PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

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PROCEEDINGS

OF THE

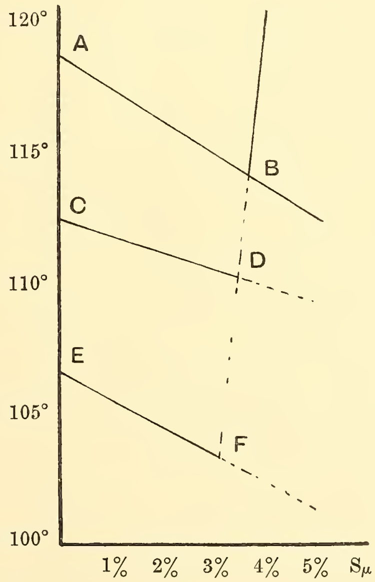
ROYAL SOCIETY OF EDINBURGH.

VOL. XXXII. 1911-12.

I.—The Freezing-Points of Rhombic Sulphur and of Soufre Nacré. By Alexander Smith and C. M. Carson.

(MS. received July 27, 1911. Read November 6, 1911.)

THE following symbols may be used to designate the well-characterised forms of sulphur: S_I, rhombic; S_{II}, ordinary monosymmetric; S_{III}, *soufre nacré*; S_{IV}, Muthmann's fourth modification; S_λ, liquid soluble sulphur; S_μ, insoluble sulphur.



S_{III}, *soufre nacré* (Gernez), is easily obtained by heating melted sulphur to 150°, placing the test-tube in a bath at 98–100°, and gently rubbing the bottom of the tube with a glass rod immersed in the sulphur. Crystals of S_{III}, of S_I, and of S_{II} appear in succession, the whole mass changing into S_{II} before final solidification.

The freezing-points with pure liquid S_λ may be called the *ideal* freezing-points; those with S_λ and S_μ in equilibrium are called the *natural* freezing-points.

Freezing-Points of S_{III}.—By melting pure sulphur at 123°, placing it in a bath at 102°, and using a thermometer instead of the glass rod, the temperature at which S_{III} crystallises from almost pure S_λ may be obtained. The ideal freezing-point, $S_\lambda \rightarrow S_{III}$, is 106·8°. After half a minute the crystallisation of S_{II} begins, and the temperature rises.

The natural freezing-point of S_{III} is observed by treating the melted sulphur with ammonia gas at 130–140°, to facilitate the formation of S_μ, and then immersing the tube in a bath at 99°. The freezing-point $S_\mu \rightleftarrows S_\lambda \rightarrow S_{III}$ is 103·4°, the depression being due to 3·1 per cent of S_μ which is in equilibrium with the S_λ at that temperature. After remaining at this point for a short time, crystals of S_I appear, with rising temperature, and finally S_{II} comes out with a still further rise in temperature.

Freezing-Points of S_I.—The sulphur is treated in the same ways as before, but the bath is kept at 108° (above the freezing-point of S_{III}). With pure S_λ the freezing-point $S_\lambda \rightarrow S_I$ is 112·8°; with the mixture of S_λ and S_μ in equilibrium (3·4 per cent. of the latter) the freezing-point $S_\mu \rightleftarrows S_\lambda \rightarrow S_I$ is 110·2°. After an interval S_{II} comes out, and the temperature rises as usual.

Freezing-Points of Various Forms of Sulphur.

Solid Phase.	Point in Diagram.	Freezing-Point.	Per cent. S_μ.
S_{II}	A, Ideal f.-p., $S_\lambda \rightarrow S_{II}$	119·25°	0·0
	B, Nat. f.-p., $S_\mu \rightleftarrows S_\lambda \rightarrow S_{II}$	114·5	3·6
S_I	C, Ideal f.-p., $S_\lambda \rightarrow S_I$	112·8	minute
	D, Nat. f.-p., $S_\mu \rightleftarrows S_\lambda \rightarrow S_I$	110·2	3·4
S_{III}	E, Ideal f.-p., $S_\lambda \rightarrow S_{III}$	106·8	minute
	F, Nat. f.-p., $S_\mu \rightleftarrows S_\lambda \rightarrow S_{III}$	103·4	3·1

Of these data, the first was determined by Smith and Holmes,* the second by Smith and Carson,† and the others in the present work.

It will be seen that the widely quoted values for the "melting-points" of the various forms of sulphur—S_I, 114·5° (Brodie); S_{II}, 120° (Brodie);

* *Proceedings*, xxiv. 299. † *Ibid.*, xxvi. 352.

S_{III}, 113·5° (Muthmann)—having been determined before the tendency of S_μ to depress the freezing (and melting) points, the influence of catalytic agents in hastening or retarding the formation of S_μ, the distinction between ideal and natural freezing-points, etc., were known, are clearly erroneous in varying degrees. So also Tammann's results and much-quoted diagram showing the effect of pressure upon the transition points of sulphur represent a cross between the ideal and the natural states. The corresponding points, especially at high pressures, which favour the formation of S_μ, would show differences of the order of 10–30°.

(*Issued separately January* 10, 1912.)

II.—The Preparation and Properties of Basic Copper Nitrate and the Hydrates of Copper Nitrate. By Alexander Charles Cumming and Alexander Gemmell.

(MS. received November 9, 1911. Read December 4, 1911.)

Anhydrous copper nitrate has never been prepared, though two hydrates are well known. The reaction between copper oxide and nitric acid would ordinarily be described by the equation:—

$$CuO + 2HNO_3 = Cu(NO_3)_2 + H_2O.$$

The substance actually formed is a hydrate of copper nitrate, and the question arises as to the source of the water if nitric acid is used, which approximates to 100 per cent. acid. The above equation cannot possibly be a correct representation of the reaction in such a case. Examination of this and similar points appeared desirable. As much of the work on the chemistry of copper nitrate dates back to an early period, little has been done to it from a physico-chemical standpoint, and some experiments have therefore been made on the condition for the formation of the hydrates and their range of stability.

Preliminary experiments confirmed the statement of previous observers that the hexahydrate was obtained by the action of dilute nitric acid on copper oxide and crystallisation below 25°, but that the trihydrate was obtained if the crystallisation was conducted above 25°, or if concentrated nitric acid was used. We found that the easiest method for the preparation of the trihydrate was to obtain it from copper oxide and concentrated nitric acid, and recrystallise it from nitric acid. The trihydrate is readily soluble in hot concentrated nitric acid, but sparingly soluble in the cold acid. The crystals were then dried on a porous tile in a vacuum desiccator over solid potash.

Non-existence of Graham's Basic Nitrate.—Graham (*Annalen der Pharmacie*, 1839, **29**, 13) states that if the nitric acid is above 1·4 specific gravity, a basic nitrate is obtained. The composition is not given, but the substance is said to be a green powder which is insoluble in water. That a basic nitrate should be obtained by the use of concentrated acid seemed remarkable and worthy of further investigation. We found by experiment that ordinary concentrated nitric acid (specific gravity 1·43) and fuming nitric acid (specific gravity 1·51) both acted on copper oxide to yield the trihydrate. No evidence conld be found of the existence of Graham's basic salt. It was noticed that the solution of the trihydrate in nitric acid had a vivid green colour, but the salt which crystallised from this solution was the blue trihydrate. It seemed so improbable that Graham should mistake the blue-coloured trihydrate for a basic salt, that we have tried experiments under as varied conditions as possible in the hope of reproducing his result, but without success. When copper oxide interacted with a relatively small quantity of fuming nitric acid the mixture became very hot, but not hot enough to decompose the trihydrate unless external heat was also supplied. In one experiment in which commercial copper oxide was employed a small amount of an insoluble basic salt was obtained, but it was found to be due to manganese, which was present as an impurity in the copper oxide.

The Interaction between Copper Oxide and Nitric Acid.—The substance obtained by the interaction of pure copper oxide and fuming nitric acid (specific gravity 1·51) was analysed after it had been dried on a tile in a desiccator over potash:—

0·886 grms. of substance left on ignition 0·2915 grms. of copper oxide.

$$Cu = 26{\cdot}2 \text{ per cent.}$$

0·217 grms. of substance gave by Crum's nitrometer method 42·0 c.c. of nitric oxide measured at 760 mm. and 120°.

$$N = 11{\cdot}6 \text{ per cent.}$$

Calculated for $Cu(NO_3)_2, 3H_2O$.

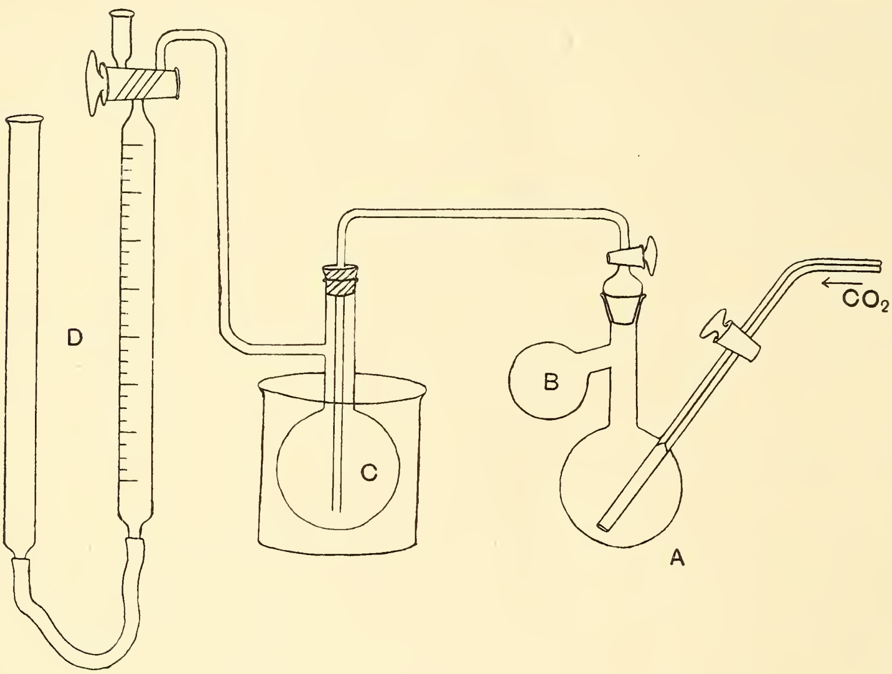
$$Cu = 26{\cdot}2 \text{ per cent., } N = 11{\cdot}6 \text{ per cent.}$$

Attention may be drawn to the fact that the trihydrate can only obtain its water of crystallisation by the decomposition of nitric acid, if the acid approximates to 100 per cent., nitric acid. The reaction is attended by the copious evolution of nitrogen peroxide fumes, resulting from this decomposition. The simplest formulation for this reaction would be

$$CuO + 6HNO_3 = Cu(NO_3)_2, 3H_2O + 2N_2O_5,$$

accompanied by a further decomposition of the nitrogen peroxide into nitrogen peroxide and oxygen.

The formation of copper nitrate trihydrate with copious evolution of nitrogen peroxide cannot well be explained in any manner other than the above, but to place the matter beyond question the presence of oxygen was proved in a special experiment. The nitric acid used was prepared by distillation of a mixture of fuming nitric acid (specific gravity 1·51) and concentrated sulphuric acid in a current of carbon dioxide. This has been shown by Roscoe (*Annalen*, **116**, 211 (1860)) to yield nitric acid which is very nearly anhydrous, and to contain only traces of nitrous acid. The copper oxide was prepared by ignition of the nitrate. The copper oxide was placed in the bulb A of the apparatus shown in the figure, and the nitric acid in the bulb B. The air was swept out of the apparatus by a current of CO_2, and the nitric acid then poured on to the copper oxide. No interaction occurred until the mixture was heated to 70°. The evolved gases were drawn by a slow stream of carbon dioxide into the empty bottle C, which was kept in a freezing mixture at a temperature below $-5°$. After passing through this cold bottle the gases no longer attacked rubber



and could, therefore, be led by a rubber tube to the gas burette D. In a preliminary experiment an evacuated apparatus was used, but the nitric acid then distilled off from the copper oxide, leaving it only slightly acted upon. This was the reason for filling the apparatus with carbon dioxide.

The gas burette was filled with concentrated potash to absorb carbon dioxide and nitrogen peroxide. In an experiment in which 1·5 grms. of copper oxide and about 8 c.c. of nitric acid were mixed, the amount unabsorbed by potash was 10·9 c.c. This was found to consist of 7·2 c.c. of oxygen and 3·7 c.c. of nitrogen. The carbon dioxide contained a little air, but the amount of nitrogen was somewhat higher than was expected, though the amount of carbon dioxide used was not known. There is no question, however, as to the formation of oxygen in the reaction. With this concentrated nitric acid the interaction with copper oxide was very sluggish, and only a small fraction of the copper oxide was attacked in the course of half an hour. The general conclusion from our experiment is that copper oxide interacts with anhydrous nitric acid at a temperature of 70° to 75°, according to the equation

$$CuO + 6HNO_3 = Cu(NO_3)_2, 3H_2O + 4NO_2 + O_2.$$

Dehydration of Copper Nitrate Hexahydrate.—Copper nitrate hexahydrate may be readily dehydrated in a vacuum desiccator by phosphorus pentoxide or solid potash. After a day or two the residue consists of copper nitrate trihydrate, which is comparatively stable, but does undergo very slow further decomposition. When the hexahydrate is desiccated nothing but water is lost until the trihydrate stage is reached. This was proved by passing a stream of air through a tube packed with the hexahydrate. The air was first purified by passing successively through caustic soda, sulphuric acid, and phosphorus pentoxide. The air then passed slowly through the tube of hexahydrate, and finally through some caustic soda solution. After several hours the caustic soda was warmed with zinc-aluminium alloy, and the gas evolved tested for ammonia. Complete absence of ammonia proved that no nitrogen compound had been given off during the dehydration of the hexahydrate.

Dehydration of Copper Nitrate Trihydrate.—Graham states that when the trihydrate was kept at 60° for some time, it slowly decomposed with formation of a basic salt, $Cu(NO_3)_2 \cdot 2Cu(OH)_2$, and that at higher temperatures the same decomposition occurred more quickly. A basic salt, $Cu(NO_3)_2 \cdot 3Cu(OH)_2$, has been prepared by many investigators (see *Dammer's Handbuch II.*, 2, 716), but the basic salt mentioned by Graham does not appear to have been made in any other way. Graham's experiment was therefore repeated. Some copper nitrate trihydrate was kept at 100° until decomposition was complete, and the residue analysed without further treatment.

0·0910 grms. yielded on ignition 0·0605 grms. of copper oxide.

$$Cu = 52{\cdot}9 \text{ per cent.}$$

0·1320 grms. gave by Crum's nitrometer method 12·0 c.c. of nitric oxide at 761 mm. and 17°.

$$N = 5{\cdot}39 \text{ per cent.}$$

Calculated for $Cu(NO_3)_2 \cdot 2Cu(OH)_2$,

$$Cu = 49{\cdot}6 \text{ per cent.}, \quad N = 7{\cdot}36 \text{ per cent.}$$

$Cu(NO_3)_2 \cdot 3Cu(OH)_2$,

$$Cu = 53{\cdot}0 \text{ per cent.}, \quad N = 5{\cdot}85 \text{ per cent.}$$

The formula $Cu(NO_3)_2 \cdot 2Cu(OH)_2$ assigned by Graham to this substance appears therefore to be incorrect.

The basic salt described above was prepared by heating the trihydrate to 100°. We found that decomposition of the trihydrate occurred even at room temperature under any conditions that led to the removal of water. For instance, when a current of dry air was passed through a tube packed with the trihydrate and then through caustic soda, a nitrogen compound could be detected within a few minutes in the soda.

When copper nitrate trihydrate was kept in a vacuum desiccator over phosphorus pentoxide for some weeks, partial dehydration took place, and on opening the desiccator a strong smell of oxides of nitrogen was noticeable. In one experiment some trihydrate was kept for seven months over phosphorus pentoxide and weighed at intervals. The decomposition had not even in this time attained completion. The residue was analysed and found to contain 53·2 per cent. of copper and 9·5 per cent. of nitrogen.

This does not agree with any simple formula, but indicates that the end-point would be a basic nitrate, and there is no reason to expect that it would differ from the basic salt prepared at a higher temperature. When solid potash was used as the dehydrating agent, a similar decomposition was found to take place.

A basic nitrate was also prepared by boiling a concentrated copper nitrate solution with copper oxide, until in the course of twenty-four hours all the black copper oxide had been replaced by a green basic salt. This was collected, washed with hot water, dried, and analysed. One sample was air-dried on tile.

0·283 grms. left on ignition 0·185 grms. of copper oxide.

0·1900 grms. gave 18·5 c.c. of nitric oxide at 760 mm. and 13°.

Found

$$Cu = 51{\cdot}8 \text{ per cent.}, \ N = 5{\cdot}87 \text{ per cent.}$$

A second sample was dried at 100°, and analysed.

0·315 grms. left on ignition 0·210 grms. of copper oxide.

0·1825 grms. gave 15·4 c.c. of nitric oxide, measured at 760 mm. and 13°.

Found

$$Cu = 53{\cdot}1 \text{ per cent.}, \ N = 5{\cdot}26 \text{ per cent.}$$

Calculated for $Cu(NO_3)_2 \cdot 3Cu(OH)_2$,

$$Cu = 53{\cdot}0 \text{ per cent.}, \ N = 5{\cdot}85 \text{ per cent.}$$

The basic nitrate prepared in this way is evidently identical with that prepared by heating the trihydrate. The same basic salt has been made in many ways and seems to be the only basic nitrate of copper.

Copper Oxide and Nitric Anhydride.—One object of these experiments was the preparation of anhydrous copper nitrate, but our results offered little hope of its isolation by dehydrating the trihydrate, even at ordinary temperatures. In all cases dehydration was accompanied by further decomposition and formation of a basic salt.

It was thought possible that copper oxide and nitric anhydride would combine, and experiments were therefore tried, to ascertain if anhydrous copper nitrate could be made in this way. The copper oxide was prepared by ignition of the nitrate, and was strongly heated immediately before use to expel any traces of moisture. Nitric anhydride was prepared from fuming nitric acid and phosphorus pentoxide by the method recommended by E. Gibson (*Proc. Roy. Soc. Edinburgh*), and obtained as hard transparent crystals, which were mixed in a sealed tube with about one-third the bulk of copper oxide. Some of the nitric anhydride still remained after several days, but no change in the copper oxide could be detected. The nitric anhydride decomposed, leaving a yellow liquid, and the decomposition apparently went on at about the same rate as with a sample of nitric anhydride sealed alone in a tube. Incidentally it may be mentioned that liquid nitrogen peroxide was also found to have no action on copper oxide. No action occurred when copper oxide and crystals of nitric anhydride were heated together in a dry test-tube until all the nitric anhydride had evaporated.

Equilibrium between the Hexahydrate and Trihydrate.—There are several references in the literature to the equilibrium conditions of the reaction

$$Cu(NO_3)_2,6H_2O = Cu(NO_3)_2\,,\,3H_2O + 3H_2O,$$

but the only recent and exact work on the subject appears to be the solubility determinations of Funk (*Z. anorg. Ch.*, 20, 412, 1899). According to his results the transition temperature is 24·5°, while the hexahydrate has a metastable melting point at 25·4°.

We obtained by the dilatometer method the following values for the transition temperature:—

$$\left.\begin{array}{l}\text{Rising temperature, } 24{\cdot}7°,\ 24{\cdot}75°,\ 24{\cdot}60°,\ 24{\cdot}60° \\ \text{Falling temperature, } 24{\cdot}65°,\ 24{\cdot}70°\end{array}\right\}\ \text{mean} = 24{\cdot}66°\ \text{(corr.)}.$$

In performing the first dilatometer experiment, it was found that there was a well-marked break at the transition temperature, and when the warming was continued there was a further abrupt break in the curve at 25·6°. This probably indicates the melting point of the (metastable) hexahydrate.

The transition temperature was also determined by the thermometric method, *i.e.* the bulb of the thermometer was imbedded in a tube containing the hexahydrate and the tube placed in a bath. The temperature of the bath was then steadily raised. The thermometer in the copper nitrate indicated a similar steady rise in temperature up to 24·60°, at which temperature it remained constant for some time, and then again rose steadily.

On cooling the temperature fell steadily to 24·65°, where it remained constant for some time, and then again fell regularly. The thermometer used in these experiments was compared with a new Reichsanstalt standard, and the necessary corrections have been applied in the above values.

Vapour Pressures of these Hydrates.—Attempts were made to determine the vapour pressure of the saturated solution, hexahydrate system, by the dew-point method (Cumming, *Jour. Chem. Soc.*, 1909, **95**, 1772), but it was found that the silver cylinder quickly became tarnished. A cylinder was therefore used which was heavily plated with gold, but a deposit formed on this also. The deposit on the gold was found to be acid in reaction, so that the vapour given off from a saturated copper nitrate solution must contain traces of acid. The vapour pressures were not constant, and it appears probable, therefore, that there is considerable hydrolysis in aqueous solution.

It was thought interesting to find the concentration of nitric acid, which would be in equilibrium with a mixture of $Cu(NO_3)_2, 3H_2O$ and $Cu(NO_3)_2, 6H_2O$. The hexahydrate is dehydrated by concentrated nitric acid, and this continued until the nitric acid was about 40 per cent. HNO_3. The trihydrate is hydrated to the hexahydrate by dilute nitric acid, and this process was found to proceed until the concentration of nitric acid had risen to about 20 per cent. HNO_3. At this stage the actions have apparently stopped, so the expected equilibrium has not so far (after several months) been attained. Further experiments, however, on the vapour pressure phenomena are contemplated.

Summary.

1. Graham states that when nitric acid of specific gravity greater than 1·4 acts on copper oxide a basic nitrate is obtained. We were unable to obtain a basic salt under these conditions.

2. Copper oxide and approximately 100 per cent. nitric acid yield copper nitrate trihydrate, nitrogen peroxide, and oxygen. The equation for the reaction is probably

$$CuO + 6HNO_3 = Cu(NO_3)_2, 3H_2O + 4NO_2 + O_2.$$

3. The only basic nitrate of copper appears to be

$$Cu(NO_3)_2 \cdot 3Cu(OH)_2.$$

The product obtained by heating the trihydrate to 100° has this composition, and is not $Cu(NO_3)_2 \cdot 2Cu(OH)_2$, as stated by Graham.

4. Dehydration of copper nitrate does not yield anhydrous copper nitrate, but results in decomposition, with formation of basic copper nitrate, whether the dehydration is performed at ordinary or higher temperatures.

5. Copper oxide does not interact with nitric anhydride.

6. The only hydrates of copper nitrate appear to be the trihydrate and hexahydrate. The transition temperature is 24·65° (±0·05) corr.

7. Concentrated nitric acid dehydrates the hexahydrate, while dilute nitric acid hydrates the trihydrate. It was not found possible to determine the concentration of nitric acid, which would be in equilibrium with both hydrates.

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University of Edinburgh.

(*Issued separately January* 10, 1912.)

III.—On the Reduction of Ferric Iron (1) by Sulphurous Acid and (2) by Zinc Dust. By Alexander Charles Cumming and E. W. Hamilton Smith.

(MS. received November 9, 1911. Read December 4, 1911.)

I. Conditions for Reduction by Sulphurous Acid and Sulphites.

So many papers have appeared on this subject that some apology seems desirable before making an additional contribution. The amount of published work on reduction with sulphurous acid is in itself an indication that many workers have found difficulties. It has been shown that the reduction does not take place in presence of large excess of hydrochloric or sulphuric acid, but the reduction will still occur while the reaction of the solution is strongly acid. On the other hand, Hillebrand ("Analysis of Silicate and Carbonate Rocks," *U.S. Bulletin*, **442**, p. 113) states if the solution after addition of sulphite is red in colour, it is too alkaline and acid must be added.

Some time ago we noticed that sulphur dioxide, passed into a boiling ferric alum solution which contained also a small quantity of free acid, did not reduce the iron even after several hours' treatment. Sulphur dioxide passed into a hot solution caused rapid and complete reduction, if ammonia had previously been added until slight precipitation occurred. While this was perfectly satisfactory from an analytical point of view, the reason was by no means obvious. A pure ferric sulphate solution is acid in reaction on account of hydrolysis, and remains acid after addition of the small amount of ammonia necessary to cause slight precipitation of ferric hydroxide. Furthermore, the precipitate was at once redissolved by the sulphurous acid, and the solution after the reduction contained free sulphuric acid, formed in the reaction, in excess of that formed by hydrolysis. Some further investigation as to the relation between degree of acidity and reduction therefore appeared desirable.

Solutions used.—The following experiments were performed with the same solutions throughout. The permanganate was decinormal, and an approximately decinormal ferric solution was prepared from ferric ammonium alum. Neither of the solutions altered in concentration during the course of the work. A few preliminary experiments had shown the ammonium sulphate had no influence on the reaction. Two sulphurous acid solutions were used, which were found by titration to be approximately 2 N. and 0·3 N. respectively.

Method of Experiment.—In order to make the experiments comparable as far as possible, certain conditions were kept constant in each case. For each titration, 25 c.c. of the ferric solution were taken, and a measured volume of sulphurous acid added. In most cases a measured volume of an acid or alkali of known concentration was added, and in all cases sufficient water was added to bring the total volume up to 60 c.c. The mixture was heated quickly to boiling. When the solution had boiled for ten minutes, 10 c.c. of dilute sulphuric acid (about 4 N.) were added, and a stream of carbon dioxide blown through the boiling liquid until all the sulphur dioxide was expelled. The complete expulsion of the sulphur dioxide required about twenty minutes.

The correct titre was found to be 24·8 c.c.

Reduction was found to be complete under the above conditions when 25 c.c. of the 2 N. sulphurous acid solution was used, whether the reduction occurred after addition of 25 c.c. of ammonia, of 25 c.c. of decinormal hydrochloric acid, or without addition of acid or alkali.

Addition of 10 c.c. of N. sulphuric acid retarded the reduction, so that the titre was 20·1 c.c.

Another set of experiments was performed with addition of varied amounts of acid and with less sulphurous acid than in the preceding experiments.

In each case 25 c.c. of the ferric solution and 25 c.c. of 0·3 N. sulphurous acid were used; a measured volume of 4 N. sulphuric acid was added, together with sufficient water to bring the total volume to 60 c.c.

Acid added = (c.c. of 4 N.)	none	1 c.c. (·2 grm.)	2·5 c.c. (0·5 grm.)	5 c.c. (1·0 grm.)	10 c.c. (2 grms.)
Titre in c.c. =	24·8	24·8	24·2	22·1 20·8	15·8 17·8

Reduction was incomplete in ten minutes if the amount of free acid exceeded 1 c.c. of 4 N. sulphuric acid.

Reduction with Sulphur Dioxide Gas.—The effect of acid on the reduction appeared to be less than in some earlier experiments, in which sulphur dioxide gas had been blown into the solution. A few experiments were tried under as nearly as possible the same conditions to ascertain the difference between reduction with the gas and with solution. In all cases the titre would be 24·8 c.c. if reduction were complete.

I. Sulphur dioxide was passed into the boiling ferric solution for ten minutes. Carbon dioxide was then blown through until no sulphur dioxide could be detected in the vapour. Titre = 24·6 c.c.

II. Sulphur dioxide was passed into the boiling solution, to which 10 c.c. of 4 N. sulphuric acid had been added. The excess of sulphur dioxide was then expelled by a current of carbon dioxide. Titre = 9·4 c.c.

III. The conditions in this experiment were the same as in II., except that the sulphur dioxide was passed into the cold solution, which was then heated. Titre = 18·5 c.c.

IV. Sulphur dioxide was passed for ten minutes into the boiling solution, to which 10 c.c. of 0·02 N. ammonia had been added. Titre = 24·8 c.c. and 24·7 c.c.

These are typical of the results obtained with sulphur dioxide gas, and leave little doubt that the difference between reduction by the gas and by a solution are to be explained by the non-absorption of sulphur dioxide by a hot acid solution.

Effect of Alkali.—Addition of a small amount of alkali appeared to accelerate the reduction, and the effect of varied quantities of alkali was therefore tried. Some experiments with sodium sulphite, sodium bisulphite, and potassium metabisulphite were also tried for comparison, as these have been recommended.

Addition of alkali caused precipitation of ferric hydroxide, but the solution remained acid until all the iron was precipitated. At or about the stage when precipitation was complete the solution was neutral to litmus. The nearer the neutral point was approached the quicker was the reduction, provided that the solution remained acid in reaction towards litmus. If the solution, after addition of the sulphurous acid or sulphite solution, was even faintly alkaline to litmus, no reduction occurred even on boiling for some time. Possibly this explains some discrepancies in work with sulphites. If an alkaline sulphite is added to a ferric solution, rapid reduction occurs if the mixed solution still reacts acid, but if so much sulphite is added that the mixed solution is alkaline, reduction only occurs after acid is added to expel the sulphur dioxide, and is therefore often incomplete.

All reducing agents increase in reducing power as the acidity is diminished or the alkalinity increased, and this reduction up to a certain point conforms to this rule. Neutrality to litmus corresponds (at least approximately) to the completion of precipitation, and it is therefore not surprising that there should be no reduction when the solution becomes alkaline, since there is no longer any iron in solution.

It becomes, therefore, a matter of indifference in practice whether sulphurous acid or a sulphite is used for reduction. The essential for rapid reduction is that the solution should contain little free acid, but the mixed solution should not be alkaline to litmus. In practice one can ensure rapid reduction by addition of sufficient alkali to cause partial precipitation of ferric hydroxide, followed by treatment with sulphurous acid.

II. Note on the Reduction of Ferric Iron by Zinc Dust and by some Zinc Alloys.

Reduction by zinc, even in the granulated form, is slow, and in many respects unsatisfactory. Devada's alloy (an alloy of zinc, copper, and magnesium) has been recommended, and appears to reduce more quickly than zinc. Aluminium in sheet form has been found to reduce quickly and completely, but leaves an objectionable amount of dross. Some experiments in this laboratory indicated that an alloy of zinc with 5 to 10 per cent. of aluminium was an efficient reducing agent for iron, and we have therefore carried out some comparative tests.

All experiments on reduction with alloys were performed in test-tubes of equal size placed in a thermostat at 60°. The rate of reduction was tested in two ways: firstly, by titration of the amount reduced in ten minutes; and, secondly, by finding the time necessary for complete reduction. The alloys were all cast in the same mould, and pieces of as nearly equal size as possible were employed. Three alloys of zinc and aluminium were tried with 5, 10, and 25 per cent. of aluminium respectively. For comparison, commercial "pure" zinc was used. A silver-zinc alloy containing 1 per cent. of silver, and a cadmium-zinc alloy containing 2 per cent. of cadmium were also tested.

The results obtained were somewhat discordant, but from a considerable number of experiments the following conclusions have been reached:—

(1) The reduction appears to be little influenced by the amount of acid, but excess of acid reduces the rate of reduction.

(2) No difference could be detected when hydrochloric acid was used instead of sulphuric acid.

(3) No difference could be detected between the rate of reduction by pure zinc and by the alloys mentioned above.

(4) Individual results varied greatly from the mean, but the variations appeared to be due to difference in the surface exposed. It appeared that the area exposed had an effect which was overwhelmingly large compared with all other factors.

Zinc dust was accordingly tried, and our results confirm those of D. J. Carnegie (*Journ. Chem. Soc.*, 53, 468, 1888), that this is an extremely useful and rapid method of production. Carnegie recommends the use of a "reversed filter," and shows that reduction with zinc dust is extremely rapid.

As our *modus operandi* differs considerably from that of Carnegie, a brief description may be given. The ferric solution should contain as little free acid as possible, the best condition being when there is just enough acid present to prevent precipitation of hydroxide or basic salts. Reduction is complete if this ferric solution is heated to boiling and poured through a narrow calcium chloride tube, packed to a depth of about four inches with fine zinc turnings, which are kept in the tube by a plug of asbestos. The solution is drawn through by suction and the filings and tube washed three or four times with a little, very dilute, hot sulphuric acid solution.

Fine zinc turnings are not always obtainable, and the following method of reduction with zinc dust was found to be equally satisfactory if the unused zinc dust was efficiently washed.

To the ferric solution in a boiling tube about 1 gramme of zinc dust was added, the mixture heated quickly to boiling, and at once poured through an ordinary filter paper, on which about 1 gramme of zinc dust had been placed. Reduction under these conditions was complete, but thorough washing with hot dilute $\left(\text{about } \frac{m}{10}\right)$ sulphuric acid was necessary to remove the last of the iron from the zinc dust. If the ferric solution contained so much acid that vigorous effervescence occurred in the cold, reduction was usually incomplete; any excess of free acid must therefore be neutralised with alkali before reduction. It is convenient to use suction for the filtration, but care must be taken that the end of the filter funnel projects at least 4 to 5 cms. below the side tube of the filter-flask, as otherwise part of the solution is carried into the pump.

Zinc dust is usually almost free from iron, and the amount used in a reduction, as above described, is so small that the error from this source will usually be negligible.

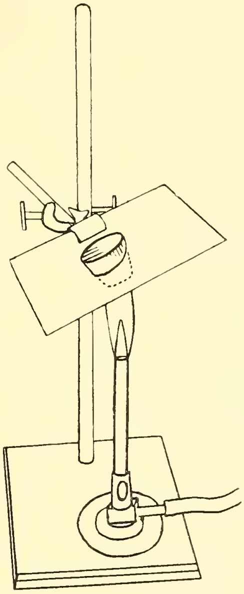
Chemistry Department,
University of Edinburgh.

(*Issued separately January* 10, 1912.)

IV.—Note on a Perforated Silica Plate for excluding Flame Gases from a Crucible during Ignition. By Alexander Charles Cumming.

(MS. received November 9, 1911. Read December 4, 1911.)

The device shown in the accompanying sketch has been in use in the Chemistry Department of the University of Edinburgh for some time. It consists of a silica plate, 5 inches square, with a hole bored in it of such



size as to admit a crucible to one-half of its depth. The silica plate is held in an inclined position by a clamp.

By this means the flame gases are excluded from the interior of the crucible during an ignition. With this device calcium carbonate in a platinum crucible is quickly reduced to oxide with a good bunsen burner, while with a Mecker burner the reduction is complete in a few minutes even when a porcelain crucible is used. The device is also useful for cases such as the ignition of nickel oxide, where there is a danger of reduction. For the estimation of sulphur in coal some such device is absolutely necessary to exclude the flame gases.

The original idea is due to J. Löwe, who used clay discs. Hillebrand

(*U.S. Geological Survey Bulletin*, **442**, 32) has suggested the use of an asbestos sheet with a perforated sheet of stout platinum foil in the centre. The silica plate has the advantage of cheapness and rigidity, and has been found in practice to be very useful in quantitative analysis.

The holes were bored on a lathe by means of a copper tube of appropriate size, the cutting end being covered with carborundum and lubricant. Doubtless the manufacturers of silica ware would supply these perforated plates.

Chemistry Department,
University of Edinburgh.

(*Issued separately January* 31, 1912.)

V.—Observations on the Body Temperature of some Diving and Swimming Birds. By Sutherland Simpson, M.D., D.Sc.

(From the Physiological Laboratory, Medical College, Cornell University, Ithaca, N.Y., U.S.A.)

(MS. received October 10, 1911. Read November 20, 1911.)

CONTENTS.

I. Introduction.

The observations recorded in the following pages were begun as far back as 1903, and at that time the intention was to investigate the influence which the different seasons of the year may have on the body temperature of warm-blooded animals. With the exception of the group of hibernating animals, birds appear to show more distinct and definite seasonal changes in their habits and bodily activities than mammals, and for this reason they were chosen as the most suitable subjects for the inquiry.

Naturally one would expect that any seasonal rhythm which might be present would be more evident in wild than in domesticated birds, the latter being shielded from the rigours of climate, and living all the year round in surroundings more or less artificial; and as several species of marine birds, which are very numerous in northern waters, are non-migratory and comparatively easily secured, it was planned to make one series of observations on these in the mating and breeding season, during the months of March and April, and another set in August and September when the birds are moulting.

Various circumstances arose, however, which made it impossible to carry out the original plan, still, many observations have been made in the course of four or five years and the records collected; and since comparatively little is known regarding the body temperature of several of the species examined, it may be of interest to some to give these records publication as they stand.

II. Previous Observations on the Subject.

The earliest temperature records of marine birds are probably those made by John Davy * on a voyage between England and Ceylon some time between 1816 and 1820. These were obtained from two species of the petrel.

"*Procellari æquinoctialis.*—In latitude N. 2° 3′, on the 8th of August, air 79°, sea 81°·5, the temperature of this bird was 103°·5, and that of another 105°·5."

"*P. capensis.*—In latitude S. 34° 1′, on the 11th of May, air 59°, sea 60°, the temperature of two birds of this kind was 105°·5."

While on a voyage around the world on a Government ship in the years 1836 and 1837, two Frenchmen, Eydoux and Souleyet,† recorded the rectal temperature of the greater albatross, the sheathbill (*Chionis minor*), and three species of petrel. Their observations were made, for the most part, in high Southern latitudes, off Cape Horn and the Cape of Good Hope, in cold weather. No statement is made as to how the birds were caught, but the thermometer was introduced into the rectum while they were still alive. The following table shows the figures they obtained. Each observation was made on a different individual:—

	Observations.	Maximum.	Minimum.	Mean.
Albatross	4	40°·3 C.	39°·6 C.	39°·9 C.
Petit albatross	1	...	...	41·2
Grand „	11	40·5	38·0	39·4
Pétrel damier	3	42·0	40·0	40·7
„ gris	2	39·7	39·5	39·6
Grand pétrel noir . . .	2	39·8	39·6	39·7
Chionis	1	...	...	40·0

A year or two subsequent to this a much more extended series of observations on the body temperature of diving and swimming birds was made by another Frenchman, Martins,‡ on a cruise in a French man-of-war to the Faroe Islands and Spitzbergen in the summer of 1838, and again in 1840. His results, however, like those of his predecessors, are open to criticism on one point in particular. He does not seem to appreciate the fact that severe muscular effort quickly raises the body temperature; at any rate he takes no account of this factor as a possible source of error in

* John Davy, *Researches, Physiological and Anatomical*, Philadelphia, 1840, p. 303.

† Eydoux and Souleyet, *Comptes rendus de l'Acad. des scien. de Paris*, t. vi., 1838, p. 458.

‡ Martins, *Jour. de Physiologie*, t. i., 1858, p. 10.

his readings. He had a special thermometer constructed for the expedition giving direct readings to the hundredth of a degree centigrade, and made *very strong*, presumably to withstand the strain put upon it while in the rectum, due to the struggles of the bird. It was not a maximal or clinical thermometer, and required to be read *in situ*.

He does not give much detail as to how the birds were secured, but they were all examined alive and uninjured with a few exceptions. Those that were wounded, unless only very slightly, were discarded. Some were caught on a baited hook and line and the rectal temperature taken immediately after they were hauled on board. All these wild birds must have been greatly agitated, and must have struggled violently when forcibly held while the thermometer was being introduced into the cloaca and rectum (where it was retained not less than three minutes) and the temperature recorded, although Martins makes no mention of this circumstance and makes no allowance for any possible error that might arise from it. Notwithstanding this, he found that the rectal temperature of these wild birds was distinctly lower than that of the domestic duck.

He also made a large number of observations on the domestic duck and goose in different parts of the country (France), and on many species of the Palmipedes confined in the Zoological Gardens in Paris, and from these he drew important deductions regarding the influence of sex, age, external temperature, and alimentation on the body temperature.

With regard to sex, he found that the mean rectal temperature of fifty drakes was 41°·915 C. and that of sixty ducks (female) 42°·264 C., showing a difference in favour of the female of 0°·349 C. While the mean figure was higher, however, the temperature of the female showed greater variation, the range (difference between the highest and the lowest readings of the whole flock) being almost double that found in the males—2°·55 C. and 1°·80 C. respectively.

The influence of age is not appreciable. Ducklings hatched in the spring are considered to have reached the age of puberty in the autumn, that is, when about six months old. In two flocks of ducks, males and females mixed, the one including thirty-one birds from four to six months old, the other thirty-seven birds from seven months to two years old, he found the mean temperature to be almost identical—42°·012 C. and 42°·011 C. respectively. Throughout the first two years of life, therefore, the temperature does not appear to be affected by the age of the individual. In the first group, even before the females had begun to lay eggs, there was a very slight sex difference in favour of the female, but not nearly so great as in the second group of older birds, as the subjoined table shows:—

31 *ducks, 6 months old and under.*		37 *ducks, from 7 months to 2 years old.*	
Mean temperature of 16 males	41°·998 C.	12 males	41°·745 C.
,, ,, 15 females	42°·025 C.	25 females	42°·278 C.
Difference	0°·027	Difference	0°·533

From a comparison of these two sets of figures it may be concluded that the temperature of the female rises after puberty, while that of the male falls.

The temperature of the surrounding medium was not found to have much effect on the body temperature of the birds. From a series of observations made on nine individuals—five ducks and four drakes,—first in winter, when the temperature of the air was below zero, and second in summer, when the outside temperature was 20° C. and over, he concluded that there was a difference of 0°·167 C. in the mean rectal temperature in favour of the winter, the figures being 42°·195 C. (winter) and 42°·028 C. (summer). He is inclined to believe, however, that this apparent rise of the body temperature in winter is not so much influenced by the temperature of the surroundings as by the increased activity of the bodily functions, since "l'époque des amours coïncide avec celle des plus grands froids de l'hiver, et l'excitation qui en resulte entretient la chaleur en activant la circulation. Je ne serais donc pas étonné qu'on né trouvât pas de différence entre la température des Palmipèdes, en hiver et en été." This, he believes, more than counteracts the fall in body temperature which might be expected to arise from the excessive cold, were that influence operating alone. It seems, however, that the number of individuals and of observations on them are much too few to form the basis of any such far-reaching conclusion.

Regarding the influence of diet, he agrees with Chossat's * findings in pigeons, viz.—that in birds deprived of solid food for five days there was a decided fall at the end of twenty-four hours; this was followed by a very slight rise during the three successive days, and at the end of the fifth day, when the experiment was stopped, there was again a slight fall.

Of the possible influence of muscular exercise on the body temperature he makes no mention, and does not take this factor into consideration at all as affecting his results in any way. This omission detracts a good deal from the value of his work, more particularly in the case of the wild birds which he examined.

In 1854 Brown-Séquard † had the opportunity of examining the body temperature of some Palmipedes in the South Atlantic off the Cape of Good

* Chossat, *Annales des sciences naturelles*, 2nd series, t. xx., 1843, pp. 181 and 294.

† Brown-Séquard, *Journ. de Physiol.*, t. i., 1858, p. 42.

Hope, and he was surprised, as the title of his communication implies ("Note sur la basse température de quelques Palmipèdes longipennes"), that the temperature should be lower than that of most other birds, considering the active habits of these Palmipedes and the vigorous and sustained flights of which they are capable. He was inclined to account for the low figures which he found by supposing that by some chance the individuals which he happened to secure had been subjected to an involuntary fast, but there was no good reason for such a supposition.

His observations were made on two species of albatross (*Diomedea exulans* and *D. chlororhynchos*), the Cape petrel (*Procellaria capensis*), three species of the Stercorariidae (jaegers and skuas)—one *Larus catarrhactes*, and two others undetermined. The albatrosses and skuas were caught with a baited hook and line, and had no other injury except that produced by the hook; the others were first wounded in the wing by a shot-gun and then captured alive. In the latter the rectal temperature was taken from ten to fifteen minutes after the birds were shot, but while they were still alive. Like his predecessors in the work, Brown-Séquard makes no statement as to whether the birds remained quiet or struggled immediately before or while the temperature was being recorded.

The mean of nine observations on as many individuals in the two species of albatross was 40°·75 C., the maximum being 41°·9 and the minimum 39°·6; in four specimens of the Cape petrel the mean was 39°·77, —maximum 40°·2, minimum 39°·4; in *Larus catarrhactes* * (*Le cordonnier*) the mean of six observations on six individuals was 40°·15, with a maximum of 40·8 and a minimum of 39°·6; and in two other specimens of the same family, species undetermined, the figures were 41°·5 and 41°·1, giving a mean of 41°·3. These observations were all made on the same day (6th April) with the temperature of the air at 23° C., in latitude 32° South.

The species examined by Brown-Séquard all belong to the order Longipennes, and the temperatures are somewhat higher than Martins found for the members of the same family in the Northern hemisphere. Brown-Séquard believes that this difference may be due to the influence of the warmer climate, Martins' observations having been made where the air temperature was in the neighbourhood of zero centigrade.

III. Methods adopted in Present Investigation.

The greater number of the birds on which the present series of observations were made were secured amongst the Orkney Islands, in the sounds

* According to the British Museum *Catalogue of Birds*, 1896, this is the common skua (*Megalestris skua*).

and bays which border the northern shores of the Pentland Firth. In these waters the marine birds are very abundant, particularly the guillemot and the shag. The cormorant appears to be somewhat scarce, and the different species of the sea-gull, although numerous, are very wary and not easily approached.

No attempt was made to catch the birds alive, since it was recognised that the excitement immediately following capture and the violent movements which they would certainly make in their attempts to regain freedom would, even in a few minutes, produce a distinct rise in the body temperature. They were shot dead at close range and the thermometer introduced into the rectum, usually in less than half a minute, and frequently within a few seconds, after death. In this short interval it is assumed that no appreciable loss of heat from the body took place, and that the figure recorded indicates the rectal temperature immediately before death; at all events, it is pretty certain that any error introduced by this method would be much smaller than what would be found if the birds had been captured and handled alive.

The shooting was done for the most part from a small sail-boat, in a smooth sea, and with a light breeze of wind. The stalking of these birds in the water requires some skill on the part of the boatman, since they are very watchful and readily take alarm. The shags and guillemots are almost always seen in the water, where they swim and dive to perfection. In flight the wings are moved rapidly, after the manner of the duck tribe, and one always has the impression that the bird is making a great effort in maintaining itself in the air. Considerable difficulty is experienced in starting; they flap along the surface of the water for some distance, and fill their wings preferably by rising into the air against the wind, and advantage is taken of this fact in getting within short gunshot range.

The shags usually fish in small flocks of from two to six or eight, or more; and when such a flock is sighted, the boat is steered, not straight for the birds, but to the windward side of them, and at such a distance as not to cause them to take alarm. Then the course is suddenly changed and the boat is held before the wind directly for the flock. When danger is scented the shag usually seeks escape by flight, not by diving, and in rising from the water against the wind it meets the boat, and is shot dead at close range. I mention this to show that the birds were not chased before being killed. Those that succeeded in diving after being wounded, and required to be followed up and killed by a second shot, are not included in the subjoined tables. No wounded bird was allowed to escape.

Practically all the birds were killed with the first shot and picked up from the water a few seconds afterwards, when a half-minute clinical thermometer (Kew certificate) was introduced into the rectum to the depth of three inches, and at the end of three minutes the temperature was recorded. Three or four thermometers were carried, so that in the event of several birds being killed at the same time there should be no delay in taking the temperature. The temperature of the air and of the sea at the surface was also recorded.

Unlike the shag and cormorant, the guillemot almost always seeks safety by diving. It is not difficult to approach, but it has such keen sight and is so quick in its movements that it frequently "dives at the flash," as the boatmen express it; that is, after seeing the flash of the gunpowder it can vanish below the surface of the water before the shot has reached the spot where it was. This is particularly true of the black guillemot or tystie.

Most of the sea-gulls were shot while flying, since they search for food and feed mainly on the wing. Their flight is so easy, however, that it does not appear to involve great muscular effort, and a gull on the wing is probably not doing more work than a shag or guillemot swimming and diving, which is really equivalent to flying under the water, since both wings and feet are used.

The solan geese were examined alive, as these birds are protected in the breeding season by the Wild Birds Protection Act, and the shooting of them is illegal. One was taken in the Orkneys on a calm day in the end of September 1907. It was not able, in the absence of wind, to raise flight from the water, and was captured after a short row. This was a young specimen, hatched probably in May.

The other six were taken off their nests on the Bass Rock, June 24, 1908. During incubation and while taking care of her young the gannet is very tame, and will sit on the nest till lifted from it. One of the assistant lighthouse-keepers, at considerable risk, descended the cliff on the east side of the Bass by means of a rope, and succeeded in securing six birds, taking them off their nests and bringing them to the top of the crag one by one, where the rectal temperature was taken by the writer. The temperature of an incubating bird is somewhat below the normal, but this is not the case after the eggs have been hatched.

One would have expected that it would be a difficult task to hold these powerful birds while the thermometer was being introduced into the rectum and the temperature recorded, but as a matter of fact they scarcely made any effort to escape; they seemed dazed and stupefied all the while, and the whole procedure was carried out without any assistance. The specimen secured in the Orkneys struggled violently when captured, and the temperature was not recorded till some hours afterwards, when it had become somewhat accustomed to its surroundings.

While on a voyage from New York to Liverpool on the R.M.S. *Mauretania*, a storm-petrel (*Procellaria pelagica*), attracted by the lights, flew on board and was caught on the deck. This was at 10.30 P.M. on July 24, 1908, in N. latitude 41° 58′, W. longitude 46° 22′—that is, in the North Atlantic, about mid-ocean. The rectal temperature was taken, and found to be only 103°·6 F. (39°·8 C.). The bird was then set at liberty. The temperature of the air and of the sea at the time was in the neighbourhood of 70° F. (21° C.). Another specimen came on board about the same time, but flew away again before it could be secured.

In the spring and autumn of 1909 one grebe and a few wild ducks of different species were shot on and in the vicinity of Lake Cayuga, in the western part of the State of New York, and the rectal temperature taken in the usual way. After the winter migration south, from October or November on to April or May, large numbers of wild ducks frequent this and the neighbouring lakes, but they are very difficult to approach and comparatively few were obtained. They were killed either while swimming, or flying immediately after having risen from the water.

In the cormorant, shag, guillemot, and sea-gull the sexes are alike in plumage, so far as I know, and cannot be easily distinguished without dissection, unless possibly by an expert ornithologist. In a number of cases, however, the birds were marked and the sex determined later by post-mortem examination, but this could not always be done. The six specimens of the solan goose taken from the nest were probably females. In all species of the duck tribe the male is more brilliantly marked than the female, and there is no difficulty in telling the one from the other.

The results obtained are presented in the following tables :—

Table I.—Records of Body Temperature of some Diving and Swimming Birds taken in the Orkney Islands and Firth of Forth, Scotland, Cayuga Lake, N.Y., U.S.A., and in the North Atlantic (Mid-Ocean).

In the Orkney Islands.

	Rectal Temperature.	Temperature of Air.	Temperature of Water.
December 25, 1903, 12.30 *to* 2 *p.m.*	...	$-2^\circ 0$ C.	$6^\circ 5$ C.
Shag (*Phalacrocoras graculus*)	39·9 C.	...	...
Cormorant (*Phalacrocoras carbo*)	40·2	...	...
Black guillemot (*Uria grylle*)	41·1	...	...
April 20, 1904, 2 *to* 4 *p.m.*	...	...	...
Shag	40·4	...	...
"	40·1	...	...
"	39·4	..	...
"	39·5	...	...
Sea-gull (*Larus canus*)	41·6	...	...
September 13, 1905, 1 *to* 5 *p.m.*	...	...	...
Shag ♂	39·4	...	...
" ♀	40·1	...	...
" ♀	40·3	...	...
"	39·5	...	...
"	39·8	...	...
"	39·7	...	...
Common guillemot (*Uria troile*) ♂	41·2	...	...
" " " ♀	40·1	...	...
" " " ♀	40·8	...	...
Black " (*Uria grylle*) ♂	40·1	...	...
" " " ♀	41·1	...	...
" " " ♀	40·8	...	...
September 16, 1905, 3 *to* 6 *p.m.*	...	13	12·1
Sea-gull (*Larus canus*)	41·9	...	...
" "	42·1	...	...
Kittiwake (*Rissa tridactyla*)	41·4	...	...
Cormorant	38 9	...	...
"	39·2	...	...
Shag	39·6	...	...
"	39·8	...	...
"	40·4	...	...
"	40·2	...	...
Common guillemot	40·5	...	...
Black "	40·7	...	...
" "	39·9	...	...
" "	41·1	...	...
September 9, 1907, 12 *to* 2 *p.m.*	...	15	11·4
Shag ♂	40·9	...	...
" ♂	39·6	...	...
" ♀	39·8	...	...
" ♀	39·6	...	...
"	39·8	...	...
"	40·8	...	...

Table I.—*continued.*

	Rectal Temperature.	Temperature of Air.	Temperature of Water.
	°	°	°
September 9, 1907, 12 *to* 2 *p.m.*	...	15	11·4
Cormorant	39·8	...	...
"	40·2	...	...
"	39·5	...	...
Black guillemot ♂	40·4	...	...
" " ♀	40·7	...	...
Sea-gull (*Larus canus*)	40·9	...	...
" "	41·4	...	...
September 10, 1907, 2 *to* 4 *p.m.*	...	21	11·8
Shag	40·7	...	...
"	39·9	...	...
"	40·9	...	...
Common guillemot	40·4	...	...
Black "	42·1	...	...
" "	41·5	...	...
" "	42·0	...	...
Sea-gull (*Larus canus*)	42·1	...	...
Kittiwake (*Rissa tridactyla*)	42·0	...	...
" "	40·9	...	...
September 21, 1907, 4 *to* 6 *p.m.*	...	8	11·4
Shag	41·0	...	...
"	40·3	...	...
Cormorant	39·9	...	...
Black guillemot ♂	40·8	...	...
" ♀	41·0	...	...
"	40·9	...	...
Sea-gull (*Larus canus*)	41·8	...	...
" "	42·1	...	...
September 23, 1907, 2 *to* 5 *p.m.*	...	14	11·2
Shag ♂	40·5	...	...
" ♀	41·2	...	...
" ♂	40·7	...	...
" ♀	39·8	..	...
" ♀	40·8	...	...
"	40·5	...	...
"	41·0	...	...
Cormorant	39·8	...	...
"	39·2	...	...
Black guillemot ♂	40·8	...	...
" ♀	40·2	...	...
September 24, 1907, 12 *to* 1 *p.m.*	...	15	11·1
Shag	41·2	...	...
"	40·7	...	...
Gannet (*Sula bassana*)	40·3	...	...
September 27, 1907, 2 *to* 6 *p.m.*	...	12	11·4
Scoter duck (*Œdemia nigra*) ♂	40·9	...	...

TABLE I.—*continued.*

	Rectal Temperature.	Temperature of Air.	Temperature of Water.
September 27, 1907, 2 *to* 6 *p.m.*	° ...	° 12	° 11·4
Scoter duck (*Œdemia nigra*) ♀ . .	41·7	...	...
Shag ♂	40·9	...	...
" ♀	40·4	...	...
" ♀	41·1	...	...
"	40·8	...	...
"	41·0	...	...
"	40·9	...	...
Cormorant (shot on wing) * . . .	42·5	...	...
Black guillemot ♂	40·6	...	...
" ♀	41·1	...	...
" ♀	41·5	...	...
" ♂	41·0	...	...
"	39·8	...	...
"	41·2	...	...
Sea-gull—lesser Blackback—(*Larus fuscus*)	41·6	...	...
Kittiwake (*Rissa tridactyla*) . . .	41·7	...	...
" " . . .	42·1	...	...
September 28, 1907, 10 *to* 12 (*noon*) . . .	...	14	11·6
Shag ♂	40·8	...	...
" ♀	40·9	...	...
" ♀	40·2	...	...
" ♀	41·1	...	...
"	41·4	...	...
"	39·8	...	...
Cormorant	40·1	...	...
"	40·3	...	...
Kittiwake	41·4	...	...
"	41·9	...	...
Sea-gull (*Larus fuscus*)	41·3	...	...
" "	42·1	...	...
" (*Larus argentatus*) . . .	42·4	...	...
September 28, 1907, 4 *to* 6 *p.m.*	...	15	11·2
Shag	40·8	...	...
Cormorant	40·1	...	...
Common guillemot ♂	39·1	...	...
" " ♀	39·8	...	...
March 14, 1908	...	9	6·7
Shag	40·9	...	...
Common guillemot	40·8	...	...
Black "	40·8	...	...
IN THE FIRTH OF FORTH OFF MUSSELBURGH.			
November 16, 1907, 2 *to* 4 *p.m.*† . . .	...	10	9·4
Kittiwake	41·8	...	...

* This bird had been flying for a considerable time, hence the high temperature. For this reason, in calculating the mean temperature of the cormorant, this figure is excluded.

† On November 23, a pair (male and female) of velvet scoter ducks (*Œdemia fusca*) were shot, but the notes of the body temperature have been lost.

Table I.—*continued.*

	Rectal Temperature.	Temperature of Air.	Temperature of Water.
January 3, 1908, 1 *to* 3 *p.m.*	...°	1°	4·7°
Common guillemot	39·5	...	...
Kittiwake (*Rissa tridactyla*)	40·8	...	...
" "	41·4	...	...
" "	42·4	...	...
" "	41·4	...	...
" "	41·4	...	...
" "	39·9	...	...
" "	42·1	...	...
" "	40·8	...	...
February 15, 1908, 1 *to* 4 *p.m.*	...	5	4·6
Razorbill (*Alca torda*) ♀	41·1	...	...
" " ♂	39·9	...	...
Common guillemot	40·1	...	...
" "	40·8	...	...
" "	40·8	...	...
On Bass Rock.			
June 24, 1908, 12 *to* 1 *p.m.*	...	16	...
Gannet (*Sula bassana*). (Taken off nest)	40·8	...	...
" "	41·4	...	...
" "	42·0	...	...
" "	42·3	...	...
" "	41·5	...	...
" "	41·8	...	...
In North Atlantic (Mid-Ocean).			
July 24, 1908, 10.30 *p.m.*	...	*21	21
Storm Petrel (*Procellaria pelagica*)	39·8	...	...
Cayuga Lake, N.Y., U.S.A.			
February 28, 1909, 2.30 *p.m.*	...	3	2·8
Whistler or American Golden-eye duck (*Clangula clangula americana*) ♂	40·4	...	...
March 18, 1909, 10 *a.m.*	...	−2	1·3
Black duck (*Anas rubripes*) ♀	41·5	...	...
" " ♂	40·9	...	...
" " ♂	40·7	...	...
April 9, 1909, 10 *to* 11 *a.m.*	...	6	3·2
Horned grebe in winter plumage (*Colymbus auritus*) ♂	40·7	...	...
Wood duck (*Aix sponsa*) ♂	42·0	...	...

* The temperature of the air and sea was about 70° F.

Table I.—*continued.*

	Rectal Temperature.	Temperature of Air.	Temperature of Water.
May 5, 1909, 12.30 *to* 1 *p.m.*	...°	18°	16·6°
Lesser blue bill duck (*Marila affinis*) ♂ .	41·6	...	...
,, ,, ,, ♂ .	42·3	...	...
,, ,, ,, ♂ .	42·6	...	...
,, ,, ,, ♀ .	42·4	...	...
November 25, 1909 (*snowstorm*), 2 *p.m.* . . .	..	0	1·2
Sawbill duck (*Merganser*, species undetermined) ♂	41·4	...	...
Sawbill duck (*Merganser*, species undetermined) ♀	41·7	...	...

Table II.—Records of Body Temperature for each Species.

Species.	Number of Observations.	Maximum.	Minimum.	Mean.
Storm petrel (*Procellaria pelagica*).	1	...°	...°	39·80°
Cormorant (*Phalacrocorax carbo*)	12	40·3	38·9	39·77
Shag (,, *graculus*)	49	41·4	39·4	40·38
Razorbill (*Alca torda*)	2	41·1	39·9	40·50
Common guillemot (*Uria troile*)	12	41·2	39·1	40·33
Black ,, (,, *grylle*)	24	42·1	39·8	40·88
Horned grebe (*Colymbus auritus*)	1	...	...	40·70
Gannet or solan goose (*Sula bassana*) . . .	7	42·3	40·3	41·44
Scoter duck (*Œdemia nigra*)	2	41·7	40·9	41·30
Whistler ,, (*Clangula clangula americana*) . .	1	...	...	40·40
Black ,, (*Anas rubripes*)	3	41·5	40·7	41·03
Wood ,, (*Aix sponsa*)	1	...	...	42·00
Lesser bluebill or scaup duck (*Marila affinis*) .	4	42·6	41·6	42·22
Sawbill duck (*Merganser*— ?)	2	41·7	41·4	41·55
Common gull (*Larus canus*)	8	42·1	40·9	41·74
Lesser blackback gull (*Larus fuscus*) . . .	3	42·1	41·3	41·67
Herring gull (*Larus argentatus*)	1	...	...	42·40
Kittiwake (*Rissa tridactyla*)	16	42·4	39·9	41·46

Table III.—Records of Body Temperature for each Order.

Order.	Number of Observations.	Maximum.	Minimum.	Mean.
		°	°	°
Tubinares (petrel)	1	...	...	39·80
Steganopodes (gannet, cormorant, shag) . .	68	42·3	38·9	40·53
Family *Sulidæ* (gannet)	7	42·3	40·3	41·44
„ *Phalacrocoracidæ* (cormorant, shag) .	61	41·4	38·9	40·07
Pygopodes (Grebe, guillemots, razorbill) . .	39	42·1	39·8	40·60
Anseres (ducks)	13	42·6	40·4	41·55
Longipennes (gulls, kittiwake)	28	42·4	39·9	41·59

Table IV.—Records showing Sex Differences in the Body Temperature.

Species.	Individuals.		Mean Rectal Temperature.	
	Males.	Females.	Male.	Female.
Shag	7	12	40°·4 C.	40°·44 C.
Common guillemot . . .	2	3	40·15	40·25
Black „ . . .	6	7	40·62	40·91
Razorbill	1	1	39·9	41·1
Ducks (several species) . .	9	4	41·52	41·82

IV. Results and Conclusions.

Influence of Sex.—With regard to the body temperature in the two sexes, there is in every species examined a difference in favour of the female,—sometimes very slight, but still a difference. In order to make a general statement in this relation, however, a greater number of individuals, and these under similar conditions with regard to environment, hour of day, etc., would require to be inspected.

Of former observers who have considered the question of sex in relation to body temperature, most find that in the female it is higher than in the male, but that the variations from the mean are greater in the female. In the human subject very little difference is believed to exist, but where a sufficient number of observations have been made it is stated that the temperature of the woman is the higher. On the authority of John Davy * a statement to the contrary is often made in the text-books. Davy found that the advantage lay with the male sex (human subject), but he only examined six individuals (three women and three men)—a number totally insufficient to enable him to arrive at such a general conclusion.

* John Davy, *Med. Times and Gaz.*, London, vol. ii., 1864, p. 337.

According to Roger * the sex influence shows itself even before the age of puberty. He found in ten boys a mean rectal temperature of 37°·107, while in fourteen girls of the same age (before puberty) the figure was 37°·191, giving a difference of scarcely one-tenth of a degree centigrade. Ogle † states that in the adult the difference in the mean temperature does not exceed half a degree Fahrenheit (about 0°·3 C.), the woman having the higher figure.

In the case of the domestic fowl, Davy ‡ found that for three cocks the mean temperature was 42°·4 C. and for three hens 42°·1 C. Here again his conclusions are probably erroneous, for the reason that they are based on the examination of an insufficient number of individuals. As already mentioned, Martins § found that in the domestic duck the male had a mean rectal temperature of 41°·91 and the female of 42°·27. He obtained his figures from fifty drakes and sixty ducks. In this relation it may be mentioned that the writer, in the course of another investigation, has made hundreds of observations on the rectal temperature of the domestic fowl, and from these it is shown that the temperature of the hen is slightly higher than that of the rooster. Similarly, in the case of wild birds belonging to orders other than those under consideration in the present paper, Simpson and Galbraith || found that the mean temperature of the female was higher than that of the male in all the species examined whose sex had been determined.

Body Temperature in Relation to Position in the Zoological Scale.—In the conclusion to an article on "The Temperatures of Reptiles, Monotremes, and Marsupials" Sutherland ¶ says: "It is clear, therefore, that there are grades of temperature, and that the mammals which are classed lowest on anatomical grounds are not only of the lowest temperature but also of the greatest range. . . . Similar, though much less complete, connecting links may be seen in the case of birds. The lowest of birds are the Ratitæ or Cursores, and these appear to have the lowest temperature. . . ." Observations on the temperature of the emu were made for him in the Melbourne Zoological Gardens, and "these are the lowest records of bird temperatures of which I know. They averaged 39°·5 C., while all the birds above the Ratitæ are invariably over 40° C." He goes on to show that if the birds were arranged in the order of their body temperatures the series would probably run parallel with the zoological series.

* Roger, Richet's *Dictionnaire de Physiologie*, t. 3, p. 96.

† Ogle, Wunderlich's *Medical Thermometry*, p. 99.

‡ Davy, *loc. cit.*

§ Martins, *loc. cit.*

|| Simpson and Galbraith, *Jour. of Physiol.*, vol. xxxiii., 1905, p. 225.

¶ Sutherland, *Proc. Roy. Soc. Victoria*, 1896, p. 57.

In the light of these remarks it is interesting to examine the mean temperature for each order as shown in Table III. Here it is seen that the Tubinares have the lowest temperature and the Longipennes the highest. Arranged according to body temperature, the order from below upwards is: Tubinares, Steganopodes, Pygopodes, Anseres, Longipennes, while the zoological series, ascending, would be Pygopodes, Longipennes, Tubinares, Steganopodes, Anseres. This is not in agreement with the idea held by Sutherland. In the mammalia, above the marsupials and monotremes, it is a well-known fact that this law does not hold good; there is no parallelism between the body temperature of an animal and its anatomical position, for in the temperature scale *Homo sapiens*, with one exception—the ass—occupies the lowest place, while in the zoological scale he is the highest. The same appears to be the case amongst the class Aves, above the Ratitæ.

It is true that only one specimen of the Tubinares was examined—the storm petrel—and it would be rash to give the number obtained from a single observation as the mean temperature of the whole order, were it not for the fact that this figure agrees closely with that found by previous observers. Eydoux and Souleyet * give 40°·7, 39°·6, and 39°·7 as the mean temperatures of three species of the petrel family, and the mean of five observations by Martins † on as many individuals of another species (*Procellaria glacialis*) is lower still, viz. 38°·76. There seems to be no doubt, then, that amongst the web-footed birds the Tubinares, or rather the family Procellariidæ of this order, have the lowest temperature.

Even families of the same order appear to differ considerably in body temperature. In the Steganopodes the Sulidæ (gannets) have a mean temperature of 41°·44 C., while for the members of the Phalacrocoracidæ which were examined (cormorant and shag) it is much lower, viz., 40°·07 C. The comparison here, however, is hardly fair, since in the gannet the observations were made on birds that were alive, although they remained quiet throughout the whole operation, while in the cormorant and shag the temperature was taken immediately after death, and it may be that this would account for the difference.

In order to find how the temperature of the wild duck compares with that of the domesticated variety, I obtained access to a small flock of mallards kept in a pen at the New York State Agricultural College, Cornell University. The colony consisted of three drakes and six ducks, from one to two years old. They were three generations from the wild form, and could be handled easily. The observations recorded below were made on April 5, 1909, between 2 and 3 p.m., with the air temperature

* Eydoux and Souleyet, *loc. cit.* † Martins, *loc. cit.*

16°·5 C. in the shade. Care was taken that they should not be unduly exercised before the thermometer was introduced.

Ducks.		Drakes.	
I.	41°·1 C.	I.	41°·6 C.
II.	42·0	II.	41·8
III.	41·6	III.	41·1
IV.	41·7	Mean	41·5
V.	41·4		
VI.	41·8		
Mean	41·6		

Here there is practically no difference between the mean temperature of the tame ducks alive and that of the wild ducks immediately after being shot. In the latter there is a greater range (minimal 40°·4, maximal 42°·6), but this one would expect considering that in the one group the observations were made on the same day and at the same hour, and in the other under widely different conditions regarding hour of day and outside temperature, etc. It is not probable, therefore, that the difference in temperature between the Sulidæ and the Phalacrocoracidæ is to be entirely accounted for by the methods adopted in taking the temperature, although it may be to some extent.

V. Summary.

Observations were made on the body temperature of a large number of diving and swimming birds of eighteen different species in the Orkney Islands and Firth of Forth, Scotland, and on and around Cayuga Lake, N.Y., U.S.A., immediately after they were killed by shooting.

1. In all the species examined, where the sex was determined, it was found that the rectal temperature of the male was slightly below that of the female.

2. Of the orders examined the highest temperatures were found in the Longipennes and the lowest in the Tubinares. When arranged according to body temperature the series does not run parallel with the zoological series.

In conclusion, I wish to express my indebtedness to Dr H. D. Reed and Dr A. H. Wright for help in identifying the specimens obtained from Cayuga Lake and the surrounding district.

(*Issued separately February* 16, 1912.)

VI.—Note upon the Structure of Ternary Alloys. By G. H. Gulliver, B.Sc., A.M.I.Mech.E., Lecturer in Engineering in the University of Edinburgh. (With One Plate.)

(MS. received November 27, 1911. Read January 22, 1912.)

In a simple class of ternary alloys the solid phases are the three component metals, each holding in solution a relatively small proportion of the other two. Any one of the three phases may be present as a separate constituent, any two may be associated together as a binary eutectic, and the three together form a ternary eutectic. The structural constituents of such a ternary alloy are—

(1) Primary crystals of one phase,
(2) Binary eutectic of two phases,
(3) Ternary eutectic of three phases.

In general the alloy contains all three constituents, but there are certain limiting compositions at which any one or any two of the constituents may be absent. In what follows it is supposed that three structural constituents are always present. Since there are three possible kinds of primary crystals, three binary eutectics, and one ternary eutectic, there are six different types of alloys when classified according to their constituents.

The lead-tin-bismuth alloys furnish an example of a ternary series of the above kind. Charpy has studied the structure of certain members of this series, and has found features which apparently correspond with the three constituents.* He has published a section of an alloy containing 5·5 per cent. of lead, 20 per cent. of tin, and 74·5 per cent. of bismuth, which, according to the equilibrium conditions determined by the same and by other investigators, should consist of primary crystals of bismuth (containing a little lead and tin in solution), together with a binary eutectic of bismuth and tin, and the ternary eutectic of bismuth, tin, and lead. In Charpy's figure can be seen three primary crystals of bismuth, each surrounded by a deep fringe of what he calls bismuth-tin eutectic; the dark ground mass is the ternary bismuth-tin-lead eutectic, of which the structure is not resolved.

The purpose of this note is to call attention to the structural arrangement of a binary eutectic in a ternary alloy. If Charpy's section be

* Charpy, *Comptes Rendus*, 1898, cxxvi. 1645.

examined more closely, dark skeleton growths are detected in the fringe surrounding each primary crystal of bismuth. These are crystals of tin, which have been attacked by the hydrochloric acid used for etching the section. The presence of the crystals of tin shows that the fringe is not the simple binary eutectic which Charpy supposed it to be.

It is advantageous to consider for a moment the conditions which prevail during the solidification of an alloy of only two metals, before going on with the case in which three are present. In the simple type to which the present observations are restricted, such a binary alloy, when solid, consists of primary crystals of one of the metals (containing a small proportion of the other in solution), in a matrix of the binary eutectic. During the course of solidification the primary crystals have grown within the liquid throughout a certain interval of temperature, at the end of which period the eutectic has crystallised around them at constant temperature. Primary crystals are free to migrate over small distances through the liquid portion of the alloy, so that a slow rate of cooling allows of the formation of large aggregates. But during the solidification of the eutectic each small crystal is virtually locked in place by those deposited around it, and the opportunity for migration is very small; the structure of a eutectic, therefore, is on a fine scale as compared with the arrangement of the primary constituent. If the rate of cooling is slow, however, some migration of the particles of the eutectic takes place, the relatively large primary crystals attracting tiny crystals of the same substance from the eutectic to themselves. A section of the cold alloy generally shows that the movement has been arrested, by the completion of solidification, before the smaller crystals have been wholly absorbed by the large ones. In alloys of strongly crystalline metals, like bismuth and antimony, the phenomenon is particularly marked. Fig. 1 is a section, somewhat imperfectly polished, of an alloy containing 60 per cent. of bismuth and 40 per cent. of tin; it shows primary crystals of bismuth in a matrix of bismuth-tin eutectic. The bismuth crystals are surrounded by a fringe of the same material, attracted from the immediately surrounding eutectic, which is accordingly left relatively rich in tin. The appearance of fig. 1 differs but little from that of Charpy's section mentioned above, so that this type of structure cannot be considered as a special feature of a ternary alloy, or one which represents the process of solidification of a ternary alloy.

When the alloy is one of three metals the primary crystals solidify under conditions similar to those in an alloy of two metals. The ternary eutectic freezes at constant temperature, like the binary eutectic of a binary alloy, and has a correspondingly fine structure. But during the period of solidification of a *binary* eutectic in a *ternary* alloy the temperature is not constant, and a portion of the mass is always liquid. The particles of the binary eutectic are therefore free to migrate, unless the rate of cooling is very rapid, and, instead of the fine structure usually associated with eutectics, the crystals of each of the two phases are able to grow to a considerable size, and have exactly the same appearance as primary crystals. Fig. 2 shows a lead-tin-bismuth alloy, differing somewhat in composition from Charpy's mixture, but having the same structural constituents. Here the large crystals of bismuth at the foot of the micrograph are the primary growth, and above them are to be seen smaller but well-formed crystals of bismuth and tin, closely intermingled; the small crystals have been formed during the second period of solidification, and represent the binary eutectic. At certain places near the foot can be seen dark skeletons of tin adjacent to large primary crystals of bismuth which must have absorbed the secondary growth of bismuth as it formed. The ground mass consists of the ternary eutectic, though its structure is not clearly apparent with such a low magnification.

Other ternary alloys, of more complex character than the above, show a corresponding arrangement of the constituents. Thus, in the lead-antimony-copper alloys, rich in lead, copper combines with antimony to form the purple compound $SbCu_2$, and there is a binary eutectic of antimony and $SbCu_2$. In an alloy containing, say, 66 per cent. of lead, 24 per cent. of antimony, and 10 per cent. of copper, the compound $SbCu_2$ is the primary solid, and it is followed by the Sb–$SbCu_2$ eutectic. A micrograph shows that both antimony and $SbCu_2$ assume primary forms, most of the white crystals of the former being traversed by the purple skeletons of the latter around which they have grown. Similarly in the tin-antimony-copper alloy known as Babbit's metal, there are apparently two primary substances —a skeleton network of the ϵ-solution of copper-tin and compact crystals of antimony.

Thus the form of the crystals in an alloy is determined largely by the physical condition of the mixture during the period of solidification; and since, in general, during the whole period of formation of a binary eutectic in a ternary alloy a portion of the mass remains liquid, the two constituents of this eutectic are able to assume the form of primary crystals.

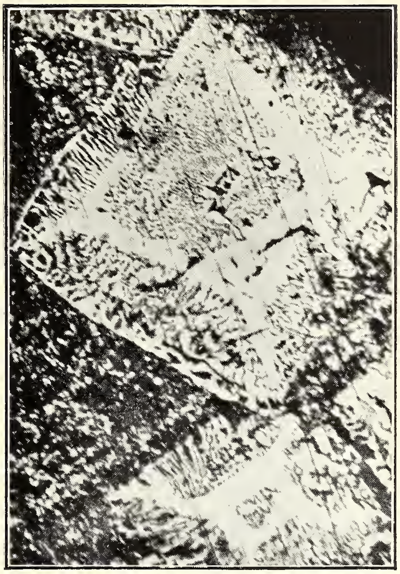


Fig. 1.

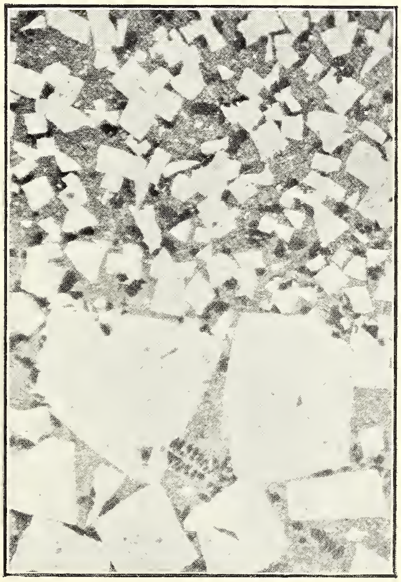


Fig. 2.

Mr G. H. Gulliver.

DESCRIPTION OF FIGURES.

Fig. 1. Tin-bismuth alloy, containing 40 per cent. of tin and 60 per cent. of bismuth, etched with nitric acid. White primary crystals of bismuth, with fringed borders. Magnified 100 diameters.

Fig. 2. Lead-tin-bismuth alloy, containing 15 per cent. of lead, 15 per cent. of tin, and 70 per cent. of bismuth, etched with nitric acid. Large white primary crystals of bismuth, smaller secondary growth of bismuth and tin, ground mass of ternary eutectic. Magnified 50 diameters.

(*Issued separately February* 16, 1912.)

VII.—The Absorption of Light by Inorganic Salts. No. V.: Copper and the Alkali Metals. By R. A. Houstoun, M.A., Ph.D., D.Sc., Lecturer on Physical Optics in the University of Glasgow.

(MS. received December 1, 1911. Read January 22, 1912.)

The research described in this article was undertaken with the double purpose of testing new apparatus and of making a rapid survey of the absorption of light by salts of the first periodic group.

The results described in the previous articles were obtained by three separate methods, viz. the thermopile in the infra-red, the spectrophotometer in the visible spectrum, and the photographic spectrophotometer in the ultra-violet. It would be a great simplification if the same apparatus could be used throughout the whole spectrum, and I therefore first of all devoted my attention to seeing if this were practicable.

The ordinary spectrograph with quartz lenses and Cornu prism is really only suitable for one purpose, viz. the photography of the ultra-violet and visible spectrum, and the attainment of sharp focus is a tedious business. The index of refraction of quartz (ordinary ray) varies from 1·65 to 1·54 within the range ordinarily used, viz. $\lambda = {\cdot}198\mu$ to $\lambda = {\cdot}768\mu$. The collimating lens thus cannot render parallel all the rays that pass through it. Also, according to theory, the Cornu prism gives a double image, unless the light is parallel and the prism is set at minimum deviation. It is therefore very creditable that the makers get such sharp spectra as they do. The instrument permits, however, of no adaptation. It cannot readily be used with a thermopile. A glass prism cannot easily be changed for the quartz one if we are working in a region where the greater dispersion of glass would be acceptable. Finally, quartz does not transmit in the far infra-red. I have therefore devoted my attention to other types of instrument.

In the fourth article of this series there is a figure showing a spectrograph in which the Wadsworth mirror-prism combination is used. This instrument has been described elsewhere.* Its advantage is, that when the mirror-prism combination is rotated, the different wave-lengths come into focus and into minimum deviation automatically at the same point

* *Knowledge*, vol. viii. p. 87; *Studies in Light Production*, The Electrician Publishing Co. (in preparation).

on the photographic plate. It has been superseded by the apparatus to be described in this paper.

With a glass prism or rock-salt prism minimum deviation is not so important as with quartz, because they do not doubly refract. With a Cornu prism we get minimum deviation only at one point on a photographic plate; also, when using a thermopile, minimum deviation is not so necessary, since the radiation receiving surface is not very narrow. I have therefore let fall the condition that there should be automatic minimum deviation, and have devised the simple universal apparatus represented

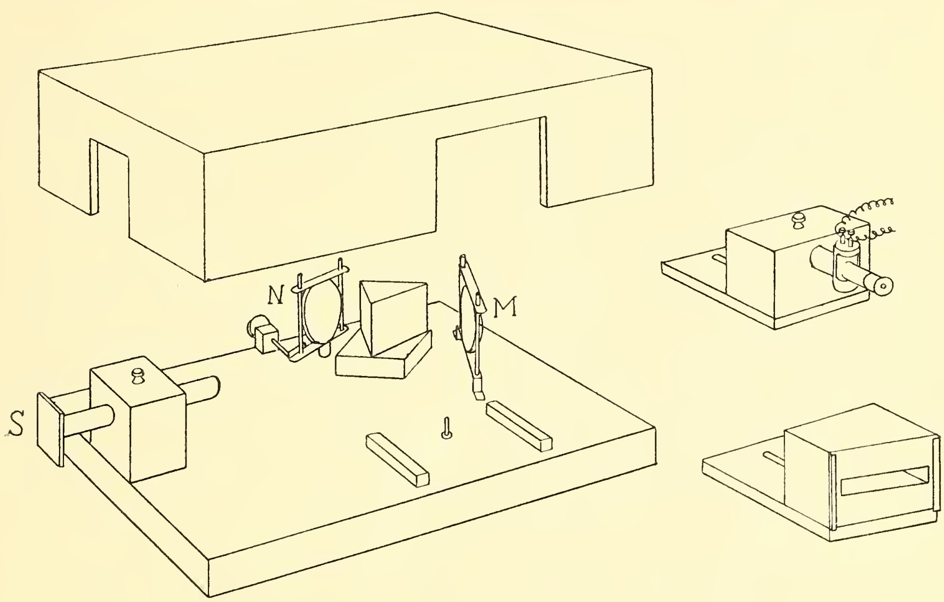


Fig. 1.

here, which is suitable for radiation measurements throughout the infra-red, visible spectrum, and ultra-violet, and incidentally as suitable for photography in the ultra-violet as the instruments on the market at present. It is with this apparatus that the observations recorded in this paper were obtained.

The sketch represents only the essentials. The slit S is attached to a brass tube sliding in a wooden block and clamped in position by a screw. The thermopile and the photographic plate carrier with the two brass grooves for holding the slide are shown at the side. They slide between the guides shown, the screw passing through the slot. They are fixed in position by tightening a nut on the screw. The light from the slit is rendered parallel by the concave mirror M, passes through the fixed prism, which is set so that the middle of the spectrum suffers minimum deviation, falls on the mirror N, and is brought to a focus on the photographic plate. The mirror N is rotated about a vertical axis through its centre by means of a micrometer screw; hence all the colours of the spectrum can be made to pass in succession across the centre of the photographic plate. The curve along which the spectrum lies can be calculated from the formula for the oblique incidence of a thin pencil on a concave mirror. It does not deviate appreciably from a straight line.

The concave mirrors had a diameter of 5 cm. and a focal length of 33·3 cm. The sides of the quartz prism were 4 cm. high and 4·6 cm. long, and the two halves were held together in a brass frame.

For the material of the mirrors I have taken nickel. Professors Hagen and Rubens have determined the reflecting power of various metals throughout the ultra-violet, visible, and infra-red, and the results show (cf. Landolt and Börnstein's Tables) that silver reflects one region in the ultra-violet very poorly while reflecting excellently in the visible and infra-red. Spiegel-magnalium reflects well throughout the whole spectrum but does not keep, and is consequently not used in optics now. We tried the alloy, 68 per cent. copper, 32 per cent tin, but rejected it on account of its brittleness. The first nickel mirrors obtained were silver on which nickel was deposited electrolytically. They were unsuitable, as was mentioned in the former paper. Then solid mirrors of 99 per cent. nickel were procured, but their polish was not good enough. Finally, a purer quality of nickel was obtained, and the mirrors made from it were satisfactory in every particular.

In the most unfavourable part of the ultra-violet nickel reflects 37·8 per cent. of the incident light. If we allow for the double reflection, only 14·3 per cent. of the incident light is finally reflected. The fraction transmitted by two quartz lenses in succession in the ordinary spectrograph cannot be greater than 80 per cent. But owing to the obliquity of the plate we have to multiply this by sin 21° in order to compare correctly the energy received per unit area of the photographic plate in the two cases. This gives 28 per cent. Hence, when quartz lenses are used, and the aperture of the instrument is the same, the illumination of the image on the photographic plate cannot be more than twice as great as when nickel mirrors are used. With nickel mirrors, owing to the better collimation, a larger aperture can be used; also I have a pair of silver mirrors for replacing the nickel mirrors for use in the visible spectrum. When all things are considered, I am therefore of opinion that the mirror spectrograph gives the brighter spectra. It has, in addition, the following advantages:—

(1) Rigidity: the only moving part is the mirror.

(2) Automatic focussing: we have only to move the plate carrier towards N until the sodium lines are in focus at a particular point on the ground glass screen, P say. Then, if N be rotated, whatever part of the spectrum falls on P is in focus there, and owing to the plate holder being cut at the proper angle the whole spectrum is in focus all over.

(3) The quartz prism can be replaced by a glass prism or rock-salt prism without any structural alterations. In that case the mirrors must of course be set at a different angle, and the angle of the plate carrier altered somewhat if the whole spectrum is to be in focus at once.

(4) The instrument can be used as monochromator, and can be employed for spectral work with a radio-micrometer or similar stationary instrument.

The apparatus was first used with the thermopile in the infra-red, a flint prism and silver mirrors being employed for this part of the work. The micrometer screw was calibrated, as described in the first article of the series, with emission lines in the visible spectrum and water bands in the infra-red. Stray heat was eliminated by means of screens before the thermopile. This method is not quite so thorough as the use of an auxiliary prism, but saves a little more energy and is above all faster in working. The other details were as formerly.

After finishing the infra-red I attempted to use the thermopile in the ultra-violet with a spark between metal electrodes as source. But with the induction coils at my disposal the deflections were not large enough to make the method practicable. I therefore fell back on photography.

The photometer already described by John S. Anderson * and myself suffers from the disadvantage that its range is limited. The illumination of each of the quartz plates cannot be altered in a greater ratio than that of one to four. In my efforts to remedy this defect I built an arrangement, the plan of which is shown in fig. 2.

Let us suppose the rays to issue from the slit of the spectrograph S. They first fall upon a quartz rhomb R, which divides them into two pencils as shown by the elevation, fig. 3. The lower pencil is reflected by the totally reflecting quartz prism P, and meets the diffusely reflecting surface B. The upper pencil keeps straight on and meets the similar diffusely reflecting surface A. These surfaces are large enough to stop all the rays issuing from the instrument. An iron arc L moves along a scale between A and B, and the illumination of these surfaces varies inversely as the square of their distances from the arc. The light from A and B forms two spectra

* *Proc. Roy. Soc. Edin.*, xxxi. p. 550 (1911).

which touch one another sharply on the photographic plate. The position of the arc was found which gives equal intensity of these spectra all over, a cell with the solution to be examined was placed in the path of one of the beams, a similar cell with water placed in the path of the other, and a

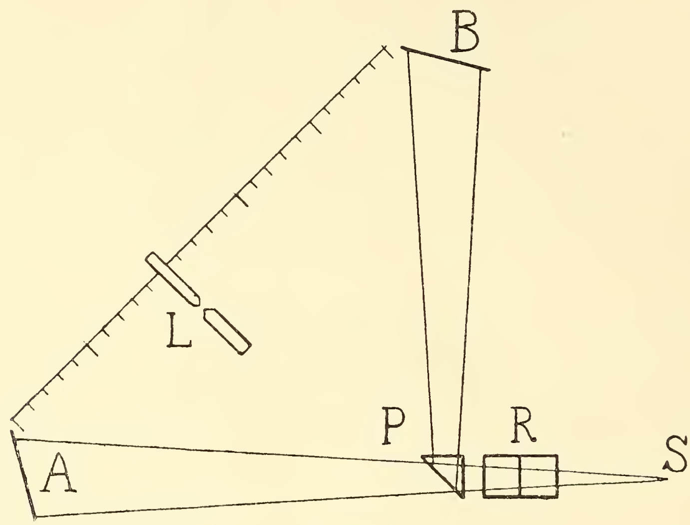


Fig. 2.

series of exposures made for different positions of the arc. Then the positions of equal density on the plate were determined, and A calculated as in the former paper. The exposures were 30 seconds.

Owing to the obliquity of the mirrors and consequent difference of focus for horizontal and vertical lines the sharp edge H of the rhomb was in

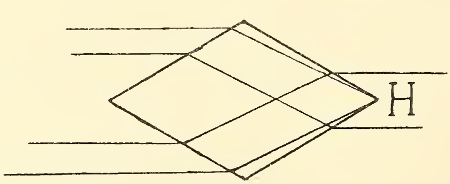


Fig. 3.

focus 3 cm. in front of the slit. Considerable difficulty was found in obtaining a suitable diffuse reflector. Plaster of Paris was tried at first, but did not reflect the ultra-violet at all. Twelve other substances were tried, and an opaque fused silica finally selected.

This method, although more convenient than that described in the fourth paper, did not prove as accurate owing to the iron arc not giving out the same quantity of light on both sides, 10 per cent. variations of the relative intensity being occasionally met with.* The arc was watched through a piece of red glass and the exposure stopped if it appeared unsymmetrical; also a telescope was mounted so as to receive the light reflected from the first surface of the Cornu prism, thus giving an image of the slit, and it was used to watch the relative intensities of the two beams.

In the arc finally used the electrodes were horizontal. The kathode slid in a tube in which it could be rotated by hand and contact was made by hand. After every exposure the electrodes were filed so as to remove the iron oxide that had formed. This was found necessary in the interests of the symmetry of the light.

It may be mentioned in passing that in a recent paper † A. H. Pfund describes a method of producing an iron arc that burns steadily for hours. It depends on the formation of a bead of oxide in a cup on the anode. I found that the symmetry of the ultra-violet radiation from this arc varied considerably even when the arc was apparently steady, and hence it could not be used.

All the salts investigated, with the exception of sodium chloride, were obtained from Kahlbaum. The curves on p. 46 give the results for cupric sulphate, cupric chloride, cupric nitrate, and cupric bromide. The absorption is so much greater in the ultra-violet that a different scale is used there. The thicknesses employed varied from 1 mm. to 4 cm., and the concentrations in grm.-mols. per litre were as follows:—

	Infra-red.	Ultra-violet.
Sulphate	$c = \cdot050$	$c = \cdot040$
Chloride	$c = \cdot0496$	$c = \cdot037$
Nitrate	$c = \cdot049$	$c = \cdot039$
Bromide	$c = \cdot0541$	$c = \cdot039$

The difference between the different curves in the infra-red is just about the error of observation. In the ultra-violet in the case of the nitrate there is just a trace of the nitrate band. The points marked with o's in the ultra-violet in the case of the sulphate are taken from a former paper.‡

* The results for the ultra-violet given in the paper are probably right to about 7 per cent. In the near ultra-violet, where a Nernst filament can be used instead of the iron arc, an accuracy of 1 or 2 per cent. can be attained.

† "Metallic Arcs for Spectroscopic Investigations," A. H. Pfund, *Astroph. Jr.*, xxvii., 1908, p. 296.

‡ "On the Absolute Measurement of Light: A Proposal for an Ultimate Light Standard," R. A. Houstoun, *Proc. Roy. Soc.*, 85 A, p. 275 (1911).

Variation of A with λ for Aqueous Solutions of Copper Salts.

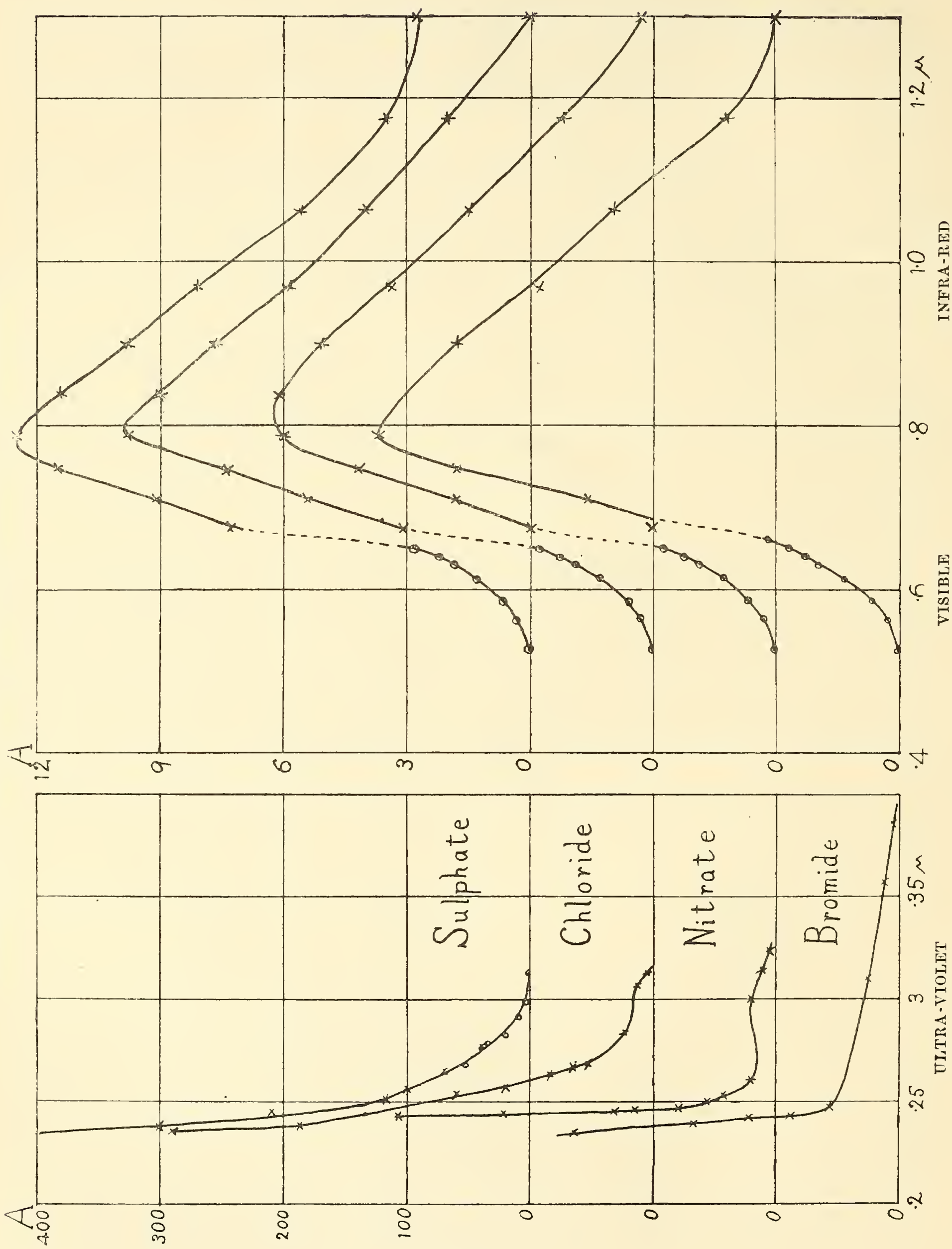


Fig. 4.

Cupric salts have been investigated with a spectrophotometer in the visible spectrum by Ewan,* Grünbaum,† and E. Müller.‡ The results in Ewan's last paper go furthest into the red, and I have plotted them with my results for the sake of completeness. Ewan's values are represented by o's. His solutions were about ten times as dilute as mine.

The work done by Ewan and Müller in the visible spectrum has shown that for dilute solutions A has the same values for all four salts, and that for strong solutions its values do not differ much from one another and from the values for the dilute solutions, except in the violet in the case of the chloride and bromide, where a marked absorption grows up with concentration and temperature. Unlike the similar change in cobalt and nickel, it is here more marked in the case of the bromide.

After finishing the four copper salts I did the sulphates of lithium, sodium, potassium, rubidium and cæsium, and then sodium chloride, sodium bromide, sodium nitrate, and silver nitrate. Aqueous solutions of these salts are of course colourless in the visible spectrum, and I found that none of them absorbed at all in the infra-red. In fact they absorbed less than water, and hence gave rise to an apparent negative absorption. The following table gives the strength of the solution used in each case and the value below which the molecular extinction coefficient must lie in the infra-red:—

	c.	A.
Lithium sulphate	1·88	·009
Sodium sulphate	·406	·02
Potassium sulphate	·425	·03
Rubidium sulphate	1·17	·004
Cæsium sulphate	1·17	·004
Sodium chloride	3·44	·002
Sodium bromide	4·98	·003
Sodium nitrate	5·69	·002
Silver nitrate	2·98	·01

The results in the ultra-violet in the case of the two nitrates are shown as curves, and in the other cases are given in a table. In the case of $AgNO_3$ *c* was ·059, and in the case of $NaNO_3$ *c* was ·118.

* *Phil. Mag.* (5), xxxiii. p. 317 (1892); *Proc. Roy. Soc.*, lvi. p. 286 (1894); *ibid.*, lvii. p. 117 (1894).

† Inaug. Diss., Berlin, 1902.

‡ *Ann. d. Phys.* (4), xii. p. 767 (1903); *ibid.* (4), xxi. p. 515 (1906).

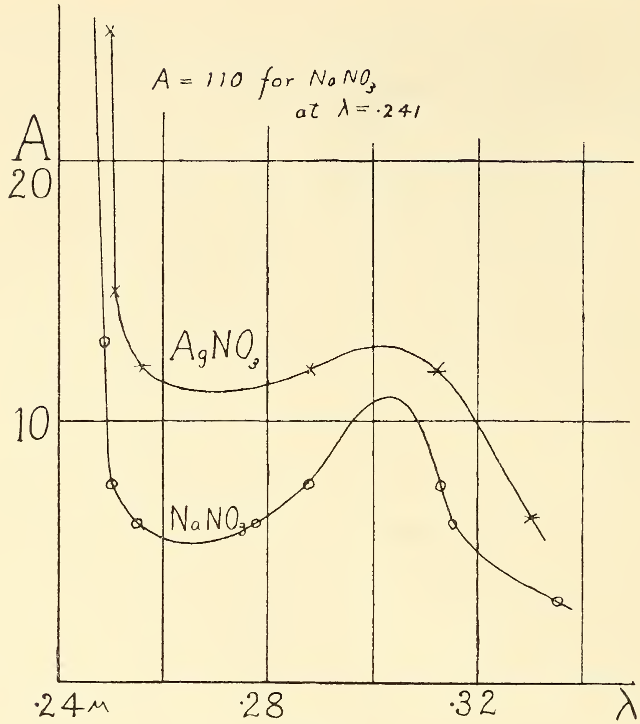


Fig. 5.

$Li_2SO_4H_2O$.			$Na_2SO_410H_2O$.			K_2SO_4.			Rb_2SO_4.			Cs_2SO_4.		
c.	λ.	A.	*c*.	λ.	A.	*c*.	λ.	A.	*c*.	λ.	A.	*c*.	λ.	A.
1·47	·231μ	< ·22	·302	·231μ	< ·9	·338	·231μ	< ·5	1·17	·239μ	= 1·1	1·17	·231μ	< ·76
	·240	= ·11		·239	= ·71		·281	= ·22		·240	·76		·236	= ·26
	·267	·05		·265	·44					·246	·45		·240	·15
				·290	·24					·267	·26			

$NaBr2H_2O$.			NaCl.		
c.	λ.	A.	*c*.	λ.	A.
·29	·238μ	= 3·1	4·49	·231μ	< ·07
	·244	2·5		·245	= ·050
	·251	1·3		·254	·050
				·277	·050
				·283	·036
				·288	·017

The sulphates were chosen for examination in the case of each of the five alkali metals because I wished to examine the effects due to the base, and our experience with cobalt sulphate and nickel sulphate showed us that the SO_4 exerts no appreciable absorption. It will be seen from the table that the absorption in the ultra-violet due to the alkali metals is very small, and in some cases it could just be measured. It is confined to the end of the range. They appear, however, to have all a band just off the range below $\cdot 235\mu$, which was the smallest wave-length I could reach with my apparatus.

The expense of the investigation was defrayed by a grant from the Carnegie Trust for the Universities of Scotland.

(*Issued separately March* 20, 1912.)

VIII.—The Absorption of Light by Inorganic Salts. No. VI.: The Cobalt Chloride Colour Change. By Alex. R. Brown, M.A., Carnegie Scholar in the University of Glasgow. *Communicated by* Dr R. A. Houstoun.

(MS. received December 1, 1911. Read January 22, 1912.)

In No. II.* of this series of papers, measurements were given of the striking change shown by cobalt chloride when an aqueous solution was heated or was altered in concentration, and it was announced that a more accurate investigation was being made in the hope of finally determining its cause. Chemists suppose the cause to be the dehydration of the hexahydrate, but the matter is not beyond doubt.

First of all, aqueous solutions were used. Except in regard to the heating arrangements the methods and apparatus were those used in the previous research. The heater in this case was a rectangular copper casting 8 cm. long, 2·5 cm. broad, 4·5 cm. high, which at one end was attached to a tin in which water was boiled. At a distance 1·2 cm. from the other end a rectangular hole was cut down through the casting, of dimensions such that it was just filled by the cell containing the solution to be heated. At each side of the hole from the base upwards a part of the casting 1·6 cm. high and 1·1 cm. broad was cut away. This permitted the transmission of light horizontally through the lower part of the vertical cell, which was of the ordinary rectangular type, and obviated the use of mirrors. As in the previous research glasses were inserted in the cell to reduce the thickness of the layer of solution examined, and the temperature of the solution was read by a thermometer.

By this method a steady temperature of about 84° could be obtained. But readings could easily be taken at points between room temperature and 84°, it being possible by judicious application and withdrawal of the bunsen burner to keep the temperature of the solution within a degree or two of any selected point. The time taken by the solution after the application of the bunsen to reach the highest temperature was slightly over 20 minutes, and on withdrawal of bunsen the times taken to cool to 50° and to room temperature were over 40 minutes and about 180 minutes respectively. Ample opportunity was thus afforded for taking readings for intermediate temperatures.

* *Proc. Roy. Soc. Edin.*, xxxi. p. 530 (1911).

The width of the spectroscope slit was less than in the previous determinations of the absorption of concentrated solutions of cobalt chloride, and independent readings could consequently be taken more closely together in the spectrum. The following table gives the effect of heating a solution of strength $c=3{\cdot}10$. Each value for the cold solution is the mean of two determinations; each value for the hot solution is the mean of three except in the case of the last three wave-lengths, when only one determination was made.

Values of A.

$c=3{\cdot}10$; $d={\cdot}046$ cm.				
λ.	Temp.	Cold solution.	Temp.	Hot solution.
434μμ	16°	2·40	84°	2·49
444		3 37		3·75
453		4·24		4·52
463		6·11		6·14
475		6·84		7·75
486		8·10		9·95
499		9·11		12·0
514		9·49		13·0
529		8·82		14·0
547		5·58		12·2
563		3·74		10·1
582		1·82		9·91
602		1·44		18·7
625		1·58	76·8°	17·4
653		1·62	70°	17·0
687		1·96	61·5°	13·2
797		1·77		

The increase in the value of A found on heating the solution to a temperature over 80° is by this method shown to be less than appeared from the method of heating the cell in hot water to about 90°, removing it, and allowing it to cool before the slit. This difference can be easily explained.

In the first place, A increases decidedly more rapidly in value between 80° and 90° than between 70° and 80°, and two values of A at 85° and 75° respectively obtained by the previous method, as the cell cooled from 90° to 70°, which it did in 3 minutes, would not give as their average the correct value of A for 80°. But there is an additional cause of the difference. In heating by the method described in the preceding pages I noticed that for some minutes after the solution had reached a steady temperature of about 84° the value of A continued to increase. This indicates that the change in absorption lagged behind the change in temperature, and that the temperature had reached a constant value some time before the absorption reached a corresponding constant value. Probably the lag exists in falling from a high temperature as well as in rising to it. Hence what we obtained by the former method as the value of A when the temperature was at 80° may be the constant value of A for a temperature of 86° or 88°. The values of A given in my table are, with the exception of the last three, constant values for a temperature of about 83° or 84°; the last three were got as the temperature was rising, it being impossible to get values of A for higher temperatures with the same thickness of solution for these wave-lengths on account of the great increase in the value of A.

