of the finger made the upper surface somewhat convex. By passing the finger all the way along the strip in this way, the latter might be gently strained throughout its entire length, the degree of straining being determined by the height out of the water to which the tissue was raised. Of course if the strip is too greatly strained, the contractile waves become blocked altogether, as described in my last paper; but shortly before this degree of straining was reached, I could generally observe that the rate of the waves was diminished. To give one instance, a contractile strip measuring 22 inches had the rate of its waves taken before and after straining of the kind described. The result was as follows:—

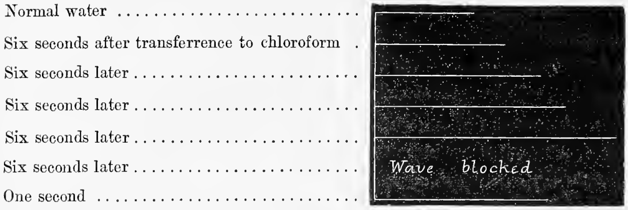
Fig. 14.



Immediately after severe handling of this kind, the retardation of contractile waves is sometimes even more marked than here represented; but I think this may be partly due to shock; for on giving the tissue a little while to recover, the rate of the waves becomes slightly increased.

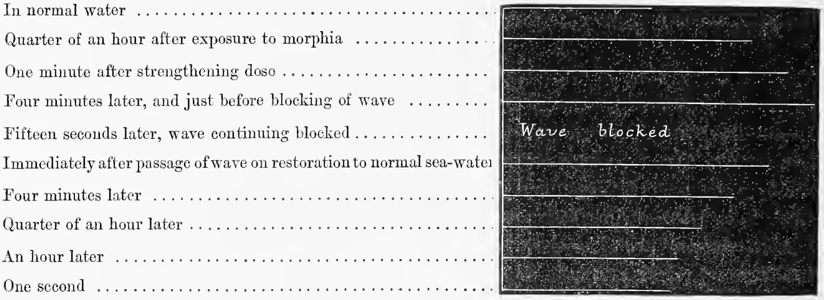
(*g*) Anæsthetics likewise have the effect of slowing the rate of contractile waves before blocking them. Taking, for instance, the case of chloroform, a narrow spiral strip 28 inches long was immersed in sea-water containing a large dose of the anæsthetic; the observations being taken at six seconds' intervals, the following were the results:—

Fig. 15.



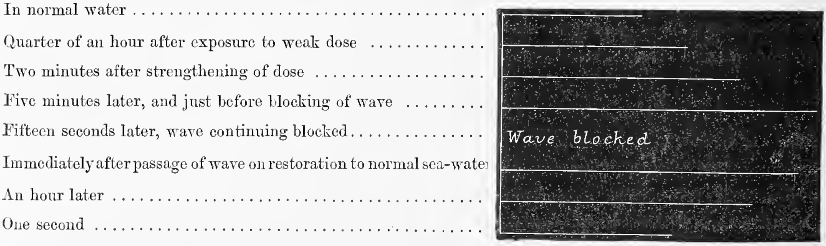
In such experiments, the recovery of the normal rate in unpoisoned water is gradual. Taking, for instance, the case of a spiral strip in morphia:—

Fig. 16.



From this it will be seen that the original rate did not fully return. Some substances, however, exert a more marked permanent effect of this kind than do weak solutions of morphia. Here, for instance, is an experiment with alcohol:—

Fig. 17.



(*h*) From these experiments, however, it must not be definitely concluded that it is the anæsthesiating property of such substances which exerts this slowing and blocking influence on contractile waves; for I find that almost any foreign substance, whether or not an anæsthetic, will do the same. That nitrite of amyl, caffein, &c. should do so, one would not be very surprised to hear; but it would scarcely be expected that strychnine, for instance, should block contractile waves; yet it does so even in doses so small as only just to taste bitter. Nay, even fresh water completely blocks contractile waves after the strip has been exposed to its influence for about half an hour, and exerts a permanently slowing effect after the tissue is restored to sea-water. These facts show the extreme sensitiveness of the nervo-muscular tissues of the Medusæ to any change in the character of their surrounding medium—a sensitiveness which we shall again have occasion to comment upon when treating of the effects of poisons.

(*i*) In conclusion, I may mention an interesting fact which is probably connected with the summation of stimuli before explained. When a contractile strip is allowed to rest for a minute or more, and a wave is then made to traverse it, careful observation will show that the passage of this wave is slower than that of its successor, provided the latter follows the former after not too great an interval of time. The difference, however, is exceedingly slight, so that to render it apparent at all the longest possible strips must be used; and even then the experimenter may fail to detect the difference, unless he has been accustomed to signalling, by which method all these observations on rate have to be made.

(B) *Stimulus-waves.*—(*a*) The rate of transmission of tentacular waves is only one half that of contractile waves, viz. 9 inches a second. This fact appeared to me very remarkable, in view of the consideration that the tentacular wave is the optical expression of a stimulus-wave, and that there can be no conceivable use in a stimulus-wave being able to pass through contractile tissue independently of a contractile wave, unless the former is able to travel more rapidly than the latter; for the only conceivable use of the stimulus-wave is to establish physiological harmony between different parts of the organism; and if this wave cannot travel more rapidly than a contractile wave which starts from the same point, it would clearly fail to perform this function.

In view of this anomaly I was led to think that if the rate of the stimulus-wave is dependent in a large degree on the strength of the stimulus that starts it, the slow rate of 9 inches a second might be more than doubled, if, instead of using a stimulus so gentle as not to start a contractile wave, I used a stimulus sufficiently strong to do this. Accordingly I chose a specimen of *Aurelia* wherein the occurrence of tentacular waves was very conspicuous, and found, as I had hoped, that every time I stimulated too gently to start a contractile wave, the tentacular wave travelled only at the rate of 9 inches a second, whereas if I stimulated with greater intensity I could always observe the tentacular wave coursing an inch or two in front of the contractile wave.

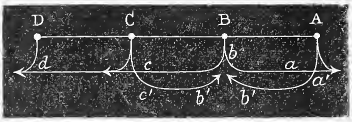
(*b*) It is remarkable, however, that in this, as in all the other specimens of *Aurelia* which I experimented upon, the reflex response of the polypite was equally long whatever strength of stimulus I applied to the swimming-bell; or, at any rate, the time was only slightly less when a contractile wave had passed than when only a tentacular wave had done so. The loss of time, however, appears to take place in the polypite itself, where the rate of response is astonishingly slow. Thus if one lobe be irritated, it is usually from four to eight seconds before the other lobes respond. But the time required for such sympathetic response may be even more variable than this, the limits I have observed being as great as from three to ten seconds. In all cases, however, the response, when it does occur, is sudden, as if the distant lobe had then for the first time received the stimulus. Moreover one lobe—usually one of those adjacent to the lobe directly irritated—responds before the other two, and then a variable time afterwards the latter also respond. This time is in most cases comparatively short, the usual limits being from a quarter of a second to two seconds. How much of these enormous intervals is occupied by the period of ganglionic latency, and how much by that of transmission, it is impossible to say; but I have determined that the rate of transmission from the end of a lobe of the polypite to a lithocyst (deducting a second for the double period of latent stimulation) is the same as the rate of a tentacular wave, viz. 9 inches a second. The presumption, therefore, is that the immense lapse of time required for reflex response on the part of the lobes is due to a prolonged period of latent stimulation on the part of the lobular ganglia, or whatever element it is that here performs the ganglionic function. And, in any case, it may be remarked, this wonderful tardiness of response on the part of the polypite of *Aurelia aurita* is strikingly similar to that of *Tiaropsis indicans*, where the rate of transmission of a stimulus in the bell, though very slow, is rapid as compared with the time that elapses before the polypite makes its sudden crouching movement. Both these cases, it will be noticed, stand in marked contrast with that of *Sarsia*, where response by the polypite to irritation of the bell or tentacles is exceedingly rapid.

V. COORDINATION.

§ 1. *Covered-eyed Medusæ.*—(*a*) From the fact that in the covered-eyed Medusæ the passage of a stimulus-wave is not more rapid than that of a contractile wave, we may be prepared to expect that in these animals the action of the locomotor ganglia is not, in any proper sense of the term, a coordinated action; for if a stimulus-wave cannot outrun a contractile wave, one ganglion cannot know that another ganglion has discharged its influence, till the contractile wave which results from a discharge of the active ganglion has reached the passive one. And this I find to be usually the case; for, as previously stated, it may usually be observed that one or more of the lithocysts are either temporarily or permanently prepotent over the others, *i. e.* that contractile waves emanate from the prepotent lithocysts, and then spread rapidly over the swimming-bell, without there being any signs of coordinated, or simultaneous, action on the part of the other lithocysts. Nevertheless in many cases such prepotency cannot, even with the greatest care, be observed; but upon every pulsation all parts of the swimming-bell seem to contract at the same instant. And this apparently perfect coordination among the eight marginal ganglia may continue for any length of time. I believe, however, that such apparently complete physiological harmony is not coordination properly so called, *i. e.* is not due to special nervous connexions between the ganglia; for, if such were the case, perfectly synchronous action of this kind ought to be the rule rather than the exception. I am therefore inclined to account for these cases of perfectly synchronous action by supposing that all, or most, of the ganglia require exactly the same time for their nutrition; that they are, further, of exactly equal potency in relation to the resistance (or excitability) of the surrounding contractile tissues; and that, therefore, the balance of force being exactly equal in the case of all, or most, of the ganglia, their rhythm, though perfectly identical, is really independent. I confess, however, that I am by no means certain regarding the accuracy of this conclusion, as it is founded on negative rather than on positive considerations. That is to say, I arrive at this conclusion regarding the cases in which such apparent coordination is observable only because in other cases such apparent coordination is *not* observable, and also, I may add, because my experiments in section have *not* revealed any evidence of nervous connexions capable of conducting a stimulus-wave with greater rapidity than a contractile wave. I therefore consider this conclusion an uncertain one; and its uncertainty is perhaps still further increased by the result of the following experiments. If a covered-eyed Medusa be chosen in which perfectly synchronous action of the ganglia is observable, and if a deep radial incision be made between each pair of adjacent ganglia, the incisions being thus eight in number and carried either from the margin towards the centre or *vice versâ*, it then becomes conspicuous enough that the eight partially divided segments no longer present synchronous action; for now one segment and now another takes the initiative in starting a contractile wave, which is then propagated to the other segments. And it is evident that this fact tends to negative the above explanation; for if the discharges of the ganglia are independently simultaneous before section, we might expect them to continue so after section. It must be remembered, however, that the form of section we are considering is a severe one, and that it must therefore not only give rise to general shock, but also greatly interfere with the passage of contractile waves, and, in general, disturb the delicate conditions on which, according to the suggested explanation, the previous harmony depended. Besides, we have before seen that for some reason or other segmentation of a Medusa profoundly modifies the rate of its rhythm. In view of these considerations, therefore, the results yielded by such experiments must not be regarded as having any conclusive bearing on the question before us. And as these or similar objections apply to various other modes of section by which I have endeavoured to settle this question, I will not here occupy space in detailing them. It seems desirable, however, in this connexion again to mention a fact briefly stated in my former paper, namely, that section conclusively proves a contractile wave to have the power, when it reaches a lithocyst, of stimulating the latter into activity; for "it is not difficult to obtain a series of lithocysts connected in such a manner that the resistance offered to the passage of the waves by a certain width of the junction-tissue is such as just to allow the residuum of the contractile wave which emanates from one lithocyst to reach the adjacent lithocyst, thus causing it to originate another wave which, in turn, is just able to pass to the next lithocyst in the series, and so on, each lithocyst in turn acting like a reinforcing battery to the passage of the contractile wave"[*]. Now this fact, I think, sufficiently explains the mechanism of ganglionic action in those cases where one or more lithocysts are prepotent over the others; that is to say, the prepotent lithocyst first originates a contractile wave, which is then successively reinforced by all the other lithocysts during its passage round the swimming-bell. In this way the passage of a contractile wave is no doubt somewhat accelerated; for I found, in marginal strips, that the rate of transit from a terminal lithocyst to the other end of the strip was somewhat lowered by excising the seven intermediate lithocysts.

(*b*) I may here state, in passing, a point of some little interest in connexion with this reinforcing action of lithocysts. When I first observed this action, it appeared to me a mysterious thing why its result was always to propagate the contractile wave in only one direction—the direction, namely, in which the wave happened to be passing before it reached the lithocyst.

Fig. 18.



For instance, suppose we have a strip A D with a lithocyst at each of the equidistant points A B C D. Suppose now that the lithocyst B originates a stimulus: the resulting contractile wave passes, of course, with equal rapidity in the two opposite directions, B A, B C (arrows $b\,a$, $b\,c$). The contractile wave $b\,a$ therefore reaches the lithocyst A at the same time as the contractile wave $b\,c$ reaches the lithocyst C; and so both A and C discharge simultaneously. What, then, should we expect to be the result? I think we should expect the wave $b\,c$ to continue on its course to D, after having been strengthened at C, and a *reflex* wave $a'\,b'$ to start from A (owing to the discharge at A) which would reach B at the same time as a similar *reflex* wave $c'\,b'$ starting from C (owing to the discharge at C); so that by the time the original wave $b\,c\,d$ had reached D, the point B would be the seat of a collision between the two reflex waves $a'\,b'$ and $c'\,b'$. And, not to push the supposed case further, it is evident that if such reflex waves were to occur, the resulting confusion would very soon require to end in tetanus. As a matter of fact, these reflex waves do not occur; and the question is, why do they not? Why is it that a wave is only reinforced in the direction in which it happens to be travelling—so that if, for instance, it happens to start from A in the above series, it is successively propagated by B C in the direction A B C D, and in that direction only, whereas, if it happens to start from D, it is propagated by the same lithocysts in the opposite direction, D C B A, and in that direction only?—the wave in the one case terminating at the

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[*] Croonian Lecture, 1876, Phil. Trans. p. 311.

lithocyst D, and in the other case at the lithocyst A. Now, although this absence of reflex waves appears at first sight mysterious, it admits of an exceedingly simple explanation. It will be remembered, from my experiments on stimulation, that the contractile tissues of the covered-eyed Medusæ cannot be made to respond to two successive stimuli of minimal, or but slightly more than minimal, intensity, unless such stimuli are separated from one another by a certain considerable interval of time. Now, when in the above illustration the contractile wave starts from A, by the time it reaches B the portion of tissue included between A and B has just been in contraction in response to the stimulus from A, while the portion of tissue included between B and C has not been in contraction. Consequently the stimulus resulting from a ganglionic discharge being presumably of minimal, or but slightly more than minimal, intensity, the tissue included between A and B will not respond to the discharge of B; while the tissue included between B and C, not having been just previously in contraction, will respond; and conversely, of course, if the contractile wave had been travelling in the opposite direction.

