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# PHYSICAL MEASUREMENTS OF SEA ICE

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# THE PROBLEM

Study the physical properties of sea ice to ascertain those properties which may be pertinent to naval operational needs.

# RESULTS

An initial study, in the Bering Sea, of such sea-ice properties as density, salinity, air-bubble distribution, size and distribution of crystal grains, and elastic moduli, was accomplished. Two sites were involved, in 1954 and 1955.

# RECOMMENDATIONS

1. Conduct a study of the variation of all parameters over one entire ice season at one location, attempting to relate the plate-wave velocity variation with the state of the sea ice and the state of the sea ice with its breaking properties.

2. Obtain heat-transfer information by making continuous measurements over a complete ice season of (1) temperature gradients through an ice sheet with a snow cover, (2) wind gradients above the ice sheet, (3) wet and dry temperature gradients above the ice sheet, and (4) the net radiation.

# ADMINISTRATIVE INFORMATION

Work was done under SW-01102, NE 121217-847.1 (NEL L6-1, Part 1), by members of the Special Research Division. This report covers work performed in February and March 1954, and March and April 1955, and was approved for publication 11 February 1958.

The authors are greatly indebted to the crew of the USCGC NORTHWIND for assistance rendered, and particularly to the UDT and EOD groups which participated in the Bering Sea Expeditions in the winter of 1954 and 1955. Appreciation is also expressed to all members of the Submarine and Arctic Research Branch, particularly Dr. W. K. Lyon for his encouragement and technical assistance; L. L. Morse for his technical assistance in the seismic instrumentation; A. C. Walker for his assistance in general instrumentation; and R. N. Rowray for his help in analyzing data.

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# INTRODUCTION

A knowledge of the physics of sea ice is essential to arctic naval operations. The measurements reported here are one phase of a major continuing study being conducted by the Navy Electronics Laboratory to furnish geophysical data on the arctic regions which will be of value in naval operations in that area.

A survey of the literature[1,2,3] (see References at end of report) in 1953, concerning the longitudinal plate-wave velocity in fresh-water ice, showed a significant variation in velocity. At that time there were no data on longitudinal plate-wave velocities in sea ice; however, since that time one article on the subject has been published,[4] indicating that if variations existed in the longitudinal plate-wave velocity in sea ice, these variations would be related to certain physical properties of the ice.

During the Winter 1954 Bering Sea Expedition and the Winter 1955 Bering Sea Expedition, measurements were made of the longitudinal plate-wave velocity. In 1954, the location of the ice sheet in which measurements were made was 60°20′ N, 168°38′ W; and in 1955 the ice sheet, although variable in position during the several days measurements were being made, occupied the position 63°05′ N, 166°09′ W. In 1955, in addition to the measurement of the longitudinal plate-wave velocity, other physical properties of the ice which might be of scientific interest were measured. The investigation of other sea-ice parameters included: (1) sampling profiles of density, temperature, salinity, air bubble, and crystal grain; (2) measurement of the longitudinal wave velocity in ice rods; (3) measurement of the frequency of air-coupled flexural waves to determine ice thickness; and (4) resistivity measurements.

The information obtained from these two surveys represents point data; that is, data which were taken at a given point in a relatively short time interval.

# ELASTIC WAVE THEORY
## Mechanical Properties

From the theory[2] of elastic waves in an isotropic medium, the longitudinal plate-wave velocity is given by the relation

$$V_p = \{E/[\rho_i(1-\sigma^2)]\}^{1/2}$$

where

$E$ = Young's modulus
$\rho_i$ = density of medium (ice)
$\sigma$ = Poisson's ratio

The shear wave velocity, $V_{sh} = (\mu/\rho_i)^{1/2}$, in which $\mu = E/[2(1+\sigma)]$ where $\mu$ = shear modulus.

If the medium is isotropic and the above two wave velocities can be detected, then the elastic properties of the medium can be described by any two elastic constants of the above equations. It should be noted that $V_p$ is a function of two elastic constants whereas $V_{sh}$ is a function of only one elastic constant. In addition, both of these wave velocities are a function of density.

The longitudinal wave velocity for rods, in which the diameter of the rod is small in comparison to its length, is given by the relation[6]

$$V_r = (E/\rho_i)^{1/2}$$

From the plate velocity and the rod velocity equations, the following relation can be written:

$$V_p = V_r/(1-\sigma^2)^{1/2}$$

which gives the plate-wave velocity in terms of the rod velocity.

The theory of flexural waves[7] in a floating ice sheet over deep water gives the relation for the phase velocity $C$ as

$$C^2 = \frac{(1/3)\pi^2\gamma^2 V_p{}^2 + \{[(\rho_w/\rho_i)g\lambda]/(4\pi^2\gamma)\}}{1 + (\rho_w/\rho_i)\,[(1-C^2)/V_w{}^2]^{-1/2}\,(2\pi\gamma)^{-1}}$$

where

$\gamma = Hf/C$
$H$ = thickness of ice sheet
$f$ = frequency of flexural wave
$C$ = phase velocity
$\rho_w$ = density of underlying water
$\rho_i$ = density of ice
$\lambda$ = wavelength in ice
$V_w$ = velocity of longitudinal wave in water
$V_p$ = velocity of longitudinal plate wave in ice
$g$ = acceleration of gravity

Flexural waves are usually generated by an explosive shot being placed in the ice or in the air above the ice. The frequency of these waves changes with the varying velocities of the shots. However, if the explosive shots are set off in the air or on the surface of the ice, the frequency remains constant. Theory[7] shows these constant-frequency waves build up to a maximum amplitude, and then fall off rapidly in amplitude. The point of maximum amplitude represents the passage of the air wave. These waves are called air-coupled flexural waves. The equation for $\gamma$ becomes

$$\gamma_a = Hf/C$$

from which the ice thickness can readily be determined.

