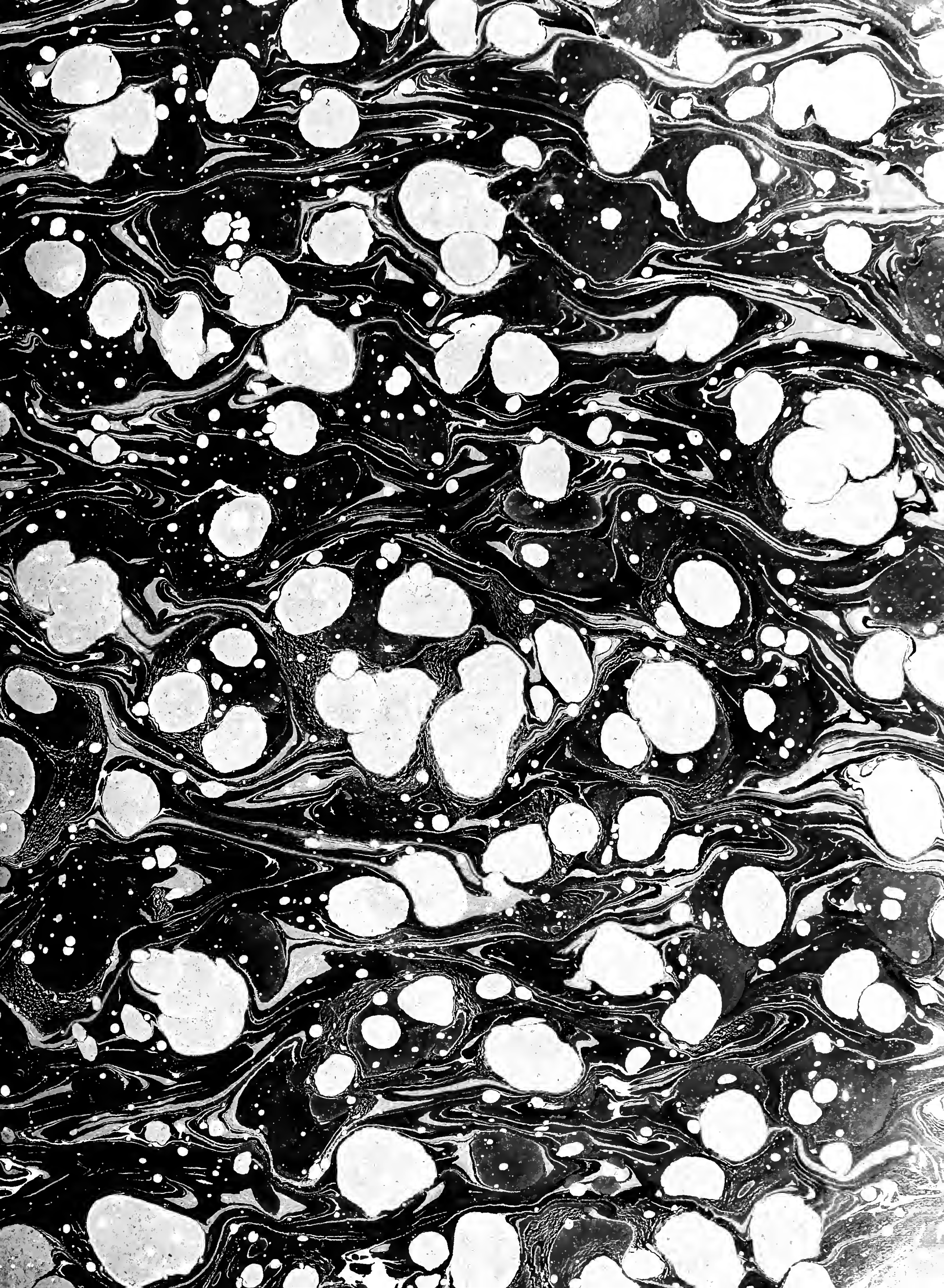

PHILOSOPHICAL
TRANSACTIONS
OF THE
ROYAL SOCIETY
OF
LONDON.

FOR THE YEAR MDCCCLXXXIII.

VOL. 174.

LONDON:
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ADJUDICATION of the MEDALS of the ROYAL SOCIETY for the year 1883, by the PRESIDENT and COUNCIL.

The COPLEY MEDAL to Professor Sir WILLIAM THOMSON, F.R.S., for (1) his discovery of the law of the universal dissipation of energy; (2) his researches and eminent services in physics, both experimental and mathematical, especially in the theory of Electricity and Thermo-dynamics.

A ROYAL MEDAL to Professor T. A. HIRST, F.R.S., for his researches in Pure Mathematics.

A ROYAL MEDAL to Professor J. S. BURDON-SANDERSON, M.D., F.R.S., for the eminent services which he has rendered to physiology and pathology, especially for his investigation of the relations of Micro-organisms to disease, and for his researches on the electric phenomena of plants.

The DAVY MEDAL to MARCELLIN BERTHELOT, For. Mem. R.S., and Professor JULIUS THOMSEN, for their researches in Thermo-chemistry.

The Bakerian Lecture "On Radiant Matter Spectroscopy: the Detection and wide Distribution of Yttrium," was delivered by Mr. W. CROOKES, F.R.S.

The Paper "On the Direct Influence of Gradual Variations of Temperature upon the Rate of Beat of the Dog's Heart," by Dr. H. NEWELL MARTIN, was appointed as the Croonian Lecture.

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PHILOSOPHICAL

TRANSACTIONS

OF THE

ROYAL SOCIETY

OF

LONDON.

FOR THE YEAR MDCCCLXXXIII.

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But the Society being of late years greatly enlarged, and their communications more numerous, it was thought advisable that a Committee of their members should be appointed to reconsider the papers read before them, and select out of them such as they should judge most proper for publication in the future *Transactions;* which was accordingly done upon the 26th of March, 1752. And the grounds of their choice are, and will continue to be, the importance and singularity of the subjects, or the advantageous manner of treating them; without pretending to answer for the certainty of the facts, or propriety of the reasonings contained in the several papers so published, which must still rest on the credit or judgment of their respective authors.

It is likewise necessary on this occasion to remark, that it is an established rule of the Society, to which they will always adhere, never to give their opinion, as a Body, upon a subject, either of Nature or Art, that comes before them. And therefore the thanks, which are frequently proposed from the Chair, to be given to the authors of such papers as are read at their accustomed meetings, or to the persons through whose hands they received them, are to be considered in no other light than as a matter of civility, in return for the respect shown to the Society by those communications. The like also is to be said with regard to the several projects, inventions, and curiosities of various kinds, which are often exhibited to the Society; the authors whereof, or those who exhibit them, frequently take the liberty to report, and even to certify in the public newspapers, that they have met with the highest applause and approbation. And therefore it is hoped that no regard will hereafter be paid to such reports and public notices; which in some instances have been too lightly credited, to the dishonour of the Society.

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LIST OF ILLUSTRATIONS.

PHILOSOPHICAL TRANSACTIONS.

I. *The Influence of Stress and Strain on the Action of Physical Forces.*[*]

By HERBERT TOMLINSON, *B.A.*

Communicated by Professor W. GRYLLS ADAMS, *M.A., F.R.S.*

Received April 5,—Read April 28, 1881.

ORIGIN AND PURPOSE OF THE INVESTIGATION.

MORE than six years ago, whilst collecting together the results of the most trustworthy measurements of the various physical constants, with a view of establishing certain relationships which I conceived to exist between them, I was much struck with the discrepancies which exist, not only between the observations of different experimenters, but also frequently between those of the same individual. Many of these discrepancies, no doubt, arise from differences in the purity of the substances employed; but, when all due allowance has been made for such a cause, there still exists a large margin, which can only be accounted for by assuming that mere alteration of molecular aggregation must modify always, and in many cases considerably, the action of physical forces. This assumption has been already abundantly justified by the experimental researches of several eminent philosophers in every part of the

---

[*] In conducting these investigations I have been aided by a grant from the Government Research Fund of £4000. For this assistance, which has and will be of the greatest service to me, I here return my grateful thanks. I feel myself also under considerable obligations to Sir WILLIAM THOMSON, whose valuable paper on "The Electrodynamic Qualities of Metals" has really formed the ground-work of this memoir. The drawings have for the most part been carefully executed by Mr. J. E. JORDAN, of the Mining Record Office; and for the intelligent carrying out of the various details of the apparatus I am indebted to Mr. KIESER, of ELLIOTT Bros., and to Mr. FURSE, the Curator of the Physical Museum at King's College, London.

domain of physical science; but, though much has been done, very much more remains to be done, and a wide field lies open before us which cannot fail to richly reward the patient explorer. I write patient explorer advisedly, because, in many cases, any difference of molecular disposition which we can effect with the means at our disposal, produces only a very minute alteration in that particular physical quality which we may be examining, and an alteration which we can only hope to measure accurately by the exercise of a large amount of perseverance and conscientious labour. But whether the effect to be observed be large or small, there is no doubt that further investigations of the kind here indicated must be made ere we can gain a true insight into the nature of the action of physical forces or into any relationships which may exist between them.

I proposed to myself, therefore, to examine as far as possible each of the various physical properties of one and the same specimen of different kinds of matter; and, further, to investigate the alterations which can be produced in these properties by stress and strain; being convinced that, by so doing, much light would be thrown on such subjects as electrical conduction, magnetic induction, thermal conduction and expansion, thermo-electricity, specific heat and elasticity. The words "stress" and "strain" are here used in their widest sense as intending to denote respectively the equilibrating application[*] of *any* physical force, and the definite alteration of form or dimensions experienced thereby by the matter acted on by the force.

In furtherance of the above-mentioned objects I procured some 60 feet of wire made of each of the various metals in common use, and also of the rarer metals, silver and platinum, and of the alloy platinum-silver, the three last in a state of chemical purity, and proceeded to determine one after the other the various physical constants of each.

In consequence of the extensive scope of these researches, several of the various parts into which the paper is divided are far from being as complete as I could wish, and hope at some future period to make them; but I venture to present them as they are to the Society, as tending to show still further than has hitherto been done the "correlation of the physical forces," and the value of this mode of investigation wherein one experiment immediately suggests several others.

PART I.—MODULI OF ELASTICITY.

"YOUNG'S MODULUS."

Description of apparatus.

The values of "YOUNG'S modulus" for the various metals were determined by a method devised by Sir W. THOMSON.[†] Wires of the same material and diameter are suspended in pairs about 1 inch apart from each other, and are attached by one

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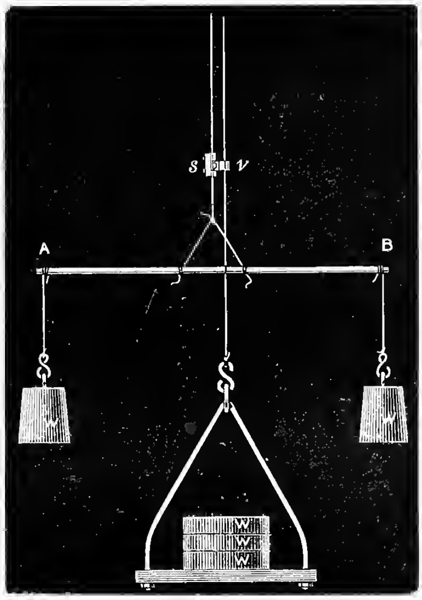
[*] 'Brit. Encyc.,' Art.: "Elasticity," p. 24.

[†] Proc. Roy. Soc., vol. xxix., p. 221.

extremity of each to the same support, the other extremities being fastened in the one case to a scale-pan, and in the other to the centre of a bar of wood or metal carrying constant equal weights at each end; the latter wire is provided with a scale, and the former with an index of some sort which, being level with and close to the scale, serves to measure any alteration of length produced by weights placed in the pan. By this simple and ingenious arrangement, any errors which might otherwise result from a slight yielding of the support, or from changes of temperature, are avoided.

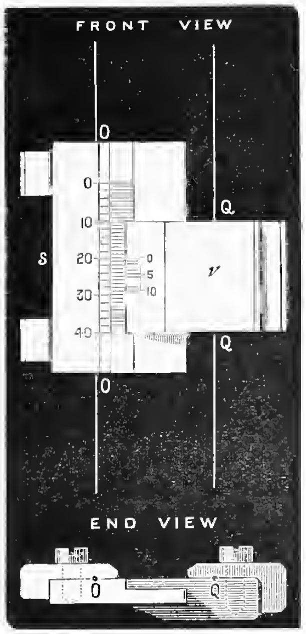
In my own particular experiments, the length of each wire between the support, which consisted of a vice firmly screwed into a stout wooden plank, and the scale and index was about 30 feet. To the extremities of the metal bar (fig. 1, A B), which was supported by one of the pair of wires, were attached two weights, each equal, in most cases, to one-fourth of the breaking-weight of the wire. To this wire was clamped by two screws a silvered metal scale (fig. 2, S), divided into half-millimetres. To the other wire was clamped in the same manner a vernier, V, reading to $\frac{1}{20}$th of a millimetre. This vernier was forked (fig. 3), so that, though capable of free up-and-down motion, it could not readily be dislodged sideways. By means of a compound microscope, an alteration of length of the wire equal $\frac{1}{100}$th of a millimetre could readily be estimated.

Fig. 1.



One-tenth size.

Figs. 2 and 3.



Scale and Vernier, full size.

Mode of experimenting.

In the case of the harder wires the one to be tested was subjected for several days to the stress produced by a weight three-fourths of the breaking-weight; the other wire, which may be called the comparison-wire, in the meantime sustaining a load equal one-half of the breaking-weight. During this time the relative positions of the scale and vernier were viewed from day to day; and when there seemed to be no alteration of these positions, the wire sustaining the heavier load was relieved from stress and allowed to rest for a period of from one to six days. Weights were then very carefully* put on to or removed from the pan attached to this wire, and the changes thereby produced in the length determined, the wire having in the meantime been permanently loaded to a sufficient extent to keep it quite straight. The softer metals—lead, tin, and even zinc and aluminium—were subjected to stresses much less in proportion to their tenacities than those used with the other metals. In their case each of the pair of wires was elongated permanently about 10 per cent. by weights, half the weight was then removed from the comparison-wire, and the two allowed to rest for four or five days. After this interval, the other wire was entirely relieved from stress, and finally was permanently loaded with a weight equal to that on the comparison-wire. As with the harder metals, the real testing did not begin till time wrought no appreciable change in the relative lengths of the two wires.

Reasons for the above precautions.

In the case of all the substances employed very great care was taken with the determinations, which often numbered more than one hundred for each of two or more different weights, and it was noticed after the experiments had been carried on for some time that the results obtained on different days frequently varied to a greater extent than could be accounted for by mere errors of observation. At first it was suspected that, in spite of the mode of suspension, a slight yielding of the upper support might be the cause of these discrepancies. Two additional wires were therefore suspended to this same support and loaded alternately with weights of 14 lbs. so as to tend to turn it about axes, respectively parallel and perpendicular to the line joining the points of attachment of the first two wires. Not the slighest effect, however, could be detected on the relative positions of the scale and vernier. The latter were then both shifted several time to parts of the wires a few inches above or below their previous positions to ascertain if any sticking between them could be observed—though in all cases care had been previously taken to adjust them so that their planes were parallel to that of the wires. But again these alterations did not seem to affect the results. The wires were now taken down and examined in case there might have

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* Several methods were tried for loading and unloading; amongst other the stress was applied by allowing measured amounts of water to flow slowly into a large but comparatively light pail attached to the wire instead of the scale-pan, the removal of the stress being accomplished by suffering the water to pass out through a piece of caoutchouc tubing attached to an orifice at the bottom of the pail, and which during the process of loading was hitched up by the side of the pail. I found, however, that quite as good results could be obtained by putting on the weights by small amounts at a time by hand; a little practice, as far as my experience goes, enables the experimenter, especially if a support be provided for the elbow, to load or unload without causing any appreciable shock.

been some slight slip of their ends through the vice. No sign, however, of anything of the kind could be detected, and on re-suspending and again testing them almost exactly the same results were obtained. Finally, the permanent loading was gradually increased in amount to determine if any error had arisen from the wires not having been sufficiently straight. It was hardly expected that this would prove to be the case, as before any of these trials had taken place the wires had been previously loaded for at least 24 hours to such an extent as to cause considerable permanent elongation. Nor indeed could any appreciable change in the measurements be detected until the weights used were such as to produce *further permanent elongation.*

When this happened, however, there was evidently an apparent alteration of elasticity. A reference to the notes made on previous occasions then revealed the fact that *whenever the above-mentioned discrepancies had occurred a similar permanent elongation had been produced.* A fresh series of experiments was therefore begun, which ended in the discovery of the following facts :—

1. In the case of a wire which has suffered permanent extension the temporary elongation which can be produced by any load becomes less as the interval between the period of permanent extension and that of applying the load becomes greater.[*]

2. This increase of elasticity is greater in proportion for great loads than for small ones.

3. The increase of elasticity takes place whether the wire is allowed to remain loaded or unloaded between the period of permanent extension and that of the testing for the elasticity.

4. The *rate of increase* of the elasticity varies considerably with different metals; with some the maximum elasticity is apparently attained in a few minutes, and with others not till some days have elapsed—iron and steel are in this last respect very remarkable.

5. The elasticity can also be increased by heavily loading and unloading several times, the rate of increase diminishing with each loading and unloading.

6. A departure from "HOOKE's law" more or less decided always attends recent permanent extension, even when the loads employed to test the elasticity do not exceed one-tenth of the breaking-weight.[†]

7. This departure is diminished very noticeably in the case of iron, and much less so with the other metals by allowing the wire to rest for some time either loaded or unloaded; it is also diminished by repeated loading and unloading.

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[*] Since writing the above I have found that the gradual increase of elasticity with time in the case of soft-iron wires has been also noticed by EWING. (Proc. Roy. Soc., 1880, No. 205, vol. xxx., p. 510.)

[†] G. WIEDEMANN and WERTHEIM have proved that there is a similar departure from "HOOKE's law" in the rigidity of metals when tested by the method of statical torsion. I shall have occasion in Part III. to refer further to Professor WIEDEMANN's valuable researches relative to torsion and magnetism: but see WIEDEMANN's 'Annalen,' 1879, vol. vi.; Phil. Mag., vol. ix., Jan. and Feb., 1880; 'La Lumière Electrique,' vol. vi., Nos. 2, 3, and 4; or WIEDEMANN's 'Galvanismus.'

The above statements apply in a greater or less degree to all the metals employed, but it will suffice perhaps to give in illustration some experiments with iron and copper wires. The pan employed in all these investigations weighed 2 kilogs., and, except in one or two instances which will be mentioned, was never detached from the wire. In the following experiments a + number will signify an increase, and a — number a decrease of length. In all cases the readings are given in half-millims., and the weight of the pan is not included in the estimate of the load.

The following series of experiments was made with an iron wire 0·62 millim. in diameter:—

Experiment I.

A weight of 12 kilogs. was put upon the pan and had the effect of permanently elongating the wire about 1½ per cent. The scale and vernier were now fixed to the wires and the following readings taken:—

No. of kilogs. on pan.	Reading of scale.	Alteration of length.	Mean values.
12	30·10		
0	5·70	−24·40	24·35*
12	30·50	+24·80	
0	6·20	−24·30	
8†	21·18		
0	5·08	−16·10	16·10
8	21·15	+16·07	
0	5·02	−16·13	
4	13·00	+ 7·98	7·95
0	5·08	− 7·92	
4	13·02	+ 7·94	

It appears therefore that the first 4 kilogs. would produce an alteration of 7·95, the second four of 8·15, and the third four of 8·25 divisions of the scale.

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* The + numbers are never taken in estimating the mean of the values of the alterations produced by the largest weight, in order to avoid the effect of permanent set.

† Scale slightly shifted by accident.

Experiment II.

The same wire having been elongated by traction to the extent of 29·4 centims. was again tried with the same weights immediately afterwards.

No. of kilogs. on pan.	Reading of scale.	Alteration of length.	Mean values.
8	21·10		
0	3·58	−17·52	17·52
8	21·10	+17·52	
0	3·58	−17·52	
4	12·08	+ 8·50	8·50
0	3·58	− 8·50	
12	31·10	+27·52	26·90
0	4·20	−26·90	
The scale-pan was now removed for 1 minute and then put on again.			
12	30·10		
0	3·12	−26·98	26·98
8	20·50	+17·38	17·37
0	3·15	−17·35	
4	11·60	+ 8·45	8·45

Taking only the last trials we see that, now, the first 4 kilogs. produce an alteration of 8·45, the second of 8·92, and the third of 9·61.

Experiment III.

The wire was still further lengthened by 11·8 centims. and the pan being on the reading of the scale was 3·40. The pan was then removed for one minute, and afterwards replaced, scale now 2·40. Again the pan was removed for 30 minutes, and replaced, scale 2·18. The trials were then renewed with the same weights as those used in the other experiments.

No. of kilogs. on pan.	Reading of scale.	Alteration of length.	Mean values.*
0	2·18		
4	10·80	+ 8·62	8·48
0	2·32	− 8·48	
8	20·30	+17·98	17·65
0	2·65	−17·65	
12	30·90	+28·25	27·50
0	3·40	−27·50	
4	11·90	+ 8·50	8·50
8	21·00	+17·60	17·50
0	3·50	−17·50	
12	30·90	+27·40	27·30
0	3·60	−27·30	

Taking the results of the last observations we obtain for the first 4 kilogs. an alteration of 8·50, for the second four 9·00, and for the third 9·80.

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* In this and the following tables the − numbers only are taken, because it was always found that after the wire had been entirely relieved from stress a small sub-permanent set was produced by each of the weights when put on for the first time after such a release.

Experiment IV.

The wire having been left for two days without weights or scale-pan was again tested with the pan on.

No. of kilogs. on pan.	Reading of scale.	Alteration of length.	Mean values.
0	3·30		
4	11·58	+ 8·28	8·23
0	3·35	− 8·23	
Scale-pan removed for 1 minute and then replaced.			
0	3·30		
8	19·80	+16·50	16·42
0	3·38	−16·42	
Pan again off and on.			
0	3·30		
12	28·10	+24·80	24·70
0	3·40	−24·70	
Pan off and on.			
0	3·30		

Here we see that the effect of the first 4 kilogs. is represented by 8·23, the second four by 8·19, and the third by 8·28.

As it had been found by preliminary experiment that the density of the wire was not permanently decreased to any extent which would introduce an appreciable error by supposing it to remain constant, we can easily make the different experiments comparable with each other by assuming that the permanent change of section is proportional to that of the length. If we do so we arrive at the following results:—

No. of experiment.	Temporary alteration of length produced by the load.	Load in kilogs.	Average alteration per 4 kilogs. on unit of area.[*]	Mean alteration per 4 kilogs. in each experiment.
I.	7·95	4	7·95	8·04
	16·10	8	8·05	
	24·35	12	8·12	
II.	8·16	4	8·16	8·41
	16·81	8	8·41	
	26·01	12	8·67	
III.	8·11	4	8·11	8·38
	16·70	8	8·35	
	26·05	12	8·68	
IV.	7·85	4	7·85	7·84
	15·66	8	7·83	
	23·56	12	7·85	

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[*] The unit of area is supposed to be that of the section of the wire in I.; the length tested was the same in all the experiments.

Remarks on the preceding experiments.

It appears from the last table that the temporary elongation produced in an iron wire by a load of given magnitude becomes greater as the permanent elongation becomes greater up to a certain limit of the latter, which limit seems to depend upon the load used to produce the temporary effect. When the above-mentioned limit has been reached further permanent elongation begins to increase the elasticity, and this increase, as other experiments have shown, is continued up to the breaking point of the wire.

The increase of elasticity produced by rest, which is very conspicuous when we compare III. and IV., is the more remarkable as it is not attended, as was at first supposed would be the case, by any appreciable permanent shortening; the latter amounting in the present instance to only ·1 millim. out of a length of 8600, actually not ·002 per cent.

This phenomenon is moreover evidently closely allied with one noted by BOTTOMLEY,[*] who has recently discovered that in the case of iron the permanent elongation which can be produced by any weight may be very largely diminished by putting on this weight in small quantities at a time with intervals of rest between, and also that the breaking stress may be considerably increased by the same process.

We may assume that the mutual attraction existing between the molecules of a wire will always tend to make them take up such positions as will give a maximum mutual attractive force. When, therefore, a wire has been permanently stretched, the molecules would immediately take up these positions were it not for coercive force; this, however, causes delay, so that if the wire were tested shortly after the permanent extension has taken place, the elasticity would be found to be less than when the molecules have had sufficient time to finally settle themselves. Nor is it necessary that any appreciable permanent contraction of the wire should attend this increase of elasticity, as the mere change of arrangement of the molecules would suffice for the purpose. To a similar cause is no doubt also due the increase of portative power which can be produced in a permanent magnet by gradually increasing its load.

Another point to be noted is the sub-permanent set which is produced in all wires; this set is greater, according as the load permanently left on the wire is greater, and also increases up to a certain limit with the time during which the stress producing the set is applied; from this it follows that the readings taken on loading a wire step by step to a certain amount will be different from those taken at the same stages of stress on unloading. This fact has already, I believe, been noticed by THOMSON; but as I wish particularly to draw attention to it in the case of iron, I give the results of one out of several experiments made on iron and copper with a view of testing the matter.

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[*] Proc. Roy. Soc., vol. xxix., p. 221.

Experiment V.

Annealed iron wire which had been very heavily loaded and unloaded a great many times on different days previous to this last experiment.

No. of kilogs. used for load.	Scale reading on loading by 4 kilogs. at a time.	Scale reading on unloading by 4 kilogs. at a time.	Difference.
0	21·22	21·22	·00
4	18·10	18·00	·10
8	15·20	15·01	·19
12	12·38	12·10	·28
16	9·45	9·22	·23
20	6·50	6·50	·00

This last experiment shows very clearly that the wire does not recover itself until all the load has been removed. Also in Experiment IV. we see that a certain amount of set disappears even with the removal of the comparatively small load of the scale-pan.

With most of the other metals the recovery of elasticity is much less marked after the first hour than is the case with iron.

The following experiments were made with a soft copper wire ·81 millim. in diameter and 630 centims. in length:—

Experiment VI.

The wire was loaded for a few minutes with a weight slightly over 8 kilogs., and, on the removal of stress, the following observations were made:—

No. of kilogs.	Time after permanent elongation.	Reading of scale.	Alteration of length.
8	4 minutes	5·80	
0	6 ,,	19·45	−13·65
8	8 ,,	5·95	+13·50
0	10 ,,	19·10	−13·15
8	12 ,,	5·80	+13·30
0	14 ,,	18·80	−13·00
8	16 ,,	5·75	+13·05
0	18 ,,	18·70	−12·95
8	20 ,,	5·68	+13·02
0	22 ,,	18·60	−12·92
8	17 hours	5·65	
0	17 hours 2 minutes	18·50	−12·85
8	,, 4 ,,	5·60	+12·90

A great part of the gradual increase of elasticity here observed is due to loading and unloading, and a similar effect may be observed in Experiments II. and III., but part is evidently due to the influence of mere rest; this is best shown by taking the differences between consecutive + or — alterations for the different times.

It will also be observed that the + and — values both here and in the experiments on iron gradually become equal under the influence of rest and loading and unloading.

Cases of aluminium and zinc.

With these metals* both the maximum temporary increase of length, caused by putting on weight, and the recovery on the removal of stress, are attained slowly in comparison with most metals. An illustration of this is afforded in the next experiment, which was made on an aluminium wire.

Experiment VII.

No. of kilogs.†	Reading of scale.	No. of minutes after putting on or taking off load.	Alteration of length in half-millims.
0	1·4		
6	18·9	1·7	+17·5
	19·2	3·0	+17·8
	19·5	5·0	+18·1
	19·8	8·5	+18·4
	20·1	12·0	+18·7
	20·4	18·0	+19·0
	20·7	26·3	+19·3
	20·8	30·0	+19·4
0	4·4	·5	−16·4
	3·7	1·5	−17·1
	3·4	6·0	−17·4
	3·0	25·0	−17·8
	1·9	1440·0	−18·9
6	19·0	1·0	+17·1
	19·3	3·0	+17·4
	19·6	5·0	+17·7
	19·9	10·5	+18·0
	20·2	18·0	+18·3
	20·5	26·0	+18·6
	20·8	40·0	+18·9
	21·1	52·0	+19·2
	21·15	60·0	+19·25
0	4·35	1·0	−16·80
	3·00	30·0	−18·15
	2·20	1440·0	−18·95

It must be observed that during the whole of this time the comparison-wire was loaded with a permanent weight equal to that on the wire which was being tested,

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* Probably tin and lead if they had been loaded sufficiently would have also behaved in this manner; it was impossible, however, to use any but very light weights in determining their elasticity, as otherwise permanent set would always have been produced.

† A weight of 4 kilogs. was kept permanently on the scale-pan.

and that the stress thus produced had been acting for several days previous to these trials.

When lesser weights were now used and only the scale-pan left on permanently the maximum alterations took place much more quickly, and the departure from "HOOKE'S law," which had before been very considerable, became comparatively slight.

The modulus was calculated from these last results.

Cases of tin and lead.

These metals are remarkable for the manner in which they run down under the influence of the slightest stress, and also for the persistence of this running down; in this latter respect they surpass aluminium and zinc. It was necessary, therefore, to keep them loaded for many days before attempting to determine their elasticity. The weights also employed for this latter purpose were very small, and, in consequence, the values of the modulus obtained by using them cannot be considered as accurate as those of the other metals; I believe, however, that they are correct within 2 per cent.

"HOOKE'S LAW."

We have seen that there is a very appreciable departure from this law shortly after permanent extension has taken place; but a departure also exists when a long rest has been allowed, both when the wire has in the meantime been heavily weighted and when it has not, even though only moderate loads be employed.

Thus, in the case of a soft copper wire capable of bearing a load of 18 kilogs., and which had been heavily loaded and frequently tested during a period of three weeks, the following observations were made:—

Experiment VIII.

Number of kilogs. in load.	Average alteration per kilog.
	Millimetre.
2	·808
4	·811
6	·816

The values here recorded are the means of about 20 observations on each weight made during the last three days of observation, and show small but decided differences.

Again, another piece of copper similar to the above was treated in the same manner, but for a period of six days, and when examined on the last day, with loads up to 12 kilogs., gave the following results as the means of seven trials with each weight:—

Experiment IX.

Number of kilogs. in load.	Average alteration per kilog.
	Millimetre.
2	·830
4	·835
6	·840
8	·847
10	·866
12	·911

It is here seen that the alteration per kilogramme rapidly increases when the loads become heavy, and yet in both these last experiments the wire recovered its original length on the removal of the load. It is evident, therefore, that in the case of annealed copper the length increases in greater proportion than the load, and this was proved to be the case whether the wire was allowed to rest loaded or unloaded.

Similar results were obtained with annealed platinum, silver, aluminium, platinum-silver, German-silver, and zinc. With soft iron, however, the case is different if the wire be weighted for some time after permanent extension has taken place. An examination of Experiment XIII. shows that under these circumstances the average alteration decreases up to a certain point as the load increases.

It remains now to consider the case of iron allowed to rest unloaded.

Experiment X.

An annealed iron wire, after having been permanently elongated by traction about 8 per cent., remained unloaded for several days, and was afterwards tested with weights up to 10 kilogs. :—

Number of kilogs. in load.	Alteration of length per kilog.
	Millimetres.
1	1·350
2	1·355
3	1·367
4	1·376
5	1·386
6	1·388
7	1·386
8	1·389
9	1·392
10	1·394

Here the average alterations of length increase with the load, though not to the same extent as with copper.

In both the last experiments with iron the recovery of the wire after the removal of the stress was so perfect that the zero position of the vernier was not shifted one-tenth of a millimetre.

Discussion of Wertheim's Experiments on Elasticity.*

The values of "Young's modulus" obtained by Wertheim by vibrations, longitudinal or transverse, are generally larger than those got by static extension; and these differences are considerably greater than those which would be produced by the heating and cooling effects of contraction and elongation.

Sir W. Thomson says† that "it is probable that his (Wertheim's) moduli, determined by static elongation, are minutely accurate; the discrepancies of those found by vibrations are probably due to imperfections of the arrangements for carrying out the vibrational method." I venture, however, to believe that the main cause of the above-mentioned discrepancies is to be found in Wertheim's *mode of proceeding* when determining the elasticity by *static extension.* The plan adopted by him was to put on a weight, take a reading with the measuring microscope, and, after removing the weight, take a second reading, the difference between these two readings being used in determining a value for the modulus. The same operations were repeated with greater and greater loads until the wire underwent very considerable permanent extension, and was in many cases broken. The mean of all the values thus obtained was taken to represent the true one.

Now, if, after considerable extension had taken place, Wertheim had repeated his trials with each of the previous weights, he would have obtained appreciably different values, and the general result would have been to give him a greater mean value for the elasticity. Moreover, my experiments have shown, as we have seen, that, even if all precautions be taken, different loads will give different values for the elasticity.

The best way of comparing the methods of static extension and longitudinal vibrations would be to determine, first, the elasticity by the former method with small loads, and then to use the latter method with the same wire under as nearly as possible the same conditions of tension. I hope at some future time to be able to make further experiments in this direction; but in some few trials with copper, iron, steel, and German-silver I have obtained values for the elasticities by the two methods which accord more nearly with each other than those got by Wertheim.

In order to ascertain whether the influence of rest—which in iron is so marked in increasing the value of "Young's modulus" as determined from static extension—would be equally or at all apparent when longitudinal vibrations are employed, several experiments were made on iron by the latter method, both the syren and Appunn's tonometer being employed for the purpose of counting the number of vibrations. Both these instruments gave very consistent results, and could be depended upon within at least $\frac{1}{2}$ per cent.; yet no difference could be detected between the note of the wire after recent permanent extension and that after a rest of 24 hours. As it was thought that perhaps the act of vibrating the wire might immediately produce the

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* Ann. de Chim. et Phys., tom. xii., 1844.

† 'Brit. Encyc.,' Art.: "Elasticity," § 77.

same effect as continued rest, a fresh pair of iron wires were suspended and tested in the usual manner with the scale and vernier; but it was ascertained that vibrating a wire under these circumstances did not produce any immediate appreciable effect on the elasticity. It would appear, therefore, that the effect of rest is not felt when the temporary elongations are very small.

Permanent Alteration of Elasticity Produced by Permanent Extension.

Experiment XI.

The same wire as in Experiment X. was further lengthened by 7, 15, 10 and 7½ centims. respectively on four separate occasions, and after each elongation the load was removed, and a rest of 24 hours allowed. After each rest the temporary alteration of length produced by 8 kilogs. was determined, the vernier after each permanent extension having been shifted to its original position.

Actual alteration observed.	Calculated alteration which would be produced on wires of the same section as in Experiment V.	Total percentage of permanent extension produced before testing.
12·88	12·88	4·0
13·02	12·87	5·1
13·32	12·87	7·5
13·50	12·84	9·0
13·65	12·83	10·2

These results are the means of several observations in each case, and show that the elasticity of copper is very slightly increased by these particular amounts of permanent extension, when the wire is allowed to rest unloaded.

A similar effect was proved to be produced on copper wire which was kept loaded for some time after permanent extension.

Experiment XII.

A piece of annealed iron wire, 860 centims. in length, was elongated by traction to the extent of 21·7 centims. so as to make it perfectly straight, and afterwards allowed to rest unloaded for two days; a set of experiments was then made which resulted in giving a mean value of 8·28 half-millims. as the alteration produced by the first 4 kilogs., 8·23 for the second, and 8·29 for the third, and an average on the whole of 8·27 for 4 kilogs.

The wire was now further lengthened by 13·3 centims., and the vernier shifted so that the same length of wire as before was under examination, and again a rest of two days allowed. On loading the wire with the same weights as before, an alteration of 8·50 was produced by the first 4 kilogs., 8·48 by the second, and 8·50 by the third, giving an average alteration of 8·49 for 4 kilogs. Allowing for the permanent diminution of section, the last alteration would correspond to 8·36 for 4 kilogs. on a wire of the same section as that previous to the second permanent elongation.

We have, in the case of this wire, therefore, a small but decided *diminution* of elasticity produced by *this amount* of permanent extension, when the wire is allowed to rest *unloaded* after the extension has taken place.

Experiment XIII.

An annealed iron wire of the same length as the previous one was tested in the same manner, except that after each permanent extension it was loaded with a weight of 20 kilogs., and this load was suffered to remain on the wire for 24 hours.

Total percentage of permanent extension ..	1·0	4·9	8·0	8·9*
Load in kilogs.	Average alteration of length in half-millims. per load of 2 kilogs. on unit area.†			
2	1·590	1·571	1·554	1·522
4	1·579	1·571	1·514	1·516
6	1·557	1·557	1·510	1·494
8	1·544	1·545	1·504	1·497
10	1·540	1·537	1·502	1·484
12	1·539	1·529	1·495	1·482
14	1·543	1·525	1·486	1·482
16	..	1·515	1·490	1·477
18	..	..	1·492	1·477
20	..	..	..	1·476

It appears, therefore, that in the case of annealed iron the elasticity is *increased* by permanent extension if the wire be allowed to remain heavily loaded for some time after such extension has taken place.

Moreover, it is remarkable that when the wire has been treated in the above-mentioned manner the average alteration per unit load *diminishes*‡ as the load

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* The wire was broken at a point about 3 inches from the scale-pan by this last extension.

† The unit area is assumed to be the area of the section of the wire after the last permanent extension; this area was ·00137 square centim.

‡ [Note added April, 1882.—It should be stated here that a load of about 6 kilogs. (not included in the loads given) was left permanently on the wire. I have since found by an indirect method (see Part II.) that, if the wire be entirely relieved from stress before beginning to test for the temporary effect of loading, the temporary elongation increases with the first few loads in greater proportion than the latter. We may say, therefore, that in the case of iron wire which has suffered very considerable permanent extension and afterwards been allowed to rest for some time either unloaded or loaded, the ratio of the temporary elongation to the load producing it first increases with the latter to a certain limit, then diminishes to a second limit, and finally begins to increase again. If, however, the wire has during the interval of rest been heavily loaded, the first limit is reached more quickly than is the case when the wire has rested unloaded; so that if, as in this instance, it is necessary to leave even a comparatively small

employed for testing *increases*, whereas with all the other annealed metals similarly treated, exactly the opposite effect is produced. This peculiarity of iron is no doubt to be attributed to its superior coercive force; and to the same cause must probably be assigned the difference between the effect of permanent extension on the elasticities of iron and copper when these metals are allowed to rest unloaded after the extension has taken place.

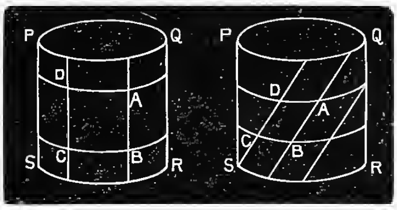
In order to examine still further the effect of leaving a heavy weight on the wire for long periods, 20 kilogs. were, after the above experiments had been made, left on the wire for one day, and then for two more days. The average increases per 2 kilogs. after each of these periods, when tested with 20 kilogs., were respectively 1·474 and 1·475 half-millims. Therefore the full effect of the loading must have been produced during the five days on which the previous trials had been made.

Finally, it should be observed that during the whole of these last experiments the wire returned to its original length on the removal of the load.

Effect of Permanent Torsion combined with Traction.

The above are the only *direct* experiments which were made of the effect of permanent extension on the value of "Young's modulus;" but having ascertained indirectly that, at any rate in the case of some metals, permanent extension will, according to its amount, produce either decrease or increase of elasticity, I was induced to make a set of observations in which torsion was combined with longitudinal traction.

Fig. 4.



Let P Q R S, fig. 4,* represent a portion of the wire in the unstrained condition; and suppose that, the upper end having been fixed, the lower end is twisted in the contrary direction to the hands of a watch, thus causing the portion A B C D to be extended along the diagonal A C and compressed along the diagonal B D; if now a load be applied at the lower end S R, this will cause the wire to twist still further or to untwist, according as the extension produced by the load along A C is greater or less than that along B D.

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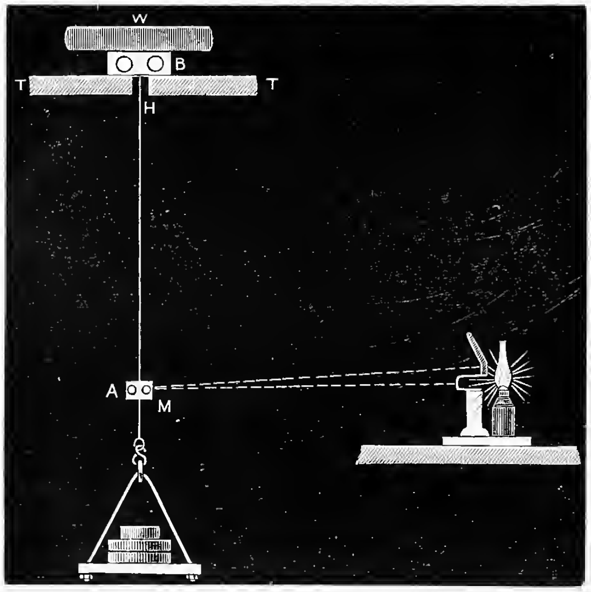
weight permanently on the wire, this weight may exceed that required for the above-mentioned limit. It is quite possible, also, that if the permanent extension and the heavy loading during rest be carried to very great excess, any load however small may exceed the first limit.]

* 'Electricity and Magnetism,' Clerk Maxwell, vol. ii., p. 86.

Mode of experimenting.

The wire to be examined passed through a small hole, H, fig. 5, made in a stout table, and was clamped at its upper extremity into a brass block, B, the latter resting on the table and being sufficiently secured by a heavy weight, W, placed on the top of it. Near the lower extremity, which was looped to receive a scale-pan, was clamped a second small block, A, to one end of which was attached a light mirror of the kind employed with reflecting galvanometers; this mirror, used in connexion with a scale and lamp, enabled the observer to detect very small differences of torsion; the distance of the scale from the mirror was 1200 scale-divisions, and the length of each wire at the commencement of each experiment was 95 centims.

Fig. 5.



In the following table will be found the results of the experiments made with the different metals, these latter being for the most part pieces of the same wires as had been used in the determinations of "YOUNG's modulus," but in each case carefully re-annealed.

TABLE I.—Number of complete turns of permanent torsion produced before the application of the load $=n$. Temporary alteration of torsion produced by the load in terms of divisions of the scale $=\alpha$; $+$ signifies further twist, $-$ untwist on loading.

a.

Iron, diameter = ·082 centim.

n.	*k.*	*a.*		
		5 mins. 0	½ hr. 0	16 hrs. 0
10	2	5·0+	5·0+	6·3+
	4	9·0+	11·5+	12·3+
	6	8·5+	13·0+	19·3+
	8	8·0+	14·0+	22·5+
	10	3·0−	14·0+	. .
		16 hrs. 0	16 hrs. 12	Average pan 2 kilogs.
50	2	12·5+	12·8+	12·8+
	4	. .	20·3+	10·2+
	6	. .	29·0+	14·5+
	8	. .	37·2+	9·3+
	10	. .	44·2+	8·8+
		16 hrs. 0		Average pan 2 kilogs.
130	2	23·0+	. .	23·0+
	4	44·5+	. .	22·3+
	6	61·0+	. .	20·3+
	8	89·5+	. .	22·4+
	10	95·0+	. .	19·0+
	5 mins. 0			
280	2	20·7+	. .	. .
	4	29·5+	. .	. .
	6	32·0+	. .	. .
	8	36·0+	. .	. .
	10	36·2+	. .	. .

b.

Platinum, diameter = ·076 centim.

n.	*k.*	*a.*	
		5 mins. 0	Average pan 2 kilogs.
50	2	75·5+	75·5+
	4	160·0+	80·0+
	6	251·5+	83·5+
		36 hrs. 2	Average pan 2 kilogs.
	2	74·0+	74·0+
	4	149·0+	74·5+
	6	237·0+	79·0+
		5 mins. 0	
100	6	322·3+	. .
		2 hrs. 6	Average pan 2 kilogs.
	2	103·5+	103·5+
	4	210·5+	105·3+
	6	327·0+	109·0+
		5 mins. 0	
150	6	412·5+	
		5 mins. 0	16 hrs. 0
250	6	477·0+	472·0+
		5 mins. 0	
300	6	514+	. .
		5 mins. 0	
348	6	522+	. .
	Broke.		

TABLE I. (continued)—Number of complete turns of permanent torsion produced before the application of the load $=n$. Temporary alteration of torsion produced by the load in terms of divisions of the scale $=\alpha$; $+$ signifies further twist, $-$ untwist on loading.

c.

Copper, diameter = ·152 centim.

n.	*k.*	*α.*		
		5 mins. 0		
50	12	·5+		
		5 mins. 0	¼ hr. 12	7 hrs. 12
150	2	. .	. .	7·5−
	4	. .	. .	15·5−
	6	. .	. .	25·5−
	8	. .	. .	. .
	10	. .	. .	43·5−
	12	42·5−	51·3−	53·5−
		5 mins. 0	1 hr. 12	14 days. 0
250	4	. .	41−	49·5−
	8	. .	87−	92·2−
	12	135−	139−	143·0−
		5 mins. 0	2 hrs. 12	Average pan 2 kilogs.
450	4	. .	103·5−	51·8−
	8	. .	200·5−	50·1−
	12	277−	304·0−	50·7−
490	Broke			

d.

Silver, diameter = ·076 centim.

n.	*k.*	*α.*
		16 hrs. 4
20	2	8+
	4	15+
120	2	83−
	4	170−
220	4	297−
320	2	285−
	4	547−
417	2	324−
	Broke	

e.

Aluminium, diameter = ·096 centim.

n.	*k.*	*α.*
		2 hrs. 2
5	2	42+
20	2	142−
120	2	513−

f.

Tin, diameter = ·098 centim.

n.	*k.*	*α.*
		5 mins. 0
5	·05	20−
10	·05	20−
20	·05	32−
40	·05	30−
80	·05	20−

g.

Lead, diameter = ·098 centim.

n.	*k.*	*α.*
		5 mins. 0
5	·05	8+
15	·05	5+
35	·05	6+
80	·05	0
	Broke	

h.

Zinc, diameter = ·098 centim.

n.	*k.*	*α.*
		5 mins. 0
20	2	110+
60	Broke	

TABLE I. (continued)—Number of complete turns of permanent torsion produced before the application of the load $=n$. Temporary alteration of torsion produced by the load in terms of divisions of the scale $=\alpha$; + signifies further twist, − untwist on loading.

k. Hard piano steel, diameter = ·082 centim.			*l.* Copper, diameter = ·096 centim.		
n.	*k.*	*a.*	*n.*	*k.*	*a.*
		5 mins. 0			5 mins. 0
5	4	171−	10	6	103+
	8	193−	20	6	153+
	12	198−	40	6	151+
		12 hrs. 12			2 hrs. 9
5	4	162−	40	6	112+
	8	192−			
	12	192−			2 hrs. 12
Same wire partially annealed					
			40	6	53+
		2 hrs. 12		10	100+
					½ hr. 13
10	4	44·5−			
	8	52·0−			
	12	47·0−	40	6	50·5+
				10	95·0+
Same wire completely annealed					
					½ hr. 14
		5 mins. 0			
			40	6	44·5+
15	4	29+		10	88·0+
	8	100+			
			Total permanent increase of length produced by the loading = 10 per cent.		
		½ hr. 8			
15	4	34+			
	8	58+			

Table I. (continued)—Number of complete turns of permanent torsion produced before the application of the load $=n$. Temporary alteration of torsion produced by the load in terms of divisions of the scale $=\alpha$; + signifies further twist, — untwist on loading.

m. Iron, a piece of the same hank as that used in *a*, but heated to a bright red and suddenly cooled.		
n.	*k.*	*a.*
		5 mins. 0
20	10	76+
120	10	160+
10 turns of torsion in the other direction.		
		5 mins. 0
	2	16+
	10	62+

n. Copper, a piece of the same wire as used in *l*.		
n.	*k.*	*a.*
		5 mins. 0
140	6	103·5+
240	6	50·0+
340	6	38·0−
		½ hr. 11
340	6	102·5−
Heated to redness by passing a burner several times up and down.		
wire cool.		5 mins. 0
340	6	50·0−
390	6	55·5−

o. Iron, a piece of the same wire as that used in *a*, but elongated 10 per cent.		
n.	*k.*	*a*
		5 mins. 0
10	6	12·2+
		24 hrs. 10
10	6	17·0+
		3 days. 0
30	6	18·0+
		12 hrs. 0
70	6	7·5+
	10	·8+
		2 days. 12
90	8	7·0−
	12	7·0−

Explanation of and remarks on Table I.

The times given in the various columns represent approximately the intervals between the imparting of the permanent torsion and the testing with the loads, whilst the numbers below the times are the number of kilogs. with which the wire was weighted during these intervals; thus $\genfrac{}{}{0pt}{}{\text{5 mins.}}{0}$ means that the wire remained unloaded for five minutes after the permanent torsion had been applied, and $\genfrac{}{}{0pt}{}{\text{16 hrs.}}{12}$ that the wire was tested 16 hours after the torsion, having in the meantime sustained a load of 12 kilogs. By unloaded we must understand that the scale-pan weighing 2 kilogs. remained on the wire, except in the cases of lead and tin, when a weight of ·05 kilog. was substituted for that of the pan.

We observe that the annealed metals may be divided into two classes; that iron, platinum, lead, and zinc, after suffering permanent torsion, *twist* temporarily[*] on loading, showing that the load produces greater temporary lengthening in the direction in which the torsion had produced extension than it does in the direction of compression. It seems, however, that if the permanent extensions previous to the loading be carried to excess, the wire will begin to *untwist* on loading; thus in *a* we observed that after 260 turns of permanent torsion the twist produced by loading has begun to diminish and in *o* that, when the wire had been considerably stretched before torsion, after 70 turns the twist is changed to untwist. Moreover, the results in *k* on hard and soft steel, combined with those in *o*, clearly show us that whether the distance between the particles be increased by mechanical means or by hardening by the process of heating and suddenly cooling[†] we ultimately arrive at a point where the twist on loading is changed to untwist.

The second class includes the metals, copper, silver, aluminium and tin; these, like the metals of the first class, at first *twist* temporarily on loading, but after a comparatively small amount of permanent torsion has been applied begin to *untwist* and continue to do so, as far as could be ascertained, until the wires will not bear any further twisting without instantly breaking; indeed, the copper in *c* was broken several times, and became so brittle and hard as to snap, like steel that has been heated to a white heat and suddenly cooled, and yet the wire apparently showed no decrease in the amount of untwisting on loading.

It will further be noticed that the average twist or untwist per kilogramme is nearly the same for the different weights employed, but other experiments on metals of the *first class* showed that when the load became excessive the average twist became less and less, and was finally converted into untwist when the load approached the breaking stress; and moreover whereas with smaller loads the *permanent* effect was in the case of both classes to cause *untwist*, with these larger stresses the wire commenced to *twist permanently:* this latter point is most easily proved with lead or tin, as comparatively small weights are required to break the wires made of these metals.

It may, I think, be fairly concluded from these and the previous experiments, that with all metals the longitudinal elasticity is *diminished* by permanent extension *carried to a certain point;* but *beyond this point increased.*

Analogous results have been obtained by THOMSON with respect to the torsional rigidity of metals[‡] and as we shall see latter on, the action of all physical forces is

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[*] It is perhaps as well to observe here that only the variations of torsion produced by *unloading* are recorded in the table.

[†] Sir W. THOMSON has already ('Brit. Encyc.,' Art.: "Elasticity," § 81) proved that hard steel wire *untwists* on loading, after suffering permanent torsion; but we see that if the steel be softened it acts like iron and *twists* on loading.

[‡] Proc. Roy. Soc., vol. xiv., p. 289, and 'Brit. Encyc.,' Art.: "Elasticity," § 78.

apparently affected in a similar manner by stress and strain whether these latter be due to mechanical force, to magnetization, or to change of temperature.

The influence of rest on the wire, whether the metal is left loaded or unloaded, is also very noticeable in the case of iron, and less but sufficiently so with the other metals; thus we see in *a* that in the case of iron after 10 turns of permanent torsion the amount of twist for 8 kilogs. is five minutes after the permanent torsion, 8 divisions, 14 divisions after a rest of half-an-hour, and 22·5 divisions after a rest of 16 hours.

TORSIONAL RIGIDITY.

The torsional rigidity of the wires was determined by the method of vibrations. The vibrators were similar to those employed by Sir W. THOMSON in his experiments on the rigidity and viscosity of metals,[*] namely, thin cylinders of sheet brass, supported by a thin, flat rectangular bar. The wire to be tested passed perpendicularly through a hole in the middle of the bar, and was there soldered. The other end of the wire was soldered into a stout iron bar, firmly held in a vice attached to a rigid support.

EFFECTS OF PERMANENT TORSION AND ELONGATION ON THE MODULUS OF RIGIDITY.

THOMSON has proved[†] that the rigidity of a wire is diminished both by permanent longitudinal extension and by permanent twist. As it seemed desirable to ascertain whether rest would restore any of the rigidity thus lost, and also whether the influences of permanent extension and torsion would be greater on vibrations through large arcs than through small ones, a series of experiments was begun of which the following are examples.

Experiment XIV.

An iron wire was considerably stretched, and the times of vibration ascertained to be—

Number of minutes after stretching.[‡]	Time of vibration.
	Seconds.
5	3·040
25	3·025
35	3·019
45	3·015
1440	3·000
2880	2·996

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[*] Proc. Roy. Soc., vol. xiv., p. 289.

[†] 'Brit. Encyc.,' Art.: "Elasticity," §§ 78, 81.

[‡] The vibrations were generally counted in each trial for about 10 minutes, and the times are reckoned from the commencement of each trial. The initial arc of vibration was in each case 10°.

Experiment XV.

A copper wire was elongated by 10 per cent., and when vibrated gave the following results :—

Number of minutes after stretching.	Time of vibration.
	Seconds.
1	4·348
11	4·337
120	4·330
1440	4·316

Experiment XVI.

An iron wire about $4\frac{1}{2}$ feet long received 50 turns of permanent twist and was then tested :—

Number of minutes after torsion.	Time of vibration.
	Seconds.
1	3·0415
180	3·0200
1440	3·0000

Experiment XVII.

A copper wire about $4\frac{1}{2}$ feet long received 50 turns of permanent twist and gave the following results :—

Number of minutes after torsion.	Time of vibration.
	Seconds.
1	4·615
2	4·500
5	4·488
1440	4·444

Experiment XVIII.

The time of vibration of a copper wire which had suffered no permanent torsion was, when vibrated through a small arc, 6·242 seconds, and when started with a twist of three revolutions* 6·316 seconds, there being thus a difference of ·074 second. The wire was now subjected to 100 turns of permanent torsion, and in a few minutes

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* Only 19 vibrations were counted in each trial, so that the amplitude of vibration might not be diminished too much.

afterwards vibrated once in 6·431 seconds through small arcs, and in 6·58 seconds when started with a twist of three revolutions. The difference between the two times is now ·149 second or twice the former difference. After a rest of 24 hours the two times became respectively 6·370 and 6·474, and the difference ·104 second.

Experiment XIX.

An iron wire, 4½ feet in length, was vibrated through different arcs for 30 seconds in each of several trials, and the means of these taken as the time of vibration.

Number of degrees in initial arc of vibration.	Mean time of oscillation.
°	Seconds = *t*.
10	1·200
90	1·250
180	1·244
90	1·228
10	1·200
360	1·210
180	1·188
720	1·250
360	1·245
180	1·220
90	1·182
10	1·200

Previous to these trials the wire had received 10 turns of permanent torsion, which had had the effect of diminishing the rigidity.

Vibrating the wire caused, in this instance, as it does in all cases where the metal has received permanent torsion, a certain amount of untwisting depending upon the amplitude of the arc of vibration.*

Experiment XX.

A piano-steel wire was vibrated several times, for about 30 seconds each time, through 1080°, and the time of vibration was found to diminish on each trial until the fourth, when it became constant. It was then vibrated through smaller and smaller arcs with the following results :—

Initial arc of vibration.	Time of vibration.
°	Seconds = *t*.
1080†	1·647
720	1·662
360	1·677
10	1·706

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[*] In consequence of this untwisting the vibrations were, in experiments of this kind, counted from the *beginning of the swing*, and not, as is usual, from the position of equilibrium.

[†] The elasticity of the wire was perfect for this degree of torsion.

Remarks on the above experiments.

It appears from Experiments XIV.–XVIII. inclusive (1) that the loss of rigidity produced by twisting or stretching a wire beyond the limits of elasticity, is partly diminished by rest; (2) that the loss is more sensible with large arcs of vibration than with small ones; and (3) that the influence of rest is more apparent in the case of large vibrations than in that of small ones.

Experiment XIX. shows that continual vibrating through large arcs has a similar effect on the rigidity to that produced on the longitudinal elasticity by heavily loading and unloading, the time of vibration through large arcs being by the former process made less, just as the temporary elongations caused by heavy loads are diminished by the latter.

Finally, Experiment XX. shows that in the case of a wire possessing great coercive force, the effect of vibrating through a large arc for several minutes actually makes temporarily the rigidity, as determined from such vibrations, *greater* than that determined from smaller vibrations: an effect analogous to that produced by leaving a wire heavily weighted for some time, when, as we have seen, the temporary effect on the length of large loads is less in proportion than of small ones.

We thus see that the effect of permanent torsion on the torsional rigidity is similar in every respect to the effect of longitudinal extension on the value of YOUNG's modulus."

Iron, aluminium, copper, and silver are the only metals which have, as yet, been tested in the above-mentioned manner, and iron, as before, is conspicuous for the large influence on it of continued rest.

In Table II. will be found embodied the results obtained in the case of each substance for the modulus of rigidity and "YOUNG's modulus," together with some other data which are further supplemented in Table III.*

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* For observations on the moduli of elasticity of nickel and carbon at the ordinary temperature of the room, and of iron, steel, nickel, and copper at the temperature at 100° C., see Part II.

TABLE II.

Name of metal.	Condition.	Specific gravity of water at 4° C. = 1. Δ.	Torsional rigidity in grms. per square centim. r.	YOUNG's modulus in grms. per square centim. e.	Ratio of lateral contraction to linear elongation. σ.
Iron (1)*	Annealed	7·759	$773{\cdot}1 \times 10^6$	1981×10^6	·281
Iron (2)	Hard drawn	7·740	$771{\cdot}1 \times 10^6$	2041×10^6	·325
Iron (3)	Hard drawn	7·520	$637{\cdot}2 \times 10^6$	1683×10^6	·321
Piano steel (1)	Hard drawn	7·814	$746{\cdot}5 \times 10^6$	1894×10^6	·269
Piano steel (2)	Hard drawn	7·784	$782{\cdot}3 \times 10^6$	1968×10^6	·259
Platinum (1)	Hard drawn	21·323	$686{\cdot}4 \times 10^6$	1443×10^6	·051
Platinum (1)	Annealed	21·300	$692{\cdot}7 \times 10^6$	1490×10^6	·076
German-silver (1)	Annealed	8·700	$493{\cdot}7 \times 10^6$	1335×10^6	·354
German-silver (2)	Annealed	8·632	$456{\cdot}2 \times 10^6$	1291×10^6	·415
German-silver (2)	Hard drawn	8·632	$389{\cdot}6 \times 10^6$	1169×10^6	·500
Copper (1)	Annealed	8·913	$440{\cdot}6 \times 10^6$	1160×10^6	·315
Copper (1)	Hard drawn	8·896	$418{\cdot}2 \times 10^6$	1449×10^6	·733
Copper (2)	Annealed	8·851	$419{\cdot}3 \times 10^6$	1218×10^6	·453
Copper (3)	Annealed	8·825	$457{\cdot}4 \times 10^6$	1143×10^6	·293
Platinum-silver (1)†	Annealed	12·623	$369{\cdot}9 \times 10^6$	1051×10^6	·420
Platinum-silver (1)	Hard drawn	12·608	$302{\cdot}3 \times 10^6$	1038×10^6	·717
Brass (1)	Hard drawn	8·396	$321{\cdot}1 \times 10^6$	$988{\cdot}4 \times 10^6$	·587
Brass (2)	Hard drawn	8·488	$332{\cdot}5 \times 10^6$	$988{\cdot}1 \times 10^6$	·504
Zinc (1)	Hard drawn	7·138	$338{\cdot}4 \times 10^6$	$766{\cdot}9 \times 10^6$	·133
Silver (1)	Annealed	10·491	$271{\cdot}8 \times 10^6$	$742{\cdot}4 \times 10^6$	·367
Silver (1)	Hard drawn	10·434	$274{\cdot}6 \times 10^6$	$764{\cdot}5 \times 10^6$	·392
Aluminium (1)	Hard drawn	2·730	$249{\cdot}8 \times 10^6$	$669{\cdot}4 \times 10^6$	·340
Aluminium (1)	Annealed	2·732	$265{\cdot}2 \times 10^6$	$673{\cdot}1 \times 10^6$	·269
Tin (1)	Drawn	7·264	$120{\cdot}9 \times 10^6$	$277{\cdot}1 \times 10^6$	·145
Lead (1)	Drawn	11·193	$74{\cdot}0 \times 10^6$	$167{\cdot}0 \times 10^6$	·136

Remarks on Table II.

The determinations of r were made in all cases with unstretched pieces of the different wires, and may for the most part be considered as extremely accurate, but in the cases of tin and lead it was found very difficult to obtain good observations on account of the great viscosity of these metals; indeed, with the former only *four* vibrations of convenient amplitude could be counted.

Tin, lead, zinc, and aluminium are placed in the order of their viscosity.

The annealed and hard drawn wires having the same numbers attached to them in the tables are not the *same pieces* but are cut from the *same hank*. I should have employed actually the same pieces in the two conditions, had I not wanted them for the purposes mentioned in Part II.

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* This metal and copper (3) I obtained through the kindness of Sir W. THOMSON; their moduli of elasticity had been carefully determined by T. GRAY, in the Physical Laboratory of Glasgow University. Iron (2), (3), steel (1), (2), and brass (2) were tested some years ago by myself with the cathetometer.

† This alloy was composed of two parts by weight of silver and one of platinum.

The values of Δ were determined very carefully, more so perhaps than was necessary. The specimens used for this purpose had not been stretched, and when in the water were well freed of air bubbles by brushing. The results are certainly correct to the third decimal place.

The ratio of lateral lineal contraction to longitudinal dilatation was calculated from the formula $\sigma=\frac{e}{2r}-1$* on the assumption of the wires being isotropic. It seems evident, however, that the values of σ thus obtained cannot claim to be even approximately correct when the metal has been rendered very hard by the process of drawing, as was the case with copper (1), platinum-silver (1), brass (1), brass (2), and German-silver (2): here we meet with apparently impossible results.

The mean value of σ for the *different* substances† employed in the *annealed* condition $=\cdot 2515$, a number closely according with that assigned by POISSON as the value of σ for *each*.

The metals copper (1), copper (2), platinum, aluminium, silver, and platinum-silver were obtained from Messrs. JOHNSON, MATTHEY, and Co. as chemically pure, and the zinc, lead, and tin wires as being as pure as could be got by the ordinary process of distillation.

ELASTICITY OF VOLUME.

If e denote the value of "YOUNG'S modulus," and σ the ratio of lateral contraction to longitudinal extension, it can easily be proved that the elasticity of volume $=\frac{1}{3}\cdot\frac{e}{1-2\sigma}$, and as e in Table II. is measured in grammes per square centimetre, it follows that the increase of volume per unit resulting from a longitudinal stress of 1 grm. per square centimetre $=\frac{1-2\sigma}{e}$.

In the following table are given the values of the volume elasticity, which will be denoted by v, and of the alteration of volume $\frac{1}{3v}$ produced by the above stress.

In the same table, in order to complete the information given in Table II., is recorded the section of each wire in square centimetres; the section of the hard-drawn metals in Table II. being approximately equal to those given here for the annealed wires.

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* THOMSON and TAIT'S Nat. Phil., p. 521.

† Copper (2) is not included in this estimate, as I have reason to believe that it was imperfectly annealed.

TABLE III.

Name of metal.	Elasticity of volume $= v = \frac{e}{3(1-2\sigma)}$.	Alteration of volume produced by longitudinal stress of 1 grm. per square centim. $= \frac{1}{3v}$.	Section in square centims.
Iron	1508×10^{6}	$221{\cdot}1 \times 10^{-12}$	6550×10^{-6}
Platinum (1)	$585{\cdot}7 \times 10^{6}$	$569{\cdot}0 \times 10^{-12}$	5178×10^{-6}
German-silver (1)	1524×10^{6}	$218{\cdot}7 \times 10^{-12}$	5731×10^{-6}
Copper (1)	1045×10^{6}	$319{\cdot}0 \times 10^{-12}$	7310×10^{-6}
Copper (3)	$920{\cdot}3 \times 10^{6}$	$362{\cdot}3 \times 10^{-12}$	18330×10^{-6}
Platinum-silver (1)	2190×10^{6}	$152{\cdot}2 \times 10^{-12}$	7681×10^{-6}
Zinc (1)*	$348{\cdot}3 \times 10^{6}$	$957{\cdot}0 \times 10^{-12}$	8144×10^{-6}
Silver (1)	$930{\cdot}3 \times 10^{6}$	$358{\cdot}3 \times 10^{-12}$	5464×10^{-6}
Aluminium (1)	$316{\cdot}0 \times 10^{6}$	$1055{\cdot}0 \times 10^{-12}$	8632×10^{-6}
Tin (1)	$130{\cdot}1 \times 10^{6}$	$2562{\cdot}3 \times 10^{-12}$	7758×10^{-6}
Lead (1)	$76{\cdot}5 \times 10^{6}$	$4360{\cdot}0 \times 10^{-12}$	7374×10^{-6}

Remarks on Table III.

There is little to be said with reference to this table except to call attention to the great alteration which takes place in the order of several of the metals, with reference to their elasticity of volume, and that occupied by them in the tables of "YOUNG'S modulus." We find, for instance, platinum, which in the latter table stands second on the list of annealed metals, here ranking as seventh, whilst the alloys, platinum-silver and German-silver, are both higher than iron, the former of the two alloys conspicuously so. It would seem, moreover, that either small reliance can be placed on the method of determining the ratio of lateral contraction to linear elongation from observations of the longitudinal elasticity and torsional rigidity, or else that the volume elasticity varies considerably with different specimens of the same metal; for instance, the mean value for the modulus of bulk elasticity in the case of the two specimens of annealed copper recorded in the last table is 982×10^{6}, whereas THOMSON† gives the corresponding value for copper as 1717×10^{6}.

PERMANENT ALTERATION OF DENSITY PRODUCED BY LONGITUDINAL TRACTION.‡

A few experiments were made with a view to determine the *permanent* alteration of density which can be produced by longitudinal traction. Two methods were adopted:

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* Zinc, tin, and lead are, though in the drawn condition, added to this list, as the process of drawing had not hardened them in any degree sufficient to make much difference in either the torsional rigidity or the modulus of elasticity.

† 'Brit. Encyc.,' Art.: "Elasticity," Table I.

‡ For observations on the alteration of density produced by torsion and hammering, see Part II.

in the one the wire was lengthened by successive loads put on for three minutes each and then removed; in the other the wire was stretched by the hand or by the aid of a lever by equal amounts each time until breaking ensued. The balance used in determining the density was an exceedingly good instrument made for me by OERTLING for the purpose of measuring the coefficients of thermal expansion of the metals by weighing them in water at different temperatures. It will suffice here to state that it was possible with this instrument to weigh an object in water to $\frac{1}{10}$th of a milligramme. The air bubbles clinging to the sides of the metals were carefully brushed off, as it was not possible to boil them off, for fear of partially annealing the wires, and the proper corrections were made for temperature and air displaced. The following experiments serve to illustrate the general nature of the results obtained by the two methods:—

Experiment XXI.

SILVER (1).

Load in kilogs. used in producing extension.	Specific gravity water at 4° C. = 1.	Total percentage of increase of length $= dl$.	Total percentage of decrease of specific gravity $= d\Delta$.	$\frac{d\Delta}{dl}$.
1	10·47691	. .	. .	. .
6	10·47561	1·15	·0124	·0108
7	10·47207	3·64	·0461	·0127
7·75	10·46754	7·28	·0892	·0123
8·25*	10·46153	9·38	·1465	·0156

Experiment XXII.

COPPER (1).

Load in measures of water, each measure = 2·5 kilogs.	Specific gravity water at 4° C. = 1.	Total percentage of increase of length $= dl$.	Total percentage of decrease of specific gravity $= d\Delta$.	$\frac{d\Delta}{dl}$.
5	8·8252	. .	. .	. .
9	8·8251	2·58	·00113	·00044
10	8·8247	4·21	·00566	·00134
11	8·8102	6·94	·1699	·02448
12	8·8076	10·74	·1993	·01856
13	8·7968	16·27	·3216	·01977

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* Wire broken.

Experiment XXIII.

IRON.

Specific gravity water at 4° C. = 1.	Total percentage of increase of length = dl.	Total percentage of decrease of specific gravity = $d\Delta$.	$\frac{d\Delta}{dl}$.
7·7849	. .	. .	. .
7·7771	9·35	·1002	·0107
7·7747	12·65	·1311	·0104
7·7730	16·87	·1529	·0091
7·7684	20·73	·2121	·0091
7·7520	25·41*	·4229	·0167

It will be observed that in all cases the ratio of the decrease of specific gravity to the increase of length at first increases to a maximum, then decreases, and again increases largely when the breaking strain has been reached. In any case, however, the alteration of density which can be produced by longitudinal traction is small, and in my own experiments never reached $\frac{1}{2}$ per cent., though several of the wires were strained to breaking.

WERTHEIM has also obtained similar results.†

RELATION BETWEEN MODULI OF ELASTICITY AND INTERMOLECULAR DISTANCE.

If we denote the specific gravity of a substance by Δ, and A represent the atomic weight, the intermolecular distance will be proportional to $\left(\frac{A}{\Delta}\right)^{\frac{1}{3}}=\alpha$.

It is natural to suppose that as α diminishes the elasticity will increase, and in fact WERTHEIM has shown‡ that is the case, and moreover that approximately "YOUNG'S modulus" varies inversely as α^7.

In the next table will be found the products of $e\times\alpha^7$ and $r\times\alpha^7$ for the annealed metals.

TABLE IV.

Metal.	Specific gravity = Δ.	Atomic weight A.	Intermolecular distance $=\left(\frac{A}{\Delta}\right)^{\frac{1}{3}}=a$.	$e\times a^7$.	$r\times a^7$.
Iron (1)	7·759	56·0	1·932	1994×10^8	778×10^8
Platinum (1)	21·300	197·4	2·100	2688×10^8	1250×10^8
Copper (1)	8·913	63·5	1·924	1133×10^8	430×10^8
Zinc (1)	7·138	65·0	2·088	1328×10^8	586×10^8
Silver (1)	10·491	108·0	2·175	1712×10^8	627×10^8
Aluminium (1) . . .	2·732	27·5	2·159	1473×10^8	580×10^8
Tin (1)	7·264	118·0	2·533	1852×10^8	812×10^8
Lead (1)	11·193	207·0	2·644	1510×10^8	669×10^8

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* Wire broke.

† Ann. de Chimie, 1844, tom. xii.

‡ Ibid.

The mean values of $e\times\alpha^7$ and $r\times\alpha^7$ are respectively 1711×10^8 and 717×10^8, and as in WERTHEIM's results, platinum and copper differ in respect to these products from the mean values, more than the other metals. I have ascertained, however, that there is greater accordance between both $e\times\alpha^7$ and $r\times\alpha^7$, in the case of the different metals, than can be obtained by taking the products of e and r with any other power of α.

THE INFLUENCE OF AN ELECTRIC CURRENT AND OF MAGNETISM ON THE TORSIONAL RIGIDITY OF METALS.

WERTHEIM has shown* that the longitudinal elasticity of metals is temporarily diminished by the passage of an electric current, independently of the alteration which would result from the elevation of temperature produced by the current: he has also proved that *long-continued* magnetization diminishes both temporarily and permanently the elasticity of iron and steel, but that if the magnetization be continued for only a short time there is no sensible effect. As it seemed desirable to supplement these observations by others on the torsional rigidity of metals, a few experiments were made with this object.

Experiment XXIV.

A copper wire, 8 feet in length and ·095 centim. in diameter, was suspended for observations of the torsional rigidity in the manner previously described. To the centre of the flat bar which carried the cylinder was soldered a piece of platinum wire, about 3 inches in length and ·05 centim. in diameter; the other extremity of this latter wire, which hung vertically downwards, dipped into a mercury cup, so that by means of the cup and a silk-covered copper wire soldered to the upper bar, from which was suspended the wire under examination, connexion could be made with a battery of GROVE's cells, in the circuit of which was placed a GAUGAIN-HELMHOLTZ's tangent galvanometer; with this galvanometer the current of one DANIELL's cell freshly charged with sulphuric acid diluted with seven parts of water and sulphate of copper solution, a deflection of 24·9° was produced through a resistance of ·71 ohm.

Observations.	Time of one vibration.	Current.	Total percentage of diminution of rigidity caused by the current.	Percentage decrease of rigidity produced by heating effect of current.	Percentage decrease of rigidity caused by the current proper.
	Seconds.				
Platinum wire out of mercury cup	5·570	0	. .	. .	. .
Platinum in cup	5·568	0	. .	. .	. .
Wire shortened	5·355	0	. .	. .	. .
	5·395	73°·4	1·54	·63	·91
	5·361	0	1·26	·63	·63

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* Ann. de Chimie, 1844, tom. xii., p. 610.

Experiment XXV.

Iron wire about 8 feet in length and ·13 centim. in diameter.

Observations.	Time of vibration.	Current.	Total percentage of diminution of rigidity caused by the current.	Percentage of diminution of rigidity produced by heating effect of current.	Percentage of diminution of rigidity produced by the current proper.
	Seconds.				
	4·153	0°	. .	. .	. .
	4·197	65°·8	2·10	1·30	·80
	4·155	0°	2·00	1·30	·70
	4·207	68°·2	2·48	1·60	·88
After three days' rest . . .	4·180	0°	. .	. .	. .
	4·177	28°·5	·14—	·05	·19—
	4·191	46°	·52	·25	·27
After three days' further rest . .	4·161	0°	. .	. .	. .
	4·252	71°	4·34	2·05	2·19
After three days' further rest . .	4·164	0°	. .	. .	. .
	4·164	16°·3	0	. .	. .

Experiment XXVI.

A piece of the same kind of iron wire as that used in the last experiment, about 35 centims. in length, was placed in the axis of a magnetizing helix, 30 centims. in length; the helix consisted of 1,200 turns of copper wire $\frac{1}{20}$th of an inch in diameter, wound round a glass tube of 3 centims. inner diameter and 3 centims. thickness.

Time of vibration.	Temperature of wire.	Current.	Total percentage of diminution of rigidity caused by the magnetizing current.	Percentage decrease due to heating effect of the current.	Percentage decrease of rigidity due simply to magnetization.
Seconds.	° C.	°			
1·739	10	0	. .	. .	. .
1·743	16	·80	·46	·28	·18
1·722	14	0	2·42	·07	2·35

Several other experiments of the same kind as the last were made with different magnetizing helices and current strengths, and all seemed to show that slight temporary diminution of rigidity is produced by high magnetizing force; but that magnetizing forces small in comparison with that indicated in the last table, and yet sufficient to cause very sensible magnetization, produced no sensible effect. The vibrations were counted for at least half an hour in each determination, and an initial arc of 10° of torsion was employed in all cases. The percentage decrease of rigidity produced by the rise of temperature caused by the current was calculated from KOHLRAUSCH's formulæ,[*] for iron

$$n=n_0(1-\cdot 000447t-\cdot 00000052t^2),$$

and for copper

$$n=n_0(1-\cdot 00052t-\cdot 00000028t^2);$$

where n_0 and n represents the rigidity at 0° C. and t° C. respectively. In Experiment XXVI. a delicate thermometer was placed near the wire, half-way down the helix, and the rise of temperature calculated from the mean of several readings taken from time to time during the passage of the current. In Experiments XXIV. and XXV. the rise of temperature was determined in the following manner:—A current of 70° was passed through the copper wire used in Experiment XXIV., and the soldered junction of a thermo-element, made of fine German-silver and iron wires, each about 2 feet in length, was kept in close contact with the copper wire by placing the latter between the two former; the other ends of the wires forming the thermo-element were connected by means of silk-covered copper wire with the terminals of a galvanometer, and after being wrapped in tissue paper and cotton-wool, were placed in a clip-stand, which was drawn on one side so as to slightly press the soldered junction against the suspended copper wire, a layer of tissue paper having been used to insulate the latter from the former. After a short time the deflection of the galvanometer became constant, and on immersing the junction of the thermo-element in water, it was found necessary to raise the temperature 8·5° C. in order to produce the same deflection as before; 8·5° C. was therefore assumed to be the rise of temperature which would be caused in the copper wire by the above-mentioned current, and the rise of temperature produced by the other currents was calculated from the assumption that the heat generated would be proportional to the square of the current strength. In the case of the iron wire it was assumed that the specific resistance of the iron would be six times that of the copper, and the rise of temperature was calculated accordingly.

An examination of these last tables shows apparently that the torsional rigidity of copper and iron is temporarily decreased by the passage of a powerful current, but is very little altered by currents of moderate intensity.[†]

Experiment XXIV. also shows that the dipping of the platinum wire into the mercury-cup did not appreciably affect the time of vibration.

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[*] 'Brit. Encyc.,' Art.: "Elasticity," § 79.

[†] I cannot place so much confidence as I could wish in the results of these particular experiments, as far as the decrease of rigidity by powerful currents is concerned, the method employed for estimating the elevation of temperature produced by the current being evidently only calculated to give a very rough approximation to the true values. I hope, however, to be in a position at some future time to attack the question in an entirely different way.

Critical points.

Several determinations of the *permanent* increase of length produced by loading were made, and led to the discovery that in all *well and carefully* annealed metals there are at least two points at which a sudden change takes place in the ratio of the load to the permanent extension produced thereby; these points have been called critical points, and it appears that changes more or less profound take place in most if not all of the physical properties of the substance when these points are attained.

As, however, the subject will be fully discussed in Part II., where it will be shown how these points can be indirectly determined more accurately than in the ordinary manner, it will suffice here to say that the existence of the first of these critical points seems to prove beyond a doubt that there is *a true limit of elasticity* for each substance, and that this is intimately connected with the value of "Young's modulus."

It would also appear[*] that at these two critical points sudden changes take place in the density of the substance.

Summary of Part I.

1. The magnitude of the temporary elongation which can be produced by any load on a wire which has experienced permanent extension is reduced by simply allowing the wire to rest either loaded or unloaded for some time after the permanent extension has taken place.

2. The length of the period of rest necessary to produce the maximum of the effect mentioned in 1 varies considerably with the nature of the metal of which the wire is made; with some metals a few minutes suffice, whilst with others, such as iron or steel, many hours are required.

3. The effect of rest mentioned in 1 is greater in proportion for large loads than for small ones, and apparently vanishes in the case of such small temporary alterations of length as are produced by causing the wire to vibrate longitudinally.

4. The magnitude of the temporary elongation which can be produced by any load on a wire which has suffered recent permanent extension is also reduced by heavily loading and unloading the wire, the rate of reduction diminishing with each loading and unloading.

5. A departure, as far as temporary elongation is concerned, from "Hooke's law," more or less decided, always ensues after recent permanent extension, even when the weights employed to produce the temporary elongation do not exceed one-tenth of the breaking-load of the wire.

6. This departure is diminished very noticeably in the case of iron, and to a greater or less extent with all metals by allowing the wire to rest for some time either loaded or unloaded; it is also diminished by repeated loading and unloading.

7. With aluminium and zinc, and probably with the more viscous metals tin and

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[*] See Experiments XXI.–XXIII. inclusive.

lead, both the maximum temporary increase of length which can be produced by any load, and the complete recovery after the removal of the load, are only attained after an interval of several hours, provided that the weights used for the load be not very small compared with the breaking-load.

8. There is a small but decided departure from "HOOKE's law," as far as temporary elongation is concerned, in all cases where the load employed to produce the elongation is of moderate amount; this is the case even when sufficient rest has been allowed to enable the wire to attain its maximum elasticity.

9. We can therefore only obtain by the method of longitudinal vibrations values for "YOUNG's modulus," which are strictly comparable with those got by the method of static extension, by experimenting when we use the latter method with very small loads, and with the wire under the same conditions of stress and strain as those occurring when the former method is adopted.

10. In the case of all metals, permanent extension, if not carried beyond a certain limit, causes, whether rest is or is not allowed after the permanent extension has taken place, a diminution in the value of "YOUNG's modulus," as determined by the method of static extension.

11. If the permanent extension be carried beyond the above-mentioned limit, further permanent increase of length increases the value of "YOUNG's modulus."

12. The limit of permanent extension mentioned in 10 varies considerably with different metals, and with the time which is allowed to elapse after the permanent extension has taken place.

13. In the cases of iron, heavy loading for some time so increases the value of "YOUNG's modulus," as determined by the method of static extension, that even when the extension would have caused, without such loading, diminution of the modulus, this diminution can be changed to an increase; with copper this is not the case.

14. With iron wire which has been heavily loaded for some time, the ratio of the temporary elongation to the load producing it becomes less as the load employed becomes greater, until a certain limit, depending upon the extent of the previous heavy loading, has been reached; whereas with most other metals, and with iron which has suffered permanent extension without allowing the load which has produced the extension to remain for some hours on the wire, the elongation increases at first in greater proportion than the load.

15. The effects on the longitudinal elasticity, and on the torsional rigidity of steel, of suddenly chilling the metal after it has been raised above a bright red heat, are similar to those produced by excessive permanent traction.

16. The loss of torsional rigidity, which is caused by twisting or stretching a wire beyond the limits of elasticity, is diminished by rest.

17. The influence of rest mentioned in 16 is greater in proportion for large arcs of vibration than for small ones, and is more noticeable with iron and steel than with most of the other metals.

18. The loss of torsional rigidity mentioned in 16 is more sensible proportionally with large arcs of vibration than with small ones.

19. Continual vibrating through large arcs has a similar effect on the torsional rigidity to that produced on "YOUNG's modulus" by heavy loading and unloading.

20. The density of a wire is very little altered by permanent extension, even if the latter be carried to the extent of breaking the wire.

21. The values of "YOUNG's modulus" obtained for the different metals are, roughly speaking, inversely proportional to the seventh powers of the mean distances between adjacent molecules in these metals.

22. The torsional rigidity of copper and iron wires seems to be temporarily decreased by the passage of a powerful electric current through the wires.

23. The torsional rigidity of iron wire seems to be temporarily diminished when the wire is subjected to a powerful longitudinal magnetizing force.

24. The effects mentioned in 22 and 23 are apparently independent of any change produced by the electric current or the magnetizing force of the temperature of the wires.

25. There are, in every well-annealed wire, two critical points at which sudden changes take place in the ratio of the permanent extension produced by longitudinal stress which is gradually increased in amount and the magnitude of the stress.

Received January 18,—Read January 26, 1882.

PART II.—ELECTRICAL CONDUCTIVITY.

The Temporary Alteration of Electrical Conductivity produced by Longitudinal Traction.

W. Thomson, in 1856,* investigated the effects of tension on electrical conductivity in copper and iron wires, and, moreover, stated that he "had very nearly established, for the case of iron at least, that the augmented resistance due to tension, either temporary or permanent, is a very little more than can be accounted for by the change of form."

In 1877† I determined in absolute units the amount of alteration produced by longitudinal traction in the resistance of steel, iron, and brass, and proved that the temporary alteration of resistance resulting from increase of length and diminution of section is with iron and steel about two-fifths and with brass four-fifths of the whole observed change.

In the above-mentioned experiments I experienced considerable difficulty in obtaining accurate results in consequence of the minuteness of the changes to be measured, and, therefore, set about devising some plan whereby the variations of resistance caused by slight changes of temperature, which had proved a source of great annoyance, might be eliminated. In this attempt I have been entirely successful, and with the arrangements described below have determined, with I believe considerable accuracy, the very small changes of resistance which can be temporarily produced by mechanical tension.

Description of apparatus.

Pieces, about $7\frac{1}{2}$ feet in length, of the same wires as used in Part I. were suspended in pairs, as in fig. 6‡, in an air chamber 4 feet in height, 4 inches inner diameter and 6 inches outer diameter; the inner of the two concentric cylinders of which the air chamber was composed being surrounded by a layer of water 1 inch thick enclosed between the two cylinders. This vessel, which was made of tinned iron, rested on a table, provided with a suitable aperture, R, and was furnished with two thermometers, T T, passing through the outer cylinder and into the axis of the inner one. The ends of the two wires were clamped into three short and stout brass blocks, A, B, C, which

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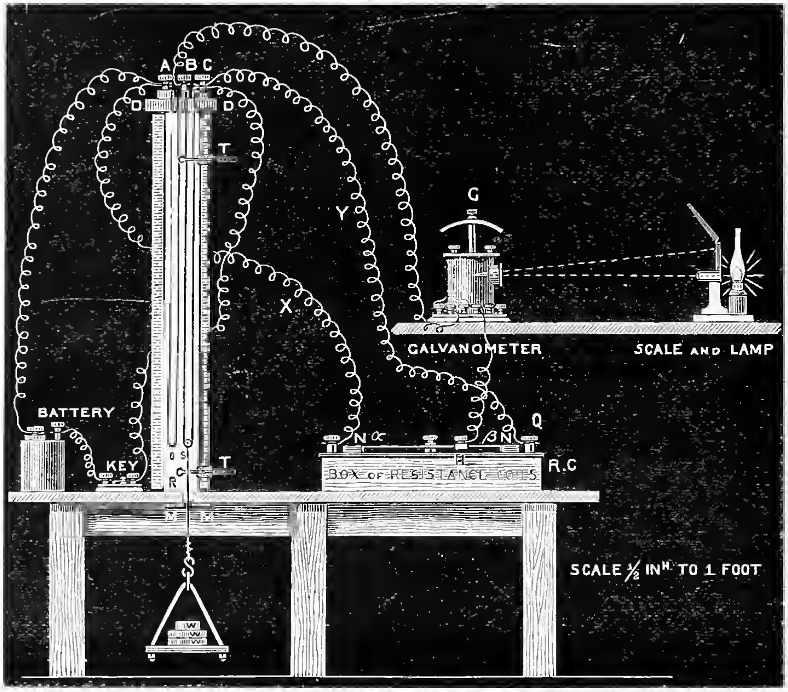
* Phil. Trans., Part IV., Feb. 28th, 1856, §§ 150–152.

† Proc. Roy. Soc., No. 183, 1877.

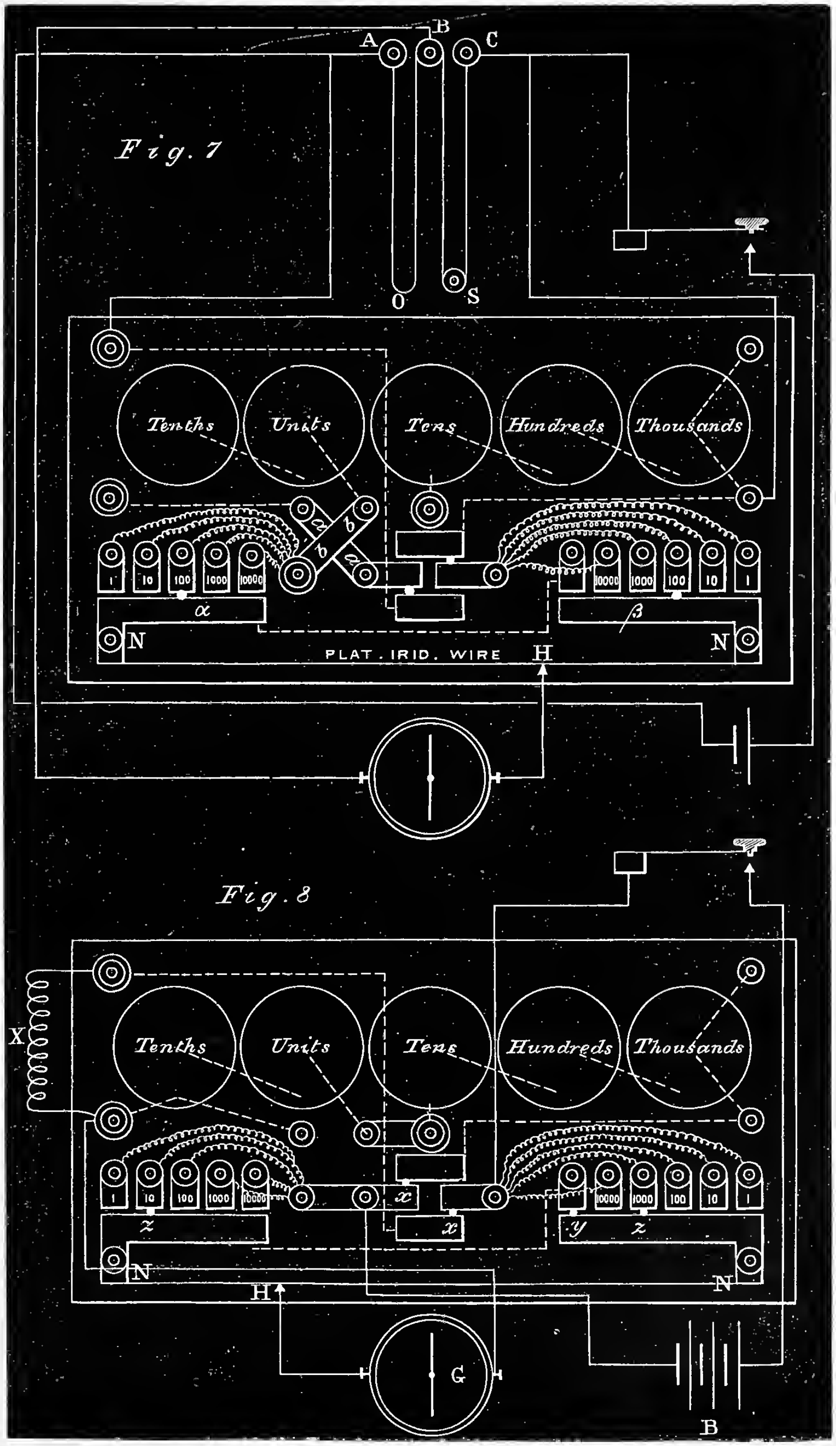
‡ In this figure the key employed to close the battery circuit is placed near the Leclanché cell and not in its actual position near the scale and lamp, in order to show the connexions more clearly.

rested upon a support of hard wood, D D, each block being separated from its neighbours by wooden partitions. The block B was double the length of A and C, and into this was clamped one end of each of the wires, the other ends being clamped into A and C. A caoutchouc-covered copper wire connected a binding-screw on A with one pole of a large-sized Leclanché and a similar wire, X, connected the same binding-screw with a set of resistance-coils (α, fig. 7). In a similar manner C was connected with the other pole of the Leclanché, and through Y with another set of resistance-coils, β; whilst B, and the sliding-piece H, which traversed a wire, N N, made in the first instance of platinum-silver and afterwards of platino-iridium, uniting α and β, were joined to two terminals of a reflecting galvanometer, G. The wire to be strained was provided with a movable pulley, S, 2 inches in diameter, to which was attached by means of a stout wire the scale-pan used in Part I., or, as in some experiments, a large pail weighing 4 kilogs. and capable of containing about 60 kilogs. of water; both wires were, before suspension, surrounded with caoutchouc-tubing silk or other insulating material. The aperture, R, could be closed by two wooden shutters, M M, having small semicircular holes cut in the centre of the extremities adjacent to each other, so as to leave only just sufficient room for the stout wire to swing freely; moreover, still further to prevent any errors which might be caused by chance currents of air, the top of the air-chamber was well covered with baize after the wires had been adjusted, and the table was also surrounded on three sides with a like material.

Fig. 6.



Figs. 7 and 8.


A
B
C
Fig. 7
O
S
Tenths
Units
Tens
Hundreds
Thousands
a
b
1
10
100
1000
10000
α
β
N
PLAT. IRID. WIRE
H
Fig. 8
X
x
y
z
G
B

It will be seen that the whole arrangement of wires, battery and galvanometer, forms, with the resistance coils α and β, a "WHEATSTONE'S balance," such that no part of the wires to be compared, namely, S and O, is out of the air-chamber. The galvanometer was provided with two sets of needles, which were astatic; the coil surrounding the upper set having a resistance of 7·487 ohms, and that surrounding the lower set a resistance of 4863 ohms, at the temperature 21° C. The wire of the former coil was divided into two equal parts, the ends of these being soldered to four terminal screws, so that, by connecting the two parts of the coil in "Multiple arc," the resistance could, if necessary, be diminished to 1·872 ohms, and the instrument rendered available for experiments requiring a galvanometer of low resistance. The upper coil only was used in this part of the enquiry, and yet the instrument proved so sensitive that, with the single cell employed for the current-motor, and with the adjusting magnet *assisting* the earth's directive force on the needles, a variation of conductivity of 1 part in 100,000* could be readily detected even in the most unfavourable case, which was that of a copper wire having a resistance of only ·0224 ohm; whilst in the majority of instances resistances were *measured within five parts in one million.*

The resistance coils were of platinum-silver, and the wire N N uniting α and β was, in the first few experiments, made of the same material, but was afterwards replaced by one of platinum-iridium; this latter wire was made expressly for me by Messrs. JOHNSON and MATTHEY, and seemed a marvel of accurate wire-drawing, as on testing it at various parts no variation of conductivity could be detected anywhere except at the two extreme ends, where it was firmly clamped into brass blocks; all the graduated portion of the wire, 470 millims. in length, seemed, within the limits of observation, to be perfectly uniform.

When required for measuring resistances in the ordinary way, the coils were arranged as in fig. 8; where X represents the wire under examination, G the galvanometer, B the battery, H the sliding-piece, and x, y, z conical plugs of brass with ebonite heads. Immediately to the right of X are a set of resistance-coils of the "dial pattern," ranging from $\frac{1}{10}$th of an ohm to 10,000 ohms; whilst below these are two sets of resistances, the arrangement and magnitude of which are sufficiently shown in the diagram; the plugs x, x serve to commute the position of X in the bridge, and the plug y to throw out, if necessary, the resistance N N, the dotted lines representing wires of extremely small resistance.

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* These beautiful instruments might, if necessary, be rendered even much more sensitive than they are by adopting a finer suspension. I have found that *a single fibre of unspun silk* is quite sufficient to sustain a weight equal to that on the suspension of many of the galvanometers which are now made, and anyone who will take the trouble to test practically, as I have done, the alteration which can be effected in the instrument by the *careful* selection of such a fibre will be surprised at the result. I feel little hesitation in asserting that my own galvanometer could be rendered *three* times as sensitive as it is now by such means; as it is, the silk suspension has been slowly untwisting for upwards of *seven years*, and still continues to do so. I have also found that keeping the galvanometer perfectly stationary, after it has once been placed in position, materially assists in preserving the magnetism of the needles.

For the purpose of this particular part of the enquiry, arrangements were made as in fig. 7, where it will be seen that the crossed pieces a a, b b, which are insulated from each other, serve to connect the tenths and units with the resistance-coils α.

Examination of the resistance-coils and of the platinum-iridium wire.

In order to test the uniformity of the platinum-iridium wire N N, two platinum-silver wires, of resistances 9 and 10 ohms respectively, were placed in the positions of O and S, fig. 7.

β was supplemented by a rheochord, and at both α and β the plugs were inserted in the 10 ohms resistance, whilst the rheochord was employed to bring H to the extreme right-hand end of the graduated portion of the wire. If, then, r denote the resistance of the ungraduated portion of N N on the right of H, together with the interposed portion of the rheochord and Y, whilst l denotes the resistance of the ungraduated part of N N on the left of H together with X, we have

$$\frac{10+l+(470-n_1)x}{10+r+n_1.x}=\frac{\text{resistance of O}}{\text{resistance of S}}=k \text{ say,}$$

or

$$10+l+(470-n_1)x=k(10+r+n_1.x) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where x is the average resistance of one division of N N, and n_1 is the number of divisions from the right-hand end of the graduated wire at which H is placed.

A resistance of $\frac{1}{10}$th ohm is now added to α from the tenths' dial-plate, and in consequence H has to be moved to n_2 divisions from the right-hand end in order to restore the balance;

$$\therefore\ 10{\cdot}1+l+(470-n_2)x=k(10+r+n_2.x) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

from (1) and (2) we obtain

$$x=\frac{{\cdot}1}{(k+1)(n_2-n_1)}.$$

Again the rheochord is adjusted so as to bring H about 10 millims. still further away from the right to, say, n_4 divisions, and on the removal of the $\frac{1}{10}$th ohm resistance H is brought back towards the right to, say, n_3 divisions; then as before

$$x=\frac{{\cdot}1}{(k+1)(n_4-n_3)}.$$

If the wire is uniform the value of x in the latter case should be equal to that of x in the former, and therefore

$$n_4-n_3=n_2-n_1$$

This was found to be so exactly the case that though H was provided with a vernier reading to $\frac{1}{10}$th of a millimetre, and the addition of the $\frac{1}{10}$th ohm required H to be shifted through about 300 millims., no difference whatever could be detected between n_4-n_3 and n_2-n_1, the galvanometer being sufficiently delicate to show an alteration of resistance equal to that of $\frac{1}{10}$th of a millimetre of N N. In the same manner each portion of the wire was tested, and it was concluded that no difference of resistance amounting to 1 in 3000 existed in any part of the graduated wire.

The uniformity of N N having been established, it was easy to compare the tenths with each other, and by determining the exact value of k, to find the values of their resistances in terms of the divisions of the platinum-iridium wire: this was accordingly done, and the values thus obtained were ascertained to accord fairly well with each other.

In order to secure still greater accuracy, the wire N N and the whole of the resistance-coils were afterwards tested by means of a second box of resistance-coils,* some eight or nine entire days having been spent in this work. These fresh trials confirmed the results obtained in the previous ones as regards the uniformity of N N and the *relative* values of the tenths; but the *absolute* values of the latter in terms of the divisions of N N were found to vary slightly on different days, as also did those of the rest of the resistance-coils. Thus the sum of the ten resistances in the units dial-plate were found on three separate days to be equal to that of 58,605, 58,739, and 58,437 divisions of N N respectively.†

From these and other experiments the average value of the resistances of each of the units at 17° C, at which temperature a unit agreed, according to KIESER, with 1 ohm, was equal to the resistance of 5859·4 millim. divisions of the platinum-iridium wire.

The units accorded very well with each other and with all the other resistances in the box except the tenths, nearly all the latter being slightly too low. The exact values, however, of each of the units and tenths were tabulated and used in calculating the results of the different experiments made in this and subsequent parts of the enquiry.

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* This box was kindly lent to me by Mr. KIESER, of ELLIOTT Bros., to whom I am also indebted for diagrams 7 and 8.

† These variations are certainly not due to errors of observation, as the results of trials made within one or two hours of each other agreed much more closely with each other; they may be attributed almost entirely to the prevalent plan of embedding resistance-coils in solid paraffin, whereby the temperature of the coils inside the box is frequently very different from that of the air outside: this plan is, I am convinced, a very bad one when great accuracy is required, as not only do the wires get heated to an extent which is very appreciable, even when only a single cell is employed, but a "PELTIER effect" is produced at the junctions of the coils and the brass-blocks which can never be properly got rid of; and thus the labour and care bestowed on these resistances are to a great extent lost. It would be better to fill the box with some such liquid as paraffin oil.

Preliminary trials and final method of experimenting.

In the first attempts which were made the galvanometer was put into circuit by means of the usual contact-piece of the sliding-block H, immediately after closing the battery; but it was soon found that the very act of pressing down the contact-piece generated small thermo-electric currents* which, lasting some minutes, frustrated all attempts at making such accurate measurements as it was hoped would ultimately be obtained. H was therefore clamped by means of a suitable spring-and-catch, with which it was provided, to any desired part of the platinum-iridium wire, and a double key was employed by means of which first the battery and then immediately afterwards the galvanometer were put into the "bridge." Here again, however, exactly the same difficulty was encountered, and by no device of covering the hand and key with cloth or silk could this source of error be entirely avoided. The double key was therefore discarded, and the galvanometer being always kept in the "bridge," a single key was used for closing the battery circuit for the brief space of time necessary for observing whether this act caused any difference of potential at the terminals of the galvanometer.

Of course the zero-point of the galvanometer needles was continually being altered by the thermo-electric currents produced by the frequent shifting and clamping of H, but this circumstance did not affect the results, and though in the case of iron a slight trouble was experienced sometimes from the "kick" of the needles due to circular magnetization, this difficulty was after a few trials surmounted, and from this point the measurements proceeded very satisfactorily.

In most instances α and β were made of about 100 ohms' resistance, and though such large resistances were, it is true, out of proportion to those of the other branches of the bridge, yet, as has been already observed, the arrangement proved of amply sufficient delicacy, and moreover rendered it impossible that any slight variations of the resistances of X and Y, which were each ·042 ohm, and of the wire N N, arising from changes of temperature, should cause any appreciable error; indeed, one great advantage of this method is that the galvanometer and the resistance-coils may be a hundred yards or more from the rest of the "bridge" without any chance of fluctuations of temperature materially influencing the result, even when the most minute variations of electrical conductivity are to be measured.

The deflections of the galvanometer were read in the usual manner by means of the image of a fine wire fixed vertically across one end of a small blackened tube, into the other end of which was fitted a lens for focussing the image of the wire on to the mirror of the galvanometer; and the end of the tube at which the wire was situated was illuminated by a paraffin lamp, placed so that the edge of the flame was in front of the wire, the reflected circle of light with the fine dark line across the centre being very

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* This fact has, I find, been also noticed by GLAZEBROOK, Phil. Mag., April, 1881, No. 68.

clearly defined on the scale, though the latter was at a distance of 6 feet from the galvanometer.*

The adjusting magnet was almost always used to *assist* the directive force of the earth's magnetism on the needles, as it was found that by so doing the shifting of the zero-point caused by the above-mentioned thermo-electric currents was considerably diminished, and at the same time the instrument was sufficiently sensitive. The wires S and O, which were made as nearly as possible of the same resistance, were, after being suspended, allowed to remain in the air-chamber for some time; β was then made 100 ohms and α adjusted, first by the resistances in the box as far as $\frac{1}{10}$th of an ohm, and finally by using the sliding-block H until no deflection of the image of the fine wire could be detected on closing the battery circuit.

As the needles of the galvanometer soon came to rest, and H could be very readily clamped and unclamped, it was possible to make the observations quickly; an interval of one minute was, however, generally allowed to elapse between two consecutive readings, as, though the battery-power was small and S and O of the same material and of the same section, yet in some cases the current evidently produced unequal heating effects in the two wires. Nor is this to be wondered at, as a difference of temperature of less than $\frac{1}{1000}$° C. would produce a sensible difference of resistance.

In ascertaining the temporary alteration of resistance caused by longitudinal traction, it was deemed advisable to adopt the same precautions as were used in determining the modulus of elasticity, as though the wires employed had been previously strained for the latter purpose and then allowed to rest for some weeks; they seemed, in some cases at least, to have become partially annealed, and it was found that the temporary alteration of electrical conductivity caused by loading was affected in precisely the same manner as the elasticity, by stress producing recent permanent extension.

Great care was taken in loading and unloading the wire S, and in the experiments which were made with the first two or three wires a large but light pail was used instead of the scale-pan. Into this vessel measured amounts of water were allowed to flow slowly through a piece of caoutchouc tubing, when it was necessary to apply stress, and the unloading was accomplished by suffering the same water to pass gently out through another piece of tubing connected with an orifice at the bottom of the pail. During the loading this latter tube was hitched up by the side of the vessel. It was found, however, that with practice quite as accurate results could be obtained by using a scale-pan in the ordinary manner, and as time was thereby saved this method was finally adopted.

Formulæ employed.

The temporary alteration of resistance which was in any case produced was so small that it could be measured by the wire joining α to β. If, therefore, A and B denote

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* The distance of the lamp and scale from the galvanometer is not drawn to the same scale in fig. 6 as for the other arrangements, for the purpose of avoiding the taking up of too much space.

the number of millimetre divisions of this wire which have a resistance equal to the branches α and β respectively, including in these the connecting wires X and Y and the parts of the divided wire on either side of H, n be the number of divisions through which it is necessary to move H in order to restore the balance of resistance when disturbed by a load W, and S be the section of the wire, then, denoting by x the increase per unit of resistance which would be produced by a unit load acting on unit area, we have within a sufficiently close degree of approximation

$$x=\frac{(\mathrm{A}+\mathrm{B})\times n\times s}{\mathrm{A}\times\mathrm{B}\times\mathrm{W}}.$$

Again, if x be multiplied by e we obtain the alteration of resistance per unit which would result from doubling the length of the wire by the application of longitudinal traction; therefore denoting this latter value by y, we have

$$y=e\times x.$$

Part of y is due to mere increase of length and diminution of section; this part $=1+2\sigma$ very nearly. Thus the alteration of the specific resistance produced by the traction

$$=y-(1+2\sigma).$$

It will be seen that x and y are calculated on the assumptions that the change of resistance is directly proportional to the stress and also to the elongation; both these assumptions were found to be nearly correct, but neither are strictly so.

The following experiment out of many will serve to show (a) that the temporary alteration of resistance is nearly but not quite proportional to the load, and (b) that it is possible to measure with considerable accuracy minute changes of electrical conductivity even when the resistance of the wire used is small.

Experiment I.

An annealed copper wire ·154 centim. in diameter and having a resistance of only ·0224 ohm was loaded by pouring 12 measures of water, each having a weight of 5825 grms., into a pail attached to the pulley on the wire. This load, which was four-fifths of the "breaking-weight," was suffered to remain on the wire for some hours and was then removed. The following consecutive observations were begun next day and extended over three days, two trials being made on each.

The numbers in the column headed "Temporary alteration of resistance" are the divisions of the platinum-silver wire N N, through which it was necessary to shift H in order to balance the effect of the load.

Number of trial.	Number of measures employed for the load.	Temporary alteration of resistance.
I.	3	188·0
	6	383·0
	9	584·0
II.	3	189·0
	6	381·0
	9	581·0
III.	3	188·0
	6	384·5
	9	583·0
IV.	3	189·5
	6	384·0
	9	583·5
V.	3	188·0
	6	385·0
	9	583·0
VI.	3	190·0
	6	384·0
	9	584·0

The mean values for three, six, and nine measures are respectively 188·8, 383·6, and 583·1, and none of the observations differ from these mean values by ·7 per cent.

Thus for the first three measures we obtain a mean alteration of 188·8, for the second 194·8, and for the third 199·5.

Experiment II.

The same wire as in the last experiment, after having been repeatedly loaded with 13 measures, was tested with the same weights as before with the following results:—

Load in measures.	Temporary alteration of resistance.	Average alteration per measure.	Difference between consecutive averages.
3	207·0	69·00	. .
6	416·7	69·45	·45
9	629·4	69·93	·48

Experiment III.

A piece of platinum (1) annealed, which had been repeatedly loaded with 12 kilogs.

Number of kilogs. in load.	Temporary alteration of resistance.	Average alteration per kilog.	Difference between consecutive averages.
4	108·5	27·13	. .
6	163·5	27·25	·12
8	219·0	27·38	·13
10	277·0	27·70	·32
12	339·0	28·25	·55

Experiment IV.

The same wire as in the last experiment, which had been further loaded and unloaded with 15 kilogs. until the recovery had become perfect for this load.

Number of kilogs. in load.	Temporary alteration of resistance.	Average alteration per kilog.	Difference between consecutive averages.
1	27·5	27·50	. .
3	83·0	27·67	·17
5	139·0	27·80	·13
7	195·5	27·93	·13
9	252·5	28·06	·13
11	310·0	28·18	·12
13	368·0	28·31	·13
15	430·0	28·67	·36

Experiment V.

A piece of silver (1) annealed, which had been previously loaded and unloaded with 8 kilogs. until the recovery had become perfect.

Number of kilogs. in load.	Temporary alteration of resistance.	Average alteration per kilog.	Difference between consecutive averages.
2	94	47·00	. .
4	189	47·25	·25
6	285	47·50	·25
8	383	47·88	·38

Experiment VI.

A piece of platinum-silver (1) rendered very hard by drawing, and which had been loaded and unloaded several times with 20 kilogs.

Number of kilogs. in load.	Temporary alteration of resistance.	Average alteration per kilog.	Difference between consecutive averages.
4	79·00	19·75	. .
8	161·80	20·23	·48
12	246·25	20·52	·29
16	330·00	20·63	·11

Experiment VII.

An annealed iron wire which had been loaded for 24 hours with 30 kilogs.

Number of kilogs. in load.	Temporary alteration of resistance.	Average alteration per 2 kilogs.	Difference between consecutive averages.
2	9·0	9·00	. .
4	20·5	10·25	1·25
6	32·2	10·73	·48
8	44·0	11·00	·27
10	56·3	11·26	·26
12	68·5	11·42	·16
14	80·8	11·54	·12
16	92·9	11·61	·07
18	105·1	11·68	·07
20	117·4	11·74	·06
22	129·8	11·80	·06
24	142·3	11·858	·058
26	155·0	11·92	·062

Remarks on Experiments I.–VII. inclusive.

All these experiments prove that the ratio of the resistance-increase to the load is not quite constant, but that the former increases in a greater proportion than the latter. This want of proportionality is seen from Experiments I.–IV. inclusive to be materially diminished, though never entirely made to vanish, by repeated heavy loading and unloading, and it appears impossible to find any single formula which will express exactly the relation between the load and the alteration of resistance for all conditions of the wire; consequently the values of x given in Table I. are calculated from observations made with two weights, the larger never exceeding one-fourth of the "breaking-weight," and being twice the smaller; and the mean of the two numbers obtained by dividing the observed alteration by the load is taken to represent the change produced by unit load, this mean being in no case different from either of the numbers by more than 1 per cent.

The columns in which are recorded the differences between the consecutive average alteration per kilogramme show that when the load employed in determining the temporary alteration of resistance approaches closely to the highest load which has been used in the preliminary operations, a comparatively rapid increase takes place in the ratio of the temporary alteration of resistance to the load.

From Experiments VI. and VII. we learn that in the case of a metal which has been rendered very hard by the process of drawing, or in the case of iron* which has been subjected to a heavy load for some time, the average alteration of resistance per

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* Probably also in the case of any metal possessing a coercive force comparable with that of iron.

unit load increases less and less rapidly up to a certain degree of stress, and afterwards begins to increase more and more rapidly, whereas with the annealed metals the average alteration increases at first by almost equal amounts.

Experiment VIII.

A piece of platinum (1) annealed was loaded and unloaded several times with 12 kilogs., and was immediately afterwards tested, with the following results:—

No. of kilogs. in load.	Temporary alteration of resistance.	Average alteration per kilog.	Difference between consecutive averages.
2	66·5	66·5	. .
4	134·0	67·0	·5
8	276·0	69·0	2·0
After a rest of 20 hours with all stress removed except that of scale-pan.			
2	67·0	67·0	. .
4	136·0	68·0	1·0
8	281·0	70·25	2·25

From the last experiment it is evident that part of the increase of elasticity which is gained by repeated heavy loading and unloading is lost by allowing the wire to rest, and, moreover, that the departure from "HOOKE's law" which we have seen to be appreciably lessened under the influence of the former cause is increased again by the latter.

In Table I. will be found the values of the specific resistance of the different metals, the increase of resistance per unit which is temporarily produced by a stress of 1 grm. per square centimetre, the increase of resistance per unit which would be caused by stress sufficing to double the length of the wire, and the increase of specific resistance per unit which would be caused by stress sufficing to double the length of the wire; the specific gravity and section of the wire are approximately the same as those recorded in Part I.

TABLE I.

Name of metal.	Condition.	Specific resistance at 12° C., *i.e.*, resistance in ohms of 1 cubic centim., between opposing faces $=\mathrm{R}$.	Increase of resistance per unit produced by stress of 1 grm. per square centim. $=x$.	Increase of resistance per unit which would be caused by stress sufficing to double the length of the wire $=y$.	Increase per unit of specific resistance which would be caused by stress sufficing to double the length of the wire $=z$.
Iron (1)	Annealed	1074×10^{-8}	2111×10^{-12}	4·180	2·618
Iron (2)*	Hard drawn	1217×10^{-8}	2100×10^{-12}	4·289	2·639
Iron (3)	Hard drawn	1201×10^{-8}	2197×10^{-12}	3·699	2·057
Piano steel (1)	Hard drawn	1653×10^{-8}	1910×10^{-12}	3·619	2·081
Piano steel (2)	Hard drawn	1882×10^{-8}	1831×10^{-12}	3·602	2·084
Platinum (1)	Hard drawn	..	2233×10^{-12}	3·341	2·239
Platinum (1)	Annealed	1434×10^{-8}	2285×10^{-12}	3·404	2·252
German-silver (1)	Annealed	2830×10^{-8}	1501×10^{-12}	1·937	0·107
German-silver (2)	Annealed	..	1545×10^{-12}	2·099	0·345
German-silver (1)	Hard drawn	2713×10^{-8}	1892×10^{-12}	2·138	0·138
Copper (1)	Annealed	$240{\cdot}8 \times 10^{-8}$	2210×10^{-12}	2·564	0·934
Copper (1)	Hard drawn	$244{\cdot}0 \times 10^{-8}$	1988×10^{-12}	2·880	..
Copper (3)	Annealed	$201{\cdot}4 \times 10^{-8}$	2324×10^{-12}	2·656	1·070
Copper (2)	Annealed	$187{\cdot}3 \times 10^{-8}$	2396×10^{-12}	2·918	1·012
Platinum-silver (1)	Annealed	3236×10^{-8}	2346×10^{-12}	2·464	0·624
Platinum-silver (1)	Hard drawn	3127×10^{-8}	2437×10^{-12}	2·530	..
Brass (1)	Hard drawn	834×10^{-8}	2302×10^{-12}	2·275	0·101
Brass (2)	Hard drawn	$656{\cdot}7 \times 10^{-8}$	2229×10^{-12}	2·203	0·231
Zinc (1)	Hard drawn	..	4406×10^{-12}	3·379	2·113
Silver (1)	Annealed	$161{\cdot}9 \times 10^{-8}$	4272×10^{-12}	3·851	1·531
Silver (1)	Hard drawn	$173{\cdot}7 \times 10^{-8}$	4561×10^{-12}	3·487	1·703
Aluminium (1)	Hard drawn	$316{\cdot}7 \times 10^{-8}$	1883×10^{-12}	1·260	−0·420
Aluminium (1)	Annealed	$311{\cdot}3 \times 10^{-8}$	1896×10^{-12}	1·276	−0·262
Tin (1)	Drawn	1166×10^{-8}	10546×10^{-12}	2·920	1·630
Lead (1)	Drawn	2142×10^{-8}	17310×10^{-12}	2·885	1·613

Remarks on Table I.

It will be seen from the above table that the specific resistance of iron is more increased by a given amount of elongation than that of any of the other metals, and that the specific resistance of aluminium is actually *decreased* by stress in the line of flow of the current; this latter fact being signified by a minus sign placed opposite aluminium in the sixth column.

It is also remarkable that the value of z for the alloys, platinum-silver, German-silver and brass should be considerably less than that of their components; and this circumstance, taken in connexion with the comparatively large increase of resistance of iron, would suggest that there is some relation between increase of electrical resistance caused by rise of temperature and that due to mechanical stress.

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* Iron (2), iron (3), steel (1), steel (2), and brass (2) were tested some years ago by a method similar to the one here described.

But the change of resistance attending a given amount of expansion caused by rise of temperature is, in the case of iron, more than a hundred times that resulting from the same amount of expansion produced by mechanical stress; and it is evident that with all metals the alteration of electrical conductivity following on alteration of temperature is due for the most part to other causes than mere contraction and expansion.

Carbon.

As it seemed desirable to extend these researches to as many substances as possible, some experiments were made with carbon rods such as are used for electric lighting purposes. These rods were between 40 and 50 centims. in length, and of different diameters, and their moduli of longitudinal elasticity could be readily determined by holding them in the centre and rubbing them along their length with a resined glove. The note obtained by the longitudinal vibrations, though, of course, very high in pitch, was quite clear and distinct, and very concordant results were obtained when the same pieces were tried at different times. The pitch of the note was determined by the syren, and the following experiment, taken at random out of my note book, will suffice to show what accuracy can be attained with this instrument* even with notes of very high pitch.

Experiment IX.

A carbon rod of length ·496 metre, rubbed longitudinally by means of a resined glove, gave a note, the lower double octave of which was taken on a monochord; the syren was then raised to the pitch of the monochord, and the number of vibrations counted for two minutes at a time.

Number of trial.	Number of vibrations recorded by the syren in two minutes.
1	6860×20
2	6822×20
3	6835×20
Mean . .	$= 6839 \times 20$

In this experiment the monochord was re-tuned at each trial, and it will be observed that the mean value does not differ from any of those forming it by so much as $\frac{3}{10}$ per cent. Of course, by extending the time of each trial and the number of observations, still greater accuracy could have been obtained.

The formula employed for calculating the elasticity is $E = \frac{n^2 \times 4 l^2 \times \Delta}{9810}$, in which l is

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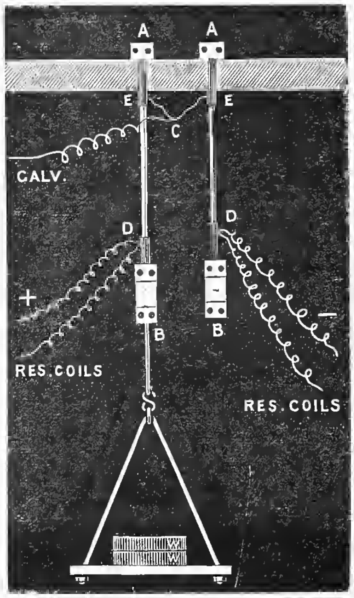
* I have to thank Mr. FURSE, the Curator of the Physical Museum at King's College, for his able assistance in this part of the work.

the length of the rod in metres, n the number of vibrations, Δ the density, and E the elasticity in kilogrammes per square millimetre.

Fig. 9.



In order to determine the alteration of resistance which could be produced by longitudinal traction, the rods were well coated for about 2 inches of their lengths with copper deposited by electrolysis, and were then arranged as shown in fig. 9. The rods passing through two holes in a table were secured at their upper ends by two clamps, A A, and at their lower extremities were fastened two other clamps, B B, either of which would also serve to carry a scale-pan suspended by an iron wire attached to its lower extremity. Two pieces of silk-covered copper wire, about 6 inches in length and $\frac{1}{30}$th of an inch in diameter, were soldered on to the deposited copper at E, and these being also joined together were connected with the galvanometer. Two other pairs of similar wire were soldered at D, and one of each pair was connected, as usual, with the resistance coils, and the other with one pole of the battery. The resistances of the silk-covered copper wires at E, which were small compared with those of the rods themselves, were calculated by determining the resistance of some 6 feet of the wire, and assuming that the resistances of the short pieces at E and the actually determined resistance of the longer piece were proportional to their lengths. The whole table was surrounded on all sides except one with baize, and the mode of proceeding the same as usual. Experiment shows that with the loads employed the alteration of resistance is nearly proportional to the load.

Experiment X.

Carbon rod, length between E and D 45 centims., diameter ·434 centim.

Number of kilogs. in the load.	Number of divisions of the platino-iridium wire through which it was necessary to move the sliding piece in order to restore the balance.	Average alteration of resistance per kilog. in terms of the division of the platino-iridium wire.
2	37·00	18·50
4	74·75	18·69
6	111·80	18·63
		Mean 18·61

In Table II. will be found some data respecting the alteration of resistance produced by loading, the values of "YOUNG's modulus," specific resistance, specific gravity, and section of each rod.

TABLE II.

Number of carbon.	Section in square centims.	Specific gravity water at 4° C=1 $=\Delta$.	Specific resistance at 15° C.	"YOUNG's modulus" in grammes per square centim. $=e$.	Increase of resistance due to a load of 1 grm. per square centim. $=x$.	Increase of resistance which would be caused by a load sufficient to double the length $=e\times x$ $=y$.	Increase of specific resistance which would be caused by a load sufficient to double the length $=z$.
1	·1411	1·518	4591×10^{-6}	$233\cdot8\times10^{6}$	11420×10^{-12}	2·470	0·970
2	·1307	1·550	4182×10^{-6}	$264\cdot3\times10^{6}$	8991×10^{-12}	2·377	0·877
3	·1480	1·552	4483×10^{-6}	$270\cdot0\times10^{6}$	9535×10^{-12}	2·575	1·075
4	·1351	1·579	3937×10^{-6}	$329\cdot0\times10^{6}$	8610×10^{-12}	2·835	1·335
5	·1216	1·581	3774×10^{-6}	$279\cdot0\times10^{6}$	7684×10^{-12}	2·144	0·644
Means	. .	1·556	4193×10^{-6}	$275\cdot2\times10^{6}$	9248×10^{-12}	2·480	·980

Remarks on Table II.

The sections of the rods were much more uniform than would perhaps have been expected; for example, in the case of No. 3, as measured by a gauge at five points equally apart, the diameter was found to be ·434, ·434, ·434, ·434, and ·433 centim., and the other rods were found to be nearly of the same uniformity as this one.

The specific gravities were determined by breaking the rods into pieces about 5 inches in length and binding a thin copper wire round them, which latter was fastened to a very fine platinum wire, so that the pieces could be weighed in air and in water.

It should be stated that carbon in this form being porous, the specific gravity as usually reckoned will be found to increase if the substance be allowed to remain in water for any time, and especially so if the carbon be boiled in water. The following experiment will show the extent of alteration of the apparent specific gravity :—

Experiment XI.

Several pieces of carbon tied together with fine copper wire, and boiled for five minutes; the pieces then taken out of the hot water, kept for five minutes in cold water, and afterwards suspended by a fine platinum wire in a large vessel filled with water at 15° C.

Weight of carbon in water.	Time after immersion in the large vessel of water, at 15° C.
4·800	5 minutes.
5·020	15 ,,
5·140	35 ,,
5·255	75 ,,
5·291	140 ,,

The pieces then boiled a second time for 40 minutes, and after having been kept in cold water for 30 minutes, again tested for loss of weight.

Weight of carbon in water.	Time after immersion in the large vessel of water, at 15° C.
5·314	5 minutes.
5·340	45 ,,
5·393	13 hours.
5·432	5½ days.
5·452	12½ ,,

As the temperature of the water varied very little during the periods of observation, we see that the apparent specific quantity went on increasing for more than 12 days, and that the total percentage of increase amounted to nearly 12. The specific gravities, however, recorded in the table are calculated from the loss of weight observed shortly after immersion in water, the pieces of carbon not having been boiled, and the section as determined by dividing the loss of weight by the length agreed very well with the section measured by the wire-gauge.

With the exception of number 5, the value of "YOUNG's modulus" increased very largely as the specific gravity increased; in fact, the value of e is roughly proportional to Δ^8.

Beetz[*] has also determined the values of e for several carbon rods, used for electric lighting, by longitudinal vibrations, and it would appear from the results obtained by him that $e \div \Delta^8$ is also fairly constant. The mean values of Beetz' results and my own are given in the next table.

Table III.

Specific gravity $= \Delta$.	Value of e in grammes per square centim.	$e \div \Delta^8$.
	Beetz.	
1·532	$152·0 \times 10^6$	501×10^4
1·547	154·7 „	466 „
1·564	174·7 „	488 „
1·580	193·5 „	498 „
1·593	205·4 „	495 „
1·631	254·8 „	509 „
	Mean value of $e \div \Delta^8 = 493 \times 10^4$	
	Tomlinson.	
1·518	$233·8 \times 10^6$	829×10^4
1·550	264·3 „	793 „
1·580	304·2 „	783 „
	Mean value of $e \div \Delta^8 = 802 \times 10^4$	

Though, however, $e \div \Delta^8$ is roughly a constant for different specimens of carbon by the same maker, the value of this constant may be evidently very different for samples from different makers, the mean value of Beetz' samples for $e \div \Delta^8$ being 493×10^4 and that of my own specimens 802×10^4.

The value of e given in Table II. for number 3 is calculated from the formula $e \div \Delta^8 = 802 \times 10^4$.

It will be noticed that the product $e \times x$, *i.e.*, the increase of resistance which would be caused by a load sufficient to double the length of the rod, is nearly a constant for the different specimens, and on referring to Table I. it will be observed that this product is much less than that for several of the metals, though the increase of resistance caused by a stress of 1 grm. per square centimetre is of course, in consequence of the small elasticity of the carbon, greater than any of the metals examined except tin and lead, whose elasticities approach much more nearly to that of this kind of carbon than any of the other substances.[†]

In calculating the alteration of specific resistance it was assumed that $\sigma = ·250$, but

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[*] Ann. der Phys. und Chem., 1881, No. 1, p. 67.

[†] The elasticity of the rods here examined is very nearly equal to that of tin.

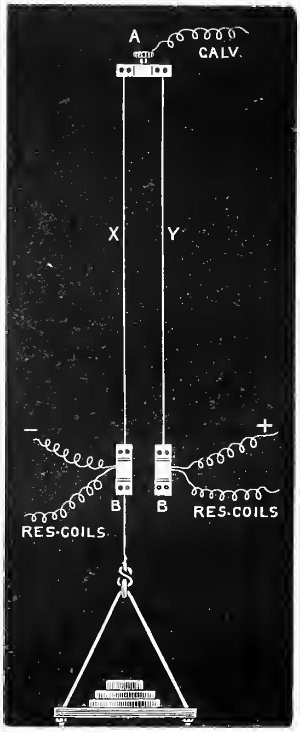
though the values given in the last column of Table II. are in consequence of the want of knowledge of σ approximate to the true ones, they must be sufficiently so to prove that not only is the total resistance of the carbon increased by loading but so also is the *specific resistance*.

The mean value of the specific resistance of the different specimens is 4193×10^{-6}, and is therefore more than 100 times greater that the corresponding number for platinum-silver, which latter metal ranks highest in the list of metals in Table I.

NICKEL.

Through the kindness of Messrs. JOHNSON and MATTHEY, who after some difficulty succeeded in drawing for me two pieces of almost pure nickel wire 8 feet in length, I was able to make some experiments on this metal, in which I obtained results so completely differing from those observed in the case of the other substances, that I may perhaps be excused for treating them in some detail.

Fig. 10.



The metal when first received was in the hard drawn condition, and in this state was tested for torsional rigidity and longitudinal elasticity, for the former in the usual manner and for the latter by longitudinal vibrations. The pieces were then suspended in the air chamber, and since it was not thought desirable to bend the wire, the following arrangements were made, which are sufficiently shown in fig. 10.

X and Y are the two wires to be compared, the clamp A rests as before on the top of the air chamber, and of the two clamps B B fastened to the lower ends of X and Y one has a stout iron wire attached to it for the purpose of a suspension for the scale-pan. Also to each of the clamps B are soldered two silk-covered copper wires $\frac{1}{40}$th of an inch in diameter and 3 feet in length, one of these serving to connect the clamp with one of the battery wires and the other with the caoutchouc-covered wire leading to the resistance coils. The wires X and Y were each $3\frac{1}{2}$ feet in length, but were not cut off from the 8 feet pieces, the remainder of the wire* being allowed to hang down on the outside of the air chamber, whilst the silk-covered copper wires fastened to the clamps B B were brought down through the hole in the table at the bottom of the air-chamber. When the experiments on the hard-drawn wire had been finished, the metal was well annealed and similar ones made on it in this new state, with this difference, however—that now the wire X was tested with larger and larger loads until it finally broke. The results of the experiments made on the wire in the hard drawn and annealed conditions will be found in Table IV.

The curves in Table V., showing the temporary alteration of resistance produced in the annealed wire by different loads after the wire had been previously stretched for some time with weights of 16, 18, 20, 22, and 24 kilogs., are drawn with their abscissæ to represent the percentage alteration of resistance on a scale of ·0025 to the millimetre, and with their ordinates the number of kilogrammes per square centimetre in the load on a scale of 25 kilogs. to the millimetre.

TABLE IV.

Condition.	Section in square centims.	Specific gravity at 20° C., water at 4° C. = 1.	Specific resistance at 20° C.	Torsional rigidity in grms. per square centim. = r.	"YOUNG'S modulus" in grms. per square centim. = e.	Ratio of lateral contraction to elongation.
Hard drawn	·005832	8·7066	1757×10^{-8}	$723·5 \times 10^{6}$	$2270·6 \times 10^{6}$	·570
Annealed .	·005832	8·7386	1736×10^{-8}	$758·5 \times 10^{6}$	$2174·6 \times 10^{6}$	·433

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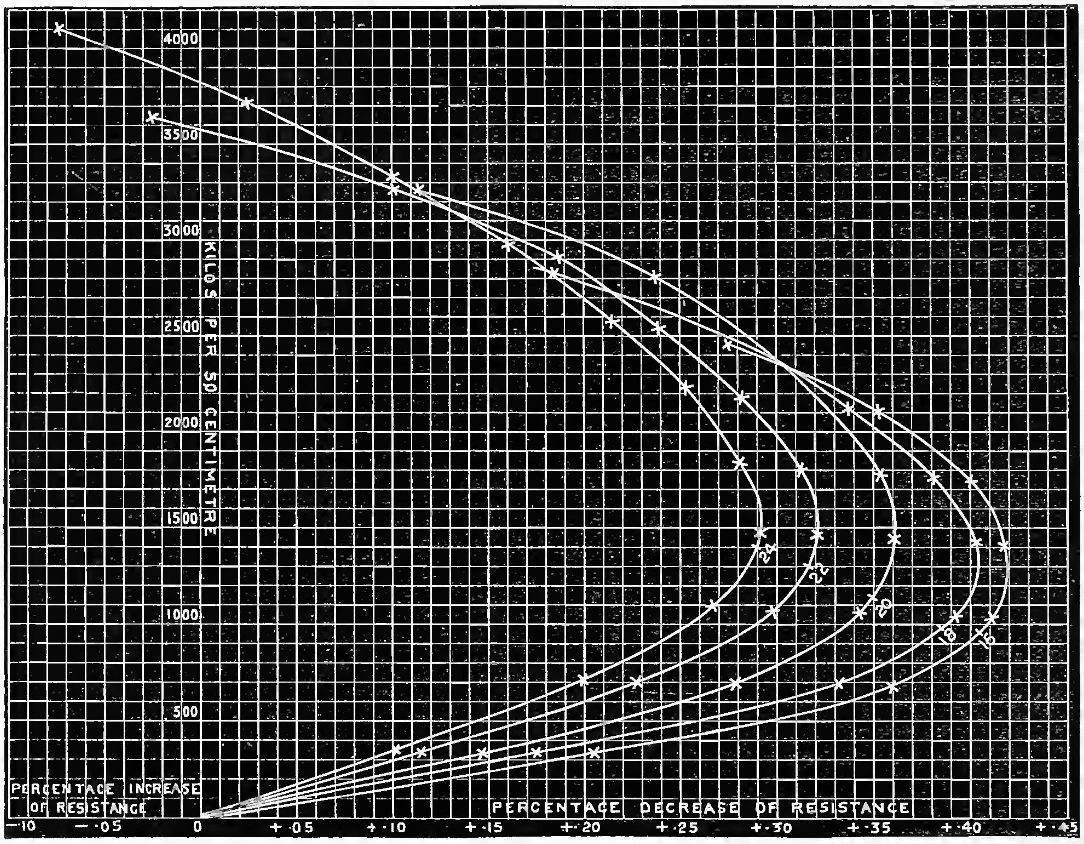
* Not shown in the figure.

Table V.—Curves showing the temporary alteration of electrical conductivity of nickel wire produced by longitudinal traction at the temperature 22° C.



Remarks on Tables IV. and V.

One of the first points to notice in Table IV. is the large value of "Young's modulus," which for both the hard drawn and the annealed conditions of the nickel exceeds the highest values obtained for pianoforte steel wire.

Again we have seen in Part I. that the value of "Young's modulus" increases as the mean distance between the molecules diminishes, and the ratio of the specific gravity to the atomic weight of nickel, which is inversely as the cube root of the mean distance between one molecule and another, is ·1493, whilst that of iron, which with copper has the ratio higher than is the case with any of the other metals mentioned in this paper, is ·1386. We have also seen that $e \times \alpha^7$ is roughly a constant for the different metals, where α is the mean distance between one molecule and another, and e is the value of "Young's modulus;" in the cases of nickel and iron this product is $\cdot 1838 \times 10^{12}$ and $\cdot 1993 \times 10^{12}$ respectively.

The ratio of the lateral contraction to the linear elongation obtained from the formula $\sigma=\frac{e}{2r}-1$ of the hard drawn nickel is of course impossible, as was the case, it will be remembered, with the value of σ with hard-drawn copper, brass, German-silver, and platinum-silver, and the wire therefore furnishes another proof that the formula cannot be applied to hard drawn metal.

The most remarkable feature, however, presented by nickel is that shown in Table V., where we learn that the resistance is absolutely *decreased* up to a certain extent of loading and then begins to increase. We see, moreover, that the *maximum decrease* becomes less and less as the wire receives more and more permanent extension, and that the point of loading where this maximum occurs gradually rises with the amount of permanent extension.[*] As might be supposed, therefore, the decreases of resistance obtained with two loads of 3 and 6 kilogs. were with the hard drawn metal much less than with the annealed one. Again, if we take the average decrease of resistance produced by a load of 1 grm. per square centimetre between the points of zero load and the load producing maximum decrease, we find it in the case of the outer curve to be 3216×10^{-12}, and the product of this number by $e=6\cdot994$, whilst the decrease of *specific* resistance attending unit increase of length, or the number corresponding to z in Table I., would amount to 8·860. All these numbers, especially the last, are very considerably greater than the corresponding *increases* of resistance obtained with any of the other metals. We thus observe that whether we regard the peculiarity of loading up to a certain point producing decrease of resistance, and after this point increase, or the comparatively enormous temporary variations of resistance produced by loading, nickel stands by itself, and the idea at once suggested itself that this abnormal behaviour of the metal might be due to the influence of circular magnetization caused by the current employed in balancing the wire and the comparison-wire. Accordingly two experiments were made of the following nature:—First, as the alteration of resistance might be only apparent and due to the fact that VILLARI's shock-currents were not the same in the stretched and unstretched wires,[†] both the galvanometer circuit and the battery circuit were kept closed, and the position of the light on the scale noted with different stretching weights: the readings taken in this way gave alterations of resistance which were exactly the same as those obtained by the usual method. Secondly, as the alterations might be really those of resistance, but due to the fact that circular magnetization, might[‡] cause an alteration of resistance in both iron and nickel, and unequally in the stretched and unstretched wires, the resistances in the bridge were so adjusted that currents of one-half and one-fourth respectively of the current which had previously

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[*] The numbers 16, 18, 20, 22, 24, on the curves represent the load which had previously been on the wire before testing for the temporary effect of loading.

[†] I did not think this likely, as I had not been able to detect anything of the kind when iron wire was used.

[‡] According to AUERBACH, but not according to experiments tried by myself.

passed through the wire in determining the resistance might now do so; but again the alterations of resistance were found to be the same as before.[*] Now if E denote the electromotive force of the LECLANCHÉ, x and y the resistances of the wire and the comparison-wire, α and β the resistances in the box, and B the resistance of the battery and connexions, the current in x will be proportional to $\frac{E}{B(1+\frac{x+y}{\alpha+\beta})+x+y}$, or since x and y were only ·44 ohm each, whilst α and β were 100 ohms each, the current in x will be nearly proportional to $\frac{E}{B+x+y}$.

E was nearly 1·5 volts, and $B+x+y$ in the case of the weakest current was made 10 ohms; therefore this current would in absolute measure of C.G.S. units be ·015. Unless, therefore, the maximum difference of alteration of resistance caused by circular magnetization in the stretched and unstretched wires had been reached by a still smaller current than this, we cannot regard the curious behaviour of nickel in respect to the effect of stress on its electrical conductivity as due to circular magnetization.

Again, another idea suggested itself, namely, that the wires being suspended vertically might cause the resistance of the stretched and unstretched wires to be altered unequally by the earth's vertical magnetic force, but this latter, it will be seen, is of too small intensity to have any effect at all comparable with that observed in the stretching; and, moreover, experiments subsequently made with the wire at right angles to the magnetic meridian, showed plainly that this was not the case. We must therefore regard the abnormal effect produced on the electrical resistance of nickel as not due to the earth's magnetic force.[†]

We have seen also that, in the case of other metals which have suffered permanent extension, rest increases the elasticity and *diminishes* the temporary *increase* of resistance which any load is capable of causing, and that with iron this effect is very apparent. It was therefore an interesting point to determine whether the *decrease* of resistance which moderate loading produces in nickel would be increased by rest. The next experiment shows that this is so.

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[*] Similar experiments with iron had also previously shown that with this metal there is no appreciable difference in the alteration of resistance produced by stretching when different current strengths are employed in the bridge.

[†] After these experiments I re-tried iron with a view to ascertain whether *very small loads* might not produce decrease of resistance, but found that the smallest load that caused any effect whatever produced as before increase of resistance.

Experiment XII.

An annealed nickel wire loaded and unloaded several times on three different occasions, with weights of 18, 20, and 22 kilogs. respectively, and then tested with equal or lesser loads.

Load in kilogs. put on and off several times $=\alpha$.	Load used in testing after $\alpha=\beta$.	Temporary alteration of resistance produced by β. $-$ Decrease of resistance. $+$ Increase of resistance.	Time after using α at which β was applied.
18	First 8 . . .	−89	$\frac{1}{2}$ hour.
	Next 10 . . .	. .	
	First 8 . . .	−95	1 ,,
	Next 10 . . .	+60	
	First 8 . . .	−99	24 hours.
	Next 10 . . .	+57	
20	First 8 . . .	−79	$\frac{1}{2}$ hour.
	Next 12 . . .	+71	
	First 8 . . .	−82·5	7 hours.
	Next 12 . . .	+67·5	
22	First 8 . . .	−72·0	$\frac{1}{2}$ hour.
	Next 12 . . .	+64·5	
	First 8 . . .	−73·5	12 hours.
	Next 12 . . .	+62·5	

We see here that rest produces a very appreciable effect, increasing the negative alteration of resistance and diminishing the positive alteration; but we notice also that as the loads α become larger and larger the influence of rest becomes less and less marked; and, lastly, that as α increases to nearly the breaking load of the wire the effects both of the moderate load to produce decrease and of the excessive load to produce increase become more and more equal, and if we turn to Table V. we see plainly that the points in the curves showing the position of maximum decrease of resistance, which at first become wider and wider apart, at length begin to close up.

All this is intelligible if we bear in mind what has been proved in Part I., namely, that the elasticity of all metals is diminished by permanent extension carried to a certain point, and beyond this point increased, and provided that we assume that *temporary** elongation without regard to the stress producing it causes increase of resistance. The question then naturally arises, do stress and the consequent temporary strain produce on the electrical conductivity of substances opposite effects?

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* Permanent elongation we shall find produces in most metals first increase of specific resistance and then decrease, but in iron and nickel first decrease and then increase.

INFLUENCE OF PERMANENT EXTENSION ON THE TEMPORARY ALTERATION OF RESISTANCE CAUSED BY LONGITUDINAL STRESS.

It has been asserted in Part I. that the elasticity of a wire is diminished by permanent extension not exceeding a certain limit. In order to furnish still further evidence in favour of this assertion some experiments were made on the influence of permanent extension on the temporary alteration of resistance produced by longitudinal stress.

Experiment XIII.

A piece of copper (3) was subjected to loads which were gradually increased, and after the removal of each load the temporary alteration of resistance was determined from the mean values got by testing with from one to five measures of water.

Number of measures used for the permanent extension.	Number proportionate to alteration of resistance of wire.	Difference between consecutive alterations of resistances.	Number proportionate to temporary alteration of resistance caused by stress of 1 grm. per square centim.	Difference between consecutive alterations of resistance.
5	860	. .	644	. .
6	862	2	648	4
7	865	3	652	4
8	878	13	658	6
9	912	34	681	23
10	952	40	692	11
11	985	33	693	1

Experiment XIV.

A similar piece of wire was loaded with from 12 to 14 measures of water, and after the removal of each load tested in a similar manner to the above.

Number of measures used for the permanent extension.	Number proportionate to alteration of resistance of wire.*	Difference between consecutive alterations of resistances.	Number proportionate to temporary alteration of resistance caused by stress of 1 grm. per square centim.	Difference between consecutive alterations of resistance.
12	1288	. .	421	. .
13	1418	130	416	−5
14	1574	156	409	−7

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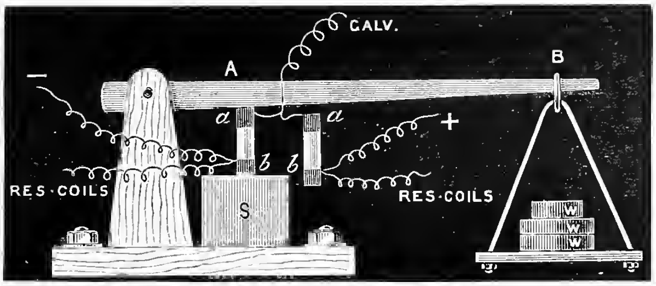
* Neither these numbers nor those in column 4 are on the same scale as the corresponding numbers in the last experiment.

The last two experiments show that with copper the temporary alterations of resistance caused by longitudinal stress increase up to a certain limit of permanent extension and then begin to decrease. We also see that the greatest changes in the values of the temporary alterations of resistance take place at those places where there are the greatest permanent changes of resistance.

THE EFFECT OF COMPRESSION ON ELECTRICAL CONDUCTIVITY.

It was anticipated that compression would in most cases produce decrease of electrical resistance, and as carbon seemed a suitable substance to experiment on, the following arrangement was made with the view of ascertaining whether compression would produce on the resistance of carbon the opposite effect to that caused by longitudinal extension :—S in fig. 11 is a block of stone on which rests upright the piece of carbon which it is desired to compress; the other piece of carbon to serve as the comparison-piece is supported near the first by a clip-stand (not shown in the figure). The two pieces of carbon were each about 4 inches in length, and were well coated with copper deposited by electrolysis for a distance of about three-quarters of an inch at the ends. To the upper ends *a*, *a* were soldered two silk-covered copper wires as in the experiments on extension, and the junction of these was connected with the galvanometer, whilst to the copper deposited on the lower extremities *b*, *b* were soldered wires to serve for connecting with the battery and resistance coils. The compression was produced by putting weights into a scale-pan attached to the end B of a lever of hard wood, which in the position of the carbon rod produced on the latter a pressure five times that of the weights in the pan.

Fig. 11.



Experiment XV.

A piece of carbon rod, 4 inches in length and ·1386 square centim. in section, was compressed by putting a weight of 2 kilogs. on to the pan at the end of the lever.

Number of trial.	Alteration of resistance as represented by the number of divisions of the platino-iridium wire through which it was necessary to move the sliding-piece in order to balance the effect of putting on or taking off 2 kilogs.
1	118·0
2	118·5
3	110·0
4	115·5
Mean	115·4

From the results of Experiment XV. it was calculated that a compression of 1 grm. per square centimetre would cause a *decrease* of resistance of 6398×10^{-12} per unit, and as the longitudinal elasticity was $267·2 \times 10^{6}$, the alteration of resistance attending an amount of compression which would suffice to halve the length of the rod would be 1·710, and of this alteration ·210 would be in the specific resistance of the rod. The specific resistance of the specimen at 15° C. was 4214×10^{-6}.

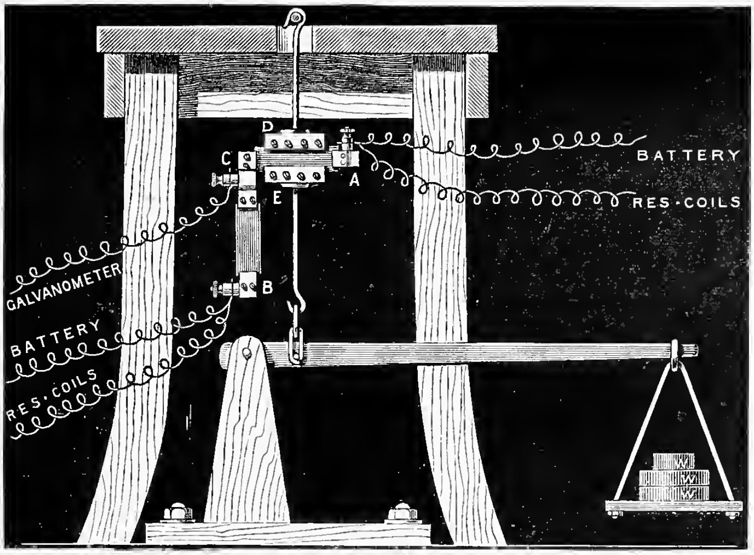
We see that the effect of compression is to diminish both the total resistance and the specific resistance of the specimen of carbon, and is therefore of an opposite nature to that of longitudinal extension.

THE ALTERATION OF ELECTRICAL CONDUCTIVITY PRODUCED BY STRESS APPLIED IN A DIRECTION TRANSVERSE TO THAT OF THE CURRENT.

It has been seen that, in the case of most metals, stress applied in the same direction as that of the current increases, both permanently and temporarily, the resistance, and it seemed probable that stress when applied in a direction perpendicular to that of the current would alter the resistance in the opposite direction. The following arrangements were therefore made with the view of testing this point:—

Strips of metal foil about 10 centims. long, 2 centims. broad, and ·01 centim. thick, were arranged in pairs as in fig. 12. D and E are stout brass clamps into which two hooked iron rods are screwed, and the strip to be examined is clamped into the brass pieces, which are 8 centims. in length, but insulated from them by means of silk. The clamp C connects the metal to be tested with a similar strip, and both strips with one terminal of the galvanometer; whilst the clamps A and B serve, as before, to connect the strips with the resistance coils and battery.

Fig. 12.



The strip to be examined was strained by means of a stout wooden lever, and the table which supported the upper hooked iron rod was surrounded with baize.

Experiment XVI.

A strip of iron foil, 10 centims. in length, 2·1 centims. in width, and of specific gravity 7·65, was subjected to a stress of 120 kilogs., and after two or three applications and removals of this load, it was found that this stress produced a *diminution of resistance* which required the sliding piece to be shifted through 20 divisions of the platino-iridium wire, and that on the removal of the stress the resistance was increased by the same amount. The load was distributed over a sectional area of ·098 square centim., and the diminution of resistance, which would be caused by a stress of 1 grm. per square centimetre, was calculated to be $123·6 \times 10^{-12}$ per unit. Inasmuch, however, as only one-third of the total resistance measured was effected by the stress, it was assumed that if the whole of the piece could have been strained to the same extent the diminution would have been 371×10^{-12}; whereas in the case of iron *wire* subjected to longitudinal traction the alteration of resistance produced by the same stress amounted to 2111×10^{-12}, or nearly six times as much as that caused by transverse traction. Again, if we assume that the value of "YOUNG'S modulus" is the same for the foil as for the wire, the total alteration of resistance caused by the transverse traction which would suffice to double the width of the strip would be ·742 per unit. Also taking the ratio of lateral contraction to elongation to be ·281, as was the case with iron (1), the total decrease of resistance due to mere change of form would be 1·562 ; and therefore on the whole there would seem to be an *increase of specific resistance* of ·82, caused by stress sufficing to double the width of the strip.

Experiment XVII.

A strip of tin-foil of nearly the same dimensions as the iron-foil used in the last experiment was tested with a load of 8 kilogs., and a permanent decrease of resistance thereby produced = ·001 per unit; a temporary decrease of ·002 per unit was caused by the application of the load, and it was calculated that the temporary decrease per unit effected by a stress of 1 grm. per square centimetre would amount to 2581×10^{-10} as compared with 1055×10^{-11}, the alteration caused by the same amount of longitudinal stress with tin wire. It would thus seem that, contrary to what takes place with the iron, the alteration produced by transverse traction is much greater (about 23 times as great) than that caused by longitudinal traction; and if we suppose the elasticity of the foil to be even much less than that of the wire, there must be a very appreciable *decrease of specific resistance ;* this latter, if the values of e and σ are the same for the foil and the wire, would be 70·0 per unit, *i.e.*, the metal would have a specific resistance of $\frac{1}{70}$th of its original specific resistance, if stress were applied transversely, sufficing to double the width of the foil. The corresponding alteration produced by the same amount of stress applied longitudinally to a wire of tin is, as we have seen, less than 2 per unit.

Experiment XVIII.

A strip of zinc-foil of nearly the same dimensions as the strips of iron and tin last used was tested with loads varying from 50 to 100 kilogs. A stress of 50 kilogs. produced a permanent decrease of resistance amounting to ·0091 per unit, 70 kilogs. a decrease of ·02 per unit, and with 100 kilogs. the foil was partly pulled away from the clamps ; before this, however, the temporary decrease of resistance produced by 50 kilogs. was ascertained to be ·028 per unit. After the accident the foil was securely reclamped, and now three trials with 50 kilogs. gave alteration of temporary resistance amounting to ·010, ·017 and ·010 respectively, the load in the second of the three trials having been allowed to remain for a longer period on the foil than was the case in the other two trials. It was then evident that the time during which the load was allowed to remain on the foil largely influenced the temporary alteration of resistance, and accordingly several experiments were made with a view to verify this fact, which was eventually abundantly established. In one of these experiments a load of 70 kilogs., after having been put on and taken off several times, was allowed to remain on for 10 minutes, when a decrease of ·0428 per unit was measured; after 20 minutes the decrease amounted to ·0557 per unit, part of the decrease produced after the first 10 minutes was permanent, but a considerable part was temporary.

The foil was now tested with loads of 12 and 20 kilogs., the latter causing a decrease of resistance represented by 300 divisions of the platino-iridium wire, and the former 180, that is, the decrease was exactly proportional to the load. From these results it was calculated that a transverse stress of 1 grm. per square centimetre would produce a decrease of resistance amounting to 12384×10^{-11} as against 4406×10^{-12}, the alteration caused by the same longitudinal stress in a zinc wire; the alteration in the former case is about 28 times that in the latter. Again, assuming that the values of e and σ are the same for the foil as for the wire, the decrease of specific resistance which would be caused by transverse strain sufficient to double the width of the strip would be 95 per unit; whilst the alteration for the same amount of longitudinal stress is about 2 per unit. The temporary decrease of resistance of zinc and tin appeared to be so very large, that it was suspected that the silk did not properly insulate the foil from the upper and lower clamps; but this did not seem to be the case, as the resistance was the same before and after clamping; nor could the stress have temporarily impaired the insulation, as if so the same large decrease would have been obtained with iron. It would thus appear that, at any rate for the metals zinc and tin, the effect on the electrical resistance of stress perpendicular in direction to the line of flow of the current is the reverse in every respect of that of stress applied longitudinally. In the case of iron we have seen that though the resistance is on the whole decreased, the mere change of form would more than account for the decrease; and that unless the amount of lengthening which any given load per square centimetre can produce is much less with iron-foil than with iron wire the specific resistance is increased.

One of the most remarkable features of these experiments is the large influence of time on the temporary alteration produced by the heavy loading of zinc and tin. A similar influence, though not to the same extent, was noticed with aluminium, zinc, and tin when great longitudinal stress was employed, and with these metals also for both directions of stress, the departure from proportionality between the load and the alteration produced thereby became very marked, though here again much more so when transverse than when longitudinal stress was applied.

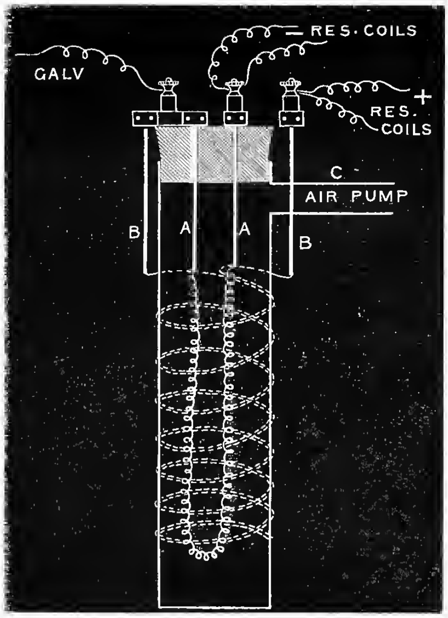
THE ALTERATION OF ELECTRICAL CONDUCTIVITY PRODUCED BY STRESS APPLIED EQUALLY IN ALL DIRECTIONS.

Unsuccessful attempts and mode of determining the lowering of the melting-point temperature of ice.

So far back as the winter of 1877 I attempted to detect and measure the effect on the electrical conductivity of wires produced by such alterations of fluid pressure as could be obtained by means of the air-pump only, being under the impression at that time that the change of conductivity caused by rise of temperature was due for the most part to mere expansion.

In the first attempt the wire to be tested, a silk-covered copper wire about 12 feet in length and $\frac{1}{40}$th of an inch in diameter, after having been well soaked in melted paraffin wax was coiled in a spiral and placed in a thin, hollow, brass tube (fig. 13), having an inner diameter of 2·8 centims. and a length of 15 centims.

Fig. 13.

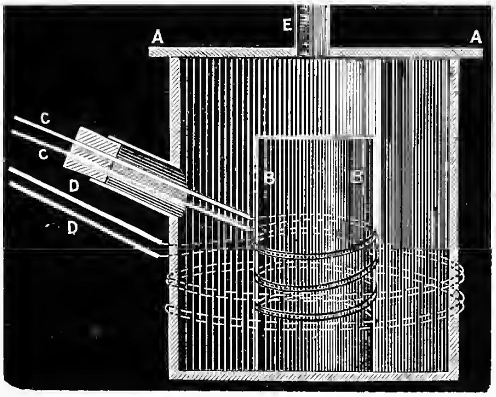


The ends of the wire were soldered to two stout copper wires A A, which passed air-tight through an indiarubber cork and served to connect the wire under examination with the comparison-wire. The latter having its ends soldered to the two stout copper wires B B, was wound double round the outside of the hollow cylinder, and was, together with B and B, secured to the cylinder by string; the comparison-wire had also been soaked in melted paraffin wax, and the four stout wires, A, A, B, B, and their junctions with the other wires, well coated with shellac varnish. The two wires, which as usual were of the same dimensions and substance, were joined up with the other parts of the bridge in the manner already described, and the tube C served to connect the cylinder with an air-pump and barometer-gauge. The whole arrangement having been placed in a box the cylinder was well packed up in sawdust, and after a sufficient time had been allowed to elapse to enable the temperature of the wires to become constant the experiments began.

This plan entirely failed, the effect of the alteration of pressure on the resistance of the wire being completely masked by the change of temperature caused by the rarefaction or condensation of the air even when a considerable time had been allowed to elapse. The condensation and rarefaction of the air would cause a change of resistance the opposite to that looked for as the result of change of pressure; and so slowly did the temperature alter after the first half-hour that I began to suspect that *increase of pressure caused increase of resistance*, and for some days actually tried to measure the *increase* of resistance apparently caused by increase of pressure. Finding, however, that the observations did not agree sufficiently with each other I filled the cylinder with water and immersed it in a large glass vessel, also filled with water, which was stirred from time to time. This plan also failed, the compression of the water causing an effect in the same direction though not to the same extent as the compression of the air had done. It was noticed, however, that after the air was let into the vessel there was *no change of resistance till a second or so afterwards.*

Finally, the following method was adopted:—The vessel containing the wire to be tested consisted (fig. 14) of a brass cylinder closed at one end, and which could be closed at the other by a brass cover A A. The vessel was about 7 inches deep and 6 inches in internal diameter; the open end and the cover which closed it having been carefully ground, so that with the help of a little grease it could be made perfectly air-tight. The wire under examination was wound double on the outside of a very thin brass cylinder B B, and the ends of the wire were soldered to the stout copper wires C, C, which latter passed through an indiarubber cork, fitting air-tight into a tube about 1 inch in diameter, which was soldered into the vessel and made an angle of 45° with the side. The comparison-wire was wound double round the outside of the large cylinder, and the two stout copper wires D, D, to which the

Fig. 14.



ends of the wire were soldered, were connected in the same way as before with C, C and the other parts of the bridge. The whole arrangement of wires and cylinders was placed in a wooden box having holes cut in one of the sides to admit of the connexions C, C and D, D passing through, and this box was in turn placed inside a larger one, so that the space between the two boxes could be filled with sawdust.

The mode of proceeding was as follows:—First, ice broken into small pieces was packed into a layer about 3 inches thick round the larger of the two cylinders, then, after an interval of about half-an-hour, both the large cylinder and B were filled with pounded ice, the cover put on, and over it placed a large sheet of paper containing sawdust. The tube E served to form by means of indiarubber tubing connexion with the air-pump, and after a sufficient time had been allowed to make the balance of the two wires constant the experiments began.

This plan seemed to act admirably. A short time after exhausting the air there seemed to be an increase of resistance, and on letting in the air again a decrease. After each operation with the air-pump the wires were tested at intervals of 10 minutes, until it was certain that the full effect on the resistance of the alteration of the pressure had been produced. Each experiment lasted more than one hour, and it was only possible to make two or three experiments before the ice had to be readjusted round the wires; as evidently after some three hours, though there might be no appreciable melting of the ice, the galvanometer showed that one or other of the wires was not quite at the temperature of the ice. The following are the results obtained:—

Experiment XIX.

Alteration of resistance represented by the number of divisions through which it was necessary to move the sliding-piece in order to restore the balance.	Alteration of pressure in millimetres of mercury.
12·35	700
13·28	721
13·00	730
12·40	718
13·20	710
12·44	707
13·46	707
12·44	707
12·84	708
12·64	708
11·32	708
13·76	708
12·74	707
12·74	707
12·74	707
12·74	710

In these experiments, the divided wire was of German-silver, each division being equal to ·00021105 ohm at the temperature of the room, and since 100 ohms were used at each end of the wire, the alteration of resistance represented by moving the sliding-piece over one division of the wire would equal $\frac{1}{\cdot 000001055}$ of the whole, the effect of one-tenth of a division was readily perceptible, and the second place of decimals in the above columns were got by taking the mean of several observations at each pressure. From the above results it was calculated that the decrease of resistance per unit produced by an increase of fluid-pressure of 1 grm. per square centimetre would be 27854×10^{-12}.

On showing these results, however, to Professor G. G. STOKES, he suggested that sufficient account had not been taken of the fact that the alteration of pressure might affect the resistance by altering the melting-point temperature of the ice, though, as I thought, the ice would not be sufficiently wet in the inner cylinder, as the temperature of the room at the time was frequently almost at 0° C., and the cylinder well surrounded by ice on the outside to affect the result in this way. Nevertheless, acting on this suggestion, I carefully determined the increase of resistance at 0° C. for a rise of 1° C., in the manner adopted by MATTHIESSEN, and found this to be ·0038587 per unit of resistance. Now, J. THOMSON was led by theoretical considerations[*] to the conclusion that the melting-point temperature of water would be lowered ·0075° C. by an increase of one atmosphere of pressure, and the matter was put to experimental test by W. THOMSON, who arrived at results agreeing almost exactly with this conclusion. If we assume one atmosphere to be equal to 76 centims. of mercury, or 1034 grms. per square centimetre, the lowering of temperature produced by a pressure of 1 grm. per square centimetre on the melting-point of ice should be ·00000725° C. Now, if the alteration of resistance was entirely due to change of temperature in the melting-point of the ice, my experiments would give a lowering of temperature for a pressure of 1 grm. per square centimetre equal to $\frac{278\cdot 54 \times 10^{-10}}{\cdot 003859}$, *i.e.*, to ·00000723° C.

This result then led me to believe that either there was no change of resistance produced by the pressure merely, or, as subsequently proved to be the case, that the alteration of pressure was too small to produce an appreciable effect even with the very delicate galvanometer which I was using. I also remembered that, when water had been employed, as in the previous experiment, as has been before mentioned, there was no appreciable effect *immediately* after the air had been let into the vessel. Moreover, I called to mind that on two occasions when the temperature of the room was almost exactly 0° C. there was no effect, when, as in the last experiments, ice had been used, *even for some* 10 *or* 20 *minutes* after using the pump; but in this case I had attributed the absence of effect to the regelation of the ice over the cylinder

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[*] Trans. Roy. Soc. Edinburgh, Jan., 1849. Cambridge and Dublin Math. Journal, Nov. 1850.

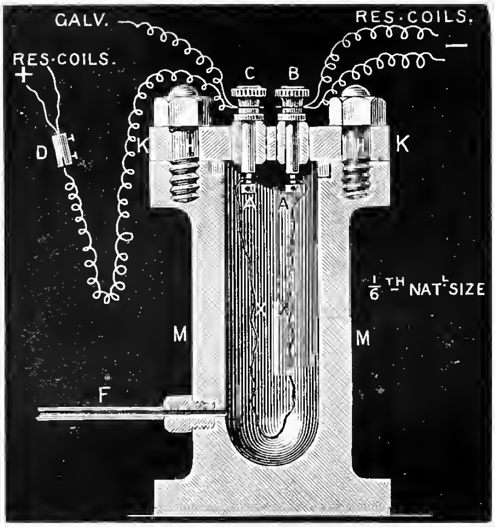
round which the wire was wrapped, and had taken the trouble several times to take off the cover and loosen the ice. I have now no doubt that the warmth of the hand in this last act had melted the ice sufficiently to allow the alteration of the melting-point temperature to be felt.

Though these experiments were failures as far as the immediate object in view was concerned, they show what a delicate and accurate thermometer the galvanometer can prove, and that an interesting investigation of the lowering of the melting-point temperature of ice* could be made by using a modification of the above apparatus suitable for pressures of two or three hundred atmospheres, such as will be described presently.

Successful Attempts and Results.

The previous trials having shown that pressures of several atmospheres must be resorted to if one was to hope to measure any alteration of resistance which might be produced by fluid-pressure, the following apparatus (fig. 15) was employed :—

Fig. 15.



M M is a strong vessel made of gun-metal (drawn to scale in fig. 15 one-sixth of the real size), C, B are two binding screws at the ends of two stout brass wires passing water-tight through a cover, K K, and separated from the latter by insulating material. This cover could be removed at will, and the ends of the wire to be tested, X, X, connected by two small binding screws (shown in the figure unscrewed) A, A, at the lower ends of the stout brass wires. After the wire had been well secured to A and A, these latter, the wire itself and those portions of the stout brass wires which projected below the cover, were well coated with paraffin wax. The vessel having been filled with

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* The effect on the resistance of the wire is so very small as to be almost neglectable, and the correction even for this can be made by data which are given further on. (See p. 77, Table VI.).

water, the cover with the wire suspended from it was put on and secured by means of three screws and nuts, two of which, H, H, are shown in the figure, the cover having been made to fit water-tight under the highest pressures employed, by means of a gutta-percha collar. The wire X and the comparison-wire were then connected up in the usual way, as shown in the figure,[*] and the whole well-covered with baize.

The first wire which was tried was a piece of the copper wire used in the last experiments, and a day having been selected at which the temperature was about 4° C., pressure was put upon the wire by means of a force-pump which formed part of a hydraulic press.[†] The pump communicated with the vessel by means of the tube F, and was capable of supplying a pressure of upwards of 5000 lbs. on the square inch. The pressure was measured by a strong spring pressure-gauge, divided so as to read to a pressure of 100 lbs. on the square inch.

On working the pump so as to increase the pressure there was, after a short time, a very decided decrease of resistance perceptible, and on removing the pressure the resistance returned to almost its former value. Several attempts were made with greater and greater pressures, which were carried up to 4000 lbs. on the square inch, and all gave indications in the same direction; but unfortunately, on raising the pressure to 5000 lbs. on the square inch, the insulating substance round one of the stout brass wires cracked, and allowed the water to come through the cover. As these trials were only intended to be preliminary, no measurements had been taken of the alteration of resistance produced, but there was now no question that in the case of copper wire increase of pressure produced decrease of electrical resistance.

These experiments could not be renewed till the following winter, when the flaw mentioned above having been repaired, some fresh trials were made with iron wire and the following measurements taken:—

Experiment XX.

Pressure in lbs. per square inch.	Number of divisions of the platino-iridium wire through which it was necessary to move the sliding piece in order to restore the balance.
900	20·0
1100	37·7
1300	25·0
Mean 1100	Mean 27·6

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[*] In the actual experiments X was secured to the vessel by string, the connexions having been well insulated by wrapping them up in paper.

[†] This pump was lent to me by the Rev. T. A. Cock, of King's College, who kindly had it put into complete working order before I used it.

These results give an alteration of 1160×10^{-12} per unit in the resistance for a pressure of 1 grm. per square centimetre, and as in the case of copper increase of pressure produces diminution of resistance.

The experiments were again put an end to by a flaw in the insulation, which was made on attempting to carry the pressure to a greater extent, and therefore the final result can only be considered as very roughly correct. Fortunately, however, there is a means of testing approximately the accuracy of this result, as it was found that at the temperature of the room (about 4° C.) the heat caused by the compression of the metal exactly balanced the effect of the pressure on the wire—so exactly, indeed, that even when pressures of 3000 lbs. per square inch were employed, *the instant after the removal of the pressure there was no change whatever in the resistance,* but in a few seconds afterwards the resistance began to decrease, and apparently attained a minimum in about half an hour. About this fact there can be no doubt, as it was repeated several times and with several different pressures. Now the change of temperature produced by an alteration of fluid pressure amounting to 1 grm. per square centimetre is for any substance equal to $\frac{T \times \alpha}{J \times x}$, where T= the absolute temperature, α the coefficient of cubical expansion, J JOULE's equivalent, and x the specific heat referred to unit volume. In this case $T=278$, $\alpha=\cdot0000342$, $J=42{,}400$, $x=\cdot842$ at the temperature 4° C.

Therefore the rise of temperature caused by an increase of pressure of 1 grm. per square centimetre would be ·000000264° C.; and since the increase of resistance of the wire for 1° C. at 4° C. had by preliminary experiments been determined to be almost exactly ·0047 per unit of resistance, the increase of resistance due to the heat of compression would be $\cdot000000264 \times \cdot0047$, or 1241×10^{-12}—a result which agrees fairly with that arrived at by direct experiment.

After making these experiments I was glad to find that CHWOLSON had succeeded in measuring the effect of fluid pressure in the cases of copper, lead, and hard brass,* and with these three metals observed that increase of fluid pressure produced decrease of electrical resistance.

In the next table are given the results of CHWOLSON's and my own experiments, in centimetre gramme units.

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* Im. Acad. of St. Petersburg Bull., March, 1881, and 'Nature,' June 2, 1881, p. 112.

TABLE VI.

Name of Metal.	Decrease of resistance per unit produced by an increase of fluid pressure of 1 grm. per square centim. = A.	Increase of resistance per unit produced by a longitudinal tension of 1 grm. per square centim. = B.	Ratio of B : A.	Decrease of resistance per unit attending a fluid pressure sufficing to halve the length of the wire.	Increase of resistance per unit attending longitudinal tension sufficing to double the length of the wire.	Decrease of specific resistance per unit attending a fluid pressure sufficing to halve the length of the wire = C.	Increase of specific resistance per unit attending longitudinal tension sufficing to double the length of the wire = D.	Ratio of C : D.
Lead . .	10638×10^{-12}	17310×10^{-12}	1·63	2·440	2·885	3·440	1·613	2·14
Copper .	$1257·0 \times 10^{-12}$	$2310·0 \times 10^{-12}$	1·84	3·470	2·713	4·470	1·005	4·45
Iron . .	$1160·0 \times 10^{-12}$	$2111·1 \times 10^{-12}$	1·82	5·269	4·180	6·269	2·618	2·39
Brass .	$1064·0 \times 10^{-12}$	$2265·5 \times 10^{-12}$	2·13	3·004	2·239	4·004	·166	2·41
Means .	. .	. .	1·83	. .	. .	. .	. .	2·85

Explanation of and remarks on Table VI.

A few words are necessary on the methods of calculating the numbers given in columns five and seven.

If e be the coefficient of longitudinal elasticity, and σ the ratio of lateral contraction to elongation, it can easily be proved that if we subject a wire to a fluid pressure of 1 grm. per square centimetre, the decrease in length per unit thereby produced will be $\frac{1-2\sigma}{e}$. If then A denote the decrease per unit of resistance produced by the pressure, it follows that the decrease attending pressure which would suffice to halve the length of the wire would be $A \div \frac{1-2\sigma}{e}$, and in this way the numbers in column five have been determined from those in column two; the values of e and σ being those given in Part I., with the exception of the value of σ for brass, which has been taken from MALLOCK's paper,* as I have reason to believe that the values of σ obtained by me for this metal are too large. Again, since the pressure would for such small amounts as are used here cause a decrease of section which would be double the decrease of length, the effect of the pressure in merely altering the dimensions would be to increase the resistance by 1 per unit. In order, therefore, to deduce the values in column seven from those in column five we have only to increase the former by 1.

It will be noticed that the *total* alteration of resistance produced by the fluid pressure is in all cases less than the alteration produced by the same amount of longitudinal stress, the ratio of the latter alteration being to that of the former as 1·83 : 1; but that the alteration of resistance when *the same change of length* is produced by the two kinds of stress is, except in the case of lead, greater when fluid pressure is

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* Proc. Royal Society, June, 1879.

employed than with longitudinal stress, and that the alteration of *specific resistance* is much greater for the former kind of stress than for the latter, the ratio being about 2·85 : 1.

The small alterations of resistance which can be produced by fluid pressure as shown in this table prove also how impossible it would have been to detect with such changes of pressure as can be effected by an air-pump any alteration of resistance due directly to compression, and therefore the value of the mode of experimenting already alluded to in determining the amount of lowering of the temperature of the freezing-point of water by pressure: in fact, the change of resistance due to the lowering of the temperature of the melting-point of ice by the pressure would be more than 22 times the change of resistance due to the pressure only.

A brief consideration also suffices to show that the alteration of resistance due to any change of temperature is in all cases very much greater than that which would follow from the same change of volume produced by mechanical stress; this will be seen at once from a glance at the next table.

TABLE VII.

Name of metal.	Coefficient of cubical expansion at 20° C.	Increase of resistance per unit caused by a rise of 1° C. at 20° C.	Rise of temperature necessary to double the volume = A.	Increase of resistance per unit caused by rise of temperature A = B.	Increase of resistance per unit if the wire could have its volume doubled by mechanical stress = C.	Ratio of B : C.
			° C.			
Lead	$\cdot 8223 \times 10^{-4}$	·00375	12,160	45·6	0·962	47·4
Copper	$\cdot 4554 \times 10^{-4}$	·00380	21,960	83·5	0·904	92·4
Iron	$\cdot 3420 \times 10^{-4}$	·00470	29,250	137·5	1·393	98·7
Brass	$\cdot 5450 \times 10^{-4}$	·00122	18,350	22·4	0·746	30·3

We see from this last table that the alteration of resistance due to any change of temperature is in the case of the four metals, lead, copper, iron, and brass, from 30 to nearly 100 times as great as that which would follow from the same change of volume produced by mechanical agency; and it would appear, again, therefore, that the increase of resistance caused by rise of temperature is principally due to other causes than mere expansion.

PERMANENT ALTERATION OF RESISTANCE PRODUCED BY LONGITUDINAL TRACTION.

Limit of elasticity.

Experiments on the permanent alteration of resistance produced by traction possess considerable advantages in determining the limit of elasticity over the methods usually adopted. In the first place, it is possible to detect much more minute elongations, even though very small lengths of the wire be employed, than would be the case if 100 feet of the wire could be tested in the ordinary way with the cathetometer, and in the second, all errors arising from the wires not being perfectly straight at the commencement of the experiments are entirely avoided.

The limit of elasticity raised by previous loading.

Robert Thalèn[*] with, no doubt, others has shown that in the cases of iron and steel the limit of elasticity can be raised by previous loading.

This was found to be so for all the metals examined, and the next experiment furnishes a fair example of the kind.

Experiment XXI.

An annealed copper wire was very carefully loaded and unloaded by allowing water to flow slowly into and out of a pail attached to it, and the permanent increase of resistance determined after each unloading. The water entered the pail from a vessel containing 5814 grms. of the liquid, and this quantity took five minutes to pass into or out of the pail.

Number of trial.	Number of measures each =5814 grms.	Number of divisions through which it was necessary to move the sliding-piece in order to restore the balance after each unloading.
1	1	14
2	1	1
3	2	31
4	1	0
5	2	2
6	3	54
7	1	0
8	2	2
9	3	9
10	4	80
11	1	0
12	2	0
13	3	0
14	4	7
15	5	122
16	1	0
17	2	0
18	3	0
19	4	0
20	5	25
21	6	235
22	1	0
23	2	0
24	3	0
25	4	0
26	5	8
27	6	29
28	7	1084

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[*] Pogg. Ann., April, 1865, and Phil. Mag., December, 1865.

Critical points.

THALÈN has also proved[*] that if a curve be drawn having for its ordinates and corresponding abscissæ lines proportional respectively to the permanent extensions and the load producing them, there is a point of maximum curvature, where the increase of length becomes suddenly large compared with the load, and suggests this point, which appears to be nearly at the same part of the curve for different specimens of iron and steel, as a suitable substitute for the so-called "limit of elasticity."

My own investigations have shown that there exist in every metal *two* such points, which I have called critical points, and these are evidently very closely related to the moduli of elasticity. The first of these critical points is one that must of necessity have escaped the notice of most observers of the elongation of wires in the usual manner, as the load required to straighten the wire sufficiently for observations made in this way is beyond this point. Now in the present investigations it is not requisite to straighten the wire, and as the increase of resistance proves to be almost exactly proportional to the permanent elongation, it is evident that the curves showing the relations between permanent extension and load will be similar to those connecting permanent increase of resistance and load.

Experiment XXI. furnishes a good example of the first critical point. It will be seen that the first critical point occurs at the seventh measure, this producing a permanent increase of resistance the ratio of which to that produced by the previous measure is appreciably greater than any of the other similar ratios.

The following experiment is a sample of a series undertaken with a view to ascertain the position of the two critical points for the different metals.

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[*] Phil. Mag., December, 1865.

Experiment XXII.

A well-annealed silver wire was suspended for trial with the scale-pan attached, and loads increasing by 1 kilog. at a time were put on for three minutes and then removed, when the permanent increase of resistance was determined for each load.

Load in kilogs.		Increase of resistance in divisions of platinum-silver wire.
	1st	40
	2nd	40
	3rd	45
	4th	60
	5th	75
	6th	80
First critical point .	7th	142
	8th	186
	9th	306
Second critical point	10th	1436
	11th	2706

Each division of the platinum-silver represents in this case an increase of resistance of only $\frac{1}{1240}$ per cent., and as the percentage of permanent elongation is half of this fraction, because the wire is decreased in section very nearly in the same proportion as it is increased in length, it follows that the total elongation produced by all the loads up to the first critical point inclusive, does not exceed ·2 per cent.

The case of iron.

The behaviour of iron under longitudinal traction is very remarkable; this metal, if the load be applied in small quantities at a time, is seen at certain points to become perfectly rigid, so that the further application of stress does not produce any further permanent elongation until the load has reached a certain value, when elongation once more commences.* Further, iron possesses three critical points at least, and may be found to have more;† these points are well shown in Experiment XXIII.

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* This was first noticed by J. T. Bottomley (Proc. Roy. Soc., No. 197, 1879), who kindly lent me some of the same kind of wire as that used by himself.

† Perhaps other metals may have more than two critical points, which might be discovered by loading the wires more gradually.

Experiment XXIII.

ANNEALED iron wire.

Load in kilogs.		Permanent increase of resistance represented by divisions of platinum-silver wire.
	1st	5
	2nd	8
	3rd	17
	4th	21
	5th	29
	6th	33
	7th	42
First critical point	8th	120
	8½	118
	9th	40
	9½	25
	10th	30
	10½	35
	11th	0
	11½	15
	12th	83
	12½	177
	13th	223
	13½	389
Second critical point	14th	2356
	14½	2028
	15th	1421
	15½	2052
	16th	36
	16½	24
	17th	304
Third critical point	17½	1603
	18th	Broke

It will be seen from the above experiment that there is a critical point at the 8th kilog., another at the 14th kilog., and a third at the 17th kilog.

The two critical points were determined in all cases by finding the two points where the ratio of the increases of resistance produced by consecutive equal loads is greatest ; the next experiment furnishes an example of the method adopted.

Experiment XXIV.

ANNEALED German-silver (2).

Load in kilogs.	Increase of resistance in divisions of the platinum-iridium wire.	Ratio of consecutive increases of resistance.
1st 2	21	
2nd 2	51	2·43[*]
3rd 2	59	1·15
4th 2	65	1·10
5th 2	90	1·38 1st critical point.
6th 2	117	1·30
7th 2	143	1·22
8th 2	326	2·28
9th 2	1083	3·32 2nd critical point.
10th 2	1156	1·07

The weight of the scale-pan, pulley, &c., was 2·5 kilogs., or 1·25 kilogs. on each part of the wire, therefore the total load at the first critical point is equal to 11·25 kilogs., and that at the second 19·25 kilogs. Now the section of the wire at the two points is ·00883 and ·00876 per square centimetre; hence the load at the first critical point is 1274 and at the second 2197 kilogs. per square centimetre.

In Table VIII. will be found the loads at the two critical points and their relation to "YOUNG'S modulus."

TABLE VIII.

Name of metal.	Value of e in grms. per square centim.	Load at the first critical point in grms. per square centim. $=\alpha$.	Load at the second critical point in grms. per square centim. $=\beta$.	Ratio $e:\alpha$.	Ratio $e:\beta$.
Iron (1)	1981×10^{6}	$2{\cdot}070\times10^{6}$	$3{\cdot}009\times10^{6}$	$9{\cdot}6\times10^{2}$	$6{\cdot}6\times10^{2}$
Nickel (1)	2175 ,,	2·200 ,,	3·100 ,,	9·9 ,,	7·0 ,,
Platinum (1)	1490 ,,	1·400 ,,	1·520 ,,	10·6 ,,	9·8 ,,
German-silver (2). . . .	1291 ,,	1·274 ,,	2·197 ,,	10·1 ,,	6·0 ,,
Copper (2)	1218 ,,	1·151 ,,	. .	10·7 ,,	. .
Copper (1)	1160 ,,	1·115 ,,	1·702 ,,	10·4 ,,	6·8 ,,
Copper (3)	1143 ,,	1·230 ,,	1·870 ,,	9·3 ,,	6·1 ,,
Platinum-silver (1) . . .	1051 ,,	1·081 ,,	1·741 ,,	9·8 ,,	6·0 ,,
Silver (1)	742 ,,	·819 ,,	1·250 ,,	9·1 ,,	6·0 ,,
Aluminium (1).	673 ,,	·730 ,,	1·120 ,,	9·2 ,,	6·0 ,,
Mean for all the different metals	. .	. .	. .	$9{\cdot}8\times10^{2}$	$6{\cdot}7\times10^{2}$

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[*] In this, as in several other experiments, the first of the ratios of consecutive increases is greater than several of those which follow, but this is probably due to the fact that the weight of the scale-pan had been acting on the wire for some hours previous to the period of actual testing.

Remarks on Table VIII.

It is evident from the last table that the loads both at the first and second critical points, in the case of each substance, bear a constant ratio to the corresponding value of "YOUNG's modulus," there being quite as much difference between the several ratios $\frac{c}{\alpha}$ and $\frac{c}{\beta}$ for the three specimens of copper as for the various metals. The time during which the load was allowed to act was in all cases the same, namely, three minutes,[*] and the wire was relieved of all weight except that of the scale-pan before determining its alteration of resistance.

Experiment XXI. having shown that the position of the critical points must be altered by the process of wire-drawing, it is necessary that very great care should be taken in annealing the wire[†] if we wish to determine the true position of these points for any substance whose particles are to be free from mutual strain previous to beginning the experiments. The theoretically correct definition of the "limit of elasticity" would be the highest load per unit of surface which a wire can bear without undergoing the slightest permanent elongation. It is clear, however, that we cannot, even with the utmost care, obtain a substance which will have its particles in a *perfectly homogeneous* condition, and it will of necessity happen that *some* of the particles are on the point of passing the elastic limit or, at any rate, that some are much nearer this limit than others, before any external stress has been applied. Consequently we can never hope to obtain the true value of the elastic limit by merely endeavouring to observe the first trace of a permanent elongation, our power to do so depending upon the delicacy of our instruments; but we can accomplish our object in all probability by determining the point at which the ratio of the elongation and the stress producing it reaches its first maximum. I would venture, therefore, to suggest, that the *first*[‡] of the two so-called critical points be taken to represent the true "limit of elasticity" of a well-annealed substance, and to agree with THALÈN that neither the method of measuring the limit of elasticity by the greatest load which will produce sensible permanent elongation, nor the purely arbitrary one adopted by WERTHEIM and others of fixing an elongation of ·0005 of the unit length as corresponding to this limit, is desirable.

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[*] Except in the case of copper (3) where five minutes was allowed.

[†] It is certainly not sufficient to heat a wire to redness *for a few minutes*, and then allow it to cool slowly; the high temperature must be maintained in some cases for a considerable time.

[‡] The *second* of the critical points is evidently the same as that suggested by THALÈN as being suitable for measuring the limit of elasticity (Phil. Mag., December, 1865), but I think that the second point is rather the precursor of breakage than of the passage of the elastic limit.

Critical points of tin and lead.

The positions of the critical points of these metals have not been satisfactorily determined though several trials were made for that purpose, and in some of these great care was taken to load by very small amounts at a time and to allow each weight to remain on the wire for a considerable period; in the following experiment each load was left on the wire for 48 hours:—

Experiment XXV.

ANNEALED lead wire.

Load in tenths of a kilog.	Increase of resistance in divisions of the platinum-iridium wire.	Ratio of consecutive increases of resistance.
1st tenth	7·0	. .
2nd	16·5	2·4
3rd	38·0	2·3
4th	49·0	1·3
5th	66·0	1·4
6th	103·0	1·6
7th	207·0	2·0
8th	540·0	2·6
9th	wire broke.	. .

If we regard the second load as the load at the first critical point, and the eighth as the load at the second critical point, we obtain for the loads at the two critical points respectively the values 48 kilogs. per square centimetre* and 74 kilogs. per square centimetre; in this case $\frac{e}{\alpha}$ would $=35\times10^2$, and $\frac{e}{\beta}=22\cdot6\times10^2$, both these numbers being much larger than the values of $\frac{e}{\alpha}$ and $\frac{e}{\beta}$ for the metals in Table VIII.

With tin, also, the loads at the first and second critical points, as far as the latter could be determined,† seemed much less in proportion to the value of e than was the case with most of the other metals.

In Table IX. the permanent increase per cent. of resistance produced by longitudinal traction is shown for each metal by a series of curves; and these curves will also show very fairly the permanent increase per cent. of length produced, since the former is very nearly equal to twice the latter.

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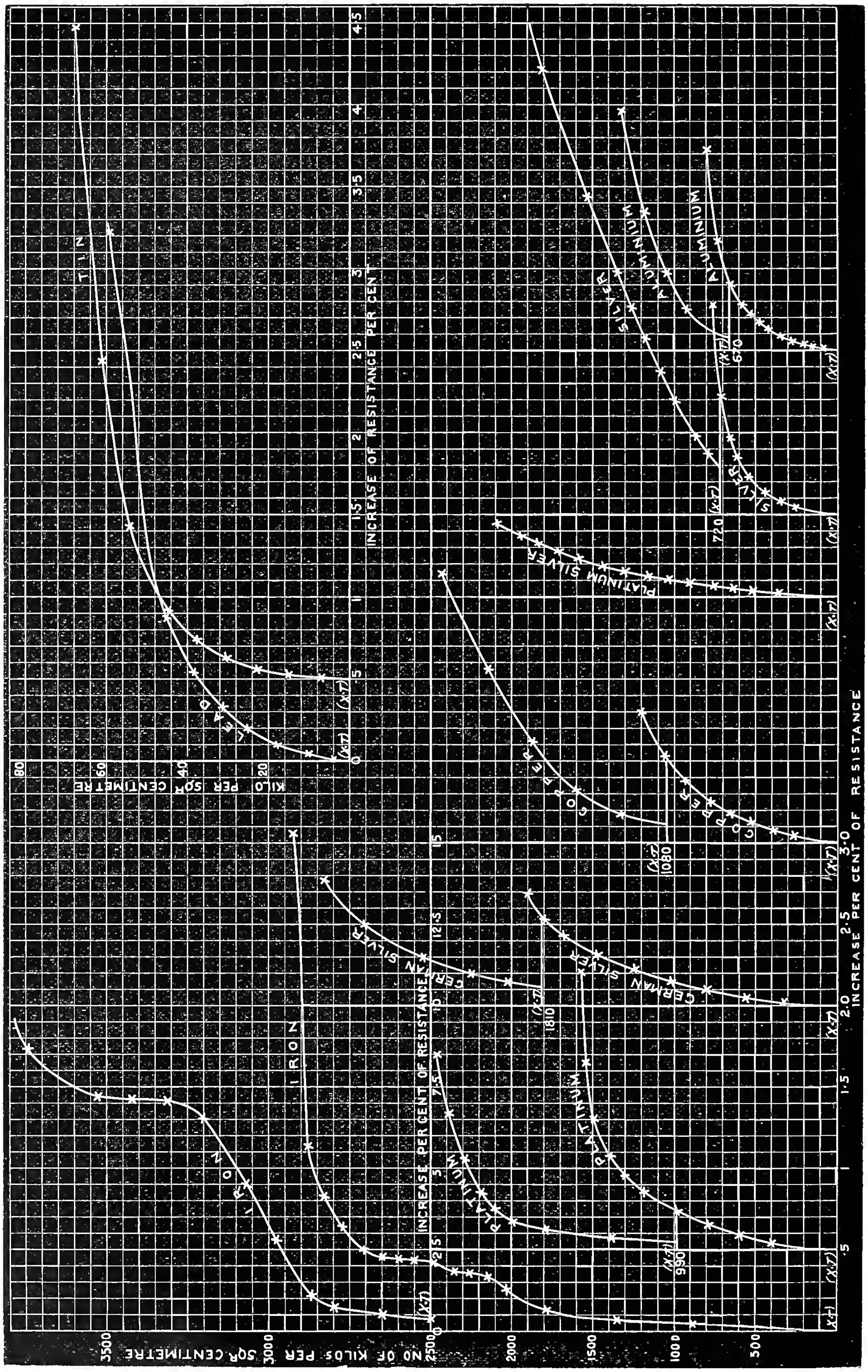
* 30 kilogs. per square centimetre added for the permanent load, consisting of pulley, &c.

† I am inclined, however, to attach very little value to the observations made on the critical points of lead and tin; very probably better results might be obtained by using wires of much greater section than those here employed.

TABLE IX.


NO OF KILOS PER SQR CENTIMETRE
500
1000
1500
2000
2500
3000
3500
INCREASE PER CENT OF RESISTANCE
.5
1
1.5
2.0
2.5
3.0
KILO PER SQR CENTIMETRE
20
40
60
80
INCREASE OF RESISTANCE PER CENT
0
.5
1
1.5
2
2.5
3
3.5
4
4.5
INCREASE PER CENT OF RESISTANCE
2.5
5
7.5
10
12.5
15
IRON
PLATINUM
GERMAN SILVER
COPPER
TIN
LEAD
SILVER
ALUMINIUM
PLATINUM SILVER
(x.T)
990
1810
1080
720
670

Explanation of and remarks on Table IX.

The curves are in most cases divided into two parts, the lower part representing the increase of resistance per cent. up to and a little beyond the first critical point, and the upper one the increase up to and beyond the second critical point. The abscissæ represent the increase of resistance per cent. for the lower curves on a scale of 40 millims. to unit increase per cent., and for the upper ones on a scale of 80 millims. to unit increase per cent.

The ordinates represent the load per square centimetre in kilogrammes, and are on a scale of 40 millims. to each 1000 kilogs. for the lower curves and of 80 millims. to each 1000 kilogs. for the upper ones.

The upper curves start from the horizontal lines drawn through their lower extremities, and the starting points on these lines are set off at distances representing on the scale for the upper curves the increase of resistance already attained: also opposite each horizontal line is placed the number of kilogrammes per square centimetre already put on to the wire. In all cases the origin of coordinates is marked X, T.

Thus, for example, the upper extremity of the lower copper curve has an abscissa $=32$ millims. and an ordinate of 48 millims, therefore the load at this point is $48\times\frac{1000}{40}$ or 2400 kilogs. per square centimetre; and the increase of resistance is $32\times\frac{1}{40}$ or 8 per cent. Similarly the upper extremity of the upper copper curve has an abscissa of 65·6 millims. and an ordinate reckoned from the horizontal line passing through the lower extremity of the curve of 56·8 millims., therefore the total load is $56{\cdot}8\times\frac{1000}{80}+1080$ or 1790 kilogs. per square centimetre, and the increase of resistance $65{\cdot}6\times\frac{1}{8}$ or 8·2 per cent.

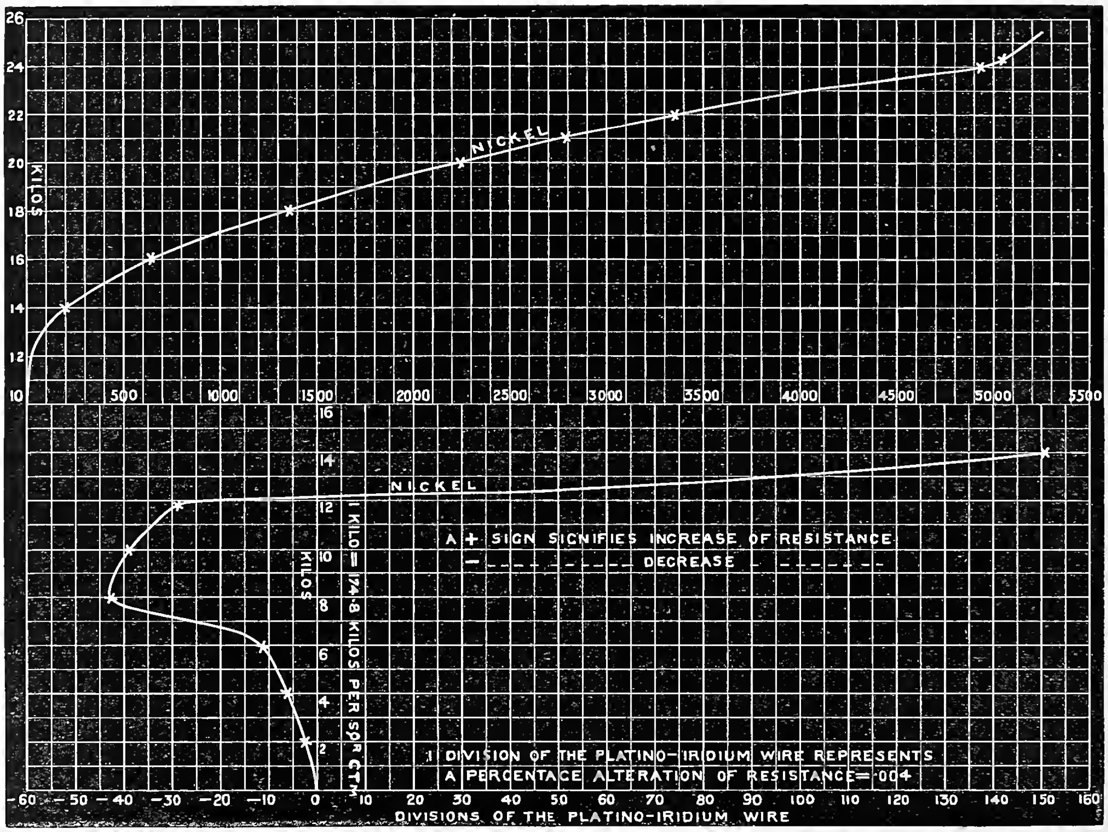
In order to include lead and tin in the same table it was found necessary to reckon the load on a scale of 2 millims. to 1 kilog. per square centimetre, and the increase of resistance on the same scale as that used for the lower curves of the other metals. Moreover, to the load registered in the table there should be added for these metals 30 kilogs. per square centimetre, this representing the permanent load on the wires.

It will be observed that there is in most cases a considerable resemblance between the forms of the upper and lower curves, and that in the case of iron the curve near the two critical points becomes very nearly a vertical straight line.

The critical points of nickel.

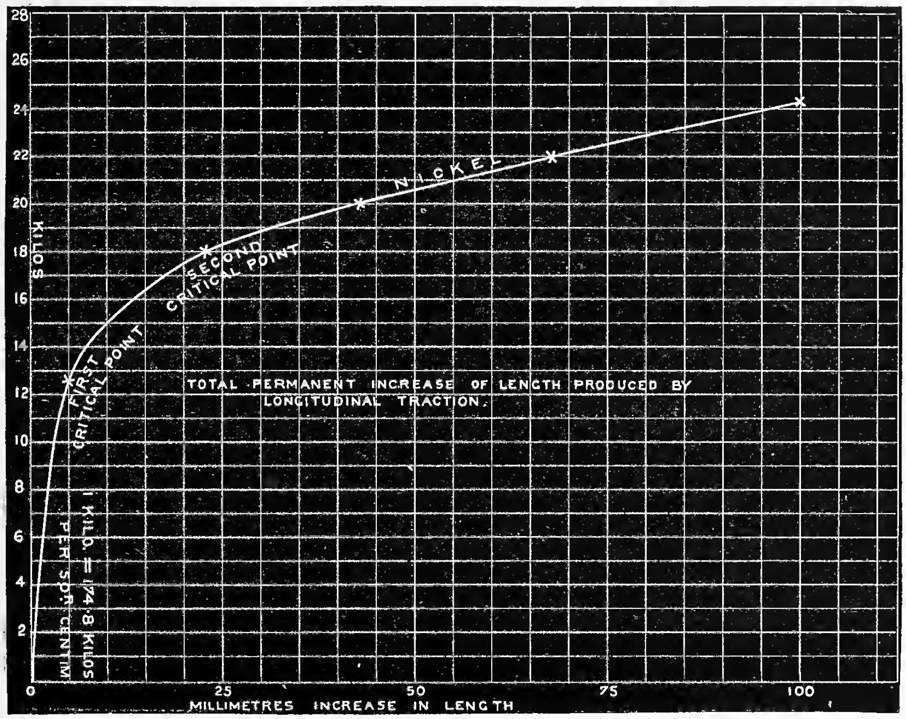
We have seen that the electrical resistance of nickel is altered in a peculiar manner by temporary longitudinal traction, and we might expect, therefore, that the effect of permanent extension would perhaps be different in the case of this metal, both in character and extent, from what it is with other substances. This is found to be so, and Table X. shows that the total permanent alteration of resistance produced by permanent extension is in the first instance of the nature of a decrease, but that, after a certain load has been reached, the resistance begins, as in other metals, to increase. The curves in Table X. are drawn with their abscissæ to represent the alteration of resistance—for the lower one on a scale of 1 division of the platino-iridium wire to the millimetre, and for the upper one on a scale of 25 divisions of the platino-iridium wire to the millimetre.

Table X.—Curve showing the total permanent alteration of electrical resistance produced by longitudinal traction from 10 to 24½ kilogs., and curve showing the permanent total alteration of electrical resistance produced by longitudinal traction from 0 to 14 kilogs.



In both curves the ordinates represent the number of kilogrammes on the wire to a scale of ·2 kilog. to the millimetre. The lower curve which represents the permanent alteration of resistance produced by loads up to 14 kilogs., shows that the resistance decreases to an extent which is nearly proportional to the load up to 6 kilogs. ; here a sudden leap is made of rapid decrease of resistance, but on reaching the next load, 8 kilogs., the curve turns, and just at the 12th kilog. there occurs a sudden and rapid increase of resistance. The upper curve which starts from the 10th kilog. shows that the rapid increase begun at the 12th kilog. is continued to an extent which is nearly proportional to the load up to the breaking point of the wire. The nature of these curves would seem to show critical points at the 6th, 8th, and 12th kilogs., but as the effect on the *specific resistance* of this metal was found to be very great in comparison with that of the other substances, it was evident that the critical points as understood to mean points of sudden increase of *length* compared with the load producing it could not safely be deduced from the curve showing the total alteration of resistance. The critical points have therefore been deduced from direct observations of the permanent increase of length produced by each load, and the results of these observations are shown in the curve of Table XI. This curve has its abscissæ representing the permanent increase of length on a scale of ·625 millim. of increase of length to the millimetre and its ordinates the load in kilogrammes on a scale of ·2 kilog. to the millimetre. From the observations of increase of length the first critical point was determined to be at the load 2200 kilogs. per square centimetre and the second at the load 3100 kilogs. per square centimetre.

Table XI.—Total permanent increase of length produced by longitudinal traction.



The Influence of Time on the Amount of Permanent Increase of Resistance or of Length which can be Produced by Longitudinal Stress.

It is well known to all who have made investigations in the subject of elasticity, that the permanent increase of length which can be produced by a given amount of longitudinal stress depends largely upon the time during which the stress is allowed to act, and as it was expected that experiments on the permanent increase of resistance would throw some light on this influence of time, several trials were made with a view of ascertaining whether a wire will, when under the action of considerable stress, show greater and greater increase of length until it breaks, or whether the time-curve of its increase of length will be asymptotic.

Experiment XXVI.

A piece of copper (3) having been loaded for some time with six measures of water was still further loaded with one measure, this being allowed to enter very slowly into the pail attached to the wire; the current from a DANIELL'S element was kept flowing through the "bridge" circuit, and the alteration of resistance observed by noting the position of the light on the scale.

Time in minutes after the completion of the loading.	Increase of resistance* for each minute in terms of the deflection of the light in scaled divisions.	Difference of increase for consecutive minutes.
1	45	. .
2	31	14
3	25	6
4	24	1
5	18	6
6	16	2
7	16	0
8	15	1
9	17	−2
10	10	7
11	13	−3
12	14	−1
13	13	1
14	7	6
15	5	2
16	8	−3
17	10	−2
18	8	2
19	5	3
20	7	−2

From the last experiment we learn that, though the wire had been so carefully loaded and the room free from vibrations of any sort, the increase of resistance did not, *shortly after loading*, take place smoothly, but the difference of increase of resistance for consecutive minutes became alternately greater and less.

In the next experiment the increase of resistance is measured by the number of divisions of the platino-iridium wire through which it was necessary to move the sliding-piece in order to restore the balance, and the times recorded are those taken to produce the increase of resistance.

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* Each scale-division here represents an alteration of resistance equal to about ten-millionths of the whole.

Experiment XXVII.

The same wire as in the last experiment was allowed to run down for a few minutes longer, and then the following observations were made :—

Increase of resistance in divisions of the platino-iridium wire.	Time in minutes.	Ratios of consecutive times.
1st . . 40	13·0	. .
2nd . . 40	19·0	1·462
3rd . . 40	24·0	1·262
4th . . 40	30·0	1·250
5th . . 40	41·8	1·394
. . . . .	. . . .	. . . .
. . . . .	. . . .	. . . .
11th . . 83·40	415	. .
Mean value of ratios = 1·342.		

It is to be noticed that here the increase of resistance takes place more regularly; and though the ratios of consecutive times are not very constant, it would seem that the times form a geometrical progression, since the time for the last increase of resistance, as calculated from the formula $t=13\times1{\cdot}342^{11{\cdot}83}$, where t denotes the time, is 422 minutes—a number agreeing sufficiently well with the observed time.

The wire was afterwards loaded more and more, and the values of the ratios of consecutive times, as determined in the same manner as in the last experiment, are recorded in Table XII.

TABLE XII.

Resistance of the wire in terms of the comparison-wire.	Total load on the wire in measures of water, each measure = 5·825 kilogs.	Calculated load in kilogs. per square centim.	Percentage value of the increments of resistances represented by 40 divisions of the platino-iridium wire.	Ratios of the consecutive times required by the loads to produce equal increments of resistance.											Mean values of ratios.	Mean values of ratios calculated for a percentage increment of resistance = ·1.
1·056	9·10	1448	·0314	1·24	2·00	1·21	1·33	1·29	1·17	1·33	1·50	1·27	1·24	1·22	1·35	2·572
1·080	9·75	1570	·0311	1·33	1·38	1·32	1·35	1·08	1·21	1·18	1·30	1·20	1·24	..	1·26	2·097
1·120	10·75	1759	·0305	1·10	1·21	1·07	1·23	1·14	1·17	1·14	1·25	1·18	1·13	1·25	1·17	1·721
1·177	11·75	1971	·0298	1·12	1·15	1·13	1·16	1·08	1·12	..	..	..	..	..	1·13	1·492

We learn from Table XII. that the ratios of consecutive times become more and more constant as the load on the wire is made larger and larger, and therefore that the velocities of increase of resistance for small equal increments of resistance form a geometrical progression.

In order to make the mean values of the common ratios comparable with each other, the common ratios calculated for equal increments of resistance of ·1 per cent. are recorded in the last column; the calculation was effected by raising the observed ratio in the last column but one to the power obtained by dividing ·1 by the percentage values of the equal increments of resistance given in the fourth column. For example, with a load of 1448 kilogs. per square centimetre, the common ratio of the geometrical progression found by the times taken for the load to increase the resistance by successive percentage amounts of ·0314 was found to be 1·35, therefore the ratio which would have ensued, if the times taken to increase through ·1 per cent. had been observed, would have been $1{\cdot}35^{\frac{\cdot 1}{\cdot 0314}}$ or 2·572.

Again, the loads in kilogrammes per square centimetre given in the third column are calculated from the resistances given in the first column in the following manner: let S_1 and S_2 be the sections of the wire before and after stretching, and let the corresponding resistances be R_1, R_2, and the lengths l_1 and l_2; then, $\frac{R_2}{R_1}=\frac{l_2}{l_1}\times\frac{S_1}{S_2}$, provided that there be no change in the specific resistance of the metal; also $l_1\times S_1=l_2\times S_2$, or $\frac{l_2}{l_1}=\frac{S_1}{S_2}$, provided that the stretching does not alter the density. But we shall see that neither the density nor the specific resistance is altered by stretching to any extent sufficient to introduce any appreciable error, therefore we have within a sufficiently close approximation $\frac{R_2}{R_1}=\left(\frac{S_1}{S_2}\right)^2$, or $S_2=S_1\times\sqrt{\frac{R_1}{R_2}}$. Thus the section S_1 having been determined, it is easy to ascertain the section after any amount of stretching. For example, the section of the wire last used was before stretching ·0183 square centim., and the resistances before and after the stretching were 1·056 and 1·080 respectively, therefore the section of the wire after stretching would be nearly $\cdot 0183\times\sqrt{\frac{1{\cdot}056}{1{\cdot}080}}$, and the actual load on the wire being 9·75 measures of water or 9·75 × 5·825 kilogs., the load per square centimetre would be $\frac{9{\cdot}75\times 5{\cdot}825}{\cdot 0183}\times\sqrt{\frac{1{\cdot}080}{1{\cdot}056}}$ kilogs. = 1570 kilogs. In all cases the resistances recorded in the first column are the means between the resistances observed at the commencement and end of the times during which the velocities of increase were noted, and the wire was allowed to run down for some time (about 20 minutes) before the observations of the velocities of increase commenced.

It is, moreover, evident from the last column in the table that the common ratio of the geometrical progression becomes less and less as the load becomes larger and larger, and we might expect to find some relation between the decrease of velocity of increase of resistance and the difference between the breaking-load and the load actually on the wire. A further examination of the results given in Table XII. shows that a relation does exist of an extremely simple nature; in fact, if we denote the breaking-load and the actual load on the wire by P and p respectively, the decrease per unit of the velocity of increase of resistance is inversely proportional to $P-p$. The value of P was carefully determined by loading pieces of the wire by moderate amounts at a time, and allowing each load to remain on the wire some ten minutes before each further addition, and measuring the diameter *close to the point of breakage* by means of a wire-gauge graduated to ·01 millim. The mean of several trials gave a value for P of 2625 kilogs. per square centimetre.

Now if t_1, t_2, t_3, &c., are the times taken to increase the resistance by successive equal and small amounts, the ratios $t_2 : t_1$, $t_3 : t_2$, &c., have been proved to be constant for the same value of p: let this constant be denoted by r; then the decrease per unit of the velocity of increase of resistance being $=\frac{1}{t_1}-\frac{1}{t_2}\div\frac{1}{t_1}$ will therefore $=\frac{r-1}{r}$, and accordingly if the above-mentioned relation holds good, $\frac{r-1}{r}\div(P-p)$ should be a constant for different values of p. The last column of the table gives the values of r for the loads given in the third column, and the values of $\frac{r-1}{r}\div(P-p)$ are for the four loads there recorded, ·000519, ·000496, ·000484, and ·000504, with a mean value of ·000501.

The question next arises, will the ratio $\frac{r-1}{r\times(P-p)}$ be the same for other metals as for copper? for if so we can calculate the breaking-load by merely loading the wire beyond the second critical point and observing the value of r; then, since $\frac{r-1}{r\times(P-p)}=·000501$,

$$P=p+\frac{r-1}{r\times·000501}.$$

With a view of ascertaining whether the breaking-load could be thus calculated, a series of experiments with different metals was begun, the mode of operating being similar to that just described in the case of copper, and the results are shown in the next table. In the same table are also given the ratios of the different moduli of longitudinal elasticity to the corresponding breaking-load.

TABLE XIII.

Metal.	Percentage value of the equal increments of resistance.	Actual load on the wire in kilogs. per square centim. $=p$.	Ratio of consecutive time calculated for an increase of ·1 per cent. of resistance $=r$.	Calculated breaking-load $P=p+\frac{r-1}{r\times\cdot000501}$.	Observed breaking-load in kilogs. per square centim.	Mean values of breaking-loads from observation and calculation.	Ratio of "YOUNG'S modulus" to breaking-load.
Copper (3)	·0314	1448	2·572	2666	2625	2625	$4·35\times10^2$
	·0311	1570	2·097	2612	. .		
	·0305	1759	1·721	2593	. .		
	·0298	1971	1·492	2628	. .		
Iron (1)	·2270	3311	1·081	3826	3816	3825	5·17 ,,
	. .	. .	. .	. .	3785		
	. .	. .	. .	. .	3867		
Zinc (2)	·9620	1418	1·014	1446	1468	1460	5·25 ,,
	. .	. .	. .	. .	1528		
	. .	. .	. .	. .	1426		
Platinum (1)	·1524	1737	1·131	1968	2061	2021	7·20 ,,
	·1487	1836	1·064	1956	2080		
	·1482	1946	1·029	2004	2099		
Silver (1)	·0609	1076	2·531	2281	2090	2185	3·44 ,,
	·0601	1192	2·015	2195	2272		
	·0582	1440	1·498	2099	2170		
Aluminium (1)	·0552	864	1·144	1295	1277	1293	5·19 ,,
	. .	. .	. .	. .	1304		
	. .	. .	. .	. .	1288		
							Mean $5·10\times10^2$

Observations on Table XIII.

We learn from the above table how very closely in all cases the calculated breaking-load agrees with the observed breaking-load, though the values of p and of the percentages of increase of resistance for which the times were observed varied considerably. What differences do exist are no greater than those between the values of the observed breaking load for different pieces of the same wire.

Since, also, the permanent increase of resistance produced when a wire is running down under the influence of a load is for small amounts nearly double the increase of length, we may determine the breaking load by observing the times taken by the load to produce successive equal increments of length amounting to ·05 per cent., and use the same formula as above.*

In estimating the breaking-load as has been before mentioned, the diameter of the wire was gauged in the immediate neighbourhood of the breaking-point; and this is necessary if we wish to determine the true breaking-load in kilogrammes per square centimetre, inasmuch as, however uniform in diameter, and however carefully annealed

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* It should be noticed that the value of r as used in the formula is greater than unity, and is obtained by dividing *the succeeding by the preceding time* of accomplishing any two consecutive increments of resistance or of length.

the wire may have been before stretching, the latter action is sure to diminish the diameter at some parts of the wire more than at others, and eventually the wire breaks at that part at which the greatest contraction has taken place.

If a wire could be obtained of perfect uniformity of diameter and substance, the contraction would gradually increase from each end to the centre where it would be greatest, and at this point the wire would break. It is interesting to watch this gradual increase of contraction from the two ends to the middle in the case of a test-bar of ductile iron or steel, and through the kindness of Sir JOSEPH WHITWORTH I was able to make the following experiment on such a bar of fluid-pressed steel, the specimen having been selected as suitable for this purpose in consequence of its great ductility.

The bar had a total length of 6 inches before stretching, but a screw was formed, 2 inches in length (see fig. 16), at each end for the purpose of securing the bar in the framework of the hydraulic press employed to stretch it. The diameter of the bar between the two screws was before testing ·7979 inch, and after each stretching the bar was removed from the press and the length and diameter re-determined: the results obtained are recorded in the next table.

Fig. 16.

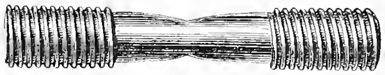

TABLE XIV.

Stress in tons per square inch.	Total permanent increase of length measured after the removal of the stress.	Total permanent contraction of diameter at the centre.	Ratio of permanent contraction of diameter to permanent elongation.
20	·1060	·025	·236
22	·1560	·030	·192
24	·2380	·050	·211
25	·3010	·053	·176
26	·5175	·098	·190
26 Second time	·7850	·228	·294
23½ Broke	·8900	·323	·363

It will be noticed that the ratio of permanent contraction of diameter to the permanent elongation is as sensibly constant as could have been expected from the way in which the diameter had to be measured, namely, by calipers,* until the

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* WHITWORTH's measuring-machine would have been employed for this purpose as it was for measuring the increase of length, had not the curving of the bar produced by the stretching rendered such an instrument unsuitable.

breaking-load is nearly reached, when the contraction begins very suddenly to increase.

Fig. 16 is from a photograph of a similar specimen, broken after stretching, and shows the gradual contraction from the ends to the centre and the sudden increase of contraction at this point. In measuring the diameter of the broken wires the gauge was placed close to the place of this sudden contraction.

It appears also from Table XIII. that there is a certain amount of relationship between the breaking-load and the modulus of longitudinal elasticity; platinum, however, having too small a breaking-load and silver too high in comparison with the modulus of elasticity when contrasted with the other metals.

It should be remembered that in the case of the former metal the loads at the critical points were less in proportion to the elasticity than was the case with the other metals, and if we regard the breaking-point as a third critical point, it would appear that these three critical points are in the case of well annealed wires related to each other roughly in the ratios of $1 : 1\frac{1}{2} : 2$ or of $2 : 3 : 4$.

The Permanent Alteration of Specific Resistance caused by Stress.

Traction.

We have seen that the alteration of specific resistance which can be temporarily produced by longitudinal traction is very small, but then the temporary lengthening is small also. Now we can with wires which have been well annealed produce, in most cases, a far more considerable permanent increase of length, and it seemed desirable to ascertain whether there would be a correspondingly large change in the specific resistance of the substance.

Three different modes of experimenting were tried:—in the first, the wire to be tested and the comparison-wire were clamped into the blocks already described, and were then stretched at full length on the floor and side by side; the short block, into which one end of the wire to be stretched was clamped, was placed behind two stout screws, which were screwed into the floor to about one half of their length, and about one inch apart; the wire passed between these screws, and the other end of it which was clamped into the longer brass block was pulled by hand until it was quite straight· a mark on the block which traversed a wooden scale divided into millimetres, and secured to the floor by screws in a position parallel to the length of the wire, serving to measure the length of the wire. It was ascertained that after a little practice the readings could be depended upon to within at least 1 millim. or about $\frac{1}{2400}$ of the whole length. After thus measuring the length, the wires were placed in the air-chamber already described, and after the usual precautions, the ratio of their resistances tested; a weight was now placed on the pulley for three minutes and then removed, when the wire was permitted to remain free from stress for 10 minutes, after which time a new determination of the ratio of the resistances of the wires was made.

The wires were now removed from the air-chamber and the one which had been stretched was remeasured. The same processes were repeated after greater and greater loads until finally the wire was broken. With some of the wires the specific gravity was determined after each stretching, but as the alteration of density was found to be very small with most of the substances, the specific gravity was determined before the stretching had commenced, and again after the wire had been considerably elongated; the density for intermediate amounts of stretching was calculated on the assumption that the change of density is proportional to the increase of length.

In the second method the wires were kept during the whole period of observation in the air-chamber; the hook on the pulley was connected by a fine copper wire, with a brass sliding piece provided with an index and capable of free vertical motion up and down a wooden scale. The lower extremity of the sliding piece was slightly weighted, so as to keep the fine copper wire, which had been previously stretched very nearly to breaking, perfectly straight. The hook on the pulley was also connected by a chain with a lever which served to elongate the wire. On commencing the experiments the weight of the lever was removed from the wire and the resistance of the latter was determined; the position of the index was then noted and afterwards the lever was used to produce the required extension, when again the wire was relieved from stress and after a few minutes the resistance and length of the wire were redetermined. In this way an alteration of length not exceeding $\frac{1}{5000}$th of the whole could be readily measured, but as it was ascertained that in the case of certain wires it was necessary *to remove entirely even the slightest constraint*, such for instance as would be caused by the small weight of the pulley or of the sliding piece, before they would attain in a sufficiently short time their ultimate resistance, a third method was adopted as follows:—

The length of the wire to be examined having been measured as in the first method, it was placed, together with the comparison-wire, at full length in a long wooden box made for the purpose, the two wires having previously been wrapped as usual in paper or surrounded by caoutchouc tubing, and after a sufficient time had elapsed, usually about 15 minutes, to enable them to assume their ultimate ratio of resistance, this latter was determined. The wires were then removed from the box, and the one to be tested stretched as in the first method to a certain extent; they were then replaced, and after the proper time their resistance ratio redetermined. At each removal of the wires from the box the connexions with the other parts of the bridge had to be disturbed; but it was ascertained, as indeed might have been expected from the mode of experimenting,* that this did not in the least affect the value of the ratio of the resistances of the two wires. A few examples will suffice to show the nature of the results obtained.

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* I have frequently removed and replaced the connexions of the wires without causing any alteration of resistance which would amount to $\frac{1}{20000}$th of the whole.

Experiment XXVIII.

A wire of annealed silver, 8 feet in length and ·085 centim. in diameter, stretched by loading.

Load in kilogs.	Total increase of length per cent. produced by the load = A.	Total increase of specific resistance per cent. produced by the load = B.	Ratio of B : A.
5	1·15	. .	. .
6	3·64	+·034	·0094
$6\frac{3}{4}$	7·28	+·274	·0376
$7\frac{1}{4}$	9·38	+·246	·0262

Experiment XXIX.

A wire of annealed copper, 8 feet in length and ·095 centim. in diameter, stretched by hand.

Total increase of length per cent. produced by stretching = A.	Total increase of specific resistance per cent. produced by stretching = B.	Ratio of B : A.
2·42	+0·395	·163
4·84	+0·649	·134
11·33	+0·373	·030
23·36	+0·296	·013

Experiment XXX.

A wire of annealed iron, 8 feet in length and ·092 centim. in diameter, stretched by means of a lever, *the stress produced by the weight of the lever being allowed to remain after the permanent extension had been completed.*

Total increase of length per cent. produced by stretching = A.	Total increase of specific resistance per cent. produced by stretching = B.	Ratio of B : A.
3·75	+·129	·034
7·96	·224	·028
11·85	·384	·032
15·74	·640	·041

Experiment XXXI.

A wire of annealed iron, 8 feet in length and ·092 centim. in diameter, stretched by hand.

Total increase of length per cent. produced by stretching = A.	Total increase of specific resistance per cent. produced by stretching = B.	Ratio of B : A.
5·74	—·105	—·0183
7·51	—·143	—·0190
9·49	—·156	—·0164
11·56	—·177	—·0153
16·36	—·150	—·0091

Experiment XXXII.

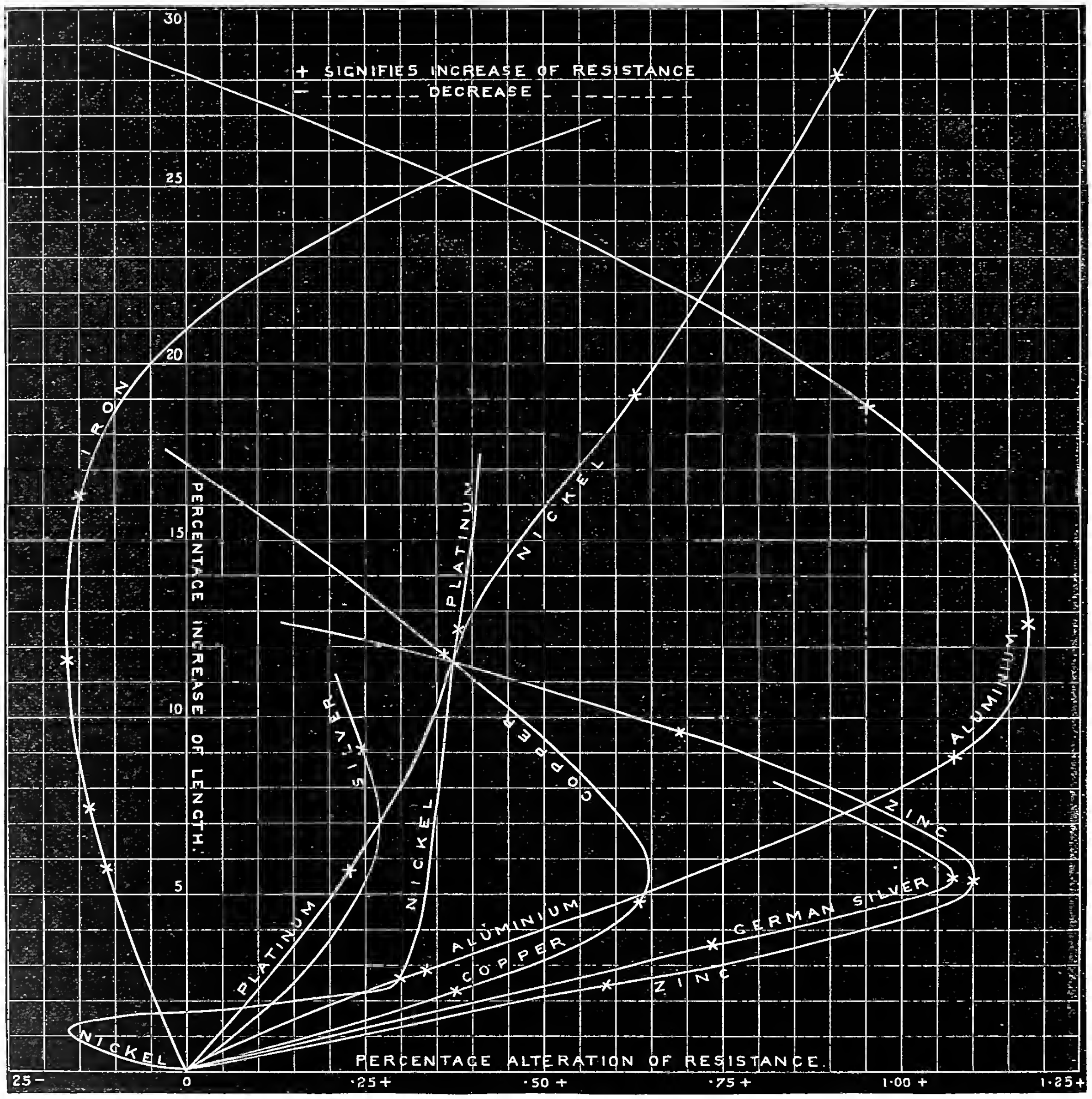
A wire of annealed nickel, 8 feet in length and ·106 centim. in diameter, stretched by loading.

Load in kilogs.	Total increase of length per cent. produced by the load = A.	Total increase of specific resistance per cent. produced by the load = B.	Ratio of B : A.
12	0·32	—0·757	—2·366
18	1·67	+1·260	0·755
20	3·06	+1·514	0·494
22	4·82	+2·052	0·426
24½	7·14	+3·634	0·509

It will be observed that of the four metals—silver, copper, iron and nickel—the two former are at first increased in specific resistance, and that the increase only continues up to a certain amount of stretching when, having reached a maximum, it begins to decrease; whilst with the two latter the first alteration of resistance is in the way of a decrease, which also after attaining a maximum begins to diminish, and with nickel there is soon a comparatively large increase. Of nine metals which have been examined, iron and nickel are the only two which in the first instance show a decrease of specific resistance, whilst the remaining seven are similar to copper and silver in respect to the change of specific resistance due to permanent extension. In the following table the effects of different amounts of permanent longitudinal extension on the specific resistance of copper, zinc, German-silver, nickel, platinum, iron, aluminium and silver are shown by a series of curves. The abscissæ of these curves represent the percentage alteration of specific resistance, and the ordinates the percentage alteration of length, the former on a scale of ·0125 percentage alteration to the millimetre, and the latter on a scale of ·25 percentage alteration to the millimetre;

with nickel, however, the alteration of resistance is so very much larger in proportion to increase of length than is the case with the other metals, that the abscissæ are represented on a scale of ·05 percentage of alteration to the millimetre, and the ordinates on a scale of ·0625 percentage alteration to the millimetre.

Table XV.—Curves showing the permanent alteration of specific resistance produced by permanent longitudinal extension.



Remarks on Table XV. and Experiments XXVIII. to XXXII. inclusive.

It should be stated that before making any observation of the resistance or length, the wire was stretched about 2 per cent. of the original length in order to render it sufficiently straight, and that the resistance and length after this stretching are taken as the starting points from which the percentage alterations are measured; in the cases, however, of German-silver and nickel this was not done, as they were deemed to be sufficiently straight without stretching, and, moreover, not capable of much extension before breaking.

The curves are evidently of parabolic shape, except perhaps in the case of nickel, whose behaviour seems to be quite abnormal. With this metal the curve after passing to the left, showing that the specific resistance is diminished by extension, takes a sharp turn round to the right, and, after passing almost vertically upwards, makes another bend afterwards, proceeding almost in a straight line. Experiment XXXII., as well as the curve for nickel also, show how much larger is the alteration of specific resistance, whether decreasing or increasing, in comparison to the alteration of length, than is the case with any of the other metals.

By comparing Experiment XXX. with Experiment XXXI., we see the effect of leaving a load on the wire after the extension has taken place; the decrease of specific resistance of iron produced by a moderate amount of extension is in this case replaced by an increase. Now we have seen in Part I. that the wire will not quite assume the length which it would have if entirely relieved from stress, but the change in resistance is much larger than can be accounted for by any change of dimensions, and in some instances the effect of a much slighter restraint than was produced by the stress of the lever sufficed to make a very appreciable difference in the specific resistance. The silver wire formed a remarkable example of the kind; with this wire the specific gravity was determined after each stretching, and the resistance was measured both before and after the former operation, the only load left on the wire being that of the pulley, which produced a stress of a quarter of a kilogramme on each half of the wire. The specific resistance after the different amount of stretchings recorded in Experiment XXVIII. was decreased ·03, ·27, ·28 and ·2 per cent. by merely removing the pulley and taking the wire down for the purpose of weighing it in water. These alterations are, it is true, not absolutely large, but if we compare them with the total alteration of specific resistance given in the third column* of the experiment, we see that they are relatively very considerable. Moreover, in this case not the slightest alteration in the length of the wire caused by the removal of the pulley could be detected; neither was the change brought about by the weighing in water, since a similar alteration was caused when the pulley was simply removed and replaced without any such weighing; nor, again, was it due to the restitution of conductivity,

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* The values given in this column are calculated from the resistance determined *after the pulley had been removed and then replaced.*

which we shall learn presently that rest causes after strain, since the resistance was decreased by the removal and replacing of the pulley in one experiment 14 hours after the permanent extension had been made; a much longer time, of course, elapsing in this case than was required in the above mentioned operations.

Again, though the curves are not capable of showing any sudden changes in the ratio of the alteration of the specific resistance, and the extension at the two critical points before alluded to, inasmuch as with most metals these points occur when a comparatively small amount of extension has taken place, yet it will be seen from Experiment XXVIII. that with the increase of length caused by the load of $6\frac{3}{4}$ kilogs. there is a sudden increase of the ratio B : A, and this load corresponds very closely with that at the second critical point of silver.

Hammering.

W. THOMSON, in 1857, experimented on the effects of hammering and permanent extension on the electrical conductivity of copper, and though no actual numbers are given in his paper, states* that "the greatest degree of brittleness produced by tension does not alter the conductivity of the metal by as much as one-half per cent." He, moreover, adds: "A similar experiment showed no more sensible effect on the conductivity of copper wire to be produced by hammering." The foregoing experiments, it will be seen, fairly bear out THOMSON's statement with reference to the small amount of alteration produced in the specific resistance of copper by longitudinal extension, and show a still smaller change in the resistances of silver and platinum. But since these experiments at the same time showed that the alteration after increasing to a maximum in one direction began to decrease, and in certain cases finally set in in the opposite direction, it seemed desirable to ascertain whether hammering would produce like effects on the specific resistance.

The third of the methods used in determining the change of specific resistance by extension was here employed, except that the wires were now lengthened by hammering them transversely. The last process was accomplished rather by a great number of comparatively small blows than by a less number of large ones, so as to hammer the wire as uniformly as possible throughout its whole length. The following are examples of the results arrived at:—

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* Proc. Roy. Soc., vol. viii., p. 553.

Experiment XXXIII.

A wire of annealed copper, 8 feet in length and ·095 centim. in diameter, hammered transversely throughout its entire length.

Total increase of length per cent. produced by hammering =A.	Total increase of specific resistance per cent. produced by hammering =B.	Ratio of B : A.
1·89	+·017	·009
3·11	+·174	·056
5·95	+·009	·002
9·66	−·209	−·022
16·76	−·530	−·032

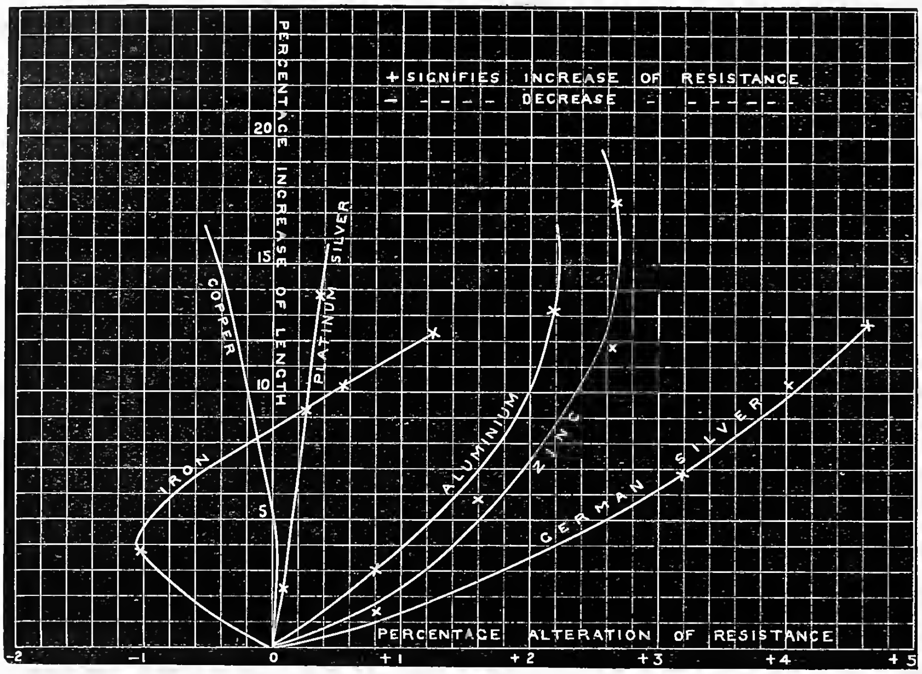
Experiment XXXIV.

A wire of annealed iron, 8 feet in length and ·092 centim. in diameter, hammered transversely throughout its entire length.

Total increase of length per cent. produced by hammering =A.	Total increase of specific resistance per cent. produced by hammering =B.	Ratio of B : A.
2·04	−0·082	−·040
3·94	−1·035	−·263
10·27	+0·554	+·054
12·31	+1·283	+·105

In the next table will be found a series of curves showing the change of specific resistance produced by hammering on the metals iron, copper, zinc, aluminium, German-silver, and platinum-silver. The abscissæ showing the percentage alteration of specific resistance are on a scale of ·05 per cent. for 1 millim., and the ordinates representing the increase of length per cent. are on a scale of ·25 per cent. for 1 millim.

TABLE XVI.—Curves showing the alteration of specific resistance produced by hammering.



Observations on the curves in Table XVI.

Experiments XXXIII., XXXIV., and Table XVI. show that the effect of hammering on the specific resistance is of a somewhat similar character to that of permanent extension. With all the metals, except iron, the specific resistance is at first increased, and this increase, after reaching a maximum, begins to diminish, but with iron the first effect is decrease of resistance which also, after attaining a maximum as the hammering is carried to a greater and greater extent, begins to diminish until finally there is a comparatively large increase of specific resistance.

The changes produced, however, by hammering, though similar in kind to those produced by longitudinal extension, are very different in amount, and a comparison of the two sets of curves and the scales on which they are formed shows that the alterations in the former case are very much greater than those in the latter.

The neutral points also, *i.e.*, the points where the curves cut the axis of ordinates, are different in the two tables, being for copper and iron much higher for the extension than for the hammering, but with zinc and German-silver lower for the extension than the hammering.

Torsion.

As it seemed desirable to supplement the observations of extension and hammering with others on torsion, with the view of ascertaining whether the strain caused by twisting a wire beyond the limits of elasticity would at all resemble the effect produced on the specific resistance by strain set up by the two former processes, a few experiments were made in which the wires having been secured at one end and stretched sufficiently to make them tight, were twisted more and more until they broke.

Experiment XXXV.

A wire of annealed zinc, 8 feet in length and ·095 centim in diameter, twisted.

Amount of torsion; the torsion of a complete revolution in a length of 1 centim. taken as unit =A.	Total increase of specific resistance per cent. produced by the torsion =B.	Ratio of B : A.
·083	+·269	3·24
·167	+·546	3·27
·250	+·376	1·27

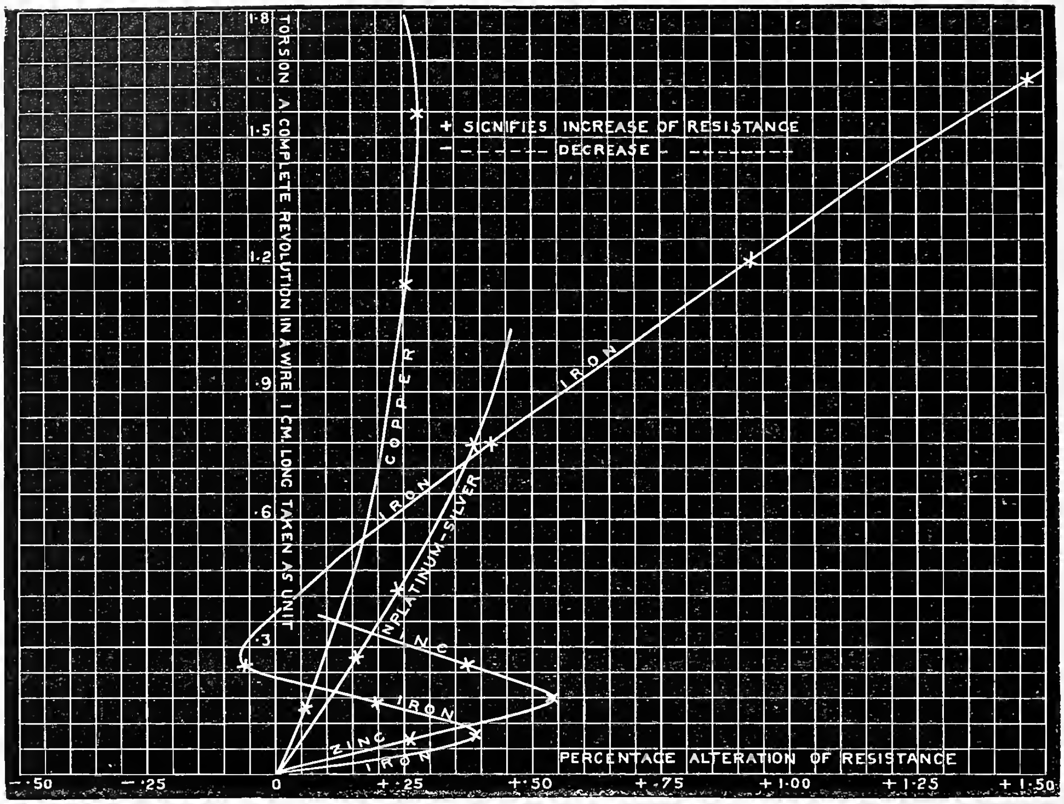
Experiment XXXVI.

A wire of annealed iron, 8 feet in length and ·092 centim. in diameter, twisted.

Amount of torsion; the torsion of a complete revolution in a length of 1 centim. taken as unit =A.	Total increase of specific resistance per cent. produced by the torsion =B. – signifies decrease of specific resistance.	Ratio of B : A.
·080	+ ·394	+4·93
·160	+ ·195	+1·22
·240	– ·053	–0·22
·320	– ·076	–0·24
·800	+ ·413	–0·52
1·200	+ ·928	–0·73
1·600	+1·454	–0·91

In Table XVII. are drawn a series of curves showing the changes produced by permanent torsion in the specific resistance of zinc, iron, copper, and platinum-silver. The abscissæ in these curves represent the percentage alteration of resistance on a scale of ·0125 to 1 millim., and the ordinates representing the torsion in terms of the torsion of a complete revolution in a length of 1 centim. of the wire taken as unit on a scale of ·015 to 1 millim.

Table XVII.—Curves showing the permanent alteration of specific resistance produced by permanent torsion.



Remarks on Table XVII.

The effect of torsion on the specific resistance is evidently of a similar nature to that of the effect of permanent extension and hammering, but the amount of alteration shown at the turning point of the curves is in all cases much less than that observed at the turning points of the curves exhibiting the results of hammering and extension. In the case of iron, the first torsion applied increases the specific resistance, but further torsion acts in the same direction as the strain caused by hammering and extension, *i.e.*, diminishes the specific resistance up to a certain point, but beyond this point increases it. It may be that the increase produced in iron by the first few turns is due to magnetic influence on the resistance, for as Thomson has observed,* the electric current instead of flowing rectilineally along the wire would flow in helical lines, and would therefore increase the resistance by longitudinal magnetization.†

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* W. Thomson, "Electrodynamic Qualities of Metals," Phil. Trans. 1879, Part I., § 229.

† W. Thomson, "Electrodynamic Qualities of Metals," Phil. Trans. 1856. Bakerian Lecture, § 146, and H. Tomlinson, Proc. Roy. Soc., June 17, 1875. [Note added May, 1882.—I have since found that *very slight* extension increases the specific resistance of iron, and am therefore inclined to reject the above hypothesis, and to believe that *very slight* strain of any kind increases the specific resistance.]

It will be noticed also that the curves like those of Tables XV. and XVI. are of parabolic shape.

THE ALTERATION OF SPECIFIC GRAVITY PRODUCED BY PERMANENT EXTENSION, HAMMERING, AND PERMANENT TORSION.

In calculating the values of the specific resistance after the wire had been subjected to any one of the above-mentioned processes, it was in the first instance assumed that there was no change in the specific gravity of the substance, and afterwards a correction applied for such change. The specific gravity of the wires in the annealed condition having been determined, the change wrought by extension, hammering, or torsion carried to an extent which was about three-fourths of that which the substance would bear without rupture was determined by a very delicate balance, and though it was not possible to dislodge any air bubbles which might be attached to the metal when immersed in water by boiling for fear of partially annealing the specimens, these bubbles were brushed off very carefully and considerable pains were taken in the weighings. A very fine platinum wire was used for the purpose of suspending the substances in water, and the vessel employed for holding the water was of considerable size so as to avoid the necessity of bending the wire to any great extent. The coils in which the wire under examination was wound, when weighed, were kept together by fine copper wire, and the weight of this in air and water together with that of the fine platinum wire was from time to time determined. The next experiments show that the change in specific gravity is roughly proportional to the amount of strain, provided this strain is not continued up to the point of rupture.

Experiment XXXVII.

Wire of annealed silver, 8 feet in length and ·090 centim. in diameter, stretched by loading.

Increase of length per cent. = A.	Decrease of specific gravity per cent. = B.	Ratio of B : A.
1·15	·0124	·011
3·64	·0461	·013
7·28	·0892	·012
9·38 (broke)	·1465	·016

Experiment XXXVIII.

A wire of annealed copper, 3 feet in length and ·095 centim. in diameter, twisted.

Number of complete revolutions of torsion = A.	Decrease of specific gravity per cent. = B.	Ratio of B : A.
400	1·16	·0029
600	1·66	·0028
800	2·13	·0027

In Table XVIII. will be found the extent of change in the specific gravity of the different metals produced by a given amount of stretching, hammering, and twisting.

TABLE XVIII.

Name of metal.	Percentage alteration of specific gravity attending a permanent increase of length of 1 per cent. produced by stretching.	Percentage alteration of specific gravity attending a permanent increase of length of 1 per cent. produced by hammering. + signifies increase of specific gravity.	Percentage alteration of specific gravity attending a permanent torsion equal to that of 1 complete revolution in a length of 1 centim. + signifies increase of specific gravity.
Platinum	−·0620	. .	. .
German-silver . .	−·0612	−·0136	. .
Nickel	−·0510	. .	. .
Zinc	−·0509	−·0146	−·01212
Iron	−·0203	−·0215	−·00283
Silver	−·0156	. .	. .
Aluminium . . .	−·0082	−·0183	. .
Copper	−·0178	+·0065	−·00252
Platinum-silver .	. .	+·0159	+·00190

In the last table a − sign signifies a decrease and a + sign an increase of specific gravity.

The specific gravity of all the annealed metals here examined is decreased by permanent extension.*

Hammering also decreased the specific gravity of all the metals subjected to this process except copper and platinum-silver; with these metals the specific gravity was slightly increased.

Permanent torsion decreased the specific gravity of copper, zinc, and iron, and increased that of platinum-silver.

Of the three processes, torsion produced the greatest maximum change in the specific gravity, thus it will be seen from Experiment XXXVIII. that the specific

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* This apparently is not always the case, as Sir W. THOMSON mentions in his article on "Elasticity," 'Brit. Encyc.,' p. 1, that a certain specimen of copper wire annealed in hot sand had its density *increased* more than 1 per cent. by longitudinal extension.

gravity of copper was decreased by the torsion more than 2 per cent.—a large amount considering the small alteration[*] which extension and hammering can produce.

The results recorded in the last table enabled the previously mentioned correction for change of specific gravity to be made in calculating the specific resistance.

For if Δ_1 and Δ_2 be the specific gravities respectively before and after stretching, hammering, or twisting, and S and x be respectively the specific resistances uncorrected and corrected for change of specific gravity, $x=\mathrm{S}\times\frac{\Delta_1}{\Delta_2}$.

Now the table furnishes the means of determining

$$\frac{\Delta_1-\Delta_2}{\Delta_1}=\alpha \text{ say};$$

and

$$\frac{\Delta_1}{\Delta_2}=\frac{1}{1-\alpha}=1+\alpha \text{ very nearly,}$$

since α is very small.

Therefore

$$x=\mathrm{S}\times(1+\alpha).$$

The correction though small was applied in all cases.

Effect of Cooling Suddenly on the Specific Resistance of Steel.

We have seen in Part I. that the effect of suddenly chilling steel heated to a high temperature produces a somewhat similar effect on the elasticity to that of excessive permanent extension, and it was concluded to be highly probable that whether the distance between the molecules be increased by mechanical strain or by the strain caused by sudden cooling, the elasticity in the direction of the line of separation of the molecules diminishes to a minimum as the separation increases, and then begins to increase. Now Barus[†] has proved that the specific resistance of steel *increases* continuously with its hardness, but Barus's experiments were made with steel heated at or above a visible red, and as the strain produced by extension, hammering, and torsion had been shown to produce up to a certain point *decrease* of resistance, it seemed a matter of some interest to ascertain whether heating the steel to a lower temperature than that of dull red and then cooling slowly would not also have the effect of decreasing the specific resistance. The following experiment was therefore tried:—

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[*] Not so much as 1 per cent. in any case which I have examined.

[†] Phil. Mag., November, 1879.

Experiment XXXIX.

A piece of annealed piano-steel wire, 8 feet in length and ·083 centim. in diameter, cooled suddenly by plunging it into cold water after it had been heated to various temperatures.

Condition.	Number proportional to specific resistance.*
Soft.	·91168
Heated and cooled, hiss not audible	·91095
Heated and cooled, hiss just audible	·91094
Heated below dull red but hiss very audible on cooling .	·91118
Heated to dull red and cooled	·91891

In this last experiment the wire was tempered when coiled, the coils being held together by wrapping fine iron wire round them, and passing a burner rapidly round the coils until it was supposed that the requisite temperature had been applied, when the wire was suddenly plunged into water at a temperature of 10° C.

This experiment, though rough as regards the mode of tempering, shows plainly that the specific resistance is *decreased* by the sudden cooling until the tempering is performed at some temperature under dull red, when the specific resistance begins to increase. It will be shown also in Part IV. that the thermo-electric properties of steel are affected in precisely the same manner, that is, that tempering beyond a dull red temperature produces opposite effects to tempering under a dull red temperature.

THE RECOVERY OF ELECTRICAL CONDUCTIVITY PRODUCED BY TIME IN WIRES WHICH ARE IN A STATE OF STRAIN.

In all the experiments described in this Part it was observed that when the wires had been subjected to stresses of any kind, whether purely mechanical or otherwise, which sufficed to produce permanent strain, they invariably gained in electrical conductivity when allowed to rest. The amount of decrease of resistance produced by rest varied however considerably with different metals, being very conspicuous in German-silver and hardly perceptible with platinum-silver.

Table XIX. shows the influence of rest in restoring the electrical conductivity of wires of German-silver, copper, iron, zinc and platinum-silver after hammering so as to increase the length about 15 per cent.; the observations being commenced 20 minutes after the hammering had been completed.

The abscissæ of the curves represent the decrease of resistance on a scale of ·0085 per cent. to 1 millim., and the ordinates on a scale of one hour to the millimetre.

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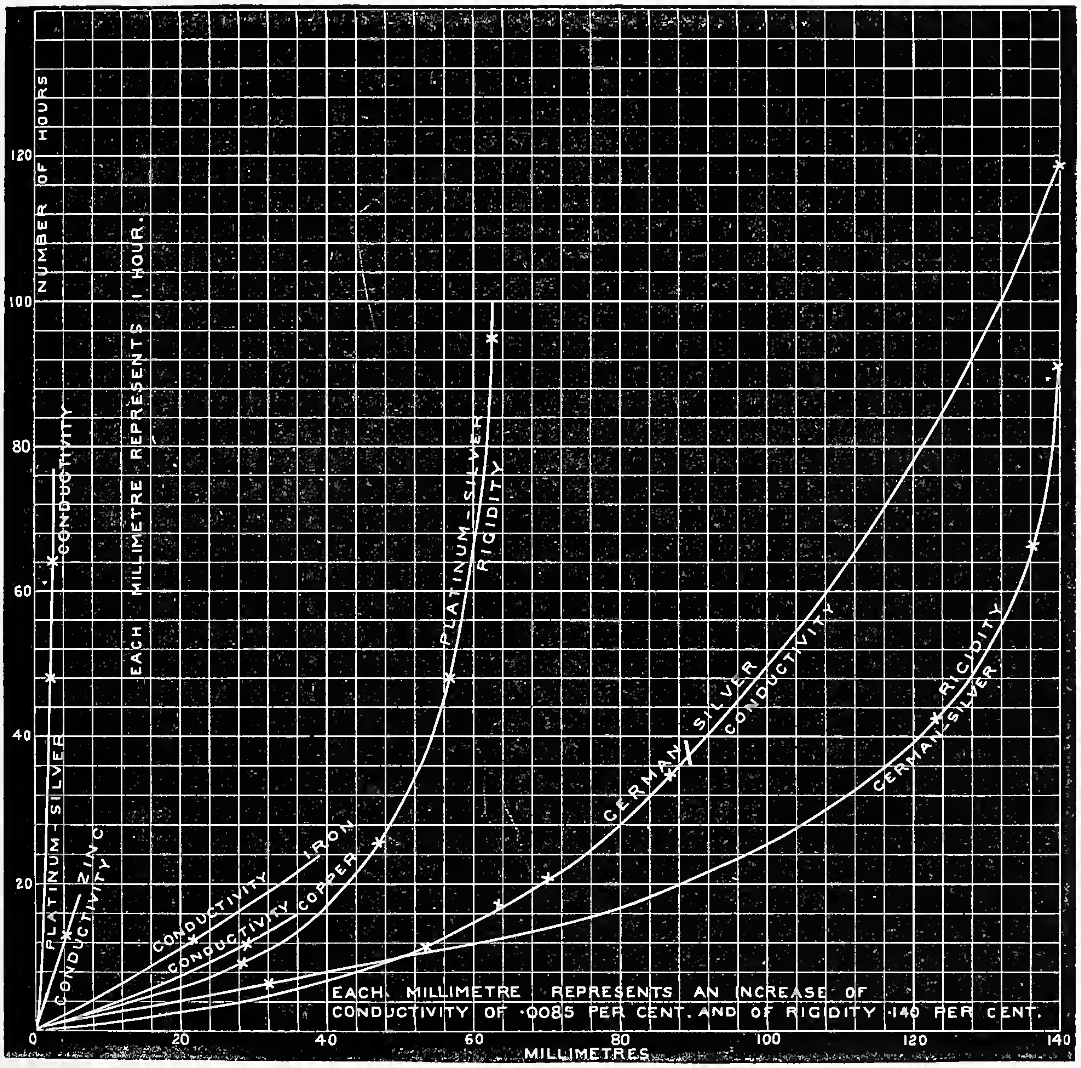
* The resistance of the comparison-wire is here taken as unit.

The tables also show a curve representing the restitution of torsional rigidity in German-silver after hammering, the abscissæ representing the increase of rigidity on a scale of ·0085 per cent. to 1 millim. and the ordinates on the same scale as for the other curves.

TABLE XIX.—Curves showing the recovery of electrical conductivity and of torsional rigidity produced by time in the case of wires which have been hammered transversely.



This table shows the marked difference between German-silver and platinum-silver in respect to the restitution of conductivity produced by rest on these metals when in a state of strain, and this large difference is not confined to strain produced by hammering, but was found to exist also in strain produced by extension, torsion, and wire-drawing, and when taken in combination with the results given in Tables XV., XVI., and XVII. on the alteration of specific resistance produced by strain, shows most conclusively the superiority of platinum-silver to German-silver in making standard resistance coils.

We have seen also in Part I. that rest, materially in some cases and to a certain extent in all, increases the elasticity of metals; now German-silver wire furnishes a conspicuous example of this; and the curve in the table representing the increase of torsional rigidity produced by rests, shows plainly that the restitution of elasticity and electrical conductivity go hand in hand. And this circumstance, when taken in conjunction with the fact that there is no change in the dimensions of the wire which would at all account for the increase of elasticity or conductivity, evidently teaches us that when we can increase the elasticity without altering the mean molecular distance, we at the same time increase the electrical conductivity.

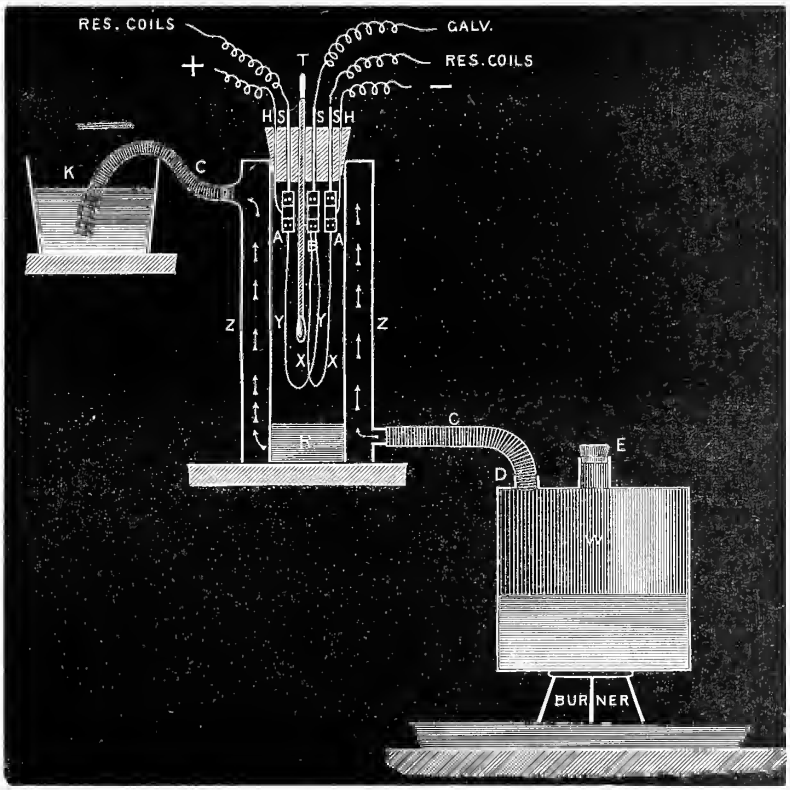
THE INFLUENCE OF PERMANENT STRAIN ON THE CHANGE OF ELECTRICAL CONDUCTIVITY PRODUCED BY ALTERATION OF TEMPERATURE.

Permanent extension, hammering, and torsion.

The previous observations on the change of specific resistance produced by extension, hammering, and torsion having given rise to the suspicion that these processes caused the wires to alter their susceptibility to change of resistance from change of temperature, and that this alteration of susceptibility bore a marked relation to the alteration of the thermo-electrical properties which are also caused by stress and strain, it was considered very advisable to further investigate the matter.

In the first few trials the large air chamber previously described was employed, but the changes wrought in the coefficient of increase of resistance from rise of temperature by the various strains were found to be so small that special precautions had to be taken to secure reliable results. Accordingly the following apparatus was made use of. In the figure Z Z is an air chamber made of two concentric brass tubes 12 inches in length. The diameter of the wider tube is 4½ inches, and of the inner tube 2½ inches, so that the two enclose between them a layer of air 1 inch thick. The two tubes are connected at the top and bottom by brass rings soldered to them, and two pieces of brass tubing soldered into the outer cylinder, one near the bottom and the other near the top, communicate by means of indiarubber tubing with a boiler, W, and a vessel of cold water, K, respectively, so that steam from the boiler can fill the entire space between the two concentric cylinders and be condensed in K. Three stout wires of copper, S S S, 4 inches in length, pass through a cork which fits tightly the inner cylinder, and their upper ends are connected with the resistance coils and galvanometer, whilst to their lower ends are attached three clamps, A A and B. To the clamps are soldered silk-covered copper wires, which, passing through the cork, are connected with the poles of the battery. The ends of the wire to be tested, X, and the comparison-wire, Y, are secured to the clamps at A A and B, the two wires having previously to clamping been enveloped in cotton tubes of the same lengths as the wires. As a further precaution, the clamps A were well wrapped up in silk, and with the wires were enclosed in a cardboard cylinder which fitted neatly into the inner brass cylinder. When it was necessary to heat the wires, jackets of several folds of baize were placed round the outside of the air chamber and also completely covered the top.

Fig. 17.



The connexions with the resistance coils of 100 ohms and the galvanometer were exactly as in previous experiments, so that from the disposition of the wires as shown in the figure, it will be seen that the influence of change of temperature on the connexions would be entirely neglectable. Moreover, the boiler and air chamber, which were some distance from the box of resistance coils, were screened from the latter, which was covered on the outside with tin foil, and only opened at the moments of actual testing for the balance between X and Y. The wires X and Y were in all cases 50 centims. long, and so disposed that they occupied the central portion of the chamber; whilst a thermometer, T, served to indicate the temperature. The air chamber stood upright on a table, and the lower portion of it, R, was packed with sawdust on which the base of the cardboard cylinder rested.*

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* It would have been better to use a cylinder of copper foil instead of the cardboard, as thereby the temperature would have been rendered more uniform throughout the length of the wire.

The mode of experimenting adopted was as follows: X and Y having been placed in position and the air chamber covered with its baize jackets, the balance between the two wires was observed, and the thermometer having been pulled sufficiently far out of the cork to enable one to take the reading of the temperature, was afterwards replaced. The water in W was then boiled, and in about half an hour the air in the chamber was found to be at 100° C., and was allowed to remain so for another half an hour, when X and Y were again balanced. The burner was next taken from under W, when a vacuum was formed in the upper part of the boiler by the steam condensing; this vacuum was at once filled by the atmospheric pressure forcing water from K, and when the space between the two brass cylinders had in a short time become filled with the water from K, the cork E was removed from the boiler, and a siphon action allowed to continue from K which was kept supplied with cold water. The jackets were then removed, and the cork E having been replaced, the whole arrangement was suffered to rest for two hours, when the thermometer indicating the temperature of the air chamber to be within a degree or so of the original temperature, the balance between X and Y was once more determined. As the processes of hammering and stretching by increasing the length of X might possibly have caused an error by making the lower portion of the wire to occupy a lower position in the chamber than the corresponding portion of Y,[*] the former wire was from time to time shortened to the same length as the latter. Also, since with all the wires it was found impossible, even by using the greatest care in annealing, to find two pieces of the same wire which would agree exactly in their co-efficients of increase of resistance, these were compared before X was subjected to strain of any sort. The following experiments illustrate the nature of the results obtained:—

Experiment XL.

An annealed copper wire, ·095 centim. in diameter, was stretched by hand permanently to different extents, and after each stretching tested for alteration of resistance from change of temperature.

Percentage increase of length.	Percentage of average temporary superior increase of resistance of stretched wire over unstretched for 1° C. between 20° C. and 100° C. + signifies superior increase of stretched wire on rise of temperature.	Percentage of average permanent decrease of resistance for 1° C. between 20° C. and 100° C. of stretched wire.
5·83	−·00046	·00104
11·40	−·00074	·00107
22·27	−·00045	·00354
32·73	+·00378	·00190

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[*] I have found it very difficult even with such an arrangement as the above to get an *exactly* uniform temperature, except in the central portion of the chamber.

Experiment XLI.

An annealed iron wire, ·0065 square centim. in section, stretched by hand.

Percentage increase of length.	Percentage of average temporary superior increase of resistance of stretched wire over unstretched for 1° C. between 20° C. and 100° C. + signifies superior increase of stretched wire on rise of temperature.	Percentage of average permanent decrease of resistance for 1° C. between 20° C. and 100° C. of stretched wire, – signifies permanent increase.
6·0	+·0071	–·00165
12·0	+·0055	+·00230

Experiment XLII.

An annealed copper wire, ·095 centim. in diameter, hammered.

Percentage increase of length.	Percentage of average temporary superior increase of resistance of hammered wire over unhammered for 1° C. between 20° C. and 100° C. + signifies superior increase of hammered wire on rise of temperature.	Percentage of average permanent decrease of resistance of hammered wire for 1° C. between 20° C. and 100° C.
1·74	–·00340	·00836
5·81	+·00117	·00603
11·05	+·00122	·00447
21·36	+·00139	·00072

Experiment XLIII.

An annealed iron wire, ·063 centim. in diameter, hammered.

Percentage increase of length.	Percentage of average temporary superior increase of resistance of hammered wire over unhammered for 1° C. between 20° and 100° C. + signifies superior increase of hammered wire on rise of temperature.	Percentage of average permanent decrease of resistance of hammered wire for 1° C. between 20° and 100° C.
6·13	+·0009	·0033
10·72	+·0000	·0056

Experiment XLIV.

An annealed copper wire, ·095 centim. in diameter, permanently twisted.

Torsion in terms of a unit taken as the torsion in a wire 1 centim. long when twisted through one revolution.	Percentage of average temporary superior increase of resistance of twisted wire over untwisted for 1° C. between 20° and 100° C. + signifies superior increase of twisted wire on rise of temperature.	Percentage of average permanent decrease of resistance of twisted wire for 1° C. between 20° and 100° C.
0·652	–·000797	. .
1·087	–·000824	·00003
3·261	–·000977	·00015
6·520	+·002097	–·003074

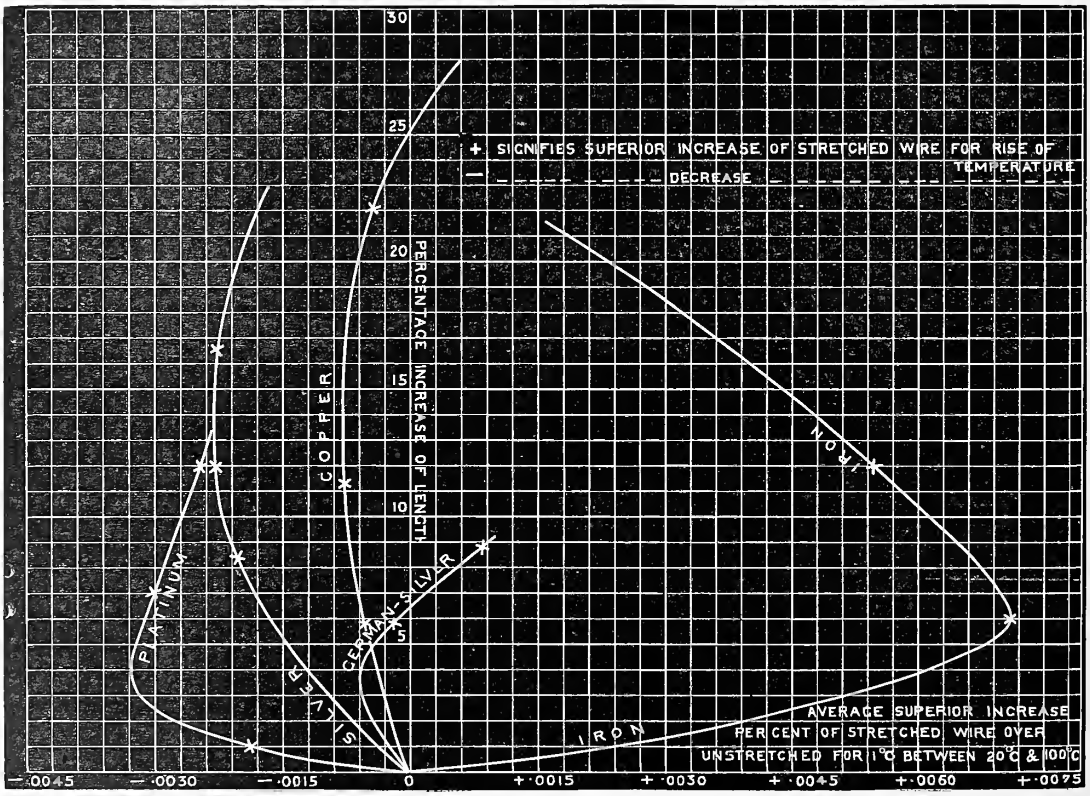
Experiment XLV.

An annealed iron wire, ·063 centim. in diameter, permanently twisted.

Torsion in terms of a unit taken as the torsion in a wire 1 centim. long when twisted through one revolution.	Percentage of average temporary superior increase of resistance of twisted wire over untwisted for 1° C. between 20° and 100° C. + signifies superior increase of twisted wire on rise of temperature.	Percentage of average permanent decrease of resistance of twisted wire for 1° C. between 20° and 100° C.
0·260	+·00474	·00025
1·090	+·00421	·00050
2·180	+·00092	·00100

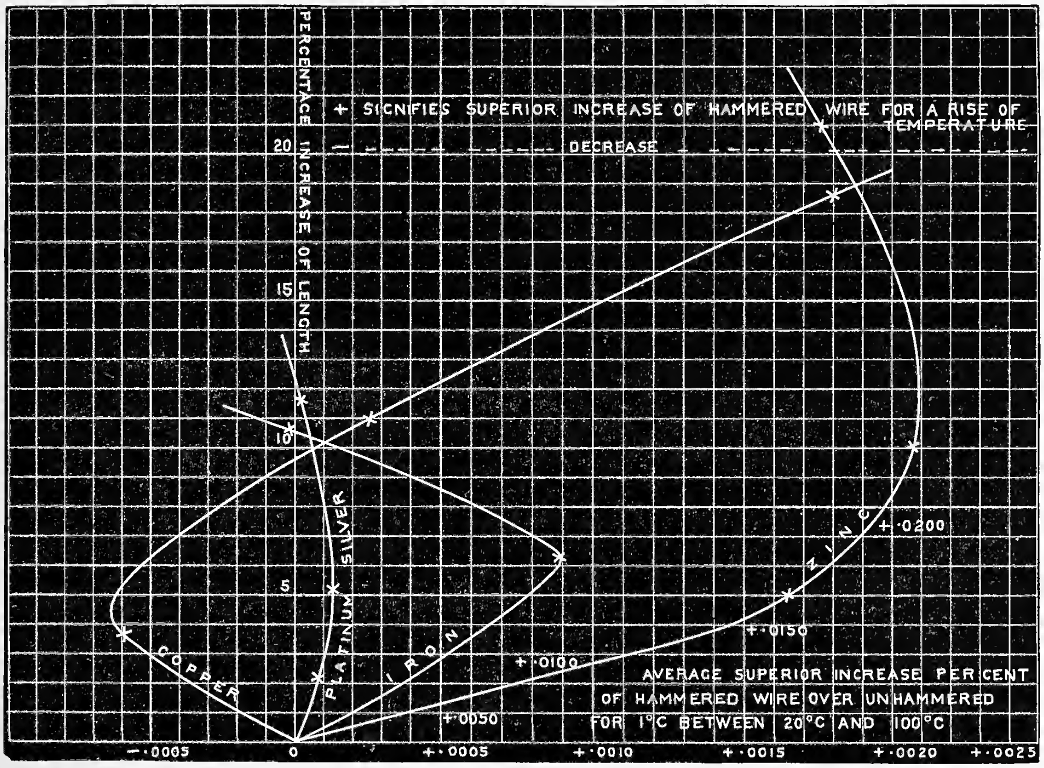
The effect of permanent extension, of hammering, and of torsion on the alteration of resistance produced by change of temperature in the case of copper, of iron, and of other metals is shown in Tables XX., XXI., and XXII.

Table XX.—Curves showing the effect of permanent extension on the temporary alteration of electrical resistance produced by change of temperature.



In Table XX. the curves have their abscissæ representing the average superior increase of resistance for a rise of 1° C. of the stretched wire on a scale of ·000075 percentage of superior increase for each millimetre, and in Table XXI. the curves are drawn with their abscissæ representing the average superior increase of resistance of the hammered wire for a rise of 1° C. on a scale of ·000025 percentage of superior increase for each millimetre. In both sets of curves the ordinates represent the percentage of permanent increase of length on a scale of ·25 percentage of increase of length for 1 millim.

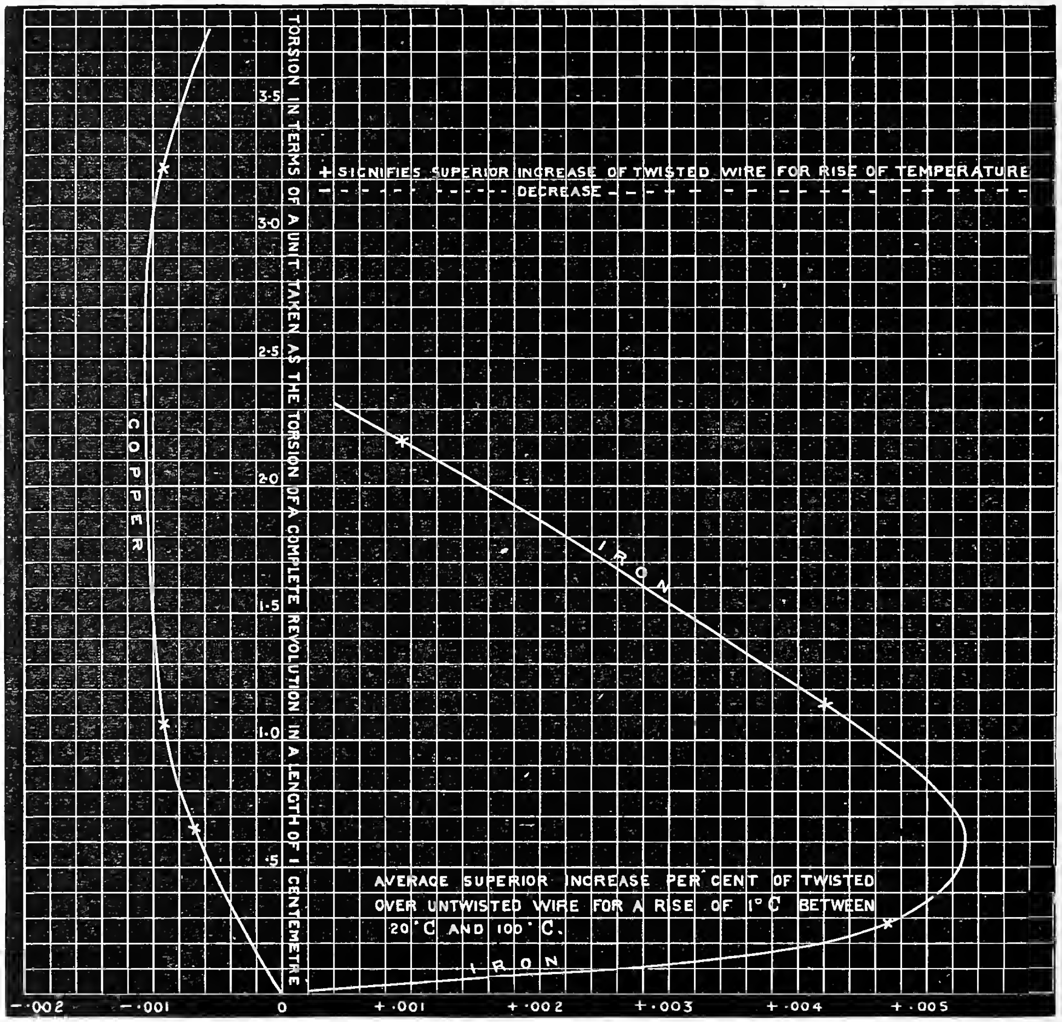
Table XXI.—Curves showing the effect of hammering on the temporary alteration of electrical resistance produced by change of temperature.



For zinc the abscissæ represent 10 times as much alteration of resistance as they do for the other metals.

In Table XXII. the abscissæ of the curves represent the superior increase of resistance of the twisted wire on a scale of ·0005 percentage of superior increase for each millimetre, and the ordinates the amount of permanent torsion in terms of a unit taken as the torsion of a complete revolution in a length of 1 centim. on a scale of ·025 unit for 1 millim. In all the tables a + sign before an abscissa signifies that the strained wire increases most on rise of temperature, and the values of these abscissæ were determined as follows:—Let $\frac{A}{B}$ and $\frac{A'}{B'}$ represent the ratios of the wire to be tested, and the comparison-wire at any two temperatures t and t'; then the average percentage superior change of resistance is $\dfrac{\left(\pm\frac{A'}{B'}\mp\frac{A}{B}\right)\times 100}{(t'-t)\times\frac{A}{B}}$, t' being the higher of the two temperatures, which were about 100° C. and 20° C.

Table XXII.—Curves showing the effect of permanent torsion on the temporary alteration of electrical resistance produced by change of temperature.



Remarks on the last experiments and tables.

The first point to be noticed is that the metals examined may be divided into two classes. In the first of these classes, which includes iron, zinc and platinum-silver,*

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* In some experiments made by H. A. Taylor (see "Report of Electrical Standards Commission," Appendix II., Brit. Ass., York Meeting, 1881), the effect of drawing platinum-silver wire to finer and

the strained wire is *most increased in resistance* by rise of temperature up to a certain limit of straining; whilst beyond this limit further strain diminishes the first effect. In the second class, which comprises copper, silver, platinum and German-silver, the strained wire is *least increased in resistance* by rise of temperature, but that, here again, after a certain limit of strain has been reached the first effect begins to be diminished. Now it will be shown in Part IV. that the metals of the first class, when subjected to a moderate amount of strain, whether the latter be produced by permanent extension, hammering, or torsion, or by other than purely mechanical means, are *thermo-electrically positive* to pieces of the same metal unstrained, and that the first effect is ultimately reversed if the strain be carried to excess; whilst the metals of the second class are, when moderately strained, *thermo-electrically negative* to unstrained pieces of the same metal, but when subjected to excessive strain become thermo-electrically positive. We are therefore led to the conclusion that there must be some close relationship between the thermo-electrical properties of metals, and their temporary alteration of resistance from change of temperature, and that *strain renders a piece of metal thermo-electrically positive or negative to a piece of the same metal unstrained according as the strained piece is caused to be less or more increased in electrical resistance by rise of temperature.*

Another point to be considered is that the metal which is *increased* in specific resistance by the strain is not always rendered *less* susceptible to change of resistance from alteration of temperature, for, as we have seen, zinc and platinum-silver are increased in specific resistance by moderate strain, and are yet at the same time rendered more susceptible to alteration of resistance from variations of temperature. With the former of the above-mentioned metals, the alteration of susceptibility to change of resistance from change of temperature is comparatively much larger than is the case of any of the other metals; whilst with platinum-silver the effect of strain in this respect is comparatively slight, and of the opposite kind to that which is produced on its two constituents.

Again, the third column in the experiments shows that the permanent effect produced by the change of temperature is not of the same nature with iron as with copper; with the former metal the resistance is in some cases actually increased by the annealing when the strain is moderate, and in those cases where a permanent decrease of resistance is caused this increases with increased strain; with the latter, on the contrary, permanent decrease of resistance after moderate straining is the result of the annealing, and this decrease after increasing to a maximum begins to become less with further strain. The difference between iron and copper in the above-mentioned respect is readily intelligible when we remember that these metals differ as regards the kind of alteration of specific resistance caused by strain, as we have seen that with iron the

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finer gauges was found to be to diminish the temperature coefficient, but it seems that Mr. TAYLOR annealed the wires after the process of drawing.

specific resistance is decreased by moderate strain but increased by excessive strain; whilst with copper, moderate strain effects increase, and excessive strain, decrease of resistance.

THE EFFECT OF TEMPORARY STRESS ON THE ALTERATION OF ELECTRICAL RESISTANCE PRODUCED BY CHANGE OF TEMPERATURE.

As permanent strain had been proved to alter the susceptibility to change of resistance from change of temperature in a manner which suggested an intimate relationship between this susceptibility and the thermo-electric properties of metals, some attempts were made to determine the effect of such temporary stress as could be caused by longitudinal traction, on the alteration of resistance produced by change of temperature, partly with a view of establishing a still closer relationship between the above-mentioned physical qualities, and partly with the object of ascertaining whether the increase of elasticity which WERTHEIM's experiments[*] seemed to have proved to be produced in iron and steel when the temperature is raised from 20° C. to 100° C., would be rendered manifest in experiments on the influence of stress on the electrical resistance. The difficulties here encountered seemed at first sight to be so great as to render it an almost impossible task to *measure* with any approach to accuracy the effect sought, unless this effect should be something very appreciable compared with the percentage alteration of resistance produced by stress at ordinary temperatures; for, as we have already seen, the increase of resistance produced by raising the temperature to 100° C. would be some hundreds of times greater than any change of resistance which can be produced in most metals by mechanical stress. After several failures, however, these difficulties were overcome, and I succeeded in measuring with almost the same accuracy the comparatively minute alterations of resistance produced by temporary longitudinal traction at the temperature of 100° C. as at the ordinary temperature of the room.

To accomplish the desired object the large air chamber used in the first part of the enquiry was provided with two tubes, one near the bottom and the other near the top; these tubes served the purpose of conveying steam into the bottom of the outer of the two cylindrical chambers of which the vessel consisted, and out again, near the top of the vessel, into a tub of cold water, a hole at the top of the chamber which had been used previously for the purpose of filling the chamber with water having been corked up after the water had been emptied out. The air chamber was wrapped round with several layers of baize, and these in turn surrounded with several layers of paper tied round with string. The top of the air chamber was also well covered in the same manner with baize and paper, and the table on which the air chamber rested was surrounded on all sides with like material with the exception of a small aperture

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[*] Ann. de Chimie et de Phys., 3^{me} série, 1844, p. 431.

through which the experimenter was enabled to adjust the weights on the scale-pan attached to the wire to be tested. The scale-pan was suspended from the pulley, which was 6 inches from the bottom of the chamber, by a wire sufficiently strong to bear the weights employed, and this wire passed through a hole in the table only just large enough to allow of free motion. The doors and windows of the room were kept shut during the testing at the higher temperature, and the usual precautions were taken to avoid any risk of permanent set or any liability to change of elasticity from testing too soon after permanent extension. In about an hour after the first entrance of the steam into the air chamber the temperature of the air at the top and bottom was nearly, if not quite, at 100° C., but it was found necessary to allow the steam to enter for three or four hours before the wire to be experimented on and the comparison-wire had assumed a sufficiently stable resistance-ratio, and even after this time there would be slow and very minute variations of this ratio first in one direction and then in the opposite. Any errors, however, which would result from slow and minute variations were got rid of in the following manner :—Let a_1, b_1 ; a_2, b_2, &c., be the apparent alterations produced by putting on and taking off the load several times in succession ; then the true alterations due to the load will be very nearly

$$\frac{a_1+a_2+2b_1}{4},\ \frac{b_1+b_2+2a_2}{4},\ \&c.$$

The following experiment will show how accurately the measurements could be made even at the temperature of 100° C.

Experiment XLV.

An annealed iron wire, 7 feet in length and ·067 centim. in diameter, was loaded and unloaded several times with a weight of 3 kilogs.; this weight was then removed and a rest of 48 hours allowed, when, on again testing with 3 kilogs., the recovery was found to be quite perfect. The wire was then heated to 100° C., and having been maintained at this temperature for several hours was again tested. Afterwards the air chamber was suffered to cool down to the original temperature of 13° C. and after a rest of 24 hours the elasticity was redetermined with the same load as before.

Number of trial.	Alteration of resistance in terms of the divisions of the platino-iridium wire produced by 3 kilogs. at 13° C.	Alteration of resistance in terms of the divisions of the platino-iridium wire at 100° C.	Alteration of resistance after cooling again to 13° C.
1	51·85	51·13	51·25
2	51·85	51·25	51·33
3	51·88	50·25	51·27
4	51·98	49·80	51·18
5	52·02	51·65	51·08
6	51·80	53·15	. .
7	51·82	52·00	. .
8	51·73	51·75	. .
9	52·38	50·33	. .
10	52·08	50·22	. .
11	51·85	50·75	. .
12	51·92	51·15	. .
13	. .	52·43	. .
14	. .	52·43	. .
Mean	51·93	51·30	51·23
Probable error per cent.	·06	·16	·03

It will be observed that even at 100° C. the probable error does not amount to ·2 per cent., and, moreover, besides the set of readings recorded in the experiment two others of a similar kind were made afterwards, giving mean values of 51·21 and 51·26, and a total mean of 51·28 as the alteration of resistance caused by a load of 3 kilogs. at 100° C., as against 51·23, the alteration produced on cooling again to 13° C.

Experiment XLVI.

The same wire was again heated to 100° C. and tested with loads of 1, 2, and 3 kilogs., and afterwards, having cooled down to 16° C., the effects of the same loads were redetermined.

Number of kilogs. in the load.	Alteration of resistance produced by the load at 100° C.	Average alteration per kilog. at 100° C.	Alteration of resistance after cooling for 24 hours to 16° C.	Average alteration per kilog. at 16° C.
1	16·89	16·89	17·40	17·40
2	35·82	17·91	34·33	17·17
3	54·99	18·33	51·62	17·21

Experiment XLVII.

The same wire was treated with a load of 6 kilogs. in exactly the same way as it had been treated when 3 kilogs. were employed, and afterwards tested with loads from 1 to 6 kilogs.

Number of kilogs. in the load.	Alteration of resistance at 12° C.	Average alteration per kilog. at 12° C.	Alteration of resistance at 100° C.	Average alteration per kilog. at 100° C.	Alteration of resistance after cooling for 24 hours to 12° C.	Average alteration per kilog. at 12° C.
1	16·96	16·96	18·60	18·60	17·42	17·42
2	33·81	16·92	37·25	18·63	35·53	17·77
3	51·97	17·32	55·92	18·57	52·50	17·50
4	69·98	17·50	75·02	18·76	70·80	17·70
5	86·03	17·21	92·84	18·57	88·45	17·69
6	102·84	17·13	111·95	18·66	106·10	17·68
Mean values	..	17·17	..	18·63	..	17·63

Experiment XLVIII.

The same wire after having been heated to 100° C., and kept at this temperature for several hours with a load of 6 kilogs. on it, was allowed to cool, and in 24 hours the alteration of resistance produced by the cooling observed.

Similar processes were employed with loads of 5 kilogs. and of 3 kilogs.

Number of kilogs. left on the wire when cooling.	Superior alteration of resistance of stretched wire produced by the cooling from 100° C. to 13° C. − signifies superior decrease of resistance of stretched wire on cooling.
6	−8·8
5	−8·0
3	−6·6

Experiment XLIX.

An annealed copper wire, 7 feet in length and ·095 centim. in diameter, treated in a manner similar to that in which the last iron wire had been treated, and tested at 12° C. and 100° C. with a load of 5 kilogs.

Temperature in degrees Centigrade.	Alteration of resistance in terms of the divisions of the platino-iridium wire produced by 5 kilogs.
12°	42·13
100	45·34
12	44·31
100	45·14

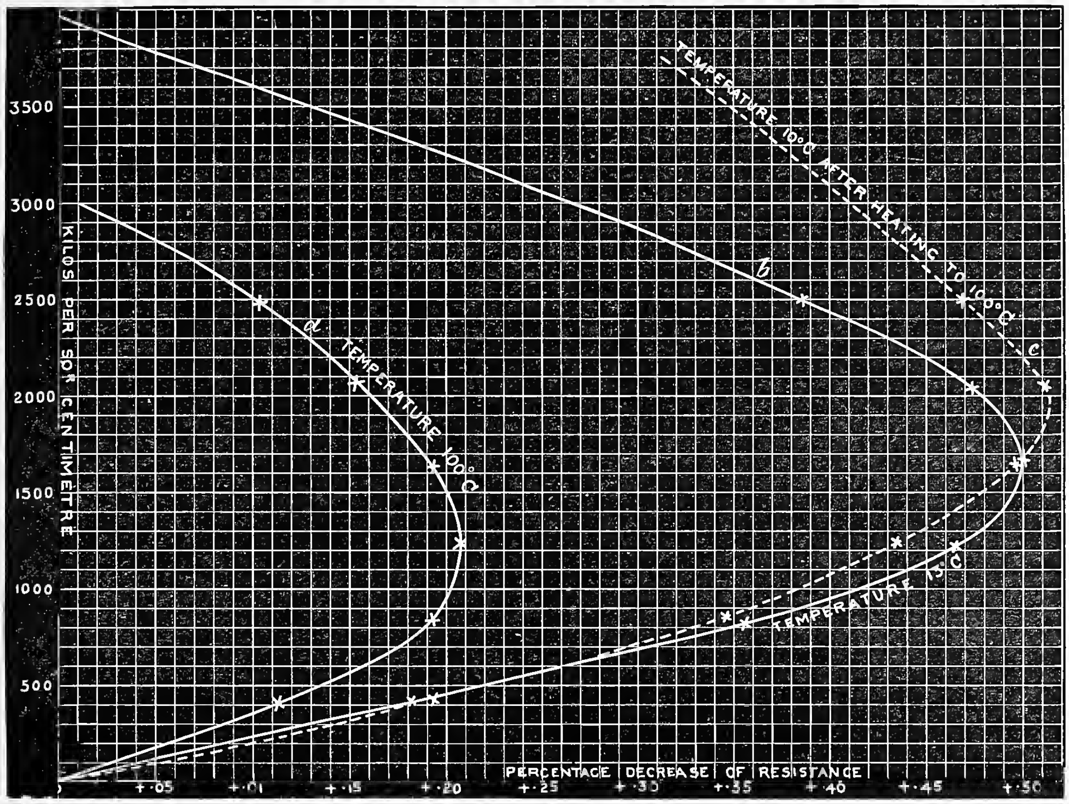
Experiment L.

A piece of the same nickel wire, which had been previously broken by testing for alteration of resistance at the ordinary temperature of the room, was annealed again and adjusted in the air chamber in the manner previously described. A load of 18 kilogs. was then suffered to remain on the wire for several minutes, and after its removal the nickel was allowed to rest unloaded for 24 hours. The wire was now tested with loads up to 12 kilogs. at the temperature of 13° C., then after heating to 100° C., and again, 24 hours afterwards, when cooled to 10° C.

Load in kilogs.	Alteration of resistance at 13° C. in terms of the divisions of the platino-iridium wire. – signifies decrease of resistance of stretched wire.	Alteration of resistance at 100° C.	Alteration of resistance after cooling to 10° C.
2	− 58·3	−36·1	− 60·6
4	−114·0	−62·8	−110·0
6	−148·4	−68·2	−137·7
8	−159·1	−62·5	−155·9
10	−151·3	−49·6	−160·7
12	−123·8	−33·8	−149·4

The effect of change of temperature on the temporary decrease of resistance produced by loading is also shown in Table XXIII.

Table XXIII.—Curves showing the temporary alteration of electrical conductivity of nickel produced by longitudinal traction at temperatures of 10° C., 13° C., and 100° C.



Remarks on the last experiments and on Table XXIII.

Experiment XLV. teaches us that though the alteration of resistance of the iron when loaded with 3 kilogs. is greater at 13° C. *before heating to* 100° C. than at 100° C.; yet, *after cooling again to* 13° C., the alteration is very slightly *less* than it had been at 100° C., and, on the whole, there is a *permanent* decrease of the temporary alteration of resistance produced by the loading. From Experiment XLVI. we learn that the second heating to 100° C. still further increases the difference of the effect of loading with 3 kilogs. at the lower and the higher temperatures; the alteration at the *higher* temperature being now more than 6·5 per cent. *greater* than at the *lower* temperature; but when the load employed is only 1 kilog., the alteration seems to be greater at the lower temperature than at the higher. Experiment XLVII. shows that when the wire was treated with 6 kilogs. there was a permanent increase of elasticity produced by the loading, and it should at the same time be noted that 6 kilogs. when first put on the wire, *barely produced a permanent increase of length*—an increase certainly not amounting to more than $\frac{1}{10}$th per cent. By comparing also columns two and six we can see that the slight annealing caused by raising to 100° C., and cooling again has diminished the elasticity, and therefore we have evidence here that *very slight extension* permanently increases the elasticity of iron; whilst in Part I. we have seen that moderate permanent extension decreases, and excessive permanent extension increases the elasticity of iron. It is evident, therefore, that we have three critical points in iron wire, and that the elasticity is first increased to a maximum, then decreased to a minimum, and finally begins to increase again as the permanent extension is gradually increased from exceedingly small amounts to the breaking point of the wire.

Again, the alteration of resistance produced by *all* the loads is now *greater* at 100° than at 12° by amounts varying from about 5 to 6 per cent.

Experiment XLVIII. also shows that when loads from 6 to 3 kilogs. are left permanently on the wire when cooling from 100° C. to the temperature of the room, the alteration of resistance decreases as we decrease the load, and bears out the previous observations that the elasticity is less at the higher than at the lower temperatures.

In Experiment XLIX. we have evidence that the slight annealing caused by raising the temperature of the copper to 100° C. permanently decreases the elasticity, and that the alteration of resistance at 100° C. is for the load employed about 2 per cent. greater than at 12° C. It is, however, when we come to Experiment L. that the most noticeable changes are seen to be produced, and when we consider the results recorded in this experiment and in Table XXII. we are led to the conclusion that the temporary alteration of susceptibility to change of resistance from changes of stress effected by raising to 100° C. is, with nickel, as remarkable as we have seen this susceptibility itself to be. The curves are drawn on the same scale as the curves for nickel at 22° C., and it will be observed that not only is the maximum diminution of resistance lessened by raising to 100° C. to an amount which is *less than one-half* of the maximum diminution at the lower temperature, but that also the load at which this maximum diminution occurs is much less; and it may well be that at a sufficiently high temperature the decrease of resistance which is observed to be produced by moderate loads at the lower temperatures would be changed to an increase. In fact, I am inclined very strongly to believe that there exists with all metals a critical temperature below which temporary stress will produce temporary decrease of resistance, and that above this temperature there is an opposite effect caused by the stress.

The value of "YOUNG's modulus" was determined for this piece of nickel by the method of static extension, in a manner to be presently described, and was equal to 2480×10^6 at the temperature of 16° C. and to 2280×10^6 at the temperature of 100° C. It will be observed that the maximum decrease of resistance as shown in Table XXIII., is appreciably greater than the maximum decrease observed with the other specimens,[*] and still further shows how the amount of this decrease depends upon the elasticity.

As regards the question whether the thermo-electric properties of the metals, as

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[*] These two specimens were received at different times from Messrs. JOHNSON, MATTHEY, & Co., and the latter shows still more remarkably than the former the large elasticity which can be obtained from nickel even in a well-annealed condition.

affected by temporary stress, be related to the susceptibilities of alteration of resistance from change of temperature in the same way as the corresponding qualities seem to be where permanent strains are concerned, it may be said to remain at present open, and only to be decided by further experiments on the effects of stress on the thermo-electric qualities of iron;[*] but as far as copper and nickel are concerned, the above question seems to be answered in the affirmative.

The Effect of Slight Mechanical Strain and of the Strain caused by raising Iron to 100° C. and afterwards Cooling, on the Torsional Rigidity of the Metal.

The above experiments had shown such an astonishing influence to be produced on the longitudinal elasticity of annealed iron by merely raising the metal to 100° C. and then cooling, that it seemed advisable to test whether or not a similar effect would be produced on the torsional rigidity of iron by a like cause, and if so, whether we can imitate the strain resulting from heating and afterwards cooling by mechanical means.

Experiment LI.

A piece of annealed iron wire was vibrated at a temperature of 13° C., and the time of a single vibration, as determined by counting the vibrations for five minutes, was found to be 1·000 second. The wire was then heated in an air chamber to 100° C., and after having been maintained at this temperature for one hour was suffered to cool, and the time of vibration found to be 0·989 second 12 hours after cooling.

Experiment LII.

A second piece of the same wire, when suspended ready for vibrating, was heated slightly by passing the flame of a Bunsen burner rather quickly up and down it several times, the vibrator being at the same time supported, so as to take off stress from the wire. The time of vibration before heating was 1·154 second, and in five minutes, 35 minutes, and 245 minutes after cooling was 1·147, 1·142, and 1·136 second respectively.

The wire was then heated to redness and cooled, when the time of vibration after five minutes was 1·156 second, and after 22 hours became 1·143 second.

Experiment LIII.

A third piece of the same wire was vibrated after different slight amounts of permanent extension had been produced; the length before stretching was 82·0 centims., and the time of vibration 1·621 second. After slightly stretching, so as to increase the length to 82·2 centims. and 84·3 centims., the time of vibration became respectively 1·614 second and 1·714 second.

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[*] This point will be fully considered in Part IV.

Experiment LIV.

A piece of annealed pianoforte steel was vibrated after different slight amounts of permanent extension.

Length of wire, in centims.	Time of vibration, in seconds.
89·00	·984
89·05	·976
89·10	·968
89·20	·968

These experiments[*] speak for themselves, and prove that the torsional rigidity is affected in a precisely similar manner to the longitudinal elasticity by raising to 100° C., and then cooling, and moreover that the strain produced by slight mechanical traction acts in a similar manner on both iron and steel to the strain produced by tempering. We see also how very quickly the increase of elasticity is changed into a decrease when the extent of strain is widened either by heat or by mechanical means. Evidently then there are for iron three critical points as regards its torsional rigidity as well as regards its longitudinal elasticity—very slight strain increasing, moderate strain decreasing, and excessive strain again increasing both these physical properties. Further, it would be interesting to determine whether cold would not produce the opposite effect on the elasticity to heat, and it seems highly probable that cooling below the temperature of the room will permanently decrease the elasticity of iron; this point, however, I hope to be in a position shortly to decide.

FURTHER DISCUSSION OF WERTHEIM'S EXPERIMENTS ON ELASTICITY.[†]

We have seen that the temporary alteration of resistance produced by any load is permanently decreased in the case of annealed iron wire by merely raising the temperature of the metal to 100° C. Now WERTHEIM'S experiments seem at first sight to prove that the elasticity of iron and steel is greater at 100° C. than at the ordinary temperature of the room; but if M. WERTHEIM had examined the elasticity after the wire tested at the higher temperature had again cooled down to the lower one, he would have found that this *apparent temporary* increase of elasticity was *really a permanent* one, and if the wire had been tested several times, first at the higher and then at the lower temperature, he would have also found, provided sufficient rest after

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[*] All these experiments were repeated several times with different specimens of iron, but invariably with the same result as regards the nature of the change.

[†] Ann. de Chimie et de Phys., 3^{me} série, 1844.

cooling had been allowed, that the elasticity of both iron and steel is *temporarily diminished* by raising the temperature to 100° C.

From WERTHEIM'S researches* we gather that in the case of iron and steel there is the following increase of elasticity between 15° to 20° C. and 100° C.

TABLE XXIV.

Metal.	Increase per cent. of elasticity between 15° to 20° and 100° C.
Annealed iron	4·90
Annealed iron wire	6·91
Annealed cast steel	2·79
Annealed English steel wire	23·20
Steel tempered blue	5·18

The increase of elasticity of the steel wire seemed so remarkable that it was determined to retry WERTHEIM'S experiments by direct observations of extension, using the same scale and vernier as have been already described in Part I., and placing the wire and the comparison-wire in the same air chamber as had been used in measuring the alteration of resistance produced by loading at different temperatures. The length of wire between the clamp, which rested on a wooden support on the top of the chamber, and the vernier was 4 feet 4 inches, and of this length 2 inches or rather more would be, roughly speaking, at the temperature of the room when the rest of the wire was heated by steam to 100° C. We must therefore multiply any observed effect on the elasticity caused by raising the temperature of the wire to 100° C. by a number which is about 4 per cent. of the apparent alteration. The air chamber rested (fig. 18) on a piece of wood, A A, 8 inches long, 6 inches broad, and $\frac{1}{2}$ inch thick, and this in turn on two stout bricks, B B, which were supported by a wooden table, D D. The wire to be examined, X, and the comparison-wire, Y, were, as usual, secured at one end of each to the same clamp, C, which rested on a piece of hard wood, E E, placed at the top of the chamber. The wires, X and Y, passed through two small holes made in E E, and also through a hole in the table, and to their lower ends were attached, in the one case, a scale-pan, and in the other a cross-bar of wood, H H, carrying constant equal weights. Exactly the same precautions were taken as have been already fully described in Part I., and, as in experiments on the alteration of resistance produced by loading, the air chamber was well covered with baize and paper.

Under these circumstances it was possible to maintain a very constant temperature of 100° C., and even had the temperature varied slightly it will be seen that no appreciable error would have been introduced, as the variation would have altered the lengths of X and Y to the same extent.

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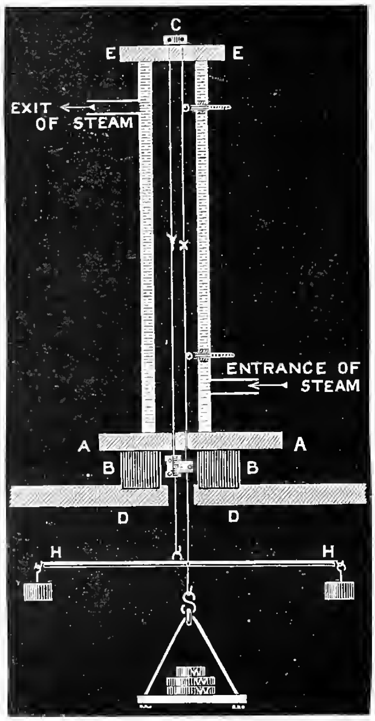
* Ann. de Chimie et de Phys., 3^{me} série, 1844, p. 431.

Fig. 18.



Experiment LV.

An annealed pianoforte-steel wire, ·085 centim. in diameter, was loaded for some hours with a weight of 10 kilogs.; this was then removed and a rest of 24 hours allowed, when a weight of 4 kilogs. was left permanently on the wire and the elasticity determined at the ordinary temperature of the room, and at 100° C., by putting on and taking off a load of 6 kilogs., the recovery being quite perfect for the load at both the higher and the lower temperatures.*

Number of trial.	Alteration of length produced by 6 kilogs. at 12° C. in half-millims.	Alteration of length produced by 6 kilogs. at 100° C.	Alteration of length produced by 6 kilogs. after cooling to 12° C.
1	1·48	1·49	1·45
2	1·50	1·49	1·46
3	1·51	1·50	1·44
4	1·51	1·48	1·44
5	1·51	1·48	1·46
6	. .	1·48	. .
Mean	1·502	1·487	1·450
Probable percentage of error . . .	·40	·27	·30

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* At the higher temperature after the *first* loading, which was not taken into consideration in estimating the elasticity.

Experiment LVI.

The same iron wire as had been used for Experiment LV. was loaded for some time with 8 kilogs.; 4 kilogs. were then removed and the wire allowed to rest for 24 hours, when the effect on the length of the wire produced by putting on and taking off 2 kilogs. was found to amount to an alteration of ·940 half-millim. at 100° C. After the action of the steam had been stopped for two hours and the temperature was about 20° C., the alteration produced by the same load amounted to ·985 half-millim., and after 20 hours rest at the temperature of 13° C. was found to be ·945 half-millim. The wire was now tested with 4 kilogs., when the alteration of length produced by this new load was 1·910 half-millim. at 13° C., and after heating for some hours to 100° C., 1·911 half-millim. The source of heat having been removed and the wire allowed to cool slowly for 20 hours, the alteration caused by the same load was 1·880 half-millim. at 13° C.

Experiment LVII.

An annealed nickel wire, ·09213 centim. in diameter, was loaded for several hours with a weight of 18 kilogs.; 10 kilogs. were then permanently left on the wire for 24 hours, and the alteration of length effected by a load of 8 kilogs. at a temperature of 16° C. amounted to 1·250 half-millim. The wire was heated to 100° C., and the alteration was now found to amount to 1·360 half-millim. After cooling down for the next five hours to a temperature of about 20° C. the alteration became 1·350 half-millim., and after three days at a temperature of 15° C. proved to be 1·250 half-millim.

Experiment LVIII.

The same copper wire as had been used in Experiment XLIX. was loaded for some hours with a weight of 11 kilogs.; 7 kilogs. were then allowed to remain on permanently and the wire suffered to rest for two days. It was then raised to the temperature of 100° C. with the weight of 11 kilogs. on, and cooled again to the temperature of the room. These operations were repeated each four times during a space of four days, and on the last of these the alteration of length produced by a load of 4 kilogs. was found to be 1·374 half-millim. at 100° C.; on cooling to 15° C. the alteration caused by the same load proved to be 1·310 half-millim., and this, or nearly the same alteration, had been found to be produced by the load at the temperature of the room, which temperature varied from 15° C. to 16° C. on each of the previous days.

The following table exhibits the difference of the alteration of electrical resistance and of length at the temperature of the room and at 100° C.

TABLE XXV.

Metal.	Percentage temporary alteration of elasticity caused by raising the temperature from 15° C. to 100° C. + signifies increase of elasticity produced by rise of temperature.		Percentage increase of alteration of resistance caused by raising the temperature from 15° C. to 100° C. − signifies decrease of alteration.
	WERTHEIM.	TOMLINSON.	
English steel wire	+23·20	−2·58	. .
Iron wire	+ 6·91	−1·64	+ 5·70
Copper	− 6·59	−4·74	+ 1·87
Nickel	. .	−8·41	−50·00

Remarks on Experiments LV.–LVIII., and on Table XXV.

It appears from the last experiments that there is really a permanent increase of elasticity produced in annealed iron and steel by merely raising the temperature to 100° C.; and what is still more remarkable, there was in the case of one specimen of iron wire, which was so ductile as to lengthen by 24 per cent. before breaking,* such a large loss of ductility that its maximum permanent elongation *barely reached* 13 *per cent. after it had been heated to* 100° *C. and allowed to cool again.* As the last discovery was made when experimenting on the effect of permanent extension on the susceptibility of the resistance of iron to change of temperature, it was thought at first that the passage of the current used in determining the electrical resistance might have some influence on the ductility, but on heating three other pieces of the same wire to 100° C., and afterwards allowing them to cool without permitting any current to flow through them, almost exactly the same change was observed; and yet in all these cases the rate of cooling was slow; so much so, indeed, that with the last specimen employed, in which special precautions had been taken to surround the small air chamber in which the wire was heated with several coatings of baize, the chamber was sensibly warm to the touch 12 hours after the cooling had commenced. How far other kinds of annealed wrought-iron might show a change of ductility from like cause I know not, but it seems evident that changes comparatively enormous can be produced in the elasticity and ductility of this metal by small alterations of temperature.

It has also been shown by Experiment XXXIX. that the electrical conductivity of annealed steel is increased by heating slightly and then cooling quickly, and it will be proved in Parts III. and IV. that there is a correspondingly large effect on the magnetic inductive capacity and on the thermo-electric properties of steel produced by the same process. It would appear, therefore, that researches of this kind might lend valuable

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* This wire I received through the kindness of Mr. J. T. BOTTOMLEY, and was especially prepared for experiments on magnetic induction carried on in the physical laboratory of Sir W. THOMSON.

aid in investigations on the liability of wrought-iron axles to fracture produced by sudden changes of the temperature of the air.

Again, it will be observed from Experiment LVI. that shortly after the iron has been heated and then cooled there is less elasticity than when a considerable rest has been allowed; and in fact we have in this case exactly the same kind of restitution of elasticity in iron as we have seen takes place after the wire has experienced mechanical extension. With nickel the increase of elasticity produced by rest after cooling is still more remarkable. Equally remarkable also is the temporary change of elasticity produced in nickel; and a comparison of the loss of elasticity produced by raising the temperature to 100° C. with the change of susceptibility to alteration of resistance from change of stress as shown in Table XXIII. affords a still further proof that stress and strain act in the contrary direction as far as electrical conductivity is concerned.

It will also be noticed in Table XXV. that the alteration of susceptibility to change of electrical resistance from change of stress is greater in iron in proportion to the alteration of elasticity when the temperature is raised to 100° C., and that the converse is the case with copper, so that there is a greater or less alteration of *specific resistance* for the loaded wire than for the unloaded, caused by rise of temperature to 100° C., according as the metal is iron or copper; and we have seen that a similar state of things occurs where the strain is that left after the removal of the stress.

THE ALTERATION OF ELECTRICAL CONDUCTIVITY PRODUCED BY MAGNETIZATION.

History of the subject and description of apparatus.

This subject has, in the case of iron, received the attention of several observers, who have in some instances differed not only as regards the amount but also as regards the nature of the change produced by magnetization on the electrical resistance. Sir W. THOMSON, in 1856,[*] was I believe the first to show that magnetization affected the electrical conductivity of iron and steel, longitudinal magnetization causing in these metals increase, and transverse magnetization decrease of resistance, and this with steel was found to be the case whether the metal was hard or soft. Shortly afterwards experimenting on nickel, THOMSON found[†] with this metal also an alteration of resistance similar in kind to that of iron, but greater in amount for the same magnetizing force. With brass, on the contrary, he failed to detect any change whatever.

Subsequently BEETZ,[‡] CHWOLSON,[§] and myself[||] pursued similar investigations.

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[*] Phil. Trans., "Electrodynamic Qualities of Metals," Part IV.

[†] Proc. Roy. Soc., vol. viii., 1857.

[‡] POGG. Ann., vol. cxxviii., p. 202 (1866).

[§] CARL'S Rep., vol. xiii., p. 232 (1877).

[||] Proc. Roy. Soc., June 17, 1875, vol. xxiii., p. 533.

BEETZ and CHWOLSON both confirmed the results of THOMSON as far as longitudinal magnetization was concerned; but with the former of the two first observers an experiment where transverse magnetization was employed, ended in giving purely negative results. The values obtained in my own experiments differed considerably from those of THOMSON as regards amount and in the case of hard steel also in nature. Moreover, it appeared that the circular magnetization which ensues when a current is passing through an iron wire caused an increase of resistance, and as we might perhaps expect circular magnetization to cause a similar effect, as regards nature, on the conductivity to transverse magnetization,* these results were not in accordance with those of THOMSON. Unfortunately I had not at the time read Sir W. THOMSON's paper, and investigations made in later years convinced me that the observations recorded in my "Preliminary Notice" were not reliable, partly because alteration of resistance from change of temperature had not been sufficiently guarded against, and partly because of a "PELTIER effect," which I have since found would, with the large battery-power employed in the circuit of the "WHEATSTONE's bridge" arrangement, vitiate the results. To my astonishment, AUERBACH, three years afterwards, published an essay† "On the Passage of the Galvanic Current through Iron," in which my own observations both as regards the magnitude and nature of the changes produced in iron and hard and soft steel by longitudinal magnetization were apparently fully confirmed. Moreover, it appeared that he also agreed with me that circular magnetization caused increase of resistence in iron; and ingeniously reasoning that, this being the case, feeble longitudinal magnetization should decrease the resistance of iron, brought forward a series of experiments which seemed fully to bear out his views. Under these circumstances it seemed to be advisable to retry some of my old experiments on iron and steel *rods*, and further to extend the enquiry to *wires* of iron and steel.

As several of the instruments used in this part of the investigation have also been employed in several experiments yet to be described in the other parts of this paper, it seems advisable to give here some description of them, as well as certain data which will be required for converting the measurements taken into absolute units.

The tangent galvanometer.

This was of the usual HELMHOLTZ-GAUGAIN pattern, made by ELLIOTT Bros., where there are two stout copper rings for measuring currents where it is desirable that the resistance of the galvanometer should be small, and three pairs of coils of finer wire for other purposes. In these investigations the former only were employed, and therefore we may consider the resistance of the galvanometer itself to be neglectable.

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* This, however, is not, I believe, the case.

† Phil. Mag., July, 1879, vol. viii., p. 1. Translated from the original essay (Leipzig, 1878), communicated by the author.

The short needle was as usual provided with an aluminium index, and by means of this, readings on a graduated circle traversed by the ends of the index could be depended upon to within $\frac{2}{10}$ths of 1°. The needle was suspended by a platinum wire $\frac{1}{1000}$th of an inch in diameter and 12 inches in length, the upper end of the wire being secured to a torsion-head provided with a vernier, which moved over a graduated circle; by means of the vernier a torsion amounting to six minutes could be measured. The fine platinum wire hung in the axis of a brass tube which was provided near its lower extremity with a small glass window, so that the illuminated image of a vertical wire focussed by a lens could be received on a small mirror and reflected back on a scale placed at a distance of 1000 of its own divisions from the mirror. The mirror was attached to a piece of stout brass wire which was connected at its lower extremity with a needle, and at its upper extremity was clamped to the lower end of the platinum wire.

The effect of the torsion on the deflection was determined by a series of careful observations made by turning the torsion-head through different angles, first in one direction and then in the opposite, and noting the corresponding deflection of the needle. Thus, in one experiment, the torsion-head having been turned through 100 degrees from left to right, there was produced a deflection of 57° 6′, and when the head was turned from right to left, the deflection was 56° 46′ on the opposite side of the zero point; the mean of the two deflections of the needle is 57°, and therefore the force of 1° of torsion of the suspension wire would $=\frac{\text{sine } 57° \times \text{H}}{(100-57)}=\cdot0195 \times \text{H}$; where H is the horizontal force of the earth's magnetic action on the needle.

In order to be able to determine in absolute units the value of the deflections of the needle, a new DANIELL'S cell of large size was charged with a saturated solution of sulphate of copper and a semi-saturated solution of sulphate of zinc. This cell was allowed to rest for two hours after having been charged, and was then short-circuited for half an hour. After this time the cell was connected up with the thick copper wires of the galvanometer, and the deflection of the needle produced, observed by means of the mirror and scale, when external resistances of 0 ohm, 10 ohms and 20 ohms were successively introduced. The deflections were reversed in each case by reversing the battery-current and the mean values of the two deflections in the different directions were for 0 ohm, 324 divisions of the scale; for 10 ohms, 110 divisions; and for 20 ohms, 30 divisions; these deflections corresponding to 8° 59′, 3° 8′, and 0° 52′ respectively. By comparing the deflection with 0 ohm and 10 ohms in circuit, the internal resistance of the battery, together with the resistance of the connecting wires, was found to be 1·055 ohm, and by comparing the deflections with 10 ohms and 20 ohms in circuit, this same resistance was calculated to be 1·058 ohm; therefore, 1·057 ohm was assumed to be the true resistance. Again, taking the electromotive force of the DANIELL thus charged to be 1·12 volt, or $1\cdot12 \times 10^8$ electromagnetic units, and the resistance of 1 ohm to be 10^9 of these units, the formula for converting the readings of the galvanometer into electromagnetic units was determined to be

$$\gamma = {\cdot}3158\ (\tan\phi + {\cdot}00839\phi\sec\phi),$$

where ϕ equals the deflection of the needle in degrees, and γ is the value of the current strength in absolute measure.

The magnetizing coils.

During a considerable part of these investigations two magnetizing coils were employed, of which descriptions are now given.

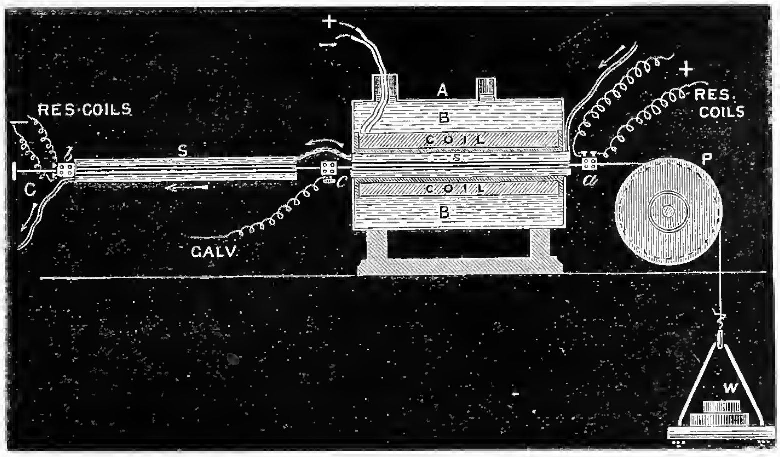
The larger of the two coils, which will be designated as the coil A, was made of about 1000 feet of cotton-covered copper wire, $\frac{1}{20}$th of an inch in diameter; the whole piece was divided into seven equal portions, and these, after having been well soaked in melted paraffin wax, were placed side by side and bound together by tape wound spirally along the whole length of the compound strand thus formed. This strand, after a further soaking in paraffin, was wound on a stout glass tube having an internal diameter of 1 inch and a length of 13 inches. The glass tube was provided at its two extremities with discs of hard wood $\frac{1}{4}$ inch thick and 6 inches in diameter, and surrounding the whole of the coil was a zinc cylinder concentric with the glass tube, which being closed at its two ends by discs of the same metal served when filled with water to keep the coil cool (fig. 19). The ends of each of the seven portions into which the whole wire was divided were connected with separate terminal screws so that the seven coils could be used either in "series" or in "multiple arc." When arranged in "series," in which form they were employed in this particular branch of the enquiry, the total resistance of the coils at 15° C. was 4·464 ohms. The compound strand was distributed along a length of 30 centims. in layers of five deep, so that the total number of turns amounted to 2100. The inner diameter of the coil was 3 centims. and the outer 7 centims.; therefore the magnetizing force at the centre and the two extremities would be respectively $4\pi \times c \times 70 \times {\cdot}986$ and $4\pi \times c \times 70 \times {\cdot}498$ absolute units, where c denotes the current strength, and the average force throughout the whole length would be $4\pi \times c \times 70 \times {\cdot}742$.

The smaller of the two coils, the coil B, was constructed as follows:—A thin tube of polished brass, with a slit running throughout its entire length, $1\frac{1}{4}$ inch internal diameter and $4\frac{1}{2}$ inches long, was covered with vulcanised caoutchouc to a depth of $\frac{1}{16}$th of an inch, and on this was wound 3 lbs. of cotton-covered copper wire, $\frac{1}{20}$th of an inch in diameter, followed by 3 lbs. of wire, $\frac{1}{10}$th of an inch in diameter. Inside the first tube was placed a second of similar kind, 1 inch in internal diameter, and connected with rings of ebonite with the first at the two ends. The second tube was concentric with the first, and of the same length, so that between the inner tube and the outer there was interposed a layer of air nearly $\frac{1}{8}$th of an inch in thickness. This arrangement was employed to prevent the heat from the magnetizing coil reaching any wire or rod placed inside. The wire forming the coil had a resistance of 1·782 ohm at the temperature of 15° C., and was distributed in 814 turns in a length of 9·7 centims. The inner diameter of the coil was 3 centims. and the outer diameter 9·0 centims., so that the magnetizing force at the centre would be $4\pi\times c\times 90{\cdot}4\times{\cdot}851$, and that of the two extremities $4\pi\times c\times 90{\cdot}6\times{\cdot}478$ absolute units; consequently the average force throughout the coil would be $4\pi\times c\times 90{\cdot}4\times{\cdot}664$. As with coil A, the whole of the wire was well soaked in paraffin wax before winding, and the insulation between one layer and another was, by determining the resistance before and after winding, ascertained to be quite perfect in both A and B.

THE EFFECT OF TEMPORARY STRESS ON THE ALTERATION OF ELECTRICAL RESISTANCE PRODUCED BY MAGNETISM.[*]

In order to test the effect of temporary and permanent extension on the change of electrical resistance produced in iron wire by magnetism, the following arrangement was adopted:—The wire, which had a total length of from 4 to 5 feet, was firmly secured at one end (fig. 19, C) to a strong upright, and was then slipped through two glass tubes of small diameter about 13 inches in length; the glass tubes were placed inside two copper vessels, S S, of the same length as themselves. These vessels each consisted of two concentric cylinders connected by soldering at the two extremities by copper rings and provided with small pipes, so that cold water could be kept running through the space between the cylinders. The outer of the cylinders was 1 inch nearly in external diameter, and the inner ¼ inch in diameter; one of the copper vessels was placed inside the magnetizing coil, A, whilst the other was supported on the table.

Fig. 19.



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[*] It should be mentioned here that in all these experiments the magnetizing coils were placed in a direction *perpendicular to the magnetic meridian.*

The wire passed over a large wooden pulley, P, and to the other end of it was attached a scale-pan for holding weights. The light clamps *a*, *b*, *c*, having been well secured to the wire, were connected up in the usual manner, as shown in the figure, with the galvanometer, battery and resistance-coils, the connecting wires in this case being for a distance of 2 feet of rather fine silk-covered copper, in order to avoid the strain which would probably have ensued from the comparatively heavy caoutchouc-covered wires ordinarily employed. The whole arrangement was then carefully covered over with baize, and water allowed to trickle from a small cistern through S, S for about an hour, the space B having been previously filled with water. After a sufficient time had elapsed to render the parts of the wire of the same temperature, the effects on the resistance of magnetism alone or magnetism combined with strain were determined. The tangent galvanometer was placed in the circuit of the coil A[*] for the purpose of measuring the current passing through the coil.

Experiment LIX.

An annealed iron wire, ·093 centim. in diameter. B.C. produced by one LECLANCHÉ. M.C.[†] by 10 GROVE cells with adjustable resistances in the external part of the circuit.

Number of kilogs. on the wire, scale-pan weighing 2 kilogs. not included.	Tangent of deflection galvanometer = tan *a*.	Alteration of resistance in terms of the platinum-silver wire = *d*; one division = an alteration of ·00081 per cent.	$\frac{d}{\tan a}$.
0	·123	48·6	395
0	·249	65·0	261
8	·249	40·0	. .
0	·105	40·0	381
4	·105	27·0	. .
6	·105	22·0	. .

Experiment LX.

Another piece of the same wire was loaded and unloaded several times with a weight of 12 kilogs., and was afterwards allowed to rest for several days unloaded. The changes of resistance were then determined by noting the position of the light on the scale, the B.C. being produced by a large DANIELL'S cell, the circuit of which, as well as that of the galvanometer, was kept closed as soon as the balance had been

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[*] In the figure the manner in which the compound strand of the coil is arranged in "series" is not shown.

[†] The current in the circuit of the "WHEATSTONE'S bridge" will be denoted by B.C. and that in the magnetizing circuit by M.C.

established. The effect of magnetization on the resistance was tested when loads of 0, 6, 10, and 12 kilogs. were on the wire. The M.C. was produced by five Grove cells.

Tangent of the deflection of the needle of the tangent galvanometer.	Load in kilogs. on the wire when under magnetization, the scale pan weighing 2 kilogs. not included.	Alteration of resistance produced by magnetization in divisions of the galvanometer-scale ; one division representing an alteration of ·0005 per cent.
. .	0	27
·247	6	19
. .	10	25
. .	12	25

Both these experiments show that the increase of resistance which was produced by the longitudinal magnetization is lessened by temporary stress up to a certain limit of the latter, and several other experiments of a similar kind proved that after the diminution of alteration of resistance caused by magnetization had reached a maximum, further temporary stress began to reverse the first effect, sometimes only just before the "breaking-load" of the wire had been reached. In no case, however, was *diminution* of resistance caused by longitudinal magnetization for the highest stress which could be put upon the wire without breaking it,* and this, too, when strengths of current of very different degrees were tried.

Experiment LXI.

An annealed nickel wire, ·105 centim. in diameter, was arranged in the same manner as the iron wire in the last experiment; but the clamps *a* and *c* (fig. 19) were placed nearer together, and just inside the coil A, so that the whole of the nickel experimented on would be under the influence of the magnetizing force. In this case the coolers, S S, were dispensed with, and, instead, the wire to be tested was provided with a solenoid of fine silk-covered copper wire, wound in two layers on a glass tube of the same length as that of the nickel wire under examination, and of a diameter such that the latter could be easily slipped inside it. This solenoid served, when required, to give the relative amounts of magnetism imparted to the steel by the different magnetizing forces. The alterations of resistance produced by the magnetism were first determined, then the galvanometer having been disconnected from the "bridge" and joined up with the solenoid, the induced currents caused by the magnetization were measured by the

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* This was rather unexpected; since Joule has shown (Phil. Mag., 1847, vol. xxx., pp. 76, 225) that whilst iron free from stress is increased in length by longitudinal magnetization, yet when loaded beyond a certain limit its length is diminished by the same cause.

"throw" of the needle, both with and without the B.C. flowing through the nickel. A preliminary set of observations had given the means of determining the amount of current which would be induced when the nickel was not in the coil, and, therefore, by subtraction the induced currents due to the nickel only could be determined. The glass tube and the fine silk-covered copper wire on it would have served to shield the nickel from any change of temperature likely to be caused by the magnetizing current; but as a further precaution, the solenoid was well wrapped up in paper so that it would just fit inside the coil A. The comparison-wire was also surrounded with glass and caoutchouc tubing, and, as with the iron, the whole arrangement was well covered over with baize.

The following values (Table XXVI.) of the alterations of resistance are the means of five or six trials with each of the various magnetizing forces employed.

TABLE XXVI.

M.C. in divisions of the scale of the tangent galvanometer α.	Throw of the galvanometer needle due to induction current caused by magnetization of the nickel when the load on the wire = 0 kilog. β.	Increase of resistance in terms of division of the platino-iridium wire when the load on the wire = 0 kilog. γ.	$\frac{\gamma}{\alpha}$.	$\frac{\gamma}{\beta}$.	Increase of resistance calculated from the formula $\gamma = a.\alpha + b.\beta$ $a = \cdot 2992$ $b = \cdot 1514$.	Increase of resistance caused by magnetism with 2 kilogs. on the wire.	Increase of resistance caused by magnetism with 6 kilogs. on the wire.	Increase of resistance caused by magnetism after the removal of the 6 kilogs. and after a rest of two days.
19	8·7	7·0	·368	·805	7·00	6·0	2·0	8·6
35	16·2	12·4	·354	·765	12·92	. .	. .	. .
53	25·6	19·1	·360	·746	19·73	. .	. .	. .
69	31·0	24·6	·356	·781	25·30	24·5	19·8	33·1
86	36·8	31·3	·364	·851	31·30	. .	. .	. .
159	56·4	57·0	·358	1·011	56·10	53·0	45·0	80·0
279	81·5	95·8	·343	1·175	95·80	99·1	85·2	131·0

It is evident from the last experiment that the increase of resistance which can be produced by magnetizing nickel wire longitudinally is diminished by temporary longitudinal stress not carried beyond a certain limit, provided the magnetizing force does not exceed a certain critical value depending upon the amount of stress applied. Thus we see that when the value of α is somewhere between 159 and 279, the load of 2 kilogs. begins to increase the alteration of resistance caused by the magnetization. Now, Sir W. THOMSON has proved*—and in the case of this particular wire I have been able to verify the fact†—that with nickel the magnetism induced by any magnetizing force is increased or diminished by stress according as the magnetizing force does or does not exceed a certain critical value. With iron, on the contrary, the induced

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* "Electrodynamic Qualities of Metals"—Part VII., Phil. Trans., Part I., 1879.

† I shall have occasion to refer to this experiment in Part III. of my paper.

magnetism is increased by stress, provided the magnetizing force does not reach a certain critical value, which however is very much less, other circumstances being the same, than is the case with nickel. It is possible, therefore, that if much smaller magnetizing forces had been employed in Experiment LIX., the increase of resistance caused by magnetism would be found to be heightened by the loads employed in that experiment.

THE EFFECTS OF PERMANENT LONGITUDINAL EXTENSION OF TORSION AND OF TEMPERING ON THE ALTERATION OF ELECTRICAL RESISTANCE PRODUCED BY MAGNETIZING.

The last experiment shows that moderate permanent logitudinal strain largely increases in the case of nickel the susceptibility to alteration of resistance from longitudinal magnetization. A similar effect is produced on iron, but as the whole point will be more fully discussed in Part III., it will suffice here to state that several experiments made according to the above plan, and also others where the comparison-wire and the wire to be tested were placed together in the magnetizing coil, proved, undoubtedly, that moderate permanent strain *increases* the susceptibility, but that this increase, after reaching a maximum, begins to decline, so that in some cases after the wire had been broken by the stress applied, the susceptibility appeared to be less than it was before the wire had been subjected to any strain. The above-mentioned maximum point depends upon the amount of magnetizing force in a manner to be hereafter described.

The effect of permanent torsion up to a certain point was to *diminish* the susceptibility to alteration of resistance from longitudinal magnetization, and the amount of diminution was independent of the direction either of the M.C. or the B.C.

The following experiment shows that in hard steel longitudinal magnetization increases the electrical resistance.

Experiment LXII.

A piece of a steel knitting-needle, 7·62 centims. in length and ·23 centim. in diameter, was hardened by heating it to a bright red and then plunging it into cold water. The piece was connected up with another of similar dimensions and similarly prepared, and both having been well covered with caoutchouc and silk, were made to form two branches of a "WHEATSTONE'S bridge." The coil B was used to impart magnetism, and with a large LECLANCHÉ for the B.C., and four GROVE'S cells for the M.C., an *increase* of resistance represented by 30 divisions of the iridio-platinum wire was obtained. The brass clamps used to connect the pieces of steel with each other, and with the other branches of the "bridge," were so massive that even in this case their resistance is neglectable; and since 30 divisions of the iridio-platinum wire would show an alteration of resistance of ·010 per cent., we may assume that this last number represents approximately the extent to which the conductivity of the steel was diminished by the longitudinal magnetization.

Moreover, the steel under examination was well within the coil, the comparison-piece being, of course, outside, and also at right angles to the coil; and it was calculated that the increase per unit of resistance would for unit magnetizing force be $\frac{\cdot 00010 \times 9\cdot 7}{814 \times \cdot 68 \times 4\,\pi \times c} = 69\cdot 8 \times 10^8$, since c, the strength of the magnetizing current here, approximately amounted to ·2 absolute unit.

It will be observed, by comparing this last result with the corresponding one in Experiment LXVIII., that the alteration of resistance produced by a given magnetizing force is very much less with the hardened steel than that caused by the same magnetizing force in a steel rod of the same diameter, but in the same condition as it was when received from the makers. Several other trials were made with the same piece of steel, in which smaller and smaller amounts of M.C. were employed, but in no case could any alteration of the nature of a *decrease* of resistance be observed.

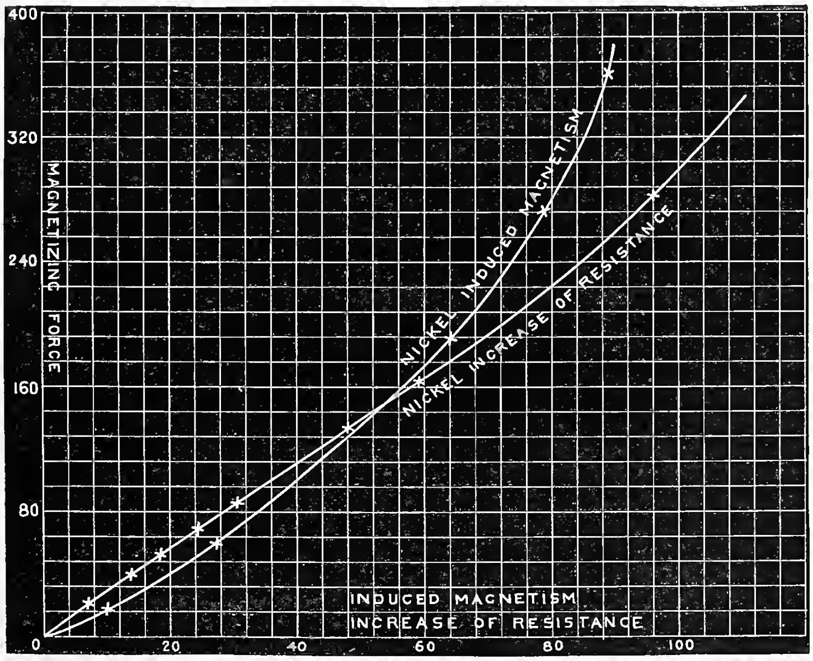
A similar experiment had been tried with pianoforte steel wire, hardened in the same way, and with various amounts of M.C., but the results were of the same nature, though much less in amount, as with the knitting-needle. It may be added that both the knitting-needle and the wire were made so hard that they were quite brittle, and with both there was a permanent as well as a temporary increase of resistance produced by the magnetization.

AN ATTEMPT TO DETERMINE RELATIONS BETWEEN THE ALTERATION OF ELECTRICAL RESISTANCE PRODUCED BY ANY MAGNETIZING FORCE, THE FORCE ITSELF, AND THE MAGNETISM INDUCED BY THE FORCE.

On consulting the fourth, fifth, and sixth columns of Table XXVI., it will be observed that the increase of resistance ensuing from magnetization depends not only upon the magnetism induced, but also upon the magnetizing force itself; and, in fact, we may say that if γ denotes the increase of resistance, whilst α and β represent the magnetizing forces and the magnetism induced respectively, $\gamma = a.\alpha + b.\beta$, where a, b are two constants. In the case of the nickel wire, α and β were measured in terms of the divisions of the scale of the tangent galvanometer, and of the scale of the THOMSON'S reflecting galvanometer respectively; whilst a and b were calculated from the observations made with $\alpha = 279$ and $\alpha = 86$. The agreement between the observed and calculated values of γ is good, and certainly quite equal to that between the different observations made with the same values of α. Columns 4 and 5 show clearly that the alteration of resistance depends in this case more on the value of the magnetizing force than on the magnetism induced, and from the fact that $\frac{\gamma}{\alpha}$ is nearly constant throughout, whereas $\frac{\gamma}{\beta}$ rapidly increases for the higher values of α, we are led to infer that the alteration of resistance would go on increasing as the magnetizing force increased, even when there might be no appreciable advance in the value of the magnetism induced.

A glance at the curves in Table XXVII. will serve to confirm the above view. These curves have their abscissæ representing both the induced magnetism and the alterations of resistance produced by the various magnetizing forces, which latter are measured by the ordinates of the curves. The induced magnetism* is represented on a scale of 1 millim. to one division of the scale of the THOMSON'S reflecting galvanometer, and the alteration of resistance on a scale of 1 millim. to one division of the platino-iridium wire. The ordinates are on a scale of 1 millim. to two divisions of the scale of the tangent galvanometer, and each of these latter divisions represents a current of ·00023 absolute unit. Each division of the iridio-platinum wire represents an alteration of resistance $=\cdot 000034$ per unit; if therefore we take the average of the first six values of $\frac{\gamma}{\alpha}$, namely, ·360, as representing the average alteration of resistance effected by a magnetizing current producing a deflection of one division on the scale of the tangent galvanometer, we find that the increase of resistance per unit produced in the nickel wire by unit magnetizing force $=\frac{\cdot 36\times\cdot 000034\times 30}{\cdot 00023\times 2100\times\cdot 75\times 4\pi}=8070\times 10^{-8}$.

TABLE XXVII.—Curves showing the increase of resistance and the amount of induced magnetism produced in nickel wire by different magnetizing forces.



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* There was no appreciable difference between the induced magnetism as determined with the B.C. flowing and that without.

Experiment LXIII.

An annealed iron wire, ·094 centim. in diameter, was arranged with the same precautions and in the same manner as the nickel wire in the last experiment; but as it was found difficult to make observations in the ordinary way in consequence of the VILLARI's "shock-currents" being very pronounced, the B.C., for which one GROVE's cell was employed, was kept flowing until the wire and the comparison-wire had assumed a sufficiently stable resistance-ratio which was very nearly equal to unity. The alteration of resistance produced by various magnetizing forces was measured by the deflection of the image of the illuminated wire on the scale, and the mode of taking the readings and the nature of the corrections to be applied for the direct action of the magnetizing coil on the galvanometer are described in Experiments LXIX. and LXXI.

The following table contains the results of this experiment:—

TABLE XXVIII.

M.C. in divisions of the scale of the tangent galvanometer. α.	Throw of the galvanometer needle due to the induction current caused by the magnetization of the iron. B.C. flowing. β.	Throw of the galvanometer needle due to the induction current caused by the magnetization of the iron. B.C. not flowing. β'.	Increase of resistance in terms of divisions of the galvanometer scale. γ.	$\frac{\gamma}{\alpha}$.	$\frac{\gamma}{\beta}$.	$\frac{\beta}{\beta'}$.	Increase of resistance calculated from the formula $\gamma=a.\alpha+b.\beta$ $a=\cdot0397$ $b=\cdot106$.
21	20	17	1·9	·091	·095	1·18	2·9
43	56	42	5·0	·116	·090	1·33	7·7
82	107	71	13·1	·160	1·220	1·51	14·6
130	134	91	19·3	·148	1·440	1·47	19·4
203	153	103	23·9	·118	1·562	1·49	23·9
383	200	. .	36·9	·096	1·845	. .	36·4
519	220	172	43·9	·085	2·000	1·28	43·9

Table XXVIII. shows that with iron as with nickel the increase of resistance can be represented by the formula $\gamma=a.\alpha+b.\beta$,[*] and, here again therefore, it seems as if the alteration of resistance would go on increasing as the magnetizing force increased even when there would be no perceptible increase of induced magnetism. The cases of iron and nickel, however, differ considerably in one respect, namely, that whereas with the latter metal the magnetizing force played the more important part in altering the

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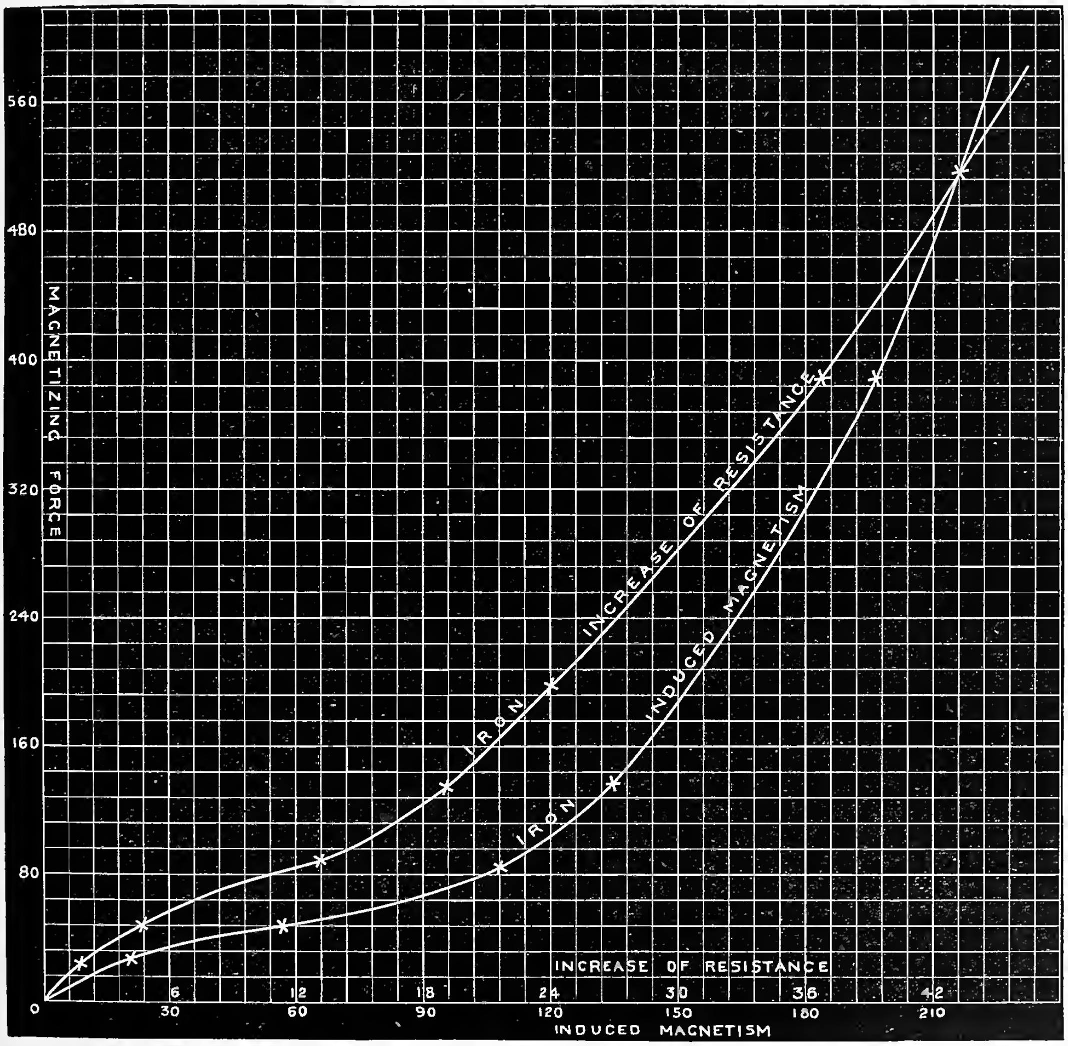
[*] The discrepancies between the observed and calculated values of γ for the first three magnetizing forces is, I believe, almost if not entirely due to the fact that, unfortunately, before any exact measurements had been made with the lower values of α, the full magnetizing force had been employed. This would not perhaps have affected the result, as far as the agreement between observation and calculation is concerned, had the induced magnetism been measured at the same time as the alteration of resistance, but this was not the case.

resistance, with the former, for moderate values of the M.C. the greater part of the alteration is effected by the induced magnetism.

Table XXIX.—Curves showing the increase of resistance and the amount of induced magnetism produced in iron wire by different magnetizing forces.



The curves in Table XXIX. showing the amount of magnetism induced and of the alteration of resistance are constructed on the same lines as those in Table XXVIII., except that in consequence of the alteration of resistance being much less and the induced magnetism greater with iron than with nickel, the alteration is represented on a scale of 1 millim. to ·3 division of the scale and the induced magnetism on a scale of 1 millim. to $1\frac{1}{2}$ divisions of the scale. Each division of the scale represents an alteration of resistance amounting to ·000023 per unit, and if we take the mean of the third and fourth values[*] of $\frac{\gamma}{\alpha}$, which may perhaps be assumed to represent approximately the average alteration of resistance for a given moderate magnetizing force, we find that with this wire the increase per unit of resistance produced by an absolute electromagnetic unit of magnetizing force would be 2335×10^{-8}.

It is desirable to draw attention also to the fact that with both iron and nickel there is no change of resistance of the nature of a *decrease* produced by magnetization, but that starting with a current from one GROVE's cell through a total external resistance of about 15 ohms[†] and with a magnetizing force not greater than 26 times the earth's horizontal magnetic force at the place, we find a continuous *increase* of resistance as the magnetizing force is increased.

THE EFFECT OF ALTERING THE STRENGTH OF THE B.C. ON THE CHANGE OF RESISTANCE PRODUCED BY ANY MAGNETIZING FORCE.

The results recorded in the last two experiments are so far at variance with AUERBACH's views already alluded to, that it seemed advisable to still further test these views by altering the strength of the B.C. whilst that of the M.C. is maintained constant.

Experiment LXIV.

A piece of the same annealed nickel wire as that used in Experiment LXI. was tested with one GROVE's cell and a total external resistance of 15 ohms for the M.C., and with from one to three GROVE's cells with no external resistance save that offered by the "bridge" and its connexions for the B.C.

Deflection of the needle of the tangent galvanometer, showing the strength of the B.C.	Increase of resistance caused by the magnetization in terms of the divisions of the iridio-platinum wire.
12°	10·1
15	10·3
17½	10·9

The numbers given in the second column are the means of several observations, and agree very fairly with each other, the difference between them being within the errors of observation, and what difference there is would show that we have a slightly *greater* increase of resistance for large values of the B.C. than for small ones.

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[*] The first two values of $\frac{\gamma}{\alpha}$ are not included, for the reason previously mentioned.

[†] That is, 10 ohms in addition to the resistance of the coil and its connecting wires.

Experiment LXV.

A piece of the same iron as had been used in Experiment LXII. was tested with one Grove's cell and no external resistance save that of the magnetizing coil for the M.C., and three Grove's cells with adjustable external resistance for the B.C.

B.C. in terms of the divisions of the scale of the tangent galvanometer.	Increase per unit of resistance. M.C. = 109 divisions of the scale of the tangent galvanometer.
130	·000300
940	·000316
1521	·000353

In this experiment, which was conducted in the same manner as Experiment LXII., there is evidently a *greater* alteration produced by magnetism when the B.C. has a high value than when it has a low one, and the differences between the different values in the second column are certainly larger than could be attributed to errors of observation. Now in Experiment LXVIII. it will be shown that with unannealed steel, and in Experiment LXI. it has been shown with annealed nickel, that there is little or no difference in the amount of alteration of resistance effected by magnetism when the B.C. is made to vary in amount; and the reason is apparent, for with the nickel and the steel there was no* appreciable difference between the induced currents caused by the magnetization of these metals when the B.C. was flowing and when it was not, whereas, if we turn to the second, third, and seventh columns of Table XXVIII., we see that there is a very appreciable difference in the case of the annealed iron wire, between the induced currents with and without the B.C.

All these experiments are in direct contradiction to those of Auerbach, but yet it was thought fit to try others with annealed pianoforte-steel wire.

Experiment LXVI.

A piece of annealed pianoforte-steel wire, 10 centims. long and ·085 centim. in diameter, was tested with various battery-power from one to four Grove's cells for the B.C., and battery-power varying from one Grove's cell with a resistance of 10 ohms in the external circuit besides the resistance of the coil A, to seven Grove's cells with no external resistance save that of A for the M.C.

In no case was a *diminution* of resistance produced by magnetization. With one Grove's cell for the B.C., and seven Grove's cells for the M.C., an increase of resistance of ·0585 per cent. was observed: lower values of the M.C. gave smaller and smaller results as the M.C. diminished. The alteration of resistance produced by unit magnetizing force was estimated in the usual manner to be 1500×10^{-8} per unit.

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* That is, no difference of such an amount as to make it seem worth while at the time to record it.

Experiment LXVII.

A strand of four pieces of the same length and of the same steel as that used in the last experiment was tested with one GROVE's cell in the B.C., and seven in the M.C., and the alteration of resistance, which, however, could not be accurately measured, was certainly not *greater* than that of the single wire when the same battery-power was employed for both the M.C. and the B.C. The change of resistance of the compound strand under the above-mentioned conditions was measured at ·040 per cent. with a probable error of 25 per cent.

THE EFFECT OF LONGITUDINAL MAGNETIZATION ON THE ELECTRICAL RESISTANCE OF A BAR OF STEEL.

As my earliest investigations recorded in the previously-mentioned "Preliminary Notice" had seemed to show that the alteration of the electrical resistance of iron and steel which can be produced by magnetization is very much greater, for the same amount of magnetizing force, when the metals are in the form of comparatively thick rods than in that of wires, the following experiment was made.

Experiment LXVIII.

A steel knitting-needle, taken in the ordinary condition, 23 centims. in length and ·233 centim. in diameter, was provided with two copper terminals, 14 centims. in length and ·410 centim. in diameter, holes having been bored $1\frac{1}{2}$ centim. in depth at one end of each terminal so as to admit the ends of the needle. A similar and similarly-furnished needle served as the comparison-piece, and the two were connected with each other and with the other parts of the "bridge" in the same manner as the wires in the last experiments had been. The clamps used, however, were more massive and the whole of the steel to be magnetized was well within the coil A, whilst the comparison-piece was outside the coil and at right angles to it. The amount of magnetism induced was measured as before, but the resistance introduced into the circuit of the galvanometer employed for this purpose had to be made very considerably greater than was the case with the iron wire, in order that the "throw" of the needle might be reduced to the proper extent. The B.C. was produced by four GROVE's cells, and the M.C. by seven GROVE's cells, each with adjustable external resistance. The following are the results obtained :—

TABLE XXX.

M.C. in divisions of the scale of the tangent galvanometer $=\alpha$.	"Throw" of the galvanometer needle due to induction current by magnetization of the steel $=\beta$.	Increase of resistance when B.C. = 19½° of tangent galvanometer, the increase measured in divisions of platino-iridium wire.	Increase of resistance when B.C. = 29°.	Increase of resistance when B.C. = 38°.	Mean increase of resistance $=\gamma$.	Increase of resistance calculated from formula $\gamma' = {\cdot}0386 \times \alpha + {\cdot}0588 \times \beta$.	$\frac{\beta}{\alpha}$.	$\frac{\gamma}{\beta}$.
23	7	1·35	. .	. .	1·35	1·30	·31	·193
108	32	6·48	. .	. .	6·48	6·05	·30	·203
152	48	8·83	8·50	8·75	8·69	8·69	·32	·181
267	82	. .	15·00	. .	15·00	15·13	·31	·183
347	102	18·88	20·30	19·00	19·39	19·39	·30	·190

From this experiment we learn that the magnetism induced in the steel is, for the magnetizing forces employed, very nearly proportional to these latter, and also that the alteration of resistance is nearly proportional to the induced magnetism, and therefore to the magnetizing force. Still more closely can the alteration of resistance be calculated from the formula $y=a.\alpha+b.\beta$, where the constants a and b, given as ·0386 and ·0588, are determined from the alterations of resistance caused by values of the M.C. equal to 347 and 152 divisions of the scale of the tangent galvanometer.

Alteration of the strength of the B.C. seems to have little or no influence on the change of resistance produced by a given amount of magnetizing force, and therefore we may assume that the product of the mean values of $\frac{\beta}{\alpha}$ and $\frac{\gamma}{\beta}$ will fairly represent the alteration which would be wrought by a current in the magnetizing coil which would suffice to deflect the needle of the tangent galvanometer through one division of the scale.

This product $={\cdot}0589$, and since a division of the iridio-platinum wire corresponds to an increase per unit of the resistance of the steel $={\cdot}000032$,* and, since also one division of the scale of the tangent galvanometer represents a current of ·00023, whilst the average magnetizing force due to unit current would in the present instance be $\frac{2100 \times {\cdot}82 \times 4\pi}{30}$, we see that the maximum increase of resistance obtained by the magnetization was ·062 per cent., and that the increase of resistance per unit for unit magnetizing force would be 1137×10^{-8}.

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* Of course correction is here, and in all similar cases, made for the resistance of the terminals, which, however, with these rods was very small.

THE EFFECT OF LONGITUDINAL MAGNETIZATION ON THE ELECTRICAL RESISTANCE OF A BAR OF NICKEL.

Experiment LXIX.

A bar of nickel, 8·3 centims. long and ·70 centim. in diameter, was soldered to two stout copper terminals, whilst a similar bar, similarly provided, served as a comparison piece. The bars were arranged in the same manner as the steel bars in the last experiment, but the magnetizing coil B was used instead of A. Before placing the nickel in B it was covered with several layers of stout caoutchouc, and the comparison piece having been furnished in like manner, the usual precautions of well covering both bars were taken. The B.C. was furnished by two GROVE'S cells and the M.C. by eight GROVE'S cells with adjustable external resistance. The circuit of the B.C. was kept closed, and the alteration caused by magnetization in the resistance of the bar was measured by the deflection of the image of the illuminated wire on the scale.*

As the resistances to be compared are in this case very small, it is advisable to show how far any measurements of alteration of resistance can be depended upon, and for this purpose the first set of readings with the smallest M.C. are given :—

Total deflection caused by passing the M.C. in terms of divisions of the galvanometer-scale. + signifies increase of resistance.†	Number of trial.
+8·50	1
+8·25	2
+7·00	3
+6·25	4
+7·75	5
+8·50	6
+8·00	7
+9·00	8
+8·25	9
+7·75	10
+7·25	11
+7·9	Mean.

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* In this, and in every other instance in which such a mode of measuring alteration of resistance was adopted, the direct action of the magnetizing coil and of the included metal core on the galvanometer when the latter was not in circuit was always determined by a separate set of experiments, as though such action was small, it could never be entirely avoided. The distance (several yards) of the coil from the galvanometer was however such that no perceptible difference in the sensibility of the latter was introduced when the M.C. was closed. Similar remarks apply to the tangent galvanometer, and the readings given are in the case of both instruments always corrected for the above mentioned direct action. For the mode of taking the readings see Experiment LXXI. on Bismuth.

† Each number is calculated from three consecutive readings in the manner described in Experiment LXXI.

The deflection due to direct action of the electromagnetic solenoid and included core when the galvanometer was disconnected from the "bridge" $= +2\cdot 9$.

Therefore the deflection due to alteration of resistance $= +5\cdot 0$.

Immediately after taking the above readings the effect of altering by ·1 ohm the side of the "bridge" adjacent to the side containing the bar was found to be a deflection of 190 divisions on the scale, and since the ratio of the resistance of the bar and its copper terminals to that of the comparison piece was $\frac{12\cdot 2}{10\cdot 0}$, it was assumed that the increase of resistance amounted to $\frac{5\cdot 0 \times \cdot 1}{190 \times 12\cdot 2}$ per unit. The deflection of the needle of the tangent galvanometer was in this case 4°, and a similar set of observations was made when the M.C. produced deflections of 6·25°, 8·25°, 10·75°, and 13°, with the results recorded below.

TABLE XXXI.

Deflection of tangent galvanometer $= \phi$.	$\tan\phi + \cdot 0084\phi \sec\phi = c$.	Increase of resistance due to magnetization in terms of divisions of the galvanometer scale $= a$.	$\frac{a}{c}$.	$\frac{a}{c^2}$.
4·00°	·104	5·0	48·1	462
6·25	·163	9·8	60·1	369
8·25	·215	17·0	79·1	368
10·75	·284	24·0	84·5	298
13·00	·343	32·5	94·4	275

It would seem that the increase of resistance produced by the magnetization varies in this case more nearly as c^2 than as c; if, however, we take the mean value of the numbers for $\frac{a}{c}$ we shall probably obtain a sufficiently close approximation to what can only be regarded as a rough measurement of the effect of magnetization on the resistance. This mean value is 73·2. The resistance of the nickel only was to that of the nickel and the connexions in the ratio of 3·8 to 12·2, and since the value of c must be multiplied by ·316 in order to obtain the value of the current in C.G.S. units, and since moreover a unit current would produce an average magnetizing force in this case of $\frac{814}{9\cdot 7} \times \cdot 7 \times 4\pi$, we see that the increase per unit of magnetizing force would on the whole be $\frac{73\cdot 2 \times \cdot 1 \times 9\cdot 7}{\cdot 316 \times 3\cdot 8 \times 190 \times 814 \times \cdot 7 \times 4\pi}$, or 4343×10^{-8}. The bar was cast, and used in the same state as sent by the makers;* when annealed the value given above would be considerably greater, as it was afterwards ascertained that annealing very largely increased the capacity for induction from moderate magnetizing forces.

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* For this bar and for the bars of cobalt and bismuth used in the next experiments I am indebted to Messrs. JOHNSON, MATTHEY, and Co. All the bars here mentioned were very nearly chemically pure.

THE EFFECT OF LONGITUDINAL MAGNETIZATION ON THE ELECTRICAL RESISTANCE OF A BAR OF COBALT.

Experiment LXX.

A bar of cobalt, 8·5 centims. long and ·75 centim. in diameter, was provided with stout copper terminals, and balanced against a similar bar. The precautions taken and the mode of experimenting were exactly the same as with the nickel bar.

TABLE XXXII.

Deflection of tangent galvanometer $=\phi$.	Tan ϕ + ·0084ϕ sec ϕ $=c$.	Increase of resistance due to magnetization in terms of divisions of galvanometer scale $=a$.	$\frac{a}{c}$.	$\frac{a}{c^2}$.
4°·00	·104	1·40	13·5	130
6·25	·163	3·90	24·0	147
13·00	·343	8·50	24·8	72

In this case the values of $\frac{a}{c}$ agree quite as well as $\frac{a}{c^2}$, and if we take the mean of the former, namely 21·1, we obtain an increase of resistance per unit attending magnetization by a unit force $=628 \times 10^{-8}$; a number, it will be noticed, only about one-seventh of that obtained in the case of the nickel. The cobalt was, like the nickel, unannealed, and annealing would have caused the effect of a moderate magnetizing force to be greater, though not so much greater as would be the case with nickel.

In both the nickel and cobalt bars there was a *permanent* increase of resistance produced by magnetization, the maximum alteration amounting in nickel to about ·125 per cent., and in cobalt to ·025 per cent.; so that the maximum permanent alteration of resistance which is caused by magnetization is much greater with nickel than cobalt.

With both bars the M.C. was found to produce very nearly the same alteration of resistance, whether passed in one direction or the other through the coil.

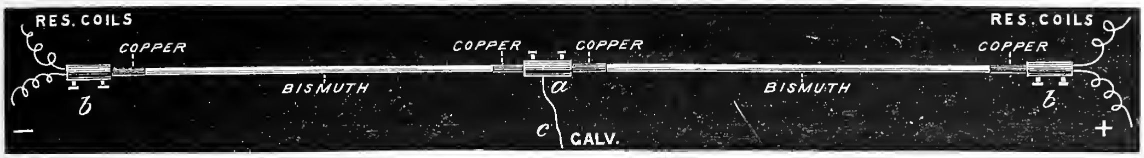
THE EFFECT OF LONGITUDINAL MAGNETIZATION ON THE ELECTRICAL RESISTANCE OF A BAR OF BISMUTH.

Experiment LXXI.

As it seemed desirable to ascertain whether diamagnetic substances would have their resistances altered in the same direction as paramagnetic ones, some experiments were made with bars of bismuth, but as the magnetization which can be imparted by an electromagnetic solenoid to bismuth is very much smaller than is the case with iron, nickel, or cobalt, it is proper that rather fuller details should be given in describing the results obtained so that the trustworthiness of these results may be duly estimated.