(*c*) Seeing that this explanation is the only one possible, and that it, moreover, follows as a deductive necessity from my experiments on stimulation, I think there is no need to detail any of the further experiments which I made with the view of confirming it. But the following experiment, devised to confirm this explanation, is of interest in itself, and on this account I shall state it. Having prepared a contractile strip with a single remaining lithocyst at one end, I noted the rhythm exhibited by this lithocyst, and then imitated that rhythm by means of single induced shocks thrown in with a key at the other end of the strip. The effect of these shocks was, of course, to cause the contractile waves to pass in the direction opposite to that in which they passed when originated by the lithocyst. Now I found, as I had expected, that so long as I continued exactly to imitate the rate of the ganglionic rhythm, so long did the waves always pass in the direction B A—A being the lithocyst, and B the other end of the strip. I also found that if I allowed the rate of the artificially caused rhythm to sink slightly below that of the natural rhythm, after every one to six waves (the number depending on the degree in which the rate of succession of my induction-shocks approximated to the rate of the natural rhythm) which passed from B to A, one would pass from A to B*.

Of course the only interpretation to be put on these facts is, that every time an artificially started wave reached the terminal ganglion it caused the latter to discharge; but that the occurrence of a discharge could not in this case be rendered apparent, because of the inadequacy of that discharge to start a reflex wave. But that such discharges always took place was manifest, both *à priori* because from analogy we may be sure that if there had happened to be any contractile tissue of appropriate width on the other side of the ganglion, the discharge of the latter would have been rendered apparent, and *à*

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* When two such waves met, they neutralized each other at their line of collision—or perhaps, more correctly, the tissue on each side of that line, having just been in contraction, was not able again to convey a contractile wave passing in the opposite direction to the wave which it had conveyed immediately before.

posteriori because, after the arrival of every artificially started wave, the time required for the ganglion to originate another wave was precisely the same as if it had itself originated the previous wave.

(*d*) In view of these results it occurred to me, as an interesting experiment, to try the effect on the natural rhythm of exhausting a ganglion thus situated by throwing in a great number of shocks at the other end of the strip. I found that after 500 single shocks had been thrown in with a rapidity almost sufficient to tetanize the strip, immediately after the stimulation ceased, the natural rhythm of the ganglion, which had previously been 20 in the minute, fell to 14 for the first minute, 18 for the second, and the original rate of 20 for the third. In such experiments the diminution of rate is most conspicuous during the first 15 or 30 seconds of the first minute. Sometimes there are no contractions at all for the first 15 seconds after cessation of the stimulating process; and in such cases the natural rhythm, when it first begins, may be as slow as one half or even one quarter its normal rate. All these effects admit of being produced equally well, and with less trouble, by faradizing the strip, when it may be even better observed how prolonged may be the stimulation without causing any thing further than such slight exhaustion of the ganglion as the above results imply*.

§ 2. *Naked-eyed Medusæ.*—(*a*) It would be impossible to imagine movements, on the part of so simple an organism, more indicative of physiological harmony than are the movements of *Sarsia*. One may watch several hundreds of these animals while they are swimming about in the same bell-jar, and never perceive, as in the covered-eyed

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* In this description I have everywhere adopted the current phraseology with regard to ganglionic action—a phraseology which embodies the theory of ganglia supplying interrupted stimulation. But although I have done this for the sake of clearness, of course it will be seen that the facts harmonize equally well with the theory of continuous stimulation which I am engaged in working out, but the publication of which is for the present postponed. (See, for an outline of the new theory, the abstract of this paper in the Proc. R. S. vol. xxv.) Indeed the fact last mentioned in the text would agree with the new better than with the old theory of ganglionic action; for my other experiments on the deganglionated tissues of *Aurelia* prove how difficult these tissues are to exhaust. Hence if the natural rhythm in the case of the above-mentioned experiments is due to the contractile, as distinguished from the ganglionic, element, we should expect the period of quiescence between cessation of the faradaic stimulus and the renewal of spontaneity to be short; because this interval, according to the new theory, is the expression of the time required for the contractile tissue to recover exhaustion sufficiently to enable the ganglionic stimulation—which had become less than minimal in relation to the diminished excitability of the contractile tissue—to become minimal. I may also state that, according to the new theory, the reason why a contractile wave is reinforced while passing a lithocyst is probably the same as that to which all reflex action is by that theory supposed to be due, viz. the molecular disturbance in an afferent nerve being of the nature of a vibration, and the continuous somewhat less than minimal stimulation supplied by the ganglion being also of the nature of a vibration, if the wave-rates in these two vibrations are supposed *synchronous*, when the afferent disturbance reaches the ganglion the amplitude of the ganglionic vibrations would be somewhat increased. Hence the stimulation, which was previously being continuously supplied by the ganglionic vibrations, and which before being augmented by the arrival of the afferent vibrations was somewhat less than minimal, would now become minimal or somewhat more than minimal. And, conversely, the facts of *inhibition* are explained by this theory of ganglionic action as due to an *interference* of vibrations the rates of which are *not* synchronous.

Medusæ, the slightest want of ganglionic coordination exhibited by any of the specimens. Moreover, that the ganglionic coordination is in this case wonderfully far advanced, is proved by the fact of members of this genus being able to steer themselves while following a light, as described in my last paper*.

In the discophorous species of naked-eyed Medusæ, however, perfectly coordinated action is by no means of such invariable occurrence as it is in *Sarsia*; for although in perfectly healthy and vigorous specimens systole and diastole occur at the same instant over the whole nectocalyx, this harmoniously acting mechanism is very liable to be thrown out of gear; so that when the animals are suffering in the least degree from any injurious conditions—often too slight and obscure to admit of discernment—the swimming movements are no longer synchronous over the whole nectocalyx; but now one part is in systole while another part is in diastole, and now several parts may be in diastole while other parts are in systole. And, as in these animals very slight causes seem sufficient thus to impair the ganglionic coordination, it generally happens that in a bell-jar containing a number of specimens belonging to different species, numerous examples of more or less irregular swimming movements are observable†.

(*b*) Taking, then, the case of *Sarsia* first, from my previous observations on the physiological harmony subsisting between the tentacles, I was led to expect that the coordination of the locomotor ganglia was probably effected by means of the same tissue-tracts through which the intertentacular harmony was effected, viz. those situated in the margin of the bell. Accordingly, I introduced four short radial cuts, one midway between each pair of adjacent marginal bodies. The coordination, however, was not

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* Removing the polypite does not interfere with this steering action; but if any considerable portion of the margin is excised, the animal seems no longer able to find the beam of light, even though one or more of the marginal bodies are left *in situ*.

† I think it is worth while to describe a peculiar irregularity of a very definite kind which is occasionally observable in *Tiaropsis diademata*. I have only noticed it in the case of tolerably vigorous specimens. It consists in the simultaneous contraction of two opposite quadrants of the nectocalyx, while the other two quadrants remain passive. The effect of the systole is therefore to convert the whole nectocalyx into an almost linear form. Diastole then follows as usual; but in the next systole the two quadrants which had previously been passive now contract, while the two which had previously contracted now remain passive. The nectocalyx, therefore, again assumes a linear form, but this time in a direction at right angles to that in which it had previously done so. Diastole then again ensues, and the third systole resembles the first one, the fourth the second, and so on—the nectocalyx alternately assuming the linear form in each of the two opposite directions. These peculiar motions may continue for a long time without interruption. Their rhythm is always perfect, and in its rate precisely resembles that which the same animal exhibits when swimming normally. The comparison is easily made, because every now and then the abnormal motions become suddenly converted into the normal ones, and *vice versâ*. The time during which a normal or an abnormal bout continues is variable. The manifestation of such abnormal swimming-bouts appears to be an idiosyncracy confined to a very small percentage of individuals; for, while one may observe a thousand specimens for any length of time without perceiving the occurrence of these peculiar movements, if there happens to be an additional specimen presenting the idiosyncracy in question, every time the observer goes to look at it he may feel that there is about an equal chance of his seeing the normal or the abnormal movements.

perceptibly impaired. I therefore continued the radial cuts, and found that when these reached one half or two thirds of the way up the sides of the inner bell (or contractile sheet), the coordination became visibly affected, and this for the first time. These experiments, however, did not satisfy me that the coordination was not chiefly, or exclusively, due to the marginal nerves; for the bell of *Sarsia* is so small, and contractile waves are in this genus so rapid, that the following hypothesis still remained open. When the whole margin of *Sarsia* is removed and the paralyzed bell stimulated, so far as the eye can judge the resulting contraction is simultaneous over the entire bell. Whether this rapid conduction of contractile influence from the seat of stimulation to all the other parts of the bell is due to muscle or to nerve, is here of no consequence; for, in view merely of the fact of such rapid conduction taking place, it follows that when the four short radial cuts are introduced, even if these cuts destroy all the nerves by which the ganglionic coordination is effected, such coordination would still *appear* to be effected in consequence of the rapid conduction of a mere contractile wave over the whole muscle-sheet from the ganglion which first happens to discharge. And, if this is the correct interpretation, we should expect the loss of coordination first to become apparent when the radial incisions reach about halfway up the bell; for, under this form of section, it is then only that a stimulus applied to the margin of a *deganglionated*, or *paralyzed*, bell can be seen to cause in the bell a rapid contractile wave as distinguished from an apparently simultaneous contraction of the entire muscle-sheet. Against this interpretation it may be urged, that even although the discharge of a ganglion thus isolated from its fellows would certainly give rise to the erroneous appearance of a coordinated discharge of all the ganglia, still, if coordination is destroyed by the short radial cuts, we should expect this destruction to become observable in consequence of the ganglia in the four quadrants of the bell discharging independently of one another as to time, and therefore as a total effect producing a flurried movement of the bell, instead of the single decided systole followed by a short but perfectly inactive period of diastole. This objection, however, though natural, is not, I think, valid; for we have seen in the last section that, in the case of *Aurelia*, a contractile wave has the effect, when it reaches a locomotor ganglion, of causing the latter to discharge [§1(*a*)]; and the same thing is therefore presumably true in the case of *Sarsia*. Consequently, when in the latter genus ganglionic coordination has been destroyed by the four short radial cuts, and when any of the separated ganglia originates a discharge, all the other ganglia will immediately afterwards do the same, because stimulated by the passage of the contractile wave. Now, as all the ganglia were previously accustomed to act in consort, the time required for their nutrition after every discharge must be nearly or quite equal in the case of each of the ganglia; so that when, after the physiological harmony has been destroyed, one of their number originates a discharge, and when, as a consequence, all the others immediately afterwards do the same, the degree of exhaustion will be nearly or quite equal in the case of all the ganglia; and therefore the time that will elapse before any of their number is again able to originate a discharge will be the same as if their physiological harmony had never been destroyed*.

In view of these considerations I tried the following experiments. Instead of beginning the radial cuts from the margin, I began them from the apex of the cone; and I found that however many of such cuts I introduced, and however far down the cone I carried them, so long as I did not actually sever the margin so long did all the divisions of the bell continue to contract simultaneously†. This fact therefore proves at least this much, that whether or not in the previous experiments true ganglionic coordination was effected through the upper zones of the bell, the margin of the bell is alone sufficient to maintain such coordination.

The next experiment I tried was to make four short radial incisions in the margin as before described, and then to continue *one* of these incisions the whole way up the bell. By careful observation I could now perceive that all the marginal ganglia did not discharge simultaneously; for when those situated nearest to the long radial cut happened to take the initiative, the resulting contractile wave, having double the distance to travel which it would have had if the long radial cut had been absent (or, as in the experiment described at the beginning of this subsection (*b*), if this radial cut had only been of the same length as the other three radial cuts), could now be followed by the eye in its very rapid course round the bell. Now the fact that in this form of section I was able to detect the passage of a *wave*, proves that the three short radial sections had destroyed the coordinated action of the marginal ganglia.

From these experiments, then, I conclude that in this genus ganglionic coordination, in the strict sense of the term, is effected exclusively by means of the marginal nerves. And as these experiments on *Sarsia* are exceedingly difficult to conduct, owing to the very rapid passage of contractile waves, it is satisfactory to find that this conclusion is further supported by the analogy which the other species of a naked-eyed Medusa afford, and to the consideration of which we shall now proceed.

(*c*) The effects of four short radial incisions through the margin of any species of *Tiaropsis*, *Thaumantias*, *Staurophora*, &c. are usually very conspicuous. Each of the quadrants included between two adjacent incisions shows a strong tendency to assume an independent action of its own. This tendency is sometimes so pronounced as to amount almost to a total destruction of contractional continuity between two or more quadrants of the bell; but more usually the effect of the marginal sections is merely that of destroying excitational continuity, or at least physiological harmony. In vigorous

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* As in the last section, I adhere in the text to the current theory of ganglionic action; but the above considerations apply equally well on the new theory. These considerations are the same as have already been employed in an abridged form with reference to *Aurelia*, at the beginning of the last section.

† This could be particularly well seen if, after the extreme apex of the cone had been removed, one of the four radial cuts was continued through the margin, and the latter was then spread out into a linear form by gently pressing the animal against the flat side of the glass vessel in which it was contained. The same experiment performed on *Aurelia* is, of course, attended with a totally different result, now one segment and now another originating a discharge which then spreads to all the others in the form of a contractile wave.

specimens this effect is sometimes only observable, or best observable, at the commencement of a swimming-bout; for in such specimens, after a greater or less number of uncoordinated contractions, the four quadrants begin to agree, as it were, upon their rhythm. This, of course, is what might be expected on the above hypothesis regarding contractile waves eliciting ganglionic discharges, and all the ganglia requiring about the same time for their nutrition; and the reason why the loss of coordination under this form of section is more marked in the discophorous species of naked-eyed Medusæ than in *Sarsia*, is doubtless merely because the passage of contractile waves is so much more slow and so much less energetic. Therefore in vigorous specimens, where these waves pass more readily, we obtain results under this form of section most conformable to those which we obtain in *Sarsia*, viz. a sort of fictitious coordination which begins to come on after a variable number of contractions have been given, and which, as just stated, is doubtless due to the fact of all the marginal ganglia requiring the same time for their recovery from the exhaustion consequent on the occurrence of each discharge.

(*d*) It is a remarkable thing that this form of section, although in actual amount so very slight, is attended with a much more pernicious influence on the vitality of the organism than is any amount of section of the general contractile tissues. Thus if a specimen of *Tiaropsis*, for example, be chosen which is swimming about with the utmost vigour, and if four equidistant radial cuts only just long enough to sever the marginal canal be made, the animal will soon begin to show symptoms of enfeeblement, and within an hour or two after the operation will probably have ceased its swimming motions altogether. The animal, however, is not actually dead; for if, while lying motionless at the bottom of the vessel, it be gently stimulated, it will respond with a spasm as usual, and perhaps immediately afterwards give a short and feeble bout of swimming. These surprisingly pernicious results are not so conspicuous in the case of *Sarsia*, although in this genus likewise they are sufficiently well marked to be unmistakable. I here append a Table to show the comparative effects of the operation in question on different species. The cases may be regarded as very usual ones, though it often happens that a longer time after the operation must elapse before the enfeebling effects become so pronounced.

