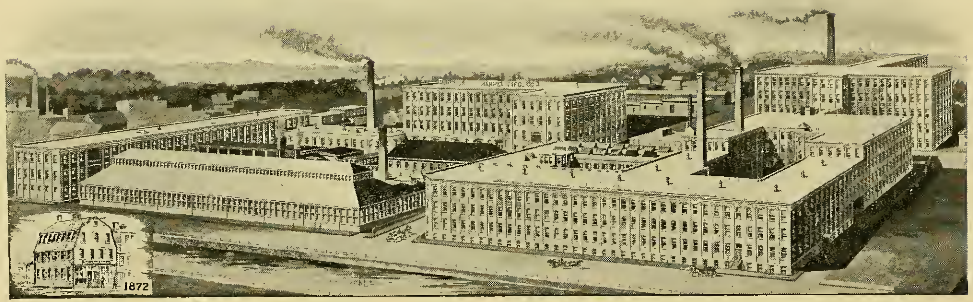

BROWN & SHARPE MFG. CO., PROVIDENCE, R. I., U. S. A.

PRACTICAL TREATISE
ON GEARING.

EIGHTH EDITION.

BROWN & SHARPE MANUFACTURING CO.
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PREFACE.

This Book is made for men in practical life; for those that would like to know how to construct gear wheels, but whose duties do not afford them sufficient leisure to acquire a technical knowledge of the subject.

CONTENTS.

PART I.

PART I.

CHAPTER I.

PITCH CIRCLE, PITCH, TOOTH, SPACE, ADDENDUM OR FACE, FLANK, CLEARANCE.

Let two cylinders, Fig. 1, touch each other, their axes be parallel and the cylinders be on shafts, turning freely. If, now, we turn one cylinder, the adhesion of its surface to the surface of the other cylinder will make that turn also. The surfaces touching each other, without slipping one upon the other, will evidently move through the same distance in a given time. This surface speed is called *linear velocity*. Original Cylinders. Linear Velocity.

TANGENT CYLINDERS.

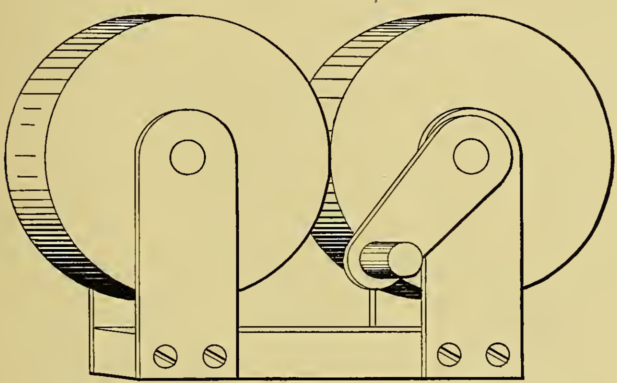


Fig. 1

Linear Velocity is the distance a point moves along a line in a unit of time.

The line described by a point in the circumference of either of these cylinders, as it rotates, may be called an arc. The length of the arc (which may be greater or less than the circumference of cylinder), described in a unit of time, is the velocity. The length, expressed in linear units, as inches, feet, etc., is the linear velocity.

The length, expressed in angular units, as degrees, is the **angular velocity.**

Angular Velocity. If now, instead of 1° we take 360°, or one turn, as the angular unit, and 1 minute as the time unit, the angular velocity will be expressed in turns or revolutions per minute.

Relative Angular Velocity. If these two cylinders are of the same size, one will make the same number of turns in a minute that the other makes. If one cylinder is twice as large as the other, the smaller will make two turns while the larger makes one, but the linear velocity of the surface of each cylinder remains the same.

This combination would be very useful in mechanism if we could be sure that one cylinder would always turn the other without slipping.

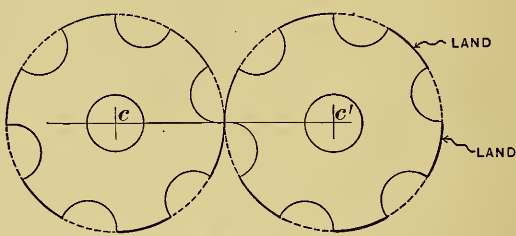


Fig. 2

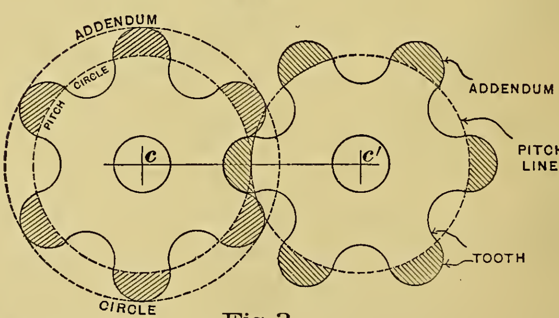


Fig. 3

In the periphery of these two cylinders, as in Fig. 2, cut equidistant grooves. In any grooved piece the places between grooves are called *lands*. Upon the lands add parts; these parts are called addenda. A land and its addendum is called a *tooth*. A toothed cylinder is called a *gear*. Two or more gears with teeth interlocking are called a *train*. A line, c c′, Fig.

Land. Addendum. Tooth. Gear. Train.

2 or 3, between the centers of two wheels is called the *line of centers.* A circle just touching the addenda is called the *addendum circle.*

Line of Centers.

Addendum Circle.

The circumference of the cylinders without teeth is called the *pitch circle.* This circle exists geometrically in every gear and is still called the pitch circle or the primitive circle. In the study of gear wheels, it is the problem so to shape the teeth that the pitch circles will just touch each other without slipping.

Pitch Circle.

Pitch Circle is also called the Primitive Circle.

On two fixed centers there can turn only two circles, one circle on each center, in a given relative angular velocity and touch each other without slipping.

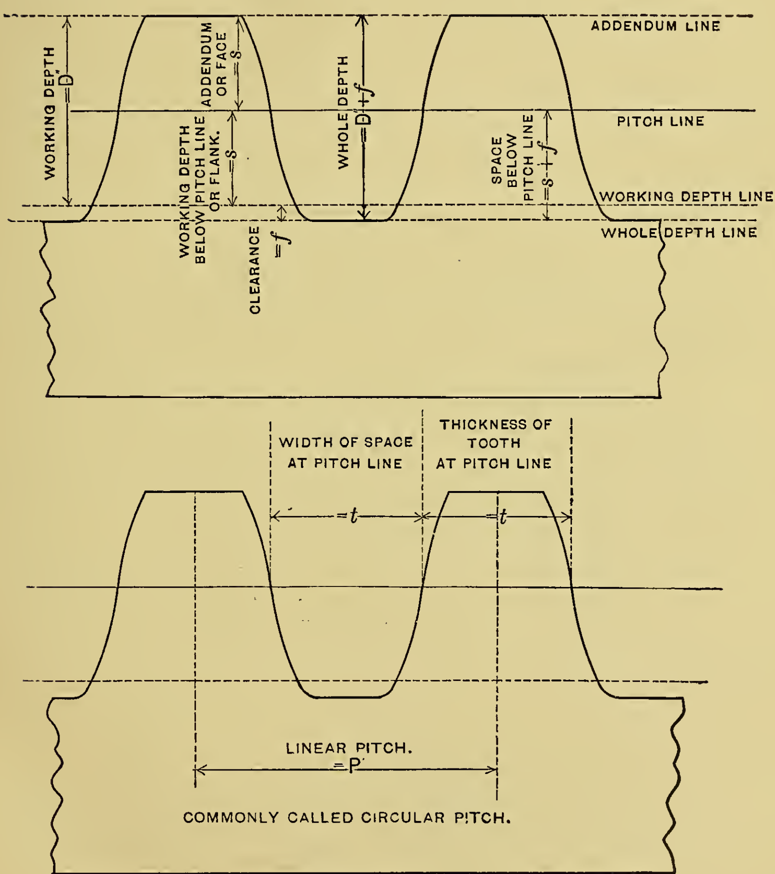


Fig. 4

Space.

The groove between two teeth is called a *space*. In cut gears the width of space at pitch line and thickness of tooth at pitch line are equal. The distance between the center of one tooth and the center of the next tooth,

Linear or Circular Pitch.

measured along the pitch line, is the *linear or circular pitch;* that is, the linear or circular pitch is equal to a

Tooth Thickness.

tooth and a space; hence, the thickness of a tooth at the pitch line is equal to one-half the linear or circular pitch.

Abbreviations of Parts for Teeth and Gear.

Let $D =$ diameter of addendum circle.
" $D' =$ diameter of pitch circle.
" $P' =$ linear or circular pitch.
" $t =$ thickness of tooth at pitch line.
" $s =$ addendum or face, also length of working part of tooth below pitch line or flank.
" $2s = D''$ or twice the addendum, equals the working depth of teeth of two gears in mesh.
" $f =$ clearance or extra depth of space below working depth.
" $s + f =$ depth of space below pitch line.
" $D'' + f =$ whole depth of space.
" $N =$ number of teeth in one gear.
" $\pi = 3.1416$ or the circumference when diameter is 1.

P' is read "P prime." D'' is read "D second." π is read "pi."

To find the Circumference and Diameter of a Circle.

If we multiply the diameter of any circle by π, the product will be the circumference of this circle. If we divide the circumference of any circle by π, the quotient will be the diameter of this circle.

Pitch Point.

The pitch point of the side of a tooth is the point at which the pitch circle or line meets the side of the tooth. A gear tooth has two pitch points.

CHAPTER II.

CLASSIFICATION--SIZING BLANKS AND TOOTH PARTS FROM CIRCULAR PITCH—CENTRE DISTANCE—PATTERN GEARS.

If we conceive the pitch of a pair of gears to be made the smallest possible, we ultimately come to the conception of teeth that are merely lines upon the original pitch surfaces. These lines are called *elements* of the teeth. Gears may be classified with reference to the elements of their teeth, and also with reference to the relative position of their axes or shafts. In most gears the elements of teeth are either straight lines or helices (screw-like lines). Elements of the Teeth.

PART I. of this book, treats upon THREE KINDS OF GEARS.

First—SPUR GEARS; those connecting parallel shafts and whose tooth elements are straight. Spur Gears.

Second—BEVEL GEARS; those connecting shafts whose axes meet when sufficiently prolonged, and the elements of whose teeth are straight lines. In bevel gears the surfaces that touch each other, without slipping, are upon cones or parts of cones whose apexes are at the same point where axes of shafts meet. Bevel Gears.

Third—SCREW OR WORM GEARS; those connecting shafts that are not parallel and do not meet, and the elements of whose teeth are helical or screw-like. Screw or Worm Gears.

The circular pitch and number of teeth in a wheel being given, the diameter of the wheel and size of tooth parts are found as follows: Sizing Blanks, &c.

Dividing by 3.1416 is the same as multiplying by $\frac{1}{3.1416}$. Now $\frac{1}{3.1416}=.3183$; hence, multiply the circumference of a circle by .3183 and the product will be the diameter of the circle. Multiply the circular pitch by .3183 and the product will be the same *part* of the diameter of pitch circle that the circular pitch is of the circumference of pitch circle. This part is called *the module of the pitch.* There are as many modules contained in the diameter of a pitch circle as there are teeth in the wheel.

A Diameter Pitch, or Module.

The Module and the Addendum measure the same, radially.

Most mechanics make the addendum of teeth equal the module. Hence we can designate the module by the same letter as we do the addendum; that is, let $s=$ the module.

$.3183\ P'=s$, or circular pitch multiplied by $.3183=s$, or the module.

Diameter of Pitch Circle.

$Ns=D'$, or number of teeth in a wheel, multiplied by the module, equals diameter of pitch circle.

Whole Diameter.

$(N+2)\ s=D$, or add 2 to the number of teeth, multiply the sum by the module and product will be the whole diameter.

Clearance.

$\frac{t}{10}=f$, or one tenth of thickness of tooth at pitch line equals amount added to bottom of space for clearance.

Some mechanics prefer to make f equal to $\frac{1}{16}$ of the working depth of teeth, or $.0625\ D''$. One-tenth of the thickness of tooth at pitch-line is more than one-sixteenth of working depth, being $.07854\ D''$.

Example.

Sizes of Blank and Tooth Parts for Gear of 30 teeth 1½ in. Circular Pitch.

Example.—Wheel 30 teeth, $1\frac{1}{2}''$ circular pitch. $P'=1.5''$; then $t=.75''$ or thickness of tooth equals $\frac{3}{4}''$. $s=1.5''\times.3183=.4775=$ module for $1\frac{1}{2}''\ P'$. (See table of tooth parts, pages 144–147.

$D'=30\times.4775''=14.325''=$ diameter of pitch-circle.

$D=(30+2)\times.4775''=15.280''=$ diameter of addendum circle, or the diameter of the blank.

$f=\frac{1}{10}$ of $.75''=.075''=$ clearance at bottom of space.

$D''=2\times.4775''=.9549''=$ working depth of teeth.

$D''+f=2\times.4775''+.075''=1.0299''=$ whole depth of space.

$s+f=.4775''+.075''=.5525''=$ depth of space inside of pitch-line.

$D''=2s$ or the working depth of teeth is equal to two modules.

In making calculations it is well to retain the fourth place in the decimals, but when drawings are passed into the workshop, three places of decimals are sufficient.

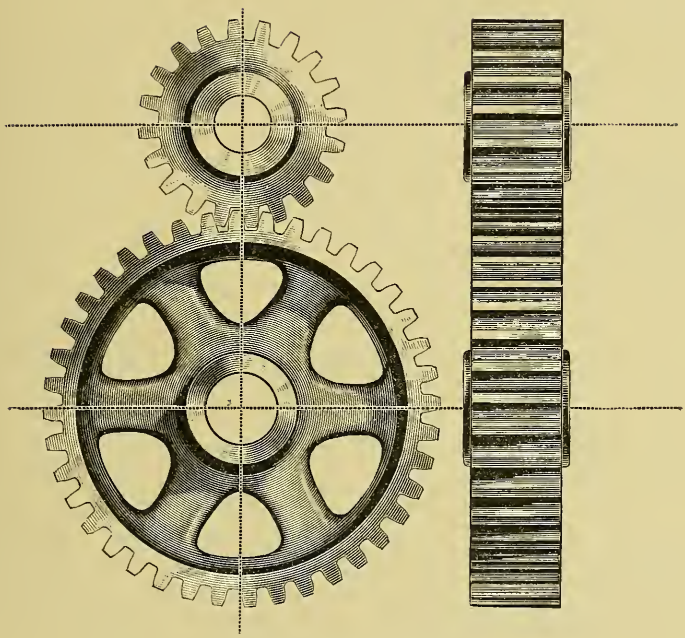


FIG. 5, SPUR GEARING.

Distance between centers of two Gears.

The distance between the centers of two wheels is evidently equal to the radius of pitch-circle of one wheel added to that of the other. The radius of pitch-circle is equal to *s* multiplied by one-half the number of teeth in the wheel.

Hence, if we know the number of teeth in two wheels, in mesh, and the circular pitch, to obtain the distance between centers we first find *s*; then multiply *s* by one-half the sum of number of teeth in both wheels and the product will be distance between centers.

Example.—What is the distance between the centers of two wheels 35 and 60 teeth, $1\frac{1}{4}''$ circular pitch. We first find *s* to be $1\frac{1}{4}'' \times .3183 = .3979''$. Multiplying by 47.5 (one-half the sum of 35 and 60 teeth) we obtain 18.899″ as the distance between centers.

Allowance for Shrinkage in Gear Castings.

Pattern Gears should be made large enough to allow for shrinkage in casting. In cast iron the shrinkage is about $\frac{1}{8}$ inch in one foot. For gears one to two feet in diameter it is well enough to add simply $\frac{1}{100}$ of diameter of finished gear to the pattern. In gears about six inches diameter or less, the moulder will generally rap the pattern in the sand enough to make any allowance for shrinkage unnecessary. In pattern gears the spaces between teeth should be cut wider than finished gear spaces to allow for rapping and to avoid having too much cleaning to do in order to have gears run freely. In cut patterns of iron it is generally enough to make spaces .015″ to .02″ wider. This makes clearance .03″ to .04″ in the patterns. Some moulders might want .06″ to .07″ clearance.

Metal Pattern Gears.

Metal patterns should be cut straight; they work better with no draft. It is well to leave about .005″ to be finished from side of patterns after teeth are cut; this extra stock to be taken away from side where cutter comes through so as to take out places where stock is broken out. The finishing should be done with file or emery wheel, as turning in a lathe is likely to break out stock as badly as a cutter might do.

If cutters are kept sharp and care is taken when coming through the allowance for finishing is not necessary and the blanks may be finished before they are cut.

CHAPTER III.

SINGLE-CURVE GEARS OF 30 TEETH AND MORE.

Single-curve teeth are so called because they have but one curve by theory, this curve forming both face and flank of tooth sides. In any gear of thirty teeth and more, this curve can be a single arc of a circle whose radius is one-fourth the radius of the pitch circle. In gears of thirty teeth and more, a fillet is added at bottom of tooth, to make it stronger, equal in radius to one-seventh the widest part of tooth space. Single Curve Teeth.

A cutter formed to leave this fillet has the advantage of wearing longer than it would if brought up to a corner.

In gears less than thirty teeth this fillet is made the same as just given, and sides of teeth are formed with more than one arc, as will be shown in Chapter VI.

Having calculated the data of a gear of 30 teeth, $\frac{3}{4}$ inch circular pitch (as we did in Chapter II. for $1\frac{1}{2}''$ pitch), we proceed as follows: Example of a Gear, N=30, P =$\frac{3}{4}''$.

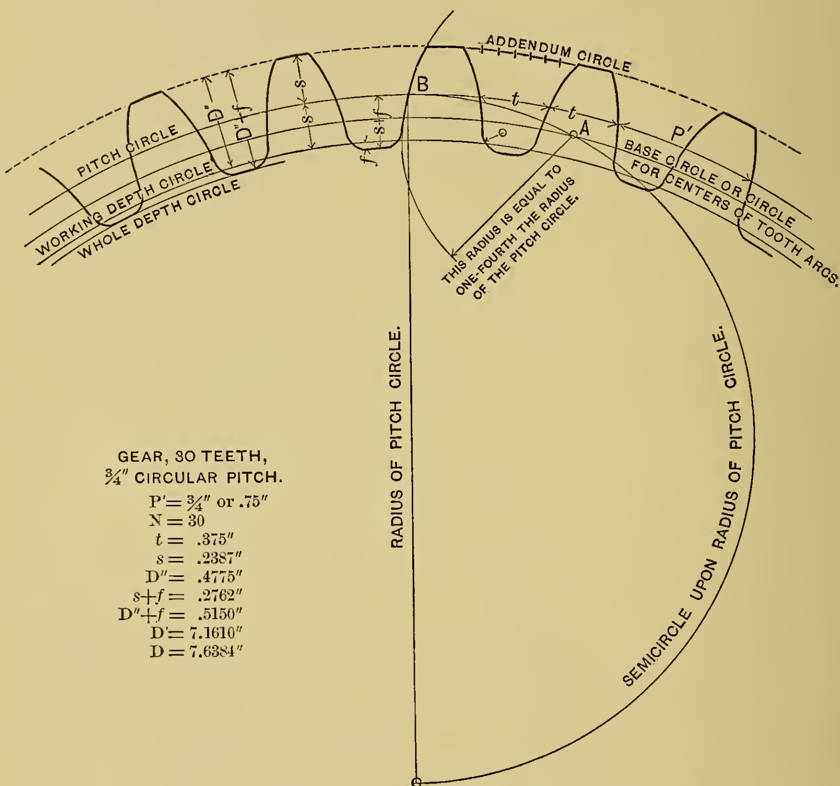
1. Draw pitch circle and point it off into parts equal to one-half the circular pitch. Geometrical Construction. Fig. 6.

2. From one of these points, as at B, Fig. 6, draw radius to pitch circle, and upon this radius describe a semicircle; the diameter of this semicircle being equal to radius of pitch circle. Draw addendum, working depth and whole depth circles.

3. From the point B, Fig. 6, where semicircle, pitch circle and outer end of radius to pitch circle meet, lay off a distance upon semicircle equal to one-fourth the radius of pitch circle, shown in the figure at BA, and is laid off as a chord.

4. Through this new point at A, upon the semicircle, draw a circle concentric to pitch circle. This last is

Fig. 6



SINGLE CURVE GEAR.

called the *base circle*, and is the one for centers of tooth arcs. In the system of single curve gears we have adopted, the diameter of this circle is .968 of the diameter of pitch circle. Thus the base circle of any gear 1 inch pitch diameter by this system is .968″. If the pitch circle is 2″ the base circle will be 1.936.″

5. With dividers set to one-quarter of the radius of pitch circle, draw arcs forming sides of teeth, placing one leg of the dividers in the base circle and letting the other leg describe an arc through a point in the pitch circle that was made in laying off the parts equal to one-half the circular pitch. Thus an arc is drawn about A as center through B.

6. With dividers set to one-seventh of the widest part of tooth space, draw the fillets for strengthening teeth at their roots. These fillet arcs should just touch the whole depth circle and the sides of teeth already described.

Peculiarity of Involute Gearing.

Single curve or involute gears are the *only* gears that can run at varying distance of axes and transmit unvarying angular velocity. This peculiarity makes involute gears specially valuable for driving rolls or any rotating pieces, the distance of whose axes is likely to be changed.

Pressure on bearings.

The assertion that gears crowd harder on bearings when of involute than when of other forms of teeth, has not been proved in actual practice.

Practice, before taking next chapter.

Before taking next chapter, the learner should make several drawings of gears 30 teeth and more. Say make 35 and 70 teeth $1\frac{1}{2}''$ P′. Then make 40 and 65 teeth $\frac{7}{8}''$ P′.

An excellent practice will be to make drawing on cardboard or Bristol-board and cut teeth to lines, thus making paper gears; or, what is still better, make them of sheet metal. By placing these in mesh the learner can test the accuracy of his work.

CHAPTER IV.

RACK TO MESH WITH SINGLE-CURVE GEARS HAVING 30 TEETH AND MORE.

Diagram, made preparatory to drawing a Rack.

This gear (Fig. 7) is made precisely the same as gear in Chapter III. It makes no difference in which direction the construction radius is drawn, so far as obtaining form of teeth and making gear are concerned.

Here the radius is drawn perpendicular to pitch line of rack and through one of the tooth sides, B. A semicircle is drawn on each side of the radius of the pitch circle.

The points A and A′ are each distant from the point B, equal to one-fourth the radius of pitch circle and correspond to the point A in Fig. 6.

In Fig. 7 add two lines, one passing through B and A and one through B and A′. These two lines form angles of $75\frac{1}{2}°$ (degrees) with radius BO. Lines BA and BA′ are called lines of pressure. The sides of rack teeth are made perpendicular to these lines.

Rack.

A **Rack** is a straight piece, having teeth to mesh with a gear. A rack may be considered as a gear of infinitely long radius. The circumference of a circle approaches a straight line as the radius increases, and when the radius is infinitely long any finite part of the circumference is a straight line. The pitch line of a rack, then, is merely a straight line just touching the pitch circle of a gear meshing with the rack. The thickness of teeth, addendum and depth of teeth below pitch line are calculated the same as for a wheel. (For pitches in common use, see table of tooth parts.)

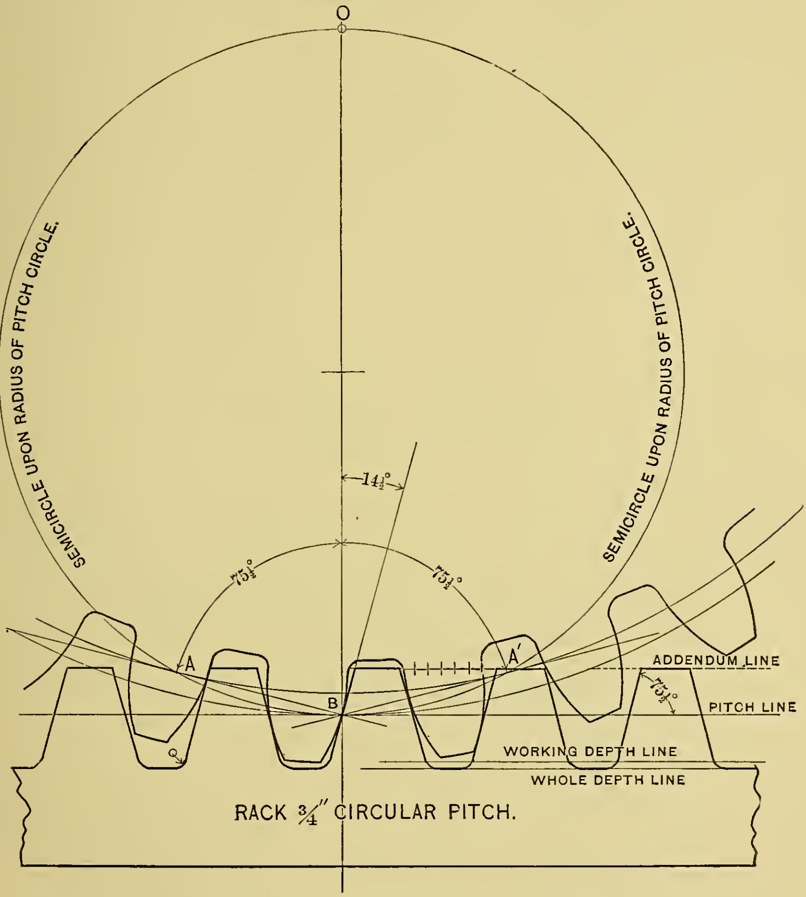
Construction of Pitch Line of Rack.

The term *circular pitch* when applied to racks can be more accurately replaced by the term *linear pitch*. Linear applies strictly to a line in general while circular pertains to a circle. Linear pitch means the distance between the centres of two teeth on the pitch line whether the line is straight or curved.

A rack to mesh with a single-curve gear of 30 teeth or more is drawn as follows:

1. Draw straight pitch line of rack; also draw addendum line, working depth line and whole depth line, each parallel to the pitch line (see Fig. 7). Rack. Fig. 7.

Fig. 7



RACK TO MESH WITH SINGLE CURVE GEAR HAVING 30 TEETH AND MORE.

2. Point off the pitch line into parts equal to one-half the circular pitch, or $= t$.

3. Through these points draw lines at an angle of $75\frac{1}{2}°$ with pitch lines, alternate lines slanting in opposite directions. The left-hand side of each rack tooth is perpendicular to the line BA. The right-hand side of each rack tooth is perpendicular to the line BA′.

4. Add fillets at bottom of teeth equal to $\frac{1}{7}$ of the width of spaces between the rack teeth at the addendum line.

Angle for sides of Rack Teeth.

The sketch, Fig. 8, will show how to obtain angle of sides of rack teeth, directly from pitch line of rack, without drawing a gear in mesh with the rack.

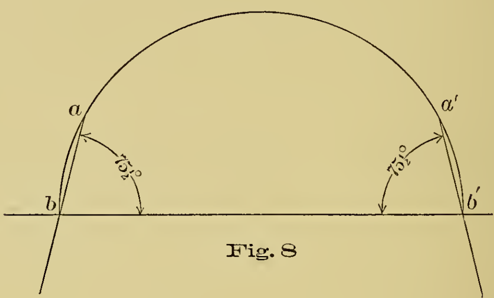


Fig. 8

Upon the pitch line $b\ b'$, draw any semicircle—$b\ a\ a'\ b'$. From point b lay off upon the semicircle the distance $b\ a$, equal to one-quarter of the diameter of semicircle, and draw a straight line through b and a.

This line, $b\ a$, makes an angle of $75\frac{1}{2}°$ with pitch line $b\ b'$, and can be one side of rack tooth. The same construction, $b'\ a'$, will give the inclination $75\frac{1}{2}°$ in the opposite direction for the other side of tooth.

The sketch, Fig. 9, gives the angle of sides of a tool for planing out spaces between rack teeth. Upon any line OB draw circle OABA′. From B lay off distance BA and BA′, each equal to one-quarter of diameter of the circle.

Draw lines OA and OA′. These two lines form an angle of 29°, and are right for inclination of sides of rack tool.

Make end of rack tool .31 of circular pitch, and then round the corners of the tool to leave fillets at the bottom of rack teeth. Width of Rack Tool at end.

Thus, if the circular pitch of a rack is $1\frac{1}{2}''$ and we multiply by .31, the product .465″ will be the width of tool at end for rack of this pitch before corners are taken off. This width is shown at x y.

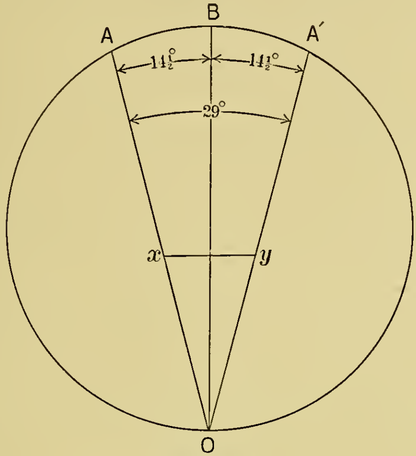
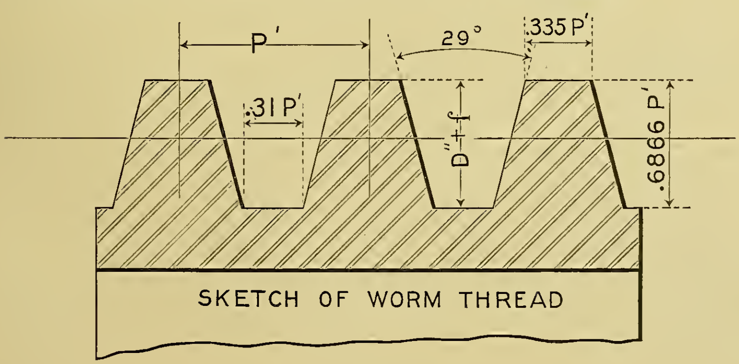


Fig. 9

A Worm is a screw that meshes with the teeth of a gear.

This sketch and the foregoing rule are also right for a worm-thread tool, but a worm-thread tool is not usually rounded for fillet. In cutting worms, leave width of top of thread .335 of the circular pitch. When this is done, the depth of thread will be right. Worm Thread Tool.



CHAPTER V.

DIAMETRAL PITCH—SIZING BLANKS AND THE TEETH OF SPUR GEARS —DISTANCE BETWEEN THE CENTRES OF WHEELS.

When it is necessary to know the Circular Pitch.

In making drawings of gears, and in cutting racks, it is necessary to know the circular pitch, both on account of spacing teeth and calculating their strength. It would be more convenient to express the circular pitch in whole inches, and the most natural divisions of an inch, as 1″ P′, $\frac{3}{4}$″ P′, $\frac{1}{2}$″ P′, and so on. But as the circumference of the pitch circle must contain the circular pitch some whole number of times, corresponding to the number of teeth in the gear, the *diameter* of the pitch circle will often be of a size not readily measured with a common rule. This is because the circumference of a circle is equal to 3.1416 times the diameter, or the diameter is equal to the circumference multiplied by .3183.

In a complete Wheel, the Pitch Circumference must contain the Circular Pitch, some whole number of times.

Pitch, in Terms of the Diameter.

In practice, it is better that the diameter should be of some size conveniently measured. The same applies to the distance between centers. Hence it is generally more convenient to assume the pitch in terms of the diameter. In Chapter II. was given a definition of the module, and also how to obtain the module from the circular pitch.

Circular Pitch and a Diameter Pitch.

We can also assume the module and pass to its equivalent circular pitch. If the circumference of the pitch circle is divided by the number of teeth in the gear, the quotient will be the circular pitch. In the same manner, if the *diameter* of the pitch circle is divided by the number of teeth, the quotient will be the module. Thus, if a gear is 12 inches pitch diameter and has 48 teeth, dividing 12″ by 48, the quotient $\frac{1}{4}$″ is the module of this gear. In practice, the module is taken in some convenient part of an inch, as $\frac{1}{2}''$ module and so on. It is convenient in calculation to designate one of these modules by s, as in Chapter II. Thus, for $\frac{1}{2}''$ module, s is equal to $\frac{1}{2}''$. Generally, in speaking of the module, the denominator of the fraction only is named. $\frac{1}{3}''$ module is then called 3 *diametral pitch*. That is, it has been found more convenient to take the *reciprocal* of the module in making calculation. The reciprocal of a number is 1 divided by that number. Thus the reciprocal of $\frac{1}{4}$ is 4, because $\frac{1}{4}$ goes into 1 four times.

Abbreviation of Module Diameter Pitch.

Reciprocal of a Number.

Hence, we come to the common definition:

Diametral Pitch.

DIAMETRAL PITCH is the number of teeth to *one inch* of diameter of pitch circle. Let this be denoted by P. Thus, $\frac{1}{4}''$ diameter pitch we would call 4 *diametral* pitch or 4 P, because there would be 4 teeth to every inch in the diameter of pitch circle. The circular pitch and the different parts of the teeth are derived from the diametral pitch as follows.

Given, the Diametral to find the Circular Pitch.

To obtain Circular Pitch from Diametral Pitch.

$\frac{3.1416}{P} = P'$, or 3.1416 divided by the diametral pitch is equal to the circular pitch. Thus to obtain the circular for 4 diametral pitch, we divide 3 1416 by 4 and obtain .7854 for the circular pitch, corresponding to 4 diametral pitch.

In this case we would write $P = 4$, $P' = .7854''$, $s = \frac{1}{4}''$.

$\frac{1''}{P} = s$, or one inch divided by the number of teeth to an inch, gives distance on diameter of pitch circle occupied by *one* tooth or the module. The addendum or face of tooth is the same distance as the module.

$\frac{1}{s} = P$, or one inch divided by the module equals number of teeth to one inch or the diametral pitch.

Given, the Diametral Pitch to find the Thickness of Tooth at the Pitch Line.

$\frac{1.57}{P} = t$, or 1.57 divided by the diametral pitch gives thickness of tooth at pitch line. Thus, thickness of teeth along the pitch line for 4 diametral pitch is .392″.

Given, the Number of Teeth in a wheel and the Diametral Pitch to find the Diameter of Pitch Circle.

$\frac{N}{P} = D'$, or number of teeth in a gear divided by the diametral pitch equals diameter of the pitch circle. Thus for a wheel, 60 teeth, 12 P, the diameter of pitch circle will be 5 inches.

Given, the Number of Teeth in a wheel and the Diametral Pitch to find the Whole Diameter.

$\frac{N+2}{P} = D$, or add 2 to the number of teeth in a wheel and divide the sum by the diametral pitch; and the quotient will be the *whole diameter* of the gear or the diameter of the addendum circle. Thus, for 60 teeth, 12 P, the diameter of gear blank will be $5\frac{2}{12}$ inches.

$\frac{N}{D'}=P$, or number of teeth divided by diameter of pitch circle in inches, gives the diametral pitch or number of teeth to one inch. Thus, in a wheel, 24 teeth, 3 inches pitch diameter, the diametral pitch is 8.

$\frac{N+2}{D}=P$, or add 2 to the number of teeth; divide the sum by the whole diameter of gear, and the quotient will be the diametral pitch. Thus, for a wheel $3\frac{2}{10}''$ diameter, 14 teeth, the diametral pitch is 5.

$D'\,P=N$, or diameter of pitch circle, multiplied by diametral pitch equals number of teeth in the gear. Thus, in a gear, 5 pitch, 8″ pitch diameter, the number of teeth is 40.

$D\,P-2=N$ or multiply the whole diameter of the gear by the diametral pitch, subtract 2, and the remainder will be the number of teeth.

$\frac{D}{N+2}=s$, or divide the whole diameter of a spur gear by the number of teeth plus two, and the quotient will be the module.

The Diametral Pitch. When we say *the diametral pitch* we shall mean the number of teeth to one inch of diameter of pitch circle, or P, $(\frac{1''}{s}=P)$.

To obtain Diametral Pitch from Circular Pitch. When the circular pitch is given, to find the corresponding diametral pitch, divide 3.1416 by the circular pitch. Thus 1.57 P is the diametral pitch corresponding to 2-inch circular pitch, $(\frac{3.1416}{P'}=P)$.

Example. What diametral pitch corresponds to $\frac{1}{2}''$ circular pitch? Remembering that to divide by a fraction we multiply by the denominator and divide by the numerator, we obtain 6.28 as the quotient of 3.1416 divided by $\frac{1}{2}$. 6.28 P, then, is the diametral pitch corresponding to $\frac{1}{2}$ circular pitch. This means that in a gear of $\frac{1}{2}$ inch circular pitch there are six and twenty-eight one hundredths teeth to every inch in the diameter of the pitch circle. In the table of tooth parts the diametral pitches corresponding to circular pitches are carried out to four places of decimals, but in practice three places of decimals are enough.

When two gears are in mesh, so that their pitch circles just touch, the distance between their axes or centers is equal to the sum of the radii of the two gears. The number of the modules between centers is equal to half the sum of number of teeth in both gears. This principle is the same as given in Chapter II., page 6, but when the diametral pitch and numbers of teeth in two gears are given, *add together the numbers of teeth in the two wheels and divide half the sum by the diametral pitch. The quotient is the center distance.*

Rule to find Distance between Centers.

A gear of 20 teeth, 4 P, meshes with a gear of 50 teeth; what is the distance between their axes or centers? Adding 50 to 20 and dividing half the sum by 4, we obtain $8\frac{3}{4}''$ as the center distance.

Example.

The term *diametral pitch* is also applied to a rack. Thus, a rack 3 P, means a rack that will mesh with a gear of 3 diametral pitch.

It will be seen that if the expression for the module has any number except 1 for a numerator, we cannot express the diametral pitch by naming the denominator only. Thus, if the addendum or module is $\frac{4}{10}''$, the diametral pitch will be $2\frac{1}{2}$, because 1 divided by $\frac{4}{10}$ equals $2\frac{1}{2}$.

Fractional Diametral Pitch.

The term *module* is much used where gears are made to metric sizes, for the reason that, the millimeter being so short, the module is conveniently expressed in millimeters. If we know the module of a gear we can figure the other parts as easily as we can if we know either the circular pitch or the diametral pitch. The module is, in a sense, an actual distance, while the diametral pitch, or the number of teeth to an inch, is a relation or merely a ratio. The meaning of the *module* is not easily mistaken.

CHAPTER VI.

SINGLE-CURVE GEARS HAVING LESS THAN 30 TEETH—GEARS AND RACKS TO MESH WITH GEARS HAVING LESS THAN 30 TEETH.

Construction, Fig. 10.

In Fig. 10, the construction of the rack is the same as the construction of the rack in Chapter IV. The gear in Fig. 10 is drawn from base circle out to addendum circle, by the same method as the gear in Chapter III., but the spaces inside of base circle are drawn as follows:

Flanks of Gears in low Numbers of Teeth.

In gears, 12 to 19 teeth, the sides of space inside of the base circle are radial for a distance, a b, equal to $\frac{3.5}{NP}$, or 3.5 divided by the product of the pitch by the number of teeth. In gears with more than 19 teeth the radial construction is omitted.

Construction of Fig. 10 continued.

Then, with one leg of dividers in pitch circle in center of next tooth, *e*, and other leg just touching one of the radial lines at *b*, continue the tooth side into *c*, until it will touch a fillet arc, whose radius is $\frac{1}{7}$ the width of space at the addendum circle. The part, b' c', is an arc from center of tooth *g*, etc. The flanks of teeth or spaces in gear, Fig. 11, are made the same as those in Fig. 10.

This rule is merely conventional or not founded upon any principle other than the judgment of the designer, to effect the object to have spaces as wide as practicable, just below or inside of base circle, and then strengthen flank with as large a fillet as will clear addenda of any gear. If flanks in any gear will clear addenda of a rack, they will clear addenda of any other gear, except internal gears. An internal gear is one having teeth upon the inner side of a rim or ring. Now, it will be seen that the gear, Fig. 10, has teeth

Internal Gear.

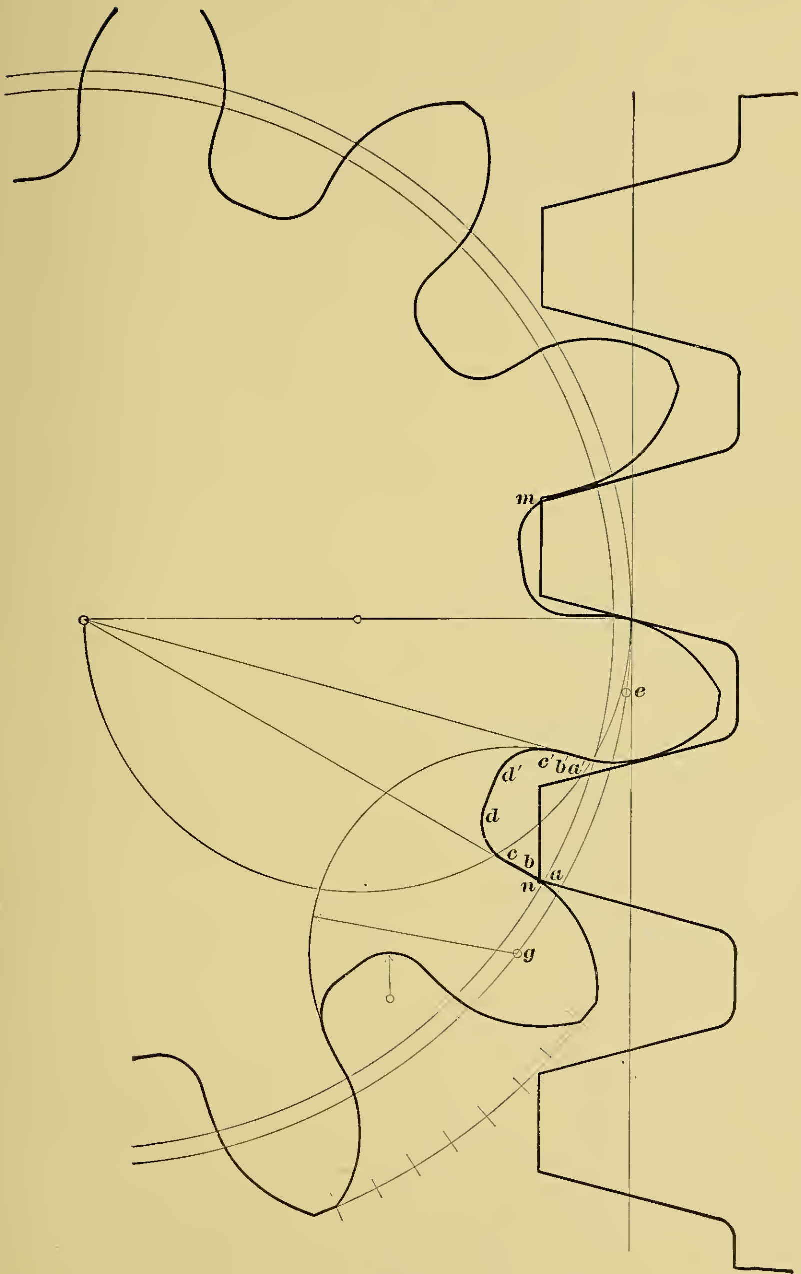


Fig. 10

too much rounded at the points or at the addendum circle. In gears of pitch coarser than 10 to inch (10 P), and having less than 30 teeth, this rounding becomes objectionable. This rounding occurs, because in these gears arcs of circles depart too far from the true involute curve, being so much that points of teeth get no bearing on flanks of teeth in other wheels.