As this effect of lag tended to make my results less reliable, and evaporation from the heated solution, though reduced greatly and to a certain extent eliminated, had not been quite prevented, I gave up the hope of obtaining measurements of the progress of the change at different temperatures, accurate enough for the determination of the law which it followed.

It is a well-known fact that solutions of cobalt chloride in ethyl alcohol are blue, and that as water is added the colour gradually changes through various shades till it reaches the colour of an ordinary aqueous solution. This change seems to be of exactly the same nature as that produced in the aqueous solution by heating. It also allows the easy and accurate measurement of the intermediate stages. I therefore turned my attention to investigating it.

In the first place, I made a mother solution, almost saturated, from which all the others were derived. Some hexahydrate chloride was carefully heated in a crucible till it lost all water of crystallization, passing through the dark-blue dihydrate state to the light-blue anhydrous state. I had then 12·3 grammes of the anhydrous salt which I dissolved in 90 c.c. of absolute alcohol of specific gravity ·794, obtaining 92 c.c. of solution. The specific gravity of the solution was ·91, and the concentration of the chloride in gramme-molecules per litre was 1·028. This is solution I, and from it I made solutions II, III, IV, and V by adding further quantities of absolute alcohol. Thereafter, by adding water to the alcoholic solutions, I made from I solutions I*a*, I*b*, I*c*, I*d*, I*e*, I*f*, I*g*, I*h*; from II solutions II*a*, II*b*, II*c*, II*d*, II*e*, II*f*; and from III solutions III*a*, III*b*, III*c*, III*d*, the water being added in such proportions that the concentration of the cobalt chloride in solution, measured in grm.-mols. per litre, decreased regularly in each series by one-twentieth or multiples of one-twentieth of its value in the purely alcoholic solution of that series.

The molecular extinction coefficient A was determined in the same way as in the second paper of the series for the dilute solutions of the chloride and other salts. That is, readings were taken on the nicol of the spectrophotometer with a cell containing the solution before the lower slit and a similar cell containing water before the upper slit, and readings were again taken with the cells interchanged. The value of d ranged from ·006 cm. in the case of solution I to 2 cm. in the case of solution V. The values of A obtained for the various solutions are given below. The concentrations of the cobalt chloride and water in grm.-mols. per litre of solution are denoted by c_1 and c_2 respectively.

Values of A.

λ.	I.	I*a*.	I*b*.	I*c*.	I*d*.	I*e*.	I*f*.	I*g*.	I*h*.
	$c_1=1·028$	·9766.	·9252.	·8738.	·8224.	·7790.	·7196.	·514.	·257.
	$c_2=0.$	3·03.	5·75.	9·35.	12·22.	15·21.	20·42.	29·51.	42·62.
434$\mu\mu$	3·33	3·83	2·59	2·22	1·49	1·95	1·70	1·10	1·24
444	3·27	4·35	2·73	2·85	2·28	2·38	2·15	1·89	1·73
453	3·28	4·84	3·87	3·74	3·07	3·50	3·21	2·56	2·45
463	4·18	4·89	4·60	4·81	4·13	4·33	4·21	3·24	3·27
475	4·99	5·51	5·51	5·74	4·64	4·93	5·05	3·83	3·95
486	5·12	6·42	6·00	7·64	5·47	5·61	5·32	4·35	4·33
499	7·41	8·12	7·38	7·83	6·67	6·20	6·24	5·26	5·26
514	9·83	9·35	9·60	8·57	7·01	6·59	6·63	5·40	5·49
529	14·2	10·7	9·95	8·48	6·08	5·90	5·64	4·01	3·63
547	30·	14·5	10·7	7·61	4·30	3·91	3·59	2·61	2·13
563	44·	24·2	15·5	10·4	3·35	2·77	2·29	1·29	1·20
582	87·	51·6	37·3	25·5	4·64	2·40	1·64	·55	·51
602	125·	90·6	64·1	38·2	6·71	2·69	1·60	·41	·43
625	182·	131·	98·	52·9	9·56	3·47	1·64	·44	·33
653	218·	154·	113·	64·7	11·2	3·56	1·98	·34	·32
687	210·	175·	125·	68·6	11·0	3·92	1·72	·29	·27
717	65·5	64·1	40·0	30·7	3·26	1·61	·58	·29	·28

[Values of A.—II.

λ.	II.	IIa.	IIb.	IIc.	IId.	IIe.	IIf.
	c_1 = ·40.	·38.	·36.	·34.	·32.	·20.	·10.
	c_2 = 0.	3·03.	5·62.	9·35.	12·22.	29·51.	42·62.
434μμ	2·77	2·84	2·85	2·43	2·11	2·22	1·68
444	2·82	3·40	3·32	3·05	2·69	2·50	2·45
453	2·97	4·28	4·03	3·62	3·42	3·20	2·76
463	3·07	5·03	5·15	4·42	4·31	4·20	3·25
475	4·53	5·51	6·01	5·24	4·85	4·50	3·90
486	6·26	6·53	6·40	5·58	5·29	5·05	4·45
499	7·53	7·85	7·19	6·92	6·15	5·81	4·95
514	10·7	9·17	8·44	6·94	6·59	6·08	5·20
529	15·5	9·01	8·01	5·99	5·87	5·07	4·20
547	25·2	10·0	6·70	3·98	3·55	3·01	2·28
563	43·6	15·3	6·47	3·15	2·19	1·95	1·08
582	94·6	28·4	10·3	2·71	1·34	1·02	·47
602	131·	47·5	14·4	2·84	1·18	·77	·36
625	168·	57·0	19·7	3·39	1·34	·71	·45
653	202·	73·3	22·2	3·62	1·29	·55	·21
687	192·	75·2	23·5	3·31	1·16	·50	·31
717	49·1	26·2	7·38	1·21	·81	·44	·19

λ.	III.	IIIa.	IIIb.	IIIc.	IIId.	IV.	V.
	c_1 = ·10.	·095.	·090.	·080.	·050.	·02.	·005.
	c_2 = 0·0.	3·17.	6·00.	11·55.	28·61.	0·0.	0·0.
434μμ	2·45	2·75	3·86	2·44	3·02	1·65	...
444	2·55	3·84	4·28	2·76	3·76	2·46	...
453	3·14	4·11	5·00	3·67	4·17	3·50	...
463	3·12	5·11	5·96	4·66	4·89	4·63	...
475	4·31	5·95	6·56	5·15	5·49	6·21	...
486	5·80	6·62	7·35	5·48	5·70	8·33	...
499	9·12	7·65	8·00	6·34	6·51	9·64	...
514	10·9	8·22	8·31	5·96	6·57	13·8	...
529	17·6	7·65	7·29	5·64	5·54	18·2	...
547	25·4	6·43	4·73	2·95	3·66	28·2	...
563	46·0	6·01	3·51	2·09	2·26	47·6	20·6
582	72·5	6·37	2·94	1·27	1·54	63·1	27·5
602	103·	8·51	2·59	1·06	1·47	80·6	36·0
625	129·	9·99	2·35	·96	1·29	87·8	41·1
653	151·	11·2	2·29	·92	1·21	98·0	44·2
687	137·	8·70	1·94	·63	·86	72·1	31·7
717	29·9	2·49	1·10	·18	·81	14·0	5·54

In order to study the data of the preceding table from the theoretical point of view, I in the first place plotted A as a function of λ. The full

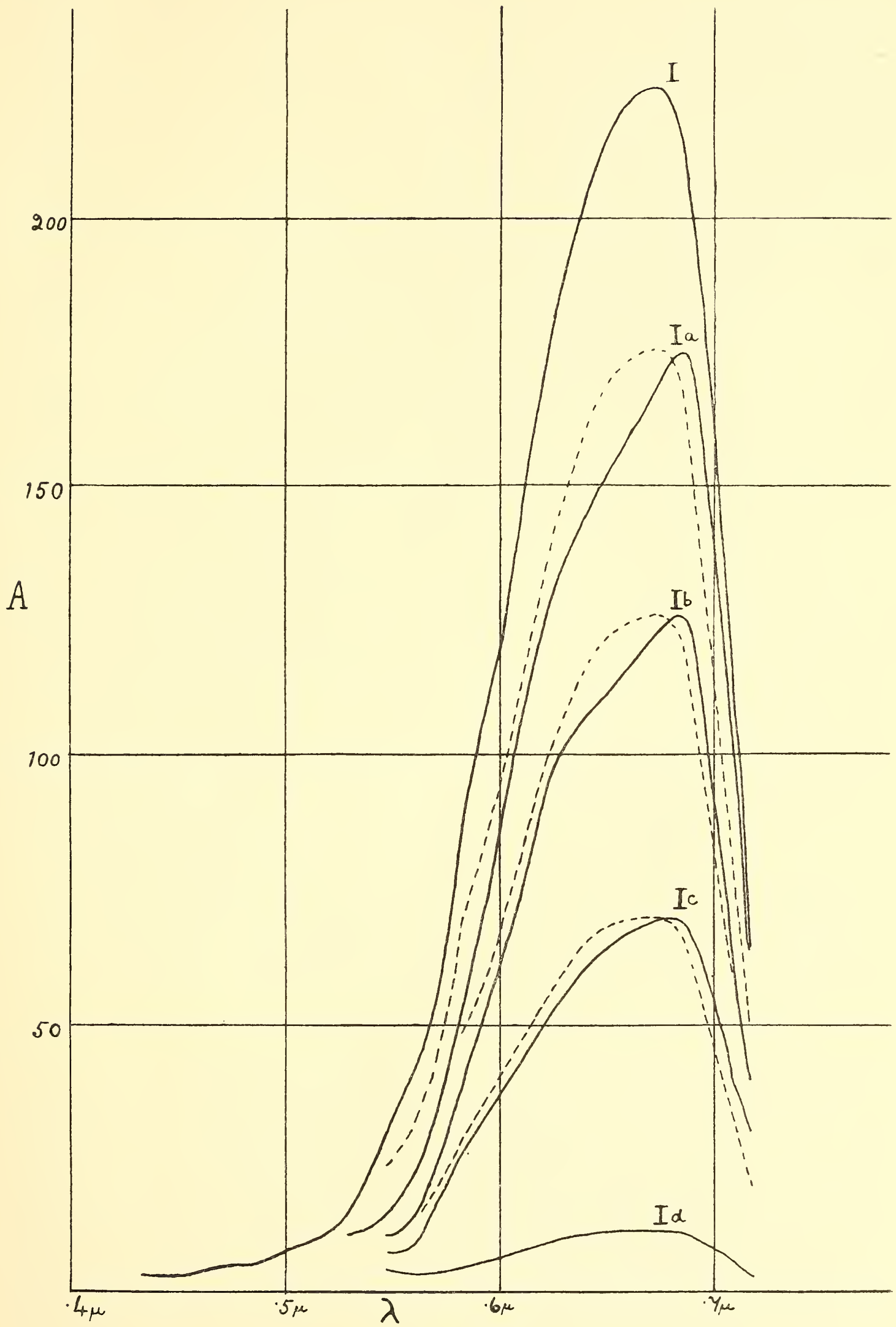


Fig. 1.

lines in fig. 1 are such curves for solutions I, I*a*, I*b*, I*c*, I*d*. It is seen from them that in the pure alcoholic solution I there is a very pronounced band with its maximum at $\lambda = 675\ \mu\mu$. As water is added the height of the band decreases in approximately the same ratio all over, and there is a slight displacement of the band as a whole toward the red. This is shown by the position of the dotted curves, which are curve I reduced in various ratios. Finally, in the case of I*e*, in which there is about 28 per cent. of water, the maximum of the band has been reduced to less than 4; and solutions I*g* and I*h* show no trace of the band, the spectrum being that of the dilute aqueous solution. Results of a similar nature were obtained with the sets of curves derived from the alcoholic solutions II and III.

It is obvious therefore that in these solutions the cobalt chloride is in two different states, one of which is characterised by this band in the red. This state I shall call the blue phase. As water is added the salt passes into that state in which it exists in dilute aqueous solution. This second state I shall call the red phase. It is characterised by a band in the green at $\lambda = 510\ \mu\mu$, but as the maximum value of A for this band in any aqueous solution not approaching saturation is only about 6, and the maximum for the band characteristic of the blue phase is about 232, the band in the green is obscured by the side of the other band if more than one-tenth of the salt is in the blue phase.

In order to find out how the distribution of the salt between the two phases varied with the quantity of water added, the maximum value of A for the band in the red was plotted as a function of the proportion of water. The results are shown by the curves in fig. 2, each solution being represented by one point. The concentration of the water in the solution in grm.-mols. per litre is denoted as before by c_2. Results are not shown for higher concentrations than $c_2 = 21$, as beyond this point practically all the chloride is in the red phase.

The maximum values of A for the three pure alcoholic solutions do not agree. This is probably due to the presence of water in the absolute alcohol employed in making up the solution, as absolute alcohol generally has from $\frac{1}{2}$ to 1 per cent of water. The curves joining the points for each series of solutions I, I*a*, ... II, II*a*, ... III, III*a*, ... become approximately straight lines when the quantity of water present in solution is small. These lines meet at a point for which $A = 232$ and $c = -{\cdot}6$. It is very probable that for pure nonaqueous solutions A varies only slightly with the concentration, there being nothing to indicate that the anhydrous chloride enters into any combination with the alcohol at room temperature. Hence from the curves it was assumed that the maximum value of A for a nonaqueous alcoholic solution is 232, and that the alcoholic solutions I, II, and III contain ·6 grm.-mol. of water per litre. The other solutions contain correspondingly more water than was stated on pp. 53 and 54.

The proportion of cobalt chloride existing in the blue phase in any solution is obviously given by $\frac{A}{232}$, and in the red phase by $1-\frac{A}{232}$, where A is the maximum value of the red band for that solution.

I next attempted to apply the law of mass action. That this law is applicable is indicated by the fact that where the proportion of alcohol to

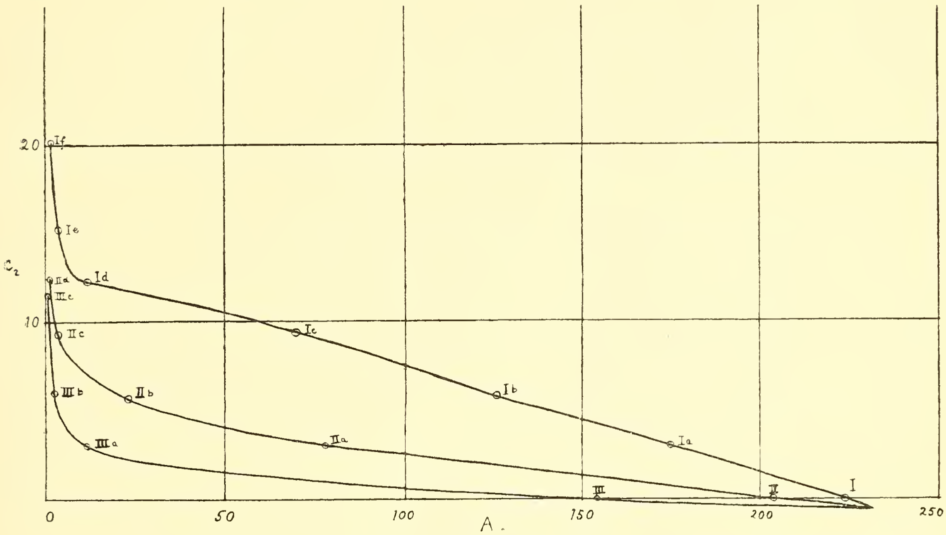


Fig. 2.

cobalt chloride is constant, as throughout each series, the amount of red phase increases with the amount of water, and where the proportion of water to cobalt chloride is approximately the same as in I*d* and II*b* and in II*d* and III*b*, the amount of blue phase increases with the amount of alcohol. It was assumed that the alcohol acted only as a diluent. This is open to question, as on bringing alcohol and water together there is a shrinkage in volume; but this does not necessarily mean chemical combination, and the assumption seems to be justified by the results. The change in the solution is obviously due to hydration, since in the blue phase there is no water at all. A question arises about the nature of the red phase. Some authorities are of opinion that it is the hexahydrate. But the absorption spectrum of the hexahydrate crystal is entirely different from that of the aqueous solution. The former has two bands, the larger with a maximum about $\lambda = 550\ \mu\mu$, and the smaller with a maximum about $\lambda =$

495 $\mu\mu$. I left the constitution of the red phase undetermined, and assumed that it was represented by the formula $CoCl_2.xH_2O$.

Let the concentrations of cobalt chloride and water in grm.-mols. per litre be c_1 and c_2 respectively. Then the concentration of the blue phase in any solution is $\frac{A}{232}c_1$, and of the red phase $\left(1-\frac{A}{232}\right)c_1$. Also the concentration of the uncombined water is $c_2-\left(1-\frac{A}{232}\right)c_1x$.

The equation of the reaction is

$$CoCl_2+xH_2O=CoCl_2\,.\,xH_2O.$$

Hence by the law of mass action

$$\left(\frac{A}{232}c_1\right)^{-1}\left\{c_2-\left(1-\frac{A}{232}\right)c_1x\right\}^{-x}\left(1-\frac{A}{232}\right)c_1=k;$$

$$\text{i.e. } c_2-\left(1-\frac{A}{232}\right)c_1x=\left(\frac{232-A}{Ak}\right)^{\frac{1}{x}}.$$

Take two solutions and let their values of A, c_1, c_2 be A′, c'_1, c'_2, and A″, c''_1, c''_2 respectively. By combining the equations for each solution we get

$$c'_2-\left(1-\frac{A'}{232}\right)c'_1x=\left\{\frac{(232-A')A''}{A'(232-A'')}\right\}^{\frac{1}{x}}\left\{c''_2-\left(1-\frac{A''}{232}\right)c''_1x\right\}$$

If A′ and A″ are approximately equal, and x has a fairly high value, the x^{th} root becomes very nearly equal to unity, and the last equation becomes

$$x=\frac{c''_2-c'_2}{\left(1-\frac{A''}{232}\right)c''_1-\left(1-\frac{A'}{232}\right)c'_1}.$$

The condition was laid down that the quantity under the root should lie between ·8 and 1·2. Only three pairs of solutions showing the red absorption band were found to satisfy this condition, and in these cases x was calculated. Data for these solutions are tabulated below, the values of c_2 being corrected so as to include the water originally in the alcohol:—

Solution.	c_1.	c_2.	A.	x.
I*c*	·8738	9·856	70	17·4
II*a*	·38	3·605	78	
I*d*	·8224	12·70	12	13·0
III*a*	·10	3·74	11·2	
I*e*	·771	15·66	3·92	13·7
II*c*	·34	9·86	3·6	
Mean value of x=14·7.				

The above calculation would seem to show that in the red phase each cobalt chloride molecule is associated with approximately fifteen molecules of water. This is the phase existing in dilute aqueous solution, and I shall meantime term it the polyhydrate.

The absorption spectra of three definite and distinct phases of cobalt chloride, viz. the anhydrous salt, the hexahydrate, and the polyhydrate, have now been investigated; and in the cases of the first and the last, values of A have been determined. No traces have so far been found of any other hydrate. With these data at my disposal I have returned to the investigation of the saturated aqueous solution.

In the first place, its spectrum though not simply that of the polyhydrate bears a great resemblance to it. The maximum of the band in the green lies practically at the same wave-length, although the values of A are increased and the shape differs slightly. There is a small band with maximum at $\lambda = 680\ \mu\mu$, probably indicating the presence of anhydrous salt. Obviously the concentration of this phase in solution cannot be more than $\frac{A}{232}$ or about $\frac{1}{120}$ of the total concentration, and this would not account for the change in the band in the green.

W. N. Hartley in his investigation of absorption spectra found that 5 c.c. of water dissolved 8 gms. of the hexahydrate at 16°, the room temperature of this research, in forming a saturated aqueous solution. This gives the proportion of 14·3 molecules of water to 1 molecule of cobalt chloride, and would enable most, if not all, of the salt to enter the polyhydrate phase. This fact and the similarity of the bands in the green for the dilute and saturated solutions have induced me to assume meantime that at least nine-tenths of the salt does go into the polyhydrate phase.

In fig. 3 I have plotted A as a function of λ for dilute and saturated solutions. The values of A for the dilute solution are taken from Paper II. of this series, and the values for the saturated solution are from this paper. These are connected by curves X and Y respectively. I then deducted nine-tenths of the ordinates of X all over the curve from the corresponding ordinates of Y. The remainders obtained are shown in the heavy line, and are a rough indication of the absorption of the remaining tenth of the chloride in solution. The curve shows clearly a band with maximum at $\lambda = 550\ \mu\mu$, and less clearly a band about $\lambda = 490\ \mu\mu$. It is to be noted that the more distinct band of the hexahydrate crystal, viz. that at $\lambda = 550\ \mu\mu$, is the more distinct here.

We thus have evidence of the presence to a slight extent of the hexahydrate in the solution. This conclusion has the support of those

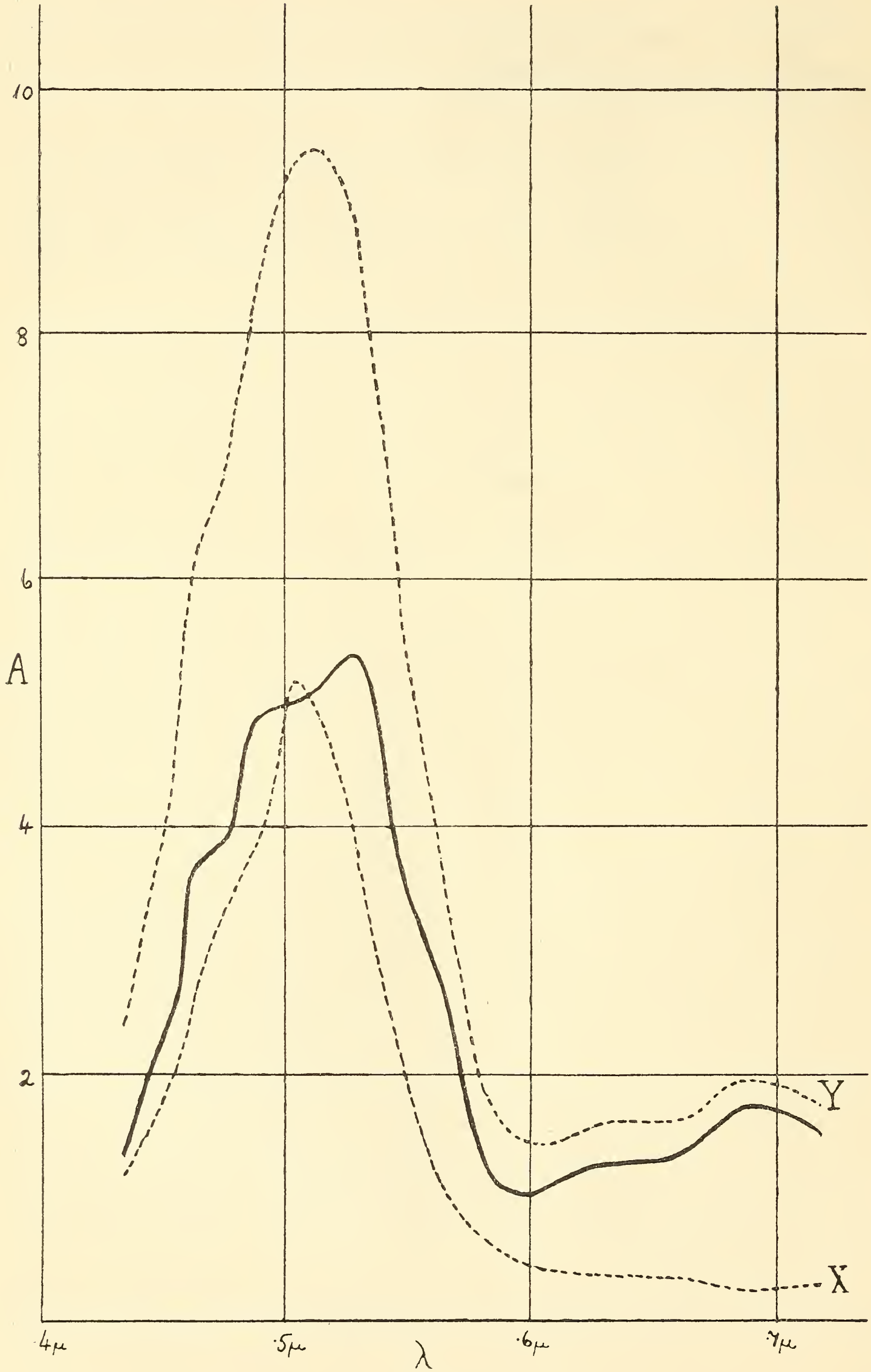


Fig. 3.

authorities who are of opinion that before a hydrate can be deposited from a solution it must be present to some extent in that solution.

The change in colour on heating the solution is mainly to be ascribed to the increase of the anhydrous phase at the expense of the polyhydrate.

This discussion of the phases in solution is interesting in connection with the hydrate theory emphasised by Jones (Carnegie Institution of Washington, Publication No. 60).

The law of mass action has not been applied to this colour change before. And in the meantime I do not consider the result quite certain, as one assumption made, viz. that there is no compound formed by the chloride and the alcohol, has yet to be tested by the use of other solvents. This work will be commenced immediately.

This investigation was carried out at the instigation of Dr R. A. Houstoun, to whom I am greatly indebted for advice and various suggestions received during its progress.

(*Issued separately March* 20, 1912.)

IX.—Observations on the Structure and Affinities of *Branchiomaldane vincenti* Langerhans. By J. H. Ashworth, D.Sc., Lecturer in Invertebrate Zoology, University of Edinburgh. (With One Plate.)

(MS. received January 22, 1912. Read January 22, 1912.)

THE genus *Branchiomaldane*, represented by a single species *vincenti*, was founded by Langerhans * to contain certain small Polychætes, which he discovered living in sand-covered tubes, among algæ, on the northern shore of Teneriffe. Professor Mesnil † has since found specimens, inhabiting mucous tubes situated on the lower side of the calcareous alga Lithothamnion, in rock-pools at St Martin, near Cape la Hague, and has given an account of some of the structural features of these worms, which he showed to be adult and hermaphrodite. Professor Fauvel ‡ considered *B. vincenti* to be a dwarf *Arenicola*, arrested in development; he therefore merged *Branchiomaldane* with *Arenicola*, and designated the worm *Arenicola vincenti*. The writer has investigated the anatomy of several specimens of this worm, kindly sent to him by Professor Mesnil, and has concluded that the genus *Branchiomaldane* should be retained; the original name of the worm—*B. vincenti*—is therefore employed in the following account. §

The form of *B. vincenti* is shown in the Plate, fig. 1, which represents a specimen 8 mm. long. Average specimens are about 8 to 11 mm. long, and ·3 to ·5 mm. in diameter at their widest part; the largest recorded specimen is 20 mm. long.

The prostomium is bluntly conical (fig. 2), or sometimes more rounded or ovoid, and overhangs the mouth. It bears groups of eyes, the number and disposition of which vary a little in different specimens. The eyes are usually arranged so as to form, on each side, an antero-lateral group, generally situated well down the side of the prostomium, and a dorsal group, near the hind margin of the prostomium.

The peristomium, which is achætous, is divided from the prostomium by a shallow groove. There is not a pocket-like nuchal invagination of

* *Nova Acta K. Leop.-Carol. Akad.*, Bd. xlii. (1881), p. 116 ; tab. v., fig. 21.

† *Bull. Sci. France Belg.*, T. xxx. (1897), p. 156 ; T. xxxii. (1899), p. 323 ; *Zool. Anz.*, Bd. xxi. (1898), p. 635.

‡ *Bull. Sci. France Belg.*, T. xxxii. (1899), p. 313 ; *Mém. Soc. Nation. Sci. Nat. Math. Cherbourg*, T. xxxi. (1899), p. 165.

§ The expenses incurred during this research have been defrayed by a grant from the Earl of Moray Endowment of the University of Edinburgh.

the dorsal epithelium, such as is present in late post-larval and in adult examples of *Arenicola.**

The next segment is almost without chætæ, and is evidently homologous with the achætous body segment of *Arenicola*, for, in both *Branchiomaldane* and *Arenicola* the œsophageal connectives unite near the middle of this segment. It is followed by the chætigerous segments, the number of which depends on the stage of growth attained; the largest number observed is fifty-one (see fig. 1). The terminal pygidium is usually bluntly conical, but its shape varies in different specimens according to the condition of extension or retraction at the time of fixation. It is slightly notched terminally, its end being four- or five-lobed.

Each chætigerous segment bears notopodial chætæ and neuropodial crotchets. The notopodia and the short † neuropodia are comparatively feebly developed, being little raised above the general surface. In all the preserved specimens examined the notopodia are not well-marked conical outgrowths, such as are present in *Arenicola*, but are only slight rounded elevations on which the aperture of the notopodial chætal sac, and the tips of the capillary chætæ, are seen. These chætæ resemble, in their general structure, those of a post-larval *Arenicola ecaudata*, but in each notopodium about one-half the chætæ have the distal fourth bent on the proximal part at an angle of about 150°. The crotchets, which are always few in number—from two or three in the first two and last segments, up to nine or ten in most of the branchial segments,—are of the same general form as those of *Arenicola*, but the rostrum is very sharply pointed. As in *Arenicola*, a crotchet of different form—with more elongate shaft, smaller rostral region, and more distally situated nodulus—is present, but for a short time only, in a few of the last-formed notopodia.

Most of the chætigerous segments are annulated. But the annulation of the anterior and middle segments is much less definite, and the number of annuli per segment is less constant than in *Arenicola*. In the larger specimens the annulation becomes obvious usually about the third segment, which is subdivided into four or five rings; each of the succeeding segments, as far back as about the twentieth, shows from five to seven rings. In the

* In a young post-larval *A. ecaudata*, 5·5 mm. long, with 46 chætigerous segments, there is no nuchal invagination, but only a faint groove; in an older example, 8·5 mm. long, with 56 chætigerous segments, there is between the prostomium and peristomium a distinct invagination of the epithelium, forming the incipient nuchal organ. Examination of sagittal serial sections of a mature specimen of *B. vincenti*, 10 mm. long, shows that a nuchal invagination is not present.

† Young examples of *A. ecaudata*, of about the same size as the specimens of *B. vincenti* described above, have considerably longer neuropodia.

following segments the numbers of annuli diminish rapidly to two—the number present in each of the last twelve to twenty segments. The specimen shown in fig. 1 is rather contracted, and some of the grooves in this, and in other specimens examined, may be due merely to folding of the body-wall, brought about by contraction of its musculature. In other specimens, killed more fully extended, the annuli are fewer, and nearly all the branchial segments are clearly bi-annulate, the larger anterior ring bearing the chætæ and the smaller posterior one the gills (fig. 4). This condition was evidently exhibited also by Langerhans' specimens (see his fig. 21 *g*), and it seems to be sufficiently constant to be considered one of the diagnostic features of this worm.

The gills of *Branchiomaldane vincenti* are of a very simple type, being composed of one, two, or three, rarely four, finger-shaped outgrowths of the body-wall, each of which contains an extension of the cœlom and a vascular loop connected with the afferent and efferent branchial vessels. In the preserved condition the gill-filaments, which are not more than ·15 mm. long in the writer's specimens, generally have an annulate appearance, due to the contraction of their muscles. In those gills in which two or more filaments are present, the latter arise either close together on the body-wall or from a short common base.

The first branchiate segment is about the twentieth, but the position of the first gill is subject to variation: for instance, in one of the specimens examined the first gill is borne on the eighteenth segment, in two on the nineteenth, in one on the twentieth, and in two on the twenty-first segment.* In most specimens a few—two to six—of the anterior and posterior gills are simple, each consisting of a single filament, but each of the other gills is composed of two or three (rarely four) filaments. The last one or two segments may be abranchiate (fig. 4).

The gills begin to arise about the time the worm attains thirty segments, and then develop rapidly; for in a specimen with thirty-six segments, gills are present on the nineteenth to thirty-fourth inclusive.

The colour of *B. vincenti* is usually pinkish, due to the blood-vessels being seen through the semi-transparent and feebly pigmented † body-wall. The epidermis contains abundant mucous cells, which secrete the envelop-

* Langerhans records examples in which the first gill was borne on the twenty-third and twenty-fourth segments respectively.

† Specimens of adult *B. vincenti* and of young *Arenicola* about the same size differ markedly in the amount of their pigmentation. The epidermis of the former usually contains only a few dark granules which are situated chiefly at the anterior end of the worm, but in young examples of *Arenicola* a considerable amount of yellow or green pigment has already been formed.

ing tube. In the mid-ventral region the epidermis is thickened and highly glandular.

The cœlom resembles, in its relations and proportions, that of an ecaudate *Arenicola*. The cœlomic fluid contains, besides the genital products described below (p. 67), numerous oval and spindle-shaped cells. Septa are present, as in *Arenicola*, at the anterior end of the first, third, and fourth segments. There are no septal pouches. The second and third septa are rather thicker and more muscular than those of specimens of *Arenicola* of the same size. Septa are also present at segmental intervals throughout the gill region. The muscles of the body-wall are arranged similarly to those of *Arenicola*: beneath the epidermis is a thin layer of circular muscles and the longitudinal muscle-bands (fig. 5). Oblique muscles occur in the segments posterior to the third septum, but have not been observed in front of this septum.

The alimentary canal (fig. 3) is of the same type as that of *Arenicola*. The mouth, which is situated antero-ventrally in the peristomium, is either a transverse slit or a narrow crescentic opening. It leads into the pharynx, which is muscular, protrusible, and well provided with blood-vessels. The œsophagus is slightly dilated posteriorly, and bears there two elongate, conical, or ovoid glands, the walls of which are simple, that is, do not present infoldings, to increase the secreting surface, such as occur in the corresponding glands of *Arenicola*. The two glands have a common stalk, and a single opening into the œsophagus. The "stomach," which has thick glandular walls, on the cœlomic face of which are numerous intersecting blood-streams, extends through nine or ten segments, and then merges into the intestine, which opens at the terminal anus. The gut contains fine débris, in which fragments of sponge-spicules, and the frustules of several species of diatoms, are the only recognisable remains of organisms.

The vascular system has not been studied in detail, as only preserved material has been available; but, so far as ascertained, the arrangement of the vessels appears to be similar to that in an ecaudate *Arenicola*. Satisfactory evidence of the presence of paired hearts is not obtainable from the sections available, but observations on the living animal would be necessary to determine definitely the absence of these organs. The efferent vessels of the sixteenth and succeeding segments open into the dorsal vessel, so that, as in the ecaudate species of *Arenicola*, all the gills return blood to this trunk. The ventral vessel is enveloped with chlorogogen cells. Many of the vessels of the branchial region attain a large size. The blood is red, due to the presence of hæmoglobin.

The brain of *B. vincenti* consists, in the specimen in which it is most highly developed, of a median anterior lobe, and a pair of large middle and of small posterior lobes. These last were not recognisable in other specimens examined. The middle dorsal region of the brain is situated near the external surface, the prostomial epithelium which overlies it being very thin. The dorsal part of the brain is cellular, the ventral part fibrous. The œsophageal connectives arise from the middle region of the brain, and, after traversing the peristomium obliquely backwards and ventrally, unite near the middle of the following achætous segment. The ventral nerve-cord, which is apparently non-ganglionated, lies immediately below the epidermis (fig. 5). Giant nerve-fibres are not present.

The eyes are situated in the sub-epidermal tissue of the prostomium. Each eye consists of a cup-shaped mass, about 6 to 10μ in diameter, of brownish pigment-spherules, in some cases with the addition of a lens, situated in the mouth of the cup. Statocysts are absent, and the nuchal groove is feebly developed (p. 62).

The published references to the nephridia of *B. vincenti* give the impression that the number of these organs is subject to variation. Professor Mesnil * stated that segmental organs, more or less darkly pigmented, were present in the fifth, sixth, seventh, and eighth chætigerous segments. In the following year † he referred to the presence of four or five pairs of nephridia. Professor Fauvel ‡ attributed to this worm four pairs of nephridia, but later § stated the number as three to five pairs, with pores in the fifth to the ninth chætigerous segments. Drs Gamble and Ashworth || found only two pairs, having openings on the fifth and sixth segments, and remarked that the internal ends appeared to be without funnels. The writer has examined the nephridia, as far as has been possible in the limited amount of material at his disposal. In each of the five specimens suitable for the study of these organs, only two nephridiopores could be seen, situated immediately *ventral* ¶ and slightly posterior to the fifth and sixth neuropodia. The aperture of the second nephridium is usually slightly larger than that of the first. That only two pairs of nephridia are present has been ascertained definitely by dissection of one specimen and examination of serial sections of two others. A diagram of the nephridia is given in the

* *Bull. Sci. France Belg.*, T. xxx. (1897), p. 158.

† *Zool. Anz.*, Bd. xxx. (1898), p. 636.

‡ *Bull. Sci. France Belg.*, T. xxxii. (1899), p. 291.

§ *Mém. Soc. Nation. Sci. Nat. Math. Cherbourg*, T. xxxi. (1899), p. 166.

|| *Quart. Journ. Micr. Sci.*, vol. xliii. (1900), p. 537.

¶ In *Arenicola* the nephridial apertures are invariably just behind the *dorsal* ends of the neuropodia.

accompanying text figure. From the funnel * of the first nephridium a short narrow tube leads into the middle or excretory portion of the organ, and this, in turn, into the terminal portion, which opens to the exterior near the fifth neuropodium. The funnel * of the second nephridium, which is considerably larger than that of the first, opens into a long, slender, convoluted tube (rather simplified in the diagram), and this merges into the main excretory portion of the organ. This part, which is moderately thick-walled † throughout, opens by a pore near the sixth neuropodium, but extends backwards as far as the eighth,‡ and in one specimen almost to the ninth neuropodium, where it ends blindly. This nephridium is sometimes dilated at its blind end, as shown, but in other cases does not present such an enlargement. The first nephridium varies in length, in the different

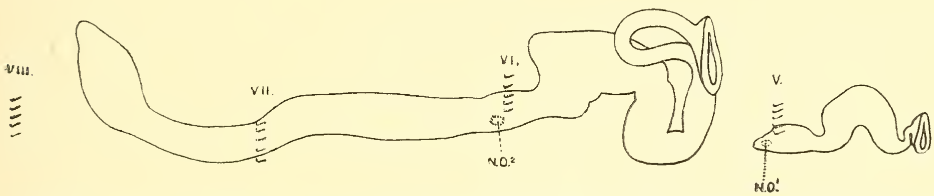


Diagram of the nephridia of the left side of an adult specimen, about 10 mm. long, seen from the inner (median) aspect. The specimen was divided mesially, the alimentary canal was dissected away from the left portion, and the nephridia drawn as they lay upon the body-wall. The crotchets of the fifth, sixth, seventh, and eighth neuropodia, and the positions of the external apertures (N.O.^{1}, N.O.^{2}) of the nephridia are indicated. × 100.

specimens examined, from ·23 to ·4 mm., and the second from ·8 to 1·0 mm. This elongate second nephridium, extending through three, or sometimes four, segments, was probably mistaken by Professors Mesnil and Fauvel for three or four separate nephridia.

The reproductive organs are situated on the cœlomic epithelium of the oblique muscles (fig. 5) and septa, and of the lateral body-wall, near the point of insertion of the oblique septum. They are not present in front of the third septum. All the specimens examined by the writer are hermaphrodite. The oöcytes fall from the gonads, when their diameter is not more than 15 to 20μ, into the cœlomic fluid, and there complete

* The funnels of both nephridia are apparently of a simple type, but they are very difficult to investigate in preserved material, and observations on living specimens are desirable in order to determine more fully the structure of these funnels.

† The wall of the nephridium contains muscle fibres, which are more highly developed in the posterior portion of the organ, especially near the blind end.

‡ Professor Mesnil (*Bull. Sci. France Belg.*, T. xxx. (1897), p. 154), stated that, in his specimens of "*Clymenides incertus*," which were undoubtedly young phases of *B. vincenti*, non-pigmented nephridia were present in the fifth, sixth and seventh chætigerous segments. Apparently the extension backwards of the second nephridium had not attained its full development in these young specimens.

their growth. Large oöcytes are very sparse in the anterior portion—in front of the eighth segment—but, in mature specimens, they practically fill the cœlomic cavities of the posterior segments (fig. 5). Each full-grown oöcyte has a thin vitelline membrane, a large amount of coarsely granular yolk, a vesicular nucleus, and a large nucleolus. When mature, the eggs are, according to Professor Mesnil, milk-white, and about ·3 mm. long and ·2 mm. broad. The volume of each egg is, therefore, four or five times that of the egg of any species of *Arenicola*. When the sizes of the parent worms are taken into account, the egg of *Branchiomaldane* is relatively of much greater magnitude. The marked disparity in the size of the eggs is correlated with a striking difference in regard to the number of eggs present in mature specimens of the two genera. Immediately previous to the breeding season, there are many thousands * of full-grown oöcytes in the cœlom of a mature female *Arenicola*, but in a mature *Branchiomaldane* the number is much smaller. An almost mature specimen of *B. vincenti* was found to contain about 120 large oöcytes, all in approximately the same phase of growth; and, judging from the small amount of free space in the cœlom, this was not far from the maximum number which could have been accommodated.

The eggs of *B. vincenti*, being much too large to escape by way of the nephridia, † can make their exit, as they do in many other Polychæta, only by rupture of the body-wall, which occurs in one or more of the posterior segments. Two of the mature specimens examined showed eggs escaping in this manner.

The stages of spermatogenesis, seen in the cœlom, are similar to those found in the *Arenicola* and other Annelids; all stages, from young spermatogonia to masses of spermatids and ripe spermatozoa, may be present in the same specimen. Several of the nephridia examined contained spermatozoa.

Professor Mesnil found eggs, in August and September 1898, around the mucous tubes in which the worms were living, and was able to show that, on hatching, the young worm has four chætigerous segments. He stated that cilia were not visible on these young specimens, except at the dorsal margin of the prostomium, that is, in the nuchal groove.

* The writer estimated the number of oöcytes in a mature female *Arenicola marina* to be about 80,000.

† The lumen of the narrow tube of each nephridium, just behind the funnel, is only about 6μ in diameter.

Systematic Position and Affinities of *Branchiomaldane*.

According to Langerhans, *Branchiomaldane* exhibits unmistakable relationship with the Maldanidæ, but he placed it in the family Thelethusæ (Arenicolidæ) on account of the presence of gills on a number of the posterior segments. *Branchiomaldane* presents, however, few features of resemblance to Maldanids; their only common characters * are the tubicolous mode of life, the possession of a non-ganglionated nerve cord, and the absence of statocysts. The parapodia and chætæ of *Branchiomaldane* exhibit a general similarity to those of certain Maldanids, but they are in still closer agreement with those of *Arenicola.* That the differences between *Branchiomaldane* and the Maldanids are much more striking than the resemblances is realised on considering the following features of the Maldanidæ—the prostomium and paired nuchal organs, the elongate non-annulate segments, often with bands of bright colour, the specialised pygidium, the simple alimentary canal without cæca, the absence of gills, and the diœcious character of each individual.

It is clear from a consideration of the anatomy of *B. vincenti* that this worm is more closely related to *Arenicola* than to any other Polychæte, and that it must be included in the family Arenicolidæ; but the writer is of opinion that the coalescence of the genera *Branchiomaldane* and *Arenicola,* recommended by Professor Fauvel, is not advisable. Professor Fauvel has pointed out that *Branchiomaldane* presents several points of resemblance to a young *A. ecaudata,* and he regards the former as furnishing a new case of "néoténie" † in Annelids. It may be noted in this connection that, as these worms live under practically identical conditions, some of the similarities may be due to convergence. Further, some of the resemblances, cited by Professor Fauvel in support of his argument, are not so close as they were believed to be; for instance, the gills of *B. vincenti* were stated to resemble those of a young *A. ecaudata,* and the nephridia of *Branchiomaldane* to have the same form as those of *Arenicola,* and to lie in the same segments as in *A. grubii* (=*branchialis* Aud. et Edw.) and *claparedii* (=*pusilla* Quatrefages). But the branchial segments of *B. vincenti* are bi-annulate, whereas in *Arenicola* they are five-ringed; the gills of the former are borne on the annulus behind the chætigerous

* An extension of a nephridium through two or more segments occurs occasionally also in the Maldanids, *e.g.*, in *Proclymene mulleri*, in which the nephridia of the eighth segment extend backwards into the tenth. See Arwidsson, I., *Zool. Jahrb. Abt. Syst.*, Suppl. Bd. ix. (1907), p. 132. But little is known of the nephridia of Maldanidæ.

† A term used by Kollman to designate the abnormal persistence of larval characters in an adult.

one, those of *Arenicola* are invariably situated on the chætigerous annulus. The nephridia of *B. vincenti* are reduced to two on each side, the second of which is elongate and extends through three or four segments; whereas the species of *Arenicola* cited have five pairs of nephridia, each of which extends through one segment only. Other striking differences are exhibited by the reproductive organs, which in *Branchiomaldane* are hermaphrodite and extensively distributed on the oblique muscles and septa; whereas *Arenicola* is invariably diœcious and its gonads are restricted to the gonadial vessels on the nephridia. The egg of *Branchiomaldane* is large and plentifully yolked and does not give rise to a ciliated larva, while the egg of all the species of *Arenicola* is much smaller, and, in those species which have been investigated, produces a free-swimming trochosphere larva. As the relationship of *Branchiomaldane* is especially with the ecaudate species of *Arenicola*, it is worthy of note that the latter possess statocysts, giant nerve-fibres, septal pouches, and a nuchal invagination, whereas all these are wanting in *Branchiomaldane*. But the important difference in regard to the position of the gills is alone sufficient, in the writer's opinion, to render necessary the maintenance of the two genera *Arenicola* and *Branchiomaldane*.

While *Branchiomaldane* presents some primitive characters, for instance, a simple conical prostomium and homonomy of its segments, it affords considerable evidence of having undergone secondary modification and retrogression. Its small size, the simple form of its gills, the absence or great reduction of certain sense-organs (statocysts and nuchal organ), the reduction in the number of nephridia, its hermaphroditism, the large size of the egg and the absence of a free-swimming larval stage—features in which *Branchiomaldane* departs from *Arenicola*—are probably correlated, to a large extent, with the much more sedentary life of the former. The reduction of sense-organs, and the reduction or modification of nephridia, are well-known secondary changes associated with the abandonment of errant for sessile habits. In particular, the occurrence of hermaphroditism affords strong evidence of departure from the primitive condition; for hermaphroditism is secondary in Polychæta, as it is in Mollusca. This seems clear for, at least, two reasons: (1) because of the few cases of hermaphroditism—only about a score of species—known in the Polychæta, and (2) because hermaphroditism is generally associated, in members of this order, with some obviously secondary modification of structure or mode of life. About half the known hermaphrodite Polychæta are tube-dwelling Sabelliformia, while most of the others are Polychætes of unusually small size and simplified structure—e.g., *Lycastis quadraticeps*, *Ophryotrocha* *puerilis.* In *Branchiomaldane,* hermaphroditism is associated with sedentary habits (as in the case of the hermaphrodite Serpulids *Spirorbis* and *Salmacina*), and with small size and comparatively simple external form, and may be safely regarded, as in the other cases mentioned above, as a secondary character.

Branchiomaldane presents an interesting parallel to the Nereid *Lycastis quadraticeps*; both worms are considerably smaller than their immediate allies; they are hermaphrodite, and their ova are much larger than those of their diœcious relatives. Dr H. P. Johnson * has pointed out that in *L. quadraticeps* the increase in the size of the ova "is not co-ordinate with complexity of organisation, or any real advance towards a higher plane of being, but rather the reverse; . . . the macroögenous Polychætes are all of puny size, comparatively simple organisation, and one † is hermaphroditic." These remarks are in accord with the grade of development exhibited by *Branchiomaldane,* which, as compared with its ally *Arenicola,* has certainly not made "any advance towards a higher plane, but rather the reverse."

The systematic position of *Branchiomaldane* may be summarised thus: the genera *Arenicola* and *Branchiomaldane* (with a single species, *B. vincenti*) together constitute the family Arenicolidæ. *Branchiomaldane* is most nearly related to the ecaudate species of *Arenicola* (*ecaudata,* Johnst. and *branchialis,* Aud. et Edw.), to young stages—about 10–12 mm. long—of which it presents some points of similarity in form and habits. But *Branchiomaldane* differs from these species of *Arenicola* in several important characters; for instance, its branchial segments are bi-annulate and the setæ and gills are borne on consecutive annuli; its nephridia are reduced to two pairs, the second of which is considerably modified; it is hermaphrodite and produces comparatively few but large eggs, and it does not possess septal pouches, statocysts, nor a nuchal invagination. Although the young stages of *Arenicola* above mentioned have, for a time, habits similar to those of *Branchiomaldane,* the former soon assume a more wandering mode of life, which they maintain henceforward; whereas *Branchiomaldane* remains sedentary, and some of its structural peculiarities are no doubt the result of retrogressive changes associated with its tubicolous habits.

* *Biol. Bull. Wood's Holl,* vol. xiv. (1908), pp. 371–386.

† Namely, *L. quadraticeps*; Dr Johnson does not appear to have been aware of the case of *Branchiomaldane.*

DESCRIPTION OF PLATE.

List of Reference Letters.

A., anus; A.B.S., achætous body segment; Ch.S.[1], first chætigerous segment; D.V., dorsal blood-vessel; E., epidermis; G.[1], first gill; Go., gonad, on oblique muscle; I., intestine; M., mouth; M.C., layer of circular muscles; M.L., band of longitudinal muscles; M.O., oblique muscle; N.C., nerve-cord; N.O.[1], N.O.[2], external openings of first and second nephridia; Not., notopodium; Nr., neuropodium; O., oöcyte; Œ., œsophagus; Œ.G., œsophageal glands; P., prostomium; Pe., peristomium; Ph., pharynx; S.[1], S.[2], S.[3], first, second, and third septa; Sp., groups of developing male cells (spermatogonia and spermatocytes); St., stomach; V.V., ventral blood-vessel (surrounded by chlorogogen cells).

All the figures are drawn from examples of *Branchiomaldane vincenti* Langerhans.

Fig. 1. Adult specimen (killed in a rather contracted condition) about 8 mm. long, with fifty-one chætigerous segments, the twenty-first to fiftieth of which are branchiate. × 35.

Fig. 2. Anterior end of adult specimen, about 10 mm. long; dorsal aspect. The groups of eyes are seen on the prostomium (P.). × 43.

Fig. 3. Anterior portion of a specimen about 10 mm. long, after staining and clearing; to show the alimentary canal and septa, which have been drawn somewhat diagrammatically. × 35.

Fig. 4. Posterior portion, including the last thirteen chætigerous segments and the pygidium, of a well-extended specimen, 10·5 mm. long. Note that the segments are bi-annulate, the anterior larger ring being chætigerous, and the posterior smaller one branchiferous. × 35.

Fig. 5. Transverse section, passing through the fortieth segment, of a mature specimen. Note the presence, in the cœlom, of numerous oöcytes, and two masses of male cells (Sp.). × 95.

(*Issued separately April* 11, 1912.)

ASHWORTH: BRANCHIOMALDANE VINCENTI.

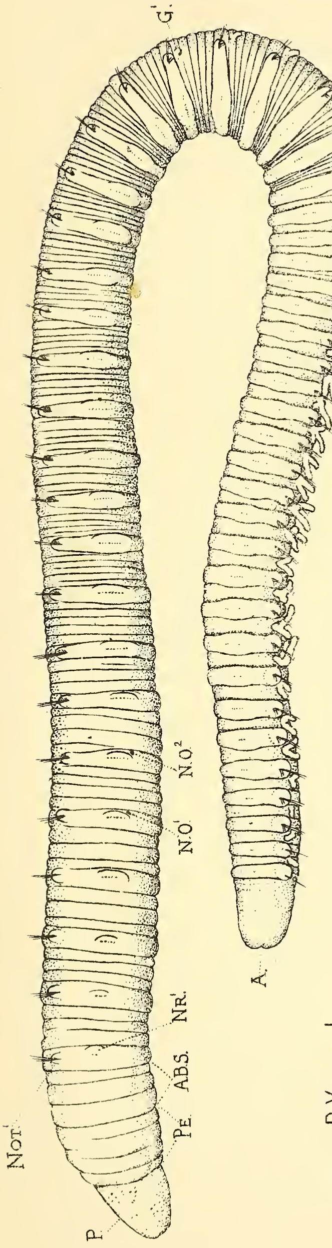


Fig. 1.

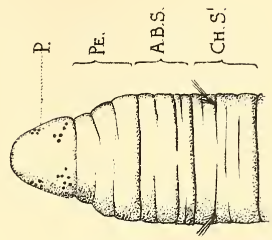


Fig. 2.

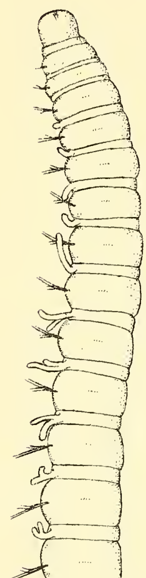


Fig. 4.

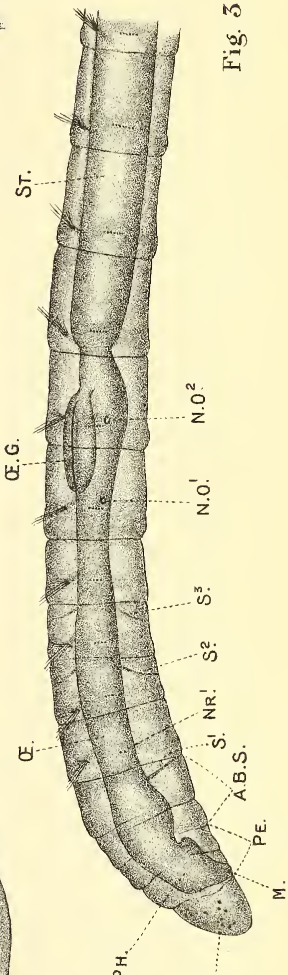


Fig. 3.

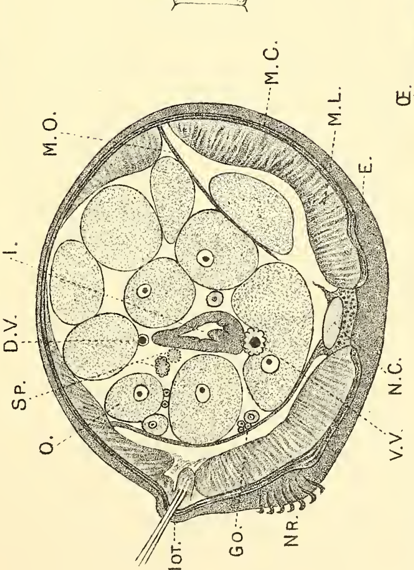


Fig. 5.

A. K. Maxwell and J. H. Ashworth, del.

M^c Farlane & Erskine, Edin^r.

X.—A Method of Measuring Mental Processes in Normal and Insane People, with special reference to Maniac Depressive Insanity.

By George Rutherford Jeffrey, M.D. (Glasg.), F.R.C.P. (Edin.), Senior Assistant Physician, Crichton Royal Institution, Dumfries; late Senior Assistant Physician, District Asylum, Ayr. *Communicated by* Sir James Crichton-Browne.

(MS. received September 5, 1911. Read November 13, 1911.)

Among the problems which confront the psychiatrist in arriving at a judgment upon a mental case, few are more difficult than the estimation of the patient's mental working capacity. A rough approximation may be attained by careful observation of the patient's daily reaction to his surroundings—by the attitude he adopts to the little problems of his daily life; but such is, at best, only a very vague and unsatisfactory measure. To impose simple tasks for the purpose of obtaining objective criteria upon which to base our judgment is unusually difficult, and patients who were incapable of comprehending the new task would have to be excluded from its action; and to interest the patients in such a task sufficiently to persuade them to do their best is often not possible. The estimation of the working capacity of a large proportion of patients would thus seem at first sight to be beyond our powers to ascertain.

Hence it is that the methods adopted by psychologists for ascertaining the mental working capacity in normal life have generally been found unpractical in insanity.

Kraepelin and his followers, in their investigations of tea, coffee, ethyl alcohol, amyl alcohol, bromides, and other medicinal substances, have, however, used a method which in their hands seems to have proved eminently satisfactory, not only for normal but also for abnormal mental states. Their test has recently been fully described (1), so I need here only briefly refer to its essential features. In it the question of the comprehension of a new mental task by the patient has been solved by utilising as the basis for the test one of the most primitive and familiar of our everyday experiences—the act of simple addition. The test consists of the addition of digits in pairs as rapidly as possible. The figures from 1 to 9 are systematically arranged, so that as far as possible the difficulty of adding each single pair is constant. Each pair, therefore, is taken to represent a constant unit of mental work, and the test enables not only the number of units of mental work done throughout a given period—

the mental working capacity for that period—to be ascertained, but also by a simple device the number of units per minute throughout the period. By plotting the number per minute of these mental units upon a curve, the progress of the mental working capacity throughout the period is objectively shown, so that we can thus demonstrate mental work in a manner analogous to that by which we can through the use of the ergograph or the dynamometer demonstrate muscular work. The mode of using the test is as follows:—The patient is required to add the digits as rapidly as possible in pairs; each investigation lasts a fixed time, usually ten or fifteen minutes. At the end of every minute the examining physician calls "Stroke," and the patient then makes a line under the last digit he has written: the number of units between any two successive lines shows the rate of work in that particular minute. To minimise time and effort in writing, to make the test an index of the intellectual, not of the muscular processes involved, only the unit of each sum is written; thus $6+7=3$, $9+8=7$. To economise space the method of continuous addition is always employed, if the patient be sufficiently intelligent to grasp it. Excepting the first and last figures in the columns, each is used twice, once added to the digit which precedes it and once to that which follows it.

8		
	1	$8+3=11$
3		
	8	$3+5=8$
5		
	2	$5+7=12$
7		
	6	$7+9=16$ (2)
9		

Besides the total number of units of work done affording an index to the mental capacity, the accuracy of the work done affords an index to the condition of the fixed associations. During the performance of the test, as might be expected, errors were frequent, some in the form of faulty addition of the right units, some in the correct addition of the wrong units, whilst others were made but subsequently corrected by the person doing the test. The nature of these errors was interesting. Thus it was found that the same error was constantly repeated by the same person, *e.g.* two sevens were persistently called 15 instead of 14, or nine and seven 17 instead of 16, etc. In this paper I do not propose to deal with the errors, but only with the question of the mental capacity, which is not influenced by errors such as I have mentioned.

The object of this research was to ascertain whether or not any definite characteristic of the mental working capacity could be detected in the disease or group of diseases described as maniac depressive insanity. I have taken this term to include "folie circulaire," and cases of periodic recurrent mania or melancholia. Most of my cases had been under my personal observation for several years, some had been under supervision in this Institution for decades. I have carefully avoided all cases in which I could not exclude the diagnosis of dementia præcox, and also all cases of a first melancholic or maniacal attack. Many of the last were undoubtedly cases of maniac depressive insanity, but I did not wish to introduce any element of uncertainty, so I thought it better to reject them, and to consider only such cases as left no possibility of doubt as regards the diagnosis. I had no difficulty in making the patients understand the method of continuous addition; they all entered into the task with an astonishing degree of zest. They discussed their daily performances, and a keen spirit of rivalry prevailed. Their interest was easily sustained for several days, and some of them were considerably disappointed when their investigation was finished; but as I early found that the majority of my cases showed a flagging interest after the fifth day, I had to limit the duration of the experiment to five days. As Maloney has suggested, I tested the patients simultaneously in groups, and took notes of their behaviour throughout each minute of the test, so that subsequently I could control and explain variations in their work. Never more than five patients were tested together. The experiment, therefore, consisted in allowing thirty cases of maniac depressive insanity to perform the "Reckoning Test," as above described, for fifteen minutes for five days. The experiments were conducted during the summer of 1911. Great care was taken that all the tests took place at exactly the same time on subsequent days, and the same uniform conditions of quiet and familiar surroundings were accurately observed during each test. As it was essential first to establish a normal with which to compare the results obtained in the insane, seven normal people were examined under the same conditions and for the same length of time.

I shall first rapidly survey various factors which influence the test. In the fifteen-minute test of any one day an accidental lapse of attention is shown by a diminution in the number of units of mental work produced during the lapse; similarly a spurt is shown by an increase. A lapse is usually followed by a spurt, and a spurt by a lapse. These variations in the output from minute to minute therefore tend largely to compensate the one for the other, so the total output is fairly characteristic of the mental capacity for the particular period selected. This period was kept constant. A person who began the test, for example, at 2.45 p.m. on one day was always examined at the same hour on the subsequent days. But as Kraepelin has shown, before the maximum output can be attained, "habituation" to the task must be achieved. This may require several days, and, as exercise also increases the output, before a true estimation of the mental capacity can be reached the test must be practised sufficiently to overcome the influence of both "habituation" and exercise. Unfortunately, as the experiments in the mental cases could be continued for only five days, I had in the normal cases also to restrict the period of observation to five days. The full effects of exercise and "habituation" had then not been entirely reached. Between the fourth and fifth day the average percentage gain in the normal test persons was 7·6 per cent; but as this gain was relatively small, and as the increase tended progressively to diminish from the second to the fifth day, the results of the fifth day closely approximate the maximum working capacity in this test.

The maximum working capacity normally varied. I therefore took the average of all of the normal test persons as a standard by which to judge the mental cases.

The gain from day to day was influenced by other factors as well as by the "habituation" and the exercise; daily dispositions, alterations in mood, in zest, in freshness, could influence the output, so the results of any one person could not be taken as typical even of his own average progress for any other than the actual five days during which he was examined. The absolute number of units gained from day to day was not an accurate index to the real progress. Thus if a test person produced 500 units in the first day and gained 100 on the second, 100 on the third, 100 on the fourth, and 100 on the fifth, a true measure of his rate of increase was evident only when the percentage increase was calculated. In this hypothetical case, although the gain per day remained equal, the rate of increase was actually 20 per cent. on the first day, 16 per cent. on the second, 14 per cent. on the third, and 12 per cent. on the fourth. Similarly, only by calculating the percentage gain could the progress of any two persons be compared; for if one person initially produced say 1000 units of mental work and another 500, and both showed an equal gain of 100 units in one day, in the first case the gain was equivalent to only 10 per cent., whereas in the second it was equivalent to 20 per cent. In order, therefore, to compare the gain from day to day not only in one but in all the test persons, normal and abnormal, I calculated the percentage increase on the work done on the previous day. Variations in mood might influence the average rate of gain in any one person. It was extremely unlikely that on corresponding days variations with the same tendency occurred in a majority of the normal test persons.

An accidental increase of rate in one was usually counterbalanced by a decrease in another. The average of all was probably a fair estimation of the normal mean rate of progress. I therefore took the average of the percentage increase in them on the second, third, fourth, and fifth days as standards by which to judge the abnormal.

I obtained a standard of normal improvability for the five days by estimating the mean percentage gain between the first and the fifth days in the normal persons.

As the moods from day to day might vary, and as "habituation" and exercise constantly tended to increase the working capacity, no one day could be taken as typical of the period. I carefully examined the *third day*,* and found that it very closely resembled in many particulars the average of the five; but in the plotting out of the typical work curve only the average of the total work of the corresponding minutes on all the five days † could be taken as representative of that minute. For accidental spurts of one day were then neutralised or depreciated by lapses of attention of another, and only when in the corresponding minutes a preponderating trend was evident in the majority of days could it stamp itself upon the curve of the mean work of the whole experiment. And as the average work curve of all the normal persons taken together further diminished the influence of such accidental variations, an accurate curve of the normal progress of mental work from minute to minute throughout the fifteen-minute interval was obtained. With this curve, the NORMAL MENTAL WORK CURVE, I compared the curves of my insane cases.

We shall first briefly consider the results yielded by the seven normal test persons.

TOTAL OUTPUT.

The total number of units of mental work performed on the first day of the task varied greatly. The results were A, 610; B, 850; C, 694; D, 814; E, 718; F, 768; and I, 642. An important factor in this variation was undoubtedly previous familiarity with figures—the arithmetical training of the person investigated. The number of units of work done in fifteen minutes varied from 610 to 850. The average output for the first day was 728. On the fifth day the corresponding results were 941, 1217, 1182, 1124, 1154, 1086, and 1045; the highest was thus 1217; the lowest 941; the average 1107. The output on the fifth day was taken as equivalent to the maximum working capacity.

* Indicated on the charts by the black curve.

† Indicated on the charts by the dotted line (when *two* curves are shown).

Improvability.

The improvement from day to day was not progressive, nor was it equal in the several test persons. The percentage increase between the first and second days varied from 11 to 37, and averaged 25; between the first and third days, from 22 to 48, and averaged 35; between the first and fourth days, from 30 to 58, and averaged 41; and between the first and fifth days, from 38 to 70, and averaged 53. The average improvement between the first and third, first and fourth, and first and fifth days varied from each other by almost equal amounts. The four test persons who on the first day yielded the lowest number of units showed emphatically the most marked improvement in the course of the five days. Their average improvement for the whole test was 61 per cent., whereas the average of the other three was only 41. With normal people, therefore, the lower the initial output, the greater apparently is the capacity for improvement. The enormous gain on the second day undoubtedly arose in part at least from increased facility in writing. The psychomotor machine in the fifteen minutes' working of the previous day had been "tuned." The tendency in the subsequent days was for the percentage increase progressively to diminish. All the persons together showed an average increase of 25·6 on the second day, 35·0 on the third, 41·9 on the fourth, and 53·0 on the fifth. (Table I.)

Table I.—Showing Initial Rate, Percentage Improvement between the First and Subsequent Days, and Percentage Rate of Variation from Day to Day.

(The percentage rate of variation is in reality the difference in the percentage gain between any two days. For full explanation see page 76.)

Persons.	Output on First Day.	Percentage Improvement between First and Subsequent Days.				Percentage Rate of Variation from Day to Day.			
		Days.				Days.			
		2	3	4	5	2	3	4	5
A.	610	20·4	32·9	40·0	54·2	20·4	10·3	5·3	10·1
B.	850	24·3	30·4	30·0	43·1	24·3	4·9	−0·3	11·3
C.	694	37·6	48·5	58·5	70·2	37·6	8·4	6·7	7·4
D.	814	17·7	22·7	34·9	38·6	17·7	5·5	8·6	2·5
E.	718	31·0	41·5	47·0	60·7	31·0	7·9	4·8	8·3
F.	768	11·4	24·5	32·1	41·4	11·4	11·8	6·0	6·9
I.	642	37·0	45·1	51·0	62·8	37·0	5·8	4·1	7·3
Average per person.		25·6	35·0	41·9	53·0	25·6	7·8	5·0	7·6

The tendency to a gradual diminution of the improvement, that is, to the attainment and maintenance of a maximum working capacity, was still better seen in the course of a second five days of the test (Table II.) undertaken by A and B, in which the variation between the first and the fifth day was only 8·9 per cent. in A and 5·4 per cent. in B. Although Mr A in the initial test had recorded 610, the lowest number of all the test persons, on the tenth day he reached 1234 units. Mr B, who in the first experiment recorded the highest total—850,—on the tenth yielded 1282. In this elementary task, therefore, the rate of improvability progressively diminishes: and an approximately uniform standard of production seems to be attainable in normal people in spite of any initial discrepancy in the mental working capacity.

Table II.—Showing Output and Percentage Variation in the Second Five Days' Test for A and B (*i.e.* G and H in Tables).

Output of A on 6th day	1151
Output of A on 10th day	1234
Output of B on 6th day	1200
Output of B on 10th day	1282

A (G in Tables).			B (H in Tables).		
Days.	Percentage Variation from Day to Day.	Percentage Variation between First and Subsequent Days.	Days.	Percentage Variation from Day to Day.	Percentage Variation between First and Subsequent Days.
7	−1·7	−1·7	7	+1·4	+1·4
8	+2·1	+0·3	8	+0·9	+2·3
9	+7·3	+7·7	9	+5·8	+8·3
10	−0·4	+7·2	10	−1·3	+6·8

As each unit of mental work has a constant difficulty, and as the time occupied in the task is essentially that required for the association processes involved (3), the time spent upon each single unit of work can be readily calculated, and its abbreviation as a result of exercise and "habituation" reckoned.

Daily Variation.

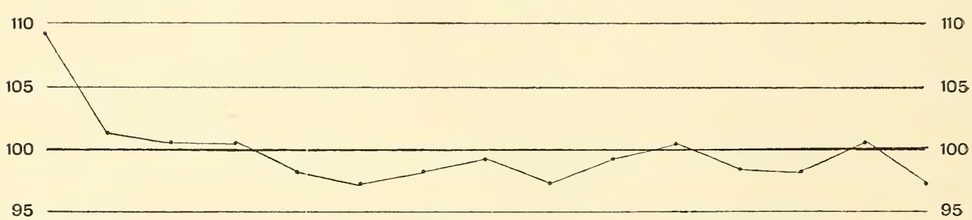
The percentage variation from day to day was markedly greatest between the first and second days; in no case did the variation on a subsequent day exceed that between the first and second. Table I. shows the percentage increase per day in the normal persons. Each increase is calculated from the total of the work produced on the previous day. Thus, the increase on the third day is stated as the percentage increase over the total output on the second day.

The Normal Mental Work Curve.

The normal mental work curve throughout the fifteen minutes varied somewhat in its details in different persons, but certain characteristics were common to all. In the various charts, the initial spurt is seen to be well marked. It varied from 3 to 25 per cent. above the average working capacity. The maximum working capacity was attained in the initial spurt in all but Case E, in whom it occurred in the eleventh minute. The maximum, other than the initial spurt, was very variable. In I it occurred in the third minute, in A and D in the fourth minute, in C in the tenth minute, in E and F in the fourteenth, and in B in the fifteenth.

Curve I.—Average Curve of the Seven Normal Persons.

Showing percentage variation from minute to minute from the average rate of the test.



The curve of the average course of the working capacity in seven normal people over five days shows several essentially characteristic points. This normal work curve (Curve I.) is probably fairly representative of the normal average course of work (*vide* p. 77). The most striking feature of the curve is the extent of the initial spurt. After the fourth minute in the interval between the fourth and the fourteenth, the amount of work was without exception less than the average. The initial spurt had a steep slope from the first minute, and rapidly sank till at the end of the second minute, as it approached the line of average, its downward progress slowed for the third and fourth minutes, and then abruptly continued until the end of the sixth minute. A tendency to spurt was evident in the fourteenth minute; but in the interval between these two spurts the rate was not constant. Similar moods do not recur at the same moment in successive days. The inconstancy was, however, somewhat regular. It consisted of two crests situated between three depressions. The duration of the first of these waves was three minutes, and of the second four. It is remarkable that in only one minute, viz. the eleventh, from the fourth to the fourteenth was the average working capacity attained. To take the middle five minutes of such a curve—as Hylan (4) suggested—would convey an erroneous impression of the working capacity. The constancy in the amount of work done from the fifth to the eleventh is no greater than in any other five-minute period between the fourth and the fifteenth minutes. Here, indeed, it is less; for the higher of the crests occurred in the eleventh minute and the deepest of the "troughs" in the sixth minute. Further, not only is it not more constant, but as has just been shown, every value recorded in the period is below the average.

I may not generalise upon this fact, as I investigated only seven normal persons, but the result at least clearly shows that Hylan's suggestion—that the middle five minutes represents best the working capacity—is not acceptable. True, the initial spurt and the terminal spurt are both avoided, but the capacity for spurting is part of the capacity for performing the task, that is to say, is part of the mental working capacity. Indeed, I have just shown that the mental capacity was usually at its maximum in the initial spurt; and that the next greatest output occurred at the close of the test time. In a daily curve, the constancy in Hylan's middle period—from the fifth to the eleventh minute—was often remarkable (see curve of Mr A's second test), but sometimes the rate during this time was not more uniform or was even less uniform than during the rest of the test (see curve of Mr A's first test). The greater tendency to spurts and to lapses of attention which Kraepelin and others have attributed to the beginning and end of the task was not very apparent. *A priori*, the period of greatest uniformity should fall, as they allege, in the mid part of the task, after the task has been properly "got under weigh," and before it has lasted long enough to induce boredom or fatigue. This middle period is really a method of estimating the results akin to that so commonly used in the investigations of the Kraepelin school—the so-called "Wahrscheinliche Mittel" estimation.

When, however, the importance of the factors conducing to variation from minute to minute, in any one day, is minimised (as by the method I have used of taking the average of each minute over a number of successive days), the results are apparently not adequately represented by the average of the middle five-minute period.

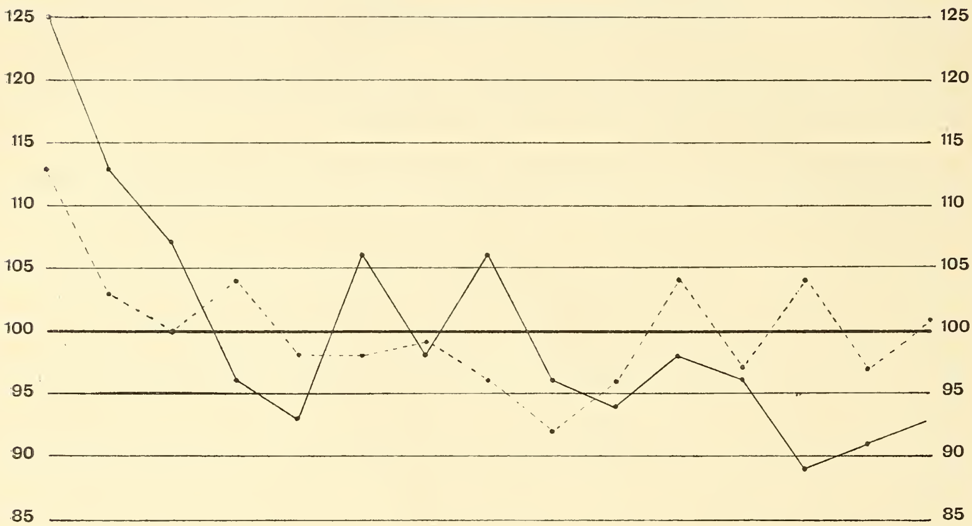
Another curious point to be observed in these curves—especially in the average curve of all the seven persons—is the absence of any evidence of the so-called "Anregung" (5), or incitation of Amberg.

These seven normal people, so far as the mental work of a single fifteen-minute period is concerned, showed their maximum output in the initial

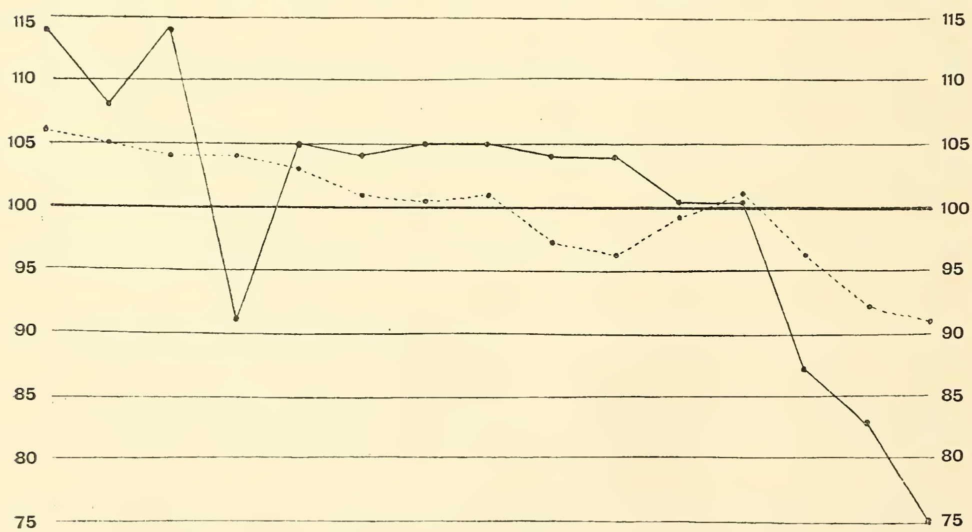
Mr A.—First Five Days' Test.

Showing percentage variation from minute to minute on the third day.

The dotted line shows the percentage variation from minute to minute on the total of the five days.



Mr A.'s Second Five Days' Test (called "Mr G." in Tables and Text).



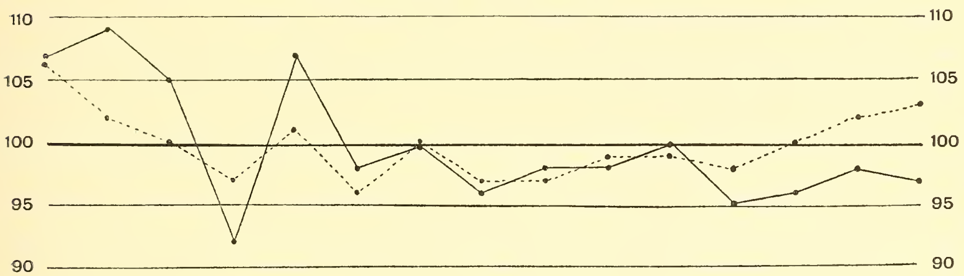
spurt. The gradual working up, "the getting under weigh," "the warming to the task" (Rivers) must have taken place each day before the task began. In fact, I had practically no evidence in my results in normal persons even of its existence.

The average normal mental work curves of A and B (G and H) in the second five minutes are most instructive. A (G in tables) showed still a high initial spurt and a tendency to maintain it for three minutes; then,

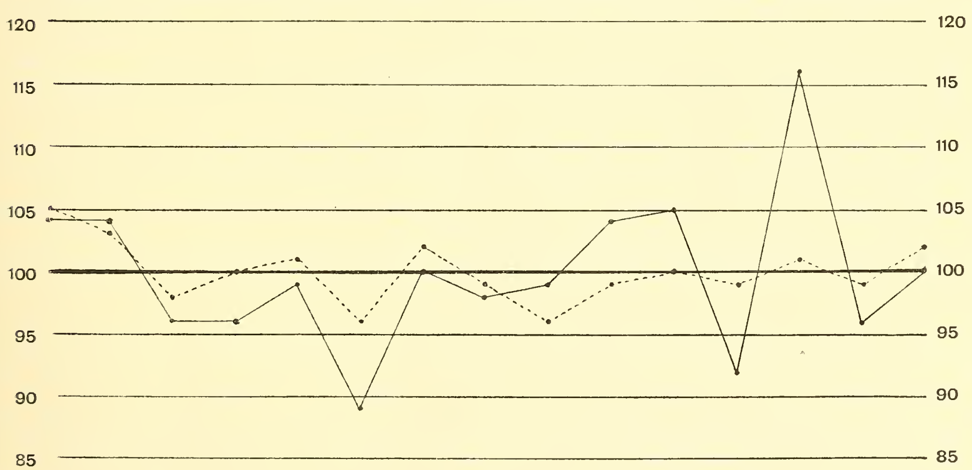
Mr B.—First Five Days' Test.

Showing percentage variation from minute to minute on the third day.

The dotted line shows the percentage variation from minute to minute on the total of the five days.



Mr B.'s Second Five Days' Test (called "Mr H." in Tables and Text).



as the effort proved too great, there was a gradual fall in the fourth and fifth minutes; then ensued an interval of five minutes in which the rate from minute to minute was almost unswervingly maintained; during the first eight minutes the attention never varied; apparently, the greatness of the effort precluded the possibility of spurting; A was working at the absolute maximum of his mental capacity, and the curve shows a plateau:

fatigue was slowly becoming more evident; it was combatted successfully for two minutes during which the rate remained at a constant and almost average level; and then the output precipitately diminished: even in the fatigue, the maximum capacity for that state was yielded, for A was incapable of summoning up energy even for the end spurt. Here, the Hylan middle period gave a good estimate of the mental working capacity, and it was certainly a period without variations.

Very different was the curve of B's (H in tables) average work from minute to minute during the second five days. The rate was in no two successive minutes uniform. The initial and terminal spurts were distinguishable; but between them the curve resembled a series of saw teeth. The average of the middle five-minute period was below the true average working capacity, and this middle period was at least as variable in its output as any other.

The mental work of a person in a given period appears to be characteristically represented by no part of that period. The output at the beginning and at the end are characteristic only of the part of the period in which it is produced. The average output, the average mental working capacity, is most closely represented by the arithmetical mean of the whole test.

In the cases of maniac depressive insanity, the age of the patients varied greatly; their previous familiarity with figures was unequal; for some had been professional men, some tradesmen, some had been engaged in household duties, and others had never had any definite employment; and some were excited, some depressed, and others were apparently enjoying a normal interval. As regards the amount of their mental working capacity, no harmony was therefore to be expected.

Total Output.

Of the thirty cases examined, the work done on the first day varied from 81 to 739 units. Case 10 was the only one whose initial output was below 100. Seven cases did less than 200 units; thirteen less than 300; nineteen less than 400; twenty-six less than 500; and twenty-nine less than 600. The average output on the first day was 339, *i.e.* 389 (53 per cent.) below the normal.

Three cases were in a state of excitement (Cases 18, 22, 28). Their maximum output was 400, 605 and 309 respectively: their maximum mental working capacity therefore averaged 438, and was 60·4 per cent. below the normal average.

Ten cases were depressed (Cases 3, 13, 15, 16, 19, 20, 21, 26, 27 and 29). Their maximum output was 312, 1075, 364, 930, 1122, 276, 520, 812, 656 and 400 respectively. Their average was 646 and was 41·6 below the normal. The degree of excitement or depression was in no case severe.

The rest of the patients (Cases 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 17, 23, 24, 25, 30) were enjoying a normal interlude. Their average was 598, and was 45·9 below the normal. Among them the lowest recorded was in Case 30, who reached the remarkably low output of 219: the highest, 1030. Even the apparently normal intervals in cases of maniac depressive insanity therefore but rarely indicate a normal mental working capacity.

Unlike the normal cases, the maximum was not always attained on the fifth day. In Case 30 the maximum was reached on the second day; "habituation" and exercise on the subsequent days were insufficient to ensure further increase, and a diminution in the output occurred. The cases were divisible into three groups, according to the day on which the maximum output occurred. In Group A—Cases 5, 15 and 29—the maximum was attained on the third day; in Group B—Cases 1, 2, 6, 8, 10, 12, 14 and 25—the maximum was reached on the fourth day; and in Group C—the remaining eighteen cases—the maximum was achieved on the fifth day.

In Group A the maximum mental working capacity varied from 364 to 674 units, and the average was 479, *i.e.* 628 (43 per cent.) below the normal.

In Group B the lowest maximum total recorded was 130, the highest 809; four were below 500; the average of the eight cases in this group was 488, which was 619 (55 per cent.) below the normal.

In Group C the smallest amount of work was done by Case 20—276 units; the largest by Case 19—1122 units: five cases, 9, 11, 13, 16 and 19, varied between 930 and 1122; they therefore showed practically the average normal mental working capacity: four produced less than 400 units, five less than 500, seven less than 600, nine less than 700, eleven less than 800, and thirteen less than 900. The average of all was 692, which was 415 (37 per cent.) lower than the normal.

Improvability.

In Group C the percentage improvement between the first and the fifth days was remarkably high: it varied from 41 (Case 22) to 153 (Case 24). Only three showed less than 50 per cent. improvement; six showed over 100 per cent.; the average improvement of the eighteen cases in this group reached the remarkable figure of 86 per cent. The normal improvability in the same period was only 53 per cent. The percentage improvement in Group C therefore exceeded the normal by 33 (Table VI.).

In Group B the percentage improvement was at its height on the fourth day: it varied between 46 and 86, and averaged 68. The normal average for the same period was 41 per cent.—an increase of 27 per cent. The percentage improvement in Group B therefore exceeded the normal by 27 (Table VI.).

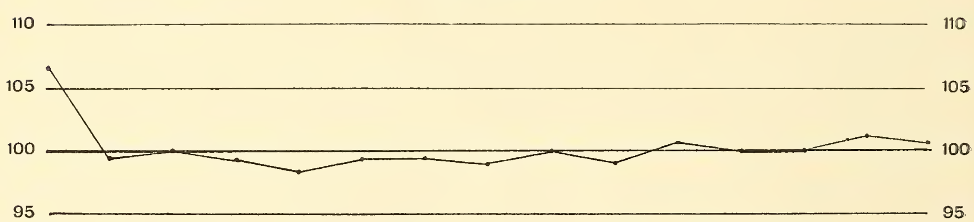
In Group A the percentage improvement was at its height on the third day; 60·8 (Case 5), 58·2 (Case 15), 60·6 (Case 29). Their average was therefore 59·8 per cent., and exceeded the normal by 24 per cent. (Table VI.).

The Mental Work Curve.

When the 30 cases of maniac depressive insanity were considered together, and their work as a group was mapped out from minute to

Curve W.—Average Curve of Thirty Cases of Maniac Depressive Insanity.

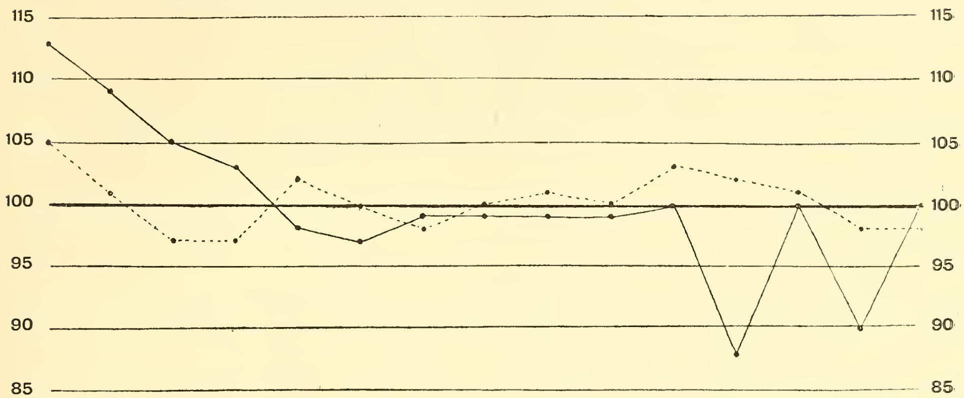
Showing percentage variation from minute to minute from the average rate of the test.



minute, Curve W was obtained. In it the initial spurt was well marked, was of very short duration, and represented the highest working capacity attained in any one minute. A tendency to spurting was observed throughout the test; the spurts were frequent, but were not of great extent; indeed, they would be more accurately described as oscillations. If the initial spurt be excepted, several other spurts were present within the fifteen-minute interval: these spurts were not sustained; the first two did not suffice to bring the output above the average level; the last three were greater, and the greatest occurred just before the end. There was no true terminal spurt. The inconstancy of the rate was such that no one period could be taken as typical of the whole. The middle five-minute period was not more uniform than the rest. And there were two periods of maximum effort—during the first minute and during the last three minutes.

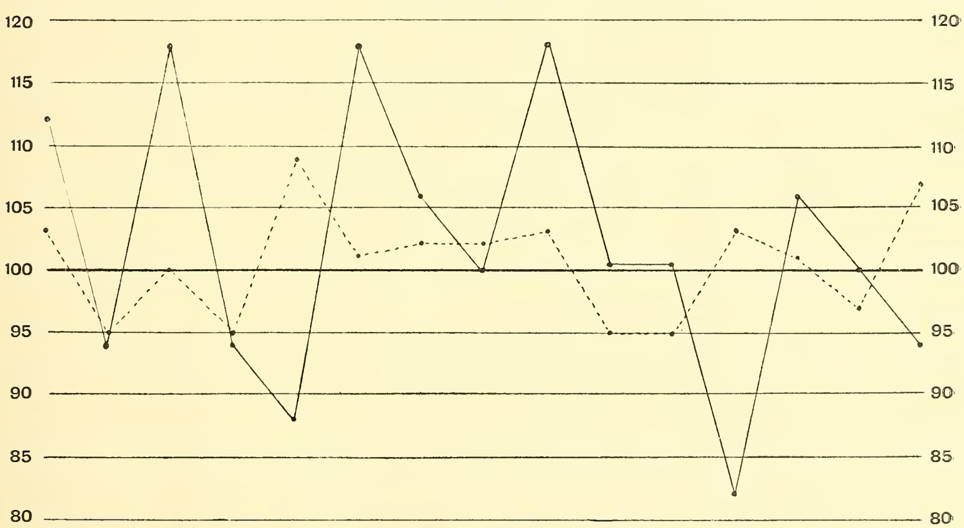
Case IX.—Maniac Depressive Curve.

Showing marked constancy of the middle five-minute period.



Case No. II.—Maniac Depressive Curve.

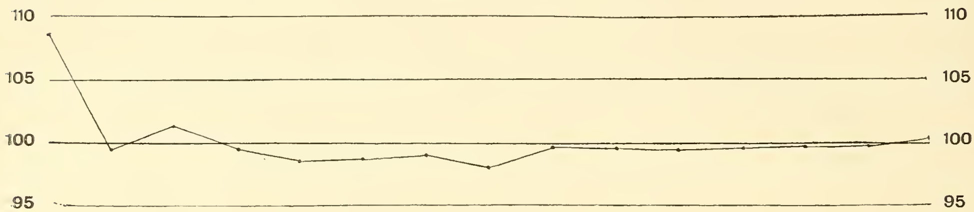
Showing marked inconstancy of the middle five-minute period as well as throughout.



In the work done by the patients in the quiescent stage of the disease (Curve X) the curve of the whole group shows some striking features. Here, again, the initial spurt was the highest. A tendency to spurt was evident throughout the test, but this was inconsiderable. After the eighth minute a higher level was attained, but even then the output only on two occasions, viz. the eleventh and fifteenth minute, reached the normal. The initial output was considerable, but rapidly fell; these patients were evidently incapable of further excessive production; they approached the normal, but could do no more. Their energy was such that it could not produce definite spurts, but what energy they had they were able to maintain, although at a subnormal level. Even the winding up of the test

Curve X.—Average Curve of the "Quiescent" Cases.

Showing percentage variation from minute to minute from the average rate of the test.

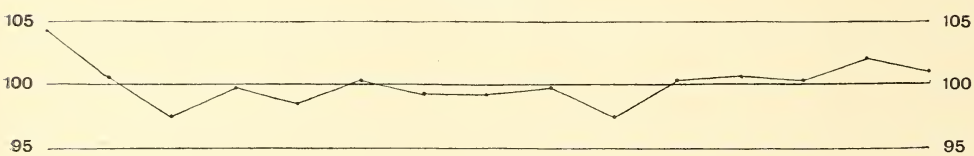


caused no definite alteration in the output of their work, for the terminal spurt was insignificant.

The curve of the mental work of the patients in the depressed phase of the disease (Curve Y) was in its first half markedly different from the last two curves. The energy with which the test began was no greater, yet the subsequent fall was markedly increased. In the reaction to this

Curve Y.—Average Curve of the "Depressed" Cases.

Showing percentage variation from minute to minute from the average rate of the test.



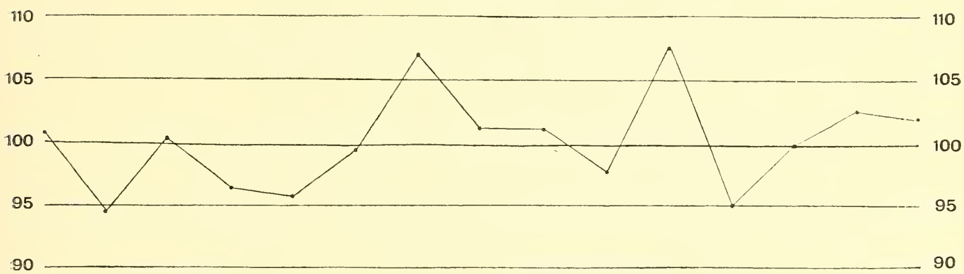
initial lapse and to subsequent lapses, a feature appeared which I am inclined to think is characteristic of this mental state. The recovery was performed in a series of steps; thus, the maximum depression of the output in the first lapse occurred in the third minute; in the fourth minute the rate increased; and after a lapse in the fifth minute, in the sixth it reached its maximum. During that time the task had apparently become progressively easier for the patient; the patient was able more and more thoroughly to concentrate; his psychomotor inertia was gradually overcome and the output tended to increase. But this effort induced a reaction, and a gradual falling off occurred, for during the four subsequent minutes, from the sixth to the tenth, the output remained below the normal.

After the tenth minute the output rose progressively for two minutes, then fell for one, once more rose and finally fell.

The curve of the excited group (Curve Z) differed totally from the others. Its characteristic was the abruptness of the spurts. One strikingly curious feature was that the excitement of beginning the task, the initial spurt, was practically non-existent. In the first minute the average amount of work throughout the whole test was almost exactly reproduced. This initial output was followed immediately by the greatest lapse, and an equally sudden recovery. The remainder of the curve consisted of a series of jerks, corresponding to violent spurts and lapses, the essential character of which was their brief duration. Their rhythmic occurrence was noteworthy. Their distribution clearly showed that in this phase of insanity the causes governing conscious effort are largely independent

Curve Z.—Average Curve of the "Excited" Cases.

Showing percentage variation from minute to minute from the average rate of the test.



of the task, its period, and its external accompaniments. There was no definite period of maximum capacity; the two main spurts reached approximately the same level. The curve seems to me wonderfully to portray the incessant and abrupt transitions of mental energy in a morbidly excited brain.

In none of these last four curves was there a terminal spurt. In the quiescent group the fifteenth minute was more productive than the fourteenth, but in it the increase was insignificant.

In none did the Hylan middle five-minute period adequately represent the maximum working capacity.

Let us now compare these curves of the mental work in morbid states with the curve obtained from the seven normal persons (Curve 1). Two striking differences are obvious in the work curve of the maniac depressive cases. First, a tendency towards a high output characterising the third five-minute interval; and, second, the small extent of the terminal spurt.

The first is probably attributable to the extreme difficulty which such patients experience in "getting under weigh" in a mental task; it is an expression of their exaggerated psychomotor inertia; and this delayed "Anregung" seems to me of definite diagnostic significance. As regards the second, the imminence of the end of the task which acts as an incentive to greater effort in normal people is either not realised owing to defective judgment of the time the test has lasted, or it is insufficient to elicit any special effort.

With these last two facts, and with emphasising the wonderful reflection of the mental state obtained on the work curve of the excited cases, I must for the moment be content. Their diagnostic value can be precisely determined only when we are in possession of similar experimental data from other mental diseases, for example dementia præcox. More results are also necessary before we can deduce the true significance of the high standard of the improvability observed in the mentally affected. But it seems probable that some at least of the abnormal degree of improvement found in these morbid mental states is due to the therapeutic action of the test, and that Maloney's assertion (6) that the Reckoning Test has in Psychiatry a therapeutic value is probably well founded.

Brief History of the Cases Examined.

Case I.—Male, aged 30, single. Had an attack of mania at the age of 20; remained well until he was 26. Since then has had mild attacks of excitement and depression with lucid intervals.

Case II.—Female, aged 55, single. Has been ill since the age of 30, with attacks of excitement and depression occurring about every twelve months, and alternating with lucid intervals varying from a few weeks to several months.

Case III.—Female, aged 48, single. During the past four years has had several severe attacks of depression and several short attacks of excitement, with a short lucid interval between the two which usually lasted for about three weeks.

Case IV.—Male, aged 60, single. First broke down mentally twenty years ago, since then has had many attacks of excitement and depression. Has been treated in various institutions, and has on several occasions been well enough to live at home.

Case V.—Female, aged 49, single. Had a severe attack of excitement at the age of 39. Recovered and broke down again five years later. Since then has been alternately excited and depressed.

Case VI.—Male, aged 47, single. Has had many attacks of excitement for which he was treated in several institutions, each attack lasting a few weeks. Following each attack of excitement there was a short period of depression.

CASE VII.—Male, aged 59, single. Has been ill for fully thirteen years, and regularly every few months passes through a definite attack of excitement, after which he becomes abnormally quiet for a few weeks.

CASE VIII.—Male, aged 23, single. First broke down mentally at the age of 17, and has been treated on at least four occasions for severe attacks of excitement, from each of which he completely recovered. During his last attack he was on several occasions depressed for a few days.

CASE IX.—Male, aged 37, single. Has been ill for seven years. Has had regularly every few months attacks of excitement and depression, alternating with lucid intervals.

CASE X.—Female, aged 53, single. Has had at least two previous attacks of excitement, and her present illness, which has lasted since the age of 39, has been characterised by recurring attacks of excitement followed by short periods of depression and lucid intervals.

CASE XI.—Female, aged 29, single. Became depressed at the age of 26. Completely recovered. Three years later had an attack of severe excitement which lasted a few months, and from which she completely recovered.

CASE XII.—Female, aged 42, single. Had her first attack of depression at the age of 20, and since then has had several attacks of excitement and depression with lucid intervals, and each lasting from a few weeks to some months.

CASE XIII.—Male, aged 55, single. Had a slight attack of depression at the age of 50, from which he recovered. Had a similar attack at the age of 55 and again recovered. For at least thirty years of his life he was liable for a few days at a time to mild phases of restlessness and excitability.

CASE XIV.—Female, aged 23, single. Had her first attack, which was one of excitement, at the age of 20, from which she recovered; became depressed three years afterwards, and again recovered after a few months.

CASE XV.—Female, aged 42, married. Had her first attack of depression at the age of 33. Nine years later again became depressed, and before completely recovering was mildly excited for a few weeks.

CASE XVI.—Female, aged 30, married. Became depressed at the age of 29, and from this she eventually recovered, although complete recovery was delayed on account of recurring attacks of depression.

CASE XVII.—Female, aged 16, single. Broke down first of all at the age of 15, although for two years previously was subject to mild attacks of restlessness and excitement. Every few weeks becomes either depressed or excited, and this is followed by a short lucid interval.

Case XVIII.—Female, aged 41, single. Had an attack of excitement several years ago. For the last two and a half years has been very unsettled, sometimes depressed, sometimes excited.

Case XIX.—Male, aged 30, single. At the age of 24 became depressed, but recovered; broke down again at the age of 29, and passed through a severe attack of depression.

Case XX.—Female, aged 49, married. At present in her fifth attack of depression. At the commencement of each of these attacks she was restless and excitable.

Case XXI.—Female, aged 52, single. Had an attack of excitement four years ago; broke down again at the age of 50, and is still depressed.

Case XXII.—Female, aged 50, single. Broke down at the age of 35, passed through an attack of excitement. On this occasion has been ill for three years, and periodically passes through attacks of excitement, which are usually followed by a short period of depression.

Case XXIII.—Female, aged 55, single. First broke down mentally at the age of 30. Since then has had many attacks of excitement and depression with lucid intervals, during which she has on several occasions been well enough to live at home.

Case XXIV.—Male, aged 43, single. Has had several attacks of excitement and depression, and from which he has always recovered.

Case XXV.—Male, aged 56, single. Had an attack of excitement at the age of 20, and another fourteen years later, from both of which he recovered. Present attack commenced at the age of 37. Since then he has had alternating attacks of excitement and depression with lucid intervals lasting for a few weeks.

Case XXVI.—Male, aged 54, widower. Has been ill since the age of 50, but for most of his life has been subject to transient "moods" of excitement and depression.

Case XXVII.—Female, aged 46, married. Became depressed at the age of 41, and since then until she recovered five years later passed through definite attacks of depression, which were succeeded by short periods of restless excitement.

Case XXVIII.—Female, aged 58, widow. Had an attack of depression at the age of 57, from which she recovered. Broke down again at the age of 58, and since then has been alternately excited and depressed.

Case XXIX.—Female, aged 28, single. At the age of 20 became depressed, but recovered. Broke down again at the age of 26, and is still depressed.

CASE XXX.—Female, aged 37, single. Has been ill since the age of 18. Had several attacks of excitement and depression with short lucid intervals during which she was well enough to live at home.

REFERENCES.

1. MALONEY, "On the Reckoning Test and its Uses in Psychiatry," *Review of Neur. and Psych.*, vol. ix., No. 7, p. 366.

2. MALONEY, *Instructions in the Use of the Reckoning Test.*

3. KRAEPELIN and others, quoted by Maloney, *Review of Neur. and Psych.*, vol. ix., No. 7.

4. "Ueber die Wirkung kurzer Arbeit's Zeiten," quoted by J. H. Wimms, "Fatigue and Practice," *Brit. Jour. Psychol.*, vol. ii. p. 153 *et seq.*, 1907–1908.

5. AMBERG, Kraepelin's *Psych. Arbeiten*, Band i., Seite 336, quoted by Maloney, *Review of Neur. and Psych.*, vol. ix., No. 7.

6. MALONEY, *ibid.*, vol. ix., No. 7.

[TABLES.

Table III.—Showing Totals and Percentage Variations per Minute in the Seven Normal Persons.

		Total for Seven Normal Persons for each Minute of Five Days.														
		1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
Total output for 5 days	33,293	2425	2246	2232	2233	2178	2156	2180	2199	2157	2199	2226	2189	2178	2236	2159
Average output for 5 days	4,756	346	321	319	319	311	308	311	314	308	314	318	312	311	319	308
Average output per minute for 5 days	317															
Percentage variation from minute to minute		+9·1	+1·2	+0·6	+0·6	−1·9	−2·8	−1·9	−0·9	−2·8	−0·9	+0·3	−1·5	−1·9	+0·6	−2·8

Table IV.—Showing same as Table III. in the Thirty Maniac Depressive Cases.

		Total for the Thirty Cases for each Minute of Five Days.														
		1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
Total output for 5 days	73,655	5226	4882	4903	4870	4810	4865	4886	4842	4910	4847	4922	4892	4900	4955	4945
Average output for 5 days	2,455	174	162	163	162	160	162	162	161	163	161	164	163	163	165	164
Average output per minute for 5 days	163															
Percentage variation from minute to minute		+6·7	−0·6	0	−0·6	−1·8	−0·6	−0·6	−1·2	0	−1·2	+0·6	0	0	+1·2	+0·6

Table V.—Showing the Average Output on the First Day, and the Days on which the Average Maximum Output occurred in the Thirty Maniac Depressive Cases.

Cases for which the average is taken.	Averages on.				
	1st Day.	2nd Day.	3rd Day.	4th Day.	5th Day.
All cases together	338				
Case 30	137	219			
Cases, 5, 15, 29	299		479		
Cases, 1, 2, 6, 8, 10, 12, 14, 25	286			488	
Remaining 18 cases	381				692

Table VI.—Showing (A) Percentage Variation of each day from First Day, and (C) Percentage Variation of each Day from Previous Day in the Thirty Maniac Depressive Cases.