# Other Physical Properties

## HEAT FLOW THROUGH A SNOW-COVERED ICE SHEET

The heat flow per unit time per unit area through a snow-covered ice sheet, which is bounded on one side by an isothermal reservoir of water and on the other side by the atmosphere, is given by the relation[8]

$$w = -k\,(\partial T/\partial Z)$$

where

$w$ = heat flux
$k$ = thermal conductivity
$T$ = temperature
$Z$ = thickness of the ice sheet

When the thermal conductivity is not constant but varies through the ice sheet, the ice sheet and the corresponding snow cover must be divided into thin layers. By a consideration of the thermal conductivity of each individual ice layer, it is possible to write the following relation for the heat conducted through the snow-covered ice sheet

$$w = \frac{T_n - T_1}{\sum_{n=1}^{\infty} (Z_n/k_n)} = \frac{T_n - T_1}{(Z_1/k_1) + (Z_2/k_2)\ \ldots + (Z_n/k_n)}$$

## RESISTIVITY OF SEA ICE

The resistivity method of measuring the resistance of sea-water ice consists of sending an electric current into the ice sheet and measuring the potential between a set of electrodes placed in the ice within the effective area of the current. The Wenner method[9] of electrode arrangements (fig. 1) was used in making the measurements.

Theoretical investigations based on the distribution of electric current in the ground are nearly all based on the assumption that the layers are horizontal and homogeneous in the horizontal plane. If the ground has uniform electrical properties for an infinite distance in a vertical direction, the resistivity, using the electrode arrangement in figure 1, is given by

$$\rho = 2\pi a\,(V/I)$$

where

$\rho$ = resistivity in ohm-centimeters
$a$ = electrode separation in centimeters
$V$ = voltage in millivolts
$I$ = current in milliamperes

For the case of nonuniform properties in a vertical direction, the equation no longer gives the true resistivity. In such a case, the resistivity determined from the measurements is called the apparent resistivity.

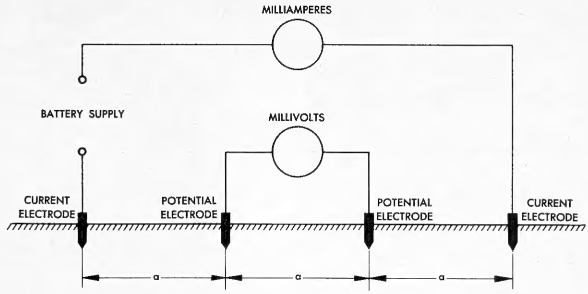


**Figure 1.** Wenner method of dc resistivity measurements.

# MEASUREMENTS, WINTER 1954 BERING SEA EXPEDITION

## Equipment and Procedures

Measurements of the longitudinal plate-wave velocity and of the ice thickness by the air-coupled flexural-wave method were made in a sea-ice sheet. The equipment consisted of Electro-technical model EVS 2-A geophones, a six-channel Brush recorder, modified Brush amplifiers with limiters, and a shot-break circuit employing a portable transmitter-receiver radio system. The icebreaker lay to in the ice sheet for a period of one to three days at three locations. At the first two stations, insufficient data were obtained because of difficulties encountered with equipment operation in cold temperatures. At the third station, satisfactory records were obtained.

Two sets of geophones, 91.44 meters (100 yards) apart, were buried in the ice for detecting the longitudinal plate-wave velocity. Each set consisted of three geophones oriented in the three vector directions. No effort was made to obtain dispersion curves, since the ship was at each location for only a short time. Hence, it was decided to standardize on 457.2 meters (500 yards) and 914.4 meters (1000 yards) as the distance from the first geophone set to the point where the explosive charges were detonated. Two additional geophones were placed on top of the ice sheet for detecting the air-coupled flexural waves. The surface of the ice sheet was, in general, very smooth. Measurements made in this ice sheet are presented in table 1. Figure 2 presents a schematic arrangement of geophones and shot-blast stations.

**TABLE 1. Seismic data taken in 1954.**

| Range (meters) | Longitudinal Plate-Wave Velocity (meters/sec) | Air-Coupled Flexural Wave Frequency (cps) | Calculated Ice Thickness (cm) | Mean Measured Ice Thickness with Probable Error (cm) | Size of TNT Charge (lbs) |
|---|---|---|---|---|---|
| 457 | 2629 ± 60 | 46.0 | 69.3 | 78.7 ± 2.5 | 10 |
| 548 | 2740 ± 60 | — | — | — | 10 |
| 912 | 2553 ± 60 | 39.4 | 78.4 | 86.9 ± 4.8 | 15 |
| 912 | 2553 ± 60 | 43.5 | 76.8 | 86.9 ± 4.8 | 15 |
| 1003 | 2495 ± 60 | 41.7 | 79.9 | 86.9 ± 4.8 | 15 |
| 910 | 2603 ± 60 | — | — | — | 40 |
| 1002 | 2615 ± 60 | 43.7 | 65.2 | 86.9 ± 4.8 | 40 |