Rounding of Addenda of Teeth.

In gear, Fig. 11, the teeth outside of base circle are made as nearly true involute as a workman will be able to get without special machinery. This is accomplished as follows: draw three or four tangents to the base circle, *i i′*, *j j′*, *k k′*, *l l′*, letting the points of tangency on base circle *i′*, *j′*, *k′*, *l′* be about $\frac{1}{3}$ or $\frac{1}{4}$ the circular pitch apart; the first point, *i′*, being distant from *i*, equal to $\frac{1}{4}$ the radius of pitch circle. With dividers set to $\frac{1}{4}$ the radius of pitch circle, placing one leg in *i′*, draw the arc, *a′ i j;* with one leg in *j′*, and radius *j′ j*, draw *j k;* with one leg in *k′*, and radius *k′ k* draw *k l*. Should the addendum circle be outside of *l*, the tooth side can be completed with the last radius, *l′ l*. The arcs, *a′ i j*, *j k* and *k l*, together form a very close approximation to a true involute from the base circle, *i′ j′ k′ l′*. The *exact* involute for gear teeth is the curve made by the end of a band when unwound from a cylinder of the same diameter as base circle.

Approximation to True Involute.

The foregoing operation of drawing tooth sides, although tedious in description, is very easy of practical application.

Rounding of Addenda of Rack.

It will also be seen that the addenda of rack teeth in Fig. 10, interfere with the gear-teeth flanks, as at *m n;* to avoid this interference, the teeth of rack, Fig. 11, are rounded at points or addenda.

Templets necessary for Rounding off Points of teeth.

It is also necessary to round off the points of involute teeth in high-numbered gears, when they are to interchange with low-numbered gears. In interchangeable sets of gears the lowest-numbered pinion is usually 12. Just how much to round off can be learned by making templets of a few teeth out of thin metal or cardboard, for the gear and rack, or, two gears required, and fitting addenda of teeth to clear flanks. However accurate we may make a diagram, it is quite

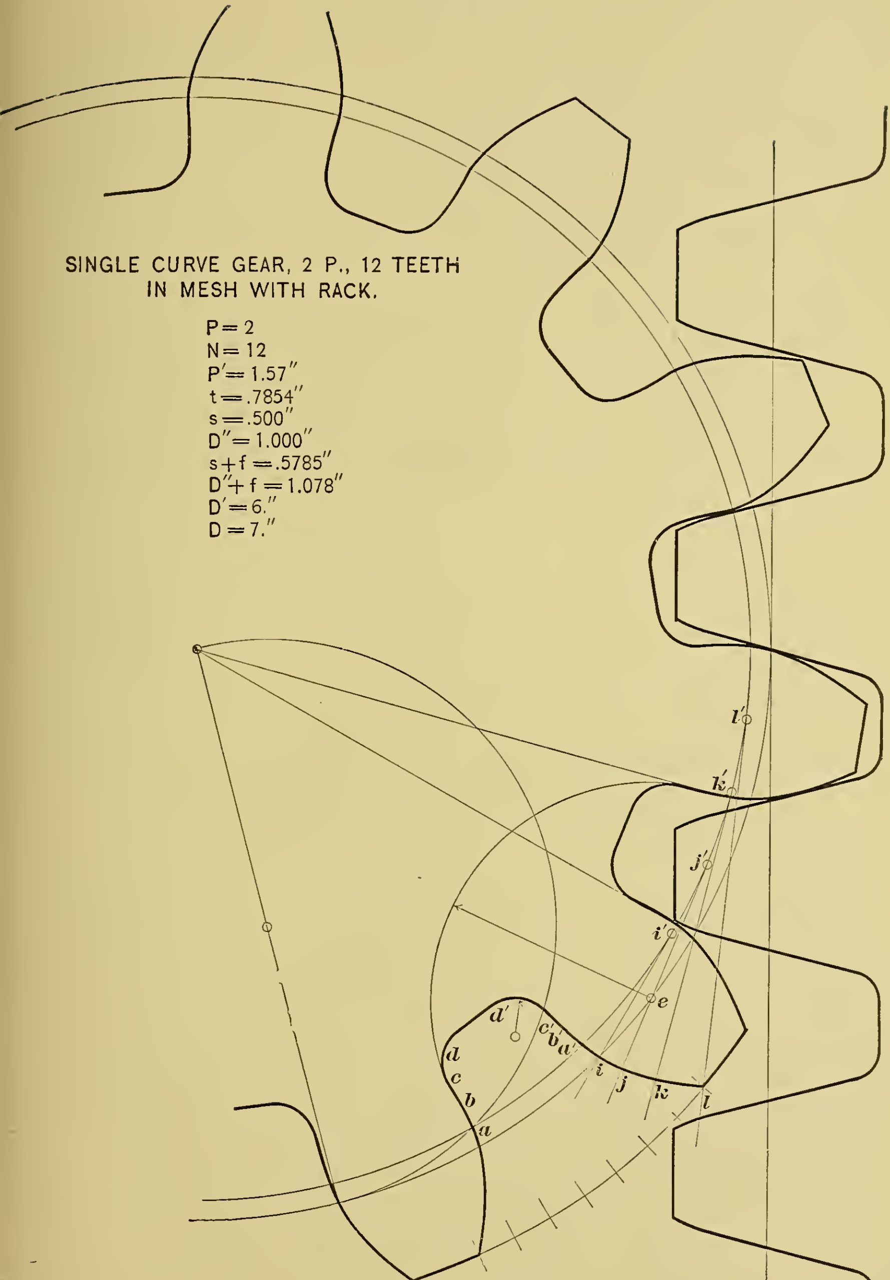

SINGLE CURVE GEAR, 2 P., 12 TEETH
IN MESH WITH RACK.
P = 2
N = 12
P′ = 1.57″
t = .7854″
s = .500″
D″ = 1.000″
s + f = .5785″
D″ + f = 1.078″
D′ = 6.″
D = 7.″
l′
k′
j′
i′
e
d′
c′
b′
a′
d
c
b
a
i
j
k
l

Fig. 11

as well to make templets in order to shape cutters accurately

Diagrams for a Set of Cutters.

It is best to make cutters to corrected diagrams, as in Fig 11. When corrected diagrams are made, as in Fig. 11, take the following:

For	12	and	13	teeth,	diagram	of	12	teeth.
"	14	to	16	"	"	"	14	"
"	17	"	20	"	"	"	17	"
"	21	"	25	"	"	"	21	"
"	26	"	34	"	"	"	26	"
"	35	"	54	"	"	"	35	"
"	55	"	134	"	"	"	55	"
"	135	"	rack,	"	"	"	135	"

Templets for large gears must be fitted to run with 12 teeth.

CHAPTER VII.

DOUBLE-CURVE TEETH—GEAR, 15 TEETH—RACK.

All Double-curve Tooth Faces are Convex.

In double-curve teeth the formation of tooth sides changes at the pitch line. In all gears the part of teeth outside of pitch line is convex; in some gears the sides of teeth inside pitch line are convex; in some, radial; in others, concave. Convex faces and concave flanks are most familiar to mechanics. In interchangeable sets of gears, *one* gear in each set, or of each pitch, has radial flanks. In the best practice, *this gear has fifteen teeth.* Gears with more than fifteen teeth, have concave flanks; gears with less than fifteen teeth, have convex flanks. Fifteen teeth is called the *Base* of this system.

Construction of Fig. 12.

We will first draw a gear of fifteen teeth. This fifteen-tooth construction enters into gears of any number of teeth and also into racks. Let the gear be 3 P. Having obtained data, we proceed as follows:

1. Draw pitch circle and point it off into parts equal to one-thirtieth of the circumference, or equal to thickness of tooth $= t$.

2. From the center, through one of these points, as at T, Fig. 12, draw line O T A. Draw addendum and whole-depth circles.

3. About this point, T, with same radius as 15-tooth pitch circle, describe arcs A K and O k. For any other double-curve gear of 3 P., the radius of arcs, A K and O k, will be the same as in this 15-tooth gear $= 2\frac{1}{2}''$. In a 15-tooth gear, the arc, O k, passes through the center O, but for a gear having any other number of teeth, this construction arc does not pass through center of gear. Of course, the 15-tooth radius of arcs, A K and O k, is always taken from the pitch we are working with.

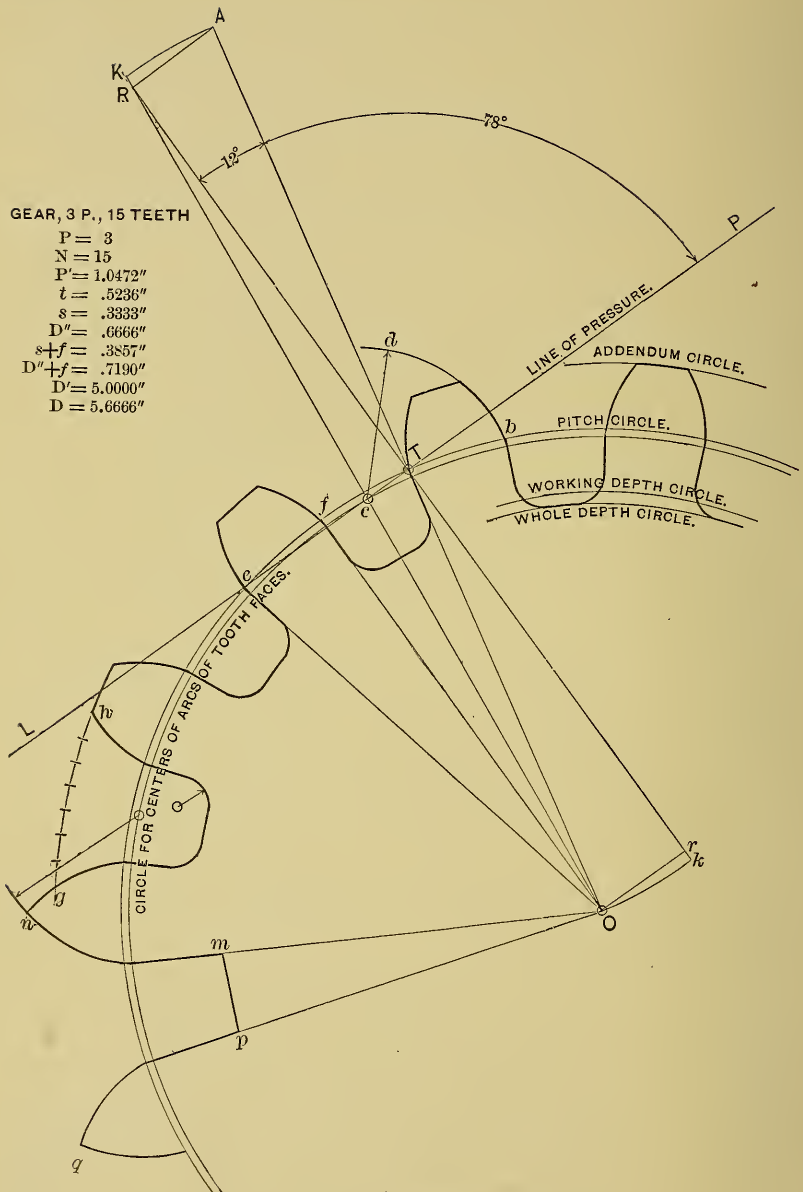


Fig. 12

DOUBLE CURVE GEAR.

4. Upon these arcs on opposite sides of line O T A, lay off tooth thickness, A K and O *k*, and draw line K T *k*.

5. Perpendicular to K T *k*, draw line of pressure, L T P; also through O and A, draw lines A R and O *r*, perpendicular to K T *k*. The line of pressure is at an angle of 78° with the radius of gear.

6. From O, draw a line O R to intersection of A R with K T *k*. Through point *c*, where O R intersects L P, describe a circle about the center, O. *In this circle one leg of dividers* is placed to *describe tooth faces*

7. The radius, *c d*, of arc of tooth faces is the straight distance from *c* to tooth-thickness point, *b*, on the other side of radius, O T. With this radius, *c b*, describe both sides of tooth faces.

8. Draw flanks of all teeth radial, as O *e* and O *f*. The *base gear*, 15 *teeth* only, has radial flanks.

9. With radius equal to one-seventh of the widest part of space, as *g h*, draw fillets at bottom of teeth.

Approximation to Epicycloidal Teeth.

The foregoing is a close approximation to epicycloidal teeth. To get exact teeth, make two 15-tooth gears of thin metal. Make addenda long enough to come to a point, as at *n* and *q*. Make radial flanks, as at *m* and *p*, deep enough to clear addenda when gears are in mesh. First finish the flanks, then fit the long addenda to the flanks when gears are in mesh.

Standard Templets.

When these two templet gears are alike, the centers are the right distance apart and the teeth interlock without backlash, they are exact. One of these templet gears can now be used to test any other templet gear of the same pitch.

Gears and racks will be right when they run correctly with one of these 15-tooth templet gears. Five or six teeth are enough to make in a gear templet.

Double-curve Rack, Fig. 13.

Double-curve Rack.—Let us draw a rack 3 P. Having obtained data of teeth we proceed as follows:

1. Draw pitch line and point it off in parts equal to one-half the circular pitch. Draw addendum and whole-depth lines.

2. Through one of the points, as at T, Fig. 13, draw line O T A perpendicular to pitch line of rack.

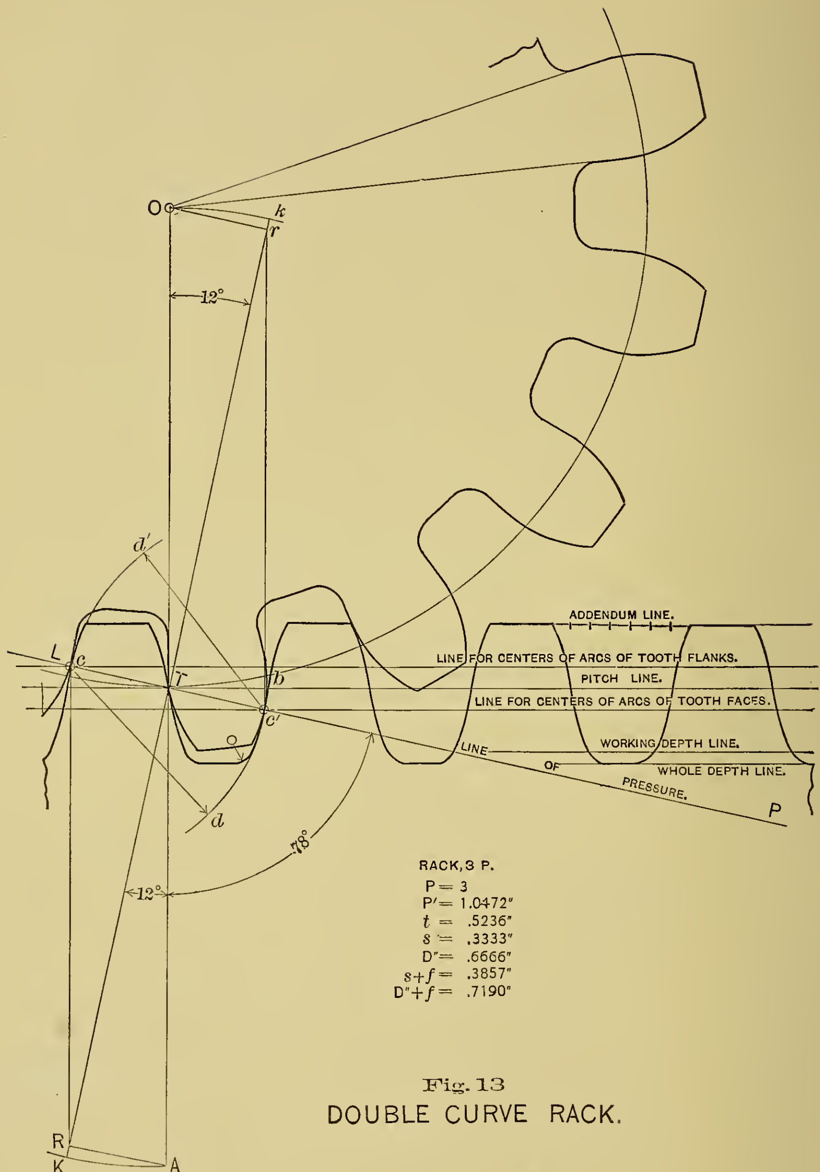


Fig. 13

DOUBLE CURVE RACK.

3. About T make precisely the same construction as was made about T in Fig. 12. That is, with radius of 15-tooth pitch circle and center T draw arcs *O k* and A K; make *O k* and A K equal to tooth thickness; draw K T *k*; draw O *r*, A R, and line of pressure, each perpendicular to K T *k*.

4. Through R and *r*, draw lines parallel to O A. Through intersections *c* and *c′* of these lines, with pressure line L P, draw lines parallel to pitch line.

5. In these last lines place leg of dividers, and draw faces and flanks of teeth as in sketch.

6. The radius *c′ d′* of rack-tooth faces is the same length as radius *c d* of rack-tooth flanks, and is the straight distance from *c* to tooth-thickness point *b* on opposite side of line O A.

7. The radius for fillet at bottom of rack teeth is equal to $\frac{1}{7}$ of the widest part of tooth space. This radius can be varied to suit the judgment of the designer, so long as a fillet does not interfere with teeth of engaging gear.

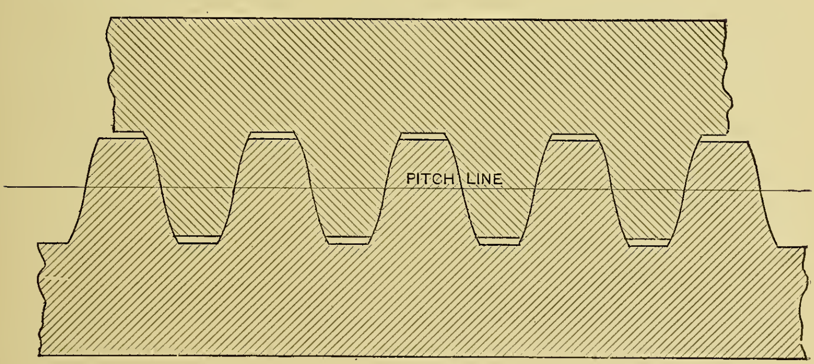


Fig. 14

Racks of the same pitch, to mesh with interchangeable gears, should be alike when placed side by side, and fit each other when placed together as in Fig. 14.

In Fig. 13, a few teeth of a 15-tooth wheel are shown in mesh with the rack.

CHAPTER VIII.

DOUBLE-CURVE SPUR GEARS, HAVING MORE AND FEWER THAN 15 TEETH—ANNULAR GEARS.

Construction of Fig. 15.

Let us draw two gears, 12 and 24 teeth, 4 P, in mesh. In Fig. 15 the construction lines of the lower or 24-tooth gear are full. The upper or 12-tooth gear construction lines are dotted. The line of pressure, L P, and the line K T *k* answer for both gears. The arcs A K and O *k* are described about T. The radius of these arcs is the radius of pitch circle of a gear 15 teeth 4 pitch. The length of arcs A K and O *k* is the tooth thickness for 4 P. The line K T *k* is obtained the same as in Chapter VII. for all double-curve gears, the distances only varying as the pitch. Having drawn the pitch circles, the line K T *k*, and, perpendicular to K T *k*, the lines A R, O *r* and the line of pressure L T P, we proceed with the 24-tooth gear as follows:

1. From center C, through *r*, draw line intersecting line of pressure in *m*. Also draw line from center C to R, crossing the line of pressure L P at *c*.

2. Through *m* describe circle concentric with pitch circle about C. This is the circle in which to place one leg of dividers to describe flanks of teeth.

3. The radius, *m n*, of flanks is the straight distance from *m* to the first tooth-thickness point on other side of line of centers, C C′, at *v*. The arc is continued to *n*, to show how constructed. This method of obtaining radius of double-curve tooth flanks applies to all gears having more than fifteen teeth.

4. The construction of tooth faces is similar to 15-tooth wheel in Chapter VII. That is: Draw a circle through *c* concentric to pitch circle; in this circle place one leg of dividers to draw tooth faces, the radius of tooth faces being *c b*.

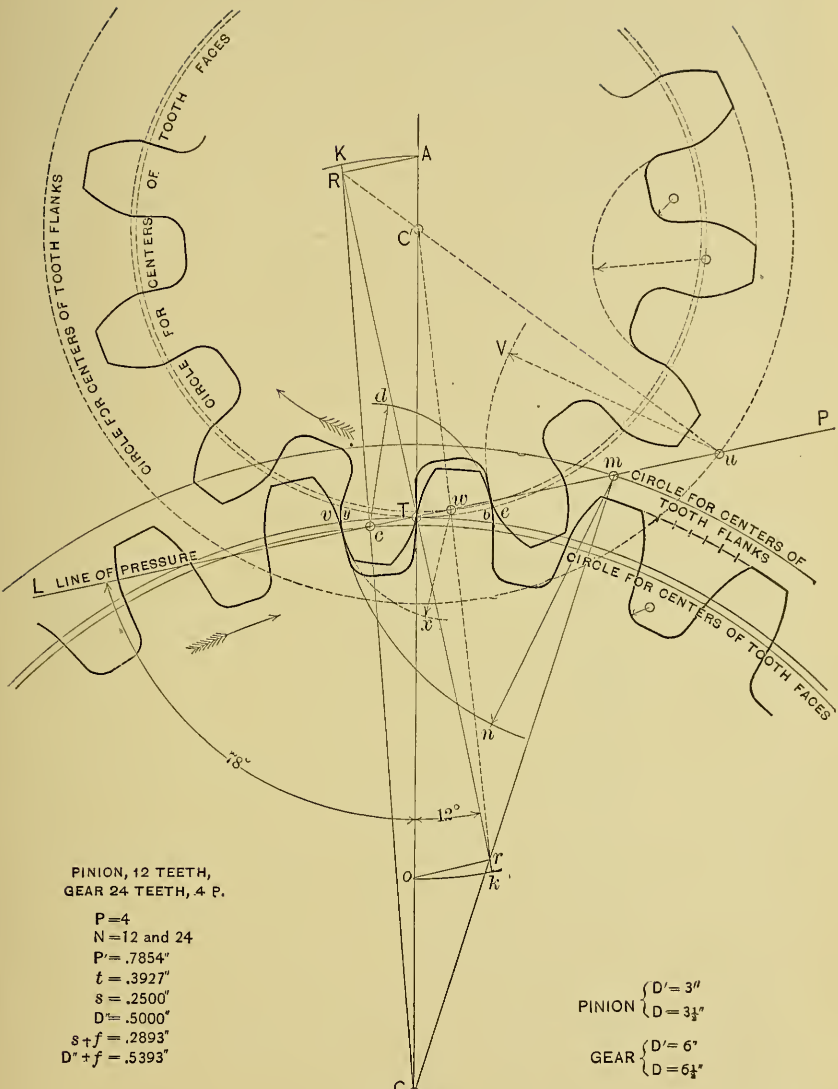


Fig. 15.

DOUBLE CURVE GEARS IN MESH.

Construction of Fig. 15 continued.

5. The radius of fillets at roots of teeth is equal to one-seventh the width of space at addendum circle.

Flanks for 12, 13 and 14 Teeth.

The constructions for flanks of 12, 13 and 14 teeth are similar to each other and as follows:

1. Through center, C′, draw line from R, intersecting line of pressure in *u*. Through *u* draw circle about C′. In this circle one leg of dividers is placed for drawing flanks.

2. The radius of flanks is the distance from *u* to the first tooth-thickness point, *e*, on the *same side* of C T C′. This gives convex flanks. The arc is continued to V, to show construction.

3. This arc for flanks is continued in or toward the center, only about one-sixth of the working depth (or $\frac{1}{3}$ *s.*); the lower part of flank is similar to flanks of gear in Chapter VI.

4. The faces are similar to those in 15-tooth gear, Chapter VII., and to the 24-tooth gear in the foregoing, the radius being *w y*; the arc is continued to *x*, to show construction.

Annular Gears.

ANNULAR GEARS. Gears with teeth inside of a rim or ring are called Annular or Internal Gears. The construction of tooth outlines is similar to the foregoing, but the *spaces* of a spur external gear become the *teeth* of an annular gear.

It has been shown that in the system just described, the pinion meshing with an annular gear, must differ from it by *at least* fifteen teeth. Thus, a gear of 24 teeth cannot work with an annular gear of 36 teeth, but it will work with annular gears of 39 teeth and more. The fillets at the roots of the teeth must be of less radius than in ordinary spur gears. An annular gear differing from its mate by less than 15 teeth can be made. This will be shown in Part II.

Annular-gear patterns require more clearance for moulding than external or spur gears.

Pinions.

In speaking of different-sized gears, the smallest ones are often called "pinions."

The angle of pressure in all gears except involute, constantly changes. 78° is the pressure angle in double-curve, or epicycloidal gears for an instant only; in our example, it is 78° when one side of a tooth reaches the line of centers, and the pressure against teeth is applied in the direction of the arrows.

The pressure angle of involute gears does not change. An explanation of the term angle of pressure is given in Part II.

We obtain the forms for epicycloidal gear cutters by means of a machine called the Odontom Engine. This machine will cut original gears with theoretical accuracy.

It has been thought best to make 24 gear cutters for each pitch. This enables us to fill any requirement of gear-cutting very closely, as the range covered by any one cutter is so small that it is exceedingly near to the exact shape of all gears so covered. 24 Double-curve Gear Cutters for each Pitch.

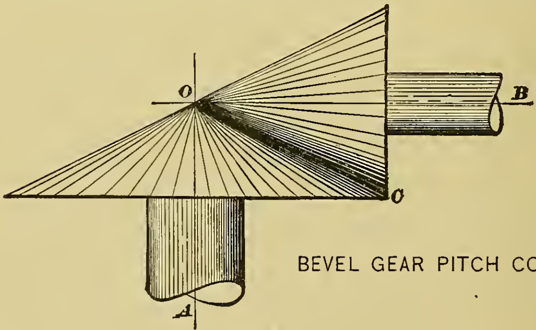
Of course, a cutter can be *exactly* right for only one gear. Special cutters can be made, if desired.

CHAPTER IX.

BEVEL-GEAR BLANKS.

Teeth of Bevel Gears formed upon frustrums of cones.

Bevel Gears connect shafts whose axes meet when sufficiently prolonged. The teeth of bevel gears are formed about the frustrums of cones whose apexes are at the same point where the shafts meet. In Fig. 16 we have the axes A O and B O, meeting at O, and the apexes of the cones also at O. These cones are called the pitch cones, because they roll upon each other, and because upon them the teeth are pitched. If, in any bevel gear, the teeth were sufficiently prolonged toward the apex, they would become infinitely small; that is, the teeth would all end in a point, or vanish at O. We can also consider a bevel gear as beginning at the apex and becoming larger and larger as we go away from the apex. Hence, as the bevel gear teeth are tapering from end to end, we may say



BEVEL GEAR PITCH CONES.

Fig. 16.

that a bevel gear has a number of pitches and pitch circles, or diameters: in speaking of the pitch of a bevel gear, we mean always the pitch at the largest pitch circle, or at the largest pitch diameter, as at b d, Fig. 17.

Fig. 17 is a section of three bevel gears, the gear o B q being twice as large as the two others. The outer surface of a tooth as m m′ is called the face of the tooth. The distance m m′ is usually called the length of the face of the tooth, though the real length is the distance that it occupies upon the line O i. The outer part of a tooth at m n is called its large end, and the inner part m′ n′ the small end.

Construction of Bevel Gear Blanks.

Almost all bevel gears connect shafts that are at right angles with each other, and unless stated otherwise we always understand that they are so wanted.

The directions given in connection with Fig. 17 apply to gears with axes at right angles.

Having decided upon the pitch and the numbers of teeth :—

1. Draw centre lines of shafts, A O B and C O D, at right angles.

2. Parallel to A O B, draw lines a b and c d, each distant from A O B, equal to half the largest pitch diameter of one gear. For 24 teeth, 4 pitch, this half largest pitch diameter is 3″.

3. Parallel to C O D, draw lines e f and g h, distant from C O D, equal to half the largest pitch diameter of the other gear. For a gear, 12 teeth, 4 pitch, this half largest pitch diameter is $1\frac{1}{2}$″.

4 At the intersection of these four lines, draw lines O i, O j, O k, and O l ; these lines give the size and shape of pitch cones. We call them "Cone Pitch Lines."

5. Perpendicular to the cone-pitch lines and through the intersection of lines a b, c d, e f, and g h, draw lines m n, o p, q r. We have drawn also u v to show that another gear can be drawn from the same diagram. Four gears, two of each size, can be drawn from this diagram.

6. Upon the lines m n, o p, q r, the addenda and depth of the teeth are laid off, these lines passing

through the largest pitch circle of the gears. Lay off the addendum, it being in these gears $\frac{1}{4}''$. This gives distance m n, o p, q r, and u v equal to the working depth of teeth, which in these gears is $\frac{1}{2}''$. The addendum of course is measured perpendicularly from the cone pitch lines as at k r.

7. Draw lines O m, O n, O p, O o, O q, O r. These lines give the height of teeth above the cone-pitch lines as they approach O, and would vanish entirely at O. It is quite as well never to have the length of teeth, or face, m m′ longer than one-third the apex distance m O, nor more than two and one-half times the circular pitch.

8. Having decided upon the length of face, draw limiting lines m′n′ perpendicular to i O, q′ r′ perpendicular to k O, and so on.

The distance between the cone-pitch lines at the inner ends of the teeth m′ n′ and q′ r′ is called the inner or smaller pitch diameter, and the circle at these points is called the smallest pitch circle. We now have the outline of a section of the gears through their axes. The distance m r is the whole diameter of the pinion. The distance q o is the whole diameter of the gear. In practice these diameters can be obtained by measuring the drawing. The diameter of pinion is 3.45″ and of the gear 6.22″. We can find the angles also by measuring the drawing with a protractor. In the absence of a protractor, templetes can be cut to the drawing. The angle formed by line m m′ with a b is the angle of face of pinion, in this pinion 59° 11′, or $59\frac{1}{6}°$ nearly. The lines q q′ and g h give us angle of face of gear, for this gear 22° 19′, or $22\frac{1}{3}°$ nearly The angle formed by m n with a b is called the angle of edge of pinion, in our sketch 26° 34′, or about $26\frac{1}{2}°$. The angle of edge of gear, line q r with g h, is 63° 26′, or about $63\frac{1}{2}°$. In turning blanks to these angles we place one arm of the protractor or templet against the end of the hub, when trying angles of a blank. Some designers give the angles from the axes of gears, but

The Whole Diameter of Bevel-Gear Blanks can be obtained by Measuring Drawings.

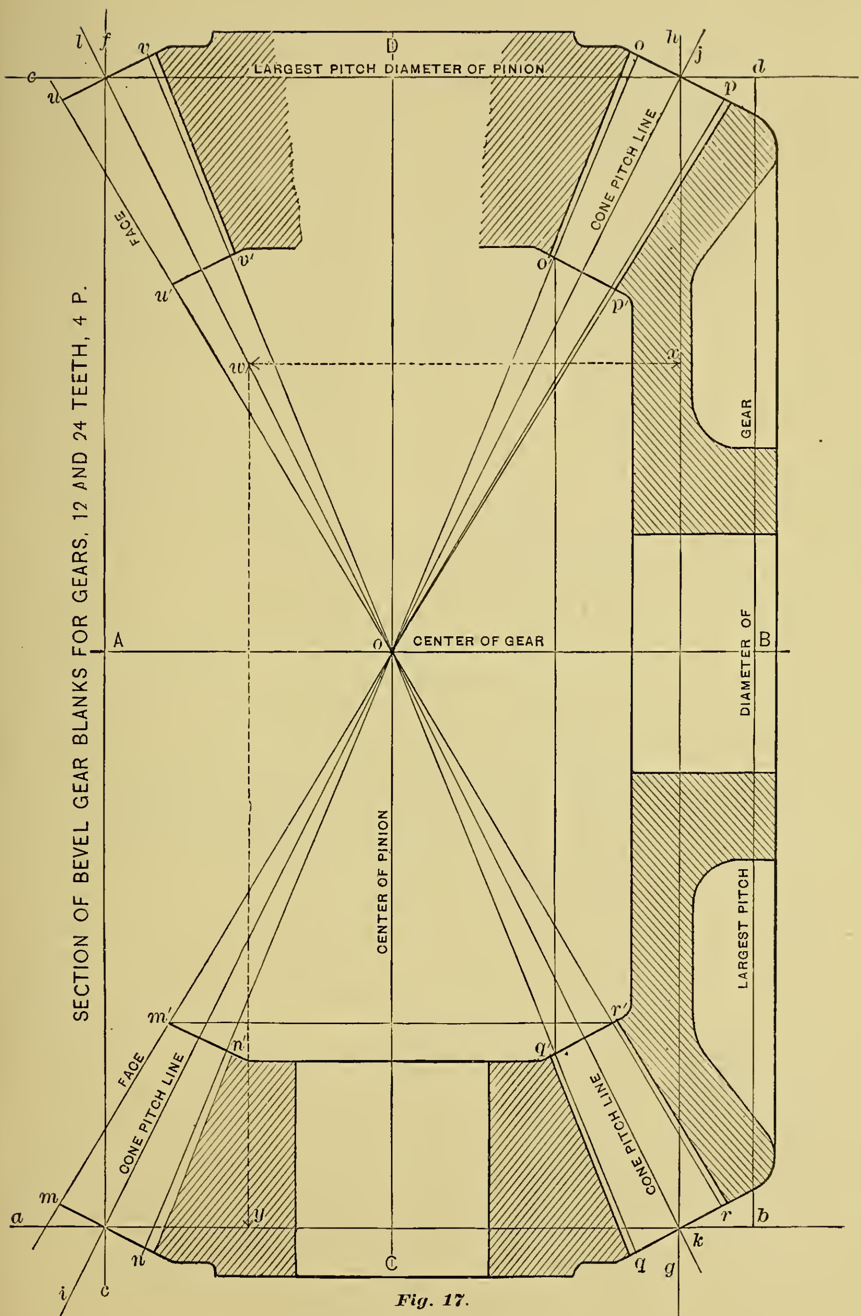

SECTION OF BEVEL GEAR BLANKS FOR GEARS, 12 AND 24 TEETH, 4 P.
LARGEST PITCH DIAMETER OF PINION
CENTER OF GEAR
CENTER OF PINION
CONE PITCH LINE
FACE
LARGEST PITCH DIAMETER OF GEAR

Fig. 17.

it is not convenient to try blanks in this way. The method that we have given comes right also for angles as figured in compound rests.

When axes are at right angles, the sum of angles of edge in the two gears equals 90°, and the sums of angle of edge and face in each gear are alike.

The angles of the axes remaining the same, all pairs of bevel gears of the same ratio have the same angle of edge; all pairs of same ratio and of same numbers of teeth have the same angles of both edges and faces independent of the pitch. Thus, in all pairs of bevel gears having one gear twice as large as the other, with axes at right angles, the angle of edge of large gear is 63° 26′, and the angle of edge of small gear is 26° 34′.

In all pairs of bevel gears with axes at right angles, one gear having 24 teeth and the other gear having 12 teeth, the angle of face of small gear is 59° 11′.

Another method of obtaining Whole Diameter of Blanks.

The following method of obtaining the whole diameter of bevel gears is sometimes preferred:

From k lay off; upon the cone-pitch line, a distance K w, equal to ten times the working depth of the teeth = 10 D″. Now add $\frac{1}{10}$ of the shortest distance of w from the line g h, which is the perpendicular dotted line w x, to the outside pitch diameter of gear, and the sum will be the whole diameter of gear. In the same manner $\frac{1}{10}$ of w y, added to the outside pitch diameter of pinion, gives the whole diameter of pinion. The part added to the pitch diameter is called the *diameter increment.*

Part II gives trigonometrical methods of figuring bevel gears: in our Formulas in Gearing there are trigonometrical formulas for bevel gears, and also tables for angles and sizes.

Construction of Bevel-Gear Blanks whose Axes are not at Right Angles.

A somewhat similar construction will do for bevel gears whose axes are not at right angles.

In Fig. 18 the axes are shown at O B and O D, the angle B O D being less than a right angle.

1. Parallel to O B, and at a distance from it equal to the radius of the gear, we draw the lines a b and c d.

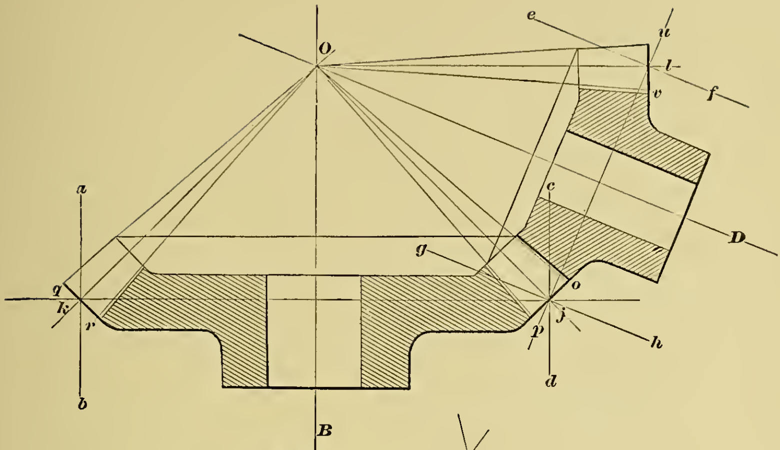
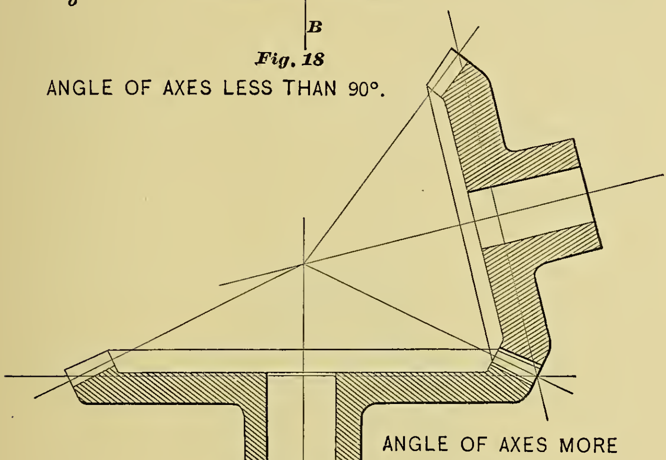


Fig. 18

ANGLE OF AXES LESS THAN 90°.



ANGLE OF AXES MORE THAN 90°.

Fig. 19

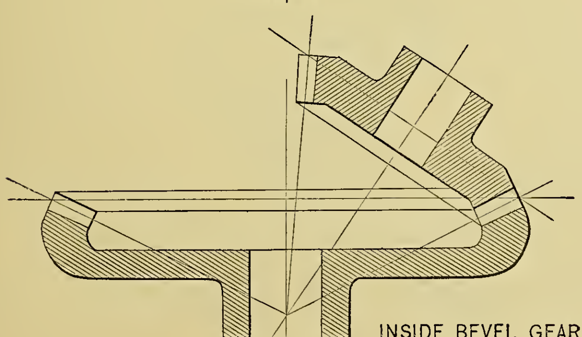


Fig. 20

INSIDE BEVEL GEAR AND PINION.

2. Parallel to O D, and at a distance from it equal to the radius of the pinion, we draw the lines e f and g h.

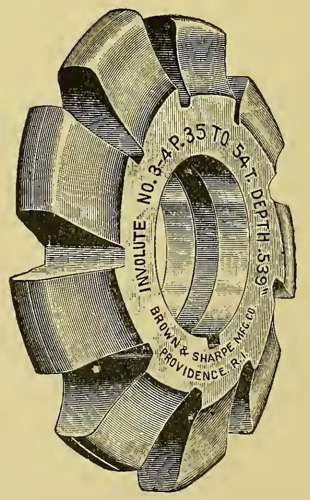
3. Now, through the point j at the intersection of c d and g h, we draw a line perpendicular to O B. This line k j, limited by a b and c d, represents the largest pitch diameter of the gear.

Through j we draw a line perpendicular to O D. This line j l, limited by e f and g h, represents the largest pitch diameter of the pinion.

4. Through the point k at the intersection of a b with k j, we draw a line to O, a line from j to O, and another from l, at the intersection j l and e f to O. These lines O k, O j, and O l, represent the cone-pitch lines, as in Fig. 17.

5. Perpendicular to the cone-pitch lines we draw the lines u v, o p, and q r. Upon these lines we lay off the addenda and working depth as in the previous figure, and then draw lines to the point O as before.

By a similar construction Figs. 19 and 20 can be drawn.



GEAR CUTTER.

CHAPTER X.

BEVEL GEARS.
FORMS AND SIZES OF TEETH.
CUTTING TEETH.

Form of bevel gear teeth.

To obtain the form of the teeth in a bevel gear we do not lay them out upon a pitch circle, as we do in a spur gear, because the rolling pitch surface of a bevel gear, at any point, is of a longer radius of curvature than the actual radius of a pitch circle that passes through that point. Thus in Fig. 21, let f g c be a cone about the axis O A, the diameter of the cone being f c, and its radius g c. Now the radius of curvature of the surface, at c, is evidently longer than g c, as can be seen in the other view at C ; the full line shows the curvature of the surface, and the dotted line shows the curvature of a circle of the radius g c. It is extremely difficult to represent the exact form of bevel gear teeth upon a flat surface, because a bevel gear is essentially spherical in its nature ; for practical purposes we draw a line c A perpendicular to O c, letting c A reach the centre line O A, and take c A as the radius of a circle upon which to lay out the teeth. This is shown at c n m, Fig. 22. For convenience the line c A is sometimes called the back cone radius.

Example, Fig. 22.

Let us take, for an example, a bevel gear and a pinion 24 and 18 teeth, 5 pitch, shafts at right angles. To obtain the forms of the teeth and the data for cutting, we need to draw a section of only a half of each gear, as in Fig. 22.