A bar of bismuth, 25 centims. in length and ·330 centim. in diameter, was provided with copper terminals of the same diameter, and connected up with a similar bar to the "bridge," as shown in fig. 20. In this figure the relative dimensions of the bismuth bars and their copper terminals, as well as the mode of establishing the "bridge," are sufficiently shown. To the binding screw, *a*, a silk-covered copper wire, *c*, was soldered to connect the bars with the galvanometer, whilst the two binding screws, *b b*, were provided with silk-covered copper wires leading to the poles of one Grove's cell and to two resistance coils of 100 ohms each in the usual manner. The coil A was employed, and the bars were arranged in the same manner and with the same precautions as the iron wire used in Experiment LIX., except that now it was necessary to dispense with the glass tubes in order that the bismuth bars could be slipped into the copper vessels through which the water flowed. The bars were, however, well varnished and covered with tissue-paper so as to ensure thorough insulation. The B.C. was kept closed and the alteration of resistance observed in the same manner as with the nickel and cobalt bars. The M.C. was produced by 13 Grove's cells, and the deflection of the needle of the tangent galvanometer was 22·5°. The readings given below are determined in each case as usual from three consecutive readings; thus, *a b c* being three consecutive readings the recorded number is $\frac{a+c+2b}{4}$.

Fig. 20.



Number of trial.	Deflection of galvanometer-needle in scale divisions. + signifies apparent increase of resistance on magnetization.
1	10·00+
2	10·00+
3	9·50+
4	10·25+
5	11·00+
6	10·50+
7	10·25+
8	10·75+
Mean	10·23+

The B.C. was now taken off and the direct action of the magnetizing coil on the galvanometer was found to produce a deflection of 8·00+. Therefore there seemed to be an increase of resistance caused by magnetization represented by 2·23 scale divisions. The B.C. was again put on, and by taking out and putting in ·1 ohm several times it was ascertained that this caused a movement of the image of the wire through 74·0 divisions. Moreover, when the M.C. was flowing, putting in or taking out ·1 ohm caused the same effect, and therefore the M.C. did not affect the sensibility of the galvanometer. In this case ·1 represented an alteration of resistance amounting to ·001 per unit, and therefore the increase of resistance from magnetization would be $\frac{2{\cdot}23 \times {\cdot}001}{74}$ per unit$=$·000031 per unit and ·0031 per cent.

The B.C. was now reversed and the following observations taken:—

Number of trial.	Deflection of galvanometer-needle in scale-divisions. + signifies apparent increase. – signifies apparent decrease.
1	7·50–
2	8·00–
3	8·00–
4	7·75–
5	7·75–
6	8·25–
Mean	7·88–

The B.C. was again taken off and the direct action of the coil on the galvanometer appeared to be now 9·5–.* Here, therefore, there would be an increase of resistance equal to that represented by 1·62 scale divisions, and the amount of increase would be ·0022 per cent. Again, the resistances of 100 ohms on two sides of the bridge were replaced by 10 ohms, so that now the arrangement having become more sensitive, a set of observations similar to the above gave an increase of resistance equal to 3·88 divisions of the scale, corresponding to an increase of ·0029 per cent. From the three sets of observations it was concluded that the electrical resistance of bismuth is *increased* by longitudinal magnetization by ·0027 per cent. for the amount of magnetizing force here employed. The increase per unit of resistance for a unit magnetizing force would be 21×10^{-8}.

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* For some reason the direct action of the magnetizing coil on the galvanometer was never quite the same for both directions of the M.C.

THE EFFECT OF LONGITUDINAL MAGNETIZATION ON THE ELECTRICAL RESISTANCE OF ZINC FOIL.

Experiment LXXII.

A piece of commercial zinc foil, 14 inches in length and ·040 millim. in thickness, was wrapped lengthwise round a soft iron bar of circular section, $\frac{1}{2}$ inch in diameter and 15 inches in length, which had previously been coated with two layers of brown paper. The width of the foil was such that when wound round the bar the edges just overlapped. The foil was secured in position by fine yet strong twine, and having been covered with two folds of brown paper the whole was placed centrally in the coil A. At the two ends the foil was cut so as to allow of these ends being clamped in the usual manner in brass blocks, and a strip of foil of similar dimensions served as the comparison-piece. The same mode of experimenting and the same precautions were taken as with the bismuth bar. Seven GROVE's cells were used for the M.C., and these produced a deflection of 15° of the needle of the tangent galvanometer. Several trials which accorded very fairly with each other showed a mean *increase* of resistance represented by 3·8 divisions of the scale. The M.C. was reversed, and again several trials showed an increase of resistance, the mean value of which was represented by 3·5 divisions. From the data obtained it was calculated that the increase of resistance amounted to ·0148 per cent. The iron core having been removed no appreciable change in the resistance of the zinc foil could be detected on passing the M.C.

THE EFFECT OF LONGITUDINAL MAGNETIZATION ON THE ELECTRICAL RESISTANCE OF COPPER WIRE.

Experiment LXXIII.

A piece of silk-covered copper wire, 12 feet in length and $\frac{1}{50}$th of an inch in diameter, was doubled backwards and forwards so as to form a bundle 1 foot in length. The whole was then well coated with shellac varnish, and when dry inserted into one of the copper coolers (fig. 19), which was placed in the coil A. A similar bundle served as the comparison-piece, and the two bundles were connected up in the usual manner with the bridge. One GROVE's cell was used for the B.C., and the deflection of the tangent galvanometer produced by the M.C. was 16°. The B.C. was kept on for 10 minutes, and whilst still on the following readings were taken :—

Number of trial.	Apparent alteration of resistance caused by magnetization. – signifies apparent decrease of resistance.
1	6·0 –
2	4·5 –
3	6·5 –
4	6·5 –
5	6·5 –
6	7·0 –
7	7·0 –
8	6·5 –
9	7·5 –
Mean	6·44–

The B.C. was now taken off, and the direct action of the coil on the galvanometer was found to be 6·50. There would therefore, on the whole, appear to be an *increase* of resistance of 0·6. The B.C. was again put on, and a similar set of observations to the above produced when the the M.C. was reversed, an apparent increase of 7·80 divisions; whilst the direct action of the coil on the galvanometer caused a deflection of 8·00 in the same direction, so that now, on the whole, there would appear to be a *decrease* of resistance represented by ·20 division of the scale. The mean result of the two sets of observations would give a decrease of resistance represented by ·07 division of the scale. Now, in this case, the galvanometer had been made so sensitive by proper use of the adjusting magnet that an alteration of ·1 ohm on one of the two sides of the bridge, containing each 100 ohms, caused a deflection of 300 divisions of the scale; accordingly 1 division of the scale would represent an alteration of resistance amounting to $\frac{1}{3000000}$ per unit, and ·07 an alteration of *less than one in four millions.* It is needless to say that this experiment shows that there is no reliable change of resistance to be detected even with the comparatively large magnetizing force employed in this case—a force which would be more than 480 times that of the earth's magnetic horizontal force at the place. The copper in this case was the ordinary copper wire usually employed for electrical purposes, but other experiments were made with chemically pure copper, and these all yielded results quite as negative as those just recorded. There is no doubt that the electrical resistance of copper is altered by magnetization; but in order to detect such alteration we should, in all probability, require the aid of a very powerful electromagnet* and a galvanometer in a very sensitive condition. Such change of resistance must be exceedingly small even with the most powerful magnetizing force that we can at the present time bring to bear—much smaller than would seem to

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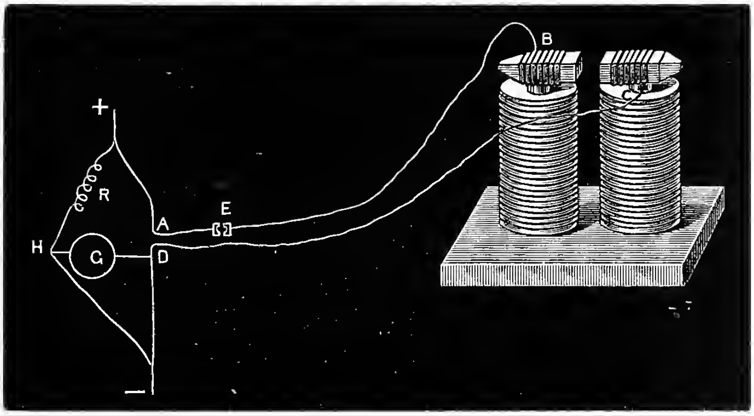
* One might, perhaps, by adopting the same plan as that used with the zinc foil, succeed in obtaining evidence of alteration of resistance in the case of copper foil.

follow from **Stewart** and **Schuster**'s experiments, some of the details of which will now be discussed.

Discussion of Stewart's and Schuster's Experiments on the Alteration of the Electrical Resistance of Copper by Magnetization.*

Stewart and **Schuster**, in a preliminary notice, bring forward the results of certain experiments which in their opinion seemed to prove that the electrical resistance of copper wire is altered by magnetization; but a glance at their mode of operating serves to show that the effects observed by them cannot be relied upon. In fig. 21 (copied from Phil. Mag., 1874) A B C D is a caoutchouc-covered copper wire several yards in length wound round the armatures of an electromagnet, and R is another resistance against which the copper wire is approximately balanced. The alteration of resistance was observed from the "*throw*" of the needle of the galvanometer G, caused by closing the B.C. by means of the contact-breaker E, first when the electromagnet was actuated by six **Grove**'s cells and then without any current in the M.C., or *vice-versâ*. The galvanometer was in such a position with reference to the electromagnet that the latter produced very little direct effect on the former, and since they obtained *momentarily* an increase of potential at D compared with the potential at H when the M.C. was passing, they inferred that the magnetization imparted by the electromagnet to the wire decreased the resistance of the copper.

Fig. 21.



These experimenters, however, seem to have entirely overlooked the fact that the powerful magnetization imparted by the magnet to the armatures would alter the capability of the latter to receive fresh magnetism. Now, when the B.C. is closed at E, the current in the coil of wire wrapped round the armatures would induce magnetism in the latter, and this would in turn send an *induced current through the caoutchouc-covered wire in the opposite direction to that of the original*; if, then, the capability of

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* Phil. Mag., May, 1874.

receiving fresh magnetism is *lessened* in the armatures by previous powerful magnetization this induced current would be *lessened*, and thus, if their mode of experimenting was adopted, the resistance of the wire would *seem* to be decreased. The following experiment was tried with a view of testing the effect of previous magnetization on the capability of receiving fresh magnetism in the case of the armature of an electromagnet.

Experiment LXXIV.

A bar of soft iron, 12 inches long $\frac{1}{2}$ inch broad and $\frac{1}{2}$ inch thick, was placed across the cores of a small electromagnet, and separated from them by a piece of tissue paper. The cores of the electromagnet were 1 inch in diameter and 5 inches in length, and were wound round with the cotton-covered copper wire $\frac{1}{16}$th of an inch in diameter, and having a resistance of nearly 1 ohm. A battery of six GROVE's cells was employed with the electromagnet, and in order to test the change of susceptibility to magnetization, some 200 turns of rather fine silk-covered copper wire were made round one end of the bar of soft iron and distributed over a length of 4 inches. The coil thus formed was connected with the galvanometer, and was further insulated from the iron core by two layers of paper. The coil was then placed inside the coil B, which itself could be placed in the circuit of one large DANIELL's cell by means of a mercury-cup. The current induced by the magnetization imparted by the coil B to the soft iron bar was measured by the "throw" of the galvanometer-needle produced when the circuit of the DANIELL's cell was closed, first without exciting the electromagnet and then with this magnet in action. The electromagnet was at such a distance from the galvanometer that no error of importance caused by the direct action of the former would be introduced. The following experiments were then made :—

Condition of electromagnet. "On" signifies magnet excited; "off" not excited.	Deflection of the image of the illuminated wire on the scale.	Difference effected in the deflection by the action of the electromagnet. + signifies increase, – signifies decrease produced by exciting the electromagnet.	Number of trial.
Off	227	. .	1
	226	. .	2
	227	. .	3
On	270	43+	1
	223	. .	2
	186	41–	3
	186	. .	4
	186	. .	5
Off	212	26–	1
	202	16–	2
	202	. .	3
On	233	31+	1
	187	15–	2
	187	. .	3
Off	212	25–	1
	200	. .	2
	198	11–	3
	198	. .	4

The current through the electromagnet was then reversed.

Condition of electromagnet.	Deflection of the image of the wire.	Difference effected in the deflection by the action of the electromagnet.	Number of trial.
On	198	. .	1
	192	. .	2
	192	. .	3
Off	250	58–	1
	212	20–	2
	212	20–	3
On	204	8–	1
	202	10–	2
	202	10–	3

It will be noticed that for both directions of the M.C. the induced current is less with the electromagnet excited than when not.

Several phenomena connected with the experiments in the paper above alluded to can be explained by referring the apparent alteration of resistance to the permanent, sub-permanent, or temporary alteration of the magnetic susceptibility of the soft iron armatures employed; and, on the whole, when we consider that, as stated by them, the electromagnet had no apparent effect to make a piece of the wire set either axially or equatorially, we must regard their results with great suspicion.*

I myself seven years ago tried the effect of an electromagnet on a copper wire coiled several times round a flat piece of hard wood, and placed between the poles of a powerful electromagnet actuated by 12 Grove's cells, but in no case could I detect the slightest real alteration of resistance. The galvanometer then employed was only able to detect an alteration of 1 in 50,000 of the resistance, and I look forward with some interest to renewed experiments in the same direction with the much more sensitive instrument which I have at present.

The Effect of Annular Magnetization on Electrical Resistance.

Fig. 22.

Let A B C D, fig. 22, be a thin slice of iron (in the plane of the paper), through which an electric current is passing in the direction of the arrows. The molecules at the upper part, A B, will tend to take up positions such that their axes are perpendicular to the plane of the paper and with their north ends above it. The magnetization imparted to the molecules will diminish from A B to the axis E F, where it will be zero, and below this axis the molecules will be impelled to place their south poles above the plane of the paper. If now H K be an independent current passing below A B C D, it will tend to reverse the magnetism of the molecules at C D and strengthen that of the molecules at A B; but since the lower molecules are nearer to H K than the upper ones, the total effect would be a partial diminution of the annular magnetism imparted by the current flowing through A B C D. If, on the contrary, the current below the wire flows from K to H, the annular magnetism would on the whole be increased.

Now, some years ago, when making attempts to discover whether the resistance of an iron wire could be altered by passing a current above or below the wire, I was led to believe that such was the case. Among several other experiments, a knitting-needle provided with copper terminals was placed upon a strip of copper 16 inches long, 3 inches broad, and $\frac{1}{20}$th of an inch thick; the strip was well varnished, and the needle was laid upon it in such a position that the axis of the needle was

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* It seemed to be the more desirable to test these results, as Auerbach twice alludes to them in his paper (Phil. Mag., July, 1879, pp. 15–17), and should they not be correct, as I cannot help feeling is probably the case, others might be misled, owing to the well-deserved reputation of these experimenters.

coincident in direction with a line drawn from end to end of the strip and bisecting the two ends. On passing a current through the strip the resistance of the wire was altered in such a manner as would make it appear that circular magnetization *decreased* the resistance. I cannot, however, place reliance on these experiments, which at the time seemed conclusive, as I have in my recent attempts not been able to verify their results when sufficient precaution was taken to avoid change of resistance from change of temperature. The following are two experiments made with the above-mentioned object:—

Experiment LXXV.

An annealed iron wire, 4 feet in length and ·085 centim. in diameter, was firmly bound by tape for a distance of 2 feet with a caoutchouc-covered copper wire $\frac{1}{16}$th of an inch in thickness. The two parts of the wire were arranged as usual to form two sides of a "WHEATSTONE'S bridge"; and whilst the B.C. was varied in different trials to very different extents and the current through the copper wire was increased from almost 0° to 50° of the tangent galvanometer, *no trace whatever* could in any case be detected of alteration of the resistance of the wire by passing a current through the copper wire, though the arrangement was sufficiently delicate at times to show an alteration of *one in one million.* At one time, indeed, it was suspected that there was an alteration, but this was afterwards traced to a slight direct action of the M.C. on the galvanometer.

Experiment LXXVI.

A strip of annealed iron foil, 8 inches in length 2 inches broad and $\frac{1}{30}$th of an inch thick, was placed upon a copper strip 12 inches long 3 inches broad and $\frac{1}{10}$th of an inch thick, the two being separated from each other by two folds of a silk handkerchief. Another strip of iron of similar dimensions served as a comparison piece, and the two iron strips were arranged as usual in the "bridge." Though the currents through the strips of both iron and copper were altered to the same extent as with the iron wire, there was still no trace of alteration of resistance caused by the current in the copper strip. It is hardly necessary to say that such arrangements as these are not favourable for bringing out the effect sought, inasmuch as the distances of the copper wire and copper strip conveying the current were both greater than the thickness of the iron itself;* but, on the other hand, if we make the iron thicker we diminish the sensitiveness of the arrangement, and we certainly cannot well diminish the distance of the copper from the iron without laying ourselves open to error from changes of resistance caused by heating.

Several attempts were now made to ascertain whether variation of the B.C. itself

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* With the wire twice as great and with the strip nearly equal.

would cause any variation in the resistance of iron wire, of which the next experiment will furnish a sample.

Experiment LXXVII.

An annealed iron wire, 14 inches long and ·85 millim. in diameter, was balanced against a platinum wire $\frac{1}{30}$th of an inch in diameter, and of nearly equal resistance. The B.C. was varied by interposing different amounts of resistance in the external circuit of one GROVE's cell, and this could be done very quickly by a suitable arrangement of mercury cups. Resistance coils of 10 ohms each were used in the two branches of the bridge, where 100 ohms resistances were generally employed.* As it was found impossible to allow the current to flow for even two or three seconds without unduly heating the wire, and as VILLARI's "shock-currents" would cause the resistance of the iron to appear greater than it should be when the B.C. was closed, and less than it should be when the B.C. was opened, the following plan was adopted :—The B.C. was, at intervals of 30 seconds, closed and *immediately* afterwards opened; and in this way it was found possible to obtain the value of the resistance-ratio of the iron and platinum so nearly, that on moving the sliding-piece to points on the iridio-platinum wire 20 millims. above or below the supposed balancing-point, a deflection of several divisions could be obtained in one direction or the other. As the contrary "shock-currents" produced by closing and immediately opening the B.C. would not quite neutralise each other's impulsive effects on the galvanometer-needle, it is obvious that the effects on the galvanometer, due simply to the fact of the sliding-piece being equal distances above or below the true balancing-point, would not produce equal deflections, but that, by taking the mean of these deflections, the true point might by easy calculation be determined. As a sample of the mode of experimenting we will take the following case :—The true balancing-point *seemed* to be 55 millims. to the right of the zero of the iridio-platinum wire, and 3 ohms were at this time in the circuit of the B.C. ; on moving the sliding-piece to 75 a deflection, on closing and immediately afterwards opening the B.C., of five divisions to the left was obtained (a left deflection would here indicate that the sliding-piece should be moved to the left in order to get the true balance). The sliding-piece was now moved to 35, and a deflection of six divisions to the right was obtained on closing and opening the B.C. From this we learn that the true balancing-point would be $35+\frac{6\times 40}{11}=57$ nearly. The resistance of 3 ohms was then removed, and similar observations gave 58 as the balancing-point. Immediately afterwards, the 3 ohms resistance having been again introduced, the point appeared to be 56. The true balancing-point when 3 ohms were in was therefore assumed to be $\frac{57+56}{2}$, and when

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* It may be perhaps as well to state here that when small resistances were being compared, 10 ohms instead of 100 ohms were generally employed, though this fact has not been always mentioned.

the 3 ohms were removed to be 58. In this way the following results were arrived at as the means of several trials :—

Resistance in the external circuit of B.C.	Position of equilibrium of the sliding-piece.	Number proportional to the B.C.
10	56·7	1·0
3	55·8	2·8
0	55·7	11·0

Now when we bear in mind that one division of the iridio-platinum wire only represents an alteration of resistance of ·0033 per cent., it appears evident that alteration of the strength of the B.C. can have but very small effect on the resistance, and even if there is any alteration it is of such a nature as to show that circular magnetization produces *decrease*, not increase of resistance.

Discussion of Auerbach's Experiments.*

The results recorded in the last few experiments are so completely at variance with my own former observations,† and with those of Auerbach that it is very desirable to attempt to account for the discrepancies. I cannot help thinking that Auerbach experimenting, as it would seem, in almost precisely the same manner as I did in 1875, may have been misled in the same way as I now believe myself to have been. We both employed copper terminals to our iron wires whose resistances were in some cases even greater than that of the wire itself, and therefore necessarily not very thick, and moreover balanced this compound wire of copper and iron against a wire of German-silver. Now when a current is passing through a wire compounded of two metals placed end to end, a "Peltier effect" is produced such that an electromotive force is developed which sends a current in the opposite direction to the original; so that, if we attempt to find the resistance of the compound wire in the usual manner by closing the battery circuit and shortly afterwards that of the galvanometer, we shall obtain an apparent value for the resistance which will depend upon the battery-power employed, upon the length of time that the battery circuit has been closed, upon the medium surrounding the wire,‡ and upon the thermo-electric power of the two metals forming the compound wire whose resistance we wish to determine. Nor is the "Peltier effect" necessarily confined to the two junctions, for since no wire can be made perfectly homogeneous

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* Phil. Mag., July, 1879.

† Ibid., June, 1875.

‡ That is whether this medium tends to preserve the inequality of temperature at the two junctions or not.

throughout, there must be unequal heating wherever two dissimilar parts of the wire join, and therefore consequently there must be developed at each such junction an opposing electromotive force. Indeed, may not the large resistance of such alloys as German-silver and platinum-silver be in a great measure due to a similar unequal heating at the junctions of the molecules of the several metals forming the alloy: with this difference, however, that here there would be an equalization of temperature the rapidity of which would be more and more approached in the case of a compound circuit made up of several pieces, say of iron and copper, as the distances between the consecutive junctions of these pieces became less and less?

Further, besides the error likely to arise from the "PELTIER effect," we must expect to encounter another, inasmuch as the heat generated in a wire by the passage of a current by no means necessarily produces the same alteration of potential at the two junctions. This may be from two causes: either different rises of temperature may be produced because the terminal at one end carries off more heat than that at the other, or because the metals which appear to be identical at the two junctions are not so.

That the "PELTIER effect" does come largely into play sometimes, we can convince ourselves by passing a current from a single cell of DANIELL for five or ten seconds through a thermopile, and then, after disconnecting the cell from the pile, putting the latter in circuit of a galvanometer; in such case a very considerable deflection can be obtained with a delicate instrument, and, indeed, we can even make the warmth of the hand, pressed against the face of one thermopile, generate such a "PELTIER effect" in a second pile connected with the first, that on disconnecting the two from each other, and then connecting the second with a reflecting galvanometer, a deflection may be obtained which can be rendered visible at a considerable distance. The following experiments will show that such errors as those above-mentioned are by no means merely theoretical.

Experiment LXXVIII.

A silk-covered German-silver wire, $\frac{1}{30}$th of an inch in diameter, was soldered at its two extremities to two copper terminals, $\frac{1}{16}$th of an inch in diameter and 6 inches in length, the whole forming a fairly accurate resistance coil of half an ohm. The coil was put for 20 seconds in the circuit of one GROVE's cell, and then, after the cell had been disconnected, and when a further period of five seconds had elapsed, connected by a mercury cup with a galvanometer; a deflection of 200 divisions was obtained, whereas, previously, there had been no perceptible deflection. After a rest of five minutes there was only a deflection of some 20 divisions. The battery was now reversed, and after a connexion with the coil for 20 seconds, a deflection of 170 divisions was obtained in the opposite direction.

Experiment LXXIX.

A piece of annealed iron wire with copper terminals, both of similar dimensions to those of the wire and terminals in the last experiment, was treated in the same manner as the German-silver, and deflections of 50 and 25 divisions, both on the same side, were obtained. The wire and its terminals were in this case well wrapped up in paper, and, as in the previous experiment, there was no sensible deflection before the GROVE's cell was used. These experiments, which are only two out of several which were made with different pairs of metals, show that with the German-silver the electromotive force generated by the "PELTIER effect" was so far greater than that due to any other cause that the deflections were nearly the same on both sides; whereas with the iron the current produced by the unequal heating of the two junctions from other causes than the "PELTIER effect" predominated, and this was found to be the case with several specimens of iron and copper.

In my experiments of 1875 I balanced iron wires or iron rods against wires of other materials, and using rather powerful electromotors (from one to six GROVE's cells) proved, as I thought, that the electrical resistance of iron increases with the intensity of the current employed in the "bridge;" but in these later investigations, in which, having a much more delicate galvanometer, I could obtain a measure of the resistance of the substance within 1 in 50,000 with a battery-power one-tenth of the smallest then used, I have been unable to detect with certainty any such change.

As for the discrepancies which exist between my present and former observations on both soft iron and hard steel, I can only attribute them to errors caused by the magnetizing coil being too close to the iron or steel to allow of sufficient protection from errors caused by heat radiated or conducted from the former, and which might increase the resistance of the metal as a whole, or cause apparent increase or decrease by unequal change of potential at the junctions of the two copper terminals with the iron or steel. At any rate, using both steel and iron of the same qualities as used then, but adopting more perfect thermal insulation and a more accurate mode of experimenting, I have been unable to detect any such considerable increase of resistance in the case of soft iron or soft steel, or any decrease of resistance of hard steel, as I did then. Now AUERBACH, with some of his specimens of iron and steel wires, obtained apparent alterations of resistance of 1, 2 and even 3 per cent.—alterations of decrease or increase which would have, in the case of specimens of a similar nature used by myself, sent the reflected image of the illuminated wire flying off the scale, whereas, instead of this, I found nothing but variations of resistance which never, with wires of a similar diameter, reached even to ·1 per cent., and this, too, with magnetizing forces which must have equalled those employed by AUERBACH.

AUERBACH, again,[*] seems to concur with BEETZ and others that the mere mechanical pull connected with magnetizing would have caused an apparent decrease of resistance

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[*] *Loc. cit.*, p. 151.

in THOMSON'S experiment on the effect of transverse magnetization on the electrical resistance of iron;[*] but when one considers the small effect of even a far larger stress than could have been produced by magnetization on the electrical resistance of iron, as shown in Table I., these objections must, I think, vanish; and further, with nickel whose resistance THOMSON has proved to be altered similarly to iron,[†] the effect of mechanical stress is of *an opposite* nature to that produced by magnetization.

REMARKS ON THE NATURE OF THE ALTERATION OF RESISTANCE WHICH IS PRODUCED BY MAGNETIZATION.

It will be observed in Table XXXIII., in which are given the values of the increase of resistance produced by unit magnetizing force, that of all the metals examined annealed nickel is the most affected,[‡] and that next in order come soft iron, soft steel, cobalt, and bismuth. Evidently the condition of the metal may largely affect the susceptibility to alteration of resistance, and from what we have previously learned the thickness may do so also, but in a direction opposite to that which was at first expected, namely, that thick wires would be less affected than thin ones when the same B.C. and M.C. were employed.

TABLE XXXIII.

Name of metals.	Condition.	Diameter in millimetres.	Increase of resistance per unit. produced by unit magnetizing force.
Iron	Annealed	0·94	2335×10^{-8}
Steel	Annealed	0·85	1500×10^{-8}
Steel	Unannealed	2·33	1137×10^{-8}
Steel	Very hard	2·33	70×10^{-8}
Nickel	Annealed	1·05	8070×10^{-8}
Nickel	Unannealed	7·00	4343×10^{-8}
Cobalt	Unannealed	7·50	628×10^{-8}
Bismuth	Unannealed	3·30	21×10^{-8}

Had the nature of the change of resistance been the same for mechanical longitudinal stress as for longitudinal magnetization in the case of *all* metals, there is nothing in the actual *amount* of alteration that might not lead us to suppose that the change of resistance from the latter cause is due to mere rotation of the molecules, as molecules, without regard to the electric currents, which, according to AMPÈRE'S hypothesis, are constantly circulating round these molecules. But when we find that with nickel

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[*] Phil. Trans., 1856, p. 741.

[†] Proc. Roy. Soc., vol. viii., 1857.

[‡] It is remarkable that the value of the "rotational coefficient" of nickel should also exceed that of the other magnetic metals.

longitudinal mechanical stress, which must cause rotation of the molecules to a certain extent, but without magnetic polarity, actually, unless carried to a very great excess, produces decrease of resistance, we are probably right in conjecturing that the change of resistance resulting from magnetization is in a great measure due to the fact that the current used in the "bridge" is encountered by a set of molecular currents circulating all more or less in the same direction, and in planes more or less at right angles to the direction of the former current as the induced magnetism is greater or less.

RELATION BETWEEN THE "ROTATIONAL COEFFICIENT" OF METALS AND THE ALTERATION OF RESISTANCE PRODUCED BY MECHANICAL STRESS.

E. H. HALL has discovered that when a strip of metal along which a current is passing is placed between the poles of an electromagnet in such a position that the lines of magnetic force are perpendicular to the plane of the strip, an electromotive force is developed in a direction at right angles both to the plane of the strip and the lines of force, and that thus the current is deflected. This deflection varies in amount and also in direction with different substances, and Professor HALL has recently read before the British Association a paper* on this subject, in which he gives a table showing the extent and direction of the deflection produced in several metals. The extent of the deflection in any substance depends among other things upon a certain constant designated by Professor HALL as the "rotational coefficient." In this table the sign + or — is prefixed to the number representing the coefficient according as the current is deflected in the same direction in which the conductor itself tends to move, or the opposite. Below is given HALL's table, and appended to it the numbers representing the increase of specific resistance per unit temporary increase of length when this latter is produced by mechanical stress. A + sign prefixed to the numbers denotes an increase, and a — sign a decrease of specific resistance.

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* 'Nature,' Nov. 10, 1881. (Abstract of a note on the above subject read by Professor E. H. HALL at the meeting of the British Association at York.)

TABLE XXXIV.

Name of metal.	"Rotational coefficient."	Temporary alteration of specific resistance per unit produced by temporary increase of length per unit. + signifies increase of resistance on application of stress.
Iron	+ 78	+2·618
Zinc	+ 15	+2·113
Lead	..	+1·613
Tin	+ 0·2	+1·630
Brass	− 1·3	..
Platinum	− 2·4	+2·239
Silver	− 8·6	+1·617
Copper	− 10·0	+1·005
Aluminium	− 50·0	−0·420
Nickel	−120·0	−8·860

Considering that HALL himself is doubtful about the order of the metals in the centre of the list, there seems to be a well-marked relation between the "rotational coefficient" and the alteration of specific resistance from temporary mechanical stress. This relationship is strikingly apparent in the case of the metals iron, zinc, copper, aluminium, and nickel, and there can be but little doubt that results of extreme interest might be obtained by observations of the effect of mechanical stress and strain on the "rotational coefficient."

THE RELATION BETWEEN ELECTRICAL RESISTANCE AND "VISCOSITY."

Whilst endeavouring to find a relation between the electrical resistances of substances and their other physical properties, I was struck with the failure which I experienced in finding any in the case of those properties which have been already examined, except one, and that is one which as yet I have not had time to examine with anything like the care which I hope at some future period to be able to bestow upon it, namely, that which has been called by Sir W. THOMSON, in the case of metals, their "viscosity." The experiments, however, which have been made show clearly that there is in all probability a very close relationship between molecular friction and electrical resistance. It is proposed to make extended observations of the diminution of amplitude of vibration of wires of considerable length whilst the oscillations of very small amplitudes are magnified by a mirror attached to the vibrator; so that small vibratory molecular displacement may be obtained, and further to examine the change of "viscosity" produced by change of temperature. It suffices, however, for the present to say that of the pure metals already examined, copper, silver, aluminium, zinc, and tin, the order of their "viscosity" is the same as that of their specific electrical resistance.

Summary of Part II.

1. The electrical resistances of iron, steel, platinum, German-silver, copper, platinum-silver, brass, zinc, silver, aluminium, tin, lead, and carbon are temporarily increased by temporary longitudinal stress, the amount of increase being nearly, but not quite, proportional to the stress.

2. The *specific* electrical resistances of all the above metals, except aluminium, is likewise temporarily increased by temporary longitudinal stress; with aluminium, however, the specific resistance is *decreased.* The total resistance and the specific resistance of nickel are both *decreased* by temporary longitudinal stress not exceeding a certain limit, whilst beyond this limit further increase of stress begins to produce increase of resistance. The alterations, both increase and decrease, are very considerably greater with this metal than those of any of the other substances examined.

3. The temporary alteration of specific resistance caused by stress is much less with the alloys German-silver, platinum-silver, and brass than with the several components of these alloys; this would suggest an apparent relation between the change of resistance caused by alteration of temperature and that due to mechanical stress; the former effect, however, is very much greater than the latter, if we regard the alterations of resistance attending the same amount of expansion in each case, and there is no doubt that the increase of resistance ensuing on rise of temperature is due almost entirely to other causes than mere expansion.

4. The elasticity of carbon rods varies considerably with different specimens, even from the same maker, and is nearly proportional in this case to the eighth power of the density. Thick rods have generally a less density than thin ones, and less elasticity.

5. The specific resistance of carbon also varies considerably with different specimens from the same maker, but there is no apparent relationship between specific resistance and elasticity.

6. The increase of resistance caused by longitudinal stress is with different specimens of carbon as with different specimens of other substances very nearly proportional to the amount of temporary elongation produced by the stress, and though with the exception of tin and lead, the total alteration of resistance resulting from a given amount of stress is less with the metals which have been examined than with carbon, this is not so with regard to the alteration of specific resistance.

7. Compression produces on the electrical resistance of substances an effect of a contrary nature to extension.

8. Stress applied in a direction transverse to that of the current produces both temporary and permanent alteration of resistance of a nature opposite to that resulting from longitudinal traction.

9. Stress applied equally in all directions diminishes the total and the specific resistance of most metals.

10. The alteration of the melting-point temperature of ice can be readily and accurately determined by observations of the change of resistance produced by fluid pressure on metal wires placed in the ice.

11. Experiments on the permanent alteration of resistance of metal wires produced by stress furnish valuable information respecting the "limit of elasticity" of metals.

12. There are two "critical points" in every metal at which sudden changes occur in the ratio of the permanent extension produced by any load and the load itself, when the latter is gradually and carefully increased. The first of these two points fixes the true "limit of elasticity," and the second the true "breaking-point" of the metal. With iron there are three, and perhaps more "critical points."

13. The "critical points" are evidently in most cases closely related to the moduli of elasticity.

14. The total resistance of most metals is permanently increased by permanent longitudinal extension, but with nickel the total resistance is permanently *decreased*, provided the extension does not pass a certain limit; beyond this limit further extension produces increase of resistance.

15. The rate at which a wire is "running down" under the influence of a load can be very advantageously studied by observing the permanent increase of resistance produced by the load.

16. If P be the "breaking-load" of a metal wire, and p be the load actually on the wire, the decrease per unit of the velocity of the increase of resistance is inversely proportional to $P-p$: so that the actual "breaking-load" of a wire can be calculated from observations of the rate of increase of resistance when a loaded wire is "running down."

17. The above-mentioned proportion holds good not only for one and the same metal but for different metals.

18. The result of experiments on the influence of permanent extension on the temporary alteration of resistance which can be produced by temporary longitudinal stress verifies the statement made in Part I. that "the elasticity of a wire is diminished by permanent extension not exceeding a certain limit, but beyond this limit increased." The effect of permanent extension on the alteration of resistance which can temporarily be produced in nickel by traction is very remarkable.

19. Permanent extension, hammering and torsion produce, even when carried to excess, very small changes in the *specific* electrical resistances of metals. Most metals have their specific resistances increased by strain caused by the above-mentioned processes, provided the strain does not exceed a certain limit: beyond this limit further strain decreases the specific resistance. In the case of iron and nickel, on the contrary, the specific resistance is at first *decreased* and afterwards increased.

20. The strain caused by heating annealed steel to a temperature slightly and very much in excess of that of the room produces effects on the specific resistance of the metal of a kind similar to those caused respectively by small and great mechanical strains.

21. The change of density which can be effected in metals by permanent extension, hammering, or torsion is small.

22. The amount of recovery of electrical conductibility which is produced by time in all metals which are in a state of strain varies considerably with the nature of the metal; with platinum-silver the amount of recovery in a given time is very small, and with German-silver comparatively very large.

23. The recovery of electrical conductivity is in all cases attended with increase of longitudinal and torsional elasticity.

24. Metals may be divided into two classes, as far as the influence of permanent strain on the susceptibility to temporary change of resistance from change of temperature is concerned. In one class the strained wire is most increased in resistance by rise of temperature up to a certain limit, whilst beyond this limit further strain diminishes the first effect. In the other class the converse takes place.

25. There is a close relationship between the thermo-electric properties of strained and unstrained metals and their susceptibility to change of resistance from change of temperature.

26. The elasticity of annealed iron or steel is not *temporarily* but *permanently* increased by raising the temperature of these metals to 100° C. The ductility of annealed iron may also be very considerably and permanently diminished by the same process. Mechanical strain influences the elasticity in the same manner as the strain caused by tempering, and we may say of both kinds of strain that in the case of iron and steel there are three "critical points"—very slight strain increasing, moderate strain diminishing, and excessive strain again increasing both the torsional and the longitudinal elasticity.

27. The temporary alteration of elasticity which is effected in the case of nickel by raising the temperature to 100° C. is very noticeable. Still more remarkable is the temporary alteration of susceptibility to change of resistance from change of stress which is produced by the same means.

28. The electrical resistances of annealed iron, annealed steel, very hard steel, nickel, cobalt, bismuth and zinc are all increased by longitudinal magnetization. The alteration of resistance produced by the magnetization of annealed nickel is very remarkable.

29. The amount of increase of resistance produced in iron and nickel by longitudinal magnetization depends not only upon the magnetism induced, but also upon the magnetizing force, in such a manner that increase of resistance will be produced by increasing the magnetizing force, even when the latter does not cause any appreciable increase of magnetism.

30. The increase of resistance which is produced by magnetization is probably not merely due to the rotation of the molecules of the magnetized substance as molecules, but to the electrical currents, which according to AMPÈRE's hypothesis are constantly circulating round the molecules.

31. The "circular" magnetization which is produced when a current flows through a wire of iron does not appreciably alter the electrical resistance of the wire.

32. The effects of temporary stress and of permanent strain on the alteration by magnetism of the resistance of an iron or nickel wire are of a similar nature to those on the alteration of the magnetic susceptibility of these metals.

33. There is a very striking relationship, both as regards amount and direction, between the alteration of specific resistance, which can be produced in a substance by longitudinal traction and the "rotational coefficient" of the substance.

34. There is evidently an intimate relationship between the "viscosity" of a metal and its specific electrical resistance.

II. *On the Specific Resistance of Mercury.*

By Lord Rayleigh, *F.R.S., Professor of Experimental Physics in the University of Cambridge, and Mrs.* H. Sidgwick.

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Our experiments on the determination of the British Association unit of electrical resistance in absolute measure are detailed in two memoirs communicated to the Society.[*] The conclusion to which they led us is that

$$1 \text{ B.A. unit} = \cdot 9865 \, \frac{\text{earth quadrant}}{\text{second}},$$

but this result differs considerably from that obtained by some other experimenters, the original Committee included. Although in the present state of the question it is not desirable that the B.A. unit should fall into disuse, there can be no question as to the importance of connecting it with the mercury unit introduced now more than twenty years ago by Siemens. It will then be possible, as recommended by the Paris Conference, to express our absolute measurements in terms of mercury, by stating what length of a column of mercury at 0° of 1 square millimetre section has a resistance of 1 ohm. Accordingly the experiments about to be described relate to the expression in terms of the B.A. unit of the resistances of known columns of mercury at 0°.

This investigation was the more necessary, as the principal authorities on the subject, Dr. Werner Siemens and Dr. Matthiessen, had obtained results differing by as much as ·8 per cent.

The earlier determinations of Siemens were vitiated by the assumption of an erroneous value (13·557) for the specific gravity of mercury, a constant which it is necessary to know in order to infer the mean section of a tube from the weight of contained mercury. The error, pointed out by Matthiessen, was afterwards[†] admitted by Siemens, who gives as the corrected expression of the relation between the two units,

$$1 \text{ mercury unit} = \cdot 9536 \text{ B.A. unit}.$$

On the other hand, the independent measurements of the resistance of mercury by Matthiessen and Hockin[‡] gave

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[*] Proceedings, April 12, 1881; Phil. Trans., 1882, Part II.

[†] Phil. Mag., xxxi., 1866.

[‡] Reprint of British Association Reports, p. 114.

1 mercury unit=·9619 B.A. unit,

the mercury unit being defined as the resistance at 0° of a column of mercury 1 metre long and 1 square millimetre in section.

Our own experiments lead us to a value not differing much from that of SIEMENS. We find

1 mercury unit=·95418 B.A. unit.

If we assume that the B.A. unit is ·98651 ohm (in accordance with our determination), we find

1 mercury unit=·94130 ohm,

the ohm being 10^9 C.G.S. The same result may be expressed in another way by saying that the ohm is the resistance of a column of mercury at 0°, 1 square millimetre in section, and 1062·4 millims. in length.

Through the kindness of Dr. C. W. SIEMENS we have had an opportunity of comparing with the B.A. units a standard mercury unit (No. 2513) issued by Messrs. SIEMENS and HALSKE. At the proper temperature (16°·7) we find that its resistance is

·95365 B.A. unit,

agreeing very closely with previous comparisons of SIEMENS' mercury measurements with the B.A. unit.

The determination of the specific resistance of mercury is simple enough in principle, though the execution is somewhat tedious, and the calculation of the results is complicated in practice by the necessity of introducing various temperature corrections. In a first sketch of the method it will be convenient to omit these corrections, which is tantamount to supposing that all the measurements are made at zero. If L be the length and s the section of the column of mercury, R its resistance, r the specific resistance of the metal,

$$R=\frac{rL}{s}, \quad \text{or} \quad r=R\frac{s}{L}$$

The length L can be measured directly, but s can only be found with the necessary accuracy from the contents. Thus if ρ be the specific gravity of mercury, and W the weight of the whole column in grammes, $\rho Ls=W$, whence $s=W/\rho L$, and

$$r=\frac{RW}{\rho L^2}$$

Apart from the temperature corrections already referred to, the simplicity of the formula is disturbed by the inevitable departure from the truly cylindrical form of the glass tubes used to contain the mercury. It is true indeed that to a first order of approximation the formula stands unaltered, as we may see if we understand by s the *mean* section of the tube. The volume is still truly expressed by sL, and the resistance is *approximately* expressed by rL/s. If, however, the squares of the variations of section cannot be neglected, the actual resistance is greater than the formula would lead us to suppose, as is evident if we imagine the section to become at one place very small.

In general we must regard s as a function of the position (x) along the tube at which it is taken. For the purposes of the present paper we may assume with sufficient approximation (see Lord RAYLEIGH'S 'Theory of Sound,' § 308)

$$R=r\int\frac{dx}{s}$$

The necessary data with respect to s are obtained by a calibration of the tube. "If a small quantity of mercury is introduced into the tube and occupies a length λ of the tube, the middle point of which is distant x from one end of the tube, then the area s of the section near this point will be $s=C/\lambda$, where C is some constant. The weight of mercury which fills the whole tube is

$$W=\rho\int s\,dx=\rho C\,\Sigma\left(\frac{1}{\lambda}\right)\frac{L}{n}$$

where n is the number of points at equal distances along the tube, where λ has been measured, and ρ is the mass of unit of volume.

"The resistance of the whole tube is

$$R=\int\frac{r\,dx}{s}=\frac{r}{C}\Sigma(\lambda)\frac{L}{n}$$

"Hence

$$WR=r\rho\,\Sigma(\lambda)\,\Sigma\left(\frac{1}{\lambda}\right)\frac{L^2}{n^2}$$

and

$$r=\frac{WR}{\rho L^2}\;\frac{n^2}{\Sigma(\lambda)\Sigma\left(\frac{1}{\lambda}\right)}$$

gives the specific resistance of unit of volume" (MAXWELL'S 'Electricity,' § 362).

In the sequel

$$\frac{1}{n^2}\Sigma(\lambda)\,\Sigma\left(\frac{1}{\lambda}\right)$$

is denoted by μ; it is a numerical quantity a little greater than unity.

Another correction is required in our method of working to take account of the resistance offered by that part of the mercury in the terminal cups, which is situated just beyond the ends of the tube. The question is identical with that of the correction necessary in calculations of pitch for the open ends of organ pipes (see 'Theory of Sound,' § 307, and Appendix A), and it scarcely admits of absolutely definite solution. We cannot, however, be far wrong in adding to the actual length of the tube ·82 of its diameter, which corresponds to the supposition that the diameter of the mercury column suddenly becomes infinite. Since, in our experiments, the whole correction only amounts to about a thousandth part, even a ten per cent. error in our estimate would scarcely be material.

Let r = resistance of a column of mercury 1 metre long and 1 square millimetre in section, at 0°, expressed in B.A. units.
R = resistance of the tube full of mercury at 0° in B.A. units.
L = length of the tube at t'° in centimetres as measured with brass rod.
l = length of a thread of mercury of nearly the length of the tube at t° as measured with brass rod.
W = weight of the same thread in grammes.
μ = coefficient correcting for conicality of tube.
$\delta\mathrm{L}$ = correction to L on account of the connecting rods not being close up to the ends of the tube $= \cdot 82 \times$ diameter of tube.
ρ = specific gravity of mercury at 0° = 13·595.
γ = cubic expansion of mercury per degree = ·0001795.
g = „ „ glass „ = ·000025.
b = linear expansion of brass „ = ·000018.
t_0 = temperature of brass measuring rod to which the lengths are corrected = 17°·2.

Then the volume of the thread at 0° = W/ρ

„ „ $t° = \dfrac{\mathrm{W}}{\rho}(1+\gamma t)$

$$\text{Mean section of the tube at } t° = \frac{\mathrm{W}(1+\gamma t)}{\rho l\{1+b(t-t_0)\}}$$

$$\text{Mean section at } 0° = \frac{\mathrm{W}(1+\gamma t)}{\rho l\{1+b(t-t_0)\}\{1+\frac{2}{3}gt\}}$$

$$\text{Length of the tube at } 0° = \frac{(\mathrm{L}+\delta\mathrm{L})\{1+b(t'-t_0)\}}{1+\frac{1}{3}gt'}$$

$$\mathrm{R}=10^{-4}.r.\mu.\frac{(\mathrm{L}+\delta\mathrm{L})\{1+b(t'-t_0)\}}{1+\frac{1}{3}gt'}.\frac{\rho l\{1+b(t-t_0)\}\{1+\frac{2}{3}gt\}}{\mathrm{W}(1+\gamma t)}$$

$$r=\frac{10^4\,\mathrm{RW}(1+\gamma t)(1+\frac{1}{3}gt')}{\rho\mu\, l\mathrm{L}(1+\frac{2}{3}gt)}\left(1-\frac{\delta\mathrm{L}}{\mathrm{L}}\right)\{1-b(t+t'-2t_0)\}$$

The value of ρ is that used by the Committee of the British Association in reducing Dr. Matthiessen's experiments (see reprint of 'Reports on Electrical Standards,' p. 114), and stated to be the mean of the values given by Kopp, Regnault, and Balfour Stewart. The values of g, γ, and b are taken from Everett's 'Units and Physical Constants'—γ being Regnault's value for the expansion of mercury. The measurements of the other quantities, which depend on the particular tube used, are given in the following table, together with the resulting value of r. The description of the means employed to obtain these data follows.

Number of observation.	Date of observation, 1882.	Number of tube.	R.	Temperature of coil F.	Temperature of second coil.	L in centimetres.	μ.	l in centimetres.	W.	t.	t'.	$\frac{\cdot 955 \times \delta L}{L}$.	$rb(t+t'-2t_0)$.	r.	Mean values of r from each tube.
1	Feb. 23 & 24	I.	·79912	12°·2 11·5	12°·1 11·5	87·771	1·00314	87·234	12·442	16°·5	16°·5	·00103	+·00002	·95386	·95416
2	,, 25 . .	I.	·79912	12·7	12·5	,,	,,	87·310	12·4545	17·2	17·2	,,	·00000	·95412	
3	,, 21 . .	I.	,,	. .	. .	,,	,,	87·035	12 4185	18·4	,,	,,	−·00002	·95424	
4	March 18 .	I.	·79920	13·7	13·75	,,	,,	87·558	12·486	20·6	,,	,,	−·00006	·95436	
5	Weighed Feb. 14	I.	·79912	. .	. .	,,	,,	87·771	12·523	16·5	16·5	,,	+·00002	·95421	
6	Feb. 24 . .	II.	·99088	12·0	. .	96·400	1·00007	96·054	12·096	16·7	16·4	·000883	+·00002	·95389	·95419
7	,, 21 to 23	II.	·99081	13·2	. .	,,	,,	95·452	12·0245	16·4	,,	,,	+·00003	·95414	
8	March 7 .	II.	·99081	11·5	. .	,,	,,	95·831	12·074	17·1	,,	,,	+·00002	·95437	
9	,, 8 .	II.	·99079	12·2	. .	,,	,,	96·151	12·113	18·0	18·0	,,	−·00003	·95436	
10	,, 30 .	II.	·99085	13·25	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	
11	,, 6 .	III.	·99711	11·2	. .	123·566	1·00046	122 218	19·620	16·2	18·7	·000778	−·00001	·95424	·95416
12	,, 10 .	III.	·99725	12·9	. .	,,	,,	123·288	19·780	18·5	,,	,,	−·00005	·95418	
13	,, 13 .	III.	·99720	12·7	. .	,,	,,	123·221	19·7665	18·4	,,	,,	−·00005	·95399	
14	,, 14 .	III.	·99725	13·4	. .	,,	,,	123·058	19·745	18·3	,,	,,	−·00005	·95425	
15	,, 22 .	IV.	·50783	13·0	12 9	194·137	1·000838	193·410	95·859	14·5	14 5	·000869	+·00009	·95440	·95427
16	,, 24 .	IV.	·50774	12·7	12·7	,,	,,	192 576	95·402	16·8	. .	,,	+·00005	·95415	

Mean of all the above values of r in B.A. units ·95418.

The mercury used for all the measurements except 10 and 14 was distilled in vacuo with an apparatus fitted up by Mr. SHAW. In order to see whether a different result might not be obtained with other mercury, some was procured from the chemical laboratory for measurements 10 and 14. For the latter a portion of this mercury was treated with nitric acid and distilled at atmospheric pressure. For measurement 10 it was treated with nitric acid, but not distilled. An accident occurred in carrying out this measurement, so that only the resistance of the column was ascertained ; but this agrees so well with the resistances found with the same tube for the other mercury, that there is no reason to suppose that any discrepancy would have appeared in proceeding with the measurement further.

The glass tubes used were supplied by CASSELLA, and were selected for uniformity of bore, so that the correction for conicality should be small. They were slender and easily broken, which made the manipulation of them difficult, and it was in fact owing to a breakage that the tube called No. I. was used so short. The measurements taken with it, at first intended to be preliminary, were, however, made with the same care as in the case of the other tubes, and the difference of length and resistance adds some variety to the data. Tubes II. and III. were cut so that their resistance should be as nearly as possible one B.A. unit. The section of tubes I., II., and III., was approximately 1 square millimetre. Tube IV. was a much larger one, introduced with a view of varying the data as much as could conveniently be done. The diameter of its bore was about 2 millims., and its length was nearly 2 metres. It was cut so as to give a resistance of about half a B.A. unit.

The ends of the tubes were ground into a convex form with emery powder on a lathe, in order that the length (L) of the bore might be measured accurately. This measurement was effected by setting two microscopes, which could be adjusted longitudinally to the exact position required by micrometer-screws graduated to $\frac{1}{10000}$ inch, so that their cross-wires should coincide with the ends of the tube. Observations were made in three or four different positions as the tube was turned round its axis, and the mean taken. After removal of the tube, a brass measuring rod belonging to the British Association was substituted for it, and the number of whole divisions corresponding most nearly to the distance between the cross-wires of the two microscopes was read off. The outstanding fraction of a millimetre was then ascertained by screwing the microscope up to the whole division and reading the difference on the screw-head. For the long tube the measuring rod was too short, and a third microscope had to be used to fix an intermediate point as a fresh departure for the scale. A thermometer laid beside the tube during the measurement gave the temperature (t') at the moment. The brass measuring rod was carefully examined, and its divisions were found to agree among themselves.

The tubes were cleaned by passing through them in succession, by means of a suction-pump, sulphuric acid, nitric acid, caustic potash, and distilled water, followed by air dried with chloride of calcium. The process with omission of the acids was in general repeated between each refilling with mercury, but it was omitted in measurement 7, and there is no record of its having been done in 1, 3, and 6.

To calibrate the tubes a short thread of mercury was inserted and moved to the various positions required, by blowing through a chloride of calcium tube. In the case of tubes I. and II., the length, λ, of the thread was measured by adjusting microscopes to its two ends, with subsequent substitution of an ivory scale divided in fiftieths of an inch. But this method was troublesome; and with tubes III. and IV. the scale was simply placed against the thread and the length read off with a magnifying-glass, a procedure which was found to give sufficiently accurate results, notwithstanding the difficulty arising from parallax owing to the thickness of the glass. The following table gives the different values of λ for each tube.

As a check upon the correction for conicality, two distinct values of μ were in some cases calculated from the alternate observations of λ, and were found to agree closely. It may not be superfluous to mention that in carrying out the computations we must work to six or seven places, although the observed values of λ themselves may not be accurate beyond the third place.

The lengths are in fiftieths of an inch.

Tube I.		Tube II.	Tube III.		Tube IV.
	80·8	104·5	135·0		171·0
80·0		104·1		134·0	172·0
	77·0	104·5	133·0		171·5
75·8		105·0		132·0	170·5
	76·0	104·5	131·5		171·5
76·4		105·2		130·5	174·5
	75·0	104·3	128·0		175·0
74·0		104·0		127·5	174·5
	73·4	104·7	126·5		175·5
73·0		104·0		126·5	176·5
	72·7	103·0	126·5		177·0
72·3		101·8		126·0	180·0
	72·5		125·0		180·5
71·9				125·5	180·7
	71·1		126·0		182·2
70·1				126·0	183·7
	69·7		126·0		183·5
68·0				126·5	182·5
	67·9		127·0		184·0
67·6				127·0	186·0
	65·9		128·5		186·5
65·3				128·0	
			128·5		
				128·0	

To find the mean section of the tubes we at first tried the method adopted by Messrs. MATTHIESSEN and HOCKIN in their experiments for the British Association. After aspirating the tube with dry air we placed it in a wooden trough full of mercury, and filled it by suction. It was then held down in the trough with iron weights till it was presumably of the same temperature as the mercury in the trough, which was taken at three places. It was then held by the fingers (previously cooled in other mercury), pressed against its two ends, and taken out of the trough, the mercury adhering to the outside was brushed off, and the contents of the tube were emptied into a small porcelain crucible and weighed. But there was no doubt that when the fingers holding the tube were bare they pressed a little way—how much it was difficult to determine—into the tube, and when they were covered with stiff leather, or other stiff material, it was difficult to get a sufficiently good hold. However, in one case (No. 5) r was calculated from the weight so obtained with leather on the fingers.

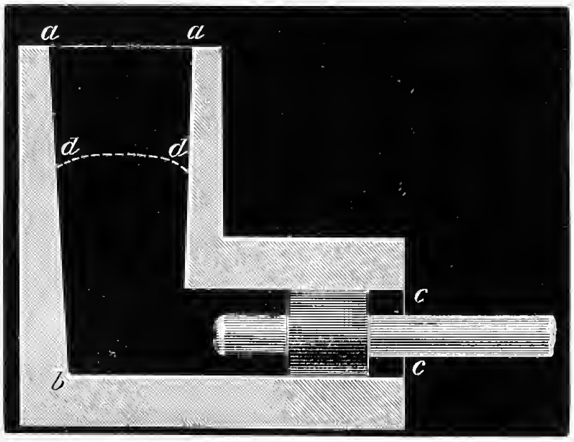
The method, followed by SIEMENS and SABINE, of screwing an iron plate up against the end of the tube, was attempted, but we did not succeed in closing the orifice sufficiently tightly in this way. Ultimately we came to the conclusion that the best results would be obtained by weighing a thread of mercury nearly as long as the tube, and of which we could ascertain the actual length by direct measurement. We thought, also, that there might be some advantage in ascertaining the volume of the mercury from the same filling as that of which the resistance had been taken, as we could not be sure that the closeness of contact between the mercury and the glass was always the same, so that the same volume of mercury would always be contained in the same length of tube, nor that the tube itself was in no way altered by the action of the caustic potash used to clean it. The plan adopted was, therefore, after measuring the resistance, to keep the tube horizontal so as to retain in it most of the mercury while the terminals were removed, and then with microscopes and divided rod to measure the thread of mercury in the same way as the tubes were measured. The length so obtained is called in the table l. The greatest difference between l and L (that in measurement 11) is scarcely over 1 per cent., and in most cases the difference is considerably less, so that, considering how nearly cylindrical the tubes were, the error in the mean section introduced by using a thread of length l instead of L is quite inappreciable. It was another advantage of our method that it avoided the necessity of filling the tube under mercury, which it would have been difficult to do with a tube so long as IV.

The only difficulty in measuring the thread of mercury arose from the convexity of its ends. This was overcome by pressing them flat with little flat-ended vulcanite pins made to fit into the tube. The curvature of the ends when free was not always the same; but it was found that the length of the mercury held with pins varied little from the number calculated on the assumption that the ends were hemispherical, namely, the length of the portion of the column of mercury which was in contact with the glass added to two-thirds of the difference between this length and that between the convex extremities. In some cases, where, owing to the pins not fitting very well or other causes, there was a difficulty in flattening the ends properly, the calculated value was used. A thermometer lay beside the tube during the measurement, so as to give the temperature t. After the measurement, the mercury was blown out into a small crucible and weighed. Care had to be taken not to leave behind minute globules, which, owing probably to the small portion of the tube unoccupied by mercury during the measuring becoming damp from the air of the room or from the fingers, tended to adhere to the glass near the ends.

In three cases (No. 5 as above mentioned and Nos. 3 and 9) the mercury weighed and measured was not that of which the resistance was taken.

No. 3 was done before it occurred to us that there might be an advantage in carrying out both operations with the same filling, and in No. 9 about one-tenth of the mercury was spilt accidentally and had to be replaced.

The equality of the arms of the balance used for the weighing was tested. The weights were compared among themselves and found to be free from appreciable error.



The terminals were composed of L-shaped pieces of ebonite, hollowed out in the manner shown (about full-size) in the figure. Each end of the tube was furnished with a short length of thick rubber tubing, by which the aperture between the glass and the ebonite was closed air-tight. As a further precaution, the space at *c c* beyond the rubber was filled up by pouring in melted paraffine wax.

After the terminals were fitted the tube was again aspirated with dry air through tubes in corks inserted at *a a*, and then filled with mercury, which was poured in to one terminal and allowed to run slowly through to the other till it stood at a considerable height, represented by *d d*, in both terminals. The tube was then placed in a wooden trough and covered with ice. Our reason for using vulcanite terminals rather than glass ones was the fear that under the influence of the ice moisture would collect on the portion of glass above the mercury and serve as a conductor. We certainly avoided all difficulty of this kind by using vulcanite. On the other hand, we probably increased a difficulty which would have existed in any case, namely, that of getting the temperature of the portion of the tube which was within the terminal down to 0°. This portion of the tube was about 2 centims. at each end, or about 5 per cent. of the length in the case of tube I., and about 2 per cent. in the case of tube IV. What the exact temperature of this part of the tube was it is impossible to say, but it was ascertained that the temperature of the mercury in the terminals with the copper connecting rods *in situ* was not higher than 5° or 6°, depending in some degree on the extent to which the ice was piled up round the cup. The mean temperature of the parts of the tubes not directly exposed to ice can hardly have been so high as 2°. Supposing it to have been 2°, and taking the case of tube I., where the largest proportion of the whole length was within the terminals, the effect would be an overestimate of r by about ·00008. In the case of tube IV. the error in r would be less than the half of this.

The tubes were connected with the resistance balance by copper rods, well amalgamated, of which one end stood on the bottom of the vulcanite terminals, so that a considerable portion of the amalgamated copper surface was in contact with the mercury. The rods were kept at a little distance from the ends of the tubes. Dr. MATTHIESSEN brought flattened copper rods up against the ends of his tubes, but this plan appeared open to objection, since it would be very difficult to secure complete contact between the copper and glass all round the edge of the orifice, especially under an opaque fluid like mercury; and any defect in such contact would render necessary an unknown correction. We preferred, therefore, to let the ends of the tube open without obstruction into the mercury cup, which may be regarded as of infinite extent by comparison. The correction necessary to take account of the resistance of the mercury beyond the ends of the tube has already been considered.

The resistance of the rods used to connect I., II., and III. with the bridge was about ·00215 B.A. unit. With tube IV. an additional rod had to be introduced to get the necessary length. This brought the resistance of the rods up to ·00291. The other end of the rods fitted into mercury cups on the resistance balance.

The balance used was one designed by Professor FLEMING (Phil. Mag., ix., p. 109, 1880), in which Professor CAREY FOSTER'S method is employed of interchanging the resistances in the two arms of the balance containing the graduated wire, so that the difference between them is expressed in terms of the wire. One thousand divisions of the graduated wire are stated by Professor FLEMING to equal ·0498 B.A. unit, and experiments of our own also showed it to be about ·05. The wire is of platinum-iridium, and as it has a high temperature coefficient compared with the platinum-silver of the standard coils, we thought it undesirable to use much over 100 divisions of it. In order to avoid this in the case of tubes I. and IV. it was necessary to introduce coils from a resistance box in multiple arc. The resistance box employed was one by Messrs. ELLIOTT Brothers. With tube I., 20 ohms from the box were used in multiple arc with the standards against which the tube was balanced, and in the case of tube IV. 24 ohms were used in multiple arc with the tube itself. Tubes II. and III. were balanced against the standard coil belonging to the British Association and deposited at the Cavendish Laboratory, called *F*. For tube IV. another of their unit coils, called the *Flat coil*, was used in multiple arc with *F*. For tube I., *F* and a five-ohm coil were used in multiple arc. The standard coils belonging to the British Association have recently been carefully compared with each other by Professor FLEMING, who has drawn out a chart in which is recorded their variation with temperature, together with their resistance in terms of the mean of their resistances at the temperatures at which they were originally considered to be correct. The values of *F* and of the *Flat coil*—both platinum-silver coils—were taken from this chart. The five-ohm coil had been compared with the British Association standards by ourselves. It was also of platinum-silver, and its temperature coefficient was assumed to be the same as that of the others.

The standard coils were immersed in water whose temperature was observed each time a resistance was measured. These temperatures are given in the table. It may be worth remarking that the resistances were taken in a different room from that in which the lengths were measured, which accounts for the difference between *t* and the temperature of the standards. The thermometer used to find all the temperatures was graduated to fifths, and was corrected by one which had been verified at Kew.

When one coil only was used to balance the tube, its terminals fitted directly into the mercury cups of the bridge, but when two were used in multiple arc their terminals were put into larger mercury cups, which were connected with the mercury cups of the bridge by short copper connecting pieces of about ·00017 ohm resistance.

All the measurements were repeated with reversed battery currents, in order to eliminate thermoelectric disturbance. The readings with battery current each way usually agreed very closely, and the mean of the two was adopted.

It will be observed that the values of R for tube IV. differ by nearly two parts in 10,000, and that there is a less proportional difference, but still an appreciable one, for the other tubes. The greatest actual difference between any two of the values in the table for the same tube is ·00014 ohm. Some small error is due to neglect of the change of resistance of the copper connecting rods and of the bridge wire with temperature. A change of 4° in the temperature of the rods would make a difference of about ·00003 ohm. There is further a probability of error in ascertaining the temperature of the standard coil. A difference of $\frac{1}{10}°$ in this also introduces a difference of ·00003 ohm in the resistance; and there is not only a probable error of perhaps $\frac{1}{10}$ in finding the temperature of the water in which the coil is immersed, but there is no certainty that the coil follows the water exactly. There is evidence, however, that the differences in R are partly due to a real difference in the resistance of different fillings of the tube—whether owing to microscopic bubbles or to a thin varying layer of air between the mercury and the glass, or to what cause, we were unable to determine.*

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* A variation in the closeness of contact between mercury and glass amounting to less than one-fifth of a wave-length of mean light would account for the difference of resistances in the two fillings of tube IV.

We found some reason for thinking that the resistance tended to diminish with time when the mercury remained long in the tube. To examine this we filled tube II. on April 3rd, and found its resistance to be ·99077. It was then left standing full of mercury till April 18th, when the resistance was ·99055. This difference can hardly be relied upon; and in any case the experiments we have tabulated cannot well be affected by any change of this kind, as the interval between the measurement of resistance and that of volume was very short, except in cases 1 and 7. In case 7 the tube stood full of mercury for two days after the resistance was taken. In case 1 the resistance was measured on two successive days, and the mean of the two values taken. The second was the lowest by ·00020, possibly owing to an error. The length was measured immediately after the last measurement of resistance.

The variations in the values of r are, as we should expect, greater than those in R, being affected by probable errors in the other data. The extreme difference amounts to less than 6 in 10,000, and the greatest divergence from the mean value is 3·3 in 10,000.

The mean value of r according to these experiments, ·95418, lies between that deduced from Dr. SIEMENS' experiments for his 1864 standard, namely, ·9534, and Dr. MATTHIESSEN's value, namely, ·9619 (Phil. Mag., May, 1865), but the difference between our value and Dr. MATTHIESSEN's, namely, ·00772, is nearly ten times as great as that between ours and Dr. SIEMENS'. We are unable to account satisfactorily for this large difference. One point, however, is worth noting. Dr. MATTHIESSEN measured the resistance of the mercury in his tubes, not at zero, but at temperatures between 18° and 19°·1 (Report of British Association Committee for 1864). To deduce the specific resistance at zero, therefore, he must have assumed the coefficient of variation with temperature, and presumably—though it is nowhere stated in the Report—he used that found from his own experiments (Phil. Trans., 1862), namely, ·074[*] per cent. per degree. Our own observations have led us to suspect that this value is too small. We made three comparisons of the resistance of tube III. in ice, and in water at approximately the temperature of the room, and one similar comparison with tube IV. The results are given in the following table. Our arrangements were not adapted for observing the resistance at other temperatures, as the open trough afforded no means of checking rapid change.

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[*] This is the value which results from the experiments made at 0° and at about 20°.

Date.	No. of tube.	Mean temperature of water in the trough.	Resistance in water.	Resistance at 0°.	Difference for 1° ÷ resistance at 0°.	Mean of the four values in the last column.
March 13 . . .	III.	12°·7	1·00814	·99720	·000863	·000861*
„ 14 . . .	III.	13·25	1·00874	·99725	·000870	
„ 28 . . .	III.	12·8	1·00810	·99720	·000854	
„ 24 . . .	IV.	12·5	·51318	·50774	·000857	

The above determined mean coincides with the value found by SCHRÖDER VAN DER KOLK,† whose observations, however, related to a much greater range of temperature. An observation by WERNER SIEMENS‡ between the temperature 18°·5 and 0° gives for the coefficient ·00090.

The difference between the coefficients ·00074 and ·00086, as applied to the reduction from 18°·7 (the mean temperature of the tubes in Dr. MATTHIESSEN's observations) to 0°, would account for about one quarter of the difference between his results and our own.

The remainder of the discrepancy may possibly be connected with the manner in which Dr. MATTHIESSEN's tubes were calibrated. Although in the description of the process a *small* column of mercury is spoken of (Reprint, p. 128), it is distinctly stated on the preceding page that the lengths of the columns of mercury were 383, 291, 245 millims. respectively, *i.e.*, nearly half the lengths of the tubes. It is possible that this may be a mistake; but if such lengths were really used, the correction for conicality would have been much underestimated, so that the specific resistance of mercury would come out too high. In the case of uniform conicality the true correction would be four times as great as that obtained by applying the formula applicable to short threads, to cases where the length is about half that of the tube.

[*January*, 1883.—The measuring rod and the weights used in the above investigation have been compared with standards verified by the Board of Trade, and the errors have been found to be negligible. But since the value of ρ employed relates to weighings *in vacuo*, a corresponding correction is called for here. On this account the final number, ·95418, should be reduced to

·95412.]

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* It should be noticed that the resistances here compared are those of the contents of a certain glass tube at various temperatures, so that the accompanying temperature variations of length and section are determined by the properties of glass and not by the properties of mercury. The results are therefore not quite comparable with those obtained in similar experiments with solid metallic wires, which are free to determine for themselves their length and section.

† POGG. Ann., cx., 1860.

‡ Ibid., cxiii., 1861.

III. *On the Ultra-Violet Spectra of the Elements.*—Part I. *Iron* (*with a map*).

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[Plates 1–5.]

Ångström's "normal solar spectrum" has served most spectroscopists as a standard of reference for wave-lengths in the visible part of the spectrum. Cornu's continuation of it, and particularly the map of the iron lines which he used in the construction of it, serves very well for such a standard up to the limit of the solar spectrum, *i.e.*, to a wave-length 2948 (U). For the region above this we have had to use Mascart's and Cornu's wave-lengths of the cadmium lines, and Cornu's wave-lengths of some magnesium lines. The intervals between those lines are, however, too great for any close approximation to the wave-lengths of intervening lines by interpolation, and, moreover, the wave-lengths did not appear to be determined with sufficient precision to serve as a standard, and the lines are ill adapted to that purpose by reason of their diffuse character. For the determination of the wave-lengths of lines in this higher region we have, therefore, been driven to form a standard for ourselves. For this purpose we have chosen the iron spectrum which had been employed by Cornu in the region which he mapped, and seemed to answer the purpose well, both from the number of lines which it presents and their characteristic grouping by which they may easily be recognised by anyone who has once become familiar with them. The wave-lengths of the most prominent lines were determined by means of a Rutherfurd diffraction grating, as detailed below, between the wave-lengths 2948 and 2327; but beyond this there is a remarkable falling off in the intensity of the iron lines, and between wave-lengths 2327 and 2135 (which is near the limit of transparency of Iceland spar) we have preferred to determine the wave-lengths of the prominent copper lines which are numerous and strong in that region. The wave-lengths of a series of lines at short intervals having thus been determined, those of the intervening lines were obtained by interpolation, and the result is shown in the map of the iron spectrum above U which accompanies this paper.

The instruments.

The goniometer used was constructed for us by HILGER, and has a circle of 18 inches diameter, graduated at intervals of 5′ by SIMMS. Fractions of 5′ are read by a microscope with a micrometer eye-piece fixed to the arm which carries the telescope.

The telescope and collimator have each an object-glass consisting of a single lens of quartz $1\frac{1}{2}$ inch diameter and a focal length of $18\frac{1}{2}$ inches for the sodium yellow light, but not more than 16 inches for the highest rays measured. The sliding tubes of both telescope and collimator are graduated in fiftieths of an inch, and alterations of focus were made on both instruments at the same time, so that the rays falling on the grating might always be nearly parallel. The graduation of the sliding tube was also used for ascertaining the distance of the photographic plate from the object-glass of the telescope. This was necessary for computing the corrections of the angular measure, as explained below. The collimator is furnished with a quartz lens, of 3 inches focal length, in front of the slit, movable to a greater or less distance, but retained by guides so that its axis may remain coincident with that of the collimator. This lens was placed about 6 inches in front of the slit, and the source of light at the same distance beyond it, so that its image was focussed on the slit.

The measurements were all made by means of photographs taken on WRATTEN and WAINWRIGHT's instantaneous dry gelatine plates. The plates ($2\frac{1}{4}$ inches by 1 inch) were held in a small slide attached to a tube which fitted the telescope in place of the eye-piece, and thus the plate could easily be turned about an axis perpendicular to its plane and coinciding with the axis of the telescope. This turning of the plate about is a matter of no small importance, as it enabled us to avoid the errors which would have arisen from measuring the distances of the lines from the irregular edge of the plate, as will be seen when the mode of measuring the photographs is described. The plates were retained in one position in the slide during exposure by three springs, of which two pressed against two edges of the plate and the other against its back.

The grating was ruled on speculum metal by CHAPMAN with RUTHERFURD's machine, and has a ruled surface of rather more than $1\frac{3}{4}$ inch in each dimension, with 17,296 lines to the inch. It is an excellent grating, but, of course, has the faults which belong to the particular machine by which it was ruled. The definition, when it has not been exposed to variations of temperature, is very good, but it has one inconvenience for our present purpose, which is, that the focus for the same ray in the spectrum of the same order does not fall at quite the same distance from the object-glass of the telescope on the two sides of the normal. The explanation of this has been given by CORNU (Comptes Rendus, lxxx., 645), who has shown that it is due to a systematic variation in the distance between the ruled lines. As the method employed by us depends upon taking angular measures of the position of the ray on both sides of the normal, and any shift of the focussing tube between the two positions would be likely to introduce serious errors, we have been obliged to be content with the photograph taken on one side being a little, though only a little, less sharp in definition than that taken on the other side. The grating was used with its plane perpendicular to the axis of the collimator, and it was brought into that position in the following way. The telescope and collimator having their axes directed as nearly as possible on to the centre of the circle, the telescope was placed opposite the collimator and the cross wires brought on to the image of the slit, and the reading of the circle taken. The grating was then placed in position and adjusted for level until the spectra occupied the middle of the field of view in all positions of the telescope. It was then adjusted in azimuth until the images of the D lines, from the light of a sodium flame in front of the slit, in the spectra of the second order, on the two sides of the normal were at equal angular distances from the axis of the collimator, as determined by the reading taken when the telescope was opposite the collimator. The grating was then clamped in that position. This adjustment had to be made by hand and was liable to disturbance in the clamping, so that it was afterwards found that the plane of the grating was not quite perpendicular to the axis of the collimator; but as the errors arising from this in the measures on the two sides nearly compensate one another, the final error in the wave-length from this cause is very small indeed.

For measuring the photographs a micrometer was constructed for us by HILGER. This is attached to the stage of a microscope and carries a small frame in which the photographic plate is held by springs. The micrometer-screw has 100 turns to the inch, and by the drum-head $\frac{1}{100}$th of a turn or $\frac{1}{10000}$th of an inch can be read. A 1 inch object-glass to the microscope was used, and measures were made by moving the plate until the lines of the photograph were successively bisected by a spider line in the eye-piece. The reading of the micrometer gave the distances between the lines.

The source of light employed was, in the first instance, the arc from a DE MERITENS magneto-electric machine, in a crucible of magnesia into which iron wire was introduced. But from the overlapping of the spectra of different orders, and the large amount of light emitted by the arc, we found that the plates were so clouded in many places that the lines could not be well seen, and we abandoned the arc for the spark between iron electrodes. This was produced by a large induction coil, worked by 5 GROVE's cells, and having a large Leyden jar connected with the secondary wire. No inconvenience arose from the overlapping of the different orders when the spark was used, because the parts of the spectra of higher and lower orders which overlapped the part of the spectrum of the fourth order to be measured were always considerably out of focus, the object glasses of telescope and collimator being uncorrected, and so the light of the lines in the overlapping spectra was diffused and produced only a faint clouding of the plate, which in no way interfered with the measure of the lines of the fourth order.

Mode of proceeding.

The first thing to be done was to obtain a focussing scale for the different angles, including the portion of the spectrum to be mapped. This was already known approximately from SARASIN's table of refractive indices of quartz, and was corrected by a series of trial plates taken at successive angles.

The electrodes, between which the spark was passed, were arranged so that the spark should pass horizontally (the slit being vertical), and at such a height that the visible image formed by the lens in front of the slit on the plates of the slit might fall just above, or partly above, the centre of the slit. The lower half of the slit was closed by a shutter, so that only the lower half of the field of view of the telescope was illuminated. The distance of the spark from the lens had next to be adjusted, as the focal length of the lens for the visible rays is very different from that for the ultra-violet which were to be photographed. This was done by estimation, as there was no need to have the image exactly focussed on the slit, so long as the slit was in the middle of the image and light enough passed through.

The telescope was then set to such an angle as would bring the line so measured nearly into the centre of the field, the focus adjusted, the photographic slide adjusted and levelled, and the plate exposed. An image of the lines was thus formed on the lower half of the plate. The slide was then turned round through 180° about the axis of the tube, so as to bring what had before been the upper side of the plate to the lower side and right to left, and again levelled. The plate was then again exposed and thus a second image of the line impressed, and one of the two images was as far to the right of the axis about which the plate had been turned as the other was to the left. Half the distance between the two images would therefore be the distance of the line from the centre of the field, and the knowledge of this would give the means of calculating the deviation of the rays producing the line from the axis of the telescope. The telescope was next turned to the corresponding angle on the other side of the collimator and the operations repeated with a second plate, but without any alterations of adjustment. The telescope was then moved through a small angle, generally 5′ or 10′, and the same operations as before repeated on both sides of the collimator, the second pair of plates being intended to serve as a check upon the first.

Similar operations were then repeated at such angular intervals as should bring in the most characteristic strong lines of iron all along the scale. Beyond the wave-length 2327, it was found that the iron lines were too faint to produce any sufficient impression on the plates. For the region beyond this up to wave-length 2135, copper electrodes were substituted for iron. This being about the limit of transparency of calcite (the material of our prisms), was the limit of our study of spectra at this time.

The measurement of the distance between the two images of a line was made by the micrometer above described, and to convert this distance into arc, it was necessary to know the distance of the plate from the centre of the object-glass of the telescope. It was found by measurement that the distance from the object-glass to the front of the photographic plate, when the sliding tube was at the 100th division, was 439 millims., and the thickness of the lens at its middle 2·5 millims., which, divided by the refractive index of quartz for the high rays observed is very nearly 1·5 millim., making the distance of the photographic plate from the optical centre of the object-glass very nearly 440·5 millims. From this the formula $\tan^{-1}\frac{D}{3068{\cdot}5+4a}$ was deduced for the angular distance of a line from the axis of the telescope when the difference of the micrometer readings of the two images of the line is D, and the number of divisions of the scale of sliding tube at which the telescope was focussed was a. To determine whether the angle so found was to be added or subtracted from the reading of the circle, all that was necessary was to observe whether the micrometer reading of the upper or lower image as seen in the microscope were the greater. The angular position of the line having thus been found on each side of the collimator, the wave-length was deduced by the ordinary formula.

The following tables give the measurements made, the calculated angles, and the wave-lengths deduced.

The quantities recorded in the several columns is as follows :—

I. The mark of the particular photographic plate.

II. The reading of the circle, giving the position of the telescope.

III. The reading of the scale of focussing tube.

IV. The measured distance of the two images of the line in hundredths of an inch.

V. One half this distance reduced to angular measure with the sign + or − according as it is to be added or subtracted from the reading of the circle to give the angular position of the line.

VI. The angular position of the line.

VII. The mean values of the angular positions so found. When several sets of plates have been taken at different times, or with varied adjustments, the means for the several sets are given separately.

VIII. The deviation from the normal to the grating of the line as deduced.

IX. The wave-length deduced or adopted. When different sets of plates give different values, these different values are inserted in brackets.

The figures belonging to different lines are separated by horizontal spaces.

In the case of plates marked with a figure less than 172 the arc was employed to give the light, in the case of all plates with the mark 172 and upwards the spark was employed.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
	° ′ ″		hundredths of an inch.	′ ″	° ′ ″	° ′ ″	° ′ ″	wave-length.
341	28 30 0	49	10·63	−11 12	28 18 48			
342	309 30 0	49	8·43	+ 8 53	309 38 53			
343	309 25 0	49	13·39	+14 6	309 39 6	309 39 0	39 19 54	2326·9
341	28 30 0	49	6·11	− 6 26	28 23 34			
342	309 30 0	49	3·96	+ 4 10	309 34 10			
343	309 25 0	49	8·72	+ 9 11	309 34 11		39 24 42	2330·9
341	28 30 0	49	4·36	− 4 35	28 25 25			
342	309 30 0	49	2·01	+ 2 7	309 32 7			
343	309 25 0	49	6·92	+ 7 17	309 32 17	309 32 12	39 26 36	2332·5
341	28 30 0	49	1·64	+ 1 44	28 31 44			
342	309 30 0	49	3·96	− 4 10	309 25 50			
343	309 25 0	49	0·87	+ 0 55	309 25 55	309 25 52	39 32 56	2337·7
341	28 30 0	49	7·92	+ 8 20	28 38 20			
342	309 30 0	49	10·37	−10 55	309 19 5			
343	309 25 0	49	5·57	− 5 52	309 19 8	309 19 6	39 39 37	2343·2
341	28 30 0	49	8·78	+ 9 15	28 39 15			
342	309 30 0	49	11·27	−11 52	309 18 8			
343	309 25 0	49	6·36	− 6 42	309 18 18	309 18 13	39 40 31	2343·9
341	28 30 0	49	13·40	+14 7	28 44 7			
342	309 30 0	49	15·89	−16 44	309 13 16			
343	309 25 0	49	11·07	−11 39	309 13 21	309 13 14	39 45 27	2348·0
262	28 45 0	50	11·67	+12 16	28 57 16			
263	28 40 0	50	16·38	+17 14	28 57 14	28 57 15		
264	309 15 0	50	13·97	−14 42	309 0 18			
265	309 10 0	50	9·18	− 9 39	309 0 21	309 0 19	39 58 28	2358·7
262	28 45 0	50	12·91	+13 35	28 58 35			
263	28 40 0	50	17·63	+18 33	28 58 33	28 58 34		
264	309 15 0	50	15·15	−15 56	308 59 4			
265	309 10 0	50	10·44	−10 59	308 59 1	308 59 2	39 59 46	2359·7
262	28 45 0	50	18·33	+19 17	29 4 17			
263	28 40 0	50	23·02	+24 13	29 4 13	29 4 15		
264	309 15 0	50	20·68	−21 45	308 53 15			
265	309 10 0	50	15·98	−16 48	308 53 12	308 53 13	40 5 31	2364·4
81	29 30 3	25	1·70	+ 1 51	29 31 54			
82	29 30 3	25	2·02	+ 2 12	29 32 15			
86	29 34 3	25	2·37	− 2 35	29 31 28	29 31 52		
84	308 59 40	25	1·52	− 1 39	308 58 1			
85	308 59 40	25	1·20	− 1 18	308 58 22	308 58 12	40 16 50	(2373·7)
100	308 53 50	25	2·47	− 2 41	308 51 9			
101	308 53 50	25	2·66	− 2 53	308 50 57	308 51 3		
102	29 40 0	25	14·64	−15 53	29 24 7			
103	29 30 0	25	5·40	− 5 52	29 24 8	29 24 7	40 16 32	2373·4
262	28 45 0	50	28·65	+30 8	29 15 8			
263	28 40 0	50	33·33	+35 3	29 15 3	29 15 6		
264	309 15 0	50	31·09	−32 42	308 42 18			
265	309 10 0	50	26·36	−27 43	308 42 17	308 42 17	40 16 25	(2373·3)

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
	° ′ ″		hundredths of an inch.	′ ″	° ′ ″	° ′ ″	° ′ ″	wave-length.
262	28 45 0	50	38·19	+40 10	29 25 10			
263	28 40 0	50	42·96	+45 11	29 25 11	29 25 11		
264	509 15 0	50	40·87	−42 59	308 32 1			
265	309 10 0	50	36·11	−37 59	308 32 1		40 26 35	(2381·6)
81	29 30 3	25	11·29	+12 14	29 42 17			
82	29 30 3	25	11·62	+12 36	29 42 39			
86	29 34 3	25	7·13	+ 7 44	29 41 47	29 42 14		
84	308 59 40	25	11·02	−11 57	308 47 43			
85	308 59 40	25	10·82	−11 44	308 47 56	308 47 48	40 27 13	(2382·1)
100	308 53 50	25	12·00	−13 1	308 40 49			
101	308 53 50	25	12·26	−13 18	308 40 32	308 40 40		2381·7
102	29 40 0	25	5·27	− 5 43	29 34 17			
103	29 30 0	25	4·10	+ 4 27	29 34 27	29 34 22	40 26 51	(2381·8)
258	308 15 0	53	16·34	+17 7	308 32 7			
259	308 20 0	53	11·86	+12 26	308 32 26	308 32 17		
260	29 45 0	53	18·78	−19 41	29 25 19			
261	29 40 0	53	13·70	−14 21	29 25 39	29 25 29	40 26 36	(2381·6)
81	29 30 3	25	18·96	+20 34	29 50 37			
82	29 30 3	25	19·14	+20 55	29 50 58			
86	29 34 3	25	14·66	+15 54	29 49 57	29 50 31		
84	308 59 40	25	18·66	−20 15	308 39 25			
85	308 59 40	25	18·45	−20 1	308 39 39	308 39 32	40 35 29	(2388·8)
258	308 15 0	53	8·56	+ 8 58	308 23 58			
259	308 20 0	53	4·01	+ 4 12	308 24 12	308 24 5		
260	29 45 0	53	11·02	−11 33	29 33 27			
261	29 40 0	53	5·89	− 6 10	29 33 50	29 33 38	40 34 46	(2388·2)
100	308 53 50	25	19·61	−21 17	308 32 33			
101	308 53 50	25	19·77	−21 27	308 32 23	308 32 28		
102	29 40 0	25	2·31	+ 2 30	29 42 30			
103	29 30 0	25	11·64	+12 38	29 42 38	29 42 34	40 35 3	2388·5
100	308 53 50	25	27·60	−29 57	308 23 53			
101	308 53 50	25	27·79	−30 9	308 23 4	308 23 47		
102	29 40 0	25	10·33	+11 12	29 51 12			
103	29 30 0	25	19·64	+21 18	29 51 18	29 51 15	40 43 44	(2395·5)
258	308 15 0	53	0·22	+ 0 14	308 15 14			
259	308 20 0	53	4·41	− 4 37	308 15 23	308 15 18		2395·4
260	29 45 0	53	2·85	− 3 3	29 41 57			
261	29 40 0	53	2·20	+ 2 18	29 42 18	29 42 7	40 43 24	(2395·2)
100	308 53 50	25	31·80	−34 30	308 19 20			
101	308 53 50	25	82·03	−34 45	308 19 5	308 19 12		
102	29 40 0	25	14·46	+15 41	29 55 41			
103	29 30 0	25	23·85	+25 53	29 55 53	29 55 47	40 48 18	(2399·2)
258	308 15 0	53	4·16	− 4 22	308 10 38			
259	308 20 0	53	8·65	− 9 4	308 10 56	308 10 47		2399
260	29 45 0	53	1·45	+ 1 31	29 46 31			
261	29 40 0	53	6·53	+ 6 51	29 46 51	29 46 41	40 47 57	(2398·9)
258	308 15 0	53	10·89	−11 25	308 3 35			
259	308 20 0	53	15·48	−16 13	308 3 47	308 3 41		
260	29 45 0	53	8·08	+ 8 28	29 53 28			
261	29 40 0	53	13·13	+13 46	29 53 46	29 53 37	40 54 58	2404·5

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
	° ′ ″		hundredths of an inch.	′ ″	° ′ ″	° ′ ″	° ′ ″	wave-length.
258	308 15 0	53	12·97	−13 36	308 1 24			
259	308 20 0	53	17·53	−18 22	308 1 38	308 1 31		
260	29 45 0	53	10·18	+10 40	29 55 40			
261	29 40 0	53	15·18	+15 54	29 55 54	29 55 47	40 57 8	2406·3
258	308 15 0	53	17·57	−18 25	307 56 35			
259	308 20 0	53	22·12	−23 11	307 56 49	307 56 42		
260	29 45 0	53	14·65	+15 21	30 0 21			
261	29 40 0	53	19·69	+20 38	30 0 38	30 0 30	41 1 54	2410·2
258	308 15 0	53	18·20	−19 4	307 55 56			
259	308 20 0	53	22·79	−23 53	307 56 7	307 56 1		
260	29 45 0	53	15·41	+16 9	30 1 9			
261	29 40 0	53	20·35	+21 20	30 1 20	30 1 15	41 2 37	2410·7
258	308 15 0	53	20·93	−21 56	307 53 4			
259	308 20 0	53	25·55	−26 46	307 53 14	307 53 9		
260	29 45 0	53	17·99	+18 51	30 3 51			
261	29 40 0	53	23·01	+24 7	30 4 7	30 3 59	41 5 25	2413·0
254	30 45 0	57	7·82	− 8 9	30 36 51			
255	30 40 0	57	3·05	− 3 11	30 36 49	30 36 50		
256	307 15 0	57	5·71	+ 5 57	307 20 57			
257	307 20 0	57	1·06	+ 1 6	307 21 6	307 21 2	41 37 54	2439·0
254	30 45 0	57	1·60	− 1 40	30 43 20			
255	30 40 0	57	3·17	+ 3 18	30 43 18	30 43 19		
256	307 15 0	57	0·67	− 0 42	307 14 18			
257	307 20 0	57	5·37	− 5 36	307 14 24	307 14 21	41 44 29	2444·3
254	30 45 0	57	0·27	− 0 17	30 44 43			
255	30 40 0	57	4·45	+ 4 38	30 44 38	30 44 41		
256	307 15 0	57	1·98	− 2 4	307 12 56			
257	307 20 0	57	6·70	− 6 59	307 13 1	307 12 58	41 45 52	2445·4
250	32 0 0	65	47·16	−48 42	31 11 18			
251	306 5 0	65	40·20	+41 31	306 46 31		42 12 23	2466·4
250	32 0 0	65	42·39	−43 47	31 16 13			
251	306 5 0	65	35·41	+36 34	306 41 34		42 17 19	2470·3
250	32 0 0	65	30·79	−31 48	31 28 12			
253	31 45 0	65	16·06	−16 35	31 28 25	31 28 18		
251	306 5 0	65	23·53	+24 18	306 29 18			
252	306 15 0	65	13·80	+14 15	306 29 15	306 29 17	42 29 31	2480·0
250	32 0 0	65	28·43	−29 22	31 30 38			
253	31 45 0	65	13·78	−14 14	31 30 46	31 30 42		
251	306 5 0	65	21·27	+21 58	306 26 58			
252	306 15 0	65	11·42	+11 48	306 26 48	306 26 53	42 31 54	2481·8
250	32 0 0	65	27·73	−28 38	31 31 22			
251	306 5 0	65	20·52	+21 12	306 26 12		42 32 35	2482·4
71	31 54 45	50	3·07	− 3 14	31 51 31			
72	306 38 55	50	5·78	+ 6 5	306 45 0			
73	306 38 55	50	5·79	+ 6 5	306 45 0	306 45 0	42 33 15	2482·9

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
	° ′ ″		hundredths of an inch.	′ ″	° ′ ″	° ′ ″	° ′ ″	wave-length.
71	31 54 45	50	2·04	− 2 9	31 52 36			
72	306 38 55	50	4·77	+ 5 1	306 43 56			
73	306 38 55	50	4·85	+ 5 6	306 44 1	306 43 59	42 34 18	2483·7
250	32 0 0	65	23·27	−24 2	31 35 58			
253	31 45 0	65	8·35	− 8 37	31 36 23	31 36 10		
251	306 5 0	65	16·24	+16 46	306 21 46			
252	306 15 0	65	6·02	+ 6 13	306 21 13	306 21 30	42 37 20	2486·1
107	306 10 10	50	17·40	+18 18	306 28 28			
108	306 0 0	50	27·46	+28 53	306 28 53			
109	306 10 10	50	17·71	+18 36	306 28 46			
110	306 20 0	50	8·64	+ 9 6	306 29 6	306 28 48		
111	32 0 5	50	11·83	−12 27	31 47 33			
112	32 0 5	50	12·02	−12 39	31 47 26	31 47 33	42 39 23	2487·7
113	31 50 10	50	2·39	− 2 31	31 47 39			
71	31 54 45	50	4·74	+ 4 59	31 59 44			
72	306 38 55	50	2·06	− 2 10	306 36 45			
73	306 38 55	50	2·08	− 2 11	306 36 44	306 36 44	42 41 30	(2489·4)
172	31 50 5	50	2·00	− 2 6	31 47 59			
173	31 40 20	50	7·41	+ 7 48	31 48 8			
174	32 0 5	50	11·67	−12 16	31 47 49	31 47 59		
175	306 23 0	50	1·04	+ 1 6	306 24 6			2489·5
176	306 33 5	50	8·37	− 8 48	306 24 17			
177	306 13 0	50	10·74	+11 18	306 24 18	306 24 14	42 41 53	(2489·7)
250	32 0 0	65	19·12	−19 45	31 40 15			
253	31 45 0	65	4·34	− 4 29	31 40 31	31 40 23		
251	306 5 0	65	11·64	+12 1	306 17 1			
252	306 15 0	65	1·79	+ 1 54	306 16 54	306 16 57	42 41 43	2489·5
250	32 0 0	65	17·91	−18 30	31 41 30			
253	31 45 0	65	3·12	− 3 13	31 41 47	31 41 38		
251	306 5 0	65	10·51	+10 51	306 15 51			
252	306 15 0	65	0·73	+ 0 45	306 15 45	306 15 48	42 42 55	2490·5
250	32 0 0	65	14·87	−15 21	31 44 39			
253	31 45 0	65	0·09	− 0 6	31 44 54	31 44 46		
251	306 5 0	65	7·42	+ 7 40	306 12 40			
252	306 15 0	65	2·41	− 2 29	306 12 29	306 12 35	42 46 5	2493·0
172	31 50 5	50	2·19	+ 2 18	31 52 23			
173	31 40 20	50	11·59	+12 11	31 52 31			
174	32 0 5	50	7·55	− 7 56	31 52 9	31 52 21		
175	306 23 0	50	3·19	− 3 22	306 19 38			2493·0
176	306 33 5	50	12·59	−13 15	306 19 50			
177	306 13 0	50	6·53	+ 6 52	306 19 52	306 19 47	42 46 17	(2493·1)
246*	305 20 0	65	51·22	+52 54	306 12 54			
247	305 25 0	65	46·37	+47 53	306 12 53	306 12 54		
248	32 45 0	65	58·23	−60 8	31 44 52			
249	32 40 0	65	53·28	−55 1	31 44 59	31 44 55	42 46 0	(2492·9)

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
	° ′ ″		hundredths of an inch.	′ ″	° ′ ″	° ′ ″	° ′ ″	wave-length.
246*	305 20 0	65	44·18	+45 38	306 5 38			
247	305 25 0	65	39·34	+40 38	306 5 38			
248	32 45 0	65	51·14	−52 49	31 52 11			
249	32 40 0	65	46·31	−47 50	31 52 11		42 53 16	(2498·6)
172	31 50 5	50	8·97	+ 9 26	31 59 31			
173	31 40 20	50	18·45	+19 24	31 59 44			
174	32 0 5	50	0·64	− 0 40	31 59 25	31 59 33		
175	306 23 0	50	10·13	−10 39	306 12 21			2498·7
176	306 33 5	50	19·56	−20 34	306 12 31			
177	306 13 0	50	0·42	− 0 27	306 12 33	306 12 30	42 53 32	(2498·8)
91	306 39 0	50	26·86	−28 15	306 10 45			
92	31 55 0	50	24·04	+25 17	32 20 17			
93	31 55 0	50	23·92	+25 10	32 20 10	32 20 14	43 4 45	2507·6
181	305 22 0	55	33·07	+34 34	305 56 34	305 56 34		
182	32 56 10	55	38·61	−40 22	32 15 48			
183	32 55 5	55	37·47	−39 10	32 15 55			
184	32 51 15	55	33·88	−35 25	32 15 50			
185	32 35 2	55	18·40	−19 14	32 15 48			
186	32 25 15	55	9·02	− 9 26	32 15 49	32 15 50	43 9 38	2511·4
246*	305 20 0	65	28·14	+29 4	305 49 4			
247	305 25 0	65	23·20	+23 54	305 48 54	305 48 59		
248	32 45 0	65	35·35	−36 31	32 8 29			
249	32 40 0	65	30·54	−31 33	32 8 27	32 8 28	43 9 44	2511·4
246*	305 20 0	65	10·95	+11 19	305 31 19			
247	305 25 0	65	5·97	+ 6 10	305 31 10	305 31 15		
248	32 45 0	65	18·43	−19 2	32 25 58			
249	32 40 0	65	13·56	−14 0	32 26 0	32 25 59	43 27 22	(2525·2)
181	305 22 0	55	16·10	+16 50	305 38 50	305 38 50		
182	32 56 10	55	22·02	−23 1	32 33 9			
183	32 55 5	55	20·32	−21 46	32 33 19			
184	32 51 15	55	17·24	−18 1	32 33 14			
185	32 35 2	55	1·72	− 1 48	32 33 14			
186	32 25 15	55	7·69	+ 8 2	32 33 17	32 33 15	43 27 13	2525·1
246*	305 20 0	65	9·81	+10 8	305 30 8			
247	305 25 0	65	4·84	+ 5 0	305 30 0	305 30 4		
248	32 45 0	65	17·35	−17 55	32 27 5			
249	32 40 0	65	12·48	−12 53	32 27 7	32 27 6	43 28 31	2526·1
246*	305 20 0	65	5·72	+ 5 54	305 25 54			
247	305 25 0	65	0·78	+ 0 48	305 25 48	305 25 51		
248	32 45 0	65	13·25	−13 41	32 31 19			
249	32 40 0	65	8·32	− 8 36	32 31 24	32 31 22	43 32 45	(2529·3)
178	305 53 10	55	19·06	−19 55	305 33 15			
179	305 47 55	55	13·90	−14 32	305 33 23			
180	305 18 0	55	14·84	+15 31	305 33 31			
181	305 22 0	55	10·92	+11 25	305 33 25	305 33 23		
182	32 56 10	55	16·92	−17 42	32 38 28			
183	32 55 5	55	15·70	−16 25	32 38 40			
184	32 51 15	55	12·23	−12 47	32 38 28			
185	32 35 2	55	3·42	+ 3 35	32 38 37			
186	32 25 15	55	12·87	+13 27	32 38 42	32 38 35	43 32 36	2529·2

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
	° ′ ″		hundredths of an inch.	′ ″	° ′ ″	° ′ ″	° ′ ″	wave-length.
246*	305 20 0	65	0·55	+ 0 34	305 20 34			
247	305 25 0	65	4·39	− 4 32	305 20 28	305 20 31		
248	32 45 0	65	8·22	− 8 29	32 36 31			
249	32 40 0	65	3·27	− 3 23	32 36 37	32 36 34	43 38 2	2533·4
246*	305 20 0	65	0·35	− 0 22	305 19 38			
247	305 25 0	65	5·[illegible]1	− 5 29	305 19 29	305 19 34		
248	32 45 0	65	7·20	− 7 26	32 37 34			
249	32 40 0	65	2·32	− 2 24	32 37 36	32 37 35	43 39 0	2534·2
246*	305 20 0	65	3·41	− 3 31	305 16 29			
247	305 25 0	65	8·27	− 8 33	305 16 27	305 16 28		
248	32 45 0	65	4·24	− 4 23	32 40 37			
249	32 40 0	65	0·65	+ 0 40	32 40 40	32 40 38	43 42 5	2536·6
178	305 53 10	55	28·22	−29 30	305 23 40			
179	305 47 55	55	23·04	−24 5	305 23 50			
180	305 18 0	55	5·71	+ 5 58	305 23 58			
181	305 22 0	55	1·74	+ 1 52	305 23 52	305 23 50		
182	32 56 10	55	7·90	− 8 15	32 47 55			
183	32 55 5	55	6·67	− 6 58	32 48 7			
184	32 51 15	55	3·20	− 3 21	32 47 54			
185	32 35 2	55	12·36	+12 55	32 47 57			
186	32 25 15	55	21·79	+22 47	32 48 2	32 47 59	43 42 5	2536·6
178	305 53 10	55	30·92	−32 19	305 20 51			
179	305 47 55	55	25·71	−26 53	305 21 2			
180	305 18 0	55	3·11	+ 3 15	305 21 15			
181	305 22 0	55	0·91	− 0 57	305 21 3	305 21 3		
182	32 56 10	55	5·40	− 5 39	32 50 31			
183	32 55 5	55	4·13	− 4 19	32 50 46			
184	32 51 15	55	0·76	− 0 48	32 50 27			
185	32 35 2	55	14·95	+15 38	32 50 40			
186	32 25 15	55	24·30	+25 24	32 50 39	32 50 37	43 44 47	2538·6
187	33 10 10	62	18·64	−19 19	32 50 51			
188	33 20 15	62	28·24	−29 16	32 50 59	32 50 55		
189	305 3 0	62	17·86	+18 31	305 21 31			
190	304 52 40	62	27·59	+28 36	305 21 16	305 21 23	43 44 46	2538·6
243	33 25 0	65	40·47	−41 48	32 43 12			
244	33 15 0	65	30·78	−31 47	32 43 13			
248	32 45 0	65	1·61	− 1 40	32 43 20			
249	32 40 0	65	3·27	+ 3 23	32 43 23	32 43 17		
245	304 50 0	65	23·15	+23 55	305 13 55			
246	304 45 0	65	27·85	+28 46	305 13 46			
246*	305 20 0	65	6·01	− 6 12	305 13 48			
247	305 25 0	65	11·01	−11 22	305 13 38	305 13 47	43 44 45	2538·6
242	33 30 0	65	32·13	−33 11	32 56 49			
243	33 25 0	65	27·25	−28 9	32 56 51			
244	33 15 0	65	17·63	−18 13	32 56 47	32 56 49		
245	304 50 0	65	9·83	+10 9	305 0 9			
246	304 45 0	65	14·56	+15 2	305 0 2	305 0 6	43 58 21	2549·1
242	33 30 0	65	31·41	−32 26	32 57 34			
243	33 25 0	65	26·58	−27 27	32 57 33			
244	33 15 0	65	16·87	−17 26	32 57 34	32 57 34		
245	304 50 0	65	8·99	+ 9 17	304 59 17			
246	304 45 0	65	13·77	+14 13	304 59 13	304 59 15	43 59 10	2549·7

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
	° ′ ″		hundredths of an inch.	′ ″	° ′ ″	° ′ ″	° ′ ″	wave-length.
242	33 30 0	65	30·64	−31 39	32 58 21			
243	33 25 0	65	25·75	−26 36	32 58 24			
244	33 15 0	65	16·12	−16 39	32 58 21	32 58 22		
245	304 50 0	65	8·19	+ 8 28	304 58 28			
246	304 45 0	65	12·89	+13 19	304 58 19	304 58 23	43 59 59	2550·3
242	33 30 0	65	15·64	−16 9	33 13 51			
243	33 25 0	65	10·74	−11 6	33 13 54			
244	33 15 0	65	1·13	− 1 10	33 13 50	33 13 52		
245	304 50 0	65	6·97	− 7 12	304 42 48			
246	304 45 0	65	2·27	− 2 21	304 42 39	304 42 43	44 15 34	2562·3
188	33 20 15	62	1·31	+ 1 21	33 21 36	33 21 36		
189	305 3 0	62	11·86	−12 18	304 50 42			
190	304 52 40	62	2·25	− 2 20	304 50 20	304 50 31	44 15 33	2562·3
242	33 30 0	65	14·50	−14 59	33 15 1			
243	33 25 0	65	9·57	− 9 53	33 15 7			
244	33 15 0	65	0·10	+ 0 6	33 15 6	33 15 5		
245	304 50 0	65	7·96	− 8 13	304 41 47			
246	304 45 0	65	3·45	− 3 34	304 41 26	304 41 36	44 16 44	2563·2
193	303 19 55	57	57·80	+60 16	304 20 11			
194	34 53 15	57	58·93	−61 27	33 51 48		44 45 48	(2585·3)
195	34 43 5	54	49·10	−51 23	33 51 42			
196	303 30 0	54	47·72	+49 57	304 9 57		44 45 52	2585·4
242	33 20 0	65	13·75	+14 12	33 44 12			
243	33 25 0	65	18·57	+19 11	33 44 11			
244	33 15 0	65	28·30	+29 14	33 44 14	33 44 12		
245	304 50 0	65	36·59	−37 47	304 12 13			
246	304 45 0	65	32·00	−33 3	304 11 57	304 12 5	44 46 4	(2585·5)
93	303 19 57	65	45·14	+46 37	304 6 34			
97	303 19 57	65	44·91	+46 26	304 6 23	304 6 29		
98	34 50 0	65	38·13	−39 23	34 10 37			
99	34 50 0	65	37·99	−39 14	34 10 46	34 10 41	45 2 6	2597·6
96	303 19 57	65	43·80	+45 14	304 5 11			
97	303 19 57	65	43·63	+45 4	304 5 1	304 5 6		
98	34 50 0	65	36·71	−37 55	34 12 5			
99	34 50 0	65	36·49	−37 41	34 12 19	34 12 12	45 3 33	(2598·7)
195	34 43 5	54	32·03	−33 32	34 9 33			
196	303 30 0	54	30·34	+31 45	304 1 45		45 3 54	(2599·0)
193	303 19 55	57	40·38	+42 2	304 1 58			2598·8
194	34 53 15	57	41·75	−43 32	34 9 43		45 3 52	(2599·0)
195	34 43 5	54	22·35	−23 24	34 19 41			
196	303 30 0	54	20·59	+21 33	303 51 33		45 14 4	(2606·6)
193	303 19 55	57	30·45	+31 45	303 51 40			2606·7
194	34 53 15	57	31·77	−33 8	34 20 7		45 14 13	(2606·8)
193	303 19 55	57	24·28	+25 19	303 45 14			
194	34 53 15	57	25·84	−26 57	34 26 18		45 20 32	(2611·5)

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
	° ′ ″		hundredths of an inch.	′ ″	° ′ ″	° ′ ″	° ′ ″	wave-length.
195	34 43 5	54	16·24	−17 0	34 26 5			
196	303 30 0	54	14·57	+15 15	303 45 15		45 20 25	2611·4
96	303 19 57	65	27·72	+28 38	303 48 35			
97	303 19 57	65	27·54	+28 27	303 48 24	303 48 30		
98	34 50 0	65	20·89	−21 34	34 28 26			
99	34 50 0	65	20·75	−21 26	34 28 34	34 28 30	45 20 0	(2611·1)
96	303 19 57	65	25·14	+25 58	303 45 55			
97	303 19 57	65	24·93	+25 45	303 45 42	303 45 49		
98	34 50 0	65	18·34	−18 57	34 31 3			
99	34 50 0	65	18·33	−18 56	34 31 4	34 31 3	45 22 37	2613·1
96	303 19 57	65	20·18	+20 50	303 40 47			
97	303 19 57	65	20·05	+20 42	303 40 39	303 40 43		
98	34 50 0	65	13·41	−13 51	34 36 9			
99	34 50 0	65	13·28	−13 43	34 36 17	34 36 13	45 27 45	(2616·9)
195	34 43 5	54	8·94	− 9 21	34 33 44			
196	303 30 0	54	7·18	+ 7 31	303 37 31		45 28 6	2617·2
193	303 19 55	57	16·92	+17 39	303 37 33			
194	34 53 15	57	18·54	−19 20	34 33 55		45 28 11	2617·2
193	303 19 55	57	6·58	+ 6 52	303 26 47			
194	34 53 15	57	8·29	− 8 39	34 44 36		45 38 55	(2625·3)
195	34 43 5	54	1·18	+ 1 14	34 44 19			
196	303 30 0	54	3·14	− 3 17	303 26 43		45 38 48	2625·2
96	303 19 57	65	9·76	+10 5	303 30 2			
97	303 19 57	65	9·45	+ 9 46	303 29 43	303 29 53		
98	34 50 0	65	2·96	− 3 4	34 46 56			
99	34 50 0	65	2·84	− 2 55	34 47 5	34 47 0	45 38 33	(2625·0)
237	34 30 0	73	7·12	+ 7 17	34 37 17			
238	34 25 0	73	11·88	+12 9	34 37 9	34 37 11		
239	303 0 0	70	18·88	+19 19	303 19 19	303 19 18	45 38 56	(2625·3)
240	303 15 0	70	4·18	+ 4 17	303 19 17			
241	34 45 0	70	7·68	− 7 53	34 37 7			
96	303 19 57	65	6·18	+ 6 23	303 26 20			
97	303 19 57	65	5·91	+ 6 6	303 26 3	303 26 12		
98	34 50 0	65	0·52	+ 0 32	34 50 32			
99	34 50 0	65	0·63	+ 0 39	34 50 39	34 50 35	45 42 12	(2627·7)
193	303 19 55	57	3·12	+ 3 20	303 23 15			
194	34 53 15	57	4·94	− 5 9	34 48 6		45 42 25	2627·9
237	34 30 0	73	10·63	+10 53	34 40 53			
238	34 25 0	73	15·41	+15 46	34 40 46	34 40 47		
239	303 0 0	70	15·44	+15 48	303 15 48	303 15 45	45 42 31	2627·9
240	303 15 0	70	0·66	+ 0 41	303 15 41			
241	34 45 0	70	4·16	− 4 16	34 40 44			

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
	° ′ ″		hundredths of an inch.	′ ″	° ′ ″	° ′ ″	° ′ ″	wave-length.
195	34 43 5	54	4·62	+ 4 51	34 47 56			
196	303 30 0	54	6·53	− 6 50	303 23 10		45 42 23	2627·9
96	303 19 57	65	2·49	+ 2 34	303 22 31			
97	303 19 57	65	2·23	+ 2 18	303 22 15	303 22 23		
98	34 50 0	65	4·34	+ 4 29	34 54 29			
99	34 50 0	65	4·45	+ 4 36	34 54 36	34 54 33	45 46 5	(2630·6)
193	303 19 55	57	0·61	− 0 38	303 19 17			
194	34 53 15	57	1·41	− 1 28	34 51 47		45 46 15	2630·7
195	34 43 5	54	8·15	+ 8 32	34 51 37			
196	303 30 0	54	10·07	−10 32	303 19 28		45 46 5	(2630·6)
237	34 30 0	73	14·27	+14 36	34 44 36			
238	34 25 0	73	18·99	+19 26	34 44 26	34 44 29		
239	303 0 0	70	11·58	+11 51	303 11 51	303 11 55	45 46 17	(2630·8)
240	303 15 0	70	2·93	− 3 1	303 11 59			
241	34 45 0	70	0·59	− 0 36	34 44 24			
195	34 43 5	54	51·32	+53 36	35 36 41			
196	303 30 0	54	53·71	−56 13	302 33 47		46 31 27	(2664·2)
232	302 30 0	76	2·77	− 2 49	302 27 11			
234	302 35 0	76	7·66	− 7 48	302 27 12	302 27 11		
235	35 30 0	76	0·16	− 0 10	35 29 50		46 31 20	2664·1
323	302 25 0	76	2·84	+ 2 54	302 27 54			
324	35 30 0	76	0·00	0 0	35 30 0		46 31 3	(2663·9)
195	34 43 5	54	53·96	+56 28	35 39 33			
196	303 30 0	54	56·38	−59 0	302 31 0		46 34 17	(2666·3)
323	302 25 0	76	0·23	+ 0 14	302 25 14			
324	35 30 0	76	2·57	+ 2 37	35 32 37		46 33 42	(2665·8)
232	302 30 0	76	5·41	− 5 31	302 24 29			
234	302 35 0	76	10·32	−10 31	302 24 29	302 24 29		
235	35 30 0	76	2·51	+ 2 34	35 32 34		46 34 3	2666·1
225	37 0 0	79	61·41	−62 22	35 57 38			
228	36 30 0	79	32·14	−32 39	35 57 21	35 57 29		
226	301 5 0	79	53·53	+54 22	301 59 22			
227	301 35 0	79	24·15	+24 32	301 59 32	301 59 27	46 59 1	(2684·4)
232	302 30 0	76	29·89	−30 28	301 59 32			
234	302 35 0	76	34·72	−35 23	301 59 37	301 59 34		2684·2
235	35 30 0	76	26·71	+27 14	35 57 14			
236	35 35 0	76	21·87	+22 18	35 57 18	35 57 16	46 58 51	(2684·3)
323	302 25 0	76	24·21	−24 41	302 0 19			
324	35 30 0	76	26·82	+27 21	35 57 21		46 58 31	(2684·0)

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
	° ′ ″		hundredths of an inch.	′ ″	° ′ ″	° ′ ″	° ′ ″	wave-length.
232	302 30 0	76	40·61	−41 24	301 48 36			
234	302 35 0	76	45·34	−46 12	301 48 48	301 48 42		
235	35 30 0	76	37·31	+38 2	36 8 2			
236	35 35 0	76	32·46	+33 5	36 8 5	36 8 4	47 9 41	2692·1
323	302 25 0	76	34·83	−35 30	301 49 30			
324	35 30 0	76	37·46	+38 11	36 8 11		47 9 21	(2691·9)
225	37 0 0	79	50·81	−51 36	36 8 24			
228	36 30 0	79	21·49	−21 50	36 8 10	36 8 17		2692·1
226	301 5 0	79	42·77	+43 26	301 48 26			
227	301 35 0	79	13·40	+13 37	301 48 37	301 48 32	47 9 52	(2692·2)
225	37 0 0	79	35·43	−35 59	36 24 1			
228	36 30 0	79	6·09	− 6 11	36 23 49	36 23 55		
226	301 5 0	79	27·19	+27 37	301 32 37			
227	301 35 0	79	2·23	− 2 16	301 32 44	301 32 41	47 25 37	2703·6
197	300 59 55	65	25·63	+26 28	301 26 23			
198	301 10 0	65	15·74	+16 16	301 26 16	301 26 20		
200	37 3 0	65	16·83	−17 23	36 45 37			
201	37 12 55	65	26·64	−27 31	36 45 24	36 45 30	47 39 35	(2713·7)
225	37 0 0	79	21·26	−21 36	36 38 24			
228	36 30 0	79	8·10	+ 8 14	36 38 14	36 38 19		2713·8
226	301 5 0	79	12·95	+13 9	301 18 9			
227	301 35 0	79	16·48	−16 21	301 18 39	301 18 24	47 39 58	(2714·0)
225	37 0 0	79	3·43	− 3 29	36 56 31			
228	36 30 0	79	26·04	+26 27	36 56 27	36 56 29		
226	301 5 0	79	5·05	− 5 8	300 59 52			
227	301 35 0	79	34·46	−35 0	301 0 0	300 59 56	47 58 16	(2727·1)
197	300 59 55	65	7·78	+ 8 2	301 7 57			
198	301 10 0	65	2·14	− 2 13	301 7 57	301 7 57		2727 0
200	37 3 0	65	0·88	+ 0 55	37 3 55			
201	37 12 55	65	8·75	− 9 2	37 3 53	37 3 54	47 57 58	(2726·9)
197	300 59 55	65	8·68	− 8 58	300 50 57			
198	301 10 0	65	18·58	−19 11	300 50 49	300 50 55		
200	37 3 0	65	17·27	+17 50	37 20 50			
201	37 12 55	65	7·59	+ 7 50	37 20 45	37 20 48	48 14 57	(2739·0)
229	37 30 0	86	16·07	−16 11	37 13 49			2739·1
230	300 35 0	86	7·97	+ 8 2	300 43 2			
231	300 30 0	86	12·87	+12 58	300 42 58	300 43 0	48 15 24	(2739·3)
229	37 30 0	86	10·99	−11 4	37 18 56			
230	300 35 0	86	2·88	+ 2 54	300 37 54			
231	300 30 0	86	7·81	+ 7 52	300 37 52	300 37 53	48 20 31	(2743·0)
197	300 59 55	65	13·79	−14 15	300 45 40			
198	301 10 0	65	23·62	−24 24	300 45 36	300 45 38		2742·8
200	37 3 0	65	22·23	+22 57	37 25 57			
201	37 12 55	65	12·47	+12 53	37 25 48	37 25 53	48 20 8	(2742·7)

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
	° ′ ″		hundredths of an inch.	′ ″	° ′ ″	° ′ ″	° ′ ″	wave-length.
197	300 59 55	65	18·35	−18 57	300 40 58			
198	301 10 0	65	28·21	−29 18	300 40 52	300 40 55		(2746·0)
200	37 3 0	65	26·68	+27 33	37 30 33			
201	37 12 55	65	17·04	+17 36	37 30 31	37 30 32	48 24 49	2746·1
229	37 30 0	86	6·46	− 6 31	37 23 29			
230	300 35 0	86	1·71	− 1 43	300 33 17			
231	300 30 0	86	3·27	+ 3 18	300 33 18	300 33 17	48 25 6	(2746·2)
229	37 30 0	86	5·78	− 5 49	37 24 11			
230	300 35 0	86	2·38	− 2 24	300 32 36			
231	300 30 0	86	2·56	+ 2 35	300 32 35	300 32 36	48 25 47	2746·7
197	300 59 55	65	18·99	−19 37	300 40 18			
198	301 10 0	65	28·85	−29 48	300 40 12	300 40 15		
200	37 3 0	65	27·38	+28 17	37 31 17			
201	37 12 55	65	17·69	+18 39	37 31 34	37 31 26	48 25 36	(2746·6)
197	300 59 55	65	22·23	−22 58	300 36 57			
198	301 10 0	65	32·12	−33 10	300 36 50	300 36 53		
200	37 3 0	65	30·57	+31 34	37 34 34			
201	37 12 55	65	20·97	+22 7	37 35 2	37 34 48	48 28 58	(2749·1)
229	37 30 0	86	2 51	− 2 32	37 27 28			
230	300 35 0	86	5·65	− 5 41	300 29 19			
231	300 30 0	86	0·74	− 0 45	300 29 15	300 29 17	48 29 6	2749·0
128	301 4 55	67	27·80	−28 39	300 36 16			
129	301 4 55	67	27·61	−28 27	300 36 28			
131	300 54 55	67	18·32	−18 53	300 36 2	300 36 15		
125	37 20 5	67	15·66	+16 18	37 36 13			
124	37 30 5	70	6·09	+ 6 15	37 36 20	37 36 17	48 30 1	2749 7
229	37 30 0	86	2·96	+ 2 59	37 32 59			
230	300 35 0	86	11·25	−11 20	300 23 40			
231	300 30 0	86	6·27	− 6 19	300 23 41	300 23 40	48 34 39	2753·0
229	37 30 0	86	6·42	+ 6 28	37 36 28			
230	300 35 0	86	14·71	−14 49	300 20 11			
231	300 30 0	86	9·74	− 9 49	300 20 11	300 20 11	48 38 9	2755·5
197	300 59 55	65	31·13	−32 9	300 27 44			
198	301 10 0	65	41·00	−42 21	300 27 39	300 27 41		
200	37 3 0	65	39·44	+40 44	37 43 44			
201	37 12 55	65	29·76	+31 23	37 44 18	37 44 1	48 38 10	2755·5
197	300 59 55	65	47·45	−49 0	300 10 55			
198	301 10 0	65	57·22	−59 6	300 10 54	300 10 54		
200	37 3 0	65	55·57	+57 23	38 0 23			
201	37 12 55	65	45·88	+48 23	38 1 18	38 0 51	48 54 58	(2767·3)
220	299 35 0	79	27·90	+28 20	300 3 20			
221	299 35 0	79	27·82	+28 15	300 3 15	300 3 18		
222	38 30 0	79	36·45	−37 1	37 52 59			
218	38 30 0	79	36·32	−36 53	37 53 7			
219	38 25 0	79	31·56	−32 3	37 52 57	37 53 2	48 54 52	2767·2
286	38 20 0	84	26·85	−27 7	37 52 53			
287	299 40 0	84	23·15	+23 23	300 3 23		48 54 45	(2767·1)

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
	° ′ ″		hundredths of an inch.	′ ″	° ′ ″	° ′ ″	° ′ ″	wave-length.
286	38 20 0	84	10·20	−10 18	38 9 42			
287	299 40 0	84	6·27	+ 6 20	299 46 20		49 11 41	2779·0
220	299 35 0	79	11·26	+11 27	299 46 27			
221	299 35 0	79	11·16	+11 20	299 46 20	299 46 24		
222	38 30 0	79	19·96	−20 16	38 9 44			
218	38 30 0	79	19·78	−20 5	38 9 55			
219	38 25 0	79	14·98	−15 13	38 9 47	38 9 51	49 11 43	2779·0
220	299 35 0	79	5·06	+ 5 8	299 40 8			
221	299 35 0	79	4·95	+ 5 2	299 40 2	299 40 5		
222	38 30 0	79	13·80	−14 1	38 15 59			
218	38 30 0	79	13·62	−13 51	38 16 9			
219	38 25 0	79	8·84	− 8 59	38 16 1	38 16 3	49 17 59	2783·4
286	38 20 0	84	3·97	− 4 1	38 15 59			
287	299 40 0	84	0·05	+ 0 3	299 40 3		49 17 58	2783·4
321	39 0 0	86	20·58	−20 43	38 39 17			
322	298 55 0	86	23·65	+23 49	299 18 49		49 40 14	2798·8
321	39 0 0	86	0·52	− 0 31	38 59 29			
322	298 55 0	86	3·26	+ 3 17	298 58 17		50 0 36	2812·9
321	39 0 0	86	25·93	+26 7	39 26 7			
322	298 55 0	86	23·26	−23 26	298 31 34		50 27 17	2831·1
281.	298 25 0	87	5·95	+ 5 59	298 30 59			
284	298 25 0	87	5·81	+ 5 51	298 30 51	298 30 55		
282	39 35 0	87	9·27	− 9 20	39 25 40			
283	39 35 0	87	9·74	− 9 48	39 25 12			
285	39 35 0	87	9·60	− 9 40	39 25 20	39 25 24	50 27 14	(2831·0)
281	298 25 0	87	0·13	− 0 8	298 24 52			
284	298 25 0	87	0·36	− 0 22	298 24 38	298 24 45		
282	39 35 0	87	3·20	− 3 13	39 31 47			
283	39 35 0	87	2·71	− 2 44	39 32 16			
285	39 35 0	87	3·58	− 3 36	39 31 24	39 31 49	50 33 32	2835·3
281	298 25 0	87	7·47	− 7 31	298 17 29			
284	298 25 0	87	7·59	− 7 38	298 17 22	298 17 25		
282	39 35 0	87	3·99	+ 4 1	39 39 1			
283	39 35 0	87	4·54	+ 4 34	39 39 34			
285	39 35 0	87	3·58	+ 3 36	39 38 36	39 39 5	50 40 50	2840·3
288	297 25 0	88	25·62	+25 45	297 50 45			
289	40 35 0	88	30·28	−30 26	40 4 26		51 6 51	(2857·8)
318	297 20 0	88	31·35	+31 30	297 51 30			2857·9
319	297 20 0	88	31·40	+31 33	297 51 33	297 51 32		
320	40 30 0	88	23·73	−23 51	40 6 9		51 7 18	(2858·1)
278	40 35 0	88	29·57	−29 43	40 5 17			
279	297 25 0	88	26·13	+26 16	297 51 16			
280	297 25 0	88	26·34	+26 28	297 51 28	297 51 22	51 6 57	(2857·8)

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
	° ′ ″		hundredths of an inch.	′ ″	° ′ ″	° ′ ″	° ′ ″	wave-length.
278	40 35 0	88	8·62	− 8 40	40 26 20			
280	297 25 0	88	5·29	+ 5 19	297 30 19		51 28 0	(2871·9)
288	297 25 0	88	4·54	+ 4 34	297 29 34			
289	40 35 0	88	9 45	− 9 30	40 25 30		51 27 58	(2871·9)
318	297 20 0	88	10·37	+10 25	297 30 25			2872·0
319	297 20 0	88	10·34	+10 24	297 30 24	297 30 25		
320	40 30 0	88	2·77	− 2 47	40 27 13		51 28 24	(2872·2)
269	40 50 0	91	21·93	−21 58	40 28 2			
270	40 55 0	91	27·00	−27 2	40 27 58	40 28 0		
271	297 10 0	91	18·96	+18 59	297 28 59			
272	297 5 0	91	23·94	+23 59	297 28 59		51 29 30	(2872·9)
288	297 25 0	88	3·02	+ 3 2	297 28 2			
289	40 35 0	88	7·86	− 7 54	40 27 6		51 29 32	(2872·9)
317	297 25 0	88	3·94	+ 3 58	297 28 58			2873·0
318	297 20 0	88	8·93	+ 8 59	297 28 59			
319	297 20 0	88	8 86	+ 8 54	297 28 54	297 28 57		
320	40 30 0	88	1·36	− 1 22	40 28 38		51 29 51	(2873·1)
278	40 35 0	88	7·12	− 7 9	40 27 51			
279	297 25 0	88	3·47	+ 3 29	297 28 29			
280	297 25 0	88	3·78	+ 3 48	297 28 48	297 28 39	51 29 36	2873·0
278	40 35 0	88	3·82	+ 3 50	40 38 50			
280	297 25 0	88	7·58	− 7 37	297 17 23		51 40 48	2880·4
269	40 50 0	91	10·94	−10 58	40 39 2			
270	40 55 0	91	15·99	−16 1	40 38 59	40 39 0		
271	297 10 0	91	7·66	+ 7 40	297 17 40			
272	297 5 0	91	12·56	+12 35	297 17 35	297 17 38	51 40 41	(2880·3)
288	297 25 0	88	8·12	− 8 10	297 16 50			
289	40 35 0	88	3·18	+ 3 12	40 38 12		51 40 41	(2880·3)
317	297 25 0	88	7·28	− 7 19	297 17 41			
318	297 20 0	88	2·56	− 2 34	297 17 26			
319	297 20 0	88	2·46	− 2 28	297 17 32	297 17 33		
320	40 30 0	88	9·75	+ 9 48	40 39 48		51 41 7	(2880·6)
269	40 50 0	91	6·50	− 6 31	40 43 29			
270	40 55 0	91	11·55	−11 34	40 43 26	40 43 27		
271	297 10 0	91	3·21	+ 3 13	297 13 13			
273	297 5 0	91	8·14	+ 8 12	297 13 12	297 13 12	51 45 7	2883·3
278	40 35 0	88	8·17	+ 8 13	40 43 13			
279	297 25 0	88	11·92	−11 59	297 13 1			
280	297 25 0	88	11·84	−11 54	297 13 6	297 13 3	51 45 5	2883·3
288	297 25 0	88	12·53	−12 36	297 12 24			
289	40 35 0	88	7·52	+ 7 34	40 42 34		51 45 5	2883·3

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
	° ′ ″		hundredths of an inch.	′ ″	° ′ ″	° ′ ″	° ′ ″	wave-length.
317	297 25 0	88	11·59	−11 39	297 13 21			
318	297 20 0	88	6·95	− 6 59	297 13 1			
319	297 20 0	88	6·77	− 6 48	297 13 12	297 13 11		
320	40 30 0	88	14·13	+14 12	40 44 12	..	51 45 31	(2883·5)
275	296 10 0	93	2·48	− 2 29	296 7 32			
276	41 50 0	93	0·91	− 0 55	41 49 5			
277	41 55 0	93	6·00	− 6 1	41 48 59	41 49 2	52 50 45	(2926·1)
267	296 10 0	93	2·85	− 2 51	296 7 9			
268	41 50 0	93	1·20	− 1 12	41 48 48	..	52 50 50	2926·0
310	41 55 0	93	10·19	+10 11	42 5 11			
311	41 50 0	93	15·00	+14 59	42 4 59	42 5 5		
312	296 0 0	93	8·16	− 8 9	295 51 51	..	53 6 37	2928·3
267	296 10 0	93	30·58	−30 33	295 39 27			
268	41 50 0	93	26·50	+26 29	42 16 29	..	53 18 31	(2943·9)
275	296 10 0	93	30·48	−30 27	295 39 33			
276	41 50 0	93	26·78	+26 45	42 16 45			
277	41 55 0	93	21·73	+21 43	42 16 43	42 16 44	53 18 36	2944·0
310	41 55 0	93	21·95	+21 56	42 16 56			
311	41 50 0	93	26·75	+26 44	42 16 44	42 16 50		
312	296 0 0	93	20·04	−20 1	295 39 59			
315	296 0 0	93	19·97	−19 57	295 40 3	295 40 1	53 18 25	(2943·9)
275	296 10 0	93	35·50	−35 28	295 34 32			
276	41 50 0	93	31·94	+31 55	42 21 55			U
277	41 55 0	93	26·89	+26 52	42 21 52	42 21 54	53 23 41	2947·3

As in the highest region we have used the lines in the spectrum of the spark between copper electrodes as lines of reference, we add here the measurements and calculated values for these lines, and for the strong magnesium lines which we have sometimes used in interpolating.

COPPER LINES.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
	° ′ ″		hundredths of an inch.	′ ″	° ′ ″	° ′ ″	° ′ ″	wave-length.
327	312 55 0	24	27·88	+30 17	313 25 17			
330	313 5 0	24	18·19	+19 46	313 24 46			
331	313 5 0	24	18·81	+20 26	313 25 26	313 25 10		
328	25 0 0	24	23·96	−26 2	24 33 58			
329	24 45 0	24	10·30	−11 11	24 33 49	24 33 53	35 34 21	2135·7
327	312 55 0	24	13·87	+15 4	313 10 4			
330	313 5 0	24	4·17	+ 4 32	313 9 32			
331	313 5 0	24	4·73	+ 5 8	313 10 8	313 9 55		
328	25 0 0	24	10·15	−11 2	24 48 58			
329	24 45 0	24	3·53	+ 3 50	24 48 50	24 48 54	35 49 30	2148·9

COPPER LINES—(continued).

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
	° ′ ″		hundredths of an inch.	′ ″	° ′ ″	° ′ ″	° ′ ″	wave-length.
297	25 50 0	31	24·22	−26 5	25 23 55			
298	312 10 0	31	23·66	+25 29	312 35 29		36 24 13	2178·8
297	25 50 0	31	13·18	−14 11	25 35 49			
298	312 10 0	31	12·24	+13 11	312 23 11		36 36 19	2189·2
325	26 0 0	31	22·94	−24 42	25 35 18			
326	311 55 0	31	25·83	+27 49	312 22 49		36 36 14	2189·2
325	26 0 0	31	20·02	−21 33	25 38 27			
326	311 55 0	31	22·98	+24 45	312 19 45		36 39 21	2191·8
297	25 50 0	31	10·32	−11 7	25 38 53			
298	312 10 0	31	9·37	+10 5	312 20 5		36 39 24	(2191·9)
297	25 50 0	31	2·37	− 2 33	25 47 27			
298	312 10 0	31	1·46	+ 1 34	312 11 34		36 47 56	2199·2
325	26 0 0	31	0·65	− 0 42	25 59 18			
326	311 55 0	31	3·19	+ 3 26	311 58 26		37 0 26	(2209·9)
291	26 25 0	37	23·62	−25 15	25 59 45			
293	26 35 0	37	33·52	−35 55	25 59 5			
296	26 25 0	34	23·27	−24 55	26 0 5			
297	25 50 0	31	9·08	+ 9 47	25 59 47	25 59 40		
298	312 10 0	31	10·19	−10 58	311 59 2			
299	311 10 0	39	46·93	+50 2	312 0 2			2209·7
294	311 25 0	37	31·70	+33 53	311 58 53			
295	311 35 0	34	22·73	+24 23	311 59 23	311 59 20	37 0 10	(2209·6)
291	26 25 0	37	15·26	−16 19	26 8 41			
293	26 35 0	37	24·69	−26 27	26 8 33			
296	26 25 0	34	14·80	−15 53	26 9 7			
297	25 50 0	31	17·55	+18 54	26 8 54			
300	26 50 0	39	39·24	−41 50	26 8 10	26 8 41		
294	311 25 0	37	22·94	+24 31	311 49 31			
295	311 35 0	34	14·04	+15 4	311 50 4			2217·5
298	312 10 0	31	18·77	−20 13	311 49 47			
299	311 10 0	39	37·91	+40 25	311 50 25	311 49 57	37 9 22	
325	26 0 0	31	7·68	+ 8 16	26 8 16			
326	311 55 0	31	5·40	− 5 49	311 49 11		37 9 33	(2217·6)
291	26 25 0	37	3·49	− 3 44	26 21 16			
293	26 35 0	37	12·99	−13 55	26 21 5			
296	26 25 0	34	3·05	− 3 16	26 21 44			
300	26 50 0	39	27·23	−29 2	26 20 58	26 21 16		
294	311 25 0	37	10·95	+11 42	311 36 42			
295	311 35 0	34	1·87	+ 2 0	311 37 0			
299	311 10 0	39	25·85	+27 33	311 37 33	311 37 5	37 22 5	2228·3
293	26 35 0	37	11·58	−12 24	26 22 36			
296	26 25 0	34	1·65	− 1 46	26 23 14	26 22 55		
294	311 25 0	37	9·51	+10 10	311 35 10			
295	311 35 0	34	0·51	+ 0 32	311 35 32			
299	311 10 0	39	24·52	+26 8	311 36 8	311 35 37	37 23 39	2229·6

Copper Lines—(continued).

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
	′ ″		hundredths of an inch.	′ ″	° ′ ″	° ′ ″	° ′ ″	wave-length.
291	26 25 0	37	11·59	+12 23	26 37 23			
292	26 30 0	38	7·30	+ 7 47	26 37 47			
293	26 35 0	37	2·15	+ 2 18	26 37 18			
296	26 25 0	34	12 00	+12 52	26 37 52			
300	26 50 0	39	11·52	−12 17	26 37 43	26 37 37		
294	311 25 0	37	4·54	− 4 51	311 20 9			
295	311 35 0	34	13·28	−14 15	311 20 45			
299	311 10 0	39	10·15	+10 49	311 20 49	311 20 34	37 33 31	2242·2
332	27 0 0	39	20·54	−21 54	26 38 6			
333	26 55 0	39	15·96	−17 1	26 37 59	26 38 2		
334	310 55 0	39	24·32	+25 56	311 20 56			
335	311 0 0	39	19·70	+20 49	311 20 49	311 20 53	37 38 34	2242·2
332	27 0 0	39	15·77	−16 49	26 43 11			
333	26 55 0	39	11·03	−11 46	26 43 14	26 43 12		
334	310 55 0	39	19·47	+20 46	311 15 46			
335	311 0 0	39	14·61	+15 27	311 15 27	311 15 37	37 43 47	2246·6
291	26 25 0	37	16·48	+17 37	26 42 37			
292	26 30 0	38	12·16	+12 59	26 42 59			
293	26 35 0	37	7·03	+ 7 32	26 42 32			
296	26 25 0	34	16·80	+18 1	26 43 1			
300	26 50 0	39	6·62	− 7 3	26 42 57	26 42 49		
294	311 25 0	37	9·45	−10 6	311 14 54			
295	311 35 0	34	18·20	−19 31	311 15 29			
299	311 10 0	39	5·31	+ 5 40	311 15 40	311 15 21	37 43 44	2246·6
332	27 0 0	39	2·98	+ 3 10	27 3 10			
333	26 55 0	39	7·80	+ 8 19	27 3 19	27 3 15		
335	311 0 0	39	4·27	− 4 31	310 55 29		38 3 53	2263·6
332	27 0 0	39	16·90	+18 1	27 18 1			
333	26 55 0	39	21·60	+23 2	27 18 2			
334	310 55 0	39	13·52	−14 25	310 40 35			
335	311 0 0	39	18·18	−19 13	310 40 47			
336	310 30 0	46	10·33	+10 55	310 40 55			
337	310 25 0	46	15·12	+15 59	310 40 59	310 40 49		
338	27 30 0	46	11·24	−11 53	27 18 7			
340	27 35 0	46	16·00	−16 55	27 18 5			
339	27 25 0	46	6·71	− 7 6	27 17 54	27 18 2	38 18 37	2276·0
291	26 25 0	37	48·59	+51 56	27 16 56			
292	26 30 0	38	44·59	+47 36	27 17 36			
293	26 35 0	37	39·99	+42 51	27 17 51			
296	26 25 0	34	48·64	+52 11	27 17 11			
300	26 50 0	39	26·44	+28 11	27 18 11	27 17 38		
294	311 25 0	37	42·01	−44 54	310 40 6			
295	311 35 0	34	50·70	−54 23	310 40 37			
299	311 10 0	39	27·31	−29 7	310 40 53	310 40 32	38 18 33	(2275·9)
336	310 30 0	46	10·33	−10 55	310 19 5			
337	310 25 0	46	5·55	− 5 52	310 19 8	310 19 6		
338	27 30 0	46	9·30	+ 9 50	27 39 50			
340	27 35 0	46	4·40	+ 4 39	27 39 39			
339	27 25 0	46	13·76	+14 33	27 39 33	27 39 41	38 40 19	2294·1

MAGNESIUM LINES.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
	° ′ ″		hundredths of an inch.	° ′ ″	° ′ ″	° ′ ″	° ′ ″	wave-length.
129	301 4 55	67	68·67	−1 10 45	299 54 10			
128	301 4 55	67	68·85	−1 10 58	299 53 57			
131	300 54 55	67	59·32	−1 1 7	299 53 48	299 54 4		
134	300 4 55	75	10·67	−0 10 53	299 54 2			
135	300 4 55	75	10·39	−0 10 36	299 54 19			
139	300 0 3	72	5·66	−0 5 47	299 54 16			
140	300 0 3	72	5·95	−0 6 6	299 53 57			
137	38 10 0	75	8·75	+0 8 56	38 18 56			
138	38 15 0	72	3·42	+0 3 30	38 18 30			
125	37 20 5	67	56·82	+0 58 32	38 18 37			
124	37 30 5	70	47·33	+0 48 35	38 18 40	38 18 41	49 12 19	2779·4
137	38 10 0	75	30·88	+0 31 31	38 41 31			
138	38 15 0	72	25·68	+0 26 18	38 41 18	38 41 25		
139	300 0 3	72	27·86	−0 28 32	299 31 31			
140	300 0 3	72	28·02	−0 28 42	299 31 21	299 31 26	49 35 0	2795·2
306	299 15 0	86	8·75	+0 8 49	299 23 49	299 23 50		
307	38 45 0	86	10·89	−0 11 1	38 33 59			
308	38 30 0	85	3·84	+0 3 52	38 33 52	38 33 55		
309	299 30 0	85	6·10	−0 6 9	299 23 51		49 35 2	2795·2
137	38 10 0	75	41·09	+0 41 56	38 51 56			
138	38 15 0	72	35·83	+0 36 42	38 51 42	38 51 49		
139	300 0 3	72	37·82	−0 38 44	299 21 19			
140	300 0 3	72	38·21	−0 39 8	299 20 55	299 21 7	49 45 21	2802·4
306	299 15 0	86	1·57	−0 1 35	299 13 25			
307	38 45 0	86	0·59	−0 0 36	38 44 24			
308	38 30 0	85	14·16	+0 14 17	38 44 17	38 44 21		
309	299 30 0	85	16·48	−0 16 37	299 13 23	299 13 24	49 45 28	(2802·4)
303	40 0 0	85	2·97	−0 3 0	39 57 0			
304	298 0 0	85	0 34	+0 0 21	298 0 21			
305	298 5 0	85	4·48	−0 4 31	298 0 29	298 0 25	50 58 18	2852·0

Remarks on the foregoing tables.

It will be observed that when two or more independent determinations have been made the wave-lengths found are often identical, rarely differ by more than ·2 of a tenth-metre. This seems to make the probable error in most cases very small—smaller than we venture to think it really may be, for there are one or two sources of error which are quite sufficient to account for a variation of ·2 of a tenth-metre in the wave-length. First the sliding tubes of the telescope, for it has a draw tube as well as the usual rack and pinion arrangement for focussing, in order to allow for the great variations in the focal distance of the uncorrected quartz lenses, and such sliding tubes have always some play, so that in the operation of reversing the photographic plate by turning the sliding tube about its axis there might easily be a small displacement.

Indeed, the measurement of plates taken successively without movement of any part of the apparatus, except the photographic slide, showed that there was such a displacement of the axis, and that it might make an error of ±10″ in the measurement of the angle, or ±·13 on an average in the value of the wave-length.

Another source of error is the want of sharp definition of some of the lines. Some of the lines are really diffuse, and in every case, as already observed, if the image is correctly focussed on the plate when the telescope is on one side of the collimator, it is always a little out of focus when the telescope is moved round to the other side. Hence there may easily be an error in measuring the distance between the lines which may easily amount to ±8″ of angular measure, or ±·1 in the value of the wave-length.

Still we do not think the probable error exceeds ±·25 of a tenth-metre.

Determination of the intermediate lines.

The wave-lengths of a sufficient number of lines of reference having been measured by the grating, the intermediate lines have been mapped by means of prisms. For this part of the work a calcite prism of 30°, cut so that one face is perpendicular to the axis of the crystal, was fixed to the end of the collimator, and a similar prism to the end of the telescope, while between them another calcite prism of 60°, cut so that the faces are equally inclined to the axis of the crystal, was maintained by a simple system of linkage at the position of minimum deviation, which is also that of single refraction. Photographs were taken at short intervals all down the scale of the spectrum of the arc and spark simultaneously. The image of the arc was focussed on the slit by the quartz lens already mentioned, and thrown just under the centre. At the same time the spark was made to pass horizontally close in front of the slit, without the interposition of a lens, but just above the centre. In this way two images were impressed on the plate overlapping one another in the middle. The distances between the lines was afterwards measured under the microscope, and the inverse squares of the wave-lengths of the intermediate lines deduced by the graphic method of interpolation between those of the lines of reference. A table of inverse squares was used for the reduction to simple wave-lengths.

The following table gives the results. In many cases there is much difference in the relative intensities of the same line in the arc and spark, and in some cases lines are visible in the photograph of the arc which are not in that of the spark, and *vice versâ.* Beyond the wave-length 2327 no spark lines seem to have made any impression on the plates, but the arc lines continue with, however, a sensible falling off in intensity up to the end of the region observed. For this region the copper lines of reference were used, as already explained, and for the highest part of it, above wave-length 2230, quartz prisms were substituted for those of calcite with advantage as regards the amount of light transmitted, but with some loss of dispersion and more of definition.

In the following table the second column gives the wave-length, and the lines marked with a "*c*" are probably carbon lines, the first gives approximately the relative intensities with which the lines are impressed on the photographic plates, 1 representing the strongest and 6 the weakest lines. It also indicates whether the line is an arc or a spark-line, *a* indicating arc-lines; *s*, spark-lines. Thus 3 *a* 1 *s* after a line indicates that its intensity in arc is 3, in spark, 1; 6 *a* indicates a line which is in arc only and of intensity 6; 2 *a s* indicates a line of intensity 2 in both arc and spark.

List of ultra-violet iron lines.

6 *a*	2167·4	4 *a*	2280 0	6 *a*	2341·2	2 *a s*	2384·2	2 *s*	2427·9
6 *a*	2171·7	6 *a*	2281·8	6 *s*	2341·6	6 *a* 5 *s*	2384·8	6 *a s*	2428·5
6 *a*	2173·4	6 *a*	2282·8	6 *s*	2341·8	6 *a*	2385·8	6 *s*	2428·7
6 *a*	2177·0	5 *a*	2283·0	1 *a s*	2343·1	4 *s*	2386·3	6 *a s*	2429·0
6 *a*	2178·0	5 *a*	2283·2	6 *a* 5 *s*	2343·6	6 *a* 4 *s*	2387·2	3 *a* 2 *s*	2429·7
6 *a*	2181·5	4 *a*	2283·6	4 *a* 3 *s*	2343·9	6 *s*	2388·0	6 *a s*	2430·5
6 *a*	2183·7	3 *a*	2284·0	4 *a*	2344·7	1 *a s*	2388·4	6 *a*	2430·7
6 *a*	2186·1	3 *a*	2287·1	3 *s*	2344·9	6 *s*	2389·2	6 *a* 2 *s*	2431·8
6 *a*	2186·8	3 *a*	2287·4	6 *a s*	2345·9	3 *a*	2389·9	2 *s*	2432·5
6 *a*	2191·3	6 *a*	2287·9	6 *s*	2346·4	5 *s*	2390·1	4 *s*	2433·2
5 *a*	2195·5	3 *a*	2288·8	1 *a s*	2347·3	6 *s*	2390·7	6 *a s*	2433·9
5 *a*	2199·3	5 *a*	2289·9	1 *a s*	2348·0	3 *a* 4 *s*	2391·3	6 *a* 3 *s*	2434·3
6 *a*	2200·0	4 *a*	2290·3	6 *s*	2349·0	6 *a s*	2392·4	5 *a* 3 *s*	2434·7
6 *a*	2200·2	6 *a*	2290·6	6 *a*	2349·5	6 *a*	2392·8	6 *a s*	2435·6
4 *a*	2207·5	4 *a*	2290·9	6 *a s*	2349·9	6 *a*	2394·1	6 *a s*	2436·0
4 *a*	2210·4	6 *a*	2291·4	6 *a* 2 *s*	2350·9	6 *s*	2394·7	5 *s*	2436·4
6 *a*	2211·4	3 *a*	2292·3	6 *a s*	2351·5	3 *a s*	2395·2	5 *s*	2436·9
4 *a*	2214·1	3 *a*	2293·6	6 *s*	2352·1	1 *a s*	2395·4	6 *s*	2437·3
3 *a*	2216·2	3 *a*	2294·2	5 *s*	2353·3	5 *s*	2396·5	4 *a*	2437·9
4 *a*	2225·2	3 *a*	2296·8	6 *a* 3 *s*	2354·1	6 *a*	2398·0	2 *s*	2439·0
3 *a*	2227·3	3 *a*	2297·6	3 *a s*	2354·6	6 *s*	2398·5	2 *a* 6 *s*	2439·4
3 *a*	2229·7	3 *a*	2298·0	5 *s*	2354·8	1 *a s*	2399·0	3 *a*	2439·8
6 *a*	2230·9	4 *a*	2298·6	6 *a* 5 *s*	2355·1	6 *a* 2 *s*	2400·0	3 *s*	2440·1
6 *a*	2240·2	3 *a*	2299·0	6 *a*	2355·6	6 *a s*	2401·0	6 *s*	2441·0
6 *a*	2242·2	6 *a*	2299·2	4 *s*	2356·7	6 *a*	2401·4	6 *a*	2441·5
6 *a*	2243·9	3 *a*	2300·0	1 *a s*	2358·7	6 *a* 5 *s*	2401·9	2 *a* 5 *s*	2442·3
5 *a*	2245·3	5 *a*	2300·4	6 *a s*	2359·2	6 *a* 5 *s*	2402·3	3 *a* 6 *s*	2443·7
4 *a*	2248·5	4 *a*	2301·0	2 *a* 1 *s*	2359·7	3 *a s*	2404·2	5 *a* 1 *s*	2444·3
4 *a*	2248·8	3 *a*	2301·4	2 *a* 1 *s*	2359·9	1 *a s*	2404·5	6 *a s*	2444·9
4 *a*	2250·5	3 *a*	2303·2	6 *a*	2360·3	6 *s*	2405·5	6 *a* 3 *s*	2445·4
6 *a*	2250·6	3 *a*	2303·4	6 *s*	2361·3	1 *a s*	2406·3	6 *s*	2445·9
6 *a*	2251·2	5 *a*	2304·4	4 *a* 3 *s*	2361·6	6 *s*	2406·6	6 *a* 3 *s*	2446·3
6 *a*	2251·6	5 *a*	2305·8	6 *s*	2362·9	6 *a*	2406·9	3 *s*	2447·1
4 *a*	2252·8	4 *a*	2306·0	3 *s*	2363·3	6 *a*	2407·3	3 *a s*	2447·5
4 *a*	2255·4	2 *a*	2308 6	3 *s*	2363·5	6 *a s*	2407·6	6 *a*	2448·1
4 *a*	2259·2	6 *a*	2309·3	2 *a* 1 *s*	2364·4	6 *s*	2408·4	6 *a*	2448·5
4 *a*	2259·8	6 *a*	2310·6	6 *a*	2365·1	1 *a s*	2410·2	4 *s*	2449·6
5 *a*	2260·4	5 *a*	2311·0	5 *s*	2365·3	1 *a s*	2410·7	6 *a* 4 *s*	2450·0
5 *a*	2260·7	6 *a*	2311·6	3 *a s*	2366·2	6 *a*	2411·4	6 *a*	2450·7
6 *a*	2262·4	6 *a*	2312·0	2 *a* 1 *s*	2368·2	1 *a s*	2413·0	6 *a* 5 *s*	2451·0
6 *a*	2262·8	2 *a*	2312·7	4 *a*	2369·1	6 *a s*	2413·8	6 *a*	2451·3
6 *a*	2263·2	6 *a*	2313·6	5 *s*	2369·6	6 *a s*	2414·8	6 *a*	2451·8
5 *a*	2264·2	6 *a*	2316·7	4 *a s*	2370·1	6 *a*	2415·4	6 *a*	2452·3
5 *a*	2264·7	5 *a*	2317·5	4 *a*	2371·1	6 *a* 5 *s*	2416·3	6 *s*	2452·9
6 *a*	2265·7	6 *a*	2317·7	4 *s*	2372·3	6 *a*	2417·1	3 *a*	2453·2
6 *a*	2266·6	5 *a*	2319·2	6 *a*	2372·7	6 *a* 2 *s*	2417·5	5 *s*	2453·5
5 *a*	2266·8	6 *a*	2319·6	4 *s*	2373·3	6 *a* 5 *s*	2418·2	6 *s*	2453·8
3 *a*	2267·2	3 *a*	2319·9	2 *a* 1 *s*	2373·4	3 *a*	2418·9	3 *s*	2454·3
6 *a*	2268·8	2 *a* 1 *s*	2326·9	6 *a*	2374·1	6 *a*	2419·4	6 *a*	2455·3
4 *a*	2270·5	6 *a*	2329·3	2 *a* 1 *s*	2374·9	6 *s*	2419·7	6 *s*	2455·7
4 *a*	2271·5	2 *a* 1 *s*	2330·9	6 *a* 3 *s*	2376·2	6 *a*	2420·0	6 *a*	2456·0
4 *a*	2271·8	2 *a* 1 *s*	2332·5	6 *a*	2376·9	6 *a*	2420·7	6 *s*	2456·4
4 *a*	2272·5	6 *a*	2333·1	6 *a*	2377·6	6 *a*	2421·3	2 *a* 5 *s*	2457·4
4 *a*	2273·8	6 *a*	2334·2	6 *s*	2378·2	6 *a* 3 *s*	2422·4	6 *a*	2458·2
4 *a*	2274·9	6 *a*	2334·5	6 *s*	2378·8	6 *a* 4 *s*	2422·9	6 *a* 1 *s*	2458·5
6 *a*	2275·2	6 *a*	2334·8	2 *a* 1 *s*	2379·0	5 *a* 1 *s*	2423·8	6 *a* 4 *s*	2460·2
4 *a*	2275·7	2 *a* 1 *s*	2337·7	2 *a* 1 *s*	2380·5	6 *s*	2424·3	6 *a*	2460·8
4 *a*	2276·9	2 *a* 6 *s*	2339·0	1 *a s*	2381·7	6 *a s*	2425·0	6 *a* 3 *s*	2461·0
4 *a*	2277·5	3 *a*	2339·3	3 *a s*	2382·7	6 *a* 5 *s*	2425·4	3 *s*	2461·4
3 *a*	2279·7	5 *a* 6 *s*	2340·0	2 *a s*	2383·0	6 *s*	2427·0	3 *a*	2161·9

List of ultra-violet iron lines—(continued).

2 a 6 s	2462·3	6 s	2507·9	5 s	2554·8	6 a	2608·2	6 a	2672·4
6 a 4 s	2462·8	5 a	2508·5	6 a	2554·9	6 a s	2608·7	6 a	2674·6
5 a	2463·4	5 s	c 2508·8	5 s	2555·2	6 a	2609·1	6 a	2675·1
4 s	2463·7	2 a 5 s	2510·6	6 a	2556·0	6 s	2609·3	6 s	2676·1
4 s	2464·5	6 a 1 s	c 2511·4	6 a	2556·6	6 a	2610·3	6 a	2677·2
3 a 4 s	2464·7	6 a	2511·6	6 s	2557·2	6 a s	2610·7	2 a 6 s	2678·5
3 s	2465·4	3 a	2512·0	6 a	2558·3	1 a s	2611·4	6 a	2679·9
5 a 2 s	2466·4	3 a 4 s	2512·2	6 s	2558·9	6 a	2612·3	5 s	2680·4
5 a	2467·2	6 a	2513·2	5 s	2559·6	1 a s	2613·3	6 a	2680·8
5 s	2467·8	3 a s	c 2514·1	6 a 4 s	2560·0	6 a	2614·0	6 a	2681·5
3 a 6 s	2468·4	6 a	2514·3	6 a	2560·3	6 a	2615·0	5 s	2682·0
3 s	2469·0	4 s	2514·7	6 a	2560·9	1 a s	2617·2	5 s	2682·4
2 s	2470·3	1 a 6 s	c 2515·8	6 a	2561·5	6 a	2617·6	6 a	2683·5
6 a	2470·5	6 a	2516·3	6 a	2561·9	6 a	2618·3	6 a 1 s	2684·2
3 a 4 s	2471·9	6 a 3 s	2516·8	2 a s	2562·3	4 s	2618·6	6 s	2685·7
2 a 5 s	2472·4	5 a	2517·4	2 a s	2563·2	5 a s	2619·9	6 a	2686·0
6 a	2472·7	3 a 6 s	2517·8	6 a	2564·2	5 s	2620·4	6 a	2686·8
5 s	2472·9	6 a	2518·5	6 a	2565·1	3 a s	2621·2	6 a	2687·3
3 a s	2474·5	3 a s	c 2518·8	4 s	2566·0	6 s	2622·6	3 a s	2688·8
6 s	2474·9	5 a 6 s	2519·3	3 a s	2566·7	3 a 6 s	2623·1	5 a	5689·3
6 s	2475·5	6 a 3 s	2520·8	5 s	2568·1	6 a	2623·6	5 a	2689·5
6 a	2475·3	6 a 3 s	2521·5	6 a s	2568·6	1 a s	2625·2	6 a	2690·9
6 s	2476·0	1 a 2 s	2522·5	6 a s	2569·4	6 a s	2626·2	6 s	2691·2
5 a 6 s	2476·5	5 a 6 s	2523·3	5 a	2570·1	6 a	2626·8	6 a	2691·7
5 s	2477·1	2 a 6 s	c 2523·9	4 s	2570·6	1 a s	2627·9	6 a 1 s	2692·1
6 s	2477·9	5 a	2524·7	6 s	2571·2	6 a 5 s	2629·2	6 s	2693·4
1 a 3 s	2478·3	6 a 2 s	2525·1	6 a	2572·5	6 a 5 s	2629·7	5 a	2694·0
6 s	2479·0	5 a 2 s	2526·0	6 s	2572·8	1 a s	2630·7	6 a	2694·4
6 a	2479·2	5 s	2526·7	6 a 3 s	2574·0	1 a s	2631·0	6 s	2694·7
2 a 6 s	2479·5	2 a 5 s	2527·1	6 a	2574·8	5 a 6 s	2632·0	5 a	2695·0
6 a 2 s	2480·0	6 a	2527·9	5 a	2575·3	6 a	2632·3	4 a	2695·6
6 s	2480·7	6 a	c 2528·1	4 a 6 s	2575·7	5 s	2632·9	6 a	2695·9
6 s	2481·3	3 a 6 s	2528·9	3 a	2576·2	6 s	2635·1	6 a	2696·6
6 a 3 s	2481·8	2 s	2529·2	4 s	2576·5	3 a 6 s	2635·5	4 s	2697·0
3 s	2482·4	3 a	2529·6	3 a s	2577·4	6 a s	2636·1	6 a	2697·7
1 a 6 s	2482·9	6 s	2529·9	6 a	2578·3	6 a	2636·6	4 a 5 s	2698·6
6 s	2483·3	4 a 6 s	2530·4	6 a	2578·7	3 s	2637·3	6 s	2699·8
3 a s	2483·7	6 a	2531·1	6 s	2578·9	4 s	2639·2	6 a 6 s	2701·2
6 a	2484·7	5 a 6 s	2532·0	6 a	2579·3	6 s	2640·7	6 a	2702·6
6 a	2485·7	6 a	2532·4	6 a	2579·5	4 a 6 s	2641·4	1 s	2703·6
5 a 2 s	2486·1	6 a	2532·6	6 a	2579·9	6 s	2641·7	3 a	2705·6
5 a	2486·4	3 a 2 s	2533·4	6 a	2580·3	3 a 6 s	2643·8	1 a 3 s	2706·0
5 a	2486·8	6 a 2 s	2534·2	6 s	2580·6	6 a 3 s	2644·9	6 a 4 s	2706·7
5 a	2487·1	2 a 3 s	2535·2	6 a	2580·9	5 a	2645·2	3 a	2708·1
1 a 3 s	2487·7	5 a 1 s	2536·6	4 a	2581·7	6 s	2645·8	4 s	2708·7
6 a	2488·7	3 a	2536·9	4 a 3 s	2582·0	4 a 6 s	2647·3	6 a s	2709·7
4 s	2489·2	4 s	2538·0	2 a 6 s	2584·0	4 s	2649·2	4 a 6 s	2710·1
1 a 2 s	2489·5	5 a 1 s	2538·6	1 a s	2585·4	6 a 5 s	2650·4	3 a 4 s	2711·2
1 a 2 s	2490·5	6 a	2539·1	3 a s	2587·5	6 a	2650·9	3 s	2711·5
2 a	2491·0	5 s	2540·4	6 s	2588·2	6 s	2652·2	6 s	2711·9
3 s	2491·1	2 a 4 s	2540·8	6 s	2590·0	6 s	2653·3	6 a	2713·5
6 a s	2492·0	4 s	c 2541·6	3 a s	2591·0	6 s	2654·4	1 a s	2713·8
3 a 1 s	2492·9	3 a 6 s	2541·7	6 a	2591·7	5 a s	2655·7	6 a	2714·4
5 a s	2493·7	6 s	2542·4	3 s	2592·2	6 a	2656·4	6 a	2714·9
6 a	2493·9	6 a 3 s	2543·0	3 a s	2593·1	6 a 3 s	2657·8	6 a 3 s	2715·7
5 a 4 s	2495·6	3 a 6 s	2543·7	6 a	2593·5	5 a	2660·8	6 a	2717·4
3 a 6 s	2496·3	4 a	2544·5	6 s	2594·5	4 a 6 s	2661·6	2 a 6 s	2718·0
6 a 2 s	2497·5	5 s	2544·9	6 a	2595·2	6 s	2662·2	1 a 3 s	2718 5
6 a 1 s	2498·7	2 a 6 s	2545·8	6 a	2596·0	5 a	2663·5	5 a	2719·7
4 s	2500·7	5 a s	2546·6	1 a s	2597·8	6 a	2664·0	1 a 4 s	2720·3
2 a 6 s	2500·9	6 s	2547·0	1 a s	2598·9	6 a 1 s	2664·2	6 s	2721·5
5 a	2501·4	6 a	2547·8	6 a	2599·7	5 a	2665·7	6 s	2721·7
6 a 2 s	2502·1	5 s	2548·4	6 a	2603·5	1 a s	2666·1	5 s	2722·3
6 a	2503·0	5 s	2549·0	6 s	2603·8	6 s	2666·7	1 a 4 s	2723·1
3 s	2503·1	5 s	2549·1	6 a	2604·4	6 a	2667·2	5 a 3 s	2724·3
3 s	2503·6	2 a	2549·2	5 s	2604·9	6 s	2668·5	5 a	2725·5
6 a s	2504·9	5 s	2549·7	5 s	2605·1	6 a	2668·7	6 s	2726·0
6 a	2505·2	5 s	2550·3	3 a	2605·3	6 s	2669·2	4 a 1 s	2727·1
3 s	2505·8	6 a s	2550·8	6 s	2605·6	6 s	2669·7	6 a	2727·5
6 a	2506·2	6 a	2552·3	5 s	2606·1	6 a	2669·9	6 a 5 s	2728·3
3 a 6 s	c 2506·6	6 a s	2552·8	6 a s	2606·5	6 s	2670·8	6 s	2729·1
3 a 6 s	2507·6	6 s	2553·4	3 a s	2606·7	6 a 5 s	2671·8	5 a 3 s	2730·2

LIST of ultra-violet iron lines—(continued).

6 *s*	2731·5	6 *s*	2764·7	6 *a s*	2803·2	6 *a* 4 *s*	2848·0	5 *a s*	2894·0
6 *s*	2732·5	6 *s*	2765·3	6 *s*	2803·8	6 *a s*	2848·2	5 *a* 4 *s*	2894·5
2 *a* 4 *s*	2733·1	6 *a*	2766·8	6 *a* 5 *s*	2804·2	6 *s*	2849·3	5 *s*	2896·7
6 *a*	2733·7	3 *a* 1 *s*	2767·2	6 *s*	2804·9	6 *s*	2855·3	6 *a*	2897·8
6 *a*	2733·9	6 *a* 4 *s*	2768·8	6 *s*	2805·4	6 *s*	2856·7	5 *a*	2898·9
3 *a* 5 *s*	2735·0	6 *a* 4 *s*	2769·1	3 *a* 6 *s*	2806·7	4 *s*	2857·9	5 *a*	2900·8
5 *a* 1 *s*	2736·5	6 *a*	2769·4	6 *a*	2807·9	6 *a*	2858·3	5 *a*	2901·3
3 *a* 6 *s*	2736·9	6 *a s*	2770·3	6 *a s*	2809·7	6 *s*	2860·9	6 *s*	2902·1
1 *a s*	2739·1	6 *s*	2771·1	6 *s*	2810 9	6 *s*	2862·1	6 *a*	2903·5
5 *s*	2741·1	2 *a* 5 *s*	2771·9	6 *a*	2811·7	6 *a*	2862·4	6 *a s*	2905·8
1 *a* 5 *s*	2742·0	6 *a*	2773·1	6 *s*	2812·2	4 *a*	2863·1	6 *a s*	2907·1
3 *a* 2 *s*	2742·8	6 *a* 5 *s*	2774·5	2 *a* 4 *s*	2812·8	4 *a*	2863·6	6 *a*	2908·2
6 *a*	2743·3	6 *s*	2776·1	6 *s*	2813·4	6 *s*	2864·7	6 *a*	2908·9
3 *a* 5 *s*	2743·7	6 *s*	2776·9	6 *a*	2815·1	5 *a*	2866·2	6 *s*	2910·5
5 *a*	2744·2	6 *s*	2777·7	6 *a s*	2817·0	6 *s*	2866·5	1 *a* 6 *s*	2911·5
2 *a* 3 *s*	2746·1	4 *a* 5 *s*	2777·9	6 *a s*	2819·0	6 *a*	2867·1	6 *a*	2913·6
2 *a s*	2746·6	6 *a*	2778·3	6 *a*	2820·4	6 *a*	2868·0	5 *a* 6 *s*	2917·4
1 *a s*	2749·0	6 *a* 2 *s*	2778·9	3 *a* 6 *s*	2822·9	3 *a* 6 *s*	2869·0	6 *a*	2920·0
2 *a* 5 *s*	2749·8	5 *a* 6 *s*	2781·6	6 *a*	2823·9	6 *s*	2870·7	6 *s*	2921·5
6 *a*	2750·6	1 *s*	2783·4	3 *a* 6 *s*	2825·1	4 *a* 5 *s*	2872·0	6 *a s*	2922·8
5 *s*	2750·8	6 *a*	2784·2	6 *s*	2827·0	4 *s*	2873·0	6 *a*	2923·2
6 *s*	2752·1	3 *s*	2785·1	6 *a s*	2827·3	3 *a*	2873·6	6 *a*	2924·7
6 *a* 1 *s*	2753·0	1 *a* 3 *s*	2788·0	6 *a* 5 *s*	2828·3	6 *a* 5 *s*	2874·9	6 *a*	2925·2
6 *a*	2753·5	6 *a*	2789·5	2 *s*	2831·0	6 *s*	2876·4	6 *a* 2 *s*	2926·0
6 *a*	2753·9	6 *s*	2790·3	3 *a* 6 *s*	2831·8	4 *a*	2876·8	1 *a* 4 *s*	2928·3
6 *a*	2754·3	6 *a*	2791·5	5 *a*	2832·4	4 *a*	2878·2	6 *s*	2931·1
1 *a s*	2755·5	6 *a*	2792·2	5 *a*	2832·8	6 *a* 5 *s*	2880·4	5 *a* 6 *s*	2932·4
3 *a*	2756·2	4 *s*	2793·3	3 *s*	2835·2	6 *a* 4 *s*	2883·3	1 *a* 5 *s*	2936·4
6 *s*	2756·9	3 *a*	2794·5	6 *s*	2836·7	6 *s*	2885·5	5 *a*	2937·3
4 *a*	2757·2	6 *s*	2796·3	5 *a* 6 *s*	2837·7	6 *a*	2885·8	4 *a* 6 *s*	2938·7
6 *a*	2759·7	5 *a s*	2797·4	6 *a*	2839·6	6 *a*	2887·3	6 *a*	2939·9
3 *a s*	2761·7	3 *a*	2797·9	6 *a* 3 *s*	2840·3	6 *s*	2887·6	2 *a*	2940·8
4 *a*	2761·9	6 *a* 4 *s*	2798·8	6 *a s*	2843·1	6 *a*	2889·2	6 *a*	2943·1
6 *s*	2762·4	6 *a*	2799·4	2 *a* 4 *s*	2843·6	6 *a*	2891·2	6 *a* 2 *s*	2944·0
4 *a*	2763·0	6 *a*	2800·1	5 *a* 4 *s*	2845·3	6 *a*	2892·0	6 *a*	2944·6
6 *s*	2763·6	3 *a*	2800·8	6 *a*	2846·5	6 *a*	2893·2	U	2947·3
6 *a*	2764·0	6 *a*	2801·8						

The accompanying map is drawn to a scale double that of ÅNGSTRÖM'S and CORNU'S maps of the solar spectrum. Those lines which are common to both arc and spark are drawn right across from top to bottom, while those which are in the arc only are not continued to the bottom, and those which are in the spark only do not begin at the top; so that the upper portion of the map represents the arc spectrum, the lower the spark spectrum, and they overlap in the middle.

We do not pretend to say that every line in this map really belongs to iron, for commercial iron wire was used to produce it, and the map may therefore probably include lines of a good many metals, certainly manganese lines in the arc, but it will not the less serve the purpose for which it has been made, namely: for reference in determining the approximate wave-lengths of lines of any spectra.

PART II.

(Received June 15, 1882.)

The account of the ultra-violet spectra of fifteen metals here recorded is a first instalment of the results of observations which we have accumulated during the past three years, but have not heretofore been able to reduce. During that time we have taken some thousands of photographs of the electric arc under various conditions, and especially in crucibles of lime and magnesia (as previously described by us), and in the presence of most of the known metals; but with the exception of CORNU'S map of the ultra-violet solar spectrum giving the chief iron lines and a few of those of other elements, up to the line U we have had little to aid us in the exploration of a new field and the assignment of the several lines to the elements producing them, and the measurement of our many photographs has cost both time and patience. Dr. W. A. MILLER long ago published an account of his photographs of the spark spectra of the elements, and Mr. HARTLEY has recently (Trans. Roy. Dublin Soc.) published photographs of the spark spectra of several elements which are a great improvement on those previously published. But those give spark spectra only, are taken with an apparatus of small dispersion, and are not reduced to scale, so that they give qualitative rather than quantitative results. The spectra which we here describe are those of the arc up to the wave-length 2200, and we give in each case the approximate wave-lengths of the lines observed. For some few of the lines of tin and aluminium the wave-lengths have been determined by means of a grating as described in the first part of this paper, but in all other cases they have been derived by interpolation from the wave-lengths of the neighbouring iron lines. In the map which accompanies this paper we have given in the top line the principal lines of iron for convenience of reference, and in the lowest line the arc lines of carbon with which it is necessary to be acquainted as they are always present, though varying much in intensity, in the arc taken between carbon electrodes. The scale of this map is one-half that of ÅNGSTRÖM'S "Normal Solar Spectrum."

We have already, in describing the visible spectra of the alkali metals and that of magnesium, called attention to probable harmonic relations between the lines. This relation manifests itself in three ways—first, by the repetition of similar groups of lines; secondly, by a law of sequence in distance, producing a diminishing distance between successive repetitions of the same group as they decrease in wave-length; and thirdly, a law of sequence as regards quality, an alternation of sharper and more diffuse groups, with a gradually increasing diffuseness and diminishing intensity of all the related groups as the wave-length diminishes.

The first relationship has long since been noticed in the case of the sodium lines which recur in pairs, and we have observed that the potassium lines between the extreme red and violet pairs are repetitions of a quadruple group, while the lithium lines (with the exception of the blue line mentioned below) are single, and one set of those of magnesium triplets. We now record a second harmonic* series of potassium lines which appear to be pairs, and the violet pair, and possibly the red pair too, belong to this series. Lithium shows a second harmonic series of single lines high up on the scale. Calcium gives a long series of well marked triplets; zinc likewise gives a series of triplets; aluminium gives pairs, and in the highest region triplets; thallium gives a series which seem to be quadruple groups with two of the four lines in each of much greater intensity than the rest. The alternations of sharper and more diffuse groups are generally apparent and are very marked in the cases of calcium and zinc. The diminishing distance and intensity and increasing diffuseness of successive repetitions of the same group as the wave-length diminishes, are in all the cases mentioned very plain. In all these cases the different lines forming a group are tolerably close to one another, so that successive repetitions of a group do not overlap one another, but it may be that in other cases the lines forming one group may be so far apart that the most refrangible line of one group may be more refrangible than the least refrangible line of the next repetition of the group; the groups and their sequence will thus be much less easily recognised.

Potassium.

The ultra-violet spectrum of potassium, so far as we have observed it, is apparently one harmonically related series of which the first member above the visible spectrum is a double line just below the solar line O; the next falls between Q and R, and the others follow at decreasing intervals, the seventh and last that we have observed falling just above U. It is only in the case of the line near O that we have been able to make sure that it consists of a pair of lines, but it is very probable that all are pairs in reality; all are strongly reversed, as might be expected from the volatility of the metal, and expanded when a fresh quantity of the metal or its compounds is introduced into the arc, so that the separation of the pairs, if such they be, could not be seen, while the more refrangible lines die away and are not recognisable as *bright* lines amongst the many lines which come out in the arc, as the alkali metal is dissipated. The line between Q and R, which is a strong line, happens to be in a region where the lines of iron, manganese, and chromium lie very closely, so that we cannot pronounce with certainty that it is a double line.

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* By an "harmonic series" of lines we merely mean a series of overtones of a fundamental vibration we do not mean that they follow the simple arithmetical law of an ordinary harmonic progression, but are comparable rather with the overtones of a bar or bell than with those of a uniform stretched string.

POTASSIUM lines.

Approximate wave-length.	Remarks.
3445·0 3443·6	Double line.
3216·5	All are easily reversed.
3101·0	
3033·0	
2992·0	The lines become weaker as they are more refrangible.
2963·4	
2942·0	

Sodium.

The sodium lines observed by us also form one apparently harmonic series with the double line, wave-length 3301, observed by CORNU. In this case also we have not been able to make out that any of the lines above 3301 are double, as when there is enough sodium present to develop them decidedly they are always more or less diffuse and reversed. Indeed, the line at 3301 is a very close pair and it is not often seen as two lines.

One line is so near to the very strong magnesium line, wave-length 2852, that the apparent development of the magnesium line by sodium was for some time an enigma to us. The sodium line is a little less refrangible than the magnesium line.

SODIUM lines.

Approximate wave-length.	Remarks.
3301·0	CORNU's double line.
2853·3	All the lines are easily reversed.
2679·0	
2593·3	

Lithium.

We have already described one apparently harmonic series of lithium lines extending into the ultra-violet up to about wave-length 3799. This series we described as all single lines though alternately sharp and diffuse. This description is correct, except that we have since found that one line of the series, namely, the strong blue line at wave-length 4604, is really a double line. When a fresh dose of lithium, or of some one of its compounds, is introduced into the arc, a second weaker line comes out on the more refrangible side of the strong blue line, and gives to it all the appearance of a reversal with an expansion of the bright wings unequally extended on the two sides. As the strong blue line is, however, often really reversed, the effect is then that of a double reversal, that is to say, the appearance is that of a broad bright band with a narrower dark band within it and a bright line in the middle of the dark band. The second line rather quickly dies out as the lithium evaporates, leaving the strong blue line comparatively permanent. We have never observed any such second line, or companion, to any of the other lines of lithium. The new series begins with a line at wave-length about 3232, and dies out with a very diffuse line at about wave-length 2359. The following is a list of the ultra-violet lines we have observed.

Approximate wave-length.	Remarks.	Approximate wave-length.	Remarks.
3984·5		2561·5	Reversed.
3913·5	Diffuse.	2475·0	,,
3862·3	Somewhat obscured by the cyanogen bands in this region.	2425·5	
3799·0		2394·5	Diffuse.
3232·0		2373·5	Very diffuse.
2741·0	Reversed.	2359·0	Very diffuse and weak.

Barium.

The barium lines are numerous, but do not fall into easily recognised harmonic series.

BARIUM lines.

Approximate wave-length.	Remarks.	Approximate wave-length.	Remarks.
3991·8		3320·9	
3908·5		3279·8	
3891·0		3261·0	
3793·5		3070·3	
3660·7		2785·1	
3598·7		2771·0	
3592·8		2739·0	
3579·1		2702·0	
3544·0		2647·0	
3524·5		2634·5	
3499·2	Very strong, reversed.	2596·7	
3419·3		2542·7	
3375·6		2347·0	Strong.
3354·8		2335·0	Very strong.
3347·7		2304·5	Strong.

Strontium.

Strontium, and its compounds, produce a line at wave-length 3705 coincident, or nearly so, with one of the lines ascribed to calcium by Cornu. We have so often observed this line much reinforced by strontium without any increase of the other calcium lines which are always present in the arc from calcium in the carbon electrodes, that we think we are justified in putting down a strontium line at this place. Two other lines of this metal are close to, but not coincident with lines of barium.

Strontium lines.

Approximate wave-length.	Remarks.	Approximate wave-length.	Remarks.
3705·0	Coincident or nearly so with a calcium line.	3458·0	Diffuse.
3653·0		3379·5	
3547·0		3364·8	
3527·0		3305·2	
3498·0		2931·1	
3464·0			

Calcium.

Cornu has mapped two calcium lines, one on either side of the solar line M, and four other lines, of which one is coincident with the solar line R, one slightly less refrangible, and the other two more refrangible. One of these lines, at wave-length 3168·5, we have never certainly seen, but the others are well developed when calcium compounds are put into the arc. Besides these we have always seen when calcium, or one of its compounds, is present in moderate quantity a series of triplets analogous to those of magnesium. Each triplet consists of two strong lines with a rather weaker line on their more refrangible side. This series appears to be harmonically related to the well-known blue triplet at wave-length 4454–24. The first repetition of this triplet occurs close to H, one line of the triplet falling below H, while the other two lines fall between H and K. The next triplet falls between N and M, and the next between O and N, and so on at decreasing intervals, the most refrangible repetitions becoming very faint and diffuse, so that in the last, a little below S, we have only been able to distinguish the strongest two lines of the triplet. The triplets are alternately diffuse and sharp, those near H, between O and N, and so on alternately, being the sharper. The diffuse triplets are stronger than the others and more easily reversed. Beyond this series we have noticed only one calcium line, and that is high up on the scale, at wave-length about 2398.

CALCIUM lines.

Approximate wave-lengths.	Remarks.	Approximate wave-lengths.	Remarks.
3967·7	H.	3285·0	Sharp triplet.
3972·3	Sharp triplet.	3273·5	
3956·0		3268·5	
3947·9		3224·5	Diffuse triplet.
3933·0	K.	3213·0	
3736·4	Mapped by CORNU.	3208·0	
3705·5	Ditto.	3181·0	Mapped by CORNU.
3644·0	Very strong. Readily reversed.	3179·0	R. ditto.
3631·0		3168·5	Ditto, not seen by us.
3623·5		3158·8	Mapped by CORNU.
3486·5	Sharp triplet.	3151·0	
3474·5		3141·0	
3468·0		3136·0	
3359·5	Very strong.	3117·5	Weak, very diffuse.
3347·5		3108·0	
3342·0		2398·0	

Zinc.

Zinc is another metal which gives a well-marked apparently harmonic series of triplets, but the different lines of each group are further separated than in the calcium or magnesium triplets. The middle line of the first triplets confounds with the sodium pair wave-length 3301, but by reason of the diffuse character of the zinc line we have not been able to decide whether the coincidence is more than approximate.

ZINC lines.

Approximate wave-lengths.	Remarks.	Approximate wave-lengths.	Remarks.
3342·0		2608·5	Diffuse.
3301·0		2582·0	
3281·0		2569·7	
3070·0		2516·0	
3035·0		2491·5	
3017·0		2480·0	
2800·0	Diffuse.	2464·5	Very diffuse.
2770·0		2440·0	
2756·0		2430·0	
2713·3			
2684·0			
2670·5			

Mercury.

As might be expected from its volatility, it is difficult to obtain lines of mercury in the arc; but one line gives a reversed image of itself at wave-length 2536·8. This line is very bright in the flame of cyanogen, containing vapour of mercury.

Gold.

Gold also gives us but few lines. The three lines we record are perhaps harmonically related.

GOLD lines.

Approximate wave-length.	Remarks.
3122·8	
2675·4	Reversed.
2427·5	Ditto.

Thallium.

Groups of thallium lines manifestly similar to one another recur, and are probably harmonically related. This recurrence is more evident in the photographs in which the lines which are expanded and reversed by the introduction of fresh metal are at once recognised. The pairs at 2921, 2710, 2609, and the lines at 2552, 2517 seem to fall into one series.

THALLIUM lines.

Approximate wave-length.	Remarks.	Approximate wave-length.	Remarks.
3775·6		2714·6	Very diffuse.
3528·3		2710·4	Reversed.
3517·8		2708·8	Strong, diffuse reversed.
3228·1		2699·7	Very diffuse.
2943·9		2665·0	Diffuse.
2921·3	Very strong.	2652·3	
2917·8		2609·4	Reversed.
2895·2		2608·6	Strong, reversed.
2825·8		2552·0	Reversed.
2826·9		2517·0	Diffuse.

Aluminium.

The spectrum of aluminium is comparatively simple. The well-known pair of lines between H and K seem to be repeated twice in the region above without much, if any, diminution of strength, but we have not observed any such lengthened sequence of repetitions of these lines as we have of the lines of magnesium and other metals. Higher on the scale we come to another series of groups which are triplets, or perhaps quadruple groups, for the first and strongest group shows a faint fourth line which we have not observed in the succeeding groups. The repetitions we record are only two, but they lie in a part of the spectrum so near the limit of transparency of calcite that it is quite possible that there may be more beyond, which will show themselves when quartz prisms are used; a strong triplet near N appearing in the spark, wave-length about 3605, 3598, 3585, does not show in the arc so far as we have observed.

ALUMINIUM lines.

Approximate wave-length.	Remarks.	Approximate wave-length.	Remarks.
2659·8 2652·0	Strong, frequently reversed.	2268·7	Strong, diffuse.
2574·5 2567·5	,, ,, ,,	2263·1	,, ,,
		2257·3	
2378·4 2373·2 2366·9	A fourth faint line close to the middle line of this group. Middle line very strong, generally reversed.	2216·0	
		2210·0	Strong, diffuse.
		2205·0	Diffuse.

Lead.

The lines of lead are numerous and strong, and many of them readily reversed. We have not yet traced any probably harmonic series amongst them.

Lead also gives some indefinite bands of continuous light about the region wave-length 2500.

LEAD lines.

Approximate wave-length.	Remarks.	Approximate wave-length.	Remarks.
4019·0		2801·1	Often reversed.
3801·0		2721·0	
3739·3		2706·1	
3683·3	Strong, often reversed.	2697·0	Middle of a very diffuse band.
3670·7		2662·7	
3639·3	,, ,, ,,	2650·5	Diffuse.
3572·0		2627·8	
3260·0	Nearly coincident with a line of tin.	2613·7	Strong, often reversed.
3238·6		2575·7	Very diffuse.
3219·6		2476·5	Strong, reversed.
3118·5		2446·1	,, ,,
2981·0 2973·5 2967·0	Weak lines.	2443·7	,, ,,
		2428·5	
		2411·5	,, ,,
2872·0		2401·8	,, ,,
2850·5	A little above the magnesium line, sometimes hidden by the expansion of the latter.	2399·4	
		2393·7	Very strong, reversed.
		2388·8	
2832·9	Very strong and diffuse.	2332·0	
2822·5	Generally reversed.		

Tin.

Tin is remarkable for the number and strength of its lines in the higher region of the spectrum, while its lines of lower refrangibility are so feeble that we have never seen any in the arc in the visible part of the spectrum. CORNU has recorded one line at wave-length 3260, and we have had no difficulty in recognising this line, but a line very nearly in that place is also developed by lead when other tin lines are not developed, and we have not been able to perceive that these lines are separable in any of our photographs. Many of the higher lines of this metal are easily reversed, indeed are almost always reversed in our photographs.

TIN lines.

Approximate wave-length.	Remarks.	Approximate wave-length.	Remarks.
3326·0		2523·5	
3260·0	Given by CORNU.	2495·5	
3175·0		2493·5	
3141·7		2483·1	
3033·0		2429·5	
3008·5		2421·5	
2986·4		2407·9	
13·1		2392·5	
2862·8		2364·7	
2839·5		2357·7	
2813·5		2354·5	
2812·5		2334·3	
2787·5		2317·0	
2784·7		2286·9	
2779·5		2282·5	
2761·5		2275·4	
2660·7		2251·0	Strong, reversed.
2636·5		2245·8	Strong.
2593·5		2231·3	
2571·0		2210·7	
2557·5		2198·7	
2546·1		2194·1	
2530·7			

ANTIMONY lines.

Approximate wave-length.	Remarks.	Approximate wave-length.	Remarks.
4032·0		2597·5	
3637·0	Close above a lead line.	2528·0	
3265·0		2426·0	
3230·8		2383·3	
3228·0		2313·0	Very strong, reversed.
3028·0		2310·0	
2876·5			

BISMUTH lines.

Approximate wave-length.	Remarks.	Approximate wave-length.	Remarks.
3595·3		2799·0	
3510·4		2780·0	
3396·2		2730·0	
3066·0	Very strong, often reversed.	2593·0	
3023·5		2524·0	
3000·0		2515·4	
2996·0		2448·0	
2937·4		2435·5	
2897·0		2431·0	
2862·0		2400·8	Very strong.
2810·0		2277·0	Weak, reversed.

Carbon.

In our map we give the carbon lines as developed in the arc. These occur in the arc taken between poles of purified graphite in air, and in nitrogen, and in carbonic acid gas, and they are always present in the arc taken in our crucibles. Most of them are also in the spark spectrum of carbon as described by us (Proc. Roy. Soc., xxxiii., 403), but some of the spark lines are not developed in the arc, and there are two lines in the arc which we did not notice in the spark.

CARBON lines.

Approximate wave-length.	Remarks.
2881·1	Not observed in spark.
2528·1	
2523·9	
2518·8	
2515·8	
2514·1	
2506·6	
2478·3	The strongest line.
2434·8	Not observed in spark.

IV. *Experiments on the Value of the British Association Unit of Resistance.*

Part I. by R. T. Glazebrook, *M.A., Fellow and Assistant Lecturer of Trinity College, Cambridge, Demonstrator of Physics in the Cavendish Laboratory, and* J. M. Dodds, *B.A., Fellow of St. Peter's College.*
Part II. by R. T. Glazebrook *and* E. B. Sargant, *M.A., Trinity College.*

Communicated by Lord Rayleigh, *M.A., F.R.S.*

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Part I.

Before leaving Cambridge, Professor Chrystal, in the summer of 1878, wound with great care two large coils of about 50 centims. in diameter to be used for a redetermination of the value of the British Association unit of resistance. A galvanometer of special construction with double coils, one of thick wire the other of thin, was also wound by him and mounted with Weber's suspension.

The coils were to be placed at a known distance apart, so that the coefficient of mutual induction could be calculated.

The apparatus remained unused till the spring of last year, when the experiments described in the present paper were commenced.

The method used is similar to those employed by Kirchhoff (Pogg. Ann., lxxvi.) and Rowland (American Journal of Science and Arts, vol. xv., 1878). The coefficient of mutual induction of the two coils is determined by calculation from the geometrical data. A current is passed through one coil, the other being in circuit with a ballistic galvanometer, and the induced current, produced when the primary current is broken or reversed, is measured by the throw of the galvanometer needle. The primary current itself being then measured by some method, we have enough data to determine in absolute measure the resistance of the secondary circuit and the galvanometer. But this resistance can be measured in terms of the B.A. unit, and hence a value obtained for the latter.

Professor Rowland measured his primary current by means of a second galvanometer, the constant of which he compared with that of the ballistic galvanometer both by direct experiment and by calculation.

In our experiments the value of the primary current was obtained by passing by means of a shunt about $\frac{1}{3000}$th part of it through the ballistic galvanometer; we thus eliminate from our equations the values of the galvanometer constants, as well as the correction which Rowland had to apply for the difference in the intensity of the horizontal component of the earth's magnetic force at the two galvanometers.

We have instead to determine exactly the ratio between two resistances of about 1 and 3000 B.A. units respectively, and to show that the heating of the wires by the current could never be such as to affect the value of this ratio appreciably. We proceed to describe the arrangement of the apparatus.

Fig. 1 gives a diagrammatic plan. A and B are the two coils, A being the primary. G is the galvanometer. K, L, M, N are four mercury cups.

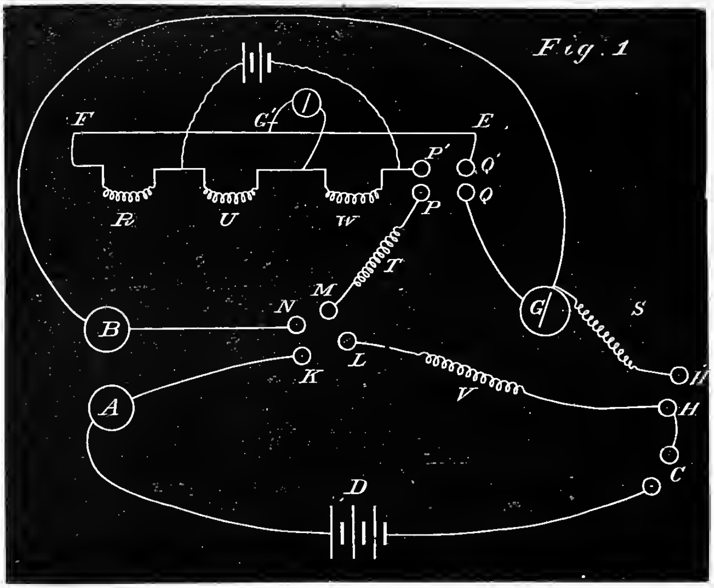
Fig. 1

P, Q, H, and H′ are also mercury cups. C is the commutator, and D the battery. Between H′ and G is a resistance, S, of about 3000 B.A. units, between H and L a second resistance, V, of about 1 unit.

Between P and M there is an adjustable resistance, T, the purpose of which will be described shortly.

The rest of the figure represents the ordinary Wheatstone's bridge arrangement for measuring the resistance of the secondary circuit. P′, Q′ are two mercury cups. E, F a divided wire. R a resistance of about 160 B.A. units (the total resistance of the secondary circuit). U and W two equal resistances of about 30 units, forming the other arms of the bridge.

Let us suppose P P′, Q Q′, and M N are connected by stout copper ┌┴┐-shaped pieces. Then our secondary circuit, broken between P and Q, forms the fourth arm of the Wheatstone's bridge, and by adjusting the variable resistance, T, the resistance of the secondary circuit can be made so nearly equal to R that the difference between them may be expressed in terms of the resistance of the bridge wire in the ordinary manner.

R then forms our standard resistance, and is the quantity actually measured in the experiments. R is the resistance of a coil of platinum-silver wire made at Professor Stuart's workshops, according to the pattern designed by Professor Fleming, of Nottingham.

The wire was supplied by Messrs. Elliott Brothers, and is that used by them in the manufacture of coils of about 100 units resistance. R, as has been stated, is about 160 units.

Now suppose that the connexions P P′, Q Q′ are broken, and that P Q is connected by means of one of the ⊓ pieces. Break M N and connect L K, then the battery circuit is complete. Now connect M N again, the secondary circuit is complete, and the current running in the primary. Reverse the commutator C, an induction current is produced in the secondary circuit, and may be measured by the throw of the galvanometer needle.

To measure the primary current M N is broken and M L and H H′ are connected.

Between H and L the primary current is divided; the resistance in the direct circuit H V L being about 1 ohm, that in the circuit H S G Q P M L about 3072 ohms, so that about $\frac{1}{3073}$ of the whole current runs through the galvanometer; the permanent deflection of the galvanometer G is observed, and from this the value of the current is calculated.

We turn now to a detailed description of the apparatus used.

The coils were wound by Professor Chrystal.

Two brass rings were carefully turned and a rectangular channel cut in the outer limb of each. A slit was cut in each ring to prevent currents in the frame (this was of course unnecessary for our experiments, but might render the coils more useful in many cases); the slit was closed with a piece of insulating material into which binding-screws connected with the wires of the coil are screwed.

The coils are lettered A and B.

In the experiments the coils are to be placed with their planes parallel and their centres on a line at right angles to their planes.

A cylindrical brass rod was carefully turned and fixed to a brass support so that it would stand in a vertical position.

Two tripod pieces of brass of the form shown in fig. 2 were constructed. The brass rod fits accurately through the aperture O in the centre of the tripod, and the plane of the arms O A, O B, O C is at right angles to the axis of the rod. The curved surfaces A A_1, B B_1, C C_1, are small portions of the same circular cylinder whose axis coincides with that of the rod, and the radius of this cylinder is the same as that of the carefully turned inner surface of the annulus on which the coils were wound.

This annulus thus would then fit on to A A_1, B B_1, C C_1 and rest on the flat surfaces A A_1 A_2 A_3, B B_1 B_2 B_3, C C_1 C_2, C_3, which are all parts of the same plane at right angles to the axis of the bar. If then we place one coil, A suppose, in this manner on the tripod, its plane is perpendicular to the axis of the bar, and its centre lies on that axis.

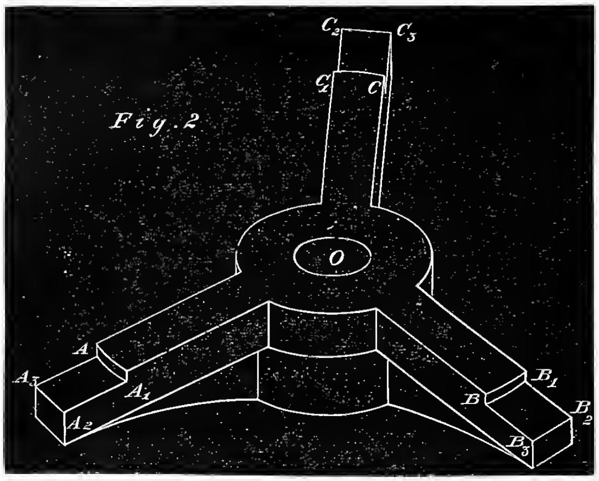

Fig. 2

To place the second coil with its plane parallel to the first we used three cylinders of brass, the ends of which had been carefully turned so as to be at right angles to their axes, while the lengths of the cylinders were as nearly as possible the same.

These three cylinders were placed vertically on the upper side of the coil A, one above each of the arms of the tripod, and the coil B rested on them; its plane thus was as nearly as may be parallel to that of A.

To bring its centre into the axis a second tripod exactly similar to the first was placed in an inverted position over it, and the coil was moved about until the flange pieces fitted inside the inner surface of the annulus as before. In this manner the coils were adjusted to the required position.

The following account of the precautions used in the winding, and the methods employed to measure the constants of the coils, is quoted from a letter of Professor CHRYSTAL'S, addressed to one of the authors of this paper (R. T. G.) :—

"The coils, stand, &c., were constructed after working drawings made by myself under the supervision of Professor MAXWELL. The immediate end in view in constructing the coils was the determination of the ohm, and this of course influenced the design of the stand. It was proposed ultimately, to use the coils as a standard instrument for producing a uniform magnetic field in which to determine galvanometer constants and the like.

"The coils were wound by myself and the then mechanic at the laboratory, Mr. FULCHER. The coils were mounted for this purpose by placing between the three armed supports"—the tripods mentioned above—"which were then braced together and mounted on an axle and stand. During the winding constant tests were taken for the insulation between the wire and the metal channel. This was the main difficulty, and wherever the slightest defect was discovered the wire was unwound for a little way and paraffin paper and paraffin used. It was found absolutely necessary, in order to secure good insulation, to cover the bottom of the channel with a ribbon of silk drawn through melted paraffin. The number of turns in each layer was separately counted and registered, and as a check a counter was attached to the axle and read at the end of the windings; the two records agreed in both cases. The resistance of the coils after winding was 84·9 and 82·7 B.A. units respectively. Four diameters were measured in every layer.

"The measurements of length were made by means of the cathetometer, and I find in my book a record of a comparison between it and the beam-compass scale, which had been tested* I believe; there are also measurements of the thicknesses of the walls of the channel and its width."

The numbers actually used in calculating the value of M were furnished by Professor CHRYSTAL from these measurements. The error between the cathetometer and beam-compass appeared in one or two measurements to amount to 1 in 2000, but in the majority of cases it was very much less, its mean value being, perhaps, 1 in 10,000, irrespective of sign. On the whole then we may, without sensible error, treat the cathetometer scale as accurate.

In another letter, Professor CHRYSTAL gives the following extract from his note-book of the direct results of the observations. He says: "You remember that each diameter is given as the mean of four.

"The first diameter is through the slit, the next 45° from it in the direction of the sun's motion, the letter on the coil being up, and so on.

"Here is the entry in my book for the fourth layer in coil A.

No. of layer.	No. of turns.	Cathetometer.	Cathetometer.	Difference.	Mean.
4	26	781·40 781·30 781·27 780·60	280·14 279·95 279·58 279·52	501·26 501·35 501·69 501·08	501·35

"You will observe in the above extract that the two intermediate diameters are greater. This happens in most layers. At any rate the diameter perpendicular to that through the slit is in the great majority of cases the greatest, as might be expected."

Professor CHRYSTAL'S measurements then gave us as the mean of four observations in different positions the value of the external diameter of each winding, and also the total number of windings in each layer. In each coil there were 30 layers and in each layer about 26 windings. In coil A the total number of windings was 797 and in coil B it was 791.

Let the external diameter of the layers be d_1, d_2, d_3, &c., and let the number of turns in a layer be $26+n_1$, $26+n_2$, &c.

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* This beam-compass has again been tested by MR. DODDS during the present year and found correct. All our measurements of length are referred to it. (Nov., 1882.)

Let t be the thickness of the wire and silk used, and let A and a be the mean radii of the coils.

$$\text{Then } 2\mathrm{A}+t=\frac{26\{d_1+d_2+\ \ldots\ +\}+n_1d_1+n_2d_2+\ \ldots}{797}$$

Now we know the diameter of the channel before the first layer was put on and also the diameter of the first layer; they were respectively

$$49{\cdot}565 \text{ and } 49{\cdot}728 \text{ centims.}$$

From this we find

$$t={\cdot}0815 \text{ centim.}$$

and finally

$$\mathrm{A}=25{\cdot}753 \text{ centims.}$$

The observations on coil B gave the same value for the thickness of the wire and covering, and we get for it

$$a=25{\cdot}766 \text{ centims.}$$

The method here adopted to determine the value of the mean radius allows for the fact that in winding one layer may sink somewhat into the one beneath.

Let the figure (fig. 3) represent a section of the coils by a vertical plane through the axis; let the coils be placed with their lettered sides down as in the figure. This we call position 1 throughout.

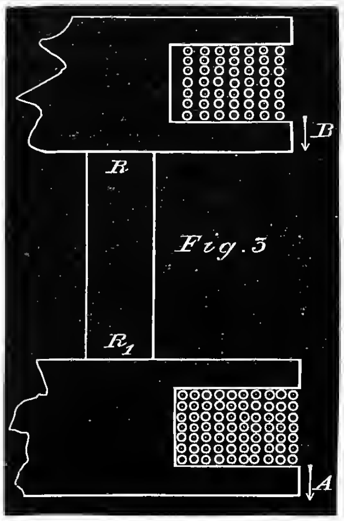


Fig. 3

In position 2 the lettered side of B was turned uppermost.

In position 3 the lettered side of A also was uppermost, while in position 4, A remained uppermost while B was again inverted.

Thus if $\downarrow$B mean that the lettered side of B was down, we have

Position 1 $\left\{\begin{matrix}\downarrow \mathrm{B}\\ \downarrow \mathrm{A}\end{matrix}\right.$ Position 2 $\left\{\begin{matrix}\uparrow \mathrm{B}\\ \downarrow \mathrm{A}\end{matrix}\right.$

Position 3 $\left\{\begin{matrix}\uparrow \mathrm{B}\\ \uparrow \mathrm{A}\end{matrix}\right.$ Position 4 $\left\{\begin{matrix}\downarrow \mathrm{B}\\ \uparrow \mathrm{A}\end{matrix}\right.$

In the figure R R_1 represents a section of one of the cylindrical rods used to separate the coils; its length and the thickness of the walls of the channel are all known. If, then, we assume that the median plane of each coil is midway between the walls of the channel, we can find the distance between the median planes; this we denote by b. But if the median plane in one coil, B for example, be not halfway between the walls but some small distance, δb suppose, nearer the lower wall, then in position 1 the value assumed for b is too large by δb, while in position 2 it is too small by δb, and by making observations in positions 1 and 2 and taking their mean we eliminate any error in the result which depends on the first power of δb. Similarly, if there be an error in the position of the median plane of the coil A of the same nature it is eliminated by inverting A.

The thicknesses of the walls of the channel as given by Professor CHRYSTAL are:—

	Coil A.	Coil B.
	centim.	centim.
Lettered side	·478	·446
Unlettered side . . .	·488	·465

Also, if $2h$, $2k$ be the radial and axial dimensions of the channel for coil A, $2h'$, $2k'$ for coil B, we have

$$h = \cdot 96 \qquad k = \cdot 95$$
$$h' = \cdot 95 \qquad k' = \cdot 95$$

Three series of brass cylinders were used to separate the coils, and the lengths of these were measured each by two observers. A pair of calipers graduated to read with a vernier to $\frac{1}{1000}$th part of an inch were found to be the most convenient instrument with which to make the measurements. The scale of the calipers was tested against the beam-compass without discovering any errors that could affect the result to as much as 1 in 10,000.

The following is the series of measurements for the longest rods used (Series C), made by R. T. G. and given as an example of the agreement of the observations.

Calling the rods α, β, γ, we found

α.	β.	γ.
inches.	inches.	inches.
9·389	9·388	9·390
9·388	9·389	9·391
9·388	9·389	9·390

It was noticed, however, that two of these rods had been slightly bruised at the end, thus producing a small lump at one point; when the rods were placed so that this lump came between the jaws of the calipers an increase of ·002 inch was observed in the length. In the calculations this greater length has been used as the true length of these two rods. In the shortest rods, series A, the difference in length, arising from a slight lump on the end of one of the rods, was somewhat greater, being about ·003 inch.

Another series of measurements made in an entirely different manner by placing first the rods and then the beam-compass beneath a pair of reading microscopes gave very closely concordant results; these measurements made the rods appear about ·001 inch longer than the measurements with the calipers. Some difference of this kind was to be expected from the difference which exists between the contact length and the sight length of a rod.

In our calculations we have taken the values given by the calipers.

Reducing them to centimetres we have for the lengths of the rods

	centims.
Series A	12·182
Series B	15·416
Series C	23·856

We have thus obtained all the dimensions requisite for the calculation of the mutual induction between the coils in the three series.

Let us call b the distance between the mean planes; b, of course, is slightly different in each of the four positions included in each series.

The calculations of M have been conducted as follows :—

If all the windings are supposed to be coincident with the mean windings, and M_0 be the mutual induction on this hypothesis

$$M_0 = 4\pi n n' \sqrt{Aa}\left\{\left(c - \frac{2}{c}\right)F + \frac{2}{c}E\right\}$$

where

$$c = \frac{2\sqrt{Aa}}{\sqrt{(A+a)^2+b^2}} \text{ and } n, n' \text{ are the number of windings;}$$

and F, E are complete elliptic integrals to modulus c (MAXWELL, vol. ii., § 701).

Appendix i., ch. xiv., to the second edition of MAXWELL'S 'Electricity' contains a table in which the logarithms of $M_0/4\pi\sqrt{Aa}$ are given for values of $\sin^{-1} c$ from 60° to 90°, proceeding by intervals of 6′.

The proper value of $\gamma = \sin^{-1} c$ is most easily obtained from the equations

$$r_1^2 = (A+a)^2 + b^2 \qquad r_2^2 = (A-a)^2 + b^2$$

$$\cos\gamma = r_2/r_1.$$

Thus when the coils are separated by the long rods

$$r_1^2=3365{\cdot}64 \qquad r_2^2=712{\cdot}36$$

whence

$$\gamma=62^\circ\ 36'\ 32''.$$

From the table

$$\log M_0/4\pi\sqrt{Aa}=\bar{1}{\cdot}5718069$$

and this gives, taking A as 25·750, a as 25·760

$$M_0={\cdot}761225\times10^8.$$

When, however, the dimensions h, k of the section are too large in comparison with b to allow us to mass all the windings together, we may use a formula of approximation due to Mr. H. J. PURKISS (Appendix ii., ch. xiv., MAXWELL, second edition).

Let the suffix $\pm h$ denote that for a is substituted $a\pm h$ in the corresponding functions, $\pm k$ that $b\pm k$ replaces b, and similarly for accented letters. Then

$$M=\tfrac{1}{6}\{M_h+M_{-h}+M_{h'}+M_{-h'}+M_k+M_{-k}+M_{k'}+M_{-k'}-2M_0\}.$$

To calculate these eight quantities in a methodical manner we notice that

$$\left.\begin{array}{l} r_{1h}^2=r_1^2+2(A+a)h+h^2 \\ r_{2h}^2=r_2^2-2(A+a)h+h^2 \\ \&c. \end{array}\right\}$$

and that consequently we have to add in each case a correcting square, and add or subtract a correcting product. The corresponding γ is found, and the rest of the calculation effected just as for the mean windings.

When the coils are separated by the long rods the greatest and least values of γ are $\gamma_{-k}=63^\circ\ 26'\ 54''$, and $\gamma_k=61^\circ\ 46'\ 56''$, the final value result being

$$M={\cdot}761921\times10^8.$$

It thus differs by nearly 1 in 1000 from the uncorrected value.

The values of b differ slightly according to the four positions of the coils, and a slight correction has to be made in the values of A, a assumed above.

It therefore becomes of importance to determine the correction in M to be made for the addition of ·001 centim. to A, a, or b.

Since $\cos\gamma=r_2/r_1$, it follows that

$$\delta_a\gamma=\delta_A\gamma={\cdot}001(A+a)/r_1^2\tan\gamma \qquad \text{in circular measure.}$$

This gives for the long rods

$$\delta_a\gamma=1{\cdot}64''.$$

Again

$$\delta_b\gamma = {\cdot}001(r_1^2 - r_2^2)/r_1^2 r_2 \tan\gamma$$

whence

$$\delta_b\gamma = -3{\cdot}16''.$$

From the tables we find that when $\gamma = 62^\circ\ 36'$, the difference in the value of $\log M_0/4\pi\sqrt{a}A$ for an addition $1''$ to γ is 77×10^{-7}; that is, since $\delta M = \delta M_0$ very approximately,

$$\delta_a M/M = \frac{\delta a}{2a} + 77 \times 10^{-7} \times 1{\cdot}64'' \times \log_e 10 = {\cdot}000048$$

$$\delta_b M/M = -77 \times 10^{-7} \times 3{\cdot}16'' \times \log_e 10 = -{\cdot}000041$$

The true values of the radii are $A = 25{\cdot}753$, $a' = 25{\cdot}766$. The correction on this account to the value of M is therefore $9 \times {\cdot}000036 \times 10^8 = {\cdot}000324 \times 10^8$.

The corrections to be applied to b are:—In position 1, $-{\cdot}008$; in position 2, $+{\cdot}016$; in position 3, $\times{\cdot}006$; in position 4, $-{\cdot}013$.

Making similar calculations for the medium and short rods we can present the values of the mutual induction thus:

TABLE giving values of the mutual coefficient of induction between the coils.

Position of coils.	Experiments A. Short rods.	Experiments B. Medium rods.	Experiments C. Long rods.
1	$1{\cdot}55636 \times 10^8$	$1{\cdot}25797 \times 10^8$	${\cdot}762107 \times 10^8$
2	$1{\cdot}55430 \times 10^8$	$1{\cdot}25649 \times 10^8$	${\cdot}761321 \times 10^8$
3	$1{\cdot}55539 \times 10^8$	$1{\cdot}25727 \times 10^8$	${\cdot}761735 \times 10^8$
4	$1{\cdot}55744 \times 10^8$	$1{\cdot}25875 \times 10^8$	${\cdot}762521 \times 10^8$
Mean of the four . .	$1{\cdot}55587 \times 10^8$	$1{\cdot}25758 \times 10^8$	${\cdot}761921 \times 10^8$
Error in M produced by an error ·001 in a or A . .	${\cdot}000063 \times 10^8$	${\cdot}000052 \times 10^8$	${\cdot}000036 \times 10^8$
Error in M produced by an error ·001 in b	$-{\cdot}000108 \times 10^8$	$-{\cdot}000078 \times 10^8$	$-{\cdot}000041 \times 10^8$

Thus the error produced by an error of ·001 centim. in b lies between ·005 and ·006 per cent., and the error in the measurement of b is certainly not more than ·001 inch or ·0025 centim.

Since the rods used to separate the coils were not exactly of the same length, the median planes cannot have been exactly parallel. The difference in the length of the rods is not as much as ·005 centim.

The radii of the coils are approximately 25 centims., and hence the angle between them is not as great as $\frac{{\cdot}005}{37{\cdot}5}$ or $\frac{1}{7500}$.

If we remember that when the coils are parallel the value of M is a maximum, so that the error due to the small angle between them depends on the square of the angle, it is clear that in our case this error is vanishingly small.

The galvanometer also was designed by Professor MAXWELL, and wound by Professor CHRYSTAL. It is also referred to in the article "Galvanometer," in the 'Encyclopædia Brittanica,' 9th edition. The description of it is taken from his account in the laboratory book, dated July, 1876.

There are two channels of rectangular section, and the following approximate dimensions:—

	inches
Depth of channel	$1\frac{1}{16}$
External diameter of bobbin	4
Breadth of channel	$0\frac{7}{8}$
Distance between channels	$0\frac{23}{32}$

Each channel contains 20 layers of thin copper wire and 16 layers of thick, making about 465 and 202 double turns respectively, so that there are 667 double turns in each channel, and about 2668 single turns on the galvanometer.

	inches.
Diameter of copper in thin wire	·014
Silk and all 82 thicknesses lie in	$1\frac{3}{8}$
Diameter of copper in thick wire	·029
Silk and all 34 thicknesses lie in	$1\frac{1}{6}$

The two thicknesses of wire were employed in order to fill the channels, and at the same time permit the resistance of the galvanometer to be reduced to the requisite amount. The ends of the wires are connected to binding screws on the bobbin marked A, B, &c., *a*, *b*, &c. A to *a* is one wire, B to *b* another. In our experiments the coils were connected up in series, the total resistance being about 60 ohms at a temperature of 13°·2 C.

The needle of the galvanometer was suspended from the WEBER suspension by three single cocoon fibres of 60 centims. in length.

The magnet was a small bar of hardened steel 1·5 centim. long, ·6 centim. broad, and ·12 centim. thick; its weight was ·708 grm. The magnet was attached by two small screws to a brass stirrup to which the mirror was fixed. A piece of brass wire 5·6 centims. long, with a screw thread cut on it, was fixed to this stirrup at right angles to the plane of the mirror, projecting equally on either side of the mirror. Two small brass cylinders could be screwed along this brass wire, and by means of them the moment of inertia and time of swing of the needle could be adjusted as required. The stirrup and mirror weighed 6·6 grms.

The galvanometer rested on a solid wooden base of about 18 centims. diameter, and this base was supported on three levelling screws.

A graduated circle is fixed to this stand, and the coils can be turned about a vertical axis, and their position read by means of a vernier. This was found useful in adjusting the coils parallel to the magnetic meridian. The galvanometer rested on a stone bracket built into the wall of the room. A scale placed approximately north and south at a distance of about 259 centims. from the magnet was reflected in the mirror and viewed through a telescope.

The scale rested on a solid wooden support on the floor of the room. The mirror, about 1·5 centim. square, was a specially good one, selected by a fortunate chance from among a number in the laboratory. The divisions of the scale were in millimetres, and after practice these could be subdivided by the eye with great accuracy to tenths. The scale itself was of paper; though this material is unsuitable for many purposes because of the changes produced by the weather in it, in our experiments these changes are of small consequence, for we require only the ratio of the throw produced by the induction current to the steady deflection produced by the permanent current; and the time which elapsed between the measurements was only a few minutes. Any shrinking or alteration of the scale will go on very approximately uniformly throughout its length and not alter the ratio of two lengths, which were never very unequal, as measured by the scale. After use the scale was carefully compared with the standard metre at the Cavendish Laboratory and the necessary correction applied to the readings.

The distance between the mirror and the scale only enters our result in the small correction necessary to reduce the scale readings so as to give the ratio of the sine of half the throw to the tangent of the deflection. It was unnecessary, therefore, to measure it with any great accuracy or to take steps to ensure its remaining the same from day to day; so long as it did not change during the half hour occupied by each experiment, all the conditions required by us were satisfied.

The resistance coils.

The standard coil R has been already referred to; the means adopted to measure its resistance will be described later.

Its value at a temperature of 14°·6 C. was found in May, 1881, to be 160·821 ohms.

The coil V used as a shunt to the galvanometer was made of thick German-silver wire. About 450 centims. of wire covered with silk were employed. The extremities of this were soldered to two stout copper rods with amalgamated ends, connexion with the rest of the apparatus being made by means of mercury cups; the ends of the rods were pressed down on to amalgamated pieces of copper at the bottom of the mercury cups.

The value of V was determined by repeated comparison at different temperatures with the B.A. unit known as "Flat coil" in Professor CHRYSTAL's report (Brit. Ass. Rep., 1876). The value of the Flat coil in mean B.A. units was taken from Professor FLEMING's recent comparison of the B.A. units at the laboratory. We have

Value of Flat coil at 15° C.	1·00003
Temperature of coefficient . . .	·00028 per 1° C
Value of V at 15° C.	1·0015
Temperature of coefficient . . .	·0003 per 1° C.

The coil itself is enclosed in a brass case and could be placed in a vessel of water. This was done during the experiment and the temperature noted by a thermometer graduated to fifths of a degree centigrade.

The coil S used with the galvanometer, when part of the battery current was sent through it, was a coil of platinum-silver wire of the ordinary form of about 3000 units resistance. It was immersed in the same vessel of water as V, and its temperature read by the same thermometer.

In our first series of observations the total value of the resistance M P Q G H, was observed and found to be 3072·38 ohms when the temperature of the coil S was 13·2.

The value of V at this temperature is 1·0011 ohms.

One extremity of the coil V dipped into the same mercury cup H as one extremity of the coil S, and the battery was also connected with this cup. The other extremity of V was connected by means of a piece of copper wire with L, the mercury cup in which the two portions of the battery current again united. This piece of copper wire was found to have a resistance of ·01556 ohm, so that the value of the resistance in the circuit H V L is 1·0167 ohms, at a temperature of 13°·2, and the currents in the two branches H V L and H G M L respectively, will be in the ratio of 3072·38 to 1·0167, so that if i be the battery current, that passing through the galvanometer will be $\frac{1\cdot0167}{3073\cdot39}i$.

T, as has been explained, was a variable resistance which could be adjusted so as to keep the difference between the resistance of the secondary circuit and the standard R sufficiently small to be measured in terms of part of the wire of the bridge E F.

During the experiments T had to be varied by somewhat over half a B.A. unit. Now T enters with S into the galvanometer circuit. The resistance, therefore, of this circuit was not quite the same during the observations, but varied by somewhat over ·25 ohm from its mean value, 3072·38 ohms.

The resistances W and U were two coils of about 30 ohms each wound on the same bobbin, and made of the same wire.

The galvanometer used with the Wheatstone's bridge was one of about 150 ohms resistance, made by Professor Stuart at the mechanical workshops, Cambridge.

Theory of the experiments.

Let $\overline{R}$ be the absolute resistance of the secondary circuit including the galvanometer, M the coefficient of mutual induction between the coils, and i the current in the primary circuit, then the total induced current produced by reversing the primary is $\frac{2Mi}{R}$.

Let β be the first throw of the galvanometer needle produced by this reversal, T the time of a complete vibration, λ the coefficient of damping, G the galvanometer constant, and H the horizontal intensity of the earth's magnetism.

Moreover let τ be the coefficient of torsion of the suspending fibre.

Then,

$$\frac{2Mi}{R}=\frac{H(1+\tau)}{G}\,\frac{T}{\pi}\left(1+\frac{\lambda}{2}\right)\sin\frac{\beta}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Again, let a current i' be passed through the same galvanometer directly afterwards, and let θ be the permanent deflection produced.

Then

$$i'=\frac{H(1+\tau)}{G}\tan\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence

$$R=\frac{2\pi M}{T\left(1+\frac{\lambda}{2}\right)}\cdot\frac{i}{i'}\cdot\frac{\tan\theta}{\sin\frac{\beta}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

But

$$\frac{i}{i'}=\frac{S+V}{V}$$

Therefore

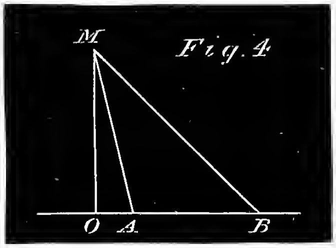
$$R=\frac{2\pi M}{T\left(1+\frac{\lambda}{2}\right)}\cdot\frac{S+V}{V}\cdot\frac{\tan\theta}{\sin\frac{\beta}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

T is, of course, the observed time of oscillation.

The correction for the finite amplitude of the swing is too small to produce any error in the result. No correction for damping or torsion is required.

In the experiments the deflections on the scale were measured, not the angles of deflection. We require, then, to reduce the scale readings to angular measure.

Let p and q be the scale values of the throw and deflection, z the distance of the point of the scale vertically below the axis of the telescope from the point which appears to coincide with the cross wire when the needle is at rest. z being measured in the same direction as p and q, let a be the distance of the scale from the mirror.



Thus in the figure (fig. 4) let M be the centre of the mirror, O the point on the scale vertically below the axis of the telescope, A the point of the scale which appears to coincide with the cross wire when the needle is at rest, and B the apparent extremity of the throw.

Now the scale was carefully set so as to be at right angles to O M, and the distance O A was always small compared with O B.

Also

$$OA=z,\ AB=p,\ AMB=2\beta,\ OM=a.$$

Hence if we neglect squares of $\frac{z}{a}$ and fourth powers of $\frac{p}{a}$ and $\frac{q}{a}$

$$\text{angle } AMO=\frac{z}{a}$$

and

$$\tan\left(2\beta+\frac{z}{a}\right)=\frac{p+z}{a}$$

Therefore

$$\sin\frac{\beta}{2}=\frac{p}{4a}\left\{1-\tfrac{1}{3}\tfrac{1}{2}\frac{p^2}{a^2}-\frac{pz}{a^2}\right\}$$

Also

$$\tan\left(2\theta+\frac{z}{a}\right)=\frac{q+z}{a}$$

hence

$$\tan\theta=\frac{q}{2a}\left\{1-\frac{q^2}{4a^2}-\frac{qz}{a^2}\right\}.$$

and

$$\frac{\tan\theta}{\sin\frac{\beta}{2}}=\frac{2q}{p}\left\{1+\frac{11p^2-8q^2}{32a^2}+\frac{(p-q)z}{a^2}\right\}$$

Now the values of p, q, z, and a were such that $\frac{(p-q)z}{a^2}$ was about ·0001, we may therefore write with sufficient accuracy

$$\frac{\tan\theta}{\sin\frac{\beta}{2}}=\frac{2q}{p}\left(1+\frac{11p^2-8q^2}{3^2}\right)$$

p and q being the scale values of the throw and deflection.

To observe these quantities accurately the following adjustments are necessary.

The scale should be parallel to the mirror when at rest.

The coils of the galvanometer should be north and south so that their plane may be parallel to the magnetic axis of the needle.

The telescope should be placed so that its axis and the normal to the mirror, when at rest, may be in the same vertical plane. If this is the case the division of the scale which appears to coincide with the vertical cross-wire will be that just below the axis of the telescope, supposing, as in our case, the telescope is placed so as to look just over the edge of the scale.

In making these adjustments the scale and telescope were first fixed so that division 250 —the middle of the scale—was vertically below the centre of the object-glass of the telescope ; the galvanometer was placed in position and levelled, and the telescope adjusted to view the image of the scale in the mirror.

Then telescope and scale were both moved until the division of the scale which coincided with the cross-wire was close to 250. When this was the case the normal to the mirror and the axis of the telescope were nearly in the same vertical plane.

The scale was then turned in a horizontal plane until its two ends, equidistant from division 250, were also equidistant from the mirror, taking care at the same time that the image of division 250, the centre of the scale, remained close to the cross-wire of the telescope.

The scale was thus put at right angles to the normal to the mirror.

A long bar magnet resting on a pivot at its centre was then supported close to the scale, and it was found that the scale was very nearly parallel to the axis of the magnet, the error was certainly not more than 20′.

Thus the scale has been set very approximately north and south, and since the mirror is very nearly parallel to the scale, it is also nearly parallel to the axis of the galvanometer needle. Hence, if we set the coils parallel to the scale or mirror they will be very nearly north and south, and their plane will be approximately parallel to the axis of the needle.

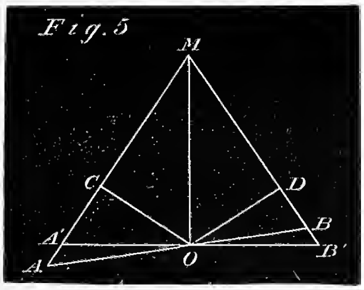
A piece of plate glass can be screwed on to the galvanometer in a position very nearly parallel to the coils. This was done, and the reflection of a lamp placed just below the telescope was observed, the galvanometer coils were turned until this reflected image was seen in the centre of the field of the telescope. Thus the galvanometer coils were placed very nearly north and south. The reading of the vernier attached to the galvanometer was noted, and by means of it the coils could readily be brought back to the same position, or placed at any required angle to the meridian.

The adjustments thus described were of course only approximate, but it is easy to show that the method of experiment eliminates any small outstanding error.

Let us suppose the coils are inclined at an angle α to the meridian. The effect of this is merely to change G into G $\cos\alpha$ in both the equations (1) and (2), and $\cos\alpha$ disappears from the resulting equation. It is better, however, that the field of force produced by the current in the galvanometer coil should be as nearly as possible uniform throughout the space through which the needle moves in the throw and deflection respectively. This condition is best satisfied if the needle when in equilibrium is parallel to the coils.

Let us now suppose that the scale is inclined at an angle γ to the plane of the mirror. Let p_1 and p_2 be the scale values of the throw to the right and left of the resting point.

Let A O B (fig. 5) be the scale, O M the normal to the mirror, draw A′ O B′ at right angles to O M.



Then A O A′$=\gamma$; let A and B be the observed extremities of the throw or deflection; θ its value in angular measure; let M A, M B, cut A′ B in A′ and B′, and let p be the true scale value of the deflection, then

$$\mathrm{A'O}=\mathrm{OB'}=p$$
$$\mathrm{AOM}=\mathrm{MOB}=2\theta$$

Draw O C, O D perpendicular to M A, M B respectively.

Then

$$\mathrm{OC}=\mathrm{OA'}\cos \mathrm{COA'}=\mathrm{OA}\cos \mathrm{COA}$$

Thus

$$p\cos 2\theta=p_1\cos(2\theta+\gamma)$$

Similarly

$$p\cos 2\theta=p_2(\cos 2\theta-\gamma)$$

Therefore

$$2p=(p_1+p_2)\cos\gamma-(p_1-p_2)\tan 2\theta\sin\gamma$$

Now p_1 was always very nearly equal to p_2, and 2θ and γ are both small. Hence very approximately indeed we have

$$p=\frac{p_1+p_2}{2}\cos\gamma$$

Similarly

$$q=\frac{q_1+q_2}{2}\cos\gamma$$

Thus, by observing deflections right and left and taking the mean, we get a value for the ratio of p/q which is independent of an error in the azimuth of the scale, much greater than anything possible in the actual experiments.

If the magnet be not parallel to the mirror the angle turned through by the mirror is still that turned through by the magnet; the fact that the magnet and mirror were very nearly parallel afforded a ready means of setting the plane of the coils in the magnetic meridian.

The time of a complete vibration was measured in the usual way by noting the times of 8 or 10 transits of the resting point over the cross wire of the telescope, then waiting for the period occupied by some 10 or 12 oscillations and again observing the times of 8 or 10 transits. The value thus determined requires reducing to that for an infinitely small arc.

Now we know that if during the observation the arc of oscillation change from α_1 to α_2, and if T′ be the observed time of oscillation, then

$$\mathrm{T}=\mathrm{T}'\left\{1-\frac{1}{8}\left(\sin^2\frac{\alpha_1}{4}+\sin^2\frac{\alpha_2}{4}\right)\right\}.$$

neglecting higher powers.

In the observations the value of α_1 was about 3°, that of α_2 about 1°·30′, the correction thus amounts to ·000025 and is quite inappreciable.

The value of λ was obtained by setting the needle vibrating, the secondary circuit being closed, and observing a series of resting points. If p_1, p_n be the amplitudes of the first and n^{th} vibration we have

$$\lambda=\frac{1}{n-1}\log_e\left(\frac{p_1}{p_n}\right)$$

Two independent observations of 17 vibrations gave

$$\frac{p_1}{p_{17}}=1{\cdot}2913 \text{ and } 1{\cdot}2909$$

Whence

$$\lambda={\cdot}0159$$

$\bar{\mathrm{R}}$ is the absolute resistance of the secondary circuit; this is very nearly but not quite equal to R, the resistance of our standard coil, and the difference between the two can be expressed in terms of the resistance of the wire of the B A bridge. This wire is 1 metre in length, and is divided into millimetres; let ρ be the resistance of 1 millim. The wire is graduated from E to F, (fig. 1); let G_1 be the position of the sliding contact piece when there is no current through the galvanometer, P P′, Q Q′, and M N being connected.

Let

$$\mathrm{EG}_1=x \text{ millim.} \qquad \mathrm{FG}_1=y \text{ millim.}$$

$\bar{\mathrm{R}}$ is the resistance of the circuit Q′ Q G B N M P P′.

Hence

$$\frac{\mathrm{R}+x\rho}{\mathrm{R}+y\rho}=\frac{\mathrm{W}}{\mathrm{U}}$$

Now interchange U and W, and let x' y' be the new values of x and y.

Thus

$$\frac{\overline{\mathrm{R}}+x'\rho}{\mathrm{R}+y'\rho}=\frac{\mathrm{U}}{\mathrm{W}}$$

But

$$x+y=x'+y'$$

Hence

$$\overline{\mathrm{R}}+x'\rho=\mathrm{R}+y\rho$$

$$\mathrm{R}=\overline{\mathrm{R}}+(x'-y)\rho$$

A number of experiments were made to determine ρ, and the value $\rho=\cdot000072$ B. A. unit was obtained.

Since ρ only comes in as a small correction, we may take one B. A. unit as 1 ohm.

Again, the value of R depends on the temperature, and our experiments required reducing to a constant temperature t_0; let t be the temperature of R at the time of experiment, R_0 the value of R at temperature t_0, and α the coefficient of increase of resistance per degree centigrade. Then we have

$$\mathrm{R}=\mathrm{R}_0\{1+\alpha(t-t_0)\}$$

Hence, finally our equation (4) becomes

$$\mathrm{R}_0\{1+\alpha(t-t_0)\}=\frac{2\pi\mathrm{M}}{\mathrm{T}\left(1+\frac{\lambda}{2}\right)}\cdot\times\frac{\mathrm{S}+\mathrm{V}}{\mathrm{V}}\times\frac{2(q_1+q_2)}{(p_1+p_2)}\left\{1+\frac{11p^2-8q^2}{32a^2}\right\}+(x'-y)\rho \quad . \quad . \quad (5)$$

The experiments were made in the following order :—

The time of swing was observed, the secondary circuit being closed as in the experiments.

The variable resistance in the secondary circuit was adjusted until the difference between $\overline{\mathrm{R}}$ and R could be measured in terms of the bridge-wire resistance, and the values of x y, x' y' determined. While this was being done a second observer read the temperatures of the coils R, S, and V, and the galvanometer G.

The connexion P P′, Q Q′ were broken, and P Q was joined. The resting point of the reflected image of the scale was observed when no current was passing through the galvanometer. This was done in the usual manner by observing five consecutive turning points.

The galvanometer needle was brought as nearly as possible to rest by the use of a damper. This consisted of a coil of wire placed near the needle, through which the current from a single LECLANCHÉ cell could be passed. By means of a second key a shunt could be introduced into this circuit so as to allow only a small fraction of the current from the battery to circulate in the coil. After a little practice the apparent oscillations of the galvanometer needle could easily be reduced to a few tenths of a millimetre of the scale.

When this had been accomplished, the battery current in the coil A was reversed and the first throw of the galvanometer needle observed. Suppose this was a throw to the right. By reversing the current at the right instant the needle could be brought very nearly to rest again, and the small swing that remained was easily destroyed by the damper. The connexions were then adjusted so that on again reversing the battery current a throw to the left was observed. A second throw to the left was observed and then a second throw to the right.

After this another reading was taken of the resting point when no current was passing.

The connexions were then altered so that a fraction of the direct current could be passed through the galvanometer, and the position of rest of the needle when the current was passing was observed. The needle was easily brought sufficiently nearly to rest in its new position by making the primary contact for a third of the time of swing, then breaking it for a second third, and finally making it again and leaving it made.

The resting point was determined while the needle was swinging from the observation of five consecutive turning points. The primary current was then reversed and a deflection in the other direction observed. After this a third reading of the resting point without any current was taken.

The connexions were again altered to observe the throws and four more were taken, one to the left, two to the right, and one to the left. A fourth observation of the resting point completed this part of the observations. The secondary circuit was then put into communication with the WHEATSTONE'S bridge, and the difference between R and $\bar{R}$ measured, giving a second series of values of x y, x' and y', and finally the thermometers were all read again.

When we had become accustomed to the work a complete set of observations, excluding the time of swing, took about 25 minutes.

In the first series of measurements the time of swing was only observed twice each afternoon—at the beginning and end of the afternoon's work. In the second series it was taken generally at the beginning of the work and after every second or third set of observations, that is to say, at intervals of somewhat less than an hour and-a-half.

Throughout the experiments one observer (R. T. G.) read the galvanometer deflections, while the other (J. M. D. in the first series, E. B. S. in the second) made or broke the various connexions as required, and noted down the scale readings as they were read out by the observer at the telescope.

To obtain from the direct results of the observations the quantities required for substitution in formula (5), the following method was adopted :—

The means of the temperatures at the beginning and end of the observation were taken as the temperatures at the time of the experiment, and the means of the values of $x\ y$, $x'\ y'$.

The alteration in temperature during the experiment was only about ·1° or ·2° C. The changes in the values of $x\ y$, &c., during the observation were produced by the variation of the temperature of the secondary coil, which being copper has a high temperature coefficient. The alteration in the value of x was rarely as much as 300 bridge divisions, and since this change went on nearly uniformly during the experiment, the mean of the two values at the beginning and end will be very accurately the true value.

In reducing the scale readings to give the throws and deflections, we had to remember that owing to the variation in the direction of the earth's magnetic force, there was a continual change going on in the resting point of the needle.

In general, this change was only ·3 or ·4 millim. during the time occupied by a set of observations, sometimes it amounted to 1 millim. or rather over, and on one or two occasions during magnetic storms the changes were so violent and sudden that we had to cease work entirely.

For determining the value of the throw the following method enables us to eliminate the effect of this change in zero.

Take the mean of the two throw readings to the right, then the mean of the two to the left, and the mean of the resting point readings, then the differences between the throw readings and the resting point, or zero readings, will give the throws right and left respectively, corrected for change in resting point. The difference between the throw readings will give the value of p_1+p_2 directly; since, however, the throws right and left ought to be the same if the adjustments are correct—it forms a test of the accuracy of the measurements to calculate p_1 and p_2 separately.

For the permanent deflection, however, in which only one observation was made on either side of the zero, the same method is not applicable. The four observations we have to consider are: zero reading, deflection to right, deflection to left, zero reading.

Suppose the zero is moving from right to left, then, if we take the mean of the zeros and consider the differences between it and the deflection reading as the deflections right and left, in each case our deflection will be too great, while, if the zero be moving in the other direction, the deflection obtained will be too small.

To obviate the difficulty we assumed that the interval of time between each two consecutive observations was the same, and that the change in zero was uniform. We then obtained by interpolation the values of the zero readings at the moments of making the deflection observations. The differences between these and the deflection readings gave then the true deflections right and left, q_1 and q_2 respectively, the whole correction being a very small fraction of the measured deflection.

The second series of throws were then treated in the same manner as the first, and a second pair of values of p_1 and p_2 obtained. These generally differed somewhat from the first, for the electromotive force of the battery—a combination of DANIELL'S cells

—was not absolutely constant throughout an experiment. Now the observations of deflection refer to a moment of time about half way between the two throws, so that the mean of the two values of p_1+p_2 will give us the value of that quantity corresponding to the value of q_1+q_2 obtained for the deflections.

For the first series of observations the difference in the values of p_1 before and after the deflection reading was sometimes, but not often, as great as 1 millim. in a throw of about 215 millims.

A copy of the observations requisite for one experiment will perhaps render the above details more complete.

OBSERVATION No. II., June 8th, 1881, 11·45 a.m. Observers, R. T. G., J. M. D.
Time of swing 23″·277.

Bridge reading value of x U, W direct 500
,, ,, x' U, W interchanged 410

Temperature.

R 13°·7 S 13°·7 Galv. 14°·5

Scale observations.

Zeros	235·5	
		227·5
	235	
		227·8
	235	

	Left.	Right.
Throws	7·0	
		455
		454·5
	7·8	

Zeros	225	
		238
	225·2	
		238
	225·2	

	Left.		Right.	
Deflections	87·2		364·8	
		94·5		377
	87·5		364·8	
		94·6		377
	87·8		365	

Zeros	237·8	
		224·2
	237·5	
		224·5
	237	

	Right.	Left.
Throws	454·2	
		6·0
		6·5
	454	

Zeros	234·5	
		227
	234·2	
		227
	234	

Temperatures.

R 13°·7 S 13°·8 Galv. 14° 8

Bridge reading value of x {Direct} 630
,, ,, x' {Interchanged} 540

Battery: four ordinary DANIELL's in series.

From these observations we obtain the following value for the zero and deflection readings :—

Zeros	231·4	231·5	230·8	230·6

	Left.	Right.
Deflections	91·0	370·9

Thus the mean zero for the first throw is 231·45, and combining this with the observations of the throw we get

$$p_1 = 454{\cdot}75 - 231{\cdot}45 = 223{\cdot}3$$
$$p_2 = 231{\cdot}45 - \quad 7{\cdot}4 \ = 224{\cdot}05$$

While for the second throw the zero is 230·7, and the values are

$$p_1 = 454{\cdot}1 - 230{\cdot}7 \ = 223{\cdot}4$$
$$p_2 = 230{\cdot}7 - \quad 6{\cdot}25 = 224{\cdot}45$$

Thus the mean value of p_1 for the experiment is

223·35

and of p_2

224·25

These numbers require correcting for scale error.

The correction to p_1 is

+1·2

that to p_2 is

+1·0

so that

$$p_1 = 224{\cdot}55 \text{ millims.}$$
$$p_2 = 225{\cdot}25 \text{ millims.}$$

The difference between them being only ·7 millim. it is clear that the adjustments are all right.

Before taking the deflections the zero reading was 231·5, after taking them it had become 230·8. Thus interpolating, the zero reading at the moment of the deflection to the left was 231·3, and we find

$$q_2 = 231{\cdot}3 - 91{\cdot}0 = 140{\cdot}3$$

while at the moment of the deflection to the right the zero reading was 231·1, and, hence,

$$q_1 = 370{\cdot}9 - 231{\cdot}1 = 139{\cdot}8$$

Correcting for scale error we have

$$q_1 = 140{\cdot}6 \text{ millims.}$$
$$q_2 = 141{\cdot}1 \text{ millims.}$$

We have also

Mean value of temperatures—

R 13°·65 S 13°·7 Galv. 14°·65

Mean value x . . . ·565
" x' . . . ·475

We proceed now to give in tabular form the results of our first series of experiments made in June, 1881.

The table will contain the values of $\frac{p_1+p_2}{2}$, $\frac{q_1+q_2}{2}$, the temperature of R, the mean values of x and x', and the time of swing. The corrections introduced by the variation in the temperature of S and the galvanometer we shall show, when we come to discuss the results, are so small that they may be neglected. We may, therefore, treat the results throughout as if the temperature of S were 13°·2, and that of the galvanometer 14°.

The battery used in all cases was four DANIELL'S cells.

The average difference between p_1 and p_2, irrespective of sign, was about ·32 millim., in one case only was it as great as ·9 millim.; it is hardly necessary, therefore, to give both p_1 and p_2 in the table.

The rods used to separate the two coils, primary and secondary, were the same, but, as has been explained, the coils were placed in each of four positions, numbered respectively I., II., III., and IV.

TABLE I.

Position.	Mean throw.	Mean deflection.	T.	Temperature R.	x.	x'.
I.	226·10	142·15	23″·277	15°·1	670	590
	226·25	142·40		15·2	583	500
	223·00	140·25		15·1	420	350
	222·70	139·70	23·250	15·2	540	470
	224·55	141·40	23·277	13·8	690	600
	224·90	140·85		13·7	440	355
II.	223·65	140·30		13·7	750	665
	222·80	140·20		13·9	505	420
	222·25	139·85		13·9	265	175
III.	218·00	137·05		14·2	770	670
	217·60	136·90		14·2	480	390
	217·50	136·70		14·3	325	240
IV.	216·05	135·85		14·3	778	680
	216·10	135·55		14·5	532	440
	215·85	135·50	23·274	14·5	400	310

The first four observations were made on June 7th, 1881, the last eleven on June 8th.

For the first four observations, therefore, we take the time of swing as 23″·264, for the last eleven as 23″·275.

The mean temperature of R during the experiments is about 14°·6; we take this then as the value of t_0. The temperature coefficient of R—a platinum-silver coil—may for the small range considered be taken as ·0003 per ohm per degree.

Thus the values of the constants in the formula (5) for R_0 are

$$t_0 = 14^\circ{\cdot}6 \qquad \alpha = {\cdot}0003$$
$$\tau = {\cdot}0007 \qquad \lambda = {\cdot}0159$$
$$a = 259 \text{ centims.}$$
$$\rho = {\cdot}000072 \text{ ohm.}$$
$$\frac{S+V}{V} = \frac{3073{\cdot}39}{1{\cdot}0167}$$

while giving p and q their mean values for the whole series of experiments, the term

$$\left\{1 + \frac{11p^2 - 8q^2}{32a^2}\right\} \text{ comes to } 1{\cdot}0018$$

As we have explained, the value of M is slightly different for each of the four positions.

The methods used to determine have been explained; the values obtained were

$$M_1 = 1{\cdot}55636$$
$$M_2 = 1{\cdot}55430$$
$$M_3 = 1{\cdot}55539$$
$$M_4 = 1{\cdot}55744$$

where the suffixes refer to the positions.

Table II. gives the results of the calculations. As the results stand there, the negative errors are fewer in number than the positive, the two greatest errors being negative.

The greatest of these is ·453, which is about 1 part in 350.

TABLE II.

Position.	R_0 in ohms.	Error from mean of set.	Mean value of R_0 for each position.	Error from mean of whole.	Percentage error.
I.	158·82	·147	158·673	·047	·029
	158·98	·307			
	158·84	·167			
	158·48	—·193			
	158·70	·037			
	158·22	—·453			
II.	158·27	—·297	158·567	—·059	—·039
	158·68	·103			
	158·76	·193			
III.	158·69	·093	158·597	—·029	—·018
	158·77	·173			
	158·33	—·267			
IV.	158·87	·241	158·629	·003	+·002
	158·51	—·119			
	158·51	—·119			

Mean value	158·626 ohms.
Mean error of mean of each from mean of whole . .	·037
Mean percentage error	·023

We shall retain the whole series of observations and take as the resistance of our standard coil R at a temperature of 14°·6 C., the value

$$158\cdot626\ \frac{\text{earth quadrants}}{\text{second}}$$

It remains now to explain the method used to determine the values of the resistances.

For this purpose the coils in a post office resistance box, made by Messrs. ELLIOTT Brothers, were compared with the standards at the Cavendish Laboratory. The 1-unit coil of the box was compared with the coil known as Flat in Professor CHRYSTAL'S report, then the 1-unit + Flat were balanced against the 2-unit coil of the box, then this 2-unit against the second 2-unit, which we will denote by 2′, then 1+2+2′ against the 5-unit coil, and so on.

In this manner all the coils between 1 and 2000 B.A. units were compared.

The British Association wire bridge was used in making the comparison.

In the ordinary use of this (CAREY-FOSTER'S method) the two coils to be compared are connected to the ends of the bridge wire and a measurement taken, the coils are then interchanged and another observation is taken, and from these two the difference between the coils is expressed directly as the resistance of a portion of the bridge wire. We, however, could not apply this method, for, calling P and Q the coils to be compared, since P and Q are coils in the same box, one end of P is always in electrical connexion with one end of Q. The following arrangement therefore was adopted :—

Two coils of known resistance were connected one to each end of the bridge wire, while P and Q formed the other arms of the bridge. The coils actually used were those marked F and G in CHRYSTAL'S report.

The sliding contact was adjusted till no current passed through the galvanometer, and its position noted.

Let $\alpha+x$ be the resistance of the portion of the wire connected with F, $\alpha-x$ of that connected with G, so that 2α is the whole resistance of the bridge wire. Let $1+\delta F$, $1+\delta G$ be the resistances of F and G at the temperature of the observation.

δF and δG are very small.

At 14°

$$\delta F=-\cdot00084 \text{ ohm}$$
$$\delta G=-\cdot00112 \text{ ,,}$$

Then we have

$$\frac{P}{Q}=\frac{1+\delta F+\alpha+x}{1+\delta G+\alpha-x}$$

Interchange F and G and let x' be the new value of x

$$\frac{P}{Q}=\frac{1+\delta G+\alpha+x'}{1+\delta F+\alpha-x'}$$

Whence $x+x'=\delta G-\delta F$.

Thus x and x' are exceedingly small, and if we neglect squares and higher powers of δF, δG, x, and x', we obtain

$$(1+\alpha)\frac{P-Q}{Q}=x-x'$$

Now $\alpha=\cdot036$ mean B.A. unit.

If then we know Q, the value of P can be found from the observations of x and x'.

Two series of observations were taken, one by R. T. G. the other by J. M. D., each observation in each series being the mean of 2 or 3.

The extreme difference between the two series was in no case more than 1 in 3000.

The box remained in the room for some time before taking the observations, and the temperature was supposed to be that of the room as indicated by a thermometer laid on the box. A small correction was made for the resistance of the copper rods which connected the box to the bridge and the plugs in the box. This was determined by one observer (R. T. G.) only, so that any error in it will affect both measurements equally. We shall show, however, shortly that it cannot affect the value of the B.A. unit as determined from our measurements. Having thus obtained the values of the resistances in the box in terms of the B.A. standard units, the value of R_0 in these units was determined by the ordinary method. We found thus the mean of several closely concordant measurements

$$R_0=160\cdot821 \text{ B.A. units}$$

the temperature being 14°·6 C.

The resistance S of our secondary circuit and galvanometer was determined in terms of the coils in the same box, and we found

$$S=3072\cdot38 \text{ B.A. units}$$

at a temperature of 13°·2 C.

Owing to the difficulty of determining the value of the resistance of the plugs in the box and the copper connecting pieces, either of these results may possibly have an error of about 1 in 1500. Now the value of S enters into the value of R_0 in absolute units and affects it in the same way.

To determine the value of the B.A. unit we require the ratio

$$\frac{R_0\dfrac{\text{earth quadrant}}{\text{second}}}{R_0 \text{ in B.A. units}}$$

Its value is

$$\frac{158\cdot626}{160\cdot821}\text{ ohms.}$$

Whence we get

$$1 \text{ B.A. unit}=\cdot98635 \text{ ohm}$$

But the error we have been considering affects in exactly the same manner the numerator and denominator of this ratio. If in consequence of it one is too great so also is the other in the same proportion. Thus the accuracy of our result is not impaired by the uncertainty of this correction. In fact, although to determine the resistance of our coil R in absolute measure we require to determine accurately a ratio of 1 to 3000, and this determination has generally been held to be one of the main objections to our method, yet to determine the value of the B.A. unit we have in addition to compare a ratio of 160 to 1. Thus, in fact, to determine the value of the B.A. unit the ratio to be compared is 160 to 3000, or about 1 to 19, and this is a much easier experiment to make.

Our 160 and 3000 have both been expressed in terms of the resistances of the box, and even though there may be some considerable error in the actual values of these resistances, the error in the ratio of any two of them is a quantity very small indeed.

In conclusion, we would refer to another objection which has been made to the method. Nearly the whole of the battery current is allowed to flow through the coil V, whose resistance is about 1 ohm; the effect of this must be to heat V and alter its resistance, thus producing error. We shall show that the error in our case is vanishingly small.

The electromotive force of the battery was at most about 5 volts, and the total resistance of the primary circuit was about 80 B.A. units. The coil V was of German-silver wire, about 450 centims. being used to make it; the wire thus was very thick, its radius being ·06 centim. The wire, silk covered, was loosely wound in a coil and enclosed in a brass case, which was immersed in water.

From these data we find that the amount of heat developed per minute in the coil will be ·055 unit.

If we suppose all this heat to be retained, the rise of temperature will be ·015° C. per 1′, and the increase of resistance ·0000045 B.A. unit, and this will be too small to affect our results. As a matter of fact, it is clearly impossible for all the heat to remain in the coil, and the correction is, *à fortiori*, too small to be considered.

During the second series of experiments a hole was bored in the brass case of the coil and a thermometer inserted. The thermometer agreed throughout in its readings with that in the water bath in which the coil was immersed.

Thus we conclude as the final result of this series of experiments that the value of the B.A. unit is ·98635 ohm.

The agreement between the individual experiments of the series is remarkable. They are, however, open to the objection that the conditions under which they were taken remained unaltered in some essential particulars. Thus the rods used to separate the primary and secondary coils were the same throughout, while the battery was also the same. It was decided, therefore, to make a second series of observations in which these quantities were varied. This was done during November and December, 1881. Mr. DODDS had left Cambridge, and his place was taken by Mr. E. B. SARGANT, of Trinity College.

PART II.

Profiting by our past experience, the arrangement of the apparatus was modified slightly.

In Part I., as has been explained, one electrode of the coil V dips into the mercury cup H, while the other is connected by means of a piece of copper wire with the cup L.

In Part II. the mercury cups and resistance coils were so placed that the second electrode of V dipped directly into the cup L; the piece of copper wire between the two, therefore, was dispensed with.

Again, in fig. 1 it will be seen that between P and M in the secondary circuit there is a variable resistance T, used to adjust the resistance of the circuit until it balanced R.

This variable resistance formed a part of one of the two paths open to the current when measuring the deflection, and, as we have said, renders the exact proportion into which the current was divided in each experiment uncertain to the amount of about 1 in 6000.

In fitting up the apparatus for Part II., T was placed between B and N. As before, the resistance of the secondary circuit could be adjusted, but that of the primary remained unaffected by alterations of T, which in this part formed no portion of it.

Three sets of rods were used to separate the primary and secondary coils, we shall call them A, B, and C, respectively. The rods A were those used in Part I.

With the rods A three different electromotive forces were used; the batteries employed being respectively four ordinary DANIELL'S, two ordinary DANIELL'S, and five THOMSON'S tray DANIELL'S.

In position B we had five THOMSON'S-DANIELL'S, and in position C five THOMSON'S-DANIELL'S and six THOMSON'S-DANIELL'S.

As before, the coils were placed in positions I., II., III., and IV., but the order of taking the observations was somewhat varied. In Part I. three observations were taken in each position without altering the coils; in Part II., however, after taking one observation in position I., one of the coils was reversed so as to bring them into position II., and an observation made; the other was then reversed, and so on, and after the four measurements had been taken the whole series was repeated. This method necessitated rather more handling of the coils than the other; it had, however, the advantage that each set of four observations was taken under more nearly similar conditions, while, in consequence of the more frequent setting of the coils, the error due to any one chance bad setting was reduced. The time of swing was observed more frequently, being taken twice and generally three times for each set of four. The times corresponding to the mean throw and deflection are given in the table, being obtained by interpolation from those actually observed.

Table III. gives the same observations for Part II. as are given in Table I. for Part I.

With reference to the table we must notice that the experiments were not always made in the order I., II., III., IV., and therefore the times of swing in the column headed T are not in order of magnitude. Thus, in the second set, the real order of the experiments was II., I., III., IV., and in this order the values of T increase uniformly by 0″·005. In the experiments with two Daniell's, experiment IV. was made on November 16th, experiment II. on November 18th, and experiments III. and I. on November 21st.

Table III.—Series A.

Battery.	Position.	Mean double throw.	Mean double deflection.	T.	Temperature R.	x.	x'.
4 Daniell's	I.	359·1	223·35	23″·384	12°·4	495	400
	II.	360·4	224·5	23·384	12·7	640	555
	III.	362·75	225·2	23·386	12·8	630	540
	IV.	361·9	225·3	23·388	13·1	615	525
4 Daniell's	I.	364·2	227·35	23·415	13·4	350	270
	II.	364·55	227·7	23·410	13·4	300	215
	III.	359·4	223·8	23·420	13·3	615	530
	IV.	352·45	220·2	23·425	13·0	565	455
5 Thomson's	I.	451·2	280·8	23·373	11·9	510	420
	II.	453·2	281·9	23·367	11·8	420	330
	III.	452·2	280·4	23·380	12·2	525	435
	IV.	450·9	280·8	23·378	12·1	515	430
2 Daniell's	I.	190·55	119·1	23·386	12·4	575	490
	II.	191·8	119·8	23·405	13·4	415	335
	III.	190·75	118·8	23·382	12·3	805	715
	IV.	191·3	119·6	23·391	13·4	570	490

Table III.—Series B.

Battery.	Position.	Mean double throw.	Mean double deflection.	T.	Temperature R.	x.	x'.
5 Thomson's	I.	374·5	287·95	23″·344	11°·6	630	545
	II.	373·25	286·5	23·355	11·9	895	800
	III.	375·1	288·3	23·322	10·8	415	335
	IV.	374·15	286·9	23·333	11·2	780	700
5 Thomson's	I.	370·9	285·6	23·406	12·6	575	485
	II.	372·0	286·3	23·405	12·5	465	385
	III.	369·15	285·15	23·407	13·0	480	380
	IV.	370·5	286·2	23·406	12·8	505	415

TABLE III.—Series C.

Battery.	Position.	Mean double throw.	Mean double deflection.	T.	Temperature R.	x.	x'.
5 THOMSON'S	I.	227·4	289·8	23″·379	12·4	560	420
	II.	229·6	292·4	23·382	12·5	655	575
	III.	228·0	290·7	23·384	12·8	705	620
	IV.	228·2	290·5	23·381	13·0	690	605
6 THOMSON'S	I.	252·9	321·9	23·389	13·6	555	485
	II.	253·1	323·4	23·391	13·3	545	475
	III.	254·7	323·9	23·393	13·2	445	340
	IV.	250·2	318·9	23·395	12·9	425	335
6 THOMSON'S	I.	243·25	310·95	23·381	13·4	950	860
	II.	242·8	309·8	23·374	12·7	680	590
	III.	240·75	307·1	23·392	14·0	915	830
	IV.	240·65	306·85	23·384	13·8	280	190

These direct experimental results require to be substituted in our formula in order that we may obtain the values of R_0. The coils were placed in the four positions I. to IV. in order to eliminate any small unknown error in the position of the mean plane.

Now our first series of experiments have been sufficient to show that this error must be exceedingly small, and the result obtained by taking the mean of the four will certainly eliminate it.

Instead, therefore, of giving the value of R_0 for each position, we shall only calculate the mean value for each set of experiments, using, of course, as our value of M, the mean of the values M_1, M_2, M_3, M_4.

The correction for damping is the same throughout the whole series of observations, the value of $1+\frac{11p^2-8q^2}{32a^2}$, of course, differs for the different currents used; it therefore is included in the table.

The value of M for each of the three series A, B, C, is also given in the table.

The value of λ found from a large series of closely concordant measurements was

$$\lambda = \cdot 01368$$

In these experiments the temperature was somewhat lower, generally, than in Part I., so that the mean value of the temperature of R was about 12° and $t_0=12°$.

The distance between the mirror and the scale was different, and we found

$$a = 218 \text{ centims.}$$

also

$$\alpha = \cdot 0003$$

$$\rho = \cdot 000072 \text{ B.A. unit}$$

α being the coefficient of increase of resistance of R per B.A. unit per degree, ρ the resistance of 1 millim. of the bridge.

The values of the resistances S and V were different from those in Part I., for the piece of copper wire which was included in the value of V in Part I. had been removed while the variable resistance T was no longer in circuit with S.

The values employed in the calculations are

$$V = 1{\cdot}00096 \text{ units.}$$
$$S = 3059{\cdot}89 \quad ,,$$

so that

$$\frac{V+S}{V} = \frac{3060{\cdot}891}{1{\cdot}00096}$$

The temperature of V and the 3000 ohms, which was in the same water bath with it, being 12°, that of the galvanometer 13°·5; this latter temperature being the mean of the temperatures of the galvanometer during the observations.

As before, we shall describe later the methods used to determine these values.

Table IV. gives the results of the calculations.

The mean value of R_0 deduced from it is

$$158{\cdot}386 \, \frac{\text{earth quadrant}}{\text{second}}$$

As we shall see afterwards, the value of R_0 in B.A. units is 160·520.

Thus

$$1 \text{ B.A. unit} = \frac{158{\cdot}386}{160{\cdot}520} \text{ ohms}$$
$$= {\cdot}986706 \text{ ohm.}$$

TABLE IV.

Series.	Battery.	M.	$\frac{11p^2-8q^2}{32a^2}$	R_0 in ohms.	Error in R_0 from mean of series.	Mean value of R_0 for series.	Error from mean.	Percentage error.
A	4 D.	$1{\cdot}55587 \times 10^8$	·00167	158·233	−·158	158·391	·005	·003
	4 D.		·00167	158·472	·081			
	5 TH.		·00263	158·296	−·095			
	2 D.		·00045	158·564	·173			
B	5 TH.	$1{\cdot}25758 \times 10^8$	·00136	158·171	−·099	158·270	−·116	−·072
	5 TH.		·00136	158·368	+·098			
C	5 TH.	${\cdot}762092 \times 10^8$	−·00019	158·397	−·061	158·458	·072	·045
	6 TH.		−·00015	158·301	−·157			
	6 TH.		−·00015	158·676	·218			

Mean of whole series 158·386 ohms.
Mean error for each series ·064.
Mean percentage error ·040.

Taking each series separately, the values we obtain are

Series	A	·98673.
,,	B	·98598.
,,	C	·98716.

The greatest difference is ·00118, or about ·12 per cent.

Again, if we refer to the Table IV., we cannot find any clear indication of an error depending on the length of the rods used to separate the coils. The values of R_0 in series B are, it is true, somewhat small; there are, however, two values in series A and one in series C which are smaller than one of those in series B.

Neither do we find any connexions between the differences and the electromotive force used. Arranging the values in order of electromotive force, we have

2 Daniell's	158·564	1 experiment.
4 ,, ...	158·352	Mean of 2.
5 Thomson's	158·209	,, 4.
6 ,, ...	158·488	,, 2.

We must notice, however, that only one experiment was made with the smallest electromotive force.

It will be instructive to arrange the results in order of magnitude, noting the series, the battery, and the date of each experiment.

We have

Date.	R_0.	Series.	Battery.	Error.	Percentage error.
Dec. 2	158·171	B	5 Thomson's .	−·215	−·135
Nov. 16	158·233	A	4 Daniell's . .	−·153	−·095
,, 21	158·296	A	4 ,, . .	−·090	−·056
,, 25	158·301	C	6 Thomson's .	−·085	−·053
Dec. 2	158·368	B	5 ,, . .	−·018	−·011
Nov. 23	158·397	C	5 ,, . .	·011	·007
,, 18	158·472	A	4 Daniell's . .	·086	·053
,, 16, 18, and 21 . .	158·564	A	2 ,, . .	·178	·111
,, 28	158·676	C	6 Thomson's .	·290	·181

An inspection is sufficient to show that there is no definite order in any column but the second with the fifth and sixth, which are consequences of it. The greatest difference between any two experiments is ·505, and this in 158 is rather less than 1 in 300, or about ·32 per cent.

Of the actual errors of each experiment from the mean, five are negative and four are positive; the mean error itself is only ·125; the mean percentage error is ·078.

The number of experiments made is too small for the calculation of the probable error to have any value, but the distribution of errors round the mean is satisfactory, and the mean percentage error is very small, if we consider the complicated nature of the observations and the variation in the important conditions.

We turn now to the measurement of the resistances of the coil used in Part II. and their comparison with the B.A. standards.

It will be remembered that in the comparison between the coil Flat of the B.A. units and the 1 ohm of the box used in Part I., the correction to be made for the copper pieces connecting the box to the wire bridge and the resistance of the plugs was thought to introduce some error, which, however, it was shown would not affect seriously the value of the B.A. unit. To reduce this error the 1-unit coil of the box was not compared directly with the B.A. standard. Lord Rayleigh had had wound two 5-unit coils and one 10-unit, which had been carefully compared by him with the B.A. units. The comparison was repeated by one of us (R. T. G.), and the differences between the two results were found to be so small that we could use either value with all the accuracy required.

Thus the values at 12° were

5 units . . .	4·99392	Lord Rayleigh.
	4·99376	R. T. G.
10 units . . .	9·98360	Lord Rayleigh.
	9·98393	R. T. G.

The second 5-unit coil was only measured by Lord Rayleigh and Professor Fleming; it belonged to the latter, and had been taken away by him before our comparison was made.

A third 5-unit, denoted afterwards by 5′, however, had been constructed for the laboratory to replace it, and its value was found to be 5·00890 at 12° C., while Professor Fleming's coil had a resistance of 5·02444. Lord Rayleigh's value of the 10 units was found by comparison with the 5 units + Fleming's 5 units in series; our value was obtained by comparison with $5+5'$. The close agreement between the two results is sufficient test of the accuracy of the comparisons.

In determining the values of the resistances of the boxes, we started from these 5 and 10-unit coils. Two boxes were used—one by Messrs. Elliott Brothers, No. 229, the other by Warden and Muirhead, No. 202. The 10-unit coils in each of these boxes were compared with our 10-unit standard, using the modification of Carey-Foster's method already employed to compare R and the resistance of the secondary circuit. Then 20 units in the box, made up in three different ways (viz.: by taking out (*a*) plug 20; (*b*) plugs 10 and 10′; (*c*) plugs 1, 2, 2, 5, and 10), was compared with the 10-unit and two 5-unit standards in series.

A large number of determinations were made both by R. T. G. and E. B. S. on different occasions. The various values obtained for the Elliott box, reduced to a temperature of 12°, are given below; the temperature is that recorded by a thermometer laid on, or on some occasions inside, the box. Each number is the mean of three or four measurements taken at the same time.

Plug out.	Value.	Observer.
10	9·9903	R. T. G.
	9·9902	R. T. G.
	9·9890	E. B. S.
10′	9·9887	R. T. G.
	9·9907	R. T. G.
	9·9908	E. B. S.
1+2+2+5	9·9870	E. B. S.
20	19·9772	R. T. G.
	19·9775	E. B. S.
10+10′	19·9769	R. T. G.
1–10	19·9756	R. T. G.

For the WARDEN and MUIRHEAD box the differences between two sets of experiments were quantities of the same order as here.

The two boxes were then placed at opposite ends of the bridge-wire, the other two arms of the WHEATSTONE's bridge being the pair of 30-ohm coils used in the previous part of the experiments.

The 50-unit plug was taken out of one box, and out of the other all the plugs from 1–20. The difference between these two nominal 50 units was thus obtained. Then plugs 1–20 were taken out of the first box, and the 50 units out of the second, and another difference obtained. In this manner the values of the coils 50, 100, 100′, and 200 were obtained.

After this the differences between the two boxes became too large to be measured in terms of the resistance of the bridge-wire, and recourse was had to the method employed in Part I., by which one coil in a box was compared with a combination of coils in the same box.

Two coils, each of about 5 ohms, were connected with the ends of the bridge-wire, while the coils to be compared, P and Q, formed the other arms of the bridge.

Let $5+\alpha$, $5+\beta$ be the resistances of the two 5-ohm coils, and let x, y, x', y' have the usual meanings.

Then

$$\frac{P}{Q}=\frac{5+\alpha+x}{5+\beta+y}$$

Intercharge the 5-ohm coils

$$\frac{P}{Q}=\frac{5+\beta+x'}{5+\alpha+y'}$$

Hence

$$\frac{P}{Q}=1+\frac{\alpha-\beta+x-y}{5}$$

$$=1-\frac{\alpha-\beta-x'+y'}{5}$$

neglecting $\left(\frac{x}{5}\right)^2$ and such terms.

Now the values of the 5 units being known, we know α and β, and we find at the temperature of the observation

$$\frac{\alpha-\beta}{5}=\cdot 0061$$

Hence

$$P-Q=Q\left\{\cdot 0061+\frac{x-y}{5}\right\}$$

and

$$P-Q=Q\left\{\frac{x'-y'}{5}-\cdot 0061\right\}$$

The values of x, y, &c., actually obtained were such as fully to justify the neglect of $\left(\frac{x}{5}\right)^2$ and such terms. By this means the values of the 500, 1000, 1000′, and 2000-unit coils in the ELLIOTT box were determined.

Having thus determined the values of the resistances of the box, they were used to determine that of R in the following manner.

R was connected with one end of the wire of the bridge, and the two boxes in multiple arc with the other, two 30-unit coils forming as before the third and fourth arms.

170 units were then taken out of the WARDEN box, and the ELLIOTT box adjusted until the difference between R and the total resistance of the compound circuit formed by the two boxes could be measured in terms of the bridge-wire. This was the case when 2920 units were out of the box. The actual value of these resistances at 12° is known from our table of resistances of the box. Making the correction for temperature and for the difference between R and the multiple arc resistance, we find

$$R_0=160\cdot 602 \text{ B.A. units.}$$

A second determination, in which 180 units were out of one box and 1490 in the other, gave

$$R_0=160\cdot 570 \text{ B.A. units.}$$

Each of these results is the mean of several experiments.

We may take, therefore, as the value of R_0 the mean

$$R_0=160\cdot 586 \text{ B.A. units.}$$

The value of S, the resistance of the circuit through which the fractional part of the current ran when the deflection of the galvanometer was observed, was determined by direct comparison with the ELLIOTT box.

The values

3060·71 B.A. units

and

3060·90 B.A. units

were obtained as the means of several determinations on two different occasions.

The mean is

$$S=3060{\cdot}80$$

The value of V was determined by comparison with the B.A. coil Flat, using the form of bridge devised by Professor FLEMING.

We found on four separate occasions the values

1·00096
1·00096
1·00095
1·00098

The temperature throughout at which these values are correct is 12°.

These methods involve the use of the resistance boxes, and, of course, there must be some uncertainty in the temperature of the coils in the interior of the box. Another series of determinations, therefore, was made of the values of R and S, in which the boxes were only used to determine the values of certain small corrections.

There is at the laboratory a coil of about 24 B.A. units—the standard used by Lord RAYLEIGH in his experiments on the value of the B.A. unit. This he had compared with the B.A. units and found that at 12° its value was 23·92820. He suggested that we should compare our standard of about 160 units directly with it, and this was done by one of us (R. T. G.).

I first determined *ab initio* the value of the 24 ohms in terms of the B.A. units. I found by a method to be afterwards described the three values

23·92807
23·92810

and

23·92850

The mean is

23·92822

which is practically identical with that found some months previously by Lord RAYLEIGH.

Now the ratio of 24 to 160 is between 1 : 6 and 1 : 7.

I therefore arranged a WHEATSTONE's bridge in the following manner:—

Two arms were the two standards I wished to compare; the coil R of about 160 units and Lord RAYLEIGH'S standard, which we will call X, of about 24.

The third arm was two single unit coils and a 5-unit arranged in series, and the fourth arm a single unit, the coil Flat.

The two single units were the B.A. units C and G of Professor CHRYSTAL'S report, their values and that of the 5-unit are accurately known in terms of the mean B.A. unit.

In all cases the electrodes of the coils were well amalgamated and rested securely on the copper discs at the bottoms of mercury cups.

The electrodes of the ELLIOTT box were connected with the two ends of the 7-unit arm, so that the box formed a shunt, and by altering the plugs in the box the effective resistance of the arm could be finely adjusted, and the ratio of the resistance of the Flat coil to that of this arm made equal to that of R to X.

Thus I found that with 164 units out of the box there was a deflection of the galvanometer of $+33·2$ scale divisions, while with 163 out the deflection was $-5·6$.

Thus the true value of the shunt is 163·143. Correcting this to the proper temperature we find that the effective resistance of this third arm is 6·70438 units, that of the Flat coil at the same temperature being ·99944 unit.

Now the temperature of R at the time of observation was 13°·3, that of X being 13°·4.

Hence

$$\frac{R_{13·3}}{X_{13·4}}=\frac{670438}{99944}$$

Whence substituting the value of X and reducing to the standard temperature 12°

$$R_0=160·523 \text{ B.A. units.}$$

A second experiment was made at a different temperature, and instead of noting the deflections of the galvanometer produced by altering the box by 1 unit a second shunt was introduced and varied until the deflection was zero: the value of this shunt was 30,000 units.

From this experiment I found

$$R_0=160·518 \text{ B.A. units.}$$

We take as the true value of R_0 deduced from these two experiments

$$R_0=160·520 \text{ B.A. units.}$$

We have now to compare directly the values of S_0 and R_0. S_0 is about 3060 units, so that the ratio of S_0 to R_0 is between 19 to 1 and 20 to 1.

Four sets of coils were, therefore, arranged for a WHEATSTONE'S bridge, two arms of which were S and R, the other two arms being 20 units—made up of the two 5 and 10-unit standards already described, while the fourth was the single coil G.

The box was used as a shunt to the 20-unit arm, and adjusted as before till the galvanometer showed no deflection, the effective resistance of this arm was found to be 19·0444 units, while at the same temperature the value of G is ·99887 unit.

Substituting the value of R at the temperature of the observation and reducing our result to the temperature 12°, we find

$$S_0 = 3059\cdot37 \text{ units}$$

A second series of observations on a different occasion gave

$$S_0 = 3059\cdot86 \text{ B.A. units}$$

The mean is

$$S_0 = 3059\cdot62 \text{ B.A. units}$$

and this is the value we have used in our calculations.*

The coils were in all cases placed in water baths and allowed to stay for some hours in them. Before making the observations the temperature was read by thermometers graduated to fifths of a degree, which were compared with each other.

To determine then the value of the B.A. unit we must use the value of $R_0 = 160\cdot520$, found in this series of measurements. Now the values of S_0 and R_0 found from the boxes were respectively

$$3060\cdot80 \text{ and } 160\cdot586$$

These differ from the values we have obtained in our last observations by 1·18 and ·066 unit respectively, or rather more than 1 in 3000. This difference would correspond to an error of about 1° in the measurement of the temperature.

Considering then the uncertainty which must attach to the temperature of the coils inside the box, it seemed best to take our last values rather than the mean of the two —we would rather regard the first series as a check upon any large error. But though this difference in the value of S will affect to the amount of 1 in 3000 the value of R_0 in absolute units, it does not affect at all sensibly the value of the B.A. unit, for this latter depends on the ratio of S_0/R_0; taking the values of S_0 and R_0 from the boxes, we have

$$\frac{S_0}{R_0} = 19\cdot0602$$

while, if we use the last values obtained for S_0 and R_0, we get

$$\frac{S_0}{R_0} = 19\cdot0607$$

the difference is only about 1 in 40,000 and does not concern us. We shall therefore put $R_0 = 160\cdot520$ ohms, the temperature being 12°.

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* The value 3059·89 used on page 255 is obtained from this by applying a temperature correction to the 60 units, the resistance of the galvanometer, which was at 13°·5.

The contrivance designed by Lord RAYLEIGH by means of which our 24-unit standard was compared with the B.A. units needs a special reference: it has been described by Lord RAYLEIGH in his second paper on the value of the B.A. unit, an abstract of which was read before the Royal Society on March 9, 1882, while the paper is published in the Phil. Trans., Part I., 1882.

Five coils each of approximately 5 ohms resistance were wound and enclosed in a box, from which the two electrodes, copper rods with amalgamated ends, of each coil protrude. By means of two series of mercury cups this system could be put either in series or in multiple arc. Then, if each coil of the series is so nearly equal to 5 units that we may neglect the square of the difference, it is easy to show that the resistance of the system in series is exactly 25 times that which it has when in multiple arc. The coil of 24 units and a single unit were arranged in series so that they could readily be put into connexion with FLEMING'S bridge. The set of five 5-unit coils in multiple arc was compared with a single unit. The connexions were rapidly altered, and the five 5 units in series were compared with the 24+1; then again adjusting the connexions, another comparison between the 25 units in multiple arc and the single was made.

In this manner a value of the 24 ohms was obtained in terms of the single ohm, the result of the comparison being as already stated.

Thus, whether we use as the values of R_0 and S_0 those found from comparison with the box or those determined by comparison with the coils, we have as values of the B.A. unit determined from this second part

Series A	·98673	mean of 4
,, B	·98598	,, 2
,, C	·98716	,, 3

While the mean result is

·986706 ohm.

The result obtained in June, 1881, Part I., as the mean of three complete sets, was

·986350 ohm.

Our discussion has shown us that the possible errors of this determination are considerably greater than that obtained in Part II. We will, therefore, give to each experiment in Part I. only half the weight of an experiment in Part II., and obtain thus as our final value for the ohm

·98665 ohm.

The value obtained by Lord RAYLEIGH in his second experiments with the rotating coil (Phil. Trans., Part I., 1882) is

·98651* ohm.

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* Since this paper was read Lord RAYLEIGH has obtained by two modifications of LORENZ' method the two values ·9867 and ·9868.

The difference, amounting as it does to about 14 parts in 100,000, is obviously less than the probable error of our result, and there can be little doubt but that the value ·9866 is only two or three parts in 10,000 from the truth. This value, as Lord RAYLEIGH has pointed out, is strikingly confirmed by JOULE's latest determinations of the mechanical equivalent of heat.

In conclusion, we would discuss further one or two possible sources of error. We have assumed that the ratio of V/S is independent of the temperature. This is not true, for S consists of two parts; one part of about 3000 units has the same temperature coefficient and is always at the same temperature as V, so that for it our assumption is justified; the other part of the galvanometer, of about 60 units, is of copper, of which the temperature coefficient is ·003, and it is not at the same temperature as V.

Now the whole range in the temperature of the galvanometer is about 3°, the range on either side of the mean 1°·5, and an error of 1°·5 in the temperature of the galvanometer we can show would affect R to the amount of about one part in 8000. Thus those values of R for which the temperature of the galvanometer differs most from the mean may be affected to the amount of 1 in 8000 by this source of error. Taking the mean of all our observations, however, the error vanishes.

Professor CHRYSTAL had warned us that he had had some difficulty in securing sufficiently good insulation between the wire of the coils A and B and the brass rings in which they were wound. We therefore tested both coils to see that this was maintained. One pole of a battery of 25 LECLANCHÉ cells was connected with the brass ring, while the other pole was put in contact with one electrode of a galvanometer of 2000 ohms resistance, the second electrode of the galvanometer being connected with the wire of the coil.

Deflections of 60 and 80 divisions of the scale were observed for the two coils A and B respectively. The same battery through 100,000 ohms when the galvanometer is shunted with 8 ohms gave a deflection of over 100 divisions, the insulation resistance therefore is considerably over

$$\frac{100{,}000 \times 2000}{8}$$

or

$$25{,}000{,}000 \text{ ohms.}$$

The error that might arise from the use of a paper scale has been discussed. In the calculations corrections have been applied to the scale readings to reduce them to the standard metre. The corrections taken were the mean of four series of observations—two by R. T. G., two by E. B. S.; and these, though made at very different times, varied only by quantities comparable with the error of an observation.

The most serious objection, however, that can be raised applies to all observations in which a ballistic galvanometer is used, and there were two points here which seemed to require special notice. Is it right to assume that on reversing the primary current the change that takes place is over in a time which is small compared with the time of swing?

Now our reversals were made with an ordinary rocking commutator dipping into mercury cups, and only occupied a small fraction of a second. We therefore made a series of experiments to see if we could find the effect produced by holding the primary circuit open for definite small periods.

On reversing in the usual rapid manner we obtained a throw which varied between 119·5 and 119·3 millims.

When our primary circuit was held open for one second the throws were 118·9, 119·3, 119·3, and 119·1, while if the contact was broken for two seconds the effect on the throw was marked. Thus if by chance in any experiment the primary circuit was open for as much as a second, so that the battery current took something over a second to get steady, it would only produce an effect of about 1 in 1000 in the result; we feel quite certain that in no case the period of break occupied more than a small fraction, from a quarter to one-sixth, of a second.

Again, it might happen that the somewhat powerful induced current passing through the galvanometer coils might alter—temporarily or permanently—the magnetic moment of our needle.

A permanent alteration would of course be indicated by variations both in the throw produced by the induction current and in the time of swing. In the actual experiments after the induction current had been passed in one direction through the coils of the galvanometer, and the throw observed, it was passed in the other direction to stop the vibrations of the needle, and it was possible that each current might have produced real permanent changes in the magnetic moment, but of exactly equal amount, so that we had observed no appreciable alterations in the time of swing which we could assign to this cause.

We therefore took a series of measurements of throws in which the current was only allowed to pass in one direction through the coils, the vibrations of the needle being stopped by means of an external damper; the times of swing also were observed at intervals.

The table below gives the result.

Time of swing.	Throw.
	115·8
23·383	
	116·0
	116·0
	115·7
	116·0
	115·9
23·389	

SECOND series.—Throws in opposite direction.

Time of swing.	Throw.
	115·5
23·389	
	115·6
	116·2
	115·8
23·392	

The throws throughout are very nearly equal; the time of swing, it is true, has increased by ·009″, but this change cannot be due to an alteration in the magnetic moment of the needle, for if the current passed in the one direction had decreased the magnetic moment and so increased the time of swing, that passed in the other direction in the second series would have increased the magnetic moment and so decreased the time of swing. The alteration in T then we must rather ascribe to a variation in the value of the horizontal intensity. This has been allowed for in the experiments by the repeated observations which were made of T. As regards the temporary effect of the induced magnetism produced by the current while it lasts, we notice that its direction would at each instant be perpendicular to the plane of the coils, so that the force between the magnet thus formed by the current and the current forming it would be along the axis of the temporary magnet and have no tendency to cause it to move. In fact, the field of force produced by the coils being uniform, the actual force on the induced magnetism will be zero.

A number of observations were also made to see if the time taken by the battery current to become steady after a reversal was appreciable. The galvanometer circuit being open the battery circuit was reversed, and then the galvanometer circuit closed at an interval of from ·25″ to ·3″ after.

If after this interval the battery current had still been varying an induction current would have shown itself in the galvanometer, but though the reversals were made in three separate experiments, 13, 7, and 10 times respectively, in time with the period of the galvanometer, no effect was produced.

We are sure, therefore, that the battery current has attained its steady value in less than ·3″ after reversal, while our former experiments have shown us that if the period of change were as great as 1″ the error produced in the throw would be less than 1 in 1000.

The direct effects of the connecting wires and commutators on the galvanometer during the experiments were carefully tested for but no result could be found. Copper wire insulated with gutta-percha was used for the connexions, and the wires in each circuit were carefully twisted up together.

Most of the apparatus employed belongs to the Cavendish Laboratory, where the experiments were conducted. Our best thanks are due to Lord RAYLEIGH for his unceasing kindness and his many valuable suggestions which have helped us satisfactorily to surmount several serious difficulties, and have added greatly to the accuracy of the results.

NOTE.

(Added December 2, 1882.)

Another source of error should have been mentioned in the previous discussion.

The external diameter of each layer was measured necessarily before the layers external to it were wound on. There will probably be a tendency in each layer to compress those below it, and thus to make the mean radius of the coil somewhat less than we have assumed.

In fact, Lord RAYLEIGH found in his experiments (Phil. Trans., Part I., 1882) that the value of the mean radius obtained from measurements made as the coil was unwound was less by 1 in 2000 than that obtained from the measurements made while winding.

Several reasons lead us to believe that any effect of the kind would be much less with our coils.

The error is produced probably by the silk being pressed over the top and bottom of the layer into the intervals between the wires—the wires being circular in section there is necessarily a space between them.

Now the amount of this free space would be proportional to the area of the cross section of the wire, while the amount of silk would depend on the circumference of this cross section.

The diameter of our wire was only about two-thirds of that used by Lord RAYLEIGH, there would, therefore, be less space in proportion to the amount of silk into which the silk could be squeezed, and the error produced would be less.

Again, if this yielding is going on, it is clear that the average thickness of a layer should appear to decrease as we get near the outside surface. A reference to the measurements shows that this is not the case. On dividing the whole series of layers into sets of five, and taking the average for each set, omitting a layer in which paraffin paper had been used, we find the values for the average thickness ·128, ·123, ·127, ·128, and ·127.

No gradual decrease is here observable. We therefore feel confident that the error was much less than in the coils wound by Lord RAYLEIGH.

A strong indirect confirmation of the accuracy of the value of the mean radius used is afforded by Lord RAYLEIGH's recent measurements by LORENZ' method.

The same coils were used as in our observations, and Lord RAYLEIGH has shown that an error in the mean radius would affect the two methods by about the same amount but in opposite directions. If δM be the error in M produced by an error δa in a the mean radius, it follows that in our case $\frac{\delta M}{M}$ is about $2\frac{\delta a}{a}$, while in LORENZ' method, M being the induction between the disc and the coils, for the arrangement adopted $\frac{\delta M}{M} = -2\frac{\delta a}{a}$ approximately—in the one case δM is of the same sign as δa, in the other the signs are opposite. The mean of the results then obtained from the two methods will be free from an error in a. But the two results are ·98665 and ·9867. They are identical to the fourth figure. Thus we infer that the error in a is probably very small.

V. *On the Normal Paraffins.*—Part IV.

By C. SCHORLEMMER, *F.R.S., and* T. E. THORPE, *F.R.S.*

Received December 5,—Read December 21, 1882.

IN the last communication made by one of us on this subject (Phil. Trans., 1880, p. 451), it was stated that we contemplated making a joint investigation on the heptane from *Pinus Sabiniana.* A large quantity of the pure heptane was treated with chlorine in the manner described in this series of communications (Phil. Trans., 1872, p. 111), and the chlorides boiling between 143° and 157°·5 were converted into the alcohols in the ordinary way. The primary alcohol boiled between 165°—170°, whilst the secondary alcohol distilled over, for the most part, between 156°—158°. The alcohols were then oxidised in the manner described in Phil. Trans., 1872, p. 121. The ketone obtained from the secondary alcohol was further oxidised by being heated with the chromic acid solution in sealed tubes at 100°. The liquid was then neutralised and the unattacked ketone was separated by distillation. In order to isolate the acids contained in the residue small quantities of sulphuric acid were added, and the acid which separated out (pentoic acid) was distilled off in a current of steam—the operation being repeated until the pentoic acid was no longer recognisable by its smell. The acetic acid contained in the residue was then separated by a further addition of sulphuric acid and distilled off by direct heating. The respective silver salts of the two acids were then prepared from the several fractions in the ordinary way. Analysis showed that the separation of the acids was complete.

I. SILVER salt from the normal alcohol.

(*a*) 0·4035 grm. salt gave 0·1842 grm. Ag=45·6 per cent.
(*b*) 0·210 „ „ 0·0953 „ =45·4 „

Calculated for silver heptoate 45·5 per cent.

II. SILVER salts from ketone from secondary alcohol.

0·2022 grm. salt A gave 0·1050 grm. Ag=51·9 per cent.
0·1491 „ „ 0·0771 „ =51·7 „

Silver pentoate =51·6 Ag.

0·235 grm. salt B. gave 0·1518 grm. Ag=64·59 per cent.

Silver acetate =64·67 per cent.

The results of the oxidation show therefore that primary heptyl alcohol and methyl-pentyl-carbinol had been formed, as in the case of heptane from petroleum.

Another portion of the mixture of the monochlorides was then heated to 100° with alcoholic solution of potash, whereby in addition to heptylene a mixture of the ethyl heptyl ethers was obtained. The heptylene, purified by repeated rectification over sodium, boiled constantly at 98°·5. It was placed in contact with an excess of fuming hydrochloric acid in the dark and in a well-closed bottle for six weeks. Heptylene from petroleum heptane was similarly treated.

Now it is remarkable that whilst hexylene from mannite combines completely with hydrochloric acid under these circumstances forming secondary hexyl chloride (Phil. Trans., 1880, p. 457), the greater part of the heptylene from *Pinus* heptane was found to be unattacked, not more than 10 per cent. of heptyl chloride having been formed. On the other hand, about one-half of the heptylene from petroleum had been converted into the chloride in accordance with the former observations made by one of us on this point.

The uncombined portions of both specimens of heptylene were again placed in contact with the fuming acid, and (the research being interrupted by other work) they were allowed so to remain for many months. At the expiration of this time it was found that the *Pinus* heptylene had united almost completely with the acid, whilst an additional quantity of the heptylene from petroleum had likewise entered into combination. It follows from this that, contrary to expectation, hydrochloric acid acting in the cold is not capable of effecting the separation of isomeric paraffins (Phil. Trans., 1880, p. 451).

It is remarkable that the *Pinus* heptylene should require so long a time to bring about its union with hydrochloric acid, since, as will be shown immediately, its constitution is exactly analogous to that of the hexylene from mannite or propyl-methyl ethylene, $C_3H_7.CH{=}CH.CH_3$, which so easily goes into combination.

In order to establish the constitution of the heptylene from *Pinus* it was oxidised by means of a solution of potassium dichromate in dilute sulphuric acid in the manner adopted by HECHT in the oxidation of hexylene from mannite (Ber. Deutsch. Chem. Ges., Bd. xi., S. 1152). The acids so formed were converted into the silver salts and analysed with the following results:—

Fraction.	Salt taken.	Silver found.	Silver pentoate 51·67.
1	0·0855	0·0443	51·81
2	0·0560	0·02875	51·34
3	0·1140	0·0590	51·75
4	0·1350	0·0700	51·84
5	0·0685	0·0350	51·09
6	0·1000	0·0540	54·00
7	0·2200	0·1370	62·27
8	0·1370	0·0855	64·60
9	0·1400	0·0910	65·00
			Silver acetate 64·67.

The first five fractions consisted of a fine crystalline powder, the last three were glistening needles.

These observations clearly indicate that only pentoic and acetic acids had been formed by the oxidation, and therefore that the *Pinus* heptylene is pure butyl-methyl-ethylene, $C_4H_9.CH{=}CH.CH_3$, which had been formed from the secondary heptyl chloride, $C_4H_9{-}CH_2{-}CHCl.CH_3$.

These results, taken in conjunction with those of former investigations, leave no doubt, therefore, that by the action of chlorine upon a normal paraffin not all the chlorides indicated by theory are formed, but only the primary and a secondary chloride which contains the group $-CHCl.CH_3$. One of us has formerly shown that by the action of bromine upon normal paraffins from petroleum only secondary bromides corresponding to chlorides are formed (Phil. Trans., 1878, Part I., p. 49). VENABLE has since shown that *Pinus* heptane is acted upon in the same way (Ber. Deutsch. Chem. Ges., Bd. xiii., S. 1649).

VI. *On a Collection of Rock Specimens from the Island of Socotra.*

By T. G. Bonney, *M.A., F.R.S., F.G.S., Professor of Geology in University College, London, and Fellow of St. John's College, Cambridge.*

Received June 12,—Read June 15, 1882.

[Plates 6–7.]

The specimens of rocks—about 500 in number—collected by Professor Bayley Balfour during his late exploration of the Island of Socotra, were forwarded to me for examination. Several of these specimens, as was to be expected under the circumstances, were in a condition unfavourable for precise determination, being often fragments from weathered surfaces and sometimes much decomposed. Each, however, has been described as far as the circumstances would admit, and microscopic slides have been prepared for me by Mr. Cuttell from about 80 of the more interesting.

As it happens, certain of these offer difficulties which in the present state of our knowledge are almost insuperable. While the use of the microscope has dispelled much confusion in our petrological ideas and supplied us in many respects with a firm basis of knowledge, it has not in every case—owing to the novelty of this mode of research and the inherent difficulties—enabled the student to feel perfect confidence in some of his conclusions, especially when he is restricted to this method of examination. Perhaps the greatest of our petrological difficulties is the distinguishing in every case between certain highly metamorphosed rocks and those of similar chemical composition which are truly igneous. It has, indeed, been maintained by some eminent geologists that certain sedimentary materials may be so altered by the combined action of water, heat, and pressure as to be converted *in situ* into a rock indistinguishable from one of those commonly held to be of igneous origin. Accordingly we read not seldom of "metamorphic granite" and of "gneiss passing into granite," for it is of these that the above opinion is commonly held. Other geologists, indeed, go yet further and make a similar assertion, not only of the more coarsely crystalline rocks such as syenite, diorite, and gabbro, but even of the more compact varieties of felstone and greenstone, which in like manner are said to afford indubitable examples of transition into beds of true sedimentary origin. As regards this view, we may admit that if a mass of clastic materials be once reduced to a molten magma its past history is obliterated; and, further, that there is no reason, so far as we know, why this melting down should not occur. In this sense, any igneous rock whatever may possibly deserve the name of metamorphic. But, while admitting the *à priori* possibility of such a change, I must confess to being sceptical as to whether any case of it has yet been fully proved to exist. I have studied not a few of the alleged instances patiently and, as I trust, without prejudice, always with the following results—either that there was no evidence which was conclusive on either side, or that to a practised eye there was very clear evidence against the asserted transition—*i.e.*, that (to refer to the instance named above) either the so-called granite was clearly only a granitoid gneiss or else that it was distinctly intrusive in the schistose rock. In these investigations the microscope is a very great help, but I freely admit that there are many cases where we cannot rely upon it alone, and must also study the rock in the field. Our knowledge at present does not enable us to pronounce upon the classificatory value of certain structures which we observe in the microscopic study of some specimens. This difficulty, however, is one which time and experience will probably remove. To admit the existence of cases where it is at present safer to suspend the judgment in no way concedes that it is impossible ultimately to arrive at a conclusion. So then, while not professing in every case from examination of hand specimens alone to decide whether a rock is a granitoid gneiss or a true granite, I believe in the distinctness of the two rocks. As it happens, some of the specimens from Socotra belong to this dubious class; and I cannot say positively, even after microscopic examination, whether in certain cases we have a granitoid gneiss or a true granite, and in others a hornblendic rock of sedimentary origin or a true diorite.

In drawing up my report on Professor BALFOUR's collection I have thought it best to give, first, a general description of the petrology of those parts of the island over which he travelled, and then to describe the more important varieties of each group of rocks with which his journey has furnished us.*

The island of Socotra is about 72 miles long from east to west and about 22 miles in breadth. The general physical features are thus described by Professor BALFOUR in his account of the island printed in the volume of reports of the British Association for the year 1881:—

"The surface features of Socotra at the present time are those of an island mountainous in the extreme. The shore line on its southern aspect is, as the map shows, a tolerably continuous one, unbroken by deep inlets or bays. On the northern side occur a few shallow bays at the mouths of the streams, which afford the only anchorage to be obtained around the island, but no one of them is safe at all seasons of the year. On all sides the hills rise with considerable abruptness over a wide area, forming bold perpendicular cliffs of several hundred feet in height, whose base is washed by the waters of the Indian Ocean, but at other places leaving plains varying in breadth up to as much as five miles between their base and the shore. On the south side of the island is the largest of these shore plains (Nogad), which, extending

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* In writing the first draft of this paper I followed the other plan, and described the specimens as they were collected by him during each section (generally representing a day) of his journey; but as I found this involve prolixity and needless repetition, I have recast the paper into its present form.

nearly the whole length of the island, is for miles covered with dunes of blown sand. On the north these plains occur chiefly at the mouths of the streams, and are the sites of the only places which may be called towns.

"The internal hilly part of the island may be roughly and shortly described as a wide undulating and intersected limestone plateau of an altitude averaging 1,000 feet, which flanks on the west, south, and east a nucleus of granitic peaks approaching 4,000 feet high. The whole of this hilly region is deeply cut into by ravines and valleys. These in the rainy season are occupied by roaring torrents, but the majority of them remain empty during the dry season. There are, however, many perennial streams on the island, especially in the central granitic region, where amongst the hills the most charming bubbling streams, dashing over boulders in a series of cascades or purling gently over a pebbly shingle, make it hard to believe that one is in such proximity to the desert region of Arabia. Few of the perennial streams reach the shore in the dry season—most of them are fiumaras.

"The eastern end of the island is most destitute of water. Here in the dry season are no rivers, and, springs being rare, it is the most arid region.

"Igneous, metamorphic, and sedimentary rocks compose the island. The fundamental rock is granitic. This crops out, as I have mentioned, towards the middle of the island, forming a series of bare pinnacles and crags, projecting, with singularly fantastic look, from the plateau below. This rock also shows on the slopes of the valleys and ravines below the compact limestone which caps it and forms the surface rock of the hill plateaux. This limestone attains in places, as seen on the cliff faces, a thickness of two or three hundred feet. Superficially, over wide areas it is rotted and broken into a jagged surface, over which progression is by no means easy, while at other spots it forms broad, smooth slabs. A shaly rock and coarse-textured purple sandstone, in beds dipping at all angles, crop out in the valleys and on the shore, whilst forming the shore-plains and the bases of the valleys is a recent breccia and conglomerate. Cutting through all these rocks, and altering them to a considerable extent, occur dykes and extensive masses of doleritic rocks and felstones, which vary much in texture."

Professor Balfour landed on the shore of Gubbet Gollonsir at the north-western extremity of the island and explored the district fringing this bay for some distance inland, so as to examine the high ground on either side of the level valley which opens out to the sea. The uplands here consist of a thick deposit of limestone, probably of middle tertiary age, but in many cases remarkably compact and hard, so that it has at times considerable resemblance to the well-known dolomites of the Italian Tyrol. This mass of limestone rests unconformably upon a group of highly crystalline gneisses, associated with diorites and perhaps with hornblende schists, which in general character correspond with the Hebridean series in the north-west of Scotland. These older rocks are frequently exposed in the beds of valleys and in the lower part of the walls of the plateau. The same description applies to the elevated districts traversed by Professor

Balfour on his way to the shore in Kurmeh Bay; but judging from the specimens which he has brought, I think it probable that some true granite is also associated with the gneissic series. The limestone, supported as usual by the latter, extends along the shore of Kurmeh Bay, and towards the eastern part it is cut (together, of course, with the older rocks) by basalt dykes. Felsite also occurs somewhere in this district, probably inland towards the south-east.

Further east along this plain, in the vicinity of Kadhab village, basalt dykes are seen cutting the limestone rock, and there are scattered boulders of granite or granitoid rock, of red felsites and rhyolites, and of a hornblendic diabase. The cliffs by the sea between Kadhab village and Hadibu Plain are composed chiefly of the usual massive limestone, which rests on a green flinty shale or argillite. The lower part of the limestone is a breccia, in which are fragments of the subjacent rock, and bands in it even resemble the latter (possibly being composed of the same material redeposited). In this neighbourhood occurs a mass of a kind of kersantite, the presence of which may account for the peculiar "baked" aspect of the shale.

Similar argillites occur along the line of strike in the valley to the south-east of Tamarida village, overlain as before by the limestone. They have a rather sharp dip to the south-south-west, and "an imperfect vertical cleavage, or possibly, jointing." Apparently beneath, but possibly intrusive in the argillites, are some compact pale grey felsites.

Inland rises the fine Haggier range whose outlines somewhat remind us of the peaks of Sinai. The bulk of the specimens from this place are reddish or pinkish felspathic granites, not generally very rich in quartz and always poor in the third constituent, generally rather coarsely crystalline, to which we may give the name of pegmatite.* This is cut by veins of a more compact granite and by at least one dyke of mica-trap. It is possible that gneiss also is present in this region. About the margin of the range the limestone rests upon the granite, and contains detritus from the latter in its lower part. The approximately highest point of the range is a pegmatite. On the south side about Adona we have the same pegmatites with a more hornblendic granite, and intrusive dykes or veins of compact purplish quartz-felsite or rhyolite.

Eastwards from Hadibu, on the way to Jebel Omhari, blocks of reddish felsite or rhyolite are shown on the plain, and the limestone is presently reached. The extreme north-west of the Girgha range is formed of pegmatites similar to those of the Haggier mountain, varying from coarse to fine in texture, and of a rather compact quartz-felsite; the latter constitutes a considerable hill mass, and is probably intrusive in the former. There is also some intrusive diabase.

Between Kami and the hamlet Ma-aber, on the Motaha, the rocks vary considerably. Granites of the usual type, compact reddish or greyish quartz-felsite or rhyolite alter-

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* There is some authority for applying this term (often used vaguely) to granites which consist almost wholly of quartz and felspar. Of these, "graphic granite" is a variety.

nate one with another. A mica-trap is also found. Evidently this is an extension of the last group of rocks. A specimen of an epidotic quartzite proves that there are, at any rate, some indications of the metamorphic series.

Following the Motaha river in a south-south-west direction we find pegmatites, coarse and fine-grained felsites, red, buff, and grey in colour, together with a greenstone so much decomposed that it is difficult to decide whether it be augitic or hornblendic. The specimens prove that the typical rocks of the Haggier range extend at least thus far to the east.

Returning to Ma-aber and continuing the section eastwards for three or four miles to Gedidery, Professor Balfour found first more red felsite, and then, near the latter place, much variation in the character and the colour of the rocks—"grey, white, yellowish, and red beds alternating." The specimens from these show that the following rocks are represented: fine-grained granite, compact felsites, diorite or, possibly, hornblendic schist and decomposed greenstone. It is also seen that the felsite, like the granite, is overlain by the usual limestone.

Between Gedidery and Gharrieh fine-grained granites and compact quartz felsites were collected, indicating that the Haggier group is prolonged as far as Khor Gharrieh, where these rocks come down to the sea. The granitic series was traced inland up the course of a stream, flowing generally north-north-west as far as the hamlet Vishas, and is cut by diorites. From this place up the Goahal Valley, in an easterly direction, crystalline rocks may be traced beneath the limestone, but some of the specimens must, I think, be gneisses, so that the ancient metamorphic series probably occurs in the eastern as well as in the western part of Socotra. Probably it could be traced still further east, but Professor Balfour, owing to indisposition, was unable to collect any specimens for the next seven miles. The limestone plateau stretches away to the eastern extremity of the island, specimens of this rock being brought from Ras Mom, a hill at the neck of the long terminal peninsula, which was the turning point of the expedition.

After retracing his steps for a few miles to the west, Professor Balfour travelled for some distance over the limestone plateau as far as Kittim, shortly after leaving which place he turned to the north-north-west, passing first over considerable masses of rhyolitic breccias, conglomerates, and grits. A specimen was also brought from a vein of calcite, containing fragments of red rhyolite and pinkish felspathic granite. Some of these fragments appear to be of volcanic origin, and there are masses of compact quartz-felsite, and of rhyolite with distinct fluidal structure, which, as Professor Balfour remarked, appeared very like lava flows. All these rocks are overlain by the plateau limestones.

From Azorah Professor Balfour turned in a west-south-west direction, following the Mitgahon gorge down to Baha, a village lying at the base of a peak called Töf, the easternmost summit of the Haggier range. Very compact quartz-felsites, and rhyolites abound. They form a conspicuous hill called Afero, which appeared to him to resemble the neck or "core" of a volcano. Near the opening of the gorge are doleritic rocks, which extend for some distance, and are, at any rate in some cases, intrusive in the granitic series which is sometimes fine-grained and is more hornblendic than usual. The red felsite also occurs.

After returning to Hadibu Professor BALFOUR crossed the Haggier range, and travelling in a general southerly direction reached the Nogad plain about the middle of the south coast of Socotra. But few specimens were collected during this part of the expedition, and the only two of interest were a red rhyolite, near Feraighey, and a rather peculiar dolerite, which will be more minutely described under their proper heads. On the plain were loose blocks of a hard conglomerate containing well-rounded pebbles (up to about 1 inch or longer in diameter) of compact red quartz-felsite and purplish rhyolite with well marked fluidal structure embedded in a silicious paste. Besides these, blocks were found of white cherty fragments, also in a silicious matrix, which contains grains of the above igneous rocks. After leaving the Nogad plain Professor BALFOUR again crossed the limestone plateau to the north coast near Gubbet Kadhab, whence he returned to Hadibu.

I now proceed to describe the more conspicuous and interesting varieties of rocks in Professor BALFOUR'S collection, taking the more crystalline and most ancient series first in order.

Gneissic rocks.

The existence of a series of highly crystalline metamorphic rocks in the Island of Socotra is indubitable, notwithstanding the difficulty already mentioned, of deciding without further examination whether certain of the specimens are to be referred to the granitoid gneisses or the granites. This series forms the floor on which rest the great limestone plateau and some other sedimentary deposits, and is well exposed in the western and to some extent in the eastern part of the island, being replaced in the central and most elevated region by true granites. The following are the principal varieties: (*a*) gneisses, not conspicuously foliated, consisting of quartz, felspar (generally of pinkish colour) and mica, black and white, and possibly hornblende. As a type of these a specimen from a hill near the opening of Gollonsir Valley was examined microscopically. The slide exhibits quartz, felspar, little elongated clusters of green hornblende with some epidote, opacite, ferrite, and a few scales of white mica. The felspar is chiefly microdine, a variety which is remarkably abundant in the Hebridean gneisses and in others of great antiquity. There is also a plagioclase, probably albite,* and some of the felspar crystals contain groups of small colourless microliths with a parallel arrangement. This structure I have also observed in some of the old gneisses from Greenland. The quartz contains small cavities (rarely with

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* In attempting to determine the species of the felspar I have made use of the optical tests given by Professors FOUQUÉ and LÉVY (among others) in their magnificent work "Roches Eruptives Françaises." I cannot, however, say that I feel great confidence in the results of the method.

bubbles), prismatic microliths of very pale green colour, and numerous colourless hair-like microliths, scattered irregularly throughout the grain. The resemblance of the specimens to examples from the Hebridean series of Scotland, the Laurentian series of America, and other Archæan rocks is very striking. [4115], one of a large series from the right bank of the upper part of the Gollonsir Valley, contains a fair proportion of black mica instead of hornblende. Apatite can be recognised, and the small, almost colourless, prismatic microliths already mentioned, some of which may be referred to this mineral, though I am of opinion that the majority are different. There is no microcline, but a good deal of plagioclase, probably both albite and oligoclase. [4124] contains hornblende with a black mica and sphene. (*b*) granitoid gneisses, consisting mainly of quartz and felspar, belonging to the group for which I have proposed the name granitoidite.* In these the quartz and felspar correspond with those described above, and the principal difference is that the rock is mainly composed of these two minerals, with an occasional grain of iron peroxide and a flake or two of iron glance or a ferruginous mica, hornblende or epidote. (*c*) hornblendic rocks, which we shall notice in the next paragraph, and (*d*) an impure quartzite from near Kami, north-east of the Haggier range. This rock consists of quartz, decomposed felspar, epidote, and perhaps a little hornblende, with, in parts, a good deal of magnetite.

Diorite and other hornblendic rocks.

I have made this division somewhat vague for two reasons: one that in rocks of igneous origin the hornblende is not unfrequently of secondary origin, having replaced augite or diallage, so that the rock is more properly a uralitic or hornblendic diabase than a true diorite: the other, that the means of making a thorough study of some difficult examples of these rocks—corresponding with those already mentioned in the gneisses and granites—are only now being obtained by me.† From the hill near the opening of Gollonsir Valley is a series of specimens, which, according to Professor Balfour, come from what appears to be a dyke in the Archæan series. Some are coarser in texture than others. Two of these [4009] and [4005] have been examined microscopically, of which the former was *in situ*, the latter from a loose block. They consist of a decomposed felspar, in which, however, the remains of the twinning characteristic of plagioclase can occasionally be discerned, of hornblende, black mica, and opacite, with some apatite, and a few grains of epidote. The hornblende is green in colour and exhibits very characteristic cleavage. The mica is sometimes altered to a greenish mineral, and often contains needles and grains of opacite, generally arranged parallel with the principal cleavage planes. In [4053] "from the slope of the hill

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* Quart. Jour. Geol. Soc., vol. xxxv., p. 322.

† It must be remembered, in excuse for this ignorance, that it is of no use to purchase specimens for study of these difficult cases, or, as a rule, to trust the statements which one finds in print. The student must collect his own specimens, and to do this it is necessary to visit distant localities and expend much time and money. Hence difficulties are but slowly removed.

above Hanigon, to the west of the former locality," the felspar is less altered, and some of it, if we may trust the optical tests, is albite, and more of the green mineral appears to be an altered mica. Among the Archæan rocks on the right bank of the upper part of the Gollonsir Valley are two dykes. One of these [4145], a dark speckled rock with the felspathic constituent weathering yellow, consists of long narrow crystals of rather decomposed plagioclastic felspar and a hornblendic mineral; some of this shows very characteristic cleavage, other crystals when cut parallel with the edges of the prism exhibit a rather fibrous structure and look as if they might be of secondary origin. One or two possibly replace magnesia-mica. There is a little quartz, epidote, and some black iron peroxide, with, perhaps, a little apatite. The other dyke has much of its hornblende in long narrow crystals with the same secondary aspect, and the felspar crystals also, which to a great extent are replaced by secondary microliths, are long and narrow like those of a dolerite. A third specimen [4197] from a rock on the bank of a stream is of the same character, but rather more porphyritic. Many of the felspar crystals show a zonal structure, and the outer part remains clear, while the inner is replaced by secondary microliths. The small extinction angle, where observable, leads me to conclude that oligoclase is the dominant felspar. In this slide magnesia-mica is recognisable.

From the tracts traversed on the east of Ghubbet Kurmeh a remarkable rock has been brought, which, with some hesitation, I class among the diorites. The compact matrix is a dark blue, almost black colour, in which are thin crystals of felspar often nearly 1″ long. With the microscope the ground mass is seen to consist of small elongated felspar crystals having a rather fibrous or "teazed" aspect, associated with much opacite and some green microliths of hornblende or chlorite, rather irregularly disseminated, and some flakes of brown mica. The larger crystals of plagioclase felspar show the greatest absorption at very small angles with the vibration planes of the crossed nicols, and parallel lamellæ extinguish almost simultaneously, so that they are probably oligoclase. The hornblende, however, has very much the aspect of a secondary product, and I am disposed to regard the rock rather as a hornblendic diabase than a true diorite.

From an isolated hill in the middle of the Gharrieh Plain (mainly limestone), which is covered with a peculiar vegetation, come some dioritic rocks (associated apparently with granites or granitoid gneisses). One of these consists of a plagioclastic felspar, which (though in part replaced by microliths) from its large angle, where unaltered, between successive extinctions (often quite 30°) is probably, in part at least, anorthite, and of hornblende, also with a secondary aspect, some chlorite, with epidote, apatite, and a dichroic fibrous lamellar green mineral, which extinguishes parallel with the lamellæ, and is more probably an altered mica than hornblende. A few grains of quartz are also present.

Other hornblendic rocks were noted during my examination of the specimens. Those described above certainly appear to be in all cases igneous rock, but it is quite possible that among them may have been some of metamorphic origin referable to the Archæan series, for, as my experience in Cornwall and Scotland has shown me, it is sometimes by no means easy to distinguish the one from the other in hand specimens.

Dolerites, basalts, &c.

The information furnished to me does not allow me to attempt any classification of this group. Some of the basalts cut the limestones and so are among the most modern known igneous rocks of the island, but whether all these augitic rocks are of the same date it is impossible to say : probably they are not.

A dolerite from the plain on the south-west of the stream in Gollonsir Valley [4051], is a rather coarsely crystalline rock, consisting mainly of whitish felspar and black augite. The former, under the microscope, proves to be well preserved labradorite, the crystals of which have fairly regular linear boundaries, and in form are rather broad oblongs. The augite, which has solidified after the felspar, is full, in one or two cases, of irregular grains of opacite, which sometimes make up almost the whole of the crystal. Incipient conversion into uralite is exhibited, crystals of augite having sometimes an irregular border of the latter mineral ; while sometimes minute scales of it are disseminated in the nearly colourless augite crystal. In one or two instances almost the whole crystal is replaced by uralite.

One of the dykes near Kadhab village [4198], I leave, after microscopic examination, with some hesitation among the basalts. It retains some traces of a clear glassy base crowded with opacite and other microliths, some acicular and colourless, which with some larger crystals are rather like anorthite, others are of a greenish-yellow colour, fibrous or filmy ; these being generally associated in irregular patches, with interspersed specks of opacite. They are most like replacements of an augitic constituent. A little hornblende can also be recognised. There are some indications of fluidal structure, and the rock evidently approaches the augite-andesite group.

[4454], one of a group of rocks of doleritic aspect, from the base of Azalin on the bank of the Hasainho, speckled whitish and blackish, consists of (*a*) labradorite in well defined crystals, evidently the first mineral to consolidate ; (*b*) a slightly brown, rather dichroic augite, which in one case approaches diallage in its close cleavage ; (*c*) a dark brown hornblende, sometimes inclining at the edges to sap-green (Plate 7, fig. 1). This paragenesis—for I think the whole aspect of the hornblende forbids the idea that it is of secondary formation (the uralites being generally green, and often quite pale)—though not very common is far from being unprecedented. It seems, for example, rather frequently in the old diabases of North Wales, and in the gabbro of Mont Colon (Pennine Alps). [4449], one of a very similar group of specimens, from the "rocks of which Azalin is composed," consists of well crystallized labradorite in good preservation, olivine, sometimes partially replaced by serpentinous microliths, opacite, augite, with a little diallage and hornblende. Some of the last mineral is certainly of secondary origin, bordering augite crystals, but one or two grains of darker colour may possibly be original constituents. The latter specimen then comes near to the true dolerites, the former to the Labrador-diorites.

From boulders in the bed of a stream-course near Fernaighey (no rock occurring *in situ*), come some specimens, of which [4499] may be taken as a type. The ground mass consists of a mass of small crystals of a white felspar and a black mineral, in which are scattered larger felspar crystals, sometimes 2 inches long. Under the microscope the rock is seen to consist of a rather decomposed labradorite, of a dull green, dirty-looking mineral associated with and occasionally replaced by opacite, probably a decomposed augite or diallage, and of a fibrous or scaly green mineral, giving bright tints with crossed nicols, also associated with grains of opacite. These aggregates have probably replaced olivine. There are also some fair sized crystals of apatite. The rock then has been an olivine-dolerite, but would now be more properly classed with the diabases.

Granites.

For the reasons given above I am unable to decide whether some of the specimens brought by Professor Balfour are true granites or only exceptional examples of the granitoid gneisses or granitoidites.* At the same time we need not hesitate to admit the existence of a considerable amount of this rock, especially in the more central part of the island, as for instance in the Haggier range. The granites of Socotra, as a rule, consist mainly of quartz and felspar, and so belong, as we have said, to the variety pegmatite. The latter mineral usually predominates and is commonly of a reddish colour. They are frequently much decomposed and not seldom show some indications of a graphic structure. It will suffice to select two or three from a large series as examples. [4249], "from the high cliff on the northern face of the Haggier range, one of the specimens typical of the rock forming the highest peaks," is a good example of the coarse red and white pegmatite. Microscopically it consists of a decomposed felspar, most, if not all, being orthoclase, quartz with numerous microlithic enclosures and minute fluid cavities, and a little of a dark green strongly dichroic mineral. Some of this is hornblende, but part of it much resembles tourmaline, to which mineral some acicular microliths, in one case in a tufted group, almost certainly belong. A second [4241] "from the slopes above Hadibu," has among it felspar, microcline, and a fair quantity of plagioclase (? oligoclase), grains of magnetite and a green chloritic mineral associated with clots of opacite, evidently replacing a magnesia-mica, associated with which is a little of a colourless mineral, possibly apatite. [4264], from the same region, is a "vein-granite, which cuts the coarser variety." It is a very finely crystalline quartz-felspar rock of a warm grey colour, containing some dark crystals about 0·25″ long. With the microscope it is seen to consist of intercrystallised quartz and felspar, the latter often exhibiting close twinning and sometimes being probably microcline.

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* Quart. Jour., Geo. Soc., vol. xxxv., p. 322.

The crystals occasionally exhibit rectilinear boundaries, but often interlace one with another and with the quartz in a very complicated way, as is not seldom the case in vein-granites. The slide contains some brown mica and tourmaline, the latter certainly a pseudomorph after the former. Two crystals retain a cleavage resembling hornblende, but behave optically as a uniaxial mineral, and when properly placed absorb the polarised beam far more completely than is usual with hornblende—a curious case of pseudomorphism, which, however, is not without parallels (Plate 7, fig. 2). [4205], an erratic near Kadhab village, doubtless from the western part of the Haggier range, is a red felspar-granite rather similar to the last, with a little iron-mica. [4207], a rather similar rock from the same region, exhibits in part a distinct micrographic structure; the decomposed felspar crystals are intimately interbanded with quartz, possibly of secondary origin; these two are most likely vein-granites. [4439], near Hesainho, in a region cut by felstone dykes, is a fine-grained pink and white granite with green specks, which under the microscope shows intercrystallised quartz and decomposed felspar (orthoclase and (?) oligoclase), with an approach to a graphic structure, besides some altered biotite—the rock is cut by a dyke of gabbro. Between these two rocks is a thin zone composed of the granite and the gabbro crushed, and to some extent mingled, though the materials of the former predominate. The constituents have been subsequently recemented, probably by deposition of quartz; this crushing was doubtless subsequent to the solidification of the newer rock. From the north-west of the Girgha range comes a series of specimens varying from coarse to moderately fine crystalline, and consisting of quartz and felspar with small quantities of hornblende and black mica, having a general resemblance to those already described from the Haggier range. Microscopic examination of one [4335] shows that the felspar is orthoclase, and perhaps microcline, with oligoclase. Small quantities of a chloritic mineral and a few microliths, possibly of tourmaline, are also present. The structure of the rock is rather abnormal for a granite, having some resemblance to the granitoid gneisses, but as there has been some local crushing and recementation, it may, notwithstanding, be a true granite.

Felstones and rhyolites.

Under this head I have retained a large number of rocks, which in some cases it would have been easy, but in others impossible, to subdivide. Petrologists are at present hardly in a position to agree upon precise definitions for the names of certain of the more acid igneous rocks, or upon the classificatory value of some of their minute structures. For example, a felstone must have a ground mass which is either microcrystalline or cryptocrystalline. But at present we cannot say whether or not these two structures correspond with different conditions in the past history of the rock, and so are of specific rather than of varietal value. Again, the latter structure, in some cases, appears to have been produced during the solidification of the mass, in others long afterwards. These, at present, it is often impossible to distinguish. Again, although we can often make a reasonable conjecture, we have no sure criteria for distinguishing between specimens taken from a dyke and from a flow, if they happen to have solidified under approximately similar circumstances. Hence, seeing that while in the present extensive series from Socotra there are some which are certainly from intrusive masses and others which are almost as certainly from lava flows, there are several which I cannot separate, either lithologically or petrologically, I have thought it better to group them under one general title, and indicate in the description what seems the most probable history of the rock from which the specimen has been taken. The majority of these Socotra felstones are of a warm red or purplish colour, though some of a grey tint also occur. They vary from minutely crystalline (approaching on this side vein-granites) to very compact subvitreous rocks which sometimes show a well marked fluidal structure. It will, perhaps, be more convenient to describe them as they occur geographically rather than to attempt to make a lithological arrangement.

The first specimen [4049] is from a dyke in the Archæan series, not far from the coast of Gubbet Gollonsir, about $1\frac{1}{2}$ mile east of Ras Bedu, at the western extremity of the northern coast of Socotra. It is a compact dull-coloured felsite, weathering a pale yellowish-brown, containing many little black crystals. Under the microscope the rock appears to have a glassy base, stained with ferrite so as to present a rather muddy aspect, in which are scattered many small and well-formed crystals, both of felspar and green hornblende; among the former orthoclase and a plagioclase, possibly albite, can be recognised; the latter crystals are generally well formed but sometimes include portions of the ground mass. This, when examined with the two nicols, is to be seen full of minute microliths of felspar, but there appears to be some remains of a glassy base. There is no free quartz. The rock then appears to be intermediate between the hornblende andesites and the sanidine (or orthoclase) trachytes. From the same neighbourhood also comes a rather granular quartz-felsite containing some hornblende. Several specimens have been examined from an interesting series obtained on the plain near Kadhab village; unfortunately none of these occur *in situ*, but, as Professor BALFOUR informs me, they doubtless come from a part of the Haggier range lying to the south-east. [4206], a compact dark felsite with paler spots and wavy bands of a more crystalline material, containing small scattered crystals of pink felspar, exhibits under the microscope a clear base (perhaps not wholly devitrified) studded with numberless minute granules and rods of opacite, indicating a fluidal structure. The spots and bands prove to be groups of spherulites crowded together with irregular interlocking edges.* There is some green hornblende, generally in clustered granules, possibly associated with minute tourmaline. The larger felspar crystals resemble orthoclase or sanidine, and there is a little free quartz. It is therefore a rhyolite, but not one of the kind which, judging from this collection, is so common in Socotra. Of this [4203, 4213, 4214] are types, "compact red felstones or

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* As in the figures of a devitrified glass: DAUBRÉE, 'Études Synthétiques de Géologie Expérimentale,' 1re Partie, pp. 170 and 171.

rhyolites," exhibiting on their weathered surfaces a beautifully distinct and rather parallel fluidal structure. This is most perfect in the second specimen. Microscopic examination shows it to be produced by ferrite-stained bands and lines of opacite granules and trichites with clearer interspaces. Small crystals of quartz, felspar, and little spherulites occur occasionally. The ground mass seems to be wholly devitrified. The first specimen shows traces of "flow brecciation" and is more unequally devitrified, with but few imperfect spherulites. The last [4214] has irregular spherulites and bands with a brush-like arrangement of microliths: it contains a little more free quartz. From "the banks of the Kereguiti stream, south of Hadibu," apparently intrusive in the green slaty rocks and the lower part of the overlying limestone, come pale buff or grey felsites. [4234] exhibits under the microscope an extremely minute devitrified structure, the slide is slightly clouded with ferrite and contains small scattered granules of quartz and felspar and minute specks of (?) hornblende. This rock is not unlike some of the most compact varieties of felstone from the Bala group of North Wales. From "the spurs of the Haggier range running towards the sea" come compact red quartz-felsites and rhyolites, generally resembling those already described. [4285] is cryptocrystalline, with a rather coraloid or arborescent structure, occasionally spherulitic; it is much stained with ferrite, contains a few scattered grains of quartz, decomposed felspar, iron peroxide, and hornblende or tourmaline, and is more probably from a dyke than from a flow.

A pinkish quartz-felsite [4331] "from the interbanded group of rocks underlying the conglomeratic base of the limestone, south of Ma-aber," has a cryptocrystalline ground mass of quartz and decomposed ferrite-stained felspar; the former mineral, which is very abundant, has at first sight a rounded or polygonal outline, giving the rock a superficial resemblance to one of fragmental origin. In this ground mass are scattered grains of quartz, crystals of felspar, and (probably) magnetite. In the eastern part of the Haggier range near Adona the coarse red pegmatite is cut by dull purplish compact felstone dykes. [4399] has been taken from a vein only about $\frac{3}{4}''$ thick. The junction with the granite is beautifully exhibited in the slide, one or two small fragments of the latter being included in the former. This exhibits a cryptocrystalline structure, almost microcrystalline in parts, which, however, is in no respect remarkable. In the other [4368], also a junction specimen, the intrusive rock is minutely devitrified, shows slight fluidal structure, and has a general resemblance to the rhyolites already described; a small band of crushed granite occurs at the junction, as shown in the figure (Plate 7, fig. 3).

The felsites, which are associated with the argillites and calcareous rocks on the banks of the Kereguiti stream (Hadibu Plain), being apparently intrusive in them, are of a different character; they are compact flinty felsites of a pale buff or grey colour. One [4234] under the microscope exhibits an extremely minute cryptocrystalline structure, the ground mass being irregularly and lightly clouded with ferrite, containing specks of a hornblendic or chloritic mineral, with grains of quartz and felspar, some of which show the twinning of plagioclase.

Rhyolites of a purplish colour, exhibiting fluidal structure, and of the former type, occur among the boulders on the plain between Girgha and Omhari, and pinkish quartz-felsites come from the extreme north-west of the Girgha range, where, in the ascent of a valley from the Hadibu plain, "red granite, white granite, reddish felstone, and intrusive dark rock, alternate one with another with marvellous rapidity, and disappear under the limestone on the northern side." One of the quartz-felsites [4285] under the microscope exhibits a rather irregular cryptocrystalline structure, approaching coralloid or aborescent, here and there spherulitic; the spherulites having an irregular outline. The slide is stained with ferrite, contains a few scattered grains and imperfect crystals of quartz, decomposed felspar, with microliths of iron-peroxide, and hornblende or tourmaline. From the group of compact felsites or rhyolites, which are members of a very variable series of rocks near Gedidery, one of the most compact, of a purplish colour [4364], exhibits under the microscope a clear base, interspersed with minute aggregated granules of ferrite, varying from a sienna-brown colour to almost black, which gives a mottled aspect to the slide. Scattered in this are crystals of sanidine and plagioclase (? albite), and other crystals, now occupied by yellowish-brown secondary minerals and clotted ferrite, but which, judging from their outlines, have in some cases almost certainly been augite, others, however, may have been biotite. The felspar crystals have a curiously rounded outline, and contain numerous enclosures of brown glass. No free quartz is visible, so that the rock must be classed with the sanidine trachytes. The compact red felsites intrusive in granites "from the side of Khor, near the village of Gharrieh," are very much of the normal type, compact reddish or pinkish felsites, containing minute specks of quartz and of a greenish mineral, with small felspar crystals. [4359], examined microscopically, exhibits one of the slightly "arborescent" cryptocrystalline structures already noticed, and is clearly related to types described above. [4350], from the same region, is very closely related to the vein-granites in structure. The ground mass is microcrystalline; the quartz, which occurs in irregular grains of variable size, has many microlithic enclosures; there are one or two distinguishable crystals of orthoclase, and clusters of small crystals of biotite.

Several specimens of compact felsites or rhyolites have been brought from the district to the west of Kittim and then north-north-west as far as Azorah, where these, together with a conglomerate or agglomerate containing a similar rock, form a large part of the hills of this district. Ten slides have been examined microscopically, but after what has been already said it will be needless to do more than indicate their distinctive features. [4443] and [4482] are microcrystalline and exhibit an imperfect micrographic structure, the former having a more porphyritic structure. It contains a good quantity of plagioclase (? albite) among its felspars and a few specks of decomposed grown mica. [4458], [4472], and [4473] are reddish rhyolites with a cryptocrystalline ground mass, and more or less of a spherulic structure. The second is a particular interesting example, longish microliths of ferrite-stained felspar being scattered, or irregularly grouped, among clear quartz and occasionally formed into true spherulites, which are often enclosed by a dark ring. The microliths frequently indicate by their grouping an incipient micrographic structure (Plate 7, fig. 4). [4408] and [4413] are rhyolites showing a very marked fluidal structure, the former affording one of the best examples of "flow brecciation" that I have ever seen; fragments of lava, often differing considerably in their minor characteristics, being entangled in the matrix of the slide, showing that the partially solidified rock from time to time has been broken up and swept along by the pressure of the still liquid mass behind (Plate 7, fig. 5). The rock has been subsequently cracked and the fissures cemented by infiltrated minerals, among which is opal. The locality of the last two rocks is indicated as "close to Azorah (almost half way between the north and south coasts)." [4446] and [4448] are rocks of a dull purple colour, with small amygdaloids (Plate 7, fig. 6). The matrix consists of very minute elongated felspar microliths in a base, rendered almost opaque with ferrite and opacite; opal, chalcedony, and a clear mineral resembling felspar occupying the vesicles of the rock. Its relations are perhaps rather with the andesites than the true trachytes. [4431], from nearer Kittim, where it is associated with conglomerates and forms low hills, looking in places much like a lava flow, differs from the others in colour, being a compact pale grey rock like some of those which I have obtained from Moneadhmore Glen, in Arran, or even from North Wales; it has a rather variable cryptocrystalline base, ferrite-stained, in which are scattered many small felspar crystals (probably orthoclase), quartz, and a chloritic mineral. Apart from the confirmatory evidence given by Professor BALFOUR, I should have felt no hesitation in asserting that some, at least, of these rocks formed parts of lava flows; they are anterior in date to the limestone beneath which they pass.

From the hill of Afero, which, Professor BALFOUR states, reminded him of a volcanic neck, west-south-west of Azorah, approaching the eastern end of the Haggier range, come more reddish compact quartz-felsites and rhyolites of the usual type. A specimen [4436] which has been examined microscopically exhibits a fluidal structure, in parts cryptocrystalline, in parts rather microcrystalline, and much ferrite-stained; but it is needless to dwell on points of only varietal interest.

The last specimens from this group of rocks examined microscopically were collected *in situ* near Feraighey, on the southern side of the Haggier range. [4440] is a very compact pale Indian-red felsite, resembling a group described above [4458, &c.], but is a little more spherulitic. [4420] is nearer to [4446] in general character; there are some curious rounded concretionary spots of opacite, and perhaps one or two minute amygdaloids.

Mica-traps.

From Socotra we have one or two representatives of this outlying and rather vague but convenient group. One [4270] is from a dyke forming the floor of a gully on the northern side of the Haggier Mountains, inland from Hadibu. The gully is excavated in the coarse reddish pegmatite, which is cut by a vein granite [4264] (p. 282). This mica-trap is a rather compact variety consisting of a reddish felspar, mica, and a dull greenish mineral. Microscopically it consists of felspar, mainly orthoclase, in which are scattered numerous microliths of opacite, hornblende and mica, with larger crystals of brown and white mica, chiefly the former, hematite and hornblende, the last being often crowded with opacite and clear microliths, perhaps apatite. The rock then is a minette and follows its usual habit of occurring in a dyke or vein.

From the shore west of Hadibu, near the argillites (p. 289), comes a very minutely crystalline rock of dull-grey colour, which when microscopically examined consists of decomposed felspar, probably plagioclase, altered magnesia-mica, iron peroxide, a little hornblende, apatite (?), and interstitial quartz. This then is a kersantite.

From between Kami and the hamlet Ma-aber, on the Motaha, comes a minutely crystalline rock [4309] associated with felspathic granitoid rocks already mentioned (p. 276), which consists of pale grey felspar, mica, and some quartz, grains of each occurring porphyritically in the ground mass. The microscope shows that a good deal of the felspar is plagioclase, some being oligoclase, but orthoclase is also present; the larger crystals often enclose numerous clear microliths, there is also much biotite and some quartz, with grains of iron peroxide. The rock then is a quartz-kersantite.

Unaltered clastic rocks.

Passing now to the comparatively unaltered clastic rocks, we have first to notice the agglomerates or conglomerates from the district near Kittim, where they occur in large masses in association with the rhyolites as already described (p. 277). There are conglomerates and breccias (generally rather decomposed) of a compact red rhyolitic rock, some of which reminded me in appearance of the specimens of late Precambrian age, which are so common to the west-south-west of Bangor, a mile or two from the town.*

[4437] has been examined microscopically. It is composed of fragments of rhyolitic lava, subangular to rather rounded, with some quartz grains, containing very minute cavities, and a very little of a cementing matrix, which is generally stained with ferrite or viridite. The lava fragments vary considerably in their structures. One, containing some small felspar microlites, has a marked fluidal structure, being banded with very dark ferrite; it is almost certainly a fragment of scoria or from the slaggy exterior of a lava flow; others exhibit various fluidal and microlithic structures. Most of them are ferrite stained, and more or less devitrified: materials derived probably from more than one source. There are also some grains of viridite, these being probably altered fragments of augite or hornblende. The rock is a rhyolitic grit, the materials of which are certainly of volcanic origin, but probably (in this particular case) have been transported by water. The resemblance already noticed to the Welsh Precambrian

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* Quart. Journ. Geol. Soc., vol. xxxv., p. 309, and Geol. Mag., Dec. ii., vol. vii., p. 298.

rocks does not disappear under the microscope, and I think we have in this group of specimens evidence of volcanoes of very considerable geologic antiquity.

The conglomerates of rhyolite and felsite, which occur as boulders on the Nogad plain (p. 278), though their materials have been derived from the same group of igneous rocks, are probably far more modern than the last. These I have not deemed it necessary to examine microscopically. Breccias and conglomerates of various kinds are not infrequent in Socotra, but generally present no special interest. Some occur just at the base of the limestone series, others may be quite modern; subærial or stream detritus, cemented by tufaceous deposit.

Argillites.

This group of rocks, found underlying the limestone near Hadibu Bay (p. 276), is one upon the exact nature of which it is difficult to decide. I am not able to satisfy myself, from the specimens which I have received, whether there is a true cleavage or not; the rock has a somewhat porcellaneous aspect, as if it were at least of palæozoic age, but as there appear to be intrusive rocks in the neighbourhood, one hesitates without further evidence to assign it to a remote geologic period. As, however, there is at the base of the limestone a breccia of fragments, seemingly in the same mineral condition, and as there seems to be an unconformity here, the interval between the two groups of rocks is probably considerable. Shaly bands are, indeed, said to occur in the lower part of the limestone, but it is possible that these may be formed of detrital material from the older group. Microscopic examination does not throw much light on the history of the latter. The specimens are composed of earthy material and minute, rather angular, fragments of a clear mineral (quartz, with possibly some felspar), and a number of exceedingly minute, doubly-refracting, rather highly-tinted granules; these are probably in some cases a variety of hornblende, in others epidote.

Limestone and dolomite.

Several specimens of the massive limestones which form so much of the upland district in Socotra are among Professor Balfour's collection. Generally they are compact in texture, with a clean subconchoidal fracture, buff, whitish, or sometimes pinkish in colour; in aspect often remarkably like the ordinary dolomitic rock of the Italian Tyrol. The weathered surfaces are sometimes curiously rough and cavernous. I have examined microscopically a few specimens from different parts of the island. The first [4168] is one of a series collected in the district extending from the head of the Gollonsir Valley towards the Kadhab Plain. These are compact in structure with a subconchoidal fracture, buff coloured and pinkish, the former being more crystalline. The specimen examined is a very pretty pinkish rock that would take a good polish. It contains numerous organisms, both tolerably perfect and fragmentary, among which foraminifera abound. Among these we can recognise[*] *Globigerina* (abundant) *Rotalinæ*—probably two forms of *Planorbulina*, one like *Ammonoides*, and the other outspread like *Mediterranensis*—*Textularia* (more than one form), *Amphistegina*, and possibly a small *Nummulites*, with fragments of mollusca, polyzoa (?), and of other organisms which I cannot recognise. The matrix is rather earthy.

A specimen [4048] from the cliff capping the ridge above Hainjou (west end of the island) is a very compact pale cream-coloured limestone. Microscopic examination shows a matrix of dusky grey material, with minutely-crystallized calcite and probably a little dolomite, which occurs also in thin veins, and some minute granules which may be glauconite. It contains several *Rotalinæ*, possibly *Planorbulina* and *Discorbina*, and perhaps *Rotalia*, together with a large fragment rather resembling a piece of a *Cristellaria*.

Buff limestones from the Kadhab Plain, [4188] and [4195]. The former contains numerous mineral fragments, apparently quartz and felspar, with organisms broken and whole, among which *Amphistegina* and *Miliola* are recognisable, and bits of the shells of mollusca. The organisms are more or less dolomitized. The latter contains many *Globigerinæ* with *Amphistegina* (small), perhaps an *Orbitoides*, and fragments of mollusca and polyzoa. This rock also is slightly dolomitized.

A specimen of limestone [4107] from the plateau west of Ras Mom, near the eastern extremity of Socotra, is a buff-coloured rock with a minute oolitic structure, the weathered surface being full of small holes. Under the microscope we found a clear dolomitic matrix full of darkish rings, circular to irregular oval in form, with a radial-fibrous structure, and as a nucleus commonly either a fragment of quartz or of a foraminifer, though sometimes they inclose simply the matrix; sometimes also the concentric structure extends to the centre of the oolitic grain.

[4456]—a very compact pale buff limestone from the top of Ras Mom, in appearance almost exactly like a bit of Schlern or Dachstein dolomite, is shown on microscopic examination to be a true dolomite. It contains numerous organisms, which, as is commonly the case, are more completely dolomitized than the matrix, and so rendered rather difficult of identification. There are many foraminifera, among them certainly *Amphistegina*, and perhaps *Nummulites*, *Globigerina*, with fragments which I believe to be polyzoa.

As regards the geologic age of these limestones, *Globegerina* has had a long existence, like *Textularia; Nummulites* began long since, and still lingers, but was most abundant in the eocene time; *Orbitoides* began in cretaceous and disappeared before the latest part of the tertiary age; *Amphistegina*, however, which is one of the most

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[*] As I have not worked much at thin sections of foraminiferal limestone, I submitted all these slides to Professor RUPERT JONES, F.R.S. With his wonted kindness, though at the time far from well, he examined them and gave me a series of notes, which are embodied in the text, so that the identifications rest upon his authority, than which it would be difficult to find a better.

abundant foraminifers in these rocks, ranges from the miocene upwards, hence it is probable that the age of these Socotra limestones is miocene.

Conclusion.

From the above remarks it would appear that in Socotra we have, as the foundation stones and "core" of the island, if the phrase be permissible, a mass of rock of very great antiquity. There is clear evidence of the presence of gneissic rocks which in their lithological characters resemble closely those which exist in the north-west of Scotland, the Malvern Hills and one or two other localities in our own island, in Northern America, and in many other parts of the globe. By whatever name these may be called, and however they may be correlated one with another, it is evident that their antiquity is enormously great, and that they had attained their present mineral condition before the earliest palæozoic rocks were deposited. Associated with these are granites, which, though of later date, are probably also of great antiquity. In the Sinai peninsula we have also gneiss, schists, and various granitic rocks. Of the latter I possess a small collection, given to me by Professor E. H. PALMER,[*] and the resemblance of some of the specimens to those of Socotra is very remarkable. In the geological notes added by the Rev. F. W. HOLLAND to the Report of the Ordnance Survey (ch. viii.) we find it stated, indeed, that the prevailing rock, in the Sinai region, is syenite (by which term probably hornblendic granite is meant), so that out of several hundred specimens he only possessed two or three of true granite. This may be, but my specimens from the summits of Serbal, of Jebel Musa, and Um Shomer closely resemble some of the Socotra rocks, especially the first and second, which are coarse reddish granites composed almost wholly of quartz and felspar (pegmatites). From the summit of Serbal also comes a finer grained granite, and I have an ordinary granite from Wady Sigillia. Other specimens, exactly as in the Socotra collection, might be either true igneous or highly metamorphic rock. In the Sinai region the old gneiss appears only to have been recognised in the northern part, where it forms an irregular trough to the north of Jebel Serbal, the higher peaks (like the Haggier range in Socotra) being granite. I miss, however, from the Socotra district, or find but feebly represented—for there is one specimen which may belong to it—the friable variably-coloured sandstones which form so marked a feature in some parts of the Sinaitic peninsula (*e.g.*, Wady Mokatteb)[†]. These, after having been assigned to more than one geological epoch, were referred by Mr. HOLLAND, on the evidence of fossils, to the

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[*] He was a member of the Sinai Survey Expedition in 1868–9. Two months after this paper was read, he was murdered by Arabs. The circumstances of his death are well known, but I may be allowed to pay a passing tribute to the memory of a singularly accomplished and learned man and very valued friend.

[†] This absence of the sandstone is probably an accident (the rock being very friable), for Professor BALFOUR mentions "purple sandstones" (see p. 275).

carboniferous. In Sinai, as in Socotra, we have huge masses of limestone, which in like way form great plateaux—as, for example, the Tih—and were deposited in an ocean, in which the well-known peaks of Sinai probably formed rocky islands, but deposition there commenced at an earlier period than we can venture with the evidence at present before us to claim for Socotra, for the limestones of Sinai are assigned to part of the cretaceous and to the eocene age; the Nummulitic limestone, for example, being finely developed as in Wady Dhaghadeh. Coraliferous beds of miocene age are, however, found in that region.* The rocks of Sinai are cut by dykes of "basalt, greenstone, and porphyrite," the first of which, as in Socotra, are probably comparatively modern, but we do not find there, so far as I can learn, representatives of the great group of the quartz-felsites and rhyolites which seem so enormously developed in Socotra and were certainly connected with active volcanos. The geological age of these cannot be determined. They are undoubtedly older than the limestone group; so that, if no part of this is earlier than the middle tertiary, they might belong to any geological period between that and the latest Precambrian, to the volcanic rocks of which they have, indeed, considerable resemblance. I am not aware that the "argillite" of Socotra—of which I can only say that it is older than the limestone—is represented in Sinai. As here, so also in Socotra, there are basalts of comparatively late geologic age—post miocene—and in the latter some compact trachytic rocks, which, however, differ from the older rhyolites, and are generally paler in colour.

We have, then, in Socotra, as it seems to me, evidence of rocks of an immense, and a land surface of a very great, antiquity. Excepting this argillite of uncertain age and limited extent, and perhaps some sandstone (also local), there is no evidence in the specimens before me to show that this island was submerged during any part of the palæozoic or mesozoic period.† During the kainozoic it undoubtedly shared in the downward movement which affected so large a portion of the globe in and about the North African and mid-Asiatic districts; but I should infer that the invasion of the sea commenced much earlier in the Sinaitic peninsula, and think it possible that the topmost peaks of the Haggier Mountains were at no time wholly submerged. As it again rose from the waves—perhaps being for a while connected with the African continent—the meteoric forces resumed their work of sculpture and the waves began their work of insulation. Since then the fauna and flora have undergone their own modifications, but in the Haggier Hills we have probably a fragment of a continental

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* Bauerman, Quart. Jour. Geol. Soc., vol. xxv., p. 37.

† "Africa south of the Sahara was probably a stable area during many of the alterations of the relative levels of land and sea of the north and of Europe."—Professor P. M. Duncan, Presidential Address to Geol. Soc. 1877, Journal, vol. xxxiii., p. 86 (Proceedings). West of the Sinai Peninsula old schists and granite crop out in Egypt, and east of it on the opposite side of the Gulf of Akabah, flanked in both cases by "Nubian sandstone." See the map attached to the Presidential Address to the Geologists' Association (delivered November 3, 1882), by Mr. W. H. Hudleston (Proceedings, vol. viii., p. 1), in which is given an admirable summary of the Geology of Palestine and the neighbouring districts.

area of great antiquity and of a land surface which may have been an "ark of refuge" to a terrestrial fauna and flora from one of the very earliest periods of this world's history.

DESCRIPTION OF PLATES.

PLATE 6.

Sketch map of Socotra.

The information at the Author's disposal has not enabled him to attempt to lay down the geological boundaries with any accuracy or to insert minor features, such as dykes, &c. It will be understood that the rock in the parts left blank (except where alluvial plains occur) is limestone of approximately middle tertiary age. Only the more important names are placed on the map.

PLATE 7.

1. Labrador diorite from base of Azalin (p. 281) × 20.
   - (*a*.) Labrador felspar, in parts rather decomposed; the striping is indicated as when a polarizing apparatus is used. This has consolidated before the hornblende.
   - (*c*.) The sap-green hornblende, within which is a crystal of iron oxide, probably ilmenite, and the pyroxenic constituent (*b*) of the text. Since writing this I have noticed that one of the two vertical sections of the mineral extinguishes when the cleavage lines coincide with the vibration planes, and is rather dichroic, giving reddish and greenish tints, according as the vibration of the polarized beam is perpendicular to or parallel with the cleavage lines of the mineral. The cross sections, though much resembling augite, are rather abnormal in some respects. Thus, though the evidence is insufficient for certainty, I think it possible that we have here the rhombic pyroxene (hypersthene ?) lately described by Mr. WHITMAN CROSS ('Amer. Jour. Science,' Feb., 1883) and Mr. TEALL ('Geol. Mag.,' Dec. ii., vol. x., p. 145).
2. Vein granite from the slopes above Hadibu (p. 283) × 20.

   The figure contains two crystals, selected from different parts of the slide, to represent those crystals which while resembling hornblende in cleavage have optical characteristics agreeing with tourmaline. They are extremely dichroic, changing from a clear sap-green to a blue-black.

3. Junction of felsite and granite. Haggier range, near Adona (p. 285) × 20.
   (*a.*) Very compact felsite.
   (*b.*) Crushed granite.
   (*c.*) Granite.
   Drawn with polarized light.
4. Rhyolite with spherulites. Azorah district (p. 287) × 40.
   Selected as a fairly typical example of a rather frequent type of rock.
5. Rhyolite exhibiting "flow brecciation," near Azorah (p. 287) × 20.
   The dark streaky part (*a*) is the general ground mass of the rock in which fragments are entangled; (*b*) exhibits small lath-like crystals of a decomposed felspar in a ferrite stained ground mass somewhat resembling that of the last slide; (*c*) exhibits a minute "speckled" devitrification structure; (*d*) with a high power exhibits minute felspar microlites crowding a dark ferrite-stained base, while in the centre of the slide is a fragment showing a well-marked fluidal structure, the lighter parts of which are devitrified. It is of course possible that some of the fragments may be entangled lapilli, but the absence of a slaggy border and the general cleanness of their edges corresponds better with the idea of their having formed parts of the flow.
6. "Trachyte" with minute amygdaloidal cavity. Near Azorah (p. 287) × 30.
   The minute felspar microliths are slightly more evident in the slide than in the drawing, and one or two of the cavities are larger than that figured, but unfortunately their contents have partly torn away in grinding.

VII. *Experiments, by the Method of* LORENTZ, *for the further Determination of the Absolute Value of the British Association Unit of Resistance, with an Appendix on the Determination of the pitch of a Standard Tuning-Fork.*

By Lord RAYLEIGH, *F.R.S., Cavendish Professor of Experimental Physics in the University of Cambridge, and Mrs.* H. SIDGWICK.

Received December 8, 1882,—Read January 11, 1883.

§ 1. IN this method, which was employed by LORENTZ in 1873,* a circular disc of metal is maintained in rotation at a uniform and known rate about an axis passing through its centre, and is placed in the magnetic field due to a battery current which circulates through a coaxal coil of many turns. The revolving disc is touched at its centre and circumference by two wires. If the circuit were simply closed through a galvanometer, the instrument would indicate the current due to the electromotive force of induction acting against the resistance of the circuit. The electromotive force corresponding to each revolution is the same as would be generated in a single turn of wire coincident with the circumference of the disc by the formation or cessation of the battery current. If this be called γ, and M be the coefficient of induction between the coil and the circumference, m the number of revolutions per second, the electromotive force is $mM\gamma$. In the actual arrangement, however, the circuit is not simply closed, but its terminals are connected with the extremities of a resistance R, traversed by the battery current, and the variable quantities are so adjusted that the electromotive force $R\gamma$ exactly balances that of induction. When the galvanometer indicates no current, the following relation, independent, it will be observed, of the magnitude of the battery current, must be satisfied—

$$R=mM;$$

and from this, M being known from the data of construction, the absolute resistance R of the conductor is determined.

One of the principal difficulties to be overcome arises from the smallness of the resistance R, necessary for a balance, even when m and M are both increased as far as possible. LORENTZ employed three resistances, ranging from ·0008 to ·002 of a mercury unit, and he evaded the necessity of comparing these small resistances with ordinary standards by constructing them of actual columns of mercury. His result was accordingly obtained directly in terms of mercury, and was to the effect that

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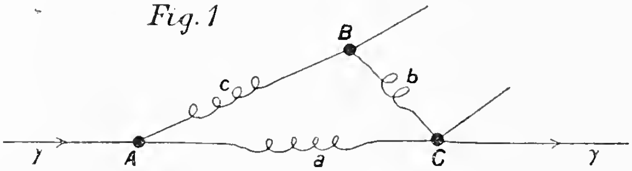
* POGG. Ann., cxlix., p. 251.

$$1 \text{ mercury unit} = \cdot 9337 \times 10^9 \text{ C.G.S.}$$

differing nearly 1 per cent. from the value (·941) obtained by ourselves.

§ 2. Under the conviction that this method offers in some respects important advantages, and influenced also by the fact that the arrangements for producing and measuring the uniform rotation necessary were ready to our hands, we determined to give it a trial, in the hope of obtaining confirmation of the results already arrived at by ourselves and by GLAZEBROOK with other methods. At first the intention was to follow LORENTZ in using for the resistance a glass tube full of mercury, with two points of which contact would be made by platinum wires passing through the glass. It appeared, however, that there would be difficulty in making the measurements with the degree of accuracy aimed at. If the wires were sealed into the glass, the section would probably be rendered irregular. An attempt was made to avoid this difficulty by using a tube from which the ends had been cut with the aid of heat. After small nicks had been filed sufficiently deep to receive the platinum wires, the ends were replaced in their original positions and secured with shellac. In this way a satisfactory uniformity of section near the points of derivation could be attained, but the measurement of the distance between these points, which is required to be known with full accuracy, was rendered difficult by the presence of the cement. It is possible that these difficulties might have been overcome, but at this point a method of shunting occurred to us, allowing the use of mercury to be dispensed with. Merely for the purpose of connecting the mercury unit with the B.A. unit or other standard of resistance, it would not be desirable to use tubes of such large bore.[*] This problem may more conveniently be taken by itself, and has already been treated by us in a former communication to the Society.[†]

Fig. 1



§ 3. In the shunt method the greater part of the main current γ passes on one side through a relatively small resistance a (see fig. 1), and the difference of potentials at the points of derivation B, C, is due to the passage of a small fraction only of the total current, the resistance $(b+c)$ being great compared with a. If at the same time b be small relatively to c, the difference of potentials is doubly attenuated. Its value for a given main current γ is found at once from the consideration that the current divides itself between the two branches in the inverse ratio of the resistances. The current through b is thus $\frac{a}{a+b+c}\gamma$, and the difference of potentials at the points

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[*] If the distance between the points of derivation were 1 metre, R = ·002 mercury unit would require a section equal to 500 square millims.

[†] Phil. Trans., 1883, p. 173.

of derivation is $\frac{ab}{a+b+c}\gamma$. The quantity $\frac{ab}{a+b+c}$ thus takes the place of R in the simple formula, and is called the effective resistance. By taking for instance $a=\frac{1}{2}$, $b=1$, $c=100$, we get an effective resistance of about $\frac{1}{200}$; and the resistances employed may be those of ordinary resistance coils, capable of accurate comparison with the standards.

§ 4. In designing the apparatus we were influenced by the fact that we had at our disposal two very suitable coils of large radius, wound some years ago by Professor CHRYSTAL, the same in fact as were used by Mr. GLAZEBROOK in his investigation by another method. By bringing the two coils close to one another and to the plane of the disc, the inductive effect is rendered a maximum. This arrangement accordingly was the one first experimented with, as being the most likely to prove successful.

The diameter of the disc is limited by two considerations. If it be too small, the whole inductive effect, and with it the sensitiveness of the arrangement, suffers. On the other hand if it be too large, the circumference enters the more intense region of magnetic force which lies near the wire, and the coefficient of induction changes its value rapidly when any alteration occurs in the mean radius of the coils, or in the diameter of the disc, and thus the final result becomes too sensitive to errors in the magnitudes of these elements. In the Phil. Mag. for November, 1882, the reader will find a calculation of the values of M for various cases, and a general comparison of the principal methods for determining absolute resistance especially in respect of errors arising in connexion with the fundamental linear measurements. For the experiments now to be described, the diameter of the disc was chosen so as to be somewhat more than half that of the coils (§§ 22, 23).

§ 5. The disc was of brass and turned upon a solid brass rod as axle. This axle was mounted vertically in the same frame that carried the revolving coil in the experiments described in a former communication to the Society* (see Plate 48), an arrangement both economical and convenient, as it allowed the apparatus then employed for driving the disc and for observing the speed to remain almost undisturbed. The coils were supported horizontally upon wooden pieces screwed on the inner side of the three uprights of the frame.

During the earlier trials, extending over the month of May, 1882, the edge of the disc was bevelled, and contact was made with it by means of a brush of fine copper wires held in a nearly vertical position. No sufficiently regular results could be obtained until the sliding surfaces were amalgamated, and even then there were discrepancies between the work of one day and that of another, whose cause was not discovered until a later period. It soon became manifest, however, that the bevelled edge would not answer the purpose, for it cut its way by degrees into the wires of the brush in such a manner as to render the effective radius uncertain. The substitution of a cylindrical for a bevelled edge promised better results. The width of the edge

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* Phil. Trans., Part II., 1882.

(equal to the thickness of the disc) was $4\frac{1}{2}$ millims. and allowed sufficient room for the contact of the brush though placed tangentially. In this way broader bearing surfaces were available, and the small extension of the contact in the direction of the axis is unobjectionable, provided everything is arranged symmetrically with respect to the middle plane of the disc.

As will presently appear, the success of the method is independent of any constant thermo-electric force at the sliding contact, but it is evident that good readings cannot be taken if the thermo-electric force changes its magnitude often and suddenly. It was found advisable to renew the amalgamation of the edge at the commencement of each day's work. The excess of mercury, if any, attaches itself to the brush, and does not appear to render the diameter of the disc uncertain.

The inner contact was made in a similar manner by a brush pressing against the shaft itself at a place a little below that at which the disc was attached. The coefficient of induction to be employed in the calculation is the difference between the coefficients for the coil and the outer and inner circles of sliding contact respectively, but the latter is quite subordinate (§ 25).

§ 6. The disc was driven by the same water-engine that was employed for the revolving coil of former determinations,* the connexion being made by a long cord passing round a wooden pulley attached to the lower part of the shaft. To the upper face of the disc was cemented a circle of paper on which were marked a series of circles of alternately black and white teeth. One observer looking through the prongs of an electro-magnetically maintained fork regulated the speed of the disc by application of the necessary friction to the driving-cord, which passed through his fingers. When one of the series of circles is seen to be stationary, a simple and easily expressed relation is established between the frequency of the fork and that of revolution. At intervals the number of beats per minute is counted between the notes of a standard fork, and (the octave of) the electric fork. There is no difficulty in thus determining the speed of rotation to within one part in 10,000. With respect to the absolute pitch of the standard fork itself, see the Appendix to this Memoir.

§ 7. When the disc is caused to rotate, and the galvanometer circuit is closed, a deflexion is observed, although the battery which generates the main current is not in action. This deflexion is due to two causes—thermo-electric force at the sliding contact, and induction dependent upon the vertical component of the earth's magnetism. Although not a direct source of error, this deflexion is better avoided, both for convenience in reading the galvanometer and because it implies the actual passage of a not insensible current through the sliding contacts and thus brings into consideration the *resistance* of these contacts. The compensation was effected by the introduction of an opposing electromotive force; for which purpose two terminals of the galvanometer circuit J, K, fig. 2, instead of being connected directly, were attached by binding screws to two points on a stout copper wire forming part of a circuit which

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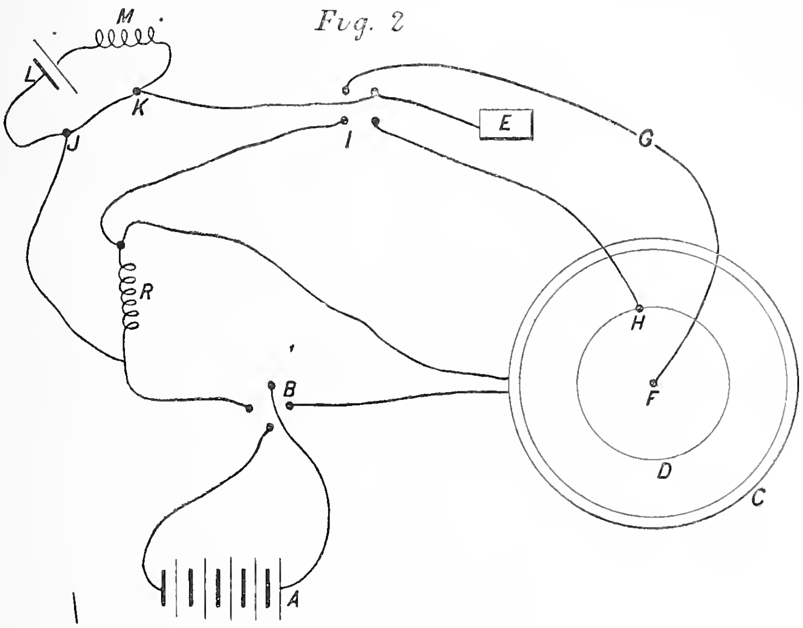
* Proc. Roy. Soc., May 5, 1881, p. 112; Phil. Trans., Part II., 1882.

included a sawdust DANIELL (L) and a resistance coil of 100 ohms (M). By shifting one of the binding screws, the galvanometer reading, in the absence of the main battery current, and after attainment of the proper speed, was made to be nearly the same as when the galvanometer contact was broken.

Fig. 2



§ 8. The general plan of the connexions and the *modus operandi* will now be intelligible from fig. 2. The poles of the battery A, consisting of 20 DANIELL cells, were connected with a mercury reversing key B, the two positions of which were distinguished by the letters E and W (east and west). From thence the current passed through the induction coils C and the equivalent resistance R, of which the details are reserved for the moment. The reflecting galvanometer, G, is placed at a considerable distance in order to avoid the direct influence of the coils, and is connected with the inner sliding contact, F. Its resistance is about $\frac{1}{2}$ ohm; and by the aid of the compensating magnet the vibrations of the needle were made slow enough to be readily observed. The terminals of the galvanometer branch, which includes also a commutator, I, are connected to the extremities of the resistance, R.

If, while the disc is maintained in uniform rotation, the reading of the galvanometer is the same whichever way the battery key may stand (correction being made, if necessary, for a direct effect upon the needle), it is a proof the contemplated balance is actually attained. In this way all disturbance from the earth's magnetism, and from thermo-electric forces whether situated at the sliding contacts, or within the resistance coils of which R is composed, or at any other part of the galvanometer circuit, is eliminated from the result. The adjustment is effected by varying a comparatively large resistance, taken from a box, and placed in multiple arc with one of the components of R.

§ 9. In actual work, however, it is not necessary, or even desirable, to hit off the balance with great accuracy. An unmistakable difference of readings when the battery key is put over, is rather an advantage than otherwise, as giving an indication that the circuits are properly closed. The plan adopted was to take a series of readings of the effect (E—W) of reversing the battery current with an effective resistance R_1, not very different from R. Single readings were liable to considerable irregularity in consequence of change in the friction at the sliding contacts, and of momentary variations in the speed. These errors cannot possibly be systematic, and are in great measure eliminated in the mean of a series. Having thus obtained the difference of galvanometer readings (E—W) corresponding to R_1, we altered the resistance in multiple arc so as to change R_1 into R_2, the difference being some such fraction as $\frac{1}{100}$ of the whole, and in such a direction that the sign of E—W is changed. The two series give by simple interpolation (after correction for the direct effect) the true value of R, that is the effective resistance corresponding to the balance. In order to get the best result relatively to the time occupied, the number of observations of E—W in each set was taken roughly in inverse proportion to the values. To diminish the influence of a progressive change in the strength of the battery current, the observations with R_2 were interspersed between those with R_1 as effective resistance. The readings were usually taken continuously, with no more delay than was necessary to allow the vibrations of the needle to become of moderate extent after each change. When they were completed, the driving cord was reversed, as well as the commutator, I, and a similar set of observations was taken with rotation in the opposite direction.