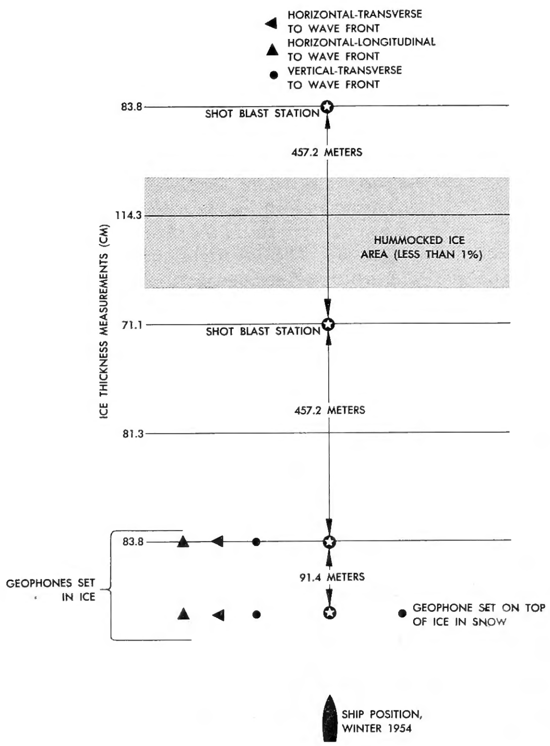


**Figure 2. Equipment layout on ice sheet (1954).**

# MEASUREMENTS, WINTER 1955 BERING SEA EXPEDITION

## Equipment and Procedures

### MECHANICAL PROPERTIES

The icebreaker USCGC NORTHWIND (WAGB-282) lay to in the ice from 23 March to 5 April. Again measurements were made of the longitudinal plate-wave velocity and of the air-coupled flexural wave frequency. The seismic equipment consisted of a Consolidated Engineering Corporation type 5-101B, 14-channel recording oscillograph. The galvanometers used in the oscillograph had a resonant frequency of 375 cps. Nine of the amplifiers were type GN amplifiers made by Engineering Laboratories, Inc., modified to give a uniform frequency response from 11 cps to 1000 cps. Five amplifiers were designed and built at NEL to provide a uniform frequency response from 3 cps to 5000 cps. Eleven geophones of Engineering Laboratories, Inc., type GS-100 were used. These geophones had a resonant frequency of 27.5 cps. Two Brush C-23 hydrophones were used to detect the water wave velocity. The shot-break signal was transmitted through field telephone wire to the recorder.

Three sets of geophones were placed 91.4 meters (100 yards) apart. Each set consisted of three geophones, one for each vector direction. In addition, two geophones were set on top of the ice sheet, and two hydrophones were placed under the ice sheet in the water. The explosive charges were detonated at a distance of 457.2 meters (500 yards) and 914.4 meters (1000 yards) from the nearest set of geophones. The general surface of the ice sheet was flat; however, the over-all ice sheet consisted of two sections which had been rafted together into one ice sheet. Thus, two sets of geophones were not in the same sheet as the one in which the explosive charges were set off. The results of these measurements are presented in tables 2 and 3. A schematic arrangement of geophones and shot-blast stations appears in figure 3.

Density profile measurements were made on a sample of ice taken from the area of Site 1 (fig. 4). The density measurements were made immediately after taking a core sample of ice by the direct method of weighing and measuring the dimensions of the sample of ice.

Salinity profiles were made at Site 1 and Site 2 (figs. 5, 6, and 7). The size of each sample was selected to assure a representative quantity. Chlorinities were determined by the titration method and then converted to salinities by the use of the Knudsen hydrographical tables.

It should be noted that errors exist in the determination of salinities by this method.[10,11] The error in this method, which is well within practical limits, is apparently less than $0.1^{0}/_{00}$.

Vertical temperature profiles through the ice sheet were made at Site 2 at the time of the explosive shots. The results are presented in figures 8, 9, and 10 for each day the tests were conducted. Thermistors were used as the thermal elements.

Profiles of the crystal grain size of a sample of ice were taken at Site 2. A graph of the distribution of grain sizes is presented in figures 11 and 12. An Eastman Kodak photo-copyer with 1.5-power magnification was used to photograph the ice samples, which were placed under a calibrated grid between cross polaroid plates.

TABLE 2. Longitudinal plate-wave velocity data taken in 1955.

| Date | Range (meters) | Longitudinal Plate-Wave Velocity (meters/sec) | Charge | |
|---|---|---|---|---|
| | | | Size (lbs) | Type |
| 3-27-1955 | 913 | 2191 ± 40 | 5 | TNT |
| | 1004 | 2132 ± 40 | 5 | TNT |
| | 1096 | 2118 ± 40 | 5 | TNT |
| | 457 | 2271 ± 40 | 1 | TNT |
| | 639 | 2147 ± 40 | 1 | TNT |
| 3-30-1955 | 457 | 2252 ± 40 | 1½ | C-3 |
| | 548 | 2231 ± 40 | 1½ | C-3 |
| | 640 | 2164 ± 40 | 1½ | C-3 |
| | 457 | 2272 ± 40 | 1 | TNT |
| | 548 | 2248 ± 40 | 1 | TNT |
| | 640 | 2217 ± 40 | 1 | TNT |
| | 458 | 2261 ± 40 | ½ | TNT |
| | 549 | 2219 ± 40 | ½ | TNT |
| | 641 | 2219 ± 40 | ½ | TNT |
| | 913 | 2193 ± 40 | 3 | TNT |
| | 1004 | 2140 ± 40 | 3 | TNT |
| | 1096 | 2154 ± 40 | 3 | TNT |
| 4-2-1955 | 457 | 2305 ± 40 | ½ | C-3 |
| | 548 | 2219 ± 40 | ½ | C-3 |
| | 639 | 2238 ± 40 | ½ | C-3 |
| | 457 | 2286 ± 40 | ½ | C-3 |
| | 549 | 2255 ± 40 | ¼ | C-3 |
| | 640 | 2266 ± 40 | ¼ | C-3 |
| | 914 | 2213 ± 40 | ⅛ | C-3 |
| | 914 | 2210 ± 40 | ⅛ | C-3 |
| | 914 | 2271 ± 40 | ¼ | C-3 |
| | 1006 | 2219 ± 40 | ¼ | C-3 |
| | 1097 | 2198 ± 40 | ¼ | C-3 |
| | 914 | 2225 ± 40 | ¼ | C-3 |
| | 1005 | 2155 ± 40 | ¼ | C-3 |
| | 1097 | 2179 ± 40 | ¼ | C-3 |

TABLE 3. Calculated ice thickness from seismic method vs measured ice thickness for 1955.