1. Draw the centre lines A O and B O, then the lines g h and c d, and the gear blank lines as described in Chapter IX. Extend the lines o′ p′ and o p until they meet the centre lines at A′ B′ and A B.

2. With the radius A c draw the arc c n m, which we take as the geometrical pitch circle upon which to lay out the teeth at the large end. The distance A′ c′

is taken as the radius of the geometrical pitch circle at the small end; to avoid confusion an arc of this circle is drawn at c″ n′ m′ about A.

3. For the pinion we have the radius B c for the geometrical pitch circle at the large end and B′ c′ for the small end: the distance B′ c′ is transferred to B c‴.

4. Upon the arc c n m lay off spaces equal to the tooth thickness at the large pitch circle, which in our example is .314″. Draw the outlines of the teeth as in previous chapters: for single curve teeth we draw a semi-circle upon the radius A c, and proceed as described in chapter III. For all bevel gears that are to be cut with a rotary disk cutter, or a common gear cutter, single curve teeth are chosen; and no attempt should be made to cut double curve teeth. Double curve teeth can be drawn by the directions given in chapters VII and VIII. We now have the form of the teeth at the large end of the gear. Repeat this operation with the radius B C about B, and we have the form of the teeth at the large end of the pinion.

5. The tooth parts at the small end are designated by the same letters as at the large, with the addition of an accent mark to each letter, as in the right hand column, Fig. 22, the clearance, f, however, is usually the same at the small end as at the large, for convenience in cutting the teeth.

When cutting bevel gears with rotary cutters, the cutting angle is the same as the working depth angle. This angle is used for two reasons: first, it is not necessary to figure the angle of the bottom; second, the inside of the teeth is rounded over a little more and this lessens the amount to be filed off at the point. When cut in this way, the line of the bottom of the tooth is parallel to the face of the mating gear and it does not pass through the cone apex or common point of the axes.

Sizes of the tooth parts.

The sizes of the tooth parts at the small end are in the same proportion to those at the large end as the line O c′ is to O c. In our example O c′ is 2″, and O c is 3″; dividing O c′ by O c we have $\frac{2}{3}$, or .666, as the ratio of the sizes at the small end to those

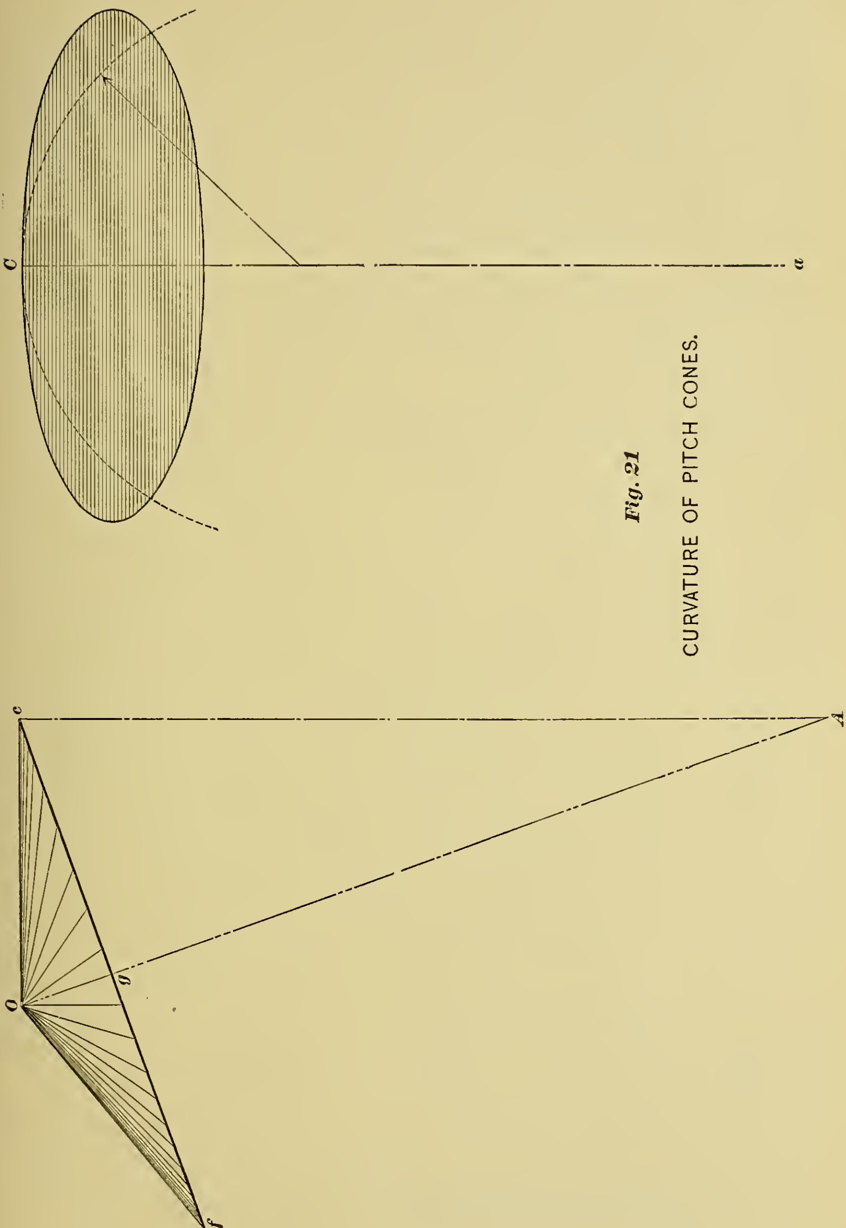


Fig. 21

CURVATURE OF PITCH CONES.

at the large: t′ is .209″ or $\frac{2}{3}$ of .314″, and so on. If the distance n m is equal to the outer tooth thickness, t, upon the arc c n m, the lines n A and m A will be a distance apart equal to the inner tooth thickness t′ upon the arc c″ n′ m′. The addendum, s′, and the working depth, D‴, are at o′ c′ and o′ p′.

6. Upon the arcs c″ n′ m′ and c‴ we draw the forms of the teeth of the gear and pinion at the inside.

Example of Cutting.

As an example of the cutting of bevel gears with rotary disk cutters, or common gear cutters, let us take a pair of 8 pitch, 12 and 24 teeth, shown in Fig. 23.

Length of tooth face.

In making the drawing it is well to remember that nothing is gained by having the face F E longer than five times the thickness of the teeth at the large pitch circle, and that even this is too long when it is more than a third of the *apex distance* O c. To cut a bevel gear with a rotary cutter, as in Fig. 24, is at best but a compromise, because the teeth change pitch from end to end, so that the cutter, being of the right form for the large ends of the teeth can not be right for the small ends, and the variation is too great when the length of face is greater than a third of the apex distance O c, Fig. 23. In the example, one-third of the apex distance is $\frac{9}{16}$″, but F E is drawn only a half inch, which even though rather short, has changed the pitch from 8 at the outside to finer than 11 at the inside. Frequently the teeth have to be rounded over at the small ends by filing; the longer the teeth the more we have to file. If there is any doubt about the strength of the teeth, it is better to lengthen at the large end, and make the pitch coarser rather than to lengthen at the small end.

Data for cutting.

These data are needed before beginning to cut:

1. The pitch and the numbers of the teeth the same as for spur gears.

2. The data for the cutter, as to its form: sometimes two cutters are needed for a pair of bevel gears.

3. The whole depth of the tooth spaces, both at

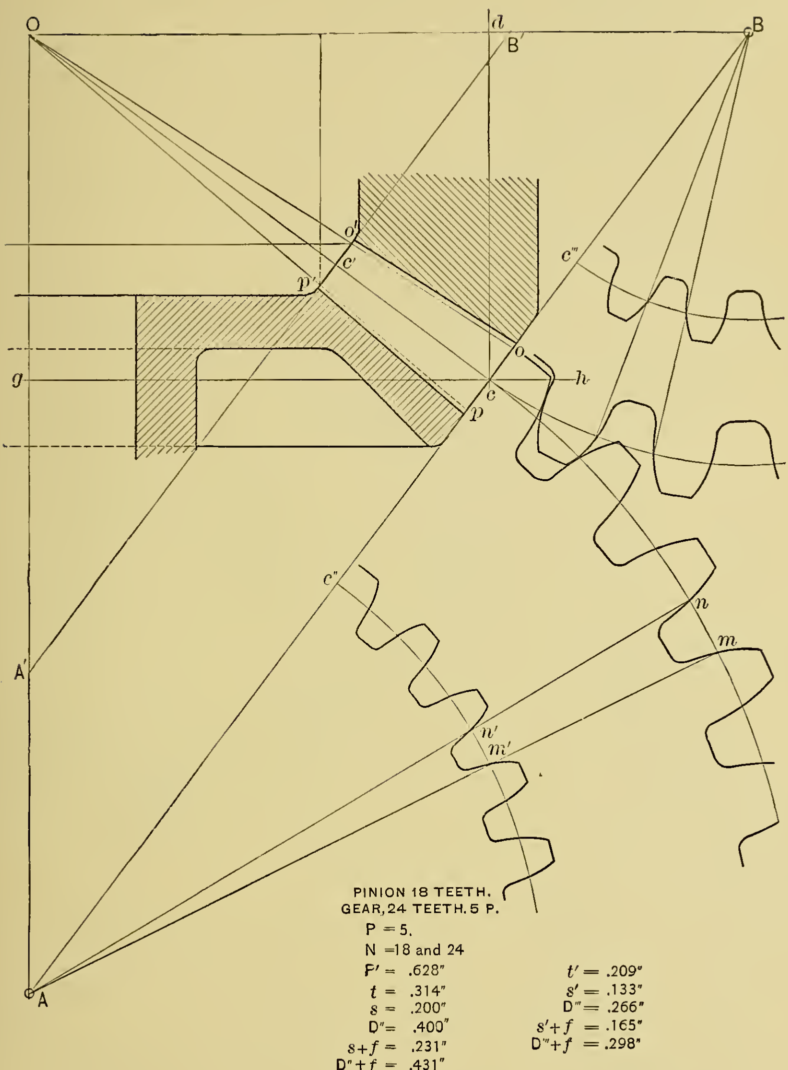


Fig. 22.

BEVEL GEARS, FORM AND SIZE OF TEETH.

the outside and inside ends; $D'' + f$ at the outside, and $D''' + f$ at the inside.

4. The thickness of the teeth at the outside and at the inside; t and t′.

5. The height of the teeth above the pitch lines at the outside and inside; s and s′.

6. The cutting angles, or the angles that the path of the cutter makes with the axes of the gears. In Fig. 23 the cutting angle for the gear c D is A Op, and the cutting angle for the pinion is B O o.

Selection of cutters.

The form of the teeth in one of these gears differs so much from that in the other gear that two cutters are required. In determining these cutters we do not have to develop the forms of the gear teeth as in Fig. 22; we need merely measure the lines A c and B c, Fig. 23, and calculate the cutter forms as if these distances were the radii of the pitch circles of the gears to be cut. Twice the length A c, in inches, multiplied by the diametral pitch, equals the number of teeth for which to select a cutter for the twenty-four-tooth gear; this number is about 54, which calls for a number three bevel gear cutter in accordance with the lists of gear cutters, pages 61 and 82. Twice B c, multiplied by 8, equals about 13, which indicates a No. 8 bevel gear cutter for the pinion. This method of selecting cutters is based upon the idea of shaping the teeth as nearly right as practicable at the large end, and then filing the small end where the cutter has not rounded them over enough.

In Fig. 25 the tooth L has been cut to thickness at both the outer and inner pitch lines, but it must still be rounded at the inner end. The teeth M M have been filed. In thus rounding the teeth they should not be filed thinner at the pitch lines.

There are several things that affect the shape of the teeth, so that the choice of cutters is not always so simple a matter as the taking of the lines A c and B c as radii.

In cutting a bevel gear, in the ordinary gear cutting

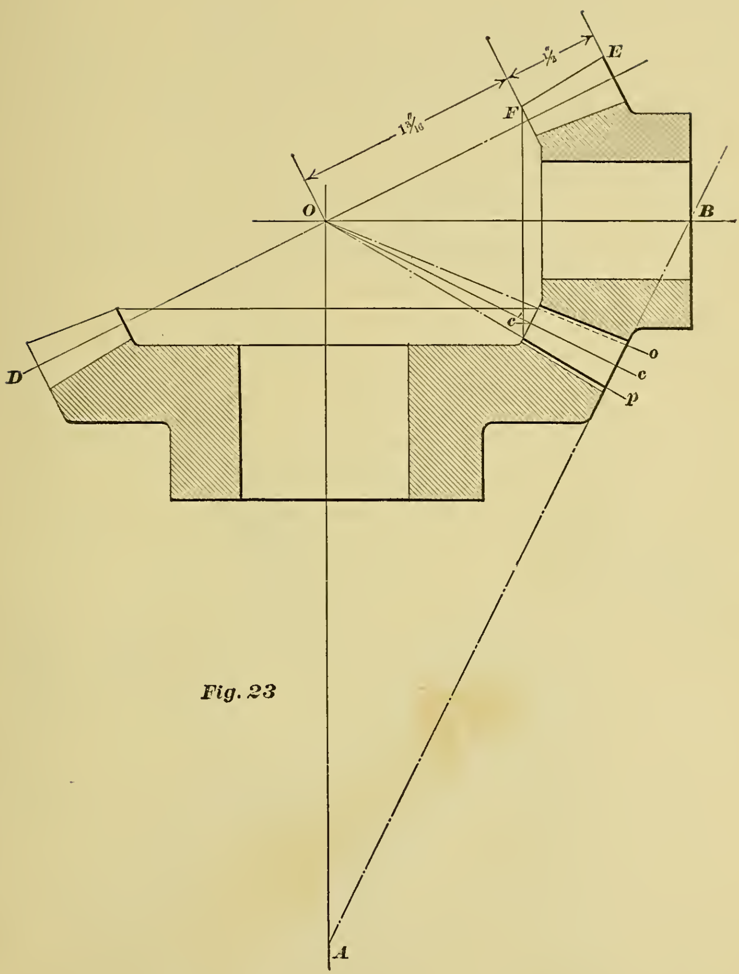


Fig. 23

BEVEL GEAR DIAGRAM FOR DIMENSIONS.

machines, the finished spaces are not always of the same form as the cutter might be expected to make, because of the changes in the positions of the cutter and of the gear blank in order to cut the teeth of the right thickness at both ends. The cutter must of course be thin enough to pass through the small end of the spaces, so that the large end has to be cut to the right width by adjusting either the cutter or the blank sidewise, then rotating the blank and cutting twice around.

Widening the space at the large end.

Thus, in Fig. 24, a gear and a cutter are set to have a space widened at the large end e′, and the last chip to be cut off by the right side of the cutter, the cutter having been moved to the left, and the blank rotated in the direction of the arrow: in a Universal Milling Machine the same result would be attained by moving the blank to the right and rotating it in the direction of the arrow. It may be well to remember that in setting to finish the side of a tooth, the tooth and the cutter are first separated sidewise, and the blank is then rotated by indexing the spindle to bring the large end of the tooth up against the cutter. This tends not only to cut the spaces wider at the large pitch circle, but also to cut off still more at the face of the tooth; that is, the teeth may be cut rather thin at the face and left rather thick at the root. This tendency is greater as a cutting angle B O o, Fig. 23, is smaller, or as a bevel gear approaches a spur gear, because when the cutting angle is small the blank must be rotated through a greater arc in order to set to cut the right thickness at the outer pitch circle. This can be understood by Figs. 26 and 27. Fig. 26 is a radial-toothed clutch, which for our present purpose can be regarded as one extreme of a bevel gear in which the teeth are cut square with the axis: the dotted lines indicate the different positions of the cutter, the side of a tooth being finished by the side of the cutter that is on the centre line. In setting to cut these teeth there is the same side adjustment and rotation of the

Teeth narrowed more at face than at root.

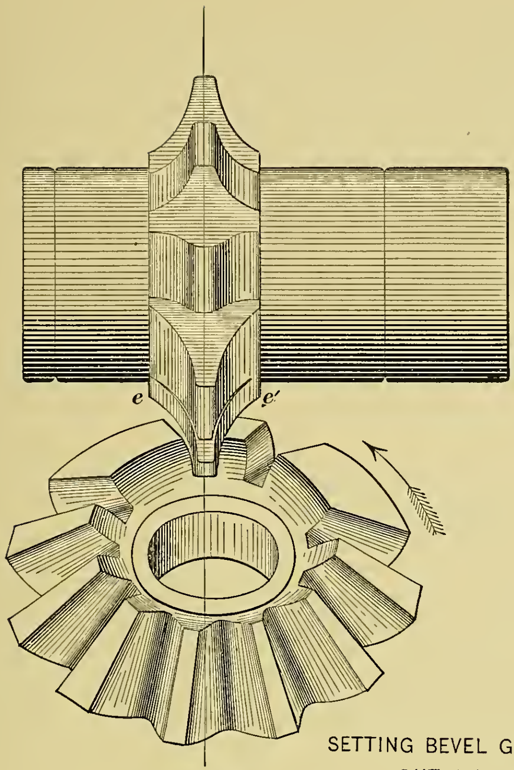


Fig. 24

SETTING BEVEL GEAR CUTTER OUT OF CENTRE.

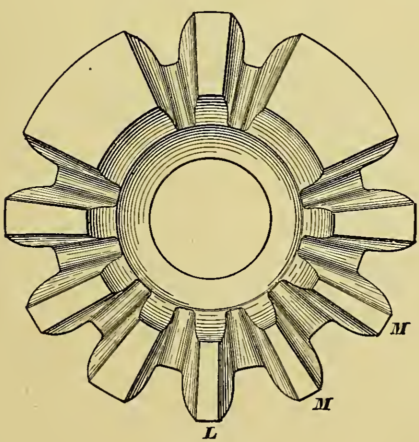


Fig. 25

spindle as in a bevel gear, but there is no tendency to make a tooth thinner at the face than at the root. On the other hand, if we apply these same adjustments to a spur gear and cutter, Fig. 27, we shall cut the face F much thinner without materially changing the thickness of the root R.

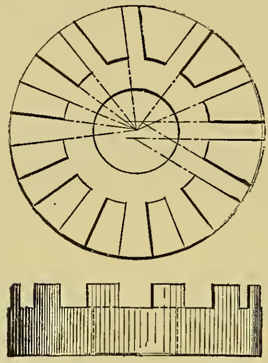


Fig. 26

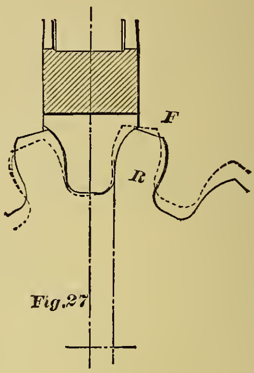


Fig. 27

Almost all bevel gears are between the two extremes of Figs. 26 and 27, so that when the cutting angle B O o, Fig. 23, is smaller than about 30°, this change in the form of the spaces caused by the rotation of the blank may be so great as to necessitate the substitution

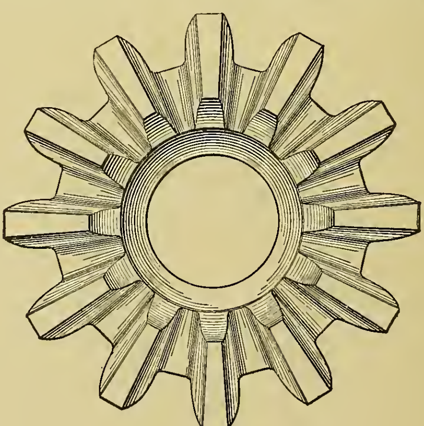


Fig. 28

FINISHED GEAR.

of a cutter that is narrower at e e′, Fig. 24, than is called for by the way of figuring that we have just given: thus in our own gear cutting department we might cut the pinion with a No. 6 cutter, instead of a No. 8. The No. 6, being for 17 to 20 teeth, cuts the tooth sides with a longer radius of curvature than the No 8, which may necessitate considerable filing at the small ends of the teeth in order to round them over enough. Fig. 28 shows the same gear as Fig. 25, but in this case the teeth have all been filed similar to M M, Fig. 25.

Filing the teeth at the small end.

Different workmen prefer different ways to compromise in the cutting of a bevel gear. When a blank is rotated in adjusting to finish the large end of the teeth there need not be much filing of the small end, if the cutter is right, for a pitch circle of the radius B c, Fig. 23, which for our example is a No. 8 cutter, but the tooth faces may be rather thin at the large ends. This compromise is preferred by nearly all workmen, because it does not require much filing of the teeth: it is the same as is in our catalogue by which we fill any order for bevel gear cutters, unless otherwise specified. This means that we should send a No. 8, 8-pitch bevel gear cutter in reply to an order for a cutter to cut the 12-tooth pinion, Fig. 23; while in our own gear cutting department we might cut the same pinion with a No. 6, 8-pitch cutter, because we prefer to file the teeth at the small end after cutting them to the right thickness at the faces of the large end. We should take a No. 6 instead of a No. 8 only for a 12-tooth pinion that is to run with a gear two or three times as large. We generally step off to the next cutter for pinions fewer than twenty-five teeth, when the number for the teeth has a fraction nearly reaching the range of the next cutter: thus, if twice the line B c in inches, Fig. 23, multiplied by the diametral pitch, equals 20.9, we should use a No. 5 cutter, which is for 21 to 25 teeth inclusive. In filling an order for a gear cutter, we do not consider

Selection of cutter when teeth are to be filed.

the fraction but send the cutter indicated by the whole number.

Later on we will refer to other compromises that are made in the cutting of bevel gears.

The sizes of the 8-pitch tooth parts, Fig. 23, at the large end, are copied from the table of spur gear teeth, pages 146 to 149.

Form of gear cutting order

The distance Oc′ is seven-tenths of the *apex distance* Oc, so that the sizes of the tooth parts at the small end, except f, are seven-tenths the large. The order for cutting these gears goes to the workmen in this form:

LARGE GEAR.

P	= 8		
N	= 24		
D″ + f	= .270″	D‴ + f	= .195″
t	= .196″	t′	= .137″
s	= .125″	s′	= .087″

Cutting Angle = 59° 10′

SMALL GEAR.

N = 12

Cutting Angle = 22° 18′

Setting the machine.

Fig. 32 is a side view of a Gear Cutting Machine. A bevel gear blank A is held by the index spindle B. The cutter C is carried by the cutter-slide D. The cutter-slide-carriage E can be set to the cutting angle, the degrees being indicated on the quadrant F.

Fig. 33 is a plan of the machine: in this view the cutter-slide-carriage, in order to show the details a little plainer, is not set to an angle.

Before beginning to cut the cutter is set central with the index spindle and the dial G is set to zero, so that we can adjust the cutter to any required distance out of centre, in either direction. Set the cutter-slide-carriage E, Fig. 32, to the cutting angle of the gear, which for 24-teeth is 59° 10′; the quadrant being divided to half-degrees, we estimate that 10′ or $\frac{1}{6}$ degree more than 59°. Mark the depth of the cut at the outside, as in Fig. 30: it is also well enough to mark the depth at the inside as a check. The thickness of the teeth at the large end is conveniently determined by the solid gauge, Fig. 29. The gear-tooth

Fig. 29

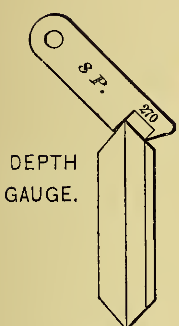


Fig. 30

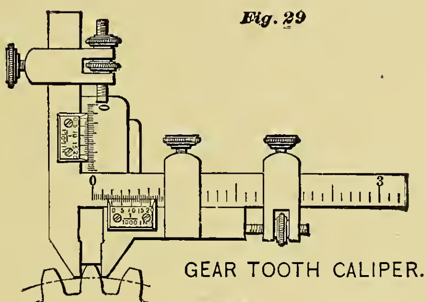


Fig. 31

vernier caliper, Fig. 31, will measure the thickness of teeth up to 2 diametral pitch. In the absence of the vernier caliper we can file a gauge, similar to Fig 29, to the thickness of the teeth at the small end.

Position of side of tooth being finished.

The index having been set to divide to the right number we cut two spaces central with the blank, leaving a tooth between that is a little too thick, as in the upper part of Fig. 25. If the gear is of cast iron, and the pitch is not coarser than about 5 diametral, this is as far as we go with the central cuts, and we proceed to set the cutter and the blank to finish first one side of the teeth and then the other, going around only twice. The tooth has to be cut away more in proportion from the large than from the small end, which is the reason for setting the cutter out of centre, as in Fig. 24.

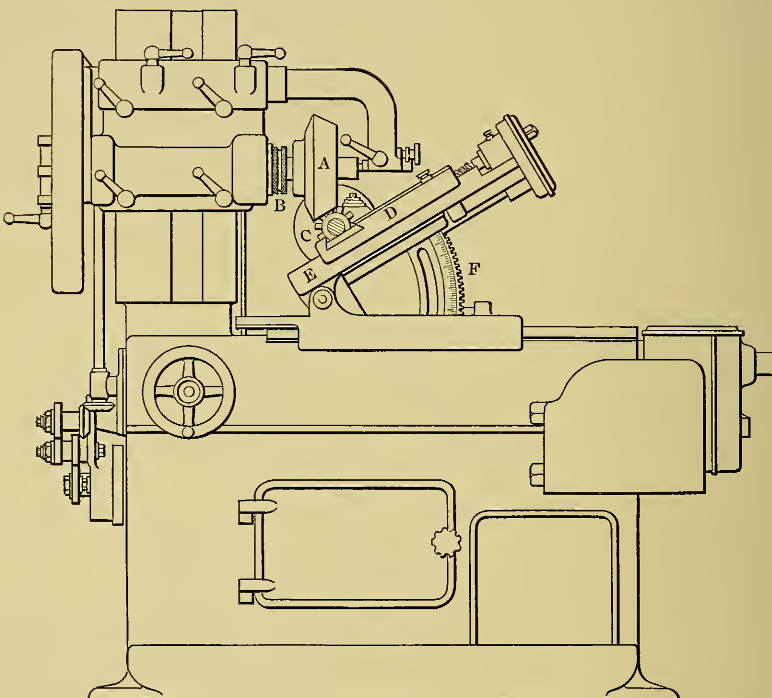


Fig. 32.

AUTOMATIC GEAR CUTTING MACHINE.

SIDE ELEVATION.

It is important to remember that the part of the cutter that is finishing one side of a tooth at the pitch line should be central with the gear blank, in order to know at once in which direction to set the cutter out of centre. We can not readily tell how much out of centre to set the cutter until we have cut and tried, because the same part of a cutter does not cut to the pitch line at both ends of a tooth. As a trial distance out of centre we can take about one-tenth to one-eighth of the thickness of the teeth at the large end. The actual distance out of centre for the 12-tooth pinion is .021″: for the 24-tooth gear, .030″, when using cutters listed in our catalogue.

Necessity of central cuts.

After a little practice a workman can set his cutter the trial distance out of centre, and take his first cuts, without any central cuts at all; but it is safer to take central cuts like the upper ones in Fig. 25. The depth of cut is partly controlled by the index-spindle raising-dial-shaft H, Fig. 33, which determines the height of the index spindle, and partly by the position of the cutter spindle. We now set the cutter out of centre the trial distance by means of the cutter-spindle dial-shaft, I, Fig. 33. The trial distance can be about one-seventh the thickness of the tooth at the large end in a 12-tooth pinion, and from that to one-sixth the thickness in a 24-tooth gear and larger. The principle of trimming the teeth more at the large end than at the small is illustrated in Fig. 24, which is to move the cutter away from the tooth to be trimmed, and then to bring the tooth up against the cutter by rotating the blank in the direction of the arrow.

Adjustments.

The rotative adjustment of the index spindle is accomplished by loosening the connection between the index worm and the index drive, and turning the worm: the connection is then fastened again. The cutter is now set the same distance out of centre in the other direction, the index spindle is adjusted to trim the other side of the tooth until one end is down nearly to the right thickness. If now the thickness of the small end is in the same proportion to the large end as Oc′ is to Oc, Fig. 23, we can at once adjust to trim the tooth to the right thickness. But if we find that the large end is still going to be too thick when the small end is right, the out of centre must be increased.

It is well to remember this : too much out of centre leaves the small end proportionally too thick, and too little out of centre leaves the small end too thin.

After the proper distance out of centre has been learned the teeth can be finish-cut by going around out of centre first on one side and then on the other without cutting any central spaces at all. The cutter spindle stops, J J, can now be set to control the out of centre of the cutter, without having to adjust by the dial G. If, however, a cast iron gear is 5-pitch or coarser it is usually well to cut central spaces first and then take the two out-of-centre cuts, going around three times in all. Steel gears should be cut three times around.

Blanks are not always turned nearly enough alike to be cut without a different setting for different blanks. If the hubs vary in length the position of the cutter spindle has to be varied. In thus varying, the same depth of cut or the exact $D'' + f$ may not always be reached. A slight difference in the depth is not so objectionable as the incorrect tooth thickness that it may cause. Hence, it is well, after cutting once around and finishing one side of the teeth, to give careful attention to the rotative adjustment of the index spindle so as to cut the right thickness.

After a gear is cut, and before the teeth are filed, it is not always a very satisfactory-looking piece of work. In Fig. 25 the tooth L is as the cutter left it, and is ready to be filed to the shape of the teeth M M, which have been filed. Fig. 34 is the pair of gears that we have been cutting; the teeth of the 12-tooth pinion have been filed.

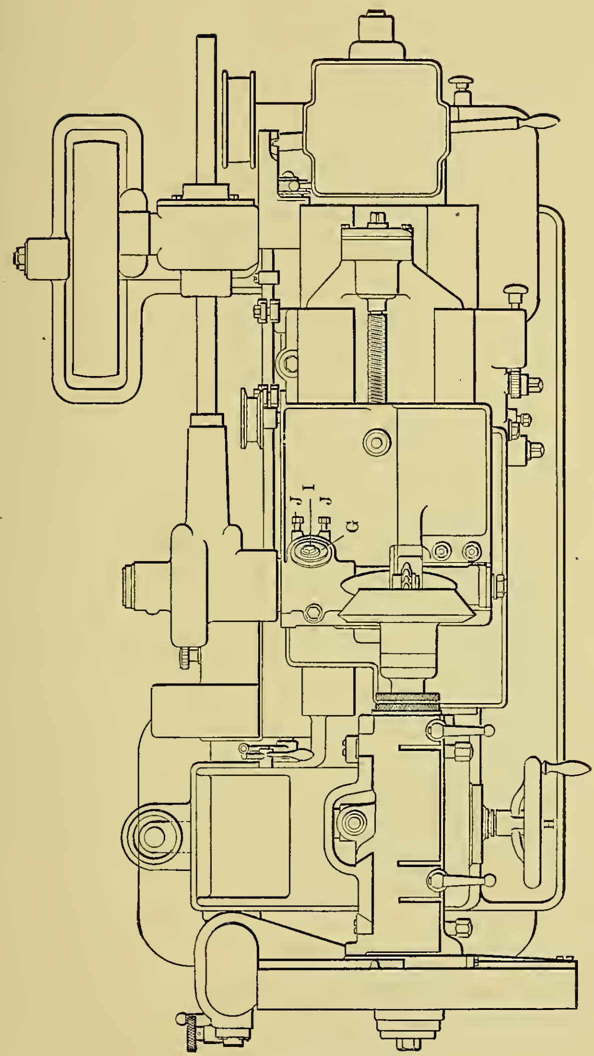


Fig. 33.

AUTOMATIC GEAR CUTTING MACHINE.

PLAN.

A second approximation.

A second approximation in cutting with a rotary cutter is to widen the spaces at the large end by swinging either the index spindle or the cutter-slide-carriage, so as to pass the cutter through on an angle with the blank side-ways, called the *side-angle*, and not rotate the blank at all to widen the spaces. This side-angle method is employed in our No. 11 Automatic Bevel Gear Cutting Machines: it is available in the manufacture of bevel gears in large quantities, because with the proper relative thickness of cutter, the tooth-thickness comes right by merely adjusting for the side-angle; but for cutting a few gears it is not much liked by workmen, because, in adjusting for the side-angle, the central setting of the cutter is usually lost, and has to be found by guiding into the central slot already cut. If the side-angle mechanism pivots about a line that passes very near the small end of the tooth to be cut, the central setting of the cutter may not be lost. In widening the spaces at the large end, the teeth are narrowed practically the same amount at the root as at the face, so that this side-angle method requires a wider cutter at e e′, Fig. 24, than the first, or rotative method. The amount of filing required to correct the form of the teeth at the small end is about the same as in the first method.

A third approximation.

A third approximate method consists in cutting the teeth right at the large end by going around at least twice, and then to trim the teeth at the small end and toward the large with another cutter, going around at least four times in all. This method requires skill and is necessarily a little slow, but it contains possibilities for considerable accuracy.

A fourth approximation.

A fourth method is to have a cutter fully as thick as the spaces at the small end, cut rather deeper than the regular depth at the large end, and go only once around. This is a quick method but more inaccurate than the three preceding: it is available in the manufacture of large numbers of gears when the tooth-face

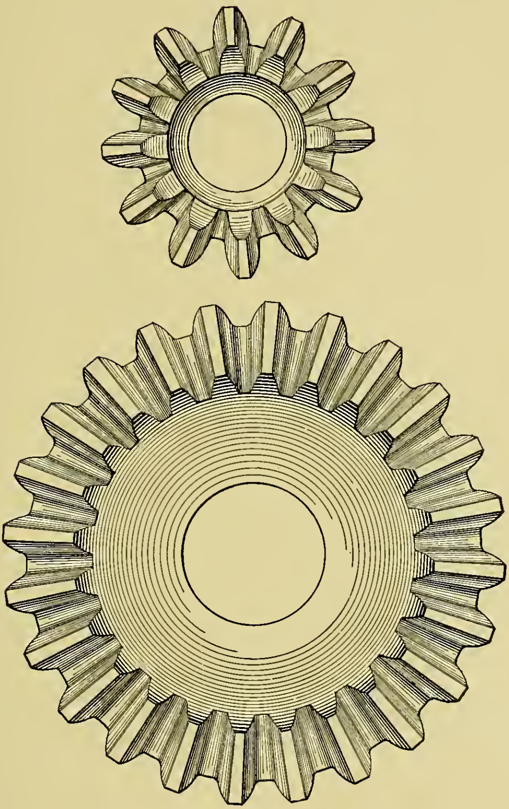


Fig. 34

FINISHED GEAR AND PINION.

is short compared with the apex distance. It is little liked, and seldom employed in cutting a few gears: it may require some experimenting to determine the form of cutter. Sometimes the teeth are not cut to the regular depth at the small end in order to have them thick enough, which may necessitate reducing the addendum of the teeth, s′, at the small end by turning the blank down. This method is extensively employed by chuck manufacturers.

A machine that cuts bevel gears with a reciprocating motion and using a tool similar to a planer tool is called a Gear Planer and the gears so cut are said to be *planed*.

Planing of bevel gears.

One form of Gear Planer is that in which the principle embodied is theoretically correct; this machine originates the tooth curves without a former. Another form of the same class of machines is that in which the tool is guided by a former.

Usually the time consumed in planing a bevel gear is greater than the time necessary to cut the same gear with a rotary cutter, thus proportionately increasing the cost.

Pitches coarser than 4 are more correct and sometimes less expensive when planed; it is hardly practicable, and certainly not economical, to cut a bevel gear as coarse as 3P. with a rotary cutter. In gears as fine as 16P. planing affords no practical gain in quality.

While planing is theoretically correct, yet the wearing of the tool may cause more variation in the thickness of the teeth than the wearing of a rotary cutter, and even a planed gear is sometimes improved by filing.

Mounting of gears.

If gears are not correctly mounted in the place where they are to run, they might as well not be planed. In fact, after taking pains in the cutting of any gear, when we come to the mounting of it we should keep right on taking pains.

Angles and sizes of bevel gears.

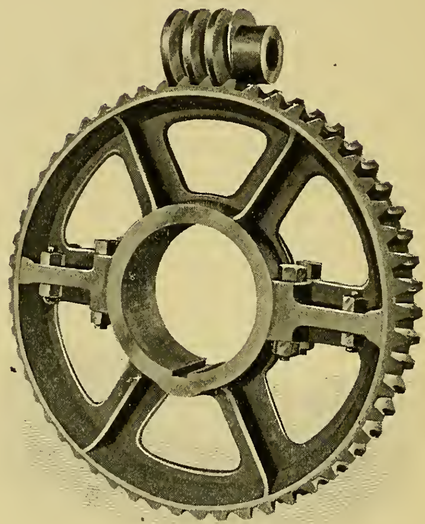
The method of obtaining the sizes and angles pertaining to bevel gears by measuring a drawing is quite convenient, and with care is fairly accurate. Its accuracy depends, of course, upon the careful measuring of a good drawing. We may say, in general, that in measuring a diagram, while we can hardly obtain data mathematically exact, we are not likely to make wild mistakes. Some years ago we depended almost entirely upon measuring, but since the publication of this "Treatise" and our "Formulas in Gearing" we calculate the data without any measuring of a drawing. In the "Formulas in Gearing" there are also tables pertaining to bevel gears.

Several of the cuts and some of the matter in this chapter are taken from an article by O. J. Beale, in the "American Machinist," June 20, 1895.

Cutters for Mitre and Bevel Gears.

Diametral Pitch.	Diameter of Cutter.	Hole in Cutter.
4	3 1-2″	1 1-4″
5	3 1-2	1 1-4
6	3 1-2	1 1-4
7	3 1-2	1 1-4
8	3 1-4	1 1-4
10	3 1-4	7-8
12	3	7-8
14	3	7-8
16	2 3-4	7-8
20	2 1-2	7-8
24	2 1-4	7-8

When each gear of a pair of bevel gears is of the same size and the gears connect shafts that are at right angles, the gears are called "Mitre Gears" and one cutter will answer for both.



WORM WHEEL.

Number of Teeth, 54. Circular Pitch, $2\frac{1}{2}$.
Throat Diameter, 44.59″. Outside Diameter 46″.

CHAPTER XI.

WORM WHEELS—SIZING BLANKS OF 32 TEETH AND MORE.

A WORM is a screw made to mesh with the teeth of a wheel called a *worm-wheel*. As implied at the end of Chapter IV., a section of a worm through its axis is, in outline, the same as a rack of corresponding pitch. This outline can be made either to mesh with single or double curve gear teeth; but worms are usually made for single curve, because, the sides of involute rack teeth being straight (see Chapter IV.), the tool for cutting worm-thread is more easily made. The thread-tool is not usually rounded for giving fillets at bottom of worm-thread. Worm.

The axis of a worm is usually at right angles to the axis of a worm wheel: no other angle of axis is treated of in this book.

The rules for circular pitch apply in the size of tooth parts and diameter of pitch-circle of worm-wheel.

The pitch of a worm or screw is sometimes given in a way different from the pitch of a gear, viz.: in number of threads to one inch of the length of the worm or screw. Thus, to say a worm is 2 pitch may mean 2 threads to the inch, or that the worm makes two turns to advance the thread one inch. But a worm may be double-threaded, triple-threaded, and so on; hence to avoid misunderstanding, it is better always to call the advance of the worm thread the *lead*. Thus, a worm-thread that advances one inch in one turn we call one-inch *lead* in one turn. A single-thread worm 4 turns to 1″ is $\frac{1}{4}$″ lead. We apply the term pitch, that is the *circular* pitch, to the actual distance between the threads or teeth, as in previous chapters. In single-thread worms the lead and the pitch are alike. In making a worm and wheel a given number of threads to Pitch of Worm Lead of a Worm-Thread.

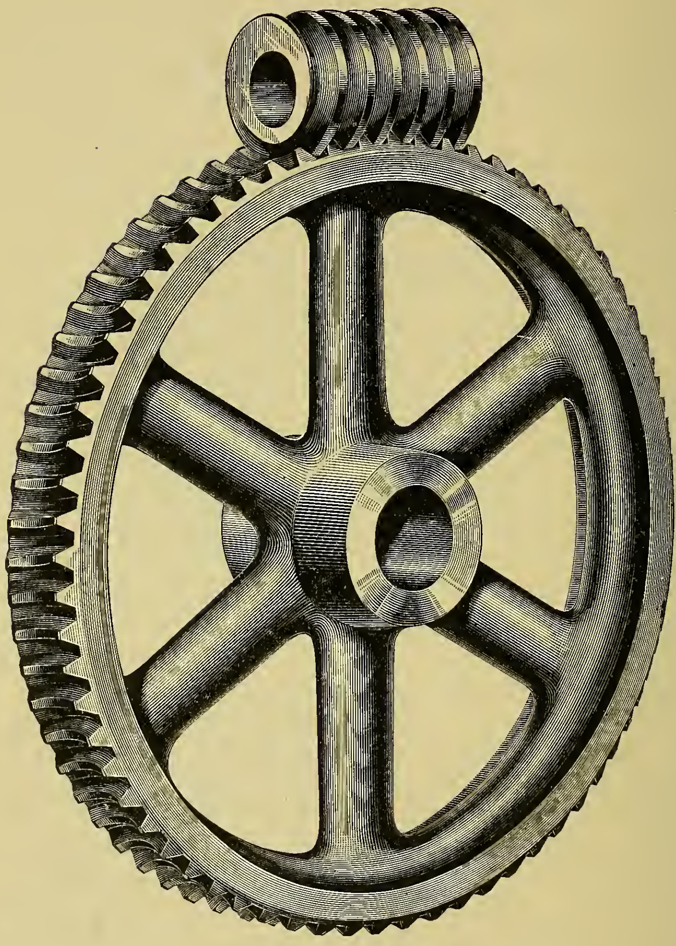


FIG. 35.—WORM AND WORM-WHEEL.

THE THREAD OF WORM IS LEFT-HANDED; WORM IS SINGLE-THREADED.