§ 10. In the earlier experiments the resistance coils composing the effective resistance were arranged as in fig. 1, in which A, B, C may be supposed to represent mercury cups, the bottoms of which were formed of amalgamated copper discs. On these discs rested the amalgamated terminals of the various resistance coils and connecting wires. The shunt *a* consisted of two unit coils in multiple arc, between which the greater part of the main current was equally divided. The magnitude of the main current was less than $\frac{1}{10}$ ampère. The resistance *b* between the points of derivation was a unit, while the third resistance *c* was alternately 105 and 106.

In reckoning the resistance of the galvanometer circuit we have to include *b*. The remainder scarcely exceeds the $\frac{1}{2}$ ohm due to the galvanometer itself. It appears therefore that the deflections obtained with the arrangement described are only one-third part as great as they would be if a quite small resistance were substituted for the unit in *b*. As the sensitiveness appeared likely to be inadequate, we afterwards replaced the unit by $\frac{1}{10}$, using for *c* a coil of ten units. As in this case the addition or subtraction of a whole ohm in *c* would make too great a difference, the adjustment was obtained by varying a comparatively large resistance placed in multiple arc with *a*.

In the light of subsequent experience it is doubtful whether this change was an improvement. The increase of galvanometer deflection was not really of much advantage, since the difficulty of getting sharp results arose from electromotive disturbances, and these were magnified in the same proportion. It would probably have been better to have retained the unit in *b*, and to have replaced the galvanometer by one of higher resistance.

§ 11. Preliminary trials having given apparently satisfactory results, we proceeded to make regular series of observations in the manner already described. We had not gone far before anomalies revealed themselves of such a character as to prove that we were not yet masters of the method. It usually happened that each day's observations agreed well together, showing that the sensitiveness was sufficient; but when we came to compare the results obtained on different days unaccountable discrepancies became apparent. The first result of the more severe criticism to which the arrangements were then subjected was to show that sufficient thought had not been given to the question of insulation. The wire composing the induction coils, or rather one extremity of it, is necessarily at a high potential, and a very moderate leakage from the coils to the frame, and thence to the disc, might cause great disturbance. Some such leakage was in fact detected on application of appropriate tests. Ebonite insulation was accordingly introduced into the supports of the coils. The battery was carefully insulated from the ground, as was also the frame carrying the revolving disc, and other precautions were taken which it is unnecessary here to detail. For the sake of definiteness one point of the galvanometer commutator was connected to earth. With these improvements tests were satisfied more severe than that of actual use, and these tests were renewed at intervals during the spinnings.

The results however still showed that some defect existed which we had not yet succeeded in detecting. It made no appreciable difference which way the disc rotated, but the means of different days' work failed to exhibit the desired accordance. Two months' work had already been spent upon the experiments, and we had begun to despair of a satisfactory issue, when it occurred to us that the connexion of the coils for compounding the effective resistance was faulty.

§ 12. By reference to fig. 1 it will be seen that the main current traverses part of the cup C, and that part of the same cup is also included in *b*. Now, although for all ordinary purposes the resistance of the parts of the cup might be neglected, in the present case it is the small effective resistance R with which it comes into comparison. If we aim at an accuracy of $\frac{1}{10000}$, we cannot afford to overlook a resistance entering in this manner, even though it may not exceed $\frac{1}{2000000}$ ohm. The discrepancies were doubtless due to small differences in the position of the wires and coils in cup C, moved as they were from day to day in order to verify the soundness of the contacts.

In order to avoid the difficulty we have only to take care that no part of *b* can possibly be traversed by the main current, and this is easily done by the introduction of another mercury cup. Fig. 3 shows the arrangement adopted. The main current enters at the cups A and D, and the greater part is taken by the two unit coils in multiple arc whose terminals rest in these cups. The galvanometer terminals are led into two other cups B and C. The ends of these are beaten flat and the legs of the $\frac{1}{10}$ rest upon them. The connexion between C and D was through a stout copper rod, which may be regarded as part of c. For the first series the connexion between A and B was through a single coil of 10 units' resistance, replaced in subsequent series by other coils giving altogether 16 and 20 units' resistance respectively.

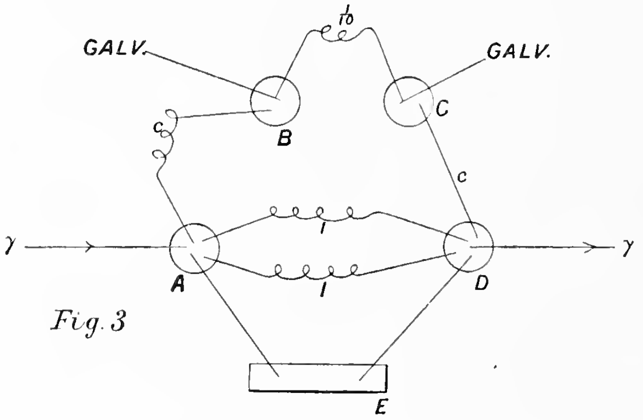


Fig. 3

To make the necessary adjustment and variation of resistance, a box, E, was placed in multiple arc with the two unit coils. The resistances taken from the box were afterwards carefully determined, but they enter into the final results in quite a subordinate manner.

§ 13. Further trials now led to the satisfactory conclusion that the defect was remedied, for the means obtained on different days agreed well together, even although the resistance coils were taken down and remounted in the interval. As we had now every reason to suppose that our experiments would have a successful issue, we proceeded to make the final adjustments preparatory to a complete series of observations.

In the first and second series the two coils were near one another, separated only by three slips of glass, and held firmly together by wooden clamps. The adjustments presented no particular difficulty. By means of an iron finger clamped to the disc and carried gradually round, it could be verified that the coils and disc were concentric and in parallel planes. The coils were gradually wedged into their places, and secured when their mean planes occupied the desired symmetrical positions relatively to the disc. It is evident that errors of maladjustment influence the result only in the second order.

§ 14. Experience in this series having shown that the arrangement was satisfactory, and that the sensitiveness was fully sufficient, we proceeded to make a second series of observations without displacement of the induction coils, but at a speed of rotation lower than before in about the ratio of 16 : 10. This, of course, entailed a corresponding change in R, which was effected by increasing the component c. An agreement between the final results of the two series would give an important confirmation, inasmuch as leakage of electricity from the main circuit into the galvanometer branch would exert a different influence in the two cases. The observations were not reduced until some time afterwards, and it then appeared that the agreement was even better than it would have been reasonable to expect.

§ 15. The final number, $\cdot 9867 \times 10^9$, expressing the value of the B.A. unit in absolute measure as determined by these two series of observations, is almost identical with that previously obtained by ourselves, and by GLAZEBROOK, using other methods. With respect to the independence of these determinations, the only thing calling for notice is the fact that the same induction coils were employed both by GLAZEBROOK and in the present investigation. In other respects there has been, we believe, scarcely any point of contact. But it is evident that an error in the measurements of mean radius of these coils must propagate itself into both results. The point to which we now wish to direct attention, is that the error of mean radius will influence the final number in *opposite directions*. In the method employed by GLAZEBROOK, an under-estimate of the mean radius would lead to an under-estimate of the induction coefficient, whereas with us it would lead to an over-estimate of that quantity. So far, therefore, as the error of mean radius is concerned, it would appear that the use of the same coils is far from impairing the value of the results. Even with respect to the number of turns, an error, if that be supposed possible, would affect the results in a different manner, for GLAZEBROOK was concerned with the *product* of the numbers for the two coils, while we evidently are concerned with the *sum*.

§ 16. In researches of this kind it is proper to calculate the influence upon the result of errors in the fundamental measurements. The value of M depends upon three linear quantities: the radius of the disc (a), the mean radius of the two coils (A), and the distance between their mean planes ($2b$). In the present case, however, the latter element enters in a very subordinate degree. From § 25 it appears that

$$\frac{dM}{M} = -1\cdot 4\,\frac{dA}{A} + 2\cdot 4\,\frac{da}{a}.$$

It has been shown* that these conditions compare favourably with those of most of the other methods that have been employed. From its nature a is much more easily measured than the diameter of a coil.

§ 17. The results deduced from the several days observations, when corrected for slight variations of temperature of the resistance coils, &c., exhibit a remarkable accordance. By reference to the tables (§ 27) the reader will see that the maximum divergence from the mean in Series I. is only about one part in 4000, while in Series II. it is even less. We were thus encouraged to carry out a modification of the method which we had had in view all along, and the results of which would be in great measure independent of those of Series I. and II.

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* Phil. Mag., Nov., 1882.

§ 18. The modification referred to relates to the position of the induction coils relatively to the disc. In the arrangement with which we have been dealing hitherto, the mean planes of the coils are nearly coincident with that of the disc, and the accuracy of the final number depends upon an exact knowledge of the mean radius of the coils. It has, on the other hand, the advantages of being practically independent of measurements parallel to the axis, and of giving the maximum coefficient of induction. In the new arrangement the coils are separated to such a distance that the *result is nearly independent of a knowledge of the mean radius.* How this may come about will be readily understood by considering the dependence of the coefficient of induction M upon A, when a and b are given. It is clear that M vanishes, both when A is very small, and also when it is very large; from which it follows that there must be some value of A for which the effect is a maximum, and therefore independent of small variations of A.

In carrying out this idea, it is not necessary to approach the above defined state of things very closely; for of course we have in reality a good approximate knowledge of the value of A. In our apparatus the distance of mean planes was about 30 centims., so that b = about 15 centims. ($A=26$, $a=16$). From the calculations in § 25 it appears that with the actual proportions

$$\frac{dM}{M}=+\cdot 12\,\frac{dA}{A}-\cdot 96\,\frac{db}{b}+1\cdot 8\,\frac{da}{a};$$

so that the error of A enters in quite a subordinate degree. The positive coefficient of dA shows that with the given coils and the given disc the separation was somewhat too great to secure the utmost independence of dA.

§ 19. The success of this arrangement depends principally upon the degree of accuracy with which b can be determined. The two rings upon which the coils are wound were held apart by three equal distance-pieces, against which they were firmly pressed by wooden clamps. The distance-pieces were hollow, of massive brass, and the terminal faces were carefully turned. Central marks upon them facilitated the adjustment of the coils into the symmetrical positions. The distance of mean planes does not however depend solely upon the distance-pieces. Even if we could assume that the mean planes are symmetrically situated relatively to the grooves in which the wire is wound, we should still have to take account of the thicknesses of the flanges. All uncertainty in this matter is eliminated by following the plan adopted by GLAZEBROOK of reversing the rings (without interchange), and then repeating the measurements. Whatever may be the situation of the mean planes and the thicknesses of the flanges, the mean result thus obtained corresponds to a distance equal to the length of the pieces *plus* half the total outside thicknesses of the rings. These quantities can all be measured with great precision, and as easily after the coils are wound as before. Full particulars are given in § 24. There can hardly be a doubt but that the determination is much more accurate than that of the mean radius of a coil; and, what is also of some importance, it admits of repetition at pleasure with comparatively little trouble.

§ 20. The sensitiveness of this arrangement was about the same as in Series II., and the table shows a good agreement among the results obtained on different days. The final number from this series is ·9868, almost the same as from Series I. and II.

The small difference of effective resistances required for balance in the two positions of the induction coils, amounting to about one part per thousand, is almost exactly accounted for by the small difference of distances of mean planes in the two cases, as deduced from Professor CHRYSTAL's measurements of the thicknesses of the flanges. In the first position (see § 24) the coils are nearer together by almost exactly one part per thousand, a difference which, according to the formula given above (§ 18), should be reproduced almost without change in M and therefore in R, the greater values of M and R corresponding to the smaller distance.

§ 21. If we combine all the results of the present investigation, giving equal weights to the two arrangements of the induction coils, we have

$$1 \text{ B.A. unit} = \cdot 98677 \times 10^9 \text{ C.G.S.}$$

With use of the ratio between the mercury unit and the B.A. unit found by us (Proc. Roy. Soc., May, 1882), this gives

$$1 \text{ mercury unit} = \cdot 94150 \times 10^9 \text{ C.G.S.};$$

or, which is the same thing, the ohm is the resistance of a column of mercury at 0° centigrade whose section is 1 square millim., and whose length is

$$1062 \cdot 14 \text{ millims.}$$

We now pass on to the details of the measurements.

DETAILS OF MEASUREMENTS.

Diameter of disc.

§ 22. Preliminary measurements of the disc while still mounted were made on August 11, 1882, with callipers by Messrs. ELLIOTT. Read by the vernier of the instrument itself the mean diameter was

$$2a = 310 \cdot 76 \text{ millims.}$$

The opening of the callipers was also determined independently by reference with the aid of microscopes to a verified scale of millimetres. In this way

$$2a = 310 \cdot 77 \text{ millims.}$$

The circumference was also measured by a steel tape, afterwards compared with the millimetre scale. Correction being made for the thickness of the tape, the result was

$$2a = 310 \cdot 84 \text{ millims.}$$

After the disc had been dismounted, the diameter could be determined more advantageously by direct observation through microscopes focussed upon its edge with subsequent reference to the standard scale. It was found (August 19, 1882) that a very appreciable difference existed between the diameter of the upper and lower faces, showing that the edge was somewhat conical. At the upper edge the diameter was 310·80, and at the lower edge 310·58. These were the extremes. At the middle of the thickness the diameter was 310·75. This departure from the truly cylindrical form was undoubtedly a defect in the apparatus, which could easily have been avoided if detected in time. When the apparatus was first set up, the success of the experiment was problematical, and a minute examination of the disc seemed premature. The diameter to be adopted is an average taken with reference to the conductivity of brush contact. The whole width of the brush being decidedly less than the thickness of the disc, and the pressure being greatest at the central parts, we decided (of course without knowing to what precise final result the estimate would lead) to take the mean of 310·75 and $\frac{1}{2}$ (310·58+310·80). Thus

$$2a = 310{\cdot}72 \text{ millims.}$$

The error due to the conicality of the edge cannot exceed one part in 5000 at the worst, and thus it appeared scarcely worth while to correct the defect and repeat the spinnings.

The diameter of the shaft at the place where the other brush contact was made, was found to be ·825 inch, or 20·96 millims.

The induction coils.

§ 23. These are the same as were used in Mr. Glazebrook's measurement, and were wound by Professor Chrystal in 1878. The following are the dimensions; for further particulars reference may be made to Mr. Glazebrook's Memoir.*

	A.	B.	Mean.
Mean radius in centims. (A)	25·753	25·766	25·760
Radial width of section ($2h$)	1·92	1·90	1·91
Axial width of section ($2k$)	1·896	1·899	1·897
Number of windings.	797	791	$\frac{1}{2}\times 1588$
Resistance (approximate) in B.A. units .	84	83	$\frac{1}{2}\times 167$

Since the coils are so nearly similar and were used symmetrically, it is sufficient to use the numbers in the last column. The section of the ring is shown in fig. 4 full size.

To find the distance of mean planes the following measurements of the thicknesses of the rims are required. They are given in centimetres.

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* Phil. Trans., 1883, p. 223.

	A.	B.
Rim (marked side)	·478	·446
Channel	1·896	1·899
Rim (unmarked side) . . .	·488	·465
Total thickness of rim . . .	2·862	2 810

Now that the rings are wound it is difficult to verify these numbers. However, the total thickness of the rings at the places touched by the distance pieces in the arrangement used for Series III. was taken, with the result

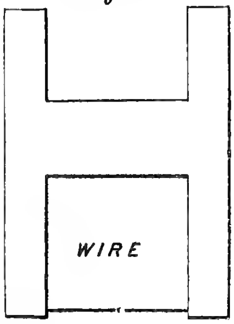
	A.	B.
Mean of three places. . . .	2·8625	2·8067

These latter values of the thicknesses will be used in the calculation of Series III.

In Series I. and II. the rings were not reversed, and we must use the numbers above given for the thicknesses of rims which were contiguous to the slips of glass; but in this case the result is not at all sensitive to changes in the distance of mean planes. The rims contiguous to the glass were for both coils the *marked* rims, of which the aggregate thickness is ·924. If we add to this the thickness of the glass strips ·454, we obtain 1·378 as the distance between the wire sections. Again, adding the mean axial width of section 1·897, we find as the distance of the mean planes

$$2b = 3{\cdot}275 \text{ centims.}$$

Fig. 4



The distance-pieces.

§ 24. The measurement of the distance-pieces used for the third series was made with great care. As only the mean is required, the three pieces were held under the microscopes in one length by a nut and a long bolt running through. Readings were taken in several positions, as the pieces were turned round, and reference was finally made to the standard scale. Two independent measurements gave 83·580 and 83·579, mean 83·5795 centims., as the aggregate length. This was further verified by measuring each piece separately with callipers, the sum of the lengths thus found being 83·582. For the mean length of these distance-pieces we take

$$27{\cdot}8598 \text{ centims.}$$

As has been already explained, the rings were used in two positions relatively to the distance-pieces, with the view of eliminating any uncertainty as to the situation of the mean planes, and of rendering the final result independent of all measurements of thickness except that of the total thicknesses of the rings. Thus the mean distance of mean planes in the two positions is

$$27{\cdot}8598+\tfrac{1}{2}(2{\cdot}8625+2{\cdot}8067)=30{\cdot}6944 \text{ centims.}$$

To compare the partial results for the two positions separately, we must use the thicknesses of the rims which were in contact with the distance pieces. In the first position these were the marked rims, and thus the distance of mean planes

$$=27{\cdot}860+{\cdot}478+{\cdot}446+1{\cdot}897=30{\cdot}681 \text{ centims.}$$

In like manner for the second position we find

$$27{\cdot}860+{\cdot}488+{\cdot}465+1{\cdot}897=30{\cdot}710 \text{ centims.}$$

The induction coefficients.

§ 25. Series I. and II. The distance (b) of the mean planes of the coils from the middle plane of the disc is

$$b=1{\cdot}637 \text{ centim.}$$

The extreme distances, required to be known for the quadrature, are

$$b+k=2{\cdot}585 \text{ centims.,} \quad b-k={\cdot}689 \text{ centim.}$$

The extreme and mean radii are

$$\mathrm{A}-h=24{\cdot}805 \text{ centims.,} \quad \mathrm{A}=25{\cdot}760 \text{ centims.,} \quad \mathrm{A}+h=26{\cdot}715 \text{ centims.}$$

while

$$a=15{\cdot}536 \text{ centims.}$$

The coefficient of induction between the disc and the middle turn of the coil, denoted by $\mathrm{M}(\mathrm{A}, a, b)$, is equal to $4\pi\sqrt{(\mathrm{A}a)}.f(\gamma)$, where $f(\gamma)$ is a function of γ given by tables.* The angle γ itself is defined by

$$\sin\gamma=\frac{2\sqrt{(\mathrm{A}a)}}{\sqrt{\{(\mathrm{A}+a)^2+b^2\}}}$$

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* MAXWELL's 'Electricity and Magnetism,' 2nd edition, § 706.

It is not necessary to give the details of the calculations, which have been carefully checked. The tabular interval being 6′, it was found desirable in many cases to proceed beyond the simple interpolation by first differences. The results are

$$\begin{aligned}M(A, a, b)&=215{\cdot}4674\\ M(A+h, a, b)&=205{\cdot}1917\\ M(A-h, a, b)&=226{\cdot}9835\\ M(A, a, b+k)&=211{\cdot}7246\\ M(A, a, b-k)&=217{\cdot}5972\end{aligned}$$

The mean coefficient for the area of the section is found by doubling the first of these values, adding in the others, and then dividing by 6.

Thus

$$M=215{\cdot}405^{*}$$

The separate values allow us to form an estimate of the effect of errors in the fundamental data. If we write

$$\frac{dM}{M}=\lambda\frac{dA}{A}+\mu\frac{db}{b}+\nu\frac{da}{a},$$

we may take approximately

$$\lambda=\frac{M(A+h, a, b)-M(A-h, a, b)}{2h}\div\frac{M}{A}=-1{\cdot}36$$

In like manner, $\mu=-{\cdot}02$, whence, since $\lambda+\mu+\nu=1$, $\nu=+2{\cdot}38$.

Series III. In this case the data remain precisely as before, except that we now have $b=15{\cdot}3472$.

We find

$$\begin{aligned}M(A, a, b)&=110{\cdot}9240\\ M(A+h, a, b)&=111{\cdot}2573\\ M(A-h, a, b)&=110{\cdot}2442\\ M(A, a, b+k)&=104{\cdot}5571\\ M(A, a, b-k)&=117{\cdot}6519\end{aligned}$$

whence

$$M=110{\cdot}926$$

Determining λ, μ, ν, as in the former case, we find

$$\frac{dM}{M}=+{\cdot}123\frac{dA}{A}-{\cdot}956\frac{db}{b}+1{\cdot}833\frac{da}{a}.$$

From these values, calculated for the circumference of the disc, we have to subtract

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* The factor expressing the number of windings is omitted.

the value (M_0) applicable to the small circuit touched by the inner brush. The area of this is $\frac{1}{4}\pi(2{\cdot}096)^2$. For the first and second series we have

$$M_0=\frac{2\pi A^2}{(A^2+b^2)^{\frac{3}{2}}}\,\tfrac{1}{4}\pi\,(2{\cdot}096)^2={\cdot}836$$

For the third series in like manner

$$M_0={\cdot}534$$

Thus finally for the first and second series

$$M-M_0=214{\cdot}569,$$

and for the third series

$$M-M_0=110{\cdot}392$$

The resistance coils.

§ 26. In all three series the resistance *b*, fig. 3, was a German-silver coil of about $\frac{1}{10}$, referred to for brevity as the $[\frac{1}{10}]$; and the resistance *a* was composed of three resistances in multiple arc, the first two being standard singles, and the third a resistance such as 7 B.A. units taken from a box. To make the necessary change, according to the plan already explained in § 9, the 7 would be replaced by 8. The value of *a* is of course determined principally by the unit resistance coils, and only secondarily by the resistance taken from the box.

The third element of the system of resistances was varied in the different series. In the first series *c* was a [10], in the second series it was [10]+[5]+[1], and in the third series [10]+[5]+[5′]. Besides the standard singles, whose values at various temperatures was already known in terms of the mean B.A. unit, we had to determine accurately the values of the $[\frac{1}{10}]$, the [10], the [5], and the [5′], as well as the small resistances of the various connecting pieces employed.

The [10] has been determined in various ways, but principally by means of the device referred to in the former paper.[*] Three German-silver wires of about 3 units each are wound on the same tube, and their terminals are so arranged that by means of a base board containing mercury cups they can be combined either in multiple arc or in series. In the former combination they are compared with a standard single, and the resistance is found to be (say) $1+\alpha$, where α is small. The coils are now without loss of time combined in series, a change which can be effected in a moment. The resistance in series is very approximately $9+9\alpha$; by the addition of the standard single it becomes $10+9\alpha$, and can now be compared with the [10]. If the difference observed be β we have $[10]=10+9\alpha+\beta$. By this method it is easy to obtain an accuracy of at least $\frac{1}{10000}$.

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[*] Phil. Trans., Part II., 1882, p. 697.

The [5]'s were determined in two ways. Five singles were combined in series and compared with one of the [5]'s; afterwards the two [5]'s were compared with one another. In the second method, which is probably preferable, the sum of the two [5]'s was found by comparison with the [10]. From the sum and difference the separate values can of course be deduced.

The measurement of the [$\frac{1}{10}$] demanded some precaution on account of its smallness. Two standard singles, the [10], and the $\frac{1}{10}$, were combined with four insulated mercury cups, and without the use of connecting pieces, so as to form a WHEATSTONE'S balance (fig. 5), care being taken to bring the associated battery and galvanometer terminals into immediate contact with the legs of the [$\frac{1}{10}$] (see § 12). To get the means of adjustment, a box, giving resistances up to 10,000, was placed in multiple arc with one of the singles. If, as was the case, the four coils be so nearly in proportion that a resistance of several hundreds from the box is needed for balance, the delicacy of the arrangement is all that can be desired. Readings are taken also with battery reversed, to eliminate thermo-electric disturbances. Especial pains were taken with the measurement of the [$\frac{1}{10}$], and of the [10], errors of which would be propagated into the results of all three series.

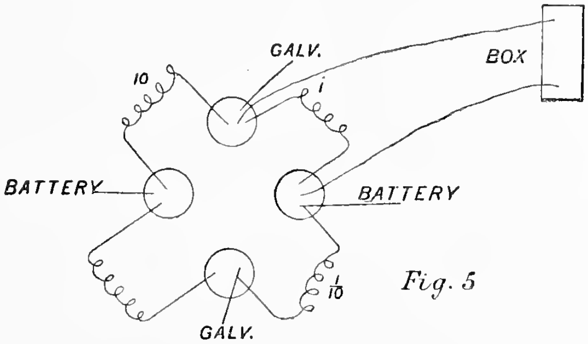

Fig. 5

§ 27. The various temperatures of the coils at the time of use, and the fluctuations from day to day, complicate the calculation of the effective resistances R_1 and R_2, which in principle is simple enough. The results are given in column II. of the Tables. Thus in the first series on July 14, when the effective resistance was ·0044076 B.A., as calculated from the values of a, b, c, for the observed temperatures of the coils, the effect (E–W) of reversing the battery key (corrected for direct effect) was -30 divisions of the galvanometer scale, the direction of rotation being positive. When the effective resistance was altered to ·0044430, the difference E–W became $+10$ divisions. From these results we infer that E–W would vanish for the effective resistance ·0044341, as given in column V. The corresponding result with negative rotation is given in column VI. These resistances relate to the actual speed of rotation determined by the frequency of vibration of the electric fork (§ 6). To render the results of different days fairly comparable, two small corrections have to be introduced, the first relating to small alterations in the relative frequencies of the two forks, as shown by the number of beats per minute (column VII.), the second to variations in the frequency of the standard fork itself, dependent upon change of temperature. The temperatures were read by a thermometer which stood between the prongs of the standard, and are given in column IX. The corrections necessary for reduction to a standard number of beats (16 per minute) and to a standard temperature (16°) are tabulated in columns VIII. and X., and the corrected results themselves in XI. and XII. In all cases the electric fork vibrated *more* quickly than the standard.

The degree of accordance in the numbers entered in these columns shows the success of the observations, so far as relates to errors of a casual character. In column XIII. the results of the positive and negative rotations are combined, so as to exhibit the total result of the day's work.

The Table, showing the results of the third series, is divided into two parts, corresponding to the two positions of the induction coils, before and after reversal (§ 19). In each position, it will be seen that two sets of observations were taken upon one of the days. Both sets, however, were complete, and in the interval between them the resistance coils were all dismounted. A similar precaution was taken at least once in each of Series I. and II.

First Series.

Coils near together.
Speed of disc about 12·8 revolutions per second.
Approximate resistances $a=\frac{1}{2}$, $b=\frac{1}{10}$, $c=10$.

Date.	Effective resistance (in B.A. units) used.	Difference of reading of galvanometer on reversal of current.		Effective resistance (in B.A. units) corresponding to zero difference in galvanometer.		Correction for change of speed of fork.				Effective resistance (in B.A. units) as finally corrected.		Means of resistances with both directions of rotation.
July, 1882.		Rotation +	Rotation −	Rotation +	Rotation −	Beats between forks.	Correction to 16 beats.	Temperature of standard fork.	Correction to 16°.	Rotation +	Rotation −	
14th „	·0044076 ·0044430	−30·0 +10·0	+32·0 − 6·3	·0044341	·0044371	17	−·0000006	° 16·7	+·0000003	·0044338	·0044368	·00443530
15th „	·0044084 ·0044438	−24·6 +10·5	+25·8 − 9·2	·0044333	·0044346	16	0	17·1	+·0000005	·0044338	·0044351	·00443445
17th „	·0044090 ·0044444	−25·4 +13·4	+27·0 −15·5	·0044322	·0044315	15	+·0000006	17·3	+·0000006	·0044334	·0044327	·00443305
18th „	·0044095 ·0044449	−27·4 +13·7	+25·9 −12·8	·0044331	·0044332	16	0	17·6	+·0000008	·0044339	·0044340	·00443395
19th „	·0044100 ·0044454	−23·9 +13·9	+27·4 −13·5	·0044324	·0044337	16	0	18·0	+·0000010	·0044334	·0044347	·00443405
20th „	·0044100 ·0044454	−24·4 +13·8	+25·4 −14·0	·0044326	·0044328	16	0	17·9	+·0000009	·0044335	·0044337	·00443360
									Means . .	·00443363	·00443450	·00443407

SECOND Series.

Coils near together.
Speed of disc about 8 revolutions per second.
Approximate resistances $a = \frac{1}{2}$, $b = \frac{1}{10}$, $c = 16$.

Date.	Effective resistance (in B.A. units) used.	Difference of reading of galvanometer on reversal of current.		Effective resistance (in B.A. units) corresponding to zero difference in galvanometer.		Correction for change of speed of fork.				Effective resistance (in B.A. units) as finally corrected.		Means of effective resistances with both directions of rotation.
August, 1882.		Rotation +	Rotation −	Rotation +	Rotation −	Beats between forks.	Correction to 72 beats.	Temperature of standard fork.	Correction to 16°.	Rotation +	Rotation −	
7th	·0027827	− 8·2	+ 9·4	·0027914	·0027918	73	−·0000004	17·6°	+·0000005	·0027915	·0027919	·00279170
„	·0028126	+20·1	−21·5									
8th	·0027821	− 8·6	+ 9·5	·0027908	·0027920	73	−·0000004	17·2	+·0000004	·0027908	·0027920	·00279140
„	·0028120	+21·0	−19·1									
9th	·0027826	− 7·9	+ 7·5	·0027912	·0027910	72	0	17·5	+·0000005	·0027917	·0027915	·00279160
„	·0028125	+19·5	−19·1									
									Means . .	·00279133	·00279180	·00279157

THIRD Series.

Coils separated.

Speed of disc about 12·8 revolutions per second.

Approximate resistances $a=\frac{1}{2}$, $b=\frac{1}{10}$, $c=20$.

Date.	Effective resistance in B.A. units used.	Difference of reading of galvanometer on reversal of current.		Effective resistance (in B.A. units) corresponding to zero difference in galvanometer.		Correction for change of speed of fork.				Effective resistance (in B.A. units) as finally corrected.		Means for each day of observations with both directions of rotation.	Means of all the observations with each direction of rotation.		Means of all observations with both directions of rotation.
August, 1882.		Rotation +	Rotation −	Rotation +	Rotation −	Beats.	Correction to 72 beats.	Temperature of standard fork.	Correction to 16°.	Rotation +	Rotation −		Rotation +	Rotation −	
								°							
14th "	·0022981 ·0023100	+ ·1 −11·0	− ·3 + 9·1	·0022982	·0022985	73	−·0000003	18·4	+·0000006	·0022985	·0022988	·00229865	·00229853	·00229910	·00229881
15th "	·0022976 ·0023095	− ·1 −11·7	− 1·9 +10·0	·0022975	·0022995	72	0	18·4	+·0000006	·0022981	·0023001	·00229910			
15th "	·0022973 ·0023092	+ 1·1 − 9·9	− ·6 +11·1	·0022985	·0022979	72	0	18·1	+·0000005	·0022990	·0022984	·00229870			
Second position of induction coils.															
16th "	·0022969 ·0023088	− ·3 −11·9	0 + 7·8	·0022966	·0022969	72	0	17·6	+·0000004	·0022970	·0022973	·00229715	·00229633	·00229650	·00229642
17th "	·0022965 ·0023084	− ·8 −11·2	+ ·7 +12·1	·0022956	·0022958	72	0	17·4	+·0000004	·0022960	·0022962	·00229610			
17th "	·0022966 ·0023085	− ·8 −10·2	+ 1·0 +12·4	·0022956	·0022956	72	0	17·4	+·0000004	·0022960	·0022960	·00229600			
												Means.	·00229743	·00229780	·00229762

§ 28. The results given in these tables are the effective resistances required to obtain a balance, expressed in terms of the B.A. unit. To reduce them to absolute measure we must multiply by 10^9, and by a factor, which we may call x, expressing the absolute value of the B.A. unit in terms of 10^9, and which it is our object to determine.

The actual value of the same quantities in absolute measure is found by multiplying the coefficients of induction $(M-M_0)$ already given (§ 25), by the number of turns in the coils 1588, and by the number of revolutions per second.

In the first series the frequency of vibration (f) of the electric tuning-fork was in the standard case (see Appendix)

$$f=\tfrac{1}{2}(128{\cdot}140+\tfrac{16}{60})=\tfrac{1}{2}\times 128{\cdot}407$$

and the number of revolutions per second is equal to $2f\div 10$. In the second and third series $2f=129{\cdot}340$, a number which in the second series is to be divided by 16, and in the third series by 10, in order to obtain the number of revolutions per second.

The equation to determine x is thus for the first series of observations

$$214{\cdot}569\times 1588\times 12{\cdot}8407=x\times {\cdot}00443407\times 10^9,$$

whence

$$x={\cdot}98674.$$

From the second series

$$x=\frac{214{\cdot}569\times 1588\times 129{\cdot}340}{16\times 10^9\times {\cdot}00279157}={\cdot}98669$$

From the third series

$$x=\frac{110{\cdot}392\times 1588\times 129{\cdot}340}{10\times 10^9\times {\cdot}00229762}={\cdot}98683$$

These are the final results already considered in § 21.

APPENDIX.

Frequency of Vibration of Standard Fork.

All our measurements, both by this method and by that of the revolving coil, being dependent upon the pitch of a standard tuning-fork, we have considered it advisable to determine this element afresh. As in the first determination,[*] a fork vibrating about 32 times per second rendered intermittent an electric current, which, passing

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[*] Proc. Roy. Soc., May, 1881, p. 137.

through the coils of small electromagnets, maintained in vibration not only the interrupter fork itself, but also a second fork of pitch about 128. After the apparatus has been a short time in operation, the vibrations of the second fork are exactly four times as quick as those of the first, independently of any precise tuning; and they give rise to audible beats when the standard fork is simultaneously excited. In the presence of extraneous noises the observation of the beats is much facilitated by the use of resonators, with one of which the ear may be connected by an indiarubber tube. The object to be aimed at is to make the intensities of the two sounds (as they reach the ear) very nearly equal. The moment of antagonism is then marked by a well defined silence, whose occurrence can be timed to within a second, although the whole duration of the beat may be 20 seconds or more. Without fresh bowing of the standard, the silences can be observed satisfactorily for at least a minute.

In the first determination the comparison between the fork of frequency 32, and the pendulum of the clock was made directly. The observer, looking over a plate carried by the upper prong of the fork, obtained 32 views per second, *i.e.*, 64 views of the pendulum in one complete vibration. The immediate subject of observation is a silvered bead attached to the bottom of the pendulum, upon which as it passes the position of equilibrium the light of a paraffin lamp is concentrated. Close in front of the pendulum is placed a screen perforated by a somewhat narrow vertical slit. If the period of the pendulum were a precise multiple of that of the fork, the flash of light which to ordinary observation would be visible at each passage, would either be visible, or be obscured, in a permanent manner. If, as in practice, the coincidence be not perfect, the flashes appear and disappear in a regular cycle, whose period is the time in which the fork gains (or loses) one complete vibration. This period can be determined with any degree of precision by a sufficient prolongation of the observations.

On account of the large number of views per second, the interval between successive visible positions of the bead, even when it is moving with maximum velocity, is rather small; and thus the adjustment of the apparatus is somewhat delicate.* In order to meet this objection, a modification has been introduced, which must now be explained.†

A few years ago it was shown almost simultaneously by La Cour and by Lord Rayleigh, that an electromagnetic engine could be accurately governed by an interrupter-fork. The construction (fig. 6) which has been found most suitable is similar to that of Froment's engine. A horizontal shaft revolving upon steel points carries a

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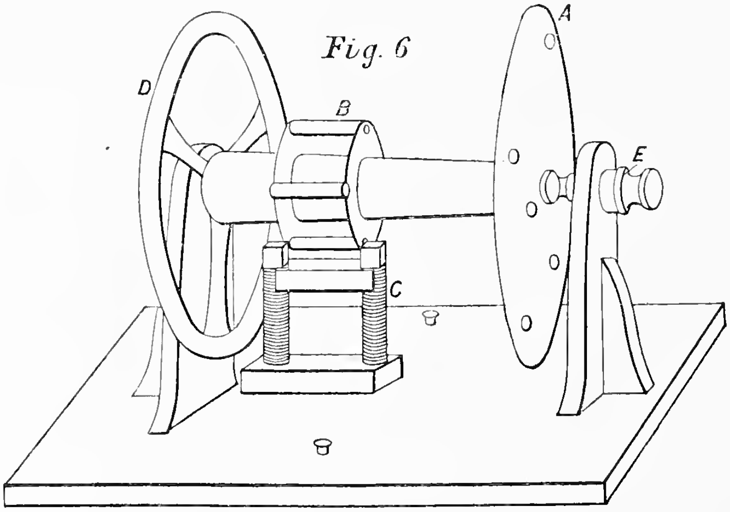
* In the earliest use of this method (" Nature," xvii., p. 12, 1877) the break-fork had a frequency of about 13, and no difficulty of this kind was experienced.

† [July, 1883.—It should be stated, however, that the wheel may easily be dispensed with, if proper care be taken in the illumination of the bead and in the management of the fork. The vibration should be vigorous, and the screens so arranged that the view past the fork at the moment of greatest elongation should be of short duration. Determinations by this method (without the wheel) have often been made successfully by students in the Cavendish Laboratory.]

number of parallel soft iron armatures, disposed symmetrically round the circumference. In the course of the revolution these armatures pass in succession between the poles of a vertical horse-shoe electromagnet, so as almost to complete the magnetic circuit. It is much better that the armatures should pass *between* the poles than *over* them, as in the most usual arrangement, for in the latter case the bearings are subjected to an unnecessary and prejudicial strain. The wheel may be used either with or without an independent driving power. In the former case the power should be very steady, and adjusted so as to give by itself nearly the speed intended. The currents from the interrupter-fork are passed also through the electromagnet of the engine, and give the force required to accelerate or retard the motion so that it may exactly synchronise with the fork, one armature passing for each complete vibration. If the independent power is in excess, the phase of the motion is such that the electromagnet is excited principally after the armatures have passed through the electromagnet; if the independent power is in defect, the electromagnet is excited principally while the armatures are approaching it. Within certain limits any necessary acceleration or retardation is obtained by suitable self-acting adjustment of phase.

Fig. 6



If when the wheel is moving steadily under the influence of the intermittent currents, a slight disturbance is communicated to it, oscillations will set in, the wheel being alternately in advance and in the rear of its proper position. In some cases these oscillations are very persistent, and interfere seriously with the utility of the instrument. To check them, a hollow ring filled with water is attached to the shaft, and revolves with it. When the rotation is perfectly regular, the water behaves as if it were a rigid body and offers no impediment to the motion, but it tends to check variations of speed of moderate period. The oscillations, when they exist, are usually audible; and in any case the behaviour of the wheel in this and other respects may be examined by looking at the interrupter-fork through a paper disc carried by the wheel and perforated symmetrically along a circle with holes equally numerous with the armatures. When all is regular, the prongs of the fork are seen in one phase only, so long as the eye retains a position fixed in space.

When the wheel runs lightly, independent driving power may be dispensed with, a sufficient amount of work being obtainable from the intermittent governing current. In the present case the whole apparatus, consisting of the two forks and the wheel, was driven by one current supplied from three GROVE cells. The only difficulty experienced is in starting the wheel. By means of string passed once round the shaft, alternately tightened for the advance and slackened for the return, it is easy to cause the wheel to achieve a speed in excess of the necessary eight revolutions per second. But it will not usually happen, every time the speed falls through the proper value, that the wheel will engage with the fork. For this purpose it is necessary that at the moment in question the phase of the wheel should be correct, within limits, which may be narrow when there is no great margin of power; and this can only happen by chance. Several attempts may be necessary before success is reached. With a little practice, however, there is no great loss of time, the ear learning to recognise, by the gradual slowing and subsequent quickening of a sort of beat, when the wheel has passed through the right speed without engagement. A fresh impulse is then given without waiting further. After a start is once effected, the wheel will usually run, keeping perfect time with the fork, until the battery is exhausted.

The wheel employed in the experiments we are now concerned with, has *four* soft iron armatures, and is governed by the interrupter-fork of frequency 32. The speed of the wheel is thus eight revolutions per second; and a single hole in a paper disc carried round with it allows eight views of the pendulum per second, the smallest number of views obtainable by direct use of the fork being 32. Altogether we may regard the frequency of the interrupter-fork as being multiplied four times precisely in the frequency of the auxiliary fork, and as divided four times precisely in the frequency of the wheel. The former is directly comparable with the standard fork, and the latter with the clock. The standard fork was screwed to the table precisely as during the electrical measurements. A thermometer placed between the prongs gave the temperature with fair accuracy.

The calculation of the results is very simple. Supposing in the first instance that the clock is correct, let a be the number of cycles per second (perhaps $\frac{1}{40}$) between the wheel and the clock. Since the period of a cycle is the time required for the wheel to gain, or to lose, one revolution upon the clock, the frequency of revolution is $8 \pm a$. The frequency of the auxiliary fork is precisely 16 times as great, *i.e.*, $128 \pm 16a$. If b be the number of beats per second between the two forks, the frequency of the standard is

$$128 \pm 16a \pm b$$

To give an idea of the magnitudes of the numbers concerned, it will be advisable to quote in detail the results of one day's observations. On October 19, with a certain loading of the interrupter-fork, the cycle of the pendulum occupied about 78 seconds, and the beats were at the rate of about six per minute. The interrupter was then *sharpened*, after which several observations were taken of the duration of five cycles of the pendulum, and of 16 beats between the forks. For the former the times found were 210, 210, 212 seconds; for the latter by simultaneous observation 58, 58, 59, 59, 59, 60, 60 seconds. The temperature, as given by the thermometer, ranged from 17°·2 to 17°·4. After the sharpening of the interrupter, the frequency both of the wheel and of the auxiliary fork was increased, so that the sign of $16a$ in the expression written above is determined to be + and that of b to be −. Using the mean values we find

$$16a = \cdot 3797, \qquad b = \cdot 2712$$

whence

$$128 + 16a - b = 128 \cdot 108$$

To this we must add ·009, making altogether 128·117, to allow for the gaining rate of the clock, which was $6\frac{1}{2}$ seconds per diem. This corresponds to a mean temperature 17°·3.

The procedure adopted was quite good enough for our purpose; but if it were desired to push the power of the method to its limit, the work should be undertaken at an astronomical observatory, and extended over the whole time required to rate the clock by observations of the stars. In this way the comparison of the period of vibration of the standard fork with the mean solar second could be effected with the same degree of accuracy as that to which the former quantity is capable of definition. Without this precaution we cannot be quite sure that the rate of the clock at the time of the observations is identical with the mean rate employed in the calculation. It is scarcely necessary to say that the uncertainty which arises under this head is common to every method by which absolute pitch could be determined.

The results obtained, including those recorded previously,[*] are given in the accompanying table. They are well represented by the formula

$$128 \cdot 140 \times \{1 - (t - 16)^0 \times \cdot 00011\},$$

in which the temperature coefficient used (·00011) is that found by M'Leod and Clarke.[†] The numbers in the fourth column are calculated from the formula.

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[*] Proc. Roy. Soc., May, 1881, p. 138.

[†] Phil. Trans., Part I., 1880.

Date.	Temperature.	Frequency by observation.	Frequency by calculation.
1881	13°	128·180	128·182
1881	14°·6	128·161	128·160
October, 1882 . .	15°·98	128·141	128·140
October, 1882 . .	17°·45	128·122	128·120
October, 1882 . .	17°·6	128·119	128·118
October, 1882 . .	17°·3	128·117	128·122

Of the small discrepancies which the table exhibits it is probable that the larger part is due to imperfect knowledge of the actual temperatures of the standard fork. The use of screens to cut off radiation from the observers would probably have effected an improvement. For the highest accuracy some sort of jacket, or chamber, would have to be contrived.

SECOND APPENDIX.

(Added July, 1883.)

On the Effect of the Imperfect Insulation of Coils.

In a former paper (Phil. Trans., 1882, Part II.) it was pointed out that the method of the revolving coil, employed by the first B.A. Committee, possesses the important advantage that it is possible to detect the existence of leakage from turn to turn, or from layer to layer, of the coil of wire. The general influence of such leakage, if undetected, upon the final number x expressing the ratio of the resistance of the coil when measured (R) in absolute units to its resistance $r\times 10^9$ as referred to B.A. units, is easily seen by supposing that one turn of the coil is simply short-circuited. The formula in C.G.S. measure is

$$x=\frac{\mathrm{R}}{r\times 10^9}=\frac{\pi^2 n^2 a\,\omega\cot\phi}{r\times 10^9}\quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (1)$$

During the revolutions the short circuited turn produces its full effect in deflecting the magnet, and error arises only in the comparison with the standard of resistance. The quantity r will evidently be under-estimated by $1/n$, and this will lead to an *over*-estimate of x, also by $1/n$. This result, however, is modified, if as in practice we take only the *difference* of effects observed when the wire contact is open and closed. The short-circuited turn will produce its effect in *both* cases, and its influence will therefore disappear from the result. For all purposes it will be virtually non-existent, and the error produced is the same as if n had simply been miscounted. The final number x will thus be over-estimated by the fraction $2/n$.

In LORENTZ's method the effect of a short circuit in the induction coil is in the same direction. M, and therefore R and x, will be over-estimated by $1/n$.

If we examine the formulæ applicable to determinations by other methods, we shall see that a similar conclusion holds good, so that in every case leakage leads to an over-valuation of x, at least whenever the result is calculated from the number of turns of wire in a coil.* Even without such an examination, it is pretty evident from consideration of the magnitudes involved that the large factor 10^9 in the denominator of the formula corresponding to (1) can only be compensated by one or more large factors expressive of the number of windings in a coil or coils. An over-valuation of these factors, due to leakage, will therefore lead to an over-valuation of x.

In carefully constructed coils serious leakage is, perhaps, not likely to occur, but its presence in a smaller degree is more probable, and is usually difficult of detection. So far as this argument applies, we may say that the smaller values of the number expressive of the B.A. unit, or of the mercury unit, in absolute measure are to be preferred to the larger.

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* The case is different when the constants of a coil of many turns are determined by electrical comparison, as for instance in KOHLRAUSCH's recent correction of the constant of his earth-inductor.

VIII. *On* Abel's *Theorem and Abelian Functions.*

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Communicated by Professor Cayley, *F.R.S.*

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The present paper is divided into two sections. The object of Section I. is to obtain an expression for an integral more general than, but intimately connected with, that occurring in Abel's theorem. The latter, as enunciated by Professor Rowe in his memoir in the Phil. Trans., 1881, is as follows:—If

$$\chi(x, y)=0$$

be a rational algebraical equation between x and y, then an expression can always be found for

$$\Sigma\int\frac{\mathrm{U}dx}{f(x)\frac{\delta\chi}{\delta y}}$$

where $f(x)$ is a function of x only, U a rational algebraical integral function of x and y, and the upper limits of the series of integrals are the roots of the eliminant with regard to y of $\chi(x, y)=0$ and a function $\theta(x, y)$.

In the case here considered two equations respectively of the degrees m and n between three variables

$$\mathrm{F}_m(x, y, z)=0$$
$$\mathrm{F}_n(x, y, z)=0$$

are given (these alone being considered, as subsequent generalisation to the case of r equations between r dependent variables and one independent variable is obvious); and an expression is obtained for

$$\Sigma\int\frac{\mathrm{U}dx}{f(x)\mathrm{J}\left(\frac{\mathrm{F}_m,\ \mathrm{F}_n}{y,\ z}\right)}$$

the upper limits of the integrals being given by the roots of the equation arrived at by eliminating y and z between F_m, F_n and an arbitrary equation

$$\mathrm{F}_p(x, y, z)=0$$

or, what is the same thing, by the co-ordinates x of the points of intersection of the three surfaces represented by F_m, F_n, F_p.

Some preliminary considerations (in connexion with §§ 92 *sqq.* of SALMON's 'Higher Algebra') are adduced in reference to the eliminants of the three equations in each of the variables; thus if X be the equation in x obtained by eliminating y and z, it is expressed in the form

$$X=B_mF_m+B_nF_n+B_pF_p$$

which afterwards proves useful. Then the ordinary case (above referred to) of ABEL's theorem is treated on the lines laid down in CLEBSCH and GORDAN's 'Treatise on the Abelian Functions;' and under the guidance of this the more general form is investigated with the result

$$\Sigma\int\frac{Udx}{f(x)J\left(\dfrac{F_m,\ F_n}{y,\ z}\right)}=\Theta\left[\frac{1}{f(x)}\right]\Sigma\left\{\frac{U}{J}\log F_p\right\}+C$$

Θ being the symbol introduced by BOOLE.

The remainder of this section is occupied with the discussion of two examples of this theorem. In the first, expressions are obtained for $E(u_1+u_2+u_3)$ and $\Pi(u_1+u_2+u_3)$, E and Π being the second and third elliptic integrals; and in the second example $E(u_1+u_2+\ldots+u_7)$ is considered.

In Section II. the addition-theorem for the functions presented in WEIERSTRASS's memoir ('Crelle,' t. lii., (1856), p. 285) is investigated. [It may be pointed out that the fundamental equations occur as natural examples of the more general form of ABEL's theorem proved in Section I.; but the equations which are obtained almost immediately are identical with those used by WEIERSTRASS, and so this case does not belong distinctively to the form of ABEL's theorem for the curve of double curvature.] For the purpose of the section use is made of the "integral-function," the partial differential coefficients of which with respect to the amplitudes give the squares of the Abelian functions. The theory is worked out at some length, and the necessary formulæ are deduced from the fundamental equations in a manner somewhat different from that of WEIERSTRASS. From the form first obtained for the sum of three integral-functions an important theorem is deduced in § 21, and a verification of this is afterwards furnished by the expansion of the two sides of the equation. It is then applied, as already mentioned, to obtain the addition-theorem for the functions.

In §§ 25, 26 is given the discussion of a particular case of the above, viz., when the functions are of the order 2, the fifteen functions being the quotients of all but one of the double theta-functions by that one. This has already formed the subject of a paper by CAYLEY in 'Crelle,' t. lxxxviii. (1878), p. 74.

SECTION I.

1. Before proceeding to the consideration of the theorem it is necessary to indicate the form in which the eliminant of three equations in three variables (or in general of μ equations in μ variables) will be used.

If we consider two equations in two variables say

$$\begin{aligned} f_n &\equiv x_0 + x_1 y + x_2 y^2 + \dots + x_n y^n = 0 \\ \phi_m &\equiv X_0 + X_1 y + X_2 y^2 + \dots + X_m y^m = 0 \end{aligned}$$

and if X be the eliminant of f_n and ϕ_m with regard to y, then we have

$$X = Af + B\phi.$$

Now X being of degree mn in x, A must be of degree $mn-n$ and B of $mn-m$; while it is sufficient that the highest power of y in A be the $(m-1)^{\text{th}}$ and in B the $(n-1)^{\text{th}}$. Write then

$$\begin{aligned} A &= A_0 + A_1 y + A_2 y^2 + \dots + A_{m-1} y^{m-1} \\ B &= B_0 + B_1 y + B_2 y^2 + \dots + B_{n-1} y^{n-1}. \end{aligned}$$

Substitute in X; since X is explicitly free from y all the coefficients of powers of y in the result must be zero. This then gives

$$\begin{array}{lll} A_0 x_1 + A_1 x_0 & + B_0 X_1 + B_1 X_0 & = 0 \\ A_0 x_2 + A_1 x_1 + A_2 x_0 & + B_0 X_2 + B_1 X_1 + B_2 X_0 & = 0 \\ A_0 x_3 + A_1 x_2 + A_2 x_1 + A_3 x_0 & + B_0 X_3 + B_1 X_2 + B_2 X_1 + B_3 X_0 & = 0 \\ \dots\dots & \dots\dots & \end{array}$$

$m+n-1$ equations to determine the ratios of the $m+n$ quantities A, B. Let

$$E = \left| \begin{array}{cccccccccc} 1 & y & y^2 & \dots & y^{m-1} & 0 & 0 & & \dots & 0 \\ x_1 & x_0 & 0 & \dots & 0 & X_1 & X_0 & & \dots & \\ x_2 & x_1 & x_0 & \dots & 0 & X_2 & X_1 & X_0 & \dots & \\ \dots & & & & & \dots & & & & \end{array} \right|$$

$$F = \left| \begin{array}{cccccccccc} 0 & 0 & \dots & & 0 & 1 & y & y^2 & \dots & y^{n-1} \\ x_1 & x_0 & 0 & \dots & & X_1 & X_0 & 0 & \dots & \\ x_2 & x_1 & x_0 & \dots & & X_2 & X_1 & X_0 & \dots & \\ \dots & & & & & \dots & & & & \end{array} \right|$$

Then A_r bears to the minor of y^r in E the same ratio as B_s bears to the minor of y^s in F: thus

$$\frac{A}{E} = \frac{B}{F}.$$

But the diagonal term in E is

$$x_0^{m-1} X_m^{\ n}$$

and E is therefore of the degree $n(m-1)$, *i.e.*, of the same degree in X as A; it is obviously of the same degree in y; hence $\frac{A}{E}$ is merely an arithmetical constant, and we may write

$$A=E, \quad B=F.$$

2. When we come to apply this method to the formation of the eliminant with regard to y and z of three equations F_1, F_2, F_3 in three variables x, y, z the result, though of similar form, viz.:

$$A_{x,1}F_1+A_{x,2}F_2+A_{x,3}F_3,$$

can in general be obtained neither so directly nor without the help of the considerations in SALMON's 'Higher Algebra,' §§ 92, *sqq.* If the three equations be each of the degree 2, the method will apply exactly as in the preceding paragraph and we obtain

$$X=A_{x,1}F_1+A_{x,2}F_2+A_{x,3}F_3$$

where

$$A_{x,r}=\alpha_{x,r}+\beta_{x,r}x+\gamma_{x,r}y\,;$$

but if the equations be not of this degree, then the following is our rule. Let F_1, F_2, F_3 be of the degrees m, n, p respectively: then we form all possible equations, which the variables satisfy, of degree not higher than $m+n+p-2$: thus we multiply F_1 by

$$y^{n+p-2},\ y^{n+p-3},\ \ldots,\ y^{n+p-3}z,\ y^{n+p-4}z,\ \ldots,\ y^{n+p-4}z^2,\ y^{n+p-5}z^2,\ \ldots$$

and so on; and so we obtain

$$\tfrac{1}{2}(n+p-1)(n+p)+\tfrac{1}{2}(p+m-1)(p+m)+\tfrac{1}{2}(m+n)(m+n-1)$$

equations from which to eliminate

$$\tfrac{1}{2}(m+n+p-1)(m+n+p)$$

quantities. But these equations are not all independent, being connected by a number of identities of the form

$$z^r y^s F_1.F_2=z^r y^s F_2.F_1$$

(where $r+s\leqq p-2$), of which there are $\frac{1}{2}p(p-1)$; there are $\frac{1}{2}n(n-1)$ of the form

$$z^{r'}y^{s'}F_1.F_3=z^{r'}y^{s'}F_3.F_1 \text{ where } r'+s'\leqq n-2$$

and $\frac{1}{2}m(m-1)$

$$z^{r''}y^{s}{}'F_2.F_3=z^{r''}y^{s''}F_3.F_2 \text{ where } r''+s''\leqq m-2,$$

and thus we have the proper number of equations. To find the eliminant X we write down the coefficients (which are, of course, functions of x) in the $\frac{1}{2}(n+p-1)(n+p)+\ldots$ equations; and reduce them to the form of a determinant by adding the coefficients in the $\frac{1}{2}p(p-1)+\ldots$ equations; then the eliminant is the quotient of the determinant formed by any

$$\tfrac{1}{2}(m+n+p-1)(m+n+p)$$

rows of the set first written down by the determinant formed from the second set after the elision of these rows.

3. To show how this can be brought into the desired form the easiest plan will be to consider an example. Let

$$F_1 \equiv Ay+Bz+C$$
$$F_2 \equiv A'y^2+F'yz+B'z^2+E'y+D'z+C'$$
$$F_3 \equiv A''y^3+D''y^2z+E''yz^2+B''z^3+F''y^2+G''yz+J''z^2+H''y+K''z+C''$$

where the coefficient of the highest powers of z and y are constants and those of other powers are functions of x such as make the order of the highest expression in the term of the same order as the equation; thus, for instance

$$F''=fx+f'$$
$$H''=hx^2+h'x+h''$$

and so on. Then we have, since $m=1$, $n=2$, $p=3$

	y^4	y^3z	yz^2	yz^3	z^4	y^3	yz	yz^2	z^3	y^2	yz	z^2	y	z	1	$yF_2.F_1$	$zF_2.F_1$	$F_2.F_1$	$F_3.F_1$
y^3F_1	A	B				C										A′			A″
y^2zF_1		A	B				C									F′	A′		D″
yz^2F_1			A	B				C								B′	F′		E″
z^3F_1				A	B				C								B′		B″
y^2F_1						A	B			C						E′		A′	F″
yzF_1							A	B			C					D′	E′	F′	G″
z^2F_1								A	B			C					D′	B′	J″
yF_1										A	B		C			C′		E′	H″
zF_1											A	B		C			C′	D′	K″
F_1													A	B	C			C′	C″
y^2F_2	A′	F′	B′			E′	D′			C′						A			
yzF_2		A′	F′	B′			E′	D′			C′					B	A		
z^2F_2			A′	F′	B′			E′	D′			C′					B		
yF_2						A′	F′	B′		E′	D′		C′			C		A	
zF_2							A′	F′	B′		E′	D′		C′			C	B	
F_2										A′	F′	B′	E′	D′	C′			C	
yF_3	A″	D″	E″	B″		F″	G″	J″		H″	K″		C″						A
zF_3		A″	D″	E″	B″		F″	G″	J″		H″	K″		C″					B
F_3						A″	D″	E″	B″	F″	G″	J″	H″	K″	C″				C

To find the eliminant we choose any 15 rows (leaving out say the y^2F_1, yzF_1, zF_2, yF_3) and form a determinant, and then divide by

$$\left|\begin{array}{cccc} E' & 0 & A' & F'' \\ D' & E' & F' & G'' \\ 0 & C & B & 0 \\ 0 & 0 & 0 & A \end{array}\right| = u.$$

(As the object is to illustrate the general case and not merely to get the result in this particular case we have not selected those rows which leave the denominator in the simplest form.) In the determinant of 15^2 constituents multiply each column by the quantity which stands at the head of it, add the results horizontally along all the rows, and replace the constituents of the last column by these new constituents which are, in order,

$$y^3F_1,\ y^2zF_1,\ yz^2F_1,\ z^3F_1,\ z^2F_1,\ yF_1,\ zF_1,\ F_1,\ y^2F_2,\ yzF_2,\ z^2F_2,\ yF_2,\ F_2,\ zF_3,\ F_3,$$

so that if we expand we have the numerator of our eliminant in the form

$$A'_{x,1}F_1+A'_{x,2}F_2+A'_{x,3}F_3$$

where the A's are determinants differing from the initial determinants in the last column alone; $A'_{x,1}$ has for its constituents there the coefficients of F_1 so long as F_1 occurs in the later form and then zeros; $A'_{x,2}$ those of F_2 where it occurs and elsewhere zeros; $A'_{x,3}$ those of F_3 where it occurs and elsewhere zeros. Moreover, we know that our eliminant is an integral function of x not extending in an infinite series; hence each of the coefficients A' must be divisible by u. If not, one of the F's (say F_1) must be so divisible; since u is a function of x only it follows that, when $u=0$, $F_1=0$ whatever z and y may be. We shall assume that such factors are removed before the investigation begins as they are useless for the purposes for which the functions are required; and hence we obtain our eliminant in the form

$$X=A_{x,1}F_1+A_{x,2}F_2+A_{x,3}F_3.$$

Similar remarks of course apply to Y and Z, the eliminants with regard to z and x, y and x.

4. We may also obtain the result as follows:

Between $F_m=0$, $F_p=0$ eliminate z and denote the eliminant by X_y; then, as we have already seen, X_y can be expressed in the form

$$X_y=\lambda_m F_m+\lambda_p F_p.$$

Between $F_n=0$, $F_p=0$ eliminate z and denote this eliminant by X'_y; then

$$X'_y=\mu_n F_n+\mu_p F_p.$$

Now X_y, X'_y are both functions of x and y; eliminating y between them and denoting the eliminant by X we have

$$\begin{aligned} X &= \rho_y X_y + \rho'_y X'_y \\ &= A_m F_m + A_n F_n + A_p F_p \end{aligned}$$

of the same form as before.

This method of expressing an eliminant obviously admits of generalisation to the case of r equations in r variables.

5. The preceding method enables us to obtain the eliminants of the equations with regard to the different variables in a particular form which is useful in the proof of the general theorem in § 7; but when the object is merely to obtain the equation giving the roots x_μ which are to form the upper limits of our integrals we should arrive at the result more easily as follows.

Obviously

$$X = \prod_{\mu=1}^{\mu=mn} F_p(x, y_\mu, z_\mu)$$

where y_μ, z_μ constitute one of the mn pairs of roots of the equations

$$F_m = 0 \quad F_n = 0$$

regarded as giving y, z in terms of x; and the product is taken over all these pairs. Now the coefficients on the right-hand side will be symmetric functions of y and z, and these can be evaluated (by the method given in SALMON'S 'Higher Algebra,' § 74) in terms of x; and there will be obtained the required equation in x.

ABEL'S *Theorem.*

6. Let

$$\chi(x, y) = 0 \quad \ldots\ldots\ldots\ldots \quad \text{(i)}$$

be an equation of the degree n which gives y in terms of x; and let

$$\theta(x, y)$$

denote a function of x of degree m—reducible to degree $n-1$ at most in y by means of (i)—the coefficients of y in which are functions of x and contain any number of arbitrary constants. Treating $\chi=0$, $\theta=0$ as two equations to determine the values of the variables, these arbitrary constants will enter into the expressions for the values of x, and will therefore vary when the latter vary. Let such a variation take place, so that

$$\frac{\delta\theta}{\delta x}dx + \frac{\delta\theta}{\delta y}dy + \delta\theta = 0 \quad \ldots\ldots\ldots\ldots \quad \text{(ii)}$$

δ operating only on the constants in θ. Moreover we have from (i)

$$\frac{\delta\chi}{\delta x}dx+\frac{\delta\chi}{\delta y}dy=0$$

or writing

$$dw=\frac{dx}{\frac{\delta\chi}{\delta y}}=\frac{dy}{-\frac{\delta\chi}{\delta x}}$$

the equation (ii) becomes

$$-dw\left(\frac{\delta\chi}{\delta x}\frac{\delta\theta}{\delta y}-\frac{\delta\theta}{\delta x}\frac{\delta\chi}{\delta y}\right)+\delta\theta=0$$

or

$$dw=\frac{\delta\theta}{\mathrm{J}(\chi,\theta)}$$

and therefore

$$\Sigma\mathrm{U}dw=\Sigma\frac{\mathrm{U}}{\mathrm{J}(\chi,\theta)}\delta\theta\,. \qquad \text{(iii)}$$

where U is any rational function of x and y, and the summation is taken over all the roots x_μ of the equation obtained by the elimination of y between χ and θ.

Let X, Y respectively denote the eliminants of χ, θ with regard to y, x; then we can express X, Y in the form

$$\left.\begin{aligned}\mathrm{X}&=\mathrm{A}\chi+\mathrm{B}\theta\\ \mathrm{Y}&=\mathrm{C}\chi+\mathrm{D}\theta\end{aligned}\right\} \qquad \text{(iv)}$$

and we write

$$\Delta=\mathrm{AD}-\mathrm{BC}.$$

Now whatever the function U may be it can be written in the form

$$\frac{\mathrm{T}}{f(x)};$$

for it must be expressible as

$$\frac{f_2(x,y_1)}{f_1(x,y_1)}$$

that is,

$$\frac{f_2(x,y_1)\prod\limits_{\mu=2}^{\mu=n}f_1(x,y_\mu)}{\prod\limits_{\mu=1}^{\mu=n}f_1(x,y_\mu)}$$

which by means of the equation $\chi=0$ is at once reduced to the above form; thus

$$\Sigma\frac{\mathrm{T}}{f(x)}\frac{dx}{\frac{\delta\chi}{\delta y}}=\Sigma\frac{\mathrm{T}}{f(x)}\frac{\delta\theta}{\mathrm{J}(\chi,\theta)} \qquad \text{(v).}$$

Each of the equations $X=0$, $Y=0$ has mn roots; if $x=x_\mu$, $y=y_{\mu'}$ be a pair which make χ and θ both vanish, these may be called congruous; but if $x=x_\mu$, $y=y_{\mu'}$ do not make χ and θ both vanish then for these

$$\Delta=0,$$

while for congruous values Δ does not vanish.

Moreover, for congruous values of x and y we at once have

$$\frac{dX}{dx}\cdot\frac{dY}{dy}=\Delta.J(\chi,\theta)$$

the congruous values being substituted for the variables, so that

$$\frac{1}{J(\chi,\theta)}=\frac{\Delta}{\frac{dX}{dx}\cdot\frac{dY}{dy}}.$$

Now in (v) the summation is for all the x's and for one of the y's, say $y_{1,\mu}$, which may be regarded in the following way. When the equation

$$\chi(x,y)=0$$

is solved for y in terms of x, there will be n roots; take one of these and denote it by y_1, which is therefore a function of x. Substitute in turn $x_1, x_2, \ldots, x_{mn}$; then we obtain for y_1 a series of values, but all derived from the single root of χ. Thus

$$\Sigma\frac{T}{x-a}\,\frac{\delta\theta}{J(\chi,\theta)}=\Sigma\frac{T_\mu}{x_\mu-a}\,\delta\theta_\mu\,\frac{\Delta_\mu}{\frac{dX}{dx}\cdot\frac{dY}{dy}}$$

since we have $x=x_\mu$, $y=y_{1,\mu}$ ($\mu=1, 2, \ldots, mn$) as the mn congruous roots. Moreover for roots other than these

$$\Delta=0$$

so that we may add on a number of vanishing terms to the right-hand side, and the removal of the restriction now gives

$$\Sigma\frac{T_\mu}{x_\mu-a}\,\delta\theta_\mu\,\frac{\Delta_\mu}{\frac{dX}{dx_\mu}\,\frac{dY}{dy_{\mu'}}}$$

(where $\mu'=1,\mu$ or $2,\mu$ or $\ldots$ or n,μ).

Moreover from (iv)

$$\Delta\chi=DX-BY$$

and therefore

$$\Delta_\mu\frac{\delta\chi}{\delta y_\mu}=-B_{\mu,\mu'}\frac{dY}{dy_{\mu'}}$$

the term corresponding to a differential of X disappearing and the others vanishing in virtue of the values assigned to the two variables; thus our expression becomes

$$-\Sigma \frac{T_\mu}{x_\mu-\alpha}\delta\theta_\mu \frac{B_{\mu\mu'}}{\frac{\delta\chi}{\delta y_{\mu'}}\cdot\frac{dX}{dx_\mu}}.$$

But the coefficient of $\frac{1}{x}$ in the expansion of $\frac{V}{X}$ in descending powers of x is

$$C_{\frac{1}{x}}\Sigma \frac{V_\iota}{(x-x_\iota)\frac{dX}{dx_\iota}}$$

$$=\Sigma \frac{V_\iota}{\frac{dX}{dx_\iota}}$$

and therefore the foregoing

$$=-C_{\frac{1}{x}}\Sigma \frac{T}{x-\alpha}\delta\theta \frac{B_{\mu'}}{X\frac{\delta\chi}{\delta y_{\mu'}}}$$

the Σ referring to the n values of y obtained from the equation (i), and the expansion being in the factors of X alone. But since we are substituting for y from (i) we have χ always zero in this, and therefore

$$X=B_{\mu'}\theta.$$

Taking now into account the expansion for the factor $\frac{1}{x-\alpha}$, we have finally

$$\Sigma \frac{T}{x-\alpha}\frac{\delta\theta}{J(\chi,\theta)}=-C_{\frac{1}{x}}\Sigma \frac{T}{x-\alpha}\frac{\delta\theta}{\theta}\frac{1}{\frac{\delta\chi}{\delta y}}+\Sigma\left[\frac{T}{\frac{\delta\chi}{\delta y}}\frac{\delta\theta}{\theta}\right]_{x=\alpha}$$

the summation in each of the terms on the right-hand side being for the n values of y. Now on the introduction of BOOLE's symbol Θ (*cf.* Phil. Trans., 1857, p. **751**), the right-hand side is merely the definition of

$$\Theta\left[\frac{1}{x-\alpha}\right]\Sigma\frac{T}{\frac{\delta\chi}{\delta y}}\frac{\delta\theta}{\theta}.$$

Let $x=\alpha$ be a root of $f(x)=0$, and in (v) expand $[f(x)]^{-1}$ in a series of partial fractions corresponding to the roots; then expressions of the form $\frac{T}{x-\alpha}\frac{\delta\theta}{J(\chi,\theta)}$ are obtained. Moreover, from the nature of the preceding fractions and the definition of the symbol Θ in connexion with them, it is obviously a distributive symbol; thus we have

$$\Sigma\frac{\mathrm{T}}{f(x)}\frac{dx}{\frac{\partial\chi}{\partial y}}=\Sigma\frac{\mathrm{T}}{f(x)}\frac{\delta\theta}{\mathrm{J}(\chi,\theta)}$$

$$=\Theta\left[\frac{1}{f(x)}\right]\Sigma\frac{\mathrm{T}}{\frac{\partial\chi}{\partial y}}\frac{\delta\theta}{\theta}$$

where the summation on the left-hand side is over the roots of the equation

$$\mathrm{X}=0$$

while on the right-hand side it is over the n roots y of the equation $\chi(x, y)=0$. The variables on the right-hand side being the arbitrary constants in θ which occur only in the factor $\frac{\delta\theta}{\theta}$, we may integrate, and we have as the result

$$\sum_{\mu=1}^{\mu=mn}\int^{x_\mu}\frac{\mathrm{T}}{f(x)}\frac{dx}{\frac{\partial\chi}{\partial y}}=\Theta\left[\frac{1}{f(x)}\right]\sum_{y=y_1}^{y=y_n}\left\{\frac{\mathrm{T}}{\frac{\partial\chi}{\partial y}}\log\theta\right\}+\mathrm{C}$$

agreeing with the form given in Professor ROWE's memoir (Phil. Trans. 1881, p. 721).

7. In the generalisation of the theorem we shall consider only two dependent variables y, z and one independent variable x; it will be seen that the work would apply, *mutatis mutandis*, to $k-1$ dependent variables and one independent. Let the variables y and z be given as functions of x by the equations

$$\mathrm{F}_m(x, y, z)=0$$
$$\mathrm{F}_n(x, y, z)=0$$

of the degrees m and n respectively. Let

$$\mathrm{F}_p(x, y, z)$$

be a function of x, y and z the coefficients of y and z in which are functions of x with any number of arbitrary constants; so that as in the simple case when z, x and y vary the constants also vary. Using the same notation as before we have

$$\frac{\partial\mathrm{F}_m}{\partial x}dx+\frac{\partial\mathrm{F}_m}{\partial y}dy+\frac{\partial\mathrm{F}_m}{\partial z}dz=0,$$

$$\frac{\partial\mathrm{F}_n}{\partial x}dx+\frac{\partial\mathrm{F}_n}{\partial y}dy+\frac{\partial\mathrm{F}_n}{\partial z}dz=0.$$

Therefore

$$\frac{dz}{\frac{\partial(\mathrm{F}_m,\mathrm{F}_n)}{\partial(x,y)}}=\frac{dx}{\frac{\partial(\mathrm{F}_m,\mathrm{F}_n)}{\partial(y,z)}}=\frac{dy}{\frac{\partial(\mathrm{F}_m,\mathrm{F}_n)}{\partial(z,x)}}=dw$$

and

$$\frac{\delta F_p}{\delta z}dz+\frac{\delta F_p}{\delta x}dx+\frac{\delta F_p}{\delta y}dy+\delta F_p=0$$

so that

$$dw.J\left(\frac{F_m,\ F_n,\ F_p}{x,\ y,\ z}\right)+\delta F_p=0$$

and therefore

$$-\Sigma T dw=\Sigma\frac{T}{J\left(\frac{F_m,\ F_n,\ F_p}{x,\ y,\ z}\right)}\delta F_p \quad \ldots\ldots\ldots \quad \text{(i).}$$

Let Z, X and Y be the eliminants of F_m, F_n and F_p respectively with regard to x and y, y and z, z and x; then as explained in § 3 we may write

$$X=A_mF_m+A_nF_n+A_pF_p$$
$$Y=B_mF_m+B_nF_\kappa+B_pF_p$$
$$Z=C_mF_m+C_nF_n+C_pF_p.$$

Each of the equations $z=0$, $x=0$, $y=0$, has mnp solutions; let those values (mnp in all) which make F_m, F_n and F_p vanish simultaneously be called congruous.

Write

$$\Delta=\begin{vmatrix} A_m & A_n & A_p \\ B_m & B_n & B_p \\ C_m & C_n & C_p \end{vmatrix}$$

so that for non-congruous values Δ is zero.

Now whatever be the value of T it can be put into the form

$$\frac{\Psi(x,\ y,\ z)}{\Phi(x,\ y,\ z)}$$

where Φ and Ψ are rational integral algebraical functions of x, y and z; and this can be expressed as

$$\frac{U}{f(x)}$$

where U is an integral function of x, y and z and $f(x)$ is a function of x only. For it is

$$\frac{\Psi(x,\ y_1,\ z_1)\prod\limits_{\mu=2}^{\mu=mn}\Phi(x,\ y_\mu,\ z_\mu)}{\prod\limits_{\mu=1}^{\mu=mn}\Phi(x,\ y_\mu,\ z_\mu)}$$

where y_μ, z_μ are a pair of values of y and z which satisfy the equations

$$F_m=0,\quad F_n=0.$$

Now the denominator will consist of symmetric functions of the y's and z's, the coefficients in its different terms involving x. These can be expressed in terms of x alone,[*] while the new term in the numerator can be expressed in terms of symmetric functions of the y's and z's and of y_1 and z_1, and thus T is reduced to the form

$$\frac{U}{f(x)}$$

and therefore

$$-\Sigma T dw = \Sigma \frac{U}{f(x)} \frac{\delta F_p}{J\left(\frac{F_m,\ F_n,\ F_p}{x,\ y,\ z}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(ii)}.$$

Let $x=\alpha$ be any root of $f(x)=0$; then, as before, we consider

$$-\Sigma \frac{U}{x-\alpha} dw,$$

that is,

$$\Sigma \frac{U}{x-\alpha} \frac{\delta F_p}{J},$$

the summation being for the mnp values of x; and a definite value of y and one of z are to be substituted in terms of x before the summation is effected.

Having these definite values of y and z (obtained from $F_m=0$, $F_n=0$) if in them we substitute in turn the mnp values of x, we shall have mnp congruous values and therefore all the congruous values. For these, as we easily see,

$$\frac{1}{J\left(\frac{F_m,\ F_n,\ F_n}{x,\ y,\ z}\right)} = \frac{\Delta}{\frac{dZ}{dz}\frac{dX}{dx}\frac{dY}{dy}}$$

and therefore

$$-\Sigma \frac{U}{x-\alpha} dw = \Sigma \frac{U}{x-\alpha} \delta F_p \frac{\Delta}{\frac{dZ}{dz}\frac{dX}{dx}\frac{dY}{dy}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(iii)}$$

the summation on each side being the same. But for all values not included in this summation we have $\Delta=0$, and therefore the restrictions on the right-hand side may be removed without altering its value, and we shall consider the summation to extend over all the roots of $F_m=0$, $F_n=0$ considered as equations in y and z and over all the roots x.

Let α denote the minor of A, β that of B, γ that of C (in each case with the same suffix) in Δ. Then we have

$$\alpha_m X + \beta_m Y + \gamma_m Z \equiv \Delta F_m$$
$$\alpha_n X + \beta_n Y + \gamma_n Z \equiv \Delta F_n$$
$$\alpha_p X + \beta_p Y + \gamma_p Z \equiv \Delta F_p.$$

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[*] *Cf.* SALMON'S 'Higher Algebra,' § 74.

Differentiating the first two of these with respect to y and z separately and then inserting the values of x, y and z as they now occur on the right-hand side of (iii) we have

$$\beta_m\frac{dY}{dy}=\Delta\frac{\delta F_m}{\delta y}$$

$$\gamma_m\frac{dZ}{dz}=\Delta\frac{\delta F_m}{\delta z}$$

$$\beta_n\frac{dY}{dy}=\Delta\frac{\delta F_n}{\delta y}$$

$$\gamma_n\frac{dZ}{dz}=\Delta\frac{\delta F_n}{\delta z}$$

and therefore

$$\begin{aligned}\Delta^2\frac{\delta(F_m, F_n)}{\delta(y, z)}&=(\beta_m\gamma_n-\beta_n\gamma_m)\frac{dY}{dy}\frac{dZ}{dz}\\&=A_p\Delta\frac{dY}{dy}\frac{dZ}{dz}\end{aligned}$$

by a known theorem in determinants; thus (iii) becomes

$$-\Sigma\frac{U}{x-\alpha}\delta w=\Sigma\frac{U}{x-\alpha}\delta F_p\frac{A_p}{\frac{dX}{dx}}\frac{1}{\frac{\delta(F_m, F_n)}{\delta(y, z)}}.$$

Now expanding in partial fractions we have

$$\frac{K}{X}=\Sigma\frac{K_\iota}{(x-x_\iota)\frac{dX}{dx_\iota}}$$

and therefore the right-hand side becomes

$$C_{\frac{1}{x}}\Sigma\left[\frac{U}{x-\alpha}\frac{\delta F_p}{\frac{\delta(F_m, F_n)}{\delta(y, z)}}\frac{A_p}{X}\right]$$

considered as expanded for the factors of X alone or, including in the expansion the term arising from $\frac{1}{x-\alpha}$, it is equal to

$$C_{\frac{1}{x}}\Sigma\left[\frac{U}{x-\alpha}\frac{\delta F_p}{\frac{\delta(F_m, F_n)}{\delta(y, z)}}\frac{A_p}{X}\right]-\Sigma\left[\frac{U\delta F_p}{\frac{\delta(F_m, F_n)}{\delta(y, z)}}\frac{A_p}{X}\right]_{x=\alpha}$$

wherein the Σ implies summation for all values of y and z in terms of x derived from the equations $F_m=0$ and $F_n=0$. Since these values are to be substituted we have

$$\begin{aligned}X&=A_mF_m+A_nF_n+A_pF_p\\&=A_pF_p\end{aligned}$$

and therefore substituting this in the above which will replace the right-hand side of (iii) and inserting the value of dw the equation becomes

$$\Sigma\frac{U}{x-\alpha}\frac{dx}{J\left(\frac{F_m,\ F_n}{y,\ z}\right)}=\Sigma\left[\frac{U}{J\left(\frac{F_m,\ F_n}{y,\ z}\right)}\frac{\delta F_p}{F_p}\right]_{x=\alpha}-C_1\underset{x}{\Sigma}\left[\frac{U}{x-\alpha}\frac{1}{J\left(\frac{F_m,\ F_n}{y,\ z}\right)}\frac{\delta F_p}{F_p}\right]$$

$$=\Theta\left[\frac{1}{x-\alpha}\right].\Sigma\left[\frac{U}{J\left(\frac{F_m,\ F_n}{y,\ z}\right)}\frac{\delta F_p}{F_p}\right]$$

by the use of BOOLE's symbol Θ as before. The summation on the left-hand side is of course over the mnp roots x; on the right-hand side it is over the mn roots y and z in terms of x of the equations $F_m=0$ and $F_n=0$. We may obviously integrate as before; and using the distributive property of Θ we obtain as our result

$$\sum_{\mu=1}^{\mu=mnp}\int^{x_\mu}\frac{U}{f(x)}\frac{dx}{J\left(\frac{F_m,\ F_n}{y,\ z}\right)}-C=\Theta\left[\frac{1}{f(x)}\right].\sum_{\mu=1}^{\mu=mn}\left\{\frac{U}{J\left(\frac{F_m,\ F_n}{y,\ z}\right)}\log F_p\right\}\quad .\ .\ .\ .\ \text{(iv)}.$$

8. The general theorem will proceed on lines not widely different from the above, and may be enunciated as follows. Let

$$\begin{aligned}&F_1\ (x_1,\ x_2,\ .\ .\ .\ ,\ x_r)=0\\&F_2\ (x_1,\ x_2,\ .\ .\ .\ ,\ x_r)=0\\&\quad .\ .\ .\ .\ .\ .\\&F_{r-1}(x_1,\ x_2,\ .\ .\ .\ ,\ x_r)=0\end{aligned}$$

be $r-1$ equations, of degrees $m_1,\ m_2,\ .\ .\ .\ ,\ m_{r-1}$ respectively, giving $x_2,\ .\ .\ .\ ,\ x_r$ in terms of x_1; and let

$$F_r(x_2,\ x_3,\ .\ .\ .\ ,\ x_r)$$

be a function of these dependent variables, the coefficients of which are functions of x_1 containing any number of arbitrary constants. Form the eliminant E of all the F's so that we shall obtain the set of roots x_1 by equating E to zero; and denote by U any algebraical rational integral function of $x_1,\ x_2,\ .\ .\ .\ x_r$. Then

$$\Sigma\int\frac{U}{f(x_1)}\frac{dx_1}{J\left(\frac{F_1,\ F_2,\ .\ .\ .\ ,\ F_{r-1}}{x_2,\ x_3,\ .\ .\ .\ ,\ x_r}\right)}=\Theta\left[\frac{1}{f(x_1)}\right].\Sigma\left\{\frac{U\log F_r}{J\left(\frac{F_1,\ F_2,\ .\ .\ .\ ,\ F_{r-1}}{x_2,\ x_3,\ .\ .\ .\ ,\ x_r}\right)}\right\}+A$$

the summation on the left-hand side being taken over all the roots of the equation $E=0$, which are assumed as the upper limits of the integrals, while on the right-hand side the summation is over all the roots of $F_1=0$, $F_2=0, \ldots, F_{r-1}=0$ considered as $r-1$ simultaneous equations giving $x_2, x_3, \ldots, x_r$ in terms of x_1.

9. Returning now to (iv) the conditions that the right-hand side should reduce to a constant are :—

1°: That $f(x)=1$, or be a factor of U;

and 2°:
$$\underset{x}{\mathrm{C}_1}\,\Sigma\frac{\mathrm{U}}{\mathrm{J}}\log\theta=0$$

which will be satisfied by
$$\underset{x}{\mathrm{C}_1}\frac{\mathrm{U}}{\mathrm{J}}=0.$$

Now J is of the order $m+n-2$ in x, and therefore the order of U may not be greater than $m+n-4$. In this case the number of terms in it will be
$$\frac{m+n-1.m+n-2.m+n-3}{1.2.3}.$$

But if the integrals $\int\frac{x^k}{\mathrm{J}}dx$ be formed they are not all independent for
$$\int\frac{\mathrm{V}_n\mathrm{F}_m}{\mathrm{J}}dx=0$$
$$\int\frac{\mathrm{V}_m\mathrm{F}_n}{\mathrm{J}}dx=0$$

where V_m and V_n are arbitrary functions of the orders $m-4$ and $n-4$ respectively, and contain
$$\frac{m-1.m-2.m-3}{1.2.3}\text{ and }\frac{n-1.n-2.n-3}{1.2.3}$$

terms. Hence the number of independent integrals in the case when the right-hand side reduces to a constant or to zero is
$$\frac{m+n-1.m+n-2.m+n-3}{1.2.3}-\frac{m-1.m-2.m-3}{1.2.3}-\frac{n-1.n-2.n-3}{1.2.3}=\tfrac{1}{2}mn(m+n-4)+1$$

This assumes that the surfaces F_m and F_n are the most general of the degrees m and n respectively and so possess no special singularities.

10. ABEL'S theorem in the more simple case applies to the intersection of plane curves. There is a fixed curve given by
$$\chi(x, y)=0$$

and integrals connected with this curve are taken, the upper limits assigned being the abscissæ of its points of intersection with another curve, the equation to which

$$\theta(x, y)=0$$

contains a number of variable parameters and therefore represents a variable curve.

But the more general form of the theorem extends the application to curves in space. We take the curve which is the intersection of two surfaces

$$\mathrm{F}_m(x, y, z)=0$$
$$\mathrm{F}_n(x, y, z)=0$$

(and which will, as a rule, be a tortuous curve), and forming the corresponding integrals we assign as the upper limits of these the ordinates x of the points of intersection of this tortuous curve with a surface the equation to which

$$\mathrm{F}_p(x, y, z)=0$$

containing a number of variable parameters represents a variable surface.

The discussion of this geometrical interpretation and of the deductions to which it leads has been carried out in a memoir by CLEBSCH ('Crelle,' t. lxiii., p. 189, 1863), wherein he proceeds from the theorems which are the forms of (iv) and (v) when the right-hand sides are zero. Example I. which follows was suggested by an analogous geometrical illustration which Professor CAYLEY gave in one of his lectures at Cambridge in the Michaelmas Term, 1881, wherein he pointed out how to obtain $\mathrm{sn}(u+v+w)$ from the analytical expression for the co-planarity of the four points of intersection of an arbitrary plane (corresponding to $\mathrm{F}_p=0$) with a fixed tortuous curve in space which was the intersection of a circular cylinder and an elliptic cylinder respectively corresponding to $\mathrm{F}_m=0$ and $\mathrm{F}_n=0$.

We now proceed to consider two examples of (iv).

11. *Example I.*

Let

$$\mathrm{F}_m\equiv y^2-(1-x^2)=0$$
$$\mathrm{F}_n\equiv z^2-(1-k^2x^2)=0$$
$$\mathrm{F}_p\equiv \mathrm{A}x+\mathrm{B}y+\mathrm{C}z-1.$$

The eliminant X is obviously

$$\mathrm{X}=\Pi\{\mathrm{A}x-1\pm\mathrm{B}(1-x^2)^{\frac{1}{2}}\pm\mathrm{C}(1-k^2x^2)^{\frac{1}{2}}\}$$

Π denoting the product of the four expressions which the above includes owing to the two double signs. It is evidently of the fourth degree in x; let the roots be x_1, x_2, x_3, x_4. As there are three arbitrary constants there will be one relation between these four roots, and this can be exhibited in the form

$$\begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix} = 0.$$

If we choose $A=0$ and one of the two, B and C, to be unity and the other zero, $X=0$ reduces to

$$x^4=0$$

and we may therefore take zero as the lower limits of all our integrals.

Let

$$y_1=+(1-x^2)^{\frac{1}{2}}$$
$$z_1=+(1-k^2x^2)^{\frac{1}{2}}$$

then $\pm y_1$ are the roots of F_m, $\pm z_1$ those of F_n. We have

$$J=4y_1z_1=4(1-x^2.1-k^2x^2)^{\frac{1}{2}}$$

and therefore by our formula (iv)

$$\sum_{\mu=1}^{\mu=4}\int_0^{x_\mu}\frac{U\,dx}{f(x)\sqrt{(1-x^2)(1-k^2x^2)}}=\Theta\left[\frac{1}{f(x)}\right]\Sigma\left\{\frac{U}{yz}\log\left(1-Ax-By-Cz\right)\right\}$$

in which the Σ on the right-hand side implies summation for the expressions obtained by the substitutions

$y=\ \ y_1$ and $z=\ \ z_1$,
$y=-y_1$,, $z=\ \ z_1$,
$y=\ \ y_1$,, $z=-z_1$,
$y=-y_1$,, $z=-z_1$.

(i.) Let $f(x)\equiv 1$, $U\equiv 1$; then the right-hand side vanishes and we have

$$u_1+u_2+u_3+u_4=0$$

where

$$x=\operatorname{sn} u.$$

Thus the preceding determinantal relation will give $\operatorname{sn}(u_1+u_2+u_3)$, which is $-x_4$, in terms of the elliptic functions of u_1, u_2, u_3.

(ii.) Let $f(x)\equiv 1$, $U=z^2=1-k^2x^2$; then we have

$$E(u_1)+E(u_2)+E(u_3)+E(u_4)$$
$$=-\underset{x}{C_1}\,\Sigma\left\{\frac{z}{y}\log\left(1-Ax-By-Cz\right)\right\}$$
$$=-\underset{x}{C_1}\left[\frac{z_1}{y_1}\log\left(\frac{1-Ax-Cz_1-By_1}{1-Ax-Cz_1+By_1}\right)+\frac{z_1}{y_1}\log\left(\frac{1-Ax+Cz_1+By_1}{1-Ax+Cz_1-By_1}\right)\right].$$

Expanding the logarithms on the right-hand side the n^{th} term gives

$$\frac{2}{2n-1}\,C_{\frac{1}{x}}\left[z_1\frac{B^{2n-1}y_1^{\,2n-2}}{(1-Ax-Cz_1)^{2n-1}}-z_1\frac{B^{2n-1}y_1^{\,2n-2}}{(1-Ax+Cz_1)^{2n-1}}\right]$$

$$=\frac{2B^{2n-1}}{2n-1}\,C_{\frac{1}{x}}\left[y_1^{\,2n-2}\frac{z_1(1-Ax+Cz_1)^{2n-1}-z_1(1-Ax-Cz_1)^{2n-1}}{\{(1-Ax)^2-C^2z_1^{\,2}\}^{2n-1}}\right]$$

$$=\frac{4B^{2n-1}C}{2n-1}\,C_{\frac{1}{x}}\left[(1-x^2)^{n-1}(1-k^2x^2)\frac{\left\{(2n-1)(1-Ax)^{2n-2}+\frac{2n-1.2n-2.2n-3}{1.2.3}(1-Ax)^{2n-4}C^2(1-k^2x^2)+\ldots\right\}}{(A^2+k^2C^2)^{2n-1}x^{4n-2}\left\{1-\frac{2A}{A^2+k^2C^2}\frac{1}{x}+\ldots\right\}^{2n-1}}\right]$$

$$=\frac{(-1)^n4k^2B^{2n-1}C}{(2n-1)(A^2+k^2C^2)^{2n-1}}\,C_{\frac{1}{x}}\left[\left\{1+\frac{2(2n-1)A}{A^2+k^2C^2}\cdot\frac{1}{x}-\ldots\right\}\right.$$
$$\left\{(2n-1)A^{2n-2}-(2n-1)(2n-2)A^{2n-3}\frac{1}{x}-k^2C^2\left(\frac{2n-1.2n-2.2n-3}{1.2.3}A^{2n-4}\right.\right.$$
$$\left.\left.\left.-\frac{2n-1\ldots 2n-4}{1.2.3}A^{2n-5}\frac{1}{x}\right)+\ldots\right\}\right]$$

$$=\frac{(-1)^n4k^2B^{2n-1}C}{(2n-1)(A^2+k^2C^2)^{2n-1}}\left[-(2n-1)\left\{(2n-2)A^{2n-3}-k^2C^2\frac{2n-2.2n-3.2n-4}{1.2.3}A^{2n-5}+\ldots\right\}\right.$$
$$\left.+\frac{2.2n-1.A}{A^2+k^2C^2}\left\{(2n-1)A^{2n-2}-k^2C^2\frac{2n-1.2n-2.2n-3}{1.2.3}A^{2n-4}+\ldots\right\}\right]$$

$$=\frac{(-1)^n4k^2B^{2n-1}C}{(A^2+k^2C^2)^{2n}}\,\frac{(A+ikC)^{2n}-(A-ikC)^{2n}}{2ikC}\ \text{after a slight reduction and writing } i=\sqrt{-1}$$

$$=\frac{(-1)^n2kB^{2n-1}}{i}\left[\frac{1}{(A-ikC)^{2n}}-\frac{1}{(A+ikC)^{2n}}\right].$$

Hence the whole coefficient as derived from all the terms in the expansion is

$$=\frac{2k}{i}\left[-\frac{B}{(A-ikC)^2}+\frac{B^3}{(A-ikC)^4}-\ldots+\frac{B}{(A+ikC)^2}-\frac{B^3}{(A+ikC)^4}+\ldots\right]$$

$$=-\frac{2k}{i}\left[\frac{B}{B^2+(A-ikC)^2}-\frac{B}{B^2+(A+ikC)^2}\right]$$

$$=-\frac{8k^2ABC}{(A^2+B^2-k^2C^2)^2+4k^2A^2C^2}$$

and this is the value of

$$E(u_1)+E(u_2)+E(u_3)-E(u_1+u_2+u_3).$$

Writing s, c, d respectively for sn$u$, cn$u$, dn$u$, the values of A, B and C are given by the equations

$$\mathrm{A}s_1+\mathrm{B}c_1+\mathrm{C}d_1=1$$
$$\mathrm{A}s_2+\mathrm{B}c_2+\mathrm{C}d_2=1$$
$$\mathrm{A}s_3+\mathrm{B}c_3+\mathrm{C}d_3=1$$

or writing

$$c_2d_3-c_3d_2+\ldots=\sigma$$
$$d_2s_3-d_3s_2+\ldots=\kappa$$
$$s_2c_3-s_3c_2+\ldots=\delta$$

$$\begin{vmatrix} s_1 & c_1 & d_1 \\ s_2 & c_2 & d_2 \\ s_3 & c_3 & d_3 \end{vmatrix}=\Delta$$

we have

$$\Delta\mathrm{A}=\sigma \qquad \Delta\mathrm{B}=\kappa \qquad \Delta\mathrm{C}=\delta$$

and then

$$\mathrm{E}(u_1)+\mathrm{E}(u_2)+\mathrm{E}(u_3)-\mathrm{E}(u_1+u_2+u_3)$$
$$=-\frac{8k^2\sigma\kappa\delta\Delta}{(\sigma^2+\kappa^2-k^2\delta^2)^2+4k^2\kappa^2\delta^2}.$$

As a verification of our formula assume

$$u_1=u_2=u_3$$

so that

$$x_1=x_2=x_3.$$

Then since the equation

$$\mathrm{A}s+\mathrm{B}c+\mathrm{C}d=1$$

has three equal roots, the values of A, B and C for our case will obviously be given by

$$\mathrm{A}s_1+\mathrm{B}c_1+\mathrm{C}d_1=1$$
$$\mathrm{A}-\mathrm{B}\frac{s_1}{c_1}-\mathrm{C}k^2\frac{s_1}{d_1}=0$$
$$\mathrm{B}\frac{1}{c_1^3}+\mathrm{C}k^2\frac{1}{d_1^3}=0:$$

for if we write $s_1+\xi$ for s_1 in the first, the coefficient of ξ must vanish, which condition gives the second equation; and similarly for the third. These last two equations give

$$\frac{\mathrm{A}}{k^2k'^2s_1^3}=\frac{\mathrm{B}}{-k^2c_1^3}=\frac{\mathrm{C}}{d_1^3}=\mu$$

and substituting in the first we find

$$\mu=\frac{1}{k'^2}.$$

From these we have at once

$$\frac{k'^4}{k^2}(\mathrm{A}^2+\mathrm{B}^2-k^2\mathrm{C}^2)=k^2k'^4s^6-d^6+k^2c^6 \quad \text{(dropping subscripts)}$$

$$=-k'^2(1-3k^2s^4+2k^4s^6)$$

$$\frac{k'^2}{k^2}\mathrm{AC}=s^3d^3.$$

Hence

$$\frac{k'^4}{k^4}\{(\mathrm{A}^2+\mathrm{B}^2-k^2\mathrm{C}^2)^2+4k^2\mathrm{A}^2\mathrm{C}^2\}=(1-3k^2s^4+2k^4s^6)^2+4k^2s^6(1-k^2s^2)^3$$

$$=1-6k^2s^4+4(k^2+k^4)s^6-3k^2s^8$$

and we therefore have to verify that

$$3\mathrm{E}(u)-\mathrm{E}(3u)=\frac{8k^2s^3c^3d^3}{1-6k^2s^4+4(k^2+k^4)s^6-3k^4s^8}.$$

Now the ordinary addition formula for E is

$$\mathrm{E}(u)+\mathrm{E}(v)-\mathrm{E}(u+v)=k^2\mathrm{sn}u\,\mathrm{sn}v\,\mathrm{sn}(u+v)$$

so that

$$\mathrm{E}(u)+\mathrm{E}(2u)-\mathrm{E}(3u)=k^2\mathrm{sn}u\,\mathrm{sn}2u\,\mathrm{sn}3u$$

$$2\mathrm{E}(u)-\mathrm{E}(2u) \qquad =k^2\mathrm{sn}^2u\,\mathrm{sn}2u$$

and hence

$$3\mathrm{E}(u)-\mathrm{E}(3u)=k^2\mathrm{sn}u\,\mathrm{sn}2u(\mathrm{sn}u+\mathrm{sn}3u).$$

But

$$\mathrm{sn}3u=\frac{3s-(4+4k^2)s^3+6k^2s^5-k^4s^9}{1-6k^2s^4+4(k^2+k^4)s^6-3k^4s^8}$$

or writing D for the denominator

$$\mathrm{D}(\mathrm{sn}u+\mathrm{sn}3u)=4s\{1-(1+k^2)s^2+(k^2+k^4)s^4-k^4s^8\}$$

$$=4s(1-k^2s^4)(1-s^2)(1-k^2s^2)$$

$$=4s(1-k^2s^4)c^2d^2.$$

Moreover

$$\mathrm{sn}2u=\frac{2scd}{1-k^2s^4}$$

so that

$$3\mathrm{E}(u)-\mathrm{E}(3u)=\frac{8k^2s^3c^3d^3}{\mathrm{D}}$$

verifying the formula as required.

(iii.) In a similar way if we write

$$\Pi(\lambda, u_1) = \int_0^{x_1} \frac{dx}{(1-\lambda^2 x^2)\sqrt{(1-x^2)(1-k^2x^2)}}$$

(so that $U=1, f(x)=1-\lambda^2x^2$) we shall obtain

$$\Pi(\lambda, u_1)+\Pi(\lambda, u_2)+\Pi(\lambda, u_3)+\Pi(\lambda, u_4)$$

$$= -\frac{\lambda}{2\sqrt{(1-\lambda^2)(k^2-\lambda^2)}} \log\left[\frac{\{(A-\lambda)^2+(B\sqrt{1-\lambda^2}+C\sqrt{k^2-\lambda^2})^2\}\{(A+\lambda)^2+(B\sqrt{1-\lambda^2}-C\sqrt{k^2-\lambda^2})^2\}}{\{(A+\lambda)^2+(B\sqrt{1-\lambda^2}+C\sqrt{k^2-\lambda^2})^2\}\{(A-\lambda)^2+(B\sqrt{1-\lambda^2}-C\sqrt{k^2-\lambda^2})^2\}}\right]$$

the values of A, B, C being those which occur in the general case in (ii.).

Let $\lambda = k\,\mathrm{sn}\,a$ so as to introduce the third elliptic integral in the form used by JACOBI; then

$$\Pi(\lambda, u) = u + \frac{\mathrm{sn}\,a}{\mathrm{cn}\,a\,\mathrm{dn}\,a}\Pi(u, a)$$

$$= u + \frac{\lambda}{\sqrt{(1-\lambda^2)(k^2-\lambda^2)}}\Pi(u, a)$$

and the form of the theorem is now obviously as follows:—

$$\Pi(u_1, a)+\Pi(u_2, a)+\Pi(u_3, a)+\Pi(u_4, a)$$

$$= \tfrac{1}{2}\log\left[\frac{\{(A+ks')^2+(Bd'+Ckc')^2\}\{(A-ks')^2+(Bd'-Ckc')^2\}}{\{(A-ks')^2+(Bd'+Ckc')^2\}\{(A+ks')^2+(Bd'-Ckc')^2\}}\right]$$

where s', c', d' stand respectively for $\mathrm{sn}\,a$, $\mathrm{cn}\,a$, $\mathrm{dn}\,a$.

12. *Example II.*

Take F_m and F_n as in example I., but now let

$$F_p \equiv Azy + (Bx+C)y + (Hx+D)z - Gx^2 - Fx - 1,$$

in effect the most general quadric relation. The eliminant X will be of the degree 8, and as there are seven arbitrary constants there will be only a single relation between the roots $x_1, x_2, \ldots, x_8$, which can be expressed in the form

$$\begin{vmatrix} c_1d_1 & s_1c_1 & s_1d_1 & s_1^2 & c_1 & d_1 & s_1 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \\ c_8d_8 & s_8c_8 & s_8d_8 & s_8^2 & c_8 & d_8 & s_8 & 1 \end{vmatrix} = 0.$$

Moreover, if we choose

$$B=H=F=G=0$$

$$-A=C=D=1$$

the equation $X=0$ is of the form

$$x^8=0$$

and we can therefore take zero for the lower limit in all our integrals. Hence we shall have

$$u_1+u_2+\ldots\qquad+u_8=0$$

where

$$x=\operatorname{sn}u$$

and the above relation will give $\operatorname{sn}(u_1+\ldots+u_7)$ in terms of the elliptic functions of $u_1,\ldots, u_7$.

Let us now find

$$\sum_{\mu=1}^{\mu=8}\int_0^{x_\mu}\sqrt{\frac{1-k^2x^2}{1-x^2}}dx\,;$$

write

$$Bx+C=w_2;\qquad Hx+D=w_1;\qquad 1+Fx+Gx^2=w_3;$$

then the right-hand side of the equation is

$$-C_{\frac{1}{x}}\left[\frac{z}{y}\log\left\{\frac{w_3-zw_1-y(Az+w_2)}{w_3-zw_1+y(Az+w_2)}\right\}+\frac{z}{y}\log\left\{\frac{w_3+zw_1+y(Az-w_2)}{w_3+zw_1-y(Az-w_2)}\right\}\right].$$

On expansion, the n^{th} term gives

$$\frac{2}{2n-1}C_{\frac{1}{x}}\left[z\frac{y^{2n-2}(Az+w_2)^{2n-1}}{(w_3-zw_1)^{2n-1}}+z\frac{y^{2n-2}(Az-w_2)^{2n-1}}{(w_3+zw_1)^{2n-1}}\right]$$

$$=\frac{2}{2n-1}C_{\frac{1}{x}}\left[\frac{zy^{2n-2}}{(w_3^2-z^2w_1^2)^{2n-1}}\{(Az+w_2)^{2n-1}(w_3+zw_1)^{2n-1}-(w_2-Az)^{2n-1}(w_3-zw_1)^{2n-1}\}\right].$$

So far as the result is concerned, the expression within the inner bracket is

$$\{w_2w_3-k^2x^2w_1+z(Aw_3+w_1w_2)\}^{2n-1}\{w_2w_3-k^2x^2w_1-z(Aw_3+w_1w_2)\}^{2n-1}$$

$$=2z\{(2n-1)(w_2w_3-k^2x^2w_1)^{2n-2}(Aw_3+w_1w_2)+\ldots\}.$$

Now

$$w_2w_3-k^2x^2w_1=x^3\left[BG-k^2H+\frac{1}{x}(CG+BF-k^2D)\right]$$

$$=x^3\left(\lambda_1+\frac{1}{x}\lambda_2\right)\text{ say ;}$$

$$\begin{aligned}\mathrm{A}w_3+w_1w_2&=x^2\left[\mathrm{AG}+\mathrm{BH}+\frac{1}{x}(\mathrm{AF}+\mathrm{BD}+\mathrm{CH})\right]\\&=x^2\left(\mu_1+\frac{1}{x}\mu_2\right)\text{ say;}\end{aligned}$$

$$\begin{aligned}w_3^2-z^2w_1^2&=x^4(\mathrm{G}^2+k^2\mathrm{H}^2)\left\{1+2\frac{\mathrm{FG}+k^2\mathrm{DH}}{\mathrm{G}^2+k^2\mathrm{H}^2}\frac{1}{x}\right\}\\&=x^4\rho_1\left(1+\frac{1}{x}\rho_2\right)\text{ say.}\end{aligned}$$

Thus the n^{th} term gives

$$\begin{aligned}&\frac{4(-1)^nk^2}{2n-1}\,\frac{1}{\rho_1^{2n-1}}\mathrm{C}_{\frac{1}{x}}\left(1-\frac{2n-1}{x}\rho_2\right)\left[(2n-1)\left(\lambda_1^{2n-2}+\frac{2n-2}{x}\lambda_1^{2n-3}\lambda_2\right)\left(\mu_1+\frac{1}{x}\mu_2\right)\right.\\&\qquad\left.-k^2\frac{2n-1.2n-2.2n-3}{1.2.3}\left(\lambda_1^{2n-4}+\frac{2n-4}{x}\lambda_1^{2n-5}\lambda_2\right)\left(\mu_1^3+\frac{3}{x}\mu_1^2\mu_2\right)+\ldots\right]\end{aligned}$$

$$\begin{aligned}&=\frac{-4(-1)^nk^2}{\rho_1^{2n-1}}\rho_2\left\{(2n-1)\lambda_1^{2n-2}\mu_1-k^2\frac{2n-1.2n-2.2n-3}{1.2.3}\lambda_1^{2n-4}\mu_1^3+\ldots\right\}\\&+\frac{4(-1)^nk^2}{\rho_1^{2n-1}}\left\{\lambda_1^{2n-2}\mu_2-k^2\frac{2n-2.2n-3}{1.2}\lambda_1^{2n-4}\mu_1^2\mu_2+\ldots\right.\\&\qquad\left.+(2n-2)\lambda_1^{2n-3}\lambda_2\mu_1-k^2\frac{2n-2.2n-3.2n-4}{1.2.3}\lambda_1^{2n-5}\lambda_2\mu_1^3+\ldots\right\}\end{aligned}$$

$$\begin{aligned}&=\frac{4(-1)^nk^2}{\rho_1^{2n-1}}\left[-\frac{\rho_2}{2ik}\{(\lambda_1+ik\mu_1)^{2n-1}-(\lambda_1-ik\mu_1)^{2n-1}\}+\frac{\mu_2}{2}\{(\lambda_1+ik\mu_1)^{2n-2}+(\lambda_1-ik\mu_1)^{2n-2}\}\right.\\&\qquad\left.+\frac{\lambda_2}{2ik}\{(\lambda_1+ik\mu_1)^{2n-2}-(\lambda_1-ik\mu_1)^{2n-2}\}\right]\end{aligned}$$

i denoting $\sqrt{-1}$. Hence summing up for all the terms and reducing we have the whole coefficient equal to

$$-\frac{4k^2\{(\rho_1^2-k^2\mu_1^2)(\mu_2\rho_1-\rho_1\rho_2\mu_1)+\lambda_1^2(\mu_2\rho_1+\rho_1\rho_2\mu_1)-2\lambda_1\lambda_2\mu_1\rho_1\}}{(\rho_1^2+\lambda_1^2-k^2\mu_1^2)^2+4k^2\lambda_1^2\mu_1^2}.$$

Now

$$\begin{aligned}\rho_1&=\mathrm{G}^2+k^2\mathrm{H}^2 &\qquad \mu_1&=\mathrm{AG}+\mathrm{BH}\\\rho_1\rho_2&=2(\mathrm{FG}+k^2\mathrm{DH}), &\qquad \mu_2&=\mathrm{AF}+\mathrm{BD}+\mathrm{CH}\\\lambda_1&=\mathrm{BG}-k^2\mathrm{H}, &\qquad \lambda_2&=\mathrm{CG}+\mathrm{BF}-k^2\mathrm{D}\end{aligned}$$

and the values of A, B, C, D, H, F, G are determined by the seven equations

$$\mathrm{A}c_\mu d_\mu+\mathrm{B}s_\mu c_\mu+\mathrm{H}s_\mu d_\mu-\mathrm{G}s_\mu^2+\mathrm{C}c_\mu+\mathrm{D}d_\mu-\mathrm{F}s_\mu=1$$

($\mu=1, 2, \ldots, 7$); and therefore the above is the value of

$$\sum_{\mu=1}^{\mu=7} \mathrm{E}(u_\mu) - \mathrm{E}\left(\sum_{\mu=1}^{\mu=7} u_\mu\right)$$

expressed in terms of the functions of the u's.

The evaluation of the corresponding expression for the sum of the third elliptic integrals presents no difficulty.

Section II.

Abelian functions, after Weierstrass.

13. The theory of these functions is detailed in a paper by Weierstrass in Crelle's Journal, t. lii., but such formulæ as may be necessary in what follows will be proved. Let

$$\left.\begin{aligned} y^2-\mathrm{P}(x) &\equiv y^2-(x-a_1)(x-a_2)\ldots(x-a_\rho)=0 \\ z^2-\mathrm{Q}(x) &\equiv z^2-(x-a_{\rho+1})(x-a_{\rho+2})\ldots(x-a_{2\rho+1})=0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (1)$$

and

$$\theta \equiv \mathrm{M}y+\mathrm{N}z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where M is of the degree ρ in x, N of $\rho-1$; say

$$\left.\begin{aligned} \mathrm{M} &\equiv x^\rho+\mathrm{M}_1x^{\rho-1}+\ldots+\mathrm{M}_{\rho-1}x+\mathrm{M}_\rho \\ \mathrm{N} &\equiv \qquad \mathrm{N}_1x^{\rho-1}+\ldots+\mathrm{N}_{\rho-1}x+\mathrm{N}_\rho \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

Then the equation for the roots x being

$$\mathrm{M}^2y^2-\mathrm{N}^2z^2=0$$

is of the degree 3ρ and involves 2ρ arbitrary constants; thus there must be ρ relations among the roots. Let these roots be denoted by $x_1, x_2, \ldots, x_\rho$; $\xi_1, \xi_2, \ldots, \xi_\rho$; $p_1, p_2, \ldots, p_\rho$; so that we may consider the ρ p's as given in terms of the x's and ξ's by the ρ relations which might be exhibited in a determinantal form. Write

$$\mathrm{R}(x)=\mathrm{P}(x)\mathrm{Q}(x)$$

and let

$$u_\mu=\tfrac{1}{2}\sum_{\lambda=1}^{\lambda=\rho}\int_{a_\lambda}^{x_\lambda}\frac{\mathrm{P}(x)dx}{(x-a_\mu)\sqrt{\mathrm{R}(x)}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

in which μ has in succession the values 1, 2, ..., ρ as also in

$$v_\mu=\tfrac{1}{2}\sum_{\lambda=1}^{\lambda=\rho}\int_{a_\lambda}^{\xi_\lambda}\frac{\mathrm{P}(x)dx}{(x-a_\mu)\sqrt{\mathrm{R}(x)}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and in

$$w_\mu=\tfrac{1}{2}\sum_{\lambda=1}^{\lambda=\rho}\int_{a_\lambda}^{p_\lambda}\frac{\mathrm{P}(x)dx}{(x-a_\mu)\sqrt{\mathrm{R}(x)}}\quad .\ .\ .\ .\ .\ .\ .\ .\ .\ .\quad (6).$$

Now the constants in M and N can be so chosen that the roots of the equation

$$\mathrm{M}^2y^2-\mathrm{N}^2z^2=0$$

shall be $a_1, a_2, \ldots, a_\rho$ each occurring thrice; for the conditions are that any one, say a_λ, shall satisfy the three equations

$$\mathrm{M}^2y^2-\mathrm{N}^2z^2=0$$

$$\mathrm{M}^2y\frac{dy}{dx}+y^2\mathrm{M}\frac{d\mathrm{M}}{dx}-\mathrm{N}^2z\frac{dz}{dx}-z^2\mathrm{N}\frac{d\mathrm{N}}{dx}=0$$

$$\mathrm{MT}+y^2\left(\frac{d\mathrm{M}}{dx}\right)^2-\mathrm{NU}-z^2\left(\frac{d\mathrm{N}}{dx}\right)^2=0$$

where T, U are functions of x. In order that a_λ may satisfy the first of these

$$\mathrm{N}=0$$

when $x=a_\lambda$; that it may satisfy the second the additional condition is that

$$\mathrm{M}=0$$

and therefore M will contain $x-a_\lambda$ as a factor. Moreover, y^2 contains $x-a_\lambda$ as a factor and therefore in M^2y^2 we shall have $x-a_\lambda$ raised to the third power; and therefore if we expand N in ascending powers of $x-a_\lambda$ the first term is of the order

$$(x-a_\lambda)^{\frac{3}{2}}.$$

Hence $\left(\frac{d\mathrm{N}}{dx}\right)^2$ has $\mathrm{A}(x-a_\lambda)$ as its first term and so vanishes when $x=a_\lambda$. But this is the additional condition that $x=a_\lambda$ should satisfy the third equation, and being satisfied it proves that a_λ may occur as a triple root provided

$$\mathrm{M}=0 \qquad \mathrm{N}=0$$

when $x=a_\lambda$. That is to say, two conditions are necessary for each root, or 2ρ in all; but as there are 2ρ disposable constants these can all be satisfied and so the truth of the proposition is established. But as N is only of the degree $\rho-1$ in x, while it has to vanish for ρ values, it must be identically zero; and we choose $\mathrm{M}=y^2$, so that the equation is $\mathrm{M}^2y^2=0$ which is obvious beforehand.

14. Applying now the general theorem from § 6 we have

$$\sum_{\lambda=1}^{\lambda=\rho}\left\{\int^{x_\lambda}\frac{\mathrm{P}(x)dx}{(x-a_\mu)\sqrt{\mathrm{R}(x)}}+\int^{\xi_\lambda}\frac{\mathrm{P}(x)dx}{(x-a_\mu)\sqrt{\mathrm{R}(x)}}+\int^{p_\lambda}\frac{\mathrm{P}(x)dx}{(x-a_\mu)\sqrt{\mathrm{R}(x)}}\right\}=\text{constant}$$

since $\frac{\mathrm{P}(x)}{x-a_\mu}$ is an integral function of x, and when $\frac{\mathrm{P}(x)}{(x-a_\mu)\sqrt{\mathrm{R}(x)}}$ is expanded in descending powers of x the highest index of x is $-\frac{3}{2}$. We have shown that $a_1, a_2, \ldots, a_\lambda$ may be regarded as triple roots of the equation for the roots, and thus we may take as the constant

$$3\sum_{\lambda=1}^{\lambda=\rho}\int^{a_\lambda}\frac{\mathrm{P}(x)dx}{(x-a_\mu)\sqrt{\mathrm{R}(x)}}.$$

Hence

$$\sum_{\lambda=1}^{\lambda=\rho}\left\{\int_{a_\lambda}^{x_\lambda}\frac{\mathrm{P}(x)dx}{(x-a_\mu)\sqrt{\mathrm{R}(x)}}+\int_{a_\lambda}^{\xi_\lambda}\frac{\mathrm{P}(x)dx}{(x-a_\mu)\sqrt{\mathrm{R}(x)}}+\int_{a_\lambda}^{p_\lambda}\frac{\mathrm{P}(x)dx}{(x-a_\mu)\sqrt{\mathrm{R}(x)}}\right\}=0$$

or

$$u_\mu+v_\mu+w_\mu=0 \quad \ldots\ldots\ldots\ldots \quad (7).$$

Now, by WEIERSTRASS's theory, given values of $u_1, u_2, \ldots, u_\rho$ imply unique values of $x_1, x_2, \ldots, x_\rho$ which are, in fact, the roots of an equation of the ρ^{th} degree whose coefficients are single-valued functions of $u_1, u_2, \ldots, u_\rho$. Every symmetrical function of $x_1, \ldots, x_\rho$ can therefore be expressed as a function $u_1, u_2, \ldots, u_\rho$, but in particular

$$(a_t-x_1)(a_t-x_2)\ldots(a_t-x_\rho)$$

(t being any of the integers $1, 2, \ldots, 2\rho+1$) is the perfect square of such a function. Write

$$\phi(x) \quad =(x-x_1)(x-x_2)\ldots(x-x_\rho) \quad \ldots\ldots\ldots \quad (8)$$

$$\left.\begin{aligned}-\mathrm{Q}(a_r) \quad &=l_r \quad (r=1, 2, \ldots, \rho)\\ \mathrm{P}(a_{\rho+s})&=l_{\rho+s}(s=1, 2, \ldots, \rho+1)\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (9)$$

then WEIERSTRASS defines

$$l_r al_r^2=\phi(a_r). \quad \ldots\ldots\ldots\ldots \quad (10)$$

for all values of r included in $1, 2, \ldots, 2\rho+1$. It is easy to verify that $x_1, x_2, \ldots, x_\rho$ are the roots of

$$\sum_{r=1}^{r=\rho}\left\{\frac{l_r al_r^2}{(a_r-x)\mathrm{P}'(a_r)}\right\}=1. \quad \ldots\ldots\ldots\ldots \quad (11)$$

for there are obviously ρ roots, and in order that x_1 may be one of these we must have

$$\sum_{r=1}^{r=\rho}\frac{(a_r-x_2)(a_r-x_3)\ldots(a_r-x_\rho)}{(a_r-a_1)(a_r-a_2)\ldots(a_r-a_\rho)}=1 \quad \ldots\ldots\ldots \quad (\mathrm{A}).$$

By a known theorem of ABEL's we have

$$\Sigma\frac{z^s}{\frac{d\chi}{dz}}=0 \text{ or } 1,$$

according as $s<$ or $=\rho-1$, the summation being extended over the ρ roots of $\chi(x)=0$; and therefore

$$\sum_{r=1}^{r=\rho}\frac{a_r^{\rho-1}}{P'(a_r)}=0$$

$$\sum_{r=1}^{r=\rho}\frac{a_r^{s}}{P'(a_r)}=0 \text{ if } s<\rho-1\geqq 0.$$

Thus the left-hand side of (A)

$$=\sum_{r=1}^{r=\rho}\left\{\frac{a_r^{\rho-1}}{P'(a_r)}-(x_2+\ldots+x_\rho)\frac{a_r^{\rho-2}}{P'(a_r)}+\ldots\right\}$$
$$=1$$

all the terms disappearing except the first and so verifying (A) and proving that $x_1, x_2, \ldots, x_\rho$ are the roots of (11).

15. Taking now our set of integrals u we have

$$\left.\begin{aligned}
0&=\tfrac{1}{2}\sum_{\lambda=1}^{\lambda=\rho}\frac{P(x_\lambda)}{(x_\lambda-a_1)\sqrt{R(x_\lambda)}}\frac{\partial x_\lambda}{\partial u_\mu}\\
0&=\tfrac{1}{2}\sum_{\lambda=1}^{\lambda=\rho}\frac{P(x_\lambda)}{(x_\lambda-a_2)\sqrt{R(x_\lambda)}}\frac{\partial x_\lambda}{\partial u_\mu}\\
&\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\\
1&=\tfrac{1}{2}\sum_{\lambda=1}^{\lambda=\rho}\frac{P(x_\lambda)}{(x_\lambda-a_\mu)\sqrt{R(x_\lambda)}}\frac{\partial x_\lambda}{\partial u_\mu}\\
&\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\\
0&=\tfrac{1}{2}\sum_{\lambda=1}^{\lambda=\rho}\frac{P(x_\lambda)}{(x_\lambda-a_\rho)\sqrt{R(x_\lambda)}}\frac{\partial x_\lambda}{\partial u_\mu}
\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (12).$$

Multiply these respectively by $\frac{l_1 a l_1^2}{P'(a_1)}, \frac{l_2 a l_2^2}{P'(a_2)}, \ldots$ and add; then, in virtue of equation (11),

$$\frac{l_\mu a l_\mu^2}{P'(a_\mu)}=-\tfrac{1}{2}\sum_{\lambda=1}^{\lambda=\rho}\frac{P(x_\lambda)}{\sqrt{R(x_\lambda)}}\frac{\partial x_\lambda}{\partial u_\mu}$$

so that if we write

$$U=\tfrac{1}{2}\sum_{\lambda=1}^{\lambda=\rho}\int_{a_\lambda}^{x_\lambda}\frac{P(x)dx}{\sqrt{R(x)}} \quad \ldots\ldots\ldots\ldots (13)$$

we have

$$\frac{l_\mu a l_\mu^2}{P'(a_\mu)}=-\frac{\partial U}{\partial u_\mu} \quad \ldots\ldots\ldots\ldots (14).$$

U is obviously a symmetric function of the x's, and is therefore a function of the u's.

16. Again solving the equations (12) regarded as giving $\frac{\delta x_\lambda}{\delta u_\mu}$ for different values of λ, we have*

$$\tfrac{1}{2}\frac{\delta x_\lambda}{\delta u_\mu}=-\frac{\sqrt{\mathrm{R}(x_\lambda)}}{\phi'(x_\lambda)}\frac{\phi(a_\mu)}{\mathrm{P}'(a_\mu)}\frac{1}{x_\lambda-a_\mu}$$

$$=-\frac{\sqrt{\mathrm{R}(x_\lambda)}}{\phi'(x_\lambda)}\frac{l_\mu a l_\mu{}^2}{(x_\lambda-a_\mu)\mathrm{P}'(a_\mu)}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15).$$

Therefore

$$\tfrac{1}{2}\sum_{\mu=1}^{\mu=\rho}\left[\frac{\phi'(x_\lambda)}{\sqrt{\mathrm{R}(x_\lambda)}}\frac{\delta x_\lambda}{\delta u_\mu}\right]=-\sum_{\mu=1}^{\mu=\rho}\frac{l_\mu a l_\mu{}^2}{(x_\lambda-a_\mu)\mathrm{P}'(a_\mu)}$$

$$=1 \text{ by } (11)$$

and

$$\sum_{\lambda=1}^{\lambda=\rho}\left\{\frac{\sqrt{\mathrm{R}(x_\lambda)}}{\phi'(x_\lambda)}\frac{1}{x_\lambda-a_s}\right\}=\tfrac{1}{2}\sum_{\lambda=1}^{\lambda=\rho}\sum_{\mu=1}^{\mu=\rho}\frac{1}{x_\lambda-a_s}\frac{\delta x_\lambda}{\delta u_\mu}$$

$$=\tfrac{1}{2}\sum_{\mu=1}^{\mu=\rho}\frac{\delta}{\delta u_\mu}\left\{\log\prod_{\lambda=1}^{\lambda=\rho}(a_s-x_\lambda)\right\}$$

$$=\tfrac{1}{2}\sum_{\mu=1}^{\mu=\rho}\frac{\delta}{\delta u_\mu}\log\phi(a_s)$$

$$=\frac{1}{al_s}\sum_{\mu=1}^{\mu=\rho}\frac{\delta al_s}{\delta u_\mu}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16')$$

where s may be any of the integers 1, 2, . . . , $2\rho+1$.

Writing

$$\sum_{\mu=1}^{\mu=\rho}\frac{\delta al_s}{\delta u_\mu}=\overline{al}_s\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

so that (16′) becomes

$$\sum_{\lambda=1}^{\lambda=\rho}\left\{\frac{\sqrt{\mathrm{R}(x_\lambda)}}{\phi'(x_\lambda)}\frac{1}{x_\lambda-a_s}\right\}=\frac{\overline{al}_s}{al_s}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Weierstrass defines

$$al_{r,s}=\frac{al_r\overline{al}_s-al_s\overline{al}_r}{a_r-a_s}$$

$$=\frac{al_r al_s}{a_r-a_s}\left[\frac{\overline{al}_s}{al_s}-\frac{\overline{al}_r}{al_r}\right]$$

$$=-\sum_{\lambda=1}^{\lambda=\rho}\left\{\frac{\sqrt{\mathrm{R}(x_\lambda)}}{\phi'(x_\lambda)}\frac{al_r al_s}{(x_\lambda-a_r)(x_\lambda-a_s)}\right\}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where r, s must be different from each other, but otherwise may be any of the integers

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* *Cf.* Scott's 'Determinants,' c. ix., §§ 11, 12.

1, 2, . . . , $2\rho+1$. Evidently $al_{r,s}=al_{s,r}$, and there are therefore $\rho(2\rho+1)$ functions $al_{r,s}$; these, together with the $2\rho+1$ functions al_s, are the functions of the theory. (They are, of course, not all independent; the complete system of relations among them may be found in the fifth section of the first chapter of the memoir already quoted.)

Further

$$\frac{1}{al_r}\frac{\partial al_r}{\partial u_s}=-\frac{1}{2}\sum_{\lambda=1}^{\lambda=\rho}\frac{1}{a_r-x_\lambda}\frac{\partial x_\lambda}{\partial u_s}$$

$$=\sum_{\lambda=1}^{\lambda=\rho}\frac{\sqrt{\mathrm{R}(x_\lambda)}}{\phi'(x_\lambda)}\frac{l_s al_s^2}{(x_\lambda-a_s)\mathrm{P}'(a_s)}\frac{1}{(a_r-x_\lambda)}$$

$$=\frac{l_s al_s^2}{\mathrm{P}'(a_s)}\frac{al_{r,s}}{al_r al_s}$$

and

$$\frac{\partial al_r}{\partial u_s}=\frac{l_s}{\mathrm{P}'(a_s)}al_s al_{r,s}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

in which

s may have any value 1, 2, . . . , ρ
r ,, ,, 1, 2, . . . , $2\rho+1$,

but r, s may not be equal. If $r\leqq\rho$, this serves as a verification of (14).

Again, since x_1, x_2, . . . , x_ρ are the roots of (11),

$$\sum_{s=1}^{s=\rho}\frac{l_s al_s^2}{(x-a_s)\mathrm{P}'(a_s)}+1=\frac{(x-x_1)(x-x_2)\ldots(x-x_\rho)}{(x-a_1)(x-a_2)\ldots(x-a_\rho)}=\frac{\phi(x)}{\mathrm{P}(x)}.$$

In this write $x=a_{\rho+r}$, ($r=1, 2, \ldots, \rho+1$); then

$$al^2_{\rho+r}=\frac{\phi(a_{\rho+r})}{l_{\rho+r}}$$

$$=1-\sum_{s=1}^{s=\rho}\frac{l_s al_s^2}{(a_s-a_{\rho+r})\mathrm{P}'(a_s)}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

which expresses $\rho+1$ functions $al_{\rho+r}$ each in terms of the ρ functions al_1, al_2, . . . , al_ρ. By (20) and (14) we have

$$al^2_{\rho+r}=1+\sum_{s=1}^{s=\rho}\frac{1}{a_s-a_{\rho+r}}\frac{\partial \mathrm{U}}{\partial u_s}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21).$$

[17. A simpler form can be given to this equation by the introduction of a series of $\rho+1$ new variables provisionally given by

$$\left.\begin{aligned} u_1 &= \sum_{r=1}^{r=\rho+1} \frac{u_{\rho+r}}{a_1-a_{\rho+r}} \\ u_2 &= \sum_{r=1}^{r=\rho+1} \frac{u_{\rho+r}}{a_2-a_{\rho+r}} \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ u_\rho &= \sum_{r=1}^{r=\rho+1} \frac{u_{\rho+r}}{a_\rho-a_{\rho+r}} \end{aligned}\right\}$$

These new u's are not fully determined: as the remaining equation necessary to determine them assume

$$f(u_{\rho+1}, \ldots, u_{2\rho+1})=0.$$

When substitution is made in U for $u_1, \ldots, u_\rho$, U will be a function of $u_{\rho+1}, \ldots, u_{2\rho+1}$; and we shall have

$$\sum_{r=1}^{r=\rho+1} \frac{\delta \mathrm{U}}{\delta u_{\rho+r}} \delta u_{\rho+r} = \sum_{s=1}^{s=\rho} \frac{\delta \mathrm{U}}{\delta u_s} \delta u_s$$

$$= \sum_{s=1}^{s=\rho} \sum_{r=1}^{r=\rho+1} \frac{1}{a_s - a_{\rho+r}} \frac{\delta \mathrm{U}}{\delta u_s} \delta u_{\rho+r}$$

and from the $(\rho+1)^{\text{th}}$ equation giving the new u's

$$\sum_{r=1}^{r=\rho+1} \frac{\delta f}{\delta u_{\rho+r}} \delta u_{\rho+r} = 0.$$

Then by the principle of indeterminate multipliers

$$\frac{\delta \mathrm{U}}{\delta u_{\rho+r}} - \sum_{s=1}^{s=\rho} \frac{1}{a_s - a_{\rho+r}} \frac{\delta \mathrm{U}}{\delta u_s} = \lambda \frac{\delta f}{\delta u_{\rho+r}}$$

for all the $\rho+1$ values of r. Multiply these $\rho+1$ equations by $u_{\rho+1}, u_{\rho+2}, \ldots$ respectively and add; then

$$\sum_{r=1}^{r=\rho+1} u_{\rho+r} \frac{\delta \mathrm{U}}{\delta u_{\rho+r}} - \sum_{s=1}^{s=\rho} u_s \frac{\delta \mathrm{U}}{\delta u_s} = \lambda \sum_{r=1}^{r=\rho+1} u_{\rho+r} \frac{\delta f}{\delta u_{\rho+r}}.$$

Let the part of U which is of the order m in the u_s's ($s \leqq \rho$) be denoted by U_m; then when expressed in terms of the $u_{\rho+r}$'s it still remains the term of order m, so that

$$\sum_{r=1}^{r=\rho+1} u_{\rho+r} \frac{\delta \mathrm{U}_m}{\delta u_{\rho+r}} = m\mathrm{U}_m = \sum_{s=1}^{s=\rho} u_s \frac{\delta \mathrm{U}_m}{\delta u_s}$$

and summing up for the terms of all orders

$$\sum_{r=1}^{r=\rho+1} u_{\rho+r} \frac{\delta \mathrm{U}}{\delta u_{\rho+r}} = \sum_{s=1}^{s=\rho} u_s \frac{\delta \mathrm{U}}{\delta u_s}$$

and therefore from the above

$$\lambda \sum_{r=1}^{r=\rho+1} u_{\rho+r} \frac{\partial f}{\partial u_{\rho+r}}=0,$$

equivalent to one of the two equations

$$\lambda=0$$

or

$$\sum_{r=1}^{r=\rho+1} u_{\rho+r} \frac{\partial f}{\partial u_{\rho+r}}=0.$$

The latter, taken with the equation

$$f(u_{\rho+1}, \ldots, u_{2\rho+1})=0,$$

implies that there is a homogeneous relation between the quantities $u_{\rho+r}$; this we may reject. The former leaves f arbitrary or non-existent, and so there would be only ρ equations to determine $\rho+1$ quantities, a difficulty, however, obviable at any time by assigning some new equation to make up the requisite number; but $\lambda=0$ simplifies the resulting equations in which it occurs, and therefore this is selected. Let us assume as our new equation

$$\frac{v}{b}=\frac{u_{\rho+1}}{b-a_{\rho+1}}+\frac{u_{\rho+2}}{b-a_{\rho+2}}+\ldots+\frac{u_{2\rho+1}}{b-a_{2\rho+1}}$$

where v is a quantity which may have a definite value assigned to it at any time, if desired. Thus we have

$$\frac{\partial U}{\partial u_{\rho+r}}=\sum_{s=1}^{s=\rho} \frac{1}{a_s-a_{\rho+r}} \frac{\partial U}{\partial u_s}$$

and therefore

$$al^2_{\rho+r}=1+\frac{\partial U}{\partial u_{\rho+r}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14')$$

similar in form to (14).

18. Let us obtain the new u's explicitly from the above equations. Writing

$$g(z)=(z-b)\mathrm{P}(z)$$

we have*

$$u_{\rho+r}=-\frac{g(a_{\rho+r})}{\mathrm{Q}'(a_{\rho+r})}\left[\frac{\mathrm{Q}(a_1)}{g'(a_1)}\frac{u_1}{a_1-a_{\rho+r}}+\ldots+\frac{\mathrm{Q}(a_\rho)}{g'(a_\rho)}\frac{u_\rho}{a_\rho-a_{\rho+r}}+\frac{\mathrm{Q}(b)}{g'(b)}\frac{\frac{v}{b}}{b-a_{\rho+r}}\right].$$

Now make b infinite, so that the assumed equation takes the form

$$u_{\rho+1}+u_{\rho+2}+\ldots+u_{2\rho+1}=v$$

and

---

* *Cf.* SCOTT's 'Determinants,' *l.c.*

$$\frac{g(a_{\rho+r})}{g'(a_s)}=\frac{\mathrm{P}(a_{\rho+r})}{\mathrm{P}'(a_s)}$$

$$\frac{g(a_{\rho+r})}{b-a_{\rho+r}}=-\mathrm{P}(a_{\rho+r})$$

$$\frac{\mathrm{Q}(b)}{bg'(b)}=1$$

so that

$$-u_{\rho+r}=\sum_{s=1}^{s=\rho}\left\{\frac{u_s}{a_s-a_{\rho+r}}\,\frac{\mathrm{P}(a_{\rho+r})}{\mathrm{Q}'(a_{\rho+r})}\,\frac{\mathrm{Q}(a_s)}{\mathrm{P}'(a_s)}\right\}-\frac{\mathrm{P}(a_{\rho+r})}{\mathrm{Q}'(a_{\rho+r})}v.$$

As a verification of these values we may deduce (14) from (14′) as follows :—

$$\sum_{s=1}^{s=\rho}\frac{\delta\mathrm{U}}{\delta u_s}\delta u_s=d\mathrm{U}=\sum_{r=1}^{r=\rho+1}\frac{\delta\mathrm{U}}{\delta u_{\rho+r}}\delta u_{\rho+r}$$

$$=\sum_{r=1}^{r=\rho+1}\sum_{s=1}^{s=\rho}\frac{\delta\mathrm{U}}{\delta u_{\rho+r}}\,\frac{l_s l_{\rho+r}}{\mathrm{P}'(a_s)\mathrm{Q}'(a_{\rho+r})}\,\frac{\delta u_s}{a_s-a_{\rho+r}}+\sum_{r=1}^{r=\rho+1}\frac{l_{\rho+r}}{\mathrm{Q}'(a_{\rho+r})}\,\frac{\delta\mathrm{U}}{\delta u_{\rho+r}}\delta v.$$

Now the quantities δu_s, δv are independent ; hence we must have

$$\frac{\delta\mathrm{U}}{\delta u_s}=\sum_{r=1}^{r=\rho+1}\frac{l_s l_{\rho+r}}{\mathrm{P}'(a_s)\mathrm{Q}'(a_{\rho+r})}\,\frac{1}{a_s-a_{\rho+r}}\,\frac{\delta\mathrm{U}}{\delta u_{\rho+r}},$$

$$0=\sum_{r=1}^{r=\rho+1}\frac{l_{\rho+r}}{\mathrm{Q}'(a_{\rho+r})}\,\frac{\delta\mathrm{U}}{\delta u_{\rho+r}}.$$

Taking the second of these, we have

$$\frac{\delta\mathrm{U}}{\delta u_{\rho+r}}=-1+al^2_{\rho+r}$$

$$=-1+\frac{\phi(a_{\rho+r})}{l_{\rho+r}}$$

and therefore

$$\sum_{r=1}^{r=\rho+1}\frac{l_{\rho+r}}{\mathrm{Q}'(a_{\rho+r})}\,\frac{\delta\mathrm{U}}{\delta u_{\rho+r}}=-\sum_{r=1}^{r=\rho+1}\frac{l_{\rho+r}}{\mathrm{Q}'(a_{\rho+r})}+\sum_{r=1}^{r=\rho+1}\frac{\phi(a_{\rho+r})}{\mathrm{Q}'(a_{\rho+r})}$$

$$=-1+1=0$$

by the theorem already quoted in the verification of equation (11). For the first summation we have

$$\sum_{r=1}^{r=\rho+1}\frac{l_s}{\mathrm{P}'(a_s)}\,\frac{l_{\rho+r}}{\mathrm{Q}'(a_{\rho+r})}\,\frac{1}{a_s-a_{\rho+r}}\,\frac{\delta\mathrm{U}}{\delta u_{\rho+r}}$$

$$=\frac{l_s}{\mathrm{P}'(a_s)}\left[-\sum_{r=1}^{r=\rho+1}\frac{l_{\rho+r}}{\mathrm{Q}'(a_{\rho+r})}\,\frac{1}{a_s-a_{\rho+r}}+\sum_{r=1}^{r=\rho+1}\frac{l_{\rho+r}al^2_{\rho+r}}{\mathrm{Q}'(a_{\rho+r})(a_s-a_{\rho+r})}\right].$$

The first term inside the bracket is the expansion in partial fractions of

$$-\frac{P(a_s)}{Q(a_s)}$$

and is therefore zero since $s \leqq \rho$; the second is

$$\sum_{r=1}^{r=\rho+1}\left\{\frac{\phi(a_{\rho+r})}{Q'(a_{\rho+r})}\,\frac{1}{a_s-a_{\rho+r}}\right\}$$

$$=\frac{\phi(a_s)}{Q(a_s)}=-a l_s^2 \quad \text{by definition}$$

so that the equation with which we began leads to

$$\frac{\delta U}{\delta u_s}=-\frac{l_s}{P'(a_s)} a l_s^2$$

that is, to equation (14).]

19. Now let

$$\left.\begin{aligned} V&=\tfrac{1}{2}\sum_{\lambda=1}^{\lambda=r}\int_{a_\lambda}^{\xi_\lambda}\frac{P(x)}{\sqrt{R(x)}}dx \\ W&=\tfrac{1}{2}\sum_{\lambda=1}^{\lambda=\rho}\int_{a_\lambda}^{p_\lambda}\frac{P(x)}{\sqrt{R(x)}}dx \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13')$$

so that V stands to the v's and W to the w's in exactly the same relation as U to the u's.

Applying now the theorem in § 6 we have

$$U+V+W=-C_{\frac{1}{x}}\tfrac{1}{2}\frac{P(x)}{\sqrt{R(x)}}\log\left(\frac{My+Nz}{My-Nz}\right)$$

$$=-C_{\frac{1}{x}}\frac{y}{z}\left[\frac{Nz}{My}+\tfrac{1}{3}\left(\frac{Nz}{My}\right)^3+\ \ldots\ \right].$$

The n^{th} term in this series gives

$$-\frac{1}{2n-1}C_{\frac{1}{x}}\left(\frac{N}{M}\right)^{2n-1}\left(\frac{z}{y}\right)^{2n-2}.$$

$$=-\frac{1}{2n-1}C_{\frac{1}{x}}\left\{\frac{N_1^{\,2n-1}}{x^n}+\text{higher powers of }\frac{1}{x}\right\}$$

so that nothing is contributed except by the first term, and we have

$$U+V+W=-N_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22).$$

The 2ρ quantities $N_1, N_2, \ldots, N_\rho, M_1, \ldots, M_\rho$ are determined by the equation

$$N_1x^{\rho-1}z+N_2x^{\rho-2}z+\ \ldots\ +N_\rho z+M_1x^{\rho-1}y+\ \ldots\ +M_\rho y=-x^\rho y$$

which is satisfied by the 2ρ values of x, viz.: $x_1, x_2, \ldots, x_\rho, \xi_1, \xi_2, \ldots, \xi_\rho$; and therefore

$$\mathrm{N}_1 \begin{vmatrix} x_1^{\rho-1}z_1, x_1^{\rho-2}z_1, \ldots, y_1 \\ x_2^{\rho-1}z_2, x_2^{\rho-2}z_2, \ldots, y_2 \\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\ x_\rho^{\rho-1}z_\rho, x_\rho^{\rho-2}z_\rho, \ldots, y_\rho \\ \xi_1^{\rho-1}\zeta_1, \xi_1^{\rho-2}\zeta_1, \ldots, \eta_1 \\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\ \xi_\rho^{\rho-1}\zeta_\rho, \xi_\rho^{\rho-2}\zeta_\rho, \ldots, \eta_\rho \end{vmatrix} + \begin{vmatrix} x_1^{\rho}y_1, x_1^{\rho-2}z_1, \ldots, y_1 \\ x_2^{\rho}y_2, x_2^{\rho-2}z_2, \ldots, y_2 \\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\ x_\rho^{\rho}y_\rho, x_\rho^{\rho-2}z_\rho, \ldots, y_\rho \\ \xi_1^{\rho}\eta_1, \xi_1^{\rho-2}\zeta_1, \ldots, \eta_1 \\ \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot \\ \xi_\rho^{\rho}\eta_\rho, \xi_\rho^{\rho-2}\zeta_\rho, \ldots, \eta_\rho \end{vmatrix} = 0.$$

20. As an example of (22) and (23) consider the elliptic functions, *i.e.*, the case in which $\rho=1$; then

$$\mathrm{N}_1 \begin{vmatrix} z & y \\ \zeta & \eta \end{vmatrix} + \begin{vmatrix} xy & y \\ \xi\eta & \eta \end{vmatrix} = 0$$

(dropping suffixes), or

$$\mathrm{N}_1(z^2\eta^2-y^2\zeta^2)=(\xi-x)y\eta(z\eta+y\zeta)$$

that is

$$\mathrm{N}_1\{(x-a_1)(\xi-a_1)-(a_1-a_2)(a_1-a_3)\}=-y\eta(z\eta+y\zeta);$$

and

$$u=\tfrac{1}{2}\int_{a_1}^{x}\frac{dx}{\sqrt{x-a_1.x-a_2.x-a_3}}$$

$$\mathrm{U}=\tfrac{1}{2}\int_{a_1}^{x}\frac{\sqrt{x-a_1}\,dx}{\sqrt{x-a_2.x-a_3}}.$$

Let

$$x=a_1+(a_2-a_1)t^2,\ k^2=\frac{a_2-a_1}{a_3-a_1};$$

then

$$u=\frac{1}{\sqrt{a_3-a_1}}\int_0^t\frac{dt}{\sqrt{1-t^2.1-k^2t^2}},$$

$$\mathrm{U}=\frac{a_2-a_1}{\sqrt{a_3-a_1}}\int_0^t\frac{t^2dt}{\sqrt{1-t^2.1-k^2t^2}}.$$

Let s, c, d denote elliptic functions of $u\sqrt{a_3-a_1}$

S, C, D ,, ,, $v\sqrt{a_3-a_1}$

σ ,, $\mathrm{sn}\{(u+v)\sqrt{a_3-a_1}\}$ *i.e.*, $-\mathrm{sn}\,w\sqrt{a_3-a_1}$.

Then

$$s^2=t^2=\frac{x-a_1}{a_2-a_1},$$

and therefore

$$x-a_1=\quad(a_2-a_1)s^2$$
$$x-a_2=-(a_2-a_1)c^2$$
$$x-a_3=-(a_3-a_1)d^2$$

and so

$$\mathrm{N}_1=\frac{(a_2-a_1)^2\sqrt{a_3-a_1}\{scd\mathrm{S}^2+\mathrm{SCD}s^2\}}{(a_2-a_1)(a_3-a_1)(1-k^2s^2\mathrm{S}^2)}$$
$$=\frac{a_2-a_1}{\sqrt{a_3-a_1}}s\mathrm{S}\sigma.$$

With the ordinary notation for the second elliptic integral we have

$$\mathrm{E}(u\sqrt{a_3-a_1})=u\sqrt{a_3-a_1}-\frac{\mathrm{U}}{\sqrt{a_3-a_1}}$$

and since

$$u+v+w=0$$

this gives

$$-\frac{1}{\sqrt{a_3-a_1}}(\mathrm{U}+\mathrm{V}+\mathrm{W})=\mathrm{E}(u\sqrt{a_3-a_1})+\mathrm{E}(v\sqrt{a_3-a_1})+\mathrm{E}(w\sqrt{a_3-a_1}$$
$$=k^2s\mathrm{S}\sigma$$
$$=\frac{\mathrm{N}_1}{\sqrt{a_3-a_1}}$$

that is

$$\mathrm{U}+\mathrm{V}+\mathrm{W}=-\mathrm{N}_1$$

agreeing with the case when $\rho=1$ of (22).

21. The evaluation of N_1 in terms of the functions can be obtained in the general case as follows.

Since $x_1, x_2, \ldots, x_\rho, \xi_1, \xi_2, \ldots, \xi_\rho, p_1, p_2, \ldots, p_\rho$ are the roots of

$$\mathrm{M}^2y^2-\mathrm{N}^2z^2=0$$

we have

$$\mathrm{M}^2y^2-\mathrm{N}^2z^2\equiv(x-x_1)(x-x_2)\ldots(x-p_\rho).$$

In this write $x=a_m$, where $m\geqq1\leqq\rho$; then

$$-\mathrm{N}^2\mathrm{Q}(a_m)\equiv(a_m-x_1)(a_m-x_2)\ldots(a_m-p_\rho)$$

that is

$$l_m\{\mathrm{N}(a_m)\}^2=l_m{}^3al_m{}^2(u)al_m{}^2(v)al_m{}^2(u+v)$$

and therefore

$$\mathrm{N}_1a_m{}^{\rho-1}+\mathrm{N}_2a_m{}^{\rho-2}+\ldots+\mathrm{N}_\rho=\pm l_mal_m(u)al_m(v)al_m(u+v).$$

Hence

$$\sum_{m=1}^{m=\rho}\frac{l_m}{\mathrm{P}'(a_m)}al_m(u)al_m(v)al_m(u+v)=\pm\sum_{m=1}^{m=\rho}\frac{\mathrm{N}_1a_m{}^{\rho-1}+\ldots+\mathrm{N}_\rho}{\mathrm{P}'(a_m)}.$$

But $a_1, a_2, \ldots a_\rho$ are the ρ roots of

$$\mathrm{P}(z)=0$$

and therefore as before

$$\sum_{m=1}^{m=\rho}\frac{a_m^{\rho-1}}{\mathrm{P}'(a_m)}=1 \text{ and } \sum_{m=1}^{m=\rho}\frac{a_m^{\rho-s}}{\mathrm{P}'(a_m)}=0(s>1\leqq\rho)$$

and so

$$\sum_{m=1}^{m=\rho}\left\{\frac{l_m}{\mathrm{P}'(a_m)}al_m(u)al_m(v)al_m(u+v)\right\}=\pm\mathrm{N}_1$$

and therefore

$$\begin{aligned}\mathrm{U}+\mathrm{V}+\mathrm{W}&=-\mathrm{N}_1\\ &=\mp\sum_{m=1}^{m=\rho}\left\{\frac{l_m}{\mathrm{P}'(a_m)}al_m(u)al_m(v)al_m(u+v)\right\}.\end{aligned}$$

On the expansion of each side in terms of the u's and v's as is done below, it is at once seen that the lower sign is the correct one; and therefore

$$\mathrm{U}+\mathrm{V}+\mathrm{W}=\sum_{m=1}^{m=\rho}\left\{\frac{l_m}{\mathrm{P}'(a_m)}al_m(u)al_m(v)al_m(u+v)\right\}.$$

This may be called the addition theorem for the integral-function; by putting $\rho=1$ and referring to the example worked out in the last section, it is at once seen to be the addition theorem for elliptic integrals of the second order.

22. In the expansion of the two sides in terms of u's and v's the first term is sufficient to indicate the correct sign in the above; but it is not uninteresting to see the agreement for terms of a higher order, and the expansion is carried on as far as the order seven in the magnitudes u.

Proceeding therefore to form the expansion of U in terms of the u's, write, with WEIERSTRASS,

$$\frac{\mathrm{P}'(a_r)}{l_r}(a_r-x_r)=s_r^2\,. \qquad \qquad (24)$$

so that

$$-\frac{\mathrm{P}'(a_r)}{l_r}dx_r=2s_rds_r$$

and

$$\frac{dx_r}{x_r-a_r}=2\frac{ds_r}{s_r}.$$

Let

$$\mathrm{a}_{r,m}=\frac{l_r}{\mathrm{P}'(a_r)}\frac{1}{a_r-a_m} \qquad \qquad (25)$$

so that

$$\frac{l_m}{\mathrm{P}'(a_m)}\mathrm{a}_{r,m}=-\frac{l_r}{\mathrm{P}'(a_r)}\mathrm{a}_{m,r};$$

then

$$x_r-a_m=(a_r-a_m)(1-\mathrm{a}_{r,m}s_r^2).$$

Substituting we have

$$\mathrm{P}(x_m)=-\frac{l_m s_m^2}{\mathrm{P}'(a_m)}\mathrm{P}'(a_m)+\left\{\frac{l_m s_m^2}{\mathrm{P}'(a_m)}\right\}^2\frac{\mathrm{P}''(a_m)}{2!}-\left\{\frac{l_m s_m^2}{\mathrm{P}'(a_m)}\right\}^3\frac{\mathrm{P}'''(a_m)}{3!}+\dots$$

$$\mathrm{Q}(x_m)=-l_m-\frac{l_m s_m^2}{\mathrm{P}'(a_m)}\mathrm{Q}'(a_m)+\left\{\frac{l_m s_m^2}{\mathrm{P}'(a_m)}\right\}^2\frac{\mathrm{Q}''(a_m)}{2!}-\dots$$

and therefore

$$\sqrt{\frac{\mathrm{P}(x_m)}{\mathrm{Q}(x_m)}}=s_m\sqrt{\frac{1-\dfrac{l_m s_m^2}{\{\mathrm{P}'(a_m)\}^2}\dfrac{\mathrm{P}''(a_m)}{2\,!}+\dots}{1+\dfrac{s_m^2}{\mathrm{P}'(a_m)}\mathrm{Q}'(a_m)-\dots}}$$

$$=s_m(1-\mathrm{A}_m s_m^2-\mathrm{B}_m s_m^4)\text{ say, } \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

correct to the fifth order. Moreover

$$d\mathrm{U}=\tfrac{1}{2}\sum_{m=1}^{m=\rho}\sqrt{\frac{\mathrm{P}(x_m)}{\mathrm{Q}(x_m)}}dx_m$$

$$=-\sum_{m=1}^{m=\rho}\frac{l_m}{\mathrm{P}'(a_m)}s_m^2 ds_m(1-\mathrm{A}_m s_m^2-\mathrm{B}_m s_m^4)$$

or

$$\mathrm{U}=-\sum_{m-1}^{m=\rho}\frac{l_m}{\mathrm{P}'(a_m)}\left[\frac{s_m^3}{3}-\frac{\mathrm{A}_m}{5}s_m^5-\frac{\mathrm{B}_m}{7}s_m^7\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

correct to the seventh order. Further

$$2du_m=\sqrt{\frac{\mathrm{P}(x_1)}{\mathrm{Q}(x_1)}}\frac{dx_1}{x_1-a_m}+\dots+\sqrt{\frac{\mathrm{P}(x_m)}{\mathrm{Q}(x_m)}}\frac{dx_m}{x_m-a_m}+\dots$$

and

$$\frac{dx_r}{x_r-a_m}=-2\mathrm{a}_{r,m}s_r ds_r(1+\mathrm{a}_{r,m}s_r^2+\mathrm{a}_{r,m}^2 s_r^4)$$

which with the help of (26) gives

$$du_m=ds_m(1-\mathrm{A}_m s_m^2-\mathrm{B}_m s_m^4)-\sum_{r=1}^{r=\rho}{}' \mathrm{a}_{r,m}s_r^2 ds_r(1+\mathrm{a}_{r,m}s_r^2+\mathrm{a}_{r,m}^2 s_r^4)(1-\mathrm{A}_r s_r^2-\mathrm{B}_r s_r^4)$$

where Σ' denotes that r may receive all values between 1 and ρ except m. Thus

$$u_m=s_m-\frac{\mathrm{A}_m}{3}s_m^3-\frac{\mathrm{B}_m}{5}s_m^5-\sum_{r=1}^{r=\rho}{}' \mathrm{a}_{r,m}\left\{\frac{s_r^3}{3}+\frac{\mathrm{a}_{r,m}-\mathrm{A}_r}{5}s_r^5\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

correct to the fifth order; and as s_m is of the order u this expansion will be sufficient for the expansion of U in (27) accurately to the seventh order. The equation (28) holds for $m=1, 2, \dots, \rho$.

Inverting it in order to obtain s in terms of u's we find that to the fifth order

$$
\begin{aligned}
s_m = u_m & \\
& + \frac{\mathrm{A}_m}{3} u_m^3 + \mathop{\Sigma_m}\limits_{r=1}^{r=\rho} \mathrm{a}_{r,m} \frac{u_r^3}{3} \\
& + \frac{5\mathrm{A}_m^2 + 3\mathrm{B}_m}{15} u_m^5 + \frac{\mathrm{A}_m}{3} u_m^2 \mathop{\Sigma_m}\limits_{r=1}^{r=\rho} \mathrm{a}_{r,m} u_r^3 + \mathop{\Sigma_m}\limits_{r=1}^{r=\rho} \frac{\mathrm{a}_{r,m}(3\mathrm{a}_{r,m} + 2\mathrm{A}_r)}{15} u_r^5 + \tfrac{1}{3} \mathop{\Sigma_m}\limits_{r=1}^{r=\rho} \mathrm{a}_{r,m} u_r^2 \mathop{\Sigma_r}\limits_{s=1}^{s=\rho} \mathrm{a}_{s,r} u_s^3 \qquad (29)
\end{aligned}
$$

where $\mathop{\Sigma_m}\limits_{r=1}^{r=\rho}$ implies summation for all values of r from 1 to ρ except m, and $\mathop{\Sigma_r}\limits_{s=1}^{s=\rho}$ for all values of s from 1 to ρ except r. Substituting this in (27) we obtain

$$
\begin{aligned}
-\mathrm{U} = \sum_{m=1}^{m=\rho} \frac{l_m}{\mathrm{P}'(a_m)} \Bigg[& \frac{u_m^3}{3} + \frac{2\mathrm{A}_m}{15} u_m^5 + u_m^2 \mathop{\Sigma_m}\limits_{r=1}^{r=\rho} \mathrm{a}_{r,m} \frac{u_r^3}{3} + \left(\frac{2\mathrm{B}_m}{35} + \frac{\mathrm{A}_m^2}{9} \right) u_m^7 \\
& + \frac{2\mathrm{A}_m}{3} u_m^4 \mathop{\Sigma_m}\limits_{r=1}^{r=\rho} \mathrm{a}_{r,m} \frac{u_r^3}{3} + \frac{u_m^2}{15} \mathop{\Sigma_m}\limits_{r=1}^{r=\rho} \mathrm{a}_{r,m}(3\mathrm{a}_{r,n} + 2\mathrm{A}_r) u_r^5 \\
& + \frac{u_m^2}{3} \mathop{\Sigma_m}\limits_{r=1}^{r=\rho} \mathrm{a}_{r,m} u_r^2 \mathop{\Sigma_r}\limits_{s=1}^{s=\rho} \mathrm{a}_{s,r} u_s^3 + \frac{u_m}{9} \mathop{\Sigma_m}\limits_{r=1}^{r=\rho} \mathop{\Sigma_m}\limits_{s=1}^{s=\rho} \mathrm{a}_{r,m} \mathrm{a}_{s,m} u_r^3 u_s^3 \Bigg] \quad . \quad . \quad . \quad (30)
\end{aligned}
$$

correct to the seventh order. In the last term inside the bracket r and s may take the same value; the double summation is in fact

$$
\left[\mathop{\Sigma_m}\limits_{r=1}^{r=\rho} \mathrm{a}_{r,m} u_r^3 \right]^2.
$$

Again

$$
\begin{aligned}
al_m^2(u) &= \frac{1}{l_m} \phi(a_m) \\
&= s_m^2 (1 - \mathrm{a}_{1,m} s_1^2)(1 - \mathrm{a}_{2,m} s_2^2) \ . \ . \ . \ (1 - \mathrm{a}_{\rho,m} s_\rho^2)
\end{aligned}
$$

(the term involving s_m^2 not occurring in the brackets)

$$
= s_m^2 \left[1 - \mathop{\Sigma_m}\limits_{r=1}^{r=\rho} \mathrm{a}_{r,m} s_r^2 + \sum_{r,\,t=1}^{r,\,t=\rho} \mathrm{a}_{r,m} \mathrm{a}_{t,m} s_r^2 s_t^2 \right]
$$

where $\mathop{\Sigma_m}\limits_{r,\,t=1}^{r,\,t=\rho}$ implies summation for all values 1, 2, . . . , ρ of r and t except m, and r and t must not have the same values. Extracting the square root we find

$$
al_m(u) = s_m \left[1 - \tfrac{1}{2} \mathop{\Sigma_m}\limits_{r=1}^{r=\rho} \mathrm{a}_{r,m} s_r^2 - \tfrac{1}{8} \mathop{\Sigma_m}\limits_{r=1}^{r=\rho} \mathrm{a}_{r,m}^2 s_r^4 + \tfrac{1}{4} \mathop{\Sigma_m}\limits_{r,\,t=1}^{r,\,t=\rho} \mathrm{a}_{r,m} \mathrm{a}_{t,m} s_r^2 s_t^2 \right] \quad . \quad . \quad . \quad (31).
$$

Let σ_m refer to $al_m(v)$, S_m to $al_m(u+v)$, so that to the first order

$$al_m(v)=\sigma_m$$
$$al_m(u+v)=\mathrm{S}_m$$

and regard σ_m and S_m as being of the same order as s_m. Then accurately to the seventh order

$$al_m(u)al_m(v)al_m(u+v)$$

$$=s_m\sigma_m\mathrm{S}_m\Big[1-\tfrac{1}{2}\sum_{r=1}^{r=\rho}{}^{m}\,\mathrm{a}_{r,m}(s_r^2+\sigma_r^2+\mathrm{S}_r^2)-\tfrac{1}{8}\sum_{r=1}^{r=\rho}{}^{m}\,\mathrm{a}_{r,m}^2(s_r^4+\sigma_r^4+\mathrm{S}_r^4)$$
$$+\tfrac{1}{4}\sum_{r,t=1}^{r,t=\rho}{}^{m}\,\mathrm{a}_{r,m}\mathrm{a}_{t,m}(s_r^2s_t^2+\sigma_r^2\sigma_t^2+\mathrm{S}_r^2\mathrm{S}_t^2)+\tfrac{1}{4}\sum_{r=1}^{r=\rho}{}^{m}\sum_{t=1}^{t=\rho}{}^{m}\,\mathrm{a}_{r,m}\mathrm{a}_{t,m}(s_r^2\sigma_t^2+\sigma_r^2\mathrm{S}_t^2+\mathrm{S}_r^2s_t^2)\Big] \quad (32)$$

where the summation in the last term in (32) is exactly as in the last term in (30). To express this in terms of u and v we must substitute the value of s in terms of u's as given by (29) and for σ and S respectively corresponding values of v and $u+v$. Let these values be inserted, both sides multiplied by $\frac{l_m}{\mathrm{P}'(a_m)}$ and the summation taken for the values $m=1$ to $m=\rho$ and compare this expression, which is

$$\sum_{m=1}^{m=\rho}\frac{l_m}{\mathrm{P}'(a_m)}al_m(u)al_m(v)al_m(u+v)\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad.\quad(\mathrm{B}),$$

with the value of U+V+W.

Firstly, they agree in the third order of quantities; for

$$-\frac{u_m^3+v_m^3+w_m^3}{3}=u_mv_m(u_m+v_m)$$

since

$$u_m+v_m+w_m=0.$$

Secondly, consider in each the terms of the order five. That in U+V+W which has $\frac{l_m}{\mathrm{P}'(a_m)}\mathrm{A}_m$ for its coefficient is

$$\tfrac{2}{15}[(u_m+v_m)^5-u_m^5-v_m^5]$$
$$=\tfrac{2}{3}u_mv_m(u_m^3+2u_m^2v_m+2u_mv_m^2+v_m^3)$$
$$=\tfrac{2}{3}u_mv_m(u_m+v_m)(u_m^2+v_m^2+u_mv_m)$$

while in (B) it is

$$\tfrac{1}{3}u_mv_m(u_m+v_m)\{u_m^2+v_m^2+(u_m+v_m)^2\}$$

and these are obviously equal.

The term in U+V+W which has

$$\tfrac{1}{3}\frac{l_m}{\mathrm{P}'(a_m)}\,\frac{l_r}{\mathrm{P}'(a_r)}\,\frac{1}{a_r-a_m},\ i.e.\ \tfrac{1}{3}\frac{l_m}{\mathrm{P}'(a_m)}\mathrm{a}_{r,m}\ \text{or}\ -\tfrac{1}{3}\frac{l_r}{\mathrm{P}'(a_r)}\mathrm{a}_{m,r},$$

for coefficient is

$$(u_m+v_m)^2(u_r+v_r)^3-(u_r+v_r)^2(u_m+v_m)^3-u_m^2u_r^3+u_r^2u_m^3-v_m^2v_r^3+v_r^2v_m^3$$

while in (B) it is

$$\begin{aligned}&(u_m+v_m)v_mu_r^3-(u_r+v_r)v_ru_m^3-\tfrac{3}{2}u_mv_m(u_m+v_m)\{u_r^2+v_r^2+(u_r+v_r)^2\}\\ &+(u_m+v_m)u_mv_r^3-(u_r+v_r)u_rv_m^3+\tfrac{3}{2}u_rv_r(u_r+v_r)\{u_m^2+v_m^2+(u_m+v_m)^2\}\\ &+u_mv_m(u_r+v_r)^3-u_rv_r(u_m+v_m)^3.\end{aligned}$$

Adding the latter up in columns, it is

$$\begin{aligned}&u_r^3[(u_m+v_m)^2-u_m^2]+v_r^3[(u_m+v_m)^2-v_m^2]+3u_mv_mu_rv_r(u_r+v_r) \text{ for first}\\ &-u_m^3[u_r+v_r)^2-u_r^2]-v_m^3[(u_r+v_r)^2-v_r^2]-3u_mv_mu_rv_r(u_m+v_m) \text{ for second}\\ &-3u_mv_m(u_m+v_m)(u_r+v_r)^2+3u_mv_mu_rv_r(u_m+v_m)+3u_rv_r(u_r+v_r)(u_m+v_m)^2\\ &\qquad\qquad-3u_rv_ru_mv_m(u_r+v_r) \text{ for third}\\ &=-u_m^2u_r^3-v_m^2v_r^3+u_m^3u_r^2+v_m^3v_r^2+(u_m+v_m)^2(u_r+v_r)^3-(u_r+v_r)^2(u_m+v_m)^3\end{aligned}$$

and therefore, to the order five, (B) and U+V+W are equal.

Thirdly, consider in the order seven the term in U+V+W which has

$$\frac{l_m}{\mathrm{P}'(a_m)}\left(\frac{2\mathrm{B}_m}{35}+\frac{\mathrm{A}_m^2}{9}\right)$$

for coefficient; it is

$$\begin{aligned}&(u_m+v_m)^7-u_m^7-v_m^7\\ &=7u_mv_m(u_m+v_m)[u_m^4+v_m^4+3u_m^2v_m^2+2u_mv_m(u_m^2+v_m^2)]\end{aligned}$$

while in B the term of order seven which is free from all the a's and is multiplied by $\frac{l_m}{\mathrm{P}'(a_m)}$ is

$$\begin{aligned}&\frac{l_m}{\mathrm{P}'(a_m)}[u_mv_m(u_m+v_m)\{u_m^4+v_m^4+(u_m+v_m)^4\}]\frac{5\mathrm{A}_m^2+3\mathrm{B}_m}{15}\\ &+\frac{l_m}{\mathrm{P}'(a_m)}\,\frac{\mathrm{A}_m^2}{9}[u_mv_m(u_m+v_m)\{v_m^2(u_m+v_m)^2+u_m^2(u_m+v_m)^2+u_m^2v_m^2\}]\\ &=\frac{l_m}{\mathrm{P}'(a_m)}\left(\frac{2\mathrm{B}_m}{5}+\frac{7\mathrm{A}_m^2}{9}\right)u_mv_m(u_m+v_m)[u_m^4+v_m^4+3u_m^2v_m^2+2u_mv_m(u_m^2+v_m^2)]\end{aligned}$$

and again these terms are equal.

I have verified the exact agreement of the two expressions for one or two others (but not for all, owing to the labour involved) of the terms of the seventh order; and this exact agreement leads us to infer the truth of the equation

$$U+V+W=\sum_{m=1}^{m=\rho}\frac{l_m}{P'(a_m)}al_m(u)al_m(v)al_m(u+v) \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

a direct proof of which has already been obtained.

23. Combining (22) and (33) we have

$$U+V+W=-N_1=\sum_{s=1}^{s=\rho}\frac{l_s}{P'(a_s)}al_s(u)al_s(v)al_s(u+v).$$

Therefore

$$\sum_{m=1}^{m=\rho}\left[\frac{\delta U}{\delta u_m}\delta u_m+\frac{\delta V}{\delta v_m}\delta v_m+\frac{\delta W}{\delta w_m}\delta w_m\right]=-\sum_{m=1}^{m=\rho}\left[\frac{\delta N_1}{\delta u_m}\delta u_m+\frac{\delta N_1}{\delta v_m}\delta v_m\right].$$

But by (7)

$$\delta u_m+\delta v_m+\delta w_m=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7')$$

so that substituting for the δw's and remembering that the δu's and δv's are independent we have

$$\left.\begin{aligned}\frac{\delta U}{\delta u_m}-\frac{\delta W}{\delta w_m}&=-\frac{\delta N_1}{\delta u_m}\\ \frac{\delta V}{\delta v_m}-\frac{\delta W}{\delta w_m}&=-\frac{\delta N_1}{\delta v_m}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34).$$

By the first of these

$$\frac{\delta^2 U}{\delta u_m\delta u_n}\delta u_n-\frac{\delta^2 W}{\delta w_m\delta w_n}\delta w_n=-\frac{\delta^2 N_1}{\delta u_m\delta u_n}\delta u_n-\frac{\delta^2 N_1}{\delta u_m\delta v_n}\delta v_n$$

and therefore by (7′)

Similarly

$$\left.\begin{aligned}\frac{\delta^2 U}{\delta u_m\delta u_n}+\frac{\delta^2 W}{\delta w_m\delta w_n}&=-\frac{\delta^2 N_1}{\delta u_m\delta u_n}\\ \frac{\delta^2 W}{\delta w_m\delta w_n}&=-\frac{\delta^2 N_1}{\delta u_m\delta v_n}\\ \frac{\delta^2 V}{\delta v_m\delta v_n}+\frac{\delta^2 W}{\delta w_m\delta w_n}&=-\frac{\delta^2 N_1}{\delta v_m\delta v_n}\\ \frac{\delta^2 W}{\delta w_m\delta w_n}&=-\frac{\delta^2 N_1}{\delta v_m\delta u_n}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

from which we see that N_1 satisfies the series of differential equations

$$\frac{\delta^2 N_1}{\delta u_m\delta v_n}=\frac{\delta^2 N_1}{\delta v_m\delta u_n}$$

of which there are $\frac{1}{2}\rho(\rho-1)$ in all.

24. Returning now to (34) and using (14) we have

$$\frac{l_m}{\mathrm{P}'(a_m)}\{al_m^2(u+v)-al_m^2(u)\}=\sum_{s=1}^{s=\rho}\frac{l_s}{\mathrm{P}'(a_s)}al_s(v)\left\{al_s(u)\frac{\delta al_s(u+v)}{\delta u_m}+al_s(u+v)\frac{\delta al_s u}{\delta u_m}\right\} \quad . (36)$$

$$\frac{l_m}{\mathrm{P}'(a_m)}\{al_m^2(u+v)-al_m^2(v)\}=\sum_{s=1}^{s=\rho}\frac{l_s}{\mathrm{P}'(a_s)}al_s(u)\left\{al_s(v)\frac{\delta al_s(u+v)}{\delta v_m}+al_s(u+v)\frac{\delta al_s(v)}{\delta v_m}\right\} \quad . (37)$$

and from these by subtraction and noticing that

$$\frac{\delta al_s(u+v)}{\delta u_m}=\frac{\delta al_s(u+v)}{\delta v_m}$$

we have

$$\frac{l_m}{\mathrm{P}'(a_m)}\{al_m^2(u)-al_m^2(v)\}=\sum_{s=1}^{s=\rho}\frac{l_s}{\mathrm{P}'(a_s)}al_s(u+v)\left\{al_s(u)\frac{\delta al_s(v)}{\delta v_m}-al_s(v)\frac{\delta al_s(u)}{\delta u_m}\right\} \quad . \quad (38).$$

Now if s be different from m

$$\frac{\delta al_s(u)}{\delta u_m}=\frac{l_m}{\mathrm{P}'(a_m)}al_m(u)al_{s,m}(u)$$

but this no longer holds when s,m are the same since $al_{m,m}$ is not a recognised function. We proceed as follows to obtain $\frac{\delta al_m(u)}{\delta u_m}$:—differentiate both sides of (20) with respect to u_m so that

$$al_{\rho+r}(u)\frac{\delta al_{\rho+r}(u)}{\delta u_m}=-\mathop{\Sigma_m}_{s=1}^{s=\rho}\left\{\frac{l_s}{a_s-a_{\rho+r}}\,\frac{al_s(u)}{\mathrm{P}'(a_s)}\,\frac{\delta al_s(u)}{\delta u_m}\right\}-\frac{l_m}{a_m-a_{\rho+r}}\,\frac{al_m(u)}{\mathrm{P}'(a_m)}\,\frac{\delta al_m(u)}{\delta u_m}$$

where $\mathop{\Sigma_m}_{s=1}^{s=\rho}$ implies that the value $s=m$ is not to be included in the summation. The equation quoted above (holding for all values of s from 1 to $2\rho+1$) when substituted in the last gives, on division by $\frac{l_m}{\mathrm{P}'(a_m)}al_m(u)$,

$$al_{m,\rho+r}(u)al_{\rho+r}(u)+\mathop{\Sigma_m}_{s=1}^{s=\rho}\left\{\frac{l_s}{(a_s-a_{\rho+r})\mathrm{P}'(a_s)}al_s(u)al_{s,m}(u)\right\}=-\frac{1}{a_m-a_{\rho+r}}\,\frac{\delta al_m(u)}{\delta u_m}$$

and (38) may now be written in the form

$$\begin{aligned}al_m^2(u)-al_m^2(v)=&\mathop{\Sigma_m}_{s=1}^{s=\rho}\frac{l_s}{\mathrm{P}'(a_s)}al_s(u+v)\{al_s(u)al_m(v)al_{s,m}(v)-al_s(v)al_m(u)al_{s,m}(u)\}\\ &-al_m(u+v)\mathop{\Sigma_m}_{s=1}^{s=\rho}\frac{l_s(a_m-a_{\rho+r})}{(a_s-a_{\rho+r})\mathrm{P}'(a_s)}al_s(v)al_{s,m}(v)al_m(u)-al_s(u)al_{s,m}(u)al_m(v)\\ &-(a_m-a_{\rho+r})al_m(u+v)\{al_m(u)al_{\rho+r}(v)al_{m,\rho+r}(v)-al_m(v)al_{\rho+r}(u)al_{m,\rho+r}(u)\} \quad (39).\end{aligned}$$

This equation holds for the values 1, 2, . . . , ρ of m and these ρ equations determine the ρ functions $al_m(u+v)$ for values 1, . . . , ρ of m in terms of functions of u and v. Moreover, r is any one of the numbers 1, 2, . . . , $\rho+1$, so that these equations can have a large variety of forms. We may thus consider the functions $al_m(u+v) (m \leqq \rho)$ as known; the $\rho+1$ functions $al_{\rho+r}(u+v)$ are given in terms of them and therefore ultimately in terms of functions of u and v by the equation

$$al^2_{\rho+r}(u+v)=1-\sum_{s=1}^{s=\rho}{}^m\left\{\frac{l_s}{(a_s-a_{\rho+r})\mathrm{P}'(a_s)}al_s^2(u+v)\right\}.$$

Treating (36) in the same manner as (38) it will yield ρ equations involving the double-suffix functions of $u+v$; this system, together with the relations between them (to which reference has already been made), will furnish the complete solution of the addition theorem for these functions.

Abelian functions of order 2.

25. Consider the particular case of the preceding for which $\rho=2$. We now have

$$\left.\begin{aligned} \mathrm{P}(x)&=(x-a_1)(x-a_2)\\ \mathrm{Q}(x)&=(x-a_3)(x-a_4)(x-a_5)\\ \mathrm{R}(x)&=\mathrm{P}(x)\mathrm{Q}(x)\end{aligned}\right\}$$

$$\left.\begin{aligned} u_1&=\tfrac{1}{2}\int_{a_1}^{x_1}\frac{x-a_2}{\sqrt{\mathrm{R}(x)}}dx+\tfrac{1}{2}\int_{a_2}^{x_2}\frac{x-a_2}{\sqrt{\mathrm{R}(x)}}dx\\ u_2&=\tfrac{1}{2}\int_{a_1}^{x_1}\frac{x-a_1}{\sqrt{\mathrm{R}(x)}}dx+\tfrac{1}{2}\int_{a_2}^{x_2}\frac{x-a_1}{\sqrt{\mathrm{R}(x)}}dx\end{aligned}\right\}$$

$$\phi(x)=(x-x_1)(x-x_2).$$

Write

$$\left.\begin{aligned} x_1-a_r&=\mathrm{a}_r\\ x_2-a_r&=\mathrm{b}_r\end{aligned}\right\}(r=1,\ 2,\ 3,\ 4,\ 5)\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

Also

$$\left.\begin{aligned} l_1,\ l_2&=-\mathrm{Q}(a_1),\ -\mathrm{Q}(a_2)\\ l_3,\ l_4,\ l_5&=\mathrm{P}(a_3),\ \mathrm{P}(a_4),\ \mathrm{P}(a_5)\end{aligned}\right\}\text{respectively}\quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Then

$$l_s al_s^2=\phi(a_s)=\mathrm{a}_s\mathrm{b}_s\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

for $s=1$, 2, 3, 4, 5; and

$$al_{r,s}=\frac{1}{(x_1-x_2)\sqrt{l_r l_s}}\left[\sqrt{\mathrm{a}_r\mathrm{a}_s\mathrm{bbb}}-\sqrt{\mathrm{b}_r\mathrm{b}_s\mathrm{aaa}}\right]\quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

the suffixes being added to the a's and b's under the radical sign so as to have 1, 2, 3, 4, 5 for the complete system under any one root-sign. Then

$$al_t al_{r,s} = \frac{1}{(x_1 - x_2)\sqrt{l_r l_s l_t}} \left[\mathrm{b}_t \sqrt{\mathrm{a}_r \mathrm{a}_s \mathrm{a}_t \mathrm{bb}} - \mathrm{a}_t \sqrt{\mathrm{b}_r \mathrm{b}_s \mathrm{b}_t \mathrm{aa}}\right]$$

and therefore

$$(a_r - a_s) al_t al_{r,s} + (a_s - a_t) al_r al_{s,t} + (a_t - a_r) al_s al_{r,t} = 0 \quad . \quad . \quad . \quad . \quad . \quad (5).$$

Again

$$l_r al_r al_{r,s} = \frac{1}{(x_1 - x_2)\sqrt{l_s}} \left[\mathrm{a}_r \sqrt{\mathrm{a}_s \mathrm{bbbb}} - \mathrm{b}_r \sqrt{\mathrm{b}_s \mathrm{aaaa}}\right]$$

and therefore

$$(a_p - a_q) l_r al_r al_{r,s} + (a_q - a_r) l_p al_p al_{p,s} + (a_r - a_p) l_q al_q al_{q,s} = 0 \quad . \quad . \quad . \quad . \quad (6)$$

in which p, q, r, s, t may be any of the numbers 1, 2, 3, 4, 5.

26. Writing $\frac{l_s}{\mathrm{P}'(a_s)} = \alpha_s$ $(s=1, 2)$, equation (39) of the last example gives

$$\begin{aligned} al_1^2(u) - al_1^2(v) = {} & \alpha_2 al_2(u+v)\{al_1(v) al_2(u) al_{1,2}(v) - al_1(u) al_2(v) al_{1,2}(u)\} \\ & + al_1(u+v)\Big[(a_1 - a_3)\{al_1(v) al_3(u) al_{1,3}u - al_1(u) al_3(v) al_{1,3}(v)\} \\ & \qquad + \alpha_2 \frac{a_1 - a_3}{a_2 - a_3}\{al_1(v) al_2(u) al_{1,2}(u) - al_1(u) al_2(v) al_{1,2}(v)\}\Big] \end{aligned}$$

$$\begin{aligned} al_2^2(u) - al_2^2(v) = {} & \alpha_1 al_1(u+v)\{al_1(u) al_2(v) al_{1,2}(v) - al_1(v) al_2(u) al_{1,2}(u)\} \\ & + al_2(u+v)\Big[(a_2 - a_3)\{al_2(v) al_3(u) al_{2,3}(u) - al_2(u) al_3(v) al_{2,3}(v)\} \\ & \qquad + \alpha_1 \frac{a_2 - a_3}{a_1 - a_3}\{al_1(u) al_2(v) al_{1,2}(u) - al_1(v) al_2(u) al_{1,2}(v)\}\Big] \end{aligned}$$

two equations which determine $al_1(u+v)$, $al_2(u+v)$.

Assuming these known we have

$$al_3^2(u+v) = 1 - \left\{\frac{\alpha_1}{a_1 - a_3} al_1^2(u+v) + \frac{\alpha_2}{a_2 - a_3} al_2^2(u+v)\right\},$$

$$al_4^2(u+v) = 1 - \left\{\frac{\alpha_1}{a_1 - a_4} al_1^2(u+v) + \frac{\alpha_2}{a_2 - a_4} al_2^2(u+v)\right\},$$

$$al_5^2(u+v) = 1 - \left\{\frac{\alpha_1}{a_1 - a_5} al_1^2(u+v) + \frac{\alpha_2}{a_2 - a_5} al_2^2(u+v)\right\}.$$

The equation (36) applied to this case is when $m=1$

$$\begin{aligned}al_1^2(u+v)-al_1^2(u)={}&\alpha_2 al_2(v)\{al_1(u+v)al_2(u)al_{1,2}(u+v)+al_2(u+v)al_1(u)al_{1,2}(u)\}\\ &-(a_1-a_3)al_1(v)\{al_1(u)al_3(u+v)al_{1,3}(u+v)+al_1(u+v)al_3(u)al_{1,3}(u)\}\\ &-\alpha_2\frac{a_1-a_3}{a_2-a_3}al_1(v)\{al_1(u)al_2(u+v)al_{1,2}(u+v)+al_1(u+v)al_2(u)al_{1,2}(u)\}\end{aligned}$$

and when $m=2$ it is

$$\begin{aligned}al_2^2(u+v)-al_2^2(u)={}&\alpha_1 al_1(v)\{al_1(u)al_2(u+v)al_{1,2}(u+v)+al_1(u+v)al_2(u)al_{1,2}(u)\}\\ &-(a_2-a_3)al_2(v)\{al_2(u)al_3(u+v)al_{2,3}(u+v)+al_2(u+v)al_3(u)al_{2,3}(u)\}\\ &-\alpha_1\frac{a_2-a_3}{a_1-a_3}al_2(v)\{al_2(u)al_1(u+v)al_{1,2}(u+v)+al_2(u+v)al_1(u)al_{1,2}(u)\}.\end{aligned}$$

A particular case of (5) is

$$(a_1-a_2)al_3al_{1,2}+(a_2-a_3)al_1al_{2,3}+(a_3-a_1)al_2al_{1,3}=0.$$

These three equations will suffice to determine $al_{1,2}(u+v)$, $al_{2,3}(u+v)$, $al_{3,1}(u+v)$; after which the other functions may be successively obtained from the equations

$$(a_2-a_3)l_4al_4al_{1,4}=(a_2-a_4)l_3al_3al_{1,3}+(a_4-a_3)l_2al_2al_{1,2} \quad . \quad . \quad . \quad . \quad (6')$$

$$(a_4-a_2)al_1al_{2,4}=(a_4-a_1)al_2al_{1,4}+(a_1-a_2)al_4al_{1,2} \quad . \quad . \quad . \quad . \quad . \quad (5')$$

$$(a_4-a_3)al_2al_{3,4}=(a_4-a_2)al_3al_{2,4}+(a_2-a_3)al_4al_{2,3} \quad . \quad . \quad . \quad . \quad . \quad (5')$$

$$(a_2-a_4)l_5al_5al_{1,5}=(a_2-a_5)l_4al_4al_{1,4}+(a_5-a_4)l_2al_2al_{1,2} \quad . \quad . \quad . \quad . \quad (6')$$

$$(a_4-a_5)al_1al_{4,5}=(a_1-a_5)al_4al_{1,5}+(a_4-a_1)al_5al_{1,4} \quad . \quad . \quad . \quad . \quad . \quad (5')$$

$$(a_3-a_5)al_4al_{3,5}=(a_3-a_4)al_5al_{3,4}+(a_4-a_5)al_3al_{4,5} \quad . \quad . \quad . \quad . \quad . \quad (5')$$

$$(a_2-a_5)al_3al_{2,5}=(a_2-a_3)al_5al_{2,3}+(a_3-a_5)al_2al_{3,5} \quad . \quad . \quad . \quad . \quad . \quad (5')$$

the figure at the end of each line denoting from which of the equations (5) and (6) the particular line has been derived.

This case has been added and all the necessary equations have been written down as a justification of the statement made at the end of § 24.

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OF THE

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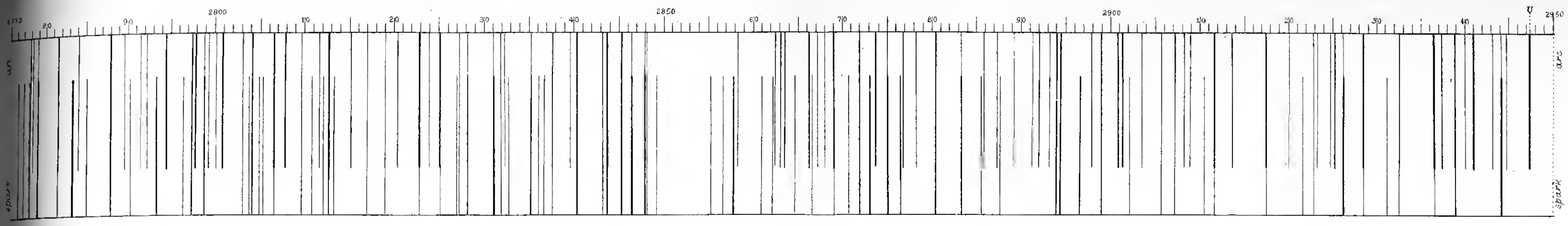
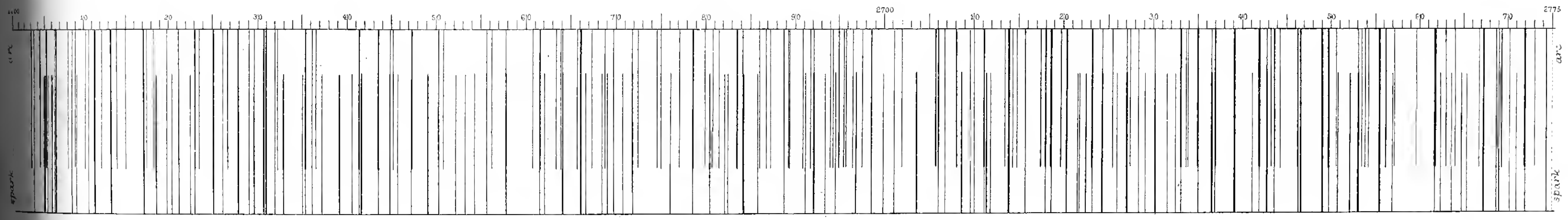
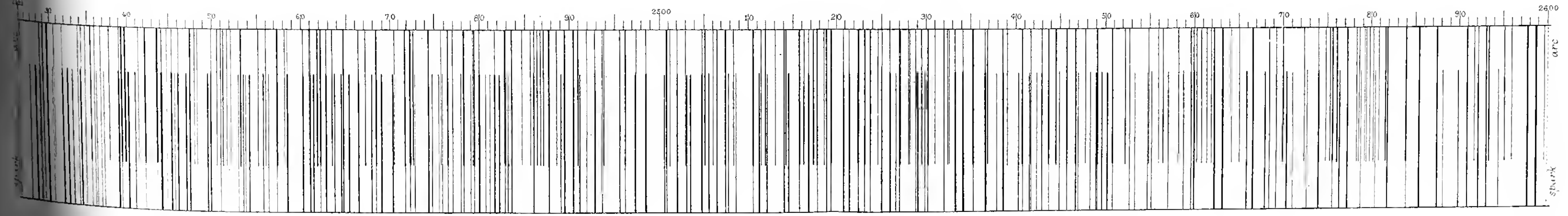
FOR THE YEAR 1883.







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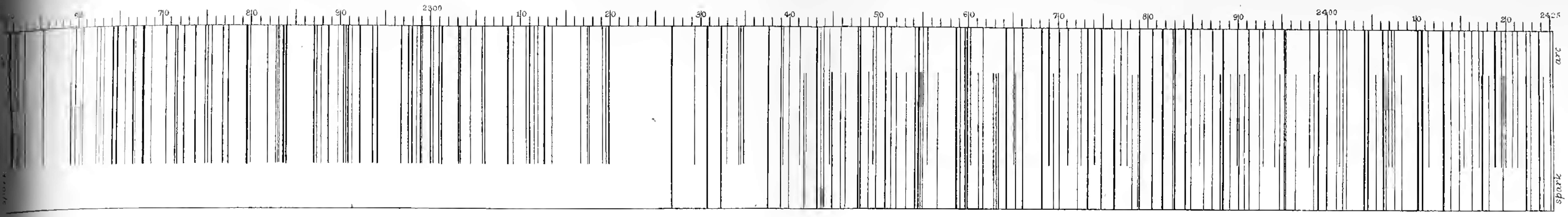

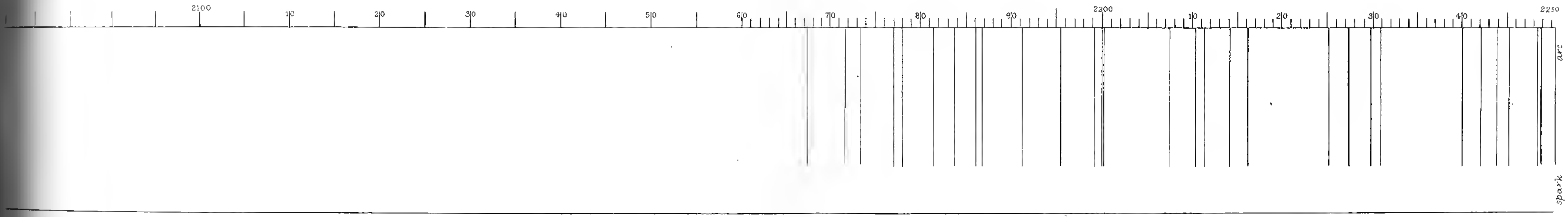

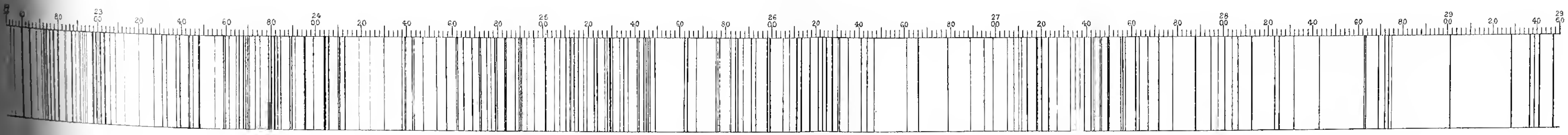

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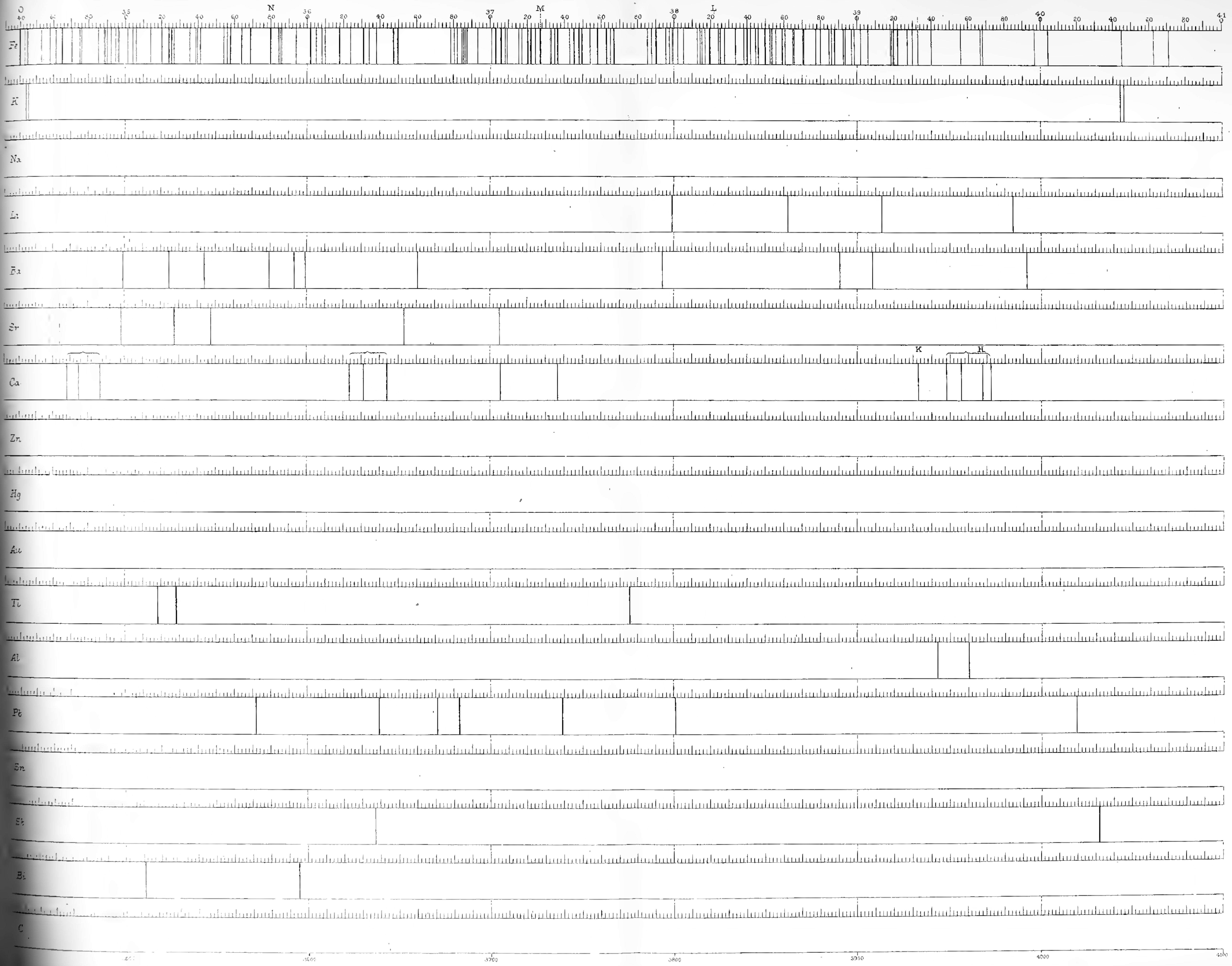

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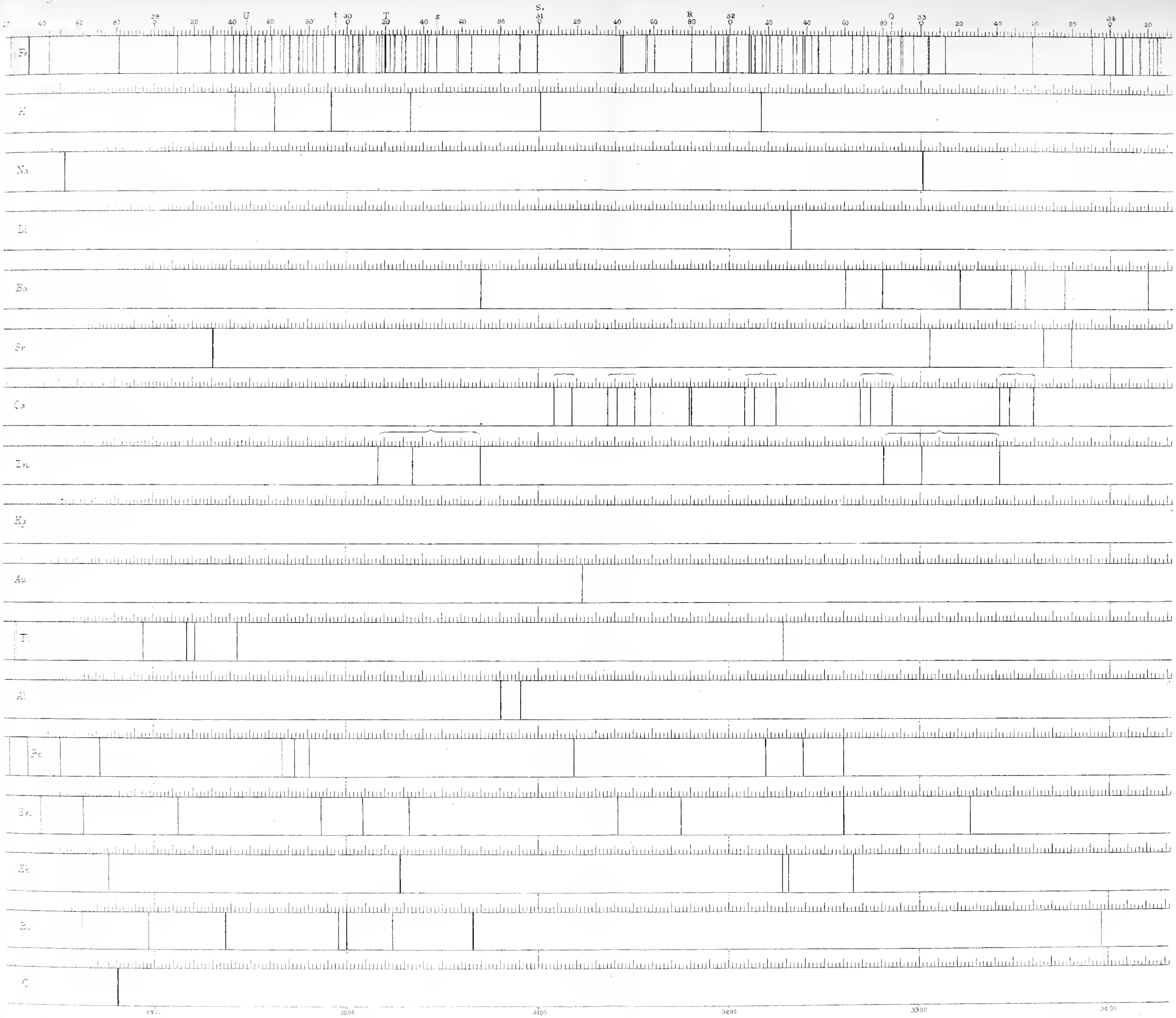

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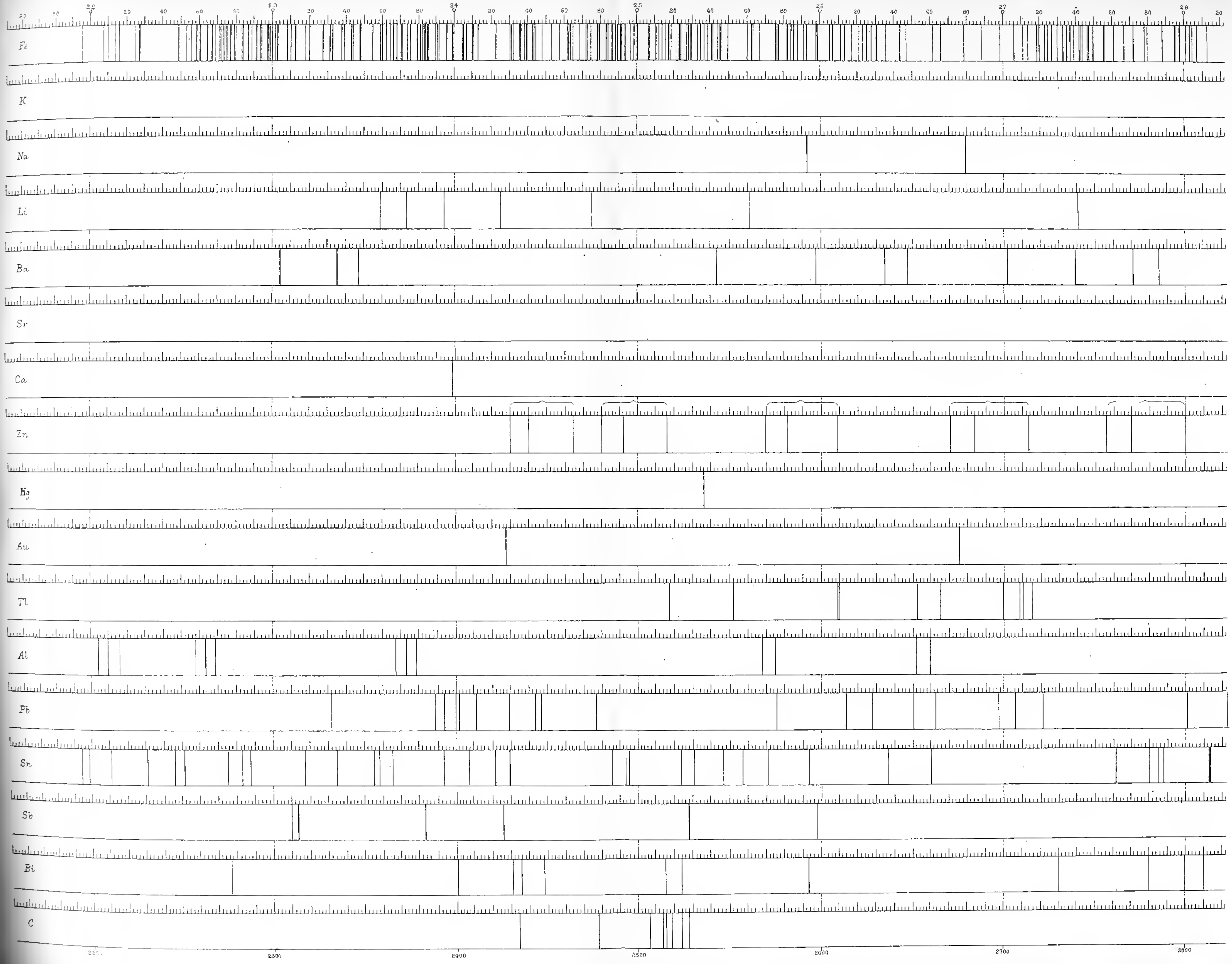

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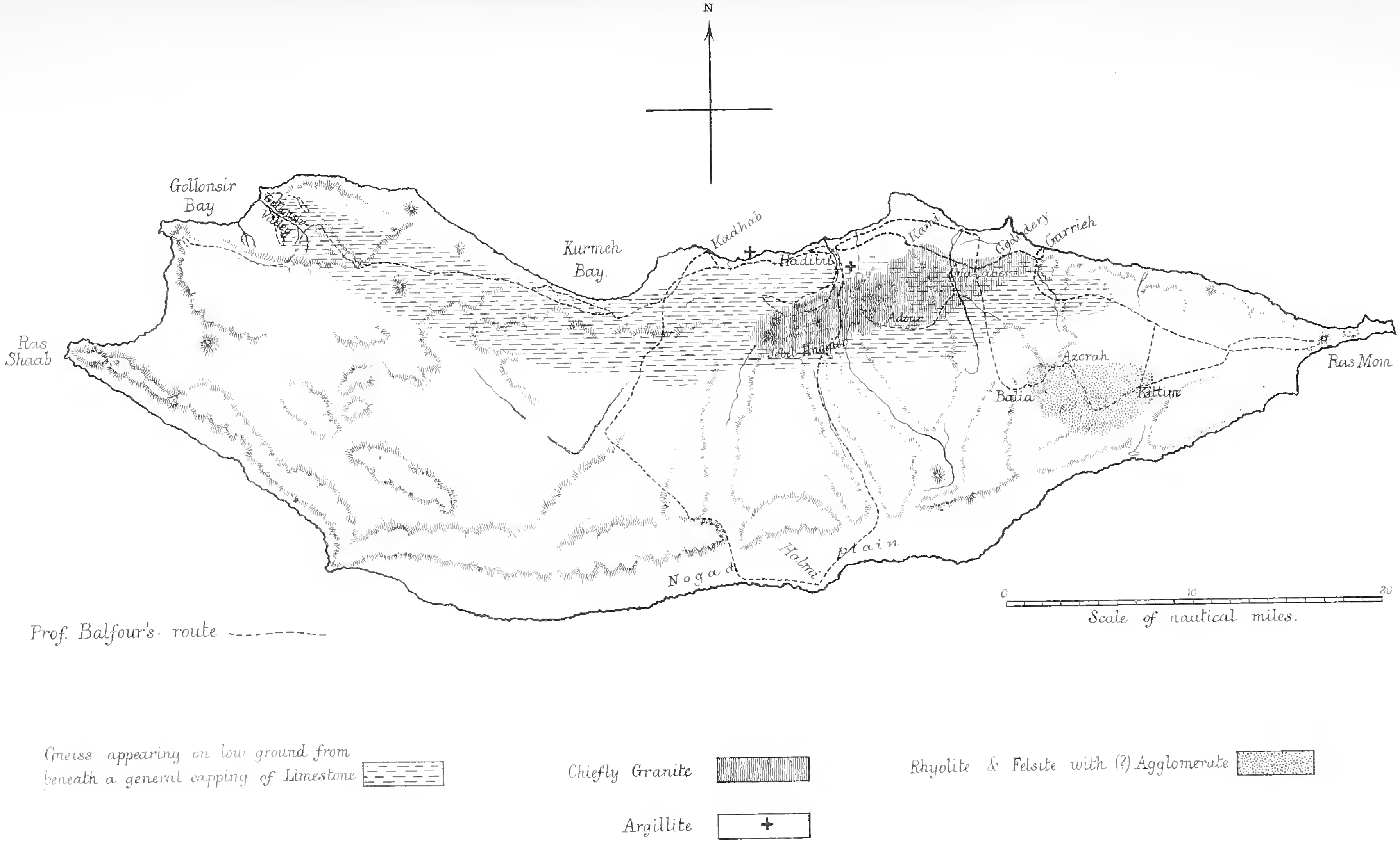

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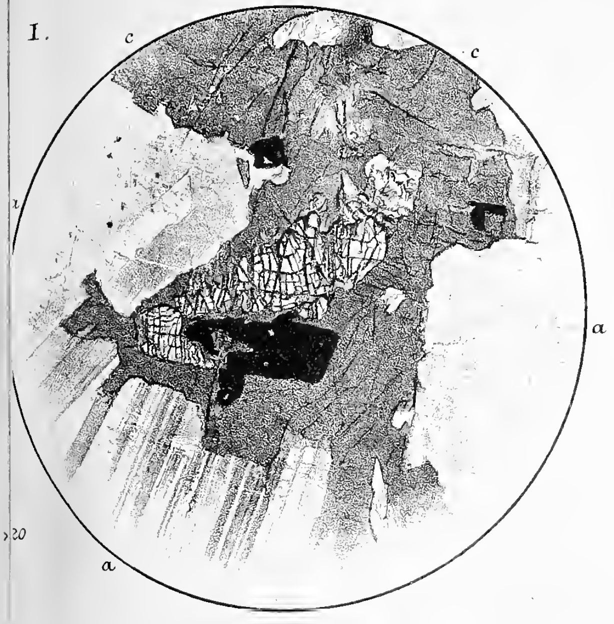

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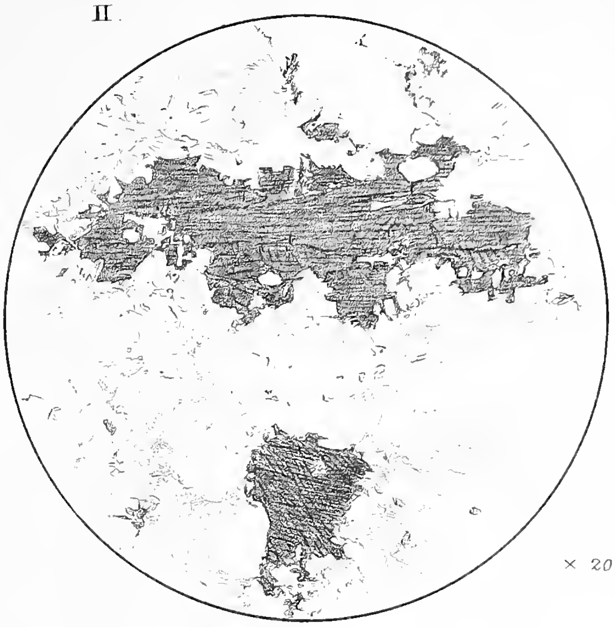

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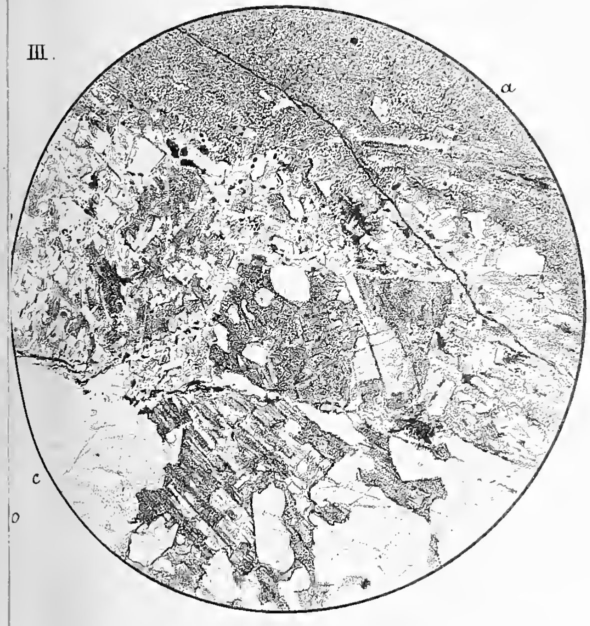

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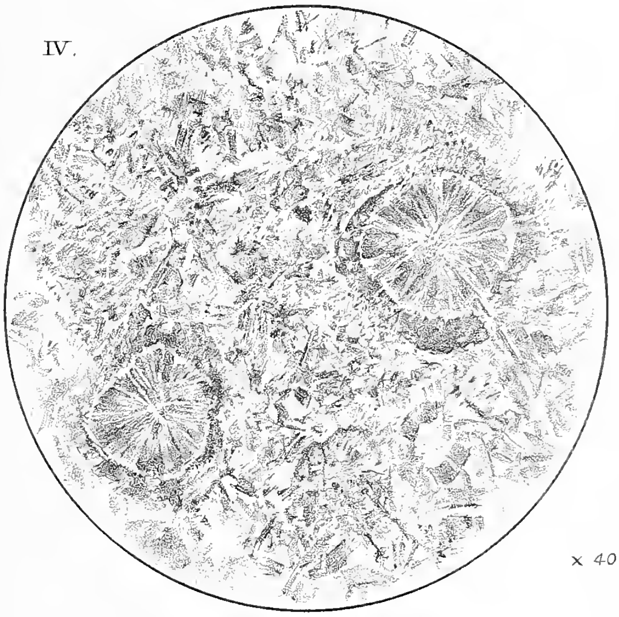

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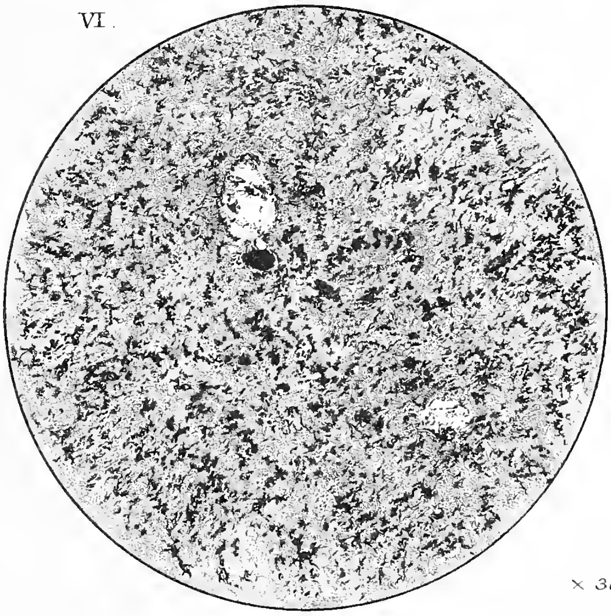

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IX. *On the Skeleton of the Marsipobranch Fishes.*—Part I. *The Myxinoids* (Myxine, *and* Bdellostoma).

By William Kitchen Parker, *F.R.S.*

Received December 14, 1882—Read January 11, 1883.

[Plates 8–17.]

Introduction.

At present, almost nothing is known of the development of these remarkable Fishes, but their structure in the adult state is of great interest; and as the other related type—the Lamprey—has received great attention lately in most of its stages, I have thought that it would be profitable to anatomists to have a detailed account of the structure of the skeleton in these lower, and less known types.

The late Professor Johann Müller left us his inestimable account of the anatomy of *Bdellostoma*, with excellent figures of the skeletal parts; but of *Myxine* he gave very few illustrations. Moreover, the absolutely accurate figures of the skull of *Bdellostoma* are small and uncoloured; they fail to show the various kinds of cartilage of which it is composed, and as this skull is so extremely unlike that of any other known vertebrate, except that of the Hag (*Myxine*), I venture to give my own (new) illustrations on a larger scale and coloured. My figures of the skeletal parts of *Myxine* will be, I believe, almost entirely new to science; and, moreover, the time seems to have arrived in which some *interpretation* of these low generalised skulls may be attempted.

This will be done by the help of what we have lately been learning of the development of the skeleton of the nearest relative of the Myxinoid—the Lamprey,—the subject of Part II. This attempt to explain the Myxinoid type of skull and skeleton generally—nearly all the cartilage in these fishes is *cephalic*—has been done by the help of our growing knowledge of the Lamprey, and also by comparison with what the writer has traced out in the early conditions of the skull in various types of Ichthyopsida, especially in the larva of *Lepidosteus*, and of a large number of Tadpoles of the *Amphibia Anura*.

The light thrown upon the Myxinoid cranio-facial apparatus by the early *chondrocranium* of other and much higher kinds of Ichthyopsida, is much greater than might have been expected, for these show, now here, now there, very remarkable archaic characters; and one kind, the Tadpole of the Nailed Toad of the Cape (*Dactylethra*), has these parts but little more specialised than in the Myxinoids; and, moreover, this type has all its cartilage (which is very copious and wild-growing), even in Tadpoles an inch long, of a peculiarly light and cellular kind—like many parts of the Myxinoid's skeleton. It is evidently *the historic representative* of an exceedingly ancient and generalised sort of Fish.

Most important help to me in this attempted interpretation has been repeated discussion of the subject with one of the very highest intellects ever devoted to Biological research—I allude to the late Professor BALFOUR; that source of light and strength is now, unhappily, lost to me.

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On the Varieties of the Connective-tissue Series found in the Skeleton of the Myxinoids.

There are four kinds of supporting-tissue in these types, namely:—

a. A very solid greenish kind of cartilage, seen in these Fishes and the Lamprey, only, as far as my experience goes; it is formed by special deposits in the softer kind.

b. Soft cartilage; colourless, and with but little inter-cellular deposit; passes insensibly in some places, suddenly in others, into hard cartilage.

c. An elastic, spongy tissue, full of large vacuoli, a degree denser than the tissue of the notochord.

d. White fibrous tissue, often exceedingly compact and strong; it may, for the present, be called *fibro-cartilage.*

On the relation of the Myxinoids to other types: their Zoological position in the "Branchiata," or "Anamniota."

Roughly, this may be expressed as follows: The Hag and *Bdellostoma* are a sort of greatly modified *Ammocœte;* they bear a relation to the Lamprey similar to that which it bears to the Anurous Amphibia. The *secular losses* among these types must have been very great indeed, for although the Marsipobranchs and the Anura—which are Marsipobranchs in their larval state—are akin to each other to a degree in which they are not akin to any other Branchiata, yet they are very far apart from each other, after all. The metamorphosis of the Tadpole lifts it far above the highest of the Marsipobranchii proper, but I feel satisfied that the Anura have only gradually become metamorphosed; and I doubt whether *all the larvæ of Pseudis* undergo that change, even now. Yet when once this change is set up, we see a very generalised and archaic Fish become *almost* a true Reptile. More than this, one kind—*Pipa*—scarcely shows a trace of gills, and all the Anura, during their metamorphosis, develop a "bladder" which is, *apparently,* the rudiment of an "Allantois;" if further research makes this supposition a settled fact, the stride made by these forms, during individual life, will be seen to be very great indeed.

It is better to call the Hag and Lamprey "Marsipobranchs" than *suctorial fishes;* the adult Lamprey, like the Tadpole, is truly suctorial, but the mouth of the *Ammocœte* or larval Lamprey, and the mouth of the Hag and *Bdellostoma,* are not modified into a circular sucking ring, but it remains as a small hooded opening, fringed with short barbels, or oral palpi. There is no cartilage whatever in the mouth of the *Ammocœte* (Plate 19, figs. 4, 5), and in the Myxinoids the only cartilage developed is as a pith to the barbels (Plate 17, figs. 1–3). All this will be explained in the sequel, but we may as well start fairly, looking upon the subjects of the present paper as greatly specialised, but not metamorphised, "Marsipobranchs," a curious variety of arrested "Sand Pride," or *Ammocœte.* In my Second Part I shall show how such a simple type is transformed into a true Sucking Fish, or Lamprey, which may be, in turn, looked upon as a sort of highly specialised, but arrested, Anurous Amphibian.

On the cranio-facial apparatus of the adult Hag-Fish (Myxine glutinosa).

All the cartilage to be found in this Fish is *cephalic,* for even the furthest rudiment of the dorsal part of the branchial basket is, like the rest of that system, supplied by a cranial nerve—the vagus; the spinal region is only supported by membrane, or strong, fibrous tissue.

Properly speaking, these Fishes, although *Craniata,* are not *Vertebrata;* they are *chordato-craniata* like the Lamprey before its metamorphosis, for neither in this, or in the large Cape species (*Bdellostoma*), can I find any cartilaginous rudiments of vertebral arches. Moreover, the *theca vertebralis* is but little enlarged where it passes, insensibly, into the *dura mater.*

The huge notochord (Plates 9 and 10, figs. 1, 2, *nc.*), with its *merely membranous* sheath, suddenly ends in a conical form between the ear capsules, its thick sheath becoming still thicker at the end, and the vacuoles and membranous bands ceasing very close to the hind part of the head, which is not definitely separate from the spinal region.

Hence the parachordal region, or investing mass (*iv.*) is very short, only one-fifth the length of the entire skull; its moieties become the trabeculæ (*tr.*) at the front third of the auditory capsules (*au.*) The right and left bands unite below (Plate 10, fig. 3) but not above (figs. 1, 2); there they do project a little behind the capsules, but, below, this is hardly perceptible. Thus, the basi-occipital cartilage is incomplete. Moreover, it is arrested behind, for there are not only no condyles, but the cartilage stops in front of the place where they would be found. The sides, or ex-occipital region, and the roof or super-occipital, are arrested, entirely; thus there is no occipital ring. At this hind part the *theca cranialis*, or "dura mater" lies on an imperfect floor of cartilage, but for the rest of its extent the trabeculæ lie rather outside than under it, and only meet and unite under the nasal capsule (*na.*) Thus the huge oval "fontanelle" reaches from end to end, and is only separated from the inferior or pituitary fontanelle by the thick lateral (trabecular) bands; yet a secondary cartilaginous part forms a partial floor to the *pro*-chordal part of the skull. The fore part of the skull has a peculiar ox-faced form, ending in two crescentic "horns;" behind these horns it narrows gently, and then widens out in an even, rounded manner, the widening being due to the facial basket-work.

The whole structure is, indeed, a generalised cranio-facial basket-work. There is no proper segmentation, but here and there the cartilage remains soft, and thus certain of the territories are marked out.

That which is so puzzling in this little unossified skull is its primitiveness and simplicity; for we are accustomed to cranial and facial structures that are differentiated from each other, and in which the facial arches are segmented into a number of parts; these can be classified and named, a typical arch being made the measure of the rest.

Here we have unenclosed land, and in such a generalised, common field, any balk, mound, stone, or bush may be useful as a landmark.*

The "horns" are not the *cornua trabeculæ;* these are suppressed in *Myxine,* but they belong to the palatine *region,* and so does at least half of the fore part of the cartilage bounding the narrow cranial cavity. The two trabeculæ are confluent in front, between

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[*] If the reader would follow the description and interpretation here attempted, it would be well for him to have the other papers of the writer before him, especially the following, namely:—"Skull of Batrachia," Part II., Phil. Trans., 1876, Plates 54–62; "Skull of Batrachia," Part III., Phil. Trans., 1881, Part I., Plates 1–44; and "Skull of *Lepidosteus*," Phil. Trans., 1882, Plates 30–38. The nomenclature shall be as uniform as I can make it.

the "prepalatine" horns (*pr.pa.*), and where they unite in front there is a wedge of soft cartilage; also the inner edge of the basal cartilage (*iv.*) behind, under the hind-brain, is soft. As to the lateral bars, the eye seizes upon a landmark; this is the "subocular fenestra" (*s.o.f.*), very familiar to us in the skulls of Tadpoles. Inside this very limited reniform membranous space the bar is purely trabecular. In front of it it is palato-trabecular (*pa., tr.*). Another familiar part can now be seen; it is behind the fenestra, and is composed of soft cartilage; this short tract is the pedicle (*pd.*), for it answers to the dorsal end of the great "suspensorium" or palato-quadrate of the Tadpole, a part always developed continuously with the basis cranii in the Anura. (See "Batrachian Skull," Part II., Phil. Trans., 1876, Plate 35, figs. 1–5, *pd.*). An oblique soft tract may be seen in front of the subocular fenestra in *Myxine;* this marks the junction of the pterygo-quadrate region with the palatine. The pre-palatine horns (*pr.pa.*) remain soft; they are large, rounded, and suddenly apiculated near their end. Now, for a while, the Tadpole's skull will fail us in our interpretation; as long as I kept, slavishly, to that chondrocranium, as my key to the skull of the Marsipobranchs, I was always falling into confusion. The Tadpole's skull, however, just when transformation is taking place, and the skull of the *suctorial larva of Lepidosteus*, greatly enlighten us at this point. (See "Skull of Batrachia," Part III., Plate 4, figs. 5–9, and "Skull of *Lepidosteus*," Phil. Trans., 1882, Plate 30, figs. 3, 7, 8.) Here we have to be cautious; for *Myxine* has its *quadrate region* suppressed at the part where the condyle is formed in the higher kinds, and there are no Meckelian or mandibular rods. Hence that huge, broad, condyle-bearing part of the enormous suspensorium of the Tadpole shown in so many of my figures, and often running up to the front of the face, is not present in *Myxine*. Moreover, I am quite persuaded that the rudiment of the quadrate region (*q.*) which does exist, is not in front, but directly below the pedicle (*pd.*), a position which is only slowly gained in the Tadpole; in *Lepidosteus* it is not far in front of the pedicle, being opposite the pituitary body ("Skull of *Lepidosteus*," Plate 30, fig. 3, *py.*, *q.c.*).

The remarkable position of the distal part of the "pier" (or suspensorium) of the mandible in the Tadpole is quite unique; it exists nowhere else but in the Anura, and depends upon the compromise, so to speak, which in them is made between a jaw-less and a jaw-bearing type. I must return to the arrested jaw-pier and its connexions with the rest of the facial basket-work, when the rest of the cranium has been described.

In the Tadpole the trabeculæ, after a time, become united together beneath the fore brain by a thin lamina of cartilage, the soft tissue gradually becoming cartilaginous; but this posterior intertrabecular tract is not found as a distinct sheet of cartilage. Afterwards, when the trabeculæ of the Tadpole have united in the ethmoidal region, a crest of cartilage appears upon it, which becomes the vertical ethmoid and septum nasi in one high tract ("Batrachia," Part III., Plate 2, fig. 1). In the Green Turtle, I have shown that the intertrabecular cartilage is found as a rounded rod between the rounded paired trabeculæ, and that afterwards they flatten out, and it rises upwards to form the ethmo-septal partition of the nasal region ('Challenger Memoirs,' vol. i., part 5, plate 2, figs. 3–7).

This is the manner of growth seen in all Reptiles above Serpents, in Birds, and in Mammals. Here, in the lowest kind of skull we know of, the median prochordal cartilage appears as two tracts, one before the other, the two quite independent of each other, and very much unlike in form and consistency; so that the skull itself in the Myxinoid is formed of separate segments, although the facial arches are not differentiated (or segmented) from the edges of the paired cranial bars.

The long oval space in front of the short parachordal tracts, reaching from there to the ethmoidal commissure (Plate 10, figs. 1, 2, 3), is imperfectly closed below by a remarkable spoon-shaped cartilage of the soft kind; this is the *hinder intertrabecula* (*p.i.tr.*). Under the proper pituitary region, we see the bowl of the "spoon," which is rostrate and perforate behind; the straight, narrow "handle" runs forwards, touching the ethmoidal commissure: it gently lessens from behind, forwards. The handle leaves a large unfloored space right and left; it is gently scooped above, and the "bowl" considerably; this lamina is of even thickness, and is quite convex below. At some height above the end of the handle, a new cartilage begins, very different from the last; this is the *front intertrabecula* (*a.i.tr.*). This bar is composed of hard cartilage; it is compressed vertically, is as wide as the handle of the "spoon," but higher than wide; it is thickest behind, where it is emarginate and acutely bilobed; and it narrows gently forwards, and is then thickened again. This "front intertrabecula" is one-sixth longer than the other, and more than its hinder fourth lies *on* the ethmoidal commissure, and *under* the long nasal labyrinth (*n.a.*, *e.n.t.*). These structures are quite unlike what is seen in the Lamprey, where the cornua trabeculæ are connate, and abort the front intertrabecula, and where the hind intertrabecula is composed of hard cartilage, and is only distinct for a very short time after metamorphosis (Plate 10, figs. 4, 5, *p.i.tr.*). The state of things seen in *Myxine* is evidently due to the *intense* specialisation of a type, which, on the whole, does not rise above the level (or *platform*) of an *Ammocœte*. Some other "novelties," quite equal to this, will be seen as we proceed.

Below the middle of the auditory capsule (Plate 9, fig. 2, *au.*), there is a round fenestra (*m.h.f.*), half as large as the reniform subocular space (*s.o.f.*); and below this, separated by a thick bar, is another oval space ($l.f^2.$), twice as large as the subocular. These, also, are nascent segmentation lines, arrested and widened out. The bar bounding the little upper space, behind (*h.m.*), is quite similar to, but wider than, the one in front—the pedicle (*pd.*); it is also composed of soft cartilage, whilst the thick bar running backwards from the pterygoid region under the reniform, and round and over the oval, fenestra, is hard cartilage. The narrow end of the oval fenestra looks upwards and forwards, and the hard bar below and in front of it, which lessens, and then widens out again, is the pterygo-quadrate bar, ending below, not in a *quadrate condyle*, but in a thin, inturned edge, somewhat rounded in outline. The hard cartilage ends, above and below, a little behind the large oval fenestra, and at its hind margin, and in the rest of the basket-work, the cartilage is soft. Like the pedicle, the short bar (*h.m.*) behind the small, round fenestra is continuous with the parachordal (*iv.*); it is the "serial homologue" of the pedicle of the suspensorium, and therefore is the head of the hyoid arch (= head of the hyomandibular). Now we begin to feel our way in this *unenclosed field;* we have lit upon some landmarks. One continuous growth of cartilage is seen running sinuously, sub-parallel with the axis, from near the fore-end of the nasal tube (*e.n.t.*) to a point beneath the 3rd spinal nerve (*sp.n.*). The fore part is what I have called the prepalatine "horn," or spike (*pr.pa.*); it is soft; then the hard tract behind it is first palatine (*pa.*), and then becomes the top of the pterygo-quadrate region (*pg.*, *q.*), passing into the "shoulder" of the hyomandibular (*h.m.*), the fore part of which answers to the wide snaggy part of that bone in an Osseous Fish, whilst the hind part corresponds to its "opercular process" for the *os operculare.* The hard cartilage of the pterygoid region is separated from that of the palatine in front, and the quadrate behind and below, by a narrow soft tract. The soft cartilage behind is first inter-hyal (*i.hy.*) then epi-hyal (*e.hy.*), and then forms the top of the 1st epibranchial (*e.br*^{1}.) The margining cartilage behind the large oval fenestra, sends back a rounded lobe into the hinder fenestra (*l.f*^{3}.)—a two-horned space; that lobate band is the "symplectic" region (*sy.*); it is soft; but the back of the quadrate region, into which it passes, is hard. The largest or hindmost two-horned fenestra although single below, is broken into two, above, by the main part of the hyoid arch—the inter-, epi-, cerato-, and hypo-hyal regions (*i.hy.*, *e.hy.*, *c.hy.*, *h.hy.*).

This arched band, bending backwards, and growing downwards and forwards, to pass into the huge basal bar, is both wider and thicker than that of an adult Frog, but unlike its counterpart in that type it is continuous with the upper or hyomandibular part of the arch. Nevertheless, its small width here suggests comparison with the hyoid of the adult, and not of the larval Frog, whose broad, short lower hyoid is suspended from the suspensorium, beneath the eye-ball, indeed under the front of the subocular space. Here the hyoid arch is curved backwards so as to lie, in the middle, below the 1st spinal nerve (*sp.n.*) whilst the arrested quadrate tract (*q.*) is directly below the middle of the auditory capsule, a position attained by the quadrate condyle of the Frog soon after metamorphosis. Where the soft cartilage of the hypo-hyal region (*h.hy.*) ends below, there the hard cartilage of the basi-hyal (*b.hy*^{p}.) begins, but there is no joint.

This continuous hyoid bar, as we have seen, passes over the last, or two-horned fenestra dividing it, above, and then riding over it. Above, this hyoid bar passes directly into the arched cartilage bounding the hind fenestra; below, that boundary of cartilage passes inside the long hyoid bar, and becomes the symplectic region. From its convex margin, behind, this hinder arch gives off two styliform outgrowths. This semicircle of soft cartilage, which forms the hinder half of the boundary of the hinder fenestra, is the 1st epi-branchial; the 1st cerato-branchial is suppressed in *Myxine*, but well-developed and distinct in *Bdellostoma* (Plate 16, figs. 1, 2, *c.br.*).

But the 1st pharyngo-branchial, or pier of the 3rd visceral arch, is well developed in both kinds; here (Plate 10, figs. 1–3; and Plate 17, figs. 1–3, *p.br*[1].) it is free from its own proper descending bar, lies obliquely inside it, and is joined to the general thickening of cartilage in the hind part of the *oval* fenestra. Thence it is very thick, and clavato-lobate, filling up nearly half of the fenestra at its inner face; it is there composed of hard cartilage. The rest is a soft, long, sinuous, inbent rod, which ends in a point some distance behind the *epi-branchial rays* (*e.br*[1].). Of the 2nd branchial arch only the upper or pharyngeal part is developed; but the relations of this and of the 1st pharyngo-branchial, enable us to determine the nature of these cartilages as to whether they are *extra*-branchials or *intra*-branchials. When the lower part of the throat of *Myxine* is removed, and the pharynx slit open for some distance (Plate 13, fig. 7), then we see that behind the pharyngeal opening of the posterior nasal canal (*p.n.c.*) there is a peculiar hood of membrane, the "pharyngeal velum" (*vl.*); it is pyriform, its narrow end is crenate, and a septum divides it behind, between the terminal folds.

When this is dissected out and examined from below (Plate 15, fig. 6), then we find that the 1st pharyngo-branchials are the supports of its outer margin, and that its swelling part has a skeleton derived from the 2nd pharyngo-branchials. The branchial pouches and clefts, *during growth, retired far away from the skeletal framework* (Plate 9, fig. 1), so that the cleft or opening of the first or *hyo-branchial* pouch lies below the *twentieth spinal nerve* (*sp.n.*); and the middle of the pericardium is below the *fortieth* (*sp.n.*). All these retired parts, pushed back, *so to speak*, by the huge lingual apparatus, are supplied by a cranial nerve, the vagus—a remarkable "prophecy" of what will take place in the retirement of the respiratory organs in the higher Vertebrata. The framework of the branchial region is left in its place, and is largely suppressed, and the parts that are developed are free to form new specialisations. The lower part of the pharyngeal velum, covered by *hypoblastic* cells, is supported by the 2nd pharyngo-branchials (Plate 15, fig. 6, *p.br*[2].), which are a pair of nbent, obliquely placed rods, thickish in front and very slender behind. At their middle they are united by a transverse bar, and this bar sends forwards two slender rods, which grow in front into large, pedate lobes. Where the slender hind part turns outwards, in the crenations of the velum, there another transverse rod is formed, thus uniting the right and left bars together; this also sends off, backwards, a small median outgrowth, and two large, lateral, multilobate outgrowths—a curious moss-like structure.

On the *right* side the rays of the 1st epi-branchial (Plate 10, figs. 1, 2, *e.br*[1].) are united at their base, and form another fenestra, so that there are *four* visceral fenestræ on the left side, and *five* on the right.

And yet this remarkable basket-work is not homologous with the curiously similar growth in the Lamprey; in that Fish the reticulation is formed outside the head-cavities; in the Hag-Fish they *must have been formed within* them, as they lie close to the hypo blastic lining of the throat.

With regard to the great inferior median bar (*b.hy.*, *b.br.*), there can be but little difficulty; for not only the Lamprey, but the Bony Gar-pike (*Lepidosteus*) also, shows us a similar huge glosso-hyal in front of the common basi-branchial bar. In the latter Fish (Phil. Trans., 1882, Plates 30–38) the basi-hyal is double, composed of two equal rounded rods, which are united along the middle. Moreover (most instructive of all the characters of that remarkable lingual skeleton), the cartilage is cut up into a number of transverse blocks by fibrous septa. This takes place in a Ganoid Fish, whose larva has a suctorial snout.

In the Myxinoids the tongue dominates the whole body; everything else yields to it, and is modified intensely by it. In the Lamprey, as well as in *Lepidosteus*, the basi-hyal becomes double in front, here it is a four-fold bar or plate; the solid cartilage being divided as it goes on expanding from behind forwards, first into two and then into four pieces (Plate 9, fig. 3); these are at once united and separated by tracts of soft cartilage or fibro-cartilage, and even by mere fibrous tissue in some places; and the two inner pieces of the terminal front part have a fenestra between them, behind. Then the bar becomes sub-carinate, but this angular projection is gradually lost, and the cartilage suddenly becomes soft, and a mere thick, *almost* fibrous web, is continued backwards behind the hypo-hyal junctions as an *anterior* basi-branchial ($b.br^1.$), which is one-third longer than the basi-hyal region.

Near the middle, this soft *fibro-cartilaginous* bar becomes keeled and alate; it then lessens gradually to a sharp point, which is gently upturned; this hinder part of the huge median bar is scooped on its upper surface; but the scooping becomes deeper and deeper, as we pass forwards to where the great fourfold basi-hyal is carinate below. The two middle pieces, in front, together form an emargination, and the outer pieces are rounded off externally. In front, the great lingual bar is twice as wide as the alate part of the basi-branchial behind. This huge beam is swung from the head, in front of its middle, by those small, soft ropes, the descending hyoid bands (*c.hy.*), which broaden and harden into the hyomandibular region (*h.m.*), above. The broad, emarginate fore-end of the huge lingual cartilage nearly reaches to the tips of the lower barbels, two pairs of which are seen protecting the slit-like mouth; whilst seven more protect the opening of the *nasal proboscis* (Plate 13, fig. 7).

The supra-lingual apparatus of Myxine.

The great basal bar (Plate 9, figs. 1–3, *b.hy.*) is only the coarser part, so to speak, of the lingual dentary apparatus; the two rows of teeth, right and left, themselves are set in a cushion of fibrous tissue, which is supported by a *supra-lingual* cartilage (Plate 12, figs. 7, 8), a *superadded* structure, the rudiment of which only re-appears in the Lamprey with one row of teeth right and left (Plate 14, fig. 9); in both cases these are peculiar to the Marsipobranchs.

When the great lingual cartilage, with the parts carried by it, is dissected out we have the structure shown in Plate 12, fig. 7; and when the teeth are removed, then the "supra-lingual" framework is seen (fig. 8), a curious *apron* with slits in it, and short strings projecting from it; it is composed of hard and soft cartilage, and of fibrous tissue ending, in front, in a horny comb; the figure shows it as considerably outspread, for display;—the sections correct this. In this hollow space lie the lingual teeth (fig. 7, *s.l.t.*); the cartilage had better be described first. This additional skeletal structure is formed in the floor and sides of the oral mucous membrane, and, with the structures it carries, reduces the cavity to a number of narrow chinks. The general outline of the supra-lingual frame-work is heart-shaped, but widely open, with projecting spurs, behind, whilst in front it is transverse, slightly emarginate, and developed into a fine comb of horny spikes. The fore part is membranous, but the cartilage creeps into this membrane, right, left, and in the middle—most there; there the fore margin of the cartilage has a small, toothed emargination in the middle and a large transverse notch, right and left. The whole cartilage tends to break up into a series of three pairs of short arches; within the hinder half there is a median bar between; these, together, form a sort of *intra-visceral* series, mimetic of the proper visceral arches, just as the *extra-visceral* framework of the Lamprey's pharynx is mimetic of such a series.

The first pair of these semi-segmented arches is wing-like, and is half separated by the next pair, which are narrow and feeble; one snag ends both of these behind; a crescentic cleft divides these for three-fourths of their extent; these two first pairs are composed of soft cartilage. The hinder pair ends in a snag, which is soft, and which is continuous in front with the root of the first snag. The side bars are twice as large as those in front of them, and are composed of hard cartilage; the median bar is soft, projects a little into the notch in front of it, and much more behind, where it reaches nearly as far backwards as the second pair of lateral processes. As it lies on the large basal beam, this hinder median part reaches as far back as the hard cartilage, and further than the setting on of the hypo-hyal ends of the hyoid arches (*h.hy.*).

In front, the horny comb helps to fill in the emargination of the basal beam.* The arrangement of the golden-coloured horny supra-lingual teeth is in a double, arched series, with a large notch behind. There are *seven* large teeth and *nine* small ones on each side; the large teeth are in front of, and outside, the others; their points look

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* There may seem to be some discrepancy between the figures of the huge basal beam—upper, lower, and lateral, and of the *sections* that illustrate its structure still further. The explanation is this: the *dissections* show the bars as invested with a strong perichondrium, and thus they *look* nearer together than they really are. The colouring of the dissections takes no account of this: the numerous sections, drawn with a *camera*, show the width of the intercartilaginous spaces.

backwards, and inwards, and they are shaped like a lancet blade, but with a slight curve, the convexity being antero-internal. A cavity can be seen in them, and also that they are made of fibres arranged featherwise. The thick horny layer is continuous at the base of the teeth; their form is due to the folding of the lining membrane of the mouth, which undergoes *corneous hypertrophy* of the cells. These structures will be described again, when we come to the *sections*. The only *antagonist* tooth is very large and canine-shaped, attached to the ethmoid below, and pointing backwards (Plate 10, fig. 3, *et.t.*); an arrangement similar to that in "Cyprynoids," where the basi-occipital, horny tooth antagonises the lower pharyngeal, true teeth.

On the branchial pouches of Myxine.

The extraordinary displacement of the branchial pouches in the Myxinoids is correlated with, or even caused by the enormous size of the lingual apparatus, the fore part of the great common basi-branchial bar being retained with the basi-hyal at the hind part of the lingual skeleton (Plate 9, figs. 1–3). But even this only reaches half way along the base of the oblique muscles that work the tongue. There is, however, a second basi-branchial bar (fig. 4, $b.br^2.$), slenderer and flatter by far, and only half the length of the main part; this is composed of hard cartilage, and after a space equal to its own length finishes the skeleton of the apparatus along the basal line.

In *Myxine* there is no skeleton to the *six pairs*[*] of cake-shaped (*placentiform*) branchial pouches, but the whole structure is membranous (see J. MÜLLER, I., plate 7, figs. 10–12; and my figure Plate 9, fig. 1).

Each pouch opens inwards into the narrow pharynx by a wide passage, and outwards by a largish tube which communicates with the opening of each succeeding tube, until they all have a common escape opening, behind. Into this space, on the *left* side, there is a sort of *fistulous passage*—the "ductus œsophago-cutaneus" (*d.œ.c.*) behind the last pouch—a sort of abortive gill-cleft, with no gill structure, the use of which I cannot see, but the morphology of which is self-evident. Behind these is the large heart (*h.*), giving off the arterial arches to the gill pouches.[†]

On the sense-capsules of Myxine.

For a detailed account of these organs, the reader is referred to MÜLLER's Memoirs; and for the auditory organs, especially, to Professor RETZIUS's magnificent volume, just published.[‡]

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[*] Mr. WELDON informs me that he has found some specimens of *Myxine glutinosa* with *seven* pairs of pouches.

[†] For details see J. MÜLLER, I., plate 7.

[‡] 'Das Gehörorgan der Wirbelthiere,' vol. i., plates 1, 2, pp. 3–12. Stockholm: 1881.

To *me* they are part of the skeleton, and therefore come into my description. The eye-balls are extremely small, and inconspicuous (Plate 13, fig. 1, *e.*, II.). The auditory capsules are very small, but not so small by far as the eye-balls; they are kidney-shaped, and look much larger below than above (Plate 10, figs. 1–3, *au.*). Indeed, below they are oval in form, with the narrow end in front, and, above, the concavity of the inner face, which is largely membranous, gives them their reniform appearance; they are composed of hard cartilage, and look like little light green seeds. They are confluent with the basis cranii and head of the hyomandibular cartilage; I shall show this in my account of the *sections.*

The nasal capsule of the Myxinoids is a unique structure; it is composed of the true olfactory organ and the vestibular region, which is proboscidiform (Plates 9, 10).

The proper olfactory organ is covered with a *grating* of cartilage (*na.*), whose bars run in a longitudinal direction.

The whole capsule is wider than long—wider than the brain cavity—is gently emarginate behind, and apiculate in front. The floor is membranous, and lies over the brain cavity behind, and the "front intertrabecula" in front. The roof and sides are enclosed by the cartilaginous grating, in which there are *nine* sub-equal bars, united fore and aft by a continuous belt of cartilage; the bars and these interspaces are nearly equal. The olfactory nerves enter the *membranous cribriform plate* by five distinct bundles (see in *Bdellostoma,* Plate 17, fig. 4—an anticipation of the Mammalian ethmoid).

The proboscidiform nasal tube is very exactly like a *Mammalian trachea,* being composed of a series of imperfect cartilaginous rings, of which I find *eleven* in *Myxine,* the last being apiculated; it projects forwards above the single narial opening; these parts will be better understood when we come to the *sections.* The upper seven *barbels* or "nasal palpi" protect this opening, and the other four the oral opening; they each have a delicate cartilaginous axis. This type of skull remained an utter enigma to me until lately, even with the great work of MÜLLER before me; and I am not aware that any one, except Professor HUXLEY, has, of late years, attempted to interpret it; nor should I have attempted now, if the task had not been lightened by my fellow workers, and if the early stages of the nearest relatives—the Lamprey, and the "Anura" in their larval stages—had not been mastered.

Of course, every determination of the nature of parts made now will be subjected to a severe and crucial test when the early stages of a true Myxinoid have been worked out. For those stages I am anxiously looking; but, meantime, this initial work will be something done; a little change of nomenclature, if needed afterwards, will be no great matter; and it is very important that this *scarcely vertebrate* type—it has no *vertebræ*—should be understood. The number of sections drawn is great, but they were all needed to make even *the worker himself* understand what lay before his eyes. The reader will need to keep the figures of the dissections (Plate 9, figs. 1–3, and Plate 10, figs. 1–3; Plate 12, figs. 7, 8; and Plate 15, fig. 6) before him whilst reading the following description of the sections; the figures of the undissected Fish (Plate 8, figs. 7–9) will also be found useful.

Vertically transverse sections of the adult Myxine glutinosa.

Before describing these sections, I may remark that the nasal passage carries the upper lip far in front of the lower; but the lower lip, even in the *Ammocœte*, is far back; whereas, in the adult Lamprey, the lower lip, *when the sucking disc is in action*, projects beyond the "anterior dorsal cartilage," and its enclosing skin, the upper lip.

The end of the external nasal passage is protected by nasal barbels, and the mouth opens further back; there are two pairs of oral barbels, the outer unciform and the inner mammillate. There are three nasal barbels on one side and four on the other; *six* thin sections of this part, one of which was drawn, show this; and, corroborative of this fact, Dr. GÜNTHER, in his 'Study of Fishes' (p. 695, fig. 320, A) gives, in a woodcut, the same number in *Myxine Australis.*

Section 1 (Plate 11, fig. 1).—Here the interior valvular opening of the external nasal duct (*e.n.*) shows four points of cartilage (*n.bb.*) cut through on one side, and three on the other; the lesser barbels are close to the passage, and the larger further out; the fore end of the prepalatine reaches to this point.*

Section 2 (fig. 2).—The nasal canal or passage (*n.p.*) is now complete, and the first ring is cut through in three places, above, and on each side (see also Plate 10, fig. 1); for this imperfect *annulus* has a rostrum. Right and left of the vertically elliptical passage, the largest nasal palpi (*n.bb.*) and the prepalatines are cut across. The median line, below, is concave; this hollow leads to the oral passage further back. The upper outline of this section shows two sub-marginal shallow grooves

Section 3 (fig. 3).—In this the sub-marginal grooves above are nearly obsolete, and the middle part of the top is slightly crested; it is slightly grooved in its broad, lower part. The prepalatine cartilages (*pr.pa.*) are now flatter and wider apart; the section is through their arch in the middle. This shows a wider but lower passage (*n.p.*), widest below, and it is also seen that the annulus just reaches the bottom, and is thickened there. Under the nasal passage, the dilated fore end of the front intertrabecula (*a.i.tr.*) is cut across; the bend downwards of its edges makes it like a *Chinese bridge.*

Section 4 (fig. 4).—Here, in this larger section, both the upper and lower surfaces are hollow in the middle; the nasal canal is becoming pyriform, with the narrow end below. The annulus is like that of the last section, but larger; but the prepalatine and front intertrabecula (*pr.pa., a.i.tr.*) are both flat in section; here the prepalatine has evidently been cut through very obliquely. The hard front intertrabecular bar is club-shaped in section in this and the next, the thick part being above.

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* The lower, larger sections of cartilage are, by mistake in this and the next figure, lettered, with the rest, as nasal barbels (*n.bb.*); they are the free ends of the prepalatine spurs.

Section 5 (fig. 5).—Here the pre-oral groove is deeper, and the front intertrabecula is here at its largest part. The true form of the prepalatine section (*pr.pa.*) is seen here. In this section, and in the 7th and 8th, we get a view of an important character in the nasal passage (*n.p.*), namely, that it has a valvular fold in it, growing down from above, and making it heart-shaped in section.

Section 6 (fig. 6).—This section is much larger than the last, and here we have the upper lip breaking into two large, sharp folds, and the palatal part of the oral vestibule wrought into two rounded folds. The valvular process of the nasal passage was not clear in this section, but we see that here the *annulus* is not thickened below, and that it turns inwards right and left. The front intertrabecula (*a.i.tr.*) is now long-oval in section, and the prepalatine (*pr.pa.*) oval. In the two folds of the upper lip (*u.l.*) we have the oral palpi (*o.bb.*) cut through, the outer longitudinally, and the inner across.

Section 7 (fig. 7).—In this section the two lobes of the upper lip (*u.l.*) are confluent, but the palpi (*o.bb.*) still come into section in a similar manner to the last; there are now five folds of the prepalatal region above, and two below; these latter look upwards, the meaning of which will soon be seen (see figs. 8–11). The valvular fold in the roof of the nasal passage is here clearly shown, and also the lessened depth of the front intertrabecula (*a.i.tr.*). The true thickness of the prepalatines (*pr.pa.*) is here shown, just at their base; and here also (see Plate 10, figs. 1–3) the ethmoidal region of the skull (*eth.*), with its attached conical yellow tooth (*et.t.*), is cut across.* The closed-in, or ethmoidal part of the skull is formed, as we saw, by the coalescence (or continuity) of the palatine bars outside, and the trabeculæ within; here, at the junction of these parts, the cartilage is high above, and gently convex below, where it carries the tooth.

Section 8 (fig. 8).—The section through the dilated hind part of the front intertrabecula (see Plate 10, fig. 2, *a.i.tr.*) was not figured; here, in front of that part, it is nearly circular in section; and because of the sinuous form of the hinder *narial annuli*, this section shows one cut into three pieces (*e.n.t.*); the nasal tube still shows its valvular character up to the proper capsule. The ethmoidal region (*eth.*) is arched; the outer part is palatine, and the middle trabecular. A cushion of fibrous tissue fills the concavity, and to it is attached the great tooth (*et.t.*), which is here cut across obliquely near its apex, showing its internal cavity. The lining of the mouth is sinuous, and the floor of it is now complete; this is the first section through the lower lip (*l.l.*), the ascending fold of which is the fore part of the *supra-lingual apparatus*. Right and left of these folds, not far from the mucous membrane of the mouth, two large hard cartilages have been caught by the razor; these are the outer front basi-hyals (*b.hy*^{a}.) (see also Plate 9, fig. 3). These bars turn upwards and out-

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* The specimen sectioned was older than the one which was dissected, and so the hard cartilage had extended further in the former; this will explain some slight discrepancies between the two sets of figures as to *colour*, and also as to the fineness or coarseness of the *dotting*.

wards, and are sub-pyriform in section, the thick end being uppermost. This 8th section is the first that takes in the lower lip, and we see how truly *Ammocœtine* these Myxinoids are.

Section 9 (fig. 9).—If this section be compared with the dissections we shall see what an elegant structure the nasal capsule (*ol.*) of *Myxine* is; in my Second Part I shall show that in the Lamprey this organ is a complete *wheel* of plates in section; here it is only half a wheel, and the folds only gently converge; this is the most simple structure of the two. Yet on this simpler platform there is a most curious specialisation of the cartilaginous capsule itself, and also of its vestibule. The accuracy of the *grating* is well shown in the sections. The two lowermost of the *nine* parallel bars are the widest; each bar has its own fold of mucous membrane, and these nearly reach to the common cavity below, which is here very large. Supporting this curious framework, we see the palato-trabecular bands (*p.tr.*, see also Plates 9 and 10); these are hard, oval, and tilted outwards, above. Then, for a short distance in front of the cranial cavity, the floor of the cranio-facial framework is open, being only floored by membrane.

A large tract of palatal tissue intervenes between this weak floor and the roof of the mouth (*m.*), the cavity of which is formed of three fissures, the outer and upper pair being semicircular, and the median, or lower, sub-oblong, but widest above, and having its sides running between deep folds. Above, the median line of the mouth is grooved; below, it is gently ridged; the lateral folds are the fore part of the *supra-lingual* framework. The proper lingual cartilages ($b.hy^{a}.$) are now seen to be *four*, instead of *two*, for the median pair have come into view (see also Plate 9, fig. 3); the *apparent distance* between the bars, here, is due to the fact that only the projecting fore end of each was cut through.

Section 10 (fig. 10).—This section is very similar to the last, but is important as showing the fusion of the two median bars ($b.hy.^{a}$); in this *old specimen* the fusion is perfect, but in the one dissected there was some appearance of distinctness along the midline, below (Plate 9, fig. 3); here the edges are thicker than the middle, which is gently scooped.

Section 11 (fig. 11).—Several new things come into this section; and here in the middle of the nasal capsule the lower cartilaginous bars are seen to have their own fold, but it is not free, so that there are only seven distinct folds in the sac. The great inferior fontanelle of the cranium bulges gently here, and the tip of the hinder intertrabecula (*p.i.tr.*) is cut across, where it supports the middle of the membranous floor. The bulging of the floor makes a great space for the nasal cavity (*n.c.*).

The palato-trabecular bands (*p.tr.*) are smaller at this part, and evenly oval in section; between the skull and the roof of the mouth there is a considerable web of fibrous tissue.

The form of the mouth cavity here is greatly modified by the horny cheek-teeth—*supra-linguals* (*s.l.t.*)—which are imbedded in a mass of fibrous stroma, and thus the wide mouth becomes a space of three fissures.

The two upper fissures are crescentic, and the form of the median space is a blunted wedge; the floor has two ridges. These sections show that the horny teeth are folds of a continuous epithelium; they are huge *papillæ* with a common horny base, and, distally, have their own cavity.

Under the folds of the mid-line the median part of the supra-lingual skeleton (*s.l.c.*) is cut through; this lies on the fused middle pair of front basi-hyals, which plate is concave under the lesser upper cartilage.

The lateral pairs of the front basi-hyals ($b.hy^a.$) are solid, oval in section, and are further apart from the middle pair than they seem to be in the bird's-eye view. (Plate 9, fig. 3; and Plate 12, fig. 7.)

Section 12 (Plate 12, fig. 1).—This section is through the hind bar of the nasal capsule (*ol.*), and the longitudinal rods of the grating are seen cut across close to the transverse bar. This section is close in front of the membranous cribriform plate, which will be described in my account of *Bdellostoma*, in which (and in *Myxine*) the olfactory nerves, as shown by MÜLLER (II., plate 2, figs. 8–12), pass into the capsule by *five* roots on each side, each root breaking into a pencil of fibres as it enters the capsule; a *Mammalian anticipation.* The palato-trabecular bands (*p.tr.*), and the *handle* of the hinder intertrabecular *spoon* (*p.i.tr.*), are cut through, and are similar to what we saw in the last section. So also are the basi-hyal bars; but, here, the median supra-lingual plate is wider, and the side wings now come into view, to support the cheek-teeth, now showing their distinctness in section.

Section 13 (Plate 12, fig. 2).—This section is very similar to the last, but it is behind the nasal capsule, and through the fore brain ($C^1.$); a large space is seen between the fore brain and the feeble floor of the skull; this is the posterior nasal (or naso-palatine) canal.

Section 14 (Plate 12, fig. 3).—Here the cranial cavity is at its widest part; the palato-trabecular bands are wider and less tilted at this part, which is in front of the small optic nerves (see Plate 13, fig. 1, *e.*, II.). The hinder intertrabecula (*p.i.tr.*) is cut across in three places, the edges of the *spoon* being caught, laterally. The supra-lingual cartilage (see fig. 8) is here continuous from side to side, being cut through in front of the foremost fissures; it forms a deep trough for the bed of the paired rows of cheek-teeth. Here (see Plate 9, fig. 3) the *four* front bars of the basi-hyal ($b.hy^a.$) are all distinct again, the section being made through the *basi-hyal fontanelle.* In the last *four* sections, the two median bars have been confluent.

Section 15 (Plate 12, fig. 4).—This section is through the fore margin of the subocular fenestra (Plate 10, figs. 1–3, *s.o.f.*), where some soft cartilage divides the *palatine* from the *pterygoid* regions (Plate 9, figs. 1, 2; and Plate 10, figs. 1–3), so that the hard cartilage (*pa.*) here is near the pterygoid; the inner is trabecular (*tr.*). Here the *spoon* of the hind intertrabecula (*p.i.tr.*) is cut across in front of its lateral attachment to the trabeculæ (*tr.*). The area on each side of the naso-palatine canal (*p.n.c.*) is the tissue through which the small optic nerves pass, but they were not caught in this section. The cheek-teeth are cut through nearer their point, and their enclosing cartilage is cut across through the first pair of fissures (fig. 8), so that it is not in *one* piece, as in the last section, but in *three.* The basi-hyal ($b.hy^{a}$.) shows some soft cartilage where the two pairs of front bars unite again, behind the median fenestra (Plate 9, fig. 3), in front of the merely double part.

Section 16 (Plate 12, fig. 5).—Here we have a section of the hinder intertrabecula (*p.i.tr.*) where it is confluent with the hind part of the trabeculæ; this is for a very short extent (Plate 10, fig. 3), but the actual union is very instructive, for in these Myxinoids this median bar retains that distinctness from the paired bars, which, in other types, is a very temporary condition. (See in the young Lamprey, Plate 10, fig. 5, *tr.*, *p.i.tr.*) At this point the basis-cranii is perfect, and being across the subocular fenestra (Plates 9, 10, *s.o.f.*) the cranium and pterygo-palatine region of the face are distinct for a short distance.

The supra-lingual skeleton (see also fig. 8) shows the depth of the middle part, and the steepness of the two sides further back across the two first fissures. The basi-hyal is now single (Plate 9, fig. 3, $b.hy^{p}$.), but this is down-bent in the middle, where two pieces have coalesced.

Section 17 (Plate 5, fig. 6).—This is also through the suborbital fenestræ (between *tr.* and *pa.*) behind the junction of the hinder intertrabecula with the trabeculæ (*p.i.tr.*, *tr.*). The pterygo-palatine bar (*pa.*) is dipping more and more towards the quadrate region; the lateral supra-lingual plates are narrower, and the basal part broader and deeper than in the last; the double basi-hyal ($b.hy^{p}$.) is narrower and deeper.

Section 18 (Plate 13, fig. 1).—In this section, behind the middle of the subocular fenestra (*s.o.f.*), a bundle of fibres is seen emerging from the brain; this is the optic nerve II., with the aborted eye-ball at its extremity. Here the trabecula (*tr.*) is a narrow rod, and the pterygo-palatine (*pa.*) is becoming flat and descending. The naso-palatine canal (*p.n.c.*) is still very wide, but the intertrabecula (*p.i.tr.*) below it is narrowing. This and the two next sections are through the hard cartilage of the hinder bars of the supra-lingual framework; the lateral plates are narrower, and the basi-hyal ($b.hy^{p}$.) similar to what was seen in the last section.

Section 19 (Plate 13, fig. 2).—This is the last section through the subocular fenestra (*s.o.f.*); it is very similar to the last, but the intertrabecula (*p.i.tr.*) is much narrower, and the pterygoid (*pg.*) is very solid; the basi-hyal ($b.hy^{p}$.) is becoming narrower, and the supra-lingual (*s.l.c.*) shows soft cartilage below, between the two hard bars (see Plate 12, fig 8).

Section 20 (Plate 13, fig. 3).—This and the next are between the first and second fenestræ, and therefore through the "pedicle" (*pd.*) of the pterygo-quadrate region (Plates 9, 10, *pg.*, *q.*); there is soft cartilage here, but with some hardened cells inside it; the trabeculæ (*tr.*) are widening towards the parachordals, and the intertrabeculæ (*p.i.tr.*) under the narrowing naso-palatine canal (*p.n.c.*) are becoming small.

The supra-lingual (*s.l.c.*) is cut through where the hard hinder bars run into soft cartilage above; the upper plate is narrow; the basi-hyal ($b.hy^{p}$.) like the last.

Section 21 (Plate 13, fig. 4).—This section was made through the second fenestra, the trabeculæ (*tr.*) are cut across, and the pedicle (*pd.*) in its hind part; the descending pterygo-quadrate bar (*pg.*) is cut through much further down. The naso-palatine canal (*p.n.c.*) is squarish, and the folds of its lining membrane are shown; under it is the fast narrowing intertrabecula (*p.i.tr.*). Here the elbow of each hard hind bar of the supra-lingual (Plate 12, fig. 8) is cut across outside one of the hinder teeth. The basi-hyal ($b.hy^{p}$.) is now in two pieces, below.

Section 22 (Plate 13, fig. 5).—This section is between the pedicle and the head of the hyomandibular, across the mandibulo-hyoid band (*m.hy.*) and the mandibulo-hyoid fenestra (*m.h.f.*) above, and through the quadrate (*q.*) below; the space between these bars is the second lateral fenestra (Plates 9, 10, $l.f^{2}$.). Inside the upper bar the front face of the auditory capsule (*au.*) is shaved off; these organs are confluent with the narrow fore part of the parachordals (Plate 10, figs. 1–3, *au.*, *iv.*), where the trabeculæ begin in front of the notochord; the intertrabecula (*p.i.tr.*) is very narrow here over the square naso-palatine canal (*p.n.c.*). One of the hinder cheek-teeth is cut through on each side, and part of the cartilaginous wall and the two hind snags. The narrowing hinder basi-hyals ($b.hy^{p}$.) are distinct.

Section 23 (Plate 13, fig. 6).—If this section be compared with the dissections (Plates 9 and 10) the great conformity of the two kinds of preparations will be seen. This and the next are the last sections in front of the notochord; the moieties of the investing mass, or parachordals, form here a mere lip to the lower edge of the *auditory shell* (*au.*), which is seen to be reniform in section, and membranous within, the walls being thickish, and composed of hard cartilage. Here the tip of the *bowl* of the intertrabecula (*p.i.tr.*) is perforated, and thus it is seen in section as two points of soft cartilage over the square naso-palatine canal (*p.n.c.*).

The mandibulo-hyoid bar (*m.hy.*) is a small oval section, a considerable distance from the capsule; much further down there is a curious hook of hard cartilage, and below it, but nearer to it, on the outside, another oval section. The hook is the first pharyngo-branchial ($p.br^{1}$.), at its enlarged end, where it partly occludes the second fenestra (Plates 9 and 10), and the bar below is the narrow neck where the pterygoid passes into the quadrate (*q.*), under the second fenestra. The basi-hyal ($b.hy^{p}$.) is like that of the last section.

Section 24 (Plate 14, fig. 1).—This section sheds light upon the dissections, and is also in turn explained by them.* Here, as in the last, the parachordals are very

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* I must here repeat the fact that these sections show the cartilage to be hard, in places where in the dissections it was soft, on account of the greater age of the specimen made into sections; where any other apparent want of conformity exists between the two sets of observations, it is due to *compression* of the specimen in cutting, for my best specimens were not well preserved, and at the best the pharynx and its *velum* are very elastic and mobile.

narrow, turned upwards, and confluent with the auditory capsules (*au.*). Under the square naso-palatine canal (*p.n.c.*) the tip of the *bowl* of the intertrabecula (*p.i.tr.*) is cut across, and near the capsule the mandibulo-hyoid bar (*m.hy.*). At a considerable distance down the side of the throat the hooked cartilage or 1st pharyngo-branchial ($p.br^1.$) is cut across, and below it the quadrate region (*q.*). The basi-hyal ($b.hy^b.$) is still double, and deep.

Section 25 (Plate 14, fig. 2).—We have now reached the notochord (*nc.*), a mere point here, and not invested by the investing mass, which is still a mere selvedge to the inner and lower edge of the auditory capsule (*au.*). This section is behind the intertrabecula, and it shows the naso-palatine canal (*p.n.c.*) still closed below. The lateral cartilages cut across are the mandibulo-hyoid band (*m.hy.*), the first pharyngo-branchial ($p.br^1.$), and the quadrate (*q.*); below, the basi-hyal is still double, and composed of hard cartilage. The bracket-shaped fold of membrane (above *vl.*, the pharynx is below *phx.*, the dotted lines are wrong) across the roof of the pharynx is the beginning of the "upper velum."

Section 26 (Plate 14, fig. 3).—This section shows the notochord (*nc.*) and the investing mass increasing, and the cavity of the auditory capsule (*au.*) laid open, and in this part the "anterior canal" is enclosed specially by cartilage. The neck and shoulder of the hyomandibular (*hm.*) is first soft, and then hard cartilage. The investing mass has grown beneath the capsule, as well as along its inner edge, below, and thus the hyomandibular which grows from it *seems to be* a mere outgrowth of the capsule.

Section 27 (Plate 14, fig. 4).—The notochord (*nc.*) is now invested by the parachordals; the hyomandibular (*hm.*) is thinner in the middle, here, than at its fore edge, as seen in the last section.

Section 28 (Plate 14, fig. 5).—This is a very instructive section, for here the naso-palatine canal (*p.n.e.*) is opening below, revealing the diagnostic character of the Myxinoids—("Hyperotreta"). The investing mass (*iv.*) now forms a neat trough for the notochord, and is, in reality, continued as a thin but confluent lamina under the auditory capsule (*au.*) where the cartilage is continuous with the soft head of the hyomandibular (*hm.*) By comparing this section with the side view (Plate 9, fig. 2, *hm.*) it will be seen that in this region the bar is soft in three places and hard in two. Inside the lower or symplectic region of the bar (*sy.*), where it runs into the quadrate region, the neck of the large 1st pharyngo-branchial ($p.br^1.$) is seen as hard cartilage. The basi-hyal ($b.hy^b.$) still retains the same character as in the sections immediately in front of it.

Section 29 (Plate 14, fig. 6).—In this section the notochord (*nc.*, *iv.*) is, *for once*, enclosed in cartilage. In the younger dissected specimen (Plate 10, figs. 1, 2,) the investing mass was not perfect anywhere above the notochord—only below (fig. 3). The auditory capsules (*au.*) are perfectly confluent with the basal plate, and here we are behind the head of hyomandibular, so that there is a space before cartilage is shown. The bar cut through is hyomandibular above (just where the opercular process is given off in Teleosteans), and symplectic (*sy.*), below. Inside the bar the 1st pharyngo-branchial ($p.{-}br^1$.) is still hard. The opening of the naso-palatine canal into the pharynx (*phx.*) is very wide here, and the round folds right and left are the sides of the "velum" (*vl.*). The basi-hyal ($b.hy^b$.) is beginning now to lose its hard cartilage; the soft cartilage at its edges is the distal part of the descending hyoid bar, the hypo-hyal region.

Section 30 (Plate 14, fig. 7).—The ends of the auditory capsules are here cut through, and the investing mass (*iv.*) is now a thick trough, in which the enlarging notochord lies. The lateral cartilages ($sy., p.br^1$.) are the same as the last, but cut through a little further backwards. The double basi-hyal now passes into the large soft elastic 1st basi-branchial above the hard paired cartilages ($b.hy^b$.). The soft end of each descending hyoid bar (*h.hy.*) is seen cut across.

Section 31 (Plate 15, fig. 1).—This section is very similar to the last, but is behind the auditory capsules, and the investing mass (*iv.*) is lesser, whilst the notochord (*nc.*) is larger. Below, the hard cartilage of the basi-hyals grows for some extent into the 1st basi-branchial (*b.h.br.*), the hypo-hyal end of the cerato-hyal bar (*h.hy.*) is still seen; it is slightly confluent with the basal bar.

Section 32 (Plate 15, fig. 2).—The notochord is now two-thirds of its average *spinal* thickness; it is almost enclosed by cartilage; this and the last are in the occipital region in front of the 1st spinal nerve (Plate 9, figs. 1, 2, *sp.n.*); here the hind brain (C^3.) begins to be *myelon.* The large flat symplectic region (*sy.*) still comes into section; above it is the inter-hyal bar (*i.hy.*), and, within it, the 1st pharyngo-branchial ($p.br^1$.). Below, the hypo-hyal is seen to be distinct from the soft elastic 1st basi-branchial ($b.br^1$.), the section of which is U-shaped and thick. The form of the two-lobed velum (*vl.*) is well shown here, with the open channel in which the naso-palatine canal ends.

Section 33 (Plate 15, fig. 3).—The notochord (*nc.*) has now acquired nearly its full size, and still it has two points of cartilage upon it, the extreme ends of the parachordals (see figs. 1, 2, *iv.*). A good distance down we get the upper band of the branchial basket-work cut through where the outer descending hyoid passes free from the hyo-mandibular region; this is the inter-hyal region (*i.hy.*). This bar re-appears again below at the bottom of the cerato-hyal as the hypo-hyal region (*h.hy.*); in some of the sections not drawn these two points passed into each other, the compression of the head curved the bar so as to make the razor pass through it in two points; and the same cause has forced the "velum" down, so that both in this and the next, the cartilages in it ($p.br^1., p.br^2$.) are lower down than in an undisturbed state. Between these small points we see the leafy part of the symplectic (see Plate 9, figs. 1, 2, *sy.*), still coming into view in section. The 1st basi-branchial ($b.br^1$.) is now a deep trough of *vacuolar* tissue, with the hypo-hyals (*h.hy.*) at its edge.

Section 34 (Plate 15, fig. 4).—This is from a considerable distance back, so that all the cartilage seen in it is mainly that of sections of the terminal rays of the basket-work (*e.br*^{1}., *p.br*^{1,2}.), with the very last part of the hyoid bar (*h.hy.*), and, above, the upper part of the 1st branchial (*e.br*^{1}.) is cut through twice. The 1st basi-branchial almost closes upon the median bands (belonging to the huge lingual muscles) that lie inside it. This section was made between the 1st and 2nd spinal nerves (Plate 9, figs. 1, 2) (*sp.n.*), and here the notochord (*nc.*) is of full size, and has no cartilage around it, whatever; the parachordals are the only *paraxial* cartilages developed; and, as in the larval Lamprey, there are no rudiments, even, of vertebræ; here, for C^3., read *my.*

Beneath the huge lingual muscles, the intra-branchial skeleton breaks out again; there (Plate 9, fig. 4) the 2nd basi-branchial (*b.br*^{2}.) reappears as a band of hard cartilage, broadish in front, and narrower behind, it is ventrally placed in relation to the first two branchial pouches; it does not support them, but lies under the "raphe" of the hinder part of the massive muscular apparatus of the tongue (*l.m.m.*).

The foregoing are all the skeletal parts I can find in this Fish, which in some respects is developed more in conformity with the higher Fishes—Cartilaginous or Bony—than the Lamprey, and in others is so very rudimentary; altogether, it evidently belongs to the *Ammocœtine* type, although greatly specialised in its own way, to its own ends.

The sections just described will be equally useful for both *Myxine* and *Bdellostoma*, for this latter larger Fish differs only in non-essentials from *Myxine;* my description of the former will merely relate to dissections, and to a general longitudinally-vertical section of the Fish.

On the skeleton of Bdellostoma Forsteri.*

On the cranio-facial apparatus of the adult Bdellostoma Forsteri.

The *theca cranialis,* or dura mater, in the hinder half, and the nasal capsule in the front half of the cranium (Plates 16, 17), have to be removed before the proper framework can be seen from above; the lingual apparatus has to be dissected from the lower part of the head before the basal part can be seen. Neither in the adult of these two types, nor in the embryo of the somewhat higher Lamprey, do we see any other explanation of the *pre-pituitary* part of the cranium than that of an outgrowth or foregrowth of the axial part of the skull. And yet the *pro*-chordal part begins

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* Dr. GÜNTHER informs me that there is but one species of this Fish, and that the kinds called *B. Forsteri* and *B. heterotrema*, by MÜLLER, are merely varieties of the same species. Dr. GÜNTHER's description of the two Myxinoid Genera is as follows:—

"MYXINE.—One external branchial aperture opening only on each side of the abdomen, leading by six ducts to six branchial sacs.

"BDELLOSTOMA.—Six or more external branchial apertures on each side, each leading by a separate duct to a branchial sac." ('Study of Fishes,' p. 695.)

MÜLLER, I., plate 7, fig. 3, gives a figure of *Bdellostoma* with *seven* pouches on the left side, besides the "ductus œsophago-cutaneous," and only six pouches on the right side. I found only six pouches on each side in my two specimens; for *plans* of these remarkable branchial organs see the same plate in MÜLLER's Memoir.

first, and is larger, often much larger, than the *para*-chordal; moreover, in most low forms the prochordal part is developed first. The notochord is very uniform in bulk along the spinal region in these large long Fishes, only gradually lessening in the caudal region. In the head, this rod (Plates 16, 17, *nc.*) suddenly becomes conical, and ends between the middle of the auditory capsules (*au.*) so that this part of it is only one-tenth the length of the entire skull. Of course in the embryo the relative length was much greater. In the embryo of the Lamprey, as I shall show in my Second Part, it is half as long as the entire skull, and in the youngest chondrocranium figured by me in *Lepidosteus* (Phil. Trans., 1882, Plate 30, fig. 3) it is *three-fifths* the length of the skull.

So that we see that in these low Myxinoids—the lowest known Vertebrata—the chondrocranium, during development, must have undergone a large amount of change, and we seem to be almost as far off as ever, in these types, in their adult state, from finding an archaic skull. Embryonic chondrocrania, in various *Ichthyopsida*, are our best guides. The investing mass (Plate 17, figs. 2, 3, *iv.*) is complete for a short distance, both above and below, and it projects, as hard cartilage, a little distance behind the auditory capsules, thus forming a rudimentary basi-occipital; but there is no occipital arch or ring. This short parachordal tract is round behind, and deeply notched in front, for the trabeculæ begin opposite the middle of the small auditory capsules. Those bands are only free from surrounding parts for a very short distance, being first (at their parachordal roots) continuous with the infero-internal edge of the auditory capsules, then on the outside with the pedicle, and in front of the short oval subocular fenestra (*s.o.f.*), for the remaining two-thirds of their length, they are continuous with the palatines (*pa.tr.*). This latter essentially compound part is very solid, and for the last sixth of the skull-length the two sides are completely united, so as to form a very solid ethmoidal region (*eth.*). The whole roof is unfinished, being only closed by the dura mater, but the depth of the auditory capsules, and the thickness of the palato-trabecular bars, give a trough-like character to this flat, unfinished skull. Below (Plate 17, fig. 3), the main bulging is formed by the hinder intertrabecula (*p.i.tr.*) and the auditory capsules, which project almost as much below as above; the skull proper is gently convex in the fore part, and in front bends downwards a little. The so-called "pituitary space," or lower cranial fontanelle, reaches from the middle of the auditory capsules to the ethmoid, and is lanceolate, but its narrow fore end is rounded. About half this membranous space is floored by the soft convex hinder intertrabecula (*p.i.tr.*), an additional cartilage, two-thirds the length of the skull, and which reaches from the trabecular roots to the ethmoidal commissure. It is a *ladle*, with its handle dilated in front, and its bowl produced behind, into a short second process or beak; the bowl occupies about a fourth of its length, and is near the hind part. The whole upper part is hollow (Plate 17, fig. 2, *p.i.tr.*), and at the middle of the bowl a short side band runs into the trabecula right and left, these bars being soft for a short space, where the junction takes place, and at this part the trabeculæ bend inwards. Exactly right and left of the *bowl* we see the regularly oval suborbital fenestræ (*s.o.f.*), each about half as large as the bowl, and having its long axis parallel with that of the skull. The front part of the skull is elegantly *ox-faced;* the bovicorn outgrowths are the soft prepalatine bars (*pr.pa.*); they are almost as long as the common palato-trabecular bar next behind them, and their length and distance apart are about equal. They have, hung about them, four pairs of *lesser horns,* three pairs acute, and the fourth double and *cervicorn;* these are the cartilages of the barbels (*n.bb.*, *o.bb.*). The rest of the face is a kind of lattice work, ending behind in free cervicorn processes. The second median element of the skull—the front "intertrabecula" (*a.i.tr.*)—is spindle-shaped and compressed; it is composed of hard cartilage, as in *Myxine;* it overlies the ethmoid in its hind half, and the nasal sac lies over it. The lower margin of the palato-facial growth is convex where the great prepalatine "horn" grows out, then concave, but is obliquely descending in the orbital region; the deepest part of the palato-quadrate tract is directly below the middle of the auditory capsule, and here the quadrate condyle *should* be found; there is none, and no mandibular rod. The lower edge of the cartilage remains soft (Plate 16, fig. 1, *q.*), and so also does the pedicle of the suspensorium (*pd.*); but there is no soft cartilage under the fore part of the subocular fenestra (*s.o.f.*) such as we saw in *Myxine* (Plates 9 and 10). The second lateral fenestra (*l.f*^{2}.) is nearly twice the size of the one in front of it; it is heart-shaped, having a round process of cartilage growing into its hind margin. A second soft "pedicle," the head of the hyomandibular (*hm.*), is seen under the hind part of the auditory capsules; between these there is a small upper fenestra (Plate 17, fig. 1, *m.hy.f.*), whose lower boundary is a thick bar of hard cartilage (*m.hy.*), the shoulder of the hyomandibular (*hm.*). The fore part of this bar is continuous with the suspensorium (its metapterygoid region), whilst, behind, it ends in soft cartilage, the inter-hyal region (*i.hy.*). The upper part of the hyomandibular in osseous Fishes is mostly very broad, projecting over the short free metapterygoid in front, and growing backwards as the "opercular process." This upper hyoid region finishes the second fenestra, behind, sending inwards a round lobe, and being also sublobate behind in three places. Below and above, the small lobes of hard cartilage pass into the soft kind, above in the inter-hyal region, and below, in the symplectic (*sy.*). From the inter-hyal region the descending epi-cerato hyal band, two-thirds the width of the hard band in front, descends *over* the hinder or third or largest fenestra (*l.f*^{3}.), the hinder half of which is enclosed by the arcuate and spiked 1st epi-branchial (*e.br*^{1}.)* There is a notch, below, between the arrested quadrate (*q.*) and the lower part of the symplectic region (*sy.*) The cartilage is all soft here, and above the junction of the two kinds the symplectic region sends a spur backwards partly filling in the lower part of the space. The long hyoid bar (*e.hy.*, *c.hy.*) is sigmoid above, and then slants forwards and downwards to its hypo-hyal region (*h.hy.*), where it is continuous with the basal bar (*b.hy*^{b}.). The lower boundary of the large posterior membranous space (*l.f*^{3}.) is

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* See Plate 17, fig. 1; in fig. 2, and in Plate 16, fig. 1, this part is lettered *c.br.* by mistake.

formed by the passing of the symplectic into the 1st epi-branchial (*sy.*, *e.br*^{1}.); this bar becomes dilated behind, has a sinuous margin in front, and sends backwards two sharp spikes or rays (non-segmented branchial rays). But this enclosing arcuate spiked bar is only the middle of the large 1st branchial arch; it has a very long pharyngo-branchial piece above (*p.br*^{1}.) and a long slender cerato-branchial (*c.br.*) below, both these rods are directed backwards and end in long, sharp styles, and both are *f*-shaped; the lower is a part *nearly* segmented off from the rest of the basket-work, still it is attached, below, to the basal bar, behind the descending hyoid. The 1st pharyngo-branchial (*p.br*^{1}.) can be seen from the side filling in the hind and lower part of the middle fenestra, and serving to give "origin" to a fan-shaped series of muscular fasciculi (Plate 16, fig. 1). That part is confluent with the hyomandibular, and is hard; the rest is soft cartilage and passes upwards a little, and then directly backwards, being enclosed in the edge of the "velum" (Plate 16, fig. 6; and Plate 17, fig. 1, *vl.*). These sigmoid rods are continuous on their inside, a little in front of their middle, with the fore end of another pair of rods, similar but smaller, and their serial homologues; these are the 2nd pharyngo-branchials (Plate 17, figs. 2, 3, *p.br*^{2}.); they have no descending part. Each bar, towards the middle, is bent towards its fellow, like the larger first pair, and the two are twice united by a cross band. The front commissural band sends forwards a pair of three-rayed rods spreading out over the others and looking forwards; and from its hind margin it gives off a single median rod, which passes directly backwards over the hinder commissure, and then breaks up into two larger sub-terminal and two smaller terminal rays. These four rays, and the points of both pairs of main pharyngo-branchials (*p.br*^{1}., *p.br*^{2}.) end in the transverse crenate hind margin of the great upper pharyngeal "velum" (Plate 17, fig. 1, *vl.*). This structure is much like that seen in *Myxine* (Plate 15, fig. 6), but in that Fish the 2nd is free from the 1st pharyngo-branchial, and the median bar of the former is absent.

The complexity of this reticulation of cartilage suggests the presence of a 3rd pharyngo-branchial rudiment, but I am not certain of its existence.

The huge basal bar is quite like that of *Myxine* (Plate 16, figs. 1, 2, 6, *b.hy.*, *b.br*^{1}.), the front part, for more than a third of the whole length, being composed of hard cartilage, and the rest of *vacuolar tissue;* this part is the 1st basi-branchial (*b.br*^{1}.), and the other basi-hyal (*b.hy.*). This is a curious piece of special "hypertrophy," for the normally single basal bar is composed of two bars behind, and four in front, all large, solid bars, oval in section. The outer of the front pieces are the largest; a small fontanelle is seen where the *six* pieces meet, and they are all connected together by tracts of soft cartilage. Looking at the structure as a whole,—first, quadruple, then double, and then single, and composed of four *varieties* of connective tissue,—we see, at once, that it is merely an enormous development of the common "basi-visceral" element. It is hollow above, all along, as in *Myxine;* in front, to hold the secondary trough-shaped cartilage (supra-lingual), and behind, to receive the tendons of some of the muscles that work the whole lingual machinery.

Close behind the hard cartilage of the basi-hyal, the hypo-hyal end (*h.hy.*) of the descending hyoid bar is there continuous both with the hard and the soft cartilage; and behind it, in its *axil*, the lesser bar, or 1st cerato-branchial ($c.br^1.$) is attached,—semi-confluent. The 2nd basi-branchial (Plate 16, fig. 3, $b.br^2.$), is a thickish rod of hard cartilage serving for the attachment of the oblique muscular mass of the lingual apparatus; it is broad in front and narrow behind, and moderately thick. MÜLLER, *who let nothing escape him*, figures a triradiate tract of soft cartilage on the "œsophago-cutaneous duct" (I., plate 7, fig. 5). I find a similar piece (Plate 16, fig. 7, *d.œ.c.*). This is extremely interesting and instructive, for it is all we have in this type—(I do not find it in *Myxine*)—of the huge "extra-branchial" basket-work of the Lamprey, which can already be seen in embryos one-fifth of an inch in length. Thus, if the Myxinoid is a sort of *Ammocœte*, it is an Ammocœte with a difference.

On the supra-lingual apparatus of Bdellostoma.

In these structures, again, we have a curious *generic* difference between *Bdellostoma* and *Myxine;* this series of intra-visceral arches is, however, only gently modified in the former from that of the latter (see Plate 12, fig. 8; and Plate 16, fig. 5).

Here, there is no differentiation of the median bar; the middle, in front, projects forwards as a triangular tongue of cartilage, but not behind the last lateral bars; the middle, there, is merely a soft narrow tract. The first pair of partly-segmented arches is the largest in this type, only the margin, all round, is soft; it ends in two soft incurved horns. The second pair of bars is soon lost in the third, which are almost as large as the first, and are quite hard, except the middle, or connective tract. I see no horny points in the front part of the common ligamentous tract in front, such as we saw in *Myxine* (Plate 12, fig. 8). A deep fissure divides the first pair of arches from the other two; they are only separated across the middle for one-third of their extent. The richly golden, lanceolate teeth (Plate 16, figs. 4 and 6), show two rows, right and left, of more equal teeth than in Myxine (Plate 12, fig. 7); there are *eleven* in each of the four rows; I counted seven on the outer, and nine on the inner rows in *Myxine.* The single upper tooth attached to the ethmoidal region of the skull is a very large "canine," hooked backwards (Plate 16, fig. 6; and Plate 17, fig. 3, *et.t.*).

On the sense-capsules and barbels of Bdellostoma.

The auditory capsules (Plate 16, figs. 1, 6; and Plate 17, figs. 1–3, *au.*) in this type also are very small, and composed of hard cartilage; they are quite confluent with the contiguous parts of the skull; seen from below (Plate 17, fig. 3, *au.*), they are of a full oval form, and their long axis is almost parallel with that of the skull, but looking a little inwards in front. Above (figs. 1 and 2, *au.*) they are reniform, on account of the deficiency of cartilage in the great "meatus internus;" these figures fail to show the involution of cartilage round the anterior canal as seen in the *sections* of these parts in *Myxine* (Plate 14, fig. 3, *au.*). The "meatus" are seen almost directly from above (fig. 2), and they are margined on their inner edge by the parachordals as they pass into the trabeculæ (*iv.*, *tr.*). The proper olfactory capsule (Plate 16, figs. 1, 6, *na.*; and Plate 17, fig. 1, *na.*) is only gently modified from that of *Myxine;* its fore margin is partly united with the last ring of the outer nasal tube (*e.n.t.*). There are only eleven imperfect annuli in *Myxine* (Plate 10, fig. 1 *e.n.t.*), in this type there are twelve, but these are not quite distinct, as in the Hag Fish; here, the first three are united, and have one small and two large fenestræ; the seventh and eighth are joined in the middle, and the twelfth, partially, with the proper capsule (*na.*). So unlike is the structure of the lips in these, to what is seen in any other types, that nothing but *re-dissection* enabled me to understand MÜLLER's perfectly accurate figures; especially was this the case in the "barbels."

Here (Plate 16, fig. 1; and Plate 17, figs. 1–3, *n.bb.*, *o.bb.*) they are much more modified than in *Myxine* (Plates 9 and 10). The upper pair (1st nasals) are united across the mid-line by a strong commissure which passes under the top of the front intertrabecula (*a.i.tr.*), and the second pair (2nd nasals) are attached to the first where they pass into the cross band.

The upper orals (*o.bb.*) are triradiate, they send an upper curved ray to join the two nasal palpi, and from their proper palpal part which grows forwards they send backwards a lobe over the 4th (or 2nd oral). This latter cartilage is bent forwards, and then sends out three short snags; its upper horn fits into the angle of the 1st oral, and its hinder horn turns backwards and downwards, nearly reaching the front of the "supra-lingual" apparatus. If we turn to the figures of the barbels in their *undissected* state, we shall see reason to believe that they all belong to the *upper lip* (Plate 8, figs. 7–9; Plate 13, fig. 7—*Myxine;* and Plate 15, fig. 5—*Bdellostoma*); in the large kind the 2nd orals are but little produced as barbels, but form an evident fold in front of the lower lip. Thus, in the fullest development of a *Myxinoid's* mouth there is no more promise of what the transformed Lamprey shows in the *lower lip* than in its own larva—the *Ammocœte.* As *Marsipobranchii,* the "Myxinoids" are more specialised, in some important respects, than the "Petromyzoids;" but as *suctorial* Fishes they are altogether below them, bearing to them a sort of *quasi-larval* relation.

Description of a longitudinally vertical section of the head of Bdellostoma.

In this section (Plate 16, fig. 6) we see many things brought into view which may help in the comprehension of this remarkable type of cranio-facial skeleton. Muscular segments and their interseptal fibrous tracts are found running over the brain up to the nasal sac (*na.*, *ol.*); the brain mass (C.) insensibly passes into the *myelon* (*my.*), which latter is supported by the large notochord (*nc.*). This figure shows the *vacuoles* ending some distance behind the thick sheath; this appearance is due to the fact that the section was not quite in the middle: the proper structure of the notochord ends opposite the middle of the ear capsules. The capsule (*au.*) is shown in section near the meatus, so that its thickness appears greater than it is in reality, as the parachordal cartilage thickens it on the inside. The folds of the nasal capsules (*na.*, *ol.*) are brought into view, and the cartilages (*na.*), both of the capsule itself (*ol.*) and of the external nasal tube (*e.n.t.*) are cut through, very near the mid-line. A continuous passage is seen from the external opening (*e.n.*) of the nasal passage (*n.p.*) to the fringed lips of the naso-palatine canal (*p.n.e.*) which opens at the mid-line at the fore part of the velum (*vl.*), here shown in section, with a point of cartilage in it, here and there, caught by the razor. The size of the cavity and the character of the folds inside it, of the fore part of the naso-palatine canal (*p.n.c.*) are seen in this section, and also that these lie *on the floor of the skull*, between the hind intertrabecula (*p.i.tr.*) and the brain (C.) The opening *out of the skull* is behind that median cartilage, between the produced "bowl" of the "spoon" (Plate 17, fig. 3, *p.i.tr.*) and the beginning of the trabeculæ (*tr.*, *iv.*). Another cavity—a space between the hinder intertrabecula and the roof of the mouth—is also seen. This is evidently artificial, and caused by the tearing away of the mucous membrane of the palate from the hinder intertrabecula. A curious coiled valve is seen behind these two openings, under the fore part of the velum, at the beginning of the pharynx; a little in front of this fold, the proper oral cavity (lined with *epiblast*, the pharynx being lined with *hypoblast*) ends. At this part, the entrance of the proper pharynx, the passage is a series of slits or fissures (see the *sections* of the skull of *Myxine*, in Plate 13). Somewhat *left* of the mid-line, this section shows only part of the front intertrabecula (*a.i.tr.*), and thus the whole of the great median tooth (*et.t.*), and the ethmoid (*eth.*), a little to the left, is seen under it. Both the ethmoid and the base of the tooth are enclosed in a special fold of membrane over the oral entrance (*m*). That space would be large, but *two additional lingual masses*, the paired rows of teeth (*s.l.t.*) and their membranous and cartilaginous settings (*s.l.c.*), fill the greater part of the space between the basi-hyal and the palatal membrane. This section being a little to left of the mid-line, shows the inner edge of the *left* bars of the basi-hyal (*b.hy.*), and as the middle bars of the front or quadruple part are shorter than those outside, the cartilage is seen to stop short of the fore end of the under face (lower lip). These cartilages are enveloped in the muscular masses that move the tongue. The upper or ethmoidal tooth (*et.t.*) is seen to turn its point backwards; in the figure it is *in front of* the paired rows of supra-lingual teeth (*s.l.t.*), but the powerful muscular apparatus works so as to make it antagonistic to the lower teeth, generally, which can be made to move backwards and forwards, and from side to side. This whole muscular mass, in my larger specimen, is sub-cylindrical, six inches long, and one inch in thickness. The reader is referred to MÜLLER's works for a description of the whole apparatus.

On the cranial nerves of Bdellostoma.

For a detailed account of these, and also of the whole nervous system, the same excellent author is referred to; but I have given an outline figure of the skull with the cranial nerves shaded, to help in the determination of the skeletal parts. My own dissection of these nerves was made with the help of MÜLLER's work ('Neurologie,' III.). I made no attempt to find any filaments of the 3rd, 4th, and 6th. The vagus nerve (Plate 10, fig. 4, X.) probably contains the fibres that in higher forms are given off as the separate glosso-pharyngeal. MÜLLER does not figure that nerve as distinct, nor can I find it. But the vagus is very large, and as its proper related structures are removed far backwards, passes directly backwards at right angles to the spinal nerves (*sp.n*$^{1-3}$.). The vagus emerges behind the auditory sac (*au.*), over the short parachordal.

The 8th or auditory nerve is not figured; it passes into the auditory capsule through the great meatus internus.

The 7th nerve (VII.) is shown; it finds its way out of the skull in front of the auditory capsule, between the "pedicle" and the head of the hyomandibular (fig. 2, *pd.*, *hm.*), and turning backwards, burrows under the "shoulder" of the hyomandibular, escaping again behind the *symplectic* region—the hind boundary of the second fenestra —and passes down with, and in front of, the descending epi-cerato-hyal bar (*e.hy.*, *c.hy.*).

The 5th or trigeminal nerve (V.) is immense, and at first sight rather difficult of interpretation. I follow MÜLLER's notation, finding nothing in it to criticise. It is seen emerging in front of the auditory capsule over the pedicle (*pd.*). It then breaks up into three branches, two of which pass *over* the subocular fenestra (*s.o.f.*), whilst one pierces that membrane behind, and passes *under* the palato-quadrate cartilage. The 1st branch, the orbito-nasal (or *ophthalmic*, V^1.), is very large, and soon breaks up into a lesser outer and a larger inner branch. The inner branch then becomes trifid. The "upper maxillary," or second branch of the trigeminal (V^2.) is *considerably less* than either of the three branches of the ophthalmic; it runs over the subocular fenestra, and the palato-quadrate cartilage. The "inferior maxillary," or third branch (V^3.), is as large as the ophthalmic, and as soon as it has passed through the suborbital fenestra it breaks up into four branches, which pass to the lower parts of the head in front.

The optic nerves (II.) are about a fourth the size of the small maxillary nerve; they pass under the ophthalmic (orbito-nasal), and are distributed to the arrested eye-ball (*e.*), enclosed in the much larger *fat body* (*c.a.*).

The olfactory nerves (I.) pass through the membranous "cribriform plate," which is in two divisions and oblique. The huge olfactory lobes give off each *five nerves*,* and these, *beginning to bifurcate*, pass in a sub-horizontal row, right and left, to the olfactory folds (*ol.*); here there is a most remarkable anticipation of the Mammalian

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* The outer or fifth bundle of the olfactory nerve lies somewhat under the fourth, and is not shown in the figure.

olfactory nerves and *cribriform plate.* If we now compare the diagram (Plate 17, fig. 4), and the meagre description just given of the cranial nerves of this great Myxinoid, we shall find many things become clear that have been for a long time confusing to the students of these types; for there has, apparently, been much *concrescence* and much *suppression* in the development of the cranial nerves in these remarkable Fishes. In the Lamprey, also, there are some very abnormal characters. The right determination of the skeletal parts is largely dependent upon a true knowledge of the cranial nerves, and, *vice-versâ,* they have to be considered in their relation to the supporting skeleton.

MÜLLER II. (" Gehörorgans," plate 3, figs. 3, 4, 5) gives upper, lower, and side figures of the brain of *Petromyzon fluviatilis,* and he enumerates the following nerves going off from it right and left, viz.: I. Olfactory (large and single); II. Optic (moderate size); III. Oculo-motor (very small); IV. Trochlearis (still smaller); V. Trigeminus; VI. Facialis; VII. Acusticus; VIII. Vagus; IX. Hypoglossus.

In the second plate figures are given of the brain of *Bdellostoma* (figs. 8, 9) and *Myxine* (figs. 11, 12).

In these closely related types we have the following cranial nerves given by MÜLLER, viz.:—

I. Olfactory (in five main nerves on each side, and these also each breaking into a small pencil of branches); II. Optic (very small in *Bdellostoma,* but paired; much smaller and only drawn on the *right side* in *Myxine*); III. Trigeminus (very large); IV. Facialis; V. Acusticus; and VI. Vagus (very large).

In his paper (III.) on the 'Neurologie' of the Myxinoids, MÜLLER (plate 2, figs. 4, 5) gives the cranium and its nerves in *Bdellostoma* from above and below. In the upper view the optic nerve is shown passing under the ophthalmic nerve, which is drawn with its first bifurcation. The larger part of the nerve is seen penetrating the suborbital fenestra, and in the lower view this lower part (3rd branch) is seen broken up into four branches, the second of which is re-split. But in the upper view there is also, behind the ophthalmic (1st branch), a small nerve passing over the palato-quadrate cartilage; this he does not name.

But in the next plate (III., figs. 2 and 3) we have this 2nd branch of the trigeminal figured on both sides. In the upper view (fig. 3) it is seen as passing over the cartilage bounding the suborbital fenestra, and in the side view (fig. 2) it is shown as running forwards along the upper surface of the palato-quadrate nearly up to the beginning of the soft "pre-palatine horn." Here it is called "Vorderen ober Ast des Trigeminus," whilst the large branches that grow from the stalk that pierced the fenestra are called "Unter Aeste desselben,"—the lower branches of the same nerve. In the upper view (fig. 3), where the 1st and 2nd branches (only) are drawn, he calls this small nerve "Hinterer oberer Hautast des Trigeminus;" whilst the branches of the great 1st division (Ophthalmic), "Vorderer ober Ast des Trigeminus," he calls "1, Hautast; 2, Nasenast; 3, Oberer Endast; and 4, Unterer Endast."

That there is no mistake here I feel certain; my own dissection shows the same pair of nerves, namely, the *small*, proper, 2nd branch of each trigeminal.*

Now in these Myxinoids we miss the 3rd, 4th, and 6th cranial nerves; also the glosso-pharyngeal or 9th nerve as a separate nerve.

In *Petromyzon*, also, MÜLLER misses the 6th or "abducens" (II., plate 3, figs. 3–5); the "upper maxillary branch" of the trigeminal is small and inconspicuous but quite as large as in *Bdellostoma*. This being the case we have an explanation of what has been a difficulty with us for years; Professor HUXLEY† speaks of "a singular anomaly" in both *Petromyzon* and the Myxinoids, in that "Both the second and third divisions of the trigeminal nerve pass through the subocular membrane, and therefore on the ventral side of the (subocular) arch;" but this error was corrected by him soon afterwards.

That the *arch* and the *membrane* (fenestra) are both the exact counterparts of what we find in the various Batrachian larvæ I cannot doubt; and there is no doubt now, of the existence of a 2nd branch to the trigeminal in *Petromyzon*; and, as we have just seen, such an *upper* branch does exist in *Bdellostoma*, between the huge 1st and 2nd branches.

Summary and conclusion of Part I. (Myxinoids).

In seeking for light upon the primordial condition of the Vertebrata one naturally looks to such forms as the Myxinoids, for in these types, even in the adult state, there are neither limbs nor vertebræ, and no distinction between head and body, except the rudiments in the head, of a cartilaginous skull, *a continuous* structure, not showing the least sign of secondary segmentation, and by far the greater part of which is in front of the notochord, or axis of the organism. But here our *gradational* work agrees with the *developmental*, for the continuous skull bars constantly arise before the secondary cartilaginous segments that are formed between the "Myomeres" behind the head. Evidently, therefore, the early "Craniata" grew supports to the enlarged and sub-divided front end of the neural axis long before anything beyond strong fibrous septa were developed between the muscular segments of the body. As for the linear growth, and the less or greater extension backwards of the main organs, —circulatory, respiratory, digestive, and uro-genital—that in the variation of the primordial form was a thing to be determined by the life and surroundings of the type. "Thereafter as *they* may be," was the tentative idea in this case.

Certainly in the Marsipobranchs and in their relations the larval "Anura," we have the most archaic "Craniata" now existing; in these the organs may be extended far backwards in a vermiform creature, as in these low Fishes, or kept well swung

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* See also RETZIUS, plates 1 and 2, and note to p. 42, in Part II.

† See "On the Nature of the Cranio-facial Apparatus of *Petromyzon*" (Jour. of Anat. and Phys. vol. 10, p. 423, a paper to which I am exceedingly indebted in working out these types).

beneath the head; the body and tail, together, forming merely a propelling organ, as is seen in Tadpoles, especially the gigantic Tadpole *of Pseudis.*

Thus we see that in low, limbless types there is no necessity for the development of more than fibrous *metameres,* but the vesicular brain, the suctorial lips, the purse-shaped gills, and the special organs of sense, these all call for support from some tissue more dense than a mere fibrous mat or web. In the *Myxinoids* we see that four special modifications of the connective tissue series are developed for the support of the properly *cephalic* organs, and two of these for them, only, so that these Fishes are chordate and craniate, but are not *vertebrate,* if we stick to the letter. At first some disappointment is felt, after careful study of these types, for notwithstanding the low level in which they *remain,* they are mere specialised Ammocœtes, keeping on the same "platform" as the larval Lamprey; yet some parts of their organisation do undergo a marvellous amount of transformation, and are, indeed, as much specialised in conformity with their peculiar habits of life as *any Vertebrates whatever, the highest not excepted.*

Yet, on the whole, the Myxinoids are a sort of *Ammocœtine* type, whilst the transformed *Ammocœte,* the Lamprey, comes nearest to the untransformed Frog or Toad,—the Tadpole. But the mere putting of this shows (*suggests* at any rate) what *losses* the fauna of the world has sustained during the evolution of the "Craniata." For us, *now,* the Myxinoids, Petromyzoids and Anurous Amphibia must all be kept "within call" of each other; but the types that have been culled out, between them, cannot be numbered.

Some other types of Fish are evidently the descendants of primordial "Marsipobranchs," notably *Lepidosteus,* the development of which has lately been made out and the results published in the Philosophical Transactions (1882, Part II.).

But the *Chimæroids* and *Dipnoi,* and, what is still more important, the *Myxinoids* themselves, have still to be followed through their early stages; if the present paper is of any value to the Morphologist, one on the embryology of these low forms would be worth many such papers.

The Myxinoids keep on the low platform of the larval Lamprey (or *Ammocœte*) in the following particulars, namely :—

a. The notochord has no paired cartilaginous vertebral rudiments in the spinal region.

b. The trabeculæ end in the ethmoidal region without growing forwards into a cornu (or two continuous cornua).

c. There are merely "barbels" round the mouth; no perfect *labial cartilages.*

d. The last character involves another, namely, that the special armature of horny teeth attached to the labials in the adult *Petromyzon* is absent.

e. The organs of vision are very feeble, and probably almost useless; in the *Ammocœte* they are arrested for a time.

f. The cranium is a mere *floor,* without side-walls or roof.

The Myxinoids come near to the adult Lamprey in the following particulars, namely:—

a. There are developed outside the skull proper, but not segmented from it, palato-quadrate and hyoidean cartilages.

b. There is a very large median cartilage belonging to both the hyoid and branchial regions.

c. The cranium acquires a floor by the development of a special hinder inter-trabecula.

d. There is a large median cartilaginous olfactory capsule.

The Myxinoids go beyond even the adult Lamprey in the following characters, namely:—

a. The facial basket-work is much more perfect, and as this is a generalised condition of the true *intra-visceral* system of cartilages, it is a very important character. There is not only an equal development of the "*suspensorium*," but the *suspensorial part* of the hyoid is developed also (it is suppressed in the Lamprey); and there is (in *Bdellostoma*) a large complete 1st branchial arch, and in both kinds pharyngo-branchial rudiments of the 2nd branchial arch.

b. The respiratory pouches are much more specialised, being carried far back under the spine.

c. There is not only a very distinct sub-cranial intertrabecula, but also a large pre-cranial or nasal median cartilage of the same nature.

d. The opening to the median olfactory sac is not a mere short membranous passage, but a long tube, encased in a series of cartilaginous (imperfect) rings.

e. Correlated with the non-development of the suctorial labial apparatus, there is an enormous development of the *lingual*, this basal bar becoming not only double, but in front quadruple; and the "supra-lingual cartilages," which are very small in the adult Lamprey, and carry only one pair of rows of small recurved teeth, in the Myxinoids are very large, and carry two pairs of rows of large teeth, with the addition of a median ethmoidal antagonist tooth.

Lastly, the greater development of the intra-visceral cartilages is correlated with the suppression of the extra-visceral basket-work, seen both in the larval and adult Lamprey, and also in the larvæ of the "Anura," generally.

EXPLANATION OF THE ABBREVIATIONS.

The Roman numerals indicate nerves or their foramina.

a.i.tr.	Front intertrabecula.
au.	Auditory capsule.
bb.	Barbels.
b.br^{1,2}.	Basi-branchials.
b.c.f.	Basi-cranial fontanelle.
b.h.br.	Basi-hyobranchial.
b.hy^{a}.	Front basi-hyal.
b.hy^{b,p}.	Hind basi-hyal.
b.p.	Buccal pouch.
br.a.	Branchial artery.
br.c.	Branchial canal.
br.d.	Branchial duct.
br.p.	Branchial pouch.
C^{1,2,3}.	Cerebrum.
c.a.	Corpus adiposum.
c.br.	Cerato-branchial.
c.hy.	Cerato-hyal.
c.tr.	Cornu trabeculæ.
cl^{1,2,3}.	Clefts.
d.m.	Dura mater.
d.œ.c.	Ductus œsophago-cutaneus.
e.	Eye
e.br.	Epi-branchial.
e.f^{2,3}.	External (*lateral*) fenestræ.
e.hy.	Epi-hyal.
e.n.t.	External nasal tube.
e.pa.	Ethmo-palatine.
eth.	Ethmoid.
et.t.	Ethmoidal tooth.
f.	Fenestra.
h.	Heart.
h.hy.	Hypo-hyal.
hm. (and *h.m.*)	Hyomandibular region.
i.vl.	Inferior velum.
l.f^{2,3}.	Lateral fenestræ.
l.l.	Lower lip.
l.mm.	Lingual muscles.
m.	Mouth.
m.h.c.	Mandibulo-hyoid-cartilage.
m.h.f.	(*m.hy.f.* and *h.m.f.*) Mandibulo-hyoid-fenestra.
m.hy.	Mandibulo-hyoid.
my.	Myelon.
na.	Nasal capsule.
n.bb.	Nasal barbels.
nc.	Notochord.
n.c.	Nasal cavity.
o.bb.	Oral barbels.
ol.	Olfactory folds.
pa.	Palatine cartilage.
pal.	Palatine mucous membrane.
p.br^{1,2}.	Pharyngo-branchials.
pg.	Pterygoid.
phx.	Pharynx.
p.i.tr.	Hinder intertrabecula.
p.n.c.	Posterior nasal canal.
pr.pa.	Pre-palatine cartilage.
p.tr.	Palato-trabecular.
q.	Quadrate region.
s.l.c.	Supra-lingual cartilage.
s.l.f.	Supra-lingual folds.
s.l.t.	Supra-lingual teeth.
sp.n.	Spinal nerve.
s.o.f.	Suborbital fenestra.
sy.	Symplectic region.
th.v.	Theca vertebralis.
u.l.	Upper lip.
vl.	Velum.

DESCRIPTION OF THE PLATES.

Plate.	Fig.		Number of times magnified.
8	1	Embryo of *Petromyzon planeri*, 7·8 millims. long; side view	33
,,	2	The same; part of upper view	33
,,	3	The same; part of lower view	33
,,	4	Larva of *Petromyzon fluviatilis*, 2 inches long (nearly); side view	5
,,	5	The same; part of upper view	5
,,	6	The same; part of lower view	5
,,	7	Adult *Myxine glutinosa*; side view	Natural size
,,	8	The same; part of upper view	Natural size
,,	9	The same; part of lower view	Natural size
,,	10	Adult *Petromyzon marinus*; lower view of head	Natural size
,,	11	Adult *Petromyzon fluviatilis*; lower view of head	2
,,	12	The same; side view of head	2
,,	13	The same; upper view of head	2
,,	14	Adult *Petromyzon planeri*; side view of head	2
,,	15	The same; lower view of head	2
,,	16	The same; upper view of head	2
9	1	Adult *Myxine glutinosa*; dissection of fore part; side view	2½
,,	2	The same dissection (part)	5
,,	3	The same; lower view	5
,,	4	The same; further back; lower view	5
10	1	Adult *Myxine glutinosa*; skull, upper view	5
,,	2	The same, with base exposed	5
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,,	4	Newly transformed *Petromyzon marinus* (4 inches long); skull, upper view	12½
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,,	6	The same; inferior labials; lower view	12½
,,	7	The same; framework of "inferior velum"	15

Plate.	Fig.		Number of times magnified.
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,,	8	Adult *Bdellostoma Forsteri* (2½ feet long); branchial region; lower view	⅔ natural size
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,,	7	The same; fore part of 1st pharyngo-branchial . . .	9
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Plate.	Fig.		Number of times magnified.
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X. *On the Skeleton of the Marsipobranch Fishes.*—Part II. *Petromyzon.*

By William Kitchen Parker, *F.R.S.*

Received January 10,—Read January 18, 1883.

[Plates 8, 10, 14, 15, and 18–26.]

In this Second Part the Marsipobranch type of skeleton will be described in several stages, as far as my materials go; afterwards, I hope to be able to show in a Third Part some of the organs in the act of transformation, the specimens for which I am now expecting.

In the present paper I shall describe :—

A. The skeleton after transformation.
  a. In the adult *Petromyzon fluviatilis.*
  b. In one-third grown *P. marinus.*
  c. In newly metamorphosed *P. marinus.*
  d. In very small (not quite transformed) young of *P. marinus.*
B. The early embryo of *P. planeri;* and
C. The larval form (or *Ammocœte*) of *P. fluviatilis,* ready for transformation.*

These stages give most of the facts we are looking for, but it will be a satisfaction to be able to explain every step in this remarkable metamorphosis.†

The friends to whom I am indebted for materials for the present paper are Professor Huxley, F.R.S., Francis Day, Esq., F.L.S., the late Frank Buckland, Esq., Professor Ray Lankester, F.R.S., the late Professor F. M. Balfour, F.R.S. (in conjunction with Osbert Salvin, Esq., F.R.S.), F. G. Penrose, Esq., and Dr. Albert Günther, F.R.S.

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* Since this paper was read I have received figures of the early embryo of *Callorhynchus* from my son, Prof. T. J. Parker, who has succeeded in finding an early stage of this important type on the shores of New Zealand. I am now satisfied that the Chimæroids are *not* nearly related to the Petromyzoids.

† The Bibliography has been given in the First Part.

A.—*The skeleton of the Lamprey after transformation.*

I shall at once take up the permanent form of the skeleton, that it may link itself on to that of the Myxinoid form described in the First Part.

a. Dissections of the young (one-third grown) of Petromyzon marinus.

The substance of which the skeleton of the Lamprey is composed is mainly the two kinds of true cartilage, soft and hard. The latter is the densest kind of hyaline cartilage seen in the Vertebrata; greenish, when looked at by reflected light, it is of a clear amber colour when seen in the mounted transparent sections; in the early embryo both kinds are similar, having very little intercellular substance. The *vacuolar* tissue is in the Lamprey confined to the notochord, the sheath of which, as in the Myxinoids, is a strong web of white fibrous tissue.

1. *The post-cephalic part of the axial skeleton.*

Here we have, after metamorphosis, roughly conical patches of hard cartilage (Plate 18, fig. 1; and Plate 22, figs. 7–9, *n.a.*) ascending the sides of the *theca vertebralis,* mounting above it, and having their bases attached to the side of the sheath of the notochord, which maintains its size and its cylindrical form between these paired rudiments, the beginnings of neural arches. This is almost the only part of the post-cephalic region in which anything but fibrous tissue is found as a skeletal structure, for the huge pharnyx is supplied with, and dominated by, a cranial nerve, right and left, the *vagus.* Yet cartilage is continued backwards, as the pharyngeal and pericardial skeleton, as far as the 13th pair of vertebral arches (Plate 18, fig. 1, *ex.br.*, *pcd.c.*, $v.a^{13}$.).

2. The *cranio-facial apparatus—cranium, proper, and ventral or intra-visceral outgrowths (rudimentary arches).*

This continuous growth of hard cartilage (mainly) has a much less antero-posterior extent than in the Myxinoids, only part of the hyoid, and none of the *intra-branchial* bars, being developed inside the temporary embryonic head-cavities and persistent aortic arches.

Nevertheless, such evident rudiments of visceral arches as can be found, albeit merely seen as basi-cranial "outgrowths" growing downwards, will be spoken of in familiar terms, the adult Myxinoids, larval Anura, and the larva of *Lepidosteus,* giving us some boldness in the determination of these ventral growths of the head.

For in all these types we have a more or less *continuous* cartilaginous growth of the cranio-facial skeleton; the facial parts being such as are developed inside the remnants of the pleuro-peritoneal space, which for a time remain as the "head-cavities."

The cranial part of the notochord (Plate 18, fig. 4, *nc.*) is twice as long as in the Myxinoids (Plates 9, 10, 16, and 17), for the invested apex can be seen projecting into the small sagittiform pituitary space ("basi-cranial fontanelle," *b.c.f.*), so as to be further forwards than the hind margin of the sub-ocular fenestra (*s.o.f.*). The apex turns upwards (fig. 5, *nc.*), and this part is invested by the hardened parachordals, which end as hard cartilage in front of the hind face of the auditory capsules (fig. 4, *iv.*, *au.*), but send backwards, below, paired spurs up to the point where, in a Tadpole, the occipital condyles would be formed. The cranial notochord becomes invested by cartilage equally above and below (Plate 22, figs. 2–6), and thence, outside, the cartilage thickens and extends right and left, until, reaching the auditory capsules, it, for a considerable extent, underfloors them. Here, however, there is no facial bar growing out from their outer sub-auditory edge as in the Myxinoids (Plates 9, 10, 16, and 17, *au.*, *iv.*, *hm.*). Where the investing mass (parachordals) passes into the trabeculæ (*iv.*, *tr.*), there the basi-cranial cartilage does extend itself into a facial outgrowth—the pedicle (*pd.*).

In front of these outgrowths the basis cranii has an elegant, narrow waist (fig. 4), bounded by the narrow hind part of the trabeculæ (*tr.*), which have a membranous tract right and left of them, and also between them (*s.o.f.*, *b.c.f.*). The middle or "pituitary" space has opened to let out the posterior nasal canal (Plate 23, fig. 1, *p.n.c.*), which passes, widening towards its enlarged *blind* end, under the cranial notochord.

From this point the trabeculæ diverge, and the space left between them is filled in by the hind intertrabecula (Plate 18, fig. 4, and Plate 10, fig. 5, *p.i.tr.*), which is everywhere confluent with them, except where vessels pass through.

The proper basis cranii is finished in front by the confluence of these three bars, and this solid ethmoidal region has a convex outline in the middle, which outline is made sinuous by a retreat of the edge, right and left, where the ethmo-palatine bars have grown on to the trabeculæ (*ep.a.*, *tr.*). This whole floor is concave below, although its middle plate was convex at first, for the pterygo-palatines, like "flying buttresses," drop down, rather suddenly, and thus form an arched, or roof-like structure. A narrow sinuous band of soft cartilage runs along the front of the skull (ethmoidal region, *eth.*), and then hard cartilage breaks out again, in the form of a large shield, which is hollow below, sinuously convex above, notched in front, and at the middle as wide as the skull with its flying buttresses. No one familiar with the skull of Tadpoles can for a moment hesitate to call this the common "trabecular cornu." In the huge Tadpole's skull (*Pseudis*, my first stage, Phil. Trans., 1881, Part I., Plate 2), the cornua are united in their hinder half, but in an older Tadpole (Plate 11, figs. 1–3) they are only separated by a notch in front, sharper, but no larger, than the emargination on the fore-edge of this so-called "posterior dorsal cartilage" of the Lamprey.

The *side* walls of the chondrocranium of the Lamprey are well developed (Plate 18, figs. 1–5), but, as in Tadpoles, and adult Anura also, the optic and trigeminal nerves (II., V.), pass out of considerable *fenestræ,* and not out of mere foramina. The orbito-sphenoidal region (fig. 5, *o.s.*) is wider than the alisphenoidal (*al.s.*), but the latter mounts up into the roof, and the two sides meet round the middle, and fore part of hind, brain (Plate 18, figs. 3, 5, *t.cr.*) The occipital *ring* does not exist, and the hard part of the basi-occipital, as we have seen, is abortively developed.

In front of the posterior sphenoidal "tegmen" there is a large pyriform fontanelle, the broad fore part of which is largely covered by the hinder part of the *double,* mammilliform nasal capsule—(*na.*) an independent structure, as in the Tadpole, larval Urodele, and Elasmobranch; this is composed of soft cartilage.

The facial outgrowths of the Lamprey's skull are confusing, because of their extreme simplicity, and because of the absence in them of the normal segmentation from the basis cranii. Yet the Lamprey is not alone in this peculiar and generalised state of the cranio-facial cartilage; it occurs in so many other Ichthyopsida, especially in the more *archaic* types, that I am strongly induced to look upon it as the retention of an *ancient* condition of the skull of the *Craniata.*

Except in the basal (ventral) region, there is no segmentation, and there the parts are so unlike those of other Fishes that some of these *basi-visceral* cartilages are of doubtful morphology; the question being whether they belong to the *deep* or *superficial* category.

Of parts formed *inside* the "head-cavities" there are only representative *regions* of the 1st and 2nd post-oral arches, and it is evident that the 1st or *maxillo-mandibular* is abortively developed below, and the hyoid abortively developed above; the basal part of the latter, as well as its sides, are clear, but the mandibular region is very doubtfully represented at all.

The only tract that can be said to belong to the proper branchial region (of "intra-branchials") is the hind part of the great lingual cartilage, behind the setting on of the lateral "cornua" of the hyoid arch.

In coming to details, I shall have to refer freely to other work of mine, in other Ichthyopsida, and also to the views written, or expressed to me in discussion, by my esteemed fellow-workers; *from one of the chief of these I can get no more counsel or help.*[*]

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[*] I have not felt myself able to stir a step without the help of Professor HUXLEY's admirable paper already referred to, on the nature of the "Cranio-facial Apparatus of *Petromyzon*" (Jour. of Anat. and Phys., vol. 10, plates 17, 18, pp. 412–429). I say this the more freely because I feel satisfied that the author of that paper will give me credit for having thrown some useful additional light upon the subject, thus necessitating a considerable divergence of opinion between us as to the meaning of some of the parts. The manner in which the late Professor BALFOUR always demanded incontrovertible proof of any view I might be holding at the time of discussion has served to make me work with extreme caution. With regard to one difficulty mentioned by Professor HUXLEY in the paper just referred to (p. 423), as to the distribution of the branches of the 5th nerve, I may mention that in two or three years after its publication

The subocular bar should be compared with that of a Tadpole during transformation, when the hinge of the mandible is below the emerging trigeminal nerve (see Phil. Trans., 1881, Plate 4, figs. 8, 9,) that is when the quadrate condyle has left the front of the head and has retreated to the postorbital region. In the adult Lamprey we have the pedicle (*pd.*) growing forwards as the pterygoid cartilage (*pg.*); these two regions form the hinder, and half the lower, boundary of the subocular fenestra (*s.o.f.*); the rest of the lower boundary is the postpalatine region (*pt.pa.*), and the front boundary the ethmo-palatine (*e.pa.*), from which grows the prepalatine spur (*pr.pa.*).

But there is no quadrate condyle, that is completely suppressed; in the Myxinoids (Plate 9, fig. 2; and Plate 16, fig. 1, *q.*) there is a rudimentary quadrate tract, but without any condyle; in these it is still further back than it would be, if developed, in the Lamprey. The slight elbowing of the subocular bar under the hind margin of the fenestra is the only sign of it in this Fish (Plate 18, fig. 1). That which, for some time, kept me from seeing this matter fairly was a comparison of the Lamprey's skull with that of a Tadpole before transformation; I then mistook the prepalatine spike for a rudimentary quadrate tract. Now if these parts in the Lamprey and the Myxinoids be compared with those of the transforming Tadpoles of the species of *Rana* (*op. cit.*, Plate 4), we shall see that it is the subservience of the free mandible (Meckelian rods) in the Tadpole to the suctorial function—they are mere carriers or supporters of the incomplete "annulus"—that makes the necessity for the forward position of the condyle of the quadrate. Indeed, nothing in morphology is more marvellous than the behaviour of those condyles, which, lying at first in the front of the head, gradually swing themselves round, and, in the Bull-frogs, come to project some distance behind it (*op. cit.*, Plate 8).

Here, in the Lamprey, the perfect *annulus*, or suctorial disk, is functionally free from cranio-facial trammels, and, being so huge, causes the once short, backwardly-placed *lower lip* to project in front of all the other structure; in the Tadpole (see in the huge *Pseudis, op. cit.*, Plate 1) it is almost as forward as in the Lamprey; but in the other kinds it is more under the face.

The forward position of the large quadrate condyles in the Tadpoles of the "Anura Phaneroglossa" masks the prepalatines for a time, turning them *inwards*, as a boundary to the internal nostrils. (See the various figures in my 3rd Memoir "On the Batrachian Skull.")

In *Dactylethra* ("Batrachian Skull," Part II., Plates 56, 57,) the condyle of the quadrate, although well in front of the head, is a small, sessile, selliform condyle on the edge of the enormous suspensorium, and the prepalatine is confluent with the cornu trabeculæ.

But in the larva of the other *Aglossal* Batrachian (*Pipa*, same memoir, Plates 60,

(namely, Nov., 1880), he himself worked out this subject afresh, and found that there was no difficulty, as a true 2*nd branch* of the trigeminal *does pass over* the subocular arch. My *later* dissections confirm this important discovery.

61,) that spike (*pr.pa.*) is free, and thus, from the first, has the directly forward position of its counterpart in the Marsipobranchs. Moreover, in the young of *Lepidosteus* (Phil. Trans., 1882, Part II., Plates 30–38, *p.pg.*) we have a prepalatine quite like that of the Myxinoids, and in this case it is the *fore-end* of the subocular bar which is primarily continuous with the basis cranii, but *afterwards* free, whereas it is the *hind part* in the embryo of the Lamprey (Plate 25, fig. 7, *pd.*, *pg.*, *tr.*)*

If we thus determine† that the Lamprey has a subocular arch like that of a transforming Tadpole, *minus* the quadrate region and condyle, we have still the question as to whether the distal (*cornual*) part of the first arch has any existence.

In the first place, it is to me a great satisfaction to find Professor HUXLEY (*op. cit.*, p. 421) saying of the "annular cartilage" that—"although its halves are united dorsally" [it] "would seem to be essentially a post-oral structure;" it certainly is, if it be homologous with the *divided* imperfect "annulus" of the Tadpole.

Now, there is a T-shaped median cartilage, whose transverse part lies directly behind the great labial ring (Plate 10, fig. 6; and Plate 18, figs. 1, 2). After determining (to *my* heart's content) the nature of the "styliform," "cornual," and lingual cartilages, as the hyoid arch, lateral and basal, Professor HUXLEY asks—"What is the *median ventral cartilage* which MÜLLER regarded as the body of the hyoid? The hyoidean arch is complete without it, and has no special connexion with it, the bent up anterior end of the lingual cartilage simply playing over it. I conceive it to be a median ventral element of the mandibular arch, notwithstanding that, in the higher Vertebrates, such an element, though the analogy of the other arches would lead us to expect its presence, is not known to occur. The third division of the trigeminal nerve passes over the expanded anterior end of this cartilage, traverses the ventral half of the annular cartilage, and runs along the anterior edge of the latter to its dorsal extremity" (p. 421). But if we look again at these cartilages in the transforming young of the great Lamprey (Plate 10, fig. 6), we shall see that the paired styloid cartilages of the lower lip are attached by their broader inner ends to the annular cartilage and are curved at their free and outer ends, so as to resemble very closely the form and relations of the small mandibles of the Tadpole (see my 3rd Paper "On the Skull of the Batrachia," Plates 2, 3, 4, 11, 15, 17, 22, 30, and 38).

Now, it occurred to me some time ago that these were the true Meckelian rods, but the late Professor BALFOUR assured me that I had no proof sufficient to sustain this view.

Then as to the median T-shaped cartilage; such a thing as a median mandibular element, well formed, but not segmented off from the Meckelian rods, does exist in "the higher Vertebrates." I have already shown it in the Green Turtle and the

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* The pharyngo-hyal element or hyomandibular is developed continuously with the skull in *Lepidosteus*, thus throwing light on the second arch of the Myxinoids.

† The difficulty with regard to the relation of the branches of the trigeminal nerve to the subocular bar and fenestra has been dealt with in the First Part (pp. 401–403).

Crocodile (see "Challenger Reports," vol. i., part 5, plate 3, fig. 6, *mk.*; plate 9, fig. 5, *mk.*; plate 10, fig. 6, *mk.*; and plate 11, fig. 2, *mk.*; also Trans. Zool. Soc., 1882, plates 63, figs. 7, 8; 65, figs. 3, 8; 66, fig. 5).

Moreover, I have recently made a large number of preparations of the visceral arches of embryo Mammals—Edentata, Insectivora, Rodentia, &c.,—and in these I find a still more developed basal rudiment to the primary mandibular arch, namely, a large, well-formed terete rod, lying on the symphysis of the "rami," and well marked as a true "basi-mandibular."

Now, considering the morphological feats performed by these Marsipobranchs, notably the two *distinct* "intertrabeculæ" of the Myxinoids, and the marvellous development and sub-division of the basi-hyal, I think it is not a great extravagance on the part of Professor HUXLEY and myself to ask them to show one "sign" more, and give us *three distinct*, rudimentary, distal mandibular elements.

I shall simply call them median and lateral distal mandibular cartilages (*m.d.m.*, *l.d.m.*), and leave their *deep* or *superficial* nature an open question.

Like the annulus they have no existence in the Ammocœte, but this is true, also, of the fore half of the subocular arch, and of all the hyoid arch.

It is evident that the Petromyzine type of skull is a great and important modification (metamorphosis) of the Myxinoid, for the larval Lamprey represents *Myxine*, just as the larval frog represents the Lamprey.

But the enormous development of the lower lip in the Lamprey has affected the jaws, half aborting them, and the hyoid arch, also, in its upper part, and causing its basal part to undergo extreme hypertrophy. The hyoid arch presents fewer difficulties than the maxillo-mandibular, but there are some remarkable modifications of this region, which are, however, greatly elucidated by what we see in the "Anura." It has no "pharyngo-hyal" element; nor has the Tadpole, at first; its distal part, which becomes the "columella auris," does not appear in our Common Frog and Toad until *two* or *three* months after metamorphosis (see my 2nd Paper "On the Batrachian Skull," pp. 622–624)—a fact discovered by Professor HUXLEY. But the *position* of the "epi-hyal" region of the hyoid arch, and its continuity with the back of the pedicle and pterygoid, is only explicable by reference to transforming and transformed "Anura."

The large broad "epi-cerato hyal" of the Tadpole is articulated to a *special* facet by a *special* condyle below the ethmoid in the antorbital region, and it is only after the swinging back, so to speak, of the quadrate region that the *epi-hyal point* is carried under the exit of the facial (*its own*) nerve (see in *Pseudis*, "Batrachian Skull," Part III., Plates 11, 12); this position is a good landmark, and shows that the quadrate of the Lamprey *should have been* post-orbital.

The epi-cerato hyal (Plate 18, fig. 5, *hy.*, *c.hy.*; and Plate 19, figs. 1–3, *e.hy.*) grows out from the back of the pedicle and pterygoid (*pd.*, *pg.*) at a right angle; a projection, the beginning of this part, is seen in the *Ammocœte* (Plate 19, figs. 4, 5, *pd.*). In the adult stage, in some Tree Frogs (*Acris Pickeringii* and *Phyllomedusa bicolor*) (see my 3rd Paper, Plate 30, fig. 2; and Plate 34, fig. 8), the epi-hyal end of the long hyoid "tape" when it has got under the emerging 7th nerve, does not, as usual, unite with the skull, but with the back of the pedicle, behind the part where the pterygoid is given off in front.

Here, as a *secondary* development, we get exactly what we have from the first in the Lamprey; and I could give numerous instances of cases of this kind; homologous parts being *continuous* with some neighbouring cartilage in one type, and becoming *confluent* in another.

The rather thick styliform (epi-cerato hyal) bar of the Lamprey curves gently upwards, as it passes downwards and backwards; it ends below in the *cornual part* or cerato-hyal (Plate 18, figs. 1 2, 5, 6, 7, *c.hy.*). This part is not segmented from the epi-hyal region; but it is quite distinct from the basal bar (*b.hy.*).

The latter bar is strongly clamped by these two horizontal growths, each of which is roughly hatchet-shaped, the blade being turned outwards from the hind part, and the handle growing forwards as a strong styloid process. The thickish blades are convex below, concave above, and a notch is seen between them and their handles; the epi-hyal grows into the top of the outer angle of the blade, which latter part is obliquely perforated in front of its middle.

The facial nerve (Plate 18, figs. 1, 5, VII.) emerges behind the pedicle (*pd.*), and then passes to the inside of the forking cartilage.

The median hyoid bar passes from the circular opening of the mouth to a point half-way between the 1st and 2nd branchial clefts (fig. 1, *b.hy.*); when dissected out it presents the appearance shown in figs. 6–8,—lower, upper, and side views. This rod is not straight, but bent sinuously from side to side, and is upturned both before and behind; it is oval in section, and has an irregular, half soft, keel in the hinder part, where it is clamped by the cerato-hyals. Narrowing into a neck towards the front part, it enlarges at the end into a head, the hind part of which is soft cartilage, and the fore part hard. That hard part is in three pieces, separated by soft synchondrosial tracts. The side pieces form a short trough, and are uncinate blades, united by the convexo-concave lower piece. These parts, which look like the mandibles of a Beetle, have a pair of supra-lingual cartilages over them (Plate 14, fig. 10); and also between the cerato-hyals there is a curious framework to the "lower velum" (Plate 18, fig. 7 *i.v.s.*); these parts I shall describe soon.

3. *On the extra-branchial basket-work.*

In the Myxinoids the only trace of *extra-visceral* cartilage found was in the "ductus œsophago-cutaneus" of *Bdellostoma* (Plate 16, fig. 7); *Petromyzon*, on the contrary, is remarkable in having the largest and most perfect framework of this kind known.

In Tadpoles, behind the three branchial openings, and in front of the first, there are "extra-branchials;" the first and last are pouches, like the bowl of a spoon, the second and third are broad bands. But these have inside them, and growing from large hypo-branchials, four pairs of short and slender cerato-branchials, *inside* the arched branches of the main branchial artery ("Batrachia," Part III., *var. loc.*). Also in Sharks (not in Skates), outside the rays of each branchial septum there is a long band of cartilage, pointed above and pedate below; there are four pairs of these, which are of the same nature as the pouches of the Tadpole and the basket-work of the Lamprey (see Trans. Zool. Soc., vol. x., plate 38, figs. 1, 2, *e.vs.*). In the Shark the typically-jointed intra-branchial arches, with their "rays," dominate; in the Tadpole the extra-branchials; in the Lamprey these latter, alone, exist. The huge development of the seven pairs of branchial pouches in the Lamprey, with their openings—*2nd* to *8th clefts*—carries the fore part of the first pouch away, backwards, from the hind margin of the hyoid; but this is a very slight displacement compared to what we saw (Part I.) in the Myxinoids.

Each main extra-branchial bar is seen to be *behind* the corresponding cleft or aperture (figs. 1, 2, *e.b.a.*$^{1-7}$, *ex.br.*$^{1-7}$); but in the adult, there is an additional bar, in front, belonging to the hyoid region, and another behind, enclosing the heart and its bag (*ex.hy.*, *pcd.c.*)* These *nine* pairs of main bars are all united below (fig. 2) into one complex cartilaginous "crate." The cartilage is of the soft kind. Above, on each side, the seven main bars run into each other, being united by a continuous growth, attached by fibrous tissue to the sheath of the notochord. Opposite the apertures each bar bends inwards, and, above and below, the inbent part gives off a crooked bar, both over and under the apertures. These cross bars join the next in front at a considerable distance from the aperture, each being strongly elbowed, and each giving off a snag, the one upwards and the other downwards. But the *cervicorn* character of these united bars is increased still further by the development of two snags in front and one behind, on each main bar, both above and below the cross bars. Then, both above and below, a common headland of cartilage unites the whole together. Nor is this all, for the lower marginal band (Plate 18, fig. 2), uniting the main bars below, bends towards the corresponding part of the other side, and coalesces with it at its convexity. This being done, again and again, there is left a row of small oval fenestræ between the junctions—*six* in all. But the hyoid part of the basket-work, the "extra-hyal" (*ex.hy.*) did not exist till after the metamorphosis (see Plate 25, fig. 8, basket work of *Ammocœte*) and the cross bars of the 1st extra-branchial (*ex.-br*^{1}.) form a loop independently of the extra-hyal. This loop, however, sends down, now, an elbowed band, which runs into the extra-hyal below its middle. The extra-hyal helps to form the grooved and perforated *quasi-sternum* on the lower face of this great respiratory pharynx (fig. 2, *ex.hy.*) and above its junction with the 1st extra-branchial (*ex.br*^{1}.) runs upwards and forwards, and is united with its own arch (epi-cerato hyal, *e.hy.*, *c.hy.*) over its junction

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* The Tadpole has no gill belonging to either the first or the second arch; therefore its three gill-slits answer to the 2nd, 3rd, and 4th gill openings of the Lamprey, whose 1st gill opening is the 2nd cleft.

with the pterygo-palatine (fig. 1). Just at its shoulder, below the emerging facial nerve (VII.), the extra-hyal sends upwards a free snag. The last extra-branchial (*ex.br*^{7}.) sends off no cervicorn processes, except from the transverse bars. It is shorter also than the rest. Behind, it joins the *extra-pericardial* (*pcd.c.*) by *three* bands, the lower being the broader (fig. 2), and each of the *two* fenestræ thus formed is made into a U-shaped space by a snag from the extra-pericardial, which also sends forwards, above them, a larger free dentate snag; the rest of the pouch is thin, roundly notched, and ends above in another similar process. Below (fig. 2), instead of a small oval fenestra, the 7th extra-branchials help the double extra-pericardial to form a large fenestra, which becomes elegantly trilobate through the growth of a right and left snag from behind. From that fenestra, up to the pair of upper processes, right and left, the two extra-pericardials are completely united together, and form a *bowl*, whose cavity is pitched forwards and a little upwards. I trust that the reader, with this description of the figures here given, will see what the writer sees, and that, seeing, he will agree with him that nothing more remarkable, and nothing more exquisite, has been revealed by the labour of Anatomists.

4. *Subsidiary cartilages in the mouth and between the mouth and pharynx.*

a. The supra-lingual cartilages.

Over and between the curious *quasi-mandibles* that grow upwards from the head of the basi-hyal (Plate 18, figs. 6–8, *b.hy.*) there is a thick pyriform cushion, right and left, with its base looking backwards. Between these two cushions, in the margin of an elliptical *supra-lingual* valley, there is a pair of rows of small, yellow, pointed, lanceolate teeth, looking inwards and backwards, a dozen or more in each row (Plate 14, fig. 9). There is a groove along the mid-line, between them, and behind this the dorsum of the tongue has crescentic cross ridges. Across the arched, broad end of the tongue there is a row of *cyclodont*, beaded, horny teeth, fifteen in all, lessening outwards, and with the middle tooth much the larger. These show themselves inside the *annulus*, on the floor of the small oral opening (Plate 8, figs. 10, 11), and are a very important part of this suctorial apparatus. But these terminal teeth of the tongue, and the radiating teeth of the annular disk, have no counterparts in the Myxinoids (Plate 8, figs. 7–9). The two small rows of sharp, supra-lingual teeth have, however, very notable counterparts, in the double rows of large teeth, in that lower group (see Plate 12, fig. 7; and Plate 16, fig. 4). The large complex supra-lingual cartilage of the Myxinoids is represented in the Lamprey by a small cartilaginous plate (Plate 14, fig. 10, *s.l.c.*), which is exposed by section in the cushion above the ascending lobes of the great basi-hyal (*b.hy.*).

b. The skeleton of the fimbriated valve between the mouth and pharynx.

This peculiar structure, formed during metamorphosis, is figured, as seen in a vertical section of the adult *P. fluviatilis* (Plate 23, fig. 1); from above, *in situ*, in a head cut through horizontally (Plate 23, fig. 2); and as removed from most of its surroundings and seen from above (Plate 14, fig. 8).

The skeleton of this valve is shown in the upper view of the basi-hyal (Plate 18, fig. 7, and in Plate 10, fig. 7), and sections of it will be described anon. This part is a fringed median projection, uniting the right and left *vela* (*vl.*), parts that are found in the newly-hatched embryo, dividing the "stomodæum" from the "archenteron" (Balfour, 'Comp. Emb.,' vol. ii., p. 74, fig. 42, *v.*). I have given a diagrammatic figure of the velum in the *Ammocœte* (Plate 25, fig. 10, *vl.*). For a description of these parts the reader is referred to Professor Huxley's paper ("Cranio-facial Apparatus of *Petromyzon*," p. 420). This curious little frame is composed mainly of two *crura* of hard cartilage, united in front by a band of soft cartilage, which sends out three pairs of filaments, and an odd one in the mid-line. These are all sinuous or *ƒ*-shaped; the two outer threads pass across and are attached to the cerato-hyals (*c.hy.*), in front of their junction with the epi-hyals (*e.hy.*); these latter are seen as cut across in the figure.

The inner of the paired threads is the longest, right and left; the others are nearly of the same length.

The hard crura are bent out, like *knees*, and these are pedate behind, with a *heel* of soft cartilage.

5. *The labial cartilages.*

The upper lip, which was a *hood* in the embryo and larva (Plate 8, figs. 1–6), overlapping and far outreaching the short, transverse lower lip, has grown very much, but not in the same degree as the lower lip (Plate 8, figs. 10–16; Plate 18, figs. 1, 2, 3; and Plate 23, fig. 1).

The upper lip has acquired a skeleton, such as we find in all the Tadpoles of the "Phaneroglossal Anura." There are five *upper labials* in this type, and the median is a shield-shaped piece, hollow below, convex above, and overlapped by the cornu trabeculæ (*u.l*^{1}., *c.tr.*); these two parts are the so-called "anterior" and "posterior dorsal cartilages." Under the lower edge of the great median upper labial, right and left, there is a small, bent, bluntish style, the "antero-lateral upper labial" (*u.l*^{2}.). Behind the large piece, right and left, there is a reniform plate of cartilage, much larger than the styloid piece, which, lying obliquely, helps to fill up the space between the two dorsal cartilages and the prepalatine; this is the "postero-lateral upper labial" (*u.l*^{3}.).

In the larva of the "Anura" (see Phil. Trans., 1881, Part I., *var. loc.*) these cartilages never *quite* agree with those of the Lamprey, although they are manifestly homologous. There may be *one*, *two*, or *four*, and they are evanescent in those types, becoming absorbed during metamorphosis, whilst *two new pairs* appear, homologous with the "adrostral cartilages" and "nasal valves" of the *Elasmobranchs.*

I have already described the cartilages that are attached to the *great ring,* doubting whether they belong to the *superficial* or the *deep* category.

There remains, now, the great *annular* cartilage to be described (Plate 18, figs. 1–2, $l.l^1$.), and the lesser pieces in the disk ($l.l^{2-4}$.). There can be no difficulty in identifying this huge, single, complete ring with the lesser, double, incomplete ring of the Tadpole, nor in determining that they are both distal superficial cartilages of the post-oral or mandibular region. I have traced the development and decadence of this suctorial structure through all its stages, *up* and *down,* in the Anura ("Skull of Batrachia," Parts I.–III.), but in this type I find no traces of it in the largest *Ammocœte* of *P. planeri* and *P. fluviatilis.* But in a very minute young of *P. marinus,* 4 inches long, (Plate 10, fig. 6, $l.l^1$.)* with the cranial elements still distinct, the chondrification is equally perfect all round, and the thickness of the ring equal. At that stage, if it had been composed of two pieces, even for *a day or two,* some signs of division would have been apparent. The actual form in the larger young (one-third grown) of the same species (Plate 18, figs. 1, 2, $l.l.^1$) is a semi-ellipse, with rounded corners at the truncated part; the narrow end is in front.

It has a thick lower edge (fig. 2) and thence is scooped upwards; the fore part (fig. 1) is much higher than the hinder; the narrow end projects much less in front of the large upper labial than *it* does beyond the cornu trabeculæ (Plate 18, figs. 1, 3, $l.l^1$., $u.l^1$., *c.tr.*). In the side view (Plate 18, fig. 1) the larger teeth, round the opening of the circular mouth, are shown *in situ;* in the dissected skull in the under view (fig. 2), the head of the tongue (lingual or basi-hyal cartilage) is seen with its teeth in the hinder part of the opening. Also in the undissected head, when seen from below (Plate 8, fig. 10), the same part is seen as a squarish mass, largely occluding the aperture; the teeth of the disk are strong and crowded behind the aperture.

In the thick edge of the disk, right and left, there are three small, hard cartilages of an irregularly oval shape (Plate 18, fig. 1, $l.l^{2-4}$.).

6. *The sense-capsules.*

The eye-balls, which are very small and obscure in the *Ammocœte* (Plate 8, figs. 4, 5) are large in the adult (figs. 11–13); in the middle species (*P. fluviatilis*) the relative size of this organ will be shown in the sections. Unlike the eye-ball of Fishes, generally, the *sclerotic,* even in *P. fluviatilis* where the eye-ball is relatively largest, is not cartilaginous but fibrous, merely, as in the Mammalia.

The auditory capsules (Plate 18, figs. 1, 3, 4, *au.*) are very similar to those of the Myxinoids (Plates 9, 10, 16, and 17), being strong, hard, oval capsules, retaining the simple form found in the newly-hatched embryo of the Anura. For in these types the

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* In that figure the distal mandibulars are lettered $l.l^2$., $l.l^3$.

anterior and posterior semicircular canals are short, wide tubes, and the horizontal canal is arrested. The meatus auditorus internus (Plate 18, fig. 5, VIII.) is scarcely larger than the optic fenestra (II.), so that, for a Fish, the capsule is very perfect inside; the space for the entrance of the auditory nerve is much more vertical than in the Myxinoids (Plates 10 and 17). The long axis of these elliptical boxes is accurately parallel with the axis of the skull; above, they stand out well from the skull, but below we see a bridge of cartilage, derived from the parachordal (*iv.*), thrown across the interspace between the capsule and the cranial floor. This scooped thin edge of the parachordal, which has become confluent with the capsule, is very distinct, for a time, in Tadpoles ("Batrachian Skull," Part III., *var. loc.*); it is continuous with the pedicle (*pd.*) in front, and in the Myxinoids (Plates 9, 10, 16, and 17) with the uppermost part of the hyoid arch.

That the *nasal capsule* (Plate 18, figs. 1, 3, 5, *na.*) is essentially double, like the trabecular cornu (*c.tr.*) is evident; it is a very exact likeness of the dilated crop of a Pigeon; its halves are a little less than the auditory capsules; they are composed of *soft* cartilage. Behind the aperture (*e.n.*) there is an emargination, followed by a widish, shallow groove; behind, the median cartilage is somewhat produced as a lobe. The cartilage runs three-fourths of the way round (Plate 21, figs. 3–5) up to the posterior nasal canal, and only forms a perfect floor behind; the aperture (*e.n.*) is membranous. These things will be best explained by the transverse sections (Plate 21). The roof is pierced by vessels, near the middle, at its hinder third (Plate 18, fig. 3, *na.*); in that figure the olfactory nerves (I.) must be supposed to be seen *through* the fontanelle (*fo.*).

Cranio-facial skeleton of newly-metamorphosed young of Petromyzon marinus. (*5 inches long*).

These were about half the length of the specimen whose skeleton has just been described; and, in development, an intermediate one between that stage and the one next to be described.

Compared with the larger specimens, we see that the cranial notochord is longer, being half as long as the cranium proper, so that the *para-* and *pro-*chordal regions are equal in length. The notochord (Plate 19, figs. 1–3, *nc.*) projects somewhat into the oval pituitary space (*b.c.f.*), which is twice as large as in the larger specimens. The inner edge of the parachordals is soft both above and below; the two are united for a short space below. The outer edges, confluent with the auditory capsules (*au.*) are of less extent, there being a considerable notch fore and aft under the outgoing trigeminal, facial, and vagus nerves (V., VII., X.). The hard cartilage is continued back, as far as the hinder outline of the auditory capsules, and is then replaced by narrowed bands of the soft kind, further than in the last stage. Behind, the *pro-*chordals (*tr.*) are narrower than the *para-*chordals (*iv.*), but they broaden out forwards; their larval form is traceable in the upper view (compare Plate 19, figs.

1 and 5, *tr.*), for the basi-cranial fontanelle (*b.c.f.*) is bounded by a thick cartilage, which thins out in the orbito-sphenoidal and ethmoidal regions (*eth.*, *o.s.*)—*post-transformation* structures. The floor in front of the basi-cranial fontanelle is due to the hind intertrabecula (*p.i.tr.*). A large round notch separates the ethmoidal and orbito-sphenoidal regions; the ethmoidal plate in front of this notch and of the thick-ribbed primary trabeculæ, is a two-winged structure, pointed in the middle, gently concave right and left, convex above, and concave below. It thickens beneath at its fore margin. There is a curious rudiment of the wall and roof, right and left; the base of the orbito-sphenoidal wall (*o.s.*) is only half the breadth of the *optic fenestra* (fig. 3, II.). The top of the wall is spiked in front, and then notched and sinuous; it bends inwards over the optic fenestra, and is thick above. The appearance, from above (Plate 19, fig. 1) of these partial walls, is like that of the pterygo-palatines (*pg.*, *pa.*) outside, but much smaller. The optic fenestra is kidney-shaped, the *hilus* being above, and the nerve (II.) passes out at the hinder third. The base of the alisphenoidal band (fig. 3, *al.s.*) is only half as large as the orbito-sphenoidal (*o.s.*), and a large uncinate foramen for the 5th and 7th nerve (V., VII.) intervenes between that band and the auditory capsule. Then the cartilage mounts over the skull cavity beyond the middle of the auditory capsules, forming a leafy blade, that grows, by its fore corner, towards its fellow of the other side, but does not meet it by a considerable space. The hind corner lies on the capsule, furthest from the mid-line, and a fenestra is seen between the *foramen ovale* (fig. 3, V.), the ear capsule, and the hind margin of the rudimentary "tegmen" (*t.cr.*).

The front outgrowth of the cranium—the great leafy hollow cornu trabeculæ (*c.tr.*)—is more than two-thirds the size of the rest of the cranio-facial structure; it is emarginate at both ends, and at its hinder part it is joined on to the ethmoid (*eth.*) by a very definite tract of soft cartilage; at that part it is slightly grooved, above.

At the junction of the *pro-* and *para*-chordals (*tr.*, *iv.*), and also at the outer margin of the ethmoid, a stout band of cartilage passes out; these are united outside, as a sort of "flying buttress," which sends forwards a free snag, and downwards, at a somewhat greater distance from the skull, a band of cartilage, equal to the size of the root; this is drawn as cut across at a little distance from its origin. The hind root is the pedicle (*pd.*); the front root the ethmo-palatine (*e pa.*); the spur is the pre-palatine (*pr.pa.*); the bar cut across is the epi-hyal (*e.hy*), with no distinct inter-hyal tract; and the main outer wing is the post-palatine (*pt.pa.*), passing into the pterygoid (*pg.*). There is the merest convexity at the junction of the two last-mentioned regions, where the *quadrate region* and *condyle* should be.

I have not figured the nasal capsule in this stage; the auditory (*au.*) is large, sub-reniform, and has a large pyriform *meatus internus* (fig. 3, VIII.), with the fundus below.

Cranio-facial skeleton of nearly metamorphosed young of Petromyzon marinus. (*4 inches long.*)

This stage is still more instructive than the last, as the metamorphic state was not over,—the *Petromyzine* skull was still in formation, and was caught in what may be called a *Myxinoid stage* (Plate 10, figs. 4, 5). Yet the suctorial (labial) cartilages were perfect (fig. 6); these have already been described, and the skeleton of the *intervelar shelf* (fig. 7), which was almost as much developed as that of the larger individuals of the same species (see Plate 18, fig. 7).

If the reader will compare this minute chondrocranium with that of *Myxine* (Plate 10, figs. 1–3), with the *larva* (Plate 19, figs. 4, 5), and with that of the transforming Tadpole of the East Indian Bull Frog (Phil. Trans., 1881, Part I, Plate 4, figs. 8, 9), he will not fail to see its true morphological signification—both *those* skulls are twice the size of *this*, yet this belongs to the largest species of Lamprey.*

Another skull which comes near to, and throws light upon this, is that of the *transforming* Tadpole of *Pseudis* ("Batrachian Skull," Part III., Plates 11, 12). Here the primary basal bands—*para-* and *pro-*chordal (Plate 10, figs. 4, 5, *iv.*, *tr.*), have their *larval* outline (see Plate 19, figs. 4, 5) perfectly distinguishable throughout. The chondrification creeps backwards, as in the early Tadpole's skull, for here the hard cartilage is seen to end some distance in front of the hind margin of the ear-capsules (*au.*); the soft cartilage runs back along the spinal notochord for some distance. The sub-auditory wings of the parachordals are less than in the last stage and are very thin; the space under the gasserian ganglion between the pedicle (*pd.*) and the capsule (*au.*) is much larger and more open than in the larger young (Plate 19, figs. 1–3). The basi-cranial fontanelle (*b.c.f.*) is now pointed in front—*sagittate;* and narrower than it will be in a few weeks.

The trabeculæ (*tr.*) in front of the pedicle (*pd.*) become half the size of the parachordals, and then widen out, both in thickness and distance, to form the *Ammocætine loop.* The depth of the floor is made by the thickness of the trabeculæ and its own concavity; and the bands are oval in section, the long axis of the oval looking upwards and inwards (see Plate 26, figs. 7–9, *tr.*). The interspace of the *loop* is accurately fitted with a cartilage, like the bowl of a spoon, but with two points, behind, instead of a handle; this bifurcation gives the lanceolate form to the basi-cranial fontanelle (*b.c.f.*).

This thin shell-like median element is the "posterior intertrabecula" (*p.i.tr.*); although only transitorily distinct, and composed of hard cartilage, it is the true counterpart of the very distinct ladle-shaped, soft cartilage, seen in the adult Myxinoids (same Plate, figs. 2, 3, *p.i.tr.*). This element is more distinct, as a rule, in the

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* I shall describe the larval skull last (Plate 19, figs. 4 and 5); yet that simple platform of a chondrocranium must be kept in sight, if the more advanced condition is to be understood. I have thought it better to take the metamorphosed skull first, as Anatomists are most familiar with it.

"Amniota" than in the "Anamniota;" especially, as I have noticed, in the Turtle and Crocodile; its outline, as distinct from that of the paired trabeculæ, can be traced in embryo Mammals, especially *Marsupials.* Here, its presence, *even for a few days,* as a distinct element, is very instructive, showing that, in some respects, the Petromyzoids lie between the Myxinoids and the Anura. *In some respects;*—but the larval Anura (see "Batrachian Skull," Part III., Plate 30, figs. 10, 11, *p.e.*) show very clearly the independence of the "anterior intertrabecula," which has no existence in *Petromyzon,* but is so remarkable in the Myxinoids (same Plate, figs. 2, 3, *a.i.tr.*). In those forms it supports the *prenasal proboscis;* in the larval Anura, with symmetrical nasal capsules, it forms the wall of partition between them, whilst the short narial passages acquire, each, a crescentic valvular cartilage, representative of the series of imperfect rings in the Myxinoids.*

In spite of the difference of form and of the character of its tissue (*hard* instead of *soft* cartilage), and of its very temporary separateness, no one can fail to see its morphological identity with that of the Myxinoids (Plate 10, figs. 2, 3, and figs. 4, 5, *p.i.tr.*).†

This small, scarcely transformed skull of the Lamprey shows exactly what has been built upon the *Ammocœtine* foundations (see Plate 19, figs. 4, 5; and Plate 10, figs. 4, 5). Outside the newly floored fore part of the skull, a flange or wing of cartilage has been added, which widens from behind forwards, until it shoots out right and left, as the ethmo-palatine (*e.pa.*) This flange runs round the looped trabeculæ in front, and thus the ethmo-palatines have an elegant, narrow, arched commissure running concentrically round the front of the early trabecular commissure.

This peculiar structure explains what was to me very anomalous in the transforming skull of the huge Tadpole of *Pseudis* ("Batrachian Skull," Part III., Plates 11, 12) where these bands are seen to pass into each other, under the ethmoidal region of the skull, and to have a distinct margin, marking the commissure off from the basis cranii, behind the transverse band. Here, however, there has been *coalescence of the basi-cranial flange with the ethmo-palatines,* for in the *Ammocœte* (Plate 12, figs. 4, 5), the subocular band of cartilage is only developed in its hinder third; in the Tadpole these parts are primarily continuous.

The *secondary* character of the cornu trabeculæ (*c.tr.*), is shown in this little skull,

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* In retaining the old *human-anatomy* names, for descriptive purposes, in special districts or regions, the morphologist is saddled with a double nomenclature. For as our conception of the development of the parts becomes more and more clear, there must of necessity arise a number of *general terms,* each of which may include several special descriptive terms. Thus, the general morphological term "intertrabecula," which I have added to RATHKE's term "trabeculæ," includes the perpendicular ethmoid, and the septum nasi; the prenasal rostrum; and the median part of the anterior and posterior sphenoidal regions.

† It is due to the late Professor F. M. BALFOUR to state that the determination of the cranial nature of this peculiar median cartilage of the Myxinoid skull is due to him, and that this was seen independently of the peculiarly conclusive instance of it in the young Lamprey. It seemed to me, for a while, to be a *median palatine* cartilage, forming a floor to the "posterior nasal canal," and putting this view forward to him, as a suggestion, his immediate answer was, "No; it is your intertrabecula."

and also its essentially double nature. It is, now, separated from the ethmoidal flange by a narrow tract of soft cartilage, which, however, expands at the median line, thus giving *two roots*, as it were, to this large leafy tract. The fore edge is ragged, and there are points of cartilage, where broad lobes will be in a week or two (see also Plate 19, figs. 1, 2, *c.tr.*). At this stage the "flying buttress" is much more like that of the late Tadpole of the Bull Frog (*op. cit.*, Plate 4, figs. 8, 9), than in the next stage (Plate 19, figs. 1, 2), for the whole framework is more outspread, and the prepalatine (*pr.pa.*) points forwards, leafy and flat.

The subocular fenestra (*s.o.f.*) is made reniform by the basi-cranial flange; its more arched outer boundary is made by the postpalatine (*pt.pa.*) passing into the pterygoid (*pg.*), which runs upwards into the pedicle (*pd.*), and downwards into the epi-hyal (*e.hy.*), without a trace of the quadrate lobe and condyle, and without a sign of segmentation. If we compare these stages of the skull of *P. marinus*, one-third grown (Plate 18); 5 inches long (Plate 19, figs. 1–3); 4 inches long (Plate 10, figs. 4, 5), we shall see that the auditory capsules are relatively larger the younger the specimen. Also that they are short-oval in the youngest, then reniform, and then short-oval again. At first the semicircular canals make no difference in the outer form; then the two wide tubes bulge the capsule out; and this is lost, afterwards, by the thickening of the walls of the capsule. The meatus internus (Plate 18, fig. 5, VIII.), is pyriform and smallish in the largest of the three, then relatively large in the next (Plate 19, fig. 3), and in the smallest (Plate 10, fig. 4), it is a large, gaping space. In making these comparisons we have to keep the particular *species* in view, for in *P. fluviatilis*, the capsules of the *Ammocœte* (Plate 19, figs. 4, 5) are already reniform.