| Date | Range (meters) | Calculated Ice Thickness (cm) | Mean Measured Ice Thickness, with Probable Error (cm) | Charge | |
|---|---|---|---|---|---|
| | | | | Size (lbs) | Type |
| 3-27-1955 | 457 | 56.8 | 69.2 ± 1.8 | 1 | TNT |
| | 457 | 62.6 | 69.2 ± 1.8 | 1 | TNT |
| | 639 | 68.0 | 105.9 ± 23.6 | 1 | TNT |
| | 639 | 66.1 | 105.9 ± 23.6 | 1 | TNT |
| | 909 | 91.3 | 69.8 ± 1.0 | 10 | TNT |
| | 1000 | 83.5 | 98.8 ± 19.6 | 10 | TNT |
| | 1092 | 72.0 | 94.8 ± 16.8 | 10 | TNT |
| 3-30-1955 | 458 | 59.5 | 69.2 ± 1.8 | ½ | TNT |
| | 641 | 59.9 | 105.9 ± 23.6 | ½ | TNT |
| | 457 | 56.8 | 69.2 ± 1.8 | 1 | TNT |
| | 457 | 74.1 | 69.2 ± 1.8 | 1½ | TNT |
| | 457 | 75.2 | 69.2 ± 1.8 | 1½ | TNT |
| | 548 | 74.9 | 114.8 ± 29.5 | 1½ | TNT |
| | 640 | 74.4 | 105.9 ± 23.6 | 1½ | TNT |
| | 640 | 76.4 | 105.9 ± 23.6 | 1½ | TNT |
| 4-2-1955 | 457 | 59.5 | 69.2 ± 1.8 | ¼ | C-3 |
| | 548 | 90.0 | 114.8 ± 29.5 | ¼ | C-3 |
| | 640 | 65.6 | 105.9 ± 23.6 | ¼ | C-3 |
| | 640 | 65.8 | 105.9 ± 23.6 | ¼ | C-3 |
| | 457 | 64.6 | 69.2 ± 1.8 | ½ | C-3 |
| | 548 | 84.2 | 114.8 ± 29.5 | ½ | C-3 |
| | 639 | 60.8 | 105.9 ± 23.6 | ½ | C-3 |
| | 1097 | 59.4 | 94.8 ± 16.8 | ¼ | C-3 |

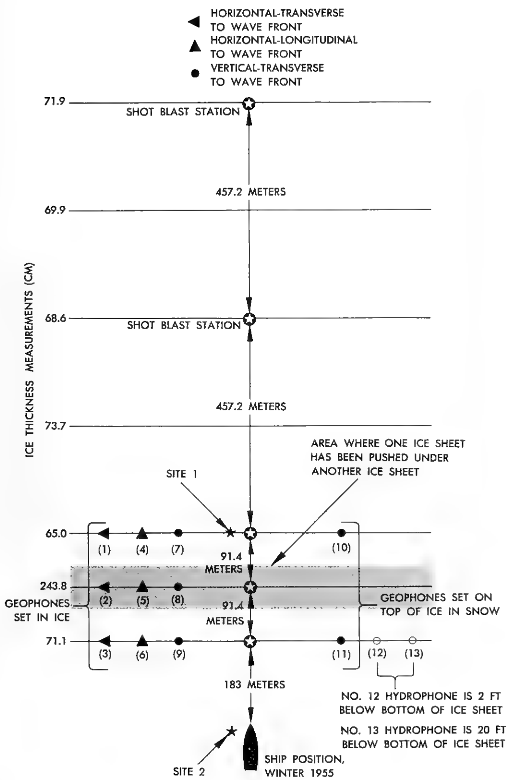


Figure 3. Equipment layout on ice sheet (1955).

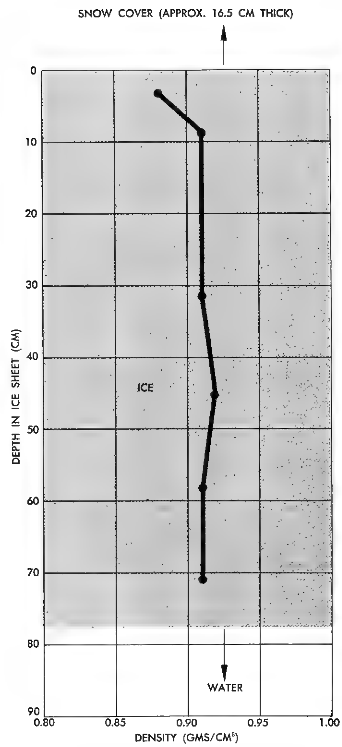


Figure 4. Density profile of sea ice sheet, site No. 1 (1955).

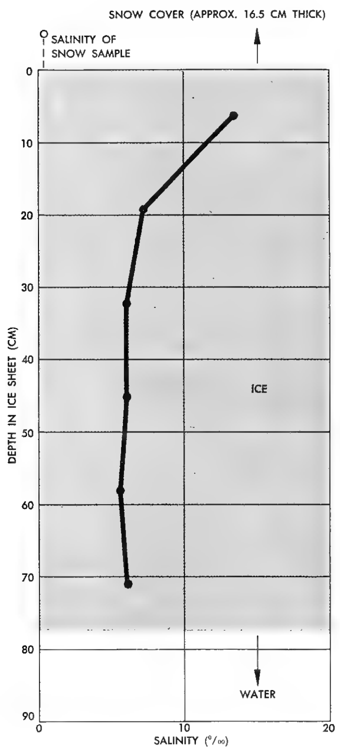


Figure 5. Sea-ice salinity profile, site No. 2 (1955).

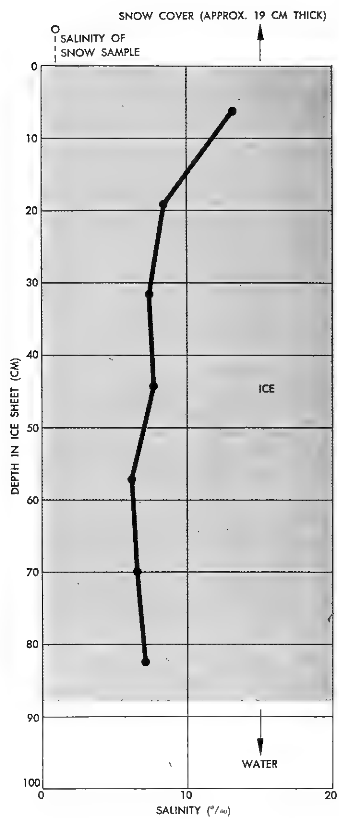


Figure 6. Salinity profile of sea ice sheet, site No. 2 (1955).