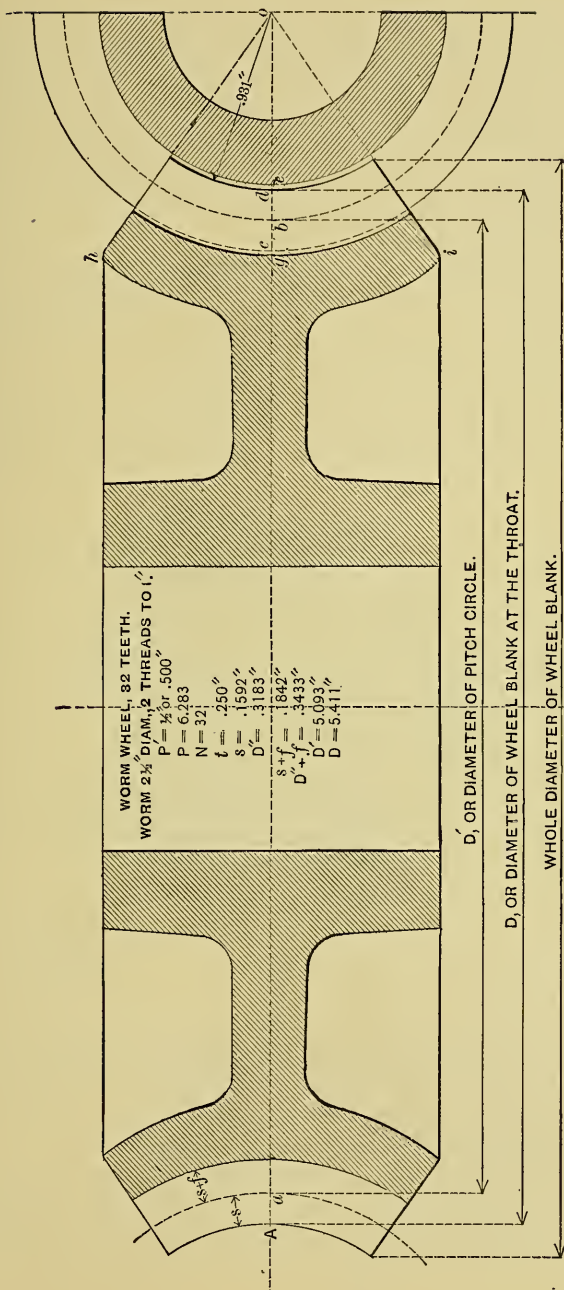


Fig. 36
WORM WHEEL BLANK.

one inch, we *divide* 1″ *by the number of threads to one inch*, and the quotient *is the circular pitch.* Hence, the wheel in Fig. 36 is $\frac{1}{2}''$ circular pitch. *Linear pitch* expresses exactly what is meant by circular pitch. Linear pitch has the advantage of being an exact use of language when applied to worms and racks. The number of threads to one inch linear, is the reciprocal of the linear pitch.

Linear pitch.

Multiply 3.1416 by the number of threads to one inch, and the product will be the diametral pitch of the worm-wheel. Thus, we should say of a double-threaded worm advancing 1″ in $1\frac{1}{3}$ turns that:

Drawing of Worm and Worm-wheel.

Lead$=\frac{3}{4}''$ or .75″. Linear pitch or $P'=\frac{3}{8}''$ or .375″. Diametral pitch or $P=8.377$. See table of tooth parts.

To make drawing of worm and wheel we obtain data as in circular pitch.

1. Draw center line A O and upon it space off the distance *a b* equal to the diameter of pitch-circle.

2. On each side of these two points lay off the distance *s*, or the usual addendum$=\frac{1}{P}''$, as *b c* and *b d.*

3. From *c* lay off the distance *c* O equal to the radius of the worm. The diameter of a worm is generally four or five times the circular pitch.

4. Lay off the distances *c g* and *d e* each equal to *f*, or the usual clearance at bottom of tooth space.

5. Through *c* and *e* draw circles about O. These represent the whole diameter of worm and the diameter at bottom of worm-thread.

6. Draw *h* O and *i* O at an angle of 30° to 45° with A O. These lines give width of face of worm-wheel.

7. Through *g* and *d* draw arcs about O, ending in *h* O and *i* O.

This operation repeated at *a* completes the outline of worm-wheel. For 32 teeth and more, the addendum diameter, or D, should be taken at the throat or smallest diameter of wheel, as in Fig. 36. *Measure sketch for whole diameter of wheel-blank.*

Teeth of Wheels finished with Hob.

The foregoing instructions and sketch are for cases where the teeth of the wheels are finished with a *hob.*

Hob.

A HOB is shown in Fig. 37, being a steel piece threaded with a tool of the same angle as the tool that threads the worm, the end of the tool being .335 of the linear pitch; the hob is then grooved to make teeth for cutting, and hardened.

Proportions of Hob.

The whole diameter of hob should be at least $2f$, or twice the clearance larger than the worm. In our relieved hobs the diameter is made about .005″ to .010″ larger to allow for wear. The outer corners of hob-thread can be rounded down as far as the clearance distance. The width at top of the hob-thread before rounding should be .31 of the linear, or circular pitch $=.31P'$. The whole depth of thread is thus the ordinary working depth plus the clearance $=D''+f$. The diameter at bottom of hob-thread should be $2f+.005''$ to $.010''$ larger than the diameter at bottom of worm-thread.

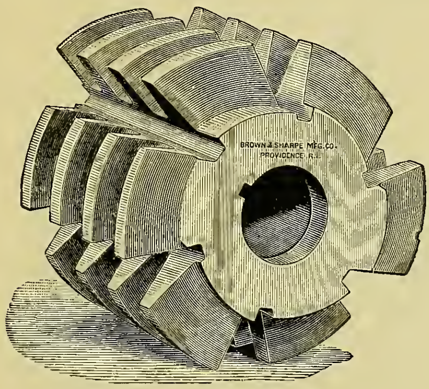


Fig. 37.—HOB.

For thread-tool and worm-thread see end of Chapter IV.

How to use the Hob.

In the absence of a special worm gear cutting machine, the teeth of the wheel are first cut as nearly to the finished form as practicable; the hob and worm-wheel are mounted upon shafts and hob placed in mesh, it is then rotated and dropped deeper into the wheel until the teeth are finished. The hob generally drives the worm-wheel during this operation. The Universal Milling Machine is convenient for doing this work; with it the dis-

Universal Milling Machine used in Hobbing.

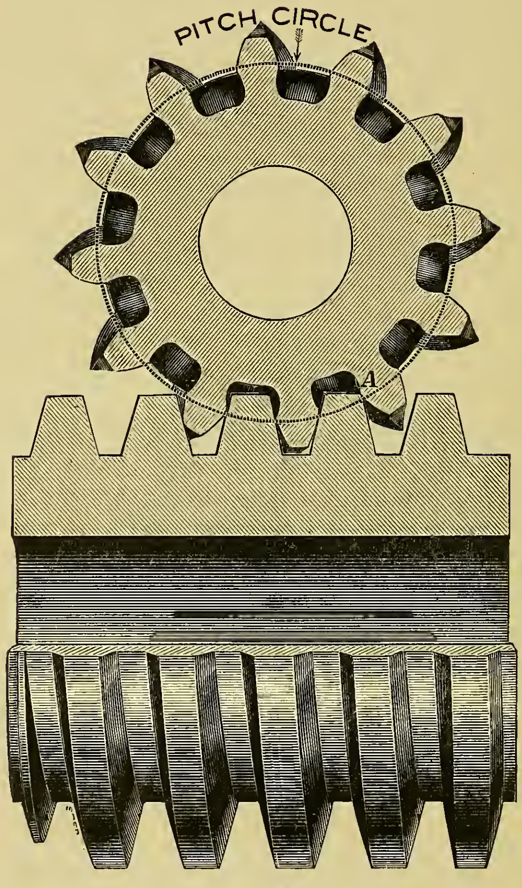


Fig. 38.

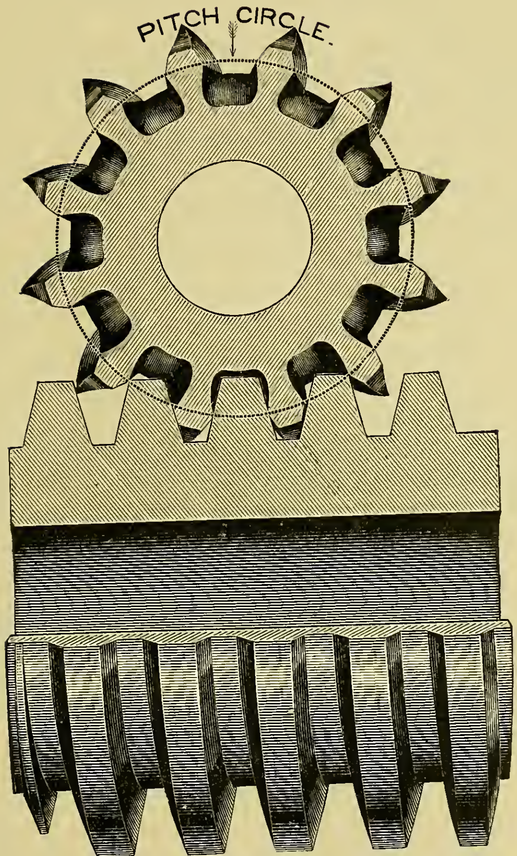


Fig. 39.

tance between axes of worm and wheel can be noted. In making wheels in quantities it is better to have a machine in which the work spindle is driven by gearing, so that the hob can cut the teeth from the solid without gashing. The object of hobbing a wheel is to get more bearing surface of the teeth upon worm-thread. The worm-wheels, Figs. 35 and 43, were hobbed.

Why a Wheel is Hobbed.

Worm-Wheel Blanks with Less than 30 Teeth.

If we make the diameter of a worm-wheel blank, that is to have less than 30 teeth, by the common rules for sizing blanks, and finish the teeth with a hob, we shall find the flanks of teeth near the bottom to be *undercut* or hollowing. This is caused by the interference spoken of in Chapter VI. Thirty teeth was there given as a limit, which will be right when teeth are made to circle arcs. With pressure angle $14\frac{1}{2}°$, and rack-teeth with usual addendum, this interference of rack-teeth with flanks of gear-teeth begins at 31 teeth ($31\frac{7}{10}$ geometrically), and interferences with nearly the whole flank in wheel of 12 teeth.

Interference of Thread and Flank.

Fig. 38.

In Fig 38 the blank for worm-wheel of 12 teeth was sized by the same rule as given for Fig. 36. The wheel and worm are sectioned to show shape of teeth at the mid-plane of wheel. The flanks of teeth are undercut by the hob. The worm-thread does not have a good bearing on flanks inside of A, the bearing being that of a corner against a surface.

Fig. 39.

In Fig 39 the blank for wheel was sized so that pitch-circle comes midway between outermost part of teeth and innermost point obtained by worm-thread.

This rule for sizing worm-wheel blanks has been in use to some extent. The hob has cut away flanks of teeth still more than in Fig. 38. The pitch circle in Fig. 39 is the same diameter as the pitch-circle in Fig. 38. The same hob was used for both wheels. The flanks in this wheel are so much undercut as to materially lessen the bearing surface of teeth and worm-thread.

Interference Avoided.

In Chapter VI. the interference of teeth in high-numbered gears and racks with flanks of 12 teeth was remedied by rounding off the addenda. Although it would be more systematic to round off the threads of a worm, making them, like rack-teeth, to mesh with interchangeable gears, yet this has not generally been done, because it is easier to make a worm-thread tool with straight sides.

Instead of cutting away the addenda of worm-thread, we can avoid the interference with flanks of wheels having less than 30 teeth by making wheel blanks larger.

The flanks of wheel in Fig. 40 are not undercut, because the diameter of wheel is so large that there is hardly any tooth inside the pitch-circle. The pitch-circle in Fig. 40 is the same size as pitch-circles in Figs. 38 and 39. This wheel was sized by the following rule: Multiply the *pitch diameter* of the wheel by .937, and add to the product four times the addendum (4 *s*); the sum will be the diameter for the blank at the throat or small part. To get the whole diameter, make a sketch with diameter of throat to the foregoing rule and measure the sketch. Fig. 40. Diameter at Throat to Avoid Interference.

It is impractical to hob a wheel of 12 to about 16 or 18 teeth when blank is sized by this rule, unless the wheel is driven by independent mechanism and not by the hob. The diameter across the outermost parts of teeth, as at A B, is considerably less than the largest diameter of wheel before it was hobbed.

In general it is well to size all blanks, as by page 66 and Figs. 36 and 38, when the wheels are to be hobbed; of course the cutter should be thin enough to leave stock for finishing. The spaces can be cut the full depth, the cutter being dropped in.

When worm-wheels are not hobbed it is better to turn blanks like a spur-wheel. Little is gained by having wheels curved to fit worm unless teeth are finished with a hob. The teeth can be cut in a straight path diagonally across face of blank, to fit angle of worm-thread, as in Figs. 41 and 44. Blank Like a Spur-Wheel.

In setting a cutter to gash a worm wheel, Figs. 42 and 45, the angle is measured from the axis of the worm-wheel and the angle of the worm thread is, in consequence, measured from the perpendicular to the axis of the worm. See Chapters V and VIII, Part II. Wheels for Gear-Cutting Machines.

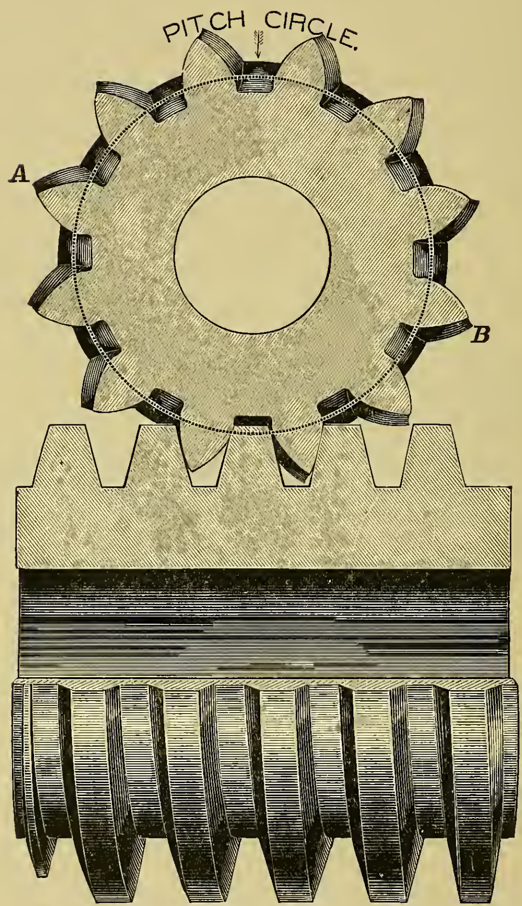


Fig. 40.

Some mechanics prefer to make dividing wheels in two parts, joined in a plane perpendicular to axis, hob teeth, then turn one part round upon the other, match teeth and fasten parts together in the new position, and hob again with a view to eliminate errors. With an accurate cutting engine we have found wheels like Figs. 42 and 45, not hobbed, every way satisfactory. As to the different wheels, Figs. 43, 44 and 45, when worm is in right position at the start, the life-time of Fig. 43, under heavy and continuous work, will be the longest. Figures 43, 44 and 45.

Fig. 44 can be run in mesh with a gear or a rack as well as with a worm when made within the angular limits commonly required. Strictly, neither two gears made in this way, nor a gear and a rack would be mathematically exact, as they might bear at the sides of the gear or at the ends of the teeth only and not in the middle. At the start the contact of teeth in this wheel upon worm-thread is in points only; yet such wheels have been many years successfully used in elevators.

Fig 45 is a neat-looking wheel. In gear cutting engines where the workman has occasion to turn the work spindle by hand, it is not so rough to take hold of as Figs 43 and 44. The teeth are less liable to injury than the teeth of Figs. 43 and 44.

The diameter of a worm has no necessary relation to the speed ratio of the worm to the worm wheel. The diameter of the worm can be chosen to suit any distance between the worm shaft and the worm-wheel shaft. It is unusual to have the diameter of the worm much less than four times the thread-pitch or linear-pitch but the worm can be of any larger diameter, five or ten times the linear-pitch, if required.

It is well to take off the outermost part of teeth in wheels (Figs. 35 and 43), as shown in these two figures, and not leave them sharp, as in Figs. 36 and 39. It is also well to round over the outer corners of the blanks for the wheels, Figs. 44 and 45. In ordering worms and worm wheels the centre distances should be given. If there can be any limit allowed in the centre distance it should be so stated.

Fig. 41.

Worm-wheel with teeth cut in a straight path diagonally across face. Worm is double-threaded.

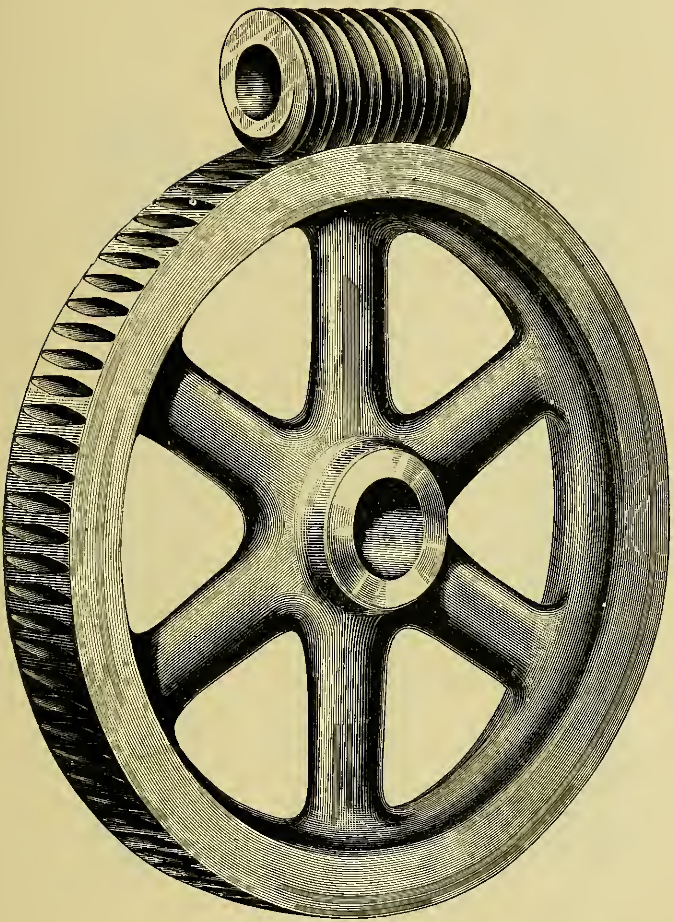


Fig. 42.

Worm and Worm-Wheel, for Gear-cutting Engine.

Fig. 43.

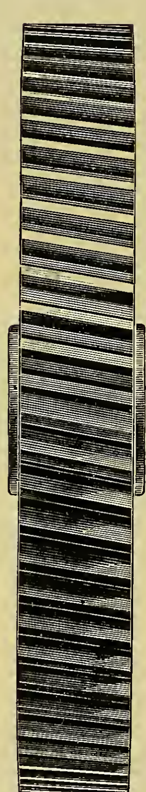


Fig. 44.

Fig. 45.

For instance, the distance from the centre of a worm to the centre of a worm wheel might be calculated at 6″ but 5 31-32″ or 6 1-32″ might answer.

By stating all the limits that can be allowed, there may be a saving in the cost of work because time need not be wasted in trying to make work within narrower limits than are necessary.

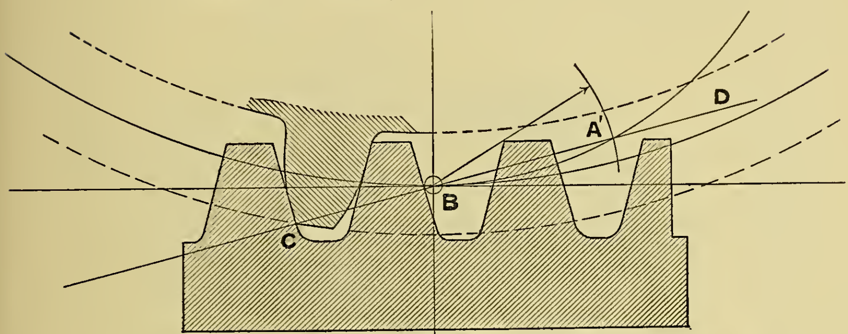


Fig. 45 A.

LENGTH OF A WORM AND OF A HOB.

In worm-wheels, like Figs. 41 and 42, having 540 teeth, worms can have bearings in ten places or along ten threads. Worms in wheels of 120 teeth bear on six threads.

In order to hob a wheel of 540 teeth, the hob must be about eleven threads long, if the worm has ten threads. For the 120 tooth wheel, the hob should have about seven threads, if the worm has six threads.

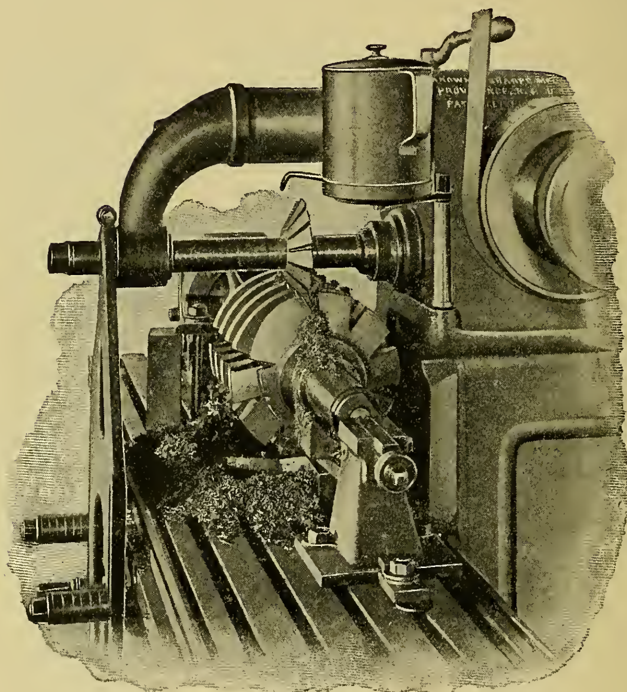
For a 30 tooth worm wheel of the form of Figs. 41 and 42, we can have only about three threads in contact and a hob four threads long, like Fig. 37, is long enough.

From the diagram, Fig. 45 A, which is similar to Fig. 7, we can tell approximately the number of threads that can bear. Let the worm move to the right and the action begins at C and ends at A′, C being the point where the line C D intersects the addendum circle of the gear and A′ being the point where the line would intersect the addendum line of the worm.

A short worm can be used in a large wheel by having the hob a little longer than the worm.

GASHING TEETH OF HOB.

10 Inches Outside Diameter.



HOBS WITH RELIEVED TEETH.

We make hobs of any size with the teeth relieved the same as our gear cutters. The teeth can be ground on their faces without changing their form. The hobs are made with a precision screw so that the pitch of the thread is accurate before hardening.

CHAPTER XII.

SIZING GEARS WHEN THE DISTANCE BETWEEN CENTRES AND THE RATIOS OF SPEEDS ARE FIXED—GENERAL REMARKS—WIDTH OF FACE OF SPUR GEARS—SPEED OF GEAR CUTTERS—TABLE OF TOOTH PARTS.

Let us suppose that we have two shafts 14″ apart, center to center, and wish to connect them by gears so that they will have speed ratio 6 to 1. We add the 6 and 1 together, and divide 14″ by the sum and get 2″ for a quotient; this 2″, multiplied by 6, gives us the radius of pitch circle of large wheel = 12″. In the same manner we get 2″ as radius of pitch circle of small wheel. Doubling the radius of each gear, we obtain 24″ and 4″ as the pitch diameters of the two wheels. The two numbers that form a ratio are called the terms of the ratio. We have now the rule for obtaining pitch-circle diameter of two wheels of a given ratio to connect shafts a given distance apart:

Center distance and Ratio fixed.

Divide the center distance by the sum of the terms of the ratio; find the product of twice the quotient by each term separately, and the two products will be the pitch diameters of the two wheels.

Rule for Diameter of Pitch Circles.

It is well to give special attention to learning the rules for sizing blanks and teeth; these are much oftener needed than the method of forming tooth outlines.

A blank $1\frac{1}{2}''$ diameter is to have 16 teeth: what will the pitch be? What will be the diameter of the pitch circle? See Chapter V.

A good practice will be to compute a table of tooth parts. The work can be compared with the tables pages 146–149.

In computing it is well to take π to more than four places, π to nine places $= 3.141592653$. $\frac{1}{\pi}$ to nine places $= .318309886$.

There is no such thing as pure rolling contact in teeth of wheels; they always rub, and, in time, will wear themselves out of shape and may become noisy.

Bevel gears, when correctly formed, run smoother than spur gears of same diameter and pitch, because the teeth continue in contact longer than the teeth of spur gears. For this reason annular gears run smoother than either bevel or spur gears.

Sometimes gears have to be cut a little deeper than designed, in order to run easily on their shafts. If any departure is made in ratio of pitch diameters it is better to have the driving gear the larger, that is, cut the follower smaller. For wheels coarser than eight diametral pitch (8 P), it is generally better to cut twice around, when accurate work is wanted, also for large wheels, as the expansion of parts from heat often causes inaccurate work when cut but once around. There is not so much trouble from heat in plain or web gears as in arm gears.

Width of Spur Gear faces. The width of face of cast-iron gears can, for general use, be made $2\frac{1}{2}$ times the linear pitch.

In small gears or pinions this width is often exceeded.

The outer corners of spur gears may be rounded off for convenience in handling. This can be provided for when turning the blank.

Speed of Gear Cutters. The speed of gear cutters is subject to so many conditions that definite rules cannot be given. We append a table of average speeds. A coarse pitch cutter for pinion, 12 teeth, would usually be fed slower than a cutter for a large gear of same pitch.

TABLE OF AVERAGE SPEEDS FOR GEAR-CUTTERS.

Diametral Pitch of Cutter.	Diameter of Cutter.	Turns per minute cutting Cast Iron.	Turns per minute cutting Wrought Iron and Steel.	Feed to One Turn of Cutter in Cast Iron.	Feed to One Turn of Cutter in Wrought Iron and Steel.	Feed per minute in Cast Iron.	Feed per minute in Wrought Iron and Steel.
2	5 in.	24	18	.025 in.	.011 in.	.60 in.	.20 in.
$2\frac{1}{2}$	$4\frac{1}{4}$ "	30	24	.028 "	.013 "	.84 "	.31 "
3	$3\frac{13}{16}$ "	36	28	.031 "	.015 "	1.12 "	.42 "
4	$3\frac{3}{8}$ "	42	32	.034 "	.017 "	1.43 "	.54 "
5	$3\frac{1}{16}$ "	50	40	.037 "	.019 "	1.85 "	.76 "
6	$2\frac{11}{16}$ "	75	55	.030 "	.016 "	2.25 "	.88 "
7	$2\frac{9}{16}$ "	85	65	.032 "	.018 "	2.72 "	1.17 "
8	$2\frac{1}{2}$ "	95	75	.034 "	.020 "	3.23 "	1.50 "
10	$2\frac{1}{8}$ "	125	90	.026 "	.014 "	3.25 "	1.26 "
12	2 "	135	100	.027 "	.017 "	3.64 "	1.70 "
20	$1\frac{7}{8}$ "	145	115	.029 "	.021 "	4.20 "	2.41 "
32	$1\frac{3}{4}$ "	160	135	.031 "	.025 "	4.96 "	3.37 "

Speed in Brass.

In brass the speed of gear-cutters can be twice as fast as in cast iron. Clock-makers and those making a specialty of brass gears exceed this rate even. A 12 P cutter has been run 1,200 (twelve hundred) turns a minute in bronze. A 32 P cutter has been run 7,000 (seven thousand) turns a minute in soft brass.

Examples from Practice.

In cutting 5 P cast-iron gears, 75 teeth, a No. 1, 6 P cutter was run 136 (one hundred and thirty-six) turns a minute, roughing the spaces out the full 5 P depth; the teeth were then finished with a 5 P cutter, running 208 (two hundred and eight) turns a minute, feeding by hand. The cutter stood well, but, of course, the cast iron was quite soft. A 4 P cutter has finished teeth at one cut, in cast-iron gears, 86 teeth, running 48 (forty-eight) turns a minute and feeding $\frac{1}{16}''$ at one turn, or 3 in. in a minute.

Hence, while it is generally safe to run cutters as in the table, yet when many gears are to be cut it is well to see if cutters will stand a higher speed and more feed.

In gears coarser than 3 P it is more economical to cut first the full depth with a stocking cutter and then finish with a gear cutter. This stocking cutter is made on the principle of a circular splitting saw for wood. The teeth, however, are not set; but side relief is obtained by making sides of cutter blank hollowing. The shape of stocking cutter can be same as bottom of spaces in a 12-tooth gear, and the thickness of cutter can be $\frac{1}{2}$ of the circular pitch, see page 40.

Keep Cutters sharp.

The matter of keeping cutters sharp is so important that it has sometimes been found best to have the workman grind them at stated times, and not wait until he can see that the cutters are dull. Thus, have him grind every two hours or after cutting a stated number of gears. Cutters of the style that can be ground upon their tooth faces without changing form are rapidly destroyed if allowed to run after they are dull. Cutters are oftener wasted by trying to cut with them when they are dull than by too much grinding. Grind the faces radial with a free cutting wheel. Do not let the wheel become glazed, as this will draw the temper of the cutter.

In Chapter VI. was given a series of cutters for cutting gears having 12 teeth and more. Thus, it was there implied that any gear of same pitch, having 135 teeth, 136 teeth, and so on up to the largest gears, and, also, a rack, could be cut with one cutter. If this cutter is 4 P, we would cut with it all 4 P gears, having 135 teeth or more, and we would also cut with it a 4 P rack. Now, instead of always referring to a cutter by the number of teeth in gears it is designed to cut, it has been found convenient to designate it by a letter or by a number. Thus, we call a cutter of 4 P, made to cut gears 135 teeth to a rack, inclusive, No. 1, 4 P.

Involute Gear Cutters.

We have adopted numbers for designating involute gear-cutters as in the following table:

No. 1	will cut	wheels from	135	teeth	to a rack		inclusive.
" 2	"	"	55	"	134	teeth	"
" 3	"	"	35	"	54	"	"
" 4	"	"	26	"	34	"	"
" 5	"	"	21	"	25	"	"
" 6	"	"	17	"	20	"	"
" 7	"	"	14	"	16	"	"
" 8	"	"	12	"	13	"	"

By this plan it takes eight cutters to cut all gears having twelve teeth and over, of any one pitch.

Thus it takes eight cutters to cut all involute 4 P gears having twelve teeth and more. It takes eight other cutters to cut all involute gears of 5 P, having 12 teeth and more. A No. 8, 5 P cutter cuts only 5 P gears having 12 and 13 teeth. A No. 6, 10 P cutter cuts only 10 P gears having 17, 18, 19 and 20 teeth. On each cutter is stamped the number of teeth at the limits of its range, as well as the number of the cutter. The number of the cutter relates only to the number of teeth in gears that the cutter is made for.

In ordering cutters for involute spur-gears two things must be given:

How to order Involute Cutters.

1. *Either the number of teeth to be cut in the gear or the number of the cutter, as given in the foregoing table.*

2. *Either the pitch of the gear or the diameter and number of teeth to be cut in the gear.*

If 25 teeth are to be cut in a 6 P involute gear, the cutter will be No. 5, 6 P, which cuts all 6 P gears from 21 to 25 teeth inclusive. If it is desired to cut gears from 15 to 25 teeth, three cutters will be needed, No. 5, No. 6 and No. 7 of the pitch required. If the pitch is 8 and gears 15 to 25 teeth are to be cut, the cutters should be No. 5, 8 P, No. 6, 8 P, and No. 7, 8 P.

Epicycloidal or Double-curve Cutters.

For each pitch of epicycloidal, or double-curve gears, 24 cutters are made. In coarse-pitch gears, the variation in the shape of spaces between gears of consecutive-numbered teeth is greater than in fine-pitch gears.

A set of cutters for each pitch to consist of so large a number as 24, was established for the reason that double curve teeth were formerly preferred in coarse pitch gears. The tendency now, however, is to use the involute form.

Our double curve cutters have a guide shoulder on each side for the depth to cut. When this shoulder just reaches the periphery of the blank the depth is right. The marks which these shoulders make on the blank, should be as narrow as can be seen, when the blanks are sized right.

Double-curve gear-cutters are designated by letters instead of by numbers; this is to avoid confusion in ordering.

Following is the list of epicycloidal or double-curve gear-cutters:—

Table of Epicycloidal or Double-curve Gear Cutters.

Cutter	A	cuts	12 teeth.	Cutter	M	cuts	27 to 29 teeth.
"	B	"	13 "	"	N	"	30 " 33 "
"	C	"	14 "	"	O	"	34 " 37 "
"	D	"	15 "	"	P	"	38 " 42 "
"	E	"	16 "	"	Q	"	43 " 49 "
"	F	"	17 "	"	R	"	50 " 59 "
"	G	"	18 "	"	S	"	60 " 74 "
"	H	"	19 "	"	T	"	75 " 99 "
"	I	"	20 "	"	U	"	100 " 149 "
"	J	"	21 to 22	"	V	"	150 " 249 "
"	K	"	23 to 24	"	W	"	250 " Rack.
"	L	"	24 to 26	"	X	"	Rack.

A cutter that cuts more than one gear is made of proper form for the smallest gear in its range. Thus, cutter J for 21 to 22 teeth is correct for 21 teeth; cutter S for 60 to 74 teeth is correct for 60 teeth, and so on.

How to order Epicycloidal Cutters.

In ordering epicycloidal gear-cutters designate the letter of the cutter as in the foregoing table, also either give the pitch or give data that will enable us to determine the pitch, the same as directed for involute cutters.

More care is required in making and adjusting epicycloidal gears than in making involute gears.

How to order Bevel Gear Cutters.

In ordering bevel gear cutters three things must be given:

1. *The number of teeth in each gear.*
2. *Either the pitch of gears or the largest pitch diameter of each gear; see Fig.* 17.
3. *The length of tooth face.*

If the shafts are not to run at right angles, it should be so stated, and the angle given. Involute cutters only are used for cutting bevel gears. No attempt should be made to cut epicyclodial tooth bevel gears with rotary disk cutters.

How to order Worm-gear Cutters.

In ordering worm-wheel cutters, three things must be given:

1. *Number of teeth in the wheel.*
2. *Pitch of the worm; see Chapter XI.*
3. *Whole diameter of worm.*

In any order connected with gears or gear-cutters, when the word "Diameter" occurs, we usually understand that the *pitch diameter* is meant. When the *whole* diameter of a gear is meant it should be plainly written. Care in giving an order often saves the delay of asking further instructions. An order for one gear-cutter to cut from 25 to 30 teeth cannot be filled, because it takes two cutters of involute form to cut from 25 to 30 teeth, and three cutters of epicycloidal form to cut from 25 to 30 teeth.

Sheet zinc is convenient to sketch gears upon, and also for making templets. Before making sketch, it is well to give the zinc a dark coating with the following mixture: Dissolve 1 ounce of sulphate of copper (blue vitriol) in about 4 ounces of water, and add about one-half teaspoonful of nitric acid. Apply a thin coating with a piece of waste.

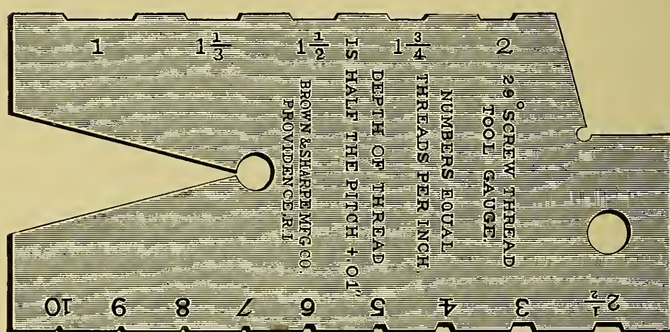
This mixture will give a thin coating of copper to iron or steel, but the work should then be rubbed dry. Care should be taken not to leave the mixture where it is not wanted, as it rusts iron and steel.

We have sometimes been asked why gears are noisy. Not many questions can be asked us to which we can give a less definite answer than to the question why gears are noisy.

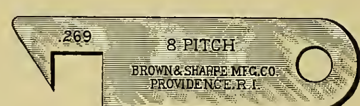
We can indicate only some of the causes that may make gears noisy, such as:—depth of cutting not right—in this particular gears are oftener cut too deep than not deep enough; (more noise may be caused by cutting the driver too deep than by cutting the driven too deep;) cutting not central—this may make gears noisy in one direction when they are quiet while running in the other direction; centre distance not right—if too deep the outer corners of the teeth in one gear may strike the fillets of the teeth in the other gear; shafts not parallel; frame of the machine of such a form as to give off sound vibrations. Even when we examine a pair of gears we cannot always tell what is the matter.

IMPROVED 29° SCREW THREAD TOOL GAUGE.

"ACME STANDARD."



DEPTH OF GEAR TOOTH GAUGES.



Depth of Gear Tooth Gauges for all regular pitches, from 3 to 48 pitch inclusive, are carried in stock.

One Gauge answers for each pitch, and indicates the extreme depth to be cut.

PART II.

CHAPTER I.

TANGENT OF ARC AND ANGLE.

In PART II. we shall show how to calculate some of the functions of a right-angle triangle from a table of circular functions, the application of these calculations in some chapters of PART I. and in sizing blanks and cutting teeth of spiral gears, the selection of cutters for spiral gears, the application of continued fractions to some problems in gear wheels and cutting odd screw-threads, etc., etc. Subjects to be explained.

A Function is a quantity that depends upon another quantity for its value. Thus the amount a workman earns is a function of the time he has worked and of his wages per hour. Function defined.

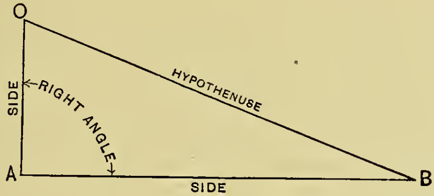


Fig. 46.

In any *right-angle triangle*, O A B, we shall, for convenience, call the two lines that form the right angle O A B the *sides*, instead of base and perpendicular. Thus O A B, being the right angle we call the line O A a side, and the line A B a side also. Right-angle Triangle.

When we speak of the angle A O B, we call the line O A the *side adjacent*. When we are speaking of the angle A B O we call the line A B the side adjacent. The line opposite the right angle is the *hypothenuse*. Side adjacent. Hypothenuse.

In the following pages the definitions of circular functions are for angles smaller than 90°, and not strictly applicable to the reasoning employed in analytical trigonometry, where we find expressions for angles of 270°, 760°, etc.

Tangent.

The Tangent of an arc is the line that touches it at one extremity and is terminated by a line drawn from the center through the other extremity. The tangent is always *outside* the arc and is also perpendicular to the radius which meets it at the point of tangency.

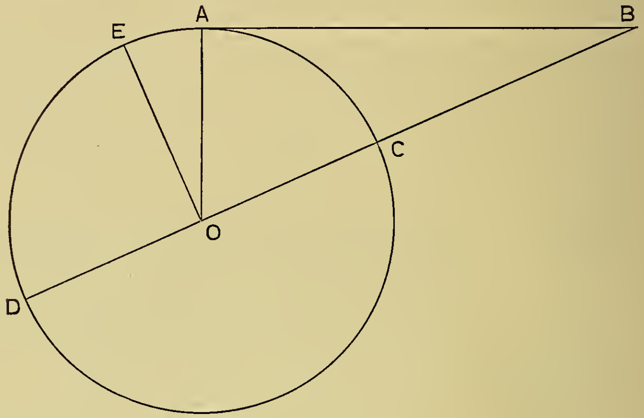


Fig. 47.

Thus, in Fig. 47, the line A B is the tangent of the arc A C. The point of tangency is at A.

An angle at the center of a circle is measured by the arc intercepted by the sides of the angle. Hence the tangent A B of the arc A C is also the tangent of the angle A O B.

In the tables of circular functions the radius of the arc is unity, or, in common practice, we take it as one inch. The radius O A being 1″, if we know the length of the line or tangent A B we can, by looking in a table of tangents, find the number of degrees in the angle A O B.

To find the Degrees in an Angle.

Thus, if A B is 2.25″ long, we find the angle A O B is 66° very nearly. That is, having found that 2.2460 is the nearest number to 2.25 in the table of tangents at the end of this volume, we find the corresponding degrees of the angle in the column at the left hand of the table and the minutes to be added at the top of the column containing the 2.2460.

The table gives angles for every 10′, which is sufficient for most purposes.

Now, if we have a right-angle triangle with an angle the same as O A B, but with O A two inches long, the line A B will also be twice as long as the tangent of angle A O B, as found in a table of tangents.

Example to find the Degrees in an Angle.

Let us take a triangle with the side O A = 5″ long, and the side A B = 8″ long; what is the number of degrees in the angle A O B?

Dividing 8″ by 5 we find what would be the length of A B if O A was only 1″ long. The quotient then would be the length of tangent when the *radius* is 1″ long, as in the table of tangents. 8 divided by 5 is 1.6. The nearest tangent in the table is 1.6003 and the corresponding angle is 58°, which would be the angle A O B when A B is 8″ and the radius O A is 5″ very nearly. The difference in the angles for tangents 1.6003 and 1.6 could hardly be seen in practice. The side opposite the required acute angle corresponds to the tangent and the side adjacent corresponds to the radius. Hence the rule:

To find the Tangent.

To find the tangent of either acute angle in a right-angle triangle: *Divide the side opposite the angle by the side adjacent the angle and the quotient will be the tangent of the angle.* This rule should be committed to memory. Having found the tangent of the angle, the angle can be taken from the table of tangents.

Complement of an Angle.

The complement of an angle is the remainder after subtracting the angle from 90°. Thus 40° is the complement of 50°.

Cotangent.

The Cotangent of an angle is the tangent of the complement of the angle. Thus, in Fig. 47, the line A B is the cotangent of A O E. In right-angle triangles either acute angle is the complement of the other acute angle. Hence, if we know one acute angle, by subtracting this angle from 90° we get the other acute angle. As the arc approaches 90° the tangent becomes longer, and at 90° it is infinitely long.

The sign of infinity is ∞. Tangent $90° = \infty$.

To lay out an Angle by the Tangent. Example, Fig. 49.

By a table of tangents, angles can be laid out upon sheet zinc, etc. This is often an advantage, as it is not convenient to lay protractor flat down so as to mark angles up to a sharp point. If we could lay off the length of a line *exactly* we could take tangents direct from table and obtain angle at once. It, however, is generally better to multiply the tangent by 5 or 10 and make an enlarged triangle. If, then, there is a slight error in laying off length of lines it will not make so much difference with the angle.

Let it be required to lay off an angle of 14° 30′. By the table we find the tangent to be .25861. Multiplying .25861 by 5 we obtain, in the enlarged triangle, 1.29305″ as the length of side opposite the angle 14° 30′. As we have made the side opposite five times as large, we must make the side *adjacent* five times as large, in order to keep angle the same. Hence, Fig. 48, draw the line A B 5″ long; perpendicular to this line at A draw the line A O 1.293″ long; now draw the line O B, and the angle A B O will be 14° 30′.