My work has been done according to what was possible with the materials at hand, first one species and then another; the reader must make the proper *specific equation.*

Solid sections of the head and branchial region of the adult Petromyzon fluviatilis.

A.—*Vertical sections.*

The various sectional views have purposely been made bald and diagrammatic, for the skeletal parts are very complex, and their relations to the surrounding parts very manifold and confusing; an exhaustive memoir on the anatomy of this type would have to be tenfold the bulk of this selected piece of work.

In the large vertical section (Plate 23, fig. 1) the segmental muscles and their deciduous septa are seen to have mounted up and overlapped the cranium as far as the external nasal opening (*e.n.*); below, the ventral muscular bands are merely indicated in the figure. The myelon and cerebrum (*my.*,* C^{1-3}.) are seen enclosed in their fibrous sheath (*theca vertebralis* and *dura mater*), and below them is the

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* The line from *my.* is too low in this figure.

large notochord (*nc.*) enclosed in its thick fibrous sheath ; it is cut through along its axial fissure (Plate 23, figs. 4–7, *nc.*), and its apex reaches to the part where the posterior nasal canal (*p.n.c.*) emerges from the skull cavity.

Below the notochord the special, narrow, long *food-pharynx* (*phx.*), is laid open; near the head this passage bends downwards *under* the nasal canal, and *over* the frimbriated shelf (see fig. 2, *i.v.s.*) the "intervelar shelf," or fringed floor. Then, enclosed in connective tissue, we see the special branchial diverticulum of the pharynx, the *branchial bronchus*, perforated by seven erect, oval, valvular apertures or clefts, each of which opens into a branchial pouch (see fig. 2, *i.b.a′.*). Below this large perforated tube, a *cul de sac*, another similar but smaller perforated tube, is seen; this is the great branchial artery (or *aorta*) (*br.a.*), giving off its secondary branchial arteries to the pouches. Below these parts we see muscles and inter-muscular septa, the structure of which does not concern us now. In front of the structure just described, where the mucous membrane is lined with cells derived from the *hypoblast*, we see the large oral or buccal involution, the epithelium of which is epiblastic. This is divided into two regions, the *labial*, or region of the *disk*, and the proper mouth cavity (*m.*), expanding into a *buccal* space right and left of the vela and intervelar valve. The first space is a low, highly ornate dome, pierced above to form the proper oral opening (see Plate 8, fig. 11), which is reduced to an arched fissure by the broad dentate end of the tongue. The oral cavity is very extensive, but it is partly taken up in front by a peculiar shelf, containing the great upper labial ($u.l^1.$), and, behind, its height is lessened by the pouched end of the posterior nasal canal (*p.n.c.*). That canal is seen to emerge from the cranial cavity in front of the notochord, and then to end between the notochord and the depressed part of the food-pharynx (*phx.*) ; it has a right and left diverticulum near its end. Narrowing in its intercranial tract, we see it widen again, under and in front of the nasal sac (*na.*), where it opens with the nasal opening (*e.n.*) ; its general form, in outline, is bracket-shaped.

The skeletal parts displayed in this section are as follows :—

The tegmen cranii (*t.cr.*) over the junction of the mid and hind brain ($C^2.$, $C^3.$) ; the nasal roof (*na.*) and a sigmoid tract under and in front of the skull ; this is intertrabecula behind, ethmoid in the middle, and cornutrabeculæ (*c.tr.*) in front ; the investing mass shows a little where it is cut through, over and under the apex of the notochord.

In the base of the mouth and throat the large basi-hyal (*b.hy.*) is halved, its soft and then its hard part, in front, is shown ; it reaches behind to the second branchial opening. The thickest part of the annular cartilage ($l.l^1.$) is above, the narrowest is below, the thin arched cartilage above and behind the upper section is the great median upper labial ($u.l^1.$). forming the skeleton of the shelf under the cornu trabeculæ. Below the basi-hyal, the median distal mandibular (*m.d.m.*), is seen close inside the proper chin, behind the fissure between the chin and the great sucking disk. The small points of cartilage cut through where the two systems of extra-branchials conjugate, below, were only figured in three places, in front (fig. 1, *ex.br.*).

Two *partial* vertical sections are figured, one (Plate 15, fig. 9) near the mid line, the other (fig. 8) more to the side. In the one nearest the middle (fig. 9) the fore part of the notochord is shown bent upwards, the cephalic part being enclosed in hard cartilage. The posterior nasal canal (*p.n.c.*) ends beneath its front spinal region; that pouch is seen to have a valvular fold inside it.

Under the pouched end of this canal, the buccal cavity is partly shown, with a small tract of the facial cartilage between it and the branchial canal; and another small point of cartilage is caught in this region, namely, part of the skeleton of the "intervelar shelf." The fore part of the branchial canal (*br.c.*) is exposed beneath the valvular end of the posterior nasal canal, and in it two oval internal branchial passages, and a *small valvular pouch* in front, at a lesser distance from the first opening than from the second. These are, in truth, the first three clefts, but the foremost, or hyomandibular opening (cl^1.), does not pass through the wall of the head, and has developed very little since it first appeared. Evidently, *it never quite closed up, or, if it did, it opened again* during metamorphosis. In the more lateral section (Plate 15, fig. 8) the investing mass shows more in the cranial end of the notochord (*nc.*, *iv.*), and beyond it, in front of the posterior nasal canal (*p.n.c.*), the basis cranii is seen. Here we have the same openings—the first *closed*, and the two next *open* clefts, and in front of these the folds of the velum (*vl.*), and part of the fimbriated "intervelar shelf." Between the lateral and sub-lateral filaments of the shelf there is seen a deep pyriform recess, this is the *lateral buccal pouch* (to the right of *b.p.*), a diverticulum of the oral mucous membrane in front of the pharynx. It will be seen that the food-pharynx is not figured in these two sections, they were made beyond or outside the wall of this narrow tube (see Plate 23, figs. 1 and 4, *phx.*). The main crus of the one side of the *intervelar* skeleton is cut through (fig. 8, *vl.*).

B.—*Horizontal sections.*

These sections help us to understand this peculiar type of cranio-facial skeleton. In the higher of the two (Plate 23, fig. 3) we see the eye-balls (*e.*) cut through their middle, and also the auditory capsules (*au.*) in which the multilocular membranous labyrinth of each side is laid open. In front, the ethmoid (*eth.*) is cut across in its most solid part, and behind it, we see the outer nasal opening (*e.n.*) cut down close to the capsule (*ol.*), and in front of the ethmoid, part of the 2nd lateral labials are seen. Behind the nasal capsule we see the fore brain (C^1.) and the cranial side walls (*o.s.*), which are sinuous and out-turned at each end, with the orbital muscles and eye-ball (*e.*) attached. Part of the pedicle is seen confluent with the auditory capsule (*au.*); the latter is open on the inner side for the entrance of the auditory nerve. The first pair of neural arches of the spine (*n.a.*[1]) are cut across, some of the notochord, and part of the first pair of branchial pouches.

In the next section, taken at a lower plane (Plate 16, fig. 2) the lower part of the eye-balls is shown, and the oral cavity is exposed with the intervelar fringed shelf (*i.v.s.*), the two-leaved "vela (*vl.*)," and the fore part of the branchial bronchus (*br.c.*) The first (arrested) cleft, and the first proper branchial opening are hidden under the folds of the "velum," but the 2nd and 3rd branchial apertures are shown clearly, and also the cavities of the three foremost pairs of pouches (*br.p.*) Both the cerato-hyal (*c.hy.*) and the stem of the intervelar skeleton (*i.v.s.*) are cut across. In front, the annulus (*l.l*[1].) is cut across, right and left, and also the labial distal mandibular cartilages (*l.d.m.*). Then close to the floor of the mouth the highest part of the basi-hyal (*b.hy.*) is caught; outside it, the lower edge of each 2nd lateral upper labial (*u.l*[3].); and close under the eye-ball the pterygo-palatine bar (*pp.g.*). [In this figure for *i.b.a*[1]. read *i.b.a*[2]., and for *br.p*[1]. read *br.p*[2].]

Vertically-transverse sections of the head of the adult Petromyzon fluviatilis.

In one rather small specimen nearly six scores of thin sections were made, stained with *eosin*, and mounted as transparent objects, in Canada balsam; of these rather more than one in four were drawn (Plates 20–22; and Plate 23, fig. 4). These reached from the sucking disk to the 2nd branchial pouch. Other sections were made by hand (solid) from the hind part of the branchial region of a very large specimen, which had been solidified in a solution of chromic acid (Plate 23, figs. 5–7). In these latter sections the muscular masses are more accurately shown than in the thin specimens that have been interpenetrated with *oils* and *resin*. These parts, however, are drawn in the figures merely to indicate the relation of the skeletal parts; it does not enter into my plan to describe them. As they appear in the figures they will be easily understood by the Anatomist. It is utterly impossible for any figures to give an adequate idea of the beauty of the transparent sections.*

1*st Section* (Plate 20, fig. 1).—This section is through the fore part of the suctorial disk,† and this, and the next, are in front of the trabecular cornu. The median upper labial (*u.l*[1].) is cut across in its narrower fore part, and beneath it the annular labial—the azygous element of the lower series (*l.l*[1].)—is seen cut across right and left of the high arched space shown by section of the great disk. This is near the front part of the annular cartilage; it is highest and thinnest at this part, and is bent upon itself below the middle. Hence in the outer view (Plate 18, fig. 1, *l.l*[1].) the upper part of the side seems to be bevelled. Below this, at a small distance, there is a somewhat lesser section of cartilage, which is not curved; this has suctorial teeth on its inside, and it lies opposite the

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* The *thin* sections were made by one of my sons, and the *camera-drawings* by another.

† To follow the meaning of the parts displayed by these sections it is necessary to keep the form of the head of the undissected Lamprey before the eye (Plate 8, figs. 11–13) and also the structure of the cranio-facial apparatus as displayed by the *dissections and solid sections* (Plates 18 and 23).

middle of the large labial muscular mass outside the cartilage (*s.d.*), the section of which is crescentic; the annular cartilage gives attachment to these fibres, above; its upper edge, only, is free. The lower cartilage is called by SCHNEIDER (see his plate 8, fig. 1, *Cd.*) "Cartilago-dentalis" ("Knorpel für die seitlichen Zähne"). I have not, elsewhere, seen this cartilage noticed, and my sections created for a time a difficulty: *dissection* of the suctorial disk, both of the young *P. marinus* and of the adult *P. fluviatilis,* gave the explanation. There is not merely *one lateral lower labial* on each side, but *three* (Plate 18, fig. 1, $l.l^{2\text{-}4}$.); they are small, irregularly oval patches of hard cartilage, lying in a profusion of large-celled simple cartilage, quite like that seen in the larva of *Dactylethra* and intermediate between the "vacuolar tissue" of the notochord (and of the huge basi-branchial bar of the Myxinoids), and the ordinary soft cartilage of Marsipobranchs generally. This large-celled simple cartilage fills the cavities of the horny teeth, and is, indeed, the kind of pithy connective tissue which goes to fill the interspaces of the higher kinds of tissue in this large spongy disk.*

2nd Section (Plate 20, fig. 2).—The first "lateral upper labial" ($u.l''$.) is now cut across under the median plate ($u.l'$.); the annular cartilage ($l.l^1$.) is now lower and more bent outwards, and the lateral lower labial ($l.l^3$.) is smaller. The fore end of the tongue and its teeth (*tg., l.t.*) is shown in the arched oral opening; here the *roof* of the oral opening is seen to be formed by the great upper labial, but the *disk* grew some distance in front of this part.

3rd Section (Plate 20, fig. 3).—In this section the remarkable imbrication of the cartilages is shown, and, looking at the side view (Plate 18, fig. 1), we see what parts have been cut across. The large overlapping trabecular cornu (*c.tr.*), the great size of which seems more remarkable in the sections than in the dissections, is thin here at its fore edge, well arched, and is much wider than the great azygous labial. That cartilage ($u.l'$.) is very similar in its sections for some distance; it is too much arched to be quite concentric with the great cornu, and its edge is thickened or ribbed, and turned slightly inwards. Following the line of its incurvation below, we see the small front lateral style ($u.l''$.), sharp above and rounded below. Here, in the upper part of this large portico, under the median labial, we see again the front outline of the great broad-ended tongue (*tg.*), which widens upwards, and has a concave upper outline. On each side of the severed part of skin, at the base of the tongue's tip, the annular cartilage ($l.l^1$.) is cut through as it thickens towards its hind part; its section is lobu-

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* Whilst describing these sections of the disk, it may be well to say that it is formed by a great hypertrophy of the *lower lip*, which grows forwards, closes over the narrowed oral opening, and unites above that opening with the upper lip (see Plate 8, figs. 11–13). Hence the cartilages behind the disk lie in the substance of the *chin* and *throat*, and evidently do not belong to the labial category. The cartilages now referred to are the "distal mandibulars," single and paired. The five cartilages under the cornu trabeculæ are formed in the upper lip, which is a large hood from the first (Plate 1, figs. 1–3). The upper and lower lips of the Lamprey are now seen to be much more (vertically) symmetrical than they seemed to be; above, there is one main azygous piece and two pairs of lesser pieces; below, one main azygous piece and three pairs of subsidiary pieces.

late above, and sharp below, the sharp part being invested with yellow horn—part of the dental series.

4th Section (Plate 20, fig. 4).—Both the cornu trabeculæ (*c.tr.*) and the median upper labial (*u.l*^{1}.) are broader than in the last, and the first lateral cartilage (*u.l*^{2}.) is cut through obliquely lower down. The annular cartilage (*l.l*^{1}.) is cut across in front of its hind margin, and shows the concavity along its upper half. The hinder teeth (*l.t.*) are shown in the hollow of the divided labial disk. At the upper edge of the annular cartilage, right and left, there is a reniform tract of cartilage; this is the head of the lateral distal mandibular (*l.d.m.*). The skeleton of the tongue (*tg.*, *b.hy'.*) is now cut across, it is a thick slab of cartilage, with both surfaces slightly concave, and the lower face of less extent than the upper.

5th Section (Plate 20, fig. 5).—This is from behind the annular cartilage, and through the hinder third of the great disk (*s.d.*). The two great roof cartilages (*c.tr.*, *u.l*^{1}.) are very similar to the last, and the first lateral upper labial (*u.l*^{2}.) is cut through near its lower end, and the foremost point of the second (*u.l*^{3}.) is cut across. The lateral distal mandibulars (*l.d.m.*) are now flat, with their upper edge turned outwards; between them, and near to them, arching over the hind part of the disk, two plates of cartilage are cut through, thick in the middle and thin outside; these are the two sides of the fore part of the "median distal mandibular" (*m.d.m.*), which is notched in its broad fore edge (see Plate 10, fig. 6) in the young *P. marinus.* Several cartilages are seen in the fore end of the tongue (*tg.*); of these the four larger tracts belong to the basi-hyal (*b.hy.*, see also Plate 18, figs. 6–8); the short anterior median (*b.hy'.*), with the pair of short ascending pieces (*b.hy''.*), and behind and under the middle piece the fore end of the main long bar (*b.hy.*), which is elliptical in section in front, and placed with its long axis upwards. This section shows, what the general view of these parts does not, namely, that when these curious *quasi-mandibles* are cut through obliquely, their common basal piece is arched a little upwards. In the curious lingual *cushions* there is a small "supra-lingual" cartilage, right and left (*s.l.c.*); its point is cut across here.

6th Section (Plate 20, fig. 6).—The use of these various sectional figures will be seen by comparing this with the last; there seems to be but little difference between them, but we miss some things seen in that, and find some new ones in this. The great leafy cornu (*c.tr.*) is wider and flatter, the median upper labial (*u.l*^{1}.) is very similar; the first lateral upper labials are gone, the section was made behind them, and the second (*u.l*^{3}.) are larger points of cartilage between the two roof-plates. The flattened distal lateral mandibulars (*l.d.m.*) are vertical here, and the median cartilage (*m.l.m.*) is a single plate, slightly bent downwards where the basi-hyal rests on it. The main basi-hyal (*b.hy.*) is now much flatter and somewhat higher; the front short middle piece is not seen, but the erect pieces (*b.hy''.*) are cut across in their hind lobe, so that there is a considerable space between them and the main bar. The little cartilages inside these (*s.l.c.*) are larger, are elliptical in section, and in the valley between the cushions that contain them, sharp yellow teeth are seen—the "supra-lingual" teeth (*s.l.t.*) (see Plate 14, fig. 9).

7th Section (Plate 20, fig. 7).—In this, the same parts are cut through as in the last, but they are different in form and position. The great cornu (*c.tr.*) is but little changed, but the median labial plate ($u.l^1.$) is considerably contracted, at this, its hinder margin. We see the wedge-like point of the second lateral upper labial ($u.l^3.$) above, and the paired distal mandibulars (*l.d.m.*) below; these are getting further apart. The median piece (*m.d.m.*) is now much narrower, being severed behind its dilated part; the main basi-hyal is as in the last, but its wings (*b.hy″.*) are vertical in this section, and so are the small supra-linguals (*s.l.c.*) between them; we still see the teeth (*s.l.t.*) between the cushions. The section of the great disk is lessening towards its hind margin.

8th Section (Plate 20, fig. 8).—The oral opening is now becoming very contracted, the great roofing cornu (*c.tr.*) is of undiminished size, but the large median labial ($u.l^1.$) only shows its postero-inferior angles, right and left. The second lateral pair ($u.l^3.$), which wedges in between the two roof-pieces, is now trilobate in section, and larger; the distal mandibulars (*m.d.m., l.d.m.*) and main basi-hyal (*b.hy.*) are much as in the last section, but the paired lobes of the basi-hyal are gone; only a small part of each supra-lingual (*s.l.c.*) is seen.

9th Section (Plate 20, fig. 9).—The roofing cornu (*c.tr.*) is narrower here, and its lower edges are thickened instead of being sharp. The upper median labial is not seen, but the second lateral piece ($u.l^3.$) is now a large slab; it is *f*-shaped in section, vertically placed, thick edged above and thin below. The distal mandibulars (*m.d.m., l.d.m.*) are smaller, and so are the supra-linguals (*s.l.c.*); the main basi-hyal (*b.hy.*) keeps its form. This section is through the last fold of the suctorial disk, which is here seen to be quite free from the rest of the section, and to have lost its lower gap.

10*th Section* (Plate 20, fig. 10).—This section is behind the lips, and only one pair of labials, the second upper lateral ($u.l^3.$) are cut across; they are thinner here than in the last. The great cornu (*c.tr.*) is one-third narrower; the distal mandibulars (*m.d.m., l.d.m.*) are getting less and less; the main basi-hyal (*b.hy.*) is much the same, and the tongue (*tg.*) is cut through behind the cartilages and teeth.

11*th Section* (Plate 20, fig. 11).—The great cornu (*c.tr.*) is gaining in thickness, but losing its width; the second lateral labials ($u.l^3.$) are cut through their hind margin; the other parts are much as in the last section. The small elevation at the top is the beginning of the nasal opening (*e.n.*).

12*th Section* (Plate 20, fig. 12).—In this section the outer nasal opening (*e.n.*) is laid open above the proximal part of the cornu trabeculæ (*c.tr.*). Here the oral cavity (*m.*) is a large oblong space, partly occluded, below, by the lingual apparatus, in the base of which we see the vertical basi-hyal (*b.hy.*). Opposite the middle of that bar the postero-inferior angle of the large lateral labial ($u.l^3.$) is cut across, and the points of the three distal mandibulars (*m.d.m., l.d.m.*) are still seen in section.

13*th Section* (Plate 21, fig. 1).—This is also through the outer nasal opening (*e.n.*) and the proximal part of the cornu (*c.tr.*); the other sections are like the rest, except that the lateral distal mandibulars are no longer seen.

14*th Section* (Plate 21, fig. 2).—The outer nasal passage (*e.n.*) is a flattened tube, vertically placed, here, for this is close in front of the nasal capsule, and through the junction of the cornu (*c.tr.*) with the solid pre-cranial plate or ethmoid; the cartilage lies over the large oral cavity (*m.*) as a thick beam, slightly arched upwards, and a little hollowed in the middle, above. The other points of cartilage are like the last.

15*th Section* (Plate 21, fig. 3).—This section is through the front third of the nasal capsule (*ol.*), showing its *nine pairs* of radiating folds or lobes, each having its own fissure, and lined with mucous membrane; the interspaces of the folds are filled with black pigment. The lower folds are shorter than the upper and lateral, and the posterior nasal canal (*p.n.c.*) is distinct; it is seen as a transversely oval space between the capsule and the ethmoid (*eth.*). The capsule itself (*na.*) is not so distinctly bilobed as in the young of *P. marinus* (Plate 18, fig. 3, *na.*), for its top is flat, and it is somewhat angulated at the sides. The cartilage of the capsule is of the softer kind, but approximates to the general hardness of the skeleton in the hind part; it does not meet below in the sections that cut through the folds,—only behind (fig. 6). It is not a mere *grating* as in the Myxinoids (Plates 10 and 17), but is a continuous sheet, dipping in towards the deeply pigmented interspaces of the folds. The cranium is cut through so as to show the continuity of the ethmoid (*eth.*) with the *ethmo-prepalatine* tract (*e.pa.*), running almost to the front spike. The whole of this arched, but flat-topped, tract is of nearly uniform thickness; the palatine bars bend a little inwards above, and then outwards below, and are rounded at their edge. Under the oral cavity (*m.*) the only cartilages cut through are the basi-hyal (*b.hy.*) and the tip of the median distal mandibular (*m.d.m.*)

16*th Section* (Plate 21, fig. 4).—This is through the middle of the nasal capsule (*na., ol.*) and the fore part of the eye-ball (*e.*). The wheel-like appearance of this organ is most perfect here, the lower folds being larger; and the lower face of the capsule, which is membranous for a slight extent, is not pushed up by the posterior nasal canal (*p.n.c.*). On each side of the ethmoid (*eth.*) there is a low wall, the beginning of the orbito-sphenoidal region, and in the middle of the descending plate of cartilage there is a fissure,—this is the fore margin of the "subocular fenestra" (*s.o.f.*); the pterygo-palatine bar (*pt.pa.*) is thickened at its lower edge. The lining of the mouth (*m.*) is quadrilobate; under it is the only other cartilage seen in this part—the basi-hyal (*b.hy.*). The third branch of the 5th nerve (V^3.) is seen in section between the subocular fenestra and the oral cavity.

17*th Section* (Plate 21, fig. 5).—This is through the back of the nasal organ, and only two pairs of upper folds are laid open, the rest of these folds being covered here by the highly pigmented stroma; the cartilaginous capsule (*na.*) reaches below to the lowest fold, right and left. The cranium at this part is four-winged, for the ethmo-palatine (*e.pa.*) is much less here, because of the large space—the subocular fenestra—(*s.o.f.*), and the orbito-sphenoidal wings (*o.s.*) are rising and diverging; in the hollow between them runs the posterior nasal canal (*p.n.c.*, outline not figured in this section). The post-palatine and pterygoid at their junction form a moderately high and thickish band; inside this, near its top, is the third branch of the trigeminal (V^3.); below the oral cavity (*m.*) is the basi-hyal (*b.hy.*).

18*th Section* (Plate 21, fig. 6).—Here we see that the apiculate hind part of the nasal capsule (see also Plate 18, fig. 3) grows vertically downwards for some extent, and that the walls, which underlaid the capsule to some degree, further forwards, have now united to form a perfect floor. The median cartilage is partly hard; it does not correspond with the *ethmo-septal* partition of higher types, but with so much of the nasal roofs as are developed downwards, back to back, to unite with the true septum. The posterior nasal canal (*p.n.c.*) is seen in the, bottom of the deep cranial trough, whose orbito-sphenoidal walls (*o.s.*) are high and diverge so as to make a wide space for the olfactory lobes (C^{1b}.), and the curious limited partition and floor of the nasal capsule (*na.*). Here the *waist* of the skull is well pinched in, and the orbits and eye-balls (*e.*) are large; large, also, is the shelving floor of the orbits, which is cartilage above and below, the intermediate space being equal in extent to both of the tracts; the upper is now trabecular (*tr.*), and the lower pterygoid (*pg.*); the space, or suborbital fenestra (*s.o.f.*), is split into two laminæ, and between them the 3rd branch of the fifth (V^3.) passes to the lower and front parts of the face.

The oral lining (*m.*) has now become five-lobed; below it, we see the vertical basi-hyal, and close to it a pair of cartilaginous points—the fore ends of the cerato-hyals (*c.hy.*) (see also Plate 18, figs. 1, 2, 6, 7).

19*th Section* (Plate 21, fig. 7).—This is through the middle of the eye-balls (*e.*), behind the nasal capsule, where the olfactory lobes (C^{1b}.) arise from the fore brain. The dura mater (*d.m.*) is thick, and still retains the form of the back part of the nasal capsule, and below this membrane's roof we see that the side walls (*o.s.*), which are gently out-turned above, are segmented below their middle, opposite the top of the posterior nasal canal (*p.n.c.*). This is the beginning of the orbito-sphenoidal fenestra (*o.s.f.*), which membrane is perforated further backwards for the optic nerve (see Plate 18, fig. 5). The subocular flange of the trabecula (*tr.*) is smaller than in the last; the remaining parts are quite similar in both sections (*pg.*, *s.o.f.*, V^3., *b.hy.*, *c.hy.*).

20*th Section* (Plate 21, fig. 8).—Here the section is through the optic nerves and the fore brain (II., see also fig. 9, C^1.), where the orbito-sphenoidal fenestra (*o.s.f.*) is wide. The dura mater (*d.m.*) is still thickened in the middle of the roof, and the orbito-sphenoids (*o.s.*) now turn in above. The trabecular flange (*tr.*) is very thin, right and left, and so also is the trabecular floor, now behind the intertrabecular region. This is the last section figured in which we shall see the posterior nasal canal (*p.n.c.*) fairly lodged *inside* the cranial cavity. The remaining parts are like what was shown in the last, but the pterygoid band (*pg.*) is broader. The fibres of the 3rd branch of the 5th nerve (V^3.) have not escaped the layers of the floor at this part.

21*st Section* (Plate 21, fig. 9).—The whole cranio-facial frame is here seen as mere patches of cartilage, three pairs of small convexo-concave tracts above, with one large trilobate, and two small pyriform, sections below. Here the cranial cavity is large, but the cerebrum (C^1.) is small; the orbito-sphenoids (*o.s.*) are separated from the trabeculæ (*tr.*) by a space (*o.s.f.*) wider than the bars themselves. These are bent in the opposite direction, the upper are convex outside and the lower concave. The pterygoid (*pg.*) is a gently-arched band; between it and the trabeculæ (*tr.*) the 3rd branch of the 5th (V^3.) is seen lying on the fenestra, ready to enter. The posterior nasal canal (*p.n.c.*) is now without a cartilaginous floor, ready to escape into the pharyngeal space; the cartilages below the buccal cavity (*m.*) are the basi- and cerato-hyals (*b.hy.*, *c.hy.*). Above these the rising floor of the mouth contains the *fimbriated intervelar shelf* (see Plate 18, fig. 7; and Plate 14, fig. 8).

22*nd Section* (Plate 22, fig. 1).—This shows a pyriform cranial cavity containing the fore brain (C^1.) at its junction with the mid brain. Here the back part of the eye-balls (*e.*) and the orbital muscles are seen, and the posterior nasal canal (*p.n.c.*) escaping through the pituitary space between the trabeculæ, which form at this part the high alisphenoidal walls (*al.s.*), in section like a pair of callipers, bent outwards, above, like blades, which are connected across the roof by the convex dura mater (*d.m.*). This section is through the hind part of the subocular fenestra; the pterygoid (*pg.*) is here thick above; the 5th nerve is not figured in this drawing. Below the lobulated oral (or *buccal*) cavity (*m.*), and between that space and the hyoid apparatus, there is a plate of hard cartilage cut across—a thinnish, sinuous tract. This is the "intervelar shelf" (*i.v.s.*), cut across behind the fimbriations. Here the cerato-hyals (*c.hy.*) are much deeper than in the last section.

23*rd Section* (Plate 22, fig. 2).—Here, if anywhere, we have an instructive section, made in the pre-auditory or post-pituitary region. The mid and hind brain (C^2., see also fig. 3, C^3.) meet here, and the latter is giving off the great trigeminal nerves (V.), with their huge gasserian ganglia. These parts are seen in the foramen ovale, under the fore face of the auditory capsule (*au.*) which is surmounted by the alisphenoid (*al.s.*), now turning inwards on to the roof of the skull. Looking at this figure and that of the dissected skull (Plate 18), we see that the razor has cut through the investing mass (*iv.*) enclosing the fore part of the notochord (*nc.*), the pedicles (*pd.*), the back part of the pterygoid (*pg.*), that part of the suspensorium (or *pterygo-palatine arcade*) which gives off the epi-hyal from its hinder margin (see Plate 18, fig. 1, *e.hy.*). Below the basis-cranii, the posterior nasal canal (*p.n.c.*) has nearly doubled its "bore;" at some distance below that tube we see, now, not a wide quinquelobate *buccal cavity*, but the plicated opening of the narrow food-pharynx (*phx.*), having under it the cartilaginous core of the intervelar shelf. Below that shelf we see the hyoid apparatus; the fissures between that musculo-cartilaginous structure and the shelf form the valvular entrance to the *branchial canal* (or "bronchus;" see also Plate 23, fig. 1). Here the cerato-hyals (*c.hy.*) are seen to be obliquely-placed plates, sinuous in form, and bent upwards, both externally and within, where they are tied to the carinate basi-hyal (*b.hy.*) (see also Plate 18, figs. 2, 6, 7).

24th Section (Plate 22, fig 3).—The cranial cavity is nearly surrounded by cartilage in this part, for the alisphenoids nearly meet as a *tegmen cranii* (*t. cr.*) over the hind part of the mid brain (C^2.). The hind brain (C^3.) is also cut across in its fore part, and the hind part of the 5th nerve (V.) is still to be seen in the foramen ovale. The auditory capsules (*au.*) have here an inner wall in front of the meatus internus. The section of the notochord (*nc.*) is larger; there is a fissure between the two parachordal plates (*iv.*) above; these plates pass, with but little thinning out, directly into the pedicles (*pd.*) The section is made through them into the hyoid bar (*e.hy*), which is gently bowed outwards, enlarges, and again lessens before it is turned forwards and inwards, as the cerato-hyal. Just above its middle an *elbow* is seen; this is the back of the pterygoid bar (*pg.*), *whence the quadrate lobe and condyle should have grown outwards and downwards.* The cerato-hyal *region* is suddenly bent upwards with its convex face downwards; each bar ends at a moderate distance from the median carinate rod (*b.hy.*). The posterior nasal canal (*p.n.c.*) is still larger than in the last section, and is sub-pentagonal in outline. Below it is the very narrow pharynx proper (food-pharynx, *phx.*), with its puckered, contracted lining, and below this is the body of the cartilage in the "intervelar shelf" (*i.v.s.*). The space below, right and left, is where the buccal cavities are passing into the branchial canal (see also Plate 23, fig. 1).

25th Section (Plate 22, fig. 4).—This is through the meatus internus, and shows the 8th nerve and its ganglion (VIII.) The capsule (*au.*) is continuous above, as in the last, with the alisphenoidal tegmen (*t.cr.*) over the hind brain (C^3.), and also, below, with the thick parachordals (*iv.*), which have now a larger notochordal section (*nc.*) within them. The basal cartilage is cut through here, behind the pedicle, but the corresponding region of the hyoid arch is suppressed, for there is no hyomandibular tract, such as we see in the Myxinoids; beyond this point there is no more *endoskeletal* cartilage in the side walls. The lower part of the epi-hyal and the cerato-hyal (*e.hy.*, *c.hy.*) are cut through, and also the basi-hyal (*b.hy.*) Above the cerato-hyal we see the crura of the intervelar shelf (*i.v.s.*) cut across the pedate end, so that the *toe* and the *foot* are separate points of cartilage (see Plate 18, fig. 7, *i.v.s.*) The section of the posterior nasal canal (*p.n.c.*) is mutilobate and large; under it the food-pharynx (*phx.*) is very small, with its mucous lining corrugated, and in the space below it we have the beginning of the special branchial canal (see also fig. 5, *br.c.*).

26th Section (Plate 22, fig. 5).—This is behind the meatus internus and the alisphenoidal roof; the hind brain is seen to be very small as compared with the cranial cavity. As in the last, the notochord (*nc.*) is roofed by the investing cartilage (*iv.*), but the floor is imperfect. The fore part of the vagus, with its ganglion (X.), is seen, and below it, that part of the extra-hyal (*ex.hy.*) which fastens on to the top of the epi-hyal bar is cut across, and, lower down, the main bar of this superficial soft band. The angle of each cerato-hyal (*c.hy.*) is cut across and the basi-hyal in its *basi-branchial region* (*b.br.*). Three cavities are severed, lined with mucous membrane, one above another, in the mid-line—these are the posterior nasal canal (*p.n.c.*), the pharynx, proper (*phx.*), and the opening of the branchial canal (*br.c.*).

27*th Section* (Plate 22, fig. 6).—This is through the hind face of the auditory capsule (*au.*) and the main mass of the vagus nerve (X.) Here the notochord (*nc.*) is floored, but not roofed, and the limited investing cartilage (*iv.*) does not reach the auditory capsule, but leaves a space for the vagus to pass through; thus the roof of the skull is gone, and most of the side walls. The extra-hyal is cut across as it goes back to join the general basket-work; the basi-hyo-branchial region is cut through (*b.hy.*), and the angles of the cerato-hyals (*c.hy.*). The posterior nasal canal (*p.n.c.*) is lessening; below it we see the food-pharynx (*phx.*), below it the branchial canal (*br.c.*), and right and left of these passages the branchial folds of the first pouch (*br.p.*)

28*th Section* (Plate 22, fig. 7) is from the fore part of the spine, and through one of the foremost pairs of neural arches (*n.a.*), which are seen to be large wedges of hard cartilage, with their upper end thick, and their lower end sharp, the section not reaching to their base. The notochord (*nc.*) has now its average size; its sheath is a thick laminar membrane, like the cornea of the Mammalian eye-ball. The other part of the endoskeleton is the basi-branchial bar (*b.br.*) imbedded in the lingual muscles; above these masses are the branchial canal (*br.c.*), the proper pharynx (*phx.*), and the posterior nasal canal (*p.n.c.*) Outside the gill pouch (*br.p.*, 1st or 2nd) the extra-branchial basket-work (*ex.br.*) is cut across in *nine* places; above, near the notochord, the longitudinal band uniting the arches above; then, on the upper convexity, one of the spurs; at a distance below this the upper, and then the lower, cross band defending the aperture; and then, below, one continuous tract of cartilage belonging to the lower part of a right and left arch, at their junction.

29*th Section* (Plate 22, fig. 8.)—This is very similar to the last section, and was taken not much further back. Here in a complete section of the neural cartilages (*n.a.*), we see that they are thickened at both ends. The posterior nasal canal (*p.n.c.*), is much folded; this is its last appearance in the sections figured; the other passages are as in the last section. The upper band of the basket-work (*ex.br.*) is cut across, also one of the snags, and one of the cross bars; the other must have been cut through, but was hidden beneath the other tissues. Below, the common junction of the bars is cut across, between the bars, and between the median fenestræ.

30*th Section* (Plate 22, fig. 9).—Here we see the neural cartilages (*n.a.*) resting upon the sheath of the notochord (*nc.*); below this the great artery (*ao.*) is cut across, but there is no posterior nasal canal between it and the pharynx (*phx.*), which is well marked off from the branchial canal (*br.c.*) The lingual apparatus and the basi-branchial (*b.br.*), is still present; this is at about the *first third* of the great branchial apparatus. Here the upper extra-branchial bands (*ex.br.*) are cut across; the uniting bands above and under the aperture; and the median inferior junction of the arches.

31*st Section* (Plate 23, fig. 4).—This is the last of the thin sections figured, and is from the beginning of the *second third* of the branchial region; both the external apertures (*e.b.a.*) are laid open. The neural arches (*n.a.*) are not so wide apart above, and they have crept some distance down the side of the notochord; under these hard tracts the soft upper bands of the basket-work (*ex.br.*) are cut across, then two of the lesser bars, then a main bar, bending inwards behind its own aperture (see Plate 18, fig. 1), and then a common inferior band at a very narrow part. The passages between the pouches are the great artery (*ao.*), the pharynx (*phx.*), and the branchial canal (*br.c.*); the median basi-hyobranchial bar is not seen, but the hind part of its huge muscular lingual apparatus is shown.

32*nd Section* (Plate 23, fig. 5).—This is the first of the transversely-vertical solid sections, made a little obliquely so as to show the external branchial aperture, the last, on one side (only) with a few of the folds (*br.p′.*). The small flat myelon (*my.*) is seen in the canal formed by the theca vertebralis (*th.v.*), and under it the notochord (*nc.*) is at its largest size. The heart (*h.*) is cut through in its fore-part, with the basket-work of the pericardial region (*pcd.c.*) cut across. The pharynx (*phx.*), at the end of the branchial canal, is larger, but is pushed out of the mid-line by the heart. This section is between two pairs of neural arches.

33*rd Section* (Plate 23, fig. 6).—The second of these solid sections is through the middle of the heart (*h.*), and shows the extra pericardial (*pcd.c.*), as an almost perfect girdle; here the opposite face of the section has been drawn, so that the pharynx (*phx.*) is seen on the other side; the tips of two neural arches (*n.a.*) have been cut through.

34*th Section* (Plate 23, fig. 7).—Here we see the fundus of the tilted *bowl*, the pericardial cartilage (*pcd.c.*); the pharynx is in the figure on the same side as in fig. 5, the same *face* of the section being drawn; only the tips of a pair of neural arches are cut across. The muscular masses in these three figures are shown in their full (uncontracted) bulk.*

On the skeleton of the embryo of Petromyzon planeri, 7·8 *mm. to* 9·5 *mm. in length.*

These small, delicate, worm-like embryos, the larger of which only, namely, those about $\frac{1}{3}$-inch long, were worked out by me, came to me through Professor F. M. Balfour, who was enabled to breed them (by artificial impregnation) through the kindness of Osbert Salvin, Esq., F.R.S. The sections were lent me by Professor Balfour, and what I have to show with regard to them may be added to his excellent account of

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* Schneider (plate 10, fig. 1) shows the cartilages of the tail-fin: these parts do not enter into my plan.

the development of this type in the 2nd vol. of his 'Comparative Embryology' (pp. 68–83, figs. 37–49).

My figures are from one of the larger of these embryos (Plate 8, figs. 1–3); in the full-figure, the side view, about *eighty* myotomes are shown; its great approach, in form, to the proper larva (or *Ammocœte*) is seen by comparing the next figures (4–6) in the same plate.

Dissection of the skull of an embryo of Petromyzon planeri, 7·8 *mm. long.*

As this is the lowest kind of primordial skull worked out at present by me, I shall compare it with that of (A) the Ray;* (B) the Sturgeon; (C) the Lepidosteus; (D) the Salmon; (E) the Axolotl; and (F) the Larval Frog and Toad.

The large notochord (Plate 25, fig. 7, *nc.*) is almost straight; in an earlier stage (BALFOUR's 'Embryology,' pp. 74, 75, figs. 42, 43) it is turned downwards at its fore end, and this bend is retained throughout life (Plate 23, fig. 1); it now reaches further forwards, relatively, than afterwards, and gradually lessens to a blunt point. Further than the hind margin of the pituitary space (*py.*) it does not go. Taking the hind margin of the auditory capsules (*au.*) as our place of measurement, behind, and the fore end of the trabeculæ (*tr.*) as another, in front, the notochord reaches exactly half way.

Measured thus some time after transformation (Plate 18, fig. 4, *nc.*, *tr.*) the notochord is about two-fifths the length of the basis cranii; thus we see that it remains as an important basi-cranial structure, but in its earliest development it shows nothing of the *cephalo-chordal* character of *Amphioxus*. The membranous sheath of the notochord is very thick at this early stage; at no stage have I found true cartilage in it, such as we see in the Elasmobranchs. The primary basi-cranial cartilages (*iv.*, *tr.*) are *para*-chordal for their hinder three-fifths, and *pro*-chordal for their front two-fifths, but the front part is thick and wide, whilst the hind part attenuates gently to a fine, pointed end; these two ends are, however, far back behind the ear-capsules; two-fifths of the parachordal length of the cartilage is behind the ear. Considering the size of the region supplied by the branches of the vagus nerve, it is not unreasonable to take all the parachordal region of the notochord as properly basi-cranial; the fact is, the head and spine are not divided off sharply—there is no joint; afterwards, when the neural arches appear in the transformed Fish, the first pair of cartilages give us the beginning of the spine.

These parts cling close to the sides of the notochord, so as to be crescentic in section (see Plate 25, figs. 3, 4, *iv.*, *nc.*) nearly to the middle of the auditory capsules (*au.*); thence they diverge further and further from the axial rod. In front of that rod, in their *pro*-chordal part, the bars (trabeculæ) are separated by a tract thrice their greatest

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* (A) Trans. Zool. Soc., vol. x., plate 35; (B) Phil. Trans., 1882, Plate 14; (C) Phil. Trans., 1882, Plate 30; (D) Phil. Trans., 1873, Plate 2; (E) Phil. Trans., 1877, Plate 22; (F) Phil. Trans., 1876, Plates 55, 56, and 60; and 1881, Part I.

width. At this part they embrace the lower part of the front of the membrano-cranium, in front of the pituitary space.

The basal bars in their pro-chordal region (*tr.*) are thickest in their middle, but do not lessen much in front, they are, in form, like a pair of *callipers*, and end in a blunt point which is turned a little towards its fellow. The bars are tilted upwards and outwards (Plate 24, figs. 1–4, *tr.*; and Plate 25, fig. 2, *iv.*).

A little in front of the auditory capsules we see on each side a small blunt hook of cartilage, turned forwards at its free end, but growing directly outwards from the trabeculæ without any sign of segmentation; these are the primordial rudiments of the pedicle and pterygoid (*pd.*, *pg.*). These *first, continuous* rudiments of the face are of great interest, as we shall see by comparing them with their counterparts in other Ichthyopsida.

So also is the condition of the primary basi-cranial bars or trabeculæ, which on their hinder part embrace the notochord as parachordals (or investing mass), but *from the first* are rather related to the fore brain as its proper supporting skeleton, than to any other part. Their union in front of the trabeculæ, and their continuation forwards, as *cornua*, are secondary matters, and vary very much in different types, but their most important character is their *continuity*, for they show no signs of being under the influence of any segmentation that may be developing in the head. Here, the basal bands are longer, but much narrower, in their parachordal than in their prochordal regions, but in the nearest relations of these *Petromyzoids*, the "Anura," they only embrace the apex of the notochord by their hinder end, and form their cornua before they unite at the mid-line. In those Amphibia the parachordal region is developed as a backward extension of the trabeculæ, but in the Urodela that part is developed independently, and the cornua trabeculæ may appear first as *ongrowths* of the trabeculæ, or afterwards as *outgrowths*. Then, as to the facial part of the skull, or upper jaw, (the upper part of the 1st branchial or visceral arch), which is greatly developed forwards over the mouth in all known Vertebrata, we see that in the Lamprey the pedicle with its hooked end or rudiment of the pterygoid cartilage is developed from the first as an outgrowth of the trabecula, and the rest of the arcade—its palatine portion growing on to the ethmoidal end of the trabecula—does not appear until after transformation.

In the Anura the whole arcade is primary, and is primarily one with the trabeculæ, both the pedicle and the ethmo-palatine tract being continuous with the basal bar.

In the Urodela nothing of the sort exists; the pedicle and its ascending process are developed as the bifid free top of the suspensorium; afterwards both unite with the skull in some kinds; in others only the ascending process. In them the palatine with its ethmo-palatine process, is an *early* free cartilage; the pterygoid process of the suspensorium is a *late* outgrowth, and only in one kind known (*Ranodon sibiricus**) does the subocular arcade *become* continuous. On the other hand in the genus *Bufo*,

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* See Wiedersheim, 'Kopfskelet der Urodelen,' 1877, plate 5, fig. 69.

amongst the Anura, after being a mere wing or flying buttress of the skull during the larval period, it becomes, in metamorphosis, broken up into separate parts, like that which is primary in the Urodela.

In the Rays there is a *pedicle*, but it is a primarily independent cartilage, the foremost of the "pharyngo-branchial" elements; in Sharks there is merely *one*, *two*, or *three* "interbranchial rays" in this part, and in all the normal "Elasmobranchs" the palatine region—which has a distinct segment in Rays but not in Sharks—is suspended to the skull by a ligament. In these types the trabeculæ are flat and wide, and chondrify first; afterwards, the parachordals become cartilaginous, but this condition runs on into the spinal region without any break, until the *occipito-atlantal* joint is formed.

In *Acipenser*, a low type of Ganoid, the whole basi-cranial region is chondrified at about the same time, but in form is similar to what is seen in the Elasmobranchs. But the upper jaw is developed quite independently of the basis-cranii; its upper part, answering to the pedicle (or metapterygoid region), is a mosaic of cartilaginous pieces, with a median row, such as is seen in the *ventral aspect*, only, of the arches, in other types.

Lepidosteus, a high Ganoid, comes nearer to the Suctorial Fishes and the Tadpole, for the palatine end of the upper jaw (palato-quadrate) is from the first continuous with the fore end of the trabecula, and so also is the pier (or *pedicle*) of the hyoid arch. In that type I find no precedence of the prochordal tracts (trabeculæ) in their chondrification; they, and the parachordal tracts, are one, and show no signs of distinction from the first.

It is therefore evident that there is to be seen, in the early chondrocranium of the various *Ichthyopsida*, a mixture of the *non-essential* and the *essential;* these have to be winnowed apart by the Morphologist. I must return to this subject in my summary; but it is manifest that the primary cranium exists merely as a pair of supporting bands from which, without segmentation, the *facial* skeleton may grow. The cartilages that arise in the intermuscular septa, in the spinal region, as rudiments of vertebræ, are a *late* product in the evolution of a Vertebrate; they possibly had no existence in countless types, the brain of which had appeared in its three main vesicles, which, becoming larger and heavier, acquired *pro-* and *para*-chordal supporting balks.

We saw (in Part I.) that one type of Fish, the Hag, more than a foot long, and as thick as a finger, with a highly complex cranio-facial apparatus, has no vertebral rudiments. This is true, also, of another (*Bdellostoma*), as thick as the wrist, and almost a yard long.

This *Myxinoid* condition remains throughout larval life in the Lampreys, and exists for a time, after the development of the basi-cranial bars, in the other *Ichthyopsida.*

In the figure given of the embryo Lamprey's skull (Plate 25, fig. 7; V., X.) I have shown the huge size of the ganglionic mass belonging to the 5th nerve (possibly that of the 7th included in this mass), and of the 10th or vagus.

The auditory capsules (*au.*) are evenly oval in form, and except where the 8th nerve enters, are covered with a thin layer of true cartilage, which, however, like the rest, is *soft*, at present.

The only skeletal part formed inside the head cavities and branchial arteries is the pedicle, with its hooked pterygoid rudiment (*pd.*, *pg.*) ; this state of things continues until the time of transformation.

But the extra-branchial basket-work is one of the first parts to be developed, as the sections now to be described show ; the pattern of the basket-work is figured from a later stage (Plate 25, fig. 8), but it merely differs from that of these embryos in size and a somewhat increased density and strength of the cartilage. The sections now to be described will be better understood by reference to a figure of BALFOUR's (vertical section of an embryo half the size of those from which these transverse sections were made) (see 'Comp. Embryol.,' p. 75, fig. 43).

Vertically transverse sections of the head and branchial region of embryos of P. planeri, 7·8 *mm. and* 9·5 *mm. in length. Magnified* 150 *and* 300 *diameters.*

1*st Section* (Plate 24, fig. 1, 150 diams. ; see also Plate 8, figs. 1–3).—This is through the fore brain (C^1.) and upper lip (*u.lp.*) of the larger embryo. Here the huge relative size of the brain is shown, filling the membrano-cranium. The parts which concern us at this part are the thin tilted ends of the trabeculæ (*tr.*), which are lanceolate in section, and are connected together by a definite tract of soft cells. In part of a similar section of the smaller embryo (Plates 24, fig. 1*a*, 300 diams.) we see that the cartilage is two cells thick in the middle, and that the intervening soft tract is in two layers near the cartilage, ready to form the trabecular commissure (see Plate 19, figs. 4, 5, *tr.*).

2*nd Section* (Plate 24, fig. 2, 150 diams.).—This is still through the upper lip (*u.lp.*), the fore brain (C^1.) is somewhat smaller at this part, the trabeculæ (*tr.*) are now oval in section, and the special layer of indifferent cells, ready to become a transverse band of cartilage, is not seen. In a partial section of the lesser embryo at this part (Plate 24, fig. 2*a*, 300 diams.), the young cartilage (*tr.*) is three cells thick.

3*rd Section* (Plate 24, fig. 3, 150 diams.).—This is through the fore brain (C^1.) behind the upper lip, and therefore through the chin (see Plate 8, figs. 1–3); the tilted trabeculæ (*tr.*) are broader than in the last section.

4*th Section* (Plate 24, fig. 4, 150 diams.).—In this section, through the hind part of the fore brain (C^1.) and the pituitary region (*py.*), the trabeculæ (*tr.*) are thicker and less tilted ; this is immediately in front of the Gasserian ganglion.

5*th Section* (Plate 24, fig. 5, 150 diams.).—This is through the mid brain (C^2.) ; the fore part of the Gasserian ganglion is cut across, and also the stem of the 5th nerve (V.) as it passes over a tract of cartilage growing directly from the cartilage (*iv.*). The notochordal apex (*nc.*) is cut across close behind the pituitary region. Another figure (Plate 25, fig. 1, 300 diams.) shows the basis-cranii and its facial outgrowths on a larger scale. The oval section of the cartilage (*iv.*) lies in a nearly horizontal position and is three cells thick, whilst the hooked rudiment of the pedicle and pterygoid (*pd.*, *pg.*) is two cells thick, with an intercalary cell or two at its bulbous end; its cells are smaller than those of the trabecular bar, but they are only a slight degree behind them in development, and there is no segmental line between them. What appears to be such a line is due to the difference in the size of the cells in the main and secondary bars. The notochord (*nc.*) is cut across; the mouth cavity (stomadæum) is very large.

6*th Section* (Plate 24, fig. 6, 150 diams.).—This is through the mid brain (C^2.) and the massive Gasserian ganglia (V.); the notochord (*nc.*) is enlarging, and the basal bars or parachordals (*iv.*) are much nearer the notochord than in the last, and they are circular in section. Here, right and left, there is a curtain hung across the back of the *stomadæum* (*m.*), partly dividing it from *the archenteron.* This is the "velum" (*vl.*), (see also Plate 25, fig. 10, *vl.*).

In a partial section, intermediate between this and the last (Plate 25, fig. 2, 300 diams.), the cartilages (*iv.*) are further from the notochord, still oval in section, and three cells thick in their middle.

7*th Section* (Plate 24, fig. 7, 150 diams.).—This is in the fore part of the hind brain (C^3.) and the fore third of the auditory capsules (*au.*); it is in front of the meatus internus; the capsules are oval in section, are a little flattened on their inner face, and slightly tilted outwards, above.

This is between the large *pre-* and *post-*auditory nerve-ganglia. The narrow terete parachordals (*iv.*) are midway between the capsules and the enlarging notochord. The space below is now *pharyngeal* (*phx.*).

A similar, but partial, section made behind the entrance of the auditory nerve (Plate 25, fig. 3, 300 diams.), shows that the membranous labyrinth (*au.*) is covered with a thin (single) layer of cartilage cells. Here the moieties of the investing mass (*iv.*), or parachordal tracts, are crescentic, and cleave close to the membranous sheath of the notochord (*nc.*).

8*th Section* (Plate 24, fig. 8, 150 diams.).—The hind brain (C^3.) is becoming smaller, for this is at the back of the auditory capsule (*au.*). Below, between the membrano-cranium and the capsule, the ganglion of the vagus (X.) is cut across. Here, also, the investing mass is in crescentic sections, cleaving close to the enlarging notochord (*nc.*).

Part of a similar but partial section (Plate 25, fig. 4, 300 diams.) shows the relations of these parts, and the form of the double row of cartilage cells, right and left.

9*th Section* (Plate 24, fig. 9, 150 diams.).—This is a very instructive section, made through the first pair of branchial pouches and their openings at the end of the hind brain (C^3.) behind the auditory capsules, but through the vagus ganglia (X.). There is still a definite tract of parachordal cartilage (*iv.*) embracing the notochord (*nc.*), and considerable tracts of the first extra-branchial (see also Plate 25, fig. 8, *ex.br*^{1}.) come into view. On one side the top of the arch is cut through, where it runs in the longitudinal upper band, and the section of that band lies close to, but is independent of the investing mass (*iv.*). Part of the arch, above, is seen on the other side and on both sides the bounding bars of the external branchial opening—on one side directly across, and on the other obliquely; in the former (the left side of the figure) the inturned middle part of the main arch is seen for some extent, and then some of its lower part. The inner wall of the first pair of branchial pouches is cut through somewhat irregularly, a thing not to be wondered at, considering the *size* of the fish sectioned.

10*th Section* (Plate 25, figs. 5 and 6, 150 and 300 diams.).—This is through the *third* or *fourth* pouch and the middle of the thyroid involution (*th.*), at the junction of the hind brain (C^3.) with the myelon. The notochord (*nc.*) is very large, and has no investing cartilage on its sides. Considerable tracts of a pair of extra-branchials come into view in this section, which is only deficient by losing the bend at each shoulder and the lowest part of each bar in the region of the thyroid body (*th.*). This section shows well what a mere lateral figure (fig. 8) cannot so well, namely, the curious inflexion of the main band immediately behind the aperture; this, however, is clearly indicated in the figure of the transformed Lamprey's basket-work (Plate 18, fig. 1). This section shows that the junction of the main bar with the upper longitudinal band takes place close to the notochord, but is quite independent of it; where the shoulder is cut across (fig. 6), there the rod shows four cells, more or less overlapping each other. The other or upper end of the bar (fig. 6) must be considered as severed obliquely (see fig. 8), taking in both the longitudinal and the descending bar at once; the edge of the sectioned part passes a little into the interspace between the notochord and aorta (*nc.*, *ao.*).

If anything remains obscure after this description, I hope to make it plain in what follows, namely, an account of the cephalic and branchial skeleton of the fully-formed and full-sized larva or *Ammocœte*.

On the skeletal structure of large larvæ of Petromyzon fluviatilis.

In following these descriptions the reader is referred to the external figure also of the *Sand-pride* or Ammocœte (larva of *P. fluviatilis*, Plate 8, figs. 4–6).

A.—*Vertical section through head and branchial region of a larva* 6 *inches long.*

In this *diagrammatic figure* (Plate 25, fig. 10) we see the hooded pouch of the mouth (see also fig. 8, *m.*) formed by the upper lip (*u.lp.*), with its moss-like growth of *inner barbels* (*bb.*). Then there is the velum (*vl.*) separating the oral cavity (*stomadæum*) from the general pharyngeal space (fore part of *archenteron*); this velum, or right and left vertical curtain, is attached above under the fore part of the auditory capsule, and is, so to speak, a production of the inner edge of the hind part of the 1st visceral fold; and behind and external to it, in the re-entering angle between it and the cheek, is the rudimentary *non-perforate* 1st *cleft.*[*] Behind this part the seven branchial pouches ($br.p^1.$–$br.p^7.$) are laid open, and their main folds shown; behind them is the heart (*h.*); this section is just *beyond* the mid-line below, for it exposes the cavities of the pouches of the *right* side. Under the muscular segments we see the myelon (*my.*) in its theca, but the brain has been removed to show the auditory capsule (*au.*) and its meatus. In front of the capsule the trigeminal (V.) passes out, and behind it, the vagus (X.). In the middle of the space between the auditory capsule and nasal sac (*ol.*) the optic nerve (II.) escapes. There is only cartilage in the mid-line in front—the trabecular commissure or ethmoid (*eth.*); over it is the nasal sac (*ol.*) with its cartilaginous roof, and a pouch is growing downwards and backwards as the beginning of the "posterior nasal canal" (*p.n.c.*); the sac opens above as the external nasal aperture (*e.n.*). The notochord (*nc.*) bends downwards in front, and reaches to the pituitary region a little in front of the ear-capsules.

This sectional *plan* will enable us to understand what follows.

B.—*Dissected head of larval* P. fluviatilis, 6 *inches long.*

The *larva* is now eighteen times the length of the *embryos* just described, and that which strikes the eye at once is that the trabeculæ (Plate 19, figs. 4, 5) have united in front to form the rudimentary ethmoid, and that the cartilage is nearly all, in the cranium proper, of the hard kind—it has passed insensibly into this sort.

The cranium is now a long loop or *staple* of cartilage; the points of which, behind the ears, are soft, but the rest all hard cartilage. The fore end turns round by a sudden convexity (*eth.*), and the sides (*tr.*) are pushed in twice, first a little, where the optic nerves (II.) emerge, and then a second time, where they clamp the notochord (*nc.*) *near*, but not *at*, its narrow bluntly-pointed end. The lanceolate space thus enclosed is only properly *pituitary* just in front of the notochord, and under the greater, fore part, of this space a floor is formed, notched, behind, under which the posterior nasal canal is beginning to creep (see Plate 25, fig. 10, *ol.*, *p.n.c.*). The lesser oval space, behind, lies lower than the front part of the proper membrano-cranium, and this hind part is the pituitary region. The under surface of the conjoined trabeculæ (Plate 19, fig 4, *tr.*) is convex, but above (fig. 5), the bars, in their fore half, are tilted, and the upper edge is growing into a low crest. This crest becomes the ethmoidal and orbito-sphenoidal walls (see Plate 19, fig. 1, *eth.*, *o.s.*); it is not well seen except in ripe larvæ (Plate 19, fig. 5); in younger specimens (Plate 26, figs. 5, 6) there is merely a tilting of the bars which have a concave upper edge.

These rudiments of cranial walls are not seen in the alisphenoidal region, here the *walls* are entirely membranous before transformation. In the parachordal region

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[*] Professor HUXLEY (Proc. Roy. Soc., Vol. xxiii. (1874), p. 129) speaks of an external opening to this cleft, but neither BALFOUR, SCOTT, nor I can find it.

(*iv.*) the notochord is exposed both above and below, and rapidly enlarging, carries the attenuated bars at their soft ends far out from the mid-line.

Beneath the meatus auditorius internus each parachordal (fig. 4, *iv.*) gives off a small wing or flap which is continuous with the middle of the infero-internal face of the capsule (*au.*); fore and aft of these flaps there is a considerable chink, or *foramen lacerum*, over which the large ganglionic masses of the *pre-* and *post-*auditory nerves (V., X.) lie.

The chinks in front of the flaps are bounded by the outgoing *pterygo-pedicle* (*pd.*, *pg.*). This is, now, a notable narrow wing of hard cartilage, right and left, with a broad proximal part, a dilated sub-bilobate terminal part growing forwards and downwards, and a narrow arcuate stalk.

Compared with the recently transformed skull (same Plate, figs. 1–3), we see that this is the pedicle, with the pterygoid region *free*, as in a Salamandrian, but with a small bud of the future epi-hyal growing from its fore lobe, behind.

The olfactory roof (*na.*) is a broad semicircular plate, convex above and concave below, and with the external aperture (*e.n.*) in the round notch; it is composed of soft cartilage.

The auditory sacs (*au.*) are reniform with the *hilus* opening as the meatus internus; they are somewhat pinched in on both sides, but most on the inner, and their long axis is parallel with that of the skull; as before mentioned, they are fastened to the basal plate by a band of cartilage, right and left—the sub-auditory wings of the parachordals (*iv.*). The capsule has a small perforation under the Gasserian ganglion (fig. 4, *au.*, V.).

C.—*Dissection of a large larva of* P. fluviatilis, *showing the branchial basket-work.*

The preparation here figured (Plate 25, fig. 8) had the superficial parts moved from the branchial region, so as to show the whole basket-work of one side, and the heart. If this figure be compared with that showing the same parts in the young of *P. marinus* (Plate 18, fig. 1) we shall see the changes produced by metamorphosis in this part. Here there is no extra-hyal,* and no extra-pericardial skeleton, but seven sinuously-vertical bars, each passing behind a cleft or external branchial passage, and each united to its fellow, before and behind, by four longitudinal tracts. The whole structure is composed of soft cartilage, now much more consistent, and with larger cells, than in the embryo. The cartilage is of a peculiarly light and spongy kind, much like that described by me some years ago in the Tadpole of *Dactylethra.*

The *cervicorn* outgrowths seen in the transformed young are not present; the upper connecting band is nearly straight, the lower is formed of a series of small arches, and these are placed right and left, back to back (Plate 25, fig. 9).

The principal bends of the main arches are, *inwards*, behind each aperture, and

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* The loop of cartilage round the 1st gill-opening is lettered *ex.hy.* by mistake.

forwards, above and below it. The first bar encloses its aperture by a simple loop of cartilage; in the skin in front of this loop we see the fore end of the *branchial groove,* in which place I have, many a time, searched in vain for the 1*st cleft,* the hyomandibular; the first open passage is the hyobranchial, and thus the fore half of the first branchial pouch must be accredited to the hyoid region. In the Tadpole the hyoid has no gill, and its three clefts answer to the second, third, and fourth of the Lamprey; the former has *three* pouches, *four* extra-branchials, and *four* rudimentary ceratobranchials inside, which form an imperfect fringed floor or septum between the food-pharynx and the respiratory cavities (see Phil. Trans., 1881, Plate 1, fig. 4).

D.—*Vertically-transverse sections of the head and branchial region of a large* (*but not ripe*) *larva of* P. fluviatilis.

1*st Section* (Plate 19, fig. 6).—The first of these sections (see also Plate 8, figs. 4–6) is through the external nasal opening (*e.n.*), and the upper lip (*u.lp.*) with its undergrowth of mass-like barbels (*bb.*); the lips are of great depth, and the top of the face is concave.

2*nd Section* (Plate 19, fig. 7).—In this the nasal roof (*na.*) is cut across behind the opening. The top of the face is still more concave than in the last, and the whole section is deeper.

3*rd Section* (Plate 26, fig. 1).—The roof (*na.*) of the nasal capsule is cut through here so as to miss the median part. The capsule is more vertical than could be shown in the dissections (Plate 19, figs. 4, 5, *na.*), and the bilobate organ of smell (*ol.*) is laid bare; the other parts are similar to what is seen in the last.

4*th Section* (partial; Plate 26, fig. 2).—Here the hinder angles of the capsule (*na.*) are cut across, and the olfactory lobes (C^{1b}.) are seen in section; barbels are still seen under the hood-like upper lip. The fore part of the nasal canal (*p.n.c.*) is shown.

5*th Section* (partial; Plate 26, fig. 3).—This is very similar to the last, but further back: this shows the angles of the nasal cartilage (*na.*) for the last time, and the large olfactory crura (C^{1b}.) are cut across their middle.

6*th Section* (partial; Plate 26, fig. 4).—The fore brain (C^{1}.) and pineal body (*pnl.*) are here severed, and the skull in its front region; the section is through the fore part of the ethmoid (*eth.*), behind the nasal sacs; the barbels (*bb.*) of the upper lip (*u.lp.*) are becoming folds. The proximal part of the posterior nasal canal (*p.n.c.*) is circular at this point.

7*th Section* (partial; Plate 26, fig. 5).—This is close behind the last, and shows the fore part of the cranial trough formed by the ethmoidal commissure of the trabeculæ. The posterior nasal canal (*p.n.c.*) is now crescentic in section, with the concavity above.

8*th Section* (Plate 26, fig. 6).—Here the fore brain (C^{1}.) rests upon the trabeculæ (*tr.*), now distinct, but tilted, ready to begin the side walls of the skull in the ethmo-presphenoidal region. The hinder blind end of the budding "posterior nasal canal" (*p.n.c.*) is cut across; in fig. 6^a, (36 diams.), the lining of this pouch is shown to be composed of round cells.

9*th Section* (partial; Plate 26, fig. 7).—This section is through the fore brain (C^1.), and the fore part of the eye-balls (*e.*). The trabeculæ (*tr.*) are now short-oval in section, and are tilted inwards, above.

10*th Section* (partial; Plate 26, fig. 8).—This is through the middle of the eye-balls (*e.*), and is quite similar to the last.

11*th Section* (partial; Plate 26, fig. 9).—This is through the mid brain (C^2.), the back of the eye-balls (*e.*), the fore part of the Gasserian ganglia (V.) and of the notochord (*nc.*). Here the trabeculæ (*tr.*) are sharp above, and inturned; they are a considerable distance from the apex of the notochord (see Plate 19, figs. 4, 5).

12*th Section* (partial; Plate 26, fig. 10).—This is the first section through the hind brain (C^3.); it also passes through the fore part of the auditory capsules (*au.*), just behind their front face. There is here a rudiment of the alisphenoidal wall cresting the capsules on the inside, and their inner wall is complete, as this is in front of the "meatus;" externally, part of the shell of the fore face of the capsule is drawn. The cartilages (*iv.*) now approach the notochord (*nc.*), and they are winged at this point; the wings are the *pterygo-pedicles* (*pd.*, *pg.*), which arch gently over the buccal cavity and thicken, externally.

13*th Section* (partial; Plate 23, fig. 8).—Here we lose the alisphenoidal crest, and the transverse facial outgrowths; the inner wall of the capsule (*au.*) is still perfect; the basal plates or parachordals (*iv.*) are oval in section.

14*th Section* (partial; Plate 23, fig. 9).—This is through the *meatus internus* or internal auditory fenestra through which the 8th nerve passes. It also shows the partial junction of the capsules with the parachordals (*au.,iv.*); those bands are now of a considerable height, and are nearer the enlarging notochord (*nc.*).

15*th Section* (Plate 26, fig. 11).—By comparing this section with the dissected skull (Plate 19, figs. 4, 5) and basket-work (Plate 25, fig. 8), everything skeletal can be identified. It is close to the back of the head, the ganglia of the vagus (X.) are cut through, and also the narrow tracts of the basal plates (*iv.*) lying against the large notochord (*nc.*); the hind brain (C^3.) still comes into view. This shows the 1st branchial pouch (*br.*p^1.) in simple outline—the 1st branchial opening (*e.b.a.*) and the loop of cartilage (*ex.br.*) enclosing it in front of the 1st extra-branchial.

We are now in the front region of the "archenteron," which is a generalised pharynx (*phx.*) permitting the passage of both the food and of the water currents into the pouches.

16*th Section* (Plate 26, fig. 12.)—This is through the back part of the 1st branchial pouch and the extreme end of the hind brain (C^3.). There is no parachordal cartilage at this part, and the top of the 1st extra-branchial arch (*ex.br.*)

is shown on each side; the sinuosities of the bar cause it to be severed at many points,—*six* on each side. The enclosing bars, and the inbent form of the main bar behind the aperture, are well seen.

17*th Section* (Plate 26, fig. 13).—This is through the forepart of the myelon (*my.*), a curiously flat structure, convex above and concave below. This, which may be through the second or third pair of pouches, shows the thyroid body (*th.*) below. Under it the lower part of an extra-branchial (*ex.br.*) is cut across, and at the apertures the cross bars and part of the inbent main bar is seen. The theca vertebralis is much larger than is necessary for the myelon (*my.*), but it is not much greater in diameter than the notochord (*nc.*), with its thick membranous sheath. Under it we see the aorta (*ao.*), and outside it other large vessels.

Summary and Conclusion.

The remarks now to be made must be considered to be a continuation (with some repetition) of those given at the end of my Ist Part.

These Fishes appear to have been given up in despair by most recent Anatomists, and yet JOHANN MÜLLER laid a solid foundation for all his successors to build upon. Anyone may be proud to become a continuator of, and a commentator on, that great and almost unerring observer.

A continuator and a commentator worthy of him has, from time to time, thrown light upon these types. I refer, of course, to Professor HUXLEY, whose researches, however, needed, what they have now to a great degree obtained, namely, the corroborating and also the correcting light derived from Embrology. Following in the footsteps of the lamented CALBERLA, Mr. W. B. SCOTT has done, and is still doing, excellent work in this way, but (before our great loss) Professor F. M. BALFOUR had shed most welcome light upon this subject.

The problem of the morphology of this group has been to me for years an irritating opprobrium, on account of its apparent insolubility; and I have again and again made advances upon it, sideways, or crabwise, fresh from the contemplation of more normal types. Those *Ichthyopsida* that undergo metamorphoses after hatching, and thus have a *larval stage* during a longer or a shorter time, are of the most service in this matter. Hence I infer that the Marsipobranchs belong to a low grade, and that even supposing the Myxinoids to be degenerate descendants of some more developed form, yet I feel quite certain that such an *archaic non-degenerate Myxinoid* would lie far below our known existing Fishes, such as the Elasmobranchs, Ganoids, and Teleosteans. But the three groups just mentioned lie far out of the way of the Marsipobranchs. The early stages of the *Anurous Batrachia* give us the best clue to them, and far off as they are from them, they are indeed their nearest existing relatives. What the early stages of the "Chimæroids" and "Dipnoi" would show, it is impossible to say; unfortunately, their early stages are not known.* With

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* See Note to p. 411.

regard to the question as to what a truly Archaic Vertebrate would be like, I feel confident that the early stages of a Myxinoid would yield us the most trustworthy evidence. Those Fishes (*Myxine* and *Bdellostoma*) are not, even in their adult condition, Vertebrata, if we speak by the letter. They are mere "*Craniate Chordata*," for, throughout life, they, like the temporary *Ammocœte*, or larval Lamprey, show no traces of cartilage in their *spinal region*, although they have a solid, complex, generalised, but rudimentary cranium. It is easy to see that the Ammocœtine form of Fish is the platform, so to speak, on which both the Myxinoids and Petromyzoids are built, and although the former rise to a lesser height above that platform, yet they do undergo a large amount of metamorphosis and are marvellously specialised in their own peculiar way. So true is this, that in the burden now, at last, laid upon me, namely, the interpretation of the Mammalian skull and the searching out the pattern of the *Prototheria* or primary Mammalia (and, if the thing be possible, getting some light upon their ancestors, the imagined *Hypotheria*), I know of no types among the *Ichthyopsida* so likely to help me in this dark work as these same Myxinoids.*

This we may say, namely, that the Myxinoids, Petromyzoids, and Tailless Amphibia, are three groups more nearly related to each other than to any known *Ichthyopsida*, and yet are far apart, in reality.

Roughly speaking, in spite of the gap made by the extinction of possibly many hundreds of *genera*, the adult Myxinoid may be said to represent a larval Petromyzoid, and the adult Petromyzoid a larval Batrachian (Frog or Toad). Indeed, these three groups might be studied in their special structure and in their structural relations to each other without the other *Ichthyopsida* once coming into mind. A knowledge of the latter does, indeed, help in this business, but they lie far out of the way, and have a specialisation and a *finish* in their structure for which we look in vain in the *permanent* and *temporary* Marsipobranchs.

Even the *Chimæroids* come so near the ordinary Elasmobranchs as to suggest that their embryology would not be so helpful as one might imagine, especially if their solid upper face has been acquired as a *secondary* modification and not a *primary* condition such as we see in the Tadpole, which is especially solid and largely continuous with the basis cranii, in the larval Aglossal types, *Dactylethra* and *Pipa*.†

Sharks, not *Skates*, retain *remnants* of the bars of the extra-branchial basket-work of

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* The reader will see that I am thinking of Professor HUXLEY's masterly paper—"On the Application of the Laws of Evolution to the arrangement of the Vertebrata, and more particularly of the Mammalia" (Proc. Zool. Soc., 1880, pp. 649–662). The author of that paper has unconsciously set the writer his work for some years to come; the "one thing" for him is to become the *continuator* of, and *commentator* upon, him who made that daring outline.

† The interposition of those remarkable Sharks, *Cestracion* and *Notidanus*, between the ordinary kinds and the Chimæroids, makes the likelihood of the solidity of the upper jaw being *primary* a very doubtful thing; I once thought otherwise, but found Mr. BALFOUR strongly set against me in this suggestion.

the Lamprey; Tadpoles show *rudiments* of the intra-branchial arches of Sharks and Skates. By far the most generalised condition of the branchial skeleton is seen in the Tadpole of the Aglossal *Dactylethra*, and the cartilage of which it is composed is similar to that of the embryo Lamprey, 7 or 8 millims. in length. In that type and in the other Aglossal form, *Pipa*, the membranous space between the *palato-quadrate arcade* and the *trabecula* is almost absent. Yet in these, as in all the Anura, there is in the 1st visceral arch a cerato-branchial element, a free Meckelian (mandibular) cartilage. Professor HUXLEY and I agree in thinking that we have found a true mandibular rudiment in the Lamprey after transformation; certainly there is none in the Myxinoids; and the *quadrate region* (*and condyle*) is quite suppressed in both the types of the Marsipobranchs (Myxinoids and Petromyzoids). Moreover, the place where this tract *should* be, is not where we find it in the larval condition of the Frog or Toad, namely, at the fore part of the face; but under the exit of the 5th nerve, where we find it in transforming Tadpoles, whose tail has become a mere stump.

This is easily accounted for; in the Tadpole the small divided suctorial (lower labial) cartilage is carried by the mandibles—it is fixed between them; these are its *arms*. During transformation, the lower lip of the Lamprey shoots forwards, and instead of lying back, as it did, under the hood-like upper lip, grows to the fore-front, and has the upper lip merely lying on the top of its upper rim in front; thus this production of the *post-oral* lip becomes the foremost part of the head.

Hence anything corresponding to mandibles in the Lamprey are carried far away from their *pier* or "suspensorium;" thus the hinge part of that pier is suppressed. The labials of the Lamprey correspond to the *temporary* labials of a Tadpole; the cartilages that appear in that region (above), during transformation, correspond to what is seen in the upper fore-face of a Shark or Skate.

The palato-quadrate arcade, with no *quadrate lobe*, and the distal cartilages attached to the suctorial disk, are all that can be accredited to the 1st visceral arch of the Lamprey. Its 2nd has no "pharyngo-hyal" element, but the lower part grows directly out of the back of the arrested suspensorium (part of the 1st arch); yet the lateral, sub-distal, and basal parts are well developed as the lingual skeleton. The only rudiment of an *intra*-branchial, behind the hyoid, is the hinder part of the large continuous basal bar ("lingual cartilage"); all the rest is *extra-visceral*, and after transformation the hyoid arch acquires an outer band of this nature. How the Myxinoids differ in these respects I have already shown, but their *intra*-visceral basket-work is dissociated from their branchial pouches, which are carried far back under the spine. Then, in their mouth, not circularly suctorial, as in the Lamprey, the dental armature, and its *special buccal skeleton*, is a huge development of parts, the like of which are feeble and subsidiary in the Lamprey.

The Myxinoids do transform beyond the Ammocœtine stage, but they appear to have suffered some deflection during the process, and to have been stopped in their attempts to rise to a higher platform; yet, full of metamorphic force, they have spent their energy on the old low level; they are, as it were, highly modified Ammocœtes, doomed to much morphological arrest, and to some degree of degradation of parts, once promising a higher development.

A study of their development will show whether these suggestions are true or untrue.

List of Abbreviations.

N.B.—Many of the abbreviations are the same as in Part I.; the following are additional.

ao.	Aorta.
al.s.	Alisphenoid.
b.p.	Buccal pouch.
br.a.	Branchial artery.
br.c.	Branchial canal.
br.p′.	Branchial folds.
c.tr.	Cornu trabeculæ.
e.ba.	External branchial aperture.
ex.br.	Extra-branchial.
ex.hy.	Extra-hyal.
i.b.a.	Internal branchial opening.
i.v.s.	Intervelar shelf.
l.d.m.	Lateral distal mandibular.
l.l.	Lower labial.
l.lp.	Lower lip.
l.t.	Lingual (and labial) teeth.
m.d.m.	Median distal mandibular.
n.a. (and *v.a.*)	Neural (or vertebral) arch.
or.m.	Orbital muscles.
o.s.	Orbito-sphenoid.
o.s.f.	Orbito-sphenoidal fenestra.
py.	Pituitary space.
s.d.	Sucking disk.
tg.	Tongue.
th.	Thyroid body.
u.l.	Upper labial.
u.lp.	Upper lip.
v.a. (and *n.a.*)	Vertebral (or neural) arch.

DESCRIPTION OF THE PLATES.

Plate.	Fig.		Number of times magnified.
18	1	*Petromyzon marinus;* one-third grown; cranio-branchial skeleton; side view	$3\frac{1}{5}$
,,	2	The same; lower view	$3\frac{1}{5}$
,,	3	Fore part of same; upper view	$3\frac{1}{5}$
,,	4	The same; lower view	$3\frac{1}{5}$
,,	5	Vertical section of the same object	$3\frac{1}{5}$
,,	6	Lingual cartilage of the same specimen; lower view .	$6\frac{2}{5}$
,,	7	The same; upper view	$6\frac{2}{5}$
,,	8	Part of the same; side view	$6\frac{2}{5}$
19	1	*Petromyzon marinus;* recently metamorphosed (5 inches long); skull; upper view	12
,,	2	The same; lower view	12
,,	3	The same (section); inner view	12
,,	4	*Petromyzon fluviatilis;* large larva (6 inches long); skull; lower view	15
,,	5	The same; upper view	15
,,	6	*P. fluviatilis;* a younger larva, but of nearly the same size; *first* of a series of vertically-transverse sections through fore part	12
,,	7	The same; *second* section	12
20	1	*Petromyzon fluviatilis* (adult); *first* of a series of vertically-transverse, thin, transparent sections	5
,,	2	The same; *second* section	5
,,	3	The same; *third* section.	5
,,	4	The same; *fourth* section	5
,,	5	The same; *fifth* section	5
,,	6	The same; *sixth* section	5
,,	7	The same; *seventh* section	5
,,	8	The same; *eighth* section	5
,,	9	The same; *ninth* section	5
,,	10	The same; *tenth* section.	5
,,	11	The same; *eleventh* section	5
,,	12	The same; *twelfth* section	5

Plate.	Fig.		Number of times magnified.
21	. .	(*As in last Plate.*)	
,,	1	The same; *thirteenth* section	5
,,	2	The same; *fourteenth* section	5
,,	3	The same; *fifteenth* section.	5
,,	4	The same; *sixteenth* section	5
,,	5	The same; *seventeeth* section	5
,,	6	The same; *eighteenth* section	5
,,	7	The same; *nineteenth* section	5
,,	8	The same; *twentieth* section	5
,,	9	The same; *twenty-first* section.	5
22	. .	(*As in last Plate.*)	
,,	1	The same; *twenty-second* section	5
,,	2	The same; *twenty-third* section	5
,,	3	The same; *twenty-fourth* section	5
,,	4	The same; *twenty-fifth* section	5
,,	5	The same; *twenty-sixth* section	5
,,	6	The same; *twenty-seventh* section	5
,,	7	The same; *twenty-eighth* section	5
,,	8	The same; *twenty-ninth* section	5
,,	9	The same; *thirtieth* section.	5
23	1	*Petromyzon fluviatilis* (adult); solid vertical section of fore part	2½
,,	2	The same; solid horizontal section of head.	2½
,,	3	The same; another similar section made on a higher plane	2½
,,	4	*Thirty-first thin* section (as in Plates 20–22)	5
,,	5	*First* similar solid section of a larger specimen	3
,,	6	The same; *second* section	3
,,	7	The same; *third* section.	3
,,	8	*Petromyzon fluviatilis* (larva, 6 inches long); *thirteenth* section (as in Plate 26)	12
,,	9	The same; *fourteenth* section	12

Plate.	Fig.		Number of times magnified.
24	1	*Petromyzon planeri* (embryo, 9·5 millims. long); *first* of a series of vertically-transverse sections of cephalic and branchial regions	150
,,	1A	Part of a similar section of an embryo 7·8 millims. long	300
,,	2	*Second* section of larger embryo	150
,,	2A	A similar section of smaller embryo	300
,,	3	*Third* section of larger embryo	150
,,	4	The same ; *fourth* section	150
,,	5	The same ; *fifth* section	150
,,	6	The same ; *sixth* section.	150
,,	7	The same ; *seventh* section	150
,,	8	The same ; *eighth* section	150
,,	9	The same ; *ninth* section	150
25	. .	(*Partly as in last Plate.*)	
,,	1	Part of the same section as in Plate 24, fig. 5. . . .	300
,,	2	Another partial section from the same series between figs. 5 and 6 of Plate 24	300
,,	3	Another partial section between figs. 7 and 8 of Plate 24	300
,,	4	A similar section near fig. 9, Plate 24	300
,,	5	Same series ; a complete section through branchial region and thyroid body	150
,,	6	Part of last section	300
,,	7	Skull of smaller embryo (7·8 millims. long) of *P. planeri* ; lower view	150
,,	8	*Petromyzon fluviatilis* (larva, 6 inches long); dissection of branchial region ; side view	5
,,	9	Part of same ; lower view	5
,,	10	The same ; vertical solid section of head and branchial region	5

Plate.	Fig.		Number of times magnified.
26	..	*Petromyzon fluviatilis* (larva, 6 inches long)	
,,	1	*Third* of a series of thin vertically-transverse sections through head and branchial region	12
,,	2	The same ; *fourth* section	12
,,	3	The same ; *fifth* section	12
,,	4	The same ; *sixth* section	12
,,	5	The same ; *seventh* section	12
,,	6	The same ; *eighth* section	12
,,	7	The same ; *ninth* section.	12
,,	8	The same ; *tenth* section.	12
,,	9	The same ; *eleventh* section.	12
,,	10	The same ; *twelfth* section	12
,,	11	The same ; *fifteenth* section.	12
,,	12	The same ; *sixteenth* section	12
,,	13	The same ; *seventeenth* section	12

XI. *On the Organization of the Fossil Plants of the Coal-Measures.*—Part XII.

By W. C. Williamson, *LL.D., F.R.S., Professor of Botany in the Victoria University, Manchester.*

Received May 4,—Read May 11, 1882.

[Plates 27–34.]

In Part IX. of this series of memoirs (Phil. Trans., Part II., 1878, pp. 319–322) I described, under the generic name of *Astromyelon,* a series of stems, transverse sections of which might readily be, and for some time were, mistaken for sections of Calamites; but I pointed out the differences which separate widely these two types of stem. Attention was directed to the almost universally decorticated condition in which the *Astromyelons* were found; the only exceptions being one or two examples (*loc. cit.*, Plate 19, fig. 7, *d*), in which the vascular zone was surrounded by a thin cellular layer.

At the recent meeting of the British Association at York, Mr. Cash, of Halifax, and Mr. Hick, of Harrogate, described a remarkable stem, discovered by Mr. Binns in the Halifax beds, and to which they gave the name of *Myriophylloides Williamsonis.*[*] I shortly afterwards suggested the desirableness of substituting the generic name of *Helophyton* as preferable to that of *Myriophylloides*, for reasons given in the note in which this suggestion was made.[†]

I subsequently received a letter from Mr. Spencer, of Halifax, telling me that he had obtained additional specimens of the new plant, which convinced him that it was a corticated state of the Oldham *Astromyelon* which I had previously described. All the specimens discovered up to the present time having been placed in my hands by my excellent auxiliaries, and undergone a careful investigation, I have no doubt about the correctness of Mr. Spencer's conclusion.

The plant proves to be a much more varied and remarkable one than I had previously thought.

The simplest form in which it has hitherto been met with is represented in fig. 1. This specimen, which is black and carbonised, is a transverse section of an axis which differs very widely from the ordinary forms of the Oldham *Astromyelon.* In its centre,

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[*] This interesting communication has since been published in the Proceedings of the Yorkshire Geological and Polytechnic Society, vol. vii., part iv., p. 400, 1881.

[†] 'Nature,' Dec. 8, 1881, p. 124.

a, we have a somewhat confused admixture of cells and vessels, constituting a central axis, surrounded by an inner cortical zone, *b*, of radially compressed cells, arranged in an approach to regular circles, and passing outwards into a less regularly disposed series of much larger cells, *c*. From these latter, isolated lines of cells, *d*, radiate, chiefly in single rows, like the spokes of a wheel. The nave of this wheel is composed of a thick zone, *e*, of large and irregular parenchymatous cells. The drawing is enlarged 70 diameters—the maximum diameter of the section being about ·08 of an inch.

Fig. 2 represents the transverse section described by Messrs. CASH and HICK at York, enlarged 55 diameters. It presents very different features from those of fig. 1 so far as its central portion is concerned. At *a* we have a large medulla, about ·04 of an inch in diameter, composed of very regular parenchyma, and closely resembling that of the Oldham *Astromyelons*. The central cells are much larger than the peripheral ones, some of them being ·005 in diameter. This medulla is surrounded by narrow, imperfectly developed, and somewhat irregular zone of vessels, *b*, arranged in radiating lines. The concentric disposition of the cells of the innermost bark, *c*, is well shown in this section, as are the radiating lines of cells, *d*, connecting this inner bark with the peripheral parenchyma *e*. Though these lines of cells have been somewhat disarranged by external pressure, we have no difficulty in tracing a continuous zone of them from *d′* to *d″*.

Fig. 3 represents a very important section for which I am indebted to Mr. SPENCER: important because it demonstrates the identity of my *Astromyelon* and the *Myriophylloides* of CASH and HICKS in the most decisive manner. In my description of the former plant I directed attention to some features very characteristic of it. The chief of these was the distinct outline and peculiar form of the conspicuous vascular wedges constituting the exogenous zone (*loc. cit.*, Plate 19, fig. 1, *b*), and the relations of these wedges to the large medulla which that zone enclosed. The laminæ of each wedge, *b*, converge at their medullary extremity, where there exists small groups of what, in my previous memoir, I designated " a few vessels of somewhat larger size than those composing the rest of the vascular zone" (*loc. cit.*, p. 320). I cannot quite satisfy myself whether these are actually vessels or liquified cells.* In either case they bear a definite relationship to the vascular wedges with which each of the several small clusters of them is associated. In fig. 3 the characteristic medulla is conspicuous at *a*. The clusters of cells or vessels just referred to are seen at *a′*, and the distinctive forms of the vascular wedges are not difficult to trace, as is shown at *b*. The exogenous zone has undergone a much greater development in this specimen than in fig. 2. The identity of the plant with the *Myriophylloides* of CASH and HICKS is shown by the retention of a small portion of the cortex. The innermost layer is imperfectly retained at *c*, but the radiating cellular laminæ are sufficiently distinct at *d*, terminating in the outer layer of cortical cells at *e*.

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* I have since obtained absolute proof that these are really vessels which may be regarded as constituting a medullary sheath.—July 6th, 1883.

Fig. 4 represents a section for which I am indebted to Messrs. CASH and HICKS. Virtually a vertical one, it has passed obliquely through the outer cortical parenchyma at its upper part and traversed the medulla at its lower end. The medullary cells are seen at *a*, corresponding closely with those represented by fig. 3, *a*, in Plate 19 of my Memoir IX.* The vessels of the exogenous zone are traversed almost radially at *b*, and more tangentially at *b′*. We next have the long, narrow, square-ended cells of the innermost bark at *c*. Above this tissue we obtain double light respecting the structure of the spokes of our vegetable wheel seen in the transverse sections. We learn that these radiating lines of cells are merely the transverse sections of long, vertical, cellular, radiating laminæ, *d*, which separate large intercellular lacunæ, *d″*, whilst at *d′*, *d′* we learn that the constituents of these laminæ are very regularly disposed cells elongated radially and having a mural arrangement. The cells of the outer cortical parenchyma, *e*, present no special peculiarities. The only additional feature noticeable in this section is a small bundle of vessels passing laterally outwards at *f*.

The three sections last described manifestly belong to the same plant in different stages of development; but I have now to direct attention to another series of specimens in my cabinet, some of which were first discovered by Mr. SPENCER, but to which others have been added by Mr. BINNS. So far as their cortical structures are concerned they are absolutely identical with those already described. Thus in each of the transverse sections, figs. 5 and 6, we have the inner cortical layer at *c*, the radiating laminæ at *d* enclosing the lacunæ at *d″*, and the outer cortex at *e*. The differences are seen in the structure of the vasculo-medullary axis—and especially of its central portion. This appears in both the sections figured to be largely if not entirely vascular, and others in my cabinet exhibit the same characteristic aspect. The exogenous vascular zone, *b*, also exhibits much less definitely the grouping of the radiating vascular laminæ into distinct wedges than is the case with the section, fig. 3. At the same time we fail to discover that grouping even in fig. 2, though the section there represented possesses the large cellular medulla so characteristic of fig. 3. In fig. 6 we find the lacunæ and radiating cellular laminæ replaced at *d‴* by a mass of coarse cellular parenchyma, and from which a vascular bundle is seen emerging at *f*.

Fig. 7 is an instructive section for which I am indebted to Mr. SPENCER. It appears to combine features seen in figs. 5 and 6 with others seen in fig. 1, and is further valuable since it illustrates the strong tendency to develop branches which seems to characterise this plant.

At the upper extremity, A, of the figure we have a nearly transverse section of an axis which has corresponded closely with fig. 1. At *a* we have an axial cluster of vessels, not arranged in any regular order, or surrounded by an exogenous zone. At *c* are the narrow elongated cells of the innermost cortex, intersected obliquely, whilst

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* I may add that I have a true vertical section of another specimen of the new plant which is almost a facsimile of the main axis of fig. 2 in Plate 19 of my ninth memoir already referred to.

the radiating cellular laminæ are seen at *d* enclosing the large lacunæ at *d''*; these, in turn, being invested by the thick outermost parenchymatous cortex, *e*.

At the opposite end, B, of the section we find another branch, intersected very obliquely, and which is evidently tending outwards, in opposite directions. We here discover a central vascular axis, *a*, dividing dichotomously into a larger one, *a'*, and a smaller one, *a''*. These branches are successively surrounded by the inner bark, *c*, the radiating laminæ, *d*, with their enclosed lacunæ, *d''*, and the outermost cortex, *e*. At C a yet smaller vascular bundle, *a'''*, is passing laterally outwards. We thus have four branches passing in as many separate directions in this one specimen. None of these branches display any indication of a distinct central medulla, though cells appear to intermingle with the irregularly grouped vessels at A, *a*. It is thus clear that this section belongs to a plant in which, as in fig. 1, we have axial vascular bundles unsurrounded by an exogenous zone, whilst in figs. 5 and 6 we have the same plant in which such a zone is fully developed.

The vessels of the vascular bundles present some peculiarities, examples of which are represented in figs. 8–14. In describing the Oldham *Astromyelon* in my memoir, Part IX., I mentioned the extreme indistinctness if not almost entire absence of all traces of structure in the walls of these vessels, though in some few there were suggestions that they had been barred. The Halifax specimens differ from the Oldham ones in this respect, though there is room for doubting whether or not the latter shows fully their original nature. In all the figures, from 8 to 13 inclusive, a considerable, and often by far the greatest, part of the walls of the vessels are homogeneous and structureless. Thus in fig. 8 we have a single vertical series of small translucent, slightly oval, areolæ, the longer axes of which cross the vessel somewhat obliquely. In fig. 11 we have a similar arrangement, only the areoles are still more oblique, and more elongated transversely. In fig. 9 the areolæ are still small but almost circular. In fig. 10 the areolæ are more irregular both in size and number, but they still occupy the central area of the vessel. In fig. 12 the larger vessel approaches more nearly to an ordinary reticulated modification of the scalariform type of vessel, whilst in the smaller one the areolæ are larger in proportion to the diameter of the vessel than in most of the other examples, but in both the striking obliquity of the areolæ, seen in figs. 8 and 11, is again observable. The forms here described are those which characterise all the specimens figured, as well as others in my cabinet, except fig. 7; the vessels in the branching vascular bundle, B, *a*, of that section resemble figs. 13 and 14, which approach nearer to the barred type ordinarily met with amongst the fossil plants of the Coal-measures; but even fig. 14 lacks the extreme regularity which ordinarily characterises these barred forms. Do these figures represent the actual state of these vessels when living, or has mineralisation destroyed the details of the structure of their walls save along certain lines? Such specimens as figs. 8, 9, and 11 are so peculiar that I incline to adopt the former explanation, and the more so because at some future period I shall have to call attention to some similarly remarkable vascular tissues which prevail in the mineral charcoals so abundant in the coals of the Carboniferous rocks.

All the specimens now described are from the Halifax beds; but I have received from Mr. BUTTERWORTH a specimen from Oldham in which the bark, though in a very imperfect state, presents the essential features of the Halifax examples.

The above descriptions show that we either have one very variable plant characterised by a bark in the middle of which are numerous large lacunæ, separated from one another by vertical radiating cellular laminæ, or that we have two or more distinct plants which have the same peculiar form of bark, but which differ in the organization of their central vasculo-cellular axes. I am inclined to regard the former of these alternatives as affording the true interpretation of these specimens. I think there can be no doubt respecting one conclusion suggested by the peculiar structure of the bark, viz.: that it indicates a plant of more or less aquatic habits. A similar structure is found in several flowering plants such as *Myriophyllum* and the petioles of *Aponogeton*. It reappears in the *Marsileæ* and *Pilularia*, amongst Rhizocarps, and it is not essentially dissimilar from that of the living Equisetums. That the plant was a Phanerogam is most improbable. It differs alike from the recent Equisetums and the fossil Calamites in the entire absence of nodal joints and medullary phragmata. The question suggests itself, may it possibly be a representative of the Marsileaceæ?

I have examined many living species of *Marsilea*, and find that on making transverse sections of their rhizomes, not too far from their growing tips, we obtain results not dissimilar from those seen in fig. 7. We constantly obtain three different, but organically united, sections. One of these is that of the main stem, in which a horseshoe-shaped vascular bundle approaches so nearly to a perfect circle as almost to enclose a central cellular axis. Another is that of the base of a leaf-petiole. In this the fibro-vascular bundle is V-shaped, as in the secondary petioles of so many ferns. The third is a root-section in which the vascular bundle is central, and enclosed within those circular zones of cortical cells that are so common in the root-structures of Cryptogams. The existence of three such dissimilar structures in one section of a stem suggests the possibility of a similar explanation being applicable to the plant under consideration. The absence of the exogenous zone from the recent *Marsileæ*, whilst it is conspicuously present in our fossil, does not militate against my suggestion, since in this, as well as in other features, the *Astromyelon Williamsonis* does not diverge more widely from the living *Marsileæ* than the arborescent Lycopods of the Coal-measures do from their dwarfed living representatives, or than the Equisetums do from the Calamites.

Fig. 15 represents a section of a small organism from Halifax, ·02 of an inch in diameter, for which I am indebted to Mr. SPENCER. It may possibly belong to the plant just described, but this is doubtful. It consists of a central area, which is divided by a crucial arrangement of cells, *a*, into four lacunæ, *b*. The rest of the organism consists of a mass of parenchymatous disarranged cells of various shapes and sizes, many of which are filled with black carbonaceous matter, *c*. I can detect no vessels in the section. The division of the central portion into four lacunar cavities reminds us of the similar divisions in the leaves of *Isoetes*.

Psaronius Renaultii.

In my memoir, Part VII., pp. 10–13, I described, under the above name, some sections which I believed to belong to an arborescent fern, and in fig. 22**, *d*, I represented the only trace of a vascular bundle hitherto met with belonging to this plant. Mr. GEORGE WILD, of the Bardsley Collieries, Ashton-under-Lyne, has since furnished me with a much finer specimen, a section of which is represented in fig. 16. The vascular bundle, *a*, is quite perfect, and corresponds both in shape and size with several of those represented by CORDA, whilst the fragments of a second similar bundle at *a′* show that the section is but a portion of a large stem in which there has been at least one circle of similar bundles. Each of these vascular zones is enclosed within a thin phlœm sheath, *b*, whilst the fundamental tissue exhibits the small gum-canals, *c*, already described; some of these, *c′*, are filled with carbonaceous matter. The size of the bundle at its greatest diameter is 1·1, the figure being enlarged about 2½ times.

Zygosporites.

In my memoir, Parts IX. and X., I described some minute objects under the above name, and whilst recognising their striking resemblance to the zygospores of several Desmideæ, I declined to follow some of my French friends who regarded them as being true Desmids. A discovery by Mr. SPENCER in the Halifax beds has justified my doing so.

Fig. 17 represents an oblong sporangium containing several of these *Zygosporites* under conditions which leave no room for doubt that they are true spores. The sporangium is about ·042 in length; fig. 17 being enlarged about 110 diameters. Fig. 18 represents the portion containing the zygospores, enlarged about 290 diameters. The spore, fig. 18, *a*, is obviously identical with the form which I designated *Z. brevipes;* but the peripheral radii in fig. 18, *b*, are longer than in my fig. 51 (Memoir, Part X., Plate 19), and approach, in that respect, fig. 54 of the same plate, to which latter form I gave the name of *Z. longipes,* indicating that these are but extreme forms of one species. Other fragments now in my cabinet leave little, if any, doubt in my mind that these spores are identical with those of the fruit figured in my memoir, Part V., Plate 5, figs. 28, 29, and 30, under the name of *Volkmannia Dawsoni.** Under these circumstances the provisional name of *Zygosporites* may be can-

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* This fruit was assigned to the genus *Volkmannia* at a time when that genus was much clearly less defined than it now is. BRONGNIART, going further even than had previously been done by UNGER,

celled, as no longer needed, the spores to which it was applied being recognized as those of *Volkmannia Dawsoni.*

Calamites.

Our knowledge of the structure of the cortical tissues of Calamites is yet very imperfect. A specimen from the Gannister bed at Moorside, near Ashton-under-Lyne, for which I am indebted to Mr. GEORGE WILD, supplies a new fact. In the memoir, Part IX., figs. 8–10 and 13, I have represented the primitive undifferentiated state of the cortical parenchyma of Calamites; and in figs. 14 and 15 of the same memoir, I have shown that a thick prosenchymatous layer was formed in the bark of some older stems. In Mr. WILD's specimen, fig. 19, the vasculo-medullary axis presents the usual features, except that the vascular wedges are more detached than ordinary owing to the partial disappearance of the cells of the primary medullary rays. The innermost bark, *a*, differs but little from the primitive state seen in the figs. 8–10 and 13 just referred to; but at *b* we have a hypodermal zone of specialised bundles of what seem to have been prosenchymatous cells. Each bundle has a triangular section, the apex being centripetal, and is from ·012 to ·008 from base to apex. The outermost bark, *c*, appears to have been a thin epidermal layer, some ·003 to ·002 in thickness. The peripheral border of this zone is sharply defined and entire, being wholly devoid of the crenulated contour so long supposed to have characterised the exteriors of sections of the Calamites. Every definite fact hitherto discovered demonstrates that those outer surfaces possessed neither longitudinal flutings nor nodal constrictions. Whether the fibrous bundles just described are the beginnings of the prosenchymatous zone shown in figs. 14 and 15 of my ninth memoir, or whether they are peculiar to some special form of Calamite, cannot now be determined. The specimen described has a diameter of ·25.

Lepidodendroid plants.

In several of my memoirs I have called attention to the gradual growth that took place in the diameter of the Lepidodendoid medulla owing to the multiplication of the medullary cells, and also to a contemporaneous increase that took place in the size of the surrounding non-exogenous vascular cylinder or "étui medullaire" of BRONGNIART, as well as in the number of its component vessels. Hitherto, however, I have failed to discover any example of *natura naturans* in either of these respects. But I found in a collection of sections submitted to me by Mr. NORMAN, of City Road,

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united *Volkmannia* with *Asterophyllites*, believing the former to be the fruiting branches of the latter. But the limits and distinctive characteristics of the genus *Asterophyllites* are themselves undefined, and, as yet, *sub judice*. Since I cannot identify my *Volkmannia Dawsoni* with any of the genera recognized by SCHIMPER or by Professor WEISS, of Berlin, and I have not, as yet, obtained so accurate a knowledge of the orientation of the sporangiophores as would alone justify me in making it the type of a new genus, the strobilus may be left provisionally where I placed it in my previous memoir.

London, a section of a *Lepidodendron* of the type of *L. Harcourtii*, in which nearly every one of the cells is in a state of simultaneous meristemic division. Fig. 20 represents a small portion of this section. The entire stem has a diameter of about 1·75; the vascular cylinder is about ·37 and the medulla about ·3 in diameter. Fig. 20, *a*, are the large vessels of the vascular cylinder. At *b* we have the thick, older, cell-walls of the medullary cells, whilst at *c* we have the newly formed septa by which each older cell is becoming divided into two or more new ones. The older cells exhibit the form of regular parenchyma; the new ones are extremely irregular in size and form, and would obviously require to undergo a considerable expansion, leading to a steady enlargement of the entire medulla, before they attained to the regular forms of the parent cells. Every fact observed thus far indicates that the vascular cylinder, *a*, develops centripetally, and that the multiplication of the medullary cells here demonstrated to have existed was preparatory to the conversion of the outermost of them into new vessels. The known facts of enlargement in the sizes of and in the number of vessels composing the vascular cylinder thus receives a probable explanation.*

Halonia.

In my memoir, Part II., p. 222, *et seq.*, I described the organization of some *Haloniæ* in which the central axis consisted, as in most of the young Lepidodendroid branches, of a central medulla surrounded by a vascular, non-exogenous cylinder. Thanks to Professor YOUNG and Mr. J. YOUNG, of the Glasgow University, I am now able to describe another interesting form of *Halonia* from the Arran deposits of Laggan Bay. In my descriptions of the Lepidodendroid plants from that locality (Memoir, Part X., p. 494) I pointed out that in *all the small* Lepidodendroid twigs, occurring so abundantly in those beds, the central axis was a solid, non-cylindrical, bundle of vessels (*loc. cit.*, Plate 14, figs. 1 and 2). Fig. 21 represents a section of the Arran *Halonia* which has a mean diameter of about ·87—a small portion of its peripheral cortical tissue having disappeared. The central axis, *a*, consists of a solid rod of barred vessels—resembling, in this respect, the young twigs with which it was found associated in the Arran beds. The surrounding cortical layers consist, as usual, of an innermost bark at *b*, composed of rather compressed but regular parenchyma, the cells of which are small, averaging about ·0012 in diameter. The space, *c*, seems to have been occupied by very similar cells, only a few of which remain. At *d* we have a middle bark, composed of larger and coarser parenchymatous cells, and at *e* is the usual prosenchymatous layer, the small cells of which are arranged in radiating series. In the specimen from which my sections were prepared the protuberances so charac-

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* Though this addition to the number of the vessels appears to be made at the centripetal border of the vascular cylinder, *a*, it must not be supposed that this addition reduces the diameter of the medulla. Unlike what occurs amongst Phanerogams, the medullæ of many *Lepidodendra* obviously continue to enlarge long after the development of the exogenous zone.—July 8, 1883.

teristic of the *Haloniæ* are sufficiently conspicuous, and in the sections we can readily distinguish between the vascular bundles going off to these protuberances, and those supplying the leaves. Three of the former are seen at *f*, *f*, *f*. They have a mean diameter of about ·004. In the case of the *Halonia* previously described, the corresponding bundle was formed by the detachment of a small but complete segment of the vascular cylinder leaving a break in the continuity of the latter. In the present case the bundle is formed in the way represented in the three transverse sections, figs. 22, 23, and 24. In all these sections, *a* represents a portion of the solid vascular axis of the branch. In fig. 22 two indentations at *b*, *b* are separating the clusters of vessels, *f*, from the main bundle. In fig. 23 the bundle, *f*, is almost entirely separated, whilst in fig. 24 it is completely detached from the axial bundle, and is moving outwards through the middle bark as in fig. 21, *f*, *f*. These bundles are distinguished from the foliar bundles by their size; fig. 25, *a*, represents a portion of the cylinder giving off a foliar bundle, *b*.

This section, drawn to the same scale as the other three, exhibits the relative sizes of these two classes of bundles. We further see that many of the larger ones proceeding to the tubercles are surrounded by small foliar bundles of their own—destined, doubtless, to supply the leaves clothing these prominences characterising the *Halonia*.

This hitherto undescribed form of *Halonia* raises anew the question whether or not Plate 10, figs. 1 and 2, of my Part X. is a younger state of the plant represented in figs. 3 and 4 of the same plate. The two are identical in every feature other than the structure of these axial bundles, which is solid in the one and a hollow cylinder enclosing medullary cells in the other. The present position of the question is as follows:—

As stated in my previous memoir, the central axis of each of the very young Lepidodendroid twigs, so abundant in the Laggan Bay deposit, consists of a solid vascular bundle having a diameter of ·012. The similar solid axis of the new *Halonia*, belonging to a branch of larger size, has a diameter of ·14. I have recently obtained from the same locality an ordinary Lepidodendroid branch of about the same dimensions as the *Halonia*, of which the axial bundle is also a solid one, with a diameter of ·14. This is the largest example I have met with having a *solid* bundle. On the other hand, the smallest of the many Lepidodendroid branches and stems from Arran which my cabinet contains, possessing a *hollow* cylindrical vascular axis, is that figured in my memoir, Part X., figs. 3 and 4. In that specimen the vascular zone surrounding the central medulla has a diameter of ·2, from which figure all these cylinders increase steadily in diameter, in the number of their component vessels, and in the dimensions of the cellular medulla which they enclose, until we reach the largest axial bundles of the arborescent stems.* The fact that all the specimens hitherto found *below* a certain diameter have the *solid* vascular bundle whilst all *above* that

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* This increase seems to be partly explained by what I have said on p. 466 about fig. 20.

diameter have the *hollow* ones, inclines me to adhere to my former opinion that these differences are merely due to age: a conclusion strengthened by the fact that *solid* axial bundles are equally absent from *every* specimen of considerable size that we have as yet obtained from any British locality. At the same time it is possible that this opinion may some day require modification.

The late Mr. BINNEY held the view that the *Haloniæ* were the roots, and not the branches, of some Lepidodendroid plants. M. RENAULT, in his recent 'Cours de Botanique Fossile,' advocates the view that some *Haloniæ* are what he terms "Stigmarhizomes" or semisubterranean creeping stems. Structural evidence compels me to reject both these conclusions.* Mr. CARRUTHERS arrived at the same conclusion as I had done from his study of some specimens which he described in 1873.† Recently visiting the museum of the Leeds Philosophical Society, I found on their shelves the magnificent and most conclusive specimen of a branching *Lepidodendron*, the terminal subdivisions of which are true *Haloniæ*, represented in fig. 26. I am indebted to Professor MIALL for an excellent cast of this specimen, from which cast Mr. BROTHERS, of Manchester, prepared the beautiful photograph copied in the above figure. It is yet more perfect than Mr. CARRUTHERS' specimen, since its lower extremity, *a*, exhibits much more markedly than his corresponding ones do the elongated foliar cicatrices characteristic of the *Lepidodendra*. At the lower portion of the branch, A, these leaf-scars have exactly the same form as those of *L. selaginoides* and *L. elegans* of LINDLEY and HUTTON. After its first bifurcation, the two branches, B, B, still retain much of their Lepidodendroid features, though the leaf-scars gradually become less elongated vertically. Towards the upper part of each of the branches, B, B, we discover the first traces of the tubercles characteristic of *Halonia*. These become yet more conspicuous and numerous in the terminal branches, C, C, where we also find that the foliar cicatrices are equilateral rhomboids, instead of the vertically elongated scars seen at A; their vertical and transverse diameters being now about equal. The difference between the two forms is shown to be due in this instance, not only to differences between the several cortical layers, but to the fact that these branches, like their parent stems, have grown more in length than in breadth. This is proven by the circumstance that we can trace the gradations from the one form to the other in the same continuous cortical surface. These Halonioid branches are obviously identical with the *Halonia tortuosa* of LINDLEY and HUTTON.

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* See memoir, Part II., p. 222, Phil. Trans., 1872. In the work referred to above, M. RENAULT endeavours to draw a distinction between *Haloniæ*, which he believes to be the subterranean rhizomes, and those which he recognizes as branches of *Lepidodendron*. He includes in the former class, which he thinks differs from his second one in the rarity with which dichotomous branching occurs in the *Haloniæ* belonging to it, the well-known *Halonia regularis*. This distinction is a purely imaginary one. *Halonia regularis* dichotomises freely.—July 7th, 1883.

† "On *Halonia* of LINDLEY and HUTTON and *Cyclostadia* of GOLDENBERG," Geol. Mag., vol. x., No. 4, April, 1873.

In my second memoir (Phil. Trans., 1872, p. 222), read in June, 1871, I said, "I have little doubt but that *Halonia* was a fruit-bearing branch of a *Lepidodendron;*" and in a note added in April, 1872,* I affirmed *absolutely*, "First, that *Halonia* belongs to the upper branches of a Lepidodendroid tree, consequently it cannot be a root;" "Secondly, we learn that *Halonia* is a specialised branch of a Lepidodendroid tree that is not itself a *Halonia*." These conclusions were further supported by Mr. CARRUTHERS in his memoir on *Halonia* and *Cyclostadia*, already referred to as published in April, 1873, and in which the author, speaking of the specimen which he then described, says, "With *Bergeria* must go *Halonia* as a separate genus, seeing that it is only a condition of *Lepidophloios;* and it may be of other Lepidodendroid plants." The specimen now described is unquestionably not a *Lomatophloios* but a true *Lepidodendron.* The figure on Plate 8 is slightly reduced in size.

Sporocarpon.

In my tenth memoir (Phil. Trans., 1880) I figured in Plate 18, fig. 39, and described (p. 510), under the name of *Sporocarpon ornatum*, a curious form of what appeared to be some Cryptogamic fruit. I have recently received two additional specimens of this organism, one from Mr. EARNSHAW, of Oldham, and the other from Mr. WILD, of Ashton-under-Lyne, which, whilst they confirm my previously expressed conviction as to the character of this object, reveal some additional features connected with it. Mr. WILD's specimen, originally a perfect circle, but slightly damaged during the preparation of the section, is represented by fig. 27. As before described, this fruit is a spherical body with an undulating peripheral outline. In both the new specimens the undulations are more regular than in that previously figured, and in the centre, *a*, of each crenulated prominence there is a canal produced by reabsorbtion of some of the parenchymatous cells. On re-examining my original specimen I find some slight traces of a similar condition. On the other hand, in both my two new examples the peripheral clusters of large cells (*loc. cit.*, fig. 39, *d*) are wanting. These circumstances suggest that the latter are examples of a more advanced stage of growth than the former. It is not improbable that this structure has been primarily developed in the interior of some parenchymatous tissue, of which the large cells, *d*, of my original figure are remnants, but which fell off after the capsule became separated from its temporary surroundings.

Dadoxylon.

In my memoir, Part VIII., I figured (*loc. cit.*, Plate 8, figs. 44 and 46; Plate 9, figs. 47 and 48) and described (p. 230) some young branches of our British form of *Dadoxylon* in which vascular bundles are given off in pairs from the medullary border

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* *Loc. cit.*, p. 225.

of the vascular cylinder, and observed respecting them, "This orientation of these small bundles from the innermost layers of the wood is suggestive of their primary relations to leaves rather than to branches;" and further pointed out that they differed only from similar leaf-bundles in living Conifers only in being in pairs instead of being single. From this I inferred that " Either two bundles went to one leaf with a double midrib, or the leaves were arranged in pairs."

Since these remarks were penned I have examined various living Conifers in hope of finding in some of them a similar organization : I have succeeded in the case of *Salisburia Adiantifolia*, but in it alone. Fig. 28 represents a transverse section of a twig of that plant of the first year's growth, made immediately below the terminal leaf-bud. At *a* we have the medulla and the xylem zone at *b* ; *c* is the cambium layer with newly forming xylem and phloëm zones.* At *d* is an inner and at *e* an outer cortical zone. Many of the cells of the medulla and of the inner bark contain large sphæro-raphides, these two tissues thus furnished being portions of the primitive fundamental tissue; the outermost bark, *e*, being an incipient cork periderm. At *f*, *f* we find the xylem ring interrupted by extensions of the medulla—which connect the latter with the cortical fundamental layer. Two vascular bundles, *g*, are given off from the vascular zone, one on each side of this extension of the medulla, and proceed outwards, diverging but slightly as they ascend towards the periphery, which they reach in pairs, as at *g′*, *g′*. In fig. 29 we have a transverse section of the petiolar base of one of the outermost leaf-scales of the leaf-bud, in which we again find a pair of these bundles, *g*, entering the leaf-scale. The prosenchymatous tracheides of each of these bundles are arranged in six or seven parallel and somewhat fan-shaped, radiating rows. In the matured leaf these two bundles obviously subdivide to form the well-known venation of the *Salisburia*.

Seeing that this twin development of the foliar bundles appears to be limited, amongst living Conifers, to the *Salisburia*, in which plant it seems to hold a definite relationship to the peculiar multinerved structure and Adiantiform contour of the leaf, may we not recognize the probability that our British carboniferous *Dadoxylons* bore some remote genetic relations to the living Gingko? No such duplex leaf-bundle appears to have been detected in any of the fine examples of *Cordaites* and their allies discovered by M. GRAND-'EURY in the St. Étienne deposits. This Salisburian form seems to be confined to our British *Dadoxylons*. That this similarity of contour in the arrangement of the leaf-bundles is not merely accidental is rendered the more probable by the fact that transverse sections of young twigs of the ancient and modern plants display corresponding identities. Thus a section of the bark of a young *Dadoxylon*† corresponds closely with the similar section of a *Salisburia*, fig. 28, in both of which the bark is separated into an outer coarsely cellular periderm and an inner one composed of much more delicate elements. This observation applies to the

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* The section was made in April.

† See Memoir VIII., fig. 34.

Salisburia, whether in its young state, when the inner bark consists, as represented in fig. 28, of undifferentiated primitive parenchyma, or, in a later stage of growth, when the bark is distinctly differentiated into an inner translucent phloëm and an external phellem layer. These correspondences between the carboniferous *Dadoxylon* and the living *Salisburia* suggests another query. The resemblances between the fossil *Trigonocarpums* of the Coal-measures and the fruits of the Gingko have long been noticed. Do not the facts just mentioned increase the probability that these *Trigonocarpums* are the fruits of *Dadoxylon?* I not only think they do, but these facts suggest the further possibility, not to say probability, that our *Dadoxylons* may be the remote Carboniferous ancestors of the oolitic *Baierias*—already recognized as the ancestral forms whence sprang the *Salisburias* of Cretaceous and yet more modern times.

I have again to recognize my obligations to Messrs. CASH, SPENCER, and BINNS, of Halifax; to Mr. HICK, of Harrogate; to Mr. WILD, of Ashton-under-Lyne; and to Messrs. BUTTERWORTH and EARNSHAW, of Oldham, for their valuable aid in searching for specimens calculated to aid my investigations.

Index to the Plates.

PLATE 27.

Astromyelon Williamsonis.

Fig. 1. Transverse section of a non-exogenous branch (?). Enlarged 70 diameters. *a.* Vasculo-cellular axis. *b* and *c.* Inner cortical zones. *d.* Radiating cellular laminæ. *e.* Outer cortical parenchyma. Mr. SPENCER.

PLATE 28.

Fig. 2. Transverse section of a stem. Enlarged 55 diameters. *a.* Large cellular medulla. *b.* Young exogenous zone, feebly developed. *c.* Innermost bark. *d.* Radiating laminæ. *d″.* Lacunæ separating the laminæ *d.* *e.* Peripheral parenchyma. Messrs. CASH and HICK.

PLATE 27.

Fig. 3. Transverse section of a stem. *a.* Medulla. *a′.* Cluster of thick-walled vessels at the inner angle of each large vascular wedge. *b.* Vascular wedges. *c.* Innermost bark. *d.* Radiating laminæ. *e.* Outer parenchyma. Mr. SPENCER.

PLATE 31.

Fig. 4. Vertical section of a stem passing obliquely through the outer cortical parenchyma at its upper end, and through the centre of the medulla at its lower one. Enlarged 40 diameters. *a.* Medullary cells. *b.* Vascular zone. *c.* Innermost cortex. *d, d′.* Radiating laminæ. *d″.* Lacunæ. *e.* Outer cortical parenchyma. Messrs. CASH and HICK.

PLATE 30.

Fig. 5. Transverse section. Enlarged 55 diameters. *a.* Central axis, chiefly or wholly composed of vessels. *b.* Exogenous zone. *c.* Inner cortex. *d.* Radiating laminæ. *d″.* Lacunæ. *e.* Outermost cortex. *f.* Vascular bundle going off to some lateral appendage. Mr. SPENCER.

PLATE 28.

Fig. 6. Transverse section of a stem, enlarged 55 diameters, but having a much thicker outer cortex. References as in fig. 5. Mr. SPENCER.

PLATE 29.

Fig. 7. Section through a branching structure. Enlarged 55 diameters. At A is an obliquely transverse section through a branch. *a.* Central axis composed of a cluster of cells and vessels. *c.* Delicate elongated cells of the inner bark. *d.* Radiating laminæ. *d″.* Lacunæ. *e.* Outer cortical parenchyma. At B is an almost longitudinal section through a dichotomising branch. References as at A. At C is a small vascular bundle, *a‴*, passing outwards. Mr. SPENCER.

PLATE 28.

Fig. 8. Form of vascular tissue. Enlarged 267 diameters.
Fig. 9. Form of vascular tissue. Enlarged 267 diameters.

PLATE 29.

Fig. 10. Form of vascular tissue. Enlarged 267 diameters.
Fig. 11. Form of vascular tissue. Enlarged 524 diameters.
Fig. 12. Form of vascular tissue. Enlarged 267 diameters.
Fig. 13. Form of vascular tissue. Enlarged 267 diameters.

PLATE 31.

Fig. 14. Form of vascular tissue. Enlarged 267 diameters.

PLATE 30.

Fig. 15. Transverse section of a small organism from Halifax. Enlarged 214 diameters. Mr. SPENCER.

Psaronius Renaultii.

Fig. 16. Transverse section of a portion of an arborescent stem. Enlarged 2½ diameters. *a, a'*. Fibro-vascular bundles. *b*. Phloëm layer investing each bundle. *c*. Small gum-canals. Mr. WILD.

PLATE 31.

Zygosporites.

Fig. 17. Sporangium from Halifax containing *Zygosporites.* Enlarged 110 diameters.

PLATE 32.

Fig. 18. Portion of fig. 17. Enlarged 290 diameters.

PLATE 33.

Calamites.

Fig. 19. Transverse section of a young Calamite from the Gannister bed at Moorside, Ashton-under-Lyne. Enlarged 20 diameters. *a*. Innermost cortex. *b*. Hypodermal zone of prosenchymatous vertical bundles. *c*. Epidermal (?) layer. *d*. Vascular wedges of the exogenous zone. *e*. Fistular medullary cavity. Mr. WILD.

Lepidodendron and Halonia.

Fig. 20. Portion of a transverse section of the medulla and vascular medullary sheath of a *Lepidodendron.* Enlarged 190 diameters. *a*. Vessels of the medullary sheath. *b*. Older medullary cells. *c*. Septa of newly-forming cells. Mr. NORMAN.

PLATE 32.

Fig. 21. Transverse section of a branch of *Halonia* from Arran. Enlarged 5 diameters. *a.* Central vascular axis. *b.* Innermost portion of the inner cortex. *c.* Outer part of the same, chiefly destroyed. *d.* Middle cortical parenchyma. *e.* Prosenchymatous cortical layer. *f, f.* Large vascular bundles supplying the Halonial protuberances. Professor YOUNG and Mr. J. YOUNG, Glasgow.

Fig. 22. Segment of the periphery of the vascular axis of another section of the specimen fig. 21. *a.* Vessels of the central axis. *b.* Constrictions of the axis separating the vessels, *f*, from it. Enlarged 54 diameters.

PLATE 33.

Fig. 23. Similar section to fig. 22, only with the clusters of vessels, *f*, completely detached from *a* to form a bundle supplying one of the Halonial tubercles. Enlarged 54 diameters.

PLATE 32.

Fig. 24. Similar section to the last, but with the tubercular bundle completely free in the inner bark. Enlarged 54 diameters.

Fig. 25. Similar section to the three last, showing the size of the *foliar* bundle, *b*, relatively to the tubercular ones.

PLATE 34.

Fig. 26. Dichotomous branch of a *Lepidodendron* terminating in smaller Halonial branches. Slightly reduced in size. *a.* Lower extremity of the branch, with oblong leaf-scars like those of *L. elegans* and *selaginoides.* *b.* Leaf-scars becoming more rhomboidal. *c.* Halonial tuberculated portions with the rhomboid leaf-scars of *Bergeria.* Professor MIALL.

PLATE 31.

Sporocarpon anomalum.

Fig. 27. Transverse section. Enlarged 55 diameters. *a.* Canal in the centre of each crenulation.

PLATE 33.

Salisburia adiantifolia.

Fig. 28. Transverse section of a young twig of the first year, made immediately below a terminal bud. Enlarged 17 diameters. *a.* Medulla, with numerous sphæroraphides. *b.* Xylem. *c.* Cambium and new fibro-vascular elements. *d.* Inner cortical zone of primitive cortex with sphæroraphides. *e.* Incipient cork periderm. *f.* Breaks in the xylem-ring where pairs of foliar bundles, *g*, are given off. *g′*. Pairs of similar bundles reaching the periphery of the bark.

Fig. 29. Transverse section of the petiolar base of a leaf-scale from the terminal bud. Enlarged 24 diameters. *g*. Pair of fibro-vascular bundles.

XII. *Experimental Researches on the Electric Discharge with the Chloride of Silver Battery.*—Part IV.

By Warren De La Rue, *M.A., D.C.L., Ph.D., F.R.S., and* Hugo W. Müller, *Ph.D., F.R.S.*

Received June 11,—Read June 14, 1883.

[Plates 35, 36.]

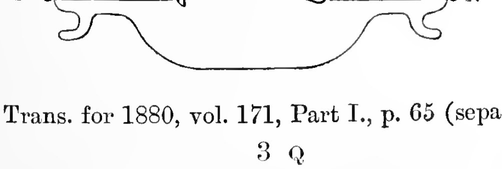
TUBE-POTENTIAL.

Pressure of least resistance dependent on the shape and dimensions of vessel.

We have already stated that the potential necessary to produce a discharge in partially exhausted tubes diminishes with the pressure until a certain pressure of minimum resistance has been reached, but as the rarefaction is increased beyond this point, then the potential has to be increased in order to produce a discharge.* The experiments on which this result was founded were made with a tube 33 inches (83·8 centims.) long and 2 inches (5·1 centims.) in diameter, and it was found that in a hydrogen partial vacuum the pressure of minimum resistance was 0·642 m.m., 845 M. Professor Stokes suggested the desirability of making experiments with a wide tube, or, still better, with a globe, as it would allow the discharge to spread laterally and diminish the resistance thereby, and, very probably, alter the pressure of minimum resistance. He considers this to be a question of importance with reference to the height of the aurora, for in the atmosphere there is ample space for lateral expansion, and it is conceivable that the least resistance may correspond to a pressure a good deal different from that which gives the least resistance in a tube.

Experiments were made in an air residue contained in a vessel in some measure resembling a prolate spheroid, 7 inches (17·8 centims.) long and 5 inches (12·7 centims.) in diameter, the distance between the terminals, one a cup, positive, the other heart-shaped, negative, was $3\frac{5}{8}$ inches (9·2 centims.) (see fig. 1). The battery employed

Fig. 1



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* Part III., Phil. Trans. for 1880, vol. 171, Part I., p. 65 (separate copy, p. 159).

consisted of 11,000 cells, giving a deflection when short-circuited of 49° with a total internal resistance of 360,800 ohms.

On June 17, 1881, the following results were obtained :—

Pressure.			Deflection.	Total resistance.	Tube-resistance.	Tube-potential V.
m.m.	M.	Cube root of M.	°	ohms.	ohms.	cells.
35·0	46,053	35·84	33·0	639,700	278,900	4796
33·5	44,079	35·32	40·0	495,000	134,200	2982
31·0	40,789	34·42	43·0	445,300	84,500	2089
27·5	36,184	33·08	40·0	495,000	134,200	2982
26·0	34,211	32·46	43·0	445,000	84,500	2089
24·0	31,579	31·61	43 0	. .	. .	. .
21·5	28,290	30·47	43·0	. .	. .	. .
19·5	25,658	29·50	43·0	. .	. .	. .
16·0	21,053	27·61	44·0	430,700	69,900	1784
13·5	17,763	26·09	45·0	415,000	54,200	1435
10·5	13,816	24·00	46·0	400,300	39,500	1085
8·5	11,184	22·36	46·5	393,500	32,700	913
6·5	8553	20·45	46·5	. .	. .	. .
4·5	5921	18·09	48·0	374,000	13,200	388
3·5	4605	16·64	48·5	367,400	6600	194
2·5	3290	14·87	47·0	386,700	25,900	737
2·0	2632	13·81	47·0	. .	. .	. .
1·5	1974	12·54	47·0	. .	. .	. .
1·0	1316	10·96	46·0	400,300	39,500	1085
0·47	618	8·52	44·0	430,700	69,900	1784
0·31	408	7·42	44·0	. .	. .	. .
0·16	211	5·95	44·0	. .	. .	. .
0·06	79	4·29	44·0	. .	. .	. .
0·03	39	3·39	44·0	. .	. .	. .
0·02	26	2·96	38·0	531,600	170,800	3534

On June 22, the following :—

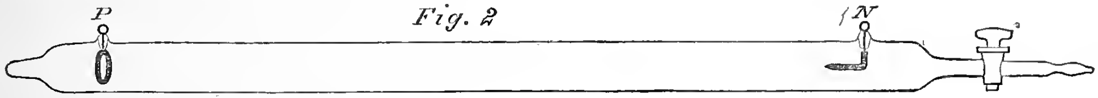
BATTERY 11,000 cells, short circuited, gave a deflection of 65°, and a total internal resistance of 193,600 ohms.

Pressure.			Deflection.	Total resistance.	Tube-resistance.	Tube-potential V.
m.m.	M	Cube root of M.	°	ohms.	ohms.	cells.
21·0	27,634	30·23	60	239,500	45,900	2108
18·0	23,684	28·72	. .	. .	. .	. .
15·0	19,737	27·03	61	230,200	36,600	1749
9·0	11,842	22·79	. .	. .	. .	. .
7·0	9211	20·96	. .	. .	. .	. .
5·0	6579	18·74	62	320,900	27,300	1360
3·0	3947	15·80	63	211,600	18,000	936
2·0	2632	13·80	. .	. .	. .	. .
1·1	1447	11·31	62	220,900	27,300	1360
0·67	882	9·59	. .	. .	. .	. .
0·17	224	6·07	59	249,500	55,900	2464
0·10	132	5·09	. .	. .	. .	. .
0·04	53	3·75	57	263,600	70,000	2920
0·16	21	2·76	51	336,200	142,600	4666

In both of these series of experiments it will be seen that the pressure of minimum resistance for an air partial vacuum in the vessel of spheroidal shape and of the dimensions mentioned is about 3 m.m., 3947 M, corresponding to the pressure of air at an altitude of 27·42 miles, at which height the aurora would from these data have a maximum brilliancy and would be visible at a distance of 499 miles. This result is very different from that before cited, which was obtained with a hydrogen tube from which the pressure of maximum brilliancy was deduced to be for air 0·379 m.m., 499 M.,* at a height of 37·67 miles and visible at a distance of 585 miles.

As no direct experiments had been made for an air residual vacuum in smaller tubes, the following investigations were made with a tube, No. 342, 1·625 inch (4·1 centims.) in diameter and 22·5 inches (57 centims.) between the terminals, one a ring positive, the other a point negative (see fig. 2, on the same scale as fig. 1).

Fig. 2



Battery 11,000 cells, deflection when short-circuited 70°, showing a total internal resistance of 151,000 ohms. At a pressure of 19 m.m. there was a glow on each terminal, but the current was not sufficient to deflect the galvanometer used until it had been lowered to 14 m.m.

Pressure.			Deflection.	Total resistance.	Tube-resistance.	Tube-potential V.
m.m.	M	Cube root of M.	°	ohms.	ohms.	cells.
14·0	18,421	26·41	46	400,300	249,200	6848
12·0	15,789	25·09	55	291,200	140,100	5291
11·0	14,474	24·37	55	. .	. .	. .
10·0	13,158	23·59	55	. .	. .	. .
9·0	11,842	22·79	56	280,400	129,300	5072
8·0	10,526	21·58	57	263,600	112,500	4693
7·0	9211	20·96	60	239,500	88,400	4061
5·0	6579	18·74	61	230,200	79,100	3780
3·0	3947	15·80	62	220,900	69,800	3477
2·0	2632	13·81	62	. .	. .	. .
0·69	908	9·68	63	211,600	60,500	3146
0·23	303	6·72	62	220,900	69,800	3477
0·15	197	5·82	60	239,500	88,400	4061
0·05	66	4·04	57	263,600	112,500	4693
0·03	39	3·39	54	302,100	151,000	5499

The pressure of least resistance is therefore 0·69 m.m., 908 M., in the tube in question, and it does not differ materially from 0·642 m.m., the pressure of least resistance in hydrogen.

A pressure of 0·69 m.m. corresponds to an atmospheric height of 34·71 miles, and an aurora at this height would be visible at a distance of 561·4 miles.

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* Proc. Roy. Soc., No. 203, 1880.

Another experiment made with a tube (fig. 3, on the same scale as fig. 1) 0·75 inch (1·9 centim.) in diameter, and 23 inches (58·4 centims.) long between two terminals, both paraboloidal in form, gave the following results :—


Fig. 3

Pressure.			Deflection.	Total resistance.	Tube-resistance.	Tube-potential V.
m.m.	M.	Cube root of M.	°	ohms.	ohms.	cells.
14	18,421	26·41	45	415,000	203,400	5392
13	17,105	25·75	46	400,300	188,700	5186
12	15,789	25·09	47	386,700	175,100	4979
11	14,479	24·37	. .	. .	. .	. .
10	13,158	23·59	48	374,000	162,400	4776
9	11,842	22·79	50	348,500	136,900	4321
8	10,526	21·58	53	313,000	101,400	3563
6	7895	19·91	56	280,400	68,800	2699
5	6579	18·74	. :	. .	. .	. .
3	3947	15·80	59	249,600	38,000	1674
2	2632	13·81	60	239,500	27,900	1281
1	1316	10·95	61	230,200	18,600	888
0·5	658	8·69	58	253,300	41,700	1811
0·08	105	4·72	47	386,700	175,100	4979
0·04	53	3·76	34	897,300	685,700	8407

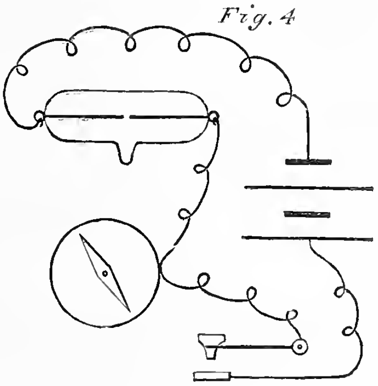
The pressure of minimum resistance in this particular case is 1 m.m., 1316 M, which is the same as that of the atmosphere at a height of 32·87 miles, where an auroral display would be visible at a distance of 546·6 miles. It is evident therefore that not only the dimensions of the tube, but possibly also the shape of the terminals, have a marked influence on the pressure of least resistance, and it is very probable that in the atmosphere where the lateral expansion is practically unlimited, the conditions of minimum resistance are different from those which exist even in large tubes.

The foregoing experiments, which show that the pressure of minimum resistance is not absolute for any particular gas, but that it varies with the size and form of the vessel containing it, seem to point to the conclusion that the carrier of electricity must be ponderable matter, and not the ether as has been suggested. Professor STOKES considers it most probable "that the presence of ponderable matter is necessary for the transfer of electricity from one place to another. At moderate exhaustions there are plenty of molecules, but they are so close that they hamper one another's motions. As the exhaustion is carried further there are still plenty of molecules, but they hamper one another less, and therefore the facility for the transfer of electricity is increased. But when the exhaustion becomes extreme, there is a loss in the facility of transfer from not having enough molecules; and as the exhaustion is still further continued, it is easily conceived that more may be lost, in point of facility of transfer, than is gained by increasing freedom in their motions. In a wide space the impediment to free motion is less than in a confined tube, and it is natural, therefore, that the effect of increasing fewness of the molecules should begin to tell at an earlier stage of exhaustion: that is, that the pressure of minimum resistance should be greater than in a tube." Later on we shall quote other experiments on the difference of potential in different parts of a tube in support of this view.

Discharge in miniature tubes.

Experiments on the discharge in miniature tubes gave most unexpected results; for example, in a tube of the dimensions represented of the natural size in fig. 4, which also represents the arrangement of the apparatus. The terminals in this tube so nearly touched, that in order to ascertain that they were not in actual contact, they were connected through a sensitive galvanometer with the opposite poles of a battery of 10 cells. The battery-power was then increased gradually to 2400 cells, when a discharge took place. After this discharge, then a single cell would pass; but if the tube was allowed to stand for a short time (5 minutes) then it required 4800 cells to reproduce a discharge. From the great heat of the discharge the terminals became red-hot and the tube very soon cracked and prevented the repetition of the experiments.

Fig. 4



But in another tube of the size shown in fig. 5 fresh experiments were made, the terminals in this tube were distant about $\frac{1}{1000}$th of an inch (0·00104) (0·0264 m.m.), the points being of the form shown magnified in fig. 6 after a discharge had taken place.

Fig. 5

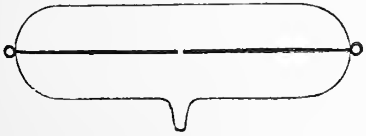
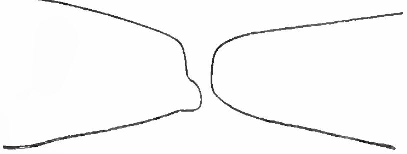


Fig. 6



After the discharge of 2240 cells had passed, then the number of elements had to be increased to 11,240 to produce a discharge, and at last even this number could not cause one except by alternating the current first in one and then in the contrary direction. Ultimately this failed to produce a discharge, but an induction coil did so;

the discharge, however, took place with more facility across the outside of the tube, at ordinary atmospheric pressure, or between the terminals of the coil which were distant $\frac{9}{16}$, 0·5625, inch (1·43 centim.).

The tube was sealed off and laid by for a few days, when it was found that 5100 cells would not pass, but that 8700 did so intermittently, and it required 10,160 to produce a continuous discharge. Subsequently 1200 cells passed, 20 would not do so even immediately after the discharge of the former number; 800 cells, 700, and then 600 passed; but after a while the last number was insufficient to produce a discharge. This tube shared the fate of the first tube and ultimately cracked. It is very possible that the strong discharge volatilized a portion of the terminals, which were of platinum, and that this condensed, or that they absorbed the residual gas so completely as to produce a vacuum too perfect to admit of a discharge taking place, and that, ultimately, sufficient of the occluded gas was again given off to render the discharge again possible.

Occlusion of gas by terminals.

The power of terminals to occlude gas and then under an electric discharge to give it off again is well exemplified by tube 48, the terminals of which are both of palladium. The tube is shown in fig. 7, one terminal is in the form of a helix, the other straight; it is 7 inches (17·8 centims.) long, and $1\frac{1}{2}$ inches (3·8 centims.) in diameter, and contains residual hydrogen. When the tube was new the terminals were bright, but by continued use in consequence of occluding the gas and giving it off again they have become mat or frosted, and porous to a certain depth. We usually cause the spiral to be negative; after the discharge of a battery of 1200 cells has passed for a few seconds the tube which was at first free from deposit, as in fig. 7, becomes coated more especially near the negative, with a mirror-like metallic deposit, as in fig. 8. After the tube has remained at rest for a few days, seven in one instance, this metallic mirror disappears entirely, being absorbed by the terminals; this shows not only that the terminals give off occluded gas and reocclude it, but that there is formed in this particular case a volatile hydrogen-palladium compound. Moreover, the vacuum appears to alter considerably during the discharge, for a stratification which is produced in the first instance ceases in a short time, but the tube regains its original condition by standing for a few days, and stratification is again produced.

Fig. 7.

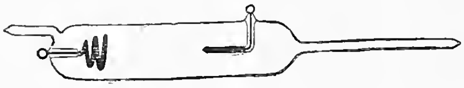
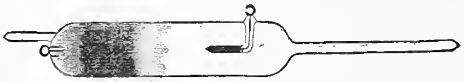


Fig. 8.

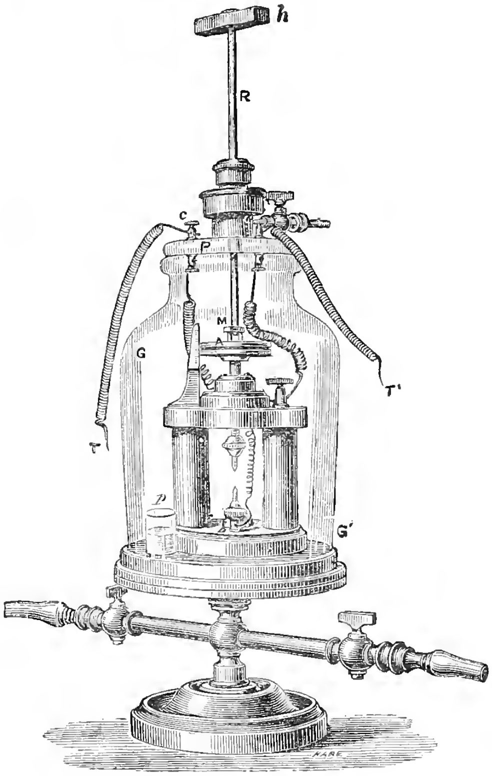


The phenomena described have been produced very many times and shown repeatedly to friends who have visited our laboratory since March 13th, 1875.

Length of spark in dry and in moist air.

Many experiments were made to ascertain the effect of saturation with aqueous vapour on the length of the striking distance; for this purpose the bell-jar and micrometer-discharger, as shown in fig. 9, were used.

Fig. 9.



The terminals employed were two paraboloidal points. The striking distance with 10,860 cells was found to be in air dried with phosphoric anhydride at a pressure of 30·345 inches (77·1 centims.) 0·57 inch (1·45 centim.).

The bell-jar was now removed and after having stood for a little while over a dish of water, was then replaced on the plate of the air-pump; in the bell-jar was placed a beaker, *p*, containing water to keep up the saturation, under these circumstances the striking distance was then found to be

	Inch.	
	0·58	(1·5 centim.)
after the lapse of 30 min.	0·58	(1·5 ,,)
,, ,, 4 hours	0·55	(1·4 ,,)
,, ,, 5 ,,	0·54	(1·4 ,,)

In another series of experiments with 8700 cells, the striking distance in dry air was found to be

	Inch.
	0·44
	0·45
	0·46
Mean . .	0·45 (1·1 centim.)
In air saturated with moisture	0·45
,, ,, ,,	0·445
Mean . .	0·447 (1·1 centim.)

From these experiments we arrive at the conclusion that the striking distance is the same in dry air as in air saturated with moisture.

Two separate currents in the same tube.

Several experiments were made in a specially constructed tube with the object of ascertaining the mutual effect of two discharges passing within it first in the same and then in a contrary direction.

Tube 347 consists of an outer tube $1\frac{3}{4}$ inch (4·4 centims.) in diameter, 31 inches (78·7 centims.) long, and $27\frac{1}{2}$ inches (69·8 centims.) between the terminals, two short pieces of tube, 9 inches (22·8 centims.) long and $\frac{1}{2}$ inch (1·27 centims.) in diameter, are fixed one at each end; the distance between the open mouths of the small tubes being 13 inches (33 centims.) (see fig. 10).

Fig. 10.

Two paraboloidal terminals are sealed in the outer tube and two precisely similar ones are sealed in the small tubes and hold them in their places.

When the discharge from a battery of 3600 cells was made to pass through the small tube a close stratification, as in Plate 35, fig. 1, was produced, which stratification was continued in the interval between the open ends of the small tubes, but of an altered form, wider apart as shown in the figure, which is a facsimile of a photograph obtained on a dry plate in one second.

When the same discharge passed through the outer large tube a continuous stratification was obtained throughout, as shown in Plate 35, fig. 2.

When the discharges from two separate batteries, each of 3600 cells, were made to pass in reverse directions, the one in the outer tube having an external resistance of 250,000 ohms inserted, the discharge in the small tubes was the strongest, and an appearance was produced as shown in Plate 35, fig. 3.

It will be noticed that the effect of this stronger discharge is to completely reverse the convexity of the strata in the middle portion of the outer discharge in the large tube, while the convexity on the two extremities of the outer discharge retains its natural direction. It would appear, therefore, that the *natural* discharge in the intervening space in the outer tube is, as it were, replaced by a non-luminous one. When the batteries were kept on for about twenty seconds, or so, these intervening strata gradually diminished, and at last disappeared entirely, as shown in Plate 35, fig. 4, where they are replaced by a non-luminous space; but it is very evident, from the shape of the strata where visible, that two discharges continued to take place in the direction proper to each from both sets of terminals; the last stratum of the small tube on the positive side curving upwards into the broad space.

On keeping the batteries in connexion for a further period another remarkable change took place, and the discharge in the larger tube appeared to gain the mastery, its strata in the interval between and above the open ends of the small tubes reappearing with a forward movement one by one at the positive side, the convexities being turned towards the negative, which is the natural direction. The strata in the small tubes did not continue beyond their extremities, and a dark space intervened. These phenomena are shown in Plate 35, fig. 5. Keeping on the discharge still longer the strata in the small tube disappeared entirely, as shown in Plate 35, fig. 6.

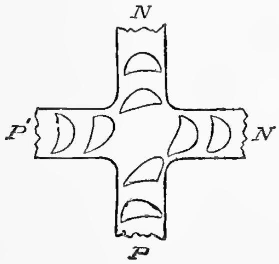
Ultimately, by continuing the discharge, both sets of the strata disappeared, as if one discharge neutralized the other, but these could be reproduced by breaking and remaking the contact with the batteries.

It is very probable when two discharges take place in the same tube in opposite directions that the column of gas becomes divided longitudinally into two layers, one of which conveys the discharge of one battery, while the other conveys that of the other battery.

Two separate currents crossing each other at right angles.

In order to ascertain the effect of two discharges from two distinct batteries crossing each other at right angles, we prepared a tube, No. 344, consisting of two tubes joined at right angles to each other, each tube being 33 inches (83·8 centims.) long and $1\frac{1}{8}$ inches (2·86 centims.) in diameter. The central portion is shown in fig. 11.

Fig. 11.



In the first place the discharge of a battery of 3600 cells was sent through one of the tubes containing air at a pressure 0·428 m.m., 563 M, causing a current of 0·01507 ampère, and producing the stratification which is represented in Plate 35, fig. 10;

which stratification is of the ordinary character, with the exception that, at the crossing of the two tubes, a wider stratum is produced. While the discharge was passing through only one of the tubes a faint illumination was also produced in the two branches of the other, but it was not stratified.

The two component tubes were subsequently connected with two separate batteries, each of 3600 cells, A and B; A with the poles P and N to the vertical component, and B with the poles P′ and N′ to the horizontal one. The effect is represented in Plate 36, fig. 15. It will be seen from the configuration of the strata at the cross, in this case when the currents were equal 0·00833 ampère, that the discharge of A goes from P towards N only as far as the cross, then turns off to the left to N′, the negative of the other battery, B, while on the other hand the discharge of B from P′ goes to the N of A battery.

The case is different if an external resistance is introduced in one of the discharges, as for instance in Plate 36, fig. 16, where, by the insertion of a resistance of 500,000 ohms in the B battery connected with the horizontal tube, the current was reduced to about one-tenth (0·00087 ampère) of that of battery A connected with the vertical tube, the discharge from A battery goes on direct from P to N and the discharge from B from P′ to N′. The different sizes of the two sets of strata render this evident. There is a bending down, however, of the strata of the weaker discharge at the cross in consequence of the action of the stronger one. One cannot but be impressed from this and other experiments before and hereinafter described by the apparent plasticity of the aggregate assemblage of molecules which constitute a stratum, for it evidently yields to external influences which modify its form. For instance, in Plate 36, fig. 17, copied from a photograph taken on a dry plate in half a second, the strata at the cross, produced by two equal batteries of 3600 cells, are so far modified in both tubes that the curvature of one stratum in each branch is made convex towards the positive instead of concave. The vacuum is of air at a pressure of 0·428 m.m., 563 M. The extremities of the vertical tube are both negative, and those of the horizontal tube positive, so that in each tube positive is opposed to positive and negative to negative.

In Plate 36, fig. 18, representing a photograph of the tube when containing hydrogen at a pressure of 0·46 m.m., with a separate discharge of 3600 cells in each tube, a very close stratification is produced. The ends of the vertical tube are both negative, those of the horizontal positive, each set of strata pursues its undisturbed course and produces a beautiful configuration at the cross where the last stratum in the two horizontal branches remains concave towards its own positive, but in the negative branch becomes convex towards the negative, yielding as it were to the influence of pressure of the strata from the positive.

In Plate 36, fig. 19, the two sets of strata follow their proper course leaving a dark interval at the cross between them. The tube contained air at a pressure of 0·428 m.m., 563 M, the batteries used being both 3600 cells, the positive of A battery was connected to the top of the vertical tube, and its negative to the left end of the horizontal tube. The positive terminal of B was connected to the right end of the horizontal tube, and its negative to the bottom of the vertical tube. The photograph from which this figure is copied was obtained in half a second.

Plate 36, fig. 20, represents the discharge through hydrogen at a pressure of 0·46 m.m., 605 M, batteries A and B, each of 3600 cells, the positive pole of A was connected with the left end of the horizontal tube, its negative to the top of the vertical, 2,600,000 ohms external resistance having been introduced in the discharge; the positive of B battery was connected to the right hand end of the horizontal tube, and its negative to the bottom. In neither set of discharges are distinct strata produced at the positive end, and only after passing the cross are such produced. It would almost seem that the resistance introduced in one pair of branches had offered an impediment to the discharge in the other.

Effect on strata of the shape of the positive terminal.

Experiments were made in order to ascertain whether the *form* of a point used as a positive terminal had any effect on that of the strata. It was found, however, that whether the terminal was carefully shaped as a paraboloid, or was simply cylindrical, the strata took precisely the same form as is shown in Plate 35, where fig. 7 represents the discharge in hydrogen from a paraboloidal point, and fig. 8 that from a cylindrical point.

Complex strata.

The complex configuration of strata in many cases appears to us to present a great difficulty in accounting for their production, and in forming a clear conception of the forces which hold the molecules composing them so persistently together. Usually there is a dark space between the negative terminal and the last stratum on the positive side, but not unfrequently the last stratum or several strata thread themselves on to the negative when it consists of a wire, as, for instance, in tube 346, shown in Plate 35, fig. 9, containing hydrogen at a pressure of 0·5 m.m., 658 M, with a battery of 3600 cells, and an external resistance 200,000 ohms, producing a current of 0·00238 ampère. A beautiful bracket-like series of strata was produced, two or three of which threaded themselves on to the negative, as is distinctly shown in Plate 35, fig. 9. Looked at from the end it could be seen that these strata had circular holes in them larger than the negative wire, so that a small dark space was left. Each stratum consists of an outer bracket convex towards the negative and an inner chord (fig. 12).

Fig. 12.

Discharge in large tubes.

In Part III., pp. 197–201, we described some experiments with a large tube 37 inches (94 centims.) long, and $5\frac{13}{16}$ inches (14·8 centims.) in diameter. We have again taken up the study of the discharge in large tubes 3 feet 2 inches (96·5 centims.) long, and 6 inches (15·24 centims.) in diameter; the resulting phenomena are shown in Plate 35, figs. 11, 12, 13, and 14. In these, which are partly copied from photographs and partly from drawings made at the time in consequence of a to-and-fro movement of the strata preventing well-defined photographs from being obtained:—

Plate 35, fig. 11, exhibits a discharge of 8700 cells with a current of 0·049 ampère in carbonic anhydride at a pressure of about 0·5 m.m., 658 M. The strata near the terminals are not large enough to fill the whole bore of the tube, but they spread and become larger as they recede, and form a sort of conical expansion up to the wall of the tube. A rapid flow was observed from the negative with a still increased velocity, at intervals, producing a gap in the flowing strata of three times the interval between them. The three strata near the negative retained their position but rotated rapidly on their axes, and when looked at obliquely appeared dark in the centre and reminded one of a smoke vortex ring.

Plate 35, fig. 12, represents the appearance in a hydrogen vacuum of about 0·5 m.m., 658 M, with the same number of cells, 8700, producing a current of 0·047 ampère. After a short time the stratification receded towards the positive, leaving a long interval devoid of any towards the negative.

Plate 35, fig. 13, represents the appearance of a vacuum of nitrogen and bromine with the same number of cells, 8700, giving a current of 0·047 ampère. After the discharge had continued for a few seconds the strata, as in the case of hydrogen, receded towards the positive end of the tube as shown in Plate 35, fig. 14.

In all the foregoing cases a very beautiful and distinct stratification was produced, in the first instance, filling the tube just as in those of smaller bore, but near the terminals the strata became gradually smaller in a conical form, the gas itself for a certain distance acting as an enclosing tube.

The dark negative space.

In the dark space near the negative end of a vacuum tube the electric discharge is undoubtedly always passing, though not evident to the eye; but its passage is rendered apparent by the great heat at times developed within it, as we have already stated in Part II.* It occurred to us to endeavour to obtain a record of its passage by means of photography, and the result shows that it is actinically dark only by comparison. Tube No. 100, containing a residue of carbonic anhydride, gave, with 2400 cells and an

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* Phil. Trans. for 1878, Vol. 169, Part I., p. 182, Exp. 47 (separate copy, p. 98).

inserted resistance of 200,000 ohms, a current of 0·00335 ampère, which produced a perfectly steady stratification during more than thirty-five minutes the tube was in connexion with the battery. In two and a-half seconds an impression of the strata was obtained on a dry plate, which is represented in Plate 36, fig. 21. The strata were then carefully covered so as to exclude all their light from the camera and to expose only the dark space; in fifteen minutes an impression was produced on the photographic plate, but it required an exposure of thirty-five minutes to produce an impression as dense as that obtained by an exposure of the strata for two and a-half seconds. The photographic intensity of the dark space is therefore 840 times less than that of the strata. The image of the dark space is shown in Plate 36, fig. 21, below that of the strata; it contains no trace of stratification, although, if any existed, it is fair to presume that it would have been depicted in consequence of the perfect steadiness of the discharge.

Potentials at different nearly aliquot parts of a column of gas, at various pressures.

The following experiments, intended to ascertain the difference of potential in different parts of a vacuum tube,* bring out instructive information, not only in reference to the relative resistances of different lengths of a column of gas at various pressures, but also in regard of the impediment presented by the terminals themselves to the passage of a discharge from gas to terminal or terminal to gas.

They were made by means of an electrometer in connexion with an induction apparatus shown in fig. 13, and described in Part III. of our researches.†

In the present instance, however, certain modifications were adopted to prevent disturbing influences and to ensure concordant results. In the first place, a trap was formed to prevent the creeping down of electricity on the inside and outside, more especially the outside of the cylindrical glass shade enclosing the induction plates. This trap consists of two hoops of tin foil,‡ half an inch in width, pasted inside and outside respectively, about 4 inches below the flat top. Both these hoops communicate to earth by means of two vertical strips of tin foil pasted on the cylinder. Before this arrangement was adopted the cylinder became slowly charged, especially in damp weather, and unduly increased the deflection of the needle of the electrometer, by acting inductively on the induction plates.

It was found very advantageous to connect the needle with a small Leyden-jar§ (L, fig. 14), presenting a coated surface of 57 square inches (3·68 square decims.), and having a capacity of 0·0016 microfarad. The jar, by increasing the capacity of the apparatus, in a great measure obviates the disturbing influence of leakage during the

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* Tube 149 with 12 aluminium rings, and tube 150 with 17 rings already described. Phil. Trans. for 1878 (vol. 169), p. 165; Part II. (separate copy, p. 81).

† Phil. Trans. for 1880, vol. 171, p. 112 (separate copy, p. 206).

‡ These are not shown in the engraving.

§ Part I. of our researches, Phil. Trans. for 1878, vol. 169, p. 99 (separate copy, p. 45).

Fig. 13.

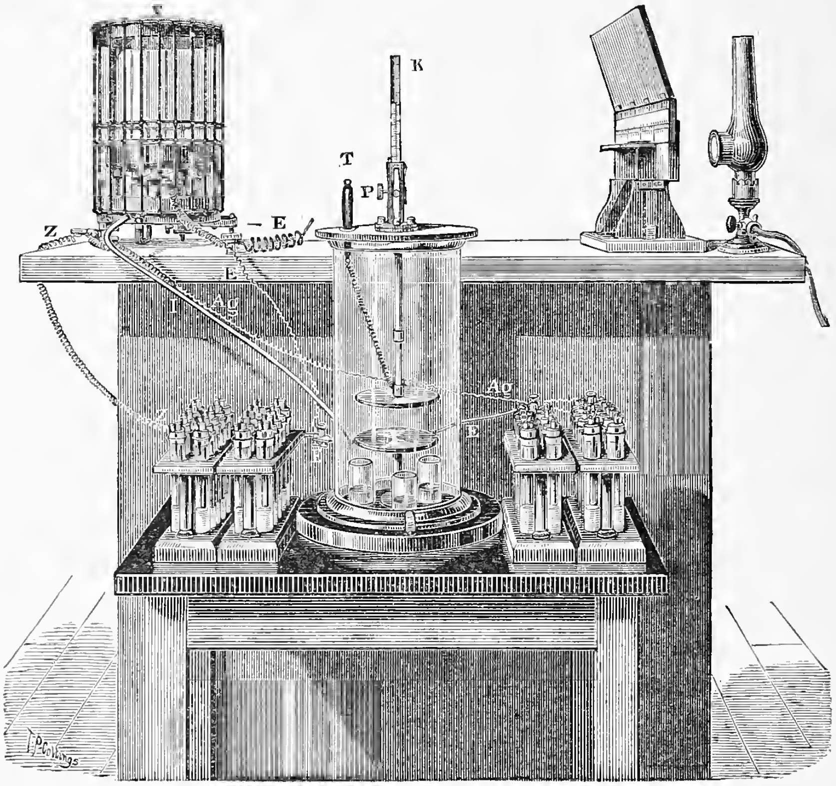
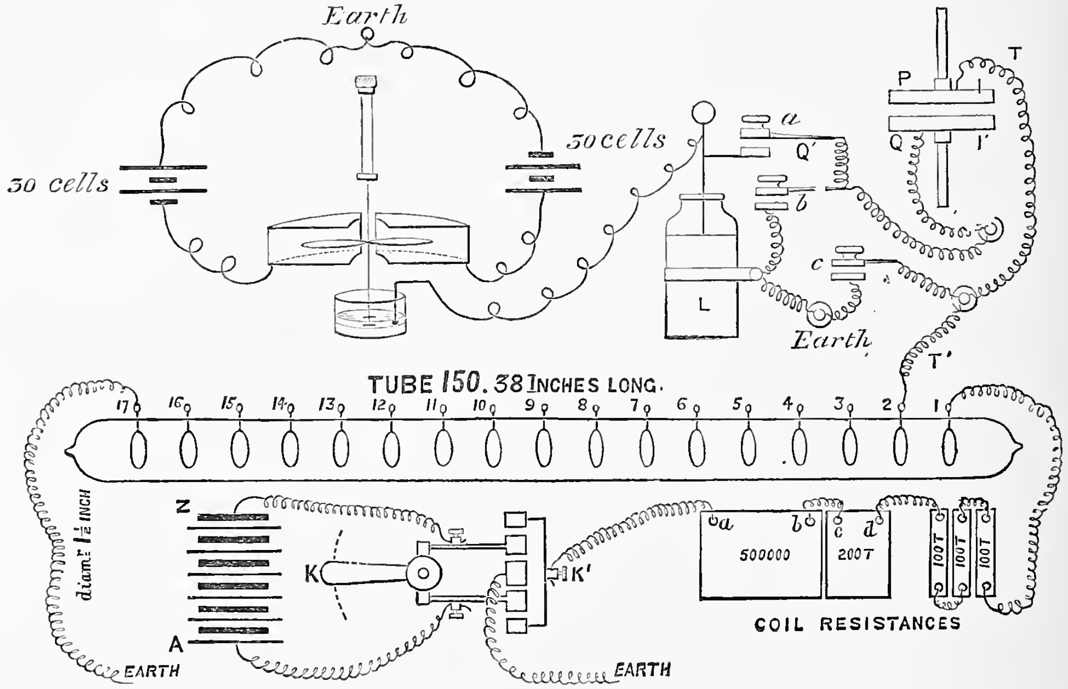

K
T
P
Z
– E
E
I
Ag
Ag
E
Z
E
J. P. Collings

Fig. 14.


Earth
30 cells
30 cells
a
Q'
b
c
L
Earth
P
T
Q
T'
T'
TUBE 150. 38 INCHES LONG.
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
diam: 1½ INCH
Z
A
K
K'
EARTH
EARTH
a
b
c
d
500000
200T
100T
100T
100T
COIL RESISTANCES

observations. For the suggestion of the Leyden-jar and the plan of observation about to be described we are indebted to Professor STOKES, by whose advice we have often profited. The arrangement of the apparatus is shown in fig. 14.

It will be seen on reference to fig. 14 that a battery of chloride of silver cells, the centre to earth, is used to charge up the opposite quadrants of the electrometer with electricity of opposite names. The number of cells is generally varied from 10 up to 60 in order to regulate the deflection, which can also be varied by varying the distance between the plates P and Q of the induction apparatus.

The needle is connected to the inside of the Leyden-jar L, the outside being permanently connected to earth. By pressing down the key *a* (which is supposed to represent a mercury connexion, as do also the keys *b* and *c*) the inside of the jar is connected with the induced plate Q; before, however, connecting the source of electricity with P, the key *b* is pressed down so as to connect the inside of the jar, the needle, and the induced plate Q to earth; this may be designated the Q system.

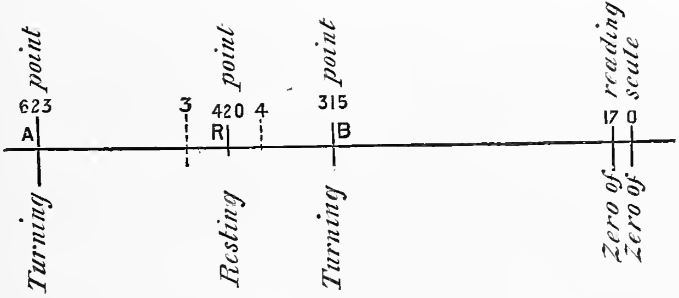
The source of electricity is now connected by means of T T′ to the inducing plate P; this may be designated the P system. No charge under this arrangement is possible in the system Q which is to earth, consequently the needle remains undisturbed.

The wire T′ is now disconnected, *b* raised to disconnect the Q system from earth; up to this point there is no deflection, but the instant that the P system is discharged to earth by pressing down the key *c* then there occurs a negative deflection; in order to decrease the leakage to a minimum the key *a* is raised so as to disconnect Q from the Leyden-jar.

In order to avoid waste of time, and disturbances of the readings by possible leakage by waiting for the needle to come to rest, the resting point was determined by means of a formula from two consecutive turning points.

The diagram fig. 15 represents the zero of the scale, the zero 17 of the readings, the first turning point A, the second turning point B, and the resting point R; 3 represents the third turning point, and 4 the fourth.

Fig. 15.



$$R=B+c(A-B)$$

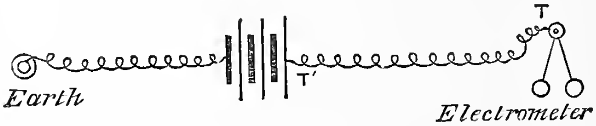
c is a factor determined once for all, by causing the needle to swing by means of an electric charge and then discharging; so that the subsequent swings are due solely to the torsion of the bifilar suspension. Four sets of observations were made and the needle allowed to come to rest at R in each series. These were very concordant, and c came out $\frac{1}{2\cdot92}=0\cdot3425$ by the formula $c=\frac{r}{1+r}$, where $1:r$ is the ratio of the length of one swing to the next, the swings being found to decrease very accurately in geometric progression. By waiting till the needle comes to rest, c may of course be determined directly, by means of the ratio in which any swing is divided by the observed resting point. Thus in fig. 16, AB : RB : : 1 : c. These two methods of determining c agreed perfectly.

The following results were obtained by making the observations on the plan just described.

TUBE 149 CO^2 (12 aluminium rings).

8540 cells. The full potential determined as in diagram fig. 16 was 251 divisions.

Fig. 16.



The positive pole of the battery was connected to No. 1 ring; the negative pole and No. 12 ring to earth, and each third ring in succession from 1 to 12 was connected with the electrometer.

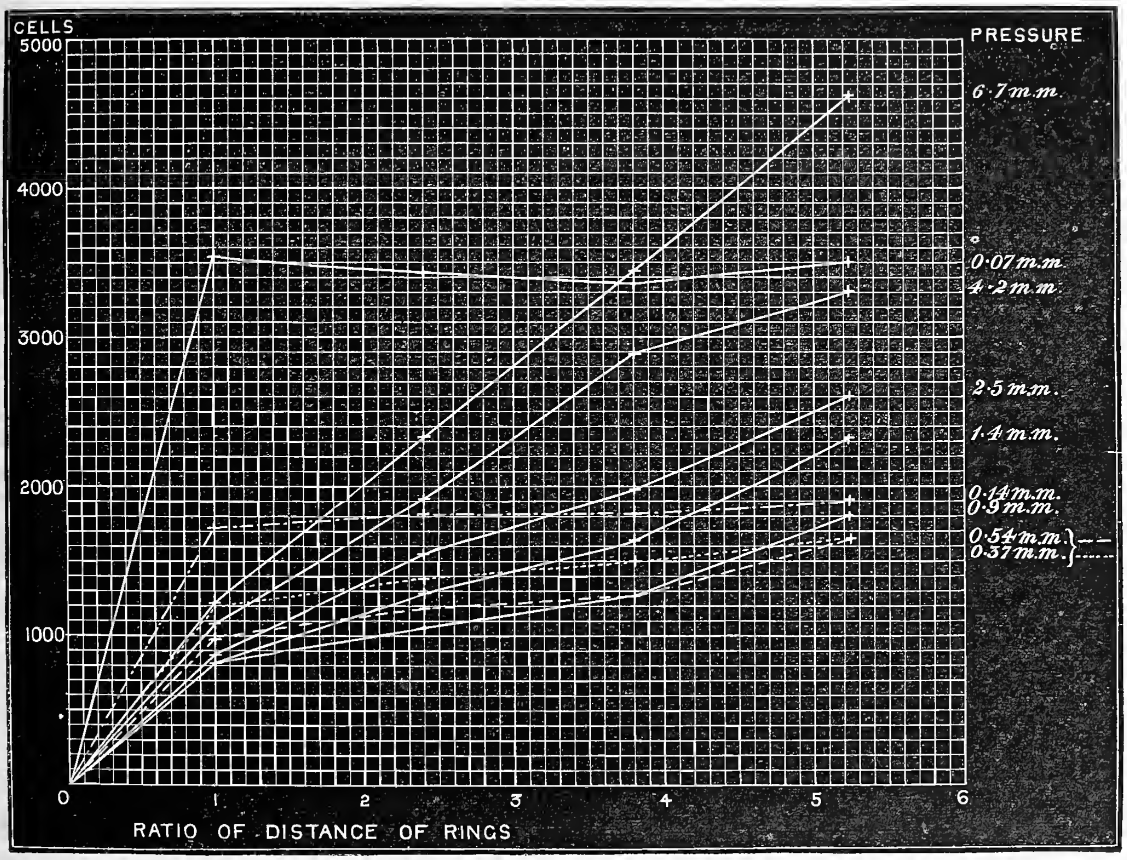
Ring.	Pressure 6·7 millimetres. Potential.			Pressure 4·2 millimetres. Potential.			Pressure 2·5 millimetres. Potential.			Pressure 1·4 millimetre. Potential.			Pressure 0·9 millimetre. Potential.		
	Divs.	Cells.	Ratio.	Divs.	Cells.	Ratio.	Divs.	Cells.	Ratio.	Divs.	Cells.	Ratio.	Divs.	Cells.	Ratio.
1	136	4267	3·78	97	3301	3·03	76	2586	3·04	68	2314	2·83	53	1803	2·21
4	101	3437	2·80	85	2892	2·66	58	1973	2·32	48	1633	2·00	37	1259	1·54
7	68	2314	1·89	56	1905	1·75	45	1531	1·80	37	1259	1·54	31	1054	1·29
10	36	1225	1·00	32	1088	1·00	25	851	1·00	24	817	1·00	24	817	1·00

Ring.	Pressure 0·54 millimetre. Potential.*			Pressure 0·37 millimetre. Potential.			Pressure 0·14 millimetre. Potential.			Pressure 0·07 millimetre. Potential.			Distance between 12th and the following rings.		
	Divs.	Cells.	Ratio.	Divs.	Cells.	Ratio.	Divs.	Cells.	Ratio.	Divs.	Cells.	Ratio.	Inches.	Centims.	Ratio.
1	49	1667	1·75	49	1667	1·40	56	1905	1·12	103	3504	0·99	22·25	57·1	5·24
4	37	1259	1·32	44	1497	1·26	51	1818	1·02	98	3334	0·94	16·20	41·1	3·81
7	35	1191	1·25	40	1361	1·14	51	1818	1·02	101	3437	0·97	10·20	25·9	2·40
10	28	953	1·00	35	1191	1·00	50	1701	1·00	104	3538	1·00	4·25	10·8	1·00

* Pressure of minimum resistance 0·54 millimetre.

The foregoing results are plotted in the diagram, fig. 17, where the abscissæ are the relative distances of the rings, as given in the last column of the above table, and the ordinates the potentials in numbers of cells.

Fig. 17.



Tube 150 CO^2 (17 aluminium rings).

8540 cells; full potential 403 divisions.

The positive pole of the battery connected to No. 1 ring; the negative pole and the 17th ring to earth. Each ring in succession was connected to electrometer.

Ring.	Pressure 3·33 millimetres.			Pressure 1·33 millimetres.			Pressure 0·8 millimetre.			Pressure 0·34 millimetre.*		
	Potential.			Potential.			Potential.			Potential.		
	Divisions.	Cells.	Ratio.	Divisions.	Cells.	Ratio.	Divisions.	Cells.	Ratio.	Divisions.	Cells.	Ratio.
1	206	4366	9·8	165	3497	7·5	112	2373	4·5	90	1907	1·3
2	168	3560	8·0	149	3158	6·8	107	2268	4·3	80	1695	1·2
3	158	3349	7·5	128	2713	5·8	99	2098	4·0	70	1484	1·0
4	148	3137	7·0	128	2713	5·8	98	2077	3·9	76	1611	1·1
5	138	2925	6·6	119	2522	5·4	94	1992	3·8	76	1611	1·1
6	124	2628	5·9	109	2310	5·0	88	1865	3·5	75	1590	1·1
7	111	2353	5·2	98	2077	4·5	82	1738	3·3	75	1590	1·1
8	99	2098	4·7	92	1950	4·2	77	1632	3·1	74	1568	1·1
9	87	1844	4·1	82	1738	3·7	71	1505	2·8	73	1547	1·1
10	77	1632	3·7	72	1526	3·3	64	1356	2·6	72	1526	1·1
11	65	1377	3·1	65	1377	2·9	58	1229	2·3	70	1484	1·0
12	53	1123	2·5	55	1166	2·5	52	1103	2·1	69	1462	1·0
13	42	890	2·0	47	996	2·1	45	954	1·8	68	1441	1·0
14	32	678	1·5	38	805	1·7	38	805	1·5	67	1420	1·0
15	26	551	1·2	29	615	1·3	31	657	1·2	67	1420	1·0
16	21	445	1·0	22	466	1·0	25	530	1·0	67	1420	1·0
17	0	..	0·0	0	..	0·0	0	..	0·0	0	..	0·0

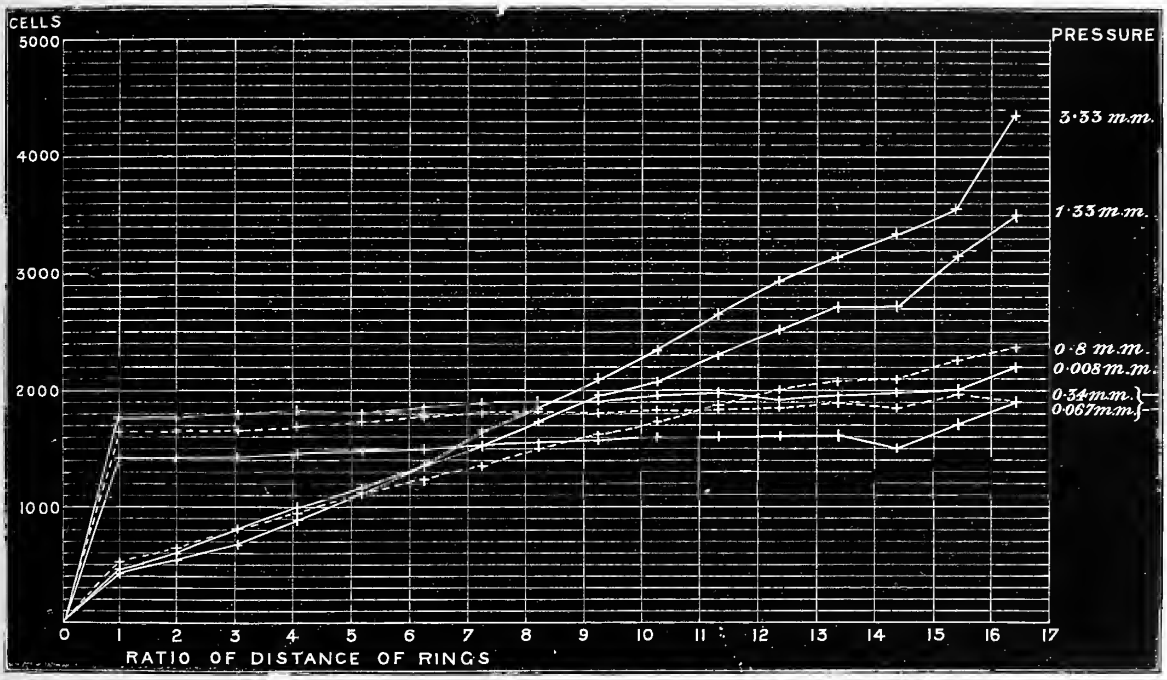
Ring.	Pressure 0·067 millimetre.			Pressure 0·008 millimetre.			Distance between 17th and the following rings.		
	Potential.			Potential.					
	Divisions.	Cells.	Ratio.	Divisions.	Cells.	Ratio.	Inches.	Centims.	Ratio.
1	90	1907	1·2	104	2204	1·2	32·10	81·5	16·44
2	93	1971	1·2	96	2034	1·2	30·10	76·4	15·41
3	87	1844	1·1	94	1992	1·1	28·20	71·6	14·44
4	89	1887	1·1	92	1950	1·1	26·02	66·1	13·34
5	87	1844	1·1	91	1929	1·1	24·15	61·3	12·37
6	86	1823	1·1	93	1971	1·1	22·05	56·0	11·30
7	86	1823	1·1	92	1950	1·1	20·00	50·6	10·24
8	85	1801	1·1	90	1907	1·1	18·00	45·7	9·22
9	86	1823	1·1	90	1907	1·1	16·10	40·9	8·24
10	85	1801	1·1	89	1887	1·1	14·12	35·8	7·24
11	83	1759	1·1	87	1844	1·0	12·12	30·8	6·22
12	82	1738	1·1	85	1801	1·0	10·02	25·4	5·13
13	79	1674	1·0	86	1823	1·0	7·92	20·1	4·06
14	79	1674	1·0	85	1801	1·0	5·90	15·0	3·03
15	78	1653	1·0	83	1759	1·0	3·90	9·9	2·00
16	78	1653	1·0	83	1759	1·0	1·95	4·9	1·00
17	0	..	0·0	0	..	0·0	..	..	..

---

* Pressure of minimum resistance 0·34 millimetre.

These results are shown in the diagram, fig. 18, in which the abscissæ are the relative distances of the rings, as given in the last column of the above table, and the ordinates the potentials in numbers of cells.

Fig. 18.



The following experiments were made in order to ascertain the effect produced on the potential of different parts of a vacuum tube connected at the ends with the opposite poles of an insulated battery when the centre of the tube was put in communication with earth, fig. 19.

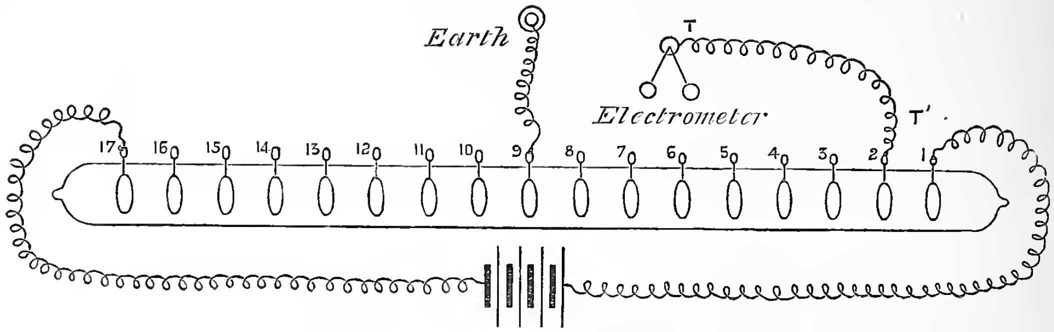
TUBE 150. CO^2.

Battery insulated 8540 cells.

The positive pole was connected to No. 1 ring; the negative to No. 17 ring. The centre ring (No. 9) to earth, and each ring in succession connected with the electrometer. Pressure about 0·5 millimetre*. The deflection was positive between 1 and 9, and negative between 9 and 17.

The diagram, fig. 19, shows the arrangement of the apparatus. The wire T, T′, was connected at T′ to each ring in succession from No. 1 to No. 17.

Fig. 19.



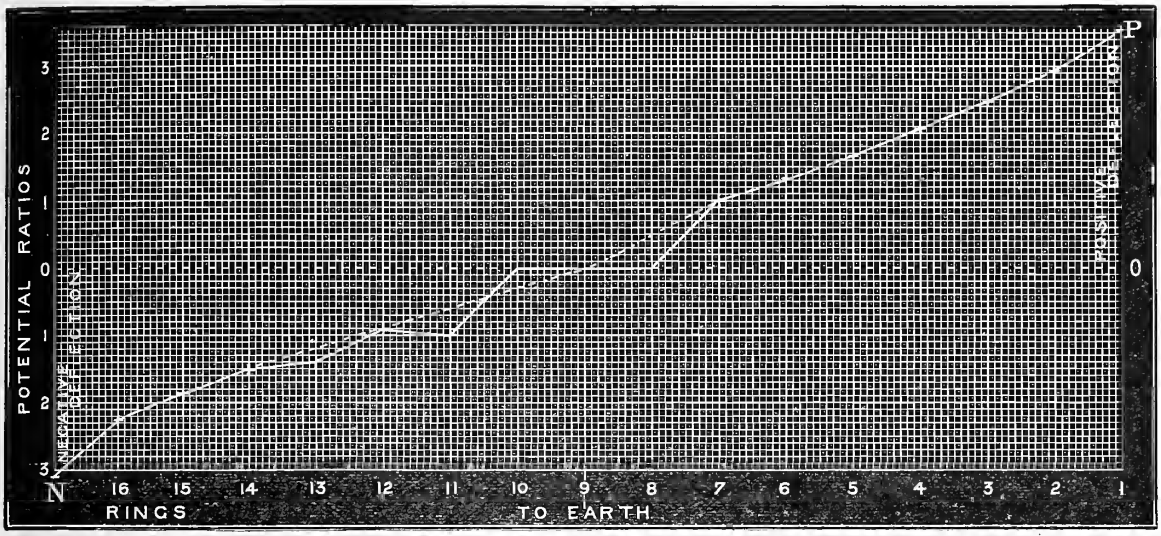
I. Ring.	Potential. II. Divisions.	Potential. III. Ratio.	Distance from 9th ring. IV. In inches.	Distance from 9th ring. V. Centims.	VI. Ratio of distances.	Column $\frac{VI}{III}$.
1	+114	3·563	15·975	40·5	4·096	1·150
2	+ 93	2·907	14·075	36·0	3·608	1·242
3	+ 79	2·469	11·925	30·2	3·058	1·239
4	+ 66	2·063	10·050	25·5	2·577	1·253
5	+ 53	1·657	7·950	20·2	2·038	1·230
6	+ 42	1·312	5·900	15·0	1·513	1·153
7	+ 32	1·000	3·900	9·9	1·000	1·000
8	. .	0·000	2·000	5·1	. .	. .
9	. .	0·000	0·000	0·0	0·000	. .
10	. .	0·000	1·900	4·8	. .	. .
11	− 42	1·000	3·850	9·8	1·000	1·000
12	− 38	0·905	5·850	14·9	1·519	1·679
13	− 59	1·405	7·950	20·2	2·065	1·470
14	− 64	1·524	9·975	25·3	2·590	1·700
15	− 78	1·858	11·975	30·4	3·111	1·674
16	− 94	2·239	13·925	35·3	3·616	1·615
17	−132	3·144	15·875	40·3	4·123	1·312

---

* The tube not being attached to the pump connected with the McLEOD gauge, the pressure could not be exactly determined.

The foregoing results are plotted down in fig. 20, in which abscissæ are the distances of the rings and the ordinates the potentials in scale divisions of the electrometer.

Fig. 20.



The following table contains the results of another experiment also with the opposite poles of the battery to opposite ends of the tube, and the centre ring No. 9 to earth. Fig. 21 represents the results graphically. Tube 150 CO_2, pressure 0·45 m.m., 4620 cells, full potential 227 divisions.

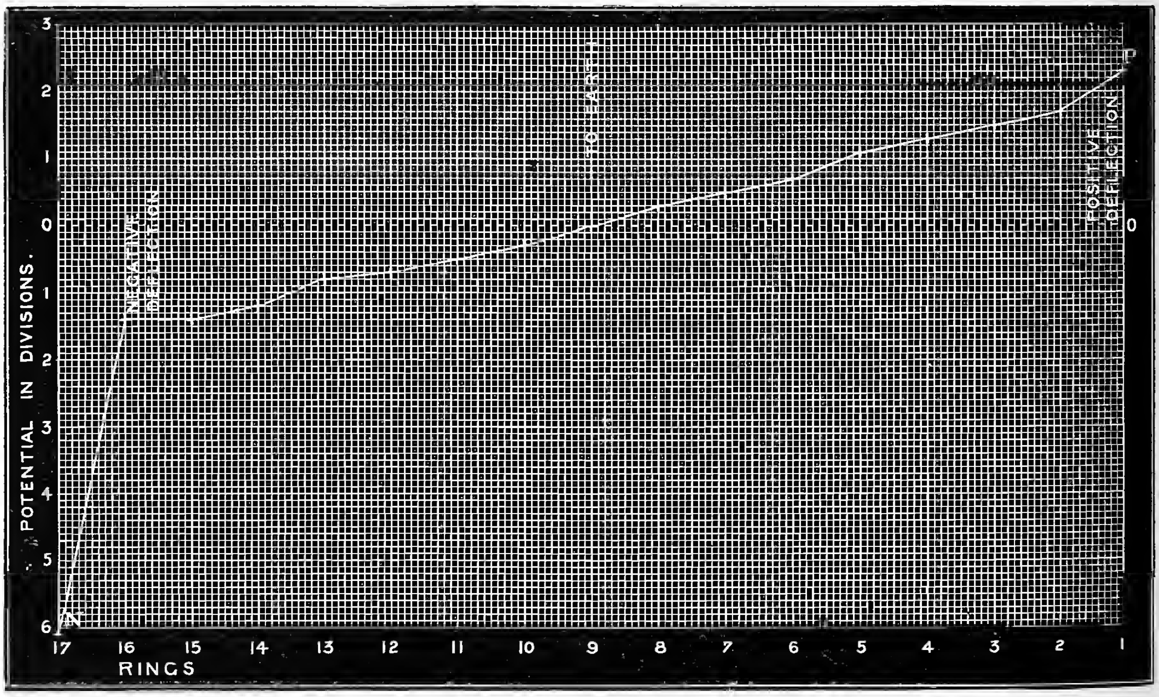
Ring.	Potential.			I.	II.	Column $\frac{II}{I}$.
	Divisions.	Cells.		Ratio of potentials.	Ratio of distances.	
Positive 1	23+	468	468	7·767	7·987	1·028
2	17+	346		5·666	7·037	1·242
3	15+	305		5·000	5·964	1·192
4	13+	264		4·333	5·024	1·160
5	11+	224		3·667	3·976	1·084
6	7+	142		2·333	2·950	1·264
7	5+	102		1·666	1·950	1·170
8	3+	61		1·000	1·000	1·000
to earth 9	0	0		0·000	0·000	0·000 Mean 1·25
10	3−	61		1·000	1 000	1·000
11	5−	102		1·666	2·026	1·531
12	7−	142		2·333	3·079	1·319
13	8−	163		2·667	4·184	1·569
14	12−	244		4·000	5·249	1·312
15	14−	284		4·667	6·302	1·350
16	14−	284		4·667	7·328	1·570
Negative 17	61−	1242	1242	20·330	8·354	0·411
		Total	1710			

When the opposite poles of the battery were connected to the two end rings and the centre ring No. 9 to earth, the deflections, as we have before said, were all positive between 1 and 9, and all negative between 9 and 17; if the connexions with the battery were made previous to the connexion of 9 to earth, and a well-formed steady stratification produced, very little change occurred in it when 9 was connected by means of a key to earth; in fact only a slight one in those strata close to the earth ring.

The curves, more especially fig. 21, show that there is no disturbance of the potentials in consequence of the earth connexion in the centre of the tube, but that the whole curve is lifted or depressed so as to cut the axis at that point. The curve (fig. 22) brings out very forcibly how far greater is the obstacle to the passage of the discharge at the negative than at the positive electrode.

We have already called attention in Part III. of our researches to the far greater obstacle presented by the negative terminal to the passage of a discharge than occurs at the positive.* We therein described an experiment with a tube in which the negative terminal was a wire 19 inches long which became more and more illuminated as the current was gradually increased.

Fig. 21.



In order to bring into strong evidence the great resistance to the passage of electricity at the negative as compared to the positive terminal, the same ring was made positive and negative alternately.

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* Phil. Trans. for 1880, vol. 171, Part I., p. 108 (separate copy, Part III., p. 202).

Tube 150. CO_2.

4640 cells at pressures 0·45 m.m. and 0·25 m.m.; in the first case battery full potential 242 divisions, in the second 213 divisions.

In the first series, at both pressures, the positive was connected with No. 1 ring; in the second, the negative was connected with No. 1 ring. No. 17 ring and the opposite pole of the battery to earth.

Ring.	Pressure 0·45 millimetre. Potential.						Pressure 0·25 millimetre. Potential.						Distance between 17th and the following rings.
	Positive to No. 1.			Negative to No. 1.			Positive to No. 1.			Negative to No. 1.			
	Divs.	Cells.	Ratio.	Divs.	Cells.	Ratio.	Divs.	Cells.	Ratio.	Divs.	Cells.	Ratio.	Ratio.
1	85	1629	1·77	61	1169	10·16	82	1786	1·34	74	1612	12·34	16·44
3	78	1496	1·62	34	652	5·66	77	1677	1·26	12	261	2·00	14·44
5	73	1400	1·52	30	575	5·00	73	1590	1·20	21	457	3·50	12·37
7	68	1304	1·42	27	518	4·50	71	1546	1·16	10	218	1·67	10·24
9	62	1189	1·29	21	402	3·50	67	1460	1·10	16	348	2·67	8·24
11	58	1112	1·21	23	441	3·83	64	1394	1·05	21	457	3·50	6·22
13	52	997	1·09	14	268	2·33	59	1285	0·97	8	174	1·33	4·06
15	48	920	1·00	7	134	1·16	59	1285	0·97	6	131	1·00	2·00
16	48	920	1·00	6	115	1·00	61	1328	1·00	6	131	1·00	1·00
17	0	0	0·00	0	0	0·00	0	0	0·00	0	0	0·00	0·00

The results are shown in the diagram, fig. 22.

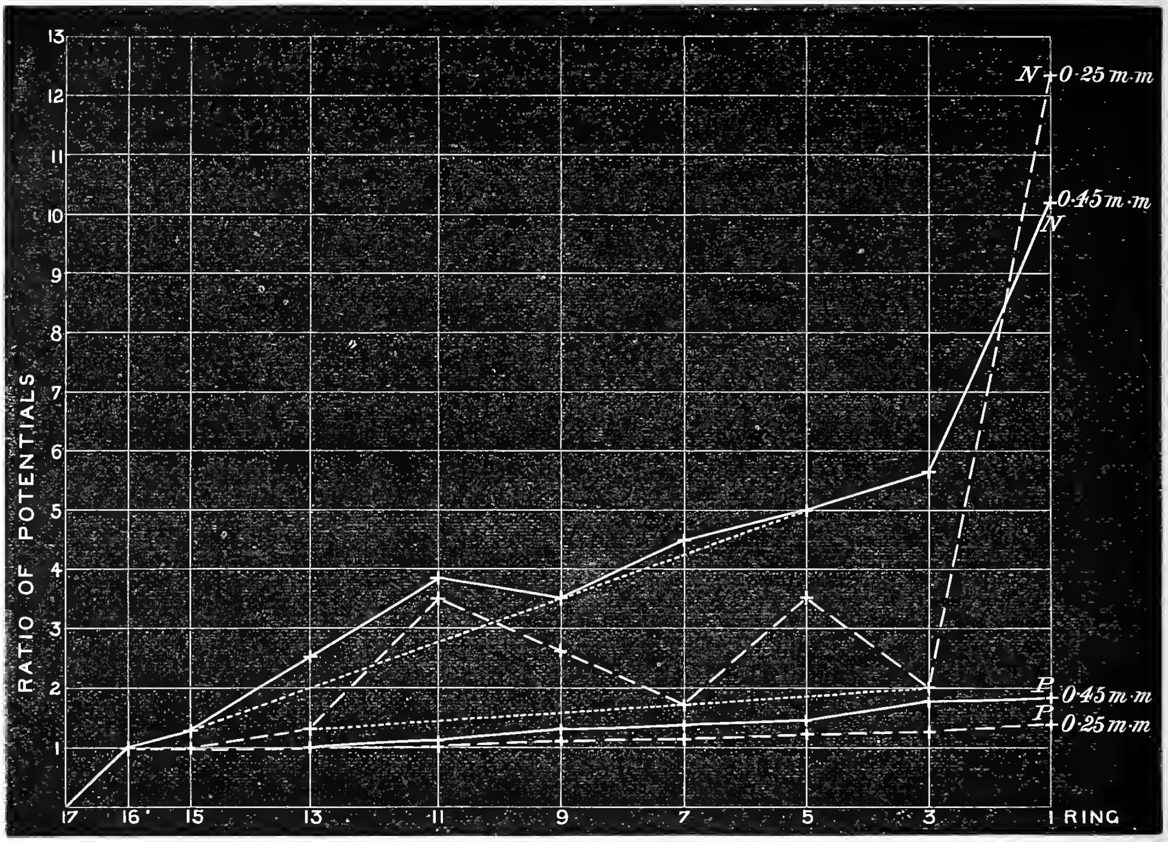
The foregoing experiments "show clearly," as Professor STOKES has pointed out to us, "that at high exhaustions nearly the whole of the energy is spent in the passage of the discharge between the gas and the negative electrode. It seems pretty clear that the electric potential energy is converted into kinetic energy.

"There appears to be no such difficulty as regards the positive. The increase of change of potential near the positive at moderate exhaustions seems fully accounted for by the contraction of the path of the discharge, as the electricity issues at first from little more than a mere point, and the path at first lies within an approximately conical boundary.

"The experiments bear in an important manner on the theoretical height of the aurora. It appears that at high exhaustions gases offer little obstacle to the passage of electricity; only there must be matter enough to carry it. On the whole the experiments tend to considerably increase the theoretical height, which accords with observation, so far as such observations can be trusted.

"How far the first issue of electricity from a charged cirrus cloud resembles the issue from a metallic electrode is a matter of speculation."

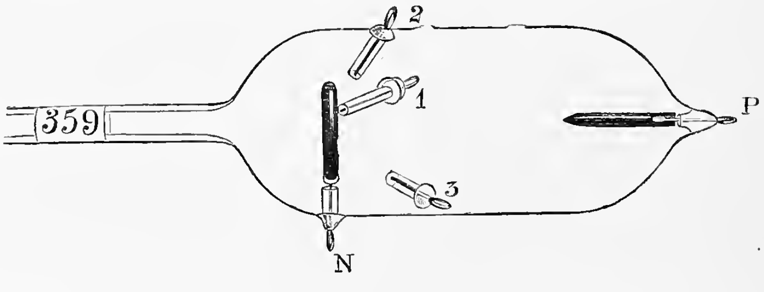
Fig. 22.



Electrical condition of a gas in a vacuum tube in the vicinity of the negative terminal.

The preceding experiments, in relation to potential at different parts of a column of gas, suggested the desirability of probing the electrical condition of a tube in the vicinity of the terminals, more especially the negative. Professor STOKES suggested that this might be conveniently done by means of wires fixed at different distances from it, provided the wires were covered with miniature glass tubes reaching nearly to their extremities. Accordingly a bulb was prepared of the form and construction represented half-size in fig. 23, in which P is a terminal in the shape of a point, to serve as the positive, N, another in the shape of a ring, to serve as the negative, both

Fig. 23.

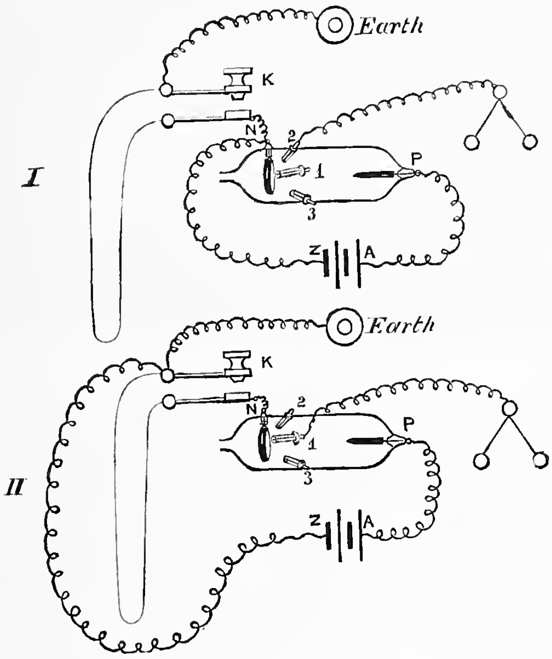


being of aluminium. Nos. 1, 2, and 3 are idle probing wires of platinum, covered with miniature glass tubes, except their extreme ends. The end of No. 1 is about 0·02, No. 2 0·2, and No. 3 0·6 inch from the ring.

In the following experiments variations were made: in one case the battery was insulated and its pole A connected with the point, and its pole Z with the ring; either a stout or a fine wire was then led from the ring, negative, to earth. At other times the battery was uninsulated, the pole A being connected to the point, and the pole Z with earth, and a stout or a fine wire led from the earth connexion to the ring. The stout wire, 0·06 inch diameter, was that ordinarily used in connecting the battery, the fine wire, 3 feet of platinum, 0·002 inch diameter, and having a resistance of 81 ohms at 19°·2 C.

Fig. 24, I. and II., shows the different arrangements.

Fig. 24.



In I. the battery is insulated, the silver end A being connected with the point, and the zinc Z with the ring; from the negative ring a stout wire is led to the lower portion of the key K, and the earth connexion is made with stout wire to the upper part of the key, so that in the position shown in the figure the communication to earth is through the fine wire, but when K is pressed down then the earth communication is through the stout wire.

II. shows the battery uninsulated, the silver end A being connected to the point, and the zinc Z to the upper part of the key. The ring is connected to earth by a stout wire, and also to the lower part of the key. In the position shown in the figure the discharge has to pass through the fine wire to earth, but when the key is pressed down it passes through the stout wire.

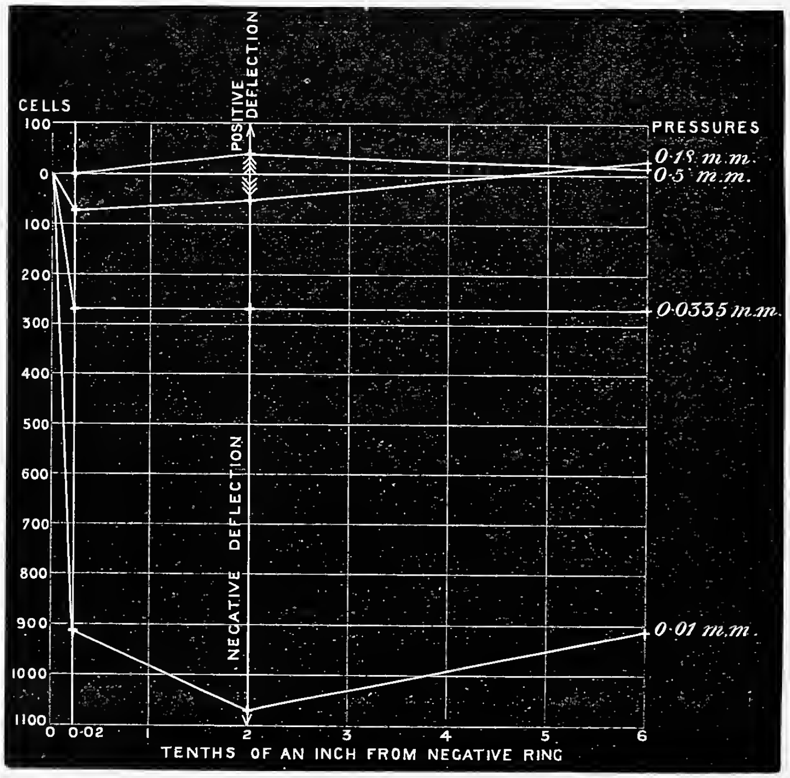
The following results were obtained with tube 359 air :—

Pressure 0·5 millimetre. Battery 1200 cells. Arrangement II. Stout wire. Potential, cells.					Pressure 0·18 millimetre. Battery 1200 cells. Arrangement II. Stout wire. Potential, cells.				
Tube.	No. 3.	No. 2.	No. 1.	Ring.	Tube.	No. 3.	No. 2.	No. 1.	Ring.
540	10+	40+	0	0	730	20+	50−	70−	0

Pressure 0·0335 millimetre. Battery 2400 cells. Arrangement II. Stout wire. Potential, cells.					Pressure 0·01 millimetre. Battery 5840 cells. Arrangement II. Stout wire. Potential, cells.				
Tube.	No. 3.	No. 2.	No. 1.	Ring.	Tube.	No. 3.	No. 2.	No. 1.	Ring.
496	273−	273−	273−	0	4627	912−	1068−	912−	0

So that in the vicinity of the negative, which was well connected to earth and gave no deflection, there was at pressure 0·18 the remarkable phenomenon of a development of a negative charge on the idle wire No. 2 of a potential equal to 50 cells, and on No. 1 of 70; this negative development increased up to a potential of as high as 1068 cells, at a pressure of 0·01 m.m. These results are shown in the curve (fig. 25).

Fig. 25.



Pressure 0·027 millimetre. Battery 5840 cells. Arrangement II.

Stout wire. Potential, cells.					Fine wire. Potential, cells.				
Tube.	No. 3.	No. 2.	No. 1.	Ring.	Tube.	No. 3.	No. 2.	No. 1.	Ring.
1794	478−	1196−	578−	0	. .	120+	120+	100+	0

Pressure 0·027 millimetre. Battery 5840 cells. Arrangement II.

Stout wire. Potential, cells.					Fine wire. Potential, cells.				
Tube.	No. 3.	No. 2.	No. 1.	Ring.	Tube.	No. 3.	No. 2.	No. 1.	Ring.
1734	619−	619−	640−	0	3921	392−	62+	186+	0

The pressure by standing during a night fell to 0·017 m.m., some of the air had evidently become partly absorbed by the terminals, and 5840 cells would no longer pass. In some of the following experiments the inside of a Leyden-jar of the capacity of 0·0049 microfarad was sometimes made to communicate with the ring (negative), the exterior of the jar being connected to earth.

Pressure 0·017 millimetre. Battery 7040 cells. Arrangement II. Stout wire.
Potential, cells.

Tube.	No. 3.		No. 2.		No. 1.		Ring.
		With jar.		With jar.		With jar.	
5134	1040−	1040−	1256−	1018−	1560−	1387−	0
4325	815−	776−	873−	892−	853−	1106−	0

Pressure 0·017 millimetre. Battery 7040 cells. Arrangement II. Fine wire.
Potential, cells.

Tube.	No. 3.		No. 2.		No. 1.		Ring.
		With jar.		With jar.		With jar.	
5134	0	0	173+	108+	195−	195−	0

The signs + and − denote the name of the electrical charge at the idle wires, and it is seen that in many cases the charge is again negative, notwithstanding that the negative ring gives no deflection.

In the following experiments the inside of the Leyden-jar was in contact sometimes with the negative, at others with the positive terminal of the tube.

Pressure 0·017 millimetre. Battery 7040 cells. Arrangement II. Stout wire.

Potential, cells.

Tube.	No. 3.		No. 2.		No. 1.		Ring.
		Jar to P.		Jar to P.		Jar to P.	
4325	1086−	466+	931−	698+	1145−	252+	0

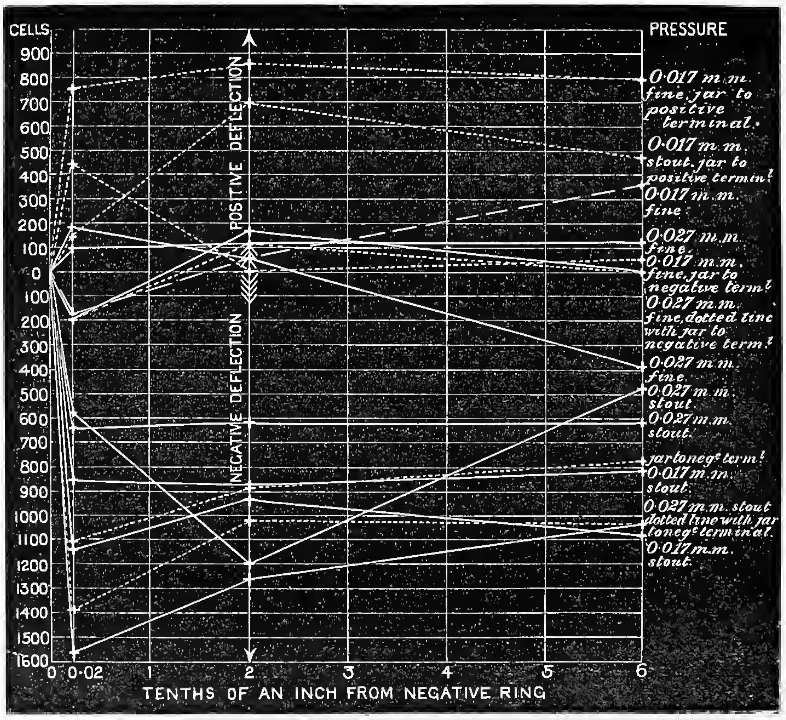
Pressure 0·017 millimetre. Battery 7040 cells. Arrangement II. Fine wire.

Potential, cells.

Tube.	No. 3.			No. 2.			No. 1.			Ring.
		Jar			Jar			Jar		
		To N.	To P.		To N.	To P.		To N.	To P.	
6874	358+	48+	787+	72+	0	859+	167−	453+	763+	0

These results are given in the curve (fig. 26).

Fig. 26.



In the following experiments the arrangement I. was employed, that is to say, the poles of an insulated battery were connected with the terminals of the tube, and an earth connexion made either by means of a stout or fine wire to the negative terminal.

Air was introduced, and at various pressures, with 2400 cells, the following results were obtained :—

Pressure 3·7 millimetres. Battery 2400 cells. Arrangement I. Potential, cells.				
Tube. Stout or fine.	No. 3. Stout or fine.	No. 2. Stout or fine.	No. 1. Stout or fine.	Ring. Stout or fine.
513+	47+	47+	47+	0

Pressure 1·7 millimetre. Battery 2400 cells. Arrangement I. Potential, cells.				
Tube. Stout or fine.	No. 3. Stout or fine.	No. 2. Stout or fine.	No. 1. Stout or fine.	Ring. Stout or fine.
513+	466+	466+	466+	0

Pressure 0·18 millimetre. Battery 2400 cells. Arrangement I. Potential, cells.					
Tube. Stout or fine.	No. 3. Stout or fine.	No. 2. Stout or fine.	No. 1.		Ring. Stout or fine.
			Stout.	Fine.	
1235+	1119+	986+	116−	140+	0

Pressure 0·1 millimetre. Battery 2400 cells. Arrangement I., mean of six experiments. Potential, cells.				
Tube. Stout or fine.	No. 3. Stout or fine.	No. 2. Stout or fine.	No. 1. Stout or fine.	Ring. Stout or fine.
1990+	1784+	1281+	384+	0

The connexion to earth made through a cork (0·5 inch diameter, and 0·5 inch long) saturated with water, and having a resistance of 4,300,000 ohms.

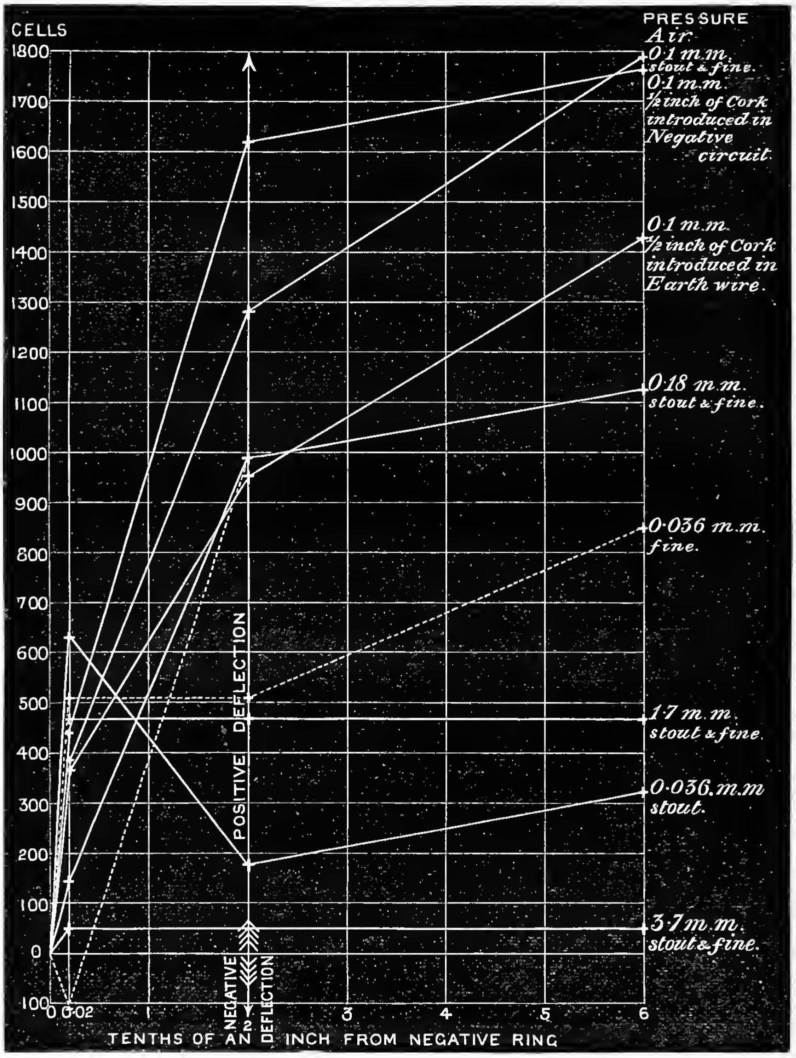
Pressure 0·1 millimetre. Battery 2400 cells. Arrangement I. Potential, cells.				
Tube.	No. 3.	No. 2.	No. 1.	Ring.
1179+	1420+	955+	367+	0

Cork introduced into circuit—

Pressure 0·1 millimetre. Battery 2400 cells. Arrangement II. Potential, cells.				
Tube.	No. 3.	No. 2.	No. 1.	Ring.
2400	1763	1616	441	2400

These results are shown in the curve (fig. 27).

Fig. 27.



Potential at the centre and periphery of a ring negative terminal.

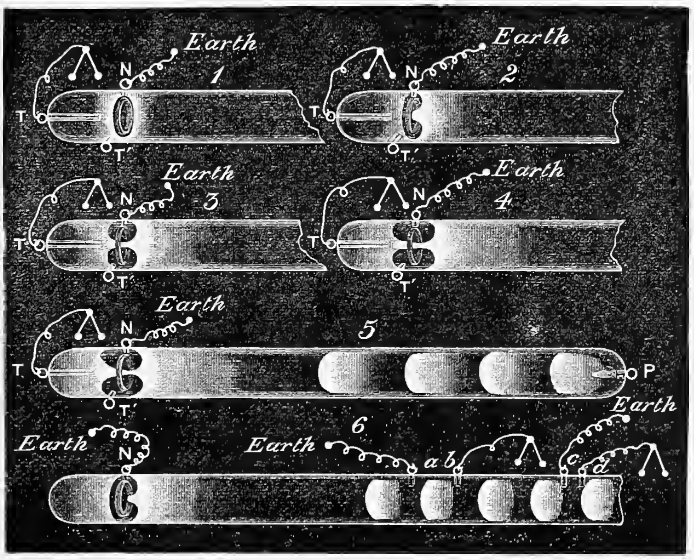
Everyone familiar with the appearance of a stratified discharge will have noticed, when the negative terminal is a ring, that as the exhaust proceeds a spindle of light approaches and at last protrudes through the interior of it, as shown in fig. 28, 5.

This spindle is thus a visible exponent of strong action among the molecules of the gas composing it, and it appeared to be of interest to probe its electrical condition. We therefore prepared a tube, No. 363, for this special object with an idle wire, surrounded by a minute glass tube, except its extremity, in the axis of the tube and projecting to a distance of $\frac{3}{8}$ of an inch (0·95 centim.) from the plane of the ring which was made negative.

The dimensions of the tube are:—Length, 15·5 inches (39·3 centims.), and 1·75 inches (4·44 centims.) in diameter, and the distance between the terminals, a point and a ring, 12 inches (30·5 centims).

A battery of 3600 cells was used, the terminal, in the shape of a point, was connected to the positive pole, and the ring and negative pole of the battery were both connected to earth. The gas used was carbonic anhydride. The full potential of the battery was found to be 313 divisions of the electrometer, that of the idle axial wire at a certain exhaust 142 divisions, equal to 1633 cells.

Fig. 28.



The exhaust was carried further, and it then required 6000 cells to produce a discharge. The tube potential was found to be equal to 2523 cells, and the idle wire, T, 2535 cells, or practically the same as the tube.

Another set of experiments was made with the same tube, and progressively greater and greater exhausts, consequently presenting different appearances of the luminosity at the negative ring as the exhaust was increased. These appearances are shown in fig. 28, Nos. 1, 2, 3, 4, 5, in which the idle wire, T, is connected with the electrometer.

		Divisions.	Ratio to tube potential = 1.
1. Idle wire,	T potential	48	= 0·52
	Tube ,,	92	
1. A little higher exhaust,	T ,,	57	= 0·60
	Tube ,,	95	
2.	T ,,	118	= 0·87
	Tube ,,	136	
2. A little higher exhaust,	T ,,	134	= 0·93
	Tube ,,	144	
3.	T ,,	143	= 0·90
	Tube ,,	158	
4.	T ,,	170	= 0·94
	Tube ,,	181	
5.	T ,,	163	= 0·95
	Tube ,,	173	

It is evident, therefore, that as the exhaust increases, and the luminous spindle becomes more developed, the potential of the idle wire, T, augments until it nearly or quite equals that of the whole tube.

In tube No. 363 there was no means of testing the potential on the outside of the negative ring under the same conditions as the centre, so that another tube, No. 364, was made in which an idle wire, T, was in the axis 0·25 inch (0·63 centim.) from the plane of the ring, and another, T′, 0·15 inch (0·38 centim.) from its periphery. The tube is 13½ inches (34·3 centims.) long, 1¾ inches (4·44 centims.) in diameter, and 9½ inches (24·1 centims.) between the terminals, and was charged with carbonic anhydride.

When 2400 cells were used, the tube potential was found to be 742 cells; the potential of the axial wire, T, was 158 cells, when that of the exterior wire, T′, was only 40 cells, so that the potential of the central wire, T, was $\frac{158}{40}=3\cdot9$ times that of the external wire T′. The potentials in relation to that of the whole tube equal 1, were found to be T wire =0·21, T′ =0·054.

Potential across a dark space and a stratum respectively.

At the suggestion of Professor Stokes, experiments were made to ascertain the potential across a dark space and also a stratum, several tubes being constructed for that object. The experiment is an extremely troublesome one, on several accounts; in the first place it is difficult to get a stratum or a space to fall exactly between a pair of idle wires, and although this may be frequently accomplished, even between the same pair, when there is a steady and slow onward motion of the strata, if the battery is left on for a time and the tube becomes gradually heated; yet even then it requires great dexterity to catch the precise moment at which to make contact with the electrometer. Part of the difficulties may possibly be due to the inherent nature of the discharge itself, which if a pulsating one of a very high order, may be caught at different phases at the instant of contact.

In tube No. 361 charged with carbonic anhydride, 32·5 inches (82·5 centims.) long, and 1·75 inches (4·44 centims.) in diameter, having two terminals, one a point, the other a ring, distant 28·25 inches (71·74 centims.), two idle glass-covered wires were fused into it. No. 1, 11·75 inches (29·84 centims.), and No. 2, 12·75 inches (32·38 centims.) from the plane of the ring which was made negative; the proper position of the idle wires was determined by that of the strata in a preliminary trial.

The battery used was 3600 cells, the negative pole and the ring terminal were connected to earth.

The tube potential was found to be =1945 cells.

When the electrometer was connected with:—

	Cells.
No. 1, touching convex side of a stratum, the potential was	= 1862
No. 2, ,, concave ,, ,, ,,	
At another pressure the tube potential	= 1784
No. 1, touching a dark space	= 1339
No. 2, ,, ,,	
No. 1, touching concave of stratum	= 1398
No. 2, ,, convex ,,	

So that when an idle wire either touched the convex or concave side of a stratum, or either side of a dark space, the potential was the same, and approached nearly that of the tube when tested at the positive terminal.

With the same tube when the two points were made to straddle a stratum or a space, one point, No. 2, connected with the electrometer, and the other, No. 1, to earth, the indications differed.

	Cells.	Stratum.	Space.
A stratum intervening, potential	= 282	= 1·32	= 1
A dark space, ,,	= 215		

Another series gave—

Stratum intervening, ,,	= 87	= 1·19	= 1
Dark space, ,,	= 73		

Another series gave results the reverse of this, namely, a greater potential when a dark space intervened, but it is possible that the connexion with the electrometer was not made precisely at the proper moment.

	Cells.		
Space	102	=	1·05
Stratum . . .	97	=	1

Taking the mean of several other sets, the balance of the results is in favour of the potential being greatest when a stratum intervenes, for example :—

	Stratum.	Space.
	1·243	1
	1·327	1
	1·081	1
Mean	1·243	1

A series of experiments were made with tube No. 362 having a hydrogen charge; a battery of 3600 cells being used, which gave a current of 0·04686 ampère through it.

This tube has four idle wires *a*, *b*, *c*, *d*, as shown in fig. 28, 6, but only two were used, namely, *c* and *d*, 1 inch (2·54 centims.) apart; and a stratum or a space was made to intervene between them. The positive pole was connected to the terminal in the shape of a point, and the negative to the ring which was also connected to earth. The following are the results :—

		Divisions.		Cells.		Divisions.		Cells.
Tube potential	=	192		1443				
Stratum ,,		111			Space	98		
		137				59		
		84				115		
		132				78		
		133				115		
		129				69		
		74				72		
		146				69		
		102				141		
		132				142		
Mean		118	=	887		96	=	722

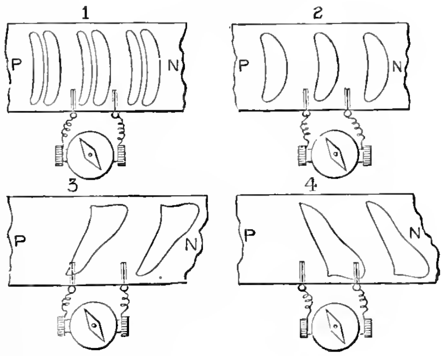
These figures may be taken as fairly representing the whole series of observations. $\frac{118}{96}=1\cdot229$, which agrees well with the ratio before cited as that of the potential of a stratum to that of a space$=1$; and we think that it may be fairly assumed that the potential is really greater when a stratum is straddled than when a space is straddled.

Eddies in an electric discharge through a vacuum tube.

On testing the discharge between two idle wires distant $\frac{5}{8}$ inch (1·6 centim.) with a THOMSON galvanometer, the current was found sometimes to go in the reverse direction to that of the main current. In order to fix with certainty this phenomenon in one's mind the current was at first sent through a fine platinum wire 26 inches (66·04 centims.) long, and 0·002 inch (0·005 centim.) in diameter, leading wires from a THOMSON galvanometer were connected $\frac{1}{2}$ inch (1·27 centim.) apart with the middle of the wire, the deflection of the galvanometer to the right showed that the current was as anticipated in the same direction in this fraction as in the whole wire. The deflection through the wire was 162 divisions with $\frac{1}{999}$ shunt, $=161{,}200$ without shunt, $=\frac{75\cdot5 \text{ volts}}{1 \text{ megohm}}$ to the right. When a moistened thread was substituted for the fine wire the effect was the same. When, however, a vacuum tube was employed, then evidence was frequently obtained of a current in a contrary direction.

With tube No. 365, 21$\frac{3}{4}$ inches (55·25 centims.) long, with 3600 cells, a series of eleven pairs of beautiful double strata like those in Plate 16, fig. 3, Part III., were obtained, fig. 29, 1; these were perfectly steady. Two idle wires, 9$\frac{3}{4}$ inches (24·8 centims.) and 10 inches (25·4 centims.) distant from the positive end and embracing the seventh pair of strata, were connected with the THOMSON galvanometer,* when the following numbers were obtained :—

Fig. 29.



	With $\frac{1}{9}$ shunt.		Without shunt.
To left	50—	=	496—
To right	56+	=	559+
To left	50—	=	496—
,,	30—	=	298—
,,	140—	=	1389—
To right	143+	=	1419+

On introducing a resistance of 500,000 ohms in the current, a pink stratification of nineteen strata was obtained like that in Plate 16, fig. 4, Part III., fig. 30, 2. The

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* The constant of this galvanometer, with the controlling magnet in the position used in the following experiments, is $C\,\frac{1 \text{ volt}}{1 \text{ megohm}}=2136$ divisions, or through 2136 ohms $\times 10^6=1$ division. The multiplier for the $\frac{1}{9}$th shunt $=9\cdot92$, that for the $\frac{1}{999}$th $=995\cdot3$.

idle wires embraced the eleventh stratum from the positive, and deflections were obtained as under with $\frac{1}{9}$ shunt :—

With $\frac{1}{9}$ shunt.		Without shunt.
To left 180−	=	1785−
To right 170+	=	1686+
,, 70+	=	694+
To left 204−	=	2023−*
To right 110+	=	1091+
,, 140+	=	1386+
,, 130+	=	1289+
,, 30+	=	297+
,, 15+	=	149+
To left 20−	=	198−

With tube No. 362 hydrogen charge and a battery of 2400 cells, the strata were tongue-shaped as in Part II.,† Plate 15, fig. 11; two touch points 1 inch (2·54 centims.) apart were connected with a THOMSON galvanometer. It was noticed that there was a strong deflection to the left (the deflection through a wire being to the right) when the base of a stratum touched the idle wire nearest the positive terminal and its apex pointed in the direction of the negative, but rose above it towards the opposite diameter of the tube, fig. 29, 3. When the apex of the tongue pointed in the reverse position so as to touch the idle wire next the negative and the base went to the other side of the tube above the idle wire near the positive, fig. 29, 4, then the deflection was in a contrary direction (that is the same as through a wire). When the tongues crossed the axis of the tube nearly at right angles, then there was an oscillation of the needle, first in one direction then in the other, according as the strata altered their position slightly with regard to the direction of the apex and base of the tongue.

The following are the readings without any shunt :—

140	left	−
20	right	+
15	left	−
210	right	+
200	,,	+
165	,,	+
150	,,	+
210	left	−

Sometimes when a deflection took place in the same direction as the current through a wire, and had arrived at its maximum, then a slow oscillation was observed in the deflection, the needle moving backwards and forwards for about 20 divisions when the full deflexion was 290, or 0·068 of the total deflection.

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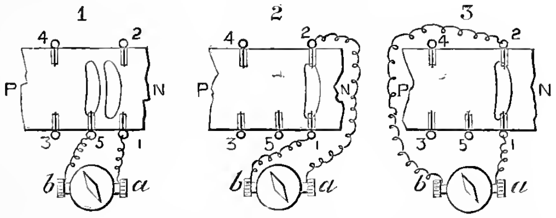
* Nearly equal to 1 volt through a megohm.

† Phil. Trans. for 1878, Vol. 169.

Another tube was made to test whether there is a current at right angles to the column of gas; namely No. 366, 21½ inches (54·61 centims.) long, and 1⅝ inches (4·1 centims.) in diameter, in it are sealed five idle wires, 1, 2, 3, 4, 5. No. 3 is 5½ inches (14 centims.) from the positive. Nos. 3, 5, 1 are all in a longitudinal line on the same side of the tube, 4 and 2 in line on the opposite side; 4 is diametrically opposite to 3, 2 diametrically opposite to 1. The tube with its wires is shown in fig. 30, 1, 2, 3; the opposite wires of the galvanometer are marked *a* and *b*.

Fig. 30.



The charge was hydrogen, and 2400 cells were used. In the first place the galvanometer was connected with two wires in a longitudinal direction, the galvanometer wire *b* to the idle wire 5, the galvanometer wire *a* to the idle wire 1; the idle wires at this time straddled both a stratum and a space. The deflection was to the right, amounting to:—

With $\frac{1}{9}$ shunt.	Without shunt.
202+	2004+
117+	1161+
62+	615+
37+	367+
22+	218+
2+	20+
82+	813+
102+	1012+
132+	1309+

The wires of the galvanometer were now reversed, *a* to 5 and *b* to 1, the deflection was now to the left, but the galvanometer connexion having been reversed, the sign was the same as in the preceding.

With $\frac{1}{9}$ shunt.	Without shunt.
98+	972+
68+	674+
83+	823+
68+	675+
58+	575+
48+	476+
38+	377+
118+	1170+

All these deflections are in the same direction as if the current went through a wire. The wires of the galvanometer were now connected, *b* to 1, *a* to 2, these being diametrically opposite, a stratum was touched at an opposite end of its diameter by each wire ; the readings were :—

With $\frac{1}{9}$ shunt.	Without shunt.	The current was from
To left 3—	30—	1 to 2
,, 8—	79—	,,

The wires were now reversed, *a* to 1 and *b* to 2, and the following observed :—

With $\frac{1}{9}$ shunt.	Without shunt.	The current was from
To the right 12+	119+	1 to 2
,, ,, 2+	20+	,,
To the left 8—	79—	2 to 1
,, ,, 3—	30—	,,
0	0	

It is evident, therefore, that there is a current sometimes in one direction, sometimes in the contrary, across a diameter of the tube, as if the motion of the molecules conveying the discharge was of an epicycloidal character. By way of comparison a tin-plate, $14\frac{1}{2}$ inches (36·8 centims.) long and $5\frac{1}{2}$ inches (14 centims.) wide, with circular ends, was made ; a connecting clamp was soldered to each end and two others across a central diameter. The direction of the current was ascertained when the poles of the battery were connected one to each end of the plate, and the opposite wires of the galvanometer also one to each end. Then the two clamps across a central diameter were connected alternately with the opposite ends of the galvanometer, the terminals remaining connected with the ends of the plate, but no deflections occurred even without a shunt.

One of the main objects of the experiments recorded in this paper was to endeavour to discover some of the obscure secrets of the electric discharge through gases. Although many more will undoubtedly have to be made before a theory can be formed which will account even for a few of the complex phenomena presented by this discharge, yet every onward step, however small, brings us nearer to the goal, and we shall consider ourselves well rewarded if we have advanced the frontier in the very smallest degree. We believe that what we have done has secured some advance, and rendered evident that electricity is always conveyed in gases by ponderable matter and not by the so-called ether ; moreover, that there are eddies in the discharge in a vacuum tube, and that it is possible that these eddies may be connected with the production of strata whose form would depend on the kind of eddy originally produced. This recalls the phenomena figured in Part III., Plate 8, fig. 34 ;[*] in this is seen an eddy of strata through small holes in a tube confining the main discharge.

We have much pleasure in thanking Professor STOKES for his many valuable suggestions, and for his kind advice generally, during the course of this investigation. To our chief assistant, Mr. JAMES FRAM, we are indebted for able and zealous co-operation, and we wish to place on record the excellent services of our junior assistant, Mr. ERNEST DAVIS. Mr. H. REYNOLDS has, as on former occasions, taken the photographs with his usual skill.

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[*] Phil. Trans. for 1880, Vol. 171.

POSTSCRIPT.

(Added July 10th, 1883.)

Our battery now consists of 15,000 cells, all of the form in which the chloride of silver is fused into rods on a flattened silver wire.[*] On the occasion of a lecture given by one of us at the Royal Institution on January 21, 1881, 14,400 cells, partly of the rod form, partly of the chloride in powder form, were used; the length of the spark with this number between paraboloidal points was 0·7 inch (17·8 millims.), and between a point and disc 0·62 inch (15·7 millims.). It will be recollected that in Part I. of our researches,[†] we obtained with 11,000 cells between two points a spark 0·62 inch long, and between a point and disc one 0·47 inch long. It does not appear, therefore, that the law of the spark being as the square of the number of cells holds good beyond a certain number; the falling off may be in part due, however, to the failure of insulation as the potential is increased.

$$\frac{14400^2 \times 0{\cdot}62 \text{ inch}}{11000^2} = 1{\cdot}060 \text{ inch (27 millims.)}$$

$$\frac{14400^2 \times 0{\cdot}47 \text{ inch}}{11000^2} = 0{\cdot}806 \text{ inch (20 millims.)}$$

Since the removal of the battery from the Royal Institution we have not charged up the whole of it, as the experiments on which we have been engaged did not necessitate more than 11,000 cells.

We have recently (May 10–July 4) repeated the experiments on the question of the polarisation of the terminals on a discharge taking place in gases, and have obtained the same results as those already described;[‡] they confirm, therefore, our previously expressed opinion,[§] that the experiments point conclusively to the deduction that the small current, which is observed when the terminals of a vacuum tube (in which a discharge has taken place) are disconnected from the battery and instantaneously connected with a galvanometer, is due solely to a static charge and not to *chemical polarisation.*

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[*] Phil. Trans., Part I., Vol. 169, 1878, p. 109, separate copy, p. 55.

[†] Ibid., p. 118, separate copy, p. 64.

[‡] Proc. Roy. Soc., No. 205, 1880, pp. 563–572.

[§] Ibid., p. 570.

INDEX TO PART IV.

PLATES 35 AND 36, REFERENCES TO TEXT.

CURVES REPRESENTING TABLES, REFERENCES TO TEXT.

XIII. *On Electrical Motions in a Spherical Conductor.*

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Communicated by J. W. L. GLAISHER, *M.A., F.R.S.*

Received March 14,—Read April 5, 1883.

THIS paper treats of the motions of electricity produced in a spherical conductor by any electric or magnetic operations outside it. The investigation was undertaken some time ago in illustration of MAXWELL'S theory of Electricity. This theory is so remarkable, more especially in the part which it assigns to dielectric media in the propagation of electromagnetic effects, that it seemed worth while to attack some problem in which all the details of the electrical processes could be submitted to calculation, although it was evident beforehand, from the researches of HELMHOLTZ* and others, that the results (so far as they are peculiar to the theory) would be of far too subtle a character to admit of comparison with experiment. In studying the mathematical character of the problem above stated I was led to a certain system of formulæ which I have since utilised in two communications to the London Mathematical Society,† and which seem likely to be of use in a great variety of physical questions.

§ 1 consists mainly of a recital of the fundamental equations and of the conditions to be satisfied at the surface of a conductor. It is assumed, in the first instance, that the magnetic susceptibility of the conductor is zero.

In § 2 is introduced the assumption that all our functions vary as $e^{\lambda t}$, where t is the time, and λ a constant. It is pointed out that this assumption is sufficiently general. The fundamental equations are then put into a mathematically convenient form. Before, however, proceeding to apply these equations as they stand, I examine the effect of assuming that the velocity (v) of propagation of electromagnetic effects in the medium surrounding the conductor is practically infinite. This assumption, which has been made by all writers (including MAXWELL himself) who have applied MAXWELL'S theory to ordinary electromagnetic phenomena, greatly simplifies the calculations without sensibly impairing the practical value of the results. If L

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* CRELLE, t. 72 (1870).

† "On the Oscillations of a Viscous Spheroid," Proc. L. M. S., Nov. 10, 1881; and "On the Vibrations of an Elastic Sphere," May 11, 1882.

stand for a linear dimension of the conductor and ρ for its specific resistance, it will appear in the sequel that when as in all practical cases λ is small compared with v/L, the error introduced by the assumption in question is of the order $\lambda\rho/v^2$. For any ordinary metallic conductor, and for any value of λ which can be appreciated experimentally, this fraction is excessively minute.

In § 3 the solutions of our equations (on the assumption above indicated) are given in the form appropriate to our present problem. These solutions are of two distinct types. Those of the first type, which are much the more important from an experimental point of view, have (I find) been discussed, though by a different method, by Professor C. NIVEN in a paper recently published.[*] As the points to which attention has been directed are for the most part sufficiently distinct in the two investigations, I have allowed the corresponding portions of my paper to stand.

In § 4 I discuss the case of electric currents started anyhow in the sphere and left to themselves. The equation which gives the "moduli" of the natural modes of decay of the first type agrees with the result obtained by Professor NIVEN.

In § 5 is studied the case of induced currents. Since any disturbance in the field (however arbitrary) can be expressed, as regards the time, by a series of simple harmonic terms, it is sufficient to consider the case when the variations in the inducing system follow the simple harmonic law. This case has moreover acquired a special interest since the invention of the telephone.

The two extreme cases, when the period of the variation in the field is very large or very small in comparison with the time of decay of free currents in the sphere, are discussed in some detail.

In § 6 the case of a thin spherical *shell* is briefly examined.

I next proceed to investigate what modifications must be introduced into the methods and the results of the preceding sections when the substance of the sphere is susceptible of magnetisation. This occupies §§ 7, 8, 9, 10.

In the remaining sections of the paper I investigate the solution of our fundamental equations, taking account of the finite value of v. The corrections to our former results are of most interest in the solutions of the second type. Although the preceding theory, based on the assumption $v=\infty$, is sufficient for all purposes of comparison with experiment, there are certain processes of (at all events) theoretical interest of which it fails altogether to give an account, viz., all those cases in which any change in the superficial electrification of the sphere takes place. For the expression of these the solutions of the second type are appropriate. There is no difficulty in working out the requisite formulæ, but in the application to the case of *free* motion a difficulty of interpretation arises which is noticed in the proper place.

1. Let us suppose that we have one or more conductors at rest in an insulating

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[*] Phil. Trans., 1882. The date of the paper is January, 1880.

medium. If F, G, H be the components of electromagnetic momentum, u, v, w those of electric current, at the point (x, y, z), we have on MAXWELL's theory

$$\left.\begin{aligned} \nabla^2\mathrm{F} &= -4\pi u \\ \nabla^2\mathrm{G} &= -4\pi v \\ \nabla^2\mathrm{H} &= -4\pi w \end{aligned}\right\} \quad \dots\dots\dots\dots \quad (1)$$

and

$$\frac{d\mathrm{F}}{dx}+\frac{d\mathrm{G}}{dy}+\frac{d\mathrm{H}}{dz}=0 \quad \dots\dots\dots\dots \quad (2),$$

where ∇^2 stands for $d^2/dx^2+d^2/dy^2+d^2/dz^2$. These equations hold good in conductors and insulators alike, provided that (as we shall assume for the present) the magnetic permeability in neither case differs sensibly from unity.

In the conductors we have, if ρ be the specific resistance,

$$\left.\begin{aligned} \rho u &= -\frac{d\phi}{dx}-\frac{d\mathrm{F}}{dt} \\ \rho v &= -\frac{d\phi}{dy}-\frac{d\mathrm{G}}{dt} \\ \rho w &= -\frac{d\phi}{dz}-\frac{d\mathrm{H}}{dt} \end{aligned}\right\} \quad \dots\dots\dots\dots \quad (3).$$

The expressions on the right-hand side of (3) are the components of electromotive force, ϕ being a function which, in the case of *steady* motion of electricity, is known by the name of the "electric potential."*

In the dielectric we have, if f, g, h be the components of electric displacement, and $1/v^2$ the specific inductive capacity, measured (like all our quantities) on the electromagnetic system,

$$\left.\begin{aligned} 4\pi v^2 f &= -\frac{d\phi}{dx}-\frac{d\mathrm{F}}{dt} \\ 4\pi v^2 g &= -\frac{d\phi}{dy}-\frac{d\mathrm{G}}{dt} \\ 4\pi v^2 h &= -\frac{d\phi}{dz}-\frac{d\mathrm{H}}{dt} \end{aligned}\right\} \quad \dots\dots\dots\dots \quad (4).$$

v is the velocity of propagation of electromagnetic effects in the dielectric medium. If this be air, v also denotes the number of electrostatic units in one electromagnetic unit of electricity.

The conditions to be satisfied at the boundary of a conductor are that F, G, H and

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* In other cases, as will be seen, this name is less appropriate.

their first derivatives must be continuous. This follows at once from the expressions for **F**, G, H in terms of the electric currents in the field, viz.,

$$\left.\begin{aligned}\mathrm{F}&=\iiint\frac{u'}{r}dx'dy'dz'\\ \mathrm{G}&=\iiint\frac{v'}{r}dx'dy'dz'\\ \mathrm{H}&=\iiint\frac{w'}{r}dx'dy'dz'\end{aligned}\right\}\quad\ldots\ldots\ldots\ldots\quad(5),$$

where r denotes the distance from the element $dx'dy'dz'$ to the point (x, y, z) at which the values of F, G, H are required. Hence if a, b, c be the components of magnetic induction, viz.,

$$\left.\begin{aligned}a&=\frac{d\mathrm{H}}{dy}-\frac{d\mathrm{G}}{dz}\\ b&=\frac{d\mathrm{F}}{dz}-\frac{d\mathrm{H}}{dx}\\ c&=\frac{d\mathrm{G}}{dx}-\frac{d\mathrm{F}}{dy}\end{aligned}\right\}\quad\ldots\ldots\ldots\ldots\quad(6),$$

these quantities will be continuous at the surface of a conductor. Conversely we may show that if F, G, H, a, b, c be continuous then the first derivatives of F, G, H will all be continuous. For this it is sufficient to prove that their derivatives in the direction of the normal will be continuous. If l, m, n be the direction-cosines of the normal, we have

$$\begin{aligned}l\frac{d\mathrm{F}}{dx}+m\frac{d\mathrm{F}}{dy}+n\frac{d\mathrm{F}}{dz}&=l\frac{d\mathrm{F}}{dx}+m\frac{d\mathrm{G}}{dx}+n\frac{d\mathrm{H}}{dx}+nb-mc\\ &=\left(m\frac{d\mathrm{G}}{dx}-l\frac{d\mathrm{G}}{dy}\right)-\left(l\frac{d\mathrm{H}}{dz}-n\frac{d\mathrm{H}}{dx}\right)+nb-mc\quad\ldots\quad(7),\end{aligned}$$

by (2), and it is easily seen from geometrical considerations that the continuity of G implies the continuity of $(m.d\mathrm{G}/dx-l.d\mathrm{G}/dy)$, and so on. Hence if F, G, H, a, b, c be continuous, the first member of (7), and the corresponding expressions for the normal derivatives of G and H, are continuous.

From this point the letters u, v, w will be used to denote solely the components of current *in the conductors*. The components of current in the dielectric are $\dot{f}$, $\dot{g}$, $\dot{h}$.

The general solenoidal conditions to be satisfied by u, v, w and f, g, h, viz.,

$$\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}=0\quad\ldots\ldots\ldots\ldots\quad(8),$$

and

$$\frac{d}{dt}\left(\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}\right)=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9),$$

require, by (3) and (4),

$$\nabla^2\phi=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10),$$

in the conductors, and

$$\nabla^2\frac{d\phi}{dt}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11),$$

in the dielectric. The superficial solenoidal condition

$$lu+mv+nw=l\frac{df}{dt}+m\frac{dg}{dt}+n\frac{dh}{dt}$$

requires, by (1), the continuity of

$$l\nabla^2\mathrm{F}+m\nabla^2\mathrm{G}+n\nabla^2\mathrm{H},$$

i.e., of

$$\left(n\frac{da}{dy}-m\frac{da}{dz}\right)+\left(l\frac{db}{dz}-m\frac{da}{dz}\right)+\left(m\frac{dc}{dx}-l\frac{dc}{dy}\right);$$

but this is implied in the continuity of a, b, c.

If $d\nu'$, $d\nu''$ be elements of a normal to the surface of a conductor, on the inside and outside respectively, we find from (3) and (4), taking account of the continuity of F, G, H,

$$-\left(\frac{d\phi}{d\nu'}+\frac{d\phi}{d\nu''}\right)=4\pi v^2(lf+mg+nh)-\rho(lu+mv+nw);$$

or, if σ denote the surface density of electricity,

$$-\left(\frac{d\phi}{d\nu'}+\frac{d\phi}{d\nu''}\right)-4\pi v^2\sigma-\rho\frac{d\sigma}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12).$$

Hence it is only when the currents are *steady* that the relation between ϕ and the free electricity in the field is the same as in electrostatics.*

If T be the kinetic and V the potential energy of the field, we have

$$\mathrm{T}=\tfrac{1}{2}\iiint(\mathrm{F}u+\mathrm{G}v+\mathrm{H}w)dxdydz$$

$$+\tfrac{1}{2}\iiint(\mathrm{F}\dot{f}+\mathrm{G}\dot{g}+\mathrm{H}\dot{h})d\xi d\eta d\zeta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13),$$

$$\mathrm{V}=2\pi v^2\iiint(f^2+g^2+h^2)d\xi d\eta d\zeta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14),$$

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* This peculiarity of MAXWELL'S theory has been pointed out by C. NIVEN, *loc. cit.*

where, for the moment, the coordinates x, y, z refer to the conductors, and ξ, η, ζ to the dielectric. Let us form the equation of energy for the case where disturbances produced anyhow in the field are left to themselves.

We have

$$\begin{aligned}\frac{d\mathrm{T}}{dt}&=\tfrac{1}{2}\iiint(\dot{\mathrm{F}}u+\dot{\mathrm{F}}u+\&c.)\,dxdydz\\&\quad+\tfrac{1}{2}\iiint(\dot{\mathrm{F}}\dot{f}+\dot{\mathrm{F}}\dot{f}+\&c.)\,d\xi d\eta d\zeta\\&=\iiint(\dot{\mathrm{F}}u+\dot{\mathrm{G}}v+\dot{\mathrm{H}}w)\,dxdydz\\&\quad+\iiint(\dot{\mathrm{F}}\dot{f}+\dot{\mathrm{G}}\dot{g}+\dot{\mathrm{H}}\dot{h})\,d\xi d\eta d\zeta\quad\ldots\ldots\ldots\quad(15)^*\end{aligned}$$

Substituting the values of $\dot{\mathrm{F}}$, $\dot{\mathrm{G}}$, $\dot{\mathrm{H}}$ from (3) and (4), we find

$$\begin{aligned}\frac{d\mathrm{T}}{dt}&=-\iiint\rho(u^2+v^2+w^2)\,dxdydz\\&\quad-4\pi v^2\iiint(f\dot{f}+g\dot{g}+h\dot{h})\,d\xi d\eta d\zeta\\&\quad-\iiint\left(u\frac{d\phi}{dx}+v\frac{d\phi}{dy}+w\frac{d\phi}{dz}\right)dxdydz\\&\quad-\iiint\left(\dot{f}\frac{d\phi}{d\xi}+\dot{g}\frac{d\phi}{d\eta}+\dot{h}\frac{d\phi}{d\zeta}\right)d\xi d\eta d\zeta\end{aligned}$$

The last two integrals disappear in virtue of the solenoidal conditions satisfied by the flow of electricity.† Hence

$$\frac{d}{dt}(\mathrm{T}+\mathrm{W})=-\iiint\rho(u^2+v^2+w^2)\,d\xi d\eta d\zeta\quad\ldots\ldots\quad(16).$$

This expresses that the electrical energy lost is equivalent to the heat generated in the conductors according to JOULE'S law.

2. Now let us suppose that F, G, H, &c., all vary as $e^{\lambda t}$. The electrical motions in the conductors and in the surrounding dielectric may be of two kinds, *free* and *forced*. In the various modes of free motion the corresponding values of λ are real and negative. In the case of forced motion the disturbing force at any point of the field may, by FOURIER'S (double-integral) theorem, be expanded, as regards the time, in a series of periodic terms. The effects of these can then be investigated separately and afterwards superposed. The value of λ corresponding to any one term is $\lambda=2\pi ip$, where p is the frequency, and $i=\sqrt{-1}$.

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* This may be deduced from (1) by GREEN'S Theorem. It is a particular case of THOMSON and TAIT, § 313 (*f*).

† MAXWELL'S 'Electricity,' § 100*a*.

On the above assumption, (2) become

$$\rho u=-\frac{d\phi}{dx}-\lambda\mathrm{F},\ \&\mathrm{c}.,\ \&\mathrm{c}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17),$$

whence eliminating u, v, w by means of (1) we obtain as the equations to be satisfied in the interior of a conductor

$$\left.\begin{aligned}(\nabla^2+k^2)\mathrm{F}&=-\frac{k^2}{\lambda}\frac{d\phi}{dx}\\ (\nabla^2+k^2)\mathrm{G}&=-\frac{k^2}{\lambda}\frac{d\phi}{dy}\\ (\nabla^2+k^2)\mathrm{H}&=-\frac{k^2}{\lambda}\frac{d\phi}{dz}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

and

$$\frac{d\mathrm{F}}{dx}+\frac{d\mathrm{G}}{dy}+\frac{d\mathrm{H}}{dz}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19),$$

where

$$k^2=-\frac{4\pi\lambda}{\rho} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20).$$

In the dielectric we have

$$\nabla^2\mathrm{F}=-4\pi\frac{df}{dt}=-4\pi\lambda f,\ \&\mathrm{c}.,\ \&\mathrm{c}.$$

$$4\pi v^2 f=-\frac{d\phi}{dx}-\lambda\mathrm{F},\ \&\mathrm{c}.,\ \&\mathrm{c}.$$

Eliminating f, g, h we obtain

$$\left.\begin{aligned}(\nabla^2+j^2)\mathrm{F}&=-\frac{j^2}{\lambda}\frac{d\phi}{dx}\\ (\nabla^2+j^2)\mathrm{G}&=-\frac{j^2}{\lambda}\frac{d\phi}{dy}\\ (\nabla^2+j^2)\mathrm{H}&=-\frac{j^2}{\lambda}\frac{d\phi}{dz}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21),$$

with

$$\frac{d\mathrm{F}}{dx}+\frac{d\mathrm{G}}{dy}+\frac{d\mathrm{H}}{dz}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22),$$

where

$$j^2=-\frac{\lambda^2}{v^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23).$$

So far our equations are exact. But it appears from various physical analogies (more especially in Acoustics), and it will be verified in the course of this paper, that when the dimensions of the conductors are small compared with j^{-1} the phenomena are sensibly the same as if j were $=0$. Now, in air, $v=3\times10^{10}$ [C.G.S.], whence $j^{-1}=v/i\lambda=3\times10^{10}/i\lambda$. Since λ is proportional to the rapidity of the electrical motions it appears that in all practical cases j^{-1} is very large. We will therefore assume for the present $j=0$, which comes to the same thing as assuming that the velocity of propagation of electromagnetic effects in the dielectric medium is practically infinite. The equations to be satisfied in the neighbourhood of the conductors then are

$$\nabla^2\mathrm{F}=0,\quad \nabla^2\mathrm{G}=0,\quad \nabla^2\mathrm{H}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

$$\frac{d\mathrm{F}}{dx}+\frac{d\mathrm{G}}{dy}+\frac{d\mathrm{H}}{dz}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25).$$

Since $l\nabla^2\mathrm{F}+m\nabla^2\mathrm{G}+n\nabla^2\mathrm{H}$ must be continuous at the surfaces of the conductors it appears at once that on the present assumption we shall have, at those surfaces,

$$lu+mv+nw=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26).$$

3. Proceeding now to the special problem of this paper, viz., the case of a solid spherical conductor surrounded by air, let us take the origin of coordinates at the centre of the sphere, and let r denote the distance of any point from the origin. It may be shown, as in the papers on the "Oscillations of a Viscous Spheroid," &c., already referred to, that the solutions of the equations (18), (19), and (24), (25) are of two distinct types, which are quite independent of one another.

First Type. We have

In the conductor:

$$\left.\begin{aligned}\mathrm{F}&=\psi_n(kr)\left(y\frac{d}{dz}-z\frac{d}{dy}\right)\chi_n\\ \mathrm{G}&=\psi_n(kr)\left(z\frac{d}{dx}-x\frac{d}{dz}\right)\chi_n\\ \mathrm{H}&=\psi_n(kr)\left(x\frac{d}{dy}-y\frac{d}{dx}\right)\chi_n\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27),$$

where χ_n is a solid harmonic of positive integral degree n, and the function ψ_n is defined by

$$\begin{aligned}\psi_n(\zeta)&=1-\frac{\zeta^2}{2.2n+3}+\frac{\zeta^4}{2.4.2n+3.2n+5}-,\ \&c.\\ &=(-)^n 3.5\ldots 2n+1.\left(\frac{d}{\zeta d\zeta}\right)^n\frac{\sin\zeta}{\zeta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28),\end{aligned}$$

from either of which forms we readily deduce

$$\psi_n'(\zeta)=-\frac{\zeta}{2n+3}\psi_{n+1}(\zeta) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29),$$

$$\psi_n(\zeta)+\frac{\zeta}{2n+1}\psi_n'(\zeta)=\psi_{n-1}(\zeta) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30),$$

$$\psi_n(\zeta)-\psi_{n-1}(\zeta)=\frac{\zeta^2}{2n+1.2n+3}\psi_{n+1}(\zeta) \quad . \quad . \quad . \quad . \quad . \quad . \quad (31),$$

The equations (27) constitute the complete solution (of the first type) of (18) and (19) subject to the condition of finiteness at the origin. In the absence of this restriction we should have to add to the right-hand sides similar terms in which n is replaced by $-n-1$.[*]

In the space surrounding the conductor we have

$$\left.\begin{aligned} \mathrm{F}&=\left(y\frac{d}{dz}-z\frac{d}{dy}\right)(\mathrm{X}_n+\mathrm{X}_{-n-1}) \\ \mathrm{G}&=\left(z\frac{d}{dx}-x\frac{d}{dz}\right)(\mathrm{X}_n+\mathrm{X}_{-n-1}) \\ \mathrm{H}&=\left(x\frac{d}{dy}-y\frac{d}{dx}\right)(\mathrm{X}_n+\mathrm{X}_{-n-1}) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32),$$

where X_n, X_{-n-1} are solid harmonics of the algebraical degrees indicated by the suffixes.

Since the values (27) of F, G, H make $(\nabla^2+k^2)\,\mathrm{F}=0$, &c., &c., it follows by (1) that the components of current inside the sphere are

$$\left.\begin{aligned} u&=\frac{k^2}{4\pi}\psi_n(kr)\left(y\frac{d}{dz}-z\frac{d}{dy}\right)\chi_n \\ v&=\frac{k^2}{4\pi}\psi_n(kr)\left(z\frac{d}{dx}-x\frac{d}{dz}\right)\chi_n \\ w&=\frac{k^2}{4\pi}\psi_n(kr)\left(x\frac{d}{dy}-y\frac{d}{dx}\right)\chi_n \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33).$$

The flow of electricity is everywhere perpendicular to the radius vector, and hence $\phi=$const., inside and outside the sphere.

From (27) and (32) we derive :—

Inside the sphere :

$$\left.\begin{aligned} a&=-\left\{(n+1)\psi_{n-1}(kr)\frac{d\chi_n}{dx}-n\frac{k^2r^{2n+3}}{2n+1.2n+3}\psi_{n+1}(kr)\frac{d}{dx}\chi_n r^{-2n-1}\right\} \\ b&=-\left\{(n+1)\psi_{n-1}(kr)\frac{d\chi_n}{dy}-n\frac{k^2r^{2n+3}}{2n+1.2n+3}\psi_{n+1}(kr)\frac{d}{dy}\chi_n r^{-2n-1}\right\} \\ c&=-\left\{(n+1)\psi_{n-1}(kr)\frac{d\chi_n}{dz}-n\frac{k^2r^{2n+3}}{2n+1.2n+3}\psi_{n+1}(kr)\frac{d}{dz}\chi_n r^{-2n-1}\right\} \end{aligned}\right\} \quad . \quad . \quad (34)\dagger$$

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[*] These terms would be required in treating the case of a hollow spherical shell.

[†] These formulæ make

$$xa+yb+zc=-n.n+1.\psi_n(kr).\chi_n$$

Outside:

$$\left.\begin{aligned} a &= -(n+1)\frac{dX_n}{dx}+n\frac{dX_{-n-1}}{dx} \\ b &= -(n+1)\frac{dX_n}{dy}+n\frac{dX_{-n-1}}{dy} \\ c &= -(n+1)\frac{dX_n}{dz}+n\frac{dX_{-n-1}}{dz} \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (35).$$

In deducing (34) we have made use of (29), (30), and of the known formula

$$x\chi_n=\frac{r^2}{2n+1}\left(\frac{d\chi_n}{dx}-r^{2n+1}\frac{d}{dx}\chi_n r^{-2n-1}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (36).$$

We have now to apply the conditions to be satisfied at the surface of the sphere. If R be the radius, the continuity of F, G, H requires

$$\psi_n(k\mathrm{R}).\chi_n=X_n+X_{-n-1} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (37).$$

The continuity of a, b, c requires

$$\psi_{n-1}(k\mathrm{R}).\chi_n=X_n \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (38),^*$$

with another condition which is, however, implied in (37) and (38). We must bear in mind that in these equations r is supposed put $=$R throughout; so that χ_n, X_n, X_{-n-1} are now *surface* harmonics, of order n.

Second type. We have

Inside the sphere:

$$\phi=\phi_n \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (39)$$

$$\left.\begin{aligned} \mathrm{F} &= -\frac{1}{\lambda}\frac{d\phi_n}{dx}+(n+1)\psi_{n-1}(kr)\frac{d\omega_n}{dx}-n\frac{k^2r^{2n+3}}{2n+1.2n+3}\psi_{n+1}(kr)\frac{d}{dx}\omega_n r^{-2n-1} \\ \mathrm{G} &= -\frac{1}{\lambda}\frac{d\phi_n}{dy}+(n+1)\psi_{n-1}(kr)\frac{d\omega_n}{dy}-n\frac{k^2r^{2n+3}}{2n+1.2n+3}\psi_{n+1}(kr)\frac{d}{dy}\omega_n r^{-2n-1} \\ \mathrm{H} &= -\frac{1}{\lambda}\frac{d\phi_n}{dz}+(n+1)\psi_{n-1}(kr)\frac{d\omega_n}{dz}-n\frac{k^2r^{2n+3}}{2n+1.2n+3}\psi_{n+1}(kr)\frac{d}{dz}\omega_n r^{-2n-1} \end{aligned}\right\} \quad . \quad (40).^\dagger$$

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* The rigorous proof of these, and of similar inferences in the sequel, may be conducted as in § 4 of the paper "On the Vibrations of an Elastic Sphere" already cited.

† These may also be written

$$\mathrm{F}=-\frac{1}{\lambda}\frac{d\phi_n}{dx}+(2n+1)\psi_{n-1}(kr)\frac{d\omega_n}{dx}-n\frac{d}{dx}\left\{\psi_n(kr)\omega_n\right\}, \text{ \&c.}$$

Outside:

$$\phi=\Phi_n+\Phi_{-n-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41).$$

$$\left.\begin{aligned} F&=\frac{d\Omega_n}{dx}+\frac{d\Omega_{-n-1}}{dx} \\ G&=\frac{d\Omega_n}{dy}+\frac{d\Omega_{-n-1}}{dy} \\ H&=\frac{d\Omega_n}{dz}+\frac{d\Omega_{n-1}}{dz} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42).$$

Here ϕ_n, Φ_n, Φ_{-n-1}, ω_n, Ω_n, Ω_{-n-1}, are solid harmonics of the algebraical degrees indicated.

These formulæ give

Inside:

$$\left.\begin{aligned} a&=-k^2\psi_n(kr)\left(y\frac{d}{dz}-z\frac{d}{dy}\right)\omega_n \\ b&=-k^2\psi_n(kr)\left(z\frac{d}{dx}-x\frac{d}{dz}\right)\omega_n \\ c&=-k^2\psi_n(kr)\left(x\frac{d}{dy}-y\frac{d}{dx}\right)\omega_n \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43);$$

Outside:

$$a=0,\ b=0,\ c=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44).$$

The sort of reciprocal relation between the formulæ (27) and (34) on the one hand, and (40) and (43) on the other, is very remarkable.

The continuity of F, G, H at the surface of the sphere implies two relations which we shall not require; whilst that of a, b, c involves

$$\psi_n(kR).\omega_n=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45).$$

This result follows also from (26), since

$$\begin{aligned} xu+yv+zw&=-\frac{1}{4\pi}(x\nabla^2F+y\nabla^2G+z\nabla^2H) \\ &=\frac{k^2}{4\pi}.n(n+1)\psi_n(kr).\omega_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46). \end{aligned}$$

4. From this point we must discuss separately the cases of free and forced motion, respectively. First let us take that of *free motion*. We assume that (no matter how) electric currents have been started in the sphere and then left to themselves.

First Type. The equations (24) must now hold not merely in the space immediately surrounding the sphere but right away to infinity. Hence we must have, in (32), $X_n=0$; and thence, by (37),

$$\psi_{n-1}(kR)=0 \quad \dots\dots \quad (47).$$

The roots of this equation in kR are all real. For the case $n=1$ we have

$$kR/\pi=1,\ 2,\ 3,\ \&c.$$

When $n=2$,

$$kR/\pi=1{\cdot}4303,\ 2{\cdot}4590,\ 3{\cdot}4709,\ \&c. \quad \dots\dots \quad (48).$$

When $n=3$,

$$kR/\pi=1{\cdot}8346,\ 2{\cdot}8950,\ 3{\cdot}9225,\ \&c. \quad \dots\dots \quad (49).$$

When the value of k for any particular mode is known, the corresponding value of λ is given by (20). If τ denote the modulus of decay, *i.e.*, the time in which the currents fall to $1/e$ of their original strength, we have

$$\tau=-\lambda^{-1}=\frac{4}{\pi}.\left(\frac{kR}{\pi}\right)^{-2}.\frac{R^2}{\rho} \quad \dots\dots \quad (50).$$

For any given mode τ is proportional to the square of the radius, and inversely proportional to the specific resistance; a result which may easily be obtained otherwise, by the method of "dimensions."

For a sphere of copper [$\rho=1642$, C.G.S.] the modulus of the slowest mode of decay is

$$\tau={\cdot}000775R^2 \text{ second},$$

the unit of R being the centimetre. For a copper sphere, of the size of the earth [$R=6{\cdot}37\times10^8$] the corresponding value of τ is very nearly 10,000,000 years.

As regards the nature of the various modes we may observe that the lines of flow of electricity inside the sphere are the intersections of the spheres $r=$const. with the cones $\chi_n/r^n=$const.; in other words, they are the contour lines of the harmonics χ_n on a series of spherical surfaces concentric with the origin. The intensity of the current at any point is proportional to $\psi_n(kr).d\chi_n/de$, when de is an elementary angle at the centre of the sphere in a plane perpendicular to the line of flow passing through the point in question. The direction of the flow changes sign as we cross either the spheres for which $\psi_n(kr)=0$, or the cones for which $d\chi_n/de=0$. The components of the magnetic induction at points outside the sphere are, by (35)

$$\left.\begin{aligned} a&=nR^{2n+1}\psi_n(kR)\frac{d}{dx}\chi_n r^{-2n-1}\\ b&=nR^{2n+1}\psi_n(kR)\frac{d}{dy}\chi_n r^{-2n-1}\\ c&=nR^{2n+1}\psi_n(kR)\frac{d}{dz}\chi_n r^{-2n-1}\end{aligned}\right\} \quad \dots\dots \quad (51).$$

The simplest and most important case is when $n=1$. This may easily be examined by making $\chi_1=x$. The lines of motion are then all circles having the axis of x as a common axis.

Second Type. It follows from (45) that we must now have

$$\psi_n(k\mathrm{R})=0 \quad \ldots \ldots \ldots \ldots \quad (52).$$

In the cases $n=1$, $n=2$, the first few roots of this equation are given by (48), (49), respectively. The values of the modulus of decay corresponding to the various values of k are to be found from (50). In the most persistent mode of the present type the value of τ for a sphere of copper is

$$\tau=\cdot000379\ \mathrm{R}^2 \text{ second.}$$

As regards the nature of the motion inside the sphere we remark in the first place that since the radial flow is zero at the surface the electric currents form closed circuits. The flow at any point may be resolved into two components, one along, the other at right angles to, the radius vector. The radial component is

$$\frac{k^2}{4\pi}n.\overline{n+1}.\psi_n(kr).\frac{\omega_n}{r} \quad \ldots \ldots \ldots \ldots \quad (53).$$

The second or transversal component is perpendicular to that cone of the series $\omega_n/r^n=$ const. which passes through the point in question; and its amount is

$$\frac{k^2}{4\pi}\{kr\psi'_n(kr)+(n+1)\psi_n(kr)\}\frac{d\omega_n}{rd\epsilon} \quad \ldots \ldots \ldots \quad (54),$$

where $d\epsilon$ denotes as before an elementary angle at the centre of the sphere in a plane perpendicular to the above-mentioned cone.

When the harmonic ω_n is *zonal*, having the axis of x (say) as axis, the nature of the motion can be very simply expressed by means of a stream-function Ψ. The motion then takes place in a series of planes through the axis of x and is the same in each such plane. If u, $\mathfrak{v}$ be the components of current parallel and perpendicular to x, viz., $\mathfrak{v}=(yv+zw)/\varpi$, where $\varpi=\sqrt{(y^2+z^2)}$, we have

$$u=\frac{1}{\varpi}\frac{d\Psi}{d\varpi},\ \mathfrak{v}=-\frac{1}{\varpi}\frac{d\Psi}{dx} \quad \ldots \ldots \ldots \ldots \quad (55),$$

where $2\pi\Psi$ is the total flux through the circle whose coordinates are (x, ϖ). Integrating (53) over the segment of the sphere of radius $r=\sqrt{(x^2+\varpi^2)}$ bounded by this circle we find

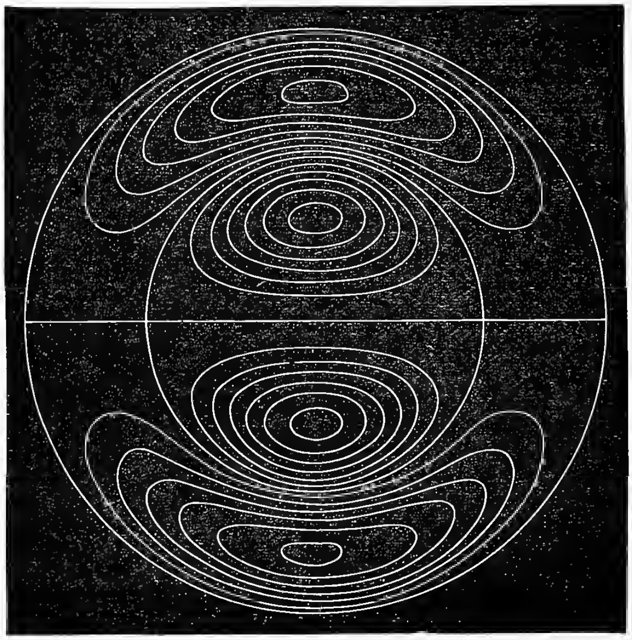
$$\Psi=\frac{k^2}{4\pi}.n.n+1.r\psi_n(kr)\int_0^\theta \omega_n \sin\theta d\theta$$

$$=-\frac{k^2}{4\pi}\psi_n(kr)\varpi\frac{d\omega_n}{d\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56).$$

Here θ denotes the colatitude (viz., $\varpi=r\sin\theta$), and ω_n is supposed expressed in terms of r, θ. The integration is effected by means of the differential equation of zonal harmonics. The most interesting case is when $n=1$. Writing $\omega_1=r\cos\theta$, we have

$$\Psi=\frac{k^2\varpi^2}{4\pi}\psi_1(kr).$$

The forms of the lines of flow ($\Psi=$ const.) corresponding to a series of equidistant values of Ψ are shown in the figure. The different systems of lines of flow are separated by the spheres for which $\psi_1(kr)=0$. The drawing includes the first two of these. In the most persistent mode the inner sphere must be taken to represent the boundary of the conductor; in the next mode the second sphere must be taken; and so on.



It appears from (44) that the currents in the sphere exercise no magnetic action in the external space. Conversely no motions of the present type can be originated by any electromagnetic operations outside the sphere. It will be shown further on that both these statements require qualification when we take account of the finite value of v.

By combining in the proper way solutions of the two types we can represent the decay of any system of currents arbitrarily given in the sphere. The determination of the harmonics χ_n, ω_n in terms of the initial circumstances, although interesting mathematically, would occupy too much space to be given in full here. It may suffice to remark that if u, v, w be any three functions satisfying the solenoidal condition (8), and if $\xi=dw/dy-dv/dz$, &c., &c., then the values of u, v, w are completely determinate throughout any spherical region having its centre at the origin when we know the values of $xu+yv+zw$ and of $x\xi+y\eta+z\zeta$ throughout that region. This is most readily seen from hydrodynamical considerations. The problem then resolves itself into the identification of the given initial values of these expressions with those which result from our formulæ, viz.,

$$xu+yv+zw=\Sigma\Sigma\frac{k^2}{4\pi}.n.\overline{n+1}.\psi_n(kr)\omega_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56\text{A}),$$

and

$$x\xi+y\eta+z\zeta=-\Sigma\Sigma\frac{k^2}{4\pi}n.\overline{n+1}.\psi_n(kr)\chi_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56\text{B}).$$

The summations here embrace all integral values of n and all admissible values of k. In (56A) these are given by $\psi_n(k\text{R})=0$, and in (56B) by $\psi_{n-1}(k\text{R})=0$. The identification can be effected by known methods.

5. Let us next proceed to consider the currents induced in the sphere by operations outside it; and for simplicity let us suppose that the changes in the field are periodic and follow the simple harmonic law. The value of λ is now prescribed, viz., it $=2\pi ip$, where p is the frequency. Hence, by (20),

$$k^2=-8\pi^2 ip/\rho,$$

and

$$k=(1-i)q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57),$$

provided

$$q^2=4\pi^2p/\rho \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58).$$

Since all our formulæ involve only even powers of q there is no loss of generality in taking q always positive.

From (45) we see that $\omega_n=0$, so that we have to deal exclusively with solutions of the *first type*. The complete solution of the problem is then given by the equations (27) and (32) in which the values of χ_n, X_{-n-1} in terms of X_n are to be obtained from the surface-conditions (37) and (38), viz., we have

$$\chi_n=\frac{1}{\psi_{n-1}(k\text{R})}\text{X}_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59),$$

$$\text{X}_{-n-1}=\left\{\frac{\psi_n(k\text{R})}{\psi_{-n-1}(k\text{R})}-1\right\}\frac{\text{R}^{2n+1}}{r^{2n+1}}\text{X}_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60).$$

The values of the functions X_n are to be found as follows. It is easily seen that if a_0, b_0, c_0 be the components of the magnetic field due to the inducing system alone, the expression $xa_0+yb_0+zc_0$ must satisfy the equation

$$\nabla^2(xa_0+yb_0+zc_0)=0$$

at all points outside the inducing system, and vanish at the origin. Hence it must admit of expansion in a series of solid harmonics of positive integral degrees, say

$$xa_0+yb_0+zc_0=\Sigma_1^{\infty}\Theta_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (61).$$

But it appears from (35) that we must have

$$xa_0+yb_0+zc_0=-\Sigma_1^{\infty}n.n+1.X_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62).$$

Comparing with (61) we find

$$X_n=-\frac{1}{n.n+1}\Theta_n.$$

For instance, let the magnetic field due to the inducing system be sensibly uniform in the neighbourhood of the sphere, say

$$a_0=I,\ b_0=0,\ c_0=0.$$

We find

$$X_1=-\tfrac{1}{2}Ix\,;\ X_2=X_3=\&c.=0.$$

The formulæ (33) for the currents in the sphere then become

$$\left.\begin{aligned} u&=0\\ v&=-D\psi_1(kr).z\\ w&=\ \ D\psi_1(kr).y \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63),$$

where

$$D=\frac{k^2I}{8\pi\psi_0(kR)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64);$$

and the disturbance (a_1, b_1, c_1) in the magnetic field, due to these currents, is given by

$$\left.\begin{aligned} a_1&=E\frac{d}{dx}\frac{x}{r^3}\\ b_1&=E\frac{d}{dy}\frac{x}{r^3}\\ c_1&=E\frac{d}{dz}\frac{x}{r^3} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65),$$

provided

$$E=-\frac{IR^3}{2}\left\{\frac{\psi_1(kR)}{\psi_0(kR)}-1\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66).$$

For the full interpretation of our formulæ it would be necessary to disentangle the real and the imaginary parts, and to discard one or the other. The results would be very complicated, even for the simplest harmonic constituent ($n=1$). There are certain cases, however, in which we can use methods of approximation, and so deduce the results of interest without much difficulty.

Thus, in the first place, let us suppose that the changes in the field are comparatively slow; more precisely, let the frequency p be very small compared with ρ/R^2. Since kR is then a small quantity, the expressions for the currents in the sphere are approximately,

$$\left.\begin{aligned} u&=-\frac{2\pi ip}{\rho}\left(y\frac{d}{dz}-z\frac{d}{dy}\right)X_n \\ v&=-\frac{2\pi ip}{\rho}\left(z\frac{d}{dx}-x\frac{d}{dz}\right)X_n \\ w&=-\frac{2\pi ip}{\rho}\left(x\frac{d}{dy}-y\frac{d}{dx}\right)X_n \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (67).$$

This is the result which we should have obtained by neglecting *ab initio* the mutual influence of the currents in the sphere. The disturbance in the field due to these currents is given by

$$\left.\begin{aligned} a_1&=J\frac{d}{dx}X_n r^{-2n-1} \\ b_1&=J\frac{d}{dy}X_n r^{-2n-1} \\ c_1&=J\frac{d}{dz}X_n r^{-2n-1} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (68)$$

where

$$J=\frac{-8\pi^2 inpR^{2n+3}}{2n+1.2n+3.\rho} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (69).$$

For spheres of the same size the disturbance is *cæteris paribus* proportional to the specific conductivity.

Next let us examine the other extreme case, where the frequency p is large compared with ρ/R^2, and consequently kR is a large number. When ζ is large, the formula (28) becomes, approximately,

$$\psi_n(\zeta)=(-)^n.3.5\ldots 2n+1.\frac{\sin\left(\zeta+n\frac{\pi}{2}\right)}{\zeta^{n+1}}.$$

Writing $\zeta = kr = (1-i)qr$, and keeping only the most important term, we find

$$\psi_n(kr) = (-)^n . 3.5 \ldots 2n+1 . \frac{e^{qr} . e^{i\left(qr + n\frac{\pi}{2}\right)}}{2i(kr)^{n+1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (70).$$

Hence the factor

$$\frac{k^2}{4\pi} \frac{\psi_n(kr)}{\psi_{n-1}(k\mathrm{R})},$$

which occurs in the expressions for the induced currents, becomes, after several reductions,

$$-\frac{(2n+1)q}{2\sqrt{2\pi\mathrm{R}}} . \left(\frac{\mathrm{R}}{r}\right)^{n+1} . e^{q(r-\mathrm{R}) + i\left\{q(r-\mathrm{R}) + \frac{\pi}{4}\right\}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71).$$

It appears from this that the disturbance inside the sphere consists of a series of waves propagated inwards from the surface with rapidly decreasing amplitude. Thus at a depth equal to the wave-length (ν, say), the amplitude is only $1/535$ of what it is at the surface. The currents are therefore almost entirely confined to a superficial stratum of thickness comparable with ν. It appears from (58) that $\nu, = 2\pi/q, = \sqrt{(\rho/p)}$. As a numerical example let $\rho = 1642$ (copper), $p = 4000$; we find

$$\nu = {\cdot}64 \text{ centimetre.}$$

The condition of the applicability of our approximation is that $2\pi\mathrm{R}$ must be large in comparison with ν.*

Since, by (70), $\psi_n(k\mathrm{R})/\psi_{n-1}(k\mathrm{R})$ is of the order $1/k\mathrm{R}$, it appears from (60) and (38) that the disturbance in the field caused by the currents in the sphere is given by

$$\left.\begin{aligned} a_1 &= -n\mathrm{R}^{2n+1} \frac{d}{dx} \mathrm{X}_n r^{-2n-1} \\ b_1 &= -n\mathrm{R}^{2n+1} \frac{d}{dy} \mathrm{X}_n r^{-2n-1} \\ c_1 &= -n\mathrm{R}^{2n+1} \frac{d}{dz} \mathrm{X}_n r^{-2n-1} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (72).$$

The magnitude of the disturbance depends therefore on the size of the sphere, but is independent of the conductivity, so long as the fundamental condition of our approximation is satisfied. The reason of this is not far to seek. The greater the conductivity the greater will be the intensity of the currents at the surface of the sphere, but the more rapid will be the rate of diminution as we pass inwards; and it is easily seen from (71) that one cause will exactly compensate the other.

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* The above results enable us to estimate what ought to be the thickness of a sheet of a given metal in order that it should act as a screen against a periodic electromagnetic action of given frequency. See the paper by Lord RAYLEIGH, cited below.

In fact, if we write

$$u'=\int^{\mathrm{R}} u dr,\quad v'=\int^{\mathrm{R}} v dr,\quad w'=\int^{\mathrm{R}} w dr,$$

where the lower limit is taken at such a depth that the currents there are insensible, we readily find that the currents are approximately equivalent to an infinitely thin spherical *current sheet* of radius R, the components of the current at any point of the sheet being given by

$$\left.\begin{aligned} u'&=-\frac{2n+1}{4\pi\mathrm{R}}\left(y\frac{d}{dz}-z\frac{d}{dy}\right)\mathrm{X}_n \\ v'&=-\frac{2n+1}{4\pi\mathrm{R}}\left(z\frac{d}{dx}-x\frac{d}{dz}\right)\mathrm{X}_n \\ w'&=-\frac{2n+1}{4\pi\mathrm{R}}\left(x\frac{d}{dy}-y\frac{d}{dx}\right)\mathrm{X}_n \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (73).^*$$

6. The foregoing methods can be readily adapted to the case of a shell bounded by two concentric spherical surfaces. The most interesting case is when the shell is infinitely thin. The free motions of the second type then decay with infinite rapidity, and there are no forced motions of this type. Hence we have practically to deal only with solutions of the first type. The theory of these has been given by Professor NIVEN, but for the sake of completeness it is here discussed from the point of view of the present paper.

Let u', v', w' be the components of the total current at any point of the shell, and let $\rho'=\rho/\delta$, where δ is the thickness of the shell. Then if all our functions vary as $e^{\lambda t}$ we shall have

$$\rho'u'=-\lambda\mathrm{F},\quad \rho'v'=-\lambda\mathrm{G},\quad \rho'w'=-\lambda\mathrm{H} \quad \ldots\ldots \quad (74).$$

In the hollow space inside the shell

$$\left.\begin{aligned} \mathrm{F}&=\left(y\frac{d}{dz}-z\frac{d}{dy}\right)\chi_n \\ \mathrm{G}&=\left(z\frac{d}{dx}-x\frac{d}{dz}\right)\chi_n \\ \mathrm{H}&=\left(x\frac{d}{dy}-y\frac{d}{dx}\right)\chi_n \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (75),^\dagger$$

whilst (32) hold for the external space. The functions F, G, H must vary continually as we cross the shell, so that

$$\chi_n=\mathrm{X}_n+\mathrm{X}_{-n-1} \quad \ldots\ldots\ldots\ldots \quad (76),$$

at the surface.

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[*] The conclusions of this section have an obvious bearing on the results obtained by Professor D. E. HUGHES in his experiments with the Induction Balance (Proc. Roy. Soc., May 15, 1879).

[†] It is here assumed that the inducing system, if any, is situate in the space external to the shell.

The first derivatives of F, G, H are, however, discontinuous, viz., if $d\nu'$, $d\nu''$ be elements of the normal drawn inwards and outwards respectively, we must have

$$\left.\begin{aligned}\frac{d\mathrm{F}}{d\nu'}+\frac{d\mathrm{F}}{d\nu''}&=-4\pi u'\\ \frac{d\mathrm{G}}{d\nu'}+\frac{d\mathrm{G}}{d\nu''}&=-4\pi v'\\ \frac{d\mathrm{H}}{d\nu'}+\frac{d\mathrm{H}}{d\nu''}&=-4\pi w'\end{aligned}\right\} \quad \ldots \quad (77),$$

which equations now replace (1). Hence, and from (74) we deduce

$$-n\chi_n+n\mathrm{X}_n-(n+1)\mathrm{X}_{-n-1}=\frac{4\pi\mathrm{R}\lambda}{\rho'}\chi_n \quad \ldots \quad (78),$$

when $r=\mathrm{R}$, the radius of the shell.

In free motion $\mathrm{X}_n=0$, and thence

$$\tau=-\lambda^{-1}=\frac{4\pi\mathrm{R}}{(2n+1)\rho'} \quad \ldots \quad (79).$$

In the case of currents induced by a system external to the shell, we find

$$\chi_n=\frac{1}{1+\lambda\tau}\mathrm{X}_n \quad \ldots \quad (80),$$

and

$$\mathrm{X}_{-n-1}=-\frac{\lambda\tau}{1+\lambda\tau}\mathrm{X}_n \quad \ldots \quad (81),$$

when τ has the value (79). The value of X_n can be found as before when the nature of the inducing system is known. Writing $\lambda=2\pi ip$ we see from (80) that if the period of the disturbance be small compared with τ the shell will almost completely shelter the enclosed region from the electromagnetic action of the external system.*

The case where the inducing system is inside the shell may be treated in a similar manner. We have to introduce a function χ_{-n-1} for the internal space, whilst X_n is zero.

7. When the magnetic permeability μ of the substance of the conductor differs sensibly from unity, the processes of the foregoing articles require some modification. The equations (1) must then be replaced by

$$\left.\begin{aligned}\nabla^2\mathrm{F}&=-4\pi\mu u\\ \nabla^2\mathrm{G}&=-4\pi\mu v\\ \nabla^2\mathrm{H}&=-4\pi\mu w\end{aligned}\right\} \quad \ldots \quad (82),$$

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* See Lord RAYLEIGH, Phil. Mag., May, 1882, p. 344.

whilst (2) and (3) are unaltered. Hence the fundamental equations (18), (19) of our method retain the same form, provided

$$k^2=-\frac{4\pi\mu\lambda}{\rho} \quad \ldots \ldots \ldots \quad (83).$$

The distribution of the induced magnetization in the conductor will be solenoidal. Hence if A, B, C be the components of this distribution, the corresponding parts of F, G, H will be

$$\frac{dN}{dy}-\frac{dM}{dz},\ \frac{dL}{dz}-\frac{dN}{dx},\ \frac{dM}{dx}-\frac{dL}{dy},$$

respectively, where

$$L=\iiint\frac{A}{r}\,dxdydz,\ M=\iiint\frac{B}{r}\,dxdydz,\ N=\iiint\frac{C}{r}\,dxdydz.$$

The integrations are supposed to extend throughout the magnetized substance, and r denotes the distance between the element $dxdydz$ and the point for which the values of L, M, N are required. Hence F, G, H are continuous at the surface, but their first derivatives, and consequently a, b, c, will be discontinuous. Let us distinguish the values of a, b, c just inside and just outside the conductor by the accents $'$ and $''$, respectively. Then the parts of a', b', c' due to the induced magnetisation are

$$-\mu\frac{dV}{dx},\ -\mu\frac{dV}{dy},\ -\mu\frac{dV}{dz},$$

and those of a'', b'', c'' are

$$-\frac{dV}{dx},\ -\frac{dV}{dy},\ -\frac{dV}{dz},$$

where V is the potential of free magnetism, viz.:

$$V=\iint(lA+mB+nC)\frac{dS}{r},$$

dS denoting an element of the surface of the body, and l, m, n the direction-cosines of the outwardly directed normal to dS, and the integration being taken over the surface of the conductor. Hence

$$a''-\frac{a'}{\mu}=4\pi l(lA+mB+nC),\ \&c.;$$

or, since $4\pi\mu A=(\mu-1)a'$, &c.,

$$\left.\begin{aligned}a'+(\mu-1)\,l\,(la'+mb'+nc')&=\mu a''\\ b'+(\mu-1)m(la'+mb'+nc')&=\mu b''\\ c'+(\mu-1)\,n\,(la'+mb'+nc')&=\mu c''\end{aligned}\right\} \quad \ldots \ldots \ldots \quad (84).$$

We notice that these conditions give

$$la'+mb'+nc'=la''+mb''+nc'' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (85),$$

as ought to be the case. In fact (85) is implied in the continuity of F, G, H.

8. Proceeding now to the case of a spherical conductor, let the origin be taken at the centre, and let r be the radius vector of any point. Let us begin with the solutions of the *First Type*, the formulæ for which are given by (27), (32), (34), (35). The continuity of F, G, H gives as before

$$\psi_n(k\mathrm{R}).\chi_n=\mathrm{X}_n+\mathrm{X}_{-n-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (86)$$

at the surface ($r=\mathrm{R}$). In applying (84) we remark that, at the surface,

$$la'+mb'+nc'=-\frac{n.n+1}{\mathrm{R}}\psi_n(k\mathrm{R}).\chi_n,$$

and hence that

$$l(la'+mb'+nc')=-\frac{n.n+1}{\mathrm{R}^2}\psi_n(k\mathrm{R}).x\chi_n,$$

$$=-\frac{n.n+1}{2n+1}\psi_n(k\mathrm{R})\left(\frac{d\chi_n}{dx}-\mathrm{R}^{2n+1}\frac{d}{dx}\chi_n r^{-2n-1}\right),$$

by (36). Hence (84) give

$$\left\{\psi_{n-1}(k\mathrm{R})+\frac{n}{2n+1}(\mu-1)\psi_n(k\mathrm{R})\right\}\chi_n=\mu\mathrm{X}_n. \quad . \quad . \quad . \quad . \quad . \quad . \quad (87),$$

with another condition which may however be shown to be included in (86) and (87).

The formulæ for the solutions of the *Second Type* are given in (40), (42), (43), (44). The surface conditions (84) yield

$$\psi_n(k\mathrm{R}).\omega_n=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (88).$$

9. In the case of *free* currents of the first type we have $\mathrm{X}_n=0$, and the equation to determine k is

$$\psi_{n-1}(k\mathrm{R})+\frac{n}{2n+1}(\mu-1)\psi_n(k\mathrm{R})=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (89).$$

When, as in iron, μ is a very large number, we have, as a first approximation,

$$\psi_n(k\mathrm{R})=0.$$

If $k\mathrm{R}=\vartheta$ be a solution of this, a second approximation is

$$k\mathrm{R}=\left\{1-\frac{1}{n(\mu-1)}\right\}\vartheta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (90).$$

When the values of $k\mathrm{R}$ have been found, the corresponding values of the modulus of decay are given by

$$\tau=\frac{4}{\pi}\left(\frac{k\mathrm{R}}{\pi}\right)^{-2}\frac{\mu\mathrm{R}^2}{\rho} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (91).$$

In iron we have $\mu=403$ (THALÉN), $\rho=9827$ C. G. S. The lowest root of (89), in the case $n=1$, is then $k\mathrm{R}=1{\cdot}4268\pi$, and the corresponding value of τ is

$$\tau={\cdot}0256\mathrm{R}^2.$$

The duration of the free currents is very much greater than in a non-magnetizable sphere of the same size and of equal conductivity. For an iron ball one foot in diameter the above value of τ is six seconds. For an iron globe of the size of the earth it would be 330,000,000 years.

The magnetic susceptibility of the substance has the effect of modifying the character, as well as the duration, of the natural modes of decay. Inside the sphere we have

$$la+mb+nc=-\frac{n.n+1}{\mathrm{R}}.\psi_n(kr)\chi_n,$$

Since, by (89), this is almost zero at the surface, the lines of magnetic induction inside the sphere are for the most part closed curves. Their forms, in the first two modes of the class $n=1$, are given by the figure of § 4. The surface of the conductor is not, however, in these two respective modes, now represented by the two spherical surfaces there shown, but rather by two concentric spherical surfaces of radii smaller (for the case of iron) by about the four hundredth part.

For the free currents of the second type we have, by (88),

$$\psi_n(k\mathrm{R})=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (92).$$

The natural modes of decay are exactly the same as when $\mu=1$, but the persistency is in each case greater in the ratio of $\mu:1$; viz., the values of τ corresponding to the various roots of (92) are given by (91).

10. In the case of *induced* currents caused by a periodic variation in the magnetic field the value of X_n is to be found in the same manner as in § 5; and χ_n, X_{-n-1} are then determined by (86) and (87). If p be the frequency,

$$k=(1-i)q$$

where now

$$q^2=4\pi^2\mu p/\rho \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (93).$$

Let us examine first the case where $k\mathrm{R}$ is small. We then have, at the surface,

$$\chi_n=\frac{2n+1.\mu}{n\mu+n+1}\mathrm{X}_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (94),$$

$$\mathrm{X}_{-n-1}=\frac{(n+1)(\mu-1)}{n\mu+n+1}\mathrm{X}_n \quad \text{. . . . .} \quad (95),$$

approximately. The currents in the sphere are then given by

$$u=-\frac{2\pi ip}{\rho}\frac{2n+1.\mu^2}{n\mu+n+1}\left(y\frac{d}{dz}-z\frac{d}{dy}\right)\mathrm{X}_n,\ v=\text{\&c.},\ w=\text{\&c.} \quad \text{. . . . .} \quad (96).$$

he principal part of the disturbance in the field, due to the presence of the sphere, is given by

$$a_1=\frac{n(n+1)(\mu-1)}{n\mu+n+1}\frac{d}{dx}\frac{\mathrm{R}^{2n+1}}{r^{2n+1}}\mathrm{X}_n,\ b_1=\text{\&c.},\ c_1=\text{\&c.} \quad \text{. . . . . .} \quad (97).$$

These terms express the effect of the induced magnetization of the sphere. The effect of the induced currents is (under the circumstances supposed) small in comparison.

Next let us take the case of $k\mathrm{R}$ large. It is to be noticed that owing to the occurrence of the factor μ in (93) this condition is satisfied by very much smaller values of the frequency than the case of a non-magnetizable substance. We then have

$$\frac{k^2}{4\pi}\psi_n(kr)\chi_n=\frac{k^2\mu}{4\pi}\cdot\frac{2n+1.\psi_n(kr)}{2n+1.\psi_{n-1}(k\mathrm{R})+(\mu-1)n\psi_n(k\mathrm{R})}\mathrm{X}_n.$$

The factor of X_n is by (70)

$$=\frac{2n+1}{4\pi}\cdot\frac{k^2\mu}{n(\mu-1)+ik\mathrm{R}}\cdot\left(\frac{\mathrm{R}}{r}\right)^{n+1}.e^{q(r-\mathrm{R})+iq(r-\mathrm{R})},$$

approximately. If we assume

$$n(\mu-1)+q\mathrm{R}=\mathrm{D}\cos\epsilon \quad \text{. . . . .} \quad (98),$$

$$q\mathrm{R}=\mathrm{D}\sin\epsilon \quad \text{. . . . .} \quad (99),$$

this may be written

$$-\frac{2n+1}{2\pi}.\frac{q^2\mu}{\mathrm{D}}.\left(\frac{\mathrm{R}}{r}\right)^{2n+1}.e^{q(r-\mathrm{R})+i\left\{q(r-\mathrm{R})+\frac{\pi}{2}-\epsilon\right\}} \quad \text{. . . . .} \quad (100).$$

From this result we draw conclusions similar to those of § 5. The depth ν within which the maximum intensity of current falls to $1/e$ of its surface values is

$$\nu=2\pi/q=\sqrt{\frac{\rho}{\mu p}}.$$

In the case of iron we have, using the same data as before, $\nu=\cdot078$ centim. for a frequency of 4000, or $\nu=\cdot78$ for a frequency of 40. The value of ν is thus, for the same frequency, very much smaller than in copper. But the integral currents induced, under the same circumstances, are much more intense in an iron sphere than in a copper sphere of the same size. Integrating (100) with respect to r through the thickness of the stratum in which the currents are sensible, we find for the components of flow at any point of the equivalent *current sheet*

$$\left.\begin{aligned} u' &= \mathrm{K}\left(y\frac{d}{dz}-z\frac{d}{dy}\right)\mathrm{X}_n \\ v' &= \mathrm{K}\left(z\frac{d}{dx}-x\frac{d}{dz}\right)\mathrm{X}_n \\ w' &= \mathrm{K}\left(x\frac{d}{dy}-y\frac{d}{dx}\right)\mathrm{X}_n \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (101),$$

where

$$\mathrm{K}=-\frac{2n+1}{2\sqrt{2\pi\mathrm{R}}}\cdot\frac{q\mathrm{R}}{\mathrm{D}}\cdot\mu . e^{i\left(\frac{\pi}{4}-\epsilon\right)} \quad \ldots \ldots \ldots \quad (102).$$

The disturbance in the field, due to the presence of the sphere, is given by

$$a_1=n\,\frac{d\mathrm{X}_{-n-1}}{dx},\ b_1=\&\text{c.},\ c_1=\&\text{c.} \quad \ldots \ldots \ldots \quad (103),$$

where

$$\mathrm{X}_{-n-1}=\left[\frac{2n+1.\mu}{\mathrm{D}}\,e^{-i\epsilon}-1\right]\frac{\mathrm{R}^{2n+1}}{r^{2n+1}}\,\mathrm{X}_n \quad \ldots \ldots \ldots \quad (104).$$

The order of magnitude of the first term within [ ] depends on the relative magnitudes of $q\mathrm{R}$ and μ. So long as $q\mathrm{R}$, though itself large, is moderately small in comparison with $n\mu$ the effect is mainly due to the induced magnetization of the sphere, and is much the same as if the substance were destitute of electrical conductivity, although the distribution of the magnetization within the sphere is very different. On the other hand when $q\mathrm{R}$ is large compared with $n\mu$ the first term in [ ] is less important, and the results approximate more to the form which they would assume in the case of infinite conductivity. The following table gives the values of D and ϵ for iron, in the case $n=1$, corresponding to various values of $q\mathrm{R}$.

qR.	10.	50.	100.	1000.
D	412	455	512	1722
ϵ	1°.23′	6°.19′	11°.16′	35°.30′

The relation between q and the frequency p is for iron

$$q=1{\cdot}27\sqrt{p}.$$

11. In the whole of the preceding investigations it has been assumed that the quantity j of § 2 may without sensible error be put $=0$. I proceed to sketch the method to be pursued when we do not make this assumption, confining myself for simplicity to the case of $\mu=1$ everywhere. The fundamental equations to be satisfied are:—for the spherical conductor (18) and (19); for the surrounding dielectric (21) and (22).

In the solution of the *First Type* the values of F, G, H and of a, b, c inside the sphere are then given by (27) and (34), respectively. Outside the sphere we shall now have

$$\mathrm{F}=\psi_n(jr)\left(y\frac{d}{dz}-z\frac{d}{dy}\right)\mathrm{X}_n+\psi_{-n-1}(jr)\left(y\frac{d}{dz}-z\frac{d}{dy}\right)\mathrm{X}_{-n-1} \quad . \quad . \quad . \quad (105),$$

where X_n, X_{-n-1} are solid harmonics of the degrees indicated. The values of G and H may be written down from symmetry. We thence find

$$a=-\left\{(n+1)\psi_{n-1}(jr)\frac{d\mathrm{X}_n}{dx}-n\frac{j^2r^{2n+3}}{2n+1.2n+3}\psi_{n+1}(jr)\frac{d}{dx}\mathrm{X}_nr^{-2n-1}\right\}+\text{terms in }\mathrm{X}_{-n-1}\,.\;(106),$$

with symmetrical formulæ for b, c. The "terms in X_{-n-1}" are to be derived from the preceding line by writing $-n-1$ for n throughout.

The continuity of F, G, H at the surface of the sphere requires

$$\psi_n(k\mathrm{R})\chi_n=\psi_n(j\mathrm{R})\mathrm{X}_n+\psi_{-n-1}(j\mathrm{R})\mathrm{X}_{-n-1} \quad . \quad . \quad . \quad . \quad . \quad (107),$$

when $r=\mathrm{R}$. The continuity of a, b, c requires in addition

$$\psi_{n-1}(k\mathrm{R})\chi_n=\psi_{n-1}(j\mathrm{R})\mathrm{X}_n+\frac{j^2\mathrm{R}^2}{2n-1.2n+1}\psi_{-n}(j\mathrm{R})\mathrm{X}_{-n-1} \quad . \quad . \quad . \quad (108).$$

In the solutions of the *Second Type* the forms of F, G, H, a, b, c inside the sphere are given as before by (40) and (43); whilst in the dielectric we shall now have

$$\phi=\Phi_n+\Phi_{-n-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (109),$$

and

$$\mathrm{F}=-\frac{1}{\lambda}\frac{d\Phi_n}{dx}-\frac{1}{\lambda}\frac{d\Phi_{-n-1}}{dx}+n+1.\psi_{n-1}(jr)\frac{d\Omega_n}{dx}-n\frac{j^2r^{2n+3}}{2n+1.2n+3}\psi_{n+1}(jr)\frac{d}{dx}\Omega_nr^{-2n-1}$$
$$+\text{ terms in }\Omega_{-n-1} \quad . \quad (110),$$

with symmetrical formulæ for G, H. The symbols Φ_n, Φ_{-n-1}, Ω_n, Ω_{-n-1} stand for solid harmonics of the algebraical degrees indicated by the suffixes. The foregoing expressions make

$$a=-j^2\psi_n(jr)\left(y\frac{d}{dz}-z\frac{d}{dy}\right)\Omega_n+\text{ terms in }\Omega_{-n-1}\quad . \quad . \quad . \quad . \qquad (111),$$

with symmetrical formulæ for b and c.

The continuity of ϕ at the surface requires

$$\phi_n=\Phi_n+\Phi_{-n-1}\quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (112),$$

when $r=\text{R}$. The continuity of F, G, H requires

$$\begin{aligned}&-\frac{\phi_n}{\lambda}+(n+1)\psi_{n-1}(k\text{R})\omega_n\\&=-\frac{\Phi_n}{\lambda}+(n+1)\psi_{n-1}(j\text{R})\Omega_n+(n+1)\frac{j^2\text{R}^2}{2n-1.2n+1}\psi_{-n}(j\text{R})\Omega_{-n-1}\quad . \quad . \qquad (113),\end{aligned}$$

and

$$-n\frac{k^2\text{R}^2}{2n+1.2n+3}\psi_{n+1}(k\text{R})\omega_n=-\frac{\Phi_{-n-1}}{\lambda}-n\frac{j^2\text{R}^2}{2n+1.2n+3}\psi_{n+1}(j\text{R})\Omega_n-n\psi_{-n-2}(j\text{R})\Omega_{-n-1}\,.\,(114).$$

Adding (113) and (114), and taking account of (112), we find

$$\begin{aligned}&\{k\text{R}\psi'_n(k\text{R})+(n+1)\psi_n(k\text{R})\}\omega_n\\&\quad=\{j\text{R}\psi'_n(j\text{R})+(n+1)\psi_n(j\text{R})\}\Omega_n+\{j\text{R}\psi'_{-n-1}(j\text{R})-n\psi_{-n-1}(j\text{R})\}\Omega_{-n-1}\qquad (115),^*\end{aligned}$$

where some reductions have been effected by means of (29) and (30).

The continuity of a, b, c requires

$$k^2\text{R}^2\psi_n(k\text{R})\omega_n=j^2\text{R}^2\psi_n(j\text{R})\Omega_n+j^2\text{R}^2\psi_{-n-1}(j\text{R})\Omega_{-n-1}\,.\quad . \quad . \quad . \qquad (116).$$

12. Let us now apply the foregoing results to the case of *free* motion. A certain relation must then hold between the surface values of X_n and X_{-n-1}, and also between those of Ω_n and Ω_{-n-1}, viz.: a relation expressing that the disturbance at infinity in the dielectric is finite. It may be shown that F, G, H are determinate when the values of $x\text{F}+y\text{G}+z\text{H}$ and of $xa+yb+zc$ are known at every point of space. Now in the first type we have $x\text{F}+y\text{G}+z\text{H}=0$, and

$$xa+yb+zc=-n.n+1.\{\psi_n(jr)\text{X}_n+\psi_{-n-1}(jr)\text{X}_{-n-1}\}\quad . \quad . \quad . \qquad (117).$$

For large values of r we have

$$\psi_n(jr)=(-)^n.3.5\ldots 2n+1.\frac{\sin\left(jr+n\frac{\pi}{2}\right)}{(jr)^{n+1}}\quad . \quad . \quad . \quad . \quad . \quad . \qquad (118).$$

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[*] Equations (109) and (115) express that the tangential components of current just outside and just inside the sphere are in the ratio of j^2 to k^2. This may also be easily deduced from the fundamental equations (18) and (21).

The last line of (28) is not a convenient expression when n is negative. But we readily deduce from (30)

$$\zeta^{-2n-1}\psi_{-n-1}(\zeta)=-\frac{1}{2n-1}\cdot\frac{d}{\zeta d\zeta}[\zeta^{-(2n-1)}\psi_{-n}(\zeta)] \quad . \quad . \quad . \quad . \quad . \quad (119);$$

and by successive applications of this formula of reduction we find

$$\psi_{-n-1}(\zeta)=\frac{(-)^n\zeta^{2n+1}}{1.3\ldots 2n-1}\cdot\left(\frac{d}{\zeta d\zeta}\right)^n\frac{\cos\zeta}{\zeta} \quad . \quad . \quad . \quad . \quad . \quad . \quad (120);$$

since, by (28), $\psi_{-1}(\zeta)=\cos\zeta$. This result, like (28), has been given in somewhat different forms by various writers.* When r is large it gives

$$\psi_{-n-1}(jr)=\frac{(-)^n(jr)^n}{1.3\ldots 2n-1}\cos\left(jr+n\frac{\pi}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (121).$$

In free motion λ is real and negative. We may therefore write $j=-i\lambda/v=i\gamma$ where γ is real, and may be taken positive. Substituting in (118) and (121), and expressing that the terms in $e^{\gamma r}$ must disappear from (117), we are led to the following relation between the surface values of X_n and X_{-n-1}

$$3.5\ldots 2n+1.X_n-i\frac{(jR)^{2n+1}}{1.3\ldots 2n-1}X_{-n-1}=0 \quad . \quad . \quad . \quad . \quad . \quad (122).$$

Similarly in the free motions of the second type we must have

$$3.5\ldots 2n+1.\Omega_n-i\frac{(jR)^{2n+1}}{1.3\ldots 2n-1}\Omega_{-n-1}=0 \quad . \quad . \quad . \quad . \quad . \quad (123).$$

The equation to determine the various values of λ is to be obtained, in the first type by elimination of χ_n, X_n, X_{-n-1} between (107), 108), and (122), and in the second type by elimination of ω_n, Ω_n, Ω_{-n-1} between (115), (116), and (123).

In all practical cases jR is exceedingly small. If we neglect all powers of jR above the second we have

$$X_n=0, \quad \Omega_n=0.$$

In the first type we then obtain

$$\psi_{n-1}(kR)=\frac{j^2R^2}{2n-1.2n+1}\psi_n(kR) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (124),$$

approximately. For a first approximation $kR=\vartheta$, where ϑ is a root of $\psi_{n-1}(\vartheta)=0$; and for a second

$$kR=\vartheta\left(1-\frac{1}{2n-1}\frac{j^2}{k^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (125).$$

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* See C. NIVEN, Phil. Trans., 1880, p. 126. Also HEINE, 'Kugelfunctionen,' t. i., § 60.

Now $j^2/k^2=\lambda\rho/4\pi v^2$, and $\lambda=-k^2\rho/4\pi=-\rho\vartheta^2/4\pi\mathrm{R}^2$, approximately. Hence (125) becomes

$$k\mathrm{R}=\vartheta\left\{1+\frac{\rho^2\vartheta^2}{16(2n-1)\pi^2v^2\mathrm{R}^2}\right\}. \qquad (126)$$

In the second type we have

$$\frac{k^2\mathrm{R}^2\psi_n(k\mathrm{R})}{k\mathrm{R}\psi'_n(k\mathrm{R})+(n+1)\psi_n(k\mathrm{R})}=-\frac{j^2\mathrm{R}^2}{n},$$

or

$$\psi_n(k\mathrm{R})=-\frac{j^2}{nk^2}k\mathrm{R}\psi'_n(k\mathrm{R}) \qquad (127),$$

to the same degree of accuracy. For a first approximation $k\mathrm{R}=\vartheta$, a root of $\psi_n(\vartheta)=0$, and for a second

$$k\mathrm{R}=\vartheta\left(1-\frac{j^2}{nk^2}\right)$$

$$=\vartheta\left(1+\frac{\rho^2\vartheta^2}{16n\pi^2v^2\mathrm{R}^2}\right) \qquad (128).$$

By combining together in the proper way solutions of this type we should be able to represent analytically the decay of any given non-uniform electrification of the surface of the sphere. The formula (128) would indicate that in any particular mode the lines of flow of electricity in the sphere are for the most part closed curves, all those which abut on the surface being confined to a stratum of thickness $\rho^2\vartheta^2/16n\pi^2v^2\mathrm{R}$. For $n=1$, and $\vartheta/\pi=1\cdot4303$, this $=1\cdot42\times10^{-22}\times\rho^2\mathrm{R}^{-1}$. In the case of any ordinary metallic conductor this would be much smaller than the dimensions of a molecule.* A result of this character cannot of course be interpreted literally. All that we can safely assert is that the currents by which the redistribution of the superficial electrification is effected are confined to a very thin film, and are probably subject to laws not yet investigated.

In the case of a globe of water [$\rho=7\cdot18\times10^{10}$ at 22° C.] the result is more intelligible; viz., the thickness of the stratum in question is then$=\cdot73\mathrm{R}^{-1}$.

13. The case of periodic induced currents [$\lambda=2\pi ip$ where p is prescribed] may be treated as follows. Let P, Q, R denote the components of electromotive force, viz.:

$$\mathrm{P}=-\frac{d\phi}{dx}-\lambda\mathrm{F},\ \mathrm{Q}=\&\mathrm{c.},\ \mathrm{R}=\&\mathrm{c.}$$

It is easily seen that if the suffix $_0$ be used to distinguish the parts of a, b, c, P, Q, R due to the inducing system, the functions $xa_0+yb_0+zc_0$ and $x\mathrm{P}_0+y\mathrm{Q}_0+z\mathrm{R}_0$ must admit of expansion (in the neighbourhood of the origin) in the forms

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[*] There is nothing peculiar to MAXWELL's theory in the order of magnitude of this result.

$$xa_0+yb_0+zc_0=\Sigma\psi_n(jr)\Theta_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (129),$$

$$xP_0+yQ_0+zR_0=\Sigma\psi_n(jr)Z_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (130),$$

where Θ_n, Z_n are solid harmonics of positive degree n. Now $xa+yb+zc$ and $xP+yQ+zR$ must differ from the above by terms representing a disturbance propagated wholly outwards. But

$$xa+yb+zc=-n.n+1.\{\psi_n(jr)X_n+\psi_{-n-1}(jr)X_{-n-1}\},$$
$$xP+yQ+zR=-\lambda.n.n+1.\{\psi_n(jr)\Omega_n+\psi_{-n-1}(jr)\Omega_{-n-1}\}.$$

The condition that

$$\psi_n(jr)\left(X_n+\frac{1}{n.n+1}\Theta_n\right)+\psi_{-n-1}(jr)X_{-n-1}$$

should represent a disturbance travelling outwards may be shown to be

$$3.5\ldots 2n+1\left(X_n+\frac{1}{n.n+1}\Theta_n\right)+i\frac{(jR)^{2n+1}}{1.3\ldots 2n-1}X_{-n-1}=0 \quad . \quad . \quad . \quad (131),$$

where in the harmonics X_n, &c., r is supposed put $=R$. Similarly we have, on the same understanding,

$$3.5\ldots 2n+1\left(\Omega_n+\frac{1}{n.n+1}\frac{Z_n}{\lambda}\right)+i\frac{(jR)^{2n+1}}{1.3\ldots 2n-1}\Omega_{-n-1}=0 \quad . \quad . \quad . \quad (132).$$

The equations (107), (108), and (131) determine χ_n, X_n, X_{-n-1} in terms of Θ_n; whilst (115), (116), and (132) determine ω_n, Ω_n, Ω_{-n-1} in terms of Z_n. Thus the complete solution of our problem is effected.

Introducing the consideration that jR is small, we find, in the solutions of the second type,

$$\Omega_n=-\frac{1}{n.n+1}\frac{Z_n}{\lambda},$$

approximately, and thence

$$k^2\psi_n(kr)\omega_n=-\frac{2n+1}{n^2.n+1}\frac{j^2}{\lambda}Z_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (133),$$

by (115) and (116). If σ_n denote the nth harmonic constituent of the surface distribution of electricity, we deduce

$$\sigma_n=\frac{1}{\lambda}\frac{d\sigma_n}{dt}=\frac{1}{4\pi v^2 R}\cdot\frac{2n+1}{n}\cdot Z_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (134).$$

For the first harmonic constituent we have the simple formula

$$\sigma_1 = \frac{3(xf'+yg'+zh')}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (135),$$

if f', g', h' denote the components of the electric displacement which would obtain at the origin if the spherical conductor were removed.

The equation (134) expresses that so long as $p\rho$ is small compared with v^2 the surface-density of electricity at any point will have at each instant the statical value corresponding to the distribution of electromotive force at that instant due to the external system. The arrangement of the currents in the sphere by which the changes in the superficial distribution are effected will however depend materially on the relation between the period of the changes in the field and the time of decay of free currents in the sphere. The discussion of this point can be conducted as in the case of the solutions of the first type, treated in § 5, and the results are analogous to those there found. When the spherical harmonics involved are zonal, the work and the interpretation are much facilitated by the use of the current-function Ψ, whose value is given by (56).

XIV. *Researches on the Foraminifera—Supplemental Memoir.*

On an Abyssal type of the Genus Orbitolites *;—a Study in the Theory of Descent.*

By WILLIAM B. CARPENTER, *C.B., M.D., LL.D., F.R.S.*

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[PLATES 37, 38.]

INTRODUCTION.

THE subject of this communication is a type of the genus *Orbitolites*—first obtained in the deep-sea dredgings of H.M. Surveying Ship "Porcupine," off the north-west of Ireland, in 1869,* and subsequently brought up from various depths in other parts of the North Atlantic and also in the Mediterranean,--which presents many points of general scientific interest; the first of these being the completeness of the transition which it establishes between the *Milioline* and the *Orbitoline* plans of growth, and the full confirmation it thus affords of the validity of the principles on which my Classification of the FORAMINIFERA is founded.

In the Monograph of the genus *Orbitolites* (1855), which constituted the First Series of my "Researches on the Foraminifera," † I embodied the results of a careful and thorough investigation of the structure and relations of all the forms under which that type was then known to me: and I showed that while the most highly developed and most specialized of these forms exhibit the *cyclical* plan of growth almost from the very commencement,—a complete zone of sub-segments being formed by gemmation from the entire periphery of the "circumambient segment" of the central "nuclear mass,"‡ and the whole disk being made up of a succession of similar concentric zones,—there are other forms in which the primary gemmation takes place from only one side of that mass, so as to impart to the early extension of the composite structure a more or less *spiral* direction,§ which only gives place to the cyclical after repeated gemmations. The transition from the one plan of growth to the other I showed to be made by the progressive widening-out of the spire, and the increase in the number of the

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* *See* Proc. Roy. Soc., vol. 18, p. 397.

† Phil. Trans., 1856, pp. 181–236.

‡ Ibid., Plate IV., figs. 5 and 6.

§ Ibid., Plate IX., figs. 1 to 4.

sub-segments (formed by the division of the principal segments) at every new stage of gemmation; so that at last the *alæ* of the spire, extending themselves on either side round the nuclear mass, meet and complete the circlet, around which new zones are then successively budded forth, as in the forms that are cyclical from their commencement.

I did not at that time feel justified in calling in question the validity of the order *Cyclostègues*, which had been instituted by M. D'ORBIGNY, for the reception of this and other types characterised by the cyclical plan of growth; but in my Second Series (presented in the following year), which contained the results of a similar investigation of the genus *Orbiculina*, I showed that the latter *always* begins life on the spiral plan of growth, which may or may not give place subsequently to the cyclical, and that the marginal portions of a full-grown cyclical *Orbiculina* cannot be distinguished from similar portions of an *Orbitolite*. From this fact I drew the conclusion* that although *Orbitolites* and *Orbiculina* had been placed by M. D'ORBIGNY in two distinct orders, *Cyclostègues*, and *Helicostègues*, "the relationship between them must be extremely close;" and ventured further to affirm that no Classification can have any claim to be considered as *natural*, in which they shall be widely separated."

To this point I reverted in the Concluding Summary appended to my Fourth Memoir,† in which I showed how completely the results of my researches were opposed to the principles on which the Classification of M. D'ORBIGNY had been framed, indicated the line of "descent with modification" by which a division of the primary segments that form the simply-chambered shell of a *Peneroplis* into sub-segments would give origin to the chamberlets of the spiral *Orbiculina*, and pointed out how gradational the transition is from the latter to the cyclical *Orbitolites*.

When I subsequently undertook, in conjunction with my friends, W. K. PARKER and T. RUPERT JONES, to frame an entirely new Classification of FORAMINIFERA on the basis of the principles I had laid down, I felt no difficulty in assenting to their view that the pedigree of this series might be traced yet further back, viz.: to those simplest forms of the *Milioline* type, whose shell is a flattened nautiloid spire altogether destitute of partitions—thus belonging to that *monothalamous* section which all previous Systematists had ranked as fundamentally distinct from the *polythalamous*. "From the undivided spiral of *Cornuspira*," I pointed out ("Introduction to the Study of the Foraminifera," p. 67), "to the regular scarcely-divided spiral of certain 'spiroloculine' forms of *Miliola*, the transition is almost insensible; and from the 'spiroloculine' we pass by easy steps to all the other forms of the *Milioline* type." Again, a subdivision of the widely-expanded spire of *Cornuspira* into segmental chambers gives us *Peneroplis*, with its septal planes perforated by a row of separate pores; while from this, it was again pointed out, the spiral *Orbiculina* might

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* Phil. Trans., 1856, p. 552.

† Ibid., 1860, p. 571.

be derived by a further division of the segments of the sarcodic body into subsegments, with a corresponding division of the primary chambers of the shell into chamberlets.

In the new specific type of *Orbitolites* I have now to describe, the whole transition which I thus hypothetically indicated, is actually presented during the successive stages of its growth. For it begins life as a *Cornuspira*, taking-on that 'spiroloculine' condition which marks the passage towards the *Milioline* type: its shell forming a continuous spiral tube, with slight interruptions at the points at which its successive extensions commence; while its sarcodic body consists of a continuous coil, with slight constrictions at intervals. The *second* stage consists in the opening-out of its spire, and in the division of its cavity at regular intervals by transverse septa, traversed by separate pores, exactly as in *Peneroplis.* The *third* stage is marked by the subdivision of the 'peneropline' chambers into chamberlets, as in the early forms of *Orbiculina.* And the *fourth* consists in the exchange of the spiral for the cyclical plan of growth, which is characteristic of *Orbitolites;* a circular disk of progressively increasing diameter being formed by the addition of successive annular zones around the entire periphery. This increase in diameter is not here accompanied (as it is in most of the other forms of the *Orbitoline* type) by a corresponding augmentation in thickness; and as the extraordinary tenuity of these disks affords an easily recognisable and (as I believe) a constant differential character of the species, I proposed in 1870[*] to designate it *Orbitolites tenuissima.*

ORBITOLITES TENUISSIMA. *Carpenter*, 1870.

The disks of *O. tenuissima* are usually almost perfectly flat (Plate 37, fig. 1), and exhibit a remarkable regularity of structure. The diameter of the largest complete specimen I have seen is not above 0·25 inch; but it is obvious from the size and curvature of the fragments which the dredges frequently contained, that they must have belonged to disks whose diameter was at least 0·6 inch, these larger specimens having come to pieces in their rough removal from the soft and tranquil ooze on which they had previously lain. This fragility depends in part upon the extreme tenuity of the disks, their thickness rarely exceeding *one three-hundredth* of an inch; and in part on the slightness of the connexion which (as I shall presently show) exists between the successive zones.[†]

When either surface of the disk of *O. tenuissima* is viewed by reflected light under a low magnifying power, its concentric zones are seen to be crossed by radial lines (Plate 37, figs. 1, 2) resembling those which pass between the septal bands

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[*] Proc. Roy. Soc., vol. 19, p. 176.

[†] In *Cycloclypeus*, the marginal portions of the disk, though of even greater tenuity, have not by any means the same fragility; partly because its *vitreous* shell-substance is much firmer than the *porcellanous* shell-substance of *Orbitolites*, and partly because a layer of it is usually continued from each new zone over the whole surface of the previously formed disk. See Phil. Trans., 1856, p. 558.

of *Peneroplis*. But when a portion of the disk is viewed under a higher power by transmitted light,—which, through the extreme tenuity of its superficial lamellæ, brings its internal structure into distinct view (Plate 38, fig. 5),—these lines are seen not to be mere surface-markings, as in *Peneroplis*, but to be the indications of internal shelly partitions, which divide each flattened annular chamber into a series of narrow chamberlets, resembling those which I formerly described as constituting the two superficial layers of the "complex" type of *Orbitolites*.* Here, however, these chamberlets form but a single plane, as in the "simple" type formerly described; and the pores by which the last-formed annulus opens at the margin of the disk are arranged in single series (Plate 37, figs. 4, 5). It is worthy of note that these pores are not round, like those of ordinary *Orbitolites*, whether of the "simple" or of the "complex" type;† but are more or less elongated in the plane of the disk—a peculiarity obviously related to its extreme compression. Similar pores are seen upon the edge of any zone from which the zone external to it has been detached by fracture; and it is obvious that they constitute the channels of communication between the cavitary system of each zone and that of the zones internal and external to it; while the marginal series brings the cavitary system of the peripheral zone (and, through it, that of every interior zone backwards to the spiroloculine "nucleus") into relation with the surrounding medium.

When a portion of the thin shelly lamella forming either surface of the disk has been removed by dilute acid, so as to lay open the cavity beneath (Plate 37, fig. 2), it is seen that each zone of chamberlets lies between two concentric rings of shell, *a*, *a*, *b*, *b*; and that the radiating partitions, *c*, while springing from the inner shell-ring, do not extend to the outer, so that a continuous gallery is there left, into which all the chamberlets open at their peripheral extremities. And when we examine the disk by transmitted light (Plate 38, fig. 5), we see it to be from this gallery—not from the chamberlets—that the pores of the shell-ring which incloses it proceed.

Whilst the structure of the concentric zones forming the peripheral portion of the disk thus corresponds in all its essential characters with that of the ordinary "simple" type described in my former Memoir, the structure of the central portion of the disk is altogether different. The spheroidal "primordial chamber" (Plate 38, fig. 3, *a*) is extremely minute, not exceeding 1-1000th inch in diameter, and from this proceeds a compressed shelly tube, which forms a nautiloid spiral around it (Plate 38, figs. 3, 5) each successive turn slightly increasing in breadth, so as closely to resemble the first-formed part of the spire of *Cornuspira*. The continuity of its cavity, however, is interrupted, usually at about every two-thirds of a turn, by a thickening of its wall (Plate 38, fig. 3, *b*), which seems to have been formed as a sort of foreshadowing of a septum at each addition to its length; and thus, as long as the growth of the shell proceeds upon the same plan, it is a 'spiroloculine' *Miliola*.

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* Phil. Trans., 1856, p. 202, Plate V., fig. 6, *c*, *c*; Plate VII., fig. 12.

† Ibid., Plate V., figs. 1 and 6, *d*, *d*.

But after making from six to eight turns (the number varying in different individuals) the spire begins to open out in the horizontal plane (Plate 38, fig. 5, *a*) without any vertical enlargement, and a complete septum is formed at the next break, marking off the first principal chamber from the previously-formed spiral tube. This septum is traversed, as in *Peneroplis,* by a variable number (*four* in the specimen here figured) of passages, which would show themselves as pores upon its external surface; but these, instead of opening into another single undivided chamber, lead into as many chamberlets, which are formed by the subdivision of the next principal chamber, *b*, by radial partitions, exactly as in *Orbiculina.* This chamber, in the individual here figured, is not separated by a completely-formed septum from the succeeding chamber, *c*, and the latter is undivided save by a single radial partition; but this is a mere individual variation,—which is of interest, however, as showing that the subdivision of the chambers into chamberlets is a secondary, not a primitive formation. The septum which closes-in the chamber *c* is traversed by 13 pores, which open into as many chamberlets formed by the subdivision of the next principal chamber; the separation of these chamberlets by radial partitions being complete for about four-fifths of the length of the chamber (that is, of the distance between its inner and its outer septum), but deficient for the outer fifth, so as to leave the continuous gallery *d*, *d*, into which all the chamberlets open at their outer ends. This chamber, it will be observed, extends itself on either side at *d′*, *d′*, so as to enclose a portion of the spiroloculine "nucleus;" and this extension is still more marked in the next chamber, whose two alæ, *e′*, *e′*, reach the ends of the transverse diameter of the original spire. The septum which separates this chamber from the preceding has the number of its pores increased to 30; and these open outwards into as many chamberlets in the next-formed chamber. As new chambers are successively added, the backward extension of their alæ is carried further and further, until (in the individual here figured, Plate 37, fig. 1) those of the ninth chamber meet at the back of the spiroloculine "nucleus," so as to enclose it all round, and the tenth chamber forms a complete ring of chamberlets, whose derivation from the undivided chamber of the 'peneropline' type is made obvious by the previous transition. With each increase in the length of the septal plane, there is a proportionate increase in the number of pores by which it is traversed, the distance between them having a very uniform average; and the number of these pores determines the number of chamberlets in the next annulus, which has thus no definite relation to that of the chamberlets in either of the last-formed or in the subsequently-formed annulus. The breadth of the zones (and, consequently, the length of their chamberlets) has a range of variation from 1-180th to 1-80th of an inch, its general average being 1-120th inch; so that a disk having a diameter of 0·6 inch (or a radius of 0·3) would be made up of about forty such concentric zones. A very narrow zone is occasionally seen to intervene between two zones of ordinary breadth; but, as I have always found this to originate in a fractured portion of the preceding zone which then formed the margin of the disk, it would seem to have been a special reparative addition.

The whole cavitary system, from the primordial chamber to the marginal annular gallery, is occupied by a continuous sarcodic body of a dark olive-green hue (Plate 38, fig. 1). Although this body may be said to consist at any one moment of a multitude of sub-segments, connected together by annular and radiating stolon-processes, yet, from what we know of its semi-fluid condition in the living animal, we may pretty confidently surmise that this subdivision is by no means permanent, but that an interchange is continually taking place between the protoplasmic contents of the inner and the outer portions of the cavitary system, so that what occupies the central spire at any moment may be transferred in no long time to the marginal annulus, and *vice versâ*.

The extreme tenuity of this sarcodic body, and the transparence of the shelly laminæ that invest it, have enabled me very distinctly to recognise, by light transmitted through the disk, the presence of nucleus-like bodies (Plate 38, fig. 2) of about 1-1750th of an inch in diameter, imbedded in its substance. As might be expected from the consideration just stated, these corpuscles are very irregularly distributed. In the specimen here figured (Plate 38, fig. 1), two of the outer half-whorls of the 'spiroloculine' centre (shown on a larger scale at *b*, *b*, *b′*, *b′*, fig. 4), are crowded with them; while in a single chamberlet, *c*, of one of the interior annuli, there are as many as five. Elsewhere, on the other hand, they present themselves with less frequency, only one or two occurring in any single chamberlet (*d*, *d*, *d*), and a large proportion of the chamberlets being entirely destitute of them. The finding of these corpuscles in the highly composite sarcode-body of *Orbitolites* is an interesting extension of the discovery, of Dr. R. HERTWIG, of corpuscles regarded by him as nuclei, in what I long since characterised as the "reticularian" type of RHIZOPODA, of which the ordinary FORAMINIFERA are the testaceous forms. This discovery was first made in the fresh-water Monothalamous *Mikrogromia*,* and subsequently extended by him to various marine Polythalamia, such as *Spiroloculina*, *Globigerina*, and *Rotalia*,† and by F. E. SCHULTZE to *Quinqueloculina*, *Lagena*, *Polystomella*, and *Planorbulina*.‡ What is the function of these corpuscles in that indefinite extension of the protoplasmic body, and the multiplication of its segments, which is so remarkable a character of this type, is not yet apparent; but that they do *not* become the centres of distinct cells separated from each other by any limiting membrane, or even of permanent segments or sub-segments, may be regarded as certain. If the nuclear character of these corpuscles be admitted, the entire composite organism thus seems to present a most interesting link of connexion between the *unicellular* and *multicellular* types; the absolute continuity of its protoplasmic substance entitling it to rank with

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* Archiv für Mikrosk. Anat., Bd. x., Supplementheft (1874) p. 1.

† Jenaische Zeitschrift, Bd. x., 1876, p. 41, &c.

‡ Archiv für Mikrosk. Anat., Bd. xiii., 1877, p. 9.

the former, whilst in the multiplication of its nuclei it obviously tends towards the latter.

The growth of this beautiful organism doubtless takes place after the completion of the first annulus, in the manner described in my former Memoir (§ 35). The sarcodic body, when enlarged by the nutriment it has appropriated, will project itself through the marginal pores, in quantity sufficient to form, by the coalescence of its separate protrusions, a continuous belt of sarcode; and in the substance of this a set of radial calcareous partitions will be deposited, commencing between the pores of the margin of the previous shell-ring, while two horizontal lamellæ are formed on the superficial planes, to constitute, as it were, the floor and ceiling of the new circle of chamberlets. These horizontal lamellæ overlap but very slightly the margin of the previous annulus (Plate 37, fig. 3, *a*, *a*); and their adhesion to it is generally so weak that the annuli readily come apart. As the subdivision of the annulus into chamberlets does not extend to its outer portion, a passage is left (seen in vertical section at *b*, *b*, *b*, fig. 3), which is occupied by a continuous ring of sarcode, as shown in Plate 38, fig. 1; and from this ring proceed the stolons which pass outwards through the pores of the septum that closes it in.

The homogeneousness of the protoplasmic substance by which the entire cavitary system is occupied, is shown (as in the types formerly described) by the completeness with which the effects of injuries are repaired, and the plan of the original fabric restored (see Phil. Trans., 1856, Plate VIII., figs. 4–9). Not only is the loss of any part of the disk repaired by the formation of a new and continuous annulus along the broken as well as the unbroken margin, so that the next and all succeeding zones follow the new contour (as shown in Plate 37, fig. 6); but a new and entire annulus of chamberlets may form itself around the whole circumference of a mere marginal fragment (fig. 7), by the enclosure of which in a subsequent succession of annuli, the discoidal form characteristic of the type is completely and characteristically restored. Owing to the transparence of these attenuated disks, I have been able to assure myself that *every part* of the margin of this fragment, whether broken or unbroken, peripheral, central, or lateral, has contributed to the formation of the first new complete annulus, by which the foundation was laid of the subsequent regular series of concentric zones; thus clearly indicating that a sarcodic extension took place from every chamberlet laid open by the fracture, as well as from the normal pores of the last-formed septal plane, and that these extensions coalesced to form a continuous ring, as in the formation of the ordinary succession of concentric annuli.

This perfect reproduction of a form of peculiar regularity, in a type of animal organisation so low that its body-substance does not show any advance upon the primitive protoplasmic condition, is, in itself, a matter of great interest. But the interest is much enhanced by the consideration that this organism begins life, and forms its first shelly envelope, upon a plan altogether different; exchanging this for its later mode of growth, by a transition so rapid as to manifest the almost sudden attainment of a much more specialised character. And when this transition has been once made, there appears no disposition whatever, in the reparation of injuries, to a reversion to the earlier plan. Now, this is a "pregnant instance" of the following "law of formation," sagaciously laid down long since by Sir JAMES PAGET:—"When, in an adult animal, a part is reproduced after injury or removal, it is made in conformity, not with that condition which was proper to it when it was first formed, or in its infantile life, but with that which is proper according to the time of life in which it is reproduced; proper, because like that which the same part had, at the same time of life, in members of former generations." And the study of this humble *Orbitolite* will be found, not only in this, but in other particulars, to justify the profound remark made by the same philosophic Pathologist,* long before the promulgation of the doctrine of "evolution," that, "if we are ever to escape from the obscurities and uncertainties of our art, it must be through the study of those highest laws of our science which are expressed in the simplest terms in the lives of the lowest orders of creation."

Geographical, Bathymetrical, and Geological distribution.

So far as is at present known, *Orbitolites tenuissima* inhabits only the North Atlantic Ocean and the seas in communication with it. The first complete specimens were obtained in the "Porcupine" dredgings of 1869, at depths of from 630 to 1,443 fathoms, between the north-west of Ireland and Rockall Bank. In the "Porcupine" expedition of 1870, however, it was brought up from a bottom of only 64 fathoms in Setubal Bay, on the coast of Portugal, and afterwards from a shallow bottom within the Mediterranean, near Carthagena. That it is an inhabitant of other parts of the Mediterranean I then inferred from the fact that I had detected fragments of it in the Foraminiferal dredgings, made at 250 fathoms by EDWARD FORBES and Lieut. (now Admiral) SPRATT in the Ægean, in 1842; and it is stated by Dr. J. GWYN JEFFREYS, in his "Report on the Biology of the 'Valorous' Cruise," that it has been dredged by the Marquis DU MONTEROSATO at from 100 to 200 fathoms' depth, off the coast of Sicily. That it might extend far to the north, would be expected from its capability of bearing the low temperature of 37° Fahr., which prevails over the deep bottom from which it was first brought up; and this expectation was verified by its presenting itself in one of the "Valorous" dredgings in Baffin's Bay (lat. 62° 6′ N., depth 1,350 fathoms, temperature 34° 6′ Fahr.), as well as at two stations in the North Atlantic, No. 12, depth 1,450 fathoms, and No. 13, depth 690 fathoms, both in the parallel of 56°. It has been only once brought up, however, in the "Challenger" expedition, viz., at Station 44, off Cape Hatteras, from a bottom of 1,700 fathoms' depth, over which creeps (there is strong reason to believe) an underflow of cold water from the Arctic basin. Several specimens have (I am informed) been since found in a

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* "Lectures on Surgical Pathology," 1849;—Lect. VII. General Considerations on Repair and Reproduction.

dredging taken by the French exploring-ship "Travailleur" in the Bay of Biscay (Fosse de Cap Breton), at a depth of 1,200 fathoms.

It would seem, therefore, that *Orbitolites tenuissima* has its proper home on the sea-bottom of the deeper parts of the North Atlantic, where the temperature ranges from 37° to 35° Fahr.; but that it also is capable of living, not only in much shallower, but also in much warmer, waters. For the temperature of the Mediterranean and Ægean, even at depths below 100 fathoms, is never less than 54°, while on the shallow bottom of Setubal Bay, and on the shore-slope near Carthagena, the summer temperature must be considerably higher.

Looking to the singular retention, in this beautiful *Orbitoline*, of the *Milioline* type from which its derivation may now be confidently affirmed, the probability seems strong that it was a very early form; and if identical with COSTA's *Pavonina italica*,* as the imperfect account given by him of that type would seem to indicate, it probably inhabited the Mediterranean during the greater part of the Tertiary period. Its persistence in the abyssal depths of the North Atlantic harmonizes well with the idea of its antiquity; those depths having been found, by the recent exploration of them, to be inhabited by many "survivals" of the Cretaceous and even earlier Faunæ. It may be remarked, finally, that the considerable diameter attained by these very fragile discs, seems a proof of the extreme tranquillity of the deep-sea bottom; since they could not otherwise have gone on growing and extending themselves, without showing more frequent marks of injury and reparation than I have observed in them.

Relation to other Orbitoline Types.

Having been requested by the late Sir C. WYVILLE THOMSON to prepare a Report on the *Orbitolites* collected in the "Challenger" Expedition, I have carefully studied the remarkable gatherings made of them on and near the summit of the Fiji reef, and also at a depth of 18 fathoms on its slope. The result of that examination now enables me to indicate with great probability the successive stages of the evolution of that highly specialised "complex" type, the derivation of which from a Milioline ancestry would have seemed—but for the completeness of the series of intermediate forms—almost inconceivable. And I can now also mark out, with more distinctness than formerly, the types of this Genus, which, in virtue of their constancy and definiteness, are entitled to rank as distinct species.

The first of these is the *O. marginalis* of Lamarck, known to him only by small Mediterranean specimens of no more than two millims. (about 0·08 inch) in diameter, but attaining on the Fijian reef a diameter of 0·2 inch, and presenting a much more characteristic aspect than is discernible in the dwarfed Mediterranean form. The well-developed "cycloline" disks of this beautiful form of the "simple" type (fig. I., 1),

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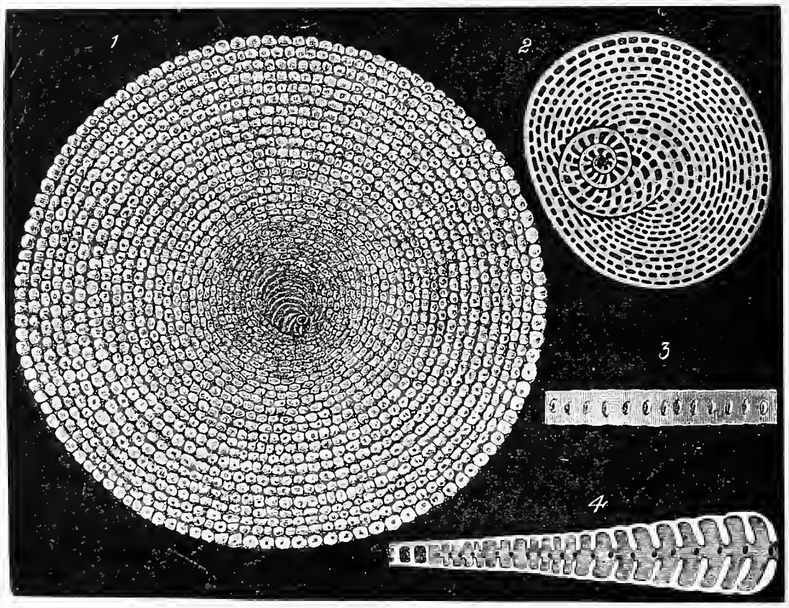
* See his "Paleontologia del Regno di Napoli," part ii., in 'Atti dell' Accad. Pontan,' vol. vii., p. 178, plate xvi., figs. 26-28.

which I formerly* differentiated only by the singleness of its row of marginal pores (fig. I., 3), I now find to be uniformly characterised by the marked eccentricity of their primordial chamber, and by the spiral direction of their early growth (fig. I., 2), which I formerly supposed to be only occasional variations. In fact, the first formed portion of these disks, like that of a young *Orbiculina adunca* (*loc. cit.*, Plate XXVIII., fig. 2), exactly resembles what a Peneropline shell would be, if its chambers, as they widen out, were to undergo division into chamberlets; thus corresponding in every essential particular with the "orbiculine" stage of *O. tenuissima.* But while we have seen that this stage, in the last-named species, is preceded by a spiroloculine coil, representing a true "milioline" stage, it has no other predecessor in *O. marginalis* than what I formerly designated as the "nucleus," consisting of a flask-shaped "primordial chamber," from the neck of which proceeds a "circumambient chamber" that passes almost completely round it (as in fig. II., 3). The morphological import of this arrangement becomes clearer when we compare the sarcodic bodies of the two types; for it is then obvious that the "circumambient segment," which springs from the "primordial segment," and then, after making a single coil around it, gives off the first "peneropline" segment, really represents the *multiple* spiroloculine coil of *O. tenuissima;* this early generalised "milioline" stage being (as it were) abbreviated with the advance towards specialisation, as we see in numberless cases elsewhere.

Fig. I.—*Orbitolites marginalis.*



The "Challenger" collection—especially that of the 18 fathoms' dredging—includes a very large number of thin flat disks, attaining a diameter of about 0·32 inch, whose surface often presents rather an "engine-turned" than an annular aspect, and which are specially characterised by the possession of a complete *double* row of marginal

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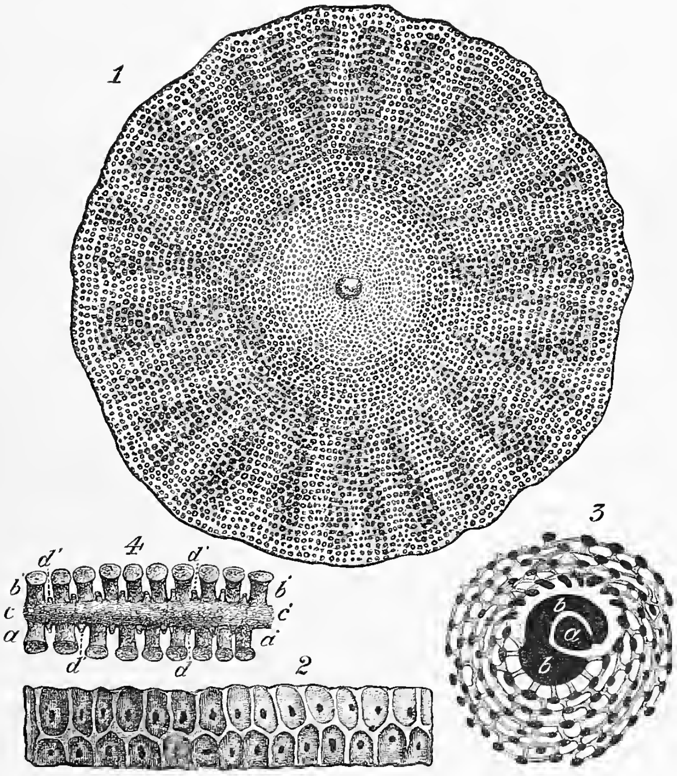
* Phil. Trans., 1856, p. 215, Plate VII., fig. 14.

pores (fig. II., 2). Both these peculiarities were noticed in my former Memoir (pp. 215, 221), but were treated as merely varietal modifications. I now find, however, that they accompany one another very constantly; and that the type is so well differentiated by them as to be fully entitled to rank as a distinct species, which I designate *O. duplex.** Notwithstanding the difference in the surface-aspect of its disks, and the doubling of their marginal pores, the sarcodic body of this species conforms in every essential particular to that of the preceding. For each of its concentric annuli consists of a single cord (fig. II., 4, *c c′*), that passes through a continuous circular gallery in the median plane of the disk, and carries a double series of columnar sub-segments (*a a′*, *b b′*), which occupy chamberlets (fig. II., 1) that extend in vertical series to the two surfaces of the disk. But each annular cord, instead of giving off (as in *O. marginalis*) a *single* stolon-process to initiate a sub-segment of the succeeding annulus, gives off *two* such processes between each pair of its own sub-segments (fig. II., 4, *d d*, *d′d′*); and these have separate passages through the septal plane—one above and the other below the annular canal, as shown in fig. II., 2,—

Fig. II.—*Orbitolites duplex.*



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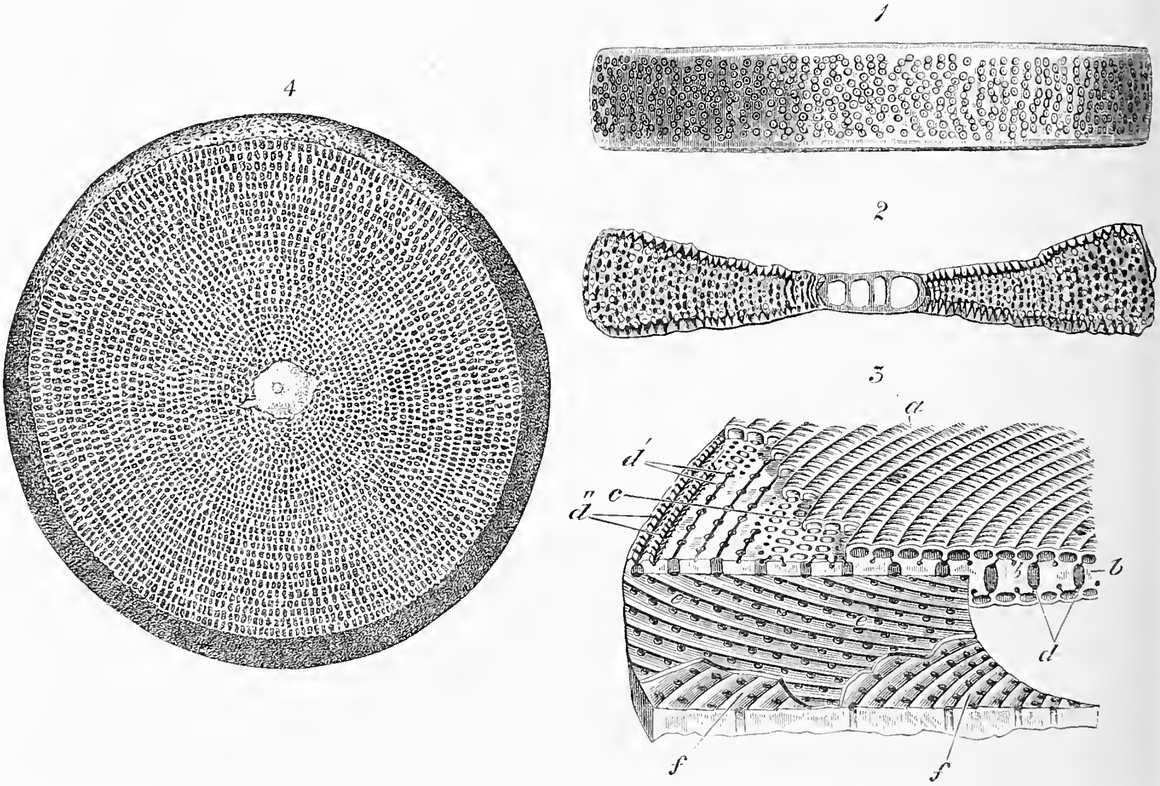
* This species, as intimated in my former Memoir, appears to be the type described by Prof. EHRENBERG (Abhandl. der König. Akad. der Wissenschaften zu Berlin, 1839) as a BRYOZOON, under the designation *Amphisorus Hemprichii.* As his conception of the generic characters of this type was fundamentally erroneous, and as he gave no diagnosis of the single species he created, I have not thought it necessary to preserve his specific designation.

opening on its external side in a double series. This arrangement, as will presently appear, is the first step in the evolution of the "complex" form of the Orbitoline type.

Another advance upon *O. marginalis* is seen in the more rapid approach of *O. duplex* to the cyclical plan, shown in the abbreviation of the early spiral stage. For the "nucleus" of *O. duplex* has but a slight eccentricity, and its circumambient segment(fig. II., 3, *b*, *b*), instead of putting forth but a single stolon-process, gives off several;[*] so that, as each of these originates a new sub-segment, a crescentic row of sub-segments is at once constituted. The row formed next in succession to this almost entirely encircles the milioline nucleus, and the third row generally completes the annulus, all further increase in the disk taking place on the cyclical plan. In *O. duplex*, therefore, we have such an abbreviation, not only of the "milioline" but also of the "orbiculine" stage, that the proper "orbitoline" type is attained at a relatively earlier period.

Fig. III.—*Orbitolites complanata.*



We come, lastly, to that "complex" form—the *O. complanata* of Lamarck—in which the special peculiarities of the *Orbitoline* type are most fully displayed. Its disks (fig. III., 4) attain not only a much larger diameter, but a relatively greater thickness, than those of either of the "simple" species; the annulations which mark their surfaces are as complete in their central as in their peripheral portions; their superficial

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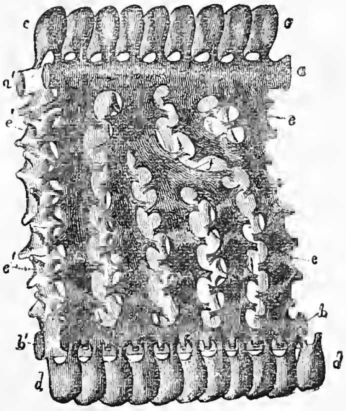
[*] Phil. Trans., 1856, Plate IV., fig. 13.

chamberlets have an elongated form (fig. III., 3, *a*), and their margins exhibit, even in the smallest (or youngest) specimens, *multiple* series of pores (fig. III., 1), indicative of that complicated arrangement of the cavitary system which I described minutely in my former Memoir.

The meaning of that arrangement is best understood by an examination of the sarcodic body left after the decalcification of the disks, which are modelled, as it were, upon it. The accompanying representation (copied from Plate IV., fig. 4 of my former Memoir) shows two annular cords, *a a′*, *b b′*, in each annular zone, instead of the single cord of *O. duplex;* and between these two cords is interposed a series of columnar sub-segments, *e e*, *e′ e′*, whose bases and summits (so to speak) are brought into continuity by them. It is of the interposed shell-substance that lodges these columnar sub-segments, that the thickness of the disk (fig. III., 2) is chiefly made up; and this is obviously in relation with the length of the columns. Between each annular cord and the nearest surface of the disk, is a series of sub-segments, *c c*, *d d*, which occupy the elongated chamberlets whose partitions are marked externally by radial lines that cross the several annuli (fig. III., 3, *a*), as in *O. tenuissima.* These partitions, however, being complete, the chamberlets have no lateral communication with each other; neither do they communicate by means of radial passages with those of the annuli internal and external to them. But each has a passage at either end through its own floor, which allows a stolon-process to pass from the sub-segment which it lodges to the annular cord beneath; each sub-segment being, therefore, in connexion with the two annular cords, and forming, as it were, a bridge between one and another, as shown in fig. IV. Except through the intermediation of these sub-segments, the annular cords of the successive zones have no connexion with each other; but the intermediate columnar sub-segments of each annulus communicate with those of the next by

Fig. IV.



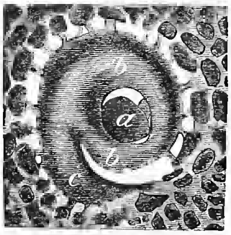
Portion of sarcodic body of *Orbitolites complanata:*—*a a′*, *b b′*, the upper and lower annuli of two concentric zones; *c c*, the upper layer of superficial sub-segments, and *d d*, the lower layer, connected with the annular cords of both zones; *e e* and *e′e′*, intermediate columnar sub-segments of the two zones.

oblique stolon-processes, that pass off alternately at regular intervals from the two sides of each column, traversing the annular septa; and the orifices of the passages in the last-formed septum, through which these stolon-processes extend themselves outwards, are seen as multiple series of pores on the margin of the disk (fig. III., 1).

The vertical section of the calcareous disk given in fig. III., 2, shows the separation of the two superficial planes of chamberlets by the interposition of the shelly fabric that gives lodgment to the intermediate sarcodic columns; while at 3 is shown diagrammatically, on a larger scale, the cavitary system of the disk, with the communication between its several parts. At *a* are seen the chamberlets of the superficial planes, which are completely closed in when not abraded; and these are shown in vertical section, above and below, at *b*, while at *c* are seen their floors, each having a pore at either end, which communicates with the annular canal beneath. The annular canals are seen at *d* in vertical section, and at *d′* and *d″* as laid open in horizontal section; the former showing how they cross the tops of the cylindrical chamberlets of the intermediate stratum, and the latter (taken a little nearer the surface) showing the manner in which they open into the pores leading to the superficial chamberlets. In the lower part of the figure, the intermediate stratum is traversed by two horizontal sections in slightly different planes, cutting across the cylindrical chamberlets, and showing the two series of oblique stolon-passages by which the chamberlets of successive annuli communicate with each other.

The nuclear mass which occupies the centre of the disk consists, as in *O. duplex*, of a "primordial segment," surrounded by a "circumambient segment," and this last (fig. V., *b*, *b*) puts forth a set of stolon-processes from its entire periphery, each of which gives origin to a columnar sub-segment; so that a complete annulus is at once constituted, thus establishing the *cyclical* plan of growth *from the very first.*

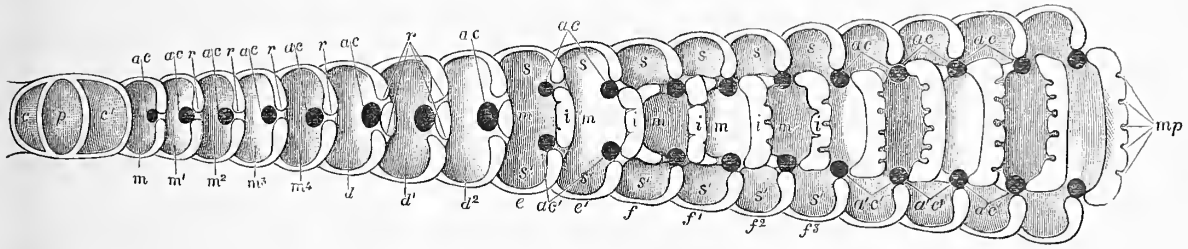
Fig. V.



The collection of specimens of *O. complanata* made on the Fiji reef contains disks of all sizes ranging from 0·04 inch to nearly 1 inch; and even in the smallest of them, whose nucleus is surrounded by only two or three annuli, the immediate assumption of the completed plan is marked by the multiplicity of the series of marginal pores. But while this may, I think, be unquestionably regarded as the *typical* condition of the species, the collection also includes an abundance of disks whose peripheral portion is characteristically "complex," whilst their central portion is no less characteristically "simple;" the passage from the one plan of growth to the other taking place at no fixed stage, but being made sometimes earlier, sometimes later, for the most part gradually, but sometimes abruptly, as I indicated in my former Memoir (§§ 57, 58).

I can now trace out more distinctly than before the successive phases of this transition; and can show how exactly the fundamental characters of *O. marginalis* and *O. duplex* are reproduced in what may be called the childhood and youth of those "sub-typical" examples of *O. complanata*, which, instead of beginning life on the "complex" plan, only attain the more elevated type in adult age. These phases are exhibited in the following vertical section (fig. VI.) taken in the radial direction, which, though representing them somewhat diagramatically, is true to nature in every essential particular. Of the successive zones traversed by the sectional plane between the circumambient chamber, $c\ c'$, which passes round the primordial chamber, p, the first five, m, m^1, m^2, m^3, m^4, are formed exactly on the type of those of *O. marginalis*; each chamberlet being connected laterally with the other chamberlets of its own zone by a single annular canal, ac, and with the chamberlets of the zones internal and external to it by the radial stolon-passages, $r\ r$. These are succeeded by three zones, d, d^1, d^2, formed upon the "duplex" plan; each chamberlet having, as in the preceding case, but a single annular canal, ac, but communicating with the chamberlet of the annulus external to it by two oblique radial stolon-passages, as shown at r. Thus, then, if the growth of this disk had been checked at the fifth zone, m^4, its margin would have presented the single row of pores characteristic of *O. marginalis;* and if at the eighth zone, d^2, it would have shown the double row characteristic of *O. duplex*. But in the next zone, e, the annular canals ac, $a'c'$, are duplicated, each of them sending off a stolon-passage into the next annulus. In this and the succeeding zone, e', however, there is no separation between the superficial portions, $s\ s'$, of the chamberlets, and their median portions, $m\ m$; and this continuity, here transitory, shows itself as the typical character of the chamberlets of the fossil *O. complanata* of the Paris basin. But in the existing *O. complanata*, a separation comes to be effected, as shown in the succeeding zones, f, f^1, f^2, f^3, by horizontal extensions of the septa $i\ i$, that are interposed between the median portions of the chamberlets, so as to form the floors of the superficial layers; while at the same time there is a shifting of their relative positions, so that the superficial chamberlets, $s\ s$, $s'\ s'$, instead of lying over or under the median portions, $m\ m$, alternate with them, and are entirely

Fig. VI.



cut off from any other communication with them than that which is afforded by the annular canals, with which each superficial chamberlet communicates at either end, by a passage which—thus traced out—is seen to be homologous with one of the double radial stolon-passages of *O. duplex*, and therefore with the single radial passage of *O. marginalis*. The septa, *i i*, which divide the median portions, *m m*, of the successive annuli, are traversed by numerous passages, which, from the lateral obliquity of their direction (fig. III., 3, *f*, *f*), scarcely show themselves in a radial section, although they debouch at the edge of the last annulus as marginal pores, *mp*.

Notwithstanding this progressive complication in the structure of the shelly disks, there is no appearance of any corresponding specialisation in the character of the sarcode body: that of the typically "complex" form showing no other advance upon the very simplest, than is marked by the duplication of the sarcodic annuli, by the separation of the superficial from the intermediate columnar sub-segments, and by the multiplication of the oblique stolon-processes which connect these last with each other, this multiplication being obviously in relation with the increasing length of the interposed columns, which shows itself in the thickening of the disk. The most marked increase in the complication of the animal body obviously consists in the duplication of the sarcodic annuli; and this may be readily conceived as a longitudinal splitting of each cord into two, with a persistence of adhesion at intervals, so that the two semi-annuli, when carried apart from one another by the interposition of the intermediate stratum, remain connected by the vertical sarcodic columns which traverse that stratum. The sub-segments which occupy the upper and under layers of surface-chamberlets are clearly shown, by their relation to the sarcodic annuli, not to be new productions, but to be homologous with the upper and under halves of the sub-segments that occupy the columnar chamberlets of the "simple" type; that homology, however, being so masked in the typically "complex" form by the displacement they have undergone, that it could not have been certainly recognised, but for the occurrence of those subtypical forms which enable the passage from the most "simple" to the most "complex" to be continuously traced-out.

I have been unable, after the most careful examination of the sarcodic bodies of *O. duplex* and *O. complanata*, to discover any indication that this progressive complication in the disposition of their parts, is accompanied by any such structural modification as might lead to the suspicion of differentiation of function. On the contrary, I find their substance to be everywhere of the same elementary character, consisting of a homogeneous protoplasm, that contains a large number of spherules of from $\frac{1}{6000}$th to $\frac{1}{8000}$th of an inch in diameter, sometimes crowded closely together, in other instances more dispersed, as shown in fig. 3, Plate IV. of my former Memoir (Phil. Trans., 1856). These spherules, when subjected to pressure, break up into a number of pellucid corpuscles, which are usually of from $\frac{1}{15000}$th to $\frac{1}{20000}$th of an inch in diameter. The absence of these spherules is a marked feature of difference in the protoplasmic body of *O. tenuissima;* on the other hand, I have not met in the higher types with those nuclear (?) bodies which I have recognised in the abyssal species (see p. 556).

The homogeneousness of the entire sarcodic body, even in the largest and most complicated forms of *O. complanata*, appears to be further indicated by the fact, that in specimens taken alive and preserved in spirit, the peripheral portion of the cavitary system is invariably found empty ; the sarcodic body, corrugated by the action of the spirit, being drawn together towards the central portion of the disk through very narrow passages of communication, which could only happen with a substance of which every part is free to move upon every other. Looking, also, to the manner in which the entire organism receives its nourishment through the marginal pores, and to the entire absence of any special means for the distribution of that nourishment, I think it may be fairly assumed that such a protoplasmic circulation goes on throughout life, as must produce a continual change in the substance of every individual sub-segment.

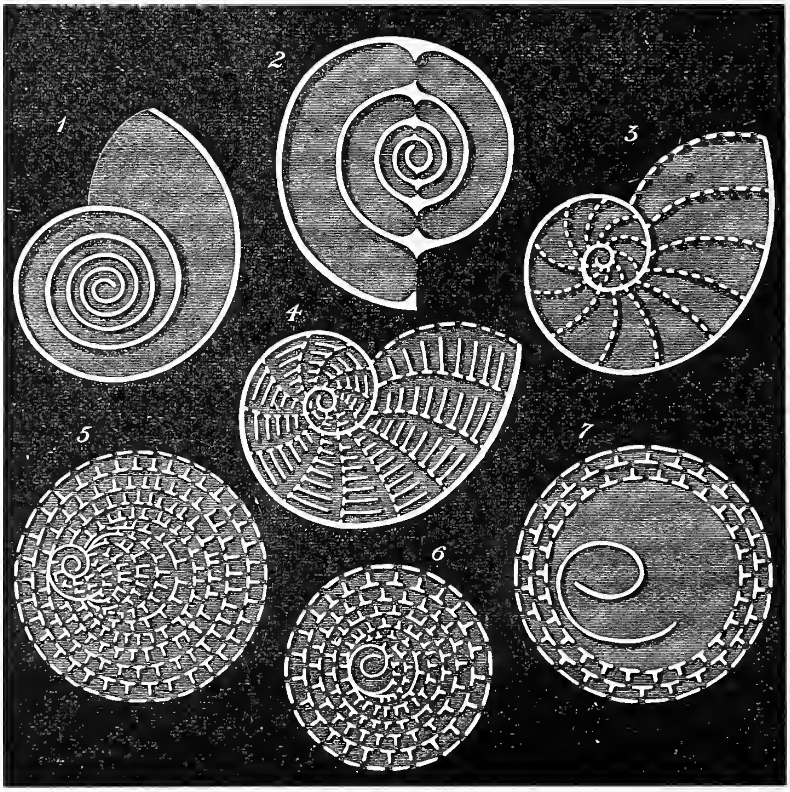
Additional evidence of this homogeneousness is afforded by the two following facts: first, that in specimens which live under conditions peculiarly favourable to their enlargement, out-growths of irregular shape, but always possessing a regular internal structure, are put forth from any part of the disk (see "Challenger" Report, Plate VII.); and second, that, as in *O. tenuissima*, every part seems equally capable of reproducing the entire disk on its characteristic plan.

Evolutionary History of the Orbitoline Type.