Day.	Case I.		Case II.		Case III.		Case IV.		Case V.		Case VI.		Case VII.		Case VIII.	
	A	C	A	C	A	C	A	C	A	C	A	C	A	C	A	C
2	25·4	25·4	16·0	16·0	9·8	9·8	−1·2	−1·2	52·7	52·7	33·7	33·7	31·8	31·8	49·1	49·1
3	42·5	13·6	19·2	2·7	24·4	13·2	16·6	18·05	60·8	5·3	47·1	10·04	64·5	24·8	68·8	13·2
4	56·9	10·1	46·3	22·6	57·5	26·6	28·6	10·3	52·7	−5·0	69·7	15·3	73·4	5·3	85·5	9·9
5	54·7	−1·4	40·3	−4·07	81·4	15·1	45·1	12·8	57·5	3·1	−27·4	−57·2	80·4	4·06	84·6	−0·4

Day.	Case IX.		Case X.		Case XI.		Case XII.		Case XIII.		Case XIV.		Case XV.		Case XVI.	
	A	C	A	C	A	C	A	C	A	C	A	C	A	C	A	C
2	34·2	34·2	29·6	29·6	53·2	53·2	52·7	52·7	15·9	15·9	34·4	34·4	48·2	48·2	63·1	63·1
3	46·5	9·1	45·7	12·3	86·0	21·3	55·6	1·8	18·8	2·5	56·1	16·1	58·2	6·7	68·1	3·0
4	68·3	14·8	60·5	10·1	98·5	6·7	86·6	18·0	30·3	9·5	72·2	10·3	51·3	−4·3	97·9	17·5
5	84·0	9·3	55·5	−3·0	111·9	6·7	78·8	−4·1	45·4	11·6	73·3	0·6	56·0	3·1	111·3	6·7

Day.	Case XVII.		Case XVIII.		Case XIX.		Case XX.		Case XXI.		Case XXII.		Case XXIII.		Case XXIV.	
	A	C	A	C	A	C	A	C	A	C	A	C	A	C	A	C
2	28·8	28·8	54·8	54·8	37·4	37·4	32·0	32·0	30·0	30·0	14·2	14·2	31·1	31·1	65·7	65·7
3	70·6	32·4	75·2	12·5	66·2	20·9	26·2	−4·3	23·3	−5·1	34·9	18·3	50·4	14·7	114·4	29·3
4	96·9	15·4	100·5	15·0	85·1	11·3	46·8	16·2	41·2	14·5	37·5	1·9	66·0	10·3	130·6	7·5
5	100·2	1·6	105·1	2·3	98·2	7·0	76·2	20·5	57·5	11·5	41·7	3·1	86·7	12·5	153·6	9·9

Day.	Case XXV.		Case XXVI.		Case XXVII.		Case XXVIII.		Case XXIX.		Case XXX.	
	A	C	A	C	A	C	A	C	A	C	A	C
2	38·4	38·4	28·5	28·5	36·3	36·3	21·0	21·0	33·3	33·3	59·8	59·8
3	47·2	6·3	48·5	15·5	78·1	30·6	68·8	31·1	60·6	20·4	37·8	−13·6
4	67·4	13·7	56·7	5·4	73·8	−2·3	105·0	21·9	37·7	−14·2	29·1	−6·3
5	66·7	−0·4	65·7	5·7	86·3	7·1	123·9	9·1	44·5	4·9	48·1	14·6

Note.—Cases 18, 22, 28, were "Excited." Cases 3, 13, 15, 16, 19, 20, 21, 26, 27, 29 were "Depressed." The remaining Seventeen Cases were in a "Quiescent" interval.

TABLE VII.—SHOWING TOTALS AND PERCENTAGE VARIATIONS PER MINUTE IN THE "EXCITED," "DEPRESSED," AND "QUIESCENT" CASES.

Excited Cases (3 in number).

	Gross Total.		Total for each Five Minutes. 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
	5450		366	343	364	349	348	361	389	367	366	354	391	345	363	373	371
Average of 3	363	Percentage variation.	+0·8	-5·5	+0·27	-3·9	-4·1	-0·5	+7	+1·1	+1·0	-2·4	+7·7	-5·	0	+2·7	+2·1

Depressed Cases (10 in number).

	26502		1840	1779	1722	1764	1737	1767	1753	1751	1765	1720	1768	1779	1768	1803	1786
Average of 10	1766	Percentage variation.	+4·19	+0·73	-2·48	-0·11	-1·64	+0·05	-0·73	-0·84	-0·05	-2·49	+0·11	+0·73	+0·11	+2·09	+1·13

Quiescent Cases (17 in number).

	41703		3020	2760	2817	2757	2725	2737	2744	2724	2779	2773	2763	2768	2769	2779	2788
Average of 17	2780	Percentage variation.	+8·6	-0·7	+1·3	-0·82	-1·9	-1·5	-1·29	-2·01	-0·03	-0·25	-0·61	-0·43	-0·39	-0·03	+0·28

TABLE VIII.—SHOWING TOTALS PER MINUTE AND NUMBER OF UNITS RECORDED PER MINUTE IN EACH OF THE NINE NORMAL "RECKONINGS."*

Name.	Days.	Fifteen-Minute Total.	Total per Minute.														
			1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
A.	1	610	43	48	43	39	43	40	41	36	32	39	39	41	40	44	42
	2	735	42	44	50	54	42	56	48	37	47	54	55	48	63	47	48
	3	811	67	61	58	52	51	57	53	57	52	51	53	52	48	49	50
	4	854	73	51	55	64	58	52	58	55	48	48	60	61	66	54	51
	5	941	77	67	56	66	64	53	60	66	62	59	67	52	56	60	76
	Total	3951	302	271	262	275	258	258	260	251	241	251	274	254	273	254	267
	Average per min. for 5 days.	263															
	Average per min. on 3rd day.	54															
B.	1	850	57	56	60	57	56	52	52	51	56	59	58	55	58	59	64
	2	1057	76	74	72	64	69	69	71	70	72	69	66	68	72	73	72
	3	1109	79	81	78	68	79	73	74	71	73	73	74	70	71	73	72
	4	1105	82	71	73	75	70	70	74	73	70	70	75	75	74	77	76
	5	1217	86	81	75	83	86	79	86	80	74	81	80	80	83	82	81
	Total	5338	380	363	358	347	360	343	357	345	345	352	353	348	358	364	365
	Average per min. for 5 days.	355															
	Average per min. on 3rd day.	74															
C.	1	694	51	42	42	45	43	41	43	44	46	49	50	47	49	53	49
	2	952	70	64	64	58	62	62	62	64	65	66	61	67	61	58	68
	3	1031	78	71	68	62	68	68	65	71	71	75	65	68	66	70	65
	4	1101	90	74	76	73	73	69	67	70	72	71	73	76	69	77	71
	5	1182	89	81	80	80	83	78	77	75	77	79	76	77	74	75	81
	Total	4960	378	332	330	318	329	318	314	324	331	340	325	335	319	333	334
	Average per min. for 5 days.	331															
	Average per min. on 3rd day.	69															

* Seven persons were tested ; but as two did the test twice, nine "reckonings" are detailed.

TABLE VIII.—*continued.*

Name.	Days.	Fifteen-Minute Total.	Total per Minute.														
			1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
D.	1	814	56	56	56	58	50	53	45	53	52	53	54	68	45	57	58
	2	956	67	68	66	64	61	58	63	64	59	61	69	60	65	67	64
	3	1009	69	61	71	73	70	64	59	76	66	64	65	65	73	64	69
	4	1096	80	74	71	73	74	73	71	75	74	72	73	78	70	72	66
	5	1124	79	82	74	81	66	71	75	75	68	81	75	71	74	77	75
	Total	4999	351	341	338	349	321	319	313	343	319	331	336	342	327	337	332
	Average per min. for 5 days.	333															
	Average per min. on 4th day.	73															
E.	1	718	50	44	51	48	48	47	51	46	49	46	45	47	46	48	52
	2	941	66	62	65	64	63	63	65	61	64	65	61	60	56	64	62
	3	1016	69	68	69	67	67	69	68	66	69	67	68	66	67	68	68
	4	1065	71	69	68	69	78	65	73	69	70	71	82	62	70	74	74
	5	1154	75	74	72	81	76	78	75	76	79	72	74	81	79	82	80
	Total	4894	331	317	325	329	332	322	332	318	331	321	330	316	318	336	336
	Average per min. for 5 days.	326															
	Average per min. on 3rd day.	71															
F.	1	768	50	47	46	47	52	50	51	51	48	48	57	58	52	56	55
	2	856	68	60	54	59	53	51	59	59	46	57	59	59	53	60	59
	3	957	76	70	65	68	59	58	56	61	63	63	63	63	61	65	66
	4	1015	73	69	68	73	58	69	69	68	67	67	63	69	65	67	70
	5	1086	82	71	74	73	69	73	70	73	69	74	76	70	71	74	67
	Total	4682	349	317	307	320	291	301	305	312	293	309	318	319	302	322	317
	Average per min. for 5 days.	312															
	Average per min. on 3rd day.	64															

G. 2nd five days' test of A.	1	1151	82	78	83	80	79	80	74	79	73	76	72	67	75	77	76
	2	1131	83	84	76	84	75	64	70	68	73	78	78	80	78	69	71
	3	1155	88	83	88	70	81	80	81	81	80	80	77	77	67	64	58
	4	1240	81	88	83	87	86	89	82	83	79	79	87	84	83	71	78
	5	1234	85	84	82	90	85	84	89	86	79	77	77	86	75	81	74
	Total	5911	419	417	412	411	406	397	396	397	384	390	391	394	378	362	357
	Average per min. for 5 days.	394															
	Average per min. on 3rd day.	77															
H. 2nd five days' test of B.	1	1200	85	83	75	81	81	75	82	77	78	80	81	80	79	78	85
	2	1217	86	81	75	83	86	79	86	80	74	81	80	80	83	82	81
	3	1228	85	85	79	79	81	73	82	80	81	85	88	75	95	78	82
	4	1300	94	88	90	85	87	89	86	90	83	82	83	87	79	89	88
	5	1282	81	89	87	87	85	83	88	86	83	82	86	89	84	85	87
	Total	6227	431	426	406	415	420	399	424	413	399	410	418	411	420	412	423
	Average per min. for 5 days.	415															
	Average per min. on 3rd day.	82															
I.	1	642	46	28	40	38	36	40	48	45	47	42	45	48	49	44	46
	2	880	65	57	57	60	53	61	58	59	58	60	60	52	55	60	65
	3	932	70	69	68	60	60	58	62	63	60	60	62	58	57	60	65
	4	970	74	73	70	64	68	60	60	68	66	64	62	57	59	62	63
	5	1045	79	78	77	73	70	76	71	71	66	69	61	60	61	64	69
	Total	4469	334	305	312	295	287	295	299	306	297	295	290	275	281	290	308
	Average per min. for 5 days.	298															
	Average per min. on 3rd day.	62															

TABLE VIII.—*continued.*

Name.	Days.	Fifteen-Minute Total.	Total per Minute.														
			1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
D.	1	814	56	56	56	58	50	53	45	53	52	53	54	68	45	57	58
	2	956	67	68	66	64	61	58	63	64	59	61	69	60	65	67	64
	3	1009	69	61	71	73	70	64	59	76	66	64	65	65	73	64	69
	4	1096	80	74	71	73	74	73	71	75	74	72	73	78	70	72	66
	5	1124	79	82	74	81	66	71	75	75	68	81	75	71	74	77	75
	Total	4999	351	341	338	349	321	319	313	343	319	331	336	342	327	337	332
	Average per min. for 5 days.	333															
	Average per min. on 4th day.	73															
E.	1	718	50	44	51	48	48	47	51	46	49	46	45	47	46	48	52
	2	941	66	62	65	64	63	63	65	61	64	65	61	60	56	64	62
	3	1016	69	68	69	67	67	69	68	66	69	67	68	66	67	68	68
	4	1065	71	69	68	69	78	65	73	69	70	71	82	62	70	74	74
	5	1154	75	74	72	81	76	78	75	76	79	72	74	81	79	82	80
	Total	4894	331	317	325	329	332	322	332	318	331	321	330	316	318	336	336
	Average per min. for 5 days.	326															
	Average per min. on 3rd day.	71															
F.	1	768	50	47	46	47	52	50	51	51	48	48	57	58	52	56	55
	2	856	68	60	54	59	53	51	59	59	46	57	59	59	53	60	59
	3	957	76	70	65	68	59	58	56	61	63	63	63	63	61	65	66
	4	1015	73	69	68	73	58	69	69	68	67	67	63	69	65	67	70
	5	1086	82	71	74	73	69	73	70	73	69	74	76	70	71	74	67
	Total	4682	349	317	307	320	291	301	305	312	293	309	318	319	302	322	317
	Average per min. for 5 days.	312															
	Average per min. on 3rd day.	64															

G. 2nd five days' test of A.	1	1151	82	78	83	80	79	80	74	79	73	76	72	67	75	77	76
	2	1131	83	84	76	84	75	64	70	68	73	78	78	80	78	69	71
	3	1155	88	83	88	70	81	80	81	81	80	80	77	77	67	64	58
	4	1240	81	88	83	87	86	89	82	83	79	79	87	84	83	71	78
	5	1234	85	84	82	90	85	84	89	86	79	77	77	86	75	81	74
	Total	5911	419	417	412	411	406	397	396	397	384	390	391	394	378	362	357
	Average per min for 5 days.	394															
	Average per min. on 3rd day.	77															
H. 2nd five days' test of B.	1	1200	85	83	75	81	81	75	82	77	78	80	81	80	79	78	85
	2	1217	86	81	75	83	86	79	86	80	74	81	80	80	83	82	81
	3	1228	85	85	79	79	81	73	82	80	81	85	88	75	95	78	82
	4	1300	94	88	90	85	87	89	86	90	83	82	83	87	79	89	88
	5	1282	81	89	87	87	85	83	88	86	83	82	86	89	84	85	87
	Total	6227	431	426	406	415	420	399	424	413	399	410	418	411	420	412	423
	Average per min. for 5 days.	415															
	Average per min. on 3rd day.	82															
I.	1	642	46	28	40	38	36	40	48	45	47	42	45	48	49	44	46
	2	880	65	57	57	60	53	61	58	59	58	60	60	52	55	60	65
	3	932	70	69	68	60	60	58	62	63	60	60	62	58	57	60	65
	4	970	74	73	70	64	68	60	60	68	66	64	62	57	59	62	63
	5	1045	79	78	77	73	70	76	71	71	66	69	61	60	61	64	69
	Total	4469	334	305	312	295	287	295	299	306	297	295	290	275	281	290	308
	Average per min. for 5 days.	298															
	Average per min. on 3rd day.	62															

Table IX.—Showing Totals per Minute and Number of Units recorded per Minute in each of the Thirty Maniac Depressive Cases.

Cases.	Days.	Fifteen-Minute Total.	Total per Minute. 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
I.	1	393	25	29	27	34	27	27	26	28	26	24	25	25	26	21	23
	2	493	40	33	36	34	32	30	35	32	32	34	33	28	28	32	34
	3	560	43	39	39	35	37	34	37	34	37	37	35	37	39	40	37
	4	617	44	45	43	39	36	40	38	45	43	37	44	41	40	38	44
	5	608	47	45	42	43	40	42	40	39	36	38	39	41	40	38	38
	Total	2671	199	191	187	185	172	173	176	178	174	170	176	172	173	169	176
	Average per min. for 5 days.	178															
	Average per min. on 3rd day.	37															
II.	1	218	8	8	11	12	22	16	16	16	16	13	16	17	15	16	16
	2	253	20	19	16	15	17	16	16	17	17	15	16	21	17	14	17
	3	260	19	16	20	16	15	20	18	17	20	17	17	14	18	17	16
	4	319	23	22	21	21	24	21	20	23	22	20	16	20	20	23	23
	5	306	23	20	22	21	20	18	22	19	18	20	20	21	21	17	24
	Total	1356	93	85	90	85	98	91	92	92	93	85	85	93	91	87	96
	Average per min. for 5 days.	90															
	Average per min. on 3rd day.	18															
III.	1	172	14	8	7	6	11	11	12	14	15	14	11	12	12	11	14
	2	189	13	12	11	16	11	14	15	13	11	15	14	11	10	7	16
	3	214	16	16	18	12	16	9	14	13	12	15	12	15	15	15	16
	4	271	15	16	17	16	15	21	21	21	19	19	19	21	16	20	15
	5	312	23	16	22	23	20	20	17	22	18	25	21	23	20	22	20
	Total	1158	81	68	75	73	73	75	79	83	75	88	77	82	73	75	81
	Average per min. for 5 days.	77															
	Average per min. on 3rd day.	14															

IV.	1	583	35	37	38	37	35	40	40	40	37	34	39	42	42	45	42
	2	576	30	44	38	34	36	34	38	39	34	44	45	41	42	36	40
	3	680	52	50	40	42	36	46	44	48	43	44	43	48	46	49	49
	4	750	50	51	52	51	52	52	51	47	50	52	57	43	48	48	46
	5	846	62	53	59	57	63	59	54	57	60	49	49	54	56	60	54
	Total	3435	229	235	227	221	222	231	227	231	224	223	233	228	234	239	231
	Average per min. for 5 days.	229															
	Average per min. on 3rd day.	45															
V.	1	419	31	28	31	31	32	27	25	28	28	26	24	32	28	24	24
	2	640	42	46	45	44	43	41	44	39	43	44	37	43	44	43	42
	3	674	44	44	45	47	44	51	45	45	42	45	45	46	44	44	43
	4	640	49	44	43	48	44	39	39	41	40	46	46	35	46	41	39
	5	660	40	43	39	45	42	51	44	45	48	43	48	45	43	43	41
	Total	3033	206	205	203	215	205	209	197	198	201	204	200	201	205	195	189
	Average per min. for 5 days.	202															
	Average per min. on 3rd day.	45															
VI.	1	350	18	18	28	33	25	25	27	27	29	29	28	15	14	16	18
	2	468	31	26	30	26	28	32	34	33	25	32	29	35	34	39	34
	3	515	28	32	31	35	29	34	34	36	38	40	35	38	38	34	33
	4	594	40	44	42	42	32	40	34	39	38	44	38	40	41	42	38
	5	254	22	19	16	15	20	19	16	18	17	17	14	12	13	16	20
	Total	2181	139	139	147	151	134	150	145	153	147	162	144	140	140	147	143
	Average per min. for 5 days.	145															
	Average per min. on 3rd day.	34															

TABLE IX.—SHOWING TOTALS PER MINUTE AND NUMBER OF UNITS RECORDED PER MINUTE IN EACH OF THE THIRTY MANIAC DEPRESSIVE CASES.

Cases.	Days.	Fifteen-Minute Total.	Total per Minute.														
			1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
I.	1	393	25	29	27	34	27	27	26	28	26	24	25	25	26	21	23
	2	493	40	33	36	34	32	30	35	32	32	34	33	28	28	32	34
	3	560	43	39	39	35	37	34	37	34	37	37	35	37	39	40	37
	4	617	44	15	43	39	36	40	38	45	43	37	44	41	40	38	44
	5	608	47	45	42	43	40	42	40	39	36	38	39	41	40	38	38
	Total	2671	199	191	187	185	172	173	176	178	174	170	176	172	173	169	176
	Average per min. for 5 days.	178															
	Average per min. on 3rd day.	37															
II.	1	218	8	8	11	12	22	16	16	16	16	13	16	17	15	16	16
	2	253	20	19	16	15	17	16	16	17	17	15	16	21	17	14	17
	3	260	19	16	20	16	15	20	18	17	20	17	17	14	18	17	16
	4	319	23	22	21	21	24	21	20	23	22	20	16	20	20	23	23
	5	306	23	20	22	21	20	18	22	19	18	20	20	21	21	17	24
	Total	1356	93	85	90	85	98	91	92	92	93	85	85	93	91	87	96
	Average per min. for 5 days.	90															
	Average per min. on 3rd day.	18															
III.	1	172	14	8	7	6	11	11	12	14	15	14	11	12	12	11	14
	2	189	13	12	11	16	11	14	15	13	11	15	14	11	10	7	16
	3	214	16	16	18	12	16	9	14	13	12	15	12	15	15	15	16
	4	271	15	16	17	16	15	21	21	21	19	19	19	21	16	20	15
	5	312	23	16	22	23	20	20	17	22	18	25	21	23	20	22	20
	Total	1158	81	68	75	73	73	75	79	83	75	88	77	82	73	75	81
	Average per min. for 5 days.	77															
	Average per min. on 3rd day.	14															

IV.	1	583	35	37	38	37	35	40	40	40	37	34	39	42	42	45	42
	2	576	30	44	38	34	36	34	38	39	34	44	45	41	42	36	40
	3	680	52	50	40	42	36	46	44	48	43	44	43	48	46	49	49
	4	750	50	51	52	51	52	52	51	47	50	52	57	43	48	48	46
	5	846	62	53	59	57	63	59	54	57	60	49	49	54	56	60	54
	Total	3435	229	235	227	221	222	231	227	231	224	223	233	228	234	239	231
	Average per min. for 5 days.	229															
	Average per min. on 3rd day.	45															
V.	1	419	31	28	31	31	32	27	25	28	28	26	24	32	28	24	24
	2	640	42	46	45	44	43	41	44	39	43	44	37	43	44	43	42
	3	674	44	44	45	47	44	51	45	45	42	45	45	46	44	44	43
	4	640	49	44	43	48	44	39	39	41	40	46	46	35	46	41	39
	5	660	40	43	39	45	42	51	44	45	48	43	48	45	43	43	41
	Total	3033	206	205	203	215	205	209	197	198	201	204	200	201	205	195	189
	Average per min. for 5 days.	202															
	Average per min. on 3rd day.	45															
VI.	1	350	18	18	28	33	25	25	27	27	29	29	28	15	14	16	18
	2	468	31	26	30	26	28	32	34	33	25	32	29	35	34	39	34
	3	515	28	32	31	35	29	34	34	36	38	40	35	38	38	34	33
	4	594	40	44	42	42	32	40	34	39	38	44	38	40	11	42	38
	5	254	22	19	16	15	20	19	16	18	17	17	14	12	13	16	20
	Total	2181	139	139	147	151	134	150	145	153	147	162	144	140	140	147	143
	Average per min. for 5 days.	145															
	Average per min. on 3rd day.	34															

TABLE IX.—*continued.*

Cases.	Days.	Fifteen-Minute Total.	Total per Minute.														
			1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
VII.	1	440	32	22	29	26	28	31	28	24	31	37	33	28	34	30	27
	2	580	38	39	31	35	42	36	39	36	41	38	37	42	39	43	44
	3	724	49	39	49	43	49	48	49	51	43	47	47	52	55	58	45
	4	763	53	48	49	54	45	51	54	52	55	51	49	48	50	55	49
	5	794	51	48	58	52	50	55	52	54	52	60	54	48	52	53	55
	Total	3301	223	196	216	210	214	221	222	217	222	233	220	218	230	239	220
	Average per min. for 5 days.	220															
	Average per min. on 3rd day.	48															
VIII.	1	436	32	24	27	27	26	25	31	24	35	26	32	32	28	34	33
	2	650	41	38	45	36	39	43	42	50	50	40	48	36	51	43	48
	3	736	56	51	55	53	45	40	51	51	52	51	48	48	50	38	47
	4	809	60	57	60	54	43	53	51	48	56	57	41	57	51	62	59
	5	805	67	53	54	51	55	54	59	44	54	55	57	43	52	57	50
	Total	3436	256	223	241	221	208	215	234	217	247	229	226	216	232	234	237
	Average per min. for 5 days.	229															
	Average per min. on 3rd day.	49															
IX.	1	552	38	36	31	35	32	36	37	36	39	35	41	41	39	35	41
	2	741	51	49	52	52	48	48	48	48	49	49	48	52	51	50	46
	3	809	61	59	57	56	53	52	53	53	53	53	54	48	54	49	54
	4	929	66	60	60	58	67	65	62	64	61	62	62	61	58	63	60
	5	1016	68	69	63	62	66	69	66	68	70	70	72	73	70	67	63
	Total	4047	284	273	263	263	266	270	266	269	272	269	277	275	272	264	264
	Average per min. for 5 days.	270															
	Average per min. on 3rd day.	54															

X.	1	81	6	4	5	5	7	4	7	4	5	5	4	6	4	8	7
	2	105	7	7	6	6	6	8	7	9	8	8	7	6	7	6	7
	3	118	10	6	4	7	8	10	10	9	10	8	6	6	10	6	8
	4	130	11	9	9	9	7	4	10	7	9	10	8	10	8	10	9
	5	126	8	8	6	11	11	9	7	9	9	9	9	8	8	9	5
	Total	560	42	34	30	38	39	35	41	38	41	40	34	36	37	39	36
	Average per min. for 5 days.	37															
	Average per min. on 3rd day.	8															
XI.	1	486	36	33	35	36	25	28	22	32	35	25	37	35	32	38	37
	2	745	50	50	51	47	46	47	51	51	48	54	53	46	48	52	51
	3	904	66	60	63	62	62	57	62	56	56	53	61	61	61	64	60
	4	965	78	64	68	66	62	62	61	61	62	67	62	62	61	64	65
	5	1030	77	79	69	66	70	67	66	67	69	64	71	64	71	64	66
	Total	4130	307	286	286	277	265	261	262	267	270	263	284	268	273	282	279
	Average per min. for 5 days.	275															
	Average per min. on 3rd day.	60															
XII.	1	345	26	19	15	21	28	18	21	26	26	27	17	26	25	30	20
	2	527	44	39	35	34	31	34	43	34	32	30	34	43	30	29	35
	3	537	40	40	40	40	40	20	36	39	36	33	37	40	25	35	36
	4	644	55	45	44	38	38	54	32	44	45	41	34	46	39	45	44
	5	617	47	46	41	50	43	30	39	39	37	46	40	35	36	47	41
	Total	2670	212	189	175	183	180	156	171	182	176	177	162	190	155	186	176
	Average per min. for 5 days.	178															
	Average per min. on 3rd day.	36															

TABLE IX.—*continued.*

Cases.	Days.	Fifteen-Minute Total.	Total per Minute. 1.	2.	3	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
VII.	1	440	32	22	29	26	28	31	28	24	31	37	33	28	34	30	27
	2	580	38	39	31	35	42	36	39	36	41	38	37	42	39	43	44
	3	724	49	39	49	43	49	48	49	51	43	47	47	52	55	58	45
	4	763	53	48	49	54	45	51	54	52	55	51	49	48	50	55	49
	5	794	51	48	58	52	50	55	52	54	52	60	54	48	52	53	55
	Total	3301	223	196	216	210	214	221	222	217	222	233	220	218	230	239	220
	Average per min. for 5 days.	220															
	Average per min. on 3rd day.	48															
VIII.	1	436	32	24	27	27	26	25	31	24	35	26	32	32	28	34	33
	2	650	41	38	45	36	39	43	42	50	50	40	48	36	51	43	48
	3	736	56	51	55	53	45	40	51	51	52	51	48	48	50	38	47
	4	809	60	57	60	54	43	53	51	48	56	57	41	57	51	62	59
	5	805	67	53	54	51	55	54	59	44	54	55	57	43	52	57	50
	Total	3436	256	223	241	221	208	215	234	217	247	229	226	216	232	234	237
	Average per min. for 5 days.	229															
	Average per min. on 3rd day.	49															
IX.	1	552	38	36	31	35	32	36	37	36	39	35	41	41	39	35	41
	2	741	51	49	52	52	48	48	48	48	49	49	48	52	51	50	46
	3	809	61	59	57	56	53	52	53	53	53	53	54	48	54	49	54
	4	929	66	60	60	58	67	65	62	64	61	62	62	61	58	63	60
	5	1016	68	69	63	62	66	69	66	68	70	70	72	73	70	67	63
	Total	4047	284	273	263	263	266	270	266	269	272	269	277	275	272	264	264
	Average per min. for 5 days.	270															
	Average per min. on 3rd day.	54															

Cases.	Days.	Fifteen-Minute Total.	1.	2.	3	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
X.	1	81	6	4	5	5	7	4	7	4	5	5	4	6	4	8	7
	2	105	7	7	6	6	6	8	7	9	8	8	7	6	7	6	7
	3	118	10	6	4	7	8	10	10	9	10	8	6	6	10	6	8
	4	130	11	9	9	9	7	4	10	7	9	10	8	10	8	10	9
	5	126	8	8	6	11	11	9	7	9	9	9	9	8	8	9	5
	Total	560	42	34	30	38	39	35	41	38	41	40	34	36	37	39	36
	Average per min. for 5 days.	37															
	Average per min. on 3rd day.	8															
XI.	1	486	36	33	35	36	25	28	22	32	35	25	37	35	32	38	37
	2	745	50	50	51	47	46	47	51	51	48	54	53	46	48	52	51
	3	904	66	60	63	62	62	57	62	56	56	53	61	61	61	64	60
	4	965	78	64	68	66	62	62	61	61	62	67	62	62	61	64	65
	5	1030	77	79	69	66	70	67	66	67	69	64	71	64	71	64	66
	Total	4130	307	286	286	277	265	261	262	267	270	263	284	268	273	282	279
	Average per min. for 5 days.	275															
	Average per min. on 3rd day.	60															
XII.	1	345	26	19	15	21	28	18	21	26	26	27	17	26	25	30	20
	2	527	44	39	35	34	31	34	43	34	32	30	34	43	30	29	35
	3	537	40	40	40	40	40	20	36	39	36	33	37	40	25	35	36
	4	644	55	45	44	38	38	54	32	44	45	41	34	46	39	45	44
	5	617	47	46	41	50	43	30	39	39	37	46	40	35	36	47	41
	Total	2670	212	189	175	183	180	156	171	182	176	177	162	190	155	186	176
	Average per min. for 5 days.	178															
	Average per min. on 3rd day.	36															

TABLE IX.—*continued.*

Cases.	Days.	Fifteen-Minute Total.	Total per Minute.														
			1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
XIII.	1	739	58	51	53	46	52	51	51	49	46	36	44	39	46	48	49
	2	857	67	67	65	56	57	60	58	54	54	57	53	53	49	53	54
	3	879	76	66	57	66	62	57	57	48	61	54	56	58	56	54	51
	4	963	70	76	66	65	64	65	60	62	67	60	59	63	56	67	63
	5	1075	80	84	76	72	74	69	70	67	68	65	70	63	72	72	73
	Total	4513	351	344	317	305	309	302	296	280	296	272	282	276	279	294	290
	Average per min. for 5 days.	301															
	Average per min. on 3rd day.	59															
XIV.	1	180	15	12	13	9	15	9	6	10	12	12	13	14	12	11	17
	2	242	18	17	18	18	15	14	20	10	16	14	13	19	17	14	19
	3	281	21	17	21	17	18	18	18	17	19	18	19	21	20	18	19
	4	310	23	20	22	18	24	24	21	17	17	24	15	19	21	20	25
	5	312	22	22	25	13	20	20	24	20	21	22	20	23	16	21	23
	Total	1325	99	88	99	75	92	85	89	74	85	90	80	96	86	84	103
	Average per min. for 5 days.	88															
	Average per min. on 3rd day.	19															
XV.	1	230	12	12	17	11	12	15	13	12	20	18	16	22	16	16	18
	2	341	18	25	19	21	21	23	24	20	21	22	28	23	22	26	28
	3	364	24	20	19	24	22	26	23	27	24	24	27	28	26	24	26
	4	348	27	24	24	22	21	21	21	23	23	22	25	21	24	27	23
	5	359	27	26	26	23	21	24	18	26	24	24	21	24	26	22	27
	Total	1642	108	107	105	101	97	109	99	108	112	110	117	118	114	115	122
	Average per min. for 5 days.	109															
	Average per min. on 3rd day.	24															

XVI.	1	440	41	34	32	26	28	27	21	27	32	26	30	30	25	32	29
	2	718	56	50	44	51	43	49	51	49	44	49	44	47	50	47	44
	3	740	58	55	47	45	47	48	48	47	48	42	52	53	50	47	53
	4	871	67	65	56	60	55	57	59	52	53	60	56	55	59	58	59
	5	930	75	73	67	63	59	63	54	57	62	57	58	61	59	62	60
	Total	3699	297	277	246	245	232	244	233	232	239	234	240	246	243	246	245
	Average per min. for 5 days.	246															
	Average per min. on 3rd day.	49															
XVII.	1	361	31	17	15	27	26	26	26	23	22	21	23	25	25	29	25
	2	465	29	32	31	27	29	28	26	30	31	27	34	34	33	39	35
	3	616	43	39	43	40	39	39	36	42	41	40	42	37	44	45	46
	4	711	73	40	50	46	45	48	43	39	49	50	47	47	45	42	47
	5	723	74	51	51	33	45	48	49	49	44	49	48	43	48	49	42
	Total	2876	250	179	190	173	184	189	180	183	187	187	194	186	195	204	195
	Average per min. for 5 days.	191															
	Average per min. on 3rd day.	41															
XVIII.	1	195	11	12	11	7	12	24	19	22	14	10	11	11	11	5	15
	2	302	16	15	13	27	23	20	27	21	34	28	24	9	13	16	16
	3	340	20	21	18	14	14	15	15	17	27	42	26	41	20	30	20
	4	391	30	23	26	28	27	23	27	27	28	24	25	17	29	26	31
	5	400	27	28	30	20	28	26	53	24	21	7	30	28	30	23	25
	Total	1628	104	99	98	96	104	108	141	111	124	111	116	106	103	100	107
	Average per min. for 5 days.	108															
	Average per min. on 3rd day.	22															

TABLE IX.—*continued.*

Cases.	Days.	Fifteen-Minute Total.	Total per Minute.														
			1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
XIII.	1	739	58	51	53	46	52	51	51	49	46	36	44	39	46	48	49
	2	857	67	67	65	56	57	60	58	54	54	57	53	53	49	53	54
	3	879	76	66	57	66	62	57	57	48	61	54	56	58	56	54	51
	4	963	70	76	66	65	64	65	60	62	67	60	59	63	56	67	63
	5	1075	80	84	76	72	74	69	70	67	68	65	70	63	72	72	73
	Total	4513	351	344	317	305	309	302	296	280	296	272	282	276	279	294	290
	Average per min. for 5 days.	301															
	Average per min. on 3rd day.	59															
XIV.	1	180	15	12	13	9	15	9	6	10	12	12	13	14	12	11	17
	2	242	18	17	18	18	15	14	20	10	16	14	13	19	17	14	19
	3	281	21	17	21	17	18	18	18	17	19	18	19	21	20	18	19
	4	310	23	20	22	18	24	24	21	17	17	24	15	19	21	20	25
	5	312	22	22	25	13	20	20	24	20	21	22	20	23	16	21	23
	Total	1325	99	88	99	75	92	85	89	74	85	90	80	96	86	84	103
	Average per min. for 5 days.	88															
	Average per min. on 3rd day.	19															
XV.	1	230	12	12	17	11	12	15	13	12	20	18	16	22	16	16	18
	2	341	18	25	19	21	21	23	24	20	21	22	28	23	22	26	28
	3	364	24	20	19	24	22	26	23	27	24	24	27	28	26	24	26
	4	348	27	24	24	22	21	21	21	23	23	22	25	21	24	27	23
	5	359	27	26	26	23	21	24	18	26	24	24	21	24	26	22	27
	Total	1642	108	107	105	101	97	109	99	108	112	110	117	118	114	115	122
	Average per min. for 5 days.	109															
	Average per min. on 3rd day.	24															
XVI.	1	440	41	34	32	26	28	27	21	27	32	26	30	30	25	32	29
	2	718	56	50	44	51	43	49	51	49	44	49	44	47	50	47	44
	3	740	58	55	47	45	47	48	48	47	48	42	52	53	50	47	53
	4	871	67	65	56	60	55	57	59	52	53	60	56	55	59	58	59
	5	930	75	73	67	63	59	63	54	57	62	57	58	61	59	62	60
	Total	3699	297	277	246	245	232	244	233	232	239	234	240	246	243	246	245
	Average per min. for 5 days.	246															
	Average per min. on 3rd day.	49															
XVII.	1	361	31	17	15	27	26	26	26	23	22	21	23	25	25	29	25
	2	465	29	32	31	27	29	28	26	30	31	27	34	34	33	39	35
	3	616	43	39	43	40	39	39	36	42	41	40	42	37	14	45	46
	4	711	73	40	50	46	45	48	43	39	49	50	47	17	45	42	47
	5	723	74	51	51	33	45	48	49	49	44	49	48	43	48	49	42
	Total	2876	250	179	190	173	184	189	180	183	187	187	194	186	195	204	195
	Average per min. for 5 days.	191															
	Average per min. on 3rd day.	41															
XVIII.	1	195	11	12	11	7	12	24	19	22	14	10	11	11	11	5	15
	2	302	16	15	13	27	23	20	27	21	34	28	24	9	13	16	16
	3	340	20	21	18	14	14	15	15	17	27	42	26	41	20	30	20
	4	391	30	23	26	28	27	23	27	27	28	24	25	17	29	26	31
	5	400	27	28	30	20	28	26	53	24	21	7	30	28	30	23	25
	Total	1628	104	99	98	96	104	108	141	111	124	111	116	106	103	100	107
	Average per min. for 5 days.	108															
	Average per min. on 3rd day.	22															

TABLE IX.—*continued.*

Cases.	Days.	Fifteen-Minute Total.	Total per Minute.														
			1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
XIX.	1	566	34	32	34	36	38	38	39	42	44	38	39	38	39	37	38
	2	778	54	55	56	55	54	52	54	50	46	49	48	48	52	51	54
	3	941	66	67	66	68	66	62	63	68	60	58	61	62	54	58	62
	4	1048	72	72	70	70	72	66	75	73	69	67	70	66	68	70	68
	5	1122	78	79	74	75	75	78	79	75	73	69	70	74	73	74	76
	Total	4455	304	305	300	304	305	296	310	308	292	281	288	288	286	290	298
	Average per min. for 5 days.	296															
	Average per min. on 3rd day.	63															
XX.	1	156	9	10	9	9	10	12	10	11	12	11	10	11	11	10	11
	2	206	14	11	12	15	17	16	14	12	14	12	11	15	14	17	12
	3	197	13	13	11	15	12	15	12	11	15	14	12	13	14	14	13
	4	229	16	16	15	16	14	14	14	15	15	14	14	20	15	17	14
	5	276	13	17	18	20	17	19	18	17	20	17	22	19	18	21	20
	Total	1064	65	67	65	75	70	76	68	66	76	68	69	78	72	79	70
	Average per min. for 5 days.	71															
	Average per min. on 3rd day.	13															
XXI.	1	330	20	18	15	22	18	27	20	14	17	23	25	27	29	28	27
	2	429	27	26	29	30	30	28	26	29	25	31	27	31	31	29	30
	3	407	29	27	27	25	28	23	30	26	27	25	29	28	30	25	28
	4	466	28	26	30	30	32	31	32	32	30	32	33	31	30	33	36
	5	520	36	33	32	34	38	37	37	35	37	35	37	35	34	33	27
	Total	2152	140	130	133	141	146	146	145	136	136	146	151	152	154	148	148
	Average per min. for 5 days.	143															
	Average per min. on 3rd day.	27															

XXII.	1	427	28	22	25	28	25	20	28	28	25	31	34	29	32	40	32
	2	488	32	26	38	31	32	34	27	37	36	26	37	30	25	38	39
	3	576	40	37	40	38	39	36	41	38	29	36	44	38	42	41	37
	4	587	43	40	40	44	33	41	39	38	40	38	35	36	40	40	40
	5	605	45	46	43	39	39	45	44	43	41	37	41	28	44	34	36
	Total	2683	188	171	186	180	168	176	179	184	171	168	191	161	183	193	184
	Average per min. for 5 days.	179															
	Average per min. on 3rd day.	38															
XXIII.	1	212	13	10	13	13	14	17	18	15	12	12	15	14	17	14	15
	2	278	18	17	19	22	17	22	18	18	17	17	19	21	18	17	18
	3	319	19	23	19	20	20	17	23	22	20	21	24	20	22	25	24
	4	352	21	22	28	21	22	28	23	17	23	26	25	23	26	23	24
	5	396	33	26	24	21	27	23	24	24	24	28	32	28	32	26	24
	Total	1557	104	98	103	97	100	107	106	96	96	104	115	106	115	105	105
	Average per min. for 5 days.	104															
	Average per min. on 3rd day.	21															
XXIV.	1	222	17	8	14	19	19	19	16	15	15	15	14	19	14	3	15
	2	368	25	28	26	22	26	21	24	26	24	23	24	23	28	22	26
	3	476	39	34	31	32	32	30	27	32	27	33	34	35	29	28	33
	4	512	40	35	38	36	33	34	35	35	35	33	34	32	34	30	28
	5	563	44	38	40	45	34	39	32	31	40	37	39	33	35	38	38
	Total	2141	165	143	149	154	144	143	134	139	141	141	145	142	140	121	140
	Average per min. for 5 days.	143															
	Average per min. on 3rd day.	32															

TABLE IX.—*continued.*

Cases.	Days.	Fifteen-Minute Total.	Total per Minute. 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
XIX.	1	566	34	32	34	36	38	38	39	42	44	38	39	38	39	37	38
	2	778	54	55	56	55	54	52	54	50	46	49	48	48	52	51	54
	3	941	66	67	66	68	66	62	63	68	60	58	61	62	54	58	62
	4	1048	72	72	70	70	72	66	75	73	69	67	70	66	68	70	68
	5	1122	78	79	74	75	75	78	79	75	73	69	70	74	73	74	76
	Total	4455	304	305	300	304	305	296	310	308	292	281	288	288	286	290	298
	Average per min. for 5 days.	296															
	Average per min. on 3rd day.	63															
XX.	1	156	9	10	9	9	10	12	10	11	12	11	10	11	11	10	11
	2	206	14	11	12	15	17	16	14	12	14	12	11	15	14	17	12
	3	197	13	13	11	15	12	15	12	11	15	14	12	13	14	14	13
	4	229	16	16	15	16	14	14	14	15	15	14	14	20	15	17	14
	5	276	13	17	18	20	17	19	18	17	20	17	22	19	18	21	20
	Total	1064	65	67	65	75	70	76	68	66	76	68	69	78	72	79	70
	Average per min. for 5 days.	71															
	Average per min on 3rd day.	13															
XXI.	1	330	20	18	15	22	18	27	20	14	17	23	25	27	29	28	27
	2	429	27	26	29	30	30	28	26	29	25	31	27	31	31	29	30
	3	407	29	27	27	25	28	23	30	26	27	25	29	28	30	25	28
	4	466	28	26	30	30	32	31	32	32	30	32	33	31	30	33	36
	5	520	36	33	32	34	38	37	37	35	37	35	37	35	34	33	27
	Total	2152	140	130	133	141	146	146	145	136	136	146	151	152	154	148	148
	Average per min. for 5 days.	143															
	Average per min. on 3rd day.	27															

XXII.	1	427	28	22	25	28	25	20	28	28	25	31	34	29	32	40	32
	2	488	32	26	38	31	32	34	27	37	36	26	37	30	25	38	39
	3	576	40	37	40	38	39	36	41	38	29	36	44	38	42	41	37
	4	587	43	40	40	44	33	41	39	38	40	38	35	36	40	40	40
	5	605	45	46	43	39	39	45	44	43	41	37	41	28	44	34	36
	Total	2683	188	171	186	180	168	176	179	184	171	168	191	161	183	193	184
	Average per min. for 5 days.	179															
	Average per min. on 3rd day.	38															
XXIII.	1	212	13	10	13	13	14	17	18	15	12	12	15	14	17	14	15
	2	278	18	17	19	22	17	22	18	18	17	17	19	21	18	17	18
	3	319	19	23	19	20	20	17	23	22	20	21	24	20	22	25	24
	4	352	21	22	28	21	22	28	23	17	23	26	25	23	26	23	24
	5	396	33	26	24	21	27	23	24	24	24	28	32	28	32	26	24
	Total	1557	104	98	103	97	100	107	106	96	96	104	115	106	115	105	105
	Average per min. for 5 days.	104															
	Average per min. on 3rd day.	21															
XXIV.	1	222	17	8	14	19	19	19	16	15	15	15	14	19	14	3	15
	2	368	25	28	26	22	26	21	24	26	24	23	24	23	28	22	26
	3	476	39	34	31	32	32	30	27	32	27	33	34	35	29	28	33
	4	512	40	35	38	36	33	34	35	35	35	33	34	32	34	30	28
	5	563	44	38	40	45	34	39	32	31	40	37	39	33	35	38	38
	Total	2141	165	143	149	154	144	143	134	139	141	141	145	142	140	121	140
	Average per min. for 5 days.	143															
	Average per min. on 3rd day.	32															

TABLE IX.—*continued.*

Cases.	Days.	Fifteen Minute-Total.	Total per Minute. 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
XXV.	1	286	19	16	13	18	21	19	23	14	23	21	23	17	20	16	23
	2	396	19	24	35	26	31	27	27	25	24	28	26	27	27	22	28
	3	421	28	31	31	31	31	25	25	27	29	30	26	28	24	27	28
	4	479	36	40	36	38	33	31	37	31	33	29	22	28	32	26	27
	5	477	40	30	30	37	34	28	29	32	30	30	34	33	27	29	34
	Total	2059	142	141	145	150	150	130	141	129	139	138	131	133	130	120	140
	Average per min. for 5 days.	137															
	Average per min. on 3rd day.	28															
XXVI.	1	490	33	32	33	29	33	37	31	39	28	30	32	33	33	36	31
	2	630	32	38	41	45	41	37	42	44	40	40	45	49	46	43	47
	3	728	41	46	41	43	45	46	57	52	48	53	48	51	50	58	49
	4	768	47	44	42	51	51	50	52	51	56	53	56	50	54	59	52
	5	812	52	48	48	52	52	53	59	55	57	50	58	56	61	52	59
	Total	3428	205	208	205	220	222	223	241	241	229	226	239	239	244	248	238
	Average per min. for 5 days.	228															
	Average per min. on 3rd day.	48															
XXVII.	1	352	20	21	23	23	28	23	23	22	21	24	25	21	24	26	28
	2	480	27	34	28	36	32	31	28	30	29	32	33	39	32	34	35
	3	627	42	36	33	42	40	43	41	45	46	41	43	46	45	44	40
	4	612	41	34	38	43	36	41	44	44	45	38	42	37	44	44	41
	5	656	40	40	37	43	43	43	40	46	48	50	45	50	45	42	44
	Total	2727	170	165	159	187	179	181	176	187	189	185	188	193	190	190	188
	Average per min. for 5 days.	181															
	Average per min. on 3rd day.	41															

XXVIII.	1	138	9	5	7	7	8	12	8	9	8	10	13	10	11	10	11
	2	177	13	10	12	5	10	10	14	12	13	12	13	11	14	13	15
	3	232	17	15	16	18	17	16	15	17	11	15	17	18	11	16	13
	4	283	15	19	22	21	20	19	15	18	17	20	21	19	19	18	20
	5	309	20	24	23	22	21	20	17	16	22	18	20	20	22	23	21
	Total	1139	74	73	80	73	76	77	69	72	71	75	84	78	77	80	80
	Average per min. for 5 days.	76															
	Average per min. on 3rd day.	15															
XXIX.	1	249	17	15	17	19	14	14	16	19	19	14	15	15	19	19	17
	2	332	21	23	21	20	20	22	22	19	22	23	24	21	24	26	24
	3	400	29	28	29	30	27	29	24	27	33	26	28	24	23	25	18
	4	343	21	19	24	21	20	21	22	26	22	23	26	21	24	24	29
	5	360	31	23	26	23	23	29	22	19	25	24	24	26	23	24	18
	Total	1684	119	108	117	113	104	115	106	110	121	110	117	107	113	118	106
	Average per min. for 5 days.	112															
	Average per min. on 3rd day.	27															
XXX.	1	137	9	7	12	7	9	4	8	11	9	7	10	12	10	12	10
	2	219	15	11	14	15	9	16	15	15	15	13	15	18	18	16	14
	3	189	20	11	11	11	13	19	12	10	15	9	12	12	10	12	12
	4	177	12	12	13	10	10	12	14	10	10	14	8	12	11	14	15
	5	203	14	14	16	16	11	20	12	15	15	15	12	14	12	10	7
	Total	925	70	55	66	59	52	71	61	61	64	58	57	68	61	64	58
	Average per min. for 5 days.	62															
	Average per min. on 3rd day.	13															

(*Issued separately April* 13, 1912).

TABLE IX.—*continued.*

Cases.	Days.	Fifteen Minute-Total.	Total per Minute. 1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.
XXV.	1	286	19	16	13	18	21	19	23	14	23	21	23	17	20	16	23
	2	396	19	24	35	26	31	27	27	25	24	28	26	27	27	22	28
	3	421	28	31	31	31	31	25	25	27	29	30	26	28	24	27	28
	4	479	36	40	36	38	33	31	37	31	33	29	22	28	32	26	27
	5	477	40	30	30	37	34	28	29	32	30	30	34	33	27	29	34
	Total	2059	142	141	145	150	150	130	141	129	139	138	131	133	130	120	140
	Average per min. for 5 days.	137															
	Average per min. on 3rd day.	28															
XXVI.	1	490	33	32	33	29	33	37	31	39	28	30	32	33	33	36	31
	2	630	32	38	41	45	41	37	42	44	40	40	45	49	46	43	47
	3	728	41	46	41	43	45	46	57	52	48	53	48	51	50	58	49
	4	768	47	44	42	51	51	50	52	51	56	53	56	50	54	59	52
	5	812	52	48	48	52	52	53	59	55	57	50	58	56	61	52	59
	Total	3428	205	208	205	220	222	223	241	241	229	226	239	239	244	248	238
	Average per min. for 5 days.	228															
	Average per min. on 3rd day.	48															
XXVII.	1	352	20	21	23	23	28	23	23	22	21	24	25	21	24	26	28
	2	480	27	34	28	36	32	31	28	30	29	32	33	39	32	34	35
	3	627	42	36	33	42	40	43	41	45	46	41	43	46	45	44	40
	4	612	41	34	38	43	36	41	44	44	45	38	42	37	44	44	41
	5	656	40	40	37	43	43	43	40	46	48	50	45	50	45	42	44
	Total	2727	170	165	159	187	179	181	176	187	189	185	188	193	190	190	188
	Average per min. for 5 days.	181															
	Average per min. on 3rd day.	41															
XXVIII.	1	138	9	5	7	7	8	12	8	9	8	10	13	10	11	10	11
	2	177	13	10	12	5	10	10	14	12	13	12	13	11	14	13	15
	3	232	17	15	16	18	17	16	15	17	11	15	17	18	11	16	13
	4	283	15	19	22	21	20	19	15	18	17	20	21	19	19	18	20
	5	309	20	24	23	22	21	20	17	16	22	18	20	20	22	23	21
	Total	1139	74	73	80	73	76	77	69	72	71	75	84	78	77	80	80
	Average per min. for 5 days.	76															
	Average per min. on 3rd day.	15															
XXIX.	1	249	17	15	17	19	14	14	16	19	19	14	15	15	19	19	17
	2	332	21	23	21	20	20	22	22	19	22	23	24	21	24	26	24
	3	400	29	28	29	30	27	29	24	27	33	26	28	24	23	25	18
	4	343	21	19	24	21	20	21	22	26	22	23	26	21	24	24	29
	5	360	31	23	26	23	23	29	22	19	25	24	24	26	23	24	18
	Total	1684	119	108	117	113	104	115	106	110	121	110	117	107	113	118	106
	Average per min. for 5 days.	112															
	Average per min. on 3rd day.	27															
XXX.	1	137	9	7	12	7	9	4	8	11	9	7	10	12	10	12	10
	2	219	15	11	14	15	9	16	15	15	15	13	15	18	18	16	14
	3	189	20	11	11	11	13	19	12	10	15	9	12	12	10	12	12
	4	177	12	12	13	10	10	12	14	10	10	14	8	12	11	14	15
	5	203	14	14	16	16	11	20	12	15	15	15	12	14	12	10	7
	Total	925	70	55	66	59	52	71	61	61	64	58	57	68	61	64	58
	Average per min. for 5 days.	62															
	Average per min. on 3rd day.	13															

(Issued separately April 13, 1912).

XI.—An Investigation into the Effects of Seasonal Changes on Body Temperature. By Sutherland Simpson, M.D., D.Sc.

(From the Physiological Laboratory of the Medical College, and the Department of Poultry Husbandry, New York State College of Agriculture, Cornell University, Ithaca, N.Y., U.S.A.)

(MS. received January 6, 1912. Read February 19, 1912.)

CONTENTS.

Introduction.

Very little is known regarding the influence of the different seasons of the year on the body temperature either of the lower animals or of man. John Davy * in 1845 published the results of daily observations on his own temperature in England, extending from August 6 to April 30 (with the exception of the month of October), and of those made at the same hour (7–8 a.m.), the highest (100° F.) was recorded on January 7, and the lowest (98·1°) on February 9. The average monthly figures were as follows :—August 98·7° F., air 61° F. ; September 98·8°, air 66° ; November 98·9°, air 51° ; December 98·7°, air 42°; January 98·8°, air 45° ; February 98·6°, air 42° ; March 98·74°, air 46° ; April 98·66°, air 54·6°. They show a remarkable constancy. A more extensive series was made at Barbadoes in the West Indies, covering a period from July 1845 to November 1848. The minimum monthly mean temperature (98·3° F.) was found in August 1845, and the maximum (98·76° F.) in August, September, and October 1848 ; there was no evidence of any regular seasonal rhythm.

These observations of Davy, however, although frequently referred to in the books, are of little value, since the temperature was taken in the mouth and not in the rectum. It has been shown by Lindhard † and others, that in the case of most individuals, readings obtained from the mouth are

* John Davy, *Phil. Trans.*, London, 1845, part ii., p. 319, and 1850, p. 437.

† Lindhard, "Investigations into the Conditions Governing the Temperature of the Body," *Danmark-Ekspeditionen til Grönlands Nordöstkyst*, 1906–1908, Copenhagen, 1910.

readily influenced by external conditions, particularly the temperature of the air, and give no reliable information regarding slight changes in the temperature of the deeper parts of the body.

In the cool season of the year, Jousset * found that the average temperature of the natives was from two- to three-tenths of a degree higher than in the warm season.

Bosanquet † made observations on his own rectal temperature, four times a day, at the same hours (9 a.m., 2, 7, and 10 p.m.) for a period of three years, with only a few interruptions. Dividing the year into thirteen lunar months, beginning in the middle of June, and taking the average temperature for each month, he obtained the following figures:—98·75° F., 98·75°, 98·80°, 98·72°, 98·80°, 98·89°, 98·92° (December), 98·90°, 98·97°, 98·90°, 98·95°, 98·82°, and 98·85°. Thus, in his case, the highest maintained average temperature occurred in the winter and early spring months, but the curve plotted out from these figures would show only a slight degree of variation throughout the year, the range being 0·25° F.

In the case of homoiothermal animals other than man, few observations have been made. W. F. Edwards ‡ made experiments upon a great number of sparrows "taken at different seasons of the year, which is preferable to keeping these creatures in captivity for any length of time. The mean temperature of these birds rose progressively from the depth of winter to the height of summer, within the limits of from two to three degrees centigrade. . . . In the month of February the mean temperature of these birds was found to be 40·8° C., in April 42°, and in July 43·77°. The temperature from this time began to decline, and followed in the same ratio in which it had increased, the sinking temperature of the year." No further statement than that just quoted is given with regard to the conditions under which the observations were made, *e.g.*, as to how the birds were caught, the hours of the day when the temperatures were taken, etc. These small birds have a marked diurnal variation,§ and it is important in comparative work of this kind that the readings be taken as nearly as possible at the same hours on successive days.

In 1838 John Davy ‖ investigated the influence of the seasons on the rectal temperature of the sheep. On January 26, at noon, the mean temperature of three ewes was 104° F. The air temperature had been

* Jousset, *Arch. de méd. nav.*, Paris, 1883, t. 40, p. 124.

† Bosanquet, *Lancet*, London, 1895, vol. i. p. 672.

‡ Edwards, art. "Animal Heat," in Todd's *Cyclopædia of Anatomy and Physiology*, vol. ii. p. 659.

§ Simpson and Galbraith, *Journ. Physiol.*, xxxiii., 1905, p. 225.

‖ Davy, *Researches*, London, 1839, vol. i. p. 208.

below freezing-point continuously for four days previously, and when the observations were made it was 29° F. in the outhouse where the sheep were confined, and into which they had been driven from the marsh the evening before. On January 29, between 1 and 2 p.m., outside temperature 46° F., the mean temperature of two sheep, also ewes, was 105° F. In relation to one of these he says that: "the second sheep was in an irritable state; the lungs abounded in granular tubercles, yet the animal was in excellent condition." The next observations were made on March 16, when the temperature of the air was 51° F. and the mean rectal temperature of four individuals (females) was 105·5°. On May 23, air 63°, the mean temperature for three individuals was 105°; on August 21, air 70°–72°, for eight individuals (two of which were lambs about fourteen weeks old), it was 105·44°, and on August 28, when the thermometer stood at 79° F. in the shade, the mean for four individuals was 105·5°. These results are not very consistent, but they appear to show that body temperature of the sheep is a little higher in the warm weather of summer than in the moderately cold weather of the English winter. It must be remembered, however, that each set of observations was made on a different group of individuals, just before they were slaughtered, and that in many respects the conditions were not similar for each group.

Present Investigation.

The causes underlying the diurnal temperature changes that are present in a greater or less degree in all homoiothermal animals have not yet been clearly established. By many it is believed that these diurnal variations are due almost entirely to influences acting on the body from without; by some it is held that they are mainly dependent on periodic or cyclical changes within the body. The question must still be regarded as an open one, although the bulk of the evidence brought forward in recent years appears to give strong support to the first view. In an important contribution to this subject, Lindhard * comes to the conclusion that the "curve of temperature variations is determined by work and mode of living, that the astronomical division of day and night is without importance in this regard, and that an inherited form is consequently out of the question, a mysterious periodicity even more so."

In the light of the "doctrine of phases" † it is not unreasonable to

* Lindhard, *loc. cit.*

† For information regarding the application of this doctrine to physiology, consult a paper by Zwaardemaker and Dakhuysen, read before the International Medical Congress at Budapest, August–September 1909, and an article in the *British Medical Journal*, February 19, 1910, p. 461.

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[*Continued on page* iii *of Cover*

suppose that there may exist in the animal body phases—diurnal, lunar, or seasonal—in which the body temperature undergoes cyclical or periodic variations comparable with the changes that take place during the œstrous cycle in the mature female, or in the mental functions of the insane in the condition known as *folie circulaire.* In this relation it is important to make a detailed study of the body temperature in association with the seasonal or other changes of habit which all animals exhibit to a greater or less degree.

Amongst homoiothermal animals no class gives greater evidence of cyclical changes than do birds; in the late summer and autumn they show distinct signs of depressed vitality, such as casting of the feathers and (in the case of the domesticated fowls) a falling off in egg-production, while in the early spring they change plumage in preparation for the mating and breeding season, and in other respects give evidence of increased activity. No systematic investigation, so far as I know, has been made on the body temperature of birds from this point of view.

In the present investigation it was at first intended to make one series of observations in the spring of the year, and another in the autumn, and to use wild birds for the purpose, since it seemed probable that seasonal changes would be more firmly established in these than in domestic species. Various circumstances arose, however, which prevented the original plan from being carried out, and although a large number of figures were collected, they could not be divided into spring and autumn groups for each species. These results have already been presented to this Society and published in its Proceedings.*

It was then resolved to make an extended and detailed study of body temperature in the domestic fowl, not only in the spring and autumn months, but throughout the whole year, and the opportunity of doing this presented itself when the writer, through the kindness of Professor Rice, obtained access to the extensive colony of hens in the Department of Poultry Husbandry at the New York State College of Agriculture. It is essential in comparative work of this kind that the conditions shall be the same throughout the year except in so far as these are influenced by the weather, and that the observations be made periodically on the same individuals, and at the same hour of the day, and for the purpose in view the arrangements in the Poultry Department were all that could be desired.

The birds were lodged in commodious wooden hen-houses of the most recent type; the upper half of the front wall which faced the south being closed in by wire-netting only, and through a small window in the back

* Simpson, *Proc. Roy. Soc. Edin.*, 1911-1912.

wall they had access to a large run outside. The pens were arranged in a row, and all were of similar construction. The floors were covered with a thick layer of straw, but no artificial heat was supplied in the winter months, so that the temperature of the air inside the pens was only a few degrees higher than that outside.

From November till March, the ground being, as a rule, covered with snow for the greater part of that time, the hens were closed in the pens and prevented from getting out into the runs. During the night and in heavy rain and snow storms folding canvas screens were used to close up the pens in front.

The food was the same all the year round, and consisted of cracked corn, wheat and oats (hard grains), and cornmeal, wheat middlings, wheat bran, oil meal, alfalfa meal, and green cut bone, all ground up together (soft food). They were also provided with mangles, oyster-shell and grit, and of course water. Each pen contained on an average about thirty hens and two roosters. Of the latter only one was free, the other being shut up in a small coop, and every two days they were interchanged.

Six groups of hens were selected for these experiments, and each group located in a separate pen, viz. I. Barred Plymouth Rocks; II. Buff Orpingtons; III. White Wyandottes; IV. Single Comb Brown Leghorns; V. White Plymouth Rocks; and VI. Rhode Island Reds. With the exception of the Leghorns, all were quiet birds, and when picked up and handled made little resistance.

After some preliminary work in the spring, the regular observations were begun in October 1909, and were continued every month on the same individuals till September 1911. From April till October the hens were usually found in the runs, and were driven into the pens before the observations were made; but in the winter months, from November till March, they were nearly always found in the pens.

Each hen was caught by an assistant, handed to the operator, and an accurate half-minute clinical thermometer was inserted into the cloaca and rectum to the depth of three inches, retained in position two minutes, and the rectal temperature then noted. The records were made, as a rule, on two successive afternoons, about the end of the third week of each month, and at the same hour for each pen throughout the series, with a few exceptions. Thus, Pen I. was examined between 1.30 and 2.30 p.m.; Pen II. between 2.30 and 3.30 p.m.; and Pen III. between 3.30 and 4.30 p.m.; and on the following day, IV., V., and VI. were gone through in the same order. The temperature of the air in the pen was recorded in each case at the same time.

In October 1909, when the work was commenced, the birds were pullets, having been hatched in the previous April or May. They were leg-branded and numbered, and twenty-five were picked from each pen for observation, except in the case of IV., where fifteen was the number started with. In the course of the first year several of these had disappeared, and by the end of the second year not more than forty-one out of the original 140 remained. A few had died, and the rest had been sent to market or disposed of in other ways.

In the accompanying tables (I. to VI.) the actual figures are given for each individual in degrees centigrade, with the date and the temperature of the air in the pen at the top of each column. The number second from the bottom in each column represents the mean temperature for all the individuals in the pen observed on that day; while the last number, printed in italics, shows the mean for those on which the observations were continued for two years.

In the Charts I. to VI., the upper curve represents the mean temperature of all the hens in each pen from which monthly records were obtained for one year; the second curve the same for those which were under observations for two years. Underneath these are plotted out curves for the air temperature in the pen and the barometric pressure as it stood at the time when the observations were made. The curve of egg-production * at the bottom will require a word of explanation. Suppose that one has a flock of one hundred hens, and that by this flock one hundred eggs are laid daily, then the egg-production is said to be 100 per cent.; if sixty eggs are laid per day by the one hundred hens, the egg production is 60 per cent., and so on. In Charts VII. and VIII. the curves are plotted from the mean figures for all the hens that were examined on the same day, that is, when the temperature and other conditions were approximately the same, for in order to minimise errors due to accidental causes, it is desirable that the averages be taken from as large a number of individuals as possible.

It is conceivable that atmospheric conditions other than temperature, such as humidity, sunshine, strength and direction of wind, etc., may affect body temperature, and in order to ascertain to what extent this is the case, I have appended a fairly complete weather table for the days on which the temperature records were made. This, by the kindness of

* This curve of egg-production is taken from a paper by Rice, Nixon, and Rogers (Bulletin 258, *College of Agriculture Publications*, Cornell University, September 1908). It was obtained from trap-nested hens living at the same station and under the same conditions as those on which the present temperature observations were made, and indeed Professor Rice informs me that this curve of egg-production is practically correct for hens anywhere in New York State.

[*For continuation see p.* 130.

TABLE I.—Pen I., Barred Plymouth Rocks, ♀. Observations made between 1.30 and 2.30 p.m.

The figures at the left margin indicate the numbers of the hens, those at the top of each column the temperature of the air in the pen when the records were made, and those opposite each number the rectal temperatures in degrees centigrade. The row of figures at the bottom represents the mean rectal temperature of the eight hens that were under observation for two years, that second from the bottom the same for all the hens.

	1909.			1910.											
	Oct. 22.	Nov. 24.	Dec. 20.	Jan. 21.	Feb. 25.	Mar. 22.	Apr. 20.	May 5.	June 28.	July 18.	Aug. 23.	Sept. 21.	Oct. 25.	Nov. 22.	Dec. 19.
	11° C.	3° C.	1° C.	8° C.	1° C.	18° C.	13° C.	16° C.	28° C.	26° C.	32° C.	23° C.	8° C.	6° C.	4° C.
1	*42·0*	*41·8*	*41·9*	*41·7*	*41·6*	*41·8*	*42·0*	*41·8*	*41·7*	*42·1*	*42·1*	*41·3*	*42·1*	*41·8*	*42·1*
2	*41·6*	*41·9*	*41·8*	*41·4*	*41·6*	*41·5*	*41·7*	*42·2*	*42·8*	*42·3*	*41·9*	*41·7*	*41·4*	*41·9*	*41·7*
3	42·0	41·8	41·7	41·3	41·7	41·9	41·8	41·4*	42·4	42·2	42·4	41·9	41·8	...	...
4	41·9	41·6	41·5	41·3	41·4	42·1	41·9	42·0	41·8	42·1	42·2	41·6	41·8	41·7	41·7
5	*42·3*	*41·9*	*41·7*	*41·8*	*41·9*	*41·8*	*41·9*	*42·1*	*42·5*	*42·5*	*42·5*	*41·4*	*41·7*	*42·0*	*41·8*
6	*42·1*	*42·1*	*42·0*	*41·4*	*41·6*	*41·5*	*41·8*	*41·7*	*42·2*	*42·0*	*42·2*	*41·7*	*41·9*	*41·7*	*41·6*
7	*42·1*	*42·0*	*41·3*	*41·3*	*41·7*	*41·9*	*41·7*	*41·8*	*42·4*	*42·2*	*42·3*	*41·9*	*41·6*	*41·6*	*41·5*
8	42·1	41·6	41·8	41·7	42·1	42·1	41·8	41·7	42·2	42·1	42·4	41·4	41·9	...	...
9	42·0	41·9	41·9	41·8	41·9	41·6	41·4*	42·0	42·2	41·9	41·9	41·7	41·9	41·9	41·7
10	41·9	42·1	42·1	41·3	41·9	42·0	41·9	42·1	42·3	42·2	42·3	41·8	41·8	...	...
11	41·5	41·9	41·8	41·4	41.4	41·5	41·8	41·7	42·5	42·6	42·3	41·8	41·9	...	...
12	41·7	41·8	41·8	41·3	41·5	41·8	41·7	41·9	42·2	41·7	41·9	41·7	42·0	42·1	...
13	41·7	41·9	41·8	41·7	41·8	41·9	41·9	41·9	42·1	42·7	41·8	41·4	41·7	41·7	41·6
14	42·1	41·8	41·9	41·4	42·0	41·6	41·8	42·0	41·9	42·1	42·2	41·4	41·5	41·7	41·4
15	41·8	41·6	41·6	41·6	41·8	41·9	41·2*	42·4	42·7	42·4	42·3	41·7	...	...	...
16	41·9	41·4	41·8	41·8	41·5	41·8	42·0	42·2	42·4	41·2*	41·9	41·6	41·6	41·6	41·6
17	*42·1*	*41·9*	*41·9*	*42·1*	*41·9*	*42·0*	*42·1*	*42·1*	*42·2*	*42·0*	*42·6*	*41·9*	*41·8*	*41·8*	*42·0*
18	*42·3*	*41·5*	*41·8*	*41·8*	*41·9*	*42 0*	*42·1*	*42·2*	*42·8*	*42·1*	*42·8*	*41·7*	*41·9*	*41·6*	*41·6*
19	42·1	41·8	42·1	41·4	41·4	41·7	41·5	41·9	42·6	41·8	42·3	42·3	42·0	42·0	41·2
20	*41·8*	*41·6*	*41·8*	*41·2*	*41·3*	*41·8*	*41·9*	*42·2*	*42·8*	*41·7*	*42·1*	*41·9*	*42·2*	*41·9*	*42·3*
21	42·1	41·7	42·0	41·6	41·7	41·6	41·8	41·9	42·3	41·9	41·9	41·5	41·8	...	...
22	41·8	41·5	41·8	41·6	41·6	41·5	41·8	41·8	42·3	41·6	42·1	41·6	41·4	..	...
23	41·9	41·6	42·0	41·5	41·5	41·9	41·7	41·6	42·2	42·0	42·6	41·5	41·9	41·5	41·4
Mean for all.	41·95	41·77	41·82	41·53	41·68	41·79	41·79	41·94	42·30	42·06	42·22	41·67	41·80	41·78	41·68
Mean for 1, 2, 5, 6, 7, 17, 18, 20	*42·04*	*41·86*	*41·78*	*41·59*	*41·69*	*41·79*	*41·90*	*42·01*	*42·42*	*42·11*	*42·31*	*41·69*	*41·82*	*41·79*	*41·82*

	1911.								
	Jan. 18.	Feb. 22.	Mar. 22.	Apr. 19.	May 24.	June 21.	July 21.	Aug. 19.	Sept. 20.
	0° C.	2° C.	14°C.	19° C.	27° C.	24° C.	30° C.	22° C.	24° C.
1	*42·1*	*42·1*	*42·0*	*41·5*	*41·7*	*41·7*	*41·8*	*41·8*	*41·9*
2	*41·8*	*41 6*	*41·8*	*40·9*	*42·1*	*40·6*	*41·9*	*41·8*	*41·7*
3	...	...	...	...	...	...	...	...	...
4	41·9	41·4	41·6	41·6	41·2*	...	...	...	...
5	*42·0*	*41·9*	*42·0*	*41·8*	*42·3*	*42·4*	*42·2*	*41·7*	*41·6*
6	*42·0*	*41·8*	*41·9*	*41·8*	*42.1*	*41·7*	*42·1*	*41·7*	*42·2*
7	*42·0*	*41·6*	*41·9*	*41·8*	*41·3**	*41·2**	*42·0*	*41·7*	*41·9*
8	...	...	...	...	...	...	...	...	...
9	41·4	42·0	41·9	41·5	41·4	...	...	...	...
10	...	...	...	...	...	...	...	...	...
11	...	...	...	...	...	...	...	...	...
12	...	...	...	...	...	...	...	...	...
13	42·2	41·7	41·7	41·7	...	...	...	...	...
14	42 0	41·5	41·5	41·6	41·7	...	...	...	...
15	...	...	...	...	...	...	...	...	...
16	42·2	41·4	41·9	...	...	...	...	...	...
17	*42·1*	*41·6*	*42·1*	*41·7*	*41·3**	*41·9*	*41·8*	*42·1*	*41·9*
18	*42·3*	*42·1*	*42·0*	*42·2*	*41·9*	*41·4**	*42·1*	*41·7*	*41·7*
19	42·1	41·7	42·3	42·3	42·0	...	...	...	...
20	*42·3*	*42·0*	*42·2*	*42·0*	*42·2*	*42·1*	*42·0*	*42·1*	*42·0*
21	...	...	...	...	...	...	...	...	...
22	...	...	...	...	...	...	...	...	...
23	42·0	41·4	41·8	41·3*	...	...	...	...	...
Mean for all.	42·03	41·72	41·91	41·69	41·77	41·62	41·99	41·82	41·86
Mean for 1, 2, 5, 6, 7, 17, 18, 20	*42·07*	*41·84*	*41·99*	*41·71*	*41·86*	*41·62*	*41·99*	*41·82*	*41·86*

* Broody.

TABLE II.—Pen II., Buff Orpingtons, ♀. Observations made between 2.30 and 3.30 p.m.

	1909.			1910.												1911.								
	Oct. 22.	Nov. 24.	Dec. 20.	Jan. 21.	Feb. 25.	Mar. 22.	Apr. 20.	May 5.	June 28.	July 18.	Aug. 23.	Sept. 21.	Oct. 25.	Nov. 22.	Dec. 19.	Jan. 18.	Feb. 22.	Mar. 22.	Apr. 19.	May 24.	June 21.	July 21.	Aug. 19.	Sept. 20.
	11° C.	3° C.	0° C.	8° C.	1° C.	17° C.	12° C.	16° C.	29° C.	26° C.	32° C.	22° C.	7° C.	6° C.	3° C.	0° C.	1° C.	14° C.	19° C.	28° C.	24° C.	31° C.	21° C.	24° C.
1	41·5	41·9	41·5	41·4	41·9	42·3	41·9	42·2	42·1	41·4*	42·6	41·7	41·6	...	...	...	..	...	...	...	...	...	...	...
2	41·8	41·4	41·2	41·8	41·4	41·9	42·1	42·1	42·1	42·0	41·9	41·8	41·5	...	...	...	...	...	...	...	...	...	...	...
3	41·6	41·8	41·2	41·9	42·1	41·5	41·8	41·8	42·3	42·2	42·6	41·7	42·1	...	...	...	...	...	...	...	...	...	...	...
4	42·0	41·5	41·8	41·4	42·0	41·8	41·8	42·1	42·4	42·1	42·5	41·7	41·6	...	...	...	...	...	...	...	...	...	...	...
5	42·0	41·9	41·4	41·8	41·5	41·9	41·9	42·0	41·6	42·7	42·4	42·2	41·5	...	...	...	...	...	...	...	...	...	...	...
6	41·7	41·8	41·5	41·8	41·4	41·5	41·5	41·8	42·6	42·7	42·1	41·8	41·6	...	...	...	...	...	...	...	...	...	...	...
7	*41·8*	*41·5*	*41·9*	*41·8*	*42·1*	*42·1*	*41·5*	*41·9*	*42·3*	*42·4*	*41·9*	*42·0*	*42·1*	*42·2*	*42·1*	*41·9*	*41·7*	*42·1*	*41·8*	*42·1*	*42·2*	*42·4*	*42·1*	*41·9*
8	*41·8*	*41·7*	*41·8*	*41·9*	*41·6*	*42·2*	*42·0*	*41·9*	*42·7*	*42·6*	*42·2*	*42·3*	*42·3*	*41·9*	*42·2*	*42·2*	*42·0*	*42·2*	*42·3*	*41·8*	*41·8*	*42·4*	*41·7*	*42·0*
9	41·7	41·5	41·7	41·5	41·5	41·4	41·8	41·9	42·4	42·6	42·7	41·9	42·0	...	...	...	...	...	...	...	...	...	...	...
10	42·0	41·8	41·5	41·4	41·5	41·8	41·8	42·0	42·1	42·1	41·8	42·0	41·7	41·9	42·2	...	...	...	...	...	...	...	...	...
11	*41·7*	*41·4*	*41·3*	*41·8*	*41·7*	*41·7*	*41·8*	*42·4*	*42·4*	*42·2*	*42·2*	*42·0*	*42·1*	*41·9*	*41·9*	*41·7*	*41·6*	*41·5*	*41·5*	*41·4**	*41·4**	*42·6*	*41·7**	*41·8*
12	*41·5*	*41·9*	*41·8*	*41·5*	*42·1*	*42·1*	*41·8*	*41·9*	*42·0*	*42·3*	*42·2*	*42·1*	*41·2*	*41·6*	*42·2*	*42·2*	*41·9*	*42·3*	*42·4*	*42·1*	*42·0*	*42·4*	*42·2*	*41·8*
13	*42·0*	*41·9*	*42·1*	*41·5*	*42·1*	*41·6*	*41·5*	*41·7*	*42·2*	*41·9*	*42·4*	*42·1*	*41·9*	*41·8*	*41·9*	*41·8*	*41·7*	*41·9*	*42·7*	*42·0*	*41·7*	*42·2*	*42·1*	*41·9*
14	41·8	41·8	41·4	41·8	41·7	41·5	41·5	41·8	41·9	41·9	42·4	41·9	42·1	42·2	...	...	...	...	...	...	...	...	...	...
15	41·8	41·5	41·4	41·3	41·8	42·1	41·8	41·9	42·1	41·2*	43·0	42·3	41·9	42·0	...	...	...	...	...	...	...	...	...	...
16	*41·9*	*42·0*	*42·1*	*41·6*	*41·4*	*41·4*	*42·0*	*41·7*	*42·6*	*42·3*	*42·2*	*42·2*	*41·8*	*41·6*	*41·5*	*41·7*	*41·5*	*41·8*	*42·0*	*41·9*	*41·9*	*42·7*	*41·9*	*42·0*
17	41·9	41·8	41·9	42·1	41·5	41·4	41·9	41·9	41·4*	42·2	41·9	42·4	41·7	..	...	...	...	...	...	...	...	...	...	...
18	*41·6*	*41·7*	*41·6*	*41·5*	*41·7*	*41·8*	*41·7*	*40·9**	*42·2*	*42·2*	*42·4*	*42·0*	*41·4*	*42·0*	*42·3*	*41·8*	*41·7*	*41·8*	*41·9*	*41·7*	*41·7*	*42·0*	*41·5**	*41·9*
19	*42·2*	*42·1*	*41·4*	*41·9*	*41·5*	*41·5*	*41·6*	*41·9*	*42·2*	*42·1*	*42·3*	*42·4*	*42·2*	*41·8*	*42·0*	*41·7*	*42·1*	*41·9*	*41·9*	*42·2*	*40·9*	*42·0*	*42·2*	*42·2*
20	*41·6*	*41·5*	*41·8*	*41·8*	*41·4*	*41·4*	*41·8*	*41·8*	*41·9*	*42·1*	*42·2*	*42·0*	*41·7*	*42·2*	*42·2*	*41·8*	*41·6*	*41·6*	*42·1*	*41·8*	*41·7*	*42·1*	*42·1*	*4·20*
21	*41·5*	*41·6*	*41·5*	*41·5*	*41·8*	*41·8*	*41·5*	*41·9*	*42·4*	*42·0*	*42·6*	*42·1*	*41·4*	*41·7*	*42·1*	*41·6*	*41·3*	*41·9*	*41·8*	*41·9*	*41·7*	*42·2*	*42·1*	*41·8*
22	41·9	42·0	42·1	42·1	41·9	41·8	41·6	42·1	42·2	42·2	42·7	42·2	42·0	...	...	...	...	...	...	...	...	...	...	...
Mean for all.	41·78	41·73	41·63	41·67	41·71	41·75	41·75	41·89	42·19	42·15	42·33	42·04	41·79	41·91	42·05	41·84	41·71	41·90	42·04	41·89	41·70	42·30	41·96	41·93
Mean for 7, 8, 11, 12, 13, 16, 18, 19, 20, 21.	*41·76*	*41·73*	*41·72*	*41·68*	*41·74*	*41·76*	*41·72*	*41·80*	*42·29*	*42·21*	*42·26*	*42·12*	*41·81*	*41·87*	*42·04*	*41·84*	*41·71*	*41·90*	*42·04*	*41·89*	*41·70*	*42·30*	*41·96*	*41·93*

* Broody.

TABLE III.—Pen III., White Wyandottes, ♀. Observations made between 3.30 and 4.30 p.m.

	1909.			1910.											
	Oct. 22.	Nov. 24.	Dec. 20.	Jan. 21.	Feb. 21.	Mar. 22.	Apr. 20.	May 5.	June 28.	July 18.	Aug. 23.	Sept. 21.	Oct. 25.	Nov. 22.	Dec. 19.
	11° C.	3° C.	0° C.	7° C.	1° C.	17° C.	12° C.	15° C.	28° C.	25° C.	32° C.	21° C.	7° C.	6° C.	2° C.
1	42·0	41·6	41·9	42·1	41·6	41·8	41·6	41·9	42·6	42·5	42·3	42·1	42·2	...	...
2	41·6	41·6	41·7	42·2	41·8	42·0	41·7	42·1	42·4	42·5	42·0	42·1	42·1	42·2	42·0
3	41·8	42·0	41·6	42·0	41·6	42·1	42·1	42·2	42·5	42·3	42·4	42·2	42·1	42·2	42·2
4	*42·1*	*42·3*	*41·5*	*41·8*	*41·7*	*41·7*	*41·9*	*42·1*	*42·7*	*42·5*	*42·3*	*41·9*	*41·9*	*42·0*	*41·8*
5	42·0	41·7	42·1	41·7	42·1	42·1	42·0	42·0	42·8	42·1	42·1	42·2	41·6	...	...
6	41·7	41·9	41·6	41·9	41·5	41·9	42·0	41·8	42·6	42·1	42·2	42·1	42·0	42·1	42·0
7	*41·9*	*42·1*	*42·1*	*42·1*	*41·8*	*41·9*	*42·1*	*42·1*	*42·7*	*42·0*	*42·2*	*42·1*	*42·1*	*42·1*	*41·9*
8	42·0	41·6	41·6	41·9	42·0	41·7	42·1	42·0	42·3	42·4	42·6	42·0	41·8	...	...
9	41·8	41·9	41·7	41·8	41·6	42·1	41·7	41·9	42·5	42·2	42·0	42·2	42·1	41·8	41·7
10	42·3	42·1	41·6	41·6	42·1	41·9	41·6	41·7	42·1	42·1	42·1	41·9	42·0	42·2	41·5
11	*42·2*	*41·7*	*41·5*	*41·4*	*41·3*	*41·5*	*41·5**	*42·1*	*42·3*	*42·3*	*42·1*	*41·8*	*42·2*	*41·9*	*41·6*
12	*42·0*	*42·1*	*41·8*	*41·8*	*41·9*	*42·1*	*42·1*	*41·9*	*42·0*	*42·1*	*43·0*	*42·1*	*41·9*	*42·1*	*41·7*
13	*41·6*	*42·1*	*42·1*	*42·0*	*42·1*	*42·1*	*42·0*	*41·9*	*42·4*	*42·2*	*42·4*	*42·4*	*41·9*	*41·8*	*41·5*
14	41·8	41·6	41·9	41·8	41·8	41·9	42·0	42·1	42·5	42·5	42·7	42·2	41·7	...	...
15	*42·1*	*42·1*	*41·6*	*41·9*	*42·0*	*41·9*	*41·8*	*41·9*	*42·2*	*42·3*	*42·6*	*41·8*	*42·1*	*41·9*	*42·0*
16	41·7	41·5	41·8	41·6	41·7	42·0	42·0	42·0	42·6	42·2	42·5	41·8	42·0	42·0	41·6
17	*42·2*	*42·1*	*42·0*	*41·7*	*41·9*	*42·0*	*41·8*	*41·7*	*42·1*	*42·0*	*42·3*	*42·1*	*41·8*	*41·9*	*41·7*
Mean for all.	41·93	41·88	41·77	41·82	41·79	41·92	41·88	41·97	42·43	42·25	42·34	42·05	41·94	42·02	41·78
Mean for 4, 7, 11, 12, 13, 15, 17.	*42·01*	*42·07*	*41·80*	*41·81*	*41·81*	*41·89*	*41·89*	*41·96*	*42·34*	*42·21*	*42·41*	*42·03*	*41·99*	*41·96*	*41·74*

	1911.								
	Jan. 18.	Feb. 22.	Mar. 22.	Apr. 19.	May 24.	June 21.	July 21.	Aug. 19.	Sept. 20.
	0° C.	1° C.	13° C.	18° C.	28° C.	23° C.	30° C.	21° C.	24° C.
1	...	...	...	...	...	...	...	...	...
2	...	...	...	...	...	...	...	...	...
3	41·8	41·5	41·7	41·8	41·7	...	...	...	...
4	*41·6*	*41·9*	*41·8*	*42·1*	*41·9*	*41·6*	*42·0*	*41·8*	*42·0*
5	...	...	...	...	...	...	...	...	...
6	...	...	...	...	...	...	...	...	...
7	*41·7*	*41·8*	*42·1*	*41·8*	*42·3*	*42·1*	*42·2*	*41·9*	*41·7*
8	...	...	...	...	...	...	...	...	...
9	42·0	42·1	41·9	41·7	...	...	...	...	...
10	41·6	41·8	...	...	...	...	...	...	...
11	*41·8*	*41·7*	*42·0*	*42·0*	*42·2*	*41·1*	*42.0*	*42·0*	*42.1*
12	*42·1*	*42·0*	*41·7*	*41·8*	*42·3*	*42·1*	*42·7*	*42·2*	*42·3*
13	*41·5*	*42·2*	*41·6*	*42·1*	*42·2*	*41·4**	*42·2*	*42·3*	*41·1**
14	...	...	...	...	...	...	...	...	...
15	*41·5*	*41·6*	*42·1*	*42·1*	*41·9*	*42·2*	*41·7*	*41·9*	*42·1*
16	...	...	...	...	...	...	...	...	...
17	*41·6*	*42·1*	*41·7*	*42·2*	*42·4*	*42·1*	*42·3*	*42·1*	*42·4*
Mean for all.	41·72	41·87	41·84	41·84	42·11	41·80	42·16	42·03	41·96
Mean for 4, 7, 11, 12, 13, 15, 17.	*41·69*	*41·90*	*41·86*	*42·01*	*42·17*	*41·80*	*42·16*	*42·03*	*41·96*

* Broody.

TABLE IV.—Pen IV., Single Comb Brown Leghorns, ♀. Observations made between 2 and 2.30 p.m.

	1909.			1910.												1911.					
	Oct. 23.	Nov. 25.	Dec. 21.	Jan. 22.	Feb. 26.	Mar. 23.	Apr. 21.	May 10.	June 27.	July 21.	Aug. 24.	Sept. 22.	Oct. 26.	Nov. 23.	Dec. 20.	Jan. 19.	Feb. 23.	Mar. 22.	Apr. 20.	May 25.	June 21.
	8° C.	1° C.	1° C.	2° C.	3° C.	15° C.	13° C.	19° C.	33° C.	28° C.	32° C.	22° C.	17° C.	8° C.	2° C.	4° C.	7° C.	13° C.	10° C.	22° C.	26° C.
1	41·9	42·0	41·9	42·0	41·8	42·0	41·9	42·2	42·4	42·4	42·2	42·2	42·3	41·9	41·8	...	...	...	...	...	...
2	*41·8*	*41·6*	*41·8*	*41·7*	*41·9*	*42·0*	*41·5*	*42·1*	*43·0*	*42·6*	*42·4*	*42·1*	*41·8*	*41·6*	*41·5*	*41·9*	*41·8*	*42·1*	*42·2*	*42·2*	*42·1*
3	42·0	41·7	42·3	41·9	42·1	41·9	42·0	42·3	41·9	42·5	42·9	42·6	42·0	...	...	...	...	...	...	...	...
4	42·1	42·0	41·8	41·5	41·9	42·1	42·2	41·7	42·7	42·6	42·1	41·8	41·7	...	...	...	...	...	...	...	...
5	*41·9*	*41·6*	*41·9*	*41·8*	*41·6*	*42·0*	*42·0*	*41·9*	*42·8*	*42·4*	*42·7*	*41·9*	*42·1*	*41·8*	*42·1*	*41·6*	*42·1*	*41·8*	*41·4*	*42·4*	*42·4*
6	42·1	42·2	42·1	42·0	41·8	41·8	41·7	42·1	42·4	42·8	42·6	42·0	41·7	...	...	...	...	...	...	...	...
7	*41·8*	*41·6*	*41·9*	*41·8*	*41·8*	*41·5*	*41·5*	*41·9*	*43·2*	*42·3*	*42·8*	*42·1*	*41·9*	*41·7*	*41·6*	*42·0*	*42·0*	*41·7*	*42·1*	*42·1*	*41·8*
8	42·1	41·9	42·0	41·9	41·6	42·2	42·0	42·2	42·8	43·1	42·8	41·9	42·0	41·8	41·4	...	...	...	...	...	...
9	*42·0*	*41·8*	*41·8*	*42·0*	*41·8*	*41·9*	*42·1*	*42·1*	*42·9*	*42·5*	*43·0*	*42·0*	*41·8*	*42·0*	*41·9*	*41·7*	*41·9*	*42·1*	*41·7*	*41·9*	*42·6*
10	*41·7*	*41·9*	*41·7*	*42·0*	*41·7*	*42·1*	*41·9*	*42·0*	*42·6*	*42·3*	*42·4*	*41·9*	*41·9*	*41·7*	*41·6*	*41·8*	*42·3*	*41·8*	*41·0**	*41·9*	*41·1*
Mean for all.	41·94	41·83	41·92	41·86	41·80	41·95	41·88	42·05	42·67	42·55	42·59	42·05	41·92	41·79	41·70	41·80	42·02	41·90	41·68	42·10	42·00
Mean for 2, 5, 7, 9, 10.	*41·84*	*41·70*	*41·82*	*41·86*	*41·76*	*41·90*	*41·80*	*42·00*	*42·90*	*42·42*	*42·66*	*42·00*	*41·90*	*41·76*	*41·74*	*41·80*	*42·02*	*41·90*	*41·68*	*42·10*	*42·00*

* Broody.

TABLE V.—PEN V., WHITE PLYMOUTH ROCKS, ♀. OBSERVATIONS MADE BETWEEN 2.30 AND 3.30 P.M.

	1909.			1910.												1911.								
	Oct. 23.	Nov. 25.	Dec. 21.	Jan. 22.	Feb. 26.	Mar. 23.	Apr. 21.	May 10.	June 27.	July 21.	Aug. 24.	Sept. 22.	Oct. 26.	Nov. 23.	Dec. 20.	Jan. 19.	Feb. 23.	Mar. 23.	Apr. 20.	May 25.	June 21.	July 21.	Aug. 21.	Sept. 20.
	8° C.	1° C.	1° C.	2° C.	3° C.	15° C.	14° C.	20° C.	32° C.	28° C.	31° C.	22° C.	15° C.	8° C.	3° C.	4° C.	5° C.	14° C.	10° C.	21° C.	26° C.	28° C.	30° C.	23° C.
1	42·0	42·7	41·9	41·7	41·9	42·1	41·8	41·9	42·7	42·4	42·6	42·1	42·2	...	...	...	...	...	...	...	...	...	...	...
2	*41·7*	*42·4*	*42·0*	*41·5*	*41·5*	*41·8*	*42·0*	*42·0*	*42·4*	*42·7*	*42·4*	*41·9*	*41·8*	*41·9*	*41·4*	*41·9*	*41·8*	*42·0*	*42·1*	*41·9*	*42·2*	*42·1*	*42·4*	*41·9*
3	41·8	42·1	41·8	41·7	41·5	41·5	41·8	41·9	42·2	42·4	42·1	42·2	...	...	...	...	...	...	...	...	...	...	...	...
4	*42·1*	*42·7*	*42·1*	*41·8*	*42·1*	*42·2*	*42·3*	*42·0*	*42·1*	*42·4*	*42·3*	*42·5*	*42·1*	*41·8*	*42·0*	*41·6*	*42·1*	*41·8*	*41·7*	*41·8*	*41·8*	*42·3*	*41·6*	*42·1*
5	41·9	42·1	41·9	41·8	41·7	41·9	41·9	42·1	43·1	42·1	42·5	42·1	41·9	41·7	41·5	...	...	...	...	...	...	...	...	...
6	42·0	41·8	41·8	41·9	41·7	41·9	41·8	42·1	42·2	42·4	42·0	41·9	...	...	...	...	...	...	...	...	...	...	...	...
7	41·5	41·7	41·6	41·8	41·8	42·1	42·1	42·1	42·3	42·5	42·2	42·1	...	...	...	...	...	...	...	...	...	...	...	...
8	41·7	42·1	41·8	41·9	41·8	41·9	41·9	42·1	42·5	42·2	43·0	42·3	42·1	...	...	...	...	...	...	...	...	...	...	...
9	41·9	41·5	41·7	41·8	41·5	42·2	41·7	42·2	43·0	42·4	42·3	42·1	...	...	...	...	...	...	...	...	...	...	...	...
10	41·7	41·7	41·7	41·9	41·7	41·9	42·0	42·2	42·9	42·4	42·9	42·0	42·0	41·8	42·1	...	...	...	...	...	...	...	...	...
11	42·0	41·7	41·8	41·7	41·5	42·2	42·1	41·7	43·2	42·4	42·4	42·2	41·8	41·7	41·9	...	...	...	...	...	...	...	...	...
12	41·9	42·0	41·7	41·7	41·7	41·8	41·8	41·7	42·4	42·6	41·9	42·1	41·7	42·1	41·6	42·0	41·6	...	...	...	...	...	...	...
13	41·7	41·5	41·7	41·8	41·7	41·9	42·0	41·8	42·4	42·2	42·7	42·3	42·3	42·1	41·9	...	...	...	...	...	...	...	...	...
14	42·0	41·7	41·7	41·9	41·5	42·1	41·8	42·4	41·9	42·1	42·6	41·9	...	...	...	...	...	...	...	...	...	...	...	...
15	41·7	41·8	41·9	41·6	41·8	42·0	42·0	42·2	42·7	43·2	42·1	41·6	41·9	41·6	41·6	...	...	...	...	...	...	...	...	...
16	*41·9*	*41·8*	*41·8*	*41·5*	*41·5*	*41·8*	*41·9*	*42·2*	*42·7*	*42·4*	*42·6*	*42·3*	*42·1*	*42·0*	*41·8*	*41·7*	*41·7*	*41·8*	*42·1*	*41·3**	*42·2*	*42·3*	*42·6*	*41·9*
17	41·6	41·6	41·7	41·5	41·7	41·9	42·1	42·3	43·0	42·2	42·8	42·2	42·1	41·9	41·7	...	...	...	...	...	...	...	...	...
18	41·9	41·6	41·8	41·9	41·7	42·2	41·9	42·1	42·8	42·1	42·0	42·1	42·0	41·7	41·8	...	...	...	...	...	...	...	...	...
19	*42·0*	*41·8*	*41·8*	*41·6*	*41·6*	*41·8*	*42·2*	*42·1*	*42·4*	*42·5*	*42·3*	*42·3*	*41·8*	*41·9*	*41·6*	*41·9*	*41·8*	*41·9*	*41·4**	*42·2*	*42·1*	*42·4*	*42·7*	*42·0*
20	41·9	41·9	41·8	41·9	41·8	41·7	41·9	42·0	42·5	42·7	42·9	42·2	42·1	42·0	42·0	...	...	...	...	...	...	...	...	...
21	41·6	41·7	41·5	41·5	41·6	41·7	41·6	42·1	42·1	42·3	42·6	42·4	41·9	41·8	41·9	41·7	41·7	41·8	42·0	41·9	...	...	...	...
22	41·9	41·8	41·9	41·7	41·5	41·8	42·2	42·0	42·3	42·1	42·5	42·1	42·2	...	...	...	...	...	...	...	...	...	...	...
23	42·0	41·9	41·7	41·9	41·8	41·9	41·6	41·9	42·8	42·5	42·3	41·6	42·0	42·1	41·9	...	...	...	...	...	...	...	...	...
Mean for all.	41·84	41·90	41·78	41·74	41·67	41·93	41·93	42·05	42·55	42·40	42·48	42·11	42·00	41·87	41·78	41·80	41·78	41·86	41·86	41·82	42·08	42·28	42·32	41·98
Mean for 2, 4, 16, 19	*41·92*	*42·17*	*41·92*	*41·60*	*41·67*	*41·90*	*42·10*	*42·07*	*42·40*	*42·50*	*42·40*	*42·25*	*41·95*	*41·90*	*41·70*	*41·77*	*41·85*	*41·88*	*41·82*	*41·80*	*42·08*	*42·28*	*42·32*	*41·98*

* Broody.

TABLE VI.—Pen VI., Rhode Island Reds, ♀. Observations made between 3.30 and 4.30 p.m.

	1909.			1910.												1911.								
	Oct. 23.	Nov. 25.	Dec. 21.	Jan. 22.	Feb. 26.	Mar. 23.	Apr. 21.	May 10.	June 27.	July 21.	Aug. 24.	Sept. 22.	Oct. 26.	Nov. 23.	Dec. 20.	Jan. 19.	Feb. 23.	Mar. 22.	Apr. 20.	May 25.	June 21.	July 21.	Aug. 21.	Sept. 20.
	8° C.	1° C.	1° C.	2° C.	2° C.	15° C.	14° C.	19° C.	32° C.	28° C.	31° C.	22° C.	16° C.	8° C.	3° C.	4° C.	6° C.	14° C.	10° C.	21° C.	25° C.	29° C.	30° C.	23° C.
1	41·5	41·8	41·5	41·7	41·5	41·8	41·8	41·5	42·2	41·9	42·3	41·7	42·1	41·9	41·8	41·6	42·0	41·6	42·2	41·8	42·1	42·2	42·3	41·9
2	41·6	42·1	41·9	41·7	41·9	41·8	42·0	41·1*	42·1	42·2	41·6	41·9	41·7	...	...	...	...	...	...	...	...	...	...	...
3	41·8	41·8	41·9	41·9	41·8	41·9	41·9	41·8	42·1	42·1	42·3	41·9	41·9	42·1	41·2	...	...	...	...	...	...	...	...	...
4	42·0	41·9	41·8	42·0	41·9	42·0	41·9	41·6	42·3	42·2	41·9	41·8	42·1	42·1	41·4	41·6	41·8	41·8	41·6	42·0	41·2*	41·6*	41·8	41·9
5	42·0	42·0	42·0	41·8	41·7	42·1	41·7	41·2*	42·3	42·2	41·1*	42·3	42·1	42·2	41·8	...	...	...	...	...	...	...	...	...
6	42·1	41·8	41·7	42·1	41·5	42·1	42·0	41·9	42·8	42·3	42·7	42·1	42·0	42·2	41·8	...	...	...	...	...	...	...	...	...
7	42·1	42·0	41·7	41·7	41·8	41·9	42·0	41·4	42·3	42·2	41·9	42·0	41·7	41·9	41·2	...	...	...	...	...	...	...	...	...
8	42·0	42·1	42·1	41·7	41·7	41·8	42·1	41·9	42·2	42·6	42·1	41·7	41·8	42·0	41·7	41·7	41·7	41·7	41·9	41·3	41·4*	42·2	41·9	42·1
9	41·8	41·9	42·0	41·8	42·0	41·5	41·8	41·7	42·4	42·4	42·3	42·4	...	...	...	...	...	...	...	...	...	...	...	...
10	41·9	42·0	42·1	41·9	41·8	42·1	41·9	42·3	42·6	42·4	43·0	42·1	...	...	...	...	...	...	...	...	...	...	...	...
11	42·0	41·9	41·7	42·1	41·9	41·8	42·3	41·4	42·0	42·3	42·2	42·0	...	...	...	...	...	...	...	...	...	...	...	...
12	41·7	41·9	41·9	41·9	41·1	42·0	41·8	41·5	42·3	42·2	42·4	42·4	42·2	41·7	41·6	41·5	42·2	41·7	41·9	42·3	41·9	42·3	42·0	41·4*
13	42·1	41·8	41·8	42·0	41·7	41·7	41·8	41·7	42·4	42·2	41·9	41·9	...	...	...	...	...	...	...	...	...	...	...	...
14	41·9	42·3	41·7	41·8	42·0	41·8	42·0	42·1	42·2	42·1	41·6	42·0	41·9	42·1	41·3	41·4	41·8	41·6	42·1	42·3	42·2	41·3*	42·5	41·8
15	42·0	41·8	42·0	41·8	41·5	41·8	41·7	42·0	42·4	42·3	42·7	42·3	42·2	42·1	41·1	...	...	...	...	...	...	...	...	...
16	42·0	42·0	41·9	41·7	41·3	41·6	41·9	42·1	42·3	42·4	42·2	42·4	41·4	42·3	41·8	41·4	42·1	41·8	41·8	42·2	42·1	42·9	42·2	41·4*
17	42·1	41·9	42·2	41·6	41·6	42·1	42·0	41·4*	42·2	42·0	42·0	41·5	41·6	41·7	41·4	41·2	41·3	41·6	41·7	41·7	42·0	42·0	42·2	42·0
18	41·9	41·7	41·8	41·7	41·9	41·9	42·1	41·5	42·4	42·4	42·2	42·1	41·9	42·1	42·1	41·7	42·1	42·0	41·8	...	...	...	...	...
19	41·8	41·8	41·7	41·8	42·1	41·8	42·1	42·0	42·3	42·3	42·1	42·1	42·2	42·0	41·4	...	...	...	...	...	...	...	...	...
Mean for all.	41·91	41·92	41·86	41·83	41·72	41·87	41·94	41·69	42·31	42·25	42·14	42·03	41·93	42·03	41·54	41·51	41·88	41·72	41·88	41·94	41·84	42·07	42·13	41·79
Mean for 1, 4, 8, 12, 14, 16, 17	41·89	41·99	41·87	41·77	41·59	41·87	41·93	41·73	42·24	42·20	42·07	41·93	41·87	41·97	41·57	41·49	41·84	41·69	41·89	41·94	41·84	42·07	42·13	41·79

* Broody.

CHART I.—Pen I., Barred Plymouth Rocks, ♀.

Shows mean rectal temperature, on dates given, of twenty-three hens for one year, and of eight for two years; also temperature of air in pen, barometric pressure, and egg-production on same dates. The figures on the right margin (degrees centigrade) refer to the lower of the two rectal temperature curves. This applies to all the charts.

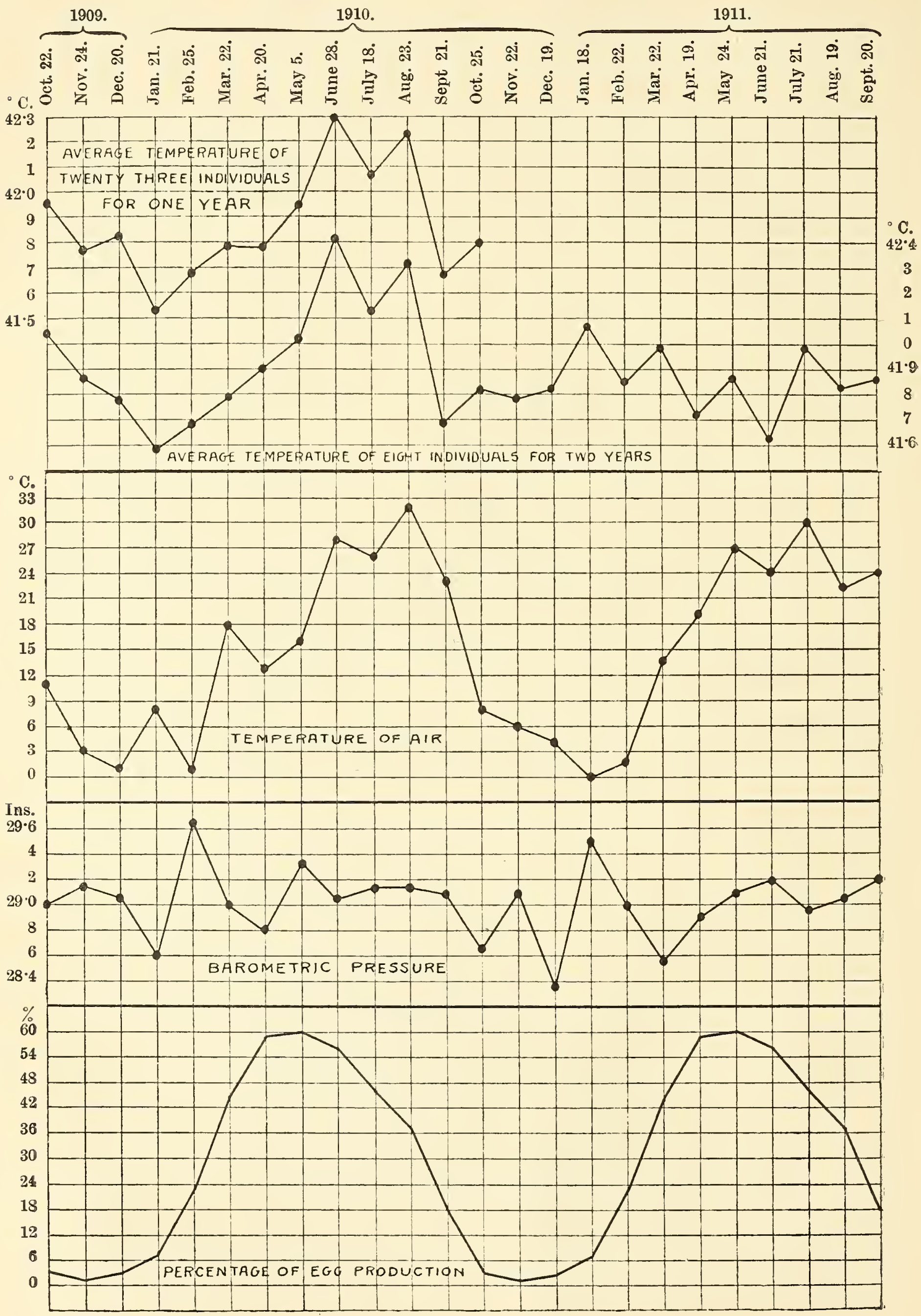


CHART II.—PEN II., BUFF ORPINGTONS, ♀.