If special accuracy is required, the tangent can be multiplied by 10; the line A O will then be 2.586″ long and the line A B 10″ long. Remembering that the acute angles of a right-angle triangle are the complements of each other, we subtract 14° 30′ from 90′ and obtain 75° 30′ as the angle of A O B.

The reader will remember these angles as occurring in PART I., Chapter IV., and obtained in a different way. A semicircle upon the line O B touching the extremities O and B will just touch the right angle at A, and the line O B is four times as long as O A.

Let it be required to turn a piece 4″ long, 1″ diameter at small end, with a taper of 10° one side with the other; what will be the diameter of the piece at the large end?

To calculate Diameter of a Tapering piece. Fig. 50.

A section, Fig. 49, through the axis of this piece is the same as if we added two right-angle triangles, O A B and O′ A′ B′, to a straight piece A′ A B B′, 1″ wide and 4″ long, the acute angles B and B′ being 5°, thus making the sides O B and O′ B′ 10° with each other.

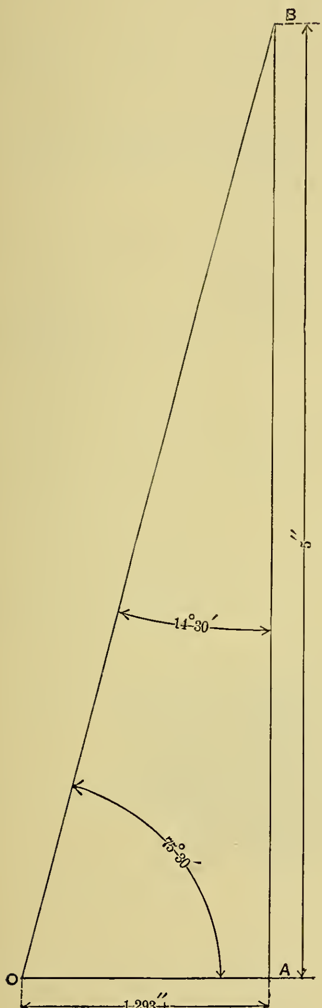


Fig. 48.

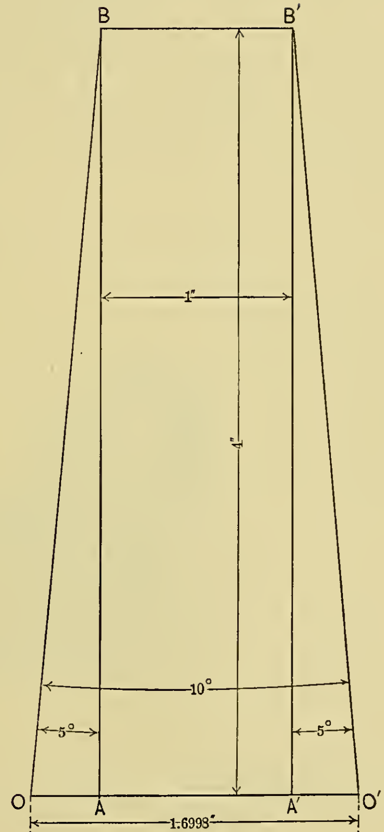
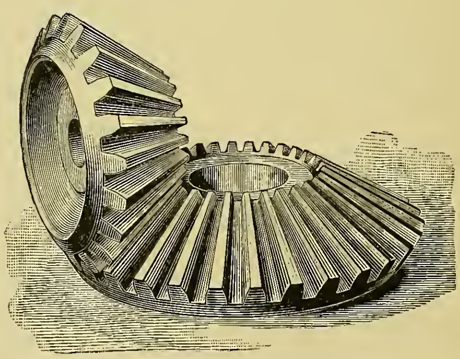


Fig. 49.

The tangent of 5° is .08748, which, multiplied by 4 , gives .34992″ as the length of each line, A O and A′ O′, to be added to 1″ at the large end. Taking twice .34992″ and adding to 1″ we obtain 1.69984″ as the diameter of large end.

This chapter must be thoroughly studied before taking up the next chapters. If once the memory becomes confused as to the tangent and sine of an angle, it will take much longer to get righted than it will to first carefully learn to recognize the tangent of an angle at *once.*

If one knows what the tangent *is,* one can tell better the functions that are *not* tangents.



CHAPTER II.

SINE—COSINE AND SECANT: SOME OF THEIR APPLICATIONS IN MACHINE CONSTRUCTION.

The Sine of an arc is the line drawn from one extremity of the arc to the diameter passing through the other extremity, the line being perpendicular to the diameter.

Another definition is: The sine of an arc is the distance of one extremity of the arc from the diameter, through the other extremity.

The sine of an angle is the sine of the arc that measures the angle. Sine of Arc and Angle.

In Fig. 50, A C is the sine of the arc B C, and of the angle B O C. It will be seen that the sine is always inside of the arc, and can never be longer than the radius. As the arc approaches 90°, the sine comes nearer to the radius, and at 90° the sine is equal to 1, or is the radius itself. From the definition of a sine, the *side* A C, opposite the angle A O C, in *any* right-angle triangle, is the sine of the angle A O C, when O C is the *radius* of the arc.

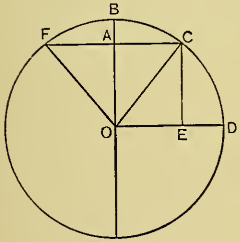


Fig. 50.

Hence the rule: *In any right-angle triangle, the side opposite either acute angle, divided by the hypothenuse, is equal to the sine of the angle.* To find the Sine.

The quotient thus obtained is the length of side opposite the angle when the hypothenuse or radius is unity. The rule should be carefully committed to memory.

Chord of an Arc.

A Chord is a straight line joining the extremities of an arc, and is twice as long as the sine of half the angle measured by the arc. Thus, in Fig. 50, the chord F C is twice as long as the sine A C.

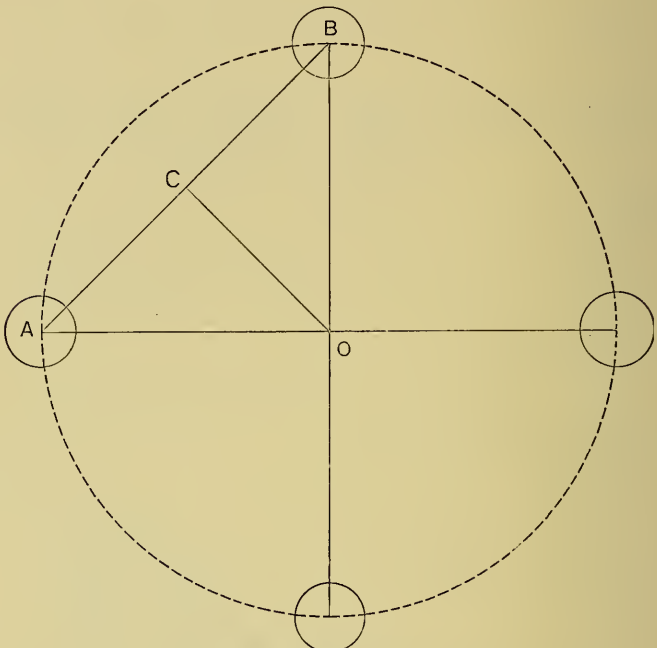


Fig. 51.

Let there be four holes equidistant about a circle 3″ in diameter—Fig. 51; what is the shortest distance between two holes? This shortest distance is the chord A B, which is twice the sine of the angle C O B. The angle A O B is one-quarter of the circle, and C O B is one-eighth of the circle. 360°, divided by 8=45°, the angle C O B. The sine of 45° is .70710, which multiplied by the radius 1.5″, gives length C B in the circle, 3″ in diameter, as 1.06065″. Twice this length is the required distance A B=2.1213″.

Example to find the Chord.

When a cylindrical piece is to be cut into any number of sides, the foregoing operation can be applied to obtain the width of one side. A plane figure bounded by straight lines is called a polygon.

Polygon.

When the outside diameter and the number of sides of a regular polygon are given, to find the length of one of the sides: *Divide* 360° *by twice the number of sides; multiply the sine of the quotient by the outer diameter, and the product will be the length of one of the sides.* To find the length of Side.

Multiplying by the diameter is the same as multiplying by the radius, and that product again by 2.

The Cosine of an angle is the sine of the complement of the angle. Cosine.

In Fig. 50, C O D is the complement of the angle A O C; the line C E is the sine of C O D, and hence is the cosine of B O C. The line O A is equal to C E. It is quite as well to remember the cosine as the part of the radius, from the center that is cut off by the sine. Thus the sine A C of the angle A O C cuts off the cosine O A. The line O A may be called the cosine because it is equal to the cosine C E.

In any right-angle triangle, the side adjacent either acute angle corresponds to the cosine when the hypothenuse is the radius of the arc that measures the angle; hence: *Divide the side adjacent the acute angle by the hypothenuse, and the quotient will be the cosine of the angle.* To find the Cosine.

When a cylindrical piece is cut into a polygon of any number of sides, a table of cosines can be used to obtain the diameter across the sides. Length of sides of Polygon.

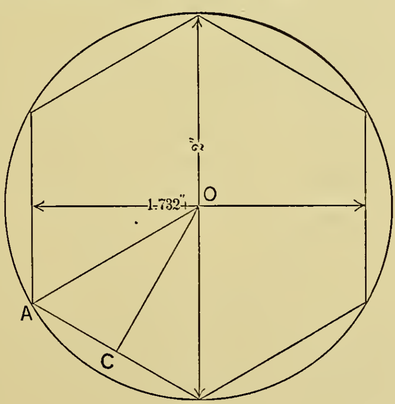


Fig. 52.

Let a cylinder, 2″ diameter, Fig. 53, be cut six-sided; what is the diameter across the sides?

The angle A O B, at the center, occupied by one of these sides, is one-sixth of the circle, =60°. The cosine of one-half this angle, 30°, is the line C O; twice this line is the diameter across the sides. The cosine of 30° is .86602, which, multiplied by 2, gives 1.73204″ as the diameter across the sides.

Of course, if the radius is other than unity, the cosine should be multiplied by the radius, and the product again by 2, in order to get diameter across the sides; or what is the same thing, multiply the cosine by the whole diameter or the diameter across the corners.

Rule for Diameter across sides of a Polygon.

The rule for obtaining the diameter across sides of regular polygon, when the diameter across corners is given, will then be: *Multiply the cosine of* 360° *divided by twice the number of sides, by the diameter across corners, and the product will be the diameter across sides.*

Look at the right-hand column for degrees of the cosine, and at bottom of page for minutes to add to the degrees.

Secant.

The Secant of an arc is a straight line drawn from the center through one end of an arc, and terminated by a tangent drawn from the other end of the arc.

Thus, in Fig. 53, the line O B is the secant of the angle C O B.

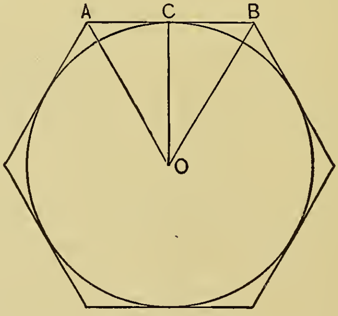


Fig. 53.

To find the Secant.

In any right-angle triangle, *divide the hypothenuse by the side adjacent either acute angle, and the quotient will be the secant of that angle.*

That is, if we divide the distance O B by O C, in the right-angle triangle C O B, the quotient will be the secant of the angle C O B.

The secant cannot be less than the radius; it increases as the angle increases, and at 90° the secant is infinity $=\infty$.

To find the Diameter across corners of a Polygon.

A six-sided piece is to be $1\frac{1}{2}''$ across the sides; how large must a blank be turned before cutting the sides? Dividing 360° by twice the number of sides, we have 30°, which is the angle C O B. The secant of 30° is 1.1547.

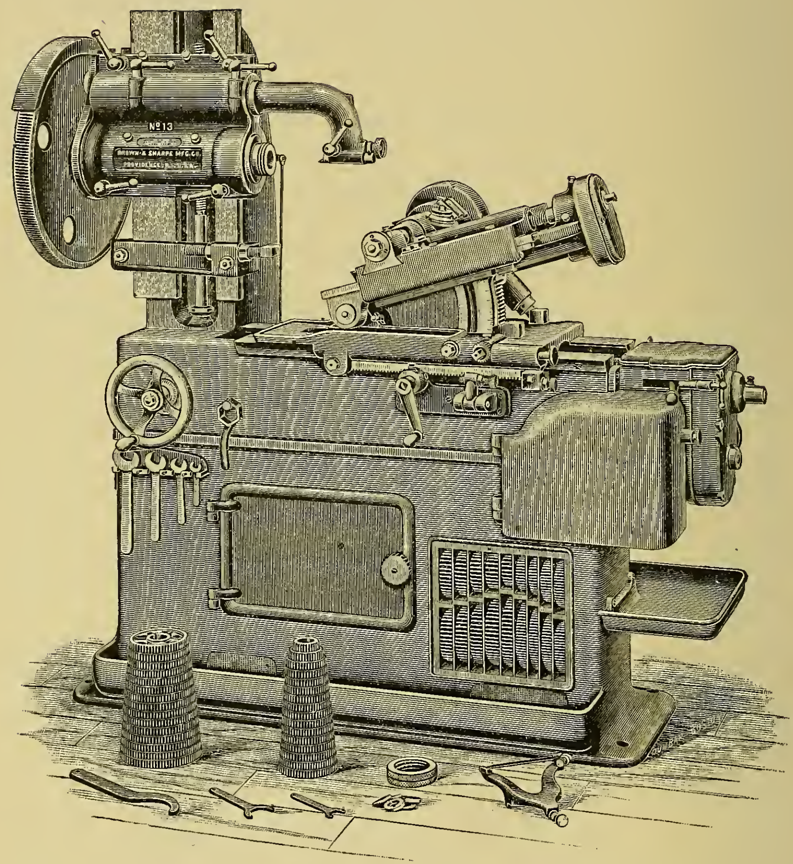
The radius of the six-sided piece is .75″.

Multiplying the secant 1.1547 by .75″, we obtain the length of radius of the blank O B; multiplying again by 2, we obtain the diameter 1.732″+.

Hence, in a regular polygon, when the diameter across sides and the number of sides are given, to find diameter across corners: *Multiply the secant of* 360° *divided by twice the number of sides, by the diameter across sides, and the product will be the diameter across corners.*

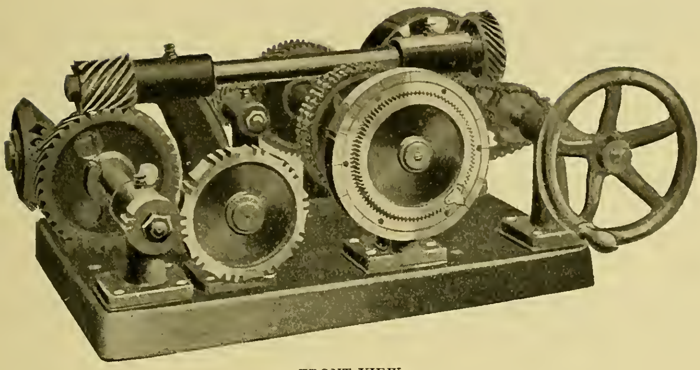
It will be seen that the side taken as a divisor has been in each case the side corresponding to the radius of the arc that subtends the angle.

The *versed sine* of an acute angle is the part of radius outside the sine, or it is the radius *minus* the cosine. Thus, in Fig. 50, the versed sine of the arc BC is AB. The versed sine is not given in the tables of circular functions: when it is wanted for any angle less than 90° we subtract the cosine of that angle from the radius 1. Having it for the radius 1, we can multiply by the radius of any other arc of which we may wish to know the versed sine.

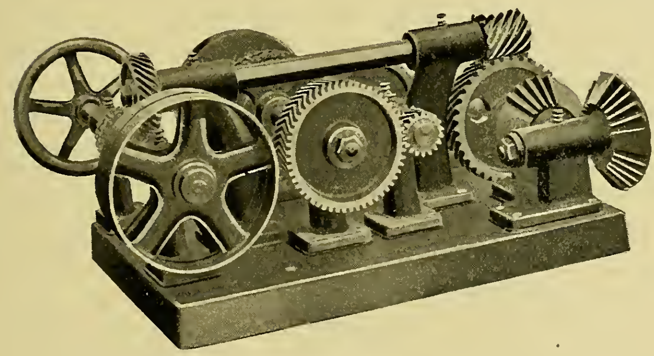


NO. 13. AUTOMATIC GEAR CUTTING MACHINE.

For Spur and Bevel Gears.



FRONT VIEW.



REAR VIEW.

GEAR MODEL.

Shows combination of six different kinds of gears.

CHAPTER III.

APPLICATION OF CIRCULAR FUNCTIONS—WHOLE DIAMETER OF BEVEL GEAR BLANKS—ANGLES OF BEVEL GEAR BLANKS.

The rules given in this chapter apply only to bevel gears having the center angle *c*′ O *i* not greater than 90°.

To avoid confusion we will illustrate one gear only. The same rules apply to all sizes of bevel gears. Fig. 55 is the outline of a pinion 4 P, 20 teeth, to mesh with a gear 28 teeth, shafts at right angles. For making sketch of bevel gears see Chapter IX., Part I.

In Fig. 55, the line O *m*′ *m* is continued to the line *a b*. The angle *c*′ O *i* that the cone pitch-line makes with the center line may be called the *center angle*.

Angle of Edge. Fig. 55.

The center angle *c*′ O *i* is equal to the angle of edge *c*′ *i c*. *c*′ *i* is the side opposite the center angle *c*′ O *i*, and *c*′ O is the side adjacent the center angle. *c*′ *i* = 2.5″; *c*′ O = 3.5″. Dividing 2.5″ by 3.5″ we obtain .71428″ + as the tangent of *c*′ O *i*. In the table we find .71329 to be the nearest tangent, the corresponding angle being 35° 30′. 35½°, then, is the center angle *c*′ O *i* and the angle of edge *c*′ *i n*, very nearly.

When the axes of bevel gears are at right angles the angle of edge of one gear is the complement of angle of edge of the other gear. Subtracting, then, 35½° from 90° we obtain 54½° as the angle of edge of gear 28 teeth, to mesh with gear 20 teeth, Fig. 55, from which we have the rule for obtaining centre angles when the axes of gears are at right angles.

Divide the radius of the pinion by the radius of the gear and the quotient will be the tangent of centre angle of the pinion.

Now subtract this centre angle from 90 deg. and we have the centre angle of the gear.

The same result is obtained by dividing the number of teeth in the pinion by the number of teeth in the gear; the quotient is the tangent of the centre angle.

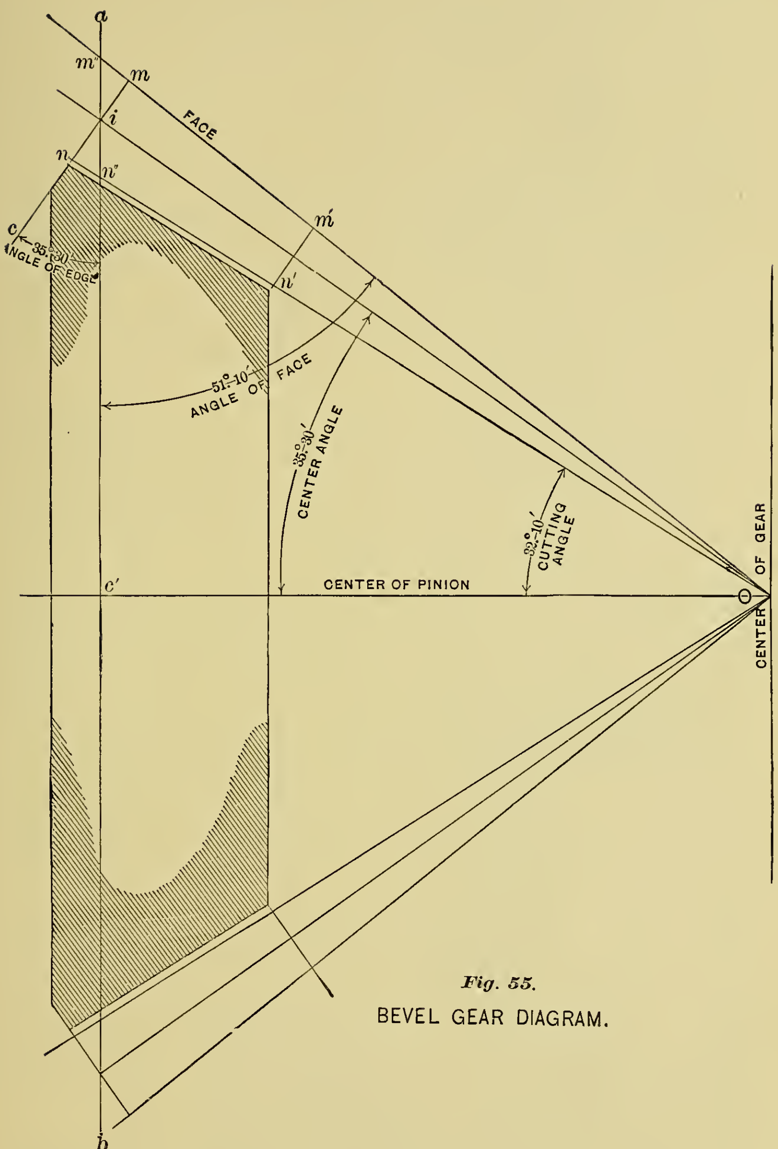


Fig. 55.

BEVEL GEAR DIAGRAM.

Angle of Face. To obtain angle of face O m'' c', the distance c' O becomes the side opposite and the distance m'' c' is the side adjacent.

The distance c' O is 3.5″, the radius of the 28 tooth bevel gear. The distance c' m'' is by measurement 2.82″.

Dividing 3.5 by 2.82 we obtain 1.2411 for tangent of angle of face O m'' c'. The nearest tangent in the table is 1.2422 and the corresponding angle is 51° 10′. To obtain cutting angle c' O n'' we divide the distance c' n'' by c' O. By measurement c' n'' is 2.2″. Dividing 2.2 by 3.5 we obtain .62857 for tangent of cutting angle. The nearest corresponding angle in the table is 32°10′.

The largest pitch diameter, k j, of a bevel gear, as in Fig. 56, is known the same as the pitch diameter of any spur gear. Now, if we know the distance b o or its equal a q, we can obtain the whole diameter of bevel gear blank by adding twice the distance b o to the largest pitch diameter.

Diameter Increment. Fig. 56. Twice the distance b o, or what is the same thing, the sum of a q and b o is called the *diameter increment*, because it is the amount by which we increase the largest pitch diameter to obtain the whole or outside diameter of bevel gear blanks. The distance b o can be calculated without measuring the diagram.

The angle b o j is equal to the angle of edge.

The angle of edge, it will be remembered, is the angle formed by outer edge of blank or ends of teeth with the end of hub or a plane perpendicular to the axis of gear.

The distance b o is equal to the cosine of angle of edge, multiplied by the distance j o. The distance j o is the addendum, as in previous chapters ($=s$).

Hence the rule for obtaining the diameter increment of any bevel gear: *Multiply the cosine of angle of edge by the working depth of teeth* (D″), *and the product will be the diameter increment.*

By the method given on page 102 we find the angle of edge of gear (Fig. 56) is 56° 20′. The cosine of 56° 20° is .55436, which, multiplied by $\frac{2}{3}$″, or the

Outside Diameter. depth of the 3 P gear, gives the diameter increment of the bevel gear 18 teeth, 3 P meshing with pinion of 12

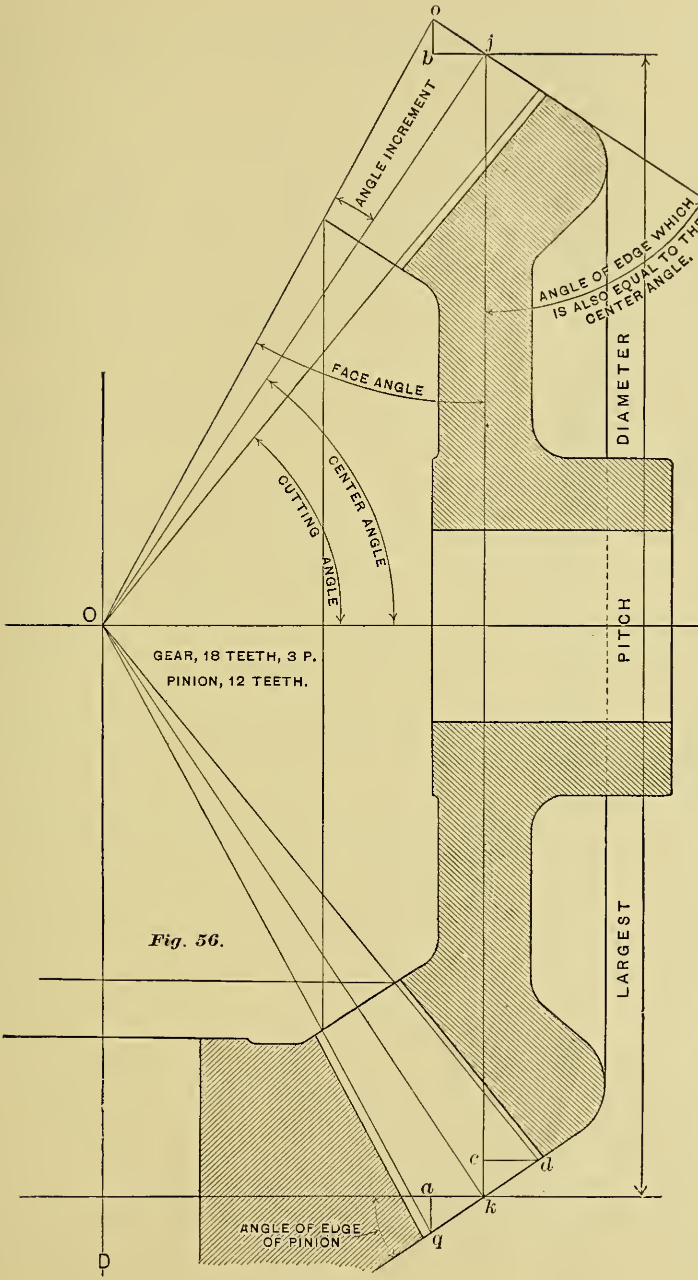


DIAGRAM.—BEVEL GEAR AND PINION.

teeth. $\frac{2}{3}$ of .55436=.369″+ (or .37″, nearly). Adding the diameter increment, .37″, to the largest pitch diameter of gear, 6″, we have 6.37″ as the outside diameter.

In the same manner, the distance *c d* is half the diameter increment of the pinion. The angle *c d k* is equal to the center angle of pinion, and when axes are at right angles is the complement of center angle of gear. The center angle of pinion is 33° 40′. The cosine, multiplied by the working depth, gives .555″ for diameter increment of pinion, and we have 4.555″ for outside diameter of pinion.

In turning bevel gear blanks, it is sufficiently accurate to make the diameter to the nearest hundredth of an inch.

Angle Increment. The small angle *o* O *j* is called the *angle increment.* When shafts are at right angles the face angle of one gear is equal to the center angle of the other gear, *minus* the angle increment.

Thus the angle of face of gear (Fig. 56) is less than the center angle D O *k*, or its equal O *j k* by the angle *o* O *j*. That is, subtracting *o* O *j* from O *j k*, the remainder will be the angle of face of gear.

Subtracting the angle increment from the center angle of gear, the remainder will be the cutting angle.

The angle increment can be obtained by dividing *o j*, the side opposite, by O *j*, the side adjacent, thus finding the tangent as usual.

The length of cone-pitch line from the common center, O to *j*, can be found, without measuring diagram, by multiplying the secant of angle O *j k*, or the center angle of pinion, by the radius of largest pitch diameter of gear.

The secant of angle O *j k*, 33° 40′, is 1.2015, which, multiplied by 3″, the radius of gear, gives 3.6045″ as the length of line O *j*.

Dividing *o j* by O *j*, we have for tangent .0924, and for angle increment 5° 20′.

The angle increment can also be obtained by the following rule:

Divide the sine of center angle by half the number of teeth, and the quotient will be the tangent of increment angle.

Subtracting the angle increment from center angles of gear and pinion, we have respectively:

Cutting angle of gear, 51°.

Cutting angle of pinion, 28° 20′.

Remembering that when the shafts are at right angles, the face angle of a gear is equal to the cutting angle of its mate (Chapter X. part 1), we have:

Face angle of gear, 28° 20′.

Face angle of pinion, 51°.

It will be seen that both the whole diameter and the angles of bevel gears can be obtained without making a diagram. Mr. George B. Grant has made a table of different pairs of gears from 1 to 1 up to 10 to 1, containing diameter increments, angle increments and centre angles, which is published in his "Treatise on Gears." "Formulas in Gearing," published by us, also contains extensive tables for bevel gearing. We have adopted the terms "diameter increment," "angle increment," and "centre angle" from him. He uses the term "*back angle*" for what we have called *angle of edge,* only he measures the angle from the axis of the gear, instead of from the side of the gear, or from the end of hub, as we have done; that is, his "back angle" is the complement of our *angle of edge.*

To lay out an Angle by the Sine.

In laying out angles, the following method may be

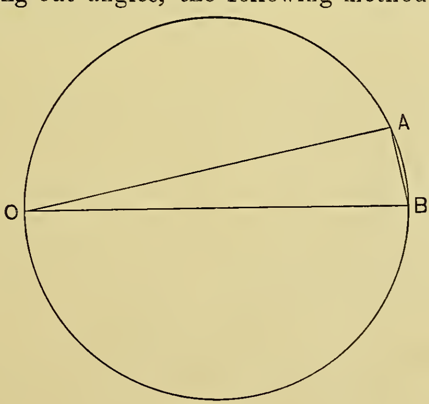


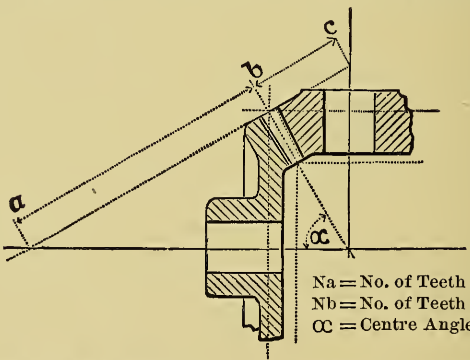
Fig. 57.

preferred, as it does away with the necessity of making a right angle: Draw a circle, A B O (Fig. 57), ten inches in diameter. Set the dividers to ten times the sine of the required angle, and point off this distance in the circumference as at A B. From any point O in the circumference, draw the lines O A and O B. The angle A O B is the angle required. Thus, let the required angle be 12°. The sine of 12° is .20791, which, multiplied by 10, gives 2.0791″, or $2\frac{8}{100}$″ nearly, for the distance A B.

Any diameter of circle can be taken if we multiply the sine by the diameter, but 10″ is very convenient, as all we have to do with the sine is to move the decimal point one place to the right.

If either of the lines pass through the centre, then the two lines which do not pass through the centre will form a right angle. Thus, if O B passes through the centre then the two lines A B and A O will form a right angle at A.

Back Cone Radius.



Measure the back cone radius *a b* for the gear, or *b c* for the pinion. This is equal to the radius of a spur gear, the number of teeth in which would determine the cutter to use. Hence twice *a b* times the diametral pitch equals the number of teeth for which the cutter should be selected for the gear. Looking in the list on page 240 the proper number for the cutter can be found.

Thus, let the back cone radius *a b* be 4″ and the diameter pitch be 8. Twice four is 8 and 8 × 8 is 64, from which it can be seen that the cutter must be of shape No. 2, as 64 is between 55 and 134, the range covered by a No. 2 cutter.

The number of teeth for which the cutter should be selected can also be found by the following formula:

$$\text{Tan. } \alpha = \frac{Na}{Nb}$$

No. of teeth to select cutter for gear $= \frac{Na}{\text{Cos. } \alpha}$ for pinion $= \frac{Nb}{\text{Sin. } \alpha}$

If the gears are mitres or are alike, only one cutter is needed; if one gear is larger than the other, two may be needed.

CHAPTER IV.

SPIRAL GEARS—CALCULATIONS FOR LEAD OF SPIRALS.

Spiral Gear.

When the teeth of a gear are cut, not in a straight path, like a spur gear, but in a helical or screw-like path, the gear is called, technically, a twisted or screw gear, but more generally among mechanics, a spiral gear. A distinction is sometimes made between a screw gear and a twisted gear. In twisted gears the pitch surfaces roll upon each other, exactly like spur gears, the axes being parallel, the same as in Fig. 1, Part I. In screw gears there is an end movement, or slipping of the pitch surfaces upon each other, the axes not being parallel. In screw gearing the action is analogous to a screw and nut, one gear driving another by the end movement of its tooth path. This is readily seen in the case of a worm and worm-wheel, when the axes are at right angles, as the movement of wheel is then wholly due to the end movement of worm thread. But, as we make the axes of gears more nearly parallel, they may still be screw gears, but the distinction is not so readily seen.

Unless otherwise stated, the shafts of screw gears are at right angles, as at A and B, Fig. 59.

The same gear may be used in a train of screw gears or in a train of twisted gears. Thus, B, as it relates to A, may be called a screw gear; but in connection with C, the same gear, B, may be called a twisted gear. These distinctions are not usually made, and we call all helical or screw-like gears made on the Universal Milling Machine *spiral gears*.

Direction of Spiral with reference to Axes. Fig. 59.

When two external spiral gears run together, with their axes parallel, the teeth of the gears must have opposite hand spirals.

Thus, in Fig. 59 the gear B has right hand spiral teeth, and the gear C has left hand spiral teeth. When the axes of two spiral gears are at right angles, both gears must have the same hand spiral teeth. A and B, Fig. 59, have right hand spiral teeth. If both gears A and B had left hand spiral teeth, the relative direction in which they turn would be reversed.

Spiral Lead.

The spiral lead or lead of spiral is the distance the spiral advances in one turn. A cylinder or gear cut with spiral grooves is merely a screw of coarse pitch or long lead; that is, a spiral is a coarse lead screw, and a screw is a fine lead spiral.

Since the introduction and extensive use of the Universal Milling Machine, it has become customary to call any screw cut in the milling machine a spiral. The spiral lead is given as so many inches to one turn. Thus, a cylinder having a spiral groove that advances six inches to one turn, is said to have a six inch spiral.

In screws the pitch is often given as so many turns to one inch. Thus, a screw of $\frac{1}{2}''$ lead is said to be 2 turns to the inch. The reciprocal expression is not much used with spirals. For example, it would not be convenient to speak of a spiral of 6″ lead, as $\frac{1}{6}$ turns to one inch.

The calculations for spirals are made from the functions of a right angle triangle.

Example, showing the nature of a Helix or Spiral.

Cut from paper a right angle triangle, one side of the right angle 6″ long, and the other side of the right angle 2″. Make a cylinder 6″ in circumference. It will be remembered (Part I., Chapter II.) that the circumference of a cylinder, multiplied by .3183, equals the diameter—$6'' \times .3183 = 1.9098''$. Wrap the paper triangle around the cylinder, letting the 2″ side be parallel to the axis, the 6″ side perpendicular to the axis and reaching around the cylinder. The hypotheneuse now forms a helix or screw-like line, called a spiral. Fasten the paper triangle thus wrapped around. See Fig. 60.

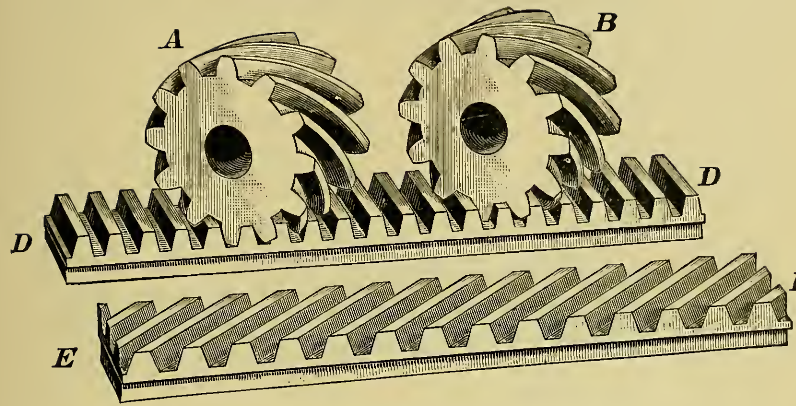


FIG. 58.—RACKS AND GEARS.

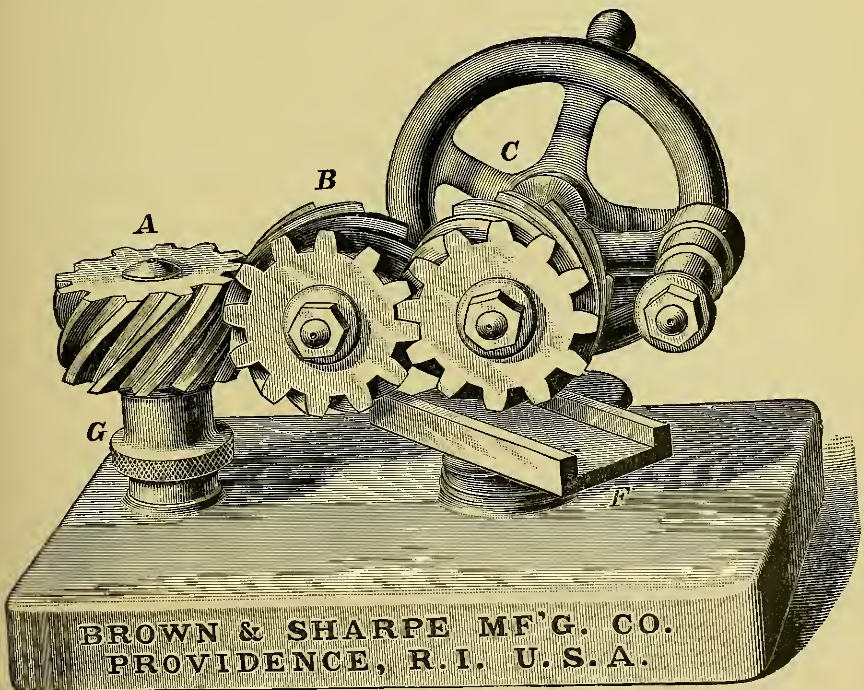


FIG. 59.—SPIRAL GEARING.

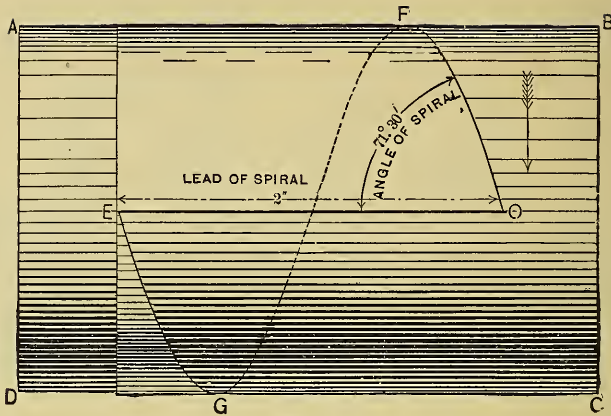


Fig. 60.

If we now turn this cylinder A B C D one turn in the direction of the arrow, the spiral will advance from O to E. This advance is the *lead of the spiral.*

The angle E O F, which the spiral makes with the axis E O, is the *angle of the spiral.* This angle is found as in Chapter I. The circumference of the cylinder corresponds to the side *opposite the angle.* The pitch of the spiral corresponds to the side *adjacent the angle.* Hence the rule for angle of spiral:

Rules for calculating the parts of a spiral.

Divide the circumference of the cylinder or spiral by the number of inches of spiral to one turn, and the quotient will be the tangent of angle of spiral.

When the angle of spiral and circumference are given, to find the lead:

Divide the circumference by the tangent of angle, and the quotient will be the lead of the spiral.

When the angle of spiral and the lead or pitch of spiral are given, to find the circumference:

Multiply the tangent of angle by the lead, and the product will be the circumference.

When applying calculations to spiral gears the angle is reckoned at the pitch circumference and not at the outer or addendum circle.

It will be seen that when two spirals of different diameters have the same lead the spiral of less diameter will have the smaller angle. Thus in Fig. 60 if the paper triangle had been 4″ long instead of 6″ the diameter of the cylinder would have been 1.27″ and the angle of the spiral would have been only 63½ degrees.

CHAPTER V.

EXAMPLES IN CALCULATION OF THE LEAD OF SPIRAL—ANGLE OF SPIRAL—CIRCUMFERENCE OF SPIRAL GEARS—A FEW HINTS ON CUTTING.

It will be seen that the rules for calculating the circumference of spiral gears, angle and the lead of spiral are the same as in Chapter I., for the tangent and angle of a right angle triangle. In Chapter IV., the word "circumference" is substituted for "side opposite," and the words "lead of spiral" are substituted for "side adjacent."

Angles of Spirals with reference to Angle of Shafts.

When two spiral gears are in mesh the angle of spiral should be the same in one gear as in the other, in order to have the shafts parallel and the teeth work properly together. When two gears both have right hand spiral teeth, or both have left hand spiral teeth, the angle of their shafts will be equal to the *sum* of the angles of their spirals. But when two gears have different hand spirals the angle of their shafts will be equal to the *difference* of their angles of spirals. Thus, in Fig. 59 the gears A and B both have right hand spirals. The angle of both spirals is 45°, their sum is 90°, or their axes are at right angles. But C has a left hand spiral of 45°. Hence, as the difference between angles of spirals of B and C is 0, their axes are parallel.

Lead in Spirals of different diameters.

If two 45° gears of the same diameter have the same number of teeth the lead of the spiral will be alike in both gears: if one gear has more teeth than the other the lead of spiral in the larger gear should be longer in the same ratio. Thus, if one of these gears has 50 teeth, and the other has 25 teeth, the lead of spiral in the 50 tooth gear should be twice as long as that of the 25 tooth gear. Of course, the diameter of pitch circle should be twice as large in the 50 tooth as in the 25 tooth gear.

In spirals where the angle is 45° the circumference is the same as the spiral lead, because the tangent of 45° is 1.

Variation in Circumference to suit a Spiral. Sometimes the circumference is varied to suit a pitch that can be cut on the machine and retain the angle required. This would apply to cutting rolls for making diamond-shaped impressions where the diameter of the roll is not a matter of importance.

When two gears are to run together in a given velocity ratio, it is well first to select spirals that the machine will cut of the same ratio, and calculate the numbers of teeth and angle to correspond. This will often save considerable time in figuring.

The calculations for spiral gears present no special difficulties, but sometimes a little ingenuity is required to make work conform to the machine and to such cutters as we may have in stock.