Thus by the combined study of *O. tenuissima* and of sub-typical examples of *O. complanata*, we are enabled to work out the whole evolutionary history of the Orbitoline type, from its simplest to its most complex form. For there can, I think, be no reasonable doubt, that the succession here presented to us in the consecutive phases of two lives, has been the genetic history of this type ; which, originating in the simplest "jelly-speck" that could form a shelly chamber, first assumed the form of a spirally-coiled undivided tube (*Cornuspira*, fig. VII., 1) ; then of a spire interrupted at intervals by imperfect partitions (*Spiroloculina*, 2) ; then of a flattened spire crossed by complete septa traversed by stolon-passages (*Peneroplis*, 3) ; then of a progressively widening spire, whose chambers are divided into chamberlets (*Orbiculina*, 4) ; then of a chamberletted disk of one storey, commencing as an orbiculine spire, but subsequently increasing by annular additions (*Orbitolites tenuissima* and *O. marginalis*, 5) ; then of a chamberletted disk, whose origin still shows in its slight eccentricity a trace of the primordial spire, and whose single storey has, so to speak, two rows of windows (*Orbitolites duplex*, 6) ; and lastly, of a "complex" disk, whose growth is cyclical from the beginning, and whose upper and lower superficial planes are separated by the interposition of an intermediate columnar structure between the duplicated annular stolons (*Orbitolites complanata*, 7). This last would seem to be the culmination of the type, which, while attaining a considerable size, has never shown, so far as is at present known, any tendency to pass into a higher form. Indeed, the typical forms of the existing *O. complanata* are in one small particular (as already shown, p. 565) more specialised than the fossil forms that were so remarkably abundant in the Middle Tertiary epoch.

It is a remarkable feature of this case, however, that all the forms through which the highest *Orbitoline* type is thus shown to have passed, continue to hold their ground at the present time, as the characteristic representatives of less specialised groups. There being every reason to regard *Cornuspiræ, Peneroplides,* and *Orbiculinæ* as distinct races, propagating themselves genetically without any essential modification, it can scarcely be supposed that every one of them is a "potential" *O. tenuissima.* So, again, as we find *O. marginalis* and *O. duplex* living and propagating under the very same conditions as *O. complanata,* I cannot regard these "simple" forms of the Orbitoline type, each of which has its characteristic plan of structure and limit of growth, as potentially "complex;" notwithstanding the exact repetition of their plans in the early stages of certain examples of the higher type. For I have never observed in the largest and best developed examples of *O. marginalis* and *O. duplex* the least tendency to assume the "complex" form; on the other hand, I have frequently found their last formed annuli deficient in internal partitions, as if their productive power had exhausted itself. It would seem, therefore, more just to

Fig. VII.



regard those sub-typical examples of *O. complanata*, which exhibit the transition I have described from the "simple" to the "complex" plan of structure, not as advanced forms of either of the two "simple" species, but as retarded forms of the highest; and a clue to the conditions of that retardation can, I think, be found in the marked inferiority I have invariably observed in the size of the original nuclear mass of these individuals. I drew attention in my former Memoir (§ 44) to the remarkable range of dimension which this mass exhibits, when a considerable number of specimens are examined; and showed that the cavity of the "primordial" and "circumambient" chambers in one individual must have been more than a hundred times as large as that of another. Now the result of the far more extended comparison of specimens which the "Challenger" collection has enabled me to make, is, that while the "nucleus" of the typical *O. complanata* (in which the cyclical plan of growth, and the "complex" structure, show themselves from the very first) is always many times larger than that of either *O. marginalis* or *O. duplex*, the "nucleus" of its sub-typical forms always bears a very close accordance in size to that of *O. duplex*, which it resembles also in the one-sided pullulation of the first sub-segments from the circumambient segment, rendering the earlier zones more or less incomplete, and the position of the "nucleus" slightly eccentric. Whether these forms genetically propagate themselves as a *race*, perpetuating an earlier stage of the evolution of the perfected type, or are merely *individuals* which have begun life as "starvelings" that do not inherit the characteristic vigour of the type, I have no adequate ground for even surmising; being only able to affirm this, that as there is no kind of constancy in the stage of growth at which the "simple" plan gives place to the "complex," there is nothing to justify a specific differentiation of this sub-typical variety. That its peculiarity may depend upon conditions less favourable to the full development of the type, seems to be indicated by the fact that, whilst the largest and most typical specimens of *O. complanata* were found in the rock-pools on the summit of the Fiji reef, where they would have the highest temperature and the greatest abundance of food, the sub-typical specimens presented themselves chiefly in the collection made by the dredge at 18 fathoms' depth.

Theory of Descent.

I propose, in the last place, briefly to examine the bearing of the remarkable case of "descent with modification," which I have thus detailed, upon the general "Theory of Descent" and of the "Origin of Species."

Those who find in "natural selection" or the "survival of the fittest" an all-sufficient explanation of the "origin of species," seem to have entirely forgotten that before "natural selection" can operate, there must be a range of varietal forms to select from; and that the fundamental question is (as Mr. DARWIN himself clearly saw, at any rate in his later years), *what gives rise to variations?* No exercise of "natural selection" could *produce* the successive changes presented in the evolutionary history of the typical *Orbitolites*, from *Cornuspira* to *Spiroloculina*, from *Spiroloculina* to *Peneroplis*, from *Peneroplis* to *Orbiculina*, from *Orbiculina* to the "simple" forms of *Orbitolites*, and from the "simple" to the "complex" forms of the last-named type. And as all these earlier forms still flourish under conditions which (so far as can be ascertained) are precisely the same, there is no ground to believe that any one of them is better fitted to survive than another. They all imbibe their nourishment in the same mode; and no one type has more power of going in search of it than another. That they are all dependent on essentially the same conditions of temperature and depth of water, is shown by their occurrence in the same marine areas. That they all equally serve as food to larger Marine Animals, can scarcely be doubted; and it is hardly conceivable that any of their devourers would discriminate (for example) between the disks of a large *O. marginalis*, a middle-sized *O. duplex*, and a small *O. complanata*, which even the trained eye of the Naturalist cannot distinguish without the assistance of a magnifying-glass.

To me, therefore, it appears that the doctrine of "natural selection" can give no account of either the origin or the perpetuation of those several types of Foraminiferal structure which form the ascending series that culminates in *Orbitolites complanata*. On the other hand, there seems traceable throughout that series a *plan* so definite and obvious, as to exclude the notion of "casual" or "aimless" variation. Between the simple spirally-coiled sarcodic cord of a young *Cornuspira*, and the discoidal body of an *Orbitolite*, with its thousands of sub-segments disposed with the most perfect symmetry, and connected together in most regular and uniform modes, who (in the absence of the intervening links) would have suspected any genetic relation—who would have ventured to construct a pedigree? And yet we find the gradations from the one to the other to be not only most complete, but often significant of further progress; many of the changes being such as seem to have no meaning except as anticipations of greater changes to come. Thus, the slight constrictions that show themselves in the first spiral coil of *O. tenuissima* (Plate 38, fig. 3) are what constitute the essential difference between the spire of *Cornuspira* and that of *Spiroloculina*; marking an imperfect septal division of the spire into chambers, which cannot be conceived to affect in any way the physiological condition of the contained animal, but which foreshadows the complete septal division that marks the assumption of the Peneropline stage. Again, the incipient widening-out of the body, previously to the formation of the first complete septum, prepares the way for that great lateral extension which characterises the next or Orbiculine stage; this extension being obviously related, on the one hand, to the division of the chamber-segments of the body into chamberletted sub-segments, and, on the other, to the extension of the zonal chambers round the "nucleus," so as to complete them into annuli, from which all subsequent increase shall take place on the cyclical plan.

In *O. marginalis*, the first spiral stage is abbreviated by the drawing-together (as it were) of the "spiroloculine" coil into a single Milioline turn of greater thickness; but the Orbiculine or second spiral stage is fully retained.

In *O. duplex,* the abbreviated Milioline centre is still retained, but the succeeding Orbiculine spiral is almost entirely dropped out, quickly giving place to the cyclical plan.

And in the typical *O. complanata* the Milioline centre is immediately surrounded by a complete annulus, so that nothing remains of the original spire save the one turn of the circumambient segment.

So, in the passage from the "simple" to the "complex" type, we have a remarkable anticipatory step in *O. duplex,* which can scarcely be supposed itself to derive any advantage from the substitution of a double for a single row of communications between the annuli, since *O. marginalis* flourishes equally well with its single row; but which forms, so to speak, a stepping-stone to a higher grade.

Everything in this history, then, shows *a well-marked progressive tendency along a definite line towards a highly specialised type of structure in the Calcareous fabric;* and this without any corresponding departure from the original homogeneity of the Animal body which forms that fabric. And as being, so far as I know, altogether unique in these peculiarities, I venture to offer this study of a humble protoplasmic organism, brought up from an ocean-depth of nearly two miles, to the consideration of those who believe with Sir JAMES PAGET, that "the highest laws of our [biological] science are expressed in the simplest terms in the lives of the lowest orders of Creation."

EXPLANATIONS OF PLATES.

PLATE 37.

Structure of Calcareous Disk of *Orbitolites tenuissima.*

Fig. 1. Surface of young disk, showing its eccentric spiroloculine "nucleus," giving origin to successive zones of orbiculine chamberlets, which gradually increase in breadth with the opening-out of the spire, until they extend completely round the nucleus; after which the successive additions are made on the cyclical plan, as concentric annuli. Magnified 25 diameters.

Fig. 2. A portion of three peripheral annuli, enlarged to 64 diameters, and partially laid open by the removal of the superficial lamella, so as to show the two annular septa, *aa*, *bb*, the chamberlets, *c*, separated by radial partitions, and the annular gallery, *d*, into which all the chamberlets open at their peripheral extremities.

Fig. 3. Vertical section of three annuli of the disk, taken in the radial direction, so as to traverse the chamberlets lengthways; *a*, *a*, junctions of two annuli, with

the annuli external to them; *b*, *b*, *b*, annular galleries traversing the septa between the chamberlets. At *a*, *a*, are seen the openings through which the sarcodic cords that occupy the annular galleries send radial extensions into the chamberlets of the succeeding annuli. Magnified 64 diameters.

Fig. 4. Internal aspect of a small portion of an annulus detached by fracture; showing the entrances to the chamberlets of that annulus through the septal plane. Magnified 64 diameters.

Fig. 5. External or peripheral aspect of a portion of a marginal annulus, showing the passages through its septal plane, as marginal pores elongated in the plane of the disk. Magnified 64 diameters.

Fig. 6. Portion of a disk, whose remainder, with the "nucleus," has been lost by injury previously to the formation of the last two annuli, which have extended themselves along the fractured margin, and into the nuclear space. Magnified 15 diameters.

Fig. 7. Incipient production of an entirely new disk, with regularly concentric annuli, from a fragment of the peripheral portion of an old one. Magnified 15 diameters.

PLATE 38.

Structure of Sarcodic Body and Calcareous Disk of *Orbitolites tenuissima.*

Fig. 1. Sarcodic body of the central portion of the disk; showing the primordial segment giving off the spiroloculine coil, the sixth turn of which, *a*, begins to open out into a peneropline form, afterwards becoming divided into rows of orbiculine sub-segments, which are connected together laterally by the continuity of the sarcodic body through the gallery at the outer end of each row, and radially by the stolon-processes that pass through the septal passages, from the gallery of the inner row into the chamberlets of the outer. Nuclear (?) corpuscles are seen irregularly distributed through the sarcodic substance. Magnified 75 diameters.

Fig. 2. Nuclear (?) bodies, as seen under a power of 450 diameters.

Fig. 3. Section of first-formed portion of the disk, laying open the primordial chamber, *a*, and the spiroloculine chambers, partially divided as at *b*, which coil round it. Magnified 125 diameters.

Fig. 4. Portion of the sarcodic body shown in fig. 1, enlarged to 125 diameters, to show the distribution of the nuclear (?) corpuscles:—*a*, expanded extremity of the last spiroloculine coil; *b*, *b*, *b′*, *b′*, portions of preceding coils, crowded with nuclear (?) corpuscles; *c*, orbiculine sub-segment, with five corpuscles; *d*, *d*, *d*, *d*, orbiculine sub-segments, each with one or with two corpuscles.

Fig. 5. Central portion of the calcareous disk, as seen by transmitted light;—*a*, expanded chamber formed by the termination of the spiroloculine coil, and

closed-in by a peneropline septum traversed by four passages; *b*, second chamber, divided by radial partition into orbiculine chamberlets; *c*, third chamber, not here separated from the second by a septum, and having only one radial partition; *d*, *d*, fourth chamber, having at *d′*, *d′*, lateral extensions which begin to enclose the spiroloculine coil; *e*, *e*, fifth chamber, with lateral extensions, *e′*, *e′*, proceeding still further backwards; these chambers, and those that succeed them, divided by radial partitions into orbiculine chamberlets. Magnified 75 diameters.

XV. *On the Affinities of* Thylacoleo.

By Professor Owen, *C.B., F.R.S., &c., Superintendent of the Natural History Department, British Museum.*

Received January 25,—Read February 1, 1883.

[Plates 39–41.]

In a former Paper on *Thylacoleo** was summed up what I then inferred from the fossil remains of the species "*carnifex*" which had reached me at that date, but acquiescence in those conclusions seemed, in the opinion of some contemporary Palæontologists, to require further evidence. I have, accordingly, omitted no opportunity of obtaining such, and the fossils so acquired form the subject of the present communication.

The locality which promised success in this quest was the limestone district of Wellington Valley, New South Wales, from one of the caves of which the first evidence of *Thylacoleo* had been obtained.†

And, here, I have to express my deep obligations to the Legislature of New South Wales for the Grants liberally voted for the further exploration of the caves and brecciated fissures of that district, and my acknowledgments of the friendly support given to my representations by the Premier, Sir Henry Parkes. The direction and supervision of these explorations were confided to the accomplished and able Curator of the Australian Museum, Sydney, Ed. V. Ramsay, F.L.S., to whom I am indebted for prompt transmission of the subjects of the present Paper, included in the results of his mission.

Dentition of the upper jaw of Thylacoleo carnifex.—Of the first or laniariform incisor the base only of the crown was the subject of fig. 1, 2, 3, Plate 11, p. 217, of the Paper above cited (1871). A detached crown of what I then inferred to have formed part of the homologous tooth is the subject of fig. 7 and 8 of Plate 11 (Paper, 1871). I have now the upper jaw with the entire dentition (Plate 39, fig. 1),

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* "On the Fossil Mammals of Australia": Part IV., Phil. Trans., 1871, p. 213.

† By the then Surveyor General of Australia, Colonel Sir Thomas Mitchell, C.B., discoverer of the fertile region which he named after his great Commander. See Appendix to his 'Three Expeditions into Eastern Australia,' 8vo., vol. ii., 1838.

of which it seems necessary to add to former figures the palatal view only for comparison with fig. 3 of Plate 11 (Paper of 1871).

The foremost tooth (Plate 39, fig. 1, *i* 1) shows an exserted crown, 1 inch 3 lines in length, curved with the convexity forward, subcompressed, conical, pointed, obtuse anteriorly, trenchant behind where the enamel forms a finely serrate ridge; it is in shape and size a tooth suggestive of powers of penetration and prehension. It is deeply inserted by a stout cement-clad fang, indicative of limited growth. The entire length of the tooth, following the curve, is 2½ inches.

The second tooth, (ib., ib., *i* 2) projects behind and partly mesiad of the base of the first, its crown is but 5 lines in length; other dimensions are shown in the figure. The working surface is bevelled off, before and behind, to a low ridge.

Next to this tooth and partially overlapping the hind and outer part of its crown is that of a larger trenchant tooth, *i* 3, 7 lines in longitudinal, 5 lines in transverse diameters, but barely exceeding the second tooth in height.

This is followed by a fourth, *p* 1, about the size of the second, *i* 2. Its crown is partly overlapped by the larger tooth, *i* 3; it is also subtrenchant lengthwise.

External to the hinder part of *p* 1, and about half the size, projects the crown of a tubercular tooth, *p* 2; it is immediately followed by a sixth tooth, *p* 3, of similar shape and size. This tooth is partly overlapped externally by the fore end of the great carnassial tooth, *p* 4.

The homologies indicated by the symbols of the five teeth crowded between *i* 1 and *p* 4 may be questioned, but that of the latter with the trenchant premolar in existing phytophagous Diprotodonts is plain. In plate 100, pp. 381–394 of the undercited work,* the eight chief modifications of the comparable Diprotodont dentition in existing Marsupials are described and figured. Of these the genera *Hypsiprymnus* and *Phascolarctos* offer the nearest approach to *Thylacoleo* in the proportions of antero-posterior to vertical extent of the crown of *p* 4. But the size of this tooth in those existing genera is much less relatively to the other teeth, especially to the bruising molars, four on each side of the upper, as of the lower jaw, which in them follow the trenchant premolar. In those genera, as in *Macropus*, *Petaurus* and *Phalangista*, three close-set incisors are lodged in the premaxillary; and, in *Macropus*, the third incisor is as much larger than the second, as is the third tooth (Plate 39, fig. 1, *i* 3) in the series of upper teeth in *Thylacoleo.*

In *Phascolarctos* and *Hypsiprymnus* a minute canine projects from the maxillo-premaxillary suture with a well-marked interval between the incisors in front and the trenchant premolar behind. In *Petaurus* and *Phalangista Cookii* two small premolars, *p* 3, *p* 2, precede the tooth *p* 4, which is in contact with the foremost of the four crushing molars.

With the five teeth, therefore, preceding the trenchant premolar, in the above-cited existing Diprotodonts may be homologized, in the aggregate, the five teeth between

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* 'Odontography,' 8vo., 1840.

the foremost tooth, *i* 1, and the carnassial, *p* 4, in *Thylacoleo*. But to which of them the term canine may be assigned is doubtful. Relative size might weigh with the largest of these intermediate teeth, *i* 3, but its trenchant character is repeated in the third incisor of the existing Kangaroos; it is not a cuspidate tooth as restored in outline at *c*, in Plate 14 of the Paper of 1871. I have nothing to alter or add to former descriptions of *p* 4 and *m* 1 (ib., ib.). In existing Diprotodonts the latter tooth has a broad tuberculate or ridged masticatory crown, and is followed by three similar molars, absent in *Thylacoleo*.

Of the extreme modification of the Diprotodont type for carnivorous work I hold the opinion expressed in the former Paper (1871), and have only to add that the retention of the seemingly functionless teeth, *i* 2–*p* 3, crowded together in the upper jaw, is significant of a principle underlying the adaptive explanation.

Dentition of the lower jaw of Thylacoleo.—In the portions of mandible of *Thyl. carnifex*, figured in Plates 12 and 13 of the Paper of 1871, the dental formula was inferred, as in that of the upper jaw, from sockets of teeth. I am now able to submit three views of the entire dentition of a fossil mandible (Plate 39, fig. 2; Plate 41, figs. 1 and 2) discovered in contiguity with the subject of fig. 1, Plate 39.

The foremost laniariform tooth, *i* 1, repeats the shape and size described (pp. 226, 227) and figured (Plate 13, figs. 4–7, Paper of 1871) from a cast transmitted to me in 1870, of a tooth in the Museum of Natural History, Sydney, which was obtained by Mr. KREFFT from "a breccia cavern, Wellington Valley"; and its characters might well condone the conjecture that it was a feline canine tooth.

Two small teeth (Plate 39, fig. 2, *p* 2, *p* 3) are wedged in between the foremost tooth, *i* 1, and the carnassial, *p* 4.

In *Phalangista vulpina* one such tooth follows the front incisor; in *Phal. Cookii* there are two; in *Petaurus* there are three denticles between the incisor, *i* 1, and the premolar, *p* 4. To two of such seemingly functionless teeth those marked *p* 2 and *p* 3, in Plate 39, fig. 2, and Plate 41, figs. 1 and 2, may be homologous. Their interest lies, as in their homotypes in the upper jaw, in the manifestation of a diprotodont dentition under its extreme functional modification in the great extinct Marsupial Carnivore.

Two sockets, indicated by *p* 2 and *p* 3, in the fossil figured in Plate 12, figs. 2 and 3, (1871), but which might have lodged the two roots of a single tooth, are now demonstrated to have held two small close-set teeth, of which the hindmost is hidden from outer view by the forepart of the carnassial, *p* 4. The foremost, *p* 2, immediately following *i* 1, is more than twice the size of *p* 3, and has a cuspidate crown (Plate 39, fig. 2); but it is a dwarf by the side of the laniariform tooth, *i* 1. To former descriptions of this tooth there seems nothing to add to the present demonstration of its place in the dental series.

The tooth, *m* 1, succeeding the carnassial, repeats in general character that described (p. 224) and figured (Plate 13, fig. 2, *m* 1, 1871) from a photograph. The difference in wear and the slight one in size may indicate the present fossil to have come from an older and somewhat (sexually ?) larger *Thylacoleo.* The apex of the anterior lobe has been worn off, and the lower hind lobe shows abrasion; but there is as little approach in relative size and conformation of crown in the present thylacolean *m* 1 to that tooth in the phytophagous Diprotodonts (Plate 41, figs. 3 and 4) as was indicated by the thylacolean subject which first came to my hands.

The tooth, *m* 2, as indicated by its socket in Plate 14 (1871), is of the same relative size, but with a more acuminate crown than is indicated by the conjectural outline there given.

The maxillary and mandibular fossils here described and figured add the entire dentition of *Thylacoleo carnifex* to the series of mammalian modifications of the dental system with which Comparative Anatomy is now enriched; and they afford sure grounds for physiological deductions as to the nature and habits of the extinct Marsupial.

Antibrachial bones of Thylacoleo.—Of the bones and portions of bone referable by size to this genus and discovered in the same cave with the jaws and teeth last described are those of the fore-arm (Plate 40), of which the ulna (fig. 4) lacks only two inches of its distal end, according to the proportions of that bone in the larger Felines: this comparison is made from its being associated with an entire radius (ib., fig. 1) of the same length as that of the Lion: both fossils form part of the same limb of the leonine Marsupial.

The articular surfaces in these fossils are as closely adapted to the divers movements of a fore-arm required for the application of the paw of a carnivore as in the Felines.

The proximal end of the radius is occupied by an articular surface (ib., fig. 2) in two continuous portions, the larger and terminal one (fig. 3, *a*) being moderately concave for adaption to the radial condyle of the humerus, the smaller convex surface (ib., *b*), bending down on the inner or ulnar border for articulation with the outer or radial concavity (fig. 5, *b*), continued from the larger and deeper trochlear surface (ib., *a*), near the humeral end of the ulna.

The proximal or humeral cavity of the radius is not circular as in herbivorous Marsupials, but is less oblong than in *Leo;* it is similarly continued upon a thick convex border, extended to form the surface *b*, fig. 3.

The shaft of the radius describes the same slight curve "radiad," or on the outer side, and maintains the same nearly uniform breadth to the distal expansion, as in the Lion. The radial, or free, border is similarly obtuse; the opposite border (*d*) is for the most part roughly trenchant. The process (fig. 1, *c*) for implantation of the *brachialis externus* muscle holds the same relative position to the proximal end as in *Leo.* At the expanded distal end the elongate tuberosity (*e*) above the produced radial or outer angle for the carpal joint shows the same oblique groove for the tendon of the *extensor carpi radialis.* The tuberosity (*f*) answering to that giving insertion to the tendon of the *supinator longus* in the Lion, is nearer the ulnar angle of the distal expansion, instead of rising midway between the terminal angles: and here I may remark that the Felines agree with the Marsupials in the presence of this muscle. In all minor modifications the leonine characters are closely repeated in the present fossil radius.

The same general correspondence of structure prevails in the ulna (fig. 4). The olecranon (*c*) offers the same development, with tuberous and ridged indications for adequate implantation of the powerful extensors of the fore-arm: it is relatively longer, but with rather less breadth than in *Leo*. The proximal articular surface has the same trochlear character (*a*) and passes uninterruptedly, but with a similar defining line, into the concave surface (fig. 5, *b*) for the corresponding side of the head of the radius. The adaption of these joints for free pronation and supination, as well as flexion and extension of the fore-paw, is as strongly marked in *Thylacoleo* as in the similarly-sized placental Felines. The few noticeable modifications indicate the derivative relation to the inplacental group.

In the existing Diprotodont Marsupials, whether climbers, flyers, burrowers or leapers, the bones of the fore-arm are freely articulated for both rotatory and flexile movements, a power which has been suggested to relate to manipulations of the nursing pouch; therefore, to be needed by the Kangaroo group as well as the rest. But in none, save the Wombat, does the proportion of the olecranon come near to that in *Thylacoleo*. The modifications of the radius and ulna for burrowing actions present differences, in number and kind, from those in *Thylacoleo* which need only a glance at the skeleton of *Phascolomys* to be appreciated.

In other Diprotodonts, especially the species *Macropus major*, for example, nearest in size to *Thylacoleo*, the olecranon is not continued beyond the trochlear cavity to the extent of that joint longitudinally: the shaft of the ulna is relatively longer and much more slender. The radius, with a circular proximal end, gives a smaller and less definite lateral articular surface to a concomitantly smaller external ('radial') offset from the trochlear articulation: the shaft of the radius is also relatively longer and more slender, and is proportionally less expanded at the distal end than in *Thylacoleo*.

Claw-phalanx of Thylacoleo.—Passing over evidences of carpal and metacarpal fossils, my remarks will, here, be limited to the characters of the terminal or ungual phalanges.

Fossil claw-bones are not few from the Thylacolean cavern, and these in shape and structure add instructive evidence of the nature of the quadruped to which, by their size, they may be attributed. By these characters a phalanx of a fore-paw may be selected: the talon which such bone supported and wielded was fully as large as that of the Lion, and indicates that it was sub-compressed, decurved and pointed.

The basal articular surface (fig. 7, *a*) shows a pair of vertical concavities divided by a mid-ridge: it occupies the breadth and greater part of the height of the base, leaving about an equal but small extent of non-articular tuberous insertional portion above and below the trochlear joint.

But the most instructive part of the phalanx is the extension from the upper and lateral borders of the base of a bony sheath (fig. 6, *b*), which overarches the proximal three-fourths of the claw-bearing part (*c*): this sheath is also continued from the sides of the broad, somewhat flattened under portion of the articular division of the phalanx, which terminates (fig. 8, *d*) where the compressed claw extends freely forward. The under surface (fig. 7) of the sheath-supporting division of the phalanx is perforated by a pair of canals, which transmitted the blood-vessels and nerves to the formative and reproductive organ of the talon.

I have not found any instance of such sheath claw-phalanx in the existing or extinct kinds of diprotodont Marsupials, other than *Thylacoleo.*

The placental Mammals which possess such claw-sheath are amongst the carnivorous, and, most conspicuously, the Feline species; also certain Edentates, more especially the great extinct Megatherioids.

The chief difference, in these placentals, is that the articular surface, in Felines, ends nearer the lower or palmar surface of the joint, which is overtopped by the prominence for insertion of the retractor tendon of the claw; while in Edentates the articular surface leaves a larger proportion of the under part of the base of the phalanx free and tuberous for insertion of the powerful muscles which deflect the claw.* In all Edentates the claw-bearing part of the sheathed-phalanx is relatively longer, thicker, and commonly less acute than in Felines.

In this comparison the ungual phalanx of *Thylacoleo* much more closely resembles that of *Felis Leo* or *Felis Tigris.*†

Mandible of Thylacoleo.—I finally submit a description and figures (Plate 41) of the fossil mandible with the Thylacoleon dentition, as the osseous evidence testifying most directly to the matter at issue.

Plate 41, fig. 1, gives the outside view of a mandible of *Thylacoleo carnifex*, in which a carnivorous modification of the dentition has been engrafted, as in the older extinct form *Plagiaulax* (ib., fig. 5), on a Marsupial and Diprotodont type.

In comparison with the mandible of the Koala (*Phascolarctos*, Plate 41, fig. 3) and Potoroo (*Hypsiprymnus*), which are selected by Professor FLOWER, F.R.S.,‡ as most nearly resembling that of *Thylacoleo*, may, first, be noted in *Thylacoleo*, fig. 1, the relative shortness of the dentigerous part of the mandible to its depth, especially at the fore end, and the outswelling wall of the socket of the great carnassial premolar (ib.,

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* 'Memoir on the Megatherium,' 4to., 1860, plate 25, fig. 1, iii.; and 'Memoir on the Mylodon,' 4to., 1840, plates 15 and 17.

† The following is CUVIER's description of this phalanx in the Feline family:—"La figure de cette phalange est celle d'un crochet fait des deux parties: l'une dirigée en avant, courbée, tranchante et pointue, reçoit l'ongle, dont la forme est à peu près la même: la base de cette première portion fait une espèce de capuchon osseux, dans lequel est reçue la base de l'ongle comme dans une gaîne." 'Leçons d'Anatomie comparée,' 8vo., ed. 1835, tome i, p. 434: "*Les dernières phalanges dans la famille des Chats.*" It is equally applicable to the subjects of figures 6 and 8 in Plate 40.

‡ "On the Affinities and probable Habits of the extinct Australian Marsupial, *Thylacoleo carnifex*, OWEN," Quarterly Journal of the Geological Society of London, March, 1868, vol. xxiv., p. 307.

fig. 1, *p* 4): next, the relative extent and depth of the depression (*d*) for the insertion of the temporal, or great biting, muscle. The outline of the termination of the coronoid process (*b*) is given on the authority of Mr. RAMSAY, from other and fragmentary portions of Thylacolean mandibles. The upper border of the process extends backward at a more open angle than in the Potoroo or Koala (ib., fig. 3), and to an extent beyond the articular condyle. The inflected angle of the jaw, *a*, characteristic of the *Marsupialia,* is so abrupt in *Thylacoleo* that only the tip is visible in an outer view (fig. 1, *a*), and this, with the angle, hardly rises above the level of the lower border of the ramus. In the Potoroo and Koala (ib., fig. 3) the angle (*a*) is relatively larger, rises higher, is less directly inflected, and the whole comes into the outer view of the ramus, as in other Marsupial Phytophagans (see fig. 4, *Phalangista;* also the Paper of 1871, p. 260, fig. 16, *Dendrolagus,* and fig. 18, *Bettongia*).

In the vegetarian Diprotodonts the ascending ramus of the jaw supporting the articular condyle, *c*, and coronoid process, *b*, is relatively narrower and loftier than in the sarcophagous kinds. (Compare fig. 3 (*Phascolarctos*) and fig. 4 (*Phalangista Cookii*) with figs. 1 and 2, Plate 41.) The latter, indeed, comes nearer than *Phascolarctos,* in retaining the small, seemingly, functionless denticles between *p* 4 and *i* 1.

But the crucial test is the shape and relative position of the articular condyle, *c*. In all existing Phytophagans it rises above the level of the molar series, in most considerably, as in the figures cited: in the existing carnivorous Marsupials, as *Thylacinus* (*loc. cit.*, p. 235, fig. 11) and *Dasyurus* (ib. ib., fig. 12) it does not rise above that level. In the shape of the condyle, the transverse much exceeding the antero-posterior diameter of its convex articular surface, and in its sessile attachment forbidding predication of a neck, carnivorous characters are seen in *Thylacoleo* which are wanting in all Marsupial Phytophagans. In all the characters in which the mandible of the Marsupial Lion agrees with that of the smaller pouched Carnivores, the resemblance is still closer to that bone in the type Carnivores of the placental series.

In the Paper of 1871 I was unable to oppose Professor FLOWER'S conjectural restoration of the mandible of *Thylacoleo* (*loc. cit.*, p. 307), according to the type of that bone in the Koala and Potoroo by other than a conjectural restoration (ib. ib., fig. 8; and Plate 12, fig. 1) in which the inferred relative position of the mandibular condyle is indicated by the curved line, *b*, in the text. The two restorations may now be tested by the figures from nature in Plate 41.

DESCRIPTION OF THE PLATES.

PLATE 39.

Fig. 1. Palatal or working surface of the teeth of the upper jaw, *Thylacoleo carnifex.*

Fig. 2. Corresponding surface of the teeth of the lower jaw, ibid.

Both figures are of the natural size.

PLATE 40.

Fig. 1. Radius, *Thylacoleo carnifex.*
Fig. 2. Proximal articular surface of the same bone.
Fig. 3. Proximal end of ditto.
Fig. 4. Ulna, *Thylacoleo carnifex.*
Fig. 5. Front view of proximal articular surface.
Fig. 6. Ungual phalanx, *Thylacoleo carnifex.*
Fig. 7. Under view of ditto.
Fig. 8. Claw-bearing portion, with one-half of the sheath removed.

(In ungual phalanges, probably of the hind jaw, the sheath is little, if at all, developed.)

PLATE 41.

Fig. 1. Outside view of mandible and teeth, *Thylacoleo carnifex.*
Fig. 2. Inside view of mandible and teeth, *Thylacoleo carnifex.*
Fig. 3. Outside view of mandible and teeth, with grinding surface of the molars, *Phascolarctos fuscus.*
Fig. 4. Inside view of mandibular ramus and teeth, *Phylangista Cookii.*
Fig. 5. Outside view of mandible and teeth, *Plagiaulax Becclesii.*

All the figures are of the natural size.

XVI. *On the Morphology and the Development of the Perithecium of* Meliola, *a Genus of Tropical Epiphyllous Fungi.*

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[Plates 42–44.]

During the course of recent researches into the nature of parasitic fungi, my attention was arrested for some time by several forms of epiphytal growths which occupy a sort of half-way position between the more pronounced endophyllous parasites, and those fungi which cannot be looked upon as requiring more than a hold-fast or shelter from their hosts. Among these are the *Meliolas*, a group established by Fries in 1825 to receive certain tropical fungi.* In the 'Annales des Sciences Naturelles' for 1851† is a memoir by Bornet on the species constituting the genus *Meliola*, in which the characters of these remarkable epiphytes are enumerated and examined, and a classification of the known forms proposed: this paper is a standing authority on the subject, and I shall have occasion to refer to it at intervals subsequently, partly to confirm some of Bornet's work, partly to add new observations and correct older views as to the nature or significance of various points.

The *Meliolas* are minute epiphyllous fungi, belonging to the *Pyrenomycetes*, the deep-brown or black mycelium of which appears as sooty patches on many and various plants in the tropics, and presents, roughly, a similar appearance to the masses of *Capnodium* or *Fumago* sometimes observed in European woods on the leaves of living plants.

Though, according to Bornet, several species must have been known under different names to the earlier botanists, the name established by Fries, and published in his revised system, was accepted by Montague and Léveillé and has persisted since: Berkeley, in England, has referred to the group in his 'Cryptogamic Botany,' and has described several species from the tropics in various papers.

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* 'Systema Orbis Vegetabilium.'

† Ser. iii., Bot., t. xvi., pp. 257, &c.

The habit of these fungi, and the fact that no true *Erysiphe* had been found among the collections of travellers in the tropics, led Fries[*] to insist strongly on the known or supposed analogies between the two genera, and, Bornet following Fries, the *Meliolas* have thus come to be regarded as replacing the *Erysiphes* in tropical countries—as, in fact, "representative species." Bornet added several facts to those already known concerning the coarser anatomy of the group; but even his excellent and systematic memoir left large gaps in the knowledge of important details, and practically nothing was known of their development or of the formation of their "fruit-bodies." These and other gaps I hope to fill up to at least a large extent in the present essay.

The appearance of this fungus as presented to the unaided eye, is much the same as that offered by *Asterina* and similar forms, and the reader may be referred to a recently published drawing of that fungus for a tolerably accurate idea of it.[†] The chief difference is that the black maculæ presented by well developed plants of *Meliola* are more decided and thicker than those of *Asterina;* all transitions are found, however, and, as with many other forms of epiphyllous *Pyrenomycetes,* it is impossible to detect exactly what fungus is present by a superficial examination.

The fungus *Meliola* may be conveniently considered as composed of a mycelium, which supports appendages and perithecia, and which arises from spores developed within the asci of the latter. Bornet considered the "receptacle" as an equally important and distinct constituent, but this is perhaps unnecessary since, as will be shown, the so-called "receptacle" can only be looked upon as a more or less accidental development, so to speak, depending and following upon the formation of the perithecium.

The mycelium, forming the chief part of the black patches found on the surface of the affected leaves, petioles &c., spreads in an irregularly stellate manner from a common centre or centres (see Plate 42, fig. 1). It is detached with comparative ease from the epidermis of the leaf, and bristles with fine, simple or branched, pointed appendages, of a black colour, which spring from the main hyphæ, and from around the subglobular perithecia which are irregularly scattered over the surface.

The main hyphæ constituting this vegetative part of the fungus, are irregularly radiating, sinuous or zigzag filaments, closely appressed to the epidermis of the leaf, &c., and composed of cylindrical joints or cells placed end to end, and branching at angles of about 45 degrees (*cf.* Plate 42, fig. 2, and Plate 43, fig. 5). Their stiff and even brittle walls are deeply coloured brown or black, and thus obscure the view of their contents: sections and reagents prove these to be finely grained protoplasm, with or without oily drops in the interior. The diameter of the hypha is equal throughout,

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[*] 'Sumna Vegetabilium,' p. 406: "Genus in foliis tropicis vulgatissimum ut *Erysiphes* in terris temperatis."

[†] Quar. Journ. Micr. Sc., October, 1882, plate 27, figs. 1 and 2. See also Bornet's beautiful figures, Ann. des. Sc. Nat., ser. iii., t. xvi., plates 21 and 22.

the apex being, as a rule, evenly rounded: the cross-septa dividing the hyphæ into cells are firmly marked, thick, and dark-coloured like the outer walls.

The main branches of the mycelium all present the same general characters described above. In many cases, however, the blunt apices of the larger hyphæ, instead of being evenly rounded, become curiously deformed by an accumulation of abnormal growths, of the nature of caps (see Plate 43, fig. 7) fitting roughly one over the other: these consist of swollen, more or less cuticularised thickenings of the cell-wall, with or without granular débris between the layers. They are evidently produced by irregularities in the forward growth of the hypha: in the moist intervals the growing apex, more delicate than the older portions of the hypha, creeps along the surface of the leaf in the normal manner; during recurrent dry and hot unfavourable periods, however, sudden hardening and stoppage of growth causes the accumulation of the caps. That unfavourable intervals in outward circumstances may produce such abnormalities is well seen in the *Saprolegniæ*, and I have in these observed the formation of successive shell-like caps of dense cellulose, more or less altered, and enclosing granular matter between the layers: the caps are coloured blue by solution of zinc-chloride and iodine, the granular débris yellow. These phenomena were by no means uncommon with the hyphæ of *Achlya* and *Saprolegnia* grown, in summer, too long in the same water; of course the pathological changes are produced by different causes in the two cases.*

Besides the main branches of the mycelium, certain small pyriform or flask-shaped outgrowths are given off at pretty regular intervals from the cylindrical cells of the larger hyphæ (see Plate 42, fig. 2, and Plate 43, figs. 3 and 4): in some cases each cell or joint gives off such a short branchlet from each side, in others from alternate sides. More rarely they are absent altogether. In all cases examined the short lateral branchlet arises as a simple bulging out of the lateral wall of the cell: as this proceeds, the bud (as it may be considered) swells out, and its cavity finally becomes separated from that of the parent branch by a firm septum. The long axis of the bud-like protuberance is very generally, though not always directed at an angle of nearly 45 degrees with that of the portion of the main hypha lying nearer the growing point (see Plate 42, fig. 2, &c.): its walls are similarly dark coloured and firm, and it contains fine grained protoplasm much as the cells of the main hyphæ. Morphologically considered, the short lateral outgrowths are undoubtedly of the nature of arrested branches.

In one form of *Meliola*, growing on the leaves of a species of Convolvulus, I have observed a second form of the lateral branchlet (see Plate 43, fig. 4), co-existing with the commoner pyriform type. In this case the outgrowth was longer, narrowed into a sort of neck, and presented the general shape of a Florence flask, seated with its bulged out body on the parent branch. In some specimens, each cell of the latter

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* There seems reason to believe that further investigation may throw light on this subject in connexion with the apposition of the cell-wall.

supported two opposite flask-shaped branchlets : in others only one, with or without a pyriform body in addition. Sometimes one or the other type occurred singly and irregularly (fig. 4).

The flask-shaped body is sometimes open at the apex, though I have never succeeded in observing anything emitted from the pore. These flask-shaped appendages recall to mind the peculiar bodies figured by WORONIN in another group of the *Pyrenomycetes* (*Sordaria*),[*] and although no grounds exist for correlating the two phenomena in detail, the fact is at least worth recording that the lateral pyriform bodies in *Meliola* are capable of subserving reproduction, as will be shown hereafter.

When the hyphæ or branchlets of this fungus are looked upon from above, and a strong light passes through from below, one often observes a minute, circular, bright spot, which appears to shine through the upper wall like a very small oil-drop; on reversing the object, so that the lower side of the hypha comes uppermost, this brilliant pore-like spot appears much more evident, and is clearly due to a thinning in the wall of the under side of the hypha, at a spot where no colouring matter is deposited in the cell-walls, and where the contained protoplasm is placed more nearly in connexion with the outside (see Plate 43, fig. 7, and Plate 44, figs. 21, 40).

BORNET apparently refers to these bright spots when he speaks of oily globules in the interior of the hyphæ,[†] though he may have been speaking of actual oil-drops developed in the dried specimens with which he chiefly worked. If BORNET's remarks refer to the bright spots here described, the facts of their appearing only on the lower wall, and not being altered by alcohol, &c., remain to be explained.

Taking all the facts into account, the view seems to recommend itself that these bright spots are the points of attachment of the hyphæ to the epidermis; if so, they are to be regarded as *haustoria* of a very rudimentary nature. The mycelium certainly is attached to the surface of the leaf, though but feebly, and it appears suggestive that alcohol specimens are more easily detached than fresh ones, possibly because the protoplasm becomes contracted and rendered brittle. No other anchoring bodies have been observed, and one notes that the position of these brilliant spots accords with that of the well-developed *haustoria* in *Asterina*,[‡] a genus of fungi at least allied to the *Meliolas*. These bright points are not always present, and in some cases seem to be normally absent. They are very generally formed at once on germination, appearing on the first short tubes put forth by the spore (fig. 40), a condition of things which may again be compared to what occurs in *Asterina*,[§] and also in *Erysiphe* and allied forms.[||] Still another point reminding us of *Asterina* and the *Erysipheæ* is the function of the pyriform branchlets; in some cases at

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[*] "Beiträge zur Morph. u. Phys. d. Pilze," DE BARY and WORONIN, ser. iii., plate 5.

[†] *Op. cit.*, p. 260, and plate 21, fig. 3.

[‡] See my description in Quar. Journ. Micr. Sc., October, 1882.

[§] BORNET, *op. cit.*, plate 28, fig. 5.

[||] DE BARY, "Beiträge zur Morph. u. Phys. d. Pilze," 1870, B. iii., plate 12, figs. 1 and 2.

least, they become detached, and act as vegetative reproductive organs or *conidia*, each putting forth bud-like processes which develop into new hyphæ. BORNET remarked the separation of these buds in *Meliola amphitricha*, and hints at their possibly serving as reproductive bodies much as the *Oidium* forms of *Erysipheæ:* since he worked with dried specimens, however, this question could not be decided.

BORNET remarks that the mycelium on the upper side of many leaves are sterile, while those below and protected from the direct rays of the sun alone support *perithecia:* this is certainly not true for the species examined by me, and, indeed, I cannot determine any difference between the upper and lower mycelia in this respect. Those on the upper surface seem quite as productive of spores, &c., as those below, and in many cases—*e.g.*, those *Meliolæ* so common on *Memecylon*—the mycelium vegetates almost exclusively on the upper surface, and is quite fertile there.

Besides the short pyriform and flask-shaped branchlets described above, the mycelium bears certain stiff, upright appendages of the nature of *setæ* (see Plate 42, figs. 1, 41, and Plate 43, fig. 8): these *setæ* spring from the cells of the hyphæ at various points in their course, and, from their position and mode of origin, are probably to be regarded, morphologically speaking, as lateral branchlets which become elongated in a direction more or less perpendicular to the plane of the leaf. Such a *seta* grows very rapidly and soon reaches its limit: the cylindrical cells composing it are relatively longer than those of the hyphæ, but resemble them in other respects (the walls being, perhaps, somewhat stiffer and more deeply coloured) and taper above, in the simple types, or become variously branched.

In most *Meliolas* the *setæ* are especially aggregated around the perithecia, forming circles of stiff radii springing from what BORNET terms the "receptacle": they are also developed, however, from various isolated points of the mycelium bearing no direct relation to the fruit-bodies.

The forms of the *setæ* vary from a simple, upright or curved filament, to structures branched like antlers, trifurcate, twisted, &c., at the tip (*cf.* Plate 43, fig. 8 and BORNET's figures*): BORNET has made use of these details in classifying the formal species, and although it is doubtful whether the more similar types are constant, there can be no objection to their use much in the same manner as the appendages of *Erysipheæ*, &c., are used to distinguish the forms of that group. BORNET regards the origin of the *setæ* at points on the mycelium as marking out places where new *perithecia* are to be developed: I cannot say that this idea is altogether a false one, but investigation of the development of the fruit-bodies seems to show that at least no necessary connexion exists between the two phenomena.

As to the function of the *setæ*, little or nothing can be stated. The earlier suggestions of SPRENGEL and FRIES (as quoted by BORNET) that they may be organs for the exit of the spores cannot be accepted: not only on the ground of the disproportion

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* *Loc. cit.*, plates 21 and 22, figs. 6, 15, 16, &c.

between the numbers, but also because the spores are too large to pass through the *setæ*, even supposing the cavity continuous and ending in an ostiolum, which is not in the case. I have often tried to discover *conidia* or other bodies in connexion with the *setæ*, but have been forced to the belief that they have no function whatever connected with spore-production. One is not now impressed with the necessity for assigning any special function to such structures: if the *setæ* are merely free-growing branches of the otherwise appressed, creeping mycelium, there is nothing surprising in the fact that some differences in form and consistency are correlated with their sub-äerial habit. This is at least no more remarkable than that the looser branches of an alga, like *Coleochæte*, should have a facies slightly differing from that of the cell series comprising the lower, creeping, appressed parts of the thallus.

The collection of *setæ* immediately around the "fruit-body" simply results, immediately, from the vigorous development of hyphæ which accompanies the later stages of formation of the perithecium: this mass of setigerous hyphæ, which seems comparable with the formation of *haustoria* and such-like organs in other fungi during the fruit development, was called the "*réceptacle*" by BORNET. As to a possible protective influence of the circles of *setæ*, the question must be left open until we know more of the conditions: in some cases, at any rate, the *setæ* do not arise until the perithecium is completely formed, and the spores nearly ripe.

The perithecium, when completely developed, is a globular or sub-globular body, consisting of a shining black or brown external case, the outer thick walls of which appear regularly embossed, and an internal mass composed of asci and spores, &c. The embossed pattern on the outer walls results from the thick-walled cells, of which it is composed, projecting at their free surfaces: where the cells join each other forming polygonal figures they do not so project.* What may be termed the base of the perithecium is sessile on the mycelium: at the opposite pole, or apex, is frequently a slight papilla, not obviously pierced by any pore. BORNET, noting this fact, imagines that the dehiscence takes place below, the whole upper part of the perithecium becoming broken away by a circular rupture at the base. In some forms, at least, the spores escape through an opening at the apex: how far this is general I do not know (Plate 42, fig. 43), but facts exist to render it probable that a minute and dilatable pore occurs in others.

Vertical sections of the mature perithecium show that within the firm, deep-coloured, external wall is a lining of softer cells, with swollen envelopes and of a more or less flattened form: this inner lining of the perithecium extends two or three cell-series deep, and is slightly yellow or pale-brown in colour (see Plate 44, fig. 33, and Plate 42, fig. 34). In the cavity thus enclosed are the groups of asci in various stages of development: these delicate, clavate sacs contain spores, or have emptied them into the semi-gelatinous, granular matrix around. With these preliminaries, I may pass on to consider and describe the development of the perithecium, as followed

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* See BORNET's figures, *loc. cit.*, plates 21 and 22.

step by step on a species of *Meliola* which I have investigated with no slight success:[*] this will be found to throw light on the morphology of these fungi from the best of sources—development—and aid in a more critical estimation of their proposed systematic position. After describing in detail the origin, mode of development and fate of the fruit and spores, I propose, therefore, to examine the relations of the *Meliolas* to *Erysiphe* and other fungi.

On examining portions of the epiphyllous mycelium bearing the short, pyriform, lateral branchlets so often referred to above, one frequently discovers specimens presenting the appearances depicted at figs. 9, 10, 11, &c. The simple pyriform body, after becoming more swollen, has suffered division into two portions or cells by a septum, usually vertical to the plane of the mycelium and leaf, and passing diagonally across the cavity with a slight curve, so as to abut on the outer walls at right angles, or nearly so. The originally unicellular protuberance becomes in this manner divided into two more or less unequal cells, and it will be shown in the sequel that these two cells have, from the first, each a different destiny in the formation of the fruit. For this reason I have indicated in the drawings, by shading, a difference which does not present itself in the natural object at this stage. The more apical cell, which is smaller and shaded darker in fig. 9 (Plate 42), may be indicated throughout by the letter A: it will be found that this cell produces the central ascogenous tissue of the young perithecium, while the other (which will be referred to as cell B) originates the outer portions of the case or perithecium wall.

Following close upon the preliminary division above described, a septum appears across the larger of the two cells, cutting the first-formed division wall at right angles, or nearly so: this is rapidly followed by another septum (Plate 43, fig. 10), and so the larger cell (B) becomes cut up into three. Following upon these, a number of further divisions in planes at right angles to the preceding are soon established (figs. 11 to 17), and at the same time, though much more slowly, one or two more division walls are formed in the cell A, thus cutting it up into a short series of about three cells (figs. 14, 15).

If the above description has been followed, it becomes clear that the division of the more rapidly growing cell, B, results in the production of a sheet of cells affixed, so to speak, to the few-celled mass resulting from the slow division of A: such being the case, and the sheet extending as new divisions are formed, the cells resulting from A become gradually enveloped more and more in those resulting from B. A comparison of the figs. 9 to 17 will facilitate matters here, and for convenience of description hereafter, and in consideration of its destiny, we may term the mass of cells produced from A the "*ascogenous core*"—or simply the core.

At a stage which may conveniently be considered the next one to fig. 11, the cells resulting from the division of B are observed to be extending as a curved layer over the "core" of cells formed by A. If, at this stage, the young fruit-body is cut off,

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[*] I must take this opportunity of thanking Professor DE BARY for kind suggestions with respect to this work.

and allowed to roll over in fluid under the microscope, the form and arrangement are found to be somewhat as sketched in fig. 12 (Plate 43), where *a* represents the view from below, *b* that from the side, and *c* an end elevation of the structure. The cell A, in fact, is becoming gradually enfolded by the layer of cells derived from B, a process which results, at a later period (Plate 43, fig. 17) in the almost complete tucking in of the "core" as the centre of a subglobular mass of cells.

As this process of "invagination by epiboly" (as it would be termed in the case of an embryo) goes on, the "core" has been more slowly cut up into cells—at first by walls perpendicular to its long axis, and then by septa in other planes at right angles—and the sub-globular body thus produced lies with the open part towards the epidermis.

After this period, two events occur: 1st, the cells of the "core," possessing very thin walls, acquire a different aspect from those of the outer shell; their finely granular protoplasm makes them appear denser and more opaque, shining through the latter until this becomes too thick to be transparent; 2nd, the open part of the growing perithecium becomes closed over, and the internal structures can no longer be made out without the aid of actual sections. At this point my observations have failed to decide which of two possible modes of growth take place: Is the covering in of the "core" completed simply by the extension and closing in of the edges of the outer layer; or are cells, cut off from the "core" below, intercalated, so to speak, into the open gap? One is led to expect by analogy that the former process takes place, but some events lead to the suspicion that such may not be the case.

At the stage corresponding to fig. 19 (Plate 43), the young perithecium appears almost opaque, very little light passing through the dark-coloured and thick outer walls; from below, however, the larger cells composing the "core" can be readily seen in the optical section, shining by means of their dense, fine-grained contents through the shell. In the next stages, the "core" can only be seen dimly through the outer envelope (Plate 44, fig. 20), even after treatment with reagents, or, as in figs. 21 and 22, after cutting or tearing off some of the outer cells.

Nothing but a fortunate vertical section through the young fruit at or near this stage will decide finally whether the lower side is covered in by the meeting of the outer shell edges, or by partial "delamination" from the lower side of the "core," and this I have not succeeded in obtaining. The thick, dark outer walls have now become so opaque, that optical sections fail to determine the course of events; and treatment with reagents does not afford evidence sufficiently satisfactory to decide the questions, since it seems impossible to remove the colouring matter. Potassic hydrate or weak acids do, it is true, render the structures a little more translucent after some time; but even the extreme resort of heating in weak chromic acid has only yielded partial results, and with this slight information on the point I have reluctantly been compelled to content myself for the present. A comparison of figs. 17 to 21 certainly suggests that the process of envelopment is completed by the outer layer of cells derived from the repeated and rapid division of the cell B, and this view may be recommended on the ground of analogies with the *Erysipheæ*, to be examined hereafter; but, while figs. 19 and 20 by no means decide the point, we shall find that in the perithecium of another species of *Meliola* (or an allied form) the construction almost certainly proceeds by continued cutting up and "delamination" of the results of division of one cell.

Be this as it may, the young perithecium now consists of the following parts:—A central "core" of delicate-walled, colourless or yellowish cells, very rich in finely granular protoplasm, and, surrounding this completely, a single layer of cells with thick, hard, dark-coloured walls (especially those on the exterior surface); the whole mass is attached to the hypha from which it originated by a very short pedicle or joint (see Plates 43, 44, figs. 19–24).

At a period slightly later than the above, the cells of the outer layer are becoming multiplied by tangential walls, and those of the inner core by radial and horizontal divisions: these processes go on for some time, until the whole perithecium is a complex of many small cells, the outer of which become firmer and darker-coloured, the inner delicate and full of fine-grained protoplasm as described.

No trace of the internal structure is, however, visible now from the outside. On isolating a perithecium at this stage—a matter of no slight difficulty, but practicable with a slender knife used under a low power of the microscope—it presents the forms shown in fig. 25 (Plate 44), on being rolled over. Above, the outer surface curves equally away from the centre, and the slightly projecting walls of the cells give it an appearance of being embossed (fig. 25, *x*.). From below (fig. 25, *y*), the object looks very different; the surface is much flattened and nearly circular, and from many of the cells are processes developing as hyphæ in all directions. These radiating processes creep close along the surface of the leaf, to which the fruit-body is also appressed, and no doubt serve to give a much firmer hold for the fruit; at first their thin walls are only of a pale brown hue, but rapidly acquire the thickness and deep colour of the fruit and mycelium. Seen from the side, the young perithecium presents the appearance sketched at fig. 25, *z*. It is these radiating anchoring hyphæ which form collectively what Bornet terms the "réceptacle," and from them, at a later period, the bristling *setæ* found around the mature fruit are developed.

From the stage just described the development of the fruit-body proceeds rapidly; but, since the objects now become of a more manageable size, I have been able, by actual sections through the perithecium embedded in spermaceti or gum, or, better still, in elder pith, to obtain some insight into the processes going on even in the centre of the mass of cells.

At stages just prior to the one last described, the central core of thin walled cells—which it will be remembered has been derived from continuous divisions of the cell A—is commencing to divide up by septa in several directions (figs. 23, 24), while the outer layers surrounding this—derived primitively from B, and, possibly, in part from

A—are divided more regularly by tangential walls, followed by radial ones at right angles as the area enlarges. As the increasing small and delicate cells of the core become formed more rapidly, a certain tendency at least to a regular arrangement can be recognised in the later stages, as shown in such sections as figs. 28 and 29, (Plate 44), and fig. 27 (Plate 42): this regularity becomes interfered with by the mutual pressure of the cells, and the outer ones, of which the walls are especially soft and swollen, become flattened and pulled in the tangential direction, and only marked by the very granular yellowish protoplasm in their diminishing cavities. In the central lower part of the core, vertical sections at this, and slightly later stages, show that certain cells, with very delicate outlines and finely granular refractive contents, maintain their larger size and upright arrangement, and are by these peculiarities well distinguished as a special group or tuft of cells (see Plate 44, fig. 28 and Plate 42, fig. 31). In oblique (Plate 44, fig. 29) and horizontal (fig. 30) sections passing through the lower third of the developing perithecium, they can also be readily distinguished by their special peculiarities, and no question can be entertained as to their significance in the formation of the essential parts of the fruit-body. This group of cells is the forerunner of the young asci, and may be termed the *Ascogonium.*

As development proceeds continuously, the outermost layers acquiring thicker and more deeply coloured walls, the above named group of upright cells become relatively larger, increasing slowly in number by a few divisions, while the diffluent, compressed cells between them and the outermost layers slowly give up their contents, and become reduced to mere granular streaks embedded in a jelly-like mass of swollen and fused cell-walls (see Plate 42, fig. 31). This process is exactly comparable to what takes place in the developing embryo-sac of certain phanerogams,* or of the pollen mother cells in the anther,† in so far as the larger cells clearly develop at the expense of material derived from those around.

The tuft of successful cells thus nourished is, in fact, the "ascogonium" of this fungus. At a slightly later stage than the one last figured, the space formerly occupied by the deliquescent remains of small cells is filled with an almost transparent semi-fluid mucus, in which a few bright granules are embedded; while the lower part of the perithecium contains a tuft of asci in various stages of development (see Plate 44, fig. 33), and which have evidently proceeded from the large cells of figs. 28 and 31 (Plates 44 and 42), which have devoured all, or nearly all, the smaller soft cells surrounding them.

Sections of perithecia at a stage between those shown in figs. 31 and 33 (Plates 42 and 44) have not been obtained, but enough evidence has been secured to enable me to conclude that the asci are the direct result of the transformation of the elongated upright cells of fig. 31 (Plate 42), which are nourished at the expense of the cells of the inner layers. Partly from the brittle nature of the outer walls, enclosing a space

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* *Cf.*, amongst others, STRASBURGER, 'Angiospermer und Gymnospermen.'

† *Cf.* STRASBURGER, 'Bau und Wachsthum der Zell-haute,' 1882.

filled with almost fluid contents, and partly from the extreme delicacy of the young asci, I have been unable to decide whether any distinct branching of the ascogenous cells precedes the formation of the definite asci : probably such is the case. We have now followed the development of the perithecium to the period when it may be considered ripe : a period of some duration, since the asci are continually and successively formed in the tuft for some time.

Fortunate sections of the perithecium wall at this stage have yielded the following information. In the centre of the apical wall, where a slight protuberance sometimes occurs, the cells of the inner wall are found to radiate towards a pale translucent spot or pore (see Plate 42, fig. 36), and although I have not been able to obtain sections exactly through this, and am therefore unable to affirm positively that it is an actual pore, there seems little doubt that this is at least the weak point through which the spores escape from the ripe perithecium, no doubt forced through by the swelling of the materials around. BORNET* believes that the perithecium opens by a circular rupture at the base : I have tried to confirm this, but failed, and am strongly persuaded that the apical spot figured is the point of exit for the spores. That a minute pore should escape observation from without is not remarkable : the reflection of the light from the black shining outer cells might easily obscure it. The general structure of these walls has already been described, and fig. 34, drawn from an extremely fortunate and very thin section, shows the details.

The very young ascus presents no features of importance to distinguish it from that of many other pyrenomycetous fungi. In its earliest state it is recognisable as a single thin-walled, club-shaped cell, tapering to a point at the lower attached end, and filled with finely granular, yellowish protoplasm (see Plate 44, fig. 37, *a.*) : sometimes a small pale, refractive nucleus-like point is seen in the protoplasm. As the young ascus grows longer, and its protoplasm increases in quantity, a fine, sharp division line makes its appearance somewhat oblique to the long axis of the whole (fig. 37, *c.*) ; this is soon followed by a second, similar longitudinal division, in a plane at right angles to the former (fig. 37, *d.*), and four well-defined masses are thus marked out. These, the young spores, do not include the whole of the protoplasm (fig. 37, *d.* and *f.*), but lie in a scanty matrix of granular matter, closely opposed face to face, and following the curve of the enlarging ascus wall on their outer walls.

As the four, almost fusiform young spores increase in size, and acquire more distinct membranous envelopes, they come to lie somewhat more loosely in the cavity of the ascus, and may cross one another in accommodation to the space at disposal. Then appear cross-septa (fig. 37, *e.*, *f.*), dividing the material of the spore into a number of compartments varying from three to five—or, in one case, a single septum only is formed—and vacuoles and granules appear in the hitherto almost homogeneous contents. As the spores ripen, their cross-septa become more firmly marked, their outer walls thicker, and, gradually brown or nearly black in colour, like the hyphæ of the

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* *Op. cit.*, p. 261.

developed mycelium; the side walls of the separate compartments also become bulged out slightly, giving the mature spore the appearance of a long oval body, constricted at intervals (see Plate 42, fig. 39). Very commonly one or two oily-looking drops accumulate in the compartments of the ripe spore.

Such is the typical mode of development of the perithecium, asci and spores. I have found no modifications of importance from a morphological point of view; it should be recorded, however, that the number of spores in the ascus varies from two to eight. Sometimes in the same perithecium one finds asci in which one, two, or three spores develop at the expense of their presumably weaker neighbours (fig. 38), in other cases the number two appears constant, only one complete division occurs in the ascus (fig. 38), while in one case to be referred to later, the asci normally produce eight two-chambered spores (Plate 42, fig. 43).

On germination, which may take place soon after their emission from the ripe perithecium, the spores seem to behave generally in the same manner; one or several simple protuberances emerge from any of the partitioned chambers (see Plate 42, fig. 40), and proceed to develop into a typical mycelium, often with a preliminary formation of the rudimentary haustoria referred to in an earlier part of this paper. This mycelium grows rapidly in moist weather, forming branches, *setæ* and fruit-bodies as before. In some seasons the leaves of various plants may be seen covered with hundreds of these young mycelia, which dry up when the atmosphere does, only to renew their growth with the rains.

Before passing on to the consideration of the pathological influence of these fungi, and of their systematic position, I will record a few details concerning a form of *Meliola* which varies somewhat from the typical cases hitherto considered; at any rate, it seems to differ more from the six or eight forms to which the above description refers, than they do among themselves.

The species to be examined has only been found on the leaves of *Pavetta indica*, and its mycelium forms more spreading and less defined patches on the leaves of that plant, than the easily recognisable sooty patches of the other *Meliolas*. The main features of its mycelium, &c., are shown in fig. 41, and differ chiefly in the delicate straggling hyphæ, with a paler brown colour and no trace of haustoria. The branching is very irregular, and somewhat like that of the form figured at fig. 3, but the short, lateral branchlets are not always ovoid, but often have sinuous, almost angular outlines, reminding one of the similar structures in *Asterina*, except that the latter bear distinct haustoria. The *setæ* are here quite simple, short, and not so hard and brittle as usual; they are also produced in smaller numbers than in the more typical species.

The greatest peculiarities, however, are offered by the fruit-bodies, or perithecia. Each of these arises as before by the successive dividing up of a short, lateral branchlet (Plate 44, fig. 42), with this difference, that the rapidly following septa permit no recognition of primitive cells destined to form the outer walls, ascogonium, &c., as before.

After a few radial, vertical, and horizontal walls have been formed, tangential septa (fig. 42, *d.*) make their appearance cutting out series of cells which are to form the outer walls, and which become firmer and more deeply coloured, from an inner cell mass which gives rise to the ascogonium much as before. Only a few asci are formed, in each of which arise eight small oval uniseptate spores, which acquire a pale brown colour as they ripen (fig. 42, *f.*, and Plate 42, fig. 43).

The mature perithecium is shaped like a pear or top, the broad end attached to the hypha by a short pedicel, the narrow free end, or apex, becoming thin and diffluent in order to allow of the escape of the spores (fig. 43). Very few or no *setæ* are formed around the perithecium, and these of the same simple type as those scattered on the mycelium (Plate 42, fig. 41). The whole structure of the fruit-body is, therefore, much simpler than that of the above described forms, and, from the semi-translucent characters of the thinner cell-walls, allows the main details to be made out by optical sections only. In some of the dark-coloured cells of freshly prepared specimens, a bluish tint is often observable; I have not seen this in any other similar form.

In no case have I succeeded in tracing a distinct alterative or destructive action of the *Meliolas* on the cells of leaves to which they are attached. In many instances, as, for example, thick leathery leaves like those of *Memecylon capitellatum*, &c., the haustoria seem to have no function beyond that of holdfasts; in others, such as *Pavetta*, *Triumfetta*, &c., attacked leaves certainly suffer from the presence of the fungus. Nevertheless, I cannot trace this to any direct action of the mycelium; the contents of the cells show no effects which can be regarded as due to the fungus mycelium directly. We must conclude, therefore, that where the life of the leaf is interfered with at all, it is indirectly; the dense crust of a well-developed *Meliola* no doubt obstructs the play of physiological functions in an obvious manner, by obscuring it from light, blocking up stomata, &c.

It is now possible to consider the question of the systematic position of these remarkable and interesting fungi. BORNET,* following FRIES and LÉVEILLÉ, places *Meliola* near the old group of *Sphaerias*, with especial reference to *Erysiphe*. I have already quoted the view of FRIES that the *Meliolas* may be considered tropical representatives of our *Erysipheæ*, and BERKELEY† takes the same position. These opinions appear to have been based simply on the resemblance in habit and the more obvious anatomical characters, and on the fact that no *Erysiphe* is known in the tropics.

The detail of structure, and especially of the development of the fruit-bodies above described, enable us to criticise these views from a somewhat firmer standpoint.

Apart from minor points of resemblance between *Meliola* and the typical *Erysipheæ*, such as the haustoria (not well developed in *Meliola*), the asci, &c., there can be no question as to certain points of agreement in the structure and development of the perithecia; nevertheless, the origin of the fruit-body in the two groups is not obviously similar, and at first sight the differences may seem greater than they really are.

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* *Op. cit.*, p. 266.

† Introd. to 'Crypt. Bot.,' p. 275.

In the typical simpler *Erysipheæ*, such as *Podosphaera*, as is well known from DE BARY's classical researches,[*] the "*carpogonium*" and "*antheridium*" arise each as a short lateral branch from separate hyphæ, at the point where two hyphæ cross: each becomes cut off by a septum, which is formed close to the parent hypha in the case of the pyriform "*carpogonium*," and about half way up the curved "*antheridium*" branch. The free end of the latter becomes closely applied to the top of the *carpogonium*, and fertilisation—possibly not complete in a physiological sense, however—is said to be complete. After this process numerous branchlets arise from the base of the *antheridium* filament (and also from the base of the *carpogonium*), grow rapidly and with numerous segments, and invest the *carpogonium*, which meanwhile begins to be (more slowly) cut up into cells.

In *Eurotium*[†] we have an essentially similar process, except in minute details, and the *antheridium* is a branch springing from the same hypha which bears the *carpogonium*, and arises just beneath the latter. Here, as before, the perithecium envelope is formed chiefly by the rapid overgrowth of cells derived from the *antheridium* branch. It is quite conceivable that a form allied to *Erysiphe* and *Eurotium*, &c., might have the unicellular *carpogonium* and *antheridium* arising quite in contact at their bases from the same branch.

If we now compare the above with the succession of events in the development of *Meliola*, the following points of analogy seem to me sound. The original pyriform branchlet—containing in itself, so to speak, the elements of the fruit-body—after the first division (Plate 42, fig. 9), may be considered as establishing morphologically an "*archecarpium*"[‡] and an *antheridial branch*—or the latter may be considered as containing in itself the *antheridium*, *plus* the elements of the perithecium wall.

If the cells A and B (fig. 9) became further developed, and diverged at their apices, we should have no difficulty in seeing these points of homology.

Thus much cannot but be allowed. The cell A resembles a true *archecarpium* in so far that it slowly produces the ascogonium and asci; the homology will not be weakened, but the contrary, if further research shows that part of the perithecium wall results from cells derived from A. The cell B so far acts as an *antheridium* branch in that it is closely applied to A, divides up more rapidly, and thus produces most—*perhaps all*—of the perithecium wall.

The above may possibly suggest some difficulties to those who have not followed the recent progress in our knowledge of sexual organs and their homologies in the lower fungi. It has of late been shown to be not improbable, but on the contrary very likely, that we should view the *Erysipheæ* as a group connecting the higher *Ascomy-*

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[*] "Beitr. z. Morph. u. Phys. d. Pilze," R. iii., 1870.

[†] *Cf.* DE BARY, *loc. cit.*

[‡] DE BARY, Beiträge IV., proposes to use this word as denoting that part of the body which becomes the ascus and pedicel in *Podosphaera*.

cetes, on the one hand, and the *Phycomycetes** (*Mucor, Peronosporeæ* and *Saprolegniæ*) on the other: the evolution of the latter group seems undoubtedly attended by a fusion of parts before separated—a withdrawal of the sexual organs, so to speak, into one another,—and De Bary has followed this out with marvellous skill and success in a number of forms passing from *Pythium,* through the *Peronosporeæ,* to certain *Saprolegniæ,* in which the male sexual organ ("*antheridium,*" "*pollinodium,*") is normally suppressed. Whether or not we suppose, with De Bary, that the *Erysipheæ* took origin from some *Peronospora*-like form, it seems reasonable to look upon *Meliola* and its immediate allies as a branch group derived from the *Erysiphe* stem, either from the ancestor of *Erysiphe* itself or from ancestors which gave rise to *Eurotium* and *Erysiphe,* and that this group has become developed in tropical lands along lines more or less parallel to those along which the European forms have proceeded in temperate climates, being, in fact—though not in the strictest sense perhaps—"representative species." Be this view entertained or rejected, I am strongly impressed with the necessity for further and closer investigation of the very remarkable group of fungi centering around or near the *Meliolæ,* since they will probably fill up yet more completely the gap—partially bridged over, it is true—between the lower and higher *Ascomycetes.*

Description of Plates.

Fig. 1. *Meliola* sp. with portion of epidermis of *Memecylon.* On the mycelium are *setæ,* branchlets, and fruit-bodies in various stages of development.—Zeiss D.

Fig. 2. Mycelium of another species of the same, found on the leaves of *Schutercia* (*Conv.*), with portion more highly magnified.—Gundl. $\frac{1}{3}$ and Zeiss D.

Fig. 3. Portion of mycelium of a species of *Meliola* on *Triumfetta* (*Tiliaceæ*).—Zeiss D.

Fig. 4. Portions of more advanced mycelium of fig. 2 more highly magnified, and showing various forms of lateral branchlets.—Zeiss J.

Fig. 5. Portions of mycelium on *Memecylon* showing mode of branching and young fruit-bodies.—Gundl. $\frac{1}{3}$ and Zeiss D.

Fig. 6. Vertical section through portion of mycelium where fruit-body is being formed. The section is not median.—Zeiss J.

Fig. 7. End of hypha with three cap-like thickenings and pore-like spot (*haustorium* ?) seen from below.—Zeiss J.

Fig. 8. Various forms of *setæ* in plan and elevation.—Zeiss D and J.

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* *Vide* De Bary, "Beitr. z. Morph. u. Phys. de Pilze," R. IV., 1881.

Fig. 9. End of hypha (with one cap-like thickening) bearing lateral pyriform branchlet which is to become a Perithecium. The first oblique septum has already appeared, the smaller cell (A) represents the ascogonium, &c., and is shaded darker; the larger one (B) will divide up more rapidly, and enclose the cell A and its progeny.

Figs. 10 and 11. Further stages in the development of the young Perithecium. The cell B is becoming divided.—Zeiss J.

Fig. 12. Young Perithecium seen from below (*a.*), from the side (*b.*), and from one end (*c*). In all, the dark cell is the one marked (A) in fig. 9; the remainder have resulted from the growth and division of the cell (B.)—Zeiss J.

Figs. 13 and 14. Slightly later stages seen from below. The cell (A) has become divided by a cross septum.—Zeiss J.

Figs. 15 and 16. Similar preparations seen from above and below.—Zeiss J.

Fig. 17. Somewhat more advanced Perithecium seen from the side. The cells resulting from the division of A ("ascogenous core") are seen through those formed by B, which are growing over them.—Zeiss J.

Fig. 18. Somewhat more advanced stage.—Zeiss J.

Fig. 19. Slightly later stage. The upper figure is seen from above, the lower from below: the latter shows the "ascogenous core."—Zeiss J.

Fig. 20. Similar preparations seen from above (lower figure) and below (upper figure.) —Zeiss J.

Figs. 21 and 22. Slightly advanced Perithecia cut by the razor. The "ascogenousc ore" is exposed at the cut parts.—Zeiss E.

Figs. 23 and 24. Similar preparations treated with chromic acid. The "ascogenous core" is seen enveloped by the cells forming the Perithecium-wall: all much swollen, and fig. 23 slightly crushed.—Zeiss J.

Fig. 25.—More advanced Perithecium seen from outside and above (*x.*), below (*y.*), and from the side (*z.*). The radiating hyphæ (réceptacle) spring from the external walls below.—Zeiss E.

Fig. 26. Portion of mycelium with young Perithecium seen from above and below.—Zeiss E.

Fig. 27. Somewhat older Perithecium. The razor has cut off one side obliquely. —Zeiss J. (camera).

Fig. 28. Vertical section through young Perithecium about this stage. The ascogenous cells in the middle are distinguished by their larger size and arrangement. —Zeiss J.

Fig. 29. Oblique (nearly horizontal and median) section through the same.—Zeiss J.

Fig. 30. Horizontal section above the base of same.—Zeiss J.

Fig. 31. Somewhat older stage in vertical section. The ascogenous cells in the centre are enlarging at the expense of those around.—Zeiss J.

Fig. 32. Portion of outer wall with disorganised cells lining it.—Zeiss J.

Fig. 33. Vertical section through nearly ripe Perithecium, showing asci and spores embedded in the gelatinous mass produced by the disorganisation of the unemployed cells.—Zeiss D.

Fig. 34. Portion of outer wall of latter in vertical section.—Zeiss J.

Fig. 35. Vertical—not median—section through ripe Perithecium (and portion of epidermis of host-plant), showing crowds of spores.

Fig. 36. Thin slice from top of similar Perithecium. A pore-like spot is seen in the centre of the radical marking.

Figs. 37, 38 and 39. Various stages in the development of the asci and spores.—Zeiss J.

Fig. 40. Germinating spores.—Zeiss D. and E.

Fig. 41. Portion of mycelium of a species of *Meliola* found on *Pavetta*, showing mycelium, *setæ*, and young Perithecia.—Zeiss J. and D.

Figs. 42 and 43. Development of Perithecia and extrusion of spores.

XVII. *On the Atomic Weight of Glucinum (Beryllium).*

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[PLATE 45.]

I. *Introductory.*

EVER since the discovery of glucinum by VAUQUELIN, in 1798, its atomic weight has been a disputed matter amongst chemists.. Its discoverer considered that its oxide was a monoide, an opinion which was however strongly opposed by BERZELIUS, who wrote the oxide Gl_2O_3 and the atomic weight 13·7 ($O=16$). The researches of AWDEJEW* and DEBRAY† again turned the scale in favour of the earlier view, and as an atomic weight of 9·2 suited the properties of the metal in the tables of periodicy constructed by MM. MENDELEEF and LOTHAR MEYER, this atomic weight has, up to quite recently, been generally accepted by chemists. As a welcome confirmation to this came a determination of the specific heat of the metal by Professor E. REYNOLDS,‡ who found that for its atomic heat to be near the normal number 6·0, its atomic weight must be 9·2 and not 13·8. Almost immediately afterwards a second determination of the specific heat was made by MM. NILSON and PETTERSON,§ who, however, obtained a result agreeing not with the lower atomic weight but with the higher.

The reasons for these conflicting opinions are to be found—first, in the anomalous position of glucinum among the elements; secondly, in the difficulties which surround the preparation of even small quantities of the free metal in a tolerably pure condition; and thirdly, in the fact that no volatile compound of glucinum is known of which the vapour density might be easily determined.

The constitution of the compounds of glucinum cannot be inferred from any physical or chemical similarities with analogous compounds of other metals. Its compounds most closely resemble those of magnesium and aluminium, but also differ from these in the most striking manner. None are isomorphous with any similar compounds of these

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* POGG. Ann., lvi., 101.

† Ann. Chim. and Phys. [3], xliv., 5.

‡ Phil. Mag. [5], iii., 38; Chem. News, xlii., 273.

§ Berl. Ber. xi., 381, 906.

two metals, if we except the result of EBELMEN,[*] who is said to have obtained the oxide isomorphous with alumina. The volatility of its chloride, its tendency to form basic compounds, its stable double fluorides with potassium and sodium, as well as the solubility of its hydrate in caustic soda or potash, all point to analogy with aluminium; while its carbonates, its double sulphate with potassium, and the greater simplicity of some of its double compounds when its oxide is written as a monoxide show similarity with magnesium.

A correct solution of this long-vexed question can therefore be only obtained either from the specific heat of the element or from the vapour density of some of its volatile compounds. Two determinations of the specific heat have been made, as referred to above, but with quite contradictory results; and, as far as I am aware, no determination of the vapour-density of glucinum chloride, nor of its compounds with ethyl and propyl, discovered by CAHOURS, has yet been made. The metal used by Mr. REYNOLDS in his experiments was prepared in a platinum crucible, and was apparently impure. His calorimeter, or as he prefers to call it, *atometer*, was "essentially a spirit thermometer with a test-tube sealed in the bulb." In the absence of any detailed description of his apparatus, and especially of the means adopted for its graduation, it cannot be definitely decided whether accurate results were possible or not. M. NILSON used iron vessels to prepare his metal and carefully determined its composition. It contained 13 per cent. of various impurities, which were allowed for in calculating the actual specific heat of the metal. The determinations were made with BUNSEN's accurate and delicate ice-calorimeter.

The researches detailed in this paper were undertaken to obtain, if possible, a purer metal than M. NILSON's, and to redetermine its specific heat. The vapour-density of volatile glucinum compounds I hope to refer to in a later paper.

II. *Extraction of glucina.*

Of the various methods recommended for the extraction of the earth from its commonest mineral beryl, I shall here only allude to two. The first consists in decomposing the finely-powdered mineral by fusion with potassium carbonate, disintegration of the fused mass with sulphuric acid, evaporation to dryness, to render the silica insoluble, concentration to separate most of alumina, as alum, and treatment with ammonium carbonate as in the second method. This process, otherwise a good one, is objectionable, because of the high temperature necessary to decompose the mineral, and because the fused mass is only very slowly attacked by the sulphuric acid. I have usually followed a different method, due to SCHEFFER,[†] which consists in decomposing the mineral by hydrofluoric acid. Briefly, the process is as follows:—The finely-powdered mineral is mixed with excess of powdered fluor spar and sulphuric acid

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[*] Ann. d. Ch. u. Pharm., lxxx., 211.

[†] Jahres. Ber. 1859, 139.

in a large leaden dish, which is furnished with a leaden lid, and is heated on a water-bath for two or three hours. The remaining sulphates are next heated in an iron crucible to expel the excess of acid and any traces of silica, then dissolved in warm water and the calcium sulphate filtered off. To the filtrate, sufficient potassium sulphate is added to form alum with the alumina present, and the solution evaporated down to crystallize. The mother-liquor, from which nearly the whole of the alumina has been thus removed, is then slowly dropped, best by a separating funnel, into a large excess of warm concentrated ammonium carbonate solution, with constant stirring. About ten times as much of the salt is necessary as the quantity of glucina supposed to be present. The whole is then placed in stoppered bottles, and allowed to stand for four or five days, after which no change appears to take place. At the end of this time the solution, which contains nearly the whole of the glucina, some alumina, and often not inconsiderable quantities of iron, is then filtered, and colourless sodium sulphide added as long as ferrous sulphide is thrown down. If the quantity of iron present produces only a coloration, but no precipitate, it is well to add a little ferric chloride, and then completely precipitate with sodium sulphide. This is the only method I know of, except fusion with acid potassium fluoride, which removes every trace of iron. In the filtered solution the ammonium carbonate is now to be decomposed by boiling, but if the concentrated solution is heated directly over the flame very violent bumping cannot be avoided. A better plan is either to dilute freely with water (to about three times the bulk), or else to warm for about two hours over a water-bath, and when most of the ammonium carbonate has been decomposed, then to boil over the flame. The final traces of the carbonate are removed by acidulating with hydrochloric acid, yielding a solution from which ammonia precipitates the hydrate. By repeating the process the glucina is obtained nearly pure. Absolutely pure it can only be prepared from the double potassium fluoride, K_3GlF_6, by fusion with acid potassium fluoride, and recrystallizing the double fluoride from hot water. The quantity of the acid fluoride required by theory (6 : 1) should be used, not only 2 : 1, as recommended by Dr. WOLCOTT GIBBS, who, I believe, first introduced the process. The purified double fluoride is then readily decomposed by the requisite quantity of concentrated sulphuric acid in a platinum dish, and the hydrate precipitated with ammonia in the usual manner.

A very good yield may be obtained from beryl by this hydrofluoric acid process, provided the mineral is sufficiently finely powdered. Fairly pure compounds of glucinum may, however, be now obtained from some continental manufacturing chemists, which only require once purifying with ammonium carbonate for all ordinary purposes.

III. *Separation of metallic glucinum.*

Metallic glucinum may be obtained either from its double fluoride with potassium or from its chloride, of which the latter is decidedly the most suitable compound.

The double fluoride, K_3GlF_6, melts at a low red-heat without decomposition.

In the fused state it readily conducts an electric current with the formation of metallic glucinum at the negative pole. The metal so obtained is, however, largely contaminated with impurities derived from the vessel in which the experiment is conducted, which is strongly attacked by the free fluorine. In one experiment with platinum electrodes—the positive electrode being a platinum crucible in which the salt was fused—the crucible lost two decigrammes in weight. The double fluoride may also be decomposed by fusion with sodium or potassium, but in this case also only a very impure product results, owing to corrosion of the vessels employed. It is possible that by using an *iron* crucible a purer metal might be obtained.

The metal used in the determination of its specific heat was prepared from its chloride, which compound was obtained in the usual way by heating a mixture of the oxide and sugar charcoal in dry chlorine. I have found it better to employ starch paste (made of pure wheaten-starch) in place of the oil which is commonly recommended for mixing the oxide and charcoal. If starch paste is used a much more compact mass is obtained after the subsequent glowing in charcoal powder, but which is still sufficiently porous to allow the chlorine to penetrate into the interior. It is advisable to employ a hard glazed porcelain tube for heating the mixture in the stream of chlorine, this being far less attacked by the chloride than hard glass. Absolute purity of the oxide used to prepare the chloride is not necessary, as, owing to the high temperature at which glucinum chloride condenses, any traces of aluminium or silicon which may be present are carried, with traces of the chloride, to the less heated portions. If, however, iron is present, it is not completely removed.

Glucinum chloride melts at about 600° (CARNELLY) to a brown liquid, of which the electric resistance is so great that it is apparently not decomposed even by a powerful current. On connecting the wires from a battery of forty quart BUNSEN cells in series with platinum electrodes dipping in fused glucinum chloride and with a galvanometer, no deflection of the needle of the instrument was observed. I can therefore confirm NILSON'S result that the chloride is practically a non-conductor of electricity. The fused or gaseous chloride is readily decomposed by sodium, but if vessels of platinum, porcelain, or glass are used the metal obtained is largely contaminated with impurities derived from the vessels. To overcome this difficulty DEBRAY* used lime. He constructed boats of a mixture of lime and alumina, placed sodium in the first and the chloride in the second, then introduced them into a tube of hard glass through which was passed a stream of dry hydrogen. The boat containing the sodium was first heated, then that containing the chloride, so that its vapour was carried by the stream of hydrogen over the fused sodium. The metallic glucinum so obtained was a compact, fibrous, crystalline mass, and was probably purer than any prepared in platinum vessels. NILSON and PETTERSON first proposed iron vessels for the reduction of the chloride. They heated equivalent quantities of the chloride and sodium in a massive iron cylinder and thus obtained a metal resembling DEBRAY'S, and comparatively pure.

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* *Loc. cit.*

I first attempted DEBRAY'S method, but owing to the difficulty of procuring compact boats of lime or of a mixture of lime and alumina of sufficient size, no good results were obtained. Nor did I get good results with NILSON'S process. I found considerable difficulty in excluding the oxygen of the air; the tubes could scarcely be unscrewed again when cold, as the sodium chloride had solidified in the thread, and when opened it was not easy to extract the metal without contamination from the outside scale. A modification of the two methods was therefore adopted, which answers admirably.

Iron boats were constructed of pieces of tubing partly filed away, and with end pieces screwed on. They were about 200 millims. long and 20 millims. in diameter. In order to prevent any contact of the reduced metal with the glass of the tubes, the boats were enclosed in slightly longer pieces of entire iron tubing, and in these cases were introduced into a wide tube of hard glass. As in DEBRAY'S method, the first boat contained about the requisite quantity of sodium, the second the glucinum chloride. When the tube had been filled with dry hydrogen, the boat containing the sodium was heated until the metal was fused, the heating was then continued backwards to the chloride, the vapour of which thus passed, mixed with the excess of hydrogen, over the molten sodium. After the somewhat violent reaction had ceased and the tube was again cold, the boat containing the reduced glucinum, together with the sodium chloride produced and the excess of sodium, was placed in a vessel of crude alcohol until hydrogen ceased to come off. It was then removed to water to dissolve out the sodium chloride. Compact crystalline masses of glucinum were thus obtained, always, however, mixed with more or less basic chloride and oxide, even when the experiment was most carefully conducted. The larger pieces of metal could be easily removed from the liquid and purified by washing with cold dilute caustic soda, which dissolves out any oxide, but is without action on the metal. The residue was then likewise treated with caustic soda, and preserved separately as being less pure. The iron boats and tubes were not attacked during the reaction, except that blisters were raised on the outer tubes which, unlike the boats themselves, were of steel. The metal was of a steel-grey colour not so white as aluminium, but considerably harder, and can be beaten into thin sheets under the hammer. Compressed in a steel mortar it yielded a compact disc of metal—0·7 gramme thus treated gave a disc 15 millims. in diameter and 3 millims. thick. Its specific gravity at 10° C. was found to be 1·84, or making allowance for the impurities present, 1·70. As it was suspected that the metal so compressed might still contain air, it was first boiled for half an hour in water before determining its specific gravity. All attempts to fuse the metal were without any good result. It remained apparently unaltered when heated to bright redness in the air, and probably became covered with a thin coating of the oxide. Heated under sodium chloride in a lime crucible by the oxyhydrogen flame imperfect fusion was obtained. When similarly heated and exposed to the air it burnt with a dazzling, bluish light. The metal dissolved readily in dilute acids leaving a minute trace of insoluble matter, quite imponderable. When 3 centigrammes were dissolved, the residue could not have been as much as 0·1 milligramme. To determine the amount of iron contained in it, 0·0341 gramme was dissolved in dilute sulphuric acid in a stream of carbonic acid, and then required 1·4 cc. of a potassium permanganate solution of which 1 cc. was equivalent to 0·32 milligramme iron, which gives the percentage of iron present to be 1·32. The oxide mixed with the metal was found by dissolving 0·0340 gramme of the metal in dilute hydrochloric and precipitating the hydrate with ammonia and weighing, after thorough washing, as oxide. The amount taken gave 0·0883 gramme oxide, corresponding to a percentage of 4·71. The composition of my metal was therefore :—

Gl	=	93·97
Gl_2O_3	=	4·71
Fe	=	1·32
SiO_2	=	traces
		100·00

That obtained by M. Nilson, the only other sample which has been analysed, had the composition :—

Gl	=	87·09
Gl_2O_3	=	9·84
Fe	=	2·08
SiO_2.	=	0·99
		100·00

I believe it is possible by using purer chloride to obtain a metal almost chemically pure by the method described above.

The position of glucinum in the electro-chemical series is a peculiar one, and deserves passing notice. In dilute acids (hydrochloric, nitric or sulphuric) glucinum is strongly electro-negative to magnesium, and feebly positive to aluminium. In caustic potash or caustic soda, the series is different, and glucinum is now negative to both metals, feebly to magnesium, strongly to aluminium ; while in ammonium carbonate the series is the same as for dilute acids. These results may be thus tabulated:—

(i.) Dilute acids and ammonium carbonate.

+Mg, Gl, Al−

(ii.) Caustic alkalies.

+Al, Mg, Gl−

IV. *Determination of specific heat.*

The most accurate method for determining the specific heat of substances of which small quantities only are available is undoubtedly that devised by BUNSEN, in which, as is well known, the quantity of ice at 0° which is melted by a given weight of the substance heated to a high temperature is measured by the diminished volume which the water produced occupies. Unless, however, the whole apparatus is kept exactly at the freezing-point, accurate results are not possible, and even under the most favourable conditions there is usually a mean error of nearly one per cent. BUNSEN originally immersed his instrument in a large box of pure freshly-fallen snow to preserve it accurately at 0°—a proceeding which has been somewhat simplified by SCHULLER and WARTHA, who coat the exterior vessel with a thick layer of ice, and work in ice-cold water. But even with this modification it is scarcely possible to use the apparatus in England, and especially on the west coast, where frost in the winter is the exception rather than the rule.

Several kinds of rough calorimeters based on the same principle have been proposed in which the heated substance produces an expansion in a liquid at a definite temperature. Among these are the *atometer* of Mr. REYNOLDS, which was used to determine the specific heat of his glucinum, and a similar form described by Professor BALFOUR STEWART.* No instrument of this description can, however, give accurate results unless most carefully shielded from external sources of heat and carefully calibrated, especially when a liquid expanding so irregularly as alcohol is used. As far as can be judged by the published results, no calibration has been attempted, and therefore the results obtained cannot be of much value.

A very simple form of apparatus has been devised by KOPP† who places the substances in a glass tube with naphtha, and heats in a bath of mercury. Considerable dexterity must be requisite to remove the heated substance from the bath of mercury to the calorimeter in exactly equal times, and a further objection is the small range of temperature through which the substances can be heated. His results, too, do not agree so closely with one another as those obtained by BUNSEN's or REGNAULT's methods, especially when only small quantities are used.

To overcome these various objections a modification of REGNAULT's method of mixtures has been adopted, which with small quantities of the substances (0·7 to 7 grammes) gives results with a mean error of one per cent. or less, and which can be easily worked by one person without assistance. The errors in REGNAULT's apparatus have been pointed out by NEUMANN and PAPE.‡ They showed how a cooling effect must be produced by an upward current of cold air when the heating arrangement was opened at the top, and explained that, in many cases, the water equivalent of the

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* Proceed. of the Phys. Soc., iv., 52, 342.

† Phil. Trans., 1865, 71.

‡ POGG. Ann., cxx., 337, 579.

casing of brass wire gauze used to contain the substances was greater than that of the substances themselves. As far as possible these errors have been avoided in the following modification of REGNAULT's method. The apparatus consists of two parts—that for raising the substance to a high temperature, and the calorimeter proper.

The *heater* is an annular brass vessel of 32 millims. internal and 64 millims. external diameter, and is 115 millims. high (Plate 45, fig. A.). Its lid screws on firmly and is made steam-tight by indiarubber packing, freed from sulphur by previous boiling in caustic soda. Three tubes pass through the lid of the heater, the central one carrying a thermometer (T_1), which gives the temperature of the interior air bath, while the other two enclose platinum wires connected with the arrangement for supporting the substance and for allowing it to fall into the calorimeter at the correct instant. All connexions in the lid are made as air-tight as possible to prevent any upward current of cold air into the heater. The substance is supported in a small platinum capsule (B) attached by a hinge to one of the platinum wires referred to above, and this capsule is held in a horizontal position by the second platinum wire, the end of which is bent at right angles to its length. To the upper end of this platinum wire is fixed a strip of wood, so that the wire can be easily turned through a small angle, the catch released, and the substance allowed to fall into the calorimeter. The heater is completely covered by a double coating of thick baize, and is placed in a wooden box (not shown in the figure). In the bottom of this wooden box is a small slider (C) which is connected with an electro-magnet and can be opened or closed at will. Two tubes (D, D) connect the annular space of the heater with a boiler containing water, placed on the other side of a tin screen, and with the air. The whole heating arrangement stands on a larger box (E) open at one side and with a long base board, and this again is supported on a suitable wooden stand.

The *calorimeter* itself (F) is a small thin platinum vessel of 50 millims. in diameter and 60 millims. in height, and containing therefore when full about 100 cc. It is furnished with an agitator of two discs of thin perforated platinum, soldered with gold to two thin platinum wires, one of which terminates about 15 millims. below the lid of the brass casing in a hook. To this hook a silken string (G) is attached, which passes over pulleys and is kept stretched by a small weight. The two discs of the agitator have perforations at one side for the bent thermometer (T_2), which gives the temperature of any liquid contained in the calorimeter. The advantages of using platinum for calorimeters have been pointed out by BERTHELOT. It always remains bright and of constant weight, and from its low specific heat corresponds to a very small equivalent of water. The weight of my calorimeter and agitator is 74·40 grammes, and its water equivalent 2·41 grammes, but it might have been made very much thinner. The calorimeter stands on three boxwood cones in a brass casing, which is fixed by three pieces of cork to a sliding board. This brass casing is 85 millims. high and 77 millims. in diameter, and its movable lid is furnished with a small slider which can be opened or closed by two iron armatures working in the two solenoids (H and H) placed one on each side. The wires from those solenoids terminate in four points of German silver at the end of the board (only one of which, J, is shown in the figure), which when the calorimeter is drawn up into its place under the heater come into contact with four springs of the same metal (K) connected with the battery and a switch. The board on which the calorimeter and its appurtenances are fixed can be moved by strings from the position shown in the figure to that exactly under the heater and back again.*

To obtain accurate readings of the different temperatures, two standard thermometers were first constructed, with a millimeter scale, and calibrated by GAY-LUSSAC'S method, using Mr. F. D. BROWN'S excellent little instrument.† These two read between about 5° and 105° C., and 1 millim. is about equivalent to 0°·2 C. They were compared with one another for every half degree between 8° and 20° C., in a large bucket of water, kept suitably agitated, and in only one instance differed by more than 0°·02 from one another. The bent thermometer (T_2) was more open, and 1 millim. was about equal to 0°·1 C. It was very carefully compared, by two series of experiments, with the two standard thermometers together, and the mean temperature taken when they differed from one another. The thermometer T_1, used for the heater, read accurately to 0°·05 C., was corrected for the boiling point, but not calibrated; the correction for the exposed thread was determined experimentally, and was found to be 0°·03 less than given by the usual formula :—

$$C = m\,(T - t)\,n.$$

The water-equivalent of the thermometer T_2 was calculated, from the weight of mercury which it contained, and the weight of glass immersed, to be 0·5 gramme, which agreed with experimental results.

Readings of the temperature to which the substance was heated are thus correct to at least 0°·1 C., and of the calorimeter to 0°·01 C.

The liquid used in the calorimeter is French turpentine, purified and redistilled. Taking the specific heat of this liquid as 0·4, and its specific gravity as 0·87, about 28 small calories are requisite to raise the temperature of 80cc. through one degree, and only a small weight of the substance is required. Thus, for silver, the quantity necessary would be $\frac{28}{80 \times 0\cdot057} = 6$ grammes (circâ), and for aluminium $\frac{28}{80 \times 0\cdot2} = 1\cdot7$ gramme (circâ) for a fall in temperature of 80° : much smaller weights than are required for any other accurate method, except BUNSEN'S. With a temperature difference of 1°·00, and reading to 0°·01, the mean error cannot well be smaller than 1 per cent., and with the instrument in its present form, this is, in fact, the limit of error. More accurate results might be possible with a still smaller calorimeter, and a still more open thermometer.

One possible objection to the use of turpentine was loss by evaporation during the

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* A battery-power of four quart BUNSEN or GROVES' cells is necessary. More convenient than this is a bichromate battery so arranged that the zincs can be lowered into the liquid when required.

† Phil. Mag. [5], xiv., 57.

experiment; this, however, was found to be too small to affect the results to any appreciable extent. The loss between two experiments, even after removing the substance, never exceeded 0·1 gramme, and a difference of 0·05 gramme would only produce a much smaller error than a difference of 0°·01 in reading the bent thermometer. In order to still further test the evaporation, the calorimeter was left open, exposed to the air of a warm room for 16 hours, and then only lost 0·26 gramme. Provided, therefore, that the agitator or substance does not rise above the level of the liquid, the loss by evaporation during an experiment can be very safely neglected.

A series of blank experiments were next made to determine whether the calorimeter gained heat during the time that it was open to receive the substance. It was found that the shortest time possible in which the whole series of operations could be performed—the calorimeter run under the heater, the substance introduced, and the calorimeter returned to its original position—was five seconds, and during this time no change could be noticed in the thermometer T_2. If, therefore, the operations follow one another promptly the gain of heat to the calorimeter may be safely said to be within the unavoidable errors of reading. All readings were of course made through a telescope.[*]

The following is the course usually adopted in the experiments. The substance, weighed to milligrammes, is introduced into the heater from below, this part of the apparatus being removed from its stand for the purpose. The heater is then returned to its position, the wooden slider adjusted, and the two tubes connected with the boiler and waste pipe respectively. During the time that the substance is heating, about 70 grammes of turpentine are poured into the calorimeter and weighed to centigrammes. The calorimeter is then placed in its brass casing, the string attached to the agitator, the thermometer and electro-magnets adjusted, and then a large plate of glass placed in front of the whole apparatus. At the end of about an hour's *vigorous* boiling the thermometer in the heater becomes stationary, starting with all cold, or in about three-quarters of an hour if the instrument has been previously used. This maximum temperature is either higher or lower than the boiling-point, according to the size of the waste pipe. In the experiments already made the waste pipe was generally small and the maximum temperature slightly higher than the boiling-point. Longer heating appears unnecessary, owing probably to the small quantities used. As soon as thermometer T_1 is constant, continuous readings of it and the other thermometer (T_2) are made at regular intervals of one minute. Thermometer T_1 will be quite constant, and if the liquid in the calorimeter is at a temperature within 1° of that of the air, the other thermometer (T_2) should not oscillate through more than 0°·01 in five minutes. At the end of a given minute T_2 is read for the last time, giving the initial temperature of the calorimeter (t), while T_1, giving the temperature to which the substance was heated (T), has been read just before. The substance is

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[*] I should state that in using large weights, as with the 10 grammes of silver, there was sometimes a little splashing, though never exceeding 0·05 gramme.

now introduced, the calorimeter returned to its original position, the agitator worked a few times and a reading made at the end of the half minute. Renewed agitations and readings are then made for every half minute until the maximum temperature (θ) is reached. The time to reach this maximum for metallic substances, with which I have as yet only experimented, is about a half to one minute.

Although it is not intended to use the calorimeter with water, as the temperature difference would be too small for the small quantities employed, three experiments were made with pure silver, and two with commercial aluminium (re-fused under sodium chloride) to test the apparatus. The following results were obtained, in which T, t and θ have the meanings given above, t' is the temperature of the air, W the weight of water, together with the water equivalent of the calorimeter, agitator and thermometer (2·91 grammes), w the weight of the substance employed, and s the required specific heat.

Silver in water.

I. $W=84{\cdot}25$, $w=10{\cdot}205$, $T=101^\circ{\cdot}9$, $t=11^\circ{\cdot}09$, $\theta=11^\circ{\cdot}71$, $t'=10^\circ{\cdot}9$.

$$s=\frac{W(\theta-t)}{w(T-\theta)}=0{\cdot}05677.$$

II. $W=84{\cdot}03$, $w=10{\cdot}205$, $T=101^\circ{\cdot}9$, $t=11^\circ{\cdot}08$, $\theta=11^\circ{\cdot}69$, $t'=10^\circ{\cdot}7$.

$s=0{\cdot}05568$.

III. $W=84{\cdot}01$, $w=10{\cdot}205$, $T=101^\circ{\cdot}9$, $t=10^\circ{\cdot}86$, $\theta=11^\circ{\cdot}47$, $t'=10^\circ{\cdot}4$.

$s=0{\cdot}05553$.

Mean specific heat $=0{\cdot}05600$; mean error $=0{\cdot}0003=0{\cdot}6$ per cent.

Aluminium in water.[*]

I. $W=85{\cdot}71$, $w=3{\cdot}502$, $T=98^\circ{\cdot}6$, $t=7^\circ{\cdot}59$, $\theta=8^\circ{\cdot}42$, $t'=8^\circ{\cdot}0$.

$s=0{\cdot}2253$.

II. $W=85{\cdot}25$, $w=3{\cdot}502$, $T=98^\circ{\cdot}5$, $t=8^\circ{\cdot}56$, $\theta=9^\circ{\cdot}37$, $t'=8^\circ{\cdot}5$.

$s=0{\cdot}2212$.

Mean specific heat $=0{\cdot}2232$; mean error $=0{\cdot}002=1$ per cent.

It will thus be evident that results comparable in accuracy with the best determinations can be obtained with the apparatus, and, I need not add, working entirely without assistance. I intend shortly to make a series of more extended experiments,

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[*] Using a wider exit tube.

and if possible to modify the apparatus so that it can be used for higher temperatures than 100°.

The specific heat of the sample of turpentine was next determined, with results as follows, T, t, θ, and t' having the same significance as before, W being the weight of turpentine used, w the weight of silver, s its specific heat (taken as 0·057), and s' the required specific heat of the turpentine.

Silver in turpentine.

I. $W=75{\cdot}30,\ w=10{\cdot}205,\ T=102^\circ{\cdot}2,\ t=10^\circ{\cdot}98,\ \theta=12^\circ{\cdot}47,\ t'=12^\circ{\cdot}8.$

$$s'=\frac{ws(T-\theta)-2{\cdot}91(\theta-t)}{W(\theta-t)}=0{\cdot}4265.$$

II. $W=75{\cdot}19,\ w=10{\cdot}205,\ T=102^\circ{\cdot}5,\ t=13^\circ{\cdot}32,\ \theta=14^\circ{\cdot}81,\ t'=14^\circ{\cdot}5.$

$$s'=0{\cdot}4165.$$

III. $W=74{\cdot}74,\ w=10{\cdot}205,\ T=102^\circ{\cdot}4,\ t=14^\circ{\cdot}56,\ \theta=16^\circ{\cdot}00,\ t'=15^\circ{\cdot}0.$

$$s'=0{\cdot}4280.$$

Mean specific heat $=0{\cdot}4236$; mean error = 0·0044 = 1 per cent.

The result obtained by Regnault for turpentine at 10° was 0·4278, and by Hirn 0·4241.

Three determinations of the specific heat of the compressed disc of glucinum referred to above were then made, yielding the following closely concordant results, in which the letters have the same significance as before, except that W is the weight of the turpentine and W′ its water equivalent + the water equivalent of the calorimeter, &c.

Glucinum in turpentine.

I. $W=72{\cdot}93,\ W'=33{\cdot}77,\ w=0{\cdot}6575,\ T=102^\circ{\cdot}1,\ t=11^\circ{\cdot}09,\ \theta=11^\circ{\cdot}85,\ t'=11^\circ{\cdot}1.$

$$s=\frac{W'(\theta-t)}{w(T-\theta)}=0{\cdot}4326.$$

II. $W=72{\cdot}67,\ W'=33{\cdot}66,\ w=0{\cdot}6568,\ T=102^\circ{\cdot}2,\ t=10^\circ{\cdot}08,\ \theta=10^\circ{\cdot}86,\ t'=9^\circ{\cdot}9.$

$$s=0{\cdot}4264.$$

III. $W=75{\cdot}12,\ W'=34{\cdot}69,\ w=0{\cdot}6565,\ T=102^\circ{\cdot}3,\ t=14^\circ{\cdot}26,\ \theta=14^\circ{\cdot}98,\ t'=14^\circ{\cdot}3.$

$$s=0{\cdot}4357.$$

Mean specific heat of glucinum $=0{\cdot}4316$; mean error = 0·0035 = 0·9 per cent.

Making allowances for the impurities which the metal contained, the true specific heat (s') of the pure metal would be :—

$$s'=\frac{0\cdot4316\times100-1\cdot32\times0\cdot114-4\cdot71\times0\cdot247}{93\cdot97}=0\cdot4453,$$

taking the specific heat of iron as 0·114, and that of glucina as 0·247. Multiplying this number by the atomic weight of glucinum when its oxide is a sesquioxide, *i.e.*, 13·65, the atomic heat becomes 6·08, proving conclusively, if DULONG and PETIT'S law is true for this metal as it is for all others, that this is the true atomic weight, and not two-thirds of this number, or 9·1.

The specific heat, as above determined, is considerably higher than that found by NILSON (0·4079), and my results might be too great for two reasons. The compressed metal was somewhat porous, and heat might have been produced by the absorption of the turpentine in its pores, and there might be a further error from hygroscopic water. The former supposition could be only decided by enclosing the metal in some impervious envelope, which has not yet been done; the latter possible error was as far as practicable avoided by drying the glucinum for two hours at 150° before each experiment. But, even supposing the above results to be erroneous to the extent of 10 per cent., it is still certain that the specific heat of the metal is nothing near 0·6, as it ought to be if the atomic weight were 9·1.

The result is unfortunate for the periodic law, and is the first serious rebuff which this useful generalisation of facts has received. Glucinum, with the atomic weight of 9·1, finds its natural position as the first element of the magnesium group; but if its atomic weight is 13·65, it falls between carbon and nitrogen, and entirely contradicts the first principles upon which the tables have been constructed. With an atomic weight of less than 12 it might be easily introduced, and it is possible that when absolutely pure such a result might be obtained. NILSON has recently made some determinations with the sulphate which reduced the atomic weight from the number previously given 13·8 to 13·65, but it does not seem quite certain that his compounds were entirely free from aluminium, which would naturally increase the atomic weight. I hope shortly to re-determine this with the purest material it is possible to obtain.

XVIII. *On the Changes which take place in the Deviations of the Standard Compass in the Iron Armour-plated, Iron, and Composite-built Ships of the Royal Navy, on a considerable change of Magnetic Latitude.*

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Communicated by Sir Frederick J. O. Evans, *K.C.B., Hydrographer of the Admiralty.*

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The period comprised between the years 1855–68 was one of active research into the magnetic character of the armour-plated and other ships of the Royal Navy and the iron ships of the Mercantile Navy.

It will be remembered that the Transactions of the Royal Society are rich in contributions to this important and interesting subject; important in a practical sense to the navigator, and of great interest as a subject of intelligent inquiry.

Among these contributions was a paper read before the Royal Society in March, 1865, "On the Magnetic Character of the Armour-plated ships of the Royal Navy, and on the effect on the compass of particular arrangements of Iron in a ship," by Frederick John Evans, Esq., Staff Commander, R.N., F.R.S., and Archibald Smith, Esq., M.A., F.R.S. This paper contained the earliest published results of the system of observation and analysis of the deviations of the compass in the ships of the Royal Navy, which, established in 1861, has been carried out to the present day.

These results showed the magnetic character of the several ships named, from the time of launching until fully equipped and at sea, and also an analysis of the semicircular deviation of those ships which had made short voyages abroad. But the change of magnetic latitude through which the ships passed was so small, and the alternative of heeling the ships in one latitude so difficult, that the authors of the paper were unable to ascertain any but approximate values of the proportions of hard and soft iron affecting their compasses.

The authors write: "The determination of the proportion of the semicircular deviation, or rather of B, which arises from vertical induction in soft iron, and that which arises from the permanent or sub-permanent magnetism of hard iron, is a matter of great interest. Theoretically it may be determined in two modes, either by observing the deviation in two different magnetic latitudes, or by observing the deviation with the ship upright and heeled over. Unfortunately there is a great want of observations under these circumstances."

During the last fifteen years long voyages into high southern magnetic inclination or dip, have been made in every class of ship in the Royal Navy, except Turret-ships, and according to the established system of the Admiralty Compass Department, the observed deviations of their compasses made in all latitudes have been analysed to obtain the values of their coefficients as shown in Table III. of this paper. From these coefficients, the constants of the hard and soft iron producing semicircular deviation at the Standard Compass positions have been computed.

A knowledge of these Constants not only provides a means of predicting for the particular ship examined the probable changes which will take place in her deviation in all parts of the navigable world, but also the power of doing the same for other ships of the same class.

Such being the case, it is thought that the Royal Society will be interested in receiving a paper treating of these subjects, commencing from the time the ships are fully equipped and ready for sea. The earlier magnetic history of the ships might have been included, but the changes which take place in their fitting, and the numerous iron bodies introduced after launching, render a comparison of the deviations observed at different stages of equipment unsatisfactory.

It is proposed to consider six classes of ships, each of which has been selected for the long range of magnetic latitude over which the ships have sailed :—

1. Iron, armour-plated.
2. Iron cased with wood.
3. Iron troop-ships.
4. Steel* and iron cased with wood.
5. Composite built.
6. Wooden ships with iron beams and vertical bulkheads.

These ships have nearly all been launched upwards of a year previous to the observations about to be discussed, and subjected to the vibration caused by steaming at high rates of speed with powerful engines. They may therefore be considered to have attained a state of magnetic stability.†

At the close of this paper will be found in Table III. a short description of the ships, the direction in which they were built, and the coefficients for each Standard Compass.

It is not intended to repeat the several mathematical formulæ by means of which

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* The vessels have iron frames, and are plated with mild steel, containing ·2 per cent. of carbon.

† See Phil. Trans., 1865, Part I., pp. 279–280.

these coefficients have been computed, as they, together with the methods of making the original observations, were so fully described in the paper on "The Magnetic Character of the Armour-plated ships of the Royal Navy," &c., 1865, to which allusion has already been made.

On comparing the results, however, in that paper with those now brought forward for discussion, a remarkable difference will be observed. In the paper for 1865, the coefficients of the semicircular deviation are those of compasses to which no mechanical correction by permanent magnets had been applied. The corresponding coefficients in the accompanying Table III. have, with the exception of the original values in England, been computed from the deviation of compasses for which a permanent bar magnet, or magnets, has been employed to annul or correct the semicircular deviation.

This correcting magnet has in every case been permanently fixed horizontally in the compass pillar, in the resultant of the magnetic forces producing semicircular deviation, and at a distance found tentatively below the card, after the several horizontal and vertical forces affecting the compass had been ascertained.

It may be asked, whether this application of correcting bar magnets of possible variable magnetic moment does not in itself introduce an element of change in the deviation in addition to those of the ship? It may be answered, with the reasonable confidence induced by fifteen years' trial, that the permanency of the magnetic moment of the magnets employed is considered to be assured.

Thus, in the ships named in the tables, the correction by magnets may be considered as the introduction into them of a permanent magnetic force acting independently on their compasses, and in opposition to the permanent magnetic forces of the ships.

It is now proposed to pass on to the chief object of this paper, which is to show the amount and direction of the changes which take place in the deviations of the standard compasses in six different classes of modern vessels in the Royal Navy, on change of magnetic latitude.

Taking the exact coefficients in Table III. in the order in which they stand we have first:—

The constant deviation.

𝔄.

Some rather large values of this coefficient are found in the tables, but it has been proved that for standard compasses placed in the central fore and aft line of the vessel where the iron is symmetrically placed with respect to that position, little or no real value from magnetic causes has been observed. An error in the bearing of the distant object used for swinging the ship, swinging her too fast, or prism error in the azimuth circle, gives fictitious values of this coefficient, which those in the table are considered to be.

Semicircular deviation.

Coefficient $\mathfrak{B}=\frac{1}{\lambda}\left(c\tan\theta+\frac{P}{H}\right)$ (approximate value in degrees $=$B) is the maximum of semicircular deviation from fore and aft forces;

$\frac{c}{\lambda}\tan\theta$ arises from soft iron;

$\frac{P}{\lambda H}$ from hard iron.

Coefficient $\mathfrak{C}=\frac{1}{\lambda}\left(f\tan\theta+\frac{Q}{H}\right)$ (approximate value in degrees $=$C) is the maximum of semicircular deviation from transverse forces;

$\frac{f}{\lambda}\tan\theta$ arises from soft iron, and is zero if the iron is symmetrically arranged;

$\frac{Q}{\lambda H}$ from hard iron.

For determining P and c separately, when $\mathfrak{B}$ has been determined in two different magnetic latitudes, the foregoing equations are put under the form,

$$\frac{P}{\lambda}+\frac{c}{\lambda}H\tan\theta=\mathfrak{B}H$$

$$\frac{P}{\lambda}+\frac{c}{\lambda}H'\tan\theta'=\mathfrak{B}'H'$$

and similarly for Q and f when $\mathfrak{C}$ has been determined in two different latitudes.

From the values of $\mathfrak{B}$ and $\mathfrak{C}$ in the table, the constants P and c, Q and f, have been calculated by the above formulæ, and the results are given in Table I., which for convenience has been placed at the end of this paper.

The explanation of this table is as follows:—

The quantities found in the columns headed "Original P" and "Original Q" are the constants P and Q, arising from the hard iron of the ship before correction by magnets, and which are mainly dependent for their values and sign upon the direction in which the ship's head lay during building. This direction is given under each ship's name.

In the columns headed "Corrected P" and "Corrected Q" are shown the constants P and Q as altered by the correcting magnets.

The quantities $\frac{c}{\lambda}\tan\theta$ and $\frac{f}{\lambda}\tan\theta$ (with their equivalents expressed in degrees) are the changing parts of the coefficients $\mathfrak{B}$ and $\mathfrak{C}$ respectively, for the South of England, which can only be corrected for all latitudes by vertical soft iron bars.

From the constants c and f in the remaining columns, the changing part of the coefficients 𝔅 and ℭ, arising from vertical induction in soft iron, may be computed whenever the magnetic inclination or dip is known, or can be taken from charts of that element.

Before further investigating the effects of a change of magnetic latitude on the coefficients, the question of how far time affects the constants P and Q requires consideration.

On looking over the values of P and Q, it will be remarked that in some ships a change takes place immediately after leaving England, which appears to be neither due to time nor change of latitude, but to another cause which will hereafter be referred to. Taking the values obtained subsequently at different times in the same geographical position it will be found—

		P.	Change.	Q.	Change.
Bellerophon.—Quebec	1874	−·002	·018	−·003	·005
	1876	−·020		−·008	
Iron Duke.—Hong-Kong	1880	−·208	·015	−·002	·035
	1881	−·193		−·037	
Northampton.—Halifax, N.S.	vi. 1880	+·024	·008	−·037	·020
	x. 1880	+·032		−·057	
Active.—Simon's Bay, C. G. Hope	x. 1874	−·120	·024	−·019	·013
	x. 1876	−·096		−·006	
Raleigh.—Spithead	ix. 1874	+·043	·014	+·026	·025
	vi. 1876	+·057		+·001	
Inconstant—Spithead	viii. 1869	−·023	·021	+·016	·001
	xi. 1871	−·044		+·017	
Himalaya.—Plymouth	vii. 1872	−·030	·009	−·061	·003
	vii. 1874	−·021		−·058	
Himalaya.—Simon's Bay, C. G. Hope	viii. 1872	−·030	·009	−·042	·039
	i. 1875	−·021		−·081	
Albatross.—Sheerness and Plymouth	iii. 1874	−·066	·002	+·015	·007
	x. 1874	−·064		+·022	
Boxer.—Esquimault, V. I.	x. 1869	−·008	·004	−·049	·019
	x. 1874	−·004		−·030	
Encounter.—Simon's Bay, C. G. Hope	iv. 1874	+·002	·050	+·022	·023
	i. 1876	−·048		−·001	

Thus, in the worst case amongst the armour-plated, iron, and composite vessels, P takes a year to alter ·015, and generally two or three years for about half that amount. A change of ·015 in P would make about a degree change in the deviation. This evidently slow change of P by time is important, as should P alter during the ship's sailing over a long range of magnetic latitude, the values of *c*, as found by the above formulæ, are correspondingly untrustworthy.

With regard to the constant Q, although more subject to change by time than P, it is comparatively of less importance, as, with few exceptions, the value of the constant *f*, which depends on the constancy of Q, is, if not zero, so small as to be neglected.

Having accepted as the result of fifteen years' trial that the correcting bar magnets are of constant magnetic moment, it would be reasonable to expect that the constants P and Q should remain unchanged in value, except the small decrease due to time. This is nearly the case, yet there are small fluctuations in them which demand notice.

It is known that if an iron vessel be placed in dock for any length of time in one direction with respect to the magnetic meridian, the values of P, and especially Q, undergo small changes dependent upon that relation. If, in addition, the vessel be subjected at the same time to concussion, from whatever cause, the change is greater. On the return of the vessel to her anchorage, or on proceeding to sea when the direction of her head varies frequently, P and Q return slowly to their original value.

It may therefore be inferred, that although P and Q as shown in the tables are for the most part due to permanent magnetism in the hard iron of the ship, there is a small part which is sub-permanent and subject to alterations from concussion, or the vibratory motion caused in the ship by powerful steam-engines when proceeding in a given direction for several days. On the removal of the cause inducing the change in P and Q they gradually return to their original values.

In the turret ships of the Royal Navy, where the standard compass is necessarily placed on a thin iron superstructure, this temporary dislocation of parts of P and Q, caused by the concussion of firing heavy guns and subsequent gradual recovery, is well known and provided for on board by constant observation for deviation of the compass.

Before considering the constants c and f, which represent the chief part of the changes which take place in the deviation of the compass on change of magnetic latitude, a few preliminary remarks appear to be necessary.

In "Contributions to Terrestrial Magnetism," No. IX.,* Sir Edward Sabine records the result of his investigations as to the effects of a change of magnetic latitude on the deviation of the Standard Compass of some wooden sailing ships of forty years ago. He concluded that their deviations were caused by vertical induction in soft iron, that they did not change directly in proportion to the dip, but there was a lagging behind proceeding from a slowness in the soft iron to part with its induced magnetism. For example, a ship passing quickly from 40° N. to 20° N., dip, would find the deviation due to vertical induction in 20° N., dip, to be that of 30° or 32° N.

In considering how far this theory applies to modern armour-plated and iron vessels, it may help to clear the question to note the rapidity with which horizontal soft iron, when magnetised by the earth's horizontal force, takes up and parts with its induced magnetism, as exemplified in swinging a ship for deviation of the compass.

In this operation—during which the direction of the ship's head passes through a complete circle—the deviation caused by horizontal induction in soft iron attains two maxima in an easterly and two in a westerly direction within an hour and a half.

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* See Phil. Trans., 1849.

Looking to this result, it hardly seems probable that vertical induction in soft iron should be slower in its action, and require perhaps days for full development.

Among the iron armour-plated ships of the tables we have, in the case of the "Triumph" and "Swiftsure" (two sister ships), experimental evidence that there is no sign of "lagging" in the changing part of their deviation, but that it alters directly as the tangent of the dip for any given position of the ship.

Both vessels, from requirements of the service, have their standard compasses placed unusually far from the stern, in a position 13 feet from the top of an armour-plated transverse bulkhead 5 inches thick. These ships were swung in the course of a few successive months, the observed deviations in each case but one corresponding with the dip at the locality, the values of which were as follows:—

$$\text{Triumph} \quad . \quad . \quad . \quad . \left\{\begin{array}{r} \theta. \\ -44^\circ \\ -34 \\ -\ 3 \\ +38 \end{array}\right\} \text{Range of } \theta \ 82^\circ.$$

$$\text{Swiftsure} \quad . \quad . \quad . \quad . \left\{\begin{array}{r} \theta. \\ +67^\circ \\ +17 \\ -11 \\ -29 \\ -52 \end{array}\right\} \text{Range of } \theta \ 119^\circ.$$

To proceed with the constants c and f.

The values of c are very valuable, not only as a means of predicting the probable change of deviation for the particular ships in which they are known, but also for ships of similar construction. As an illustration of the similarity of this constant in certain ships, the following examples are given where sister ships are bracketed:—

	c.
Triumph	$+\cdot 106$
Swiftsure	$+\cdot 110$
Northampton	$-\cdot 021$
Nelson	$-\cdot 014$
Shah	$-\cdot 005$
Raleigh	$-\cdot 006$
Comus	$+\cdot 034$
Cleopatra	$+\cdot 032$
Carysfort*	$+\cdot 044$
Boxer	$+\cdot 023$
Pert	$+\cdot 026$
Firefly	$+\cdot 005$
Wrangler	$+\cdot 002$

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* The "Carysfort" is a sister ship to "Comus" in every respect, with the exception that the former has no vertical iron shaft through which the screw is raised. In the "Comus" and "Cleopatra" this shaft is 30 feet from the compass.

From the position of the standard compass in the "Northampton" and "Nelson," 83 feet from the stern, a positive value of c might have been expected; but there is a transverse armoured bulkhead, 8 inches thick, the top of which is 30 feet nearer the stern than the compass in each vessel, which is the probable cause of the minus sign for c.

It is a subject for further inquiry as to what extent iron masts contribute to the values of c when the Standard Compass is placed near them. A series of experiments was made near Athens, where the magnetic dip is approximately 53° N., and at Singapore, in 13° S. dip, on board the "Ruby."* The Standard Compass of this ship is placed at 6 feet 9 inches from the iron mizenmast. The results showed that at three different parts of the mast (on a level with the compass, and a few feet above, and below that level) the effects of transient induction were very small, and that the mast acted almost entirely as a permanent magnet.

The remaining constant of the semicircular deviation, f, is one which has hitherto been accepted as zero, from the iron in the transverse section of the ships of the Royal Navy—except turret ships—being considered as placed symmetrically with respect to the Standard Compass.

In the Table I., however, there are five ships for which values of f have been discovered, and one, the "Triumph," in which it reaches an amount which could not be disregarded in a forecast of that vessel's deviation for any given geographical position. From Table I. the following values of f have been collected:—

	f.	$\frac{f}{\lambda}\tan\theta$ in England.
Bellerophon.	−·005	−·012=0° 42′
Triumph.	−·013	−·038=2° 10′
Swiftsure	+·004	+·010=0° 35′
Active	+·004	+·010=0° 35′
Himalaya	+·006	+·015=0° 52′
Comus	−·004	−·010=0° 35′

From the peculiar construction of some of the later types of armour-plated ships the constant f will probably attain higher values than those hitherto experienced.

Quadrantal deviation.

Coefficient $\mathfrak{D}=\frac{a-e}{2\lambda}$ (approximate value in degrees =D) is the maximum of quadrantal deviation from soft iron symmetrically placed.

Coefficient $\mathfrak{E}=\frac{d+b}{2\lambda}$ (approximate value in degrees =E) is the maximum of quadrantal deviation from soft iron unsymmetrically placed.

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* These experiments were made by Navigating Lieutenant HENDERSON, R.N., of the "Ruby."

On again referring to the paper "On the Magnetic Character of the Armour-plated Ships, &c.," of 1865, we read at page 275, "D and E do not change with a change of geographical position."

As regards $\mathfrak{D}$ this is fully confirmed by the results in the tables. Time alone appears to cause a gradual change in this coefficient during the first two or three years after launching, after which it remains remarkably permanent.

Coefficient $\mathfrak{E}$ has no real value in the ships under discussion.

Coefficient λ.

$\lambda=1+\frac{a+e}{2}$ is a factor generally less than 1, giving the northern component of the mean directive force of the needle, or "mean force to north."

λ, as might be expected from its close connexion with $\mathfrak{D}$, appears to be affected solely by lapse of time similarly to $\mathfrak{D}$; for example, in the "Malabar" (a sister ship to the "Euphrates" of the tables), a valuable series of observations was made between England and Bombay,* the results of which are here recorded.

			λ.
Malabar.—	Spithead,	17 vi. 67	·861
	Aden,	10 i. 70	·861
	Bombay,	$\frac{11}{13}$ ii. 70	·906
	Suez,	xi. 70	·907
	Spithead,	5 ix. 71	·932
	Spithead,	28 x. 78	·930

Each of the above values of λ is the mean of several observations conducted under favourable circumstances, and the observations, as far as they go, confirm the conclusions drawn from other ships in England. The ship's visit to the heat of the tropics seems to have accelerated the change in λ.

Coefficient μ.

$\mu=1+k+\frac{\text{R}}{\text{Z}}$, in which k represents the vertical force caused by vertical induction in the soft iron of the ship; R the vertical force from the hard iron.

The values of μ contribute largely in many ships to the heeling error. For the purpose of eliminating the values of k from R observations in widely different magnetic latitudes are still required.

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* By Staff Commander J. C. Richards, R.N.

In order to show the relative proportions of hard to soft iron affecting the standard compasses of the ships named in the tables—which are cruisers liable to be sent on long voyages—the values of $\sqrt{P^2+Q^2}$ and $\sqrt{c^2+f^2}$ have been placed together here—

	$\sqrt{P^2+Q^2}$	$\sqrt{c^2+f^2}$
Iron armour-plated ships—		
Bellerophon	·216	·005
Iron Duke	·416	·060
Triumph	·039	·106
Swiftsure	·058	·110
Northampton	·345	·021
Nelson	·287	·024
Iron vessels cased with wood—		
Active	·541	·016
Shah	·173	·005
Raleigh	·282	·006
Inconstant	·238	·039
Iron troop-ships—		
Euphrates	·359	·027
Himalaya	·281	·009
Orontes	·219	·029
Steel and iron ships cased with wood—		
Comus	·238	·034
Carysfort	·263	·044
Cleopatra	·076	·032
Composite vessels—		
Ruby	·500	·027
Gannet	·185	·024
Albatross	·232	·029
Boxer	·196	·023
Pert	·244	·026
Firefly	·332	·005
Wrangler	·375	·002
Wooden ships with iron beams, &c.—		
Encounter	·234	·018
Sapphire	·233	·045

On looking through the above values of $\sqrt{P^2+Q^2}$, it will be noticed that large differences occur in ships of similar construction.

It will be remembered that all iron and composite vessels are large magnets, generally of widely different forms. The values of $\sqrt{P^2+Q^2}$ are, therefore, chiefly dependent upon the position which the standard compass occupies on board these ships, considered as magnets.

Again, each iron body introduced during equipment into that great magnet, the ship, tends to modify in one direction or another its action upon the compass.

Consequently, if the compass be moved from the stern along the central longitudinal line of a ship towards her bow, it will be subjected to the influence of forces varying from those of repulsion or attraction, to zero, and then to those of attraction or repulsion.

For example, the "Iron Duke" and the "Triumph"—although not sister ships—are alike in many points, and built nearly in the same direction.

	$\sqrt{P^2+Q^2}$.	Distance of standard compass from stern.
Iron Duke . . .	·416	81 feet.
Triumph . . .	·033	105 ,,

It might at first sight be inferred from these results, that the position of the compass in "Triumph" is better than in the "Iron Duke." Keeping in view the object—always much desired—of so placing the compass as to have, when corrected, small changes of deviation on change of magnetic latitude, it will be seen this is not the case.

A reference to the values of $\sqrt{c^2+f^2}$, representing the changing part of the deviation, shows that in the "Iron Duke" the value is +·060; in the "Triumph" it is +·106. Thus, the "Triumph's" compass would be improved as regards deviation by moving it further towards the stern and away from the armour-plated bulkhead causing the large value of c, and opposing the increased value of P which would ensue, by a bar magnet.

These considerations tend to show the importance of the long-established regulations with regard to the placing of the standard compass in ships of the Royal Navy, which provide, that the best possible position with regard to surrounding iron shall be selected for it, subject to the interests of the ship as an engine of war.

General Conclusions.

The following general conclusions have especial reference to the Standard Compass positions in those vessels mentioned in the tables, and to all others of similar types.

1. A large proportion of the semicircular deviation is due to permanent magnetism in hard iron.

2. A large proportion of the semicircular deviation may be reduced to zero, or corrected for all magnetic latitudes, by fixing a hard steel bar magnet or magnets in the compass pillar in opposition to and of equal force to the forces producing that deviation.

3. A very small proportion of the semicircular deviation is due to sub-permanent magnetism, which diminishes slowly by lapse of time.

4. The sub-permanent magnetism produces deviation in the same direction as the permanent magnetism in hard iron, except when temporarily disturbed, (1) by the ship's remaining in a constant position with respect to the magnetic meridian for several days, (2) by concussion, (3) or by both combined, when the disturbance is intensified.

5. To ascertain the full value of changes in the sub-permanent magnetism, observations should be taken immediately on removal of the inducing cause.

6. In the usual place of the standard compass the deviation caused by transient vertical induction in soft iron is small, and of the same value (nearly) for ships of similar construction.

7. The preceding conclusions point to the conditions which should govern the selection of a suitable position for the standard compass with regard to surrounding iron in the ship.

TABLE I.—Values of the Constant Parameters P and c, Q and f, at Standard Compass Positions in Her Majesty's Ships.

Geographical position.	Date of observation.	Original P.	Corrected P.	$\frac{c}{\lambda}\tan\theta$ in England.	c.	Original Q.	Corrected Q.	$\frac{f}{\lambda}\tan\theta$ in England.	f.
Armour-plated ships.									
BELLEROPHON.—Built S. 51° E. Distance of standard from stern 61 feet, from iron mast 15 feet.									
Spithead . . .	3 xi. 73	+·024	−·004	0	0	+·215	+·025	−·012 = −0° 42′	−·005
Halifax, N.S. .	5 viii. 74	..	+·031	..	..	..	−·002		
Quebec. . . .	16 ix. 74	..	−·002	..	..	..	−·003		
Trinidad . . .	1 iii. 76	..	−·020	..	..	..	−·003		
Halifax, N.S. .	21 vi. 76	..	−·020	..	..	..	−·010		
Quebec. . . .	24 viii. 76	..	−·020	..	..	..	−·008		
IRON DUKE.—Built S. 39° W. Distance of standard from stern 81 feet, from iron mast $17\frac{1}{2}$ feet.									
Plymouth. . .	20 i. 71	+·407	..	+·158 = +9° 5′	+·060	−·166	..		
" . . .	16 ix. 71	..	−·019		..	..	+·048		
Gibraltar . . .	x. 71	..	−·047	..	..	..	−·008		
6° N. 96° E. . .	10 xii. 71	..	−·047	..	..	..	+·097		
38° N. 136° E. .	22 viii. 72	..	−·110	..	..	..	+·010		
49° N. 140° E. .	viii. 73	..	−·099	..	..	..	+·028		
30° N. $133\frac{1}{2}$° E. .	27 vii. 74	..	−·123	..	..	..	+·054		
$12\frac{3}{4}$° N. 46° E. .	11 iii. 75	..	−·126	..	..	..	−·016		
Plymouth. . .	27 vii. 75	..	−·136	..	..	..	+·042		
Belfast. . . .	5 ix. 76	..	−·138	..	..	..	+·034		
Plymouth. . .	18 vii. 78*	+·402*	−·122†	..	..	−·108	−·022†		
Singapore. . .	xi. 78	..	−·193	..	..	..	+·031		
Yokohama . .	vii. 79	..	−·193	..	..	..	−·021		
Hong Kong . .	28 xii. 80	..	−·208	..	..	..	−·002		
" . .	31 xii. 81	..	−·193	..	..	..	−·037		
TRIUMPH.—Built S. 45° W. Distance of standard from stern 105 feet, from iron mast $27\frac{1}{2}$ feet.									
Spithead . . .	15 v. 78	−·033	−·289	+·304 = +17°40′	+·106	−·020	+·044	−·038 = −2° 10′	−·013
Equator, $24\frac{1}{2}$° W.	20 vi. 78	..	−·222		..	..	−·012		
23° S. 39° W.. .	1 vii. 78	..	−·214	..	..	..	+·009		
$40\frac{1}{2}$° S. $78\frac{3}{4}$° W. .	7 vii. 78	..	−·194	..	..	..	−·001		
Valparaiso. . .	x. 78	..	−·190	..	..	..	−·007		
$9\frac{1}{4}$° S. $80\frac{3}{4}$° W. .	7 ii. 79	..	−·209	..	..	..	−·001		
20° N. 157° W. .	4 iv. 79	..	−·195	..	..	..	−·003		
SWIFTSURE.—Built S. 56° W. Distance of standard from stern 107 feet, from iron mast $28\frac{3}{4}$ feet.									
Plymouth . . .	8 v. 82	−·057	−·233	+·304 = +17°40′	+·110	+·010	−·016	+·010 = 0° 35′	+·004
1° S. 26° W. . .	22 vi. 82	..	−·182		..	..	−·007		
23° S. 43° W. .	5 vii. 82	..	−·227	..	..	..	−·086		
Monte Video . .	23 vii. 82	..	−·236	..	..	..	−·031		
Sandy Point, Magellan Straits .	4 viii. 82	..	−·239	..	..	..	−·016		

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* Previous to this date the ship had been under repair at Birkenhead.

† After re-correction by a permanent magnet.

TABLE I.—Values of the Constant Parameters P and c, Q and f, at Standard Compass Positions in Her Majesty's Ships (continued).

Armour-plated ships (continued).

NORTHAMPTON.—Built S. 6° E. Distance of standard from stern 83 feet, from iron mast 22 feet.

Geographical position.	Date of observation.	Original P.	Corrected P.	$\frac{c}{\lambda}\tan\theta$ in England.	c.	Original Q.	Corrected Q.	$\frac{f}{\lambda}\tan\theta$ in England.	f.
Sheerness . . .	1 xi. 79	+·345	..	−·060	−·021	−·008	−·002		
Spithead . . .	18 xii. 79	..	+·050	= −3° 25′	..	..	−·002		
27½° N. 62¾° W. .	23 i. 80	..	+·060	..	..	..	−·053		
St. Lucia . . .	20 ii. 80	..	+·041	..	..	..	−·073		
Bermuda . . .	8 v. 80	..	+·082	..	..	..	−·090		
Halifax, N.S. .	27 vi. 80	..	+·024	..	..	..	−·037		
Rimouski, River St. Lawrence .	21 viii. 80	..	+·046	..	..	..	−·046		
Halifax, N.S. .	1 x. 80	..	+·032	..	..	..	−·057		
Dominica . . .	22 i. 81	..	+·046	..	..	..	−·004		

NELSON.—Built S. 21° E. Distance of standard from stern 83 feet, from iron mast 23 feet.

Geographical position.	Date of observation.	Original P.	Corrected P.	$\frac{c}{\lambda}\tan\theta$ in England.	c.	Original Q.	Corrected Q.	$\frac{f}{\lambda}\tan\theta$ in England.	f.
Sheerness . . .	28 vii. 81	+·281	..	−·043	−·014	−·060	..		
Plymouth . . .	28 ix. 81	..	+·032	= −2° 30′	..	..	+·027		
Madeira . . .	11 x. 81	..	+·048	..	..	..	+·016		
Simon's Bay, C. G. Hope . . .	14 xi. 81	..	+·048	..	..	..	+·040		

Iron ships cased with wood.

ACTIVE.—Built N. 33° E. Distance of standard from stern 69 feet, from iron mast 16½ feet.

Geographical position.	Date of observation.	Original P.	Corrected P.	$\frac{c}{\lambda}\tan\theta$ in England.	c.	Original Q.	Corrected Q.	$\frac{f}{\lambda}\tan\theta$ in England.	f.
Portsmouth . .	13 x. 73	−·541	−·120	+·039	+·015	+·024	..	+·010	+·004
,, . .	15 x. 73	..	−·120	= +2° 15′	..	..	−·019	= +0° 35′	
Simon's Bay, C. Good Hope . .	x. 74	..	−·120	..	..	..	−·019		
,, . .	x. 75	..	−·094	..	..	..	−·023		
,, . .	xi. 76	..	−·096	..	..	..	−·006		

SHAH.—Built S. 70° E. Distance of standard from stern 70 feet, from iron mast 17½ feet.

Geographical position.	Date of observation.	Original P.	Corrected P.	$\frac{c}{\lambda}\tan\theta$ in England.	c.	Original Q.	Corrected Q.	$\frac{f}{\lambda}\tan\theta$ in England.	f.
Spithead . . .	8 vii. 76	+·003	..	−·012	−·005	+·173	..		
,, . . .	4 x. 76	..	+·009	= −0° 42′	..	..	+·083		
Esquimault, Vancouver's Island.	x. 77	..	−·017	..	..	..	−·067		
Coquimbo . . .	8 iii. 78	..	−·017	..	..	..	−·002		
Panama . . .	11 v. 78	..	−·010	..	..	..	+·002		

RALEIGH.—Built S. 51° E. Distance of standard from stern 65½ feet, from iron mast 8¾ feet.

Geographical position.	Date of observation.	Original P.	Corrected P.	$\frac{c}{\lambda}\tan\theta$ in England.	c.	Original Q.	Corrected Q.	$\frac{f}{\lambda}\tan\theta$ in England.	f.
Sheerness . . .	6 vi. 74	+·195	..	−·017	−·006	+·203	..		
,, . . .	8 vi. 74	..	+·029	= −1° 0′	..	..	+·054		
Spithead . . .	18 ix. 74	..	+·043	..	..	..	+·003		
Simon's Bay, C. Good Hope . .	iii. 75	..	+·043	..	..	..	+·026		
Bombay . . .	29 x. 75	..	−·006	..	..	..	−·010		
Spithead . . .	14 vi. 76	..	+·057	..	..	..	+·001		

TABLE I.—Values of the Constant Parameters P and *c*, Q and *f*, at Standard Compass Positions in Her Majesty's Ships (continued).

Geographical position.	Date of observation.	Original P.	Corrected P.	$\frac{c}{\lambda}\tan\theta$ in England.	*c*.	Original Q.	Corrected Q.	$\frac{f}{\lambda}\tan\theta$ in England.	*f*.
Iron ships cased with wood (continued).									
INCONSTANT.—Built S. 38° W. Distance of standard from stern 79 feet, from iron mast 13 feet.									
Spithead . . .	19 viii. 69	+·297	..	+·107 = +6° 10′	+·039	−·124	..		
,, . . .	20 viii. 69	..	−·023		..	..	+·016		
Plymouth . . .	21 viii. 70	..	−·071	..	..	..	+·020		
Spithead . . .	18 xi. 71	..	−·044	..	..	..	+·017		
Rio de Janeiro .	9 i. 72	..	−·044	..	..	..	−·015		
Spithead . . .	12 x. 80	+·238	..	..	..	+·010	..		
,, . . .	14 x. 80	..	−·094*	..	..	..	−·005*		
Melbourne . .	vii. 81	..	−·150	..	..	..	+·031		
Simonoseki . .	15 xi. 81	..	−·150	..	..	..	−·041		
Simon's Bay, C. Good Hope . .	v. 82	..	−·131	..	..	..	−·020		
Iron troopships.									
EUPHRATES.—Built N. 67° W. Distance of standard from stern 96 feet, from iron mast 33¾ feet.									
Birkenhead . .	16 v. 67	−·239	..	+·079 = 4° 30′	+·027	−·267	..		
,, . .	18 v. 67	..	−·226		..	..	+·019		
Spithead . . .	18 vi. 67	..	−·172	..	..	..	+·004		
Simon's Bay, C. Good Hope . .	31 vii. 67	..	−·159	..	..	..	+·092		
Suez	10 i. 68	..	−·156	..	..	..	+·097		
Spithead . . .	11 x. 71	−·245	..	..	..	−·089	..		
HIMALAYA.—Built . . . . . . Distance of standard from stern 64 feet.									
Spithead . . .	8 iii. 67	−·077	..	+·017 = +1° 0′	+·007	+·270	..	+·015 = +0° 52′	+·006
,, . . .	15 iii. 67	..	−·048		..	..	+·035		
Plymouth . . .	2 vii. 72	..	−·030	..	..	..	−·061		
Simon's Bay, C. Good Hope . .	viii. 72	..	−·030	..	..	..	−·042		
Plymouth . . .	9 xi. 74	..	−·021	+·015 = +0° 52′	+·006	..	−·058		
Simon's Bay, C. Good Hope . .	18 i. 75	..	−·021		..	..	−·081		
Plymouth . . .	18 vi. 75	..	−·031	..	..	..	−·081		
ORONTES.—Built N. 68° W. Distance of standard from stern 66 feet, from iron mast 9 feet.									
Birkenhead . .	26 i. 76	−·084	..	+·074 = +4° 15′	+·029	−·202	..		
Spithead . . .	2 iii. 76	..	−·042		..	..	+·018		
,, . . .	5 ix. 79	..	−·058	..	..	..	+·084		
Simon's Bay, C. Good Hope . .	x. 79	..	−·058	..	..	..	+·034		
Mauritius . . .	xi. 79	..	−·044	..	..	..	+·094		

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* After re-correction by a permanent magnet.

TABLE I.—Values of the Constant Parameters P and c, Q and f, at Standard Compass Positions in Her Majesty's Ships (continued).

Geographical position.	Date of observation.	Original P.	Corrected P.	$\frac{c}{\lambda}\tan\theta$ in England.	c.	Original Q.	Corrected Q.	$\frac{f}{\lambda}\tan\theta$ in England.	f.
Steel and iron ships cased with wood.									
COMUS.—Built S. 11° W. Distance of standard from stern 50 feet, from iron mast 7¾ feet.									
Sheerness . . .	14 xi. 79	+·246	..	+·089 = +5° 7′	+·034	−·029	..	−·010 = −0° 35′	−·004
,, . . .	18 xii. 79	+·234	+·009		..	+·041	+·022		
Simon's Bay, C. Good Hope . .	20 iii. 80	..	−·042	..	..	..	+·038		
Hong Kong . .	12 vii. 80	..	−·042	..	..	..	+·038		
Singapore . . .	13 xii. 81	..	−·017	..	..	..	+·035		
CARYSFORT.—Built S. 11° W. Distance of standard from stern 50 feet, from iron mast 7·9 feet.									
Sheerness . . .	5 x. 80	+·262	..	+·111 = +6° 23′	+·044	−·014	..		
,, . . .	6 x. 80	..	−·099		..	..	−·024		
Simon's Bay, C. Good Hope . .	1 iv. 81	..	−·133	..	..	..	+·030		
Kobé, Japan . .	10 xi. 81	..	−·133	..	..	..	+·007		
CLEOPATRA.—Built S. 11° W. Distance of standard from stern 49½ feet, from iron mast 6¾ feet.									
Plymouth . . .	15 ix. 80	+·057	..	+·082 = +4° 43′	+·032	+·050	..		
,, . . .	17 ix. 80	..	−·033		..	..	+·031		
Monte Video . .	31 xii. 80	..	−·074	..	..	..	−·028		
Hong Kong . .	20 xii. 81	..	−·074	..	..	..	+·039		
Composite vessels.									
RUBY.—Built N. 3° W. Distance of standard from stern 50 feet, from iron mast 6¾ feet.									
Sheerness . . .	30 vi. 77	−·486	..	+·077 = +4° 25′	+·027	−·109	..		
,, . . .	3 vii. 77	..	−·086		..	..	−·009		
Gulf of Xeros .	iii. 78	..	−·083	..	..	..	+·003		
Singapore . . .	vi. 78	..	−·083	..	..	..	+·018		
Rangoon . . .	x. 79	..	−·101	..	..	..	−·028		
Zanzibar . . .	12 iii. 80	..	−·076	..	..	..	−·009		
GANNET.*—Built S. 76° E. Distance of standard from stern 38 feet.									
Sheerness . . .	29 iv. 79	+·122	..	+·065 = +3° 45′	+·024	+·139	..		
,, . . .	30 iv. 79	..	−·012		..	..	+·010		
Coquimbo . . .	x. 79	..	−·059	..	..	..	−·032		
Esquimault, Vancouver Island .	vii. 81	..	−·059	..	..	..	−·014		
ALBATROSS.*—Built S. 50° E. Distance of standard from stern 37½ feet.									
Sheerness . . .	17 xii. 73	+·209	..	+·073 = +4° 12′	+0·29	+·100	..		
,, . . .	20 iii. 74	..	−·066		..	..	+·015		
Rio de Janeiro .	5 vii. 74	..	−·066	..	..	..	−·017		
Plymouth . . .	10 x. 74	..	−·064	..	..	..	+·022		
Taboga	27 x. 75	..	−·091	..	..	..	−·006		
Pisco	15 xii. 76	..	−·093	..	..	..	+·002		
Esquimault, Vancouver Island .	29 x. 77	..	−·093	..	..	..	+·016		

* These vessels have wooden masts.

TABLE I.—Values of the Constant Parameters P and *c*, Q and *f*, at Standard Compass Positions in Her Majesty's Ships (continued).

Geographical position.	Date of observation.	Original P.	Corrected P.	$\frac{c}{\lambda}\tan\theta$ in England.	*c*.	Original Q.	Corrected Q.	$\frac{f}{\lambda}\tan\theta$ in England.	*f*.
Composite vessels (continued).									
BOXER.*—Built S. 60° W. Distance of standard from stern $30\frac{1}{2}$ feet.									
Greenhithe	30 xi. 68	+·124	−·004	+·062	+·023	−·150	−·022		
Rio de Janeiro	20 ii. 69	..	−·016	= +3° 35′	..	..	−·105		
Esquimault, Vancouver's Island	5 x. 69	..	−·008	..	..	..	−·049		
Komax, Vancouver's Island	24 iv. 71	..	−·029	..	..	..	−·058		
Esquimault	30 xii. 74	..	−·004	..	..	..	−·030		
Coquimbo	13 v. 75	..	−·004	..	..	..	−·003		
PERT.*—Built S. 65° W. Distance of standard from stern $29\frac{3}{4}$ feet.									
Devonport	3 ii. 70	+·170	..	+·070	+·026	−·174	..		
„	8 ii. 70	..	−·056	= +4° 0′	..	..	−·016		
Elephant Bay, W. C. Africa	29 xii. 70	..	−·056	..	..	..	−·051		
Rio de Janeiro	29 xii. 71	..	−·043	..	..	..	−·054		
FIREFLY.*—Built N. 26° E. Distance of standard from stern $15\frac{3}{4}$ feet.									
Plymouth	24 vi. 79	−·332	+·004	+·012	+·005	..	+·015		
Simon's Bay, C. Good Hope	29 v. 80	..	+·004	= +0° 42′	..	..	+·001		
Gaboon River, W. C. Africa	30 viii. 80	..	+·032	..	..	..	+·046		
WRANGLER.*—Built N. 45° E. Distance of standard from stern $15\frac{3}{4}$ feet.									
Plymouth	26 v. 81	−·183	−·017	+·006	+·002	+·323	+·004		
Simon's Bay, C. Good Hope	31 xii. 81	..	−·017	= +0° 20′	..	..	+·027		
Wooden corvettes with iron beams and iron vertical bulkheads from keel to lower deck.									
ENCOUNTER.—Built S. 76° E. Distance of standard from stern 48 feet. Masts wooden.									
Sheerness	2 ix. 73	+·182	..	+·046	+·018	+·148	..		
„	9 ix. 73	..	+·016	= +2° 40′	..	..	+·034		
Simon's Bay, C. Good Hope	11 iv. 74	..	+·002	..	..	..	+·022		
„	15 x. 74	..	−·002	..	..	..	+·007		
„	20 i. 76	..	−·048	..	..	..	−·001		
Grenada	30 x. 76	..	−·048	..	..	..	−·014		
SAPPHIRE.—Built East. Distance of standard from stern $47\frac{3}{4}$ feet, from iron mast $5\frac{1}{2}$ feet.									
Plymouth	21 viii. 75	+·202	..	+·117	+·045	+·115	..		
„	24 viii. 75	..	−·093	= +6° 45′	..	..	+·012		
Simon's Bay, C. Good Hope	30 xi. 75	..	−·093	..	..	..	+·013		
Wellington, N.Z.	xii. 76	..	−·107	..	..	..	+·001		
Sydney, N.S.W.	viii. 77	..	−·152	..	..	..	−·033		

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* These vessels have wooden masts.

TABLE II.—Table of Terrestrial Magnetic Elements for the Year 1880, used in the computations.

Geographical position.	Inclination θ.	Approximate annual change.	Absolute horizontal force in British units.	Approximate annual change.
Greenwich	+67° 36′	− 1′·7	3·915	+·005
Gibraltar	+55 43	− 4·5	5·259	+·007
Maderia	+56 0	− 5·0	5·282	+·006
Gaboon River	−15 0	−12·0	6·50	−·012
Elephant Bay	−39 12	..	5·40	..
Simon's Bay, C. Good Hope	−56 28	− 5·0	4·270	−·007
Quebec	+77 0	..	2·97	..
Rimouski	+77 52	..	2·85	..
Halifax, N.S.	+74 48	− 1·8	3·388	+·002
Bermuda	+65 0	..	5·115	+·002
St. Lucia	+43 50	..	6·75	..
Dominica	+45 48	..	6·67	..
Grenada	+41 20	..	6·85	..
Trinidad	+39 40	..	6·89	..
Rio de Janeiro	−10 46	+ 3·0	5·66	−·016
Monte Video	−29 10	+ 8·5	5·745	−·017
Sandy Point, Magellan Straits	−52 20	+11·0	6·056	−·004
27½° N., 62¾° W.	+60 0	..	5·70	..
Equator, and 24½° W.	+15 7	− 9·0	6·40	−·007
23° S., 39° W.	−11 52	+ 2·0	5·51	−·016
40½° S., 78¾ W.	−44 8	..	6·20	..
1° S., 26° W.	+16 42	− 9·0	6·40	−·007
Valparaiso	−33 39	+ 5·0	6·078	−·018
Coquimbo	−29 12	+ 4·0	6·14	−·017
Pisco	− 7 44	..	6·69	−·013
Panama	+31 57	..	7·560	−·004
Esquimault, Vancouver Island	+71 54	..	4·21	..
9¼ S., 80¾° W.	−- 3 15	..	7·0	..
20 N., 157° W.	+37 58	..	6·64	..
Komax, Vancouver Island	+71 37	..	4·27	..
Gulf of Xcros.	+55 23	..	5·49	..
Suez	+40 47	− 1·0	6·50	..
Zanzibar	−36 0	..	6·205	..
Mauritius	−56 23	− 1·0	5·158	−·002
Bombay	+19 29	+ 1·2	8·10	+·003
Rangoon	+17 44	..	8·15	..
6° N., 96° E.	− 5 10	..	8·30	..
Singapore	−13 11	− 0·2	8·251	+·004
Hong Kong	+32 40	+ 4·2	7·794	+·006
Yokohama	+49 12	+ 3·4	6·417	..
Kobé, Japan	+48 48	+ 3·5	6·56	..
Simonoseki	+48 40	+ 4·0	6·64	..
49° N., 149° E.	+62 30	..	5·00	..
38° N., 136° E.	+53 5	..	6·19	..
34° N., 133½° E.	+46 8	..	6·60	..
12¾° N., 46° E.	+ 5 52	..	7·50	..
Melbourne	−67 5	Stationary	5·117	−·002
Sydney, N.S.W.	−62 44	Stationary	5·754	−·003
Wellington, N.Z.	−65 12	− 1·0	5·266	..

The + sign indicates north inclination or dip, the − sign that of south dip. The correction for annual change of the inclination or dip is to be applied algebraically.

TABLE III.—Coefficients.

Place.	Date.	Approximate coefficients. A.	B.	C.	D.	E.	Exact coefficients. 𝔄.	𝔅.	ℭ.	𝔇.	𝔈.	Maximum of semicircular deviation. $\sqrt{B^2+C^2}$ Mean horizontal force of ship. $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$ Amount.	Direction.	Mean force to north. λ.	$\frac{1}{\lambda}$.	Coefficients of horizontal induction. Head-ward. a.	To star-board. e.	Part of 𝔇 from Fore and aft induction.	Transverse induction.	Mean vertical force. μ.	Heeling coefficient to *windward*.	Heeling coefficient for Vertical induction in transverse iron.	Vertical force and induction in vertical iron.	Variable part of vertical force. $\frac{g}{\tan\theta}$.	g.
		° ′	° ′	° ′	° ′	° ′						°	°					° ′	° ′		° ′	° ′	° ′		
Iron armour-plated ships.																									
BELLEROPHON.—Iron screw ship, armour-plated; 7550 tons; 6520 H.P.; 13 guns. Built, Chatham; head S., 51° E.; launched, 3 iii. 66.—Standard Compass.																									
Spithead	3 xi. 73	...	From	quad	rant.	...	...	+·028	+·233	+·098	...	13⅓	·235	83											
,,	4 xi. 73	+ 1 11	*− 0 16*	*+ 1 3*	+ 5 12	− 0 32	+·020	*−·005*	*+·017*	+·090	−·009	*1¼*	*·018*	*109*	·878	1·139	−·043	−·201	− 1 22	+ 6 4	1·056	+ 0 42	+ 0 33	+ 0 9	+·016	+·040
Halifax, N.S.	5 viii. 74	− 0 8	*+ 2 20*	*− 0 50*	+ 6 9	...	−·002	*+·041*	*−·014*	+·107	...	*2¾*	*·044*	*344*											
Quebec	16 ix. 74	− 0 38	*− 0 11*	*− 2 26*	+ 5 16	+ 1 30	−·011	*−·003*	*−·042*	+·092	+·026	*2½*	*·041*	*265*											
Trinidad	1 iii. 76	+ 0 1	*− 0 45*	*− 0 21*	+ 5 39	+ 0 26	+·001	*−·013*	*−·006*	+·098	+·008	*0½*	*·015*	*199*											
Halifax, N.S.	21 vi. 76	− 0 28	*− 1 30*	*− 1 35*	+ 6 11	− 0 37	−·008	*−·026*	*−·028*	+·108	−·011	*2¼*	*·037*	*224*											
Quebec	24 viii. 76	− 0 13	*− 1 40*	*− 1 25*	+ 4 30	+ 0 39	−·004	*−·029*	*−·025*	+·078	+·011	*2*	*·032*	*286*											
IRON DUKE.—Iron double screw ship, armour-plated; 6010 tons; 4270 H.P.; 14 guns. Built at Pembroke; head S., 39° W.; launched, 1 iii. 70.—Standard Compass.																									
Plymouth	20 i. 71	+ 0 55	+34 18	−11 24	+ 5 45	+ 0 46	+·016	+·598	−·180	+·100	+·013	36	·624	343											
,,	16 ix. 71	+ 1 9	*+ 7 31*	*+ 3 11*	+ 6 23	− 0 2	+·020	*+·137*	*+·052*	+·112	−·001	*8¼*	*·147*	*21*	·925	1·081	+·028	−·178	+ 0 54	+ 5 29	·826	+ 0 1	+ 0 29	− 0 28		
Gibraltar	x. 71	+ 0 13	*+ 3 32*	*− 0 23*	+ 6 0	− 0 5	+·004	*+·065*	*−·007*	+·104	−·001	*3½*	*·066*	*354*											
6° N., 96° E.	10 xii. 71	+ 0 33	*− 1 37*	*+ 2 57*	+ 5 40	+ 0 39	+·009	*−·029*	*+·048*	+·099	+·011	*3½*	*·066*	*121*											
38° N., 136° E.	22 viii. 72	− 0 59	*+ 0 35*	*+ 0 27*	+ 5 17	+ 0 22	−·017	*+·010*	*+·007*	+·092	+·006	*0¾*	*·012*	*35*											
49° N., 140° E.	viii. 73	+ 0 53	*+ 2 12*	*+ 1 27*	+ 4 48	− 0 10	+·015	*+·040*	*+·024*	+·084	−·003	*2½*	*·047*	*31*											
34° N., 133½° E	27 vii. 74	+ 0 30	*− 0 26*	*+ 2 0*	+ 4 45	+ 0 25	+·009	*−·007*	*+·033*	+·083	+·007	*2*	*·034*	*102*											
12¾° N., 46° E.	11 iii. 75	− 2 25	*− 3 23*	*− 0 36*	+ 5 3	+ 0 34	−·042	*−·062*	*−·009*	+·088	+·010	*3½*	*·063*	*189*											
Plymouth	27 vii. 75	− 0 12	*+ 0 38*	*+ 2 45*	+ 4 47	+ 0 13	−·003	*+·011*	*+·046*	+·083	+·004	*3*	*·047*	*76*											
Belfast Lough	5 ix. 76	− 0 27	*+ 1 0*	*+ 2 15*	+ 4 16	+ 0 7	−·007	*+·018*	*+·038*	+·074	+·002	*2¼*	*·042*	*64*											
Plymouth*	18 vii. 78	Fro	m four	cardin	al poin	ts.	...	+·593	−·117	...	...	36	·605	349											
,,	,,	+ 0 8	*+ 1 26*	*− 1 25*	+ 4 57	+ 0 21	+·002	*+·026*	*−·024*	+·086	+·006	*2*	*·035*	*317*											
Singapore	xi. 78	+ 0 1	*− 6 17*	*+ 1 1*	+ 4 56	+ 0 14	+·001	*−·114*	*+·016*	+·086	+·004	*6½*	*·115*	*172*											
Yokohama	vii. 79	− 1 40	*− 3 0*	*− 0 52*	+ 5 3	+ 0 31	−·029	*−·054*	*−·014*	+·088	+·009	*3*	*·056*	*194*											
Hong Kong	28 xii. 80	− 0 46	*− 3 57*	*− 0 2*	+ 4 52	+ 0 34	−·013	*−·072*	*−·001*	+·085	+·010	*4*	*·072*	*181*											
,,	31 xii. 81	− 1 8	*− 3 35*	*− 1 12*	+ 4 23	+ 0 49	−·020	*−·064*	*−·020*	+·079	+·014	*3¾*	*·067*	*197*											
TRIUMPH.—Iron screw ship, armour-plated; 6640 tons; 5110 H.P.; 14 guns. Built on River Tyne; head S., 45° W.; launched, 29 ix. 70.—Standard Compass.																									
Plymouth	18 iii. 73	− 1 9	+20 53	−10 0	+ 6 45	− 0 5	−·020	+·377	−·161	+·117	−·001	23¼	·410	337											
,,	,,	+ 0 18	*+ 1 40*	*− 0 23*	+ 7 12	+ 0 33	+·005	*+·031*	*−·007*	+·125	+·009	*1¾*	*·032*	*347*	·839	1·192	−·056	−·266	− 1 56	+ 9 7	·881	+ 0 26	+ 0 47	− 0 21	+·067	+·167
Spithead†	15 v. 78	...	From S.	W. qua	drant.	...	...	+·266	−·063	...	...	15	·273	347											
,,	,,	− 1 23	*− 2 7*	*+ 0 51*	+ 5 51	− 0 53	−·024	*−·039*	*+·014*	+·102	−·015	*2¼*	*·041*	*160*											
Equator, long. 24½° W.	20 vi. 78	− 2 29	*− 7 0*	*− 0 56*	+ 7 6	− 0 58	−·043	*−·129*	*−·015*	+·124	−·017	*7*	*·130*	*187*											
23° S., 39° W.	1 vii. 78	− 3 31	*−11 13*	*+ 0 38*	+ 6 41	− 0 44	−·061	*−·205*	*+·010*	+·116	−·013	*11¼*	*·205*	*177*											
40½° S., 78¾° W.	7 viii. 78	+ 0 49	*−14 51*	*+ 0 55*	+ 6 39	− 1 19	+·014	*−·271*	*+·015*	+·116	−·023	*15*	*·272*	*177*											
Valparaiso	ix. x. 78	− 0 29	*−12 14*	*+ 0 14*	+ 6 30	− 0 20	−·008	*−·224*	*+·004*	+·113	−·006	*12¼*	*·224*	*179*											
9½° S., 80½° W.	7 ii. 79	+ 1 31	*− 7 28*	*− 0 11*	+ 6 55	+ 0 18	+·026	*−·138*	*−·003*	+·120	+·005	*7½*	*·138*	*181*											
20° N., 157° W.	4 iv. 79	− 1 51	*− 2 2*	*− 0 51*	+ 6 28	− 0 42	−·032	*−·037*	*−·014*	+·112	−·012	*2½*	*·040*	*201*											
Esquimault.	v. 79	...	From S.	W. qua	drant.	...	...	*+·103*	*+·015*	...	...	*5½*	*·104*	*8*											

* After repair at Birkenhead. Compass re-corrected at Plymouth. † Compass re-corrected on this day.

NOTE.—The values of B. and C., &c., printed in italics have been computed from the tables of deviation observed after the compass had been corrected by magnets.

TABLE III.—Coefficients (continued).

Place.	Date.	Approximate coefficients.					Exact coefficients.					Maximum of semicircular deviation. $\sqrt{B^2+C^2}$ Mean horizontal force of ship. $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$			Mean force to north. λ.	$\frac{1}{\lambda}$.	Coefficients of horizontal induction.		Part of $\mathfrak{D}$ from		Mean vertical force. μ.	Heeling coefficient to *windward*.	Heeling coefficient for		Variable part of vertical force.	
		A.	B.	C.	D.	E.	$\mathfrak{A}$.	$\mathfrak{B}$.	$\mathfrak{C}$.	$\mathfrak{D}$.	$\mathfrak{E}$.	Amount.		Direction.			Headward. *a*.	To starboard. *c*.	Fore and aft induction.	Transverse induction.			Vertical induction in transverse iron.	Vertical force and induction in vertical iron.	$\frac{g}{\tan\theta}$.	*g*.
		° ′	° ′	° ′	° ′	° ′						°		°					° ′	° ′		° ′	° ′	° ′		
Iron armour-plated ships (continued).																										
SWIFTSURE.—Iron, screw ship, armour plated; 6910 tons: 4910 H.P.; 18 guns. Built on River Tyne; head S., 56° W; launched, 15 vi. 70.—Standard Compass.																										
Plymouth	17 vi. 72	From southern and west semicircles.					...	+·386	−·186	+·122	...	25¼	·430	337	·834	1·199	−·064	−·268	− 2 11	+ 9 13	1·184	+ 1 21	+ 0 48	+ 0 33	+·062	+·155
,,	8 v. 82	...	From N.E. quadrant.			...	...	+·239	...	+·108	ass.	12½	·239	...												
,,	,,	+ 0 33	*+ 5*	*− 0 22*	+ 6 12	+ 0 22	+·009	*+·038*	*−·006*	+·108	+·006	*2*	*·038*	*351*	·876	1·141	−·030	−·218	− 0 56	+ 7 8	1·103	+ 0 52	+ 0 36	+ 0 15	+·066	+·158
1° S., 26° W.	22 vi. 82	+ 1 5	*− 5 0*	*− 0 13*	+ 6 19	+ 0 46	+·019	*−·092*	*−·004*	+·110	+·013	*5*	*·092*	*183*							*·872*			*− 0 21*		
23° S., 43° W.	5 vii. 82	− 4 1	*−11 8*	*− 4 10*	+ 5 44	+ 0 17	−·070	*−·202*	*−·068*	+·100	+·005	*12*	*·213*	*199*												
Monte Video	23 vii. 82	− 3 31	*−14 6*	*− 1 40*	+ 4 49	− 0 9	−·061	*−·255*	*−·027*	+·084	−·002	*14¼*	*·255*	*185*												
Sandy Point, Magellan Straits	4 viii. 92	− 1 54	*−18 32*	*− 1 7*	+ 6 3	− 1 34	−·033	*−·337*	*−·018*	+·105	−·027	*18½*	*·337*	*183*												
NORTHAMPTON.—Iron double screw ship, armour-plated; 7630 tons; 6070 H.P.; 12 guns. Built at Glasgow; head S., 6° E.; launched, 18 xi. 76.—Standard Compass.																										
Sheerness	1 xi. 79	− 0 44	+20 1	− 0 37	+ 3 38	− 0 6	−·013	+·354	−·010	+·063	−·002	20	·354	359	·834	1·199	−·114	−·218	− 3 54	+ 7 32	·903	+ 0 21	+ 0 37	− 0 16	+·055	+·132
Spithead	18 xii. 79	+ 0 37	*+ 0 1*	*− 0 6*	+ 4 3	− 0 1	+·011	*+·001*	*−·002*	+·070	−·001	*0¼*	*·002*	*260*												
27½° N., 62¾° W.	23 i. 80	+ 1 13	*+ 0 21*	*− 2 39*	+ 3 44	− 0 16	+·021	*+·006*	*−·044*	+·065	−·004	*2¾*	*·044*	*278*												
St. Lucia, S.W. Point	20 ii. 80	+ 0 58	*+ 0 17*	*− 3 4*	+ 3 30	+ 0 22	+·017	*+·005*	*−·051*	+·061	+·006	*3¼*	*·051*	*276*												
Bermuda	8 v. 80	− 1 53	*+ 1 12*	*− 4 58*	+ 5 3	− 0 40	−·034	*+·022*	*−·082*	+·088	−·012	*5*	*·085*	*285*												
Halifax, N.S.	27 vi. 80	+ 0 36	*− 3 10*	*− 3 2*	+ 3 40	− 1 32	+·010	*−·057*	*−·051*	+·064	−·027	*4¼*	*·077*	*222*												
Rimouski River, St. Lawrence	31 viii. 80	− 1 38	*− 2 17*	*− 4 32*	+ 3 42	− 0 31	−·028	*−·041*	*−·076*	+·064	−·009	*5*	*·087*	*241*												
Halifax, N.S.	1 x. 80	− 1 1	*− 2 32*	*− 4 50*	+ 3 40	− 0 15	−·018	*−·045*	*−·081*	+·064	−·004	*5¼*	*·093*	*241*												
Dominica	22 i. 81	− 1 37	*+ 0 23*	*− 0 10*	+ 4 1	− 0 18	−·028	*+·007*	*−·003*	+·070	−·005	*0½*	*·008*	*336*												
NELSON.—Iron, double screw ship, armour-plated; 7630 tons; 6640 H.P.; 12 guns. Built at Glasgow; head S., 21° E; launched 4 xi. 76.—Standard Compass.																										
Sheerness	28 vii. 81	− 0 3	+16 36	− 4 7	+ 4 0	+ 0 7	−·001	+·296	−·072	+·070	+·002	17	·305	347	·830	1·204	−·112	−·228	− 3 52	+ 7 52	·952	+ 0 31	+ 0 39	− 0 8		
Plymouth	28 ix 81	− 0 23	*− 0 13*	*+ 1 57*	+ 4 7	− 0 5	−·007	*−·004*	*+·033*	+·071	−·001	*2*	*·033*	*97*												
Madeira	11 x. 81	− 0 6	*+ 0 55*	*+ 0 51*	+ 3 47	− 0 24	−·002	*+·016*	*+·015*	+·066	−·007	*1¼*	*·022*	*43*												
Simon's Bay, C. Good Hope	14 xi. 81	− 0 44	*+ 4 35*	*+ 2 34*	+ 3 37	− 0 12	−·013	*+·082*	*+·044*	+·063	−·003	*5¼*	*·093*	*28*												
Iron ships cased with wood.																										
ACTIVE.—Iron cased with wood; 3080 tons; 4130 H.P.; 12 guns. Built at Blackwall; head N., 33° E; launched, 13 ii. 69.—Standard Compass.																										
Portsmouth	28 ii. 70	From deviation and force on one point					...	−·540	−·064	+·085	ass.	32	·544	187												
,,	7 iii. 70	− 0 10	*− 2 35*	*+ 0 42*	+ 4 0	− 0 8	−·003	*−·046*	*+·012*	+·070	−·002	*3*	*·048*	*165½*	·904	1·106	−·033	−·159	−1 4	+5 4	1·096 *·854*	+0 44 *+0 2*	+0 26 +0 26	+0 18 *−0 24*	... ...	ass.
,,	13 x. 73	...	From quadrant			...	...	−·537	−·016	+·059	...	32½	·537	182	·941	1·063	−·004	−·114	−0 7	+3 34	1·052	+0 26	+0 18	+0 8	...	
,,	15 x. 73	+ 0 24	*− 4 59*	*− 0 34*	+ 3 17	− 0 5	+·007	*−·089*	*−·010*	+·057	−·001	*5*	*·090*	*186*							*·949*	*+0 1*	*+0 18*	*−0 17*	...	
Simon's Bay	x. 74	− 1 27	*− 7 51*	*− 1 31*	+ 2 41	+ 0 59	−·025	*−·140*	*−·025*	+·047	+·017	*8*	*·142*	*190*												
,,	x. 75	− 1 16	*− 6 33*	*− 1 33*	+ 2 14	− 0 18	−·022	*−·115*	*−·028*	+·039	−·005	*6¾*	*·118*	*194*												
,,	xi. 76	− 2 12	*− 6 33*	*− 0 42*	+ 2 44	− 0 18	−·038	*−·117*	*−·012*	+·048	−·005	*6½*	*·118*	*186*												

TABLE III.—Coefficients (continued).

Place.	Date.	Approximate coefficients.					Exact coefficients.					Maximum of semicircular deviation. $\sqrt{B^2+C^2}$ Mean horizontal force of ship. $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$		Mean force to north. λ.	$\frac{1}{\lambda}$.	Coefficients of horizontal induction.		Part of 𝔇 from		Mean vertical force. μ.	Heeling coefficient to *windward*.	Heeling coefficient for		Variable part of vertical force.	
		A.	B.	C.	D.	E.	𝔄.	𝔅.	ℭ.	𝔇.	𝔈.	Amount.	Direction.			Headward. *a*.	To starboard. *e*.	Fore and aft induction.	Transverse induction.			Vertical induction in transverse iron.	Vertical force and induction in vertical iron.	$\frac{g}{\tan\theta}$.	*g*.
		° ′	° ′	° ′	° ′	°						°	°					° ′	° ′		° ′	° ′	° ′		
Iron ships cased with wood (continued).																									
SHAH.—Iron cased with wood; 6050 tons; 7480 H.P.; 26 guns. Built at Portsmouth; head S. 70° E.; launched, 10 ix. 73. Standard Compass.																									
Spithead	8 vii. 76	+ 0 8	− 0 33	+11 25	+ 5 44	+ 0 15	+·002	−·009	+·188	+·100	+·004	11½	·188	93											
,,	4 x. 76	− 1 29	*− 0 8*	*+ 5 25*	+ 5 5	+ 0 33	−·026	*−·002*	*+·090*	+·089	+·009	*5½*	*·090*	*91*	·923	1·083	+·005	−·159	+0 12	+4 54	·988	−0 24	+0 26	−0 2	+·011	+·027
Esquimault	x. 77	+ 0 8	*− 1 49*	*− 4 6*	+ 5 15	+ 0 20	+·002	*−·033*	*−·068*	+·091	+·006	*4⅓*	*·076*	*244*											
Coquimbo	8 iii. 78	...	*− 0 31*	*− 0 2*	+ 5 37	+ 0 30	...	*−·009*	*−·001*	+·095	+·009	*0⅓*	*·009*	*186*											
Panama	11 v. 78	− 0 2	*− 0 31*	*+ 0 5*	+ 5 15	+ 0 38	−·001	*−·009*	*+·001*	+·091	+·011	*0½*	*·009*	*174*											
RALEIGH.—Iron cased with wood; 5200 tons; 5640 H.P.; 22 guns. Built at Chatham; head S. 51° E.; launched, 1 iii. 73. Standard Compass.																									
Sheerness	6 vi. 74	+ 0 39	+11 46	+15 15	+ 8 0	− 0 28	+·011	+·217	+·244	+.140	−·008	20	·326	48											
,,	8 vi. 74	+ 0 39	*+ 1 1*	*+ 3 59*	+ 6 33	+ 0 35	+·011	*+·018*	*+·065*	+·114	+·010	*4¼*	*·067*	*74*	·833	1·200	−·072	−·262	− 2 29	+ 9 2	·930 *·772*	+ 0 33 *+ 0 6*	+ 0 47	−0 14 *−0 41*	−·017	−·042
Spithead	18 ix. 74	− 0 6	*+ 2 1*	*+ 0 14*	+ 6 55	− 0 36	−·002	*+·035*	*+·004*	+·120	−·010	*2¼*	*·037*	*6*											
Simon's Bay	iii. 75	− 1 14	*+ 3 17*	*+ 1 44*	+ 6 35	− 0 55	−·021	*+·059*	*+·028*	+·116	−·016	*3¾*	*·066*	*26*											
Bombay	29 x. 75	+ 1 14	*− 0 16*	*− 0 27*	+ 5 59	+ 0 18	+·021	*−·006*	*−·006*	+·105	+·005	*0⅓*	*·008*	*225*											
Spithead	14 vi. 76	+ 0 58	*+ 2 53*	*+ 0 3*	+ 5 19	+ 1 4	+·017	*+·052*	*+·001*	+·094	+·019	*3*	*·053*	*1*											
INCONSTANT.—Iron cased with wood; 5780 tons: 7360 H.P.; 16 guns. Built at Pembroke; head S. 38° W.; launched, 12 xi. 68. Standard Compass.																									
Spithead	19 viii. 69	+ 0 56	+24 54	− 8 40	+ 4 55	− 0 22	+·016	+·441	−·140	+·085	−·006	26⅓	·463	342	·890	1·123	−·035	−·185	−1 6	+ 6 2	·827	+ 0 2	+ 0 31	− 0 29	+·042	+·105
,,	20 viii. 69	+ 0 13	*− 4 38*	*+ 1 6*	+ 4 52	+ 0 15	+·004	*+·083*	*+·018*	+·085	+ 004	*5*	*·085*	*12*											
Plymouth	21 viii. 70	+ 0 48	*+ 1 30*	*+ 1 23*	+ 4 48	+ 0 3	+·014	*+·027*	*+·023*	+·084	+·001	*2*	*·035*	*40*											
Spithead	18 xi. 71	+ 0 7	*+ 3 8*	*+ 1 8*	+ 4 13	− 0 4	+·002	*+·057*	*+·019*	+·074	−·001	*3¼*	*·060*	*19*											
Rio de Janeiro	9 i. 72	+ 0 42	*− 2 23*	*− 0 40*	+ 4 22	+ 0 24	+·012	*−·042*	*−·011*	+·076	+·007	*2¼*	*·044*	*195*											
Spithead	12 x. 80	...	From N	.W. qua	drant.	...	...	+·374	+·011	+·068	...	22	·375	1											
,, *	14 x. 80	+ 0 55	*+ 0 5*	*− 0 24*	+ 3 54	+ 0 16	+·016	*+·001*	−·006	+·068	+·005	*0⅓*	*·006*	*280*							·860	+ 0 3	+ 0 25	− 0 22		
Melbourne	6 vii. 81	+ 0 18	*−13 5*	*+ 1 37*	+ 3 40	− 0 17	+·005	*−·234*	*+·027*	+·064	−·005	*13*	*·236*	*173*											
Simonoseki	15 xi. 81	+ 0 55	*− 2 50*	*− 1 37*	+ 3 55	...	+·016	*−·050*	*−·027*	+·068	...	*3¼*	*·058*	*208*											
Simon's Bay	4 v. 82	− 0 26	*−11 22*	*− 1 18*	+ 2 47	+ 0 35	−·008	*−·202*	*−·021*	+·048	+·010	*11½*	*·203*	*186*											
Iron troopships.																									
EUPHRATES.—Iron screw; 6211 tons; 3900 H.P. Built at Birkenhead; head N. 67° W; launched, 24 xi. 66.—Standard Compass.																									
Birkenhead	16 v. 67	...	From	semi	circle.	...	...	−·201	−·314	+·098	...	21¾	·370	237											
,,	18 v. 67	+ 1 40	*− 8 4*	*+ 1 21*	+ 5 39	+ 0 43	+·029	*−·147*	*+·022*	+·098	+·013	*8¼*	*·149*	*171⅓*	·851	1·175	−·066	−·232	− 2 11	+ 7 50	1·197	+ 1 22	+ 0 44	+ 0 38		
Spithead	18 vi. 67	− 0 9	*− 6 43*	*+ 0 16*	+ 5 45	− 0 9	−·003	*−·123*	*+·005*	+·100	−·003	*6¾*	*·123*	*177½*											
Simon's Bay	31 vii. 67	− 3 17	*− 9 4*	*+ 5 44*	+ 5 24	− 1 5	−·057	*−·165*	*+·095*	+·094	−·010	*10¾*	*·141*	*150*											
Suez	10 i. 68	+ 0 9	*− 4 33*	*+ 3 59*	+ 3 40	− 0 21	+·002	*−·081*	*+·067*	+·064	−·006	*6*	*·105*	*140*											
Spithead	11 x. 71	...	From	quad	rant.	...	...	−·166	−·105	+·067	...	11¼	·196	212											

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* Compass re-corrected on this day.

TABLE III.—Coefficients (continued).

Place.	Date.	Approximate coefficients. A.	B.	C.	D.	E.	Exact coefficients. 𝔄.	𝔅.	ℭ.	𝔇.	𝔈.	Maximum of semicircular deviation. $\sqrt{B^2+C^2}$	Mean horizontal force of ship. $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$ Amount.	Direction.	Mean force to north. λ	$\frac{1}{\lambda}$.	Coefficients of horizontal induction. Head-ward. a.	To star-board. e.	Part of 𝔇 from Fore and aft induction.	Transverse induction.	Mean vertical force. μ.	Heeling coefficient to *windward*.	Heeling coefficient for Vertical induction in transverse iron.	Heeling coefficient for Vertical force and induction in vertical iron.	Variable part of vertical force. $\frac{g}{\tan\theta}$.	g.
		° ′	° ′	° ′	° ′	° ′						°		°					° ′	° ′		° ′	° ′	° ′		

Iron troopships (continued).

HIMALAYA.—Iron screw; 4690 tons; 2500 H.P. Built, 1854, at Blackwall.—Standard Compass.

Place.	Date.	A.	B.	C.	D.	E.	𝔄.	𝔅.	ℭ.	𝔇.	𝔈.	$\sqrt{B^2+C^2}$	Amount.	Direction.	λ	$\frac{1}{\lambda}$	a	e	Fore and aft	Transverse	μ	Windward	Transverse iron	Vertical iron	$\frac{g}{\tan\theta}$	g
Spithead . . .	8 iii. 67	+ 0 6	− 3 48	+17 50	+ 3 3	+ 0 29	+·002	−·067	+·299	+·053	+·008	18	·307	103												
,, . . .	15 iii. 67	+ 1 20	− *2 6*	+ *3 2*	+ 2 25	+ 0 35	+·023	−*·037*	+*·052*	+·042	+·010	3½	*·063*	*125*	·945	1·058	−·015	−·095	− 0 28	+ 2 53	1·167	+ 0 41	+ 0 15	+ 0 26	+·151	+·377
Plymouth . . .	2 vii. 72	+ 1 13	− *0 55*	− *2 52*	+ 3 0	+ 0 14	+·021	−*·016*	−*·049*	+·052	+·004	3	*·051*	*252*	·950	1·053	...	−·100	...	+ 3 0	1·119	+ 0 34	+ 0 16	+ 0 18	+·007	+·017
Simon's Bay . .	viii. 72	− 0 50	− *2 31*	− *2 53*	+ 2 57	− 0 18	−·014	−*·046*	−*·049*	+·051	−·005	3½	*·067*	*227*												
Plymouth . . .	9 xi. 74	+ 0 25	− *0 23*	− *2 42*	+ *2 49*	+ 0 9	+·007	−*·018*	−*·046*	+·049	+·003	2¾	*·047*	*260*												
Simon's Bay . .	18 i. 75	− 0 56	− *1 37*	− *4 56*	+ 2 58	− 0 2	−·016	−*·030*	−*·085*	+·052	−·001	5¼	*·091*	*251*												
Plymouth . . .	18 vi. 75	+ 0 36	− *0 51*	− *4 0*	+ 2 46	− 0 12	+·010	−*·016*	−*·069*	+·048	−·004	4¼	*·071*	*257*												

ORONTES.—Iron screw; 5920 tons; 2570 H.P. Built at Birkenhead; head N. 68° W; launched, 22 xi. 62.—Standard Compass.

Place.	Date.	A.	B.	C.	D.	E.	𝔄.	𝔅.	ℭ.	𝔇.	𝔈.	$\sqrt{B^2+C^2}$	Amount.	Direction.	λ	$\frac{1}{\lambda}$	a	e	Fore and aft	Transverse	μ	Windward	Transverse iron	Vertical iron	$\frac{g}{\tan\theta}$	g
Birkenhead . .	26 i. 76	− 0 13	− 0 54	−12 57	+ 3 5	+ 0 8	−·004	−·016	−·218	+·054	+·002	13	·219	266												
Spithead . . .	2 iii. 76	+ 1 1	+ *1 41*	+ *1 12*	+ 3 31	+ 0 18	+·017	+*·030*	+*·020*	+·061	+·005	2	*·040*	*34*												
,, . . .	5 ix. 79	− 0 46	+ *0 46*	+ *5 24*	+ 3 17	+ 0 4	−·013	+*·013*	+*·091*	+·057	+·001	5½	*·092*	*82*												
Simon's Bay . .	x. 79	− 3 21	− *5 40*	+ *2 1*	+ 3 34	− 0 12	−·058	−*·102*	+*·034*	+·062	−·002	6	*·108*	*162*												
Mauritius . . .	xi. 79	− 0 56	− *4 21*	+ *4 28*	+ 3 39	+ 0 13	−·016	−*·078*	+*·076*	+·064	+·004	6¼	*·109*	*136*												

Iron and steel ships cased with wood.

COMUS.—Iron and steel screw corvette, cased with wood; 2380 tons; 2450 H.P.; 14 guns. Built, Glasgow; head S. 11° W; launched, 3 iv. 78.—Standard Compass.

Place.	Date.	A.	B.	C.	D.	E.	𝔄.	𝔅.	ℭ.	𝔇.	𝔈.	$\sqrt{B^2+C^2}$	Amount.	Direction.	λ	$\frac{1}{\lambda}$	a	e	Fore and aft	Transverse	μ	Windward	Transverse iron	Vertical iron	$\frac{g}{\tan\theta}$	g
Sheerness . . .	14 xi. 79	...	From N.W. quadrant.			...	...	+·354	−·041	+·098	...	20¾	·354	353	·937						·998	+ 0 25	+ 0 26	− 0 1		
,, . . .	15 xi. 79	+ 0 50	− *0 3*	+ *0 49*	+ 6 9	+ 0 34	+·014	−*·001*	+*·013*	+·107	...	0¾	*·014*	*94*	·932	1·073	+·033	−·169	+ 1 0	+ 5 9	·977	+ 0 22	+ 0 26	− 0 04		
,, . . .	18 xii. 79	...	From north semicircle.				...	+·342	+·030	+·107	...	20¼	·348	5							*·796*	− *0 5*		− *0 31*		
,, * . . .	19 xii. 79	+ 0 17	+ *5 25*	+ *0 52*	+ 6 9	+ 0 15	+·005	+*·099*	+*·014*	+·107	+·004	5½	*·100*	*8*												
Simon's Bay . .	20 iii. 80	+ 0 21	− *5 17*	+ *2 38*	+ 5 52	− 0 19	+·006	−*·097*	+*·044*	+·102	−·005	6	*·106*	*155*												
Hong-Kong . .	12 vii. 80	+ 0 7	+ *0 2*	+ *1 6*	+ 6 8	+ 0 37	+·002	+*·001*	+*·018*	+·107	+·011	1	*·018*	*87*												
Singapore . . .	10/13 xii. 81	− 0 17	− *0 1*	+ *1 18*	+ 5 40	+ 0 42	−·605	−*·001*	+*·019*	+·099	+·012	1¼	*·019*	*90*												

CARYSFORT.—Iron and steel screw corvette, cased with wood, 2380 tons; 2400 H.P.; 14 guns. Built, Glasgow; head S. 11° W.; launched, 26 ix. 78.—Standard Compass.

Place.	Date.	A.	B.	C.	D.	E.	𝔄.	𝔅.	ℭ.	𝔇.	𝔈.	$\sqrt{B^2+C^2}$	Amount.	Direction.	λ	$\frac{1}{\lambda}$	a	e	Fore and aft	Transverse	μ	Windward	Transverse iron	Vertical iron	$\frac{g}{\tan\theta}$	g
Sheerness . . .	5 x. 80	From southern semicircle.					...	+·387	−·015	...	...	21	·387	358												
,, . . .	6 x. 80	− 0 29	+ *0 22*	− *1 30*	+ 5 23	+ 0 18	−·008	+·006	−·025	+·094	+·005	1½	*·026*	*284*	·950	1·052	+·039	−·139	+ 1 13	+ 4 10	·897	+ 0 5	+ 0 21	− 0 26	+·040	+·096
Simon's Bay . .	1 iv. 81	− 0 50	−*10 58*	+ *1 48*	+ 5 8	− 0 8	−·014	−*·198*	+*·029*	+·089	−·002	11	*·200*	*172*												
Kobé, Japan . .	10 xi. 81	− 0 50	− *1 51*	+ *0 14*	+ 5 0	− 0 18	−·014	−*·033*	+*·004*	+·087	−·005	1¾	*·033*	*173*												

CLEOPATRA.—Steel and iron screw corvette, cased with wood; 2380 tons; 2610 H.P.; 14 guns. Built, Glasgow; head S. 11° W.; launched, 1 viii. 78.—Standard Compass.

Place.	Date.	A.	B.	C.	D.	E.	𝔄.	𝔅.	ℭ.	𝔇.	𝔈.	$\sqrt{B^2+C^2}$	Amount.	Direction.	λ	$\frac{1}{\lambda}$	a	e	Fore and aft	Transverse	μ	Windward	Transverse iron	Vertical iron	$\frac{g}{\tan\theta}$	g
Devonport . . .	15 ix. 80	From southern semicircle.					...	+·143	+·054	+·109	...	8¾	·153	21												
,, . . .	17 ix. 80	+ 0 25	+ *2 32*	+ *2 5*	+ 6 16	− 0 35	+·007	+*·046*	+*·034*	+·109	−·010	3¼	*·057*	*37*							·942	+ 0 17	+ 0 27	− 0 10		
Monte Video . .	23/31 xii. 80	− 0 26	− *4 2*	− *1 17*	+ 6 26	+ 0 34	−·007	−*·074*	−·021	+·112	+·010	4¼	*·077*	*196*												
Hong-Kong . .	20 xii. 81	− 0 6	− *1 0*	+ *1 15*	+ 6 19	+ 0 48	−·002	−*·018*	+*·021*	+·110	+·014	1½	*·028*	*130*												

* Compass re-corrected on this day.

TABLE III.—Coefficients (continued).

Place.	Date.	Approximate coefficients.					Exact coefficients.					Maximum of semi-circular deviation. $\sqrt{B^2+C^2}$ Mean horizontal force of ship. $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$		Mean force to north. λ.	$\frac{1}{\lambda}$.	Coefficients of horizontal induction.		Part of 𝔇 from		Mean vertical force. μ.	Heeling coefficient to *windward*.	Heeling coefficient for		Variable part of vertical force.	
		A.	B.	C.	D.	E.	𝔄.	𝔅.	ℭ.	𝔇.	𝔈.	Amount.	Direction.			Headward. *a.*	To starboard. *e.*	Fore and aft induction.	Transverse induction.			Vertical induction in transverse iron.	Vertical force and induction in vertical iron.	$\frac{g}{\tan\theta}$.	*g.*
		° ′	° ′	° ′	° ′	° ′						°	°					° ′	° ′		°	° ′	° ′		
Composite vessels.																									
Ruby.—Composite screw corvette; 2120 tons; 1830 H.P.; 12 guns. Built, Hull; head N. 3° W.; launched, 10 viii. 76.—Standard Compass.																									
Sheerness	30 vi. 77	From east semicircle.				...	...	−·409	−·128	...	...	25½	·430	197											
,,	3 vii. 77	− 0 56	− *1 19*	− *0 45*	+ 5 48	− 0 53	−·016	−*·024*	−*·011*	+·101	−·015	*1½*	*·026*	*205*	·853	1·172	−·061	−·233	− 2 2	+ 7 50	1·444	+ 1 22	+ 0 41	+0 41	−·020	−·050
Gulf of Xeros	iii. 78	− 0 44	− *1 18*	+ *0 7*	+ 6 9	− 0 6	−·013	−*·024*	+*·002*	+·107	−·002	*1¼*	*·024*	*176*							*·941*	+ *0 30*		−*0 10*		
Singapore	vi. 78	+ 0 45	− *2 59*	+ *0 38*	+ 5 50	+ 0 15	+·013	−*·055*	+*·010*	+·102	+·004	*3*	*·056*	*170*												
Rangoon	x. 79	− 0 33	− *2 32*	− *0 58*	+ 5 0	+ 0 6	−·010	−*·046*	−*·016*	+·087	+·002	*2¾*	*·049*	*199*												
Zanzibar	12 iii. 80	+ 0 11	− *4 20*	− *0 26*	+ 4 42	+ 0 5	+·003	−*·079*	−*·007*	+·082	+·001	*4¼*	*·079*	*185*												
Gannet.—Composite screw sloop; 1130 tons; 1110 H.P.; 6 guns. Built, Sheerness; head S. 76° E.; launched, 31 viii. 78.—Standard Compass.																									
Sheerness	29 iv. 79	− 0 52	+11 20	+ 9 17	+ 3 22	+ 0 11	−·015	+·201	+·155	+·059	+·003	14½	·253	38	·895	1·117	−·052	+·158	− 1 39	+ 5 2	·864	+ 0 3	+ 0 24	− 0 21		
,,	30 iv. 79	− 0 43	+ *2 54*	+ *0 42*	+ 3 47	+ 0 7	−·012	+*·052*	+*·012*	+·066	+·002	*3*	*·053*	*13*												
Coquimbo	x. 79	+ 0 25	− *3 15*	− *1 24*	+ 3 55	+ 0 36	+·007	−*·058*	−*·023*	+·068	+·010	*3½*	*·063*	*202*												
Esquimault	vii. 81	− 0 2	+ *1 17*	− *0 53*	+ 3 26	− 0 21	−·001	+*·024*	−*·015*	+·060	−·006	*1½*	*·028*	*328*												
Albatross.—Composite screw sloop; 940 tons; 840 H.P.; 4 guns. Built, Chatham; head S. 50° E.; launched, 27 viii. 73.—Standard Compass.																									
Sheerness	17 xii. 73	− 0 38	+16 33	+ 6 17	+ 3 59	− 0 24	−·011	+·295	+·105	+·070	−·007	17¾	·315	20	·951	1·051	+·017	−·115	+ 0 34	+ 3 25	·820	− 0 10	+ 0 18	− 0 28	+·034	+·085
,,	20 iii. 74	− 1 31	+ *0 19*	+ *0 55*	+ 4 1	− 0 6	−·026	+*·005*	+*·016*	+·070	−·002	*1*	*·017*	*72*												
Rio de Janeiro	5 vii. 74	− 0 39	− *2 55*	− *0 45*	+ 4 14	+ 0 6	−·011	−*·053*	−*·012*	+·074	+·002	*3*	*·054*	*193*												
Plymouth	10 x. 74	+ 0 10	+ *0 10*	+ *1 23*	+ 3 28	− 0 41	+·002	+*·003*	+*·023*	+·060	−·012	*1½*	*·023*	*82*												
Taboga	27 x. 75	− 0 20	− *1 44*	− *0 12*	+ 3 44	+ 0 9	−·006	−*·031*	+*·003*	+·065	+·003	*1¾*	*·031*	*186*												
Pisco	15 xii. 76	+ 0 31	− *3 28*	+ *0 4*	+ 2 55	+ 0 41	+·009	−*·061*	+*·001*	+·051	+·012	*3½*	*·061*	*179*												
Esquimault	29 x. 77	− 0 47	+ *0 16*	+ *0 56*	+ 3 26	− 0 6	−·014	+*·005*	+*·016*	+·060	−·002	*1*	*·017*	*72*												
Boxer.—Composite double screw gun vessel; 603 tons; 590 H.P.; 4 guns. Built, Deptford; head S. 60° W.; launched, 25 i. 68.—Standard Compass.																									
Greenhithe	30 xi. 68	...	From N.E. quadrant			...	...	+·195	−·161	+·072	...	14¾	·253	320											
,,	,,	+ 0 11	+ *3 11*	− *1 27*	+ 3 39	+ 0 20	+·003	+*·057*	−*·024*	+·064	+·006	*3½*	*·062*	*337*												
Rio de Janeiro	20 ii. 69	− 0 6	− *0 49*	− *4 15*	+ 3 50	− 0 24	−·002	−*·014*	−*·071*	+·067	−·007	*4¼*	*·072*	*259*												
Esquimault	⅕ x. 69	− 0 8	+ *3 42*	− *2 53*	+ 2 33	− 0 20	−·002	+*·065*	−*·049*	+·050	−·006	*4¾*	*·082*	*323*												
Komax, Vancouver Island	24 iv. 71	+ 1 33	+ *2 52*	− *3 27*	+ 2 25	+ 0 19	+·027	+*·051*	−*·059*	+·042	+·005	*4½*	*·078*	*311*												
Esquimault	30 xii. 74	− 2 52	+ *4 4*	− *1 46*	+ 3 18	+ 0 24	−·050	+*·073*	−*·030*	+·057	+·007	*5*	*·079*	*338*												
Coquimbo	13 v. 75	− 1 05	− *0 59*	− *0 5*	+ 2 20	− 0 10	+·019	−*·017*	−*·002*	+·041	−·002	*1*	*·017*	*186*												
Pert.—Composite double screw gun vessel; 603 tons; 500 H.P.; 4 guns. Built, Glasgow; head S. 65° W.; launched, 22 vi. 68.—Standard Compass.																									
Devonport	3 ii. 70	...	From N.E. quadrant			...	...	+·251	−·186	+·065	...	18	·312	323											
,,	8 ii. 70	+ 1 0	+ *0 46*	− *1 04*	+ 3 50	− 0 31	+·017	+*·013*	−*·017*	+·067	−·009	*1¼*	*·021*	*307*	·939	1·065	+·002	−·124	+0 5	+3 45						
Elephant Bay, W. Coast of Africa	29 xii. 70	+ 3 9	− *3 30*	− *2 15*	+ 2 59	...	+·055	−*·062*	−*·038*	+·052	...	*4¼*	*·073*	*202*												
Rio de Janeiro	29 xii. 71	− 0 7	− *2 0*	− *2 13*	+ 3 15	− 1 35	−·002	−*·036*	−*·038*	+·057	−·028	*3*	*·052*	*227*												

TABLE III.—Coefficients (continued).

Place.	Date.	Approximate coefficients.					Exact coefficients.					Maximum of semi-circular deviation. $\sqrt{B^2+C^2}$			Mean force to north.	$\frac{1}{\lambda}$.	Coefficients of horizontal induction.		Part of $\mathfrak{D}$ from		Mean vertical force.	Heeling coefficient to *windward*.	Heeling coefficient for		Variable part of vertical force.	
		A.	B.	C.	D.	E.	$\mathfrak{A}$.	$\mathfrak{B}$.	$\mathfrak{C}$.	$\mathfrak{D}$.	$\mathfrak{E}$.	Amount.	Mean horizontal force of ship. $\sqrt{\mathfrak{B}^2+\mathfrak{C}^2}$	Direction.	λ.		Head-ward. *a*.	To star-board. *e*.	Fore and aft induction.	Transverse induction.	μ.		Vertical induction in transverse iron.	Vertical force and induction in vertical iron.	$\frac{g}{\tan\theta}$.	*g*.
		° ′	° ′	° ′	° ′	° ′						°		°					° ′	° ′		° ′	° ′	° ′		
Composite vessels (continued).																										
FIREFLY.—Composite screw gunboat; 455 tons; 470 H.P.; 4 guns. Built, Glasgow; head N. 26° E.; launched, 28 vi. 77.—Standard Compass.																										
Plymouth . . .	24 vi. 79	...	From	S.W. qu	adrant	...	...	−·330	...	...	...	18	·330	180												
,, . .	,,	+ 0 45	+ *0 55*	+ *0 57*	+ 3 45	+ 0 18	+·013	+*·016*	+*·016*	+·065	+·005	*1¼*	*·023*	*45*												
False Bay, C. of Good Hope . .	29 vi. 80	− 0 23	− *0 14*	+ *0 4*	+ 4 6	+ 0 4	−·006	−*·004*	+*·001*	+·071	+·001	*0¼*	*·004*	*166*												
Gaboon R. . .	30 viii. 80	+ 0 32	+ *1 3*	+ *1 42*	+ 3 40	+ 0 45	+·009	+*·019*	+*·029*	+·064	+·013	*2*	*·035*	*57*												
WRANGLER.—Composite screw gun vessel; 465 tons; 480 H.P.; 4 guns. Built, Barrow-in-Furness; head N.E.; launched, 5 x. 80.—Standard Compass.																										
Plymouth . . .	26 v. 81	− 0 37	−10 28	+20 28	+ 3 15	− 0 30	−·011	−·183	+·339	+·057	−·009	23	·385	117	·970	assu	med.				·943	+ 0 7	+ 0 15	− 0 15		
,, . . .	,,	− 0 4	− *0 41*	+ *0 15*	+ 4 38	− 0 31	−·001	−*·012*	+*·004*	+·081	−·009	*0¾*	*·013*	*162*												
Simon's Bay . .	31 xii. 81	− 0 52	− *1 5*	− *1 32*	+ 5 2	+ 0 9	−·015	−*·020*	+*·026*	+·088	+·002	*2*	*·033*	*232*												
Wooden ships with iron beams and vertical bulkheads.																										
ENCOUNTER.—Wood corvette, iron beams and iron transverse bulkheads from keel to lower deck. Built at Sheerness; head S. 75° E.; launched, 6 xi. 69.—Standard Compass.																										
Sheerness . . .	2 ix. 73	− 0 3	+13 24	+ 9 27	+ 4 12	+ 0 20	−·001	+·240	+·157	+·073	+·006	16½	·287	33	·940	1·063	+·008	−·128	+ 0 20	+ 3 53	·844	− 0 4	+ 0 20	− 0 24	+·002	+·005
,, . . .	9 ix. 73	+ 0 35	+ *3 32*	+ *2 8*	+ 4 14	− 0 11	+·010	+*·064*	+*·036*	+·074	−·003	*4*	*·074*	*29*												
Simon's Bay . .	11 iv. 74	+ 1 18	− *1 26*	+ *1 19*	+ 3 40	+ 1 0	+·020	−*·026*	+*·022*	+·064	+·017	*2*	*·034*	*140*												
,, . . .	15 x. 74	+ 0 36	− *1 10*	+ *0 27*	+ 4 43	+ 0 28	+·010	−*·021*	+*·008*	+·082	+·008	*1⅓*	*·022*	*160*												
,, . . .	20 i. 76	− 1 9	− *4 12*	− *0 12*	+ 2 48	− 0 59	−·020	−*·075*	−*·001*	+·050	−·017	*4½*	*·076*	*185*												
Grenada . . .	30 x. 76	+ 0 36	− *0 42*	− *0 37*	+ 3 20	− 0 22	+·010	−*·014*	−*·009*	+·059	−·006	*1*	*·016*	*218*												
SAPPHIRE.—Wood corvette, iron beams and iron transverse bulkheads from keel to lower deck. Built at Devonport; head E.; launched, 24 ix. 74.—Standard Compass.																										
Devonport . . .	21 viii. 75	− 0 22	+18 44	+ 7 24	+ 4 7	− 0 25	−·006	+·333	+·123	+·072	−·007	20	·352	20	·938	1·066	+·005	−·129	+ 0 9	+ 3 58	·831	− 0 6	+ 0 21	− 0 27	+·013	+·032
,, . . .	24 viii. 75	− 0 17	+ *1 3*	+ *0 45*	+ 4 9	− 0 17	−·005	+*·018*	+*·013*	+·072	−·005	*1⅓*	*·022*	*36*												
Simon's Bay . .	30 xi. 75	− 0 5	− *8 54*	+ *0 49*	+ 3 55	+ 0 20	−·001	−*·160*	+*·013*	+·068	+·006	*9*	*·160*	*175*												
Wellington, N. Z.	xii. 76	− 0 49	−*10 43*	+ *0 5*	+ 3 0	− 0 52	−·014	−*·188*	+*·001*	+·052	−·015	*11*	*·189*	*179*												
Sydney, N. S. W.	viii. 77	− 0 10	−*11 20*	− *1 25*	+ 3 48	+ 0 15	−·003	−*·203*	−*·024*	+·066	+·004	*12*	*·204*	*187*												

XIX. *Pelvic Characters of* Thylacoleo carnifex.

By Professor Owen, *C.B., F.R.S., &c.*

Received April 13,—Read April 26, 1883.

[Plate 46.]

Since the communication of the 1st February, 1883, "On the Affinities of *Thylacoleo*," I have received, through the favour of the Trustees of the Museum of Natural History, Sydney, and the care of the accomplished keeper, E. P Ramsay, Esq., F.L.S., a second consignment of the palæontological results of his exploration of the Breccia Caverns of Wellington Valley, New South Wales.

Besides additional confirmation of the dental, mandibular, antibrachial, ungual, and other osteal characters of *Thylacoleo*, these accessory specimens afford further evidence of the carnivorous modifications of parts of the skeleton. Of these a well-marked one is yielded by the pelvis (Plate 46, fig. 1).

I may premise that this part in *Felis* and *Macropus* shows the following differences:—

In *Macropus major* (Plate 46, fig. 2) the ilium, *i*, extends above, or anterior to, the acetabulum, *a*, into an elongate body of a triedral form: the two outer facets are deeply hollowed transversely, are separated from each other by a sharp well-produced ridge, and each is divided from the flattened and roughened mesial, or sacral, third surface by similar ridge-like productions bounding the respective margins of the outer non-articular surfaces. In *Felis* (Plate 46, fig. 3) the superacetabular body of the ilium, *i*, is a relatively broader plate of bone with the slightly concave outer surface undivided. The ilium, in *Macropus*, bends outward to its free tuberous end and contracts thereto. In *Felis* it slightly expands to the ridge-shaped free border. In *Macropus* the contour of the acetabular margin approaches a triangular figure, the truncate apex being forward or next the ilium; the basal part of the margin is cleft above the canal or groove leading from the ischium, *s*, to the bottom of the articular cavity, *a*. In *Felis* the contour of the acetabulum, *a*, is sub-circular; the iliac border is not produced into an angle. The synovial glandular pit is broader and shallower, and the mesial boundary of the canal leading thereto is not produced; the lateral boundary alone arches over part of that canal.

A marked difference between *Macropus* and *Felis* is the relative length and slenderness of the ischium, *s*, as it recedes from the acetabular region to the terminal tuberosity, s'. In the Marsupial it contributes four-fifths of the outer border of the relatively large obturator vacuity, *o*. In *Felis* the shorter, broader, and thicker body of the ischium, as it descends or retrogrades to the tuberosity, s', contributes but half of the outer border of the relatively smaller obturator foramen, *o*, the anterior third of each border running parallel with the acetabulum.

The portion of pelvis from the Wellington cave (Plate 46, fig. 1) corresponds in size, and includes the parts to which the above-defined characters of the carnivorous and herbivorous Mammals can be respectively compared. The ilium, *i*, is lamellar in form, not a triedral bar. The acetabulum, *a*, has a wide and shallow sub-circular synovial cavity, *p*, the entry to which is not encroached upon by the mesial and distal acetabular border, but is partially overarched thereby posteriorly or laterally.

The tuberosity, fig. 1, *t*, for the origin of the superacetabular tendon of the rectus femoris muscle is rather more prominent than in *Felis*, fig. 3, *t*; in *Macropus*, fig. 2, *t*, it is almost flat, and is indicated by a roughened surface.* The distance of this surface from the acetabulum is the same in the Kangaroo as is that of the process, *t*, in the cave-fossil; in the Lion it is nearer to the acetabular border.

In *Macropus major* a subquadrate process, *i*, with a basal breadth of one inch, extends for seven lines forward from the ilio-pubic commencement of the brim of the pelvis. This process does not relate to the attachment of a marsupial bone, it answers rather to the "anterior inferior spine" of the human ilium, and gives attachment to the main origin of the "rectus femoris;" it may relate, in both, to the action of that muscle in maintaining the erect, bipedal posture.

The "anterior inferior spine" is represented by a narrow rough slightly produced ridge from the same position in the leonine pelvis, and is not more developed in the cave-fossil. In this the facets, fig. 1, *m*, at the fore-end of the ischio-pubic symphysis for the attachment of marsupial bones, are present: but such surface is not upon an outwardly produced epiphysial bone, as in the full-grown *Macropus major*.

But now it may be asked—"Why is the comparison confined to the largest of the existing herbivorous Marsupials?" To which I reply that size seems to relate to the sphere of activity in which Marsupials obtain their vegetable food. Those that seek it in trees, like the arboreal Phalangers and Koalas, do not exceed a Cat or Rabbit in size, while the grazers attain a bulk which equals that of *Thylacoleo carnifex*, and the vegetarian contemporaries of that extinct species much exceeded in size the Boomer Kangaroo (*Macropus major*). Among the cave remains of the prey of *Thylacoleo* is part of a gnawed pelvis twice the size of that of the Boomer, but closely repeating the Macropodal characters of that part of the skeleton in the existing Kangaroos.†

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* In man it is a groove, and gives attachment to the so-called "reflected tendon" of the rectus femoris.

† 'Fossil Mammals of Australia,' 4to., vol. ii., pl. cxxx. (*Palorchestes*).

I have, however, pursued my comparisons of the Thylacolean pelvis with that of smaller Marsupial vegetarians, and also with the pelvis of the existing carnivorous Marsupials, the largest of which is small in comparison with *Thylacoleo.*

In all these existing Marsupials the triedral character of the elongate ilium is retained, but in a much less marked degree than in *Macropus.* The outer dividing angle or ridge is feebly developed, and the facets it divides are not excavated, not in any of them, at least, as in the Kangaroos. The anterior end of the ilium bends outward in the Koala, and, in a greater degree, in the Wombat. The dasyurine pelvis, figs. 4, 5, resembles in the minor indication of such bend that of the cave-fossil; and also shows the relative shortness of the ischium as it extends from the acetabulum, and the characters of that cup. But both *Thylacinus* and *Diabolus* differ from *Thylacoleo* in the minor development of the superacetabular process, *t*, in the size of which the great pouched Carnivore resembles the Felines. Both, however, repeat in the mere roughened indication of the strong antacetabular process, *i*, in the Kangaroos, the character by which the Felines resemble *Thylacoleo.* The triangular facets, *m*, at the fore-end of the ischio-pubic symphysis, for the attachment of the pouch-bones are as in *Thylacoleo.*

Thus it was seen that in the points in which the Fossil differs from the Feline, it adheres to the Marsupial type, especially as shown by the larger still existing pouched Carnivores; as, for example, on the shorter extent of the ischium below the acetabulum and in its tuberous end being continued more squarely mesiad, instead of bending forward to join the pubis, as in *Felis* (fig. 3, *o*).

On the whole the fossil pelvis most resembles the dasyurine modification of this part of the skeleton, but by characters too nearly similar to those in the equipedal Diprotodonts to sanction a reference of *Thylacoleo* to the Polyprotodont group.

One character by which the larger Carnivore differs from the existing Marsupials, and agrees with the placental Carnivores, viz.: the superacetabular tuberosity, *t, e.g.*, indicates a disposition of powerful limb-muscles in harmony with their frequent actions in a predatory quadruped engaged in pulling down a larger vegetable-feeder. In the differential modifications of the pelvis of the terrestrial pouched grazers and browsers may be seen relations to muscular developments needed for the transfer of locomotive power, in rapid flight, to one pair of limbs, the hinder ones, in which the larger terminal horn-sheaths of the digits approach the character of hoofs. The smaller fore-paws retain the unguiculate structure, relating, in the female, to the economy of the pouch.

If the foregoing details be deemed tedious, or seem superfluous, in addition to the deductions as to the life-work of the extinct Marsupial founded on its skull, its teeth, its fore-limbs, I must plead the deserved reputation as a Comparative Anatomist of the distinguished Member of the Royal Society, who doubts the carnivority of the

Thylacoleo;* and the testimony which Professor FLOWER cites† in support of his doubts, may also have weight with many readers:—"Mr. GERARD KREFFT, the able Curator of the Australian Museum, Sydney, in the 'Annals and Magazine of Natural History,' vol. xviii., ser. 3, p. 146, 1866, gives his opinion that '*this famous Marsupial Lion was not more carnivorous than the Phalangers of the present time.*'"

The value of this testimony I leave to the judgment of the Fellows of the Royal Society of Sydney, and of the Linnean Society of the same capital of New South Wales.‡

ADDENDUM.

(Added August, 1883.)

[Since the Report by the Referees on the preceding Paper was submitted to the Council of the Royal Society, I have received from the Secretary the following remark made by one of the Referees:—

"I think the value of the Paper would be enhanced if it contained a distinct statement whether the reference of the fossil to *Thylacoleo* is inferred or based on such circumstantial evidence as leaves no room for doubt in respect to its identification."

The following is submitted in compliance with the Secretary's suggestion. The circumstantial evidence consists in the absence of any other fossils, showing a carnivorous type of dentition, agreeing in size with limb-bones of like indication and corresponding size, save jaws and teeth, of a *Thylacoleo*. The arrival, subsequent to the reading of the Paper, of Mr. ED. W. RAMSAY, F.L.S., as Superintendent of the Australian Department of the Fisheries Exhibition, has added, by verbal details and supplementary specimens, direct confirmation of the deductions from his previous transmission of the described fossils. If no other specimens than the limb-bones had been received they would have impressed me with the conviction that a Carnivore exceeding in size the existing Thylacine, in the degree in which the Lion surpasses the Wolf, had co-existed with the Diprotodonts, Nototheriums, and other large phytophagous Marsupials.

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* "What was the particular form of food associated with the most singular dentition of *Thylacoleo*, it would be hazardous to do more than conjecture. As the flora of the country in which this strange animal existed has probably undergone as great a change as the fauna, it is not unlikely that the material on which it subsisted has passed away with the creature itself. It may have been some kind of root or bulb; it may have been fruit; it may have been flesh."

† "On the Affinities and probable Habits of the extinct Australian Marsupial, *Thylacoleo carnifex* OWEN," Quarterly Journal of the Geological Society, vol. xxiv., 1868.

‡ Proceedings of the Linnean Society of New South Wales, 8vo., 1883, p. 187 ("On Tooth-marked Bones of Extinct Marsupials," by CHARLES DE VIS, B.A.).

The extirpating cause of the *Felis spelæa* together with the huger Herbivores (*Elephas, Rhinoceros, Megaceros, Bos primigenius*), represented by remains in British caverns, may be inferred to have operated in relation to the analogous evidences in Australia. That cause I conceive to have been Prehistoric Man.]

Description of the Plate.

PLATE 46.

Fig. 1. Os innominatum, *Thylacoleo carnifex,* nat. size.
Fig. 2. Ib. *Macropus major,* $\frac{1}{3}$ nat. size.
Fig. 3. Ib. *Felis Leo,* do.
Fig. 4. Ib. *Thylacinus cynocephalus,* $\frac{1}{2}$ nat. size.
Fig. 5. Ib. *Diabolus ursinus,* do.

XX. *The Limiting Thickness of Liquid Films.*

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Communicated by R. B. Clifton, *M.A., F.R.S., Professor of Experimental Philosophy in the University of Oxford.*

Received March 6,—Read April 19, 1883.

[Plate 47.]

The experiments described in this Paper are an extension of our previous investigations on the properties of liquid films. The interest and the difficulty of such inquiries increase as the thickness of the films diminishes, and culminate when they are sufficiently thin to show the black of the first order of Newton's rings. We can in that case only infer from the colour that the thickness is less than a certain possible maximum. Our knowledge as to the real value of this maximum is, we venture to think, very uncertain, but it furnished, we believe, previous to our own investigations, the only clue to the thickness of a black liquid film.

In a Paper on the "Thickness of Soap Films" (Proc. Roy. Soc., 1877, No. 182, p. 345), we were however able to show, for the particular liquid and apparatus used :—

i. That the variations in thickness of the black portions of the films were but a small fraction of that thickness.

ii. That the thickness was independent of the breadth of the black ring.

iii. That it was also independent of the thickness of that portion of the film which appeared to the naked eye to be in immediate contact with it.

We also proved, on the assumption that the specific resistance of the liquid in the film was identical with that of the same liquid in mass, that the average thickness of the black films observed must have been 12×10^{-6} millims.

In a more recent Paper (Phil. Trans., 1881, p. 447) we have shown that this assumption is correct for thicknesses greater than 374×10^{-6} millims., below which the number of our observations was insufficient to enable us to arrive at a reliable conclusion. It was also shown that very slight changes in the temperature or hygrometric state of the air produce great variations in the composition of films formed of a mixture of soap solution and glycerine.

In order, therefore, to investigate further the properties of very thin films, it was necessary that the temperature and hygrometric state of the air in contact with the films should be more completely under control. An apparatus by which this end is obtained has been devised, and we have with it repeated our observations on the electrical resistance of black soap films, with all the advantages gained by the use of the electrometer instead of the galvanometer which was previously employed (see Phil. Trans., 1881, p. 457). These experiments were sufficient to test the constancy of the thickness of black films, but would not alone afford a trustworthy measure of its absolute value. If NEWTON'S value of the thickness corresponding to the beginning of the black be accepted as correct, a black film must be at least ten times thinner than the thinnest for which we have directly proved that the specific resistance is the same as that of the liquid in mass. It was therefore uncertain whether the physical properties of films of such different thicknesses were the same, and it was necessary to check by some independent method the absolute thicknesses deduced from the electrical experiments. The thickness of a single black film is indeed so small that it is probably impossible to measure it by any direct optical method. We have, however, succeeded in determining optically the average thickness of a number of such films, and have thus obtained the required independent confirmation of the results of the electrical observations.

We propose therefore to describe (1) the electrical, (2) the optical experiments, and finally to compare the results with each other, and with those already referred to which were obtained some years ago.

I. *Electrical experiments.*

The liquid employed in the experiments of which we have already published a description, was invariably PLATEAU'S *liquide glycérique*, to which was added 3, 5, or 7 per cent. of potassium nitrate. This liquid is admirable for many purposes. Films made with it are very persistent, thus allowing long continued observations to be made upon them, but they cannot always be depended upon to thin so far as to exhibit the black of the first order. Sometimes the black appears, and extends to a distance of two or three millimetres, or even more; at other times no trace of black is seen after several hours. We have not on any subsequent occasion been able to secure with a *liquide glycérique* a formation of black at all comparable in extent with that obtained in several of the experiments described in our first Paper above referred to. It was necessary for our purpose to discover a liquid from which films could be made possessing the two-fold property of persisting and of becoming black with tolerable regularity. Moreover, the black must extend to a distance of at least 11 millims. from the top, to admit of observations of any value being made upon it by the electrometer method. Experiments extending over many months were carried out with the object of discovering such a liquid, and a number of different solutions were examined, consisting of mixtures of glycerine, water and oleate of soda in varying proportions, potassium nitrate being in all cases added to increase the conductivity. It was found at last that a plain soap solution not containing any glycerine answered the purpose best, and the solution employed in the experiments about to be described had the following composition:—

	Grammes.
Oleate of soda	1·44
Potassium nitrate	2·88
Distilled water	100

Specific resistance.—The specific resistance of the soap solution was determined by the method described in our previous paper (Phil. Trans., *loc. cit.*).

At 12° C the specific resistance was 43·9 ohms per centimetre cube.
15 ,, ,, 40·8 ,, ,,
18·8 ,, ,, 37·0 ,, ,,

We may therefore assume without important error the specific resistance at the temperatures:—

	13°	14°	15°	16°	17°	18°
to be	43	42	41	40	39	38

Refractive index.—The refractive index of the solution was 1·337 at the temperature 16° C.

Description of the apparatus.—The apparatus constructed for us by Messrs. Elliott Bros. is represented, half size, in Plate 47, fig. 1. It consists of a box made of thick glass plates, bevelled at the edges, and cemented together. The internal dimensions are 10 centims. square by 16·25 centims. high. The lid is a glass plate 11·4 centims. square, which closes the box airtight by means of a little grease. To it are attached all the essential parts of the apparatus. The lid with its fittings is shown in plan in fig. 3.

The soap film is supported between two platinum cylinders B and F (fig. 1), each 32·5 millims. in diameter. B is screwed to the end of a brass tube A, with a rack running along its length, which passes through a hole in the centre of the lid. It can be raised or lowered by means of a pinion C (figs. 1 and 3). That part of it which extends above the lid is enclosed in a larger tube in such a way that it can be moved up or down without establishing any connexion between the inside of the box and the outer air. D is a brass rod terminating below in a stout platinum wire E, which is bent at right angles and carries the short cylindrical ring F, perforated with holes. The rod D passes through an ebonite sheath G cemented to the lid. H is an ebonite milled head by which the ring F can be moved laterally as well as up and down.

The needles are supported in the following manner:—J is a brass tube accurately fitting the ebonite sheath, K, and terminating in a milled head, L. It supports a rectangular prism of ebonite, through which pass two straight gold wires, the so-called needles. The extremities of the latter are soldered to fine insulated copper wires, which pass through a hole in the tube, seen at N, and are connected at the top of the tube to the binding screws, *c*, *c*. A glass shield, cemented to the ebonite clip P, and perforated with holes through which the needles pass without touching the glass, is employed to protect the pillar which supports the needles from the spray caused by bursting of the films. The tube J can not only be turned about its own axis, but the hole through which the sheath K passes being slotted, as shown at W, fig. 3, can be moved to and fro parallel to itself to a limited extent. By these means the needles can be inserted in the film in any suitable position. Only two needles are represented in the figure. There were three, but as a rule two only were used. *a* and *b* are binding screws for connecting the upper and lower supports of the film respectively with the circuit.

On the left of figs. 1 and 3, and in fig. 2 is shown the arrangement for saturating the inside of the box with moisture. It consists of an endless strip of linen O, passing over an ebonite roller Q, and kept stretched by an ebonite roller R below, the latter being weighted by a core of lead. The frame S carrying the strip can be raised or lowered by means of a nut T working on a screw fastened to the frame, and thus the linen can always be made to touch the liquid with which the bottom of the box is covered. The bevelled wheels seen at U, and the screw-head V, show how the upper roller is rotated so as to bring all parts of the linen in succession in contact with the liquid. The strip becomes elongated when wet, but by the nut T it can be drawn up to a suitable height. All the fittings connected with this part of the apparatus are of ebonite.

The hair hygrometer and the thermometer are not shown in fig. 1. They are supported on a single frame which is attached to the lid at Z, fig. 3. X is a plug closing a hole through which the liquid is introduced into the box.

To ensure constancy of temperature the film-box was placed in the centre of a glass tank full of water at the temperature of the room. The tank is made of thick glass plates, bevelled at the edges and cemented together. It is 30 centims. high by 25 centims. square. In the centre of it is fitted a square glass case of the same height as the tank, open top and bottom, and just large enough in cross section to allow the film-box to slide into it down to a fixed support. When the film-box is in its place it is surrounded by 7·5 centims. of water on every side except top and bottom. It was necessary to leave the top exposed in order to be able readily to move the needles or the linen strip, but the space underneath was filled up with cotton wool. By these means the temperature inside the film case can be maintained constant for many hours together.

The principle of the method of investigation employed was the same as that adopted in our previous experiments. A current from a battery of 9 LECLANCHÉ cells was passed through a film from top to bottom, and also through a box of resistance coils containing a resistance of one megohm. The binding screws forming the terminals of this resistance could be connected with the electrometer. The gold needles could also be connected with the electrometer. Thus the difference of potential between the two needles in the film could be compared with that between two other points in the same circuit separated by a known resistance, and the resistance of the film between the two needles thence determined. The independent difference of potential between the needles, when they were in the film, but when no current was passing, was troublesome, but was reduced to a minimum by carefully cleaning them with nitric acid at the beginning of each day's work. Every part of the circuit was carefully insulated, and the insulation of the two gold needles was specially tested at the end of each set of observations. If the result of the test was not satisfactory, the preceding observations were discarded. In the case of no experiment given below did the insulation-resistance of the needles fall below 300 megohms.

The observations were carried out in the following manner. The glass case having been thoroughly cleaned and the hygrometer adjusted to stand at from 40° to 50° on the scale, when in a saturated space, the cover with its fittings was introduced into its place, and the apparatus thus enclosed was placed in the centre of the glass water-tank. The ebonite plug X, fig. 3, was then removed, a tube terminating in a small funnel introduced, and about 20 centims. of the soap solution poured in. This quantity was sufficient to cover the bottom of the box to a height of about 3 or 4 millims. The tube was withdrawn and the plug replaced. The screw-head V was then rotated so as to bring every part of the endless band into the liquid. This operation was subsequently repeated from time to time to ensure the linen remaining thoroughly wet. The apparatus was then left to itself for 30 or 40 minutes, which interval was as a rule sufficiently long to allow the hygrometric state to become constant. The hygrometer index rose at first rapidly and afterwards more slowly to a limiting position which it steadily maintained. On one occasion the indication was observed at intervals during 36 hours, and did not vary more than a tenth of a scale division. The new apparatus has, in fact, enabled us to overcome the chief difficulties encountered with the old. We can now maintain the temperature and hygrometric state of the space round the film constant as long as we wish. To proceed:—The electrical connexions having been completed, the lower ring F was turned to one side by the button H, and the pinion C rotated until the mouth of the upper cup just touched the liquid. When it was raised, a plane film was formed over its mouth, and this, after the ring F was restored to its position, was blown out into a cylinder in the usual way. The needles were inserted in the film and adjusted so that their position could be easily observed by the cathetometer. The length of the cylindrical film was usually about 34 or 35 millims., and the upper needle was placed about 5 millims. below the cup B. The distance between the points where the needles pierced the film varied in different experiments between 4·6 and 5·3 millims. There were, indeed, as has been stated, three needles on the same support, but only the first and second were used, the third being bent on one side and not touching the films. On one occasion, however, the first and third needles were used, the middle one being bent back.

A mass of liquid having the form of a distorted circle, with its longer axis vertical, formed around each needle at the point where it entered the film. The horizontal wire of the cathetometer telescope was made to touch this circle at the top and bottom, and the mean reading was taken as that proper to the needle. The ratio between the mean diameter of these liquid masses and their distance apart was a necessary datum in applying a correction to the direct results. This correction was necessitated by the fact that the equipotential lines, which in the undisturbed film are horizontal circles, are distorted in the neighbourhood of the liquid masses formed by the insertion of the needles.

If a be the mean radius of these circles of liquid,
b their distance apart,

then calculation shows that the observed resistance of the film between the needles must be increased by the following percentages:—

$$\text{If } \frac{b}{a} = 6, \quad 7, \quad 8, \quad 9, \quad 10$$
$$\text{percentage to be added} = 6{\cdot}1, \ 4{\cdot}3, \ 3{\cdot}4, \ 2{\cdot}6, \ 2{\cdot}2.$$

The films, although they were all made from the same solution and thinned under apparently identical conditions, behaved very differently one from another. Sometimes a ring of black was seen to form a few minutes after the film was blown, and to extend rapidly downwards. At other times half an hour elapsed before any black appeared. The passage of the electric current has a considerable effect in retarding the initial formation of the black, and sometimes prevents it altogether. When the black is once formed, the passage of the current appears sometimes to check its growth, but this effect is not always observed. As a rule the circuit was not completed until the film was in a condition suitable for electrical measurements, or, in other words, until the black had extended to a distance of 1·5 or 2 millims. below the second needle. Few films reached this stage, and we considered ourselves fortunate if in a day's work we succeeded in making trustworthy observations on a single film. If the black reached the second needle it not unfrequently continued to spread far below. On three or four occasions the film became black from top to bottom, a distance of 34 millims. The phenomenon of a cylindrical soap film, 32·5 millims. in diameter and 34 millims. long, black throughout its entire area, is a very remarkable one. Under these circumstances, so little light is reflected from any part of the film, that it is difficult to say at first sight whether a film is present or not.

The following table (Table I.) contains the results of our observations of 13 films. No results have been omitted from the table excepting such as we knew to be affected with error owing to defective insulation of the needles or other causes.

TABLE I.

Date and number of film.	Temperature.	Distance between needles. (*b*)	Mean radius of liquid mass round each needle. (*a*)	Number of observations from which the result is derived.	Length of *black* when film broke.	Time the film lasted.	Time occupied in electrical observations.	Resistance of black (ohms per millim.).	Percentage to be added.	Resistance corrected for disturbance of equipotential lines.†	Resistance of liquid in mass at temperature of observation (ohms per c.c.).	Thickness of black (10^{-6} millims.).
	°	millims.	millim.		millims.	minutes.	minutes.					
1882.												
Dec. 21, III.	14·5	4·95	0·6*	6	14	35	15	320,600	3·0	330,200	41·5	12·31
„ 26, III.	14·2	4·82	0·6*	8	27	. .	. .	299,000	3·4	309,200	42·0	13·30
„ 26, IV.	14·5	4·65	0·6*	5	Throughout	. .	. .	294,000	3·6	304,600	41·5	13·34
„ 26, V.	14·8	4·625	0·6*	12	Throughout	. .	. .	297,000	3·6	307,700	41·2	13·11
„ 27, IV.	15·6	10·13	. .	5	30	. .	. .	273,000	. .	273,000	40·4	14·49
1883.												
Jan. 8, I.	14·4	5·42	0·59	1	15	25	3	317,000	2·5	324,920	41·6	12·54
„ 8, II.	14·7	4·85	0·55	5	20	53	11	504,000	2·8	518,100	41·3	7·81
„ 8, III.	14·8	4·85	0·56	10	Throughout	75	30	546,000	2·9	561,800	41·2	7·18
„ 10, VI.	14·0	4·625	0·65	5	Throughout	73	18	368,000	4·2	383,500	42·0	10·73
„ 20, VII.	16·1	4·815	0·53	4	30	48	18	316,000	2·6	324,200	39·9	12·05
„ 20, VIII.	16·3	4·84	0·63	6	33	50	28	340,900	3·7	353,500	39·7	11·00
Feb. 8, II.	14·8	5·30	0·62	6	13	90	20	319,000	3·0	328,600	41·2	12·28
„ 8, III.	15·0	5·32	0·70	11	29	180	120	310,700	3·7	322,200	41·0	12·47
											Total . .	152·61
											Mean . .	11·74

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* Approximately; the diameter was not accurately measured.

† The correction is only carried to four significant figures.

Our observations show that the black, at least in a cylindrical film, does not become thinner by lapse of time or by increase of area. In illustration of this point we may refer to Film III., Jan. 8, and Film III., Feb. 8.

Film III., Jan. 8.—The measurements began when the length of the black portion was about 11 millims., and were continued until the whole area of the film was black. The first measurement was made 45 minutes after the film was formed.

The resistance of the film between the needles

50 minutes after the film was formed was 2·63 megohms
60 ,, ,, ,, 2·69 ,,
72 ,, ,, ,, 2·67 ,,

The mean of all the observations was 2·672.

Film III., Feb. 8.—This film, after being formed, was left to itself for an hour. At the end of this time about 15 millims. were black, and observations were at once commenced and were continued at intervals during two hours, at the end of which the film burst. The extreme length of the black was 29 millims.

After 1^h 20^m the resistance was 1·644
2^h 20^m ,, ,, 1·607
2^h 50^m ,, ,, 1·680

The mean being 1·654.

Although in any given film the thickness of the black appears to be fairly constant, it will be seen from the above table that the thicknesses vary a good deal in different films. The most serious deviations from the mean occurred on Jan. 8 in Films II. and III., the values deduced from these being 7·82 and 7·19. We have no reason however to think the experiments on this day less trustworthy than others, as none of the precautions usually taken to ensure accuracy were neglected. Defective insulation at the needles, involving a deviation of the circuit, would have resulted in an increased and not a diminished value of the thickness. The fact that Film I. of the same date yielded the value 12·53, which does not differ much from the mean of the others, precluded the possibility that a different liquid had by some mischance been used. The number 12·53, however, had been derived from a single observation, and hence might appear to be less trustworthy than the others. To set all doubt on this point at rest, the specific resistance of the liquid used was without delay redetermined, and was, as was expected, found to be normal.

II. *Optical experiments.*

The object of these experiments was, as has been already stated, the measurement of the mean thickness of a number of black soap films by an optical method.

A FRESNEL'S optical bank, of the pattern devised by Professor CLIFTON, was fitted with the apparatus necessary to produce interference bands by means of thick plates. The plates were specially prepared by Messrs. ELLIOTT Bros., and as their thickness was 18 millims., a considerable separation of the two interfering rays could be obtained. The angle between the two plates of the compensator could be altered, so that the sensitiveness of the instrument was under control and the angular motion of the whole compensator could be read off correct to 1′ by means of a vernier. A small brass table was placed between the mirrors. It was carried by one of the sliders of the bank, and could be raised or lowered by rackwork. To this a brass plate, 460 millims. long by 50 millims. broad and about 3 millims. thick, was firmly clamped. It was furnished with two pairs of brass V-pieces, in which glass tubes about 400 millims. long and 18 millims. in internal diameter were placed. The ends of the tubes were ground and closed with plates cut from the same piece of plate glass. The requisite adhesion between the plates and tubes was obtained by slightly moistening the extremities of the latter.

All the different parts of the apparatus were marked, so that after each readjustment they could be readily replaced in the positions they previously occupied. With this precaution it was possible to remove the tubes and set them up again many times in succession without displacing the interference bands from the field of view.

The light employed was that of an oil lamp, and to prevent disturbance by heat from this, the apparatus was set up in front of a draught closet within which the lamp was placed. The window was then closed and the air surrounding the apparatus was thus completely cut off from that in the neighbourhood of the flame. A large screen of stout pasteboard prevented any light or radiant heat from the lamp or from the window of the draught closet falling upon any part of the apparatus except the first mirror and its immediate surroundings.

Before performing an experiment the interiors of the tubes were thoroughly moistened with the liquid to be used, into which one of the extremities of each of them was then dipped. On withdrawing them plane films were formed in the tubes, which, if they were then inverted, ran a little way down them. A second film could then be formed in each by again dipping the ends in the liquid, and so on until the tubes contained between 50 and 60 films apiece. Being thus charged they were placed in the V-pieces, closed with the glass plates, and left undisturbed until the films had thinned sufficiently to make an observation possible.

During the whole of this process, each film, with the exception of the first and last formed, was only directly exposed to the air for the few seconds which elapsed before the next in order was made. A considerable quantity of liquid was retained between the films, so that when the tubes were closed by the glass plates the whole of the air within them must speedily have become saturated. In this saturated space the films remained for at least half an hour if formed of plain soap solution, or at least an hour if formed of *liquide glycérique* before they were ready for observation. The constitution of the films therefore cannot possibly have differed much from that of the liquids from which they were formed, and even if this were not so the changes in the refractive indices would have been too small to produce appreciable errors (see Phil. Trans., 1881, p. 485). When the films were thinning the field of view was traversed both by bands of colour due to the interference produced by the thin films, and by others due to the thick plates. To prevent confusion we propose to restrict the term interference *bands* to the first of these, and to call the second *fringes*. The instrument was so adjusted that the fringes were vertical and widely separated. Cross wires were introduced into the middle of the field of view, the compensator was placed in the vertical position and the central black fringe, exhibited when white light was used, was brought up to the vertical wire by slightly altering the orientation of the mirrors. A sheet of ruby glass was then interposed between the lamp and the mirrors, and the angles through which it was necessary to turn the compensator, to bring up to the vertical wire the fringes which were right and left of the central one, were measured. The mean of these values was taken as the angle corresponding to a wave length of red light, or according to our previous measurements to 615×10^{-6} millims. (Phil. Trans., 1881, p. 454).

The red glass having been removed, the "zero" or reading of the compensator when the central black fringe touched the vertical at its intersection with the horizontal cross wire was determined. It was then necessary to break a number of films in one of the tubes without in the least shaking or disturbing the apparatus. For this purpose one or two stout sewing needles had been enclosed with the films. A strong electromagnet was now used to move these, and a known number of films having thus been broken, the displacement of the black line was determined. In breaking the films care was always taken to leave if possible two or three unbroken at each end of the tube. Disturbance by the irruption of air from the outside, if the contact between the ends of the tubes and the glass plates was imperfect, was thus prevented.

If then

T be the average thickness of a film,
n the number of films broken,
μ the refractive index of the liquid,
λ the wave-length of red light,
δ the angular displacement of the compensator necessary to restore the central fringe, after the rupture of the films, to the position it previously occupied; and
α the angle (as above defined) corresponding to a wave-length of red light,

we have the equation

$$n(\mu-1)\mathrm{T}=\frac{\delta}{\alpha}\lambda.$$

When the tubes were placed in position the colours of the bright transmitted bands were, owing to the large number of films, very vivid, and the dark bands were very obscure. The fringes which had previously been made vertical were seen crossing the bright bands in a more or less sloping direction, but were completely lost in the darker portions of the field. When the films had thinned sufficiently to show by reflected light the white and black of the first order, the passage from the one tint to the other was, as is usual, so sudden as to appear discontinuous. The corresponding dark and light transmitted bands were very intense and the boundary between them was also perfectly definite and sharp. The fringes visible in the bright portion of the field (corresponding to the black seen by reflected light) were for the greatest part of their length vertical, but at first they often displayed a very considerable curvature at their lower extremities. In such cases they crossed the boundary at a very small angle and were lost in the dark band. The direction of the curvature was different on different occasions. The accompanying figure is a reproduction of a sketch on an enlarged scale made at a time when the phenomenon was very marked. The cause of the curvature was evidently an increasing difference of thickness between the films in the two tubes in the neighbourhood of the limits of their black portions.

As all did not thin at precisely the same rate, this limit was in different films at different vertical elevations, and it might at first sight seem probable that the curvature was due to the fact that the boundaries between the white and black were in the one tube lower than in the other. The interfering rays in a given horizontal plane would thus, in the one case, traverse black films only, while in the other they would also pass through some white ones, and, as the number of these would increase rapidly as the vertical elevation of the plane diminished, a distortion of the fringes similar to that observed might have been produced.

If this had been the true explanation, we should have expected either that the intensity of the illumination would have increased gradually on passing from the dark to the bright transmitted bands, or that a marked discontinuity would have been observed in the curved fringes at the points where the number of the white films traversed increased. Neither of these phenomena were observed. The dark part of the field was very intense close to the boundary and the curves were unbroken.

Another hypothesis which would serve to explain the phenomenon is that the black portions of the films increased in thickness near the junction with the white, and that this increase was different in the two tubes. No evidence of any such change of thickness was ever given by the electrical experiments, but it is, of course, possible that the behaviour of the cylindrical and plane films might in this respect be different.

Let n and n' be the number of the films in the two tubes. Let the mean thicknesses of the films in the first tube at the levels where the fringes crossed the boundary into the dark (transmitted) band, and where they became vertical, be t_1 and t_2, and in the second let them be t_1' and t_2'. Then the difference of the paths of the interfering rays would be

$$\{(nt_1-n't_1')-(nt_2-n't_2')\}(\mu-1)=\frac{\delta}{a}\lambda$$

where δ is the angular displacement of the compensator corresponding to the distance between the centre of one of the vertical fringes and the point where it cuts the boundary. Now since t_1 and t_1' are greater than t_2 and t_2' respectively, $\lambda\delta/(\mu-1)na$ is an inferior limit of the quantity t_1-t_2.

The following table gives the data by which the value of this limit was determined on several occasions. Lengths are given in terms of millionths of a millimetre.

TABLE II.—$\lambda=615$, $\mu-1=0{\cdot}4$.

δ/a.	n.	t_1-t_2.
1·26	36	53·8
0·75	49	23·5
0·92	31	45·6
0·84	58	22·2

This table shows that a very considerable change of thickness is necessary to account for the phenomenon. If the average thickness of a black film be taken as 12×10^{-6} millims., the films in one tube at the boundary of the black must have been from 3 to 5 times as thick as elsewhere.

This explanation is not without its difficulties. The thickness of the "beginning of the black," when the proper correction for the refractive index is made, is, according to NEWTON'S tables, 36×10^{-6} millims., whereas one of our experiments (if we adopt the above hypothesis) shows the average thickness of the thickest black parts of the film to be $(53{\cdot}8+12)\times10^{-6}=65{\cdot}8\times10^{-6}$ millims.

This discrepancy may, perhaps, be explained by the fact we have already referred to (Phil. Trans., 1881, p. 453) that measures on the diameters of NEWTON'S rings are of little value near the central black patch.

It is, however, difficult to understand why all the films in one tube should behave so differently from those in the other, while, if it be assumed that the difference is caused by a few films only, it becomes necessary to extend the limits of the black through an improbable range. The phenomenon was always transient, and the whole of the curvature took place within a distance of one or two millimetres.

The measurement of the shift of the zero was a matter of some nicety. The field of view was scarcely large enough to make it convenient to take the readings when the vertical wire bisected the interval between two dark fringes. The observations were therefore made by causing them to touch the vertical cross wire, and, as their outline was irregular, it required some care to decide upon the particular phenomenon which should be called contact. Two readings might easily differ by a degree or more, *i.e.*, by from $\frac{1}{13}$th to $\frac{1}{14}$th of a wave length. As a rule the errors were much less, and the mean of five readings was always taken. Another reading as large or as small as the largest or smallest would hardly ever have altered the mean by more than 6′, and we think the extreme possible error of the mean is not more than 10′. The most serious difficulty was due to the instability of the zero. The two tubes were mounted, as has been described, side by side, and the distance travelled by the interfering rays in unenclosed air was not more than 30 centims. Owing, however, either to slight changes in temperature or hygrometric state, or to some other undetected cause, the zero was continually moving. The motion was generally though not always in one direction. It was very variable in amount. Sometimes it was negligible, sometimes it produced a change of 4° or 5° in as many minutes. It is evident that the thickness of the films, given by any particular experiment, would be greater or less than the true value according as the motion of the zero was, or was not, in the same direction as that produced by the rupture of the films. The observed facts were in accordance with this, and the numbers obtained were generally larger or smaller than the mean according to the direction of the motion of the zero before the experiment. If therefore the motion remained constant during observations made on the two tubes, the results given by each would be oppositely affected, and the mean value would be correct. Even if, as was the case, it was impossible to ensure such constancy, the error of the mean would probably be much less than that of the individual observations. In all our experiments, therefore, the result of each was taken to be the mean of the numbers obtained by breaking the films first in one tube, and then in the other.

The following are the details of the last experiment performed with the *liquide glycérique*. The zero was on this occasion remarkably steady.

The ruby glass having been placed in front of the lamp, the angles through which it was necessary to turn the compensator to bring the fringes to the right and left of the central dark fringe into the position it previously occupied, were measured. These were always taken in the order—centre, left, centre, right, centre. The readings right and left were compared with the means of the two readings for the centre between which they were taken, to reduce the effect of any zero movement which might be in progress.

The angles given by two such sets of measures were

13° 41′, 13° 31′, 13° 25′, and 13° 59′ ; mean 13° 39′.

The ruby glass was then removed and the experiment proceeded with as follows:—

Films counted.—Tube I., 59. II., 53.

Position of zero.—Five readings, when the central black fringe was in the standard position, gave

1° 50′, 1° 58′, 1° 46′, 1° 45′, and 2° 13′ respectively.

Films counted again.—Tube I., 58. II., 53.

Position of zero read once more 1° 37′.

Mean of six readings 1° 52′.

Films broken in Tube I., then counted

Tube I., 4. II., 53.

New position of zero determined five times

6° 42′, 6° 11′, 6° 58′, 7° 12′, 7° 15 .

Films counted and found unaltered.

Position of zero read once more 7° 10′.

Mean of six readings 6° 55′.

Films broken in Tube II.—A slight delay occurred here, as some of the films did not break easily. When the operation was completed they were counted again

Tube I., 4. II., 4.

New position of zero determined five times

3° 26′, 3° 24′, 3° 2′, 3° 18′, 3° 26′.

Mean of five readings 3° 19′.

Hence the zero shifted 6° 55′−1° 52′=5° 3′=303′ when 54 films were broken in Tube I., and 6° 55′−3° 19′=3° 36′=216′ when 49 films were broken in Tube II.

As therefore $\alpha=13° 39'=819'$, and $\mathrm{T}=\delta\lambda/\alpha n(\mu-1)$ we get from the first tube (in millionths of a millimetre)

$$\mathrm{T}=303\times615/819\times54\times0{\cdot}4=10{\cdot}5,$$

and from the second

$$\mathrm{T}=216\times615/819\times49\times0{\cdot}4=8{\cdot}3.$$

Mean value T=9·4.

This result is less than usual. During the experiments on the first tube the zero was very steady, and the value obtained was almost exactly equal to the mean from all the experiments. Before the observations on the second tube, the zero showed a tendency to rise, and the slight delay which followed may have given time for a motion sufficient to reduce the second value to that actually found.

The following tables give the results of the experiments.

The films formed of plain soap solution did not last so well as those made of *liquide glycérique*. The numbers broken in the experiments are therefore smaller. Several experiments failed, as too many films broke before they were ready for observation, or, as some slight disarrangement of the apparatus was observed, which would invalidate the observations.

Two observations on the soap solutions gave results so widely different from the rest that they are omitted as probably incorrect. With these exceptions all the results obtained are included in the tables.

Column I. gives the number of minutes which elapsed after the tubes were set up, before the films were broken in tubes 1 and 2 respectively.

Column II. gives the number of films broken in each tube.

Column III. the average thickness of a film in millionths of a millimetre.

TABLE III.—Soap Solution.

I.		II.		III.
1.	2.	1.	2.	
40	50	21	30	14·4
30	40	32	46	11·7
160	180	34	29	11·9
30	35	23	26	11·4
90	95	36	33	10·6
37	40	17½*	18½*	11·8
35	45	17	13	13·2†
33	40	24	20	10·3†
33	40	34½*	32	13·4
				Mean 12·1

TABLE IV.—Liquide Glycérique.

I.		II.		III.
1.	2.	1.	2.	
87	93	35½*	31	10·6
85	93	27	30	10·2
80	85	28	32	12·5
60	63	52	51	10·7
65	70	50	49	10·2
75	82	49	45	11·0
65	63	54	49	9·4
				Mean 10·7

It will be observed that there is no relation between the time which had elapsed since the formation of the films and their thickness.

* Films broke during measurements.

† Zero very unsteady.

III. *Summary of results.*

We are now able to sum up the results of our experiments. Observations have been made upon three liquids, the properties of which are given in the following table.

Date of observation.	Nature of liquid.	Percentage of KNO_3 in water of solution.	Refractive index.	Specific resistance at 15°.
1877	Liquide glycérique	3	1·395	214
1883	,, ...	5	1·397	166
,,	Soap solution without glycerine .	2·88	1·337	41

The first and third liquids were examined electrically; the former by the galvanometer, the latter by the electrometer method. The second and third liquids were examined by the optical method.

In no case was there any evidence, when the liquid films were cylindrical, of a change in the thickness of the black portion. In the case of the plane films formed in the tube, the optical observations indicated an increase in the thickness of the black near its lower extremity. The evidence on this head is however doubtful. Whenever the area of the black portion of the film became somewhat extended the phenomenon, which may indicate a difference between the thicknesses of its various parts, disappeared. There seems, therefore, no doubt that in the case of films formed as in our experiments, the black portions assume a particular thickness either at, or soon after, their first formation, and that this remains unaltered either by lapse of time or by alterations in the dimensions of the black area.

Although, however, our observations prove that this thickness is practically constant for any one film, they indicate considerable variations in its magnitude for different films. The differences between the numbers given by the optical method are perhaps not much in excess of the probable error of experiment, but in the case of the electrical observations they far exceed it. They may be partly due to slight changes in constitution; but the following reasons negative the supposition that this is the only, or indeed an important, cause.

In the first place, the constitution of a *liquide glycérique* is more difficult to maintain unaltered than that of a soap solution. It is, however, in films formed with the latter that the greatest apparent variations in thickness occur. This, on the other hand, is in accord with the fact that films formed without glycerine are, as is proved by the colour phenomena they display, more uncertain and irregular in their behaviour than those made of the standard solution. Again, if a change of constitution took place, we should probably have detected it by progressive changes in the calculated thickness, which would in reality have been due to alterations in the specific resistance. Finally, the absolute constancy of the hygrometer and thermometer seem to preclude the possibility of any considerable change of constitution even in a black film. On the whole then we think that the thickness of the black portions is really different in different films. These differences are in general relatively small. According to NEWTON, the thickness of the "beginning of the black" would be 36×10^{-6} and 37×10^{-6} millims. for the *liquide glycérique* and the soap solution respectively. Apart, therefore, from our previous knowledge of molecular magnitudes, any thickness less than these would be equally probable; but the following table of the results obtained proves that both methods concur in showing that the average thickness is about $11{\cdot}6\times10^{-6}$ millims., while the electrical experiments show that the "probable error," or divergence of the thickness of any given film from the mean value, is $1{\cdot}2\times10^{-6}$ millims.

TABLE V.

Liquide glycérique.		Soap solution.	
Electrical method (1877).	Optical method (1883).	Electrical method.	Optical method.
12·2	10·6	12·32	14·4
11·9	10·2	13·32	11·7
12·0	12·5	13·36	11·9
11·6	10·7	13·13	11·4
12·0	10·2	14·46	10·6
. .	11·0	12·58	11·8
. .	9·4	7·82	13·2
. .	. .	7·19	10·3
. .	. .	10·74	13·4
. .	. .	12·07	. .
. .	. .	11·01	. .
. .	. .	12·29	. .
. .	. .	12·46	. .
Mean 11·9	10·7	11·74	12·1

Into the causes of the variations in the thickness of different black films we do not now propose to enquire. The above observations prove that they are comparatively small. The fact that the boundary between the black and coloured portions of a film is always well defined, and that there must therefore be a very sudden change of thickness in passing from the one to the other seems to point to a region of instability in the neighbourhood of the beginning of the black, such that films, the thicknesses of which are included within it, thin very rapidly to below its lower limit. Very short-lived films made of ordinary soap and water sometimes exhibit a grey tint, intermediate to the white and black of the first order, but persistent films, as far as our experience goes, never do. The foregoing observations seem to fix the limit of this region at about $14{\cdot}5 \times 10^{-6}$ millims., and to prove that films generally thin to below but not to very much below it, so that the thickness of black soap films rarely differs from $11{\cdot}6 \times 10^{-6}$ millims., by more than one or two millionths of a millimetre. It has never been observed to fall below $7{\cdot}2 \times 10^{-6}$ millims., and thus, without attaching any theoretical importance to the term, this thickness seems to be practically the *limiting thickness* of such liquid films as we have studied.

We conclude by summarizing the results arrived at in this and our former papers with respect to black soap films :

(1.) Persistent soap films, which thin sufficiently to exhibit the black of the first order of NEWTON'S rings, invariably display an apparent discontinuity in their thickness at the boundary of the black and coloured portions.

(2.) The whole of the black region at the time of or very soon after its formation is of a uniform thickness.

(3.) This thickness remains unaltered in any film, whether the coloured parts of the film are thinning or thickening, increasing or diminishing in extent.

(4). It is different for different films, but no connexion has been traced between its magnitude and the time which elapsed between the first formation of the film, and the first appearance of the black, or between either and the time of observation.

(5.) The mean values of this thickness are the same to within a fraction of a millionth of a millimetre, whether the films be plane or cylindrical, in contact with metal or with glass, formed of soap solution alone or with the addition of more than two-thirds of its volume of glycerine.

(6.) Two completely independent methods of measuring the thickness of the black portions of the films give concordant results.

(7.) The mean value of the thickness, calculated by giving equal weight to the results of the electrical and optical experiments, is $11{\cdot}6 \times 10^{-6}$ millims., the extreme values being $7{\cdot}2 \times 10^{-6}$ and $14{\cdot}5 \times 10^{-6}$ millims.

The smaller of these quantities is therefore a limiting thickness to which a soap film in air saturated with the vapour of the liquid from which it is formed rarely attains, and below which none of the films observed by us have thinned.

XXI. *The Direct Influence of Gradual Variations of Temperature upon the Rate of Beat of the Dog's Heart.*

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[Plates 48, 49.]

Introduction.

In the year 1881 I briefly described (1) a method of experimenting by which the heart and lungs of a Dog or Cat could be completely isolated physiologically from the remainder of the body of the animal, and kept alive some hours for study in an apparently normal condition, the heart beating regularly and maintaining a good arterial pressure. Since then I have been at work investigating the influence of various conditions upon the pulse-rate of Dogs' hearts so isolated; while under my supervision several of my pupils have been engaged in studying the work done in a unit of time by such hearts under different external conditions.

As regards the effects of variations of arterial pressure upon the pulse-rate of the isolated Dog's heart, my results have already been published (2); and detailed observations as to the influence of variations in venous pressure will shortly be printed. But in so far as the influence of temperature variations upon the cardiac rhythm is concerned, only a brief preliminary announcement (3) has been made. In the present paper I propose to give a full account of my experiments upon this subject, which is one that, apart from and in addition to its purely physiological interest, has considerable practical importance in connexion with inquiries as to the immediate cause of the quick pulse so constantly found in warm-blooded animals suffering from fever.

Almost all that we have hitherto known concerning the direct influence of temperature changes upon the cardiac rhythm is derived from experiments made upon cold-blooded animals, especially Frogs. As regards these animals all observers are agreed that as the temperature of the heart is gradually raised from near zero to about 40° C., its beat is quickened. It is clear, however, that we can only argue with much reserve from the heart of the Frog to that of the Mammal when the direct effect of temperature variations is concerned. The Frog can hardly be said to have any normal temperature, and has but slightly developed temperature-regulating physiological mechanisms; its healthy temperature varies from a very low point in midwinter to 32° C. or above on a Baltimore summer day. The Mammal, on the contrary, is constructed to maintain a definite normal temperature, which does not vary beyond very narrow limits; a departure of even a couple of degrees from this normal is always the sign or the cause of pathological processes. We find the Mammalia, accordingly, provided with highly complex temperature-regulating mechanisms, in possessing which they differ very sharply from the Amphibia. While it might therefore be expected *à priori* that the Frog's heart is so constructed as to work better at those warm temperatures at which the general nervo-muscular apparatus of the animal is most active, and the calls upon the organs of nutrition greatest, the discovery that such is actually the case and that the warmed hearts of Frogs beat quicker than cold, does not justify us in forthwith concluding that the Mammalian heart, placed in and adapted to the needs of an animal with only one healthy temperature, would behave in like manner. This doubt concerning the validity when extended to warm-blooded animals of arguments based on experiments made with the hearts of Frogs is increased when we call to mind the fact that an elevation, within physiological limits, of the temperature of the medium to which a cold-blooded animal is exposed increases its tissue metamorphoses, as evidenced by a greater excretion of carbon oxide, while exactly the reverse is the case in respect to the Mammal. Recalling the wonderful physiological adaptation of the organs of animals to the conditions under which they live, we might almost expect that increased temperature (not reaching pathological limits) of the blood carried to it would lead to a slowing of the beat of the Mammalian heart in correlation with the diminished oxidations then occurring in the body generally, and its consequently diminished nutritional demands.

There are still other reasons why the direct application to the Mammalian heart of the results of experiments upon Frogs is unsatisfactory. It has been shown (4) that the muscular tissue of the Amphibian heart differs considerably in histological characters from that of the Mammalian: with this difference in minute structure quite important functional differences may be associated. Moreover, the Mammalian heart is known to be far more under the control of extrinsic nerve centres than is that of the Frog. Though the heart of the latter animal receives cardio-inhibitory fibres through the vagus, their centre of origin is not usually in action, as shown by the fact that cutting the vagi does not lead to pulse-quickening; exactly the reverse and to a very marked extent is the case in the Dog (see especially V. BEZOLD) (5), and also in Man as shown by the phenomena observed in cases of atropin poisoning. In addition, the Mammalian heart receives from the cerebro-spinal centre accelerator nerve fibres, and the existence of any such pulse-quickening fibres in connexion with the Frog's heart is at present doubtful. Consequently, bearing in mind that greater division of physiological duties which characterises the higher animal, we may justifiably doubt whether the simple relation of higher temperature (within limits) and quicker pulse found in the Frog and dependent only on the properties of the heart itself, may not be entirely absent in the isolated heart of the higher animal, which we know to have its rate of beat under normal circumstances controlled by a highly specialised set of extrinsic nerve centres.

The above considerations, taken in connexion with the fact that "fever" can hardly be said to exist in an animal with so variable a normal temperature as the Frog exhibits, made it very desirable to study directly the influence of temperature variations upon the pulse-rate of the Mammalian heart.

The experiments hitherto made upon Mammalia do not really solve the question whether the quicker pulse of the warmer animal is due to a direct or an indirect action (*i.e.*, one exerted by extrinsic nerve centres) upon the heart. BERNARD, WALTHER, HORWATH, and no doubt others have found a slow pulse in artificially cooled animals; the same phenomenon has been observed in hybernating Mammals during their winter sleep. As regards the effect of heightened temperature upon the pulse-rate, BRUNTON (6) has showed that when Rabbits are heated the heart beats quicker. But when a whole animal is warmed or cooled we are not justified in concluding that because the heart beats quicker or slower therefore the temperature change has directly influenced the rhythm of that organ. Not only may temperature changes indirectly affect the heart through its extrinsic nerves, but they may also so alter tissue metamorphosis in various organs as to essentially modify the composition of the blood flowing through the heart; and we know that very slight alterations in the chemical composition of that liquid may profoundly influence the heart. Before we are entitled to state positively that changes of temperature directly influence the rhythm of the heart of the warm-blooded animal, we must have data based on experiments made with the hearts of such animals cut off from all possible control through extrinsic nerve centres, and supplied with nutriment of constant composition. The only experiments known to me which approach the fulfilment of such conditions are those made by several observers (SCHENK (7), WERNICKE (8), CLELAND (9)) on the influence of temperature changes upon the rate of pulsation of the hearts of embryo Chicks during the first three days of incubation. Such experiments afford, however, even a less safe ground for conclusions as to the adult Mammalian heart than do the experiments upon Frogs' hearts above referred to. The heart of three-day Chick embryos is but a protoplasmic mass, little differentiated, presenting neither definite muscular or nervous tissue, and without any developed controlling extrinsic nerves. From the fact that such a mass of hardly-differentiated embryonic cells contracts more frequently when warm than when cold, we cannot safely conclude that the adult heart, with its fully developed muscular and nervous tissues, and placed under the governance of nerve centres located outside it in the body, would, if isolated, respond in like manner to similar temperature changes.

While experiment upon the isolated hearts of Frogs, Fishes, and Bird embryos, combined with the changes in the pulse-rate observed when Mammals are heated or cooled, have led to a general consensus of opinion among physiologists that gradual and moderate increases of temperature quicken the Mammalian pulse by direct action upon the cardiac tissues, and moderate diminutions of temperature similarly slow the pulse, the proof that the action of such temperature changes was exerted directly upon the heart itself did not seem satisfactory, for the reasons above stated. Hence the investigation described in the following pages was undertaken.

The method.

The fundamental idea upon which all my work on the isolated Mammalian heart has been based is to occlude all vessels of the systemic circulation except those supplying the heart itself, while leaving the pulmonary circulation intact. The heart and lungs being supplied with blood alone retain their vitality; all extraneous nerve centres getting no blood soon die with the remainder of the animal. Moreover, the blood supplied to the heart passes through no organ of the body but the lungs, and in these it undergoes simple and well understood changes; no sudden chemical alteration in it due to the products of the abnormal activity or commencing death of muscle, gland, or brain is possible. As the blood flows around through heart and lungs time and again, it no doubt experiences a gradual deterioration due to loss of foods and gain of wastes from those organs; but this change is gradual and uniform, and if a sufficient quantity of blood be used, the accumulation of wastes (carbon dioxide being carried off by the lungs as in normal conditions) and the deterioration in nutritive quality do not for some hours alter its constitution to an extent which in any way interferes with the forcible, regular, and normal beat of the heart. The means adopted for renewing the blood circulating through heart and lungs, as also for maintaining constant arterial and venous pressures and for regulating the temperature of the blood not having as yet been published in detail, and the method also having been much modified since the preliminary account was published, it is necessary to describe with some minuteness the operation of isolating the heart and the apparatus employed for subsequently keeping it alive under approximately normal and readily controllable conditions. I do this the more readily as the present form of the apparatus is the result of more than a year's experience and the accumulated improvements suggested by several workers (among whom special acknowledgment is due to my friends and pupils W. H. Howell and F. Donaldson), so that it now leaves little to be desired in the convenience with which it admits of keeping a heart under conditions in which venous pressure, arterial pressure, and temperature are readily ascertained and controlled. So far as the present series of experiments (those relating to the effect of temperature changes upon the pulse-rate of the isolated heart) is concerned, Dogs only have been used, and defibrinated strained Calf's blood has been the medium employed to nourish the isolated heart.

The animal having been placed under the influence of chloroform, ether, morphia, or curaré, the further course of an experiment was as follows :—

After tracheotomy the pneumogastro-sympathetic trunks were divided on each side of the neck with the object of saving the heart from the results of the powerful excitation of the cardio-inhibitory centre in the medulla oblongata, which usually occurs later, when the blood-supply of the brain is cut off. A cannula was also placed in the cardiac end of each common carotid artery, the arteries being clamped on the cardiac sides of the cannulæ. Next, the first pair of costal cartilages and the bit of sternum lying between them were cut away, and artificial respiration commenced; then the internal mammary arteries were tied as they pass forwards from the subclavians to the breast bone. The whole front and sides of the thorax were next cut away, and the right subclavian artery dissected out and tied just above the point at which it separates from theright carotid. The superior vena cava was then prepared, and ligatures placed loosely around it ready for subsequently occluding the vessel and tying in a cannula.

Proceeding now to the left side of the chest, the subclavian artery is ligated, and, the left lung being gently held aside, the aorta is isolated and cleared near the diaphragm. A ligature is placed loosely around the vessel, just beyond its arch, and a strong clamp tightened on it to the distal side of this ligature. An aperture having been made in the thoracic aorta, near its posterior end, a cannula of the form represented in Plate 48, fig. 4, and filled with defibrinated strained Calf's blood, is inserted into the vessel, and, the aortic clamp being removed, is pushed up to the left end of the aortic arch, where the ligature above-mentioned is tied tightly around it. These aortic cannulas are made of thin brass tubing, and are kept at hand of several sizes, so that one can always be found which fits tightly into the aorta of the animal, and is closely clasped by the elastic walls of that vessel. The cannula has on its distal end the bit of rubber tubing, *v*, on which is the clamp, *w*, which is screwed tight when the tube is filled with defibrinated blood before its insertion into the artery.

So far all the systemic arteries but the coronaries of the heart are occluded. Each common carotid has a cannula in it; both subclavians are ligated below the point at which they give off any branch, and the aortic cannula is tied in at a level of the vessel, just beyond its arch, at which it has given off no bronchial or intercostal branches.* As one consequence, violent dyspnœic symptoms usually occur in spite of the steadily maintained artificial respiration, being of course due to the want of a

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* Sometimes in young Dogs a minute branch is given off from the innominate artery to the thymus. This was sometimes tied, but usually neglected, as it is difficult to get at, and the amount of blood drained off by it is trivial, and when both venæ cavæ are tied cannot get back to the heart.

supply of fresh blood to the respiratory nerve centre. To complete the preliminary operation the inferior vena cava is tied above the diaphragm, and the right lung being pushed towards the median line, the vena azygos is ligated near its junction with the superior cava; the latter vessel is then tied below the point where the innominate and internal mammary veins join it by tightening one of the ligatures already described as placed loosely around it.

The next step is to wash out the blood contained in the heart and lungs, and replace it by defibrinated blood. For this purpose the cannula z (Plate 48, fig. 3), connected with the MARIOTTE flask U, filled with defibrinated Calf's blood at the temperature 38° C., is inserted into the cardiac end of the superior cava, and tied there. The clamp on the tubing connecting the flask with the cannula is opened, and blood from the flask allowed to enter the right auricle. The clamps on each carotid, and on the aortic cannula, are then opened in turn for a short time so as to wash out all blood already in heart and lungs, and replace it with the defibrinated blood.

This having been done, and the clamps again closed, the animal, still tied on the dog-holder, is transferred to the warm moist chamber represented in outline in Plate 48, fig. 1; in this chamber it is thenceforth fed steadily with defibrinated blood of known temperature, supplied at a known and controllable pressure, and from the chamber it pumps out blood against a known and readily varied aortic resistance. The structure, contents, and preliminary preparation of the warm chamber have next to be described. It is 125 centims. long, 65 centims. wide, and 65 centims. high. It has no bottom, but when in use sits in a shallow iron trough (not represented in the figure) filled with water, and raised on supports which admit of BUNSEN burners being placed under it, by whose means the air in the chest is kept moist and warm. The roof, sides, and the end, A, are glazed; the end, B, is of wood, and perforated by apertures through which several tubes pass. The object of glazing most of the walls of the chamber is to enable a ready view to be had of what is going on inside it; this is apt to be interfered with by condensation of water on the glass during the course of experiment; this drawback may, however, be nearly entirely obviated by smearing the inside of the glass with glycerine.

In the chest are two MARIOTTE'S flasks, C and D, each of a capacity of about four litres. The flasks are entirely similar in all respects, but for the sake of clearness in the diagram the tubes only have been represented in connexion with C, while the water-jacket which surrounds each flask is only indicated with D. This jacket, E, is merely a cylindrical tinned-iron bucket, somewhat wider than the flask. It is filled with water, and has, in connexion with it, a syphon by which it can be readily emptied, and a supply tube through which it can be filled. The syphon and supply tube have been omitted in the figure. Their ends pass outside the warm chamber, so that the water in the jackets can be changed without opening the box. As the flask empties of blood when in use, it tends to float up in the water of the vessel E. This is prevented by the collar *a*, which fits round the neck of the flask, and is attached by the bars *b*, *b*, to the upper edge of E.

In connexion with C are shown the tubes which pass through the air-tight cork of each MARIOTTE's flask. These are four in number. Two (*c*, *d*) are used when the flask is to be filled with blood; the other two (*e*, *f*) are employed when the flask is at work supplying the heart. When C is to be filled, the tubes *e* and *f* are closed by clamps or stopcocks put on the pieces of rubber tubing attached to their upper ends. The clamp *g* on the rubber tube attached to the upper end of *d* (which tubing, as shown in the figure, passes through an aperture, G, in the roof of the chamber) is opened, as is also the stopcock *h*, which is placed on the course of the tube leading from *c* to the funnel F. Meanwhile the corresponding stopcock *h*′, on the tube leading to the flask D, is closed. Defibrinated blood poured into F then enters the flask C through *c*, and the air which it displaces is driven out through *d*. C having been four-fifths filled, the stopcock *h* and the clamp *g* are closed, and the clamp *i* opened.

From the tube *f*, which dips deepest into the flask C, leads the rubber tube *k*; this passes through the end B of the warm chamber, and the next part of its course is shown in Plate 48, fig. 2, where *k* is seen to lead to the stopcock *l*, which is connected with one limb of the Y-piece *m*, another limb of which is attached to the corresponding tube *k*′, leading from the flask D. The remaining limb of the Y-piece leads to the rubber tube *n*, which is seen again in fig. 1, after entering the warm chamber. There *n* is seen to be continuous with the T-piece *o*, in the vertical limb of which is the thermometer *p*. Beyond the T-piece is the stopcock *q*, which ends in the rubber tubing *y*.

The flask C having been filled, we next go to the flask D, in connexion with which much of the details of the tubing have been omitted; but in all respects the flasks C and D and their connexions are similar. In the lettering of the figures whenever a connexion of C is indicated by a letter, the corresponding connexion of D is indicated by the same letter with a dash: *h* of C answers to *h*′ of D; *d* of C to *d*′ of D; *i* of C to *i*′ of D, and so forth; so that a detailed description of the tubes connected with D is unnecessary.

To fill D the stopcock *h* is closed, and *h*′ opened, as is also *g*′, while *i*′ and *k*′ are closed. Defibrinated blood poured into F then enters the flask D, and is added until that flask is about one-fourth filled. Then the stopcock *h*′ is closed, the clamp *g*′ screwed up, and the clamp *i*′ opened. The further course of *k*′ is seen in Plate 48, fig. 2, where it is shown as joining *k* at *m*; it therefore ends also in the stopcock and rubber tubing *q* and *y*.

So far we have got the flask C four-fifths full of defibrinated blood and the flask D one-fourth full. It remains to fill the tube *f* and its fellow, and the system of tubes leading from both of them to the stopcock *q*, which during an experiment is connected with the superior vena cava, and has to supply the heart steadily with defibrinated blood. The tube *f* and its fellow have to act as syphons, and therefore the lower ends of *f* and *f′* must be above the level of the exit of *q*. To secure this, both flasks, C and D, are suspended by cords *r*, *r′*, which support each flask and its water-jacket. These cords pass over pulleys borne on a framework, H, I, J, K, attached to the roof of the warm chamber, and each passes at its distal end round a fastener, *s*; by means of these cords the Mariotte's flasks can be raised and maintained at any desired level within the warm chamber. In the series of experiments here described both flasks were raised to the same height, although in fig. 1, C has been drawn lower than D for the sake of showing their connexions more clearly in the drawing.

To fill the syphon connected with C, the stopcock *h* and the clamp *g* are closed; the clamp *i* is left open, as is the stopcock *l* (Plate 48, fig. 2), while *l′* is kept shut. Then *q* (fig. 1) is opened, and suction applied to the end of *y*; blood then flows out of C through *f*, while air enters through *e*; and this blood is supplied to *y* under constant pressure.

The cock *q* is now closed, as also *h′* and the clamp *g′*. The clamp *i′* is left open, the stopcock *l* (Plate 48, fig. 2) closed, and *l′* opened. When *q* is now once more opened and suction applied to *y*, blood from D passes out by the tube *k′* and reaches *q* through the tube *n* (fig. 1). D now, like C, behaves as a Mariotte's flask, and supplies blood to *y* under a constant pressure. If both flasks be raised to the same height above the level of the superior vena cava, with which *y* (as will be described immediately) is connected, we can supply a heart with blood from either flask at will. When the stopcock *l* (Plate 48, fig. 2) is open and *l′* closed, the heart is fed from the flask C; when *l* is closed and *l′* opened, the blood is derived from D.

The flasks and the syphon tubes are filled as above described before the operation on the Dog is commenced, and the stopcocks so arranged (*l* open and *l′* closed) that on opening *q* blood will be drawn from C.

The water-jackets around each flask being filled, the gas burners under the trough which supports the warm chamber are lighted. From time to time *q* is opened, and blood from C let flow through it. When the temperature of this blood, as indicated by the thermometer *p*, is about 37° C., the gas is turned low and the operation on the Dog, described above, is proceeded with. While the flasks are warming *g* and *g′* are left open to allow some of the air in each flask to escape as it becomes expanded by the heat. Just before transferring the animal to the warm chest, *g* is screwed up, but *g′* left open; *h′* is also opened, and care is taken that *h* is shut. Under these circumstances C supplies blood to *y* when *q* is opened, while D (only one-fourth filled) is cut off from all connexion with *q* but is ready to receive any blood poured into it from the funnel F, or flowing to it through the tube L.

When the animal is transferred to the chamber the portable Mariotte's flask (Plate 48, fig. 3) is carried along with it by an assistant, and still supplies the heart with blood. A bit of brass tubing, *u* (fig. 1), inserted into the lower end of the tube, *t*, is now connected with the distal end of the piece of rubber tubing, *v* (Plate 48, figs. 1 and 3), attached to the distal end of the aortic cannula. The clamp *w* is then opened wide and the left ventricle pumps into and fills the tube *t*, from whose distal end the blood enters the funnel *x*; from this funnel it passes along L to the stopcock *h'* and thence to the flask D. The tube *t* has a bore at least as wide as that of the thoracic aorta of the animal, so that the heart pumps freely into it.

Next, the superior vena cava cannula *z* (Plate 48, fig. 3) is slipped out of the rubber tube connecting it with the portable MARIOTTE's flask U, and quickly inserted into *y* (fig. 1), care being taken that *y* is first filled with blood. The stopcock *q* being then opened, the heart is steadily supplied with blood from C. This blood, after traversing the lungs, is driven out of the left ventricle through *t*, and flows back to D, where it collects; accordingly as C empties D fills. When C is nearly exhausted the stopcock *h'* is closed, and also the clamp *g'*; *i'* is opened, as is also the stopcock *l'* (Plate 48, fig. 2). Simultaneously *h* and *g* are opened, and *i* and *l* closed. D now becomes the feeding and C the recipient flask. When D in turn is empty and C full the reverse steps to those above described make C the supplying and D the receiving flask; and so on as often as necessary in the course of an experiment. As all the clamps and stopcocks lie outside the warm chamber the connexion of the flasks with the heart can be changed when desired without opening the chamber. During an experiment the tube L and the part of *t* outside the warm chamber are kept wrapped in raw cotton, as also the funnel *x*; and the openings G and G' are loosely covered with damp cloths.

To return to the steps immediately following the placing of the animal in the warm chest: *y* having been connected with the superior vena cava, the bellows hitherto used are disconnected from the tracheal cannula, and over this is slipped the delivery tube of one of the convenient respiration engines, driven by water pressure, manufactured by the Cambridge Scientific Instrument Company; this engine henceforth maintains uniform artificial respiration: its delivery tube is not represented in the figure, but enters the warm chest through an aperture in its back. Next a clamp is placed on the left subclavian artery, close to its origin. The vessel is opened between this and the ligature previously placed on it, and the bulb of a thermometer inserted into the artery. The clamp being removed, the thermometer is pushed down until its bulb projects into the aortic arch, and is then firmly tied in that position.

Finally the cannula M is placed in the right carotid of the Dog and the cannula N in the left, and the clamps on those vessels removed. These cannulas are in connexion with the lead tubes O and P, which pass out through the end B of the warm chamber, and are connected with manometers. One manometer is a FICK's spring manometer, and is used for indicating the pulse-rate; the other is a mean pressure mercury manometer, after MAREY, having in its bend a stopcock which is nearly closed, so that each pulse-beat is hardly visible on the tracing, but the mean pressure at any time in the carotid is indicated.* The pens of both manometers write over

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* In some of the earlier experiments only a mercury manometer was used. Owing to the doubts

one another on the paper of a LUDWIG's large kymograph. Below them, in the same vertical line, a chronograph pen inscribes seconds.

As soon as the carotid cannulas are inserted the front of the warm chamber (which had been removed to admit of placing the animal inside and performing the above described manipulations) is replaced. The gas burners below the trough supporting the warm chamber are turned up, and a pause made before beginning observations until the air in the chamber, which has been much cooled while the front was away, is again heated up to about 38° C.; and also until at least twenty minutes have elapsed since the complete occlusion of all the systemic circulation except that through the coronary vessels. Before the lapse of this time all signs of any activity of the extra-cardiac nerve centres cease, and the physiologically isolated heart is ready for experiment under conditions in which venous pressure, arterial pressure, and the temperature of the blood flowing through it are under very complete control.

By raising or lowering the MARIOTTE's flasks, C and D, venous pressure (*i.e.*, the pressure under which blood enters the right auricle) can be varied within wide limits. In the experiments described in the present paper it was always kept at that exerted by the weight of a column of defibrinated Calf's blood 15 centims. in height, except when the contrary is expressly stated. Aortic pressure can be varied by sliding the support Q, which carries with it the exit of the aortic outflow tube, up or down the vertical rod R. Only the lower part of this rod is represented in the figure; its upper end reaches to the ceiling of the room. In most experiments the height of Q was arranged so that the mean pressure in the carotid was about 100 millims. of mercury. I had supposed before trial that I could in this way keep mean arterial pressure absolutely constant. But in spite of the small resistance offered by the wide aortic cannula and the wide system of tubes leading from it to the outflow point S, it turned out that the pressure as measured in the carotid (and therefore in the aortic arch) did not depend entirely and simply on the difference of level between the root of the aorta and the aperture of S. The left ventricle pumped out so much blood as to get up some elastic tension in the aortic arch and the arterial stumps still connected with it, and the pressure due to this was added to that dependent on the height of the column of blood against which the heart worked and on friction in the outflow tubes through which it was driven. So long as the heart works with sufficient force to pump blood up to and out of S the resistance due to the weight of the column of blood to be lifted remains the same; if the rate of flow be slower, the resistance, and therefore the increased pressure due to friction, will be diminished, but in such wide tubes probably only to a trivial extent. When, however, any cause, such as change in temperature, deterioration in quality of the blood supply, impediment in the pulmonary flow, or gradual death of the isolated heart, influences the amount of blood pumped out in the unit of time by

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which have been cast upon the accuracy of this instrument when a very slow or a very quick pulse is to be recorded, the FICK manometer was subsequently added. It turned out, however, that this was unnecessary; the pulse-rate recorded by both manometers was exactly the same.

the left ventricle, then the elastic reaction due to distension of the stumps of the great arteries is altered. Hence, even while a heart is pumping blood freely out through the exit S, kept at a constant height, variation of arterial pressure, as measured in the carotid, may occur to the extent of 10 millims. of mercury pressure. Such variations will be noticed in some of the protocols of experiments given in this paper; but fortunately they in no way affect the question here considered, viz.: the influence of changes of temperature upon the rate of beat of the isolated heart. I have previously shown (2) that slow variations of arterial pressure between the limits of 30 and 150 millims. of mercury do not in the least influence the pulse-rate of the isolated Dog's heart, provided venous pressure and the composition and temperature of the blood be kept constant.

Venous pressure and, approximately, arterial pressure being kept uniform the temperature of the blood alone was altered in the experiments below described. The variation was effected in two ways. First by pouring a little heated (50° C.) or cooled (10° C.) blood into the funnel F from which it entered the flask not in use at that moment, and warmed or cooled the blood already in it. Then this flask was used to feed the heart and the other as the recipient, by opening and closing the proper clamps and stopcocks. This method was rarely used, as it sometimes produced secondary effects, due to the comparatively sudden changes of temperature in the blood supplied to the heart. A more gradual and uniform alteration in the temperature of the blood was secured by changing the water in the jackets around the MARIOTTE's flasks. Some hot water and some water cooled by ice to 5° or 10° C. were always kept at hand during an experiment. If a series of heating observations was to be made, some of the water already in the jackets was syphoned off, and replaced with warm, care being taken that the temperature never rose above 60° C., so as to avoid all risk of coagulating any of the proteids of the defibrinated blood: more hot water was added from time to time if necessary. To initiate a series of observations as to the effect of cooling, the iced water was of course employed. The best results were obtained when the temperature of the water in the jackets did not differ by more than 20° from the temperature of the blood in the flask. When either MARIOTTE's flask was in use the rapid bubbling through its contents of the air entering by the tubes *e* and *e'* ensured their thorough mixture.

Having waited, then, for the death of extrinsic nerve centres, and until the thermometer *p* had during some minutes indicated a tolerably even temperature, the water around the MARIOTTE's flasks was cooled or heated, and a series of observations commenced. The initial temperature usually lay between 37° and 38°, but, as will be seen in the experiment protocols which follow, was sometimes higher or lower. Tracings of arterial pressure and pulse-rate were taken at intervals varying from one to five minutes. When the tracing was completed an assistant immediately opened a small door in the front of the warm chamber, and read off the temperature of the blood flowing through the heart.

As regards this temperature, the question arose which thermometer to use; that, *p*, in the inflow tube, or that pushed down the left subclavian to the aortic arch. The former gave the temperature of the blood flowing through the cavity of the right heart; the latter the temperature of the blood in the left auricle and ventricle and aorta, and accordingly in the coronary arteries supplying the cardiac capillaries. *À priori*, there seemed little doubt that it would be the temperature of this latter blood, brought as it was into close relation with every muscle fibre and ganglion cell in the heart, which would exert an influence on the cardiac rhythm, if any did. Experiment soon confirmed this. Both thermometers were read in several experiments, and it was always found that the pulse-rate changes followed much more closely the variations of temperature indicated by the instrument in the subclavian. In most cases, accordingly, only the reading of this thermometer was undertaken, as it was very desirable to reduce to a minimum the time during which the door of the warm chamber was open.

The temperature observed was written on the kymograph paper over the tracing, along with the time at which the latter had been taken. After a pause, another tracing was taken, time and temperature noted as before, and so on throughout the experiment, which was usually continued until the heart began to show symptoms of weakened or abnormal action.

The roll of tracings was subsequently gone over carefully, and on the graphic record of each observation periods of twenty seconds marked out; the pulses during that time were counted, and the mean arterial pressure measured. The results were then put in tabular form, the actual pulses counted being multiplied by three, so as to give the rate per minute instead of the number of beats in twenty seconds. In the "detailed results" given below, six such tables are printed; as curves present very quickly and accurately to the apprehension the general outcome of long columns of figures, charts have also been constructed (Plate 49) giving the curves of temperature variation and pulse-rate change during two of these experiments.

Detailed results of experiments.

Before proceeding to the following tables, which give the actual figures as to pulse-rate and temperature for several experiments, a few words of explanation are desirable with reference to some three or four points.

First, it will be noted that for normal temperatures (38-39° C. in the left ventricle of the Dog, according to CLAUDE BERNARD) (10) the pulse is very fast. This is undoubtedly due to the section of both pneumogastrics, cutting off the heart from control by the extrinsic cardio-inhibitory centre, which is normally very active in the heart of the Dog. Upon atropin paralysis of the peripheral pneumogastric connexions with the heart V. BEZOLD and BLŒBAUM (5) found in this animal the pulse-rate sometimes increased 80 per cent. An increase of 80 per cent. above the average will more than account for the quickest pulse observed by me at normal temperatures.

Second, it will be seen that, quite independent of any changes of temperature, the heart beats slower towards the end of an experiment than it did at the beginning, although its action may still be regular and each pulsation powerful. This is undoubtedly due to altered nutrition resulting from the use of Calf's blood, as it was not observed, or at least not until much later in my earlier experiments (2), when Dog's blood was employed. In consequence of this gradual and progressive slowing of the pulse it might be objected in a cooling experiment that any observed diminution of the rate of heart-beat was dependent on other conditions than cooling of the organ. To meet this objection, in most instances after a series of cooling observations a series of heating has been made on the same heart, and these show that in every case the heart beats much quicker when again warmed. This makes it clear that the slow pulse previously observed was not due merely to progressive malnutrition of the isolated heart, but was mainly dependent on the lower temperature to which the organ was exposed. Taking for example Experiment I., we find that at 1^h 34^m P.M. the heart beat 246 times a minute at the temperature of 37°·8 C. Forty-three minutes later (at 2^h 17^m P.M.) it beat only 217 times per minute at the temperature 38°·1 C; but meantime the pulse-rate, at 1^h 57^m M.P., had been down to 73 per minute, the temperature being 27°·8 C. This slow pulse being followed twenty minutes later by one nearly three times as fast cannot of course have been conditioned by any progressive diminution of functional capacity dependent on the prolonged use of Calf's blood; this becomes still more obvious when, later on in the same experiment, we find a second cooling accompanied by a slower pulse, and a second heating by a quicker.

Third, it may be noted that in no case does any one of the experiments given last longer than two hours, and that, with one exception (Experiment VI.), it is stated that the observations had ceased because of some obvious abnormality in the heart's action. In my earlier experiments with isolated hearts a practically normal beat often lasted for four hours or longer. They were however carried out on a different plan, which allowed of the use of defibrinated Dog's blood to nourish the heart. Instead of permitting the left ventricle to pump blood out through a wide aortic cannula, the only exit left was through a narrow cannula in one carotid, and, in correspondence with this fact, the tube supplying the superior cava was also narrow. In the present series of experiments the widest possible cannulæ was placed in the aorta and vena cava, and all the tubing attached to these, and the stopcocks upon it, had a bore as wide as that of the cannulæ. Under such circumstances the heart pumps round three or four litres of blood in a very few minutes, and with a smaller amount the stopcocks and clamps used to make the flasks C and D alternately feeding and recipient, would have to be changed at such short intervals as to make it impossible to carry on any uniform series of consecutive observations. With the original method 1000 to 1500 cubic centims. of whipped blood was enough for convenient use, and this quantity it was possible to obtain from Dogs. When four litres or more of blood are wanted it becomes practically impossible to use Dog's blood, and so some other had to be selected. After several trials Calf's blood was chosen. This blood, however, nourishes the heart less satisfactorily, and hence the earlier indications of commencing death.

With respect to the choice of blood I add a few words which may be of aid to any one desirous to repeat my experiments. It is important to have it from quite young Calves; that is to say, from animals which are still suckling: a point of itself of some interest when considered in connexion with the well known fact that the chemical composition of the urine of the nurslings of Herbivora shows that their nutritional processes agree in the main with those of adult Carnivora, and differ essentially from those of the adults of their own species. In spite of all care I used to be frequently disappointed by the death of the isolated heart before any satisfactory number of observations could be carried out upon it, even in cases when I could think of no cause for the failure. Light broke upon me when the laboratory attendant, whose duty it was to bring the blood from the slaughter-house, remarked one day that it seemed to him that we nearly always got on better when he did not get the blood from "wharf calves." On questioning, I found that "wharf calves" was the term employed by Baltimore butchers to indicate animals which, though still young enough to yield veal, were of such age that they had long ceased to live on milk. Since the blood of such Calves has been rejected the percentage of failures has considerably decreased. It is hardly necessary to add that care must be taken that no extraneous matter enters the blood during its collection. Baltimore butchers stun the Calves and then cut their throats, and while the blood flows out vomiting frequently occurs and sends the contents of the stomach into the collecting pail. The blood from each animal has therefore to be collected separately, so that the quantity already obtained may not be rendered unfit for use by admixture with matters from the stomach of another animal.

Even with the best obtainable Calf's blood, however, the results are not as satisfactory as with Dog's blood. Not only does the heart die sooner, but other changes occur which shorten the time during which an experiment can be carried on. The most marked of these is lung œdema, which nearly always takes place in the course of an hour and a half, to such an extent as to seriously impede the pulmonary circulation and the æration of the blood in the lungs. In consequence, the supply of blood to the left heart is hindered, and the right heart becomes gorged, and its auricle finally paralyzed; and this, of course, puts an end to an experiment. Another trouble which is apt to occur when Calf's blood is used is considerable pericardial exudation, often to such an extent as to seriously interfere with the beat of the heart. This difficulty may, however, be readily avoided by cutting a small hole in the pericardium as soon as the heart is placed in the warm chamber. A third difficulty met with when Calf's blood is employed is more serious. Many observers have noted on the isolated Frog's heart, supplied with various nutrient liquids, a gradual increase in the bulk of the organ in the course of a prolonged experiment; this increase being due, apparently, to an alteration in the elastic modulus of the cardiac muscle. The same phenomenon is observed when a Dog's heart is fed with Calf's blood. Gradually the systolic size of the organ increases, until at last, even at the height of its systole, the heart very nearly fills the pericardiac sac. During the subsequent diastole there is, therefore, but little opportunity for the organ to expand and receive blood. When this state of things takes place, one sees on the tracings that a good arterial pressure is still maintained, and that the heart rhythm is regular, but the height of each pulse curve is much diminished; and on looking at the exit (Plate 48, fig. 1, S) of the aortic outflow tube, it is seen that the quantity of blood expelled at each systole is markedly decreased. If the heart be then examined it will be found so distended as to tightly fill the pericardium, and if the latter be carefully cut away the pulse-rate remains unaltered; but the heart now does again nearly, or quite, its original work: the pulse-curves on the tracing regain their previous extent, and the gush from the aortic outflow tube at each systole becomes as great as it was before the occurrence of the distension of the heart. The impediment to the heart's action, due to this expansion, may be avoided either by cutting away the pericardium before beginning a set of observations or by removing it later when it begins to interfere with the heart's action. Both methods have been used in the course of the experiments whose results are given in the present paper. In selecting special examples for publication it seemed best, however, to include, mainly, cases in which the normal state of things had been interfered with as little as possible; and in none of the tables which follow was the pericardium cut away before the commencement of the observations, and in only two cases (Experiments II. and IV.) during their progress. It seemed desirable to include these for the purpose of showing that, although the heart's effective work is much diminished when it has become so distended as to fill the pericardium, yet its rate and force of beat are unaltered.

The ill results of pulmonary œdema above described may be obviated to a great extent by pricking numerous holes in the lungs with a fine needle. This allows the liquid collected in the air cells and small bronchial tubes to escape, and relieves the pressure on the pulmonary capillaries, while it also allows air to reach the air cells. This operation in no way affects the general result so far as pulse-rate is concerned, the chief objection to it being the loss of blood due to trickling from the wounds. To avoid objections, only one case (Experiment IV.) in which the lungs were so pricked is included in the experiments detailed in the present communication.

Before leaving this question of the troubles attending the use of Calf's blood, I may state that some considerable experience has led me to the conclusion that the drawbacks more than balance the advantages, at least in so far as most experiments are concerned. If I had to repeat the investigation here described, I should certainly tie the aorta just beyond its arch, and connect the outflow tube *t* with the left carotid instead of with the aorta; pulse-rate and mean pressure could then be recorded by manometers placed in the right subclavian and carotid arteries, and in correlation with the narrowed outflow orifice, the feeding tube, *n*, of the heart could be narrowed. Under such circumstances much less blood would be pumped around in a given time, and it would be possible to obtain the quantity requisite for carrying on an experiment from Dogs instead of from Calves. Pulmonary œdema and loss of cardiac elasticity would then occur much later. Of course in other cases, as when, for example, the greatest amount of blood which could be forced out from the left ventricle in a systole was to be sought, or the work done by the left ventricle under varying conditions, it would be necessary to use the wide tubes and stopcocks which I have above described, and these would almost necessarily lead to the use of other than Dogs' blood for the nourishment of the isolated heart.

Fourth, as a final remark before proceeding to give experiment protocols, I call attention to the fact that in the following tables it will be seen that now and then a slight rise of temperature occurs in the course of a cooling experiment, or a slight fall in the course of a heating. Such breaks were nearly always due to the necessity of changing the feeding MARIOTTE's flasks from time to time. While C is emptying and D filling, it is not possible to ensure that when D is in turn connected with the heart, the blood in it shall always be exactly of such temperature as to fit into the series of cooling or heating observations which had been carried on with C. An endeavour was always made to make the observations with the alternate flasks regularly consecutive as regards changes of temperature, and it will be seen that, in most cases, this was attained. When it was not, the resulting temporary rises or falls of temperature serve only to verify the general result; a slight and transitory heating in the course of a general cooling experiment quickens the pulse, and *vice versâ*.

I now give, in tabular form, the results of six experiments.

Experiment I.

April 24, 1882.—The Dog used weighed 5790 grms. and was chloroformed during the operation of isolating the heart. Venous pressure throughout equal to that exerted by a column of defibrinated Calf's blood 15 centims. in height. Arterial pressure, measured in the right carotid, varied between 97 and 104 millims. of mercury. All the systemic vessels but those of the coronary system of the heart were occluded at 12^{h} 50^{m} P.M.

Number of observation.	Time, P.M.		Temperature, centigrade, indicated by thermometer passed through left subclavian to aortic arch.	Pulse-rate per minute.	Remarks.
	h.	m.			
1	1	33	37·5	240	
2	1	34	37·8	246	
3	1	38	35·5	204	
4	1	40	34 8	191	
5	1	42	33·8	178	
6	1	44	32·0	153	
7	1	46	31·5	148	
8	1	48	30·5	129	
9	1	50	29·9	119	
10	1	52	29·0	105	
11	1	54	28·0	82	
12	1	57	27·8	73	
13	1	58	28·3	79·5	
14	2	01	29·0	83	
15	2	04	29·6	88	
16	2	06	31·1	129	
17	2	07	33·0	155	
18	2	08	33·9	168	
19	2	10	35·5	190	
20	2	11	37·0	207	
21	2	13	37·9	223	
22	2	14	36 8	203	
23	2	15	37·3	209	
24	2	17	38·1	217	
25	2	19	39·5	233	
26	2	20	40·5	240	
27	2	22	39·8	225	
28	2	26	39·5	219	
29	2	28	38·0	198	
30	2	30	36·8	181	
31	2	31	36·0	179	
32	2	34	34·5	159	
33	2	36	34·5	160	
34	2	38	32·8	131	
35	2	39	31·8	114	
36	2	41	30·8	87	
37	2	43	30·3	84	
38	2	45	30·5	80	Irregular.
39	2	46	30·8	81	Regular.
40	2	47	31·3	84	Slightly irregular.
41	2	49	31·5	87	Regular.
42	2	51	32·1	118·5	
43	2	53	32·5	126	
44	2	55	33·1	135	
45	2	59	36·5	184	
46	3	00	36·3	160	
47	3	02	35·8	167	
					The pulse now became very irregular, and its rate fell rapidly in spite of a supply of warmer blood to the heart.

The results of Experiment I. are represented graphically on Plate 49. Each division along the abscissa corresponds to two minutes of time. The level of the abscissa line answers to a temperature of 25° C. and to a pulse-rate of 60 per minute. The continuous curve represents the pulse variations during the experiment. Each division on the height of ordinates drawn from any point of the pulse curve to the abscissa answers to ten pulse beats more than 60 per minute. The dotted curve represents the temperature variations. Each division of height in ordinates drawn from it to the abscissa represents one degree centigrade above 25°. It will be observed that the curves of temperature and pulse-rate fall and rise together throughout the experiment.

Experiment II.

April 27, 1882.—The Dog weighed 5550 grms. Chloroform and ether administered during the operation of isolating the heart. Venous pressure that exerted by a column of defibrinated Calf's blood 15 centims. in height. All the systemic vessels but those supplying the heart itself were ligated at $3^h\ 10^m$ P.M. The animal was transferred to the warm chamber at $3^h\ 15^m$ P.M., and then decapitated and a stout wire run down the spinal canal as far as the lumbar region before any observations as to pulse-rate were made.

Number of observation.	Time, P.M.	Arterial pressure in left carotid, in millims. of Hg.	Temperature in aortic arch.	Pulse-rate per minute.	Remarks.
	h. m.				
1	3 30	100	38·0	237	
2	3 33	102	37·9	234	Since last observation one carotid cannula had slipped out and been replaced.
3	3 40	99	38·5	241·5	
4	3 42	98	38·5	244	
5	3 44	98	41·5	273	
6	3 46	101	40·5	258	
7	3 47	99	40·9	261	
8	3 48	100	42·0	267	
9	3 52	100	42·0	265·5	
10	3 53	99	42·5	265·5	
11	3 54	100	42·5	250	
12	3 55	97	39·5	222	Flask changed since last observation; hence the rapid alteration of temperature.
13	3 57	97	37·0	198	
14	3 58	97	36·0	189	
15	3 59	97	35·5	175·5	
16	4 00	98	34·7	169	
17	4 01	97	34·0	162	
18	4 02	98	34·0	165	
19	4 04	98	33·9	153	
20	4 05	99	32·9	144	
21	4 06	100	32·5	140	
22	4 09	98	31·7	124·5	
23	4 11	100	30·1	105	Pericardium cut away since last observation.
24	4 12	100	30·0	105	
25	4 15	101	29·9	97	
26	4 17	101	29·5	88	
27	4 19	101	29·0	84	
28	4 21	103	28·5	76	
29	4 23	104	27·5	75	
30	4 26	108	27·3	66	
31	4 27	112	28·0	66	
32	4 30	108	28·1	69	
33	4 32	108	29·5	111	
34	4 33	108	31·5	129	
35	4 34	107	32·5	162	
36	4 35	109	34·0	183	
37	4 36	105	35(?)	150	
38	4 37	107	34·1	144	
39	4 39	105	33·5	135	
40	4 41	106	33·5	133·5	
41	4 43	105	34·0	135	Pulse irregular.
42	4 44	102	34·7	117	Pulse regular.
43	4 46	. .	. .	. .	Heart's beat now became very irregular and experiment was discontinued.

Experiment II. presents two points of special interest: in the first place the brain was removed and the cervical and dorsal spinal cord destroyed before the observations commenced, so that an additional security was obtained that no cerebro-spinal centres were influencing the pulse rate. In the second place it is one of the cases in which the heart became considerably distended during the course of the experiment, so that the pericardium had to be cut away. As will be seen, this did not at all affect the general result.

Experiment III.

May 3, 1882.—Dog weighed 6000 grms. Narcotised by subcutaneous injection of acetate of morphia before the operation of isolating the heart was commenced. Venous pressure at first that due to a column of whipped blood 10 centims. high, and afterwards to a column 15 centims. in height. Heart isolated at 12h 55m P.M.

Number of observation.	Time, P.M.	Carotid pressure in millims. of Hg.	Temperature. C.° in aortic thermometer.	Pulse-rate per minute.	Remarks.
	h. m.				
1	1 20	110	34·5	151·5	Venous pressure 10 centims.
2	1 23	110	34·9	162	
3	1 25	110	36·1	185	
4	1 27	110	36·1	186	
5	1 29	110	37·9	211	
6	1 30	110	39·3	225	
7	1 31	110	40·0	232·5	
8	1 33	111	40·5	235	
9	1 35	110	40·3	222	
10	1 38	110	38·5	202·5	
11	1 41	109	37·0	184·5	Venous pressure raised to 15 centims. between observations 10 and 11.
12	1 43	110	36·9	195	
13	1 45	110	35·5	168	
14	1 46	112	33·9	152	
15	1 49	110	33·5	156	
16	1 51	110	32·7	142	
17	1 53	110	32·1	129	
18	1 55	111	30·0	102	
19	1 57	111	29·1	94·5	
20	1 58	110	28·9	87	
21	1 59	96	28·0	67·5	Heart weakens and ceases to pump round before next observation.
22	2 06	42	31·5	63	
23	2 08	. .	. .	. .	
24	2 10	. .	. .	. .	Heart beat irregular and experiment discontinued.

Experiment IV.

May 10, 1882.—Dog weighed 10,300 grms. Narcotised by subcutaneous injection of acetate of morphia before commencing the operation of isolating the heart. Venous pressure that due to a column of defibrinated Calf's blood 15 centims. in height. Heart isolated at 12^h 25^m P.M.

Number of observation.	Time, P.M.		Carotid pressure in millims. of Hg.	Temperature in aortic arch.	Pulse-rate per minute.	Remarks.
	h.	m.				
1	12	45	118	34·1(?)	158	It seems almost certain that the reading of the thermometer in observation 1 was a degree out, and should be 35·1°.
2	12	47	117	34·5	151	
3	12	49	116	35·1	157	
4	12	51	116	35·5	156	
5	12	53	116	37·1	183	
6	12	54	116	39·0	195	
7	12	55	116	38·0	180	
8	12	56	116	38·3	181·5	
9	12	58	116	38·5	184·5	
10	12	59	116	38·5	183·0	
11	1	00	116	38·0	172·5	
12	1	02	112	37·5	166·5	
13	1	03	113	36·5	157·5	
14	1	05	112	35·0	137·0	
15	1	06	114	34·0	127·5	
16	1	08	114	33·5	126·0	
17	1	09	112	33·5	126·0	
18	1	11	111	32·7	114	
19	1	13	110	31·9	108	
20	1	14	111	31·5	102	
21	1	15	113	31·0	99	
22	1	17	113	30·5	92	
23	1	20	112	31·5	110	
24	1	22	112	32·5	119	
25	1	24	110	33·9	133·5	
26	1	27	110	34·0	129	Lungs pricked since last observation.
27	1	30	113	34·5	139·5	
28	1	32	114	35·0	140	
29	1	34	114	35·5	148	
30	1	37	(?)	36·5	179	Pericardium cut away since last observation.
31	1	38	101	38·0	192	
32	1	39	105	39·0	198	
33	1	40	107	39·6	199	
34	1	42	108	39·0	189	
						After this the heart suddenly ceased to pump round, and its right auricle was seen to be paralysed. The lungs were extremely œdematous.

The chart on Plate 49 represents graphically the results of the preceding experiment.

Experiment V.

May 22, 1882.— Dog weighed 5605 grms. Chloroform administered while the heart was being isolated. Venous pressure at first that due to a column of defibrinated Calf's blood 10 centims. in height, then doubled. Temperatures taken both in inflow tube (by thermometer *p*, Plate 48, fig. 1) and in the aortic arch by a cannula thrust down the left subclavian artery. Heart isolated at 1^{h} 30^{m} P.M. The mean temperature given in the sixth column is obtained by adding together the inflow and outflow temperatures and dividing by 2. It does not really represent the mean temperature of the heart, as while the inflow temperature is that of the blood in right auricle and ventricle, and the outflow (aortic) temperature that in left auricle and ventricle, the latter is also the temperature of the blood circulating in the walls of the heart itself.

Number of observation.	Time, P.M.	Carotid pressure in millims. of Hg.	Inflow temperature.	Outflow (aortic) temperature.	Mean temperature.	Pulse-rate per minute.	Remarks.
	h. m.						
1	1 55	94	36·3	36·5	36·4	227	Venous pressure 10 centims.
2	2 00	94	37·3	37·3	37·3	234	
3	2 05	94	37·3	37·5	37·4	238	
4	2 10	94	37·5	36·7	37·1	225	
5	2 12	93	38·5	37·5	38·0	231	
6	2 14	96	39·5	39·0	39·2	249	Venous pressure raised to 20 centims.
7	2 16	96	38·0	38·5	38·2	244·5	
8	2 18	96	38·5	38·3	38·4	238·5	
9	2 20	97	39·0	38·7	38·8	241	
10	2 23	94	39·1	39·0	39·0	244	
11	2 25	94	40·3	39·7	40·0	249	
12	2 27	95	40·7	40·1	40·4	252	
13	2 30	97	40·0	39·9	39·9	252	
14	2 33	92	40·0	39·7	39·8	243	
15	2 35	92	40·0	39·5	39·7	233	
16	2 38	52	(?)	(?)	. .	102	Pulse very suddenly slowed and became somewhat irregular and experiment discontinued.

Experiment VI.

May 22, 1882.—Dog weighed 1140 grms. Chloroformed while the heart was being isolated. Venous pressure throughout that due to a column of defibrinated Calf's blood 20 centims. in height. Heart isolated at 3h 40m P.M.

Number of observation.	Time, P.M.	Carotid pressure in millims. of Hg.	Temperature of aortic blood.	Pulse-rate per minute.	Remarks.
	h. m.				
1	4 05	101	37·5	173	
2	4 07	99	36·9	170	
3	4 09	98	36·7	163·5	
4	4 14	98	36·7	156	
5	4 17	97	36·0	145·5	
6	4 20	97	35·1	132	
7	4 22	98	34·6	126	
8	4 23	98	34·1	117	
9	4 25	99	33·5	114	
10	4 28	97	33·0	108	
11	4 30	97	32·3	105	
12	4 32	96	32·5	91	
13	4 33	98	31·5	85	
14	4 34	98	31·1	85·5	
15	4 36	96	30·5	73·5	
16	4 37	96	30·1	76	
17	4 39	98	29·9	68	
18	4 41	95	29·5	69	
19	4 43	98	29·3	61	
20	4 45	98	28·9	63	
21	4 47	96	28·7	55·5	
22	4 51	98	28·7	61	
23	4 55	94	28·5	54	
24	4 57	97	28·5	54	
25	5 00	97	28·3	48	
26	5 03	96	28·1	52	
27	5 05	96	27·7	43	
28	5 08	94	27·5	28	Pulse irregular but each beat powerful.
29	5 10	90	27·6	24	
30	5 12	90	27·6	24	
31	5 14	92	27·6	21	
32	5 16	82	27·3	19·5	Heart ceases to pump blood to top of aortic outflow tube.
33	5 18	67	27·3	21	
34	5 21	58	27·1	21	
35	5 23	52	27·3	18	
36	5 25	49	27·3	18	
37	5 27	51	27·5	21	
38	5 30	45	28·0	19·5	
39	5 35	70	28·0	40·5	Pumps round again.
40	5 37	92	28·3	48	
41	5 40	93	28·5	58	
42	5 42	94	28·7	66	
43	5 43	96	29·1	73	
44	5 45	95	29·9	76	
45	5 49	93	30·5	82·5	
46	5 51	93	31·5	99	
					Experiment now discontinued. Heart still beating regularly and forcibly.

The above experiment is remarkable for the very slow pulse observed throughout. Even at 37°·5 the pulse was only 173 per minute, whereas in most isolated hearts it is over 200 at that temperature. When the temperature was brought down to near 27° the extraordinary slow pulse of 18 per minute resulted; a pulse so slow that although each beat was powerful the left ventricle pumped out in each minute less blood than was drained off from the aorta by the coronary arteries, so that the level of the blood in the aortic exit tube fell lower and lower until the carotid pressure finally came down to 41 millims. of mercury. On again heating the blood supplied to the heart the organ regained completely its functional activity. Before the cooling (observation 13) the pulse rate at the temperature 31°·5 was 85 per minute, and pressure in the carotid was 98 millims. of Hg. After the cooling, on again heating, we find at the same temperature (observation 46) a pulse of 99 per minute and a carotid pressure of 93 millims. of mercury. It is unfortunate that the experiment was not continued, but the exceptionally slow pulse obtained was not recognised until the tracings were counted out the next day, and as it was the second experiment of the same date I was fatigued and stopped so soon as I had satisfied myself that reheating the blood had quickened the pulse, instead of going on as usual until the heart began to show signs of commencing death.

Conclusions.

As regards the question which the preceding experiments were primarily designed to answer, their results are decisive. They make it clear that the Mammalian heart when quite cut off from all extraneous nervous control, and when supplied with blood which has not been altered in composition by products of abnormal tissue change, due to abnormal heating or cooling of other organs of the body, does beat quicker when warmer blood is supplied to it, and slower when it gets cooler blood. In this respect the heart of the Dog behaves quite like that of the Frog. In spite of the greater division of physiological duties in the body of the Mammal, and the greater subjection of the Mammalian heart to control from special extrinsic nerve centres, the Dog's heart in its own neuro-muscular apparatus is so constituted as to have its rate of periodic activity directly controlled by its temperature. To account for the quick pulse of fever we need therefore assume no paralysis of extrinsic cardio-inhibitory nerve centres and no excitation of cardio-accelerator. The warmed Mammalian heart beats quicker because of its own physiological properties.

In addition to the above main question, several subsidiary points have some light thrown upon them.

(1.) The rate of beat of the Mammalian heart does not directly depend upon the temperature of the blood reaching the right auricle, except in so far as this influences the temperature of the blood pumped out by the left ventricle and supplied to the coronary arteries. It is not the temperature of the blood in its cavities which influences the rate of beat of the Dog's heart, but the temperature of the blood sent to its capillaries. In other words, temperature changes do not influence the pulse-rate by stimulating afferent nerves in the endocardium which then act upon cardio-motor ganglia, but they act directly upon the muscle fibres or nerve cells of the organ.

(2.) A second subsidiary fact illustrated by the preceding experiments is that the heart of the Dog can be nourished for some time and kept in a good state of functional activity when fed only with Calf's blood: but this blood is far less satisfactory than Dog's blood, its use soon leading to pulmonary œdema and alteration of the elastic modulus of the cardiac muscular tissue.

(3.) As a third point of interest it may be noted that no clotting takes place in defibrinated blood circulated for some hours through the living heart and lungs. Such blood contains an abundance of fibrino-plastin (paraglobulin) and fibrin ferment, together with the quantity of salines necessary for the formation of fibrin if fibrinogen were present. Fibrinogen, therefore, is produced in other organs of the body than heart and lungs. By further experiments in which the isolated heart shall be connected with various other isolated organs and pump blood through them I hope to discover in what organs fibrinogen is produced.

It would have added much to the interest of the research described in the preceding pages if determinations had been made as to the highest and lowest temperatures at which the Dog's heart would beat, and I had hoped when commencing the investigation to have discovered those temperatures. It turned out, however, that with the method of work described in the preceding pages this was not possible. When the heart is considerably cooled, for example, it pumps around so little blood that the amount sent out at each systole of the left ventricle is less than that carried off by the coronary arteries. Under these circumstances the coronary system is mainly supplied with warm blood derived from the column of liquid accumulated in the aortic outflow tube (Plate 48, fig. 1, *t*.) As a consequence, the blood in the right heart comes to be of a very different temperature from that circulating in the cardiac capillaries, and the result is irregular and inco-ordinate action of the right and left sides of the heart, and a total cessation of all circulation. Quite similar results follow warming of the blood supplied to the right auricle to near the death temperature. Consequently I have not been able to discover the temperature limits of the vitality of the Dog's heart. Some preliminary experiments, carried on in a different manner, lead me to hope that the question as to the highest and lowest temperature at which a Dog's heart will beat can be solved; but my work in that connexion is not yet ready for publication.

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OF THE

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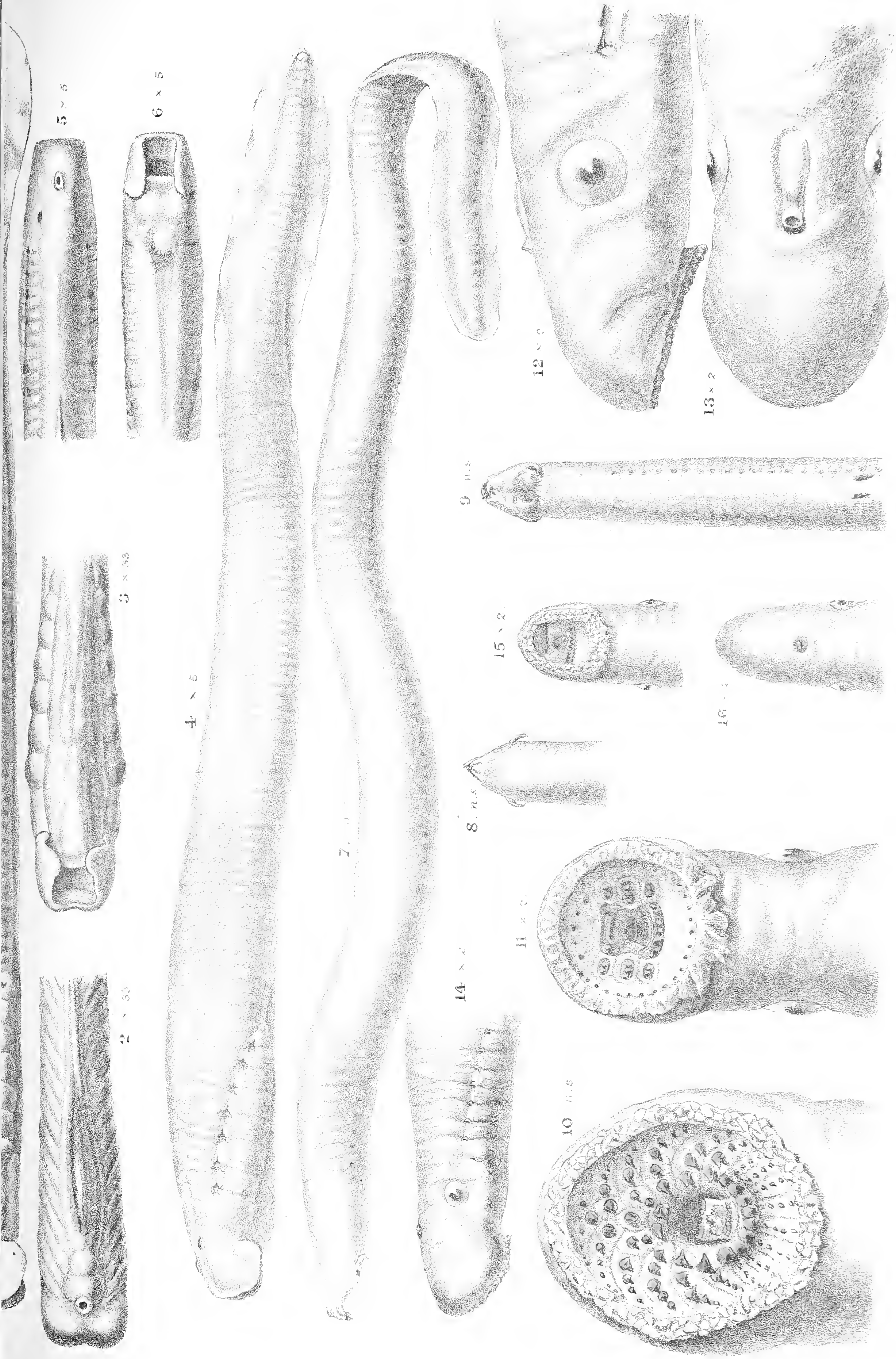
FOR THE YEAR 1883.



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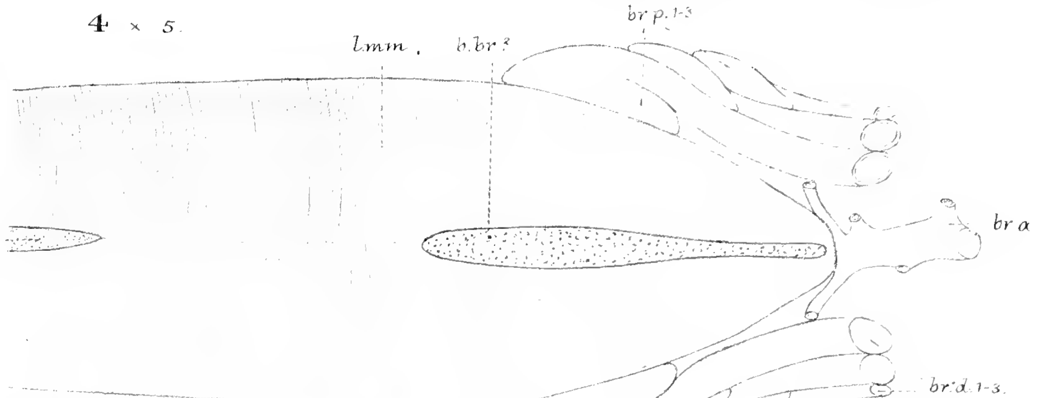
MARSIPOBRANCH FISHES

West Newman imp.



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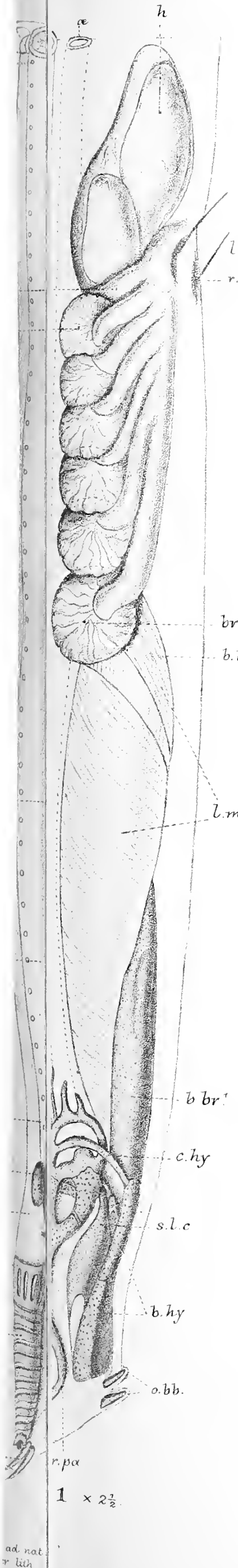

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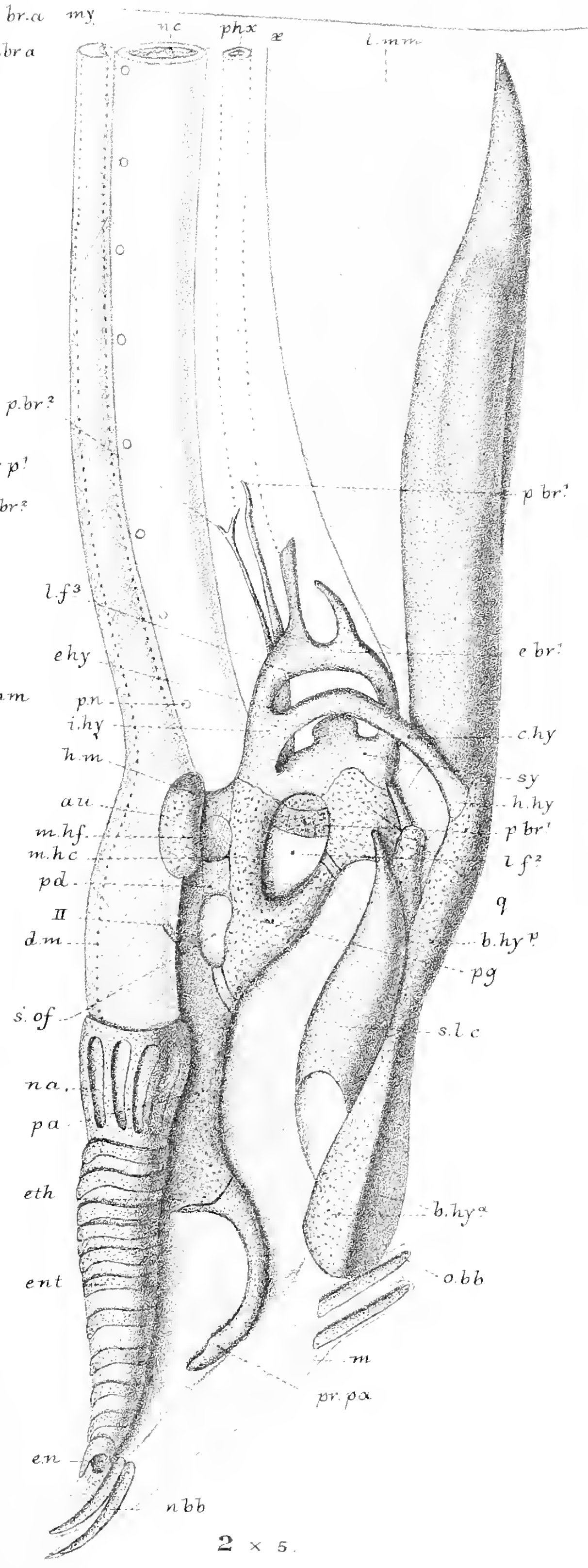

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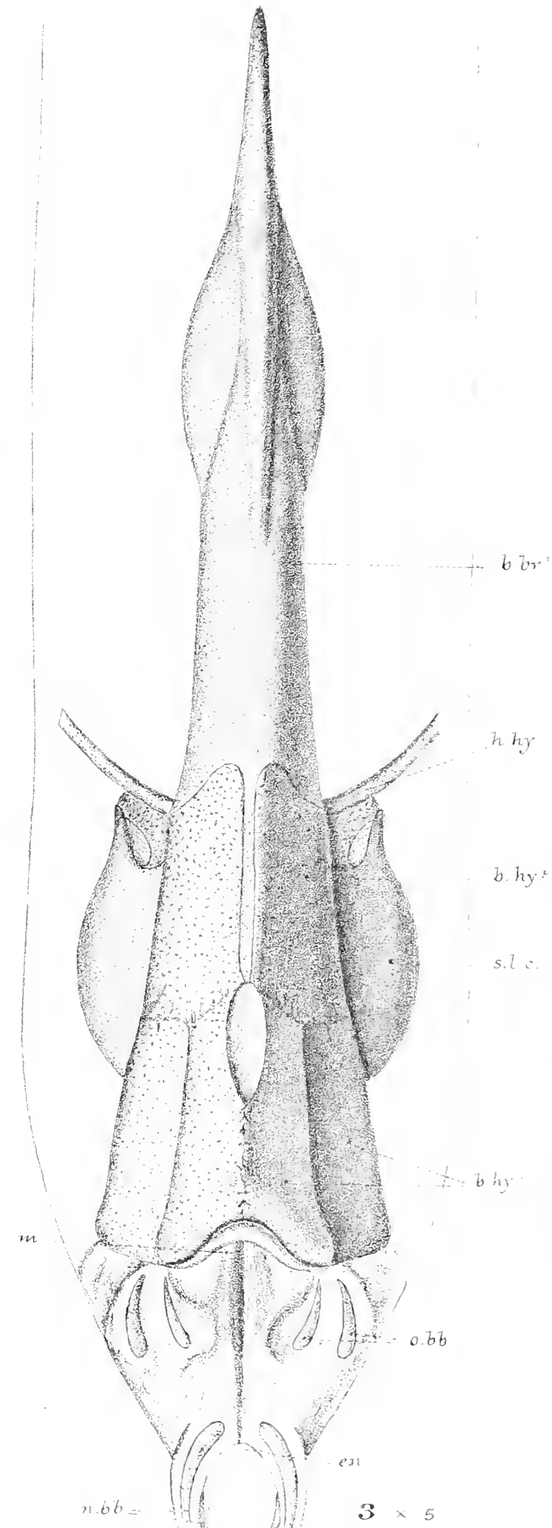

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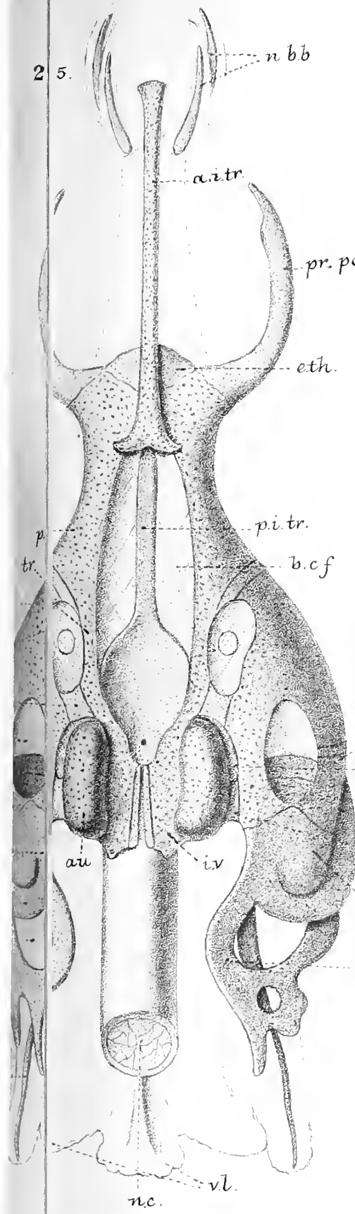

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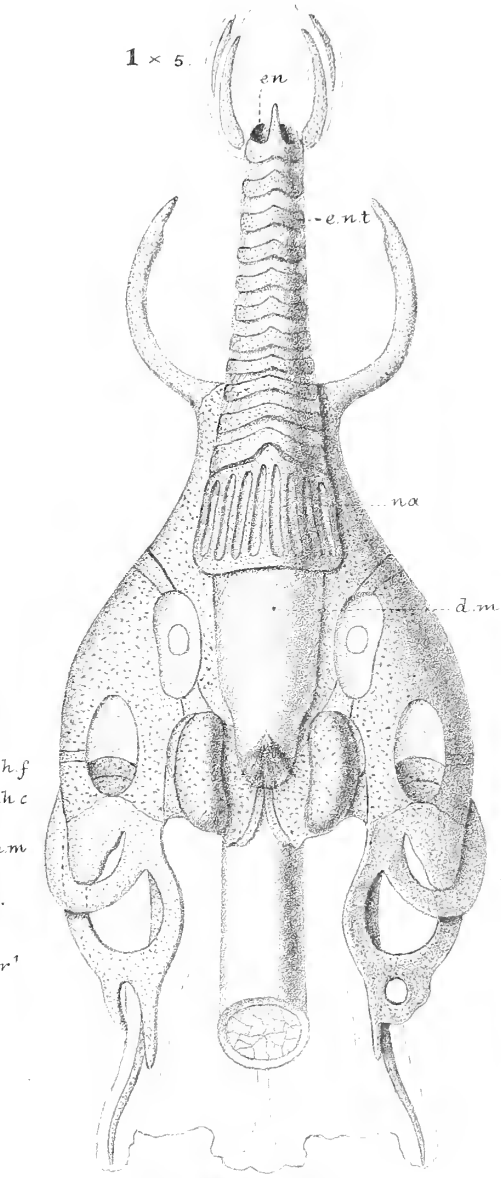

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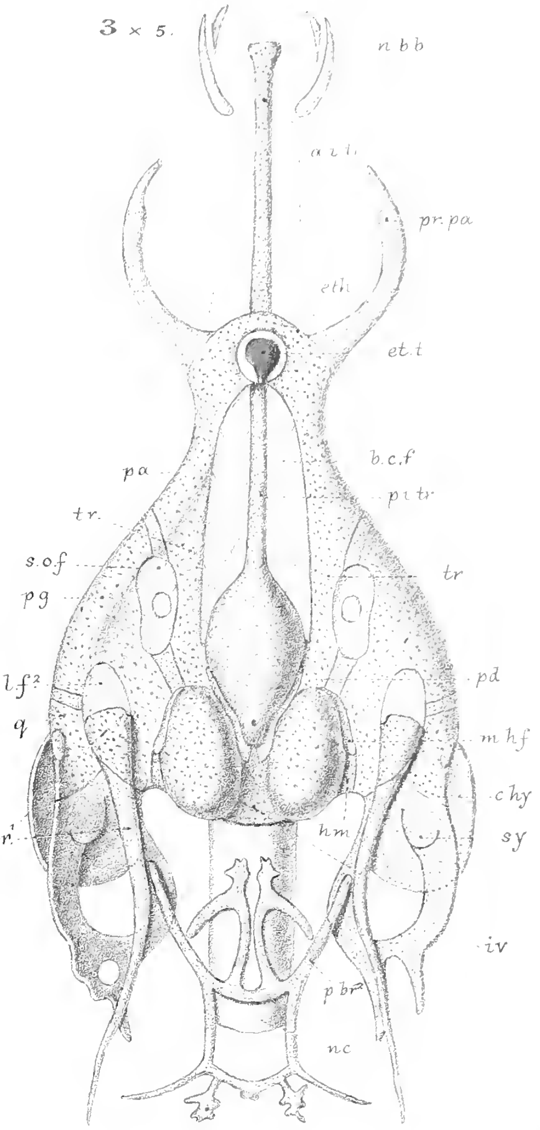

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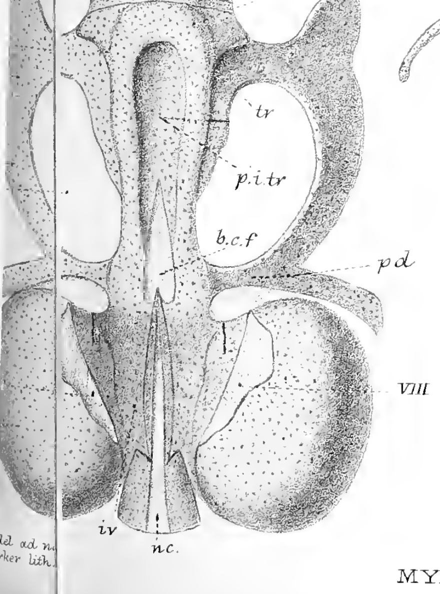

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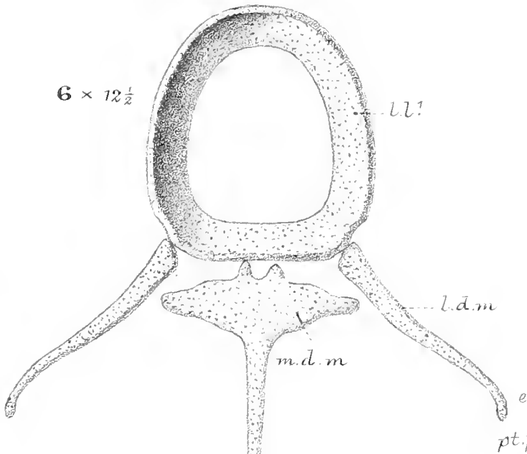

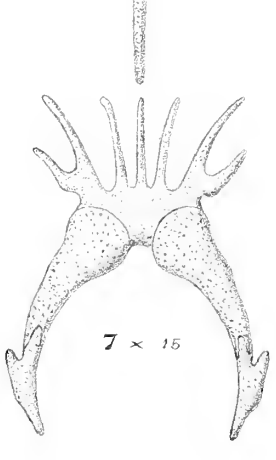

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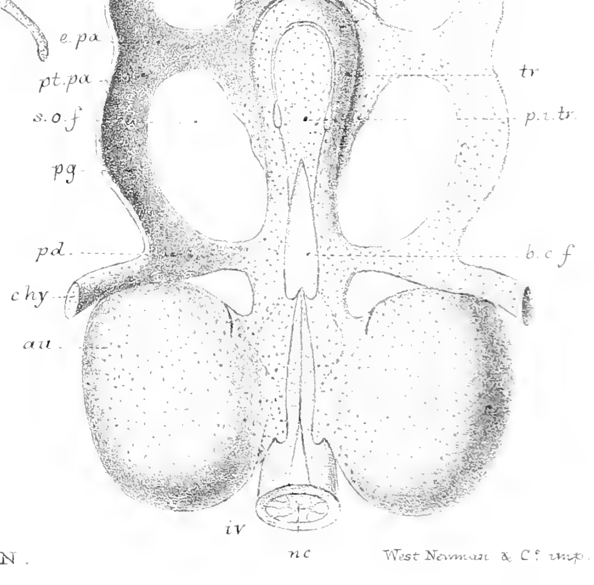

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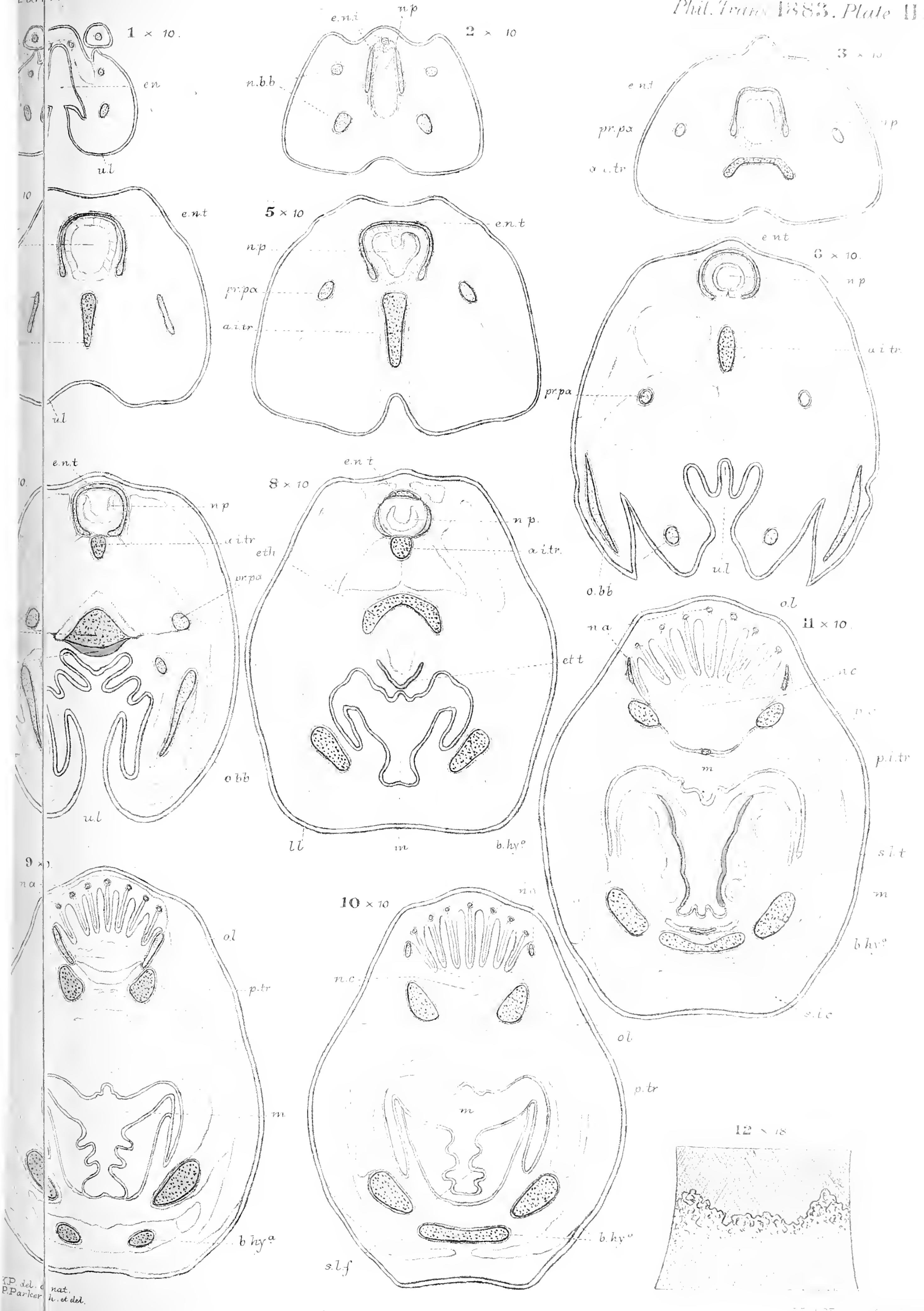

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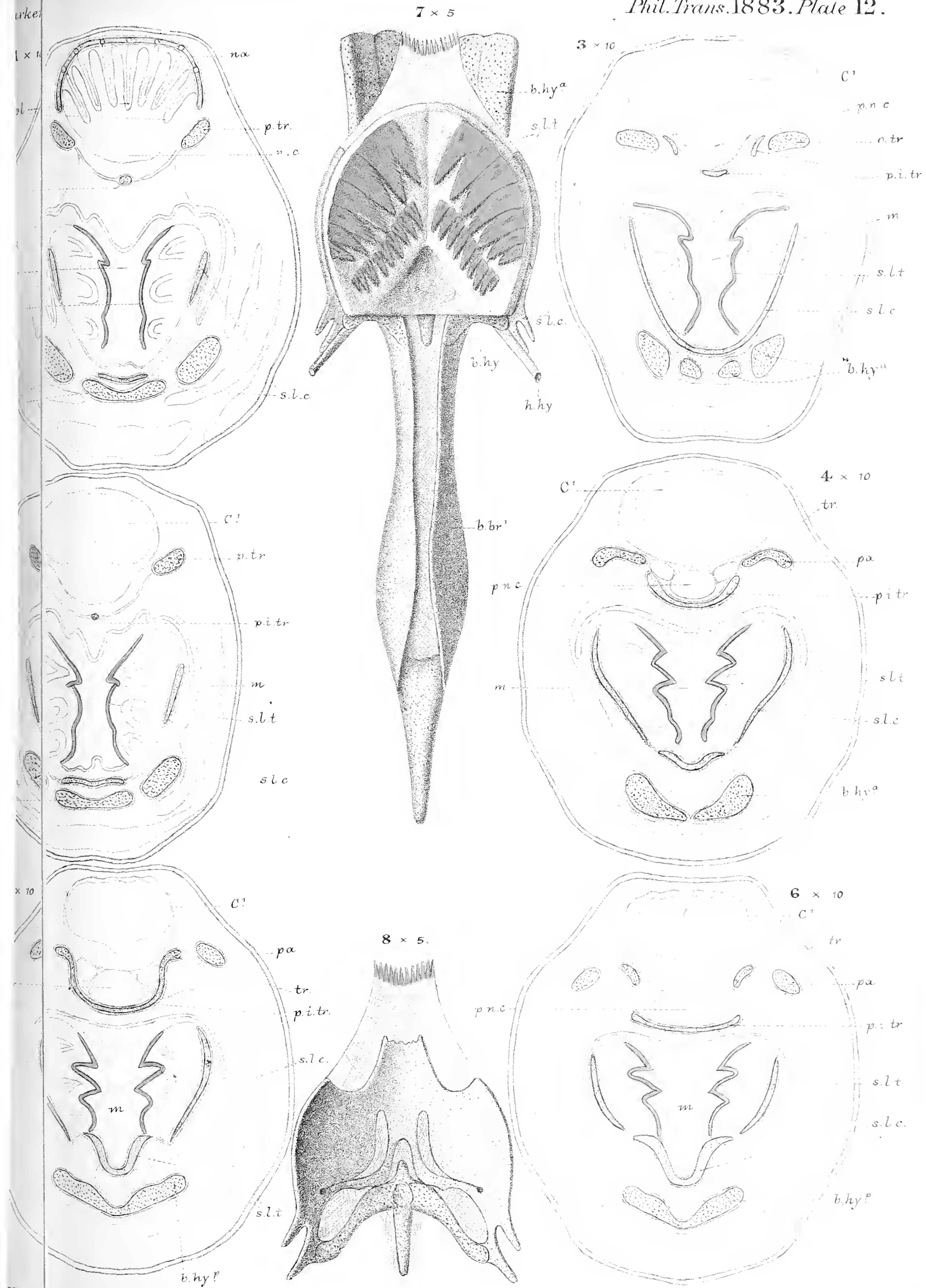

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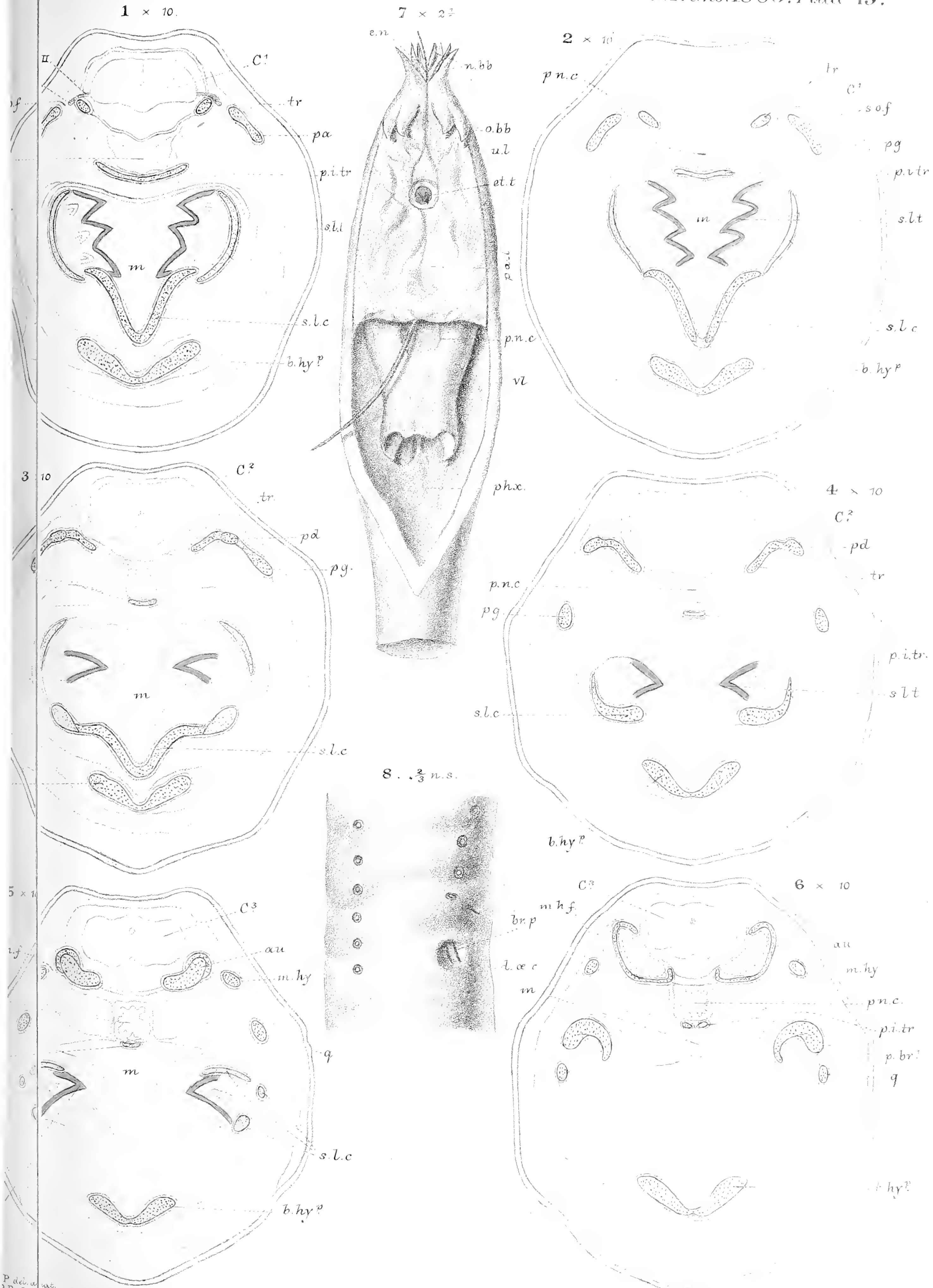

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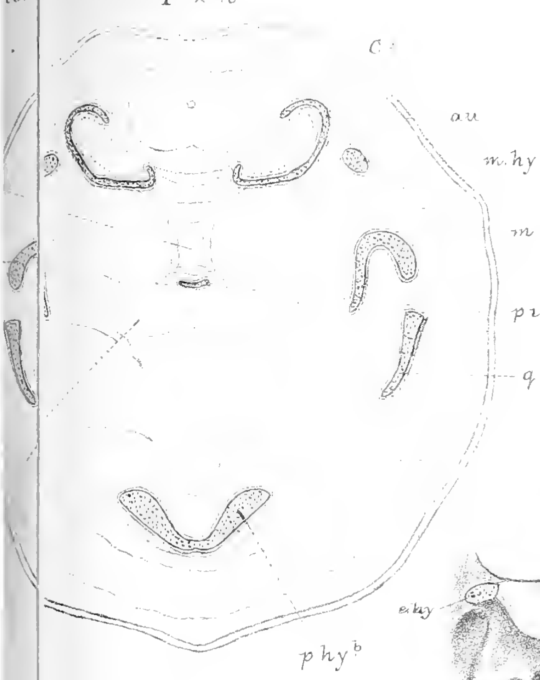

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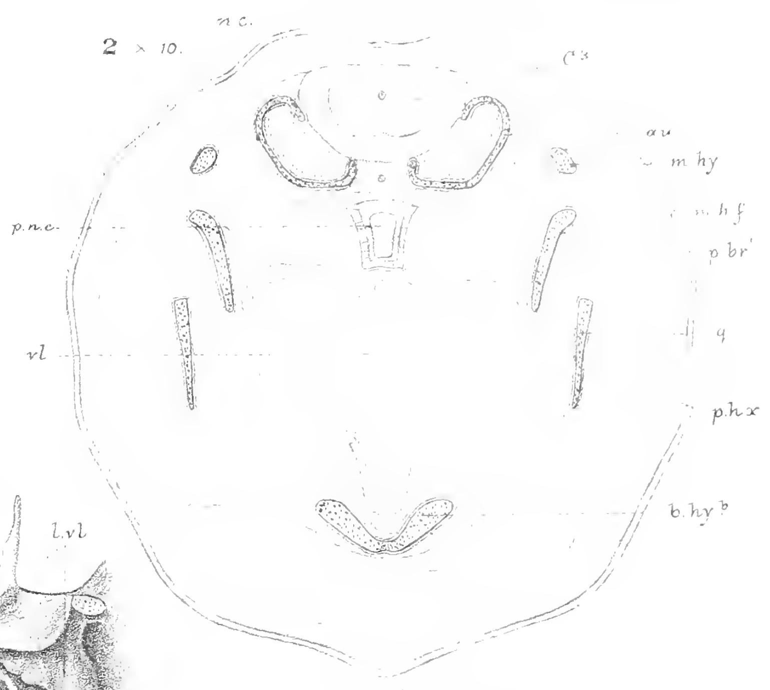

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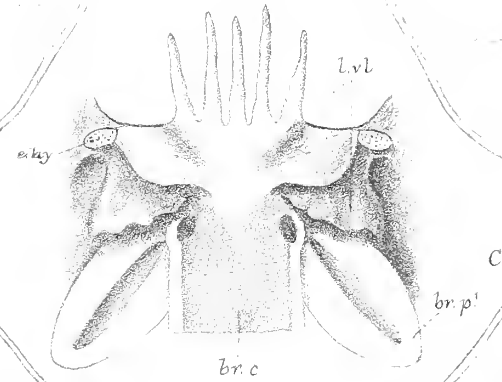

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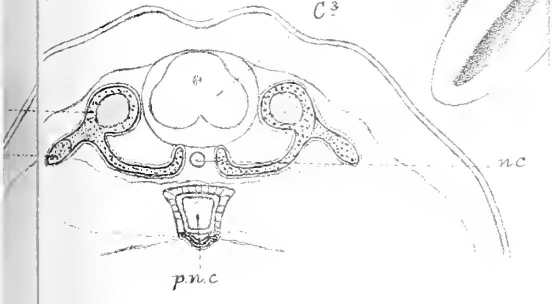

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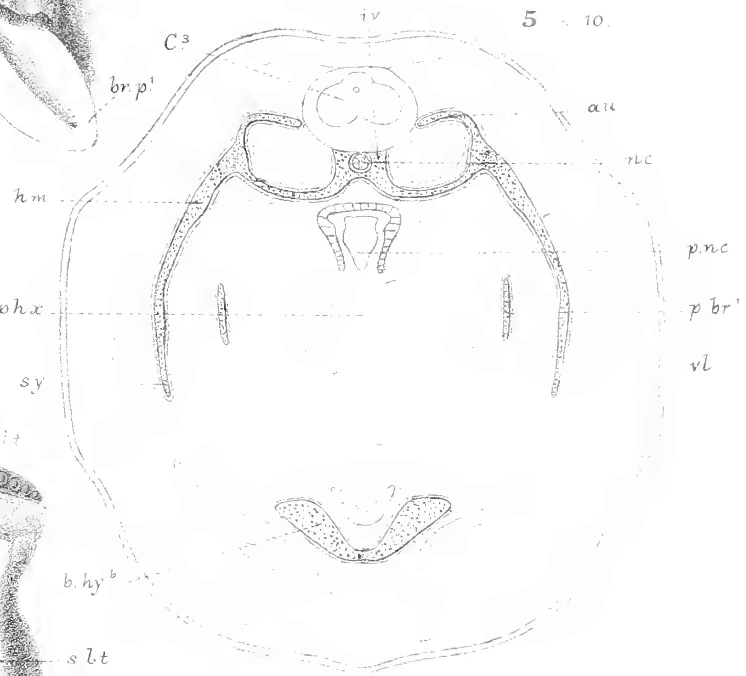

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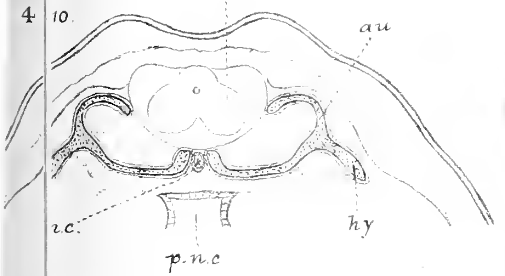

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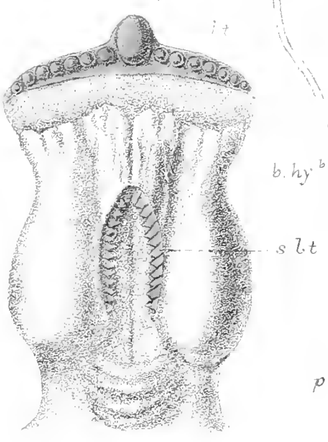

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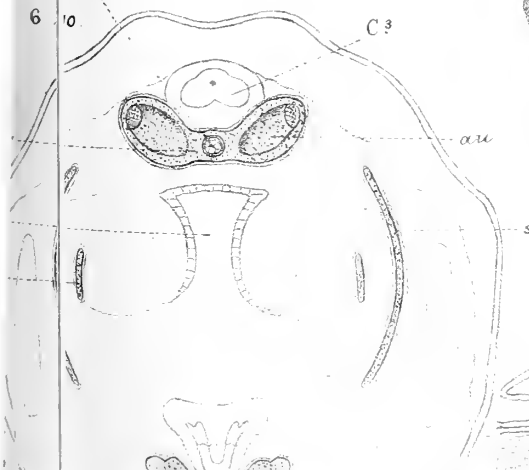

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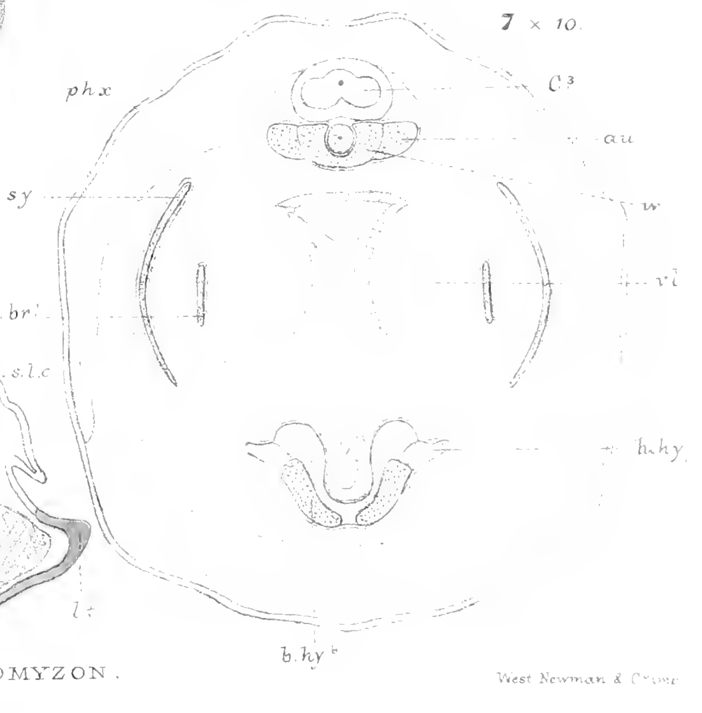

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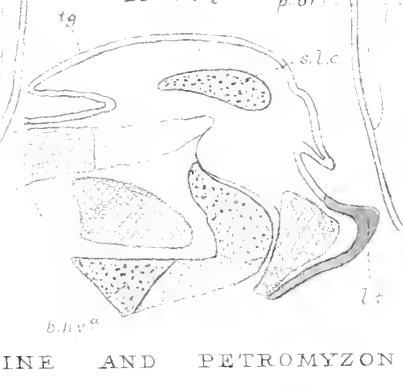

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MYXINE AND PETROMYZON.

West Newman & Co. imp.