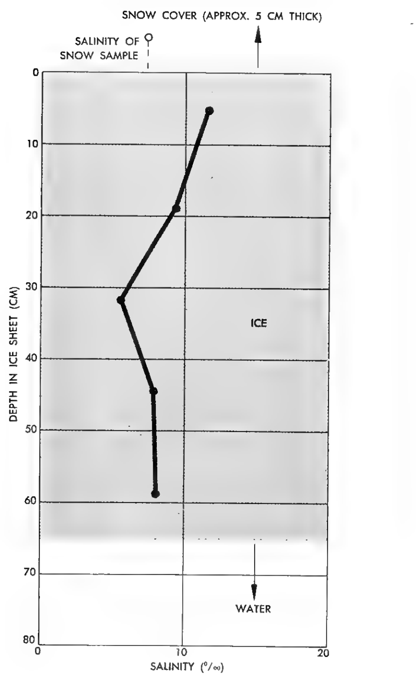


**Figure 7. Salinity profile of sea ice sheet, site No. 1 (1955).**

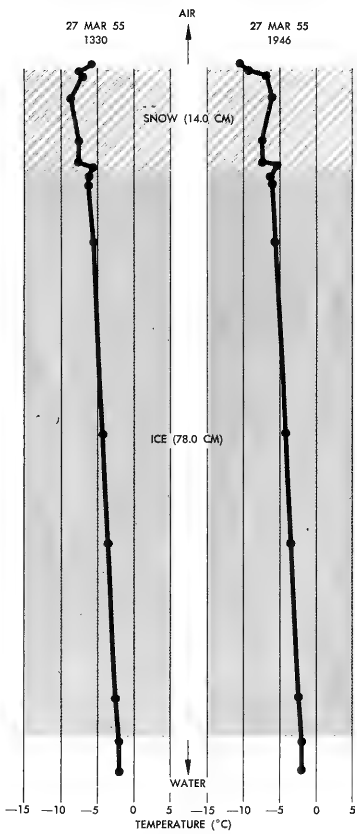


Figure 8. Temperature profile through ice sheet and snow cover, site No. 2 (27 Mar 55).

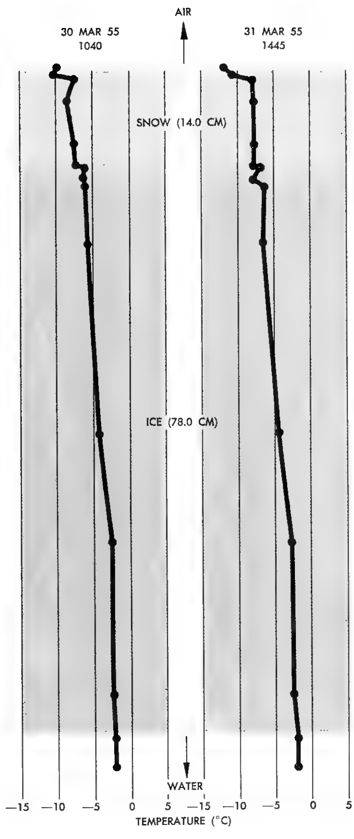


Figure 9. Temperature profile through ice sheet and snow cover, site No. 2 (30 and 31 Mar 55).

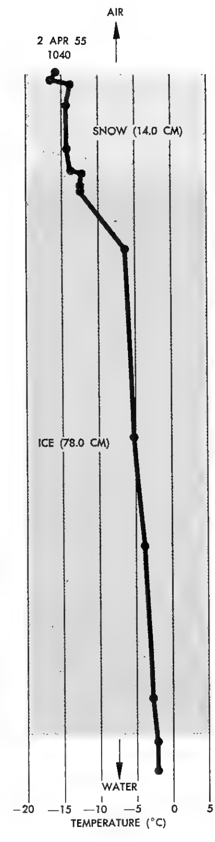


Figure 10. Temperature profile through ice sheet and snow cover, site No. 2 (2 Apr 55).