Name of species.	Number of contractions during five minutes before operation.	Number during one minute after operation.	Number during five minutes after operation.	Ultimate effects.
Tiaropsis diademata	57	11	0	Permanent rest.
—— indicans	148	23	0	„ „
—— polydiademata	102	18	0	„ „
—— oligoplocama	131	39	0	„ „
Sarsia tubulosa	144	56	14	„ „

This decided effect of so slight a mutilation will not perhaps appear to other physiologists so noteworthy as it appears to me; for no one who has not witnessed the experiments can form an adequate idea of the amount of mutilation of any parts, other than their margins, which the Medusæ will endure without suffering even from the effects of shock. Another point worth mentioning with regard to the operation we are considering is that not unfrequently the interruptions of the margin, which have been produced artificially, begin to extend themselves through the nectocalyx in a radial direction; so that in some cases this organ becomes spontaneously segmented into four quadrants, which remain connected only by the apical tissue of the bell. I do not think that this is due to the mere mechanical tearing of the tissues as a consequence of the swimming motions, for the latter seem too feeble to admit of their producing such an effect.

(*e*) In conclusion, I may state that I have been able temporarily to destroy the ganglionic coordination of *Sarsia* by submitting the animals to severe nervous shock. The method I employed to produce the nervous shock without causing mutilation was to take the animal out of the water for a few seconds while I laid it on a small anvil, which I then struck violently with a hammer. On immediately afterwards restoring the Medusa to sea-water, spontaneity was found to have ceased, while irritability remained. After a time spontaneity began to return, and its first stages were marked by a complete want of coordination; soon, however, coordination was again restored. But this experiment by no means invariably yielded the same result. Spontaneity, indeed, was invariably suspended for a time; but its first return was not invariably, or even generally, marked by an absence of coordination, even though I had previously struck the anvil a number of times in succession. I was therefore led to try another method of producing nervous shock; and this I found a more effectual method than the one just described. It consisted in violently shaking the *Sarsia* in a bottle half filled with sea-water. I was surprised to find how violent and prolonged such shaking might be without any part of the apparently friable organism, except perhaps the tentacles and polypite, being broken or torn. The subsequent effects of shock were remarkable. For some little time after their restoration to the bell-jar, the *Sarsiæ* had lost, not only their spontaneity, but also their irritability; for they would not respond even to the strongest stimulation. In the course of a few minutes, however, peripheral or muscular irritability returned, as shown by responses to nipping of the nervo-muscular sheet. The animals were now in the same condition as when anæsthesiated by caffein or other central nerve poison; but in a few minutes later central or reflex irritability also returned, as shown by single responses to single nippings of the tentacles. Last of all spontaneity began to return, and was in some few cases conspicuously marked by a want of coordination—all parts of the margin originating impulses at different times with the result of producing a continuous flurried, or shivering, movement of the nectocalyx. After a time, however, these movements became coordinated; but in most cases when a swimming-bout had ended and a pause intervened, the next swimming-bout was always inaugurated by a period of shivering before coordination became established. This effect might last for a long time; but eventually it, too, disappeared—the swimming-bouts then beginning with coordinated action in the usual way.

VI. POISONS.

The last subject of which I shall treat in the present communication is the effects of various muscle- and nerve-poisons on the locomotor system of Medusæ. In my former paper I dealt briefly with this subject; but, having this year bestowed a considerable amount of additional labour upon it, I shall now discuss it at somewhat greater length. The subject appears to me one of considerable importance, seeing that the effects of the muscle- and nerve-poisons have never, as yet, been systematically tried on the Medusæ—animals that present us with the earliest appearance both of muscle and of nerve. I make this remark in order to excuse myself for burdening this already over-lengthy paper with a number of additional facts, which will be of interest only to the physiologist.

§ 1. *Chloroform.*—My observations with regard to the distribution of nerves in *Sarsia* led me to investigate the order in which these connexions are destroyed, or temporarily impaired, by anæsthetics. The results, I think, are worth recording. In *Sarsia*, then, the following phases always mark the progress of anæsthesia by chloroform &c.:—1. Spontaneity ceases, as described in my former paper. 2. On now nipping a tentacle, pulling the polypite, or irritating the bell, a *single* locomotor contraction is given in answer to every stimulation. (In the unanæsthesiated animal a *series* of such contractions would be the result of such stimulation.) 3. After locomotor contractions can no longer be elicited by stimuli, nipping a tentacle or the margin of the bell has the effect of causing the polypite to contract. 4. After stimulation of any part of the nectocalyx (including tentacles) fails to produce response in any part of the organism, the polypite will continue its response to stimuli applied directly to itself.

§ 2. *Nitrite of Amyl.*—On *Sarsia* the effect of this agent is much the same as that of chloroform—the description just given in § 1 being quite as applicable to the effects of the nitrite as to those of chloroform. Before the loss of spontaneity supervenes, the rate of the rhythm is increased, while the strength of the pulsations is diminished.

Tiaropsis diademata, from the fact of its presenting a very regular rhythm and being but of small size, is a particularly suitable species upon which to conduct many experiments relating to the effects of poisons. On this species the nitrite in appropriate (*i. e.* in very small) doses first causes irregularity and enfeeblement of the contractions, together with quickening of the rhythm. After a short time, a gradual cessation of the swimming-motions becomes apparent—these motions dying out more gradually, for example, than they do under the influence of chloroform. Eventually each pulsation is marked only by a slight contraction of the muscular tissue in the immediate neighbourhood of the margin. If the dose has been stronger, however, well-marked spasmodic contractions come on and obliterate such gradual working of the poison. In all cases irritability of all parts of the animal persists for a long time after entire cessation of spontaneous movements—perhaps for three or four minutes in not over-poisoned animals; but eventually it too disappears. On being now transferred to normal sea-water, the process of recovery is slower than it is after anæsthesiation by chloroform. It is interesting, moreover, to observe, that just as the power of coordination was the first thing to be affected by the nitrite, so it is the last thing to return during recovery.

§ 3. *Caffein.*—The effects of caffein on *Sarsia* may be best studied by immersing the animals in a saturated sea-water solution of the substance. In such solutions the Medusæ float to the surface, in consequence of their lower specific gravity. I therefore used shallow vessels, in order that the margins of the nectocalyces might rest in the level of the water that was thoroughly saturated. The immediate effect of suddenly immersing *Sarsia* in such a solution is very greatly to increase the rate of the pulsations, and, at the same time, to diminish their potency. The appearance presented by the swimming-motions is therefore that of a fluttering nature; and such motions are not nearly so effectual for progression as are the normal pulsations in unpoisoned water. This stage, however, only lasts for a few seconds, after which the spontaneous motions begin gradually to fade away. Soon they altogether cease, though occasionally one among a number of *Sarsiæ* confined in the same saturated solution will continue, even for several minutes after the first immersion, to give one or two very feeble contractions at long intervals. Eventually, however, all spontaneity ceases on the part of all the specimens; and now the latter will continue for a very long time to be sensitive to stimulation. At first *several* feeble locomotor contractions will be given in response to each stimulus; and as on the one hand these contractions never originate spontaneously, while, on the other hand, *paralyzed Sarsiæ* never respond to a single stimulus with more than a single contraction, these multiple responses must, I think, be ascribed to a state of exalted reflex irritability. After a longer exposure to the poison, however, only a single response is given to each stimulus; and still later all irritability ceases. On now transferring the *Sarsia* to unpoisoned water, recovery is effected even though the previous exposure has been of immensely long duration, *e. g.* an hour.

An interesting point with regard to caffein-poisoning of *Sarsia* is, that as soon as spontaneity ceases the tentacles and polypite lose their tonus and become relaxed to their utmost extent. This is not the case with anæsthesiation by chloroform, even when pushed to the extent of suspending irritability. If, however, *Sarsiæ* which have been anæsthesiated to this extent in chloroform be suddenly transferred to a solution of caffein, the tentacles and polypite may soon be seen to relax, and eventually these organs lose their tonus as completely as if the anæsthesia had from the first been produced by the caffein. Moreover in this experiment the irritability, which had been destroyed by the chloroform, returns in the solution of caffein—provided the latter be not quite saturated—though spontaneity of course remains suspended throughout.

The effects of graduating the doses of caffein may be stated in connexion with another species, viz. *Tiaropsis diademata.* In a weak solution the effects are a quickening of the pulsations (*e. g.* from 64 to 120 per minute) together with a lessening of their force. On slightly increasing the dose, the pulsations become languid, and prolonged pauses supervene. If the dose is again somewhat strengthened, the pulsations become weaker and weaker, till they eventually cease altogether. The animal, however, is now in a condition of exalted reflex irritability; for its response to a single stimulus consists not merely, as in the unpoisoned animal, of a single spasm, but also, immediately after this, of a series of convulsive movements somewhat resembling swimming-movements destitute of coordination. If the strength of the solution be now again increased, a stage of deeper anæsthesiation may be produced, in which the Medusa will only respond to each stimulation by a single spasm. In still stronger solutions the only response is a single feeble contraction; while in a nearly saturated solution the animal does not respond at all. But even from a saturated solution *Tiaropsis diademata* will recover when transferred to unpoisoned water.

§ 4. *Strychnia.*—In my former paper I confined my remarks with reference to strychnia to the effects of this poison on a species of covered-eyed Medusa. It seems desirable, therefore, to supplement these remarks with a few additional ones regarding the effects of this poison on the naked-eyed Medusæ. In the case of *Sarsia* the symptoms of strychnia-poisoning are not well marked, from the fact that in this species convulsions always take the form of locomotor contractions. The symptoms, however, are in some respects anomalous. They are as follows:—First of all the swimming-motions become considerably accelerated, periods of quiescence intervening between abnormally active bouts of swimming. By-and-by a state of continuous quiescence comes on, during which the animal is not responsive to tentacular irritation, but remains so to direct muscular irritation, giving one response to each direct stimulus. The tentacles and polypite are much relaxed. In a sea-water solution just strong enough to taste bitter, this phase may continue for hours; in fact till a certain opalescence of the contractile tissues—which it is a property of strychnia, as of most other reagents, to produce—has advanced so far as to place the tissues beyond recovery. If the exposure to such a solution has not been very prolonged, recovery of the animal in normal water is rapid. In a specimen exposed for 2½ hours to such a solution, recovery began in half an hour after restoration to normal water, but was never complete. In all cases, if the poisoning is allowed to pass beyond the stage at which response to direct muscular irritation ceases, the animal is dead.

On *Tiaropsis indicans* this poison has the effect of causing a general spasm, which would be undistinguishable from that which in this species results from general stimulation of any kind, were it not that there is a marked difference in one particular. For in the case of strychnia-poisoning the spasm, while it lasts, is not of uniform intensity over all parts of the nectocalyx; but now one part and now another part or parts are in a state of stronger contraction than other parts, so that, as a general consequence, the outline of the nectocalyx is continually changing its form. Moreover, in addition to these comparatively slow movements, there is a continual twitching observable throughout all parts of the nectocalyx. Each individual twitch only extends over a small area of the contractile tissue; but in their sum their effect is to throw the entire organ into a sort of shivering convulsion, which is superimposed on the general spasm. After a time the latter somewhat relaxes, leaving the former still in operation, which, moreover, now assume a paroxysmal nature—the convulsions consisting of strong shudders and frequent spasms with occasional intervals of repose.

In the case of *Tiaropsis diademata* the action of strychnia is very similar, with the exception that there is no *continuous* spasm, although *occasional* ones occur amid the twitching convulsions. After a time, however, all convulsions cease, and the animal remains quiescent. While in this condition its reflex excitability is abnormally increased, as shown by the fact that even a gentle touch will bring on, not merely a single responsive spasm, as in the unpoisoned animal, but a whole series of successive spasms, which are often followed by a paroxysm of twitching convulsions. The condition of exalted reflex irritability is thus exceedingly well marked. Recovery in normal water at this stage is rapid, the motions being at first characterized by a want of coordination, which, however, soon passes off.

§ 5. *Veratrium.*—In *Sarsia* the first effect of this poison is to increase the number and potency of the contractions; but its later effect is just the converse, there being then prolonged periods of quiescence, broken only by very short swimming-bouts consisting of feeble contractions. The feebleness of the contractions gradually becomes more and more remarkable, until at last it is with great difficulty that they can be perceived at all; indeed the progressive fading away of the contractions into absolute quiescence is so gradual that it is impossible to tell exactly when they cease. During the quiescent stage the animal is for the first time insensible both to tentacular and to direct stimulation of the contractile tissues. That the gradual dying out of the strength of the contractions is not altogether due to the progressive advance of central paralysis, would seem to be indicated by the fact that contractions in response to direct stimulation of the contractile tissues are no more powerful at any given stage of the poisoning than are either responses to tentacular stimulation or the spontaneous contractions. Still, as we shall immediately see, in the various species of *Tiaropsis*, irritability persists after cessation of the spontaneous contractions. In *Sarsia* the nervous connexions between the tentacles and polypite, and also between the tentacles themselves, are not impaired during the time that the bell is motionless; and even when the irritability of the bell has quite disappeared as regards any kind of stimulation, the polypite and tentacles will continue responsive to stimuli applied either directly to themselves, or to any part of the nervo-muscular sheet of the bell.

The convulsions due to the action of veratrium are well marked in the various species of the genus *Tiaropsis.* They consist of violent fluttering motions without any coordination; but there are no spasms, as in the case of strychnia-poisoning. After the convulsions have lasted for some time, a quiescent stage comes on, during which the animal remains responsive to stimulation, though not abnormally so. Recovery in unpoisoned water is rapid, the movements being at first marked by an absence of coordination.

§ 6. *Digitalin.*—The first effect of this poison on *Sarsia* is to quicken the swimming-motions, and then to enfeeble them progressively till they degenerate into mere spasmodic twitches. The polypite and tentacles are now strongly retracted, while the nectocalyx is drawn together so as to assume an elongated form. The latter is now no longer responsive either to tentacular or to direct stimulation; but the tentacles and polypite both remain responsive to stimuli applied either directly to themselves or to the nervo-muscular tissue of the bell. Death always takes place in very strong systole; and as this is an exceedingly unusual thing in the case of *Sarsia*, there can be no doubt that, in this respect, the action of the digitalin is different on the Medusæ from what it is on the heart.

On the various species of *Tiaropsis* digitalin at first causes acceleration of the swimming-movements, with great irregularity and want of coordination. Next strong and persistent spasms supervene, which give the outline of the nectocalyx an irregular form; and every now and then this unnatural spasm gives place to convulsive swimming-motions. Evidently, however, the spasm becomes quite persistent and excessively strong. The polypite of *Tiaropsis indicans* crouches to its utmost, and the animal dies in strong systole.