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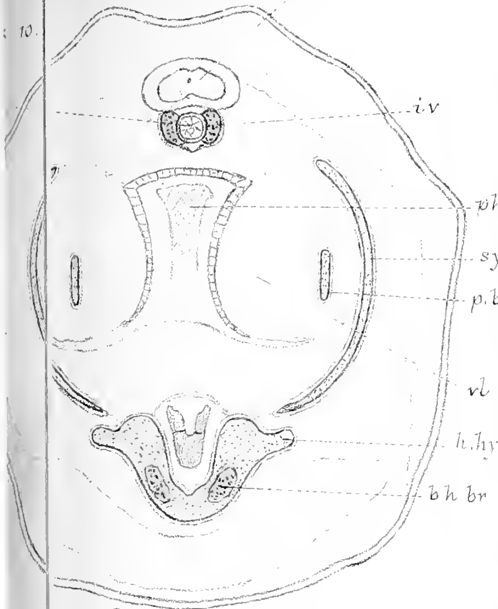

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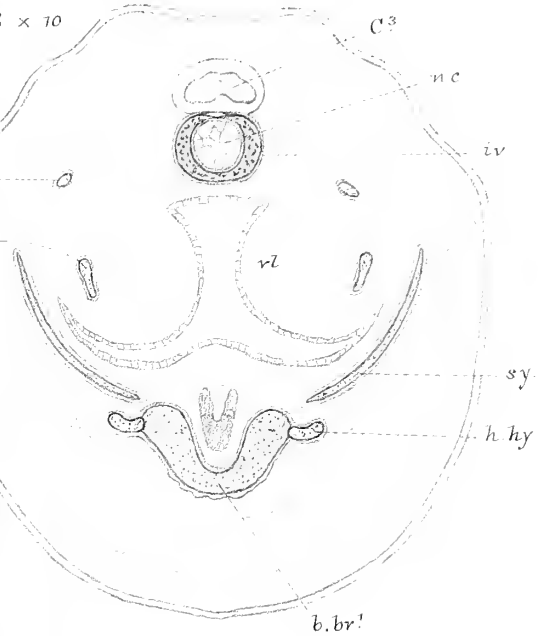

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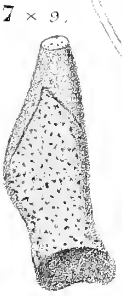

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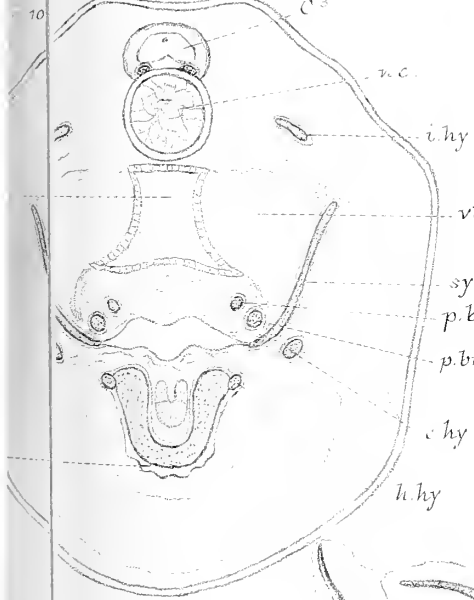

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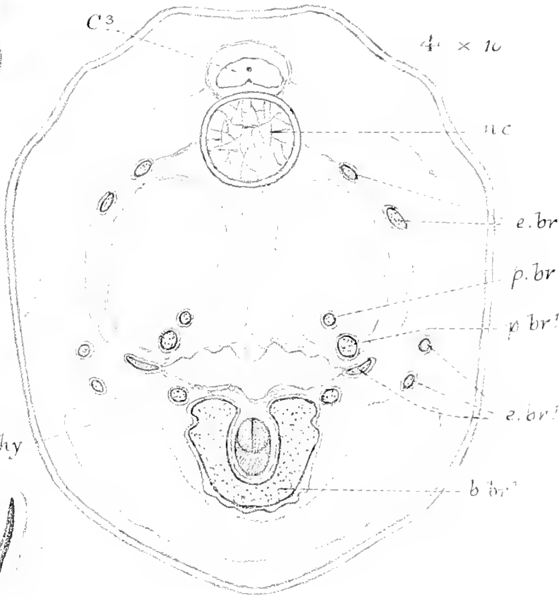

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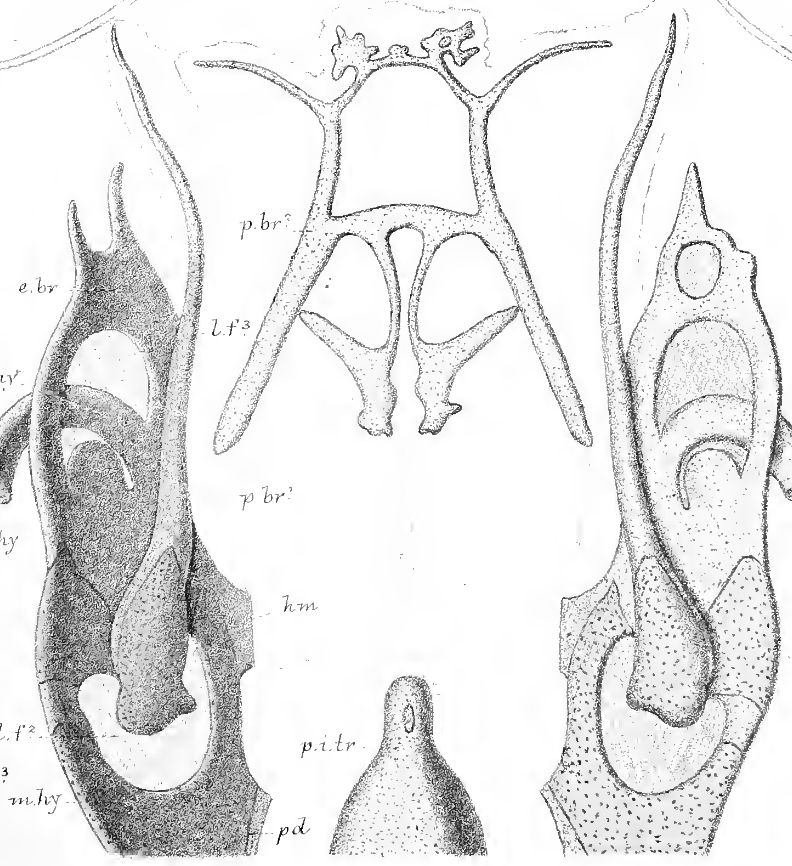

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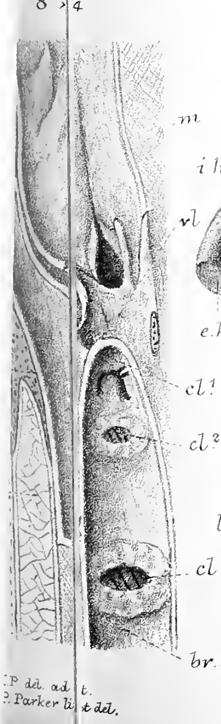

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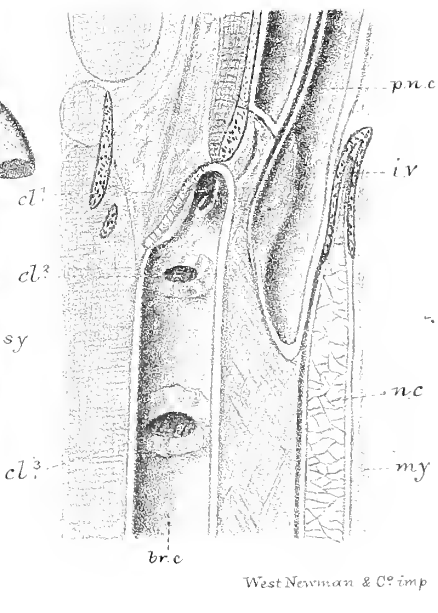

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BDELLOSTOMA, MYXINE, AND PETROMYZON

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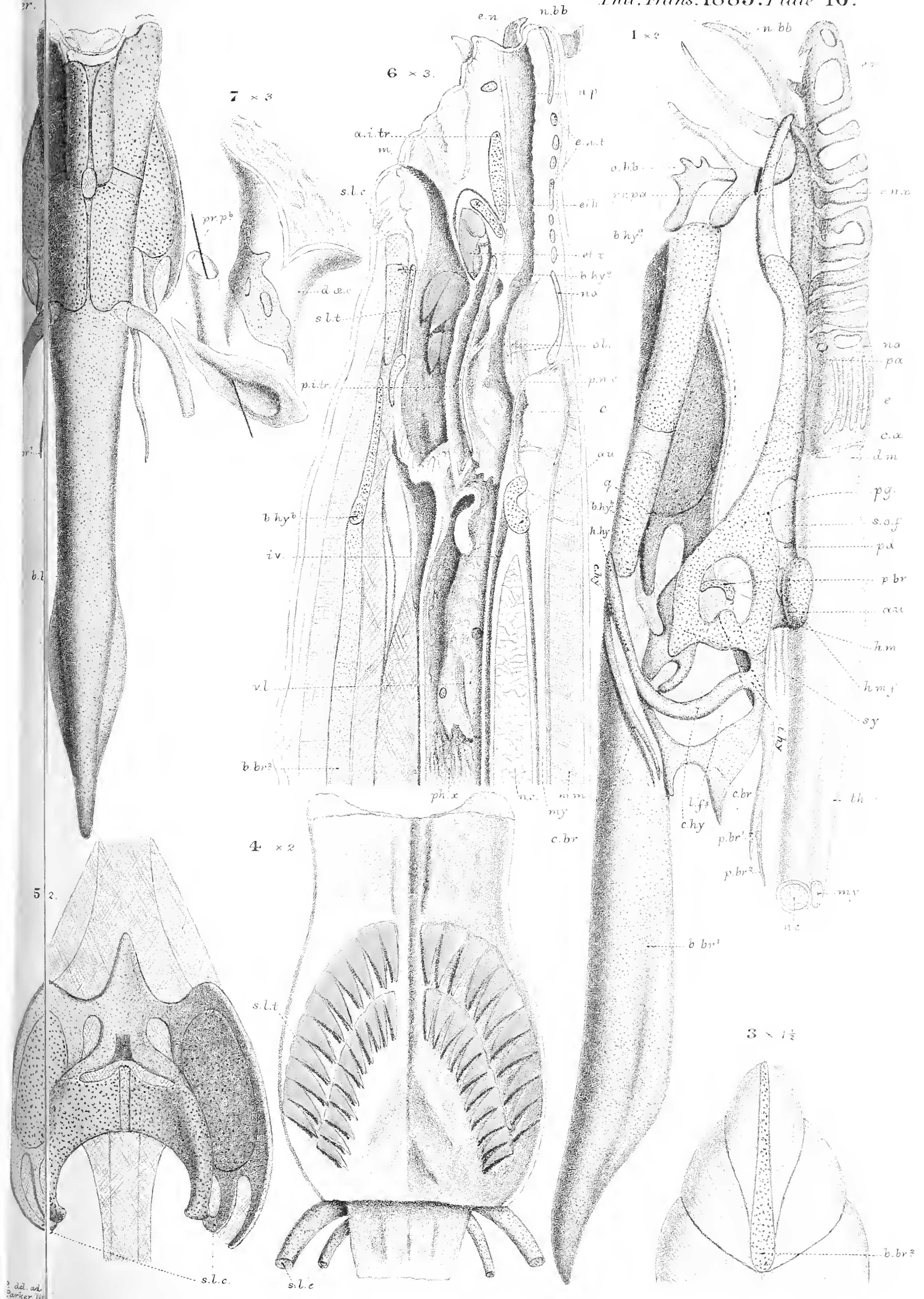

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BDELLOSTOMA FORSTERI.

West Newman & Co. imp

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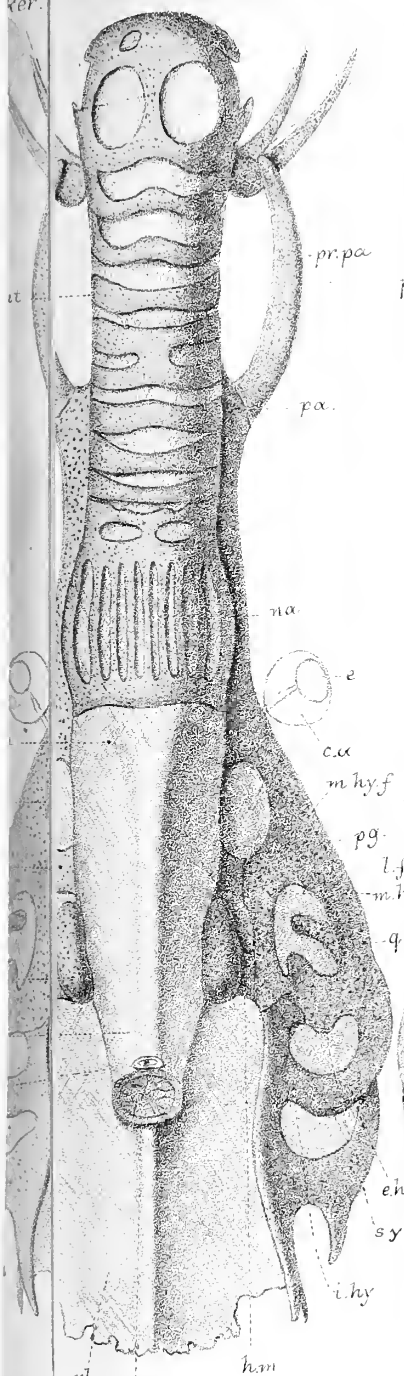

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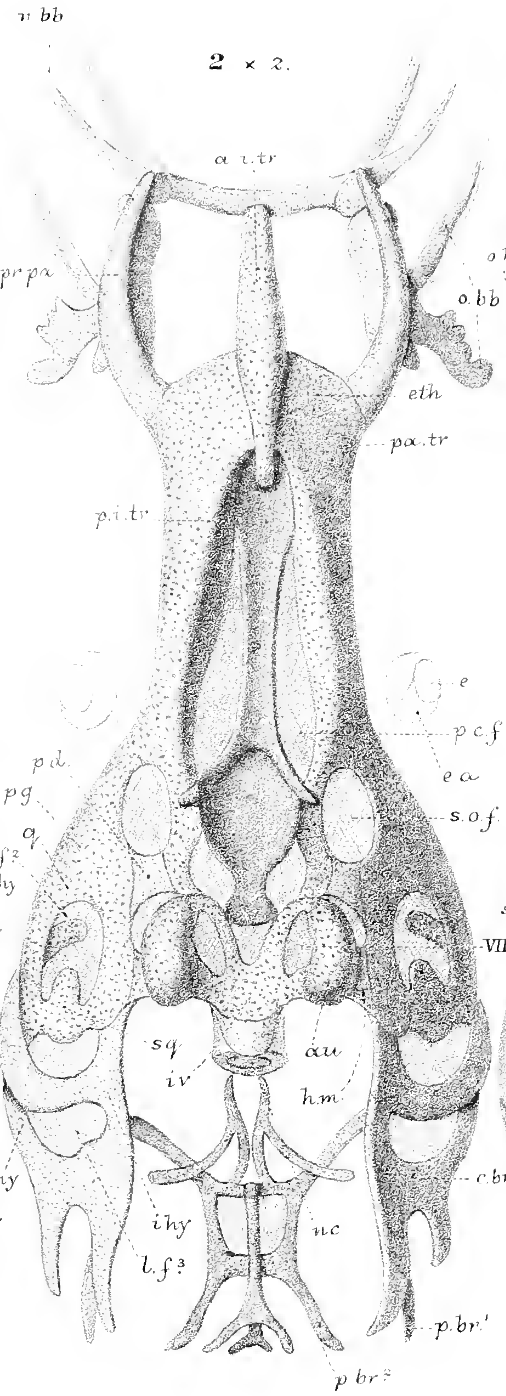

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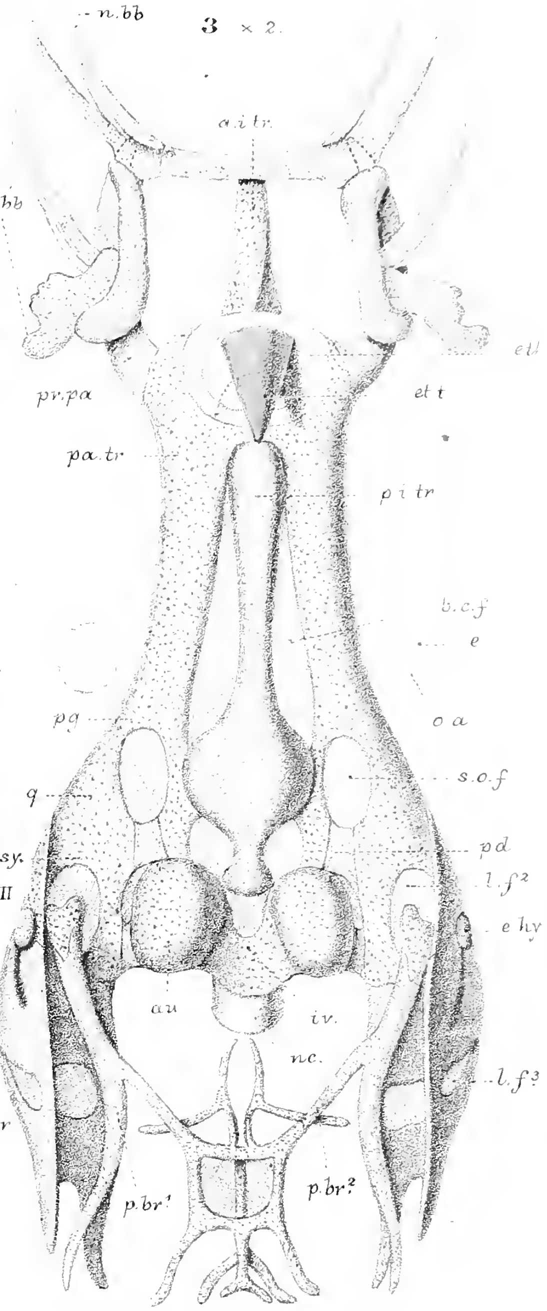

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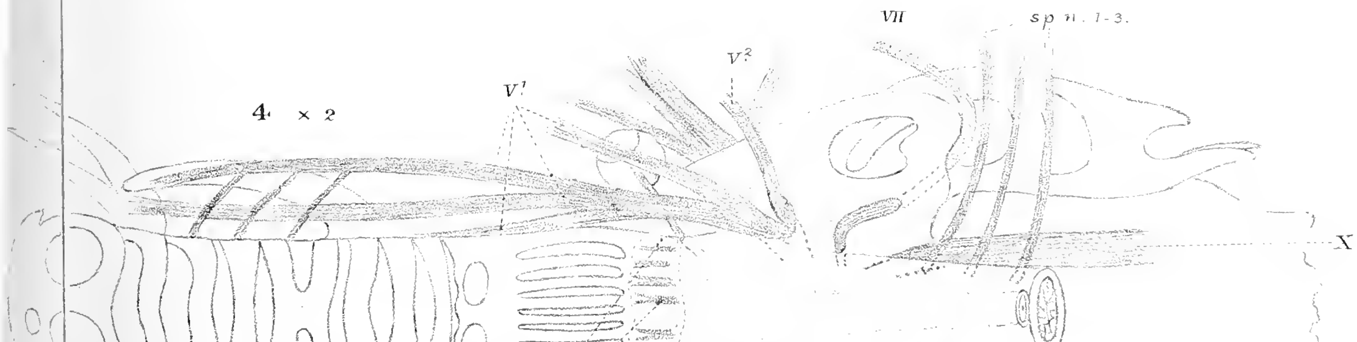

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BDELLOSTOMA FORSTERI.

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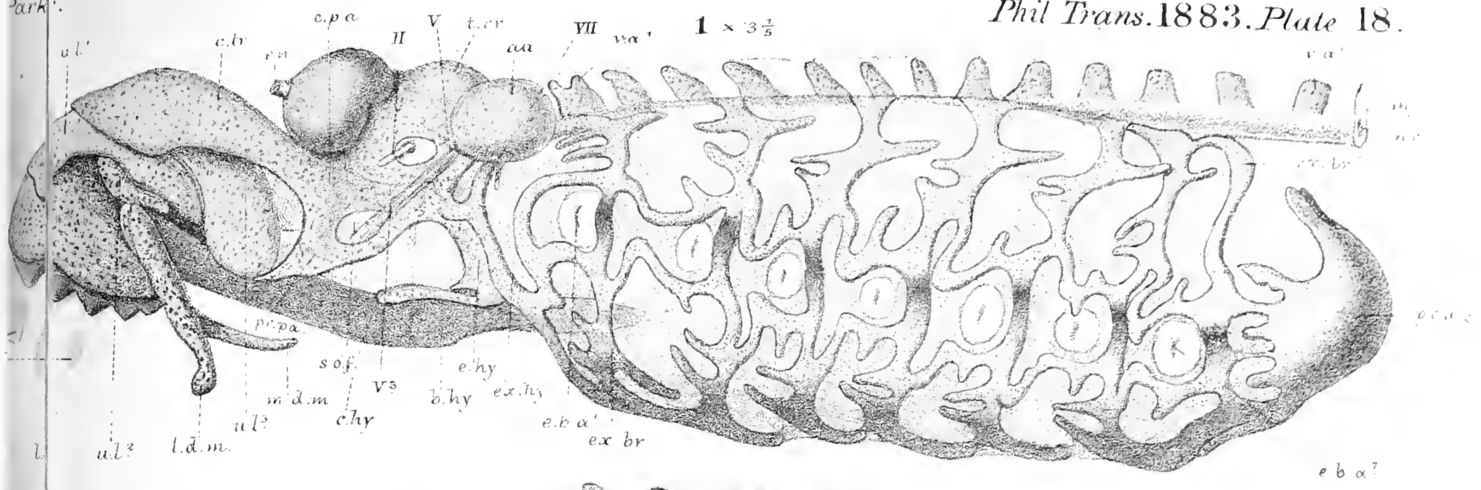

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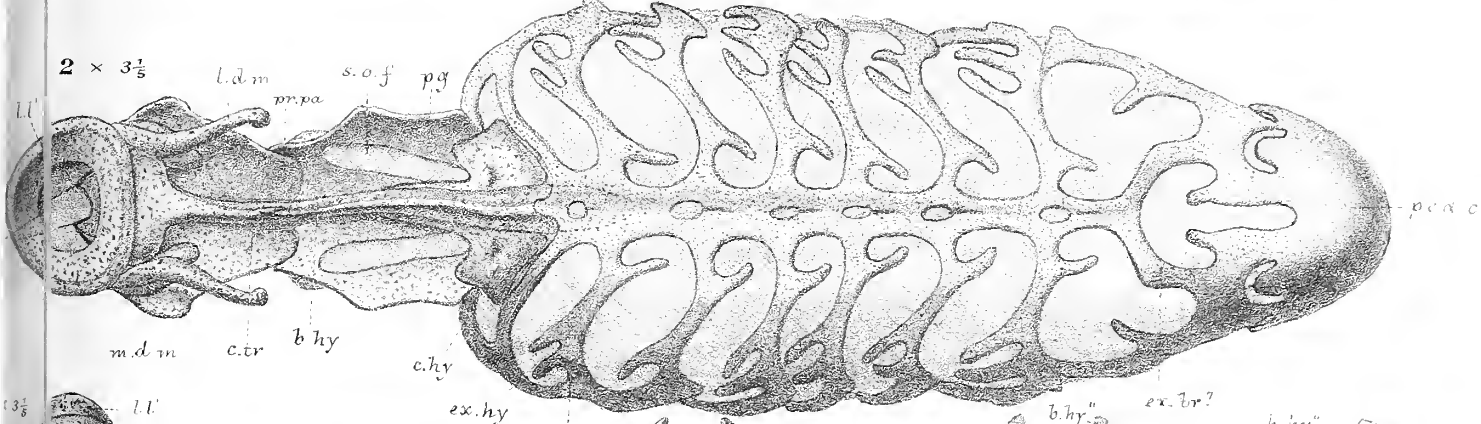

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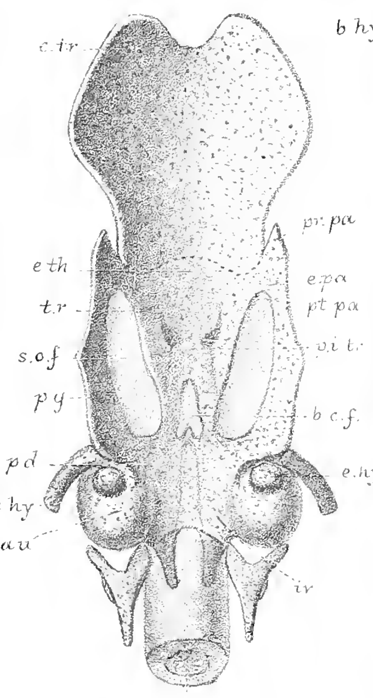

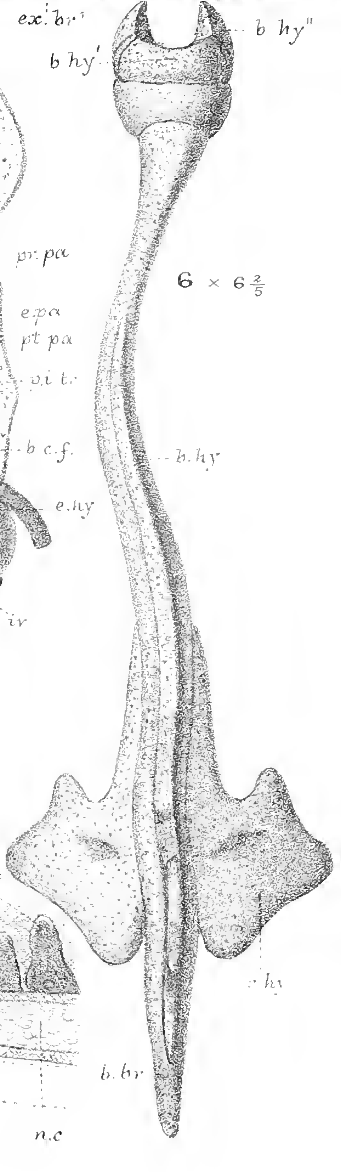

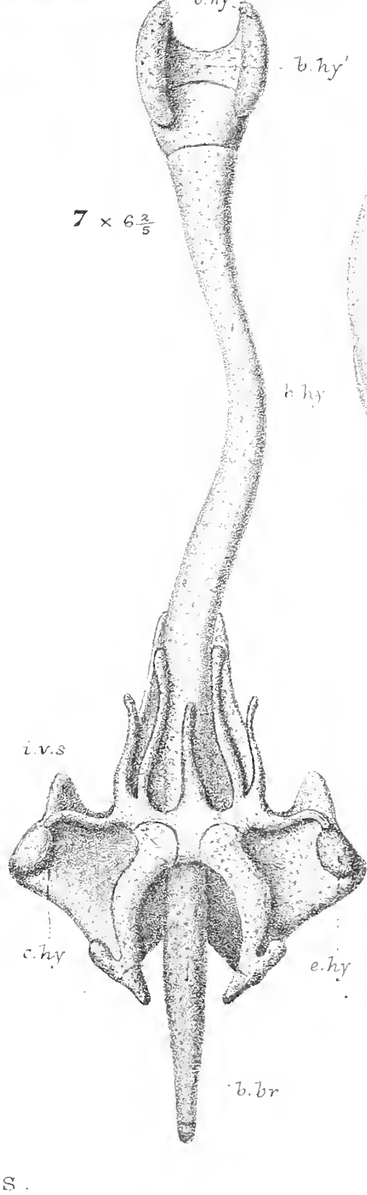

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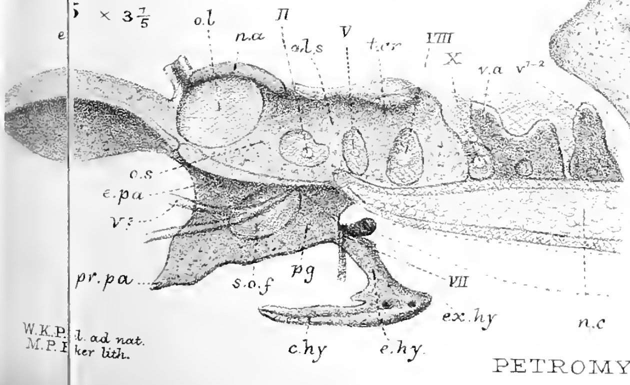

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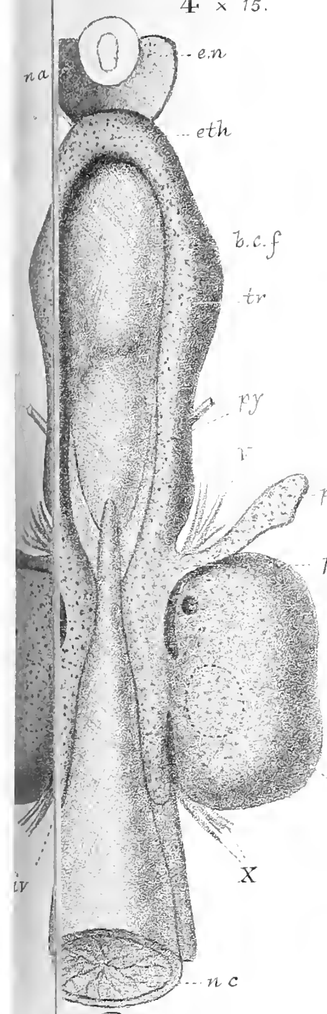

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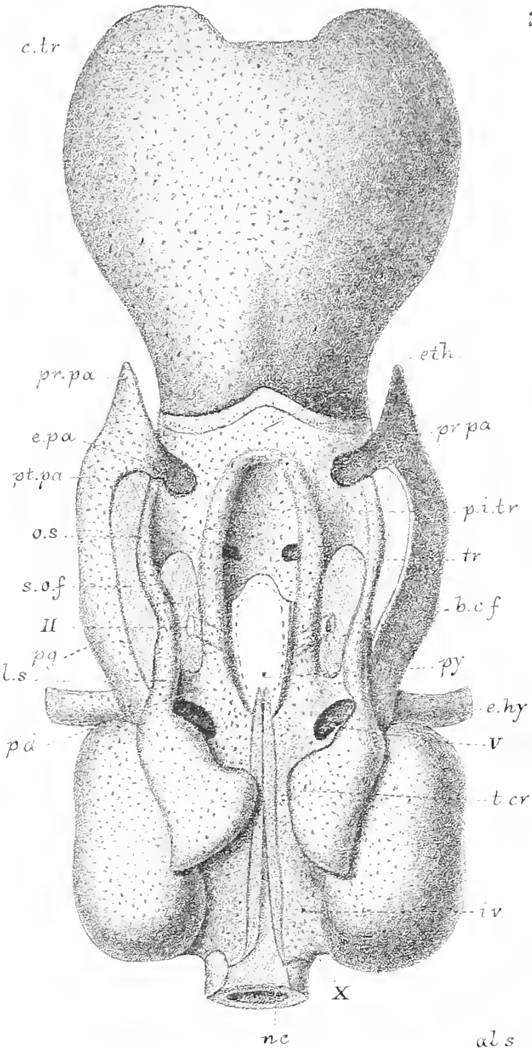

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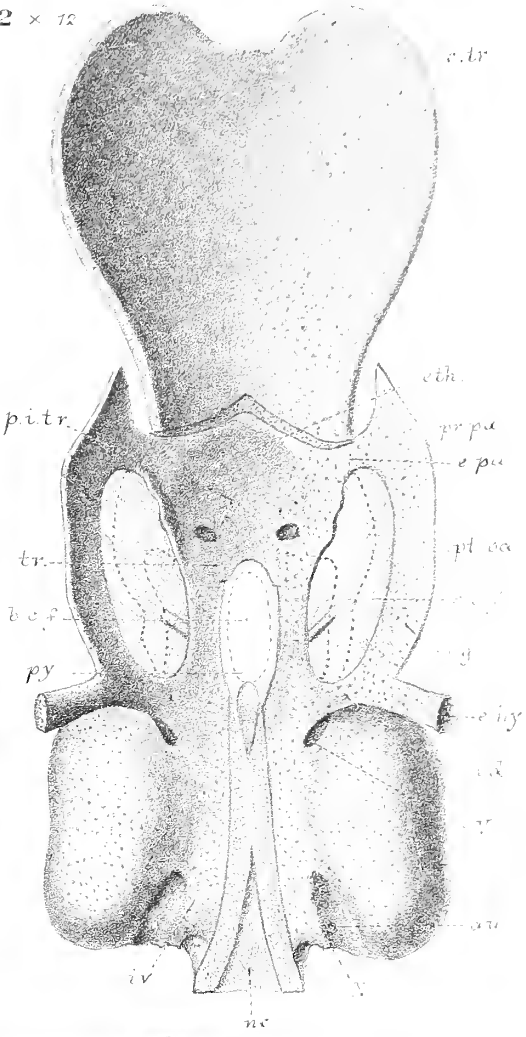

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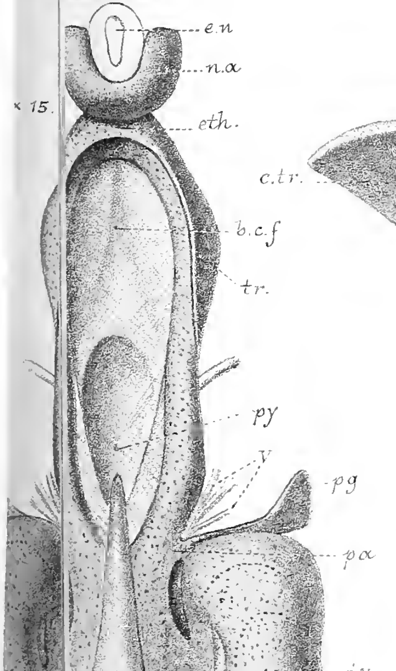

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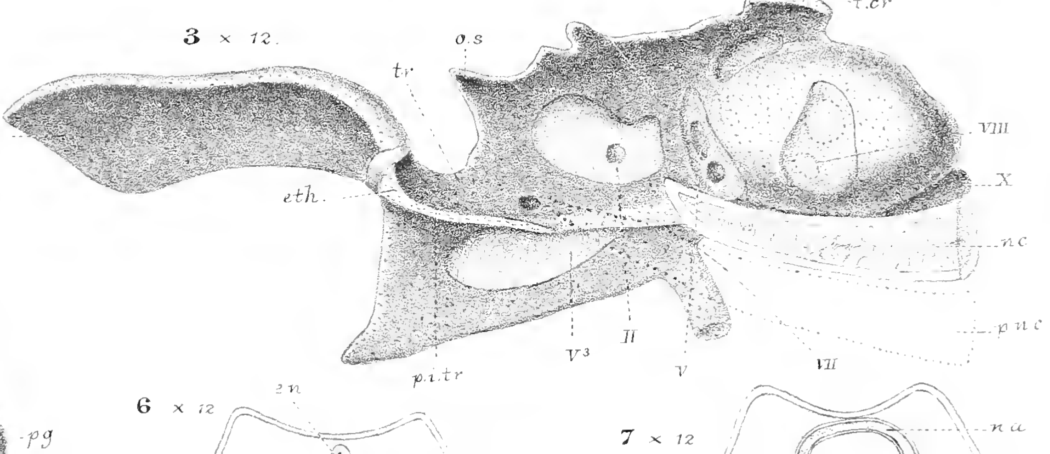

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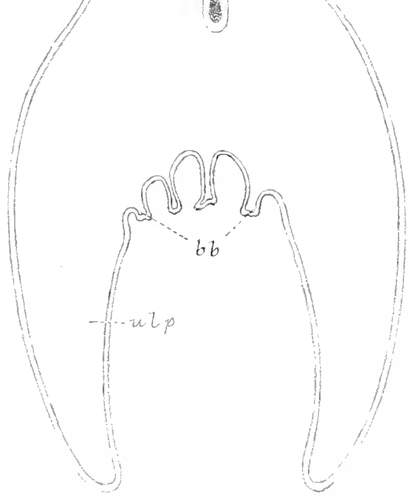

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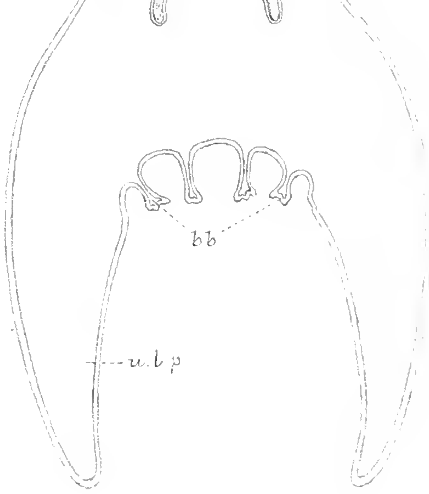

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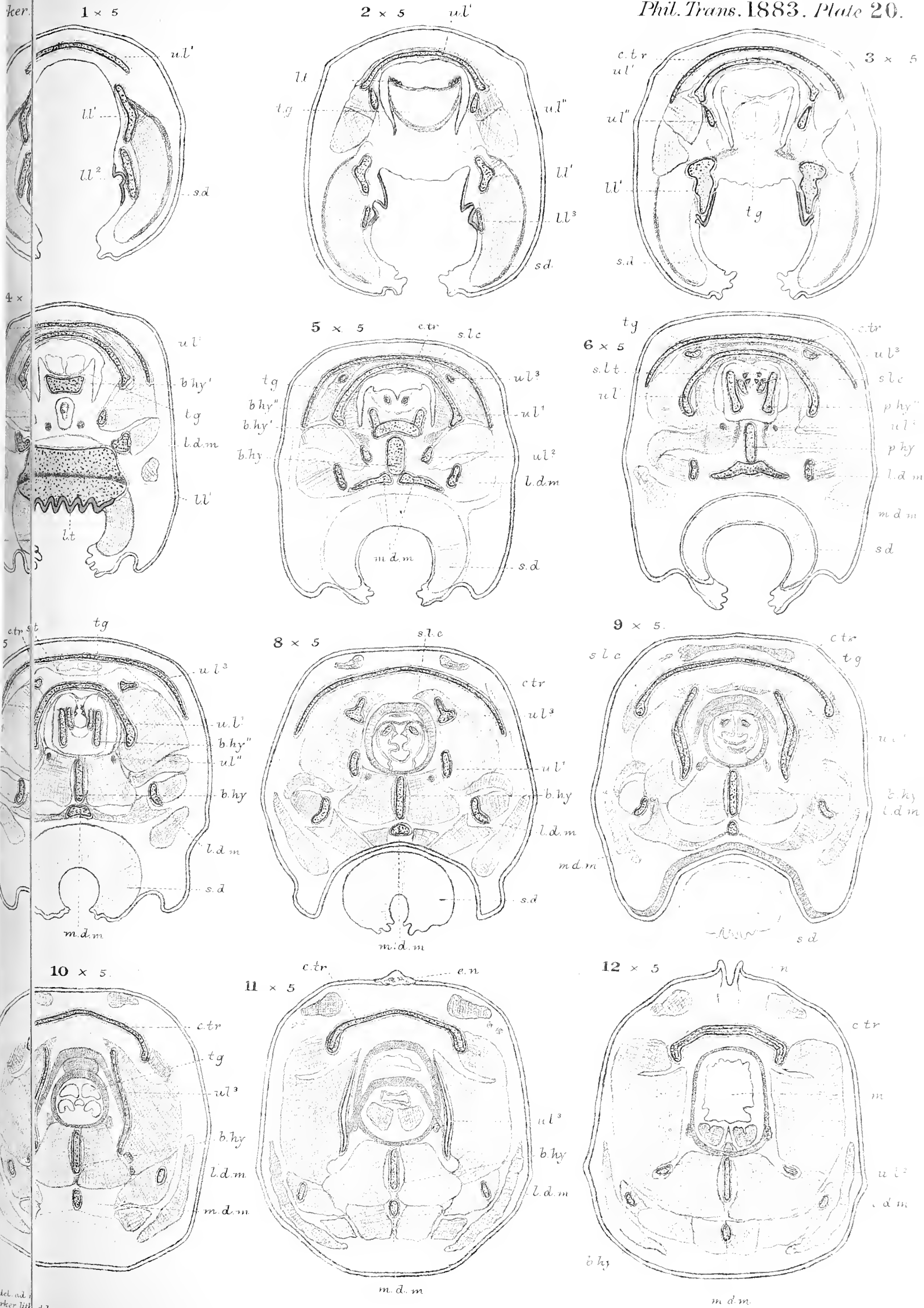

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PETROMYZON FLUVIATILIS.

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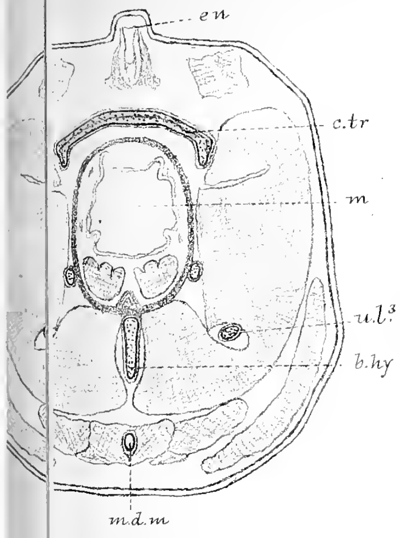

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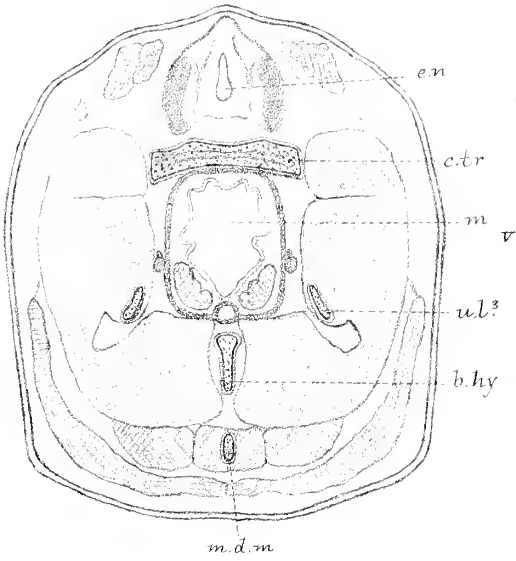

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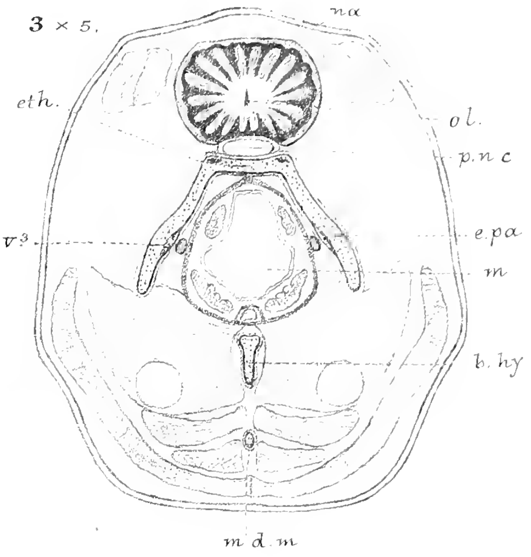

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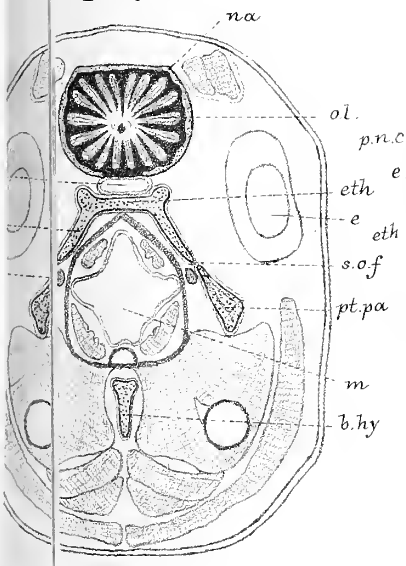

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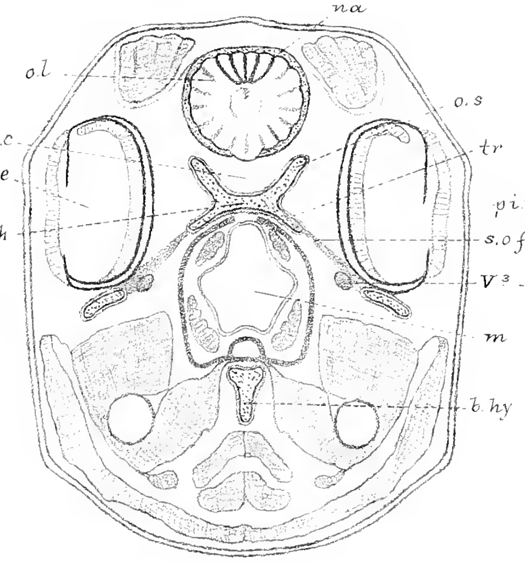

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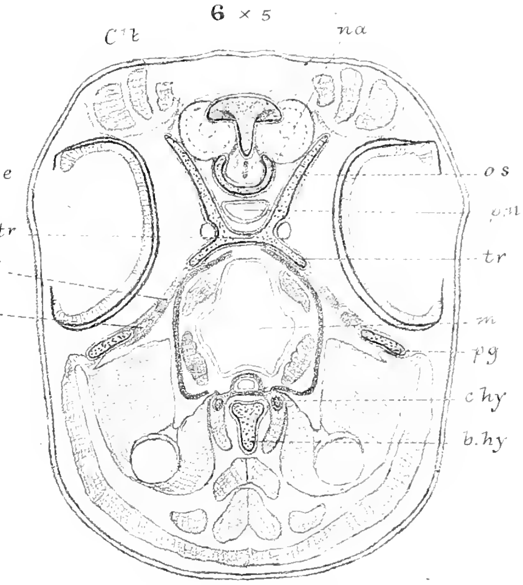

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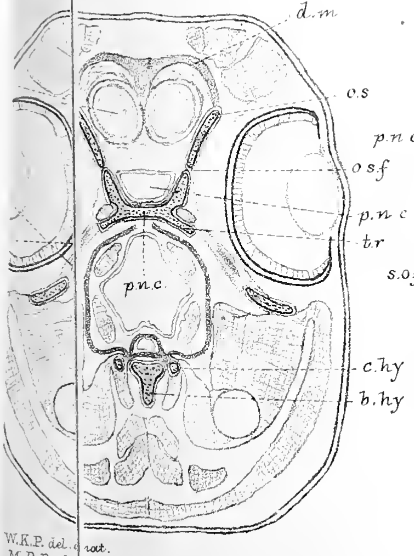

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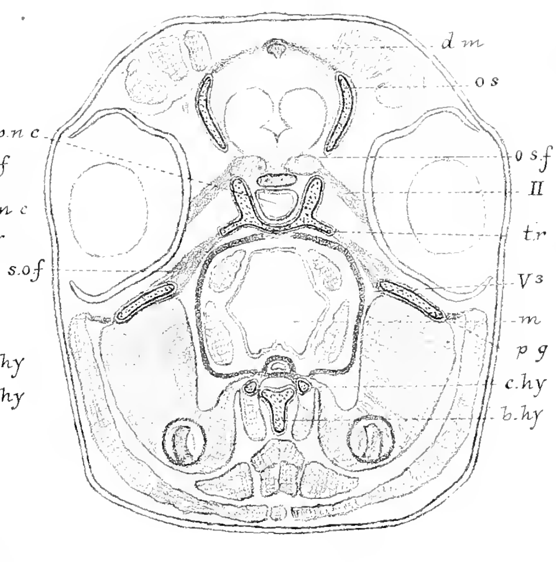

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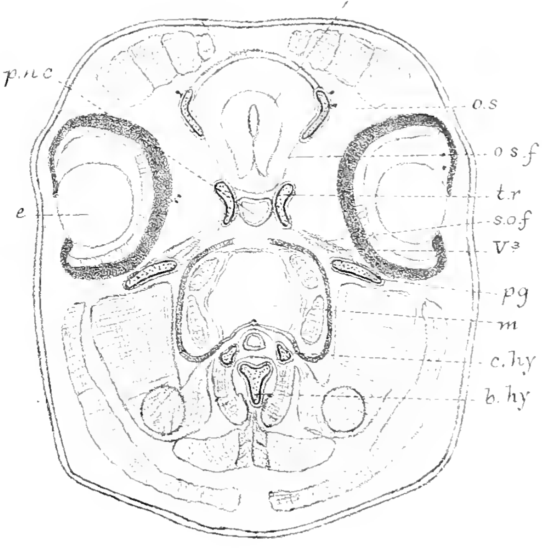

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PETROMYZON FLUVIATILIS.

West Newman & Cº imp.

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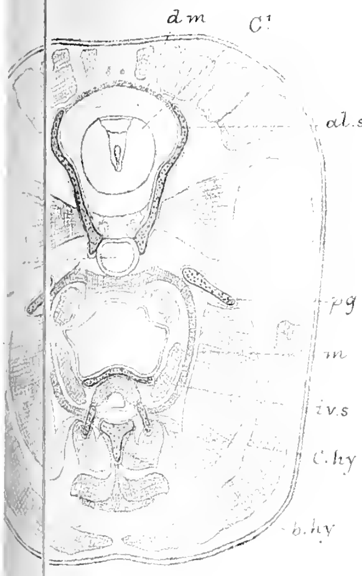

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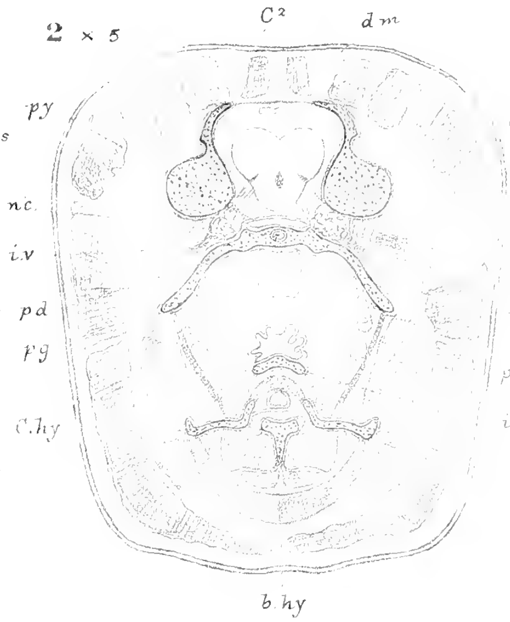

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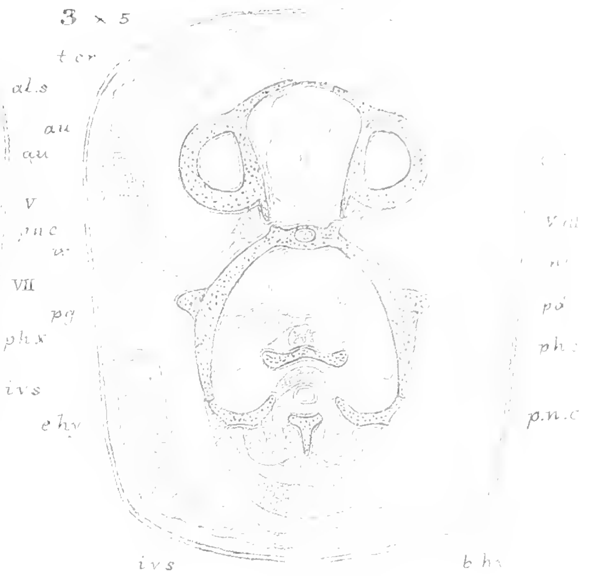

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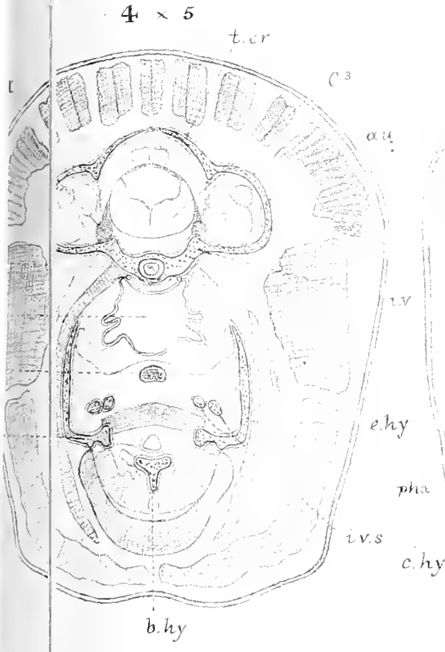

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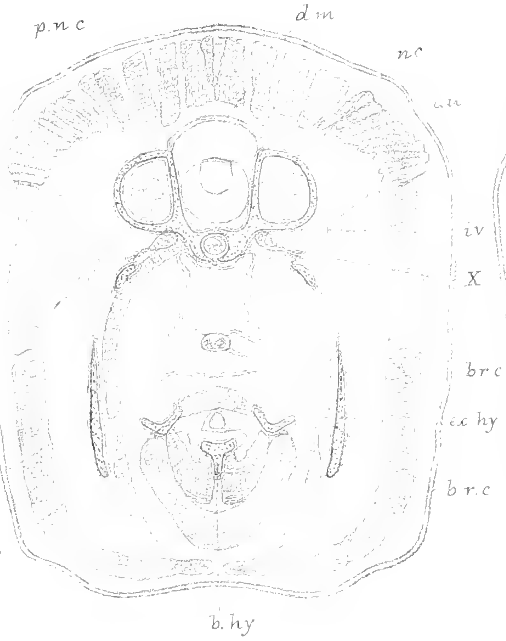

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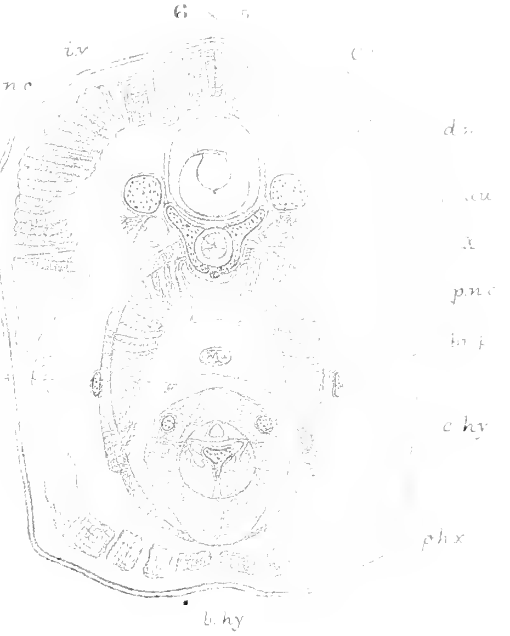

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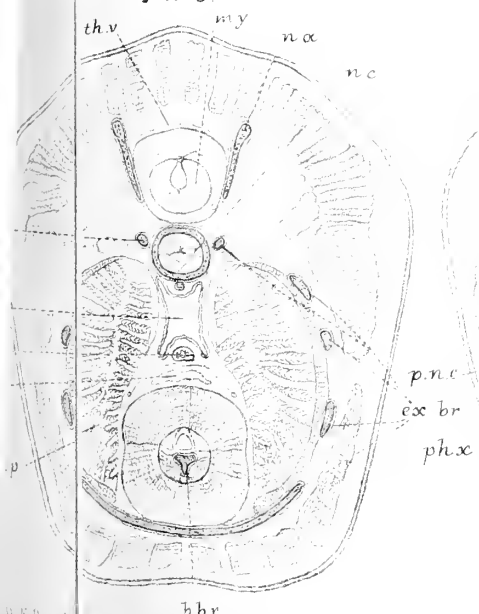

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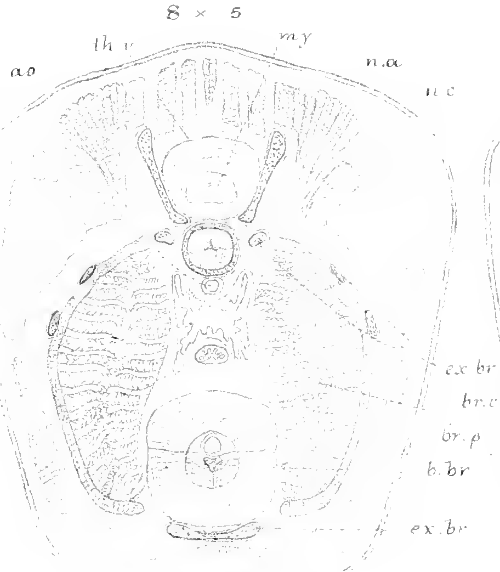

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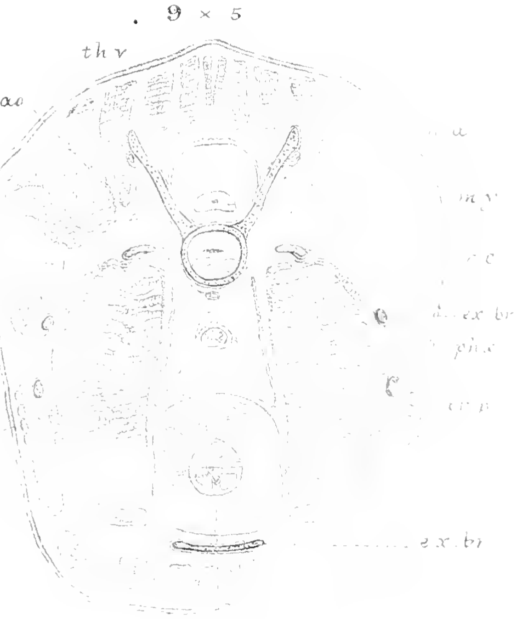

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PETROMYZON FLUVIATI

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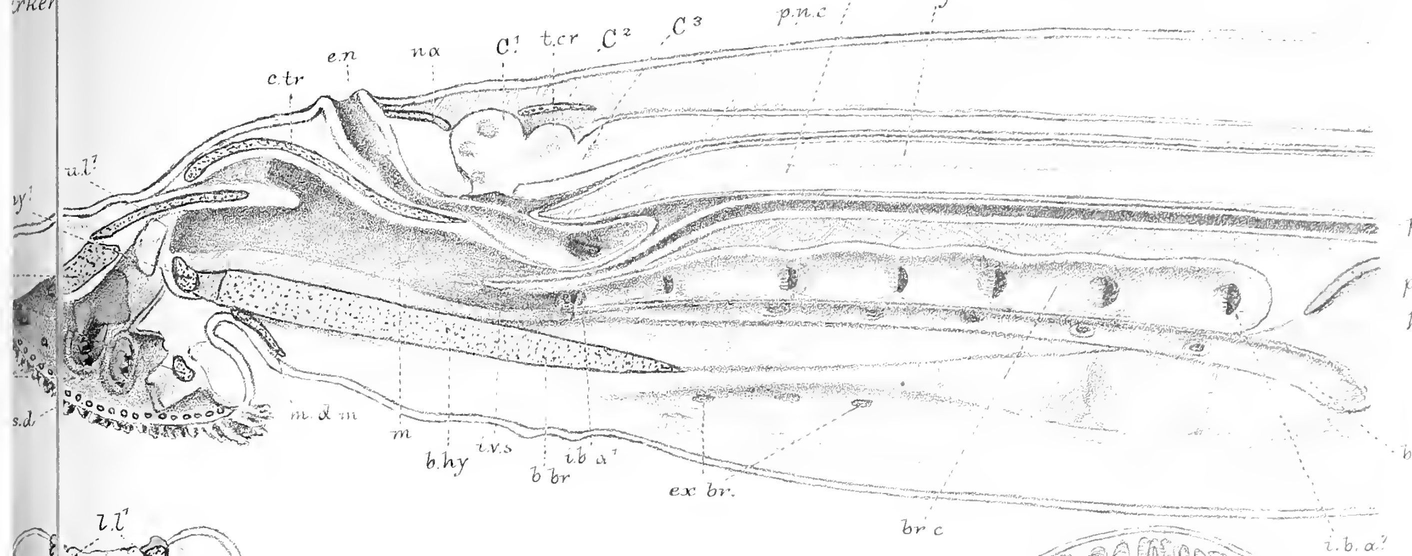

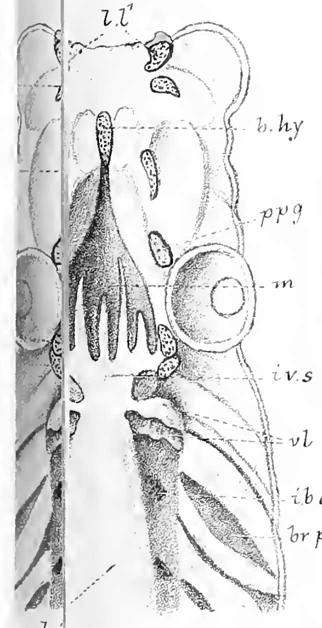

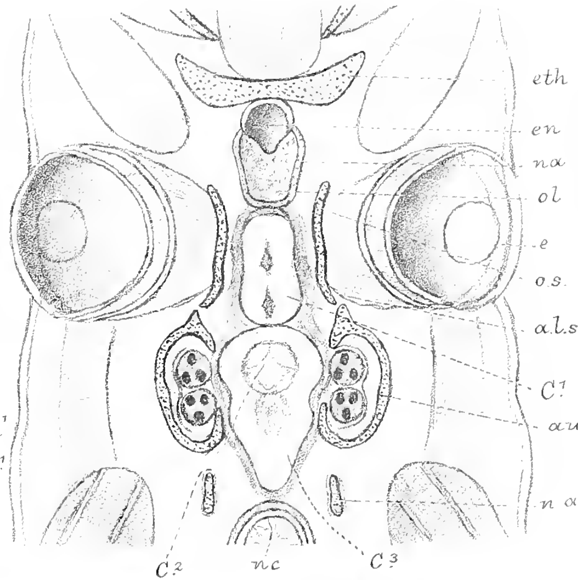

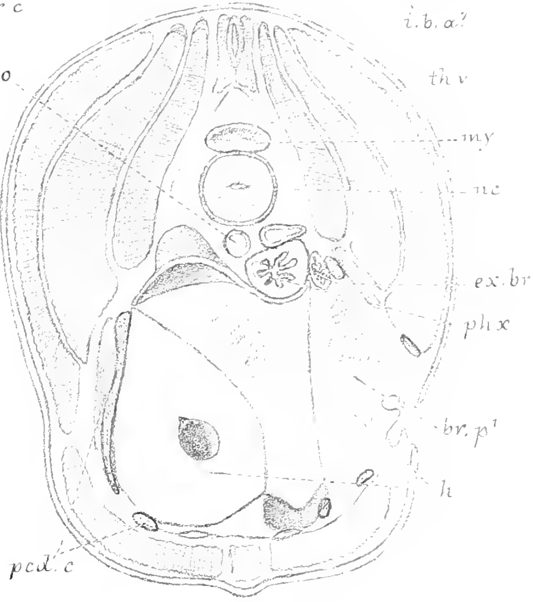

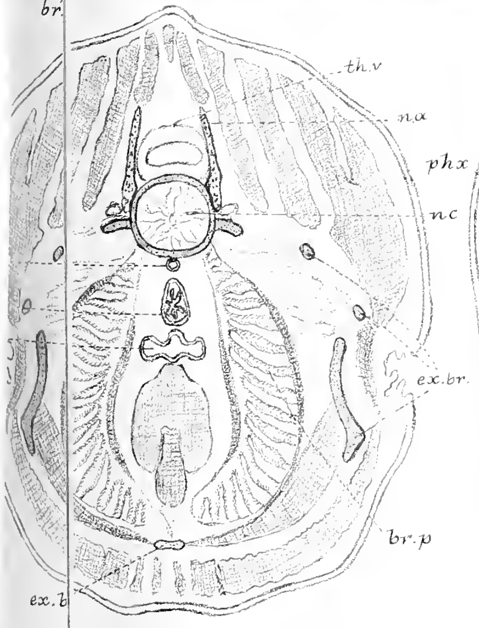

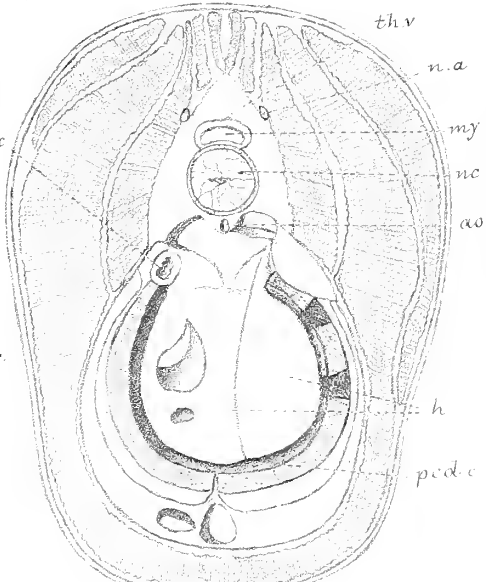

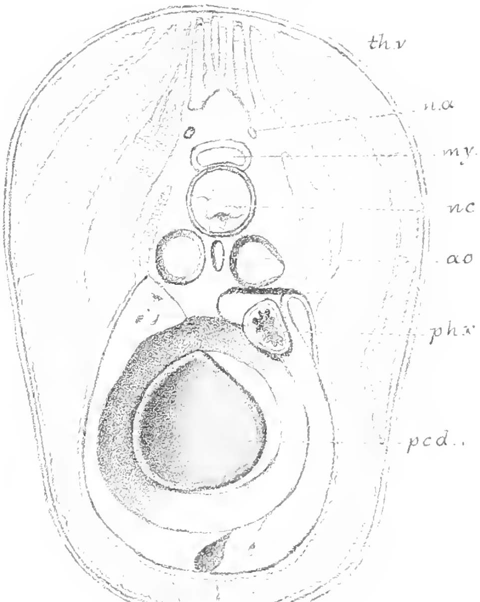

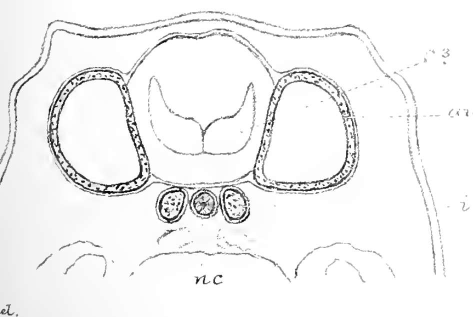

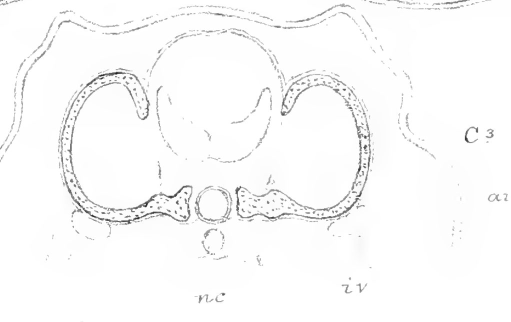

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PETROMYZON FLUVIATILIS.

West Newman & C°

Phil. Trans. 1883. Plate 24.

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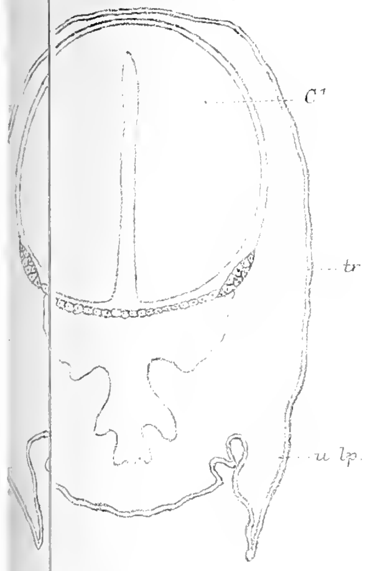

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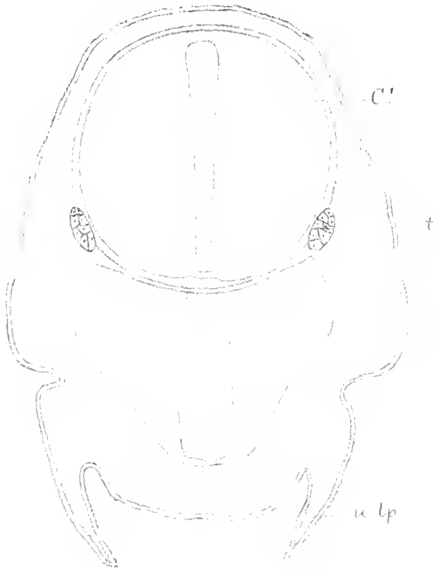

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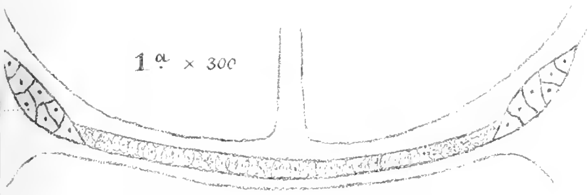

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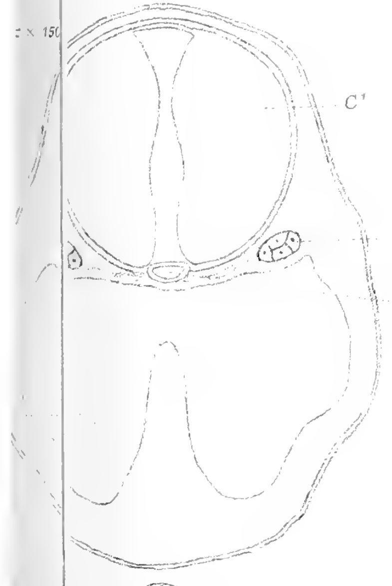

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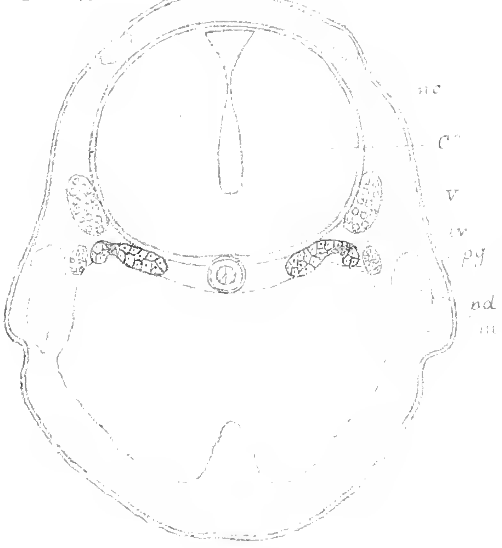

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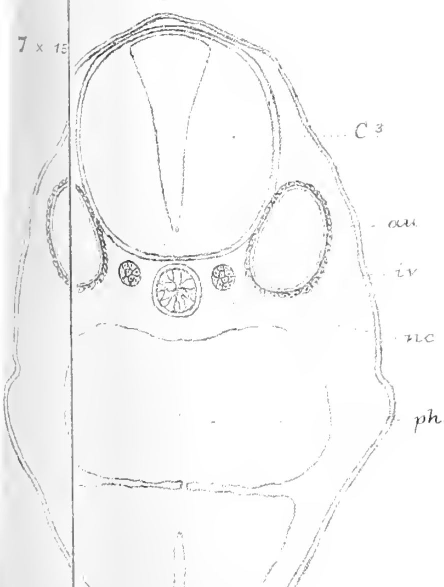

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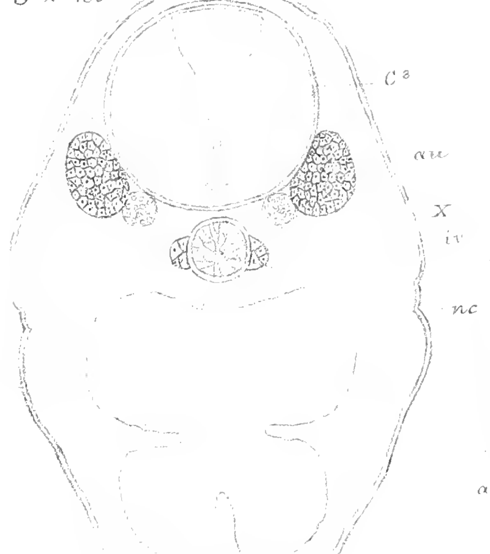

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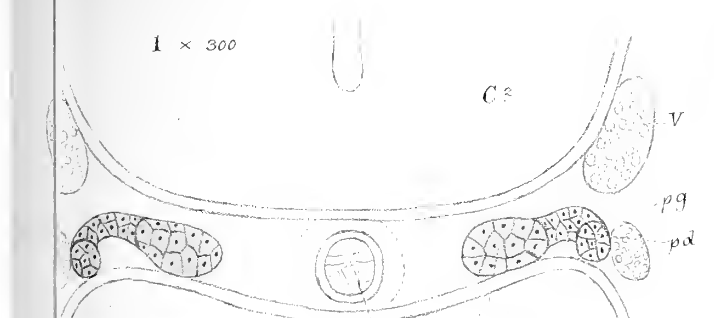

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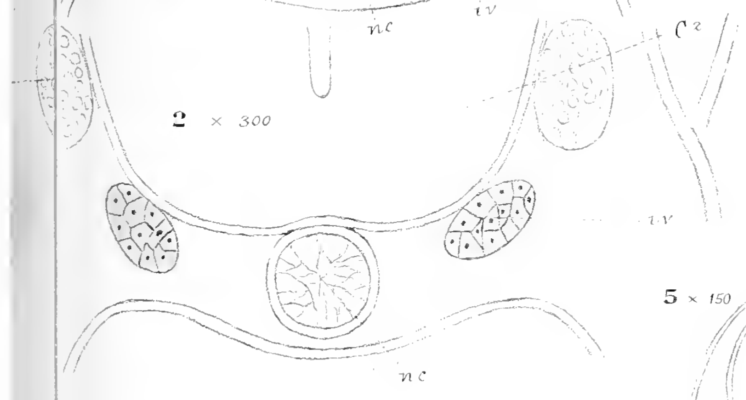

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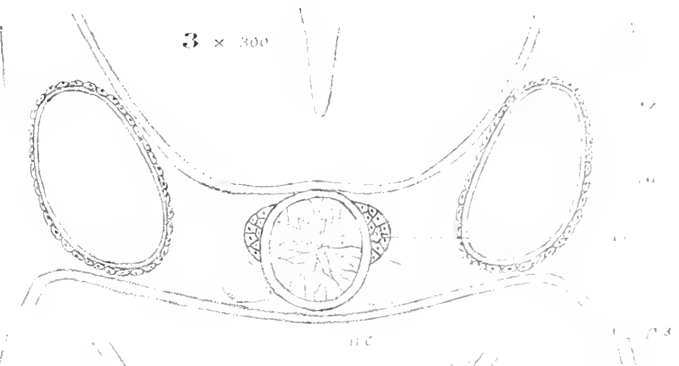

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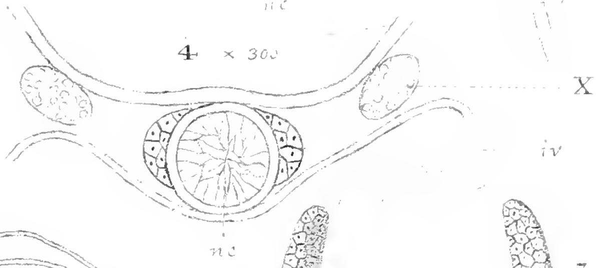

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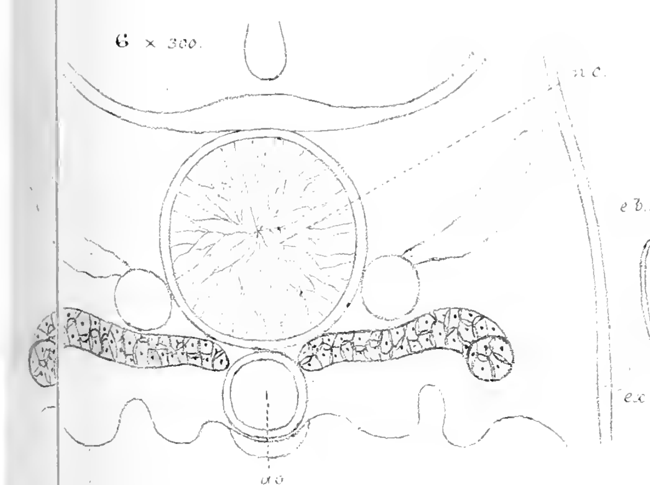

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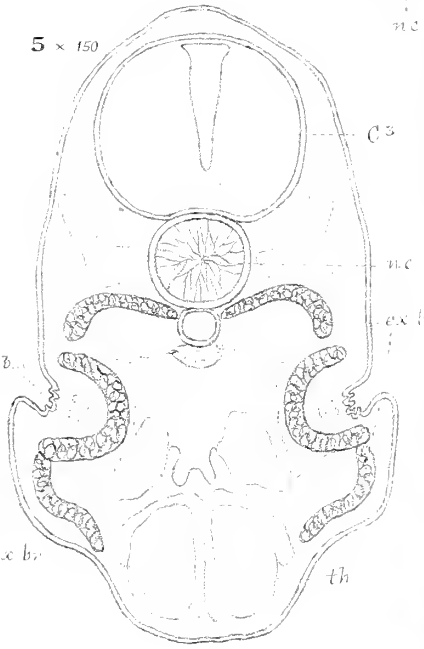

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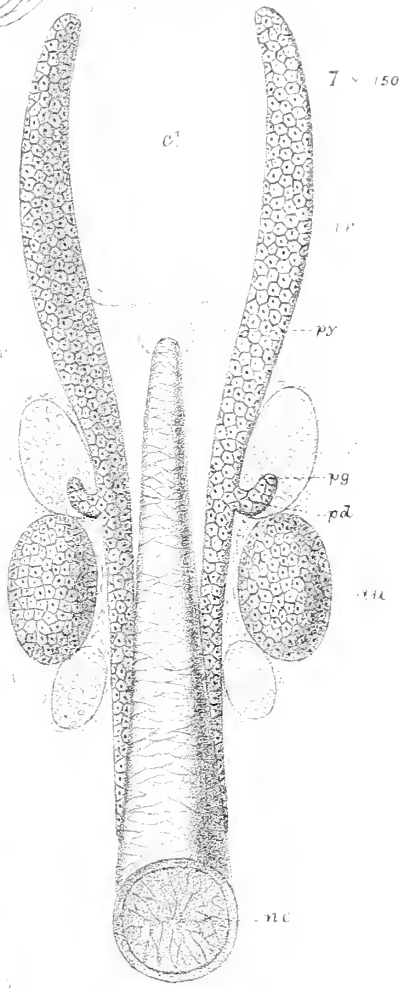

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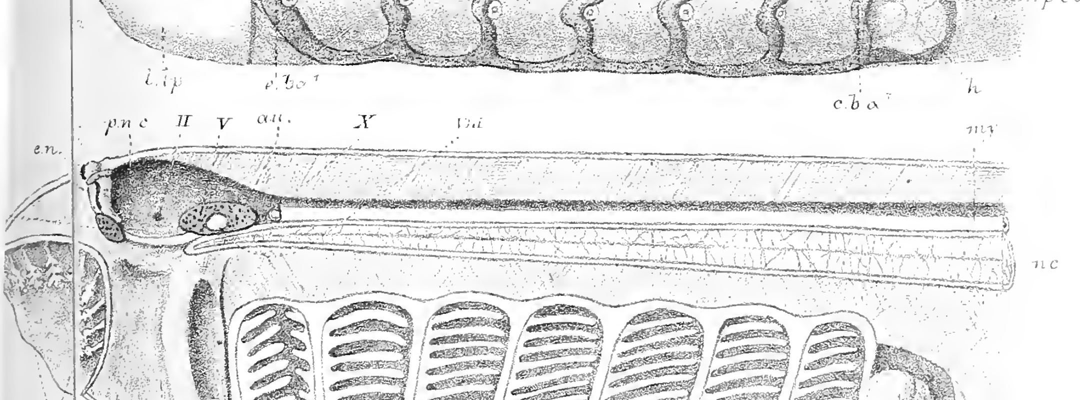

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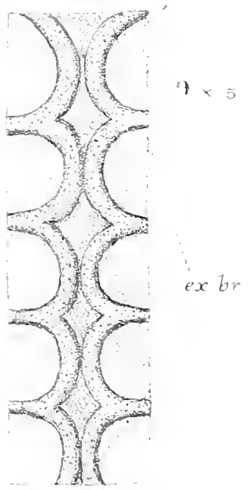
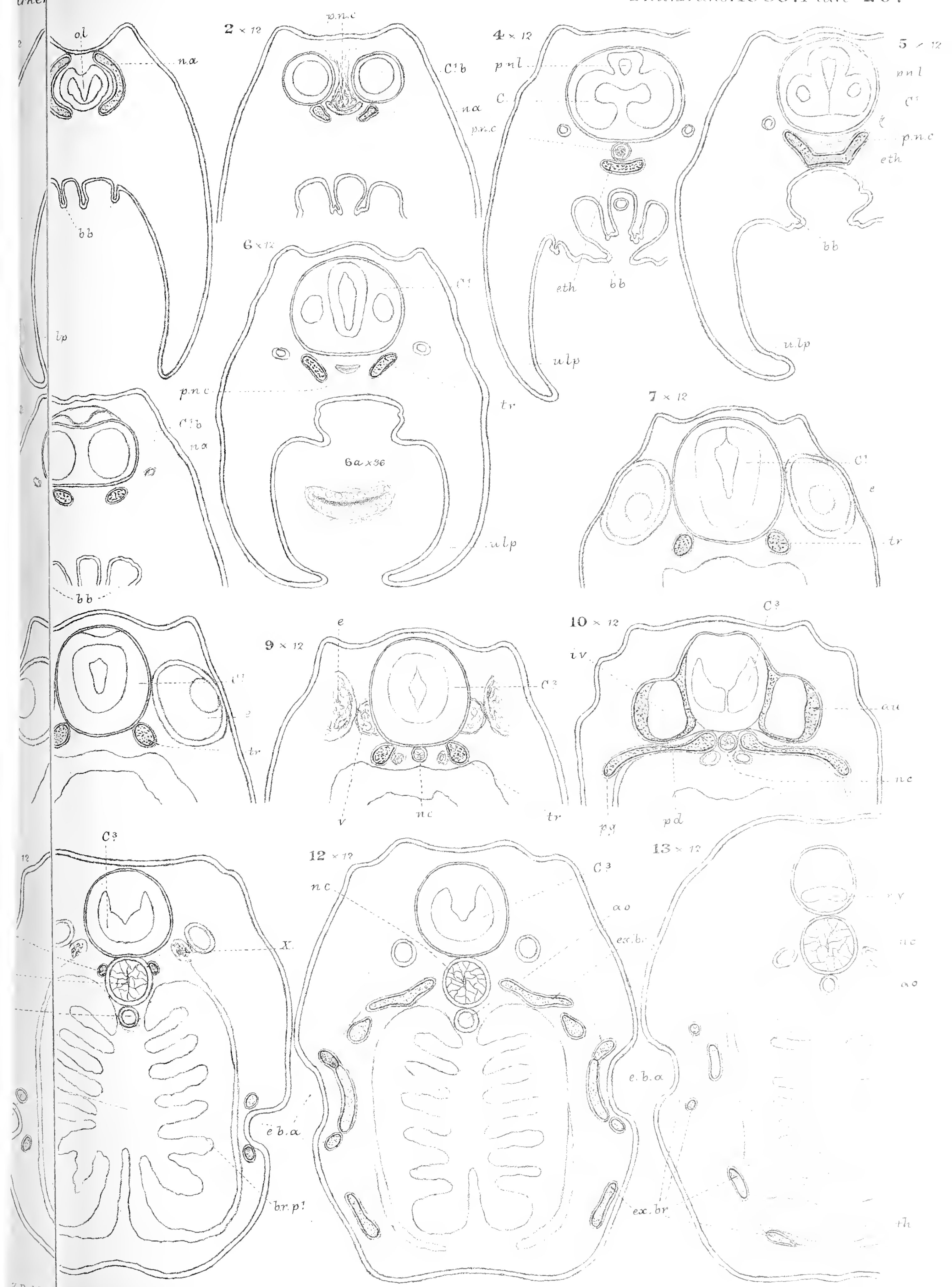


K.P. del. ad. nat.
.P. Parker lith. et del.

PETROMYZON.

West, Newman & Co. lith.



K.P. del. ad nat.
T.P. Parker lith et del.

PETROMYZON FLUVIATILIS.

West Newman & Co imp.

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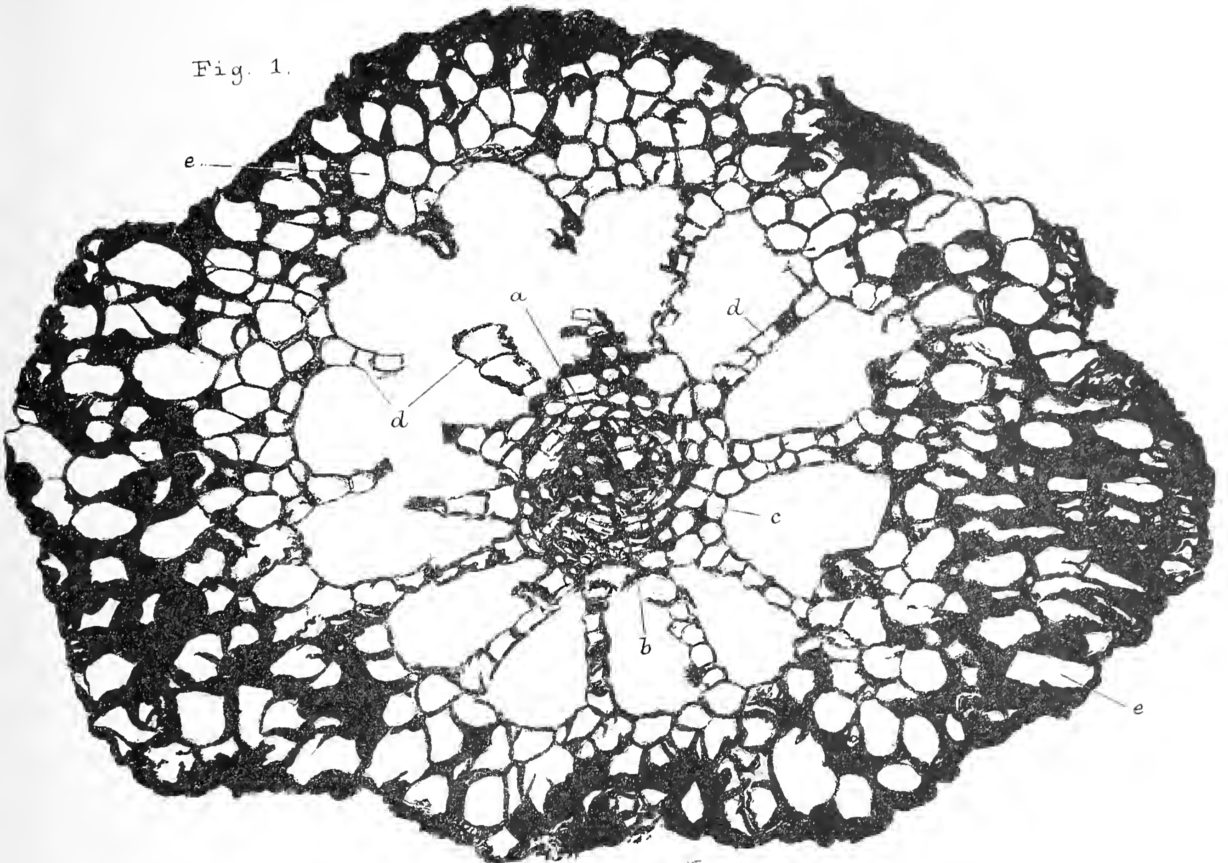

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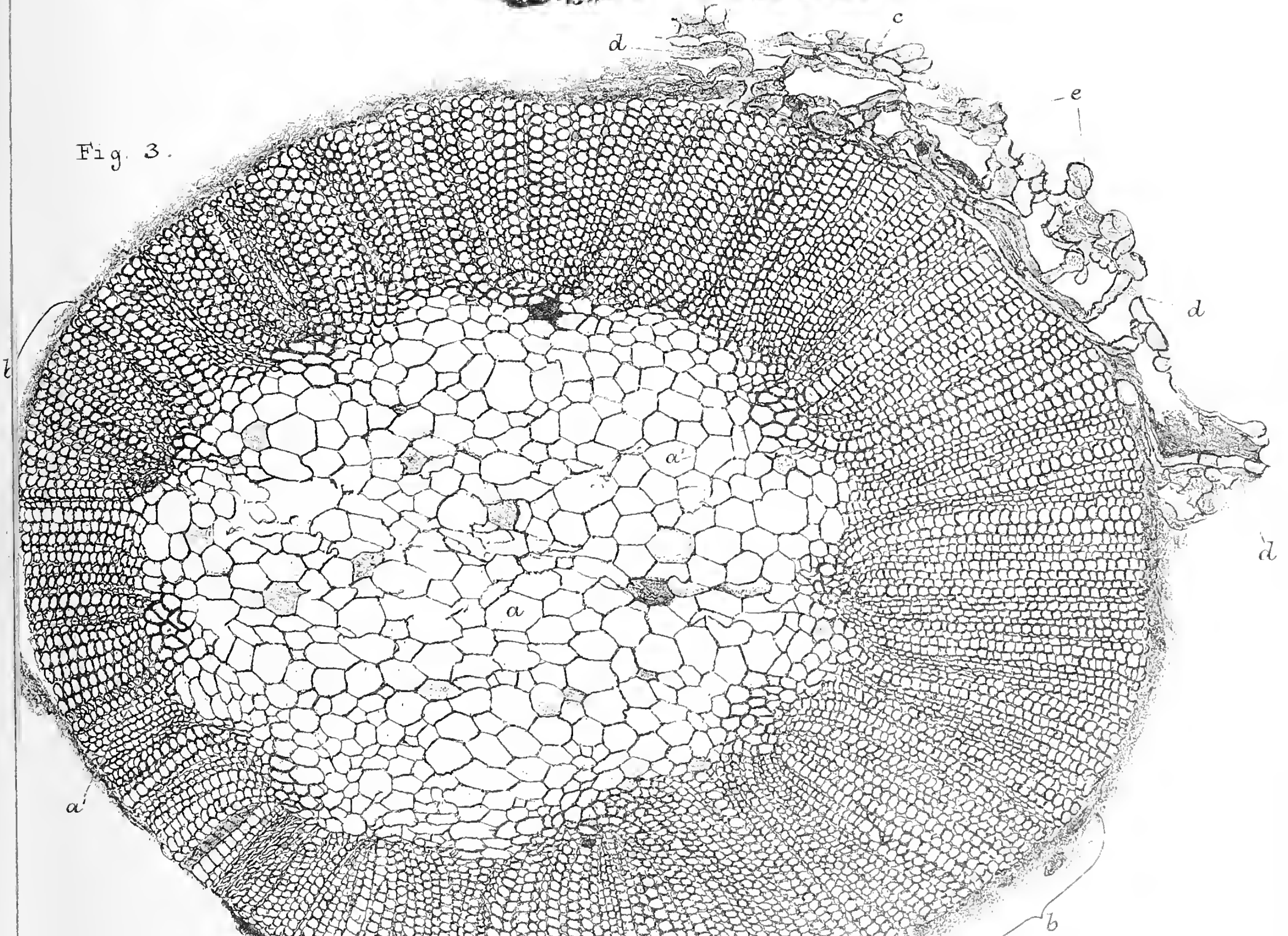

W. C. Williams , Del.

Whiteman & Bass. Photo Lith. London

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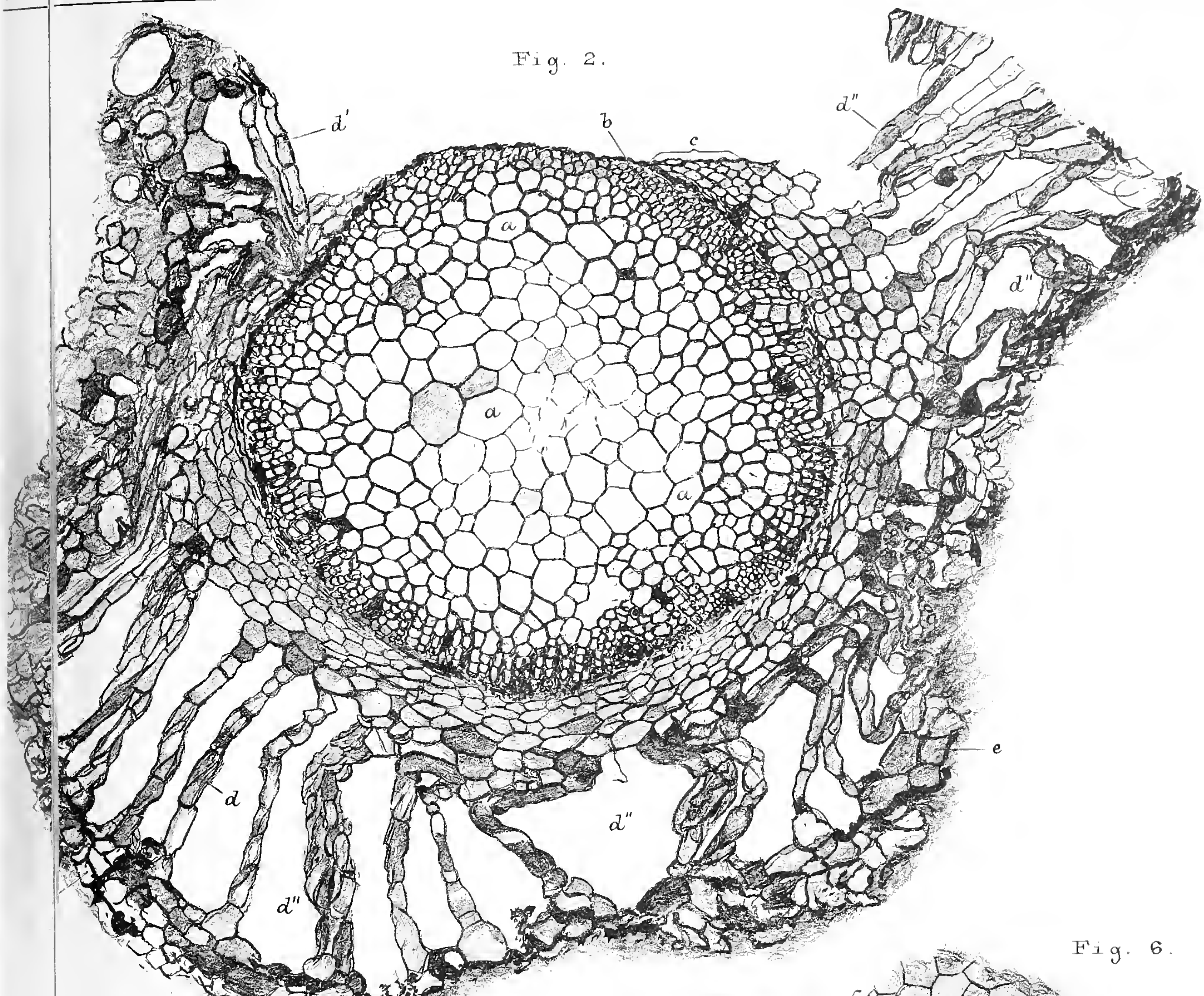

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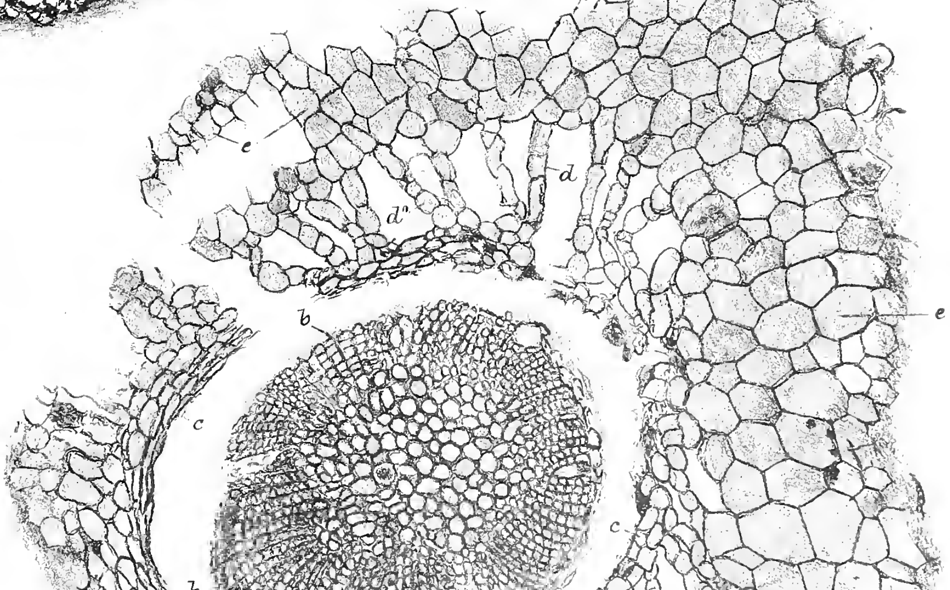

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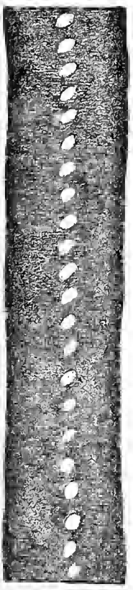

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Whiteman & Bass Photo-Lith. London

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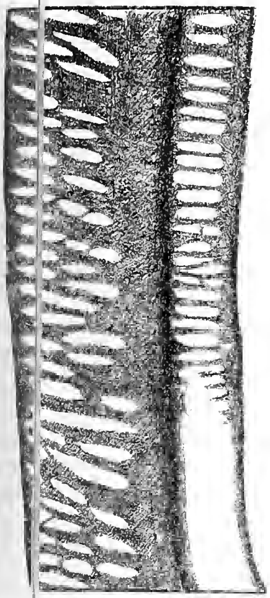

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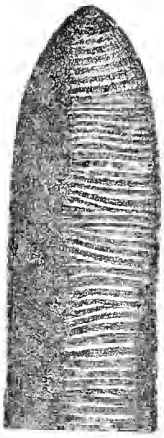

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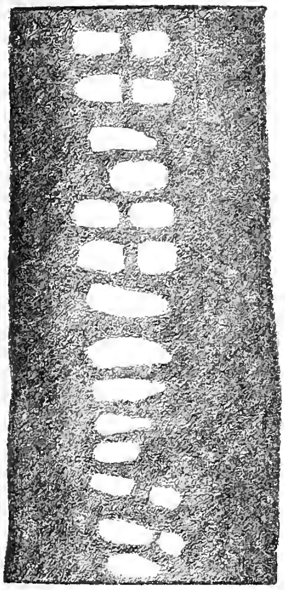

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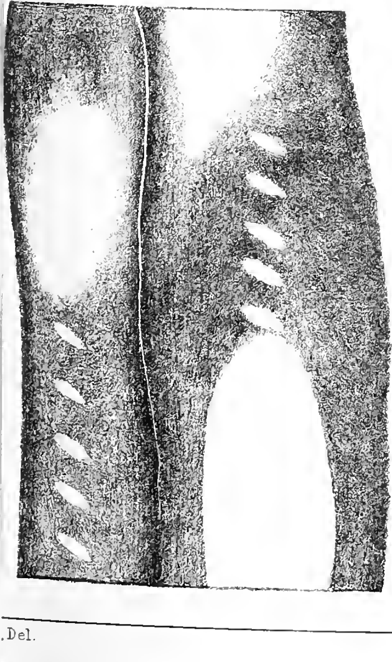

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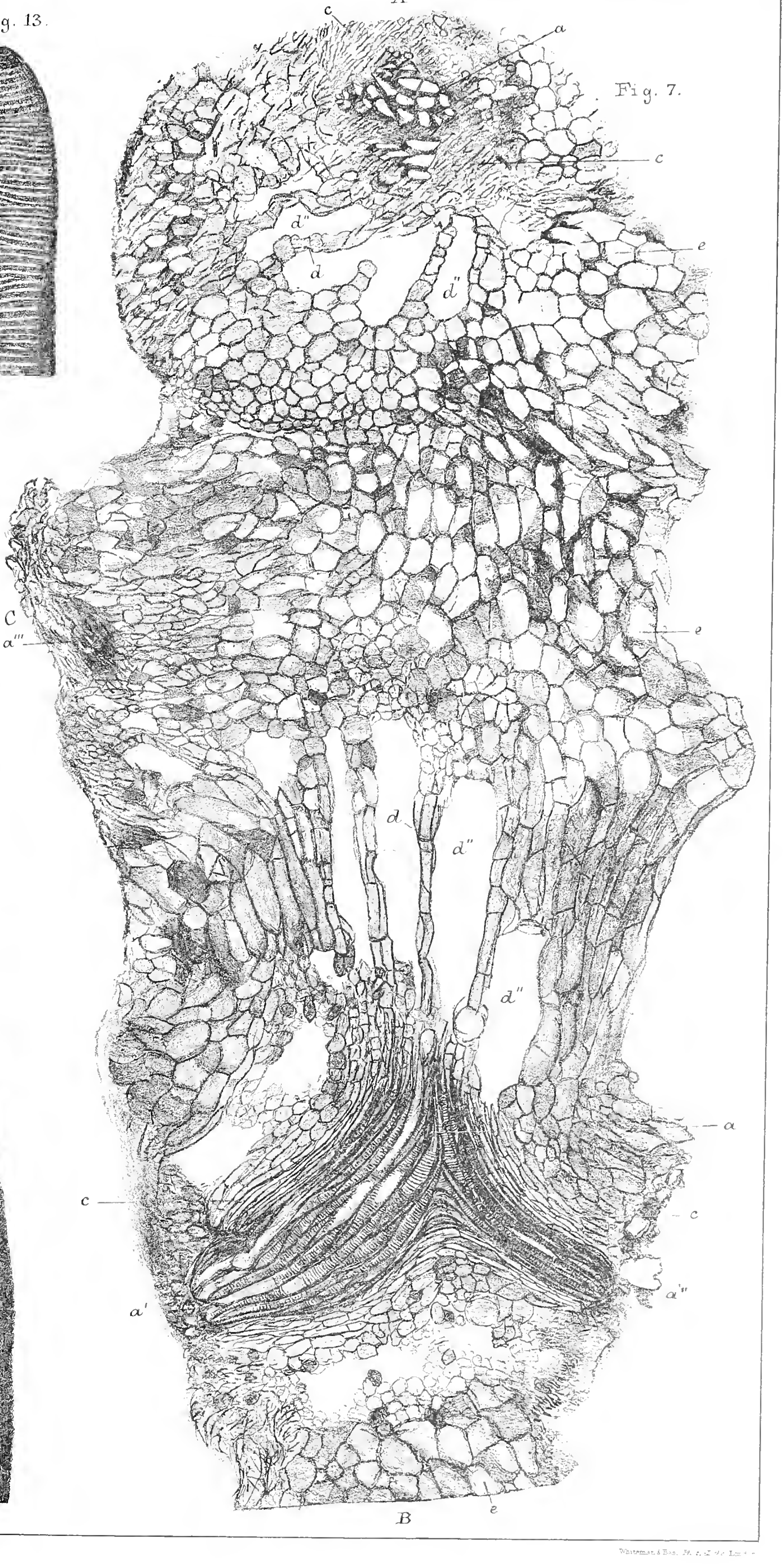

W. C. Williams, Del.

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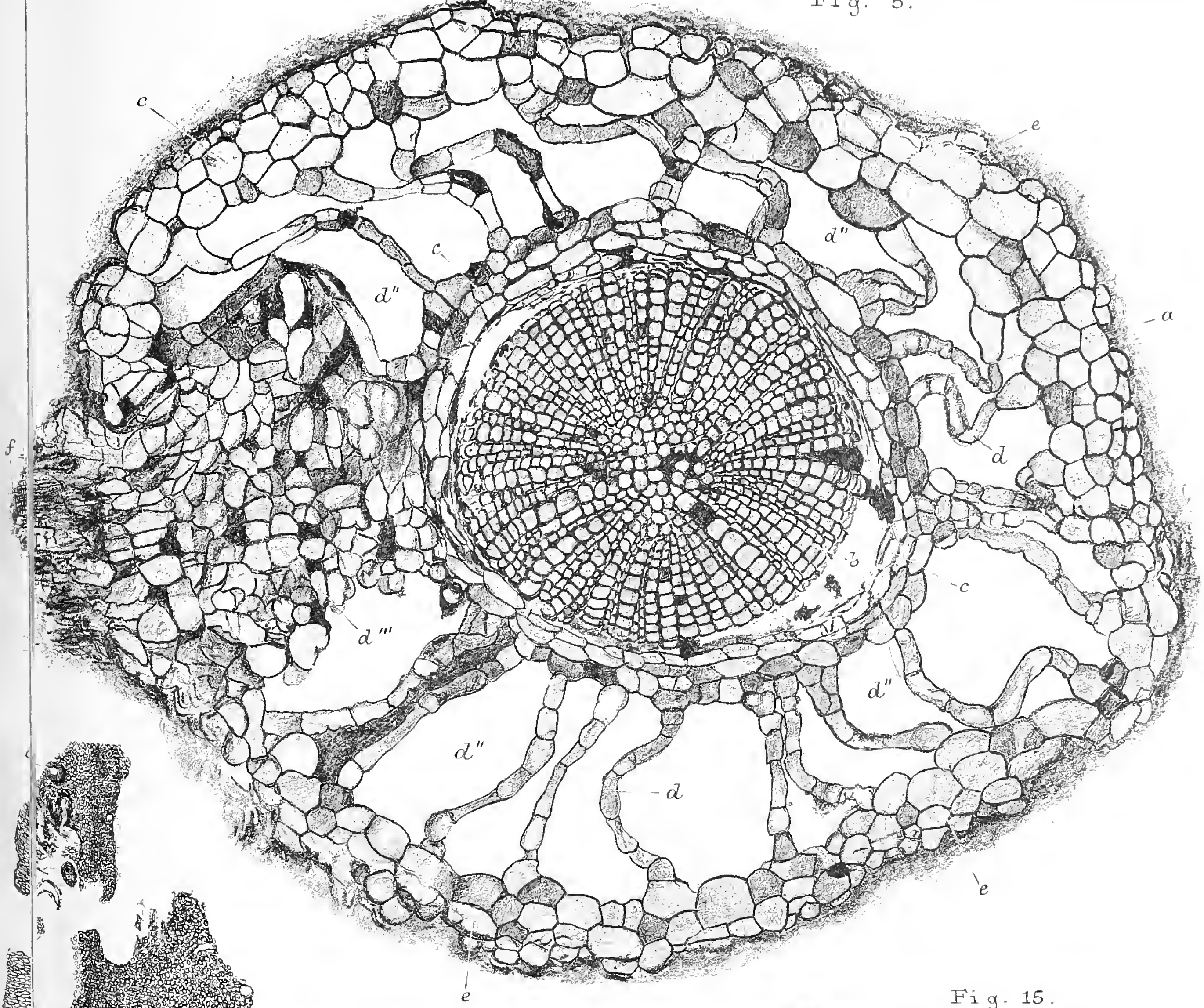

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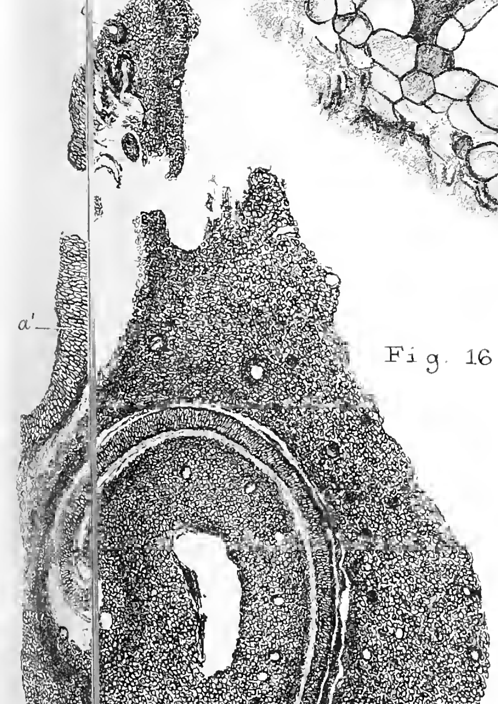

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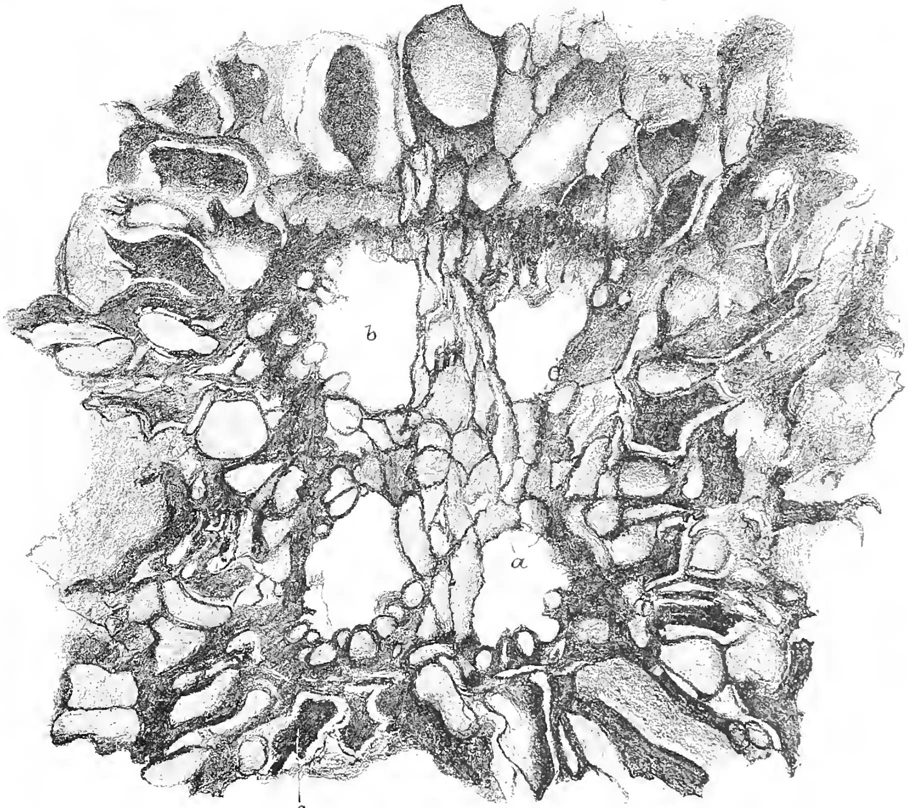

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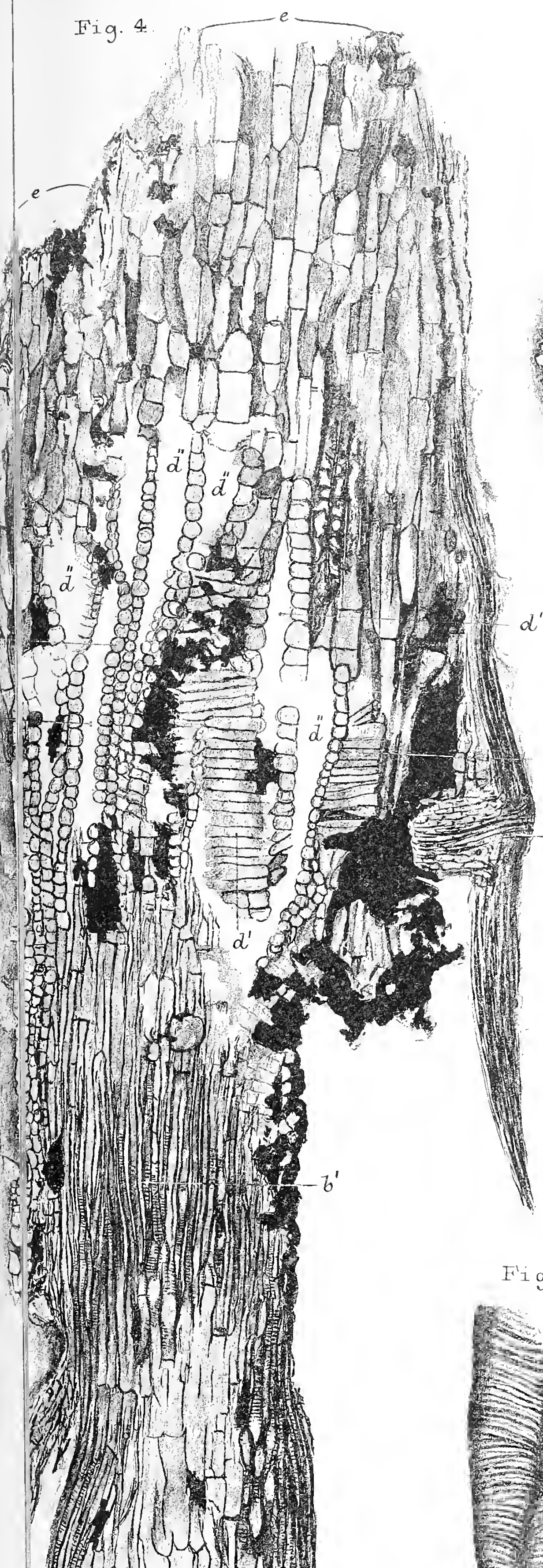

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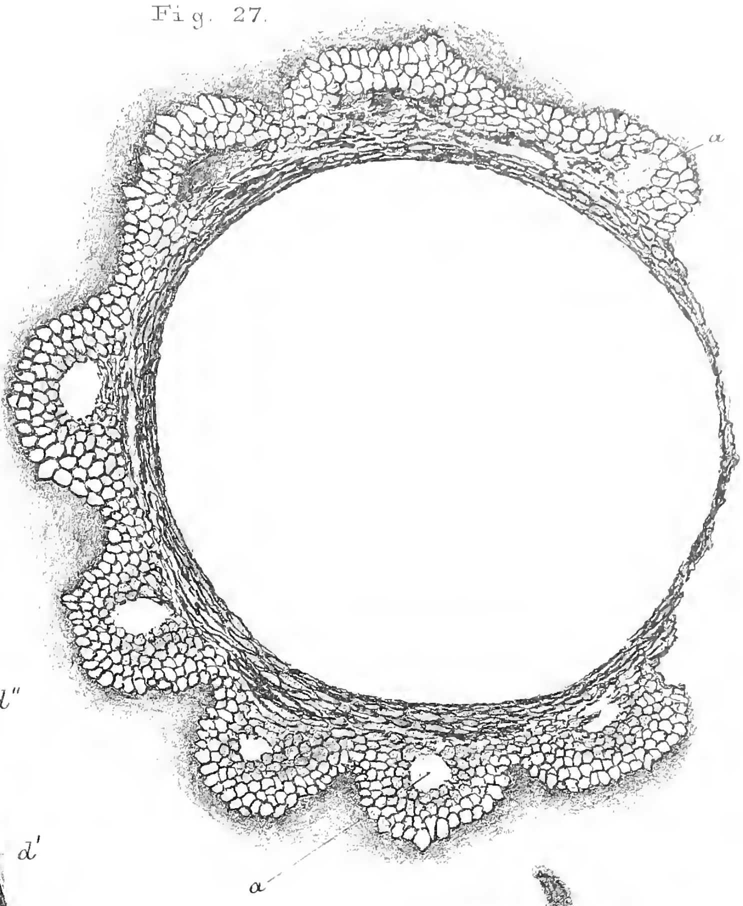

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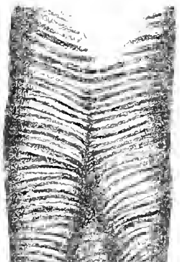

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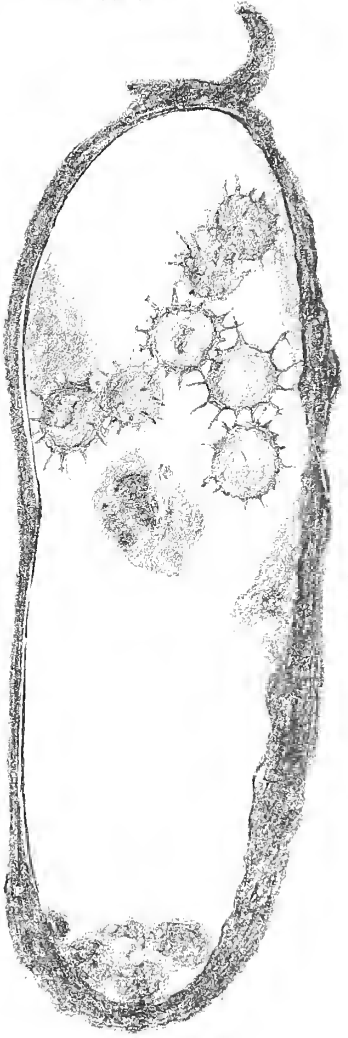

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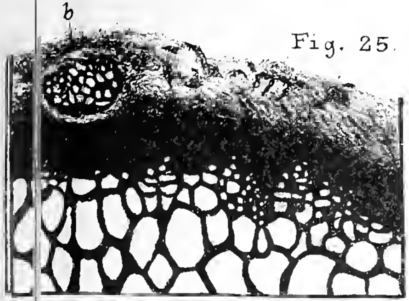

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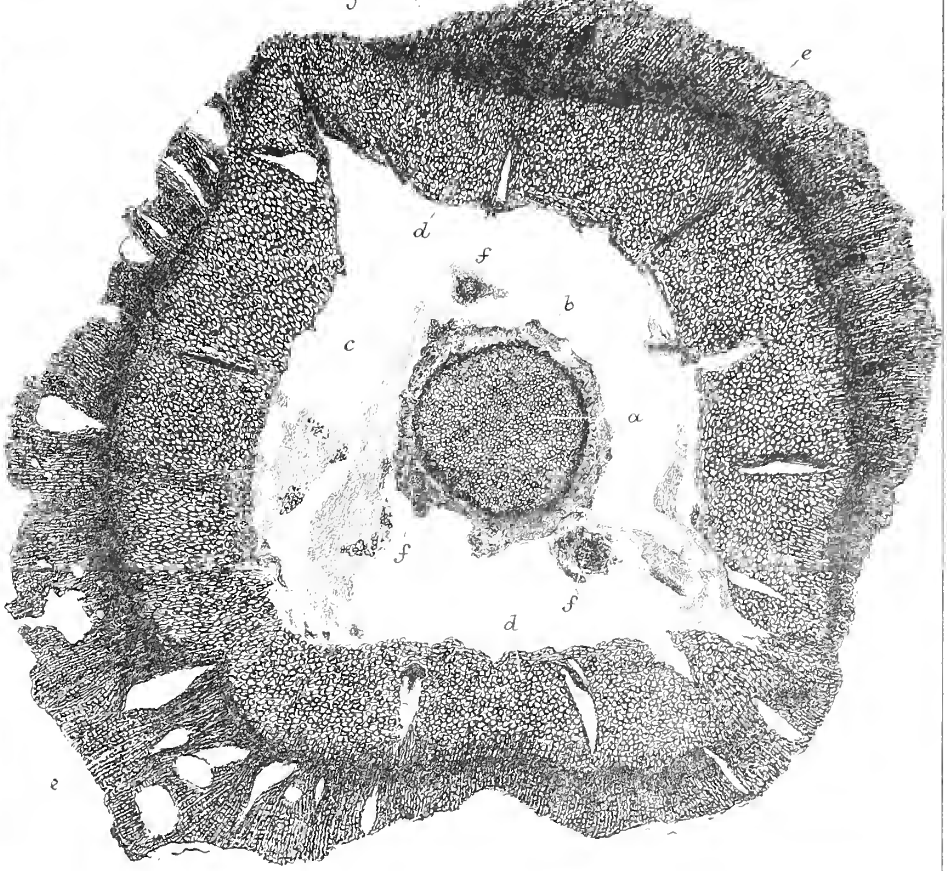

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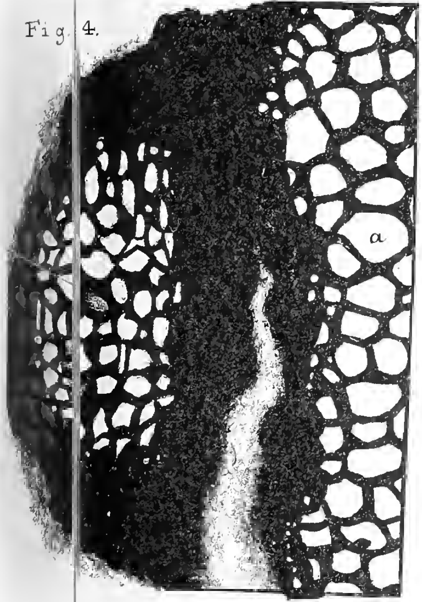

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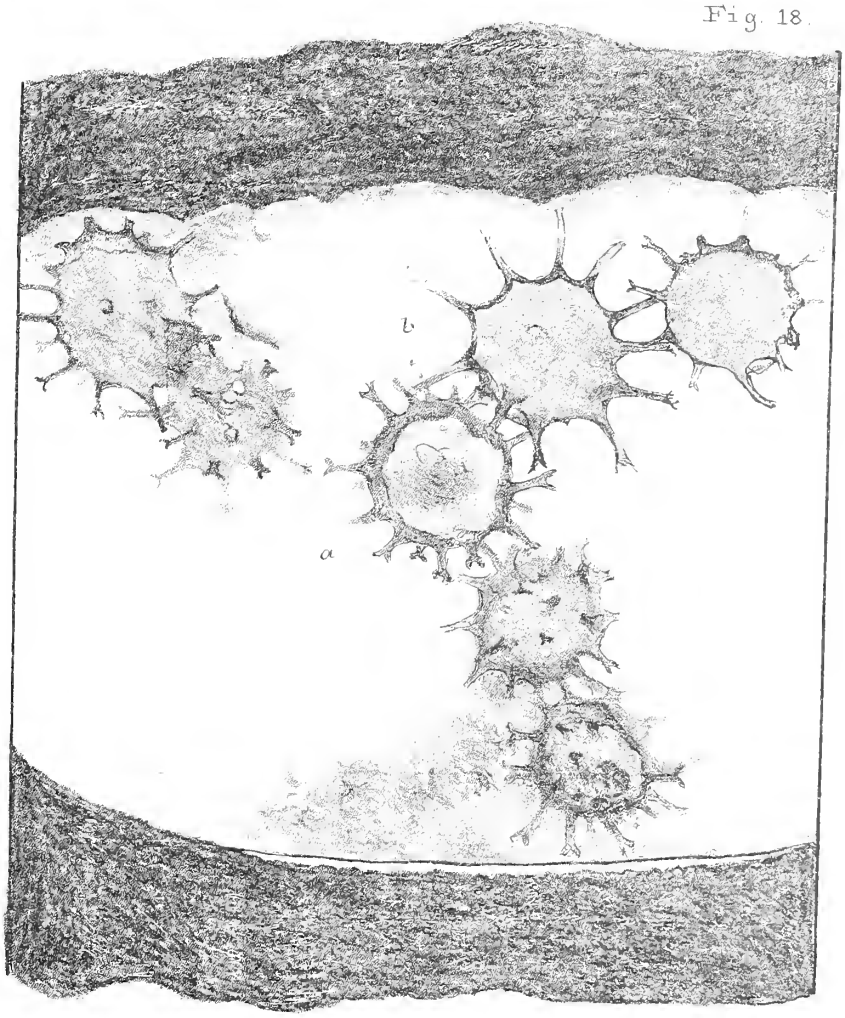

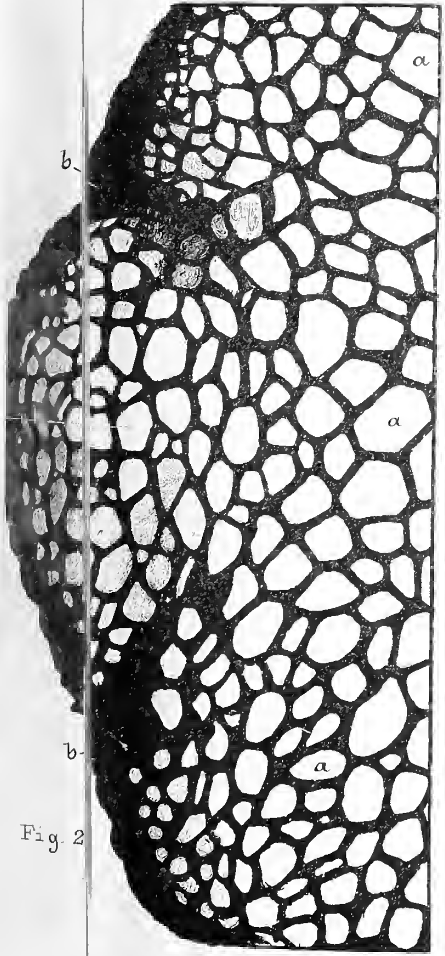

Fig. 2

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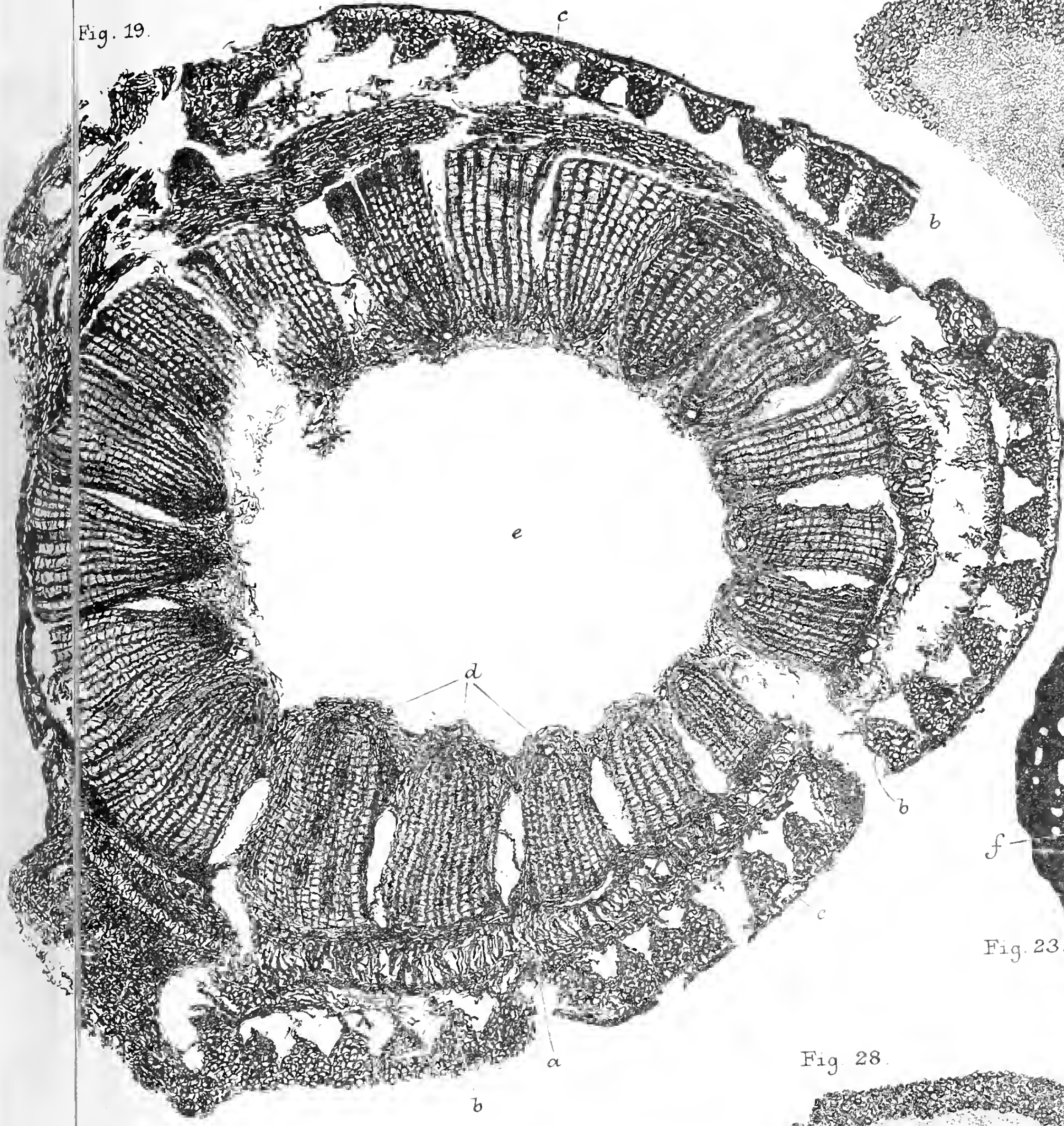
Fig. 19.



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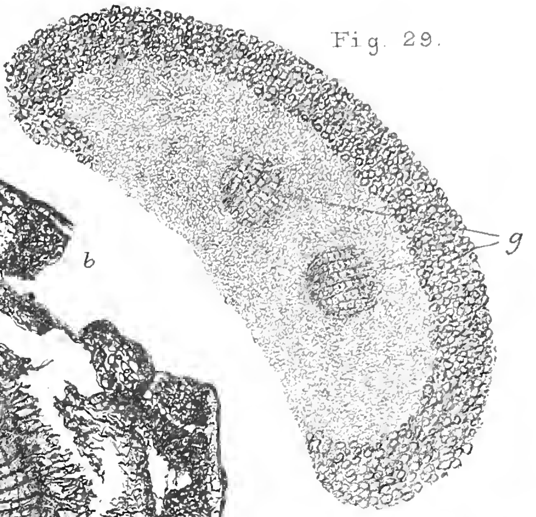

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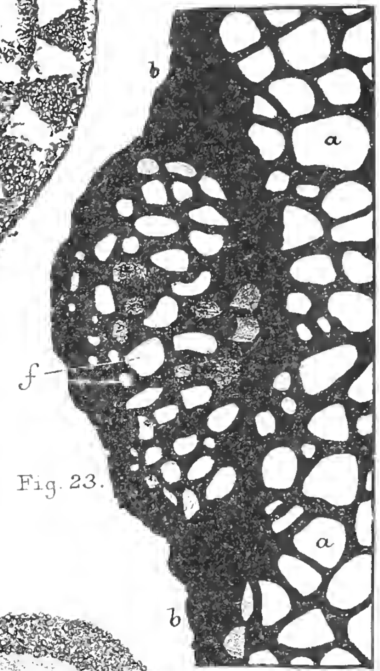

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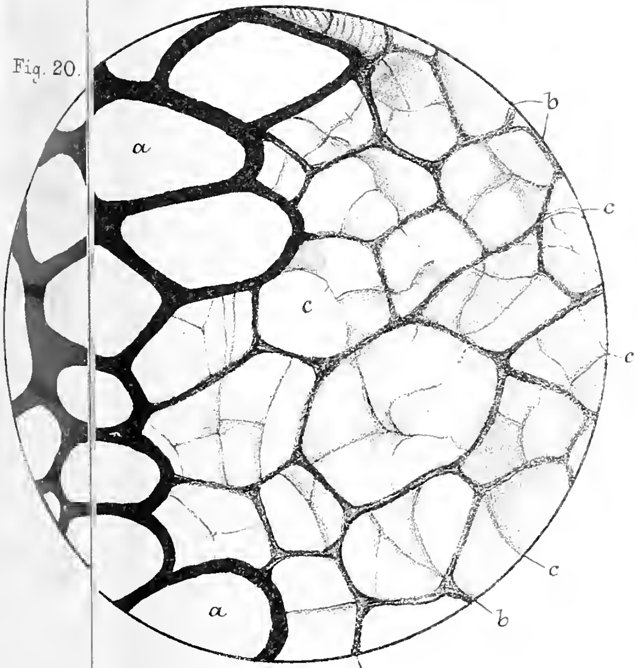

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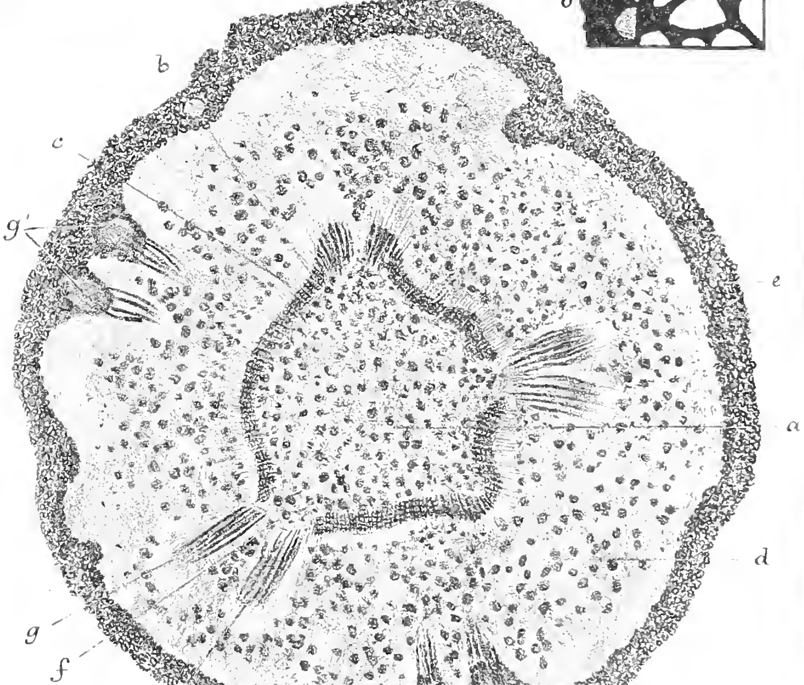

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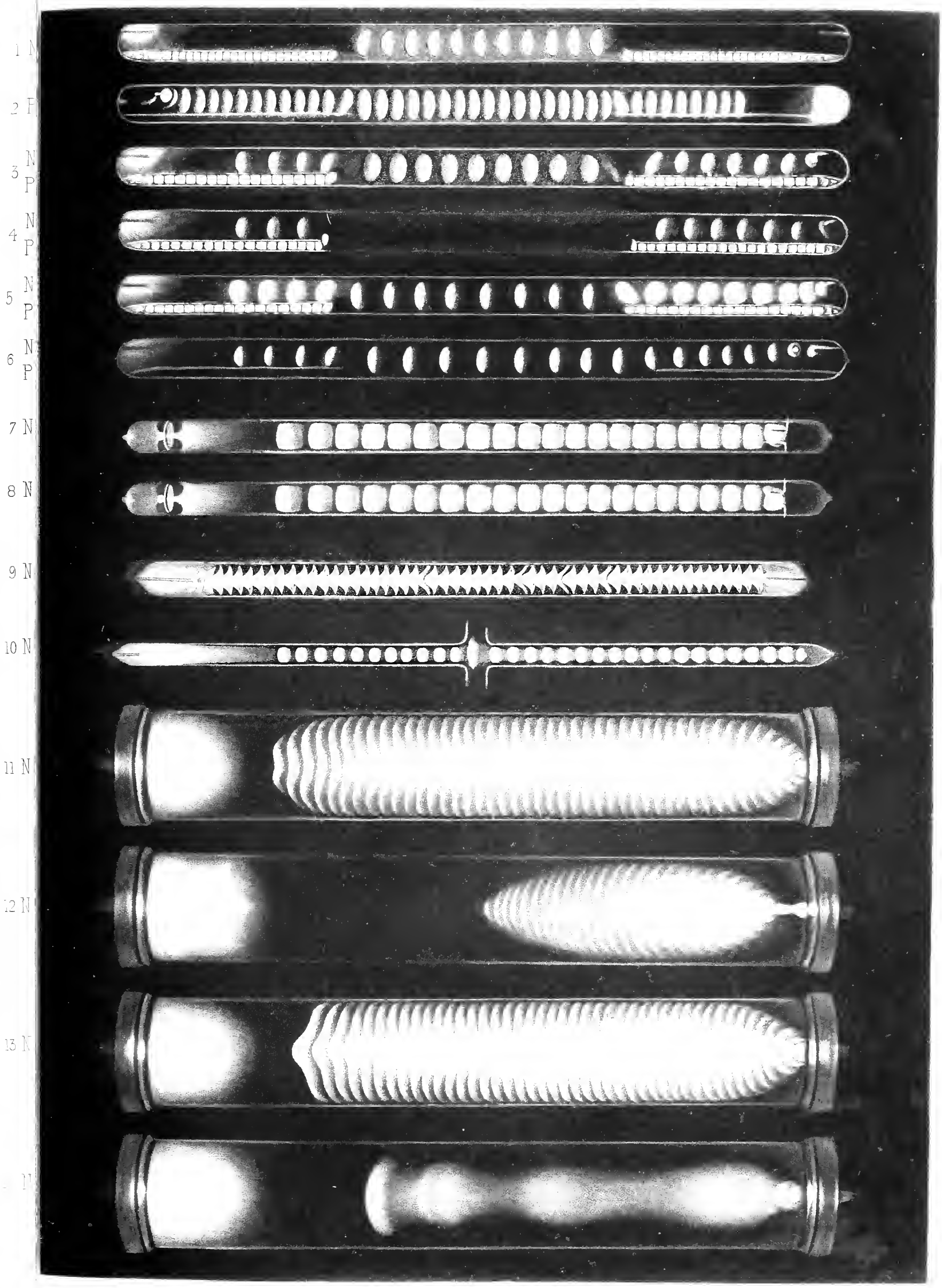
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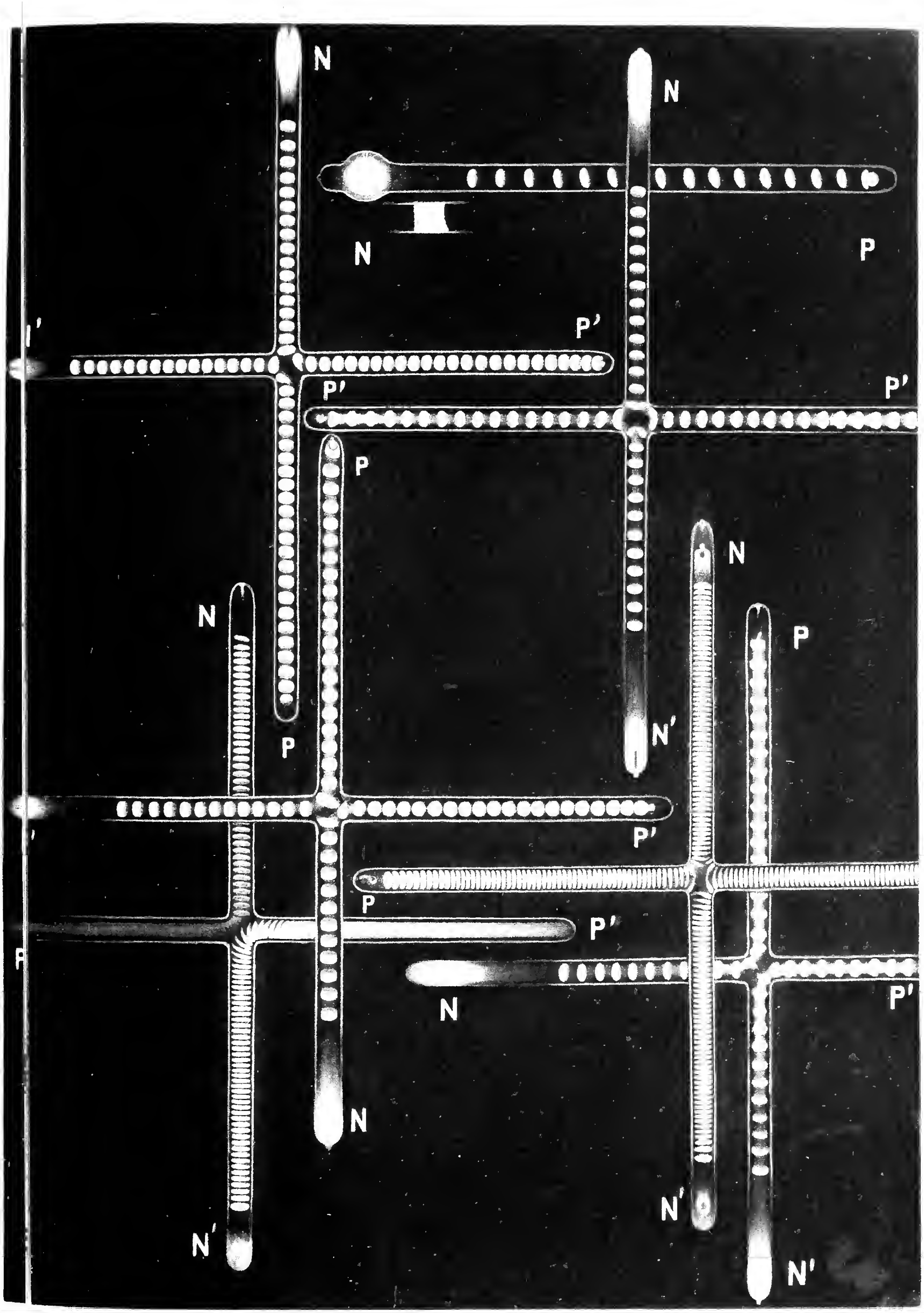
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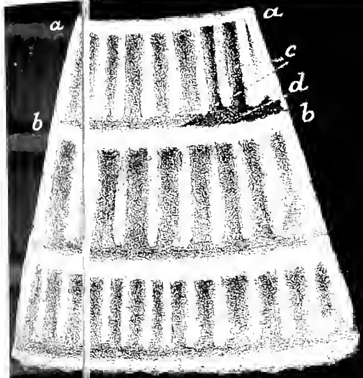

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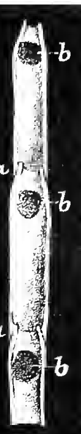

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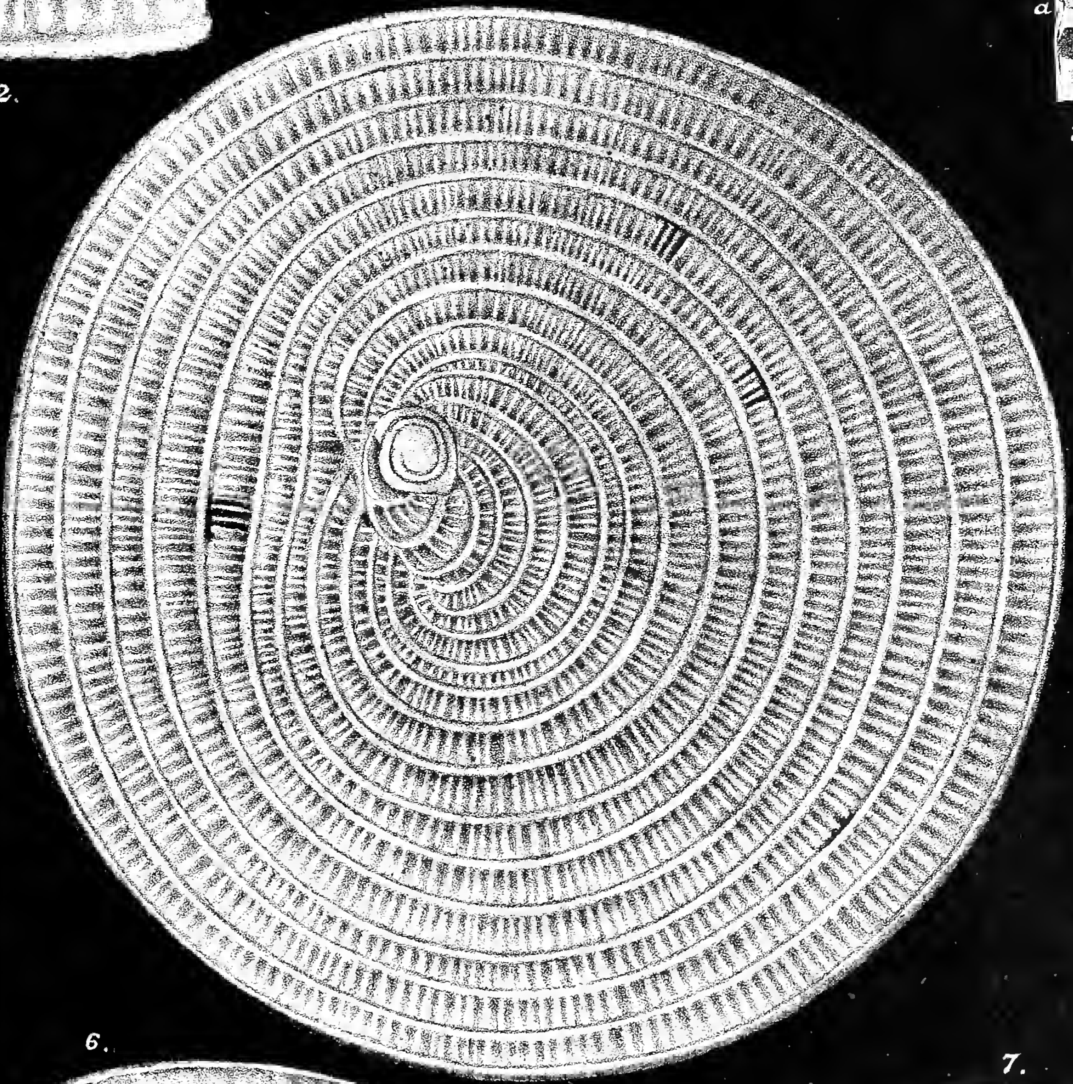

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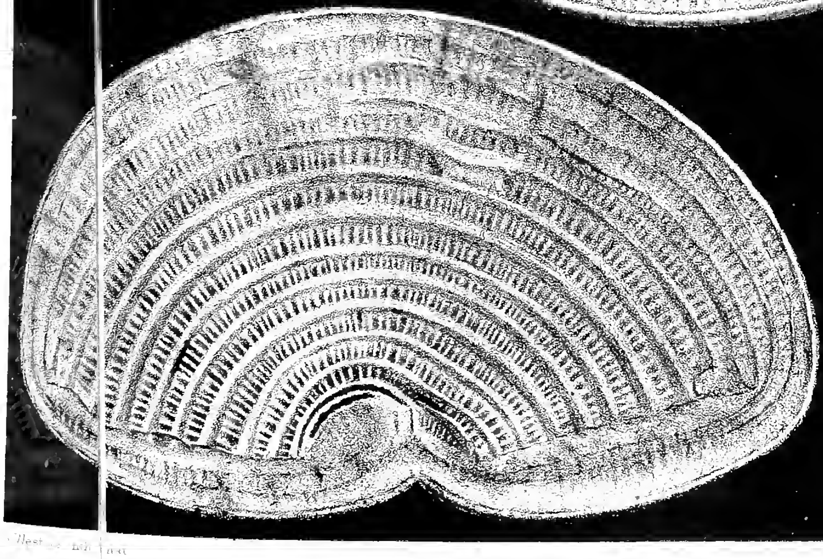

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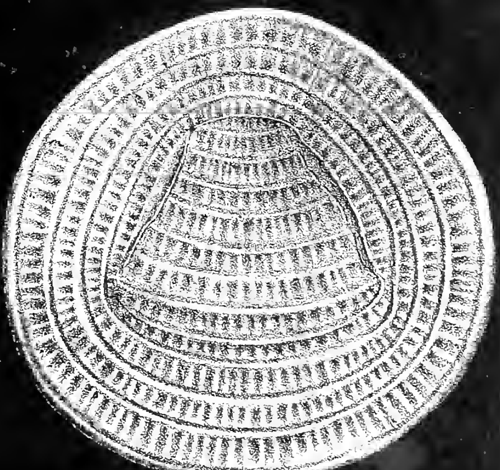
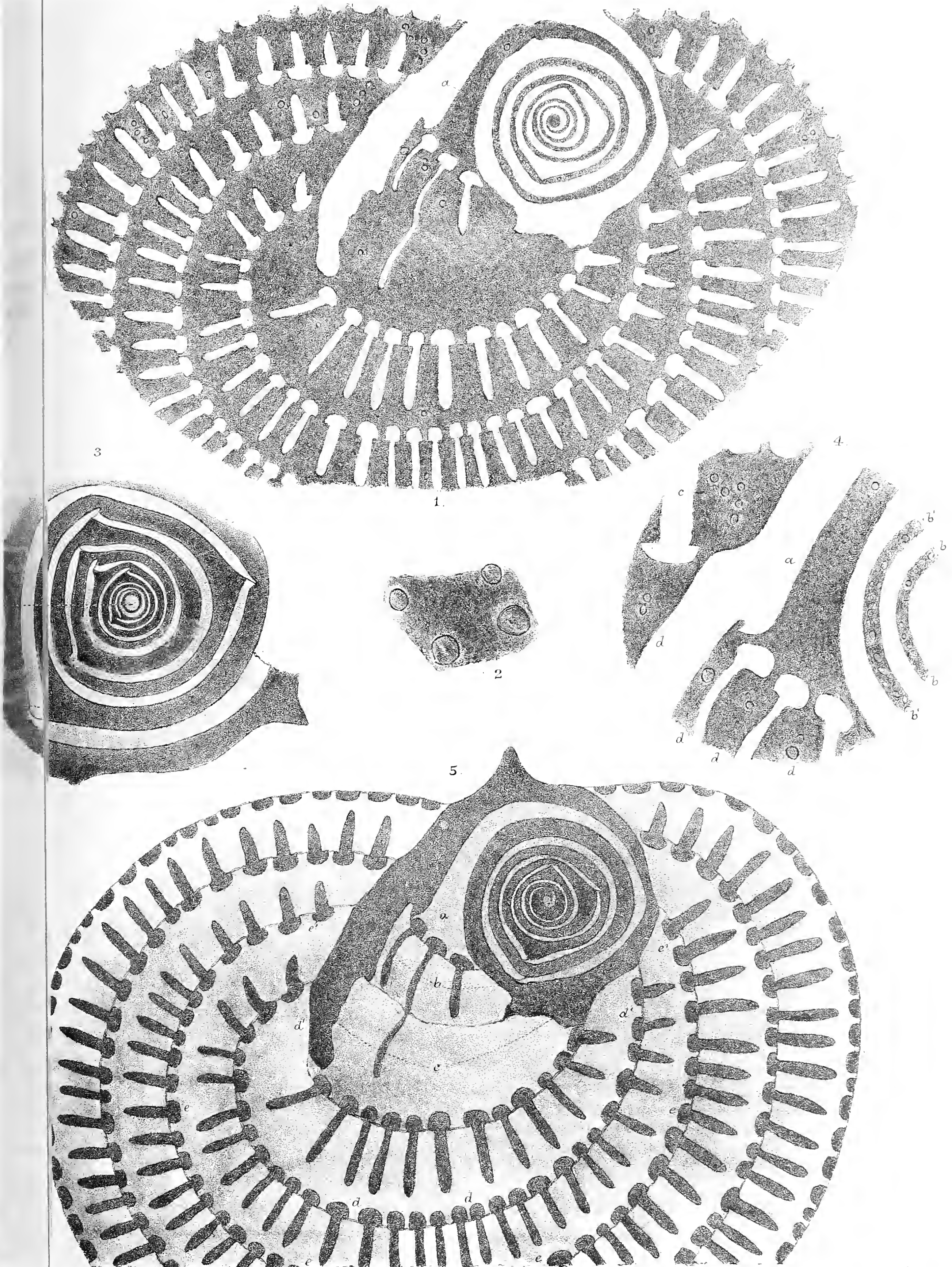


ORBITOLITES TENUISSIMA.



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West Newman & Co imp.

ORBITOLITES TENUISSIMA

THYLOCOLEO CARNIFEX

West, Newman & Co.

Fig. 1.

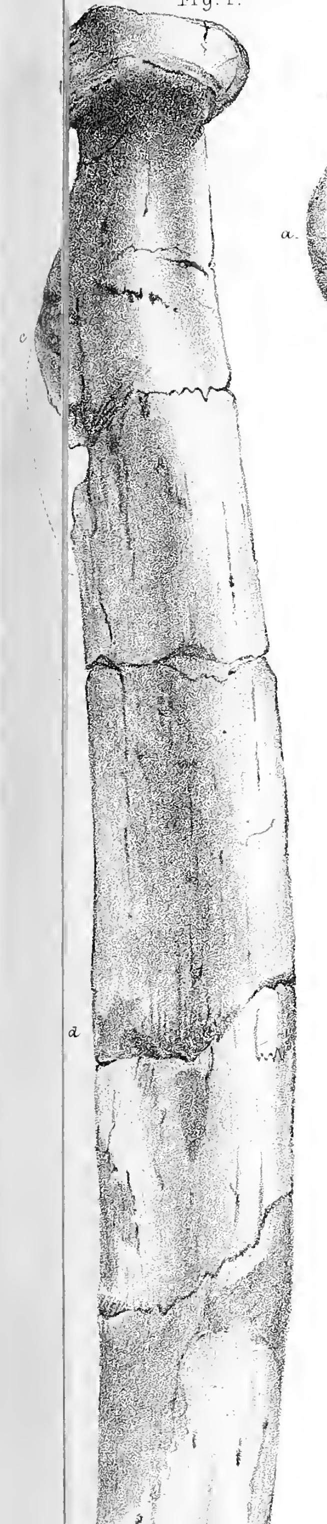


Fig. 2

Fig. 3.

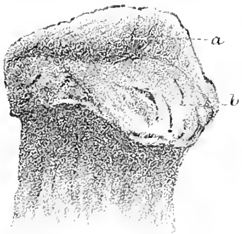
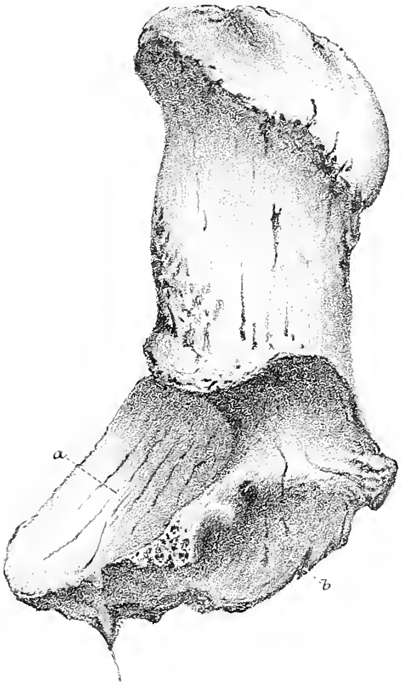


Fig. 4

Fig. 5.



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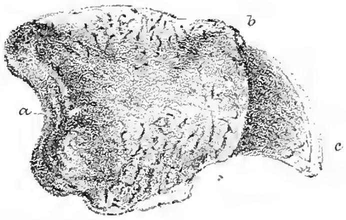

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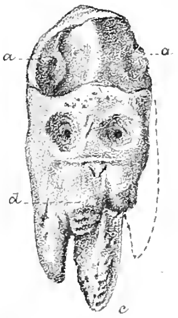
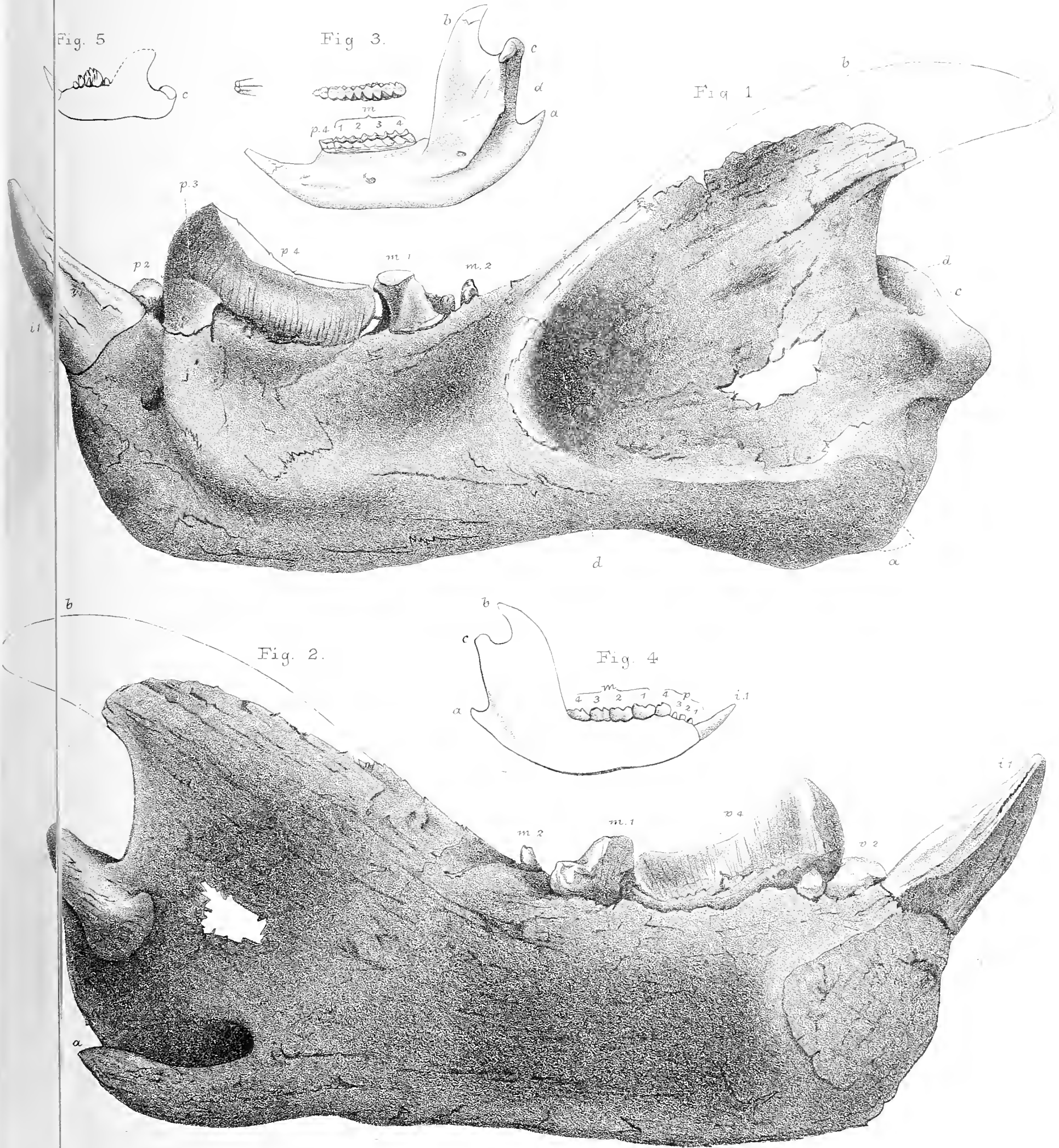


Fig. 8.

THYLOCOLEO CARNIFEX.

West, Newman & Co. lith.



West Newman & Co. lith

THYLOCOLEO CARNIFEX.

Ward

Phil. Trans. 1883. Plate 42.

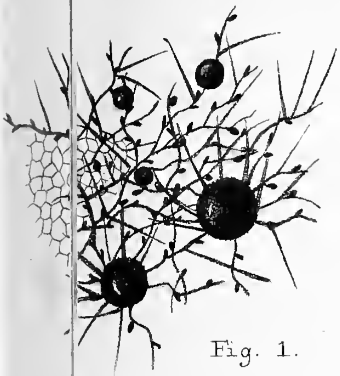
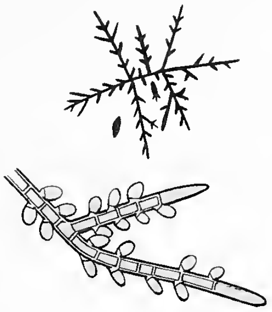


Fig. 1.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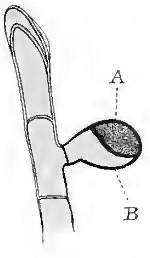

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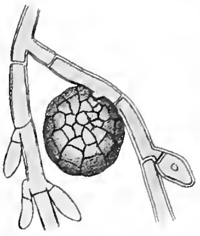
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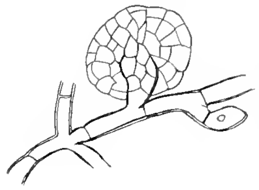

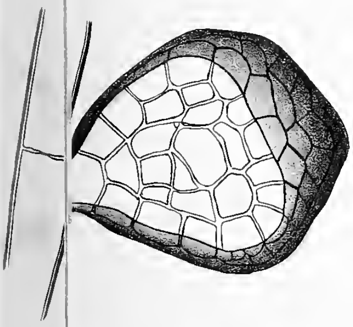
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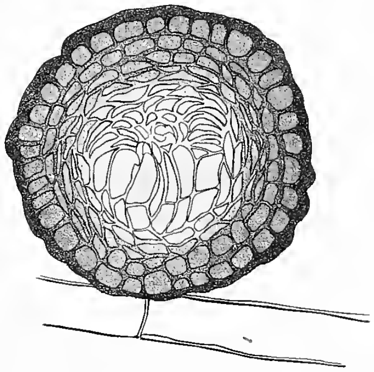
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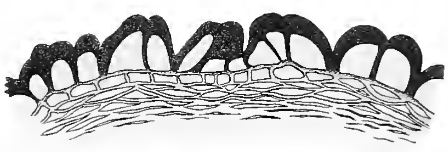
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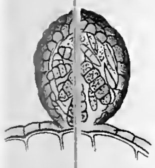
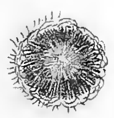

F . 35.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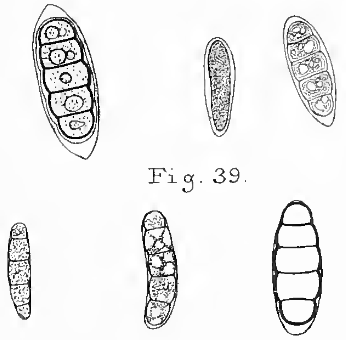
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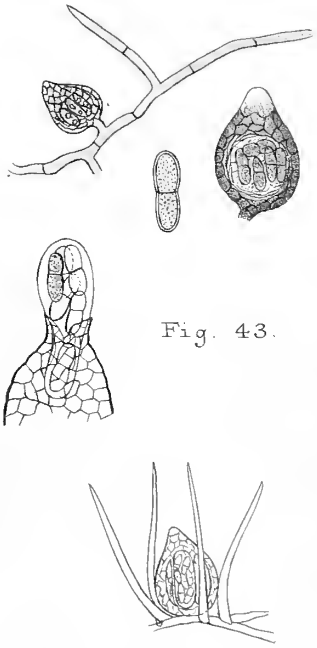
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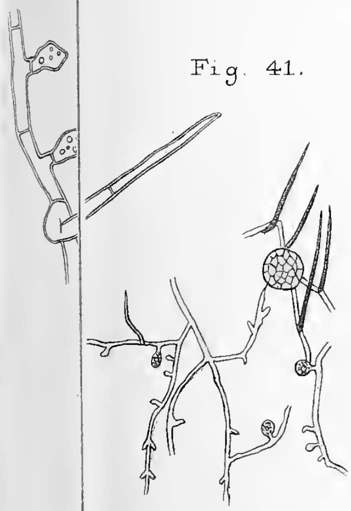
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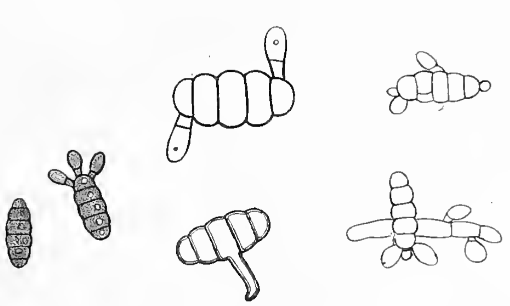
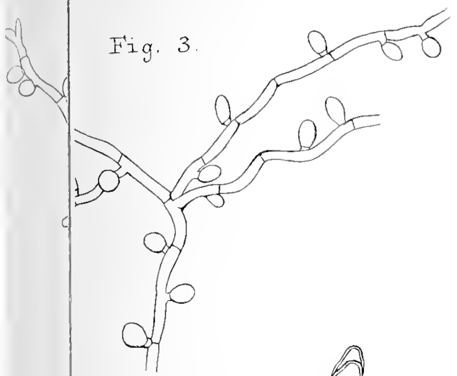


Fig. 40.

West Newman & C° chr. lith.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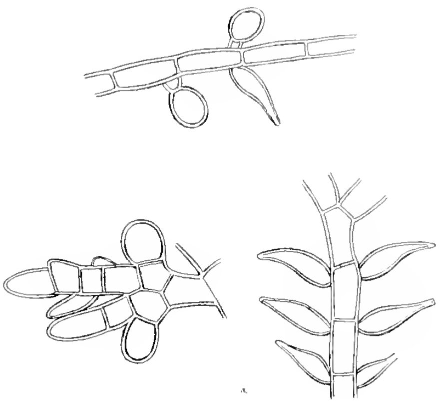
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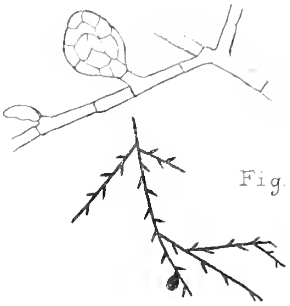
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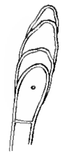
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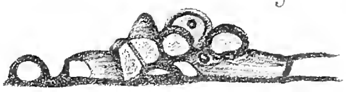


Fig 6.


Fig. 10.


Fig. 11.

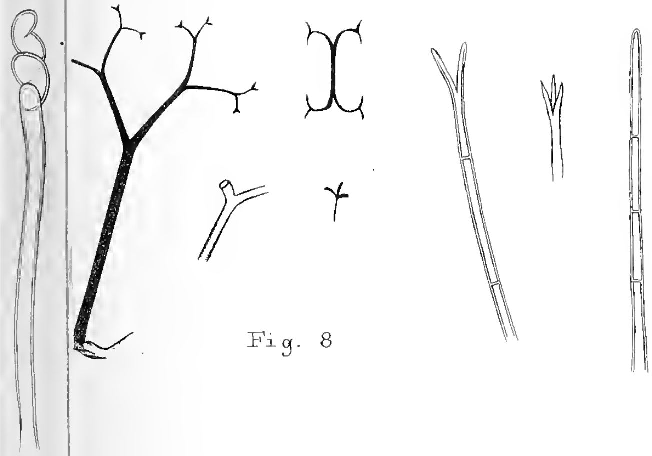
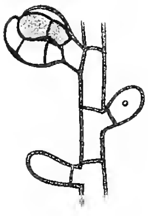


Fig. 8

g. 12.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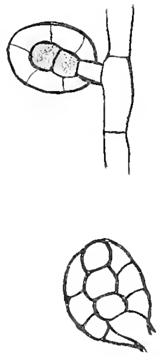
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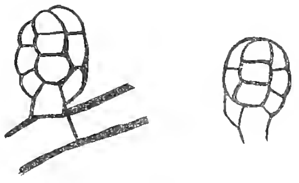
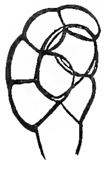


Fig. 15.


Fig 16.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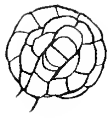
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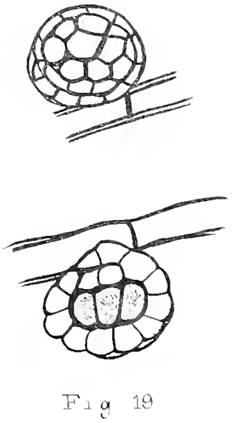
Fig 19

West, Newman & C° lith

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Phil. Trans 1883. *Plate* 44.

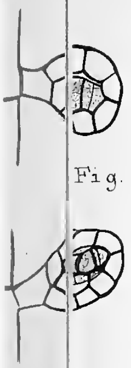
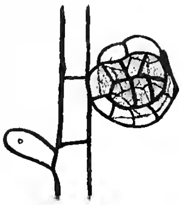


Fig. 20

Fig. 21.



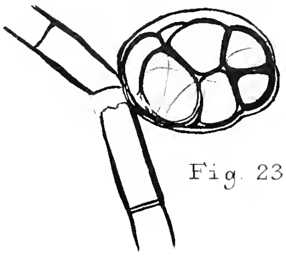
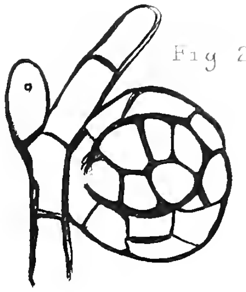


Fig. 23.

Fig 24



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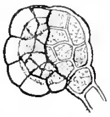


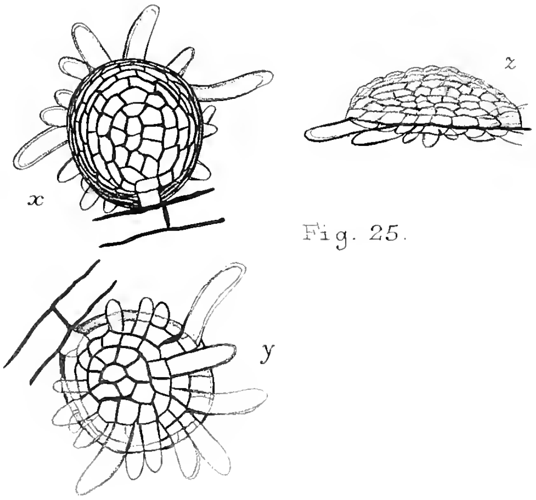
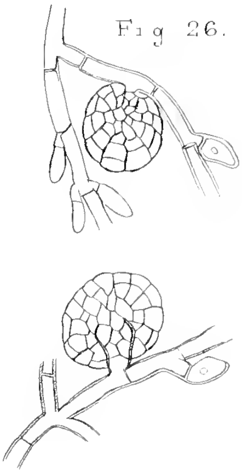


Fig. 25.

Fig 26.



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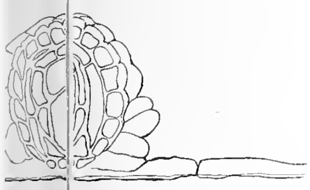

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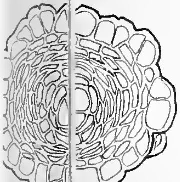

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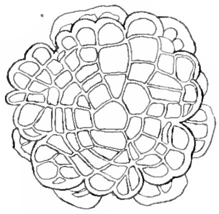


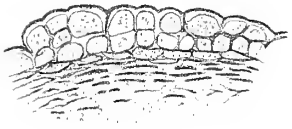
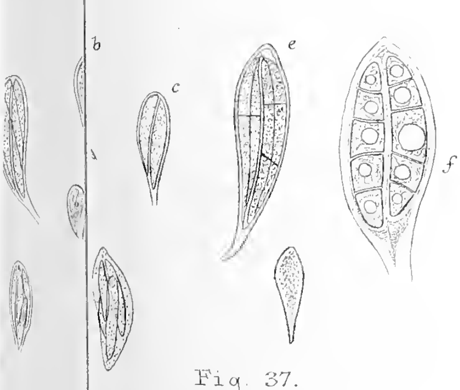


Fig. 32.

Fig 33

Fig. 38.



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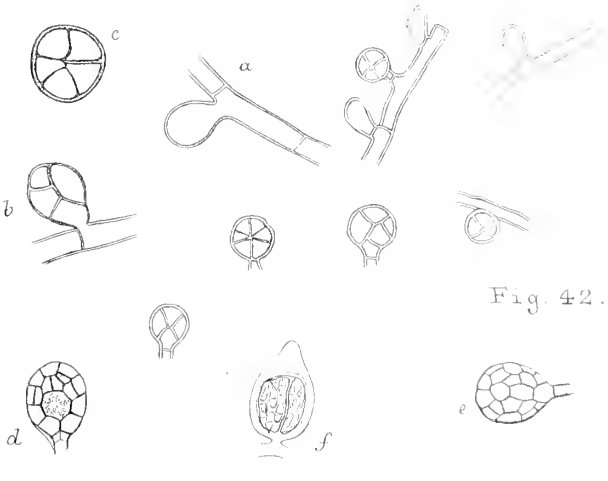

Fig. 42.

West Newman & C° lith

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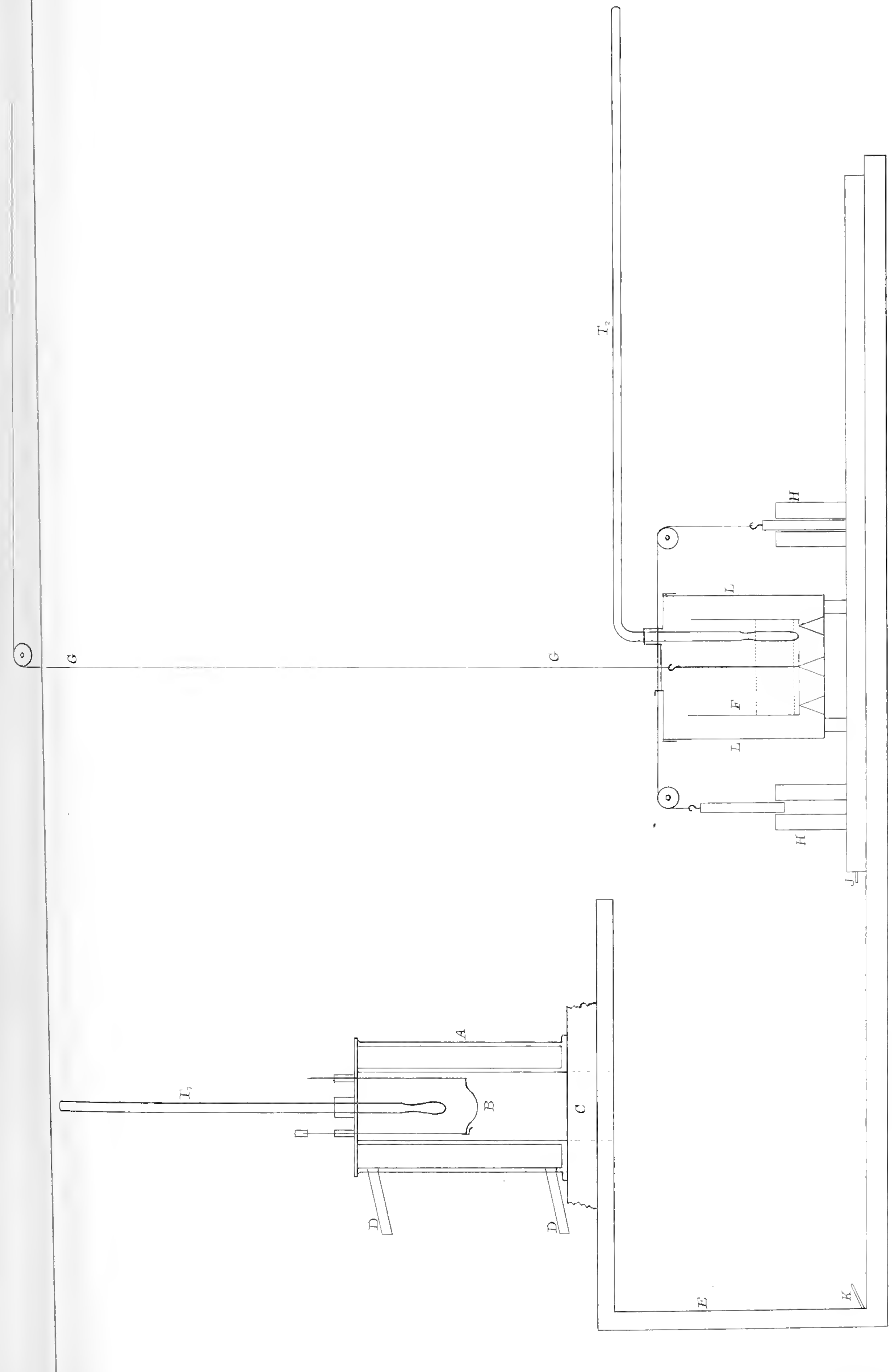
Phil. Trans. 1883. Plate 45.



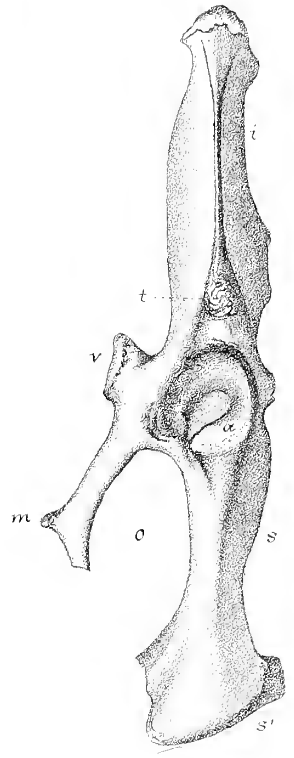
Atomic weight of Glucinum: Calorimeter for determining specific heat of the metal.

(Scale: 5 m/m = 1 c/m.)

West Newman & Co. lith.

Fig. 1.

Fig. 2.



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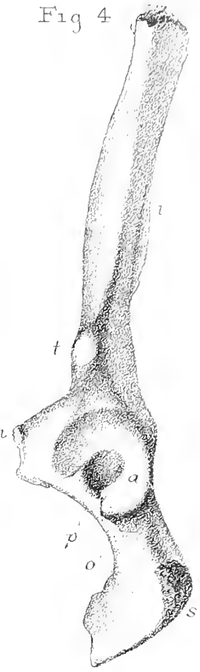


Fig 5

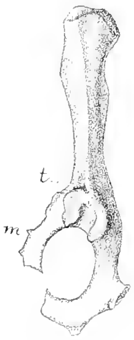
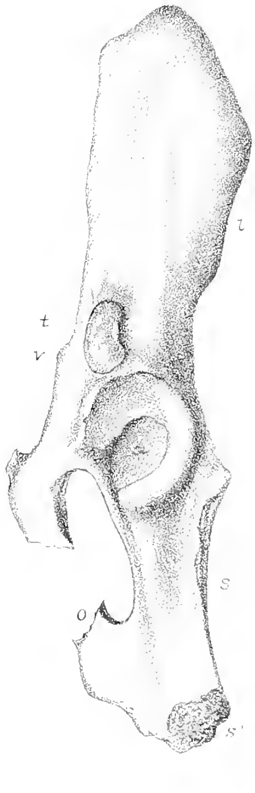


Fig. 3.



West, Newman & C^o. imp

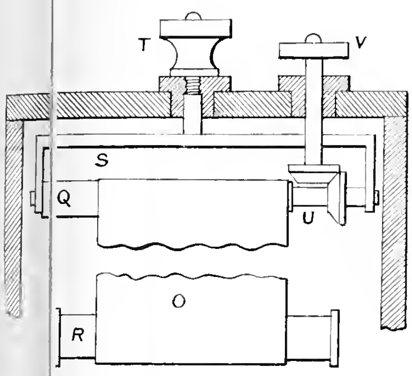


Fig. 2.

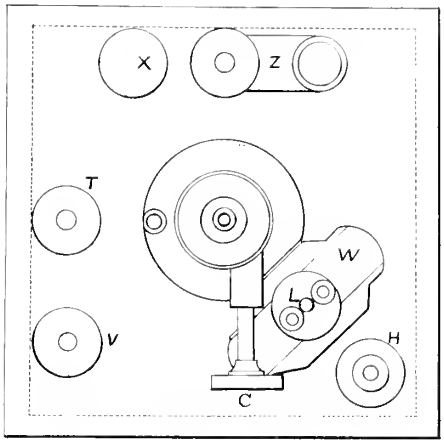


Fig. 3.

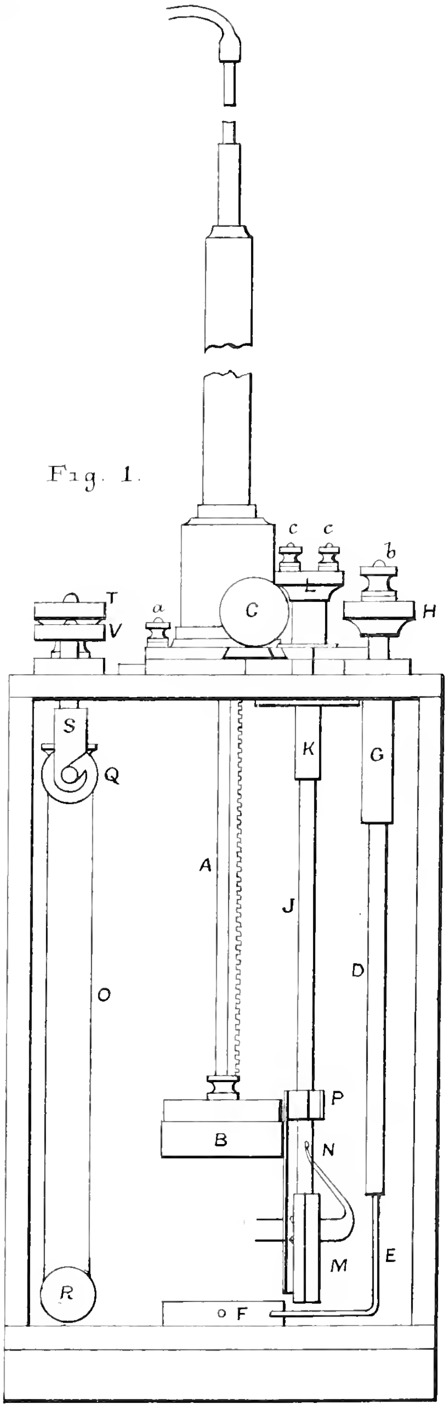


Fig. 1.

West Newman & C° lith

Fig 2.

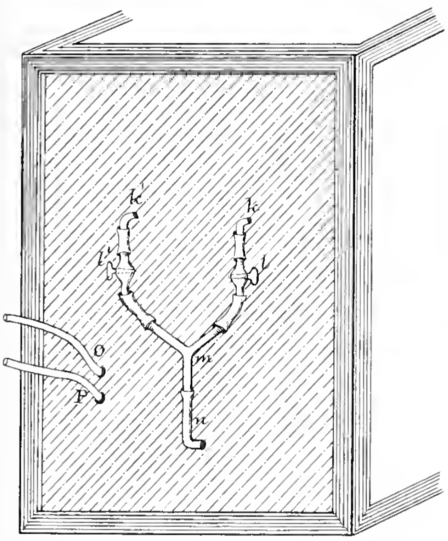


Fig 3.

Fig. 4.

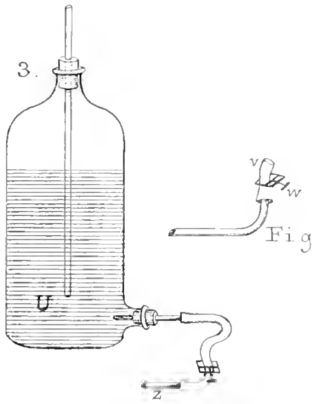
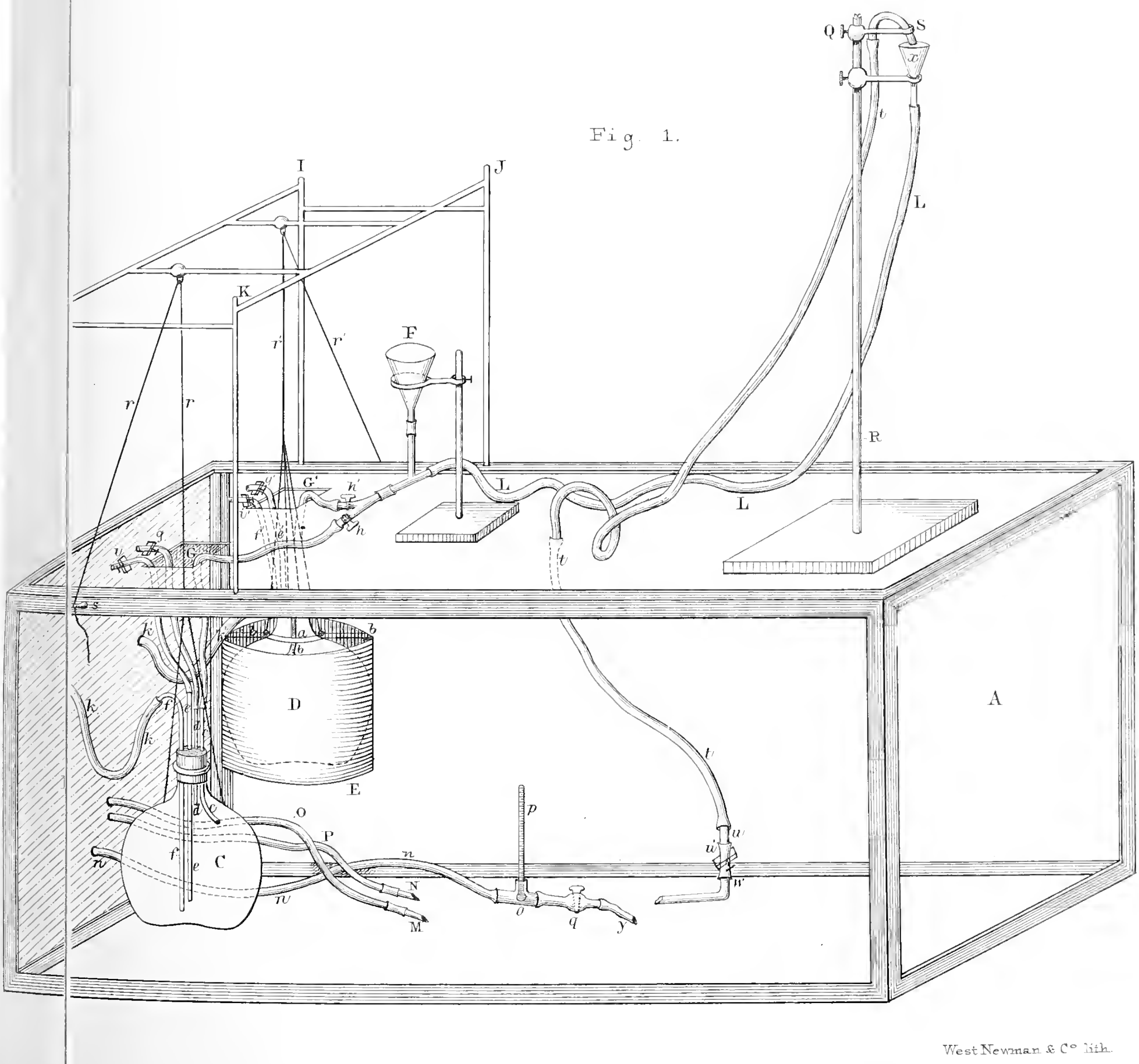


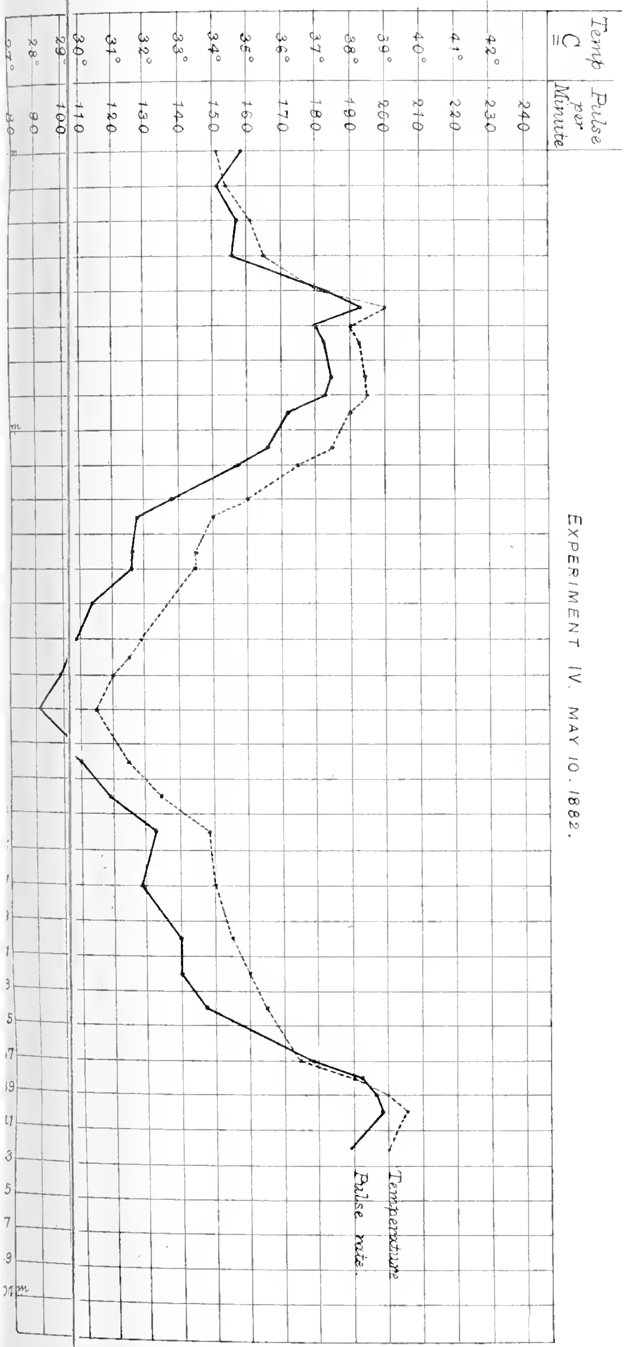
Fig. 1.

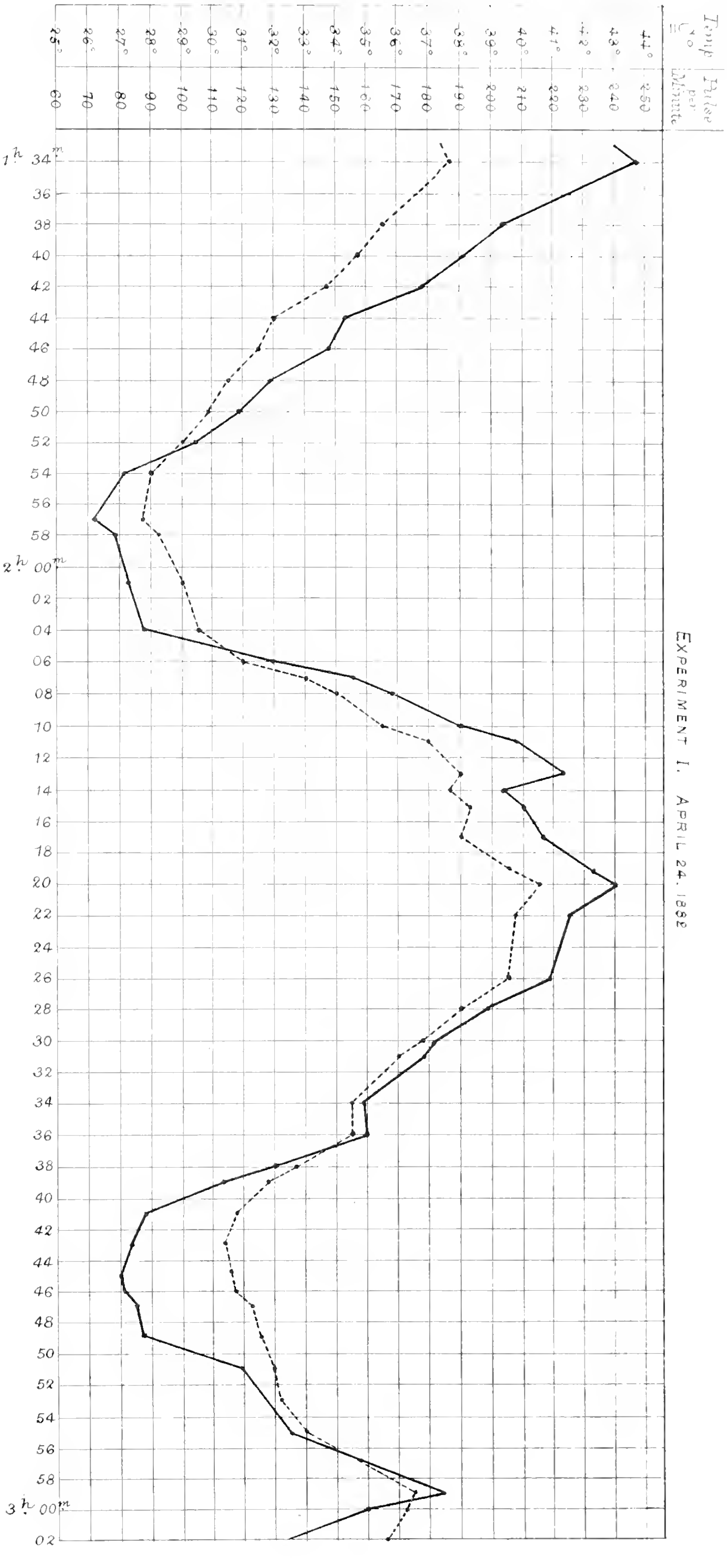


West Newman & C° lith.

rtin.

Phil. Trans. 1883. Plate 49.


EXPERIMENT IV. MAY 10. 1882.


EXPERIMENT I. APRIL 24. 1882

West Newman & Cº lith.

THE PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY.

	£	s.	d.
1801. Part I...	0	17	0
—— Part II...	0	17	6
1802. Part I...	0	11	0
—— Part II...	0	17	6
1803. Part II...	0	13	6
1804. Part I...	0	10	6
—— Part II...	0	12	6
1805. Part I...	0	10	0
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1806. Part I...	0	13	6
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1812. Part I...	0	17	6
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1813. Part I...	0	14	0
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—— Part II...	1	0	0
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1826. Part I...	1	2	6
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—— Part III...	2	0	0
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1827. Part II...	0	18	0
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—— Part II...	1	1	6
1840. Part I...	0	18	0
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1841. Part I...	0	10	0
—— Part II...	1	10	0
1842. Part I...	0	16	0
—— Part II...	1	2	0
1843. Part I...	0	10	0
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—— Part II...	1	10	0
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—— Part IV...	1	12	0
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—— Part II...	0	16	0
1848. Part I...	1	0	0
—— Part II...	0	14	0
1849. Part I...	1	0	0
—— Part II...	2	5	0
1850. Part I...	1	10	0
—— Part II...	3	5	0
1851. Part I...	2	10	0
—— Part II...	2	10	0
1852. Part I...	1	0	0
—— Part II...	2	5	0
1853. Part I...	0	18	0
—— Part II...	0	12	0
—— Part III...	1	2	0
1854. Part I...	0	12	0
—— Part II...	0	16	0
1855. Part I...	0	16	0
—— Part II...	1	6	0
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—— Part II...	1	4	0
—— Part III...	1	4	0
1857. Part I...	1	8	0
—— Part II...	1	4	0
—— Part III...	1	2	0
1858. Part I...	1	8	0
—— Part II...	3	0	0
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—— Part II...	2	5	0
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—— Part II...	2	1	6
1861. Part I...	1	3	0
—— Part II...	1	5	0
—— Part III...	1	7	6
1862. Part I...	2	14	0
—— Part II...	3	0	0
1863. Part I...	1	14	0
—— Part II...	1	7	6
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—— Part II...	1	7	6
—— Part III...	1	10	0
1865. Part I...	2	2	0
—— Part II...	1	5	0
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—— Part II...	2	7	6
1867. Part I...	1	3	0
—— Part II...	1	15	0
1868. Part I...	2	5	0
1868. Part II...	2	0	
1869. Part I...	2	10	
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PART III.

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XXII. *On the Ciliated Groove (Siphonoglyphe) in the Stomodæum of the Alcyonarians.*

By SYDNEY J. HICKSON, *B.A. (Cantab.), B.Sc. (Lond.), Assistant in the Anatomical Laboratory, Oxford.*

Communicated by Professor H. N. MOSELEY, *F.R.S.*

Received May 23,—Read June 14, 1883.

[PLATES 50, 51.]

MANY years ago the distinguished naturalist GOSSE (2) described two ciliated grooves in the stomach of the Sea Anemones, the function of which is to keep up the circulation of sea-water whilst the animals are retracted, and which he called the gonidial canals ("demicanaux" of HOLLARD (8) and "schlund-rinnen" of the German writers).

These grooves are situated on what are now known as the dorsal and ventral sides of the stomach, and their presence has been confirmed and their histology more thoroughly investigated by R. and O. HERTWIG in their great work 'Die Actinien' (7).

Concerning them these authors say: "An unseren Durchschnitten durch die verschiedenen Actinienarten sind sie überall leicht zu erkennen und schienen sie sich uns durch eine besonders starke Bewimperung auszureichen."

In the genus *Cerianthus* only one groove is present (HAIME) (6), and this is of great depth.

In consequence of the histological difficulties which attend the investigation of the *Alcyonarians*, their minute anatomy has not been very thoroughly investigated, and the presence of the ciliated groove on the ventral side of the stomodæum has been generally overlooked. The text-books of comparative anatomy do not mention it, nor can I find any reference to it in any memoirs except that of the HERTWIGS (*l.c.*), where it is simply stated to be present on the ventral side of the stomodæum, and a rough diagrammatic sketch given of it in *Alcyonium;* and in a paper on *Sarcodictyon* by GOSSE (3).

In the present communication I shall give the result of a series of investigations carried on during the last twelve months, for the purpose of tracing this ciliated grove through the various genera of *Alcyonaria*, and in referring to the various parts I shall throughout employ the following terms: I shall call the hollow communication between the mouth and the body-cavity, which is formed by an invagination of the epiblast, the stomodæum; the portion of it that lies in the region of the mouth, the outer portion; and the portion that communicates with the body-cavity I shall call the inner portion; the general cavity of the polyps, which is sometimes short, but sometimes enormously elongated, I shall call the body-cavity; and in referring to the dimorphic forms I shall use the terms "autozooids" and "siphonozooids" which Professor MOSELEY (15) has introduced in place of the terms "polyps" and "zooids" used by Professor KÖLLIKER and others.

In *Alcyonium palmatum* the stomodæum presents in transverse section a rhombic-shaped cavity, with long lateral walls and short dorso-ventral walls; it is wide in its outer portion, but becomes considerably narrowed towards its opening into the body-cavity, so that one might describe it as a funnel which is laterally compressed (Plate 50, figs. 1 and 2).

The ciliated groove, commencing about one-third of the total depth of the stomodæum from the mouth, is continued along its ventral side as far as its opening into the body-cavity.

The stomodæum is lined by a columnar ciliated epithelium, varying in thickness from ·004 millim. to ·002 millim.

The ciliated groove is distinguished by the following peculiarities: the epithelium is thicker than it is over the rest of the stomodæum, being at least ·005 millim. thick, the free edges of the columnar epithelium cells of which it is composed are very clear and definite, and the cilia are remarkably long and strong, being from ·006 millim. to ·007 millim long.

Of the genus *Alcyonium* I have examined three other species, namely, *A. digitatum*, *A. pulmonaria*, and *A. pachyclados*, but owing to the larger number of spicules in these species, they are not so favourable for histological manipulation.

An examination of living specimens of our English *Alcyonium digitatum* revealed the fact that the cilia of the groove, moving almost simultaneously, produce a current of water from without inwards, whereas the cilia lining the rest of the stomodæum produce currents in the opposite direction. Thus a complete circulation is brought about in the polyps; the inward current produced by the cilia of the groove is probably aided by the cilia of the ventral mesenterial filaments, whilst an outward current, commencing on the dorsal and lateral mesenteries, is taken up by the action of the cilia lining the general surface of the stomodæum.

It is evident from anatomical and physical considerations that the chief part of the propulsion of these currents is performed by the cilia lining the groove on the ventral side of the stomodæum, and consequently I propose to call this groove "the siphonoglyphe."

I may summarise the characters of the siphonoglyphe in the genus *Alcyonium* as follows:—

The siphonoglyphe is present along the inner two-thirds of the ventral wall of the stomodæum (Plate 50, fig. 2), it is formed by that portion of the wall of the stomodæum which lies between the attachment of the two ventral mesenteries only, it is marked by a distinct thickening of the epithelium, and the cilia are long and strong.

In the genus *Clavularia*, in which the non-retractile polyps stand apart from one another on a flat creeping stolon, the siphonoglyphe is marked by a considerable thickening of the wall of the stomodæum, but the cilia are comparatively short and delicate. Moreover, the siphonoglyphe does not extend over so great an area as it does in *Alcyonium*, for in a series of transverse sections no trace of it can be found in the outer two-thirds, but only in the inner third of the stomodæum.

In the genera *Spongodes* and *Nepthya* the siphonoglyphe has about the same area and extent that it has in *Alcyonium*. My specimens of *Nepthya* are not very well preserved, but, although the general histology is not easy to make out, the general features of the siphonoglyphe are quite manifest. My specimens of *Spongodes* are, however, beautifully preserved, and in them the cilia which line the general surface of the stomodæum may be easily seen. In most *Alcyonaria* which have been preserved in spirit these cilia are not easy to observe.

In the genus *Briareus* a very definite siphonoglyphe is present with long and delicate cilia.

In *Tubipora* the siphonoglyphe is well marked, and provided with a dense armature of long and strong cilia.

I was fortunate enough to obtain a fine specimen of the rare *Cœlogorgia palmosa* from Zanzibar. This genus, which resembles many of the Gorgonian Alcyonarians in the complex branching of its axis, and in the fact that the polyps are not completely retractile, differs from them in that there is no axial skeleton, but a single large canal runs along the axes of the colony and its branches. In transverse section the cavity of the stomodæum presents the appearance of a short T, the cross portion of the T being the siphonoglyphe (Plate 50, fig. 3). The siphonoglyphe is beset with cilia, which, though very much longer than those of the rest of the stomodæum, are shorter and more delicate, comparatively speaking, than those found on the siphonoglyphe of other forms.

Passing on now to those forms which exhibit the phenomenon of dimorphism.

In the genus *Sarcophyton*, the dimorphism of which was first pointed out by Kölliker (9), and subsequently confirmed by Moseley (14), the autozooids possess a siphonoglyphe which is not so well marked as it is in the preceding genera. It is only present along the inner third of the stomodæum, it never occupies a greater portion of the wall of the stomodæum than that between the two ventral mesenteries, there is no marked thickening of the epithelium of the stomodæum, and the cilia themselves are neither so numerous nor so strong as they are in other cases (Plate 50, fig. 4). In the siphonozooids, however, the siphonoglyphe is very well marked; there is a decided thickening of the epithelium; it extends along nearly the whole of the ventral side of the stomodæum, from the mouth to the body-cavity; it extends round the wall of the stomodæum frequently as far as the insertion of the ventro-lateral mesenteries, and the cilia are characteristically long and strong (Plate 50, fig. 5).

It is exceedingly probable, from these facts, that the great part of the circulation in *Sarcophyton* is carried on by the siphonozöoids, whilst the autozooids carry on the same function to a much smaller extent.

The genus *Paragorgia* is also dimorphic, a fact which seems to have been previously overlooked, and here we find a condition concerning the siphonoglyphe slightly different from that found in *Sarcophyton.*

After a careful examination of numerous autozooids, both in the retracted and expanded condition, I could find no trace of a true siphonoglyphe. There seems to be no marked thickening of the epithelium on the ventral side of the stomodæum, nor are the cilia markedly longer or stronger in that region (Plate 50, fig. 6).

In the siphonozooids, however, the siphonoglyphe is remarkably strong, and extends as far round the stomodæum as the insertion of the latero-ventral mesenteries, whilst the long cilia reach half-way across its lumen (Plate 51, fig. 8).

In *Paragorgia*, as in *Siphonogorgia*, the ova are borne by the siphonozooids, and frequently they are so full of them as to cause the stomodæum to be pushed to one side and the mesenteries to be broken (Plate 51, fig. 7).

A similar condition to this was found in the genus *Heteroxenia.* No trace of a siphonoglyphe could be seen in the autozooids, whilst a well-marked one was found in the stomodæum of the siphonozooids.

Amongst the Pennatulida I have examined two genera, *Pennatula* and *Renilla.* In the autozooids of *Pennatula* I could find no siphonoglyphe, nor in the autozooids of *Renilla.* KÖLLIKER (9) does not mention any ciliated groove in any of the numerous Pennatulids he examined, nor does MARSHALL (17) in *Pennatula, Funiculina* and *Virgularia.* I think therefore I am justified in concluding that the siphonoglyphe does not exist in the autozooids of the Pennatulida. In the siphonozooids, however, a well-marked siphonoglyphe exists; in *Pennatula* it occupies a considerable portion of the stomodæum (Plate 51, fig. 10), and is armed with numerous long and strong cilia; in *Renilla* it is remarkable for its enormously long cilia, which stretch right across the lumen of the stomodæum (Plate 51, fig. 9).

Thus it will be seen that in the dimorphic Alcyonarians the siphonoglyphe has a tendency to disappear from the autozooids and to become very prominent in the siphonozooids. In *Sarcophyton* it still remains in the autozooids, but considerably diminished both in size and importance; in *Paragorgia, Heteroxenia,* and *Pennatulida* it has completely disappeared from them, whilst in all forms a well-marked siphonoglyphe is present in the siphonozooids.

In the genus *Heliopora,* some specimens of which Professor MOSELEY has kindly placed at my disposal, a siphonoglyphe of moderate dimensions is present.

Amongst the Gorgonidæ I have examined only two genera, *Villogorgia* and *Primnoa,* and in neither of these could I find any trace of a siphonoglyphe. In the genus

Villogorgia, although my specimens were most excellently preserved, I could find no cilia on any part of the stomodæum. I do not wish to assert for a moment that they do not exist, but if they do they must be exceedingly minute to show no trace in preserved specimens. The epithelial cells are filled with minute highly refracting particles, which conceal their outlines in exactly the same way as MARSHALL has described in the stomodæum of *Funiculina* (17, p. 14).

An interesting feature in the stomodæum of *Villogorgia* is the presence of a deep groove on the dorsal side (Plate 51, fig. 11), which is not beset, as the siphonoglyphe is, with long cilia. The epithelium of this dorsal groove is thinner than it is in other parts of the stomodæum. I have found nothing that corresponds with this dorsal groove in any other Alcyonarian.

In the genus *Primnoa,* again, I could find no trace of a siphonoglyphe, either in fully grown polyps or young buds (Plate 51, fig. 12).

Von KOCH does not describe a siphonoglyphe in any of the numerous Gorgonidæ he has examined (*Isis, Gorgonia, Sclerogorgia,* &c.), so that it seems to me probable that it does not exist in the stomodæum of Gorgonidæ, or, at any rate, in those Gorgonidæ with polyps that are not completely retractile.

It must be remembered in reference to Von KOCH's evidence that this author did not describe a ciliated groove in *Tubipora,* where it is undoubtedly present; but in this genus the stomodæum of the retracted polyps is so folded and creased that unless exceedingly thin sections are made it is easily overlooked, whereas in the non-retractile polyps of many Gorgonidæ where the stomodæum is not much folded a simple series of transverse sections would show it at once were it present.

General observations on the presence of the siphonoglyphe in the Alcyonaria.

In the three genera which have been described of simple *Alcyonaria* which do not form colonies, namely—*Monoxenia* (HÆCKEL, 5), *Hartea* (WRIGHT, 20), and *Haimea* (M. EDWARDES, 18), no siphonoglyphe has been described, and considering the small area that the circulation of these animals has to traverse, it seems to me probable that it does not exist in them. Where, however, a wider circulation was introduced, owing to the formation of complicated colonies, the aid to the circulation afforded by a siphonoglyphe became necessary.

In *Clavularia,* in which the colony consists of a number of polyps standing on a thin stolon, the circulation is not very extensive, and consequently we find that the siphonoglyphe is not very strong. In genera such as *Alcyonium, Spongodes, Nephthya,* &c., where there are long body-cavities and a considerable amount of gelatinous sarcosoma, a stronger circulation is necessary, and consequently we find that the siphonoglyphe has assumed more important proportions.

As long as the siphonoglyphe is confined to that portion of the wall of the stomodæum which lies between the two ventral mesenteries there is probably but little interference with the other functions of the stomodæum; but when the necessities of the circulation require a stronger propulsive power than would be supplied by such a siphonoglyphe, certain of the polyps are arrested in their development in order that they may supply that additional power, and the colony becomes dimorphic.

The dimorphic *Alcyonaria* invariably present a considerable amount of fleshy sarcosoma, or else large spaces in which a circulation of sea-water is maintained, and it is usually the case that the circulation is entirely maintained by individuals which have become specially modified for that purpose—the siphonozooids.

Following this line of reasoning it is not difficult to understand the absence of the siphonoglyphe in the Gorgonidæ. In these animals there is always present a hard axis which may be either horny or else horny and calcareous. This axis frequently occupies the greater bulk of the colony (*e.g.*, *Primnoa*), so that the sarcosoma remains as a thin bark covering it. The result of this arrangement is that the canal system does not traverse so large an area as it does in such forms as *Paragorgia*, &c., in which there is no solid axis to the colony. The less the extent of the area supplied with canals the less the need of a strong propulsive arrangement, and consequently the siphonoglyphe is proportionately useless and disappears, the diminished circulation being carried on by the ordinary cilia of the stomodæum.

In the genus *Cœlogorgia* we have an example of an Alcyonarian which resembles the Gorgonidæ in many respects, but differs from them in the important fact that a single large canal occupies the position of the solid axis of the other forms. We must suppose that there is a constant circulation going on in this axial canal as well as in the ordinary canals of the colony, and corresponding with this we find a well-marked siphonoglyphe in the polyps for carrying on this more extensive circulation.

In the genus *Solenogorgia* described by GENTH (1) there is a somewhat similar condition, large canals being present in the axis of the colony and again in *Solenocaulon* described by GRAY (4). A re-examination of these genera would probably reveal the fact that their polyps possess a well-marked siphonoglyphe such as we find in the genus *Cœlogorgia*.

The genus *Heliopora* presents us with a condition which is not so easy to understand. The sarcosoma of the other forms of *Alcyonaria* is here represented only by a delicate layer of tissue covering the skeleton, but at the same time, owing to the large amount of space occupied by the cœnenchymal tubes, there must be a considerable amount of fluid constantly circulating throughout the colony. It is, therefore, somewhat surprising to find but a feebly-developed siphonoglyphe in the stomodæum of the polyps. It may be, however, as Professor MOSELEY has suggested to me, that a rapid circulation would be of no particular advantage to a colony which possesses but a small amount of living sarcosoma, or indeed by hurrying away the food particles, it might be positively disadvantageous to it. This may possibly account for the feeble siphonoglyphe.

There is however a considerable difficulty in accounting for the presence of a siphonoglyphe in the polyps, whilst holding the view that the cœnenchymal tubes represent siphonozooids in which the stomach, mesenteries, &c., have degenerated.

The tendency of the dimorphic forms is, as I have pointed out, to throw the siphonic function upon the siphonozooids and to eliminate it from the autozooids.

If, for any reason, it was of advantage to any dimorphic Alcyonarian to diminish the power of the circulation, this would be done by the gradual atrophy of the siphonoglyphe in the autozooids, and were this diminution insufficient the siphonozooids, or their siphonoglyphe alone, would become smaller and smaller. An example of this kind of process is presented by *Renilla*. In this genus there is but a small amount of fleshy sarcosoma, but there are large canal spaces which occupy the greater part of the colony, and here we find, owing probably to the need of only a feeble current, very small siphonozooids. In *Pennatula*, which presents a considerable quantity of sarcosoma, the siphonozooids are comparatively large.

In *Heliopora*, on the view that the cœnenchymal tubes represent siphonozooids, we should have to suppose that the siphonozooids became smaller and smaller, then completely atrophied, and subsequently a siphonoglyphe re-appeared in the autozooids. This would obviously necessitate a stage in their history in which there was no siphonoglyphe, which would be a condition very difficult to understand.

It is also difficult to believe that the mouth, stomach, and mesenteries would have all completely disappeared in this way, for even in the lowermost depths of the long body-cavities of such forms as *Tubipora*, *Alcyonium*, &c., two or more ridges remain, indicating the position of the mesenteries, and we should at least expect to find some such trace of the mesenteries in the degenerate siphonozooid.

Remarks on the classification and phylogeny of the Alcyonaria.

At present it can hardly be said that the classification of the *Alcyonaria* is in a satisfactory condition for two reasons, firstly, because no serious attempt has yet been made to trace the probable steps of the phylogeny of the group, and secondly because it is based on external zoological differences between genera rather than on the general features of their anatomy.

Taking the classification in CLAUS's 'Grundzuge der Zoologie' as the one most generally adopted, we find such obvious incongruities as the following: the grouping together of such colonial genera as *Alcyonium*, *Clavularia*, &c., with the simple isolated *Haimea*, *Hartea*, &c., the position of *Paragorgia* amongst the Gorgonidæ, and so on. Recently, G. von KOCH (13) has suggested a classification that is based on the varieties of the skeleton, but it seems to me that the Pennatulidæ and Gorgonidæ are not so closely related as to justify their position in the same division of the same group (*Axifera*).

In presenting the following speculations on the phylogeny of the *Alcyonaria*, I am fully aware that the great difficulties of this group are only just beginning to be appreciated, but I do so in order to point out the part which the presence or absence of a siphonoglyphe may play in the arrangement of the group, and some other points upon which the classification may turn.

There can be little doubt, I think, that the ancestral form of the *Alcyonaria* was not colonial, but was a simple isolated individual differing but slightly from the isolated genera which exist at the present day.

The fact that the three genera of isolated *Alcyonaria* are remarkably rare, present but few species, and have a wide geographical distribution (Monoxenia, coast of Arabia, Haimea, Fiji Islands, and Hartea, west coast of Ireland), point to the conclusion that they are the representatives of an ancient group which may have been much larger than it is now.

It might therefore be advisable to separate these genera as a distinct group, which might be called the PROTO-ALCYONARIA.

The next step in the phylogeny was the formation from such an isolated ancestor of a colony. The formation of the colonies may have taken place in two ways: first, by the formation of buds from the first formed polyp; and secondly, by the intermediation of a stolon upon which the young buds were formed.

A colony formed in this second way would with slight modifications give us a form such as our modern *Clavularia* or *Cornularia*.

In the genus *Tubipora* there is a stolon which I shall point out in a subsequent paper is very similar to the stolon of *Clavularia*. *Tubipora* might in fact have been derived from a *Clavularia*-like ancestor, in which the following modifications took place: The polyps became considerably elongated, and the spicules of the body-wall fused together to form a hard tubular support for them. These long polyps then became connected by canals which eventually joined together to form the horizontal platforms traversed by a network of the canals, and from them new polyps budded as they do from the original stolon.

If this reasoning is subsequently proved to be accurate it will be necessary to separate those forms with a stolon from the rest of the *Alcyonaria* into a separate group, which might be called the STOLONIFERA.

In the great majority of the *Alcyonaria* we have sufficient evidence, I think, to prove that they are formed by budding from the first-formed polyps which usually remain in the centre of the colony.

Taking a hypothetical ancestor, x, which probably had a conformation somewhat as follows: A central large polyp from which sprung, in a fan-shaped manner, a number of lateral buds of which those nearest the central polyp were the largest, we should have the rest of the *Alcyonaria* formed from it by modifications in several directions. In one direction we have the well-marked group of the *Pennatulida*. This group probably sprang from the ancestral stock at a very remote period, as is shown by the changes which have taken place in the central polyp, the arrangement of the subsequently formed polyps upon it, and the complete and universal dimorphism of the colonies. In another direction arose the ancient group of which *Heliopora* is a survival. This group, which was formerly placed amongst the *Tabulata*, was probably very rich in genera and species in palæozoic times, but it is gradually becoming extinct.

In another direction arose the modern genus *Alcyonium* and its numerous allies. This genus differs from the ancestral form x chiefly in the fact that the polyps are capable of being retracted within the sarcosoma, but in other respects is probably more closely related to it than any other genera. The fact that the polyps are capable of being retracted is not one of very great importance, for we find both in this family and in the Gorgonidæ, that nearly allied genera differ from one another in this respect. The dimorphic genus, *Sarcophyton,* is probably closely related to *Alcyonium.* The presence of dimorphism is not sufficient to warrant the supposition that they sprang from different stocks, for this condition occurs in so many widely different genera that it is reasonable to suppose that it was introduced more than once in the course of the evolution of the group.

As an example of this we find that the genus *Xenia,* which probably followed another line of evolution from the hypothetical ancestor, is not dimorphic, whereas the genus *Heteroxenia,* very similar to it in other respects, is dimorphic.

The lines which evolution took in producing the large number of genera of *Primnoaceæ, Gorgonaceæ,* &c., are much more difficult to make out, but the following represents perhaps as near an approximate to them as our present knowledge permits.

Taking *Siphonogorgia* as a form intermediate between the ancestral type and the true Gorgonidæ, we find that the chief diversion lies in the fact that the colony has assumed an arborescent shape, and a support for it is produced by a more copious development of spicules in the axial portions of the colony. The body-cavities of the polyps, however, remain long, as they were in the ancestral form. Most probably there was another stage between *Siphonogorgia* and this ancestral form which was not dimorphic. From *Siphonogorgia, Paragorgia* differs chiefly in the fact that the body-cavities of the polyps have become reduced in length, and a complicated system of canals occupies the position which they formerly occupied. If *Corallium* is dimorphic, as RIDLEY (19) and MOSELEY (16) consider it to be, it was derived from an ancestor similar to *Paragorgia* in which, by a fusion of the spicules, a solid rod occupies the axis of the colony. The rare genus *Pleurocorallium* differs from *Corallium* in the fact that the polyps are not retracted into the cœnenchym, but this condition may be simply due to a more copious development of spicules in the walls of the polyps, thereby offering a physical difficulty to the retraction of the polyps.

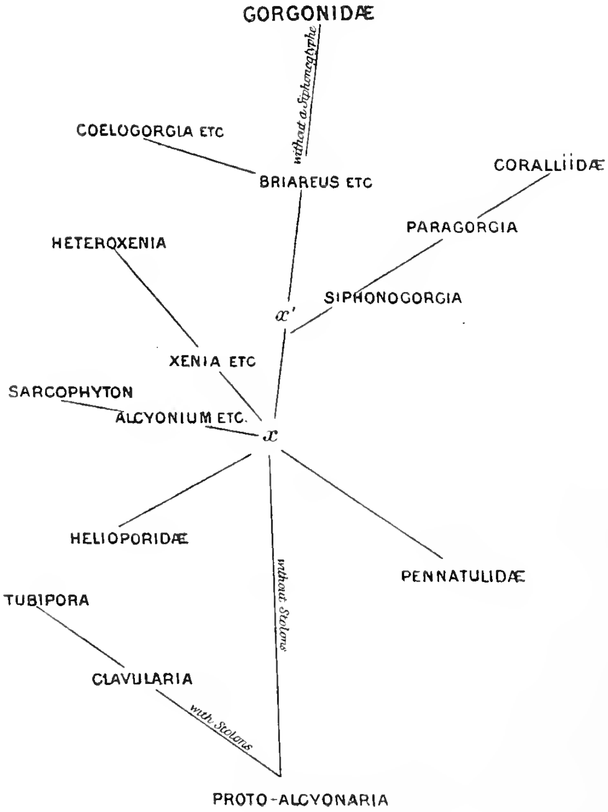
Passing by the form which I suppose at one time existed similar to *Siphonogorgia* but not dimorphic, we should obtain such forms as *Briareus,* in which the body-cavities of the polyps are short; there is no solid axis and no dimorphism, and from such genera, *Cœlogorgia, Solenogorgia,* &c., might be obtained by the development of large canals in the axis of the colony, or again by the development of horny or calcareous axes, we should obtain the remaining families of Gorgonacæ, Primnoaceæ, &c.

In the Primnoaceæ I have shown that there is no siphonoglyphe, and the negative evidence of other authors affords us strong evidence for supposing that it is absent in the Gorgonaceæ. In both these families there is only a thin layer of cœnenchym covering the solid axis. If these two facts are mutually dependent upon one another, as I suppose them to be, there is sufficient reason for separating those forms which have only a thin cœnenchym covering the axis, and into which the polyps are never completely retracted, and in which there is no siphonoglyphe, into a distinct group.

This group might retain the name of the Gorgonidæ, whilst other genera which do not possess these characters and which were formerly included in the Gorgonidæ, might be placed in another group of the *Alcyonaria* altogether.

Provisionally, then, the *Alcyonaria* might be divided into the following groups :—

1. The Proto-Alcyonaria, containing the simple isolated genera.
2. The Stolonifera containing those forms with stolons such as *Tubipora, Clavularia, Sarcodictyon,* &c.
3. The Pennatulida constituted exactly as it is at present.
4. The Gorgonidæ, containing the Primnoaceæ, Gorgonaceæ, and the other families which possess no siphonoglyphe.
5. The Alcyonidæ containing all the remaining Alcyonarians.



In conclusion I should mention that my researches have been carried on in the morphological laboratory of the Oxford University Museum, and I had the great advantage of using a very valuable collection of Alcyonarians brought by Dr. GULLIVER from Zanzibar; and to the beautiful state of preservation in which I found them, many of my best results are due.

My best thanks are due to Professor LANKESTER for some excellent specimens of *Paragorgia*, *Villogorgia* and *Briareus*, and I am also deeply indebted to Professor MOSELEY, who freely placed his numerous preparations at my disposal, and whose constant aid and advice have been of invaluable assistance to me.

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Description of the Plates.

The following lettering is used throughout.

st.	Stomodæum.
si.	Siphonoglyphe.
siph.	Siphonozooid.
aut.	Autozooid.
m.f.	Mesenterial filament.
sp.	Spicules in the body-wall.
sp′.	Spicules in the walls of the stomodæum.
sp″.	Spicules in the sarcosoma.
c.c.	Gastrovascular canals.
ep.	Epithelium.
p.	Used in figs. 11 and 12 for the spaces left after the skeletal plates have been dissolved in acid.
ov.	Ova.
t.	Tentacles.
v.	Ventral side.
d.	Dorsal side.

Fig. 1. A diagrammatic sketch of a longitudinal section through a polyp of *Alcyonium digitatum,* showing the wall of the siphonoglyphe on the ventral side with its long cilia pointing towards the body cavity, and the wall of the stomodæum on the dorsal side, which is not so thick, and beset with but small cilia. The arrows indicate the directions the currents of water take in a living polyp.

Fig. 2. Transverse section through a polyp of *Alcyonium palmatum* in the middle region of the stomodæum. The siphonoglyphe is seen on the ventral side of the stomodæum, armed with its long cilia, and in its substance are seen imbedded the long spicules.

Fig. 3. Transverse section through a polyp of *Cœlogorgia pulmosa.* The lumen of the stomodæum is seen to be T-shaped in section, the cross portion of the T forming the siphonoglyphe is armed with long, delicate cilia, and the rest of the stomodæum with short, dense cilia.

Fig. 4. Transverse section through an autozooid of *Sarcophyton pulmo* (semi-

diagrammatic). The siphonoglyphe is here feebly marked, and armed with comparatively short cilia.

Fig. 5. Section through a portion of a colony of *Sarcophyton* in a plane at right angles to the long axes of the polyps. A number of siphonozooids are seen in transverse section, separated from one another by a fleshy sarcosoma, traversed in all directions by the gastrovascular canals. The siphonoglyphe in all is seen to be well developed and situated on the same side of the stomodæum. Portions of two autozooids are seen at the sides of the drawing.

Fig. 6. Transverse section through the stomodæum of an autozooid of *Paragorgia arborea.* The stomodæum is seen to be thrown into folds, and the epithelium is armed with short cilia throughout. There is no marked thickening of the epithelium, nor lengthening of the cilia on the ventral side.

Fig. 7. Transverse section through a siphonozooid of *Paragorgia* (semi-diagrammatic). The siphonoglyphe, of very large proportions, is seen in the stomodæum. The gastrovascular canals are seen anastomosing in the sarcosoma. The spicules are omitted.

Fig. 8. Vertical section through a portion of a colony of *Paragorgia arborea.* A number of siphonozooids are seen in longitudinal section, some of which contain ova. The branching canal system is represented as it appears in section, and numerous spicules are seen imbedded in the substance of the sarcosoma.

Fig. 9. Transverse section through the stomodæum of a siphonozooid of *Renilla.* The siphonoglyphe has not a very great extent, but is armed with enormously long cilia, which extend across the lumen of the stomodæum.

Fig. 10. Transverse section through a siphonozooid of *Pennatula.* The siphonoglyphe here is of considerable size, and armed with long cilia.

Fig. 11. Transverse section through a polyp of *Villogorgia.* This drawing was kindly done for me by Mr. G. C. Bourne, of New College, Oxford, and accurately represents the appearance of one of my sections through a polyp of *Villogorgia,* which was stained in borax carmine after decalcification by means of nitric acid. The epidermic cells lining the stomodæum are not easily differentiated from one another, owing to the numerous highly refracting particles which they contain. The cilia lining the stomodæum cannot be seen with the highest power. There is no siphonoglyphe, but a deep groove (*g.*) runs down the dorsal side of the stomodæum.

Fig. 12. Transverse section through a polyp of *Primnoa lepadifera.* The stomodæum is lined by small cilia, but no siphonoglyphe is present. No trace of the dorsal groove of *Villogorgia* can be seen.

XXIII. *On the Determination of the Number of Electrostatic Units in the Electromagnetic Unit of Electricity.*

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Communicated by Lord Rayleigh, *F.R.S.*

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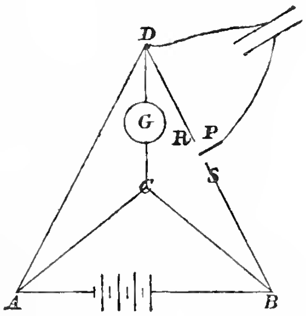
The values which various physicists have found for "v," the number of electrostatic units in the electromagnetic unit of electricity, differ so widely from each other that it seems important that additional experiments should be made in order to help to determine the value of this important constant. Six determinations of "v" have been published. The first determination was made by Weber, who measured the capacity of a condenser, both electrostatically and electromagnetically. Hockin and Professors Ayrton and Perry have also determined "v" in this way. Maxwell determined it by balancing the electrostatic attraction between two discs maintained at different potentials against the repulsion between electric currents circulating at the back of the discs, the currents being derived from the battery which maintained the discs at different potentials. Sir William Thomson and Mr. Shida have determined it by measuring an electromotive force both electrostatically and electromagnetically.

The following method was employed in this investigation: it is a very slight modification of the method described in § 776 of Maxwell's 'Electricity and Magnetism.' In a Wheatstone's bridge, A B C D, with the galvanometer at G, and the battery between A and B, the circuit B D is not closed, but the points B and D are connected with two poles, R and S, of a commutator, between which a travelling piece, P, moves backwards and forwards; P is connected with one plate of a condenser, the other plate of which is connected with D. Thus when P is in contact

Fig. 1.



with S, the condenser will be charged, and until it is fully charged, electricity will flow into it from the battery; this will produce a momentary current through the various arms of the bridge. When the moving piece P is in contact with R, the two plates of the condenser are connected, and the condenser will discharge itself through D R, and as the resistance of D R is infinitesimal in comparison with the resistance of any other circuit, the discharge of the condenser will not send an appreciable amount of electricity through the galvanometer. Thus, if we make the moving piece P oscillate quickly from R to S, there will, owing to the flow of electricity to the condenser, be a succession of momentary currents through the galvanometer. The resistances are so adjusted that the deflection of the galvanometer produced by these momentary currents is balanced by the deflection due to the steady current through the galvanometer, and the resultant deflection is zero. When this is the case there is a relation between the capacity of the condenser, the number of times the condenser is charged and discharged per second, and the resistances in the various arms of the bridge.

As the investigation of this relation given in MAXWELL's 'Treatise on Electricity and Magnetism' is only an approximation, it may be worth while to give here an exact investigation of the relation between the capacity of the condenser and the resistances in the arms of the bridge; though we shall find that when the resistances have the values which they had in the present investigation, MAXWELL's formula is very nearly correct. This relation will enable us to calculate the electromagnetic measure of the capacity of the condenser.

Let $\dot{x}$ be the current in A B
$\dot{y}$,, ,, B S
$\dot{z}$,, ,, D C

then the currents in

$$A\,C=\dot{x}-(\dot{y}+\dot{z})$$

$$A\,D=\dot{y}+\dot{z}$$

$$C\,B=\dot{x}-\dot{y}$$

Let b be the resistance of A B
a ,, ,, A C
c ,, ,, A D
g ,, ,, D C
d ,, ,, B C

The resistances of D R, S B are so small in comparison with the other resistances that they may be neglected.

The Dissipation Function

$$=\tfrac{1}{2}\{b\dot{x}^2+a(\dot{x}-(\dot{y}+\dot{z}))^2+c(\dot{y}+\dot{z})^2+g\dot{z}^2+d(\dot{x}-\dot{y})^2\}$$

The Potential Energy

$$=\tfrac{1}{2}\frac{y^2}{\mathrm{C}}$$

where C is the capacity of the condenser.

Thus if E be the electromotive force of the battery, we have, neglecting the self-induction of the resistance coils in the circuit,

$$b\dot{x}+a(\dot{x}-(\dot{y}+\dot{z}))+d(\dot{x}-\dot{y})=\mathrm{E}$$

$$-a\{\dot{x}-(\dot{y}+\dot{z})\}+c(\dot{y}+\dot{z})-d(\dot{x}-\dot{y})+\frac{y}{\mathrm{C}}=0$$

$$-a(\dot{x}-(\dot{y}+\dot{z}))+c(\dot{y}+\dot{z})+g\dot{z}=0$$

or

$$(a+b+d)\dot{x}-(a+d)\dot{y}-a\dot{z}=\mathrm{E}$$

$$-(a+d)\dot{x}+(a+c+d)\dot{y}+(a+c)\dot{z}+\frac{y}{\mathrm{C}}=0$$

$$-a\dot{x}+(a+c)\dot{y}+(a+c+g)\dot{z}=0$$

To solve these equations, assume

$$\dot{x}=u+p\epsilon^{-\lambda t}$$

$$\dot{y}=q\epsilon^{-\lambda t} \qquad\qquad y=\frac{q}{\lambda}(1-\epsilon^{-\lambda t})$$

$$\dot{z}=w+r\epsilon^{-\lambda t}$$

where t is measured from the instant when the moving piece P first touches S.

Substituting we get

$$w=\frac{a\mathrm{E}}{(a+c+g)(a+b+d)-a^2}$$

$$u=\frac{(a+c+g)\mathrm{E}}{(a+c+g)(a+b+d)-a^2}$$

$$\frac{q}{\lambda\mathrm{C}}=\frac{\{(a+d)(a+c+g)-(a+c)a\}\mathrm{E}}{(a+c+g)(a+b+d)-a^2}.$$

$$r=\frac{q\{(a+b+d)(a+c)-a(a+d)\}}{a^2-(a+b+d)(a+c+g)}$$

therefore

$$\frac{r}{\lambda}=-\frac{\mathrm{CE}\{(a+b+d)(a+c)-a(a+d)\}\{(a+d)(a+c+g)-a(a+c)\}}{\{(a+c+g)(a+b+d)-a^2\}^2}$$

But r/λ is the quantity of electricity that flows through the galvanometer whilst the condenser is being charged. If the condenser is charged and discharged n times in a second, the quantity of electricity which flows through the galvanometer in one second is nr/λ, and if this is to balance the steady current, we must have

$$n\frac{r}{\lambda}+w=0$$

or

$$n\mathrm{C}=\frac{\{(a+c+g)(a+b+d)-a^2\}a}{\{(a+b+d)(a+c)-a(a+d)\}\{(a+d)(a+c+g)-a(a+c)\}}$$

or

$$n\mathrm{C}=\frac{a\left\{1-\dfrac{a^2}{(a+c+g)(a+b+d)}\right\}}{cd\left\{1+\dfrac{ab}{c(a+b+d)}\right\}\left\{1+\dfrac{ag}{d(a+c+g)}\right\}}$$

Now in the actual experiment the resistances a, b, c, d, g had about the following values :—

$a=$ 1,200 B.A. units.
$b=$ 2,500 „
$c=$100,100 „
$d=$900,000 „
$g=$ 11,000 „

So that in this case the formula $n\mathrm{C}=a/cd$ is correct to within 0·1 per cent., and it is the one we shall use to calculate the electromagnetic measure of the capacity of the condenser.

With these values of the resistances we find that λ is greater than 5000, thus the time constant of the system is very small compared with the time during which the plates of the condenser are connected together, so that the condenser is completely discharged each time.

The electrostatic measure of the capacity must be calculated from the geometrical constants of the condenser. It was necessary to use a guard ring in order to simplify the calculation, and to avoid the influence of the irregular distribution of electricity near the edges of the condenser, but as a condenser with a guard ring could not be worked by the commutator, the capacity of the guard ring condenser had to be compared experimentally with that of a condenser without a guard ring which could be worked by the commutator.

The investigation thus divides itself naturally into three parts :—

First, the theoretical calculation of the electrostatic capacity of the guard ring condenser. For this purpose it was necessary to determine the geometrical constants of the guard ring condenser.

Secondly, the comparison of the capacity of this guard ring condenser with that of a condenser without a guard ring.

Thirdly, the determination by the method just described of the electromagnetic measure of the capacity of the condenser without a guard ring.

I shall describe these parts separately.

Part I.

The calculation of the electrostatic capacity of the guard ring condenser.

The condenser, which was designed by Lord Rayleigh, is represented in section in fig. 2, and in plan in fig. 3.

Fig. 2.

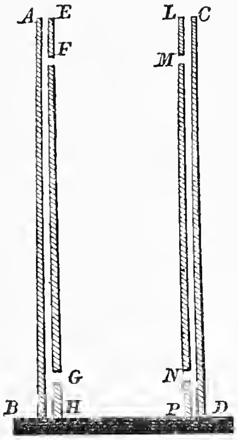
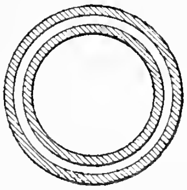


Fig. 3.



B H P D is a thick ebonite board placed in an approximately horizontal position, in this board two concentric circular grooves are cut. A cylindrical brass ring, H P, whose external diameter is about 23 centims., and whose height is about 10 centims., fits into the smaller of these grooves. Three pieces of ebonite carefully ground down to the same thickness (about 3 centims.), are placed at equal intervals on the top of this ring. On these the brass cylinder F G M N is placed; this cylinder is of exactly the same diameter as the cylindrical ring H P, and is about 60 centims. long. The cylinders, G F M N and H P, are placed so that their axes are coincident; this is tested by placing a straight-edge against the sides of the cylinder. On the top of this cylinder three pieces of ebonite are placed, and upon the top of these a cylindrical ring E L, similar to the one at the bottom; another brass cylinder, A B D C, made in three pieces, two rings similar in dimensions to the rings H P E L, and a long middle piece of the same length as the cylinder F G M N is then fitted over the other cylinders, the bottom ring fitting into the outer groove in the ebonite board; the internal diameter of this cylinder is about 25 centims. The distance between the cylinders at the top is tested by observing how far a wedge, whose vertical angle is very small, sinks down between the cylinders. When the system is properly adjusted, the variation in the distance is only a small percentage of its mean value.

The dimensions of this condenser were ascertained in the following way:—The length of the cylinder was measured by beam compasses, and the diameters of the inner and outer cylinders by callipers; the difference of these readings was not, however, taken as the distance between the cylinders, for though the error made in determining the diameter of either cylinder may be a small fraction of either diameter, yet since the diameters are nearly equal, it may not be a small fraction of their difference. The distance between the cylinders was determined by fastening the middle pieces of the two cylinders down to a flat board by a thin layer of shellac, and then filling the space between them with water which had been boiled a few hours before the experiment so as to be in a condition to absorb any air-bubbles that might be formed. The quantity of water required to fill this space was carefully weighed. This gives the volume of the water, and knowing the length of the cylinder and the diameter of one of them, the difference of the diameters can be calculated.

The results of these measurements are:—

LENGTH of cylinder, measured by beam compasses.

60·97
60·965
60·97

Mean 60·968 centims.

INTERNAL diameter of outer cylinder, measured by callipers.

9·986
9·989
9·992

Mean 9·989 inches, or 25·372 centims.

EXTERNAL diameter of inner cylinder, measured by callipers.

9·254
9·255
9·250

Mean 9·253 inches, or 23·50 centims.

WEIGHT of water required to fill the space between the cylinders.

4406·8	grammes at	17·5° C.
4404·6	,,	13·5°
4401	,,	12·2
4403	,,	11·5
Mean 4405·1 grammes.		

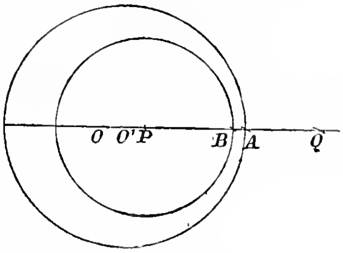
Since the greatest difference in temperature does not affect the result by one part in a thousand, the correction for temperature is neglected.

We find from these numbers that the distance between the cylinders is ·941 centim.

When the distance between the cylinders was measured by hair dividers, the least distance was found to be ·826 centim., the greatest ·984 centim., giving ·79 centim. as the distance between the axes of the cylinders.

Since the axes of the cylinders are not quite coincident, we cannot calculate the capacity by the ordinary formula. We proceed to investigate a formula which will hold in this case.

Fig. 4.



Let the figure represent a section of the cylinders by a plane perpendicular to their axes. Let O be the centre of the section of the cylinder O A, O′ the centre of the section of O′ B. Let O A$=a$, O′ B$=b$.

Let P and Q be inverse points with respect to both circles, so that

$$\mathrm{OP.OQ}=a^2$$

$$\mathrm{O'P.O'Q}=b^2$$

Then if $\phi=\mathrm{A}-\mathrm{B}\log r_1/r_2$, when r_1, r_2 are the distances of a point T from P, Q respectively, ϕ will satisfy LAPLACE'S equation and will be constant over both cylinders. Thus ϕ will be the potential of the electrical distribution, and by comparison with the ordinary form for the potential of an electrified cylinder we see that $\frac{1}{2}$ B will be the quantity of electricity per unit length upon either cylinder. Let the outer cylinder be connected with the earth so that its potential is zero, and let the potential of the inner cylinder be V.

Then we have

$$0=\mathrm{A}-\mathrm{B}\log\frac{\mathrm{PA}}{\mathrm{QA}}$$

$$\mathrm{V}=\mathrm{A}-\mathrm{B}\log\frac{\mathrm{PB}}{\mathrm{QB}}$$

therefore

$$\mathrm{V}=\mathrm{B}\left\{\log\frac{\mathrm{PA}}{\mathrm{QA}}-\log\frac{\mathrm{PB}}{\mathrm{QB}}\right\}=\mathrm{B}\log\frac{\mathrm{PA.QB}}{\mathrm{QA.PB}}$$

$$\mathrm{B}=\frac{\mathrm{V}}{\log\dfrac{\mathrm{PA.QB}}{\mathrm{QA.PB}}}$$

but $\frac{1}{2}$ B is the quantity of electricity per unit length upon either cylinder, and since the capacity is the quantity of electricity divided by the difference of potential, the capacity of the two cylinders

$$=\frac{1}{2}\frac{l}{\log\frac{\text{PA.QB}}{\text{QA.PB}}}$$

where l is the length of either cylinder.

Let

$$\text{OP}=x \qquad \text{OO}'=c$$

then

$$\text{OQ}=\frac{a^2}{x}. \qquad \text{PA}=a-x. \qquad \text{QA}=\frac{a^2}{x}-a.$$

therefore

$$\frac{\text{PA}}{\text{QA}}=\frac{x}{a}$$

similarly

$$\frac{\text{QB}}{\text{PB}}=\frac{b}{x-c}$$

Since O′P.O′Q$=b^2$ we have

$$(x-c)\left(\frac{a^2}{x}-c\right)=b^2$$

therefore

$$(a^2+c^2-b^2)-c\frac{a^2}{x}-cx=0$$

or

$$x^2+x\frac{b^2-(a^2+c^2)}{c}+a^2=0$$

Solving we find that

$$x=\frac{ca^2}{a^2-b^2}\left\{1+\frac{b^2c^2}{(a^2-b^2)^2}\right\}$$

approximately, supposing that as in our condenser $\frac{c^2}{a^2-b^2}$ is small.

therefore

$$x-c=\frac{cb^2}{a^2-b^2}\left\{1+\frac{a^2c^2}{(a^2-b^2)^2}\right\}$$

$$\log\frac{\text{PA.QB}}{\text{QA.PB}}=\log\frac{x.b}{a(x-c)}=\log\frac{a\left(1+\frac{b^2c^2}{(a^2-b^2)^2}\right)}{b\left(1+\frac{a^2c^2}{(a^2-b^2)^2}\right)}$$

so that the capacity of the condenser

$$=\frac{1}{2}\frac{l}{\log\left\{\frac{a\left(1+\frac{b^2c^2}{(a^2-b^2)^2}\right)}{b\left(1+\frac{a^2c^2}{(a^2-b^2)^2}\right)}\right\}}$$

$$=\frac{1}{2}.\frac{l}{\log\frac{a}{b}-\frac{c^2}{a^2-b^2}} \text{ approximately.}$$

Substituting the values of $a-b$ and b given above we find

$$\log\frac{a}{b}=\cdot 07705$$

$$\frac{c^2}{a^2-b^2}=\cdot 00027$$

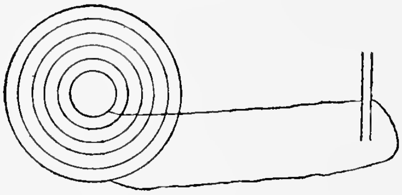
and the electrostatic measure of the capacity of the condenser is consequently 396·8.

PART II.

The comparison of the capacity of this condenser with that of one without a guard ring.

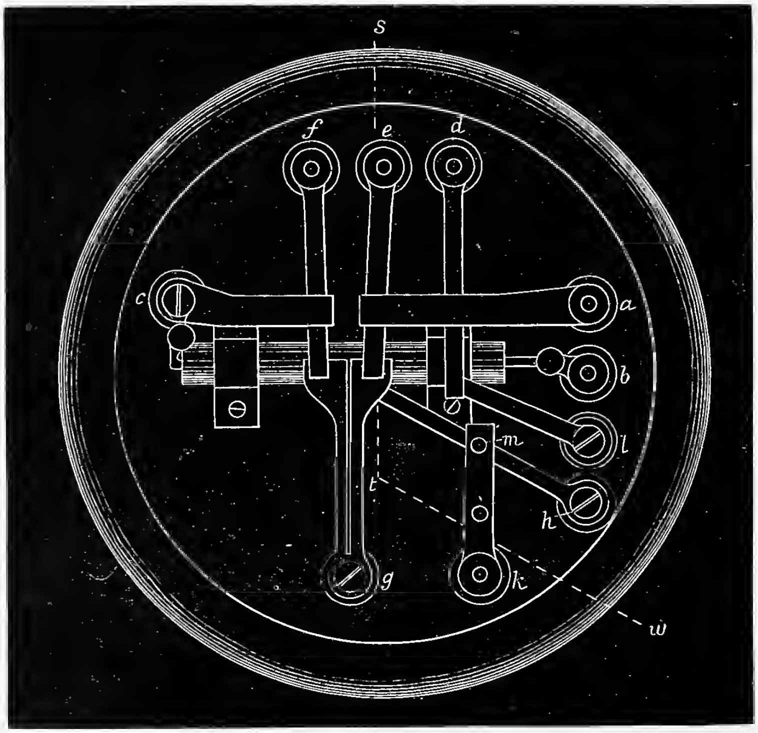
As this condenser could not be worked by a commutator on account of the guard ring, it was necessary to compare its capacity with the capacity of a condenser without a guard ring. At first it was intended to compare the guard ring condenser with one of considerably greater capacity. Such a condenser was constructed, having a number of brass discs separated by thin pieces of shellac, the alternate discs being electrically connected, a weight was placed upon the disc at the top to keep the system steady; and the system was placed in a vessel formed by putting a bell-jar on a surface plate. There were two openings into this vessel, one of these was connected with a water pump; the other with the air outside the jar by a series of tubes filled with cotton wool and chloride of calcium, to free the air passing through them from dust and moisture; air was then pumped through the vessel for about 24 hours, and both openings were then closed. The capacity of this condenser was compared with that of the guard ring condenser, by connecting one plate of each condenser to earth, and the other with two points, P and Q, of a battery circuit; resistance boxes being placed between P and Q. A point O of the circuit between P and Q was then put to earth, and the resistance in the parts O P, O Q, so adjusted that when the charges of the two condensers were sent simultaneously into an electrometer there was no deflection of the needle, showing that the charges in the two condensers were then equal and opposite. In this case, the capacities of the condensers, whose plates were connected with P and Q respectively, would bear the same ratio to each other as the resistance in O Q bears to the resistance in O P. With the battery-power obtainable, this method however was found not to be sufficiently sensitive, as the resistance in either of the arms O P, O Q, could be altered by about ·75 per cent. without appreciably disturbing the equilibrium of the needle of the electrometer when the charges of the condenser were sent into it. It was therefore decided to make a condenser without a guard ring equal in capacity to the guard ring condenser, and employ the method given in § 229 of MAXWELL'S 'Electricity and Magnetism,' to determine when the two condensers were of equal capacity; this method can be made much more sensitive than the one just described, as the Leyden-jar used in MAXWELL's method can easily be raised by an electrophorus to a very high potential. The new condenser consisted of several co-axial tubes represented in section in fig. 5, The alternate tubes were connected together, and the two series connected with opposite plates of a very fine plate condenser, which was very kindly lent to me by the Rev. COUTTS TROTTER, Fellow of Trinity College.

Fig. 5.

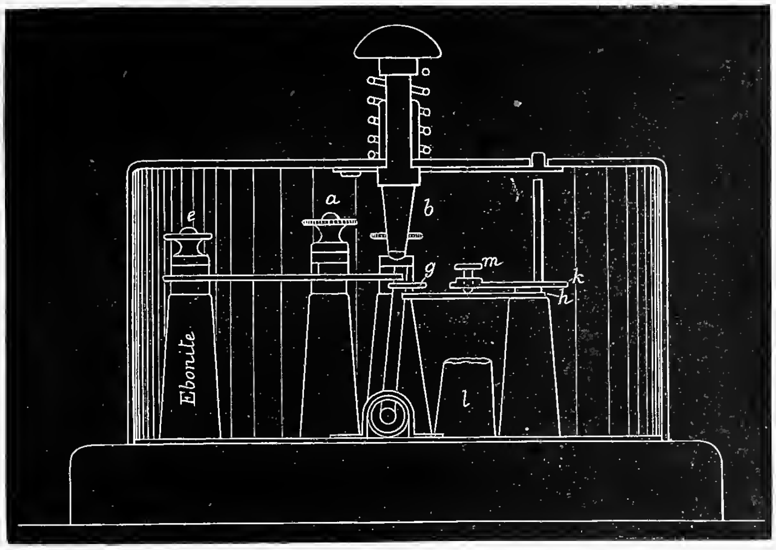


A rough adjustment could be made by altering the number of tubes connected together, while the fine adjustment was effected by altering the distance between the plates of the plate condenser by means of a finely cut screw. The equality of this condenser and the guard ring condenser was tested by the method given in § 229 of MAXWELL's 'Electricity and Magnetism,' using a key which was very kindly lent to me by Dr. JOHN HOPKINSON, F.R.S., and which had been used by him for a similar purpose. I quote the description given of it by him in his paper on the "Electrostatic Capacity of Glass," Part II., p. 360, Phil. Trans., Part II., 1881 :—

" Fig. 3.



"Fig. 4.



" Section on line s, t, w.

"*a*, *c* are stiff insulated horizontal contact bars connected to the two poles of the battery. *d*, *e*, *f* are insulated springs normally touching *a* and *c* on the under side. *d* is connected by a wire to the guard ring, *e* to the plate of guard ring condenser, *f* to the sliding condenser. *b* is an insulated binding screw connected with *c* for the purpose of more conveniently introducing the battery wire. *l* is a spring connected to earth. *k* is a stiff insulated piece carrying an adjustable point *m*, normally in contact with the upper side of the insulated spring *h*. From *k* a wire leads to the quadrant of the electrometer. *k* can at any moment be put to earth by a spring key. The insulated spring *g* has its end between *e*, *f*, and *h*, and is normally in contact with neither. The springs *d*, *e*, *f* can be simultaneously bent downwards by an insulated plunger. When this plunger is struck downwards we have the following operations effected in a fraction of a second—

1°. *d* and *e* are in contact with *a*.
f in contact with *c*.

2°. *d*, *e*, and *f* insulated.

3°. *d* connected to *l*.
e, *f*, and *g* connected together.

4°. *e*, *f*, *g*, *h*, *k* connected together.

5°. Connexion of *k* and *h* broken.

* * * * * * * *

"The whole switch, binding screws and all, is covered with a brass cover connected to earth and provided with apertures for the connecting wires. The ebonite legs which carry the pieces *a*, *b*, *c*, *d*, *e*, *f*, *g*, *k* are attached to a brass base plate, so that if any leakage occur from *a*, *b*, *c*, *d*, *e*, or *f*, it shall be to earth and not to the electrometer."

The connexions are made in the following way. Let A denote the outer cylinder of the guard ring condenser, B the guard ring pieces, and C the inner cylinder; let A′ and C′ denote the plates of the other condenser; T the armature of the Leyden-jar, which is not connected with the earth. Then A is connected with the earth; B to A′ and to *d* of the key, C to *e* of the key, C′ to *f* and T to *a*; *b* and *l* are connected with the earth, and *h* is connected with the electrometer.

Before the plunger is pushed down A is put to earth; B and C to T; A′ to T; C′ to earth.

When the plunger is pushed down, before it reaches *e* and *g*, A is to earth. B and C are charged and disconnected. A′ and C′ have equal and opposite charges.

When the plunger is pushed down a little further, so that *d* comes into contact with *l*, B and A′ are put to earth, so that the charges on C and C′ are free to flow into the electrometer when the plunger goes a little further and strikes *h*.

If the capacities of the two condensers are equal, the charges on C and C′ are equal, and of opposite signs, and when they flow together into the electrometer, their combined effect will be zero. The distance between the plates of the plate-condenser was altered, until the needle of the electrometer was not deflected when the plunger of the key was pushed down. This method was found to be very sensitive; if after a balance had been obtained, the capacity of one condenser was altered by 1 per cent., the quantity of electricity sent to the electrometer was sufficient to drive the spot of light off the scale.

The insulation of the two condensers and the key was tested several times, both electrostatically and by attempting to pass a current through them. If either condenser was charged, and the key put in electrical connexion with it, the loss of charge in five minutes was not quite 2 per cent., so that the loss in the small time required to push the plunger down is quite negligible. When the condensers and the key were put in circuit with a battery of 150 DANIELL's elements, no current could be detected with a galvanometer whose resistance was 11,000 ohms.

PART III.

The determination of the electromagnetic measure of the capacity of the condenser without the guard ring.

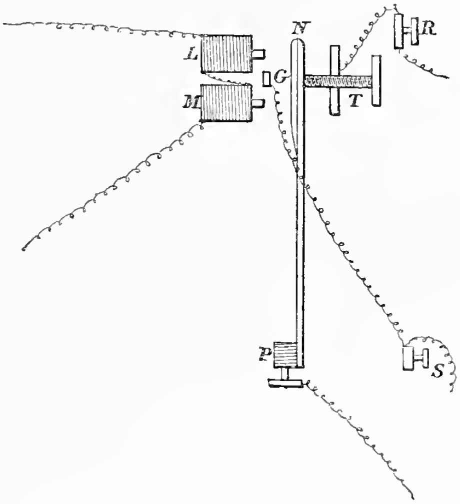
This was effected by the method described at the commencement of this paper.

The galvanometer used had a resistance of about 11,000 ohms. It was insulated by placing it upon pieces of glass coated with paraffin.

The battery consisted of 150 DANIELL's cells, coupled up with 25 LECLANCHÉ. The DANIELL's cells were put into trays, containing 10 each. The resistance of the battery was about 2500 ohms. The insulation of the battery gave a considerable amount of trouble, but the following plan was found successful. The case containing the trays was placed on glass supports about 2 inches thick covered with a thin layer of paraffin, while each tray was insulated from the case by pieces of ebonite.

The commutator was one which had been previously used by Lord Rayleigh, and had been designed by him.

Fig. 6.



The current from two Groves' cells passes first through a tuning-fork interruptor, and then through the coils L M of an electromagnet. P N is a strip of brass with a piece of iron attached to it. When there is no current passing through the electromagnet, the elasticity of the rod P N makes it press against a screw T, which is electrically connected with a binding screw R: when the current passes through the electromagnet the magnet attracts the iron attached to the rod P N and brings it into contact with the stop G, which is electrically connected with the binding screw S. The letters P, R, S indicate the same points in this figure as in fig. 1. All the places where contact is made by the vibrating piece P N are covered with platinum, and the whole arrangement is fastened down to an ebonite board. As the current passes intermittently through the coils L N of the electromagnet, the vibrating piece P N strikes alternately against the parts G and T; when it strikes against G the opposite plates of the condenser are connected with the two poles of the battery; when it strikes against T the condenser is discharged (see fig. 1).

This commutator was found to work extremely well. When it was in good order the spot of light reflected from the mirror of the galvanometer through which the intermittent current passed never moved off one division of the scale, and the only thing by which the deflection could be distinguished from one due to a steady current was a slight indistinctness in the edge of the image of the spot of light.

The speed of the tuning-fork interruptor was found by comparing it with that of the standard fork used by Lord Rayleigh in his determination of the ohm in absolute measure. The standard fork vibrates about 128 times per second, while the tuning-fork used in this investigation vibrates about 32 times per second. This fork was used to drive another of about four times its frequency, and the number of beats per second between this driven fork and Lord RAYLEIGH'S standard fork was counted. At the temperature of 15° C. there were 12 beats in 20 seconds between the two forks, and the standard fork vibrated more slowly than the other. The standard fork makes 128·15 vibrations per second, so that if n be the number of vibrations per second of the fork used to drive the commutator, we have

$$4n \times 20 - 12 = 128{\cdot}15 \times 20$$
$$n = 32{\cdot}1875.$$

The observations.

The observations consisted of two parts. The capacity of the movable condenser had to be adjusted until it was equal to the capacity of the guard ring condenser. This was ascertained by the method described in Part II.; and then this adjustable condenser was put in the WHEATSTONE'S bridge as in fig. 1, and the resistances of the arms of the bridge adjusted so that the deflection of the galvanometer due to the steady current was just balanced by the deflection due to the intermittent current arising from the flow of electricity to the condenser when the movable piece P was in contact with S. The resistances in the arms A D, B C (fig. 1) were kept constant, and the adjustment was effected by altering the resistance in A C.

The steady current, when it was not balanced by the current arising from the charging of the condenser, produced a deflection of the dot of light reflected from the mirror of the galvanometer of about 120 scale divisions, and as a fine wire was placed before the lamp of the galvanometer and focussed on the scale, readings could easily be made to quarter of a division.

The following are the results of the observations, and it may be worthy of remark that, as many of the pieces of apparatus used were required for the ordinary work of the laboratory, the whole arrangement had to be taken down and put together again between each determination. This must have had the effect of getting rid of a good many accidental errors, and taking it into consideration the following numbers seem as near together as could be expected for such complex observations. The resistances are given in B.A. units.

RESISTANCES in the various arms of the WHEATSTONE'S bridge, when there was no deflection of the galvanometer.

	Resistance in the arm A D.	Resistance in B C.	Resistance in A C.
1.	899,666	99,920	1294
2.	899,666	99,920	1285
3.	899,930	99,950	1297
4.	899,700	99,925	1287
5.	899,700	99,925	1297

The mean of these correct to 1/10 per cent. is

899,700	99,925	1292

According to Lord RAYLEIGH'S determination of the ohm the B.A. unit

$$=\cdot 987\times 10^{9}$$

so that from the formula $nC=a/cd$ we find that the electromagnetic measure of the capacity of the condenser $=\cdot 4517\times 10^{-19}$.

The electrostatic measure of the capacity of the same condenser is 396·8.

So if v be the ratio of the electrostatic unity of electricity to the electromagnetic

$$v^2=\frac{396\cdot 8\times 10^{19}}{\cdot 4517}$$

$$v=2\cdot 963\times 10^{10} \text{ in C.G.S. units.}$$

Some experiments were made with a tuning-fork vibrating 44 times a second; the results of those were found to agree very closely with those obtained when the tuning-fork vibrated 32·18 per second. The above experiments were made in the Cavendish Laboratory, Cambridge, and I have much pleasure in thanking Lord RAYLEIGH for the very valuable advice which he gave to me throughout the investigation, as well as for his kindness in designing several of the more important pieces of apparatus.

XXIV. *The Development of Renilla.*

By Edmund B. Wilson, *Ph.D., Fellow of Johns Hopkins University.*

Communicated by Professor Huxley, *F.R.S.*

Received October 5,—Read December 14, 1882.

[Plates 52–67.]

The observations recorded in the following pages were made in the course of three summers' work at the marine laboratory of the Johns Hopkins University, organised and directed by Professor W. K. Brooks, and located for the past three years at Beaufort, N.C., where the material for this paper was collected.

The abundance of *Renilla reniformis* (Cuv.), at Beaufort, suggested the desirability of a careful study of its embryology, and this was rendered still more apparent by the studies which Mr. Mitsukuri had made upon the anatomy of the adult organism.

I was therefore much pleased when a lucky accident, during the summer of 1880, put me in possession of a few very young colonies. Subsequent search over the ground resulted in the discovery of a considerable number of young colonies in various stages of growth, and I was thus enabled to make a rather full study of the growth of the colony from the simple primary or axial polyp up to the adult organism with its secondary polyps in a state of full sexual maturity. A single specimen, finally, of the ciliated larva was taken at the surface and kept in the aquarium until the free-swimming life was abandoned and the characteristic tentacles and spicules made their appearance.

An outline of the general results of these observations was published in the American Journal of Science for December, 1880, a full description being however deferred in the hope of procuring still earlier stages for a study of the embryonic development. This hope was happily realised in the following summer, when two or three lots of fertilised eggs were obtained; and, finally, in the season of 1882, the eggs were procured in considerable abundance, and a very satisfactory study of the phenomena of development was effected.

During the latter season the eggs of a Gorgonian, *Leptogorgia virgulata* (Edw. and Haime), were procured—though in small numbers only—and I have studied to some extent the development of this polyp also. The material was however scanty, and the development, so far as observed, closely similar to that of *Renilla;* hence these observations will be described only for the sake of comparison and in connexion with those relating to *Renilla*.

I have been unable to overcome entirely certain technical difficulties, and my results will therefore be found inconclusive on a few points. Furthermore, the unexpected close of the spawning period during the last season's work brought to an unwelcome close my observations on the earlier stages of development, and I have not been able, for this reason, to follow in detail the phenomena of the fertilisation of the egg and the behaviour of the segmentation-nuclei during the early stages. Still I venture to hope that my observations form a decided advance on what is now known of the development of the Alcyonaria. KOWALEVSKY's well-known observations on *Sympodium, Clavularia, Alcyonium,* and *Gorgonia* form the basis of almost the whole of our knowledge of the subject; and these observations, though of great interest, were published in a very condensed form, and were in part rendered inaccessible to many zoologists through their publication in the Russian language. They indicated that the early stages of Alcyonarian polyps would well repay more extended observation, and this expectation has perhaps been realised in the case of *Renilla*.

So far as the Pennatulacea are concerned, nothing is known of the embryonic development, and only the most meagre accounts exist concerning the mode of budding and formation of the colony. The latter phenomena however involve questions of much interest on account of the highly specialised nature of the colony as expressed in the marked polymorphism of its members and in the remarkable relations of symmetry existing between them.

For the foregoing reasons it seems to me desirable to publish these observations without further delay, since I can see in the future no near opportunity of making them complete.

Before considering the phenomena of development it will be useful to glance for a moment at some of the structural features of the adult *Renilla* and their relation to the characteristics of other Alcyonaria. For a full description the reader is referred to the well-known papers of KÖLLIKER* and EISEN,† who have described in some detail the structure of *Renilla reniformis* and *R. amethystina*.

Renilla is a genus of Pennatulacea, a group which forms the highest division of the Order Alcyonaria. The organism, when adult, is a community or colony, the members of which consist of an axial polyp and a large number of secondary polyps produced by the budding of the axial or primary individual and organically united with it. The colony has the form of a reniform disc with a deep sinus at one side into which is inserted a flexible peduncle which roots the organism in the sand. The polyps are arranged in radiating lines over the surface, projecting upwards over the general

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* "Anatomisch-Systematische Beschreibung der Alcyonarien, Erste Abtheilung, Die Pennatuliden." Abdruck a. d. Abhandlungen d. Senkenb. Naturforsch. Gesellschaft, Bd. vii., viii. Frankfort, 1872.

† "Bidrag til Kännedom om *Renilla*." Kongl. Svensk. Vet. Handl., Bd. xiii.

surface but lying nearly horizontally at the margin, where new polyps continually make their appearance. Each polyp may be retracted into a "cell" which is morphologically the basal part of the polyp and forms a part of the disc.

The polyp has eight septa or mesenteries, eight pinnate tentacles, and eight mesenterial or gastric filaments, of which the dorsal pair are more slender and of different structure from the others. The two lateral pairs of septa bear the reproductive organs, male or female, as the case may be, the sexes being separate. The septa are provided with delicate longitudinal muscles by which the retraction of the polyp into its cell is effected. These muscles are always placed on the ventral sides of the septa, so that the dorsal gastric chamber contains no muscles, while the ventral chamber contains them on both sides. Thus we observe a marked bilateral symmetry in the arrangement of all the internal organs, which is further emphasized in the dorso-ventral elongation of the mouth and œsophagus. This symmetry is expressed also in the arrangement of the calyx-teeth, which are conical projections from the walls of the gastric chambers at the level of the upper face of the disc. The ventral chamber is always destitute of a tooth, the dorsal chamber always bears one, and the lateral teeth are symmetrically arranged with respect to the dorso-ventral axis.

Besides the large sexual polyps there are other forms known as the *rudimentary individuals*, or in Kölliker's terminology, as the *zooids*. These are microscopic in size, have no tentacles, no mesenterial filaments, no reproductive organs, and commonly only two calyx-teeth—those, namely, on the ventro-lateral chambers. The zooids possess, in fact, only septa, mouth, and œsophagus, the latter being richly ciliated within. Two distinct forms of zooids exist. One of these is represented by a single large zooid, placed near the middle of the disc on the dorsal side, and provided with the full number of calyx-teeth. It is for the most part through the mouth of this zooid that the water is discharged which circulates through the cavities of the colony. For this reason I shall call it the *exhalent zooid*, a name which seems preferable to Kölliker's term "Haupt zooid." The other zooids are arranged in groups or clusters on the dorsal sides of the polyp-cells in the median line; there are usually four such groups on each cell. It is their function to draw water from the exterior into the cavities of the colony, as may be shown by adding finely pulverised carmine to the water of the aquarium. Minute but powerful currents may thus be seen setting into the open mouths of the zooids. The zooids, like the sexual polyps, exhibit a marked bilateral symmetry in the disposition of all their organs; the mouth and œsophagus are elongated in the dorso-ventral plane, the gastric chambers are of different sizes and symmetrical arrangement, and the two calyx-teeth occupy corresponding positions on the sides of the median plane.

The colony as a whole is also bilaterally symmetrical to a very striking degree. This is more obvious in young specimens, but is always clearly marked even in the largest colonies. Each polyp has its counterpart on the opposite side of the colony, and the dorso-ventral axes of the two polyps have the same inclination to that of the axial polyp, since the ventral chamber, which bears no calyx-teeth, is always directed outwards towards the margin of the disc. Thus the secondary polyps stand at all angles from 0° to 90° with the axial polyp; those at the sides are placed so that their dorso-ventral axes form right angles with that of the axial polyp, while those directly in front of the axial polyp coincide with it in direction.

Lastly we may note the structure of the peduncle. Its cavity is divided into a dorsal and a ventral chamber by a horizontal partition which is pierced along its sides and at its lower extremity with openings by which the chambers are put in communication. Both chambers end blindly in front, but communicate by small openings with the adjacent polyp-cells. The upper canal communicates with the exterior through the exhalent zooid already described. The horizontal partition appears to split anteriorly into a dorsal and a ventral plate, between which lies the posterior part of the body—*i.e.*, part of the cell—of the axial polyp.

The structure of the body-wall in the peduncle, where it is most fully developed, is as follows (after EISEN). Beginning with the exterior there are: (1) external epithelium; (2) a thick layer of connective tissue containing the spicules; (3) a layer of fibrous connective tissue free from spicules; (4) longitudinal muscles; (5) circular muscles; (6) internal epithelium. In other parts of the body the arrangement is somewhat different since the amount and structure of the connective tissue varies in different parts of the body and the spicules are absent from the walls of the free portions of the polyps.

Many of the structural features of *Renilla* are common to other Alcyonaria. The polyps always exhibit more or less of bilateral symmetry in the elongation of the mouth, disposition of the septa and septal muscles, grouping of the mesenterial filaments and arrangement of the reproductive organs. In all but one or two cases colonies are formed by processes of asexual multiplication, and these not uncommonly show traces of bilateral symmetry. Among the Pennatulacea bilaterality is always more or less marked, culminating in the *Renillaceæ*, where the symmetry is nearly complete.

The definite relation between the dorso-ventral axes of the secondary and primary polyps has been observed in a few other Alcyonaria, but observations on this point are very scanty. KÖLLIKER, in his great work on the Pennatulids, has described something similar among the more typical forms, and it is highly probable that further investigation would show that in all Pennatulacea definite relations of this sort exist. MOSELEY observed in *Heliopora* and *Sarcophyton*[*] that the dorsal sides of the polyps face in a definite direction; and according to HAACKE[†] the polyps of *Madrepora* have a like disposition. As pointed out further on, this matter is one of much theoretical interest in connexion with the law of budding in *Renilla*.

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[*] Phil. Trans., Vol. 166, 1876.

[†] "Zur Blastologie der Korallen," Jena. Zeitschrift, Bd. XIII

The polymorphism of the Pennatulids was first observed by VERRILL in 1864 in *Renilla*, and was afterwards shown by KÖLLIKER to be of general occurrence in the group. It occurs also in some of the Alcyonacea, as in *Sarcophyton* (MOSELEY) and *Heteroxenia* (KÖLLIKER), and Professor VERRILL has informed me of his discovery of rudimentary individuals in two species of *Paragorgia*, members of the Gorgonacea.

For our knowledge of the embryology of the Alcyonaria we are almost entirely indebted to KOWALEVSKY's well-known researches, though LACAZE-DUTHIERS, many years earlier, made a few observations on the development of *Corallium*. In 1873 KOWALEVSKY gave some account of the embryological development of *Alcyonium digitatum* and *Gorgonia verrucosa*,* and in 1879 published a brief account of the early development of *Sympodium coralloides* and *Clavularia crassa*,† which he studied in conjunction with MARION. KÖLLIKER has given a brief account of the development of the buds in *Halisceptrum*, and DALYELL published fragmentary notes on the early development of *Virgularia*.‡

Even less is known in regard to the development of the colony in the Pennatulacea. FRITZ MÜLLER observed in 1864 the simple axial polyp of *Renilla* and gave a few notes upon its structure. KÖLLIKER figures a very young colony of *Pteroides* and gives a few notes concerning the young stages of *Kophobelemnon*. WILLEMÖES-SUHM has also described and figured one or two of the early stages of the colony in *Umbellularia*.§ A thorough study of the mode of budding has, however, never been made; and the observations just mentioned, though of interest, are too incomplete to be of great value.

In all of the Alcyonaria thus far studied the germ-layers appear to be differentiated through some process of delamination. Among other polyps, however—as we know from the observations of KOWALEVSKY, LACAZE-DUTHIERS, METSCHNIKOFF, JOURDAN and others—some forms undoubtedly pass through a typical invaginate gastrula stage, while others appear to develop as delaminate planulæ. BALFOUR states on the authority of KLEINENBERG,|| that in a number of Zoantharia the segmentation is unequal, "indicating, perhaps, the occurrence of an epibolic gastrula." I shall, however, show further on, that inequality in cleavage is by no means a certain indication of epibolic invagination.

With this brief sketch of the anatomy and embryology of the Alcyonaria, in which

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* "Untersuchungen über die Entwickelung der Cœlenteraten, Nachrichten der Kaiserl., Gesellsch. der Freunde der Naturkenntniss der Anthropologie und Ethnographie." Moskau, 1873 (Russian). Abstract in HOFFMANN and SCHWALBE's 'Jahresbericht,' 1875, Bd. ii.

† 'Zoologischer Anzeiger,' No. 38, 1879.

‡ 'Rare and Remarkable Animals of Scotland,' vol. ii., pp. 181–190, *t.* KÖLLIKER.

§ 'Annals and Magazine of Natural History,' vol. xv., 1875.

|| 'Comparative Embryology,' vol. i.

only those features have been mentioned which appear of interest in connexion with the following study of *Renilla,* we may pass to a description of the observations.*

I.

SEGMENTATION OF THE EGG AND FORMATION OF THE GERMINAL LAYERS.

§ 1. *External features of segmentation.*

Renilla, like most other Alcyonaria, is diœcious, and on account of the rather marked difference in colour between the ova and spermatic capsules, the sexes may usually be distinguished by external examination. During the months of May, June, and July many *Renillas* were found with the cavities of the polyps packed with the lead-coloured ovaries or the whitish spermatic capsules. The egg or mass of sperm-cells is enclosed in a very distinct follicle of ciliated entoderm cells which is ruptured at the time of spawning, the eggs being thus discharged into the gastric cavity and thence passed out to the exterior.

The eggs make their exit through the mouths of the sexual polyps, and the time occupied in spawning is very short. They are vomited forth in great masses, together with a considerable quantity of mucus, by a reversed peristaltic movement of the œsophagus, the entire colony being usually in a state of complete expansion. The mass of eggs is often held for some time clasped in the tentacles before being thrown off into the water. All of the polyps in the central part of the disc spawn simultaneously; those near the edge of the disc often do not spawn with the others,

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* It is perhaps worth while to describe briefly the methods employed in the preparation of the embryos. For sections of the early stages the most satisfactory method is that recommended by Bobretsky and so successfully employed by Mayer, Hatschek, and others. The eggs were heated in sea-water to about 60° C., and maintained at that temperature for two or three minutes in order to coagulate thoroughly the protoplasm. They were then hardened for twenty-four hours in potassium bichromate, washed two hours in sea-water, and then gradually hardened in alcohol (50 per cent. three hours, 75 per cent. three hours, 90 per cent. six hours, and then transferred to absolute alcohol). After standing twenty-four hours in picro-carmine, and again soaking a few hours in absolute alcohol, they were embedded in paraffin and vaseline and cut with the sledge microtome.

For sections of later stages the embryos were exposed for a few minutes to very dilute osmic acid ($\frac{1}{20}$ per cent., or less) until a barely perceptible brown tint was produced. After thorough washing they were transferred to weak, strong and absolute alcohol, and stained and embedded as before.

For isolation of the muscle-fibres and other elements of the tissues, the method recommended by the Hertwig brothers was employed. The larvæ were placed for ten or fifteen minutes in a mixture of equal parts of $\frac{1}{20}$ per cent. osmic acid and $\frac{1}{5}$ per cent. acetic acid in sea-water, then thoroughly washed and soaked for several days in $\frac{1}{5}$ per cent. acetic acid in sea-water. They were then stained *in toto* and teased in glycerine.

With other methods of hardening I have had no success. Bobretsky's method is highly to be recommended for the early stages, and affords very clear and satisfactory preparations.

perhaps because they are younger and less mature. The ovaries of polyps which had recently spawned were usually found to contain considerable quantities of immature eggs. Hence it seems probable that there may be several successive broods of egg in a single year, since the spawning season extends over two or three months.

It is a rather curious fact that the eggs are always laid at very nearly the same hour of the day, viz., about 6 A.M. Large numbers of *Renillas* were kept in aquaria, and the act of spawning was several times observed. In a single case only the spawning took place as early as half-past five and it was never observed to occur later than 7 A.M. This regularity appears to be independent of temperature, although this has a very important influence on the rate of development; for the hour was the same on cold and warm days. It is not unlikely that marine animals are more regular in such habits than has been suspected. A similar case is that of *Lucifer*, which, as Dr. Brooks has observed, deposits its eggs always at the same hour, viz.: from 9 to 10 P.M.

During the discharge of the eggs by the females the males pour out the spermatic fluid in a milky cloud rising from the colony. The male element is apparently discharged, like the eggs, through the mouths of the sexual polyps. The spermatozoa are of the ordinary tailed form with pyriform heads, and swim with great activity. Fertilisation is effected in the water.

When first discharged the eggs are usually more or less distorted by pressure during their passage through the œsophagus; within a few minutes, however, they become perfectly spherical, and have an average diameter of about ·35 mm. They are of stony opacity, so that the germinal vesicle is invisible, and are destitute of any proper limiting membrane, though the peripheral layer of the vitellus is clearer and less granular than the rest. The entire substance of the vitellus is densely packed with deutoplasm granules, which upon rupture of the egg appear as clear yellowish spherules. Polar cells were never observed.

It will be convenient to describe first those changes which are visible from the exterior, leaving to the next section an account of the corresponding internal changes as discovered in sections. In a third section a review of the facts will be given, together with a discussion of their significance.

The segmentation of the egg in *Renilla* is remarkable for the surprising amount of individual variation of which it is capable. So great is this variation that it is safe to say that no two eggs ever develop in precisely the same way; and although most of the variations may be arranged in a definite series, some of them are so irregular that they seem to follow no definite law. No one indeed without actually following the entire development of some of these eggs would suppose them capable of normal development. For a long time, in fact, I passed by some of the less usual forms as due to abnormal or pathological changes, and only after repeated and careful study was able to convince myself that these peculiar embryos gave rise to active larvæ, differing in no visible respect from those which had developed along the more usual course. The matter appeared to me of such interest and importance that I gladly availed myself of the aid of two of my fellow workers at the laboratory—Mr. H. L. Osborn and Dr. J. Meredith Wilson—in order to study as completely as possible the various forms of development. A large number of eggs, produced at different times by different individuals, were kept under continuous observation from the time of fertilisation up to an advanced stage of the segmentation; they were then proved to be capable of full and normal development by isolation in small glass vessels until the free-swimming larval stage was attained. We were thus enabled to determine with all possible certainty the fact that at least five or six well-marked modes of yolk-cleavage, with many minor variations, may occur as normal phenomena of development, that the segmentation may be at first equal or unequal, complete or partial, regular or irregular, and that a great amount of variation exists in the duration of the various stages of activity and quiescence.

The interval between fertilisation and the first cleavage varies greatly, and is in general greater when the temperature is low. Segmentation may begin within ninety minutes after fertilisation, or it may be delayed three or four hours beyond this. It was found that the longer this preliminary quiescence continued, the more apt the eggs were to pass through the less usual modes of development, while those which developed promptly were as a rule of the two common types about to be described.

1. I will first describe a common, though not the most frequent, mode of development illustrated by figs. 1 to 18. The egg having remained perfectly spherical from the time of fertilisation, becomes of irregular outline, and in two or three minutes divides into eight equal segmentation spheres. These are at first imperfectly separated (fig. 1), but soon become exceedingly distinct (fig. 2). In the individual figured the spheres swelled up slightly one minute later (fig. 3), then gradually flattened together somewhat, and the egg passed into a slightly-marked quiescent period or "resting stage" (figs. 4, 5). This continued fifteen minutes, when the spheres again swelled up, and each divided into equal parts (fig. 6) so that the embryo consisted of sixteen spheres. These again flattened together somewhat, and a second resting stage ensued (fig. 7) which continued for twenty minutes. The slight swelling of the spheres shown in fig. 3 is not accompanied by any visible cleavage. It is probably attendant upon some internal change, which may, perhaps, be a division of the nuclei —possibly of the spheres also—in a plane parallel with the surface. It is certain that such cleavages take place sooner or later, but I have not been able to trace the connexion between them and the external signs of activity (see § 2).

The segmentation now proceeded with great regularity, and appeared from the exterior to be regular and complete. Each stage of division, during which the spheres are swollen and rounded, was followed by a period of quiescence, in which the spheres were flattened and more closely pressed together. This regular alternation of rest and activity continued for a long time, until the spheres had become very small and the embryo had begun to elongate. It is clearly shown in the series of figures from 1 to 18, of which the first fourteen are from one individual (time, 115 minutes), the last four from another specimen (time, 32 minutes). The intervals of time between the successive visible cleavages were somewhat irregular, as shown in the following statement:—

Between figures	2	and	5	9 minutes.
„	5	„	7	45 „
„	7	„	9	29 „
„	9	„	11	12 „
„	11	„	13	23 „
„	13	„	15	Not observed.
„	15	„	17	27 minutes.

In the eggs of many animals the periods follow one another with great uniformity, and the irregularity in the present case is therefore somewhat unusual. It depends perhaps on the fact that the embryo is solid, and that during the whole segmentation the cleavages take place not only in planes at right angles to the surface, but also in planes parallel to it. The latter cleavages would not be visible externally, but might retard the surface cleavages at certain periods. This is apparently the true explanation of the long delay of forty-five minutes between figs. 5 and 7; for, as we shall see in the following section, the delamination, by means of which the layers are separated, takes place at this period when the embryo consists of sixteen spheres.

2. The mode of segmentation which has been described occurred with slight variations in rather less than one-third of all the eggs studied. In the most usual case, however, the eight-sphere stage is entirely passed over, and the egg divides at once into sixteen spheres at the first cleavage.

This mode of cleavage, illustrated by figs. 30 to 37, is, except in the first stage, quite like the cleavage into eight spheres. The egg is at first perfectly spherical, then becomes irregular in form, with a wavy outline, and at length falls at once into sixteen spheres (fig. 33), which are, as a rule, of equal size. Though very distinct at first, they soon flatten together, and the egg passes into a resting stage (fig. 35), which continues for ten to twenty minutes. This quiescent period, though only slightly marked in the specimen figured, is sometimes very pronounced, so that the embryo may be nearly or quite indistinguishable from the unsegmented egg. The subsequent development is very regular, and is like the first case.

As noted above, the spheres are usually of equal size. It is, however, a common occurrence for the segmentation to be more or less unequal, as shown in figs. 38 to 44. In this case the embryo presents externally the appearance of an epibolic gastrula, consisting of macromeres and micromeres. In the eight-sphere stage, also, embryos were sometimes observed to consist of four large spheres, capped by smaller ones, exactly as in the early stages of many epibolic gastrulas. In the sixteen-sphere stage there are often three or four larger spheres which are always placed at one pole of the egg, and are not separated by smaller spheres.

The first cleavage into eight spheres may be incomplete and irregular as well as unequal. Thus the egg shown in figs. 45 to 48 divided at first incompletely into eight (fig. 45), and then passed into a somewhat marked quiescent stage (fig. 46) of fifteen minutes' duration, one of the spheres retaining its prominence, as shown in the figure. It then divided into sixteen nearly equal spheres (figs. 47, 48), and its subsequent development was regular. A somewhat similar case is illustrated by figs. 49 to 58. In this individual the first resting stage (figs. 51, 52) was very marked, the outlines of the spheres became quite invisible, and the embryo could only be distinguished from the unsegmented egg by its slightly irregular outline. In this individual it is shown, further, that the spheres do not necessarily divide simultaneously, though this is usually the case. The sphere marked *a.* did not divide at the third general cleavage (fig. 55), but delayed until the next, or fourth, cleavage, when it divided into two spheres, *a.a.*

In these cases of slightly unequal cleavage it was in several instances observed that the smaller spheres sometimes increased considerably in size, after the cleavage was apparently complete, so as to reduce the inequality considerably. This is rendered possible, perhaps, by the circumstance to be afterwards described, that the earlier cleavages do not extend to the middle of the ovum, and the spheres are continuous at first with a central solid unsegmented mass. Or it may possibly be due to a re-arrangement of the material of the spheres, such as a change from a vertical to a horizontal elongation.

3. In the third form of segmentation to be described, of which a single case only was observed by Dr. WILSON, the egg divided at the first visible cleavage into thirty-two (±) spheres, passing over both the eight-sphere and the sixteen-sphere stages. The segmentation was somewhat unequal and became more so in later stages owing to the more rapid multiplication of the spheres at one pole. The egg developed perfectly, however, and produced a larva which appeared to be quite normal.

In the three forms of segmentation so far described, a certain number of individuals were observed to undergo considerable changes of form fifteen to twenty-five minutes before actual cleavage took place. The eggs became slightly irregular, with wavy outlines, as if about to segment; but within a few minutes they became again perfectly spherical, and remained so until the actual segmentation began. This was observed only once preliminary to the eight-sphere cleavage, and occurred in the single example of the thirty-two-sphere division. About one-fourth of those which divided at once into sixteen underwent the preliminary change.

There can be little doubt that these preliminary changes of form are attendant upon divisions of the nuclei. The egg appears to make an effort, so to speak, at cleavage, but has not sufficient energy to complete the division of the vitellus. I shall return to this point further on. It is perhaps worth noting that the interval (fifteen to twenty-five minutes) between the preliminary change and the first cleavage is nearly always considerably greater than the ordinary resting stages (eight to eighteen minutes).

4. In a fourth form of segmentation (figs. 19 to 24), of which a single example only was observed, the egg was divided at first into two equal parts by a horizontal cleavage, and then incompletely into eight by two partial vertical furrows at right angles to each other and to the horizontal furrows (figs. 19, 20). The vertical furrows started from the horizontal one at four equi-distant points, and travelled about half-way towards the upper and lower poles. They stopped abruptly at these points however, and the egg passed into a very marked resting stage (fig. 21), during which the form was nearly spherical and the furrows could only be seen at the points of union with each other. At the next cleavage the egg divided into about sixteen spheres of different sizes (see fig. 22). The spheres remained sharply marked and rounded for twelve minutes, then flattened together slightly, but five minutes later swelled slightly and each divided into two with beautiful regularity (fig. 24). The subsequent development was regular and normal.

5. In one case an egg was observed to divide into two nearly equal parts, and then passed into a marked resting stage (figs. 25–27). In several other cases eggs were observed divided into four equal parts (figs. 28 and 29). Unluckily, the subsequent development was not followed in either case, and I cannot state whether these eggs were normal. In any case they are interesting, as filling out the series of different modes of segmentation of which the eggs are capable. In view of the great variation which certainly does exist it seems not improbable that these forms are capable of normal development.

6. In the cases so far described, the entire mass of the vitellus segments at the same time or nearly so. In several instances, however, segmentation began at one pole of the egg, leaving a large mass undivided at the opposite pole. These eggs had exactly the appearance of undergoing a partial segmentation, like that of *Pyrosoma*, or some Teleostean fishes. Thus in the egg shown by figs. 59 to 62 segmentation began with the formation of four small spheres at one pole of the egg, which then passed into a very marked resting stage. At the next cleavage (figs. 61, 62) the unsegmented portion broke up into about twelve spheres, of which two or three were somewhat larger than the others (fig. 62). The egg is now closely similar to that shown in fig. 38 which was directly derived from the unsegmented egg, and its subsequent development calls for no remark. Figs. 63 to 67 represent a similar case. In this instance the

small spheres at the second cleavage gradually extended downwards, being successively constricted off from the unsegmented mass. The first resting stage (fig. 64) was much less marked than in the individual last described, and in some individuals of this type the first period of quiescence is not attended by any flattening of the spheres, though a considerable pause always follows the formation of the first four or five small spheres.

7. Lastly, I may describe a very peculiar segmentation, shown by figs. 68 to 72. The egg when first observed consisted of three large spheres and four much smaller ones. One of the latter soon divided, and the egg passed into a slightly marked resting stage (fig. 70). At the next cleavage both large and small spheres divided (figs. 71 and 72) without apparent regularity, and the inequality still remained marked. In later stages the spheres gradually became more uniform in size, the embryo developed normally, and on the following day the free-swimming larva could not be distinguished from those produced by more usual forms of development.

Review.

The egg may divide at the first cleavage into two, four (?), eight, sixteen, or thirty-two spheres, which may be equal or unequal in size. In some cases the egg undergoes a preliminary change of form some time before cleavage, without, however, dividing, and returning afterwards to a spherical form. The cleavage into eight parts may be irregular and incomplete, and at the next cleavage sixteen spheres are formed.

Cleavage may begin at one pole of the egg with the formation of four or five small spheres, and (usually) after a quiescent period the remainder of the vitellus breaks up at once or progressively into spheres of approximately the same size as those first formed, and the egg passes into the sixteen-sphere stage.

Lastly, the segmentation may be very irregular as well as very unequal, and follows no discernible order.

I have described the various forms of segmentation in what may seem wearisome detail, since the existence of so wide a range of variation in segmentation is quite unprecedented, so far as recorded observations show. In the eggs of many animals the course of the segmentation appears to be remarkably constant, and the various cleavages follow one another with almost mathematical regularity. So far as I am aware, BROOKS was the first to point out, in the case of the Oyster, in 1879, that the eggs of the same species, or even of the same individual, may normally undergo more than one mode of development. He described in the Oyster two forms of segmentation, of which one was clearly derived by an abbreviation of the other. Intermediate forms were not, however, observed, and the eggs could not be said to exhibit variation except in one definite direction. In *Renilla* the eggs vary in many directions, and the different forms of development must be due to varying structural arrangements within the egg.

This fact of extremely early variation is, I believe, one of great importance. It is evident that a structural variation in one of the segmentation spheres must make itself felt, to a greater or less extent, in the structure and development of the cells derived from it, and may therefore appear ultimately as symmetrical or correlated variations in the larva or adult organism.

Leptogorgia, like *Renilla,* is diœcious, and the eggs are fertilised in the water after their discharge from the parent. The eggs are slightly smaller (·30 millim.), and of a rosy tint, but are otherwise quite similar to those of *Renilla.* They are discharged in the same manner through the mouths of the polyps, and at the same hour of the day, viz., 6 A.M. Unlike *Renilla,* the eggs are discharged in small numbers only, each polyp producing, so far as could be ascertained, only two or three ripe eggs at a time. The polyps in all parts of the colony discharge their eggs nearly simultaneously.

The segmentation is closely similar to the most common mode of *Renilla,* but differs in some rather interesting particulars. Owing perhaps to the scarcity of material, variations in the segmentation were not observed; but only three or four eggs were kept under continuous observation from the time of fertilisation. In all these cases the egg underwent slight changes of form about an hour before the beginning of segmentation, returning afterwards to an almost perfectly spherical form. The interval between this change of form (which is undoubtedly, as in *Renilla,* the expression of an attempt at cleavage) and the beginning of actual segmentation is much greater than in *Renilla.*

At the first cleavage the egg divides into sixteen very distinct equal spheres, which soon flatten together very completely, and a strongly marked quiescent period follows, during which the embryo can scarcely be distinguished from the unsegmented egg. This continues for about twenty minutes, when the spheres again swell up and become very distinct but *do not divide.* This condition continues for several minutes when the spheres again flatten down, and a second resting stage occurs which is rather less marked than the first.

Unluckily, I did not succeed in procuring satisfactory sections of this stage, since the methods of hardening employed with *Renilla* proved useless for *Leptogorgia.* There is every reason to believe, however, reasoning from analogy, that this swelling of the spheres is accompanied by a division of their substance; and this division can only be in a plane parallel with the surface—in other words, it must be a delamination cleavage. The delamination in *Renilla,* as we shall see, takes place when the embryo consists of sixteen segmentation spheres, but with considerable irregularity, and I have not been able to connect it certainly with any external sign of activity. In *Leptogorgia* all of the spheres appear to divide at nearly the same moment, the delamination being nearly as regular as in *Gorgonia* or *Liriope.*

The egg shown in figs. 73 to 106 developed nearly in the manner just described, but with the important difference that the delamination cleavage appeared to take place when the embryo consisted of thirty-two (±) instead of sixteen spheres. When first observed the egg consisted of thirty-two (±) spheres (fig. 73), which afterwards flattened together very completely (fig. 74). Fifteen minutes later the spheres swelled up and became very prominent (fig. 75), but the embryo passed into another quiescent stage (fig. 76) without visible division of the spheres.

Hence it would seem that the period at which delamination occurs is not invariable, or it may be that it takes place at different periods in different parts of the same egg. The number of eggs observed was not large enough to determine this interesting and important point.

The remainder of the segmentation (figs. 77 to 85) is closely similar to *Renilla* and does not call for special remark. The periods of activity and quiescence alternate with great regularity, and have approximately the same duration as in *Renilla*. Although I have examined a considerable number of eggs (probably fifty or sixty), they were never found to consist of less than sixteen spheres or undergoing "partial" segmentation ; and although some inequality was observed, this was never so marked as in *Renilla*. It would therefore seem that the form of segmentation is more firmly fixed than in *Renilla*.

§ 2. *Internal phenomena of segmentation.*

a. The unsegmented egg.

In the fresh state no trace of a germinal vesicle can be seen in the unsegmented egg. A series of sections shows, however, that a large vesicle is present (fig. 86) containing a very distinct germinal spot. In immature eggs the vesicle lies near the centre of the egg, but in the ripe egg it is situated near the periphery of the vitellus. It is enclosed in a delicate but very distinct membrane, and has a somewhat reniform shape with the concave side turned outwards. The interior appears to be filled with a finely granular substance, which stains intensely. No protoplasmic reticulum can be seen, and if present its meshes must be of exceeding fineness so as to produce the appearance of a fine granulation. The germinal spot is of a rounded form and lies near the centre of the germinal vesicle. It is of high refrangibility and stains intensely with picro-carmine. Under a high magnifying power it is seen to consist of a lighter clear peripheral layer, enclosing a number of spheroidal bodies, which are separated by a reticulum of deeply staining substance.

The body of the vitellus consists of a fine protoplasmic network, closely packed with rounded granules of deutoplasm, which are scarcely affected by the staining fluid. A rather narrow peripheral zone of the vitellus does not take the staining fluid, and is of a more finely granular structure than the rest of the vitellus. This zone is faintly visible in fresh eggs when flattened under the compressor, and it persists until a late stage of the segmentation. In some preparations, however, it does not appear, and sometimes it is marked in one part of a section and invisible in other parts (see fig. 94).

As shown in fig. 86, the germinal vesicle lies in contact with this clear peripheral layer, which extends inwards slightly to meet it. In the section following that which is figured, the peripheral layer actually bends inwards so as to form a slight funnel-shaped depression leading inwards towards the germinal vesicle. Possibly this may be due to shrinkage; for I have never observed such a depression in fresh eggs. More probably it should be regarded as a kind of micropyle through which the spermatozöon enters the egg. The metamorphosis of the germinal vesicle, consequent upon fertilisation, was not followed.

As already stated, there is a considerable interval between fertilisation and the first visible cleavage. Sections through the egg show that, although the vitellus is apparently inactive, the nuclei are rapidly multiplying. The egg, which at first contains a single nucleus, becomes polynuclear and passes into the condition of a polyplast or syncytium, each nucleus corresponding to one of the future segmentation spheres, as shown by later sections.

I did not succeed in following completely the progressive multiplication of the nuclei, and can only assert that they become more numerous up to the time of cleavage, when each sphere contains a single nucleus. Many sections were obtained containing two nuclei, several with three, and a few in which four nuclei were visible. By making series of consecutive sections through the ova, it is possible to determine approximately the number of nuclei. I have thus observed the egg with four nuclei, others with four amphiasters, representing the multiplication of four nuclei into eight, and others containing eight separate nuclei. In others the number of nuclei is still greater, and in one case I was able to count sixteen nuclei, as described below. The nuclei do not always divide simultaneously, for I have, in several cases, observed eggs containing ordinary nuclei, and also typical amphiasters, with their characteristic spindles and star-shaped heads. In some cases an amphiaster and an undivided nucleus appear in the same section. In all these cases the eggs were perfectly spherical before treatment with reagents, and showed no sign of division.

Fig. 87 represents a section through a spherical unsegmented egg, a few minutes before its fellows divided into sixteen spheres. The irregularity of form is a result of shrinkage which, however, affects only the external form, leaving the substance of the vitellus uniform, and quite free from shrinkage cavities. This section is from one side of the egg, and contains two distinct amphiasters. Passing inwards, the second section contains two nuclei, and the third one. Four nuclei appear in the fourth, and four in the fifth, which is represented in fig. 88. Three of the latter are simple, while the fourth is elongated, and apparently about to become an amphiaster. Two nuclei appear in the sixth section, one in the seventh, one in the eighth, and a single amphiaster in the ninth and last. The four nuclei of sections 4 and 5 have the same relative positions, and are doubtless identical: the nucleus of No. 3 corresponds with one of No. 2; and one of No. 6 with that of No. 7. I find, counting each amphiaster as two nuclei, the egg contains fifteen nuclei, and, counting the elongating nucleus of No. 5 as two, we have a total of sixteen nuclei, corresponding with the number of spheres formed at the first cleavage.

The segmentation nuclei bear no resemblance to the germinal vesicle or egg-nucleus. When in the quiescent state they appear as intensely stained finely granular areas, shading off, insensibly, into the surrounding mass of the vitellus, and without enclosing membrane or nucleoli. In later stages, when the vitellus has undergone division, they sometimes appear as small vesicles, containing a clear substance, and a very deeply stained nucleolus. The yolk-granules are almost always disposed in radiating lines about the nucleus, but this appearance varies greatly, and is sometimes scarcely discernible. I have no new observations to offer on the phenomena attending their multiplication, since the abundance of deutoplasm obscures the structure of the nuclei and amphiasters. Nuclei were, however, observed in every stage of division, and I will briefly describe their transformation: The nucleus becomes slightly elongated, then decidedly so, and the radiate arrangement of the surrounding granules is very marked (fig. 88). In a slightly later stage, the nucleus has a dumb-bell shape, with the vitelline granules radiating from each extremity Still later, the typical amphiaster form is attained, with two deeply stained nuclear areas, surrounded by very marked radiating lines of granules, and connected by a striated spindle. The stars then move apart, the spindle becomes attenuated. In later stages, during the cleavage process, the body of the cell splits into two at this stage of the amphiaster, the line of division passing at right angles to the spindle, near its middle point. In the unsegmented egg, the two stars simply move apart, and the spindle entirely disappears.

After the division of the amphiaster is completed, the two new nuclei assume the ordinary appearance, and the radiating arrangement of the yolk-granules becomes less marked.

The nuclei are at first situated near the centre of the egg, but as the time for cleavage approaches they travel towards the periphery, where the first segmentation spheres are to make their appearance.

b. The cleavage process.

The first division of the vitellus (fig. 89) consists in the formation of rounded prominences over its surface, of which each contains one of the nuclei derived from the continued division of the segmentation nucleus, and is therefore the equivalent of a cell (*i.e.*, a segmentation sphere). These spheres are however entirely fused together, and there is at first no trace of lines of division between them. The egg is still a polyplast or syncytium though the vitellus is being acted upon by forces which tend to split it up into separate portions corresponding in number with the nuclei. It is important to note that whether eight or sixteen spheres are formed at the first cleavage, each contains a single nucleus only; for this shows that division of the vitellus does not always occur at the same stage in the division of the nuclei.

The prominences soon become very marked, increasing in size at the expense of the central mass and becoming at length of a pyriform shape. The egg then consists of a central unsegmented mass containing no nuclei, and surrounded by partly formed spheres, each containing a single nucleus and connected by a broad isthmus with the central mass. In some cases, at any rate, the embryo now passes into a resting stage, as shown in fig. 95. The spheres flatten together and are separated by very distinct narrow clear spaces, which terminate abruptly some distance from the centre, thus leaving the central mass quite unsegmented, and continuous with the mass of the partially-formed spheres. In the figure some of the spheres appear to be completely separated from the central mass, and this may perhaps be the case with some of them. Others however are certainly not separated from the central mass.

In the second stage of activity, or perhaps in some cases in the first, the spheres increase still further at the expense of the central mass, which becomes at length reduced to a very small remnant (fig. 92), to which the spheres are attached by narrow necks. Finally even this remnant disappears, and the completely formed spheres extend to the very centre of the embryo. A small mass of granular matter still remains in the middle of the embryo, and the spheres are attenuated at their inner ends (fig. 93). No segmentation cavity exists at this stage, but the inner ends of the cells soon become evenly rounded and a small segmentation cavity is formed (fig. 94), in which a quantity of granular *débris* usually remains. The spheres are destitute of cell-membranes, but are separated by a small quantity of intercellular substance. Their substance is completely similar to that of the unsegmented egg, the nuclei have the same appearance as in the latter, and are situated in the outer halves of the cells. The clear peripheral zone observed in the unsegmented egg is still very distinct in some specimens, but in others cannot be seen. It does not follow the lines of cleavage into the interior of the egg.

c. *Formation of the layers.*

The egg is now in the condition of a blastula in which the cells are not yet differentiated into ectoderm and entoderm. In the next change—which constitutes perhaps the most important epoch in the development of the larva—the ectoderm and entoderm are separated by a process of delamination; *i.e.*, the inner end of each sphere separates as an entoderm cell from the outer portion which remains as an ectoderm cell. A careful study of my sections taken in connexion with the external appearances, leaves no room for doubt that this is the mode in which the layers are separated; but it is clear that the cells do not in all cases perform the delamination cleavage simultaneously. On the contrary there appears to be much irregularity in this process, which is not surprising in view of the other remarkable variations in the segmentation which have been described. Thus, in the same specimen, some of the cells may be undivided and contain simple nuclei; others contain delamination amphiasters (*i.e.*, those whose long axes are radially directed); while others have completely divided into ectodermic and entodermic moieties. Moreover, delamination cleavages may be in progress in some of the cells, while in others the cleavages are taking place in vertical planes. This is shown, for instance, in fig. 99, where two of the cells (*a*, *b*) contain delamination amphiasters, as shown by the direction of their long axes, while a third cell (*c*) is about to divide in a vertical (or radial) plane, as shown by the position of the amphiaster.

This suggests the interesting question as to whether delamination cleavages really take place in all of the cells, or may not rather be limited to the cells over a certain area. My sections are inconclusive on this point, which is of great importance in its bearing on the mode of transition between the invaginate and delaminate modes of development. (See BALFOUR'S 'Comparative Embryology,' vol. ii., p. 280, and my paper on the early stages of some polychætous annelides in 'Studies from the Biological Laboratory of the Johns Hopkins University,' vol. ii., No. 2, 1882; compare also the very interesting observations of CLAUS on "Die Entwickelung des Aequoriden-Eies," Zool. Anzeiger, No. 112, June, 1882.)

In fig. 94 one of the cells (*a.*) is in the act of cleavage, and the direction of the amphiaster and the form of the cell indicate that the cleavage is in a horizontal plane—*i.e.*, is a delamination cleavage. Fig. 96 represents a section (osmic acid) through an egg, a little later, in which the inner portions of several of the spheres are separating, or have recently separated, as entoderm cells. Unluckily, the nuclei do not appear in the section, which is furthermore somewhat disfigured by shrinkage cavities.

In a few cases I have observed at a much earlier period divisions of the nuclei, which may possibly represent delamination cleavages. Such a case is shown in fig. 98. The egg is about to divide into sixteen spheres, but contains two amphiasters, which have the same position as the true delamination amphiasters already described. It seems possible that the inner star of each amphiaster is destined to form the nucleus of an entoderm cell, and the outer star that of an ectoderm cell when division of the vitellus takes place. I have not traced this out, however, and the appearance may be open to a quite different interpretation. It is, however, certain that there is a good deal of variation in the delamination process, and the embryos do not display the beautiful regularity in this respect which has been described in some Cœlenterate eggs. As already mentioned, *Leptogorgia* appears to differ from *Renilla* in this respect, since the period of delamination is sufficiently marked to produce a special "active stage," represented externally by the simultaneous swelling of all the spheres.

At the close of the delamination process, the egg consists of a solid mass of cells in which every trace of the segmentation cavity has disappeared. As shown in figs. 99 and 100, the ectoderm does not at first form a distinct layer, the cells dovetailing with those of the central entodermic mass. As the egg passes into the resting stage, however (fig. 97), the ectoderm becomes pretty well defined as a single layer of large cuboidal cells. The central mass is composed of large rounded polygonal entoderm cells, which differ little in structure from those of the ectoderm.

§ 3. *General considerations and comparison with other forms.*

With the formation of the germ-layers the segmentation may be regarded as finished, and it may be useful to review the facts in comparison with other forms, in order to appreciate their significance.

Examples of the continued division of the segmentation nucleus before cleavage of the vitellus are very common, but in most cases the nuclei become far more numerous before cleavage occurs than in the ovum of *Renilla*. In the case of the Isopod *Asellus* (VAN BENEDEN) the segmentation is entirely similar to one of the forms observed in *Renilla*, the nuclei multiplying to the number of eight, and the vitellus then dividing at once into eight spheres. In view of the total dissimilarity of the adult forms, this identity in segmentation is a striking instance of the independence of the yolk-cleavage from the adult structure; and it would be clear, from this case alone, that the particular form of the segmentation may be wholly determined by secondary or adaptive causes.

This fact is rendered especially conspicuous from the astonishing amount of variation shown in the *Renilla* segmentation. This variation concerns not only small details, but also features which are usually held to be characteristic of quite different types of development. Hence we can see how readily the form of segmentation might be acted on by natural selection, for advantageous variations would certainly tend to be preserved and harmful ones destroyed. It must, however, be admitted that the action of heredity appears to have little precision in this case, for the most unlike variations appear in the eggs of the same parent, and I have not observed that any particular variation occurs more frequently in the eggs of particular individuals.

We may now inquire, What is the direct cause of the variations in the yolk-cleavage? As we have seen, the nuclei divide, so far as can be determined, in the ordinary course, and sooner or later the vitellus follows. It is highly probable that the division of the nuclei is in all cases nearly regular, and the variations of the yolk-cleavage depend upon the varying activity of the vitellus, either as a whole or in its various parts. There seems to be always a tendency to the cleavage of the vitellus simultaneously with the division of the nuclei, but this tendency varies in force or meets with varying resistance. As described above, the vitellus seems sometimes to make abortive attempts to divide simultaneously with the nuclei, these efforts being expressed in temporary changes of form in the vitellus, but not resulting in complete cleavages. In other cases the attempt is partially successful, as where the vitellus divides incompletely into eight (fig. 45). Sooner or later the tendency gathers energy enough to carry out a complete segmentation. The egg may be able to do this at the first division of the segmentation nucleus into two, or may be unable to effect it until six successive divisions of the nuclei have taken place, and the egg therefore divides into thirty-two spheres at the first cleavage.

In searching for the cause of these variations in the activity of the vitellus, the idea at once suggests itself that it lies in the variations of the amount and distribution of the deutoplasm. It has been pretty clearly established by the researches of late years that the protoplasmic and deutoplasmic constituents of the vitellus are, in a certain sense, antagonistic to each other in their influence upon the rate of development. The protoplasm is the active part, while the deutoplasm, *as such*, is inert, and, until absorbed and converted into protoplasm, exercises a retarding influence upon development.

The egg of *Renilla* is heavily laden with deutoplasm spheres, which, as we shall see, long remain inert, and are not converted into protoplasm until a late stage of development. If we suppose—and the assumption appears fully justifiable—that the amount and distribution of the deutoplasm in the vitellus are subject to slight variation, most of the variation receives a simple explanation. It is, of course, possible, or even probable, that the activity of the protoplasm may vary also; but since the two constituents of the vitellus are, as it were, counterbalanced against each other, a variation in the amount or activity of the protoplasm must have the same effect as the opposite variation in the deutoplasm, and hence we may for the sake of simplicity consider the amount of deutoplasm alone.

The researches of FLEMMING, STRASBURGER, and others have within a few years clearly shown that the division of the nucleus of a cell produces, or is at any rate closely associated with, a tendency to division in the body of the cell. If then the deutoplasm of an egg be scanty, this tendency may be strong enough at the first division of the nucleus to overcome the inertia of the mass of the vitellus and the egg divides into two cleavage spheres at the start. This condition is permanently retained in the eggs of many animals, but in *Renilla* occurs only as a rare variation. With an increasing amount of deutoplasm, equally distributed, the cleavage of the vitellus is longer and longer delayed, though the ineffectual efforts of the vitellus may be expressed in slight changes in the form of the ovum.

Bearing these considerations in mind it is exceedingly interesting to compare the various modes of development of *Renilla* with those of other animals, and especially of certain forms existing among the Arthropods.

In *Lucifer*, as described by BROOKS (Phil. Trans., 1882), the egg is transparent and nearly destitute of deutoplasm. The segmentation is regular and total, the nuclei and bodies of the spheres divide regularly and simultaneously into two, four, eight, &c., as far as the segmentation can be followed, and the spheres remain perfectly distinct from one another. In *Palæmon*, described by BOBRETSKY (whose Russian paper I know only from German abstracts), the deutoplasm is abundant, but the segmentation is regular and total at first. Late in the development, however, the inner ends of the high columnar cells ("yolk pyramids") fuse together to form a homogeneous yolk. *Penæus* (HAECKEL) undergoes a similar development, but the fusion of the inner ends of the spheres occurs at a far earlier stage when only four spheres are formed. Whether these are at first distinct was not determined.

In *Eupagurus* and a number of other Decapods studied by MAYER (Jenaische Zeitschrift, Bd. xi., 1877) a curious condition exists which is intermediate between the preceding forms and *Renilla.* The nucleus divides regularly into two, four, and eight, but without a concomitant cleavage of the vitellus. After the formation of the eight nuclei, however, the vitellus divides into two, four, and eight complete spheres, each of which contains one of the nuclei. In the next stage sixteen spheres are formed, but their inner ends no longer extend to the middle of the egg, the spheres having fused to form a yolk-mass as in *Penæus* or *Palæmon.* Finally in *Asellus,* already referred to, the nuclei multiply as in *Eupagurus* to the number of eight, but the vitellus then divides *at once* into eight partially-formed spheres, without undergoing the previous divisions into two and four. This condition is characteristic of about one-third of the *Renilla* eggs, but in most cases the division of the vitellus is retarded until sixteen nuclei are formed.

In rare cases cleavage is delayed until thirty-two nuclei are formed, and here again we find that this condition, though a rare variation in *Renilla,* is permanent and normal in another group of animals, namely, the Araneina (HUB. LUDWIG, Zeitsch. Wiss. Zool., Bd. xxvi., 1876). In this well-known case the nuclei multiply to the number of thirty-two before the vitellus actually divides, though a partial segregation of its material is effected. From this condition the step is not great to the eggs of the Insecta and Acarina, where a still larger number of nuclei are formed before cleavage begins.

There can be no doubt that the regular division of the vitellus in geometrical progression into two, four, &c., spheres, is in general to be regarded as the most primitive mode of development, the process being only a special case of cell-division. In a number of polyps, both of the Alcyonarian and Actinarian types, as in *Clavularia,* or some species of *Actinia* (KOWALEVSKY), this primitive mode of development is still retained. Hence, if we regard the most frequent mode of segmentation in *Renilla*—namely, direct division into sixteen spheres—as the normal mode, the occasional division into eight, four, or two may be regarded as cases of reversion to conditions which were once the prevalent modes of development. On the other hand, the single observed case of division into thirty-two spheres shows that while the sixteen-sphere cleavage has been pretty well established, a tendency to further abbreviation still exists. We cannot doubt that if any change of condition should render a further concentration of development advantageous, this tendency or capability would come into play, and a segmentation like that of the Insecta might be produced.

As the various forms of regular cleavage may be explained as the result of variations in the amount of equally distributed deutoplasm (or in the activity of the protoplasm), so we may in part explain the various forms of unequal segmentation as due to variations in the distribution of the deutoplasm. As shown in figs. 55–57 certain spheres may be slower in their development than others, so that their descendants are larger, a fact long since observed by ALLMAN in Hydroid eggs. This is probably caused by the presence of a larger amount of deutoplasm than common, though possibly to the tardy division of the nuclei. In the forms of "partial" segmentation shown in figs. 59–67, the large unsegmented mass must contain a number of fully formed nuclei, since it breaks up almost at once into several spheres. Hence the delay is caused, apparently, by some obstacle in the vitellus, which we may suppose to be an especially great amount of deutoplasm in one half of the egg, as is normally the case in the entodermic pole of an epibolic gastrula. It is possible in this case also to suppose that the delay is due to tardy multiplication of the nuclei, but this explanation seems less probable than the other. In some cases the small spheres are gradually constricted off from the unsegmented part, and the egg may pass into a resting stage, leaving a number of spheres only half formed (see figs. 59, 60, 63, 64). This fact strongly indicates that there is some resistance to be overcome in the vitellus, for there can be little doubt that the half-formed spheres contain fully developed nuclei.

There are a number of other facts which point in the same direction. The first formed cleavage furrows penetrate very slowly towards the centre of the ovum and, in some cases at least, do not reach the centre during the first stage of activity. The segmentation is at first, therefore, of the type which BALFOUR has termed *centrolecithal,* the egg consisting of a peripheral layer of partially-formed cells and a solid central yolk-mass. The egg differs somewhat in structure, however, from a typical centrolecithal ovum; for the central yolk-mass, so far as can be determined, does not contain at this period a greater proportion of deutoplasm than the peripheral parts, though it does so at a later stage. The failure of the cleavage furrows to reach the centre of the egg seems to be due either to the resistance being greater in the central parts, or to the exhaustion of the energy of the protoplasm before the inertia of the entire mass of deutoplasm has been overcome.

It is interesting to compare *Renilla* in this respect with *Clavularia* on the one hand and *Alcyonium* on the other, as described by KOWALEVSKY. In *Clavularia* the resistance of the entire mass of deutoplasm would seem to be less than in *Renilla,* as the egg divides completely and regularly from the first. In *Alcyonium,* on the other hand, the resistance in the central mass is greater than in *Renilla,* and the segmentation does not affect the central portions of the egg for some time. Irregular protoplasmic protuberances separate themselves from the yolk to form segmentation spheres, which after a time arrange themselves in a simple regular ectodermic layer. The central mass remains, for a considerable time, quite unsegmented, but finally breaks up into large rounded entoderm cells. Hence it appears that the cleavages do not reach the centre of the egg until the delamination takes place; and in this case the cause seems pretty clearly to lie in the greater abundance of deutoplasm in the central portion of the egg.

The principle that unequal distribution of deutoplasm produces unequal rates of development in different parts of the egg, will not, however, account for some of the forms of unequal segmentation in *Renilla*. When, for instance, the egg divides into four larger and four smaller spheres, the former do not contain a greater number of nuclei than the latter, since at the following cleavage all are divided alike into two parts, and further, we have seen that the inequality existing at first may be considerably reduced without the occurrence of any visible cleavage. It is improbable that the cause is simply a lack of precision in the action of the vitellus, since the arrangement of the spheres is constant, so far as observed, the larger spheres being at one pole of the egg and the smaller spheres at the other. The resemblance of the egg at this stage to an epibolic gastrula has already been noted, and the idea naturally suggests itself that this resemblance may be due, not to accident, but to actual reversion of the gastrula *form*, though the essential features of the development are entirely different from those of the gastrula. There are a number of facts which indicate the derivation of the delaminate planula from an epibolic gastrula like that of *Euaxes;* and if the planula has had such an origin, it is not improbable that it might occasionally revert to the original unequal form of segmentation.

§ 4. *Changes of external form and further histological differentiation.*

At the close of segmentation the embryo is roughly spherical in form, varying considerably in outline. As development proceeds the body elongates slightly so that a longer axis (antero-posterior) can usually be made out, but the larvæ both of *Renilla* and *Leptogorgia* assume the most irregular and strange forms (figs. 100^a, 100^b, 100^c, 107). Occasionally a larva develops very regularly, preserving a nearly even oval outline until the cilia make their appearance. But in far the greater number large irregular prominences and depressions make their appearance over the whole surface of the embryo, and the form becomes so strangely modified that it is difficult to believe the shrunken and distorted larvæ capable of further development. In fact I unhesitatingly considered them at first as abnormal or dying specimens. No two of them have the same form, and they sometimes appear almost like huge *Amœbæ* with short rounded pseudopodia extended in various directions. Nevertheless the larvæ are perfectly normal, as I repeatedly proved by isolating them in small vessels and following their development. A regular oval form is once more gradually assumed (fig. 101), and most of the larvæ of twenty-four hours show no trace of the strange changes of form through which they have passed. The various prominences and processes are not capable of active movement, and the change of form is exceedingly slow. I am entirely unable to say what the significance of this curious change of form may be, and can hardly find a parallel to it in the development of other animals.

The rate of development varies exceedingly in different individuals, being sometimes twice as rapid as in other cases. In nearly all instances, however, the embryo acquires a dense and uniform covering of cilia when about twenty-four hours old, the body having meanwhile assumed an oval form. The cilia do not at first possess the power of movement, but in a few hours become actively vibratile and propel the larva through the water. As the cilia assume their functional activity the form of the body becomes pyriform, the future oral end being the larger. This form is sometimes marked in the larva of twenty-four hours (fig. 101), but the difference between the oral and aboral extremities is usually less conspicuous than in the specimen figured.

The swimming movements become very active in the thirty-six-hours' larva, and are very characteristic. The larva swims with the aboral end directed forwards, revolving at the same time on the longitudinal axis. The larger (oral) end simply revolves about its centre while the smaller end describes a circle, so that the larva advances by a kind of cork-screw movement. Many of the larvæ swim actively about, but most of them crowd to the surface, where they arrange themselves in rows about the edge with their smaller ends turned upwards and outwards, and the swimming movements entirely cease. A very similar habit was observed by Lacaze-Duthiers in the larvæ of *Astroides*.

By the end of the third day the body becomes elongated (fig. 103), and exceedingly contractile and changeable in form. The larva may be at one moment of a worm-like elongation and at the next instant contract to a short rounded form as in fig. 105. The cilia begin to disappear and the larva swims very sluggishly near the bottom of the aquarium. During the fourth day the cilia entirely disappear and the larvæ sink to the bottom, attaching themselves loosely by means, apparently, of a mucous-like secretion. The larval life is now ended and the tentacles and spicules soon make their appearance (see § 11).

Leptogorgia agrees in the main with *Renilla*, but the development takes place more slowly. The embryo, after passing through the period of distortion, becomes of a regular oval form and acquires a uniform coating of cilia. The aboral end soon becomes slightly smaller and the larva swims with the same peculiar cork-screw movement observed in *Renilla*. The larvæ have the same habit of arranging themselves in rows at the surface of the water. On the fourth day the larvæ are much elongated (fig. 112), and possess the power of active contraction. The larval life is not ended until about the sixth or seventh day, when the cilia disappear, the larva sinks to the bottom and once more assumes a short rounded form (fig. 113), and the eight septa become faintly visible about the eighth day. Some of the larvæ attach themselves firmly by the aboral end, but others remain free as long as they were kept under observation (seven weeks). In one case two larvæ, originally quite distinct, became attached to each other near their oral ends (fig. 114). The union became very complete in a day or two, and no line of division between them could be made out. The larvæ were kept for a fortnight, but underwent very little change, and finally died. I believe their union was due simply to accidental adhesion, and has no significance bearing upon the formation of the colony. Kowalevsky observed in *Alcyonium* that numbers of the larvæ fused together in a similar manner, but their subsequent history was not followed. It is very probable that in this case also the union was accidental and was produced by the crowding of the larvæ in small aquaria.

The formation of the septa and tentacles will be described in the following section, and we may now consider the internal histological changes which have been in progress during the stages just described.

At the close of segmentation, the embryo (fig. 97) is a solid planula consisting of a central mass of large rounded cells, enclosed by a layer of cuboidal ectoderm cells. As development proceeds, the cells of both layers continually decrease in size by multiplication, and those of the ectoderm gradually assume a marked columnar form. At the same time, the character of the cell-contents changes somewhat, the deutoplasm spheres disappearing from the ectoderm cells, which accordingly appear less coarsely granular, and remaining only in the central cells, where they continue to be very distinct, until a short time before the appearance of the digestive cavity.

The structure of an embryo of the stage superficially shown by figs. 12 and 13, is well shown in fig. 118. The section figured is from *Leptogorgia*, chosen on account of its good state of preservation; but it agrees in nearly all respects with sections through the corresponding stage of *Renilla.* The outer envelope consists of a single layer of cuboidal cells, in many of which are visible large rounded nuclei. The cells are destitute of membranes. Their contents are granular, but destitute of distinct deutoplasm spheres, and are scarcely stained by the picrocarmine. The peripheral zone of earlier stages is not visible, but in *Renilla* sections of this stage, it appears very clearly, as shown in fig. 119, taken from a somewhat later stage.

The central part of the embryo is occupied with a solid mass of large rounded entoderm cells, or, as they may for the present be called, *central* cells. The latter are enclosed by delicate but distinct membranes, which separate them sharply from each other, and from the surrounding ectoderm. Nuclei are visible in many of them; and some of the larger ones, being in course of division, contain two nuclei. The character of the cell-contents varies somewhat in different parts of the central mass. The more centrally placed cells are closely packed with clear spherules of deutoplasm left unstained by the carmine, between which is a kind of network of finely granular, deeply stained matter. The nuclei appear as clear vesicles, surrounded by deeply stained, finely granular areas. Passing towards the outer portions of the central mass, the deutoplasm spheres become less numerous, disappearing almost entirely in the outermost cells which adjoin the ectoderm.

It is important to notice this early differentiation in the distribution of the deutoplasm; for it indicates either that the deutoplasm is more abundant here even in early stages, when no difference between the central and peripheral parts of the egg is apparent to the eye, or that the protoplasm of the outer portions is more active, and hence assimilates more rapidly the deutoplasm. Either alternative lends support to the view suggested at p. 744, to account for the failure of the earlier cleavage furrows to reach the centre of the egg.

A comparison of figs. 118 and 97 shows that the central mass in the later stage is somewhat greater than in the earlier; and an examination of a number of sections indicates that this difference is a constant one. Hence it seems probable that more than one delamination cleavage may take place, that the central mass may from time to time receive accessions from the outer layer through the occurrence of horizontal cleavages. I have not been able to demonstrate this, though some of my sections give indications of such a process. In some cases the ectoderm cells appear elongated, and as if about to divide in the horizontal plane. It is certain, as will subsequently appear, that such cleavages occur in the ectoderm until a late period, though in later stages, when the supporting lamella is formed, the cells thus produced remain, of course, ectodermic. There seems to be no reason why such cleavages occurring at an early stage should not produce entoderm cells, and such, I am inclined to think, is actually the case.

After the stage shown in fig. 118, however, the cleavages take place for a considerable period mainly in vertical planes, so that the columnar form of the ectoderm cells becomes more and more marked.

The structure of the embryo may be far less regular than is indicated by fig. 118, since the cells often multiply more rapidly over one half of the embryo, and the division of the central cells is often irregular.

Sections through the *Renilla* embryo of about four and a half hours are represented in figs. 119 and 120. The embryo has the same general characters as in the last stage figured, but the cells have largely increased in number. The ectoderm cells have a definitely columnar form, and consist of a granular substance which is not, apparently, enclosed in cell membranes. They are separated by narrow, clear spaces which contain, apparently, a small quantity of intercellular substance. The central cells, on the other hand, are surrounded by definite membranes, which appear in the sections as narrow, dark lines.

There is still no indication of a definite membrane separating the ectoderm from the central mass. The cells of the two layers are to some extent dovetailed together, and have nearly the same structure. Here and there in the ectoderm are rounded or pyriform cells which appear to be in course of division. The clear peripheral zone still appears distinctly at the outer ends of the ectoderm cells. In some specimens it bears a fringe of fine filaments which appear like cilia, but are in reality the remnants of the spermatozoa with which the embryo remains covered for a considerable period.

In the embryo of eight hours (figs. 122–124) the ectoderm layer is sharply differentiated from the central mass, but the latter has undergone very slight change except in the further division of its cells. The ectoderm cells have now a high columnar form, though here and there rounded cells may be observed (fig. 122). At their inner ends, where they are usually somewhat expanded, they abut against the cells of the central mass, and in many parts of the sections are separated from the latter by an irregular, scarcely defined membrane (this is represented as too distinct in the figures). In some sections the membrane does not appear. This membrane appears to be the first rudiment of the supporting lamella ("Stützlamella") which is so characteristic a structure among the polyps and hydroids, but the main body of the lamella is formed somewhat later, as described in the next section. The ectoderm cells now stain very differently from those of the central mass, and the peripheral zone has disappeared. The substance of the cells appears scarcely coloured, while the nuclei are deeply stained. The central cells, on the other hand, stain deeply, so as to be very sharply differentiated from the ectoderm in colour as well as in form. The deutoplasm spheres have the same distribution as in earlier stages, being very abundant and clearly defined in the more centrally placed cells (fig. 124) and becoming indistinct and scanty, or quite disappearing in the outer cells. There is still not the least trace of a digestive cavity.

I am unable to say what the significance of the peripheral zone may be. A very similar zone is described by HOFFMANN in the ectoderm cells of *Tetrastemma* at an early stage and by RABL in the ectoderm of *Unio*. It is possibly concerned in the formation of the cilia, but this seems improbable, since it disappears in *Renilla* long before the cilia are formed.

In this stage the sections of the embryo sometimes appear exactly as if taken from an epibolic gastrula, for the ectoderm cells may be very different both in size and form on opposite sides of the embryo. This appearance is however entirely deceptive, and is produced simply by the tardy multiplication of the cells over one half of the embryo.

§ 5. *Formation of the entoderm and appearance of the digestive cavity.*

The embryo has thus far remained quite solid with no trace of a digestive cavity. For some hours longer this condition continues, the only change consisting in the multiplication of the cells of both layers. About the twentieth hour, however, or at the time when the cilia make their first appearance, peculiar changes become evident in the central cells which are the forerunner of the formation of the proper entoderm and the stomach cavity. The deutoplasm spheres disappear completely from the central cells, which then have a coarsely and irregularly granular appearance with very distinct membranes and deeply stained nuclei. The central mass is still solid, however, and the cells are all of the same irregularly rounded form. In a few hours a very perceptible difference can be seen between the outer and the inner cells of the central mass. Those which lie just beneath the ectoderm (figs. 125 and 126, *en.*) become much clearer, their substance stains very little, and many of them assume a slightly columnar form. Their nuclei are very distinct, of a slightly oval form, and very deeply stained. The cells are in some parts of the larva arranged in a single layer, but in other parts seem to be placed two or more deep. It is difficult in this, as in subsequent stages, to say whether this appearance of several layers in the entoderm may not be due to the sections being always more or less oblique in different parts of the section; but I believe, after examination, that they do form several layers in some parts of the embryo at this stage. This layer of clear cells is the permanent or true entoderm.

The central cells, on the other hand, remain rounded and very granular, and stain more deeply than the entoderm cells. They become ultimately disorganised and are absorbed as food by the true entoderm cells; hence they may hereafter conveniently be termed the *yolk-cells.*

The yolk-cells form at first a solid mass which is directly continuous with the entoderm cells surrounding it. Soon, however (fig. 125), the yolk-cells become more loosely connected, and considerable cavities appear in the central mass in which the yolk-cells often lie quite disconnected from the other cells or united in groups of two or more cells. It is difficult to gain a clear idea of the changes which bring about this condition. Apparently the entire larva increases somewhat in size while the membranes of the yolk-cells become partially disorganised. In parts of the yolk-mass the cell-contents with their nuclei seem actually to drop out of the cell-membranes, which remain as a delicate network (fig. 126) in which the form of the cells is still perfectly preserved. Possibly this occurs only as a result of rough handling after the sections are made. Still I believe it may be in part a normal occurrence and that some at least of the free naked cells in the yolk-cavities may have been thus liberated. Others of the free cells have at first delicate cell-membranes, but these afterwards disappear.

The yolk-cells are rounded, but vary greatly in form and size. Most of them are still distinctly nucleated, but the nuclei are less sharply defined and have less regular outlines than those of the entoderm cells. The cell-substance contains no deutoplasm spheres, and consists of a granular substance which stains irregularly and in some places not at all. Besides the yolk-cells there is a considerable quantity of granular substance in the form of small balls or masses lying in the yolk-cavities, and here and there may be seen a deeply stained free nucleus surrounded by a small quantity of granular matter. It is probable that the granular matter is derived from the breaking down of the yolk-cells, but it is difficult to say how far these appearances are the result of normal phenomena of disintegration, and how far due simply to mechanical injuries produced by manipulation. The general features of a section of this stage (twenty-two and a half hours) are well shown in fig. 125.

Still later the yolk-mass becomes completely disorganised, breaking up into a kind of *débris* in which several distinct elements can be recognised (fig. 127). There are: firstly, rounded cells with distinct nuclei and membranes, which are simply free yolk-cells; second, similar but usually smaller cells which have no membrane; third, free nuclei which are usually associated with a small quantity of granular matter; fourth, small rounded granular bodies, about one-fourth the size of the yolk-cells and destitute of nuclei; and fifth, still smaller granules, apparently produced by the disintegration of the preceding.

The entoderm (*en.*) has now a very different appearance from that of the last stage. The cells are columnar, with very distinct oval nuclei, which are always situated in the outer half of the cells; but the cell-contents are dark and opaque, being densely packed with granules. The cell-outlines are thus more or less obscured, and though always distinct towards the outer part of the cell may be quite invisible towards the base. The granulation is of quite different appearance in the inner and outer parts of the cells. In the basal (*i.e.*, outer) part the granules are fine and closely packed, and are left nearly or quite unstained, while in the inner ends of the cells the granulation is coarse and irregular, and stains more readily. This difference is so constant that in most specimens the basal granulation forms a pretty distinct narrow zone extending around the entire entoderm. The cells are in some parts of the sections only one layer deep, but in other parts the entoderm consists of several layers and varies greatly in thickness.

The general features of a twenty-nine-hours' larva are shown in fig. 128, and a portion of the body-wall, more highly magnified, in fig. 129. The body is now distinctly elongated and the oral end can be distinguished by its greater size. The entoderm is composed of high columnar cells, and is everywhere much thicker than the ectoderm. The gastric cavity is clearly defined and the yolk-mass is greatly reduced in bulk. Under a high power (fig. 129) the yolk-cells are found to have nearly disappeared, though here and there one may still be recognised. The yolk is almost entirely composed of the naked granular spheroidal bodies described in the last stage. They vary a good deal in size, but are on the average rather larger than the nuclei of the entoderm cells. They appear to have had their origin in the breaking up of the yolk-cells, though some of them are perhaps small yolk-cells which have lost their nuclei.

The entoderm cells (*en.*) are much elongated and present some interesting characters. Towards their bases they are filled as before with fine granules, which stain very slightly, and are arranged in a distinct zone encircling the entire larva. Their inner portions (apical) present a confused coarsely granular appearance, entirely unlike that of the basal granulation. The cells seem to contain rounded granular masses, which have the same appearance as the smaller spheroidal bodies of the yolk, which lie outside the cells in the stomach cavity. As in the last stage, the granules are so abundant that it is difficult to make out the outlines of the cells, which only appear clear and well defined at their inner ends.

The yolk gradually disappears as development progresses and the larva rapidly increases in size. Much variation exists in the length of time required for absorption, but it is always complete, so far as I have observed, by the forty-eighth hour, and the gastric cavity is left empty, or sometimes containing a small quantity of a delicate *débris*, which appears to be the remains of the membranes of the yolk-cells. After the yolk is completely absorbed the contents of the entoderm cells again change their character, as shown in fig. 130 (fifty-two hours). They become once more clear, and the basal granulation nearly disappears. The cells still contain a considerable quantity of granules, but the contrast with the preceding stages is marked. The cell-outlines consequently become much more distinct. The nuclei are deeply stained, very conspicuous, and are situated always in the inner parts of the cells.

Conclusions.

Although I have made many sections of larvæ prepared by various methods while yolk-absorption was in progress, I have failed to obtain decisive evidence as to the precise *modus operandi* by which the yolk-cells or their remains are absorbed by the entoderm cells. This failure is due to the excessive minuteness and delicacy of the tissues which renders it extremely difficult to make satisfactory preparations of them. But a careful study of the sections inclines me to the belief that the smaller particles of the yolk-*débris* are engulphed bodily by the entoderm cells Amœba-fashion, the process of digestion being completed within the body of the cell : that, in other words, the young *Renilla* is nourished by a form of intra-cellular digestion. As we have seen, the cells are at first clear and nearly destitute of granules. They become granular, however, and increase in size as soon as the disintegration of the yolk-cells begins, and their granular appearance continues until the absorption of the yolk is completed, when they become again clearer. The large, coarse granules in their inner ends (*i.e.*, those turned towards the yolk-mass) have the same appearance as the small yolk-granules lying just outside the cells, and the entoderm often contain rounded granular masses which are very similar, though with less distinct outlines, to the yolk-spheroids of the digestive cavity. The spheroids may often be observed to lie directly upon the entoderm cells, and the inner ends of the latter are sometimes produced into small amœboid processes reaching out into the digestive cavity, though this is rare.

These appearances suggest, though they do not prove, that the yolk-granules and spheroids pass bodily into the cells. I have never seen them in the act of passing into the cells, but the technical difficulties are great, and the other considerations seem of sufficient weight to warrant the provisional acceptance of the view advanced above.

This conclusion, if well-founded, is of interest in connexion with recent discoveries in regard to intra-cellular digestion in Cœlenterata and Turbellaria. The occurrence of such a form of digestion in the sponges has long been a familiar fact, and the more recent researches of METSCHNIKOFF, CLAUS, GEGENBAUR, PARKER and RAY LANKESTER have shown that an essentially similar mode of digestion occurs in the adults of many Cœlenterata belonging to the higher groups, namely: in *Hydra*, Hydroid polyps, Hydromedusæ, Acalephs, Actiniæ, Ctenophora and Siphonophora. METSCHNIKOFF showed in 1878 that the same remarkable process takes place in a number of fresh water Turbellaria, and he has ascribed to it an important phylogenetic significance. He points out the fact that intra-cellular or amœboid digestion is confined, so far as known, to the most primitive groups of the Metazoa and in the Cœtenterates seems to be the normal and most frequent if not the only process. The digestive functions of an entoderm cell in these cases are identical with those of a unicellular Protozoa, and METSCHNIKOFF is inclined to consider the former as an actual survival of the latter—a physiological character which was originally present in all Metazoa then existing, and has only been lost in higher forms. LANKESTER even suggests that the absorption of unsaponified fats in the highest Metazoa may possibly be a last relic of the primitive mode of digestion.

None of the writers on this subject have pointed out the identity of the process with that of the absorption of the yolk in *Astacus*, described by REICHENBACH in 1877 (Zeitschrift für Wiss. Zool., Bd. xxix., 1877). In this case the amœba-like action of the entoderm cells was observed with the greatest clearness. The cells put forth large pseudopodia, and actively engulph the yolk-granules which were observed in every stage of the passage from the yolk-mass into the cell-bodies. The ingestion takes place, it is true, at the basal instead of the apical end of the cells, since the yolk lies outside the archenteron; but this circumstance does not tell against the identity of the process with that of adult Cœlenterata and Turbellaria and of the larval *Renilla*. WOLFSON has observed a similar process in the yolk-absorption of *Lymnæus*, and in this case the nutriment, as in *Renilla*, is contained within the archenteron (see Bulletin de l'Académie Impériale des Sciences de Saint-Pétersbourg, tom. xxvi., pp. 79–99, 1880. Lu le 9 Octobre, 1879).

It is interesting to find the embryonic entoderm cells exhibiting this primitive mode of digestion, though it is clearly to be regarded simply as an adaptation connected with the presence of a large amount of food-yolk. Still the idea is suggested that the amœba-like ingestion of food in the larva may perhaps be due to a kind of reversion, the reappearance in the larva of a feature which, in the case of *Astacus* and *Lymnæus* at least, has become quite dormant in the adult. Whether it exists in the adult *Renilla* I have been unable to determine, but it cannot be observed in the young transparent colonies (see p. 786).

Whatever be the mode of absorption, the granular basal zone, so often referred to, appears to be a reserve store of food-material—either the actual remains of the ingested food-granules, or a new store of granules laid up for future use by the protoplasm after being richly fed. It would seem that the cell packs away its reserve supply of food in its basal part, leaving the apical or inner end free to continue the active work of feeding; so that there is in a sense a physiological division of labour within the cell. It is noteworthy that the entoderm nucleus is invariably situated in the inner part of the cell which contains the coarser granules. This position of the entoderm nuclei appears to be not uncommon in embryos where the gastric cavity is filled with food material (compare *Lumbricus*, *t.* KLEINENBERG, and *Planorbis*, *t.* RABL); and RABL has suggested in the case of *Planorbis* that they play a part in the absorptive activity of the cells. I have not been able to discover any such function in the nuclei.

§ 6. *Comparison with other forms.*

Upon comparing the formation of the digestive cavity in *Renilla* with that of other Anthozoa, we find, in some cases, a close agreement, but in other cases the phenomena are entirely different. All of the Alcyonarian forms, so far as known, excepting *Monoxenia*, agree in their general features with *Renilla*, developing as solid delaminate planulas, in which the gastric cavity is hollowed out by the disintegration and absorption of a central mass of yolk-cells, and the latter are at first indistinguishable from the true or permanent entoderm cells. *Gorgonia*, according to KOWALEVSKY, is an exception to the rule; for the embryo contains a central cavity surrounded by a layer of ciliated rounded cells, which are in turn enclosed in a layer of columnar true entoderm cells. The ciliated cells are believed by KOWALEVSKY to be absorbed, and are considered as homologous with the yolk-cells of other forms. It is noteworthy that *Leptogorgia*, though far more nearly allied to *Gorgonia* than to *Renilla*, agrees entirely in development with the latter, and does not have a permanent segmentation cavity.

Among the Zoantharia, the greater number of forms agree with the Alcyonaria in developing as solid delaminate planulæ in which the gastric cavity is formed by the absorption of a central yolk-mass. A few forms, on the other hand, viz.: *Cerianthus* (KOWALEVSKY, JOURDAN), *Actinia equina L.* (JOURDAN), and perhaps an allied *Actinia*, and probably *Caryophyllium* (KOWALEVSKY), develop as invaginate gastrulæ. BALFOUR states, on the authority of KLEINENBERG, that in some of the apparently delaminate types the segmentation is unequal, which "probably indicates an epibolic gastrula." While the occurrence of epibolic gastrulæ among these forms is by no means improbable, it cannot be accepted on this evidence alone; for the segmentation of *Renilla* shows that such an inference may be entirely false.

It is a curious fact that in two at least of the invaginate forms, viz.: *Actina equina* (JOURDAN), and *Cerianthus* (KOWALEVSKY), a yolk-mass is formed in the gastric cavity some time after the invagination has occurred, though no traces of it exist in earlier stages. Thus, of the former species JOURDAN states: "L'espace entre les cloisons est toujours occupé par une masse probablement vitelline, et qu'on croirait exsudée des tissus de la larve; cette masse nutritive est formée par de grosses vésicules semblables à des cellules adipeuses et par des noyaux fortement colorés par les réactifs."* KOWALEVSKY regards the yolk-mass of *Cerianthus* as a secretion of the deeper layers of the entoderm, and considers its elements as fat globules. In both cases the yolk-mass is eventually resorbed. If the origin of the yolk-mass is correctly described by these eminent observers, it is clearly not homologous with that of the Alcyonarian forms.

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* Ann. d. Sci. Nat., 6^{me} série, tome x., p. 129.

The phenomena of the yolk-absorption have not been carefully studied, and it is therefore impossible to draw any general conclusions in regard to the significance of the processes described for *Renilla.* An examination of JOURDAN's descriptions and figures of the larvæ of *Balanophyllia regia* (GOSSE) leaves in my mind little doubt that in this case also the yolk is ingested *Amœba*-fashion by the entoderm cells, though JOURDAN himself puts an entirely different interpretation on his own observations, as may be seen from the following extracts. He says of the entoderm cells at an early stage (*l.c.,* p. 134): "Elles sont très volumineuses, allongées, contiennent des nucléoles fortement colorés par les réactifs *et de grandes vésicules hyalines.* Au centre de la masse vitelline constituant l'endoderme, ces cellules disparaissent, les vésicules hyalines persistent seules" (the italics are my own). In later stages, when six or more septa have appeared: "Sur les coupes transversales, les grandes cellules situées au bord externe de l'entoderme des larves vermiformes ont disparu; les vésicules hyalines persistent et forment la totalité de la masse entodermique." In later stages, however, when the yolk is absorbed, as shown by his figure of the adult entoderm (*l.c.,* fig. 110, plate 15), the cells come into view again, having the same form as before, but rarely containing the "vésicules hyalines."

It appears in the highest degree improbable that the entoderm cells should completely disappear to be subsequently re-developed in precisely the same form. A far more credible conclusion is that the yolk-vesicles are taken bodily into the cells in such numbers as finally to obscure the cell-outlines entirely. The entoderm then seems to have disappeared and only makes its re-appearance when the yolk-vesicles have been assimilated by the protoplasm of the cells. This conclusion is strengthened by the fact that in JOURDAN's figure of the larva of *Actinia equina* (*l.c.,* fig. 119, plate 16) the entoderm cells are figured, before the absorption of the yolk has begun, as clear, well defined, and destitute of yolk-vesicles, while the gastric cavity is completely filled with "vésicules hyalines" precisely like those of *Balanophyllia.* This condition, according to my view, precedes one like the earlier stage of *Balanophyllia* in which absorption has recently begun and in which the entoderm cells resemble those of *Astacus,* as figured by REICHENBACH.*

In all known Alcyonarians the central mass, though at first unsegmented, does sooner or later divide into cells, although many of these perform no active function, become disorganised, and serve only as food for other cells. This indicates that the yolk-cells are the descendants of cells which were once of structural significance; for otherwise their formation and subsequent disintegration would seem to be a sheer waste of energy. They are identical in origin and structure with the permanent entoderm cells, and are undoubtedly homologous with the latter. Hence we may infer that the yolk-cells were originally functional entoderm cells in which deutoplasm accumulated to such an extent that they became devoted solely to the storing of food for the embryo. The remaining entoderm cells retained their functional

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* Zeitschr. für Wiss. Zool., Bd. xxix., 1877.

activity as digestive cells, and by an early development of this function in the embryo became capable of digesting the yolk-cells precisely as if the latter were foreign food-matters introduced through the mouth. How such a two-fold specialisation of the entoderm cells was possible is shown in the embryo of *Gorgonia;* for in this case the yolk-cells still persist in an apparently functional state, being ciliated and surrounding a central cavity. Only a step before this is the planula of *Gorgonia* or *Liriope* (METSCHNIKOFF and FOL) in which the central cavity exists from the first and all the delaminated entoderm cells persist as such.

If we push this speculation further and inquire after the causes which originally determined that some of the primitive entoderm cells should persist as such while others became yolk-cells, we encounter a very broad question, which it would be hardly profitable to enter upon here, since it belongs too exclusively at present to the region of pure speculation. The question is of the same nature, for instance, as that concerning the influence which determines the survival of a particular cell of the germinal epithelium of the ovary, as an ovum, while its neighbours are absorbed, or remain as simple epithelial cells.* We can only say that the differentiation probably stood in some relation with the relative position of the cells; for only the peripheral cells persist as entoderm cells. This suggests that the divergence may have depended upon, or is at least now directly determined by, differences in the supply of oxygen afforded to the cells—in other words is due to respiratory differences. The peripheral cells being nearer to the exterior, must command a more plentiful supply of oxygen, and in this respect have a decided advantage over the inner cells. This may be enough to determine the survival of the former and the disintegration of the latter.

According to a theory of WEISSMANN'S, the cells of the ovary (in *Leptodora*) attain a certain "maximal development," which is a critical point in the life of a cell. If it receive an additional impulse, though a very slight one, it continues to develop into an ovum at the expense of its less fortunate neighbours. If, on the other hand, it does not receive this impulse, the cell loses its power of development and is absorbed by the developing ova. The determining impulse is believed by WEISSMANN to be a slight advantage of nutrition which is potent because acting at a critical moment. Such a theory of "maximal development" would seem to apply well in the present case, but the impulse to development does not seem to be in any way connected with general nutrition but only with the supply of oxygen. The theory, though resting perhaps on a rather slender basis, has the merit of showing how a very slight difference in the supply of oxygen might determine the survival or the degeneration of the cells.

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* See on this point WEISSMANN, "Ueber die Bildung von Wintereiern bei *Leptodora hyalina*," Zeitz. f. Wiss. Zool., Bd. xxvii., 1876, who has given an elaborate discussion of the question in the case of the ova of the Cladocera.

§ 7. *Changes in the ectoderm and formation of the supporting lamella.*

In the larva of eight hours, as already described, there is a delicate sinuous membrane lying between the ectoderm and entoderm, upon which the cells of the former are planted as upon a basement membrane. This is perhaps the first beginning of the characteristic supporting lamella, but it is far less well defined and less conspicuous than in later stages, and the great bulk of the lamella is formed somewhat later by a peculiar transformation of the inner ends of the ectoderm cells. It is difficult to determine the origin of this preliminary membrane, but appearances indicate that it is secreted by the expanded bases of the ectoderm cells. The membrane varies much in appearance and is sometimes quite invisible even in much later stages. It is often apparent in one part of a section and quite invisible in other parts, while the true lamella, once formed, is remarkably constant and distinct.

The ectoderm cells of this stage have a high columnar form, which, though ultimately lost, is retained throughout the succeeding stages until a late period. At intervals, however, the cells rapidly proliferate (fig. 131, twenty-eight hours), and the columnar form may at these times be temporarily lost, the cells assuming various rounded forms and becoming in many cases entirely separated from the underlying entoderm cells. The division of the cells takes place both in horizontal and vertical planes, so that the ectoderm gradually becomes several layers deep. At the close of a period of proliferation most of the cells resume the high columnar form, some of them extending through the entire thickness of the ectoderm, others extending inwards from the surface and terminating by attenuated extremities without reaching the entoderm. Others, again, are placed with their broader end—which contains the nucleus—lying near the bottom of the ectoderm, and others still are of a fusiform shape with the thickest part containing the nucleus, near the middle of the ectoderm. The structure of the ectoderm at this stage is very like that of *Heliopora* (MOSELEY, Phil. Trans., Vol. 166, 1876).

Besides the columnar cells there are others of a rounded form with centrally placed nuclei, which lie in the deeper parts of the ectoderm or in the narrow clear space which often separates the layers; they often lie directly on the outer ends of the entoderm cells. These never return to the columnar form and persist throughout the entire development. They give rise to elements of the so-called mesoderm, some of them becoming the matrices for the development of the spicules, and others remaining as peculiar rounded cells which are possibly nerve-cells.

In the larva of about twenty-two hours (figs. 125, 132), the basal ends of the columnar ectoderm cells undergo a peculiar change of form and structure. They separate completely from the entoderm, become smoothly rounded, the character of the granulation changes, and they stain less readily than before. At the same time a large quantity of a finely granular substance makes its appearance in the space between the ectoderm and entoderm (figs. 125, 133). This space is sometimes very wide on account of the shrinkage of the central mass, but even in this case is sometimes nearly filled with the granular matter. The appearance of the granular mass varies greatly in different specimens and in different parts of the same section. It may be very abundant and of a loose flocculent character in one part, while elsewhere it gradually disappears and is replaced by a definite membrane lying between the ectoderm and entoderm, which is unmistakably the supporting lamella. In favourable specimens the granular mass may be traced around the section, becoming more and more closely compacted until it passes directly into the supporting lamella.

These facts leave no doubt that the supporting lamella is derived from the granular mass, which becomes compacted together to form a definite membrane. The granular mass probably never has naturally any considerable thickness, being compacted into the membrane as soon as it is formed. The loose flocculent character is probably produced by the action of the reagents which causes the material of the supporting lamella to swell up, while the central mass at the same time shrinks away from the ectoderm, forming the cavity in which the granular mass lies.

In fig. 133, which will illustrate the appearance of a section at this period, there are parts of the section where neither granules nor lamella appear, other parts where the outer ends of the entoderm cells are covered only by their own cell-membranes outside of which is a small quantity of granular matter; while in other portions a pretty distinct lamella is formed with abundant granular matter outside of it. The entoderm cells show no change at any time during the formation of the lamella. This indicates that the ectoderm alone is concerned in the production of the granular matter which forms the lamella, and this conclusion is confirmed by a study of the ectoderm cells. The inner ends of the cells are rounded and swollen and often terminate in knob-like swellings (fig. 132, *b.*), attached to the bodies of the cells by narrow necks. These swollen inner ends then separate from the bodies of the cells and lie in the deeper parts of the ectoderm or in the space between the two layers. In some specimens, of which fig. 132 is a good example, the lower part of the ectoderm is closely packed with these rounded bodies, of which some are still attached to the cells, but most are free. The substance of which these balls are composed is quite like the granular substance, and the balls may be seen in various stages of disintegration.

Hence we may conclude that the material of the supporting lamella is derived from the disintegrated granular balls which have separated from the ectoderm cells. It is possible that the swollen inner ends do not normally separate bodily from the ectoderm cells and that this is accidentally done in making the sections. This is hardly probable, however, since the outlines of the granular masses are usually regular, and they are found free in large numbers. The granular bodies appear in some cases to discharge their contents without breaking down and losing their form. I conclude this from the occasional presence of clear, rounded bodies in the granular mass of the same form and size as the granular bodies. These clear bodies (several of which are shown in fig. 133) appear to be surrounded by delicate membranes and become in some cases incorporated into the substance of the supporting lamella. They are not to be confounded with the rounded nucleated cells which are sometimes also found in the granular mass and may likewise become incorporated into the lamella. Besides the latter, cellular elements derived from the entoderm may in some cases enter into the composition of the lamella. Now and then an entoderm cell (see figs. 125 and 132) may flatten down, become incapable of development and become incorporated with the mass of the lamella.

After the secretion of the granular matter is completed the cells re-assume their high columnar form with their inner ends often resting upon the lamella as on a basement membrane (figs. 127–130). The latter appears as a very distinct, narrow, structureless membrane sharply separated from the ectoderm and entoderm. Outside of it is usually a narrow, clear space, but the granular matter has entirely disappeared. This condition of the ectoderm is maintained until the sixtieth or seventieth hour, when the ectoderm totally changes its character. The lamella remains unchanged up to the latest stage of the colony observed, without increasing in thickness or undergoing visible change of structure.

Review.

The material of the lamella is derived from the cells of the ectoderm by a peculiar form of cuticular secretion, which consists in the separation of rounded granular masses from the inner ends of the cells. The formation of these bodies is a process entirely different from cell-division since the nuclei do not divide, and they remain quite unchanged during the process. The granular bodies in most cases disintegrate, but sometimes appear to discharge their contents as in ordinary secretion. Possibly the ectoderm cells may also in some cases discharge the contents of their swollen basal ends without the separation of a part of the cell, but this must, I believe, be exceptional. The mode of secretion described is a very anomalous one, and appears to stand midway between the disintegration and discharge of an entire cell during secretion, as in the formation of "goblet-cells" in mucous glands, and the more usual forms of secretion in which the product exudes from the cell without the destruction of the latter.

The formation of the supporting lamella in other forms has not been worked out with sufficient care to afford any basis for comparison. KOWALEVSKY concluded that the lamella in *Actinia* is secreted by the entoderm, since it penetrates into the septa, which are entirely entodermic. This is, however, an unwarrantable conclusion; for, as will be shown later, the lamella of the radial septa has an entirely different origin from that of the peduncular septum, and both differ in origin from that of the body-wall. It is evident that the supporting lamella, though probably containing cellular elements derived both from the ectoderm and entoderm, is not in any sense a special mesodermic layer, but has only the significance of a structureless cuticular membrane separating and supporting the two fundamental layers of the body.

II.

DEVELOPMENT OF ORGANS AND TISSUES.

The larva now consists of a layer of ectoderm and entoderm separated by the supporting lamella and enclosing the gastric cavity. The latter has as yet no communication with the exterior and shows no trace of division into the eight radiating chambers characteristic of all Alcyonarian polyps. Within a few hours—usually between the fortieth and fiftieth—the œsophagus is formed, though it is not perforated until a far later period, and the gastric cavity is divided into chambers through the appearance of radiating septa. These structures develop simultaneously, but it will be convenient to follow their formation separately, and the same plan will be followed in describing the development of other organs.

§ 8. *Formation of the œsophagus and mouth.*

The œsophagus usually makes its appearance in the larva of about forty hours as a solid invagination of ectoderm at the larger end of the body (fig. 134). The high columnar ectoderm cells at this point change their form entirely and rapidly multiply, and are pushed into the body of the larva as a solid plug (fig. 134, *st.*). The invaginated cells are very small, rounded, possess distinct but very small nuclei, and are so closely packed together that their outlines can scarcely be distinguished except in very thin sections. They differ widely from the entoderm cells, being far smaller, staining more deeply and of a different tint, and their nuclei are much smaller. In some cases the œsophagus contains from the first a very small cavity extending inwards from the exterior, but it is usually quite solid. As the plug of ectoderm is pushed in, it carries before it the supporting lamella and the entoderm, the cells of the latter multiplying at the same time. These entodermic cells assume a high columnar form, with their long axes directed towards the œsophagus (fig. 134).

The ectodermic plug grows rapidly backwards and assumes a somewhat pyriform shape from the expansion of its lower extremity (fig. 136), and the entoderm, which everywhere covers it, often becomes much thickened, especially at its lower end. A narrow cavity (fig. 135) then appears in its centre, communicating at the anterior extremity with the exterior, but still ending blindly below. The cavity appears to be formed by the giving way of the central cells, aided to some extent, perhaps, by absorption. In this stage the cells towards the outer opening sometimes have an obscurely columnar form, but towards the inner end of the invagination are small and rounded as before. The cavity at its very first appearance, as shown in transverse sections, is greatly elongated in a particular direction, which is always the same in relation to the septa and is shown by the later development to coincide with the dorso-ventral axis.

The œsophagus remains in this condition for a considerable period (twenty to twenty-five hours), during which the only change consists in the clear definition of the cavity and the expansion of its lower end (fig 136). In most cases the lower angles of the cavity are prolonged downwards so that the cavity has a distinct Y-shape; this form is sometimes much more pronounced than in the figure. The cavity then breaks through, thus placing the gastric cavity for the first time in communication with the exterior.

I have made many sections, longitudinal and transverse, through the œsophagus at this period, a study of which leaves little doubt that great variation exists in the formation of the mouth, as in so many other features of the development. The most common mode is illustrated by figs. 137–140. The wall of the œsophagus thins away by absorption at one of the lower angles of the Y (fig. 131) and finally breaks away at this point (fig. 139). At the opposite side the mass of tissue forming the bottom of the œsophagus still remains attached to the lateral wall of the œsophagus and to the edges of the septa which have meanwhile been formed. As the septa grow backwards this mass of tissue (which for the sake of convenience I shall call the *œsophageal plug*) is carried down with them, being attached to the edges of one or more of them (fig. 140), sometimes by a narrow neck. The mass of tissue is then gradually absorbed and the œsophagus is left in free communication with the gastric cavity. In several of my specimens, at this stage, a large mass of tissue may be observed lying in the gastric cavity below the œsophagus. This is quite similar in appearance to the œsophageal plug, and is, I believe, identical with it. Hence it would appear that in some cases absorption takes place all around the œsophageal plug, which finally drops out bodily into the gastric cavity and is there absorbed as if it were food or yolk-material. In a number of specimens, one of which is shown in fig. 141, a still different mode was observed. Absorption here begins near the middle of the bottom of the œsophagus between the two arms of the Y-shaped cavity and the opening at length breaks through at this point, leaving the remains of the œsophageal plug attached to the lips of the œsophagus where they are absorbed.

During these changes the layer of ectoderm forming the bottom of the œsophageal cavity becomes indistinct, and in most cases the supporting lamella which separates it from the underlying mass of entoderm disappears. The cells of both layers in the plug change their character and are no longer differentiated by the staining fluid, so that the œsophageal plug appears to be composed of uniform confused granular cells. In one of my specimens (fig. 159) the greater part of the œsophageal plug seems to have been absorbed, leaving the supporting lamella stretching across the œsophageal cavity. Below this is a mass of delicate *débris*, which is apparently the last remains of the œsophageal plug.

Review.