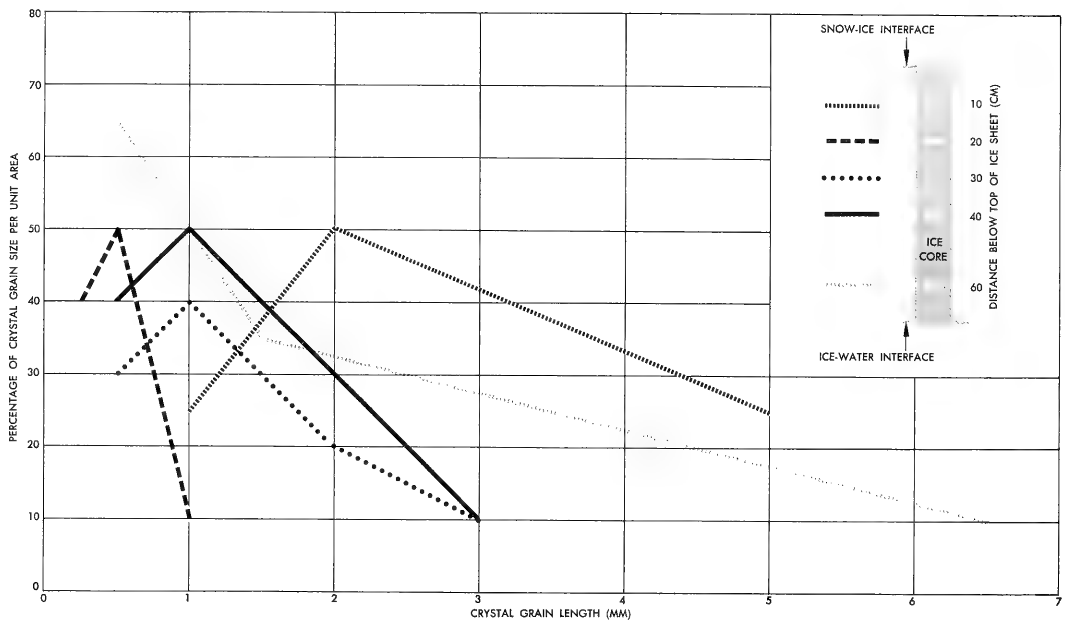


**Figure 11. Crystal grain size distribution for ice samples of *X-Y*-axis orientation.**

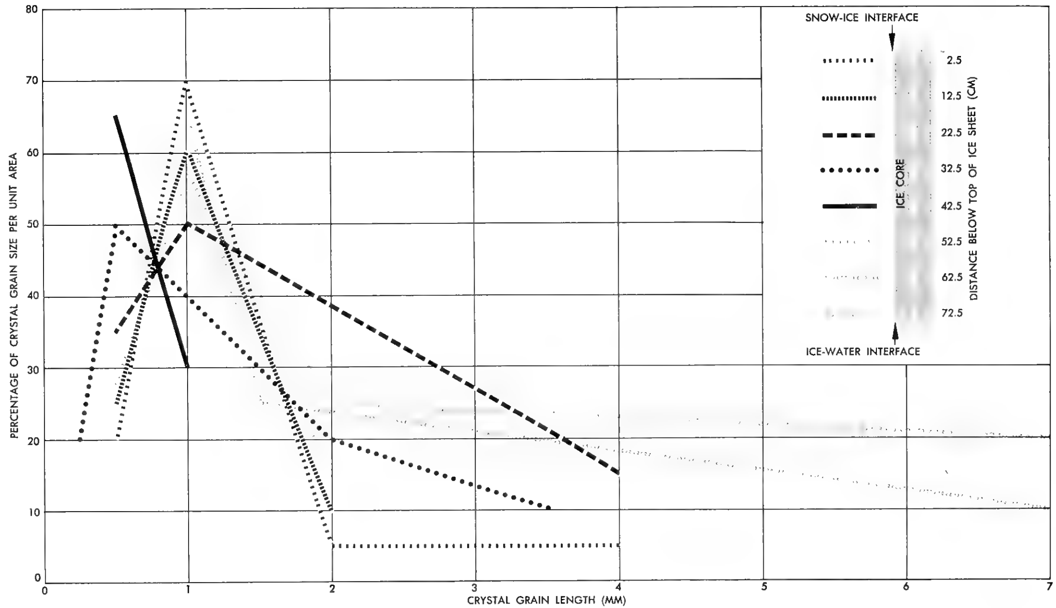


**Figure 12.** **Crystal grain size distribution for ice samples of *Z*-axis orientation.**

Profiles of the air-bubble distribution in ice were made at Site 2. Figures 13 and 14 show the distribution of air bubbles for various samples of ice. The Eastman Kodak photo-copyer was used again to photograph the ice samples for air-bubble content. A low-power microscope was used to photograph the smaller air bubbles, with the ice samples placed under a grid graduated in 0.5 mm.

The longitudinal wave velocity of a sea-ice rod was measured by the resonant-rod method. The radius of the rod, obtained by coring, was small in comparison to its length. The rod was allowed to reach a temperature equilibrium over a period of two or three days. A Hewlett-Packard audio signal generator, model 205 AG, was used to drive a balanced armature which, in turn, was coupled to a 2-inch-diameter plate attached to one end of the ice rod. An earphone was attached to the other end of the rod to detect the waves generated. The signal from the earphone was fed into a Ballantine vacuum-tube voltmeter.

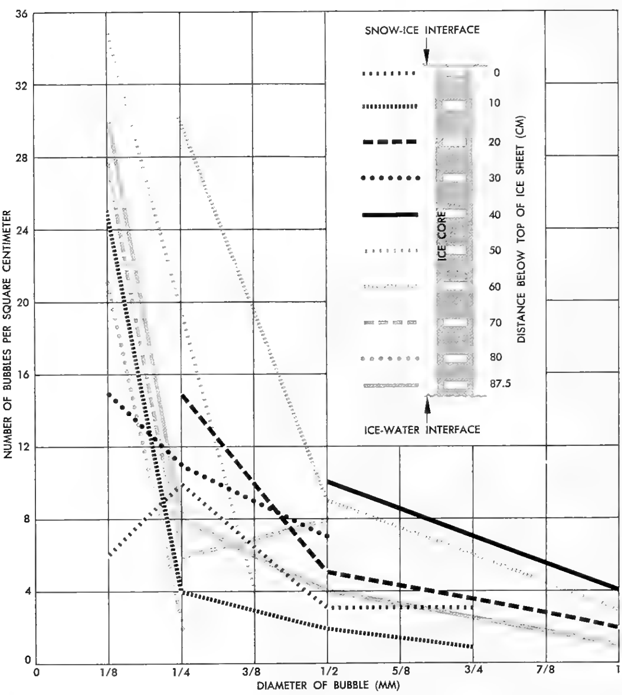


**Figure 13.** Air-bubble size distribution for ice samples of *X-Y*-axis orientation, site No. 2.

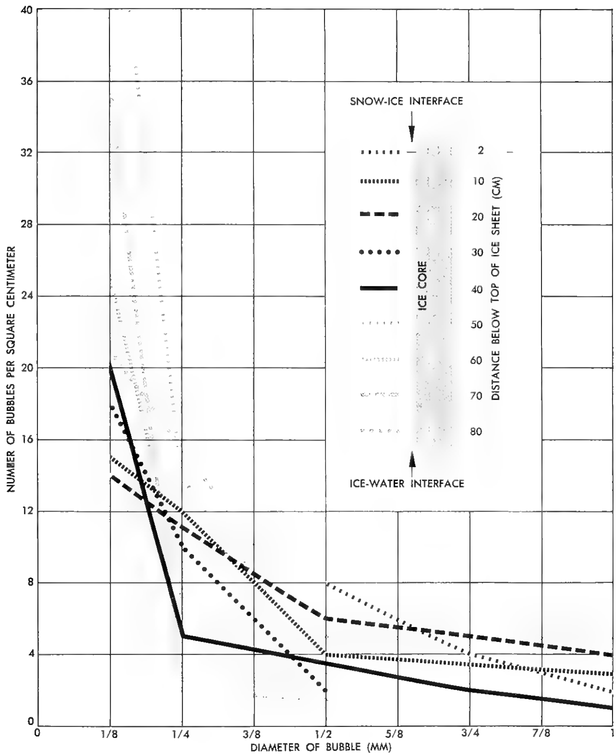


**Figure 14. Air-bubble size distribution for ice samples of *Z*-axis orientation, site No. 2.**

## OTHER PHYSICAL PROPERTIES

HEAT FLOW MEASUREMENTS THROUGH AN ICE SHEET. Since the thermal gradients from the water through the ice sheet, the snow cover, and the atmosphere were measured for the state of the ice, it was decided to measure the conducted heat flow per unit time per unit area through a sea-ice sheet with a snow cover. The measurements were made at Site 2 and the data are presented in figure 15.