Let it be required to make two spiral gears to run with a ratio of 4 to 1, the distance between centres to be 3.125″ ($3\frac{1}{8}$″), the axes to be parallel.

By rule given in Chapter XII., Part I., we find the diameters of pitch circles will be 5″ and $1\frac{1}{4}$″. Let us take a spiral of 48″ lead for the large gear, and a spiral of 12″ lead for the small gear. The circumference of the 5″ pitch circle is 15.70796″. Dividing the circumference by the lead of the spiral, we have $\frac{15.70796}{48} = .32724$″ for tangent of angle of spiral. In the table the nearest angle to tangent, .32724″, is 18° 10′.

As before stated, the angle of the teeth in the small gear will be the same as the angle of teeth or spiral in the large gear.

A difference in Angles at top and bottom of Spiral Grooves. Now, this rule gives the angle at the pitch surface only. Upon looking at a small screw of coarse pitch, it will be seen that the angle at bottom of the thread is not so great as the angle at top of thread; that is, the thread at bottom is nearer parallel to the centre line than that at the top.

This will be seen in Fig. 61, where A O is the centre line; *b f* shows direction of bottom of thread, and *d g* shows direction of top of thread. The angle A f b is less than the angle A g d. The difference of angle being due to the warped nature of a screw thread.

A cylinder 2″ diameter is to have spiral grooves 20° with the centre line of cylinder; what will be the lead of spiral? The circumference is 6.2832″. The tangent of 20° is .36397. Dividing the circumference by the tangent of angle, we obtain $\frac{6.2832}{.36397} = 17.26''+$ for lead of spiral. Example in calculation of Lead of Spiral.

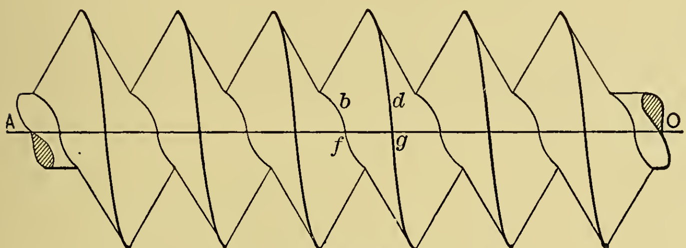


Fig. 61.

In Chapter XI, part I, it is stated that, when gashing the teeth of a worm-wheel, the angle of the teeth across the face is measured from the line parallel to the axis of the wheel.

To obtain this angle from the worm, divide the lead by the pitch circumference of the worm, and the quotient will be the tangent of the angle that the thread makes with a plane perpendicular to the axis.

CHAPTER VI.

NORMAL PITCH OF SPIRAL GEARS—CURVATURE OF PITCH SURFACE—FORM OF CUTTERS.

Normal to a Curve.

A Normal to a curve is a line perpendicular to the tangent at the point of tangency.

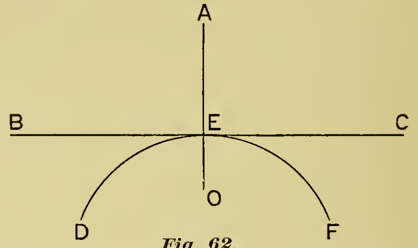


Fig. 62.

In Fig. 62, the line B C is tangent to the arc D E F, and the line A E O, being perpendicular to the tangent at E the point of tangency, is a *normal* to the arc.

Fig. 63 is a representation of the pitch surface of a spiral gear. A′ D′ C′ is the circular pitch, as in Part I. A D C is the same circular pitch seen upon the periphery of a wheel. Let A D be a tooth D C and a space. Now, to cut this space D C, the path of cutting is along the dotted line *a b*. By mere inspection, we can see that the *shortest* distance between two teeth along the pitch surface is not the distance A D C.

Let the line A E B be perpendicular to the sides of teeth upon the pitch surface. A continuation of this line, perpendicular to all the teeth, is called the *Normal Helix*. The line A E B, reaching over a tooth and a space along the normal helix, is called the *Normal Pitch, or the normal linear pitch.*

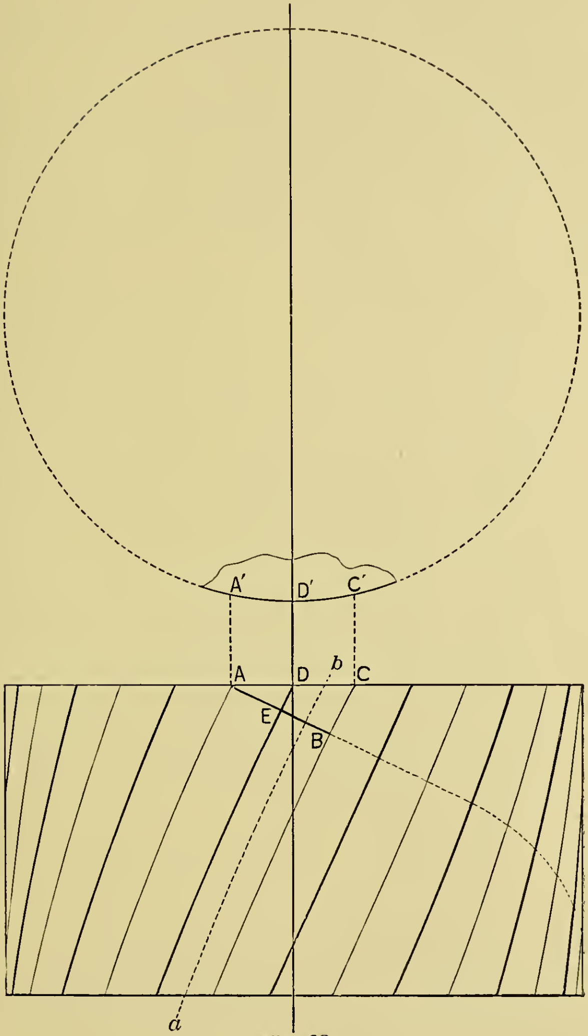


Fig. 63.

Normal Pitch.

The *Normal Pitch* of a spiral gear is then: *The shortest distance between the centers of two consecutive teeth measured along the pitch surface.*

In spur gears the normal pitch and circular pitch are alike. In the rack D D, Fig. 58, the linear pitch and normal pitch are alike.

Cutter for Spiral Gears.

From the foregoing it will be seen that, if we should cut the space D C with a cutter, the thickness of which at the pitch line is equal to one-half the *circular* pitch, as in spur wheels, the space would be too wide, and the teeth would be too thin. Hence, spiral gears should be cut with thinner cutters than spur gears of the same circular pitch.

The angle C A B is equal to the angle of the spiral. The line A E B corresponds to the *cosine* of the angle C A B. Hence the rule: *Multiply the cosine of angle of spiral by the circular pitch, and the product will be the normal pitch.* One-half the normal pitch is the proper thickness of cutter at the pitch line.

To find Normal Pitch.

If the normal pitch and the angle are known, *Divide the normal pitch by the cosine of the angle and the quotient will be the circular pitch.*

This may be required in a case of a spiral pinion running in a rack. The perpendicular to the side of the rack is taken as the line from which to calculate angle of teeth. That is, this line would correspond to the axial line in a spiral gear; and, when the axis of the gear is at right angles to the rack, the angle of the teeth with the side of the rack is obtained by subtracting this angle from 90°.

The angle of the rack teeth with the side of the rack can also be obtained by remembering that the cosine of the angle of spiral is the sine of the angle of the teeth with the side of the rack.

The addendum and working depth of tooth should correspond to the *normal pitch,* and not to the circular pitch. Thus, if the normal pitch is 12 diametral, the addendum should be $\frac{1}{12}''$, the thickness .1309″, and so on. The diameter of pitch circle of a spiral gear is calculated from the *diametral* pitch. Thus a gear of 30 teeth 10 P would be 3″ pitch diameter.

But if the normal pitch is 12 diametral pitch, the blank will be $3\frac{2}{12}''$ diameter instead of $3\frac{2}{10}''$.

It is evident that the normal pitch varies with the angle of spiral. The cutter should be for the normal pitch. In designing spiral gears, it is well first to look over list of cutters on hand, and see whether there are cutters to which the gears can be made to conform. This may avoid the necessity of getting a new cutter, or of changing both drawing and gears after they are under way. To do this, the problem is worked the reverse of the foregoing; that is: Normal Pitch varies.

First calculate to the next finer pitch cutter than would be required for the diametral pitch. To make Angle of Spiral conform to Cutters given.

Let us take, for example, two gears 10 pitch and 30 teeth, spiral and axes parallel. Let the next finer cutter be for 12 pitch gears. The first thing is to find the angle that will make the normal pitch .2618″, when the circular pitch is .3142″. See table of tooth parts. This means (Fig. 63) that the line A D C will be .3142″ when A E B is .2618″. Dividing .2618″ by .3142″ (see Chap. IV.), we obtain the cosine of the angle C A B, which is also the angle of the spiral, $\frac{.2618}{.3142}=.833$.

The same quotient comes by dividing 10 by 12, $\frac{10}{12}=.833+$; that is, divide one pitch by the other, the larger number being the divisor. Looking in the table, we find the angle corresponding to the cosine .833 is 33° 30′. We now want to find the pitch of spiral that will give angle of $33\frac{1}{2}°$ on the pitch surface of the wheel, 3″ diameter. Dividing the circumference by the tangent of angle, we obtain the pitch of spiral (see Chap. V.) The circumference is 9.4248″. The tangent of 33° 30′ is .66188, $\frac{9.4248}{.66188}=14.23$; and we have for our spiral 14.23″ lead.

When the machine is not arranged for the exact pitch of spiral wanted, it is generally well enough to take the next nearest spiral. A half of an inch more or less in a spiral 10″ pitch or more would hardly be noticed in angle of teeth. It is generally better to take the next longer spiral and cut enough deeper to bring center distances right. When two gears of the same size are in mesh with their axes parallel, a change When exact Pitch cannot be cut.

of angle of teeth or spiral makes no difference in the correct meshing of the teeth.

Spiral Gears of Different Sizes of Mesh. But when gears of different size are in mesh, due regard must be had to the spirals being in pitch, proportional to their angular velocities (see Chapter V.)

We come now to the curvature of cutters for spiral gears; that is, their shape as to whether a cutter is made to cut 12 teeth or 100 teeth. A cutter that is right,

Shape of Cutter. to cut a spur gear 3″ diameter, may not be right for a spiral gear 3″ diameter. To find the curvature of cutter, fit a templet to the blank along the line of the normal helix, as A E B, letting the templet reach over about one normal pitch. The curvature of this templet will be nearer a straight line than an arc of the addendum circle. Now find the diameter of a circle that will approximately fit this templet, and consider this circle as the addendum circle of a gear for which we are to select a cutter, reckoning the gear as of a pitch the same as the normal pitch.

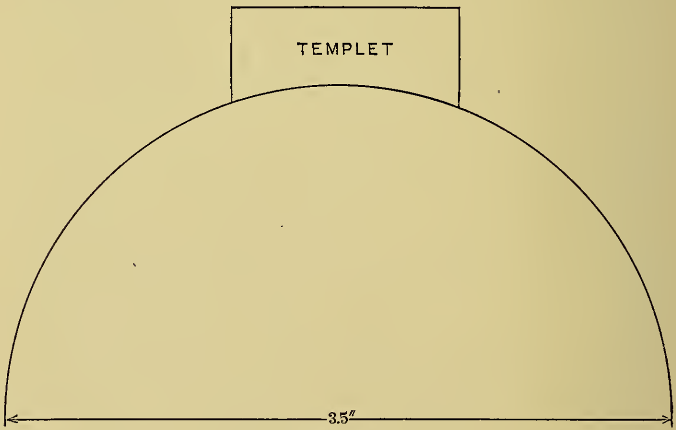


Fig. 64.

Thus, in Fig. 64, suppose the templet fits a circle $3\frac{1}{2}''$ diameter, if the normal pitch is 12 to inch, diametral, the cutter required is for 12 P and 40 teeth. The curvature of the templet will not be quite circular, but is sufficiently near for practical purposes. Strictly, a flat templet cannot be made to coincide with the normal helix for any distance whatever, but any greater refinement than we have suggested can hardly be carried out in a workshop.

This applies more to an end cutter, for a disk cutter may have the right shape for a tooth space and still round off the teeth too much on account of the warped nature of the teeth.

The difference between normal pitch and linear or circular pitch is plainly seen in Figs 58 and 59.

The rack D D, Fig. 58, is of regular form, the depth of teeth being $\frac{11}{16}$ of the circular pitch, nearly (.6866 of the pitch, accurately). If a section of a tooth in either of the gears be made square across the tooth, that is a normal section, the depth of the tooth will have the same relation to the thickness of the tooth as in the rack just named.

But the teeth of spiral gears, looking at them upon the side of the gears, are thicker in proportion to their depth, as in Fig. 59. This difference is seen between the teeth of the two racks D D and E E, Fig. 58. In the rack D D we have 20 teeth, while in the rack E E we have but 14 teeth; yet each rack will run with each of the spiral gears A, B or C, Fig. 59, but at different angles.

The teeth of one rack will accurately fit the teeth of the other rack face to face, but the sides of one rack will then be at an angle of 45° with the sides of the other rack. At F is a guide for holding a rack in mesh with a gear.

The reason the racks will each run with either of the three gears is because all the gears and racks have the same normal pitch. When the spiral gears are to run together they must both have the same normal pitch. Hence, two spiral gears may run correctly together though the circular pitch of one gear is not like the circular pitch of the other gear.

If a rack is to run at any angle other than 90° with the axis of the gear it is well to determine the data from a diagram, as it is very difficult to figure the angles and sizes of the teeth without a sketch or diagram.

CHAPTER VII.

CUTTING SPIRAL GEARS IN A UNIVERSAL MILLING MACHINE.

A rotary disk cutter is generally preferable to a shank cutter or end mill on account of cutting faster and holding its shape longer. In cutting spiral grooves, it is sometimes necessary to use an end mill on account of the warped character of the grooves, but it is very seldom necessary to use an end mill in cutting spiral gears.

Proving the Setting of the Machine.

Before cutting into a blank it is well to make a slight trace of the spiral with the cutter, after the change gears are in place, to see whether the gears are correct. If the material of the gear blanks is quite expensive, it is a safe plan to make trial blanks of cast iron in order to prove the setting of the machine, before cutting into the expensive material.

The cutting of spiral gears may develop some curious facts to one that has not studied warped surfaces. The gears, Fig. 59, were cut with a planing tool in a shaper, the spiral gear mechanism of a Universal Milling Machine having been fastened upon the shaper. The tool was of the same form as the spaces in the rack D D, Fig. 58. All spiral gears of the same pitch can be cut in this manner with one tool. The nature of this cutting operation can be understood from a consideration of the meshing of straight side rack teeth with a spiral gear, as in Fig. 58. Spiral gears that run correctly with a rack, as in Fig. 58, will run correctly with each other when their axes are parallel, as at B C, Fig. 59; but it is not considered that they are quite correct, theoretically, to run together when the gears have the same hand spiral, and their axes are at right

Fig. 65

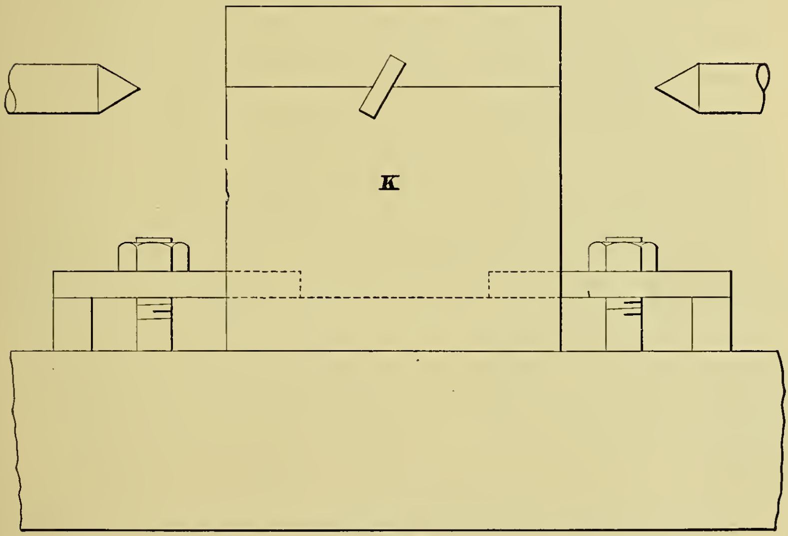


Fig. 66

angles, as A B, Fig. 59, though they will run well enough practically. The operation of cutting spiral teeth with a planer tool is sometimes called *planing* the teeth. Planing is an accurate way of shaping teeth that are to mesh with rack teeth and for gears on parallel shafts; this method has been employed to cut spiral pinions that drive planer tables, but has not been found available for general use.

It is convenient to have the data of spiral gears arranged as in the following table: Data.

	Gear.	Pinion.
No. of Teeth		
Pitch Diameter		
Outside Diameter		
Circular Pitch		
Angle of Teeth with Axis . .		
Normal Circular Pitch . . .		
Pitch of Cutter		
Addendum s		
Thickness of Tooth t . . .		
Whole Depth D″+f		
No. of Cutter		
Exact Lead of Spiral . . .		
Approximate Lead of Spiral . .		
Gears on Milling Machine to Cut Spiral		
Gear on Worm		
1st Gear on Stud		
2nd Gear on Stud		
Gear on Screw		

A spiral of any angle to 45° can generally be cut in a Universal Milling Machine without special attachments, the cutter being at the top of the work. The cutter is placed on the arbor in such position that it can reach the work centrally after the table is set to the angle of the spiral. In order to cut central, it is generally well enough to place the table, before setting it to the angle, so that the work centres will be central with the cutter, then swing the table and set it to the angle of the spiral.

For very accurate work, it is safer to test the position of the centres after the table has been set to the angle. Central Setting.

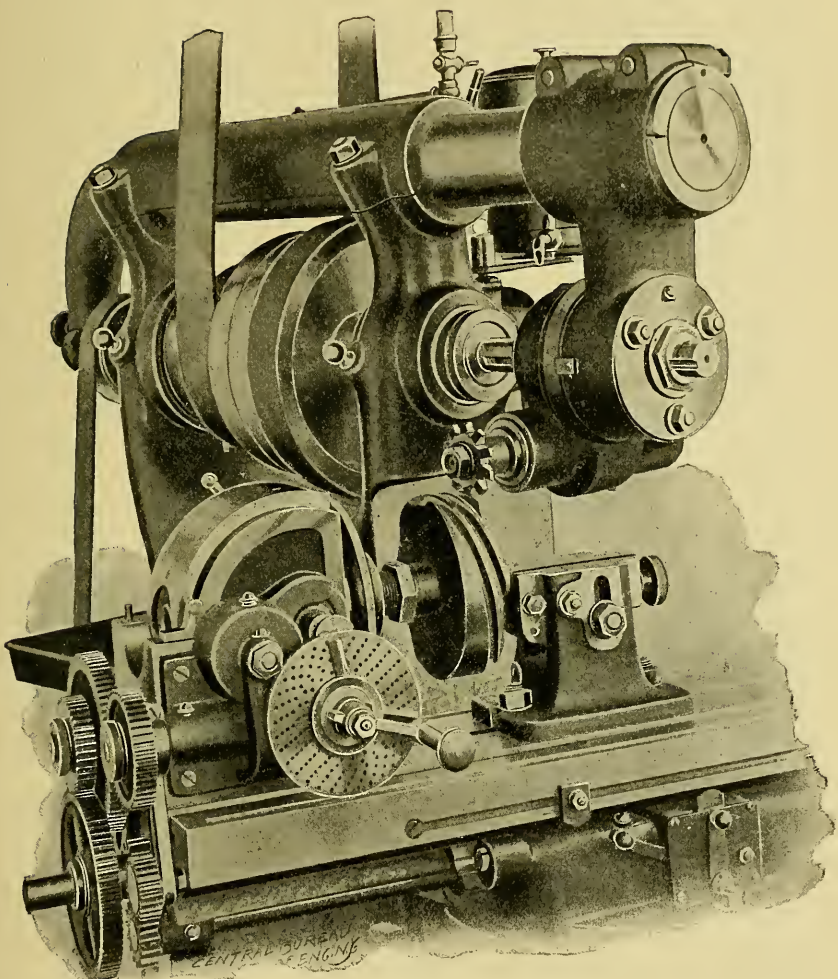


Fig. 67.

USE OF VERTICAL SPINDLE MILLING ATTACHMENT
IN CUTTING SPIRAL GEARS.

This can be done with a trial piece, Fig. 65, which is simply a round arbor with centre holes in the ends. It is mounted between the centres, and the knee is raised until the cutter sinks a small gash, as at A. This gash shows the position of the cutter; and if the gash is central with the trial piece, the cutter will be central with the work. If preferred, the arbor can be dogged to the work spindle; and the line B C drawn on the side of the arbor at the same height as the centres; the work spindle should then be turned quarter way round in order to bring the line at the top. The gash A can now be cut and its position determined with the line.

In cutting small gears the arbor can be dogged to the work spindle; the distance between the gear blank and the dog should be enough to let the dog pass the cutter arbor without striking.

A spiral gear is much more likely to slip in cutting than a spur gear.

For gears more than three or four inches in diameter it is well to have a taper shank arbor held directly in the work spindle, as shown in Figs. 67 and 68; and for the heaviest work, the arbor can be drawn into the spindle with a screw in a threaded hole in the end of the shank.

After cutting a space the work can be dropped away from the cutter, in order to avoid scratching it when coming back for another cut. Some workmen prefer not to drop the work away, but to stop the cutter and turn it to a position in which its teeth will not touch the work. To make sure of finding a place in the cutter that will not scratch, a tooth has sometimes been taken out of the cutter, but this is not recommended. The safest plan is to drop the work away.

Angle greater than 45°

In cutting spiral gears of greater angle than 45°, a vertical spindle milling attachment is available, as shown in Figs. 67 and 68.

In Fig. 67 the cutter is at 90° with the work spindle when the table is set to 0, so that the proper angle at which the table should be set, is the *difference* between the angle of the spiral and 90°. Thus, to cut a 70°

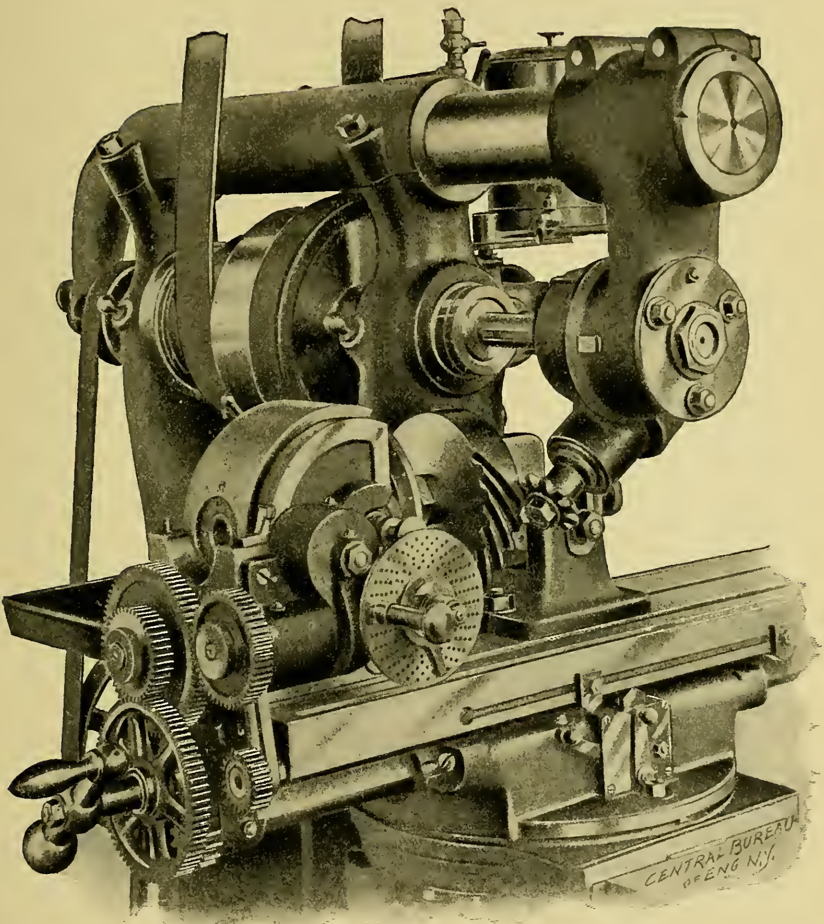


Fig. 68.

USE OF VERTICAL SPINDLE MILLING ATTACHMENT
IN CUTTING SPIRAL GEARS.

spiral, we subtract 70° from 90°, and the remainder, 20°, is the angle to set the table. In cutting on the top, Fig. 67, the attachment is set to 0.

In Fig. 68 the cutter is at the side of the work; the table is set to 0, and the attachment is set to the *difference* between 90° and the required angle of spiral.

In setting the cutter central it is convenient to have a small knee as at K, Fig. 66. A line is drawn upon the knee at the same height as at the centres. The cutter arbor is brought to the angle as just shown, and a gash is cut in the knee. When the gash is central with the line, the cutter will be central with the work.

The cutter can be set to act upon either side of the gear to be cut, according as a right hand or a left hand spiral is wanted. The setting in Fig. 68 is for a right hand spiral.

If the gear blank were brought in front of the cutter, and the reversing gear set between two change gears, the machine would be set for a left hand spiral.

For coarser pitches than about 12 P diametral, it is well to cut more than once around, the finishing cut being quite light so as to cut smooth.

CHAPTER VIII.

SCREW GEARS AND SPIRAL GEARS—GENERAL REMARKS.

The working of spiral gears, when their axes are parallel, is generally smoother than spur gears. A tooth does not strike along its whole face or length at once. Tooth contact first takes place at one side of the gear, passes across the face and ceases at the other side of the gear. This action tends to cover defects in shape of teeth and the adjustment of centres. Working of Spiral Gears.

Since the invention of machines for producing accurate epicyloidal and involute curves, it has not so often been found necessary to resort to spiral gears for smoothness of action. A greater range can be had in the adjustment of centers in spiral gears than in spur gears. The angle of the teeth should be enough, so that one pair of teeth will not part contact at one side of the gears until the next pair of teeth have met on the other side of the gears. When this is done the gears will be in mesh so long as the circumferences of their addendum circles intersect each other. This is sometimes necessary in gears for rolls.

Relative to spur and bevel gears in Part I., Chapter XII., it was stated that all gears finally wore themselves out of shape and might become noisy. Spiral gears may be worn out of shape, but the smoothness of action can hardly be impaired so long as there are any teeth left. For every quantity of wear, of course, there will be an equal quantity of backlash, so that if gears have to be reversed the lost motion in spiral gears will be as much as in any gears, and may be more if there is end play of the shafts. In spiral gears there is end pressure upon the shafts, because of the screw-like action of the teeth. This end pressure is sometimes balanced by putting two gears upon each shaft, one of right and one of left hand spiral. End Pressure upon Shafts of Spiral Gears.

The same result is obtained in solid cast gears by making the pattern in two parts—one right and one left-hand spiral. Such gears are colloquially called "herring-bone gears."

In an internal spiral gear and its pinion, the spirals of both wheels are either right-handed or left-handed. Such a combination would hardly be a mercantile product, although interesting as a mechanical feat.

In screw or worm-gears the axes are generally at right angles, or nearly so. The distinctive features of screw gearing may be stated as follows:

Distinctive features of Screw Gearing.

The relative angular velocities do not depend upon the diameters of pitch-cylinders, as in Chapter I., Part I. Thus the worm in Chapter XI., Fig. 35, can be any diameter—one inch or ten inches—without affecting the velocity of the worm-wheel. Conversely if the axes are not parallel we can have a pair of spiral or screw gears of the same diameter, but of different numbers of teeth. The direction in which a worm-wheel turns depends upon whether the worm has a right-hand or left-hand thread. When angles of axes of worm and worm-wheel are oblique, there is a practical limit to the directional relation of the worm-wheel. The rotation of the worm-wheel is made by the end movement of the worm-thread.

The term worm and worm-wheel, or worm-gearing, is applied to cases where the worms are cut in a lathe, and the shapes of the threads or teeth, in axial section, are like a rack and the pitch is measured on a line parallel to the axis. The shape usually selected is like the rack for a single curve or involute gear. See Chap. IV, Part I. Worms are sometimes cut in a milling machine.

If the form of the teeth in a pair of screw gears is determined upon the normal helix, as in Chap. VI., the gears are usually called Spiral Gears.

If we let two cylinders touch each other, their axes being at right angles, the rotation of one cylinder will have no tendency to turn the other cylinder, as in Chapter I., Part I.

We can now see why worms and worm-wheels wear out faster than other gearing. The length of worm-thread, equal to more than the entire circumference of worm, comes in sliding contact with each tooth of the wheel during one turn of the wheel. Why Worm Wheels wear so fast.

The angle of a worm-thread can be calculated the same as the angle of teeth of spiral gear; only, the angle of a worm thread is measured from a line or plane that is perpendicular to the axis of the worm.

When a multiple threaded worm is cut in a milling machine and the angle of the thread is less than 75° with the axis of the worm, it may be desirable to work by the normal pitch. *The normal pitch can be obtained by multiplying the thread-pitch by the sine of the angle of the thread with the axis.*

CHAPTER IX.

CONTINUED FRACTIONS—SOME APPLICATIONS IN MACHINE CONSTRUCTION.

Definition of a Continued Fraction.

A continued fraction is one that has unity for its numerator, and for its denominator an entire number plus a fraction, which fraction has also unity for its numerator, and for its denominator an entire number plus a fraction, and thus in order.

The expression, $\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{5}}}$ is called a continued fraction. By the use of continued fractions, we are enabled to find a fraction expressed in smaller numbers, that, for practical purposes, may be sufficiently near in value to another fraction expressed in large numbers. If we were required to cut a worm that would mesh with a gear 4 diametral pitch (4 P.), in a lathe having 3 to 1-inch linear leading screw, we might, without continued fractions, have trouble in finding change gears, because the circular pitch corresponding to 4 diametral pitch is expressed in large numbers: $4\ P = \frac{7854}{10000}\ P'$.

Practical use of Continued Fractions.

This example will be considered farther on. For illustration, we will take a simpler example.

What fraction expressed in smaller numbers is nearest in value to $\frac{29}{146}$? Dividing the numerator and the denominator of a fraction by the same number does not change the value of the fraction. Dividing both terms of $\frac{29}{146}$ by 29, we have $\frac{1}{5\frac{1}{29}}$, or, what is the same thing expressed as a continued fraction, $\cfrac{1}{5+\cfrac{1}{29}}$. The continued fraction $\cfrac{1}{5+\cfrac{1}{29}}$ is exactly equal to $\frac{29}{146}$. If now, we reject the $\frac{1}{29}$, the fraction $\frac{1}{5}$ will be larger than $\cfrac{1}{5+\cfrac{1}{29}}$, because the denominator has been diminished, 5 being less than $5\frac{1}{29}$. $\frac{1}{5}$ is something near $\frac{29}{146}$ expressed in smaller numbers than 29 for a

Example in Continued Fractions.

numerator and 146 for a denominator. Reducing $\frac{1}{5}$ and $\frac{29}{146}$ to a common denominator, we have $\frac{1}{5}=\frac{146}{730}$ and $\frac{29}{146}=\frac{145}{730}$. Subtracting one from the other, we have $\frac{1}{730}$, which is the difference between $\frac{1}{5}$ and $\frac{29}{146}$. Thus, in thinking of $\frac{29}{146}$ as $\frac{1}{5}$, we have a pretty fair idea of its value.

There are fourteen fractions with terms smaller than 29 and 146, which are nearer $\frac{29}{146}$ than $\frac{1}{5}$ is, such as $\frac{15}{76}$, $\frac{16}{81}$ and so on to $\frac{28}{141}$. In this case by continued fractions we obtain only one approximation, namely $\frac{1}{5}$, and any other approximations, as $\frac{15}{76}$, $\frac{16}{81}$, &c., we find by trial. It will be noted that all these approximations are smaller in value than $\frac{29}{146}$. There are cases, however, in which we can, by continued fractions, obtain approximations both greater and less than the required fraction, and these will be the nearest possible approximations that there can be in smaller terms than the given fraction.

In the French metric system, a millimetre is equal to .03937 inch; what fraction in smaller terms expresses .03937″ nearly? .03937, in a vulgar fraction, is $\frac{3937}{100000}$. Dividing both numerator and denominator by 3937, we have $\frac{1}{25\frac{1575}{3937}}$. Rejecting from the denominator of the new fraction, $\frac{1575}{3937}$, the fraction $\frac{1}{25}$ gives us a pretty good idea of the value of .03937″. If in the expression, $\frac{1}{25+\frac{1575}{3937}}$, we divide both terms of the fraction $\frac{1575}{3937}$ by 1575, the value will not be changed. Performing the division, we have

$$\cfrac{1}{25+\cfrac{1}{2+\cfrac{787}{1575}}}.$$

We can now divide both terms of $\frac{787}{1575}$ by 787, without changing its value, and then substitute the new fraction for $\frac{787}{1575}$ in the continued fraction.

Dividing again, and substituting, we have:

$$\cfrac{1}{25+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{787}}}}$$

as the continued fraction that is exactly equal to .03937.

In performing the divisions, the work stands thus:

```
3937) 100000 (25
       7874
       -----
       21260
       19685
       -----
        1575) 3937 (2
              3150
              ----
               787) 1575 (2
                    1574
                    ----
                       1) 787 (787
                          787
                          ---
                           0
```

That is, dividing the last divisor by the last remainder, as in finding the greatest common divisor. The quotients become the denominators of the continued fraction, with unity for numerators. The denominators 25, 2, and so on, are called incomplete quotients, since they are only the entire parts of each quotient. The first expression in the continued fraction is $\frac{1}{25}$ or .04—a little larger than .03937. If, now, we take $\frac{1}{25+\frac{1}{2}}$, we shall come still nearer .03937. The expression $\frac{1}{25+\frac{1}{2}}$ is merely stating that 1 is to be divided by $25\frac{1}{2}$. To divide, we first reduce $25\frac{1}{2}$ to an improper fraction, $\frac{51}{2}$, and the expression becomes $\frac{1}{\frac{51}{2}}$, or one divided by $\frac{51}{2}$. To divide by a fraction, "Invert the divisor, and proceed as in multiplication." We then have $\frac{2}{51}$ as the next nearest fraction to .03937. $\frac{2}{51}=.0392+$, which is smaller than .03937. To get still nearer, we take in the next part of the continued fraction, and have

$$\cfrac{1}{25+\cfrac{1}{2+\cfrac{1}{2}}}.$$

We can bring the value of this expression into a fraction, with only one number for its numerator and one number for its denominator, by performing the operations indicated, step by step, commencing at the last part of the continued fraction. Thus, $2+\frac{1}{2}$, or $2\frac{1}{2}$, is equal to $\frac{5}{2}$, Stopping here, the continued fraction would become

$$\cfrac{1}{25+\cfrac{1}{\frac{5}{2}}}.$$

Now, $\frac{1}{\frac{5}{2}}$ equals $\frac{2}{5}$, and we have $\frac{1}{25+\frac{2}{5}}$. $25\frac{2}{5}$ equals $\frac{127}{5}$; substituting again, we have $\frac{1}{\frac{127}{5}}$. Dividing 1 by $\frac{127}{5}$, we have $\frac{5}{127}$. $\frac{5}{127}$ is the nearest fraction to

.03937, unless we reduce the whole continued fraction

$$\cfrac{1}{25+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{787}}}}$$ which would give us back the .03937 itself.

$\frac{5}{127}=.03937007$, which is only $\frac{7}{100000000}$ larger .03937. It is not often that an approximation will come so near as this.

Practical use of the foregoing Example.

This ratio, 5 to 127, is used in cutting millimeter thread screws. If the leading screw of the lathe is 1 to one inch, the change gears will have the ratio of 5 to 127; if 8 to one inch, the ratio will be 8 times as large, or 40 to 127; so that with leading screw 8 to inch, and change gears 40 and 127, we can cut millimeter threads near enough for practical purposes.

The foregoing operations are more tedious in description than in use. The steps have been carefully noted, so that the reason for each step can be seen from rules of common arithmetic, the operations being merely reducing complex fractions. The reductions, $\frac{1}{25}$, $\frac{2}{51}$, $\frac{5}{127}$, etc., are called *convergents*, because they come nearer and nearer to the required .03937. The operations can be shortened as follows:

Example.

Let us find the fractions converging towards .7854″, the circular pitch of 4 diametral pitch, $.7854=\frac{7854}{10000}$; reducing to lowest terms, we have $\frac{3927}{5000}$. Applying the operation for the greatest common divisor:

```
3927) 5000 (1
      3927
      ----
      1073) 3927 (3
            3219
            ----
             708) 1073 (1
                   708
                   ---
                   365) 708 (1
                        365
                        ---
                        343) 365 (1
                             343
                             ---
                              22) 343 (15
                                  22
                                  ---
                                  123
                                  110
                                  ---
                                   13) 22 (1
                                       13
                                       --
                                        9) 13 (1
                                            9
                                           --
                                            4) 9 (2
                                               8
                                               -
                                               1) 4 (4
                                                  4
                                                  -
                                                  0
```

Bringing the various incomplete quotients as denominators in a continued fraction as before, we have:

$$\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{15+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\frac{1}{4}}}}}}}}}$$

Now arrange each partial quotient in a line, thus:

1	3	1	1	1	15	1	1	2	4
1	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{7}{9}$	$\frac{11}{14}$	$\frac{172}{219}$	$\frac{183}{233}$	$\frac{355}{452}$	$\frac{893}{1137}$	$\frac{3927}{5000}$

Now place under the first incomplete quotient the first reduction or convergent $\frac{1}{1}$, which, of course, is 1; put under the next partial quotient the next reduction or convergent $\frac{1}{1+\frac{1}{3}}$ or $\frac{1}{1\frac{1}{3}}$, which becomes $\frac{3}{4}$.

1 is larger than .7854, and $\frac{3}{4}$ is less than .7854.

Having made two reductions, as previously shown, we can shorten the operations by the following rule for next convergents: *Multiply the numerator of the convergent just found by the denominator of the next term of the continued fraction, or the next incomplete quotient, and add to the product the numerator of the preceding convergent; the sum will be the numerator of the next convergent.*

Proceed in the same way for the denominator, that is multiply the denominator of the convergent just found by the next incomplete quotient and add to the product the denominator of the preceding convergent; the sum will be the denominator of the next convergent. Continue until the last convergent is the original fraction. Under each incomplete quotient or denominator from the continued fraction arranged in line, will be seen the corresponding convergent or reduction. The convergent $\frac{11}{14}$ is the one commonly used in cutting racks 4 P. This is the same as calling the circumference of a circle 22-7 when the diameter is one (1); this is also the common ratio for cutting any rack. The equivalent decimal to $\frac{11}{14}$ is .7857 ×, being about $\frac{3}{10000}$ large. In three settings for rack teeth, this error would amount to about .001″

For a worm, this corresponds to $\frac{14}{11}$ threads to 1″; now, with a leading screw of lathe 3 to 1″, we would want gears on the spindle and screw in a ratio of 33 to 14.

Hence, a gear on the spindle with 66 teeth, and a gear on the 3 thread screw of 28 teeth, would enable us to cut a worm to fit a 4 P gear.

CHAPTER X.

ANGLE OF PRESSURE.

In Fig. 69, let A be any flat disk lying upon a horizontal plane. Take any piece, B, with a square end, *a b*. Press against A with the piece B in the direction of the arrow.

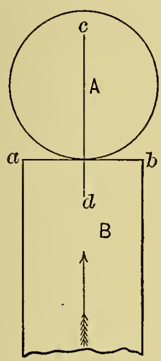


Fig. 69.

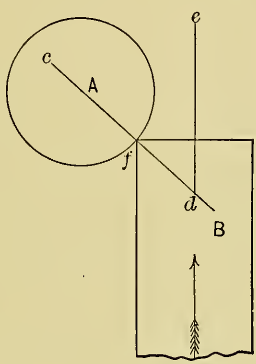


Fig. 70.

It is evident A will tend to move directly ahead of B in the normal line *c d*. Now (Fig. 70) let the piece B, at one corner *f*, touch the piece A. Move the piece B along the line *d e*, in the direction of the arrow.

It is evident that A will not now tend to move in the line *d e*, but will tend to move in the direction of the normal *c d*. When one piece, not attached, presses against another, the tendency to move the second piece is in the direction of the normal, at the point of contact. This normal is called the *line of pressure*. The angle that this line makes with the path of the impelling piece, is called the *angle of pressure*.

Line of Pressure.

In Part I., Chapter IV., the lines B A and B A′ are called lines of pressure. This means that if the gear drives the rack, the *tendency* to move the rack is not in the direction of pitch line of rack, but either in the direction B A or B A′, as we turn the wheel to the left or to the right.

The same law holds if the rack is moved in the direction of the pitch line; the *tendency* to move the wheel is not directly tangent to the pitch circle, as if driven by a belt, but in the direction of the line of pressure. Of course the rack and wheel *do* move in the paths prescribed by their connections with the framework, the wheel turning about its axis and the rack moving along its ways. This pressure, not in a direct path of the moving piece, causes extra friction in all toothed gearing that cannot well be avoided.

Although this pressure works out by the diagram, as we have shown, yet, in the actual gears, it is not at all certain that they will follow the law as stated, because of the friction of teeth among themselves. If the driver in a train of gears has no bearing upon its tooth-flank, we apprehend there will be but little tendency to press the shafts apart.

Arc of Action.

The arc through which a wheel passes while one of its teeth is in contact is called the *arc of action*.

Base of System of Interchangeable Gears.

Until within a few years, the base of a system of double-curve interchangeable gears was 12 teeth. It is now 15 teeth in the best practice (see Chapter VII., Part I.)

The reason for this change was: the base, 15 teeth, gives less angle of pressure and longer arc of contact, and hence longer lifetime to gears.

CHAPTER XI.

INTERNAL GEARS.

In Part I., Chapter VIII., it is stated that the space of an internal gear is the same as the tooth of a spur gear. This applies to involute or single-curve gears as well as to double-curve gears.

The sides of teeth in involute internal gears are hollowing. It, however, has been customary to cut internal gears with spur gear-cutters, a No. 1 cutter generally being used. This makes the teeth sides convex. Special cutters should be made for coarse pitch double-curve gears. In designing internal gears, it is sometimes necessary to depart from the system with 15-tooth base, so as to have the pinion differ from the wheel by less than 15 teeth. The rules given in Part I., Chapters VII. and VIII., will apply in making gears on any base besides 15 teeth. If the base is low-numbered and the pinion is small, it may be necessary to resort to the method given at the end of Chapter VII., because the teeth may be too much rounded at the points by following the approximate rules. The base must be as small as the *difference* between the internal gear and its pinion. The base can be smaller if desired.