§ 7. *Atropin.*—In the case of *Sarsia* atropin causes convulsive swimming-motions. The systoles next become feeble, and finally cease. The nectocalyx is now somewhat drawn together in persistent systole, with the polypite and tentacles strongly retracted. Muscular irritability remains after tentacular irritability has disappeared, but it is then decidedly enfeebled.

In the various species of *Tiaropsis* the convulsions are strongly pronounced. They begin as mere accelerations of the natural swimming-motions, but soon grow into well-marked convulsions, consisting of furious bouts of irregular systoles following one another with the utmost rapidity, and wholly without coordination. Occasionally these movements are interrupted by a violent spasm, on which strong shuddering contractions are superimposed.

§ 8. *Nicotin.*—On dropping *Sarsia* into a sea-water solution of nicotin of appropriate strength, the animal immediately goes into a violent and continuous spasm, on which a number of rapidly succeeding minute contractions are superimposed. The latter, however, rapidly die away, leaving the nectocalyx still in strong and continuous systole; tentacles and polypite are retracted to the utmost. Shortly after cessation of spontaneity, the bell is no longer responsive to tentacular stimulation, but remains for a considerable time responsive to direct stimulation of its own substance; eventually, however, all irritability disappears, while the tentacles and polypite relax. On transferring the animal to normal water, muscular irritability first returns, and then central, as shown by the earlier response of the bell to direct than to tentacular stimulation; but if the animal has been poisoned heavily enough to have had its muscular irritability suspended, it is a long time before central irritability returns. Soon after central irritability has returned, the animal begins to show feeble signs of spontaneity, the motions being exceedingly weak, with long intervals of repose; but the degree of such feebleness depends on the length of time during which the animal has previously been exposed to the poison; thus in a specimen which had been removed from the poison immediately after the disappearance of reflex irritability had supervened, recovery began in ten minutes after re-immersion, and was complete in half an hour.

In *Tiaropsis* the symptoms of nicotin-poisoning are also well marked. When gradually administered, the first effect of the narcotic is a complete loss of coordination in the swimming-motions. A slight increase of the dose brings about a tonic spasm, which differs from the natural spasm of these animals—(*a*) in being stronger, so that the nectocalyx becomes ball-shaped rather than square, (*b*) in being much more persistent, and (*c*) in undergoing variations in its intensity from time to time, instead of being a contraction of uniform strength; thus the spasm temporarily affects some parts of the nectocalyx more powerfully than other parts, so that the organ may assume all sorts of shapes. Such distortions proceed even further under the influence of nicotin than under that of strychnine, &c. Sometimes, for instance, one quadrant will project in the form of a pointed promontory; at other times two adjacent or opposite quadrants will thus project, and occasionally all four will do so, the animal thus becoming star-shaped. Sometimes, again, one quadrant will be less contracted than the other three, while at other times more or less slight relaxations affect numerous parts of the bell, its margin being thus rendered sinuous, though more or less violently contracted in all its parts. This state of violent spasm lasts for several minutes, when it gradually passes off, the nectocalyx relaxing into the form of a deep bowl and remaining quite passive, except that every now and then one part or another of the margin is suddenly contracted in a semilunar form. By-and-by, however, even these occasional twitches cease, and the animal is now insensible to all kinds of stimulation. Recovery in normal water is gradual, and marked in its first stage by the occasional retractions of the margin last mentioned. At about this stage also, or sometimes slightly later, the animal first becomes responsive to stimulation; and it is interesting to note that the response is performed, not by giving a general spasm as would the unpoisoned animal, but by folding in the part irritated—an action which very much resembles, on the one hand, the spontaneous convulsive movements just described, and, on the other, the response which is given to stimulation by the unpoisoned bell when gently irritated after removal of its margin. After these stages there supervenes a prolonged period of quiescence, during which the animal remains normally responsive to stimulation. Spontaneity may not return for several hours, and, after it does return, the animal is in most cases permanently enfeebled. Indeed, on all the species of Medusæ, nicotin, both during its action and in its subsequent effects, is the most deadly of all the poisons I have tried.

§ 9. *Alcohol.*—The solution must be strong to cause complete intoxication. The first effect on *Sarsia* is to cause a great increase in the rapidity of the swimming-motions—so much so, indeed, that the bell has no time to expand properly between the occurrence of the successive systoles, which, in consequence, are rendered feeble. These motions gradually die out, leaving the animal quite motionless. The nectocalyx is now responsive to stimuli applied at the tentacles, and sometimes two or three contractions will follow such a stimulus, as if the spontaneity of the animal were slightly aroused by the irritation. Soon, however, only one contraction is given in response to every tentacular irritation, and by-and-by this also ceases—the Medusa being thus no longer responsive to central stimulation. It remains, however, for a long time responsive to stimulation of the nervo-muscle sheet; indeed the strength of the alcohol solution must be very considerable before loss of muscular irritability supervenes. It may thus be made to do so, however; and on then transferring the animal to normal water, recovery begins in from three minutes to a quarter of an hour. The first contractions are very feeble, with long intervals of repose; but gradually the animal returns to its normal state.

The above remarks apply also to *Tiaropsis*. In *T. indicans* the polypite recovers in normal water sooner than does the nectocalyx. Both in *Sarsia* and *Tiaropsis* the polypite and tentacles are retracted while exposed to alcohol, and, after transference to normal sea-water, the animals float on the surface—presumably in consequence of their having imbibed some of the spirit. The period during which floatation lasts depends, (*a*) on the strength of the alcohol solution used, and (*b*) on the time of exposure to its influence. It may last for an hour or more; but in no case is recovery complete till some time after the floatation ceases.

§ 10. *Cyanide of Potassium.*—On *Sarsia* the first effect is to quicken the contractions and then to enfeeble them. The animal assumes an elongated form, as already described under atropin. Spontaneity ceases very rapidly even in weak solutions; and for an exceedingly short time after it has done so, the bell continues responsive both to tentacular and to direct stimulation. For a long time after the bell ceases to respond to any kind of stimulation, the nervous connexions between the tentacles and between the tentacles and polypite remain intact, as also do the nervous connexions of these organs with all parts of the bell. This interesting fact is rendered apparent, first, by stimulating a tentacle and observing that all the four tentacles and the polypite respond; and, second, by irritating any part of the nervo-muscular sheet of the bell and observing that while the latter does not respond, both the tentacles and the polypite retract. Recovery from this stage occupies several hours.

In the case of *Tiaropsis* the convulsions are, as usual, more pronounced, being marked by the occurrence of a gradually increasing spasm, which differs from a normal spasm in the respects already described under strychnia. In all the species both of *Sarsia* and *Tiaropsis*, the polypite and tentacles are retracted during exposure to this poison.

§11. *Remarks.*—The above are some among the poisons which I have tried; but to avoid undue length I will not proceed further with the list on the present occasion. Enough, I think, has been said to show how surprising is the resemblance between the actions of these various poisons on the Medusæ and on the higher animals. When the physiologist bears in mind that in *Sarsia* we have the means of testing the comparative influence of any poison on the central, peripheral, and muscular systems respectively*, he will not fail to appreciate the value of the foregoing observations. In reading over the whole list he will meet with an anomaly here and there; but, on the whole, I do not think he cannot fail to be satisfied with the wonderfully close adherence which is shown by these elementary nervous tissues to the rules of toxicology that are followed by nervous tissues in general. In one respect, indeed, there is a conspicuous and uniform deviation from these rules; for we have seen that in the case of every poison mentioned more or less complete recovery takes place when the influence of the poison has been removed, even though this has acted to the extent of totally suspending irritability. In other words, there is no poison in the above list which has the property, when applied to the Medusæ, of destroying life till long after it has destroyed all signs of irritability. What the cause of this uniform peculiarity may be is, of course, conjectural; but I may suggest two considerations which seem to me in some measure to mitigate the anomaly. In the first place, we must remember that in the Medusæ there are no nervous centres of such vital importance to the organism that any temporary suspension of their functions is followed by immediate death. Therefore, in these animals, the various central nerve-poisons are at liberty, so to speak, to exert their full influence on all the excitable tissues without having the course of their action interrupted by premature death of the organism, which in higher animals necessarily follows the early attack of the poison on a vital nerve-centre. Again, in the second place, we must remember that the method of administering the above-mentioned poisons to the Medusæ was very different from that which we employ when administering them to other animals; for, in the case of the Medusæ, the nervo-muscular tissue is spread out in the form of an exceedingly tenuous sheet, so that when the animal is soaking in the poisoned water every portion of the excitable tissue is equally exposed to its influence; and that the action of a poison is greatly modified by such a difference in the mode of its administration has recently been proved by Professor GAMGEE, who found that when a frog's muscle is allowed to soak in a solution of vanadium &c. it loses its irritability, while this is not the case if the poison is administered by means of the circulation.

In conclusion, I may observe that in the case of all the poisons I have tried, the time required for recovery after the animal is restored to normal water varies immensely. The variations are chiefly determined by the length of time during which the animal has been exposed to the influence of the poison, but also, in a lesser degree, by the strength of the solution employed. To take, for instance, the case of caffein or chloroform, if *Sarsiæ* are transferred to normal water after they first cease to move, a few seconds are enough to restore their spontaneity; whereas if they are allowed to remain

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* The method of comparison consists, as will already have been gathered from the perusal of the foregoing sections, in:—first, stimulating the tentacles, and observing whether this is followed by such a discharge of the attached ganglion as causes the bell to contract; next, stimulating the bell itself, to ascertain whether the muscular irritability is impaired; and, lastly, stimulating either the tentacles or the bell, to observe whether the reciprocal connexions between tentacles, bell, and polypite are uninjured.

in the poisoned water for an hour, they may not move for one or two hours after their restoration to unpoisoned water. In consequence of such great variations occurring from these causes, I was not able to compare the action of one poison with that of another in respect of the time required for effects of poisoning to pass away.

§ 12. *Fresh Water.*—As fresh water exerts a very deadly influence on the Medusæ, this seems the most appropriate place for describing its action. Such a description has already been given by Professor L. AGASSIZ, but it is erroneous. He writes:—"Taking up in a spoonful of sea-water a fresh *Sarsia* in full activity, when swimming most energetically, and emptying it into a tumbler full of fresh water of the same temperature, the little animal will at once drop like a ball to the bottom of the glass and remain for ever motionless—killed instantaneously by the mere difference of the density of the two media"*. As regards the appearance presented by *Sarsia* when subjected to "this little experiment," the account just quoted is partly correct; but Professor AGASSIZ must have been over-hasty in concluding that, because the animals seemed to be thus "killed instantaneously," such was really the case. Nothing, indeed, could be more natural than his conclusion; for not only is the contrast between the active swimming-motions of the *Sarsia* in the sea-water and their sudden cessation of all motion in the fresh water very suggestive of instantaneous death, but, a short time after immersion in the latter, their contractile tissues, as Professor AGASSIZ observed, become opalescent and whitish. Nevertheless, if he had taken the precaution of again transferring the *Sarsia* to sea-water, he would have found that the previous exposure to fresh water had not had the effect which he ascribes to it. After a variable time his specimens would have resumed their swimming-motions; and although these might have had their vigour somewhat impaired, the animals would have continued to live for an indefinite time—in fact quite as long as other specimens which had never been removed from the sea-water. Even after five minutes' immersion in fresh water, *Sarsia* will revive feebly on being again restored to sea-water, although it may be two or three hours before they do so; they may then, however, live as long as other specimens. In many cases *Sarsia* will revive even after ten minutes' exposure; but the time required for recovery is then very long, and the subsequent pulsations are of an exceedingly feeble character. I never knew a specimen survive an exposure of fifteen minutes†. In not a few cases, after immersion in fresh water, the animal continues to pulsate feebly for some little time; and, in all cases, irritability of the contractile tissues persists for a little while after spontaneity has ceased. The opalescence above referred to principally affects the polypite, tentacles, and margin of the nectocalyx. While in fresh water the polypite and tentacles of *Sarsia* are strongly retracted.

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* Mem. American Acad. Arts and Sciences, 1850, page 229.

† The covered-eyed Medusæ survive a longer immersion than the naked-eyed—*Aurelia aurita*, for instance, requiring from a quarter to half an hour's exposure before being placed beyond recovery. Moreover the cessation of spontaneity on the first immersion is not so sudden as it is in the case of the naked-eyed Medusæ—the pulsations continuing for about five minutes, during which time they become weaker and weaker in so gradual a manner that it is hard to tell exactly when they first cease.

Thinking it a curious circumstance that the mere absence of the few mineral substances that occur in sea-water should exert so profound and deadly an influence on the nervo-muscular tissues of the Medusæ, I was led to try some further experiments to ascertain whether it is, as AGASSIZ affirms, to the mere difference in density between the fresh and the sea-water, or to the absence of the particular mineral substances in question, that the deleterious influence of fresh water is to be ascribed. Although my experiments lead to no very instructive conclusion, they are, I think, worth stating.

I first tried dissolving chloride of sodium in fresh water till the latter was of the same density as sea-water. *Sarsiæ* dropped into such a solution continued to live for a great number of hours; but they were conspicuously enfeebled, keeping for the most part at the bottom of the vessel, and having the vigour of their swimming-motions greatly impaired. The tentacles and polypite were strongly retracted, as in the case of exposure to fresh water, and the tissues also became slightly opalescent. Thinking that perhaps a fairer test would be only to add as much chloride of sodium to the fresh water as occurs in sea-water, I did so; but the results were much the same. On now adding sulphate of magnesium, however, to the amount normally present in sea-water, the *Sarsiæ* became more active. I next tried the effects of chloride of sodium dissolved in fresh water to the point of saturation, or nearly so. The *Sarsiæ*, of course, floated to the surface, and they immediately began to show symptoms of torpidity. The latter became rapidly more and more pronounced, till spontaneity was quite suspended. The animals, however, were not dead, nor did they die for many hours—their irritability continuing unimpaired, although their spontaneity had so completely ceased. The tentacles and polypite were exceedingly relaxed, which is an interesting fact, as being the converse of that which occurs in water containing too small a proportion of salt. Lastly, to give the density hypothesis a still more complete trial, I dissolved various neutral salts and other substances, such as sugar &c., in fresh water till it was of the density of sea-water; but in all cases, on immersing *Sarsiæ* in such solutions, death was as rapid as that which followed their immersion in fresh water.