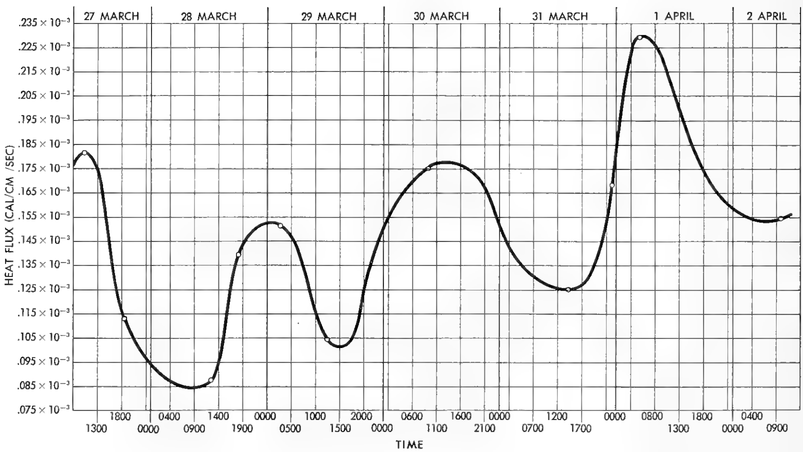


**Figure 15. Conducted heat flow through snow-covered ice sheet, site No. 2.**

RESISTIVITY MEASUREMENTS IN SEA ICE. Resistivity measurements were made in the ice sheet at Site 2. Brass rods were driven into the ice sheet to serve as electrodes. A switching device, which was designed to reverse the polarity of the electrodes at each successive reading, was used to reduce the polarizing effect on the electrodes. A 6-volt and a 12-volt lead storage battery were used as the power supply for the system.

# SUMMARY

## Mechanical Properties

In comparing the results of 1954 and 1955 in tables 1 and 2, a difference of 300 to 400 meters/sec exists in the longitudinal plate-wave velocity. In Oliver, Crary, and Cotell's work,[4] variations in the plate velocity are of the order of 500 meters/sec. These variations are significant and can be attributed only to differences in the internal structure of the sea-ice sheet. In comparing the longitudinal plate-wave velocity obtained by seismic methods (table 2) with the longitudinal plate-wave velocity obtained by excitation of a long rod (table 4), a difference

TABLE 4. Resonant rod measurements of sea ice for 1955.

| Rod Length (meters) | Resonant Frequency (cps) | Rod Velocity (meters/sec) | Longitudinal Plate-Wave Velocity (assuming $\sigma = 0.33$) (meters/sec) | Young's Modulus E $\times 10^{10}$ dynes/cm$^2$ |
|---|---|---|---|---|
| 0.791 | 1930 | $3050 \pm 10$ | $3240 \pm 10$ | 8.4 |
| 0.812 | 1850 | $3000 \pm 10$ | $3180 \pm 10$ | 8.1 |
| 0.651 | 2330 | $3040 \pm 10$ | $3220 \pm 10$ | 8.3 |
| 0.644 | 2370 | $3040 \pm 10$ | $3220 \pm 10$ | 8.3 |
| 0.666 | 2200 | $2930 \pm 10$ | $3110 \pm 10$ | 7.7 |
| 0.595 | 2520 | $3000 \pm 10$ | $3180 \pm 10$ | 8.1 |
| 0.970 | 1475 | $2830 \pm 10$ | $3000 \pm 10$ | 7.2 |
| Temperature Range: —18°C to —15°C | | | | |

in the plate velocity of the order of 700 to 800 meters/sec is obtained. These results are contrary to the results of Ewing, Crary, and Thorne, Jr.[2] on lake ice, in which they found no significant difference in the plate-wave velocity when determined either by use of thin rods or by the seismic method. Poisson's ratio, which is not critical in this relation, is assumed to be 0.33. The rods of ice were obtained with the long axis normal to the ice sheet. The rods were stored for several days at a temperature between —18°C and —15°C. The temperature gradient in the ice sheet was —8°C at the top of the sheet to —2°C at the bottom of the sheet, as is shown in figures 8, 9, and 10. The difference in temperature of the order of 10°C between the ice sheet and the rod might indicate that the temperature of the sea ice has more than a second-order effect on the elastic constants, whereas temperature appears to have only a second-order effect with fresh-water ice.[12] Another reason for the discrepancy between the longitudinal plate-wave velocity of the ice sheet and the longitudinal plate-wave velocity as calculated from the rod velocity might be due to attenuation and dispersion. In this case, the faster high-frequency waves would be attenuated first[4] in the ice sheet as they emanate from the shot point; hence, at various distances from the shot point, a change in the velocity would be detected. However, with the short length of the ice rods, the high-frequency waves should be detected. Malmgren[13] reports sea-ice densities between 0.857 gm/cm$^3$ and 0.923 gm/cm$^3$, a difference of 0.066 gm/cm$^3$. The difference in density could contribute to a variation in the longitudinal plate-wave velocity of the order of 150 meters/sec. The measurements presented in this report do not show such a large variation of density. It is evident that although density variation contributes to a change in the longitudinal plate-wave velocity, it is not the major contributor. In 1954 and 1955, measurements were made in smooth and in rough ice. An indication of the smoothness of the ice is shown by the probable error of ice thickness measurements from tables 1 and 2.

One set of geophones was placed in an area of rafted ice in 1955. The plate velocities from this geophone set were lower. The size of the charge which was used appeared to have no effect on the plate velocity, provided it was sufficiently large to generate the waves.

Attempts to measure shear wave velocities were unsuccessful, as it was impossible to identify positively the shear wave from the complex waves recorded.