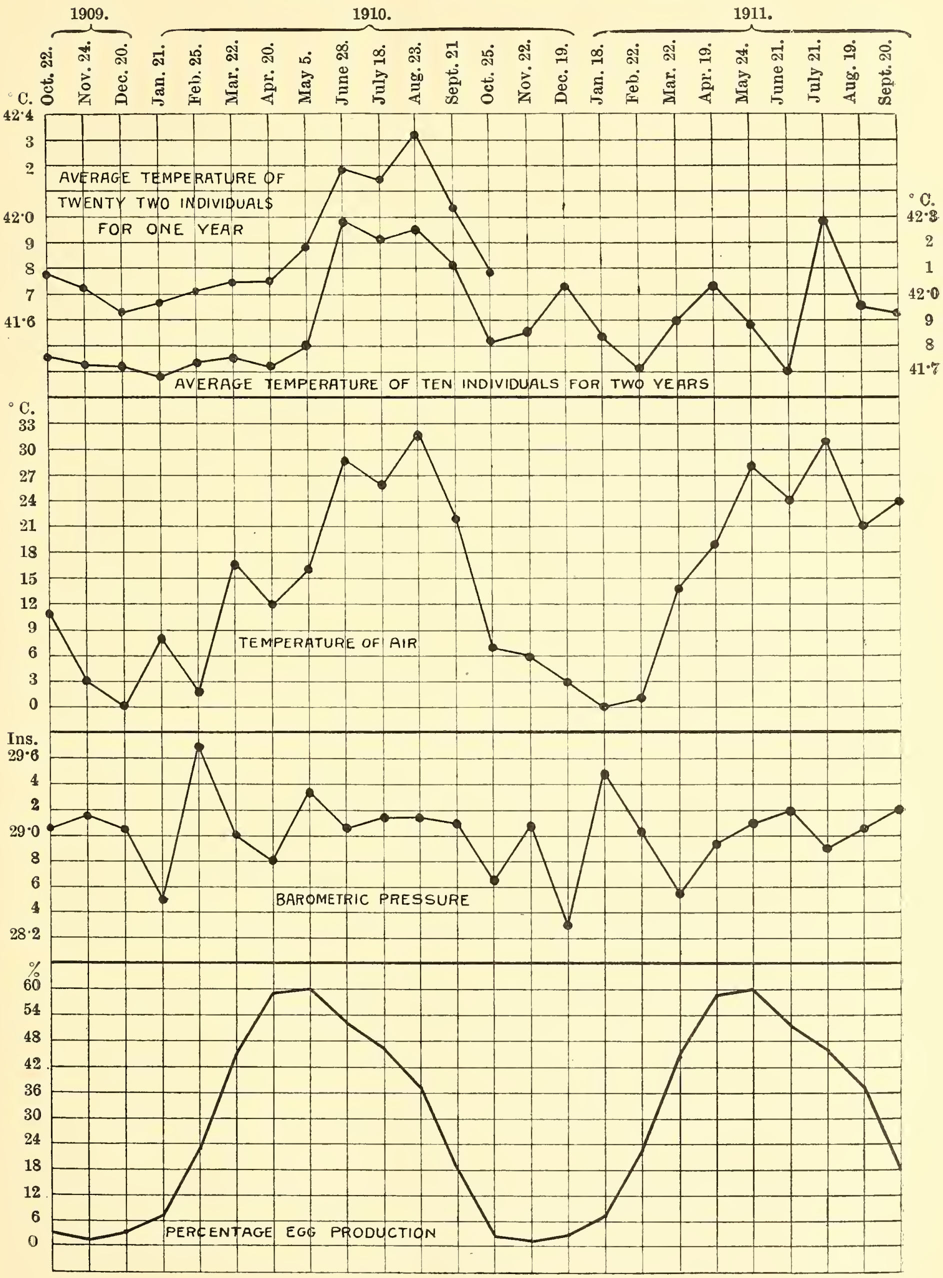


CHART III.—Pen III., White Wyandottes, ♀.

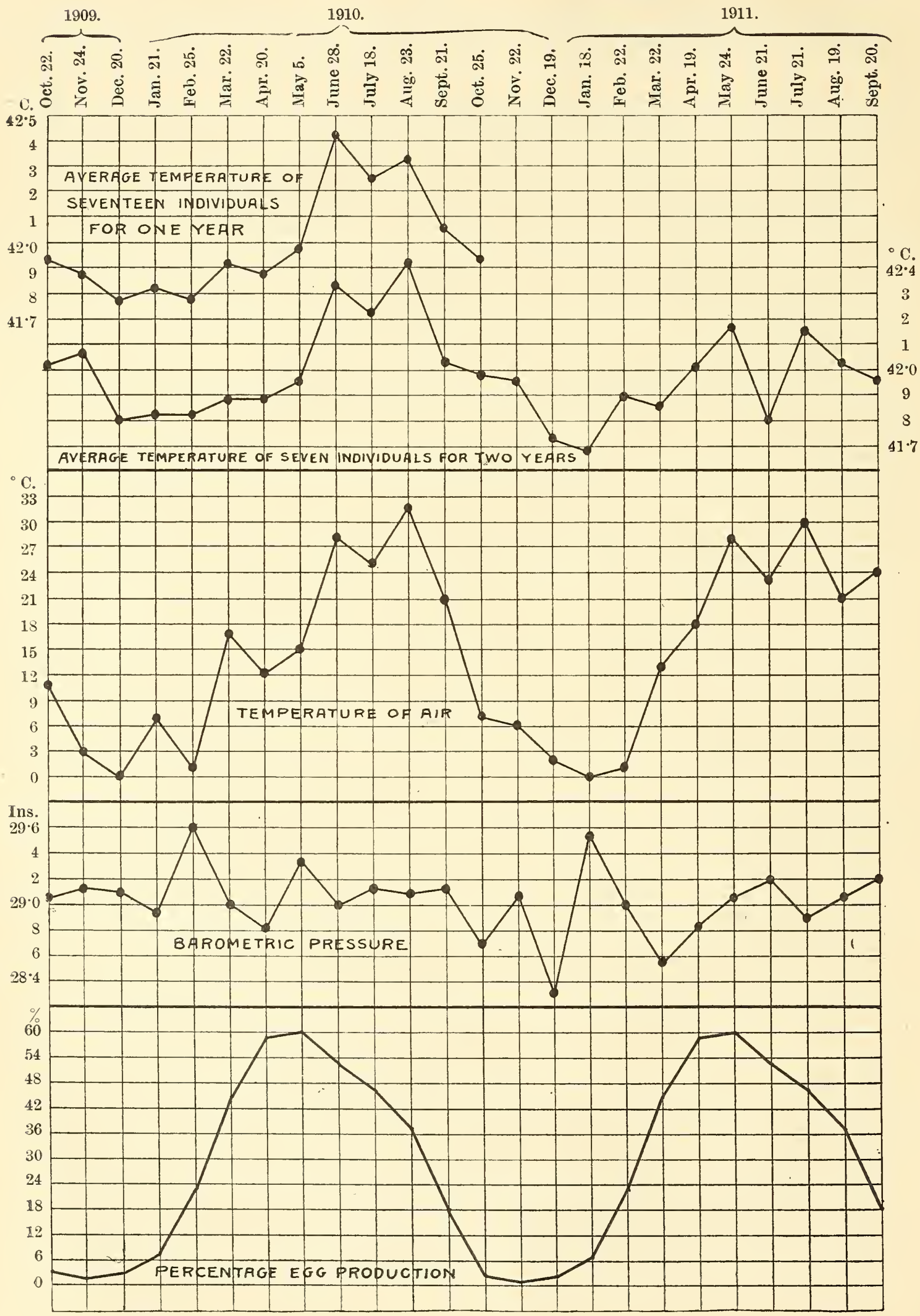


CHART IV.—PEN IV., SINGLE COMB BROWN LEGHORNS, ♀.

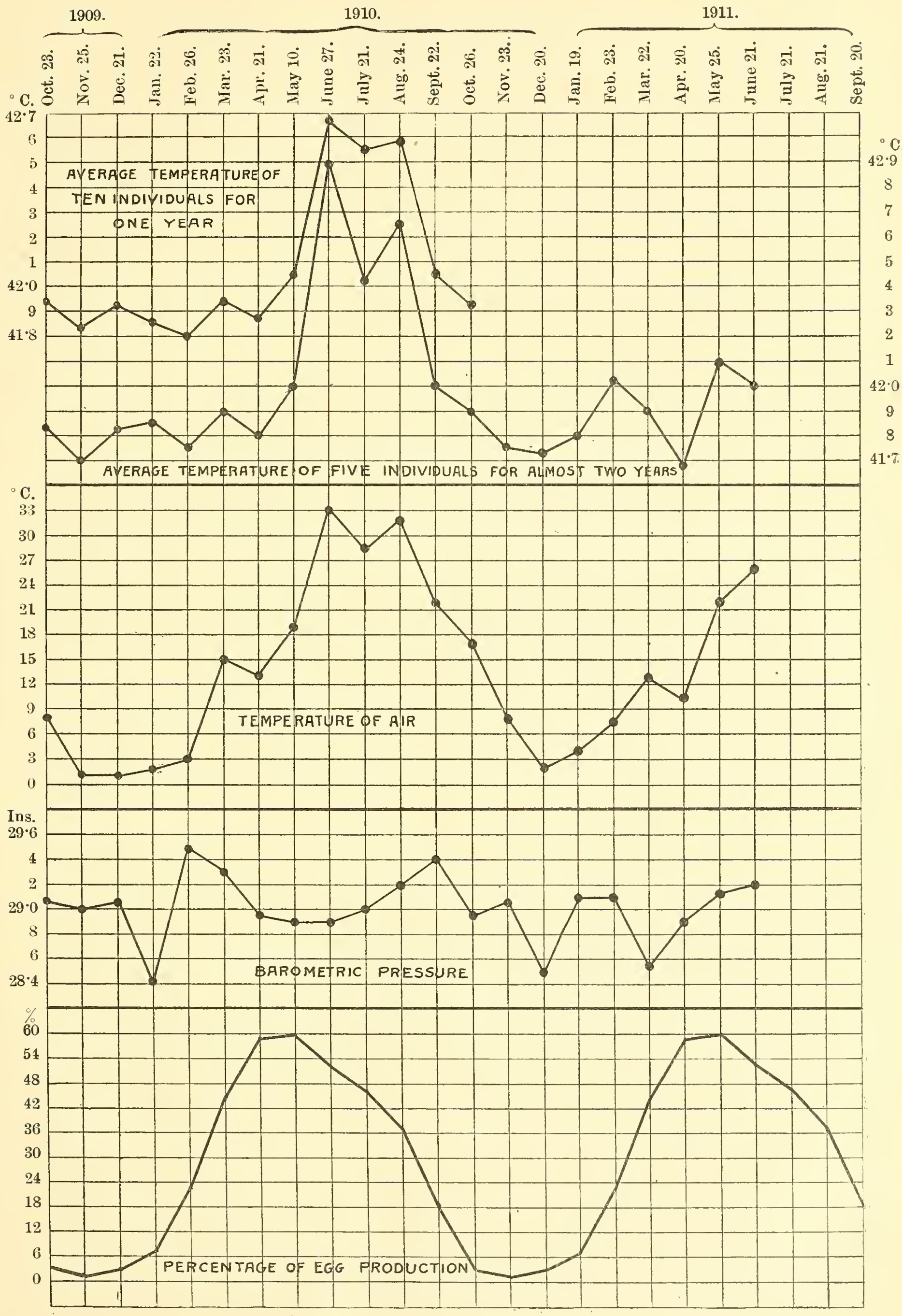


CHART V.—Pen V., White Plymouth Rocks, ♀.

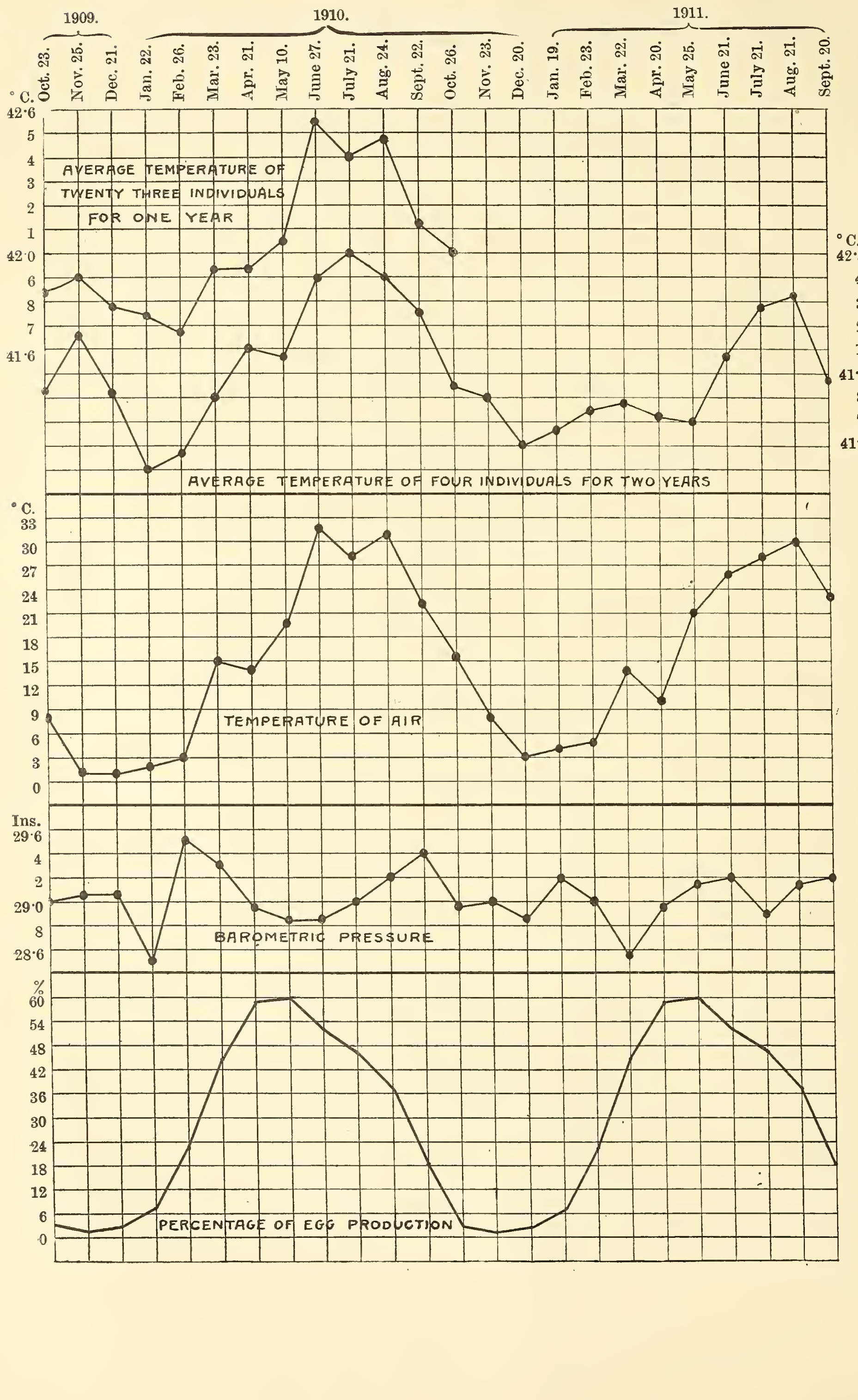


CHART VI.—PEN VI., RHODE ISLAND REDS, ♀.

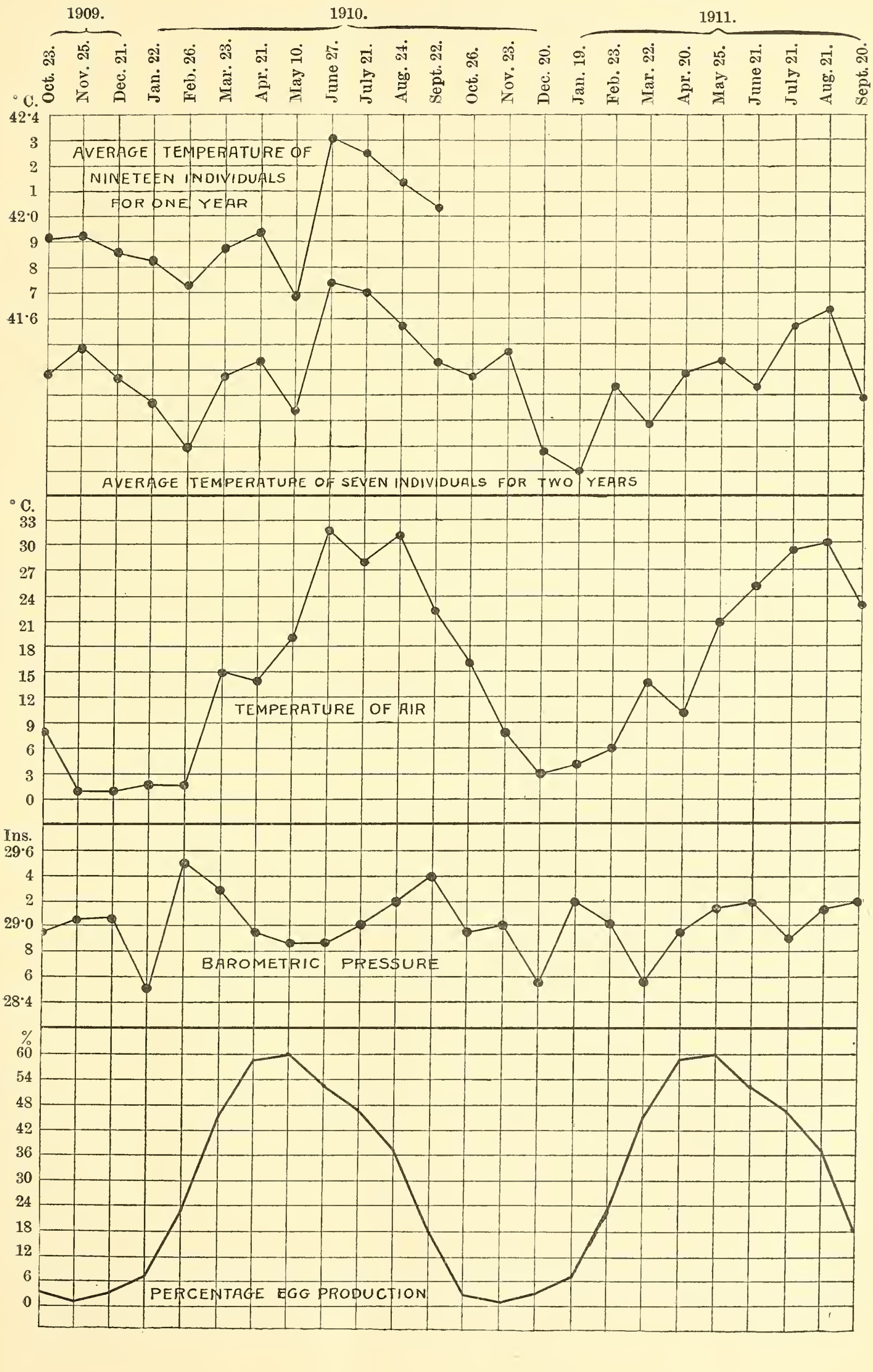


CHART VII.

Shows curves of mean rectal temperature of hens in Pens I., II., and III. Observations made on all on same day between 1.30 and 4.30 p.m.

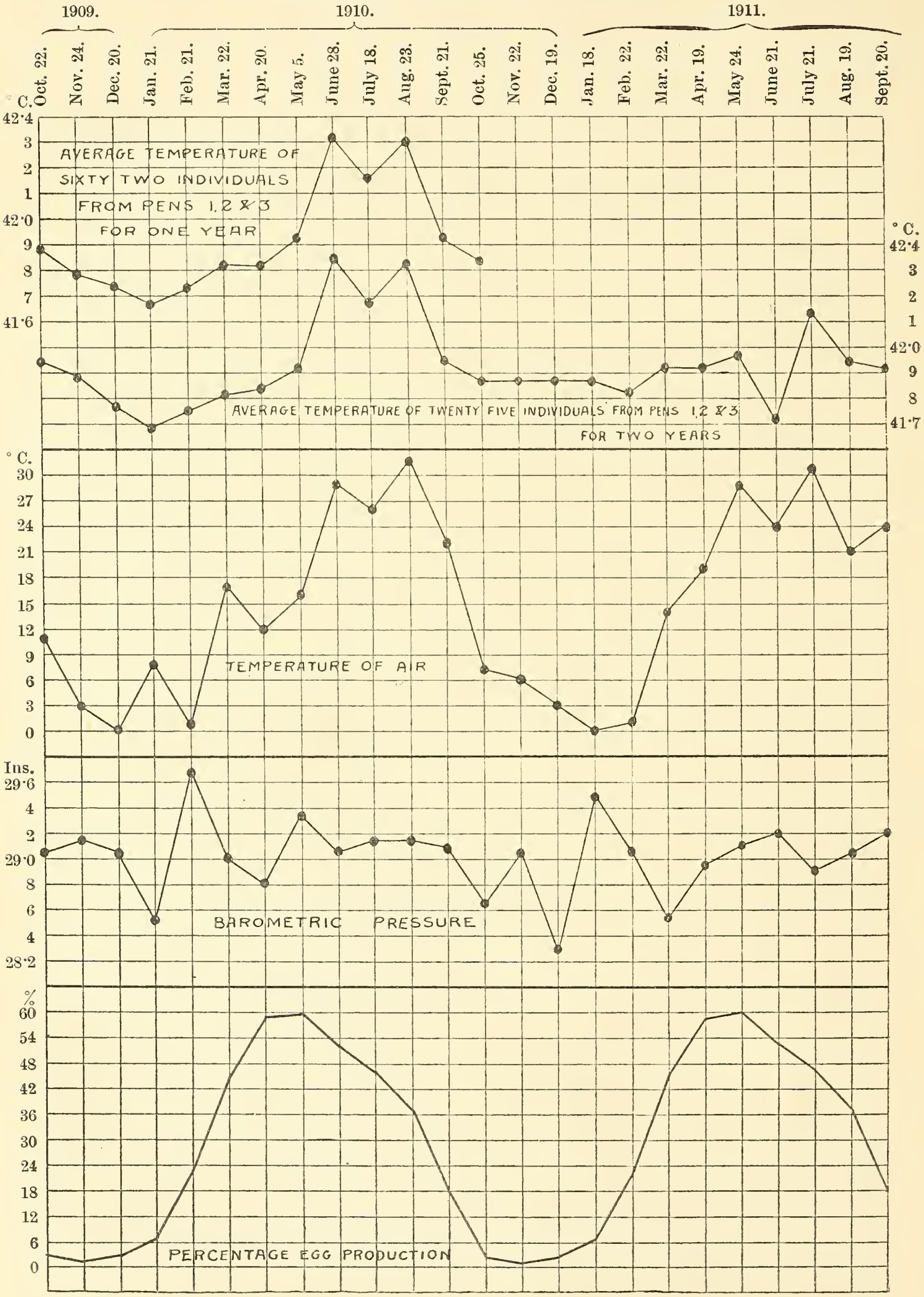
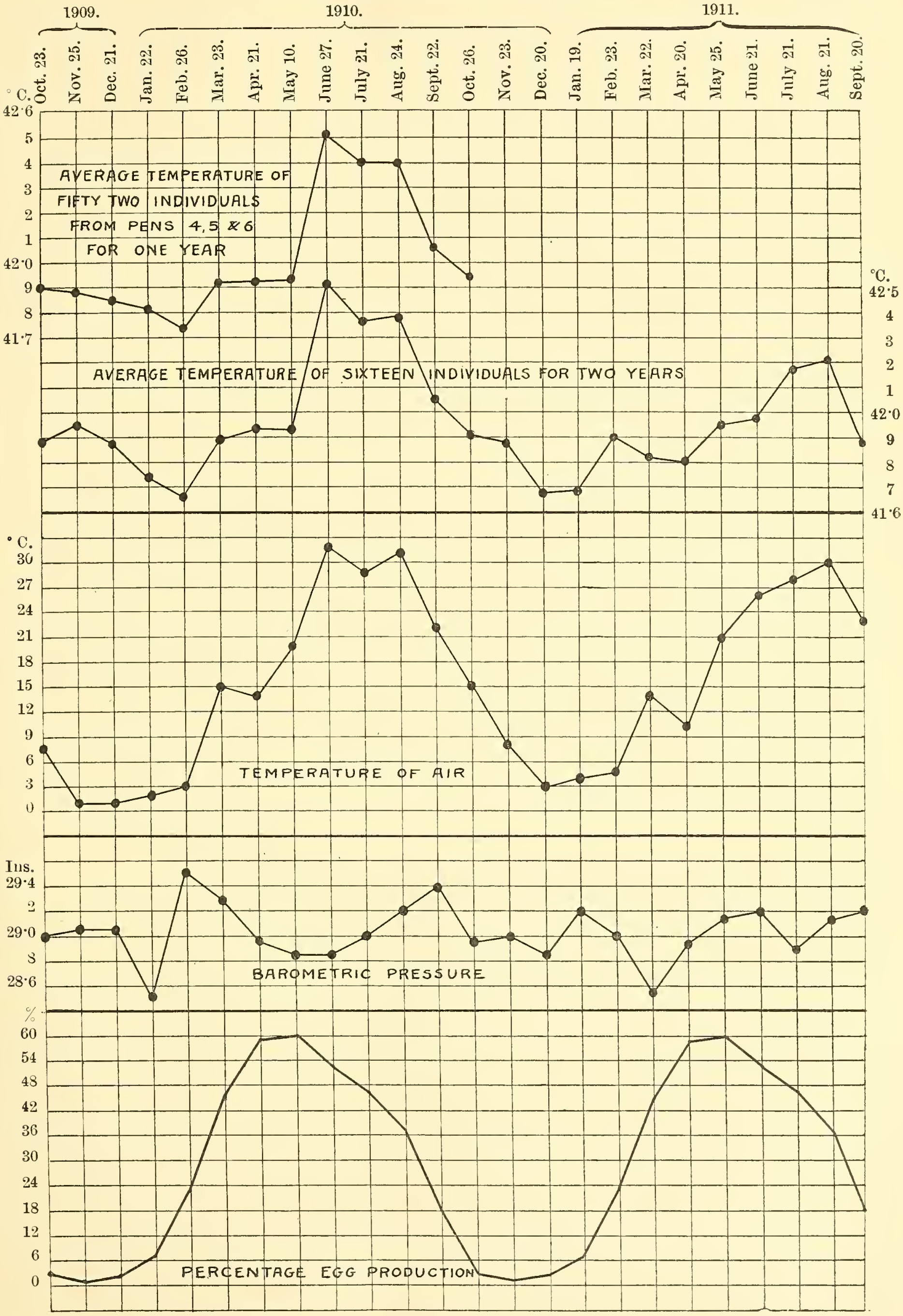


CHART VIII.

Shows curves of mean rectal temperature of hens in Pens IV., V., and VI. Observations made on all on same day between 2 and 4.30 p.m.



Dr Wilson, I was permitted to extract from the records of the Ithaca Station of the United States Weather Bureau, and it was a fortunate circumstance that this station should be located on the top of the Agricultural College Buildings, only about 150 yards distant from the hen pens. The percentage of humidity is not available for the afternoon, only a single daily observation having been made, and that at 8 a.m.

Results and Discussion.

From an examination of the tables and charts it will be found that the body temperature curve, in a general way, rises and falls with the temperature of the air, but the relationship is not a close one, unless when the external temperature is very low or very high. Considering the records of the whole pen for the first year only, the Buff Orpingtons and White Wyandottes have the lowest rectal temperature in December, the Barred Plymouth Rocks in January, the Brown Leghorns, White Plymouth Rocks, and Rhode Island Reds in February; December, January, and February are therefore the months when the body temperature is lowest, as also that of the external air. It would be erroneous to conclude from this, however, that the low external temperature is the only cause of the fall in body temperature, since two other factors come into operation, viz. muscular exercise and sunlight. As mentioned previously, during the winter months the fowls, either from choice or compulsorily, remain indoors for the most part, the ground being covered with snow, and have not the same opportunity for exercise as in the summer months, when they prefer to be outside in the run scratching amongst the dry earth. In November, December, and January the sunshine is also at a minimum, and both these influences may have an effect on the body temperature.

In all cases the highest body temperature is found in June, July, and August, when the weather is at its hottest, and the rise is a very distinct one. Here the important factor is evidently the external temperature, since the hens are under practically the same conditions with regard to exercise from March till November, and by consulting the weather table it will be seen that the hottest days in June, July, and August were to a large extent sunless days. For example, the total sunshine on May 5, and September 21, 1910, was 14·2 and 12·2 hours respectively, as against 8·8 on June 28, and 6·8 on August 23. It was observed on days when the thermometer stood in the neighbourhood of 90° F. that slight exertion was sufficient to cause in the hens distressful symptoms, such as panting with open mouth, and rapid respiration.

When we compare the body temperatures on dates about equal distances on the two sides of the hot summer weather, when all the external conditions are approximately the same, *e.g.* April with October, and May with September, we find that there is practically no difference. The mean for all the birds (114) of which the records were kept for one year was as follows:—April 41·86° F., October 41·90°, May 41·93°, September 41·99°. It would appear, therefore, that a high external temperature has a distinct influence on the body temperature of the hen, and that this is the main factor in raising it in the hot weather which usually prevails in the State of New York in June, July, and August. It must be kept in mind that this applies only to the hours when the observations were made, viz. from 1.30 to 4.30 p.m. when the temperature of all homoiothermal animals attains its maximum. Whether the mean temperature for the twenty-four hour period is affected in the same sense we are unable to say. It is not so clear that the fall in body temperature in the winter months is due to the cold weather experienced in December, January, and February, since other factors (lack of opportunity for much muscular exercise, and diminished sunlight) are in operation at the same time.

In the hen the body temperature does not appear to be affected in any way by the barometric pressure, although it has been stated that a relationship does exist between the two in the human subject. It is difficult to understand, however, how this can be.

A weather condition more likely to influence body temperature would be humidity, but for this it has not been possible to plot a curve, since the daily records were taken at 8 a.m., and from this it could not be estimated with any degree of accuracy what the humidity would be in the afternoon at the hours when the temperature observations were made.

When the curves obtained from the reduced number of hens from which records were collected over two years are examined, it will be found that the second year agrees with the first fairly closely, until we come to the summer months, and here the curve of mean temperature is very irregular and does not rise as it did the former summer, although the external temperature is practically the same. The chief reason for this irregularity and the apparently low temperature is to be found in the fact that in the second summer, when the birds were two years old, a large proportion of them happened to be broody on the days when the temperatures were being taken, and in this condition, as is well known, the body temperature is always low. This reduced the mean for the whole. For example, in Pen VI. (Rhode Island Reds), on June 21, 1911, two out of seven were found sitting in a corner of the room on one or two eggs; these had temperatures

Weather Table for Days on which Observations on Temperature of Hens were made.

	Shade temperature range from 1 to 5 p.m. in ° Fahr.	Barometer range from 1 to 5 p.m. in inches.	Humidity at 8 a.m. (100 = saturation).	Precipitation from Midnight to Midnight in inches.	Snow on Ground at 8 p.m. in inches.	Actual Hours of Sunshine.	Possible Hours of Sunshine.	Percentage of Sunshine.	Prevailing Direction of Wind for 24 hours.	Average hourly Velocity of Wind in miles.
1909. October 22	51–48	29·00–29·05	93	·03	0	0	10·8	0	SW.	10·3
23	43–39	29·05–28·95	77	·21	0	0	10·7	0	NW	5·2
November 24	30–28	29·15–29·10	86	·03	T	·4	9·5	4	NW.	13·0
25	26–27	28·95–29·05	93	·34	4·0	0	9·5	0	NW.	21·6
December 20	20–23	29·10–29·05	72	0	1·5	0	9·1	0	SW.	7·3
21	24–23	29·10–29·05	84	0	1·0	0	9·1	0	SW.	8·2
1910. January 21	43–45	28·65–28·50	80	·39	1·0	0	9·6	0	SE.	19·5
22	33–28	28·35–28·50	83	T	1·0	1·2	9·7	12	SE.	30·1
February 25	21–18	29·65–29·60	100	T	9·5	10·5	11·0	95	E.	5·2
26	29–34	29·55–29·50	66	T	9·5	6·4	11·1	58	SE.	27·0
March 22	62–60	29·05–29·00	75	T	0	2·0	12·2	16	SE.	13·5
23	54–59	29·35–29·30	82	0	0	12·3	12·3	100	NW.	7·0
April 20	51–46	28·80	84	·02	0	4·4	13·6	32	SE.	10·6
21	57–55	28·95	86	·04	0	5·8	13·6	43	SE.	6·7
May 5	52–54	29·35	88	0	0	14·2	14·2	100	NW.	9·2
10	62–63	28·95–28·80	84	·03	0	10·7	14·4	74	NW.	10·3
June 27	89–85	28·95–28·85	64	·01	0	6·5	15·2	43	NW.	10·4
28	82–78	28·95	87	0	0	8·8	15·2	58	NW.	7·0

1910. July	18	72–73	29·15	70	0	0	13·9	14·9	93	NW.	7·2
	21	80–76	29·00	60	T	0	9·3	14·8	63	SW.	7·9
August	23	80–83	29·15–29·20	73	0	0	6·8	13·6	50	S.	13·3
	24	86–81	29·20	93	0	0	6·5	13·5	48	S.	12·3
September	21	73–67	29·10–29·20	68	0	0	12·2	12·2	100	NW.	8·9
	22	60–66	29·4	73	0	0	11·7	12·2	96	NW.	5·5
October	25	44–41	28·65–28·70	83	·70	0	0	10·6	0	SE.	9·3
	26	56–47	28·95	80	0	0	6·5	10·6	61	NW.	6·0
November	22	39–40	29·05–29·10	86	T	0	0	9·6	0	NW.	6·2
	23	44–41	29·20–28·95	89	·05	T	6·5	9·6	68	SE.	12·1
December	19	30–29	28·35–28·30	94	T	3·0	1·6	9·1	18	SW.	9·6
	20	25–22	28·50–28·55	95	·06	3·6	2·7	9·1	30	NW.	7·1
1911. January	18	26–22	29·55–29·50	90	0	0·4	9·6	9·6	100	SE.	9·6
	19	37–30	29·10–29·20	76	T	0	2·3	9·6	24	W.	12·4
February	22	34–27	29·00	64	·18	6·5	2·3	10·9	21	NW.	15·1
	23	35–31	29·15–28·95	60	T	5·8	10·9	11·0	99	SW.	13·6
March	22	58–52	28·55–28·60	40	·03	0	0	12·2	0	NW.	21·3
April	19	64–61	28·90–28·80	80	·06	0	0	13·5	0	SE.	6·1
	20	56–52	29·10–29·05	90	·10	0	0	13·6	0	NW.	9·4
May	24	78–73	29·10–29·05	88	·14	0	4·4	14·8	30	NW.	8·1
	25	66–68	29·15	80	0	0	5·1	14·9	34	NW.	6·5
June	21	72–73	29·20	69	0	0	11·8	15·2	78	NW.	9·4
July	21	80–84	28·95–28·85	74	·06	0	8·9	14·8	60	SW.	5·5
August	19	70–68	29·05	80	0	0	8·3	13·7	61	NW.	13·7
	21	82–80	29·20–29·15	65	0	0	12·8	13·6	94	NW.	5·9
September	20	70–65	29·2	81	0	0	11·6	12·3	94	NW.	6·8

Weather Table for Days on which Observations on Temperature of Hens were made.

	Shade temperature range from 1 to 5 p.m. in ° Fahr.	Barometer range from 1 to 5 p.m. in inches.	Humidity at 8 a.m. (100 = saturation).	Precipitation from Midnight to Midnight in inches.	Snow on Ground at 8 p.m. in inches.	Actual Hours of Sunshine.	Possible Hours of Sunshine.	Percentage of Sunshine.	Prevailing Direction of Wind for 24 hours.	Average hourly Velocity of Wind in miles.
1909. October 22	51–48	29·00–29·05	93	03	0	0	10·8	0	SW.	10·3
23	43–39	29·05–28·95	77	·21	0	0	10·7	0	NW	5·2
November 24	30–28	29·15–29·10	86	·03	T	·4	9·5	4	NW.	13·0
25	26–27	28·95–29·05	93	·34	4·0	0	9·5	0	NW.	21·6
December 20	20–23	29·10–29·05	72	0	1·5	0	9·1	0	SW.	7·3
21	24–23	29·10–29 05	84	0	1·0	0	9·1	0	SW.	8·2
1910. January 21	43–45	28·65–28·50	80	·39	1·0	0	9·6	0	SE.	19·5
22	33–28	28·35–28·50	83	T	1·0	1·2	9·7	12	SE.	30·1
February 25	21–18	29·65–29·60	100	T	9·5	10·5	11·0	95	E.	5·2
26	29–34	29·55–29·50	66	T	9 5	6 4	11·1	58	SE.	27·0
March 22	62–60	29·05–29·00	75	T	0	2·0	12·2	16	SE.	13 5
23	54–59	29·35–29·30	82	0	0	12·3	12·3	100	NW.	7·0
April 20	51–46	28·80	81	·02	0	4·4	13·6	32	SE.	10·6
21	57–55	28·95	86	·04	0	5·8	13·6	43	SE.	6·7
May 5	52–54	29·35	88	0	0	14·2	14·2	100	NW.	9·2
10	62–63	28·95–28·80	84	·03	0	10·7	14·4	74	NW.	10·3
June 27	89–85	28·95–28·85	64	·01	0	6·5	15·2	43	NW.	10·4
28	82–78	28·95	87	0	0	8·8	15·2	58	NW.	7·0
1910. July 18	72–73	29·15	70	0	0	13·9	14·9	93	NW.	7·2
21	80–76	29·00	60	T	0	9·3	14·8	63	SW.	7·9
August 23	80–83	29·15–29·20	73	0	0	6·8	13·6	50	S.	13·3
24	86–81	29·20	93	0	0	6·5	13·5	48	S.	12·3
September 21	73–67	29·10–29·20	68	0	0	12·2	12·2	100	NW.	8·9
22	60–66	29·4	73	0	0	11·7	12·2	96	NW.	5·5
October 25	44–41	28·65–28·70	83	·70	0	0	10·6	0	SE.	9·3
26	56–47	28·95	80	0	0	6·5	10·0	61	NW.	6·0
November 22	39–40	29·05–29·10	86	T	0	0	9 6	0	NW.	6·2
23	44–41	29·20–28·95	89	·05	T	6 5	9·6	68	SE.	12·1
December 19	30–29	28·35–28·30	94	T	3·0	1·6	9·1	18	SW.	9·6
20	25–22	28·50–28·55	95	·06	3·6	2·7	9·1	30	NW.	7·1
1911. January 18	26–22	29·55–29·50	90	0	0·4	9·6	0·6	100	SE.	9·6
19	37–30	29·10–29·20	76	T	0	2·3	9·6	24	W.	12·4
February 22	34–27	29·00	64	·18	6·5	2 3	10·9	21	NW.	15·1
23	35–31	29·15–28·95	60	T	5·8	10·9	11·0	99	SW.	13·6
March 22	58–52	28·55–28·60	40	·03	0	0	12·2	0	NW.	21·3
April 19	64–61	28·90–28·80	80	·06	0	0	13·5	0	SE.	6·1
20	56–52	29·10–29·05	90	·10	0	0	13·6	0	NW.	9·4
May 24	78–73	29·10–29·05	88	·14	0	4·4	14·8	30	NW.	8·1
25	60–68	29·15	80	0	0	5·1	14·9	34	NW.	6·5
June 21	72–73	29·20	69	0	0	11·8	15·2	78	NW.	9·4
July 21	80–84	28·95–28·85	74	·06	0	8·9	14·8	60	SW.	5·5
August 19	70–68	29·05	80	0	0	8·3	13·7	61	NW.	13·7
21	82–80	29·20–29·15	65	0	0	12·8	13·6	94	NW.	5·9
September 20	70–65	29·2	81	0	0	11·6	12·3	94	NW.	6·8

of 41·2° and 41·4°—much below the general average—and this brought the mean figure for the seven down to 41·84°, as compared with 42·24° for the same birds on June 27, 1910. If all the birds in this condition had been excluded, the figures would be practically the same as on the corresponding dates of the previous year. Another cause of the irregularity is the small number of individuals under observation, since the larger the number from which the mean figures are obtained the smaller is the effect of individual and accidental variations on the mean temperature.

We now come to the most important point, and that is a consideration of the effect of the annual curve of egg-production on the body temperature curve. The curve of egg-production may be taken as the vitality curve of the hen, since it is functionally most active when this curve reaches its maximum and least so at its minimum. The curve of egg-production attains its highest point in April and May; the temperature curve in June, July, and August. Here the hot weather of summer is a disturbing influence, since but for this the body temperature curve might have fallen parallel with the egg-production curve, but if we compare the body temperature in May when egg-production is at its highest, with that in October when it is almost at its lowest, we find that the two figures are nearly identical. The mean for 114 hens in May is 41·93°, and for October 41·90° and on the days in these months (1910) on which the hens were examined the weather conditions were fairly comparable,—if anything, both as regards sunlight and external temperature, they were in favour of May (see weather table, pp. 132–133).

It would thus appear that in the domestic fowl the annual cyclical change represented by the egg-production or vitality curve does not affect appreciably the body temperature, and that, if the effect of muscular exercise be excluded, its variations are due to influences acting on the body from without rather than from within.

The upper and lower limits, and the annual temperature range for the different breeds examined, are as follows, counting all the birds on which observations were made for one year:—

Rhode Island Reds	42°·31 – 41°·69 = 0°·62	range
White Wyandottes	42·43 – 41·77 = 0·66	,,
Buff Orpingtons	42·33 – 41·63 = 0·70	,,
Barred Plymouth Rocks	42·30 – 41·53 = 0·77	,,
Brown Leghorns	42·67 – 41·80 = 0·87	,,
White Plymouth Rocks	42·55 – 41·67 = 0·88	,,

Although there is little difference amongst the six varieties, it may be stated, in a general way, that the heaviest and most lethargic birds have the lowest temperature, and the smallest and most active birds the highest. Compare in this relation the Barred Plymouth Rocks with the Brown Leghorns.

Summary.

Monthly observations, extending over one year, were made on the rectal temperature of 114 domestic fowls (Gallus gallus, ♀) and records from forty-one of these were obtained for two years. Six different breeds were used, each located in a separate pen, all under similar conditions, and the mean temperatures for each group were plotted out to form an annual temperature curve. It was found that—

1. The lowest temperatures occur in December, January, and February, and the highest in June, July, and August, corresponding in a general way with the temperature of the external air.

2. Barometric pressure does not appear to have any influence on the body temperature of the hen.

3. The curve of egg-production does not coincide with the annual temperature curve, the former reaching its highest level in April and May, the latter in June, July, and August.

If we compare the mean rectal temperature at two periods of the year when the external or weather conditions are approximately the same (April–May and September–October), but when the vitality of the birds, as indicated by the curve of egg-production, moulting, etc., is at a maximum and minimum respectively, we find that the figures are practically identical. This would seem to show that cyclical bodily changes have little effect on body temperature as compared with outside influences.

In conclusion I should like to express my sense of indebtedness to Professor Rice for granting me permission to carry on the work in his department, and to Mr W. G. Krum for assistance while it was in progress.

(*Issued separately April* 29, 1912.)

XII.—The Theory of Circulants from 1861 to 1880. By Thomas Muir, LL.D.

(MS. received September 23, 1911. Read November 20, 1911.)

Roberts, M. (1861).

[Question 581. *Nouv. Annales de Math.*, xx. p. 139. Solution by E. Beltrami in (2) iii. (1864 February) pp. 64–66.]

Roberts' theorem concerns the circulant C whose elements are the terms of the expansion of

$$\frac{(t+1)^n-1}{t},$$

and is to the effect (1) that there are no odd powers of t in the development of the circulant, and (2) that if in the said development ξ be put for t^2, the equation in ξ

$$C_{t^2=\xi} = 0$$

has for its roots the squares of the differences of the roots of the equation

$$x^n-1=0.$$

If $\omega_1, \omega_2, \ldots$ be the n^{th} roots of 1 we have identically

$$(t-\omega_1)(t-\omega_2)\ldots(t-\omega_n) = t^n-1$$

and therefore also

$$\left\{t-(\omega_1-\omega_r)\right\}\left\{t-(\omega_2-\omega_r)\right\}\ldots\left\{t-(\omega_n-\omega_r)\right\}\equiv(t+\omega_r)^n-1,$$

where the r^{th} factor on the left is simply t itself. Hence the expression

$$\frac{(t+\omega_1)^n-1}{t}\cdot\frac{(t+\omega_2)^n-1}{t}\cdots\frac{(t+\omega_n)^n-1}{t}$$

consists of $n(n-1)$ factors which if suitably combined in pairs are replaceable by $\frac{1}{2}n(n-1)$ factors of the form $t^2-(\omega_r-\omega_s)^2$. But the said expression being equal to C by Spottiswoode's theorem (which, however, Beltrami does not assume *) the desired result at once follows.

* For his mode of proof see under Baltzer (1864).

Zehfuss, G. (1862).

[Anwendungen einer besonderen Determinante. *Zeitschrift f. Math. u. Phys.*, vii. pp. 439–445.]

Zehfuss proves Spottiswoode's result by multiplying the rows in order by $\theta_r^n, \theta_r^{n-1}, \ldots, \theta_r$ respectively, and the columns in order by $1, \theta_r, \theta_r^2, \ldots, \theta_r^{n-1}$, a procedure which amounts to multiplying the determinant by $\theta_r^n(1 \cdot \theta_r \cdot \theta_r^2 \ldots \theta_r^{n-1})^2$, that is, by 1. Addition of the rows is then all that is wanted to reach the desired result.

The rest of the article (pp. 441–445) is devoted to the "Anwendungen," namely, (1) to Eisenstein's expression

$$x^3 + \mathrm{DD}'y^3 + \mathrm{DD}''z^3 - 3\mathrm{D}xyz,$$

where the letters denote complex numbers and $\mathrm{D} = \mathrm{D}'\mathrm{D}''$; and (2) to a letter of Jacobi's on cyclotomy and the theory of integers.

Baltzer, R. (1864).

[Theorie und Anwendung der Determinanten. . . . 2[te] vermehrte Aufl. viii + 224 pp., Leipzig.]

With Baltzer (§ 11, 1, 2, 3) the determinant

$$\begin{vmatrix} a_0 - y & a_1 & a_2 & \ldots & a_{n-1} \\ a_{n-1} & a_0 - y & a_1 & \ldots & a_{n-2} \\ a_{n-} & a_{n-1} & a_0 - y & \ldots & a_{n-3} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_1 & a_2 & a_3 & \ldots & a_0 - y \end{vmatrix},$$

or
$$\mathrm{C}(a_0 - y, a_1, a_2, \ldots, a_{n-1}) \text{ say,—}$$

which, of course, is not more general than

$$\mathrm{C}(a_0, a_1, a_2, \ldots, a_{n-1}),$$

—is openly reached (p. 92) by eliminating x dialytically * from the equations

$$y = a_0 + a_1x + a_2x^2 + \ldots + a_{n-1}x^{n-1} \quad \text{and} \quad 1 = x^n.$$

* Namely, by using on the first equation the multipliers $x, x^2, \ldots, x_{n-1}$, and substituting 1 for x^n wherever the latter turns up, exactly as Beltrami did. The same result, however, is reached by following Bezout's "abridged method." On the other hand, the application of Sylvester's dialytic method unmodified entails, as we know, the performance of multiplication on *both* equations, and gives an eliminant of a higher order. For example, in the case of $n = 3$ it gives

$$\begin{vmatrix} a_2 & a_1 & a_0 & \cdot & \cdot \\ \cdot & a_2 & a_1 & a_0 & \cdot \\ \cdot & \cdot & a_2 & a_1 & a_0 \\ 1 & \cdot & \cdot & -1 & \cdot \\ \cdot & 1 & \cdot & \cdot & -1 \end{vmatrix},$$

where we have to increase the 4th column by the 1st, and the 5th by the 2nd, before we can reach $\mathrm{C}(a_0, a_2, a_1)$.

He does not, however, note, as Beltrami did in a special case early in the same year, that by using another method of elimination, namely, Euler's of 1748, the eliminant is found to be

$$\prod_{r=1}^{r=n}\left(a_0 - y + a_1\theta_r + a_2\theta_r^2 + \ldots + a_{n-1}\theta_r^{n-1}\right)$$

where θ_r is an n^{th} root of 1: and thus he fails to bring out the fact that Spottiswoode's result of 1853 is nothing more nor less than the statement of the equality of those two eliminants.* Instead of this he performs the operation

$$\text{col}_1 + \theta_r\text{col}_2 + \theta_r^2\text{col}_3 + \ldots + \theta_r^{n-1}\text{col}_n$$

and so arrives at a practically equivalent result, namely, that the equation

$$C(a_0 - y, a_1, a_2, \ldots, a_{n-1}) = 0$$

is satisfied by putting

$$y = a_0 + a_1\theta_r + a_2\theta_r^2 + \ldots + a_{n-1}\theta_r^{n-1}.$$

Two other points worth noting are (1) his calling $C(a_0, a_1, a_2, \ldots, a_{n-1})$ the *norm* of $a_0 + a_1\theta_r + a_2\theta_r^2 + \ldots + a_{n-1}\theta_r^{n-1}$, in accordance with an extension of a usage of Gauss', and (2) his statement that of the n^n terms got by working out the product

$$\prod_{r=1}^{r=n}\left(a_0 + a_1\theta_r + a_2\theta_r^2 + \ldots + a_{n-1}\theta_r^{n-1}\right)$$

only the $1.2.3\ldots n$ terms of the determinant remain. In regard to the former it has to be remarked that θ_r must then be restricted to stand for a *primitive* n^{th} root of 1, and in regard to the latter that even some of the $1.2.3\ldots n$ terms of the determinant do not remain.

Baltzer, R. (1870).

[Theorie und Anwendung der Determinanten . . . . 3[te] verbesserte Aufl. viii + 242 pp., Leipzig.]

In addition to a few changes in phraseology this edition contains the fresh theorem that *in every term of* $C(a_0, a_1, a_2, \ldots, a_{n-1})$ *the sum of the suffixes is divisible by* n. The proof is based on the fact that the suffix of any element (i, k) is either $-i+k$ or $n-i+k$ according as i is less † or greater than k; for from this it follows that in the case of any term

$$(1, r)(2, s)(3, t) \ldots\ldots$$

* See *History*, ii. pp. 369–370.

† He means "not greater."

the sum of the suffixes differs from a multiple of n by

$$-1-2-3-\ \dots\ +r+s+t+\ \dots\ ,$$

that is by 0.

STERN, M. A. (1871).

[Einige Bemerkungen über eine Determinante. *Crelle's Journ.*, lxxiii. pp. 374–380.]

Stern opens with what he means to be merely a fresh proof of Spottiswoode's result: what he actually obtains, however, is something more important, namely, not merely the establishment of the fact that

$$a_1 + a_2\theta_r + a_3\theta_r^2 + \ \dots\ + a_n\theta_r^{n-1}$$

is a factor of

$$C(a_1, a_2, \dots, a_n),$$

but the further fact that the cofactor is

$$A_1\theta_r^n + A_2\theta_r^{n-1} + A_3\theta_r^{n-2} + \ \dots\ + A_n\theta_r,$$

where $A_1, A_2, \dots$ are the signed complementary minors of the elements of the first row of C. By way of proof he notes that if the multiplication of these two factors be performed, the coefficient of θ_r^n in the product is

$$A_1a_1 + A_2a_2 + \ \dots\ + A_na_n, \quad i.e.\quad C;$$

and the coefficient of any other power of θ_r being equal to the product of $A_1, A_2, \dots, A_n$ by a row of C other than the first must vanish. The proof is thus seen to be based on the identities

$$\left.\begin{array}{lcl} a_1A_1 + a_2A_2 + \ \dots\ + a_nA_n & = & C \\ a_nA_1 + a_1A_2 + \ \dots\ + a_{n-1}A_n & = & 0 \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot & & \\ a_2A_1 + a_3A_2 + \ \dots\ + a_1A_n & = & 0 \end{array}\right\}$$

which Stern also uses to obtain by means of addition the special case

$$\begin{aligned} (a_1 + a_2 + \ \dots\ + a_n)(A_1 + A_2 + \ \dots\ + A_n) &= C, \\ \text{or, say,}\quad \Sigma a\,.\,\Sigma A &= C. \end{aligned}$$

Differentiating, he next obtains from this, with the help of a result of Cremona's,

$$\left.\begin{array}{l} \Sigma A + \Sigma a\,.\,\dfrac{\partial\Sigma A}{\partial a_h} = nA_h, \\ \Sigma A + \Sigma a\,.\,\dfrac{\partial\Sigma A}{\partial a_\kappa} = nA_\kappa, \end{array}\right\}$$

$$\text{and } \therefore\quad \Sigma a\,.\left(\frac{\partial\Sigma A}{\partial a_h} - \frac{\partial\Sigma A}{\partial a_\kappa}\right) = n(A_h - A_\kappa),$$

from which he concludes that

$$\mathrm{A}_h - \mathrm{A}_\kappa \text{ contains } \Sigma a \text{ as a factor.}$$

Thenceforward he specialises. Taking the case where n is prime, and where, therefore, the n roots are $1, \theta, \theta^2, \ldots, \theta^{n-1}$, he denotes the product of all Spottiswoode's factors except

$$a_1 + a_2 + \ldots + a_n \qquad \text{and} \qquad \theta^n a_1 + \theta^{n-1} a_2 + \ldots + \theta a_n$$

by

$$b_1 + b_2\theta + b_3\theta^2 + \ldots + b_n\theta^{n-1}$$

and establishes the result

$$\mathrm{C} = \Sigma a \,.\, \frac{n\Sigma ab - (\Sigma a)^{n-1}}{n-1},$$

where Σab stands for $a_1b_1 + a_2b_2 + \ldots + a_nb_n$. The expressing of the b's in terms of the a's is not attempted. Further specialisation affects the a's, the investigation (pp. 377–380) becoming more intimately associated with the theory of integers.

Glaisher, J. W. L. (1872).

[On functions with recurring derivatives. *Proceedings London Math. Soc.*, iv. pp. 113–116.]

The functions referred to are

$$1 + \frac{x^n}{n\,!} + \frac{x^{2n}}{(2n)\,!} + \ldots .$$

$$x + \frac{x^{n+1}}{(n+1)\,!} + \frac{x^{2n+1}}{(2n+1)\,!} + \ldots .$$

$$\cdots\cdots\cdots\cdots$$

$$\frac{x^{n-1}}{(n-1)\,!} + \frac{x^{2n-1}}{(2n-1)\,!} + \frac{x^{3n-1}}{(3n-1)\,!} + \ldots .$$

which are readily seen to satisfy the differential equation

$$\frac{d^n u}{dx^n} = u,$$

the derivate of the first function being the last, the derivate of the second being the first, and so on. Denoting them by $\phi_0, \phi_1, \phi_2, \ldots, \phi_{n-1}$, Glaisher of course obtains at once

$$\left.\begin{array}{rcl} \phi_0 + \phi_1 + \phi_2 + \ldots + \phi_{n-1} &=& e^x \\ \phi_0 + \theta_1\phi_1 + \theta_1^2\phi_2 + \ldots + \theta_1^{n-1}\phi_{n-1} &=& e^{\theta_1 x} \\ \cdots\cdots\cdots\cdots & & \\ \phi_0 + \theta_{n-1}\phi_1 + \theta_{n-1}^2\phi_2 + \ldots + \theta_{n-1}^{n-1}\phi_{n-1} &=& e^{\theta_{n-1} x} \end{array}\right\}$$

where $1, \theta_1, \theta_2, \ldots, \theta_{n-1}$ are the n^{th} roots of 1, and thence by multiplication and the use of Spottiswoode's theorem

$$\begin{vmatrix} \phi_0 & \phi_1 & \phi_2 & \cdots & \phi_{n-1} \\ \phi_{n-1} & \phi_0 & \phi_1 & \cdots & \phi_{n-2} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \phi_1 & \phi_2 & \phi_3 & \cdots & \phi_0 \end{vmatrix} = e^{x(1+\theta_1+\cdots+\theta_{n-1})} = e^0 = 1.$$

Similarly $\psi_0, \psi_1, \psi_2, \ldots, \psi_{n-1}$ being the functions got from the ϕ's by making the even-numbered terms of the latter negative, and therefore being solutions of the equation

$$\frac{d^n u}{dx^n} = -u,$$

it is found that

$$\begin{vmatrix} \psi_0 & \psi_1 & \psi_2 & \cdots & \psi_{n-1} \\ -\psi_{n-1} & \psi_0 & \psi_1 & \cdots & \psi_{n-2} \\ -\psi_{n-2} & -\psi_{n-1} & \psi_0 & \cdots & \psi_{n-3} \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ -\psi_1 & -\psi_2 & -\psi_3 & \cdots & \psi_0 \end{vmatrix} = 1.$$

It is properly pointed out that the case of the ϕ's where $n=3$ had been obtained in 1827 by Louis Olivier (see *Crelle's Journ.*, ii. pp. 243–251).

Günther, S. (1875).

[Lehrbuch der Determinanten - Theorie, ... viii + 236 pp., Erlangen.]

Günther devotes two pages (§ 11, pp. 93-95) to the subject, but they contain nothing fresh save the proposal to call the determinant "doppelt-orthosymmetrisch," a quite unsuitable name which accurately describes a very different special form.*

* A determinant which is doubly orthosymmetric can have only two different elements, and must have all its odd-numbered columns identical, and all its even-numbered columns identical. This is readily seen on starting with the first two elements and then carrying out the requirements of double orthosymmetry. For example

$$\begin{vmatrix} a & b & a & b \\ b & a & b & a \\ a & b & a & b \\ b & a & b & a \end{vmatrix}.$$

Baltzer, R. (1875).

[Theorie und Anwendung der Determinanten. . . . 4te verbesserte Aufl. . . . . viii+247 pp., Leipzig.]

The only fresh matter in this edition is the statement that in the case of the evanescent determinant

$$C(a_0 - \phi, a_1, a_2, \ldots, a_{n-1})$$

where

$$\phi = a_0 + a_1\theta + a_2\theta^2 + \ldots + a_{n-1}\theta^{n-1}$$

the signed complementary minors of the elements of any row form an equirational progression whose constant multiplier is θ. In illustration of this we may add that the adjugate of

$$C(-b\theta - c\theta^2, b, c),$$

if we write M for $b^2\theta^2 + bc + c^2\theta$, is

$$\begin{vmatrix} M & \theta M & \theta^2 M \\ \theta M & \theta^2 M & M \\ \theta^2 M & M & \theta M \end{vmatrix} \quad i.e. \quad M^3 \begin{vmatrix} 1 & \theta & \theta^2 \\ \theta & \theta^2 & 1 \\ \theta^2 & 1 & \theta \end{vmatrix} \quad i.e. \quad 0.$$

Nicodemi, R. (1877).

[Intorno ad alcune funzioni piu generali delle funzioni iperboliche. *Giornale di Mat.*, xv. pp. 193–234.]

The section which concerns determinants (pp. 205–210) contains an already known proof of Spottiswoode's theorem, this theorem being thereupon used, as Glaisher had already done (1872), to obtain a generalisation of

$$\begin{vmatrix} \cosh x & \sinh x \\ \sinh x & \cosh x \end{vmatrix} = 1.$$

Scott, R. F. (1878).

[On some theorems in determinants. *Messenger of Math.*, viii. pp. 33–37.]

In § 2 of this paper there are five fresh results, the first two being, if $\lambda = \log C(a_1, a_2, \ldots a_n)$,

$$C(a_1, a_2, \ldots, a_n) \times C\left(\frac{\partial\lambda}{\partial a_1}, \frac{\partial\lambda}{\partial a_2}, \ldots, \frac{\partial\lambda}{\partial a_n}\right) = n^n,$$

$$\text{and Hessian of } \lambda = (-1)^{\frac{1}{2}n(n-1)} \cdot n^n.$$

In establishing these, the requisite differentiations are performed on the linear factors of the circulant. The next two are merely stated, namely,

$$C(1^2, 2^2, \ldots, n^2) = (-1)^{n-1} \cdot \frac{(n+1)(2n+1)}{12} \cdot n^{n-2} \cdot \left\{(n+2)^n - n^n\right\},$$

$$C\left(1, n-1, \tfrac{1}{2}(n-1)(n-2), \ldots, 1\right) = \begin{cases} 2^{n-1} & \text{when } n \text{ is odd.} \\ 0 & \text{when } n \text{ is even.} \end{cases}$$

The last gives the evaluation of a circulant in which the elements are the cosines (or sines) of the angles $a, a+b, a+2b, \ldots, a+(n-1)b$, namely,

$$\frac{\{\cos a - \cos(a+nb)\}^n - \{\cos(a-b) - \cos(a-\overline{n-1}\,.\,b)}{2(1-\cos nb)}.$$

This is reached by using the exponential expression for the cosine.

Glaisher, J. W. L. (1877–78).

[Sur un déterminant. *Assoc. franç. pour l'avancem. des sci.*, vi. pp. 177–179.]

[On the values of a class of determinants. *Report . . . British Assoc. . . .* xlvii. p. 20.]

[On the factors of a special form of determinant. *Quart. Journ. of Math.*, xv. pp. 347–356.]

As has been already pointed out, the annexing of $-x$ to the diagonal elements of the circulant $C(a_1, a_2, \ldots, a_n)$, does not alter the determinant as regards generality. If, however, the order of the last $n-1$ rows be reversed, thus producing a determinant equal to $(-1)^{\frac{1}{2}(n-1)(n-2)}C$ and symmetric with respect to the primary diagonal, the annexing of $-x$ to the elements of the said diagonal produces a determinant requiring fresh investigation. This requirement Glaisher supplies.

Having found that

$$\begin{vmatrix} a-x & b & c \\ b & c-x & a \\ c & a & b-x \end{vmatrix} = -\{x-(a+b+c)\} \cdot \left\{ \begin{array}{l} x^2-(a+\omega b+\omega^2 c) \\ \quad\cdot(a+\omega^2 b+\omega c) \end{array} \right\},$$

$$\begin{vmatrix} a-x & b & c & d \\ b & c-x & d & a \\ c & d & a-x & b \\ d & a & b & c-x \end{vmatrix} = \{x-(a+b+c+d)\} \cdot \{x-(a-b+c-d)\} \cdot \left\{ \begin{array}{l} x^2-(a+bi+ci^2+di^3) \\ \quad\cdot(a-bi+ci^2-di^3) \end{array} \right\},$$

and that the cofactor of $x-(a+b+c+d+e)$ in the next case was a quadratic in x^2, he surmised the existence of a general proposition including the three, and stated his surmise at the meetings of the two Associations mentioned above. The immediate result was that Professor J. C. Adams formally established the anticipated identity. Taking a set of equations whose eliminant is the determinant in question, for example

$$\left.\begin{aligned}
\kappa x &= \kappa a + \lambda b + \mu c + \nu d + \xi e\,, \\
\lambda x &= \kappa b + \lambda c + \mu d + \nu e + \xi a\,, \\
\mu x &= \kappa c + \lambda d + \mu \epsilon + \nu a + \xi b\,, \\
\nu x &= \kappa d + \lambda e + \mu a + \nu b + \xi c\,, \\
\xi x &= \kappa e + \lambda a + \mu b + \nu c + \xi d\,,
\end{aligned}\right\}$$

Adams used on them the multipliers $1, \theta, \theta^2, \theta^3, \theta^4$ (θ being an imaginary fifth root of 1), thus obtaining by addition

$$x(\kappa + \theta\lambda + \theta^2\mu + \theta^3\nu + \theta^4\xi) = (a + \theta b + \theta^2 c + \theta^3 d + \theta^4 e)(\kappa + \theta^{-1}\lambda + \theta^{-2}\mu + \theta^{-3}\nu + \theta^{-4}\xi).$$

Similarly he obtained

$$x(\kappa + \theta^{-1}\lambda + \theta^{-2}\mu + \theta^{-3}\nu + \theta^{-4}\xi) = (a + \theta^{-1}b + \theta^{-2}c + \theta^{-3}d + \theta^{-4}e)(\kappa + \theta\lambda + \theta^2\mu + \theta^3\nu + \theta^4\xi),$$

and then from the two by multiplication

$$x^2 = (a + \theta b + \theta^2 c + \theta^3 d + \theta^4 e)(a + \theta^{-1}b + \theta^{-2}c + \theta^{-3}d + \theta^{-4}e).$$

This last being free of $\kappa, \lambda, \mu, \nu, \xi$ it followed that

$$\begin{aligned}
x^2 - (a + \theta b + \theta^2 c + \theta^3 d + \theta^4 e)& \\
\cdot (a + \theta^{-1}b + \theta^{-2}c + \theta^{-3}d + \theta^{-4}e)&
\end{aligned}$$

was a factor of the determinant with which he started, and therefore that the said determinant was equal to

$$\begin{aligned}
&- \{x \ - (a + b + c + d + e)\} \\
&\cdot \left\{\begin{aligned} x^2 - (a + b\theta + c\theta^2 + d\theta^3 + e\theta^4) \\ .\,(a + b\theta^4 + c\theta^3 + d\theta^2 + e\theta) \end{aligned}\right\} \\
&\cdot \left\{\begin{aligned} x^2 - (a + b\theta^2 + c\theta^4 + d\theta + e\theta^3) \\ .\,(a + b\theta^3 + c\theta + d\theta^4 + e\theta^2) \end{aligned}\right\}.
\end{aligned}$$

Glaisher himself properly points out that since θ is of the form $\cos \frac{2}{5}m\pi + i \sin \frac{2}{5}m\pi$, Adams' quadratic factor must be of the form

$$x^2 - (\mathrm{A} + \mathrm{B}i)(\mathrm{A} - \mathrm{B}i) \qquad i.e. \quad x^2 - (\mathrm{A}^2 + \mathrm{B}^2),$$

a result which is in accordance with the fact of the reality of the roots of Lagrange's determinantal equation.

Glaisher's theorem, it should also be noted, is another instance of the assertion of the equivalence of two different forms of an eliminant.

Three results of computation made in the course of the paper are also worth recording, but in a shorter notation, namely,

$$C(a, b, c, d, e) = \sum^{0} a^5 - 5\sum^{0} a^3(be+cd) + 5\sum^{0} a(b^2e^2+c^2d^2) - 5abcde,$$

$$\frac{C(a, b, c, d, e)}{a+b+c+d+e} = \sum^{0} a^4 - \sum^{0} a^3(b+c+d+e) + \sum^{0} a^2(b^2+c^2)$$
$$+ \sum^{0} a^2(2bc+2bd-3be-3cd+2ce+2de) - \sum^{0} abcd,$$

and

$$\begin{vmatrix} 1 & b & c & d & e \\ 1 & c-x & d & e & a \\ 1 & d & e-x & a & b \\ 1 & e & a & b-x & c \\ 1 & a & b & c & d-x \end{vmatrix} = x^4 - x^2(2\sum^{0} a^2 - \sum^{0} ab - \sum^{0} ac) + \frac{C(a, b, c, d, e)}{a+b+c+d+e}.$$

where, as before,

$$\sum^{0} a^3(be+cd) = a^3(be+cd) + b^3(ca+de) + c^3(db+ea) + \ldots$$

Glaisher, J. W. L. (1878).

[On a special form of determinant, and on certain functions of n variables analogous to the sine and cosine. *Quart. Journ. of Math.*, xvi. pp. 15–33.]

Part of this paper (§§ 8, 9, 19, 20) is a continuation of the paper of 1872, and the other part a continuation of that of 1877–78.

The proof that the product of two circulants is expressible as a circulant is effected by substituting Spottiswoode's equivalent for each. Similarly there is established, though somewhat imperfectly, the fresh theorem

$$C(a_1, a_2, \ldots, a_{2n}) = C(B_1, B_2, \ldots, B_n),$$

where

$$B_1 = (a_1, a_2, \ldots, a_{2n} \between a_1, -a_{2n}, a_{2n-1}, \ldots, -a_2)$$
$$B_2 = (a_1, a_2, \ldots, a_{2n} \between a_3, -a_2, a_1, \ldots, -a_4)$$
$$\cdots\cdots\cdots\cdots\cdots\cdots$$
$$B_n = (a_1, a_2, \ldots, a_{2n} \between a_{2n-1}, -a_{2n-2}, a_{2n-3}, \ldots, -a_{2n}),$$

the special point to be noted being that since $x^{2n} = (x^2)^n$ the $(2n)^{\text{th}}$ roots of 1 are the square roots of the n^{th} roots of 1 and therefore may be grouped in pairs whose sums vanish. For example, such a pair of *sixth* roots of 1 being $\theta, -\theta$, the product of the corresponding pair of factors of $C(a, b, c, d, e, f)$ is

$$(a+\theta b+\theta^2c+\theta^3d+\theta^4e+\theta^5f)(a-\theta b+\theta^2c-\theta^3d+\theta^4e-\theta^5f),$$

the development of which we might obtain by changing it into

$$(a+\theta^2c+\theta^4e)^2 - (b+\theta^2d+\theta^4f)^2\theta^2$$

and then squaring, etc., but which is better investigated by seeking the cofactors of

$$\theta^0 \text{ or } \theta^6, \quad \theta^1 \text{ or } \theta^7, \quad \theta^2 \text{ or } \theta^8, \quad \theta^3 \text{ or } \theta^9, \quad \theta^4 \text{ or } \theta^{10}, \quad \theta^5$$

in the result of the multiplication. These are found to be

$$\begin{array}{lll}
(\ \ a, -f, \quad e, -d, \quad c, -b \between a, b, c, d, e, f) & i.e. & a^2 - 2bf + 2ce - d^2, \\
(-b, \quad a, -f, \quad e, -d, \quad c \between \qquad\qquad\quad\ ) & i.e. & 0, \\
(\ \ c, -b, \quad a, -f, \quad e, -d \between \qquad\qquad\quad\ ) & i.e. & 2ac - b^2 - 2df + e^2, \\
(-d, \quad c, -b, \quad a, -f, \quad e \between \qquad\qquad\quad\ ) & i.e. & 0, \\
(\ \ e, -d, \quad c, -b, \quad a, -f \between \qquad\qquad\quad\ ) & i.e. & 2ae - 2bd + c^2 - f^2, \\
(-f, \quad e, -d, \quad c, -b, \quad a \between \qquad\qquad\quad\ ) & i.e. & 0,
\end{array}$$

thus bringing clearly out their law of formation. The product itself is evidently

$$(a^2 - 2bf + 2ce - d^2) + \theta^2(2ac - b^2 - 2df + e^2) + \theta^4(2ae - 2bd + c^2 - f^2),$$

where θ^2 is one of the third roots of 1, and we are consequently entitled to conclude that

$$\mathrm{C}(a, b, c, d, e, f) = \mathrm{C}(a^2 - d^2 - 2bf + 2ce, -b^2 + e^2 + 2ca - 2df, c^2 - f^2 - 2db + 2ea).$$

Glaisher actually uses this to compute the final development of the six-line circulant. After correcting three misprints * and effecting condensation by employing a symbol for "alternating cyclic sums" we find the said development to be

$$\begin{aligned}
\sum^{0}(\pm a^6) - 3\sum^{0}\left\{\pm a^4(bf + ce + d^2)\right\} \\
+ 2\sum^{0}\left\{\pm a^3(c^3 + 3b^2e + 3f^2c + 6bcd + 6def)\right\} \\
+ 9\sum^{0}(\pm a^2c^2e^2) + 9\sum^{0}\left\{\pm a^2(b^2f^2 - 2c^2df)\right\},
\end{aligned}$$

where, for example, $\sum^{0}\left\{\pm a^2(b^2f^2 - 2c^2df)\right\}$ stands for

$$a^2(b^2f^2 - 2c^2df) - b^2(c^2a^2 - 2d^2ea) + c^2(d^2b^2 - 2e^2fb) - \ldots,$$

but where, nevertheless, $\sum^{0}(\pm a^2c^2e^2)$ does not stand for

$$a^2c^2e^2 - b^2d^2f^2 + c^2e^2a^2 - d^2f^2b^2 + e^2a^2c^2 - f^2b^2d^2$$

but merely for $a^2c^2e^2 - b^2d^2f^2$.†

The rest of the paper (§§ 10–18) does not contain anything fresh so far as circulants are concerned.

* $2e^3a^3$, $12c^3bcd$, $12a^3def$ should be $2c^3a^3$, $12a^3bcd$, $12c^3def$.

† This is the reason why we do not combine the two into $\sum^{0}\left\{\pm a^2(b^2f^2 + c^2e^2 - 2c^2df)\right\}$.

Menesson, (1878).

[Solutions des questions proposées (Question 185). *Nouv. Correspondance Math.*, iv. pp. 185–187.]

Not only is Spottiswoode's result here explicitly obtained by equating the two forms of the eliminant of

$$\left.\begin{aligned} a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1x + a_0 &= 0 \\ x^n - 1 &= 0 \end{aligned}\right\}$$

which were referred to under Baltzer (1864), but the parallel result is also stated, namely, that

$$C(a_0, a_1, a_2, \ldots, a_{n-1}) = a_{n-1}^n(\beta_1^n - 1)(\beta_2^n - 1) \ldots (\beta_{n-1}^n - 1)$$

where $\beta_1, \beta_2, \ldots, \beta_{n-1}$ are the roots of the first equation.

Minozzi, A. (1878).

[Sopra un determinante. *Giornale di Mat.*, xvi. pp. 148–151.]

Minozzi's purpose is to find the final development of a circulant, not from the determinant form, but from Spottiswoode's product. His first point is that, multiplication of the n factors having been performed, the terms of the resulting expression must be of the form

$$\mathrm{A}a_0^{e_0}a^{e_1}a_2^{e_2} \ldots a_{n-1}^{e_{n-1}}$$

where the e's are positive integers whose sum is n. His next is that any a entering into this, say a_k, must be accompanied by one of the θ's raised to the power k, and that therefore the form of A is known. His third is that A, being thus of necessity a symmetric function of the roots of the equation $x^n - 1 = 0$, can be calculated, and that all the more readily by reason of the fact that so many of the simple symmetric functions are equal to 0. For example, the coefficient of $a_0^2a_1a_3$ in $C(a_0, a_1, a_2, a_3)$ is

$$\sum \theta_1^0\theta_2^0\theta_3^1\theta_4^3,$$

i.e.
$$\theta_3^1\theta_4^3 + \theta_3^3\theta_4^1 + \theta_2^1\theta_4^3 + \theta_2^1\theta_3^3 + \theta_2^3\theta_4^1 + \theta_2^3\theta_3^1$$
$$+ \theta_1^1\theta_4^3 + \theta_1^1\theta_3^3 + \theta_1^1\theta_2^3 + \theta_1^3\theta_4^1 + \theta_1^3\theta_3^1 + \theta_1^3\theta_2^1,$$

i.e.
$$(\theta_1 + \theta_2 + \theta_3 + \theta_4)(\theta_1^3 + \theta_2^3 + \theta_3^3 + \theta_4^3) - (\theta_1^4 + \theta_2^4 + \theta_3^4 + \theta_4^4),$$

i.e.
$$0.0 - 4.$$

Minozzi is careful to note that, in accordance with Baltzer's theorem of 1870, it is only necessary to calculate A when the sum of the suffixes of the a's is 0 or a multiple of n. For example, in the case of $C(a_0, a_1, a_2, a_3)$ the coefficients would have to be found for

$$\begin{array}{llll} a_0a_0a_0a_0\,, & a_0a_0a_1a_3 & a_0a_2a_3a_3 & a_3a_3a_3a_3\,. \\ & a_0a_0a_2a_2 & a_1a_1a_3a_3 & \\ & a_0a_1a_1a_2 & a_1a_2a_2a_3 & \\ & a_1a_1a_1a_1\,, & a_2a_2a_2a_2\,, & \end{array}$$

Lemonnier, H. (1879).

[Calcul d'un déterminant. *Nouv. Annales de Math.*, (2) xviii. pp. 518–524.]

The determinant in question is the circulant in which the elements are in equidifferent progression. The two modes of procedure followed are like Baehr's of 1860, but neither is so good.

Glaisher, J. W. L. (1879 January).

[Theorems in Algebra. *Messenger of Math.*, viii. pp. 140–144.]

One of the two theorems is connected with determinants, and is to the effect that if the $2n$ quantities $a_1, a_2, \ldots, a_{2n}$ be connected by the n relations

$$\left.\begin{array}{lllll} (a_1\,, & -a_{2n}\,, & a_{2n-1}\,, \ldots, -a_2 & \between\ a_1, a_2, a_3, \ldots, a_{2n}) & = 1 \\ (a_3\,, & -a_2\,, & a_1\,, \ldots\ldots, -a_4 & \between\ \qquad\qquad\qquad\qquad\ ) & = 0 \\ (a_5\,, & -a_4\,, & a_3\,, \ldots\ldots, -a_6 & \between\ \qquad\qquad\qquad\qquad\ ) & = 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ (a_{2n-1}\,, & -a_{2n-2}\,, & a_{2n-3}\,, \ldots, -a_{2n} & \between\ \qquad\qquad\qquad\qquad\ ) & = 0 \end{array}\right\}$$

then

$$C(a_1, a_2, a_3, \ldots, a_{2n}) = 1\,,$$

and the elements of the first row of C are respectively equal to the complementary minors of the elements of the first column.

The truth of the first part of the theorem is a direct consequence of the same author's theorem of the preceding year, our illustration of which makes clear that if

$$2ac - b^2 - 2df + e^2 = 0 \quad \text{and} \quad 2ae - 2bd + c^2 - f^2 = 0\,,$$

we must have

$$C(a, b, c, d, e, f) = (a^2 - d^2 - 2bf + 2ce)^3\,,$$

and with a third condition

$$C(a, b, c, d, e, f) = 1.$$

To establish the second part we have to take along with the n conditioning equations the n identical equations

$$\left.\begin{array}{llllll}
(a_2\,, -a_1 & , a_{2n} & , \ldots , -a_3 & \between\ a_1, a_2, a_3, \ldots, a_{2n}) & = & 0 \\
(a_4\,, -a_3 & , a_2 & , \ldots , -a_5 & \between\qquad\qquad\qquad\qquad) & = & 0 \\
\cdots & \cdots & \cdots & \cdots & & \cdot \\
(a_{2n}, -a_{2n-1}, & a_{2n-2}, & \ldots , -a_1 & \between\qquad\qquad\qquad\qquad) & = & 0
\end{array}\right\}$$

thus obtaining a set of $2n$ equations having $a_1, a_2, a_3, \ldots, a_{2n}$ as quasi unknowns and $(-1)^n C$ as the determinant of their coefficients. The usual expression for an unknown as the quotient of two determinants leads at once to the desired result. Thus when $n=3$ the set of equations is

$$\left(\begin{array}{cccccc}
a & -f & e & -d & c & -b \\
b & -a & f & -e & d & -c \\
c & -b & a & -f & e & -d \\
d & -c & b & -a & f & -e \\
e & -d & c & -b & a & -f \\
f & -e & d & -c & b & -a
\end{array}\ \between\ a, b, c, d, e, f\right) = 1, 0, 0, 0, 0, 0.$$

LIST OF AUTHORS

whose writings are herein dealt with.

(*Issued separately May* 17, 1912.)

XIII.—"On the Singular Solutions of Partial Differential Equations of the First Order." By H. Levy, M.A., B.Sc., Carnegie Scholar. (*Communicated by* D. Gibb, M.A., B.Sc.)

(MS. received March 4, 1912. Read May 6, 1912.)

Section I.—Synthetic Treatment.

The complete integral of the differential equation

$$\phi(xyzpq)=0$$

is a relation among the variables, which includes as many arbitrary constants as there are independent variables. But it is important to distinguish carefully between differential equations which have been formed by the elimination of constants from some complete primitive, and those whose origin is quite unknown, or which may have been constructed by some method totally different from the first.

In the original case, the differential equation can always be integrated in finite terms, while in the latter, only under the conditions laid down in Cauchy's Existence Theorem can an integral be obtained, and even then usually as an infinite series.

These conditions demand that $\phi(xyzpq)=0$ be an irreducible integral function of p and q, and a synectic function of all its arguments. Let $z=\phi(xy)$ be any integral, regular in the neighbourhood of (x_0y_0), and let z_0, p_0, q_0 be the values of $z, p,$ and q corresponding to $x=x_0$, $y=y_0$, then if $\mathrm{P}=\partial\phi/\partial p$, $\mathrm{Q}=\partial\phi/\partial q$, and $\mathrm{P}_0\neq 0$, the differential equation can be put in the form

$$p=\chi(xyzq)$$

where χ is synectic near x_0, y_0, z_0, and such that

$$p_0=\chi(x_0y_0z_0q_0).$$

It follows by Cauchy's Existence Theorem * that the equation has one integral and only one, which reduces to

$$z=\phi(x_0y) \text{ when } x=x_0.$$

Hence the supposed integral will always be furnished by Cauchy's Existence Theorem, unless it involves

$$\mathrm{P}=0, \quad \mathrm{Q}=0.$$

* Goursat's "Leçons sur l'integration des Équations aux derivées partielles."

Any integral which satisfies these latter two conditions, we call a singular solution.

If such a solution therefore exists, we may obtain it by the elimination of p and q between the three equations

$$(a)\ \phi = 0, \quad (b)\ \phi_p = 0, \quad (c)\ \phi_q = 0.$$

There are two fundamentally distinct methods by which the problem of the singular solution may be attacked. In the first place, we may treat the question analytically by discussing the particular forms of singularity that may exist in the general equation $\phi(xyzpq) = 0$; or, secondly, we may apply a synthetic method, viz. given the singularities which are to occur, construct the general differential equation which possesses them, imposing in the latter case such restrictions upon the function as to ensure that it admits of an integral.

In the earlier part of this paper I propose to treat this problem from the synthetic point of view.

Darboux in his "Mémoire sur les Solutions Singulières des Équations aux Derivées Partielles du Premier Ordre" (*Mémoires présentés à l'Académie des Sciences*, tome xxvii.), gives the following differential equation as the general form of an equation possessing the singular solution $z = 0$

$$2\phi(xy)z + Ap^2 + 2Bpq + Cq^2 + Dpz + Eqz + Fz^2 = 0,$$

A, B, C, D, E, F being arbitrary functions of x, y, z, p, and q.

Although this is apparently true merely for the particular case where the plane of x, y is the singular solution, yet the more general case where the surface $f(xyz) = 0$ is the singular integral can be immediately deduced from the above by the substitution

$$z = f(xyZ),$$

giving

$$p = f_x + fZp', \quad q = f_y + fZq',$$

where

$$p' = \frac{\partial Z}{\partial x}, \qquad q' = \frac{\partial Z}{\partial y},$$

and omitting the dashes and writing z for Z we finally deduce that the differential equation of the first order, having the singular solution $f(xyz) = 0$, is given by

$$\phi \equiv A(p + f_x/f_z)^2 + 2B(p + f_x/f_z)\ (q + f_y/f_z) + C(q + f_y/f_z)^2 + Df(xyz) = 0 \quad . \quad (1)$$

where A, B, C, D are arbitrary functions of x, y, z, p and q, such that ϕ is an irreducible and synectic function of these arguments.

This latter transformation suggests at once that a differential equation possessing a singular solution may often be considerably simplified in form by the substitution

$$Z = \text{singular solution},$$

and thus facilitate its integration.

Among the various cases that may occur in the derivation of the singular solution of $\phi(xyzpq)=0$, there is one of special importance, viz. when the expressions for p and q derived from

$$\phi_p = 0 \text{ and } \phi_q = 0$$

render

$$\phi \equiv 0,$$

i.e. when the three equations are equivalent to only two independent equations.

In this case, if p and q satisfy the symbolical relation

$$\begin{vmatrix} p & q & -1 \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ p & q & -1 \end{vmatrix} = 0 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

the integral of the equation

$$pdx + qdy - dz = 0$$

will furnish a onefold of singular solutions. On referring to equation (1), it is at once evident that this corresponds to the case where $D=0$, for then the derivation of the singular integral leads to

$$p = -f_x/f_z, \quad q = -f_y/f_z,$$

and these satisfy the three conditions

$$\phi = 0, \quad \phi_p = 0, \quad \phi_q = 0,$$

while the integral of

$$pdx + qdy - dz = 0$$

is

$$f(xyz) = c.$$

Hence the general Partial Differential Equation of the first order which has the given onefold of Singular Solutions $f(xyz)=c$ is

$$\phi \equiv A(p + f_x/f_z)^2 + 2B(p + f_x/f_z)\,(q + f_y/f_z) + C(q + f_y/f_z)^2 = 0 \quad . \quad . \quad (3)$$

where A, B, and C are arbitrary functions of x, y, z, p and q such that ϕ is irreducible as regards p and q, and is a synectic function of its variables.

The following method of constructing such equations was given by the late Professor Chrystal in his class lectures.

Commencing with an equation $\phi(xyzpq)=0$, having a singular solution $\chi(xyz)=0$, then $\phi(xyzpq)=\chi(xyz)$ will become an identity when the values of p and q derived from it by differentiating partially with respect to p and q be substituted, so that this last equation will have a onefold of singular solutions.

To a certain extent this was working in the dark. It was necessary, in the first place, to obtain a function ϕ having a definite singular solution, either by guessing one, or by eliminating two arbitrary parameters from an equation having a Lagrangian Singular Solution, in either case usually entailing a great deal of labour. In the second place, it was extremely difficult by this method to work to an equation possessing the particular singular onefold that might be desired. On considering equation (1) it is seen that had the term associated with D been any other than $f(xyz)$, there would have been no singular solution, nevertheless on putting $D=0$, the resulting equation would still possess a singular onefold. But Professor Chrystal's process of putting the singular solution to the right-hand side of the equation is really equivalent to putting $D=0$. Hence, for the success of his method, we need not necessarily commence with an equation possessing a definite singular solution, but one which merely determines p and q by partial differentiation, to satisfy the symbolical relation (2).

The following examples will serve to illustrate the foregoing:—

Example 1.—Suppose $f(xyz)=c$ is given by $x^2+y^2-cz^2=1$, a system of conicoids.

$$\therefore\ f_x/f_z=-xz/(x^2+y^2-1),\quad f_y/f_z=-yz/(x^2+y^2-1).$$

Hence an equation with the required onefold would be

$$(x^2-1)\left(p-\frac{xz}{x^2+y^2-1}\right)^2+2xy\left(p-\frac{xz}{x^2+y^2-1}\right)\left(q-\frac{yz}{x^2+y^2-1}\right)+(y^2-1)\left(q-\frac{yz}{x^2+y^2-1}\right)^2=0$$

which gives

$$(px+qy-z)^2-p^2-q^2=\frac{-z^2}{x^2+y^2-1}.$$

We shall return to this example later, as it is one given by Professor Chrystal as illustrative of an equation possessing a onefold.

Example 2.—Let the onefold be given by

$$z=\alpha x+\beta y+c$$

where α and β are constants, and c is arbitrary.

Hence ϕ may be written as

$$\Sigma A(p-\alpha)^r+\Sigma B(q-\beta)^s+\Sigma C(p-\alpha)^t(q-\beta)^{t'}=0\ . \qquad . \qquad . \qquad (4)$$

where

r and s are >1,

t and t' are $\geqslant 1$.

If the coefficients in this equation are functions of p and q only, so that x, y, and z are quite absent, the complete integral is given by

$$z = lx + my + n,$$

l and m being connected by equation (4), where p and q have been replaced by these quantities respectively.

This condition is fulfilled when

$$l = \alpha, \quad m = \beta,$$

and hence the singular onefold is a particular case of the complete integral. It is important to remark that in many cases the equation might be reduced to the form (4) by a substitution, and in such cases also the singular onefold is included in the complete integral.

Example 3.—In illustration suppose X, Y, Z to be functions of x, y, and z respectively, then the differential equation

$$\Sigma(pZ + X)^m(qZ + Y)^n = 0 \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where

$$(1)\ m \text{ and } n \geqslant 1$$

or

$$(2)\ m = 0\ n > 1$$

or

$$(3)\ m > 1\ n = 0$$

has a onefold of singular solutions which is included in the complete integral.

For example,

$$(p - x)^2(q - y)^2 - (p - x)^2 - (q - y)^2 = 0$$

when reduced by the substitution

$$Z = z - \frac{x^2}{2} - \frac{y^2}{2}$$

gives $x^2 + y^2 - 2z = c$ as a onefold of singular solutions, and

$$z = \frac{x^2 + y^2}{2} + \alpha x + \beta y + c$$

where $\alpha^2\beta^2 - \alpha^2 - \beta^2 = 0$ as complete integral.

On considering equation (3), it seems that when p and q appear to integral powers, the lowest degree possible with a given onefold of singular solution is the second, the coefficients being functions of x, y, z only. It can easily be shown, however, that every such equation is reducible as regards p and q, that is to say, it is impossible for an irreducible equation of the second degree in p and q to have a singular onefold. For, by what has already been said, it must be completely represented by

$$\psi_1(f_z p + f_x)^2 + 2\psi_2(f_z p + f_x)(f_z q + f_y) + \psi_3(f_z q + f_y)^2 = 0 \quad . \quad . \quad (6)$$

where ψ_1, ψ_2, and ψ_3 are any functions of x, y, and z. But this may evidently be written in reduced form

$$\psi_1 f_z p + (\psi_2 \pm \sqrt{\psi_2^2 - \psi_1\psi_3})f_z q = -\psi_1 f_x - (\psi_2 \pm \sqrt{\psi_2^2 - \psi_1\psi_3})f_y \quad . \quad . \quad (7)$$

Hence Lagrange's Subsidiary System gives

$$\frac{dx}{\psi_1 f_z} = \frac{dy}{(\psi_2 \pm \sqrt{\psi_2^2 - \psi_1\psi_2})f_z} = \frac{dz}{-\psi_1 f_x - (\psi_2 \pm \sqrt{\psi_2^2 - \psi_1\psi_3})f_y}$$
$$= \frac{f_x dx + f_y dy + f_z dz}{0}$$

Hence one integral is

$$f(xyz) = c_1.$$

Suppose

$$u(xyz) = c_2$$

is another integral when the upper sign is taken, and

$$v(xyz) = c_3$$

when the lower sign is taken with the square root, then the complete integrals of the two reduced parts are

$$f(xyz) + \mathrm{A}u = \mathrm{B},$$
$$f(xyz) + \mathrm{A}'v = \mathrm{B}'.$$

Hence in this case the so-called singular onefold—and it satisfies Cauchy's conditions—is the infinity of surfaces common to both reduced parts.

As an example we may take the case already quoted, *Example* 1—

$$(xp + yq - z)^2 - p^2 - q^2 = \frac{-z^2}{(x^2 + y^2 - 1)}$$

which, being of the second degree in p and q, reduces to

$$p(x^2 - 1) + q(xy \pm \sqrt{x^2 + y^2 - 1}) = xz \pm \frac{yz}{\sqrt{x^2 + y^2 - 1}}.$$

Example 4.—

$$(px - 2z)^2 + qz(px + qy - 2z) = 0$$
$$\phi_p = 0 \quad \text{gives} \quad 2x(px - 2z) + xzq = 0$$
$$\phi_q = 0 \quad \text{gives} \quad z(px + qy - 2z) + yzq = 0$$

which are equivalent to

$$2px + qz - 4z = 0, \quad z = 0,$$

and

$$2px + qz - 4z = 0, \text{ and } px + qy - 2z = 0.$$

Hence $z=0$ is a singular solution, and $z=ax^2$ is a singular onefold giving $z=0$ for $a=0$.

But the original equation may be written in the form

$$(px-2z)^2+(px-2z)qz+q^2yz=0,$$

which is reducible as regards p and q to

$$2xp+(z+\sqrt{z^2-4yz})q=4z.$$

$\therefore$ Lagrange's Subsidiary System is

$$\frac{dx}{2x}=\frac{dy}{z+\sqrt{z^2-4yz}}=\frac{dz}{4z},$$

two integrals of which are

$$z=c_1x^2 \text{ and } \sqrt{z-4y}+\sqrt{z}=c_2.$$

Hence the solution is

$$z=x^2f(\sqrt{z-4y}+\sqrt{z}),$$

and the singular onefold is given by $f=$ arbitrary constant.

It has already been shown that $f(xyz)=c_1$ is one integral of the equation

$$\psi_1 f_z p+(\psi_2+\sqrt{\psi_2{}^2-\psi_1\psi_3})f_z q=-\psi_1 f_x-(\psi_2+\sqrt{\psi_2{}^2-\psi_1\psi_3})f_y \quad . \quad (7)$$

Suppose the total expression

$$\psi_2{}^2-\psi_1\psi_3$$

be given, but the separate functions ψ_1, ψ_2, and ψ_3 be not given, then we can easily find expressions for ψ_1, ψ_2 and ψ_3, so that

$$\psi_2{}^2-\psi_1\psi_3=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

should be a solution of equation (7), for we need merely calculate p and q from (8), insert their values in (7), and then choose ψ_1 and ψ_2, so that the resulting equation should be an identity.

Hence equation (7) is a general form for a linear equation of the first order having any given integral $f(xyz)=c_1$, and a given surface $\psi_2{}^2-\psi_1\psi_3=0$ as a solution which is a locus of branch points. In the last example discussed, in fact, it can easily be seen that $z=4y$ is a solution of the equation, yet it is not contained in the final solution.

Example 5.—$f(xyz)=c=z-2y$ is to be an integral,

$$\therefore f_x=0, \quad f_y=-2, \quad f_z=1,$$

then (7) becomes

$$\psi_1 p+(\psi_2+\sqrt{\psi_2{}^2-\psi_1\psi_3})q=2(\psi_2+\sqrt{\psi_2{}^2-\psi_1\psi_3}),$$

or

$$(\psi_2-\sqrt{\psi_2{}^2-\psi_1\psi_3})p+\psi_3 q=2\psi_3.$$

If

$$\psi_2^2 - \psi_1\psi_3 = z - x - y = 0,$$

then

$$p = 1, \quad q = 1,$$
$$\therefore \ \psi_2 + \psi_3 = 2\psi_3,$$
$$\therefore \ \psi_2 = \psi_3 = 1 \text{ say}.$$
$$\therefore \ (1 - \sqrt{z - x - y})p + q = 2,$$

an example discussed by Professor Chrystal in a memoir (*Trans. R.S.E.*, vol. xxxvi., 1892).

Example 6.—Suppose

$$f(xyz) = c_1 = z,$$
$$\therefore \ f_x = 0, \quad f_y = 0, \quad f_z = 1,$$

$\therefore$ (7) gives

$$\psi_1 p + (\psi_2 + \sqrt{\psi_2^2 - \psi_1\psi_3})q = 0.$$

Let

$$\psi_2^2 - \psi_1\psi_3 = 0 = (1 - z^2)\,(x^2 + y^2 + z^2 - 1)$$

$z^2 = 1$ satisfies our equation identically.

$$x^2 + y^2 + z^2 - 1 = 0$$

gives

$$p = -x/z, \quad q = -y/z,$$

and the equation on substituting these values,

$$x\psi_1 + y\psi_2 = 0.$$

This will be satisfied by

$$\psi_1 = x^2 + z^2 - 1, \quad \psi_2 = xy.$$

Hence the required equation is

$$(x^2 + z^2 - 1)p + [xy + (1 - z^2)^{\frac{1}{2}}(x^2 + y^2 + z^2 - 1)^{\frac{1}{2}}]q = 0,$$

an example discussed by Goursat.

The reducible equation from which this may be supposed to be derived is

$$(x^2 + z^2 - 1)p^2 + 2xypq + (y^2 + z^2 - 1)q^2 = 0 \quad . \quad . \quad . \quad (9)$$

On testing this for singular solutions we get

$$\left.\begin{array}{r} (x^2 + z^2 - 1)p + xyq = 0 \\ xyp + (y^2 + z^2 - 1)q = 0 \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

which together satisfy (9), and give for consistency

$$\begin{vmatrix} x^2 + z^2 - 1 & xy \\ xy & y^2 + z^2 - 1 \end{vmatrix} = 0,$$

i.e.

$$(x^2 + y^2 + z^2 - 1)\,(1 - z^2) = 0,$$

which also satisfies (10), and may be regarded as a singular solution.

It has been seen that a differential equation may possess one or more, but a finite number, of singular solutions, or it may possess a onefold infinity of such. It would be interesting to investigate whether it would be possible for an equation to contain both types of phenomena, where the singular surface is distinct from the onefold. There is no reason *a priori* why this should not be possible.

For such to be the case, it is evident that (1) must be expressible in the form (3) or *vice versa.*

Let $f(xyz)=c$ be the onefold, and $g(xyz)=0$ the given singular solution, then the equation

$$\phi[(p+f_x/f_z),\ (p+g_x/g_z)\ .\ .\ .\ .]+g(xyz)\psi[(p+f_x/f_z),\ (q+f_y/f_z)]=0 \quad . \quad (11)$$

where ϕ is a function both of $(p+f_x/f_z)$, etc., and $(p+g_x/g_z)$, etc., of the type of ϕ in equation (3), and ψ is a similar function of $p+f_x/f_z$ and $q+f_y/f_z$ only, is readily seen to have $g(xyz)=0$ as a singular solution and $f(xyz)=c$ as a singular onefold.

An interesting case of this is the following:—

$$\phi=\mathrm{A}(p+f_x/f_z)^2{}_1\psi+2\mathrm{B}(p+f_x/f_z)(q+f_y/f_z){}_1\psi{}_2\psi+\mathrm{C}(q+f_y/f_z)^2{}_2\psi=0 \quad . \quad (12)$$

where A, B, and C are any functions of x, y, z, p, and q as in equation (3), and ${}_1\psi$ and ${}_2\psi$ are functions of x, y, and z which satisfy the conditions

$$\begin{vmatrix} {}_1\psi_y & f_y \\ {}_1\psi_z & f_z \end{vmatrix}=0 \qquad \begin{vmatrix} {}_2\psi_x & f_x \\ {}_2\psi_z & f_z \end{vmatrix}=0,$$

then evidently both

$${}_1\psi=0 \quad \text{and} \quad {}_2\psi=0$$

will satisfy

$$\phi=0, \quad \phi_p=0, \quad \phi_q=0,$$

and the equation considered will have $f(xyz)=c$ as a singular onefold, and ${}_1\psi=0$, ${}_2\psi=0$ as singular solutions.

These examples will serve to emphasise the possibility of the existence of numerous solutions of a differential equation, which are not even suggested by the complete primitive, and even though such singularities may only very occasionally be encountered, it is well to bear in mind that they do exist.

In the case of ordinary and partial differential equations of the first order, these have been more or less thoroughly analysed at various times by different investigators, but the field for higher orders than the first remains still practically an unexplored region.

That such solutions do exist can easily be demonstrated by application of methods analogous to those pursued at the beginning of this paper for equations of the first order.

The importance of this will be seen not to be overestimated if it is remembered that it is really from among equations of higher orders that mathematics receives its most frequent application in physics.

The singular solutions of partial differential equations of the first order have been fruitfully discussed by Lagrange and others from yet another point of view, viz. as regards their derivation from the complete primitive.

The Lagrangian Singular Solution of the Partial Differential Equation whose twofold integral is

$$f(xyzab)=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

is obtained by eliminating a and b between

$$f=0, \quad f_a=0, \quad f_b=0.$$

If $\psi(xyz)=0$ be this singular solution, then it is easily shown that (13) is of the form

$$f\equiv(a-A)^2\psi_1+2(a-A)(b-B)\psi_2+(b-B)^2\psi_3+a\psi(xyz)=0 \,. \quad . \quad (14)$$

where ψ_1, ψ_2, and ψ_3 are functions of $xyzab$, and A and B are functions of xyz. This enables us at once to verify that every member of (14) touches the singular solution at a point, if we remember that any point on it is given by the three equations

$$\psi=0, \quad A=a, \quad B=b,$$

and $\psi=0$, $A=a$ meet (14) bis where $B=b$ meets it, while $\psi=0$, $B=b$ meet it bis where $A=a$ meets it.

This has, however, been thoroughly discussed by Professor Hill in his paper to the Royal Society "On the Locus of Ultimate Intersections of Lines and Surfaces," 1892.

In the special case where $a=0$ in (14),

$$f=0, \quad f_a=0, \quad f_b=0,$$

are satisfied by

$$A=a, \quad B=b,$$

and there is no Lagrangian Singular Solution corresponding to these values, although there may exist one for other values of a and b.

If ψ_1, ψ_2, ψ_3 are functions purely of x, y, and z, then there does not exist any Lagrangian Singular Solution at all. In all cases it can easily be shown that every member of

$$(a-A)^2\psi_1+2(a-A\,(b-B)\psi_2+(b-B)^2\psi_3=0 \quad . \quad . \quad . \quad (15)$$

has a unodal line given by

$$A=a, \quad B=b.$$

We shall utilise these facts to investigate the nature of the $p-q$ discriminant of the partial differential equation of the first order.

Section II.—Geometrical Treatment.

Dr J. H. M. Wedderburn in his paper "On the Isoclinal Lines of a Differential Equation of the First Order" (*Proc. R.S.E.*, vol. xxiv.) made use of the idea of isoclinal lines in a plane to investigate the nature of the p-discriminant locus of an ordinary differential equation. In what follows I propose to extend the conception of isoclinals to space of three dimensions, and by means of it, to investigate the nature of the Cauchian Singular Solution, especially the case of the onefold of these solutions.