VII. GENERAL SUMMARY.

The "fundamental experiment" of paralyzing nectocalyces by removing their margin has this year been repeated a vast number of times on various species of naked-eyed Medusæ, and always with the same result. In the case of the covered-eyed Medusæ, I have confirmed such of my previous observations as required confirmation, from the fact of their having been at variance with those of Dr. EIMER. I have continued to find that the lithocysts are the exclusive seats of spontaneity, so far as the "primary movements" are concerned. I have failed to detect the slightest evidence of spontaneity on the part of the contractile zones. I am still unable to say that the "secondary contractions" appear to me either more "feeble," "inefficient," or "local" than the primary ones; neither can I modify my previous statements as to the time during which these secondary movements persist; for instead of "generally ceasing after a few moments" or "hours," they usually last for several days—in fact until the tissues begin to decompose. I am able, however, to agree with Dr. EIMER's statement that, other things equal, the unmutilated tissues preserve their irritability longer than those from which lithocysts have been removed; and I am able to agree with his statement that the rate of the natural rhythm has a tendency to bear an inverse proportion to the size of the individual, though I find that size is not the only factor in determining such rate. But his further statements with regard to the duration of the natural pauses bearing a direct proportion to the number and strength of the previous contractions, require, I think, to be taken with reserve; for both in *Aurelia* and in *Sarsia*, although a *general* relation of this kind may be observed, I have not found it to be so precise as Dr. EIMER affirms. Similarly, although it is true that the smaller segments of a divided *Aurelia* show a *general* tendency to contract less frequently than the larger segments, I have not found evidence of the precise relation which Dr. EIMER describes. On the other hand, I have found evidence of two additional factors which in these segmentation experiments are as important in determining the rate of the rhythm as is the size of the segments. These factors are (*a*) the potency of the lithocysts and (*b*) the time that elapses after the segmentation has been performed; for at first the smaller segments, if they contain prepotent lithocysts, have a quicker rhythm than the larger segments, but afterwards the rhythm of the former progressively slows more rapidly than does that of the latter. Although, however, the endurance is thus less in the case of a prepotent lithocyst in a small segment than is the endurance of less potent lithocysts in a large segment, the endurance of the former towards deprivation of oxygen is greater than is that of the latter; for, under such circumstances, the prepotent lithocyst will continue to act rhythmically in water which is stale enough to cause the less potent lithocysts to act irregularly, or not to act at all.

Other forms of mutilation—such as cutting off the polypite or portion of the swimming-bell—causes, first acceleration of the rhythm, and then a progressive decline to a certain point below the original rate. The rate then remains stationary at this point, but may again be made temporarily to rise and permanently to fall by removing another portion of the swimming-bell. In these experiments the rhythm, besides becoming permanently slowed, is also often rendered permanently irregular. Again, paring down the contractile tissues from around a single lithocyst has the effect, when the tissue is greatly reduced, of giving rise to enormously long periods of inactivity. During such a period, however, stimulation may initiate a bout of rhythmical contractions, to be followed by another prolonged pause. These facts tend to show that the apparently automatic action of the lithocysts is really due to a constant stimulation supplied by other parts of the organism.

Temperature exerts a profound influence on the rate of the rhythm. This influence may be best observed within moderate limits of variation; for water below 20° suspends spontaneity and even irritability, while water above 70° permanently slows the rhythm after having temporarily quickened it. But water between 50° and 60° permanently quickens the rhythm during the time that Medusæ, which have been removed from colder water, are exposed to its influence. In very cold water the loss of spontaneity is a gradual though rapid process, as is also its return in warmer water. After having been frozen solid, *Aurelia* will recover on being thawed out, but the original rate of rhythm was not observed fully to return.

Oxygen accelerates the rhythm, while carbonic acid retards it, and in strong doses destroys both spontaneity and irritability. Deficient aëration of the water containing the Medusæ causes irregularity of their rhythm, as well as the occurrence of pauses. The latter become more and more pronounced the longer the water is left without being changed, till at last spontaneity altogether ceases; but on now restoring the animals to fresh sea-water, their recovery is surprisingly sudden.

As regards stimulation, I have proved that a few drops of hot water allowed to run over the excitable tissues of Medusæ cause a responsive contraction. Single mechanical or chemical stimuli applied to paralyzed swimming-bells of covered-eyed Medusæ frequently produces in response a small series of rhythmical contractions.

Light acts as a powerful stimulus to some species of Medusæ. Vigorous *Sarsiæ* respond to a single flash by entering on a bout of swimming; but less vigorous specimens only give a single contraction. The stimulus has been proved to be light *per se*, and not the sudden transition from darkness to light. Moreover a constant flood of light seems to act towards *Sarsia* as a constant stimulus. After removal of their marginal bodies, *Sarsiæ* no longer respond to luminous stimulation, while, on the other hand, any contractile tissue left adhering to the excised marginal bodies will continue to do so. *Tiaropsis polydiademata* responds to luminous stimulation as it responds to all other kinds of stimulation, viz. by going into a spasm; but the time that elapses between the occurrence of the stimulus and the occurrence of the response is very much longer in the case of luminous than in that of any other kind of stimulation. I find that this time is not, properly speaking, a period of latent stimulation; for a single flash requires to be of the same duration as a continuous flood of light in order to procure a response. Hence the period represents the time during which a certain summation of stimulating influence is taking place in the ganglia.

The period of latent stimulation in the case of *Aurelia* is greatly modified by certain conditions. Of these, temperature exerts the greatest influence; but, as a novel fact of great interest, the most important influence from a physiological point of view is that of the summation of stimuli, which, however, I need not again describe. At the bottom of a staircase the latent period is $\frac{5}{8}$ second, while at the top of a staircase it is only $\frac{3}{8}$ second. Summation of stimuli also greatly increases the amplitude of the contractions; so that it both develops in the tissue a state of expectancy and arouses it into a state of increased activity. My results in this connexion differ from those of Dr. BOWDITCH in that (*a*) the medusoid tissue has a shorter memory for the occurrence of a previous stimulation than has the cardiac; and (*b*) in the medusoid tissue a stimulus which at the bottom of a staircase is of less than minimal intensity, is able at the top of a staircase to give rise to a contraction of very nearly maximum intensity. This latter fact proves that the staircase action is, at any rate in part, dependent on the process of stimulation as distinguished from that of contraction. Moreover, that the effect is one pervading the whole extent of the irritable tissues, is proved by another fact, viz. that when at any stage during the construction of a staircase the electrodes are suddenly shifted to any other part of the irritable tissues, the next contraction will rank as the next step in the staircase. Other experiments have conclusively proved that the staircase action obtains in the case of the natural ganglionic stimulation in exactly the same way, both quantitatively and qualitatively, as it does in the case of artificial stimulation.

The excitable tissues of *Aurelia* may be thrown into tetanus by means of strong faradaic stimulation. That the tetanus is due to summation of contractions may be proved by throwing in single shocks at one end of a spiral strip with a key, and observing that if the shocks are thrown in with a certain rapidity, the resulting contractile waves overlap one another, so giving rise to persistent spasm. In such experiments, however long the strip may be, and however complicated the time relations between successive stimuli are made, whatever disturbances are set up at one end of the strip are faithfully transmitted to the other.

Reflex action has been proved to occur in various species of Medusæ. In *Sarsia* definite nervous connexions of constant occurrence have been shown to exist between the tentacles, but not between the tentacles and the polypite. Section of the nervo-muscular sheet proves that in the case of this genus physiological harmony may, as a rule, be easily destroyed, although it occasionally happens that such is not the case.

In *Tiaropsis indicans* the pointing action of the polypite is manifested by a portion of any size which may be cut from that organ. The ganglionic or localizing function is therefore diffused throughout the tenuous contractile tissues of the latter. This function admits of being destroyed, as regards any portion of the bell, by introducing a short incision between that portion and the base of the polypite; but after this the polypite, though no longer able to localize the seat of irritation, continues able to perceive that a stimulus is being applied in the bell somewhere. This shows that while the connexions on which the localizing function depend are radial, there are other connexions between the bell and polypite which are not radial. The latter connexions will endure a considerable amount of section before they are destroyed, particularly in the case of the tracts occupied by radial tubes. On carrying a spiral incision three fourths of the way round the nectocalyx, so as to leave one radial tube intact, and on then irritating any part of the spiral strip, it may be observed that the polypite usually points to the unsevered radial tube.

The spasmodic movements of *Staurophora laciniata* usually occur only when either the margin or the radial tubes are irritated. Although in this species ordinary contractile waves are easily blocked by section, spasmodic waves will continue to pass after very severe forms of section. Now it is a remarkable thing that the contractile tissues, although themselves incapable of *originating* a spasm in response to stimulation, are nevertheless so wonderfully capable of *conducting* a spasm when this has been originated by irritation of the slender tissue-tracts above named. It is as though every fibre of the general contractile tissue were capable of liberating energy in either of two very different ways, and that whenever one part of the general mass is made to liberate its energy in one of these two ways, all the other parts of the mass do the same—and this no matter how far through the mass the liberating process may have to extend. And to say that it is the ganglionic element in the margin which, to recur to our previous metaphor, here acts as a detonator, is not to explain the facts; for although it would be an interesting thing to know that a ganglion-cell may be able to originate two different kinds of impulse according as it liberates its energy spontaneously or in answer to direct stimulation, this knowledge would merely serve to transfer the questions which now apply to the marginal and radial tube-tissues in general to the ganglionic tissues in particular. Again, the supposition of the ganglia acting as detonators when themselves directly irritated, would not explain why it is that the *contractile tissues* are capable of two such very different kinds of *response*. Anæsthetics block spasmodic waves, but not till they have suspended spontaneity, and even destroyed muscular irritability as regards direct stimulation.

In *Aurelia aurita* the passage of a tentacular wave marks the passage of a stimulus-wave. Such waves may be started more readily by stimulating some tracts than by stimulating others, though there is no constancy as to position of these tracts in different individuals. The case of stimulus-waves in this particular, therefore, resembles that of contractile waves, which, as explorations by graduated stimuli show, may also be more readily started from some tracts than from others. Again, the two cases resemble one another in the still more important particular of the astounding degree to which the tissues may be mutilated without their physiological connexions being destroyed. For excitational continuity being thus shown as difficult to destroy, in the case of this Medusa, as is contractional continuity, we are led to conclude, for reasons which I need not repeat, that both these functions are probably dependent on the same tissue-elements. And, in any case, the fact that the essentially nervous function of maintaining excitational continuity is able to persist in these primitive nervous tissues after they have been submitted to the severest possible forms of section, is a fact the significance of which, it seems to me, can scarcely be overrated. The fact itself cannot be explained by KLEINENBERG's theory of double-function cells; for sometimes contractile waves will become blocked by section before the tentacular waves, and sometimes *vice versâ*. We seem, therefore, driven upon the theory of a nerve-plexus whose constituent elements are capable of vicarious action in almost any degree. This theory is supported by the results of explorations with graduated stimuli, and also by the consideration that in *Sarsia*, which is a more highly integrated form than *Aurelia*, the supposed plexus is so far differentiated that vicarious action on the part of its constituent elements is usually possible in but a low degree. Again, *Tiaropsis* appears to stand midway between *Sarsia* and

Aurelia in respect of the degree to which integration of its structures has advanced; and, as we have seen, it likewise occupies an intermediate position in respect of the degree in which vicarious action of the supposed nervous elements is possible.

Contractile waves in *Aurelia aurita* travel at the rate of 18 inches per second, if the temperature of the water is about that of the sea; but the rate is greatly modified by temperature, straining, anæsthetics, and various foreign substances. Stimulus-waves only travel at the rate of 9 inches per second, if the stimulus which starts such a wave is not strong enough at the same time to start a contractile wave; but if the stimulus is strong enough to start both waves, they both travel at about the same rate.

There appears to be no further coordination among the lithocysts of the covered-eyed Medusæ than such as arises from contractile waves coursing rapidly from one of the number and, as it passes the others, causing them successively to discharge; but, in the case of the naked-eyed Medusæ, true coordination has been proved to occur between the marginal ganglia, and the tracts through which it is effected have been proved to be the marginal nerves. Slightly cutting the margin of a naked-eyed Medusa exerts a very deleterious influence upon the vigour of the animal; and violent nervous shock, while it always suspends both spontaneity and irritability, will sometimes also destroy coordination for a considerable time after spontaneity returns.

The poisons whose effects I have described are chloroform, nitrite of amyl, caffein, strychnia, veratrium, digitalin, atropin, nicotin, alcohol, and cyanide of potassium. The details of this part of the inquiry are rendered particularly interesting from the fact that in the case of *Sarsia* we have the means of testing the comparative influence of any poison on the central, peripheral, and muscular systems; but it is needless here to repeat details. In general, it will be remembered that the effects of the various poisons on these respective systems are almost uniformly such as occur in the case of the higher animals. In one important particular, indeed, the actions of nearly all the poisons differ from their actions on the higher animals; for there is no poison in the above list which has the property, when administered to the Medusæ, of destroying life till long after it has destroyed all signs of irritability. I have sought to explain this apparent anomaly by the considerations, first, that the Medusæ present to the action of the central-nerve poison no nerve-centres of vital importance to the organism; and, second, that my method of administering the poisons to the Medusæ was very different from that which is usually employed when administering these poisons to vertebrated animals.

Fresh water acts as a deadly poison to the Medusæ. The naked-eyed species usually cease their movements the instant they touch the fresh water, and are killed by it after an exposure of a few minutes; but the covered-eyed species are slightly more tolerant of its influence. The cause of this deadly influence exerted by fresh water depends on the absence of sea-water, and not on the difference of density between the former and the latter. Chloride of sodium alone, added in appropriate amount to fresh water, deprives the latter, to a very great extent, of its deleterious influence; but this is not the case with any other substance which I have tried. Brine acts as an anæsthetic, and, in depriving the tentacles and polypite of their muscular tonus, exerts an effect the opposite of that which is exerted by fresh water.

Postscript.

Having been in private correspondence with Prof. Eimer during the time that this paper was passing through the press, I now (April 9th, 1878) learn from him that all the differences which have hitherto existed between our statements admit of being reconciled. At his request, therefore, and in order to explain the character of the reconciliation, I append this Postscript.

The points wherein Prof. Eimer's observations did not perfectly agree with mine, or mine with his, were three in number:—

1st. We were not at one regarding the alleged rule that when an *Aurelia* is artificially segmented the rate of the rhythm which is manifested by each of the segments severally bears a direct proportion to the size of the segment, and this in a ratio such that the sum of the contractions which are performed by all the severed segments in a given interval of time is equal to the number of contractions which the unmutilated organism had previously performed in a similar interval of time. Dr. Eimer explains that further observation on his part has not tended to confirm this rule, and therefore that he desires his previous statements on this subject to be considered as withdrawn.

2nd. With regard to the character of the "secondary contractions" which always supervene in the covered-eyed Medusæ after removal of their lithocysts, our results have not hitherto been in full agreement. Dr. Eimer has described these contractions as irregular, inefficient, and feeble contractions, which are of a local nature and rarely last any considerable time after removal of the lithocysts—generally ceasing after a few moments, or, at any rate, after several hours. On none of these points did my observations altogether harmonize with this description, and I am therefore most glad to learn from Dr. Eimer that his further observations have now brought his views on all these points into precise agreement with my own.