Tables 1 and 3 show the results of ice-thickness measurements determined from air-coupled flexural wave frequencies. When compared with measured ice thickness, the results for an area of smooth ice show, in general, an ice thickness approximately 10 centimeters less than the measured value. The difference probably is due to the effect of the bottom layer of the ice sheet which is a weak slush type of ice. For data taken in an area of rafted ice, the results from calculated ice thickness are less accurate. The probable error in these tables gives an indication of the unevenness of the ice-sheet surface.

Efforts were made in 1955 to collect data on the independent parameters which would aid in defining the state of sea ice. State of the ice is defined here as being a function of a set of independent parameters, such that when these parameters have some given value, the physical properties of the ice are fixed; that is, the ice has a given state. If one of these parameters should vary, then the ice would assume a different state. If the sea ice state can be defined by some dependent parameter $p$; and further, if $p$ is some function of $n$ independent parameters such that

$$p = f(q_1, q_2, q_3, \ldots q_n)$$

then any infinitesimal change in one or more of the independent parameters results in a change of the sea-ice state.

At NEL, studies are being made on sea ice in an effort to determine which are the independent parameters. In the work covered in this report, the parameters which have been considered are density, salinity, air-bubble distribution, crystal grain size, crystal grain distribution, and the elastic moduli.

.The two salinity profiles taken at Site 2 are in agreement. Site 1 and Site 2 did not appear to be located in the same ice sheet, and the variation in the salinity profiles at these two sites seems to indicate that the thermal history of each sheet was quite different.

A density profile was made only at Site 1. The profile shows a distinct variation in density but there appears to be no correlation with any of the other parameters. The average density through the ice sheet is 0.90 gm/cm$^3$. This value is used in all calculations in the report.

The profiles of air-bubble distribution, as presented in figures 13 and 14, show that the size of the maximum number of air bubbles per unit area is below 0.1 mm. The general shape of the curves does not appear to have any correlation with depth in the ice sheet.

The profiles of the crystal grains, as presented in figures 11 and 12, show the relation between the size of the crystal grain and the size distribution per unit area. For nearly all depths in the ice sheet, the highest percentage of crystal grain size lies in the 0.5 mm to 1.0 mm region.

# Other Physical Properties

## CONDUCTED HEAT FLOW THROUGH AN ICE SHEET WITH A SNOW COVER

Figure 15 shows the conducted heat flow per unit area from the water through the ice sheet, the snow cover, and into the atmosphere. The data cover a period of one week from 27 March to 2 April 1955. Resistance-wire thermometers and Western Electric type 14B thermistors served as the thermal elements and measurements were made with a Leeds and Northrup type S bridge. Malmgren's[14] values for the thermal conductivity of ice are used in the calculations. The thermal conductivity value for snow is the average value found in Dorsey.[15] The measurements represent a preliminary heat-transfer study,, in which the convective heat flux, the evaporative heat flux, and the heat of ice formation have not been measured.

## RESISTIVITY MEASUREMENTS IN SEA ICE

The apparent resistivity measurements in sea ice represent a preliminary survey of techniques and necessary instrumentation. The apparent resistivities (table 5) show a wide variation with increase in electrode spacing. The ice thickness in the area of these measurements showed an average value of 84.0 cm (33 inches). The

TABLE 5. Apparent dc resistivity measurements.

| Separation of Electrodes *a* (feet) | Separation of Electrodes *a* (cm) | Potential *E* (mv) | Current *I* (ma) | Resistivity 191 *a*(*E*/*I*) (ohm-cm) |
|---|---|---|---|---|
| 0.25 | 7.62 | 1150 | 78 | 705 |
| 0.50 | 15.24 | 820 | 70 | 1120 |
| 0.50 | 15.24 | 725 | 58 | 1187 |
| 0.75 | 22.86 | 500 | 48 | 1490 |
| 1.0 | 30.48 | 300 | 40 | 1432 |
| 1.0 | 30.48 | 575 | 82 | 1340 |
| 1.5 | 45.72 | 180 | 34 | 1517 |
| 1.5 | 45.72 | 330 | 63 | 1500 |
| 2.0 | 60.96 | 325 | 80 | 1550 |
| 2.0 | 60.96 | 300 | 88 | 1300 |
| 3.0 | 91.44 | 100 | 64 | 895 |
| 3.0 | 91.44 | 125 | 70 | 1023 |
| 4.0 | 121.92 | 70 | 69 | 773 |
| 4.0 | 121.92 | 30 | 75 | 305 |
| 6.0 | 182.88 | 15 | 90 | 191 |
| 6.0 | 182.88 | 10 | 102 | 112 |

results indicate the effect of salinity variations in the ice sheet on the apparent resistivity values. As electrode spacing is increased, the distribution of electric current spreads vertically downward into the ice sheet and to the water beneath the ice. The low resistivity value with a minimum electrode separation of 7.62 cm (3 inches) indicates the high salinity content of the ice at or near the surface. The apparent resistivity calculations indicate a maximum value at an electrode spacing of approximately 60.9 cm (2 feet). As electrode separation is increased beyond 60.9 cm (2 feet), the apparent resistivity falls off rapidly, indicating the shunting effect of the sea water path of higher conductivity. The measurements were taken within a temperature range of —5°C to —2°C. Because of the short period of time in which field measurements could be made, it was impossible to obtain a more complete temperature-resistivity profile. In comparing the data obtained with the results of a more complete study of another group,[16] it appears that the measurements are reasonably consistent with those of the other group within a temperature range common to the results of each group.

## RECOMMENDATIONS

1. A study should be made of the variation of all parameters over one entire ice season at one location. In conjunction with this survey, an attempt should be made to relate the plate-wave velocity variation with the state of the sea ice and the state of the sea ice with its breaking properties.

2. Continuous measurements over a complete ice season should be made of: (1) the temperature gradients through an ice sheet with a snow cover, (2) wind gradients above the ice sheet, (3) wet and dry temperature gradients above the ice sheet, and (4) the net radiation. This would give the heat transfer through an ice sheet by three different methods. From these measurements the thermal conductivity of the sea-ice sheet, the surface albedo, and the exchange coefficients at the air-snow or ice boundary can be determined.

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