The differential equation

$$\phi(xyzpq)=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

besides defining a twofold family of integral surfaces, also defines a family obtained by regarding p and q as arbitrary constants in (16). Such a family we call the isoclinal surfaces of (16), and they are given by

$$\phi(xyzab)=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Any member of (17) may be regarded as generated by moving from point to point of contiguous members of the twofold solution of (16), for which $p=a$, $q=b$.

Hence there exists, associated with every member of (17), a twofold infinity of infinitesimal planes all perpendicular to the direction

$$(a\big/\sqrt{a^2+b^2+1}, \quad b\big/\sqrt{a^2+b^2+1}, \quad -1\big/\sqrt{a^2+b^2+1}).$$

In what follows these shall be referred to as the *integral elements* of (17).

This family furnishes an evident method of describing any member of the twofold solution of (16). For starting at any arbitrary chosen point on any member (a_1, b_1) of (17) we draw an integral element, perpendicular to the direction $(a_1 : b_1 : -1)$ which in general meets each of an infinity of contiguous isoclinals $(a_1+\partial a_1, b_1+\partial b_1)$ in an infinitesimal line. Proceeding now to draw integral elements perpendicular to the direction $(a_1+\partial a_1; b_1+\partial b_1 : -1)$ from each of these to the neighbouring isoclinals, we gradually trace out an integral surface of (16). Hence by commencing from every one of the twofold of points on any given isoclinal, and proceeding as shown above from surface to surface, we map out the twofold integrals of (16).

Now the Cauchian Singular Solution of (16) is the same as the Lagrangian Singular Solution of (17), being derived by exactly the same process; moreover, the latter is touched at any point by some member of the twofold (17).

Suppose $PRR_1R_2SS_1S_2$ (fig. 1) to be any surface (ab) touching E the envelope at the point P, then we may suppose the part of this (ab) isoclinal near P to be formed by an infinite number of curves RPS, R_1PS_1, R_2PS_2, . . . all touching E at P. Each of these lines is cut at all points right up to P by parallel integral elements, which coalesce at P without in general crossing E.

Now all round P there is a onefold infinity of points P′P″ . . . . at

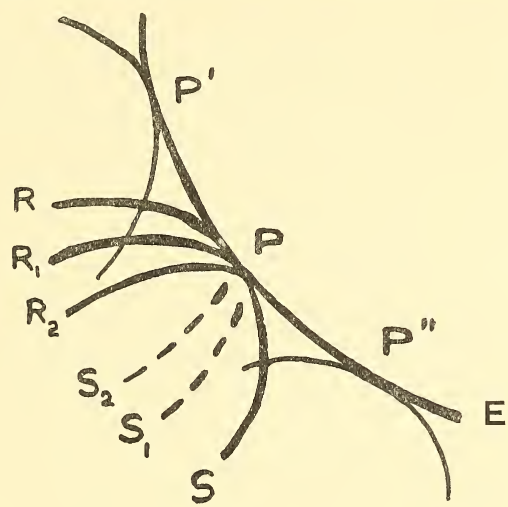


Fig. 1.

which the envelope E is touched by neighbouring isoclinals, whose integral elements are slightly inclined to those of RPS.

Hence the integral surface enveloped by the elements of corresponding points on the PP′P″ . . . . isoclinals converges to P and stops, forming a conical point at which the tangent cone has collapsed into a line.

Hence the Cauchian Eliminant is in general a locus of such points.

If the direction of the envelope E is also the direction of the integral surfaces, then E is the envelope singular solution. Now $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the envelope are given by

$$\phi_x + \frac{\partial z}{\partial x}\,\phi_z = 0$$

and

$$\phi_y + \frac{\partial z}{\partial y}\,\phi_z = 0,$$

and hence the necessary and sufficient conditions for a Cauchian Singular Solution are

$$\phi_x + p\phi_z = 0, \quad \phi_y + q\phi_z = 0.$$

By a similar process exactly we can deduce the geometrical interpretation of the Cauchian Eliminant, corresponding to various singular loci on the isoclinals. In particular, consider the case where each isoclinal possesses a unodal line; the system will then be given by equation (15).

If

$$A = -\psi_x/\psi_z, \quad B = -\psi_y/\psi_z,$$

the differential equation obtained by substituting p for a, and q for b in

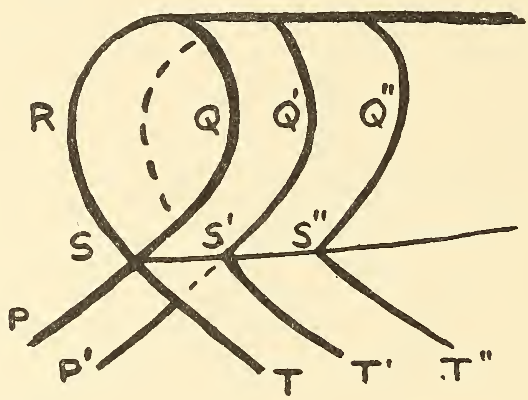


Fig. 2.

(15), will be the general form of an equation with a onefold of singular solutions

$$\psi(xyz) = c.$$

Each isoclinal surface, therefore, has a unodal line given by

$$\psi_x + a\psi_z = 0, \quad \psi_y + b\psi_z = 0.$$

We may suppose every such surface in the neighbourhood of the unodal line SS′S″ . . . . to be formed by a system of curves PQRST, P′Q′R′S′T′ . . . . etc. (fig. 2).

Across PSQ and RST a system of integral elements pass, and through S there are two, which therefore touch, and have in general different curvatures; the same is true for S′S″ . . . . and intersecting as they do on the same isoclinal, the tangent planes at SS′S″ . . . . are all parallel. Round S there is a onefold of other points $S_1S_2S_3$ . . . . say, lying on neighbouring unodal lines, at which the integral elements differ infinitely little in direction from that at S.

Hence, moving from S along the tangent plane to the osculating surfaces, we reach a onefold infinity of other corresponding points $S_1S_2S_3$ . . . . and from each of these we may proceed by drawing new integral elements. Thus, commencing at S, we trace out a surface which is touched at each point by two integral surfaces, and corresponding to every one of the onefold infinity of points S′S″ . . . . on the unodal line a similar surface can be constructed.

Finally, therefore, we trace out a onefold of surfaces, touched at every point by two integral surfaces, and these constitute the onefold of singular solutions.

The unodal lines

$$\psi_x + a\psi_z = 0, \quad \psi_y + b\psi_z = 0,$$

will then be the loci of the points of contact of parallel tangent planes, and will therefore be isoclinal lines for the onefold in question.

The unodal lines being given by

$$a = -\psi_x/\psi_z, \quad b = -\psi_y/\psi_z,$$

the direction cosines of the normal to the onefold are

$$-\psi_x/\psi_z : -\psi_y/\psi_z : -1,$$

and the singular onefold is therefore obtained by integrating

$$-\frac{\psi_x}{\psi_z}dx - \frac{\psi_y}{\psi_z}dy - dz = 0,$$

giving

$$\psi(xyz) = c.$$

It follows that if we regard the normals to the integral surfaces at any point in space as forming a cone, each such cone has a double generator.

(*Issued separately May* 17, 1912.)

XIV.—On General Dynamics:—I. Hamilton's Partial Differential Equations and the Determination of their Complete Integrals. By Professor A. Gray, F.R.S.

(MS. received February 19, 1912. Read February 19, 1912.)

THE present paper contains the first part of a series of notes on general dynamics which, if it is found worth while, may be continued. In § 1 I have shown how the first Hamiltonian differential equation is led up to in a natural and elementary manner from the canonical equations of motion for the most general case, that in which the time t appears explicitly in the function usually denoted by H. The condition of constancy of energy is therefore not assumed. In § 2 it is proved that the partial derivatives of the complete integral of Hamilton's equation with respect to the constants which enter into the specification of that integral do not vary with the time, so that these derivatives equated to constants are the integral equations of motion of the system.*

In § 3 the alternative form of differential equation, also given by Hamilton, and the equations which flow from it, are dealt with in a similar manner; while § 4 contains some remarks on the dual system thus obtained and its applications.

1. *Derivation of the first partial differential equation from the canonical equations.*—Let T denote the kinetic energy of a system of connected particles, V the potential energy, and H as usual the value of the expression $\Sigma(p\dot{q}) - \mathrm{T} + \mathrm{V}$. Here q denotes any one of the k generalised co-ordinates which with the equations of connection define the configuration of the particles at time t, and p denotes the corresponding element of generalised momentum. V is the potential energy, or more properly the function of $q_1, q_2, \ldots, q_k$ from which the generalised forces at time t are given by the partial derivatives $-\partial \mathrm{V}/\partial q_1, -\partial \mathrm{V}/\partial q_2, \ldots, -\partial \mathrm{V}/\partial q_k$. In the general case here considered V may be also an explicit function of the time t.

The complete solution of any dynamical problem regarding such a system gives

$$\left.\begin{array}{l} q_1 = f_1(a_1, a_2, \ldots, a_k, b_1, b_2, \ldots, b_k, t) \\ q_2 = f_2(a_1, a_2, \ldots, a_k, b_1, b_2, \ldots, b_k, t) \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ q_k = f_k(a_1, a_2, \ldots, a_k, b_1, b_2, \ldots, b_k, t) \end{array}\right\} \quad . \quad . \quad . \quad (1)$$

* This discussion was suggested by a passage regarding a particular case in Darboux, *Théorie des Surfaces*, t. ii., § 563.

Here $a_1, a_2, \ldots, a_k, b_1, b_2, \ldots, b_k$ are constants of values to be assigned by the initial circumstances. For example, the as may be the initial co-ordinates, and the bs the initial velocities, or the initial momenta.

Differentiation of (1) with respect to t gives $\dot{q}_1, \dot{q}_2, \ldots, \dot{q}_k$. From these values and the expression of T in terms of $\dot{q}_1, \dot{q}_2, \ldots, \dot{q}_k$, namely,

$$\begin{aligned} \mathrm{T} = \tfrac{1}{2}(\mathrm{A}_{11}\dot{q}_1^2 &+ 2\mathrm{A}_{12}\dot{q}_1\dot{q}_2 + \ldots + 2\mathrm{A}_{1k}\dot{q}_1\dot{q}_k \\ &+ \mathrm{A}_{22}\dot{q}_2^2 + 2\mathrm{A}_{23}\dot{q}_2\dot{q}_3 + \ldots + 2\mathrm{A}_{2k}\dot{q}_2\dot{q}_k \\ &+ \ldots\ldots + \mathrm{A}_{kk}\dot{q}_k^2 \\ &+ \mathrm{A}_1\dot{q}_1 + \mathrm{A}_2\dot{q}_2 + \ldots + \mathrm{A}_k\dot{q}_k + \mathrm{A}, \end{aligned} \quad (2)$$

where the As are functions of the qs and t, we find $p_1, p_2, \ldots, p_k$ $(=\partial\mathrm{T}/\partial\dot{q}_1, \partial\mathrm{T}/\partial\dot{q}_2, \ldots, \partial\mathrm{T}/\partial\dot{q}_k)$ in the form

$$\left.\begin{aligned} p_1 &= g_1(a_1, a_2, \ldots, a_k, b_1, b_2, \ldots, b_k, t) \\ p_2 &= g_2(a_1, a_2, \ldots, a_k, b_1, b_2, \ldots, b_k, t) \\ &\ldots\ldots \\ p_k &= g_k(a_1, a_2, \ldots, a_k, b_1, b_2, \ldots, b_k, t) \end{aligned}\right\} \quad (3)$$

Now by means of (1) we can express *any* k constants, let us say the bs, in terms of $q_1, q_2, \ldots, q_k, t$, and the remaining constants, thus obtaining by substitution in (3)

$$\left.\begin{aligned} p_1 &= \mathrm{G}_1(q_1, q_2, \ldots, q_k, t, a_1, a_2, \ldots, a_k) \\ p_2 &= \mathrm{G}_2(q_1, q_2, \ldots, q_k, t, a_1, a_2, \ldots, a_k) \\ &\ldots\ldots \\ p_k &= \mathrm{G}_k(q_1, q_2, \ldots, q_k, t, a_1, a_2, \ldots, a_k) \end{aligned}\right\} \quad (4)$$

The canonical equations are of the type

$$\dot{p} = -\frac{\partial\mathrm{H}}{\partial q}, \quad \dot{q} = \frac{\partial\mathrm{H}}{\partial p}, \quad (5)$$

where, however, H is supposed to be expressed as the function of the ps, the qs, and t, which we obtain by finding $\dot{q}_1, \dot{q}_2, \ldots, \dot{q}_k$, in terms of the ps, the qs, and t, from the equations

$$p_1 = \frac{\partial\mathrm{T}}{\partial\dot{q}_1}, \; p_2 = \frac{\partial\mathrm{T}}{\partial\dot{q}_2}, \ldots, \; p_k = \frac{\partial\mathrm{T}}{\partial\dot{q}_k}, \quad (6)$$

derived from (2), and substituting in (2) and in $\Sigma(p\dot{q})$.

From the ps as expressed in (4) we find for *any* p

$$\frac{dp}{dt} = \frac{\partial p}{\partial q_1}\dot{q}_1 + \frac{\partial p}{\partial q_2}\dot{q}_2 + \ldots + \frac{\partial p}{\partial q_k}\dot{q}_k + \frac{\partial p}{\partial t}, \quad (7)$$

which by the canonical equations (5) can be written

$$\frac{\partial\mathrm{H}}{\partial q} + \frac{\partial\mathrm{H}}{\partial p_1}\frac{\partial p}{\partial q_1} + \frac{\partial\mathrm{H}}{\partial p_2}\frac{\partial p}{\partial q_2} + \ldots + \frac{\partial\mathrm{H}}{\partial p_k}\frac{\partial p}{\partial q_k} = -\frac{\partial p}{\partial t}. \quad (8)$$

But by direct differentiation of H, regarded, as stated above, as a function of the ps, the qs, and t, we obtain

$$\frac{\partial H}{\partial q}+\frac{\partial H}{\partial p_1}\frac{\partial p_1}{\partial q}+\frac{\partial H}{\partial p_2}\frac{\partial p_2}{\partial q}+\ldots\ldots+\frac{\partial H}{\partial p_k}\frac{\partial p_k}{\partial q}=\left(\frac{\partial H}{\partial q}\right). \quad . \quad . \quad . \quad (9)$$

The q in this equation corresponds to the p of the last, and the $\partial H/\partial q$ in brackets on the right is the partial derivative of H with respect to q, taken after the values of the ps in H have been replaced by their expressions in terms of the qs and t as given in (4). Subtracting now (8) from (9), we get

$$\frac{\partial H}{\partial p_1}\left(\frac{\partial p_1}{\partial q}-\frac{\partial p}{\partial q_1}\right)+\frac{\partial H}{\partial p_2}\left(\frac{\partial p_2}{\partial q}-\frac{\partial p}{\partial q_2}\right)+\ldots\ldots+\frac{\partial H}{\partial p_k}\left(\frac{\partial p_k}{\partial q}-\frac{\partial p}{\partial q_k}\right)=\left(\frac{\partial H}{\partial q}\right)+\frac{\partial p}{\partial t}. \quad . \quad . \quad (10)$$

There are of course k such equations.

The left side of (10) at once suggests the idea of a function S of the co-ordinates, $q_1, q_2, \ldots, q_k$, the time t, and k constants (either the as, say, or k other constants $c_1, c_2, \ldots, c_k$, related to the as) such that

$$p_1=\frac{\partial S}{\partial q_1}, \; p_2=\frac{\partial S}{\partial q_2}, \; \ldots, \; p_k=\frac{\partial S}{\partial q_k}\,; \quad . \quad . \quad . \quad . \quad . \quad (11)$$

for the left side of (10) will then vanish identically, since the quantities $\partial H/\partial p_1$, $\partial H/\partial p_2$, $\partial H/\partial p_k$, which are the velocities $\dot{q}_1, \dot{q}_2, \ldots, \dot{q}_k$, are supposed to be finite, and $\partial^2 S/\partial q_1 \partial q_2 = \partial^2 S/\partial q_2 \partial q_1$, etc.

If, moreover, S be such that

$$\frac{\partial S}{\partial t}=-H, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where H is now supposed to be expressed in terms of $q_1, q_2, \ldots, q_k, t$, and the constants $a_1, a_2, \ldots, a_k$, then $\partial H/\partial q$ taken on this supposition is the quantity denoted by $(\partial H/\partial q)$ in (10), and we have, dropping the brackets,

$$-\frac{\partial H}{\partial q}=\frac{\partial}{\partial q}\frac{\partial S}{\partial t}=\frac{\partial}{\partial t}\frac{\partial S}{\partial q}=\frac{\partial p}{\partial t}\,; \quad . \quad . \quad . \quad . \quad . \quad (13)$$

so that the right-hand side of (10) also vanishes.

Taking then H in its usual form $H(p_1, p_2, \ldots, p_k, t, q_1, q_2, \ldots, q_k)$ and replacing the ps by their values, the partial derivatives of S with respect to the qs, we obtain as the necessary and sufficient condition fulfilled by the function S, the differential equation

$$\frac{\partial S}{\partial t}+H\left(\frac{\partial S}{\partial q_1}, \frac{\partial S}{\partial q_2}, \ldots, \frac{\partial S}{\partial q_k}, t, q_1, q_2, \ldots, q_k\right)=0. \quad . \quad . \quad (14)$$

The function $S(q_1, q_2, \ldots, q_k, t, a_1, a_2, \ldots, a_k)$, including as it does k arbitrary constants, is the "complete integral" of this equation, and fulfils

k conditions of the form expressed in (10) which result from the canonical equations. If the k constants in S are not "distinct," being connected by a relation or relations, S will still be a function of $q_1, q_2, \ldots, t$, and a smaller number of distinct constants, which satisfies (10), and now represents a family of solutions.

It will be noticed that

$$\frac{dS}{dt} = \frac{\partial S}{dt} + \Sigma \frac{\partial S}{\partial q}\dot{q} = \Sigma(p\dot{q}) - H = L, \quad . \quad . \quad . \quad . \quad (15)$$

where L is what is sometimes called the Lagrangian function or the "kinetic potential." S is Hamilton's "principal function." It may be determined by the equation

$$S = \int_0^t \{\Sigma(p\dot{q}) - H\}dt, \quad . \quad . \quad . \quad . \quad . \quad (16)$$

or otherwise, as will be noticed in the sequel.

2. *Proof that the partial derivatives of the complete integral with respect to the constants are themselves constants.*—We calculate the time-rate of variation of the partial derivative of S with respect to any one of the k distinct constants, say a_j. We have

$$\frac{d}{dt}\frac{\partial S}{\partial a_j} = \frac{\partial}{\partial t}\frac{\partial S}{\partial a_j} + \sum_i\left(\dot{q}_i \frac{\partial}{\partial q_i}\frac{\partial S}{\partial a_j}\right) = \frac{\partial}{\partial t}\frac{\partial S}{\partial a_j} + \sum_i\left(\frac{\partial H}{\partial p_i}\frac{\partial}{\partial q_i}\frac{\partial S}{\partial a_j}\right), \quad . \quad . \quad (1)$$

where $\partial H/\partial p_i$, derived from H expressed in terms of the ps, the qs, and t, is put for $\dot{q}_i$, by the canonical equation for $\dot{q}_i$. Now (1) can be written

$$\frac{d}{dt}\frac{\partial S}{\partial a_j} = -\frac{\partial H}{\partial a_j} + \sum_i \frac{\partial H}{\partial p_i}\frac{\partial p_i}{\partial a_j} = 0, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

for the as only enter into H by the replacement of the ps by the corresponding partial derivatives of S. We have therefore by (2)

$$\frac{\partial S}{\partial a_j} = c_j, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

a constant. If a_j be an initial co-ordinate, c_j is $-b$ if b_j be the corresponding initial momentum.

There are k such equations, and these are the finite equations of motion. For they enable the k co-ordinates $q_1, q_2, \ldots, q_k$ to be expressed in terms of t and the $2k$ constants $a_1, a_2, \ldots, a_k, b_1, b_2, \ldots, b_k$, as in (1), § 1.

If, however, $H = h$, a constant, so that $H = h$ is a first integral of the equations of motion, then h becomes one of the constants in the expression of S, which has now the form given by

$$S = -ht + W(q_1, q_2, \ldots, q_k, a_1, a_2 \ldots, a_{k-1}, h), \quad . \quad . \quad (4)$$

where W denotes a function of the quantities within the brackets. We have $p_i = \partial W/\partial q_i$, and $\partial S/\partial h = -t + \partial W/\partial h = \text{const}$. The partial derivative $\partial S/\partial h$ is thus constant; it enables the time of passage from the initial configuration to that at the instant considered to be found. See § 4 below.

3. *Derivation of Hamilton's second differential equation.*—It is only necessary to substitute in the canonical equations $H' = -H$ to obtain as typical equations

$$\dot{q} = -\frac{\partial H'}{\partial p}, \quad \dot{p} = \frac{\partial H'}{\partial q}, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which q, p are related to H′ as p, q are to H in (5), § 1. It might be inferred that a function S′ of $p_1, p_2, \ldots, p_k, t$ and the constants $a_1, a_2, \ldots, a$ which gives k equations of the form

$$q_i = \frac{\partial S'}{\partial p_i}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

from which the integrals of the canonical equations can be deduced, may be found by the differential equation *

$$\frac{\partial S'}{\partial t} + H'\left(\frac{\partial S'}{\partial p_1}, \frac{\partial S'}{\partial p_2}, \ldots, \frac{\partial S'}{\partial p_k}, p_1, p_2, \ldots, p_k, t\right) = 0. \quad . \quad (3)$$

A formal proof of this theorem may be constructed in precisely the same manner as (14), § 1, was established. By means of (3), § 1, the constants $b_1, b_2, \ldots, b_k$ are expressed in terms of $p_1, p_2, \ldots, p_k, t$ and the constants $a_1, a_2, \ldots, a_k$. Thus we get for a typical q (q_i say)

$$q_i = F_i(p_1, p_2, \ldots, p_k, t, a_1, a_2, \ldots, a_k). \quad . \quad . \quad . \quad (4)$$

From the qs as thus expressed we find

$$\dot{q}_i = \frac{\partial q_i}{\partial p_1}\dot{p}_1 + \frac{\partial q_i}{\partial p_2}\dot{p}_2 + \ldots + \frac{\partial q_i}{\partial q_k}\dot{q}_k + \frac{\partial q}{\partial t}, \quad . \quad . \quad . \quad (5)$$

which by the canonical equations (1) can be written

$$\frac{\partial H'}{\partial p_i} + \sum_j \frac{\partial H'}{\partial q_j}\frac{\partial q_i}{\partial p_j} = -\frac{\partial q_i}{\partial t}. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This equation corresponds to (8), § 1.

Direct differentiation of H′ regarded as a function of the ps, the qs, and t gives

$$\frac{\partial H'}{\partial p_i} + \sum_j \frac{\partial H'}{\partial q_j}\frac{\partial q_j}{\partial p_i} = \left(\frac{\partial H'}{\partial p_i}\right), \quad . \quad . \quad . \quad . \quad . \quad (7)$$

* The duality represented by this equation and (14), § 1, is one of the first results of the contact transformation theory of dynamics. For the addition of $\{\Sigma(\dot{p}q) + H\}dt$ to $\{\Sigma(p\dot{q}) - H\}dt$ gives the perfect differential $\Sigma(pdq + qdp)$. Hence we get $S' = \int\{\Sigma(\dot{p}q) + H\}dt$, and so $\partial S'/\partial t = H$, $q = \partial S'/\partial p$.

where the expression on the right denotes the partial derivative of H′ with respect to p_i taken after the values of the qs in H′ have been supposed replaced by their expressions as given in (4). Subtracting (25) from (26) we obtain

$$\sum_j \frac{\partial \mathrm{H}'}{\partial q_j}\left(\frac{\partial q_j}{\partial p_i}-\frac{\partial q_i}{\partial p_j}\right)=\left(\frac{\partial \mathrm{H}'}{\partial p_i}\right)+\frac{\partial q_i}{\partial t}. \quad . \quad . \quad . \quad . \quad (8)$$

This equation suggests a function S′ of the momenta $p_1, p_2, \ldots, p_k, t, a_1, a_2, \ldots, a_k$ such that

$$q_1=\frac{\partial \mathrm{S}'}{\partial p_1},\ q_2=\frac{\partial \mathrm{S}'}{\partial p_2},\ \ldots\ldots,\ q_k=\frac{\partial \mathrm{S}'}{\partial p_k}. \quad . \quad . \quad . \quad (9)$$

With these values of the qs the left-hand side of (8) vanishes. If, moreover, S′ be such that

$$\frac{\partial \mathrm{S}'}{\partial t}=-\mathrm{H}', \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where H′ is supposed expressed in terms of the ps, the as, and t, $\partial \mathrm{H}'/\partial p_i$ taken on this supposition is the quantity which appears in (8) as $(\partial \mathrm{H}'/\partial p_i)$, and we have

$$-\frac{\partial \mathrm{H}}{\partial p_i}=\frac{\partial}{\partial p_i}\frac{\partial \mathrm{S}'}{\partial t}=\frac{\partial}{\partial t}\frac{\partial \mathrm{S}'}{\partial p_i}=\frac{\partial q_i}{\partial t}, \quad . \quad . \quad . \quad . \quad . \quad (11)$$

so that the right-hand side of (8) vanishes. If, then, we replace the qs in H, as usually expressed, by the values given in (9) we obtain the partial differential equation

$$\frac{\partial \mathrm{S}'}{\partial t}-\mathrm{H}\left(p_1, p_2, \ldots, p_k, \frac{\partial \mathrm{S}'}{\partial p_1}, \frac{\partial \mathrm{S}'}{\partial p_2}, \ldots, \frac{\partial \mathrm{S}'}{\partial p_k}, t\right)=0. \quad . \quad . \quad (12)$$

The function $\mathrm{S}'(p_1, p_2, \ldots, p_k, t, a_1, a_2, \ldots, a_k)$ referred to above is the complete integral of this equation, and fulfils all the conditions which result from the canonical equations.

As in § 2 (for S) it can be shown that the partial derivatives of S′ with respect to the as are all constants if these constants are distinct.

4. *Relations of the two forms of the principal function. Calculation of the functions in different cases. Derivation of the modified Lagrangian function for ignoration of co-ordinates.*—An equation similar to (15), § 1, holds for S′ also. For we have

$$\frac{d\mathrm{S}'}{dt}=\frac{\partial \mathrm{S}'}{\partial t}+\sum\left(\frac{\partial \mathrm{S}'}{\partial p}\dot{p}\right)=\mathrm{H}+\Sigma(q\dot{p}) \quad . \quad . \quad . \quad . \quad (1)$$

so that

$$\mathrm{S}'=\int_0^t\{\mathrm{H}+\Sigma(q\dot{p})\}dt. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Taken along with (15), § 1, this gives

$$\frac{d}{dt}(\mathrm{S}+\mathrm{S}')=\Sigma(p\dot{q}+\dot{p}q), \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

so that

$$S+S'=\Sigma(pq)-\Sigma(p_0q_0). \quad . \quad . \quad . \quad . \quad . \quad (4)$$

We have also, of course,

$$S-S'=\int_0^t\{\Sigma(p\dot{q}-\dot{p}q)-2H\}dt; \quad . \quad . \quad . \quad . \quad (4')$$

but this can hardly be regarded as differing from (4). If we integrate $-\Sigma(\dot{p}q)dt$ by parts we get $\Sigma pq+\int\Sigma(p\dot{q})dt$, so that (5) becomes (4).

Besides the relation already known $\partial S/\partial t=-\partial S'/\partial t$, we obtain from (4) the equation (taking now S, S′ as the "indefinite integrals")

$$\frac{\partial S}{\partial a_j}=-\frac{\partial S'}{\partial a_j}=b_j, \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where b is a constant. For, having regard to the variables in terms of which S and S′ are expressed according to their definitions, we have by (4)

$$\frac{\partial S'}{\partial a_j}=\sum\left(p\frac{dq}{\partial a_j}\right)-\left\{\frac{\partial S}{\partial a_j}+\sum\left(\frac{\partial S}{\partial q}\frac{\partial q}{\partial a_j}\right)\right\}=-\frac{\partial S}{\partial a_j}. \quad . \quad . \quad . \quad (6)$$

It is here assumed, however, that the constant a_j is not a momentum which is constant in consequence of the non-appearance of the corresponding co-ordinate in H. If a_j be such a momentum a, and the co-ordinate be q_j, we shall have

$$\frac{\partial S'}{\partial a}=\sum\left(p\frac{\partial q}{\partial a}\right)+q_j-\left\{\frac{\partial S}{\partial a}+\sum\left(\frac{\partial S}{\partial q}\frac{\partial q}{\partial a}\right)\right\}=q_j-\frac{\partial S}{\partial a}. \quad . \quad . \quad (7)$$

Thus while $\partial S/\partial a$ is a constant, $\partial S'/\partial a$ is not: an example is given below. I have not seen this theorem before, probably because S′ is little used. A more complete account of these constants is deferred.*

In the determination of S or S′ in actual cases I prefer, as a rule, to employ (16) of § 1, or (2) of the present article, rather than to set up the related differential equations for $\partial S/\partial q$, or $\partial S'/\partial p$, and solve them, which is the usual practice. Of course the two processes are fundamentally the same, but the calculation of the time-integrals, in the equations referred to, is the more intelligible, at least to the student. It may be noticed that (16), § 1, gives at once, as a matter of course, when certain co-ordinates

* *Note added April* 18.—Reciprocal dynamical theorems are easily derived by means of the two functions. Thus, let the constants in S, S′ be the as (the initial co-ordinates). Then, for the initial momentum b_i, we have $b_i=-\partial S/\partial a_i=\partial S'/\partial a_i$, and therefore $\partial b_i/\partial p_j[=\partial(\partial S'/\partial p_j)/\partial p_j]=\partial q_j/\partial a^i$: also $\partial b_i/\partial q_j[=-\partial(\partial S/\partial q_j)/\partial a_i]=-\partial p_j/\partial a_i$. If the constants in both functions be the bs (the initial momenta), two other theorems are obtained in a similar way, namely, $\partial a_i/\partial p_j=-\partial q_i/\partial b_i$; $\partial a_i/\partial q_j=\partial p_i/\partial b_i$.

I find, however, that this mode of deriving (two of) these relations has been anticipated by v. Helmholtz (*Crelle*, 1886), who also gave physical interpretations. Applications are given by Lamb (*Lond. Math. Soc.*, 1888).

$q_1, q_2, \ldots, q_g$ are absent from the expression for H, and when, therefore, $\partial H/\partial q_1, \partial H/\partial q_2, \ldots, \partial H/\partial q_g$ are all zero (so that $\dot{p}_1, \dot{p}_2, \ldots, \dot{p}_g$ are all zero, and $p_1, p_2, \ldots, p_g$ are constants $\alpha_1, \alpha_2, \ldots, \alpha_g$),

$$S = \alpha_1 q_1 + \alpha_2 q_2 + \ldots + \alpha_g q_g + \int \{\Sigma(p\dot{q}) - H\} dt, \quad . \quad . \quad (8)$$

where $\Sigma(p\dot{q})$ refers to the remaining co-ordinates, $q_{g+1}, \ldots, q_k$, and H is expressed in terms of the ps (in this case $\alpha_1, \alpha_2, \ldots, \alpha_g, p_{g+1}, \ldots, p_k$) and t, with the co-ordinates $q_{g+1}, \ldots, q_k$. Thus S, determined by the complete integral of (14), § 1, has the form, with remaining constants $a_1, a_2, \ldots, a_{k-g}$,

$$S = \alpha_1 q_1 + \alpha_2 q_2 + \ldots + \alpha_g q_g + \Phi(q_{g+1}, \ldots, q_k, t, \alpha_1, \alpha_2, \ldots, \alpha_g, a_1, a_2, \ldots a_{k-g}). \quad (9)$$

If H is a constant, h, the function Φ has the form

$$\Phi = -ht + \Psi(q_{g+1}, \ldots, q_k, \alpha_1, \ldots, \alpha_{g-1}, h, a_1, \ldots, a_{k-g}). \quad . \quad (9')$$

If the zero of time be $t = 0$, we have

$$t = \frac{\partial}{\partial h}(\Psi - S) = \frac{\partial \Psi}{\partial h} - \gamma, \quad . \quad . \quad . \quad . \quad . \quad (10)$$

since $\partial S/\partial h$ is a constant, γ say.

In (9′) the g constants, $a_{g+1}, \ldots, a_k$, have been replaced by $\alpha_1, \ldots, \alpha_g$, obtained by putting $p_1 = \alpha_1, \ldots, p_g = \alpha_g$ in (4) of § 1, and eliminating a_{g+1}, etc.

As an example of the process of finding S and S′ we take the case of a planet the co-ordinates of which at any instant are the radius vector r, the heliocentric longitude ϕ, and the co-latitude θ. Here H has a constant value h (which is negative for elliptic motion), and the potential energy is $\mu m/r$, where m is the mass of the planet. We easily find

$$2h = m\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2 \sin^2\theta \,.\, \dot{\phi}^2 - 2\frac{\mu}{r}\right), \quad . \quad . \quad . \quad (11)$$

or, with $q_1 = r$, $p_1 = m\dot{r}$, $q_2 = \theta$, $p_2 = mr^2\dot{\theta}$, $q_3 = \phi$, $p_3 = mr^2 \sin^2\theta \,.\, \dot{\phi}$,

$$2h = \frac{1}{m}\left(p_1{}^2 + \frac{1}{r^2}p_2{}^2 + \frac{1}{r^2 \sin^2\theta}p_3{}^2\right) - 2\frac{\mu m}{r}. \quad . \quad . \quad . \quad (11')$$

The co-ordinate q_3 does not appear in this equation, and therefore we can write $p_3 = \alpha$, a constant. Thus by (16), § 1, we obtain, to a constant,

$$S = -ht + \alpha\phi + \int m(\dot{r}^2 + r^2\dot{\theta}^2)dt = -ht + \alpha\phi + \int m\dot{r}dr + \int mr^2\dot{\theta}d\theta. \quad . \quad (12)$$

But

$$\dot{r}^2 + r^2\dot{\theta}^2 = 2\left(\frac{h}{m} + \frac{\mu}{r}\right) - \frac{\alpha^2}{m^2 r^2 \sin^2\theta}. \quad . \quad . \quad . \quad (13)$$

The value which this gives for $\dot{r}^2$ when θ is constantly $\frac{1}{2}\pi$ at once suggests putting (K = a constant)

$$\dot{r}^2 = 2\left(\frac{h}{m} + \frac{\mu}{r}\right) - \frac{K^2}{r^2}, \quad r^2\dot{\theta}^2 = -\frac{\alpha^2}{m^2r^2\sin^2\theta} + \frac{K^2}{r^2}. \quad . \quad . \quad (14)$$

These give

$$m\dot{r} = \pm m\sqrt{2\left(\frac{h}{m} + \frac{\mu}{r}\right) - \frac{K^2}{r^2}}, \quad mr^2\dot{\theta} = \pm m\sqrt{K^2 - \frac{\alpha^2}{m^2\sin^2\theta}}. \quad . \quad (15)$$

We have thus obtained for $m\dot{r}$ a function of r, and for $mr^2\dot{\theta}$ a function of θ, while the value of $m(\dot{r}^2 + r^2\dot{\theta}^2)$ is left unaltered. The value of the constant K^2 thus introduced is settled by the conditions of the problem. In point of fact mK is by the second of (15) the angular momentum of the planet about the sun. Of course, $K^2 > \alpha^2/m^2\sin^2\theta$.

Equation (12) becomes by (15) (ambiguities of sign understood)

$$S = -ht + \alpha\phi + m\int dr\sqrt{2\left(\frac{h}{m} + \frac{\mu}{r}\right) - \frac{K^2}{r^2}} + m\int d\theta\sqrt{K^2 \quad \frac{\alpha^2}{m^2\sin^2\theta}}, \quad (16)$$

which gives the value of S reduced to quadratures. The three necessary constants involved in S are h, α, K^2. Partial differentiation of this expression with respect to h, gives the time t taken by the system to pass from the configuration at time $t=0$, to that at the instant under consideration: the partial derivatives with regard to α and K^2 give the path.

The usual process is to write (11) in the form

$$\left(\frac{\partial S}{\partial q_1}\right)^2 + \frac{1}{q_1^2}\left(\frac{\partial S}{\partial q_2}\right)^2 + \frac{1}{q_1^2\sin^2 q_2}\left(\frac{\partial S}{\partial q_3}\right)^2 - 2m\left(h + \frac{\mu}{q_1}\right) = 0. \quad . \quad . \quad (17)$$

To solve this differential equation the variables are separated by splitting it into the three,

$$\left(\frac{\partial S}{\partial q_1}\right)^2 + \frac{K^2}{q_1^2} - 2m\left(h + \frac{\mu}{q_1}\right) = 0, \quad \left(\frac{\partial S}{\partial q_2}\right)^2 + \frac{\alpha^2}{\sin^2 q_2} - K^2 = 0, \quad \frac{\partial S}{\partial q_3} = \alpha, \quad . \quad (18)$$

which are then solved separately. It is easy to see that the result of integrating

$$dS = \frac{\partial S}{\partial t}dt + \frac{\partial S}{\partial q_1}dq_1 + \frac{\partial S}{\partial q_2}dq_2 + \frac{\partial S}{\partial q_3}dq_3 \quad . \quad . \quad . \quad . \quad (19)$$

agrees with (16), but the process is not quite so direct and clear. The only difference, however, is that in the first process the terms $-ht + \alpha\phi$ come as a matter of course; for the others $m\dot{r}$, $mr^2\dot{\theta}$ are determined in the first process, and their equivalents $\partial S/\partial q_1$, $\partial S/\partial q_2$ in the second. The ambiguities of sign which are common to both methods correspond of course to different conditions of the problem, and must be carefully considered when applications are worked out in detail.

We now find the function S′ for the same problem. Starting from (2) of the present article,

$$S' = \int_0^t \{H + \Sigma(q\dot{p})\} dt,$$

we get for the same limits of integration

$$S' = ht + \int q_1 dp_1 + \int q_2 dp_2, \quad . \quad . \quad . \quad . \quad (20)$$

since, as we have seen, $p_3\ (= mr^2 \sin^2\theta \,.\, \dot{\phi})$ is a constant, denoted above by a. But by (14), since $q_1 = r$, $p_1 = m\dot{r}$, $q_2 = \theta$, $p_2 = mr^2\dot{\theta}$, we have

$$q_1^2 - \frac{2m^2\mu}{p_1^2 - 2mh} q_1 + \frac{m^2 K^2}{p_1^2 - 2mh} = 0, \quad \sin^2 q_2 = \frac{a^2}{m^2 K^2 - p_2^2}, \quad . \quad . \quad (21)$$

The first of these gives

$$q_1 = \frac{m}{p_1^2 - 2mh} \left\{ \mu \pm \sqrt{\mu^2 - \frac{K^2}{m^2}(p_1^2 - 2mh)} \right\}, \quad . \quad . \quad . \quad (22)$$

and the second

$$q_2 = \sin^{-1}\left(\pm \frac{a}{\sqrt{m^2 K^2 - p_2^2}} \right). \quad . \quad . \quad . \quad . \quad (22')$$

Thus we obtain for S′

$$S' = ht + \int \frac{m dp_1}{p_1^2 - 2mh} \left\{ \mu \pm \sqrt{\mu^2 - \frac{K^2}{m^2}(p_1^2 - 2mh)} \right\} + \int dp_2 \sin^{-1}\left(\pm \frac{a}{\sqrt{m^2 K^2 - p_2^2}} \right). \quad (23)$$

In the usual mode of discussion we should begin with the differential equation, written down from (11),

$$(p_1^2 - 2hm)\left(\frac{\partial S'}{\partial p_1}\right)^2 - 2\mu m^2 \frac{\partial S'}{\partial p_1} + p_2^2 + \frac{a^2}{\sin^2 \frac{\partial S'}{\partial p_2}} = 0, \quad . \quad . \quad . \quad (24)$$

which is then split into the two,

$$(p_1^2 - 2mh)\left(\frac{\partial S'}{\partial p_1}\right)^2 - 2\mu m^2 \frac{\partial S'}{\partial p_1} + m^2 K^2 = 0, \quad \frac{\partial S'}{\partial p_2} = \sin^{-1}\left(\pm \frac{a}{\sqrt{m^2 K^2 - p_2^2}} \right). \quad (25)$$

From this $\partial S'/\partial p_1$ is found by solving the quadratic, and with $\partial S'/\partial p_2$ and $\partial S'/\partial t$ is used to form the expression on the right of

$$dS' = \frac{\partial S'}{\partial p_1} dp_1 + \frac{\partial S'}{\partial p_2} dp_2 + \frac{\partial S'}{\partial t} dt, \quad . \quad . \quad . \quad . \quad (26)$$

in which the variables are separated. Equation (23) is then found by integration.

We obtain here a good example of the theorem stated in (7′) above. From (16) we obtain

$$\frac{\partial S}{\partial a} = \phi - \int \frac{a d\theta}{\sqrt{K^2 - \dfrac{a^2}{m^2 \sin^2\theta}}} \frac{1}{m^2 \sin^2\theta},$$

But by (15)

$$r^2\dot{\theta} = \frac{p_2}{m} = \sqrt{\mathrm{K}^2 - \frac{\alpha^2}{m^2 \sin^2\theta}}, \text{ and } \alpha = mr^2 \sin^2\theta \,.\, \dot{\phi}.$$

Hence

$$\frac{\partial \mathrm{S}}{\partial \alpha} = \phi - \int \dot{\phi} dt = 0, \quad . \quad . \quad . \quad . \quad . \quad (27)$$

if ϕ, as it is here taken, is zero when $t=0$. This gives

$$\frac{\partial \mathrm{S}'}{\partial \alpha} = \phi - \frac{\partial \mathrm{S}}{\partial \alpha} = \phi, \quad . \quad . \quad . \quad . \quad . \quad (28)$$

the value of the co-ordinate associated with α.

This may be verified by differentiation of S′ with respect to α, as follows. From (23) we get

$$\frac{\partial \mathrm{S}'}{\partial \alpha} = \int \frac{dp_2}{\sqrt{m^2\mathrm{K}^2 - p_2{}^2 - \alpha^2}} = \int \frac{dp_2 \tan\theta}{\alpha}, \quad . \quad . \quad . \quad . \quad (29)$$

since $m^2\mathrm{K}^2 - p_2{}^2 = \alpha^2 / \sin^2\theta$. But we have also from this expression

$$p_2 dp_2 = \frac{\alpha^2}{\sin^2\theta} \frac{d\theta}{\tan\theta},$$

so that

$$\frac{\tan\theta}{\alpha} dp_2 = \frac{\alpha d\theta}{p_2 \sin^2\theta} = \frac{mr^2 \sin^2\theta \,.\, \dot{\phi} \,.\, d\theta}{mr^2 \sin^2\theta \,.\, \dot{\theta}} = \dot{\phi} dt, \quad . \quad . \quad . \quad (30)$$

Thus we get, as in (28),

$$\frac{\partial \mathrm{S}'}{\partial \alpha} = \int \dot{\phi} dt = \phi,$$

reckoning from the same zero as before.

Finally we notice that (9) leads directly to the modified Lagrangian function L_1, to be used when certain co-ordinates (q_1, q_2, ... , q_g, above), which are represented in H by their velocities only, are ignored. We have

$$\mathrm{L}_1 = \mathrm{L} - \alpha_1 q_1 - \alpha_2 q_2 - \ldots\ldots - \alpha_g q_g, \quad . \quad . \quad . \quad . \quad (31)$$

where L denotes the unmodified function. For if we write

$$\mathrm{S}_1 = \mathrm{S} - \alpha_1 q_1 - \alpha_2 q_2 - \ldots\ldots - \alpha_g q_g, \quad . \quad . \quad . \quad . \quad (32)$$

any q of suffix higher than g enters only in S_1, so that S_1 is the principal function for the non-ignored co-ordinates. It is understood, of course, that wherever any of the velocities $\dot{q}_1, \dot{q}_2, \ldots, \dot{q}_g$ occur in L_1 they are to be replaced by their values in terms of $q_{g+1}, q_{g+2}, \ldots, q_k, \dot{q}_{g+1}, \dot{q}_{g+2}, \ldots, \dot{q}_k$, given by the equations

$$\frac{\partial \mathrm{T}}{\partial \dot{q}_1} = \alpha_1, \quad \frac{\partial \mathrm{T}}{\partial \dot{q}_2} = \alpha_2, \ldots\ldots, \quad \frac{\partial \mathrm{T}}{\partial \dot{q}_g} = \alpha_g, \quad . \quad . \quad . \quad . \quad (33)$$

derived from T as expressed in (2), § 1.

(*Issued separately May* 17, 1912.)

XV.—The Fata Morgana. By Professor F. A. Forel, Morges, Switzerland.

Abstract of an Address delivered before the Society on July 14, 1911; translated by Professor C. G. Knott.

Among optical phenomena which originate over the surface of water there is one so ill-defined and ill-observed as to be still mysterious; till now it has received no valid explanation. The Italians call it the Fata Morgana. Under conditions still lacking precise description, there appear on the far side of the Straits of Messina certain fantastic visions, fortresses and castles of unknown cities, which seem to emerge from the sea, soon to vanish again. These are the "palaces" of the "fairy Morgana," which appear and disappear at the capricious stroke of the magician's wand.

Most of the accounts of the phenomenon are founded on the extravagant description and the amazing picture published in 1773 by the Dominican friar Don Antonio Minasi, professor of botany at the Roman College of Sapienzia. This drawing, with its incoherent groupings of castles and boats, reflected and refracted at random in a manner quite inconsistent with physical possibilities, was largely the creation of the distorted imagination of an artist who did not understand in the least the wonderful illusion.

To appreciate the confusion which reigns in the scientific world in regard to this phenomenon, the reader need only refer to the chapter on the Fata Morgana in Pernter's admirable work on *Meteorological Optics* (Vienna, 1910). There he will be impressed with the uncertainty of the conclusions reached by the author after a study of the insufficient and contradictory documents to which he had access.

Professor V. E. Boccara of Reggio has endeavoured to bring some little order into the question,* but with doubtful success, as a critical examination of his drawings and diagrams will probably prove.

In 1854 Charles Dufour rendered a signal service by his recognition of the phenomenon of the Fata Morgana on the Lake of Geneva;† and his

* "La Fata Morgana," *Mem. della Societa degli spettroscopisti Italiani*, xxxi., Catania, 1902; contains a full bibliography of the question, with extracts from the principal observations.

† "Mirages et réfractions anormales sur le lac Léman," *Bull. Soc. Vaud. Sc. nat.*, xxxii. 271, Lausanne, 1853–56.

descriptions and explanation were free from the exaggerations and contradictions of his predecessors. He showed the illusion to a pupil of his, a small schoolboy of thirteen; and since then, following the instructions of my beloved master, I have seen the Fata Morgana again and again every springtime, some four or five hundred times in all. I can therefore speak of it as a familiar friend.

In my monograph, *Le Léman* (Lausanne, 1895), and in my note of 1896,* I have endeavoured to lay down the determining conditions of the phenomenon. Since then I have taken a further step in the co-ordination of the facts, and I propose to give to-day the principal features of the mirage, which may perhaps serve as a better foundation for the mathematical study of the phenomenon, which I leave in the care of my mathematical and physical friends.†

The physical conditions which determine the appearance of the Fata Morgana are always the same. When, under the bright skies of spring or early summer, the lake lies calm or slightly rippled by light and intermittent puffs of air, there appears at times during the afternoon when the air is warmer than the water a phantom-like transformation of the scenery of the opposite coast. Over a horizontal stretch of twenty or thirty degrees, the familiar details of the coast-line, which may be from ten to thirty kilometres distant, become strangely transformed. Resting on the water horizon, and bounded above by another horizontal line at a height of a few minutes of arc, there comes into view a vertically striped band or striated zone (*zone striée*), seemingly composed of rectangles placed side by side, of varied tints and hues. It might be compared to the distant cliffs of Dover as seen by travellers crossing the Channel; or to a great city built on the shore, with blocks of houses ranged along the quays, a Genoa, a Naples, or a Constantinople, miraculously rearing itself in a region known to be occupied by a few

* "Réfractions et Mirages: passage d'un type à l'autre," *Bull. Soc. Vaud. Sc. nat.*, xxxii. 271, Lausanne, 1898.

† Refraction phenomena are not always presented in nature so clearly or so simply as might be desired. There may be no doubt as to the general nature of the mirage, which may nevertheless be difficult to interpret. The phenomena originate at a far distance, in air more or less saturated with water vapour, and frequently masked by a veil of fog. The observations are beset with great difficulties. Had it not been so I should not have spent more than fifty years in arriving at the present explanation. The observation of refraction phenomena over the surface of a lake demands an intimate acquaintance with all its characteristics. An occasional traveller spending a few days on its shores cannot be familiar enough with the lake scenery to enable him to know what changes, if any, may have occurred. Prolonged residence in the neighbourhood of the lake and a keen interest in the ever-shifting illusions are essential to a complete study of the phenomena.

scattered huts. Such are the palaces which the fairy Morgana creates before our wondering eyes.

The position of its first appearing varies with each occasion. It comes into view now here, now there. It does not remain steady, but moves more or less slowly in one or other direction, never lingering in the same neighbourhood for more than ten or twenty minutes. As the mirage passes on towards another part of the horizon, the coast-line recovers its ordinary aspect.

The phenomenon as a whole does not last long. Barely an hour will be consumed as it moves from one end to the other of the half circle of the opposite coast. I have never seen it before noon, and never after six o'clock in the evening.

The Fata Morgana is visible only to an observer whose eye is a few metres above the level of the lake. The best height would seem to be from two to four metres (six to twelve feet). A shift of a foot or two above or below the best position in any given case is sufficient to make the phenomenon disappear. This limited range in the position of the eye which ensures the visibility of the mirage at once explains the astonishing scarcity of good observations.

Following Dufour, we are in the habit of speaking of the Fata Morgana as being due to abnormal refraction, although we know perfectly well that there is nothing abnormal in natural phenomena. Given the conditions, the consequences necessarily follow.

"Abnormal" though we call it, the Fata Morgana comes between two phenomena which we shall call normal, because they are frequent and easily observed. It succeeds the one and precedes the other.

Under the category of optical refractions in air over the surface of a lake, I regard as "normal" phenomena those which accompany refraction in air over water whose temperature is either higher or lower than that of the superposed air. When the air is cooled by contact with cold water, the successive layers of air rise in temperature from below upwards. This might be called the *direct* thermal gradient. When, on the other hand, the lower layers are warmed by contact with warmer water, the thermal gradient is *inverse,* the temperature in the air falls as the height increases through a limited stratum of air.

The mirage phenomena which accompany *refraction over warm water* are the most frequent. This condition holds throughout the whole day during autumn and winter, and during the morning hours in springtime and part of summer. The air is then colder than the surface on which it rests; the thermal gradient is of the inverse type, and the curve of a refracted ray of light is concave above. The characteristic optical accompaniments are:

1. A depression of the plane of the apparent horizon of the lake * below its normal position; the apparent horizon is lower than the true horizon.
2. The apparent exaggeration of the rotundity of the earth, which becomes evident to the eye, although normally it is unrecognisable.
3. The approach of the circle of the horizon, much less distant than it ought to be according to the height of the observer's eye above the surface of the water.
4. The apparent exaggeration of the crests of waves, which show like crenations along the line of the horizon.
5. The phenomenon of "mirage": "the mirage of the desert." Objects lying low over the surface of the water and situated beyond the circle of the horizon are seen as inverted images below Bravais' "ligne de partage." † These images lie in the zone which separates the "ligne de partage" from the apparent horizon of the lake.

These details become more marked as the difference of temperature between the cold air and the warm water is increased.

The phenomena associated with *refraction over cold water* are rarer than the preceding. They appear only during the afternoon hours of warm days in spring and summer, and occasionally in the morning hours of very hot days in the height of summer. In this case the air is warmer than the water, and the lower layers of air cooled by contact with the water are characterised by a thermal gradient of the direct type. The curve of a refracted ray of light is concave below. The characteristic optical accompaniments are:

1. An apparent elevation of the plane of the horizon above the normal position; the apparent horizon is higher than the true horizon.
2. The apparent concavity of the surface of the lake, resembling a broad valley rising with gentle slopes towards the margin.
3. The apparent extension of the circle of the horizon; distant boats, which to the observer's eye should have been on the circle of the

* The true horizon is the tangent cone to the surface of the lake, the vertex of the cone being at the eye of the observer, and the calculation being made on the assumption of no refraction; the circle of the true horizon is the curve of contact of the cone with the surface of the lake. The cone and circle of the apparent horizon are similarly defined in terms of the rays of light as they enter the eye after having been displaced by atmospheric refraction. These cones are so flat that they may be spoken of as planes.

† This is the line which separates the erect and inverted images in the usual mirage.

horizon or beyond it, appear to be at the bottom of the illusory valley of water well within the apparent circle of the horizon.

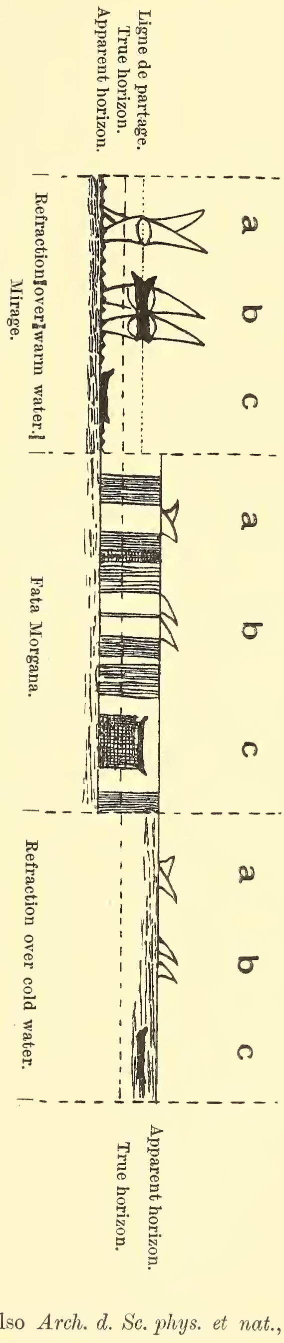
4. The visibility of the lower parts of the opposite coast, which normally should have been hidden by the rotundity of the water surface. For example, the Château de Chillon at 34 kilometres distance appeared to the spectator to be resting on the quay of Morges.
5. The reduction in relative height or dwarfing of the lower parts of the opposite coast.

These details are seen only when the phenomenon becomes established. They may be preceded by what I have called the *mirage over warm water* (*Le Léman*, p. 532, etc.). I do not dwell on it here, because it is not necessary for the purpose of this lecture.

I propose to confine my attention to the principal normal accompaniments of these refractions, namely, the depression or elevation of the apparent horizon according as the water surface is warmer or colder than the air above.

When investigating the variations of the apparent horizon,* I have carefully determined the position of the true horizon by telescopic measurement of the positions of a mountain top and its reflection in an artificial horizon. Midway between these positions, after correction for the dip of the horizon, lies the true horizon. The hill chosen was the Dent d'Oche, which is distant 24·4 kilometres from Morges, and rises beyond the opposite coast to about 2221 metres above sea-level.

Referred to this true horizon, the apparent horizon has been found to vary between the



a and *b*, vessels beyond the circle of the horizon; *c*, boat within the circle. Fata Morgana shifts from right to left.

* *Bull. Soc. Vaud. Sc. nat.*, xxxv. 25, Lausanne, 1899; see also *Arch. d. Sc. phys. et nat.*, viii. 373, Geneva, 1899.

extreme positions of nearly 8 minutes of arc above and fully 4 minutes of arc below.

Let us now study the sequence of phenomena on a fine day towards the end of spring or the beginning of summer, when, as hour follows hour, the air being at first colder than the water surface becomes warmer, while the temperature of the water remains comparatively constant. The temperature of the air, lower than that of the water in the morning, becomes equal to it towards midday, and exceeds it in the afternoon. Consequently the two normal types of refraction succeed each other in time. Let us suppose, for example, that the temperature of the morning air is 15° C., that of the water 18° C. The phenomena that are associated with refraction in air over warm water are in all their perfection ; the apparent horizon of the lake is depressed below the true horizon. As the day progresses, the air heats rapidly under the powerful action of the solar radiation ; the water also grows warmer, but at a slower rate because of its great thermal capacity. The temperature of the air soon equals that of the water, and is not long in passing it. In the afternoon the water may have risen in temperature to 20° C., and the air to 26° or 28° C. The phenomena associated with refraction in air over cold water are developed, the apparent horizon is elevated, the surface of the lake appears concave.

But the transformation from the one type to the other does not take place slowly or progressively. The depression of the horizon due to refraction over warm water does not diminish little by little until its value is zero; and the elevation of the horizon does not grow little by little, starting from zero when the temperatures of the two media are equal, and attaining a maximum when the difference of temperature is the greatest. The transformation does not occur simultaneously over the whole lake. The change takes place suddenly at each region, and successively from point to point. At a particular instant there may appear at different parts of the lake the two types of phenomena clearly recognisable.

It is at such an instant that the Fata Morgana appears, represented, as I have already said, by a striated zone of rectangles in juxtaposition. Its features are at first quite disconcerting. When first recognised, it may appear in any azimuth. It moves slowly, in one or other direction, along the shores of the lake. It was from a study of the manner of the shifting in position that I gained the key to the explanation of the phenomenon, which may be summarised under four heads:—

I. The Fata Morgana has its origin in the region between the two regions where the opposite types of refraction rule ; in the one region the morning conditions still hold, while in the other region the afternoon conditions have been established.

II. The lower limit of the striated or ribbed zone of the Fata Morgana is continuous with the depressed line of the horizon which is associated with refraction in air over warm water; the upper limit, with the elevated line of the horizon accompanying the refraction over cold water.

III. The Fata Morgana shifts always from the region where the refractions are over cold water towards that where the refractions over warm water still hold sway. The effect is due to the refraction over cold water invading the scenery point after point.

IV. On the rare occasions on which I have observed the first appearance of the Fata Morgana, I have always seen it at one of the extremities of the circle of the horizon.*

The general conclusions may be stated in these words:—

(*a*) The Fata Morgana is made manifest at the region where the morning type of refraction in air over warm water is being transformed into the afternoon type of refraction over cold water.

(*b*) At this region the eye of the observer placed at a convenient height sees simultaneously and in superposition both the depressed and the elevated horizons associated with the two types of refraction.

(*c*) Bright objects on the lower parts of the opposite coast are stretched and drawn out in height between the two momentarily coexistent false horizons of the lake, and, by forming rectangles in juxtaposition, give the appearance of the banded or ribbed structure of the striated zone. In my memoir of 1896 I showed that the transition from the one type to the other does not take place slowly and progressively; that even when towards the middle of the day the temperature of the air becomes equal to that of the water, and ere long slightly exceeds it, the depression of the apparent horizon and other mirage phenomena associated with the refraction over warm water persist for some little time. During the persistence of this mirage over the cold water there must be an unstable equilibrium due to the thermal stratification in the lower layers of air. The rapid transformation from this instability to the stability associated with the direct thermal gradient is the determining factor in the production of the Fata Morgana. The suddenness of its appearing and its brief transitory character are at once explained.

* If my hypothesis is sound, the instability which leads to the Fata Morgana may occur at the middle of the stretch round which the phenomenon is seen as well as at the extremities. In such a case we should see the Fata Morgana, at first single, splitting into two moving in opposite directions, the one to the right, the other to the left. This possible variation in the details of the illusion I have searched for in vain. Should it ever be observed, it will be an *experimentum crucis*, establishing the sufficiency of my hypothesis.

In this discussion I have limited myself to the salient features which are indispensable to the presentation of my theory. As to the identity of the phenomenon here described with that observed at the Straits of Messina I could deduce several details, such as I have given in my book *Le Léman*. For example, a ship sailing in the banded zone of the Fata Morgana, a steamer, or a house of known form, may be deformed so as to be absolutely unrecognisable. Although never bent into the impossible physical positions shown in Minasi's drawing, they may have their relative dimensions altered in an incredible manner. May we not find some excuse for the artist in the fact that, under the restrictions of his art, he attempted to draw a scene whose extraordinary distortions he could not explain? The vertical extension of objects situated in the banded zone, the flattening of objects above the upper limit of this zone, the multiplication of images of the same object, the superposition of erect and inverted images, etc., etc., are so astonishing and so irrational that we may well forgive the superstitions of early observers, amazed and perturbed by these fantastic illusions.

The accompanying figure illustrates my hypothesis; it needs no further explanation.

(*Issued separately May* 25, 1912.)

XVI.—The Sun as a Fog Producer. By Dr John Aitken, F.R.S.

(MS. received March 2, 1912. Read March 18, 1912.)

I fear that most, on reading the above title, will think it is either a printer's or an author's error, and that the title ought to have been "The Sun as a Fog Disperser." We are familiar with the sun's power of clearing away fogs; we so often see fogs clear away as the sun rises, that we seem to have shut our eyes to another side of the question, which is, that under certain conditions the very opposite of this happens, the air thickening and fogging as the sun rises. Some years ago it was noticed at Falkirk, especially during the winter months, that on many mornings on which the air was clear before sunrise it gradually thickened to a dense haze, or to a dense fog, while in pure country air the observer never noticed similar changes. The question naturally suggested itself, what was the cause of this difference between the air at Falkirk and that of the country? A series of meteorological observations was therefore begun to ascertain whether this occasional fogging after sunrise was merely an accident, or if it had any connection with the sun; that is, to ascertain whether its appearance was what we call fortuitous, or if it appeared according to any law in nature.

As we are to be dealing with haze and fog in this communication, it may be as well to state, as near as we can, what we mean by the words "haze" and "fog." No air can be called absolutely transparent in the accurate sense of the word. All distant scenery is always more or less veiled by what we term haze. This haze is composed of an enormous number of very fine particles of dust floating in the air. If the air be dry and the number of particles small, their hazing effect is slight; but as their number increases, so does their hazing effect. In addition to this, however, the relative humidity of the atmosphere has a great influence on the density of the haze. When the humidity increases the particles condense vapour on their surfaces and so increase in size and in their hazing effect, till when the air is saturated they grow large and form fog particles, and their power of shutting out the landscape is enormously increased. The dust particles may thus produce a haze of greater or less thickness according to their number, or they may produce a fog, but there is no hard-and-fast line between what we call a haze and a fog; we usually call it fog when very thick and damp, but even here the boundary line is unsatisfactory, as we have dry fogs. So, again, there is no hard-and-fast line between fogs and clouds. Fogs are generally composed of a greater number of smaller particles than clouds ; but cumulus clouds are very much like fogs in this respect. I have shown in a previous communication that they also are composed of closely packed small particles, and it is only after a time that they become fewer and larger by the evaporation of the smaller particles and the condensation of the vapour on the larger ones. In these phenomena we have a gradual change from haze (the effect of which is mainly due to dust and very little to water) to fog (the effect of which is mostly due to the water condensed on the dust) and to the obscuring effect of cloud (which is almost entirely due to water); but between these three domains there are no hard-and-fast boundaries.

The conditions which seemed likely to determine the appearance of the morning fogs, if they are a sun effect, had to be sought for in the state of the air at the time and place. As all fogs require some water for their formation, the humidity of the air had to be observed while they were forming; and as their formation is probably connected with some impurities in the atmosphere, observations had to be made also on the direction and velocity of the wind, as such records would show whether the air coming to the place of observation was pure, or was contaminated with the pollutions of the densely inhabited areas in most directions. It was also necessary to take the velocity of the wind, as it gives the degree of concentration of the impurities. Along with these observations, the transparency of the air was noted; also the amount of cloud on the eastern horizon, to show the amount of sunshine. Two observations on transparency were generally made—the first one early, generally before sunrise, if possible; and the second at 9.30 a.m., or later in some cases. The transparency of the air was observed by noting the amount of haze or fog on a hill 400 feet high and about 300 feet above the place of observation, at a distance of three-quarters of a mile to the south-south-west. When the air was very thick, nearer objects were used for the purpose. From these observations on the haze the limit of visibility was calculated.

Table No. I. gives samples of the observations made for this investigation. They are taken from the weather book, which also contains the other usual meteorological observations. Only a few of the observations taken when the air thickened are given in the table, and it is unnecessary to record the great majority of the others, as on most mornings the air did not lose its transparency after sunrise. In all those cases in which it did not lose its transparency it was found that this was due, either to the absence of sunshine; to the amount of wind preventing the accumulation of impurities in the air; to the dryness of the air, making it unsuitable for the formation of

TABLE No. I.*

Year.	Month.	Day.	Wind.		Temperatures.				Sunshine.	Limit of Visibility in Miles.	
			Direction.	Force.	Night Min.	Dry.	Wet.	Diff.		8 a.m.	9·30.
1909	Feb.	6	SW	·2	32	35	34	1	1	2	1
,,	,,	7	SW	·2	25	29	28	1	1	3	1
,,	,,	8	SSW	·5	32	37	35·2	1·8	0	4	4
,,	,,	13	WNW	·2	28	33	32	1	1	10	10
,,	,,	16	SW	·2	31·5	36	34	2	1	10	1·3
,,	,,	19	SW	·2	30	34·2	33	1·2	1	2	1
,,	Apr.	2	NE	·2	26	36	34	2	1	3	1
,,	Nov.	8	SW	·5	23	35	32·5	2·5	$\frac{3}{4}$	10	1
,,	,,	21	WSW	·2	30	37	36	1	$\frac{1}{4}$	100	2
,,	,,	22	WNW	·2	27	35	31·5	3·5	$\frac{1}{2}$	4	4
,,	Dec.	9	S	·1	31	40	38	2	0	10	10
,,	,,	21	SSW	·2	23	24	?	?	1	4	0·25
,,	,,	27	SW	·1	33	34·5	34	0·5	$\frac{1}{2}$	5	0·3
1910	Jan.	4	WSW	·2	38	38·5	37·5	1	1	100	1
,,	,,	21	WNW	·2	26	32	?	?	1	100	100
,,	Feb.	4	SW	·2	32·5	35	33	2	1	3	1
,,	,,	15	SSW	·2	33	36	34·8	1·2	$\frac{1}{2}$	3	1·3
,,	Mar.	29	SW	1·	27·5	37	35·3	1·7	1	5	0·25
,,	Nov.	20	W	·2	24	36	32	4	1	100	100
1911	Jan.	12	NNW	·2	32·8	34	32	2	1	10	10
1912	Feb.	20	WNW	·2	38·8	41	39	2	1	100	100

dense haze or fog; or to its blowing from a pure direction. In many of the observations not here recorded the increase in haze was slight, but it was observed that on all the mornings the increase was directly proportional to the humidity and sunshine, and inversely to the velocity of the wind and dryness of the air, and that when the wind was from an impure direction the thickening invariably took place when the conditions were favourable. With north-westerly winds, however, this thickening did not take place unless the velocity of the wind fell to nearly calm, in which case, the impurities thrown into the air in the immediate neighbourhood not being carried away, a thickening took place. If the velocity of the wind was 1 or over on the usual scale, little thickening took place even when the wind was from an impure direction; but even with a force of 1, considerable thickening took place if the air was very damp. Clear sun at sunrise gave the maximum effect, but sunlight through clouds also acted on the air, though much less powerfully. The humidity and the sunshine are the

* In the above table it will be noticed that in all but one case the force of the wind was less than 1 on the usual scale (1–12), that is, there was generally only a gentle drift. In the column under "Sunshine" 1 indicates a cloudless sky, that is, continuous sunshine.

two most powerful factors in these phenomena. If the wet-bulb depression be more than 2°, the thickening is seldom great, and decreases with increase of dryness as the day advances. The worst conditions of all are: cloudless sunrise, wet-bulb depression 1° or under, and absence of wind after blowing from an impure direction. Under these conditions the air invariably thickens either to a very thick haze or a dense fog, according to the humidity. If the air be nearly saturated, the sun soon gets hazed out and a dense fog is formed. Of course, on some mornings there are already dense fogs formed before sunrise, due possibly to night radiation, or they may be a heritage from the previous day; and on these occasions, though the sun may not be responsible for their appearance, yet it causes them to become denser. Again, there may be fogs at sunrise, and these may tend to clear though the sun be shining if the wind should happen to rise and clear away the impure air.

The history of these sun fogs varies greatly. If the wind does not rise they generally remain all day if the sky keeps cloudless. Sometimes they only thicken for a time, and as the day advances and the air gets drier they begin to dissolve, and sometimes the wind comes and clears them away.

Returning to Table I., it will be noticed that no wet-bulb readings are given when the temperature is under 32°, as readings under these conditions are of little value. It will be seen from the column for velocity of wind that the velocity was on most occasions small, generally only a fraction of 1 on the usual scale of wind velocities. It should also be noted that, as shown in the columns for temperatures, the temperature was always higher at the hour at which the observations were made than it had been during the night. The fogging therefore would not be due to the air becoming colder. An examination of the observations in the table shows that on the days on which there was no sunshine there was no thickening, even when the other conditions were favourable, and that winds from west to north do not thicken, as they bring to the place of observation pure air, blowing as they do from thinly populated districts. The air, however, from all other directions comes from densely populated areas, and contains a large amount of the products of combustion of coal, and other impurities. I have shown in a previous paper * that at the place of observation the air is nine times more hazed when the wind is southerly than when it blows from the north-west quadrant, this being due to the impurities thrown into the air in its passage over the densely inhabited parts. The effect of sunshine on these impurities is shown in all the observations given in Table I. when the wind was southerly, the limit of visibility being in every case greatly reduced when

* *Proc. Roy. Soc. Edin.*, January 1893.

there was any sunshine—the limit in one case being only $\frac{1}{100}$ after sunning compared with what it was before. It will be noticed from the table that the air was often thick before sunrise with the wind from the pure direction, but the history of these cases shows that the wind had previously been from an impure direction, and the sun would appear to have worked up all its impurities, since further sunshine had no effect on it, as may be seen from the table.

With regard to the barometrical conditions under which these fogs are formed, I find that nearly all the cases entered in the table took place in anticyclonic conditions. But it is doubtful if this has any real significance, further than that it is generally under these conditions that the air circulation becomes slow enough to allow of the accumulation of impurities. In cyclonic areas the circulation is usually quick enough to keep the air fairly pure. The position of the centre of the anticyclone with regard to the position of the place of observation does not seem to be of any importance, as these fogs formed with the centre either to the north, south, east, or west. The densest of the fogs, however, took place when there was no general circulation, but light airs moving in different directions at different parts of the country, a condition eminently suitable for the accumulation of impurities in the atmosphere.

Summing up the results of these observations, it has been found :—*First*, that when the wind is slight and brings moist air from an impure direction, sunshine invariably destroys the clearness of the air and causes a thick haze or a fog. *Second*, that air from pure directions is not thickened by sunshine.* *Third*, that when there is no sunshine there is no thickening of the air though the air be damp and from an impure direction. *Fourth*, that high winds, by preventing the accumulation of impurities, tend to check the formation of haze by sunshine, and clear away sun-formed haze. *Fifth*, that when there is more than three degrees difference between the dry- and wet-bulb thermometers the sun only causes a slight increase in haze.

It seems probable that the so-called summer or heat haze may also be due to the action of sunshine on the impurities in the atmosphere. Thunder

* An instance of the effect of local impurities on the formation of these fogs was seen on 23rd January of this year at Shandon, on the Gareloch. The wind during the night was very light and from the north, but fell calm in the morning, and the sky was cloudless. Over the whole Clyde area a dense fog formed, while over the Gareloch the air remained perfectly clear, the distant mountains being quite sharp. The only difference in the air over the two areas was that while the air over the Gareloch remained pure, that over the Clyde was much polluted by the products of the fires of the towns on the banks of the Clyde. The air over the Clyde might have been slightly warmer and damper than the other ; but as the Gareloch is tidal, this difference can have been but slight.

haze may also have this origin, as it forms during periods of calm weather, and the rising damp impure air probably causes the thickening under the influence of sunshine, as this thickening takes place before the air arrives at the cloud level. Although I have in the above generally used the word "sunshine," it must not be understood that only direct sunshine acts in these cases; light through the clouds also acts, but much less powerfully, and only very feebly if the clouds be dense.

It does seem very curious that we should have been seeing these fogs all our lives and yet should never have observed any connection between them and sunshine—just as with a great many other things which we see every day, till they become so familiar that we do not think they require any explanation. Now, I believe we can predict with a fair degree of confidence what is going to be the state of the air on any morning on which we have the necessary meteorological information. However clear the air may be before sunrise, if it be damp, the sky cloudless, and the air either town air or from an impure direction, then a dense haze or fog will form unless the wind rises.

Though this investigation clearly shows that the sun produces certain kind of fogs, yet it is by no means here contended that it is to be censured for their appearance. It would rather appear that it is doing its best to show us the state of pollution into which our modern civilisation has brought our atmosphere, as it only inflicts these fogs on the areas upon which man has thrown the waste products of his industries and converted the atmosphere into a vast sewer, as a penalty for something wrong in his methods.

The Cause of Sun-formed Fogs.

As these fogs are only formed in air which has come from densely populated parts of the country, it seemed probable that they are formed by the light acting on some of the impurities thrown into the atmosphere in its passage over these areas. In previous papers I have shown that the sun is capable of forming nuclei of cloudy condensation—that is, small dust particles out of many different gases; so that it seemed possible that the sun was here doing something of the same kind, and converting some of the gaseous impurities in the air from polluted districts into solid or liquid particles. And the question comes to be, what are the impurities in the polluted air out of which the sun can form nuclei?

But before going further it may be as well to point out that there are two kinds of nuclei. One kind has no affinity for water vapour and, though it condenses water in even unsaturated air, that action is only a surface one; so that, though high humidities increase the hazing effect of this dust, they only do so to a slight extent. For instance, with a wet-bulb depression of 3° the hazing is only twice what it is with a depression of 8°. The other kind of nucleus, however, has an affinity for water vapour, and condenses it into minute drops in even unsaturated air, and so causes dense fogs. The first kind is the most common, and it is the one which generally forms haze in pure districts. The second is generally found in abundance in inhabited areas, and is the true fog-former, since it forms fogs in even unsaturated air; and as it causes condensation in unsaturated air, we will call its action spontaneous condensation. As these nuclei have an affinity for water, each particle tends to persist—unlike cloud particles, the smaller drops of which tend to evaporate and hand over their vapour to the larger ones, so causing a decrease in their number. The fog particles have no such tendency, as they each tend to hold their own share of water, since any tendency for the particles to evaporate is checked by the concentration of the impurities in the smaller ones and by dilution in the larger, thus increasing the affinity in the smaller and decreasing it in the larger. Nuclei with affinity for water, therefore, tend to persist and retain their numbers and fogging effect.

In a previous communication, read before this Society in February 1890, there is described a somewhat rough method of finding the condensing powers of the different kinds of dust. It is shown that the dust produced by burning magnesium begins to condense vapour at about the same temperature as does a glass surface—that is, at the dew-point. Dust from gunpowder smoke condensed at a temperature 5° above the dew-point; dust from burning sodium condensed at a temperature as much as 17° above the dew-point; and the ordinary dust in the atmosphere condensed at about 2·3° above the dew-point. There would no doubt be water condensed on all these kinds of dust at all degrees of dryness, and these figures only show at what degree of dryness the dust began to show a visible change, due to a great increase in the amount of condensed vapour. The figure for dust in ordinary air corresponds very much with what we find from observations on the action of dust in the atmosphere—namely, a quick thickening of the air when the wet-bulb depression goes under 2°.

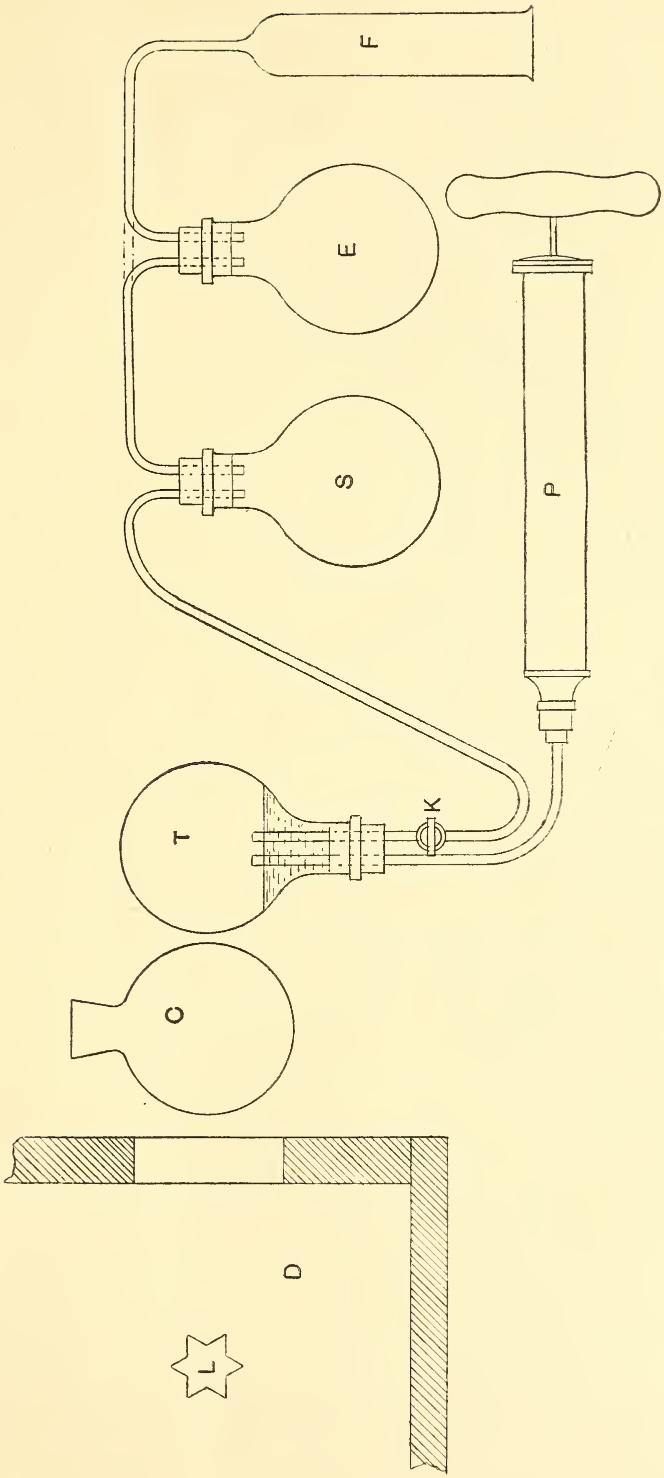
From the above it is evident that it is not enough to show that the sun can form nuclei out of any particular impurity: we must also find whether or not the nuclei have any affinity for water vapour. In the one case the nuclei will only form haze more or less dense, and not a fog, in unsaturated air. Only those nuclei which have an affinity for water can be called true fog-formers in unsaturated air.

Apparatus for Testing Air.

The apparatus used in this investigation was the same as that described in a previous communication to this Society,* and is here reproduced (fig. 1). It consists of the test-flask T, in which the air is tested for fine dust or nuclei of cloudy condensation. This flask is connected by a tube with the pump P. It is also connected by another tube with the flask S, which in turn is connected with E as shown, while E is connected with the filter F. In the test-flask T is a little water to moisten the air. The interior of this flask is illuminated by the incandescent-gas light L, placed in a dark lantern which has a vertical adjustable slit on the side next T. The light is concentrated on the test-flask T by means of the condenser C, which consists of a globular glass flask filled with water. By working the pump P air is drawn through the filter F into the flasks, and the pump is worked till all dusty air is taken out. When testing the air for nuclei the stopcock K is closed and a short stroke of the pump is made to expand the air in T, the inside of T being examined while the expansion is being made to see if any cloudy condensation is taking place. This examination can be most easily done if the test-flask be surrounded by dark surfaces and the room be as dark as convenient for attending to work. A magnifying lens at the level of the flask, and fixed pointing in a direction at about an angle of 140 degrees with the direction of the light, and focussed on the centre of the flask, will be found to be of assistance in some cases where the nuclei are few and small. If the filter has been packed with sufficient cotton-wool, or whatever is used in its place, and all the joints are tight, there will be no condensation in the test-flask when the stopcock is closed and a short stroke of the pump made to expand the air. If condensation persists after sufficient pumping has been done, then either the filter is defective and more cotton-wool must be added, or some of the joints are faulty and must be put right before a test can be made.

As it is necessary in this investigation to test not only whether nuclei are produced or not, but also whether the nuclei have any affinity for water vapour—that is, whether they are only haze producers or are true fog nuclei—the usual method by expansion was used for testing the presence of all kinds of nuclei; and the method of testing whether they have an affinity for water or not was to draw the air with the nuclei into the test-flask T as usual, but now the stopcock K is not closed nor is any expansion made with the pump. The nuclei are simply left in the damp air and allowed to remain a few minutes, and are then examined to see if they

* *Proc. Roy. Soc. Edin.*, vol. xxx., Part IV., No. 3.


F
E
S
P
K
T
C
D
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form any cloudy condensation without expansion. For this examination the magnifying lens will be necessary, as the cloud particles so formed are often very small. This is a somewhat delicate experiment, because any inequality in the temperature of the sides of the flask, or of the water in it, will give rise to mixtures of airs of different temperatures and humidities, which by mixing will cause supersaturation and condensation on nuclei which have no affinity for water vapour. It is therefore necessary to prevent all unequal heating of the test-flask, and the light should be as carefully filtered from heat rays as possible, and only allowed to enter the flask for as short a time as is necessary for examination. A better plan, however, is to use the light of the sky, if there is nothing in the flask on which it can act. For making these tests it has been found convenient to surround the flask with a small box painted black inside, with a small opening about 1 or 2 cm. diameter for admitting the light, and a larger one for seeing what is taking place in the flask.

With all these precautions, one is never quite certain that any condensation taking place in the flask where there is no expansion may not be due to inequalities of temperature. It therefore seemed desirable to make some tests in air that was not quite saturated with water vapour. At first it was thought that the simplest method of doing this would be to mix a certain proportion of dry air with the saturated air. But as the humidity of the air of the laboratory, as well as its temperature, is a variable quantity, constant degrees of dryness could not be depended on without the use of complicated apparatus. It was then thought that if a little chloride of sodium were dissolved in the water in the test-flask, its presence would lower the vapour-pressure in the flask; and if it was found that the presence of the solution of this salt in the flask had no effect of itself, or on the gases tested, its use was permissible. A saturated solution was first tried, but it was found to reduce the vapour pressure too low for anything but powerful fog-producers making themselves visible. The effect of the presence of the salt on ordinary condensation produced by expansion was so interesting that it may be referred to here. The vapour-pressure of a saturated solution is not too low to prevent the formation of a dense cloud in ordinary air, if sufficient expansion be made; but the interesting point was that the cloud did not remain so long a time as usual, but rapidly vanished, being gone in a second. The affinity of the salt for water is so great that it robbed the nuclei of their water, which evaporated, diffused, and was absorbed by the salt solution. The change took place with marvellous rapidity, pointing to the rate of diffusion of water vapour in air being very great; and this experiment is a simple method of illustrating this.

It was thought it might be interesting to see to what distance this very rapid diffusion took place. The above experiment was made in a globular flask 8 cm. diameter, and, as stated, the whole cloud disappeared in a second. A flask 13 cm. diameter was now tried. When the condensation was made in this flask all the cloud did not disappear at once, but a small ball of cloud was left in the centre of the flask. This small cloud fell quickly in a mass to the bottom, where it rapidly disappeared. The fall of the cloud being due to the friction of the cloud particles on the air giving it a quasi-greater density than the surrounding air, its fall was much quicker than that of the small drops through the air, and the changed shape of the cloud on striking the water showed it had moved in a mass.

A saturated solution of salt being unsuitable for these tests on the condensing power of different nuclei, experiments were then made with weaker solutions. If 1 part of saturated solution was mixed with 19 parts of water in the test-flask, it was found to meet the requirements, there being no spontaneous condensation with ordinary air, even with slight inequalities of temperature. This will be partly due to the air not being saturated, but part will be due to the salt on the sides of the flask checking evaporation, even when slightly heated owing to increased concentration of the salt by loss of vapour.

Some measurements were made on the effect of the salt in lowering the vapour tension. Two similar flasks about 10 cm. diameter were used. In one was placed a concentrated solution of salt, and in the other some tap water. In each of the flasks was hung a wet-bulb thermometer. The wet-bulb in the flask with tap water was required to act as a control of the reading of the thermometer in the flask with salt water. The result was that the evaporation caused by the salt kept the temperature of its wet-bulb 3·5° lower than the other. As the vapour in the flask had to diffuse some distance to the salt water, it was thought that if the salt water was nearer the bulb the difference in the readings might be greater. Accordingly, two similar cylindrical vessels, about 3·5 cm. diameter and 18 cm. deep, were lined with blotting-paper. One of the cylinders contained the saturated solution of salt, the other only water. Both cylinders were plunged in a vessel of water to keep the temperature as steady as possible. The result was that the wet-bulb surrounded by salt fell only 3·6° below the other, or but a fraction of a degree lower than when larger vessels were used, a result which was indicated by the rapid disappearance of the cloud particles in 10-cm. flasks previously described. It should be mentioned that for these tests two exactly similar thermometers were used, having equal-sized bulbs and equally covered with muslin ; and also that the thermometers were changed from the one vessel to the other at each test, so as to check any difference in them.

Sulphur Oxides.

The impurities in polluted air which first called for attention were the sulphur oxides, as previous investigations have shown them to be powerful producers of nuclei of both kinds. These gases are thrown in abundance into the air of polluted areas, being the products of the combustion of the sulphur in coal. The first of these oxides tested was sulphurous oxide, SO_2, and sulphurous acid, H_2SO_3. In these experiments both the fumes of burning sulphur and the solution of it in water were used. As both these forms of the compound gave very much the same reactions, the latter was for convenience most generally used, and we shall here consider their action to be alike, unless when specially referred to, and we will for simplicity use the symbol SO_2 for both.

One generally associates sulphurous oxide with fumes and fogs, yet I find that SO_2, so long as it is in good company,—that is, in pure air—shows no tendency to produce any kind of nuclei. It can be kept in pure air and water vapour for a long time without producing a single nucleus of condensation. But unfortunately this gas has a great tendency to form, from our point of view, undesirable associations. It seems to be ever on the outlook for something with which it may combine, or ready to take the smallest hint from outside influences to change and combine with its previously unattractive neighbour, when it falls from its high estate of a free-moving gaseous molecule to the condition of a solid or liquid particle confined to brownian movements, and probably ends its independent existence in a fog particle, or possibly in a raindrop, after which all independent existence ceases.

The study of SO_2 as a nucleus-producer is an extremely interesting one. It is so unstable that there are many gases which make nuclei with it, and also many outside influences which change it from the gaseous condition to a solid or liquid particle. In experimenting with SO_2 a little of the ordinary commercial acid mixed with a great deal of water is put in the flask S (fig. 1) to keep up a supply of the gaseous acid. In the flask E was placed a solution of any gas it was desired to add to the SO_2 in S. When it was required to test the effect of outside influences on the SO_2 the flask E was removed and the filter F connected direct with S. As already stated, we can have SO_2 along with filtered air drawn into the test-flask T and expanded without the SO_2 giving any nuclei of condensation, not a drop being seen on expansion; and accordingly it may be kept a long time without producing any nuclei if it is kept under certain conditions. We shall therefore first consider some of the outside influences which make the SO_2 a nucleus-producer, as we have to guard against them when considering the effects of other gases on it. The first of these influences we will consider is

Light.

The SO_2 only remains free from nuclei if we keep it in the dark. If the flask S is exposed to light, especially sunshine, then a change is effected which converts the acid into an active nucleus-producer. If some of the gas be sunned in S and drawn into the test-flask and expanded, a very dense condensation takes place. Some of these sun-formed nuclei are of the kind that form spontaneous condensation, showing not only that the sunshine has formed nuclei, but that some of them have an affinity for water. It has also been found that the sun makes more nuclei of spontaneous condensation when the products of burning sulphur are used in place of the solution. The products of the burnt sulphur were in this test drawn into the apparatus through the filter. Further, it has been found that this action of light is a *cumulative one*, the particles growing in size under the continued influence of light. If only a short exposure to light be given, then the particles are so small that they require a considerable supersaturation to make them nuclei, and it is necessary that the action of the light should be continued a minute or two, according to its strength, to increase the growth of the particles to such an extent that they become active nuclei with only slight supersaturation. This change of the SO_2 is also produced by the light of burning magnesium, but only very slightly by light from an incandescent gas mantle.

These experiments on the action of light on SO_2 vapour remind one of Prof. Tyndall's work on the decomposition of some vapours by the electric light. This action on SO_2 is, however, evidently something different from decomposition. There is the growth of the nuclei to be explained; and, further, if the SO_2 were decomposed the sulphur nucleus would have no affinity for water. It would rather seem that the effect is due to some combination of the SO_2 molecules taking place with other gases present, which will be referred to later on.

The conditions of the experiments on sunlight and SO_2 were now changed. In place of the apparatus shown in fig. 1, the apparatus designed for showing radio-activity, described in a paper read before this Society,*

* *Proc. Roy. Soc. Edin.*, vol. xxix., Part V., No. 30.

was used. In this apparatus the successive condensations always take place in the same air. The air is enclosed at the top of a vertical tube by a water seal, the same air being always subjected to the successive expansions and condensations. When working with this apparatus, a little of the solution of SO_2 was put in the water to keep up a supply of sulphurous vapour. After all dust particles had been got rid of by successive expansions and condensations, so that no condensation took place in the dark with slight expansion, the tube was sunned, when, as was previously found, condensation took place on expansion. But most unexpectedly the condensation was observed to decrease in density with every succeeding shower, till at last all condensation ceased; though there was plenty of SO_2 in the air, the sun no longer had any effect on it. Why this difference? In the other apparatus condensation always followed after sunning. The conditions in the two methods of testing are, however, quite different. In the flask arrangement, fig. 1, fresh supplies of filtered air were constantly supplied, but in the tube apparatus the same air was used over and over again; so that it looked as if something else was necessary to make the SO_2 active with sunshine, and as if this something was all used up after a number of showers had been made. The stoppage of all action seemed to indicate that the repeated showers had removed something from the air, and that the sun was unable to form nuclei from SO_2 in air so purified.

If the above explanation was correct, then it might be possible to take this something out of the ordinary impure air and make it also incapable of forming nuclei with SO_2 and sunshine when using the apparatus shown in fig. 1—that is, with successive quantities of filtered but in this case purified air, in place of simply filtered air. For the purpose of purifying as well as filtering the air, a cotton-wool filter saturated with caustic soda * was tried, and it was found that it purified the air; the SO_2 no longer

* I have previously advocated the filter method of purifying gases in place of bubbling them through solutions, as its effects are far more powerful than those of the ordinary one, which has a comparatively small action unless the gas has a very strong affinity for the contents of the solution; and even then it is far from being as perfect as the filtering process, and is practically useless for the experiments here described. Caustic solutions are not pleasant to work with, and the fingers may suffer if one attempts to pack the filter with wool soaked in caustic. The following method of preparing such a filter, however, presents no difficulties. The filter is first packed with cotton-wool, or any other suitable substance, in the dry state. The narrow end of the filter is then fitted into an india-rubber stopper which fits any bottle; an air-pump is also connected with the bottle. Some of the caustic solution is now poured into the upper open end of the filter and the pump worked. This draws the solution down through the cotton and thoroughly wets it. The pump should be worked quickly at the finish to draw out as much solution as possible, otherwise there will be bubbling produced by the filtered air, and nuclei made. These, however, may be checked by a second filter. The solution in the filter can be renewed when necessary in the same way.

responded to the action of sunshine, just as it ceased to respond when the same air was frequently used in the tube apparatus. This experiment showed that there was something in ordinary air having an affinity for soda which was necessary for making the SO_2 active as a nucleus-producer with sunshine.

Pure Country Air.

Having found that sunshine has no effect on SO_2 in air purified either by successive showers of SO_2 nuclei, or by being passed through cotton-wool soaked in caustic soda, it was thought it might be interesting from a meteorological point of view to know whether there was any air in nature so pure, or at least having that kind of purity that it will not form nuclei of condensation with SO_2 after being sunned. When visiting Loch Awe, Argyllshire, in June last, some tests were made of the air of that place for the purpose of getting information on this point. At that place, or indeed anywhere in the Highlands, we cannot expect to find pure air when the wind blows from impure directions. At Loch Awe only the winds from west to north are pure; from all other directions they bring impure air from inhabited areas of the surrounding country. These impurities are carried great distances. The air from the Lowlands of Scotland sends vast quantities of impurities to the wilds of the Highlands. At Loch Awe, with north-westerly winds, the number of dust particles is low, from 100 to 200 particles per c.c.; but when the wind comes from any other direction the number goes up to at least ten times as many, and often to much higher figures.

The apparatus used at Loch Awe was the same as that already described and shown in fig. 1, with the exception that the lamp for illuminating the test-flask was not used, because the products of combustion would have polluted the air. For illumination the light of the sky was used. The test-flask being enclosed in the blackened box already described, the light of the sky was found to give ample illumination for the purpose.

The tests were begun on 24th June, and though the wind was from a northerly direction the air was never very pure, owing to local pollution. The number of dust particles was very variable, owing to the irregular mixing of this pollution. With this air the SO_2 always gave a slight condensation after sunning, but the amount varied greatly. The effect, however, was very different from what I had ever seen before. Instead of the dense fog given by the air of Falkirk, sometimes only a slight shower would fall in the flask. On the 25th and 26th the direction of the wind shifted slightly, and the local pollutions no longer came to the window of the room where the tests were being made. The air was very clear and pure. The number of particles on these days was from 182 to 207 per c.c. With this air the SO_2 gave no condensation after being sunned; it acted exactly like the artificially purified air. To make sure that there was no mistake in these tests, and that the failure of the pure air to give any cloudy condensation with SO_2 was not due to any lack of other conditions, the following experiments were made. The window as well as the door of the room was closed, and a wax match was burned in the room to make some of the pollution of inhabited areas. This raised the number of particles in the room from 200 to 50,000 per c.c. This air was drawn into the apparatus and sunned; on testing, it gave a very dense condensation, showing that the sun was strong enough, and that there was enough SO_2 in the flask ready to form an alliance so soon as its partner put in an appearance, though unable to form nuclei in the pure air.

Some experiments were made in this pure air with the products of burning sulphur to see if the sulphur produced the something necessary to make its products active with sunshine. A little sulphur was burned and the products mixed with the air in the room and drawn into the sunning flask, from which the SO_2 solution had been removed. After exposure to light this always gave a slight condensation, showing that some action had been produced by the light. It was found, however, that if a match was burned in the room at the same time, or a little after, a very much denser condensation took place, which goes to show that the burning sulphur does not produce sufficient of the something necessary to make all the SO_2 active; and it seems possible that the small amount of condensation produced by burning sulphur alone may have been due to the combustion of organic matter in the sulphur, or produced in the act of lighting it. Though this was done in the passage outside the room, some of the products may not have been excluded.

The experiments with solution of SO_2 were continued on the 27th and 28th of the month; but by this time the wind was getting westerly and south-westerly, and the air was losing its transparency, while the number of particles had risen to from 1000 to 2000 per c.c. With this air there was always some condensation in the test-flask, though never so dense as that given by the air at Falkirk; if, however, a match or a piece of paper was burned in the room the density became very great. This semi-pure air could also be purified by passing it through the caustic filter, showing that the slight showers given by it and SO_2 are probably due to the same cause as the denser condensation given by the air of Falkirk.

Silica Flask.

I have here put these experiments on pure country air in their historical order, though from the physical point of view it may not be the best. In making these experiments with SO_2 and sunshine, glass flasks had been generally used A silica one had occasionally been experimented with, but little difference in its action from that of the glass flasks had been noticed; and as good working results had always been obtained with glass, the use of the silica flask was not continued, since broken glass is more easily replaced than broken silica. Having found that neither purified air nor the pure air of Loch Awe gave any nuclei with SO_2 and sunshine—that the SO_2 in pure air was, so to speak, dead, as far as its power for harm with sunshine was concerned—one felt inclined to ask if all the conditions of the experiments were satisfactory; did not this new knowledge about pure air call for a review of all the conditions of the tests? And the glass flasks at once came under suspicion, as they stop certain of the sun's rays. Experiments were therefore made with silica flasks. This was done on my return from Loch Awe, and only the impure air of that place, filtered through the caustic solution, was used. The result was now different. While the sun's rays could form no nuclei in pure air after passing through glass, they proved to be powerful nucleus-producers after their passage through silica.

The experiments with impure and pure air at first seemed to point to some other impurity in the air being necessary to make the SO_2 an active nucleus-producer with sunshine. Now we find, by using a flask more transparent than glass to the ultra-violet rays, that the sun can act on SO_2 without, so far as we know, the presence of any other impurity. However, these experiments made with glass flasks are not without their teaching from a meteorological point of view. We see that the sun's rays, if unchecked in any way, can produce nuclei out of SO_2 in pure air; but the experiments made in glass show that if the rays of shorter wave-length are held back by glass, haze, or clouds, the sun will have no effect unless there are some other impurities in the air in addition to SO_2. In confirmation of this it may be stated that if there is only a good light and no direct sunshine the SO_2 and pure air in silica flasks give very little condensation, showing that clouds stop the rays that make SO_2 active in pure air, while good light without sunshine makes SO_2 active in impure air. We must always remember that when there is SO_2 in the atmosphere there are always plenty of the other impurities.

The impurity in the atmosphere necessary to make SO_2 more active with diffused light has not been identified. It does not seem to be carbonic acid gas; at least, that gas liberated from its solution in water has no effect. It may, however, be contended that a gas in that condition is, so to speak, dead, and not so chemically active as one freshly prepared. The other impurity may possibly be carbonic oxide; but as there seems to be no method of preparing that gas free from nuclei, or other objectionable impurities, direct tests have not been made. The following experiments, however, were tried with the products of perfect and imperfect combustion, in the hope that they might throw some light on this point. The air entering the filter was drawn from the top of a flame of a bunsen burner. The supply of air to the burner was first set to give perfect combustion—that is, non-luminous. In that condition almost all the carbon would be in the form of CO_2. These products were drawn into the apparatus through the filter, mixed with SO_2, and sunned. On testing, only a slight condensation took place, such as was obtained without the products, the air on that day being very pure, as it came from the north-west. The air-supply to the burner was now cut off, so as to produce a luminous flame, and its products were now drawn into the apparatus, sunned, and tested as before, when a very dense condensation resulted. These tests would seem to indicate that the CO_2 of perfect combustion is not the impurity in the air required to increase the activity of the SO_2 and make it a nucleus-producer with filtered sunlight. The experiments also indicate that it is probably the CO of imperfect combustion that is the partner in the formation of the nuclei. This conclusion is evidently not final, as there are so many other gases besides CO given off in imperfect combustion, and it may be one of them. The above experiment was made in another way for confirmation. The windows of the room were closed and a bunsen flame burned for a short time and the products mixed with the air of the room. On testing this air with SO_2 it showed little or no increase in condensing power over the slight condensation it previously gave. Ordinary gas-jets were now lighted and allowed to burn a short time, and the air in the room mixed. On testing this air it gave a great increase in the density of condensation. Both these ways of testing show that the impurity required to make SO_2 active is some product of imperfect combustion.

Radio-activity.

Another outside influence which acts on SO_2 and makes it a nucleus-producer is radio-activity. If we keep the flask S in the dark and bring near it a small quantity of any radio-active substance, the acid becomes a active nucleus-producer, the test-flask showing a dense fog on expansion; and at the same time there is also produced a little spontaneous condensation. It may be here stated that this apparatus will be found to be a convenient instrument for showing and measuring radio-activity. The action of radio-activity on the SO_2 is, like that of light, a *cumulative one.* If we only allow the radium, or whatever substance we are using, to act for a short time, the nuclei are small and do not give any condensation unless high expansions are used; but if time be given for the radium to act, the nuclei can be grown to such a size that they cause condensation with the slightest supersaturation, showing that they have grown to the size of ordinary dust nuclei. One might imagine that in this case the radio-activity had converted the SO_2 into SO_3 by its action on the water vapour; but this is not the whole case, because most of these nuclei have no affinity for water, and only condense when the air is slightly supersaturated. The small amount of spontaneous condensation nuclei may be due to the production of SO_3. There is a curious and interesting point connected with these nuclei, similar to what was found with light. Working on the above method—that is, with the apparatus as in fig. 1—in which filtered air is drawn into the apparatus, so that fresh supplies are used for each test, there is no difficulty in growing to the size of dust particles the nuclei produced by radio-activity. But if we make the experiment with the tube apparatus, already referred to, in which the same air is used over and over again, it is found that after a time it is impossible to grow the particles to such a size that they will form nuclei with very slight expansion. Even after a long exposure to radium no condensation is produced without an expansion similar to that required for condensation on ions. It was noticed, however, that if the cloud particles produced under these conditions were evaporated by compression, they always left nuclei of dust-like size, as slight expansion afterwards gave nearly as dense a cloud as the one dissolved.

The time of exposure to the action of radium is not given in these experiments, because the strength of the salt is not known. It may, however, be stated that the radium salt used is capable of discharging an ordinary gold-leaf electroscope in a few seconds if held near it. With one minute's exposure to this tube held close to the flask the SO_2 nuclei were still too small to form nuclei with very slight expansion, and some minutes' action was required to make them large enough to give condensation with the slightest expansion. The rays from an X-ray tube were also found to act very powerfully on the SO_2, causing very dense condensation on expansion.

While treating of the action of radio-activity we may refer to the action of emanations from radio-active substances, though they ought perhaps properly to come in under the effects of other gases on SO_2. When testing the effects of emanations the flask E is replaced and the bottle (without its stopper) containing the radio-active substance is placed in it, so that the emanations may escape and be carried with the filtered air into the flask S and there mixed with SO_2. Most of the substances tested gave very slight condensation, and the emanations from the radium salt, which gave dense condensation by the action of its β and γ rays, gave only a very slight effect. The only substance tested that gave a strong action was a thorium hydroxide. A bottle containing a small quantity of it, placed in the flask E, gave a constant and a dense condensation in the test-flask. In this case it is the α rays that are responsible for the very dense fogging. The same bottle of thorium seemed to have little effect in discharging an electroscope, but its emanations discharged it quickly. The probable reason for the great difference between the radium and the thorium emanations is that there was much more of the latter salt than of the former, and also that thorium emanations break down about 6000 times more quickly than the radium. In these experiments with thorium emanations it is not necessary to use the flask E. The bottle with the thorium may be put in a flask connected with the outer end of the filter, when the emanations are drawn into S through the filter. The emanations come quickly through the cotton-wool—that is, they are but little absorbed by the filter, and they can be quickly washed out with air; unlike most gases, the first part of which is all absorbed and it is hardly possible to wash them all out afterwards by continued pumping, while the emanations, being inactive, pass freely through. The action of the radio-active bodies is probably due to their ionising properties, but the nuclei which cause spontaneous condensation are probably formed by the oxidation of the SO_2 by the peroxide of hydrogen formed by the β rays in the moist air, and also by the action of α and β rays converting the oxygen into ozone.