3rd. Lastly, the difference of opinion which existed between us with regard to the exact seat of spontaneity in the covered-eyed Medusæ has also been happily removed. It appears that I have hitherto been under some misapprehension as to the precise area of tissue which Dr. Eimer intended to denote by his term "contractile zone;" for while I have hitherto supposed that by the term "contractile zone" Dr. Eimer intended to denote "the crescent-shaped interruption of the margin in which the lithocyst, together with its gelatinous hood, is situated"*, I now learn that I was incorrect in this supposition. From Prof. Eimer's letter I do not quite recognize the precise tissue-area which he endeavours to describe as the area to which his term "contractile zone" is

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* Quoted from the Postscript to my Croonian Lecture, 1876, Phil. Trans. p. 309.

applicable; but this is a matter of no moment, as he leaves no doubt that the area in question is confined to what I may call the microscopical vicinity of the lithocyst. No doubt, therefore, the difference between our experimental results is to be attributed merely to the different manner in which we performed the operation of excising the lithocysts: for while conducting my experiments I supposed that the question which had to be settled was whether the spontaneity of the animal proceeded from the large "crescent-shaped interruptions" or from the very minute lithocysts; and, accordingly, in removing the latter alone, I was careful to remove them well *from* their roots—thus, no doubt, disturbing the "nerve-epithelium" which Mr. Schäfer has figured as occupying the microscopical vicinity of the lithocysts. On the other hand, Dr. Eimer, in conducting his experiments, must have had a completely different question in view, and therefore, in excising the lithocysts, he no doubt took scrupulous care to remove them well *above* their roots, so leaving the "nerve-epithelium" intact. Thus our apparent want of agreement on this point is shown to have arisen merely from a want of understanding with regard to the term "contractile zone."

It only remains for me to express my sincere satisfaction that the few and comparatively immaterial points with regard to which Prof. Eimer and myself were not previously quite agreed are thus so completely disposed of, and hence that in all respects our published results, in so far as they are common, are now in full accord.

G. J. R.

INDEX

TO THE

PHILOSOPHICAL TRANSACTIONS

FOR THE YEAR 1877.

LONDON

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MAGNETIC SURVEY _ *between the Equator and the parallel of 40°S. _ Epoch 1840-45.* Declination.

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MAGNETIC SURVEY _ *between the Equator and the parallel of 40° S. _ Epoch 1840-45.* Inclination.

Sabine.

Phil. Trans. 1877. Plate 18.

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(Sabine)

MAGNETIC SURVEY – *between the Equator and the parallel of 40° S. – Epoch 1840–45.* Intensity.

Phil. Trans. 1877. Plate 19.

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Fig 1

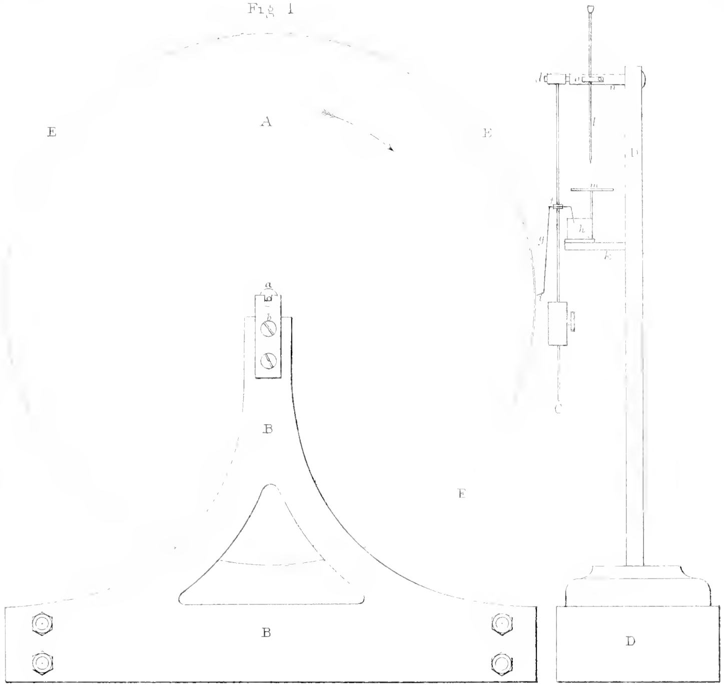
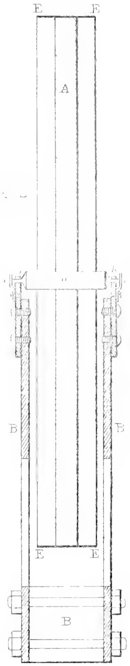


Fig 2



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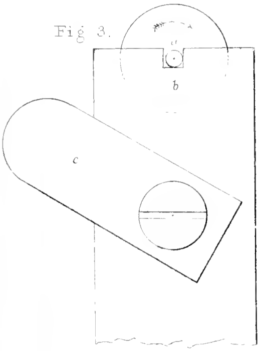

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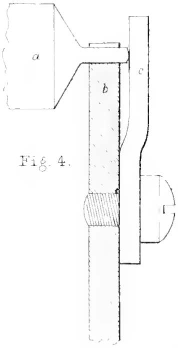

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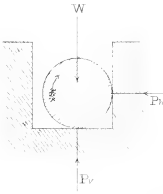

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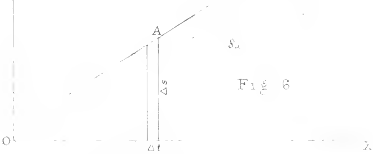

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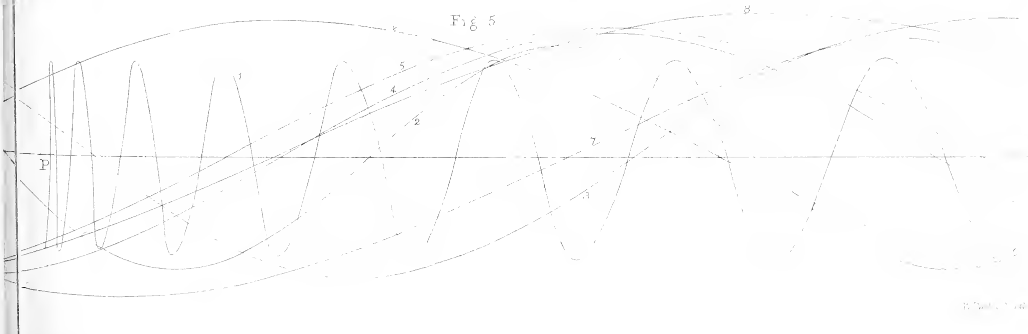

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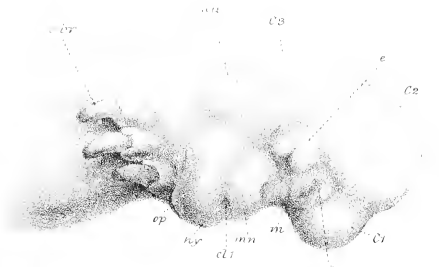

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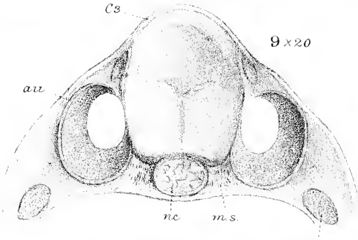

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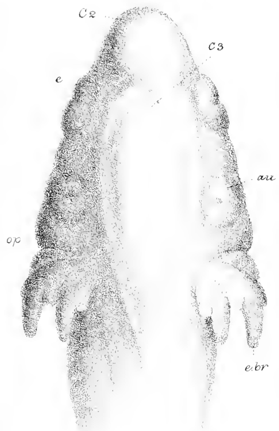

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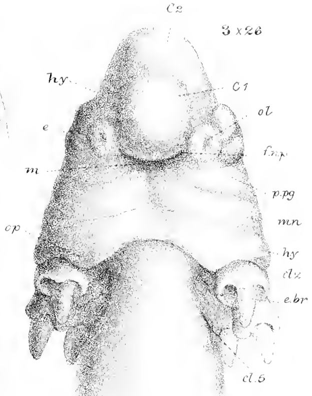

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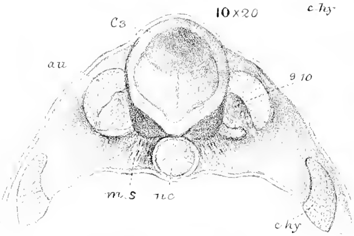

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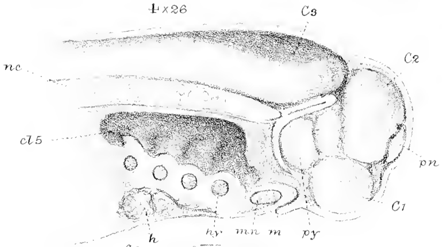

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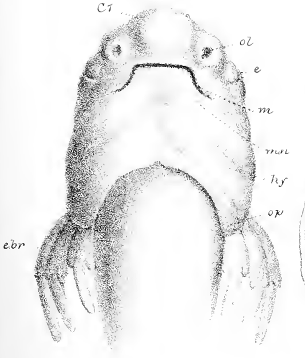

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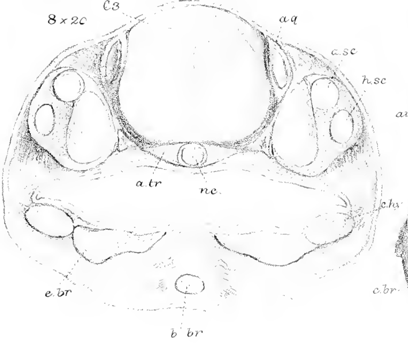

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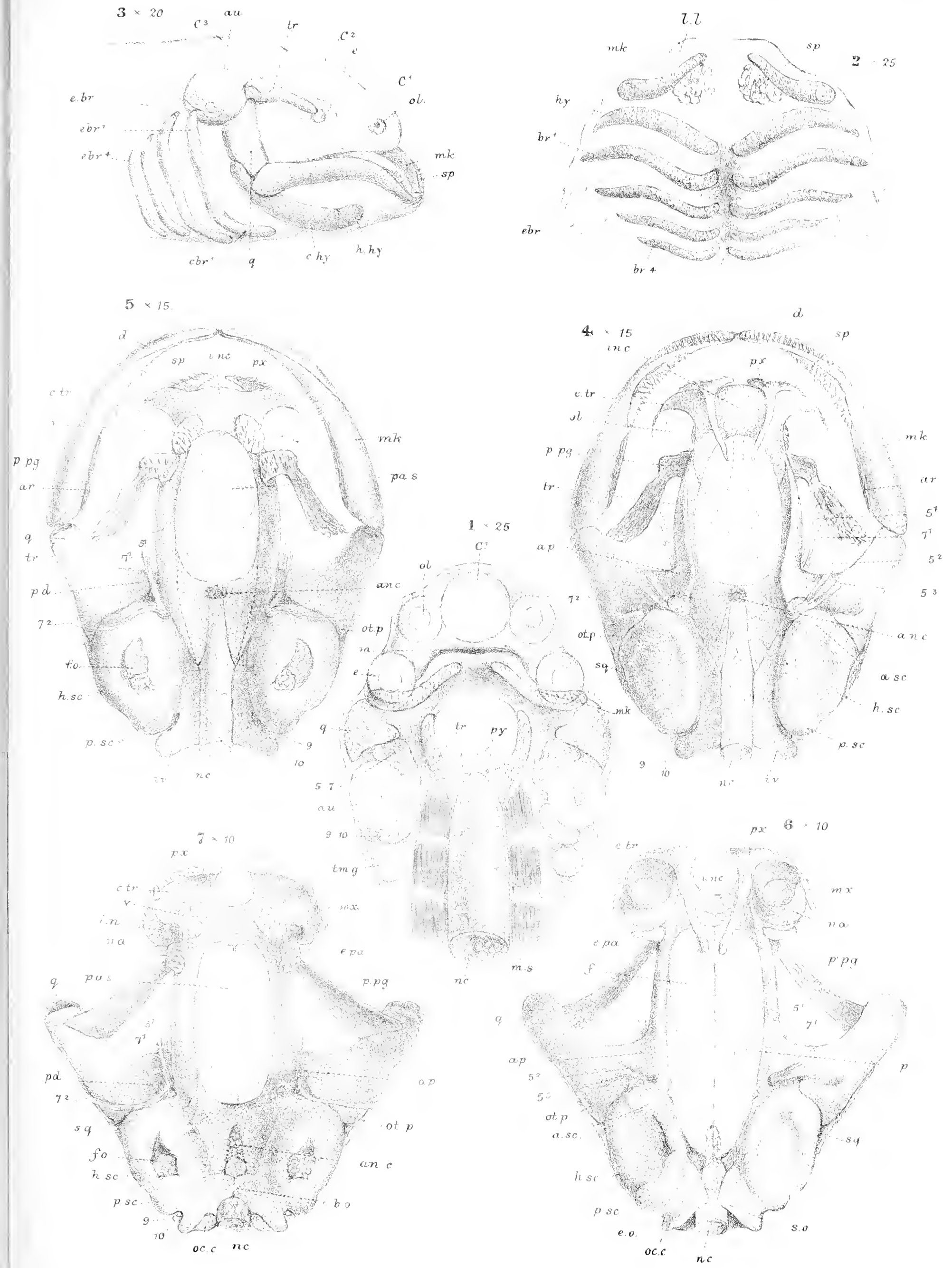

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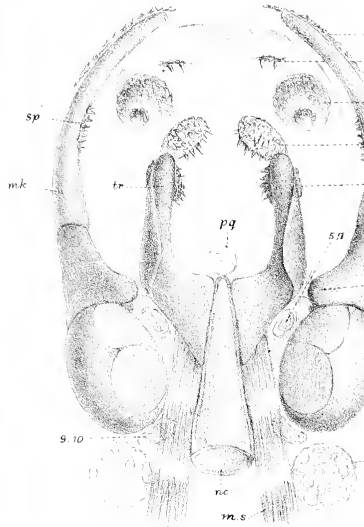

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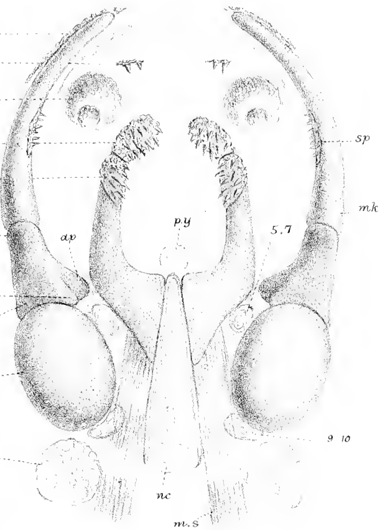

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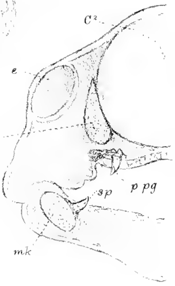