The earlier view, according to which the œsophagus is to be regarded as a stomach, opening below into the body-cavity, is now entirely abandoned. In view of its embryological history, the œsophagus is apparently a true stomodæum, comparable to that of the higher Metazoa. The general occurrence of this structure—which, so far as the evidence at command shows, is homologous throughout all the groups in which it is found—is a very striking fact which probably has an important phylogenetic significance. Its universal occurrence among the Anthozoa and complete absence from the Hydrozoa is a strong argument in favour of the more primitive nature of the latter group. From the fact that the Anthozoa are the most primitive group in which the stomodæum appears, it might be concluded that this group represents the stock from which the higher Metazoa have descended. It seems, however, much more probable that the line of descent has been through some primitive Turbellarian form which, in common with the polyps, derived the stomodæum from a still earlier group. What this origin of the stomodæum was is still an unsolved problem. The hypothesis that the stomodæum is to be regarded as the introverted manubrium of a Hydrozoan, though a plausible one, has no embryological facts in its favour, and can hardly be accepted without additional evidence.

§ 9. *Development of the septa.*

The septa make their appearance at about the same time with the stomodæum, and are well developed within a few hours. As we shall see below, the eight radial septa of the anterior part of the body, which are characteristic of all Alcyonaria, differ entirely in structure and mode of origin from the peduncular septum, a structure which is found in the Pennatulacea alone. Hence it will be convenient to describe separately the development of the two forms of septa.

a. Formation of the radial septa.

Although the peduncular septum makes its appearance some time before the radial septa, it is preferable to describe the development of the latter first. They make their appearance simultaneously at the oral extremity of the larva at the time when the stomodæal invagination takes place, and gradually extend thence backwards about to the middle of the body. Although I have made many sections through the septa at the time of their first appearance, and have given special attention to the matter, I have not been able to discover any difference in the time of their appearance. In later stages, as described further on, they are of different lengths, and the differences are perfectly constant. This is, however, the only indication of a regular succession in the development of the septa, and in the earlier stages no difference can be observed.

The septa appear upon longitudinal section (fig. 136, fifty-two hours) as thick plates of entoderm cells (*s.s.*) extending downwards from the oral end and ending by free edges below. Inwardly they are continuons with the entoderm covering the stomodæum; outwardly they join the entoderm of the body-wall. In transverse section (fig. 142) they are seen to radiate at nearly equal intervals from the stomodæum. The centre of each is occupied by a delicate supporting lamella, continuous outwardly with that of the body-wall, and inwardly with that which separates the ectodermic and entodermic layers of the œsophagus. The entoderm cells are arranged upon both sides of the lamella in a thick irregular layer. They are of an elongated pyriform shape, and are so large and closely packed as to fill up entirely, in most cases, the spaces between the septa. In one or two of the compartments, however, a small space appears near the middle, the cells radiating towards it in all directions from the septa, body-wall, and œsophagus. These spaces constantly increase in size as development proceeds, and form the radiating chambers which surround the stomach. The cells are so closely packed at first that a longitudinal section in nearly any plane gives the appearance of fig. 136, the entoderm having the appearance simply of being greatly thickened in the oral region.

Anteriorly the septa extend quite across the gastric cavity from the œsophagus to the body-wall, as shown in the figure. Behind the œsophagus their inner edges are free and the septa appear in transverse section as low ridges which scarcely rise above the level of the general layer of entoderm. They may, however, be readily recognised by the presence of the central layer of supporting lamella and the radiating disposition of the cells over them. This is shown in fig. 143, which represents a section from the same larva (forty-eight hours) with fig. 142 taken farther back at the lower end of the œsophagus. Three of the septa still reach the œsophagus (*œ.*), two are barely united with it, and two are separated from it by considerable intervals. As development proceeds the septa become constantly thinner and the intervening chambers increase correspondingly in size. This is effected partly through the increasing size of the larva and in part by a change of form in the entoderm cells covering the septa, which become far less elongated. Fig. 144 represents a section through the anterior part of a four days' larva in which the radiating chambers have attained a considerable size. Fig. 145 is from the same specimen at the posterior end of the œsophagus; this corresponds very closely with the earlier stage shown in fig. 143. Fig. 146 is from the same specimen still further back, showing the free septa. The bilateral arrangement of the septa is strikingly shown in the symmetrical disposition of the septa of different widths (see p. 764). The entoderm cells have entirely changed their form, being now more or less flattened, or even forming in some places a flat pavement epithelium. On the edges of the septa have appeared the mesenterial filaments (*f.f.*) but a description of these may conveniently be deferred to the following section.

I have studied carefully the young septa for evidence of the participation of the ectoderm in their formation, but am led to conclude that they are formed almost exclusively from the entoderm, though in some cases a few ectoderm cells may make their way into the outer parts of the septa. In the youngest septa observed, the supporting lamella almost always appears as a simple membrane joining the lamella of the body-wall nearly at a right angle, and sometimes without interrupting its outline (fig. 147). In most cases, however, the lamella of the body-wall bends inwards slightly at the point where the septum meets it, and ectoderm cells with conspicuous nuclei may sometimes be seen lying directly in the angle thus formed (fig. 148.) (This figure is from the peduncular septum, but answers equally well for the radial septa.) In a very few cases the lamella appears to be actually infolded to some extent at the base of the septum, and ectoderm cells pass into the space thus formed, and thus come to lie within the body of the septum. In still other cases this fold appears to close up, forming a small triangular space at the root of the septum in which one or two ectoderm cells appear, as shown in fig. 149, *n*. These never extend far out into the septum, however, and the greater portion of the lamella of the latter is secreted, as I believe, by the bases of the entoderm cells.

The question as to whether the lamella of the septum is double, and contains ectoderm cells invaginated from the exterior, is one of much theoretical interest, since, if this be the case, the septa are to be regarded as actual infoldings of the entire body-wall, and not as simple entodermic ridges. LACAZE-DUTHIERS in his beautiful memoirs on the development of polyps,[*] expressly states that both of the layers of the body-wall participate in the formation of the septa, and he figures in the larvæ of *Astroides calycularis* ectoderm cells with numerous nematocysts passing directly into the body of the septum. On the other hand, he is strenuously opposed by KOWALEVSKY, who maintains that the entoderm alone is concerned in the formation of the septum. My own observations throw no new light on this interesting question; for although the great bulk of the septum with its lamella is in *Renilla* certainly entodermic, yet the occasional entrance of a few ectoderm cells into the base of the septum may indicate that an invagination of ectoderm originally occurred, in connexion with a special development of the underlying entoderm, but was subsequently nearly or completely lost. The matter is certainly worth further investigation in other polyps, for it is difficult to believe that LACAZE-DUTHIERS's figures rest upon no other basis than pure imagination.

b. Arrangement of the septa.

The septa are grouped about the œsophagus with a definite relation to the dorso-ventral axis, as shown in transverse sections (fig. 142). The cavity of the œsophagus is elongated in the dorso-ventral axis, and its angles are opposite two compartments, which may in KÖLLIKER's terminology be called the dorsal and ventral chambers. On each side of the œsophagus are, therefore, three chambers which are called respectively the dorso-lateral, median lateral or simply lateral, and ventro-lateral chambers. Following the same terminology, the septa may be designated as dorsal, dorso-lateral, ventro-lateral and ventral, respectively, there being four on each side of the œsophagus.

This bilateral grouping of the septa becomes very conspicuous in transverse sections

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[*] Arch. de Zool. Exp. et Géner, tome i., ii.

below the œsophagus in later stages. In the four days' larva (fig. 146) the septa are clearly seen to be arranged in pairs on opposite sides of the dorso-ventral axis. The dorsal septa (*d.s.*) are very narrow and widely separated, and have no mesenterial filaments on their edges, the dorso-lateral (*d.l.s.*) are much wider, and are thickened at their edges to form the mesenterial filaments; the ventro-lateral septa (*v.l.s.*) are widest of all, and the ventral septa (*v.s.*) are about equal to the dorso-lateral.

When the septa are sufficiently far advanced to be visible from the exterior upon rendering the larva transparent by reagents or by compression in the fresh state, they are found to have a remarkable and definite arrangement. This arrangement is apparent at a very early stage, and remains unchanged as far as the development can be followed. Hence it will be convenient to describe it from a somewhat older specimen (figs. 103, 104, four days). The dorsal septa (*d.s.*) extend backwards for about one-fourth the length of the body, where they are joined by the dorso-lateral septa (*d.l.s.*). From their point of union the peduncular septum (*p.s.*) extends backwards to the aboral end of the body. The ventro-lateral septa (*v.l.s.*) extend backwards some distance beyond the point of union of the above-mentioned septa, and then bend upwards to join the peduncular septum at the point *u* (fig. 104). In some cases it is difficult to trace the septum up to the peduncular septum, especially when the larva is fully expanded. In fact, I completely overlooked their connexion in my earlier paper, and described the septum as terminating freely below. In some specimens this appears to be actually the case, though it is difficult to make sure of it, but in every case the line of longitudinal muscles accompanying the septum (see p. 780) is continued up to the peduncular septum. The ventral septa (*v.s.*) are of nearly the same length as the dorsal, and in some specimens appear to terminate freely below. In most cases, however, careful examination during a half-contracted state of the larva shows that the lower ends of the septa bend towards one another and unite in the median ventral line. From their point of union a band of longitudinal muscles extends backwards in the median line of the body. In specimens where the septa themselves do not actually join, the lines of accompanying muscles bend towards one another and unite in the same way that those of the ventro-lateral septa join the peduncular septum.

The arrangement of the septa shows, therefore, a very marked bilateral symmetry, the septa being disposed according to their width, length, and relations to each other, in pairs which are symmetrically placed with reference to the dorso-ventral plane.

c. *Formation of the peduncular septum.*

The peduncular septum has a quite different mode of origin from the radial septa, though it is continuous with the latter at their earliest appearance. It makes its appearance at about the fortieth hour at the *posterior* end of the body, sometimes, at any rate, before the stomodæum or the radial septa are formed. A longitudinal section through this part of the body of a forty-hour larva is shown in fig. 150. The rudiment of the peduncular septum (*p.s.*) appears as a rounded mass of entoderm cells, at the base of which is a delicate supporting lamella running inwards from the lamella of the body-wall, and becoming insensibly lost among the cells. The anterior part of the stomach still contains a considerable quantity of unabsorbed yolk, and the stomodæum is just beginning to be formed. From this point the septum grows rapidly forwards, ending by a free edge in front. As the septum extends forwards its lateral portions grow more rapidly than the middle, so that the free edge becomes deeply concave in front. I have no figures of this in early stages where it is most pronounced; it is shown in fig. 136 at *e*, where it has extended very far forwards.

By reason of this structure of the septum, the posterior part of the body is completely divided into a dorsal and ventral chamber (fig. 154); while farther forwards, in front of the edge of the septum, the gastric cavity is undivided, and a section shows only the lateral forward extensions of the septum (fig. 154, *a.*, *a.*). These have exactly the appearance of two independent septa, situated on opposite sides of the body. If they be traced forwards they are found to be continuous with the dorsal septa at their point of union with the dorso-lateral pair, as explained at p. 765. The free edge gradually extends forwards until it reaches the point at which the lateral portions join the radial septa, and then remains stationary for a long period. Its subsequent development is described at p. 795.

A transverse section through the peduncular septum behind its free edge (fig. 151) shows that it is composed mainly of two thick layers of clear, rather ill-defined entoderm cells, separated by a peculiar membrane (*ax.*). At the sides this membrane appears like the ordinary lamella of the septa, and joins the lamella of the body-wall. Towards the middle, however, the membrane splits into two layers enclosing a narrow space in which appear numbers of conspicuous nuclei similar to those of the entoderm cells. Cell-outlines can only faintly be distinguished, but there can be no doubt that the nuclei belong to cells which are enclosed in the lamella and may conveniently be termed the *axial cells.* These cells are confined to the central portion of the septum behind the free edge. The forward extension of the lateral parts of the septum show no trace of anything like the axial cells in their lamella. As development proceeds, the axial cells become more and more flattened between the enclosing layers of lamella and at length nearly or quite disappear (fig. 173, *p.s.*). The lamella of the peduncular septum has then the same appearance as that of the radial septa in which no axial cells were ever observed.

In order to ascertain the origin of the axial cells it is necessary to study sections of still earlier stages of development. Fig. 153 represents a longitudinal section of a somewhat younger larva (forty-eight hours). The entoderm cells forming the main mass of the septum are here very distinct and of a high columnar form. The axial cells are larger and their outlines are more distinct. Their appearance is more clearly shown in fig. 136, from another specimen. The lamella is simple behind but splits further forwards into two delicate membranes between which lie the axial cells. The latter are clear, with very delicate, rounded or polygonal outlines, and with very conspicuous intensely stained nuclei, which are quite similar to those of the entoderm cells (*en.*). Towards the free edge of the septum (*e.*) the two layers of the lamella disappear and the axial cells become confounded with the entoderm cells.

The latter point is most clearly shown in transverse sections taken just behind the free edge of the septum (fig. 152). We find here that the lamella of the lateral portion is simple as in the radial septa, but further inwards the lamella splits into two layers between which lie a number of closely packed axial cells. Still farther inwards the layers of the lamella entirely disappear and the axial cells graduate insensibly into the rounded entoderm cells which form the edge of the septum. In the section immediately behind this, the layer of axial cells can be traced quite across from one side to the other, but they lie several cells thick in the middle and are scarcely distinguishable from the adjoining entoderm cells. In sections further forwards the septum entirely disappears and the body of the larva consists of an unbroken layer of ectoderm and entoderm enclosing a nearly solid mass of yolk.

These sections show very clearly that as the septum grows forwards the entoderm cells of which it is composed arrange themselves in three layers. The two outer layers persist as the entodermic covering of the peduncular septum, and form its main bulk; the cells of the middle layer atrophy, flatten together, and form the axial cells. The two layers of lamella which enclose the axial cells are no doubt secreted by the adjoining entoderm cells; the appearances indicate that these membranes are simply the confluent and much thickened membranes of the cells.

As in the case of the radial septa, I have studied with care the possibility of ectodermic cells passing into the septum at its lateral parts where it joins the body-wall, but have been unable to find decisive evidence of such a process. The sections show exactly the same appearances as those of the radial septa. The lamella of the septum sometimes joins that of the body-wall abruptly, without any infolding of the latter; in other cases the lamella of the body-wall is somewhat infolded, and the angle thus formed contains ectoderm cells; in other cases, again, a small triangular space appears at the root of the septum, enclosing one or two cells. The latter are quite similar to the ectoderm cells which appear in the last-described case, and seem to have been introduced from the outside. In rare cases, one of which is carefully represented in fig. 156, the lamella has the appearance of folding in so as to leave a narrow connexion between the cleft containing the axial cells and the ectodermic layer. From these appearances I conclude that the ectoderm cells may in some cases actually pass into the septum by an infolding of the lamella, but they can never do so in considerable numbers, and take only the most insignificant part in the formation of the septum. Far the greater part of the peduncular septum, as of the radial septa, is formed from entoderm cells alone. Misled by certain theoretical considerations, I was at first strongly inclined to regard the axial cells as ectodermic in origin, having been invaginated from the exterior in a fold of the lamella. More careful study entirely disproved this view. The entodermic origin of the axial cells is placed beyond all doubt, by the fact that typical entodermic spicules are sometimes developed in them. These are unmistakable in form and optical characters, and are never developed in ectoderm cells.

In its earlier stages the peduncular septum forms a complete partition, extending from side to side, and reaching the posterior end of the body. At a later period it becomes perforated along its sides, and at its posterior extremity by rounded openings, which place the chambers of the peduncle in communication. The posterior opening (fig. 155, *p.*) becomes very large, and the lateral openings (*o.*) also increase in size, until the septum has the appearance of being suspended by narrow threads from the lateral walls of the peduncle (see figs. 181, 182). The lateral openings subsequently become much reduced in size, or even close entirely (figs. 206, 207), but the posterior opening remains permanently in the adult, and has been described and figured by KÖLLIKER.

In *Leptogorgia* the eight radial septa are visible when the larva ceases to swim, and attaches itself to the bottom (fig. 113). So far as could be determined, they develop simultaneously, and extend throughout the entire length of the body, without joining one another, or otherwise departing from a strictly radial disposition. They have, however, the same bilateral arrangement with respect to the œsophagus as in *Renilla*. The mouth and œsophagial cavity are distinctly elongated in a definite plane, which may by analogy be regarded as the dorso-ventral. Nothing like the peduncular septum in its fully formed condition was observed, but there is an accumulation of entoderm cells at the aboral end of the larva (fig. 116), developed in connexion with the axis, which is very similar to the peduncular septum in its earliest stages.

Review.

The radial septa and the peduncular septum are structures widely different from one another in structure and origin. The former have a simple cuticular supporting lamella, consist of two strata of entoderm cells, arise at the anterior extremity of the body and grow backwards; the latter, on the other hand, has a double supporting lamella, consists of three layers of entoderm cells, arises at the posterior end of the body and grows forwards.

The eight radial septa are of universal occurrence among the *Alcyonaria*, have in all cases the same grouping about the œsophagus, possess an entirely similar musculature, and for these reasons are clearly homologous throughout the group. I had strong hopes that a careful study of the early development of the radial septa might give some indication of the relation in which they stand to the septa of other groups of polyps. The result is, however, a purely negative one, and affords absolutely no new basis for speculation upon the systematic affinities of the *Alcyonaria*. Their development is greatly condensed and abbreviated, and shows not the slightest indication of any such remarkable and regular sequence as that which LACAZE-DUTHIERS has shown to characterise the development of the septa in various representatives of the Zoantharia.[*] In this respect *Renilla* agrees with all the Alcyonaria whose embryonic development has been investigated, though observations on this matter are so scanty as to afford no satisfactory basis for comparison. In the case of *Alcyonium*, KOWALEVSKY was unable to make out the succession of the septa, but he states that it seemed to be analogous to that of the Zoantharia, as described by LACAZE-DUTHIERS. This statement is, however, too vague to be of any value.

The lateral forward extensions of the peduncular septum (fig. 154) have precisely the same structure as the ordinary septa, and they are continuous anteriorly with the dorsal pair of septa. Hence there can be little doubt that the peduncular septum is to be regarded as formed by the union of the dorsal pair of radial septa, beginning at the posterior end and extending thence forwards. It is highly probable that all of the septa in *Renilla* originally extended to the posterior extremity of the body; for this is the case in the larval *Leptogorgia* and in nearly all other polyps (*Cerianthus* excepted). The six ventral septa have ceased to extend as far as the posterior extremity, but the primitive condition has been retained by the dorsal septa and they have furthermore united by their inner edges to form a flat plate, the peduncular septum.

The axial cells, according to this view, are to be regarded as having been formed along the line of union between the two septa by a peculiar arrangement of the entoderm cells in this region (see fig. 152). Before considering the cause of such an arrangement, it is necessary to look for the homologue of the peduncular septum in other Pennatulids. As has already been stated, no homologous structure is known to exist, except in the Pennatulacea; but its homologies in this group appear tolerably clear although they cannot be determined with certainty without further embryological investigation. KÖLLIKER has described with great care the structure of the peduncle in many species of Pennatulids. The most usual and typical structure is as follows. The cavity of the peduncle is divided by four septa into four chambers of which two occupy a lateral position, the third is dorsal, and the fourth ventral. The four septa meet in the middle of the peduncular cavity, forming a central mass within which lies the axis enclosed in an epithelial sheath. Toward the posterior end the two lower septa become free from the body walls and run out upon the hinder end of the axis which lies free in the peduncular cavity. A part of each upper septum likewise extends out upon the free extremity of the axis, but the remaining parts of the upper septa fuse together to form a single transverse septum which runs backwards to the tip of the peduncle and thus divides the latter at its posterior end into a dorsal and a ventral chamber. (For a full description of this very peculiar arrangement, which can scarcely be described without figures, see KÖLLIKER's 'Pennatuliden,' p. 23).

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[*] Arch. d. Zool. Exp., tome i., ii.

What the relation of these four septa is to the axial polyp is quite unknown, owing to the complete lack of anatomical studies of very young Pennatulids. Through what process the lower end of the axis comes to lie free in the peduncular cavity is also unknown. But it seems highly probable, as KÖLLIKER remarks (*l.c.*, p. 270) that the peduncular septum of *Renilla* is homologous with the single horizontal septum (*septum transversale* of KÖLLIKER) of the posterior part of the peduncle in other Pennatulids. This homology appears especially clear in the case of *Renilla amethystina* (VERRILL); for in this case, the extreme anterior part of the peduncle is divided as in the Pennatulidæ into four chambers, which clearly correspond to the four longitudinal canals of the latter. A section through this part of the peduncle (which is situated near its anterior end and forms in reality a part of the disc) is very similar except in the lack of an axis to a section through the four chambers of *Pennatula* (see fig. 72, plate 8, KÖLLIKER). The two additional chambers of *Renilla amethystina* are laterally placed, and are developed apparently as a pair of cavities in the substance of the peduncular septum. The four partitions which thus arise are all continuous behind with the single horizontal septum.

This comparison appears to me to be well founded, though it cannot be proved so without the aid of further embryological studies. If it is so, *Renilla amethystina* is a perfect connecting link, so far as the structure of the peduncle goes, between *R. reniformis* and the axis-bearing Pennatulids.

In all the latter forms the axis, when present, is suspended by these four septa, and it is difficult to understand their appearance in *Renilla amethystina,* except on the supposition that in this form an axis once existed, but was subsequently lost. In *R. reniformis,* the four septa also have disappeared, leaving only the peduncular septum as the representative of the *septum transversale.* This view is supported by the development of the colony which, as pointed out in section 19, indicates the derivation of *Renilla* from an axis-bearing form, resembling the *Bathyptileæ.*

As a matter of fact, we find the axis developed in very different degrees in the various genera of the Pennatulids; and in certain of the *Veretillidæ,* as *Clavella* or *Cavernularia,* the axis is very small or, even in some species of the same genera, quite absent. Whether the rudimentary condition or total want of an axis in these forms is due to the gradual loss of an axis cannot be determined; but the probabilities certainly appear to be in favour of such a view, since these genera have a much less primitive structure in some other respects than some of the axis-bearing forms. We are perhaps able to get some idea of how the axis might be gradually lost. Since the axis ends at some distance from the tip of the peduncle, a certain amount of movement is still permitted to the latter; and the great development of the peduncular muscles indicates that this power of movement must be an important factor in the life of the organism. In *Renilla* the power of movement is of vital importance (see p. 784), and an axis would be of no conceivable use. It is easily conceivable that the power of movement might become of paramount importance to one of the axis-bearing forms, and the presence of a rigid axis would in such a case be disadvantageous. Hence we can see how, by natural selection, the posterior muscular part of the peduncle might be constantly increased in size and importance, accompanied by a corresponding reduction of the axis. If this process were continued until the axis disappeared, a condition would result like that shown in *Renilla amethystina*, and by a further reduction the structure of *R. reniformis* would be attained, in which the dorsal and ventral chambers and the *septum transversale* alone remained.

The foregoing considerations strongly suggest that the peculiarities in the structure and formation of the peduncular septum may be in some way a result of the former existence of an axis. It is, however, useless to speculate on this matter so long as the development of the axis in the typical Pennatulids is unknown; and in regard to this, in KÖLLIKER's words, "mangeln alle und jede Erfahrungen."

Two entirely different views of the Pennatulid axis have been entertained. KÖLLIKER, on the one hand, regards it as of mesodermic origin, the mesodermic elements being supposed to be originally derived from the entoderm. To quote his own words ('Pennatuliden,' p. 428): "Anders bei der Kalkaxe, denn hier spielt ein osteoblastenähnliche Zellenlage, deren Abstammung von dem Entoderma zwar wohl sicher vermuthet werden darf, aber noch nicht nachgewiesen ist, eine Hauptrolle." On the other hand, KOCH considers the axis as probably ectodermic in its origin. This author, while admitting the so-called axis of a certain division of the Gorgonacea (Pseudaxonia) to be mesodermic, has given very strong reasons for the belief that the true axis of many Alcyonaria is secreted by a layer of epithelial cells directly derived from the ectoderm. KÖLLIKER himself observed that in some of the Pennatulida (*Pteroides, Virgularia*) the axis is surrounded by a distinct epithelial layer, and KOCH has shown that this is the case not only in other Pennatulacea, but also in those Gorgonida which possess a true axis. KOCH's observations are conclusive that this epithelial layer, in the fixed Gorgonians, consists of invaginated ectoderm cells which secrete the axis as a cuticular structure. This "axis-epithelium" of the fixed Gorgonians is identical in structure with that of the Pennatulids, and the latter is believed by KOCH, though from analogy only, to be also ectodermic, its original connexion with the exterior having been lost.

In the face of such conflicting views as to the nature of the axis, it is impossible to determine its real relation to the peduncular septa and the *septum transversale*. Without definite knowledge on this point, it is clearly premature to frame any definite hypothesis as to the significance of the peduncular septum of *Renilla*, and the solution of this problem can only be found by studying the embryology of the axis-bearing Pennatulids.

§ 10. *Development of the mesenterial filaments.*

The mesenterial filaments are visible as soon as the larva becomes sufficiently transparent as dark granular thickenings on the edges of the septa at their upper portions where they join the œsophagus. They may be seen while the larva is still swimming, but their arrangement can be made out only after the larva has attached itself and the body has begun to elongate. It is then apparent that they vary in length and have a definite disposition. Those of the dorsal septa are very short indeed (fig. 177), or in some cases may not be visible at all when the others are well developed. They appear as knob-like prolongations of the lip of the œsophagus attached to edges of the dorsal septa. The dorso-lateral filaments (*d.l.f.*) are much longer, extending along the edges of the septa nearly to the buds (p^1.) which have now appeared at the point where the dorsal and dorso-lateral septa unite. The ventro-lateral filaments (*v.l.f.*) are still longer and extend down to the level of the buds or beyond them. The ventral filaments, finally, are very short, being intermediate in length between the dorsal and dorso-lateral filaments.

This grouping of the filaments is quite constant and exists at a very early stage. It is extremely difficult to determine whether these varying lengths represent the actual succession of the filaments since the latter are in their early stages closely contracted together and their arrangement cannot be made out. This grouping persists for a long time and the dorsal filaments remain permanently shorter than the others, and of different structure as KÖLLIKER has observed. (The dorsal filaments are in many cases longer than the others, but this is, in *Renilla* at least, only apparent, and is due to the fact that they never become convoluted like those of the lateral and ventral septa.) All of the filaments except the dorsal pair increase rapidly in length and very soon become folded back and forth and variously convoluted (see figs. 183, 205). This is a result of the circumstance that the filaments increase in length much more rapidly than the septa which bear them, and they are necessarily therefore thrown into folds or "gathers."

The dorsal filaments grow backwards very slowly and are never thrown into transverse folds (see figs. 183, 204). They are less opaque than the other filaments, with a darker central line, and are of much less diameter than the others. These differences are permanent and persist in the adult. The dorsal filaments always remain in connexion with the œsophagus, and appear like long narrow prolongations of the latter down upon the edges of the septa. The other filaments, though at first extending quite up to the œsophagus, soon become more or less widely separated from the œsophagus, fading insensibly away a short distance below the lips of the latter.

In transverse sections the filaments appear as simple thickenings of the entoderm at the edges of the septa, which differ in appearance from the remaining entoderm of the septa only in being more granular. The supporting lamella may be traced out nearly to the middle of the thickening where it fades away and disappears.

The filaments appear to arise near the lips of the œsophagus, growing thence downwards along the septa. This suggests the possibility of ectodermic elements from the stomodæum entering into their composition, and I have made many longitudinal sections for the study of this point. Fig. 157 represents a longitudinal section through a larva of 100 hours (the mouth being fully formed) and the remains of the œsophageal plug (probably) being attached to a septum at *pl.* To the left is a mesenterial filament (*f.*) clearly outlined and well differentiated from the rest of the septum by its more intense colour and granular appearance. Above, the ectoderm of the stomodæum may be very clearly distinguished from the entoderm by its less granular appearance and different colour. Following the ectoderm of the stomodæum downwards, it passes insensibly into the entoderm of the filament without any indication of a limit between them. On the right side, however, the ectoderm is separated below by a faint rounded outline, below which the entoderm is slightly thickened and more granular. The large granular mass is possibly a filament but more probably the œsophageal plug. The same general features are shown in fig. 158, and the ectoderm at the left side of the stomodæum becomes entirely continuous below with the entoderm of a mesenterial filament (*f.*).

From these sections it might be concluded that the filaments are actually downgrowths from the stomodæum. In some cases, however, the filament appears to have at first no connexion with the œsophagus. This is shown for instance in fig. 159, where the filament (*f.*) on the left side ends in front by a definite rounded outline and has no connexion with the œsophagus. It is possible that this thickening on the septum is not really a filament but a part of the œsophageal plug. Nearly conclusive evidence is however afforded by the section shown in fig. 137. In this specimen there is a thickening on the edge of a septum (*f.*) which is probably the beginning of a mesenterial filament before the cavity of the œsophagus has broken through, and there is no possibility of any communication with the stomodæal ectoderm.

In *Leptogorgia* the filaments become visible shortly after the attachment of the larva. Two of them are much shorter than the others and are borne by a pair of septa which enclose one of the chambers at the angles of the elongated mouth; they are in all probability homologous with the dorsal pair of filaments in *Renilla*. The six other filaments are much longer and are equal to one another in length. This arrangement was maintained almost unchanged for seven weeks when the young polyps were killed.

Conclusions.

The mesenterial filaments are at first purely entodermic structures, formed as thickenings on the edges of the septa. After the absorption of the bottom of the œsophagus the ectoderm of the stomodæum becomes directly continuous with the entoderm of the edges of the septa and mesenterial filaments. Hence the possibility certainly exists of the filaments or septa containing elements derived from the ectoderm. This must be borne in mind in considering the origin of the sexual elements which subsequently make their appearance in the walls of the dorso-lateral and ventro-lateral septa ; for their derivation from the ectoderm is brought within the bounds of possibility. All writers agree that both ova and spermatozoa in the Anthozoa are derived from entoderm, and this has usually been regarded as beyond all question. The probabilities certainly appear to be very strongly in favour of this view, but it must, I think, be admitted that the possibility of an ectodermic origin for the sexual elements is not entirely excluded. [See Appendix.]

§ 11. *Changes of external form, appearance of the tentacles, and general histological changes.*

When the larva abandons its free-swimming life and settles upon the bottom it has a more or less elongated form, and the posterior part of the body is very extensible and changeable in shape. The ectoderm and entoderm have undergone little change. The entoderm cells are large and clear, with scanty coarse granules, and very distinct large oval nuclei. The ectoderm cells still retain their high columnar form, and have a finely granular contents, which stains slightly. The cells are planted on the lamella, and many of them extend through the entire thickness of the ectoderm. Besides the columnar form, there are other more or less rounded cells in the deeper layers of the ectoderm.

The body now elongates rapidly (figs. 176–178), and the ectoderm undergoes a great change. The columnar cells lose their form, become rounded, proliferate rapidly, and lose their connexion with the lamella. The outermost cells finally become flattened or fusiform, and form a thin layer covering the exterior of the body (see figs. 173–175). At the same time a considerable amount of clear gelatinous matter is formed, which sometimes entirely separates the cells from each other (figs. 160, 161), and forms the greater part of the ectoderm. In the deeper parts of the ectoderm appear rounded cells of various forms. Here and there are very large, deeply-stained, oval cells (fig. 166, *sp.*) ; others are nearly spherical, and groups of four deeply-stained cells are occasionally seen (fig. 166). Besides these, long fusiform cells may in some places be seen lying on the outer side of the lamella. These characters appear especially in the middle and posterior parts of the body. In the anterior region transitional forms may be seen, and at the extreme anterior end the columnar form is long retained.

The entoderm cells undergo meanwhile little change; they vary greatly in appearance according to the state of contraction of the body. During contraction they are of a high columnar form but when the body is fully extended they become much shortened or even flattened.

Leptogorgia presents the same general histological characters at this stage but the ectoderm cells at the aboral end by which the larva attaches itself retain a high columnar form and very granular structure, and secrete a yellowish cement by which the young polyp is firmly attached. The entoderm is also much thickened at this point. This thickening of the layers is shown at *a*, in figs. 115 to 117. The cement substance is undoubtedly to be regarded as the first rudiment of the axis, which is therefore an ectodermic product. It is probable that budding takes place from the basal part of the young polyp so that the colony has at first the form of a flattened plate or encrustation covering the object to which it is attached by the cement secreted by the bases of the polyps. This may be inferred from the structure of the adult colony, but I did not succeed in observing the budding in young stages although the young polyps were kept for seven weeks in the aquarium. The individual shown in figs. 115 to 117 did not attach itself, and the thickening of the layers at the base was much greater than in those which became attached. Upon making a longitudinal section through this specimen, when seven weeks old, the dark mass, *a*, was found to contain a solid yellow horny mass composed of a substance quite like the cement by which other individuals were attached. The basal part of the wall of the body seems to have been invaginated and the cement then secreted in the cavity thus formed. The polyp therefore appeared to have an internal axis, but this must be regarded as an unusual condition which probably occurs only when the larva fails to attach itself.

The tentacles, in both genera, appear soon after the attachment of the larva as conical outgrowths from the anterior ends of the radial chambers (figs. 115 and 176). I have not observed the least difference in the time of their appearance though I have observed them in every stage of development and in many different individuals. In this respect, as in the formation of the septa, the development of the Alcyonaria is more abbreviated than that of the Zoantharia; for in many representatives of the latter group the tentacles, like the septa, develop in regular sequence.

The tentacles are at first quite simple, with no indication of pinnæ. The latter soon make their appearance along the sides of the tentacles, a new pair being formed, roughly speaking, every day. The new pinnæ are formed near the base of the tentacle and are carried outwards by the longitudinal growth of the latter. The formation of pinnæ ceases after about ten to twelve pairs have appeared and the growth of the tentacles is arrested. The pinnæ are somewhat irregularly disposed and the paired arrangement often disappears towards the tip of the tentacle. Those in the middle of the tentacle are always longer than the basal or apical ones. The pinnæ are formed as simple diverticula from the tentacle and consist accordingly of a layer of ectoderm and entoderm separated by the lamella and enclosing a prolongation from the cavity of the tentacle. The tips of the tentacles and pinnæ are often slightly swollen from the accumulation of minute thread-cells at these points.

§ 12. *Development of the spicules and calyx-teeth.*

The spicules of *Renilla*, as Eisen observed, are of two different forms, and I have found these to have an entirely different origin. The large elongated spicules which give the colony its beautiful purple colour and stiffen its walls are produced entirely by the ectoderm. On the other hand the small transparent oval spicules, which occur in small numbers only, are developed in the entoderm alone.

The former make their appearance soon after the attachment of the larva on each side of the middle region of the body near the first pair of buds. They have the appearance of delicate transparent rod-like bodies which are at first quite colourless. They increase slowly in number, extending backward along the line of the peduncular septum on each side. It is only when the formation of the colony is well advanced that they acquire a purple colour and begin to extend forwards towards the oral extremity and upwards and downwards around the body. They gradually extend over the whole area of the body which therefore acquires a delicate purple tint, except towards the tip of the peduncle which remains white. The spicules become very scanty or quite disappear towards the anterior extremity and assume a peculiar arrangement at their upper limit. In each compartment some distance behind the bases of the tentacles they arrange themselves in lines radiating backward and sidewise from a small central area which ultimately forms the tip of a calyx-tooth (fig. 185, *cx.*). At the same time this region becomes elevated so as to form a low conical prominence which in later stages gradually grows out into a hollow pointed diverticulum from the chamber, its walls being stiffened by long spicules; this is a calyx-tooth. When the crown of tentacles is retracted the tooth lies at the anterior end and forms a hard pointed prominence projecting forwards. Calyx-teeth appear on all of the chambers except the ventral one where the formation of a tooth is a rare exception.

In transverse sections the needle-shaped spicules are found to lie in the lower layers of the ectoderm outside the lamella, and a study of the smallest spicules shows that they are formed in the interior of rounded cells lying in the ectoderm. It is difficult to demonstrate the spicule-cells, even in the earliest stages of the spicules, and I have never seen them with certainty after the spicules have attained any considerable size. They cannot be isolated by teasing, and when *in situ* it is difficult to distinguish them from the surrounding cells. By staining the tissues deeply with eosin the bodies of the cells may occasionally be clearly distinguished. Figs. 171^a to 171^g show different forms of the cells containing very young spicules. In some of the cells nuclei appear: in others they are invisible. The calcareous matter first appears as an irregular elongated mass in the protoplasm of the cell and shows to the eye no trace of crystalline structure. As a rule there is only a single concretion in a cell, but the spicules are occasionally formed from two centres, as in fig. 171^a. As the spicule increases in size the enclosing layer of protoplasm becomes very thin and I have never been able to demonstrate it in spicules of one-fourth the full size. The largest spicule figured (*g.*) is not more than one-eighth the length of a fully formed spicule.

The entodermic spicules (figs. 172^a to 172^n) make their appearance at a much earlier period than those of the ectoderm. They first appear in the lateral portions of the peduncular septum as oval, highly refractive bodies, which are plainly visible from the exterior (fig. 181). They become in time very numerous and are arranged in an irregular longitudinal band on each side of the septum. They appear also in the entoderm of the lateral walls of the body and are especially numerous near the posterior end. They become easily detached from their points of origin and may often be seen suspended in the fluid which circulates in the cavities of the young polyp. Most of the spicules of the peduncular septum are formed in its two outer layers, but it often happens that a few are developed in the axial cells and when the latter atrophy are left embedded in the lamella (see p. 766).

The entodermic spicules also are formed in the interior of cells which may much more readily be demonstrated than the ectodermic spicule-cells. The cells, as shown in fig. 172, are variable in form and usually contain distinct nuclei. The calcareous matter is first deposited in the form of very minute rounded nodules which, as in the case of the ectoderm spicules, may be clearly brought into view by examination with polarised light. Examined by ordinary transmitted light they appear in their earliest stages as transparent, scarcely visible bodies: or they may be quite invisible. If, however, they be examined with the polariser, and the upper prism be rotated, they come into view with the greatest clearness; and by a proper adjustment of the prisms both the cell and the calcareous nodules come clearly into view. The spicule-cells may contain only one nodule, or two or three may be present. In the latter case each nodule appears in some cases to give rise to an independent spicule. In other cases spicules may be seen more or less closely united in groups of two, three, or four, and it is probable that each such group is developed within a single cell. The form of the nodules varies exceedingly, being spherical, oval, or irregularly angular. A not uncommon appearance is shown in fig. 172^n. The spicule has an oval form and its substance refracts the light in such a way as to produce two darker lozenge-shaped areas at the ends. The fully formed spicules are usually of a smoothly rounded oval form but are in many cases obscurely angular at the ends (see figs. 172^n). This is not definite enough, however, to admit of comparison with inorganic crystals.

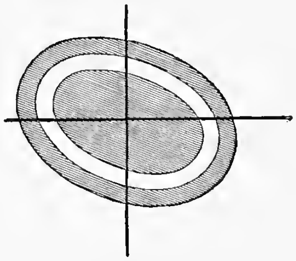
If the spicules be treated with dilute acid, the calcareous matter is dissolved with effervescence, leaving a nearly transparent organic basis which accurately retains the form of the spicules. In some cases, at least, their organic basis is formed before the calcareous matter is deposited. I have sometimes seen cells resembling the spicule-cells, and containing clear bodies quite similar to the calcareous nodules, but destitute of calcareous matter.

In order to determine the molecular structure of the spicules, I submitted a number of them to Professor B. K. EMERSON, the well-known mineralogist of Amherst

College, for examination with the polariscope. He kindly undertook to examine them, and the following statement of his results is quoted verbatim:—

"I find the smaller spicules to polarise perfectly and very brightly, and if the spicules are a carbonate, it is probably aragonite rather than calcite, because of the bright colour it affords.

"Each spicule is made up of a core of crystalline material surrounded by a thin layer of non-polarising matter, and this by an outer layer, slightly thicker, and also of equal thickness, which polarises as does the core.



"The crystalline axes uniformly run parallel to the long and short diagonals, as in the sketch, so that the core is a crystal probably of the rhombic system, and the outer layer is controlled in its position by this core and has parallel axes with it.

"The larger spicules I can best explain by saying that for a moment, on examining the slides, I supposed the small ones to be cross sections of the former (larger) ones. They are like bones filled with marrow under the microscope, the bone and the marrow representing the core and the outer layer of the small ones, and these being separated by an amorphous (in one case there were two amorphous layers) layer. In this case, the long axis of the spicule is a crystallographic axis. And bone and marrow are *orientirt* alike."

From this, the very interesting fact appears that the spicules are formed by a true process of crystallisation, though the form and structure of the crystals are modified, probably by reason of their deposition in an organic viscous medium. This point, as noted below, is one of much theoretical interest.

In *Leptogorgia* the characteristic spicules appear in the ectoderm soon after the attachment of the larva. They are quite irregularly distributed, and extend up into the bases of the tentacles. No entodermic spicules were observed.

Review.

My observations on the development of the spicules are in accord with those of KOWALEVSKY on the spicules of *Sympodium* (Zool. Anzeiger, No. 38, 1879), and as KOWALEVSKY points out, the process is quite similar to the formation of spicules in the mesoderm cells of sponges observed by SCHULTZE and METSCHNIKOFF. Their mode of development strongly recalls the formation of inorganic crystals in the interior of vegetable cells, and possibly indicates the origin of the spicular skeleton. This question presents serious difficulties under the theory of natural selection, for it is impossible to see how the occasional appearance of minute calcareous nodules in the tissues can originally have been of any use to the organism.

Everyone is familiar with the formation of crystals of lime salts in the interior of vegetable cells, where they perform no function as supporting organs, and are apparently mere by-products of the activity of the protoplasm. In this respect the entodermic spicules of *Renilla* resemble the deposits in many vegetable cells; for they are of no use to the colony as supporting organs, and unless we consider their present condition as having been acquired through degeneration, they must originally have been developed without reference to such a function. From the analogy of the deposits in vegetable cells, and in the entoderm cells of *Renilla,* it seems not improbable that the ectodermic spicules of *Renilla* had originally no function as supporting organs, having been formed simply as by-products of the activity of the protoplasm under peculiar conditions, such, for instance, as a superabundance of lime salts in the water. If, however, calcareous nodules once made their appearance in any considerable quantity in the tissues, they might serve as supporting organs, and be developed through natural selection to almost any extent. They might thus attain the great size and functional importance of the ectodermic spicules of *Renilla* or other Pennatulids, or by agglutination come to form a compact skeleton as in *Tubipora.*

It is remarkable to find so wide a difference between the skeletons of Alcyonaria and Zoantharia, as must exist if KOCH's recent conclusions as to the skeleton of *Asteroides calycularis* are well founded;[*] and it seems probable that the skeleton has been quite independently acquired in the two groups. The present considerations will, of course, apply to the Alcyonarian skeleton only.

§ 13. *Development of the muscular system.*

The larva of three days is very changeable in form (figs. 104, 105), showing that contractile elements have made their appearance; and careful examination of specimens rendered transparent by reagents reveals the presence of numerous short delicate unstriated muscle-fibres underlying the ectoderm. These are found to have a definite and constant arrangement, which will be described before considering the histology of the tissue.

a. Distribution.

The muscle-fibres are from the first arranged in two systems, viz.: a layer of longitudinal fibres, and a layer of circular fibres, which ultimately come to lie outside the former. The circular fibres first appear in the posterior half of the body in a broad

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[*] Mittheilungen aus der Zool. Station zu Neapel, Band iii., Heft iii., 1882, pp. 284–292.

sheet which nearly encircles the body, but is interrupted at certain points where the longitudinal fibres are situated. As seen in surface view (figs. 160, 161 *c.m.*) they appear as delicate transparent fibres which are quite disconnected from one another. The sheet of circular fibres extends at first no further forwards than the first pair of buds; but as development proceeds the fibres extend forwards to the oral extremity. In the tentacles they seem never to be developed.

The longitudinal fibres differ from the circular in being at first arranged in definite tracts. These correspond in part with the septa, a narrow band of fibres following the line of attachment of each of the radial septa, and of the peduncular septum on each side of the body; these may be termed the septal tracts. Besides these there are two median tracts extending forwards, above and below, from the posterior extremity of the body. The dorsal median tract extends forwards nearly to the first pair of buds and there terminates. The ventral median tract bifurcates at its anterior extremity, and the two branches become continuous with the tracts of the ventral septa (see fig. 176 and p. 765). This arrangement of the longitudinal muscles is strongly bilateral, the median plane corresponding with the dorso-ventral axis of the body. From these primitive tracts the longitudinal fibres gradually extend laterally until they form an unbroken sheet lying within the circular fibres. They also extend forwards into the tentacles, and out towards their tips.

In transverse sections the longitudinal fibres appear as small dark spots lying in the basal part of the entoderm just within, and in contact with the supporting lamella. The entoderm cells covering the median tracts always show a fan-shaped arrangement, which is especially marked in younger stages, while the tract is still very narrow, as shown in fig. 162 (dorsal tract). Fig. 164 represents the ventral tract of a young specimen, and fig. 163 the same tract of an older individual.

The septal tracts (fig. 165) are divided into two parts by the lamella of the septum. In early stages these two parts lie at the base of the septum; but as development proceeds the fibres on the *ventral* side gradually extend out into the septum, until at length they cover a broad tract on the ventral face of the lamella. They also extend for some distance out upon the ventral face of the peduncular septum (fig. 167). (The presence of longitudinal muscles in the lateral portions of the peduncular septum is mentioned by KÖLLIKER, 'Pennatuliden,' p. 274.) These muscles form the retractors of the polyp, which have therefore the arrangement characteristic of all the Alcyonarian polyps which have thus far been examined. (*Cf.* KÖLLIKER, EISEN, LINDAHL, MOSELEY, and HAACKE.)

The circular muscles can scarcely be seen in transverse sections, but are here and there visible, as in fig. 169. In longitudinal sections they are plainly visible (fig. 168) as a series of dark spots within the supporting lamella. They are somewhat irregularly placed, and are not grouped in definite tracts.

b. Histology.

By macerating the tissues in HERTWIGS' mixture of osmic and acetic acids (see p. 728) the entodermic elements can be teased apart, and the muscle-cells completely isolated. We find thus that each muscle-fibre is developed from the base of an entoderm cell, the fibre and cell together constituting an "epithelio-muscular" cell or—to adopt the more convenient term proposed by CLAUS—a myoblast.

The myoblasts are of exceedingly diverse forms, as illustrated by the series of figures (170^a to 170^p). There is great variation in the length of the fibre, depending apparently on the age of the fibre, since the shorter ones are often no thicker than the longer.

The fibres taper towards both ends and are sometimes thrown into transverse folds. They consist of a homogeneous highly refractive substance which differs entirely from the body of the cell. The latter is composed of a granular substance, and contains a distinct rounded nucleus. As to the form of the cell, every gradation may be observed between a regular columnar cell planted on the fibre (fig. 170^d) and a slight accumulation of protoplasm surrounding a nucleus, which is closely applied to the side of the fibre (fig. 170^p). In all of the forms a delicate layer of granular protoplasm often extends for a considerable distance along the fibre (see figs. $170^{g,\,n,\,p}$).

From these appearances I conclude that the body of the cell always extends at first to the surface of the entoderm, the myoblast being at this stage a typical "epithelio-muscular" cell. As the fibre increases in size the body of the cell sinks into the entoderm and diminishes in size, the myoblast then becoming an "intra-epithelial" muscular cell (HERTWIGS). Finally, the myoblast is wholly buried in the entoderm, the cell-body dwindles away and a "sub-epithelial" cell results—*i.e.*, an ordinary nucleated muscle-fibre. This course of development is, however, only inferred from the perfect series of forms shown among the myoblasts, since the outlines of the cells cannot be distinguished in the sections with sufficient clearness to follow their development.

The fibres of both layers are at first arranged in simple flat sheets. Later, the lamella is thrown into folds, so that both systems of muscles assume a more or less arborescent form in sections. The foldings take place in a very peculiar manner, such that those of the longitudinal layer alone are visible in transverse sections, and, *mutatis mutandis*, those of the circular layer in longitudinal sections. I have not followed in detail the development of these folds, since it has been very thoroughly studied by the HERTWIG Brothers in the Actiniæ.

Review.

Both systems of muscles, circular and longitudinal, are formed in the entoderm alone, and an ectodermic musculature is entirely wanting, with the possible exception of some of the muscles of the tentacles. No other case of a purely entodermic musculature, so far as I am aware, is known to exist, though in most of the Actiniæ the ectodermal muscles are very feebly developed, as JOURDAN and the Brothers HERTWIG have shown. This result can, however, hardly occasion surprise, in view of the astonishing amount of variation in the musculature of polyps. KOWALEVSKY states that the longitudinal muscles of *Alcyonium* are of entodermic origin, but refers the circular muscles doubtfully to the ectoderm. Beyond these, observations on the embryonic development of the muscles in Alcyonaria are wanting.

In their mode of development the muscle-fibres agree with other Cœlenterata with exception of the Ctenophora. They are developed in the form of epithelio-muscular cells or myoblasts, which have the same form and structure as those of other Anthozoa, as described especially by the Brothers HERTWIG and by JOURDAN. The myoblasts do not, however, retain this form permanently, as is the case with many polyps. The cell-bodies become reduced to a small quantity of protoplasm enclosing a nucleus, and the myoblast is situated beneath the epithelial layer as in the medusæ (HERTWIGS), some hydroids (KOROTNEFF, CIAMICIAN), and a number of polyps (HERTWIGS, JOURDAN). It is, however, possible that some of the epithelio-muscular cells may retain this form permanently.

III.

DEVELOPMENT OF THE COLONY.

The primary polyp of *Renilla*, produced by sexual reproduction from the egg, begins at a very early age to produce secondary polyps by budding, and thus builds up a community or colony of individuals organically united together. Although this process is a very common one among the Anthozoa, and is all but universal among the Alcyonaria, it possesses a special interest in the case of *Renilla*, on account of the very early period of life at which the power of asexual reproduction is developed, and more especially from the remarkably definite and constant structural relations existing between the members of the community.

§ 14. *Development and functions of the first pair of sexual polyps.*

When the larva is no more than seventy-two hours old, and is still actively swimming through the water, a pair of buds make their appearance on the dorsal side, just above the point where the dorsal and dorso-lateral septa join each other and the peduncular septum. This position of the buds is entirely constant, and I have never seen the least variation from it in the many hundreds of specimens examined.

The buds, as shown at p^1. in figs. 103, 104, appear as slight rounded elevations, with darker centres and without visible septa or mouth-openings. Upon making a longitudinal section through the bud at this stage (*i.e.*, a section transverse to the axial polyp) we find that the darker centre is produced by an ingrowth of ectoderm (fig. 173, *st.*) which forms the first rudiment of the œsophagus, and is therefore a stomodæum. The lamella (*sl.*) is pushed inwards for some distance so as to form a kind of pouch, filled with a solid mass of ectoderm. The lamella at the bottom of the stomodæum then becomes perforated, so that the ectoderm becomes continuous with the entoderm, though there is still no cavity in the œsophagus. The process is fundamentally like the formation of the stomodæum in the axial polyp, but the mouth-opening is differently formed. The cavity of the œsophagus appears as a funnel-shaped depression at the inner end of the stomodæum opening within into the gastric cavity of the axial polyp and terminating outwardly in the solid plug of ectodermic tissue which forms the stomodæum. The cells of this plug are small and rounded, without definite arrangement. Farther inwards the ectoderm cells assume an irregularly columnar arrangement on either side of the cavity, and at the inner end of the stomodæum become definitely columnar and graduate insensibly into the entoderm cells around the lips of the œsophagus.

The cavity of the œsophagus soon breaks through to the exterior, forming a small oval opening, the mouth, which gradually becomes elongated in the dorso-ventral plane until it has the form of a long cleft. The cells of the stomodæum become at the same time of a high columnar form, and cilia make their appearance at the inner ends of those on the ventral side. By the action of these cilia strong currents are drawn into the colony through the mouths of the buds which are held widely open. These currents may be readily shown by adding finely-powdered carmine to the water, when the particles may be seen to be sucked with force into the mouths of the buds. In this manner large quantities of water are sucked into the cavity of the axial polyp, whose body may thus become greatly distended. When a sufficient amount of water has been taken in, the mouths of the buds are tightly closed, and the water is thus retained. The water thus taken in is kept in active circulation by means of the cilia which cover the entoderm. The currents, which are rendered plainly visible by the particles suspended in the fluid, follow a definite course. In the upper chamber of the peduncle the current sets always backwards, and the fluid flows thence into the lower chamber through the openings along the sides, and at the posterior end of the peduncular septum. In the lower chamber the current flows forwards into the anterior part of the gastric cavity.

It is by means of the fluid contained in the gastric cavity that the young polyp is enabled to effect the active creeping movements which it now performs. The *modus operandi* is as follows. The anterior part of the body being well distended, an active peristaltic contraction of the circular muscles takes place and the fluid is thus forced backwards into the posterior region (which may now be termed the peduncle). The latter consequently becomes much elongated, somewhat as the ambulacral "foot" of an Echinoderm is protruded, and the body is pushed forwards a short distance. The circular muscles then relax and the longitudinal ones contract in such a manner as to pull the posterior region forwards towards the anterior part which adheres to the bottom. By the constant repetition of this process the whole organism moves slowly forwards. The creeping movements are very irregular, since the action of the muscles is not uniform. The longitudinal muscles frequently contract more on one side than on the other, so that the body sways and twists about from side to side, often turning completely over and undergoing all kinds of contortions. Nevertheless the organism often creeps for a considerable distance and may even crawl up the perpendicular sides of a glass vessel. The same power of active movement is possessed by the adult colony, and the conditions under which the organism lives are obviously such as to render this power of vital importance to the creature. Living as it does on shifting beds of sand, the colony would be buried and smothered were it not for this power of creeping. If, however, a *Renilla* colony be covered with sand in the aquarium, it soon works its way to the surface and the polyps are enabled to expand in the water.

In the vital necessity of the power of movement lies no doubt the explanation of the very early appearance of the buds. If the young polyp, upon abandoning its free-swimming life and settling in the sand, possessed no means of taking in water and thus of creeping, it would be very apt to be smothered in the shifting sand. By the very early appearance of the buds the young polyp is enabled to imbibe water and to creep as soon as the sedentary life is assumed, and is thereby preserved from destruction.

This view receives a strong confirmation upon comparing *Leptogorgia* with *Renilla* in this respect. The former does not possess the power of creeping but attaches itself at an early age to solid objects in situations where it is not likely to be buried. Precisely as we should expect under the foregoing view, the buds of *Leptogorgia* do not appear at an early period. In my specimens, in fact, they had not made their appearance at the end of nearly two months (!), whereas in *Renilla* they appear at the end of three days.

As the buds become older and more fully developed they gradually cease to perform the function of imbibing water. It is however assumed by younger buds and is in turn lost by the latter as they become older. Throughout the entire life of the organism this function is performed by the sexual polyps in their early stages. The function is lost, so far as I have observed, as the bud becomes mature and is adapted to perform the functions of nutrition and reproduction. This may readily be demonstrated by placing a contracted colony in a vessel of water containing finely divided carmine. The water is forcibly sucked in through the mouths of all of the young marginal buds, but never through the adult polyps. This function is performed by the zooids during their entire existence; so that the latter structures are physiologically identical with the young sexual polyps (see § 21).

As may be seen in fig. 173, the bud lies at first almost entirely inside the primary polyp, projecting inwards from the body wall and forming only a very slight prominence on the exterior. As development proceeds the bud is pushed outwards so as to form an obtusely conical elevation on the exterior (fig. 178, p^1.). At the same time the entoderm grows downwards from the tip of the bud in eight radiating plates (fig.

178[c]) stretching between the wall of the œsophagus and the lateral wall of the bud. These are the septa. They have the same structure as in the young primary polyp, consisting of two layers of entoderm cells separated by a delicate supporting lamella which joins that of the body-wall. As the septa are formed the outer wall of the bud becomes divided into eight lobes (fig. 184[a]) which correspond with the eight chambers of the body. As the bud grows outwards (*cf.* fig. 205,) the septa grows inwards (*i.e.*, downwards toward the axial polyp) so that their lower extremities remain at about the level of the body-wall of the primary polyp.

The septa when first formed stand at nearly equal intervals from each other, though those on the dorsal side are often a little more crowded than the others. As the bud develops farther the septa assume a definite arrangement as shown in fig. 184[a]. The ventral pair approach more closely so that the ventral chamber, which is opposite one end of the elongated mouth, becomes distinctly narrower than the two adjoining ventro-lateral chambers. As shown by the later development, the narrow ventral chamber is homologous with the ventral chamber of the axial polyp and we are thus enabled to determine the orientation of the young polyp. We find that the dorso-ventral axis of the bud has a constant position with reference to the primary polyp, which is shown in fig. 184[a]; *a–p* represents the long axis of the primary polyp, and *d–v* the dorso-ventral axis of the bud. The latter cuts the former nearly at a right angle, but is always inclined slightly forwards (*a* represents the anterior extremity of the axial polyp).

I will add a brief account of the habits of the young colony at this stage.

If the creature be left to itself it gradually comes to a state of rest, burying the peduncle in the sand. The body always assumes nearly the same position, the dorsal side (as determined by the interior structure) being held upwards and the buds extending horizontally on either side. The anterior part of the main polyp, with its crown of tentacles, is directed obliquely upwards and forwards. This position is maintained throughout all the following stages, and this is, I believe, a fact of the greatest importance which stands in causal connexion with the bilateral symmetry of the organism.

The tentacles of the axial polyp may at this stage be entirely retracted into the anterior part of the body. This is effected by the invagination of the oral end of the body, the tentacles being at the same time strongly contracted. When fully expanded they are held nearly horizontally with the outer portion curving gently backwards. When the polyp is hungry the tentacles are moved actively back and forth, somewhat after the fashion of a *Synapta*, but without regularity. If supplied with food, such as Gasteropod veligers, the tentacles close eagerly upon it, and it is held for some time closely clasped by them. They are then taken into the œsophagus and passed in a bolus down to its lower portion where they remain for some time, the lower opening of the œsophagus remaining tightly closed. The bolus is at length suddenly passed into the stomach and retained during digestion in its upper portion. I was unable to discover any indication of intra-cellular digestion. The contents of the veliger shells were dissolved out and were then circulated through the gastric cavity in the form of oil globules. The empty shells were finally ejected through the œsophagus by a reversed peristaltic action.

In *Leptogorgia,* which was fed with oyster larvæ, the process was slightly different. The larvæ were passed into the œsophagus until a large bolus was accumulated at the lower end. The bolus was then passed into the stomach and closely clasped by the short mesenterial filaments. It was thus held for two or three hours, and its remains were finally ejected through the mouth. This seems to indicate that the filaments are intimately concerned in the process of digestion; but, as before, I could not determine the mode of action of the cells.

This observation is interesting, taken in connexion with KRUKENBERG's physiological studies upon the nature of the filaments in the Actiniæ.* From experiments on artificial digestion he is led to conclude that the mesenterial filaments are mainly or entirely concerned in the act of digestion—so far, at least, as proteid matters are concerned—and my observations seem to point in the same direction.

§ 15. *Arrangement and succession of the sexual polyps.*

I have not succeeded in raising the young colonies in the aquarium beyond the stage shown in fig. 178, and my observations on the later stages were made from specimens procured in the sand, which were found in every stage of development. Hence I cannot give the rate of development, since the young colonies develop very slowly or not at all when kept in aquaria. Large numbers of them were examined and the succession of the buds was found to be nearly constant in early stages though somewhat variable in later ones.

The buds develop always symmetrically in pairs with wonderful regularity, as the accompanying series of figures will show. The appearance of the first pair has already been described.

The second pair invariably appear just behind the first (fig. 182, p^2.), and their mode of development is quite like that of the first pair. As soon as the dorso-ventral axis can be distinguished, they are found to be placed like those of the first pair, though the obliquity is less marked, and the axis of the buds often form a right angle with the long axis of the primary polyp. The second pair are at first quite disconnected from the first pair, but soon fuse to some extent with them, the buds being separated by a thin partition wall which terminates by a free edge below (fig. 204, *e*.). The third pair are formed some time after the second, a short distance in front of and obliquely below the first. As before, they are at first quite separate from the other buds, but soon fuse with the first pair (see figs. 204 to 207). The

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* Vergleichend-physiologische Studien an den Küsten der Adria, Erste Abtheilung, 1880.

fourth pair (Fig. 185, p^4.) arise in front of and slightly below the third in the same manner as the other buds.

Up to this point the order of succession is almost invariable. The sequence in the appearance of the remaining buds is subject to considerable variation, though their position is definite and constant. They make their appearance in the angles between the buds already formed, and in the angles between these and the primary polyp; this will be rendered clear by an inspection of figs. 186 to 188.

In fig. 185 a fifth pair (p^5.) have appeared between the first and second. In fig. 186 two additional pairs (p^6. and p^7.) have appeared; one pair (p^6.) are placed in the posterior angles between p^2. and the primary polyp, and the other (p^7.) are between p^1. and p^3.

Fig. 187 is a still older colony in which five new pairs, besides the seven of fig. 186, have appeared.

Of these new pairs p^{12}. are placed in front of the entire series, while the remaining four (p^{11}., p^{10}., p^9., and p^8.) are placed in the angles between p^3.–p^4., p^1.–p^5, p^5.–p^2., and p^2.–p^6., respectively.

Fig. 188 is a still later stage with the tentacles retracted.

The buds are designated as before. Only one additional pair has appeared (p^{13}.), but those of the last stage have greatly increased in size, as may be seen by comparing the corresponding buds marked p^{10}. in the two figures.

The colony has now assumed the form of a flattened disc, with polyps in various stages of growth situated all around the edge. This form results from the circumstance that the secondary polyps grow out laterally away from the primary polyps, and the younger polyps borne in their angles are thus carried further and further away from the centre of the group. The longitudinal axes of the secondary polyps radiate in every direction from the central point. The posterior part of the axial polyp (*ped.*) may now be recognised as the peduncle of the colony.

It has already been noted that the third bud lies a little below the level of the first, and the fourth a little below the third. The buds are therefore arranged on each side in an oblique line, extending forwards and downwards. This line is continued by succeeding buds so that the anterior buds finally come to lie partly on the lower side of the axial polyp, as at p^{12}. in fig. 188. The two lines of buds finally meet one another at the ventral side of the axial polyp. The latter meanwhile bends gradually upwards so that the two lines of buds are kept nearly horizontal, and when they meet are situated at the anterior edge of the disc, and not at its lower side. The axial polyp is thus cut off entirely from the edge, and now rises from the upper side of the disc. This process will be rendered clear by an inspection of fig. 189, where *ax.* designates the axial polyp, and p^{12}., p^{12}., the foremost pair of lateral buds which have united behind the axial polyp at the point x. In the angle between p^{12}., p^{12}. has appeared a median bud which completes the outline of the disc in front.

The portion of the axial cell which is included in the disc, forms the "polyp-cell" and its free portion is what is usually termed the polyp. The latter may be entirely invaginated into the former by the action of the longitudinal muscles of the septa. The calyx-teeth, which have meanwhile increased greatly in length, are situated just at the upper surface of the disc, and when the polyp is retracted they radiate from the opening of the cell (see fig. 189).

The foregoing account of the enclosure of the axial polyp will apply equally well to the secondary polyps. On account of the continual appearance of young buds in the angles between older ones, each of the latter is bordered by younger polyps on each side. The latter gradually extend downwards, and finally meet behind the older polyp which at the same time bends upwards, and thus becomes enclosed within the disc. This is shown in fig. 188, where p^7. and p^{10}. are already extending behind p^1.

As mentioned at p. 789, the ventral compartment of the polyp never has a calyx-tooth, a fact which is rendered conspicuous when the polyps are retracted (see fig. 189). When the polyp turns upwards and is enclosed in the disc, the ventral chamber necessarily comes to be situated on the outer side of the polyp or away from the centre of the group.

By the union of the lateral lines of buds the outline of the disc is completed at front, and the marginal buds now form an unbroken series from one side to the other behind the axial polyp. At the posterior part of the disc, however, the outline is never completed and a permanent sinus remains in which the peduncle is attached. This is due to the cessation of the formation of lateral buds in the posterior angle after three or four buds have been formed (see fig. 189).

§ 16. *Formation of organs in the secondary polyps.*

The early development of the bud, including the formation of the œsophagus and septa, has already been described; but we have still to consider the development of organs in later stages. The bud agrees in the main with the primary polyp, but there are certain important differences in the sequence of development of certain organs.

a. Development of the calyx-teeth.

As the bud grows outwards the outer ends of the chambers grow out into obtusely conical projections, which ultimately form the calyx-teeth, though they are at first closely similar to tentacles. As observed by KÖLLIKER and EISEN, they are formed in definite sequence, and I can in the main confirm the accounts of these authors. This sequence cannot be observed in the appearance of the calyx-teeth of the primary polyp, and is only obscurely shown in the buds which are first formed. As the colony increases in size, however, the sequence becomes very marked, especially in the posterior parts of the disc where the calyx-teeth are usually longer than elsewhere.

Figs. 190 to 193 illustrate the most usual succession of the teeth. The first to develop are those of the ventro-lateral chambers (fig. 190). These are often enormously elongated, especially in the posterior parts of the disc, and they remain for a long time distinctly longer than the others, as may be seen in fig. 193. After a considerable interval they are followed by the calyx-tooth of the dorsal chamber (fig. 191). The lateral teeth appear nearly simultaneously; but so far as I have observed, the median-lateral teeth usually precede somewhat the dorso-lateral. This is sometimes quite decided (fig. 192); but in a few cases the dorso-lateral teeth are first to appear. According to KÖLLIKER this is the rule, and he states that when five teeth are present, the missing teeth are always those of the median-lateral chambers. The ventral chamber very rarely develops a tooth. In younger buds, when all of the teeth are formed, they usually increase pretty regularly in length from above downwards, and this gradation is more marked usually in posterior parts of the disc. In later stages, the difference gradually becomes less, until the teeth are of nearly equal length. In the rare cases of the appearance of a ventral tooth this is always smaller than the others. The calyx-teeth vary greatly in length in different colonies. We note, finally, that the calyx-teeth are usually all formed before the tentacles appear, whereas the reverse is true of the primary polyp.

It is surprising to find this regular succession in the appearance of the calyx-teeth, which must be structures much younger, phylogenetically, than the tentacles. It may perhaps depend upon the circumstance that the polyps are in early stages placed side by side, so that the upper and lower calyx-teeth are more directly exposed to the environment. This does not, however, account for the absence of a tooth on the ventral chamber, and in our ignorance of the functions of the teeth in early stages, it is useless to speculate on the matter. In the mature bud the teeth probably serve as an armature for the mouths of the polyp-cells, since they are then stiffened with spicules and must form an effective defence. They can hardly perform such an office, however, in the young buds and in the zooids, though their early and ample development in both these cases indicates that they must perform some function. The very brilliant and beautiful phosphorescence of the colony appears to have its principal seat in the calyx-teeth of the young buds; but this can hardly throw any light upon their function.

Whatever be their function, the sequence in the development of the calyx-teeth seems to stand in no relation with the definite succession of the tentacles in some Zoantharia, but is dependent upon some special unknown conditions peculiar to the *Renilla* colony.

b. Development of the tentacles.

The formation of the tentacles agrees entirely with that of the tentacles of the primary polyp, and calls for no special remark. They make their appearance simultaneously, after the formation of the calyx-teeth, as conical outgrowths of the compartments between the mouth and the calyx-teeth. They are at first simple, but soon become pinnate, the pinnæ developing somewhat irregularly in pairs at the bases of the tentacles.

c. Development of the mesenterial filaments.

The mesenterial filaments are formed in essentially the same manner as in the primary polyp, appearing as thickenings on the edges of the septa. They differ however in one striking feature from the filaments of the primary polyp, viz.: in the order of their appearance. The dorsal filaments in the latter are last to appear and slowest in development, whereas in the buds they are in many cases first to appear, and in all cases develop at first more rapidly than the other six. This agrees entirely with Kölliker's observations on the development of the secondary polyps in *Halisceptrum* ('Pennatuliden,' p. 161) in which the dorsal filaments are well formed before a trace of the other six can be made out.

This remarkable contrast between the development of the filaments of the primary and secondary polyps shows clearly that in searching for the relations between the various groups of polyps, as indicated by their embryology, we are not justified in comparing the egg-development of one form with the bud-development of another, or in taking the structure of the bud as any necessary indication of the succession of the parts in the egg-embryo. That the importance of this principle has been unconsciously disregarded will, I think, be clear from the following citations.

Moseley writes in his admirable paper on *Heliopora* and *Sarcophyton* (Phil. Trans. Vol. 166, 1876, p. 121): "It seems extremely difficult to reconcile the extraordinary succession of the mesenteries in the development of the Zoantharians, discovered by Lacaze-Duthiers, with the facts presented by Alcyonarians. Did the development of the eight mesenteries of Alcyonaria correspond with that of the first eight mesenteries formed in Actiniadæ, the first mesenteries formed would be either the lateral dorsal or lateral ventral; but these are those which are most rudimentary in the zooids of *Sarcophyton*. Moreover the mesenterial filaments of the two lateral pairs of septa are in the development of Actiniadæ the first to appear, and not the dorsal, which are longest in the Alcyonarian polyps and most persistent in the zooids. Apparently, however, development in Alcyonarians follows a different course."

These words seem clearly to imply that the greater length of the dorsal filaments and their persistence in the zooids indicates their earlier development in the embryo.

Kölliker states explicitly ('Pennatuliden,' p. 427): "Die Septa und Septula sind Falten des Entoderm und entstehen wahrscheinlich alle zugleich (ich), dagegen bilden sich in erster Linie nur an zweien derselben Verdickungen des Entoderma (Mesenterial-filamente) und später erst treten solche auch an den andern 6 Septa gleichzeitig auf." At p. 434, he even extends this statement so as to apply to the entire group of Alcyonaria: "Bei den Alcyonarien treten nun allerdings auch *zwei* Mesenterial-filamente früher als die anderen auf, allein diese stehen *dicht beisammen*, und bilden sich nach allem, was wir wissen, alle acht Septa auf einmal."

Nevertheless, in *Renilla* at least, the exact reverse of what is indicated by the passages cited is actually the case; and the presumption is that the same holds true in *Sarcophyton*, *Halisceptrum* and other Alcyonaria.

The facts presented by the bud-development of *Renilla* tend to show that a definite sequence in the appearance of symmetrically repeated parts may very readily be acquired or modified through the action of secondary causes which are, however, for the most part too obscure to be recognised.

§ 17. *Development of the zooids.*

a. The exhalent zooid.

The exhalent zooid (*ex.* in all the figures) makes its appearance some time after the appearance of the first pair of secondary polyps and always before the second pair are developed. It occupies always the same position, viz.: on the median line of the dorsal compartment a short distance in front of the pair of buds (fig. 181). Its early development is in all respects identical with that of the sexual polyps, and when the septa are well established they are found to have the same arrangement as in the latter. The ventral chamber is very narrow and remains always without a calyx-tooth. As the zooid increases in size a short calyx-tooth appears on each of the other chambers, and these are developed simultaneously so far as observed (figs. 188, 189). The zooid is in this stage closely similar to the mouth of the cell of a sexual polyp when the latter is contracted (*cf.* fig. 189). The zooid remains in essentially the same condition during its whole existence, but the calyx-teeth become much more elongated and the ventral chamber becomes so small as almost to disappear. No tentacles are ever developed and I have never observed the least rudiment of them.

We find that the dorso-ventral axis of the zooid, which may at once be determined by the elongation of the mouth and the position of the ventral compartment, coincides with the long axis of the primary polyp; and furthermore that the ventral side is turned towards the posterior part of the latter. This relation of the axis is constant, though the axis of the zooid sometimes forms a small angle with the long axis of axial polyp. It sometimes happens that two exhalent zooids are formed. In this case one of them is usually placed in the normal position and the other lies at one side with its axis more or less oblique. In one case the zooid was devoid of a mouth-opening.

b. The inhalent zooids.

An especial interest attaches to the development of these zooids on account of the curious and constant relations existing between their axis and between these and the axis of the sexual polyps.

The zooid develops in quite the same manner as the young sexual polyp or the exhalent zooid, but never progresses beyond the stage in which two calyx-teeth (those of the ventro-lateral chambers) are formed. The zooid is therefore structurally as well as physiologically (see p. 784) identical with the young sexual polyp. In the fully-developed zooid the œsophagus is of an oval form, elongated slightly in the dorso-ventral axis, and connecting with the exterior through an oval mouth. The inner wall of the œsophagus is covered on its ventral side with powerful cilia, by the action of which water may be drawn in from the exterior in precisely the same manner as by the young sexual polyps. The mouth is furnished with a sphincter muscle by which it may be tightly closed when the cavities of the colony are sufficiently distended with water. As already described, the sexual buds, as they increase in size, gradually cease to perform the function of drawing in water. The zooids, however, retain this function permanently and have been specialised for this purpose alone since they have neither tentacles, mesenterial filaments, nor reproductive organs. The sexual buds hand over their early function, as it were, to the zooids as they become themselves adapted to play another part in the economy of the organism.

As shown in fig. 202 the two calyx-teeth of the zooids become greatly elongated and in some specimens, especially in the posterior parts of the disc, may attain an enormous development. Their walls are soft and flexible and are considerably thickened towards the tips where the cells assume a columnar form. It seems very probable that they may perform tactile functions, but I have been unable to demonstrate this in living specimens.

The chambers have the usual arrangement, there being always a somewhat narrow ventral chamber enclosed between two wide ventro-lateral ones. The five upper chambers are always smaller than the ventro-lateral ones and are nearly equal in size. It is therefore always easy to distinguish the dorso-ventral axis of the zooid, which for the sake of brevity I shall call simply the *axis.*

The zooids are produced in pairs like the sexual polyps, though with less regularity. The first pair (fig. 185, z^1.) make their appearance on the dorsal side of the axial polyp near the bases of the first pair of sexual polyps, at the time when four or five pairs of sexual polyps have appeared. Behind these there appear two or three pairs of zooids somewhat irregularly placed on the upper side of the axial polyp. They are arranged (*cf.* figs. 188, 189) on either side of a longitudinal space which remains permanently free from zooids and is very conspicuous in the fully-formed colony. KÖLLIKER has termed this area the *keel* (Kiel), and it is of common occurrence among the Pennatulida. In the adult colony it extends forwards from the posterior sinus about half-way across the disc. The exhalent zooid is placed at its anterior end and groups of inhalent zooids border it on either side. The axes of these zooids are very irregularly placed, but as a rule the ventral side of the zooid is turned towards the posterior part of the colony.

The remaining zooids appear on the dorsal side and in the median line of the sexual polyp-cells, and a constant relation exists between the axis of the zooids and of the polyps on which they are placed. For the sake of convenience I shall term these zooids *dorsal zooids* to distinguish them from the *marginal zooids* which border the keel.

Four dorsal zooids, to begin with, make their appearance on the upper side of each polyp-cell. They are formed successively, proceeding from the base of the polyp outwards towards the oral extremity, as may be seen upon comparison of figs. 187, 188, 189. In fig. 187 the polyp p^5. has a single dorsal zooid, and p^2. has two. In fig. 189 the polyp p^3. has three zooids and p^7. has four.

The bilateral arrangement of the zooids is well shown by fig. 187, in which the positions of the zooids are accurately represented. With two exceptions each zooid has its counterpart on the opposite half of the colony. The exceptions are the marginal zooid *zm.*, and the dorsal zooid *zd.*, which appear on the right side only.

The zooids are sometimes formed on very young sexual buds, as at p^4. in fig. 187. This recalls the very early appearance of the power of budding in the axial polyp. Upon examining the axis of a dorsal zooid we find that in many cases it coincides with the long axis of the sexual polyp on which it is seated, and where it does not the axis of the zooid forms less than a right angle with that of the polyp. Moreover, the ventral chamber of the zooid is always placed at that end of the axis which is turned towards the basal part of the polyp and therefore towards the centre of the colony. There is a strong tendency in the zooid to assume a position on the secondary polyp corresponding with the position of the exhalent zooid with respect to the primary polyp (see p. 791); and the variations from this position caused by the greater or less obliquity of the axis must be considered as departures from the type. Upon the axial polyp only one zooid as a rule, though sometimes two, appears in front of the exhalent zooid.

Multiplication of the zooids.

The zooids have thus far been described as if remaining simple, as is really the case up to the stage shown in fig. 188. Soon after this, however, the zooids themselves become centres of multiplication and each zooid becomes the parent of a whole group of secondary zooids. It is therefore necessary to distinguish primary and secondary zooids as we have recognised primary and secondary sexual polyps.

The axis of the primary dorsal zooid, as we have seen, stands in a definite relation to that of the sexual polyp. The axes of the secondary zooid, on the contrary, show no direct relation to those of the sexual polyp *but to those of the primary zooid.* Hence we must regard the latter as the real parent of the secondary zooids, though these appear to arise as buds on the dorsal side of the polyp-cell and not directly upon the primary zooid. We must, at any rate, grant that the primary zooid is a centre of force which controls the development of the secondary zooids, and it will be convenient for our purpose to consider the latter as the progeny of the former.

The multiplication of the zooids varies exceedingly, as we might expect from their rudimentary structure and great numbers, but the variation affects only the number and arrangement of the zooids, leaving the relations between their axes unaltered. Figs. 194 to 203 illustrate the multiplication from a simple zooid (fig. 194) to a group of eighteen. The figures, it will be understood, are not drawn from different stages of an individual group but represent a number of different groups in various stages of development.

In what may be regarded as the typical case a group of four zooids is first formed (fig. 199). The upper one (*d.*), situated at the dorsal side of the primary zooid (*p.*), is usually first to appear (figs. 195–197,) but the lateral zooids (*l.l'.*) may appear, singly or together, before the upper one (see fig. 198). In the group of four the primary zooid is distinguished by its greater size and by the possession of calyx-teeth on the ventro-lateral chambers. The ventral chambers of the zooids (*v.*) are turned away from the centre of the group. Thus the axes of the lateral zooids form an angle of 90° with that of the primary zooid, and the axis of the upper zooid is 180° from that of the primary zooid.

New zooids now make their appearance in irregular succession in the angle between the four already formed (figs. 200–202) so that the group then consists typically of eight zooids. The same relation of the axes holds good for the new zooids —*i.e.*, the ventral chamber is turned outwards, or away from the centre of the group. The superiority in size of the primary zooid is still marked and its calyx-teeth are very well developed. In most, though not in all, cases the upper zooid also acquires a pair of calyx-teeth as shown in the figures, and sometimes one of the lateral zooids also (fig. 202, *l.*). Most of the secondary zooids remain however without calyx-teeth, though the ventro-lateral chambers are always larger than the others. The ventral chamber is always very narrow in the zooids which have calyx-teeth, but in the other zooids it is often scarcely narrower than the ventro-lateral chambers. The axis of the zooid can however be always recognised by the elongation of the mouth and the crowding of the six upper septa.

Many of the subsequently formed zooids develop in the same manner as those already described, appearing in the angles between pre-existing zooids and having their ventral chambers turned away from the centre of the entire group. In some cases, however, the secondary zooids become in their turn centres of multiplication, thus forming minor groups which repeat, more or less completely, the formation of the primary group. This is shown in fig. 203. The primary zooid of the system is marked *p.*, and above it lies the dorsal secondary zooid (*d.*) corresponding with the upper zooid in fig. 199. A considerable number of the lateral zooids are simple and their axes are related to that of the primary zooid (as may be seen from the position of their ventral chambers *v.v.*). At *x.*, however, is a group of four zooids which are arranged about a centre of their own and form a secondary group quite similar to the primary group shown in fig. 199. The principal zooid (p^{11}.) of this secondary group has the usual position with respect to the primary group and appears to correspond with the zooid *x*. of fig. 202. The three other zooids are evidently placed with reference to p^{11}. and not to p^{1}. Thus the zooid *d''*. has its ventral chamber turned *towards* the centre of the main group and *away from* that of the secondary group. Hence *d''*. is the offspring of p^{11}. and the grandchild of p^{1}.

There appear to be in fig. 203 two other secondary centres, but each is represented by two or three zooids only. Thus *a*, *b*, and *c* seem to be arranged about the centre *y*, while *d'*, *e*, and perhaps *f*, are arranged about a centre at *z*. It is rare to find the secondary groups completely or symmetrically formed, and in many, perhaps most, cases no secondary centres can be certainly identified. In fact, I have seen only two cases in which the secondary groups were as perfectly formed as at *x*. in fig. 203.

Review.

The close correspondence between the mode of budding of the zooids and of the sexual polyps must already have struck the attention of the reader. If the group of four zooids shown in fig. 199 be compared with the group consisting of three sexual polyps and the exhalent zooid (fig. 181) the composition of the two groups is found to be the same. If the axial polyp in fig. 181 be turned upwards at its anterior part, as actually happens at a later stage, it will have the same relation to the exhalent zooid as that existing between the lower and upper zooids of fig. 199, and the two lateral buds in fig. 181 when turned upwards have precisely the same position as the lateral zooids in fig. 199.

Similarly, we may compare the group of eight zooids (fig. 202) with the groups of eight shown in figs. 184, 205 and 206 (these are seen from the ventral side so that the dorsal member of the group, the exhalent zooid, does not show directly). It is scarcely necessary to remind the reader that I do not mean that the corresponding members of the two groups are homologous with each other, but only that they have been produced by a similar form of asexual multiplication.

Summing up these results, we find that the multiplication of the zooids conforms to a definite law, which upon comparison is found to be identical with that which rules the budding of the sexual polyps.

§ 18. *Closure of the peduncular canals.*

We have finally to describe the manner in which the two canals of the peduncle become closed in front and thus complete the canal-system. As described in the introduction, these canals are in the adult completely closed in front, whereas in the young they communicate freely with the gastric cavity of the axial polyp.

a. The dorsal canal.

The closure of the dorsal canal is effected by the free edge of the peduncular septum growing forwards and finally uniting with the dorsal wall just anterior to the exhalent zooid. This is illustrated by figs. 204 to 207. In fig. 204 the free edge (*e.*) is still some distance behind the zooid (*ex.*). In the next figure it has advanced further forwards and in fig. 207 has nearly passed the zooid. The edge finally unites with the dorsal wall at about the stage of fig. 187 and the canal is completely closed. These stages in the forward movement of the edge of the septum are shown also in dorsal view in figs. 181, 186 and 185.

As the septum travels forwards its edge stretches between the bases of the dorsal pair of septa and the latter gradually extend down upon the lower side of the peduncular septum in a manner which it is difficult to describe, and is scarely shown in the figures. In fig. 181 the edge may be seen stretching between the dorsal septa far behind the dorsal mesenterial filaments (*d.f.*). In fig. 205 the edge of the septum and the filaments have nearly met. In fig. 206 the lower ends of the dorsal filaments lie below the peduncular septum, and in fig. 207, finally, the filaments and septa lie for more than half their length on the lower side of the peduncular septum.

b. The ventral canal.

The closure of this canal is effected by an entirely different process which I have not been able to follow out completely. It has already been mentioned (p. 786) that the partition between the first two sexual buds on each side ends below by a free edge (fig. 204). From this point a delicate flap or fold of membrane extends for a short distance inwards on the under side of the peduncular septum (fig. 204, *fl.*). The latter bends rather suddenly upwards at this level to terminate by the free edge (*e*). in front. In a later stage these flaps extend still further inwards so as nearly to meet on the under side of the septum. The edge of the flap also extends obliquely upwards and forwards across the base of the bud (fig. 205). Still later the two flaps unite below the peduncular septum and form a single membrane extending entirely across the upper part of the ventral canal and ending by a smooth round edge below (fig. 206). At the sides the membrane is slung to the body-wall by fibrous strands like those which suspend the peduncular septum (see p. 768), and it now extends nearly across the base of the bud.

In the latest stage observed (fig. 207) the membrane extends at the sides nearly around the body and has united with delicate irregular outgrowths from the ventral and ventro-lateral septa. Thus the ventral canal is separated by an incomplete partition from the anterior part of the axial polyp where the septa, mesenterial filaments and other organs are situated. There is still, however, a very large rounded opening in the middle of the partition through which the currents of the gastric fluid still flow. In later stages this opening closes up completely, probably by the approximation and union of the edges of the membrane, but I have been unable to follow this since the walls of the body become very opaque through the appearance of great numbers of spicules.

In the adult the peduncular septum appears to be split horizontally in front into two layers between which lies the cavity of the axial polyp. The development of the parts shows that this is not really the case. The upper layer alone is a direct continuation of the septum, while the lower layer is a secondary formation produced by outgrowths from the walls of the body and the septa.

The specimen represented in fig. 207 shows an interesting abnormal condition of the tentacles which deserves mention. When first discovered the two lower lateral tentacles on the left side of the axial polyp were aborted, possessing only a single pair of rudimentary pinnæ. *The two corresponding tentacles of the first lateral bud on the same side were aborted in a precisely similar manner* (see the figure). The specimen was kept alive for a fortnight, but unfortunately died before the other buds had acquired their tentacles. Meanwhile the aborted tentacles grew to about half the size of the normal ones.

The rudimentary condition of the two corresponding tentacles in the primary and secondary polyps may have been due to accidental mutilation, but the chances against such a coincidence seem very great. If on the other hand it were due to the inheritance by the bud of a mutilation or monstrosity in the parent the case would be very interesting.

IV.*

GENERAL CONSIDERATIONS.

§ 19. *The systematic relations of* Renilla.

In reviewing the development of the *Renilla* colony, we are naturally led to inquire whether the arrangement and succession of the buds throws any light on the relations of *Renilla* to other members of the Pennatulacea. Here, as in the case of the homologies of the organs of the individual, the basis for comparison is very narrow on account of the imperfect state of our knowledge. Fortunately, however, the evidence is enough to show how the mode of budding in *Renilla* may readily be reduced to the ordinary type as exhibited in the penniform Pennatulids, as for instance in *Pennatula* or *Pteroides.*

Kölliker was so fortunate as to obtain a very young colony of *Pteroides Lacazii* (Köll.), a representative of the typical Penniformes, and his valuable and interesting observations, when compared with my own on *Renilla,* are enough to show that the mode of growth is essentially the same in these widely different forms. In *Pteroides* ('Pennatuliden,' p. 356, plate xxiii., figs. 214, 215) as in *Renilla* there is a primary or axial polyp which produces paired lateral buds; the order of their appearance was not determined nor was it ascertained whether new buds are interpolated between older ones. The young colonies of *Renilla* and *Pteroides* are in this stage essentially alike, as may be seen on comparison of KÖLLIKER's fig. 214 (*Pteroides*) with my figs. 186 or 187. In both, the axial polyp terminates in the median line in front and the structure of the colony is strictly bilateral.

The subsequent history of the axial polyp was not followed nor has this ever actually been done save in *Umbellularia*,[*] which is an exceedingly aberrant form, and may for the present be left out of consideration. Inferring its history, however, from a study of the adult forms, KÖLLIKER makes the following general statement (*l.c.*, p. 420) : " Die typische Bau dieser Stöcke ist ohne Kenntniss ihrer Entwicklung nicht zu verstehen, und bemerke ich daher vor Allem, das der erste aus dem Embryo hervorgehende Polyp, den ich den Haupt, oder axial Polypen nenne, wahrscheinlich nicht überall in derselben Weise sich verhält. Bei den Einen Formen, wie bei den *Veretilliden, scheint derselbe sich zu erhalten* und später, wie die secundär aus ihm entstandenen Individuen, einfach als Geschlectsthier zu wirken. Bei andern Abtheilungen dagegen, wie bei den *Pennatuleen*, und *Renillaceen, verkümmert der axial Polyp schon früh* und stellt gewissermassen *ein rein* vegetatives Individuum dar, dessen Function erlischt, sobald eine gewisse Zahl secondäre Einzelthiere gebildet sind. Sei dem wie ihm wolle, so bilden sich auf jeden Fall die späteren Einzelthiere *als seitliche Knospen* au dem ersten Polypen und beruht auf einer fortgesetzten solchen Knospenbildung wesentlich die Entstehung der ganzen Colonie."

This statement must be slightly modified, so far as the Renillaceæ are concerned; for the axial polyp does not in this case ăbort but remains, as in the *Veretillidæ*, as a sexual-feeding polyp (KÖLLIKER's statement is evidently made under the assumption that the exhalent zooid—"*Hauptzooid*"—is the aborted axial polyp, a view which has been shown to be erroneous).

Furthermore it is not certain that the axial polyp, even in the elongated *Penniformes* and *Virgularieæ*, remains at the anterior end, increasing in length throughout the growth of the colony; for WILLIMÖES-SUHM observed (*l.c.*) that in *Umbellularia* the axial polyp does not retain its original position in the median plane, but becomes bent to one side so as to assume a lateral position, its former place being taken by one of the secondary lateral buds. Still, the evidence seems to be upon the whole in favour of the view that the primary polyp, whether remaining functionally active or becoming aborted, does retain its median position in the elongated Pennatulids and forms the central axis of the community.

In the simple elongated Pennatulids—as in the *Bathyptileæ* of KÖLLIKER—the axial polyp produces a series of simple lateral buds on each side, which have a bilaterally symmetrical arrangement, and remain simple throughout the life of the organism. From this condition, as KÖLLIKER fully shows, a nearly complete series may be formed on the one hand through the *Protoptileæ*, *Funiculineæ*, *Virgularinæ* to the typical *Penniformes*, and on the other hand through the *Kophobelemnonicæ* to the *Veretillidæ*.

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[*] WILLEMÖES-SUHM, Ann. and Mag. of Nat. Hist., vol. xv., 1875

Upon comparison we find that the colonies of *Pteroides* and *Renilla*, though widely different in their adult state from each other and from the *Bathyptileæ*, pass through a stage of development which precisely corresponds with the permanent condition of the latter group. Obviously this fact tells strongly in favour of the derivation of both the *Penniformes* and the *Renillaceæ* from the *Bathyptileæ* or a representative group, and this is the conclusion which upon the whole appears to me most probable. So far as the *Penniformes* are concerned this conclusion is simply a reiteration of KÖLLIKER'S conclusions, but in regard to the *Renillaceæ* it is entirely different. KÖLLIKER'S view is as follows ('Pennatuliden,' p. 450):—

"Nur zu den *Renilliden* führt keine Brücke von den jetzt lebenden Pennatuliden aus und müssen wir zum Verständnisse derselben auf eine noch nicht beobachtete Urform, ähnlich den jugendlich von FRITZ MÜLLER beobachteten *Renillen* oder den Cornularien unter den Alcyoniden, zurückgehen, die der Kürze halber *Archiptilum* heissen mag. Dieses *Archiptilum* wäre also als ein freier einfacher Polyp nach Art der Edwardsien aber mit der innern Organisation der Alcyonarien zu denken und liesse ich an ihm schon eine solche Differenzirung annehmen, dass ein Stiel und ein Kiel zu unterscheiden wäre. Aus solchen Archiptileen oder weiteren Umbildungen derselben könnte man dann einerseits durch besondere Art der Knospenbildung die *Renilliden*, anderseits die Protoptileen und die Bathyptileen ableiten und wäre im ihnen das vereinigende Band der ganzen Ordnung gegeben. Die Abkunft der Archiptileen selbst anlangend, so werden wir naturgemäss auf die Hydroidpolypen geführt und kann es nach dem, was wir über den Bau von *Hybocodon*, *Tubularia*, und *Cormorpha* wissen, keine Schwierigkeiten machen, von denselben aus den Uebergang zu den gekammerten Anthozoen zu finden wie dies auch Haeckel angedeutet hat. Diese Protanthozoen würden dann in weiterer Linie zu den Urtypen der verschiedenen Abtheilungen der Korallthiere und somit auch zu den Archiptileen sich entwickelt haben."

The development of the *Renilla* colony shows, however, that it is unnecessary to go back further than the *Bathyptileæ*, so far at least as the mode of budding is concerned. The peculiar form of the colony is a result primarily of the circumstance that *the longitudinal growth of the axial polyp ceases at an early stage*, while the two series of lateral buds continue to extend forwards until they enclose the axial polyp; and secondarily of the fact that new lateral buds are constantly interpolated between those already formed, and that the lateral buds fuse with each other to some extent. If we imagine the axial polyp in fig. 189 to become greatly elongated, so as to separate the older buds from one another, and thus to leave room for the younger buds between them, we should have a colony similar to the *Bathyptilum*, *i.e.*, a long central axis with a single series of lateral buds on each side.

KÖLLIKER'S view of the derivation of *Renilla* involves one serious difficulty on any monophyletic theory of descent. The original simple progenitor of the Alcyonaria cannot have possessed the *septum transversale* or the four peduncular septa of the higher Pennatulids, since these are structures peculiar to the Pennatulacea, and do not exist so far as known in the simple young of other Alcyonaria (*cf. Leptogorgia*, p. 768). It is hard to avoid the conclusion that these structures are intimately connected with the formation of the peculiar internal axis in those Pennatulids (including all but *Renilla*) which possess such a structure. In *Renilla reniformis* the *septum transversale* alone is developed (as the peduncular septum), but in *Renilla amethystina*, as described by KÖLLIKER and EISEN, four partitions appear in the anterior part of the peduncle (see p. 770), which appear to be homologous with the four peduncular septa by which the axis is suspended in the axis-bearing Pennatulids. If, then, the latter forms and *Renilla* have independently arisen from the *Archiptilum*, which possesses no axis, it is impossible to account for the presence of the four peduncular septa in some species of *Renilla*. Whereas, if *Renilla* is descended from an axis-bearing form resembling *Bathyptilum* the occasional appearance of four peduncular septa presents no difficulty (compare § 9).

As KÖLLIKER has shown, the lateral pinnæ (*Blätter*) of the *Penniformes* are probably derived from simple lateral buds by the appearance of a series of dorsal buds upon the latter :—

"In der That lehren die Pennatuliden mit Blättern, dass jedes Blatt anfänglich nur aus wenigen, wahrscheinlich ursprünglich nur aus Einem Polyp besteht und dass die übrigen Individuen nach und nach an der *Dorsalseite* desselben *aus ihm hervorbilden*, was theils durch *Theilungen*, theils durch *Knospenbildungen* aus ihm geschieht" ('Pennatuliden,' p. 430).

At first thought it might seem probable that this dorsal series of buds is represented in *Renilla* by the series of dorsal zooids which always appear on the upper median line of the sexual polyps. But upon examination we find that the axes of the zooids are differently placed from those of the polyps. The ventral chambers of the former face inwards (towards the centre of the disc), whereas those of the polyps in the pinnæ of the *Penniformes* face backwards; it seems therefore improbable that they can correspond. As we shall see in a following section, it is doubtful whether the zooids of *Renilla* are homologous with (*i.e.*, directly descended from) sexual polyps. The representatives of the dorsal polyps of the pinnæ, if present at all in *Renilla*, are rather to be sought in those lateral buds of *Renilla* which do not arise directly upon the body of the axial polyp.

Summary.

The development of the colony in *Renilla* indicates its ancestral origin from a form resembling the *Bathyptileæ* from which have also been derived along different lines of descent the *Pennatuleæ* on the one hand, and the *Kophobelemnonieæ* and *Veretillidæ* on the other. In the course of this transformation an axis has probably been lost, the only indication of it at present being the persistence of the *septum transversale* (peduncular septum) and in some species of the four peduncular septa. No decisive evidence on the latter question can be adduced until the development of the axis is made known.

§ 20. *Bilateral symmetry of* Renilla.

The very striking bilateral symmetry, both of the individual polyps and of the entire community, is constantly brought before our notice in studying the anatomy and development of *Renilla;* and it is impossible to leave the subject without considering briefly the significance and origin of this symmetrical arrangement of parts.

Reviewing the symmetry of the individual, we find that is expressed, firstly, in the existence of a dorso-ventral axis, represented by certain median unpaired parts, viz.: the elongated œsophagus and mouth, the ventral chamber devoid of a calyx-tooth, the dorsal chamber with a well-developed calyx-tooth, and the dorsal and ventral median areas of longitudinal muscles. All the remaining parts are bilaterally arranged with respect to this axis, viz.: the tentacles, calyx-teeth, septa, mesenterial filaments, reproductive organs, and the septal areas of longitudinal muscles. The tentacles have a nearly perfect radiate arrangement, but the arrangement of the other organs is, to say the least, as much bilateral as radiate. The bilaterality of the calyx-teeth is strongly expressed in their mode of development, since, with the exception of the dorsal tooth, they appear in successive pairs. The septa are arranged in pairs of different lengths, and are joined together at their lower ends in a strictly bilateral arrangement. The bilaterality of the mesenterial filaments is nearly as marked as that of the septa on account of their arrangement in pairs of different length, their structure and rate of development. The reproductive organs have a strictly paired arrangement, appearing only on the dorso-lateral and ventro-lateral septa. The longitudinal muscles of the septa finally show a marked bilateral symmetry in their arrangement, being always placed on the ventral sides of the septa.

The bilaterality of many of these parts must be of comparatively recent acquisition; for in other and lower polyps it is less evident or entirely wanting. Traces of bilateral symmetry are found in nearly all polyps, but in most of the lower forms (Zoantharia) radial symmetry, more or less complete, predominates. In the higher forms the radiating parts assume a more definitely bilateral arrangement which is very marked in the Alcyonaria and reaches its culmination in *Renilla.*[*] Hence there can be no doubt that the bilateral structure is, in part at least, due to a rearrangement of parts which were formerly radially symmetrical. The bilateral symmetry is, as it were, built upon a basis of radial symmetry; and traces of the latter, more or less pronounced, may accordingly be seen in the bilateral arrangement of most of the parts of *Renilla.* Thus it exists almost unmodified in the grouping of the tentacles, in the septa has partly given place to a bilateral arrangement, and in the reproductive organs is scarcely or not at all to be recognised.

[*] See especially HAACKE, "Blastologie der Korallen," Jena. Zeitschr., Bd. xiii., 1879, and HAECKEL, "Generelle Morphologie" und "Studien z. Gastræa-theorie," Jena. Zeitschr., Bd. viii., ix., 1874–5.

In the community produced by the asexual multiplication of the individual, the bilateral symmetry is very nearly perfect. Such departures from perfect symmetry as do exist are inconstant, and appear to be due simply to slight inequalities of growth produced by varying conditions of nutrition. In the adult colony a middle plane is clearly marked by the form of the disc, position and internal structure of the axial polyp, position of the exhalent zooid and of the "keel," and the insertion of the peduncle. On either side of this axis the polyps and zooids are disposed with great regularity. Each sexual polyp has its exact counterpart on the opposite side of the disc, the axis of the two polyps making the same angle with the long axis of the colony. The groups of zooids also correspond pretty closely on the two sides, though less perfectly than the polyps.

The budding of the colony is at first strictly bilateral with surprisingly small variation; and this is true both of the polyps and of the zooids. In later stages the polyps assume a radiating arrangement, as may be seen in figs. 187 and 189, and a radial symmetry is therefore feebly indicated in the disc. This is however due simply to the cessation of growth in the long axis—*i.e.*, the axial polyp—and stands in no relation whatever to the radial symmetry of the individual. In this case we have a slightly marked secondary radial symmetry superimposed upon a primary bilateral form; and in this respect the symmetry of the colony exactly reverses the symmetry of the individual.

It seems clear therefore that the symmetry of the colony has been acquired independently of the symmetry of the individual, and it will be advantageous to consider separately the origin of the symmetry in the two cases.

If we examine the position of the individual polyps, we observe that they are so placed as to have a bilaterally symmetrical environment *which corresponds with the bilateral arrangement of their parts.* Below, they rest upon the sand; above, they are exposed to the water; so that the dorsal and ventral sides are very differently conditioned. The lateral conditions are, however, identical, since each polyp is closely united with a similar polyp on each side. It is impossible to avoid the conclusion that the bilateral environment stands in causal connexion with the bilateral structure, and the probabilities seem strongly in favour of the view that the bilateral structure—or, at least, some of its features—is a result of the environment. This view is in harmony with the prevailing general theories of symmetry which have been especially and independently developed by HAECKEL and SPENCER; namely, in SPENCER'S language, that the form of symmetry depends ultimately on the nature and distribution of the incident forces acting upon the organism. These theories are so familiar as to need no review here, and I will only refer to HAECKEL'S views concerning the ancestry of the Cœlenterata as developed in the papers upon the "Gastræa theory" to which reference has been made.

According to HAECKEL'S theory the Cœlenterate series has been evolved from a primitive ancestral "*Protascus*," immediately derived from the *Gastræa* by the attachment of the latter at the base and the gradual acquisition of a radiate structure as a result of the equality in all directions of the lateral conditions. This theory leaves unexplained the bilateral symmetry which appears in a greater or less degree in all polyps, and HAACKE ("Blastologie der Korallen," *l.c.*) has endeavoured to explain this as the result solely of the formation of colonies. This author holds that in solitary forms like the Actiniæ the bilateral symmetry is due to descent from colony-building species, and he believes that the paired development of the septa is thus to be explained, though in precisely what manner he is unable to say.

Without accepting in all details HAACKE's views, which are only a special application of the environment theory of HAECKEL and SPENCER, it appears to me highly probable that the nature of the environment of the individual polyps in the colony will satisfactorily explain their bilateral structure. It is, of course, impossible to explain exactly how the bilaterality of the various organs is related to the bilaterality of the environment on account of our imperfect knowledge of the functions of these organs and of the laws of growth. But we cannot admit that the perfect correspondence between structure and environment is due to mere accident, and the only alternative is to regard it as the result of adaptation in the organism.

An obvious objection to this view is that it may be putting the cart before the horse; for there may be laws of nutrition or of growth, dependent upon a bilateral structure already existing, which limit the production of buds to the sides of the axial polyp. But we have seen that the zooids—which are undeveloped buds—are produced in the dorsal sides of the polyps in *Renilla*, and KÖLLIKER has shown that in other Pennatulids the zooids may appear anywhere upon the polyps. All parts of the sexual polyps therefore possess equally the power of producing buds, and hence the circumstance that each polyp is laterally united with two other individuals depends on the general form of the colony and not upon any limitation existing in the laws of growth in the individual.

Passing now to a consideration of the symmetry of the colony, we find here the same general conditions as in the individual. The young colony, as we have seen, (p. 785) assumes a definite position as soon as it begins its sedentary life, and this position is maintained during the entire existence of the organism. In this habitual position of the colony, with the peduncle rooted in the sand and the disc expanded upon the surface, the dorsal and ventral sides are quite differently conditioned, while the sides are similarly conditioned. The conditions of nutrition within the colony being equally distributed, the rate of growth must tend to be equal upon the two sides, and any modifying agency must, so far as we can see, tend to be equal upon both. There seems to be no reason to doubt that such an equality of lateral conditions, if maintained for a long period of time, would ultimately produce as perfect a bilaterality as that of *Renilla*. It is unfortunate that so little is known of the habits of other Pennatulids in which the bilateral symmetry is less marked than in *Renilla*. It is probable that a study of these forms with reference to the relation between them and their environments would throw further light upon the influence exercised by the environment upon the mode of budding and thus upon the symmetry of the colony.

§ 21. *Polymorphism of* Renilla.

Polymorphism has been definitely recognised as existing in the Pennatulacea since the publication of KÖLLIKER's great work so often cited in the foregoing pages, but the existence of "rudimentary individuals" was observed in *Renilla* by VERRILL many years earlier.[*]

We may distinguish in *Renilla* at least four kinds of individuals, viz.: *a*, the axial polyp; *b*, the secondary sexual polyps; *c*, the exhalent zooid; and *d*, the inhalent zooids. Possibly two classes of the latter should be recognised, viz.: zooids which possess a pair of calyx-teeth and those which are devoid of these structures.

The question now arises whether these various forms of individuals are to be regarded as morphologically equivalent—that is, whether all are to be considered as the direct descendants of originally similar individuals which have become modified in various directions for the physiological division of labour. There can be no doubt concerning the nature of the secondary sexual polyps, for these are identical in all essentials with the axial polyp. With the various forms of zooids, however, the case is different; for we have here to consider whether these are the aborted and rudimentary descendants of sexual polyps or are new formations which have never had a more highly organised structure than at present. To put the question in a concrete form we may inquire: Did the zooids during their past history ever possess tentacles, mesenterial filaments, and reproductive organs which were gradually lost as the polyps became specialised for the performance of a single function only, or had the zooids, when first developed in the colony, the same imperfect polypoid structure as at present?

The problem is the same as that presented by the Siphonophora, and in the latter case has given rise to the two totally different views with which everyone is familiar. On the one hand LEUCKART, VOGT, HAECKEL, CLAUS, and others regard the various parts of the Siphonophora (*Nectocalyces, Polypites, Hydrophyllia,* &c.) as the variously modified direct descendants of individuals which were once fully developed, though organically connected together. On the other hand we have the view especially urged by HUXLEY and METSCHNIKOFF, that these parts are only organs which never existed as fully formed individuals.

At first thought it would appear tolerably clear that the zooids of *Renilla* must have acquired their present structure simply through having degenerated from individuals resembling the sexual polyps. They agree closely with the latter in general structure, the differences consisting for the most part in the absence of organs

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[*] "Revision of the Polyps of the Eastern Coast of the United States," Bull. Mus. Comp. Zool., Cambridge: 1864.

like the tentacles or mesenterial filaments, which could be of no use as the polyp gradually became exclusively adapted to the performance of a single function (taking in or discharging water). There are some structural details in the rudimentary zooid which seem scarcely explicable if not due to direct inheritance from a fully developed polyp. Such characters are the absence of a calyx-tooth from the ventral chamber and the presence of two long calyx-teeth on the ventro-lateral chambers. In some of the Pennatulids, according to KÖLLIKER, the zooids even possess a pair of mesenterial filaments on the two dorsal septa, and the presence of such rudimentary organs in the zooids would seem to be a strong indication of their descent by degeneration from sexual polyps.

A moment's consideration shows however an insurmountable difficulty in the way of this view. The zooids are far too numerous to have ever been represented by full-sized polyps, for there would not have been room for them on the colony. The dorsal zooids on a single polyp number from 20 to 70 or more in different species of *Renilla*, and it is obvious that even a far smaller number of full-sized polyps could not possibly have stood upon the dorsal side of a single individual. The same difficulty exists in many other Pennatulids, as in *Veretillum* or in some species of *Pennatula* (*e.g.*, *P. rubra*), where almost the entire ventral surface is covered with closely set zooids.

Hence the sexual polyps and the zooids cannot be regarded as equivalent members of the community, for they are not divergent modifications of identical ancestral forms. The zooids are new formations, acquired after the rest of the colony was established. In this case the question as to the "individuality" of the zooid is merely a verbal one; for if descent be made the criterion we cannot consider them such, and yet they are absolutely indistinguishable from young polyps. The interesting point is that buds may appear in a colony which never attain full development as ordinary individuals but are arrested at an early stage, before they have acquired all of their organs, and made to play a part in the physiological division of labour. If polymorphism thus produced may occur in the Pennatulid community, there is no reason why it may not occur in the Siphonophora, and it is possible that some of the members of the latter organism may have had such an origin. These members may be called "individuals" or "organs which simulate individuals," according to our fancy, the distinction being merely verbal.

Such a view would perhaps partially reconcile the conflicting views respecting the nature of Siphonophora referred to at p. 804. It is admitted by the advocates of the polymorphism theory that some of the structures of the Siphonophora—as, for instance, the tentacles—are not to be regarded as modified individuals ("Persons" of HAECKEL) but are simply organs belonging to the true individuals, though they cannot be distinguished from the latter by their ontogenetic development. It is not improbable that other members of the organism—for instance, the hydrophyllia or pneumatocysts—may have the significance of imperfectly developed buds which owe their origin not to degeneration from more highly organised individuals but to arrest of development at an early age. The possibility still remains that some other members—for instance, the feeding polyps or the nectocalyces — may be the direct descendants of fully-developed functional individuals which have become adapted to different functions in the physiological division of labour.

The possibility must be borne in mind that the various members of a compound organism are not necessarily of morphological equivalence — which is simply a convenient term to denote identity of ancestral origin—and that, according as the members are or are not equivalent, different forms of polymorphism are to be distinguished. In some cases, as among some Hydroida, the polymorphism seems clearly the result of a physiological division of labour among members which were originally completely and similarly developed as individuals. Such communities alone can be regarded as polymorphic in the sense in which this term was originally applied by LEUCKART to the Siphonophora. The polymorphism of *Renilla* and other Pennatulid colonies has probably had in part a different origin and such cases must be clearly distinguished from typical polymorphism. For example, in some Pennatulids two distinct forms of secondary polyps may be recognised, viz.: feeding polyps possessing tentacles and sexual polyps destitute of tentacles. These two forms are probably to be regarded as differently modified descendants of sexual polyps like those of *Renilla*, in which the functions of nutrition and reproduction were united. To this extent the colony is therefore polymorphic in the ordinary sense. The remaining members of the community, viz.: the zooids, have however, probably had a different origin, since they are buds which never attain to complete development and never did so in the past.

The zooid is in every respect—physiological as well as anatomical—identical with the young bud which is destined to form a sexual polyp. Moreover the zooid may in some Pennatulids under some circumstances actually develop into a polyp, as KÖLLIKER states, and I have myself observed. The zooid is to be regarded therefore as a bud in a state of arrested development, which has however acquired the power of asexual multiplication.

We must therefore consider the difficult question as to the agency which originally caused the arrest of development in the buds. How, it may be asked, can in the first place a bud have been produced identical in all respects with the buds which are to form mature polyps, and yet incapable of full development ?

It is perhaps impossible to give a complete answer to this question, but the key to the solution of the problem lies possibly in the fact that the zooid, although in an embryonic state, possesses nevertheless the power of asexual multiplication. As pointed out on a preceding page, the secondary zooids of a group are to be regarded as offspring of the primary zooid and not directly of the sexual polyp on which they are placed. We may therefore explain their rudimentary structure as the result of inheritance from the primary zooid, and hence have only to consider how the latter can have been produced.

It has already been stated that the primary zooid is almost always larger and more perfectly developed than the secondary zooids derived from it. If, then, the secondary zooid owes its rudimentary structure to inheritance from a slightly more advanced bud, may not the primary zooid, as Dr. W. K. BROOKS has suggested to me, have been produced by the multiplication of a still more perfect bud, like the zooid, for instance, of *Halisceptrum* which possesses a pair of mesenterial filaments ? This in turn may have been formed by the multiplication of a more highly organised bud, and so on until a fully developed polyp stood at the beginning of the series. This will be rendered more clear by an illustration, in the consideration of which it is necessary to bear clearly in mind the fact that the immature bud of the sexual polyp performs the same function as the zooid and that this function is of vital importance to the organism.

Suppose a secondary bud, A, to give rise by asexual multiplication to a tertiary bud, B, which remained longer in a rudimentary state and developed less perfectly than A, and hence performed more perfectly the function of taking in water. In a succeeding generation B gives rise to still more rudimentary individuals, C, and so on through many generations until true zooids, permanently rudimentary, were produced. The functions of the rudimentary and of the fully developed individuals being entirely different, the intermediate or transitional forms would perform both functions less perfectly. They would therefore tend to disappear by natural selection until a colony would result like *Renilla* in which no well-marked transitional forms existed. Such a process is widely different from direct degeneration since each stage of the series is not represented in the preceding stage. Thus in the foregoing illustration C is not represented in the preceding stage by B, but is an entirely new individual produced as a bud upon B; and this is true of each succeeding stage. If, then, the ancestral history of a zooid could be followed backward from one generation to another we should not find it becoming more and more highly organised, but a point would be reached when it would entirely disappear.

This view is perhaps of too speculative a nature to be accepted without reserve, but it has at least the merit of showing how structures like zooids, of considerable complexity, might suddenly arise without direct descent from or the gradual modification of any corresponding structures in a preceding generation.

In regard to the nature of such structures as the zooids, HUXLEY's definition of the "organs" of the Hydrozoa appears to me most satisfactory. They are, namely :—"Organs which tend more or less completely to become independent existences or zooids." (The term zooid is here used in a general sense and not in the special sense employed in this paper.) A careful distinction must, however, be drawn between these " organs " and those which are due to the direct degeneration or other modification of complete individuals; and the possibility must be borne in mind that these different kinds of structures may co-exist in the so-called polymorphic communities.

Beaufort, N.C., August 1, 1882.

[*Appendix.*—During the passage of this paper through the press, I have discovered in several genera of Alcyonaria that the dorsal filaments are, in fact, ectodermic downgrowths from the stomodæum, whereas the six others are strictly entodermic structures. My failure to recognise this fundamental difference was due to the circumstance that the entodermic filaments become at an early stage perfectly continuous with the stomodæal ectoderm, like the dorsal filaments, and my most favourable sections happened to be in every case through the entodermic filaments. For a description of these new observations I must refer to a forthcoming paper in the 'Mittheilungen aus der Zoologischen Station zu Neapel.'—Naples, September 20, 1883.]

EXPLANATION OF FIGURES.

The following lettering is used uniformly in all the figures. Other reference letters are explained for the separate figures. The figures of sections are with a few exceptions drawn with the camera. Those of the segmenting eggs and of the external appearances of the colony are free-hand.

al.	Gastric cavity.
ax.	Axial cells of peduncular septum.
c.	Central cells.
c.m.	Circular muscles.
ch.	Radial chamber.
cx.	Calyx-teeth.
d.c.x.	Dorsal calyx-tooth.
d.l.ch.	Dorso-lateral chamber.
d.l.f.	Dorso-lateral mesenterial filaments.
d.l.s.	Dorso-lateral septa.
d.s.	Dorsal septa.
d.f.	Dorsal mesenterial filaments.
e.	Free edge of peduncular septum.
ec.	Ectoderm.
en.	Entoderm.
ex.	Exhalent zooid.
f.	Mesenterial filament.
l.m.	Longitudinal muscles.
œ.	Œsophagus.
p^1., p^2., p^3., &c.	Sexual polyps, or buds destined to become such, numbered in the order of their appearance.
ped.	Peduncle.

p.s.	Peduncular septum.
s.	Septa.
s.l.	Supporting lamella.
sp.	Spicule or spicule cell.
st.	Stomodæum.
t.	Tentacle.
v.	Ventral chamber or ventral side.
v.s.	Ventral septa.
v.cx.	Ventral (ventro-lateral) calyx-teeth.
v.f.	Ventral mesenterial filaments.
v.l.s.	Ventro-lateral septa.
v.l.f.	Ventro-lateral mesenterial filaments.
v.l.ch.	Ventro-lateral chamber.
y.	Yolk.
z.	Inhalent zooids.

PLATE 52.

Figs. 1 to 14. Segmentation of an egg which divides at first into eight spheres. Time as follows :—Fig. 1, 8.50 A.M. ; 2, one minute ; 3, two m.; 4, seven m.; 5, ten m.; 6, twenty-five m.; 7, thirty-four m.; 8, fifty-five m.; 9, sixty-three m.; 10, sixty-eight m.; 11, seventy-five m.; 12, ninety m.; 13, ninety-eight m.; 14, one hundred and fifteen m. × 85.

Figs. 15 to 18. Continuation of segmentation, from another specimen. Time as follows :—Fig. 15, 10 A.M.; 16, ten m.; 17, twenty-seven m.; 18, thirty-two m. × 85.

Figs. 19 to 24. Unusual mode of segmentation. Time as follows :—Fig. 19, 7.55 A.M.; 20, two m.; 21, ten m.; 22, twenty m.; 23, twenty-three m. ; 24, forty m. × 65.

Figs. 25 to 27. Division of an egg into two spheres. × 65.

Figs. 28, 29. Egg divided into four spheres. × 65.

PLATE 53.

Figs. 30 to 37. Segmentation of an egg which divided at once into sixteen spheres. Time not recorded. × 65.

Figs. 38 to 41. Unequal segmentation beginning with sixteen spheres. Time as follows :—Fig. 38, 9.7 A.M.; 39, same in different position ; 40, thirty-three m.; 41, thirty-six m. × 65.

Figs. 42 to 44. Another specimen illustrating unequal segmentation beginning with sixteen spheres. Time:—Fig. 42, 9.20 A.M.; 43, same in different position; 44, thirty m. × 65.

Figs. 45 to 48. Segmentation of an egg which first divided incompletely into eight spheres, and afterwards completely into sixteen. Time:—Fig. 45, 9.5 A.M.; 46, five m.; 47, twenty-one m.; 48, thirty-three m. × 65.

Figs. 49 to 58. Segmentation of an egg which first divided irregularly and incompletely into eight. The sphere marked *a* failed to divide with the others at the third cleavage but divided at the fourth (57). Time:—Fig. 49, 10.29 A.M.; 50, four m.; 51, six m.; 52, fifteen m.; 53, different position, seventeen m.; 54, thirty-one m.; 55, forty-one m.; 56, forty-eight m.; 57, fifty-six m.; 58, sixty-eight m. × 65.

Figs. 59 to 62. "Partial" or progressive form of segmentation. Time:—Fig. 59, 10.6 A.M.; 60, eight m.; 61, nineteen m.; 62, same in different position. × 65.

Figs. 63 to 67. Segmentation similar to the last. Time:—Fig. 63, 11.35 A.M.; 64, nine m.; 65, thirteen m.; 66, nineteen m.; 67, twenty-one m. × 65.

PLATE 54.

Figs. 68 to 72. Very unequal and irregular form of segmentation. Time:—Not recorded. × 65.

Figs. 73 to 76. Segmentation of *Leptogorgia*. Time:—Fig. 73, 10.14 A.M.; 74, three m.; 75, twenty-one m.; 76, thirty-six m. × 70.

Figs. 77 to 85. Further segmentation of *Leptogorgia*; another specimen. Time:—Fig. 77, 10.5 A.M.; 78, fifteen m.; 79, twenty-four m.; 80, thirty-five m.; 81, seventy-eight m.; 82, ninety-five m.; 83, one h. fifty-five m.; 84, two h. five m.; 85, two h. twenty-five m. × 70.

Fig. 86. Section through unfertilised egg showing germinal vesicle and spot, *d*. × 85.

Fig. 87. Section through egg immediately before segmentation. × 85.

Fig. 88. Section through the same egg, separated from the last section by three intervening ones. × 85.

Fig. 89. Section through an egg in the act of division into sixteen spheres, directly from the unfertilised egg. × 85.

Fig. 90. Section through an egg which from the exterior appeared to consist of eight spheres. × 85.

Fig. 91. Section through an egg with sixteen superficial spheres and a central unsegmented mass. × 85.

Fig. 92. Similar to the last but with the central mass much reduced. × 85.

Fig. 93. The spheres now extend to the centre of the egg and the central mass has nearly vanished. × 85.

Fig. 94. Blastula, with distinct segmentation cavity. One of the spheres, *a*, is apparently undergoing a delamination cleavage. × 85.

Fig. 95. Egg in resting stage with unsegmented central mass. × 190.

PLATE 55.

Fig. 96. In this embryo the inner ends of the spheres are separating or have just separated from the outer portion. × 190.

Fig. 97. Delamination completed. × 145.

Fig. 98. Unsegmented egg or one in the act of division. To show vertical amphiasters. × 85.

Fig. 99. Section through an egg in which the delamination is partially accomplished but is in progress in the cells *a* and *b*. The section is incomplete below but complete above. × 85.

Fig. 100. Later stage; the last to show delamination still in progress. The section is complete above and below but incomplete on the sides. × 85.

Figs. 100^a., 100^b., 100^c. Three larvæ of about twelve hours to show irregularity in form. × 65.

Fig. 101. Free-swimming larva of about twenty-four hours. × 45.

Fig. 102. A slightly older larva under compression, showing septa. The ventro-lateral septa could not be followed up to the peduncular septum as it ordinarily can. × 45.

Fig. 103. Larva of about three and a-half days' showing septa and buds. Dorsal view. × 45.

Fig. 104. The same larva, from left side. × 45.

Fig. 105. Same specimen shown in figs. 103 and 104 in a state of strong contraction. × 45.

Fig. 106. *Leptogorgia.* Same embryo shown in figs. 77 to 85, ten minutes later than fig. 85. × 70.

Fig. 107. Another specimen two hours later in the irregular stage. × 70.

Fig. 108. The same specimen twenty hours later. × 70.

Fig. 109. *Leptogorgia* two days old. × 70.

PLATE 56.

Fig. 110. The same, three days. × 110.

Fig. 111. The same as last under compression showing ectoderm and stomodæum, *st.* × 110.

Fig. 112. The same, four days old. × 110.

Fig. 113. The same, eight days; recently attached to bottom. × 50.

Fig. 114. Two larvæ united together. × 50.

Fig. 115. The same, eleven days. At *a* is the posterior thickened region which secretes the axis. × 50.

Fig. 116. The same individual, twelve days. × 50.

Fig. 116^a. Part of wall of body more highly magnified to show spicules in ectoderm.

Fig. 117. The same, thirteen days. Mesenterial filaments well developed. × 50.

Fig. 118. Section through *Leptogorgia* embryo of six hours. × 160.

Fig. 120. Section through *Renilla* embryo of about four hours. × 290.

PLATE 57.

Fig. 119. Section through *Renilla* embryo of four and three-quarter hours.

Fig. 121. Section through an embryo of eight and a-half hours. × 290.

Fig. 122. Portion of a section of same stage more highly magnified. × 880.

Fig. 123. Same as last. × 880.

Fig. 124. Yolk cells from last with deutoplasm spherules. × 880.

Fig. 125. Longitudinal section through embryo of twenty-two and a-half hours. × 195.

PLATE 58.

Fig. 126. Portion of same more highly magnified. × 350.

Fig. 127. Part of a section through a twenty-five-hours' embryo. × 350.

Fig. 128. Longitudinal section through a twenty-nine-hours' embryo. × 165.

Fig. 129. Part of the same more highly magnified. × 350.

Fig. 130. Part of section through a fifty-two-hours' larva. × 350.

Fig. 131. Part of section of a twenty-eight-hours' embryo to show proliferation of ectoderm. × 350.

Fig. 132. Part of section through an embryo further advanced than the last to show formation of supporting lamella; *a* represents a single ectoderm cell with swollen base, *b*; *b b* are the rounded bodies which have separated from the ectoderm cells; *c* is one of the rounded cells from the deeper parts of the ectoderm. × 350.

Fig. 133. Part of longitudinal section through a twenty-six-hours' embryo to show formation of supporting lamella.

Fig. 134. Longitudinal section through a forty-hours' larva to show formation of stomodæum. × 230.

PLATE 59.

Fig. 135. Longitudinal section through a forty-eight-hours' larva. × 230.

Fig. 136. Vertical longitudinal section through a fifty-two-hours' larva. × 165.

Fig. 137. Longitudinal section through a seventy-five-hours' larva. Œsophagus still closed. × 140.

Fig. 138. Similar section through a slightly later stage (100 hours). × 140.

Fig. 139. A similar section of a sixty-two-hours' embryo; the œsophagus has just broken through. × 140.

Fig. 140. Longitudinal section of seventy-five-hours' larva. Mouth fully formed, but "œsophageal plug" (*pl.*) still adhering to the edge of one of the septa. × 140.

Fig. 141. Sixty-hours' larva. Mouth breaking through bottom of stomodæum. × 140.

Fig. 142. Transverse section through anterior part of forty-eight-hours' larva. × 315.

PLATE 60.

Fig. 143. Transverse section from same specimen further back at the level of lower end of œsophagus. × 315.

Fig. 144. Transverse section through anterior part of a four-days' larva. × 315.

Fig. 145. From same larva at lower end of œsophagus. × 315.

Fig. 146. Transverse section of four-days' larva posterior to œsophagus. × 350.

Fig. 147. Part of a transverse section through a forty-eight-hours' larva showing a septum and general histology. × 880.

Fig. 147^a. Portion of ectoderm from last. × 880.

PLATE 61.

Fig. 148. Section through lateral attachment of peduncular septum to body-wall. Forty-eight hours. × 350.

Fig. 149. Section through radial septum and part of body-wall. Forty-eight hours. × 350.

Fig. 150. Longitudinal section through posterior part of body of a forty-hours' larva to show formation of peduncular septum. × 135.

Fig. 151. Transverse section through posterior part of a forty-eight-hours' larva showing peduncular septum. × 315.

Fig. 152. Transverse section through the free-edge of the peduncular septum. Forty-eight hours. × 180.

Fig. 153. Part of longitudinal section through peduncular septum, highly magnified, to show axial cells. × 350.

Fig. 154. Transverse section in front of peduncular septum. Forty-eight hours. × 315.

Fig. 155. Longitudinal section just above peduncular septum. × 220.

Fig. 156. Transverse section through peduncular septum at its attachment to the body-wall. Forty-five hours.

PLATE 62.

Figs. 157, 158, 159. Longitudinal sections through 100-hours' larvæ showing continuity of stomodæal ectoderm with the mesenterial filaments. × 140.

Figs. 160, 161. Surface views of four-days' larvæ to show muscular fibres. × 880.

Fig. 162. Transverse section through the dorsal median tract of longitudinal muscles. Four days. × 880.

Fig. 163. Transverse section through ventral median tract of longitudinal muscles. × 880.

Fig. 164. Corresponding section through a younger specimen. × 880.

Fig. 165. Transverse section through septum and a septal tract of longitudinal muscles. × 880.

PLATE 63.

Fig. 166. Transverse section through septal tract below the septum.

Fig. 167. Transverse section through body-wall and peduncular septum to show longitudinal muscles in the ventral side of the latter.

Fig. 168. Part of longitudinal section through the body-wall showing the circular muscles. × 880.

Fig. 169. Transverse section through body-wall showing circular muscles in longitudinal section. × 880.

Fig. 170. Various forms of myoblasts from the entoderm. × 700.

Fig. 171. Cells from the deeper layers of the ectoderm with spicules developing in their interior. × 700.

Fig. 172. Entoderm cells with spicules in course of formation. × 700.

PLATE 64.

Fig. 173. Section through a four-days' larva passing through one of the buds (p^1.). × 315.

Fig. 174. Similar section passing through bud at p^1.; *a*, lateral forward extension of peduncular septum. × 315.

Fig. 175. Transverse section through four-days' larva passing through both buds. × 315.

Fig. 176. Ventral view of four-and-a-half-days' young polyp. × 56.

Fig. 177. The same specimen one day later from the left side. × 56.

PLATE 65.

Fig. 178. Young polyp of about nine days from right side. × 30.

Fig. 178^a. Tentacle. × 45.

Fig. 178^b. Bud in profile view. × 90.

Fig. 178^c. Bud from above. × 90.

Fig. 179. Right lateral view of part of a somewhat older polyp. × 45.

Fig. 180. Dorsal view of last. × 45.

Fig. 181. Dorsal view of polyp with recently developed exhalent zooid. × 30.

Fig. 182. Dorso-lateral view of polyp with two pairs of buds. × 30.

Fig. 183. Lateral view of polyp with third pair of buds just appearing. × 40.

Fig. 183^a. Dorsal view of peduncle partly contracted to show bands of circular muscles.

Fig. 183^b. Upper view of bud p^3.

Fig. 183^c. Same bud in lateral optical section.

Fig. 184. Ventral view of part of polyp with three pairs of buds. × 40.

Fig. 184^a. Oral view of p^1.

PLATE 66.

Fig. 185. Dorsal view of young colony having five pairs of buds and three zooids. × 40.

Fig. 186. Dorsal view of part of young colony with seven pairs of lateral buds and three pairs of zooids. × 60.

Fig. 187. Dorsal view of fully expanded colony with twelve pairs of lateral buds and numerous zooids. × 30.

Fig. 188. Dorsal view of colony in a state of contraction with thirteen pairs of lateral buds. × 30.

Fig. 189. Dorsal view of left half of the disc of a young colony. The zooids have multiplied to form groups represented by small circles. × 15.

PLATE 67.

Figs. 190 to 193. Series illustrating development of calyx-teeth. From a mature colony. × 20.

Figs. 194 to 203. Series illustrating the multiplication of a single primary zooid (*p.*) to a group of eighteen. × 80.

Fig. 204. Ventral view in optical section of the bases of the first two pairs of lateral buds to show the partition between them and its free-edge (*fl.*) below. × 50.

Fig. 205. Ventral view in optical section of an older colony to show further development of the lateral folds (*fl.*). × 50.

Fig. 206. Similar view of a still older specimen in which the two folds have met to form a single fold beneath the peduncular septum. × 50.

Fig. 207. Similar view of older specimen in which the closure of the ventral canal is well advanced. × 50.

XXV. *On the Continuity of the Protoplasm through the Walls of Vegetable Cells.*

By WALTER GARDINER, *B.A., late Scholar of Clare College, Cambridge.*
Communicated by W. T. THISELTON DYER, *C.M.G., F.R.S.*

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[PLATES 68–70.]

In Professor SACHS' latest publication the following remarkable passage occurs:[*] "Every plant, however highly organised, is fundamentally a protoplasmic body forming a connected whole, which as it grows on, is externally clothed by a cell membrane, and internally traversed by innumerable transverse and longitudinal walls." The above statement, both as being the outcome of pure physiological thought, and invested as it is with the authority of so distinguished a botanist, cannot fail to be very striking, on account of its forcible suggestiveness, and any observations which demonstrate an actual continuity in organs of large extent, must be of interest to show the truth of SACHS' remarks in a sense somewhat more literal than his own.

At the time of writing, the instances of the existence of any such continuity of the protoplasm were but few. SACHS[†] himself in 1863, and HANSTEIN[‡] in the following year, had proved that in sieve-tubes an actual perforation of the sieve plate did take place, and that by means of the sieve-pores a connexion between the contents of neighbouring cells was established. Their results in this direction were fully confirmed by WILHELM,[§] JANCZEWSKI,[||] and RUSSOW.[¶]

But it was not until the year 1880 that any further steps were made, when TANGL[**] demonstrated that in the ripe endosperm cells of *Strychnos Nux-vomica*, *Phœnix dactylifera*, and *Euterpe oleracea* the cell-walls were perforated by fine protoplasmic threads. His observations were in the main confirmed by STRASBURGER,[††]

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[*] 'Vorlesungen über Pflanzen-Physiologie,' p. 102.

[†] SACHS' 'Flora,' 1863, p. 68.

[‡] HANSTEIN, 'Die Milchsaftgefässe.' Berlin, 1864, p. 23.

[§] 'Zur Kentniss des Siebröhrenapparates Dicotyler Pflanzen.' Leipzig, 1880.

[||] 'Études comparées sur les tubes cribreux.' Cherbourg, 1881.

[¶] 'Sitzber. Dorpater Naturf. Ges.,' April 23. Also in the same journal, 1882, pp. 257–327.

[**] "Ueber offene Communication zwischen Zellen des Endosperms." PRINGSHEIM'S 'Jahrbücher für Wiss. Bot.,' vol. xii., 1880.

[††] 'Bau und Wachsthum,' p. 23, *et seq.*

whose general results in connexion with the mode of formation of the cell-wall had impressed him so strongly that the relations existing between the protoplasm and the cell-wall were of the most intimate kind, that he had devoted a special chapter in his work to the consideration of the probability of the perforation of the cell-wall by protoplasmic threads.* In this chapter he distinctly states that although he had not himself been able to establish the existence of any general continuity between vegetable cells, yet that he had but little doubt that such a relation did actually occur.

In a preliminary note published in the 'Quarterly Journal of Microscopical Science' for October, 1882,† I stated that I had succeeded in demonstrating that the continuity of the protoplasm of adjacent cells in the pelvinus of *Mimosa pudica* was maintained by protoplasmic filaments which passed through pits in the cell wall, and later on‡ I showed that the same occurs in *Robinia* and *Amicia*.

Subsequent to the publication of my first results, and previous to the present communication, appeared a most important paper by RUSSOW.§ In this paper the author states that in the bast-parenchyma cells, and in the phlœm medullary-ray cells of many of the Amentaceæ, *e.g.*, *Populus, Salix, Quercus, Betula, Corylus;* in *Fraxinus, Syringa, Olea, Æsculus, Acer;* in the *Abietineæ,* and further in *Cucurbita pepo* and *Lappa tomentosa,* a treatment of thin sections with Chlor. Zinc Iod. demonstrates that a communication between adjacent cells is established by means of pits, the pit membrane being perforated by fine protoplasmic threads.||

In the following paper I propose to deal more fully than I have hitherto done with my researches upon pulvini; to treat of the methods I employed, and also to give an account of my investigations as to the structure of endosperm cells, which were undertaken with the view of controlling my results with pulvini. I think that these investigations will succeed in proving not only that perforation of the cell-wall by protoplasmic threads does actually take place, but also that such perforation is of very frequent occurrence.

Methods.

Preservation of material.—As it was a point of primary importance that the material for an investigation of this kind should be preserved with the least possible change, I instituted a number of experiments with the view of ascertaining which of the various reagents commonly in use was the most reliable and what precautions were necessary to insure the most successful result.

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* *Loc. cit.*, 'Die Wegsamkeit der Zellhaüte,' p. 246, *et seq.*

† GARDINER, "Open Communication between the Cells in the Pulvinus of Mimosa pudica."

‡ Proc. Roy. Soc., November 11, 1882.

§ 'Sitzber. d. Dorpater Nat Gesellsh.,' 1882, p. 350. See STRASBURGER's remarks, 'Sitzb. d. Niedenrrh Ges.,' December 4, 1882. I now find (Jan. 16th, 1884), that RUSSOW's paper was read at the January meeting of the Dorpat Society.

|| With FROMMAN's and ELSBERG's results I have already dealt. See 'Quart. Jour. Micr. Sci.,' April, 1883. GARDINER "On some Recent Researches on the Continuity of the Protoplasm through the Walls of Vegetable Cells."

In my paper "On the Continuity of the Protoplasm in the Motile Organs of Leaves"* I stated that when the plasmolytic condition is induced in a cell, the contracted primordial utricle does not lie free in the cell cavity, but is connected to the cell-wall by numerous fine threads of protoplasm. Since these threads are exceedingly thin and easily ruptured, the value of a preservative agent can be readily tested by observing with what degree of success it can fix the protoplasm of such a cell, and can preserve unbroken the delicate threads. (See Plate 70, figs. 34, 35, 36, and 37.)

For this purpose thin transverse sections of the pulvinus of *Robinia pseudacacia* were rapidly cut in water, and treated for about five minutes with a 10 per cent. solution of sodium chloride. The excess of salt was quickly washed out with water, and the sections were exposed in a watch-glass with frequent stirring to the action of the fluid to be experimented upon, mounted and examined.

The following are the principal results of those experiments :—

With absolute alcohol all the threads were broken, great contraction taking place, attended by great alteration in the shape of the rounded central mass of protoplasm, which now assumed an irregular as opposed to a regular spherical form with a smooth contour.

With 1 per cent. osmic acid in the same way the sharply rounded contour gave place to an irregular, uneven outline, and general swelling of the protoplasm occurred. All the strings were broken. The nucleus, however, was well preserved, though somewhat swollen. It is possible that either a stronger solution of the acid or osmic acid vapour would be more successful.

One per cent. chromic acid, with the exception perhaps of an alcoholic solution of corrosive sublimate, gave the least satisfactory results. None of the threads were preserved, and the nucleus and protoplasm had undergone great alteration of form.

A saturated watery solution of picric acid, on the other hand, gave very satisfactory results indeed. With this reagent the nucleus was especially prominently brought into view, and the protoplasm had undergone the least change. Though in many cases obvious shrinking was produced, yet as a rule the rounded contour was well preserved, and many threads remained unbroken (see Plate 70, fig. 38). Silver nitrate after plasmolysis with nitre, and gold chloride were also tried, but with little success.

As a result of these experiments it would appear that none of these reagents are entirely successful. In every case the protoplasm, even if killed at once, undergoes more or less shrinking, attended with great alteration of form. My results as to absolute alcohol agree with those of FLEMMING, who also finds that saturated picric acid, and 1 per cent. chromic acid, are preferable fixing agents for nuclear investigation. As to chromic acid our results differ. But whatever the reagent used, it is quite apparent that it is easier to deal with young cells, full of protoplasm, with very

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* Proc. Roy. Soc., November 11, 1882.

small vacuoles, or no vacuoles at all, than with large full-grown cells when large vacuoles are present.

In the latter case there is every opportunity for contraction, and there is moreover always a tendency to a dilution of the fixing fluid by the cell-sap. This being the case, any successful results with full-grown cells may be regarded as very favourable evidence for the efficacy of the reagent employed. In order to eliminate any doubt as to whether the salt solution influenced the result, analogous experiments were made with fresh tissue. The great drawback to the thorough efficiency of picric acid is that it wets the tissue with some difficulty and only penetrates after some time. This fact becomes very apparent when large pieces of tissue are used at any time of difficult permeability attendant on peculiar histological structure. A saturated solution of picric acid in absolute alcohol to some extent obviates this difficulty, but it is not so successful as a saturated watery solution, although it appears to be a valuable reagent for ordinary work.

With regard to other manipulative details, it is, as mentioned above, important to cut up the material into small pieces, and also to place it at once upon cutting in the preservative medium. My usual plan, in fact, was to cut off the pulvini and allow them to drop, then and there, into picric acid, in order to avoid any loss of water due to evaporation, which as far as delicate investigations are concerned will soon very gravely affect the whole cell-equilibrium. After treatment with picric acid for about 24 hours the material is removed, rapidly washed with water, and placed in alcohol, the latter being changed until the yellow coloration of the picric acid is no longer obtained. Any method of preservation is, however, very imperfect. Not only is appreciable contraction produced, but a great amount of rigidity of the protoplasm occurs due to coagulation and death. These considerations and results determined me to use fresh material, which I employed afterwards all through the investigation.*

* In connexion with the experiments upon fresh material the results obtained with Spirogyra are of ome interest. They confirm those alluded to in the text. Absolute alcohol was shown to be an utter ailure. Watery picric acid was the best reagent, preserving the lenticular form of the nucleus, and demostrating the threads going to the chlorophyll bands with great success. A saturated solution of picric acid in absolute alcohol is to be preferred next, but it causes definite shrinking. The great point, however, that these experiments made evident was that throughout the entire process of preservation and staining it is necessary to keep all the solutions as nearly as possible of the same density and to avoid any rapid diffusion. Thus if it be required to put up a preparation of Spirogyra, one can first fix the cell with saturated watery picric acid. Then wash in dilute alcohol and stain with either dilute ammonia-hæmatoxylin or a dilute alcoholic solution of one of the aniline dyes. Any dense staining solution will at once cause shrinking. But after this point comes the difficulty. Dilute or strong glycerine will at once cause great shrinking, whatever be the precautions employed, and the only way which is apparently left open to adopt is to mount in such a medium as camphor water, which will cause swelling, in a dilute solution of potassium acetate or calcium chloride, or, still better, in dilute alcohol. To the latter there is the obvious objection that it will act upon most of the varnishes that are used to surround the cover glass and so work its way out. I should suggest as a varnish in this case, a strong solution of gelatine in glacial acetic acid, but hitherto I have not been able to try whether it would work. These results are, however, worth consideration.

Method of preparation.—Experiments have shown that in order to demonstrate in the most satisfactory manner the perforation of the cell wall by protoplasmic threads, it is usually necessary that the wall should be either swollen or dissolved.

Both these methods have already been successfully made use of in the case of sieve tubes by SACHS, who employed, as his reagent, strong sulphuric acid; and by HANSTEIN, who used Chlor. Zinc Iod. In both cases iodine served as a stain for the protoplasm. In investigating the subject of protoplasmic continuity, I have made use of both these methods, but with important modifications. Sulphuric acid is naturally by far the more powerful: strongly swelling or dissolving the cell wall, and laying bare, as it were, the protoplasm to the action of staining reagents, while Chlor. Zinc Iod., on the other hand, when possible, is always preferable, on account of its less vigorous action, attended with less distortion of relative arrangement.

The method used by SACHS for demonstrating the actual perforation of the sieve-plate is essentially based upon the difference of reaction of strong sulphuric acid towards the cell-wall and the protoplasm. The former is partially dissolved, or excessively swollen, while the latter remains but little acted upon, and can be readily stained and examined. The usual plan has been to mount a thin section of tissue in dilute iodine solution, and when sufficiently stained, strong sulphuric acid was run in, and the observation was made. Or the section was first stained with iodine, and then mounted in sulphuric acid. But there are some objections to this method. First, the sulphuric acid is run in, once for all, and thus its action cannot be regulated. Secondly, the iodine from its very colour is not a sufficiently deep stain. Further, the cellulose blue produced; the precipitation of the iodine; and the rapid disintegration of the tissue due to the powerful action of the acid; cause the method to be only satisfactory in such cases as sieve-tubes, where the continuity is pronounced and the material favourable, for here the cell-walls easily dissolve, and the middle lamella is but little developed.

The modification I have adopted has been to divide the process into two parts, and to substitute aniline colours for the iodine. I propose to give a detailed account of the whole process.

A thin section of fresh material is taken up on a platinum spatula; the water is removed with blotting-paper, and a drop of strong sulphuric acid is dropped upon it by means of a glass rod. When the acid has been allowed to act for a determinate time (some seconds), depending on the nature of the tissue and the extent to which the action is required to be carried, the section is rapidly washed by immersing the spatula in a quantity of water contained in a large watch-glass, at the same time stirring, so as to wash out the acid as quickly as possible, and stop its action. Thus the sulphuric acid can be kept entirely under control. After about two changes of water the section may be at once stained, or put into alcohol for future use.

The length of time that the acid requires to act naturally varies with the nature of the material used. Thick-walled tissue requires longer treatment than thin-walled, and the permeability and peculiar characteristics of the cell-wall in question must be taken into consideration; the difference of reaction being in different cases very great. If, however, the action be properly regulated, the cell-wall will be much swollen; the protoplasm will undergo a certain amount of contraction, but, at the same time, will not be withdrawn from the cell wall at those points where any intimate union exists between the wall and the protoplasm. The middle lamella will, of course, not be destroyed. If the action be allowed to proceed further, the protoplasm itself will be attacked, the cell-wall will begin to dissolve, the middle lamella will also swell; and when in this condition will stain very deeply with reagents, thus making any satisfactory observation impossible.

Experiment shows that unless the action is decidedly forced, the cell wall, though apparently dissolved, does not in reality undergo complete solution, but is only swollen and diffluent. That this is the case may be proved by treating a washed out section with Chlor. Zinc Iod., when the ordinary blue cellulose reaction will be obtained.

The probable action of the sulphuric acid upon sections of the fresh material may now be dealt with. In the first place, the protoplasm is apparently at once killed, although, at the same time, decided shrinking occurs, owing to the great dehydrating power of the reagent. This shrunken appearance is, however, somewhat magnified, because, in addition to the contraction produced by the rapid abstraction of water, the protoplasm has also been squeezed and pressed upon, on all sides, by the swelling cell-wall. But the point which must be especially strongly brought into prominence, is the fact that during the swelling any close relation which may exist between the protoplasm and the cell-wall appears to be maintained, at least where such relation is at all pronounced. Thus in cases where reactions with Chlor. Zinc Iod. and iodine show that the closing membrane of a pitted cell is perforated by fine protoplasmic threads, it will be found that when such a cell is treated with sulphuric acid, the protoplasm projecting into the pit, and especially that portion of it abutting on to the closing membrane, will firmly adhere to the latter, and will resist, without rupturing, a very considerable strain; and even if rupture should at length take place, it will seldom, if ever, occur close to the pit membrane. In attempting to explain the appearances produced by the action of strong sulphuric acid, one must clearly bear in mind that there are two factors to be considered, viz.: the rôle of the protoplasm and the rôle of the cell wall. At the same time there is going on, not only a shrinking of the one, but a swelling of the other. Two principal objections may be very reasonably brought forward to explain the fact, that the protoplasm adheres to the pit. First, it may be said, that the protoplasm is retained and even injected into the pit by the pressure of the swelling wall. That this objection will not hold is apparent from the fact that the same phenomenon occurs in the case of cells which have been cut into. Furthermore, the swollen wall frequently does not abut directly on to the protoplasm, but a considerable space intervenes between the two. Again, by the action of strong dehydrating agents a further shrinking of the protoplasm may be induced. The second and more important objection is, that the narrowing of the pit, on account of the swelling, imprisons and firmly embraces the protoplasm in the pit cavity. In answer to this, it may be urged that the shrinking of the protoplasm takes place more quickly than the swelling of the wall, and that the protoplasm projecting into the pit would have time to withdraw before being imprisoned. In deep pits of small diameter, it is indeed possible that the narrowing of the cavity does play some definite part, but whether this be so or not, experiment proves that the protoplasm also adheres to pits which are shallow, and moreover possess sloping sides. In this case, any such explanation could hardly be brought forward. Lastly, it must be remembered that the action of the sulphuric acid is carefully regulated, and is not carried to an extreme limit, and that the results obtained with this reagent have been fully confirmed with Chlor. Zinc Iod. All the preceding remarks as to the action of sulphuric acid apply only to the cases in which fresh material is used, since here the protoplasm has not been rendered brittle by any preliminary treatment with reagents, and consequently has undergone as little alteration as possible, and will not break when any slight tension is set up.

After treatment with sulphuric acid, and washing out with water, the section may be stained with iodine, as in the usual process; but I used in preference, and with greater success, analine dyes, especially the violet and blue.

In my earlier experiments I used HOFMANN'S violet (*Trimethyl rosanilin*) as the staining reagent. In the first place, HOFMANN'S violet is a dye which, of all others, is extremely rapid in its action, quickly and thoroughly permeating the tissues. Again, it works extremely well with sulphuric acid, being soluble in, and hardly affected by this reagent, as far as all its staining properties are concerned. Thus one need not take such care to wash out the acid before staining; for, although when the proportion of acid is large the HOFMANN'S violet is temporarily turned green, yet on subsequently washing with water before mounting in glycerine, the violet colour is restored. The whole process may, indeed, be done in one operation, for the solid dye may be dissolved in strong sulphuric acid; the mixture furnishes a dark brown-yellow solution. The section is now simply treated with the mixture, and then washed well with water. The above method gives extremely satisfactory results with sieve-tube preparations; and, moreover, any lignified tissue which happens to be present is coloured gold yellow, as in the ordinary aniline sulphate reaction.

To the use of HOFMANN'S violet there is, however, the great objection that the whole of the tissue—protoplasm, cell-wall, middle lamella, and pit-membrane—is stained. If, however, the stained section be treated for some long time (three to four days), with dilute glycerine the dye in the cell-wall, middle lamella, and pit-membrane dissolves out, whereas that staining the protoplasm remains but little acted upon. This lengthy manipulation is an obvious objection, but nevertheless HOFMANN'S violet often gives extremely satisfactory preparations, and by mounting the section in strong glycerine, the middle lamella may be made almost transparent; and when in such a condition will no longer present any hindrance to successful observation.

But the better and more reliable reagent is HOFMANN'S blue.* As a result of numerous experiments I am able to state that this dye is a particularly satisfactory reagent for staining the protoplasm alone, and as such is of extreme value for botanical research, and supplies a long-felt want. I find that it works best after treatment with picric acid, and that unless the solution in alcohol be too strong or the staining be decidedly forced, there will be little if any coloration of structures other than protoplasm. But when HOFMANN'S blue is used, the washing-out of the sulphuric acid must be carefully attended to, for the two will not work together as in the case of HOFMANN'S violet. After staining, the section is well washed with water and mounted in dilute glycerine. Such was the method I used in my investigation upon the structure of pulvini. Having thus dealt at some length with sulphuric acid, I must now proceed to describe in the same way Chlor. Zinc Iod.

The action of this reagent is well known. It causes a swelling up of the cell-wall, and at the same time colours the cellulose blue. It is, however, much less violent in its action than sulphuric acid, causing but little distortion of form or displacement of relative arrangement. There is simply a slow and regulated swelling. Sections may be at once treated with Chlor. Zinc Iod., or may be first stained with iodine which helps as it were to accentuate its differentiating powers. The easy manipulation attending the use of Chlor. Zinc Iod., its high refractive index, and the satisfactory manner in which its gradual action may be observed, cause it to be one of the most valuable reagents employed in botanical research. For the demonstration of the presence of protoplasmic threads running through the thickness of the cell wall. TANGL† first used Chlor. Zinc Iod. in his investigation upon the endosperm cells of *Strychnos, Phœnix,* and *Areca.* The sections were first stained with iodine and then

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* Under the somewhat loose term aniline blue, are frequently included and described by writers a number of salts which are obviously perfectly distinct, both as regards chemical and physical characters. For example, there is soluble or water blue, insoluble blue, gentiana blue, phenylene blue, benzyl blue, methylene blue, cyanine or chinoline blue, HOFMANN'S blue, besides others, Bavarian blue, Capri blue, &c., some of which are patented products of the various aniline-dye manufacturers. Frequently any given dye obtained from one maker will absolutely differ in staining properties from that of the same name obtained elsewhere. This being so, it is necessary to state very clearly the exact name and maker of any of the so-called aniline blues that may be made use of. I first used HOFMANN'S blue at the Würzburg laboratory, and to the kindness of Professor SACHS I am indebted for the information that it is known as HOFMANN'S blau (anilin blau), and may be obtained from MORELLI, Druggist, &c., Semmel Strasse, Würzburg. A tolerably strong solution is made in 50 per cent. alcohol, to which is added a drop or two of acetic acid. After staining, the section is washed with water and mounted in dilute glycerine or glycerine jelly. I find that when dyes are dissolved in solutions containing a higher percentage of alcohol than that named above, most dyes will lose their selective power for particular structures, and will begin to stain all tissues alike. These results are confirmed as regards animal tissues by Dr. MAYER, of the Naples Zoological Station. See Mt. Zool. Stat., Neapel ii. (1880), pp. 1–27.

† *Loc. cit.*

mounted in Chlor. Zinc Iod. The cell-wall is coloured yellow; the protoplasm and the protoplasmic threads a dark brown. The success of the reaction depends upon the fact that when cellulose has experienced a loss of water, and has become dry as in the case of ripe endosperms and other dry tissues, the Chlor. Zinc Iod. will not at first give the usual blue coloration. It is only after some considerable time that the section will begin to turn blue, or if the cell-wall be very thick or very dry, the blue colour may not be produced at all, but only a yellow brown, which is frequently increased in depth by the precipitation of iodine attending such lengthy action. Sections of a germinating seed of *Phytelephas* furnish an excellent proof that the assumption of the blue colour on treatment with Chlor. Zinc Iod. depends upon hydration, for whereas the normal cells will give a yellow colour, those which are being encroached upon by the absorbent foot of the growing embryo, and are being broken down and at the same time thoroughly wetted, will give in a peculiarly characteristic manner the customary blue cellulose reaction.

But usually in sections of ripe endosperms the cell walls become yellow, and the protoplasm colours dark brown. In most cases nothing can be seen at first of any threads in the cell-wall, but after some time (varying from a quarter of an hour to one hour) they gradually come into view and are moreover apparently increased in size by the gradual precipitation of iodine upon them due to the action of the Chlor. Zinc Iod.

One great objection to this method is that when fresh tissue or thin walled tissue is used, the ordinary blue cellulose reaction occurs which totally obscures the threads from view, and makes all observation of no avail. Moreover, no permanent preparations can be made. My first idea was to employ the same modification as I had adopted in the case of sulphuric acid and use HOFMANN'S blue instead of iodine. With this, however, I at first experienced some difficulty. TANGL* found in *Strychnos* that when he had swollen the walls with water and could see the threads with dilute iodine solution, he was unable to stain them with any ordinary dye, such as hæmatoxylin or carmine. In the same way I found that after the action of iodine and Chlor. Zinc Iod. I could not succeed in staining the threads with any solution of aniline colours. In consequence of this I made a number of experiments with *Strychnos*. I found that although no dyes would demonstrate the threads, yet with solutions of such coloured bodies as gold chloride, picric acid, chromic acid, and iodine, they became more or less clearly apparent. It will be noticed that all these substances are well-defined crystalloids, whereas most of the aniline colours are inclined to be colloidal or, at least, are crystallised with some difficulty. This suggested that the whole phenomenon was simply a matter of diffusion, the solutions of crystalline bodies apparently permeating the substance of the cell wall (crystalloid), but especially diffusing into the protoplasm (colloid), and in the same way the solution of the colloidal aniline dyes diffusing but little or not at all into the colloidal protoplasm. In consequence of this conclusion I made the experiment of dissolving the solid HOFMANN'S

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* *Loc. cit.*

blue in a 50 per cent. alcoholic saturated solution of picric acid, under the supposition that the latter might mechanically carry with it into the tissue the dissolved aniline dye, and that on washing out the picric acid with water, the protoplasmic threads would be left stained. Such treatment I found to be perfectly satisfactory, the threads running through the cell-walls and, indeed, the whole of the protoplasm being stained blue, while the cell-wall either remained quite uncoloured or, if the action was forced, coloured but slightly, and to a much less extent than the protoplasmic threads which could still be easily recognised.

As regards the action of picric acid, experiment seems to show that in addition to being one of the most valuable preservative media, it also restrains the coloration of the cell-wall by solutions of such a dye as HOFMANN's blue, which though a special stain for the protoplasm, will upon lengthy action stain the cell-wall also. Thus, if two sections be stained, one of alcohol material and the other of material which has been treated with picric acid previous to preservation in alcohol, in the former the cell-wall will be definitely stained, while in the latter little, if any, coloration will occur. Thus, the action of the picric acid is of twofold significance, not only serving as a vehicle for the passage of the HOFMANN's blue into the minute protoplasmic filaments, but also restraining at the same time the coloration of the cell-wall.

It now only remains for me to describe in detail my method of manipulation with Chlor. Zinc Iod. and the picric acid solution of HOFMANN's blue. Sections of fresh material are cut in water and placed in ordinary iodine solution until they are well stained. They are then taken out by means of a platinum lifter, the iodine solution is removed with blotting paper, and they are mounted in Chlor. Zinc Iod. In those cases where the blue colour is not produced until after some time, it is usually possible to see something of any threads that may be present, and thus many very conclusive observations may be made prior to staining with picric-HOFMANN's-blue. In fact, treatment with iodine will often bring out clearly many points that HOFMANN's blue will not, such as a satisfactory demonstration of the passage of the threads through the substance of the middle lamella, where the lamella is well developed. Further, the threads appear thicker than with HOFMANN's blue, and in any case the treatment will give some idea of what one may expect to see after staining with the aniline dye. The time that Chlor. Zinc Iod. requires to act depends greatly upon the character of the tissue. With many dry endosperms and other cells with thickened walls it will take as long as twenty-four hours to thoroughly permeate the tissue. In my own experiments I was in the habit of mounting in Chlor. Zinc Iod. on one morning and staining on the following day. If not allowed to act for a sufficiently long time, it will be found that while some portions of the walls are swollen, others are hardly acted upon, and that the difference of refractive index of these two portions will give a very confusing appearance to the whole section, and very greatly hinder successful observation. Experiment alone can decide the time required for the complete action;

the only point of importance requiring attention is that the reagent should be allowed to act long enough.

After treatment with Chlor. Zinc Iod. the section is well washed in water until the blue or brown colour (as the case may be) has disappeared. It is then placed for about a quarter of an hour in the picric-HOFMANN's-blue solution, and after being well washed is mounted in glycerine or glycerine jelly.

As before mentioned, the staining solution is made as follows: To 100 cub. cent. of strong alcohol (*e.g.* about 90 per cent. strength) is added an equal bulk of distilled water. The resulting solution is saturated with picric acid, and HOFMANN's blue is added, until the liquid is of a dark blue-green colour. It is then filtered.

I used the method with Chlor. Zinc. Iod. almost entirely in my researches upon the structure of endosperm cells. I did not discover the picric-HOFMANN's-blue modification until I had finished my work with pulvini. Thus, the results with pulvini rest mainly on the sulphuric acid modification.

On the nature of the pit membrane.

If a thin section of almost any tissue be treated with Chlor. Zinc Iod. it will be seen that the walls of almost every cell are distinctly pitted.* These pits are brought into prominence from the fact that whereas the thicker unpitted portions of the cell wall give a well-defined cellulose blue reaction, the thin pit-closing membranes stain but slightly, or in some cases apparently remain quite colourless.† Indeed, so common is this pitting, which the above-mentioned reaction demonstrates, that it would be a statement little short of the truth to say that every cell whatsoever, is pitted to a greater or less degree. Moreover, the closing membrane of the pit itself may also be pitted, as in the seed of *Lupinus hirsutus*, &c. As a rule, it is only in the case of thin walled cells that it is necessary to apply any reagent to bring this pitting into view, for the more the cell wall increases in thickness the more pronounced does the pitting become, until its appearance is at length so marked that we are accustomed to speak of it as a pitted cell *par excellence.* It is a point of special interest to note that

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* Cells of *Strychnos*, *Dioscorea*, and *Tamus* are notable exceptions.

† Even in cases where an *en face* view of the pit will give the impression that no coloration has occurred, a transverse section will show that in reality it is slightly stained, and contrasts as markedly with the deep blue stain of the thick wall as the same blue staining of a young cambium cell does with that of the mature, full-grown cell which is subsequently produced. The pits in the parenchymatous cells of the petiole of *Cycas revoluta* are of special interest here. There are, as it were, two systems of pits. The larger, which are arranged in rows up the sides of the cell, face the intercellular spaces, and stain deep blue with iodine and Chlor. Zinc Iod. The smaller pits between the communicating cell walls, on the other hand, do not stain perceptibly when viewed from above. (See Plate 68, fig. 12.) DE BARY mentions pits of *Cycas* and *Encephalartos* which give a callus reaction. (See 'Verg. Anatomie,' p. 125. See also RUSSOW's important paper, "Ueber Tüpfelbildung," &c. Sitzber. der Dorpat Natur., 1882, pp. 350-389. RUSSOW is inclined to think that no staining of the pit membrane occurs.)

in two adjoining cells [*] any unequal thickening that may occur always takes place symmetrically on either side of the first formed cell-wall, and in such a way that the two pits which are formed in consequence are exactly opposite one another.

Among many other examples, the thickened cells of hard endosperms and the parenchymatous tissue of all pulvini exhibit this structure to a high degree, and since it was probable that by means of these pits a communication between adjacent cells was established, the study of the nature of the pit-membrane becomes one of great importance. The result of experiments with various staining reagents may now be detailed.

As at first mentioned, Chlor. Zinc Iod. usually stains the pit membrane but little.[†] Instead of treating the section with this reagent alone, better and more decisive results may be obtained by first soaking the tissue in iodine, then rapidly washing to get rid of the extraneous iodine which would otherwise be precipitated over the tissue, and then mounting in Chlor. Zinc Iod., or the section may be first treated with Chlor. Zinc Iod., then washed and mounted in iodine solution. This gives good results in cases where protoplasm is left sticking to the pits as in the parenchyma cells of the pulvinus of *Amicia* or the endosperm cells of *Bomarea.*

Methyl violet gives very striking, and at the same time is apt to give very deceptive results. When a washed out section of pitted tissue that has been exposed to the action of sulphuric acid is treated with this reagent the whole of the tissue becomes rapidly stained. The protoplasm is coloured a deep purple; the cell-wall is stained violet; and the closing membrane and sides of the pit are brought into prominence since they assume a purple colour, somewhat lighter than that of the protoplasm. The middle lamella also stains deeply. Now in a much-pitted tissue, *e.g.*, that of a pulvinus, the cell wall after treatment with sulphuric acid usually becomes much swollen, causing an elongation and at the same time a narrowing of the pits, and may, moreover, in its swollen condition closely invest and surround the protoplasm. When such a section is treated with methyl violet, the deeply stained tubular pits, being placed symmetrically opposite one another on either side of the common cell wall, abut on the shrunken and similarly stained protoplasm, and give the impression that a distinct and well-defined continuity exists from cell to cell. Thus in *Amicia* the most beautiful and striking appearance is produced which is further heightened by the fact that processes from the main protoplasmic mass usually go for some distance into the pits. (See Plate 68, fig. 10.) If, however, the section be treated for some time with dilute glycerine, the colour is dissolved from the cell wall and the pits and the protoplasm alone remains stained, thus making the real state of things apparent.

The reaction with methylene blue is perhaps the most characteristic. When a section is stained with this reagent before treatment with sulphuric acid, the cell wall and the pit membrane will be deeply coloured, the protoplasm being left unstained.

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[*] I exclude from this statement such cases as that of a cell adjoining a vessel, &c. See 'SACH'S Text-book,' English edition, 1882, p. 26.

[†] In old cells with thick pit membranes the staining of Chlor. Zinc Iod. is, however, very apparent.

If, however, the section be first treated with the acid, then washed, and stained with methylene blue, only the closing membrane and the sides of the pits will be stained (see Plate 68, fig. 8), unless the action of the sulphuric acid be forced. Both the protoplasm and the rest of the cell-wall undergo scarcely any coloration. *Thus methylene blue, apart from its great value as a stain for cell-wall, becomes by this modification a reagent for pit membrane. Naturally HOFMANN's blue stains neither the cell-wall nor the substance of the pit membrane. The whole results of my investigation appear to point to the conclusion that the staining of this reagent is specially confined to the protoplasm. In the case of many palm endosperms, where after the action of Chlor. Zinc Iod. and picric-HOFMANN's-blue threads can be observed going through the closing membrane of the pits, it is the threads which are specially stained, and are in consequence defined from the substance of the pit membrane. Indeed, so characteristic is the staining of HOFMANN's blue, that experiment seems to point to the conclusion that in those cases where a staining of the pit membrane occurs, such staining points to the presence of protoplasm.

There is, however, another special structure which is also stained by HOFMANN's blue, and which can be distinguished from protoplasm by its solubility in strong sulphuric acid, viz.: the callus of sieve-tubes. It was RUSSOW† who first used aniline blue as a reagent for callus, and even combined it with Chlor. Zinc Iod. As I did not know what particular blue RUSSOW used, I made a number of experiments with the various blues I had at my disposal,‡ with the result that the special staining of the callus was confined to two of them, viz.: HOFMANN's blue and water or soluble blue, one of which it is pretty certain that RUSSOW employed. Water blue is only second to HOFMANN's blue in that it also especially stains protoplasmic structures. Now the properties of callus are somewhat peculiar.§ WILHELM showed that it was soluble in sulphuric acid, and insoluble in ammoniacal oxide of copper. In the former respect it resembles cellulose, and indeed its mode of formation—arising as symmetrical warts on either side of the cell wall—as described by JANCZEWSKI,|| and confirmed by STRASBURGER,¶ certainly give some colour to this idea.** On the other hand, unlike cellulose it

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* After keeping the section for a long time in dilute glycerine, staining of the protoplasm does take place since the glycerine dissolves the dye. This solution ultimately stains the protoplasm.

† RUSSOW, 'Sitzber. Dorpater Nat. Ges.,' 1881, April 23, and 'Bot. Ztg.,' 39, 1881, p. 723.

‡ HOFMAN's blue stains protoplasm and callus. Soluble or water blue, ditto. Benzyl blue, protoplasm cell-wall and callus, like rest of cell wall. Insoluble aniline blue, *i.e.*, solution in spirit, as benzyl blue. Neither of the latter appear to stain the pits. Methylene blue, stains cell-wall and pit membrane Phenylene blue resembles methylene blue. Both water blue, methylene blue, and HOFMANN's violet may be obtained at MARTINDALE's, New Cavendish Street, Portland Place, London. The rest of the dyes that I used were obtained from the Actien Gesellshaft für Anilin Fabrication, Berlin.

§ 'Beiträge zur Kentniss des Siebröhrenapparatus Dicotyler Pflanzen.' Leipzic, 1880.

|| 'Études comparées sur les tubes cribreux.' Cherbourg, 1881.

¶ 'Bau und Wachsthum,' p. 56, *et seq.*

** This was first noted by WILHELM. *Loc. cit.*, p. 16.

is insoluble in ammoniacal cupric oxide, and moreover it gives with Chlor. Zinc Iod. not the customary blue but an intense red-brown coloration. Lastly, the result with HOFMANN'S blue appears to point to a protoplasmic character, opposed to which conclusion is the fact that it dissolves in sulphuric acid. Thus the question appears to be, whether it is related to protoplasm or to cellulose, or whether it consists of a modified cellulose basis permeated by a protoplasmic structure.* This, however, minute study of development alone can decide, but the point I wish to bring forward is the fact that it is coloured by dyes which especially stain the protoplasm.

There is a curious parallelism in the action of callus towards HOFMANN'S blue and of pit membrane towards methylene blue, after treatment with the same reagent (viz.: sulphuric acid) which may perhaps be worth mention.

If a section of a second year stem of *e.g.*, *Vitis vinifera*, be treated with HOFMANN'S blue it will be found that both the protoplasm and the callus will be stained. If, however, sulphuric acid be allowed to act before staining, the callus will naturally be dissolved and will no longer colour, and only the protoplasm will be left stained.

If in the same way a section of pitted tissue, *e.g.*, pulvinus of *Robinia*, be treated with methylene blue, both the cell wall and the pit membrane become coloured. But if the section be first treated with sulphuric acid, the swollen or dissolved cell wall will remain unstained and only the closing membrane and the sides of the pits will alone be stained blue.

Now, if it be allowed that callus may be regarded as altered protoplasm, it might be suggested from the foregoing reactions that cell wall is to be looked upon as altered pit membrane, or rather that pit membrane is to be regarded as consisting of cell wall that has retained its original properties and has undergone comparatively little chemical change. However, I prefer at present to draw no definite conclusions from these observed phenomena but merely desire to put forward the facts.†

On the structure of pulvini.

Having thus treated of the methods employed, and made some remarks as to the nature of the pit membrane, I am now in a position to proceed with the description of my investigation of the structure of pulvini. This work was commenced in the Würzburg laboratory in the month of July, 1882, under the direction, and at the suggestion, of Professor SACHS.

I studied in detail the pulvini of *Mimosa pudica*, *Robinia pseudacacia*, *Amicia*

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* See RUSSOW'S observations on Abies Picta, 'Stzb. d. Dorpat. Naturf. Gesell.,' 1881, p. 70. Also STRASBURGER, 'Bau und Wachsthum,' p. 60. RUSSOW ('Stzb. d. Dorpat. Naturf. Gesell.,' Feb. 17th, 1882), like myself, in contradistinction to JANCZEWSKI ('Mem. de la Soc. des Sc. Nat. et Math. de Cherbourg.,' vol. xxxiii., p. 209, 1882), believes that the reactions of callus point essentially to its protoplasmic nature.

† I find in reality that the above reaction of the pit membrane with methylene blue takes place in consequence of the fact that the membrane is more resistant than the rest of the cell-wall. Whether this is in consequence of the presence of protoplasm in its structure remains to be proved. This STRASBURGER also found to be the case. See 'Bau und Wachsthum,' p. 16.

zygomeris, and *Phaseolus multiflorus*. I do not intend in the present paper to enter into any discussion with regard either to the nature of irritability or the phenomena of movement of which these plants serve as illustrations, but merely to confine myself to such structural detail as is necessary for the clear comprehension and significance of my results. The principal literature of the subject has been collated by PFEFFER,[*] to whose researches and those of SACHS[†] we owe the greater part of our knowledge of what is one of the most interesting phenomena of plant life.[‡]

Mimosa pudica.—As a rule the main pulvini at the base of the petiole of the leaf were chiefly made use of on account of their larger size and consequent easier manipulation. The secondary and tertiary pulvini, however, gave the same result. Thin longitudinal and, as far as possible, axial sections of the fresh material were taken, since the unequal contraction and puckering-up of the tissue due to the tensions produced by the violent action was not so great as in transverse sections.

As regards its anatomical structure the pulvinus shows a thin vascular bundle surrounded by a thick layer of parenchymatous cells. The epidermis is not pronounced and the epidermal cells have undergone very little, if any cuticularisation. For the most part confined to the underside of the pulvinus are several long stiff multicellular hairs.

Immediately under the epidermis the cells are small, as are those immediately surrounding the bundle, and between these two layers occur the cells of maximum size (see Plate 68, figs. 1 and 3). From the hypodermal cells inwards the intercellular spaces, which are at first inconspicuous, become more and more apparent, until in these cells around the bundle itself a system of large communicating air spaces exist (Plate 68, figs. 3 and 4). The vascular bundle is arranged on the concentric type, the phlœm being outermost and surrounding the xylem. In the phlœm the walls of the prosenchymatous cells are greatly thickened and very highly refractive; the middle lamellæ between them are also almost inconspicuous (see Plate 68, fig. 1), the structure of which is similar to that of *Mimosa*. The cell-walls of the upper half of the pulvinus are thicker than those of the lower, which moreover is the side towards which the bending takes place, and this rule is followed in the secondary and tertiary pulvini also, viz.: that the cells of the side which becomes concave on bending, have always thinner walls than the side which becomes convex, so that whereas in the main pulvini the underside has the thinner walls, in the pulvini of the leaflets the reverse is the case. The parenchymatous cells each contain a number of chlorophyll granules and a nucleus. One or more drops of tannin are also present,[§] which can be well seen,

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[*] PFEFFER, 'Physiologishe Untersuchungen,' 1873, i. id.; 'Die periodischen Bewegungen der Blattorgane,' 1875; see also 'Pflanzen Physiologie,' 1880.

[†] SACHS', 'Handb. der Exp. Phys.,' 1866, p. 479, *et seq.*

[‡] See also DARWIN, 'Movements of Plants,' 1880.

[§] I could not detect the special pellicle mentioned by PFEFFER. See SACHS' Text-book, p. 889.

by staining the section with methyl violet, and washing with alcohol. The dye is then dissolved from everything but the tannin drops. With osmic acid they also stain a blue-black and with chromic acid a brown-yellow. The latter reagent, however, affects the protoplasm as well, and thus does not allow the individual drops to be distinguished.

On treating with Chlor. Zinc Iod. it becomes apparent that the parenchymatous cells are freely pitted, each such pit being so little stained as to appear quite transparent, thus presenting a marked contrast to the ordinary deep cellulose blue of the rest of the cell-wall. The pits, as a rule, are somewhat shallow and of small diameter except in those cells bordering on the vascular bundle, which from their peculiar configuration in consequence of the presence of large intercellular spaces exhibit on their walls pits of much larger size (Plate 68, fig. 4). The pits are greater in number on the longitudinal than on the transverse walls. The thin pit membrane between two cells is, except under very favourable circumstances, extremely difficult to observe in the unstained condition. It is perhaps brought out most clearly by staining the protoplasm with HOFMANN's blue when the unstained wall will be seen as a thin colourless membrane separating the protoplasm of one cell from that of the other.

Even with the most favourable section there is no indication of the existence of any connexion between the protoplasm of neighbouring cells. Deep staining with iodine or with HOFMANN's blue shows the outline of the protoplasm to be well defined and sharply limited by the cell-wall at all points.

But if the wall be swollen with sulphuric acid, and after washing stained with iodine, methyl-violet and glycerine, or HOFMANN's blue, it will become apparent that a definite communication between the cells does exist, and that such communication is established by means of the pits (Plate 68, fig. 5). The appearance of a well-prepared section is extremely characteristic, reminding one to some extent of a gold chloride preparation of corneal connective-tissue cells. The protoplasm, as it has contracted away from the cell-wall, has adhered to the membranes of the pits, at those points in the cell-wall where pits are present; and in consequence, the whole section presents the appearance of a number of stained and interconnected irregularly-shaped stellate masses, for the narrow processes of any one mass unite at their apices with those proceeding from the neighbouring masses, thus exhibiting a well-defined reticulate arrangement. The reason that the processes proceeding from the masses of two contiguous cells are opposite one another obviously depends upon the symmetrical development of pits on either side of the cell-wall (Plate 68, fig. 3). But that the relation between two such processes is of the most intimate character is quite evident from the fact that in many instances it appears that an optical continuity exists between them, thus establishing a means of communication between cell and cell (Plate 68, fig. 5).

Successful sections are somewhat difficult to prepare, for if the sulphuric acid does not act sufficiently long, the cell-wall is either little or not at all swollen, and when in this condition cannot be permeated by the dye, and if the action has been allowed to proceed too far, the protoplasm is attacked and the delicate connexions soon become obliterated. Moreover, at the same time the middle lamella becomes swollen and will deeply stain, which of all things is to be avoided. A regulated action of the acid gives the best results, and if the protoplasm be not sufficiently shrunk to show up the processes to the best advantage, the section need only be mounted in strong glycerine which will soon bring about the desired effect. Even in one and the same preparation, though a successful one, the acid may have acted unequally, due, it may be, to varying thickness of the section, and thus the different results produced by the acid may be observed at the same time. However, in a well-prepared section, where the action of the acid has been properly regulated, plain examples of continuity are apparent. Upon longer treatment, the further shrinking of the protoplasm causes a greater tension to be exerted upon the processes and rupture ensues. This frequently occurs, but the rupture nearly always takes place on one or both sides of the point where the thin thread-like process crosses the middle lamella, and seldom at the point itself. Thus the threads cannot be said to be merely pulled out of the pits, for rupture takes place in such a manner that a longer or shorter length still remains in the pit cavity.

Finally, when the action of the acid has been carried too far, the processes appear to have been partially destroyed, and but few can be traced as far as the swollen and now deeply-stained middle lamella. Many of the processes appear to be directly and uninterruptedly continuous from cell to cell, whilst others are swollen at the point where they cross the middle lamella. In other cases between the two ends of the strongly-stained processes there is a lighter-stained portion, which connects the two This lighter-stained area exhibits a haziness and appears to be somewhat indistinct, although well defined from the rest of the swollen cell-wall, and clear enough not to be confounded with the middle lamella (Plate 68, fig. 5). Again, when the protoplasm is but slightly contracted, and but little tension has been exerted on the threads, the point of junction of the two threads is both slightly swollen and also coloured darker than the rest. In spite, however, of the fact that in several cases direct continuity appears to exist, I am strongly of opinion, both from analogy and from such appearances as I last described, that in reality a sieve-plate arrangement occurs. It must be borne in mind that the difficulties of examination are great, both on account of the smallness of the pits and the thinness of the pit membranes, but in any case we cannot imagine that the threads go bodily through the pit, for were it so, the pits would not possess a closing membrane, and ordinary staining would soon demonstrate the existence of the protoplasm, by which the pit was perforated.

Although PFEFFER's[*] results appear to prove that it is the underside of the pulvinus which is especially sensitive, I have not been able to establish any difference between them as far as histological evidence goes. Nor is this to be greatly wondered at, for the method for such a discrimination is essentially rough, and one would hardly

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[*] See SACHS' 'Text book,' p. 889.

expect such physiological differences to be made apparent by a somewhat coarse histological treatment. Certain cells occur scattered about in the tissue which are both larger and stain more deeply than their neighbours, but the latter phenomenon may be, and probaby is, caused by the presence of tannin. In the parenchyma of both the upper and lower side of the pulvinus the connexion appears to be more pronounced in the cells of the middle layer than it is in those either next the epidermis or next the vascular bundle; and since the cells are more freely pitted on the longitudinal than on the transverse walls more connexions exist through those of the one than of the other.

With regard to the middle lamella there is some difficulty, unless very careful preparation is adopted. It will in any case stain, the depth of the staining depending upon the action of the acid and of the dye, and if the treatment with the one or with the other be forced, the great coloration of the lamella will so obstruct the view that it will be impossible to see with certainty whether or not a distinct continuity of the protoplasmic processes occurs. The difficulty may, however, in a great measure be removed by long treatment with strong glycerine, which both dissolves the greater portion of the colouring matter from the lamella, and at the same time renders it sufficiently transparent for a decisive observation to be made.

The bast fibres of *Mimosa* are of peculiar interest with regard to this question. The middle lamellæ between these cells are so little developed that they are recognised with some difficulty, and which is an important fact, do not stain at all. Consequently, the additional factor of difficulty, that the presence of a well-developed middle lamella involves, is here done away with. Each bast cell is freely pitted, the pits of neighbouring cells being placed symmetrically opposite one another. When treated with sulphuric acid and stained in the usual manner the following appearance is produced. The pit membranes being somewhat thick have distinctly swollen, and in so doing have increased the distance from one another of the ends of the protoplasmic processes projecting into the pit cavity. All the processes are deeply stained, and between each symmetrically opposite pair is a small less stained portion traversing the pit membrane, which from its reactions must be protoplasm. Thus it stains with iodine, and when coloured with methyl violet is not dissolved by glycerine (Plate 68, fig. 6). It is also well brought out by HOFMANN's blue, the staining characters of which have already been sufficiently dwelt upon. That it is not callus is clear from the fact that it does not dissolve in sulphuric acid. The structure traversing the pit membrane is somewhat difficult to observe, both on account of its very small size and its want of definition. Indeed, it rather presents the appearance of a small blue cloud between the ends of the deeply-stained and well-defined processes.

The protoplasmic processes projecting into the pits are broad at their extremities, and are at the same time more deeply coloured at that point. They gradually taper off from the bottom of the pit inwards, widening again as they join the general protoplasm of the cell. Each pair of processes with their above-mentioned broad ends and the cloud between them forcibly suggest a sieve-tube arrangement, and from analogy, as I shall point out later on, I believe that such is the case. I have, however, with the highest powers at my disposal been unable to resolve the stained structure traversing the pit membrane into fine lines as I had hoped to do, although the whole appearance is most strongly suggestive of a striation, the direction of which is parallel to the long axis of the pit. However, it seems certain that there is a protoplasmic communication which can be plainly seen, and is not complicated by the presence of a stained middle lamella.

Thus it appears that from the epidermal cells right up to the last living bast fibre which impinges on the first dead vessel a direct continuity from cell to cell has been established, and that such a pulvinus may be regarded as a connected whole.

Robinia pseudacacia.—As in *Mimosa*, thin axial, longitudinal sections of the main pulvini were examined. Fresh material was used in every case, and after treatment with sulphuric acid the sections were stained with either HOFMANN'S violet and glycerine, or with HOFMANN'S blue. In fundamental structure the pulvinus of this plant resembles that of *Mimosa*. Rough examinations show that it is much larger, and that its surface is quite smooth and free from hairs. The cells do not appear to be so freely pitted, nor is tannin so abundant. In many of the cells which are scattered about the tissue, and are smaller than their neighbours, are crystals of calcium oxalate, which can be well seen embedded in the protoplasm of the containing cell. The cells in certain cases possess more than one nucleus. The nuclei are large and well developed, and are brought into prominent view in the case of tissue which has been previously treated with picric acid.

After treatment with sulphuric acid it can be seen that, in a well prepared section, the cells present very much the same appearance as those of *Mimosa* (Plate 68, fig. 7). The continuity existing between the processes is not as pronounced as in the former case, and the appearance of threads going straight and uninterruptedly through the pits is not so frequent (Plate 68, fig. 7). On the contrary, there is more indication of the existence of a sieve-plate-arrangement, which is very marked in those cases which admit of successful observation. Frequently at the point of junction of two processes there is a distinct and well defined swelling which stains perceptibly lighter than the very darkly stained threads, which it connects one with the other. It can clearly be distinguished from the pit membrane and the middle lamella, and can almost certainly be resolved into a striated appearance, although the observation cannot perhaps be regarded as perfectly conclusive or perfectly satisfactory. In consequence of the presence of fewer pits on the cell walls the interconnecting protoplasmic processes are fewer in number than in *Mimosa*. The bast fibres present the same appearance as those of *Mimosa*, although the appearance, on the whole, is not so marked. The secondary pulvini display essentiallyt he same structure as the main pulvinus of the whole leaf, with the exception that the number of tannin cells is very much greater.[*] When stained with chromic acid the protoplasm of the tannin cells exhibits a distinct appearance of reticulation, but from what cause I am at present ignorant.

Amicia zygomeris.—The pulvinus of this highly interesting plant was pointed out to me by Professor SACHS as well worthy of investigation. As a most striking example of both periodic and irritable movements this plant has apparently escaped general observation. It is particularly sensitive to alternations of day and night, and assumes the sleep position long before even such plants as *Robinia.* If violently shaken the leaves will, after a time, fall, and will be similarly affected some time after being cut and placed in water: the large size of the leaves rendering the least movement very conspicuous. Since it was the secondary pulvini that were especially movable, and they were, at the same time, of a comparatively large size, I used them in preference to still larger main pulvinus.

The chief characteristics of the pulvinus tissue of *Amicia* are the thinness of the walls of the parenchymatous cells, the extremely unlignified character of the vascular bundle and the remarkable development of a system of large pits, which is in this case extremely pronounced. The whole tissue is very succulent, and easily admits of thin sections being cut (Plate 68, fig. 9).

On treating with iodine and Chlor. Zinc Iod. the pits are, as usual, markedly brought into view. From the contrast of the deep blue coloration of the walls with that of the pits, it appears, at first sight, that no staining of the latter has taken place. Sections transverse to the pit, however, show that both a very slight staining of the pit-membrane has occurred, and that the membrane is extremely thin. In some of the pits small masses of protoplasm may be recognised sticking to the pit-membrane, being brought into view in consequence of their brown staining reaction. The pit-membrane is well stained by methylene blue. Scarcely any difference can be detected between the thickness of the cell-walls on the upper and under sides of the pulvinus. Except just beneath the epidermis, and next the vascular bundle, the cells are relatively large. The layer of protoplasm (primordial utricle) lining the cell-wall is thin, and the central vacuole is large. In consequence of this, very great shrinking of the protoplasm is possible, and experience shows that the successful preservation of this tissue is extremely difficult. Any reagents causing the least diffusion very soon affect the protoplasm, and the only at all successful treatment is brought about by

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[*] The fact deserves notice that in the cases where the protoplasm displays any great activity of function, the cells of such a tissue usually contain tannin. For example, pulvini of *Mimosa Robinia*, *Desmodium*, &c., leaf of *Dionœa*, *Drosera.* Again in galls, where a stimulation of the protoplasm followed by rapid growth occurs. Notice that in *Robinia* it is the pulvini of the leaflets that move more than the main pulvinus, which have the greater quantity of tannin. The effect of tannin in producing aggregation is dealt with by A. F. W. SCHIMPER ('Bot. Zeit.,' 14, 1882) On Tannin. See also GARDINER "On the General Occurrence of Tannin in the Vegetable Cell, and a Possible View of its Physiological Significance," Proc. Camb. Phil. Soc., vol. iv., pt. vi., pp. 387–394, and Bot. Central. Bd. xvi., No. 48, p. 258.

means of saturated, watery, picric acid. Absolute alcohol is quite unsatisfactory. In the same way sulphuric acid causes very great contraction, the processes being usually ruptured, and nearly always pulled perceptibly from the pit membrane.

The cell-walls possess that peculiar semi-horny structure which is equally shared by so many of the Leguminosæ, and swell greatly with sulphuric acid. The delusive and at the same time very beautiful effects obtained by staining a section after treatment with sulphuric acid with methyl violet have already been dealt with under the head of pit membrane (Plate 68, fig. 10). As there mentioned, the bottom and sides of the pits are markedly stained by this reagent in a way somewhat similar to that of the protoplasm, and at first sight the appearance suggests that the cells are freely connected—the one with the other—by unbroken protoplasmic threads. The whole structure is remarkably like that of an enlarged representation of free cell formation. However, on treating the section with glycerine, all the deception disappears with the solution of the colouring matter, and it will then become apparent that in reality the connexion is neither so well defined nor so pronounced as in the case of *Mimosa* and *Robinia* (Plate 68, fig. 11). In no instance, so far as I can ascertain, do the processes approximate to one another in the unbroken way in which they appear to do in *Mimosa*. In the larger cells occupying the middle layer between the epidermis and vascular bundle, the protoplasm is either entirely pulled from the pit membrane, or the processes which at first connected the protoplasm of the pit with the general protoplasmic mass are ruptured so as to leave a short portion only sticking to the pit membrane. In the four last layers of cells which abut on to the vascular bundle where the cells are smaller with thicker walls and smaller vacuoles, it will be seen that although shrinking has taken place, yet the whole appearance of contraction is not so great although the protoplasm projecting into the pits has, as a rule, been pulled from the pit membrane. In the swollen pit membrane between the two symmetrically placed processes the same stained structure is apparent as occurs in the bast fibres of *Mimosa*, although the whole appearance is much more marked (Plate 68, fig. 11). The stained portion as before, and more markedly, suggests an appearance of striation, but with the strongest powers at my disposal, consistent with clear definition, I was unable to resolve the structure into fine threads. All that one can say is that its reactions point to its protoplasmic nature. In such of the other cells of the larger celled tissue as could be favourably observed, the same structure was present. It may here be mentioned that in cases, *e.g.*, *Phœnix* (Plate 69, fig. 13), where with a high power a sieve plate arrangement can be seen, and the threads clearly made out; with a low power the same appearances are produced as in the case in point, or as in bast fibres, and it seems probable that here a sieve plate arrangement does in reality occur.

The staining of that portion of the pit membrane which colours with aniline blue or which is left stained by methyl violet after prolonged action of glycerine, must not be confounded with the coloration of the bottom and sides of the pit which occurs with methyl violet alone or with methylene blue after the action of sulphuric acid. In the latter instance the pit, as a whole, stains. In the former it is the staining of a substance other than pit membrane which runs through the latter, and which, by its different reactions, is to be separated from the pit membrane itself. Its reactions, as before-mentioned, point to a protoplasmic character.

Experiments were made with other pulvini and other organs of similar character, the results of which are detailed below. The experiments were somewhat hurried as the season was late, and although, to the best of my belief, the results are accurate, yet I do not regard them as perfectly conclusive, and I must work over the subject in detail on a future occasion.

Phaseolus multiflorus appears to be connected as *Amicia*.

Desmodium gyrans resembles *Mimosa* in structure.

Dionæa muscipula.—Sections of the tissue next the vascular bundle showed the cells to be connected as in *Mimosa*. In the epidermal and sub-epidermal layers this structure was especially evident, and some processes were seen uniting the glands with the cells.

Stamens of Cynara.—The lengthy oblong cells surrounding the central bundle appeared connected one to another principally through their end walls, in a manner almost exactly resembling that of a sieve tube. Apparently some connexion between them also took place through the side walls.

Tendrils.—In the oblong cells of the tendrils of *Bryonia*, a similar sieve-tube-like arrangement appeared to occur, especially on the end walls.

On the structure of endosperm cells.—From some points of view I could not regard the results I had obtained with pulvini as either perfectly satisfactory or perfectly conclusive. In spite of a probability little short of certainty, some doubt still remained; for it could be brought forward, that in the first place the results had been obtained by means of an extremely powerful reagent, with whose action we were by no means intimately acquainted; and, secondly, that we had no such examples of the general perforation of the pit-membrane by protoplasmic threads. And even allowing that the pit-membrane was traversed by fine threads, the great question that required answering was—Do these threads in reality cross the middle lamella, or is it only a case of the membrane itself being pitted, and the threads running up to the lamella, but no further?

In order, therefore, to put my results on as firm a basis as possible, it was necessary to experiment with my methods upon any such cases as might exist, where the passage of protoplasmic threads through the cell-wall was a confirmed fact, or to endeavour to establish, in a manner which admitted of no doubt, other instances of the existence of similar phenomena.

The first and most obvious examples of the occurrence of the perforation of the cell-wall are naturally afforded by sieve-tubes, and, in consequence, I began by investigating the results produced upon such structures by the reagents which I had employed in the case of pulvini.

In this direction I found that the method was in every way peculiarly adapted to show the intimate structure of sieve-tubes. In the course of my investigations on pulvini I had frequent opportunities for observing sieve-tubes, *e.g.*, in *Mimosa*, *Robinia*, &c. In both the above-named cases the sieve-tubes are very small, but treatment with sulphuric acid, and subsequent staining with methyl violet and glycerine, or HOFMANN's blue, brought out these structures very successfully, and defined in an extremely clear manner the very fine threads connecting the contents of neighbouring tubes. The sieve-tubes of *Dahlia variabilis*, *Ricinus communis*, and *Phaseolus multiflorus* were also investigated. In *Ricinus* the youngest sieve-cells where perforation had not yet taken place were clearly demonstrated. In *Phaseolus* the general occurrence of a lenticular highly refractive body in the sieve-tube cavity was noticed, but I must defer a description of it until a future occasion.

But in the end the fact became apparent that although the results obtained with sieve-tubes gave very valuable proof of the success of the method I had adopted, yet that their structure could not be exactly compared to that of the parenchymatous cells of pulvini. Thus, in sieve-tubes, the cell-walls tend to assume a soft and somewhat mucilaginous character, and in them the middle lamella is but little developed, and the whole wall readily dissolves in sulphuric acid.

In the cells of the pulvinus, on the other hand, the walls greatly resist the action of the acid, and the development of the middle lamella is essentially pronounced.

There was, however, still one road left open, and that was to investigate the structure of thickened endosperm cells where all the requisite conditions were present, and what was of greater importance still, where the pit membrane was extremely thick, and would be likely to show plainly the existence of threads traversing its substance.

Some results had already been obtained in this direction, for TANGL,* in 1880, in his paper on "Open Communication between the Cells of Endosperms," had shown that in *Strychnos Nux-vomica*, *Phœnix dactylifera*, and *Areca oleracea*, a communication between the protoplasm of neighbouring cells was established by means of fine protosplasmic threads running through the cell-wall. In *Strychnos* the walls were thick and devoid of pits, and the presence of the threads was not confined to any particular portions of the cell-wall, but they occur over the whole area. In *Phœnix* and *Areca*, on the other hand, it was by means of pits that the connexion was brought about; the pit membrane being perforated in a manner very much resembling that which takes places in a sieve-tube.

TANGL's results with *Strychnos* were fully confirmed by STRASBURGER,† but in the case of *Phœnix* and *Areca*, he states that he was unable to see the threads with the clearness conveyed by TANGL's figure, and although he says that the pit membrane of *Phœnix* is demonstrably porous, yet the general tone of his statements

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* *Loc. cit.*

† *Loc. cit.*

give one the idea that he has not been fully able to satisfy himself as to the structure by direct successful observation.

I then resolved to repeat for myself TANGL'S experiments, and also to investigate in as thorough a manner as possible the endosperm tissue of other species of Palms, and of other seeds of a similar nature. This work was carried on in the Jodrell Laboratory of the Royal Gardens, Kew, during the first three months of the present year.

Of the Order Palmæ I have examined the seeds of typical representatives of a great number of the genera, and I have, in addition, investigated the structure of the endosperm of members of the following Orders, viz.: Leguminosæ, Rubiaceæ, Myrsineæ, Cornaceæ, Loganiaceæ, Hydrophyllaceæ, Iridaceæ, Amaryllidaceæ, Dioscoriaceæ, Melanthaceæ, Liliaceæ, Smilaceæ, and Phytelephasieæ.

A mere glance at the foregoing list will be sufficient to show that a very large number of seeds were required, although from the great resources of the Royal Gardens I found no difficulty in obtaining typical representatives of any of the genera, and I cannot speak too highly of the great kindness I received on every side, from the Kew authorities, both in rendering me every assistance, and enabling me to obtain whatever material I was in need of for my investigation. Especially do I owe a debt of gratitude to W. T. THISELTON DYER, Esq., the Assistant Director, not only for the help I always received from him, but also for the kindly interest he took in my work all along.

Of the methods I employed I have already spoken in the earlier part of this paper. The usual plan I adopted was to cut with a razor, wetted with water, thin sections of the seeds, which were then stained with iodine and mounted in Chlor. Zinc Iod. Usually they could be examined at this stage; the exceptions being in those cases where the pit membrane rapidly assumed the blue cellulose coloration. After the prolonged action of Chlor. Zinc Iod. they were washed in water stained with picric-HOFMANN'S-blue, and after a second washing in water were mounted permanently in glycerine (strong or dilute) or glycerine jelly.

In certain cases some slight modification of this process had to be resorted to, which was occasioned by the peculiar characteristics of the tissues in question. Thus, for example, in such endosperms as *Strychnos* or *Tamus*, where great swelling takes place upon treatment with water, the sections were cut in alcohol, stained with alcoholic iodine, and after treatment with Chlor. Zinc Iod. were washed with dilute alcohol; stained, and mounted in strong glycerine after having been well stirred in glycerine, on taking out of the staining fluid, instead of washing with water, although usually quick washing with water will succeed equally well. Again, where the pit membrane was thin, and taking up water soon became coloured blue with Chlor. Zinc, Iod., and would only for a short time retain its primary yellow coloration: such tissue was also stained with dilute alcoholic iodine.

The strength of the iodine must be altered as the nature of the material requires, Thus *Phytelephas* or *Lodoicea* require a strong solution of iodine, while *Ruscus* or *Colchicum* will quickly assume a dark yellow with a solution of a comparatively weak strength. Treatment with sulphuric acid was also resorted to, not only as an alternative method, but also for the purpose of confirming my results with pulvini; although, from the very great thickness of the walls and the consequent enormous swelling which occurs, it was found that, as a rule, Chlor. Zinc Iod. was the preferable reagent for ordinary use.

On repeating the observations already made upon *Strychnos Nux-vomica* my results fully confirmed those of TANGL and STRASBURGER in every particular; and in thin and carefully-prepared sections it can be plainly seen that the threads do cross the middle lamella. Like TANGL, I was unable to stain the threads with reagents in the usual manner; and, in consequence, I instituted those experiments which led me to adopt that particular modification of dissolving HOFMANN's blue in picric acid, and using it as a stain, which I have already dwelt upon in the earlier part of this paper. When by the use of alcohol the extreme swelling which takes place upon treatment with water is prevented, sections may be stained with picric-HOFMANN's-blue, and after mounting in strong glycerine may be successfully observed.

With regard to *Strychnos potatorum*, I am disposed to agree with STRASBURGER that a sieve-plate-arrangement does exist between the pits, for a striation could certainly be made out. However, the seeds I had to work upon were extremely old, and as such I look upon the results obtained with them as unsatisfactory.

As regards the structure of *Phœnix dactylifera*, when treated with iodine and Chlor. Zinc Iod. I came to the same conclusions as Professor STRASBURGER* that, although a striation could be observed, the threads were not nearly so clear as TANGL's drawing represents, and, indeed, were made out with difficulty. After treatment with sulphuric acid, washing, and then iodine-staining, they were defined much more clearly; but the best and in every way most satisfactory results were obtained by staining the washed-out sections with HOFMANN's violet and glycerine. In the latter case the stained protoplasm was contracted, and running through the pit membrane could be seen well-coloured threads presenting a distinct sieve-plate-arrangement (Plate 69, fig. 13).

In the same way *Areca oleracea*, usually known as *Euterpe oleracea*, at first gave a very feeble result when treated in the usual way; but, after a great number of trials and the use of strong iodine, and a prolonged action of Chlor. Zinc Iod., delicate threads could be plainly observed, which, moreover, appeared to cross the little developed middle lamella.

Having obtained the above-named results I commenced the examination of a number of palm seeds and of other seeds possessing a similar structure, in the hope of being in the end able to make some statements as to their general histology and to determine how far such a structure was of general occurrence. The following is the list of the palm seeds examined. I am indebted to Sir JOSEPH HOOKER for kindly

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* *Loc. cit.*

looking over this list for me and not only making several valuable alterations, but also adding the authority for each species.

Arecineæ.
Areca triandra. Roxb.
Areca Catechu. L.
Stevensonia grandifolia. Duncan.
Rhopalostylis sapida. W. and D.
Howea Belmoreana. Ben.
Kentia costata. Ben.
Archontophœnix Cunninghamii. W. and D.
Euterpe oleracea. Mart.
Euterpe edulis. Mart.
Hyophorbe Verschaffeltii. Wendl.
Synechanthus fibrosus. Wendl.
Didymosperma distichum. H.f.
Pinanga latisecta. Bl.
Heterospathe elata. Scheff.
Caryota urens. L.
Manicaria saccifera. Gærtn.
Cyrtostachys Renda. Bl.
Calyptrogyne Swartzii. H.f.
Calyptrocalyx spicatus. Bl.
Chamædorea tinella. Wendl.
Prestœa pubigera. H.f.
Ceroxylon andicola. H. and B.
Oncosperma horridum. Seem.
Lepidocaryeæ.
Calamus calicarpus. Griff.
Mauritia flexuosa. Linn f.
Calamus fissus. Bl.
Plectocomia Himalyana. Griff.
Pirgafetta elata. Becc.
Raphia Hookeri. M. and W.
Borasseæ.
Latania Loddigesii. Mart.
Lodoicea Sechellarum. Mart.
Geonoma vaga. Griseb and Wendl.
Bentinckia Conda-panna. Berry.
Corypheæ.
Thrinax, sp.
Corypha elata. Roxb.
Licuala Rumphii. Bl.
Livistona Hoogendorpii. T. and B.
Washingtonia filifeva. Wendl.
Sabal umbraculifera. Mart.
Rhapidophyllum Hystrix. W. and D.
Cocoineæ.
Cocos nucifera. L.
Cocos flexuosa. Mart.
Bactris, sp.
Astrocaryum rostratum. H.f.
Syagrus botryophora. Mart.
Martinezïa Aiphanes. Kl.
Maximiliana cariboea. Gr. and W.
Desmoncus, sp.
Martinezïa caryotifolia. H. and K.
Guilelma speciosa. Mart.
Diplothemium, sp.
Phytelephasieæ.
Phytelephas macrocarpa. R. and P.
Phœniceæ.
Phœnix dactylifera. L.

In all the above seeds a direct means of communication between the cells of the endosperm was observed.

Confining myself at first to the Palmæ, I would point out that in their structure the various seeds present every possible modification both of thickness or thinness of the pit membrane, of clearness or difficulty of observation, of variations in the size of the cell, and in degree of development of the middle lamella.

In making the subsequent observations with reference to the study of a number of instances of one and the same phenomenon, I propose to deal with the subject in a somewhat general manner and to illustrate my statements by such typical examples as will best serve my purpose.

I. *Development.*—In no case have I worked out the development of any of the seeds that I have examined. On account of want of time and opportunity, it is, therefore, a subject which must be reserved for another occasion. I would only draw attention here, to the striking similarity which the arrangement of the protoplasmic threads joining the cells of such endosperms as *Strychnos, Tamus*, or *Dioscorea* presents to the same arrangement of achromatin fibres which accompanies the development of the similar structure in *Agrimonia Eupatoria*,* and the close resemblance of the barrel form, so beautifully shown by *Heterospathe, Bentinckia*, or *Lodoicea*, to the like form assumed by the fibrillæ between the dividing nuclei in such endosperms as *Caltha palustris*,† or, indeed, in cell division in general. As TANGL‡ remarks, it seems as if the fibrillæ persisted during the subsequent cellulose formation and deposition. The appearance, perhaps, suggests that such is the case, and that the particles of cellulose have been deposited around the threads. It may also be noticed that no instance of a reticulate arrangement of the threads has been observed. In any case it is apparent, of course, that grave alterations must be occasioned by subsequent growth and increase in size of the cells, but anything certain development alone can decide.

II. *Structure of young endosperm cells.*—A number of observations were made upon the young endosperms of *Archontophœnix Cunninghamii, Sabal umbraculifera*, and *Rhopalostylis sapida.* In all these cases it was found that when the cell was still living, as could be seen from the presence of a well-developed nucleus, the connexion between the cells was fully maintained, and therefore that communication had existed in any case from a very early period (Plate 69, figs. 14 and 15).

As the cells grow older profound changes take place in the protoplasm, which usually result in the death of the cell. In order to ascertain the fate of the nucleus, portions of ripening seeds of *Archontophœnix elegans* and *Rhopalostylis sapida* were treated for twenty-four hours with saturated watery picric, and well washed with alcohol, until the yellow colour of the acid had quite disappeared. Sections were cut, which were stained with hæmatoxylin, and mounted in dilute glycerine. It was then apparent that well stained nuclei were present in the cells occupying the central portion of the seed (Plate 69, fig. 15), and as one gradually traced the staining effects from within, outwards, it was seen that the nearer the periphery, the less conspicuous became the cell-nucleus, until in the outermost layers no trace of a nucleus could be detected; its substance staining less and less, and its outline becoming more and more badly defined. Thus it apparently suffers a complete disorganisation.

Along with changes of the nucleus proceed alteration of the protoplasm. In many seeds—*e.g., Phytelephas*—but little protoplasmic substance appears to remain in the cell. Oil very frequently occurs as a cell content, and sometimes is present in large quantities, especially in the Cocoineæ—*e.g., Cocos, Bactris*, &c. Small crystals may also occur, and in such examples as *Diplothemium, Syagrus*, and *Corypha* aleurone grains are met with. In the cases which I examined with special reference to the

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* STRASBURGER, 'Zellbildung und Zelltheilung,' Tafel I., fig. 15.

† *Loc. cit.*, Tafel II., fig. 31.

‡ *Loc. cit.*

question the cells appeared to be quite dead, and as such they are simply preyed upon by the growing embryo. Thus, all the changes which result in their subsequent breaking down proceed from the embryo itself.

Special experiments were made with sulphuric acid, in order to observe its action in cases where a continuity was known to exist. In fresh living cells treatment with sulphuric acid, and staining with methyl violet and glycerine or HOFMANN'S blue, showed that, although the protoplasm had contracted, those portions projecting into the pits still adhered to the pit membrane, and that the threads of protoplasm running through the pit membrane were continuous on either side with the above-mentioned symmetrically opposite processes (figs. 14 and 15). The processes, in fact, appear to be held to the pit membrane by the threads in question in all cases where the continuity is pronounced. Under a low power the individual threads could not be distinguished, and the appearance then presented was that of two darkly-stained threads united by a lighter-stained area running between them—in fact, the very appearance presented by *Mimosa* and *Robinia** (*cf.* figs. 5, 7, 14, and 15). In the case of ripening seeds, the protoplasm may be made to contract slightly from the membrane, and then a similar phenomenon is induced to that which occurs in *Amicia* and the bast cells of *Mimosa* (*cf.* figs. 11 and 13), although in them it is not occasioned by loss of vitality, but rather from the fact that the threads are probably extremely fine and the continuity not so pronounced as it is in the case of the parenchyma cells. In fully-ripe seeds where the cells are dead, the protoplasm always contracts away from the cell-wall, and a similar state of things usually occurs when the cell has been killed by the action of reagents (see figs. 16 and 22). Thus, both my method and my results have received very satisfactory confirmation and elucidation.

III. *General results with ripe endosperms.*—As a rule, most of the seeds I examined were either one or, at most, two years old. I also made use of some museum specimens, but decided to reject the results I obtained with them, as I had reason to believe that in many cases those results were abnormal. As regards their favourable or unfavourable character as material for showing the perforation of the cell-wall by protoplasmic threads, seeds greatly differ one from another. In the first place, it may be stated, as a general rule, that the thicker the pit membrane the easier can the threads be distinguished. In very thin pit membranes the observation of such threads as may cross it requires great precaution and care; there is nothing, so to speak, for the eye to catch upon, and one has to detect a line within a line. It is this very fact that causes endosperm tissue to be so favourable for such an investigation as the present one; for here, not only, as a rule, are both the cells and the pits unusually large, but, what is much more important, the pit membranes are thick. In many cases, however, this is not the case, and an examination of such examples as *Manicaria*, *Mauritia*, or *Caryota* is quite sufficient to prove that the

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* The results with *Bomarea* also confirm this.

successful observation of the threads crossing a thin pit membrane is a matter of extreme difficulty; and it also serves to show that in other cases where the pits are very small and the membranes very thin this difficulty is so increased as to become almost an impossibility.

But apart from any consideration of the pit membrane, the ease or difficulty of observation also appears to depend greatly upon the peculiar characteristics of the seed itself. For instance, as I stated at the outset, *Phœnix dactylifera* and *Euterpe oleracea* are inclined to be unfavourable material. In *Euterpe edulis*, on the other hand, the connecting threads can be easily demonstrated. Of numerous other instances, *Geonoma*, *Plectocomia*, *Areca triandra*, *Areca catechu*, and *Cocos nucifera*, afford examples of cases where difficulties of observation occur.

Among the most favourable material for examination are the endosperms of *Bentinckia*, *Stevensonia*, *Thrinax*, *Heterospathe*, *Syagrus*, *Corypha*, *Howea*, and *Lodoicea* (see figs. 16, 17, 18, 22, and 25). The degree of development of the middle lamella varies greatly. As a rule, in thickened endosperms it attains but little development, or, even if this be not the case, it stains but little, and its refractive index varies only slightly from that of the general cell-wall. In *Calamus*, *Sabal*, *Raphia*, and *Ptychosperma*, the lamella is decidedly pronounced. In such cases as *Stevensonia* and *Calamus* (Plate 69, fig. 24) both the middle lamella and the threads are well developed; and though in them there is some difficulty in determining whether the middle lamella is actually perforated by the protoplasmic threads, yet, as a rule, careful examination and preparation will decide that in the vast majority of cases it can be seen that such perforation does occur, and such examples as *Heterospathe*, *Kentia*, *Mauritia*, or *Bentinckia* do away with all possible doubt (figs. 16, 19, 22, 23). As to the manner in which the communication between the endosperm cells is established, experiment shows that there are two possible ways which essentially depend upon the configuration of the cell.

In such exceptional cases as *Strychnos*, *Tamus* (Plate 70, fig. 33), and *Dioscorea*, where the walls are extremely thick, and, at the same time, devoid of pits, the communicating protoplasmic threads run through the cell-wall. A section of such an endosperm exhibits the threads, which are seen freely perforating the wall, except at the corners of the cell, at the point where the junction of several cells occurs.

The usual mode of union, however, is by means of pits. As I have mentioned elsewhere, the presence of pits in the cell-wall, due to unequal thickening, is of almost universal occurrence, and it is through the closing membranes of such pits that the protoplasmic threads run. This, in fact, appears to be by far the most common and typical way in which the continuity of the protoplasm of adjacent cells is brought about.

In other, and perhaps less frequent cases, examples of both modes of connexion occur; the communication taking place not only through the pits, but through the walls as well. I have observed that this happens in *Kentia Belmoriana*, *Kentia costata*, *Lodoicea*, *Bentinckia*, and *Asperula* (see figs. 16, 17, 19, 31); but I am led to believe that such union is of much more general occurrence. In all the foregoing examples, the threads running through the walls are more especially obvious in the cells just below the surface, and gradually become less and less visible, as one approaches the central tissue of the seed. This appears to me to be simply an arrangement for insuring that every facility should be given for the passage of nutritive material from without inwards, and also that it should have opened to it as many channels as possible. It is obvious, for instance, that the amount of plastic formative substance required for building up such a tremendous endosperm as that of *Lodoicea* must be very considerable, and even supposing its growth to be slow, the drain on the nutritive material must be large, and the rate of its flow must be very great. Consequently the increased facilities for easy transmission must be of great advantage. And not only in the development of the endosperm, but also upon germination, is this structure of great use to the plant, for at that period the outer cell-layers will have become very dry, and consequently the difficulty of their being broken down by the absorbent foot of the cotyledon will be increased. But at the same time, owing to the greater development of a system of channels in them, they are more easily permeated and wetted by the cell sap holding in solution the ferment which will bring about their final disorganisation.

The form presented by the aggregate of threads traversing the pit membrane is usually that of the well-known basket or barrel-shape which is met with in connexion with nuclear division. In many instances, and especially in *Bentinckia*, the shapely sweep of the curving threads, and the graceful arrangement of the whole thread-complex is extremely striking and beautiful (Plate 69, fig. 16). In other cases the bending of the curve is not so marked, and in very thin pit membranes—*e.g.*, *Synechanthus*, *Livistona*, &c.—the threads appear to be altogether straight.

In the instances where the threads go through the cell-wall their direction is seldom straight, but usually bent, and resembling in arrangement the appearance presented by the achromatic fibres during free-cell-formation.

Every variation occurs both as regards the size of the cells, the distribution of the pits, and the number and thickness of the threads. Thus, whereas the cells of *Caryota urens* and *Lodoicea* (Plate 69, fig. 19) are large, those of *Thrinax* and *Geonoma* are small. In such endosperms as *Manicarea* and *Chamædorea* the pits are very numerous, while in *Washingtona* but few are present. In *Calyptronoma* the threads are few and somewhat stout, while in *Oncosperma* they are very numerous and fine. In *Bentinckia* and *Heterospathe* they are also many in number. The threads are made very conspicuous by staining with iodine and treatment with Chlor. Zinc Iod., for the latter reagent appears to cause a decided precipitation of iodine upon them as well as upon the general protoplasm which is accompanied by an increase in apparent diameter. That this is actually the case may be demonstrated either by reversing the operation and staining with iodine after treatment with Chlor. Zinc Iod. and subsequent washing, or by staining with picric-HOFMANN'S-blue. (Compare Plate 69, figs. 22 and 23.)

As a rule nothing can be seen of the threads when a section of endosperm tissue is mounted and stained in the usual manner. But to this statement *Bentinckia* affords an exception, for here an appearance of striation can be detected, and in *Stevensonia* staining with HOFMANN'S violet alone makes the threads apparent. Treatment with iodine, picric acid, or with a mixture of iodine and glycerine will also often bring them into view, *e.g.*—*Lodoicea* (Plate 69, fig. 20), *Latania*, and *Bentinckia*.

Experiments with the object of injecting the threads with colouring solutions met with no success. Pieces of the endosperms of *Latania* and *Calamus* were fitted into a bored india-rubber cork, which was then tightly fastened into one end of a manometer tube, the shorter arm of which contained the solution of the colouring matter, and the longer held the mercury by means of which the injection-pressure was induced. First, a solution of water-blue in water was employed, and as this caused swelling of the wall a solution of insoluble blue in alcohol was used in preference. However, when exposed to the pressure of a column of mercury of sixty inches no injection occurred.

Besides the particular methods I have chosen for the elucidation of this subject, many others were tried with little or no success. Sections of *Bentinckia*, as being favourable material, were treated in the usual way with solutions of gold chloride and silver nitrate, but with no result. In every case it was found that it was necessary to swell the cell-wall before staining. After swelling with Chlor. Zinc Iod. and washing, silver nitrate was again tried, and this time with some small amount of success. I adopted a modification of treating the section with sulphuretted hydrogen-water, after exposure in a 2 per cent. solution of silver nitrate for half-an-hour, and subsequent washing, instead of reducing the silver by the action of light, as in the usual process. The result was perhaps better, but still far from satisfactory, and would be quite inapplicable in any case where the threads were not particularly well developed. Some sections, after swelling, were treated with an alcoholic solution of tannin, and when washed were shaken up with a solution of ferric chloride. The wall then coloured the usual blue-black, and the colourless threads could be seen fairly well. Other sections, again, were soaked in a solution of ferric chloride, and after washing were treated with a solution of potassium ferrocyanide. In this case the threads were less clearly defined than with tannin and iron. Lastly, sections were treated for some time with a solution of corrosive sublimate, in the hope that an insoluble compound might be formed with the remains of the cell protoplasm. After washing the section was shaken up in sulphuretted hydrogen-water, but with no good result. Thus all these experiments pointed to the fact that my methods, if not perfectly satisfactory, were at least fairly successful.

In concluding the subject of Palm endosperms I might make a few remarks upon some particular examples which appear to be of equal interest.

In all the seeds nearly related to *Calamus* the structure is very typical (Plate 69, fig. 24). I have already noticed the great development of the middle lamella in these endosperms. Another interesting fact is, that in *Calamus* and *Metroxylon* a well-marked cubical crystal is present, imbedded in the wall of each cell. It seems as if there had been a period in the life of the cell when the protoplasm had required to get rid of some of the calcium oxalate resulting from the metabolic activity of the protoplasm. This was consequently thrown down in the form of a crystal which adhered to the cell wall, and in the subsequent thickening which occurred, was gradually covered in until it was at length surrounded on all sides by the cellulose.

Lodoicea sechellarum is of interest, not only as affording one of the clearest examples of the perforation both of the wall and pit membrane, but also because of its very unique distribution (Plate 69, fig. 19).

In *Oncosperma* the threads are excessively fine, and certainly suggest the extreme probability of the existence of threads which are so delicate as to be invisible (Plate 69, fig. 21). In fact, I am inclined to believe that this really is so in such endosperms as *Cocos nucifera*. To this seed *Martinezia caryctifolia* presents a useful transition. With iodine it can be seen that very fine threads do go through the almost smooth walls, but upon treatment with picric-HOFMANN's-blue the individual threads cannot be distinguished, and only a blue coloration occurs. In *Cocos*, which has essentially the same structure, I was unable to observe threads, though I cannot doubt that such threads do exist. In all the *Cocoineæ* the walls are thin and must be carefully examined. They are, however, of extreme value, both from the point of view of analogy and comparison. In *Syagrus* (Plate 69, fig. 25) and *Desmoncus* the threads are well seen. *Heterospathe elata* is a particularly favourable endosperm for demonstrating the perforation of the middle lamella, which here is but little developed (Plate 69, figs. 22 and 23). The threads appear very clearly with iodine. In *Phytelephas*, although the walls are extremely thick the pits are small, the pit membrane somewhat thin, and the threads are demonstrated with difficulty. The cells contain but little remains of the protoplasm, and several results have induced me to think that the amount of solid matter in the perforating thread channels is so small that the channels are practically empty (Plate 70, fig. 26).

These results seem to show that in all the Palmæ the structure of the endosperm cells is similar.

Endosperms other than those of Palms.—As I have remarked elsewhere, the endosperm of Palm seeds is particularly favourable material for an investigation of the perforation of the cell wall by protoplasmic filaments. And, speaking generally, in the examination of most endosperms other than those of the Palmæ, additional difficulties are presented which greatly interfere with successful observation. Especially does it become apparent that the thickness of the pit membrane is not nearly so great, and this fact both increases the difficulties of making out the threads, and in consequence of the rapid blue coloration of such a thin membrane causes the observations with iodine and Chlor. Zinc Iod. frequently to be almost valueless and often an impossibility. It is in such cases that my staining method comes to be so important.

Often it would seem that the threads are so excessively fine that they cannot be resolved as separate filaments, and the appearance presented by the whole aggregate of threads crossing the pit membrane is simply that of a blue coloration. In this direction the results with *Bomarea* are of special interest, as they tend to give weight to the view that my experiments have led me to adopt, viz.: that a well defined blue coloration, after the action of Chlor. Zinc Iod. and picric-HOFMANN's-blue, points to the presence of protoplasmic threads in the cell-wall.

Professor STRASBURGER* states, in the case of the endosperm cells of *Ornithogalum* and in the pith cells of *Taxodium*, that the pit membranes are demonstrably porous, and that a striation can be observed crossing the membrane upon action with iodine and Chlor. Zinc Iod. He also represents this striation in figs. 17, 18, 19, and 23, Tafel I.; and again in fig. 23, Tafel II., he shows that a similar striation may be seen in the closing membrane of the pits in the thickened cells of the seed-coat of *Viscum*.

As far as regards *Ornithogalum* I can fully confirm his results, and the fact of the existence of a similar structure in *Taxodium* and *Viscum* is one of great value.

Staining with picric-HOFMANN's-blue, subsequent to the action of iodine and Chlor. Zinc Iod., will demonstrate in the case of *Ornithogalum* that the pit membrane is distinctly blue, while the rest of the cell-wall is practically colourless, and will also bring out more clearly the striation of the pit membrane, due to the presence of threads. This, however, is by no means a favourable case.

Sections of the endosperm cells of *Bomarea oligantha*, after swelling and staining, gave me good results. If examined in a somewhat cursory manner it is at once observed that the pit membrane is well coloured and distinctly delineated from the rest of the cell-wall (Plate 70, fig. 27). In some instances it can be observed that instead of the whole pit membrane being uniformly coloured, it may be traversed by one or two coloured bands which run through the otherwise colourless substance of the membrane (refer to figure). In favourable instances well defined striation can be seen. In an *en face* view of the pits it becomes evident that the pit membrane exhibits essentially the same appearance as that presented by a sieve-plate (Plate 70, fig. 28), and the appearance of the two coloured bands is explained from the fact that in some instances the whole membrane is not necessarily perforated, but that the perforation and hence the sieve-plate structure may be confined to particular areas of the membrane of the pit. This particular distribution of perforating areas also explained the appearance of bifurcation, which is sometimes presented by the apex of much contracted protoplasmic

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* 'Bau und Wachsthum,' p. 16, *et seq.*

processes when pulled out of the pit cavity in consequence of the action of strong sulphuric acid, or other dehydrating agents.

The staining results with *Bomarea* appear to me to give great support to the idea that a pronounced coloration of the pit membrane by picric-HOFMANN's-blue after the action of iodine and Chlor. Zinc Iod. gives evidence of the presence of protoplasmic threads in the cell wall and therefore of perforation.

In *Ruscus*, although the cells are large, the pit membranes are very thin and quickly coloured with iodine and Chlor. Zinc Iod. After staining in the usual manner fairly well defined threads can be seen (Plate 70, fig. 29). The same is the case with *Iris* and *Xiphium*.

Colchicum is a particularly plain example of perforation of the pit membranes, which are here somewhat thick. Both with iodine and with HOFMANN's blue the individual threads are easily distinguished (Plate 70, fig. 30). Of Dicotyledons exhibiting a similar structure, *Ardisia polycephala* is an example of some interest on account of its peculiar reaction with iodine. With a dilute solution of this reagent the substance of the cell-walls give a blue reaction, exactly resembling that of starch. Stronger solutions rapidly cause a dark brown coloration. The seeds of *Ardisia crenulata* behave in the same way. The same has been observed in the seed of *Pæonia*, and has been long known in the case of the phlœm of *Lycopodium;* the so-called fungus cellulose ; and (when the iodine solution is of a certain strength) in mucilage cells. In *Nemophila*, although the cells are small, an appearance of striation is plainly evident (Plate 70, fig. 32). The structure of the horny seeds of certain of the *Rubiaceæ, e.g., Coffea, Galium,* and *Asperula* (Plate 70, fig. 31) is of some interest. The cell-walls present a somewhat crumpled appearance, and there is no definite arrangement in their shape. After treatment with strong iodine and a lengthy action of Chlor. Zinc Iod., a system of fine threads is clearly brought into view. Where the wall is pitted, the threads go through the pits, or, if not, through the thick wall, as the case may be. This was observed in *Asperula* only. The rest were not examined in detail.

The structure of the seeds of *Tamus* and *Dioscorea* are very important as affording additional confirmation of TANGL's results with *Strychnos.* The thick walls of these seeds present no pits, and are of the same transparent horny nature as those of *Strychnos Ignatia.* After treatment with iodine and Chlor. Zinc Iod. the very numerous threads which freely perforate the entire thickness of the cell-wall gradually come into view, and resemble in both arrangement and properties those of *Strychnos.* The fact that the threads cross the middle lamella is even better demonstrated in *Tamus* than in the former instance, for here the development of the lamella is not so great. The cell-walls soon swell very strongly, and in so doing the threads are broken up into a number of points, as TANGL has observed, and in the swollen portion of the wall at last become invisible (Plate 70, fig. 33). In *Dioscorea* the threads are much finer than in either *Strychnos* or *Tamus.* In both instances threads can be observed uniting all the cells, including of course those directly below the surface. In this respect they differ from *Strychnos* as far as their demonstrable character goes.

In such of the *Cæsalpiniæ* as possess endosperms a similar pitted structure of the cells occurs. The existence of threads was observed only in *Bauhinia*, but I cannot doubt that other leguminous seeds of the same structure will show the same occurrence of threads, *e.g.*, *Sophora Japonica* and *Gleditchia* mentioned by Von MOHL.*

Subjoined is a list of the seeds examined. In those whose names are printed in italics it was actually observed that there was a protoplasmic continuity from cell to cell. The rest were not examined in detail.

- *Leguminoseæ.*
    - *Bauhinia variegata.*
- *Rubiacea.*
    - *Asperula odorata.*
    - Galium aparine.
    - Galium spurium.
    - Coffea Arabica.
    - Sherardia arvensis.
- *Myrsineæ.*
    - Ardisia crenulata.
    - *Ardisia polycephala.*
- *Cornaceæ.*
    - *Aucuba Japonica.*
- *Loganiaceæ.*
    - *Strychnos Nux-vomica.*
    - *Strychnos Ignatia.*
    - *Strychnos potatorum* (?)
- *Hydrophyllaceæ.*
    - Nemophila discoidalis.
    - *Nemophila parviflora.*
    - Phacelia pimpernelloides.
- *Melanthaceæ.*
    - *Colchicum speciosum.*
- *Liliaceæ.*
    - Asparagus officinalis.
    - Asparagus sp.
    - *Ornithogalum tenuifolium.*
    - *Ornithogalum narbonense.*
    - Yucca, sp.
- *Smilaceæ.*
    - Polygonatum Japonicum.
    - *Ruscus aculeatus.*
- *Iridaceæ.*
    - *Iris pseudacorus.*
    - *Xiphium vulgare.*
    - Iris ochroleuca.
- *Dioscoreaceæ.*
    - *Dioscorea dæmonorum.*
    - *Tamus communis.*
- *Amaryllidaceæ.*
    - *Bomarea oligantha.*

The above results have established not only that protoplasmic threads do perforate the cell-wall, and thus bring adjacent cells into communication with one another, but that such perforation is of very frequent occurrence. My results with endosperm ells have fully confirmed those which I obtained with pulvini, and have both elucidated the structure that occurs in those organs, and given every support to the methods that I employed in their investigation. It would thus appear that not only in the endosperms of Palms, but in those of other plants in general, the cells are placed in connexion one with the other. It may be objected that I have used thick walled endosperms in every instance. I gave my reasons for so doing, and although I have not as yet examined the structure of thin walled endosperm cells, I have but little doubt that the same means of communication takes place in them also, for every range of difference of thickness of the cell-walls occurs, not only in the same order but

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* Von MOHL, 'Vegetable Cell,' English translation, p. 33.

in families of that order, that differ but little one from another. Russow's results are also of especial value here.

Results with Plasmolysis.—At an early stage in this investigation certain phenomena in connexion with experiments upon the preservation of tissues forced themselves upon my notice. What was especially striking was the different result which was obtained when different tissues were treated with the same reagent, and under the same conditions.

Thus, upon examination of sections of the pulvini of *Mimosa*, *Robinia*, and *Amicia*, which had been all carefully preserved in absolute alcohol, it will be seen that the degree with which the protoplasm is contracted from the cell-wall varies greatly in the three cases. In the cells of *Mimosa* the protoplasm will have undergone but little contraction, and the whole tissue will show signs of successful preservation. In *Robinia*, on the other hand, an appreciable contraction has evidently taken place, and in *Amicia* this state of things has attained a maximum, for almost every cell exhibits the much shrunken protoplasm lying freely in the cell cavity, and separated on all sides from the cell-wall. Since in every instance the cells are full grown, and are under equal conditions, it would seem probable that the protoplasm is held closer to the cell-wall in some cases than in others.

After having obtained my results with *Mimosa*, *Robinia*, and *Amicia*, it seemed the more probable that the above appearances were in reality a consequence of the intimate union between the cell-wall and protoplasm which my investigations had shown to exist, and the pronouncedness of which appeared to vary.

In consequence of these and other considerations, I was led to study, in a detailed manner, the effect of plasmolysing such cells, since it seemed to be almost certain that the phenomena accompanying such a condition would afford additional confirmation of the results I had already obtained with somewhat powerful reagents.[*] According to De Vries[†] when the plasmolytic condition is induced in a cell by means of dilute dehydrating agents, the protoplasm (primordial utricle) separates entirely from the cell-wall, and appears as a much contracted vesicle lying freely in the cell cavity.

On the other hand, both Pringsheim[‡] and Nägeli[§] had noticed that in certain cases the protoplasm appears to separate with some difficulty from the cell-wall, and that it was frequently connected to it by means of one or more threads in those cases where great contraction had taken place.

It had also been long known that in filamentous *Algæ*,[||] the protoplasm upon contraction is often connected to the cell-wall by threads. These, however, may be rather described as isolated cases, for no generalisations were made, nor was any

[*] Proc. Roy. Soc., Nov. 11, 1882.

[†] 'Untersuchungen über die Mechanischen Ursachen der Zellstrehung.' Leipzic, 1877.

[‡] 'Bau und Bildung der Pflanzenzelle.' 1854.

[§] 'Pflanzenphysiologishe Untersuch.' 1855.

[||] Hofmeister, 'Die Pflanzenzelle.' 1867.

particular attention drawn to the fact; on the contrary, it has been generally accepted that on plasmolysis the protoplasm is quite free from the cell-wall.

However, in repeating these experiments I find that in all the cases I have examined the contracted protoplasm is always connected to the cell-wall by means of very numerous protoplasmic threads.

The above phenomena were also discovered subsequently and independently by BOWER,* whose excellent paper on the subject appeared shortly after my own. My experiments were first made upon pulvini, but were afterwards extended to tissues in general (figs. 34, 35, 36, 37).

The most detailed observations were made upon transverse sections of the pulvini of *Amicia zygomeris* and *Robinia pseudacacia*, after treatment with 2·5 per cent., 5 per cent., and 10 per cent. solution of sodium chloride, but since the results obtained in other cases differ so little, one may describe the phenomena which accompany plasmolysis in general terms.

If a dilute solution of salt be employed, *e.g.*, 2·5 per cent., the protoplasm will gradually contract away from the cell-wall, and will at length frequently appear to lie quite freely in the cavity. In other cases the protoplasm will adhere to the cell-wall at certain points. But if the section be examined for some time, it will be seen that delicate strings of protoplasm will gradually come into view, and increase in number, until at length the contracted protoplasmic mass will present the appearance of a sphere suspended in the cell cavity by innumerable fine protoplasmic strings (Plate 70, fig. 37).

If contraction be rapidly brought about by means of a stronger solution, *e.g.*, 10 per cent., it will be observed that the protoplasm experiences some difficulty in separating from the cell-wall, and may even become divided up during the process into two or more portions (Plate 70, fig. 35), each of which rapidly assumes a spheroidal shape; also several somewhat thick threads may be seen connecting the protoplasm to the cell-wall or the protoplasmic masses to one another (figs. 35 and 36). Subsequently the finer threads come into view. I am inclined to believe that it is these thicker threads which have been hitherto seen, and that the finer threads have, up till now, escaped observation; and although, as BOWER† remarks, the difference between the thicker and the finer threads is only one of degree, yet the importance of the observation is in no way diminished thereby.

The thicker threads frequently present nodal swellings of a perfectly spherical form. These spherical nodes may either abut on to the cell-wall or may occupy any other position upon the thread. When, by chance, rupture of the threads occurs, part contracts to the central protoplasmic mass, and part forms a small sphere on the side of the cell-wall.

The first indication of the existence of the fine threads is afforded by an appearance

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* Quart. Jour. Micr. Sci. Jan. 1883.

† *Loc. cit.*

of striation, which gradually becomes more and more defined until distinct threads can be observed. At first the diameter of the threads gradually diminishes from the protoplasm to the cell-wall, so that it is impossible to trace the thread over the whole of its course (Plate 70, fig. 40); but after some time it comes more clearly into view, until at length it is apparent that it extends right up to the wall in question.

The thickness of the threads varies greatly. Up to a certain point, more and more threads come into view the longer the cell is observed, until at length the appearance presented will be that of a central contracted sphere of protoplasm from which radiate out to the cell-wall numerous fine threads, some of which are of an appreciable size, other smaller though still well defined, and others so difficult to see that their presence is only indicated by a faint striation traversing the space between the protoplasm and the cell-wall.

The phenomenon of the gradual definition of the threads appears to suggest that a thickening of their substance has taken place, and as BOWER* has observed, this in reality does occur.

He has seen also that the nodal swellings appear to travel from the protoplasm to the cell-wall, and is of opinion that the thickening of the threads is due in a great measure to a drawing out of fresh substance from the main protoplasmic body. He also suggests that lateral coalescence of the strings may occur. My view of the case, however, differs from his. It is certain that at first the protoplasm quickly contracts, owing to the rapid diffusion which occurs. The water diffuses from the cell vacuole into the salt-solution, much more quickly than the salt-solution diffuses into the water, so that the contraction of the protoplasm reaches its maximum when it has lost the greatest amount of water. After a time osmosis ceases, but not until the strength of the fluid, both inside and outside of the protoplasm, is the same. And in the subsequent equilibrium which occurs, the protoplasm, which had before suffered an abnormal contraction, owing to the rapid loss of the water it had contained; now takes up in exchange a small quantity of the salt-solution, and the ultra shrinking (so to speak) is relieved,† and a definite swelling of the protoplasm takes place. Thus the tension on the threads is no longer so great, and, owing to their elastic character, they thicken up and are thus brought into view. Subsequently they cease to thicken, and by the time the shrunken protoplasm has regained its equilibrium they become quite lax. Both BOWER and myself have observed that, after some length of time has elapsed, the threads execute lateral vibrations which are possibly caused by currents due either to diffusion or to temperature.

It seems probable that the action of the salt-solution, unless very dilute, causes grave changes to take place in the protoplasm. Ordinary cells do not give much evidence of this, since on washing out with water they regain their usual appearance.

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* *Loc. cit.*

† It loses, in fact (if I may be allowed to use the expression), some of its water of constitution, and takes up in its stead the salt-solution.

If, however, *Spirogyra* cells be plasmolysed, it will be seen that the whole structure has been much affected, for the chlorophyll bands will no more resume either their accustomed appearance or arrangement, and a general swelling of the cell takes place.

The strings of protoplasm which normally traverse the cell vacuole in ordinary living cells frequently exhibit the same appearances as those which are presented by plasmolysed threads, and nodal swellings may also occur. I have observed this particularly well in the hypodermal cells of potato tubers.*

I have also frequently noticed that as a result of plasmolysis many chlorophyll grains will tend to aggregate around the nucleus as if some connexion with the latter existed, such as PRINGSHEIM observed in *Spirogyra.*†

The point of special interest to me was to ascertain whether these threads bore any relation to the pits. As I stated in my paper before the Royal Society, I have observed several well defined instances in which threads do go to pits, and in Plate 70, fig. 34, which is a made-up figure embodying in one representation the results of numerous individual cases, I have attempted to illustrate such appearances. In one instance, where plasmolysis had been quickly induced by means of a strong salt solution, two spheres of protoplasm occupied the two opposite pit depressions, from each of which a thread ran to the main protoplasmic mass. However, numerous experiments have convinced me that no reliance can be placed upon the results obtained by plasmolysis, as giving any certain clue to the existence of protoplasmic continuity. With this opinion BOWER also agrees.‡ In fact, the greater proportion of threads bear no relation to pits, and in such an experiment as plasmolysing a hair of *Primula sinensis,* it is seen that as many threads go to the longitudinal as to the transverse walls, and are thus present on the free walls, as well as those separating contiguous cells (see also Plate 70, fig. 39).

As I mentioned in the earlier part of my paper, my efforts to fix and stain these plasmolytic figures did not meet with perfect success, although picric acid gave very satisfactory results. I am, however, inclined to think that additional shrinking was produced by the use of glycerine, and the method deserves another trial. As a result of the staining, both the threads and the protoplasm were well brought into view, but a very great proportion of the threads were ruptured, and appeared as little spheres attached to the cell-wall. I was unable to trace the protoplasm into the cell-wall, but at that time I had not adopted my plan of staining with picric-HOFMANN's-blue. It is possible that with this reagent some results may be obtained. In my paper before the Royal Society,§ I stated that I had succeeded in showing the passage of the protoplasm through the cell-wall when the wall was left intact, and not swollen by reagents; the method consisting in treating thin sections of fresh material with saturated picric

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* See figure of cell of hair of *Cucurbita*, SACHS' 'Vorlesungen,' p. 752.

† PRINGSHEIM, 'Uber Lichtwirkung und Chlorophyllfunction.' Leipzic, 1881. Tafel XIV., fig. 4.

‡ *Loc. cit.*

§ *Loc. cit.*

acid. There are two mistakes in that statement. First I should have said "treating plasmolysed sections:" and what is of more importance, I am inclined to believe that my observations were not perfectly trustworthy. I had two particularly plain instances of an apparent passage of protoplasm through the cell-wall, one of which I have represented in Plate 70, fig. 38. Although it still seems perfectly clear and plain, I am almost convinced that some abnormal appearance has been produced, either by distortion of the section, or owing to the fact that, intersecting the two coloured protoplasmic threads, are thin pit membranes which I cannot resolve.

As regards plasmolysis, numerous tissues were examined, and in all the same occurrence of strings was observed. Both BOWER and myself believe that the phenomenon is universal. As definite instances where actual observations were made I may mention the pulvini of *Mimosa*, *Phaseolus*, *Rhynasia*, *Oxalis*, *Biophytum*, *Apios*, *Desmodium*, *Maranta* and *Marattia;* various roots, *e.g.*, *Beta;* petioles, *e.g.*, *Primula* and *Ficus;* leaves, *e.g.*, *Primula;* young endosperm cells, *e.g.*, *Rhopalostylis*, *Sabal*, and *Ancuba*. Stems and other structures examined from time to time gave the same results. These results, taken in conjunction with those of BOWER, make it extremely probable that the same phenomenon is displayed by every living cell whatsoever.

In attempting to explain these appearances which accompany plasmolysis one has only hypothesis to offer. BOWER[*] suggests two views—(1) that the main mass of protoplasm on retreating may leave the cell-wall still completely lined with a thin film of protoplasm; (2) that the peripheral part of the protoplasm being entangled as a network among the deposited microsomata may, on the contraction of the main mass, be drawn out at the points of entanglement into fine strings like those observed; while the surface of the wall is left free, and not covered by a film of protoplasm.

But it seems to me that all the above phenomena may be explained from the mere fact that the cell-wall is so perfectly wetted (to use a physical phrase) by the protoplasm; for as STRASBURGER's[†] results show, the connexion between the cell-wall and the protoplasm is one of the most intimate description, even if any direct perforation of the cell-wall by protoplasmic filaments be left out of the question. The very same effects may be obtained with stringy mucus adhering to a glass tumbler. My results have certainly shown that the connexion between protoplasm and cell-wall is much closer than was imagined to be the case; but I am inclined to doubt whether the existence of protoplasmic threads in the cell-wall at all influences the phenomenon of plasmolysis, for they are equally well displayed over the whole surface of the wall, and bear no relation even to such pits as those occurring in the young endosperm cells of *Archontophœnix* and *Rhopalostylis*, where well pronounced continuity is known to occur. But I am bound to admit that it is a question of hypothesis against hypothesis, and I look forward with interest to the results of

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[*] *Loc. cit.*

[†] 'Bau und Wachstum,' p. 246.

plasmolysing such a cell as *Tamus communis* (Plate 70, fig. 33). In concluding the subject I should like to state my views as to the reason why plasmolysis does not give any reliable assistance to the subject of the perforation of the cell-wall by protoplasmic threads.

When the protoplasm separates from the cell-wall in consequence of the action of dehydrating agents, it always tends to assume a spheroidal form, in consequence of the action of the two forces of pressure and tension, which endeavour to bring about a state of equilibrium. Now the pulling force that the living protoplasm must exhibit in contracting from the cell wall and assuming its spheroidal condition must be very considerable. As we have seen from the appearance presented by such sections as Plate 70, fig. 40, there is a tendency on separation for the protoplasm to adhere rather to the main protoplasmic mass than to the cell-wall, and in consequence of this the protoplasm of the fine filaments going through the cell-wall will tend to be pulled out of its canal, and thus the thread proceeding from it will be no thicker than one which arises from the general cell-wall, and will therefore not be especially apparent. In instances where plasmolysis is very rapidly induced, the protoplasm quickly contracts, and even becomes divided up into several masses. Then it may possibly happen that, owing to the particular combination of forces, a minute sphere of protoplasm may be retained, sticking to the pit membrane (as in Plate 70, fig. 34), although it may equally well adhere to the cell-wall (as in Plate 70, fig. 35).

But with such strong reagents as sulphuric acid the case is different. Owing to the rapid death of the protoplasm, the assumption of that spheroidal form attended with the exhibition of the usual rending force between the protoplasm and the cell-wall is prevented. The factor of life no longer asserts itself, and the contraction produced is now merely a mechanical shrinking, in consequence of dehydration, and the separation tends to take place rather between protoplasm and cell-wall than between protoplasm and protoplasm. Thus any intimate union which may exist between the protoplasm of the cell and the protoplasm running through the cell-wall tends to be maintained, and if sufficiently pronounced is made evident.*

I am now in a position to bring my paper to a conclusion.

I have succeeded in demonstrating that in living tissues a means of communication between adjacent cells exists. My results have been confirmed by RUSSOW, whose valuable contribution I have already mentioned. The wide field that this discovery opens is so great, and the whole bearing of the subject is so enormous, that it would be useless for me either to attempt to sketch its significance or indicate the important inferences which arise therefrom, in the present paper. We are now in a position if not to understand, at least to get a clearer insight into, such phenomena as the downward movement of a sensitive leaf upon stimulation, of the wonderful action of a germinating embryo on the endosperm cells, even those which are far

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* In connexion with this subject, see DE BARY's figure of the sieve-tube of *Vitis* after the action of iodine and potassic iodide (*loc. cit.*, fig. 75, p. 186).

removed from it, and finally of the whole cell mechanism. The passage of protoplasm from cell to cell, which numerous observations have showed must occur, can now be explained, and the mere fact of the possibility of this taking place increases very materially our knowledge as to general mechanics of the vegetable cell.

Although I am aware of the danger of rushing to conclusions, I cannot but remark that when these results—which were foreshadowed by SACHS and HANSTEIN when they discovered the perforation of the sieve-plate—are taken in connexion with those of RUSSOW, it appears extremely probable that not only in the parenchymatous cells of pulvini, in phlœm parenchyma, in endosperm cells, and in the prosenchymatous bast-fibres, is continuity established from cell to cell, but that the phenomena is of much wider, if not of universal occurrence.

Finally, I have to acknowledge the many kindnesses I have received during this investigation. Of Professor SACHS' kindness to me it is impossible for me to speak sufficiently highly. The mere fact that it was at his suggestion that this work was undertaken will show how much I owe him. To my friend and former teacher, Dr. S. H. VINES, I am indebted for much valuable advice. Especially must I also express my most sincere gratitude to my friend, Dr. D. H. SCOTT, not only for his valuable criticisms and suggestions, but for the many assistances that he has given me in every possible way during the whole of this difficult work.

NOTE.

(Added January 12th, 1884.)

Since the communication of the above I have written two more papers on the same subject, viz.:

1. "On the Continuity of the Protoplasm through the Walls of Vegetable Cells," Proc. Roy. Soc., December 20, 1884, which deals with the confirmation of my methods and the further establishing of my results. Since in the endosperm cells of *Bentinckia Conda-panna* the threads can be seen by merely mounting a section in dilute glycerine, such a preparation is taken as normal, and can be then compared with similar sections, in the preparation of which reagents have been employed. Such comparison is in every way satisfactory. I have further confirmed the existence of a continuity in *Dionœa*, and have established that in the parenchymatous cells of the leaf bases of *Aucuba Japonica* and *Prunus lauro-cerasus* distinct threads can be made out, crossing the pit-closing membrane. I then make some remarks as to the function of the threads.

2. "On the Continuity of the Protoplasm through the Walls of Vegetable Cells," Arbeiten des Botanischen Instituts in Würzburg. Bd. III., Heft I.

This is a fairly complete paper, embodying all the results I have obtained up to the present time.

Description of Plates.

PLATE 68.

Fig. 1. Transverse section of the pulvinus of *Trifolium repens* which in its principal details resembles that of *Mimosa pudica*. (× 55.)

Fig. 2. Longitudinal section of a portion of the pulvinus of *Trifolium repens*, showing the cells immediately beneath the epidermis. (× 105.)

Fig. 3. Longitudinal section of a portion of the pulvinus of *Mimosa pudica*, showing the cells immediately beneath the epidermis. The intercellular spaces are small and badly developed. (× 235.)

Fig. 4. Cells of the pulvinus of *Mimosa pudica* which are situated immediately around the vascular bundle. The intercellular spaces are large and conspicuous. (× 235.)

Fig. 5. Cells of the pulvinus of *Mimosa pudica*, situated midway between the epidermis and the vascular bundle, after treatment with sulphuric acid, staining with methyl violet, and washing with dilute glycerine. The protoplasmic contents are shrunken and deeply coloured. The remains of the middle lamellæ can be seen. Certain of the processes appear to join uninterruptedly from cell to cell. In others between the two darkly stained ends is a lighter stained area uniting the two. The latter is believed to be the typical and only true means of continuity. (× 550.)

Fig. 6. Portions of two bast-cells from the pulvinus of *Mimosa pudica* after treatment with sulphuric acid, and staining with methyl violet and glycerine. (× 1020.)

Fig. 7. Cells of the pulvinus of *Robinia pseudacacia*, situated as in *Mimosa* (fig. 5), after treatment with sulphuric acid and staining with methyl violet and glycerine. The typical mode of connexion between adjacent cells is better seen than in *Mimosa*. The appearances of an uninterrupted continuity are not so frequent. (× 550.)

Fig. 8. Cells of the pulvinus of *Robinia pseudacacia* after treatment with sulphuric acid and staining with methylene blue. The bottom and sides of the pits are stained. (× 105.)

Fig. 9. Longitudinal section of a portion of the pulvinus of *Amicia zygomeris*. In certain of the cells conspicuous pits are apparent. (× 235.)

Fig. 10. Cells of the pulvinus of *Amicia zygomeris*, situated midway between the epidermis and the vascular bundle, after treatment with sulphuric acid, staining with methyl violet, and mounting in dilute glycerine. Between the adjoining masses of the much shrunken protoplasm are numerous fine stained processes uniting the two. These in reality represent the stained

bottoms and sides of the much swollen and resistent pits. The swollen cell-wall abuts directly on to the protoplasm. By long treatment with dilute glycerine all the colour becomes dissolved from the pits, and the protoplasmic masses are then left fairly isolated one from another, or by prolonged treatment with sulphuric acid the resistent pits become swollen, and then stain like the rest of the wall. (× 1020.)

Fig. 11. Cells of the pulvinus of *Amicia zygomeris* situated immediately around the vascular bundle, where the cell-walls are thick and the pits deep and well developed, after treatment with sulphuric acid and staining with methyl violet and glycerine. The protoplasmic processes tend to adhere to the pit-membrane, and between any two contiguous processes is a lighter stained area. (× 1020.)

Fig. 12. A cell from the rachis of the leaf of *Cycas revoluta*, treated with iodine and Chlor. Zinc Iod. The pits opposite the intercellular spaces stain deep blue, whereas those separating the contents of adjacent cells are but feebly coloured. (× 235.)

PLATE 69.

Fig. 13. A cell from the ripe endosperm of *Phœnix dactylifera* after treatment with sulphuric acid and staining with methyl violet and glycerine. Some portions of the wall remain but little acted upon. The protoplasmic processes of the main shrunken mass have separated with difficulty from the pit-closing membrance, and the protoplasmic threads which traverse that structure, and normally abutted on to the ends of the protoplasmic processes of the pits, are well stained and brought into view. Compare figs. 6 and 11. (× 550.)

Fig. 14. Young endosperm cell of the seed of *Archontophœnix Cunninghamii* (*Seaforthia elegans*) after treatment with sulphuric acid and staining with methyl violet and glycerine. The pit processes of adjacent cells are united by fine protoplasmic threads, after the manner of a sieve-tube. This compares with figs. 5 and 7. (× 550.)

Fig. 15. Young endosperm cell of *Rhopalostylis sapida* (*Areca sapida*) after treatment with sulphuric acid, methyl violet, and glycerine. The processes from the shrunken protoplasm which enter the pits adhere to the pit-closing membrane, and the opposite processes of adjacent cells are united to one another and held in position by delicate protoplasmic threads perforating the pit-closing membrane. (× 550.)

Fig. 16. Cells of the ripe endosperm of *Bentinckia Conda-panna* after treatment with Chlor. Zinc Iod. and staining with picric-HOFMANN's-blue. The proto-

plasmic cell contents have undergone degeneration, and many oil drops are present. Traversing both the thick pit-closing membranes, and also the general cell-walls, are complexes of fine protoplasmic threads. (× 550.)

Fig. 17. Portions of cells of ripe endosperm of *Howea Belmoreana* (*Kentia Belmoreana*), cell contents not shown, treated with Chlor. Zinc Iod. and picric-HOFMANN's-blue. The threads traverse the pit-closing membranes and the general cell-walls. (× 550.)

Fig. 18. Cells of ripe endosperm of *Howea Belmoreana*, with protoplasmic cell contents. Treated as before. (× 550.)

Fig. 19. Portions of cells of the ripe endosperm of *Lodoicea Sechellarum* (the double cocoa-nut) treated with iodine and Chlor. Zinc Iod. Protoplasmic threads traverse the pit-membranes and general cell-wall. Cell contents not shown. (× 550.)

Fig. 20. Piece of cell-wall of same mounted in a mixture of iodine and glycerine. Threads fainter. Those traversing the unpitted portion of the wall do not appear to perforate as far as the free surface. The delicate channels containing protoplasm are widest in the region of the middle lamella, and colour with little or no swelling. In consequence of incomplete swelling the remaining portions of the threads are not visible. (× 550.)

Fig. 21. Portions of cells of the ripe endosperm of *Oncosperma horridum* treated with iodine and Chlor. Zinc Iod. The threads are very fine; much finer than it is possible to represent them in a drawing. (× 550.)

Fig. 22. Cells of the ripe endosperm of *Heterospathe elata* treated with Chlor. Zinc Iod. and picric-HOFMANN's-blue. Middle lamella but little developed. Threads fairly thin. (× 550.)

Fig. 23. Portion of same treated with iodine and Chlor. Zinc Iod., showing that by such treatment the threads appear much thicker, owing to the precipitation of iodine upon them. (× 550.)

Fig. 24. Ripe endosperm cell of *Calamus callicarpus* treated with Chlor. Zinc Iod. and picric-HOFMANN's-blue. Cell contents in this and many following omitted. Middle lamella well developed. Embedded in the cell-wall are crystals of calcium oxalate. (× 550.)

Fig. 25. Portions of ripe endosperm cells of *Syagrus botryophora* treated as before. Pits shallow and little developed. (× 550.)

PLATE 70.

Fig. 26. Cell of ripe endosperm of *Phytelephas macrocarpa* (vegetable ivory) treated with iodine and Chlor. Zinc Iod. (× 550.)

Fig. 27. Portion of cell-wall of almost ripe endosperm of *Bomarea oligantha* treated with Chlor. Zinc Iod. and picric-Hofmann's-blue. In the case of one of the pit-closing membranes the badly developed threads are not present over the whole of its surface. (× 550.)

Fig. 28. *Én face* view of same. In the upper of the two pits the threads run as in the last described example. In the lower they are distributed equally over the surface. The figure is badly drawn, for in the upper pit the sections of the threads should have been more plainly apparent, and in the lower the unstained portions should have been represented stained, and *vice versâ*. (× 1020.)

Fig. 29. Portion of cell-walls of ripe endosperm of *Ruscus aculeatus* treated with Chlor. Zinc Iod. and picric-Hofmann's-blue. Pit-membranes thin. Threads badly developed and seen with difficulty. (× 550.)

Fig. 30. Portion of cells of ripe endosperm of *Colchicum speciosum* treated as before. Threads much better defined. (× 550.)

Fig. 31. Portion of cell-walls of ripe endosperm of *Asperula odorata* treated with iodine and Chlor. Zinc Iod. Threads traverse pit-membranes and walls. (× 550.)

Fig. 32. Portion of cell-walls of ripe endosperm of *Nemophila parviflora* treated with Chlor. Zinc Iod. and picric-Hoffmann's-blue. Cells small. Pit-membranes thin. Threads difficult to see. (× 550.)

Fig. 33. Portion of cell-walls of ripe endosperm of *Tamus communis* treated with iodine and Chlor. Zinc Iod. Part of the wall much swollen and coloured blue, in consequence of the usual cellular reaction. In this swollen area the threads can no longer be detected. In the lower half of the figure the apparently unswollen walls are commencing to swell, and the protoplasmic threads are breaking up into small points, instead of presenting the appearance of lines, as in the walls of the upper half of the section, which are still fairly intact. (× 550.)

Fig. 34. Cell of pulvinus of *Robinia pseudacacia* after treatment with a 10 per cent. solution of common salt. Appearance presented some two hours after plasmolysis. Certain of the threads can be seen going to pits. (× 550.)

Fig. 35. Cell of same tissue examined about ten minutes after mounting in 10 per cent. salt solution. (× 550.)

Fig. 36. Similar cell treated in the same way. Examined half an hour after treatment. (× 550.)

Fig. 37. Cells of pulvinus of *Apios tuberosa* treated with 5 per cent. salt solution. Examined three hours after plasmolysis. (× 440.)

Fig. 38. Cells of pulvinus of *Apios tuberosa* after treatment with 10 per cent. salt solution, saturated watery picric acid, and Hofmann's blue. Some of the threads are fairly preserved. Two thick threads appear to perforate

two pits, and unite the protoplasmic masses of the neighbouring cells as stated in text (refer to paper). (× 550.)

Fig. 39. Cell of the lamina of *Trichomanes pyxidiferum* as seen ten minutes after plasmolysis with a 10 per cent. salt solution (after BOWER). The threads appear to have no fixed relation to the pits. (× 550.)

Fig. 40. Similar cell as seen two hours after plasmolysis (after BOWER). (× 550.)

XXVI. *Supplement to former Paper, entitled—"Experimental Inquiry into the Composition of some of the Animals Fed and Slaughtered as Human Food."—Composition of the Ash of the entire Animals, and of certain separated parts.*

By Sir John Bennet Lawes, *Bart., LL.D., F.R.S., F.C.S., and*
Joseph Henry Gilbert, *Ph.D., LL.D., F.R.S., V.P.C.S.*

Received June 13,—Read June 21, 1883.

In our former paper (Phil. Trans., Part II., 1859), we considered the analytical results which had then been obtained illustrating the actual and comparative composition of certain collective portions, and of the entire bodies, of animals of the farm, of different descriptions, and in different stages of growth and fatness. The results were given relating to ten animals, namely—a fat calf, a half-fat ox, a fat ox, a fat lamb, a store sheep, a half-fat old sheep, a fat sheep, a very fat sheep, a store pig, and a fat pig. The constituents which had been determined were—the total fat (by melting, expression, and ether-extraction), the total nitrogen, and the total mineral matter (ash). These were given in detail for certain separated parts, and in summary for all those parts collectively which are usually classed by the butcher as "carcass," for all those collectively classed as "offal," and for the entire animal (fasted live-weight). At that time the analyses of the ashes of the different animals, and their separated parts, were not completed. It is the object of this supplementary paper to record the results of forty complete ash-analyses, and to indicate their connexion with the main inquiry, and their importance as an element of it. To do this it will be desirable in the first place briefly to summarise the results and conclusions previously given.

From the data above referred to, the composition of some of the separated parts, and of the entire bodies, of the ten animals was given, so far as the total mineral matter, the total nitrogenous substance, the total fat, the total dry substance, and the water are concerned.

From these results the composition of the increase in weight, during the fattening period, of numerous animals was estimated. Also, in numerous cases in which the amount and the composition of the food consumed had been determined, the relation of the constituents stored up in the increase to those so consumed was calculated. Finally, the relation of the non-nitrogenous, or non-flesh forming, to the nitrogenous constituents in animal food and in bread was compared.

For the study of the subject from a more physiological point of view, the actual weights, and the percentage proportion in the entire body, of the individual organs, and of certain more arbitrarily separated parts, were determined. To this end in all 326 animals were experimented upon—namely, 2 calves, 2 heifers, and 14 oxen; 249 sheep, in five classes as to age, maturity, fatness, and mode of feeding; and 59 pigs, in seven classes, arranged chiefly according to the food consumed. The following is a very condensed summary of some of the results obtained in this part of the inquiry:—

Table I.—Percentage (in fasted live-weight) of certain collective parts.

	Oxen.	Sheep.	Pigs.
Stomachs and contents	11·61	7·43	1·28
Intestines and contents	2·74	3·53	6·24
Total	14·35	10·96	7·52
Heart and aorta, lungs and windpipe, liver, gall bladder and contents, pancreas and spleen	2·96	3·30	3·01
Blood	4·01	3·97	3·63
Total	6·97	7·27	6·64

These facts are of considerable interest viewed in connexion with the great difference in the character of the food of the different animals; the ruminants consuming such a large proportion of fibre, much of which is indigestible; and the well-fed pig but little indigestible matter, and a relatively large proportion of starch, the primary transformations of a large part of which are supposed to take place after leaving the stomach, and more or less throughout the intestinal canal. With the great variations which the figures show in the proportion of the receptacles and first laboratories of the food, with their contents, the further elaborating organs (if we may so say), with their fluids, appear to bear a much more uniform relation by weight to the entire body in the different descriptions of animal.

The results further showed that whilst during the fattening process the total "carcass" parts increased both in actual weight and in percentage in the entire body, the remaining parts, constituting the so-called "offal," also increased in actual weight, but in a much less degree than the carcass parts, and they actually diminished in percentage proportion to the total live-weight.

The following is a summary of the composition of the ten animals analysed:—

Table II.—Summary of the Composition of the Ten Animals Analysed.

	Mineral matter (Crude ash).	Dry nitrogenous substance.	Fat.	Total dry matter.	Water.	Contents of stomachs and intestines (in moist state).
Per Cent. in Carcass.						
Fat Calf	4·48	16·6	16·6	37·7	62·3	
Half-fat Ox	5·56	17·8	22·6	46·0	54·0	
Fat Ox	4·56	15·0	34·8	54·4	45·6	
Fat Lamb	3·63	10·9	36·9	51·4	48·6	
Store Sheep	4·36	14·5	23·8	42·7	57·3	
Half-fat old Sheep	4·13	14·9	31·3	50·3	49·7	
Fat Sheep	3·45	11·5	45·4	60·3	39·7	
Very fat Sheep	2·77	9·1	55·1	67·0	33·0	
Store Pig	2·57	14·0	28·1	44·7	55·3	
Fat Pig	1·40	10·5	49·5	61·4	38·6	
Per Cent. in Offal (Sum of parts excluding Contents of Stomachs and Intestines).						
Fat Calf	3·41	17·1	14·6	35·1	64·9	
Half-fat Ox	4·05	20·6	15·7	40·4	59·6	
Fat Ox	3·40	17·5	26·3	47·2	52·8	
Fat Lamb	2·45	18·9	20·1	41·5	58·5	
Store Sheep	2·19	18·0	16·1	36·3	63·7	
Half-fat old Sheep	2·72	17·7	18·5	38·9	61·1	
Fat Sheep	2·32	16·1	26·4	44·8	55·2	
Very fat Sheep	3·64	16·8	34·5	54·9	45·1	
Store Pig	3·07	14·0	15·0	32·1	67·9	
Fat Pig	2·97	14·8	22·8	40·6	59·4	
Per Cent. in Entire Animal (Fasted Live-weight).						
Fat Calf	3·80	15·2	14·8	33·8	63·0	3·2
Half-fat Ox	4·66	16·6	19·1	40·3	51·5	8·2
Fat Ox	3·92	14·5	30·1	48·5	45·5	6·0
Fat Lamb	2·94	12·3	28·5	43·7	47·8	8·5
Store Sheep	3·16	14·8	18·7	36·7	57·3	6·0
Half-fat old Sheep	3·17	14·0	23·5	40·7	50·2	9·1
Fat Sheep	2·81	12·2	35·6	50·6	43·4	6·0
Very fat Sheep	2·90	10·9	45·8	59·6	35·2	5·2
Store Pig	2·67	13·7	23·3	39·7	55·1	5·2
Fat Pig	1·65	10·9	42·2	54·7	41·3	4·0

We must refer to our former paper for the detailed discussion of the composition of the animals, and their different parts, of which the foregoing Table gives a very condensed view. We need only call attention here to some of the most prominent indications.

It will be observed that there is a very much larger proportion of total fat than of total nitrogenous substance, in all the animals excepting the calf; that the percentage of nitrogenous substance diminishes, and that of the fat greatly increases, as the animals mature; also that the percentage of the total mineral matter decreases as the animals mature.

It is obvious that the increase during the fattening period will consist in still less proportion of nitrogenous substance, and in still greater proportion of fat. In fact the amount of fat stored up may be 8 or 10 times as much as that of the nitrogenous substance; and in the case of very fat pigs even more. The proportion of the total mineral matter, like that of the nitrogenous substance, is also much less in the fattening increase of the animal, than in the entire body.

Calculation further showed that the proportion of the nitrogenous substance of the food which was finally retained was very small. For example, sheep fattening on a good mixed ration will probably so retain in increase less than 5, or even less than 4 per cent. of the nitrogenous substance consumed in their food. If, however, the food is low in nitrogenous substance, more than 5 per cent. of that consumed may be stored up. In the case of pigs a larger proportion of the nitrogenous substance of the food is stored up, perhaps on the average 7½ per cent. If the food be low in nitrogen, consisting chiefly of cereal grain for example, perhaps nearly 10 per cent., or if high in nitrogen perhaps not more than 5 per cent. of that consumed will be finally retained.

The amount of fat stored up was shown to be very much greater than the amount of ready formed fat in the food. Fat was, therefore, largely formed within the body; and the results led to the conclusion that it was largely produced from carbohydrates.

It has been stated that the amount of mineral matter stored up in fattening increase is very small. Further, the proportion of that consumed which is retained depends so much on the character of the food that no general estimate can be safely given. The amount is at any rate almost immaterial, and the proportion will probably be always considerably less than that of the consumed nitrogenous substance retained. In connexion with this point it may be mentioned that in the case of each of the oxen and sheep the amount of mineral matter to one of nitrogenous substance was almost exactly 0·3 in the collective carcass parts, but it was lower in the other parts, and in the entire bodies. The results which it is the special object of the present communication to put on record will throw more light on the mineral composition of the animals.

Before closing this summary statement, and entering upon the special subject-matter of the present paper, brief reference should be made to some conclusions of importance to which the consideration of the composition of the animals as so far given, led.

It was estimated that of the total nitrogenous substance, and of the total fat, of the bodies of the animals, the following proportions would be consumed as human food :—

TABLE III.

	Per cent. consumed as human food.	
	Of the total nitrogenous compounds of the body.	Of the total fat of the body.
Calves	60	95
Oxen	60	80
Lambs	50	95
Sheep	50	75
Pigs	78	90

Thus, not only do the bodies of the fattened animals contain much more fat than nitrogenous substance, but a much larger proportion of the total fat than of the total nitrogenous substance is estimated to be consumed as human food. It results that, taking the average of the fat and the very fat animals, nearly four times as much dry fat as dry nitrogenous substance would be so consumed.

Finally, a comparison of the composition of the estimated consumable portions of the fattened animals, with that of wheat-flour bread, led to the conclusion that, taking into consideration the much higher oxidable capacity of the fat of the animal food than of the starch of the bread, the animal food contributed a considerably higher proportion of non-nitrogenous substance, reckoned as starch, to one of nitrogenous substance, than bread. We said :—" It would appear to be unquestionable, therefore, that the influence of the introduction of our staple animal foods, to supplement our otherwise mainly farinaceous diet, is, on the large scale, to reduce, and not to increase, the relation of the assumed flesh-forming material, to the more peculiarly respiratory and fat forming capacity, so to speak, of the food consumed."

It was concluded that the admitted advantages of a mixed animal and vegetable diet were essentially connected with the amount, the condition, and the distribution of the fat in the animal portions of the food ; that concentration and digestibility were probably elements in the explanation of the facts ; that the liberal distribution of the ready-formed fat with the transforming nitrogenous matters throughout the body, will modify the character of the changes constantly going on ; and that the difference in the condition of the nitrogenous substance in the animal and vegetable foods, has also to be taken into account.

Quantity and Composition of the Mineral Matter (Ash) in certain separated parts, and in the entire bodies, of the ten animals analysed.

In our former paper the actual quantity of ash was given for the bones, and for certain soft parts separately, of the carcass ; also for each separate internal organ, and for other separated parts constituting the offal (Appendix-Tables I.–X., pp. 580–589); and the percentage of ash in each separated part is given in Appendix-Table XII., p. 591. At that time the ashes had not been analysed; but the work has now been long completed, and the results have only waited for leisure for adequate discussion. It is not proposed even now to treat the subject exhaustively, but to submit the results obtained with so much explanation and comment as will suffice to give a clear idea of their character, to indicate some of their most important bearings, and to direct the further study of them.

The ashes that have been analysed are, for each of the ten animals—1. Of a proportional mixture of all carcass parts; 2. Of a proportional mixture of all offal parts; 3. Of a proportional mixture of all parts, both carcass and offal, representing the ash of the entire animal.

As separated by the butcher, there is but little difference in the apportionment of the different parts to the carcass and offal respectively, in the case of oxen and sheep; but whilst with these animals the head and feet go with the offal, in the case of the pigs they are weighed with the carcass. Accordingly, the head and feet of the pigs were separately treated, and the ashes of these parts separately analysed. In the Tables, for the sake of comparison with the results for the other animals, those relating to the head and feet of the pigs are not included with the carcass; but they can, of course, be reckoned either with the carcass or with the offal as may be desired.

In the case of the oxen and sheep, the portions yielding carcass-ash are—the greater part of the skeleton, the flesh, the kidneys, and the fat membrane of the parts. In the case of the pigs the skin also is included with the carcass. The offal parts yielding ash are—the stomachs and intestines (without contents and washed), the heart, aorta, lungs, windpipe, blood, liver, pancreas, thymus gland, the glands about the throat, the spleen, the bladder, gall-bladder, bile, brains, tongue, head flesh, head bones, head skin and ears, pelt, hair or wool, leg bones, feet and hoofs, tail flesh, tail bones, diaphragm, &c.

Of the ashes from the carcass parts twelve complete analyses have been made; that is, one for the carcass of each of the ten animals, and two duplicates. The duplicates are of the fat ox, and of the fat sheep, carcass ashes.

Of the ashes from the offal parts, seventeen complete analyses have been made; one for the offal parts of each animal, one for the head and feet ash of each of the two pigs, and five duplicates.

Of the ashes representing the entire bodies of the animals, eleven analyses have been made; that is one for each animal, and one duplicate.

In all, therefore, forty complete ash analyses have been made; and there have frequently been duplicate determinations of individual constituents. The detailed results of the analyses are given in Appendix-Table I. (p. 885) for the ashes of the carcass parts; in Appendix-Table II. (p. 886) for the ashes of the offal parts; and in Appendix-Table III. (p. 887) for the entire animal ashes. In the upper division of each Table the actual analytical results are given; in the middle division the same calculated to exactly 100; and in the lower division the results are calculated to 100 excluding sand and charcoal—that is, showing the composition of what may be called the *pure ash*.

In E. Wolff's two volumes—'Aschen-Analysen'—he excludes carbonic acid, as well as sand and charcoal, in calculating the composition of what he terms "Reinasche." This exclusion could hardly be avoided in arranging for comparison the recorded results of various analysts, in many of which carbonic acid was not included; and from the point of view of the chemical statistics only, of crops and other products, it is of little consequence. As, however, in many cases, the amount of carbonic acid represents, more or less exactly according to circumstances, the quantity of base which has been in combination with organic acids, its amount, and the differences in its amount, in different descriptions of ash, are indications of considerable interest. Obviously, in the case of ashes of such heterogeneous mixtures as those now in question, the record is of less importance from this point of view; whilst in some of the animal matters carbonates doubtless exist as such. But, as in other cases it is important to include the carbonic acid among the constituents of the pure ash, it is included here also for the sake of uniformity of plan.

It is freely admitted that results relating to carbonic acid require very careful consideration, if misinterpretation is to be avoided. Not that the determination of the amount of it actually existing in an ash is a matter of difficulty in experienced hands; but, according to the character of the ash, and to the conditions of the incineration, more or less of the carbonates may have been converted into more fixed salts, or the carbonic acid may be expelled and the ash causticised.

It is in fact very difficult, if not impossible, with some descriptions of ash, such for example as contain much silica, or phosphates with less than three of fixed base, so to conduct the incineration as to retain what may be termed the normal amount of carbonic acid. Indeed, after an ash has been kept for some time, and has acquired water, and perhaps regained carbonic acid, it is in some cases extremely difficult finally to heat it before weighing out for analysis, so as to ensure, on the one hand the expulsion of all water, and on the other the retention of the normal amount of carbonic acid. These points have been very fully investigated in connexion with the analyses of about 700 ashes, of various products, of known history, prepared at Rothamsted.

In the ashes of the mixed animal matters the amount of carbonic acid is in all cases small; but the differences in the amounts obtained according to the methods of preparation for analysis well illustrate the difficulties involved. Thus, in five of the carcass ashes, and in eight of the entire animal ashes, carbonic acid was determined:—1, in the ash some time after preparation and without re-ignition (but calculated on re-ignited ash); 2, after re-ignition preparatory to weighing out for the determination of other constituents; 3, after treatment with ammonium carbonate and exposure to very low red heat. The average amount in the five carcass ashes was—determined in the not re-ignited ash 2·59, in the re-ignited ash 0·87, and in the ash treated with ammonium carbonate and very gently heated 1·52; the average amount in the eight entire animal ashes was—determined in the not re-ignited ash 2·20, in the re-ignited ash 0·87, and in the ash treated with ammonium carbonate 1·48. After much comparative study of the results, the determinations after treatment with ammonium carbonate have been adopted throughout the series of animal ash analyses. Judging, however, from the recorded amounts of carbonates in numerous analyses of bones, and also from the analytical results themselves, as will be seen further on, it seems very probable that even the amounts so determined are too low. On the other hand, it is obvious that they are higher than in the ash in the re-ignited condition as weighed out for the determination of other constituents, and the generally high totals which the actual analyses show, are largely due to this cause. In fact, if the determinations of carbonic acid in the re-ignited ash had been entered in the Tables, not only would the totals have ranged considerably lower, but those for the carcass and entire animal ashes would range lower than those for the offal ashes, which is as it should be considering that fluorine was not determined in the ashes.

Thirty-seven of the forty animal ash analyses recorded in this paper were made, chiefly in the Rothamsted Laboratory, by Mr. RICHTER, now of Charlottenburg, Berlin; the remaining three were made by Mr. R. WARINGTON. Partly in the Rothamsted Laboratory and partly at Charlottenburg, Mr. RICHTER has conducted nearly the whole of the several hundred ash analyses above referred to, as well as numerous investigations of method, with a view to testing the limits of accuracy of previous work, and to attain greater accuracy in future. Besides the points already alluded to, he has analysed mixtures of precipitates obtained in precisely the same way in series of analyses, to determine their degree of purity, and so on. It is not intended to go into these matters of detail of method on this occasion. Satisfactory evidence will, however, be afforded in the course of the discussion of the results as to the degree of accuracy and trustworthiness of the analyses; and a comparison of the duplicates given in the Appendix-Tables I., II., and III. will afford further evidence on the point.*

Before considering the composition of the ashes it will be well to show at one view the amount, and to some extent the distribution, of the total ash in the different animals. The following Table (IV.) shows the amount of total ash in 100 fasted live-weight of each of the animals, and the proportion of the whole yielded by the carcass parts and the offal parts respectively. The first three columns show the amounts of crude ash, and the second three the amounts of pure ash. As already stated, the contents of stomachs and intestines are not included in the parts analysed.

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* It need only further be remarked in reference to the analyses as such, that the subsequent investigations of method referred to above, lead to the conclusion that the phosphoric acid determinations made by the magnesium process (nearly the whole) may perhaps be too high, to an extent not exceeding 0·35 per cent. The actual results obtained are, however, recorded in the Tables; nor would any of the conclusions drawn be affected were the supposed correction adopted.

Table IV.—Percentage of Crude Ash, and of Pure Ash, in the fasted Live-Weight.

	Crude ash.			Pure ash.		
	From carcass parts.	From offal parts.	From total parts.	From carcass parts.	From offal parts.	From total parts.
	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
Fat Calf	2·782	1·018	3·800	2·772	1·006	3·779
Half-fat Ox	3·603	1·061	4·664	3·568	1·044	4·612
Fat Ox	3·019	0·901	3·920	2·997	0·882	3·879
Fat Lamb	2·173	0·763	2·936	2·162	0·719	2·881
Store Sheep	2·325	0·839	3·164	2·317	0·747	3·064
Half-fat Old Sheep	2·214	0·959	3·173	2·207	0·848	3·055
Fat Sheep	1·982	0·829	2·811	1·970	0·700	2·670
Very fat Sheep	1·748	1·155	2·903	1·744	1·123	2·867
Store Pig	1·708	0·961	2·669	1·699	0·954	2·652
Fat Pig	1·062	0·587	1·649	1·054	0·581	1·635

When referring to the amounts of crude ash as given in Table II., attention was called to the fact that the percentage of mineral matter, like that of the nitrogenous substance, decreases as the animals mature. This is more clearly seen in the figures in Table IV. relating to the pure ash. Thus, comparing the fat ox with the half-fat ox, there is not only a lower percentage of pure ash in the entire animal, but a lower proportion of the whole contributed both by the carcass parts and the offal parts. Comparing, again, the store sheep, the fat sheep, and the very fat sheep, there is a considerably lower percentage of mineral matter (pure ash) contributed from the carcass parts of the fat than of the store sheep, and less still from those of the very fat sheep. There is also less from the offal parts of the fat sheep than of the store sheep; but there is a considerable excess in the case of the offal parts of the very fat sheep; and, in consequence, some excess in the percentage in the entire animal. Lastly, comparing the store pig and the fat pig, the latter shows a considerably lower proportion of mineral matter from carcass parts, from offal parts, and from all parts.

Referring to the Appendix-Tables I., II., and III. (pp. 885-87), for any further details, the following Table, V., shows the percentage composition of the *pure ash* (that is, excluding sand and charcoal), of the classified parts and of the entire bodies of the ten animals analysed. The upper division of the Table gives the results for the ash of the carcass parts, the middle division for that of the offal parts, and the lower division for the ash of the entire bodies of the animals (excluding contents of stomachs and intestines). When duplicate analyses have been made the mean results only are here given. At the head of each division of the Table are given the percentages of crude ash and of pure ash, not as in Table IV. in each case calculated to the weight of the entire body, but to the weight of the collective parts to which the division refers.

It should be further explained that, for comparison with the results relating to the same parts of the other animals, the composition of the ash of the collective offal parts of the two pigs is calculated from the analyses of the ash of the parts exclusive of the head and feet, and of that of the head and feet, the details of which are given in Appendix-Table II. Again, it will be observed that the results relating to the entire animal ash of the very fat sheep are given in brackets, the figures not being those of the actual analysis, but calculated from the results of the analyses of the ash of the carcass parts and of the offal parts separately. The results of the actual analysis of the entire animal ash are given in the Appendix-Table III.; but although there is no reason to doubt the accuracy of the analysis, there can be no doubt that there has been some omission of parts in making the mixture for burning to ash. Some item rich in potash has obviously been omitted.

Table V.—Percentage of Crude Ash, and of Pure Ash (excluding Sand and Charcoal), and Percentage Composition of the Pure Ash.

	Fat Calf.	Half-fat Ox.	Fat Ox.	Fat Lamb.	Store Sheep.	Half fat old Sheep.	Fat Sheep.	Very fat Sheep.	Store Pig.	Fat Pig.
Collective Carcass Parts.										
	Per cent.	Per cent.	Per cent	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.
Crude ash	4·48	5·56	4·56	3·63	4·36	4·13	3 45	2·77	2·57	1·40
Pure ash	4·46	5·51	4 53	3·61	4·34	4·12	3·43	2·76	2·56	1·39
Peroxide of iron	0·39	0·62	0·56	0·43	0·36	0·49	0·40	0·39	0·63	0·64
Lime	43·93	46·89	47·02	46·83	45·43	46·21	46·65	47·36	40·35	38·59
Magnesia	2·09	1·71	1·70	1·79	1·86	1·76	1·81	2·05	2·13	2·08
Potash	5·90	4·87	4·54	4·62	5·18	5·07	4·65	3·78	8·47	9·68
Soda	3·08	2·60	2 59	2·47	2·97	2·65	2·80	2·74	3·72	4·40
Phosphoric acid	41·54	40·00	40·40	40·37	40·36	40·62	40·84	41·00	40·02	40·19
Sulphuric acid	1·03	0·66	0·69	0·81	1·24	0·50	0·53	0·47	1·96	1·26
Carbonic acid	1·14	1·80	1·68	1·82	1·40	1·84	1·47	1·63	1·17	1·26
Chlorine	1·02	0·75	0·88	0·93	1·46	1·02	0·93	0·70	1·81	2·25
Silica	0·11	0·27	0·14	0·14	0·07	0·07	0·13	0·04	0·15	0·17
Total	100 23	100·17	100·20	100·21	100·33	100·23	100·21	100·16	100·41	100 52
Deduct O = Cl.	0·23	0·17	0·20	0·21	0·33	0·23	0·21	0·16	0·41	0·52
Total	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00
Collective Offal Parts (excluding Contents of Stomach and Intestines).										
Crude ash	3·41	4·05	3·40	2·45	2·19	2·72	2·32	3·64	3·07	2·97
Pure ash	3·37	3·98	3·33	2·31	1·95	2·40	1·96	3·54	3·04	2·93
Peroxide of iron	1·10	1·32	1·78	2·41	3 68	3·73	4·87	2·09	0 90	1·31
Lime	41·39	44·51	41·16	35·91	36·42	37·35	35·22	36·97	41·77	41·07
Magnesia	1·68	1·42	1·28	1·67	1·77	1 57	1·81	1·69	1·79	1 59
Potash	4·46	3·10	4·80	9·28	7·25	7·37	7·89	8·23	5·60	5·99
Soda	6·53	5·56	6·41	6 91	6·99	5·58	6·03	7·29	4·81	4·86
Phosphoric acid	39·26	38·12	39·27	34·86	33·60	35·24	33·15	35·07	40·87	39·85
Sulphuric acid	1·19	1·23	1·59	3·42	2·87	3·17	3·36	1 82	1·23	1·50
Carbonic acid	1·14	1·76	0·90	0·39	0 92	0·99	1·07	1·81	0·67	1·40
Chlorine	3·80	3·30	3·07	4·74	5·31	3·38	3·72	4·76	2·58	2·99
Silica	0·31	0·41	0·43	1·48	2·40	2·38	3·72	1·34	0·34	0·28
Total	100·86	100·73	100·69	101·07	101·21	100 76	100·84	101·07	100·56	100·84
Deduct O = Cl.	0·86	0·73	0·69	1·07	1·21	0·76	0·84	1·07	0·56	0·84
Total	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00
Entire Animal. Fasted Live-weight (but Contents of Stomach and Intestines excluded).*										
Crude ash	3 80	4·66	3·92	2·94	3·16	3·17	2·81	2·90	2·67	1·65
Pure ash	3 77	4·61	3·88	2·88	3·06	3·06	2·69	2·86	2·65	1·64
Peroxide of iron	0·53	0·97	0·41	0·84	1·24	1·35	1·00	(1·05)	0·91	0·76
Lime	43·95	45·26	46·62	44·57	43·12	44·39	44·61	(43·29)	40·58	38·49
Magnesia	2·20	2·03	1·53	1 82	1·82	1·72	1·79	(1·90)	2·01	2·04
Potash	5·40	4·41	4·46	5·74	5·64	5·27	5·53	(5·53)	7·39	8·57
Soda	3·82	3·08	3·04	3·58	3·90	3·35	3·56	(4·52)	4·16	4·36
Phosphoric acid	40·37	40·22	39·80	38·96	38·96	39·15	38·72	(38·68)	40·12	40·14
Sulphuric acid	1·08	0·86	0·79	1·18	1·78	1·06	1·01	(0·99)	2·33	2·15
Carbonic acid	1·34	1·97	2·13	1·53	1·09	1·83	1·67	(1·70)	0·60	1·20
Chlorine	1·55	1·24	1·47	1·86	2·31	1·61	1·61	(2·30)	2·22	2·78
Silica	0·12	0·24	0·08	0·33	0·67	0·63	0·86	(0·56)	0·18	0·14
Total	100·36	100·28	100·33	100·41	100·53	100·36	100·36	(100 52)	100·50	100·63
Deduct O = Cl.	0·36	0·28	0·33	0·41	0·53	0·36	0·36	(0·52)	0·50	0·63
Total	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

* See p. 874, in reference to the entire animal ash of the very fat sheep.

The first point to notice in the analytical results is that the amount of iron peroxide is much higher in the ash of the offal parts than in that of the carcass parts, and that it is much higher in the offal ash of the sheep, than in that of either the oxen or the pigs. This is doubtless due to adventitious matter in the wool, which it was extremely difficult to clean. Indeed, alumina was found, clearly indicating the presence of ferruginous clay. Further, the amount of ferric oxide (as also that of silica) has a very obvious relation to the amount of "sand" found in the ashes. Notwithstanding, therefore, that the offal ash of the animals included that of the blood, the amount of ferric oxide found in the offal ashes must not be relied upon. Reference to Appendix-Table III. will show, however, that the ashes of the offal parts of the pigs, exclusive of the head and feet, do contain a very high percentage of ferric oxide; but if, as in the case of the oxen and sheep, the ash of the head and feet, with its very low percentage of ferric oxide, be included in the collective offal ash, the percentage of ferric oxide in the so-reckoned offal ash of the pig is much lower than in that of the sheep.

The records of the amounts of ferric oxide in the ashes of the carcass parts are only very little open to the same objection as in the case of the offal ashes; but it is obvious that the high percentage in the latter will unduly raise the amount in the entire animal ashes.

Referring to the more important constituents, it is at once seen that the animal ashes consist very largely of phosphate of lime. In the case of ashes of crude products, and particularly of mixed animal substances like those now under consideration, it would be out of place to attempt to arrange the constituents as salts. But it may nevertheless be useful to indicate the general relation of base to acid in the ashes. The lime and magnesia may be taken as essentially, though of course not exclusively, representing the bases of the bone-ash; whilst the potash and soda may in the same general, though not in an exclusive sense, be classified as the flesh and blood bases. Again, by far the larger proportion of the phosphoric acid will be due to the bones; whilst some of it as such, and probably some as the product of the oxidation of phosphorus in the burning, will be connected with the nitrogenous constituents in the non-bony portions of the body. The sulphuric acid, again, will in part be due to the oxidation of sulphur in the burning.

It may be stated, by way of illustration only, that if the phosphoric acid found be calculated as wholly tribasic, the lime of the ashes, excepting in the case of the pigs, would be nearly, and the lime and magnesia together, quite, or more than, sufficient to combine with the whole of the phosphoric acid. The potash and soda again, would be considerably more than sufficient to combine with the sulphuric acid, chlorine, and carbonic acid. It is thus indicated that, notwithstanding some of the phosphoric acid found may be due to the oxidation of phosphorus, and some of the sulphuric acid to the oxidation of sulphur, in the burning, there is upon the whole, according to the above mode of reckoning, an excess of base in the ashes of the ruminants, whilst figures obtained in the same way do not indicate a similar result in the case of the pigs. In the ashes of the pigs the phosphoric acid is considerably in excess of the quantity required to give tribasic phosphates with the whole of the lime and magnesia; the former of which is in relative deficiency. The result is that, upon the whole ash of the pigs, the figures show a deficiency rather than an excess of base, especially in the case of the fatter animal.

The consistency of the result with the ruminants, and again the consistency, but in the opposite direction, with the pigs, would lead to the conclusion that the indication is not simply due to the conditions of incineration, or to error of any kind. As bearing upon the point it is in the first place to be borne in mind that fluorine was not determined in the ashes; but certainly its amount would not be sufficient to turn the scale. Then there is the question whether organic acid salts, and carbonates existing as such, are adequately represented by the amount of carbonic acid determined in the ashes; and there is the further question whether sulphuric acid, and possibly phosphoric acid, may have been reduced, or sulphuric acid or chlorine expelled, in the burning.

According to direct experiments of WAY and OGSTON (Jour. Roy. Ag. Soc. Eng., vol. ix.), sulphuric acid is expelled by silica, but not by acid phosphates, in incineration. They conclude, however, that there is no loss of phosphoric acid; nor of chlorine in careful burning. Others have concluded that the expulsion of both sulphuric acid and chlorine is dependent both on the character of the ash, and on the conditions of the incineration; and in this Mr. RICHTER's experience leads him to concur. On the other hand it has to be considered whether the phosphoric acid and sulphuric acid found in the ashes are not in excess of the amounts existing as such in the substances burnt. On this point WAY and OGSTON and others have long ago concluded that sulphur is oxidated in incineration in very variable amounts according to circumstances; and quite recently GROUVEN has concluded that sulphur is converted into sulphuric acid in the ordinary methods of incineration in free air, and that under the same circumstances sulphates existing in the organic substance burnt are not reduced. GROUVEN concludes that about half of the sulphur may be converted into sulphuric acid in the burning. Again, according to Dr. VOELCKER's experiments (Rep. Brit. Ass., 1857, abstract, p. 60) at any rate a large proportion of the sulphur and phosphorus is not oxidated in the incineration; and FRESENIUS has obtained similar results so far as phosphorus is concerned. Lastly, with regard to the question whether sulphuric acid is reduced by charcoal in the burning, Mr. RICHTER has found in some parallel experiments with wheat grain ashes high sulphuric acid with high charcoal, and low sulphuric acid with low charcoal; but on the other hand he has not observed sulphuretted hydrogen on dissolving such ashes in hydrochloric acid.

It may be stated that the foregoing observations as to the relation of base and acid in the ashes apply generally to those of the collective carcass, of the collective offal, and of the entire bodies of the different descriptions of animal. But where, as in the case of the pigs, the ash of the head and feet, and that of the other offal parts, were analysed separately, the ash of the former, due largely to bone, showed some excess of base even calculating the whole of the phosphoric acid as tribasic, whilst the ash of the other offal parts (the soft parts, blood, &c.) showed, on the same mode of reckoning, about one and a half time as much acid as base; indeed of phosphoric acid alone there is, so reckoned, very much more than is equivalent to the total bases. Here then there is evidence that the ash of the soft parts contains phosphoric acid with less than three of fixed base, and probably some due to the oxidation of phosphorus. In further elucidation of the point in question it may be stated that, although the oxen and sheep show a higher percentage of total nitrogenous substance than the pigs, yet the amount of pure ash yielded from the non-bony parts is higher in proportion to that from the bones in the case of the pigs than in that of the ruminants. That is to say, there is with the pigs a higher proportion of the ash due to parts containing more potash and soda, and less lime and magnesia as base; and so far as phosphoric acid may have existed in the animal substance in combination with potash and soda as ortho-phosphates with water or ammonia also as base, the calculation of the whole of the phosphoric acid of the ash as tribasic (as in our illustration) would necessarily show a relative deficiency of base.

Examination of the Table will show, as might be expected, that the ash of the carcass parts contains a much higher percentage of potash than of soda. This is the case with both the ruminants and the pigs. But with the relative deficiency of lime in the carcass-ash of the pigs, there is a higher percentage of both potash and soda than in that of the ruminants. The distinction between the different animals on these points is chiefly due to the less proportion of ash from bone in the case of the pigs; but it may in part be due to the thick skin being included with the carcass in the case of the pigs, whilst in that of the other animals the skin is not so included.

The ash of the offal parts, including that of the blood, but comparatively little of that of bones, contains, in the case of the ruminants, generally a much higher percentage of both potash and soda than that of the carcass parts, but the proportion of soda to potash is much greater. In the offal ash of the pigs on the other hand (which does not include the ash of the skin) the percentage of both potash and soda is considerably lower than in that of the sheep, and the soda considerably lower than in that of the oxen also.

Reference to Appendix-Table II. (p. 886) will show that in the ash of the offal parts of the pigs excluding the skin and the head and feet, there is only between 3 and 4 per cent. of lime, but about 25 per cent. of potash, and nearly 15 per cent. of soda; whilst in the ash of the head and feet there is nearly 50 per cent. of lime, only between 1 and 2 per cent. of potash, and between 2 and 3 per cent. of soda. Again, as above referred to, there was a considerable excess of acid, especially phosphoric, in the ash of the non-bony portions.

Comparing the percentage composition of the ashes of the entire bodies of the different animals, the chief points of distinction are that, in the case of the pigs there is a lower percentage of lime, a higher percentage of potash and soda, and a higher percentage of sulphuric acid than in the corresponding ash of the ruminants. There is also generally a somewhat higher percentage of phosphoric acid in the entire animal ash of the pigs and the oxen than in that of the sheep.

With these few remarks suggested by a consideration of the percentage composition of the different ashes, we turn now to the bearing of the results as brought to view on applying them to calculate the amount, and as far as practicable the distribution, of the several constituents in a given live-weight of the different animals.

Accordingly, there is given in Table VI. (p. 880), not as before the quantity in 100 of ash, but the quantity in lbs. of each ash constituent, in the actual weight of the collective carcass parts, in the actual weight of the collective offal parts, and in the actual weight of all parts of each of the ten animals. The results are given in more detail in the upper portions of Appendix-Tables IV., V., and VI. (pp. 888-90); in IV. for the calf and oxen, in V. for the lamb and sheep, and in VI. for the pigs. There will be found, besides the amounts in the carcass parts, and in the offal parts respectively, those in the entire animal—first, by addition of the quantities in the carcass and offal parts; secondly, calculated from the direct analysis of the entire animal ashes; and thirdly, the mean of the two last quantities. For the composition of the entire animal, as given in Table VI. (p. 880), this *mean result* is adopted.

Again, in Table VII. (p. 881), is given the quantity of each constituent, not in the actual weight of the separated parts, and the entire bodies of the animals, but calculated in each case to 1,000 lbs. fasted live-weight; thus giving a comparative view of the composition of a given live-weight of the different animals, so far as the mineral or ash constituents are concerned. The particulars are given in detail in the lower divisions of the Appendix-Tables IV., V., and VI.

In the Tables VI. and VII. (pp. 880-81), as in former ones, the upper division gives the results for the carcass parts, the middle division those for the offal parts, and the lower division those for all parts collectively.

Before commenting on these Summary-Tables, we would call attention to the close accordance which the Appendix-Tables IV., V., and VI. show in the mineral composition of the entire bodies, calculated in the one case by the addition of the constituents determined separately in the carcass and in the offal parts, and in the other from the direct analysis of the ash from all parts. It is to be observed that this accordance is satisfactory confirmation not only of the correctness of the ash analyses, but of the preparation of the proportional mixtures of the different parts for burning, representing, respectively, the collective carcass parts, the collective offal parts, and the mixture of all parts. The result of the comparison will, we think, be found very satisfactory in every case excepting that of the entire animal ash of the very fat sheep, to the probable source of error in which reference has already been made (p. 874).

Table VI.—Quantities, in lbs., of Pure Ash, and of each Ash Constituent, in the Collective Carcass Parts, in the Collective Offal Parts, and in the Entire Body (fasted live-weight) of each Animal.

	Fat Calf.	Half-fat Ox.	Fat Ox.	Fat Lamb.	Store Sheep.	Half-fat Old Sheep.	Fat Sheep.	Very Fat Sheep.	Store Pig.	Fat Pig.
Collective Carcass Parts.										
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Fresh weight	160·560	797·688	939·375	50·500	52·063	56·259	73·063	159·250	62·403	140·546
Pure ash	7·173	43·945	42·531	1·831	2·262	2·319	2·505	4·399	1·598	1·949
Peroxide of iron	0·027	0·275	0·240	0·008	0·008	0·011	0·010	0·017	0·010	0·012
Lime	3·151	20·606	19·998	0·857	1·027	1·071	1·169	2·083	0·345	0·752
Magnesia	0·150	0·754	0·724	0·033	0·042	0·041	0·045	0·090	0·034	0·041
Potash	0·423	2·139	1·932	0·085	0·117	0·117	0·117	0·166	0·135	0·189
Soda	0·221	1·141	1·105	0·045	0·067	0·061	0·070	0·120	0·060	0·085
Phosphoric acid	2·980	17·574	17·174	0·739	0·913	0·942	1·023	1·804	0·640	0·783
Sulphuric acid	0·074	0·293	0·296	0·015	0·027	0·012	0·013	0·021	0·031	0·025
Carbonic acid	0·082	0·790	0·715	0·033	0·032	0·043	0·037	0·072	0·019	0·025
Chlorine	0·073	0·328	0·373	0·017	0·033	0·024	0·023	0·031	0·029	0·044
Silica	0·008	0·120	0·060	0·003	0·002	0·002	0·003	0·002	0·002	0·003
Total	7·189	44·020	42·617	1·835	2·269	2·324	2·510	4·406	1·605	1·959
Deduct O = Cl.	0·016	0·075	0·086	0·004	0·007	0·005	0·005	0·007	0·007	0·010
Total	7·173	43·945	42·531	1·831	2·262	2·319	2·505	4·399	1·598	1·949
Collective Offal Parts (excluding Contents of Stomachs and Intestines).										
Fresh-weight	77·114	322·766	376·036	26·331	37·433	37·110	45·408	80·113	29·492	36·541
Pure ash	2·604	12·869	12·522	0·603	0·730	0·891	0·890	2·839	0·893	1·069
Peroxide of iron	0·029	0·170	0·223	0·015	0·027	0·033	0·043	0·059	0·008	0·014
Lime	1·077	5·728	5·153	0·217	0·266	0·333	0·313	1·050	0·373	0·439
Magnesia	0·044	0·183	0·161	0·010	0·013	0·014	0·016	0·048	0·016	0·017
Potash	0·116	0·399	0·601	0·056	0·053	0·066	0·070	0·234	0·050	0·064
Soda	0·170	0·715	0·802	0·042	0·051	0·050	0·054	0·207	0·043	0·052
Phosphoric acid	1·022	4·905	4·916	0·210	0·244	0·314	0·295	0·996	0·365	0·426
Sulphuric acid	0·031	0·160	0·200	0·021	0·021	0·028	0·030	0·051	0·011	0·016
Carbonic acid	0·030	0·226	0·113	0·002	0·007	0·009	0·010	0·051	0·006	0·015
Chlorine	0·099	0·425	0·385	0·028	0·039	0·030	0·033	0·135	0·023	0·032
Silica	0·008	0·052	0·054	0·009	0·018	0·021	0·033	0·038	0·003	0·003
Total	2·626	12·963	12·608	0·610	0·739	0·898	0·897	2·369	0·898	1·078
Deduct O = Cl.	0·022	0·094	0·086	0·007	0·009	0·007	0·007	0·030	0·005	0·009
Total	2·604	12·869	12·522	0·603	0·730	0·891	0·890	2·839	0·893	1·069
Entire Animal, Fasted Live-Weight (but Constituents of Contents of Stomachs and Intestines excluded).*										
Fresh-weight	258·750	1232·000	1419·000	84·406	97·625	105·063	127·156	239·363	93·938	185·000
Pure ash	9·765	56·818	55·094	2·438	2·991	3·217	3·411	7·238	2·491	3·022
Peroxide of iron	0·054	0·499	0·346	0·022	0·036	0·044	0·044	(0·076)	0·021	0·025
Lime	4·257	26·026	25·428	1·082	1·291	1·418	1·505	(3·133)	1·014	1·177
Magnesia	0·204	1·043	0·886	0·044	0·055	0·055	0·061	(0·138)	0·050	0·060
Potash	0·533	2·521	2·496	0·141	0·170	0·177	0·189	(0·400)	0·185	0·256
Soda	0·382	1·802	1·790	0·087	0·117	0·110	0·123	(0·327)	0·104	0·135
Phosphoric acid	3·969	22·668	22·015	0·950	1·161	1·259	1·322	(2·800)	1·002	1·211
Sulphuric acid	0·105	0·471	0·465	0·032	0·051	0·037	0·039	(0·072)	0·050	0·053
Carbonic acid	0·122	1·068	1·004	0·036	0·036	0·055	0·052	(0·123)	0·020	0·039
Chlorine	0·162	0·730	0·782	0·045	0·070	0·053	0·056	(0·166)	0·053	0·080
Silica	0·014	0·155	0·080	0·010	0·020	0·021	0·033	(0·040)	0·005	0·005
Total	9·802	56·983	55·272	2·449	3·007	3·229	3·424	(7·275)	2·504	3·041
Deduct O = Cl.	0·037	0·165	0·178	0·011	0·016	0·012	0·013	(0·037)	0·013	0·019
Total	9·765	56·818	55·094	2·438	2·991	3·217	3·411	(7·238)	2·491	3·022

* See p. 874, in reference to the entire animal ash of the very fat sheep.

TABLE VII.—Quantities, in lbs., of Pure Ash, and of each Ash Constituent, in 1,000 lbs. Fasted Live-Weight in each case.

	Fat Calf.	Half-fat Ox.	Fat Ox.	Fat Lamb.	Store Sheep.	Half-fat o'd Sheep.	Fat Sheep.	Very Fat Sheep.	Store Pig.	Fat Pig.
COLLECTIVE CARCASS PARTS.										
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Fresh-weight	621	647	662	598	533	536	575	630	664	760
Pure ash	27·742	35·643	29·973	21·682	23·158	22·094	19·715	17·402	17·003	10·539
Peroxide of iron . . .	0·104	0·223	0·169	0·095	0·082	0·105	0·079	0·067	0·106	0·065
Lime	12·187	16·713	14·093	10·148	10·514	10·204	9·200	8·240	6·863	4·066
Magnesia	0·580	0·611	0·512	0·391	0·430	0·391	0·354	0·356	0·362	0·222
Potash	1·636	1·735	1·361	1·006	1·198	1·115	0·921	0·656	1·436	1·022
Soda	0·855	0·925	0·779	0·533	0·686	0·581	0·551	0·475	0·638	0·460
Phosphoric acid . . .	11·526	14·254	12·103	8·751	9·347	8·974	8·051	7·136	6·810	4·234
Sulphuric acid	0·286	0·238	0·208	0·178	0·287	0·114	0·102	0·083	0·330	0·135
Carbonic acid	0·317	0·641	0·504	0 391	0·328	0·410	0·291	0·285	0·202	0·135
Chlorine	0·282	0·266	0·263	0·201	0·338	0·229	0·181	0·123	0·309	0·238
Silica	0·031	0·098	0·042	0·035	0·020	0·019	0·024	0·008	0·021	0·016
Total . . .	27·804	35·704	30·034	21·729	23·230	22·142	19·754	17·429	17·077	10·593
Deduct O = Cl. . .	0·062	0·061	0·061	0·047	0·072	0·048	0·039	0·027	0·074	0·054
Total . . .	27·742	35·643	29·973	21·682	23·158	22·094	19·715	17·402	17·003	10·539
COLLECTIVE OFFAL PARTS (EXCLUDING CONTENTS OF STOMACHS AND INTESTINES).										
Fresh weight	298	262	265	312	383	353	357	317	314	197
Pure ash	10·063	10·447	8·824	7·145	7·469	8·476	6·998	11·234	9·488	5·756
Peroxide of iron . . .	0·112	0·138	0·157	0·177	0·276	0·314	0·338	0·234	0·086	0·076
Lime	4·162	4·650	3·631	2·571	2·722	3·168	2·461	4·155	3·960	2·363
Magnesia	0·170	0·148	0·114	0·118	0·133	0·133	0·126	0·190	0·170	0·091
Potash	0·448	0·324	0·424	0·664	0·542	0·628	0·550	0·926	0·532	0·344
Soda	0·657	0·580	0·565	0·498	0·522	0·476	0·425	0·819	0·457	0·280
Phosphoric acid. . . .	3·949	3·982	3·464	2·488	2·496	2·987	2·319	3·941	3·878	2·294
Sulphuric acid	0·120	0·130	0·141	0·249	0·215	0·266	0·236	0·202	0·117	0·087
Carbonic acid	0·116	0·184	0·080	0·024	0·072	0·086	0·079	0·202	0·064	0·080
Chlorine	0·383	0·345	0·271	0·332	0·399	0·285	0·259	0·534	0·245	0·172
Silica	0·031	0·042	0·038	0·107	0·184	0·200	0·259	0·150	0·032	0·017
Total . . .	10·148	10·523	8·885	7·228	7·561	8·543	7·052	11·353	9·541	5·804
Deduct O = Cl. . .	0·085	0·076	0·061	0·083	0·092	0·067	0·054	0·119	0·053	0·048
Total . . .	10·063	10·447	8·824	7·145	7·469	8·476	6·998	11·234	9·488	5·756
ENTIRE ANIMAL, FASTED LIVE WEIGHT (BUT CONSTITUENTS OF CONTENTS OF STOMACHS AND INTESTINES EXCLUDED).*										
Fresh-weight†	919	909	927	910	916	889	932	947	978	957
Pure ash	37·759	46·094	38·826	28·876	30·615	30·634	26·836	28·636	26·501	16·320
Peroxide of iron . . .	0·207	0·405	0·244	0·261	0·369	0·419	0·343	(0·301)	0·218	0·133
Lime	16·463	21·114	17·919	12·808	13·214	13·503	11·844	(12·395)	10·792	6·359
Magnesia	0·788	0·846	0·611	0·515	0·558	0·524	0·484	(0·546)	0·532	0·324
Potash	2·061	2·045	1·759	1·664	1·735	1·681	1·483	(1·582)	1·963	1·380
Soda	1·477	1·461	1·261	1·030	1·197	1·043	0·968	(1·294)	1·101	0·727
Phosphoric acid . . .	15·349	18·390	15·514	11·257	11·883	11·988	10·404	(11·077)	10·660	6·544
Sulphuric acid	0·406	0·382	0·328	0·386	0·522	0·352	0·307	(0·285)	0·532	0·288
Carbonic acid	0·470	0·367	0·708	0·427	0·369	0·529	0·409	(0·487)	0·213	0·208
Chlorine	0·625	0·592	0·552	0·533	0·722	0·505	0·437	(0·657)	0·570	0·432
Silica	0·054	0·126	0·056	0·119	0·205	0·204	0·255	(0·158)	0·053	0·028
Total . . .	37·900	46·228	38·952	29·000	30·774	30·748	26·934	(28·782)	26·634	16·423
Deduct O = Cl. . .	0·141	0·134	0·126	0·124	0·159	0·114	0·098	(0·146)	0·133	0·103
Total . .	37·759	46·094	38·826	28·876	30·615	30·634	26·836	(28·636)	26·501	16·320

* See p. 874, in reference to the entire animal ash of the very fat sheep.

† Excluding evaporation, and contents of stomachs and intestines.

Not much stress should be laid on the exact quantities of the total ash, or of the individual mineral constituents, in the actual weights of the particular animals analysed, as shown in Table VI., as the actual weights and condition of animals coming under similar designations may vary considerably. Subject to the reservation here implied, it may be stated that a calf weighing 160 lbs. carried off less than 10 lbs. of total mineral matter; oxen weighing from 1,200 to 1,400 lbs. from 55 to 60 lbs.; a fat lamb about 2½ lbs.; a store sheep under 3 lbs; a fat sheep from 3¼ to 3½ lbs.; and a very fat sheep of nearly 240 lbs. live-weight, twice as much, or more than 7 lbs. The pigs again, contained less than sheep in proportion to their weight.

The calf carried off about 4 lbs. phosphoric acid=between 8 and 9 lbs. of phosphate of lime, little more than half-a-pound of potash, and immaterial amounts of other mineral constituents. The oxen carried off between 22 and 23 lbs. phosphoric acid=less than 50 lbs. of phosphate of lime, and about 2½ lbs. of potash. The fat lamb carried off less than 1 lb. phosphoric acid=only about 2 lbs. of phosphate of lime, the store sheep and an ordinary fat sheep rather more=between 2½ and 3 lbs. phosphate of lime, whilst the amount of potash in any of these animals would only be from 2¼ to 3 ounces. There would be proportionally greater variation in the actual weight of pigs sold off the farm than of sheep; and, for this reason, it is especially in their case, though it is so in that of the other animals also, better to consider the amount of mineral constituents lost to the farm in them in relation to a given live-weight rather than in the actual live-weight.

Table VII. which shows the amount of the different constituents in carcass, in offal, and in the entire body, *of* 1,000 *lbs. fasted live-weight,* of the different animals, is much more instructive.

In the first place 1,000 lbs. live-weight of calves or oxen is seen to carry off much more mineral matter than 1,000 lbs. live-weight of lambs or sheep, and 1,000 lbs. live-weight of pigs much less than sheep. In the particular cases in question, there were 46 lbs. of total mineral matter per 1,000 lbs. live-weight of the lean ox of less actual weight, and scarcely 39 lbs. in an equal weight of the fatter animal. The difference is in the right direction, but doubtless somewhat excessive; the fatter and heavier animal having actually less total mineral matter. Whilst 1,000 lbs. live-weight of oxen may thus contain 40 lbs. or even nearly 50 lbs. of mineral matter, the same weight of sheep will carry off only about 30 lbs. or less, and the same live-weight of pigs less still, and sometimes very much less. In all cases by far the larger proportion of the total mineral matter is in the collective carcass parts; and in the case of the pigs the proportion so distributed would be much greater than the Table shows, as there the head and feet are included with the offal, whilst in practice they are weighed with the carcass.

Referring to the amounts of the most important mineral constituents, whilst 1,000 lbs. live-weight of calves or oxen may carry off from 30 to 40 lbs. of phosphate of lime, the same weight of sheep would carry off only about 26 lbs. or less, and an equal live-weight of pigs considerably less still. With each description of animal the quantity of phosphate is less in a given live-weight of the fatter than of the leaner individuals ; and this is especially so in the case of the pigs. In round numbers it may be said that 1,000 lbs. live-weight of oxen will carry off only 2 lbs., or less, of potash ; 1,000 lbs. of sheep from $1\frac{3}{4}$ to $1\frac{1}{2}$ lb. ; and 1,000 lbs. of pigs about the same ; in each case the less, the fatter the animal. Of the potash, as of the phosphoric acid, by far the larger proportion of the whole is in the carcass parts. The constituent coming next in amount is soda ; but with oxen the quantity in 1,000 lbs. live-weight does not reach $1\frac{1}{2}$ lb., with sheep it is only about 1 lb., and with pigs about the same, or less in the fat condition.

It may be said with regard to each description of animal that a given live-weight will contain less of every constituent the more it is matured or fattened.

So far as the practical bearings of the subject are concerned, it will be seen that the production and sale of the animals of the farm carries off comparatively immaterial amounts of mineral constituents, but an equal weight of oxen more than the same weight of sheep, and an equal weight of sheep more than the same weight of pigs. Again, four-fifths of the whole, or even much more, will be phosphate of lime, and the amount of potash very small. The loss to the land, or to the manure from purchased food, will, however, be considerably more with growing than with only fattening animals.

It is obvious, indeed, that the amount of mineral constituents lost to the farm by mere fattening increase will be almost insignificant. We have elsewhere estimated that the *increase* of oxen and sheep over the final four or six months of the fattening period, will not contain more than about $1\frac{1}{2}$ per cent. of mineral matter ; that of pigs over the usually shorter period not more than 1 per cent., and in the case of very fat animals less still.

As conveying a somewhat more definite idea on the point, the amount of some of the most important mineral constituents that would be removed from an acre of fair average pasture and arable land, in animal increase and in some other products, may be compared. Such estimates can obviously be only approximate, and the quantities will be subject to considerable range of variation. Taking them as such, it may be stated in general terms that—of phosphoric acid an acre would lose more in milk, and four or five times as much in wheat or barley grain, or in hay, as in the fattening increase of oxen or sheep. Of lime the land would lose about twice as much in the animal increase as in milk, or as in wheat or barley grain, but, perhaps, not more than one-tenth as much as in hay. Lastly, of potash an acre would yield only a fraction of a pound in animal increase, six or eight times as much in milk, perhaps twenty or thirty times as much in wheat or barley grain, and more than one hundred times as much in hay.

The loss to the land in the animal increase is, in fact, chiefly in phosphate of lime, in amount varying from 5 to 10 lbs. per acre. In milk the loss is higher in phosphoric acid, less in lime, and more in potash. In wheat and barley grain the loss of phosphoric acid is several times as great, and it is chiefly as phosphate of potash; whilst in hay the loss in phosphoric acid is much the same as in wheat or barley grain, but that of both lime and potash is very much greater than in any of the other products.

It is freely granted that the results which have been brought forward are calculated to suggest rather than to answer questions of interest from the point of view of the physiologist. He will ask why the selection of parts submitted to analysis was not more detailed. The answer must be that the agricultural aspects of the subject were necessarily those which guided the course of the investigation; and that, although it would have been carried out in more detail had it been practicable to do so, the pressure of other equally essential work has enforced the limitation which has been adopted. The execution of 40 complete ash-analyses is indeed a matter of no small labour; and however much we may regret that we have not been able to give a wider scope to the inquiry, we must be satisfied that the results do at least form a substantial contribution to the chemical statistics of the feeding of the animals of the farm for human food.

Appendix-Table I.—Percentage Composition of the Ash of the Collective Carcass Parts of Ten Animals.

	Fat Calf.	Half-fat Ox.	Fat Ox.			Fat Lamb.	Store Sheep.	Half-fat old Sheep.	Fat Sheep.			Very fat Sheep.	Store Pig.	Fat Pig.
			Analysis 1.	Analysis 2.	Mean.				Analysis 1.	Analysis 2.	Mean.			
1. Actual Analyses of Crude Ash.														
Iron peroxide	0·39	0·62	0·56	0·56	0·56	0·43	0·36	0·49	0·40	0·40	0·40	0·39	0·63	0·64
Lime	44·25	46·57	46·95	47·00	46·98	46·88	45·60	46·08	46·54	46·73	46·63	47·67	40·42	38·89
Magnesia	2·11	1·70	1·75	1·65	1·70	1·79	1·87	1·76	1·79	1·81	1·80	2·06	2·13	2·10
Potash	5·94	4·84	4·54	4·53	4·53	4·63	5·20	5·05	4·64	4·67	4·66	3·80	8·49	9·76
Soda	3·10	2·58	2·55	2·64	2·60	2·47	2·98	2·64	2·72	2·87	2·80	2·76	3·73	4·43
Phosphoric acid	41·85	39·72	40·42	40·29	40·35	40·42	40·51	40·51	40·77	40·89	40·83	41·28	40·09	40·50
Sulphuric acid	1·04	0·66	0·65	0·73	0·69	0·81	1·25	0·50	0·52	0·54	0·53	0·47	1·96	1·27
Carbonic acid	1·15	1·79	1·68	1·68	1·68	1·82	1·40	1·83	1·47	(1·47)	1·47	1·64	1·17	1·27
Chlorine	1·03	0·74	0·86	0·89	0·88	0·93	1·47	1·02	0·93	0·93	0·93	0·71	1·81	2·27
Silica	0·11	0·27	0·14	0·14	0·14	0·14	0·07	0·07	0·12	0·14	0·13	0·04	0·15	0·17
Sand	0·35	0·98	0·69	0·74	0·71	0·49	0·36	0·30	0·61	0·61	0·61	0·26	0·41	0·76
Charcoal	Trace	Trace	Trace	Trace	Trace	None	Trace	None	Trace	Trace	Trace	Trace	Trace	Trace
Total	101·32	100·47	100·79	100·85	100·82	100·81	101·07	100·25	100·51	101·06	100·79	101·08	100·99	102·06
Deduct O=Cl	0·23	0·17	0·19	0·20	0·20	0·21	0·33	0·23	0·21	0·21	0·21	0·16	0·41	0·52
Total	101·09	100·30	100·60	100·65	100·62	100·60	100·74	100·02	100·30	100·85	100·58	100·92	100·58	101·54
2. Composition of Crude Ash, Calculated to exactly 100.														
Iron peroxide	0·38	0·62	0·56	0·56	0·56	0·43	0·36	0·49	0·40	0·40	0·40	0·39	0·62	0·63
Lime	43·77	46·43	46·67	46·69	46·68	46·60	45·26	46·07	46·40	46·34	46·37	47·23	40·19	38·30
Magnesia	2·09	1·70	1·74	1·64	1·69	1·78	1·86	1·76	1·78	1·79	1·79	2·04	2·12	2·07
Potash	5·88	4·82	4·51	4·50	4·51	4·60	5·16	5·05	4·63	4·63	4·63	3·77	8·44	9·61
Soda	3·07	2·57	2·54	2·62	2·58	2·45	2·96	2·64	2·71	2·85	2·78	2·73	3·71	4·36
Phosphoric acid	41·40	39·60	40·16	40·03	40·09	40·18	40·21	40·50	40·65	40·54	40·59	40·90	39·86	39·89
Sulphuric acid	1·03	0·66	0·65	0·73	0·69	0·81	1·24	0·50	0·52	0·54	0·53	0·47	1·95	1·25
Carbonic acid	1·14	1·78	1·67	1·67	1·67	1·81	1·39	1·83	1·46	1·46	1·46	1·63	1·16	1·25
Chlorine	1·02	0·74	0·86	0·88	0·87	0·92	1·46	1·02	0·93	0·92	0·93	0·70	1·80	2·24
Silica	0·11	0·27	0·14	0·14	0·14	0·14	0·07	0·07	0·12	0·14	0·13	0·04	0·15	0·17
Sand	0·34	0·98	0·69	0·74	0·72	0·49	0·36	0·30	0·61	0·60	0·60	0·26	0·41	0·75
Charcoal	Trace	Trace	Trace	Trace	Trace	None	Trace	None	Trace	Trace	Trace	Trace	Trace	Trace
Total	100·23	100·17	100·19	100·20	100·20	100·21	100·33	100·23	100·21	100·21	100·21	100·16	100·41	100·52
Deduct O=Cl	0·23	0·17	0·19	0·20	0·20	0·21	0·33	0·23	0·21	0·21	0·21	0·16	0·41	0·52
Total	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00
3. Composition of Pure Ash (that is excluding Sand and Charcoal) Calculated to 100.														
Iron peroxide	0·39	0·62	0·56	0·56	0·56	0·43	0·36	0·49	0·40	0·40	0·40	0·39	0·63	0·64
Lime	43·93	46·89	47·00	47·04	47·02	46·83	45·43	46·21	46·69	46·62	46·65	47·36	40·35	38·59
Magnesia	2·09	1·71	1·75	1·65	1·70	1·79	1·86	1·76	1·80	1·81	1·81	2·05	2·13	2·08
Potash	5·90	4·87	4·54	4·54	4·54	4·62	5·18	5·07	4·65	4·66	4·65	3·78	8·47	9·68
Soda	3·08	2·60	2·55	2·64	2·59	2·47	2·97	2·65	2·73	2·86	2·80	2·74	3·72	4·40
Phosphoric acid	41·54	40·00	40·46	40·33	40·40	40·37	40·36	40·62	40·90	40·79	40·84	41·00	40·02	40·19
Sulphuric acid	1·03	0·66	0·65	0·73	0·69	0·81	1·24	0·50	0·52	0·54	0·53	0·47	1·96	1·26
Carbonic acid	1·14	1·80	1·68	1·68	1·68	1·82	1·40	1·84	1·47	1·46	1·47	1·63	1·17	1·26
Chlorine	1·02	0·75	0·86	0·89	0·88	0·93	1·46	1·02	0·93	0·93	0·93	0·70	1·81	2·25
Silica	0·11	0·27	0·14	0·14	0·14	0·14	0·07	0·07	0·12	0·14	0·13	0·04	0·15	0·17
Total	100·23	100·17	100·19	100·20	100·20	100·21	100·33	100·23	100·21	100·21	100·21	100·16	100·41	100·52
Deduct O=Cl	0·23	0·17	0·19	0·20	0·20	0·21	0·33	0·23	0·21	0·21	0·21	0·16	0·41	0·52
Total	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

APPENDIX-TABLE II.—Percentage Composition of the Ash of the Collective (and some separated) Offal Parts (excluding Contents of Stomachs and Intestines) of Ten Animals.

	Fat Calf.			Half-fat Ox.				Fat Ox.	Fat Lamb.	Store Sheep.	Lean Sheep.	Half-fat old Sheep.	Very fat Sheep.			Store Pig (excluding head and feet).			Fat Pig (excluding head and feet).	Head and feet.	
	Analysis 1.	Analysis 2.	Mean.	Analysis 1.	Analysis 2.	Analysis 3.	Mean.						Analysis 1.	Analysis 2.	Mean.	Analysis 1.	Analysis 2.	Mean.		Store Pig.	Fat Pig.
1. ACTUAL ANALYSES OF CRUDE ASH.																					
Iron peroxide	1·16	1·06	1·11	1·12	1·49	1·29	1·30	1·73	2·29	3·29	3·33	4·16	2·09	2·01	2·05	4·25	4·65	4·45	5·70	0·20	0·28
Lime	41·57	41·71	41·64	44·52	43·37	44·05	43·98	40·09	34·15	32·56	33·31	30·08	36·21	36·21	36·21	3·50	3·39	3·44	3·65	49·90	49·83
Magnesia	1·71	1·66	1·68	1·57	1·62	1·03	1·41	1·25	1·59	1·58	1·40	1·55	1·67	1·64	1·66	1·61	2·52	2·08	1·32	1·77	1·65
Potash	4·45	4·53	4·49	3·00	3·11	3·08	3·06	4·68	8·83	6·48	6·57	6·74	8·05	8·08	8·06	23·17	23·53	23·35	24·04	1·66	1·59
Soda	6·61	6·53	6·57	5·37	5·41	5·70	5·49	6·24	6·57	6·25	4·98	5·15	7·14	7·14	7·14	14·25	14·50	14·38	13·58	2·67	2·72
Phosphoric acid	39·47	39·53	39·50	37·80	37·28	37·91	37·66	38·25	33·16	30·04	31·43	28·32	34·32	34·36	34·34	38·24	38·16	38·20	34·62	41·18	40·88
Sulphuric acid	1·20	1·20	1·20	1·28	1·22	1·16	1·22	1·55	3·25	2·57	2·83	2·87	1·77	1·78	1·78	2·53	2·34	2·44	2·70	0·98	1·12
Carbonic acid	1·27	1·02	1·15	1·74	(1·74)	(1·74)	1·74	(0·88)	(0·37)	0·82	0·88	0·91	1·72	1·82	1·77	(0·00)	(0·00)	(0·00)	0·26	0·76	1·59
Chlorine	3·81	3·85	3·83	3·39	3·36	3·04	3·26	2·99	4·51	4·75	3·02	3·18	4·67	4·66	4·66	9·80	7·87	8·83	9·88	1·25	1·33
Silica	0·30	0·31	0·31	0·40	0·33	0·46	0·40	0·42	1·41	2·15	2·12	3·18	1·36	1·25	1·31	1·21	1·02	1·12	1·10	0·14	0·16
Sand	1·22	1·11	1·16	1·49	1·93	1·43	1·62	2·09	5·81	10·99	11·71	15·72	2·75	2·80	2·77	3·62	3·78	3·70	3·81	0·29	0·46
Charcoal	Trace	Trace	Trace	Trace	Trace	Trace	Trace	None	Trace	Trace	Trace	Trace	Trace	Trace	Trace	0·54	0·27	0·40	1·78	Trace	Trace
Total	102·77	102·51	102·64	101·68	100·86	100·89	101·14	100·17	101·94	101·48	101·58	101·86	101·75	101·75	101·75	102·75	102·03	102·39	102·44	100·80	101·61
Deduct O=Cl	0·86	0·87	0·87	0·72	0·76	0·69	0·72	0·67	1·02	1·08	0·68	0·72	1·05	1·05	1·05	2·21	1·78	2·00	2·66	0·28	0·30
Total	101·91	101·64	101·77	100·96	100·10	100·20	100·42	99·50	100·92	100·40	100·90	101·14	100·70	100·70	100·70	100·54	100·25	100·39	99·78	100·52	101·31
2. COMPOSITION OF CRUDE ASH, CALCULATED TO EXACTLY 100.																					
Iron peroxide	1·14	1·04	1·09	1·11	1·49	1·29	1·30	1·74	2·27	3·28	3·30	4·11	2·08	2·00	2·04	4·23	4·64	4·44	5·71	0·20	0·28
Lime	40·79	41·04	40·91	44·10	43·32	43·96	43·79	40·29	33·84	32·43	33·01	29·74	35·96	35·96	35·96	3·48	3·38	3·43	3·67	49·64	49·19
Magnesia	1·68	1·64	1·66	1·56	1·62	1·03	1·40	1·26	1·57	1·57	1·39	1·53	1·66	1·63	1·65	1·63	2·51	2·07	1·32	1·76	1·63
Potash	4·37	4·46	4·42	2·97	3·11	3·07	3·05	4·70	8·75	6·45	6·51	6·67	7·99	8·02	8·00	23·05	23·47	23·26	24·09	1·65	1·57
Soda	6·49	6·43	6·46	5·32	5·40	5·69	5·47	6·27	6·51	6·23	4·94	5·09	7·09	7·09	7·09	14·17	14·47	14·32	13·61	2·66	2·68
Phosphoric acid	38·72	38·89	38·80	37·44	37·24	37·83	37·50	38·44	32·86	29·92	31·15	28·00	34·08	34·12	34·10	38·04	38·07	38·05	34·69	40·97	40·35
Sulphuric acid	1·18	1·18	1·18	1·27	1·22	1·16	1·22	1·56	3·22	2·56	2·81	2·84	1·76	1·77	1·76	2·52	2·33	2·43	2·71	0·97	1·11
Carbonic acid	1·25	1·00	1·13	1·72	1·74	1·74	1·73	(0·88)	(0·37)	0·82	0·87	0·90	1·71	1·81	1·76	(0 00)	(0·00)	(0·00)	0·26	0·76	1·57
Chlorine	3·74	3·79	3·77	3·36	3·36	3·03	3·25	3·01	4·47	4·73	2·99	3·15	4·64	4·63	4·64	9·75	7·85	8·80	9·90	1·24	1·31
Silica	0·30	0·31	0·31	0·39	0·33	0·46	0·40	0·42	1·40	2·14	2·10	3·15	1·35	1·24	1·29	1·20	1·02	1·11	1·10	0·14	0·16
Sand	1·20	1·09	1·14	1·48	1·93	1·43	1·61	2·10	5·76	10·95	11·61	15·54	2·73	2·78	2·76	3·60	3·77	3·68	3·82	0·29	0·45
Charcoal	Trace	Trace	Trace	Trace	Trace	Trace	Trace	None	Trace	Trace	Trace	Trace	Trace	Trace	Trace	0·54	0·27	0·41	1·78	Trace	Trace
Total	100·86	100·87	100·87	100·72	100·76	100·69	100·72	100·67	101·02	101·08	100·68	100·72	101·05	101·05	101·05	102·21	101·78	102·00	102·66	100·28	100·30
Deduct O=Cl	0·86	0·87	0·87	0·72	0·76	0·69	0·72	0·67	1·02	1·08	0·68	0·72	1·05	1·05	1·05	2·21	1·78	2·00	2·66	0·28	0·30
Total	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00
3. COMPOSITION OF PURE ASH (THAT IS, EXCLUDING SAND AND CHARCOAL), CALCULATED TO 100.																					
Iron peroxide	1·15	1·05	1·10	1·12	1·52	1·31	1·32	1·78	2·41	3·68	3·73	4·87	2·13	2·05	2·09	4·41	4·84	4·62	6·05	0·20	0·28
Lime	41·29	41·49	41·39	44·75	44·18	44·60	44·51	41·16	35·91	36·42	37·35	35·22	36·97	36·97	36·97	3·63	3·52	3·58	3·87	49·78	49·41
Magnesia	1·70	1·65	1·68	1·58	1·65	1·04	1·42	1·28	1·67	1·77	1·57	1·81	1·70	1·68	1·69	1·70	2·62	2·16	1·40	1·77	1·63
Potash	4·42	4·51	4·46	3·02	3·17	3·12	3·10	4·80	9·28	7·25	7·37	7·89	8·22	8·25	8·23	24·04	24·46	24·25	25·52	1·66	1·58
Soda	6·56	6·50	6·53	5·40	5·51	5·77	5·56	6·41	6·91	6·99	5·58	6·03	7·29	7·29	7·29	14·78	15·07	14·92	14·42	2·66	2·70
Phosphoric acid	39·20	39·32	39·26	38·00	37·97	38·38	38·12	39·27	34·86	33·60	35·24	33·15	35·04	35·10	35·07	39·67	39·67	39·67	36·75	41·08	40·53
Sulphuric acid	1·19	1·19	1·19	1·29	1·24	1·17	1·23	1·59	3·42	2·87	3·17	3·36	1·81	1·82	1·82	2·63	2·43	2·53	2·87	0·98	1·11
Carbonic acid	1·26	1 01	1·14	1·75	1·77	1·76	1·76	0·90	0·39	0·92	0·99	1·07	1·75	1·86	1·81	(0·00)	(0·00)	(0·00)	0·28	0·76	1·58
Chlorine	3·78	3·83	3·80	3·41	3·42	3·08	3·30	3·07	4·74	5·31	3·38	3·72	4·77	4·76	4·76	10·17	8·18	9·18	10·49	1·25	1·32
Silica	0·30	0·31	0·31	0·40	0·34	0·47	0·41	0·43	1·48	2·40	2·38	3·72	1·39	1·28	1·34	1·26	1·06	1·16	1·17	0·14	0·16
Total	100·85	100·86	100·86	100·72	100·77	100·70	100·73	100·69	101·07	101·21	100·76	100·84	101·07	101·07	101·07	102·29	101·85	102·07	102·82	100·28	100·30
Deduct O=Cl	0·85	0·86	0·86	0·72	0·77	0·70	0·73	0·69	1·07	1·21	0·76	0·84	1·07	1·07	1·07	2·29	1·85	2·07	2·82	0·28	0·30
Total	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·09	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

APPENDIX-TABLE III.—Percentage Composition of the Ash of the Entire Bodies (excluding Contents of Stomachs and Intestines) of Ten Animals.

	Fat Calf.	Half-fat Ox.	Fat Ox.	Fat Lamb.	Store Sheep.	Half-fat old sheep.	Fat Sheep.			Very fat Sheep.*	Store Pig.	Fat Pig.
							Analysis 1.	Analysis 2.	Mean.			
1. ACTUAL ANALYSES OF CRUDE ASH.												
Iron peroxide . .	0·52	0·96	0·41	0·83	1·20	1·32	0·70	1·22	0·96	0·13	0·91	0·77
Lime	43·40	44·85	46·08	43·86	41·90	43·29	42·75	42·75	42·75	53·25	40·43	38·68
Magnesia . . .	2·17	2·01	1·51	1·79	1·77	1·68	1·66	1·77	1·72	1·75	2·00	2·05
Potash	5·33	4·37	4·41	5·65	5·48	5·14	5·30	5·29	5·30	0·37	7·36	8·61
Soda	3·77	3·05	3·00	3·52	3·79	3·27	3·36	3·45	3·41	1·72	4·14	4·38
Phosphoric acid .	39·86	39·86	39·33	38·34	37·86	38·18	37·21	37·09	37·11	40·88	39·97	40·34
Sulphuric acid. .	1·07	0·85	0·78	1·16	1·73	1·03	0·94	0·99	0·97	0·32	2·32	2·16
Carbonic acid . .	1·32	1·95	2·11	1·51	1·06	1·78	(1·29)	1·61	1·61	2·04	0·60	1·21
Chlorine. . . .	1·53	1 23	1·45	1·83	2·24	1·57	1·53	1·54	1·54	0·22	2·21	2·79
Silica.	0·12	0·24	0·08	0·32	0·65	0·61	0·81	0·83	0·82	0·04	0·18	0·14
Sand	0·79	1·11	0·89	1·53	3·25	3·32	3·93	4·27	4·10	0·51	0·66	0·79
Charcoal . . .	Trace	Trace	Trace	None	Trace	None	Trace	None	Trace	Trace	Trace	Trace
Total . .	99·88	100·48	100·05	100·34	100·93	101·19	99 48	100·81	100·29	101·23	100·78	101·92
Deduct O=Cl	0·35	0·28	0·33	0·41	0·51	0·35	0·34	0·35	0·35	0·05	0·50	0·63
Total . .	99·53	100·20	99·72	99·93	100·42	100·84	99·14	100·46	99·94	101·18	100·28	101·29
2. COMPOSITION OF CRUDE ASH, CALCULATED TO EXACTLY 100.												
Iron peroxide . .	0·52	0·96	0·41	0·83	1·19	1·31	0·71	1·21	0·96	0·13	0·91	0·76
Lime	43·60	44·76	46·21	43·89	41·73	42·93	43·12	42·55	42·78	52·63	40·32	38·19
Magnesia . . .	2·18	2·00	1·52	1·79	1·76	1·66	1·67	1·76	1·72	1·73	1·99	2·02
Potash	5·36	4·36	4·42	5·66	5·46	5·10	5·35	5·27	5·31	0·37	7·34	8·50
Soda	3·79	3·04	3·01	3·52	3·77	3·24	3·39	3·44	3·41	1·70	4·13	4·32
Phosphoric acid .	40·04	39·78	39·44	38·37	37·70	37·86	37·53	36·92	37·14	40·40	39·86	39·83
Sulphuric acid. .	1·07	0·85	0·78	1·16	1·72	1·02	0·95	0·99	0·97	0·32	2·31	2·13
Carbonic acid . .	1·33	1·95	2·12	1·51	1·06	1·77	(1·30)	1·60	1·60	2·01	0·60	1·20
Chlorine. . . .	1·54	1·23	1·45	1·83	2·23	1·56	1·54	1·53	1·54	0·22	2·20	2·76
Silica.	0·12	0·24	0·08	0·32	0·65	0·61	0·82	0·83	0·82	0·04	0·18	0·14
Sand	0·80	1·11	0·89	1·53	3·24	3·29	3·96	4·25	4·10	0·50	0·66	0·78
Charcoal . . .	Trace	Trace	Trace	None	Trace	None	Trace	None	Trace	Trace	Trace	Trace
Total . .	100·35	100·28	100·33	100·41	100·51	100·35	100·34	100·35	100·35	100·05	100·50	100·63
Deduct O=Cl	0·35	0·28	0·33	0·41	0 51	0·35	0·34	0·35	0·35	0·05	0·50	0·63
Total . .	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00
3. COMPOSITION OF PURE ASH (THAT IS, EXCLUDING SAND AND CHARCOAL), CALCULATED TO 100.												
Iron peroxide . .	0·53	0·97	0·41	0·84	1·24	1·35	0·74	1·27	1·00	0·13	0·91	0·76
Lime	43·95	45·26	46·62	44·57	43·12	44·39	44·90	44·44	44·61	52·89	40·58	38·49
Magnesia . . .	2·20	2·03	1·53	1·82	1·82	1·72	1·74	1·84	1·79	1·74	2·01	2·04
Potash	5·40	4·41	4·46	5·74	5·64	5·27	5·57	5·50	5·53	0·37	7·39	8·57
Soda	3·82	3·08	3·04	3·58	3·90	3·35	3·53	3·59	3·56	1·71	4·16	4·36
Phosphoric acid .	40·37	40·22	39·80	38·96	38·96	39·15	39·08	38·56	38·72	40·61	40·12	40·14
Sulphuric acid. .	1·08	0·86	0·79	1·18	1·78	1·06	0·99	1·03	1·01	0·32	2·33	2·15
Carbonic acid . .	1·34	1·97	2·13	1·53	1·09	1·83	(1·35)	1·67	1·67	2·02	0·60	1·20
Chlorine. . . .	1·55	1·24	1·47	1·86	2·31	1·61	1·61	1·60	1·61	0·22	2·22	2·78
Silica.	0·12	0·24	0·08	0·33	0·67	0·63	0·85	0·86	0·86	0·04	0·18	0·14
Total . .	100·36	100·28	100·33	100·41	100·53	100 36	100·36	100·36	100 36	100·05	100·50	100·63
Deduct O=Cl	0·36	0·28	0·33	0·41	0·53	0·36	0·36	0·36	0·36	0·05	0·50	0·63
Total . .	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00	100·00

* There has doubtless been some omission of parts in making the mixture for burning to ash in this case, see p. 874.

APPENDIX-TABLE IV.—Mineral Constituents (Ash), in lbs., *per Animal*, and *per* 1,000 *lbs. Fasted Live-Weight.*

1. In Collective Carcass Parts. 2. In Collective Offal Parts. 3. In Entire Animal.

Entire Animal—first by addition of amounts in Carcass and Offal, second by direct analysis, third mean.

In "Offal Parts," and in "Entire Animal," in each case exclusive of contents of stomachs and intestines.

CALF AND OXEN.

	Fat Calf.					Half-fat Ox.					Fat Ox.				
	Carcass.	Offal.	Entire animal.			Carcass.	Offal.	Entire animal.			Carcass.	Offal.	Entire animal.		
			By addition.	By direct analysis.	Mean.			By addition.	By direct analysis.	Mean.			By addition.	By direct analysis.	Mean.
1. "FRESH-WEIGHTS," AND CONSTITUENTS, PER ANIMAL. FRESH-WEIGHTS OF CARCASS AND OFFAL=SUM OF PARTS; ENTIRE ANIMAL=FASTED LIVE-WEIGHT.															
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Fresh-weight	160·6	77·1	258·8			797·7	322·8	1232·0			939·4	376·0	1419·0		
Pure ash	7·173	2·604	9·777	9·753	9·765	43·945	12·869	56·814	56·822	56·818	42·531	12·522	55·053	55·135	55·094
Iron peroxide	0·027	0·029	0·056	0·051	0·054	0·275	0·170	0·445	0·552	0·499	0·240	0·223	0·463	0·228	0·346
Lime	3·151	1·077	4·228	4·287	4·257	20·606	5·728	26·334	25·719	26·026	19·998	5·153	25·151	25·706	25·428
Magnesia	0·150	0 044	0·194	0·214	0·204	0·754	0·183	0·937	1·149	1·043	0·724	0·161	0·885	0·846	0·866
Potash	0·423	0·116	0·539	0·527	0·533	2·139	0·399	2·538	2·505	2·521	1·932	0·601	2·533	2·459	2·496
Soda	0·221	0·170	0·391	0·373	0·382	1·141	0·715	1·856	1·747	1·802	1·105	0·802	1·907	1·674	1·790
Phosphoric acid	2·980	1·022	4·002	3·937	3·969	17·574	4·905	22·479	22·858	22·668	17·174	4·916	22·090	21·941	22·015
Sulphuric acid	0·074	0·031	0·105	0·105	0·105	0·293	0·160	0·453	0·488	0·471	0·296	0·200	0·496	0·434	0·465
Carbonic acid	0·082	0·030	0·112	0·131	0·122	0·790	0·226	1·016	1·120	1·068	0·715	0·113	0·828	1·179	1·004
Chlorine	0·073	0·099	0·172	0·151	0·162	0·328	0·425	0·753	0·707	0·730	0·373	0·385	0·758	0·807	0·782
Silica	0·008	0·008	0·016	0·012	0·014	0·120	0·052	0·172	0·138	0·155	0·060	0·054	0·114	0·045	0·080
Total	7·189	2·626	9·815	9·788	9·802	44·020	12·963	56·983	56·983	56·983	42·617	12·608	55·225	55·319	55·272
Deduct O=Cl	0·016	0·022	0·038	0·035	0 037	0·075	0·094	0·169	0·161	0·165	0·086	0·086	0·172	0·184	0·178
Total	7·173	2·604	9·777	9·753	9·765	43·945	12·869	56·814	56·822	56·818	42·531	12·522	55·053	55·135	55·094
2. "FRESH-WEIGHTS," AND CONSTITUENTS, PER 1,000 LBS. FASTED LIVE-WEIGHT. "FRESH-WEIGHTS"—EXCLUDING EVAPORATION, AND CONTENTS OF STOMACHS AND INTESTINES.															
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Fresh-weight	621	298	919			647	262	909			662	265	927		
Pure ash	27·742	10·063	37·805	37·712	37·759	35·643	10·447	46·090	46·098	46·094	29·973	8·824	38·797	38·854	38·826
Iron peroxide	0·104	0·112	0·216	0·197	0·207	0·223	0·138	0·361	0·448	0·405	0·169	0·157	0·326	0·161	0·244
Lime	12·187	4·162	16·349	16·577	16·463	16·713	4·650	21·363	20·865	21·114	14·093	3·631	17·724	18·115	17·919
Magnesia	0·580	0·170	0·750	0·827	0·788	0·611	0·148	0·759	0·932	0·846	0·512	0·114	0·626	0·596	0·611
Potash	1·636	0·448	2 084	2·038	2·061	1·735	0·324	2·059	2·032	2·045	1·361	0·424	1·785	1·733	1·759
Soda	0·855	0·657	1·512	1·442	1·477	0·925	0·580	1·505	1·417	1·461	0·779	0·565	1·344	1·179	1·261
Phosphoric acid	11 526	3·949	15·475	15·223	15·349	14·254	3·982	18·236	18·544	18·390	12·103	3·464	15·567	15·462	15·514
Sulphuric acid	0·286	0·120	0·406	0·406	0·406	0·238	0·132	0·368	0·396	0·382	0·208	0·141	0·349	0·306	0·328
Carbonic acid	0·317	0·116	0·433	0·507	0·470	0·641	0·184	0·825	0·909	0·867	0·504	0·080	0·584	0·831	0·708
Chlorine	0·282	0·383	0·665	0·584	0·625	0·266	0·345	0·611	0·574	0·592	0·263	0·271	0·534	0·569	0·552
Silica	0·031	0·031	0·062	0·046	0·054	0·098	0·042	0·140	0·112	0·126	0·042	0·038	0·080	0·032	0·056
Total	27·804	10·148	37·952	37·847	37·900	35·704	10·523	46·227	46·229	46·228	30·034	8·885	38·919	38·984	38·952
Deduct O=Cl	0·062	0·085	0·147	0·135	0·141	0·061	0·076	0·137	0·131	0·134	0·061	0·061	0·122	0·130	0·126
Total	27·742	10·063	37·805	37·712	37·759	35·643	10·447	46·090	46·098	46·094	29·973	8·824	38·797	38·854	38·826

APPENDIX-TABLE V.—Mineral Constituents (Ash), in lbs., *per Animal*, and *per* 1,000 *lbs. Fasted Live-Weight.*

1. In Collective Carcass Parts. 2. In Collective Offal Parts. 3. In Entire Animal.

Entire Animal—first by addition of amounts in Carcass and Offal, second by direct analysis, third mean.

In "Offal Parts," and in "Entire Animal," in each case exclusive of contents of stomachs and intestines.

LAMB AND SHEEP.

	Fat Lamb.					Store Sheep.					Half-fat old Sheep.					Fat Sheep.					Very fat Sheep.				
	Carcass.	Offal.	Entire animal.			Carcass.	Offal.	Entire animal.			Carcass.	Offal.	Entire animal.			Carcass.	Offal.	Entire animal.			Carcass.	Offal.	Entire animal.		
			By addition.	By direct analysis.	Mean.			By addition.	By direct analysis.	Mean.			By addition.	By direct analysis.	Mean.			By addition.	By direct analysis.	Mean.			By addition.	By direct analysis.	Mean.
1. "FRESH-WEIGHTS," AND CONSTITUENTS, PER ANIMAL. WEIGHTS OF CARCASS AND OFFAL=SUMS OF PARTS; ENTIRE ANIMAL=FASTED LIVE-WEIGHT.																									
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Fresh-weights	50·5	26·3	84·4			52·1	37·4	97·6			56·3	37·1	105·1			73·1	45·4	127·2			159·3	80·1	239·4		
Pure ash	1·831	0·603	2·434	2·442	2·438	2·262	0·730	2·992	2·990	2·991	2·319	0·891	3·210	3·224	3·217	2·505	0·890	3·395	3·427	3·411	4·399	2·839	7·238	7·238	7·238
Iron peroxide	0·008	0·015	0·023	0·021	0·022	0·008	0·027	0·035	0·037	0·036	0·011	0·033	0·044	0·044	0·044	0·010	0·043	0·053	0·034	0·044	0·017	0·059	0·076	(0·076)	0·076
Lime	0·857	0·217	1·074	1·089	1·082	1·027	0·266	1·293	1·289	1·291	1·071	0·333	1·404	1·432	1·418	1·169	0·313	1·482	1·529	1·505	2·083	1·050	3·133	(3·133)	3·133
Magnesia	0·033	0·010	0·043	0·044	0·044	0·042	0·013	0·055	0·054	0·055	0·041	0·014	0·055	0·055	0·055	0·045	0·016	0·061	0·062	0·061	0·090	0·048	0·138	(0·138)	0·138
Potash	0·085	0·056	0·141	0·140	0·141	0·117	0·053	0·170	0·169	0·170	0·117	0·066	0·183	0·170	0·177	0·117	0·070	0·187	0·190	0·189	0·166	0·234	0·400	(0·400)	0·400
Soda	0·045	0·042	0·087	0·087	0·087	0·067	0·051	0·118	0·116	0·117	0·061	0·050	0·111	0·108	0·110	0·070	0·054	0·124	0·122	0·123	0·120	0·207	0·327	(0·327)	0·327
Phosphoric acid	0·739	0·210	0·949	0·952	0·950	0·913	0·244	1·157	1·165	1·161	0·942	0·314	1·256	1·262	1·259	1·023	0·295	1·318	1·327	1·322	1·804	0·996	2·800	(2·800)	2·800
Sulphuric acid	0·015	0·021	0·036	0·029	0·032	0·028	0·021	0·049	0·053	0·051	0·012	0·028	0·040	0·034	0·037	0·013	0·030	0·043	0·035	0·039	0·021	0·051	0·072	(0·072)	0·072
Carbonic acid	0·033	0·002	0·035	0·037	0·036	0·032	0·007	0·039	0·033	0·036	0·043	0·009	0·052	0·059	0·055	0·037	0·010	0·047	0·057	0·052	0·072	0·051	0·123	(0·123)	0·123
Chlorine	0·017	0·028	0·045	0·045	0·045	0·033	0·039	0·072	0·069	0·070	0·024	0·030	0·054	0·052	0·053	0·023	0·033	0·056	0·055	0·056	0·031	0·135	0·166	(0·166)	0·166
Silica	0·003	0·009	0·012	0·008	0·010	0·002	0·018	0·020	0·020	0·020	0·002	0·021	0·023	0·020	0·021	0·003	0·033	0·036	0·029	0·033	0·002	0·038	0·040	(0·040)	0·040
Total	1·835	0·610	2·445	2·452	2·449	2·269	0·739	3·008	3·005	3·007	2·324	0·898	3·222	3·236	3·229	2·510	0·897	3·407	3·440	3·424	4·406	2·869	7·275	(7·275)	7·275
Deduct O=Cl	·004	·007	·011	·010	0·011	·007	·009	·016	·015	0·016	·005	·007	·012	·012	0·012	·005	·007	·012	·013	0·013	·007	·030	·037	(0·037)	0·037
Total	1·831	0·603	2·434	2·442	2·438	2·262	0·730	2·992	2·990	2·991	2·319	0·891	3·210	3·224	3·217	2·505	0·890	3·395	3·427	3·411	4·399	2·839	7·238	(7·238)	7·238
2. "FRESH-WEIGHTS," AND CONSTITUENTS, PER 1,000 LBS. FASTED LIVE-WEIGHT. "FRESH-WEIGHTS"—EXCLUDING EVAPORATION, AND CONTENTS OF STOMACHS AND INTESTINES.																									
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Fresh-weights	598	312	910			533	383	916			536	353	889			575	357	932			630	317	947		
Pure ash	21·682	7·145	28·827	28·924	28·876	23·158	7·469	30·627	30·603	30·615	22·094	8·476	30·570	30·697	30·634	19·715	6·998	26·713	26·960	26·836	17·402	11·234	28·636	28·636	28·636
Iron peroxide	0·095	0·177	0·272	0·249	0·261	0·082	0·276	0·358	0·379	0·369	0·105	0·314	0·419	0·419	0·419	0·079	0·338	0·417	0·268	0·343	0·067	0·234	0·301	(0·301)	0·301
Lime	10·148	2·571	12·719	12·897	12·808	10·514	2·722	13·236	13·193	13·214	10·204	3·168	13·372	13·634	13·503	9·200	2·461	11·661	12·028	11·844	8·240	4·155	12·395	(12·395)	12·395
Magnesia	0·391	0·118	0·509	0·521	0·515	0·430	0·133	0·563	0·553	0·558	0·391	0·133	0·524	0·524	0·524	0·354	0·126	0·480	0·488	0·484	0·356	0·190	0·546	(0·546)	0·546
Potash	1·006	0·664	1·670	1·658	1·664	1·198	0·542	1·740	1·730	1·735	1·115	0·628	1·743	1·619	1·681	0·921	0·550	1·471	1·495	1·483	0·656	0·926	1·582	(1·582)	1·582
Soda	0·533	0·498	1·031	1·030	1·030	0·686	0·522	1·208	1·187	1·197	0·581	0·476	1·057	1·028	1·043	0·551	0·425	0·976	0·960	0·968	0·475	0·819	1·294	(1·294)	1·294
Phosphoric acid	8·751	2·488	11·239	11·276	11·257	9·347	2·496	11·843	11·924	11·883	8·974	2·987	11·961	12·016	11·988	8·051	2·319	10·370	10·439	10·404	7·136	3·941	11·077	(11·077)	11·077
Sulphuric acid	0·178	0·249	0·427	0·344	0·386	0·287	0·215	0·502	0·542	0·522	0·114	0·266	0·380	0·324	0·352	0·102	0·236	0·338	0·275	0·307	0·083	0·202	0·285	(0·285)	0·285
Carbonic acid	0·391	0·024	0·415	0·439	0·427	0·328	0·072	0·400	0·338	0·369	0·410	0·086	0·496	0·562	0·529	0·291	0·079	0·370	0·448	0·409	0·285	0·202	0·487	(0·487)	0·487
Chlorine	0·201	0·332	0·533	0·533	0·533	0·338	0·399	0·737	0·706	0·722	0·229	0·285	0·514	0·495	0·505	0·181	0·259	0·440	0·433	0·437	0·123	0·534	0·657	(0·657)	0·657
Silica	0·035	0·107	0·142	0·095	0·119	0·020	0·184	0·204	0·205	0·205	0·019	0·200	0·219	0·190	0·204	0·024	0·259	0·283	0·228	0·255	0·008	0·150	0·158	(0·158)	0·158
Total	21·729	7·228	28·957	29·042	29·000	23·230	7·561	30·791	30·757	30·774	22·142	8·543	30·685	30·811	30·748	19·754	7·052	26·806	27·062	26·934	17·429	11·353	28·782	(28·782)	28·782
Deduct O=Cl	·047	·083	·130	·118	0·124	·072	·092	·164	·154	0·159	·048	·067	·115	·114	0·114	·039	·054	·093	·102	0·098	·027	·119	·146	(·146)	0·146
Total	21·682	7·145	28·827	28·924	28·876	23·158	7·469	30·627	30·603	30·615	22·094	8·476	30·570	30·697	30·634	19·715	6·998	26·713	26·960	26·836	17·402	11·234	28·636	(28·636)	28·636

APPENDIX-TABLE VI.—Mineral Constituents (Ash), in lbs., *per Animal*, and *per* 1,000 *lbs. Fasted Live-Weight.*

1. In Collective Carcass Parts. 2. In Collective Offal Parts. 3. In Entire Animal.

Entire Animal—first by addition of amounts in Carcass and Offal, second by direct analysis, third mean.

In "Offal Parts" and in "Entire Animal," in each case exclusive of contents of stomachs and intestines.

PIGS.

	Store Pig.						Fat Pig.					
	Carcass.	Offal, excluding Head and Feet.	Head and Feet.	Entire Animal.			Carcass.	Offal, excluding Head and Feet.	Head and Feet.	Entire Animal.		
				By Addition.	By direct Analysis.	Mean.				By Addition.	By direct Analysis.	Mean.
1. "FRESH-WEIGHTS," AND CONSTITUENTS, PER ANIMAL. WEIGHTS OF CARCASS AND OFFAL=SUM OF PARTS; ENTIRE ANIMAL=FASTED LIVE-WEIGHT.												
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Fresh-weights	62·4	20·3	9·2	93·9			140·5	23·2	13·4	185·0		
Pure ash	1·598	0·156	0·737	2·491	2·491	2·491	1·949	0·198	0·871	3·018	3·025	3·022
Iron peroxide	0·010	0·007	0·001	0·018	0·023	0·021	0·012	0·012	0·002	0·026	0·023	0·025
Lime	0·645	0·006	0·367	1·018	1·011	1·014	0·752	0·008	0·431	1·191	1·164	1·177
Magnesia	0·034	0·003	0·013	0·050	0·050	0·050	0·041	0·003	0·014	0·058	0·062	0·060
Potash	0·135	0·038	0·012	0·185	0·184	0·185	0·189	0·050	0·014	0·253	0·259	0·256
Soda	0·060	0·023	0·020	0·103	0·104	0·104	0·085	0·028	0·024	0·137	0·132	0·135
Phosphoric acid	0·640	0·062	0·303	1·005	0·999	1·002	0·783	0·073	0·353	1·209	1·214	1·211
Sulphuric acid	0·031	0·004	0·007	0·042	0·058	0·050	0·025	0·006	0·010	0·041	0·065	0·053
Carbonic acid	0·019	0·000	0·006	0·025	0·015	0·020	0·025	0·001	0·014	0·040	0·037	0·039
Chlorine	0·029	0·014	0·009	0·052	0·055	0·053	0·044	0·021	0·011	0·076	0·084	0·080
Silica	0·002	0·002	0·001	0·005	0·005	0·005	0·003	0·002	0·001	0·006	0·004	0·005
Total	1·605	0·159	0·739	2·503	2·504	2·504	1·959	0·204	0·874	3·037	3·044	3·041
Deduct O=Cl.	0·007	0·003	0·002	0·012	0·013	0·013	0·010	0·006	0·003	0·019	0·019	0·019
Total	1·598	0·156	0·737	2·491	2·491	2·491	1·949	0·198	0·871	3·018	3·025	3·022
2. "FRESH-WEIGHTS," AND CONSTITUENTS, PER 1,000 LBS. FASTED LIVE-WEIGHT. "FRESH-WEIGHTS"—EXCLUDING EVAPORATION, AND CONTENTS OF STOMACHS AND INTESTINES.												
	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
Fresh-weights	664	216	98	978			760	125	72	957		
Pure ash	17·003	1·663	7·825	26·491	26·512	26·501	10·539	1·068	4·688	16·295	16·346	16·320
Iron peroxide	0·106	0·075	0·011	0·192	0·245	0·218	0·065	0·065	0·011	0·141	0·124	0·133
Lime	6·863	0·064	3·896	10·823	10·761	10·792	4·066	0·043	2·320	6·429	6·290	6·359
Magnesia	0·362	0·032	0·138	0·532	0·532	0·532	0·222	0·016	0·075	0·313	0·335	0·324
Potash	1·436	0·405	0·127	1·968	1·958	1·963	1·022	0·269	0·075	1·366	1·400	1·380
Soda	0·638	0·245	0·212	1·095	1·107	1·101	0·460	0·151	0·129	0·740	0·713	0·727
Phosphoric acid	6·810	0·661	3·217	10·688	10·632	10·660	4·234	0·394	1·900	6·528	6·560	6·544
Sulphuric acid	0·330	0·043	0·074	0·447	0·617	0·532	0·135	0·033	0·054	0·222	0·351	0·288
Carbonic acid	0·202	0·000	0·064	0·266	0·160	0·213	0·135	0·005	0·075	0·215	0·200	0·208
Chlorine	0·309	0·149	0·096	0·554	0·585	0·570	0·238	0·113	0·059	0·410	0·454	0·432
Silica	0·021	0·021	0·011	0·053	0·053	0·053	0·016	0·011	0·006	0·033	0·022	0·028
Total	17·077	1·695	7·846	26·618	26·650	26·634	10·593	1·100	4·704	16·397	16·449	16·423
Deduct O=Cl.	0·074	0·032	0·021	0·127	0·138	0·133	0·054	0·032	0·016	0·102	0·103	0·103
Total	17·003	1·663	7·825	26·491	26·512	26·501	10·539	1·068	4·688	16·295	16·346	16·320

XXVII. THE BAKERIAN LECTURE.—*On Radiant Matter Spectroscopy: The Detection and wide Distribution of Yttrium.*

By WILLIAM CROOKES, *F.R.S.*

Received May 24,—Read May 31, 1883.

Introduction.

1. IN March, 1881, I sent to the Royal Society a preliminary notice of some results I had obtained when working on the molecular discharge in high vacua.* When the spark from a good induction coil traverses a tube having a flat aluminium pole at each end, the appearance changes according to the degree of exhaustion. Supposing atmospheric air to be the gas under exhaustion, at a pressure of about 7 millims. a narrow black space is seen to separate the luminous glow and the aluminium pole connected with the negative pole of the induction coil. As the exhaustion proceeds this dark space increases in thickness, until, at a pressure of about 0·02 millim. (between 20 and 30 M.)†, the dark space has swollen out till it nearly fills the tube. The luminous cloud showing the presence of residual gas has almost disappeared, and the molecular discharge from the negative pole begins to excite phosphorescence on the glass where it strikes the side. There is great difference in the degree of exhaustion at which various substances begin to phosphoresce. Some refuse to glow until the exhaustion is so great that the vacuum is nearly non-conducting, whilst others begin to become luminous when the gauge is 5 or 10 millims. below the barometric level. The majority of bodies, however, do not phosphoresce till they are well within the negative dark space. This phosphorogenic phenomenon is at its maximum at about 1 M., and, unless otherwise stated, the experiments now about to be described were all tried at this high degree of exhaustion.