Neither the radio-activity of the atmosphere nor the emanations from the ground seem likely to play any part in these sun-fogs, as they are always present night and day. With regard to the possible radio-activity of the sun, we require more information before we can say whether or not it plays any part in the phenomenon. It is not likely that either the α or the β rays can penetrate through our atmosphere; and as we can keep photographic plates with only a covering of black paper, the γ rays cannot be of much importance, even if they exist.

Electricity.

The last of the outside influences to which I shall refer is electricity. The apparatus used for testing the effect of the electric discharge is the same as that represented in fig. 2 in the paper previously referred to,* and consists simply of an arrangement of wires for making a point discharge, in the flask S. The other arrangements are all as before. It is found that if an electric discharge be made in air containing SO_2, an enormous number of nuclei are produced, which give a very dense fog on expansion; and, further, many of the nuclei cause condensation in unsaturated air. One cannot help wondering to what extent the high-tension electricity now so much used for stimulating plant life, and high-tension transmission for other purposes, may affect the air in causing the formation of fine dust out of the impurities in the atmosphere of inhabited areas.

Gases which Act on SO_2.

Turning now to the effect of the presence of other gases on SO_2. As previously stated, neither oxygen, nitrogen, nor water vapour has any effect on it; but there are a number of other gases in the atmosphere, especially of polluted districts, about which some information is desirable, as these gases act on the SO_2 without the aid of light or other outside influence. Along with the SO_2 the combustion of coal throws into the atmosphere a number of other gases. Amongst these there is

Ammonia.

When ammonia and air are tested in the apparatus they are found to form no nuclei after being acted on by light, or radio-activity. But if we put a little weak solution of SO_2 in the flask S, and some weak ammonia in E, we get constant condensation on expansion, and also some spontaneous condensation. As this action, however, can take place in the dark, it is probable that any nuclei produced in this way will have already been formed before sunrise, so that ammonia does not seem to have anything to do with the formation of these morning fogs, though they will add to the early morning haze. Another important gas given off during combustion is

Hydrogen Peroxide.

If we put a little of the ordinary peroxide solution in the flask E in place of the ammonia, and test it alone with filtered air, it will be found to give

* *Proc. Roy. Soc. Edin.*, vol. xxxi., Part IV., No. 31.

no nuclei after being exposed to either light or radio-activity. But if we now place some SO_2 in S while the peroxide is in E, then on the gases mixing a dense condensation takes place on expansion, and there is also a dense condensation without expansion, showing that there has been produced an immense number of nuclei with a strong affinity for water. This is probably due to the formation of SO_3 by the action of the peroxide. We shall see later that it is possibly peroxide of hydrogen that is responsible for the thickness of the sun-fogs. Of course, all the peroxide formed by the combustion of the coal will combine at once with the SO_2 and be responsible for part of the early morning haze, but the oxidising agents formed during the combustion do not seem to be enough to oxidise all the SO_2.

Ozone.

Ozone is another constituent of the atmosphere requiring attention. It has been found that if we introduce ozonised air, from an ordinary ozone tube, into the apparatus, it reacts powerfully on the SO_2 and produces a very dense condensation after being sunned.

Products of Combustion.

Having shown how unstable SO_2 is when prepared by artificial processes, let us now examine its character when produced along with the other gases in our fires and furnaces during the combustion of coal. In testing these products the same apparatus was used as before, and shown in fig. 1. The products were collected from a coal fire and put into a large glass flask about 1 foot in diameter. For collecting the gases a long glass tube about 5 mm. diameter was used. The tube was held with one end over a bright, smokeless part of the fire, while the upper end entered the glass flask. The natural heat convection carried the gases into the flask and displaced the air. The flask was then removed to the apparatus, and a tube connected with the open end of the filter dipped into the flask among the products, the top of the flask being stopped with a plug of cotton-wool to prevent the escape of the products and at the same time allow air to enter. The pump was then worked to draw the products through the filter into the apparatus in the usual way. The following are the results of the tests. If very strong products were used—that is, only air that had passed through the fire unmixed with other air,—then there was constant condensation in the test-flask without light and without expansion. That is, if strong products, even when free from dust, are provided with plenty of moisture, they seem to be able to set up a spontaneous condensation without any outside influence. If, however, the products are mixed with air, they are incapable of forming nuclei in the dark ; but after they have been exposed to sunshine they give a very dense condensation on expansion. This condensation after sunning continued for long, with each sample taken from the flask, and after a great deal of pumping had been done and products in the flask had been reduced to a very weak mixture. Part of the persistency in the condensation would no doubt be due to some of the gases absorbed at first by the filter getting free when purer air was passing, as it is found that a filter through which strong products have been passed is difficult to cleanse by pumping air through it, and can never be used again for any other purpose. The products from the fire can be kept in the large flask for hours and retain their activity if kept in the dark. These tests show that the sulphur products of our fires react to light in the same way as the SO_2 was found to do.

Other tests were made on the products of the combustion of coal, but this time without filtering them. In these tests the filter was removed and the pipe conveying the products was connected with the sunning-flask. The tests showed that the products gave rise to a considerable amount of spontaneous condensation in the test-flask, where they met with saturated air. If the filter was now introduced it stopped nearly all the spontaneous condensation, because it stopped not only the nuclei but all the oxidising gases in the products, as they cannot pass through the filter; and if the gases were sunned a dense form of condensation took place on expansion, but very little spontaneous condensation. If, however, peroxide of hydrogen was added to the filtered products of the fire in the flask, a very dense condensation took place on expansion, and also a good deal of spontaneous condensation, just as was found when testing SO_2.

Let us now try to picture to ourselves what takes place in the atmosphere in which the products of our fires are mixed. During the burning of the coal the sulphur in it is oxidised to sulphurous acid, and much of it soon changes to sulphuric acid. The latter, having a strong affinity for water, will form a number of nuclei. Some of these acids will also combine with the ammonia formed during combustion and produce many nuclei. Further, if there is any ozone formed during the combustion it also will help to make more nuclei. But it would appear that, in spite of all these different gases produced during combustion acting on the sulphur acids and tending to change them from the gaseous to the solid or liquid state, there still seems to remain an enormous amount of these sulphur products in the atmosphere in a gaseous form, and it is probably on these that the sun acts, converting them into fine dust. No attempt has been made to determine what is the exact chemical composition of this sensitive gas. All that is shown is that it acts in every way like SO_2. But, further, the sun acts not only on these gases, but also on the other constituents of the atmosphere, producing substances which act powerfully on the SO_2, and not only make nuclei with it, but also that worst kind, the nuclei of spontaneous condensation. Probably the worst of the gases produced by sunshine is the peroxide of hydrogen. This gas is generally admitted to be produced by the ultra-violet rays of the sun. It is found in dew and in rain, but not in dew formed during the night—only in that condensed after sunrise. These same ultra-violet rays are also credited with the production of ozone in the atmosphere. It would thus appear that during sunshine there are constantly being produced both peroxide of hydrogen and ozone, two powerful oxidising agents; and we have seen in a previous part of this communication that these gases act powerfully on SO_2, making with it nuclei which cause condensation in unsaturated air, and at the same time increasing the number of the ordinary nuclei. The presence of the first form of nuclei will therefore add much to the thickness even of air which has some degree of dryness. To these influences acting on SO_2 we may possibly add the effect of any electric discharge taking place in the atmosphere should the sun's action be shown to increase the current.

Turning now briefly to the question of the amount of these sulphur products in our atmosphere. Is there sufficient of them to account for the results attributed to them? No attempt has been made to ascertain their amount in the atmosphere; but even if we knew, it would not help us much, as we do not know how many particles go to a gram. So we can here only fall back on what is taking place in our fires to see how much sulphur is burned there along with the coal. The consumption of coal in this country is something like 200,000,000 tons per annum; and supposing we allow 1 per cent. as the amount of sulphur in them, which is not a high estimate, then there are burned something like 2,000,000 tons of sulphur every year, which gives an average consumption of over 5000 tons per day, more or less, according to the season, etc. The quantity seems enormous, one might almost say incredible, and it also represents a colossal waste, to say nothing of what might be its money value. From the quantity of sulphur burned in our fires, there are evidently enough sulphur products to pollute the atmosphere from Land's End to John o' Groat's House.

In connection with this question of the power of the products of our fires to produce sufficient haze after being acted on by sunshine to account for all the haze in the air, it may be as well here to recall the results of the tests made on the products of burning coal. In these experiments it was seen that a very small amount of the products provided material for the production of great quantities of fine dust under the influence of sunshine. The quantity of matter required is evidently extremely small when we consider that a filter which has once been used to filter the products of the combustion of coal will afterwards give off enough material to the air passing through it to cause fogging after sunning in a very great quantity of air. The hazing effect of these very small particles is probably mainly due to the water condensed on them by surface action.

There is a point which I fear may have been lost sight of, and which it is very necessary should be kept in mind—namely, that in these experiments with the products from coal the question of smoke is entirely eliminated, as the products were filtered. Here, therefore, comes an important conclusion —namely, that all the nuclei produced by sunshine acting on the impurities will still come into existence and haze and fog the atmosphere, even though our laws succeed in stopping all smoky chimneys.

In addition to the impurities thrown into the atmosphere in this country, we must remember that enormous quantities are also added by our neighbours across the seas, with the result that the air coming from them, with winds blowing from east, south-east, and south, comes to us already much polluted. East winds coming to the east coast of Scotland are always highly charged with fine dust, and, unless when very dry, are heavily hazed when they arrive on our shores. The south winds, which are more impure than the east on the east coast, have their impurities mainly from our own fires, though they are far from being pure on their arrival on our southern shores. The only really pure air that comes to this country is that from the Atlantic on our west and north coasts.

At Falkirk there are no sun fogs when the wind is from the north-west quadrant, because the air from that direction is pure and almost free from the products of combustion; but it seems probable that at many places in this country, especially inland ones, sun fogs will be produced with winds from all directions, as they bring with them the products of combustion from the fires and furnaces in every direction.

Products of Combustion of Purified Gases.

Having shown that the products of combustion of sulphur, whether burned alone or in the combustion of coal, give rise to very great numbers of nuclei, and also produce nuclei which condense water in unsaturated air, we will now turn to the consideration of the products of the other constituents of coal—namely, carbon and hydrogen—and see if they have any nucleus-producing power. For these tests we will use the products of combustion of ordinary household gas, and see whether they have any of the objectionable properties of the sulphur products. In testing the products from the gas the same methods were used as previously; but before proceeding a word of caution may be useful here, as to the method of collecting these gases. It is necessary that all parts of the apparatus that will get heated by the gases should be previously highly heated to thoroughly cleanse them; because, if this is not done, there is a risk of something being given off from the heated surfaces which may invalidate the results. In these tests a chimney 2 feet high and 3 inches diameter was placed over the flame, the products being drawn from the top of the chimney and passed through a coil of metal pipe to cool them, and then passed to the testing apparatus. The products from the perfect and the imperfect combustion of the gas were tested—that is, from non-luminous and luminous flames.

The products, as before, were put to two different tests. First, to find if they gave any true fog nuclei—that is, nuclei with the power of condensing water in unsaturated air. The method of testing this was to draw the products direct and without being filtered into the silica sunning-flask, where they were exposed to the sunshine. The gases were then drawn into the test-flask, in which there was some water; but no expansion was made in this test, the gases being simply left to see if there were any nuclei in them having an affinity for water vapour and capable of causing condensation without supersaturation. When this was done it was found that neither the sunned products from perfect combustion nor those from imperfect combustion gave any nuclei of spontaneous condensation. As oxidising agents other than those formed during combustion were found to act powerfully on SO_2, these products were also tested in the same way. A little peroxide of hydrogen was put in the sunning-flask to act along with light on the gases; but even with the aid of the peroxide the sun produced no nuclei of spontaneous condensation. In this test no salt was added to the water in the test-flask to reduce the vapour tension; the air would therefore be fully saturated, and probably in some places a little supersaturated.

In tests of this kind it is sometimes difficult to say whether or no there is any spontaneous condensation, as sometimes the particles are extremely minute and individually invisible, even with the aid of a lens, and owing to the unavoidable reflections of the light in the flask it is sometimes difficult to say whether any reflected light comes from the particles, or

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MODEL INDEX.

Schäfer, E. A.—On the Existence within the Liver Cells of Channels which can be directly injected from the Blood-vessels. Proc. Roy. Soc. Edin., vol. , 1902, pp.

Cells, Liver,—Intra-cellular Canaliculi in.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

Liver,—Injection within Cells of.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

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[*Continued on page* iii *of Cover*.

from the sides of the flask. In cases of that kind it has been found useful to employ another lens to concentrate the light. This lens is held in the hand, and by its movements any cone of light it may form on the particles is caused to move, and we then know whether any suspected illumination is due to reflections from the flask or to particles in the air.

A better form of test-flask has lately been used. It is simply a hollow prism-shaped bottle, one of the kind used for liquid prisms in spectrum observations. The flat vertical sides of the prism give fewer reflections, and the worked glass of the sides of the prism allow of clearer vision; while the angle at which the two sides are set is an advantage for observing, as it directs the line of vision somewhat in the direction of the source of light and not at right angles to it. The movable condensing lens is also used with this prism-shaped flask, partly on account of the advantage of having a movable cone, and partly because it gives a brighter illumination. For observing, a double convex lens about 2 inches diameter and 2 inches focus, or a microscope with a 1-inch objective, will be found suitable.

The second test to which the products from the gas flame were put was made in order to see if they produced any kind of nuclei which would cause condensation in supersaturated air after being exposed to sunshine. We have previously seen that the filtered products of burning sulphur and also the products from an ordinary clear fire gave dense condensation in supersaturated air after being sunned, though they gave none without the action of light. The products of this gas were therefore submitted to the same test. After leaving the cooling coil they passed through the filter into the silica flask, where they were sunned, being afterwards drawn into the test-flask, when they were expanded. It was found that the sunshine had produced no effect on them, no condensation taking place. Some peroxide of hydrogen was now added to the products, but even with it and sunshine supersaturation showed that no nuclei had been produced.

These tests were made with the products of both luminous and non-luminous flames. They show that the products of purified gas do not form nuclei of spontaneous condensation, and that the filtered products do not form nuclei of any kind, behaving in both ways quite differently from the sulphur products.

In experiments of this kind it is advisable always to have a check test on the results, in order to make sure that the apparatus is working correctly. For this purpose, after making each of the above tests on the gas products, a very little sulphur was burned and the products allowed to enter with the gas products into the apparatus and produce condensation. This was done in order to make certain that all the conditions were the same as those which showed the sulphur products to be such objectionable nuclei-producers.

When the results of these tests of the products of combustion of purified gas are compared with those obtained with sulphur, it is evident that, while the sulphur products give an abundance of nuclei of spontaneous condensation and very great quantities of nuclei without affinity for water vapour, and therefore tend to produce dense haze and fogs under the influence of sunshine, the products from the purified gas, on the other hand, as they neither form nuclei of spontaneous condensation nor other nuclei under the influence of sunshine, do not seem to play any part in these sun fogs; though we have seen that one of the products assists in the formation of the nuclei formed by the SO_2.

Dust and Temperature.

In a communication read before this Society on February 3, 1890, there is discussed the question of the relation between the amount of dust in the atmosphere and the temperature. It was shown that on many occasions when the temperature was high the number of particles was great, also that on these occasions the night radiation was checked, owing to the amount of dust in the air checking the loss of heat by radiation. It was shown, also, that the radiation is generally strongest just after sunset—due, probably, to the dust particles being dry and small at that time, and that as the night progresses the particles tend to grow with the increasing dampness of the air, their larger size accounting for the decrease in the radiations. It was shown that the connection between the amount of dust and sunshine was most marked in summer, as one might expect, owing to the longer hours of sunshine at that season. Then in the paper referred to the question is put, "Is this increase of dust particles with sunshine a case of cause and effect? or are both due to the same cause? or is it merely a coincidence?" This investigation of the effects of sunshine on the products of combustion now helps us to answer these questions. It would appear that both the high temperature and the high dust were due to the sunshine, which would be abundant on warm days, the sunshine converting the impurities in the atmosphere into fine dust, and also heating the air, and the heating power of the sun would be increased by the presence of the high dust contents of the air, making it a better absorber of the heat rays.

Prevention of Sun Fogs.

In previous papers I have frequently referred to the great nucleus-producing powers of the products of the combustion of sulphur. The conclusions then arrived at seem to be here confirmed—namely, that these products are one of the principal causes of haze, and of the dense fogs of large cities and great manufacturing centres; that they play no mean second part along with smoke in these phenomena. At present the importance of these products from burning sulphur does not seem to receive anything like the attention it requires. Though smoke-abatement societies are everywhere active at the present time, yet no attention is given to the invisible products of the sulphur in the coal. Now, no one would disparage the efforts of these societies to reduce the smoke which casts a pall over our cities and manufacturing centres; yet this investigation points to the fact that even if they succeeded in stopping all smoke, the atmosphere would still be densely hazed in all conditions, and thickly fogged when the air was damp. The task of getting rid of the sulphur products is not so simple as that of getting quit of the smoke. Of course, one can easily in imagination make conditions in which both smoke and sulphur products would be kept out of the atmosphere. If we were only allowed to use gas in our fires, and all the gas were purified from sulphur, etc., then we might look for a purified air. It would, however, be necessary for the retorts, or whatever was used for distilling the coal, to be heated with purified gas, and the resultant coke of the retorts made into water-gas and purified, as coke contains a great deal of sulphur, which is oxidised and passes over with the other gases. Any such treatment of our coal, however, seems for the present to be outside practical engineering, and the remedy for the condition of our atmosphere would appear in the meantime to be rather hopeless. Most of the undesirable conditions of life, however, are yielding to patient investigation, and there seems no reason why this one should not at least be mitigated, if the subject were taken up by some institution or body of investigators interested in the subject. So far as one can see, the chemist is the one likely to indicate the line of research. For instance, can he not tell us of some substance which when burned with coal will combine with the sulphur products and prevent their escape into the atmosphere—something that will give more promise of success than the lime treatment? That idea is merely suggested because it seems to be the simplest and the least likely to rub against our conservative ways of doing things; and unless the substance was very expensive, the country would not object to an Act of

Parliament to make its use compulsory. I think that if the country were thoroughly awake to the deleterious effects of the sulphur products, the concentration of many minds on the subject would be likely to result in some remedy being found.

Other Considerations.

Though this investigation clearly points to the great power of sulphur products in forming both haze and fog, yet it is evident we have not brought sufficient evidence to prove that these products alone, along with sunshine, are the cause of morning sunshine fogs. We have, so to speak, detected the products leaving their home by the fireside, or furnace, fully armed for harm; but something more is required to show that they really bring about the fogging in the manner here suggested. A burglar may leave his home fully supplied with the implements of his profession, but that is no proof that he has really done any damage; and though the law may punish him, yet in science we must be sure the offence has been committed, as it is quite possible the accused may have left his home with the power of doing harm, but may with the passage of time lose this power. Some change may come over the sulphur products which may alter them and make them insensitive to the action of sunshine. In order to complete the evidence against them we must find them prowling at night with all their powers for evil. We can hardly look for them during the day, as the sun will be acting on them and making them into nuclei; but at night, when there is nothing which we know of to change them, we ought to be able to detect their presence in the air; since we have here assumed that it is the impurities that have been collecting during the quiet nights that the sun acts on, causing the formation of thick haze and fog.

The detection of these impurities in the air is a very difficult matter. In the experiments described the quantities of matter are exceedingly small, but in the free atmosphere they are very much less. If we use the test for the presence of nuclei of spontaneous condensation—that is, by adding peroxide of hydrogen and exposing to light—then the particles are too few to form a cone of light in the test-flask, and the individual particles are frequently too small to be seen with a lens. Then if we test the air for sulphur products by first filtering it, and then adding the peroxide and acting on them with light to see if they are capable of producing cloudy condensation on supersaturation by expansion, we are at once met with a difficulty in the action of the filter. All filters absorb some of the gas, and the air coming through at any particular time is not in the same condition as when it entered, but is affected by the condition of the air which previously passed through. If the previous air was purer, the sample will be too pure; if it was more impure, the sample will be too impure. And, further, it is doubtful whether small quantities of sulphurous acid are not entirely destroyed by the filter. So these methods of testing do not seem likely to give a satisfactory answer to our question.

The other method of investigating is to test the air for nuclei of spontaneous condensation. First note the number of particles the air gives when drawn into the test-flask and saturated with water, then add to the air some hydrogen peroxide and expose to light, and see if there is any change in the number of particles formed by spontaneous condensation. If so, then it may be concluded that there is in the air some gas capable of producing fog under the influence of sunlight. A third method of testing is to use a dust-counter and first find the number of particles in the air, then add the peroxide and expose to light and see if the number is increased. If it is, then some oxidisable gas will be present.

The action of the sulphurous acid and the sunshine may, however, not be so simple as at first appears. There may be no SO_2 in the atmosphere after the products of the fires are much diluted with air. Still, there are great quantities of some form of sulphur products present, as all air, both night and day, in polluted areas gives when tested a great number of nuclei of spontaneous condensation, while pure air, and even pure air to which the products of a gas flame have been added, give none. The sun will, of course, act on the products of our fires in a concentrated condition while they leave the chimney-tops; but as the morning fogs are fairly uniform in density, over considerable areas, it rather looks as if the thickening took place in the air which had left the chimneys some time before and become evenly diffused.

Radiation Fogs.

Before concluding, it may be as well to inquire whether there may not be some other possible explanation of these fogs which form at sunrise. For instance, may it not be possible that they are due to radiation? It is well known that the radiation from the earth is stronger just after sundown than it is later, owing to the air then being drier and the hazing effect of dust less; and one may ask, may not the drying of the upper air at sunrise increase the radiation in the morning, and so cause fogging by the air getting cooled by radiation? This at first sight may seem a possible explanation, but an examination of the morning temperatures gives no support to the idea. While these fogs are forming the air temperature is rising, as shown by the figures in Table I., and the radiation temperature on the grass is also rising quickly. The radiation temperature, of course, rises because the thickening of the air has made it less diathermanous. As the temperature of the air is rising it must be receiving its heat from the sun, since its temperature is above that of the surface of the ground, as shown by the temperature in the screen being always higher than on the grass during the formation of these fogs. Since the temperature of the air is rising, its relative humidity will be decreasing, and if some other influence were not at work the air ought to become clearer and not more hazed. But even if we suppose that radiation is in some mysterious way responsible for these fogs, yet it could not make them without the aid of the sulphur products, because these fogs form in air which is not saturated, and it is only with the aid of the sulphur products that fogs can be formed in unsaturated air. We may therefore dismiss the radiation theory, unless radiation can be shown to act in some way not at present known. Of course, it is not here contended that radiation cannot produce fogs, as it is well known that it can; but in the absence of sulphur products it can only do so by cooling the air to the saturation point, a condition not necessary for the formation of these sun-fogs.

From Dr Shaw's recent book on *Forecasting Weather*, it appears that the tendency for fogs to form in the morning has been observed by others. At page 287 he says: "The morning is the most favourable time for the formation of anticyclonic fogs, because if the sky has been clear, radiation during the night will have reduced the temperature of objects on the surface of the ground, and a cold layer of air will be found at the surface." This, however, gives no explanation of why these fogs do not form before sunrise, and only begin to form after the sun is up, and the temperature of the air and of the ground is rising; nor does it explain why they are only formed in polluted air, or how they are formed in unsaturated air.

There is a peculiarity of these sun-fogs which has been frequently observed; and as it helps to confirm the explanation offered of their formation, it may be here referred to. During these sun-fogs, which often continue for days in succession, and persisted in a marked way last October, it has been observed that they frequently tend to get less thick as the night advances, and to be clearer the next morning, only to thicken again after sunrise. This clearing during the night would seem to be caused by the sun ceasing to shine and keep up the supply of nuclei, while the light winds tend to clear away those made during the previous day.

It is not here contended that this investigation has gone into the whole question of the origin of these sun-fogs. We have only dealt with the effects of the products of burning sulphur, but evidently there is a possibility of some of the other gases produced during combustion, other than those here referred to, playing some part in their formation, either alone or in combination with the sulphur compounds.

It may be as well to call attention to the great difficulties met with in experiments of the kind here described. The extreme minuteness of the quantities of matter involved, the difficulty in getting purity of materials and vessels, and the many complicated influences at work, make experimenting of this kind full of pitfalls; so that much of what is here advanced is likely to have more than the average number of errors usual to physical investigations, and, though every care has been taken to make as many check experiments as possible, yet some of the conclusions may not be final. We can therefore only look on this communication as a first attempt to solve the relation between sunshine and a certain class of fogs.

(*Issued separately July* 1, 1912.)

XVII.—The Molecular Theory of Magnetism in Solids.

By Professor W. Peddie.

(Read February 5, 1912. MS. received March 4, 1912.)

1. It is well known that the molar theory of magnetism, in which the magnetic medium is regarded as being continuous in its ultimate structure and properties, is of very limited applicability to the elucidation of magnetic phenomena. The fact of the impossibility of isolating one kind of magnetism from the other by any process based on inductive action makes evident the essentially molecular nature of magnetism. This was recognised even in the early development of the theory by Poisson, who presumed that the molecules only became magnetic under the action of an external field. Weber recognised the necessity for the ascription of permanence to the magnetic quality of the molecule in order to account for the phenomena of magnetic saturation; and his theory was modified by Maxwell so that it might also include in its range the phenomena of residual magnetisation. In Maxwell's discussion of Weber's theory, we have the first explicit statement of the distinctive feature of the modern theory of molecular magnetism.

He says: "Let us now suppose that a magnetic force X is made to act on the iron in the direction of the axis of x, and let us consider a molecule whose axis was originally inclined a to the axis of x. If this molecule is perfectly free to turn, it will place itself with its axis parallel to the axis of x, and if all the molecules did so, the very slightest magnetising force would be found sufficient to develop the very highest degree of magnetisation. This, however, is not the case. The molecules do not turn with their axes parallel to x, and this is either because each molecule is acted on by a force tending to preserve it in its original direction, or because *an equivalent effect is produced by the mutual action of the entire system of molecules.*"

Maxwell then points out that Weber adopts the former postulate as being the simplest, and he next proceeds to make the modification above referred to. There can be no doubt that Maxwell's own preference would have been for the latter postulate, but the former was sufficient for his purpose in connection with the magnetic data available at the time. The first application of his idea was made by Ewing, who explained by its means the general form of the magnetisation curve of iron, and showed that a magnetic body, consisting of groups of four molecules in square arrangement, but randomly oriented on the whole, would possess residual magnetisation of the magnitude observed in iron.

2. Very valuable experimental development of the subject has been made by Weiss, who showed that crystals of magnetite, which crystallises in forms belonging to the cubic system, exhibit magnetic quality having cubic symmetry. It is by no means necessary that the magnetic and the structural symmetries of a magnetic crystal should be identical when non-magnetic constituents are also present; and it is of interest to note in this connection that Weiss finds that the magnetic quality of pyrrhotine does not present true hexagonal symmetry.

Weiss has also developed the theory of molecular magnetism in its application to crystalline structures, on the assumption that the internal field acting on any one magnet is uniform, and has successfully explained the observed phenomena. In particular, he has explained the existence of the "magnetic plane" observed in pyrrhotine as the result of an internal demagnetising field in the direction normal to that plane.

3. In two former papers (*Proc. R.S.E.*, 1905 and 1907) I have worked out and evaluated expressions for the internal fields acting on the poles of co-directed molecular magnets in a cubic arrangement, and have compared the results with Weiss's observations on magnetite. In particular it appears that the cubic arrangement of most open order cannot, while that of the homogeneous closest-packed order can, give results in agreement with the observed facts.

It is very desirable that a general development of the theory should be given, in order that those general results, which hold, with mere modifications of detail, in many particular cases, should appear. The object of this paper is to furnish such a discussion, and to apply its results specially to the cases of magnetite and pyrrhotine and to the general question of the dependence of magnetic quality upon molecular configuration.

4. Consider an ideal magnet of semi-length a and pole-strength m. The centre of this magnet being at the origin, the force F at the point (r, θ), θ being measured from the south to north direction of the axis, has a radial component

$$\mathrm{F}\cos\phi = \frac{m}{r^2}\left(1-\frac{a}{r}\cos\theta\right)\left(1+\frac{a}{r}\left(\frac{a}{r}-2\cos\theta\right)\right)^{-3/2} - \frac{m}{r^2}\left(1+\frac{a}{r}\cos\theta\right)\left(1+\frac{a}{r}\left(\frac{a}{r}+2\cos\theta\right)\right)^{-3/2},$$

and a transverse component

$$\mathrm{F}\sin\phi = \tfrac{1}{2}\frac{ma\sin\theta}{r^3}\left[\left(1+\frac{a}{r}\left(\frac{a}{r}-2\cos\theta\right)\right)^{-3/2} + \left(1+\frac{a}{r}\left(\frac{a}{r}+2\cos\theta\right)\right)^{-3/2}\right].$$

By expansion we find

$$\left(1+\frac{a}{r}\left(\frac{a}{r}-2\cos\theta\right)\right)^{-3/2} = \sum_0^\infty{}_n \frac{1}{2^n}\frac{a^n}{r^n}\sum_0{}_p \frac{(-)^p\,\lfloor\underline{2(n-p)+1}\,\cos^{n-2p}\theta}{\lfloor\underline{n-p}\;\lfloor\underline{n-2p}\;\lfloor\underline{p}}.$$

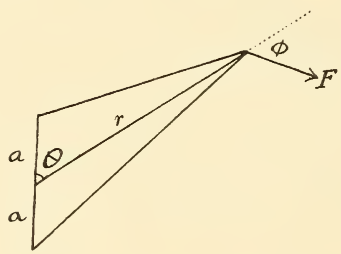
On multiplication by $\left(1 - \frac{a}{r}\cos\theta\right)$ the right-hand side becomes

$$\sum_0^\infty {}_n \frac{1}{2^n}\frac{a^n}{r^n}\sum_0 {}_p \frac{(-)^p \cos^{n-2p}\theta}{p!}\left\{\frac{(2(n-p)+1)!}{(n-p)!\,(n-2p)!} - \frac{(2(n-p)-1)!}{(n-p-1)!\,(n-2p-1)!}\right\},$$

the upper limit for p being $(n-1)/2$ when n is odd, while, when n is even, it is $n/2$ in the first term of the bracket, and $(n-2)/2$ in the second term. From this the similar sum, with $-\cos\theta$ replacing $\cos\theta$, has to be subtracted. Thus n must be odd, and the final sum becomes double of the above. Hence

$$\mathrm{F}\cos\phi = \frac{\mathrm{M}}{r^3}\sum_1^\infty {}_n \frac{1}{2^{n-1}}\left(\frac{a}{r}\right)^{n-1}\sum_0^{\frac{n-1}{2}} {}_p \frac{(-)^p(2(n-p)-1)!}{p!\,(n-p-1)!\,(n-2p-1)!}\cdot\frac{n+1}{n-2p}\cdot\cos^{n-2p}\theta,$$

where $\mathrm{M} = ma$ is the magnetic moment of the magnet.



The corresponding expression for the other component is

$$\mathrm{F}\sin\phi = \frac{\mathrm{M}\sin\theta}{r^3}\sum_0^\infty {}_n \frac{1}{2^n}\frac{a^n}{r^n}\sum_0^{n/2} {}_p \frac{(-)^p(2(n-p)+1)!}{p!\,(n-p)!\,(n-2p)!}\cos^{n-2p}\theta,$$

where n is even. So, writing n for $n-1$ in the expression for $\mathrm{F}\cos\phi$, we obtain, with n always even,

$$\mathrm{F}\cos(\theta+\phi) = \frac{\mathrm{M}}{r^3}\sum_0^\infty {}_n \frac{1}{2^n}\frac{a^n}{r^n}\sum_0^{n/2} {}_p \frac{(-)^p(2(n-p)+1)!}{p!\,(n-p)!\,(n-2p)!}\cos^{n-2p}\theta\left[\frac{2(n-p)+3}{n-2p+1}\cos^2\theta - 1\right],$$

$$\mathrm{F}\sin(\theta+\phi) = \frac{\mathrm{M}}{r^3}\sum_0^\infty {}_n \frac{1}{2^n}\frac{a^n}{r^n}\sum_0^{n/2} {}_p \frac{(-)^p(2(n-p)+1)!}{p!\,(n-p)!\,(n-2p)!}\,\frac{2(n-p)+3}{n-2p+1}\cos^{n-2p}\theta\cos\theta\sin\theta.$$

These are respectively expressions for the components of force, at (r, θ), taken parallel to, and transverse to, the direction of the magnet, and can be identified with

$$\mathrm{F}\cos(\theta+\phi) = \frac{\mathrm{M}}{r^3}\sum_0^\infty {}_\mu \left(\frac{a^2}{r^2}\right)^\mu 2(\mu+1)\mathrm{P}_{2(\mu+1)},$$

$$\mathrm{F}\sin(\theta+\phi) = \frac{\mathrm{M}}{r^3}\sum_0^\infty {}_\mu \left(\frac{a^2}{r^2}\right)^\mu \sin\theta\,\mathrm{P}'_{2(\mu+1)},$$

where $n = 2\mu$, and P is the zonal harmonic in $\cos\theta$ of order $2(\mu+1)$.

5. These quantities represent also the similar components, at the origin, due to a similarly disposed magnet at (r, θ).

Hence, by summation for all values of (r, θ) we obtain the value of the total components of force at the origin, due to any given distribution of co-directed magnets.

6. If now we simplify and make definite the problem by taking the case of a homogeneously and rectangularly arranged infinite system of equal and co-directed magnets, one of which is situated at the origin, we can obtain the values of the parallel and transverse components of force at the poles of the magnet placed at the origin by means of the operator $a\left(\alpha\frac{\partial}{\partial x}+\beta\frac{\partial}{\partial y}+\gamma\frac{\partial}{\partial z}\right)$, where α, β, γ are the direction cosines of the axis. If we denote the operator by Δ, and the term, in the parallel component of force, which involves the power 2μ of the ratio a/r, by $L_{2\mu}$, the value of this term at a pole is

$$L'_{2\mu} = L_{2\mu} + \frac{1}{2!}\Delta^2 L_{2(\mu-1)} + \frac{1}{4!}\Delta^4 L_{2(\mu-2)} + \ . \ . \ . + \frac{1}{(2\mu)!}L_0 .$$

To evaluate this we have the conditions

$$\Delta r = a\cos\theta ,$$
$$\Delta\cos\theta = \frac{a}{r}\sin^2\theta ,$$
$$\Delta\sin\theta = -\frac{a}{r}\sin\theta\cos\theta ,$$
$$r^2 = x^2 + y^2 + z^2 ,$$
$$\cos\theta = \frac{\alpha x + \beta y + \gamma z}{r} .$$

By means of the known relations

$$(2n+1)\sin^2\theta\, P'_n(\cos\theta) = n(n+1)(P_{n-1}(\cos\theta) - P_{n+1}(\cos\theta) ,$$
$$(2n+1)\cos\theta\, P_n(\cos\theta) = nP_{n-1}(\cos\theta) + (n+1)P_{n+1}(\cos\theta) ,$$

it is easily found that

$$\frac{1}{2p!}\Delta^{2p}L_{2(\mu-p)} = \frac{(2\mu+1)!}{2p!\,(2\mu-2p+1)!}L_{2\mu} .$$

So

$$L'_{2\mu} = \sum_0^{\mu}{}_p \frac{(2\mu+1)!}{2p!\,(2\mu-2p+1)!} . L_{2\mu} .$$

But this sum is the sum of the even coefficients in the expansion of $(1+x)^{2\mu+1}$. Therefore

$$\begin{aligned} L'_{2\mu} &= 4^{\mu}L_{2\mu} , \\ &= \frac{M}{r^3}\left(4\frac{a^2}{r^2}\right)^{\mu} 2(\mu+1)P_{2(\mu+1)} . \qquad . \quad . \quad . \quad . \quad (1) \end{aligned}$$

Similarly, using T_{2m} to represent the term in $F \sin(\theta+\phi)$ which involves the power 2μ of the ratio a/r, and employing the other known relations

$$\sin^2\theta . P''_n(\cos\theta) = 2\cos\theta P'_n(\cos\theta) - n(n+1)P_n(\cos\theta),$$
$$(2n+1)\cos\theta P'_n(\cos\theta) = nP'_{n+1}(\cos\theta) + (n+1)P'_{n-1}(\cos\theta),$$
$$(2n+1)P_n(\cos\theta) = P'_{n+1}(\cos\theta) - P'_{(n-1)}(\cos\theta),$$

we get

$$T'_{2\mu} = 4^\mu T_{2\mu},$$
$$= \frac{M}{r^3}\left(4\frac{a^2}{r^2}\right)^\mu \sin\theta P'_{2(\mu+1)} . \qquad (2)$$

Finally, therefore,

$$F\cos(\theta+\phi) = \sum . \frac{M}{r^3}\sum_0^\infty{}_\mu\left(4\frac{a^2}{r^2}\right)^\mu 2(\mu+1)P_{2(\mu+1)}, \qquad (3)$$

$$F\sin(\theta+\phi) = \sum . \frac{M}{r^3}\sum_0^\infty{}_\mu\left(4\frac{a^2}{r^2}\right)^\mu \sin\theta P'_{2(\mu+1)}, \qquad (4)$$

where the first Σ in each expression indicates summation for all the constituent magnets in the infinite system, omitting the one at the origin. If, in any crystalline system of molecular magnets, the arrangement is not singly homogeneous, so that products and odd powers of co-ordinates do not vanish on summation, these expressions require modification, for odd powers of Δ would appear in the developments of $L'_{2\mu}$ and $T'_{2\mu}$.

7. The most powerful term in (3) and (4) respectively is the first. Each successive term is of order two higher in the ratio a/r than the preceding. Therefore the effect of the first term is felt at much greater distances than that of any other, and is more strongly felt at any one distance. Consequently that term, in each series, is of greatest importance in the determination of the so-called ferromagnetic quality of a system of molecular magnets.

In the expression (3), for the component of the internal field in the direction of magnetisation, the first term is

$$\sum . \frac{M}{r^3}2P_2 = \sum . \frac{M}{r^3}(3\cos^2\theta - 1) . \qquad (5)$$

If ρ, $p\rho$, $q\rho$ represent respectively the distances between successive planes normal to the three principal rectangular directions in the space-lattice determined by the magnet-centres, we have

$$r^2 = \rho^2(\lambda^2 + p^2\mu^2 + q^2\nu^2),$$

where λ, μ, ν are integers to which all values, positive and negative, from 1 to ∞ are to be given, and we may presume $q \nless p \nless 1$. Also, if α, β, γ be the direction cosines specifying the magnetisation with reference to the principal axes,

$$\cos\theta = \frac{a\lambda + p\beta\mu + q\gamma\nu}{\sqrt{\lambda^2 + p^2\mu^2 + q^2\nu^2}}.$$

Hence, by expression of (5), there results

$$L'_0 = \frac{M}{\rho^3}[A(3a^2 - 1) + B(3\beta^2 - 1) + C(3\gamma^2 - 1)], \quad . \quad . \quad . \quad (6)$$

where

$$A = \sum . \frac{\lambda^2}{(\lambda^2 + p^2\mu^2 + q^2\nu^2)^{5/2}}, \quad B = \sum . \frac{p^2\mu^2}{(\lambda^2 + p^2\mu^2 + q^2\nu^2)^{5/2}}, \quad C = \sum . \frac{q^2\nu^2}{(\lambda^2 + p^2\mu^2 + q^2\nu^2)^{5/2}}.$$

The terms involving products of λ, μ, ν vanish by symmetry, since positive and negative values alike occur.

Equation (6) indicates an ellipsoidal distribution of the parallel component of the internal field.

The first term in the transverse component of the internal field is

$$T'_0 = 3M\sum\frac{\sin\theta\cos\theta}{r^3}.$$

The component of this in the direction of the principal axis denoted by λ is

$${}_\lambda T'_0 = 3\frac{M}{\rho^3}\Sigma . [\lambda - a(a\lambda + p\beta\mu + q\gamma\nu)](a\lambda + p\beta\mu + q\gamma\nu)(\lambda^2 + p^2\mu^2 + q^2\nu^2)^{-5/2},$$

whence

$$\left.\begin{aligned} {}_\lambda T'_0 &= \frac{M}{\rho^3}a[A(1 - a^2) - B\beta^2 - C\gamma^2], \\ {}_\mu T'_0 &= \frac{M}{\rho^3}\beta[B(1 - \beta^2) - C\gamma^2 - Aa^2], \\ {}_\nu T'_0 &= \frac{M}{\rho^3}\gamma[C(1 - \gamma^2) - Aa^2 - B\beta^2]. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (7)$$

8. The quantities B and C become respectively equal to A when $p = q = 1$; they are equal to each other when $p = q$. The quantity A steadily decreases as either p or q increases. So also B and C steadily decrease as q and p respectively increase. When q differs little from unity, say $q = 1 + \epsilon$, with $p = 1$, we have, to the first order in ϵ,

$$C = \frac{1 + 2\epsilon}{3}\Sigma . (\lambda^2 + \mu^2 + \nu^2)^{-3/2} - \frac{5}{3}\epsilon\Sigma . \nu^4(\lambda^2 + \mu^2 + \nu^2)^{-7/2}.$$

But the latter term is less than $\frac{5}{3}\epsilon\frac{1}{3}\Sigma . \nu^2(\lambda^2 + \mu^2 + \nu^2)^{-5/2}$, so that $\partial C/\partial\epsilon$ is positive and greater than $\frac{1}{3}\Sigma . (\lambda^2 + \mu^2 + \nu^2)^{-3/2}$. Similarly,

$$\mathrm{B} = \sum \cdot \frac{\mu^2}{(\lambda^2+\mu^2+\nu^2)^{5/2}}\left(1 - 5\epsilon\frac{\nu^2}{\lambda^2+\mu^2+\nu^2}\right),$$

$$\frac{\partial \mathrm{B}}{\partial \epsilon} = -5\sum \cdot \frac{\mu^2\nu^2}{(\lambda^2+\mu^2+\nu^2)^{7/2}}.$$

So that $\partial\mathrm{B}/\partial\epsilon$ is negative, and this result is independent of the value of p.

Results of this kind, along with considerations of symmetry, enable us to obtain general information regarding the magnetic action of a homogeneous crystalline arrangement (referable to rectangular axes) of magnets, without actual evaluation of the constants A, B, C. It should be noted also that, in the application to solids, M may be a function of the temperature. In the absence of knowledge regarding the constitution of a magnetic molecule, it may possibly have to be regarded as a function of the resultant magnetic field intensity.

From (6) we get the principal values of the internal field in the directions in which λ, μ, ν are taken:

$$\left.\begin{aligned} {}_\lambda\mathrm{L}'_0 &= \frac{\mathrm{M}}{\rho^3}[2\mathrm{A} - \mathrm{B} - \mathrm{C}], \\ {}_\mu\mathrm{L}'_0 &= \frac{\mathrm{M}}{\rho^3}[-\mathrm{A} + 2\mathrm{B} - \mathrm{C}], \\ {}_\nu\mathrm{L}'_0 &= \frac{\mathrm{M}}{\rho^3}[-\mathrm{A} - \mathrm{B} + 2\mathrm{C}]. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (8)$$

Hence

$${}_\lambda\mathrm{L}'_0 + {}_\mu\mathrm{L}'_0 + {}_\nu\mathrm{L}'_0 = 0, \quad . \quad . \quad . \quad . \quad . \quad (9)$$

which shows that the internal force must have a demagnetising effect when the magnetisation is along one principal axis at least.

From (6) and (7) we see that there is zero internal field when $p = q = 1$, *i.e.* when $\mathrm{A} = \mathrm{B} = \mathrm{C}$. This is the case of a cubic crystalline arrangement. Hence we perceive the possibility of metals which crystallise on the cubic system having strong magnetic properties. (It must be remembered that this zero field is due to co-orientation of all the magnets.)

If $p = 1$, $q > p$, A and B remain equal, while C increases; hence ${}_\lambda\mathrm{L}'_0$ and ${}_\mu\mathrm{L}'$ become negative, while ${}_\nu\lambda'_0$ becomes positive. Provided that q is sufficiently greater than p, the same condition holds even if $p > 1$. Thus, in a crystal containing magnetic molecules in ortho-rhombic arrangement subject to the above limitations, the axis along which the spacing is widest is a *Direction of Easy Magnetisation*, while magnetisation is difficult in directions lying in the perpendicular plane. Experimental exemplification of this condition is still wanting.

If $q \geqq p > 1$, provided that the excess of q over p be not too great, we may have ${}_\nu\mathrm{L}'_0$ and ${}_\mu\mathrm{L}'_0$ both positive, while ${}_\lambda\mathrm{L}'_0$ is negative. Thus, in a crystal containing magnetic molecules in ortho-rhombic arrangement subject to these conditions, the axis along which the spacing is closest is a direction of difficult magnetisation, while the plane perpendicular to it is a *Plane of Easy Magnetisation.* Extension of experimental evidence on this point is desirable, but the existence of a magnetic plane has been shown by Weiss in the case of pyrrhotine, and has been explained by him as the result of an internal demagnetising field.

9. By means of (8), it is easy to deduce

$$L'_0 = {}_\lambda L'_0\alpha^2 + {}_\mu L'_0\beta^2 + {}_\nu L'_0\gamma^2, \qquad (10)$$

which gives the parallel component of the internal field in the direction (α, β, γ) in terms of the principal parallel components. From this equation it follows that (9) is invariant relative to any set of rectangular axes suiting the crystalline symmetry, and that the conditions $A=B=C$, or $A=B$ provided that the new λ, μ axes lie in the plane of the old λ, μ axes, etc., are also invariant. But the condition of correspondence to crystalline symmetry is very restrictive. Thus, if we take $\gamma=0$, $\alpha^2=\beta^2=\frac{1}{2}$, we get $L'_0=\frac{1}{2}{}_\nu L'_0$ for both 45° lines, and therefore $A=B$ necessarily for these two lines as axes; so that the invariance is restricted to pairs of axes relatively to which the crystalline symmetry is identical.

The special case in which $2B=A+C$ is of interest. Here ${}_\mu L'_0=0$, ${}_\lambda L'_0=-{}_\nu L'_0$, so that there is no internal field in the direction of the μ axis, while magnetising and demagnetising forces respectively exist along the λ and μ axes. The surface represented by (10) encloses, in the plane $\beta=0$, an area whose boundary is $r={}_\lambda L'_0 \cos 2\theta$, alternate lobes having opposite signs. In the plane $\gamma=0$, the trace is $r={}_\lambda L'_0 \cos^2\theta$.

Equations (10) and (9) show that the lines $\alpha^2=\beta^2=\gamma^2$ are lines of zero parallel component. The surface represented by (10) has three lobes situated in the spaces marked out by these lines. The elliptic cone, given by equating the left-hand side of (10) to zero, passes through the lines and separates the lobes.

In fig. 1 sections of (10), with the values ${}_\lambda L' = -2$, ${}_\mu L'_0=-1$, ${}_\nu L'_0=+3$, the ν-axis being a magnetic line, are given. The curves represent one quadrant of the section by the planes $\gamma=0$ and $\alpha=0$. L'_0 is negative in the former case; in the latter case, in the loop next the ν-axis, it is positive. If we took ${}_\lambda L'_0=-3$, ${}_\mu L'_0=1$, ${}_\nu L'_0=2$, the signs of L'_0 in the respective regions would be reversed, and the plane perpendicular to the λ-axis would be a magnetic plane.

10. From (2) we find

$$T'_0 = \sum \cdot \frac{M}{r^3} \sin\theta P'_2 = 3M \sum \cdot \frac{\sin\theta\cos\theta}{r^3}.$$

Using the preceding notation, the component of this in the direction indicated by λ is

$$
\begin{aligned}
{}_{\lambda}\mathrm{T}'_0 &= 3\frac{\mathrm{M}}{\rho^3}\sum \cdot \frac{\lambda - \alpha(\alpha\lambda + \beta\mu + \gamma\nu)}{(\lambda^2 + p^2\mu^2 + q^2\nu^2)^{5/2}}(\alpha\lambda + \beta\mu + \gamma\nu) \\
&= 3\frac{\mathrm{M}}{\rho^3}[\mathrm{A}\alpha(1 - \alpha^2) - \mathrm{B}\alpha\beta^2 - \mathrm{C}\alpha\gamma^2] \\
&= 3\frac{\mathrm{M}}{\rho^3}[\mathrm{A} - (\mathrm{A}\alpha^2 + \mathrm{B}\beta^2 + \mathrm{C}\gamma^2)]\alpha\,, \qquad \qquad (11)
\end{aligned}
$$

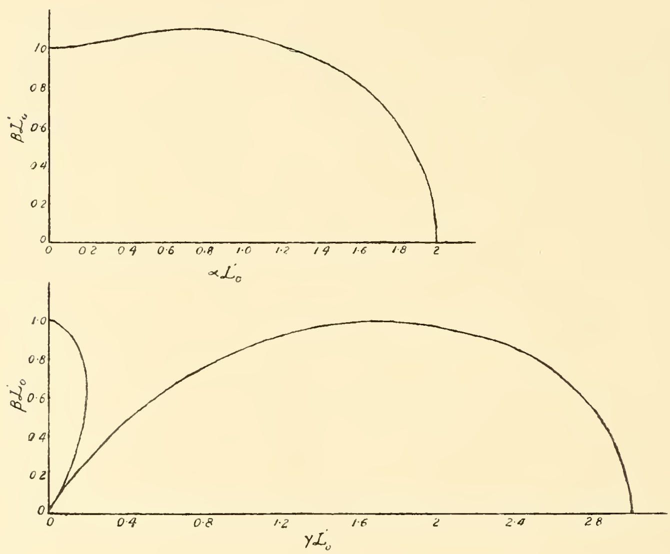


Fig. 1.

and the similar expressions for ${}_{\mu}\mathrm{T}'_0$, ${}_{\nu}\mathrm{T}'_0$ can be at once obtained by cyclical interchange of A, B, C; α, β, γ. Hence, from the condition

$$
\mathrm{L}'_0 = -\frac{\mathrm{M}}{\rho^3}[(\mathrm{A} + \mathrm{B} + \mathrm{C}) - 3(\mathrm{A}\alpha^2 + \mathrm{B}\beta^2 + \mathrm{C}\gamma^2)]
$$

along with (8), we have

$$
\begin{aligned}
{}_{\lambda}\mathrm{T}'_0 &= ({}_{\lambda}\mathrm{L}'_0 - \mathrm{L}'_0)\alpha\,, \\
{}_{\mu}\mathrm{T}'_0 &= ({}_{\mu}\mathrm{L}'_0 - \mathrm{L}'_0)\beta\,, \\
{}_{\nu}\mathrm{T}'_0 &= ({}_{\nu}\mathrm{L}'_0 - \mathrm{L}'_0)\gamma\,.
\end{aligned}
$$

These results indicate a very simple geometrical construction for the principal components of the transverse force by means of the surface (10), whose radii have the values L'_0. Draw the spheres of radii ${}_{\lambda}\mathrm{L}'_0$, ${}_{\mu}\mathrm{L}'_0$, ${}_{\nu}\mathrm{L}'_0$, which are the principal radii of (10), and draw the radius-vector of (10) in the direction (α, β, γ). The differences between the radii of the spheres and the radius-vector of (10) give, when projected respectively upon the principal axes, the principal components of the transverse force when the magnetisation is in the direction of that radius-vector.

But it is more useful to analyse the transverse force into its two components P and Q respectively in, and perpendicular to, a plane containing the direction of magnetisation and the ν-axis, *i.e.* the γ-axis. Let δ indicate the trace of this plane on the (α, β) plane, the trace making an

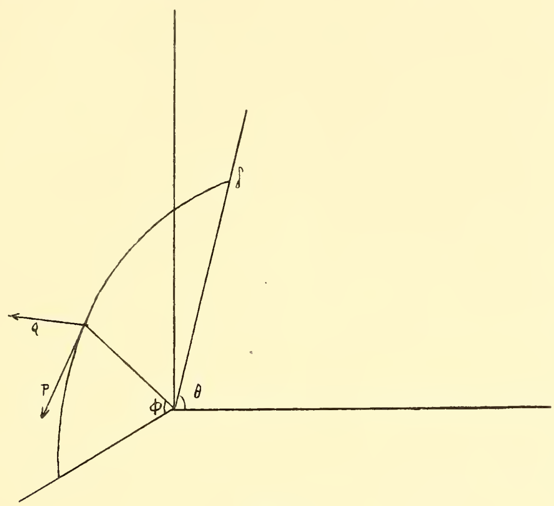


Fig. 2.

angle θ with the α-axis. Writing the expressions of type (11) in the forms $3M/\rho^3 . [(A-B)\beta^2+(A-C)\gamma^2]\alpha$, etc., we have

$$Q = 3\frac{M}{\rho^3}[-\{(A-B)\beta^2+(A-C)\gamma^2\}\alpha \sin\theta + \{(B-C)\gamma^2+(B-A)\alpha^2\}\beta\cos\theta],$$

$$R = 3\frac{M}{\rho^3}[\{(A-B)\beta^2+(A-C)\gamma^2\}\alpha\cos\theta + \{(B-C)\gamma^2+(B-A)\alpha^2\}\beta\sin\theta],$$

where R is the component along δ. With the conditions $\gamma = \cos\phi$, $\alpha = \sin\phi\cos\theta$, $\beta = \sin\phi\sin\theta$, and noting that the component parallel to γ is ${}_\nu L'_0 = 3M/\rho^3[(C-A)\alpha^2+(C-B)\beta^2]\gamma$, so that $P = {}_\nu L'_0 \sin\phi - R\cos\phi$, we find

$$\left.\begin{aligned} P &= \frac{3M}{\rho^3}[(C-A)\cos^2\theta + (C-B)\sin^2\theta]\sin\phi\cos\phi, \\ Q &= \frac{3M}{\rho^3}(B-A)(1+\cos^2\phi)\sin\phi\cos\theta\sin\theta. \end{aligned}\right\} \quad . \quad . \quad (12)$$

These formulæ show that the component of the internal field transverse to the direction of magnetisation vanishes, as the parallel component (§ 8) does, when $A=B=C$; so that the magnets, when in the cubic or isometric arrangement, are absolutely free from mutual control so far as the most powerful term in the zonal harmonic expansion is concerned. In the tetragonal system ($C = B = A$), the transverse component lies in a plane passing through the unique principal axis (ν). This condition also holds (§ 11) in the hexagonal system. Its value is then $P = 3M/\rho^3 . (C - A) \sin\phi \cos\phi$, which vanishes only when the magnetisation is in the direction of the greatest axis or lies in the plane of the two other principal axes. Therefore, although the parallel component of the internal force is zero in the directions given by $\alpha^2 = \beta^2 = \gamma^2$, magnetisation in these directions could

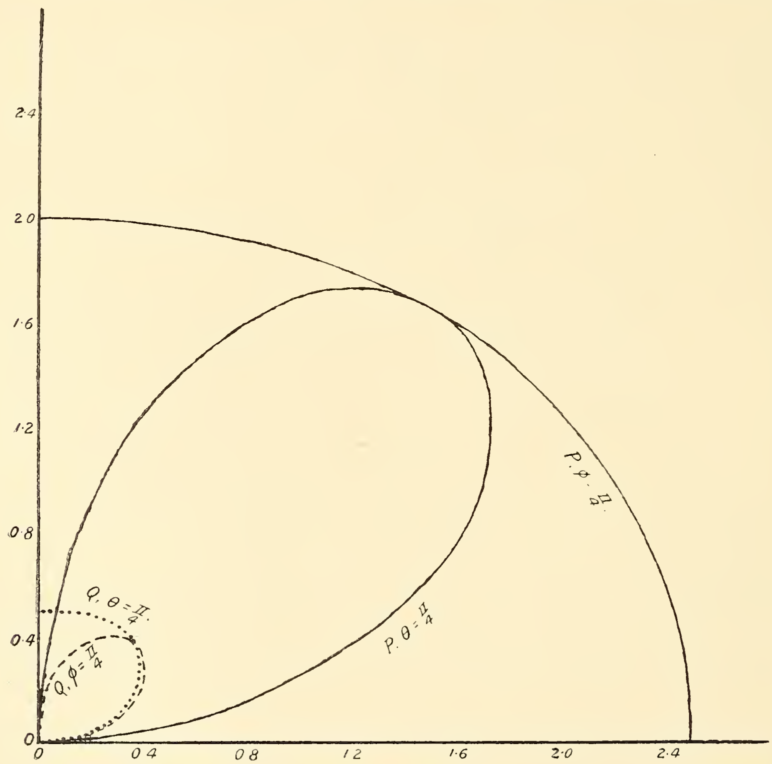


Fig. 3.

only be maintained by the action of a transverse component of the external field balancing $P = \sqrt{2}M/\rho^3 . (C - A)$. In the more general case of the orthorhombic system ($C > B > A$), the formulæ show that no direction of magnetisation is stable under the action of the internal forces of the first order alone except the direction of the greatest principal axis; for, in every other case, the internal field has a component transverse to the direction of magnetisation. In fig. 3 values of P and Q are shown, for the value $\pi/4$ of θ and ϕ, with the same numerical values of the constants as were assumed in § 9.

It is of importance to note the symmetry of the expressions for the transverse component when the magnetisation is in any one of the three principal planes. The symmetry is identical with regard to either principal axis in one plane, and is therefore *not influenced by difference of scale of the space-lattice in the three principal rectangular directions.*

11. In the hexagonal system we shall consider the ν-axis to be the normal to the plane of the hexagonal arrangement, and the λ-axis to be along a line of closest grouping in the hexagonal plane (fig. 4). Here we have $r^2 = \rho^2(\lambda^2 + 3\mu^2 + q\nu^2)$, so that $p = \sqrt{3}$ and ρ is half of the least distance between centres of magnets in the hexagonal arrangement. Using the same notation as before, we have equations (6), (8), (9), and (10) holding with respect to the parallel component of force, and the structural symmetries will determine relations amongst A, B, and C. Thus the condition

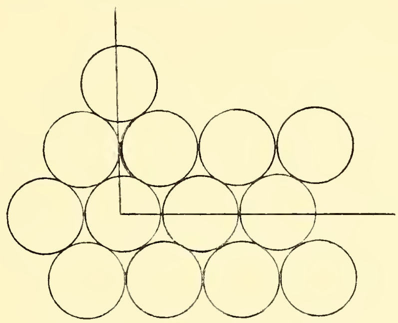


Fig. 4.

of identity of arrangement in the primary plane relatively to the λ-axis or a line inclined to it at 60° makes (6) take the same value with $\alpha = 1$, $\beta = 0$, $\gamma = 0$ as with $\alpha = 1/2$, $\beta = \sqrt{3}/2$, $\gamma = 0$; from which at once

$$A = B$$

independently of the value of C. Hence, by (12), we have $Q = 0$,

$$P = \frac{3M}{\rho^3}(C - A)\sin\phi\cos\phi .$$

The axis of hexagonal symmetry is therefore a direction of stable or unstable magnetisation according as C is > or < A.

12. To deal with the cases in which the space-lattice belongs to the monoclinic or triclinic groupings, it is convenient to refer the system of magnets to the (in general) oblique λ, μ, ν axes. Here, as before, $x = \rho\lambda$, $y = \rho p\mu$, $z = \rho q\nu$; and, l, m, n being now direction *ratios*, we have $x = lr$, $y = mr$, $z = nr$, along with

$$l(l + mZ + nY) + m(m + nX + lZ) + n(n + lY + mX) = 1 ,$$

or, say,

$$ll' + mm' + nn' = 1\,, \quad . \quad . \quad . \quad . \quad . \quad (13)$$

X, Y, and Z being respectively the cosines of the angles between the y and z, z and x, x and y, axes. Hence

$$r^2 = \rho^2[\lambda(\lambda + p\mu Z + q\nu Y) + p\mu(p\mu + q\nu X + \lambda Z) + q\nu(q\nu + \lambda Y + p\mu X)]\,,$$

or, say, as in (13),

$$r^2 = \rho^2(\lambda\lambda' + p\mu(p\mu)' + q\nu(q\nu)')\,.$$

Also, a, β, γ being the direction ratios of the magnetisation, the cosine of the angle between r and (a, β, γ) is

$$\cos\theta = \frac{1}{r}(ax' + \beta y' + \gamma z') = \frac{\rho}{r}(a\lambda' + \beta(p\mu)' + \gamma(q\nu)') = \frac{\rho}{r}(\lambda a' + p\mu\,.\,\beta' + q\nu\,.\,\gamma')\,.$$

Hence, with $r^2 = xx' + yy' + zz'$, applying the oblique-axes operator

$$\Delta = \left(a\frac{\partial}{\partial x} + \beta\frac{\partial}{\partial y} + \gamma\frac{\partial}{\partial z}\right),$$

we readily verify the results of § 5, now referred to oblique axes,

$$\Delta r = a\cos\theta\,.$$
$$\Delta \cos\theta = a/r\,.\,\sin^2\theta\,,$$
$$\Delta \sin\theta = -a/r\,.\,\sin\theta\cos\theta\,.$$

Therefore the former expressions for $T'_{2\mu}$ and $L'_{2\mu}$ are directly applicable to oblique axes when the modified values of $\cos\theta$ and $\sin\theta$ are taken. The value of L'_0 becomes, since $\Sigma\,.\,(\lambda\mu) = 0$, etc., as before,

$$L'_0 = \frac{M}{\rho^3}[A'(3a'^2 - 1) + B'(3\beta'^2 - 1) + C'(3\gamma'^2 - 1)]\,, \quad . \quad . \quad (14)$$

where

$$A' = \sum\,.\,\frac{\lambda^2}{[\lambda\lambda' + p\mu(p\mu)' + q\nu(q\nu)']^{5/2}}, \quad B' = \sum\,.\,\frac{p^2\mu^2}{[\lambda\lambda' + p\mu(p\mu)' + q\nu(q\nu)']^{5/2}},$$
$$C' = \sum\,.\,\frac{q^2\nu^2}{[\lambda\lambda' + p\mu(p\mu)' + q\nu(q\nu)']^{5/2}}\,.$$

Putting $\beta = 0$, $\gamma = 0$, $a = 1$ we get

so also

$$\left.\begin{aligned} {}_aL'_0 &= \frac{M}{\rho^3}[2A' + B'(3Z^2 - 1) + C'(3Y^2 - 1)]\,; \\ {}_\beta L'_0 &= \frac{M}{\rho^3}[A'(3Z^2 - 1) + 2B' + C'(3X^2 - 1)]\,, \\ {}_\gamma L'_0 &= \frac{M}{\rho^3}[A'(3Y^2 - 1) + B'(3X^2 - 1) + 2C']\,, \end{aligned}\right\} \quad . \quad . \quad . \quad (15)$$

instead of (8).

If, on the other hand, we put $\beta'=0$, $\gamma'=0$, $\alpha'=1$, etc., we find

$$\left.\begin{aligned}{}_{\alpha'}L'_0&=\frac{M}{\rho^3}(2A'-B'-C'),\\ {}_{\beta'}L'_0&=\frac{M}{\rho^3}(-A'+2B'-C'),\\ {}_{\gamma'}L'_0&=\frac{M}{\rho^3}(-A'-B'+2C'),\end{aligned}\right\}\quad . \quad . \quad . \quad . \quad (16)$$

which correspond exactly to (8), and exhibit $\alpha'=1$, $\beta'=0$, $\gamma'=0$; $\alpha'=0$, $\beta'=1$, $\gamma'=0$; $\alpha'=0$, $\beta'=0$, $\gamma'=1$; as the three principal directions of the internal field. If we take $A' \not> B' \not> C'$, we see that, except in special cases, ${}_{\alpha'}L'_0$ is negative; ${}_{\beta'}L'_0$ is negative if $B'-A' < C'-B'$, positive if $B'-A' > C'-B'$; and ${}_{\gamma'}L'_0$ is positive. By (13) these axes are given by the pure strain

$$\begin{aligned}\alpha'&=\alpha \quad +\beta Z+\gamma Y,\\ \beta'&=\alpha Z+\beta \quad +\gamma X,\\ \gamma'&=\alpha Y+\beta X+\gamma.\end{aligned}$$

Their direction ratios are $\alpha=1$, $\beta=(XY-Z)/(1-X^2)$, $\gamma=(XZ-Y)/(1-X^2)$; and so on, the others being got by cyclical interchange of α, β, γ, and X, Y, Z, simultaneously. It is important to remark that these direction ratios of the principal axes for the internal field *depend only on the angles between the crystalline axes, and not at all on the scale of spacing* of the centres of magnets in any direction.

The component L'_0, parallel to the magnetisation, now vanishes when $\alpha'^2=\beta'^2=\gamma'^2=1/3$. These conditions give

$$\alpha\{3-2(X+Y+Z)-(X^2+Y^2+Z^2)+2(XY+YZ+ZX)\}$$
$$=\sqrt{3}\{1+Y+Z+X(X-Y-Z)\},$$

β and γ being got by cyclical interchange. These give, as we see they should from the condition $\alpha\alpha'+\beta\beta'+\gamma\gamma'=1$, the result $\alpha+\beta+\gamma=\sqrt{3}$.

In the case of the transverse component in the clinic systems, the equations (7) are replaced by

$${}_{\lambda}T'_0=3\frac{M}{\rho^3}\sum.\frac{\lambda-\alpha[\alpha\lambda'+\beta(p\mu)'+\gamma(q\nu)']}{(\lambda\lambda'+p\mu(p\mu)'+q\nu(q\nu)')^{5/2}}(\alpha\lambda'+\beta(p\mu)'+\gamma(q\nu)');\ \text{etc.}$$

$$\text{or}\ {}_{\lambda}T'_0=3\frac{M}{\rho^3}\sum.\frac{\lambda-\alpha(\lambda\alpha'+p\mu.\beta'+q\nu.\gamma')}{(\lambda\lambda'+p\mu(p\mu)'+q\nu(q\nu)')^{5/2}}(\lambda\alpha'+p\mu\beta'+q\nu\gamma');\ \text{etc.}\quad . \quad . \quad (17)$$

$$=3\frac{M}{\rho^3}\sum.[\lambda^2(\alpha'-\alpha\alpha'^2)-p^2\mu^2\alpha\beta'^2-q^2\nu^2\alpha\gamma'^2](\lambda\lambda'+p\mu(p\mu)'+q\nu(q\nu'))^{-5/2}$$

$$=\frac{3M}{\rho^3}[A'(\alpha+\beta Z+\gamma Y)(1-\alpha^2-\alpha\beta Z-\alpha\gamma Y)-B'\alpha(\alpha Z+\beta+\gamma X)^2$$
$$-C'\alpha(\alpha Y+\beta X+\quad)^2],$$

replacing (11) with similar expressions for ${}_{\mu}T'_0$ and ${}_{\nu}T'_0$.

The various equations show that the internal field is distorted from its symmetrical disposition when the crystalline axes are inclined, while retaining its general features relative to them.

In the subsequent sections, crystalline arrangements referable to rectangular axes will alone be considered.

13. The term of next order in the parallel component of the internal force is

$$L'_2 = 2\frac{Ma^2}{\rho^5}[35\cos^4\theta - 30\cos^2\theta + 3](\lambda^2 + p^2\mu^2 + q^2\nu^2)^{-5/2}.$$

With $\cos\theta = (\alpha\lambda + \beta \,.\, p\mu + \gamma \,.\, q\nu)/(\lambda^2 + p^2\mu^2 + q^2\nu^2)^{1/2}$, this becomes

$$L'_2 = \frac{2Ma^2}{\rho^5}[(D\alpha^4 + E\beta^4 + F\gamma^4) + 2(G\alpha^2\beta^2 + H\beta^2\gamma^2 + I\gamma^2\alpha^2) - (A_2\alpha^2 + B_2\beta^2 + C_2\gamma^2) + J]; \tag{18}$$

which, if we consider the cubic system, reduces to

$$\begin{aligned} L'_2 &= \frac{2Ma^2}{\rho^5}[D(\alpha^4 + \beta^4 + \gamma^4) + 2G(\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2) - A_2 + J] \\ &= \frac{2Ma^2}{\rho^5}[(D - G)(\alpha^4 + \beta^4 + \gamma^4) + G - A_2 + J], \end{aligned}$$

where

$$D = 35\sum \cdot \frac{\lambda^4}{(\lambda^2 + \mu^2 + \nu^2)^{9/2}},\quad G = 35\sum \cdot \frac{3\lambda^2\mu^2}{(\lambda^2 + \mu^2 + \nu^2)^{9/2}},\quad A_2 = 30\sum \cdot \frac{\lambda^2}{(\lambda^2 + \mu^2 + {}^2\nu)^{7/2}},$$

$$J = \sum \cdot \frac{3}{(\lambda^2 + \mu^2 + \nu^2)^{5/2}}.$$

We readily deduce $3A_2 = 10J$, $2G + 3D = 35/3 \,.\, J$. Hence, finally,

$$L'_2 = \frac{Ma^2}{\rho^5}(G - D)\left[\frac{6}{5} - 2(\alpha^4 + \beta^4 + \gamma^4)\right]. \qquad . \quad . \quad . \quad . \tag{19}$$

The corresponding term in the transverse component is

$$T'_2 = \frac{Ma^2}{\rho^5}\sin\theta\cos\theta\,(70\cos^2\theta - 30)(\lambda^2 + p^2\mu^2 + q^2\nu^2)^{-5/2},$$

and the component of this in the direction of the λ-axis is (*cf.* § 7)

$${}_\lambda T'_2 = \frac{Ma^2}{\rho^5}\sum \cdot \frac{[\lambda - \alpha(\alpha\lambda + p\beta\mu + q\gamma\nu)]}{(\lambda^2 + p^2\mu^2 + q^2\nu^2)^3}\left\{\frac{70(\alpha\lambda + p\beta\mu + q\gamma\nu)^3}{(\lambda^2 + p^2\mu^2 + q^2\nu^2)^{3/2}} - \frac{30(\alpha\lambda + p\beta\mu + q\gamma\nu)}{(\lambda^2 + p^2\mu^2 + q^2\nu^2)^{1/2}}\right\}.$$

In the cubic system this becomes

$$\text{so}\qquad \left.\begin{aligned} {}_\lambda T'_2 &= 2\frac{Ma^2}{\rho^5}(G - D)\,\alpha[(\alpha^4 + \beta^4 + \gamma^4) - \alpha^2], \\ {}_\mu T'_2 &= 2\frac{Ma^2}{\rho^5}(G - D)\beta[(\alpha^4 + \beta^4 + \gamma^4) - \beta^2], \\ {}_\nu T'_2 &= 2\frac{Ma^2}{\rho^5}(G - D)\,\gamma[(\alpha^4 + \beta^4 + \gamma^4) - \gamma^2]. \end{aligned}\right\} \qquad . \quad . \quad . \quad . \tag{20}$$

Hence, in the plane $\gamma=0$, T'_2 becomes ${}_\gamma T'_2=2\dfrac{Ma^2}{\rho^5}(G-D)\alpha\beta(\alpha^2-\beta^2)$. Fig. 5 represents this curve and the curve (19) on the same scale, the latter being in dashes, the former in full lines. The dotted curve refers to § 15.

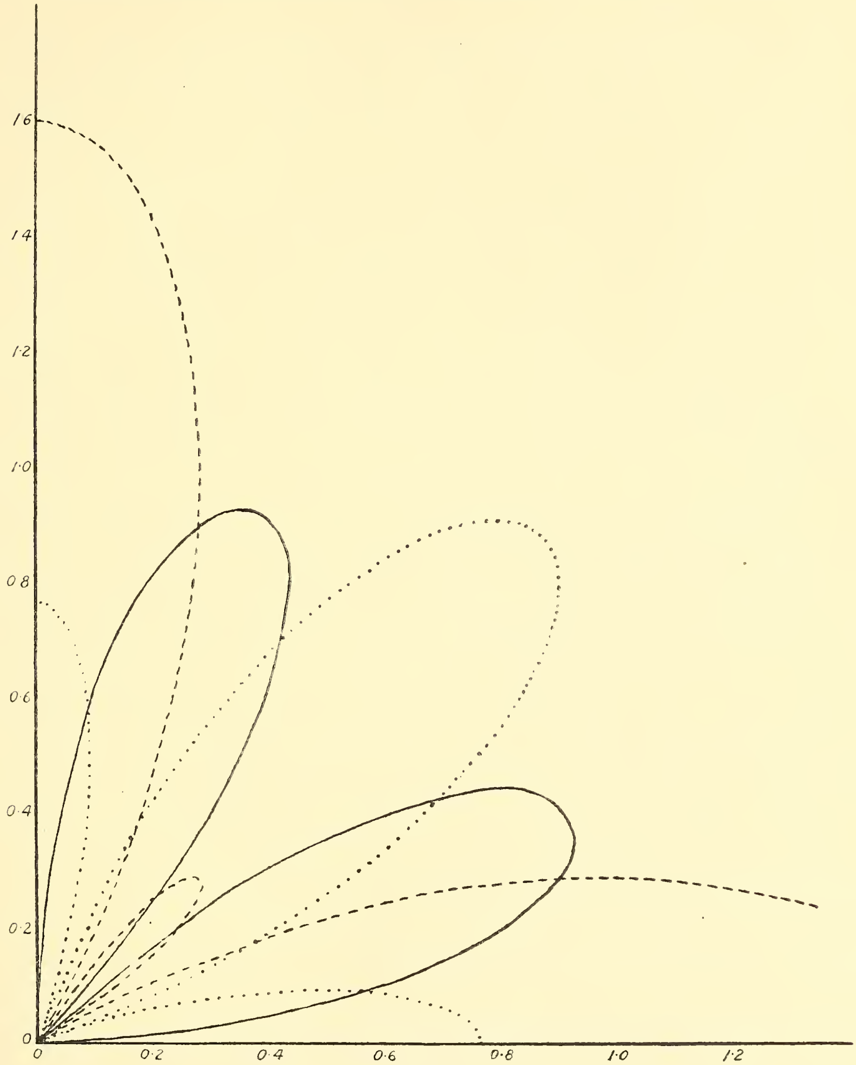


FIG. 5.

If $G > D$, (19) and (20) show that the internal force opposes the magnetisation when that is parallel to a principal (quaternary) axis in the cubic arrangement (*e.g.* $\alpha=1$, $\beta=0$, $\gamma=0$). On the other hand, the internal force is positive when the magnetisation is along a binary axis (*e.g.* $\alpha=\beta=1/\sqrt{2}$, $\gamma=0$), or a ternary axis (*e.g.* $\alpha=\beta=\gamma=1/\sqrt{3}$).

Again, if we write $\alpha^2 = 1/3 - \xi$, $\beta^2 = 1/3 - \eta$, $\gamma^2 = 1/3 + \xi + \eta$, where ξ and η are small, we find ${}_\lambda T'_2 = P\xi$, ${}_\mu T'_2 = P\eta$, ${}_\nu T'_2 = P(\xi + \eta)$. In these expressions P is a positive quantity. Hence the transverse force acts so as to turn the system of co-directed magnets into coincidence with the ternary axis. Similarly, we find that the transverse force always tends to turn the system from coincidence with a quaternary axis. It acts more towards, or from, a binary axis according as the displacement of the direction of magnetisation from the binary axis is, on the whole, towards a quaternary, or a ternary, axis.

From these conditions we see that, when $G > D$, the ternary axes are the only directions of stable equilibrium for the co-directed system of magnets when the external field is removed. On the other hand, if $D > G$, the quaternary axes are directions of stable equilibrium.

14. To determine the values of L'_2, and the components of T'_2, in the hexagonal system, we have to put $p = \sqrt{3}$, $q = q$, in the preceding section if we choose the axes as in § 11. Equation (18) is then subject to the conditions

$$D = 35\sum \cdot \frac{\lambda^4}{(\lambda^2 + 3\mu^2 + q^2\nu^2)^{9/2}}, \quad E = 35\sum \cdot \frac{9\mu^4}{(\lambda^2 + 3\mu^2 + q^2\nu^2)^{9/2}}, \quad F = 35\sum \cdot \frac{q^4\nu^4}{(\lambda^2 + 3\mu^2 + q^2\nu^2)^{9/2}};$$

$$G = 35\sum \cdot \frac{9\lambda^2\mu^2}{(\lambda^2 + 3\mu^2 + q^2\nu^2)^{9/2}}, \quad H = 35\sum \cdot \frac{9q^2\mu^2\nu^2}{(\lambda^2 + 3\mu^2 + q^2\nu^2)^{9/2}}, \quad I = 35\sum \cdot \frac{3q^2\nu^2\lambda^2}{(\lambda^2 + 3\mu^2 + q^2\nu^2)^{9/2}};$$

$$A_2 = 30\sum \cdot \frac{\lambda^2}{(\lambda^2 + 3\mu^2 + q^2\nu^2)^{7/2}}, \quad B_2 = 30\sum \cdot \frac{3\mu^2}{(\lambda^2 + 3\mu^2 + q^2\nu^2)^{7/2}}, \quad C_2 = 30\sum \cdot \frac{q^2\nu^2}{(\lambda^2 + 3\mu^2 + q^2\nu^2)^{7/2}};$$

$$J = \sum \cdot \frac{3}{(\lambda^2 + 3\mu^2 + q^2\nu^2)^{5/2}}.$$

Relations amongst these quantities are at once given by the conditions of molecular symmetry in the directions $\alpha = 1$, $\beta = \gamma = 0$, and $\alpha = 1/2$, $\beta = \sqrt{3}/2$, $\gamma = 0$, and those of symmetry in the directions $\alpha = \gamma = 0$, $\beta = 1$, and $\alpha = \sqrt{3}/2$, $\beta = 1/2$, $\gamma = 0$. Expressing these conditions in (18) we find, under limitation of the problem to the plane $\gamma = 0$,

$$D - A_2 + J = \frac{D}{16} + \frac{9E}{16} + \frac{3G}{8} - \frac{A_2}{4} - \frac{3B_2}{4} + J,$$

$$E - B_2 + J = \frac{9D}{16} + \frac{E}{16} + \frac{3G}{8} - \frac{3A_2}{4} - \frac{B_2}{4} + J.$$

Hence $D - E = A_2 - B_2$, and $G - E = D - G$, and therefore, in the hexagonal system,

$$L'_2 = \frac{2Ma^2}{\rho^5}(D - A_2 + J), \quad . \quad . \quad . \quad . \quad . \quad (21)$$

that is, *the parallel component of this order, is absolutely constant in the principal plane*. Its sign depends on the relative values of D, J, and A_2.

Similarly, the expression for ${}_\lambda T'_2$ reduces to

$$ {}_\lambda T'_2 = 2\frac{Ma^2}{\rho^5}(D - G)[\alpha^2\beta^2 - \beta^4 - \beta^2]\alpha \equiv 0, \qquad (22)$$

so that *the component of the internal field transverse to the direction of magnetisation is absolutely zero.*

15. The expression for the next order term in the parallel component of the internal field is

$$\begin{aligned} L'_4 &= 6\frac{M}{\rho^3}\left(\frac{a^2}{\rho^2}\right)^2 \sum \cdot (\lambda^2 + p^2\mu^2 + q^2\nu^2)^{-7/2} 16 P_2, \\ &= 6\frac{M}{\rho^3}\left(\frac{a^2}{\rho^2}\right)^2 \sum \cdot \left[231\frac{g^6}{f^{13/2}} - 315\frac{g^4}{f^{11/2}} + 105\frac{g^2}{f^{9/2}} - 5\frac{1}{f^{7/2}}\right], \end{aligned}$$

where $g = \alpha\lambda + p\beta\mu + q\gamma\nu$, $f = \lambda^2 + p^2\mu^2 + q^2\nu^2$. On expansion the sum is

$$\begin{aligned} &231\left\{ \alpha^6\sum\frac{\lambda^6}{f^{13/2}} + \beta^6\sum\frac{p^6\mu^6}{f^{13/2}} + \gamma^6\sum\frac{q^6\nu^6}{f^{13/2}} + 90\alpha^2\beta^2\gamma^2\sum\frac{p^2q^2\lambda^2\mu^2\nu^2}{f^{13/2}} \right. \\ &\quad + 15\left[\alpha^4\left(\beta^2\sum\frac{\lambda^4p^2\mu^2}{f^{13/2}} + \gamma^2\sum\frac{\lambda^4q^2\nu^2}{f^{13/2}}\right) + \beta^4\left(\gamma^2\sum\frac{p^4q^2\mu^4\nu^2}{f^{13/2}} + \alpha^2\sum\frac{p^4\mu^4\lambda^2}{f^{13/2}}\right)\right. \\ &\qquad \left.\left. + \gamma^4\left(\alpha^2\sum\frac{q^4\nu^4\lambda^2}{f^{13/2}} + \beta^2\sum\frac{q^4p^2\nu^4\mu^2}{f^{13/2}}\right)\right]\right\} \\ &- 315\left\{ \alpha^4\sum\frac{\lambda^4}{f^{11/2}} + \beta^4\sum\frac{p^4\mu^4}{f^{11/2}} + \gamma^4\sum\frac{q^4\nu^4}{f^{11/2}} + 6\left[\alpha^2\beta^2\sum\frac{p^2\lambda^2\mu^2}{f^{11/2}} + \beta^2\gamma^2\sum\frac{p^2q^2\mu^2\nu^2}{f^{11/2}} \right.\right. \\ &\qquad \left.\left. + \gamma^2\alpha^2\sum\frac{q^2\nu^2\lambda^2}{f^{11/2}}\right]\right\} \\ &+ 105\left\{ \alpha^2\sum\frac{\lambda^2}{f^{9/2}} + \beta^2\sum\frac{p^2\mu^2}{f^{9/2}} + \gamma^2\sum\frac{q^2\nu^2}{f^{9/2}} \right\} - 5\sum\cdot\frac{1}{f^{7/2}} \qquad (23) \end{aligned}$$

In the special case of the cubic system this becomes

$$\begin{aligned} &231\{(\alpha^6+\beta^6+\gamma^6)A'_3 + 15[\alpha^4(\beta^2+\gamma^2) + \beta^4(\gamma^2+\alpha^2) + \gamma^4(\alpha^2+\beta^2)]B'_3 + 6\alpha^2\beta^2\gamma^2E'_3\} \\ &\qquad - 315\{(\alpha^4+\beta^4+\gamma^4)C'_3 + 6(\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2)D'_3\} + 30R'_0, \end{aligned}$$

where

$$A'_3 = \sum\frac{\lambda^6}{f^{13/2}},\ B'_3 = \sum\frac{\lambda^4\mu^2}{f^{13/2}},\ C'_3 = \sum\frac{\lambda^4}{f^{11/2}},\ D'_3 = \sum\frac{\lambda^2\mu^2}{f^{11/2}},\ E'_3 = \sum\frac{\lambda^2\mu^2\nu^2}{f^{11/2}},\ R'_3 = \sum\frac{1}{f^{11/2}}.$$

In the case of magnetisation parallel to a principal plane ($\gamma = 0$) we get, by means of the conditions $\alpha^4 + \beta^4 + 2\alpha^2\beta^2 = 1 = \alpha^6 + \beta^6 + 3\alpha^2\beta^2$, the result

$$\frac{231}{2}[3(\alpha^4+\beta^4)(A'_3 - 5B'_3) - (A'_3 - 15B'_3)] - 315[(\alpha^4+\beta^4)(C'_3 - 3D'_3) + 3D_3'] + 30R'_3.$$

Now, by expression respectively of the quantities $(\lambda^2+\mu^2+\nu^2)\lambda^4$, $(\lambda^2+\mu^2+\nu^2)\lambda^2\mu^2$, and $(\lambda^2+\mu^2+\nu^2)$, each divided by the proper power of f, we obtain

$$\begin{aligned} C'_3 &= 2B'_3 + A'_3, \\ D'_3 &= 2B'_3 + E'_3, \\ R'_3 &= 3C'_3 + 6E'_3; \end{aligned}$$

and no other independent relations than these connect the six quantities A'_3, B'_3, C_3', D'_3, E'_3, and R'_3.

We can therefore express the result in terms of A'_3, C'_3, and R'_3, and thus get

$$L'_4 = \frac{3M}{\rho^3}\left(\frac{a^2}{\rho^2}\right)^2 \tfrac{1}{2}[231A'_3 - 315C'_3 + 30R'_3](21(\alpha^4+\beta^4) - 17) \quad . \quad . \quad . \quad (24)$$

The dotted curve in fig. 5 represents (24) on an arbitrary scale. The relative sizes of its lobes are strongly contrasted with those of the similar curve characteristic of the cubic arrangement.

The transverse component, of this order, is

$$\sum \cdot \frac{M}{r^3}\left(\frac{a^2}{r^2}\right)^2 \sin\theta \,.\, P'_6.$$

$$= \frac{M}{\rho^3}\left(\frac{a^2}{\rho^2}\right)^2 \sum . \sin\theta\cos\theta[231(6\cos^4\theta) - 315(4\cos^2\theta) + 105(2)](\lambda^2+p^2\mu^2+q^2\nu^2)^{-7/2}.$$

The λ-component of this is (*cf.* § 13)

$$\xi = \frac{M}{\rho^3}\left(\frac{a^2}{\rho^2}\right)^2 42\sum \cdot \left[\frac{\lambda - \alpha(\alpha\lambda + p\beta\mu + q\gamma\nu)}{f^{1/2}}\right]\frac{(\alpha\lambda + p\beta\mu + q\gamma\nu)}{f^{1/2}}\left[33\frac{(\alpha\lambda+p\beta\mu+q\gamma\nu)^4}{f^2} - 30\frac{(\alpha\lambda+p\beta\mu+q\gamma\nu)^2}{f} + 5\right]f^{-7/2}.$$

In the cubic system this reduces to

$$\alpha \cdot \frac{M}{\rho^3}\left(\frac{a^2}{\rho^2}\right)^2 42\{33[(\alpha^4(1-\alpha^2) - \beta^6 - \gamma^6)A'_3 - 15(\alpha^4(\beta^2+\gamma^2) + \beta^4(\gamma^2+\alpha^2) + \gamma^4(\alpha^2+\beta^2))B'_3 + 5(\beta^4+\gamma^4+2\alpha^2(\beta^2+\gamma^2))B'_3 + 30(1-3\alpha^2)\beta^2\gamma^2E'_3] - 30[(\alpha^2(1-\alpha^2) - \beta^4 - \gamma^4)C'_3 - 6(\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2)D'_3 + 3(\beta^2+\gamma^2)D'_3]\}.$$

On limitation of the investigation to the plane $\gamma = 0$ we have

$$\xi = \beta^2\alpha \,.\, 42\frac{M}{\rho^3}\left(\frac{a^2}{\rho^2}\right)^2\{33[(\alpha^2-\beta^2)A'_3 - 15\alpha^2B'_3 + 5\beta^2(1+\alpha^2)B'_3] - 30[(\alpha^2-\beta^2)C'_3 - 6\alpha^2D'_3 + 3D'_3]\}.$$

By interchange of α and β we obtain the μ-component η, and the resultant transverse component of the internal force, acting in the plane $\gamma = 0$ in the direction from the λ-axis to the μ-axis, is $\eta\alpha - \xi\beta$. By evaluation of this we get the result

$$_{\gamma}T'_4 = 21\frac{M}{\rho^3}\left(\frac{a^2}{\rho^2}\right)^2\alpha\beta(\beta^2-\alpha^2)(231A'_3 - 315C'_3 + 30R'_3)\,, \quad . \quad . \quad (25)$$

the prefix γ referring to the condition $\gamma = 0$.

It is to be noted that the bracket involving A'_3, etc., is identical with the corresponding bracket in (24). Although, by means of the above equations connecting A'_3, B'_3, etc., it is possible to assign various inequalities limiting these quantities, it is not possible thereby to determine the sign of $231A'_3-315C'_3+30R'_3$. Its sign can only be found by summation with respect to the given values of λ, μ, ν, which depend upon the special space-lattice involved. Its form is identical with that of the full-line curve in fig. 5.

A point of special interest lies in the fact, deducible from (20), that ${}_\gamma T'_2$ and ${}_\gamma T'_4$ have the same geometrical form, and in the fact that ${}_\gamma L'_2$ and ${}_\gamma L'_4$ also have the same geometrical form.

16. Equation (23) becomes applicable to the hexagonal system by the substitution $p=\sqrt{3}$. Taking the case $\gamma=0$, we get

$$
{}_\gamma L'_4 = 6\frac{M}{\rho^3}\left(\frac{a^2}{\rho^2}\right)\{231[\alpha^6P+\beta^6Q+15\alpha^2\beta^2(\alpha^2S+\beta^2T)] \\
-315[\alpha^4X+\beta^4Y+6\alpha^2\beta^2X']+105(\alpha^2W+\beta^2W')-5N\} \quad (26)
$$

if we write $P=\Sigma.\lambda^6f^{-13/2}$, $Q=\Sigma.27\mu^6f^{-13/2}$, etc. In the general case (23) contains twenty coefficients, between which ten independent relations, each involving four coefficients, are obtainable by evaluation of the quantities $(\lambda^2+3\mu^2+q^2\nu^2)\lambda^4$, $(\lambda^2+3\mu^2+q^2\nu^2)q\mu^4$, $(\lambda^2+3\mu^2+q^2\nu^2)q^4\nu^4$, $(\lambda^2+3\mu^2+q^2\nu^2)3\lambda^2\mu^2$, $(\lambda^2+3\mu^2+q^2\nu^2)3q^2\mu^2\nu^2$, $(\lambda^2+3\mu^2+q^2\nu^2)q^2\nu^2\lambda^2$, $(\lambda^2+3\mu^2+q^2\nu^2)\lambda^2$, $(\lambda^2+3\mu^2+q^2\nu^2)3\mu^2$, $(\lambda^2+3\mu^2+q^2\nu^2)q^2\nu^2$, and $(\lambda^2+3\mu^2+q^2\nu^2)$, each divided by the proper power of $(\lambda^2+3\mu^2+q^2\nu^2)$. So far, therefore, as these relations go, the ten coefficients P, Q, etc., are independent. But the conditions of symmetry of the hexagonal arrangement in the plane $\gamma=0$ show that ${}_\gamma L'_4$ has equal values with $\alpha=1$, and $\alpha=1/2$, and that it has also equal values with $\alpha=0$ and $\alpha=\sqrt{3}/2$. From these relations we find

$$
\left.\begin{aligned}
&\frac{33}{16}(P-Q+S-T)-3(X-Y)+(W-W')=0\,,\\
&11(P+Q-5(S+T))-10(X+Y-6X')=0\,.
\end{aligned}\right\} \quad . \quad . \quad . \quad (27)
$$

whence we may express P and Q, say, in terms of the remaining eight quantities

Equation (26) shows that the parallel component of the internal field in the plane $\gamma=0$ is symmetrical in the sextants 0° to 60°, etc.; and has maxima or minima in the directions 0°, 60°, etc., and minima or maxima in the directions 30°, 90°, etc.

In this case the transverse component is

$$
{}_\gamma T'_4 = 42\frac{M}{\rho^3}\left(\frac{a^2}{\rho^2}\right)^2\alpha\beta[33\{\beta^4Q-\alpha^4P+5(\alpha^4S-\beta^4T)+2\alpha^2\beta^2(T-S)\} \\
-30\{\beta^2Y-\alpha^2X+3(\alpha^2-\beta^2)X'\}+5(W'-W)] \quad (28)
$$

This vanishes at $\alpha=0$, $\beta=0$; and, with the conditions (27), it vanishes also at 30° and 60°. The transverse component is thus representable by a *symmetrical twelve-looped curve* in the plane $\gamma=0$. The sign of any one loop of this component, and the maximal or minimal nature of the parallel component in, say, the direction $a=0$, are only determinable by numerical evaluation of the coefficients.

17. In the case of the cubic system, in a face plane of the cubic lattice, *i.e.* with $\gamma=0$, we have by (6) and (19) with $a=\cos\theta$

$$_{\gamma}L'_2=0\,,\quad T'_0=0\,.$$

And, by (19) and (20),

$$_{\gamma}L'_2=\frac{Ma^2}{\rho^5}(G-D)\left(\sin^2 2\theta-\frac{4}{5}\right),$$

$$_{\gamma}T'_2=\frac{Ma^2}{2\rho^3}(G-D)\sin 4\theta\,.$$

Also, by (24) and (25),

$$_{\gamma}L'_4=-\frac{3M}{\rho^3}\left(\frac{a^2}{\rho^2}\right)^2\cdot\frac{21}{4}[231A'_3-315C'_3+30R'_3]\left(\sin^2 2\theta-\frac{8}{21}\right),$$

$$_{\gamma}T'_4=-\frac{3M}{\rho^3}\left(\frac{a^2}{\rho^2}\right)^2\cdot\frac{7}{4}[231A'_3-315C'_3+30R'_3]\sin 4\theta\,.$$

The latter pair have the *same geometrical form* as the preceding pair. Calculation of the numerical values of G and D (*Proc. R.S.E.*, 1905, 1907) shows that G is greater than D. Evaluation of A'_3, C'_3, and D'_3 has not yet been carried out, but the multiplier a^2/ρ^2 in the latter pair probably makes $_{\gamma}L'_4$ and $_{\gamma}T'_4$ small relatively to $_{\gamma}L'_2$ and $_{\gamma}T'_2$ apart from the smallness of A'_3, etc.

In an octahedral plane (plane normal to a ternary axis of the cubic system), the variable part of the parallel component of the internal field is proportional to $a^2\beta^2\gamma^2$. For, under the condition $a+\beta+\gamma=0$ which then subsists, we have $2(a^4+\beta^4+\gamma^4)=4(a^2\beta^2+\beta^2\gamma^2+\gamma^2a^2)=1$ and $2(a^6+\beta^6+\gamma^6)=1-3a^2\beta^2\gamma^2$. Referring now to rectangular axes in the plane $a+\beta+\gamma=0$, and measuring θ, counter-clockwise from a binary axis in that plane, we find that the variable part of the component L'_4 of the internal field parallel to the direction of magnetisation is proportional to

$$\sin^2\theta(3-4\sin^2\theta)^2.$$

Similarly, L'_2 is constant and L'_0 is zero.

In Weiss's experimental investigation of magnetite no observations were made in an octahedral plane, but such observations have been carried out by his pupil V. Quittner (*Ann. d. Physik*, 30, 1909). In the paper above referred to (*Proc. R.S.E.*, 1905), the molecular theory was applied to the elucidation of Weiss's results on plates cut perpendicular to a quaternary and a binary axis respectively. In that paper it was presumed that the crystalline arrangement in the cubic space-lattice was the one which gave to each molecular magnet twelve nearest neighbours, and the general nature of the experimental results was shown to be in agreement with the theory. [In that paper the following errata have to be noted: § 6, second last formula, for 15 read 30; § 9, for 44 read 49, and for $A>B$ read $A<B$. The force parallel to a quaternary axis is negative, which agrees with Weiss's theoretical postulate made in explanation of the peculiar form of the curve of magnetisation along a quaternary axis.] In the second paper (*Proc. R.S.E.*, 1907), a similar investigation was given in respect to the arrangement in which each magnet has six nearest neighbours, and the results were found to be opposed to the experimental data; so that this arrangement cannot exist in magnetite. Nor can it occur in crystals of metallic iron, for these were found by Weiss to have magnetic properties similar to those of magnetite. The only other cubic crystalline arrangement (not yet investigated) is that in which each magnet has eight nearest neighbours.

Quittner has carried out more extended observations on plates cut parallel to the cubic faces and the octahedral faces in fields of various strengths, and certain peculiarities are brought out. For the general elucidation of these, the existence of internal fields of the types above discussed are sufficient. The normal results are due to the cubic crystalline structure, and the abnormalities are due to modifications of that structure by crystalline flaws—incipient foliation in definite planes. This explanation of the abnormalities was adopted by Weiss and strengthened by Quittner. The molecular theory bears it out fully. The abnormalities are sometimes present, sometimes absent, even in different parts of the same crystal.

Fig. 6 is reproduced from Quittner's paper. The ordinates represent intensity of magnetisation in the plane of a cubic face; and various curves, corresponding to different strengths of field, are given. In weak fields, the regular plate shows no directional variation of magnetisation. This is probably due to the existence of thermal motions in the molecules which can temporarily free some molecules from the control of an internal field and so leave them subject to the external field alone. At higher fields maxima appear in the directions of the diagonals and minima in the directions of the axes. Now the value of ${}_{\gamma}L'_2$ given at the commencement of this section, with the condition $G>D$, shows that the internal force along a quaternary axis is negative, while it is positive along a diagonal.

Thus the variation of magnetisation exactly follows the theoretical variation of the field. As saturation is approached these variations tend to disappear. The effect of ${}_{\gamma}L'_4$, if present, is similar to that of ${}_{\gamma}L'_2$.

In the irregular plate an abnormality appears, even at low fields, in the existence of a maximum along one diagonal and a minimum along the other. But, in a truly cubic arrangement, ${}_{\gamma}L'_0$ is zero. Therefore the effect must either be due to deformation of the crystal by relative elongation along the former diagonal (§ 8), or it must be caused by internal flaws in planes perpendicular to the latter diagonal, which would reduce the permeability in the direction of the diagonal itself.

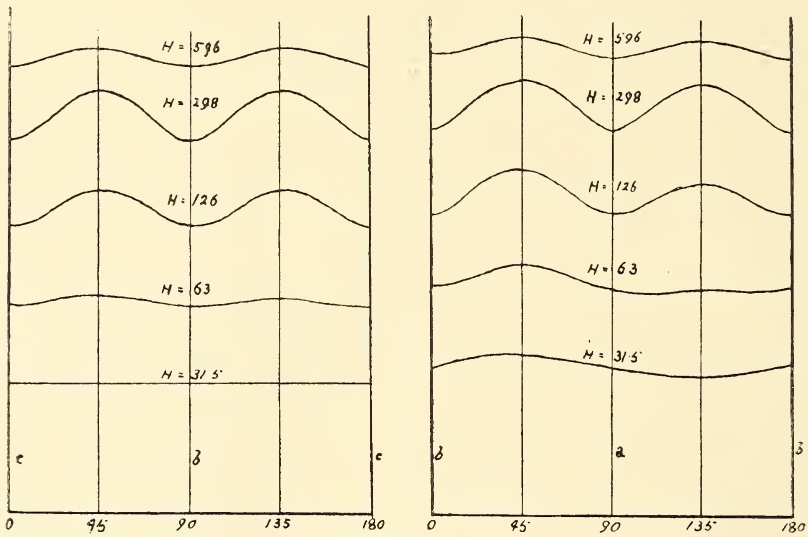


Fig. 6.

Quittner's diagrams of the transverse effect are not reproduced here. Zero values occur in its variation where stationary values of the parallel effect occur, just as the theory indicates; and its maxima and minima also follow the theoretical indications, both as regards the normal and the abnormal conditions. In fig. 5 the values of ${}_{\gamma}L'_2$, ${}_{\gamma}T'_2$, and ${}_{\gamma}L'_4$ are represented respectively by $r = 2\ (\sin^2 2\theta - \frac{4}{5})$ (dashes), $r = \sin 4\theta$ (full line), and $r = 2\ (\sin^2 2\theta - \frac{8}{21})$ (dotted curve). If r and θ be represented by rectangular co-ordinates, the correspondence between the variations of r and the variations of the magnetisation in Quittner's curves becomes at once evident.

18. The left-hand part of fig. 7 shows Quittner's observations on the parallel component of magnetisation in the case of an octahedral plate. In weak fields an effect corresponding in type to L'_0 appears. In stronger fields, three maxima and three minima occur. The same takes place in

the strongest field, but maxima and minima are as nearly as possible interchanged.

In the right-hand side of the same figure, the curves $r = \cos 2\theta - \cos 4\theta \pm 2\cos 6\theta$ are represented. The correspondence in general form between these curves and Quittner's curves (4) and (3), respectively, is very evident. Now an expression $\cos 2\theta + p \cos 4\theta + q \cos 6\theta$ is proportional to $\sin^6\theta + \mathrm{P}\sin^4\theta + \mathrm{Q}\sin^2\theta$, where P and Q are functions of p and q; and an expression $\mathrm{L}'_0 + p'\mathrm{L}'_2 + q'\mathrm{L}'_4$—the variable parts of L'_0, L'_2, and L'_4 (§ 17) being proportional respectively to $\sin^2\theta$, $\sin^2 2\theta$, and $\sin^2\theta(3-4\sin^2\theta)^2$—is, apart from a constant, proportional to $\sin^6\theta + \mathrm{P}'\sin^4\theta + \mathrm{Q}'\sin^2\theta$, where P′ and Q′ are functions of p' and q'. Hence the presumption is very strong

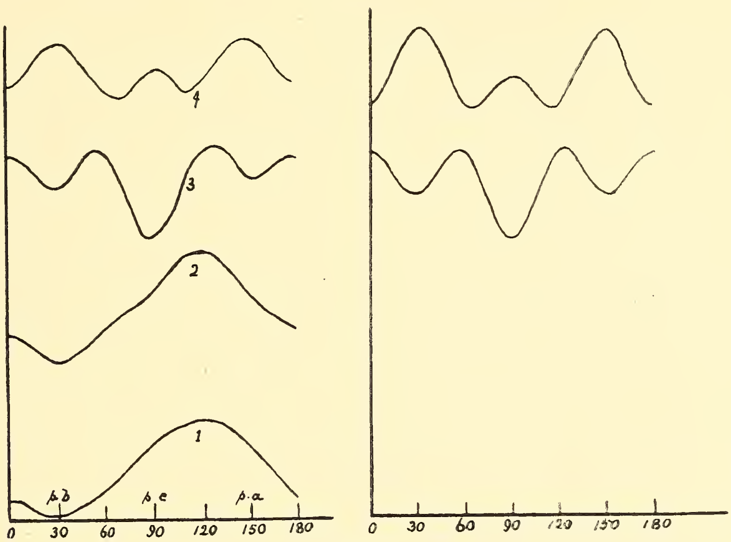


FIG. 7.

that the variation of the magnetisation is due to three constituents of the form L'_0, L'_2, and L'_4 in the expression for the internal field.

Therefore, if the crystalline structure is truly cubic, since the true terms L'_0 and L'_2 *in an octahedral plate* are not variables (§ 17), the actual terms L'_0 and L'_2, now postulated, can only be due to deformation or to false structure caused by flaws. The change in sign of the term involving $\cos 6\theta$, which means a change in sign of the term L'_4, could only, in a truly cubic arrangement, be due to a change from one to another cubic arrangement, *e.g.* a transformation, in high fields, from the cubic molecular arrangement with twelve nearest neighbours to that with six nearest neighbours (*Proc. R.S.E.*, 1907). But, if this transformation occurred, maxima and minima, in a cubic face, should appear respectively in the direction of a quaternary axis and a binary axis; and Quittner's results show that the opposite occurs. The most probable hypothesis seems to be that the term $+\cos 6\theta$ is due to the actual crystalline structure; and that, in low fields, its effect is overbalanced by a similar effect, of opposite sign, due to the existence of crystalline flaws.