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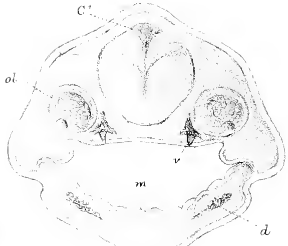

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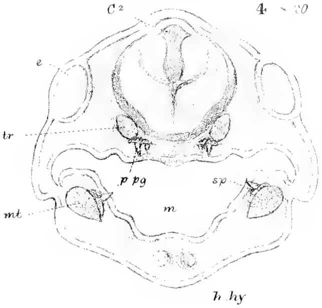

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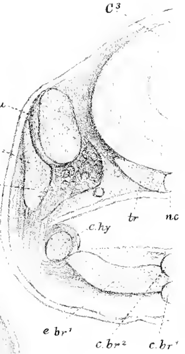

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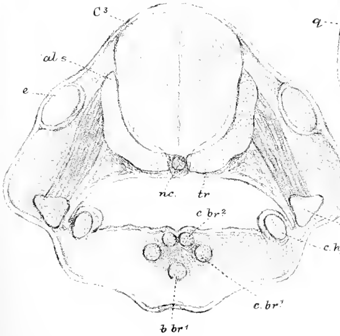

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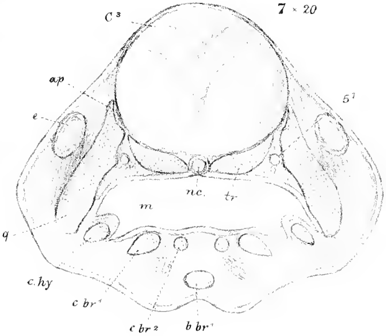
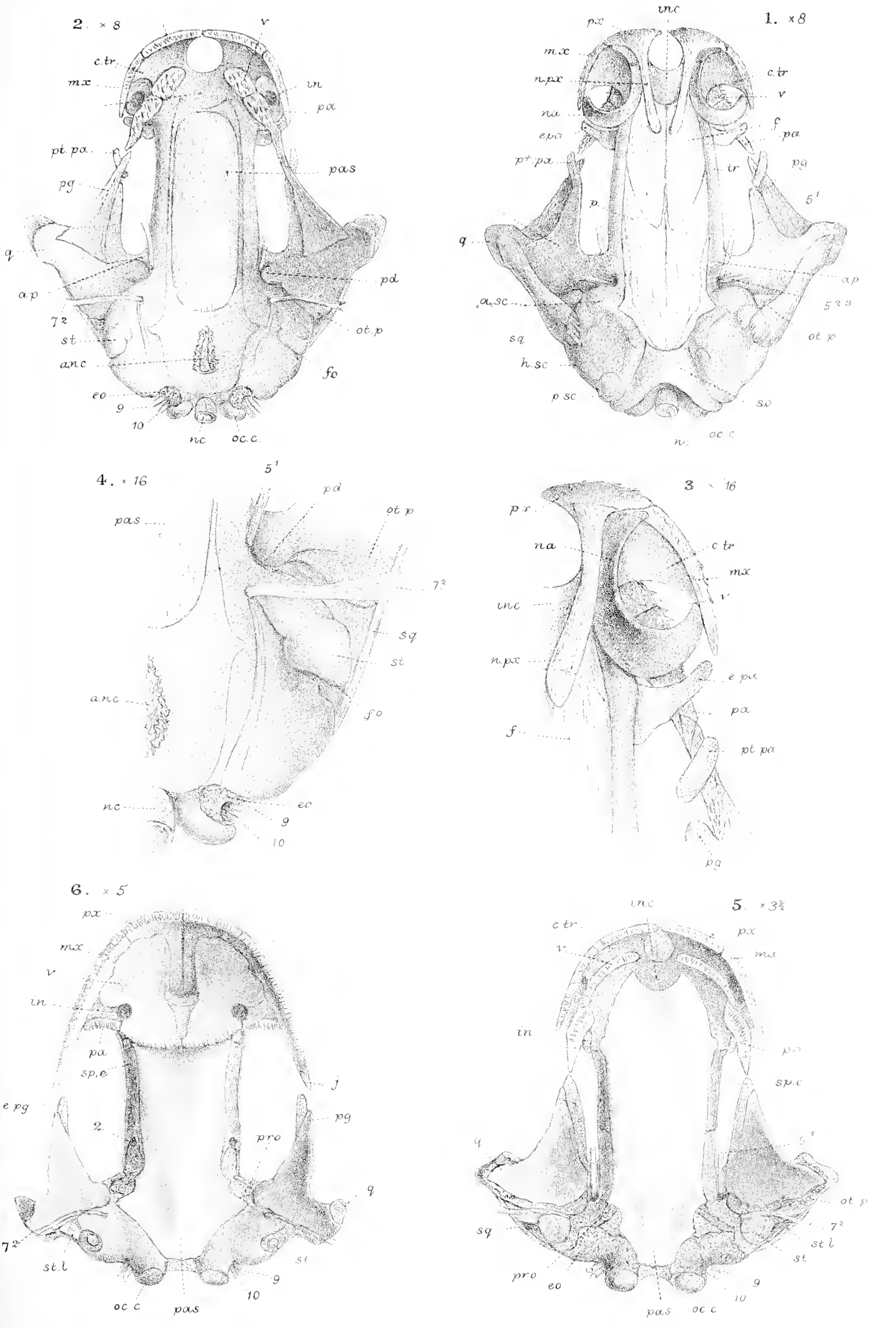


W. K. P. del. at nat
G. West lith.

W. West & Co. imp.

Siredon. 3rd Stage.

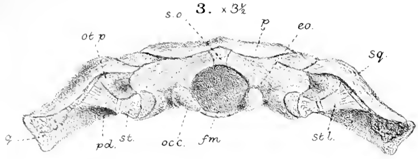


W K P del. ad nat.
G West lith.

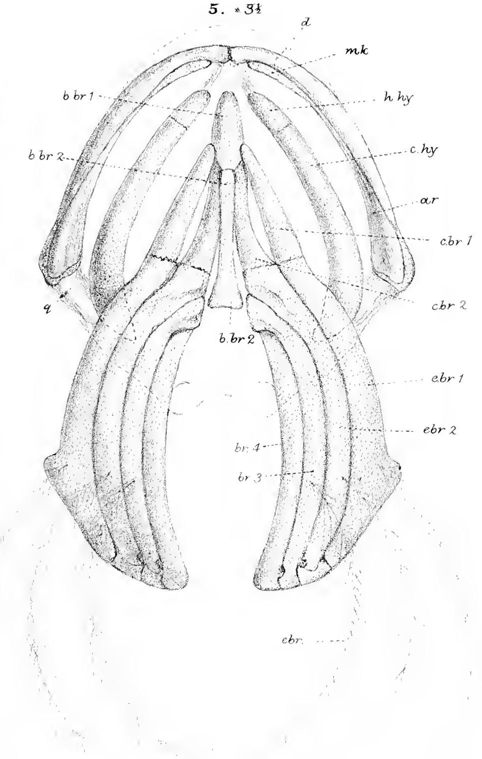
W West & C imp.

Siredon. Figs. 1–4 6th *Stage.* 5. 9th *Stage.* Amblystoma, Fig. 6.

3. × 3½



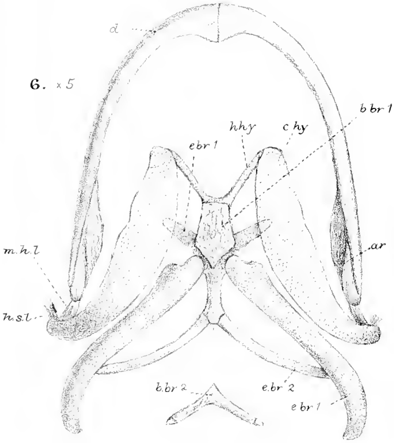
5. × 3½



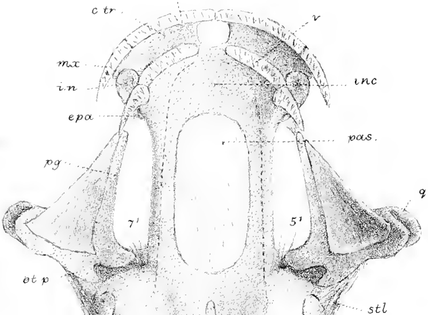
7. × 6

4. × 3½

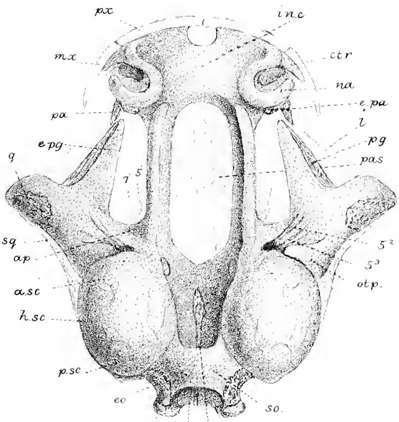
6. × 5



2. × 5.



1. × 5.

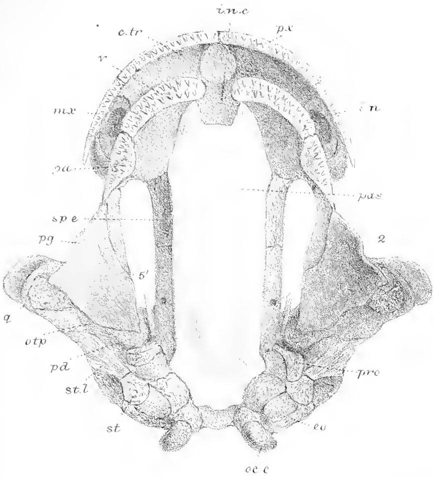


W.K.P. del. ad nat.
G. West lith.

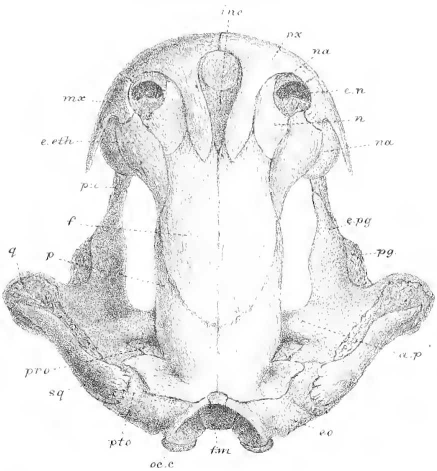
W West & Co imp.

Siredon. Figs, 1, 2, 7th Stage. 3-5. 8th Stage. Amblystoma Figs. 6-7.

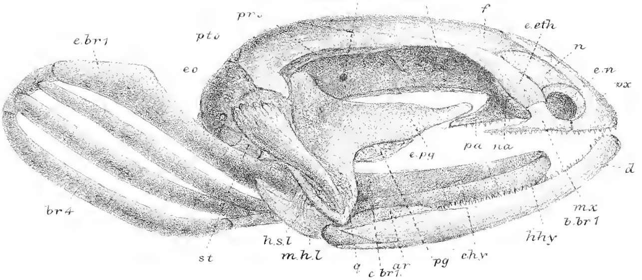
2 × 3½



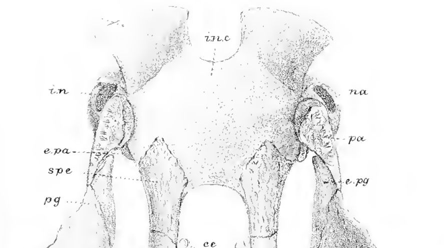
1 × 3½



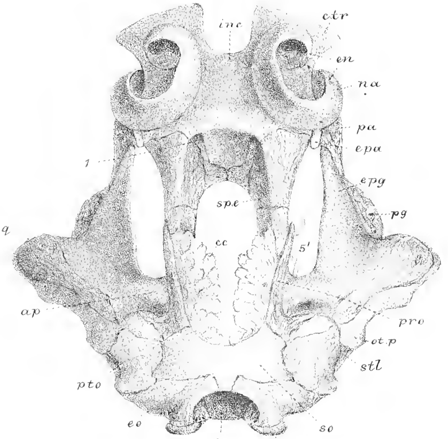
5 × 3½



4 × 3½



3 × 3½

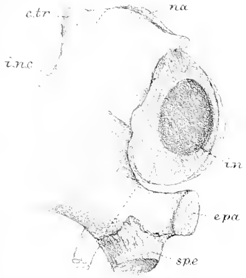


W.K.P. del. ad nat. G. West lith.

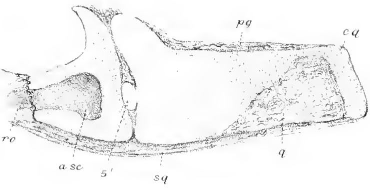
W West & Co. imp.

Siredon 8th Stage.

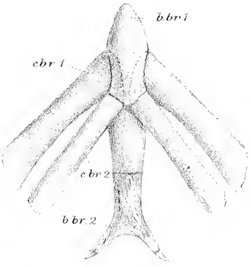
3 x 7

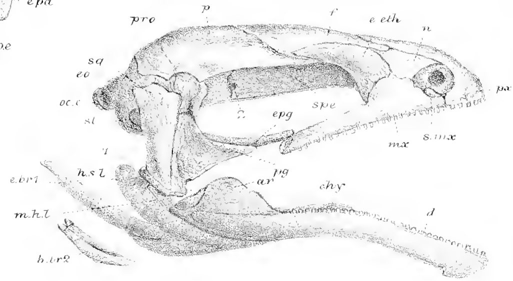


4 x 7



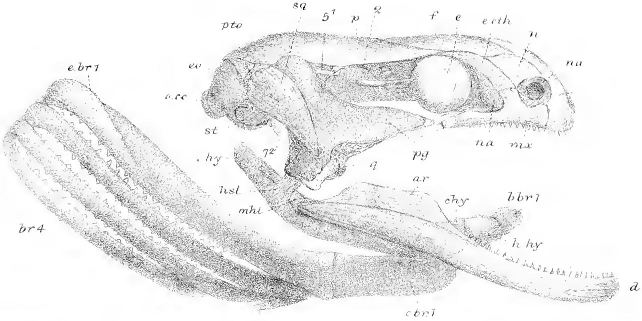
5 x 3½



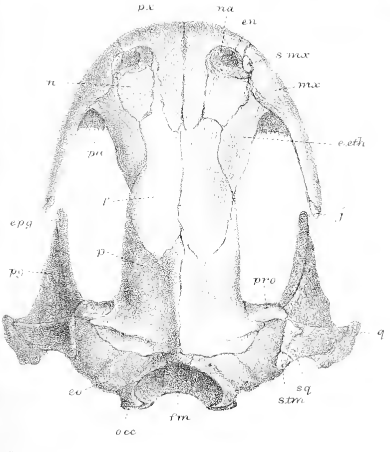


7 x 5

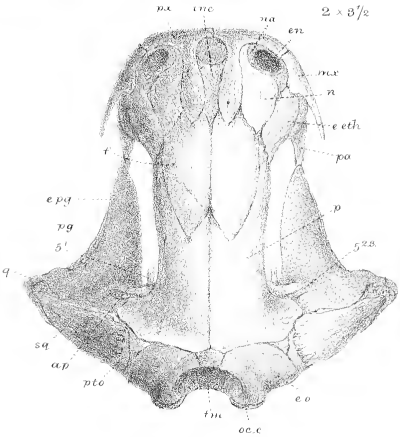
1 x 3½



6 x 5



2 x 3½

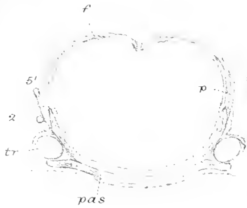


W.K.P. ad nat del. G. West lith.