Under the influence of this discharge, which I have ventured to call radiant matter, a large number of substances emit phosphorescent light, some faintly and others with great intensity. On examining the emitted light in the spectroscope most bodies give a faint continuous spectrum, with a more or less decided concentration in one part of the spectrum, the superficial colour of the phosphorescing substance being governed by this preponderating emission in one or other part of the spectrum.

Sometimes, but more rarely, the spectrum of the phosphorescent light is discontinuous, and it is to bodies manifesting this phenomenon that my attention has been

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* Proceedings of the Royal Society, No. 213, 1881.

† M. = one-millionth of an atmosphere.

specially directed for some years past, considerable interest attaching to a solid body whose molecules vibrate in a few directions only, giving rise to spectrum lines or bands on a dark background.

The citron band spectrum.

2. For a long time past I have been haunted by a bright citron-coloured band or line appearing in these phosphorescent spectra, sometimes as a sharp line, at others as a broader nebulous band, but having always a characteristic appearance and occurring uniformly in the same spot. This band I first saw in the summer of 1879, and from that date down to a comparatively short time ago all my efforts to clear up the mystery proved vain. By chemical means it was not difficult to effect a partial separation of a certain mineral or earthy body into two parts, one giving little or no citron band, the other giving one stronger than the original band; and by again treating this latter portion by appropriate chemical means, the citron band-forming body could frequently be still more concentrated; but further than this for a long time it seemed impossible to go. I soon came to the conclusion that the substance I was in search of was an earth, but on attempting to determine its chemical properties I was baffled. A more Proteus-like substance a chemist never had to deal with. In my preliminary note, above referred to, speaking of the possibility that some of these spectrum-forming bodies might be new chemical elements, I said—"The chemist must be on his guard against certain pitfalls which catch the unwary. I allude to the profound modification which the presence of fluorine, phosphorus, boron, &c., causes in the chemical reactions of many elements, and to the interfering action of a large quantity of one body on the chemical properties of another which may be present in small quantities."

3. This warning was not unnecessary. No Will-o'-the-Wisp ever led the unwary traveller into so many pitfalls and sloughs of despond as the hunt for this phantom band has entrapped me. I have started with a large quantity of substance which, from preliminary observations, promised to be a rich mine of the desired body, and have worked it up chemically to a certain point, when the citron band vanished, and could not be again detected in either precipitate or filtrate. Half-a-dozen times in the last four years the research has been given up as hopeless, and only a feeling of humiliation at the thought of a chemist being beaten by any number of anomalies made me renew each time the attack. Likewise, the tedious character of the research made a long continuance of failures very disheartening. To perform a spectrum test, the body under examination must be put in a tube and exhausted to a very high point before the spectroscope can be brought to bear on it. Instead of a few minutes, many hours are occupied in each operation, and the tentative gropings in the dark, pre-eminently characteristic of this kind of research, have to be extended over a long period of time.

4. I soon found that the best way to bring out the band was to treat the substance

under examination with strong sulphuric acid, drive off excess of acid by heat, and finally to raise the temperature to dull redness (10). The anhydrous sulphate thus left frequently showed the citron band in the radiant matter tube, when before this treatment the original substance showed nothing (75).

Examination of calcium compounds.

5. My first idea was that the band might be due to a compound of lime. Much chemical evidence tended to support this view. I have already explained that the chemical extraction was rendered very difficult by the fact of the citron band so frequently turning up both in the precipitate and the filtrate. By neglecting the portion showing least citron band, and separating all the elements present which gave little or none, I could generally concentrate the citron band into a solution which—according to our present knowledge of analytical chemistry—should contain little else than the earths, alkaline earths, and alkalies. Ammonia added to this solution would precipitate an earth (11, 14), and in the filtrate oxalic acid would precipitate an insoluble oxalate (7, 13).

The citron band hovered between these two precipitates, being sometimes stronger in one and sometimes in the other. It was also to be detected, but more faintly, in the residue left after evaporating to dryness and igniting the filtrate from the oxalate.

I frequently obtained no precipitate with ammonia, and then the oxalate gave the band brilliantly; and occasionally the ammonia precipitate when formed gave little or no citron band. I was, however, generally sure to find it in the insoluble oxalate, and sometimes it was very brilliant, being accompanied by two bright green bands and a fainter red band.

6. At this time one of the minerals which showed the citron band most strongly was a phosphorescent apatite from Ireland; and knowing the difficulties of separating the last traces of phosphoric acid from the earths, I explained the foregoing facts by the presence of small quantities of phosphoric acid, which gave rise to the ammonia precipitate; the bulk of the citron body not being precipitated by ammonia, but coming down as oxalate; whilst a little of this oxalate would remain dissolved in the ammoniacal salts present, and so appear with the alkalies.

I tested this hypothesis in every imaginable way, by mixing small quantities of phosphoric acid with salts of lime and other earths, in the endeavour to imitate the conditions occurring in the native minerals, and so educe the citron band; but I was unable to get any precipitate giving the citron band when I started with materials which did not originally give it.

7. A sufficient quantity of precipitated oxalate (5) having in course of time been accumulated, I attempted its purification. It was ignited, dissolved in dilute hydrochloric acid, and rendered slightly alkaline with ammonia and ammonic sulphide. The liquid was boiled to a small bulk, keeping it alkaline, and was then set aside in

a warm place: a slight flocculent precipitate formed. This was filtered, and the filtrate re-concentrated. The clear strong solution should now contain nothing but barium, strontium, and calcium, with traces of elements from previous groups which might be soluble in the precipitants employed or in the ammoniacal salts present (for we know that the word *insoluble* applied to a precipitate is not an absolute term, and in minute analysis allowance must be made not only for the slight solubility of precipitates in the reagents present, but also for the power possessed by most precipitates of carrying down with them traces of soluble metallic salts from solution). Besides these, it was possible that a hitherto unrecognised element might be present, to which the citron band was due. By the ordinary process of analysis I could, however, only detect the presence of calcium and strontium.

8. The concentrated ammoniacal solution was added to an excess of a boiling solution of ammonic sulphate, and the whole was set aside for twenty-four hours; the precipitate which had formed was filtered off and washed with a saturated solution of ammonic sulphate. The precipitate was found to consist of strontic sulphate. On testing this in a radiant matter tube the citron band was very decided, although much fainter than in the original oxalate. The filtrate was diluted largely, heated, and precipitated with a hot solution of ammonic oxalate; it was then allowed to stand for some time, when a bulky white calcic oxalate came down. This was filtered and washed. Tested in the radiant matter tube, after ignition and treatment with sulphuric acid, it gave the citron band, far exceeding in brightness the spectrum of the original oxalate.

9. So far all the chemical evidence went to show that the band-forming substance was calcium, and further tests tried with the purified oxalate confirmed this inference. Every analytical test to which it was subjected showed lime, and nothing but lime; all the salts which were prepared from it resembled those of lime, both physically and chemically; the flame spectrum gave the calcium lines with extraordinary purity and brilliancy; and finally, the atomic weight, taken with great care, came out almost the same as that for calcium, 39·9 as against Ca 40.

10. I now sought for the citron band amongst other calcium minerals. The preliminary testing was simple. The finely powdered mineral was moistened with strong sulphuric acid, the action being assisted by heat, and the mass was then raised to dull redness (4). It was then put into a radiant matter tube and the induction spark passed through after the exhaustion had been pushed to the required degree.

Treated in this manner most native compounds of lime gave the citron band. A perfectly clear and colourless crystal of Iceland spar converted into sulphate gave it strongly, native calcic phosphate less strongly, and a crystal of arragonite much more brightly. A stalactite of calcic carbonate from the Gibraltar caves gave the band almost as well as calcite, as also did cinnamon stone (lime alumina garnet), iron slag from a blast-furnace, commercial plaster of Paris, and most specimens of ordinary burnt lime.

The citron band not due to calcium.

11. Evidence stronger than this in favour of the view that the citron band was an inherent characteristic of calcium could scarcely be; but, on the other hand, there was evidence equally conclusive that the band was not essential to calcium. The ammonia precipitate (5) sometimes gave the citron band with great strength and purity, and although I had not yet obtained this in quantities sufficient for a detailed examination, it was easy to decide that it contained no phosphoric, silicic, or boric acid, fluorine, or other body likely to cause the precipitation of lime in this group. This precipitate must therefore be an earth, and the more carefully I purified it from lime and other substances, the more brilliantly shone out the citron band, and the more intense became the green and red bands.

Another stubborn fact was this :—Starting with a lime compound which showed the citron band, I could always obtain a calcic oxalate which gave the band stronger than the original substance; but if I started with a lime compound which originally gave no citron band, I could never by any means, chemical or physical, constrain the lime or the earthy precipitate to yield a citron band.

12. Among the minerals tried was eudialyte, a silicate of zirconium, iron, calcium, and sodium, containing about 10 per cent. of lime. No citron band could be detected on testing the original mineral or any of the constituents separated from it on analysis. This, and a lump of common whiting (levigated chalk), were for some time my only sources of lime which gave no citron band.

13. The only explanation that I could see for this anomaly was that the elusive citron band was caused by some element precipitated with the calcic oxalate, but present in a quantity too small to be detected by ordinary chemical means. I then thought that were I to fractionally precipitate the solution of lime, the band-forming body might be concentrated in one or the other portion. Accordingly the calcic oxalate (7, 8, 9) was ignited and dissolved in hydrochloric acid, and fractionally precipitated in three portions with ammonic oxalate, the first and third portions being comparatively small. They were dried, ignited with sulphuric acid, and tested in the radiant matter tube. All three portions showed the citron band, but the portion which came down first gave the band decidedly the strongest, and the third portion precipitated showed it weakest. This therefore pointed to a difference between calcium and the body sought for. The process, however, was not satisfactory, and I was driven to seek some other method.

14. A portion of an ammonia precipitate found to give the citron band very well (5, 11), was dissolved in dilute sulphuric acid, and the solution evaporated down. Crystals were formed which were difficultly soluble in hot water, but appeared more soluble than calcic sulphate.

A large quantity of the calcic oxalate (7, 8, 9) was ignited with sulphuric acid at a dull red heat, and the resulting calcic sulphate was finely ground and then boiled in a very small quantity of water—not sufficient to dissolve the one-hundredth part of it. The mass was thrown on a filter, and the small quantity of clear liquid which came through was precipitated with ammonic oxalate. The resulting white precipitate was ignited with sulphuric acid, and tested in the radiant matter tube. For the sake of comparison a portion of the calcic sulphate remaining on the filter was also put in a radiant matter tube. The sulphate from the aqueous extract gave the citron band far more brilliantly than the calcic sulphate from the filter. I found, however, that it was impossible, by any amount of washing or boiling out, to deprive the calcic sulphate of all power of giving the citron band, although it was possible in this way to weaken its intensity considerably.

Experiments with calcic sulphate.

15. Supposing that the substance giving the citron band formed a sulphate more soluble in water than calcic sulphate, it was anticipated that repeated washings with cold water would extract some of it, which might then be detected more easily. About four pounds' weight of commercial plaster of Paris, which showed very faint traces of the citron band, were mixed with water and rapidly poured on a large filter. Before the mass solidified a slight saucer-like depression was made in the upper part, and a few ounces of water were poured on. This ran through slowly, and it was then poured back and the exhaustion repeated several times. The aqueous extract was then evaporated to dryness, ignited with sulphuric acid, ground in a mortar with small successive quantities of water, the liquid boiled, filtered, and precipitated, first with ammonia, and the filtrate with ammonic oxalate. These precipitates both showed the citron band very fairly, far more intensely than it was seen in the original calcic sulphate. The green and red bands were also visible.

The same mass of plaster of Paris was then washed, as before, with a little dilute hydrochloric acid passed through several times, and this extract was treated in the same way by evaporation and extraction with water, and the filtrate precipitated, first with ammonia, and then with ammonic oxalate. In these precipitates the citron band, together with the green and red bands, were much more brightly manifest than in the precipitates from the aqueous extract.

Wide distribution of the citron band-forming body.

16. These experiments are conclusive in proving that the citron band is not due to calcium, but to some other element, probably one of the earthy metals, occurring in very minute quantities, but widely distributed along with calcium, and I at once commenced experiments to find a more abundant supply of the body sought for. Amongst other substances tested I may note the following as giving a more or less decided citron band in the spectrum when treated with sulphuric acid in the manner indicated above (10):—Crystallised barytic chlorate, heavy spar, common limestone, strontic nitrate, native strontic carbonate, crystallised uranic nitrate, commercial

magnesic sulphate, commercial potassic sulphate, Wagnerite (magnesic phosphate and fluoride), zircon, cerite, and commercial ceric oxalate.

Examination of zircon for the citron band.

17. Some specimens of zircon treated in the above manner appeared sufficiently rich to make it probable that here might be found an available source of the citron-band-yielding body. I found it in crystals from Green River, North Carolina, from Ceylon, from Expailly, from Miask (Oural), and from Brevig, and having a good supply of North Carolina zircons I started working up these in the following manner:—

The finely-powdered zircons were fused with sodic fluoride, and the melted mass powdered, boiled with sulphuric acid, and filtered. The solution was precipitated with excess of ammonia, the precipitate well washed and dissolved in hydrochloric acid, and the solution made nearly neutral. A little zirconic oxychloride sometimes separated on evaporation; this was filtered off. An excess of sodic thiosulphate was now added, and the whole boiled for some time until a portion of the filtrate gave no further precipitate on boiling again with sodic thiosulphate. The precipitated zirconic thiosulphate was worked up for zirconia; it was found to be quite free from the substance giving the citron band. The solution filtered from the zirconic thiosulphate was precipitated with ammonia, and the brown gelatinous precipitate was well washed. The filtrate was precipitated with ammonic oxalate, which brought down much calcic oxalate. This showed the citron band, but not strongly. The brown gelatinous precipitate was dissolved in nitric acid. Argentic nitrate was added to separate chlorine, and the filtrate from the argentic chloride was boiled down with nitric acid and excess of metallic tin to separate phosphoric acid. The clear solution, separated from the stannic oxide, phosphate, &c., was boiled down with hydrochloric acid to remove nitric acid, and then saturated with hydric sulphide to separate silver and tin.

18. The filtrate from the sulphides was freed from hydric sulphide by boiling, and was then mixed with tartaric acid and excess of ammonia, to precipitate any yttria that might be present, together with FORBES's zirconia β[*] (jargonia?). On standing for some hours this gave a small quantity of a precipitate, which was separated by filtration; it was tested in the radiant matter tube, and found not to give the citron-band spectrum (44). To the filtrate ammonic sulphide was added to precipitate the iron. The black precipitate was filtered off, and the filtrate evaporated to dryness, and ignited to destroy the organic matter. The residue, heated with sulphuric acid and ignited, gave the citron spectrum very brightly. This would probably be the earth which FORBES calls zirconia γ.[†]

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[*] 'Chem. News,' vol. 19, p. 277.

[†] *Loc. cit.*

19. For many years chemists have suspected that what is known as zirconia might be a compound. SVANBERG* found that zircons from different localities varied in specific gravity, and the earth or earths obtained by fractional precipitation with oxalic acid had not the same properties, the hydrogen equivalents of the metals of the earths of the different fractions varying from 17·01 to 27·3, the metal of the earth hitherto recognised as zirconia being 22·4.† He considered zirconia to contain two different earths, the oxalate of one being less soluble in acid than that of the other, and their sulphates differing in crystalline form and solubility. He proposed the name "noria" for one of the earths, retaining that of zirconia for the other. The researches of BERLIN, on the other hand, seem to disprove this.

20. Remembering the remarkable result produced in the absorption spectrum of some jargons by the presence of a minute trace of uranium,‡ I tried numerous experiments with this metal, adding small quantities of it to zirconia, lime, thoria, ceria, &c., but in no case could I educe the citron-band spectrum by this means.

I may condense a year's work on zircon,—more than ten pounds weight of crystals from North Carolina having been worked up—by stating that the result was comprised in about 300 grains of an earthy residue (18), and about two ounces of oxalate, chiefly calcium; the former gave the citron band very well. The process as detailed above is given, since by this means a very large quantity of zircons was worked up, affording me the material which ultimately enabled me to solve the problem which at one time seemed almost hopeless.

The zirconia prepared from these zircons when tested sometimes showed the citron band, especially after precipitation as an oxychloride. Zirconia precipitated as thiosulphate did not yield the citron band (28). A zirconia rich in citron band, fractionally precipitated by ammonia, yielded precipitates of increasing richness, the last fraction showing the citron band strongly.

21. The calcic oxalate obtained from zircon gave unsatisfactory results, so attention was directed to the earthy residue (18). This was found to be of highly complex character, containing thoria (which had escaped precipitation as thiosulphate), ceria, lanthana, didymia, yttria, and probably some of the newly-discovered rarer earths.

Examination of cerite for the citron band.

22. The position of the citron band in the spectrum falls exactly on the strongest absorption band of didymium, so that a piece of didymium glass or cell of solution of the nitrate entirely obliterates the citron band. This naturally suggested that the band was due to didymium.

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* POG. Ann., vol. 65, p. 317.

† SVANBERG's numbers for these earths are 938 to 1320 (M_2O_3), the earth hitherto recognised as zirconia being 1140; oxygen being 100. For the sake of uniformity I have recalculated his equivalents for the metals on the $O = 16$ scale, taking the formula as M_2O (see note 1, par. 40).

‡ 'Chem. News,' vol. 19, pp. 121, 142, 205, 277; vol. 20, pp. 7, 104; vol. 21, p. 73.

Cerite was accordingly the next mineral experimented on. The powdered mineral tested in the tube in the original way gave a good citron band. It was made into a paste with sulphuric acid, and after all action had ceased it was extracted with cold water. The earths were then precipitated with ammonic oxalate, and the oxalate ignited. The fawn-coloured powder was then converted into sulphate, dissolved in water, and the cerium metals precipitated by long digestion with excess of potassic sulphate. When no didymium bands could be detected in a considerable thickness of the supernatant liquor it was assumed that all the cerium metals were down, and the liquid was filtered.

23. The precipitated double sulphates were dissolved in hydrochloric acid, and the earths precipitated as oxalates. After ignition and treatment with sulphuric acid, the mixed ceria, lanthana, and didymia were tested in the radiant matter tube, but the merest trace only of citron band was visible.

24. This experiment proved the inadequacy of the didymium explanation (22), and further tests showed that not only could I get no citron band in pure didymium compounds, but the spectrum entirely failed to detect didymium in many solutions of the earth which gave the citron band brilliantly.

25. Attention was now turned to the solution filtered from the insoluble double sulphates from cerite (22). Potash in excess was added to the filtrate, and the flocculent precipitate filtered off, and after well washing was converted into sulphate, and tested in a radiant matter tube. The spectrum, of extraordinary brilliancy, was far brighter than any I had hitherto obtained. Unfortunately, however, the quantity was too small to be subjected to very searching chemical analysis.

Examination of thorite and orangite.

26. Search was next made amongst other minerals rich in the rarer earths. Thorite, another disputed mineral, was finely powdered, treated with sulphuric acid, and tested in the radiant matter tube. It gave the citron spectrum most brilliantly—equal, in fact, to the mixture of earths obtained from zircons (18, 21) at so great an expenditure of time and trouble. Orangite treated in the same manner gave almost as good a spectrum. Pure thorinic sulphate prepared by myself was found not to give the citron band, but three specimens prepared and given to me by friends all gave it, so it was not unlikely that in thorite and orangite might at last be found a good source of the long-sought element—that in fact the body I was hunting for, if not thorina, might possibly be Bahr's hypothetical wasium. Having obtained about 2 lbs. of orangite and thorite, they were worked up as follows :—

27. The finely-powdered mineral was heated for some time with strong hydrochloric acid, and when fully gelatinised and all action had ceased, it was evaporated to dryness to render the silica insoluble ; then extracted with water slightly acidulated with hydrochloric acid, boiled, and filtered. Hydric sulphide was passed through the

filtrate for some time. The flask then corked was set aside for twenty-four hours and filtered. The filtrate was evaporated to a small bulk, nearly neutralised with ammonia, and then boiled for some time with excess of sodic thiosulphate. This precipitated the thorina, alumina, zirconia, and titanic acid, whilst it left in solution the metals of the cerium and yttrium groups. The filtrate was boiled down to a small bulk, when a further precipitation took place: this was filtered off and added to the first thiosulphate precipitate. To the clear filtrate excess of ammonic oxalate was added, and the whole allowed to rest twenty-four hours. The precipitated oxalates were filtered, washed, ignited, dissolved in hydrochloric acid, and the excess of acid evaporated off. The aqueous solution was then mixed with a large excess of freshly precipitated baric carbonate, and set aside for twenty-four hours with frequent shaking (29). This would precipitate much of the cerium, and any iron or alumina which might have escaped previous treatment. The liquid was filtered from the precipitate produced by baric carbonate, and the clear solution, which would contain nothing but barium, and some of the yttrium and cerium metals, was treated as described further on (30).

28. The thiosulphate precipitate tested in the radiant matter tube gave no citron band, nor did it seem possible to detect this band on testing the purified thorina obtained from this precipitate, nor from the alumina or zirconia from the same precipitate. This confirmed the results obtained when working up zircons, that sodic thiosulphate did not precipitate the citron band-forming body.

29. The barium precipitate (27) was dissolved in hydrochloric acid, the baryta separated with sulphuric acid, and the solution precipitated with ammonic oxalate. The ignited precipitate, which amounted to 0·223 per cent. of the mineral taken, contained the cerium metals. On testing in a radiant matter tube it gave the citron band only moderately well—not nearly so strong as the original thorite and orangite. The iron and alumina in the filtrate from the ceric oxalates were likewise precipitated and tested; they showed a faint trace of citron band.

30. The solution (27) filtered from the barium precipitate was freed from baryta by sulphuric acid, precipitated with ammonic oxalate, and the precipitate washed and ignited; it amounted to only 0·125 of the mineral taken. Tested in the radiant matter tube it showed the citron band about as well as the corresponding earth from the barium precipitate.

This was disheartening, for after having started with a mineral which gave the citron band well, and having hunted the citron band as it were into a corner, the only result was two trifling precipitates showing the citron band less intensely than did the raw material itself. The experiment, however, proved one thing: the band-forming substance was not thorina. The occurrence of this spectrum must therefore be due to some other element present in small quantity in thorite and orangite.

31. The two mixtures of earths—the one from the barium precipitate (29) and the other from the barium filtrate (30)—which showed the citron line moderately well,

were dissolved in sulphuric acid, the solution neutralised as nearly as possible with potash, and digested for several days with excess of potassic sulphate. The solution, which at first showed the didymium bands, was then found to be free from didymium.

32. The insoluble double sulphates were filtered and washed with a cold saturated solution of potassic sulphate. The precipitate was boiled for some time in ammonia, filtered, dissolved in hydrochloric acid, and precipitated with ammonic oxalate. This precipitate was ignited and tested in the radiant matter tube. It gave scarcely a trace of citron band (23). The earth was further purified by the potash and chlorine method, and was found to consist principally of ceric oxide.

33. The solution filtered from the insoluble potassio-ceric sulphate (31) was boiled with ammonia and ammonic sulphide. A small quantity of a white flocculent earth came down—too small a quantity to weigh. Tested in a radiant matter tube, it gave the citron band better than either of the above precipitates, showing that by this treatment the body had been concentrated (25).

34. It seemed possible that the earth sought for might be present in larger quantity in the thorite, but that it had been gradually carried down mechanically or by mass-action rather than chemically, by the numerous operations it had undergone before getting it to the final stage. Therefore a fresh quantity of thorite was extracted with hydrochloric acid. The solution was precipitated with potassic sulphate, taking the usual precautions to secure complete precipitation. A bulky precipitate ensued, which contained the thorina and cerium earths. These were separated and tested, and found to give only a faint citron band.

35. The solution of earthy sulphates soluble in potassic sulphate was precipitated with ammonic oxalate. The precipitate ignited with sulphuric acid, and tested in a radiant matter tube, gave the citron spectrum with great brilliancy (25, 33).

Chemical facts connected with the citron body.

36. Certain chemical facts concerning the behaviour of the sought-for element which came out during the course of the tentative trials already described had considerably narrowed the list amongst which it might probably be found. All the evidence tended to show that it belongs to the group of earthy metals, consisting of aluminium, beryllium, thorium, zirconium, cerium, lanthanum, didymium, and the yttrium family, together with titanium, tantalum, and niobium. The sought-for earth is insoluble in excess of potash (25); this excludes aluminium and beryllium. It is not precipitated by continued boiling with sodic thiosulphate (17, 27); this excludes aluminium, thorium, and zirconium. Fused with acid potassic sulphate, the resulting compound is readily soluble in cold water; this excludes tantalum and niobium. Evaporating to dryness with hydrochloric acid and heating for some time does not render the mass insoluble in water (27); this excludes titanium and silicium. It is easily soluble in an excess of a saturated solution of potassic sulphate (25, 33, 34); this excludes thorium, the cerium group, some of the numerous members of the

yttrium group, and zirconium. The only remaining elements among which this elusive body would probably be found are those members of the yttrium family which are not precipitated by potassic sulphate.

37. On the other hand, the body giving the citron band spectrum did not behave like one of the known earths. A rich residue was fused with sodic carbonate, and the mass extracted with water. The insoluble residue, on testing in the usual way, was rich in citron band, but subsequent treatment of the aqueous solution gave me an earth which also gave the citron band strongly.

An acid solution of the citron body was precipitated by ammonia and ammonic chloride. The earth was not completely precipitated, but after a long boiling some remained in solution. I have since ascertained that the detection of the citron-band body in solution under these circumstances is only owing to the marvellous delicacy of the test, which carries our powers of recognition far beyond the resources of ordinary chemistry.

38. Besides obtaining indirect evidence that the citron band was not due to certain elements, I tried special experiments with each substance, brought to the highest possible state of purity. In many cases I detected more or less traces of citron band; but I had come to the conclusion, abundantly warranted by facts, that this citron band was an extraordinarily sensitive test of the presence of the element causing it; and the minute chemistry of many of these earthy metals being insufficiently known, it was not surprising that traces of one of them should adhere to another in spite of repeated attempts to purify it out. With each successive fractional precipitation the citron band became fainter, showing that with perseverance the last trace would probably disappear. The time this process would have occupied, in my opinion, seemed not worth the little additional evidence it would have afforded.

39. Taking into consideration the extremely small quantity of phosphorescent material which had so far been obtained, all these experiments justified me in assuming that the body sought for not only belonged to the group of earths, but also most probably to the sub-group not precipitated by potassic sulphate to which yttria belongs. As, however, the number of these metals has increased so much within the last few years, and as the quantity of material which I had up to the present at my disposal was too small to admit of a satisfactory chemical examination being made of it, search was commenced among other sources known to be rich in these metals. Besides, not only did the majority of the substances I had up till now obtained in anything like quantity indicate the citron band earth to belong to the yttria group (33, 34, 36), but also that either the earth itself showed an absorption band in the spectroscope, or was invariably accompanied by one which did. On the other hand, I had a certain amount of evidence that the earth sought for did not show a band in the spectroscope (24); but remembering the extremely small quantity of very impure substance experimented with, the evidence on this point was not at all conclusive.

The sought-for body one of the yttrium family.

40. The yttria earths form a somewhat numerous family. Fortunately for chemists, a mineral rich in yttria earths—samarskite—has been found in large quantity in Mitchell County, North Carolina, and to this mineral I accordingly now directed my attention.

The following list of elements of the yttrium and its allied families, said to occur in samarskite and similar minerals, may be considered complete to the present time.

Name.	Absorption Spectrum.	Hydrogen equivalent of Metal. (1) (Type of Oxide M_2O.)
Cerium	No	47·1 (2)
Columbium(3)	Yes	—
Decipium	Yes	57·0 (4)
Didymium	Yes	48·5 (5)
Didymium β	Yes	47·0 (6)
Erbium	Yes	55·3 (7)
Holmium(8)	Yes	54·0 (9)
Lanthanum	No	46·0 (10)

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(1) As it is at present doubtful whether the oxides of several of the metals in this table belong to the type M_2O, M_2O_3, or MO, I have, for the sake of uniformity and simplicity, in calculating the values from the composition of their salts, by which these metals are chiefly discriminated, taken the type of oxide to be M_2O.

(2) BÜHRIG, 'J. Pr. Chem.,' ser. 2, vol. xii., p. 209.

(3) Dr. J. LAWRENCE SMITH in a paper read before the United States National Academy of Sciences in 1879, announced the discovery in Samarskite of two new elements, which he named Columbium and Rogerium ('Nature,' vol. xxi., p. 146). I have failed to find any further notice of these elements. This Columbium must not be confounded with the well-known Columbium, sometimes called Tantalum.

(4) DELAFONTAINE, 'Comptes Rendus,' vol. lxxxvii., p. 632, vol. xciii., p. 63; 'Chemical News,' vol. xxxviii., p. 223, vol. xliv, p. 67.

(5) CLÈVE, 'Bull. Soc. Chim.,' ser. 2, vol. xxi., p. 246; BRAUNER, 'Comptes Rendus,' vol. xciv., p. 1718; 'Chemical News,' vol. xlvii., p. 175.

(6) CLÈVE, 'Comptes Rendus,' vol. xciv., p. 1528; 'Chemical News,' vol. xlv., p. 273. BRAUNER, 'Comptes Rendus,' vol. xciv., p. 1718; 'Chemical News,' vol. xlvi., p. 16.

(7) CLÈVE, 'Comptes Rendus,' vol. xci., p. 381; 'Chemical News,' vol. xlii., p. 199. LECOQ DE BOISBAUDRAN, 'Comptes Rendus,' vol. lxxxix., p. 516; 'Chemical News,' vol. xl., p. 147.

(8) Called by SORET, the first discoverer, "X." Subsequently CLÈVE discovered the same metal and called it holmium. SORET has now adopted CLÈVE's name. 'Comptes Rendus,' vol. lxxxix., p. 708, and vol. xci., p. 378; 'Chemical News,' vol. xl., p. 224, and vol. xlii., p. 199. LECOQ DE BOISBAUDRAN, 'Comptes Rendus,' vol. lxxxix., p. 516; 'Chemical News,' vol. xl., p. 147.

(9) CLÈVE, 'Comptes Rendus,' vol. lxxxix., p. 478; 'Chemical News,' vol. xl., p. 125.

(10) BRAUNER, 'Comptes Rendus,' vol. xciv., p. 1718; 'Chemical News,' vol. xlvi., p. 16.

Name.	Absorption Spectrum.	Hydrogen equivalent of Metal. (Type of Oxide M_2O).
Mosandrum	No	51·2(11)
Philippium (12)	No	—
Rogerium(13)	Yes	—
Samarium	Yes	50·0(14)
Scandium	No	14·7(15)
Terbium	No	49·5(16)
Thorium	No	58·4
Thulium	Yes	56·5(17)
Ytterbium	No	57·9(18)
Yttrium	No	29·7(19)
Yttriun α	No	52·2(20)
Yttrium β	Yes	49·7(21)
Zirconium	No	22·5

(11) LAWRENCE SMITH, 'Comptes Rendus,' vol. lxxxvii., pp. 145, 146, 148. MARIGNAC, *ibid.*, vol. lxxxvii., p. 281. DELAFONTAINE, in October, 1878 (*ibid.*, vol. lxxxvii., p. 600), considers mosandrum a mixture of terbium, yttrium, erbium, didymium, and philippium. Subsequently, however, LAWRENCE SMITH, in November, 1878 (*ibid.*, vol. lxxxvii., p. 831), adduces chemical and other reasons to show that his mosandrum is not a mixture, but a true element. A year later, September 1, 1879 (*ibid.*, vol. lxxxix., p. 480), LAWRENCE SMITH repeats the claim for mosandrum to be classed with the elements.

(12) DELAFONTAINE, 'Comptes Rendus,' vol. 87, p. 559; Chemical News,' vol. 38, p. 202; 'Jour. Chem. Soc.,' vol. 36, p. 116.

(13) *See* Note (3) to Columbium, *ante*.

(14) LECOQ DE BOISBAUDRAN, 'Comptes Rendus,' vol. lxxxviii., p. 322, and vol. lxxxix., p. 212; 'Chemical News,' vol. xxxix., p. 115, and vol. xl., p. 99. BRAUNER, 'Chemical News,' vol. xlvii., p. 175; CLÈVE, 'Comptes Rendus,' vol. xcvii., p. 94; 'Chemical News,' vol. xlviii, p. 39.

(15) NILSON, 'Comptes Rendus,' vol. xci., p. 118; 'Chemical News,' vol. xlii., p. 83. CLÈVE, 'Comptes Rendus,' vol. lxxxix., p. 419; 'Chemical News,' vol. xl., p. 159.

(16) MARIGNAC, 'Ann. Chim. et Phys.,' ser. 5. vol. xiv., p. 247; 'Journ. Chem. Soc.,' vol. xxxvi., p. 113. DELAFONTAINE, 'Ann. Chim. et Phys.,' ser. 5, vol. xiv., p. 238; 'Journ. Chem. Soc.,' vol. xxxvi., p. 114.

(17) CLÈVE, 'Comptes Rendus,' vol. lxxxix., p. 478, and vol. xci., p. 328; 'Chemical News,' vol. xl., p. 125, and vol. xlii., p. 182. THALÈN, 'Comptes Rendus,' vol. xci., p. 376; 'Chemical News,' vol. xlii., p. 197.

(18) MARIGNAC, 'Comptes Rendus,' vol. lxxxvii., p. 578; 'Chemical News,' vol. xxxviii., p. 213. NILSON, 'Comptes Rendus,' vol. lxxxviii., p. 642, vol. xci., p. 56; 'Chemical News,' vol. xlii., p. 61.

(19) CLÈVE, 'Comptes Rendus,' vol. xcv., p. 1225; 'Chemical News,' vol. xlvii., p. 4. 'Bull. Soc. Chim.,' vol. xxxix., p. 120; 'Chemical News,' vol. xlvii., p. 143.

(20) MARIGNAC, 'Comptes Rendus,' vol. xc., p. 899; 'Chemical News,' vol. xli., p. 250.

(21) This is almost certainly identical with LECOQ DE BOISBAUDRAN'S samarium. See MARIGNAC, 'Comptes Rendus,' vol. xc., p. 899; 'Chemical News,' vol. xli., p. 250. SORET, 'Comptes Rendus,' vol. xci., p. 378; 'Chemical News,' vol. xlii., p. 199.

41. Some of these claimants will certainly not stand the test of further scrutiny. Thus samarium and yttrium β are in all probability identical; and I should scarcely have included philippium, as ROSCOE[*] has conclusively proved that this is a mixture of terbium and yttrium, and my own results (61) confirm those of ROSCOE. Moreover, others of these so-called elements will probably turn out to be mixtures of known elements. But in the confessedly very imperfect state of our knowledge of the chemistry of these metals it is not safe for me in this research to assume that any one of them will surely not survive. The complete list as it stands will therefore be taken to contain all hitherto claimed as new, although it is almost certain to include too many.

The sought-for body has no absorption spectrum.

42. In the second column "Yes" or "No" indicates whether the solutions give an absorption spectrum when examined by transmitted light. Now could I definitely settle whether solutions of the citron-band body gave an absorption spectrum or not, I could at once eliminate a whole class of elements.

This was not difficult to determine. I have already said (22, 24) that spectroscopic examination entirely failed to detect didymium in many solutions of the earth which gave the citron band strongly. This was not always the case. In early days of this research I frequently obtained absorption bands innumerable when the citron-band body was known to be present; but as I became better acquainted with the chemical reactions of the new earth I gradually succeeded in eliminating one after the other those metals yielding absorption spectra. The earth from zircons (18, 21) gave the most satisfactory results in this respect. This, after removing the little didymium present, gave but a trace of an absorption spectrum, which from its general appearance was probably due to erbia. The earth obtained from cerite (25), which gave the citron spectrum with great brilliancy, on the other hand yielded no absorption spectrum; and generally I may say that, whenever I started with a sufficient quantity of an earth giving both citron-band spectrum and absorption spectrum, I could, by appropriate chemical means, always separate it into three portions,—one which gave the citron-band spectrum with great brilliancy, and showed in concentrated solution a very faint absorption spectrum, and frequently none at all; another which gave very little citron-band spectrum, but a good absorption spectrum; and a third intermediate portion—about four-fifths of the whole—which gave both citron band and absorption spectrum. This portion, by repetition of the treatment, could again be split up in the same way, and the operation repeated as often as the stock of material held out.

43. Having definitely settled the question that the metal giving the citron-band spectrum was not one of those giving an absorption spectrum, the possible elements become materially narrowed to the following list :—Cerium, lanthanum, mosandrum, scandium, terbium, thorium, ytterbium, yttrium, yttrium α, and zirconium.

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[*] 'Jour. Chem. Soc.,' vol. 41, p. 277.

Of these the potassic sulphate reaction (36) excludes cerium, lanthanum, scandium, thorium, yttrium α, and zirconium, so there are left only the following:—

Mosandrum,
Terbium,
Ytterbium,
Yttrium.

44. Certain chemical reactions for a long time made me dismiss yttrium from the list of likely bodies. In my analysis of zircons (18), towards the latter part of the process, I used the following process to separate the iron:—The solution, mixed with tartaric acid and excess of ammonia, was allowed to stand for some time. A small quantity of a precipitate gradually formed, which was filtered off, and it was this filtrate, after separating the iron with ammonic sulphide, that yielded the greatest quantity of substance giving the citron band. Now one of the methods of separating yttria from alumina, berylla, thoria, and zirconia is to precipitate it as tartrate in the presence of excess of ammonia, the other earths remaining in solution. **Fresenius** says:—"The precipitation ensues only after some time, but it is complete."

The precipitate thus obtained with tartaric acid and ammonia should therefore contain all the yttria: *it gave no citron band whatever in the radiant matter tube;* whilst the residue, which should be free from yttria (18), proved for a long time the only source of material wherewith to investigate the chemical properties of the body giving the citron spectrum.

45. Another reason which made me, at this stage of the research, pass over yttria, was that I had already tested this earth in the radiant matter tube. In a paper on "Discontinuous Phosphorescent Spectra in High Vacua," read before the Royal Society, May 19th, 1881,* I said—"Yttria shows a dull greenish light giving a continuous spectrum" (75).

For these reasons I for a long time omitted yttria from my list of possible bodies, and considered that the earth, if not a new one, might turn out to be either mosandra, terbia, or ytterbia.

Analysis of samarskite.

46. A very large quantity (about 15lbs. weight altogether) of samarskite was worked up, partly by the hydrofluoric acid method of **Lawrence Smith**,† and partly by fusion with potassic bisulphate. The niobic and tantalic acids after purification were found to give no citron band spectrum.

These methods both gave as a result a large quantity of mixed earths containing most, if not all, of the bodies enumerated in par. 40. Tested in the radiant matter tube, this material gave the citron spectrum very brilliantly. It was dissolved in hydrochloric acid, neutralised as nearly as possible with ammonia, and boiled with

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* Proc. Roy. Soc., No. 213, 1881.

† 'Comptes Rendus,' vol. 87, p. 146.

sodic thiosulphate. This precipitated the thoria, zirconia, and alumina. In this precipitate some of the scandia might also be found, if present in quantity, but as scandic thiosulphate is not completely precipitated, and the earth is present only in minute traces, not much scandia, it is probable, was thus carried down.

This thiosulphate precipitate, treated in the usual way with sulphuric acid, gave no citron band in the radiant matter tube.

47. The filtrate from the thiosulphate was precipitated hot with excess of ammonia, and the precipitate after washing treated with sulphuric acid, dried, and heated till fumes of sulphuric acid disappeared. The sulphate, whitish with a very pale rose tint, was finely ground, and dissolved with frequent agitation in the smallest possible quantity of cold water—an operation which required much time. The solution was then precipitated with potassic sulphate, taking all necessary precautions to keep the liquid well saturated with potassic sulphate. This operation was allowed to go on for about ten days, when the precipitated double sulphates were filtered off and slightly washed with a saturated solution of potassic sulphate. The precipitate contained cerium, lanthanum, didymium, didymium β, decipium, samarium, scandium, yttrium α, yttrium β, together with any thorium and zirconium which might have escaped the thiosulphate treatment.

48. The filtrate from the double sulphates was precipitated hot with ammonia, which brought down the erbia, holmia, mosandra, terbia, thulia, ytterbia, and yttria. The small quantity of manganese in solution was in this operation completely thrown out.

49. The insoluble double sulphates (45) were dissolved in hydrochloric acid, precipitated hot with ammonia, washed till free from potassium salts, re-dissolved, precipitated as oxalates, ignited, and set aside for further examination. On testing in the radiant matter tube this mixture of oxides was found to be practically free from citron band.

50. The ammonia precipitate from the sulphates soluble in potassic sulphate (46) was well washed till free from potassium salts, and dissolved in excess of nitric acid. The concentrated solution gave an absorption spectrum showing lines belonging to erbium and allied metals. Having already proved that the body I was seeking was not one of those metals which gave an absorption spectrum (42, 43), my first object was to find some method by which I could roughly separate this mixture of earths into two portions, one giving absorption bands, and the other having no action on the transmitted spectrum. I found this was possible by taking advantage of the different solubility of the oxalates in nitric acid.

51. The highly acid solution of the nitrates was fractionally precipitated in the following manner:—

To the boiling liquid a solution of ammonic oxalate was added drop by drop. The precipitate at first formed re-dissolved on stirring. The cautious addition of ammonic oxalate was repeated until the precipitate refused to dissolve entirely, but left the hot liquid somewhat milky. It was then rapidly cooled with constant stirring, which brought down a heavy crystalline oxalate. This was filtered off, and called oxalate A. The filtrate, again heated to boiling, was precipitated in exactly the same way with a further quantity of ammonic oxalate till the hot liquid became opalescent. On cooling and stirring, a farther quantity of oxalate came down. The filtrations and precipitations were repeated until no more precipitate could be obtained. Usually I could get twelve or thirteen fractionations in this manner; towards the end the solution did not get milky, and it had to stand sometimes twenty-four hours before much oxalate came down.

52. The fractions first precipitated by oxalic acid gave very strong absorption bands when the concentrated solutions of the oxides were examined by transmitted light. The fractions last precipitated showed the absorption bands only faintly.

53. These operations gave me oxalates from A to L. These, ignited, with free access of air, were then each dissolved in nitric acid, and again separately fractionated as oxalates. The result was about 150 precipitates, ranging from A_1 A_2 . . . A_{12}, B_1 B_2 . . . B_{12}, to L_1 L_2 . . . L_{12}.

These, after ignition, were separated into five lots according to order of colour, and the fractionation of each of the five lots repeated as already described; the series of operations now closely resembling those of PATTINSON's process for desilvering lead. This gave me about sixty lots. This time the hydrogen equivalent of the metal of each lot was taken by converting the oxalate into sulphate and estimating the sulphuric acid, assuming M_2O to be the type of oxide (40, note 1). The result was a series of earths having hydrogen equivalents (M) ranging from about 48 to 33. The earths were now sorted into high, low, and intermediate, those giving intermediate H equivalents being re-fractionated with repeated H equivalent estimation, the highest and lowest being each time separated and added to the former high and low lots.

54. The ultimate result of about five hundred fractional precipitations gave me a mixture of earths having an H equivalent $M=48$, and showing a strong absorption spectrum (56); a mixture having an H equivalent $M=33$, having no absorption spectrum (65); and intermediate earths.

In the radiant matter tube all these fractions gave the citron-band spectrum well, but that of the earth of lowest equivalent was much the brightest, and that of the highest equivalent the least intense.

55. Three methods are available for the partial separation of these earths and for the complete purification of any one of them. The formic acid process (56, 57) is best for separating terbia, as terbic formate is difficultly soluble in water, the other formates being easily soluble.

Fractional precipitation with oxalic acid (63, 64, 65) separates first erbia, holmia, and thulia, then terbia, and lastly yttria. This is the only method which is applicable for the separation of small quantities of terbia from yttria.

Fusing the nitrates (60, 68, 69) separates ytterbia, erbia, holmia, and thulia from yttria. It is not so applicable when terbia is present, and is most useful in purifying the gadolinite earths. This process is the only one known for separating ytterbia from yttria.

Selection must be made of these methods according to the mixture of earths under treatment, changing the method as one earth or the other becomes concentrated on one side or thrown out on the other. Each operation must be repeated many times before even approximate purity is attained. The operations are more analogous to the separation of members of homologous series of hydrocarbons by fractional distillation than to the separations in mineral chemistry as ordinarily adopted in the laboratory.

Preparations of pure terbia.

56. The mixture of high equivalent earths (54) richest in terbia, erbia, holmia, and thulia was treated as follows :—

The earths were dissolved in dilute formic acid, and the solution heated for some time. A white powder of terbic formate separated. This was filtered off, the solution containing the more easily soluble formates evaporated to dryness, and ignited. In this way the $M=48$ earths were separated into two lots, one rich in terbia and the other rich in erbia, &c. The treatment with formic acid was again repeated on both lots, and the crude terbia finally purified as follows :—

57. The crude terbia from all the operations was systematically treated by the formic acid process, keeping the liquid so dilute that only a portion of the terbic formate separated out each time. The syrupy solution of formates was treated as described further on (60). The hydrogen equivalent of the terbium was taken each time; latterly it kept pretty constant at 49·5. The terbia was also tested in the radiant matter tube. At first the citron spectrum was very strong; gradually, however, it got fainter and fainter under the repeated formic treatment, until finally the spectrum became so weak as to satisfy me that it was due only to impurity in the terbia, and that, had the material been sufficient to stand against the extravagant process of purification adopted, I should finally have got a terbia giving no citron-band spectrum. (Subsequent examination (87) showed me that this terbia did not contain more than 1-5000th part of yttria.)

58. A concentrated solution of the purest terbia obtained in this way, when examined by the spectroscope, showed no absorption lines whatever: proving the absence of erbium, holmium, and thulium.

59. The hydrogen equivalent (49·5) would not definitely show the absence of ytterbium (57·9) and yttrium (29·7); but these would have been separated by the formic acid treatment, terbic formate requiring 30 parts of water for its solution, whilst yttric and ytterbic formates dissolve in less than their own weight of water. Moreover, it was not probable that the terbia contained an appreciable quantity of any of these earths as an impurity, for neither the oxalic acid, the fusing nitrate, nor the formic acid process of fractionation produced any change in the atomic weight, 49·5.

Preparation of mixed erbia, holmia, and thulia free from other earths.

60. The filtrate from the terbic formate (57), rich in erbia, and containing besides terbia, holmia, thulia, and yttria, was now treated by converting it into nitrates, evaporating to dryness, and submittting the mass to careful fusion, stopping the operation when the liquid mass began to evolve nitrous fumes. The erbic, holmic, and thulic nitrates decomposing before the yttric nitrate, extraction with water gave an insoluble residue rich in erbia, holmia, and thulia, and a filtrate rich in yttria. The insoluble residue was dissolved in nitric acid, again evaporated to dryness, and fused. These operations were repeated eight or ten times, with the result of raising the H equivalent of the erbium metals to about 56·8, but the citron-band spectrum remained strong for some time after. It, however, ultimately disappeared. A concentrated solution of this erbic, &c., nitrate showed a beautiful and intense absorption spectrum. I did not attempt any separation of erbium, holmium, and thulium from each other, as the evidence here obtained is sufficient to show that the element giving the citron-band spectrum is not one of these three metals. Likewise I had far too little material to enable me to enter on a work of such difficulty with any prospect of success.

Philippia.

61. The so-called philippia was sought for in the portion of earths intermediate between the terbia and yttria (54). These were dissolved in dilute formic acid, and the solution, filtered from some terbic formate which would not dissolve, was carefully evaporated down to a small bulk, filtering off the terbic or other difficultly soluble formates as they deposited. The clear concentrated solution was then set aside over sulphuric acid to crystallise. In the course of a few days brilliant rhombic prisms crystallised out, having exactly the appearance described by DELAFONTAINE.* The finest of these crystals were picked out, dried on blotting-paper, and analysed. The hydrogen equivalent was found to be $M = 38·2$. The citron-band spectrum in the radiant matter tube was very brilliant. The solution decanted from these crystals was evaporated to a syrupy consistency, filtered from insoluble terbic formate which deposited, and treated for yttria (65).

Some of the best rhombic crystals were added to cold water acidulated with formic acid, and gently heated, but all attempts to dissolve and re-crystallise them failed. A large quantity of an insoluble formate separated, and the mother-liquor on concentration again deposited shining rhomboidal crystals. On attempting to re-crystallise these, they again deposited an insoluble white powder. The mother-liquor was found to contain a large quantity of yttria, and the white insoluble formate on ignition gave an earth having the atomic weight and chemical behaviour of terbia. This entirely

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* 'Comptes Rendus,' vol. 87, p. 559; 'Chemical News,' vol. 38, p. 202; 'Journ. Chem. Soc.,' vol. 36, p. 116.

corroborates Professor ROSCOE's conclusions,* that DELAFONTAINE's philippia is nothing but a mixture of yttria and terbia.

Mosandra.

62. The chemical characters of this earth are so little known that I could not attempt to search for it. But as the citron band-forming earth always appeared concentrated amongst those whose double sulphates were most soluble in potassic sulphate,—and, of these, amongst those having the palest colour and lowest atomic weight,—it was scarcely conceivable that the earth I was in search of should ultimately prove to be one whose properties did not in any case correspond to these,—of a dark orange yellow colour, forming a difficultly soluble double potassic sulphate, and having the very high equivalent of $M = 51{\cdot}2$; these being the properties ascribed to mosandra by the discoverer, Professor LAWRENCE SMITH.

Separation of terbia and yttria from erbia, holmia, and thulia.

63. The mother-liquors, from which as much terbic formate as possible had been separated by the process above described (56, 57), were now evaporated down with nitric acid till all the formates were decomposed, and the highly acid solutions of nitrates were fractionally precipitated with oxalic acid (51, 52, 53).

64. The erbic, holmic, and thulic oxalates come down first; then the terbic oxalate; lastly the yttric oxalate (53). After repeated fractional precipitations I at last succeeded in obtaining a mixture of yttria and terbia of a golden colour, which gave a very brilliant phosphorescent spectrum in the radiant matter tube, but showed no trace of absorption band when the concentrated solution of the nitrates was examined in the spectroscope.

Separation of terbia and yttria.

65. The crude yttria was now added to the mixture of earths (54) having a hydrogen equivalent $M = 33$, and the whole submitted again to fractionation with oxalic acid, in a somewhat modified manner.

An excess of strong nitric acid was added to the solution of mixed terbic and yttric nitrates, and the solution was heated to the boiling point. Strong oxalic acid solution was added drop by drop till a faint permanent precipitate was produced. Strong nitric acid was now added, a drop at a time, till the solution again became clear, and the whole was allowed to cool very slowly without agitation. On cooling, an oxalate crystallised out in brilliant prisms. These contained nearly all the terbia with some of the yttria, whilst the mother-liquor contained most of the yttria with a little terbia. The filtrate was treated with more oxalic acid, a fresh crop of crystals being produced,

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* 'Jour. Chem. Soc.,' vol. 41, p. 277.

when the crystals were ignited, and the resulting earths re-treated with nitric acid and oxalic acid. After repeated fractionations I finally obtained in this manner a perfectly white yttria and a terbia containing a small quantity of yttria. This terbia was added to the crude terbia from previous operations, and purified as already described (57).

These operations gave me two earths,—yttria and terbia,—which, from the constancy of their H equivalents, were taken to be pure. The earths giving absorption spectra and having H equivalents other than 29·7 and 49·5, include erbia, holmia, and thulia. This portion was not further examined for the purposes of this investigation.

Ytterbia.

66. Before considering it finally proved that the substance forming the citron-band spectrum was yttria, it was necessary to prepare ytterbia and ascertain its behaviour in the radiant matter tube, this earth and yttria being the only remaining earths to which the citron spectrum could possibly belong.

The two metals have hydrogen equivalents—ytterbium 57·9 and yttrium 29·7. The chemical reactions are also sufficiently different to render their separation a matter of no very great difficulty.

67. Gadolinite is said by NILSON to contain most ytterbia, so this mineral was chosen in preference to samarskite. The crude earths were first purified from all the earths whose sulphates are difficultly soluble in potassic sulphate (22, 25, 31 to 36), then by the formic acid process from terbia (56, 57), and lastly by fractional precipitation with oxalic acid from the erbia earths (65). There remained an almost white yttria, which gave the citron-band spectrum very brilliantly. Now, gadolinite contains only about 0·1 per cent. of ytterbia, and about 35 per cent. of yttria; therefore the ytterbia to yttria in this mixture was somewhat in the proportion of 1 to 300, and it gave the citron-band spectrum as brillantly as I had ever seen it. The probability was that the earth forming nearly the whole was the one giving the spectrum.

68. Ytterbic nitrate decomposes on fusion almost as easily as erbic nitrate (60), whilst yttric nitrate resists decomposition much more energetically.* Fusion of the nitrates is also the best process for throwing out the erbia, holmia, and thulia, and is therefore the best for purifying gadolinite yttria, as this mineral is rich in the erbia earths and contains little terbia.

The gadolinite yttria was converted into nitrate, fused for a short time, and extracted with water. The portions soluble and insoluble in water were again separately submitted to this treatment, until at last a colourless earth was obtained, the nitrate of which decomposed easily on fusion, and another whose nitrate resisted decomposition when exposed for a long time to nearly a red heat (70).

The earth from the easily decomposed nitrate gave at first a faint citron-band

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* MARIGNAC, 'Comptes Rendus,' vol. 90, p. 902.

spectrum, evidently due to impurity. On repeating the operation several times I at last succeeded in obtaining a white earth which gave only the merest trace of citron-band spectrum. Its hydrogen equivalent, 58·0, and its chemical properties showed that it was probably MARIGNAC's ytterbia. Subsequent experiments satisfied me that this earth did not contain more than 1-10,000th part of yttria (84, 87). The extreme tediousness of the chemical operations necessary to obtain this high degree of purity, and the long time they require, prevented me from pushing these results beyond what was necessary to prove the special point at issue.

Purification of yttria.

69. The white earth obtained in the operation described at par. 65 might still contain traces of terbia, together with erbia, holmia, and thulia. I had relied on the absence of absorption spectrum as proving the absence of erbia, holmia, and thulia, but this test is not a very delicate one, and a final purification was therefore attempted. The decomposition of the fused nitrates was now the process relied on for this final purification, the yttric nitrate resisting nearly a red heat without decomposition, whilst the erbic, holmic, and thulic nitrates are decomposed at a much lower temperature. The operation was carried on as described at par 60.

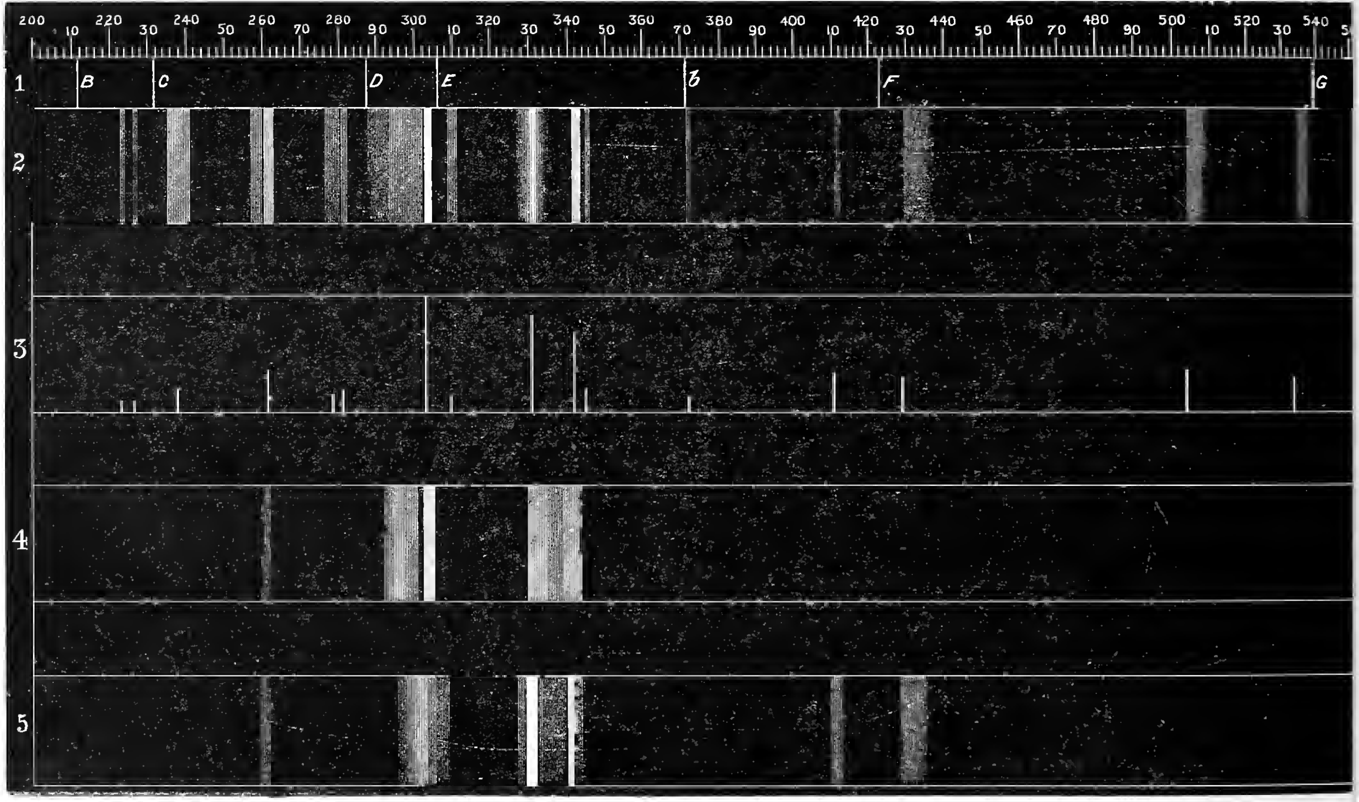
The yttric nitrate left undecomposed, after repeated fusions, was now fused at a higher temperature, extracted with water, filtered from insoluble residue, and the operation repeated on the filtrate. After several such operations the H equivalent of the yttria was taken at every succeeding operation, and the spectral appearance in the radiant matter tube was also examined. The equivalent gradually got down to 31·0, but the spectra did not vary very much; that from the earth of lowest equivalent being, however, the most brilliant.

70. The yttric nitrate, prepared from gadolinite and freed from ytterbia by the fusion of the nitrates (68), was converted into oxalate and ignited. The resulting yttria was quite white, and on testing in the radiant matter tube gave a spectrum absolutely identical with that given by the zircon (18), cerite (25), thorite and orangite (33, 34), and samarskite (64, 69), yttrias. Pure yttria was also prepared from yttro-tantalite, euxenite, allanite, tyrite, and also from plaster of Paris (15) and common limestone. In no case could I detect any difference in the position or intensity of the lines shown by their phosphorescent spectra.

The phosphorescent spectrum of yttria.

71. The spectrum shown by pure ignited yttric sulphate in a radiant matter tube is one of the most beautiful objects in the whole range of spectroscopy. The lines are not so sharp as those given by spark spectra, but are more like the flame spectra

of the alkaline earths. The spectrum is best seen under low dispersion and not too narrow a slit. The accompanying cut gives an accurate map of the spectrum. I have given in line No. 1 the position of the principal FRAUNHOFER lines for comparison of position. Line No. 2 gives the position of the bands, and No. 3 the relative intensities represented by the heights of the ordinates. The numbers along the top refer to a scale of squared oscillation frequencies, or of the squared reciprocals of wave lengths.



72. Commencing at the red end, two narrow faint bands are seen at 2245 and 2275, followed by a stronger and broader red band extending between 2355 and 2415. Another faint band occurs between 2577 and 2610, followed after a very narrow black interval by a stronger reddish-orange band extending to 2627. Another faint orange band occurs at about 2800, with edges too indistinct for measurement. At about 2940 a faint yellow band appears, extending to about 3025. The strong citron-coloured band follows closely from 3028 to 3049; and a little further on, between 3100 and 3120, a much fainter citron band is seen. Two characteristic green bands follow after a dark interval; the first, very bright, extending between 3312 and 3320, but shading off on each side; the second somewhat fainter, but more sharply defined than the first, extending from 3420 to 3440; there is also a third faint green band, between 3460 and 3467. At 3730 is the centre of a narrow and faint

bluish-green band; at 4110 to 4125 in a blue band; and at 4296 another blue band commences, and, extending a short distance, fades away so gradually as to render measurement of the further side impossible. At 5052 and 5351 are two violet lines, but they are not sufficiently sharp to enable accurate measurements to be taken.

I have carefully compared the spark spectrum given by yttric chloride with the phosphorescent spectrum, and have not found any similarity between them, neither have I detected any discontinuity of spectrum on examining the faint light shown by yttrium compounds in BECQUEREL's phosphorescope.

73. The above description applies to the spectrum shown either by pure yttria or by an earth tolerably rich in yttria. When traces are present the citron band only is seen. A little more yttria brings out the first and then the second green band, and finally, as the proportion of yttria increases, the red and blue bands appear (80 to 86).

Circumstances modifying the yttria spectrum.

74. In the early days of this investigation I frequently found that an earthy mixture which by one mode of treatment gave no spectrum, would give a good citron band by a modification of the treatment, and I gradually found that I was most likely to get the spectrum when the body had been treated with sulphuric acid and then ignited to dull redness (4). Not knowing the circumstances governing the appearance of the citron band, it would not then have been safe to have altered this mode of treatment. Now, however, having ascertained the earth to which the spectrum was due, and having a considerable quantity at my disposal, experiments were tried on other methods of treating yttria before exposing it in the radiant matter tube.

75. Pure yttria precipitated by ammonia from the sulphate was dried at a temperature below redness and tested. It did not phosphoresce in the slightest degree, and, necessarily, no citron-band spectrum was to be seen. The yttria was removed from the tube, converted into sulphate, heated to redness, and again tested. It now gave the citron band magnificently. This shows what apparently trivial circumstances will alter the whole course of an investigation. In 1881, when searching for discontinuous phosphorescent spectra, I tried a similar experiment with pure precipitated yttria (45), and entirely missed its citron-band spectrum. Had I first treated the yttria with sulphuric acid, instead of testing the earth itself in the radiant matter tube, the results would have been very different, and this research would probably have never been undertaken.

76. Yttria was now prepared by igniting the precipitated oxalate at a red heat. On testing in the radiant matter tube it phosphoresced with feeble intensity, the light being about one-twentieth of that given by the ignited sulphate under similar conditions. The citron band was almost as sharp as the sodium line, and was shifted one division towards the blue end, now occupying the position 3050 to 3060, its

former place 3028 to 3049 being quite dark. The appearance is shown in line No. 4. On superposing this spectrum and that from the ignited sulphate the displacement of the citron bands was clearly observed; with a very narrow slit the two bands were seen not to touch. The two green bands were visible, but very hazy and indistinct, and only to be resolved into bands with difficulty. The yttria was now removed from the tube, ignited to a bright red heat, and re-tested. The spectrum was a little stronger than that given by the yttria ignited at a lower temperature, but in other respects the general appearance and measurements were unchanged. No alteration was caused by subsequent ignition to a white heat.

77. Pure yttric sulphate ignited to a bright white heat gave a spectrum corresponding to the oxide (76); the sulphate having been decomposed by the high temperature.

78. Yttric phosphate was precipitated, washed, and dried at a heat below redness, and introduced into the radiant matter tube. It phosphoresced faintly, giving the citron band hazy and faint, extending from about 3010 to 3060. The red bands were faint, and the green bands, especially the first one, were stronger than usual. The salt was now removed from the tube, and heated to redness. It became of a grey colour, and now phosphoresced with a beautiful green light. The citron band was still broad and faint, but the green bands were very bright and distinct, and the red band between 2610 and 2627 was also stronger. The spectrum No. 5 shows the appearance.

Heating the phosphate before the blowpipe made little change in the character of the phosphorescence. It was moistened with sulphuric acid, heated to a dull redness, and then tested, but no further change was produced in the spectrum. This experiment shows that the citron-band test for yttrium is far less delicate in the presence of phosphoric acid than in its absence.

Occurrence of yttria in Nature.

79. It is an old and probably a true saying that every element could be detected everywhere had we sufficiently delicate tests for it. Early observations (10, 16) had prepared me for the wide distribution of the element giving the citron band, and no sooner had the exquisite sensitiveness of this spectrum test forced itself on my notice than I sought for yttrium in other minerals. Facts which I had noticed in connexion with the variation of the appearance of the citron spectrum, according to the quantity of yttrium present, showed that it might be possible to devise a process for the rough quantitative estimation of yttrium, and after several experiments this was ultimately carried out in the following manner:—

The calcic carbonate which was found to give no citron band (12) was boiled in a quantity of nitric acid insufficient to dissolve it. The solution was filtered from the insoluble residue, diluted to a convenient bulk, and standardised: 14·91 grains of solution contained 1 grain of calcium. This operation was performed in a room in which had been no yttria compound, and the chemicals and apparatus were new, and had not been taken into the general laboratory. A portion of the standard solution was precipitated with ammonic oxalate, and the calcic oxalate ignited and treated with sulphuric acid. Tested in the radiant matter tube it gave no citron band.

Pure yttric sulphate was dissolved in water to such a strength that 3,000 grains of solution contained 1 grain of yttrium.

80. The solutions were mixed together in the proportion of 1 of yttrium to 100 of calcium, evaporated to dryness, and ignited with sulphuric acid, and the residue tested in a radiant matter tube. The spectrum was bright, the citron band, the two green bands, the blue, and the red bands showed distinctly.

81. A mixture was now prepared in the proportion of 1 of yttrium to 500 of calcium, and tested as above. The citron band was strong, but the green bands were fainter; the blue bands were still visible.

82. A mixture containing 1 of yttrium to 1000 of calcium was next prepared. In the radiant matter tube the citron band was almost as strong as in the last experiment, but the edges were not so sharp, the blue bands were faint, and the green bands had disappeared.

83. A mixture containing 1 of yttrium to 5000 of calcium tested in the radiant matter tube showed the citron band still very bright, but hazy about the edges. No other bands were seen.

84. A mixture of 1 yttrium and 10,000 of calcium was now tried. The citron band was still decided, but not at all sharp.

85. One of yttrium to 100,000 of calcium was next prepared and tested. The citron band was faint, but easily seen. It could not, however, be obtained at all sharp, and appeared broader than usual.

86. A mixture of 1 of yttrium and 1,000,000 of calcium was lastly prepared, and tested in the radiant matter tube. The citron band was very faint, but there was no mistaking its presence, and with care I have no doubt a smaller quantity than 1 in 1,000,000 could be detected. This, however, appears to be near the limit of the test.

87. These seven tubes were mounted on a board, so that connexion with the induction coil could rapidly be made to either of them; and various minerals, &c., were prepared and tested in radiant matter tubes (10). By comparing their spectra with those of the standard tubes I could, after a little practice, determine roughly the proportion of yttrium present, supposing the test not to be interfered with by the presence of phosphoric acid (78).

88. The following are some of the most interesting results obtained in this way:—

		Parts.
Pink coral (one particular specimen)	One part of yttrium in	200
Strontianite	One ,, ,,	500
Stilbite	One ,, ,,	500
Hydrodolomite, from Vesuvius	One ,, ,,	500
Witherite	One ,, ,,	1000
Arragonite	One ,, ,,	2000
Chondrodite (Humite), from Vesuvius. . . .	One ,, ,,	4000
Egyptian syenite (Cleopatra's Needle) . . .	One ,, ,,	7000
Calcite	One ,, ,,	10,000
Natrolite	One ,, ,,	10,000
Ox bone	One ,, ,,	10,000
Meionite (Vesuvius)	One ,, ,,	10,000
Meteorite (Alfianello, Feb. 16, 1883)	One ,, ,,	100,000
Brevicite	One ,, ,,	200,000
Prehnite	One ,, ,,	500,000
Thomsonite	One ,, ,,	500,000
Vesbine, mixed with lava, from Vesuvius . .	One ,, ,,	700,000
Dolomite	One ,, ,,	1,000,000
Tobacco ash	One ,, ,,	1,000,000
Leucite, from Vesuvius	Less than one ,,	1,000,000
Nepheline, from Vesuvius	None	
Meteorite (Dhurmsala, 1860)	None	
Analcite	None	
Phenakite	None	
Chrysolite	None	
Haüynite	None	
Turquoise	None	

Indications of other spectrum-yielding elements.

89. Throughout the course of this paper I have devoted myself only to the citron-band spectrum. I do not, however, wish it to be thought that no other spectra were obtained. On the contrary, I have repeatedly seen indications of another very beautiful spectrum characterised by a strong red and a double orange band, and, more rarely, of a third spectrum distinct from the other two. These I am investigating, but not yet having obtained definite results I forbear from saying any more about them. I hope that they may bear sufficiently good fruit to be worthy of presentation at some future time to the Royal Society.

XXVIII. *On a new Crinoid from the Southern Sea.*

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[PLATE 71.]

TOWARDS the end of last year I received from Mr. MURRAY a small collection of "Challenger" *Comatulæ* which had been retained by the late Sir WYVILLE THOMSON, and was found among his other collections after his death. It included several duplicates of forms already known to me, among them being three fairly perfect specimens of a type which I had only been able to study from a mutilated calyx. There were also eight or nine new species of *Antedon,* all from stations which had already yielded *Comatulæ,* two of them abundantly so. Lastly, there was an apparently insignificant little specimen from a depth of 1,800 fathoms at Station 158 in the Southern Sea.[*] It has five simple arms, and appeared at first sight to be merely a young individual of *Eudiocrinus semperi,*[†] which was dredged at Stations 164 and 169. Upon closer examination, however, I found that the specimen, although a true *Comatula,* and resembling *Eudiocrinus* in having but five arms, presents two characters which occur in no other Neocrinoid. In fact it is only among some of the older Palæocrinoids that similar features are to be met with, and I have no hesitation in saying that this is by far the most remarkable of all the Crinoids obtained by any of the recent deep-sea exploring expeditions.

Under these circumstances I propose to distinguish the type by the generic name *Thaumatocrinus,*[‡] with the specific designation *renovatus.*

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[*] Station 158, March 7, 1874, lat. 50° 1′ S., long. 123° 4′ E.; depth 1,800 fathoms; bottom temperature 3° C., Globigerina-ooze. The only other *Comatula* obtained at this station was *Promachocrinus abyssorum.*

[†] This is the *Ophiocrinus semperi* of my preliminary report (Proc. Roy. Soc., No. 194, 1879, p. 385). Owing to the previous employment of *Ophiocrinus* by SALTER and also by ANGELIN, I have proposed *Eudiocrinus* for the recent type to which this name was given by SEMPER (Journ. Linn. Soc. Zool., vol. xvi., p. 493).

[‡] *θαῦμα,* a marvel.

The total width of the calyx across the disc is barely 2 millims.; and the height of the centrodorsal and radials together is about the same. The former (Plate 71, figs. 1–4, *cd*) is rounded below, with its central canal completely closed up, so that it must have been detached for some little time from the remainder of the stem. The bases of half a dozen cirri are attached to it, and there are pits for the reception of two or three more. In the largest stump which is preserved (Plate 71, figs. 1, 3, *c*) the first two joints are quite short, as is usually the case, but the third reaches a length of 1·5 millim., so that the cirri must have been very like those of *Eudiocrinus semperi.* Except in this respect, however, and in the presence of five undivided arms, there is no further resemblance between the two types; for *Eudiocrinus* has a rosette, and consequently no basals appear externally. The radials are also only partially visible, owing to the extension of the centrodorsal over their lower surfaces, and the oral plates of the larva do not persist in the adult. But in *Thaumatocrinus* (Plate 71, figs. 1–4) there are relatively large basals (*b*), which completely separate the centrodorsal (*cd*) from the radials (*r*). This is itself an unusual feature in any *Comatula,* as will be explained further on.

Upon these five basals there rests a ring of ten arched plates, five of which (*r*) bear the arms, and are evidently the radials. But they *are not in contact laterally,* as is the case in every other five-rayed Neocrinoid with which I am acquainted, for they alternate with five smaller plates (*i, i*), which rest upon the truncated apices of the basals, while the radials rest in the angles formed by the adjacent sides of every two contiguous basals. Four of these five interradial plates terminate in a free edge at the margin of the disc; but the fifth, that on the anal side, bears a small tapering appendage of four or five joints, the last of which seems to end freely (Plate 71, figs. 2, 4, 5, *aa*).

The arms are composed of somewhat elongated joints, the second of which (Plate 71, figs. 1, 2, 4, b_2) bears the first pinnule. This is on the right side in three arms, and on the left in the other two. A similar variation occurs in *Eudiocrinus semperi,* but I do not attach much importance to it. The pinnules are very slender and delicate.

The disc resembles that of *Hyocrinus.* Its central portion is occupied by a relatively large oral pyramid (Plate 71, fig. 5, *o*), while between this and the margin are two or three irregular rows of small anambulacral plates (Plate 71, figs. 1–5, *an*), some of them extending up on to the lower part of the long anal tube (*at*). The large size and comparatively dense appearance of these orals indicates that they are not undergoing the process of resorption as those of other *Comatulæ* do. In some species (*e.g., Ant. dentata,* SAY = *A. sarsi,* DUB. and KOR.) the process is completed long before the end of the pentacrinoid stage; but in *Ant. rosacea* the orals persist in a partially resorbed condition after the loss of the stem, though they soon disappear completely.

Thaumatocrinus is thus the only *Comatula* yet known in which the oral plates of the larva persist, as they do in *Hyocrinus* and *Rhizocrinus.* The other peculiarities which it exhibits are (1) the appearance of a closed ring of basals on the exterior of the calyx; (2) the separation of the radials by inter-radial plates; (3) the presence of a jointed arm-like appendage on the inter-radial of the anal side.

The persistence of the oral and basal plates of the larva, together with the small size of the specimen, might be thought to indicate that *Thaumatocrinus* is merely a type in which the resorption of the orals and the metamorphosis of the basals into a rosette take place unusually late. But as I have already pointed out, the condition of the centrodorsal, and of the cirri which it bears, is evidence that the specimen must have been detached from its stem for some little time, while the orals show no signs of any approaching resorption. The existence of the interradials also decidedly indicates that the basals on which they rest form a permanently closed ring on the exterior of the calyx. This is the case in but a very few *Comatulæ*, either recent or fossil.

Schlüter mentions a Cretaceous species in which it occurs;* while there are several forms, both of Cretaceous and of Jurassic age, in which the basal ring is incomplete, and the radials partly rest on the centrodorsal. But the only recent type in which the basals remain visible on the exterior of the calyx is the curious genus *Atelecrinus*;† and here they are very small in proportion to the radials. In all other recent *Comatulæ* the basals disappear from the exterior of the calyx towards the end of Pentacrinoid life, and become transformed into the rosette. Some species remain much longer in the Pentacrinoid stage than others; so that of two calices of equal size, the one may be still attached to a stem, the top joint of which bears but a few rudimentary cirri, and have large basals; while the other has a centrodorsal bearing 15 or 20 cirri, and concealing both the basals, and a part of the first radials. *Antedon rosacea* and *A. dentata* (*A. sarsi*, Dub. and Kor.) are two excellent instances of this difference, the latter attaining a length of 40 millims. in the Pentacrinoid state. A similar condition is presented by a Pentacrinoid which was dredged by the "Porcupine," and is probably to be referred to *Ant. eschrichti*, or to *Ant. quadrata*.‡ It is stouter and altogether more robust than any larva of *Ant. dentata* which I have seen; and though its radials are as large as those of the free *Thaumatocrinus*, yet its basals are actually higher than those of the single specimen of the latter type; while the centrodorsal on which they rest has merely a few imperfect cirrus-stumps, and is scarcely larger than the stem-joints below it. On the other hand, another larva from near Ascension (S. 344; 420 fathoms) has equally large radials resting directly on the centrodorsal, which bears about eight well-developed cirri; but the basals have already disappeared from the exterior of the calyx. An earlier stage in the development of this same larva is shown in Plate 71, fig. 6, for comparison with *Thaumatocrinus*. Although the calyx and arms are well developed, the basals (*b*)

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* Zeitschr. d. Deutsch. Geol. Gesellsch., Jahrg. 1878, p. 66.

† Bull. Mus. Comp. Zoöl., vol. ix., No. 4, 1881, p. 16, pl. i., figs. 1–7.

‡ This is the *Ant. celtica* of Marenzeller, and of Duncan and Sladen; *non* Barrett.

separate the radials (r) from the centrodorsal (cd), which is, as yet, but slightly differentiated from the stemjoints below it. In all cases, however, the basals become concealed very soon after the conclusion of the Pentacrinoid stage, if not before. I can find no trace of them in various abyssal *Comatulæ*, which are no larger than *Thaumatocrinus renovatus*, and suspect therefore that in this type they persist through life as they do in *Atelecrinus*. Were they really only larval basals, and destined to have been eventually transformed into a "rosette," *Thaumatocrinus* would present a still more curious combination of characters than it actually does.

Both the persistence of the basals and the considerable development of the orals are characters which, either singly or combined, would cause the type to be regarded as one of no little interest; but they are altogether cast into the shade by the other peculiarities of the calyx, viz., the complete separation of the radials by relatively large interradial plates, and the presence of the anal appendage.

It might perhaps be suggested that the ten-rayed *Promachocrinus* affords some explanation of the first of these points. In this genus* the basals only exhibit a pentamerous symmetry, for the rosette into which they become transformed is connected with a basal star of five rays only, just as in any other *Comatula*. These five basal rays are attached to the dorsal surfaces of five out of the ten radials so as to partially separate them from the centrodorsal. These radials, therefore, are really interradial in position, and so correspond to the five interradial plates of *Thaumatocrinus*. But here the resemblance ceases; for the "interradial radials" of *Promachocrinus* precisely resemble the five true radials with which they alternate, and the arms borne by the two sets of plates are indistinguishable. I cannot, therefore, regard *Promachocrinus* as anything but a very regular variation of the usual pentamerous symmetry of the Crinoids.

Failing *Promachocrinus*, there is no other Neocrinoid which presents anything like the remarkable morphological condition of *Thaumatocrinus*. To find a corresponding developmental stage we must go back to a very early period in the ontogeny of a Crinoid, *i.e.*, one but little later than the appearance of the rudiments of the lowest arm-plates. The radials first appear as isolated plates in the spaces "where the upturned angles of two oral plates are opposed to the bevelled off upper angles of two adjacent basals."† They gradually increase in size, and ere long come to form a nearly complete circle, two of them being separated for a time by the anal plate. This is eventually lifted out from between them, but the radials of *Ant. rosacea* do not come into complete lateral contact until after the appearance of the first whorl of cirri. In the larva represented in Plate 71, fig. 6, however, the cirri do not appear until the radials have met laterally, and the arm-bases are well developed. The radials of a mature *Comatula*, therefore, form a closed ring of five plates; and any interradials which may subsequently appear are limited to the angles between adjacent second

* Proc. R. S., No. 194, 1879, p. 385; see also Journ. Linn. Soc. Zool., vol. xv., p. 214, pl. 12, fig. 28.

† C. Wyville Thomson, "On the Embryogeny of *Antedon rosaceus*," Phil. Trans., 1865, p. 528.

and third radials, though sometimes attaining a considerable relative size and importance, as in *Guettardicrinus* and some species of *Apiocrinus*.

It is well known that many peculiarities which are merely transitory in young larvæ of the Neocrinoids, are permanently retained in some of the Palæocrinoids. This is the case, for example, with the primitive position of the anal plate within the ring of (first) radials of the larval *Antedon*. Thus in the Devonian genus *Hexacrinus* (AUSTIN), and in some allied forms from the Carboniferous limestone among the *Platycrindæ*, two of the five radials are separated permanently by a single large anal plate; and the still earlier condition, before the radials have come into lateral contact at all, finds a parallel in the remarkable genus *Reteocrinus*,* from the Trenton and Hudson River groups (Lower Silurian) of North America. In this type, however, the radials are separated by what Messrs. WACHSMUTH and SPRINGER † describe as an "interradial series resting directly upon the basals, consisting of a very large number of minute pieces of irregular form, and without definite arrangement." A similar development of small irregular plates between the rays occurs in many Neocrinoids, both stalked and free, but the interradial series always commence at the level of the second or third radials, and are completely separated from the basals by the ring of united first radials.

Now in *Thaumatocrinus* we not only find the primitive lateral separation of the radials to be permanent, as in *Reteocrinus*, but instead of the small and irregular interradials which rest on the basals of that type, *Thaumatocrinus* has one relatively large plate between every two radials (Plate 71, figs. 1–4, *i*, *i*). This is, as it were, a further development of the embryonic condition, but in a new direction. It is, however, one which is not to be found in any Neocrinoid, either recent or fossil, and it is only among certain of the Palæozoic *Rhodocrinidæ* that a similar peculiarity presents itself. Messrs. WACHSMUTH and SPRINGER have grouped the genera in which it occurs into a special section, *Rhodocrinites*.‡ They are *Lyriocrinus* (HALL); *Rhipidocrinus* (BEYRICH); *Thylacocrinus* (OEHLERT); *Anthemocrinus* (W. and S.); *Rhodocrinus* (MILLER); and *Ollacrinus* (CUMBERLAND). All of them have a ring of ten plates resting on the basals, viz., the radials and five interradials of about the same size. This is well shown in the diagram of the calyx of *Thylacocrinus* (Plate 71, fig. 7), which I have copied from that given by OEHLERT.§

While resembling the *Rhodocrinites* in having five large plates separating the radials, *Thaumatocrinus* differs from them, and from most Palæocrinoids, in the absence of any higher series of calicular interradial plates resting upon the first series which

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* Of BILLINGS, *emend.* WACHSMUTH and SPRINGER.

† "Revision of the Palæocrinoidea," Part II., p. 192. From the Proceedings of the Philadelphia Academy, July 26, 1881, p. 366.

‡ *Ibid.*, pp. 182–184.

§ "Description de deux nouveaux genres de Crinoïdes du terrain dévonien de la Mayenne." Bull. Soc. Géol. de France. 3ᵉ Ser., Tom. vii., pl. 1., fig. 2.

separate the radials. Except on the anal side, these primary interradial plates of *Thaumatocrinus* end simply in a free rounded edge at the margin of the disc (Plate 71, figs. 1–3, 5, *i*), which is doubtless due to the simplicity of the arms; for these become free almost at once, and are not connected laterally by much perisome, in which higher orders of interradials could be supported. But in the presence of the appendage on the azygous interradial (Plate 71, figs. 2, 4, 5, *aa*), *Thaumatocrinus* bears a remarkable resemblance to *Reteocrinus.* The latter genus was established by BILLINGS on some badly preserved fragments from the Trenton limestone of Ottawa.[*] MEEK, and WETHERBY have since described some species of *Glyptocrinus* presenting very similar characters to those of BILLINGS' genus, and have noted the resemblance between them; while WETHERBY[†] subsequently came to the conclusion "that several forms of our so-called *Glyptocrinus* should be referred to this genus." WACHSMUTH and SPRINGER[‡] have accordingly reconstructed *Reteocrinus,* and have proposed as type of the genus *Glyptocrinus nealli* (HALL), a proceeding for which they have been severely criticised by MILLER.[§]

The original examples of BILLINGS' type species were so imperfectly preserved, that the distinctive characters of his genus were incompletely known. As, however, WACHSMUTH and SPRINGER, like WETHERBY, fully believed *Glyptocrinus nealli* to be a *Reteocrinus,* I do not see how they could have redefined the genus better than by selecting such a well-known species as their type.

Taking *Reteocrinus* then as defined by WACHSMUTH and SPRINGER, we find that its posterior interradial area is wider than the other four, "with a conspicuous row of decidedly larger and more prominent special anal plates along the median part." BILLINGS gives a good figure of this structure in *R. stellaris,*[||] and speaks of it as follows: "If this series of joints constitute a true arm, then there must be six arms in this species." The same feature appears, though less prominently, in *R. nealli* (HALL) sp., and in *R. baeri* (MEEK) sp., and also in *R. richardsoni* (WETHERBY), though in *R. gracilis* (WETHERBY) and *R. cognatus* (MILLER) sp., it appears to be absent. It is unusually distinct, however, in *Xenocrinus penicillus* (MILLER)[¶] (Plate 71, fig. 8), a type which closely resembles *Reteocrinus* in general appearance; and I fully agree with Messrs. WACHSMUTH and SPRINGER[**] in thinking that it should

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[*] 'Canadian Organic Remains.' Decade iv., p. 63.

[†] "Description of new Fossils from the Lower Silurian and Subcarboniferous Rocks of Ohio and Kentucky." Journ. Cincinnati Soc. Nat. Hist., vol. iv., April, 1881, p. 7 (of separate copy).

[‡] Revision. II., p. 191.

[§] "Description of Two New Genera and Eight New Species of Fossils from the Hudson River Group." Journ. Cincinnati Soc. Nat. Hist., vol. v., April, 1882, pp. 12, 13 (of separate copy).

[||] *Op. cit.*, p. 64, pl. 9, fig. 4*a*.

[¶] "Description of Some New and Remarkable Crinoids and other Fossils of the Hudson River Group, and notice of *Strotocrinus bloomfieldensis.*" Journ. Cincinnati Soc. Nat. Hist., vol. iv., April, 1881, pl. 1, fig. 3, and pl. 4, fig. 4, pp. 71–73.

[**] Revision. II., p. 185.

be associated with the *Rhodocrinidæ* rather than with the *Actinocrinidæ* to some of which (*e.g.*, *Melocrinus*) it would be allied, owing to the presence of a tetramerous base. Wachsmuth and Springer point out that "no Actinocrinoid has ever been discovered in which the interradial field, except at the azygous side, extends to the basal disc." But Miller's description, which is borne out by his figures (one of which is reproduced in Plate 71, fig. 8), runs as follows: "The azygous area is remarkably large, and covered in the central part by a vertical series of plates having about the same size as the regular radial series, and upon each side of the vertical series there is a depressed area covered by small plates having a tubercle in the central part, as in the regular interradial areas. There are seven plates, each having a length about twice as great as its width, in the vertical series, from the basal plate upon which the series rests to the top of the vault. This vertical series is continued to the top of the proboscis, and contains in its entire length more than fourteen plates. It has such strong resemblance to the radial series, except as to the branching at the secondary radials, that the general appearance of the body is that of a species having six radial series."

There can, I think, be no reasonable doubt that the anal appendage of *Thaumatocrinus,* although free laterally, owing to the simplicity of the rays, is homologous with the vertical series of plates in the anal interradius of *Reteocrinus* and *Xenocrinus;* and it is not a little curious to find a character which died out some time before the Mesozoic epoch recurring in a recent *Comatula.* I am quite at a loss as to the probable function of this anal appendage in *Thaumatocrinus,* but it seems to differ from that of the Palæocrinoids in one point, for Miller describes it in *Xenocrinus* as continued to the top of the proboscis, which is not the case in *Thaumatocrinus.* The lower part of the anal tube bears plates, but they are continuous with those covering the disc over which the anal appendage arches, without, however, forming any connexion with the plates in question.

It is difficult to consider the existence of interradials and of the anal appendage of *Thaumatocrinus* as instances of atavism, for no known Neocrinoid presents any similar characters, and it is a long way back from a recent *Comatula* to a Palæozoic Crinoid. The appendage soon disappeared, both the genera possessing it being of Lower Silurian age; but Crinoids with the interradials resting on the basals persisted into the Carboniferous period, and possibly also some with an anal appendage. Nothing of the kind is visible, however, in any genus of Neocrinoids, so that the reappearance of these characters in such a specialised type as a *Comatula* is not a little surprising. Associated with them we find the distinctly embryonic characters of persistent basal and oral plates, the latter occurring in no other *Comatula,* together with the simplicity of the undivided arms.

Thaumatocrinus is thus a type of unusual interest, and should be sought for carefully in any future deep-sea explorations. It is evident that the possibilities of the abyssal fauna are by no means exhausted yet.

The presence of the oral pyramid in *Thaumatocrinus,* as in *Hyocrinus,* suggests the idea that the little specimen obtained from a depth of 2,325 fathoms, at Station 223 in the east Pacific, may, perhaps, be related to the former genus. Sir WYVILLE THOMSON* spoke of it as *Hyocrinus bethellianus* (?), with the remark, "It is certainly in many respects very unlike the adult *H. bethellianus*, but it may possibly turn out to be the young of that species." No figure of it is to be found either in the "Atlantic" or in any of the numerous plates which were drawn at Edinburgh under Sir WYVILLE's direction, and it is to be feared that this "beautiful little thing" has been mislaid, as Mr. MURRAY has been unable to discover it among the material which was in Sir WYVILLE's hands at the time of his death.

The discovery of *Thaumatocrinus* restores the numerical equality between the living genera of *Comatulæ* and of stalked Crinoids, and raises their joint total to twelve. Species of every genus, except *Holopus*, have been obtained by the various British expeditions for deep-sea exploration, as shown in the following table :—

Family.	Genera.	Number of species obtained.	Remarks.
Comatulidæ . .	*Antedon*, FREM.	83	Seven obtained by the "Porcupine" and the "Triton." The rest by the "Challenger."
	Actinometra, MÜLL.	52	"Challenger." One by the "Porcupine."
	Promachocrinus, P.H.C.	3	„
	Eudiocrinus, P.H.C.	3	„
	Atelecrinus, P.H.C.	2	„
	Thaumatocrinus, P.H.C.	1	„
Pentacrinidæ .	*Pentacrinus*, MILLER	5	One obtained by the "Porcupine;" one species doubtful, perhaps representing a new genus.
	Metacrinus WY. TH., and P.H.C.	10	One founded on stem fragments only.
Bourgueticrinidæ	*Rhizocrinus*, SARS.	2	"Challenger" and "Porcupine."
	Bathycrinus, WY. TH.	3	One obtained by the "Porcupine."
Hyocrinidæ . . .	*Hyocrinus*, WY. TH.	1	Besides one doubtful young specimen.

General considerations.

The peculiarities of *Reteocrinus*, as well as of *Thaumatocrinus*, have suggested certain morphological considerations bearing on the various classifications of the *Rhodocrinidæ* that have hitherto been proposed.

One cannot help wondering where the circular commissure of *Reteocrinus* was situated. In ordinary Crinoids with directly contiguous radials, each of them is traversed by a portion of the circular canal in which the commissure is lodged. But

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* "Notice of New Living Crinoids belonging to the *Apiocrinidæ*." Journ. Linn. Soc. Zool., vol. xiii., p. 55.

in *Reteocrinus* and in *Xenocrinus* (Plate 71, fig. 8) the isolated radials are quite narrow, and the interradial spaces separating them are sometimes twice their breadth. How did the circular commissure traverse these spaces? Whether there were canals in the radials or not, the interradial portions of the commissures must have been freely exposed to the body-cavity at the bottom of the calyx, for it is not likely either that the commissure was absent, or that it was situated within the ring of basal plates. In this respect also, therefore, *Reteocrinus* presents an embryonic feature, for in the earlier stages of Pentacrinoid life the axial cords simply lie on the ventral surface of the radials and brachials, without having any channels hollowed out in these plates for their reception, as is subsequently the case; and whatever was the case with the radials and basals of *Reteocrinus*, it is improbable that the irregular interradial plates which were crossed by the cords were in any way grooved for their reception.* These may fairly be regarded as corresponding to the numerous irregular plates which occur upon the disc and between the rays of many Neocrinoids. Those of *Reteocrinus*, however, do not stop at the level of the second radials, but extend right down to meet the basals. Sometimes there appear to be only one or two between every two radials, *e.g.*, *R. gracilis*, but in other cases the number seems to be larger, and the plates can hardly be regarded as the complete morphological equivalents of the larger and more regular single interradials which occur in the *Rhodocrinidæ*.

The interradial portions of the circular commissure must have passed over the ventral surface of these large plates, if not actually piercing them. The same must be the case in *Thaumatocrinus*, and unless its radials are different from those of all other Neocrinoids, the axial cords must be lodged in canals, which is probably also true for the interradials. In any case, however, the relation of these plates to the axial cords shows that they belong, like the radials and basals, to the radial system, rather than to the perisomatic. It is not easy to make out their homologies in other Echinoderms, but they are perhaps represented in the disc of an Ophiurid by the proximal row of intermediate plates, while the interradials generally correspond to the distal rows.

The morphological differences involved in the separation or lateral union of the

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* Müller, and more recently Zittel, have considered the presence or absence of canals within the calyx-plates as affording an important character which distinguishes the Palæozoic from the younger Crinoids. There are many Palæocrinoids, however, in which these canals are present, *e.g.*, *Allagecrinus*, *Platycrinus*, and all forms with true articular facets on the distal faces of the radials. *Platycrinus* has, nevertheless, been placed by Professor Chapman in his division *Emedullata*, the calyx and arm-plates of which are "without internal canals" (See "A Classification of Crinoids," read before the Royal Society of Canada, May 26, 1882). Strangely enough, *Marsupites* is placed in the same division, although any specimen with a good articular surface on the radials shows the opening of the central canal as distinctly as possible, and the canal actually pierces the substance of the plate, not ending abruptly on its ventral surface, as in the radials of *Cupressocrinus*.

(first) radials seem to have been first noticed by ZITTEL.[*] For he made them a fundamental distinction between the two families of *Glyptocrinidæ* and *Rhodocrinidæ*, in which he placed several genera that had been somewhat scattered in previous classifications. To the latter he referred types with a more or less depressed or spherical calyx, in which the lowest interradials rest directly on the basals, and form, together with the radials, a ring of ten alternating plates, *e.g.*, *Rhodocrinus* and *Ollacrinus*. In the *Glyptocrinidæ*, on the other hand, the calyx is higher, and the lowest interradials rest upon the upper edges of contiguous radials. This family includes *Glyptocrinus* and *Glyptaster* (HALL), with *Eucrinus* (ANGELIN), and also *Thylacocrinus* (OEHLERT), which seems somewhat out of place; for it has a large globular calyx, and five large interradials, which completely separate the radials from one another (Plate 71, fig. 7).

Messrs. WACHSMUTH and SPRINGER[†] express considerable doubt whether the differences between these two families in ZITTEL's classification, "even if they were persistent, can be deemed sufficient for a family distinction. *Thysanocrinus* of the *Rhodocrinidæ* has generally at four sides the first interradial disposed between the first and second radials; while in *Thylacocrinus*, according to OEHLERT's figure (Plate 71, fig. 7), all five first interradials rest directly upon the basals." I must confess that I cannot quite follow this argument. The *Thysanocrinus* referred to is HALL's type of that name, which WACHSMUTH and SPRINGER subsequently place under *Dimerocrinus* (PHILLIPS); and in their generic diagnosis of it they say, "Interradial areæ composed of but few plates, the first one large, placed between the second radials, and leaning partly against the third, with two small plates above. Posterior, or anal area wider, the first plate in line with the first radials, and of the same size." *Thylacocrinus*, on the other hand, is said to have "Interradials numerous, the lower one resting directly on the basals;" *i.e.*, all five interradials meet the basals, and not that on the anal side only. This difference is further recognised by WACHSMUTH and SPRINGER, for they place the two genera in different sections of their family *Rhodocrinidæ;* and I do not, therefore, see the force of their doubts respecting the persistency of the characters in this portion of ZITTEL's classification. In fact, they make great use of the position of the lowest interradials in defining their subdivisions of the family.

According to their general description of the *Rhodocrinidæ*, "In most of the genera the first interradial rests directly upon the truncate upper side of the basals, thereby separating the first radials all round. In others, however, only the first plate of the posterior or anal side is supported by a basal, that of the other four sides being placed against the upper corners of the first, and between the second radials, the former producing an almost pentahedral, the latter a more or less bilateral symmetry." This

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[*] 'Palæontologie,' pp. 374–376.

[†] Revision. II., pp. 181, 182.

is perfectly true, and the distinction sharply marks off the section of bilateral *Glyptasterites* from the pentahedral *Rhodocrinites.*

The section *Glyptocrinites,* however, is somewhat heterogeneous. Its calyx is said to be almost perfectly pentahedral with the "interradial areas depressed, the first plate resting either directly upon the basals, or between the second and third radials, without special anal plate beneath their line." Only three genera are included in this section, and Messrs. WACHSMUTH and SPRINGER seem to have been somewhat uncertain about so grouping them; for they remark (p. 183), "It might have been not out of the way if we had placed the genus *Glyptocrinus* in a group by itself, as it differs from *Archæocrinus* and *Reteocrinus,* with which it has been associated, *and from all other Rhodocrinidæ,*[*] in having the first plate at each interradial side placed between the second radials."

In *Archæocrinus* (W. and S.), on the other hand, the lowest interradials rest directly upon the basals, as is also the case with the small and irregular interradials of *Reteocrinus.* Both these genera, therefore, have isolated radials and a pentahedral symmetry ("somewhat bilateral" in *Reteocrinus*) just as in the *Rhodocrinites.* But the latter lack the "rounded strongly elevated ridges" which distinguish the radials of *Archæocrinus* and *Reteocrinus.* This, however, is merely a character in the superficial ornamentation of the calyx; and it seems to me of altogether minor importance as compared with the morphological differences between the lateral union and the isolation of the radials. In this last feature *Archæocrinus* and *Reteocrinus* resemble the *Rhodocrinites,* and if the limits of that section could not be enlarged to receive them, they might very well be left in a group by themselves, distinguished by their ornamentation.

But they are out of place by the side of *Glyptocrinus,* with *all* its radials united laterally. It thus represents a comparatively late ontogenetic condition, not even the radials of the posterior side being separated by an anal plate as in the *Glyptasterites.* There are, doubtless, close affinities and remarkable transition forms between *Glyptocrinus* and *Reteocrinus,* as asserted by Messrs. WACHSMUTH and SPRINGER. But these depend very largely upon the characters of the rays and arms, which are of a comparatively subordinate value; while the lateral separation of the radials in the last named genus, and in the *Rhodocrinites,* is a fact of considerable importance in Crinoid Morphology.

It must be remembered also that *Glyptocrinus* has decided affinities with some of the earlier *Actinocrinidæ,* certain species appearing to be without under-basals. In fact, according to the American authors, "It is a question whether that genus, at least partly, should not be arranged with the other group altogether."

Thus, then, I would divide the *Rhodocrinidæ* (W. and S.) into groups as follows:—

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[*] The italics are mine.

I. Radials completely separated laterally, either by single interradial plates, or by groups of small ones.	
α. No ridges on the radials	*Rhodocrinites* (W. and S.)
β. With ridges on the radials	*Archæocrinus.* *Reteocrinus.*
II. The two posterior radials separated by an anal plate which rests on a basal	*Glyptasterites* (W. and S.)
III. Radials in contact all round the calyx	*Glyptocrinus.*

I cannot help suspecting, however, that ZITTEL's arrangement of these genera into two families, *Glyptocrinidæ* and *Rhodocrinidæ*, is the most natural one; though I should place *Thylacocrinus* in the latter, and not in the former as he has done.

POSTSCRIPT.

(Added February 1, 1884.)

During the past year the peculiarities of *Thaumatocrinus* have naturally been much in my mind, and I have been led to believe that the structure which I have called the anal appendage is represented in other Palæocrinoids besides the two Silurian genera already mentioned.

WACHSMUTH and SPRINGER* describe *Taxocrinus* and *Onychocrinus* as having a small lateral proboscis in the anal area, which consists of a series of from two to six narrow quadrangular plates, longitudinally arranged, and resting on the upper surface of a basal. MEEK and WORTHEN† spoke of it in *Onychocrinus* as "really looking very much like a diminutive arm rising from the anal area;" and they subsequently found the remainder of the anal interradius to be occupied by a great number of minute irregular plates, which pass gradually upwards into those of the "vault," just as in *Reteocrinus* and *Xenocrinus.* If *Thaumatocrinus* were a larger type, with plated perisome between the rays, as in *Pentacrinus asteria*, its tapering anal appendage would be in the same condition as that of *Onychocrinus*, becoming merged above into the general plating of the anal interradius.

I do not think, therefore, that WACHSMUTH and SPRINGER are quite correct in describing *Taxocrinus* and *Onychocrinus* as having "a small lateral tube." That the arm-like series of plates supported the lower portion of the anal interradius is doubtless true. But I do not imagine the plates to have been in any way hollowed on their inner sides for the reception of the hind-gut. This undoubtedly opened to the exterior at a higher level, through a regular anal tube just as in other Crinoids.

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* Revision. I., pp. 46–53.

† 'Palæontology of Illinois,' vol. ii., p. 243; vol. iii., p. 494.

Excellent figures of the anal series in *Taxocrinus* are given by SCHULTZE[*] and ANGELIN,[†] while that of *Onychocrinus* is well represented by MEEK and WORTHEN.[‡] These figures may be advantageously compared with those of *Thaumatocrinus* on Plate 71.

Since the preceding paper was presented to the Society in April, 1883, the discussion between Messrs. MILLER and WACHSMUTH respecting the nature of *Reteocrinus* has been carried on with considerable vigour.

Stimulated by MILLER'S criticisms, Messrs. WACHSMUTH and SPRINGER were able (with the help of Mr. W. R. BILLINGS) to demonstrate a considerable amount of resemblance between *Reteocrinus stellaris* (BILLINGS), and *Glyptocrinus nealli* (HALL). Both types have (1) the under-basals visible externally, (2) the radials separated laterally by the lowest interradials, which rest on the basals, and (3) a prominent median row of plates in the anal interradius; though WACHSMUTH and SPRINGER do not lay much stress upon the last point. "*Reteocrinus* is readily identified by its highly elevated radial ridges, and depressed interradial spaces, filled with numerous small plates of irregular arrangement, and extending between the first radials down to the basals; by its under-basals, often well developed; its strongly marked bilateral symmetry; and by its ten primary arms as a rule."[§]

MILLER replied by giving a detailed comparison of *Glyptocrinus nealli* and *Reteocrinus stellaris,* and believed himself to have found such great differences between them, "that it is doubtful whether they should even be classified in the same family."[||] He lays much stress upon differences in the general aspect of the cup and arms; a little more so, perhaps, than is necessary, considering the poor state of preservation of the Canadian specimens. Two points, however, seem to me to be of greater importance. In all the species which have been lately referred to *Reteocrinus* by WACHSMUTH and SPRINGER, the under-basals are poorly developed, or perhaps even absent; while the third radial is the axillary. But in *R. stellaris* there are quite large under-basals, and the fourth radial is the axillary; and I have some doubt, therefore, as to the advisability of referring to this little-known generic type a number of species which do not present these characters, more especially the latter one. They all agree, however, with *Reteocrinus stellaris* in a feature which both WACHSMUTH and SPRINGER and I myself regard as specially distinguishing *Reteocrinus* from *Glyptocrinus,* viz.,

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[*] "Monographie der Echinodermen des Eiflerkalkes." Denkschr. d. Wiener Akad. Bd. xxvi., 1866. Taf. iv., figs. 2, 2*b*, 3, 4*b*.

[†] "Iconographia Crinoideorum," &c. Stockholm, 1878. Tab. xviii., fig. 8. Tab. xx., figs. 9, 13, 16. Tab. xxiii., fig. 5.

[‡] 'Palæontology of Illinois,' vol. v., pl. xiv., fig. 4.

[§] "Remarks on *Glyptocrinus* and *Reteocrinus,* two genera of Silurian Crinoids." Amer. Journ. Sci., vol. xxv., April, 1883, pp. 265-266.

[||] "Response to the Remarks of Messrs. WACHSMUTH and SPRINGER on the genera *Glyptocrinus* and *Reteocrinus.*" Amer. Journ. Sci., August, 1883, p. 112.

the separation of the first radials by the calyx-interradials. WALCOTT[*] has recently pointed out that a new species, which he has described as possessing this character, "departs from the typical form of *Glyptocrinus*," and he suggests its reference to another generic type. MILLER, however, regards this striking difference in the position of the lowest interradial as of no systematic value whatever, even for specific classification. For he identifies *Reteocrinus gracilis* (WETHERBY), with a type previously described by himself as *Glyptocrinus angularis*, and since recognised by WACHSMUTH as a true *Glyptocrinus*. The lowest interradials rest "between the upper sloping sides of the first radials;" while in *R. gracilis* the radials are widely separated laterally, and the lowest plates of the irregular interradial series rest upon the basals. This feature also occurs in four species which are referred by MILLER[†] to *Glyptocrinus*, although in *G. decadactylus*, which he takes as his type, "the regular interradial areas have one plate resting upon the primary radials!"

But MILLER goes even further than this. He establishes a new genus, *Gaurocrinus*, for types hitherto described under *Glyptocrinus*, but possessing a dicyclic base; and he refers to it five species, two of which are new. In one of these, and in the three species previously known, the lowest interradials rest upon the basals. But in *Gaurocrinus splendens*, n.sp., the large hexagonal basals are "not truncated upon the upper face by an interradial." MILLER'S mode of classification, therefore, totally disregards such important morphological differences as the separation or lateral union of the primary radials; and I cannot believe that it will find acceptance among philosophical palæontologists.

Gaurocrinus differs from *Glyptocrinus* in having a dicyclic base,[‡] that of *Glyptocrinus*

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[*] "Descriptions of new species of Fossils, from the Trenton Group of New York." 35th Ann. Report N. Y. State Mus. Nat. Hist., p. 2 (of separate copy).

[†] "*Glyptocrinus* redefined and restricted, *Gaurocrinus Pycnocrinus*, and *Compsocrinus* established." Journ. Cincinn. Soc. Nat. Hist., Dec., 1883, vol. vi., pp. 217-228.

[‡] It is much to be regretted that Mr. MILLER still uses the empirical and utterly irrational nomenclature, which is now being gradually replaced by a system based upon sound morphological considerations. He remarks that "the policy of changing the nomenclature may well be doubted, and ought not to be entered upon without the clearest conviction that, by so doing, error of some kind is being eradicated." No better illustration of such an error could be found than his statement that *Gaurocrinus* "is primarily distinguished from *Glyptocrinus* by possessing five sub-radials." This name was given by DE KONINCK to the so-called parabasals of MÜLLER, "afin de faire comprendre qu'ils alternent avec les radiales;" and since "the presence or absence of sub-radial plates is regarded of special generic importance" by Mr. MILLER, we are led to conclude that the "basals" of *Glyptocrinus* (MILLER), in which sub-radials are absent, do *not* alternate with the radials. But this is exactly contrary to the fact! Sub-radial plates, alternating with the radials, *are invariably present;* and it is the radially situated under-basals which may be "present or absent." In the former case Mr. MILLER calls them basals, which name he also gives to the sub-radial plates when there are no under-basals.

This method is doubtless both "easy" and "expressive." But it unfortunately implies an homology between the radially situated plates of one genus (*e.g.*, *Poteriocrinus*), and plates which are interradial in another (*e.g.*, *Platycrinus*); and this is utterly opposed to the fundamental principles of morphology, not only in the Crinoids, but also in the Echinoderms generally.

being monocyclic only. It is curious, however, that MILLER should make *Glyptocrinus decadactylus* the type of a genus, the speciality of which is the presence of a monocyclic base. For HALL, MEEK, and WACHSMUTH[*] have all described it as being dicyclic, *i.e.*, as possessing under-basals.

It has been abundantly proved within the last few years that the presence or absence of under-basals upon the exterior of the calyx is a character which is generally of very little value for systematic purposes. *Encrinus* and *Heterocrinus* afford good instances of this. The late Mr. MEEK, whose judgment Mr. MILLER will hardly question, was fully aware of the great amount of variation in the cup of *Heterocrinus*, some species having one series of plates beneath the radials, and others, two; while he further recognised that the upper row of the dicyclic base represents the single row of the monocyclic forms.

The same is the case with *Glyptocrinus* and its allies, as was recognised by WACHSMUTH and SPRINGER, though MILLER will not admit it. One would like to know whether he would rearrange the species of *Heterocrinus* on the principles which have led him to separate *Gaurocrinus* from *Glyptocrinus*.

DESCRIPTION OF THE FIGURES.

PLATE 71.

The following lettering is used throughout all the figures:—*aa*, anal appendage; *an*, anambulacral plates; *at*, anal tube; *b*, basals; b_2, second brachial; *c*, cirrus; *cd*, centrodorsal; *i*, interradial plates; *o*, orals; *r*, radials.

Figs. 1–5.—*Thaumatocrinus renovatus.* All × 15.

Figs. 1 and 2. Side views, radial. In 1, the right anterior, and in 2, the right posterior ray faces the observer.

Figs. 3 and 4. Side views, interradial. Fig. 3 shows the left anterior interradius with its single large interradial (*i*) and anambulacral plates (*an*).

Fig. 4 shows the posterior or anal interradius, in which the interradial plate bears the jointed anal appendage (*aa*).

Fig. 5. View of the disc from above, showing the anal tube and appendage, the oral pyramid, and the marginal zone of anambulacral plates.

Fig. 6. Radial view of an *Antedon*-Pentacrinoid from S. 344, showing the lateral union of the radials. × 15.

Fig. 7. Calyx of *Thylacocrinus*; after OEHLERT.

Fig. 8. Anal side of *Xenocrinus penicillus*, showing the anal appendage (*aa*); after MILLER.

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[*] Revision. II., p. 7.

XXIX. *An Experimental Investigation of the Circumstances which determine whether the Motion of Water shall be Direct or Sinuous, and of the Law of Resistance in Parallel Channels.*

By OSBORNE REYNOLDS, *F.R.S.*

Received and Read March 15, 1883.

[PLATES 72–74.]

SECTION I.

Introductory.

1. *Objects and results of the investigation.*—The results of this investigation have both a practical and a philosophical aspect.

In their practical aspect they relate to the *law of resistance to the motion of water in pipes,* which appears in a new form, the law for all velocities and all diameters being represented by an equation of two terms.

In their philosophical aspect these results relate to the fundamental principles of fluid motion; inasmuch as they afford for the case of pipes a definite verification of two principles, which are—*that the general character of the motion of fluids in contact with solid surfaces depends on the relation between a physical constant of the fluid and the product of the linear dimensions of the space occupied by the fluid and the velocity.*

The results as viewed in their philosophical aspect were the primary object of the investigation.

As regards the practical aspect of the results it is not necessary to say anything by way of introduction; but in order to render the philosophical scope and purpose of the investigation intelligible it is necessary to describe shortly the line of reasoning which determined the order of investigation.

2. *The leading features of the motion of actual fluids.*—Although in most ways the exact manner in which water moves is difficult to perceive and still more difficult to define, as are also the forces attending such motion, certain general features both of the forces and motions stand prominently forth, as if to invite or to defy theoretical treatment.

The relations between the resistance encountered by, and the velocity of, a solid body moving steadily through a fluid in which it is completely immersed, or of water moving through a tube, present themselves mostly in one or other of two simple forms. The resistance is generally proportional to the square of the velocity, and when this is not the case it takes a simpler form and is proportional to the velocity.

Again, the internal motion of water assumes one or other of two broadly distinguishable forms—either the elements of the fluid follow one another along lines of motion which lead in the most direct manner to their destination, or they eddy about in sinuous paths the most indirect possible.

The transparency or the uniform opacity of most fluids renders it impossible to see the internal motion, so that, broadly distinct as are the two classes (direct and sinuous) of motion, their existence would not have been perceived were it not that the surface of water, where otherwise undisturbed, indicates the nature of the motion beneath. A clear surface of moving water has two appearances, the one like that of *plate glass*, in which objects are reflected without distortion, the other like that of *sheet glass*, in which the reflected objects appear crumpled up and grimacing. These two characters of surface correspond to the two characters of motion. This may be shown by adding a few streaks of highly coloured water to the clear moving water. Then although the coloured streaks may at first be irregular, they will, if there are no eddies, soon be drawn out into even colour bands ; whereas if there are eddies they will be curled and whirled about in the manner so familiar with smoke.

3. *Connexion between the leading features of fluid motion.*—These leading features of fluid motion are well known and are supposed to be more or less connected, but it does not appear that hitherto any very determined efforts have been made to trace a definite connexion between them, or to trace the characteristics of the circumstances under which they are generally presented. Certain circumstances have been definitely associated with the particular laws of force. Resistance, as the square of the velocity, is associated with motion in tubes of more than capillary dimensions, and with the motion of bodies through the water at more than insensibly small velocities, while resistance as the velocity is associated with capillary tubes and small velocities.

The equations of hydrodynamics, although they are applicable to *direct motion, i.e.*, without eddies, and show that then the resistance is as the velocity, have hitherto thrown no light on the circumstances on which such motion depends. And although of late years these equations have been applied to the theory of the eddy, they have not been in the least applied to the motion of water which is a mass of eddies, *i.e.*, in *sinuous motion*, nor have they yielded a clue to the cause of resistance varying as the square of the velocity. Thus, while as applied to waves and the motion of water in capillary tubes the theoretical results agree with the experimental, the theory of hydrodynamics has so far failed to afford the slightest hint why it should explain these phenomena, and signally fail to explain the law of resistance encountered by large bodies moving at sensibly high velocities through water, or that of water in sensibly large pipes.

This accidental fitness of the theory to explain certain phenomena while entirely failing to explain others, affords strong presumption that there are some fundamental principles of fluid motion of which due account has not been taken in the theory. And several years ago it seemed to me that a careful examination as to the connexion between these four leading features, together with the circumstances on which they severally depend, was the most likely means of finding the clue to the principles overlooked.

4. *Space and velocity.*—The definite association of resistance as the square of the velocity with sensibly large tubes and high velocities, and of resistance as the velocity with capillary tubes and slow velocities seemed to be evidence of the very general and important influence of some properties of fluids not recognised in the theory of hydrodynamics.

As there is no such thing as absolute space or absolute time recognised in mechanical philosophy, to suppose that the character of motion of fluids in any way depended on absolute size or absolute velocity, would be to suppose such motion without the pale of the laws of motion. If then fluids in their motions are subject to these laws, what appears to be the dependance of the character of the motion on the absolute size of the tube and on the absolute velocity of the immersed body, must in reality be a dependance on the size of the tube as compared with the size of some other object, and on the velocity of the body as compared with some other velocity. What is the standard object and what the standard velocity which come into comparison with the size of the tube and the velocity of an immersed body, are questions to which the answers were not obvious. Answers, however, were found in the discovery of a circumstance on which sinuous motion depends.

5. *The effect of viscosity on the character of fluid motion.*—The small evidence which clear water shows as to the existences of internal eddies, not less than the difficulty of estimating the viscous nature of the fluid, appears to have hitherto obscured the very important circumstance that *the more viscous a fluid is, the less prone is it to eddying or sinuous motion.* To express this definitely—if μ is the viscosity and ρ the density of the fluid—for water $\frac{\mu}{\rho}$ diminishes rapidly as the temperature rises, thus at 5° C. $\frac{\mu}{\rho}$ is double what it is at 45° C. What I observed was that the tendency of water to eddy becomes much greater as the temperature rises.

Hence connecting the change in the law of resistance with the birth and development of eddies, this discovery limited further search for the standard distance and standard velocity to the physical properties of the fluid. To follow the line of this search would be to enter upon a molecular theory of liquids, and this is beyond my present purpose. It is sufficient here to notice the well known fact that

$$\frac{\mu}{\rho} \text{ or } \mu'$$

is a quantity of the nature of the product of a distance and a velocity.

It is always difficult to trace the dependance of one idea on another. But it may be noticed that no idea of dimensional properties as indicated by the dependance of the character of motion on the size of the tube and the velocity of the fluid, occurred to me until after the completion of my investigation on the transpiration of gases, in which was established the dependance of the law of transpiration on the relation between the size of the channel and the *mean range* of the gaseous molecules.

6. *Evidence of dimensional properties in the equations of motion.*—The equations of motion had been subjected to such close scrutiny, particularly by Professor STOKES, that there was small chance of discovering anything new or faulty in them. It seemed to me possible, however, that they might contain evidence which had been overlooked, of the dependance of the character of motion on a relation between the dimensional properties and the external circumstances of motion. Such evidence, not only of a connexion but of a definite connexion, was found, and this without integration.

If the motion be supposed to depend on a single velocity parameter U, say the mean velocity along a tube, and on a single linear parameter c, say the radius of the tube; then having in the usual manner eliminated the pressure from the equations, the accelerations are expressed in terms of two distinct types. In one of which

$$\frac{\mathrm{U}^2}{c^3}$$

is a factor, and in the other

$$\frac{\mu\mathrm{U}}{\rho c^4}$$

is a factor. So that the relative values of these terms vary respectively as U and

$$\frac{\mu}{c\rho}.$$

This is a definite relation of the exact kind for which I was in search. Of course without integration the equations only gave the relation without showing at all in what way the motion might depend upon it.

It seemed, however, to be certain if the eddies were owing to one particular cause, that integration would show the birth of eddies to depend on some definite value of

$$\frac{c\rho\mathrm{U}}{\mu}$$

7. *The cause of eddies.*—There appeared to be two possible causes for the change of direct motion into sinuous. These are best discussed in the language of hydrodynamics, but as the results of this investigation relate to both these causes, which, although the distinction is subtle, are fundamentally distinct and lead to distinct results, it is necessary that they should be indicated.

The general cause of the change from steady to eddying motion was in 1843 pointed out by Professor STOKES as being that under certain circumstances the steady motion becomes unstable, so that an indefinitely small disturbance may lead to a change to sinuous motion. Both the causes above referred to are of this kind, and yet they are distinct, the distinction lying in the part taken in the instability by viscosity.

If we imagine a fluid free from viscosity and absolutely free to glide over solid surfaces, then comparing such a fluid with a viscous fluid in exactly the same motion—

(1.) The frictionless fluid might be instable and the viscous fluid stable. Under these circumstances the cause of eddies is the instability as a perfect fluid, the effect of viscosity being in the direction of stability.

(2.) The frictionless fluid might be stable and the viscous fluid unstable, under which circumstances the cause of instability would be the viscosity.

It was clear to me that the conclusions I had drawn from the equations of motion immediately related only to the first cause; nor could I then perceive any possible way in which instability could result from viscosity. All the same I felt a certain amount of uncertainty in assuming the first cause of instability to be general. This uncertainty was the result of various considerations, but particularly from my having observed that eddies apparently come on in very different ways, according to a very definite circumstance of motion, which may be illustrated.

When in a channel the water is all moving in the same direction, the velocity being greatest in the middle and diminishing to zero at the sides, as indicated by the curve in fig. 1, eddies showed themselves reluctantly and irregularly; whereas when the water on one side of the channel was moving in the opposite direction to that on the other, as shown by the curve in fig. 2, eddies appeared in the middle regularly and readily.

Fig. 1.

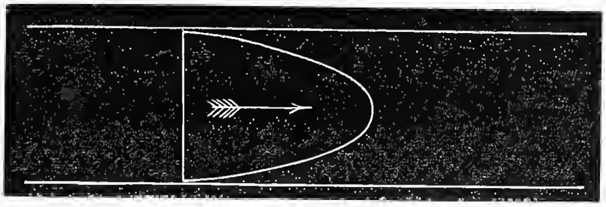
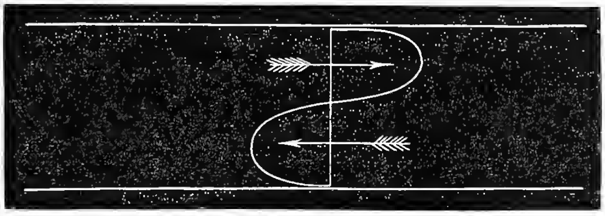


Fig. 2.



8. *Methods of investigation.*—There appeared to be two ways of proceeding—the one theoretical, the other practical.

The theoretical method involved the integration of the equations for unsteady motion in a way that had not been accomplished and which, considering the general intractability of the equations, was not promising.

The practical method was to test the relation between U, $\frac{\mu}{\rho}$, and c; this, owing to the simple and definite form of the law, seemed to offer, at all events in the first place, a far more promising field of research.

The law of motion in a straight smooth tube offered the simplest possible circumstances and the most crucial test.

The existing experimental knowledge of the resistance of water in tubes, although very extensive, was in one important respect incomplete. The previous experiments might be divided into two classes: (1) those made under circumstances in which the law of resistance was as the square of the velocity, and (2) those made under circumstances in which the resistance varied as the velocity. There had not apparently been any attempt made to determine the exact circumstances under which the change of law took place.

Again, although it had been definitely pointed out that eddies would explain resistance as the square of the velocity, it did not appear that any definite experimental evidence of the existence of eddies in parallel tubes had been obtained, and much less was there any evidence as to whether the birth of eddies was simultaneous with the change in the law of resistance.

These open points may be best expressed in the form of queries to which the answers anticipated were in the affirmative.

(1.) What was the exact relation between the diameters of the pipes and the velocities of the water at which the law of resistance changed?

Was it at a certain value of

$$c\mathrm{U}\,?$$

(2.) Did this change depend on the temperature, *i.e.*, the viscosity of water? Was it at a certain value of

$$\rho\,\frac{\mathrm{U}}{\mu}\,?$$

(3.) Were there eddies in parallel tubes?

(4.) Did steady motion hold up to a critical value and then eddies come in?

(5.) Did the eddies come in at a certain value of

$$\frac{\rho c\mathrm{U}}{\mu}\,?$$

(6.) Did the eddies first make their appearance as small and then increase gradually with the velocity, or did they come in suddenly?

The bearing of the last query may not be obvious; but, as will appear in the sequel, its importance was such that, in spite of satisfactory answers to all the other queries, a negative answer to this, in respect of one particular class of motions, led me to the reconsideration of the supposed cause of instability.

The queries, as they are put, suggest two methods of experimenting :—

(1.) Measuring the resistances and velocities of different diameters, and with different temperatures of water.

(2.) Visual observation as to the appearance of eddies during the flow of water along tubes or open channels.

Both these methods have been adopted, but, as the questions relating to eddies had been the least studied, the second method was the first adopted.

9. *Experiments by visual observation.*—The most important of these experiments related to water moving in one direction along glass tubes. Besides this, however, experiments on fluids flowing in opposite directions in the same tube were made, also a third class of experiments, which related to motion in a flat channel of indefinite breadth.

These last-mentioned experiments resulted from an incidental observation during some experiments made in 1876 as to the effect of oil to prevent wind waves. As the result of this observation had no small influence in directing the course of this investigation, it may be well to describe it first.

10. *Eddies caused by the wind beneath the oiled surface of water.*—A few drops of oil on the windward side of a pond during a stiff breeze, having spread over the pond and completely calmed the surface as regards waves, the sheet of oil, if it may be so called, was observed to drift before the wind, and it was then particularly noticed that while close to, and for a considerable distance from the windward edge, the surface presented the appearance of *plate glass;* further from the edge the surface presented that irregular wavering appearance which has already been likened to that of sheet glass, which appearance was at the time noted as showing the existence of eddies beneath the surface.

Subsequent observation confirmed this first view. At a sufficient distance from the windward edge of an oil-calmed surface there are always eddies beneath the surface even when the wind is light. But the distance from the edge increases rapidly as the force of the wind diminishes, so that at a limited distance (10 or 20 feet) the eddies will come and go with the wind.

Without oil I was unable to perceive any indication of eddies. At first I thought that the waves might prevent their appearance even if they were there, but by careful observation I convinced myself that they were not there. It is not necessary to discuss these results here, although, as will appear, they have a very important bearing on the cause of instability.

11. *Experiments by means of colour bands in glass tubes.*—These were undertaken early in 1880 ; the final experiments were made on three tubes, Nos. 1, 2, and 3. The diameters of these were nearly 1 inch, $\frac{1}{2}$ inch, and $\frac{1}{4}$ inch. They were all about 4 feet 6 inches long, and fitted with trumpet mouthpieces, so that water might enter without disturbance.

The water was drawn through the tubes out of a large glass tank, in which the tubes were immersed, arrangements being made so that a streak or streaks of highly coloured water entered the tubes with the clear water.

The general results were as follows :—

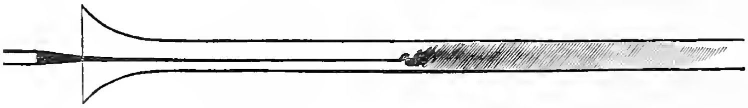
(1.) When the velocities were sufficiently low, the streak of colour extended in a beautiful straight line through the tube, fig. 3.

Fig. 3.

(2.) If the water in the tank had not quite settled to rest, at sufficiently low velocities, the streak would shift about the tube, but there was no appearance of sinuosity.

(3.) As the velocity was increased by small stages, at some point in the tube, always at a considerable distance from the trumpet or intake, the colour band would all at once mix up with the surrounding water, and fill the rest of the tube with a mass of coloured water, as in fig. 4.

Fig. 4.



Any increase in the velocity caused the point of break down to approach the trumpet, but with no velocities that were tried did it reach this.

On viewing the tube by the light of an electric spark, the mass of colour resolved itself into a mass of more or less distinct curls, showing eddies, as in fig. 5.

Fig. 5.

The experiments thus seemed to settle questions 3 and 4 in the affirmative, the existence of eddies and a critical velocity.

They also settled in the negative question 6, as to the eddies coming in gradually after the critical velocity was reached.

In order to obtain an answer to question 5, as to the law of the critical velocity, the diameters of the tubes were carefully measured, also the temperature of the water, and the rate of discharge.

(4.) It was then found that, with water at a constant temperature, and the tank as still as could by any means be brought about, the critical velocities at which the eddies showed themselves were almost exactly in the inverse ratio of the diameters of the tubes.

(5.) That in all the tubes the critical velocity diminished as the temperature increased, the range being from 5° C. to 22° C.; and the law of this diminution, so far as could be determined, was in accordance with Poiseuille's experiments. Taking T to express degrees centigrade, then by Poiseuille's experiments,

$$\frac{\mu}{\rho} \propto \mathrm{P} = (1 + 0\cdot0336\,\mathrm{T} + 0\cdot00221\,\mathrm{T}^2)^{-1}$$

taking a metre as the unit, U_s the critical velocity, and D the diameter of the tube, the law of the critical point is completely expressed by the formula

$$\mathrm{U}_s = \frac{1}{\mathrm{B}_s}\,\frac{\mathrm{P}}{\mathrm{D}}$$

where

$$\mathrm{B}_s = 43\cdot79$$
$$\log \mathrm{B}_s = 1\cdot64139$$

This is a complete answer to question 5.

During the experiments many things were noticed which cannot be mentioned here, but two circumstances should be mentioned as emphasizing the negative answer to question 6. In the first place, the critical velocity was much higher than had been expected in pipes of such magnitude, resistance varying as the square of the velocity had been found at very much smaller velocities than those at which the eddies appeared when the water in the tank was steady; and in the second place, it was observed that the critical velocity was very sensitive to disturbance in the water before entering the tubes; and it was only by the greatest care as to the uniformity of the temperature of the tank and the stillness of the water that consistent results were obtained. This showed that the steady motion was unstable for large disturbances long before the critical velocity was reached, a fact which agreed with the full-blown manner in which the eddies appeared.

12. *Experiments with two streams in opposite directions in the same tube.*—A glass tube, 5 feet long and 1·2 inch in diameter, having its ends slightly bent up, as shown in fig. 6, was half filled with bisulphide of carbon, and then filled up with water and both ends corked. The bisulphide was chosen as being a limpid liquid but little heavier than water and completely insoluble, the surface between the two liquids being clearly distinguishable. When the tube was placed in a horizontal direction, the weight of

Fig. 6.

the bisulphide caused it to spread along the lower half of the tube, and the surface of separation of the two liquids extended along the axis of the tube. On one end of the tube being slightly raised the water would flow to the upper end and the bisulphide fall to the lower, causing opposite currents along the upper and lower halves of the tube, while in the middle of the tube the level of the surface of separation remained unaltered.

The particular purpose of this investigation was to ascertain whether there was a critical velocity at which waves or sinuosities would show themselves in the surface of separation.

It proved a very pretty experiment and completely answered its purpose.

When one end was raised quickly by a definite amount, the opposite velocities of the two liquids, which were greatest in the middle of the tube, attained a certain maximum value, depending on the inclination given to the tube. When this was small no signs of eddies or sinuosities showed themselves; but, at a certain definite inclination, waves (nearly stationary) showed themselves, presenting all the appearance of wind waves. These waves first made their appearance as very small waves of equal lengths, the length being comparable to the diameter of the tube.

Fig. 7.

When by increasing the rise the velocities of flow were increased, the waves kept the same length but became higher, and when the rise was sufficient the waves would curl and break, the one fluid winding itself into the other in regular eddies.

Whatever might be the cause, a skin formed slowly between the bisulphide and the water, and this skin produced similiar effects to that of oil on water; the results mentioned are those which were obtained before the skin showed itself. When the skin first came on regular waves ceased to form, and in their place the surface was disturbed, as if by irregular eddies, above and below, just as in the case of the oiled surface of water.

The experiment was not adapted to afford a definite measure of the velocities at which the various phenomena occurred; but it was obvious that the critical velocity at which the waves first appeared was many times smaller than the critical velocity in a tube of the same size when the motion was in one direction only. It was also clear that the critical velocity was nearly, if not quite, independent of any existing disturbance in the liquids; so that this experiment shows—

(1.) That there is a critical velocity in the case of opposite flow at which direct motion becomes unstable.

(2.) That the instability came on gradually and did not depend on the magnitude of the disturbances, or in other words, that for this class of motion question 6 must be answered in the affirmative.

It thus appeared that there was some difference in the cause of instability in the two motions.

13. *Further study of the equations of motion.*—Having now definite data to guide me, I was anxious to obtain a fuller explanation of these results from the equations of motion. I still saw only one way open to account for the instability, namely, by assuming the instability of a frictionless fluid to be general.

Having found a method of integrating the equations for frictionless fluid as far as to show whether any particular form of steady motion is stable for a small disturbance, I applied this method to the case of parallel flow in a *frictionless* fluid. The result, which I obtained at once, was that flow in one direction was stable, flow in opposite directions unstable. This was not what I was looking for, and I spent much time in trying to find a way out of it, but whatever objections my method of integration may be open to, I could make nothing less of it.

It was not until the end of 1882 that I abandoned further attempts with a frictionless fluid, and attempted by the same method the integration of a viscous fluid. This change was in consequence of a discovery that in previously considering the effect of viscoscity I had omitted to take fully into account the boundary conditions which resulted from the friction between the fluid and the solid boundary.

On taking these boundary conditions into account, it appeared that although the tendency of viscoscity through the fluid is to render direct or steady motion stable, yet owing to the boundary condition resulting from the friction at the solid surface, the motion of the fluid, irrespective of viscoscity, would be unstable. Of course this cannot be rendered intelligible without going into the mathematics. But what I want to point out is that this instability, as shown by the integration of the equations of motion, depends on exactly the same relation

$$\mathrm{U} \propto \frac{\mu\rho}{}$$

as that previously found.

This explained all the practical anomalies and particularly the absence of eddies below a pure surface of water exposed to the wind. For in this case the surface being free, the boundary condition was absent, whereas the film of oil, by its tangential stiffness, introduced this condition; this circumstance alone seemed a sufficient verification of the theoretical conclusion.

But there was also the sudden way in which eddies came into existence in the experiments with the colour band, and the effect of disturbances to lower the critical velocity. These were also explained, for as long as the motion was steady, the instability depended upon the boundary action alone, but once eddies were introduced, the stability would be broken down.

It thus appeared that the meaning of the experimental results had been ascertained, and the relation between the four leading features and the circumstances on which they depend traced for the case of water in parallel flow.

But as it appeared that the critical velocity in the case of motion in one direction did not depend on the cause of instability with a view to which it was investigated, it followed that there must be another critical velocity, which would be the velocity at which previously existing eddies would die out, and the motion become steady as the water proceeded along the tube. This conclusion has been verified.

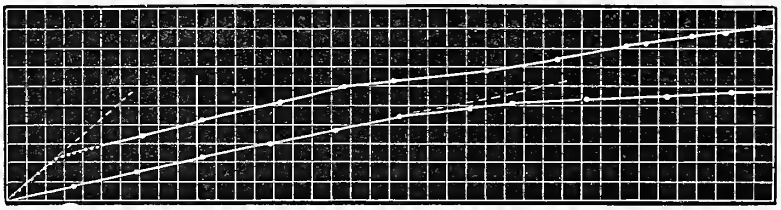
14. *Results of experiments on the law of resistance in tubes.*—The existence of the critical velocity described in the previous article could only be tested by allowing water in a high state of disturbance to enter a tube, and after flowing a sufficient distance for the eddies to die out, if they were going to die out, to test the motion.

As it seemed impossible to apply the method of colour bands, the test applied was that of the law of resistance as indicated in questions (1) and (2) in § 8. The result was very happy.

Two straight lead pipes No. 4 and No. 5, each 16 feet long and having diameters of a quarter and a half inch respectively were used. The water was allowed to flow through rather more than 10 feet before coming to the first gauge hole, the second gauge hole being 5 feet further along the pipe.

The results were very definite, and are partly shown in fig. 8, and more fully in diagram 1, Plate 74.

Fig. 8.



(1.) At the lower velocities the pressure was proportional to the velocity, and the velocities at which a deviation from the law first occurred were in exact inverse ratio of the diameters of the pipes.

(2.) Up to these critical velocities the discharge from the pipes agreed exactly with those given by POISEUILLE's formula for capillary tubes.

(3.) For some little distance after passing the critical velocity, no very simple relations appeared to hold between the pressures and velocities. But by the time the velocity reached 1·2 (critical velocity) the relation became again simple. The pressure did not vary as the square of the velocity, but as 1·722 power of the velocity, this law held in both tubes and through velocities ranging from 1 to 20, where it showed no signs of breaking down.

(4.) The most striking result was that not only at the critical velocity, but throughout the entire motion, the laws of resistance exactly corresponded for velocities in the ratio of

$$\frac{\mu}{\rho c}$$

This last result was brought out in the most striking manner on reducing the results by the graphic method of logarithmic homologues as described in my paper on Thermal Transpiration. Calling the resistance per unit of length as measured in the weight of cubic units of water i, and the velocity v, $\log i$ is taken for abscissa, and $\log v$ for ordinate, and the curve plotted.

In this way the experimental results for each tube are represented as a curve; these curves, which are shown as far as the small scale will admit in fig. 9, present exactly the same shape, and only differ in position.

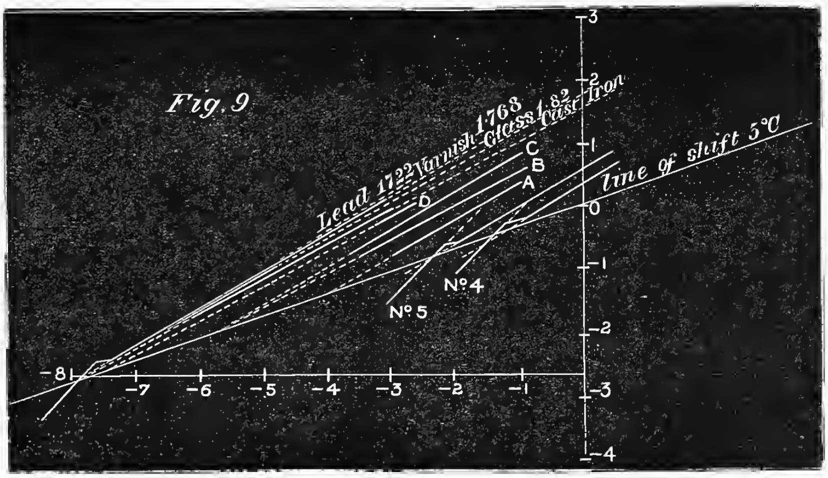


Fig. 9

Pipe.	Diameter. m.
No. 4, Lead	0·00615
,, 5, ,, ...	0·0127
A, Glass	0·0496
B, Cast iron	0·188
D, ,, ...	0·5
C, Varnish	0·196

Either of the curves may be brought into exact coincidence with the other by a rectangular shift, and the horizontal shifts are given by the difference of the logarithm of

$$\frac{D^3}{\mu^2}$$

for the two tubes, the vertical shifts being the difference of the logarithms of

$$\frac{D}{\mu}$$

The temperatures at which the experiment had been made were nearly the same, but not quite, so that the effect of the variations of μ showed themselves.

15. *Comparison with* DARCY'S *experiments.*—The definiteness of these results, their agreement with POISEUILLE'S law, and the new form which they more than indicated for the law of resistance above the critical velocities, led me to compare them with

the well known experiments of DARCY on pipes ranging from 0·014 to 0·5 metre in diameter.

Taking no notice of the empirical laws by which DARCY had endeavoured to represent his results, I had the logarithmic homologues drawn from his published experiments. If my law was general then these logarithmic curves, together with mine, should all shift into coincidence, if each were shifted horizontally through

$$\frac{D^3}{P^2}$$

and vertically through

$$\frac{D}{P}$$

In calculating these shifts there were some doubtful points. DARCY's pipes were not uniform between the gauge points, the sections varying as much as 20 per cent., and the temperature was only casually given. These matters rendered a close agreement unlikely. It was rather a question of seeing if there was any systematic disagreement. When the curves came to be shifted the agreement was remarkable. In only one respect was there any systematic disagreement, and this only raised another point; it was only in the slopes of the higher portions of the curves. In both my tubes the slopes were as 1·722 to 1; in DARCY's they varied according to the nature of the material, from the lead pipes, which were the same as mine, to 1·92 to 1 with the cast iron.

This seems to show that the nature of the surface of the pipe has an effect on the law of resistance above the critical velocity.

16. *The critical velocities.*—All the experiments agreed in giving

$$v_c = \frac{1}{278} \frac{P}{D}$$

as the critical velocity, to which corresponds as the critical slope of pressure

$$i_c = \frac{1}{47700000} \frac{P^2}{D^3}$$

the units being metres and degrees centigrade. It will be observed that this value is much less than the critical velocity at which steady motion broke down; the ratio being 43·7 to 278.

17. *The general law of resistance.*—The logarithmic homologues all consist of two straight branches, the lower branch inclined at 45 degrees and the upper one at n horizontal to 1 vertical. Except for the small distance beyond the critical velocity these branches constitute the curves. These two branches meet in a point on the curve at a definite distance below the critical pressure, so that, ignoring the small portion of the curve above the point before it again coincides with the upper branch, the logarithmic homologue gives for the law of resistance for all pipes and all velocities

$$A \frac{D^3}{\theta^2} i = (B \frac{D}{\theta} v)^n$$

where n has the value unity as long as either number is below unity, and then takes the value of the slope n to 1 for the particular surface of the pipe.

If the units are metres and degrees centigrade

$$A=67{,}700{,}000$$
$$B=396$$
$$P=(1+0·0336\ T+0·000221\ T^2)^{-1}$$

This equation then, excluding the region immediately about the critical velocity, gives the law of resistance in POISEUILLE's tubes, those of the present investigation and DARCY's, the range of diameters being

from 0·000013 (POISEUILLE, 1845)
to 0·5 (DARCY, 1857)

and the range of velocities

$$\left.\begin{array}{l}\text{from } 0·0026\\ \text{to } 7·\end{array}\right\} \text{metres per sec., 1883.}$$

This algebraical formula shows that the experiments entirely accord with the theoretical conclusions.

The empirical constants are A, B, P, and n; the first three relate solely to the dimensional properties of the fluid summed up in the viscosity, and it seems probable that the last relates to the properties of the surface of the pipe.

Much of the success of the experiments is due to the care and skill of Mr. FOSTER, of Owens College, who has constructed the apparatus and assisted me in making the experiments.

SECTION II.

Experiments in glass tubes by means of colour bands.

18. In commencing these experiments it was impossible to form any very definite idea of the velocity at which eddies might make their appearance with a particular tube. The experiments of POISEUILLE showed that the law of resistance varying as the velocity broke down in a pipe of say 0·6 millim. diameter; and the experiments of DARCY showed this law did not hold in a half-inch pipe with a velocity of 6 inches per second.

These considerations, together with the comparative ease with which experiments on a small scale can be made, led me to commence with the smallest tube in which I could hope to perceive what was going on with the naked eye, expecting confidently that eddies would make their appearance at an easily attained velocity.

19. *The first apparatus.*—This consisted of a tube about $\frac{1}{4}$ inch or 6 millims. in diameter. This was bent into the siphon form having one straight limb about 2 feet long and the other about 5 feet (Plate 72, fig. 10).

The end of the shorter limb was expanded to a bell mouth, while the longer end was provided with an indiarubber extension on which was a screw clip.

The bell-mouthed limb was held vertically in the middle of a beaker with the mouth several inches from the bottom as shown in figs. 10 and 10′.

A colour tube about 6 millims. in diameter also of siphon form was placed as shown in the figure, with the open end of the shorter limb just under the bell mouth, the longer limb communicating through a controlling clip with a reservoir of highly coloured water placed at a sufficient height. A supply-pipe was led into the beaker for the purpose of filling it; but not with the idea of maintaining it full, as it seemed probable that the inflowing water would create too much disturbance, experience having shown how important perfect internal rest is to successful experiments with coloured water.

20. *The first experiment.*—The vessels and the siphons having been filled and allowed to stand for some hours so as to allow all internal notion to cease, the colour clip was opened so as to allow the colour to emerge slowly below the bell (Plate 73, fig. 11).

Then the clip on the running pipe was opened very gradually. The water was drawn in at the bell mouth, and the coloured water entered, at first taking the form of a candle flame (Plate 73, fig. 12), which continually elongated until it became a very fine streak, contracting immediately on leaving the colour-tube and extending all along the tube from the bell mouth to the outlet (fig. 10). On further opening the regulating clip so as to increase the velocity of flow, the supply of colour remaining unaltered, the only effect was to diminish the thickness of the colour band. This was again increased by increasing the supply of colour, and so on until the velocity was the greatest that circumstances would allow—until the clip was fully open. Still the colour band was perfectly clear and definite throughout the tube. It was apparent that if there were to be eddies it must be at a higher velocity. To obtain this about 2 feet more were added to the longer leg of the siphon which brought it down to the floor.

On trying the experiment with this addition the colour band was still clear and undisturbed.

So that for want of power to obtain greater velocity this experiment failed to show eddies.

When the supply pipe which filled the beaker was kept running during the experiment, it kept the water in the beaker in a certain state of disturbance. The effect of this disturbance was to disturb the colour band in the tube, but it was extremely difficult to say whether this was due to the wavering of the colour band or to genuine eddies.

21. *The final apparatus.*—This was on a much larger scale than the first. A straight tube, nearly 5 feet long and about an inch in diameter, was selected from a large number as being the most nearly uniform, the variation of the diameter being less than 1-32nd of an inch.

The ends of this tube were ground off plane, and on the end which appeared slightly the larger was fitted a trumpet mouth of varnished wood, great care being taken to make the surface of the wood continuous with that of the glass (Plate 73, fig. 13).

The other end of the glass pipe was connected by means of an indiarubber washer with an iron pipe nearly 2 inches in diameter.

The iron pipe passed horizontally through the end of a tank, 6 feet long, 18 inches broad and 18 inches deep, and then bent through a quadrant so that it became vertical, and reached 7 feet below the glass tube. It then terminated in a large cock, having, when open, a clear way of nearly a square inch.

This cock was controlled by a long lever (see Plate 73) reaching up to the level of the tank. The tank was raised upon tressels about 7 feet above the floor, and on each side of it, at 4 feet from the ground, was a platform for the observers. The glass tube thus extended in an horizontal direction along the middle of the tank, and the trumpet mouth was something less than a foot from the end. Through this end, just opposite the trumpet, was a straight colour tube three-quarters of an inch in diameter, and this tube was connected, by means of an indiarubber tube with a clip upon it, with a reservoir of colour, which for good reasons subsequently took the form of a common water bottle.

With a view to determining the velocity of flow, an instrument was fitted for showing the changes of level of the water in the tank to the 100th of an inch (Plate 72, fig. 14). Thermometers were hung at various levels in the tank.

22. *The final experiments.*—The first experiment with this apparatus was made on 22nd February, 1880.

By means of a hose the tank was filled from the water main, and having been allowed to stand for several hours, from 10 A.M. to 2 P.M., it was then found that the water had a temperature of 46° F. at the bottom of the tank, and 47° F. at the top. The experiment was then commenced in the same manner as in the first trials. The colour was allowed to flow very slowly, and the cock slightly opened. The colour band established itself much as before, and remained beautifully steady as the velocity was increased until, all at once, on a slight further opening of the valve, at a point about two feet from the iron pipe, the colour band appeared to expand and mix with the water so as to fill the remainder of the pipe with a coloured cloud, of what appeared at first sight to be of a uniform tint (fig. 4, p. 942).

Closer inspection, however, showed the nature of this cloud. By moving the eye so as to follow the motion of the water, the expansion of the colour band resolved itself into a well-defined waving motion of the band, at first without other disturbance, but after two or three waves came a succession of well-defined and distinct eddies. These were sufficiently recognisable by following them with the eye, but more distinctly seen by a flash from a spark, when they appeared as in fig. 5, p. 942.

The first time these were seen the velocity of the water was such that the tank fell 1 inch in 1 minute, which gave a velocity of $0^{\mathrm{m}}\cdot627$, or 2 feet per second. On slightly closing the valve the eddies disappeared, and the straight colour band established itself.

Having thus proved the existence of eddies, and that they came into existence at a certain definite velocity, attention was directed to the relations between this critical velocity, the size of the tube, and the viscosity.

Two more tubes (2 and 3) were prepared similar in length and mounting to the first, but having diameters of about one-half and one-quarter inch respectively.

In the meantime an attempt was made to ascertain the effect of viscosity by using water at different temperatures. The temperature of the water from the main was about 45°, the temperature of the room about 54°; to obtain a still higher temperature, the tank was heated to 70° by a jet of steam. Then taking, as nearly as we could tell, similar disturbances, the experiments which are numbered 1 and 2 in Table I. were made.

To compare these for the viscosity, POISEUILLE's experiments were available, but to prevent any accidental peculiarity of the water being overlooked, experiments after the same manner as POISEUILLE's were made with the water in the tank. The results of these however agreed so exactly with those of POISEUILLE that the comparative effect of viscosity was taken from POISEUILLE's formula

$$P^{-1}=1+0\cdot03368\,T+0\cdot000221\,T^2$$

where $P \propto \mu$ with the temperature and T is temperature centigrade.

The relative values of P at 47° and 70° Fah. are as

1·3936 to 1

while the relative critical velocities at these temperatures were as

1·45 to 1

which agreement is very close considering the nature of the experiments.

But whatever might have been the cause of the previous anomalies, these were greatly augmented in the heated tank. After being heated the tank had been allowed to stand for an hour or two, in order to become steady. On opening the valve it was thought that the eddies presented a different appearance from those in the colder water, and the thought at once suggested itself that this was due to some source of initial disturbance. Several sources of such disturbance suggested themselves—the temperature of the tank was 11° C. above that of the room, and the cooling arising from the top and sides of the tank must cause circulation in the tank. A few streaks of colour added to the water soon showed that such a circulation existed, although it was very slow. Another source of possible disturbance was the difference in the temperature at the top and bottom of the tank, this had been as much as 5°.

In order to get rid of these sources of disturbance it was necessary to have the tank at the same temperature as the room, about 54° or 55°. Then it was found by several trials that the eddies came on at a fall of about 1 inch in 64 seconds, which, taking the viscosity into account, was higher than in the previous case, and this was taken to indicate that there was less disturbance in the water.

As it was difficult to alter the temperatures of the building so as to obtain experiments under like conditions at a higher temperature, and it appeared that the same object would be accomplished by cooling the water to its maximum density, 40°, this plan was adopted and answered well, ice being used to cool the water.

Experiments were then made with three tubes 1, 2, 3, at temperatures of about 51° and 40°. All are given in Table I.

TABLE I.

Experiments with Colour Bands—Critical Velocities at which Steady Motion breaks down.

Pipe No. 1, glass.—Diameter 0·0268 metre ; log diameter $\bar{2}$·42828.
,, No. 2, ,, ,, 0·01527 ,, ,, $\bar{2}$·18400.
,, No. 3, ,, ,, 0·007886 ,, ,, $\bar{3}$·89783.
Discharge, cub. metre=·021237 ; log =$\bar{2}$·32709.

Date, 1880.	Reference Number.	Pipe.	Temperature, centigrade.	Time of discharge.	Velocity, metres.	log time.	−log P.	log V.	log B_s.
1 March	1	No. 1.	8·3	60	0·6270	1·77815	0·11242	$\bar{1}$·79729	1·66200
3 ,,	2	,,	21	87	0·4325	1·93959	0·25654	$\bar{1}$·63593	1·67930
25 ,,	3	,,	15	70	0·5374	1·84500	0·19198	$\bar{1}$·73035	1·64936
21 April	4	,,	12	60	0·6270	1·77815	0·15712	$\bar{1}$·79729	1·61730
,,	5	,,	13	64	0·5878	1·80618	0·16882	$\bar{1}$·76926	1·64464
,,	6	,,	13	67	0·5614	1·82617	0·16882	$\bar{1}$·74927	1·65363
,,	7	,,	13	64	0·5878	1·80618	0·16882	$\bar{1}$·76926	1·64464
,,	8	,,	5	54	0·6967	1·73239	0·06963	$\bar{1}$·84305	1·65898
,,	9	,,	5	52	0·7235	1·71600	0·06963	$\bar{1}$·85940	1·64269
22 ,,	10	,,	10	62	0·6068	1·79239	0·13319	$\bar{1}$·78305	1·65546
,,	11	,,	11	64	0·5870	1·80613	0·14525	$\bar{1}$·76931	1·65716
25 March	12	No. 2.	22	155	0·7476	2·19033	0·26710	$\bar{1}$·87367	1·67523
23 April	13	,,	11	110	1·052	2·04139	0·14525	0·02261	1·64814
,,	14	,,	11	108	1·072	2·03342	0·14525	0·03058	1·64017
,,	15	,,	4	83	1·396	1·91907	0·05621	0·14493	1·61486
,,	16	,,	4	83	1·396	1·91907	0·05621	0·14493	1·61486
,,	17	,,	4	83	1·396	1·91907	0·05621	0·14493	1·61486
,,	18	,,	6	86	1·348	1·93449	0·08278	0·12951	1·59371
,,	19	,,	6	85	1·362	1·92941	0·08278	0·13459	1·59863
24 ,,	20	No. 3.	11	220	1·967	2·34242	0·14525	0·29392	1·66300
,,	21	,,	10·5	224	1·932	2·35024	0·13920	0·28610	1·67687
,,	22	,,	11	218	1·982	2·33845	0·14525	0·29789	1·65903
,,	23	,,	11	116	2·004	2·33445	0·14525	0·30189	1·65503
25 ,,	24	,,	4	164	2·637	2·21484	0·05621	0·42150	1·62446
,,	25	,,	4	172	2·517	2·23552	0·05621	0·40082	1·64514
,,	26	,,	6	176	2·460	2·24551	0·08278	0·39083	1·62856
,,	27	,,	6	176	2·460	2·24551	0·08278	0·39083	1·62856
,,	28	,,	6	174	2·488	2·24054	0·08278	0·39580	1·62359
,,	29	,,	6	177	2·446	2·24791	0·08278	0·38837	1·63102

This gives the mean value for log B_s 1·64139 ; and B_s=43·79.

In reducing the results the unit taken has been the metre and the temperature is given in degrees centigrade.

The diameters of the three tubes were found by filling them with water.

The time measured was the time in which the tank fell 1 inch, which in cubic metres is given by

$$Q = \cdot 021237$$

In the table the logarithms of P, v, and B_s are given, as well as the natural numbers for the sake of reference.

The velocities v have been obtained by the formula

$$v = \frac{4\,Q}{\pi\,D^2}$$

B_s being obtained from the formula

$$B_s = \frac{P}{vD}$$

The final value of B_s is obtained from the mean value of the logarithm of B_s.

23. *The results.*—The values of log B_s show a considerable amount of regularity, and prove, I think conclusively, not only the existence of a critical velocity at which eddies come in, but that it is proportional to the viscosity and inversely proportional to the diameter of the tube.

The fact, however, that this relation has only been obtained by the utmost care to reduce the internal disturbances in the water to a minimum must not be lost sight of.

The fact that the steady motion breaks down suddenly shows that the fluid is in a state of instability for disturbances of the magnitude which cause it to break down. But the fact that in some conditions it will break down for a large disturbance, while it is stable for a smaller disturbance shows that there is a certain residual stability so long as the disturbances do not exceed a given amount.

The only idea that I had formed before commencing the experiments was that at some critical velocity the motion must become unstable, so that any disturbance from perfectly steady motion would result in eddies.

I had not been able to form any idea as to any particular form of disturbance being necessary. But experience having shown the impossibility of obtaining absolutely steady motion, I had not doubted but that appearance of eddies would be almost simultaneous with the condition of instability. I had not, therefore, considered the disturbances except to try and diminish them as much as possible. I had expected to see the eddies make their appearance as the velocity increased, at first in a slow or feeble manner, indicating that the water was but slightly unstable. And it was a matter of surprise to me to see the sudden force with which the eddies sprang into existence, showing a highly unstable condition to have existed at the time the steady motion broke down.

This at once suggested the idea that the condition might be one of instability for disturbance of a certain magnitude and stable for smaller disturbances.

In order to test this, an open coil of wire was placed in the tube so as to create a definite disturbance as in Plate 72, fig. 15.

Eddies now showed themselves at a velocity of less than half the previous critical velocity, and these eddies broke up the colour band, but it was difficult to say whether the motion was really unstable or whether the eddies were the result of the initial disturbance, for the colour band having once broken up and become mixed with the water, it was impossible to say whether the motion did not tend to become steady again later on in the tube.

Subsequent observation however tended to show that the critical value of the velocity depended to some extent on the initial steadiness of the water. One phenomenon in particular was very marked.

Where there was any considerable disturbance in the water of the tank and the cock was opened very gradually, the state of disturbance would first show itself by the wavering about of the colour band in the tube; sometimes it would be driven against the glass and would spread out, and all without a symptom of eddies. Then, as the velocity increased but was still comparatively small, eddies, and often very regular eddies, would show themselves along the latter part of the tube. On further opening the cock these eddies would disappear and the colour band would become fixed and steady right through the tube, which condition it would maintain until the velocity reached its normal critical value, and then the eddies would appear suddenly as before.

Another phenomenon very marked in the smaller tubes, was the intermittent character of the disturbance. The disturbance would suddenly come on through a certain length of the tube and pass away and then come on again, giving the appearance of flashes, and these flashes would often commence successively at one point in the pipe. The appearance when the flashes succeeded each other rapidly was as shown in Plate 72, fig. 16.

This condition of flashing was quite as marked when the water in the tank was very steady as when somewhat disturbed.

Under no circumstances would the disturbance occur nearer to the trumpet than about 30 diameters in any of the pipes, and the flashes generally, but not always, commenced at about this distance.

In the smaller tubes generally, and with the larger tube in the case of the ice-cold water at 40°, the first evidence of instability was an occasional flash beginning at the usual place and passing out as a disturbed patch two or three inches long. As the velocity was further increased these flashes became more frequent until the disturbance became general.

I did not see a way to any very crucial test as to whether the steady motion became unstable for a large disturbance before it did so for a small one; but the general impression left on my mind was that it did in some way—as though disturbances in the tank, or arising from irregularities in the tube, were necessary to the existence of a state of instability.

But whatever these peculiarities may mean as to the way in which eddies present themselves, the broad fact of there being a critical value for the velocity at which the steady motion becomes unstable, which critical value is proportional to

$$\frac{\mu}{\rho c}$$

where c is the diameter of the pipe and $\frac{\mu}{\rho}$ the viscosity by the density, is abundantly established. And cylindrical glass pipes for approximately steady water have for the critical value

$$v=\frac{\mathrm{P}}{\mathrm{B}_s \mathrm{D}}$$

where in metres $\mathrm{B}_s=43{\cdot}79$ about.

Section III.

Experiments to determine the critical velocity by means of resistance in the pipes.

24. Although at first sight such experiments may appear to be simple enough, yet when one began to consider actual ways and means, so many uncertainties and difficulties presented themselves that the necessary courage for undertaking them was only acquired after two years' further study of the hydrodynamical aspect of the subject by the light thrown upon it by the previous experiment with the colour bands. This has been already explained in Art. 13. Those experiments had shown definitely that there was a critical value of the velocity at which eddies began if the water were approximately steady when drawn into the tube, but they had also shown definitely that at such critical velocity the water in the tube was in a highly unstable condition, any considerable disturbance in the water causing the break down to occur at velocities much below the highest that could be attained when the water was at its steadiest; suggesting that if there were a critical velocity at which, for any disturbance whatever, the water became stable, this velocity was much less than that at which it would become unstable for infinitely small disturbances; or, in other words, suggesting that there were two critical values for the velocity in the tube, the one at which steady motion changed into eddies, the other at which eddies changed into steady motion.

Although the law for the critical value of the velocity had been suggested by the equations of motion, it was, as already explained, only at the beginning of this year that I succeeded in dealing with these equations so as to obtain any theoretical explanation of the dual criteria; but having at last found this, it became clear to me that if in a tube of sufficient length the water were at first admitted in a high state of disturbance, then as the water proceeded along the tube the disturbance would settle down into a steady condition, which condition would be one of eddies or steady motion, according to whether the velocity was above or below what may be called the real critical value.

The necessity of initial disturbance precluded the method of colour bands, so that the only method left was to measure the resistance at the latter portion of the tube in conjunction with the discharge.

The necessary condition was somewhat difficult to obtain. The change in the law of resistance could only be ascertained by a series of experiments which had to be carried out under similar conditions as regards temperature, kind of water, and condition of the pipe; and in order that the experiments might be satisfactory, it seemed necessary that the range of velocities should extend far on each side of the critical velocity. In order to best ensure these conditions, it was resolved to draw the water direct from the Manchester main, using the pressure in the main for forcing the water through the pipes. The experiments were conducted in the workshop in Owens College, which offered considerable facilities owing to arrangements for supplying and measuring the water used in experimental turbines.

25. *The apparatus.*—This is shown in Plate 72, fig. 17.

As the critical value under consideration would be considerably below that found for the change for steady motion into eddies, a diameter of about half an inch (12 millims.) was chosen for the larger pipe, and one quarter of an inch for the smaller, such pipes being the smallest used in the previous experiments.

The pipes (4 and 5) were ordinary lead gas or water pipes. These, which owing to their construction are very uniform in diameter and when new present a bright metal surface inside, seemed well adapted for the purpose.

Pipes 4 (which was a quarter-inch pipe) and 5 (which was a half-inch) were 16 feet long, straightened by laying them in a trough formed by two inch boards at right angles. This trough was then fixed so that one side of the trough was vertical and the other horizontal, forming a horizontal ledge on which the pipes could rest at a distance of 7 feet from the floor; on the outflow ends of the pipes cocks were fitted to control the discharge, and at the inlet end the pipes were connected, by means of a T branch, with an indiarubber hose from the main; this connexion was purposely made in such a manner as to necessitate considerable disturbance in the water entering the pipes from the hose. The hose was connected, by means of a quarter-inch cock, with a four-inch branch from the main. With this arrangement the pressure on the inlet to the pipes was under control of the cock from the main, and at the same time the discharge from the pipes was under control from the cocks on their ends.

This double control was necessary owing to the varying pressure in the main, and after a few preliminary experiments a third and more delicate control, together with a pressure gauge, were added, so as to enable the observer to keep the pressure in the hose, *i.e.*, on the inlets to the pipes, constant during the experiments.

This arrangement was accomplished by two short branches between the hose and the control cock from the main, one of these being furnished with an indiarubber mouthpiece with a screw clip upon it, so that part of the water which passed the cock might be allowed to run to waste, the other branch being connected with the lower end of a vertical glass tube, about 6 millims. in diameter and 30 inches long, having a bulb about 2 inches diameter near its lower extremity, and being closed by a similar bulb at its top.

This arrangement served as a delicate pressure gauge. The water entering at the lower end forced the air from the lower bulb into the upper, causing a pressure of about 30 inches of mercury. Any further rise increased this pressure by forcing the air in the tubes into the upper bulb, and by the weight of water in the tube. During an experiment the screw clip was continually adjusted, so as to keep the level of the water in the glass tube between the bulbs constant.

26. *The resistance gauges.*—Only the last 5 feet of the tube was used for measuring the resistance, the first 10 or 11 feet being allowed for the acquirement of a regular condition of flow.

It was a matter of guessing that 10 feet would be sufficient for this, but since, compared with the diameter, this length was double as great for the smaller tube, it was expected that any insufficiency would show itself in a greater irregularity of the results obtained with the larger tube, and as no such irregularity was noticed it appears to have been sufficient.

At distances of 5 feet near the ends of the pipe, two holes of about 1 millim. were pierced into each of the pipes for the purpose of gauging the pressures at these points of the pipes. As owing to the rapid motion of the water in the pipes past these holes, any burr or roughness caused in the inside of the pipe in piercing these holes would be apt to cause a disturbance in the pressure, it was very important that this should be avoided. This at first seemed difficult, as owing to the distance—5 feet—of one of the holes from the end of pipes of such small diameter the removal of a burr, which would be certain to ensue on drilling the holes from the outside, was difficult. This was overcome by the simple expedient suggested by Mr. Foster of drilling holes completely through the pipes and then plugging the side on which the drill entered. Trials were made, and it was found that the burr thus caused was very slight.

Before drilling the holes short tubes had been soldered to the pipes, so that the holes communicated with these tubes; these tubes were then connected with the limbs of a siphon gauge by indiarubber pipes.

These gauges were about 30 inches long; two were used, the one containing mercury, the other bisulphide of carbon.

These gauges were constructed by bending a piece of glass tube into a U form, so that the two limbs were parallel and at about one inch apart.

Glass tubes are seldom quite uniform in diameter, and there was a difference in the size of the limbs of both gauges, the difference being considerable in the case of the bisulphide of carbon.

The tubes were fixed to stands with carefully graduated scales behind them, so that the height of the mercury or carbon in each limb could be read. It had been anticipated that readings taken in this way would be sufficient. But it turned out to be desirable to read variations of level of the smallness of $\frac{1}{1000}$th of an inch or $\frac{1}{40}$th of a millimetre.

A species of cathetometer was used. This had been constructed for my experiments on Thermal Transpiration, and would read the position of the division surface of two fluids to $\frac{1}{10000}$th inch (Phil. Trans. 1879, p. 789).

The water was carefully brought into direct connexion with the fluid in the gauge, the indiarubber connexions facilitating the removal of all air.

27. *Means adopted in measuring the discharge.*—For many reasons it was very desirable to measure the rate of discharge in as short a time as possible.

For this purpose a species of orifice or weir gauge was constructed, consisting of a vertical tin cylinder two feet deep, having a flat bottom, being open at the top, with a diaphragm consisting of many thicknesses of fine wire gauze about two inches from the bottom; a tube connected the bottom with a vertical glass tube, the height of water in which showed the pressure of water on the bottom of the gauze; behind this tube was a scale divided so that the divisions were as the square roots of the height. Through the thin tin bottom were drilled six holes, one an eighth of an inch diameter, one a quarter of an inch, and four of half an inch.

These holes were closed by corks so that any one or any combination could be used. The combinations used were:

Gauge No. 1. The $\frac{1}{8}$ inch hole alone.
No. 2. The $\frac{1}{4}$ inch hole alone.
No. 3. A $\frac{1}{2}$ inch hole alone.
No. 4. Two $\frac{1}{2}$ inch holes.
No. 5. Four $\frac{1}{2}$ inch holes.

According to experience, the velocity with which water flows from a still vessel through a round hole in a thin horizontal plate is very nearly proportional to the area of the hole and the square root of the pressure, so that with any particular hole the relative quantities of water discharged would be read off at the variable height gauge. The accuracy of the gauge, as well as the absolute values of the readings, was checked by comparing the readings on the gauge with the time taken to fill vessels of known capacity. In this way coefficients for each one of the combinations 1, 2, 3, 4, 5 were obtained as follows:—

TABLE II.

No. of Gauge.	Readings on Gauge.	Time.	Quantity.	Coefficient.	Logarithmic coefficient.
		Seconds.	c.c.		
Gauge No. 1	19·55	61	1160	·966	$\bar{1}$·985
ib.	—	59	1160		
No. 2	5·3	54	1160	4·055	·608
ib.	15·3 full	—	A	4·055	—
No. 3	15	360	A	16·220	1·210
No. 4	15	178	A	32·440	1·511
No. 5	15	90	A	64·880	1·812

From this table it will be seen that the absolute values of the coefficients were obtained from experiments on the gauges No. 1 and No. 2, the coefficients for the gauges 3, 4, and 5 being determined by comparison of the times taken to fill a vessel of unknown capacity, which stands in the Table as A. The relative value of these coefficients came out sensibly proportional to the squares of the diameters of the apertures.

For the smaller velocities it was found that the gauge No. 1 was too large, and in order not to delay the experiment in progress, two glass flasks were used : these are distinguished as flasks (1) and (2) ; their capacities, as subsequently determined with care, were 303 and 1160 c.c. The discharge as measured by the times taken to fill these flasks are reduced to c.c. per second by dividing the capacities of the flasks by the times.

28. *The method of carrying out the experiments.*—This was generally as follows :— My assistant, Mr. FOSTER, had charge of the supply of water from the main, keeping the water in the pressure gauge at a fixed level.

The tap at the end of the tube to be experimented upon being closed, the zero reading of the gauge was carefully marked, and the micrometer adjusted so that the spider line was on the division of water and fluid in the left hand limb of the gauge. The screw was then turned through one entire revolution, which lowered the spider line one-fiftieth of an inch ; the tap at the end of the pipe was then adjusted until the fluid in the gauge came down to the spider line ; having found that it was steady there, the discharge was measured.

This having been done, the spider line was lowered by another complete revolution of the screw, the tap again adjusted, and so on, for about 20 readings, which meant about half an inch difference in the gauge. Then the readings were taken for every five turns of the screw until the limit of the range, about 2 inches, was reached. After this, readings were taken by simple observation of the scale attached to the gauge. In taking these readings the best plan was to read the position of the mercury or carbon in both limbs of the gauge, but this was not always done, some of the readings entered in the notes referred to one or other limb of the gauge, care having been taken to indicate which.

In the Tables III., IV., and V. of results appended, the noted readings are given and the letters *r*, *l*, and *b* signify whether the reading was on the right or left limb, or the sum of the readings on both limbs.

The readings marked *l* and *r* are reduced by the correction for the difference in the size of the limbs as well as the coefficient for the particular fluid in the gauge.

Thus it was found with the mercury tube that when the left limb had moved through 39 divisions on the scale the right had moved through 41, so that to obtain the sum of these readings the readings on the left, or those marked *l*, had to be multiplied by 2·05 and those on the right by 1·95.

With the bisulphide of carbon gauge, 11 divisions on the left caused 9 on the right, so that the correction for the reading on the left was 1·8 and on the right 2·2.

29. *Comparison of the pressure gauges.*—The pressures as marked by the gauges were reduced to the same standard by comparing the gauges; thus ·25 of the left limb of the mercury corresponded with 24 inches on both limbs of the bisulphide. Therefore to reduce the readings of the bisulphide of carbon to the same scale as those of the mercury they were multiplied by

$$\frac{\cdot 25 \times 20\cdot 5}{24} = 0\cdot 0213$$

This brought the readings of pressure to the same standard, *i.e.*, $\frac{1}{1000}$th of an inch of mercury, but these were further reduced by the factor 0·00032 to bring them to metres of water.

As it was convenient for the sake of comparison to obtain the differences of pressure per unit length of the pipe, the pressures in metres of water have been divided by 1·524, the length in metres between the gauge holes, and these reductions are included in the tables of results in the column headed *i*.

From the discharges as measured by the various gauges having been reduced to cubic centimetres, the mean velocity of the water was found by dividing by the area of the section of the pipe.

30. *Sections and diameters of the pipes.*—The areas were obtained by carefully measuring the diameters by means of fitting brass plugs into the pipes and then measuring the plugs. In this way the diameters were found to be—

Diameter, No. 4 pipe, ·242 inch, 6·15 millims.
,, No. 5 pipe, ·498 inch, 12·7 millims.

These gave the areas of the sections—

Section, No. 4 pipe, 29·7 square millims.
,, No. 5 pipe, 125 square millims.

The discharge in cubic centimetres divided by the area of section in square millimetres gave the mean velocity in metres per second as given in the Tables III., IV., and V.

The logarithms of i and v are given for the sake of comparison.

31. *The temperature.*—The chief reason why the water from the main had been used was from the necessity of having constant temperature throughout the experiments, and my previous experience of the great constancy of the temperature of the water in the mains, even over a period of some weeks.

At the commencement of the experiments the temperature of the water when flowing freely was found to be 5 C. or 41° F., and it remained the same throughout the experiments. Nevertheless, a fact which had been overlooked caused the temperature in the pipes to vary somewhat and in a manner somewhat difficult to determine.

This fact, which was not discovered until after the experiments had been reduced, was that the temperature of the workshop being above that of the main, the water would be warmed in flowing through the pipes to an extent depending on its flow. The possibility of this had not been altogether overlooked, and an early observation was made to see if any such warming occurred, but as it was found to be less than half a degree no further notice was taken until on reducing the results it was found that the velocities obtained with the very smallest discharges presented considerable discrepancies in various experiments; this suggested the cause.

The discrepancies were not serious if explained, so that all that was necessary was to carefully repeat the experiments at the lower velocities observing the temperatures of the effluent water. This was done, and further experiments were made (see Art. 33).

TABLE III.

Experiments on Resistance in Pipes made January 29, 1883.

Pipe No. 4, lead.—Diameter (as measured 0·242 inch), 6·15 millims. Length: total, 16 feet; to first gauge hole, 9·6 feet; between gauge holes (5 feet), 1·524 metres. Water from the Manchester Main.

Reference number.	Pressures.			Discharges.									Temperature.		Slope of pressure in water.	Velocity in metres per second.		
	Mercury in water.	Bisulphide of carbon in water.	Reduced to metres of water.	Time in seconds taken to fill flask.				Velocity through orifice in thin plate.				Reduced to c.m. per second.	Centigrade.	Fahrenheit.				
				1. 303 c.c.	2. 1160 c.c.	3. 500 c.c.	4. 1000 c.c.	Gauge No. 2.	Gauge No. 3.	Gauge No. 4.	Gauge No. 5.				i.	v.	log i.	log v.
1	20	..	0·0131	130	..	..	..	..	..	..	..	2·33	12	..	0·0086	0·0785	$\bar{3}$·935	$\bar{2}$·895
2	40	..	0·0262	69	..	..	..	..	..	..	..	4·40	..	..	0·01720	0·1480	$\bar{2}$·236	$\bar{1}$·170
3	60	..	0·0393	45	..	..	..	..	..	..	..	6·73	11	..	0·0258	0·2265	$\bar{2}$·412	$\bar{1}$·355
4	80	..	0·0524	34	..	..	..	..	..	..	..	8·91	..	..	0·0345	0·3000	$\bar{2}$·537	$\bar{1}$·477
5	100	..	0·0656	28	..	..	..	..	..	..	..	10·70	10	..	0·0430	0·3640	$\bar{2}$·634	$\bar{1}$·561
6	120	..	0·0787	23	..	..	..	..	..	..	..	13·2	9	..	0·0516	0·4426	$\bar{2}$·713	$\bar{1}$·646
7	140	Unsteady	0·0918	21	..	..	..	..	..	..	..	14·5	8	..	0·0602	0·4865	$\bar{2}$·780	$\bar{1}$·687
8	160	..	0·1040	..	80	..	..	..	..	..	..	14·5	7	..	0·0682	0·5106	$\bar{2}$·834	$\bar{1}$·708
9	160 l	..	0·1040	..	76	..	..	..	..	..	..	15·2	6	..	0·0682	0·5106	$\bar{2}$·834	$\bar{1}$·708
10	180	..	0·1181	..	71	..	..	..	..	..	..	16·3	..	..	0·0774	0·5483	$\bar{2}$·889	$\bar{1}$·739
11	200	..	0·1313	..	71	..	..	..	..	..	..	16·3	..	..	0·0861	0·5483	$\bar{2}$·935	$\bar{1}$·739
12	220	..	0·1443	..	69	..	..	..	..	..	..	16·8	..	..	0·0946	0·5650	$\bar{2}$·976	$\bar{1}$·752
13	240	..	0·1574	..	67	..	..	..	..	..	..	17·3	..	..	0·1033	0·5822	$\bar{1}$·014	$\bar{1}$·765
14	260	..	0·1707	..	66·5	..	..	..	..	..	..	17·4	..	..	0·1120	0·5862	$\bar{1}$·049	$\bar{1}$·768
15	280	..	0·1837	..	64	..	..	..	..	..	..	18·1	6	..	0·1206	0·6096	$\bar{1}$·081	$\bar{1}$·785
16	300	..	0·1968	..	61·5	..	..	..	..	..	..	18·8	..	..	0·1292	0·6339	$\bar{1}$·111	$\bar{1}$·802
17	320	..	0·2099	..	60	..	..	..	..	..	..	19·3	..	..	0·1378	0·6520	$\bar{1}$·139	$\bar{1}$·813

TABLE III. (continued).

Reference number.	Pressures.			Discharges.									Temperature.		Slope of pressure in water.	Velocity in metres per second.		
	Mercury in water.	Bisulphide of carbon in water.	Reduced to metres of water.	Time in seconds taken to fill flask.				Velocity through orifice in thin plate.				Reduced to c.m. per second.	Centigrade.	Fahrenheit.				
				1. 303 c.c.	2. 1160 c.c.	3. 500 c.c.	4. 1000 c.c.	Gauge No. 2.	Gauge No. 3.	Gauge No. 4.	Gauge No. 5.				*i.*	*v.*	log *i.*	log *v.*
18	320	..	0·2099	..	..	..	..	4·7	..	..	..	19·1	..	..	0·1378	0·6413	$\bar{1}$·139	$\bar{1}$·807
19	400	..	0·2613	..	54	..	..	5·3	..	..	..	21·5	..	..	0·1714	0·7228	$\bar{1}$·234	$\bar{1}$·859
20	500 *l*	..	0·3274	..	..	..	..	6·0	..	..	..	24·3	..	..	0·2148	0·8185	$\bar{1}$·332	$\bar{1}$·913
21	700	..	0·4592	..	..	..	..	7·4	..	..	..	30·0	..	..	0·3014	1·033	$\bar{1}$·479	0·014
22	1000	..	0·6562	..	..	..	..	9·4	..	..	..	38·1	..	..	0·4306	1·283	$\bar{1}$·634	0·108
23	1500	..	0·9355	..	..	..	..	11·7	..	..	..	47·5	5	..	0·6138	1·268	$\bar{1}$·788	0·103
24	2000	..	1·2480	..	..	..	..	13·6	..	..	..	55·1	..	..	0·8185	1·854	$\bar{1}$·913	0·268
25	2500	..	1·5560	..	..	..	..	15·8	..	..	..	64·2	..	..	1·021	2·158	0·009	0·334
26	3000	..	1·8710	..	..	..	..	17·5	..	..	..	71·0	..	..	1·228	2·388	0·089	0·378
27	3500	..	2·1830	..	..	..	..	19·1	..	..	..	79·1	..	..	1·433	2·661	0·156	0·425
28	4000 *r*	..	2·4950	..	..	..	..	20·1	..	..	..	81·1	..	..	1·637	2·729	0·214	0·436
29	4000	..	2·4950	..	..	..	..	..	4·9	..	..	79·5	..	..	1·637	2·674	0·214	0·427
30	5000	..	3·1120	..	..	..	..	..	5·7	..	..	92·5	..	..	2·042	3·112	0·310	0·493
31	6000	..	3·7420	..	..	..	..	..	6·5	..	..	105·0	..	..	2·455	3·540	0·390	0·549
32	7000	..	4·2660	..	..	..	..	..	7·1	..	..	115·0	..	..	2·865	3·873	0·457	0·588
33	8000	..	4·9890	..	..	..	..	..	7·7	..	..	125·0	..	..	3·274	4·198	0·515	0·623
34	8000 *l*	..	5·1290	..	..	..	..	..	8·0	..	..	130·0	..	..	3·444	4·467	0·537	0·650
35	9000	..	5·9030	..	..	..	..	..	8·6	..	..	139·0	5	..	3·873	4·689	0·588	0·671

TABLE IV.

Conditions the same as in Table III., except the temperatures at the lower velocities.

Reference number.	Pressures.			Discharges.									Temperature.		Slope of pressure in water.	Velocity in metres per second.		
	Mercury in water.	Bisulphide of carbon in water.	Reduced to metres of water.	Time in seconds taken to fill flask.				Velocity through orifice in thin plate.				Reduced to c.m. per second.	Centigrade.	Fahrenheit.				
				1. 303 c.c.	2. 1160 c.c.	3. 500 c.c.	4. 1000 c.c.	Gauge No. 2.	Gauge No. 3.	Gauge No. 4.	Gauge No. 5.				*i*.	*v*.	log *i*.	log *v*.
36	20	..	0·01313	..	..	227	..	..	..	..	..	1·750	10	50	0·008591	0·0740	$\bar{3}$·934	$\bar{2}$·869
37	40	..	0·02625	..	..	131	..	..	..	..	..	3·289	8	46·4	0·01718	0·1390	$\bar{2}$·235	$\bar{1}$·143
38	60	..	0·03936	..	..	80	..	..	..	..	..	4·966	7	44·6	0·02577	0·2100	$\bar{2}$·411	$\bar{1}$·322
39	80 } l	..	0·05249	..	..	61	..	..	..	..	..	6·517	6	42·8	0·03436	0·2755	$\bar{2}$·536	$\bar{1}$·440
40	100	..	0·06562	..	..	50·5	..	..	..	..	..	7·871	5	41	0·04296	0·3327	$\bar{2}$·633	$\bar{1}$·522
41	120	..	0·07871	..	..	..	86	..	..	..	..	11·650	5	41	0·05153	0·3918	$\bar{2}$·712	$\bar{1}$·593
42	140	..	0·09184	..	..	..	76	..	..	..	..	13·16	5	41	0·06296	0·4426	$\bar{2}$·779	$\bar{1}$·646
43	160	..	0·1040	..	..	..	66	..	..	..	..	15·18	5	41	0·06808	0·5106	$\bar{2}$·833	$\bar{1}$·708
44	180	Unsteady	0·1181	..	..	..	62	..	..	..	..	16 15	5	41	0·07727	0·5433	$\bar{2}$·888	$\bar{1}$·735
45	200	..	0·1313	..	..	..	61	..	..	..	..	16·41	5	41	0·08591	0·5521	$\bar{2}$·934	$\bar{1}$·742
46	220	..	0·1443	..	..	..	60	..	..	..	..	16·52	5	41	0·09441	0·5560	$\bar{2}$·975	$\bar{1}$·745
47	240	..	0·1574	..	..	..	58	..	..	..	..	17·26	5	41	0·1031	0·5808	$\bar{1}$·013	$\bar{1}$·764
48	280 } l	..	0·1837	..	..	..	55	..	..	..	..	18·20	5	41	0·1203	0·6124	$\bar{1}$·080	$\bar{1}$·787
49	320	..	0·2099	..	..	..	52	..	..	..	..	19·06	5	41	0·1375	0·6413	$\bar{1}$·138	$\bar{1}$·807
50	360	..	0·2250	..	..	..	50	..	..	..	..	20·00	5	41	0·1473	0·6730	$\bar{1}$·168	$\bar{1}$·828
51	400	..	0·2625	..	..	..	47	..	..	..	..	21·29	..	41	0·1718	0·7162	$\bar{1}$·235	$\bar{1}$·855

TABLE V.

Pipe No. 5, lead.—Diameter (as measured 0·498 inch), 12·7 millims. Length: total, 16 feet; to first gauge hole, 9·6 feet; between gauge holes (5 feet), 1·524 metres. Water from the Manchester Main.

Reference number.	Pressures.			Discharges.									Temperature.		Slope of pressure in water.	Velocity in metres per second.		
	Mercury in water.	Bisulphide of carbon in water.	Reduced to metres of water.	Time in seconds taken to fill flask.			Velocity through orifice in thin plate.					Reduced to c.m. per second.	Centigrade.	Fahrenheit.				
				1. 303 c.c.	2. 1160 c.c.	3. 500 c.c.	Gauge No. 1.	Gauge No. 2.	Gauge No. 3.	Gauge No. 4.	Gauge No. 5.				i.	v.	log i.	log v.
52	. .	1	0·001219	. .	. .	. .	4·5	. .	. .	. .	. .	4·346	12	. .	0·00080	0·0346	$\bar{4}$·902	$\bar{2}$·539
53	. .	2	0·002438	. .	. .	. .	8·4	. .	. .	. .	. .	8·110	. .	. .	0·00159	0·0646	$\bar{3}$·203	$\bar{2}$·810
54	. .	3	0·003656	. .	. .	. .	11·2	. .	. .	. .	. .	9·841	. .	. .	0·00239	0·0784	$\bar{3}$·379	$\bar{2}$·894
55	. .	4	0·004876	. .	. .	. .	16·4	. .	. .	. .	. .	15·85	11	. .	0·00319	0·1262	$\bar{3}$·504	$\bar{1}$·101
56	. .	5	0·006082	. .	. .	. .	. .	4·4	. .	. .	. .	17·83	. .	. .	0·00398	0·1420	$\bar{3}$·600	$\bar{1}$·152
57	. .	6	0·007312	. .	. .	. .	. .	5·3	. .	. .	. .	21·48	10	. .	0·00478	0·1711	$\bar{3}$·680	$\bar{1}$·233
58	. .	7 } l	0·008532	. .	. .	. .	. .	6·0	. .	. .	. .	24·33	. .	. .	0·00558	0·1937	$\bar{3}$·747	$\bar{1}$·287
59	. .	8	0·009750	. .	. .	. .	. .	7·0	. .	. .	. .	28·38	8	. .	0·00638	0·2260	$\bar{3}$·805	$\bar{1}$·354
60	Un-steady	9	0·01097	. .	. .	. .	. .	7·0	. .	. .	. .	28·38	. .	. .	0·00717	0·2260	$\bar{3}$·856	$\bar{1}$·354
61	. .	10	0·01219	. .	. .	. .	. .	7·6	. .	. .	. .	30·84	7	. .	0·00798	0·2455	$\bar{3}$·902	$\bar{1}$·390
62	. .	11	0·01340	. .	. .	. .	. .	8·0	. .	. .	. .	32·44	. .	. .	0·00877	0·2583	$\bar{3}$·943	$\bar{1}$·412
63	. .	20 b	0·01365	. .	. .	. .	. .	8·0	. .	. .	. .	32·44	. .	. .	0·00893	0·2583	$\bar{3}$·951	$\bar{1}$·412
64	. .	12	0·01463	. .	. .	. .	. .	8·4	. .	. .	. .	34·05	. .	. .	0·00957	0·2710	$\bar{3}$·981	$\bar{1}$·433
65	. .	13	0·01582	. .	. .	. .	. .	8·6	. .	. .	. .	34·84	. .	. .	0·01036	0·2774	$\bar{2}$·015	$\bar{1}$·443
66	. .	14 } l	0·01707	. .	. .	. .	. .	8·8	. .	. .	. .	35·65	. .	. .	0·01117	0·2838	$\bar{2}$·048	$\bar{1}$·453
67	. .	15	0·01837	. .	. .	. .	. .	9·0	. .	. .	. .	36·48	6	. .	0·01203	0·2905	$\bar{2}$·080	$\bar{1}$·463

TABLE V. (continued).

Reference number.	Pressures.			Discharges.									Temperature.		Slope of pressure in water.	Velocity in metres per second.		
	Mercury in water.	Bisulphide of carbon in water.	Reduced to metres of water.	Time in seconds taken to fill flask.			Velocity through orifice in thin plate.					Reduced to c.m. per second.	Centigrade.	Fahrenheit.				
				1. 303 c.c.	2. 1160 c.c.	3. 500 c.c.	Gauge No. 1.	Gauge No. 2.	Gauge No. 3.	Gauge No. 4.	Gauge No. 5.				*i.*	*v.*	log *i.*	log *v.*
68	..	40	0·02729	..	..	..	..	10·8	..	..	..	43·76	..	..	0·01787	0·3484	$\bar{2}$·252	$\bar{1}$·542
69		60	0·04093	..	..	..	..	13·6	..	..	..	55·09	..	..	0·02680	0·4386	$\bar{2}$·428	$\bar{1}$·642
70	.	80	0·05458	..	..	..	..	16·3	..	..	..	66·07	..	..	0·03573	0·5261	$\bar{2}$·553	$\bar{1}$·721
71	..	100	0·06824	..	..	..	..	18·2	..	..	..	73·80	..	..	0·04467	0·5875	$\bar{2}$·650	$\bar{1}$·769
72		120	0·08185	..	..	..	..	20·2	..	..	..	81·85	..	..	0·05483	0·653	$\bar{2}$·739	$\bar{1}$·814
73	..	120	..	..	..	..	..	..	4·8	..	..	77·81	5	..	..	0·620	..	$\bar{1}$·792
74	..	140	0·9550	..	..	..	..	..	5·2	..	..	84·34	..	..	0·06252	0·672	$\bar{2}$·796	$\bar{1}$·827
75	..	160 } *l*	0·1092	..	..	..	..	..	5·8	..	..	93·98	..	..	0·07145	0·749	$\bar{2}$·854	$\bar{1}$·874
76	..	170	0·1159	..	..	..	..	..	6·0	..	..	97·28	..	..	0·07586	0·775	$\bar{2}$·880	$\bar{1}$·889
77	..	180	0·1228	..	..	..	..	..	6·3	..	..	102·1	..	..	0·08036	0·813	$\bar{2}$·905	$\bar{1}$·910
78	..	190	0·1295	..	..	..	..	..	6·5	..	..	105·2	..	..	0·08473	0·838	$\bar{2}$·928	$\bar{1}$·923
79	..	200	0·1365	..	..	..	..	..	6·6	..	..	107·0	..	..	0·08934	0·852	$\bar{2}$·951	$\bar{1}$·930
80	..	216	0·1433	..	..	..	..	..	6·8	..	..	110·2	..	..	0·09376	0·878	$\bar{2}$·972	$\bar{1}$·943
81	..	220	0·1500	..	..	..	..	..	7·0	..	..	113·6	..	..	0·09818	0·904	$\bar{2}$·992	$\bar{1}$·956
82	..	230	0·1567	..	..	..	..	..	7·3	..	..	118·4	..	..	0·1026	0·942	$\bar{1}$·011	$\bar{1}$·974
83	..	240	0·1637	..	..	..	..	..	7·5	..	..	121·7	..	..	0·1072	0·993	$\bar{1}$·030	$\bar{1}$·986
84	250 } *l*	..	0·1637	..	..	..	..	..	7·5	..	..	..	5	..	0·1072	..	$\bar{1}$·030	..
85	300	..	0·1968	..	..	..	..	..	8·6	..	..	139·4	..	..	0·1289	1·110	$\bar{1}$·110	0·045

TABLE V. (continued).

Reference number.	Pressures.			Discharges.										Temperature.		Slope of pressure in water.	Velocity in metres per second.		
	Mercury in water.	Bisulphide of carbon in water.	Reduced to metres of water.	Time in seconds taken to fill flask.			Velocity through orifice in thin plate.					Reduced to c.m. per second.		Centigrade.	Fahrenheit.				
				1. 303 c.c.	2. 1160 c.c.	3. 500 c.c.	Gauge No. 1.	Gauge No. 2.	Gauge No. 3.	Gauge No. 4.	Gauge No. 5.					*i.*	*v.*	log *i.*	log *v.*
86	400 *l*	. .	0·2625	. .	. .	. .	. .	. .	10·3	. .	. .	166·8		. .	. .	0·1718	1·328	$\bar{1}$·235	0·123
87	500	. .	0·3281	. .	. .	. .	. .	. .	11·7	. .	. .	189·7		. .	. .	0·2148	1·511	$\bar{1}$·332	0·179
88	1,000	. .	. .	. .	. .	. .	. .	. .	11·5	. .	. .	186·3		. .	. .	. .	1·483	. .	0·171
89	1,500	. .	0·4798	. .	. .	. .	. .	. .	14·2	. .	. .	230·2		. .	. .	0·3141	1·833	$\bar{1}$·497	0·263
90	2,000	. .	0·6398	. .	. .	. .	. .	. .	17·3	. .	. .	280·6		. .	. .	0·4188	2·234	$\bar{1}$·622	0·349
91	2,000	. .	. .	. .	. .	. .	. .	. .	. .	8·3	. .	269·2		. .	. .	. .	2·143	. .	0·331
92	3,000	. .	0·9595	. .	. .	. .	. .	. .	. .	10·8	. .	350·0		. .	. .	0·6281	2·787	$\bar{1}$·798	0·445
93	5,000	. .	1·596	. .	. .	. .	. .	. .	. .	18·7	. .	479·8		. .	. .	1·045	3·820	0·019	0·582
94	7,000 *v*	. .	2·239	. .	. .	. .	. .	. .	. .	. .	8·7	605·4		. .	. .	1·466	4·820	0·166	0·683
95	7,000	. .	2·239	. .	. .	. .	. .	. .	. .	. .	10·2	563·7		. .	. .	1·466	4·488	0·166	0·652
96	9,000	. .	2·878	. .	. .	. .	. .	. .	. .	. .	11·4	660·7		. .	. .	1·884	5·261	0·275	0·721
97	11,000	. .	3·516	. .	. .	. .	. .	. .	. .	. .	12·4	736·3		. .	. .	2·302	5·875	0·362	0·769
98	13,000	. .	4·150	. .	. .	. .	. .	. .	. .	. .	13·5	803·6		5	. .	2·717	6·398	0·434	0·806
99	15,000	. .	4·798	. .	. .	. .	. .	. .	. .	. .	13·7	875·0		. .	. .	3·141	6·967	0·497	0·843
100	15,500	. .	4·955	. .	. .	. .	. .	. .	. .	. .	. .	887·2		5	. .	3·244	7·064	0·511	0·849

32. *The results of the experiments.*—A considerable number of preliminary experiments were made until the results showed a high degree of consistency. Then a complete series of experiments were made consecutively with each tube. The results of these are given in Tables III. and V.

33. *The critical velocities.*—The determination of these, which had been the main object of the experiments, was to some extent accomplished directly during the experiments, for starting from the very lowest velocities, it was found that the fluid in the differential gauge was at first very steady, lowering steadily as the velocity was increased by stages, until a certain point was reached, when there seemed to be something wrong with the gauge. The fluid jumped about, and the smallest adjustment of the tap controlling the velocity sent the fluid in the gauge out of the field of the microscope. At first this unsteadiness always came upon me as a matter of surprise, but after repeating the experiments several times, I learnt to know exactly when to expect it. The point at which this unsteadiness is noted is marked in the tables.

It was not, however, by the unsteadiness of the pressure gauge that the critical velocity was supposed to be determined, but by comparing the ratio of velocities and pressures given in the columns v and i in the tables. This comparison is shown in diagram I., Plate 74, the values of i being abscissæ and v ordinates. It is thus seen that for each tube the points which mark the experiments lie very nearly in a straight line up to definite points marked C, at which divergence sets in rapidly.

The points at which this divergence occurs correspond with the experiments numbered 6 and 59, which are immediately above those marked unsteady.

Thus the change in the law of pressure agrees with the observation of unsteadiness in fixing the critical velocities.

According to my assumption, the straightness of the curves between the origin and the critical points would depend on the constancy of temperature, and it was the small divergences observed that suggested a variation of temperature which had been overlooked. This variation was confirmed by further experiments, amongst which are those contained in Table IV. These showed that the probable variation of the temperature was in Table III. from 12° C. to 9° C. at the critical point, and from 12° C. to 8° C. in Table V., which variations would account for the small deviation from the straight.

It only remained, then, to ascertain how far the actual values of v_c, the velocity at the critical points, corresponded with the ratio $\frac{\mu}{D}$ or $\frac{P}{D}$.

For tube 4 from the Table III.

$$D=0^{m}\cdot00615$$

$$v_c=0^{m}\cdot4426$$

at 9° C.; at this temperature

$$P=\cdot757$$

see p. 952.

Hence putting

$$B_c = \frac{P}{v_c D}$$

we have

$$B_c = 279{\cdot}7$$

Again, for tube 5, Table V.

$$D = {\cdot}0127$$

$$v_c = {\cdot}2260$$

at 8° C.; at which temperature

$$P = {\cdot}7796$$

whence

$$B_c = 272{\cdot}0$$

The differences in the values of B_c thus obtained would be accounted for by a variation of a quarter of a degree in temperature, and hence the results are well within the accuracy of the experiments.

To each critical velocity, of course, there corresponds a critical value of the pressure. These are determined as follows.

The theoretical law of resistance for steady motion may be expressed

$$A_c D^2 i = B_c P v$$

and multiplying both sides by $\frac{D}{P^2}$

$$\frac{A_c D^3 i}{P^2} = B_c \frac{D}{P} v$$

This law holds up to the critical velocity, and then the right hand number is unity if B_c has the values just determined.

$$A_c = \frac{P^2}{D^3 i_c}$$

by Table III.

$$i_c = {\cdot}0516$$

$$P^2 = {\cdot}573$$

$$D^3 = {\cdot}000,000,232$$

which give

$$A_c = 47,750,000$$

By Table V.

$$i_c = {\cdot}00638$$

$$P^2 = {\cdot}607$$

$$D^3 = {\cdot}00000205$$

which give

$$A_c = 46,460,000$$

which values of A_c differ by less than by what would be caused by half a degree of temperature.

The conclusion, therefore, that the critical velocity would vary as

$$\frac{\mu}{D}$$

is abundantly verified.

34. *Comparison with the discharges calculated by* POISEUILLE's *formula.*—POISEUILLE experimented on capillary tubes of glass between ·02 and ·1 millim. in diameter, and it is a matter of no small interest to find that the formula of discharges which he obtained from these experiments is numerically exact for the bright metal tubes 100 times as large.

POISEUILLE's formula is—

$$Q=1836{\cdot}724(1+0{\cdot}0336793\,T+0{\cdot}000220992\,T^2)\frac{HD^4}{L}$$

T = temperature in degrees centigrade.
H = pressure in millims. mercury.
D = diameter in millims.
L = length in millims.
Q = discharge in millims. cubed.

Putting

$$i=\frac{13{\cdot}64H}{L}$$

$$P=1+(0{\cdot}336793\,T+0{\cdot}000220992\,T^2)^{-1}$$

$$v=\frac{4Q}{\pi D^2}$$

and changing the units to metres and cubic metres this formula may be written

$$47700000\frac{D^3}{P^2}i=278\frac{D}{P}v$$

the coefficients corresponding to A_c and B_c.

The agreement of this formula with the experimental results from tubes 4 and 5 is at once evident. The actual and calculated discharges differ by less than 2 per cent., a difference which would be more than accounted for by an error of half a degree in the temperature.

35. *Beyond the critical point.*—The tables show that, beyond the critical point, the relation between i and v differs greatly from that of a constant ratio; but what the exact relation is, and how far it corresponds in the two tubes, is not to be directly seen from the tables.

In the curves (Plate 74, diagram I.) which result from plotting i and v, it appears that after a period of flatness the curves round off into a parabolic form; but whether they are exact parabolæ, or how far the two curves are similar with different parameters, is difficult to ascertain by any actual comparison of the curves themselves, which, if plotted to a scale which will render the small differences of pressure visible, must extend 10 feet at least.

36. *The logarithmic method.*—So far the comparison of the results has been effected by the natural numbers, but a far more general and clearer comparison is effected by treating the logarithms of l and v.

This method of treating such experimental results was introduced in my paper on Thermal Transpiration (see Phil. Trans., Part II., 1879, p. 753).

Instead of curves, of which i and v are the abscissæ and ordinates, log i and log v are taken for the abscissæ and ordinates, and the curve so obtained is the logarithmic homologue of the natural curve.

The advantage of the logarithmic homologues is that the *shape* of the curve is made independent of any constant parameters, such parameters affecting the position of all points on the logarithmic homologue similarly. Any similarities in shape in the natural curves become identities in shape in the logarithmic homologues. How admirably adapted these logarithmic homologues are for the purpose in hand is at once seen from diagram II., Plate 73, which contains the logarithmic homologues of the curves for both pipes 4 and 5.

A glance shows the similarity of these curves, and also their general character. But it is by tracing one of the curves, and shifting the paper rectangularly until the traced curve is superimposed on the other, that the exact similarity is brought out. It appears that, without turning the paper at all, the two curves almost absolutely fit.

It also appears that the horizontal and vertical components of the shift are—

Horizontal shift	·913
Vertical shift	·294

which are within the accuracy of the work respectively identical with the differences of the logarithms of $\frac{D^3}{P^2}$ and $\frac{D}{P}$ for the two tubes.

37. *The general law of resistance in pipes.*—The agreement of the logarithmic homologues shows that not only at the critical velocities but for all velocities in these two pipes, pressure which renders $\frac{D^3}{\mu^2}i$ the same in both pipes corresponds to velocities which render $\frac{D}{\mu}v$ the same in both pipes. This may be expressed in several ways. Thus if the tabular value of i for each pipe plotted in a scale be multiplied by a number proportional to $\frac{D^3}{P^2}$ for that particular pipe and the values of v by a number proportional to $\frac{D}{P}$, then the curves which have these reduced values of i and v for abscissæ and ordinates will be identical.

A still more general expression is that if

$$i = F(v)$$

expresses the relation between i and v for a pipe in which $D = 1$, $T=0$, $P=1$.

$$\frac{D^3 i}{P^2} = F\left(\frac{Dv}{P}\right)$$

expresses the relation for every pipe and every condition of the water.

The determination of the relation between circumstances of motion and the physical condition of the water in such a general form was not contemplated when the experiments were undertaken, and must be considered as a result of the method of logarithmic homologues which brought out the relation in such a marked manner that it could not be overlooked. Nor is this all.

It had formed no part of my original intention to re-investigate the law of resistance in pipes for velocities above the critical value, as this is ground which had been very much experimented upon, and experiments seemed to show that the law was either indefinite or very complex—a conclusion which did not seem inconsistent with the supposition that above this point the resistance depended upon eddies which might be somewhat uncertain in their action. But although it was not my intention to investigate laws, I had made a point of continuing the experiments through a range of pressures and velocities very much greater I think than had ever been attempted in the same pipe.

Thus it will be noticed that in the larger tube the pressure in the last experiment is four thousand times as large as in the first. In choosing the great range of pressures I wished to bring out what previous experiments had led me to expect, namely, that in the same tube for sufficiently small pressures the pressure is proportional to the velocity, and for sufficiently great pressures, the pressure was proportional to the square of the velocity. Had this been the case not only would the lowest portion of the logarithmic homologues up to the critical point have come out straight lines inclined at 45 degrees, but the final portion of the curve would have come out a straight line at half this inclination, or with a slope of two horizontal to one vertical.

The near approach of the lower portions of the curve to the line at 45° led me, as I have already explained, to discover that the temperatures had risen at the lower velocities, and to make a fresh set of experiments, some of which are given in Table IV., in which, although the temperatures were not constant, they were sufficiently different from the previous ones to show that the discrepancy in the lower portions of the curves might be attributed to variations of temperature, and the agreement with the line of 45° considered as within the limits of accuracy of experiment.

When the logarithms of the upper portions of the curve came to be plotted, the straightness and parallelism of the two lines was very striking.

There are a few discrepancies which could not be in any way attributed to temperature, as with so much water moving this was very constant, but on examination it was seen that these discrepancies marked the changes of the discharge gauges. The law of flow through the orifices not having been strictly as the square roots of the heights, the manner in which the gauges had been compared forbade the possibility of there being a general error from this cause; the middle readings on the gauge were correct, so that the discrepancies, which are small, are mere local errors.

This left it clear that whatever might be their inclination the lines expressed the laws of pressures and velocities in both tubes, and since the lines are strictly parallel, this law was independent of the diameter of the tube. This point has been very carefully examined, for it is found that the inclination of these lines differs decidedly from that of 2 to 1, being 1·723 to 1, and so giving a law of pressures through a range 1 to 50 of

$$i \propto v^{1\cdot 723}$$

This is different from the law propounded by any of the previous experimenters, who have adhered to the laws

$$i = v^2$$

or

$$i = \mathrm{A}v + \mathrm{B}v^2$$

That neither of these laws would answer in case of the present experiments was definitely shown, for the first of these would have a logarithmic homologue inclined at 2 to 1, and the second would have a curved line. A straight logarithmic homologue inclined at a slope 1·723 to 1 means no other law than

$$i \propto v^{1\cdot 723}$$

I have therefore been at some pains to express the law deduced from my experiments on the uniform pipes so that it may be convenient for application. This law as already expressed is simply

$$\frac{\mathrm{D}^3}{\mathrm{P}^2} i = f\left(\frac{\mathrm{D}v}{\mathrm{P}}\right)$$

where f is such that

$$x = f(y)$$

is the equation to the curve which would result from plotting the resistance and velocities in a pipe of diameter 1 at a temperature zero.

The exact form of f is complex, this complexity is however confined to the region immediately after the critical point is passed.

Up to the critical point

$$\mathrm{A}_c \frac{\mathrm{D}^3}{\mathrm{P}^2} i = \mathrm{B}_c \frac{\mathrm{D}v}{\mathrm{P}}$$

After the critical point is passed the law is complex until a velocity which is 1·325 v_c is reached. Then as shown in the homologues the curve assumes a simple character again

$$\mathrm{A} \frac{\mathrm{D}^3}{\mathrm{P}^2} i = \left(\mathrm{B} \frac{\mathrm{D}v}{\mathrm{P}}\right)^{1\cdot 723}$$

that is, the logarithmic homologue becomes a straight line inclined at 1·723 to 1.

Referring to the logarithmic homologues (Plate 73, diagram II.), it will be seen that although the directions of the two straight extremities of the curve do not meet in the critical point, their intersection is at a constant distance from this point which in the logarithmic curves is, both for ordinates and abscissæ,

$$0{\cdot}154$$

These points o are therefore given by

$$\log\frac{D^3 i_c}{P^2}=\log\frac{D^3 i_o}{P^2}+0{\cdot}154$$

$$\log\frac{Dv_c}{P}=\log\frac{Dv_o}{P}+0{\cdot}154$$

Therefore putting

$$A=\frac{P^2}{D^3 i_o},\ B=\frac{P}{Dv_o}$$

$$\log A=\log A_c+0{\cdot}154$$

$$\log B=\log B_c+0{\cdot}154$$

and by the values of A_c and B_c previously ascertained (Art. 33, p. 971),

$$\log A=8{\cdot}8311 \qquad A=67{,}700{,}000$$

$$\log B=2{\cdot}598 \qquad B=\quad 396{\cdot}3$$

We thus have for the equation to the curves corresponding to the upper straight branches

$$A\frac{D^3}{P^2}i=\left(B\,\frac{Dv}{P}\right)^{1{\cdot}723}$$

And if n have the value 1 or 1·722 according as either member of this equation is $<$ or $>$ 1 the equation

$$A\frac{D^3}{P^2}.i=\left(\frac{BDv}{P}\right)^n$$

is the equation to a curve which has for its logarithmic homologue the two straight branches intersecting in o, and hence gives the law of pressures and velocities, except those relating to velocities in the neighbourhood of the critical point, and these are seldom come across in practice.

By expressing n as a discontinuous function of $B_c\frac{Dv}{P}$ the equation may be made to fit the curve throughout.

38. *The effect of temperature.*—It should be noticed that although the range is comparatively small, still the displacement of the critical point in Tables III. and IV. is distinctly marked. The temperatures were respectively 9° C., 5° C.

$$\text{At } 9^\circ \log P^{-1}=0{\cdot}12093$$

$$\text{At } 5^\circ \log P^{-1}=0{\cdot}06963$$

$$\text{Difference}\quad ={\cdot}05130$$

This should be the differences in the values of log v_c in Tables III. and IV. The actual difference is ·062. Also the differences in log i_c should be the differences in P^2 or ·10260, whereas the actual difference is ·121.

The errors correspond to a difference of about 1° C., which is a very probable error.

It would be desirable to make experiments at higher temperature, but there were great difficulties about this which caused me, at all events for the time, to defer the attempt.

SECTION IV.

Application to DARCY'S *experiments.*

39. DARCY'S *experiments.*—The law of resistance came out so definitely from my experiments that, although beyond my original intention, I felt constrained to examine such evidence as could be obtained of the actual experimental results obtained by previous experimenters.

The lower velocities, up to the critical value, were found, as has already been shown (Art. 35), to agree exactly with POISEUILLE'S formula.

For velocities above the critical values the most important experiments were those of DARCY—approved by the Academy of Sciences and published 1845—on which the formula in general use has been founded. Notwithstanding that the formula as propounded by DARCY himself could not by any possibility fit the results which I have obtained, it seemed possible that the experiments on which he had based his law might fit my law. A comparison was therefore undertaken.

This was comparatively easy, as DARCY'S experimental results have been published in detail.

Altogether he experimented on some 22 pipes, varying in diameter from about the size of my largest, $0^m{\cdot}0014$ up to $0^m{\cdot}5$. They were treated in several sets, according to the material of which they were composed—wrought iron gas pipes, lead pipes, varnished iron pipes, glass pipes, new cast iron and old rusty pipes.

The method of experimenting did not differ from mine except in scale, the distance between DARCY'S gauge points being 50^m instead of 5 feet in my case. The great length between DARCY'S gauge points entailed his having joints in his pipes between these points, and the nature of his pipes was such as to preclude the possibility of a very uniform diameter. His experiments appear to have been made with extreme care and very faithfully recorded, but the irregularity in the diameters, which appears to have been as much as 10 per cent., and the further irregularity of the joints, preclude the possibility of the results of his experiments following very closely the law for uniform pipes. Another important matter to which DARCY appears to have paid but little attention was temperature. It is true that in many instances he has given the temperature, but he does not appear to have taken any account of it in his discussion of his results, although it varied as much as 20° C. in the cases where he has given it, and as his pipes, 300 metres long, were in the open air, the effect of the sun on the pipes would have led to still larger differences.

The effect of these various causes on his results may be seen, as he took the precaution to use two pressure gauges on separate lengths of 50^m of his pipes, and the records from these two gauges by no means always agree, particularly for the lower velocities. In one case the results are as wide apart as 15 and 7, and often 10 or 15 per cent. In arriving at tabular values for i he has taken the mean of the two gauges.

Taking these things into account, I could not possibly expect any close agreement with my results; still, as experiments on pipes of such large diameters are not likely to be repeated, at any rate with anything like the same care and success, they offered the only chance of proving that my law was general.

40. *Reduction of the experimental results.*—Rejecting all the experiments on rusty and rough pipes, *i.e.*, selecting the lead, the varnished, the glass, and new cast iron pipes, which ranged from half-an-inch to twenty inches diameter, I had the logarithmic homologues drawn. These are shown on diagram III., Plate 74. In the case of two of the smaller pipes the smallest velocity is well below the critical point, and in several of the other pipes the smallest velocity is near the critical velocity. This accounts for the lower ends of the logarithmic curves being somewhat twisted; for the remainder of the logarithmic homologues are nearly straight; some are slightly bent one way and some another, but they are none of them more bent than may be attributed to experimental inaccuracy.

The inclinations of the upper ends of the lead and bituminous pipes is 1·746, slightly greater than mine; but in the cases of the glass pipes and the cast iron pipes the slopes are 1·82 and 1·92 respectively.

So much appeared from the logarithmic homologues themselves, but the most important question was, would the curves agree with the results calculated from the formula

$$A\frac{D^3}{P^2}i=\left(B\frac{D}{P}v\right)^n$$

41. *Comparison with the law of resistance.*—In applying this test I was at first somewhat at a loss on account in some cases of the want of any record of the temperature, and the doubt as to such temperatures as had been recorded being the temperature of the water in the pipes between the gauges.

The dates at which the experiments were made to a certain extent supplied the deficiency of temperature, the temperatures given fixing the law of temperature, so that the probable temperature could be assumed where it was not given.

Assuming the temperature, the values of

$$i_o=\frac{P^2}{AD^3}$$

$$v_o=\frac{P}{BD}$$

were calculated for each tube, using the values of A and B as already determined, log i_o and v_0 are the co-ordinates of O the intersection of the two straight branches of the logarithmic curves, so that the application of the formula to the results was simply tested by continuing the straight upper branches of the logarithmic homologues to see whether they passed through the corresponding point O.

The agreement, which is shown in diagram III., Plate 74, is remarkable. There are some discrepancies, but nothing which may not be explained by inaccuracies, particularly inaccuracies of temperature.

42. *The effect of the temperature above the critical point.*—It is a fact of striking significance, physical as well as practical, that while the temperature of the fluid has such an effect at the lower velocities that, *cœteris paribus*, the discharge will be double at 45° C. what it is at 5° C., so little is the effect at the higher velocities that neither DARCY or any other experimenter seems to have perceived any effect at all.

In my experiments the temperature was constant, 5° C. at the higher velocities, so that I had no cause to raise this point till I came to DARCY'S result, and then, after perplexing myself considerably to make out what the temperatures were, I noticed the effect of the temperature is to shift the curves 2 horizontal to 1 vertical, which corresponds with a slope of 2 to 1, and so nearly corresponds with the direction of the curves at higher velocities that variations of 5° or 10° C. produce no sensible effect; or, in other words, the law of resistance at the higher velocity is sensibly independent of the temperature, *i.e.*, of the viscosity.

Thus not only does the critical point, the velocity at which eddies come in, diminish with the viscosity, but the resistance after the eddies are established is nearly, if not quite, independent of the viscosity.

43. *The inclinations of the logarithmic curves.*—Although the general agreement of the logarithmic homologues completely establishes the relations between the diameters of the pipes, the pressures and velocities for each of the four classes of pipes tried, viz., the lead, the varnished pipes, the glass pipes, and the cast iron, there are certain differences in the laws connecting the pressures and velocity in the pipes of different material. In the logarithmic curves this is very clearly shown as a slight but definite difference between the inclination of the logarithmic homologues for the higher velocities.

The variety of the pipes tried reduces the possible causes of this difference to a small compass. It cannot be due to any difference in diameters, as at least three pipes of widely different diameters belong to each slope. It is not due to temperature. This reduces the cause for the different values of n to the irregularity in the pipes owing to joints and other causes, and the nature of the surfaces.

The effect of the joints on the values of n seems to be proved by the fact that DARCY'S three lead pipes gave slightly different values for n, while my two pipes without joints gave exactly the same value, which is slightly less than that obtained from DARCY'S experiments.

Darcy's pipes were all of them uneven between the gauge points, the glass and the iron varying as much as 20 per cent. in section. The lead were by far the most uniform, so that it is not impossible that the differences in the values of n may be due to this unevenness.

But the number of joints and unevenness of the tarred pipes corresponded very nearly with the new cast iron, and between these there is a very decided difference in the value of n. This must be attributed to the roughness of the cast iron surface.

44. *Description of Diagram* III.

Diagram III.—In this diagram the experiments of Poiseuille and Darcy are brought into comparison with those of the present investigation.

In consequence of the number of lines, the general aspect of the diagram is somewhat confused, but such confusion vanishes so soon as it is clearly perceived that each line of dots indicates the logarithmic homologue for some particular pipe as determined by experiment, reduced and plotted in exactly the same manner as for diagram II.; DD and EE being exact repetitions of the logarithmic homologue for pipes 4 and 5, on a somewhat smaller scale.

It is at once apparent from diagram III. how, for the most part, the experiments have been well below or well above the critical values. In the small pipes of Poiseuille the velocities were below the critical values, and hence lie in straight lines inclined at 45°.

The smallest pipe on which Poiseuille's experimented had a diameter of 0·014 millim.; only one experiment, marked A, is shown in the diagram, as the remaining three extended outside the range of the plate. They fall exactly on the dotted line through A, and do not reach the critical value.

The same is true of all the rest of Poiseuille's experiments except those made on a much larger pipe, diameter 0·65 millim., hence it is thought sufficient to plot only one, namely BB.

CC shows the experimental results obtained with the pipe 0·65 millim. diameter, and these reach the critical value as given by the formula, and then diverge from the line.

It is important to notice, however, that the points are not taken directly from Poiseuille's experiments, which have been subjected to a correction rendered necessary by the fact that Poiseuille did not measure the resistance by ascertaining the pressure at two points in the pipe, but by ascertaining the pressure in the vessels from which and into which the water flowed through the pipe, so that his resistance includes, besides the resistance of the pipe, the pressure necessary to impart the initial velocity to the water. This fact, which appears to have been entirely overlooked, had a very important influence on many of Poiseuille's results. Poiseuille endeavoured to ascertain what was the limit to the application of his law, and, with the exception of his smallest tubes, succeeded in attaining velocities at which the results were no longer in accordance with his law.

When I first examined his experiments I expected to find these limiting velocities above the critical velocities as given by my formula. In all cases, however, they were very much below, and it was then I came to see that POISEUILLE had taken no account of the pressure necessary to start the fluid.

It then became interesting to see how far the deviations were to be explained in this way.

In pipes of sensible size the pressure necessary to start the fluid lies between

$$\frac{v^2}{2g} \text{ and } 1{\cdot}505\,\frac{v^2}{2g}$$

according to whether the mouthpiece is trumpet-shaped or cylindrical. POISEUILLE states that he was careful to keep both ends of his pipe cylindrical, hence according to the law for mouthpieces of sensible size, the pressures which he gives should be corrected by $1{\cdot}505\,\frac{v^2}{2g}$.

This correction was made, and it was then found that with all the smaller tubes POISEUILLE'S law held throughout his experiments, and with the larger pipe it held up to the critical value and then diverged in exact accordance with my formula, as shown by the line CC.

DARCY'S experiments in the case of three tubes F, G, I fall below the critical value, and in all these cases agree very well with the theoretical curve as regards both branches.

This, however, must be looked upon as accidental, as at the lower velocities DARCY had clearly reached the limit of sensitiveness of his pressure gauges; thus, for instance, the experiment close by the letter F is the mean of two readings which are respectively 7 and 15; there is a tendency throughout the entire experiments to irregularity in the lower readings which may be attributed to the same cause, and this seems to explain the somewhat common deviation of the one or two lower experiments from the line given by the middle dots.

A somewhat similar cause will explain cases of deviation in the one or two upper experiments, for the discrepancy in the two gauges here again becomes considerable.

For these reasons the intermediate experiments were chiefly considered in determining the slopes of the theoretical lines.

These slopes were obtained as the mean of each class of tubes :—

Lead jointed	1·79
Varnished	1·82
Glass	1·79
New cast iron	1·88
Incrusted pipe	2·
Cleaned pipe	1·91

and then in the cases in which the temperature was given, I, J, L, M, N, the points O having being determined by the formula,

$$\begin{aligned}\text{Log}\ i_0 &= 2\log P - 3\log D - 7{\cdot}851\\ \text{Log}\ v_0 &= \ \ \log P - \ \ \log D - 2{\cdot}598\end{aligned}$$

the lines having the respective slopes were drawn through these points and in all cases agreed closely with the experiments.

In the cases where the temperature was not given the values of log i_o and log v_0 were calculated for 5° C., these are shown along the line marked "line of intersections at 5°," through these points lines are shown drawn at an inclination of 2 to 1, which are the lines on which O would lie whatever might be the temperature. These with the respective slope lines were drawn so as most nearly to agree with the experiments, these intersect the lines at 2 to 1 in the points O which indicate the temperatures, and considering the extremely small effect of the temperature these are all very probable temperatures with the exception of G, H, and S, in which cases O is above the line for 5° C. This indicates strongly that in these cases there must have been a small error, 2 or 3 per cent., in determining the effective diameter of the pipes.

It seemed very probable that roughness in the pipes, such as might arise from incrustation or badly formed joints, would affect the logarithmic homologues, and for this reason only the smoother classes of pipes were treated; but with a view to test this idea, the homologues Q and R, which related to the same incrusted pipe before and after being cleaned were drawn, and their agreement is such as to show that for such pipe the effect of incrustation is confined to the effect on the diameter of the pipe, and on the value of n which it raises to 2. This, however, was a large pipe and the velocities a long way above the critical velocity, so that it is quite possible that the same incrustation in a smaller pipe would have produced a somewhat different effect.

The general result of this diagram is to show that throughout the entire range—from pipes of $0^{\cdot m}000014$ to $0^{\cdot m}5$ in diameter, and from slopes of pressure ranging from 1 to 700,000—there is not a difference of more than 10 per cent. in the experimental and calculated velocities, and with very few exceptions the agreement is within 2 or 3 per cent., and it does not appear that there is any systematic deviation whatever.

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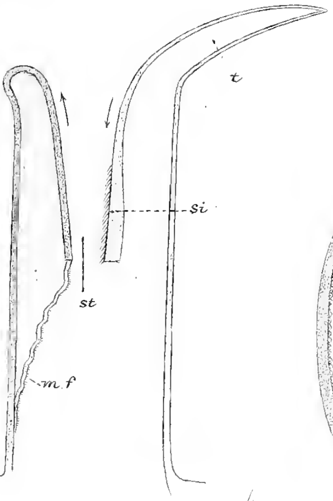

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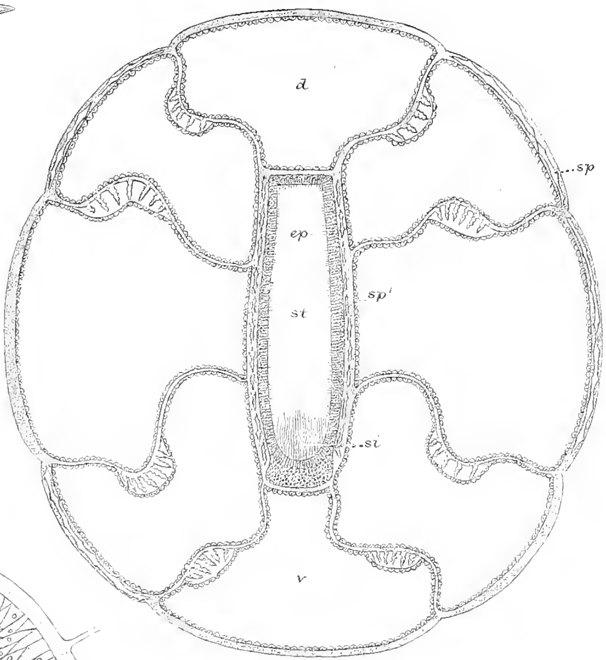

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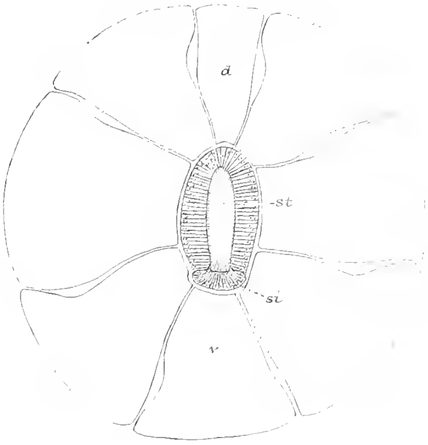

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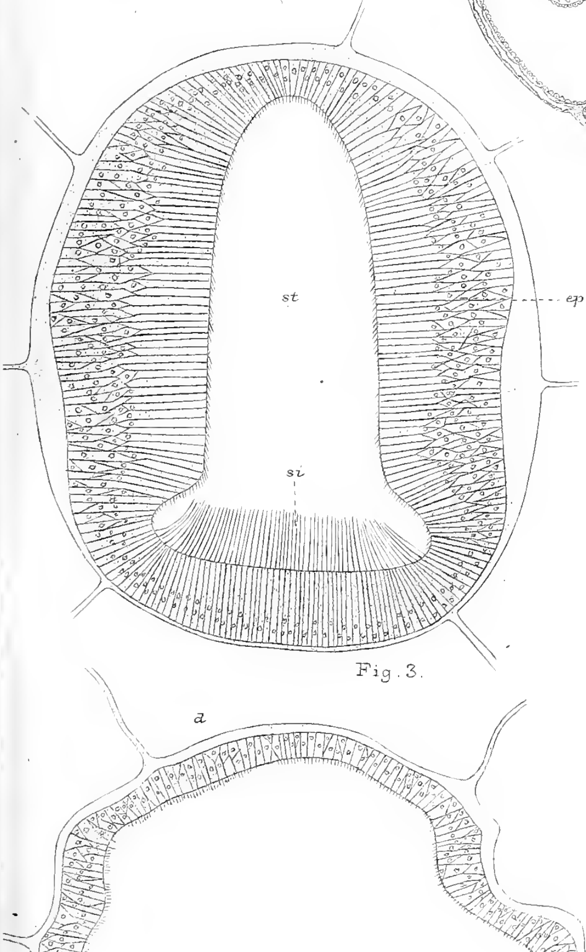

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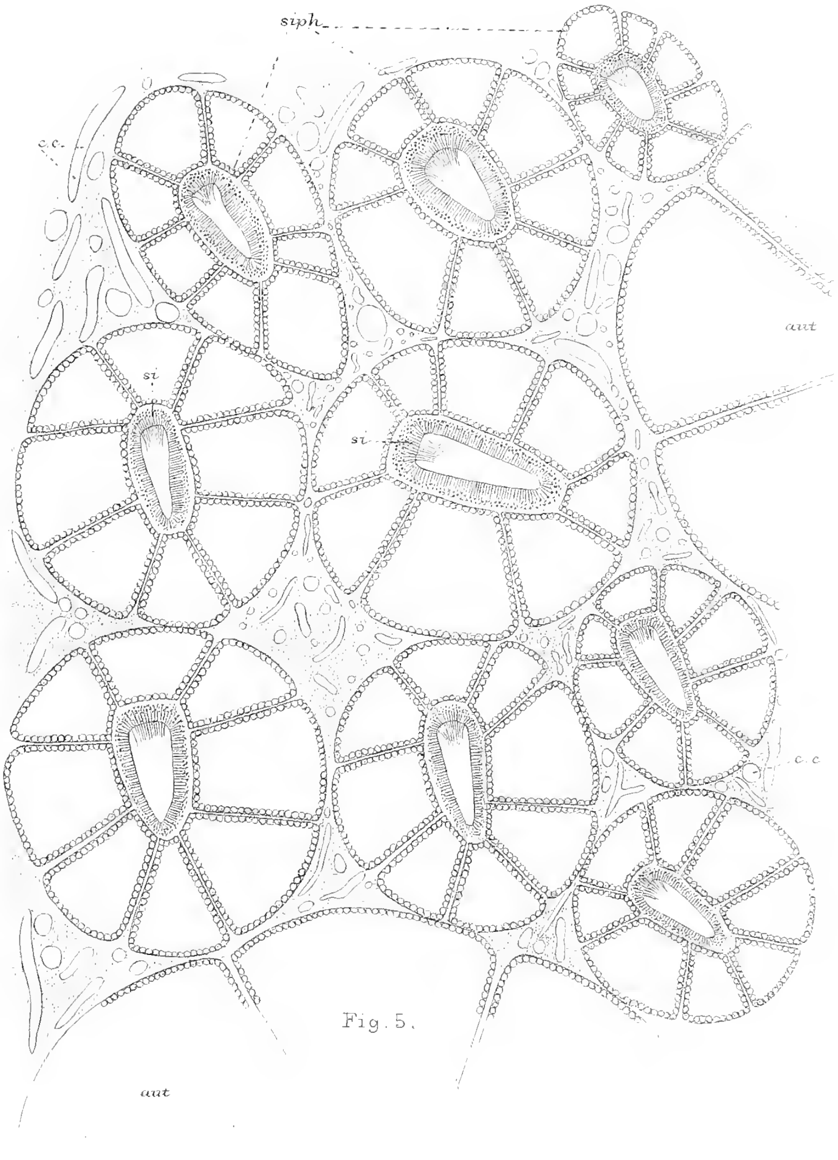

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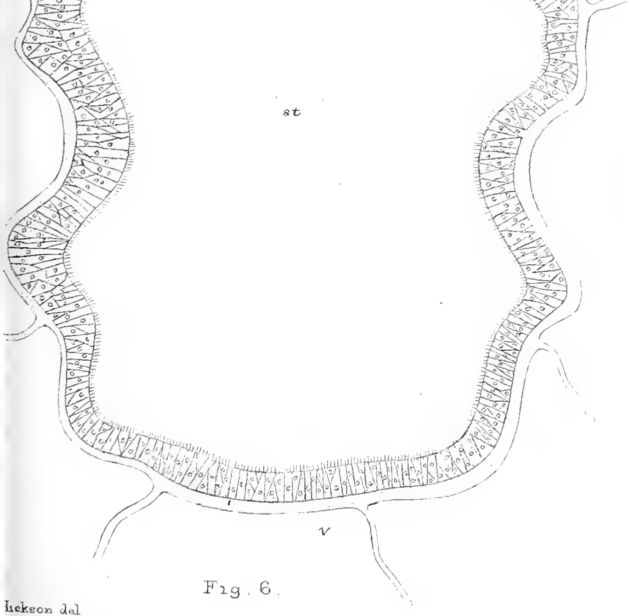

Fig. 6.

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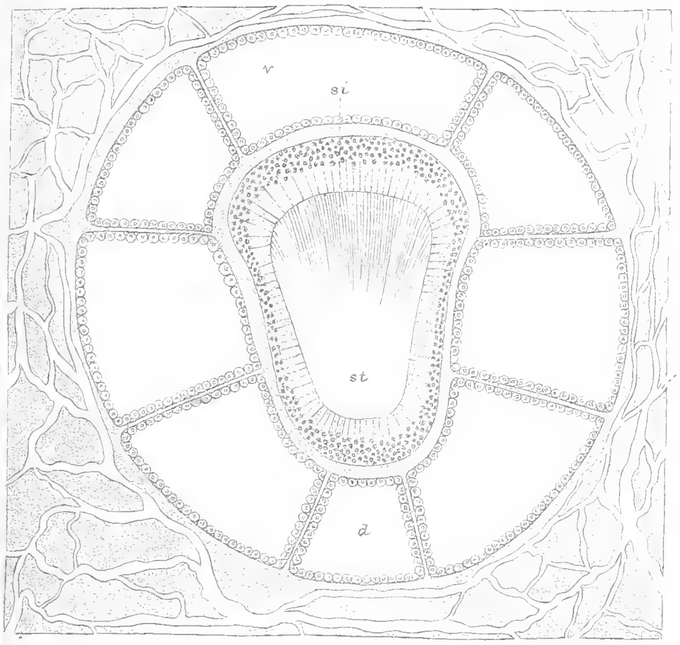

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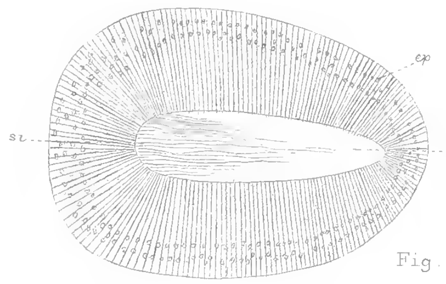

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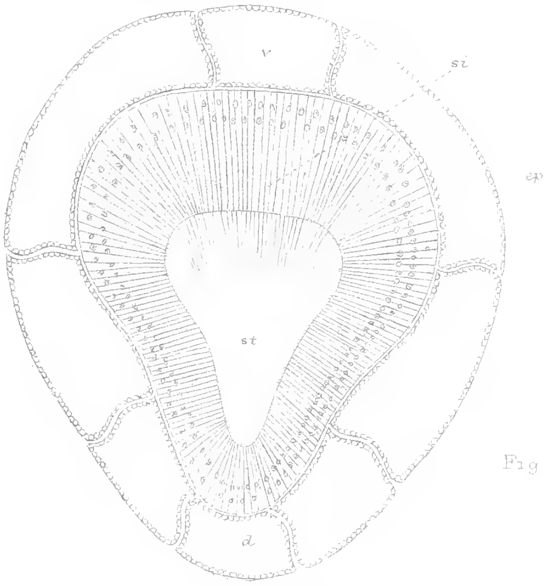

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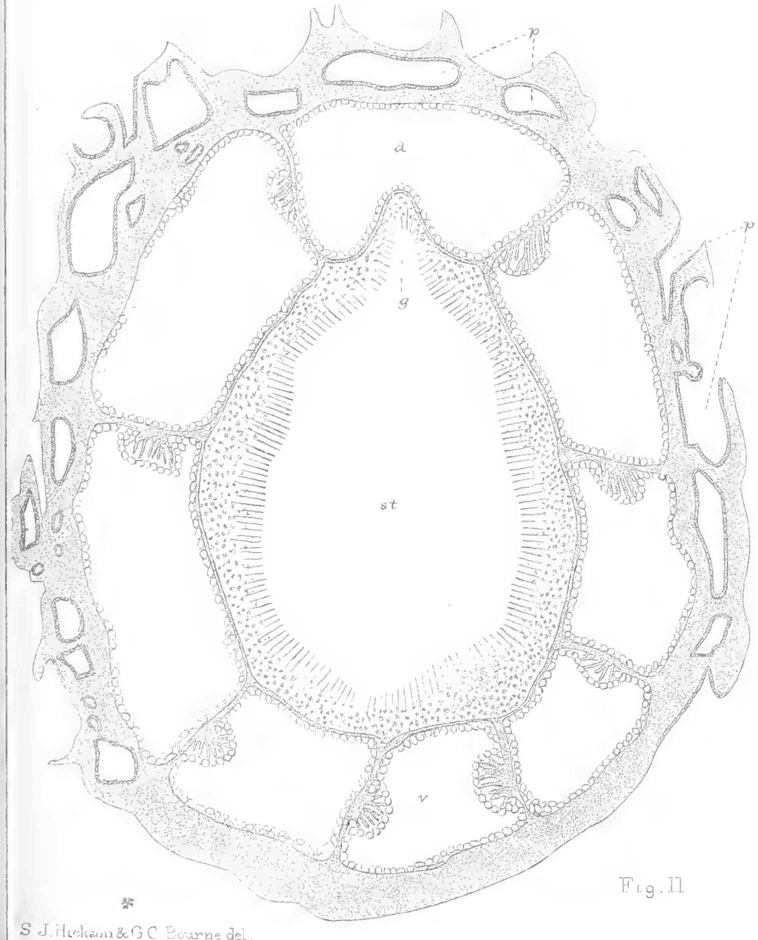

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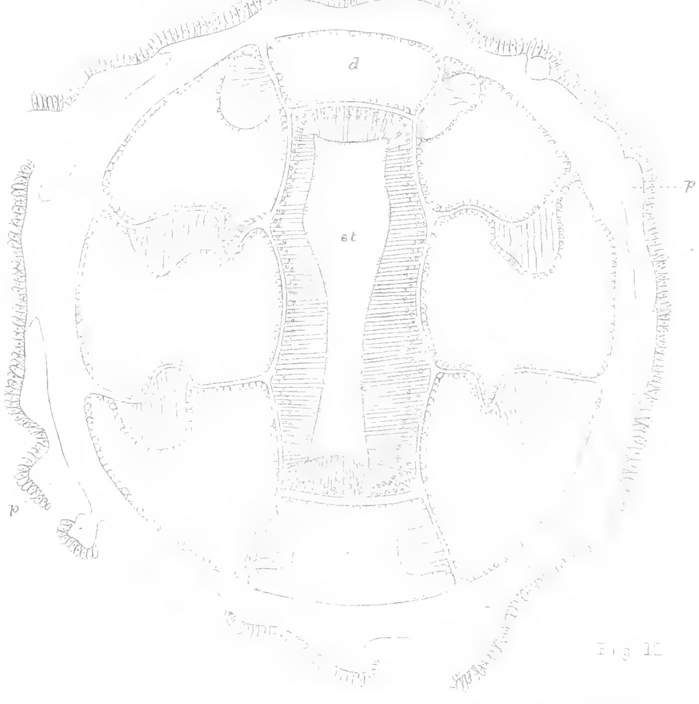

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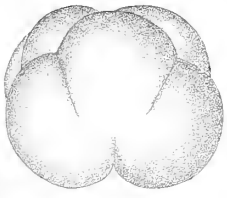
T. H. Wesley lith.
West, Newman & Co. imp.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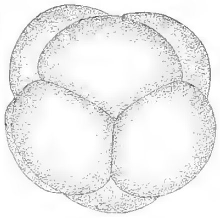
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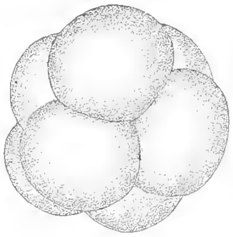
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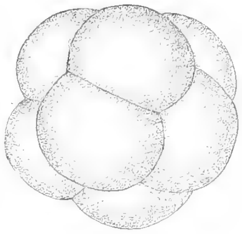
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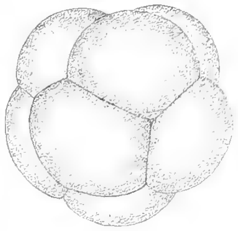
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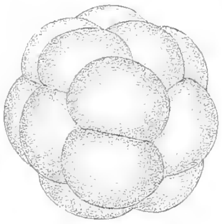
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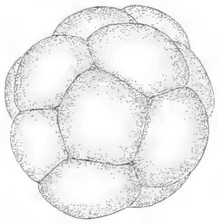
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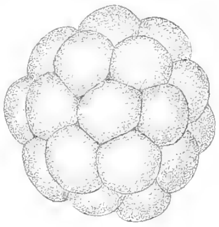
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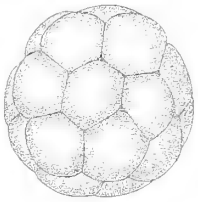
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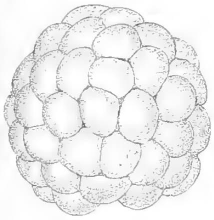
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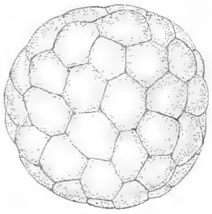
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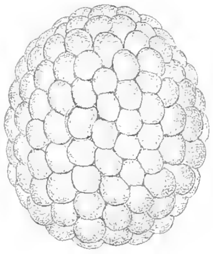
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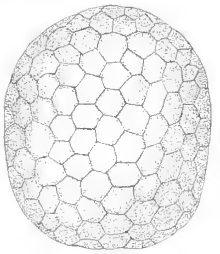
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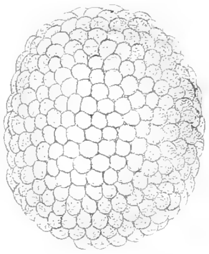
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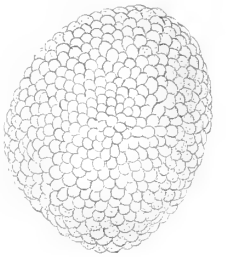
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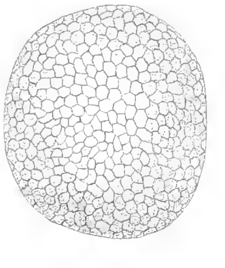
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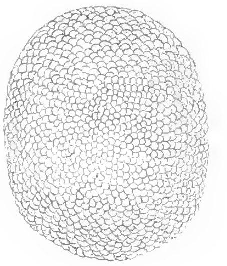
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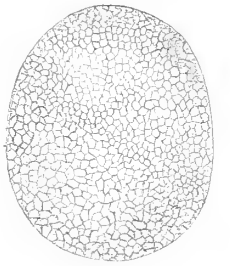
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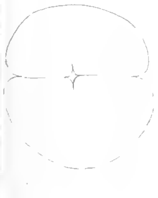
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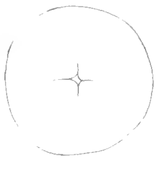
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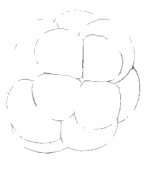
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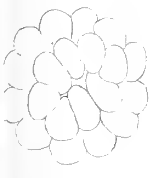
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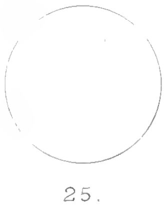
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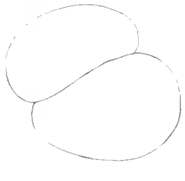
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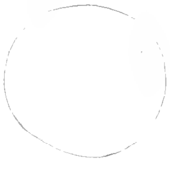
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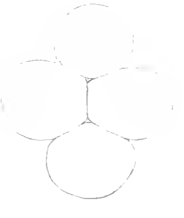
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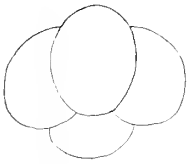

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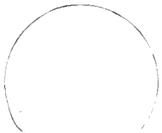

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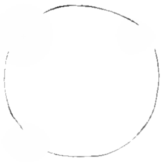

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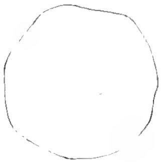

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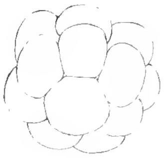

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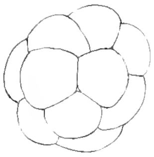

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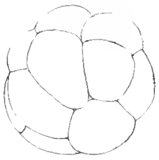

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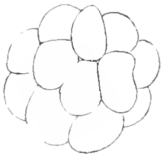

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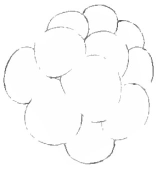

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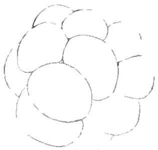

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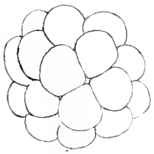

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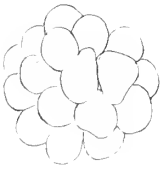

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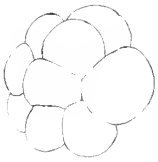

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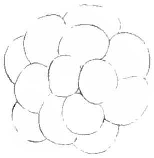

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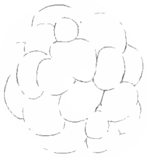

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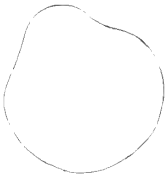

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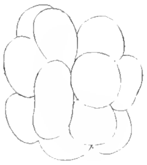

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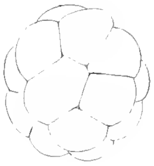

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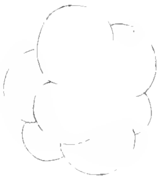

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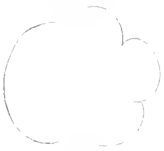

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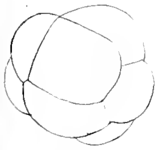

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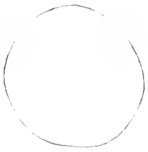

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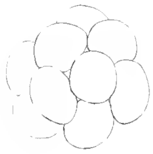

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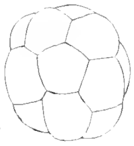

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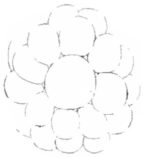

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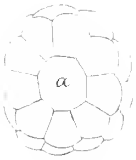

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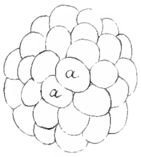

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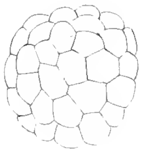

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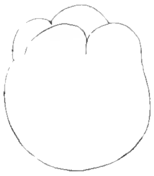

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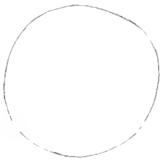

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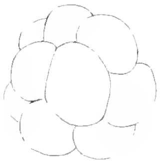

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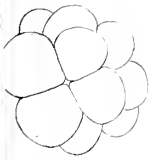

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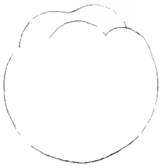

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West Newman & Cº lith.

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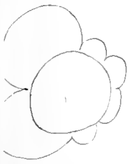
Phil. Trans. 1883. Plate 54.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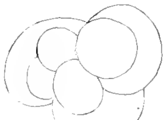
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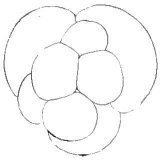
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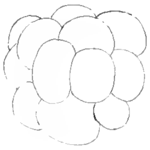
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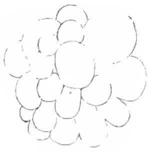
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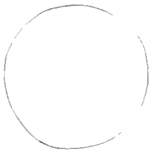
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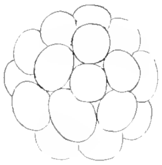
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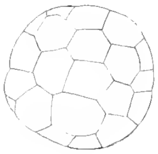
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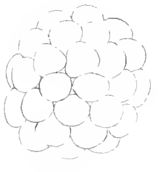
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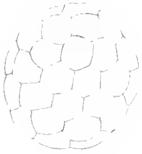
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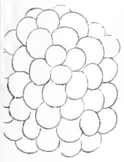
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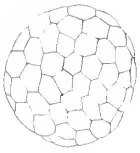
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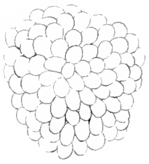
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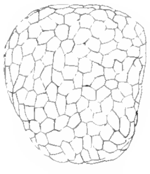
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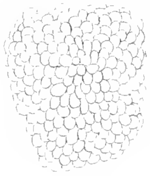
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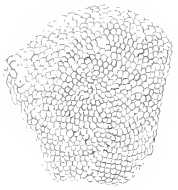
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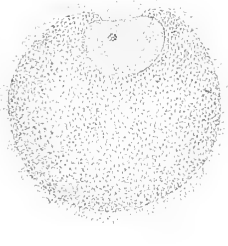
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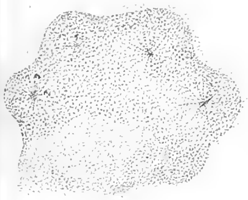
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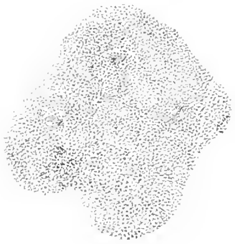
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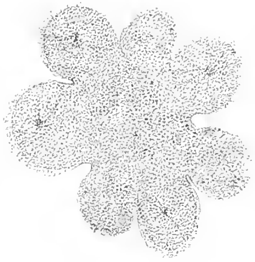
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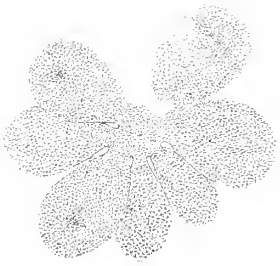
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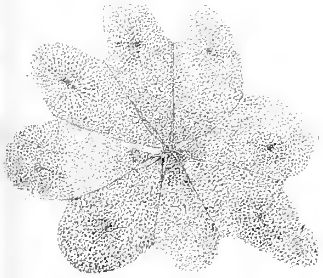
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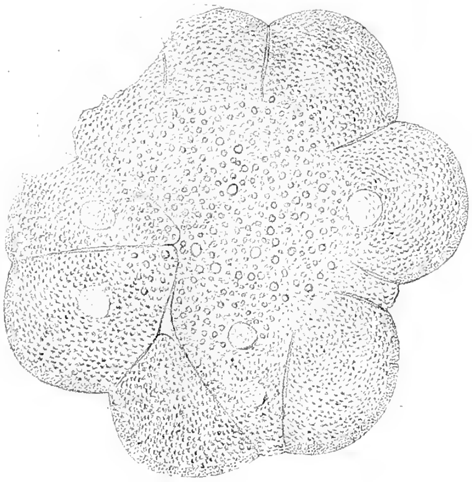
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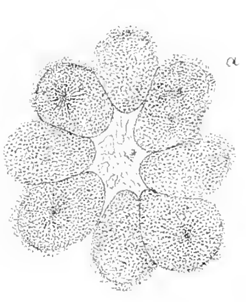


94.

West Newman & Co. lith

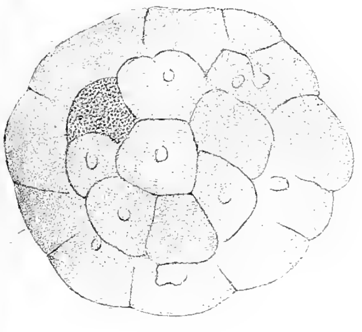
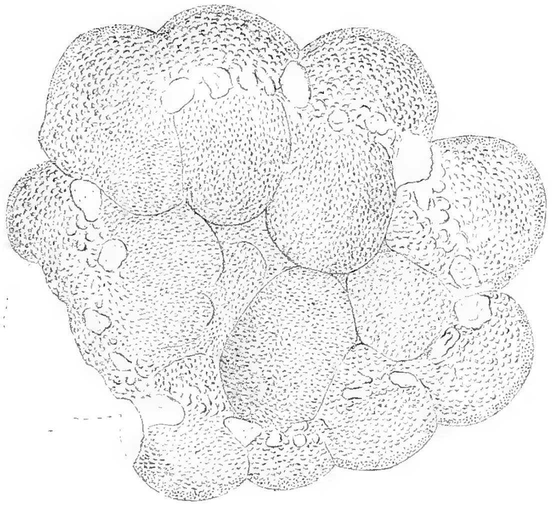


Fig. 97.



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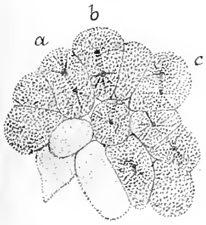

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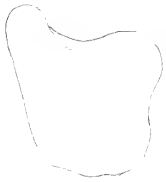

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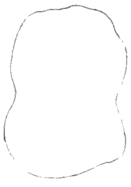

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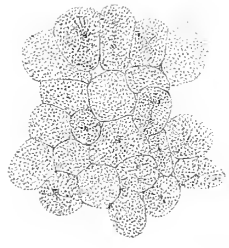

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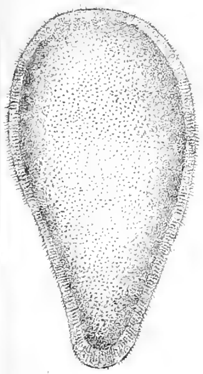

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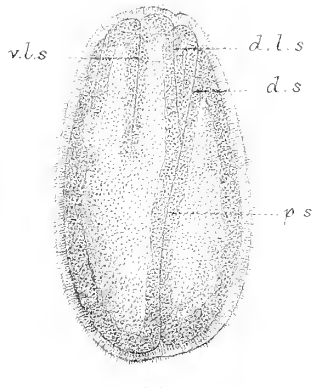

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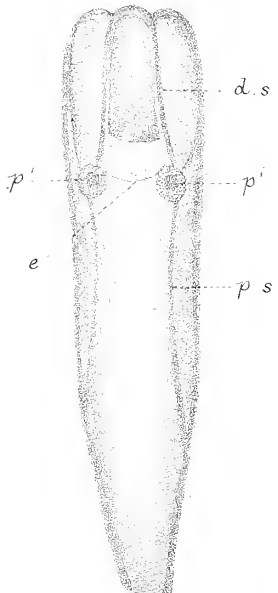

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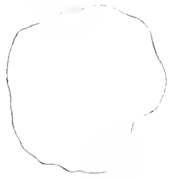

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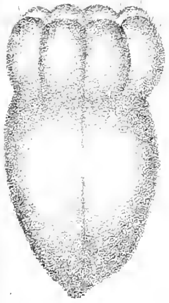

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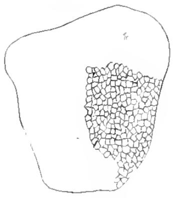

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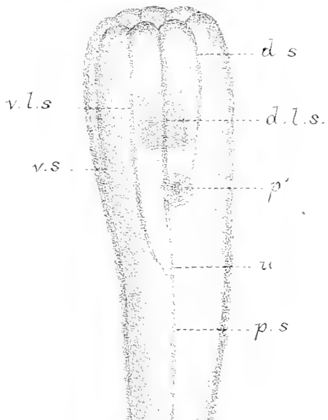

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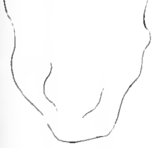

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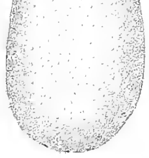

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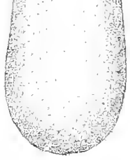

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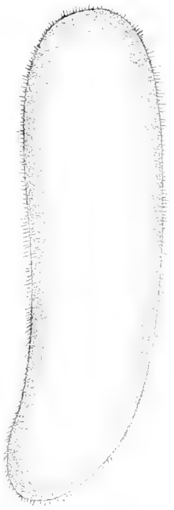
Fig. 110.



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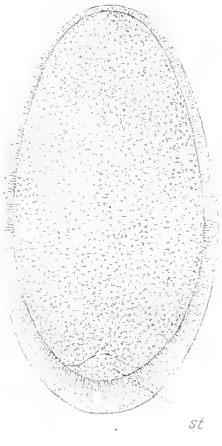
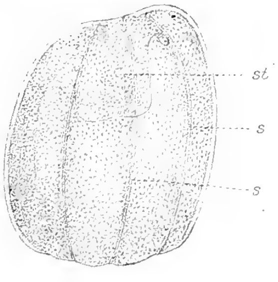


Fig. 112.

Fig. 113.



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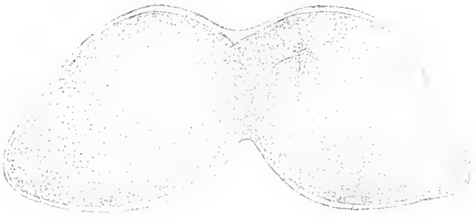

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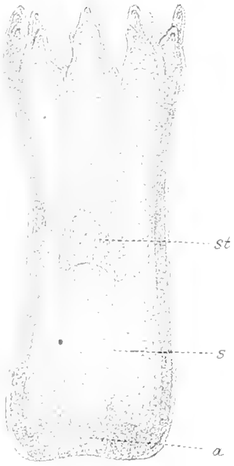


Fig. 117.

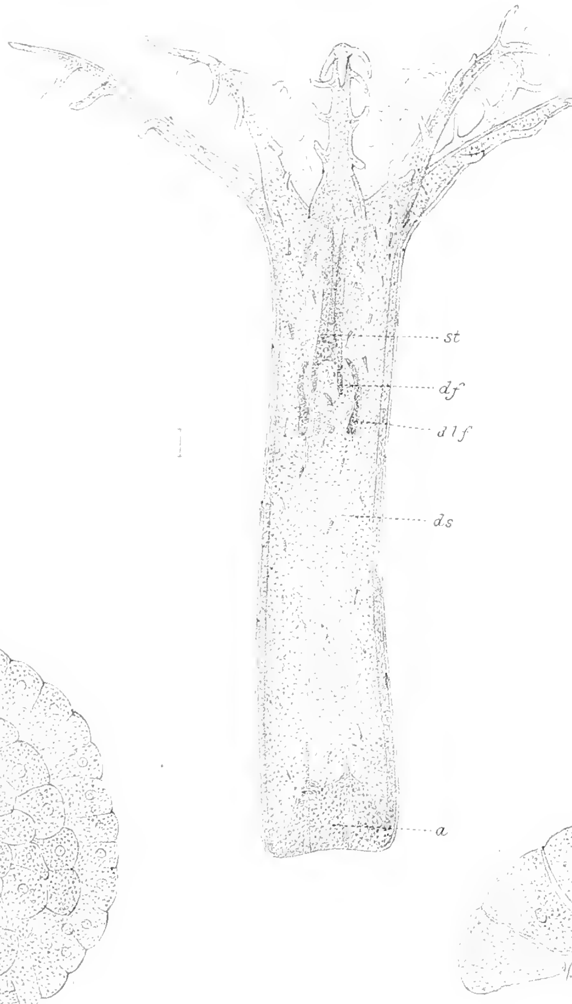
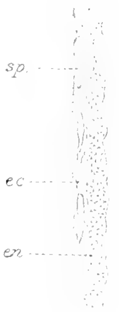


Fig. 116 a.



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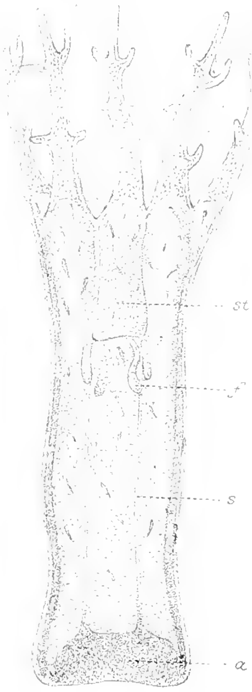

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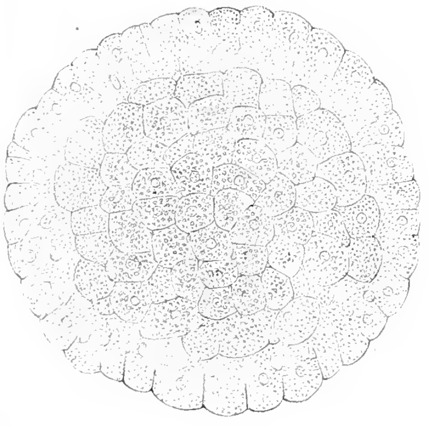

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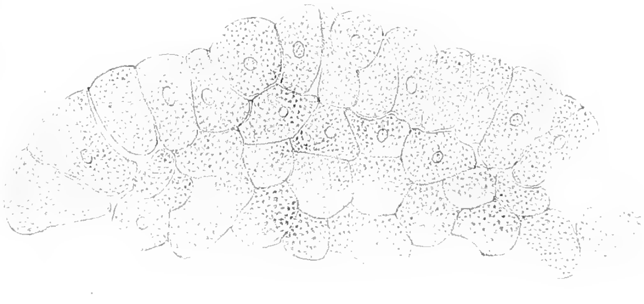
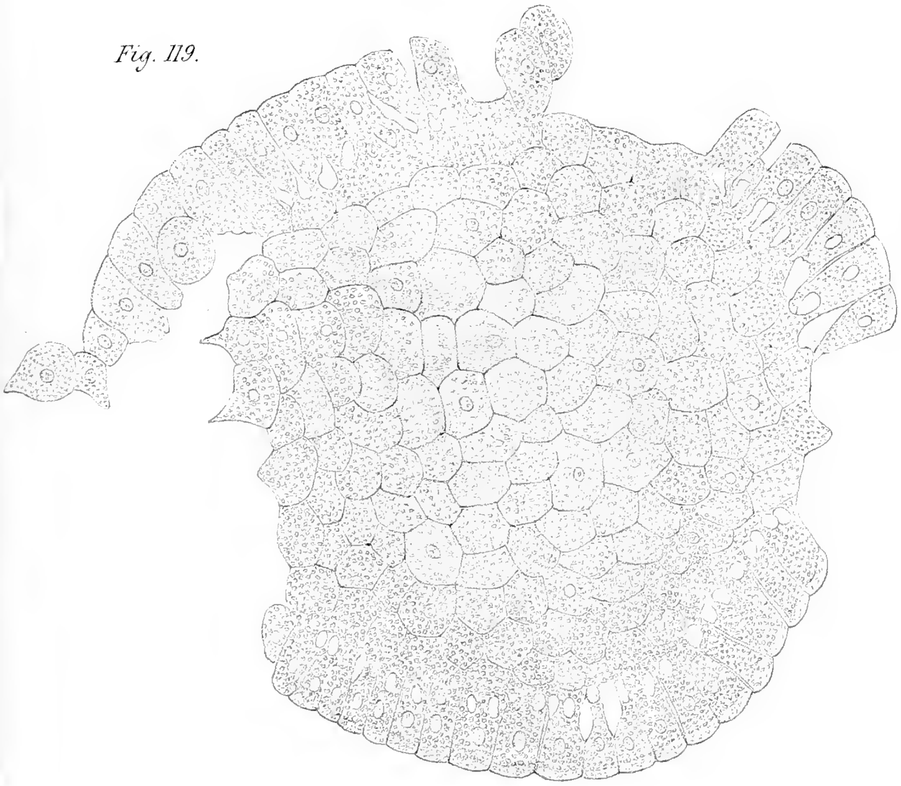


F. Huth, Lith^r Edin^r



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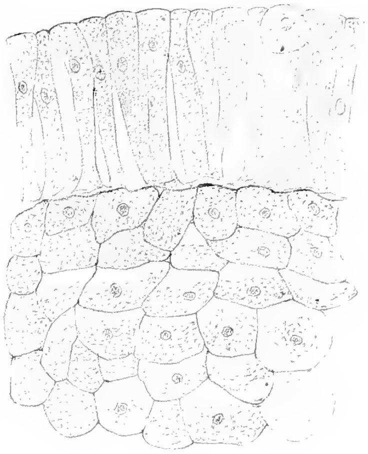

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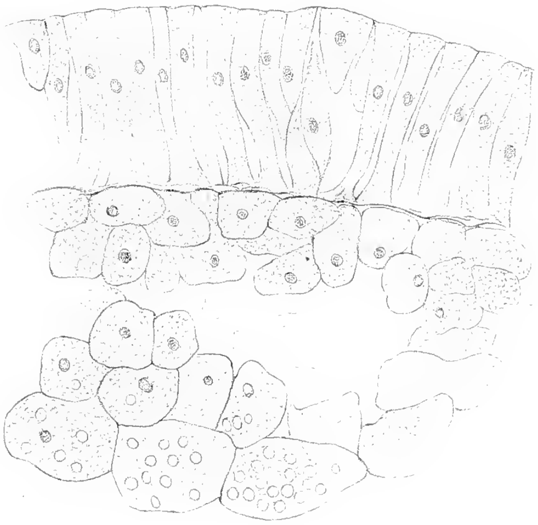

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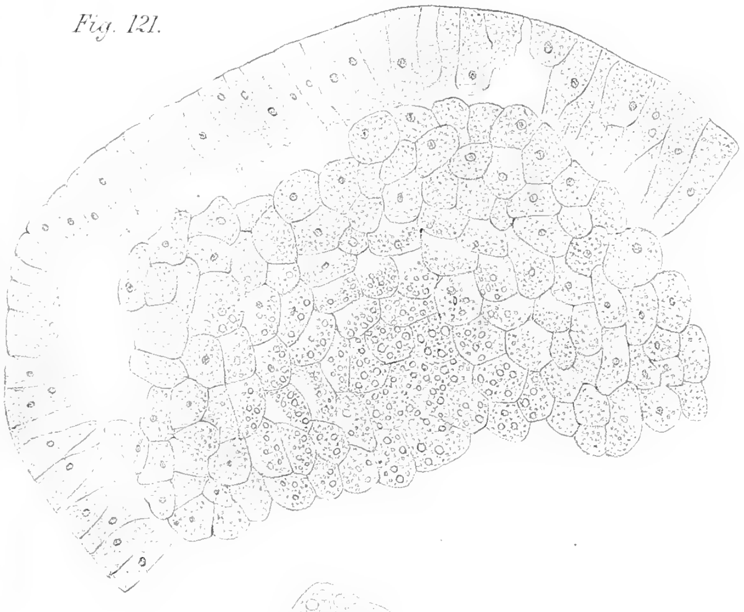

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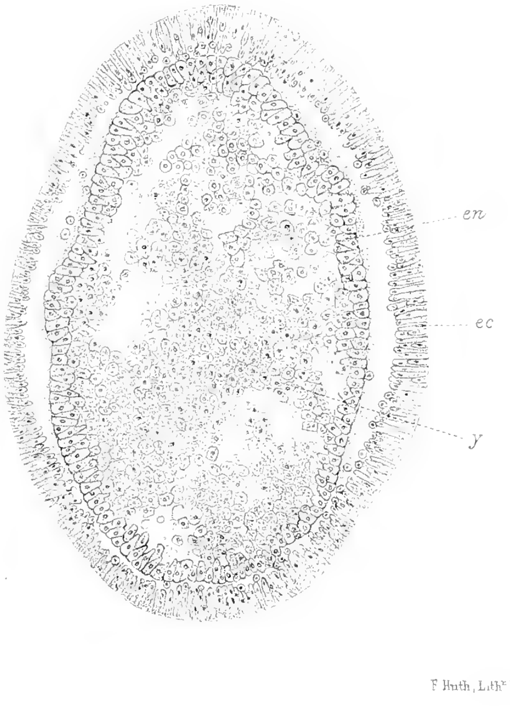

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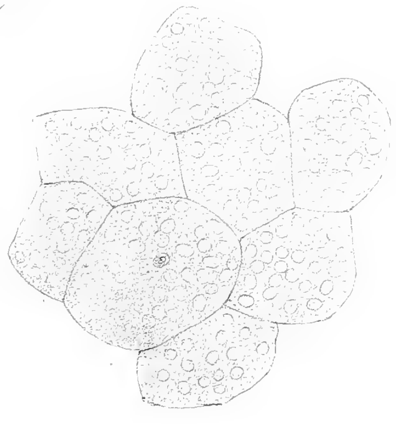


Fig. 124.

F Huth, Lith^{r} Edin^{r}

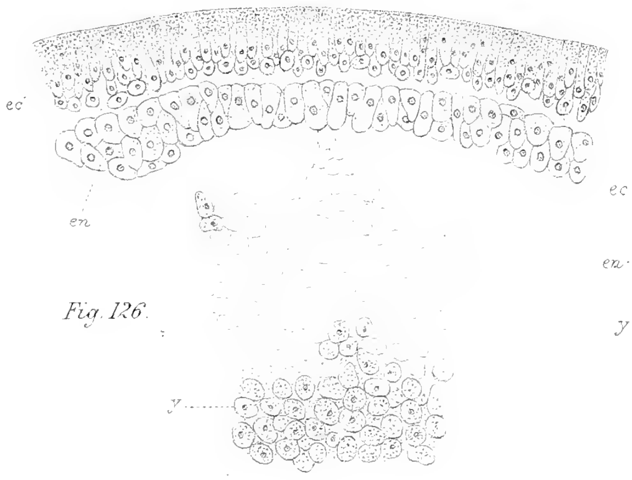
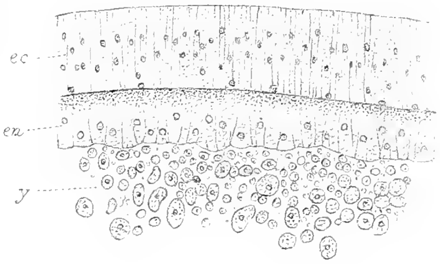


Fig. 126.

Fig. 127.



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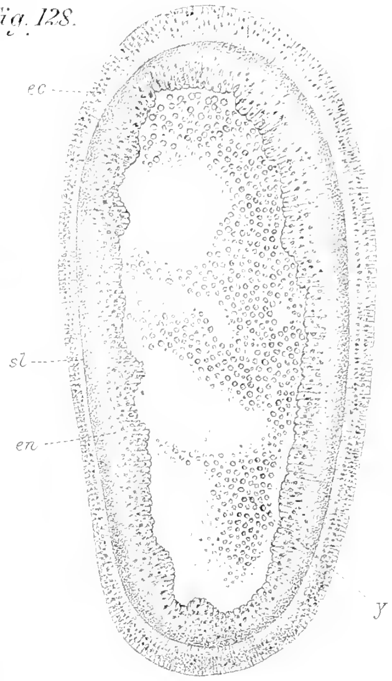

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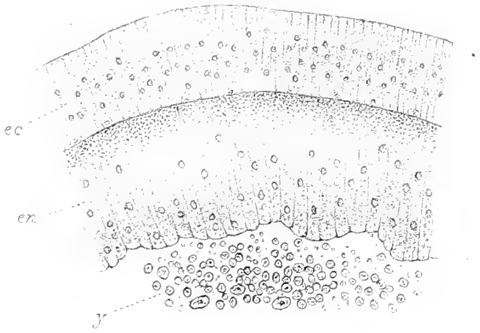

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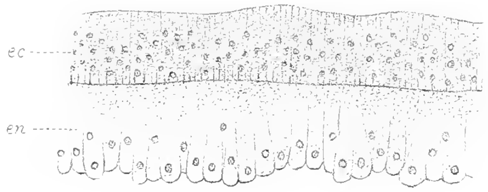


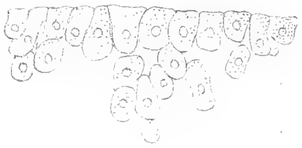
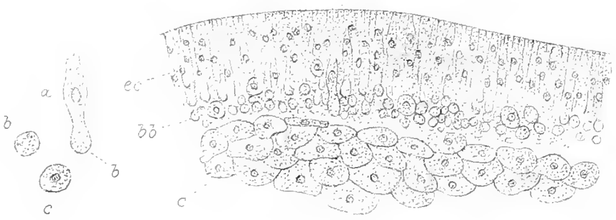


Fig. 134.

Fig. 132.



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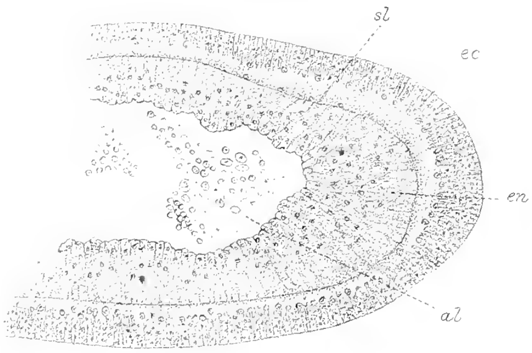

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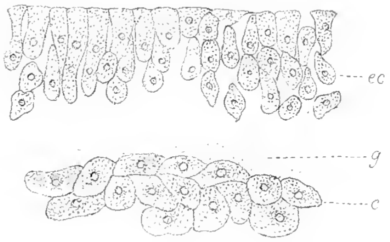


F Huth, Lithr Edinr.

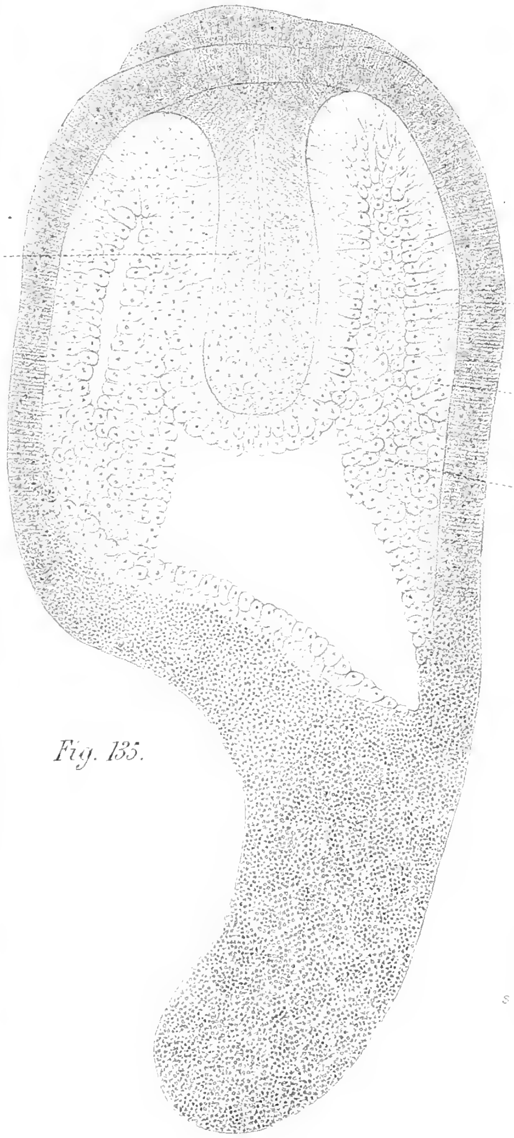


Fig. 135.

Fig. 136.

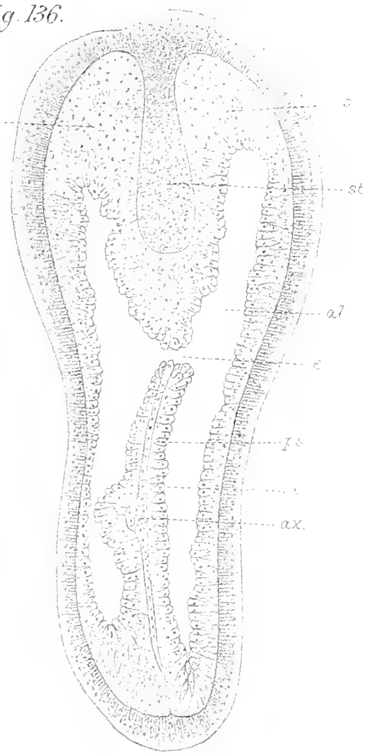


Fig. 137.

Fig. 138.

Fig. 141.

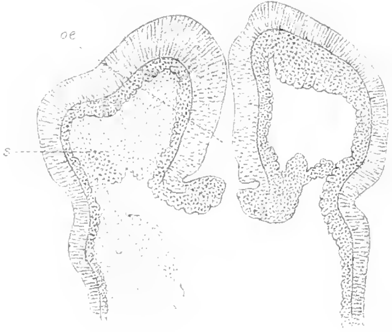
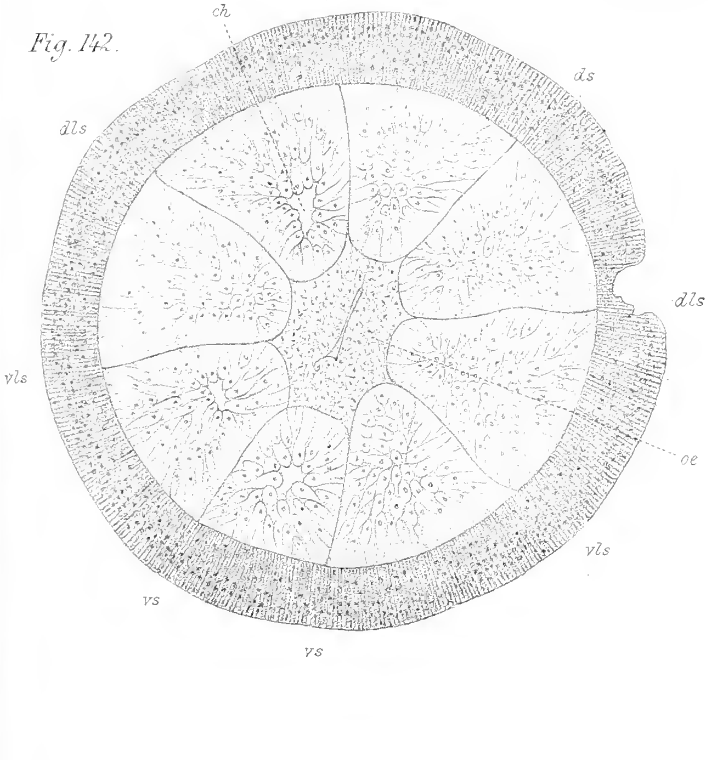


Fig. 139.

Fig. 142.



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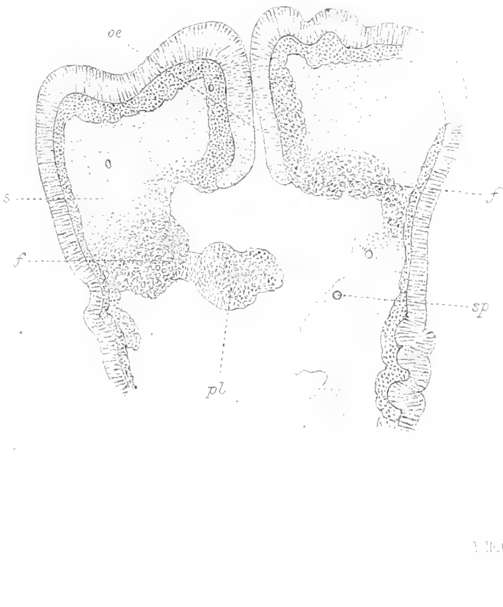


[illegible] Edin^r

Fig. 143.

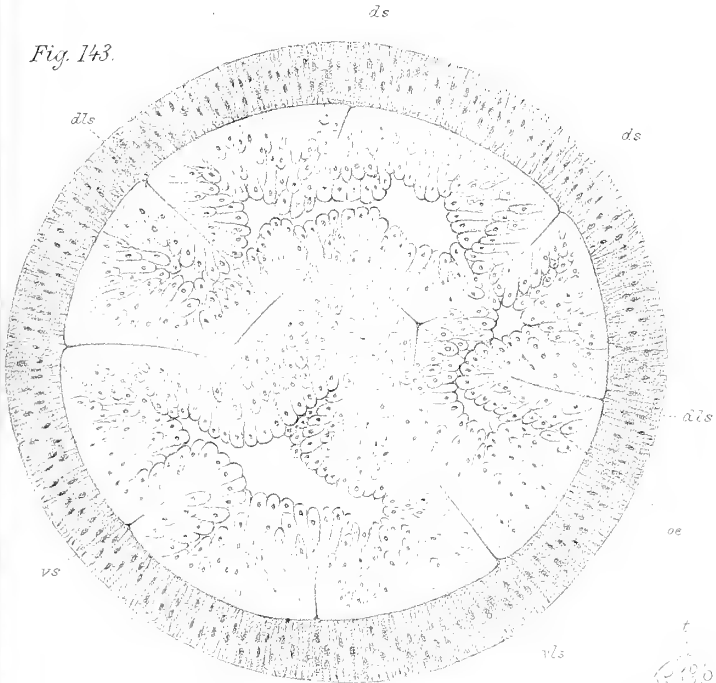
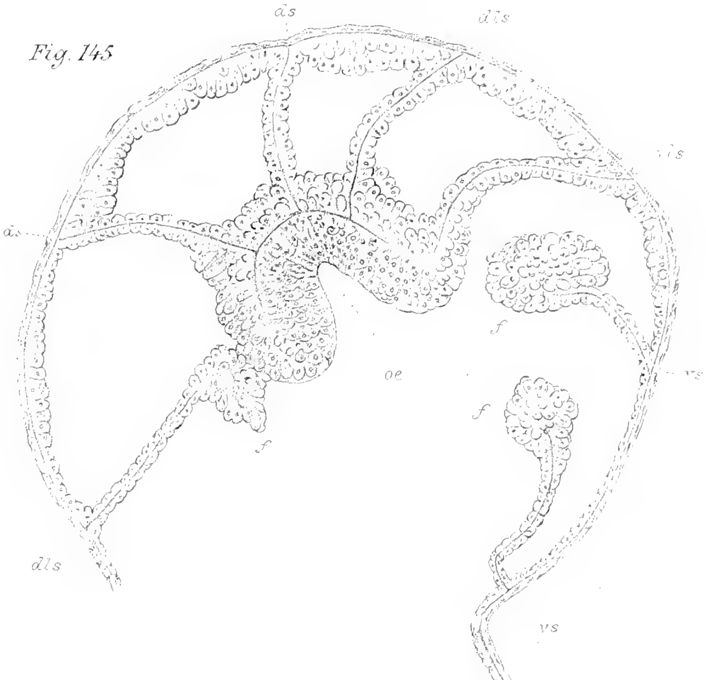


Fig. 145



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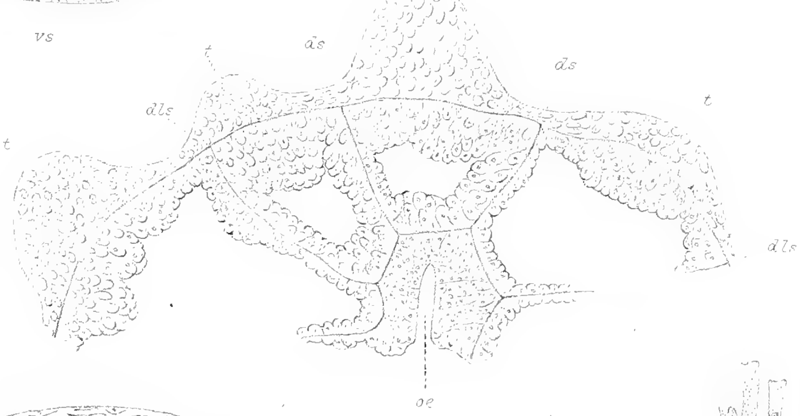

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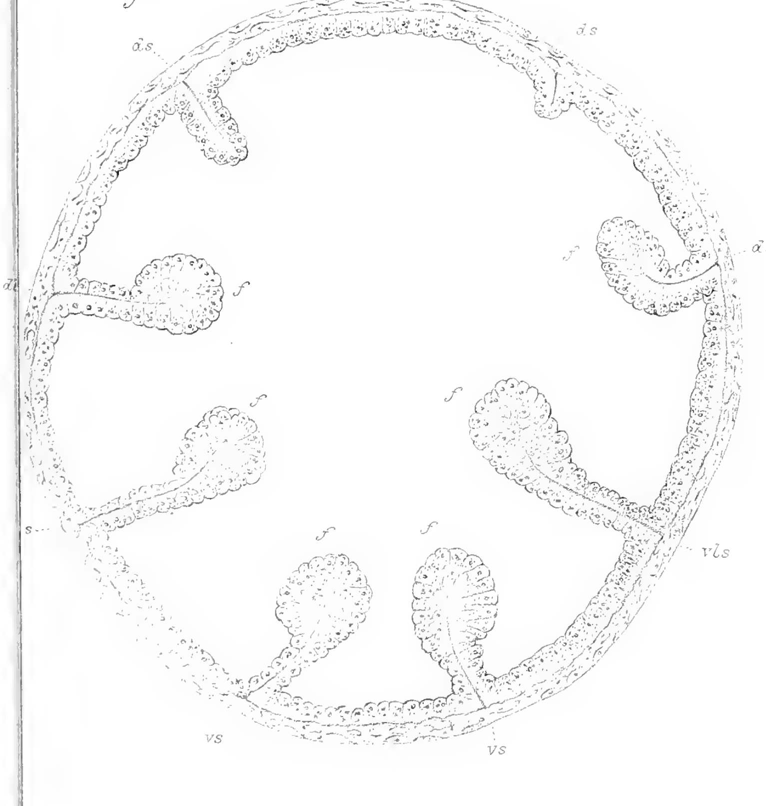
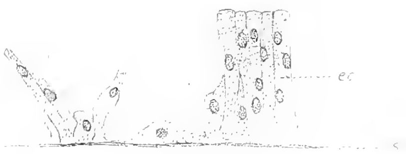


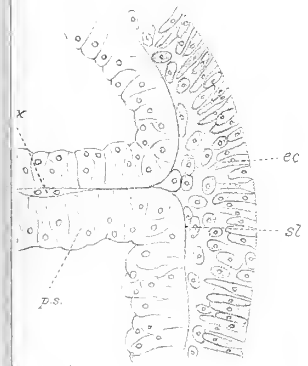
Fig. 147.

Fig. 147 a



F. Huth, Lith^r Edin^r

Fig. 148.



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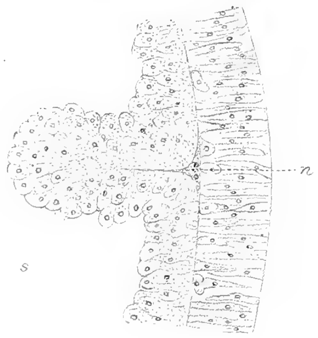

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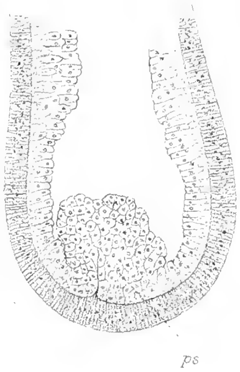

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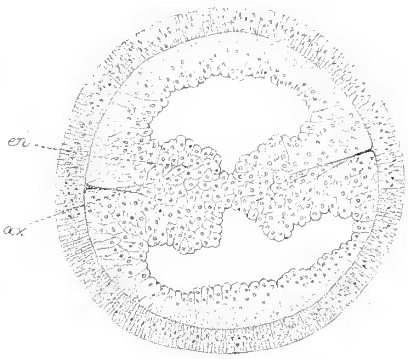

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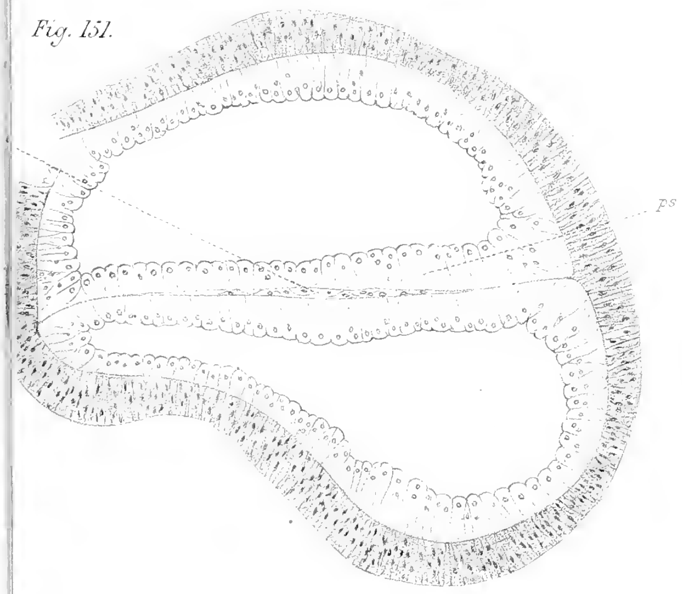

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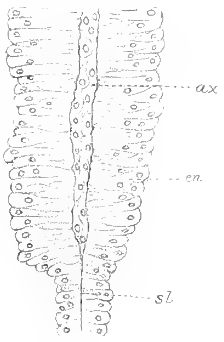


Fig. 156

Fig. 154.

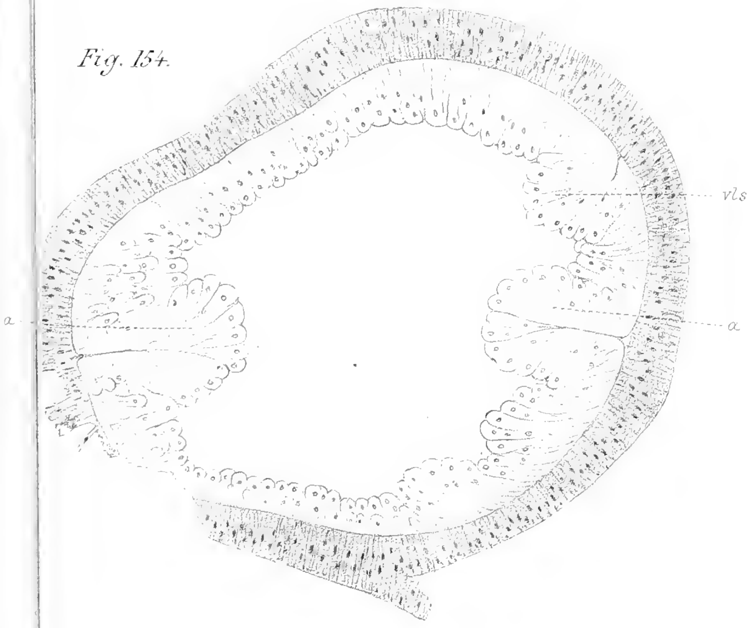


Fig. 155.

F Huth, Lith^r Edin^r

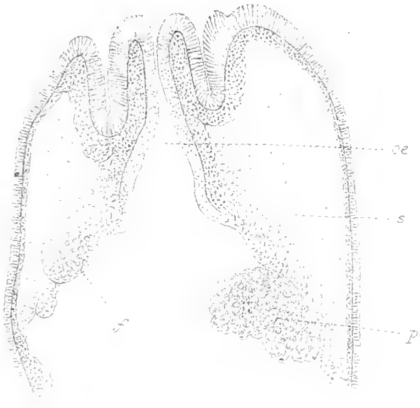


Fig. 157.

Fig. 158.

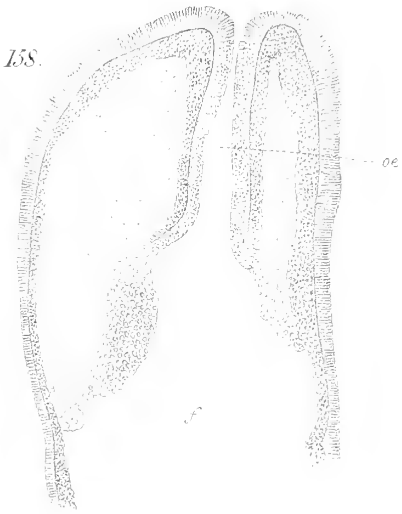


Fig. 159.

Fig. 160.

Fig. 162.

Fig. 163.

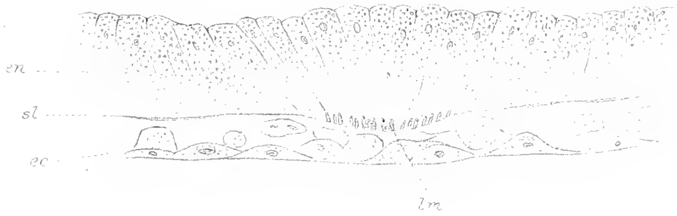
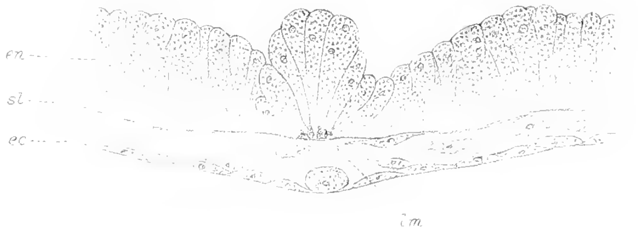


Fig. 161.

Fig. 164.



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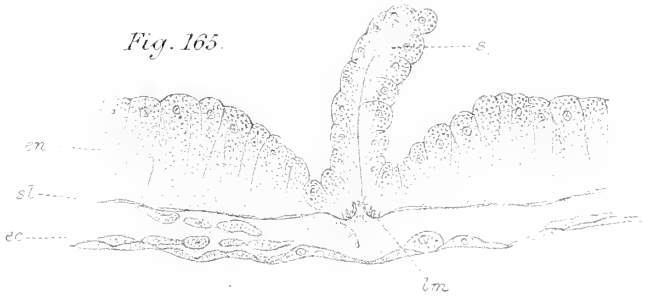
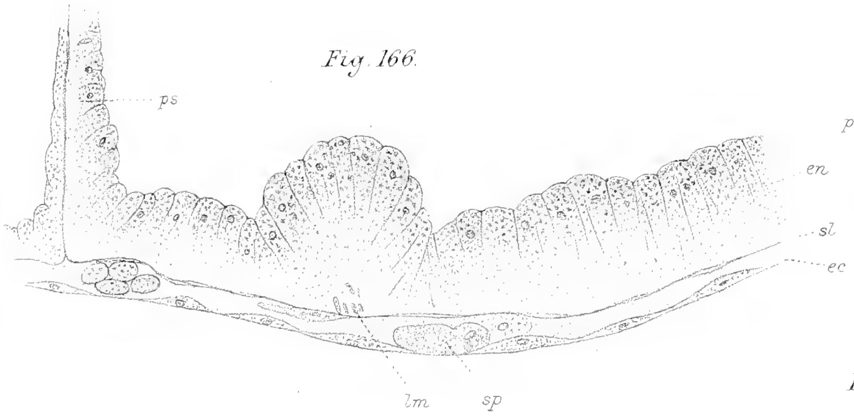


F. Huth, Lith.r Edin.r

Fig. 166.



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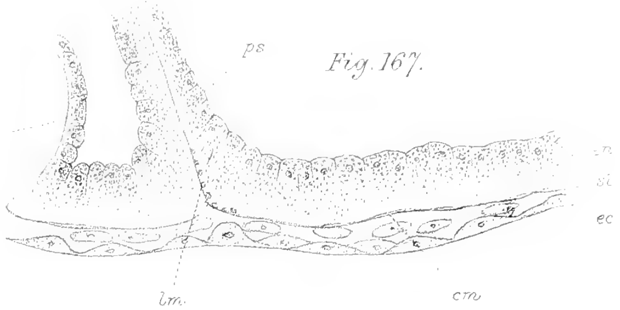

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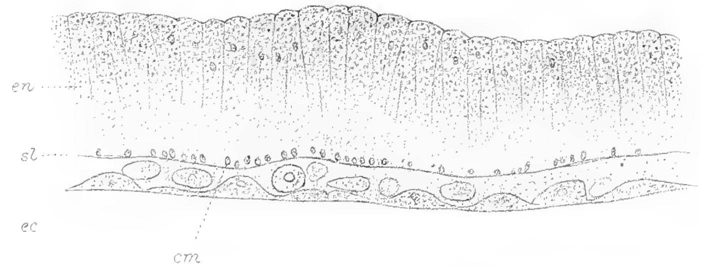

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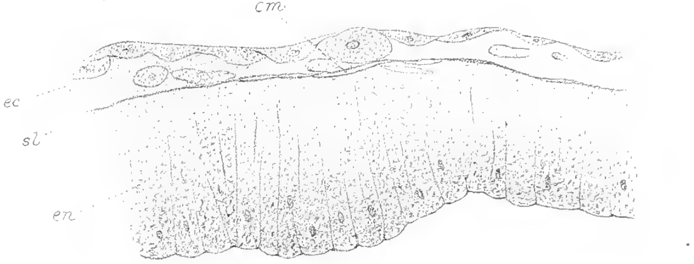

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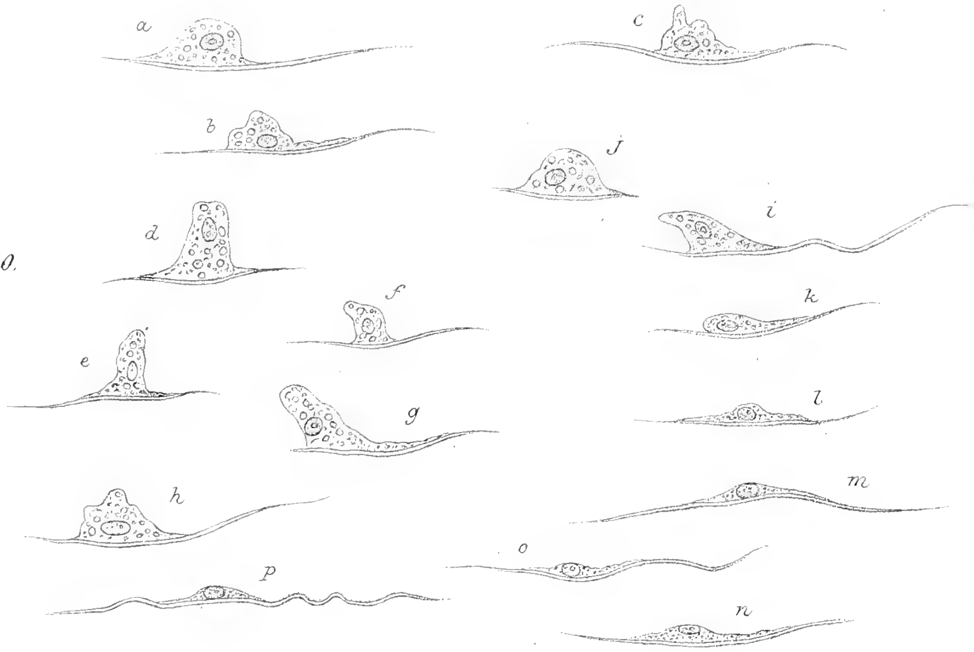

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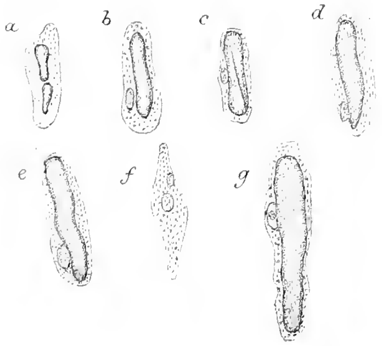

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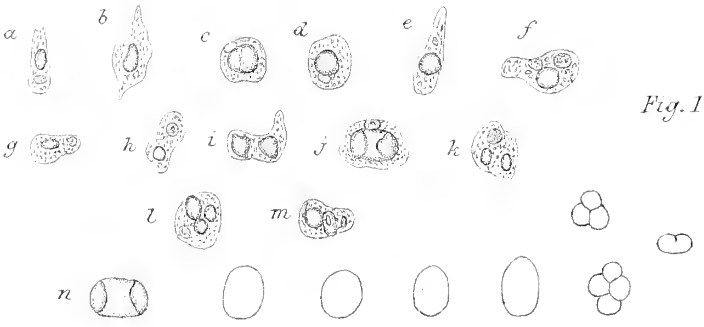

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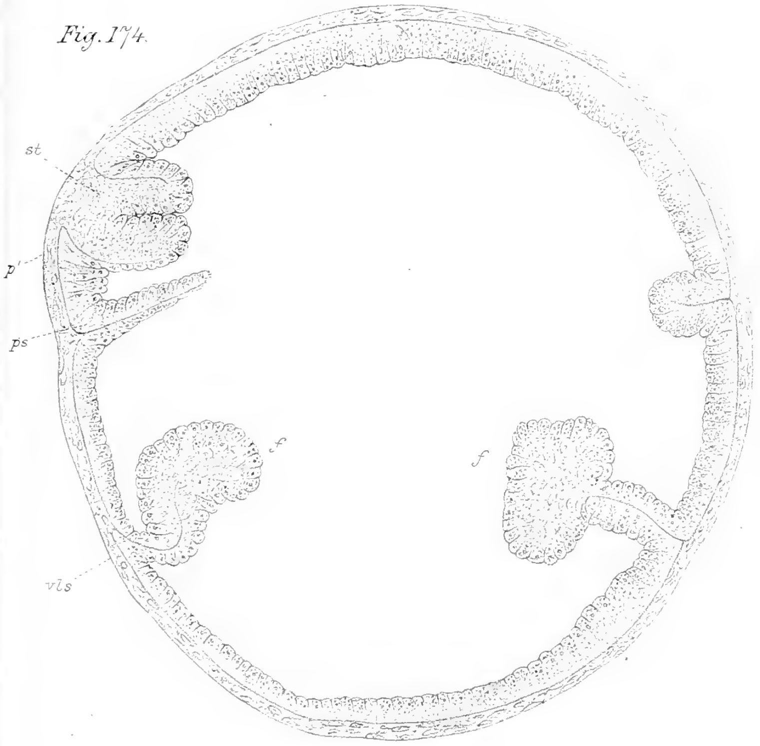
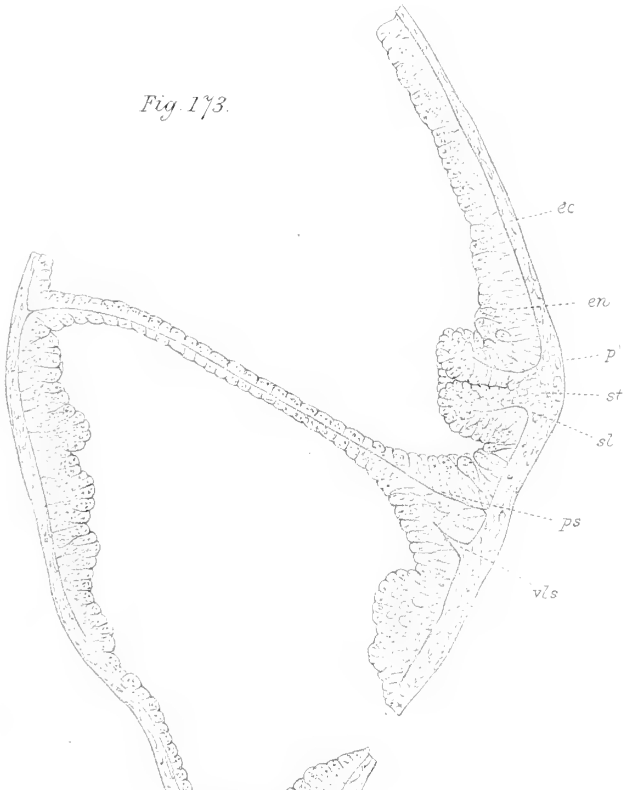


Fig. 175.

Fig. 173.



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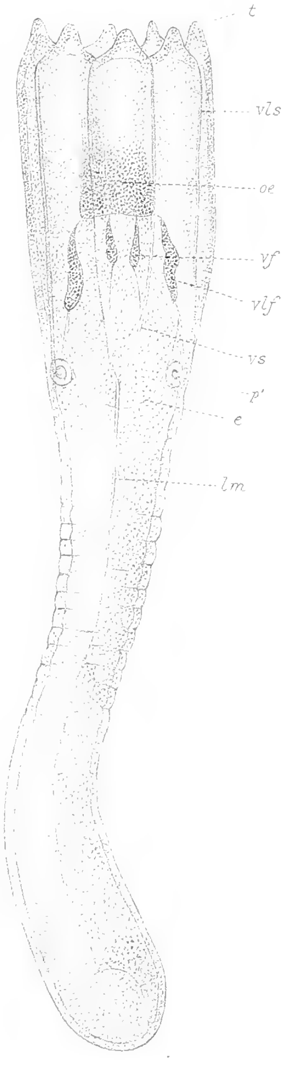

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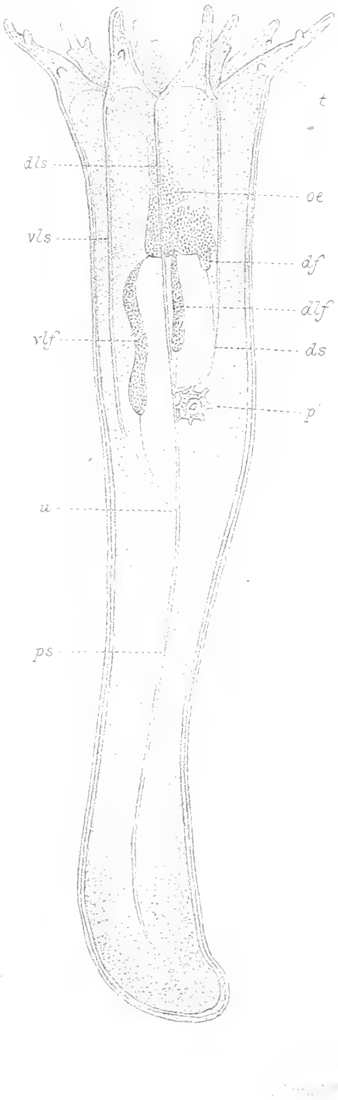
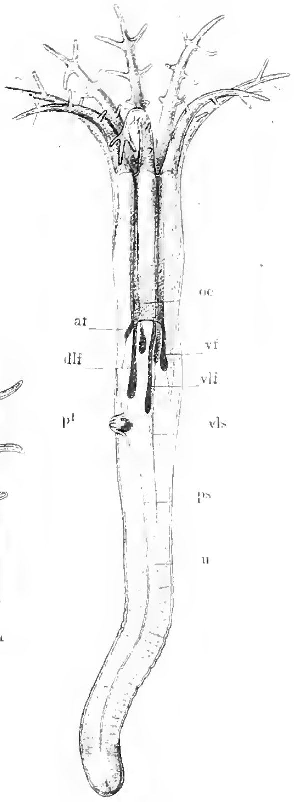
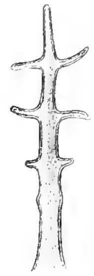




178

x40



178a

x60

61

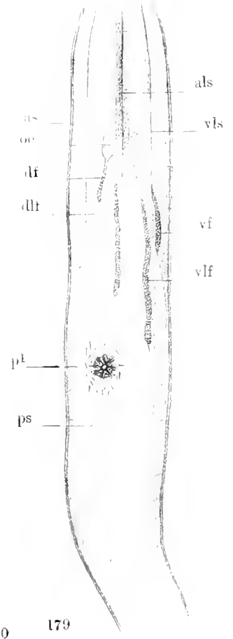
178b

x120

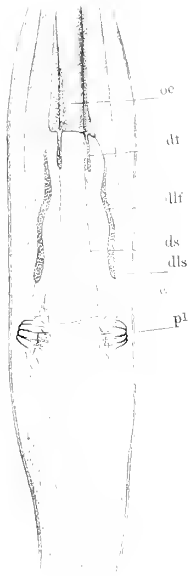
62

178c

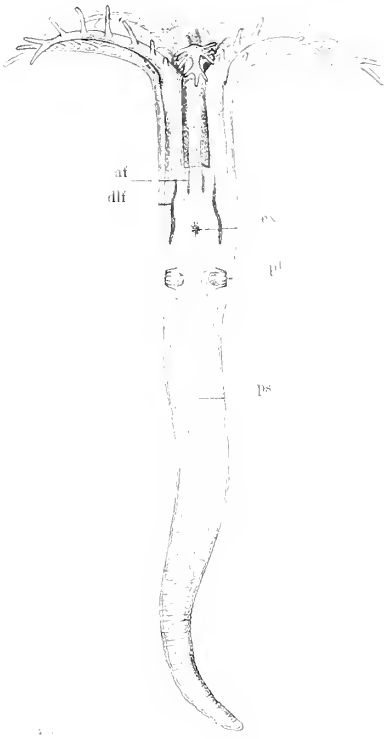
x120



x60 179

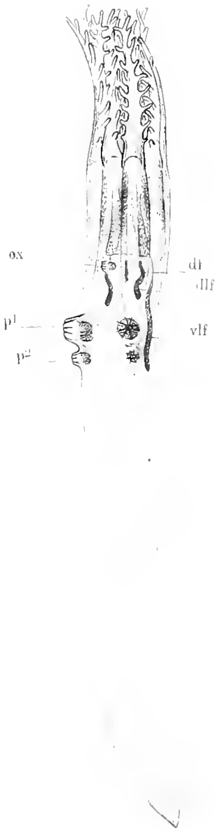


180



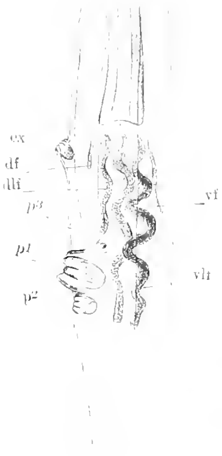
x40

181



182

x30

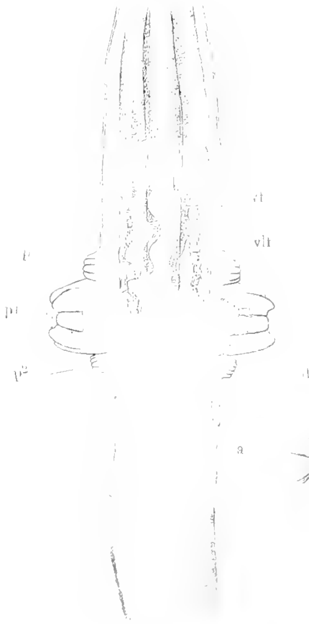


183

x40

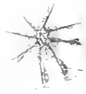
183a

183b



x50

184

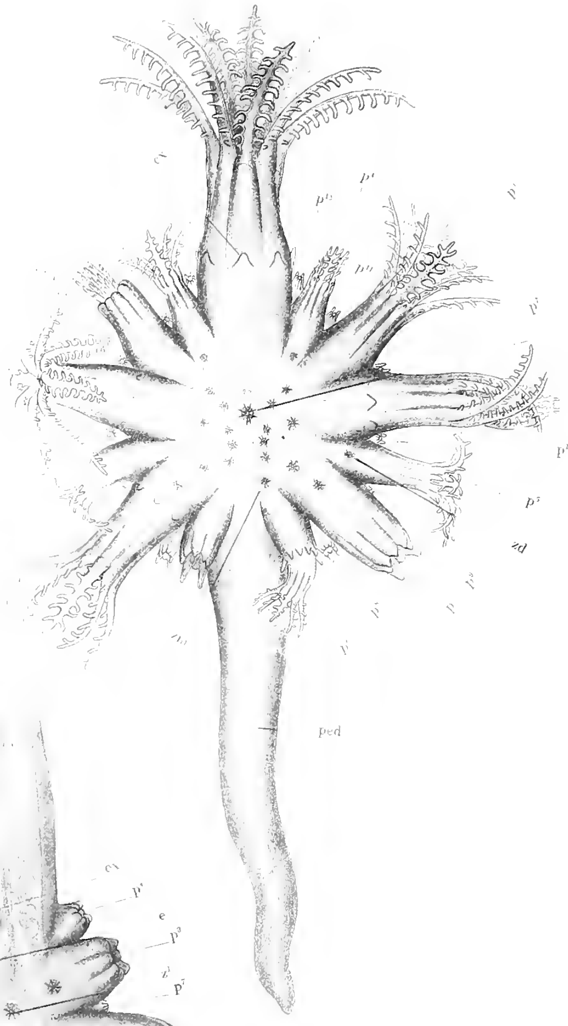


184

Photographed and Printed by the Photomechanical Printing Company 1a Lansdowne Road, Tottenham

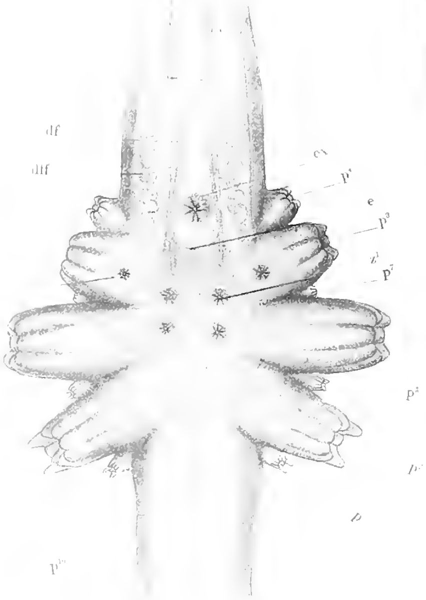
185

x40



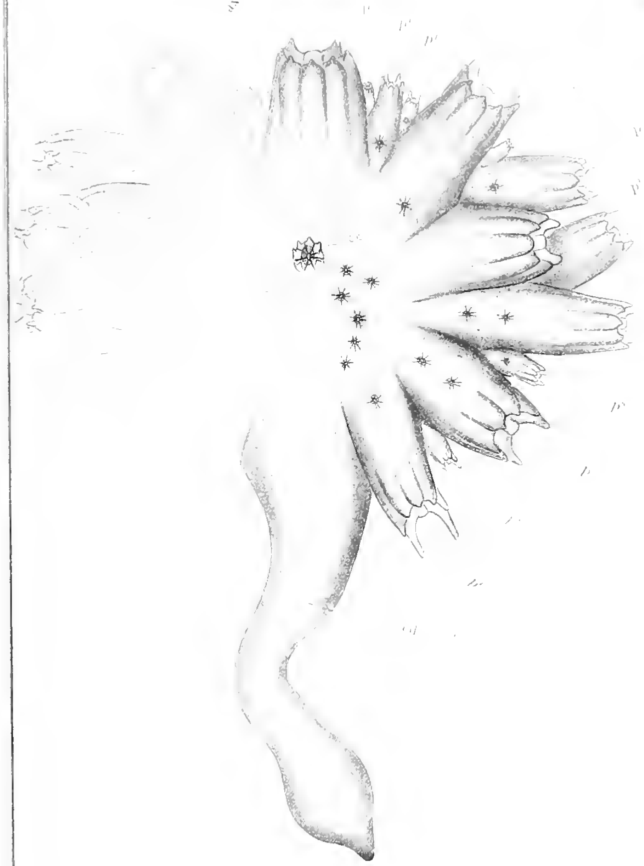
187

x30



186

x60



188

x30

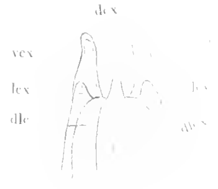
189

x15

YOGRAPHED AND PRINTED BY THE PHOTOMECHANICAL PRINTING COMPANY 1A LANSDOWNE ROAD TOTTENHAM

190

191

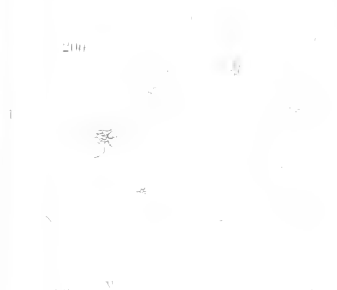


192

x20

200

201



x80

207

206

x50

x50

itographed and Printed by the Photomechanical Printing Company 1a Lansdowne Road Tottenham

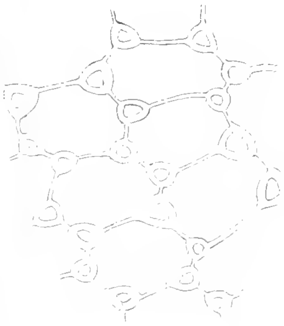


Fig 4

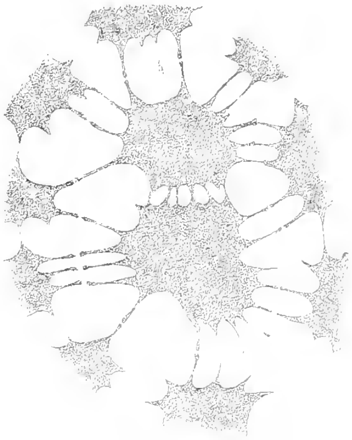


Fig 7

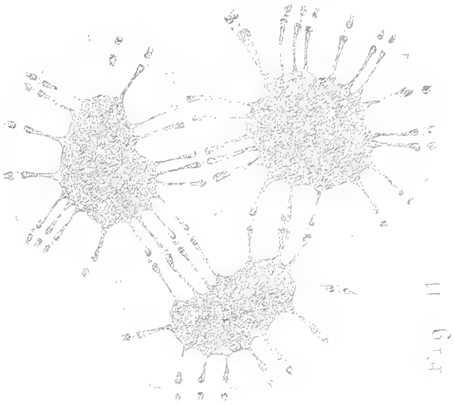
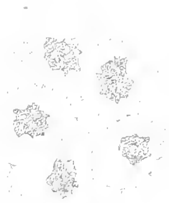


Fig 11


Fig 3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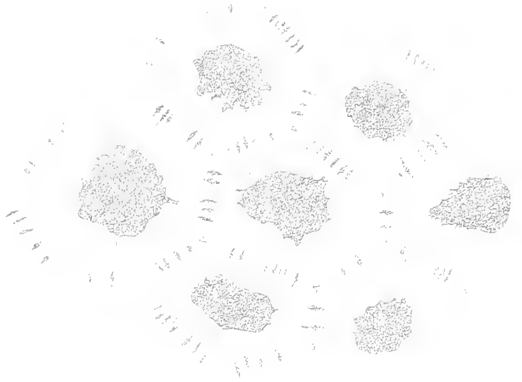
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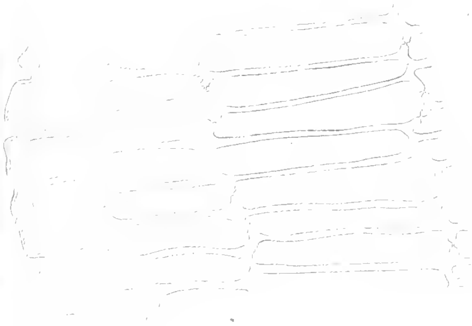
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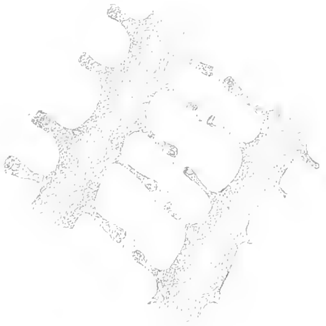
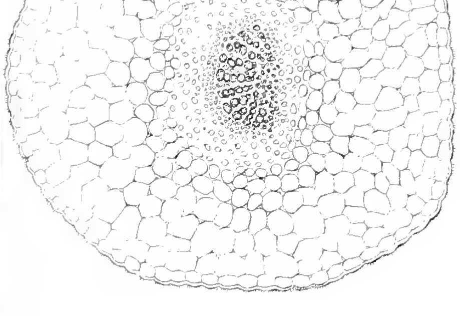


Fig 6


Fig 12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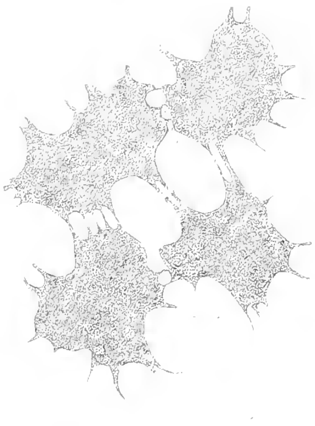
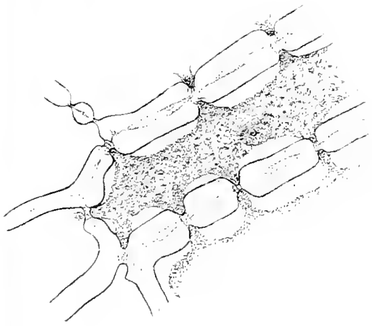


Fig 5


Fig 9

W Gardiner del

West, Newman & Co lith.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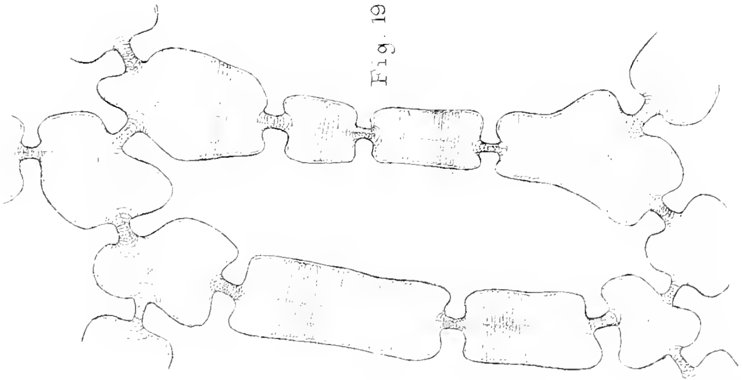
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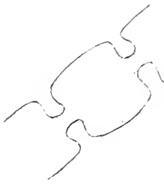
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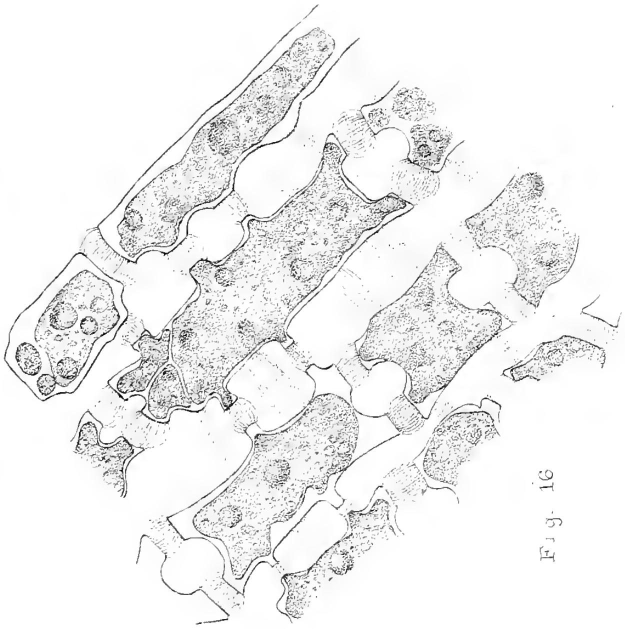
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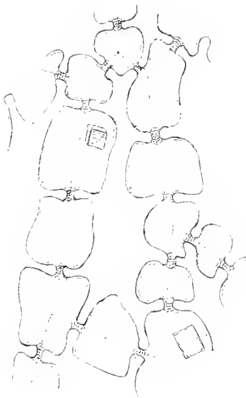
Fig. 25.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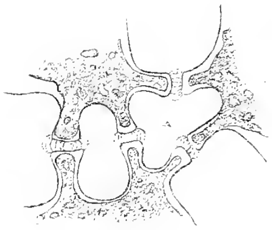
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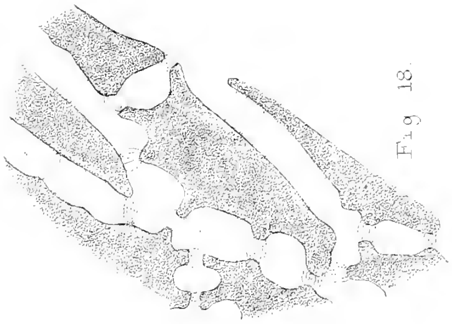
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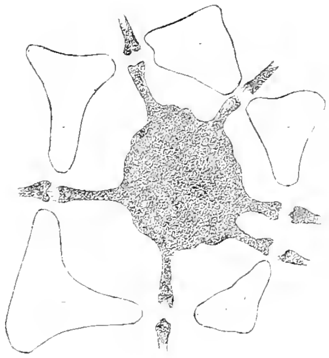
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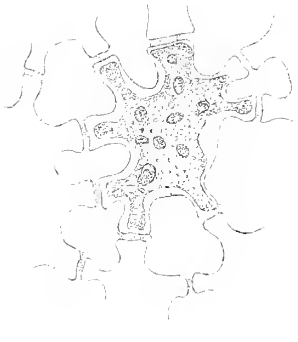
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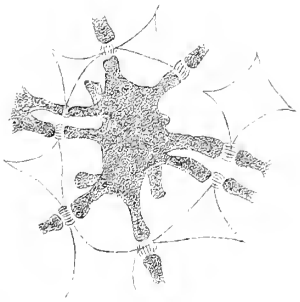
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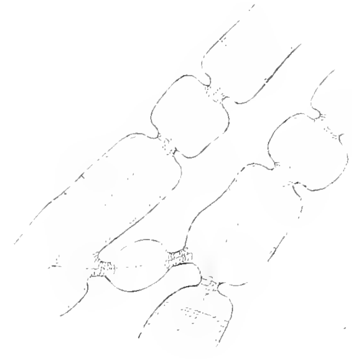
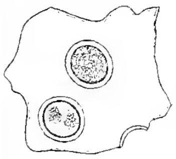


Fig. 17.


Fig. 21.

W. Gardiner del.

West, Newman & Co. lith.



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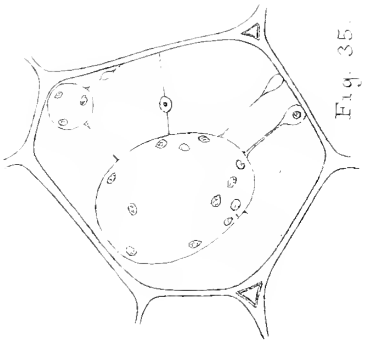

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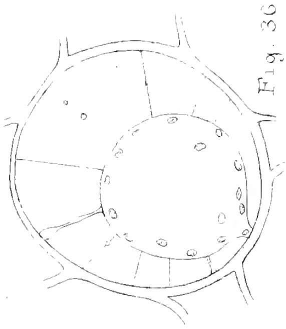

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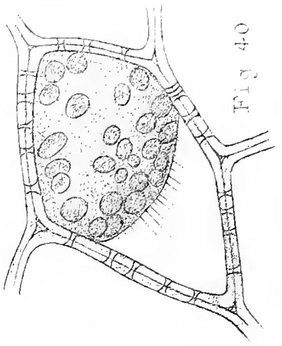

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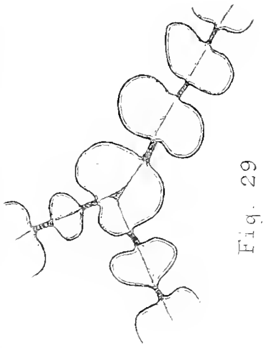

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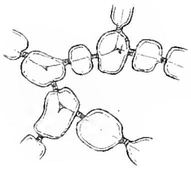

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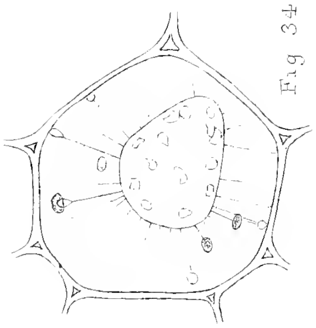

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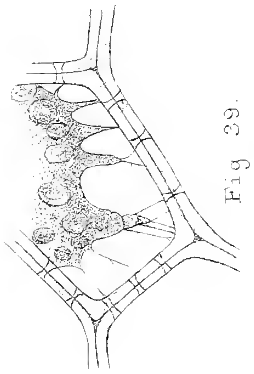
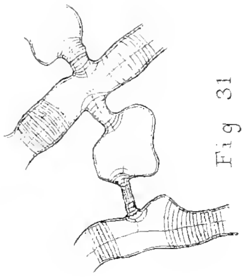


Fig. 39.

Fig. 27.



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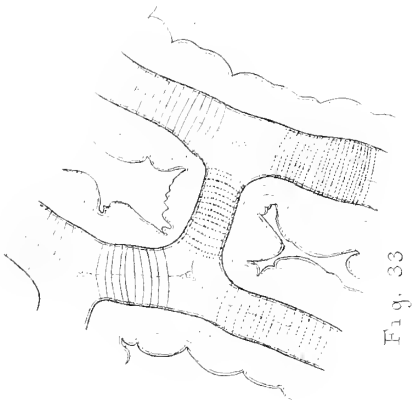

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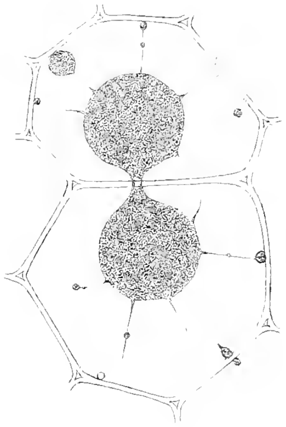

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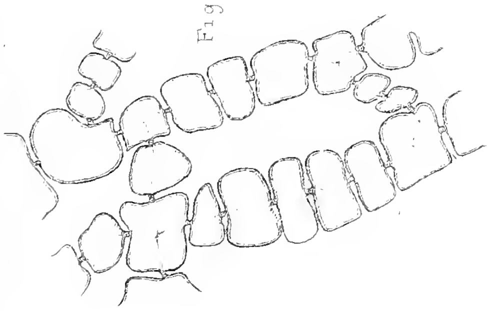

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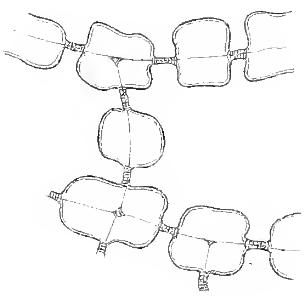

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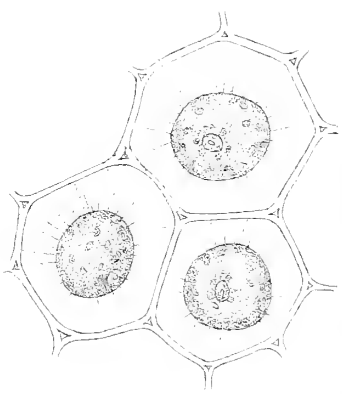
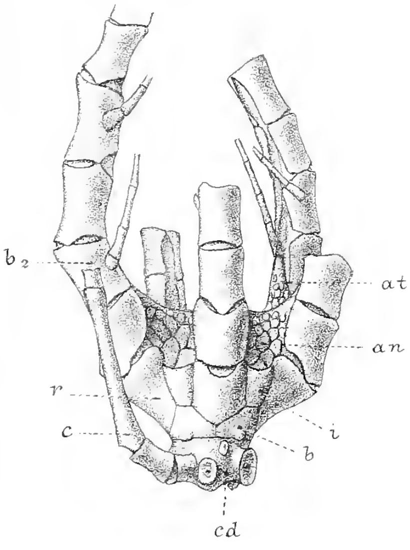


Fig. 37.

W. Gardiner del.

West Newman & Cº lith.



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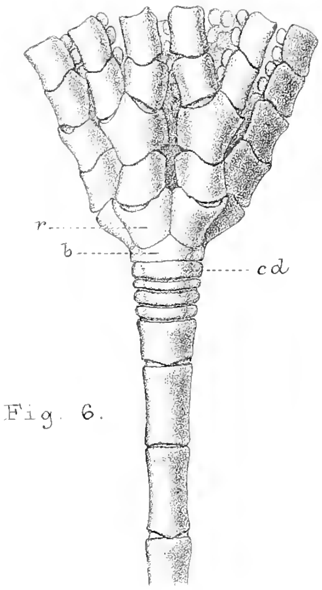

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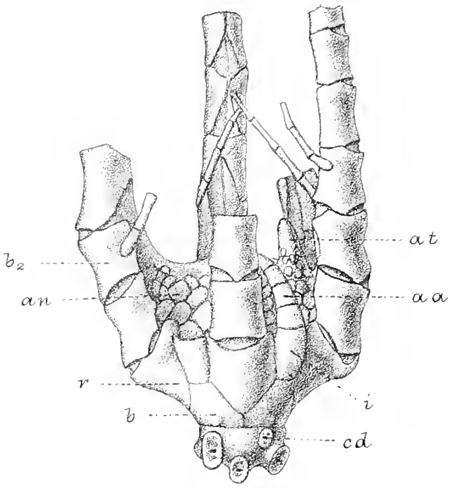

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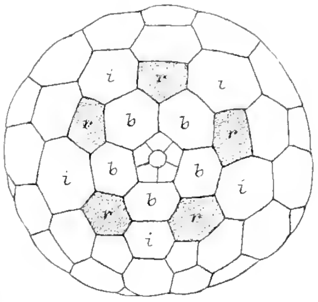

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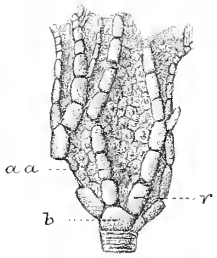

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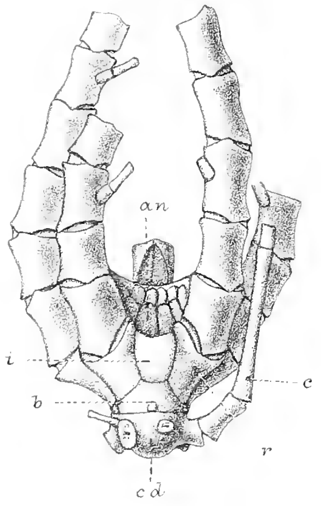

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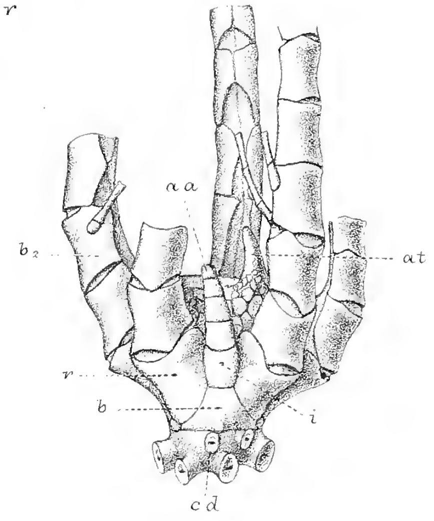

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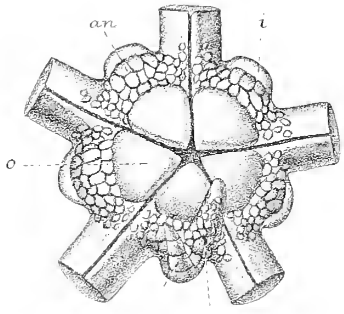
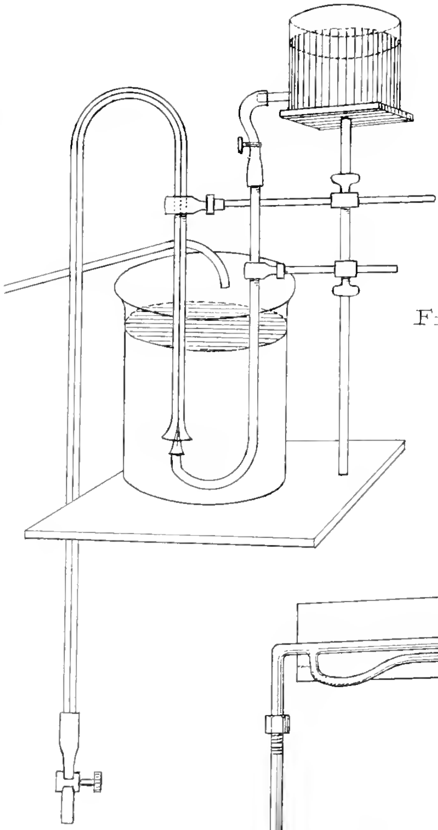


Fig. 5.

Berjeau & Highley del. et lith.

West, Newman & Co. lith.

THAUMATOCRINUS RENOVATUS, Nov. gen. et sp.



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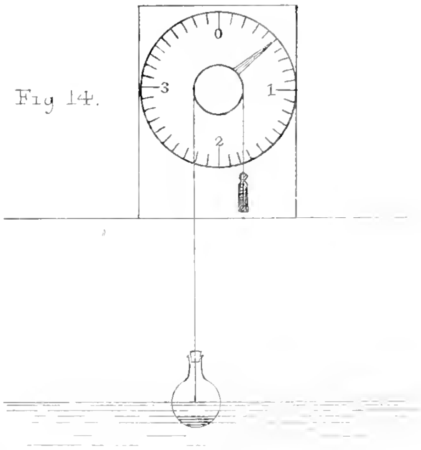

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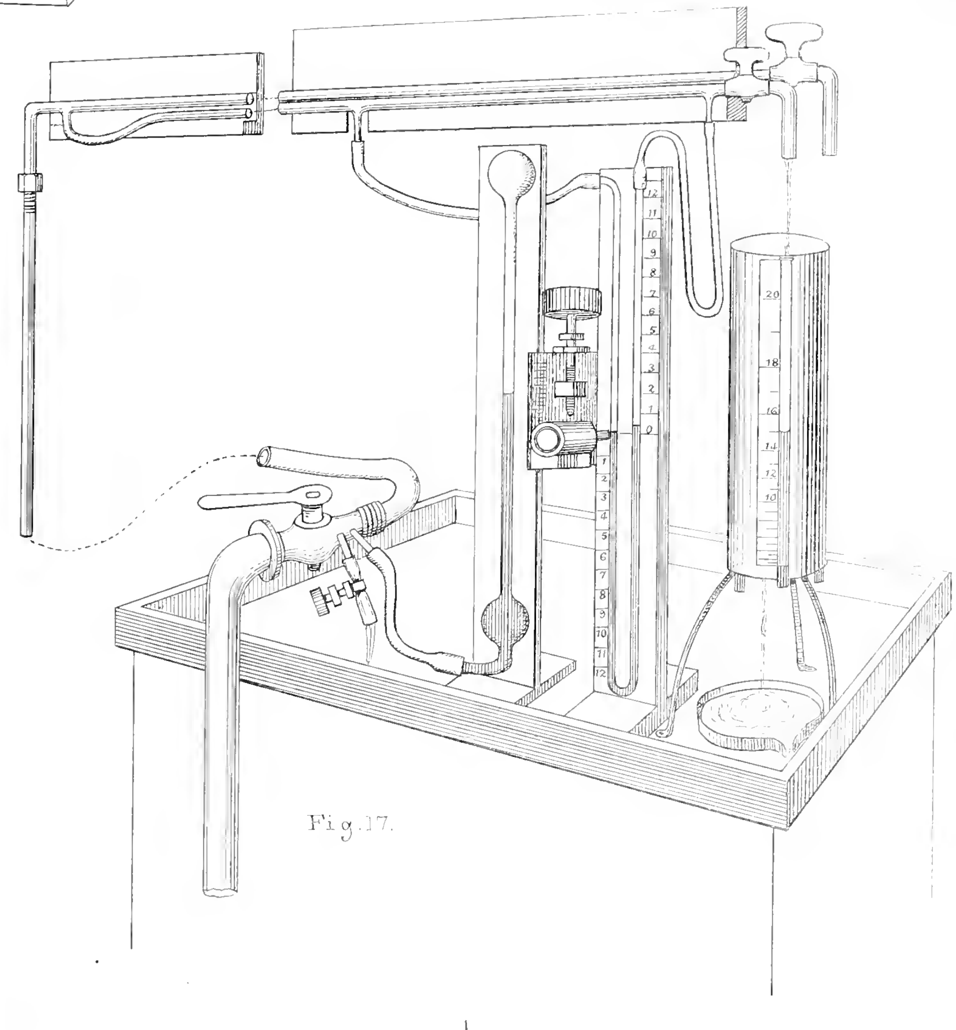

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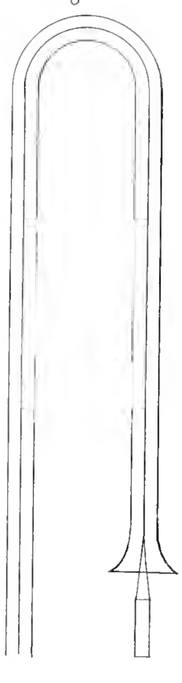

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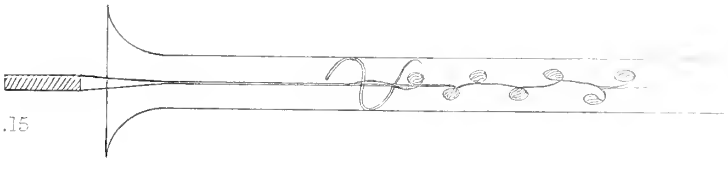

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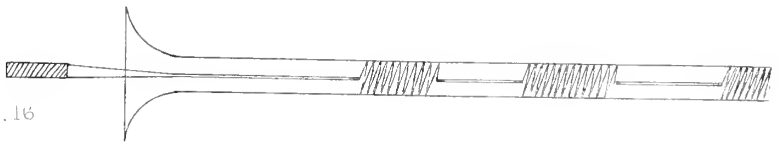

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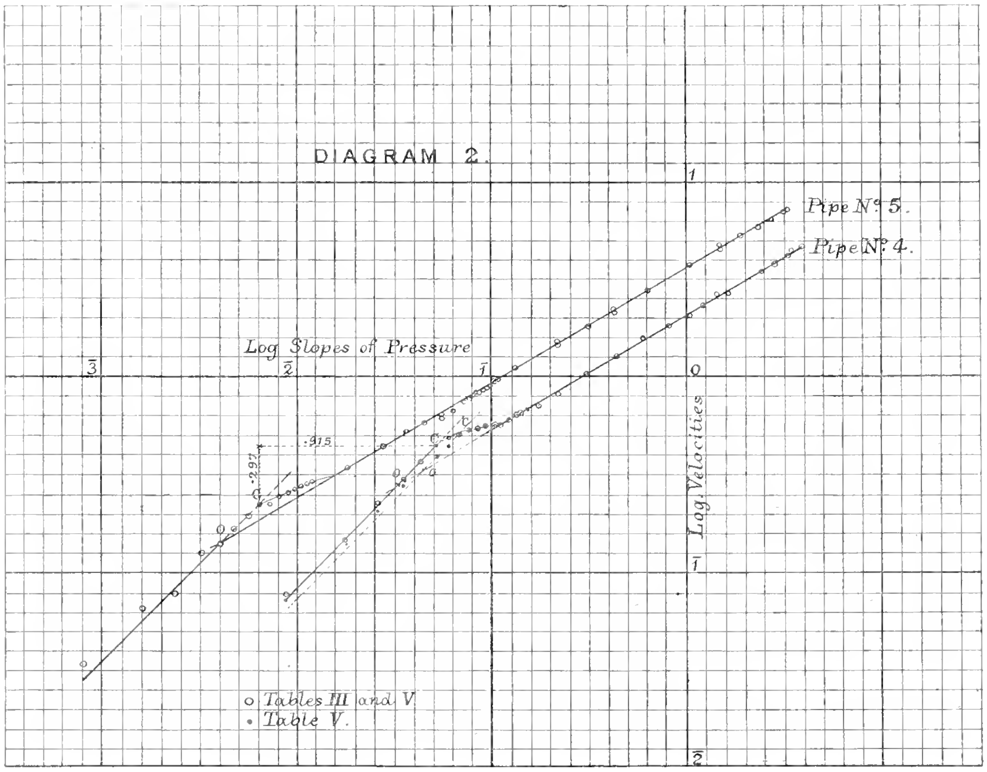
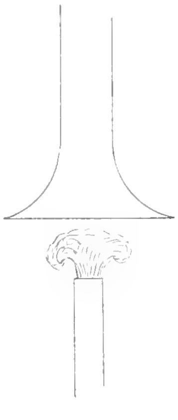


DIAGRAM 2.

Fig. 11.



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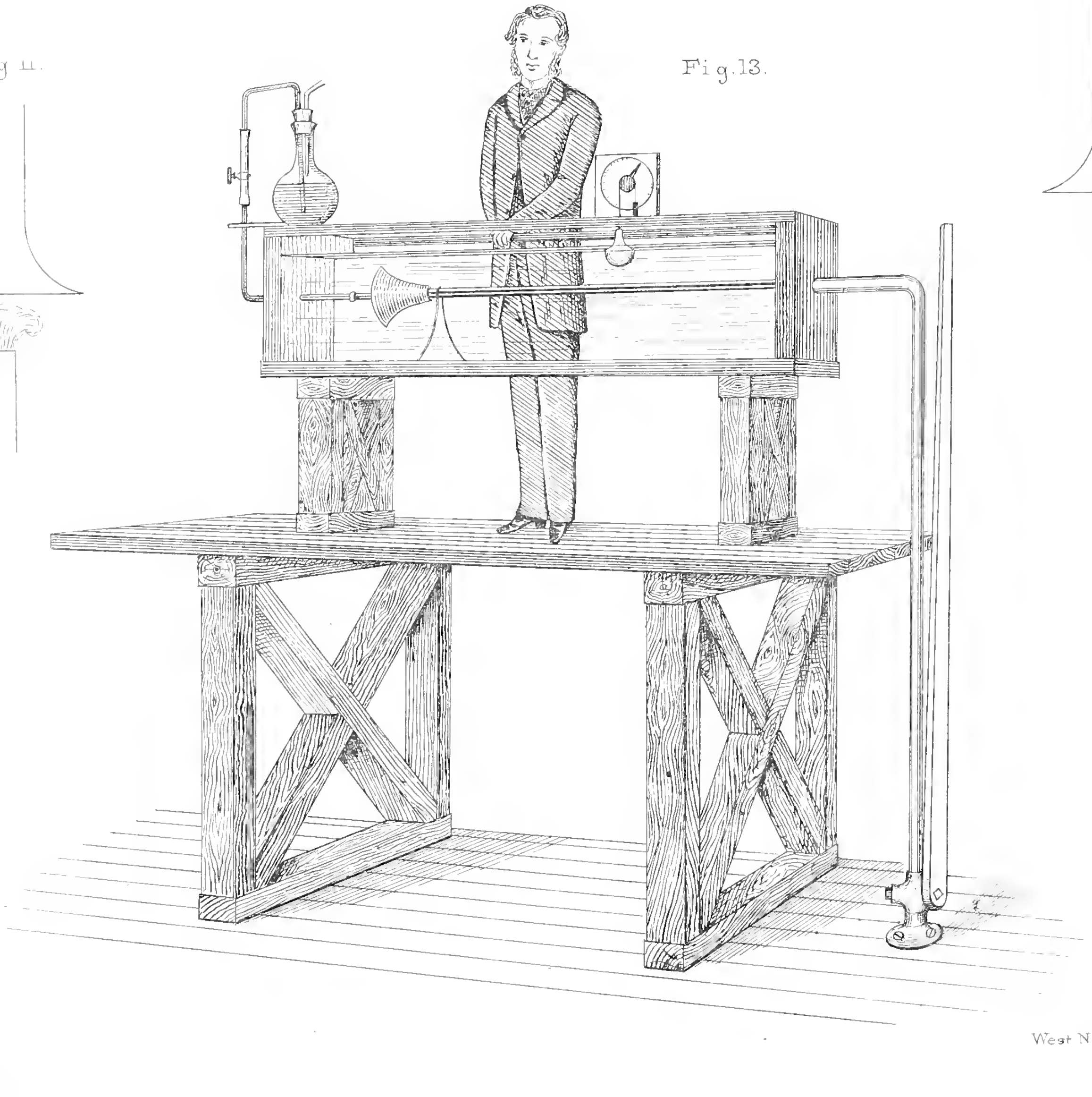

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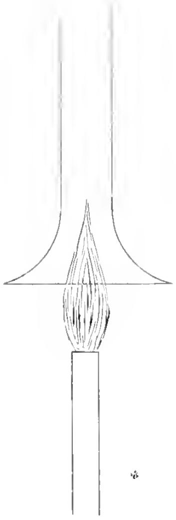


West Newman & Co lith.

DIAGRAM 1.

Curves of Pressure and Velocity in Pipes
N° 4. Diameter 0m·00615 at 5°c. and
N° 5 Diameter 0m·012,7

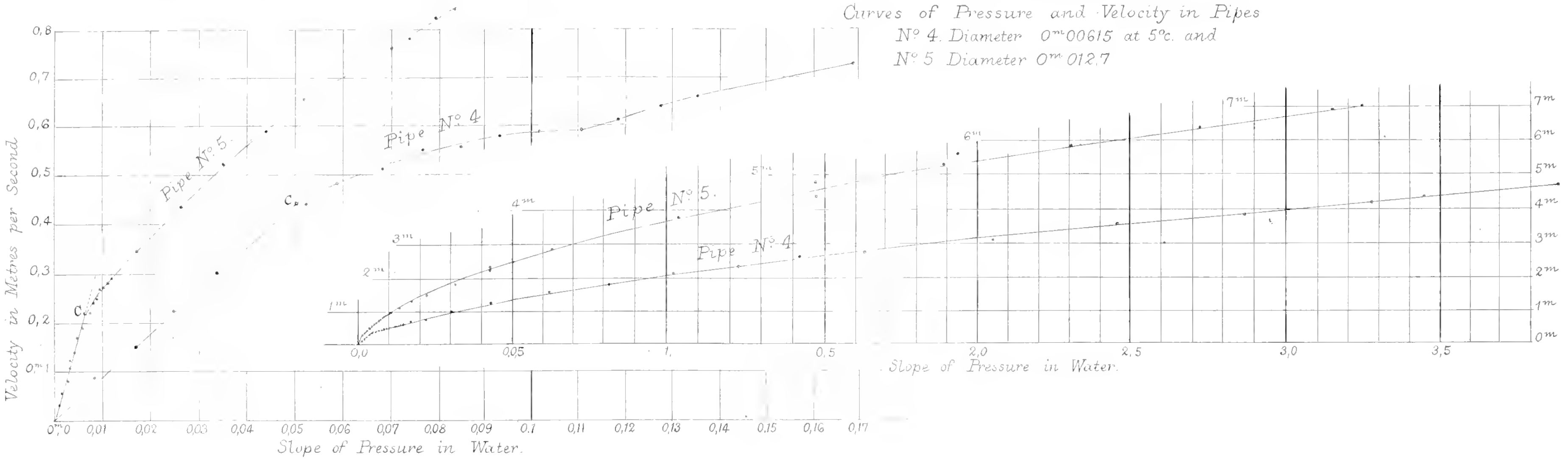
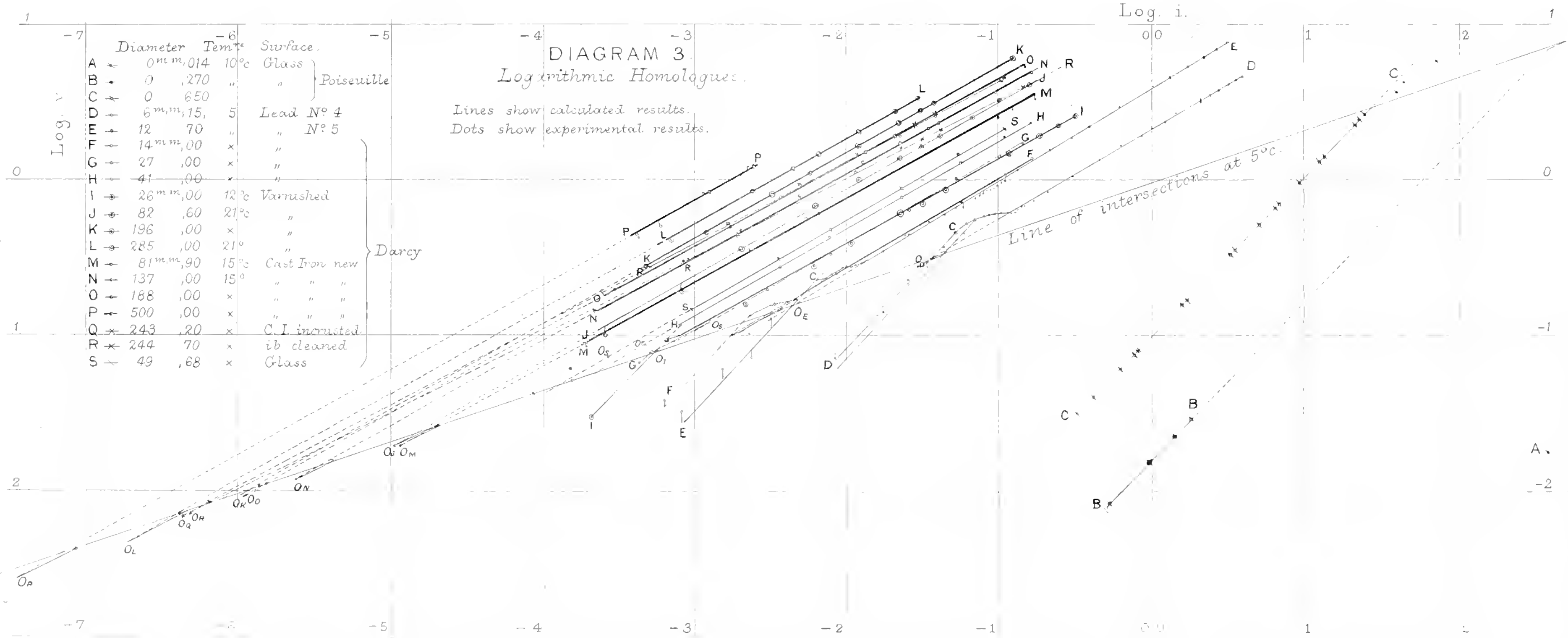


DIAGRAM 3

Logarithmic Homologues.

Lines show calculated results.
Dots show experimental results.



		Diameter		Temp.re	Surface.	
A	·—	0mm,	014	10°c	Glass	Poiseuille
B	•—	0	,270	"	"	
C	✕—	0	650			
D	○—	6m,m,	15,	5	Lead N° 4	
E	•—	12	70	"	" N° 5	
F	○—	14mm,	00	×	"	Darcy
G	○—	27	,00	×	"	
H	○—	41	,00	×	"	
I	⊕—	26mm,	00	12°c	Varnished	
J	⊕—	82	,60	21°c	"	
K	⊙—	196	,00	×	"	
L	⊕—	285	,00	21°	"	
M	○—	81m,m,	90	15°c	Cast Iron new	
N	·—	137	,00	15°	" " "	
O	•—	188	,00	×	" " "	
P	•—	500	,00	×	" " "	
Q	✕—	243	,20	×	C.I. incrusted	
R	✕—	244	70	×	ib cleaned	
S	·—	49	,68	×	Glass	

West Newman & Co lith

THE PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY.

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
1801. Part I...	0	17	0	1834. Part I...	0	17	0	1852. Part I...	1	0	0	1868. Part II...	2	0	0
—— Part II...	0	17	6	—— Part II...	2	2	0	—— Part II...	2	5	0	1869. Part I...	2	10	0
1802. Part I...	0	11	0	1835. Part I...	1	2	0	1853. Part I...	0	18	0	—— Part II...	3	3	0
—— Part II...	0	17	6	—— Part II...	0	14	0	—— Part II...	0	12	0	1870. Part I...	1	10	0
1803. Part II...	0	13	6	1836. Part I...	1	10	0	—— Part III...	1	2	0	—— Part II...	1	18	0
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—— Part II...	0	12	6	1837. Part I...	1	8	0	—— Part II...	0	16	0	—— Part II...	2	5	0
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—— Part II...	1	1	0	—— Part II...	0	14	0	1865. Part I...	2	2	0	—— Part II...	2	0	0
1831. Part I...	1	10	0	1849. Part I...	1	0	0	—— Part II...	1	5	0	—— Part III...	2	10	0
—— Part II...	1	12	0	—— Part II...	2	5	0	1866. Part I...	1	14	0	—— Part IV...	1	0	0
1832. Part I...	1	1	0	1850. Part I...	1	10	0	—— Part II...	2	7	6	1883 Part I...	1	10	0
—— Part II...	2	0	0	—— Part II...	3	5	0	1867. Part I...	1	3	0	—— Part II...	2	10	0
1833. Part I...	1	1	0	1851. Part I...	2	10	0	—— Part II...	1	15	0	—— Part III...	1	12	0
—— Part II...	2	18	0	—— Part II...	2	10	0	1868. Part I...	2	5	0				

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