Weiss postulated the existence of flaws or incipient fissures parallel to the planes of the cubic faces. Seeing that the internal field parallel to the cubic edges is negative, these flaws may not be necessary to explain the more rectilinear rise of magnetisation under increasing field parallel to the cubic edges than of that parallel to the binary or the ternary axes; but (*Proc. R.S.E.*, 1905, p. 1061) they readily explain the relative order of

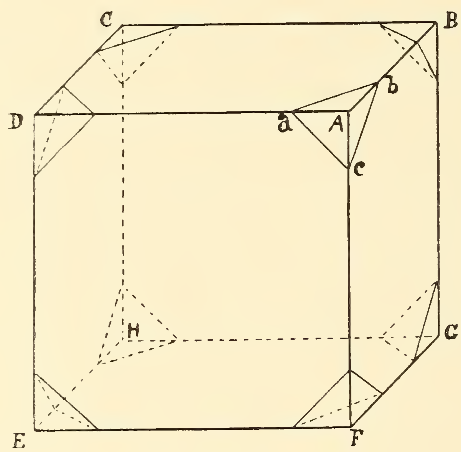


Fig. 8.

magnitude observed by Weiss in the case of residual magnetisation in these directions. They cannot account for the maximum and minimum which appear, in weak fields, in the abnormal plate, fig. 5. To explain these we must postulate an excess of flaws in planes perpendicular to the 135° diagonal. Foliation parallel to the dodecahedral planes or to the octahedral planes could produce that effect. Quittner, pointing out the readiness with which magnetite cleaves in octahedral planes, postulated incipient splitting parallel to them.

In fig. 8, let AD, AB, AC be Quittner's axes a, b, c. Foliation parallel to any of the octahedral planes at D, B, E, and G (the pair D, G are identical, as also are the pair B, E) diminishes the permeability in the direction ab in the cubic face DAB; and this gives the result observed in fig. 5, in the plane of the axes a, b, at $H = 31{\cdot}5$. Also, if the foliation occurs equally in each of these four octahedral planes, the result is to diminish equally the permeability along the diagonals in the face FAB. This may occur to an extent practically sufficient to annul that normal excess of permeability along the diagonals over that along the edges, which is due to the molecular structure, and is exhibited in the value of ${}_{\gamma}L'_2$. Hence we have a possible explanation of the rectilinearity of the curve, at $H=31{\cdot}5$, in the plane of the axes b and c, fig. 6. At higher fields, magneto-striction may tend to efface the fissures and allow the structural effect ${}_{\gamma}L'_2$ to become dominant, as the diagram shows.

Similarly, equal foliation parallel to the two octahedral planes at the corners A or H, C or F, preserves symmetry in the plane of the axes b and c; while, if it be not so marked as the preceding foliation, it will not obliterate its effect in the plane of the axes a and b.

Foliation parallel to any of the other octahedral planes intersects the plane at the corner A in lines which are perpendicular to one or other of the projections of the axes a, b, c on the plane. It thus tends to make the permeability in the directions of these projections small, and so gives rise to a $\cos 6\theta$ term in the magnetisation having its minima in the directions of the projections of the quaternary axes. This is exactly the nature of the effect which would be due to the internal field in an octahedral plane in a non-foliated molecular arrangement with twelve nearest neighbours. And, if the foliation perpendicular to the projection of the c axis be more effective than that perpendicular to the other two axes, a $\cos 2\theta$ term, with its minimum in the direction of the c axis, which the analysis of the curves in fig. 7 shows to exist at the higher fields, results. The only argument against this explanation of the existence of the $\cos 2\theta$ term in the value of the magnetisation in an octahedral plane is that, if the foliation in planes perpendicular to the diagonal DG be more effective than that in planes perpendicular to the diagonal BE, the $\cos 2\theta$ term in the plane ab would have its minimum in the direction of the diagonal AG, while fig. 6 exhibits the reverse effect. We shall see immediately how that adverse result may be overbalanced.

Octahedral foliation cannot give rise to a $\cos 4\theta$ term in an octahedral plane. For the production of this effect by foliation, we have to postulate foliation parallel to dodecahedral planes.

Foliation parallel to the dodecahedral planes which pass perpendicularly through the diagonals of the face ABCD might be sufficient to practically annul the structural effect ${}_{\gamma}L'_2$ in that face at low fields, while magneto-striction would diminish the foliation effect at higher fields and allow the structural effect to preponderate, as fig. 6 shows. We have already seen that the same effect may be produced by octahedral foliation. If dodecahedral foliation perpendicular to the face ABGF takes place, and if that parallel to AG exceeds that parallel to BF, a term involving $\cos 2\theta$, with its maximum in the direction AG, as exhibited in fig. 6, will appear in the effect parallel to ABGF in weak fields. The diagram shows that it persists to fairly high fields, where it becomes relatively obliterated, presumably in consequence of electro-striction. If the effect be sufficiently strong, it may overbalance the opposite effect, already alluded to, due to a possible octahedral foliation.

Now the equal foliations in the dodecahedral planes parallel to AC and BD produce unequal effects in the octahedral plane at A. For the former cuts that plane normally, while the latter does not. This gives a $\cos 2\theta$ term with its maximum along the projection of the a axis. And the dodecahedral foliations parallel to AG and BF give one with its maximum along the projection of the c axis: the latter being stronger than the former if the foliation parallel to AG be in excess. The two effects together combine into a $\cos 2\theta$ effect with its maximum close to 120° (fig. 7) and its minimum near the projection of the b axis, as Quittner observed.

Foliation perpendicular to the quaternary axes could annul the structural $\cos 4\theta$ effect in a cubic face in weak fields, and, by becoming less at high fields, could allow the structural effect to show. But any want of equality in this foliation would give rise to a $\cos 2\theta$ term with its maximum parallel to a quaternary axis, of which there is no experimental evidence. Nor can that foliation give a $\cos 4\theta$ term in an octahedral plane. But it gives, in an octahedral plate, a $\cos 6\theta$ term having its minima in the directions of the projections of the axes on the octahedral plane, agreeing with the structural effect and the effect of octahedral foliation. The $\cos 6\theta$ term observed at low fields in the octahedral plane must be due to dodecahedral foliation, which gives maxima along the projections of the axes, and which, diminishing as the field increases, allows the structural effect to preponderate ultimately.

Thus the normal and abnormal effects observed in magnetite are deducible from theory, the latter being due to foliation.

19. The mineral pyrrhotine, which crystallises in hexagonal, or quasi-hexagonal, form, has been investigated experimentally by Weiss; and he has also given a theoretical explanation in terms of the action of an internal field. He found that a magnetic plane (§ 8) existed. Fig. 9, which is copied from his paper (*Journ. de Phys.*, iv., 1905), exhibits the general nature of his results. The upper curve represents the magnetisation parallel to the external field, and the lower curve represents the magnetisa-

tion transverse to it. In each case there is evidence of a $\cos 2\theta$ term with a weak $\cos 6\theta$ term superposed upon it. The former corresponds to a rhombohedral structure, L'_0 and T'_0, either real or induced by foliation. From §§ 11, 14, 16, we see that no term of the forms $\cos 2\theta$ and $\cos 4\theta$ can appear in the case of a truly hexagonal structure, while a term of the form $\cos 6\theta$ is apparent.

Weiss found it possible to obtain by fracture a crystal in which the $\cos 6\theta$ effect was almost absent, and he regarded the substance as constituted crystallographically of three conjoined rhombohedral crystals which thus simulated hexagonal form—a view adopted also by some crystallographers. This is a thoroughly plausible view, quite in consonance with the molecular theory. But another possible view, *so far as magnetic quality is concerned*, is that the $\cos 6\theta$ term is due to a real hexagonal structure; in which case

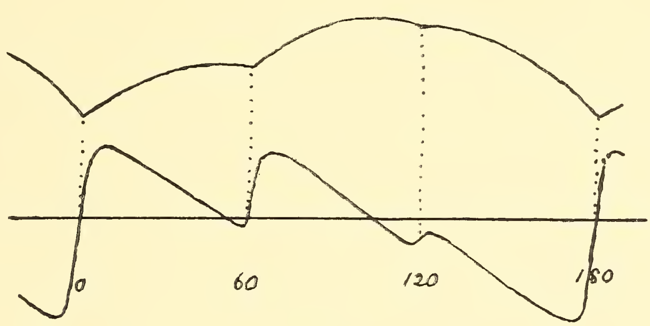


Fig. 9.

the $\cos 2\theta$ term must be induced by foliation. Indeed, the $\cos 6\theta$ effect may, in part at least, be due to foliation also.

Some evidence of variability in the constitution of the rhombohedral effect is given by Weiss's observations. In the right-hand part of fig. 10, copied from his paper, curves of the component of magnetisation parallel to the external field are given, four curves being shown at different values of the field.

When we put $\gamma = 0$, L'_0 takes the form $A \sin^2\theta + B \cos^2\theta$ (eq. 10), and (eqs. 18, 23) L'_2 and L'_4 each take the form $C + D \sin^2\theta + E \sin^2 2\theta$: so the internal field up to this order, in the rhombohedral arrangement, is of the form

$$C + D \sin^2\theta + E \sin^2\theta .$$

The four full-line curves in the left-hand side of fig. 10 have the equations $1 - 0{\cdot}1 \sin^2\theta - \frac{1}{6} \sin^2 2\theta$, $1 - 0{\cdot}2 \sin^2\theta - \frac{1}{6} \sin^2 2\theta$, $1 - 0{\cdot}5 \sin^2\theta - \frac{1}{6} \sin^2 2\theta$, and $1 - 0{\cdot}8 \sin^2\theta - \frac{1}{6} \sin^2 2\theta$, the numerical values having been chosen merely so as to indicate the general nature of the correspondence between the form of these curves and the form of Weiss's curve. It must, of course, be remembered that the general equation just given represents the value of that part of an internal field which is parallel to the direction of magnetisation supposed to have attained its constant saturation value, while Weiss's curves represent the component of magnetisation in the direction of the external field supposed to be constant. But it is the variation of the internal field which causes the variation of the magnetisation, so the general nature of the two sets of curves should be very similar.

The chief difference between the two sets lies in the breadth of the minimum in the neighbourhood of 90°. Suppose that the sin 2θ term were

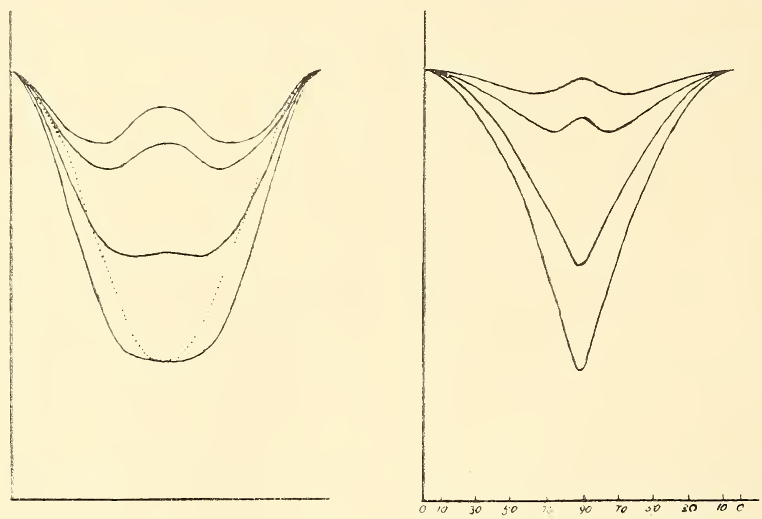


Fig. 10.

absent. In this case, with the same maximum and minimum values, the curve would be $1-0{\cdot}8\sin^2\theta$, which is represented by the dotted line. The marked flatness at the apex of the lower full curve is therefore due to the L'_4 effect. Hence if, at the lower fields, the $\sin^2 2\theta$ term were reversed in sign, the apex of the curves would be very sharp and the sides hollow. This might occur if the sign of the true structural term were negative, while a similar term of positive sign preponderated at the lower fields until increasing magneto-striction caused its relative obliteration. But the law connecting magnetisation with field intensity is very determinative. If the lower left-hand curve represented the total field (internal plus external) with the line of zero field lying not much below its vertex, and if the magnetisation were proportional to the square of the field, the curve of magnetisation would have a very sharp apex and would correspond fairly well to Weiss's lower curve.

20. In the preceding investigation it has been postulated that the molecular magnets are all co-directed. This implies that the external field has been strong enough to turn them all so as to form a homogeneous crystalline assemblage, and a single infinite homogeneous assemblage has alone been considered. In the paper *Proc. R.S.E.*, 1905, a brief consideration of the effect of plane boundaries has been given. But the process can readily be applied to cases in which sub-groups of the magnets are differently oriented. Thus magnets in alternate plane layers might be oppositely directed and be in stable equilibrium. If such a compound arrangement corresponded to a condition of greater exhaustion of internal potential energy than that above considered, we could have the system of magnets in a condition practically "non-magnetic," and, at a definite temperature, the two conditions might conceivably be of equal probability; so that, above that temperature, the one might prevail, while below it the other might prevail. In this way the demagnetisation of iron, regarded as a congeries of crystals, may be explainable. Again, below that temperature, either condition might be possible, as, *e.g.*, is the case with some iron-nickel alloys. This is indeed probably more to be expected in the case of two interpenetrating and distinct, independently homogeneous, crystalline arrangements, than it is in the case of two interpenetrating and identical arrangements, together forming one single homogeneous arrangement, even if the magnetic moments of the molecules in each individual set were different.

The factor M, the magnetic moment, which appears in the formulæ, will effectively depend upon temperature; and its form can readily be found from statistical considerations. It will also depend upon the inherent structure of the magnetic molecule.

21. The persistence of the earth's magnetism, in spite of the high temperature supposed to exist in its interior, may possibly be attributable to the maintenance, under extreme pressure, of a magnetic condition which, under ordinary pressures, could not persist at very high temperatures. This might very conceivably occur if the pressure stress were not isotropic. For the preceding discussion shows that the powerful terms L'_0 and T'_0, which do not exist in cubic crystals, would then become effective. A numerical estimate shows that these terms would become as efficient as the terms L'_2 and T'_2 if the cubic crystals were made tetragonal under a stress difference amounting to some tens of tons' weight per square cm. To account for axial polarity in this way, we see from § 8 that there must be

an excess of pressure perpendicular to the axis. Tidal stress, if effective, would give a semi-diurnal effect with minima following the tidal crest; and a fortnightly variation. [Otani (*Proc. Math. Phys. Soc.*, Tokyo, Jan. 1910) has found that compression initially decreases the intensity of magnetisation of rocks in the line of compression.]

22. The preceding investigation must also apply to the piezo-electric effect in crystals, and determine relations between the internal electric field and the external stress which induces the polarity; and it will also have applications to electro- and magneto-optical effects.

(*Issued separately July* 11, 1912.)

XVIII.—On the Torsional Oscillations of Magnesium Wire.

By G. P. Seaman. *Communicated by* Professor W. Peddie.

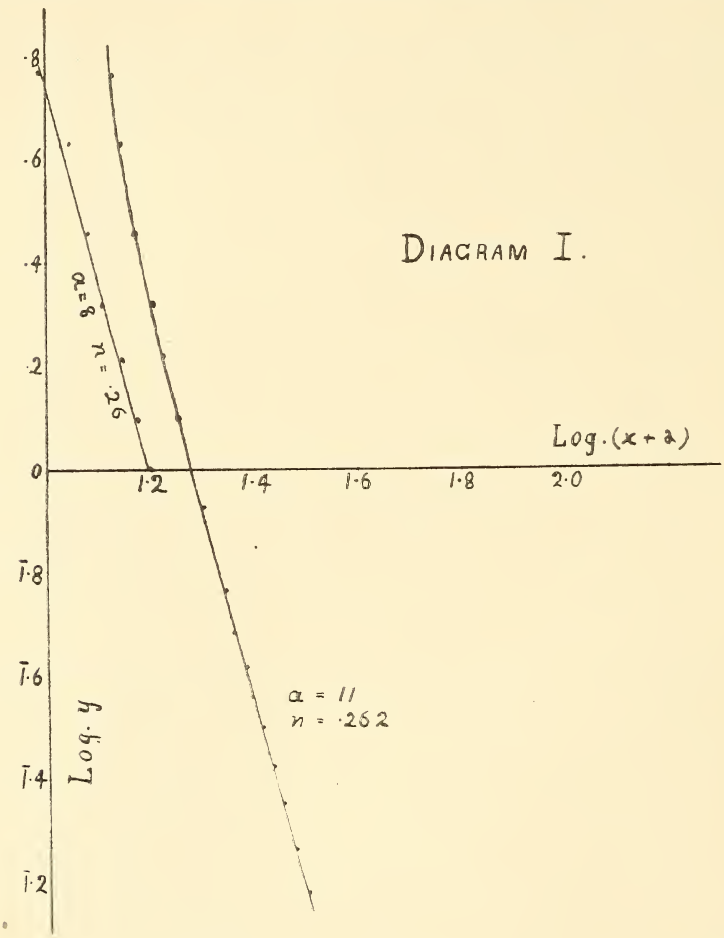
(MS. received Feb. 5, 1912. Read Feb. 5, 1912.)

It has been shown by numerous experiments (see *Proc. Roy. Soc. Edin.*, 1911) that the equation $y^n(x+a)=b$ holds very accurately for wires of different materials, where $y=$ range of oscillation, $x=$ number of oscillations, n, a, and b, constants depending on the metal under the given conditions. In the following experiments a magnesium wire was subjected to various conditions, *e.g.* extensional strain, torsion, change of temperature, etc., and in each case the above law was tested. The method of procedure was as follows:—A magnesium wire of known dimensions and under ordinary physical conditions was fixed at its upper end to a clamp, and to its lower end was attached centrally a long horizontal heavy cylinder. To the cylinder a circular scale divided into millimetres was attached concentrically with the wire. When the upper end of the wire was turned round the cylinder was made to rotate round the wire as an axis, and the reading on the scale, when the maximum angular deflection was reached, was taken by means of a telescope. Similarly, consecutive maxima were read until the oscillations became small, when the zero was taken, and thus successive values of y were found.

The experiments were directed towards finding the effects, on the constants n, a, and b, of the subjection of the wire to (1) original conditions in the unheated state, (2) repeated rotational strain in the unheated state, (3) repeated extensional strain in the unheated condition, (4) a temperature of 100° C., (5) a temperature of 250° C., (6) a temperature of 350° C., (7) a temperature of 450° C., (8) a temperature of 500° C. The same wire was then immersed in liquid air for some time, but this produced no evident effect upon the constants. Since we have $n\log y+\log(x+a)=\log b$, if $\log y$ be plotted against $\log(x+a)$ the points will, if the formula hold, lie on a straight line provided the proper value of a be chosen; and the tangent of the angle which this line makes with the $\log y$ axis will give n, whence b can be calculated. In each case a duplicate set of observations was taken so as to confirm the results.

Numerical Results.

(i.) In the case of magnesium wire under normal physical conditions, it was found, on plotting $\log y$ against $\log(x+a)$, that the whole graph did not represent a straight line. For large oscillations there was a distinct curvature of the graph with a certain value of a, while, for the same value of a, the graph for small oscillations was quite straight. Hence it was necessary to find a different value of a for the first part of the curve in order, if possible, to make that part also a straight line. It was found that



the formula was applicable, with slightly different values of the constants, to the two parts of the graph (see Diagram I.).

	a.	n.
First experiment . . .	8	·253
	11	·266
Second experiment . . .	8	·259
	11	·262

The two experiments give practically identical results.

(ii.) Attempts were next made to stretch the magnesium wire, but this was found to be impossible, as the wire broke even when a very small amount of stretching was produced. The magnesium wire was next subjected to rapidly alternating rotational strain for different periods of time, and the results were as follows:—

	Time.	*a.*	*n.*
First experiment . . .	30 min.	8	·289
		11	·289
Second experiment . . .	210 min.	8	·232
		11	·228

Thus it appears that the twisting does not affect the constants to a great extent. The change in the value of n in the two cases is quite certain, but the values of n got in successive experiments may differ to some extent in the second place after the decimal point.

(iii.) Another portion of the wire was next fixed between two clamps, and was subjected to periodic extensional stress (see J. B. Ritchie, *Proc. Roy. Soc. Edin.*, 1911). The results were as follows:—

Time.	*a.*	*n.*
30 min.	8	·247
	13	·208
180 min.	8	·239
	13	·233

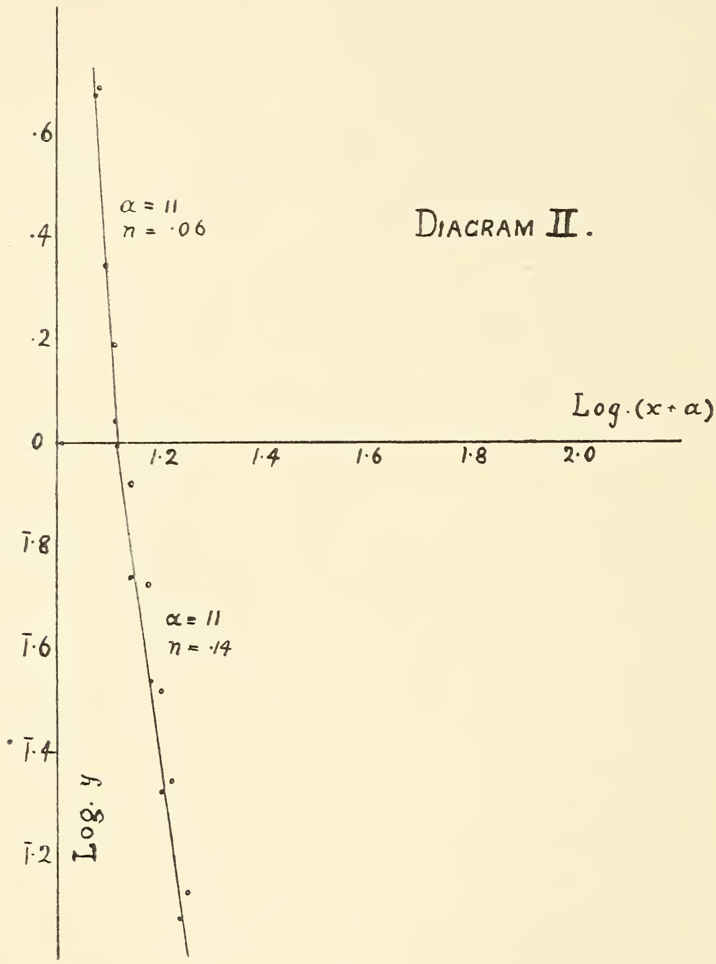
As it is quite possible that there may be a slight error even in the second figure after the decimal point, these values differ so little from one another and from the preceding that we may say that the effect of fatigue on unheated magnesium wire does not decidedly alter the constants.

(iv.) A length of the wire was next placed in an electric furnace and heated to various temperatures. The following were the results:—

Temperature.	*a.*	*n.*
100° C.	8	·243, ·269
	11	·276, ·279
250° C.	8	·243
	11	·250
350° C.	8	·190
	11	·280
450° C.	11	·112, ·121
	11	·266, ·260
500° C.	11	·060, ·080
	11	·140, ·138

The second series of values under the heading n refer to the results of duplicate experiments. The constants are not greatly affected until a temperature of about 350° C. is reached. At higher temperatures the value of n falls continuously in the case of small oscillations. In the case of large oscillations it seems to rise at first and then fall. Generally speaking, the results are similar to those found by Mr Ritchie in the case of brass wire.

Diagram II. gives a few of the observed data at the temperature 500° C. The circles refer to the duplicate experiment.



(v.) Finally, the same wire which had been heated was placed in liquid air for half an hour, and the following were the results:—For large oscillations, $a = 1$, $n = \cdot 089$; for small oscillations, $a = 11$, $n = \cdot 014$. Thus the cooling of the wire practically left the constants unchanged.

The apparatus used for producing the rotational and extensional strains was described by Mr Ritchie, *Proc. Roy. Soc. Edin.*, 1910–11, p. 440, and was constructed for Dr Peddie by means of a Royal Society grant.

(*Issued separately July* 11, 1912.)

XIX.—On the use of Antiseptics in Autolysis of Animal and Vegetable Matter. By Dorothy Court, B.Sc., Carnegie Research Fellow. *Communicated by* Dr E. Westergaard.

(MS. received March 18, 1912. Read June 3, 1912.)

In a previous publication (1) allusion was briefly made to the difficulty which had been experienced in preserving sterility in material during a series of auto-digestion experiments. Contrary to expectation, it was found that a mixture of ground barley and water, when incubated at 37° for a few days, developed, even in the presence of chloroform or toluol in excess, a strong butyric acid fermentation.

Since this question was one not only of vital interest to the work in hand but also of great general importance, it was decided to carry out a series of experiments in the endeavour to determine the position with regard to the use of antiseptics in the study of biochemical processes.

The study of enzymatic activity is rendered particularly difficult by the fact that the presence of the active agents—the enzymes—is, as yet, only recognisable by the results of their action. Consequently, the study of enzymes is, in the meantime, confined to the examination of the reactions with which they associated, and it is generally agreed that these must be of such a nature that the action of the enzyme can be regarded as catalytic, and also that the catalysis and the agent producing it must be separable from life. The third generally accepted condition—the specific nature of the catalyst—does not concern the present paper, and the same may be said of the other condition which may or may not be introduced into the definition, viz. the capacity of producing anti-bodies. Provided, then, that it has been shown that all the conditions as to the nature of the reaction, which are demanded in the definition, have been fulfilled, it still remains to be shown that the reaction is neither the result of the presence of surviving protoplasm of the organism in which the enzyme is supposed to be produced, nor of bacteria or other micro-organisms. It is with the purpose of settling this point that recourse is made to the use of antiseptics.

The majority of these reactions seem to take place, under the natural conditions, in heterogeneous colloid systems, and in all probability they are, in the first instance at least, of the nature of adsorption processes. It is found, moreover, that the activity is more or less rapidly destroyed by almost any manipulation or treatment, a fact which is fully in accordance with the previous statements. In view of these facts, it is obviously imperative that any investigation as to enzymatic processes should be carried out as far as possible under conditions resembling those existing in the natural environment.

The ideal method would therefore be to follow the chemical changes occurring in the various liquids, tissues, or organs of the living animal or plant; a method which is, however, full of practical difficulties, and is further open to the objection that it is impossible, under such conditions, to differentiate between the activity of living protoplasm and that of purely catalytic agents. This objection may also be urged when similar observations are made on entire organs removed either *in vivo* or post mortem, while, on the other hand, a more or less complete disintegration of the tissues and the protoplasmic structure introduces an almost equally great difficulty in the probability of the presence of micro-organisms of various descriptions. This difficulty is so much the greater in that there is scarcely any form of enzymatic activity which is not possessed by some, or all, of the more commonly occurring bacteria. Consequently, if the occurrence of a reaction is to be taken as a criterion of the presence of an enzyme, it is an essential condition that the experiments must be carried through with absolute certainty of sterility in the material. This is at once the most vital and vulnerable point in experimental enzymology, and it is impossible to pay too much attention to it.

In the case of liquids, sterility may be secured by means of filtration, but such cases are comparatively rare, and the method is open to the objection that adsorption compounds may be, and very probably are, formed with the filtering material (porcelain, etc.).

Most frequently, therefore, recourse is made to the use of antiseptics, although this method is also, at the best, open to objection.

Leaving out of count, for the present, the fact that the conditions of reaction in a disintegrated tissue are not comparable with the natural ones where reacting material and reaction products may constantly and gradually be added and removed by diffusion, and the fact that it is inconceivable that a substance however chemically indifferent can be added to a colloid system without causing some change in reaction, conductivity, specific surface, etc., there yet remain two essential points to consider, viz. the efficiency of the antiseptic as such and its possible action on the enzyme itself, the substrate, or the reaction products.

The ideal antiseptic is, of course, one which combines highest efficiency with minimal influence on the other conditions. It is obvious that it is almost as undesirable to study enzymatic catalysis in the presence of an agent which positively catalyses the same reaction, as to study it under conditions which retard or prevent it; and as it is impossible to obtain an absolute value for the activity of an enzyme, since no method has, as yet, been found by which enzyme may be obtained in a state of purity, it is in the meantime difficult to say what is acceleration and what is retardation.

These requirements are obviously of equal importance and of almost equal difficulty in fulfilment. However, since sterility is a fundamental condition, it is accordingly necessary in choosing an antiseptic for enzymatic work to see that this is fulfilled while at the same time the other conditions are complied with as far as possible.

In view of the great and constantly increasing importance of enzymology, it is only to be expected that a very considerable number of publications on the subject of antiseptics in this connection should have appeared; but it is somewhat unfortunate that the vast majority deal only with the influence of the antiseptic on the reaction, the efficiency being taken as a foregone conclusion, a fact which may probably account for a large number of apparently contradictory results.

The literature on the former part of the question is so great that a review of it in a short paper like the present is quite out of the question, and would, moreover, serve no purpose, since it has been very efficiently dealt with up to 1907 by Vandevelde in the *Biochemische Zeitschrift*, vol. iii., 1907, and more recently in Abderhalden's *Biochemische Arbeitsmethoden*, and by Samuely in Oppenheimer's *Handbuch der Biochemie*.

In connection with zymase it has been very fully investigated by Buchner (2), and with regard to the digestive enzymes a multitude of references is found throughout medical and physiological literature.

The position is, however, very different with regard to the other side of the question. Statements such as: "The material remained sterile," "Toluol was used as antiseptic," are of frequent occurrence, but these can scarcely be considered convincing unless supported by more complete information. As an example of the little attention this side of the subject has received, it may be mentioned that in the otherwise eminently valuable monograph: *On the Nature of Enzyme Action*, by Bayliss, London, 1911, under the heading "Antiseptics," is found the solitary statement, "On the whole, toluene appears to be the least injurious." No mention whatever is made of the efficiency, and little more can be extracted from the other handbooks.

The explanation lies in the fact that an investigation of this kind is extremely hard to carry out; and as the efficiency of the different antiseptics varies in the case of different media, it is practically inexhaustible. Further, it has to be kept in mind that the negative results of sub-cultures does not in itself constitute a positive proof of the sterility of the material from which they are made. Another point of some importance arises from the fact that absolute sterility is not necessarily required, but only absence of development, a fact worthy of consideration since it is obviously undesirable in cases such as those in question to employ any stronger antiseptic agent than absolutely necessary.

The main part of the literature dealing with antiseptics from the point of view of efficiency is found in the publications of Salkowski and his co-workers, while more or less isolated statements occur throughout a large number of publications on both plant and animal physiology. The conclusions thus arrived at, far from representing anything like a complete survey of the question, rather emphasise the necessity for more detailed and exhaustive inquiry, especially since many previously accepted antiseptics have been shown to be far from reliable, and since, even in the best cases, the efficiency seemed to vary with a large number of other factors.

The literature on this point is naturally confined to the examination of a small number of antiseptics, since the large majority of germicides are out of the question for biochemical work owing to their too active nature, either chemically or otherwise. For this reason, the employment of inorganic acids and bases, as well as salts, is undesirable, since it is scarcely to be expected that an ionisable substance may be introduced into the condition under which these biochemical reactions take place without causing considerable disturbance in the said conditions. The same applies to a number of organic substances, while, even in the case of some of the less active compounds such as alcohol, their presence tends to disturb the colloid conditions and may even cause coagulation.

Formaldehyde, hydrocyanic acid, benzoic and salicylic acids are, to some extent, also open to objection in this way; but, as these have been fairly extensively used for the purpose, it seemed desirable to subject them to examination, in order, at least, to determine the limits of their applicability.

The main bulk of the literature concerns itself, then, with the following antiseptics: chloroform, formaldehyde, hydrocyanic, benzoic, and salicylic acids, thymol, toluol, and mustard oil. More or less isolated references are frequently met with in physiological and medical literature as to the employment of some antiseptic without any further remark as to the efficiency, the probability being that this was taken for granted and not demanding investigation.

With regard to the use of chloroform, Muntz (3) states that chloroform is capable of destroying micro-organisms without influencing enzyme action. This is later confirmed, with some reservations as to the latter point, by Salkowski (4) and Fokker (5). Kaufmann (6) found chloroform capable of sterilising bacterial cultures if present in sufficient quantity. For this purpose a solution of chloroform in water was not sufficiently strong, the addition of pure chloroform being necessary to produce sterility. Later, Kikkoji (7) finds that chloroform water provides an efficient medium for the autodigestion of calf liver, provided the relation of solid material to liquid is not greater than $\frac{1}{10}$. The presence of excess chloroform appears capable of preventing the growth of bacteria even when the amount of solid material is much greater—even $\frac{1}{3}$; but it is noted that this excess seems to retard the progress of digestion. Salkowski (8) states that it is possible to keep easily decomposable liquids for as long as a year in presence of chloroform, provided the evaporation of chloroform is prevented by efficient stoppering. The somewhat contradictory results of Vandevelde (9) are ascribed by Salkowski to the neglect of this precaution.

In connection with the preservation of material obtained from ground-up seeds, Miss White (10) states that the mere addition of chloroform was found insufficient, sterility being only obtained after soaking the seeds for some time in chloroform and subsequently grinding while still wet.

As a general conclusion from these statements, chloroform appears to be fairly efficient, provided the necessary precautions are taken. It may also be mentioned here that the results of the experiments which form the subject of the present paper point to a similar conclusion: that of all the antiseptics examined chloroform is probably the one most to be recommended for general use.

With regard to the use of formaldehyde, Kikkoji (7) finds that in all concentrations upwards from ·03 per cent. bacterial life seems to be destroyed. This concentration (·03 per cent.) is recommended by Kikkoji as the optimum, as the progress of enzyme action is retarded with the increasing concentration of the antiseptic. Price (11) gives the concentration of ·06 per cent. as completely destructive to bacterial life.

In working on the plant proteases, Vines (12) finds hydrocyanic acid to be the most desirable antiseptic for the purpose, thymol being discarded in its favour.

The efficiency of benzoic and salicylic acids has been investigated by Navassart (13), who states that a saturated solution of benzoic acid remains free from bacterial life, while in concentrations lower than this it is not efficient. Saturation and half-saturation are efficient in the case of salicylic acid. Confirmatory results have been put forth by Kikkoji (7) and Yoshimoto (14). These writers recommend the use of these substances in preference to chloroform, as the progress of digestion appears, in comparison, to be retarded by the latter.

Navassart (13) and Yoshimoto (14) also give some results obtained in the employment of mustard oil. In all concentrations upwards from $\frac{1}{8}$ saturation this antiseptic was apparently capable of maintaining sterility in the material.

With regard to toluol, a multitude of references as to its use in various researches may be found, though any inquiries as to its efficiency are extremely infrequent.

Hildebrandt (15) finds it possible to obtain sterility in liquids by prolonged shaking with excess of toluol, and to maintain this sterility by covering the surface with a layer of toluol. The reliability of toluol is, however, questioned by Vandevelde (9), who, on this ground, throws doubt on the value of a considerable amount of previously published work.

Thymol is recommended by Lewin (16) as being capable of delaying putrefaction in milk, egg white, and other easily putrefiable material for considerable periods. It has also been used to some extent by Stoclasa and Schittenhelm, and in the earlier researches of Vines, but in none of these cases is its use supported by any bacteriological evidence. Later, Kaufmann (6) decides, on the result of a bacteriological investigation, that thymol is incapable of preventing development, while Vandevelde (9) declares thymol to have no influence whatever on bacterial life.

The experiments I now proceed to describe were rendered necessary for the purpose of the research in which I am engaged, and the present paper does not, of course, pretend to deal in anything like a complete way with this vast and important subject. It is merely a record of my observations.

The investigation falls naturally into two stages, viz. :—

1. The efficiency of the antiseptic.
2. The influence of efficient antiseptics on enzyme actions.

With regard to the latter part of the question, it seemed preferable to employ, in the experiments, such enzymes only as had been fairly well studied, but it is evident that the whole problem is practically inexhaustible. With regard to the efficiency of the antiseptics, the question may again be divided into two—namely, the antiseptic value in clear homogeneous liquids, and again, in non-homogeneous suspensions such as milk, or egg albumen, or the even less homogeneous masses obtained by disintegrating animal or vegetable tissue. The following antiseptics were included in the experiment:—

1. Chloroform.	7. Phenol.	13. Thymol.
2. Bromoform.	8. Benzoic acid.	14. Naphthalene.
3. Benzol.	9. Salicylic acid.	15. Camphor.
4. Nitrobenzol.	10. Eucalyptus oil.	16. Mustard oil.
5. Toluol.	11. Creosote.	17. Formaldehyde.
6. Xylol.	12. Cymol.	18. Hydrocyanic acid.

A solution of iodoform in aceton was also employed in a few cases, but was given up owing to practical difficulties which were experienced in following the reactions in its presence.

Of the eighteen antiseptics, the following had apparently no appreciable influence on the development of bacteria in freshly prepared, clear beef-extract, which had been saturated with the various antiseptics, and to which had been added a drop of canal water—bromoform, cymol, naphthalene, camphor, mustard oil, benzol, toluol, xylol. The possible influence of excess of the antiseptics was then tried in the case of benzol, toluol, xylol, with the result that, although the development of bacteria was distinctly retarded in all cases, the antiseptics could not, nevertheless, be regarded as efficient. This was shown by preparing plates of alkaline, pepton-meat-extract gelatine from the material before digestion and after periods of two days and four days respectively. A continued increase was shown in the number of colonies in all cases. All these antiseptics were accordingly discarded. Similar results were obtained in the cases of nitrobenzol, phenol (·2 per cent.), eucalyptus oil, creosote, thymol, which were also excluded from the later experiments. There were consequently only the five following antiseptics left for further examination: chloroform, formaldehyde, hydrocyanic acid, benzoic acid, and salicylic acid. These were tested in the following materials: milk, egg yolk, egg white, minced fish, minced pancreas, pancreas ground to pulp with sand, barley ground with water. In the first instance, the pancreas was put through a mincing machine; but as it was found that none of the antiseptics were capable of preserving sterility, it was decided to repeat the experiment with more finely disintegrated material. In the subsequent experiments, the pancreas was first put through the mincer, and afterwards reduced to a practically homogeneous mass by trituration with sand in a mortar, and ultimately passed through a fine sieve. The large amount of air which was necessarily introduced during these manipulations was afterwards removed by means of a vacuum pump. The results of the tests are shown in the accompanying table, in which – means that the experiment gave a negative result, while + in the first column indicates decomposition of the material, in the second column presence of bacteria, and in the third and fourth columns a distinct increase in the number of bacteria during incubation, as shown by the number of colonies appearing in gelatine plates which were made at definite intervals.

The influence of these five antiseptics on the action of taka-diastase, pancreatin, yeast invertase, yeast endotryptase, and pepsin was now tried, with the following results:—

Taka-diastase acting on 2 per cent. soluble starch solution for one hour at 20° C. In the presence of the various antiseptics, the following number of cubic centimetres of the reaction product were required to reduce 5 c.c. Fehling's solution:—

Chloroform,	1 per cent.	8·5
Formaldehyde,	·1 ,,	8·1
Hydrocyanic acid,	·1 ,,	8·3
Benzoic acid,	·1 ,,	8·0
Salicylic acid,	·1 ,,	10·9

Yeast invertase acting on a 10 per cent. cane-sugar solution for 30 minutes at 50° C. The following number of cubic centimetres were required to reduce 5 c.c. Fehling's solution:—

Chloroform,	1 per cent.	6·1
Formaldehyde,	·1 ,,	7·53
Hydrocyanic acid,	·1 ,,	5·55
Benzoic acid,	·1 ,,	5·72
Salicylic acid,	·1 ,,	5·74

Pancreatin.—In this case only chloroform and formaldehyde were used, as the efficiency of the other antiseptics seemed very doubtful in the presence of an alkaline reaction. The substrate in this case was a 20 per cent. solution of pepton roche, the progress of peptolysis being estimated by the amount of precipitated tyrosin. In the case of formaldehyde, no action seemed to have taken place at all, while in the case of chloroform a considerable amount of tyrosin was deposited.

In the case of *yeast endotryptase* the method adopted was identical. In this case no action was obtained in the presence of any of the antiseptics with the exception of chloroform.

Pepsin.—For this purpose a 1 per cent. solution of edestin in dilute

	Milk.				Egg yolk.				Egg white.				Fish (minced).				Pancreas (minced).				Pancreas (ground).				Barley (ground).			
	Macroscopic	Microscopic.	Plates. 1	Plates. 2	Macroscopic.	Microscopic.	Plates. 1	Plates. 2	Macroscopic.	Microscopic.	Plates. 1	Plates. 2	Macroscopic.	Microscopic.	Plates. 1	Plates. 2	Macroscopic.	Microscopic.	Plates. 1	Plates. 2	Macroscopic.	Microscopic.	Plates. 1	Plates. 2	Macroscopic.	Microscopic.	Plates. 1	Plates. 2
Chloroform—																												
1 per cent.	–	–	–	–	+	+	+	+	–	–	–	–	+	...	+	+	+	...	...	...	+	+	not made		+	+	not made	
2 „	–	–	–	–	+	+	+	+	–	–	–	–	+	...	+	+	+	...	...	...	–	+	–	–	–	+	–	–
4 „	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	+	...	...	...	–	+	–	–	–	?	–	–
8 „	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	+	...	...	...	–	+	–	–	–	?	–	–
Formol—																												
·05 per cent.	–	–	–	–	–	+	+	+	–	+	–	–	–	...	–	–	+	...	...	...	–	+	–	–	+	+	not made	
·1 „	–	–	–	–	–	?	–	–	–	–	–	–	–	...	–	–	+	...	...	...	–	+	–	–	–	+	–	–
·2 „	–	–	–	–	–	–	–	–	–	–	–	–	–	...	–	–	+	...	...	...	–	+	–	–	–	+	–	–
·4 „	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	+	...	...	...	–	+	–	–	–	+	–	–
Hydrocyanic acid—																												
·05 per cent.	...	...	...	...	–	+	+	+	–	–	+	+	...	...	...	...	+	...	...	...	+	+	+	+	+	+	not made	
·1 „	+	+	+	+	–	–	–	–	–	–	–	–	–	...	–	–	+	...	...	...	+	+	+	+	–	+	–	–
·2 „	+	+	+	+	–	–	–	–	–	–	–	–	–	...	–	–	+	...	...	...	–	+	–	–	–	?	–	–
·4 „	...	...	...	...	–	–	–	–	...	...	...	...	...	...	...	...	+	...	...	...	–	+	–	–	–	?	–	–
Salicylic acid—																												
·1 per cent.	+	+	...	...	–	–	–	–	–	+	+	+	+	...	...	...	+	...	...	...	+	+	...	...	+	+	+	+
·15 „	+	+	...	...	–	–	–	–	–	+	+	+	+	...	...	...	+	...	...	...	+	+	...	...	–	+	+	+
·2 „	+	+	...	...	–	–	–	–	–	+	+	+	+	...	...	...	+	...	...	...	+	+	+	+	–	+	+	+
·4 „	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	+	...	...	...	–	+	+	+	–	?	+	+
·8 „	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	+	...	...	...	–	+	+	+	–	?	+	+
Benzoic acid—																												
·1 per cent.	+	...	...	...	–	–	–	–	–	–	–	–	+	...	...	...	+	...	...	...	+	...	...	...	+	...	...	...
·15 „	+	...	...	...	–	–	–	–	–	–	...	...	+	...	...	...	+	...	...	...	+	...	...	...	+	...	...	...
·2 „	+	...	...	...	–	–	–	–	–	...	...	...	+	...	...	...	+	...	...	...	+	...	...	...	+	...	...	...
·4 „	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	+	...	...	...	–	+	+	+	–	+	+	+
·8 „	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	+	...	...	...	–	–	–	–	–	+	–	–

hydrochloric acid was digested with a pepsin extract at 37°. A series of twelve digestions were made up for each antiseptic. At successive periods of half an hour each, one of these was withdrawn and saturated with sodium chloride. The progress of the reaction may be estimated by the decrease in precipitable material. In this case the necessary comparison was made by noting the time required to bring the members of the different series to the same state of clearness. The following table gives the number of half-hour periods which elapsed before the liquid reached its maximum clearness :—

Chloroform,	1	per cent.	10
Formaldehyde,	·1	„	7
Hydrocyanic acid,	·1	„	3
Benzoic acid,	·1	„	7
Salicylic acid,	·1	„	7

In the case of chloroform, the digestion appeared to proceed rapidly in the earlier stages, but an opalescence remained which was not appreciably reduced on continued digestion. Whether this was due to the presence of edestin or not is impossible to say.

General Conclusions.

The results of the experiments show :—

1. It is essential that the material should in all cases be as homogeneous as possible.

2. None of the antiseptics examined can be indiscriminately employed, and it is necessary in every case to choose the antiseptic with due regard to the material as well as to the nature of the enzyme which is to be examined.

3. It is not permissible to depend upon the efficiency of any one antiseptic in autolysis experiments, and the bacteriological examination is in all cases necessary.

References.

(1) *Proc. Roy. Soc. Edin.*, vol. xxxi. p. 342.

(2) Buchner, *Ber.*, 30, 31, 32; *Zeitschr. f. Phys. Ch.*, 44; *Bioch. Zeitsch.*, 4.

(3) Muntz, *C.R.*, 80, 1250 (1875).

(4) Salkowski, *Deutsche mediz. Wochenschrift* (1888), No. 168; *Zeit. f. klin. Medizin* (1890), Festschrift.

(5) Fokker, *Fortschritte d. Med.* (1891), No. 3.
(6) Kaufmann, *Zeit. f. phys. Ch.*, 39 (1903).
(7) Kikkoji, *Zeit. f. phys. Ch.*, 63 (1909).
(8) Salkowski, *Zeit. f. phys. Ch.*, 63 (1909).
(9) Vandevelde, *Bioch. Zeitschrift*, Band iii., 1907.
(10) White, *Proc. Roy. Soc.*, B., 550, 1909.
(11) Price, *Cent. f. Bacteriologie*, II. Abt., Bd. xiv., No. 314.
(12) Vines, *Annals of Botany*, 16.
(13) Navassart, *Zeit. f. phys. Ch.*, 72.
(14) Yoshimoto, *Zeit. f. phys. Ch.*, 58.
(15) Hildebrandt, *Beitr. Chem. Physiol.*, 5 (1904).
(16) Lewin, *C. mediz. Wiss.*, 1875.

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XX.—Report on Rock Specimens dredged by the "Michael Sars" in 1910, by H.M.S. "Triton" in 1882, and by H.M.S. "Knight Errant" in 1880. By Dr B. N. Peach, F.R.S. (With Folding Map, Text Figs., and Nine Plates.)

(MS. received March 18, 1912. Read May 13, 1912.)

I have been entrusted by Sir John Murray, K.C.B., with the examination of some of the coarser materials, more especially the rock-fragments, obtained from the bed of the Atlantic by Sir John Murray and Dr Hjort during the recent *Michael Sars* Expedition in 1910, and also with the re-examination of the collections dredged from the Wyville Thomson ridge and the Faroe Banks during the former expeditions of H.M. ships *Triton* in 1882 and *Knight Errant* in 1880.*

The present report is a summary of my examination, the more detailed description of the specimens obtained by the *Michael Sars* being reserved for the forthcoming official report of the expedition.

The material obtained by the *Michael Sars* will be discussed first, and as that from Station 95 is not only the most abundant but also the most interesting, it will now be dealt with.

Station 95. 26th July 1910.

Lat. 50° 22′ N., long. 11° 44′ W.; depth 1797 m. (981 fms.).

The material from this station, which lies about 230 miles south-west of Mizen Head in Ireland, naturally falls under two categories, viz. (1) over 200 rock fragments which may be considered to owe their presence on the bed of the sea to natural agencies; (2) a large quantity of furnace-slag ("clinker") or cinder, a few fragments of coal, and three small pieces of glazed pottery with "willow pattern" in blue, all of which have fallen from ships. There is also the cannon bone of a small ox, which may have dropped from a floating carcase, although there is still more likelihood of its having been thrown overboard from a cattle ship.

I. Distributed by Natural Agencies.

The rock-fragments from this station, all of which have been only partially embedded in globigerina ooze, are thus shown to be loose and removed from their parent rocks. They furnish examples of the three

* *Proc. Roy. Soc. Edin.*, vol. xi., 1882, p. 638.

great divisions into which geologists have classified rocks, viz. (*a*) Sedimentary, (*b*) Metamorphic, and (*c*) Igneous.

(*a*) *Sedimentary Rocks.*—Many of the sedimentary rock-specimens contain characteristic fossils, and their relative geological age can be determined with certainty, while many others can be classified by their lithological peculiarities with a considerable amount of accuracy and even their probable source indicated.

A group of over forty specimens of greenish-grey greywacke sandstones, dark shales, and black lydian-stone, identical in lithological character with rocks that floor a large portion of the Southern Uplands of Scotland and the northern central part of Ireland, may with considerable confidence be referred to the great Silurian Formation, though no fossils were found in them. Of these, twenty-three specimens (48·93 per cent.) are well glaciated (Plate I. figs. 1 and 2), six (12·76 per cent.) subangular and ice-moulded, seven (14·9 per cent.) rolled and well rounded (Plate II. figs. 1–6), and only eleven (23·6 per cent.) angular.

To the Devonian formation are to be assigned four specimens of purple conglomeratic grits, with stained quartzite and acid igneous pebbles, identical in lithological character with some of the "Glengariff Grits" of the Dingle Peninsula in the south-west of Ireland. These specimens show irregular fracture faces and are not glaciated, nor did they yield any organic remains.

Carboniferous rocks are well represented by over forty specimens ranging from 4½ to 2 inches in diameter. Sixteen of these are of limestone and calcareous shales with encrinites, and two are of limestone with chert, all of types identical with rocks *in situ* in Galway, Clare, and the centre of Ireland. One sandstone fragment was met with, crowded with species of *Schizodus* and *Edmondia* in the manner in which they occur in the Lower Carboniferous rocks of the Solway region of southern Scotland, and Londonderry and Tyrone in Ireland (Plate III.).

The other specimens are chiefly sandstones and sandy shales, many of which contain plant remains, and one with *Spirorbis carbonarius.* A few fragments of yellow sandstone and sandy shale like the above but without plants, doubtfully included with the Carboniferous rocks, could be matched from almost any Carboniferous area in Britain or Ireland.

Most of the limestone fragments show traces of having been glaciated; only one is well rolled, and three specimens, including those with chert, are angular. Of the sandstones and sandy shales the larger proportion have been glaciated, one is rounded, only three are angular, and a few are too friable to afford evidence as to their original condition.

The next series of identifiable fragments, thirteen in number, are of Cretaceous age. Nine of these are of chalk and four are chalk flints. One of the chalk fragments (4 inches in diameter) consists of ochreous chalk, containing numerous fossils, including the remains of a small species of *Exogyra*, a *Dentalium*, and several polyzoa and foraminifera. It has been glaciated before deposition and afterwards cariously weathered by solution. One specimen of hard chalk, 2½ inches in diameter, contains lines of well-rounded sand grains, like the base of the chalk where it overlies the Triassic sandstones below the Antrim Volcanic Plateau. It is well glaciated and faceted. Four smaller well-glaciated fragments of hard grey chalk also occur. Two specimens of finely crystalline white marble, like the chalk of Antrim where it has been invaded by intrusive igneous rocks, are also met with. One of these (5 inches in diameter) is well glaciated and faceted on one side, and shows a ragged fractured surface on the other. This specimen has evidently formed part of a larger boulder which has been glaciated and subsequently broken prior to transportation, for it bears evidence of having been embedded nearly edge on, and must have lain for some time with about two-thirds of its surface exposed (Plate IV.).

Of the four specimens of flint, varying from two to three inches in diameter, the largest is a portion of a dark flint nodule with part of its original coating or "skin" still remaining. Glacial markings are found both on the skin and on the fracture faces. It has been still further flaked since being glaciated, but at so remote a date that a well-marked "patina" has been developed over the newer faces. The remaining three flints are whitish, like those of Antrim, are well-rolled portions of fractured nodules, and show "bulbs of percussion" or "chatter-marks" where they have been hurtled together in a torrent, or where they have been subjected to wave action (Plate V. fig. 2).

Five small specimens cannot with any certainty be classed with the above. One of these, of coarse-grained dolomite, like a vein-rock, is ice-moulded. Such veins occur in the Carboniferous Limestones of Ireland; in fact, a thin vein of like character occurs in one of the limestone blocks from the present locality. Two are ice-moulded fragments of vein quartz which have come from a region of schistose rocks in a low grade of metamorphism. Two well-rounded or rolled pebbles occur; one is of vein quartz and shows "bulbs of percussion" (Plate II. fig. 5), and the other is of white crystalline limestone or dolomite (Plate II. fig. 6).

(*b*) *Metamorphic Rocks.*—Under this category are grouped two distinct types of rocks, viz. (1) a set of highly crystalline, coarsely banded gneisses, and (2) a set consisting of both sedimentary and intrusive igneous rocks of low-grade metamorphism.

(1) There are nineteen gneiss fragments, seventeen of which are ice-moulded and glaciated, the other two being angular. The largest is about 4 inches, the rest being of nearly uniform size, between 2 and 3 inches in diameter. They contain *orthogneisses* (foliated igneous rocks), like most of the Lewisian gneisses, and *paragneisses* (metamorphosed sediments), like the muscovite-biotite gneisses and quartz-biotite-granulites of the Moine series of Scotland. Rocks of this type make up a large part of the north of Ireland and floor large tracts of southern Scandinavia.

Dr Flett sends the following note on his examination of the microscopic slides of the gneisses referred to :—

"Among the gneisses sliced, several are of distinctly Lewisian type. They are (57)* pyroxene-gneiss, (61) epidotic hornblende-gneiss; (55) biotite-hornblende-gneiss; (59) biotite-gneiss [orthogneiss]; (58) muscovite-biotite-gneiss. Two others are distinctly like the coarser gneisses of the Moine series. They are (56) biotite-gneiss rather crushed, and (60) biotite-gneiss or granulite somewhat coarse-grained. (54) is a granulitic gneiss which has more resemblance to the Moine rocks than to the Lewisian. All these rocks could be matched in the North-West Highlands [of Scotland] without any difficulty. (62) is a sericitic granite-gneiss of doubtful origin."

(2) The specimens to the number of twenty-one that come under this heading are more varied in character than the gneisses. One small glaciated fragment, $2\frac{1}{2}$ inches in diameter, more altered than the rest, is a puckered black graphitic schist like that occurring along the belt which traverses the Central Highlands of Scotland and is continued into the Donegal Highlands of Ireland. Similar rocks also occur in Connemara.

The most conspicuous type of rock is a green lustrous puckered slate or phyllite, showing that it has been subjected to pressure in more than one direction (as shown in Plate V. fig. 1). There are eight specimens, varying from 2 to 5 inches in longest diameter. They are for the most part flat and thin, showing foliation and joint faces, and with sharp corners and edges. Two thicker specimens are glaciated, one of which is only a part of a broken-up boulder.

Two small angular specimens of blue slate show that their cleavage is at right angles to the bedding planes. One piece of purple slate is glaciated.

Four fragments of flaser greywacke or grit, one of which is glaciated, and an angular specimen of coarse pebbly grit or conglomerate, make up the tale of schists of sedimentary origin.

* The numbers refer to those of the thin slices.

Metamorphosed igneous rocks are represented by an angular specimen ($8+4+\frac{1}{2}$ inches) of sheared fine-grained grey felsite with small ochreous spots, and three small specimens of flaser chloritic epidiorite filled with carbonates, one of which is angular and the others glaciated.

An assemblage of rocks such as those referred to under heading 2 of the Metamorphic series occurs all along the Highland border in Scotland, and in the Londonderry, Donegal, and Connemara regions of Ireland. Green and purple slates of Carboniferous age also occur in the south-west of Ireland along the line of country affected by the Hercynian system of folding.

(*c*) *Igneous Rocks.*—Among the material from this Station there are about fifty fragments of igneous rock, ranging from 2 to 4 inches in greatest diameter. Of these more than one-half are holocrystalline and intrusive, while twenty-one are lava-form and one fragmental.

Among the holocrystalline rocks are seven small specimens of granite, two of which are glaciated, four angular, and one rounded. According to Dr Flett, who has examined them microscopically, they resemble the "Newer" granites of Scotland and Ireland, and are unlike those of Tertiary age. One angular specimen (4 inches in diameter) of nephiline syenite, unlike any known rock of the kind in Western Europe, has been described by Lady MacRobert, who states that "it cannot at present be compared with any known syenite of the North Atlantic basin." * †

Two small fragments of lamprophyre resemble the dyke rocks which are so plentifully distributed round the plutonic centres in Scotland and Ireland.

Twenty specimens of dolerite occur, mostly angular and with sharp edges and angles; five only are glaciated, and one, of rather different type from the others, is egg-shaped and well rolled. Rocks like these occur plentifully in southern Scotland, in the north of Ireland, and also in the south of England.

The effusive or lava-form rocks are chiefly olivine-bearing basalts, and are usually amygdaloidal. There are twenty small specimens, mostly showing irregular fracturing, and only five of them are glaciated. Similar rocks occur in the Tertiary Volcanic Plateau in North Ireland.

Among the material is a square block of spongy fragmental rock, enclosing pieces of pumice and numerous insect fragments, that may have been floated off from some oceanic island and carried by the warm-water current, and, becoming waterlogged, sunk at this locality.

* *Geol. Mag.* (Jan. 1912), Dec. V., vol. ix. p. 1.

† Professor Ramsay of Helsingfors, who saw the specimen, mentioned to me in conversation that he had this year (1911) observed a similar rock in place in the east of the Kola Peninsula.

All the igneous rocks have been examined microscopically by Dr Flett, who has kindly supplied the following description:—

"Of the igneous rocks that were sliced for microscopic examination one much decomposed specimen may be a basaltic tuff, and several belong undoubtedly to the quartz-dolerite group, but the remainder are olivine basalts and olivine-free basalts. Some of them are rich in olivine, but in most that mineral is rather scarce, and it is never in a good state of preservation. As a rule the specimens are somewhat decomposed, and none of them is absolutely fresh, so that there is no probability that they are of Recent age; this practically excludes the possibility that they have been derived from the modern volcanoes of Iceland. Taking the series as a whole, they have many features that make it unlikely that they have been derived from the volcanic rocks of Carboniferous age in the west of Scotland (Clyde Valley, etc.). On the other hand, they have marked affinities with the Tertiary volcanic rocks of the Inner Hebrides and the north of Ireland. The quartz-dolerites are no exception to this, for members of this class occur as intrusive sills and dykes among the Tertiary volcanic rocks. This conclusion rests only on the general characters of the collection, for among these rocks there is not one that shows peculiarities by which its source can be established beyond doubt."

The table on the next page shows the number and condition of the rock specimens from this Station. A glance at this table makes it at once apparent that such a range of formations as our collection represents could hardly occur in place within the small space swept by the trawl.

Mode of Occurrence of the Rock Specimens.—A close study of the specimens shows that at the time they were dredged all, without exception, had been only partially embedded in the ooze of the sea-floor. This is apparent from the disposition of the attached organisms, such as siliceous sponges, serpulæ, and horny worm-tubes and hydroids and arenaceous foraminifera, as well as from a slight coating of manganese oxide, which occur on the exposed parts, but are absent from the embedded surfaces. The manganese coating is most pronounced near the junction of the two areas.

The examination in the "Challenger" office of the deposit from the floor of the ocean at this locality shows it to be a "globigerina ooze" with 63·17 per cent. of calcium carbonate, leaving a residue of 36·83 per cent. In the calcareous part the remains of pelagic and bottom-living foraminifera, echinoid spines, ostracods, and coccoliths have been observed, but it is evident that warm-water surface foraminifera have been its chief source. The residue is shown to be made up of 1 per cent. of siliceous organisms (viz. radiolaria and sponge spicules), and mineral matter which makes up the remaining 35·83 per cent. Of this, 2 per cent. is composed of angular and rounded fragments of quartz and felspar with a mean diameter of 0·09 mm.

Table of Rock Specimens from Station 95.

			Angular.	Rounded.	Sub-angular Ice-moulded.	Glaciated.	Total.
Sedimentary	Cretaceous	Chalk with exogyra	...	...	...	1	
		Chalk with sand grains	...	...	...	1	
		Chalk-flints	...	3	...	1	12
	? Cretaceous	Fine-grained grey limestone	...	...	...	4	
		Fine-grained white marble	...	...	...	2	
	Carboniferous	Limestone with encrinites	1	1	15	1	
		Limestone with chert	2	...	...	...	
		Limestone with *Schizodus* and *Edmondia*	...	...	1	...	39
		Sandstone with encrinites	1	...	...	...	
		Sandstone with *Spirorbis*	1	...	...	...	
		Sandstone with plants	1	...	...	1	
	? Carboniferous	Sandstone grit, sandy shale	2	1	6	5	
	Devonian	Purple grit and conglomerate	4	...	...	...	4
	Silurian	Greywacke	4	7	...	22	
		Shale	2	...	...	1	
		Lydian stone	...	...	...	1	45
	? Silurian	Dark shale	1	...	...	1	
		Greywacke, quartz-veined	4	...	...	2	
	Uncertain	Vein quartz	1	...	2	...	6
		Dolomitic vein stuff	1	...	1	1	
Igneous	Intrusive	Granite	4	1	...	2	7
		Syenite	1	...	...	...	1
		Lamprophyre	...	...	...	2	2
		Dolerite	17	1	...	5	23
	Lavaform	Basalt	17	...	1	6	24
	Pyro-clastic	Volcanic agglomerate	1	...	...	...	1
Metamorphic	Highly	Gneiss	2	...	3	14	19
	Slightly	Graphitic schist	...	...	...	1	
		Slate and phyllite	7	...	...	3	17
		Flaser greywacke	4	...	...	2	
		Flaser epidiorite	2	...	...	1	3
		Flaser felsite	1	...	...	...	1
			81	14	29	80	204
		Percentages	39·71	6·86	14·21	39·21	
				Ice-moulded		14·21	%
		The combined ice-moulded and glaciated =				53·42	%

The remainder consists of amorphous clayey matter with minute mineral particles and fragments of siliceous organisms.

The sounding-tube brought up from the bottom at this Station a roll about 9 inches in length, showing that it had pierced the deposit to a depth much beneath that reached by any of the stones captured by the trawl.

From these facts it is at once apparent that the stones are not the debris of rocks *in situ*. Moreover, more than half of them (53·42 per cent.) are either ice-moulded or well glaciated, and the proportion of angular and rounded fragments to the glaciated ones is not much in excess of what is to be found in our "tills" or boulder-clays, and very much less than that occurring in the moraines of Alpine glaciers.

The peculiar assemblage of stones can nearly all be matched by rocks occurring *in situ* in the west of Scotland and in northern and western Ireland. It is further significant that the very formations that are absent from those areas, although covering large parts of England, are equally unrepresented in our collection.

The researches of glacialists have now made out, from the direction of ice-markings on the solid rocks and the distribution of boulders, that ice, having its origin in the west of Scotland, forced its way on to the north of Ireland, and there combining with the native ice, split up into two branches, one of which filled the Irish Sea and made its way down so far south as to override part of Pembrokeshire on the one side and on the other to impinge upon the Irish shore to at least as far south-west as Cork. The other branch, passing westwards, made its way into the Atlantic, and reached the deep water at the edge of the continental shelf as portion of the ice-sheet that enveloped the greater part of Britain and the whole of Ireland at the period of maximum glaciation.

It is to this period that we must look for the peculiar distribution of most of our rock-fragments by means of floating ice, for the evidence already adduced shows that the stones were dropped from above into globigerina ooze. Some small proportion of our material may have been floated off from our shores during the later part of the glacial period by pack-ice or ice-foot, which might account for the presence of so many angular, evidently frost-riven, splinters of dolerite; while some of the rolled stones may even have been distributed by floating river ice drifted seawards during sudden thaws. A few of the smaller stones may have been transported by floating seaweed. Some of the material may even have been brought by part of the "polar pack" since glacial times, as an occasional small floe has been observed to approach within a few hundred miles of our shores, and it is perhaps to this source that we must look for the presence of the specimen of nepheline syenite.

Some of the glaciated stones have nests of clay attached to them in which are rounded grains of quartz over one millimetre in diameter, much larger than those found in the ooze, which average only 0·09 mm. in diameter, showing that the clay is no part of the ooze, but must have been transported with the stone, and is of the nature of a boulder clay. The faceting of many of the striated stones shows that they were in all probability embedded in a *moraine profonde* sufficiently tenaceous to hold them in place while the ice passed over them and cut facets upon them (Pl. IV.).

The presence of the globigerina ooze indicates that at the period of the distribution of the stones there was a surface warm-water current from the west, and that at Station 95 the water could not have been shallow but probably much the same depth as at present, showing that the continental shelf had already been drowned.

The observations of Messrs Wright and Muff * prove that in the Cork district of Ireland there is a rock-notch and beach not far above present sea-level which are covered up by the till left by the combined Scottish and Irish ice during the period of maximum glaciation. This notch has also been recognised in Wales by Dr Jehu,† and in the Western Islands of Scotland by Mr Wright,‡ indicating that in these regions the relative level of land and sea is much the same now as it was at the beginning of the Glacial Period.

The absence at Station 95 of gabbro fragments like those dredged in great profusion from the "Porcupine Bank," § which is evidently the worn-down stump of a great Tertiary volcanic centre comparable with that of Mull, Skye, or St Kilda, also goes to indicate that the bank was "drowned" and out of reach even of ice at the period of distribution of our material.

The conclusion arrived at regarding the stones from Station 95 may appear to be at variance with that set forth by Messrs Cole and Crooke in their memoir on "Rock Specimens dredged from the Floor of the Atlantic off the Coast of Ireland," || who infer that the material examined by them is mainly the debris of rocks in place on the sea-floor. The difference, however, is more apparent than real. A great deal of their material comes from what is part of the "continental shelf" or its immediate margin. They may therefore be dealing in part with an old land surface and the delta of rivers that flowed across it, which had been pushed forward on to the "Atlantic Rise," the Porcupine Bank being at the time a volcanic pile rising out of the deep water as an island in process of being cut down to

* W. B. Wright and H. B. Muff, "The Pre-glacial Raised Beach of the South Coast of Ireland," *Scient. Proc. Roy. Dublin Soc.*, N.S., vol. x. p. 250, 1904.

† Jehu, *Trans. Roy. Soc. Edin.*, vol. xlvii. (1909), p. 27.

‡ W. B. Wright, *Geol. Mag.*, Dec. V., vol. viii. (1910), pp. 98–109.

§ A. J. Grenville Cole and T. Crooke, "On Rock Specimens dredged from the Floor of the Atlantic off the Coast of Ireland," *Mem. Geol. Survey of Ireland* (1910), pp. 4–10, and map.

|| *Op. cit.*, pp. 26–27, and map.

the level of the sea at that time. Although the shelf was in all probability drowned prior to the on-coming of the great ice-sheet, yet during its maximum extension the ice must have treated this shallow area as a land surface and only ceased at the deep water along the "Atlantic Rise," beyond which the distribution of material would be by floating ice. Messrs Cole and Crooke show that a considerable number of glaciated

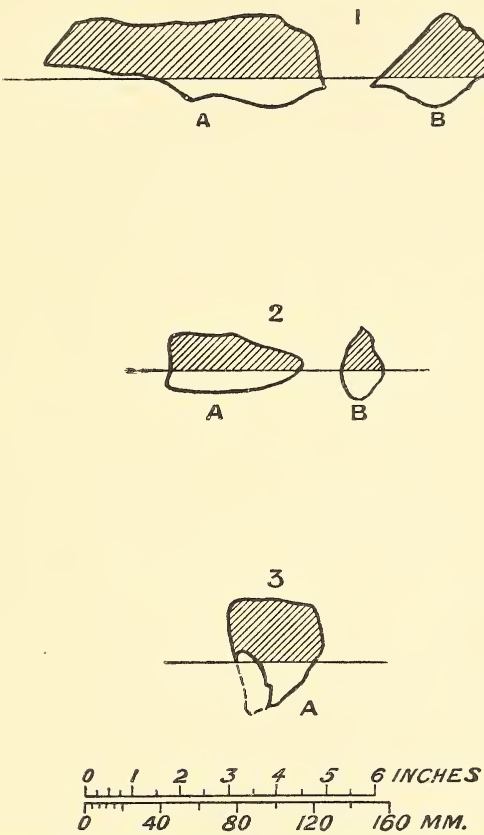


FIG. 1.—Showing relative proportions of specimens embedded in Ooze (white) to exposed portions (hatched). A, side on; B, end on.

stones occur in their material, and believe that they may represent "distributed moraine material." * The difference therefore is one of degree only. The assemblage of stones in the material examined by Messrs Cole and Crooke from their Stations 18 to 22 as shown on their map † is remarkably like that from Station 95 of the *Michael Sars*, so that the conclusion is unavoidable that there is a distinct connection between the distribution of the material in the two adjacent areas, both just beyond the edge of the continental shelf.

* *Op. cit.*, p. 22. † *Op. cit.*, pp. 21–26.

The manner in which the stones lay on the sea-bed at the time of their capture is well worth consideration. Most of them have had only about one-third of their bulk embedded in the ooze, and many of them even less (text figs. 1 and 2). As has been already stated, this is shown by the coating of manganese and the presence of adherent organisms on the exposed surfaces. Not only is this the case, but many of the more elongated specimens are embedded on end (text fig. 3), and in the case of flat specimens, on edge (text fig. 4), although in general they are nearer the horizontal. From a consideration of these facts it is highly improbable that they could have been dropped on to the present surface of the ooze, which must be of

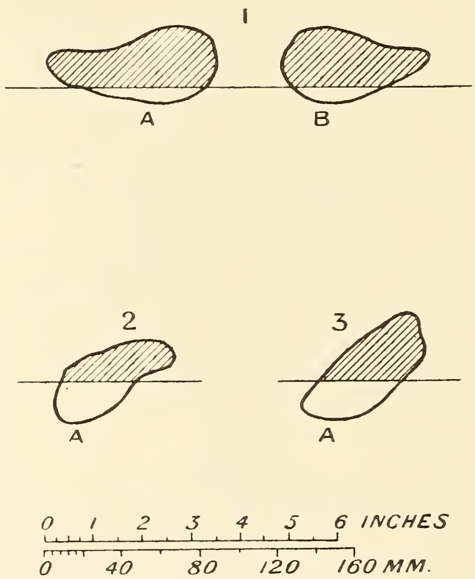


FIG. 2.—Explanation as in text fig. 1.

considerable consistency to support them in such precarious positions as some of them had assumed (text fig. 5).

A more feasible explanation is that they had dropped on to a flocculent ooze, into which they sank, being arranged in such a way as to offer the least resistance to friction, till they reached a layer of sufficient tenacity to prevent further sinking. At first they would be totally buried. Their presence at the surface is probably due to a current that is just strong enough not only to sweep away the falling pelagic organisms that mainly go to form the ooze, but also to denude some of the looser top deposit and partially to expose the stones. It might be argued that solution of the calcareous organisms would bring about the same effect; but the ooze at the Station has already been shown to contain 35·83 per cent. of clay and

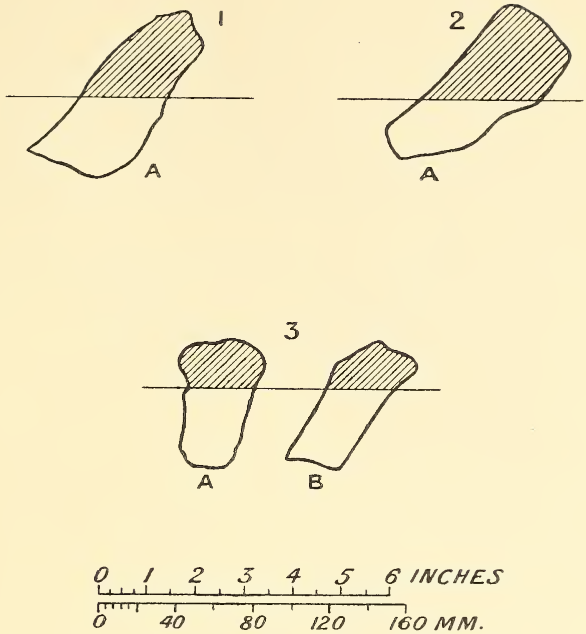


Fig. 3.—Explanation as in text fig. 1.

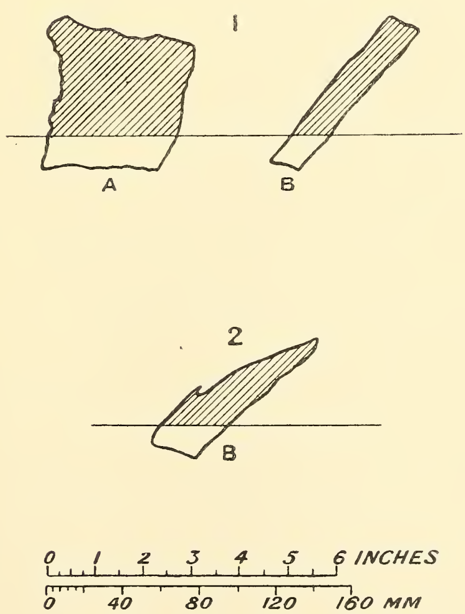


Fig. 4.—Explanation as in text fig. 1.

minerals insoluble in hydrochloric acid. It is clear that solution must have gone on to a considerable extent by the manner in which the glaciated limestone fragments have been cariously weathered after deposition as displayed on both their exposed and embedded sides. But the theory that the work has been done by current-action rather than by solution receives additional support from the fact that nearly all the flat stones,

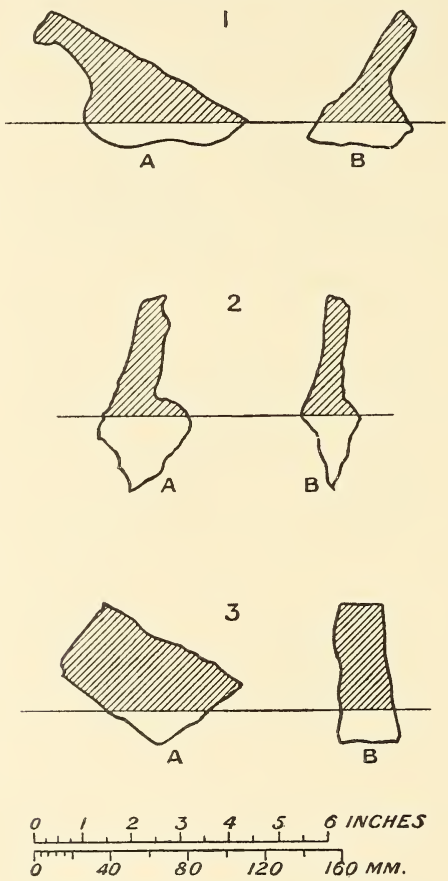


Fig. 5.—Explanation as in text fig. 1.

even when they have lain on their sides, have had parts of their under sides exposed. If the stones could be oriented, the direction of current would also be indicated (text fig. 6).

The explanation above offered may also account for the curiously even and comparatively small size of the stones (on an average $3 \times 2 \times 2$ inches), which seem to have been assorted according to their relative weight compared with their superficial area. It is only the flat and thin stones that reach a length of 4 inches or more. The larger stones with a greater cubic content may have been buried sufficiently deep to escape capture by the trawl, which appears to have swept the surface only and not to have penetrated the ooze. The trawl itself may have acted selectively and brought up the more exposed specimens.

Such an explanation would further account for the thinness of the coating of manganese found on the exposed parts of the stones.

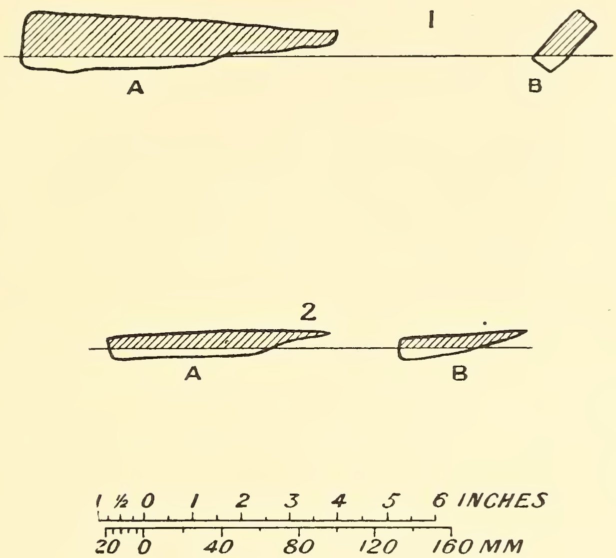


Fig. 6.—Explanation as in text fig. 1.

II. Distributed by Human Agency.

Under this category of material distributed by human agency are included 157 specimens of furnace-clinker and cinders, the largest of which measures about 5 inches cube. There are also five fragments of unburnt coal, both anthracitic and bituminous, the largest being $4 \times 2 \times 2$ inches.

One specimen represents the metacarpal (cannon bone) of a small ox (Pl. VII. fig. 1).

Another specimen consists of three pieces of an earthenware jar with the willow pattern in blue.

Considering the short time that has elapsed since steamships came into existence it is astonishing that such a large proportion of the dredged material should consist of furnace clinker, but it must be remembered that Station 95 is in the direct route of "Atlantic liners." From the large number of specimens belonging to both categories obtained from Station 95 it may be thought that the sea-floor is thickly strewn with such material, but it must be taken into consideration that the trawl was at work for some hours and dragged for some considerable distance.

Station 4. 10th April 1910.

Lat. 48° 27′ N., long. 8° 36′ W.; depth 1000 m. (547 fms.).

The specimens from this locality consist of a piece of a moulded coal briquette ($4 \times 3\frac{1}{2} \times 3$ inches), three fragments of furnace clinker (the largest $4\frac{1}{2} \times 3 \times 2$ inches), and one small piece of coal showing a sort of "cone in cone" structure.

The piece of briquette, which is nearly a cube, has serpulæ attached to all sides but one, upon which it has evidently lain, and seems not to have sunk into the ooze. All the above material has doubtless fallen overboard from ships. Briquettes made of coal-dust compacted by tar have been in use for only a very short time.

According to the investigations made in the "Challenger" office, the deposit at this Station is globigerina ooze with 70 per cent. calcium carbonate and a large percentage of bottom-living foraminifera. Mineral particles with a mean diameter of 0·12 mm. make up half the residue.

Station 10. 19th to 21st April 1910.

Lat. 45° 26′ N., long. 9° 20′ W. (Bay of Biscay); depth 4700 m. (2657 fms.).
Globigerina Ooze.

The material from this Station consists of 339 rock specimens varying from $\frac{1}{8}$ inch to over 2 inches in greatest diameter, 70 pieces of furnace clinker and cinder, and 10 small fragments of coal.

Of the rock specimens about 200 were individually determined, while the rest, which are mostly under $\frac{1}{4}$ inch in diameter, were either too small or too much decomposed for correct determination, but small though these fragments be, examination with the lens shows that many of them are glaciated.

Sedimentary Rocks.—Among the sedimentary rocks represented are two small fragments of greywacke like those from Station 95, one of which is striated and the other ice-moulded.

One small fragment, glaciated on one side, is like the "Glengariff Grits" of Devonian age from the Dingle Peninsula in the south-west of Ireland.*

* Rock specimens of this type have been identified in the material brought up from parts of the sea-floor lying between the present Station and Station 95 in connection with the laying of the "Atlantic cables." They also occur as boulders on the Scilly Isles.

Carboniferous rocks are represented by seven specimens of dark encrinital limestone like the Carboniferous Limestone of Ireland, seven small beautifully glaciated specimens of parrot coal or dark carbonaceous shale. Twelve fragments of dark micaceous sandstone, resembling some of the fossiliferous sandstones from Station 95, may belong to this formation.

A few small flakes of dark shale, probably a fragment of a larger piece, also occur. One of these contains a fragment of what appears to be a cycadaceous leaf, and is probably derived from jurassic rocks.

The cretaceous formation is represented by two specimens of decomposing splintery rock like a hardened greensand, over forty fragments of chalk and hard grey limestone, thirteen of which show grains of glauconite. Most of the above are glaciated. Some of the hard limestone may be of Cainozoic age, like that recorded by Messrs Cole and Crooke * from the dredgings off the coast of Ireland, close to Station 95 of the *Michael Sars.*

Metamorphic Rocks.—The collection contains seventeen small specimens of granulitic muscovite-biotite gneiss, like that from Station 95. Of these thirteen are subangular and ice-moulded, and four well striated. There are three pieces of hornblende-gneiss, one subangular and two glaciated. These latter are like some of the Lewisian gneisses of the north-west of Scotland. One small fragment of chlorite schist is a sheared epidiorite, like those from Station 95.

Igneous Rocks.—The plutonic rocks are represented by six fragments of granite, like those from Station 95, eleven of diorite and gabbro, and one of quartz-syenite, all glaciated; dyke rocks by fourteen fragments of dolerite; and volcanic rocks by thirteen specimens of slaggy basalt.

The above assemblage of rock specimens, most of which are glaciated, is almost identical with that from Station 95, leading to the conclusion that they are from the same sources and distributed by the same agencies.

The conditions under which they occur on the sea-floor seem to be somewhat different from those occurring at Station 95, as most of the specimens show a more pronounced coating of manganese oxide. One specimen of limestone has a comparatively large manganese concretion attached to its exposed side.

The investigation in the "Challenger" office shows that the deposit at this Station is globigerina ooze, of which the sounding-tube brought up a roll about 5 inches long. The ooze consists of 66·11 per cent. of calcium carbonate and a residue of 33·89 per cent. of clay and mineral matter. The smallness of the rock fragments may be due in part to causes similar to those suggested for the nearly uniform size of the stones from Station 95.

* *Op. cit.*, p. 23.

The smaller fragments would not sink so deep as the larger ones, and would therefore be the sooner exposed.

The material distributed by human agency consists of seventy pieces of furnace clinker and cinder, ten fragments of coal, the largest of which is only $1\frac{1}{2}$ inches in greatest diameter, which seems to indicate that this locality is in the direct route of steamboats.

Station 48. 31st May 1910.

Lat. 28° 54′ N., long. 24° 14′ W.; depth 5000–5500 m.
Globigerina Ooze (?).

Among the material from this Station there are two chips of dark chalk-flint with white crust denoting the original outside of the nodules from which they were derived. The edges of the chips are rubbed, which is suggestive of their having been glaciated.

Metamorphic rocks are represented by one small specimen of foliated epidiorite or hornblende schist ($1 \times \frac{3}{4} \times \frac{1}{2}$ inch), which has been ice-moulded, but is now covered with manganese oxide.

The igneous rock fragments are volcanic, four of which are thoroughly decomposed basalt rocks (the largest $1\frac{1}{4} \times \frac{3}{4} \times \frac{1}{2}$ inch); but the chief feature of the material consists of sixty-seven fragments of rolled oceanic pumice.

The foliated epidiorite and the chalk flints, and in a less degree the basalt fragments, appear to connect up this material with that from Stations 10 and 95, as if this Station were still within the influence of the northern drift ice. The pumice fragments, which are of the rolled type so widely distributed over the bed of the ocean, as is shown by the work of former expeditions, more especially by that of the *Challenger*, must have had quite another history. A study of them brings out many points of interest. They are all small and more or less rounded, but in the process of attrition to which they have been obviously subjected, the harder portions, such as the sanidine crystals, have been left projecting above the more fragile cellular parts. Hence it is clear that the shape of the fragments has been greatly determined by the projecting crystals as shown in Pl. VIII. figs. 1 and 2.

Some of the pumice fragments enclose pieces of glassy rock with porphyritic crystals, evidently representing portions of the cooled crust of the lava-flow broken up and involved in the still liquid scum. The pumice must have emanated from some volcanic centre producing acid lavas, probably an oceanic island which, like Krakatoa, during violent eruption, sent off great floes of floating pumice, the fragments of which were hurtled against each other till they were more or less rounded. They were then carried forward by winds and currents, and eventually scattered and floated on till waterlogged. The material under consideration was probably borne to this Station by the southern descending branch of the Gulf Stream.

The manner in which the crystals stand out from the cellular matrix further suggests the idea that, under such a pumice floe as that sent off by Krakatoa, deposits like the so-called "crystal tuffs" may have been produced.

The organism such as *Stephanoscyphus* attached to the specimens from this locality show that some of the stones must have been embedded "end on."

Three small pieces of furnace clinker and cinder, the largest only $\frac{3}{4}$-inch in diameter, were found among the material. The much corroded auditory bulla of a large finner whale was also obtained from this locality (Pl. VII. fig. 2).

Station 88. 18th July 1910.

Lat. 45° 26′ N., long. 25° 45′ W.; depth 3120 m. (1703 fms.).
Globigerina Ooze.

No rock fragments, but only some pieces of coal evidently dropped from ships, were found among the material submitted to me from this Station, but a fragment of limestone (5×3 inches in diameter) is recorded from the material examined in the "Challenger" office, and the sounding-tube brought up a roll of ooze 14 inches in length. The residue of the ooze, insoluble in hydrochloric acid, is 26·34 per cent., consisting of 20 per cent. "minerals (0·15 mm. in mean diameter) angular and rounded; quartz, orthoclase, mica." This large amount of continentally-derived material evidently shows that the Station is or has been within the sphere of floating ice.

In the material examined by me the only specimens that appear to have been brought by natural agency are four pieces of teredo-bored wood, the largest $2\frac{1}{4}$ inches long and quite thin. The wood is endogenous, and has the appearance of that of a palm. It has evidently floated about till it has been thoroughly riddled by some boring animal, probably a species of *Teredo*. It may have been derived from some tropical or subtropical source, and has probably been brought by the Gulf Stream from the West Indies or South America.

The other material from this locality consists of three pieces of furnace clinker (from 2 to 4 inches in diameter), a fragment of anthracite and one of bituminous coal, all evidently dropped from ships. One knotted piece of wood-charcoal ($1\frac{1}{2} \times 1\frac{1}{2} \times 1$ inch) shows twelve distinct lines of growth, showing that it comes from a country of strongly contrasted seasons. This specimen may have either been floated from off the land or been thrown overboard from a ship.

Station 23. 5th and 6th May 1910.

Lat. 35° 32′ N., long. 7° 7′ W.; depth 1215 m. (664 fms.).

The only material examined by me from this locality consists of two pieces of furnace clinker which are parts of the same specimen and the largest of which is $6\frac{1}{2} \times 3 \times 2\frac{1}{2}$ inches. It has lain on the bottom sufficiently long to have a simple coral attached to it. In the crevices of the cinder were found several species of foraminifera, pteropods (chiefly species of *Hyalea*), and some examples of cumaceous and amphipodous crustacea. In addition to the material examined by me the trawl brought up a very large amount of pteropods, foraminifera, and lamellibranch shells.

Station 24. 6th and 7th May 1910.

Lat. 35° 34′ N., long. 7° 35′ W.; depth 1615 m. (883 fms.).

The only material from this Station consists of three pieces of furnace clinker or cinder, the largest about $3\frac{1}{2}$ inches long. These have been partially embedded in a reddish ooze containing foraminifera and pteropods. Attached to the exposed surfaces of the cinders are siliceous sponges, serpulæ, and two species of lamp-shells, *Terebratula cranium* and *Terebratulina caput-serpentis.* One valve of a thick-shelled lamellibranch bored by a carnivorous gasteropod is embedded in the ooze.

The study of the clinkers from the last two Stations, situated just outside the Straits of Gibraltar, and the attached organisms, suggests that the Stations are under the influence of the strong under-current which sets out of the Mediterranean through the Straits.

The presence of the strongly calcareous shells inhabiting shallow water from this and the preceding Station 23, some of which are bored by gasteropods, seems to indicate that this area has undergone subsidence at no very distant period of time.

Station 25b. 8th May 1910.

Lat. 35° 46′ N., long. 8° 16′ W.; depth 2055 m. (1122 fms.).

From this Station the material consists of an oyster shell and four specimens of a large *Balanus B. porcatus* (Pl. IX. fig. 1).

The larger ones seem to have lain longer on the bottom than the others, as their bases are cariously etched by solution, and one of them is more deeply etched and its colour more faded than the other. The two narrower specimens are much fresher. All have lain sufficiently long on the bottom to be partially embedded in the ooze, and for arenaceous foraminifera and a small species of *serpula* to attach themselves on both the inside and the outside of the parts not embedded.

The oyster shell has certainly been brought by human agency, as it is not the European *Ostrea edulis* but a fine characteristic specimen of the American "Blue Point" that has only been an object of import into Europe since fast "Atlantic liners" began to run between the two continents, about the end of last century. Both valves of the shell are preserved, and when brought up, the hinge cartilage must have been unbroken, and is still quite fresh, while the inside nacre still retains its sheen (Pl. IX. fig. 2).

Shallow-water polyzoa and an agglutinated sandy worm-tube are still attached to the valve that lay uppermost during life, which is indicated by its other valve having been attached to some hard substance, evidently another oyster shell. The organisms have therefore been brought from America with the oyster, for no organisms like those attached to the *Balani* have been found on the shell, which evidently has not lain long enough, at this Station, for their growth.

Station 70. 30th June 1910.

Lat. 42° 59′ N., long. 51° 51′ W.; depth 1100 m. (601 fms.).

The material from this Station consists of seven rock fragments. The largest of these ($2 \times 1\frac{1}{2} \times 1$ inches) is of greenish, fine-grained, hardened mudstone, and is well glaciated. One small piece, just over 1 inch in diameter, of a light grey limestone, is polished and striated on one side only, the others showing fresh fractures. It is evidently only a portion of a boulder. One layer of the limestone is full of small organisms, mostly foraminifera, and shows sections of what appear to be pteropods on the fracture-faces. The parent rock is therefore in all probability of Tertiary age. A tiny piece of calcareous sandstone is apparently from the same source as the limestone.

One small glaciated fragment of gneissose amphibolite (1 inch in diameter) represents the metamorphic rocks.

Igneous rocks are represented by one specimen (1 inch in diameter) of gabbro or coarse-grained dolerite and two small glaciated fragments ($\frac{1}{2}$ inch in diameter) of basic rock, much epidotised basalt, which probably represent Tertiary lavas.

Adhering to one side of the largest stone is sandy mud—not an organic ooze—in which it appears to have been partially embedded. On the exposed side it bears sessile foraminifera, both arenaceous and hyaline, and some dried-up horny-looking patches, liked dried tunicates. The stones are all remarkably fresh, and do not seem to have lain long in their present position.

As this Station is in the direct route of the icebergs and pack-ice brought down by the cold current to this latitude almost yearly, the source of the striated stones is probably to be looked for in the Arctic regions, whence such an assemblage could easily emanate. Fossiliferous Tertiary strata associated with basic volcanic rocks resting directly upon an old gneissose floor are found in West Greenland. No organic ooze appears to be forming at this locality, and the sandy mud in which the stones are embedded is in all probability supplied by the floating ice.

Station 101. 6th August 1910.

Lat. 57° 41′ N., long. 11° 48′ W.; depth 1853 m. (1013 fms.).
Hard clay bottom.

Only one specimen of a reddish fine-grained flaggy sandstone measuring $4 \times 3 \times \frac{3}{4}$ inches comes from this Station. It is an ellipsoidal disc, glaciated round its edges, and is evidently only a portion of a large egg-shaped striated boulder which has split along its original bedding plane.

Rocks exactly similar in lithological characters to this specimen occur in the Old Red Sandstone formations as developed in Shetland, Orkney, and Caithness. Several glaciated boulders of identical character have been brought from stations in the dredgings made along the Wyville Thomson ridge which connects the north of Scotland with the Faroe Banks. These stations are intermediate between Shetland, Orkney, and Caithness and Station 101. Reference will be made to this and other points based on the study of the finer deposits from Station 100 when the distribution of the material from the dredgings of H.M.S. *Knight Errant* and *Triton* are treated of.

Rock Specimens from North Rona collected during the Cruise of H.M.S. "Knight Errant."

Two groups of rocks are represented in the twelve specimens from North Rona, viz. a series of banded hornblende-biotite gneisses with associated granite and pegmatite, all of the Cape Wrath type of the Lewisian Gneiss, evidently representing the "Country Rock" of the island,

and five portions of separate rounded pebbles of coarse pebbly arkose of the Cape Wrath type of the Torridon sandstone. Specimens Nos. 3 and 4 * are granular flaggy biotite-gneiss with some hornblende, representing the acid portion of the fundamental complex. No. 1 is a dark grey granular medium-grained hornblende-gneiss with very dark hornblende, more abundant than the interspersed felspar and quartz, and is from one of the more basic bands. No. 6 consists of four fragments of a coarse-grained dark rock made up of dark hornblende and a very black biotite (haughtonite), evidently from one of the characteristic "basic knots"; some of the pieces have giant quartz and felspar on their edges, evidently portions of pegmatite veins, both of which are common at Cape Wrath. No. 5 is a granular or granitoid biotite-gneiss or granite, of which No. 7 is a coarser-grained variety with some hornblende. These probably represent granites injected into the original fundamental complex but which are now incorporated along with it to make up the Cape Wrath type of the Lewisian gneiss. Specimens numbered (2) consist of four portions of blocks of reddish-brown felspathic sandstone or arkose (Torridonian), from 2 to 4 inches in greatest diameter. Two of the pieces are ice-moulded or glaciated; the others are portions of rolled blocks. No. 8 is a rounded pebble of coarse grit or fine conglomerate containing pebbles, many of which are of the red spherulitic felsite, a common feature of the Torridon conglomerates of the Cape Wrath region. These Torridonian fragments are evidently not *in situ*. They need not have come from the mainland, but may have been derived from an under-sea extension of the Torridon rocks of Cape Wrath.

H.M.S. "Knight Errant."

Station 2. 28th July 1880.

Lat. 60° 29′ N., long. 8° 19′ E.; depth 375 fms.

The material from this station on the Wyville Thomson ridge consists entirely of stones, about forty in number, ranging from $\frac{1}{2}$ inch to 12 inches in longest diameter.

Most of the stones are glaciated, some of them distinctly striated, some subangular with angles rubbed off (ice-moulded), while a few are rounded. Only some of the larger stones show traces of having been partially embedded in a fawn-coloured sandy boulder clay. The greater number are more or less covered all over with polyzoa, serpulæ, and sponges, as if they had lain on a hard bottom or loosely piled upon each other. Their

* The numbers refer to the *Knight Errant* collection only.

appearance suggests that they had all at one time been embedded in the clay, but had been washed out of it by a current which was not powerful enough to roll and round them, but only to scrub them with the smaller particles washed out of the clay.

That the clay had great tenacity is shown by the position of the largest

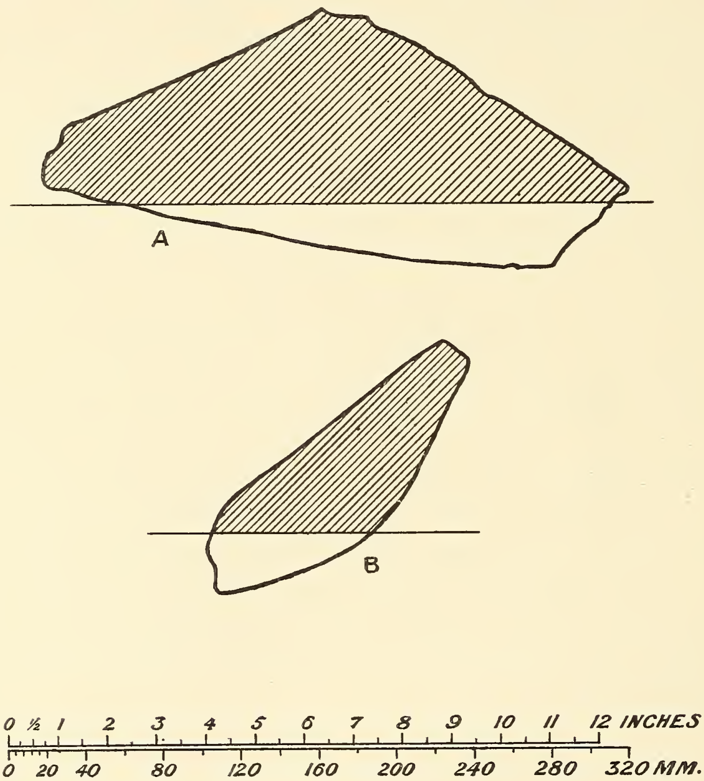


Fig. 7.—Striated Boulder of Red Sandstone. The embedded portion plain, the exposed part hatched. A, side on ; B, end on.

boulder, which though only slightly embedded and almost on edge, had not toppled over.

Fourteen of the stones are from sedimentary rocks. Of these, six almost certainly belong to the Old Red Sandstone formation as developed in Sutherland, Caithness, Orkney, and Shetland. The largest of these ($12 \times 7\frac{1}{2} \times 2\frac{3}{4}$ inches) and a smaller one are of a dark, reddish-brown fine-grained hard micaceous sandstone, like some members of the Brenista flags of Southern Shetland (text fig. 7). Two small well-glaciated fragments of grey flagstone are identical with the well-known "Caithness flags" of Caithness and Orkney. The remaining specimens are of red micaceous shale or flag and a small piece of red grit, which could be matched with rocks from any of the great subdivisions of the Old Red Sandstone from the north of Scotland. Outliers of the Old Red Sandstone which could supply such fragments occur as far west along the north coast of Sutherland as Tongue and Eilean nan Ron, and may extend under the sea much further to the northwards.

A considerable number of small specimens are undoubtedly of Secondary age. Of these, two angular fragments of a dark calcareous sandstone with grains of magnetite, one with the cast of a belemnite, a piece of dark oolitic limestone, and a fragment of calcareous sandstone like some of the Brora rocks, appear to belong to members of the Jurassic series.

Two small specimens of ochreous weathered limestone, full of casts of foraminifera, probably represent some Cretaceous limestone.

The remaining sedimentary pebbles are of uncertain age. One is of a well-rounded hard green grit, probably a pebble out of a Torridonian or Old Red Sandstone conglomerate, the other a small, well-glaciated fragment of hard greenish grit or conglomerate with quartz veins (probably Torridonian), and a rounded pebble of dark crystalline limestone, which may be a metamorphic marble.

The metamorphic rocks are represented by twelve fragments of granulitic gneiss or schist of Moine type, the two largest of which are angular and the rest well glaciated. Some of them are quartz granulites with white mica, in a comparatively low stage of metamorphism, in which some of the original grains of quartz are only peripherally granulitised, indicating that they come from near the western limit of these schists and close to the Moine thrust-plane which traverses North Sutherland in a north-north-east direction and passes out to sea a little to the east of Loch Erriboll Others, completely granulitised and with biotite, are in a higher stage of metamorphism, like the rocks further to the east. Among the metamorphic rocks is a small glaciated fragment of augen-gneiss, a foliated igneous rock with eyes of felspar. It resembles a foliated "Cansip Porphyry," and probably comes from the zone of "mylonised" rocks in immediate proximity to the Moine thrust-plane.

The igneous rocks are represented by nine specimens, all of which are glaciated. Of these, one is of diorite, one of quartz felsite with porphyritic felspar and quartz blebs, set in a fine-grained green ground-mass. The remaining seven specimens are of dolerite. Such dolerites are widely distributed in central Scotland, but are rare in Sutherland, Caithness, Orkney, and Shetland.

H.M.S. "Knight Errant."

Station 3. 3rd–4th August 1880.

Lat. 59° 12′ N., long. 5° 57′ W.; depth 53 fms.

The material from this Station consists of about 25 lb. weight of sand, with pebbles from a little over the size of sand-grains up to nearly 3 inches in longest diameter, and a lot of decayed shells. The largest stones appear as if they had been glaciated, but had been subsequently scrubbed and polished by the passage of sand, the joint faces being still apparent. The smaller stones appear to have been rolled, but only those under ½ inch in diameter. Organisms such as polyzoa and serpulæ are still attached to them, sometimes on all their sides, but more frequently one side is bare of them, showing that they had lain on, or had been partially embedded in, the sand. Numerous shells of sublittoral or shallow-water habitat, such as *Pecten, Astarte, Mactra, Pectunculus, Aporrhais, Fusus,* and *Trochus* occur plentifully with the sand and stones. All were dead when dredged. Only a few of the valves of the lamellibranchs are whole, and none together. Most of them are thoroughly riddled with boring sponges and much decayed by solution, suggesting that they are not in their natural habitat. Only the calcareous tubes of *Ditrupa,* a tubicolar worm, are fresh, as if the animal had been living when caught by the dredge.

A very large proportion of the stones from this locality are of sedimentary origin. The Torridonian rocks are represented chiefly by dark chocolate-coloured arkoses with pebbles stained red. Some are coarse-grained and epidotic, but most of them are fine-grained. There are 43 fragments above 1 inch in diameter, the largest only 1½ inches, while there are no less than 142 fragments below 1 inch. None of the fragments is well rounded, but all are polished as if by attrition of sand. To this formation may also belong ten small pebbles of red-stained gneiss, probably derived from a red conglomerate. There are 22 small specimens of quartzite, presumably of Cambrian age, one or two of which are somewhat rounded, the others are like glaciated stones that have had their markings obliterated by sand. The Cambrian quartzite passes out to sea at the mouth of Loch Erriboll on the north coast of Sutherland.

The Old Red Sandstone is mostly represented by over seventy fragments of the characteristic and unmistakable "Caithness flagstones," some of which are calcareous. The largest is below 2 inches in diameter, but the majority are under ½ inch. Were specimens below ¼ inch to be counted, hundreds of fragments of these flagstones would have to be recorded.

Among the larger specimens eight are of a reddish and yellowish sandstone, like the Upper Old Red Sandstone of Dunnet Head and Hoy.

A specimen of limestone with crinoids probably represents the Carboniferous Limestone, while two specimens of clay ironstone, one filled with the shells of lamellibranchs, being a regular "mussel band ironstone," are probably of Carboniferous age. The furthest north occurrences of such rocks *in situ* on the east coast of Scotland is in Fife, where they pass out to sea at the "East Neuk."

The Jurassic formation is probably represented by five fragments of fossiliferous rock. One of these is a fine-grained dark grey limestone with lamellibranchs and other fossils, another dark limestone has casts of corals like *Montlivaultia*, one contains a reticulate sponge, while another is oolitic, like a specimen from Station 2.

To the Cretaceous formation belong two small pieces of chalk, seventeen chips of chalk flint and several of hard grey sandy rock crowded with fossils, one showing the section of a cidarid spine, and, more doubtfully, one specimen of cherty rock with worm-tubes, and another fragment of flinty rock with sponge spicules.

Jurassic and Cretaceous rock capable of supplying the assemblage of stones from this haul and that from Station 2 occur either *in situ* or as large ice-transported masses on both shores of the Moray Firth, and are known to extend out to sea to the east.

Metamorphic rocks are represented by eighty-seven small pieces of hornblende-gneiss, hornblende-biotite-gneiss and biotite-gneiss of Cape Wrath type of the Lewisian Gneiss. The Moine schists are represented by about fifty specimens, chiefly quartz-granulites with muscovite, like rocks near the Moine thrust-plane. One is a "frilled schist" like that occurring with the "Mylonites." Some, however, are in a higher state of crystallisation, and represent the biotite-granulites and muscovite-biotite-gneiss that occur further to the east.

Igneous rocks bear a very small proportion to the others. Only two are plutonic, one of granite, and one of gabbro. Dyke rocks are represented by one specimen of porphyrite. There are fourteen fragments of dolerite, some of which bear amygdules filled with zeolites and are probably basic lavas, while others are more probably intrusive.

Of doubtful age and origin are eleven specimens of vein quartz and one of reddish marble.