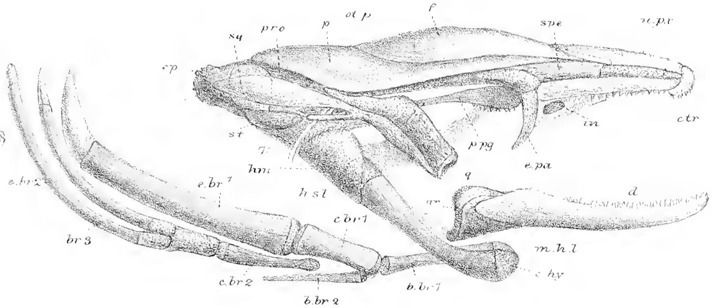
W. West & Co imp.

Siredon Figs. 1–5 9th Stage; Amblystoma Figs 6, 7.

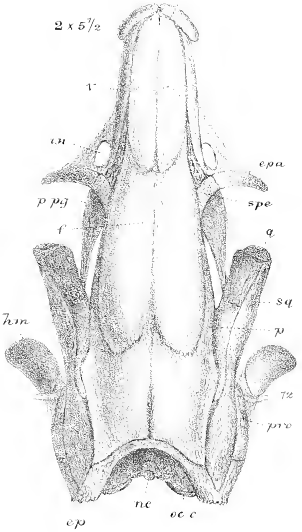
8 x 5½



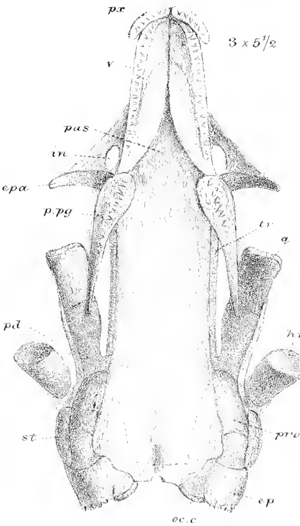
1 x 5½



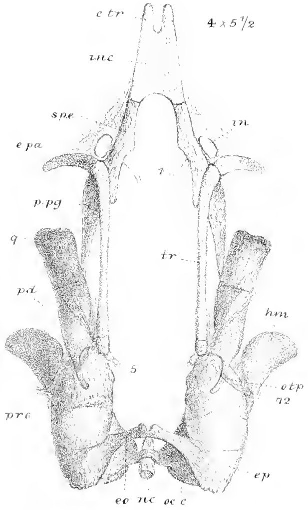
2 x 5½

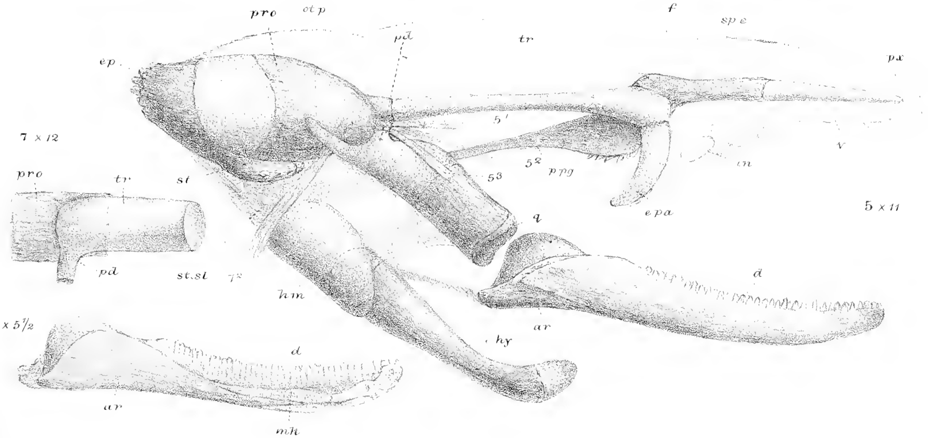


3 x 5½



4 x 5½





5 x 11

7 x 12

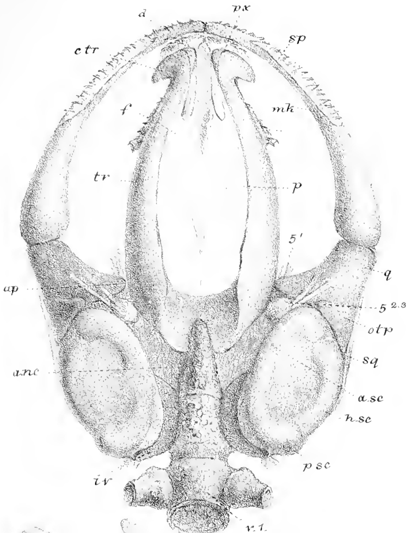
6 x 5½

W.K.P. del. ad nat. G. West lith.

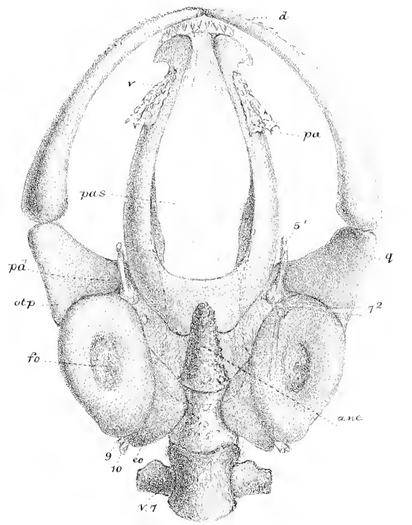
W. West & Co. imp.

Proteus anguinus.

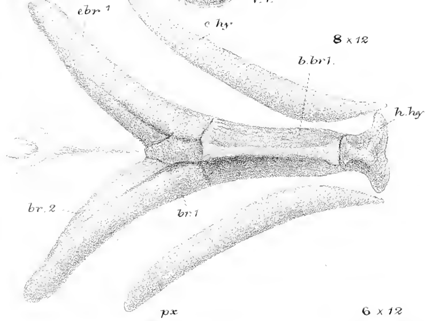
1 × 24



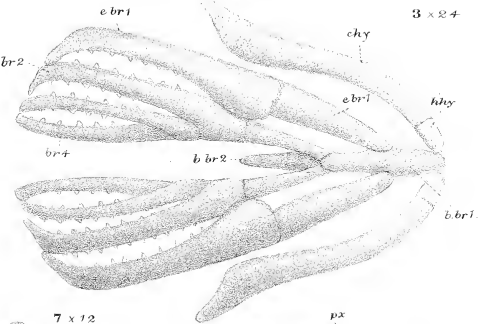
2 × 24



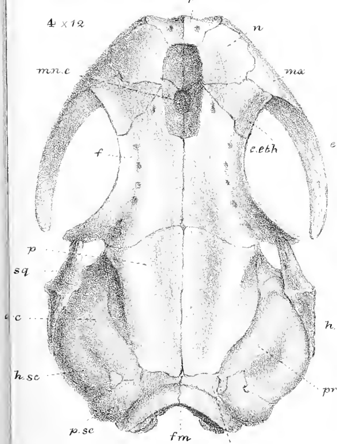
8 × 12



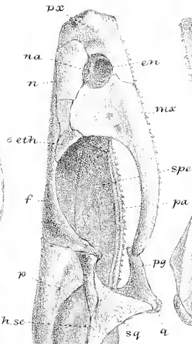
3 × 24



4 × 12

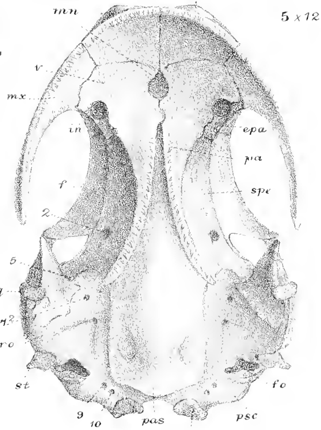


6 × 12



7 × 12

5 × 12



W.K.P. del. ad nat. G West lith.

W West & Co. imp.

Seironota perspicillata; *Figs* 1–3 *larva* 4—8 *adult*

Fig. 1

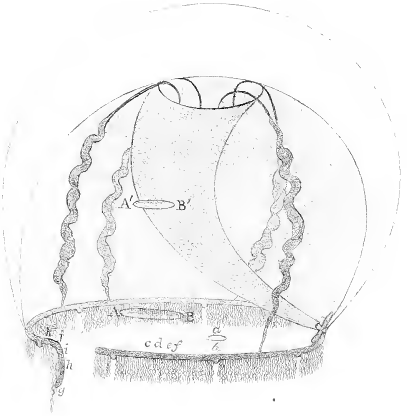


Fig. 2

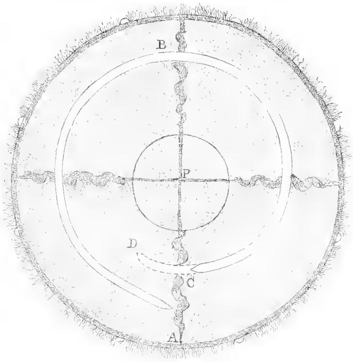
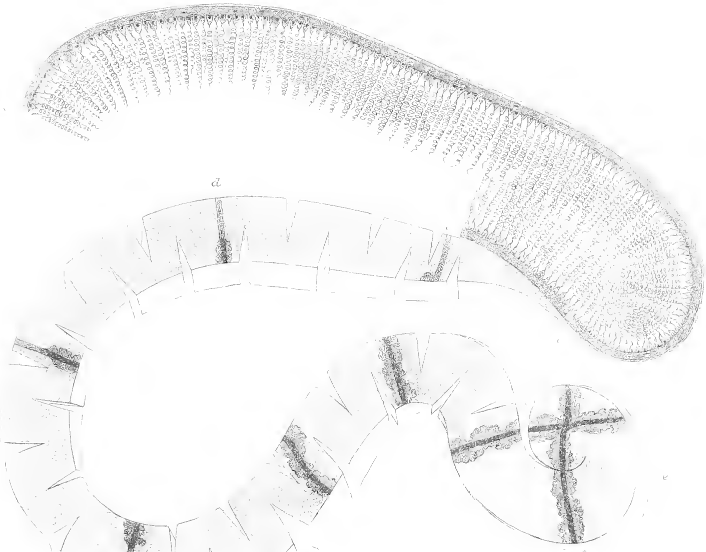
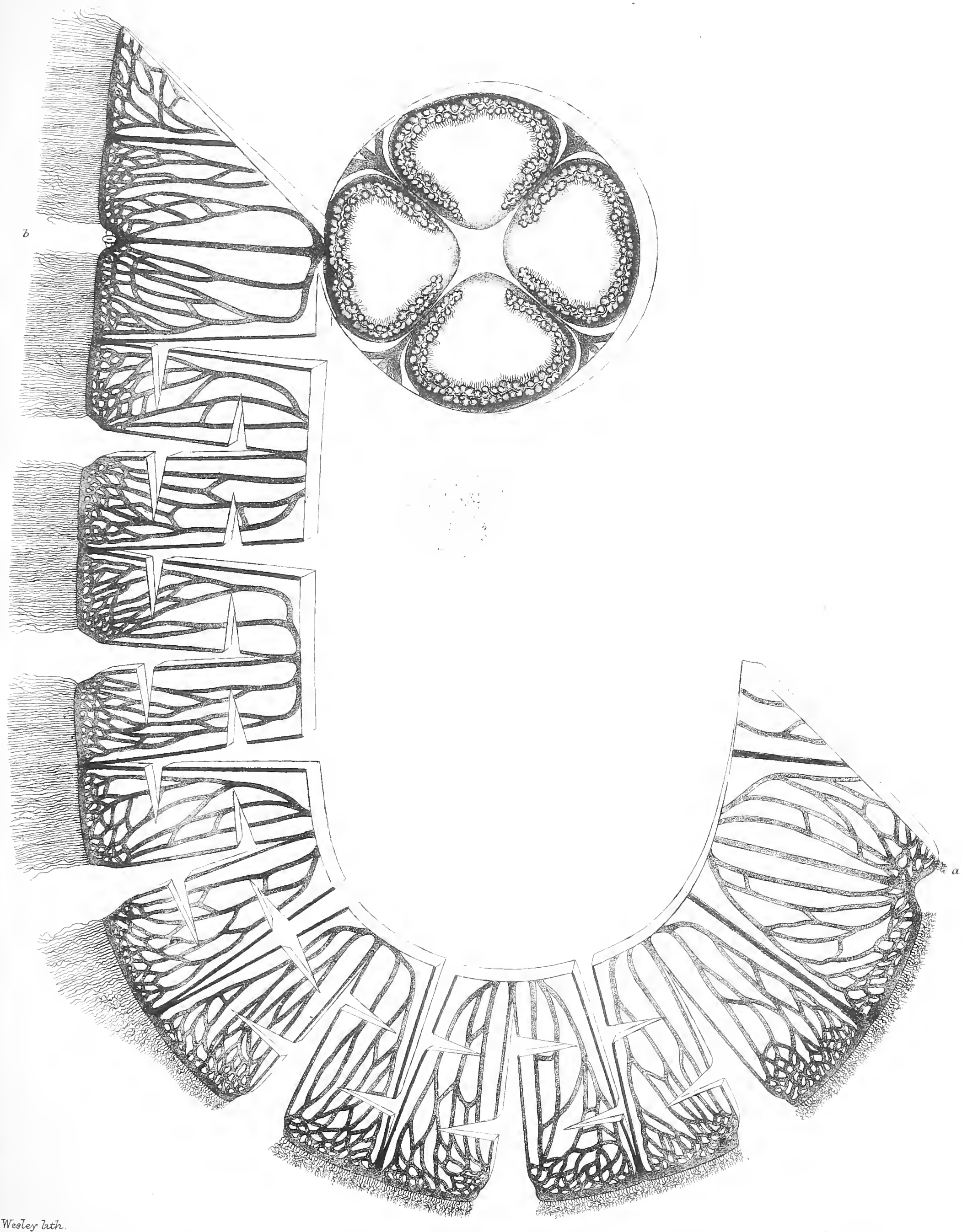


Fig. 3



W.H.Wesley, del. W.West & Co. imp.



W.H. Wesley lith.

W West & Co imp.