Special Cutters for coarse Pitch.

Base for Internal Gear Teeth.

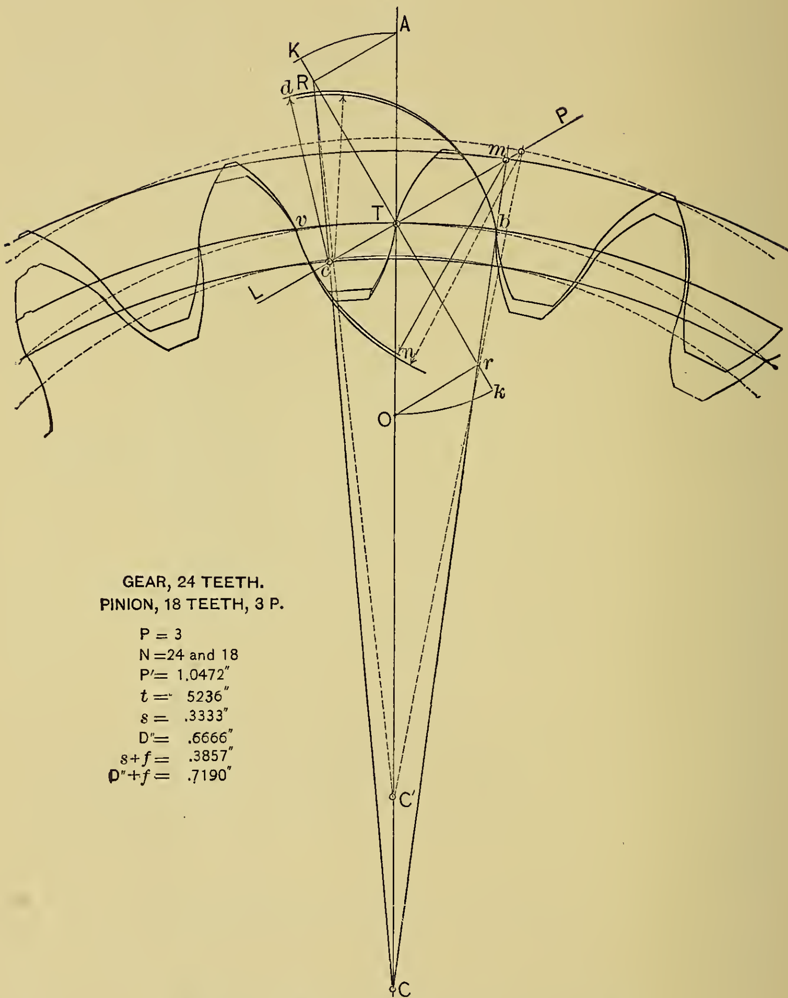
Let it be required to make an internal gear, and pinion 24 and 18 teeth, 3 P. Here the base cannot be more than 6 teeth.

In Fig. 71 the base is 6 teeth. The arcs A K and O *k*, drawn about T, have a radius equal to the radius of the pitch circle of a 6-tooth gear, 3 P, instead of a 15-tooth gear, as in Chapter VIII., Part I.

The *outline* of teeth of both gears and pinion is made similar to the gear in Chapter VIII. The same

Description of Fig. 67.

Fig. 71.



INTERNAL GEAR AND PINION IN MESH.

letters refer to similar parts. The clearance circle is, however, drawn on the *outside* for the internal gear. As before stated, the spaces of a spur wheel become the *teeth* of an internal wheel. The teeth of internal gears require but little for fillets at the roots; they are generally strong enough without fillets. The teeth of the pinion are also similar to the gear in Chapter VIII., substituting 6-tooth for 15-tooth base. To avoid confusion, it is well to make a complete sketch of one gear before making the other. The arc of action is longer in internal gears than in external gears. This property sometimes makes it necessary to give less fillets than in external gears.

In Fig. 71 the angle K T A is 30° instead of 12°, as in Fig. 12. This brings the line of pressure L P at an angle of 60° with the radius C T, instead of 78°. A system of spur gears could be made upon this 6-tooth base. These gears would interchange, but no gear of this 6-tooth system would mesh with a double-curve gear made upon the 15-tooth system in Part 1.

CHAPTER XII.

STRENGTH OF GEARING.

We have been unable to derive from our own experience, any definite rule on this subject but would refer those interested to "Kent's Mechanical Engineers' Pocket Book," where a good treatment of the subject can be found.

We give a few examples of average breaking strain of our Combination Gears, as determined by dynamometer, the pressure being measured at the pitch line. These gears are of cast iron, with cut teeth.

Diametral Pitch.	Face.	No. Teeth.	Revolutions per Minute.	Pressure at Pitch Line.
10	1 1-16	110	27	1060
8	1 1-4	72	40	1460
6	1 9-16	72	27	2220
5	1 7-8	90	18	2470

These are the actual pressures for the particular widths given.

If we take a safe pressure at 1-3 of the foregoing breaking strain, we shall have for

10 Pitch	353 1-3	Lbs. at the	Pitch Line.
8 "	486 2-3	"	"
6 "	740	"	"
5 "	823 1-3	"	"

The width of the face of a gear is in good proportion when it is $2\frac{1}{2}$ times the circular pitch.

TOOTH PARTS.

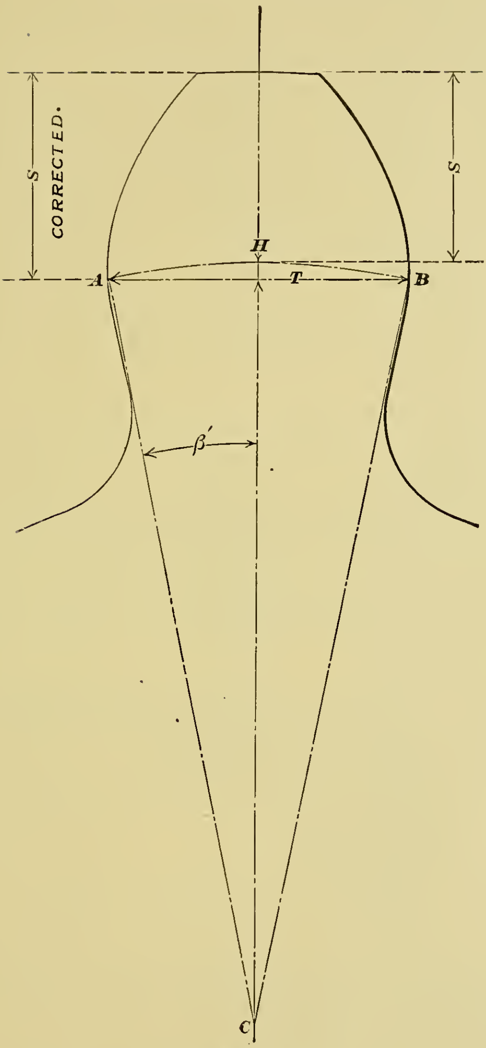


Fig. 73.

GEAR TOOTH 1 P

The dimensions of tooth parts as given in the tables, pages 144 to 147, are correct according to the definition of tooth parts, pages 4 and 16; but, as the pitch line of gears is curved, the thickness of a tooth will not be measured on the pitch line if the caliper is set to the figures given in the tables mentioned. To measure the teeth accurately on the pitch line, the caliper must be set to the chordal thickness and the depth setting to the pitch line must be to the corrected *s*, as explained and tabulated. If the gear blank is not of the correct diameter, the proper allowance must be made in setting the caliper for the depth. It is utterly useless to be guided by the outside of a gear blank when the outside diameter is not right. The measuring of the tooth thickness is well enough, as a check, but it is oftentimes as well first to make sure that the spaces are cut to the right depth.

Fig. 73 is a sketch of a gear tooth of 1 P. In measuring gear teeth of coarse pitch accurately the chordal thickness of the tooth, ATB, must be known, because it may be enough shorter than the regular tooth-thickness AHB, or t, to require attention. It may be also well to know the versed sine of the angle β', or the distance H, in order to tell where to measure the chordal thickness.

Chordal Thicknesses of Teeth of Gears, on a Basis of 1 Diametral Pitch.

N = Number of teeth in gears.
T = Chordal thickness of Tooth. $T = D' \sin. \beta'$
H = Height of Arc. $H = R\ (1 - \cos. \beta')$
D$'$ = Pitch Diameter.
R = Pitch Radius.
$\beta' = 90°$ divided by the number of teeth.

NOTE.—For any pitch not in the following tables to find corresponding part:—Multiply the tabular value for one inch by the circular pitch required, and the product will be the value for the pitch given.

Example: What is the value of s for 4 inch circular pitch? $.3183 = s$ for $1''$ P$'$ and $.3183 \times 4 = 1.2732 = s$ for $4''$ P$'$.

The expression "Addendum and $\frac{1''}{P}$" (addendum and the module) means the distance of a tooth outside of pitch line and also the distance occupied for every tooth upon the diameter of pitch circle.

CHORDAL THICKNESSES OF TEETH OF GEARS.

INVOLUTE.

Cutter.	T	H	Corrected S for Gear.
No. 1 —135 T — 1 P	1.5707	.0047	1.0047
No. 2 — 55 T — 1 P	1.5706	.0112	1.0112
No. 3 — 35 T — 1 P	1.5702	.0176	1.0176
No. 4 — 26 T — 1 P	1.5698	.0237	1.0237
No. 5 — 21 T — 1 P	1.5694	.0294	1.0294
No. 6 — 17 T — 1 P	1.5686	.0362	1.0362
No. 7 — 14 T — 1 P	1.5675	.0440	1.0440
No. 8 — 12 T — 1 P	1.5663	.0514	1.0514
11 T — 1 P	1.5654	.0559	1.0559
10 T — 1 P	1.5643	.0616	1.0616
9 T — 1 P	1.5628	.0684	1.0684
8 T — 1 P	1.5607	.0769	1.0769

EPICYCLOIDAL.

Cutter.	T	H	Corrected S for Gear.
A — 12 T — 1 P	1.5663	.0514	1.0514
B — 13 T — 1 P	1.5670	.0474	1.0474
C — 14 T — 1 P	1.5675	.0440	1.0440
D — 15 T — 1 P	1.5679	.0411	1.0411
E — 16 T — 1 P	1.5683	.0385	1.0385
F — 17 T — 1 P	1.5686	.0362	1.0362
G — 18 T — 1 P	1.5688	.0342	1.0342
H — 19 T — 1 P	1.5690	.0324	1.0324
I — 20 T — 1 P	1.5692	.0308	1.0308
J — 21 T — 1 P	1.5694	.0294	1.0294
K — 23 T — 1 P	1.5696	.0268	1.0268
L — 25 T — 1 P	1.5698	.0247	1.0247
M — 27 T — 1 P	1.5699	.0228	1.0228
N — 30 T — 1 P	1.5701	.0208	1.0208
O — 34 T — 1 P	1.5703	.0181	1.0181
P — 38 T — 1 P	1.5703	.0162	1.0162
Q — 43 T — 1 P	1.5705	.0143	1.0143
R — 50 T — 1 P	1.5705	.0123	1.0123
S — 60 T — 1 P	1.5706	.0102	1.0102
T — 75 T — 1 P	1.5707	.0083	1.0083
U —100 T — 1 P	1.5707	.0060	1.0060
V —150 T — 1 P	1.5707	.0045	1.0045
W—250 T — 1 P	1.5708	.0025	1.0025

SPECIAL.

No. Teeth.	T	H	Corrected S for Gear.
9 T — 1 P	1.5628	.0684	1.0684
10 T — 1 P	1.5643	.0616	1.0616
11 T — 1 P	1.5654	.0559	1.0559

DIAMETRAL PITCH.

"NUTTALL."

Diametral Pitch is the Number of Teeth to Each Inch of the Pitch Diameter.

To Get	Having	Rule.	Formula.
The Diametral Pitch.	The Circular Pitch.	Divide 3.1416 by the Circular Pitch	$P=\frac{3.1416}{P'}$
The Diametral Pitch.	The Pitch Diameter and the Number of Teeth	Divide Number of Teeth by Pitch Diameter	$P=\frac{N}{D'}$
The Diametral Pitch.	The Outside Diameter and the Number of Teeth	Divide Number of Teeth plus 2 by Outside Diameter	$P=\frac{N+2}{D}$
Pitch Diameter.	The Number of Teeth and the Diametral Pitch	Divide Number of Teeth by the Diametral Pitch	$D'=\frac{N}{P}$
Pitch Diameter.	The Number of Teeth and Outside Diameter	Divide the product of Outside Diameter and Number of Teeth by Number of Teeth plus 2 .	$D'=\frac{D\,N}{N+2}$
Pitch Diameter.	The Outside Diameter and the Diametral Pitch . . .	Subtract from the Outside Diameter the quotient of 2 divided by the Diametral Pitch	$D'=D-\frac{2}{P}$
Pitch Diameter.	Addendum and the Number of Teeth.	Multiply Addendum by the Number of Teeth	$D'=s\,N$
Outside Diameter.	The Number of Teeth and the Diametral Pitch	Divide Number of Teeth plus 2 by the Diametral Pitch	$D=\frac{N+2}{P}$
Outside Diameter.	The Pitch Diameter and the Diametral Pitch	Add to the Pitch Diameter the quotient of 2 divided by the Diametral Pitch	$D=D'+\frac{2}{P}$
Outside Diameter.	The Pitch Diameter and the Number of Teeth	Divide the Number of Teeth plus 2 by the quotient of Number of Teeth and by the Pitch Diameter	$D=\frac{N+2}{\frac{N}{D'}}$
Outside Diameter.	The Number of Teeth and Addendum .	Multiply the Number of Teeth plus 2 by Addendum	$D=(N+2)\,s$
Number of Teeth.	The Pitch Diameter and the Diametral Pitch	Multiply Pitch Diameter by the Diametral Pitch	$N=D'P$
Number of Teeth.	The Outside Diameter and the Diametral Pitch . . .	Multiply Outside Diameter by the Diametral Pitch and subtract 2.	$N=DP-2$
Thickness of Tooth.	The Diametral Pitch.	Divide 1.5708 by the Diametral Pitch	$t=\frac{1.5708}{P}$
Addendum.	The Diametral Pitch.	Divide 1 by the Diametral Pitch, or $s=\frac{D'}{N}$	$s=\frac{1}{P}$
Root.	The Diametral Pitch.	Divide 1.157 by the Diametral Pitch	$s+f=\frac{1.157}{P}$
Working Depth.	The Diametral Pitch.	Divide 2 by the Diametral Pitch.	$D''=\frac{2}{P}$
Whole Depth.	The Diametral Pitch.	Divide 2.157 by the Diametral Pitch	$D''+f=\frac{2.157}{P}$
Clearance.	The Diametral Pitch.	Divide .157 by the Diametral Pitch	$f=\frac{.157}{P}$
Clearance.	Thickness of Tooth.	Divide Thickness of Tooth at pitch line by 10	$f=\frac{t}{10}$

CIRCULAR PITCH.

"NUTTALL."

Circular Pitch is the Distance from the Centre of One Tooth to the Centre of the Next Tooth, Measured along the Pitch Line.

To Get	Having	Rule.	Formula.
The Circular Pitch.	The Diametral Pitch.	Divide 3.1416 by the Diametral Pitch	$P'=\frac{3.1416}{P}$
The Circular Pitch.	The Pitch Diameter and the Number of Teeth	Divide Pitch Diameter by the product of .3183 and Number of Teeth	$P'=\frac{D'}{.3183\ N}$
The Circular Pitch.	The Outside Diameter and the Number of Teeth	Divide Outside Diameter by the product of .3183 and Number of Teeth plus 2	$P'=\frac{D}{.3183\ N+2}$
Pitch Diameter.	The Number of Teeth and the Circular Pitch	The continued product of the Number of Teeth, the Circular Pitch and .3183	$D'=NP'.3183$
Pitch Diameter.	The Number of Teeth and the Outside Diameter	Divide the product of Number of Teeth and Outside Diameter by Number of Teeth plus 2 . . .	$D'=\frac{ND}{N+2}$
Pitch Diameter.	The Outside Diameter and the Circular Pitch	Subtract from the Outside Diameter the product of the Circular Pitch and .6366	$D'=D-(P'.6366)$
Pitch Diameter.	Addendum and the Number of Teeth.	Multiply the Number of Teeth by the Addendum	$D'=N\ s$
Outside Diameter.	The Number of Teeth and the Circular Pitch	The continued product of the Number of Teeth plus 2, the Circular Pitch and .3183 . . .	$D=(N+2)P'.3183$
Outside Diameter.	The Pitch Diameter and the Circular Pitch	Add to the Pitch Diameter the product of the Circular Pitch and .6366	$D=D'+(P'.6366)$
Outside Diameter.	The Number of Teeth and the Addendum	Multiply Addendum by Number of Teeth plus 2	$D=s\ (N+2)$
Number of Teeth.	The Pitch Diameter and the Circular Pitch	Divide the product of Pitch Diameter and 3.1416 by the Circular Pitch	$N=\frac{D'\ 3.1416}{P'}$
Thickness of Tooth.	The Circular Pitch.	One-half the Circular Pitch . .	$t=\frac{P'}{2}$
Addendum.	The Circular Pitch.	Multiply the Circular Pitch by .3183, or $s=\frac{D'}{N}$	$s=P'\ .3183$
Root.	The Circular Pitch.	Multiply the Circular Pitch by .3683	$s+f=P'\ .3683$
Working Depth.	The Circular Pitch.	Multiply the Circular Pitch by .6366	$D''=P'\ .6366$
Whole Depth.	The Circular Pitch.	Multiply the Circular Pitch by .6866	$D''=P'\ .6866$
Clearance.	The Circular Pitch.	Multiply the Circular Pitch by .05	$f=P'\ .05$
Clearance.	Thickness of Tooth.	One-tenth the Thickness of Tooth at Pitch Line	$f=\frac{t}{10}$

GEAR WHEELS.

TABLE OF TOOTH PARTS—CIRCULAR PITCH IN FIRST COLUMN.

Circular Pitch.	Threads or Teeth per inch Linear.	Diametral Pitch.	Thickness of Tooth on Pitch Line.	Addendum and Module.	Working Depth of Tooth.	Depth of Space below Pitch Line.	Whole Depth of Tooth.	Width of Thread-Tool at End.	Width of Thread at Top.
P′	$\frac{1''}{P'}$	P	*t*	*s*	D″	*s*+*f*	D″+ *f*	P′×.31	P′×.335
2	$\frac{1}{2}$	1.5708	1.0000	.6366	1.2732	.7366	1.3732	.6200	.6700
$1\frac{7}{8}$	$\frac{8}{15}$	1.6755	.9375	.5968	1.1937	.6906	1.2874	.5813	.6281
$1\frac{3}{4}$	$\frac{4}{7}$	1.7952	.8750	.5570	1.1141	.6445	1.2016	.5425	.5863
$1\frac{5}{8}$	$\frac{8}{13}$	1.9333	.8125	.5173	1.0345	.5985	1.1158	.5038	.5444
$1\frac{1}{2}$	$\frac{2}{3}$	2.0944	.7500	.4775	.9549	.5525	1.0299	.4650	.5025
$1\frac{7}{16}$	$\frac{16}{23}$	2.1855	.7187	.4576	.9151	.5294	.9870	.4456	.4816
$1\frac{3}{8}$	$\frac{8}{11}$	2.2848	.6875	.4377	.8754	.5064	.9441	.4262	.4606
$1\frac{1}{3}$	$\frac{3}{4}$	2.3562	.6666	.4244	.8488	.4910	.9154	.4133	.4466
$1\frac{5}{16}$	$\frac{16}{21}$	2.3936	.6562	.4178	.8356	.4834	.9012	.4069	.4397
$1\frac{1}{4}$	$\frac{4}{5}$	2.5133	.6250	.3979	.7958	.4604	.8583	.3875	.4188
$1\frac{3}{16}$	$\frac{16}{19}$	2.6456	.5937	.3780	.7560	.4374	.8156	.3681	.3978
$1\frac{1}{8}$	$\frac{8}{9}$	2.7925	.5625	.3581	.7162	.4143	.7724	.3488	.3769
$1\frac{1}{16}$	$\frac{16}{17}$	2.9568	.5312	.3382	.6764	.3913	.7295	.3294	.3559
1	1	3.1416	.5000	.3183	.6366	.3683	.6866	.3100	.3350
$\frac{15}{16}$	$1\frac{1}{15}$	3.3510	.4687	.2984	.5968	.3453	.6437	.2906	.3141
$\frac{7}{8}$	$1\frac{1}{7}$	3.5904	.4375	.2785	.5570	.3223	.6007	.2713	.2931
$\frac{13}{16}$	$1\frac{3}{13}$	3.8666	.4062	.2586	.5173	.2993	.5579	.2519	.2722
$\frac{4}{5}$	$1\frac{1}{4}$	3.9270	.4000	.2546	.5092	.2946	.5492	.2480	.2680
$\frac{3}{4}$	$1\frac{1}{3}$	4.1888	.3750	.2387	.4775	.2762	.5150	.2325	.2513
$\frac{11}{16}$	$1\frac{5}{11}$	4.5696	.3437	.2189	.4377	.2532	.4720	.2131	.2303
$\frac{2}{3}$	$1\frac{1}{2}$	4.7124	.3333	.2122	.4244	.2455	.4577	.2066	.2233
$\frac{5}{8}$	$1\frac{3}{5}$	5.0265	.3125	.1989	.3979	.2301	.4291	.1938	.2094
$\frac{3}{5}$	$1\frac{2}{3}$	5.2360	.3000	.1910	.3820	.2210	.4120	.1860	.2010
$\frac{4}{7}$	$1\frac{3}{4}$	5.4978	.2857	.1819	.3638	.2105	.3923	.1771	.1914
$\frac{9}{16}$	$1\frac{7}{9}$	5.5851	.2812	.1790	.3581	.2071	.3862	.1744	.1884

To obtain the size of any part of a circular pitch not given in the table, multiply the corresponding part of 1″ pitch by the pitch required.

TABLE OF TOOTH PARTS.—*Continued.*

CIRCULAR PITCH IN FIRST COLUMN.

Circular Pitch.	Threads or Teeth per inch Linear.	Diametral Pitch.	Thickness of Tooth on Pitch Line.	Addendum and Module.	Working Depth of Tooth.	Depth of Space below Pitch Line.	Whole Depth of Tooth.	Width of Thread-Tool at End.	Width of Thread at Top.
P′	$\frac{1''}{P'}$	P	*t*	*s*	D″	*s*+*f*	D″+*f*.	P′×.31	P′×.335
$\frac{1}{2}$	2	6.2832	.2500	.1592	.3183	.1842	.3433	.1550	.1675
$\frac{4}{9}$	$2\frac{1}{4}$	7.0685	.2222	.1415	.2830	.1637	.3052	.1378	.1489
$\frac{7}{16}$	$2\frac{2}{7}$	7.1808	.2187	.1393	.2785	.1611	.3003	.1356	.1466
$\frac{3}{7}$	$2\frac{1}{3}$	7.3304	.2143	.1364	.2728	.1578	.2942	.1328	.1436
$\frac{2}{5}$	$2\frac{1}{2}$	7.8540	.2000	.1273	.2546	.1473	.2746	.1240	.1340
$\frac{3}{8}$	$2\frac{2}{3}$	8.3776	.1875	.1194	.2387	.1381	.2575	.1163	.1256
$\frac{4}{11}$	$2\frac{3}{4}$	8.6394	.1818	.1158	.2316	.1340	.2498	.1127	.1218
$\frac{1}{3}$	3	9.4248	.1666	.1061	.2122	.1228	.2289	.1033	.1117
$\frac{5}{16}$	$3\frac{1}{5}$	10.0531	.1562	.0995	.1989	.1151	.2146	.0969	.1047
$\frac{3}{10}$	$3\frac{1}{3}$	10.4719	.1500	.0955	.1910	.1105	.2060	.0930	.1005
$\frac{2}{7}$	$3\frac{1}{2}$	10.9956	.1429	.0909	.1819	.1052	.1962	.0886	.0957
$\frac{1}{4}$	4	12.5664	.1250	.0796	.1591	.0921	.1716	.0775	.0838
$\frac{2}{9}$	$4\frac{1}{2}$	14.1372	.1111	.0707	.1415	.0818	.1526	.0689	.0744
$\frac{1}{5}$	5	15.7080	.1000	.0637	.1273	.0737	.1373	.0620	.0670
$\frac{3}{16}$	$5\frac{1}{3}$	16.7552	.0937	.0597	.1194	.0690	.1287	.0581	.0628
$\frac{2}{11}$	$5\frac{1}{2}$	17.2788	.0909	.0579	.1158	.0670	.1249	.0564	.0609
$\frac{1}{6}$	6	18.8496	.0833	.0531	.1061	.0614	.1144	.0517	.0558
$\frac{2}{13}$	$6\frac{1}{2}$	20.4203	.0769	.0489	.0978	.0566	.1055	.0477	.0515
$\frac{1}{7}$	7	21.9911	.0714	.0455	.0910	.0526	.0981	.0443	.0479
$\frac{2}{15}$	$7\frac{1}{2}$	23.5619	.0666	.0425	.0850	.0492	.0917	.0414	.0446
$\frac{1}{8}$	8	25.1327	.0625	.0398	.0796	.0460	.0858	.0388	.0419
$\frac{1}{9}$	9	28.2743	.0555	.0354	.0707	.0409	.0763	.0344	.0372
$\frac{1}{10}$	10	31.4159	.0500	.0318	.0637	.0368	.0687	.0310	.0335
$\frac{1}{16}$	16	50.2655	.0312	.0199	.0398	.0230	.0429	.0194	.0209
$\frac{1}{20}$	20	62.8318	.0250	.0159	.0318	.0184	.0343	.0155	.0167

To obtain the size of any part of a circular pitch not given in the table, multiply the corresponding part of 1″ pitch by the pitch required.

GEAR WHEELS.

TABLE OF TOOTH PARTS—DIAMETRAL PITCH IN FIRST COLUMN.

Diametral Pitch.	Circular Pitch.	Thickness of Tooth on Pitch Line.	Addendum and $\frac{1''}{P}$	Working Depth of Tooth.	Depth of Space below Pitch Line.	Whole Depth of Tooth.
P	P′	t	s	D″	$s+f$.	D″+f.
$\frac{1}{2}$	6.2832	3.1416	2.0000	4.0000	2.3142	4.3142
$\frac{3}{4}$	4.1888	2.0944	1.3333	2.6666	1.5428	2.8761
1	3.1416	1.5708	1.0000	2.0000	1.1571	2.1571
$1\frac{1}{4}$	2.5133	1.2566	.8000	1.6000	.9257	1.7257
$1\frac{1}{2}$	2.0944	1.0472	.6666	1.3333	.7714	1.4381
$1\frac{3}{4}$	1.7952	.8976	.5714	1 1429	.6612	1.2326
2	1.5708	.7854	.5000	1.0000	.5785	1.0785
$2\frac{1}{4}$	1.3963	.6981	.4444	.8888	.5143	.9587
$2\frac{1}{2}$	1.2566	.6283	.4000	.8000	.4628	.8628
$2\frac{3}{4}$	1.1424	.5712	.3636	.7273	.4208	.7844
3	1.0472	.5236	.3333	.6666	.3857	.7190
$3\frac{1}{2}$	.8976	.4488	.2857	.5714	.3306	.6163
4	.7854	.3927	.2500	.5000	.2893	.5393
5	.6283	.3142	.2000	.4000	.2314	.4314
6	.5236	.2618	.1666	.3333	.1928	.3595
7	.4488	.2244	.1429	.2857	.1653	.3081
8	.3927	.1963	.1250	.2500	.1446	.2696
9	.3491	.1745	.1111	.2222	.1286	.2397
10	.3142	.1571	.1000	.2000	.1157	.2157
11	.2856	.1428	.0909	.1818	.1052	.1961
12	.2618	.1309	0833	.1666	.0964	.1798
13	.2417	.1208	.0769	.1538	.0890	.1659
14	.2244	.1122	.0714	.1429	.0826	.1541

To obtain the size of any part of a diametral pitch not given in the table, divide the corresponding part of 1 diametral pitch by the pitch required.

TABLE OF TOOTH PARTS—*Continued.*

DIAMETRAL PITCH IN FIRST COLUMN.

Diametral Pitch.	Circular Pitch.	Thickness of Tooth on Pitch Line.	Addendum and $\frac{1''}{P}$	Working Depth of Tooth.	Depth of Space below Pitch Line.	Whole Depth of Tooth.
P.	P′.	*t.*	*s.*	D″.	*s*+*f*.	D″+*f*.
15	.2094	.1047	.0666	.1333	.0771	.1438
16	.1963	.0982	.0625	.1250	.0723	.1348
17	.1848	.0924	.0588	.1176	.0681	.1269
18	.1745	.0873	.0555	.1111	.0643	.1198
19	.1653	.0827	.0526	.1053	.0609	.1135
20	.1571	.0785	.0500	.1000	.0579	.1079
22	.1428	.0714	.0455	.0909	.0526	.0980
24	.1309	.0654	.0417	.0833	.0482	.0898
26	.1208	.0604	.0385	.0769	.0445	.0829
28	.1122	.0561	.0357	.0714	.0413	.0770
30	.1047	.0524	.0333	.0666	.0386	.0719
32	.0982	.0491	.0312	.0625	.0362	.0674
34	.0924	.0462	.0294	.0588	.0340	.0634
36	.0873	.0436	.0278	.0555	.0321	.0599
38	.0827	.0413	.0263	.0526	.0304	.0568
40	.0785	.0393	.0250	.0500	.0289	.0539
42	.0748	.0374	.0238	.0476	.0275	.0514
44	.0714	.0357	.0227	.0455	.0263	.0490
46	.0683	.0341	.0217	.0435	.0252	.0469
48	.0654	.0327	.0208	.0417	.0241	.0449
50	.0628	.0314	.0200	.0400	.0231	.0431
56	.0561	.0280	.0178	.0357	.0207	.0385
60	.0524	.0262	.0166	.0333	.0193	.0360

To obtain the size of any part of a diametral pitch not given in the table, divide the corresponding part of 1 diametral pitch by the pitch required.

Natural Sines and Cosines.

INTERNATIONAL CORRESPONDENCE SCHOOLS.

′	0°		1°		2°		3°		4°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.00000	1.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	1.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	1.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	1.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	1.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	1.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	1.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	1.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	1.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	1.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	1.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	89°		88°		87°		86°		85°		′

′	5°		6°		7°		8°		9°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	84°		83°		82°		81°		80°		

′	10°		11°		12°		13°		14°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	
′	79°		78°		77°		76°		75°		′

′	15°		16°		17°		18°		19°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.94580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	74°		73°		72°		71°		70°		

′	20° Sine	20° Cosine	21° Sine	21° Cosine	22° Sine	22° Cosine	23° Sine	23° Cosine	24° Sine	24° Cosine	′
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	69°		68°		67°		66°		65°		

′	25° Sine	25° Cosine	26° Sine	26° Cosine	27° Sine	27° Cosine	28° Sine	28° Cosine	29° Sine	29° Cosine	′
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	64°		63°		62°		61°		60°		

′	30°		31°		32°		33°		34°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	59°		58°		57°		56°		55°		

′	35°		36°		37°		38°		39°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.5 802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.5 826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	49°		48°		47°		46°		45°		

Natural Tangents and Cotangents.

International Correspondence Schools.

′	0°		1°		2°		3°		4°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.00000	Infinite	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	.07022	14.2411	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	.07051	14.1821	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	.07080	14.1235	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	.07110	14.0655	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	.07139	14.0079	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	.07168	13.9507	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	.07197	13.8940	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	.07227	13.8378	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	.07256	13.7821	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	.07285	13.7267	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	.07314	13.6719	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	.07344	13.6174	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	.07373	13.5634	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	.07402	13.5098	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	.07431	13.4566	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	.07461	13.4039	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	.07490	13.3515	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	.07519	13.2996	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	.07548	13.2480	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	.07578	13.1969	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	.07607	13.1461	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	.07636	13.0958	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	.07665	13.0458	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	.07695	12.9962	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	.07724	12.9469	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	.07753	12.8981	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	.07782	12.8496	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	.07812	12.8014	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	.07841	12.7536	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	.07870	12.7062	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	.07899	12.6591	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	.07929	12.6124	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	.07958	12.5660	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	.07987	12.5199	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	.08017	12.4742	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	.08046	12.4288	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	.08075	12.3838	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	.08104	12.3390	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	.08134	12.2946	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	.08163	12.2505	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	.08192	12.2067	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	.08221	12.1632	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	.08251	12.1201	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	.08280	12.0772	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	.08309	12.0346	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	.08339	11.9923	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	.08368	11.9504	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	.08397	11.9087	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	.08427	11.8673	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	.08456	11.8262	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	.08485	11.7853	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	.08514	11.7448	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	.08544	11.7045	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	.08573	11.6645	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	.08602	11.6248	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	.08632	11.5853	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	.08661	11.5461	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	.08690	11.5072	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	.08720	11.4685	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	.08749	11.4301	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	89°		88°		87°		86°		85°		

′	5°		6°		7°		8°		9°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	.15838	6.31375	60
1	.08778	11.3919	.10540	9.48781	.12308	8.12481	.14084	7.10038	.15868	6.30189	59
2	.08807	11.3540	.10569	9.46141	.12338	8.10536	.14113	7.08546	.15898	6.29007	58
3	.08837	11.3163	.10599	9.43515	.12367	8.08600	.14143	7.07059	.15928	6.27829	57
4	.08866	11.2789	.10628	9.40904	.12397	8.06674	.14173	7.05579	.15958	6.26655	56
5	.08895	11.2417	.10657	9.38307	.12426	8.04756	.14202	7.04105	.15988	6.25486	55
6	.08925	11.2048	.10687	9.35724	.12456	8.02848	.14232	7.02637	.16017	6.24321	54
7	.08954	11.1681	.10716	9.33155	.12485	8.00948	.14262	7.01174	.16047	6.23160	53
8	.08983	11.1316	.10746	9.30599	.12515	7.99058	.14291	6.99718	.16077	6.22003	52
9	.09013	11.0954	.10775	9.28058	.12544	7.97176	.14321	6.98268	.16107	6.20851	51
10	.09042	11.0594	.10805	9.25530	.12574	7.95302	.14351	6.96823	.16137	6.19703	50
11	.09071	11.0237	.10834	9.23016	.12603	7.93438	.14381	6.95385	.16167	6.18559	49
12	.09101	10.9882	.10863	9.20516	.12633	7.91582	.14410	6.93952	.16196	6.17419	48
13	.09130	10.9529	.10893	9.18028	.12662	7.89734	.14440	6.92525	.16226	6.16283	47
14	.09159	10.9178	.10922	9.15554	.12692	7.87895	.14470	6.91104	.16256	6.15151	46
15	.09189	10.8829	.10952	9.13093	.12722	7.86064	.14499	6.89688	.16286	6.14023	45
16	.09218	10.8483	.10981	9.10646	.12751	7.84242	.14529	6.88278	.16316	6.12899	44
17	.09247	10.8139	.11011	9.08211	.12781	7.82428	.14559	6.86874	.16346	6.11779	43
18	.09277	10.7797	.11040	9.05789	.12810	7.80622	.14588	6.85475	.16376	6.10664	42
19	.09306	10.7457	.11070	9.03379	.12840	7.78825	.14618	6.84082	.16405	6.09552	41
20	.09335	10.7119	.11099	9.00983	.12869	7.77035	.14648	6.82694	.16435	6.08444	40
21	.09365	10.6783	.11128	8.98598	.12899	7.75254	.14678	6.81312	.16465	6.07340	39
22	.09394	10.6450	.11158	8.96227	.12929	7.73480	.14707	6.79936	.16495	6.06240	38
23	.09423	10.6118	.11187	8.93867	.12958	7.71715	.14737	6.78564	.16525	6.05143	37
24	.09453	10.5789	.11217	8.91520	.12988	7.69957	.14767	6.77199	.16555	6.04051	36
25	.09482	10.5462	.11246	8.89185	.13017	7.68208	.14796	6.75838	.16585	6.02962	35
26	.09511	10.5136	.11276	8.86862	.13047	7.66466	.14826	6.74483	.16615	6.01878	34
27	.09541	10.4813	.11305	8.84551	.13076	7.64732	.14856	6.73133	.16645	6.00797	33
28	.09570	10.4491	.11335	8.82252	.13106	7.63005	.14886	6.71789	.16674	5.99720	32
29	.09600	10.4172	.11364	8.79964	.13136	7.61287	.14915	6.70450	.16704	5.98646	31
30	.09629	10.3854	.11394	8.77689	.13165	7.59575	.14945	6.69116	.16734	5.97576	30
31	.09658	10.3538	.11423	8.75425	.13195	7.57872	.14975	6.67787	.16764	5.96510	29
32	.09688	10.3224	.11452	8.73172	.13224	7.56176	.15005	6.66463	.16794	5.95448	28
33	.09717	10.2913	.11482	8.70931	.13254	7.54487	.15034	6.65144	.16824	5.94390	27
34	.09746	10.2602	.11511	8.68701	.13284	7.52806	.15064	6.63831	.16854	5.93335	26
35	.09776	10.2294	.11541	8.66482	.13313	7.51132	.15094	6.62523	.16884	5.92283	25
36	.09805	10.1988	.11570	8.64275	.13343	7.49465	.15124	6.61219	.16914	5.91236	24
37	.09834	10.1683	.11600	8.62078	.13372	7.47806	.15153	6.59921	.16944	5.90191	23
38	.09864	10.1381	.11629	8.59893	.13402	7.46154	.15183	6.58627	.16974	5.89151	22
39	.09893	10.1080	.11659	8.57718	.13432	7.44509	.15213	6.57339	.17004	5.88114	21
40	.09923	10.0780	.11688	8.55555	.13461	7.42871	.15243	6.56055	.17033	5.87080	20
41	.09952	10.0483	.11718	8.53402	.13491	7.41240	.15272	6.54777	.17063	5.86051	19
42	.09981	10.0187	.11747	8.51259	.13521	7.39616	.15302	6.53503	.17093	5.85024	18
43	.10011	9.98931	.11777	8.49128	.13550	7.37999	.15332	6.52234	.17123	5.84001	17
44	.10040	9.96007	.11806	8.47007	.13580	7.36389	.15362	6.50970	.17153	5.82982	16
45	.10069	9.93101	.11836	8.44896	.13609	7.34786	.15391	6.49710	.17183	5.81966	15
46	.10099	9.90211	.11865	8.42795	.13639	7.33190	.15421	6.48456	.17213	5.80953	14
47	.10128	9.87338	.11895	8.40705	.13669	7.31600	.15451	6.47206	.17243	5.79944	13
48	.10158	9.84482	.11924	8.38625	.13698	7.30018	.15481	6.45961	.17273	5.78938	12
49	.10187	9.81641	.11954	8.36555	.13728	7.28442	.15511	6.44720	.17303	5.77936	11
50	.10216	9.78817	.11983	8.34496	.13758	7.26873	.15540	6.43484	.17333	5.76937	10
51	.10246	9.76009	.12013	8.32446	.13787	7.25310	.15570	6.42253	.17363	5.75941	9
52	.10275	9.73217	.12042	8.30406	.13817	7.23754	.15600	6.41026	.17393	5.74949	8
53	.10305	9.70441	.12072	8.28376	.13846	7.22204	.15630	6.39804	.17423	5.73960	7
54	.10334	9.67680	.12101	8.26355	.13876	7.20661	.15660	6.38587	.17453	5.72974	6
55	.10363	9.64935	.12131	8.24345	.13906	7.19125	.15689	6.37374	.17483	5.71992	5
56	.10393	9.62205	.12160	8.22344	.13935	7.17594	.15719	6.36165	.17513	5.71013	4
57	.10422	9.59490	.12190	8.20352	.13965	7.16071	.15749	6.34961	.17543	5.70037	3
58	.10452	9.56791	.12219	8.18370	.13995	7.14553	.15779	6.33761	.17573	5.69064	2
59	.10481	9.54106	.12249	8.16398	.14024	7.13042	.15809	6.32566	.17603	5.68094	1
60	.10510	9.51436	.12278	8.14435	.14054	7.11537	.15838	6.31375	.17633	5.67128	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	84°		83°		82°		81°		80°		

′	10°		11°		12°		13°		14°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.17633	5.67128	.19438	5.14455	.21256	4.70463	.23087	4.33148	.24933	4.01078	60
1	.17663	5.66165	.19468	5.13658	.21286	4.69791	.23117	4.32573	.24964	4.00582	59
2	.17693	5.65205	.19498	5.12862	.21316	4.69121	.23148	4.32001	.24995	4.00086	58
3	.17723	5.64248	.19529	5.12069	.21347	4.68452	.23179	4.31430	.25026	3.99592	57
4	.17753	5.63295	.19559	5.11279	.21377	4.67786	.23209	4.30860	.25056	3.99099	56
5	.17783	5.62344	.19589	5.10490	.21408	4.67121	.23240	4.30291	.25087	3.98607	55
6	.17813	5.61397	.19619	5.09704	.21438	4.66458	.23271	4.29724	.25118	3.98117	54
7	.17843	5.60452	.19649	5.08921	.21469	4.65797	.23301	4.29159	.25149	3.97627	53
8	.17873	5.59511	.19680	5.08139	.21499	4.65138	.23332	4.28595	.25180	3.97139	52
9	.17903	5.58573	.19710	5.07360	.21529	4.64480	.23363	4.28032	.25211	3.96651	51
10	.17933	5.57638	.19740	5.06584	.21560	4.63825	.23393	4.27471	.25242	3.96165	50
11	.17963	5.56706	.19770	5.05809	.21590	4.63171	.23424	4.26911	.25273	3.95680	49
12	.17993	5.55777	.19801	5.05037	.21621	4.62518	.23455	4.26352	.25304	3.95196	48
13	.18023	5.54851	.19831	5.04267	.21651	4.61868	.23485	4.25795	.25335	3.94713	47
14	.18053	5.53927	.19861	5.03499	.21682	4.61219	.23516	4.25239	.25366	3.94232	46
15	.18083	5.53007	.19891	5.02734	.21712	4.60572	.23547	4.24685	.25397	3.93751	45
16	.18113	5.52090	.19921	5.01971	.21743	4.59927	.23578	4.24132	.25428	3.93271	44
17	.18143	5.51176	.19952	5.01210	.21773	4.59283	.23608	4.23580	.25459	3.92793	43
18	.18173	5.50264	.19982	5.00451	.21804	4.58641	.23639	4.23030	.25490	3.92316	42
19	.18203	5.49356	.20012	4.99695	.21834	4.58001	.23670	4.22481	.25521	3.91839	41
20	.18233	5.48451	.20042	4.98940	.21864	4.57363	.23700	4.21933	.25552	3.91364	40
21	.18263	5.47548	.20073	4.98188	.21895	4.56726	.23731	4.21387	.25583	3.90890	39
22	.18293	5.46648	.20103	4.97438	.21925	4.56091	.23762	4.20842	.25614	3.90417	38
23	.18323	5.45751	.20133	4.96690	.21956	4.55458	.23793	4.20298	.25645	3.89945	37
24	.18353	5.44857	.20164	4.95945	.21986	4.54826	.23823	4.19756	.25676	3.89474	36
25	.18384	5.43966	.20194	4.95201	.22017	4.54196	.23854	4.19215	.25707	3.89004	35
26	.18414	5.43077	.20224	4.94460	.22047	4.53568	.23885	4.18675	.25738	3.88536	34
27	.18444	5.42192	.20254	4.93721	.22078	4.52941	.23916	4.18137	.25769	3.88068	33
28	.18474	5.41309	.20285	4.92984	.22108	4.52316	.23946	4.17600	.25800	3.87601	32
29	.18504	5.40429	.20315	4.92249	.22139	4.51693	.23977	4.17064	.25831	3.87136	31
30	.18534	5.39552	.20345	4.91516	.22169	4.51071	.24008	4.16530	.25862	3.86671	30
31	.18564	5.38677	.20376	4.90785	.22200	4.50451	.24039	4.15997	.25893	3.86208	29
32	.18594	5.37805	.20406	4.90056	.22231	4.49832	.24069	4.15465	.25924	3.85745	28
33	.18624	5.36936	.20436	4.89330	.22261	4.49215	.24100	4.14934	.25955	3.85284	27
34	.18654	5.36070	.20466	4.88605	.22292	4.48600	.24131	4.14405	.25986	3.84824	26
35	.18684	5.35206	.20497	4.87882	.22322	4.47986	.24162	4.13877	.26017	3.84364	25
36	.18714	5.34345	.20527	4.87162	.22353	4.47374	.24193	4.13350	.26048	3.83906	24
37	.18745	5.33487	.20557	4.86444	.22383	4.46764	.24223	4.12825	.26079	3.83449	23
38	.18775	5.32631	.20588	4.85727	.22414	4.46155	.24254	4.12301	.26110	3.82992	22
39	.18805	5.31778	.20618	4.85013	.22444	4.45548	.24285	4.11778	.26141	3.82537	21
40	.18835	5.30928	.20648	4.84300	.22475	4.44942	.24316	4.11256	.26172	3.82083	20
41	.18865	5.30080	.20679	4.83590	.22505	4.44338	.24347	4.10736	.26203	3.81630	19
42	.18895	5.29235	.20709	4.82882	.22536	4.43735	.24377	4.10216	.26235	3.81177	18
43	.18925	5.28393	.20739	4.82175	.22567	4.43134	.24408	4.09699	.26266	3.80726	17
44	.18955	5.27553	.20770	4.81471	.22597	4.42534	.24439	4.09182	.26297	3.80276	16
45	.18986	5.26715	.20800	4.80769	.22628	4.41936	.24470	4.08666	.26328	3.79827	15
46	.19016	5.25880	.20830	4.80068	.22658	4.41340	.24501	4.08152	.26359	3.79378	14
47	.19046	5.25048	.20861	4.79370	.22689	4.40745	.24532	4.07639	.26390	3.78931	13
48	.19076	5.24218	.20891	4.78673	.22719	4.40152	.24562	4.07127	.26421	3.78485	12
49	.19106	5.23391	.20921	4.77978	.22750	4.39560	.24593	4.06616	.26452	3.78040	11
50	.19136	5.22566	.20952	4.77286	.22781	4.38969	.24624	4.06107	.26483	3.77595	10
51	.19166	5.21744	.20982	4.76595	.22811	4.38381	.24655	4.05599	.26515	3.77152	9
52	.19197	5.20925	.21013	4.75906	.22842	4.37793	.24686	4.05092	.26546	3.76709	8
53	.19227	5.20107	.21043	4.75219	.22872	4.37207	.24717	4.04586	.26577	3.76268	7
54	.19257	5.19293	.21073	4.74534	.22903	4.36623	.24747	4.04081	.26608	3.75828	6
55	.19287	5.18480	.21104	4.73851	.22934	4.36040	.24778	4.03578	.26639	3.75388	5
56	.19317	5.17671	.21134	4.73170	.22964	4.35459	.24809	4.03076	.26670	3.74950	4
57	.19347	5.16863	.21164	4.72490	.22995	4.34879	.24840	4.02574	.26701	3.74512	3
58	.19378	5.16058	.21195	4.71813	.23026	4.34300	.24871	4.02074	.26733	3.74075	2
59	.19408	5.15256	.21225	4.71137	.23056	4.33723	.24902	4.01576	.26764	3.73640	1
60	.19438	5.14455	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	79°		78°		77°		76°		75°		

′	15°		16°		17°		18°		19°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.26795	3.73205	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.26826	3.72771	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.26857	3.72338	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.26888	3.71907	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.26920	3.71476	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.26951	3.71046	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.26982	3.70616	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.27013	3.70188	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.27044	3.69761	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.27076	3.69335	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.27107	3.68909	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.27138	3.68485	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.27169	3.68061	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.27201	3.67638	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.27232	3.67217	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.27263	3.66796	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.27294	3.66376	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.27326	3.65957	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.27357	3.65538	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.27388	3.65121	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.27419	3.64705	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.27451	3.64289	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.27482	3.63874	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.27513	3.63461	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.27545	3.63048	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.27576	3.62636	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.27607	3.62224	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.27638	3.61814	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.27670	3.61405	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.27701	3.60996	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.27732	3.60588	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.27764	3.60181	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.27795	3.59775	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.27826	3.59370	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.27858	3.58966	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.27889	3.58562	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.27921	3.58160	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.27952	3.57758	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.27983	3.57357	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.28015	3.56957	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.28046	3.56557	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.28077	3.56159	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.28109	3.55761	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.28140	3.55364	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.28172	3.54968	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.28203	3.54573	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.28234	3.54179	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.28266	3.53785	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.28297	3.53393	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.28329	3.53001	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.28360	3.52609	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.28391	3.52219	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.28423	3.51829	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.28454	3.51441	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.28486	3.51053	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.28517	3.50666	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.28549	3.50279	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.28580	3.49894	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.28612	3.49509	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.28643	3.49125	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	74°		73°		72°		71°		70°		

′	20° Tang	20° Cotang	21° Tang	21° Cotang	22° Tang	22° Cotang	23° Tang	23° Cotang	24° Tang	24° Cotang	′
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	.44558	2.24428	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	.44593	2.24252	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	.44627	2.24077	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	.44662	2.23902	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	.44697	2.23727	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	.44732	2.23553	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	.44767	2.23378	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	.44802	2.23204	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	.44837	2.23030	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	.44872	2.22857	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	.44907	2.22683	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	.44942	2.22510	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	.44977	2.22337	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	.45012	2.22164	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	.45047	2.21992	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	.45082	2.21819	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	.45117	2.21647	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	.45152	2.21475	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	.45187	2.21304	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	.45222	2.21132	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	.45257	2.20961	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	.45292	2.20790	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43230	2.31271	.45327	2.20619	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	.45362	2.20449	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	.45397	2.20278	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	.45432	2.20108	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	.45467	2.19938	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	.45502	2.19769	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	.45538	2.19599	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	.45573	2.19430	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	.45608	2.19261	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	.45643	2.19092	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	.45678	2.18923	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	.45713	2.18755	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	.45748	2.18587	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	.45784	2.18419	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	.45819	2.18251	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	.45854	2.18084	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	.45889	2.17916	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	.45924	2.17749	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	.45960	2.17582	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	.45995	2.17416	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	.46030	2.17249	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	.46065	2.17083	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	.46101	2.16917	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	.46136	2.16751	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	.46171	2.16585	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	.46206	2.16420	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	.46242	2.16255	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	.46277	2.16090	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	.46312	2.15925	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	.46348	2.15760	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	.46383	2.15596	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	.46418	2.15432	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	.46454	2.15268	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	.46489	2.15104	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	.46525	2.14940	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	.46560	2.14777	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	.46595	2.14614	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	.46631	2.14451	0
′	Cotang 69°	Tang 69°	Cotang 68°	Tang 68°	Cotang 67°	Tang 67°	Cotang 66°	Tang 66°	Cotang 65°	Tang 65°	′

′	25°		26°		27°		28°		29°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	.55431	1.80405	60
1	.46666	2.14288	.48809	2.04879	.50989	1.96120	.53208	1.87941	.55469	1.80281	59
2	.46702	2.14125	.48845	2.04728	.51026	1.95979	.53246	1.87809	.55507	1.80158	58
3	.46737	2.13963	.48881	2.04577	.51063	1.95838	.53283	1.87677	.55545	1.80034	57
4	.46772	2.13801	.48917	2.04426	.51099	1.95698	.53320	1.87546	.55583	1.79911	56
5	.46808	2.13639	.48953	2.04276	.51136	1.95557	.53358	1.87415	.55621	1.79788	55
6	.46843	2.13477	.48989	2.04125	.51173	1.95417	.53395	1.87283	.55659	1.79665	54
7	.46879	2.13316	.49026	2.03975	.51209	1.95277	.53432	1.87152	.55697	1.79542	53
8	.46914	2.13154	.49062	2.03825	.51246	1.95137	.53470	1.87021	.55736	1.79419	52
9	.46950	2.12993	.49098	2.03675	.51283	1.94997	.53507	1.86891	.55774	1.79296	51
10	.46985	2.12832	.49134	2.03526	.51319	1.94858	.53545	1.86760	.55812	1.79174	50
11	.47021	2.12671	.49170	2.03376	.51356	1.94718	.53582	1.86630	.55850	1.79051	49
12	.47056	2.12511	.49206	2.03227	.51393	1.94579	.53620	1.86499	.55888	1.78929	48
13	.47092	2.12350	.49242	2.03078	.51430	1.94440	.53657	1.86369	.55926	1.78807	47
14	.47128	2.12190	.49278	2.02929	.51467	1.94301	.53694	1.86239	.55964	1.78685	46
15	.47163	2.12030	.49315	2.02780	.51503	1.94162	.53732	1.86109	.56003	1.78563	45
16	.47199	2.11871	.49351	2.02631	.51540	1.94023	.53769	1.85979	.56041	1.78441	44
17	.47234	2.11711	.49387	2.02483	.51577	1.93885	.53807	1.85850	.56079	1.78319	43
18	.47270	2.11552	.49423	2.02335	.51614	1.93746	.53844	1.85720	.56117	1.78198	42
19	.47305	2.11392	.49459	2.02187	.51651	1.93608	.53882	1.85591	.56156	1.78077	41
20	.47341	2.11233	.49495	2.02039	.51688	1.93470	.53920	1.85462	.56194	1.77955	40
21	.47377	2.11075	.49532	2.01891	.51724	1.93332	.53957	1.85333	.56232	1.77834	39
22	.47412	2.10916	.49568	2.01743	.51761	1.93195	.53995	1.85204	.56270	1.77713	38
23	.47448	2.10758	.49604	2.01596	.51798	1.93057	.54032	1.85075	.56309	1.77592	37
24	.47483	2.10600	.49640	2.01449	.51835	1.92920	.54070	1.84946	.56347	1.77471	36
25	.47519	2.10442	.49677	2.01302	.51872	1.92782	.54107	1.84818	.56385	1.77351	35
26	.47555	2.10284	.49713	2.01155	.51909	1.92645	.54145	1.84689	.56424	1.77230	34
27	.47590	2.10126	.49749	2.01008	.51946	1.92508	.54183	1.84561	.56462	1.77110	33
28	.47626	2.09969	.49786	2.00862	.51983	1.92371	.54220	1.84433	.56501	1.76990	32
29	.47662	2.09811	.49822	2.00715	.52020	1.92235	.54258	1.84305	.56539	1.76869	31
30	.47698	2.09654	.49858	2.00569	.52057	1.92098	.54296	1.84177	.56577	1.76749	30
31	.47733	2.09498	.49894	2.00423	.52094	1.91962	.54333	1.84049	.56616	1.76629	29
32	.47769	2.09341	.49931	2.00277	.52131	1.91826	.54371	1.83922	.56654	1.76510	28
33	.47805	2.09184	.49967	2.00131	.52168	1.91690	.54409	1.83794	.56693	1.76390	27
34	.47840	2.09028	.50004	1.99986	.52205	1.91554	.54446	1.83667	.56731	1.76271	26
35	.47876	2.08872	.50040	1.99841	.52242	1.91418	.54484	1.83540	.56769	1.76151	25
36	.47912	2.08716	.50076	1.99695	.52279	1.91282	.54522	1.83413	.56808	1.76032	24
37	.47948	2.08560	.50113	1.99550	.52316	1.91147	.54560	1.83286	.56846	1.75913	23
38	.47984	2.08405	.50149	1.99406	.52353	1.91012	.54597	1.83159	.56885	1.75794	22
39	.48019	2.08250	.50185	1.99261	.52390	1.90876	.54635	1.83033	.56923	1.75675	21
40	.48055	2.08094	.50222	1.99116	.52427	1.90741	.54673	1.82906	.56962	1.75556	20
41	.48091	2.07939	.50258	1.98972	.52464	1.90607	.54711	1.82780	.57000	1.75437	19
42	.48127	2.07785	.50295	1.98828	.52501	1.90472	.54748	1.82654	.57039	1.75319	18
43	.48163	2.07630	.50331	1.98684	.52538	1.90337	.54786	1.82528	.57078	1.75200	17
44	.48198	2.07476	.50368	1.98540	.52575	1.90203	.54824	1.82402	.57116	1.75082	16
45	.48234	2.07321	.50404	1.98396	.52613	1.90069	.54862	1.82276	.57155	1.74964	15
46	.48270	2.07167	.50441	1.98253	.52650	1.89935	.54900	1.82150	.57193	1.74846	14
47	.48306	2.07014	.50477	1.98110	.52687	1.89801	.54938	1.82025	.57232	1.74728	13
48	.48342	2.06860	.50514	1.97966	.52724	1.89667	.54975	1.81899	.57271	1.74610	12
49	.48378	2.06706	.50550	1.97823	.52761	1.89533	.55013	1.81774	.57309	1.74492	11
50	.48414	2.06553	.50587	1.97681	.52798	1.89400	.55051	1.81649	.57348	1.74375	10
51	.48450	2.06400	.50623	1.97538	.52836	1.89266	.55089	1.81524	.57386	1.74257	9
52	.48486	2.06247	.50660	1.97395	.52873	1.89133	.55127	1.81399	.57425	1.74140	8
53	.48521	2.06094	.50696	1.97253	.52910	1.89000	.55165	1.81274	.57464	1.74022	7
54	.48557	2.05942	.50733	1.97111	.52947	1.88867	.55203	1.81150	.57503	1.73905	6
55	.48593	2.05790	.50769	1.96969	.52985	1.88734	.55241	1.81025	.57541	1.73788	5
56	.48629	2.05637	.50806	1.96827	.53022	1.88602	.55279	1.80901	.57580	1.73671	4
57	.48665	2.05485	.50843	1.96685	.53059	1.88469	.55317	1.80777	.57619	1.73555	3
58	.48701	2.05333	.50879	1.96544	.53096	1.88337	.55355	1.80653	.57657	1.73438	2
59	.48737	2.05182	.50916	1.96402	.53134	1.88205	.55393	1.80529	.57696	1.73321	1
60	.48773	2.05030	.50953	1.96261	.53171	1.88073	.55431	1.80405	.57735	1.73205	0
	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	
′	64°		63°		62°		61°		60°		′

′	30° Tang	30° Cotang	31° Tang	31° Cotang	32° Tang	32° Cotang	33° Tang	33° Cotang	34° Tang	34° Cotang	′
0	.57735	1.73205	.60086	1.66428	.62487	1.60033	.64941	1.53986	.67451	1.48256	60
1	.57774	1.73089	.60126	1.66318	.62527	1.59930	.64982	1.53888	.67493	1.48163	59
2	.57813	1.72973	.60165	1.66209	.62568	1.59826	.65024	1.53791	.67536	1.48070	58
3	.57851	1.72857	.60205	1.66099	.62608	1.59723	.65065	1.53693	.67578	1.47977	57
4	.57890	1.72741	.60245	1.65990	.62649	1.59620	.65106	1.53595	.67620	1.47885	56
5	.57929	1.72625	.60284	1.65881	.62689	1.59517	.65148	1.53497	.67663	1.47792	55
6	.57968	1.72509	.60324	1.65772	.62730	1.59414	.65189	1.53400	.67705	1.47699	54
7	.58007	1.72393	.60364	1.65663	.62770	1.59311	.65231	1.53302	.67748	1.47607	53
8	.58046	1.72278	.60403	1.65554	.62811	1.59208	.65272	1.53205	.67790	1.47514	52
9	.58085	1.72163	.60443	1.65445	.62852	1.59105	.65314	1.53107	.67832	1.47422	51
10	.58124	1.72047	.60483	1.65337	.62892	1.59002	.65355	1.53010	.67875	1.47330	50
11	.58162	1.71932	.60522	1.65228	.62933	1.58900	.65397	1.52913	.67917	1.47238	49
12	.58201	1.71817	.60562	1.65120	.62973	1.58797	.65438	1.52816	.67960	1.47146	48
13	.58240	1.71702	.60602	1.65011	.63014	1.58695	.65480	1.52719	.68002	1.47053	47
14	.58279	1.71588	.60642	1.64903	.63055	1.58593	.65521	1.52622	.68045	1.46962	46
15	.58318	1.71473	.60681	1.64795	.63095	1.58490	.65563	1.52525	.68088	1.46870	45
16	.58357	1.71358	.60721	1.64687	.63136	1.58388	.65604	1.52429	.68130	1.46778	44
17	.58396	1.71244	.60761	1.64579	.63177	1.58286	.65646	1.52332	.68173	1.46686	43
18	.58435	1.71129	.60801	1.64471	.63217	1.58184	.65688	1.52235	.68215	1.46595	42
19	.58474	1.71015	.60841	1.64363	.63258	1.58083	.65729	1.52139	.68258	1.46503	41
20	.58513	1.70901	.60881	1.64256	.63299	1.57981	.65771	1.52043	.68301	1.46411	40
21	.58552	1.70787	.60921	1.64148	.63340	1.57879	.65813	1.51946	.68343	1.46320	39
22	.58591	1.70673	.60960	1.64041	.63380	1.57778	.65854	1.51850	.68386	1.46229	38
23	.58631	1.70560	.61000	1.63934	.63421	1.57676	.65896	1.51754	.68429	1.46137	37
24	.58670	1.70446	.61040	1.63826	.63462	1.57575	.65938	1.51658	.68471	1.46046	36
25	.58709	1.70332	.61080	1.63719	.63503	1.57474	.65980	1.51562	.68514	1.45955	35
26	.58748	1.70219	.61120	1.63612	.63544	1.57372	.66021	1.51466	.68557	1.45864	34
27	.58787	1.70106	.61160	1.63505	.63584	1.57271	.66063	1.51370	.68600	1.45773	33
28	.58826	1.69992	.61200	1.63398	.63625	1.57170	.66105	1.51275	.68642	1.45682	32
29	.58865	1.69879	.61240	1.63292	.63666	1.57069	.66147	1.51179	.68685	1.45592	31
30	.58905	1.69766	.61280	1.63185	.63707	1.56969	.66189	1.51084	.68728	1.45501	30
31	.58944	1.69653	.61320	1.63079	.63748	1.56868	.66230	1.50988	.68771	1.45410	29
32	.58983	1.69541	.61360	1.62972	.63789	1.56767	.66272	1.50893	.68814	1.45320	28
33	.59022	1.69428	.61400	1.62866	.63830	1.56667	.66314	1.50797	.68857	1.45229	27
34	.59061	1.69316	.61440	1.62760	.63871	1.56566	.66356	1.50702	.68900	1.45139	26
35	.59101	1.69203	.61480	1.62654	.63912	1.56466	.66398	1.50607	.68942	1.45049	25
36	.59149	1.69091	.61520	1.62548	.63953	1.56366	.66440	1.50512	.68985	1.44958	24
37	.59179	1.68979	.61561	1.62442	.63994	1.56265	.66482	1.50417	.69028	1.44868	23
38	.59218	1.68866	.61601	1.62336	.64035	1.56165	.66524	1.50322	.69071	1.44778	22
39	.59258	1.68754	.61641	1.62230	.64076	1.56065	.66566	1.50228	.69114	1.44688	21
40	.59297	1.68643	.61681	1.62125	.64117	1.55966	.66608	1.50133	.69157	1.44598	20
41	.59336	1.68531	.61721	1.62019	.64158	1.55866	.66650	1.50038	.69200	1.44508	19
42	.59376	1.68419	.61761	1.61914	.64199	1.55766	.66692	1.49944	.69243	1.44418	18
43	.59415	1.68308	.61801	1.61808	.64240	1.55666	.66734	1.49849	.69286	1.44329	17
44	.59454	1.68196	.61842	1.61703	.64281	1.55567	.66776	1.49755	.69329	1.44239	16
45	.59494	1.68085	.61882	1.61598	.64322	1.55467	.66818	1.49661	.69372	1.44149	15
46	.59533	1.67974	.61922	1.61493	.64363	1.55368	.66860	1.49566	.69416	1.44060	14
47	.59573	1.67863	.61962	1.61388	.64404	1.55269	.66902	1.49472	.69459	1.43970	13
48	.59612	1.67752	.62003	1.61283	.64446	1.55170	.66944	1.49378	.69502	1.43881	12
49	.59651	1.67641	.62043	1.61179	.64487	1.55071	.66986	1.49284	.69545	1.43792	11
50	.59691	1.67530	.62083	1.61074	.64528	1.54972	.67028	1.49190	.69588	1.43703	10
51	.59730	1.67419	.62124	1.60970	.64569	1.54873	.67071	1.49097	.69631	1.43614	9
52	.59770	1.67309	.62164	1.60865	.64610	1.54774	.67113	1.49003	.69675	1.43525	8
53	.59809	1.67198	.62204	1.60761	.64652	1.54675	.67155	1.48909	.69718	1.43436	7
54	.59849	1.67088	.62245	1.60657	.64693	1.54576	.67197	1.48816	.69761	1.43347	6
55	.59888	1.66978	.62285	1.60553	.64734	1.54478	.67239	1.48722	.69804	1.43258	5
56	.59928	1.66867	.62325	1.60449	.64775	1.54379	.67282	1.48629	.69847	1.43169	4
57	.59967	1.66757	.62366	1.60345	.64817	1.54281	.67324	1.48536	.69891	1.43080	3
58	.60007	1.66647	.62406	1.60241	.64858	1.54183	.67366	1.48442	.69934	1.42992	2
59	.60046	1.66538	.62446	1.60137	.64899	1.54085	.67409	1.48349	.69977	1.42903	1
60	.60086	1.66428	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	59°		58°		57°		56°		55°		

′	35°		36°		37°		38°		39°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.70021	1.42815	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.70064	1.42726	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.70107	1.42638	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.70151	1.42550	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.70194	1.42462	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.70238	1.42374	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.70281	1.42286	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.70325	1.42198	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.70368	1.42110	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.70412	1.42022	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.70455	1.41934	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.70499	1.41847	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.70542	1.41759	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.70586	1.41672	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.70629	1.41584	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.70673	1.41497	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.70717	1.41409	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.70760	1.41322	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.70804	1.41235	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.70848	1.41148	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.70891	1.41061	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.70935	1.40974	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.70979	1.40887	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.71023	1.40800	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.71066	1.40714	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.71110	1.40627	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.71154	1.40540	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.71198	1.40454	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.71242	1.40367	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.71285	1.40281	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.71329	1.40195	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.71373	1.40109	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.71417	1.40022	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.71461	1.39936	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.71505	1.39850	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.71549	1.39764	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.71593	1.39679	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.71637	1.39593	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.71681	1.39507	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.71725	1.39421	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.71769	1.39336	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.71813	1.39250	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.71857	1.39165	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.71901	1.39079	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.71946	1.38994	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.71990	1.38909	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.72034	1.38824	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.72078	1.38738	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.72122	1.38653	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.72167	1.38568	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.72211	1.38484	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.72255	1.38399	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.72299	1.38314	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.72344	1.38229	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.72388	1.38145	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.72432	1.38060	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.72477	1.37976	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.72521	1.37891	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.72565	1.37807	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.72610	1.37722	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	.96625	1.03493	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	.96681	1.03433	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	.96738	1.03372	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	.96794	1.03312	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	.96850	1.03252	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	.96907	1.03192	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	.96963	1.03132	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	.97020	1.03072	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	.97076	1.03012	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	.97133	1.02952	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	.97189	1.02892	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	.97246	1.02832	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	.97302	1.02772	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	.97359	1.02713	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	.97416	1.02653	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	.97472	1.02593	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	.97529	1.02533	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	.97586	1.02474	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	.97643	1.02414	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	.97700	1.02355	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	.97756	1.02295	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	.97813	1.02236	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	.97870	1.02176	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	.97927	1.02117	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	.97984	1.02057	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	.98041	1.01998	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	.98098	1.01939	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	.98155	1.01879	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	.98213	1.01820	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	.98270	1.01761	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	.98327	1.01702	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	.98384	1.01642	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	.98441	1.01583	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	.98499	1.01524	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	.98556	1.01465	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	.98613	1.01406	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	.98671	1.01347	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	.98728	1.01288	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	.98786	1.01229	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	.98843	1.01170	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	.98901	1.01112	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	.98958	1.01053	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	.99016	1.00994	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	.99073	1.00935	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	.99131	1.00876	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	.99189	1.00818	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	.99247	1.00759	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	.99304	1.00701	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	.99362	1.00642	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	.99420	1.00583	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	.99478	1.00525	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	.99536	1.00467	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	.99594	1.00408	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	.99652	1.00350	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	.99710	1.00291	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	.99768	1.00233	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	.99826	1.00175	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	.99884	1.00116	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	.99942	1.00058	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	1.00000	1.00000	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	49°		48°		47°		46°		45°		

NATURAL SECANT.

Deg.	0′	10′	20′	30′	40′	50	60′	
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0001	1.0001	89
1	1.0001	1.0002	1.0002	1.0003	1.0004	1.0005	1.0006	88
2	1.0006	1.0007	1.0008	1.0009	1.0010	1.0012	1.0013	87
3	1.0013	1.0015	1.0016	1.0018	1.0020	1.0022	1.0024	86
4	1.0024	1.0026	1.0028	1.0030	1.0033	1.0035	1.0038	85
5	1.0038	1.0040	1.0043	1.0046	1.0049	1.0052	1.0055	84
6	1.0055	1.0058	1.0061	1.0064	1.0068	1.0071	1.0075	83
7	1.0075	1.0078	1.0082	1.0086	1.0090	1.0094	1.0098	82
8	1.0098	1.0102	1.0106	1.0111	1.0115	1.0120	1.0124	81
9	1.0124	1.0129	1.0134	1.0139	1.0144	1.0149	1.0154	80
10	1.0154	1.0159	1.0164	1.0170	1.0175	1.0181	1.0187	79
11	1.0187	1.0192	1.0198	1.0204	1.0210	1.0217	1.0223	78
12	1.0223	1.0229	1.0236	1.0242	1.0249	1.0256	1.0263	77
13	1.0263	1.0269	1.0277	1.0284	1.0291	1.0298	1.0303	76
14	1.0306	1.0313	1.0321	1.0329	1.0336	1.0344	1.0352	75
15	1.0352	1.0360	1.0369	1.0377	1.0385	1.0394	1.0402	74
16	1.0402	1.0411	1.0420	1.0429	1.0438	1.0447	1.0456	73
17	1.0456	1.0466	1.0475	1.0485	1.0494	1.0504	1.0514	72
18	1.0514	1.0524	1.0534	1.0544	1.0555	1.0565	1.0576	71
19	1.0576	1.0586	1.0597	1.0608	1.0619	1.0630	1.0641	70
20	1.0641	1.0653	1.0664	1.0676	1.0687	1.0699	1.0711	69
21	1.0711	1.0723	1.0735	1.0747	1.0760	1.0772	1.0785	68
22	1.0785	1.0798	1.0810	1.0823	1.0837	1.0850	1.0863	67
23	1.0863	1.0877	1.0890	1.0904	1.0918	1.0932	1.0946	66
24	1.0946	1.0960	1.0974	1.0989	1.1004	1.1018	1.1033	65
25	1.1033	1.1048	1.1063	1.1079	1.1094	1.1110	1.1126	64
26	1.1126	1.1141	1.1157	1.1174	1.1190	1.1206	1.1223	63
27	1.1223	1.1239	1.1256	1.1273	1.1290	1.1308	1.1325	62
28	1.1325	1.1343	1.1361	1.1378	1.1396	1.1415	1.1433	61
29	1.1433	1.1452	1.1470	1.1489	1.1508	1.1527	1.1547	60
30	1.1547	1.1566	1.1586	1.1605	1.1625	1.1646	1.1666	59
31	1.1666	1.1686	1.1707	1.1728	1.1749	1.1770	1.1791	58
32	1.1791	1.1813	1.1835	1.1856	1.1878	1.1901	1.1923	57
33	1.1923	1.1946	1.1969	1.1992	1.2015	1.2038	1.2062	56
34	1.2062	1.2085	1.2109	1.2134	1.2158	1.2182	1.2207	55
35	1.2207	1.2232	1.2257	1.2283	1.2308	1.2334	1.2360	54
36	1.2360	1.2386	1.2413	1.2440	1.2466	1.2494	1.2521	53
37	1.2521	1.2548	1.2576	1.2604	1.2632	1.2661	1.2690	52
38	1.2690	1.2719	1.2748	1.2777	1.2807	1.2837	1.2867	51
39	1.2867	1.2898	1.2928	1.2959	1.2990	1.3022	1.3054	50
40	1.3054	1.3086	1.3118	1.3150	1.3183	1.3216	1.3250	49
41	1.3250	1.3283	1.3317	1.3351	1.3386	1.3421	1.3456	48
42	1.3456	1.3491	1.3527	1.3563	1.3599	1.3636	1.3673	47
43	1.3673	1.3710	1.3748	1.3785	1.3824	1.3862	1.3901	46
44	1.3901	1.3940	1.3980	1.4020	1.4060	1.4101	1.4142	45
	60′	50′	40′	30′	20′	10′	0′	Deg.

NATURAL COSECANT.

NATURAL SECANT.

Deg.	0′	10′	20′	30′	40′	50′	60′	
45	1.4142	1.4183	1.4225	1.4267	1.4309	1.4352	1.4395	44
46	1.4395	1.4439	1.4483	1.4527	1.4572	1.4617	1.4662	43
47	1.4662	1.4708	1.4755	1.4801	1.4849	1.4896	1.4944	42
48	1.4944	1.4993	1.5042	1.5091	1.5141	1.5191	1.5242	41
49	1.5242	1.5293	1.5345	1.5397	1.5450	1.5503	1.5557	40
50	1.5557	1.5611	1.5666	1.5721	1.5777	1.5833	1.5890	39
51	1.5890	1.5947	1.6005	1.6063	1.6122	1.6182	1.6242	38
52	1.6242	1.6303	1.6364	1.6426	1.6489	1.6552	1.6616	37
53	1.6616	1.6680	1.6745	1.6811	1.6878	1.6945	1.7013	36
54	1.7013	1.7081	1.7150	1.7220	1.7291	1.7362	1.7434	35
55	1.7434	1.7507	1.7580	1.7655	1.7730	1.7806	1.7882	34
56	1.7882	1.7960	1.8038	1.8118	1.8198	1.8278	1.8360	33
57	1.8360	1.8443	1.8527	1.8611	1.8697	1.8783	1.8870	32
58	1.8870	1.8959	1.9048	1.9138	1.9230	1.9322	1.9416	31
59	1.9416	1.9510	1.9606	1.9702	1.9800	1.9899	2.0000	30
60	2.0000	2.0101	2.0203	2.0307	2.0412	2.0519	2.0626	29
61	2.0626	2.0735	2.0845	2.0957	2.1070	2.1184	2.1300	28
62	2.1300	2.1417	2.1536	2.1656	2.1778	2.1901	2.2026	27
63	2.2026	2.2153	2.2281	2.2411	2.2543	2.2676	2.2811	26
64	2.2811	2.2948	2.3087	2.3228	2.3370	2.3515	2.3662	25
65	2.3662	2.3810	2.3961	2.4114	2.4269	2.4426	2.4585	24
66	2.4585	2.4747	2.4911	2.5078	2.5247	2.5418	2.5593	23
67	2.5593	2.5769	2.5949	2.6131	2.6316	2.6503	2.6694	22
68	2.6694	2.6883	2.7085	2.7285	2.7488	2.7694	2.7904	21
69	2.7904	2.8117	2.8334	2.8554	2.8778	2.9006	2.9238	20
70	2.9238	2.9473	2.9713	2.9957	3.0205	3.0458	3.0715	19
71	3.0715	3.0977	3.1243	3.1515	3.1791	3.2073	3.2360	18
72	3.2360	3.2653	3.2951	3.3255	3.3564	3.3880	3.4203	17
73	3.4203	3.4531	3.4867	3.5209	3 5558	3.5915	3.6279	16
74	3.6279	3.6651	3.7031	3.7419	3.7816	3.8222	3.8637	15
75	3.8637	3.9061	3 9495	3.9939	4.0393	4.0859	4.1335	14
76	4.1335	4.1823	4.2323	4.2836	4.3362	4.3901	4.4454	13
77	4.4454	4.5021	4.5604	4.6202	4.6816	4.7448	4.8097	12
78	4.8097	4.8764	4.9451	5 0158	5.0886	5.1635	5.2408	11
79	5.2408	5.3204	5.4026	5.4874	5.5749	5.6653	5.7587	10
80	5.7587	5.8553	5.9553	6.0588	6.1660	6.2771	6.3924	9
81	6.3924	6.5120	6.6363	6.7654	6.8997	7.0396	7.1852	8
82	7.1852	7.3371	7.4957	7.6612	7.8344	8.0156	8.2055	7
83	8.2055	8.4046	8.6137	8.8336	9.0651	9.3091	9.5667	6
84	9.5667	9.8391	10.127	10.433	10.758	11.104	11.473	5
85	11.473	11.868	12.291	12.745	13.234	13.763	14.335	4
86	14.335	14.957	15.636	16.380	17.198	18.102	19.107	3
87	19.107	20.230	21.493	22.925	24.562	26.450	28 653	2
88	28.653	31.257	34.382	38.201	42 975	49.114	57.298	1
89	57.298	68.757	85.945	114.59	171.88	343.77	∞	0
	60′	50′	40′	30′	20′	10′	0′	Deg.

NATURAL COSECANT.

DECIMAL EQUIVALENTS OF PARTS OF AN INCH.

$\frac{1}{64}$	.01563	$\frac{21}{64}$	.32813	$\frac{45}{64}$	.70313
$\frac{1}{32}$	.03125	$\frac{11}{32}$	.34375	$\frac{23}{32}$	.71875
$\frac{3}{64}$	.04688	$\frac{23}{64}$	.35938	$\frac{47}{64}$	.73438
1-16	.0625	3-8	.375	3-4	.75
$\frac{5}{64}$	.07813	$\frac{25}{64}$	.39063	$\frac{49}{64}$	.76563
$\frac{3}{32}$	.09375	$\frac{13}{32}$	.40625	$\frac{25}{32}$	.78125
$\frac{7}{64}$	.10938	$\frac{27}{64}$	.42188	$\frac{51}{64}$	.79688
1-8	.125	7-16	.4375	13 16	.8125
$\frac{9}{64}$	.14063	$\frac{29}{64}$	.45313	$\frac{53}{64}$	.82813
$\frac{5}{32}$	.15625	$\frac{15}{32}$	.46875	$\frac{27}{32}$	.84375
$\frac{11}{64}$	.17188	$\frac{31}{64}$	.48438	$\frac{55}{64}$	.85938
3-16	.1875	1-2	.5	7-8	.875
$\frac{13}{64}$	.20313	$\frac{33}{64}$	.51563	$\frac{57}{64}$	.89063
$\frac{7}{32}$	.21875	$\frac{17}{32}$	.53125	$\frac{29}{32}$	.90625
$\frac{15}{64}$	.23438	$\frac{35}{64}$	.54688	$\frac{59}{64}$	.92188
1-4	.25	9-16	.5625	15-16	.9375
$\frac{17}{64}$	.26563	$\frac{37}{64}$	.57813	$\frac{61}{64}$	.95313
$\frac{9}{32}$	.28125	$\frac{19}{32}$	.59375	$\frac{31}{32}$	.96875
$\frac{19}{64}$	.29688	$\frac{39}{64}$	.60938	$\frac{63}{64}$	.98438
5-16	.3125	5-8	.625	1	1.00000
		$\frac{41}{64}$	.64063		
		$\frac{21}{32}$	.65625		
		$\frac{43}{64}$	.67188		
		11-16	.6875		

TABLE OF DECIMAL EQUIVALENTS

OF

MILLIMETRES AND FRACTIONS OF MILLIMETRES.

mm.	Inches.	mm.	Inches.	mm.	Inches.	mm.	Inches.
$\frac{1}{100}$ =	.00039	$\frac{33}{100}$ =	.01299	$\frac{64}{100}$ =	.02520	$\frac{95}{100}$ =	.03740
$\frac{2}{100}$ =	.00079	$\frac{34}{100}$ =	.01339	$\frac{65}{100}$ =	.02559	$\frac{96}{100}$ =	.03780
$\frac{3}{100}$ =	.00118	$\frac{35}{100}$ =	.01378	$\frac{66}{100}$ =	.02598	$\frac{97}{100}$ =	.03819
$\frac{4}{100}$ =	.00157	$\frac{36}{100}$ =	.01417	$\frac{67}{100}$ =	.02638	$\frac{98}{100}$ =	.03858
$\frac{5}{100}$ =	.00197	$\frac{37}{100}$ =	.01457	$\frac{68}{100}$ =	.02677	$\frac{99}{100}$ =	.03898
$\frac{6}{100}$ =	.00236	$\frac{38}{100}$ =	.01496	$\frac{69}{100}$ =	.02717	1 =	.03937
$\frac{7}{100}$ =	.00276	$\frac{39}{100}$ =	.01535	$\frac{70}{100}$ =	.02756	2 =	.07874
$\frac{8}{100}$ =	.00315	$\frac{40}{100}$ =	.01575	$\frac{71}{100}$ =	.02795	3 =	.11811
$\frac{9}{100}$ =	.00354	$\frac{41}{100}$ =	.01614	$\frac{72}{100}$ =	.02835	4 =	.15748
$\frac{10}{100}$ =	.00394	$\frac{42}{100}$ =	.01654	$\frac{73}{100}$ =	.02874	5 =	.19685
$\frac{11}{100}$ =	.00433	$\frac{43}{100}$ =	.01693	$\frac{74}{100}$ =	.02913	6 =	.23622
$\frac{12}{100}$ =	.00472	$\frac{44}{100}$ =	.01732	$\frac{75}{100}$ =	.02953	7 =	.27559
$\frac{13}{100}$ =	.00512	$\frac{45}{100}$ =	.01772	$\frac{76}{100}$ =	.02992	8 =	.31496
$\frac{14}{100}$ =	.00551	$\frac{46}{100}$ =	.01811	$\frac{77}{100}$ =	.03032	9 =	.35433
$\frac{15}{100}$ =	.00591	$\frac{47}{100}$ =	.01850	$\frac{78}{100}$ =	.03071	10 =	.39370
$\frac{16}{100}$ =	.00630	$\frac{48}{100}$ =	.01890	$\frac{79}{100}$ =	.03110	11 =	.43307
$\frac{17}{100}$ =	.00669	$\frac{49}{100}$ =	.01929	$\frac{80}{100}$ =	.03150	12 =	.47244
$\frac{18}{100}$ =	.00709	$\frac{50}{100}$ =	.01969	$\frac{81}{100}$ =	.03189	13 =	.51181
$\frac{19}{100}$ =	.00748	$\frac{51}{100}$ =	.02008	$\frac{82}{100}$ =	.03228	14 =	.55118
$\frac{20}{100}$ =	.00787	$\frac{52}{100}$ =	.02047	$\frac{83}{100}$ =	.03268	15 =	.59055
$\frac{21}{100}$ =	.00827	$\frac{53}{100}$ =	.02087	$\frac{84}{100}$ =	.03307	16 =	.62992
$\frac{22}{100}$ =	.00866	$\frac{54}{100}$ =	.02126	$\frac{85}{100}$ =	.03346	17 =	.66929
$\frac{23}{100}$ =	.00906	$\frac{55}{100}$ =	.02165	$\frac{86}{100}$ =	.03386	18 =	.70866
$\frac{24}{100}$ =	.00945	$\frac{56}{100}$ =	.02205	$\frac{87}{100}$ =	.03425	19 =	.74803
$\frac{25}{100}$ =	.00984	$\frac{57}{100}$ =	.02244	$\frac{88}{100}$ =	.03465	20 =	.78740
$\frac{26}{100}$ =	.01024	$\frac{58}{100}$ =	.02283	$\frac{89}{100}$ =	.03504	21 =	.82677
$\frac{27}{100}$ =	.01063	$\frac{59}{100}$ =	.02323	$\frac{90}{100}$ =	.03543	22 =	.86614
$\frac{28}{100}$ =	.01102	$\frac{60}{100}$ =	.02362	$\frac{91}{100}$ =	.03583	23 =	.90551
$\frac{29}{100}$ =	.01142	$\frac{61}{100}$ =	.02402	$\frac{92}{100}$ =	.03622	24 =	.94488
$\frac{30}{100}$ =	.01181	$\frac{62}{100}$ =	.02441	$\frac{93}{100}$ =	.03661	25 =	.98425
$\frac{31}{100}$ =	.01220	$\frac{63}{100}$ =	.02480	$\frac{94}{100}$ =	.03701	26 =	1.02362
$\frac{32}{100}$ =	.01260						

10 mm. = 1 Centimeter = 0.3937 inches.
10 cm. = 1 Decimeter = 3.937 inches.
10 dm. = 1 Meter = 39.37 inches.
25.4 mm. = 1 English Inch.

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Practical Treatise
on Gearing

Brown & Sharpe Mfg. Co.
Providence, R. I., U. S. A.