H.M.S. "Triton."

Station 5. 10th August 1882.

Lat. 60° 20′ 15″ N., long. 8° 8′ W.; depth 433 to 285 fms.

The materials from this Station resemble those from Station 2 of the *Knight Errant* in their condition and their mode of arrangement. Most of the stones appear to have been glaciated and subsequently scoured by sand but not rolled, while the attached organisms show that they were not embedded but rested on small points only, and usually on their more flattened sides. They are mostly small, only one attaining the length of 4 inches; the others are all under 2 inches.

Torridonian rocks are represented by only one small specimen of grit or arkose.

To the Old Red Sandstone probably belong one subangular fragment of yellow sandstone like the upper Old Red Sandstone of Hoy or Dunnet Head, four fragments of grey sandstone with carbonaceous streaks, all resembling some of the Lerwick sandstones of Shetland.

Metamorphic rocks are represented by one specimen of fine-grained granulite and one of muscovite-biotite gneiss like the Moine rocks. One small pebble of phyllite or fine-grained mica-schist is like the rocks of the Cliff Hills in the south of Shetland.

Igneous rocks are represented by one rolled pebble of fine-grained diorite and one of pink red-stained granite, which has been glaciated. These have in all probability been derived from a conglomerate of Old Red Sandstone age.

H.M.S. "Triton."

Station 13. 31st August 1882.

Lat. 59° 51′ 20″ N., long. 8° 18′ W.; depth 570 fms.

The material from this locality resembles that from Station 5, but is more abundant.

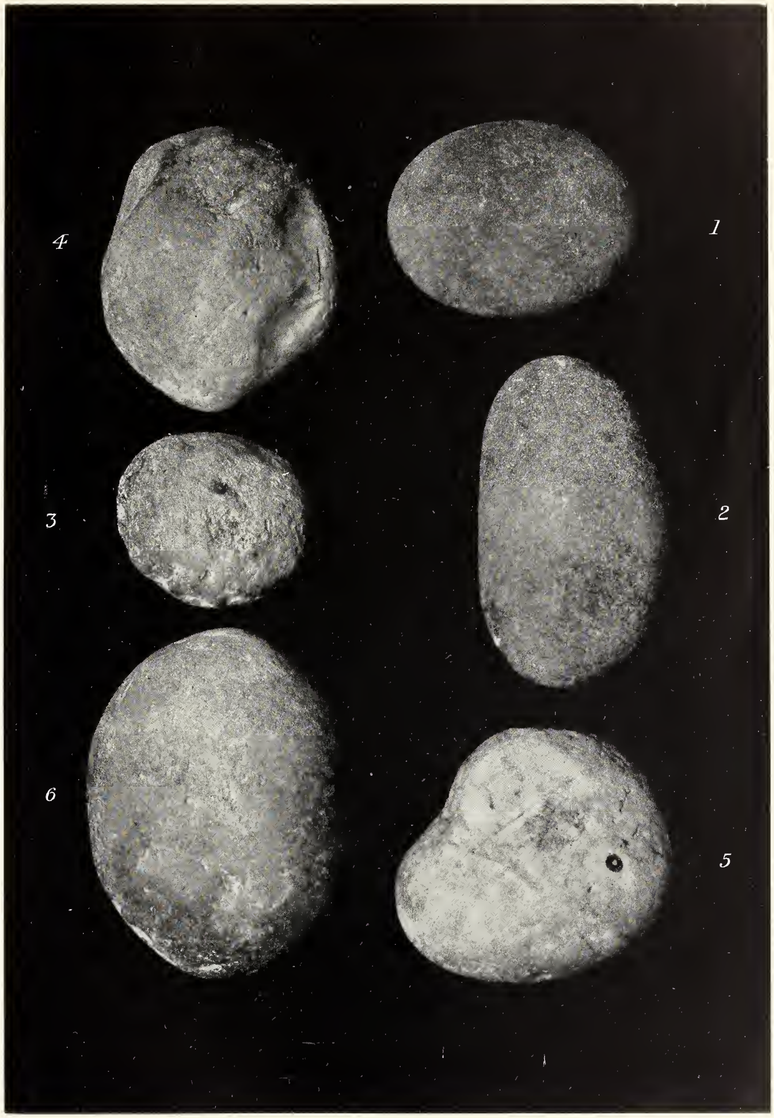
The Torridonian rocks contribute fourteen specimens, twelve being of the characteristic arkose and two detached pebbles of reddened gneiss.

The Cambrian quartzite supplies eight small subangular fragments, the largest only ¾ inch in longest diameter.

The Old Red Sandstone is represented by sixteen small flattish rolled pebbles of "Caithness flagstone" type and fifteen of hard grey sandstone, like some of the harder beds intercalated with the flags in Caithness and Orkney, and one of friable red sandstone.

Fig. 1.—Glaciated Boulder of dark Silurian Shale. (Nat. size.)

Fig. 2.—Glaciated Boulder of fine-grained Silurian Greywacke. (Nat. size.)



Rolled Pebbles a little less than natural size.

FIGS. 1, 2.—Silurian Greywacke. FIG. 3.—Encrinital Limestone (Carboniferous).
FIG. 4.—Chalk Flint. FIG. 5.—Vein Quartz. FIG. 6.—Dolomite.

Portions of Boulder of Carboniferous Sandstone with *Schizodus* and *Edmondia*. (About nat. size).

Glaciated and Faceted Boulder of Hard Chalk. Shows bulb of percussion.

Fig. 1.—Angular fragment of puckered Green Phyllite, showing evidence of having been compressed in two different directions. (About nat. size.)

Fig. 2.—Rolled Pebble of Chalk-Flint, showing "Chatter-marks." Part of Pl. II. fig. 4 enlarged.

Specimen 2, Text fig. 5, nat. size.—Cherty Encrinital Limestone. The dark "manganese" stain, indicated by arrow, marks the line between the embedded and exposed surfaces.

Fig. 1.—Portion of "Cannon-bone" of Ox.

Fig. 2.—Whale's "Earbone."

Fig. 1.—Fragments of Oceanic Pumice rounded by attrition with projecting crystals and harder parts. (About nat. size.)

Fig. 2.—Specimen 1 of fig. 1, enlarged to show crystal of Sanidine.

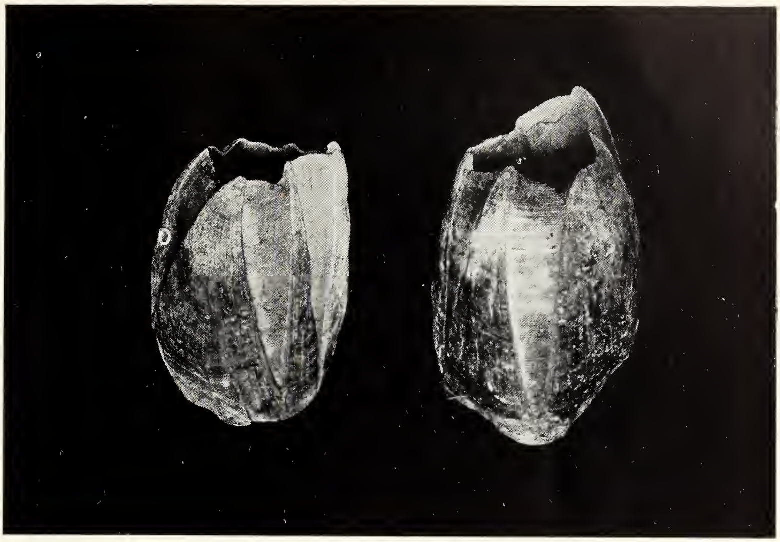


Fig. 1.—*Balanus Porcatus*, with Serpula and bottom Foraminifera attached.

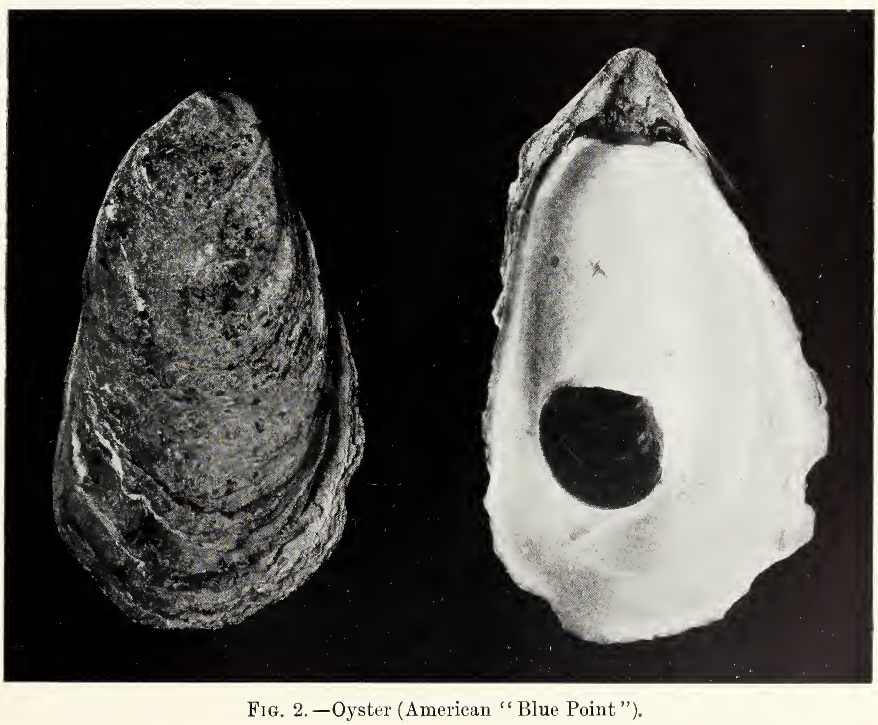
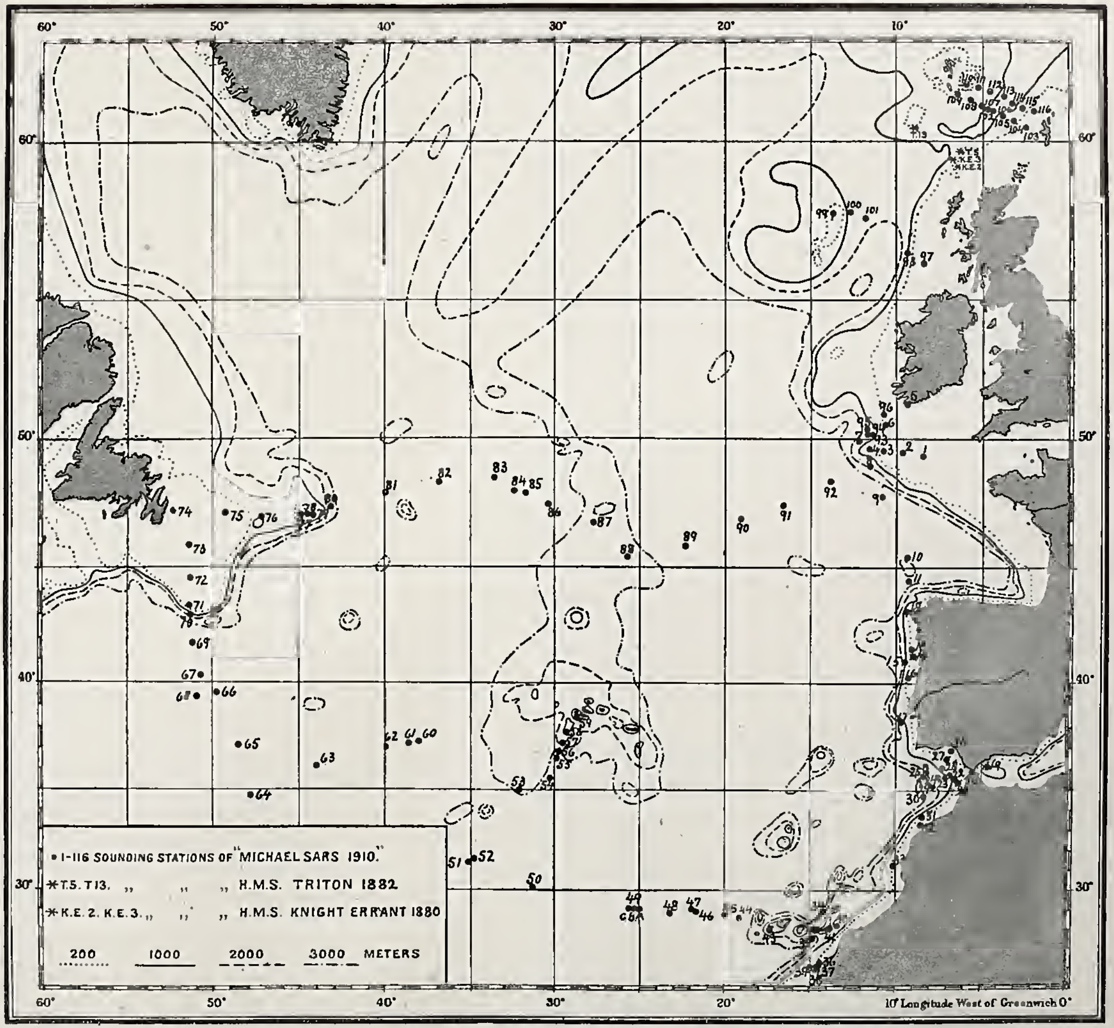


Fig. 2.—Oyster (American "Blue Point").

ORTH ATLANTIC DEEP SEA EXPEDITION 1910.

THE "MICHAEL SARS" NORTH ATLANTIC DEEP SEA EXPEDITION 1910.



DR B. N. PEACH.

Jurassic rocks are probably represented by eight small fragments of fossiliferous sandy limestone, like those from Station 3 of the *Knight Errant*.

The Cretaceous formation yields one small specimen of chalk and a piece of calcareous rock with casts of sponge spicules.

Of doubtful age and origin are two specimens of white marble.

To the Lewisian gneiss may belong eight specimens of granular hornblende and biotite-gneiss of Cape Wrath type, and one of pegmatite with felspars converted into agalmatolite in the manner that happens on the mainland where the Cape Wrath assemblage of gneisses is overlaid by Cambrian quartzites. A specimen of chloritic schist with bladed actinolite suggests a metamorphosed calcareous sediment. Rocks of this latter type occur with the Moine schist and Dalradian rocks of the mainland and of the Shetland Isles.

Igneous rocks are represented by two small fragments of granite, one with biotite and one with hornblende. No less than fifteen specimens of dolerite and basalt occur; one of olivine dolerite measures 4 inches in diameter.

The mode of occurrence of the stones from the dredgings of H.M.S. *Knight Errant* and *Triton* along the Wyville Thomson ridge and the Faroe Bank suggests that they have been embedded in a kind of boulder clay, and have been exposed on the sea-bottom by strong currents that have passed over the ridge. Not only is there the warm-water drift passing eastwards, driven forward by the prevalent westerly winds, but strong tidal currents press backwards and forwards daily over a great part of the ridge.

The stones are or have been for the most part glaciated, and many of them still retain their striations. The portions of the ridge examined appear to be of the nature of a *moraine profonde*.* This deposit was probably produced by the great ice-sheet which, emanating from the mainland of Scotland, combined with the Scandinavian ice and, filling the North Sea, passed westwards over Caithness, Orkney, and Shetland to beyond the Wyville Thomson ridge, receiving further accessions of ice from North Sutherland by the way, before reaching water sufficiently deep to break it up into icebergs.† The assemblage of stones in the dredgings strongly favours this view. The Lewisian gneisses, Torridonian arkoses, Cambrian quartzites, and "Moine schists" from the rocks of the

* *Proc. Roy. Soc. Edin.*, vol. xi., 1882, p. 649.

† *Brit. Assoc. Report for* 1885, Trans. of Sections: "Further Evidence of the Extension of the Ice in the North Sea during the Glacial Period," by B. N. Peach and J. Horne.

mainland of Sutherland or from their extension northwards, now either under sea, or in the Northern Isles, are well represented. In addition are found the peculiar locally developed types of the Old Red Sandstone rocks of Caithness, Orkney, and Shetland, which form a considerable percentage of the dredged stones, while Secondary rocks of types unknown in the west of Scotland but which are to be found *in situ* on the east coast, and are known to occur below sea and also in South Sweden and Denmark, along the path of the great ice-sheet, occur plentifully at all the stations. The few Carboniferous rocks and the dolerites may have come along with the ice from the Carboniferous areas of Central Scotland, or the extension of those rocks eastwards under the North Sea.

A like assemblage of fossiliferous Secondary and Carboniferous boulders was met with in the shelly boulder clays which are found in Caithness and Orkney,* but rocks of these types are apparently absent from Shetland.†

The evidence obtained by the *Michael Sars* from Stations 100 and 101 is also in keeping with this view and affords it additional support. The specimen of sandstone from Station 101 is like the Brenista flags of Shetland,‡ while the examination of the blue mud from Station 100 in the "Challenger" office shows that it contains 30 per cent. of minerals, the quartz grains being "tinged red" or with "green chloritic tinge." The red quartz grains are in all probability derived from Torridonian or Old Red Sandstone, and the green from the Lewisian gneiss where it has undergone decomposition in pre-Torridonian time, or from the "Epidotic Grits" so common in the basement members of the Torridonian system. "Tourmaline, biotite, and orthoclase," the other minerals found in the mud at these Stations, also point to some source such as the Lewisian gneiss. Fragments of black and red slaggy lava and volcanic glass also occur, suggesting that they had come from Iceland or Faroe. The commingling of these materials suggests the work of floating ice beyond the limit of the great ice-sheet.

The occurrence of such a number of dead and decaying shells of shallow-water habitat at Station 3 (*Knight Errant*) in fifty-three fathoms affords evidence of a widespread subsidence of the land and of the sea-bottom. The absence of the higher raised beaches in Caithness, Orkney, and Shetland, together with the occurrence of submerged peat-mosses within the same area, and also in the north of the island of Lewis, all point to such

* B. N. Peach and J. Horne, "The Glaciation of the Orkney Islands," *Quart. Journ. Geol. Soc.*, vol. xxxvi. p. 648, 1880. "The Glaciation of Caithness," *Proc. Roy. Phys. Soc. Edin.*, vol. vii. p. 307, 1881.

† B. N. Peach and J. Horne, "The Glaciation of the Shetland Isles," *Quart. Journ. Geol. Soc.*, vol. xxxv. p. 317, 1879.

‡ See *ante*.

sinking. The Faroes supply additional evidence of subsidence in the absence of raised beaches, and in the great depth of the sounds with precipitous sides intervening between the islands. The great depths to which the seaward extension of the Iceland fjords cut the surrounding submerged area corresponding to the "continental shelf," and the sunken shell banks between Iceland and Jan Mayen, which according to Dr Nansen indicate that the region there has gone down no less than 2600 metres,* all point to a subsidence in post-glacial time which begins at the north of Scotland and increases towards the north-west. If, then, during the maximum glaciation, the combined Scandinavian and Scottish ice-sheet made its way out over the Wyville Thomson ridge, and if the continuation of that ridge which connects Faroe and Iceland with Greenland stood much higher than at present, the confluent ice-sheets emanating from Europe on the one side and from Greenland and Iceland on the other may have met and formed a barrier preventing the warm water entering into the polar basin by that route. The whole of the Gulf Stream would then have had to make its way southwards. The distribution of the material dredged from the Atlantic by the *Michael Sars* favours such a supposition.

* Brogger, "Norges Geologiske Undersögelse," No. 31 (1901), pp. 94–96. Brogger's summary in English, p. 683.

(*Issued separately August* 2,1912.)

XXI.—Transverse Induction Changes in Demagnetised and Partially Demagnetised Iron in relation to the Molecular Theory of Magnetism. By James Russell.

(Read March 4, 1912. MS. received March 26, 1912.)

In a former communication* the æolotropy of iron, demagnetised by the method of decreasing reversals, was shown. During the early stages of induction the permeability is greater to the re-application of a magnetising force in the same direction—whether positive or negative—as that used in the immediately preceding demagnetising process, than to a transverse force. The demagnetisation is complete in the sense that there is no external polarity, and also in the sense that perfect symmetry exists to the subsequent application of the same positive or negative directional field. This symmetry being uni-directional, æolotropy immediately appears if the subsequent magnetising force contains a transverse component.

The hypothesis was advanced that on the completion of the demagnetising process by decreasing reversals of a directional force *ab*, a preponderance of the molecules might set equatorially in reference to this force. If the subsequent magnetising field makes an angle other than zero with *ab*, the number of molecules lying in the most advantageous position—*i.e.* at right angles to the new field—is reduced and the deflecting moment becomes less and less as the angle approximates to 90°. A general explanation is thus afforded, in terms of the theory of rotatable molecular magnets, why iron in the early stage of induction is not isotropic to a magnetising field having a component at right angles to that used in the immediately preceding demagnetising process.

It follows as a deduction, either from the above hypothesis or from the fact of the æolotropy of demagnetised iron,† that, when the angle between the demagnetising force and that subsequently applied is other than 0° and 90° the application of the latter ought

First.—To develop a transverse induction component which will tend to disappear as saturation values are reached; and which ought

Second.—To change sign if either the direction of the subsequently

* "Magnetic Shielding in Hollow Iron Cylinders," *Trans. Roy. Soc. Edin.*, vol. xl. pp. 649–654 (on "Magnetic Æolotropy").

† "The Molecular Condition of Iron demagnetised by Various Methods," *Proc. Roy. Soc. Edin.*, vol. xxiv. p. 544.

applied field be reversed or the direction of the demagnetising force be rotated so that its tangent changes sign. If both these changes be made simultaneously, the sign of this theoretical effect ought to remain unaltered.

The experimental results confirmed the above deductions, notwithstanding the fact that the direction of the demagnetising force did not remain constant during the demagnetisation process. This variation, the result of the demagnetising effect of the ends of the hollow cylinder used, was so considerable that general conclusions only were published. The complete paper, as then written, was held over, and the present communication now takes its place. Differences between the two will be made apparent as this paper proceeds.

Objects of Investigation.

I. To investigate the induction changes which occur at right angles to the magnetising field when the iron has been previously

(*a*) demagnetised by decreasing reversals of a force having a transverse component;

(*b*) left residually magnetised by a method of partial demagnetisation by decreasing reversals of a force having a transverse component.

II. To co-ordinate the experimental results with the theory of rotatable molecular magnets based upon as simple assumptions as possible.

Experiments I*a* repeat those referred to in the paper mentioned above, the direction of the demagnetisation remaining more constant owing to both the magnetic circuits being completed in iron. The substitution of partial demagnetisation, I*b*, for residual magnetisation produced in the usual way will be discussed under the heading "Æolotropy" at a later stage.

Molecular Theory.

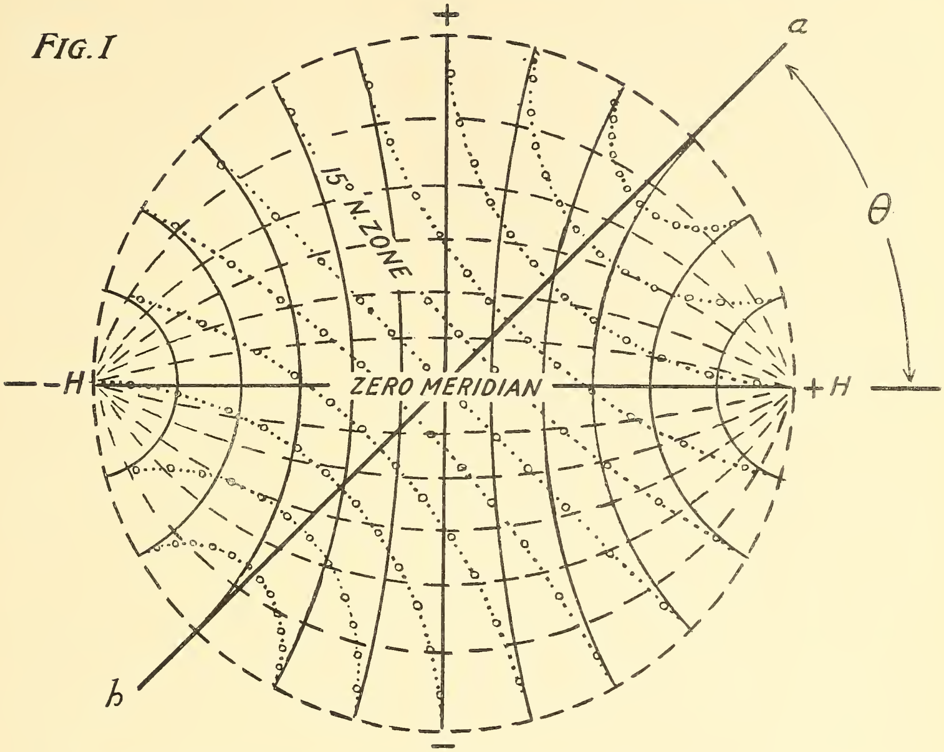
Weber's theory of magnetic induction assumes that the molecules of iron, or other magnetic materials which have never been magnetised, are magnets pointing at random and equally stable, on the whole, in all directions. To express this, Maxwell supposes a sphere to be described and a radius drawn from the centre parallel to the direction of each of the molecules in the mass. Varying this illustration, the magnetic axes of the molecules (or the resultant axes of stable molecular groups) may be regarded as diameters of a sphere with north and south polarity. It is supposed that any unit of area on the surface of this sphere contains an equal number of N. and S. poles, that the extremities of the molecules (diameters) are equally spaced on this surface and equally stable in all directions.

After iron has once been magnetised and demagnetised by decreasing reversals all of these assumptions no longer hold, however perfectly the process may have been carried out. The process of demagnetisation, however, is such that for reasons of symmetry any unit area on the surface of the hypothetical sphere contains an equal number of N. and S. poles. Further, the distribution and stability of the molecules must be symmetrical in reference to the directional demagnetising force, *i.e.* zonal. For different parallels of latitude either the distribution or the stability or both must vary.

Consider (fig. 1), the stereographic projection of a hemisphere on the plane of the paper containing the demagnetising force *ab* and the subsequent magnetising field H H. The dotted curves are parallels of latitude in reference to *ab*, which may now be supposed to have demagnetised the iron by decreasing reversals. The little circles represent the equal distribution and stability of the molecules along these parallels. The numerical values to be given to these circles for different parallels need not be considered at present. Meanwhile let each little circle represent one molecule of N. and one molecule of S. polarity of known latitude and longitude in reference to *ab*. When referred to the polar axis H H the new latitude (or the complementary polar distance) and longitude of each molecule can be calculated, or read off if Mr Blaikie's device for solving spherical triangles be used. A modification * of this apparatus facilitated the calculation involved. Let it be assumed that on the application of the positive field +H, each molecule leaves its known position and takes up a final position which, on the average, lies on the same meridian. Each N. and S. molecule experiences, after its rotation towards its final position under any given force +H, a transverse vertical change of orientation which will be zero for those molecules rotating along the zero meridian (horizontal), increasing, however, according to the sine law as the great circles through which the molecules rotate approximate to 90° on either side. If the rotation of the N. molecules towards (up) and from (down) the equator at right angles to H H, in the upper half of the hemisphere be regarded as positive and negative respectively, in the

* A circular disc of cardboard, on which was drawn the stereographic projection of a hemisphere, could rotate from beneath on the surface of a wooden board. Over this arrangement a piece of transparent paper was securely stretched. Obviously the new co-ordinates of any series of known points marked on the paper could, on the rotation of the disc through any required angle, be immediately read off. Mr Blaikie's apparatus has been exhibited at the Royal Society of Edinburgh and other societies, but no account of his ingenious device appears to have been published. Dr Dyson, Astronomer Royal, kindly showed me Mr Blaikie's device at the Royal Observatory, Edinburgh, which suggested the above modification of the more complete apparatus.

FIG. I


+
a
θ
15° N. ZONE
--H
ZERO MERIDIAN
+H
b
−

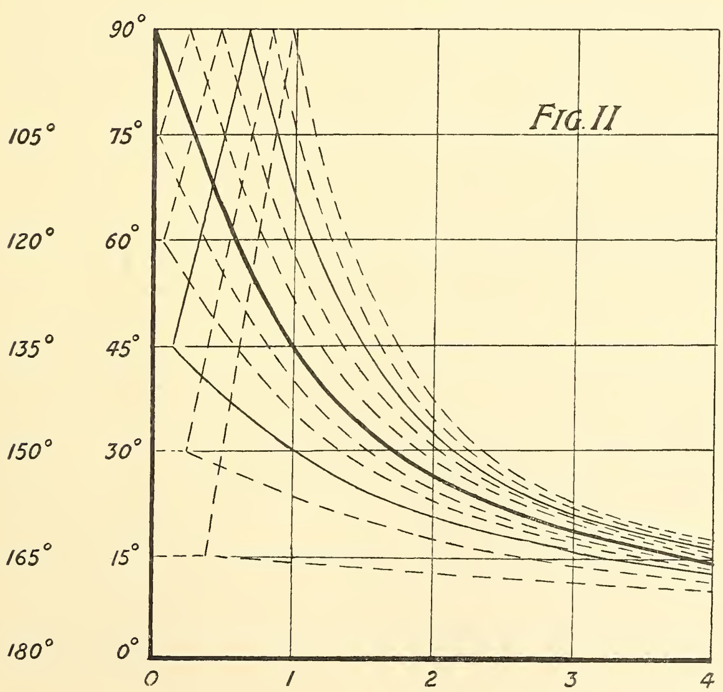


FIG. II
90°
105°
75°
120°
60°
135°
45°
150°
30°
165°
15°
180°
0°
0
1
2
3
4

lower half of the hemisphere the opposite relationship will hold. Further, on the above assumption, the rotation of the S. molecules in the upper and lower halves of the hemisphere will have the same effect as the rotation of the N. molecules in the lower and upper halves of the hemisphere respectively. A little consideration will also show that if the hemisphere—as in the present case—be bounded by the plane containing *ab* and H H, the positive and negative vertical changes of position of the N. molecules in the upper and lower halves of the hemisphere as above defined, expresses fully what is occurring in the whole sphere irrespective of the polarity of the molecules. For the future, therefore, N. molecules only need be considered.

In iron which has been demagnetised by heat, the position of the molecules in the upper and lower halves of the hemisphere is, by hypothesis, each the reflection of the other, and after rotation by the field the opposite changes of the vertical components of the molecular magnetic moments will neutralise each other. Such iron experiences no transverse induction change when magnetised.

On the other hand, in the case of iron demagnetised by reversals, unless *ab* coincides with, or is at right angles to, H H the position or stability or both of the molecules in zones will not be symmetrical in the upper and lower halves of the hemisphere, and after rotation by the field the changes of the vertical components of the molecular magnetic moments will not neutralise each other. It is this which constitutes, on the molecular theory, transverse induction changes in iron previously demagnetised by decreasing reversals.

If it now be assumed that a definite law connects the initial and final angular positions of a molecule with the field strength, the transverse vertical change due to each molecule can be calculated. The law (arbitrary) upon which the present calculations are based is, for those molecules lying equatorially in reference to the magnetising field, that the cotangent of the polar distance is equal to the strength of the field. Thus (full line curve of fig. 2), when the deflecting field is unity, the polar distance of the molecules will be 45°, finally approximating to 0° for high values of the field measured as abscissæ. For molecules lying at other angles than 90° the law is given by the various curves of fig. 2, which are so drawn that they reach the asymptote at higher values of the magnetising field the greater the initial polar distance of each molecule. These vertical ordinates have double values, and the (initially) ascending or descending curves must be taken according as the angles are greater or less than 90°.

If the transverse vertical components of induction, due to the rotations of each molecule for the same parallels of latitude in reference to the polar demagnetising force *ab* (fig. 1), be summed up for values of $+H$ increasing from zero, curves may be plotted for the various zones with which the experimental results may be compared. These curves may also be compounded with each other as may be found necessary.

An example of the numerical calculations involved is shown in the annexed table, the demagnetising angle being $\theta = 45°$, the zone 15° N.

$\theta = 45°$. Zone $= 15°$ N. parallel.

			Horizontal Ordinate of Fig. 2.													
			0		·2.		·4		·6		·8		1·0		1·25	
	Longitudes.	Sin of Longitudes.	Polar Dist.	Sin P.D. × sin long.	Polar Dist.	Sin P.D. × sin long.	Polar Dist.	Sin P.D. × sin long.	Polar Dist.	Sin P.D. × sin long.	Polar Dist.	Sin P.D. × sin long.	Polar Dist.	Sin P.D. × sin long.	Polar Dist.	Sin P.D. × sin long.
Upper half of zone.	82°	·99	119°	·87	110°	·93	94°	·99	78°	·87	67°	·91	57°	·83	47°	·72
	65	·91	117	·81	108	·87	93	·91	77	·89	66	·83	56	·76	46	·66
	51	·78	111	·73	99	·77	86	·78	73	·75	62	·69	54	·63	45	·55
	38	·62	103	·60	93	·62	79	·61	68	·58	58	·52	50	·48	42	·41
	27	·45	95	·45	85	·45	73	·43	62	·40	54	·36	47	·33	40	·29
	16	·28	86	·28	75	·27	65	·25	57	·23	49	·21	44	·19	38	·17
	5	·09	74	·09	66	·08	58	·08	51	·07	45	·06	40	·06	35	·05
	Summation			3·83	...	3·99	...	4·05	...	3·89	...	3·58	...	3·28	...	2·85
	Transverse change			0	...	·16	...	·22	...	·06	...	− ·25	...	− ·55	...	− ·98
Under half of zone.	4°	·07	297°	− ·06	301	− ·06	309	− ·05	314	− ·05	319	− ·05	324	− ·04	328	− ·04
	17	·29	307	− ·23	310	− ·22	316	− ·20	320	− ·19	324	− ·17	328	− ·15	331	− ·14
	31	·51	317	− ·35	317	− ·35	323	− ·31	326	− ·28	329	− ·26	332	− ·24	335	− ·22
	50	·77	325	− ·44	325	− ·44	328	− ·41	330	− ·29	333	− ·35	335	− ·33	337	− ·30
	76	·97	329	− ·50	329	− ·31	331	− ·47	333	− ·44	335	− ·41	337	− ·38	339	− ·35
	Summation			− 1·58	...	− 1·57	...	− 1·44	...	− 1·35	...	− 1·24	...	− 1·14	...	− 1·05
	Transverse change			0	...	·01	...	·14	...	·23	...	·34	...	·44	...	·53
Total transverse change				0	...	·17	...	·36	...	·29	...	·09	...	− ·11	...	− ·35

parallel, and the magnetising field positive. The seven double columns correspond to the abscissæ of fig. 2 from zero to 1·25 and represent values of the magnetising field. The higher values between 1·25 and 4 are omitted on account of the space that would be required. The polar distance and longitude of the equally spaced molecules on the zone selected (fig. 1) are determined when referred to the polar field $+H$. The longitudes are entered in the vertical column on the left, the polar distances in the zero column above referred to. Reference to fig. 2 determines for increasing ordinate values the polar distances of the molecules, which are entered in their respective columns ·2, ·4, ·6, etc. The sines of these angles into the sines of the longitude occupy the adjacent columns. The summation of these, each diminished by the summation of the first column (original position of the molecules), gives the change of the transverse components of the molecular moments for the various values of the horizontal ordinates (positive field) increasing from zero. The lowest horizontal column gives the summations of the upper and under halves of the zone. A zone, however, may occur in one half of the hemisphere only.

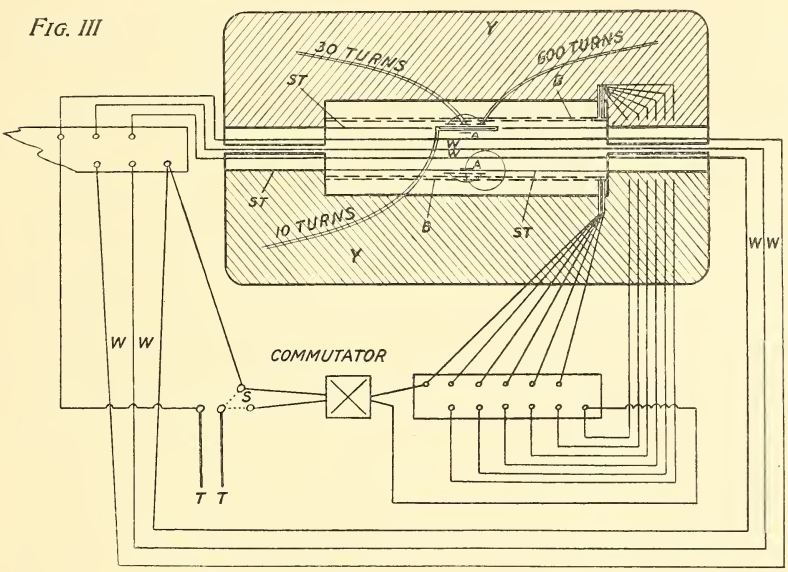
Apparatus.

The thin iron cylinders previously used, formed from sheet transformer iron, were in these experiments replaced by a length of drawn steel tube. The steel tube, inserted into a solenoid of covered copper strip, surrounded a bundle of copper wires along its axis. By means of these conductors the tube could thus be subjected to a *circular*, a *longitudinal*, or, when the central wires and solenoid are joined in series, to a *spiral* magnetising force. This last constitutes the *demagnetising force*, and may be inclined at any angle θ to that due to the central wires which constitutes the *subsequent demagnetising field* H_c. An exploring coil of many turns of covered copper wire surrounding the central portion of the steel tube measures the longitudinal, *i.e.* the transverse induction change.

The circular induction B_c is wholly completed in the steel, while the longitudinal induction completes its magnetic circuit in part through the steel and in part through a massive iron yoke. The reluctance of the latter is, owing to its large sectional area relative to that of the steel tube, negligible so long as its permeability remains high. When the longitudinal induction is great, and consequently the permeability of the yoke reduced, this induction in the tube may fall below its calculated value. A correction may thus require to be made on the calculated value of the demagnetisation angle θ due to the combined fields.

Figs. 3 and 3*a* show the arrangements in further detail and are drawn to scale. The former is a sectional elevation, the external connections being diagrammatic only. Y is a massive Hopkinson yoke of annealed Lowmoor iron, split in two. The experimental steel tube S T rests on the under half, while the upper half of the yoke secures it in position by its own weight. The hole in the double yoke was bored true and fits the steel tube accurately. Two small holes were drilled in the shell of the tube on a line parallel to its axis, each 2·6 cms. from the centre, through which

FIG. III



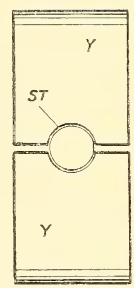


FIG. IIIa

10 turns of fine wire were threaded to form an exploring coil for measuring the circular induction. A pasteboard cylinder fits loosely over the steel tube. Two exploring coils of fine wire and two layers of copper strip forming a solenoid were wound on the former. The first consists of 30 turns for the purpose of measuring the longitudinal induction B_L due to the solenoid producing the longitudinal field H_L. This coil of comparatively few turns is only used when a current is flowing in the solenoid and measures not only the longitudinal induction in the steel tube, but the longitudinal field in the air space which it encloses. The ratio of the sectional area of this space to that enclosed by the tube is 42 : 30, and, as it is relatively large in comparison with the sectional area of the shell of the steel tube, a compensating coil A A of 42 turns was introduced within and secured to the inside of the tube. These two coils are in the same plane and, being con-

nected in series so that they mutually oppose each other, measure the longitudinal induction in the shell of the steel tube only, so long as the field in the air space may be regarded as of equal intensity within and without the steel tube. A departure from this condition may occur when, for instance, the yoke is highly saturated, but even then the error introduced will be small.

The second exploring coil of 600 turns is too sensitive to advantageously measure the longitudinal induction when produced by the longitudinal field (solenoid). It is, however, essential for the main purpose of this investigation, and it measures, after the iron has been demagnetised by the spiral field, the longitudinal induction B_L (now the transverse component), when the circular field H_c due to the central wires is increased by increments from zero. Under these circumstances no longitudinal field exists in the area enclosed by the shell of the steel tube, and no compensating coil is required.

The magnetising solenoid consists of two layers of copper strip B B of rectangular section wound in six lengths, their ends dipping into mercury cups as shown. These ends may be connected in series, three series two parallel, two series three parallel, or all in parallel. The strip was of a width sufficient to permit a close winding of 3·68 turns per cm. of lengths of both layers. In this way it was arranged that the solenoid and 36 of the central wires along its axis produced in the shell of the steel tube the same value of longitudinal (H_L) and circular (H_c) fields per ampère respectively.

The circular field is produced by the closely packed straight copper wires lying lengthwise within the steel tube and symmetrical about its axis. The ends of these wires dip into a series of mercury cups, three wires only being shown in the diagram. They can be connected up in any way, but for obvious reasons the combinations are limited to those symmetrical about the axis of the steel tube.

When 36 of the central wires and solenoid are in series, the iron can be demagnetised by a spiral field at an angle $\theta = 45°$ with that (H_c) subsequently applied, the two-way switch S having cut the solenoid out of the circuit. A sufficiently large number of other combinations can be obtained to demagnetise the steel tube at various angles. A commutator, reversing the current in the central wires relative to the solenoid, changes a right-handed into a left-handed spiral force in the shell of the steel tube.

The terminals T T are in circuit with 16 secondary cells, a Weston milliampère meter with shunts, resistances, commutators, revolving rheostat, and revolving current reverser, so that the various operations necessary may be conveniently performed. The revolving reverser and rheostat are used to demagnetise the iron.

Each exploring coil could, as required, be connected in series with a calibrating coil in the known field of a long solenoid and a Broka galvanometer arranged ballistically. When used with the coil of 600 turns without resistances 1 mm. scale division = one-third of a C.G.S. unit of induction (B_L) in the steel tube.

Further data are as follows:—

Length of experimental steel tube	33 cms.
Average thickness of shell	·0646 cm.
Average radius (to middle of shell) . . .	1·56 cms.
Sectional area of steel enclosed by—	
(1) 10 turns exploring coil measuring circular induction B_c	0·399 sq. cm.
(2) 30 turns exploring coil measuring longitudinal induction, B_L	0·633 sq. cm.
(3) 600 turns exploring coil measuring transverse (longitudinal) induction change, dB_L	0·633 sq. cm.

Value of circular field at radial distance of 1·56 cms. from axis of steel tube:—

Per wire per ampère . . .	$H_c = 0{\cdot}128$ C.G.S. units.
Per 37 wires per ampère . .	$H_c = 4{\cdot}74$,, ,,
Per 36 wires per ampère . . .	$H_c = 4{\cdot}61$,, ,,

Value of longitudinal field due to solenoid, the 6 strips being in series, $H_L = 4{\cdot}61$ C.G.S. units.

Demagnetised Iron.

(1) *Experimental.*

The magnetic quality of the drawn steel tube is indicated by the following measurements of a hysteresis loop many times repeated.

	H.	B.
At cyclic extremes	35	14,000
Residual magnetisation	0	10,600
Coercive force	6	0

The continuous line curves of fig. 4 show the transverse (longitudinal) induction changes dB_L which occur when the circular field H_c is increased by increments from zero to a maximum, the steel tube having been previously demagnetised by the spiral force making a calculated angle of $\theta = 18^{\circ}\cdot 4$ with H_c. If demagnetisation takes place at a calculated value of $\theta = 71^{\circ}\cdot 6$ (complementary angle), the broken line curves result. The orientation of the spiral demagnetising field is indicated by continuous and broken line arrows for $\theta = 18^{\circ}\cdot 4$ and $71^{\circ}\cdot 6$ respectively. If the tangents of those angles, given by the ratios between the calculated values of the longitudinal and circular fields, be positive, the spiral demagnetising force may be considered right-handed, and if negative left-handed.

If the demagnetising force be right-handed, the transverse induction change occurs in the first or third quadrants according as the subsequent field is positive or negative. On the other hand, if the demagnetising force be left-handed, the transverse change occurs in the fourth or second quadrant as the subsequent field is positive or negative.

In this respect, therefore, the results previously arrived at are confirmed. The curves, however, are more symmetrical in the four quadrants than those previously obtained without the massive yoke.

The curves plotted in fig. 4 are typical of a series of experiments for other calculated angles of the demagnetising force, giving equally symmetrical results. Fig. 5 gives the complete series, but the average of the readings obtained in the four quadrants are plotted in the first quadrant, the demagnetising spiral force being taken as right-handed and the subsequent field (H_c) positive. The highest curve, so far as observed, is that obtained when the demagnetising angle is $\theta = 45^{\circ}$. When rotated in either direction the curves take lower values, and finally vanish as θ approximates to 0° or 90°.

It will be noted that the curves obtained for pairs of the calculated complementary angles do not coincide. The angle at which demagnetisation takes place is determined by the actual ratios of the longitudinal and circular *inductions* in the shell of the steel tube. It was found experimentally that, when the longitudinal component of the combined inductions remained small, and consequently the magnetic reluctance of the iron yoke negligible, the angle at which demagnetisation actually took place coincided with that calculated from the field ratios so nearly that no correction is required when $\theta = 9^{\circ}\cdot 4$, $18^{\circ}\cdot 4$, and $26^{\circ}\cdot 5$.

On the other hand, it was found that when the longitudinal component was large, and consequently the magnetic reluctance increased at high values of the combined fields, the angle at which demagnetisation took place did not remain constant during the demagnetisation process. Fig. 6 shows the variation which occurs as the spiral alternating induction decreases from a maximum of $B = 15{,}000$, as determined by the ratios of the longitudinal

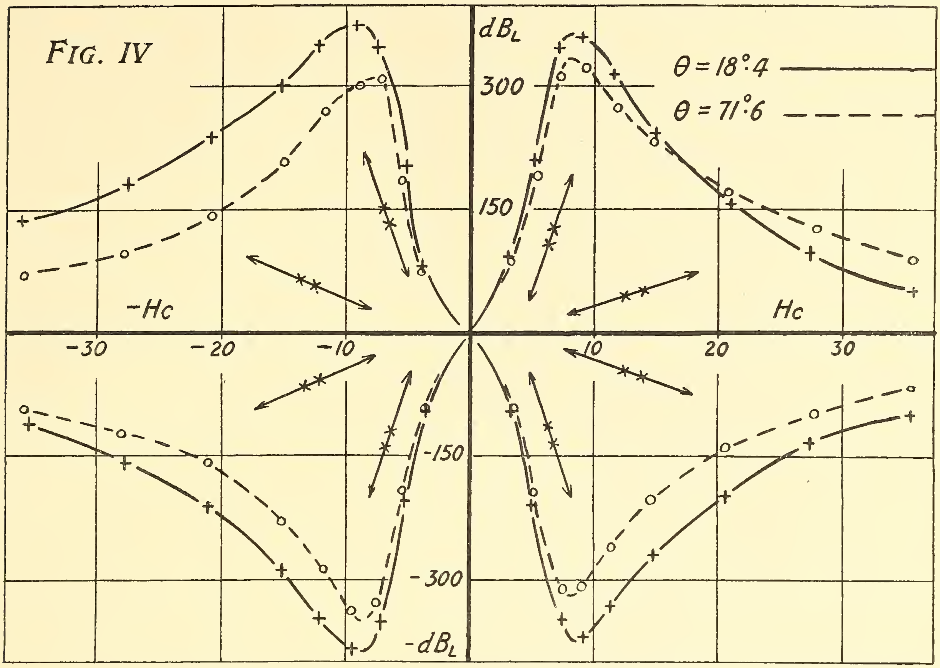


FIG. IV
dB_L
300
150
-150
-300
-dB_L
θ = 18°·4
θ = 71°·6
-Hc
Hc
-30
-20
-10
10
20
30

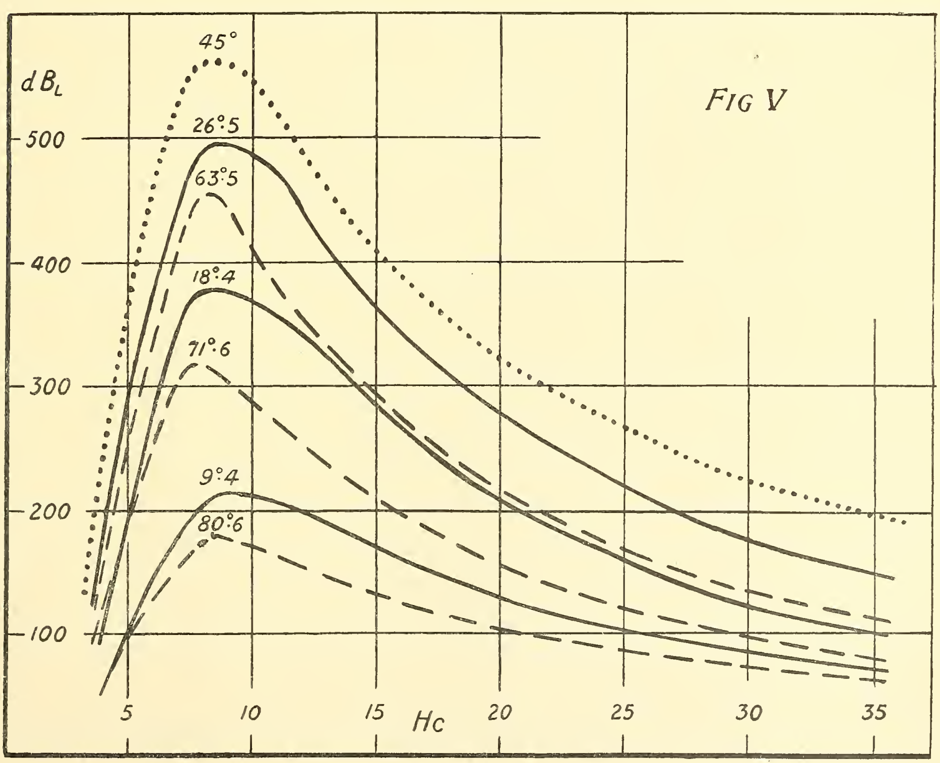
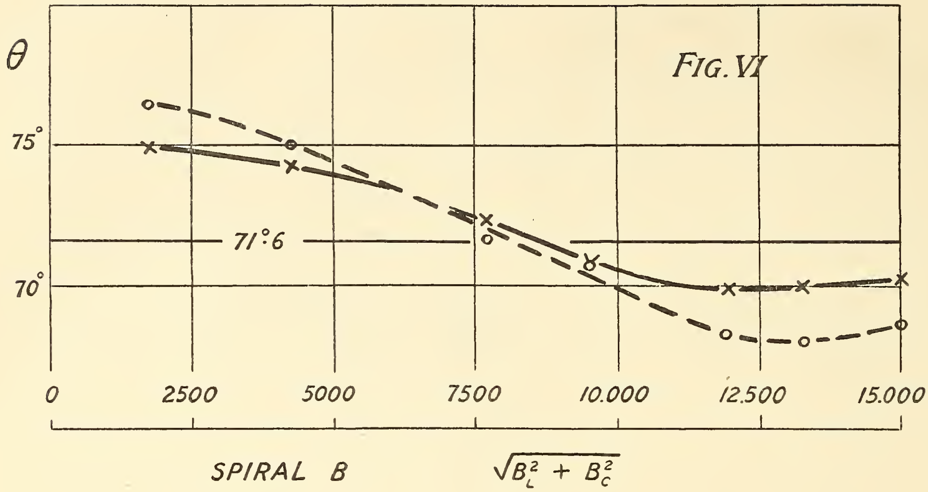
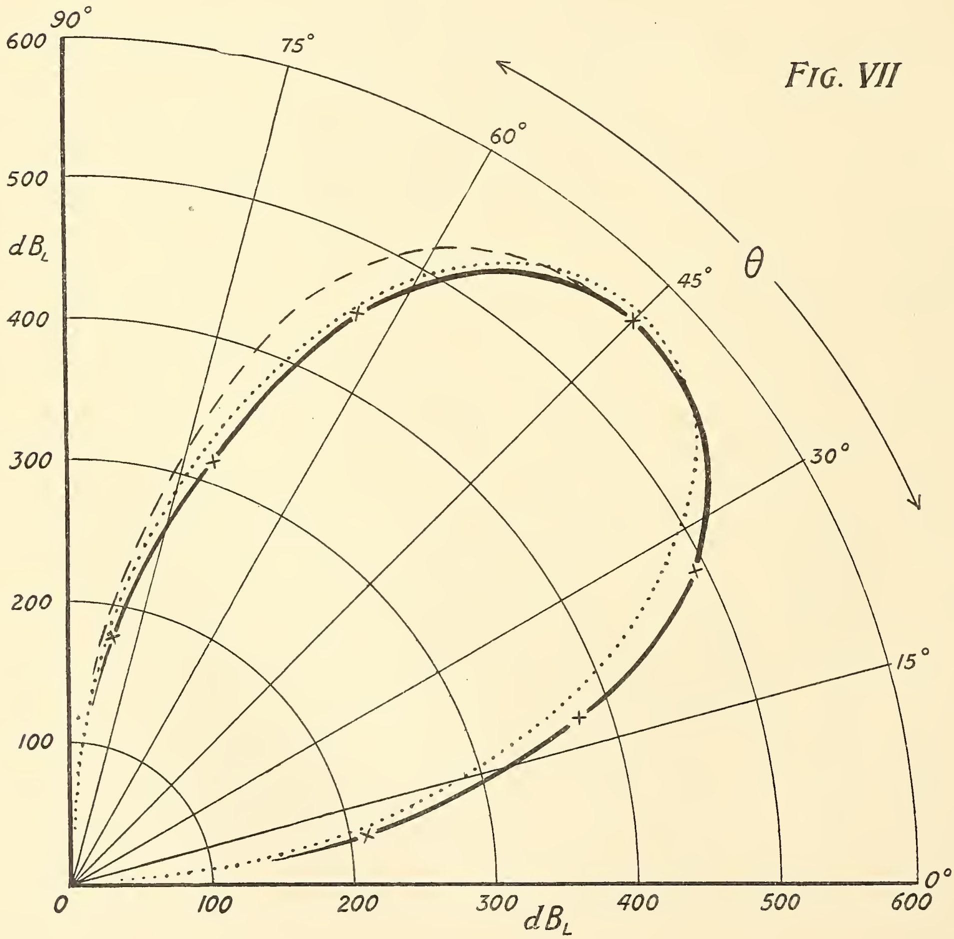


FIG V
dB_L
500
400
300
200
100
45°
26°·5
63°·5
18°·4
71°·6
9°·4
80°·6
5
10
15
Hc
20
25
30
35


θ
FIG. VI
75°
71°·6
70°
0
2500
5000
7500
10.000
12.500
15.000
SPIRAL B
$\sqrt{B_L^2 + B_C^2}$


90°
600
75°
FIG. VII
60°
500
dB_L
45°
θ
400
300
30°
200
15°
100
0
100
200
300
400
500
600
0°
dB_L

and circular induction (continuous curve), and by the ratios of the longitudinal and circular residual magnetisation (broken curve), at a sufficient number of stages. Between $B = 15{,}000$ and 8000 demagnetisation is proceeding at a less angle than the calculated value of $\theta = 71^\circ{\cdot}6$, at lower values than $B = 8000$ at a higher angle. The same result holds for $\theta = 63^\circ{\cdot}5$ and $80^\circ{\cdot}6$.

The maximum value of each curve of transverse induction change (fig. 5) may be plotted radially against the demagnetisation angle. The continuous line curve (fig. 7) shows the experimental result for the calculated angles θ, the dotted curve given by $\sin\theta\cos\theta$, symmetrical in the quadrant, being added for comparison. But the angle of demagnetisation does not remain constant during the process for the three higher angles. What each stage contributes to the demagnetisation is unknown. If it could be assumed that the decreasing reversals between $B = 15{,}000$ and 8000 have a less effect on the maxima of the various curves than reversals at lower values, the result would be more symmetrical, the dash-line curve taking the place of the continuous curve for $\theta = 63^\circ{\cdot}5$, $71^\circ{\cdot}6$, and $80^\circ{\cdot}6$.

Reverting again to fig. 5, it may be observed that the curves closely resemble differential permeability (dB/dH) curves. $B - H$ induction curves were plotted for the various angles of θ, but they are not here reproduced, their co-ordination with the transverse effects not being pursued in this communication. It may be stated, however, that for low values of H the co-directional induction B_c is first delayed, afterwards accelerated the closer the demagnetisation angle approximates to $\theta = 90^\circ$. Under the same conditions the transverse curves are more peaked for the higher than the lower angles of θ. When plotted against B_c these differences are not obvious.

(2) *Theoretical.*

The co-ordination of the above experimental results with the hypothesis advanced will now be attempted.

The various diagrams of fig. 8 show the results calculated by the methods already indicated. The ordinates represent the changes of the vertical (transverse) components of the molecular moments in terms of the summations of the products of sines, as illustrated by the table on p. 297. The abscissæ (positive field) have the same signification as in fig. 2.

Diagrams (1) to (5) show the results calculated for double parallels of latitude as marked on each diagram. They represent the contributions

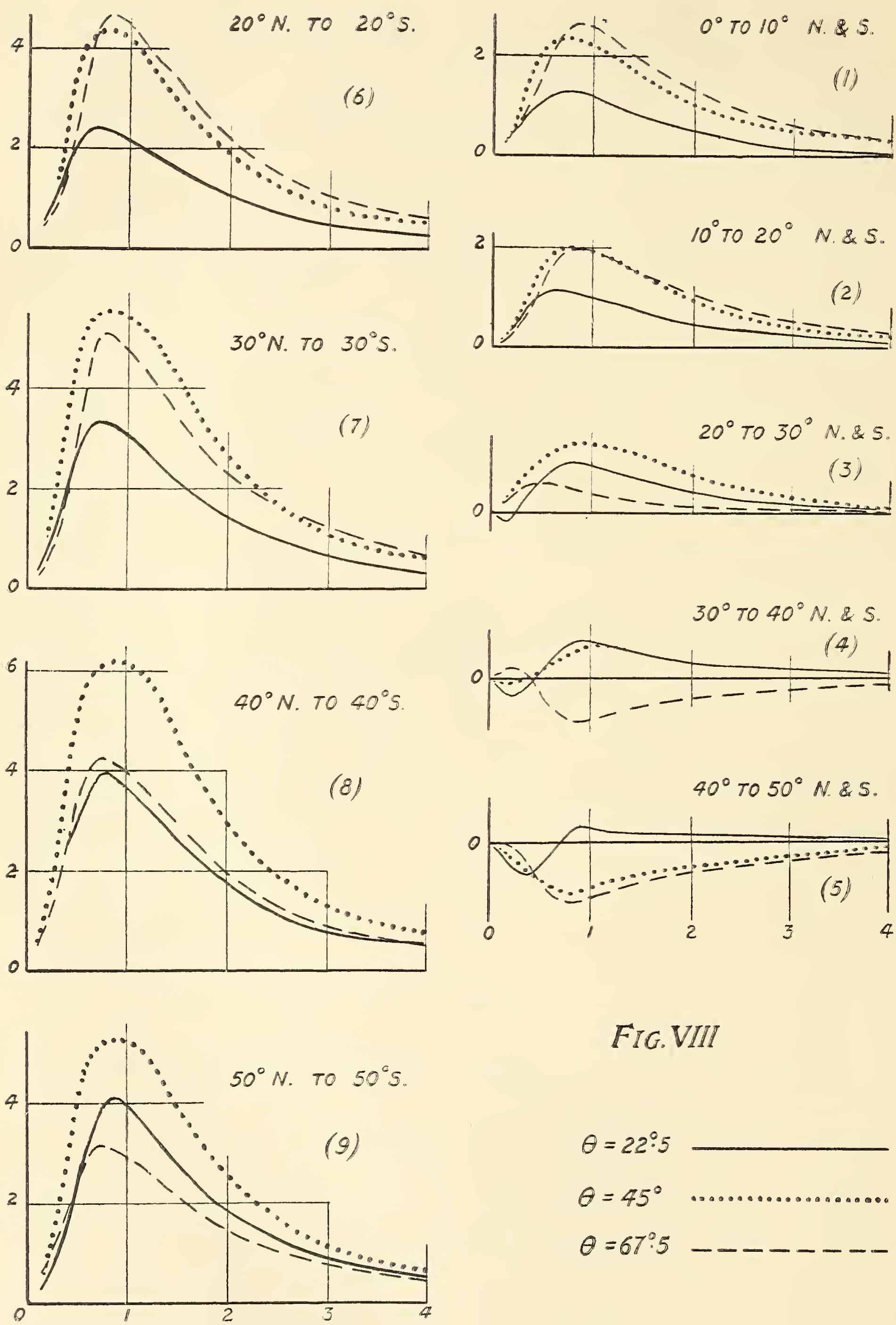

20° N. TO 20° S.
(6)
30° N. TO 30° S.
(7)
40° N. TO 40° S.
(8)
50° N. TO 50° S.
(9)
0° TO 10° N. & S.
(1)
10° TO 20° N. & S.
(2)
20° TO 30° N. & S.
(3)
30° TO 40° N. & S.
(4)
40° TO 50° N. & S.
(5)
θ = 22°·5
θ = 45°
θ = 67°·5

FIG. VIII

made by each double zone towards the total transverse effect, when the angle θ (fig. 1) $= 22^\circ{\cdot}5$, 45°, and $67^\circ{\cdot}5$—continuous, dotted, and dash-line curves respectively. As each zone has been reduced by the cosine of its angular distance from the equator, belts with *equal* molecular surface distribution are being considered.

None of the diagrams represent the experimental results. But as they are successively summed up, so that broader and broader equatorial belts are taken, the results improve. The curves for the three angles of θ change their relative positions and approximate to the experimental results in diagrams (8) and (9). The former deals with an equatorial belt between 40° north and south parallels, the latter with a belt between 50° north and south parallels. Between these positions the curves for the complementary angles $\theta = 22^\circ{\cdot}5$ and $67^\circ{\cdot}5$ will practically coincide, the maximum values of both falling considerably below the maximum of the curve for $\theta = 45^\circ$, in very much the same way as in the experimental diagrams figs. 5 and 7.

So far as the transverse effects are concerned, therefore, the experimental results may be co-ordinated with a broad equatorial belt of equally spaced molecules in reference to the demagnetising (polar) force.

But on the assumptions which have been made this conclusion does not allow a sufficiently large saturation value for the induction co-directional with the subsequent magnetising force H_c. The number of molecules in the equatorial belt between 10° north and south parallels having been taken as 12, their number in a broad belt, say between 45° north and south parallels, may be considered as approximately 98, *i.e.* proportional to area. This number will also represent the saturation value of the co-directional induction (B_c) when all the molecules are in line with H_c ($98 \times \cos 0^\circ$). From fig. 8 (8) and (9) the transverse component in terms of the summation of sines may be taken as 5·75 when $\theta = 45^\circ$. The ratio of these two theoretical effects is thus 17, while the same ratio as determined experimentally is $B_c/B_L = 27$, the saturation value of the co-directional induction being $B_c = 15{,}000$ and the transverse induction being $B_L = 560$ when $\theta = 45^\circ$.

The reason of this discrepancy is obvious. In the absence of any known law connecting the distribution of the molecules in demagnetised iron with latitude, and as a first trial only, zones between 0° and 50° north and south latitude have been considered to the exclusion of zones between higher parallels. If these latter had been taken into account, and as their transverse effect is opposite to that for zones at lower latitudes, as can be at once seen from the curves of fig. 8 (1) to (5), or even by inspection of diagram fig. 1, it is evident that there must be a closer distribution of molecules in those lower zones (equatorial) to obtain on summation a transverse effect corresponding to the experimental results. But this increase in the number of molecules considered would also increase the theoretical ratio between the co-directional and transverse effects, a nearer approximation to the experimental ratio B_c/B_L being thus obtained. The co-ordination of the transverse and co-directional inductions at all stages of the subsequent magnetising force must also be considered.

Meanwhile it may be concluded that the experimental results can be co-ordinated with a preponderance of molecules within a broad equatorial belt in reference to the demagnetising (polar) force.

Æolotropy.

In the paper on the "Molecular Condition of Iron demagnetised by Various Methods," a comparison was instituted between the changes involved in the process by which iron passes from the condition in which it is left demagnetised by decreasing reversals and by annealing, as residual magnetisation is superposed upon both these conditions. By putting on and withdrawing, say, a positive field the iron was left residually magnetised, but in a condition more susceptible to negative than positive change. This simple method introduced a more complicated form of magnetic æolotropy than that which results from demagnetisation by decreasing reversals.

In the latter case the iron is isotropic in reference to the positive or negative application of the same directional force producing the demagnetisation, but æolotropic in all other directions.

In the former case the iron is æolotropic to subsequent positive and negative applications of the same directional force producing the residual magnetisation, and it is also æolotropic in all other directions. A condition of this kind does not satisfy even approximately one of the theoretical assumptions made—that on the whole the molecules rotate along great circles of longitude, the *initial* direction of molecular rotation depending largely upon the previous history of the iron.*

In this investigation therefore a method of partial demagnetisation by decreasing reversals is adopted. The steel tube is in the first place subjected, by means of the revolving commutator, to reversals of the maximum demagnetising force. On the commutator being stopped, the maximum value of H is left positive. The field is withdrawn and reversed to a smaller negative value of, say, $-H_1$. Demagnetisation from this maximum

* "The Shift of the Neutral Points, etc.," *Proc. Roy. Soc. Edin.*, vol. xxix. pp. 30–34.

now proceeds in the usual way by reversals decreasing to zero. As the value of $-H_1$ on the first reversal is taken less, the residual magnetisation becomes greater and *vice versa*. This method supplies a wide range of stable residual magnetisation more nearly approximating to the less complex æolotropy of complete demagnetisation by reversals, and to a condition such that the theoretical assumption referred to immediately above may be more nearly fulfilled.

Partially Demagnetised Iron.

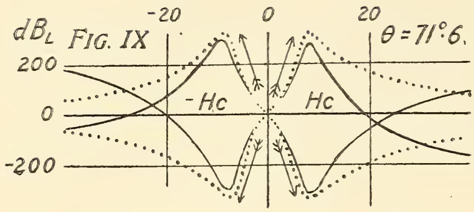
(1) *Experimental.*

In figs. 9 and 10 the dotted curves reproduced from fig. 4 show the transverse effects typical of complete demagnetisation by reversals when the calculated angle $\theta = 71^\circ{\cdot}6$. The arrows now represent not only the orientation of the demagnetising angle θ as formerly, but the polarity of the residual magnetisation due to partial demagnetisation. If it be arranged that the residual magnetisation has a component coinciding with the field H_c subsequently applied, the continuous approximately symmetrical curves of fig. 9 are obtained. If, on the other hand, it be arranged that the residual magnetisation has a component opposite to the field H_c, the continuous curves of fig. 10, likewise symmetrical in the four quadrants, are the result.

These results are quite distinct and indicate the departure of the transverse induction curves from those of complete demagnetisation for the same angle θ when the residual magnetisation is small.

The above experiments are typical of many others for the same demagnetisation angle of $\theta = 71^\circ{\cdot}6$, throughout a wide range of residual magnetisation between $B = 100$ and 10,000 in C.G.S. units. The curves are plotted in fig. 11, the orientation and polarity of the residual magnetisation being indicated by the arrows. Those on the right are the average of four practically symmetrical curves obtained under the conditions already described for fig. 9. Those on the left are the average of four practically symmetrical curves obtained under the conditions already described for fig. 10. In both cases the orientation of θ is right-handed and the polarity of the residual magnetisation positive, but a component coincides with or directly opposes the subsequent field according as H_c is positive (curves on the right) or negative (curves on the left).

Fig. 12 gives the results of another set of experiments obtained in the same way when the demagnetisation angle $\theta = 18^\circ{\cdot}4$, the calculated complementary angle.


dBL
FIG. IX
θ = 71°·6.
-20
0
20
200
0
-200
-Hc
Hc

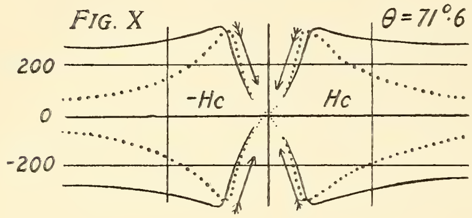


FIG. X
θ = 71°·6
200
0
-200
-Hc
Hc

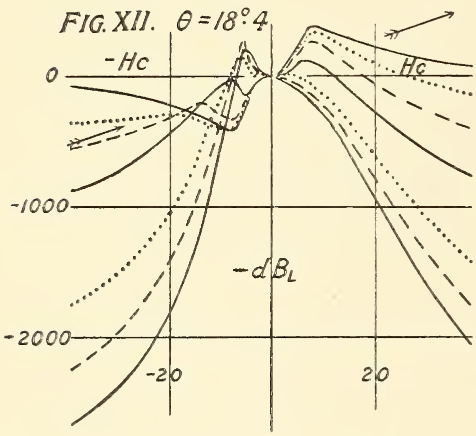
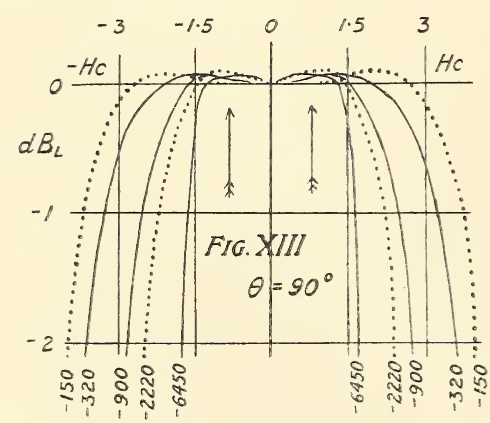
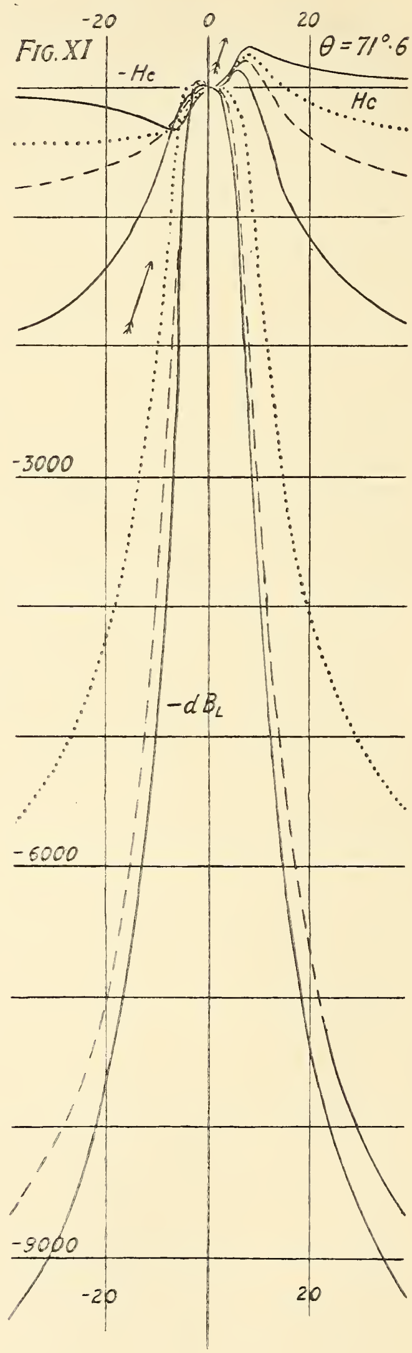


FIG. XII. θ = 18°·4
-Hc
Hc
0
-1000
-2000
-dBL
-20
20


-3
-1·5
0
1·5
3
-Hc
Hc
0
dBL
-1
-2
FIG. XIII
θ = 90°
-150
-320
-900
-2220
-6450
-6450
-2220
-900
-320
-150


-20
0
20
FIG. XI
θ = 71°·6
-Hc
Hc
-3000
-dBL
-6000
-9000
-20
20

In both figs. 11 and 12 the curves on the left ($-H_c$) cross the original curves of complete demagnetisation and each other as the residual magnetisation is increased. In fig. 10 this crossing is sufficiently well marked, and is in fact an early stage of the large initial rise (figs. 11 and 12) from the third to the second quadrant which occurs when the residual magnetisation is sufficiently increased and the subsequent magnetising field negative.

On the other hand, the curves on the right do not appear to cross, fig. 9 for weak magnetisation being in this respect typical of the whole series. Attention, however, may be called to the fact that when the residual magnetisation is small, the curves nearly coincide with that of complete demagnetisation which may indicate crossings close to the origin.

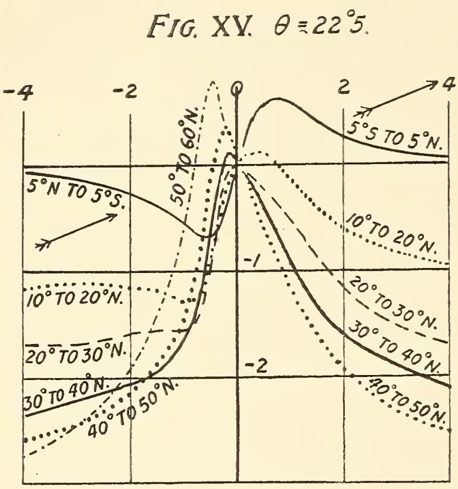
When $\theta = 90^\circ$ (the curves of complete demagnetisation having vanished in the horizontal axis) small initial crossings are obtained, and to show them the ordinates have been expanded one thousand times in fig. 13. Note that the curves on the right and left are now symmetrical about the vertical axis.

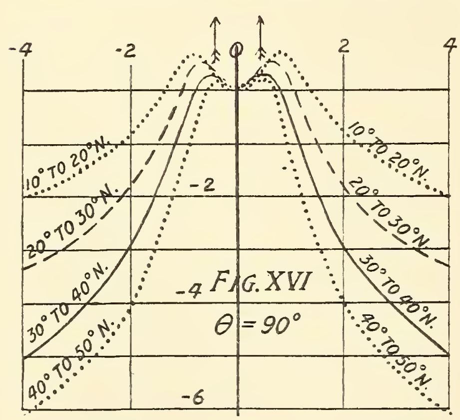
The residual magnetisation for each pair of curves ($+H$ and $-H$) was not directly measured (figs. 11, 12, and 13), but may be approximately obtained from the consideration that the final negative values of the various curves, when H_c is sufficiently increased, is equal to the residual magnetisation into $\sin\theta$—the angle at which partial demagnetisation had been performed.

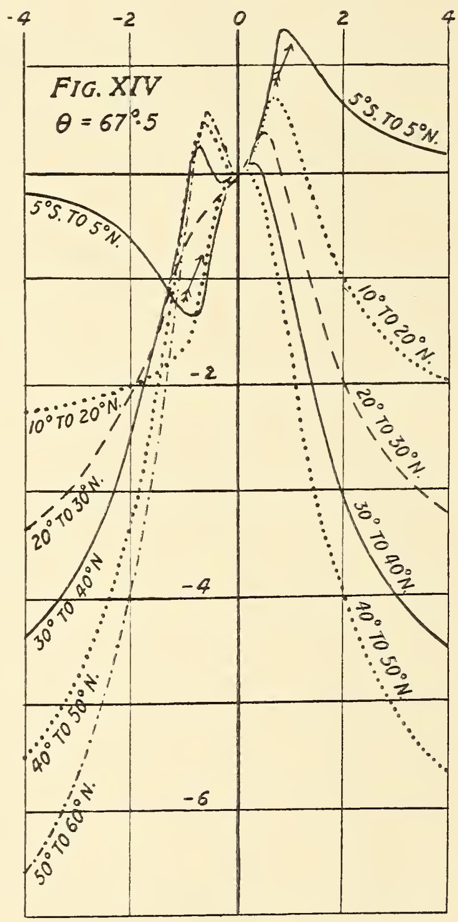
It may be mentioned that when the steel tube is left *residually magnetised in the ordinary way*, the crossings of the curves when H_c is negative (figs. 11 and 12) are less marked and their rise into the second quadrant either entirely absent ($\theta = 71^\circ{\cdot}6$) or very small ($\theta = 18^\circ{\cdot}6$)—an entirely similar result to that obtained in the previous investigation. The simple withdrawal of the positive field leaves the iron more susceptible to negative than positive change. On the other hand, if the steel tube be left residually magnetised by the withdrawal of the positive field, followed by the application and withdrawal of a reduced negative field, the iron may be left positively magnetised but more susceptible to positive than to negative change. Under this condition it was found that the rise of the transverse curves into the second quadrants is readily obtained. The production of the residual magnetisation by the method of partial demagnetisation adopted in the present investigation eliminates both these experimental extremes—the result of the more complex form of magnetic æolotropy already referred to.

(2) *Theoretical.*

Figs. 14, 15, and 16 exhibit the results calculated from the hypothesis of zonal distribution. The ordinates have the same signification as in fig. 8, which dealt with double parallels of latitude illustrative of the demagnetised condition. In the present case the polarity of the residual







magnetisation (positive) is represented by *single* zones of north latitude as shown by the arrows. The zones contain an equal number of molecules and the parallels bounding each zone are given in the diagrams.

The curves must be regarded as a first approximation, dealing as they do with single zones departing from a narrow equatorial belt, not from a broad equatorial belt between wide parallels found to be typical of complete demagnetisation by reversals. Nevertheless the general similarity of the theoretical and experimental curves is obvious and may be traced in considerable detail.

In fig. 14 $\theta = 67^\circ{\cdot}5$, the contribution made by each zone towards the transverse effect when H_c is positive (fig. 1) is shown by the curves on the right; when the rotation is towards $-H_c$, by the curves on the left. Fig. 15 represents in the same way the results calculated for the complementary angle $\theta = 22^\circ{\cdot}5$. The curves on the left of both these figures cross each other, and in their initial stages finally pass from the third to the second quadrants for zones bounded by higher parallels, as they do in the corresponding experimental curves of figs. 11 and 12 when the residual magnetisation is increased. For higher zones (50° to 60° N. and others not here reproduced) the rise into the second quadrant is greater for the smaller (fig. 15) than for the larger (fig. 14) complementary angle. This phenomenon is well marked in the experimental curves. For reasons of symmetry, however, the positive portions of these curves must decrease as θ is sufficiently reduced, and finally vanish with the transverse effects of which they form a part when $\theta = 0^\circ$.

But the final negative values of the transverse effects due to zones of north latitude (positive residual magnetisation) must vary as $\sin\theta$. The positive portions of the curves do not therefore necessarily vanish when $\theta = 90^\circ$, the angle at which the transverse effects reach their maxima. For reasons of symmetry, however, the differences between the curves on the right and left must decrease when θ is sufficiently increased, each becoming the reflection of the other in the vertical axis when $\theta = 90^\circ$.

Fig. 16 shows the curves calculated for the four north zones indicated when $\theta = 90^\circ$. The curves are now symmetrical about the vertical axis; the equatorial zone typical of complete demagnetisation has vanished in the horizontal axis, above which all the other zones cross each other. The vertical rise is thrust towards the origin the further the zone is removed from the equator.

These crossings were not obtained in the calculations for the curves on the right of fig. 15 when $\theta = 22^\circ{\cdot}5$. They are indicated in the theoretical curves on the right of fig. 14 (the scale being too contracted to do more than this) when $\theta = 67^\circ{\cdot}5$, and these crossings appear to have their counterpart in the close spacing of the corresponding experimental curves of fig. 11 in their initial stages when the residual magnetisation is small. For this reason calculations were made for $\theta = 90^\circ$, and finally these theoretical deductions were at a later stage experimentally verified in fig. 13, already shown on p. 310. No doubt the initial increase of the theoretical curves (fig. 16) indicates a much greater induction change than that obtained in the experimental verification (fig. 13), which is indeed minute, not being greater than $B=0{\cdot}1$ C.G.S. unit. But, as already indicated, narrow zones only, so far as this part of the investigation is concerned, have been considered, and under this limitation the general relationship of the various curves is as close as could have been expected.

It may therefore be concluded that, when iron is left residually magnetised by the method of partial demagnetisation by reversals, the displacement in either direction of the broad equatorial belt found to be distinctive of complete demagnetisation by the same method, can on this view be readily traced in the experimental results.

In conclusion I desire to express my indebtedness to the Royal Society of London for placing at my disposal Government grants for the purpose of prosecuting these researches.

(*Issued separately August* 9, 1912.)

XXII.—Preliminary Note on the Effect of Vibration upon the Structure of Alloys. By G. H. Gulliver, B.Sc., A.M.I.Mech.E., Lecturer in Engineering in the University of Edinburgh. (With Two Plates.)

(Read May 6, 1912. MS. received July 5, 1912.)

Of late years a great deal of experimental work has been carried out with the object of determining the effect upon various metals of a load applied and removed many times in quick succession. The chief object of such researches is to determine the maximum stress which can be applied and removed indefinitely without causing fracture of the piece. Little attention has been given to any change in the structure of the metal, except in cases where there has been well-marked permanent distortion. The writer accordingly undertook some simple preliminary experiments in order to ascertain the effect upon the structure of a few alloys of numerous small blows, each of insufficient force to cause a permanent deformation of the specimens.

Before the internal structure of metals and alloys had been investigated there was a strong belief among engineers that shock and vibration caused crystallisation of a metal, and that belief is widely held even at present. A piece of metal, after long service in a situation where it has been exposed to much vibration, may fail in one of two ways—abruptly or gradually. In the latter case a crack starts at some point of weakness or defect, and spreads gradually through the piece, until the section is so much reduced that the rest breaks abruptly; the earlier developed part of the broken surface is smooth and shows little or no structural detail, whereas the later part is rough and often crystalline in appearance. When the complete fracture has occurred suddenly, due probably to an excessive shock, the whole of the broken surface may have a coarsely crystalline appearance; this is especially true of wrought iron. It is a common observation that shock fractures of wrought iron are apt to be crystalline; the apparent degree of coarseness of the broken surface, though not an accurate measure, gives a rough indication of the scale of structure of the metal. There is always the possibility that the coarse structure existed before the piece was subjected to shock, though the apparent degree of coarseness is not infrequently such as to make this doubtful; in days when metals under ordinary conditions were not regarded as crystalline the possibility was not admitted.

In the light of later knowledge the older observations, of little value individually, when taken collectively give ground for supposing that vibration favours an increase in the size of metallic crystals, so that a piece after long-continued vibration may be expected to have a coarser structure than it possessed originally. This is no matter for surprise, for in a body not in a state of perfect internal equilibrium any molecular agitation tends in the direction of equilibrium. With a pure metal, not subject to allotropic change, there can be no lack of chemical equilibrium, and there is only the physical instability of crystal size upon which the effect of vibration may be manifested. Experience strongly suggests that in any

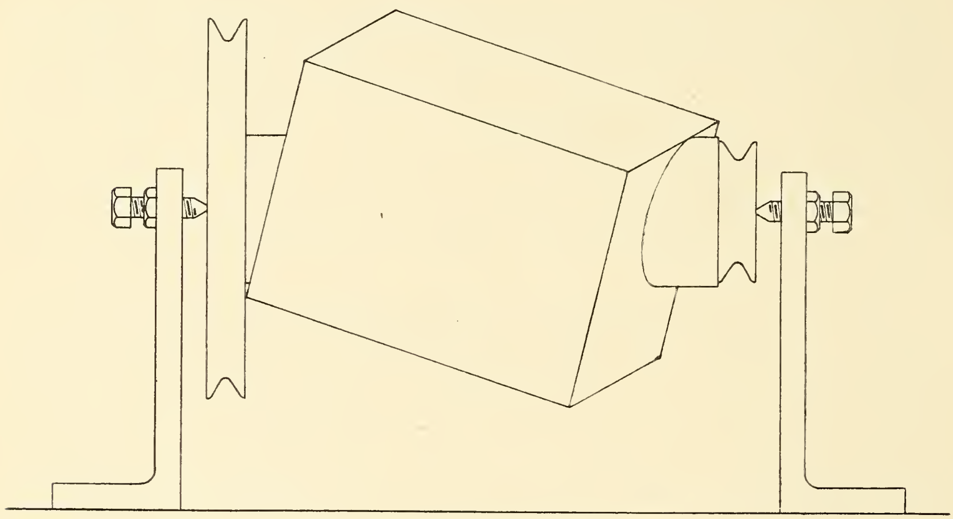


Fig. 1.—Wooden tumbling barrel.

mass of uniform metal there is a tendency for all the crystals to assume a parallel orientation—that is, to become a single crystal. In ordinary circumstances there is sufficient resistance to prevent the accomplishment of such a rearrangement, even when a practically unlimited time is available, but every shock to which the piece is subjected, by producing a momentary reduction of the resistance, may allow of some slight rearrangement.

The general effect of vibration in increasing the scale of the structure of a metal being considered, though not proved, as highly probable, it seemed useful to the writer to investigate, in a preliminary manner, the effect upon alloys in which there was a chemical, or physico-chemical, as well as a purely physical instability, so as to determine whether an appreciable change of structure could take place in a reasonable time for experiment. To this end a bar of gun-metal, containing 90 per cent. of copper and 10 per cent. of tin, was cast in an iron mould about 4 inches long and $\frac{3}{8}$ inch square section. This alloy, when in a state of physico-chemical equilibrium at the atmospheric temperature, consists of uniform crystals of a homogeneous solid solution of tin (or a copper-tin compound) in copper, usually designated α. Rapid cooling from the liquid state, as by pouring the melted mixture into a cold iron mould, prevents the attainment of that condition, and the chill cast alloy is formed of crystal skeletons of inhomogeneous α, each surrounded by a network of a material of somewhat indefinite composition and constitution, but richer in tin than the skeletons (fig. 2). Chilled castings of other alloys were prepared, but no definite results were obtained from them; these alloys were: copper 70, zinc 30; copper 90, aluminium 10; and aluminium 95, zinc 5 per cent.

In order to apply the numerous small blows required, the chill cast pieces, sawn into $\frac{3}{8}$-inch cubes, were put into a wooden box, about 5 inches by 5 inches by 7 inches, mounted as shown in fig. 1, and rotated, by means of a small water motor, at a speed which averaged 80 revolutions per minute. The apparatus was run, as a rule, only during the daytime for periods of 6 to 8 hours. In a tumbling box of this kind, if the speed of rotation is too low, the pieces slide down the sides of the box and receive no appreciable blow, whereas if the speed is too high they remain in stationary contact with the sides, on account of centrifugal action. At the right speed of rotation the pieces are carried to nearly the top of the box, and then drop freely to the bottom; each piece therefore receives two blows per revolution. About two dozen of the metal cubes were put into the box together; but since some were harder than others, the softer pieces suffered considerable distortion, especially in their surface layers, by impact with the harder pieces. Much wear took place, chiefly through the exfoliation of the crushed surface layers; the aluminium-zinc alloy suffered the greatest wear, the copper-zinc and the copper-tin showing less, and the copper-aluminium very little. Thus the original intention of imparting vibration without distortion was defeated, but the results obtained with the copper-tin alloy are not without interest.

Fig. 2 is a section of the chill cast gun-metal, already described: the dark primary α-skeletons are small, and their outlines are irregular. Pieces were withdrawn from the rattling box after various periods of rotation, cut through the middle, polished, and etched, exactly as the original unshaken sample, with a solution of ferric chloride acidified with hydrochloric acid. After only $\frac{1}{4}$ million revolutions no change was definitely detectable, but figs. 3, 4 and 5, taken after $\frac{3}{4}$, $1\frac{1}{2}$, and 3 million revolutions, respectively, show a progressive increase in the size of the skeletons and a greater regularity of arrangement. The increase in the size of the crystal grains, or aggregates of uniform orientation, is more clearly shown when the pieces are etched more vigorously, though such treatment gives specimens only suitable for low-power examination; some of the grains appear dark and others bright, and the limits of most of them are well defined. Figs. 6 to 9 were obtained by more deeply etching the specimens represented by figs. 2 to 5 respectively; the average size of crystal grain, as determined from the bright areas of fig. 6, shows a continuous increase through figs. 7, 8, and 9, the final linear dimensions, after three million revolutions, being 5 or 6 times as great as in the original alloy. As regards the piece which experienced three million rotations, the scale of the structure varies a good deal over the prepared surface, and fig. 9 represents about the coarsest part of the area; the other three pieces figured were sensibly uniform. There is no doubt as to the increase in the *average* size of crystal grain throughout the series.

Since the photographs illustrate four separate pieces of metal, it may be well asked whether the change of structure is real or only apparent—in other words, whether or not the structure of all four pieces was sensibly identical at the beginning. Unfortunately, the rattled pieces were not examined microscopically before rotating, but other pieces cut from the same bar showed approximately the same scale of structure as figs. 2 and 6. If the larger grains of the rotated pieces had been due to slower cooling of the metal in the corresponding portions of the original casting, one would have expected their skeleton structure to be coarser also. Now, an inspection of figs. 2 to 5 shows that as the size of the skeletons increases, their branches become distinctly finer—a result which is not so astonishing if the growth has taken place within the *solid* metal.

If the change of structure is real, it may be due to causes other than vibration—namely, distortion and rise of temperature caused by the blows. The rattled pieces of the copper-tin alloy gave but little evidence of distortion except in the immediate neighbourhood of their external surfaces. On the contrary, the copper-zinc, in which exfoliation was very marked, showed dark striations upon the primary skeletons even in the centre of the piece. The occurrence of distortion is known to be favourable to subsequent recrystallisation, and it may be that the growth noticed would have been less if deformation of the pieces had been prevented. But, on the other hand, the crystals did not exhibit any noticeable growth when the pieces were not rotated. The period of vibration lasted for five months after the date of casting the specimens. A comparison of rattled and unrattled pieces was made at the end of that time, and again after three years of quiescence. No change in the structure of any of the specimens during the latter period could be detected. Hence, even if distortion did facilitate growth during vibration, the conclusion that the chief factor in the change was the vibration itself is not invalidated. The atmospheric temperature during the period of vibration varied between 50° and 70° F. (10° and 21° C.), but the limits may have been somewhat wider during the succeeding three years. As regards the heat developed during rotation, the rise of temperature shown by an ordinary thermometer after the tumbling box had run for several hours was less than 1° F.; it follows from this that any local rise of temperature at a point just receiving a blow must have been small. It was clearly impossible to make any measurement of local heating, but, with a weight of less than $\frac{1}{50}$ lb. falling through a height of about 6 inches, a local rise of more than 10° C. is extremely unlikely.

The preliminary experiments, therefore, without being regarded as in any way conclusive, may be held to indicate a change in the structure of chill cast gun-metal under the influence of vibration. If this be really the case, the remarkable result is the growth of inhomogeneous skeletons in preference to the formation of a uniform solid solution—a result which would show that the forces due to the physical instability are of greater effect than those due to lack of chemical equilibrium. So far as can be judged from examination of the few specimens available, the apparent growth of a crystal grain takes place as the result of a piecemeal change in the orientation of surrounding grains—an action which would involve not merely the rotation of molecules, but the migration of appreciable masses in order that a larger and continuous dark-etching skeleton should be formed. It is true that a similar process occurs to some extent during annealing, though this aspect of the matter does not appear to have been remarked hitherto; the change in the direction of chemical uniformity is more marked in annealing, but the increase of crystal size proceeds whether the aggregates are homogeneous or not.

For most of the apparatus used in preparing and examining the specimens the author is indebted to the Earl of Moray Endowment of the University of Edinburgh.

DESCRIPTION OF FIGURES.

Gun-metal, containing 90 per cent. of copper and 10 per cent. of tin, chill cast, etched with mixture of ferric chloride and hydrochloric acid. Dark primary skeletons of α-solution, and white tin-rich network. Magnified 100 diameters.

Fig. 2. Original alloy.

Fig. 3. After $\frac{3}{4}$ million revolutions in tumbling barrel.

Fig. 4. After $1\frac{1}{2}$ million revolutions.

Fig. 5. After 3 million revolutions.

Same specimens, more deeply etched, showing light and dark crystal grains. Magnified 50 diameters.

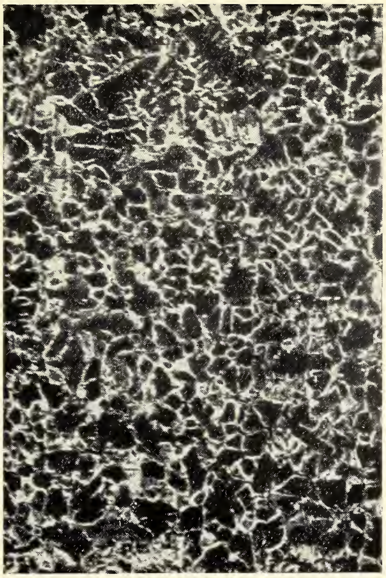
Fig. 6. Original alloy.

Fig. 7. After $\frac{3}{4}$ million revolutions.

Fig. 8. After $1\frac{1}{2}$ million revolutions.

Fig. 9. After 3 million revolutions.

(*Issued separately August* 9, 1912.)



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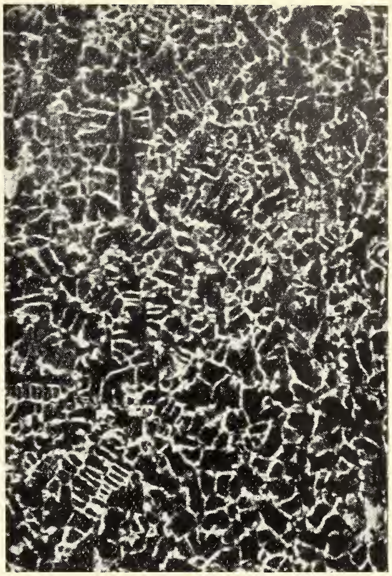

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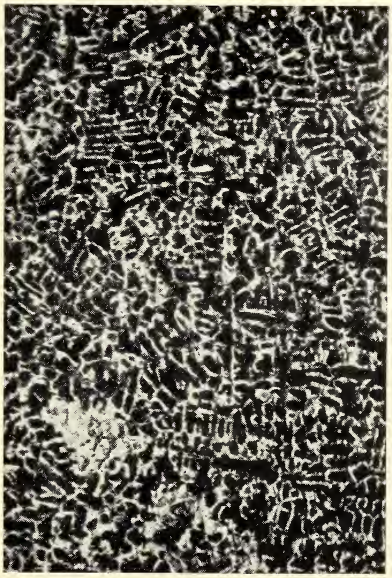

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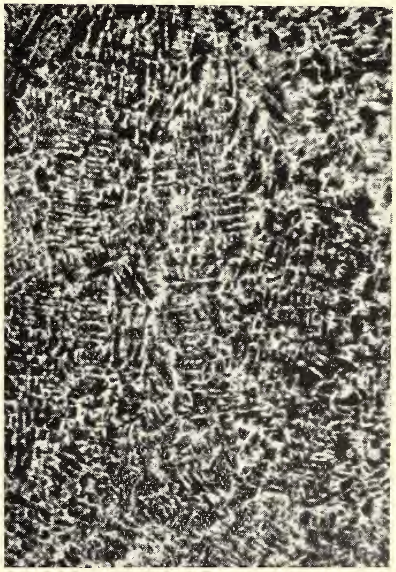

Fig. 5.

Fig. 6.

Fig. 7.

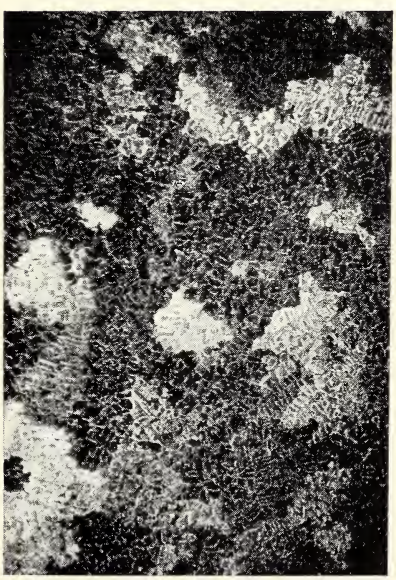

Fig. 8.

Fig. 9.

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E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

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XXIII.—On a Continuous-current Motor-Gyrostat with accessories for demonstrating the Properties and Practical Applications of the Gyrostat.

By **James G. Gray**, D.Sc., Lecturer on Physics in the University of Glasgow; and **George Burnside**, Mechanical Assistant to the Professor of Natural Philosophy in the University of Glasgow.

(Read May 6, 1912. MS. received July 8, 1912.)

No branch of dynamics or physics provides a more profitable or interesting subject of study to the student of physics or engineering than that which deals with the phenomena presented by spinning-tops. Within the last few years the most interesting form of top, namely the gyrostat, has advanced from being a piece of lecture-room apparatus, or a toy, to a position of great practical importance. In the dirigible torpedo, Brennan's monorail car, the Schlick device for rendering a ship steady at sea, and in the gyrostatic compass, the results are obtained by utilising the properties of the gyrostat. Undoubtedly, too, the gyrostat is destined to play an important part in many inventions still to be brought to light.

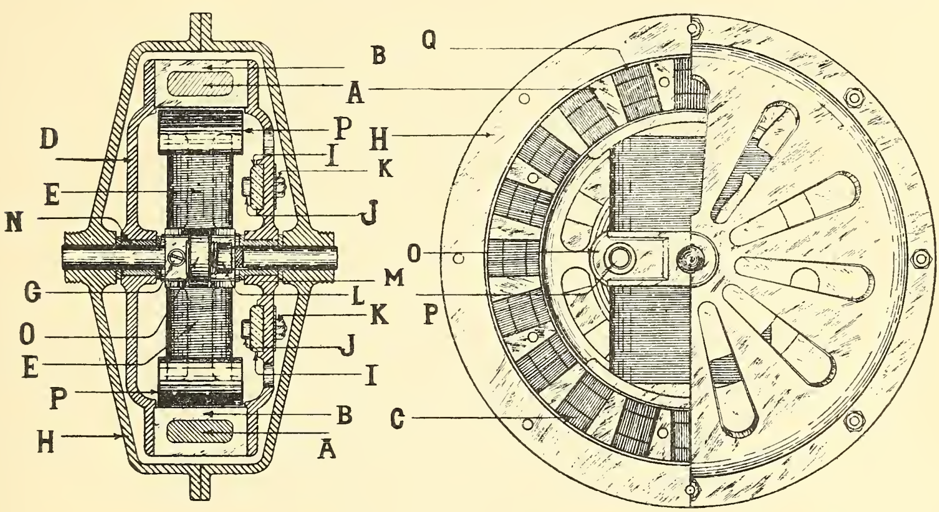
But, quite apart from the important part played by the gyrostat in practical affairs, the study of spinning-tops is well worth while pursuing for its own sake. To be effective, such study should not be confined to the mathematical treatment of the phenomena; lectures should be accompanied throughout by experiments carried out with actual tops and gyrostats. Unfortunately, the gyrostatic apparatus available up to the present time is the reverse of satisfactory, and experiments carried out by means of it are, even when the experimenter possesses great skill and experience, far from being effective.

The gyrostat, or gyroscope, in its simplest form consists of a heavy fly-wheel mounted in a suitable framework. If the gyrostat is to be effective the flywheel or rotor must have a large moment of inertia, the framework in which the flywheel rotates must be light, and means must be provided of spinning the flywheel with great angular velocity. In gyrostats, as constructed by instrument and toy makers, up to the present time, the spinning body consists of a heavy wheel whose axis terminates in steel pivots which revolve in steel or brass bearings carried in the frame. The spin is obtained by the ancient device of passing one end of a string through a hole in the spindle of the wheel, winding up the string on the spindle, and then drawing it off by the application by hand of a large force to the free end of the string. Such a gyrostat has serious defects, great skill and experience are called for on the part of the experimenter, and even then the spin obtained is far from considerable. Again, in the process of spinning considerable stresses are applied at the pivots; as a general rule a gyrostat spun in the manner above described deteriorates with each successive spin, and is soon rendered useless unless the pivots are repaired or replaced. Further, in consequence of friction at the pivots, the spin rapidly falls off, and no attempt can be made by its means to do more than demonstrate qualitatively the more obvious properties of the gyrostat. Experiments of a quantitative nature are quite out of the question.

It is the object of the present paper to describe a continuous-current motor-gyrostat and accessories which have been invented and constructed by the authors, the work being carried out in the workshops of the Natural Philosophy Department of the University of Glasgow. In designing the gyrostat the following points were kept continually in view:—(1) The distribution of matter in the flywheel should be such as to make the moment of inertia as large as possible for a given mass; (2) the mass of the non-spinning parts should be reduced to a minimum; (3) the design should be such as to allow of the angular velocity being high; (4) there should be effective means of keeping the motor cool; (5) the gyrostat and accessories should be such as to allow of experiments, both qualitative and quantitative, being carried out with perfect ease and safety.

The construction of the motor-gyrostat will be seen from figs. 1 and 2. Fig. 1 shows the instrument in section and side elevation. The armature-core A, B is made of malleable cast iron or mild steel, and has the form of a ring A, B, provided with equally spaced cavities on which are wound the armature coils Q. The coils are all wound in the same direction, the end of one coil being connected to the beginning of the next, so as to form a complete circuit. The junctions between the coils are connected to a series of commutator studs J through the medium of clamping nuts K. The armature ring is supported centrally from the shaft by means of two magnalium discs D, perforated to allow of the armature and field magnets being cooled by air circulation. The discs are fitted with gun-metal bushes N so as to form good working surfaces on the shaft, upon which they rotate. Experiment has shown that this form of bearing is superior to ball-bearing races. The magnalium discs are recessed near their outer edges to fit tightly into the inner periphery of the armature, and are fixed in position by means of countersunk screws, let in flush with the surfaces of the discs so that the rotating parts may be as free from projections as possible. The armature ring together with the magnalium discs form a balanced rotor of large moment of inertia; and magnalium being very light material, practically all the mass of the rotating system is concentrated in the ring. On one of the magnalium discs is mounted the commutator, consisting of phosphor-bronze studs J, each of which is held in position by a nut K. The commutator is mounted on a ring of insulating fibre I, arranged concentrically with the shaft. A phosphor-bronze body connecting each of the studs with its nut is enclosed in an insulating sleeve.

The space within the armature ring and its supporting discs is occupied by the field magnet system P, and brush gear L. Placing the field magnets in this way within the armature ring enables the mass of the stationary part



Section through Motor-Gyrostat. Side elevation of Motor-Gyrostat.

Fig. 1.

to be reduced to a minimum; and, further, the non-rotating mass is situated near the axis, the most convenient position. The field magnet is composed of malleable cast iron or mild steel, and carries two windings, one on each side of the shaft. The field magnet casting is provided with two projections O, one on either side of the magnet coils. Each such projection is adapted to receive the brush gear L and brushes; the latter are mounted parallel with the shaft. The brushes make contact with the phosphor-bronze studs, which project inwards from the magnalium disc supporting the armature ring as already described. By means of two screws G the field magnet is clamped to the shaft, which is fixed in this type of motor. The shaft extends out to an equal distance on both sides of the field magnet casting; one end of it is bored out to allow of the connections from the field magnets and brushes being brought to the outside of an internal casting H made of magnalium (cast in halves) fitted together over the motor, each half being provided with a flange at its outside to allow of the two halves being screwed together.

In this type of motor-gyrostat the only rotating parts are this Gramme-ring armature, the commutator, and the two discs supporting the armature ring. The field magnet, the shaft, and the outside casing are stationary. The shaft projects slightly on both sides of the casing; each projecting part has a thread cut upon it to receive a knurled nut. The two nuts (these are not shown in the figure) are screwed against the outside casing, and thus fix the latter to the shaft.

To keep the motor-gyrostat as cool as possible the outside casing is

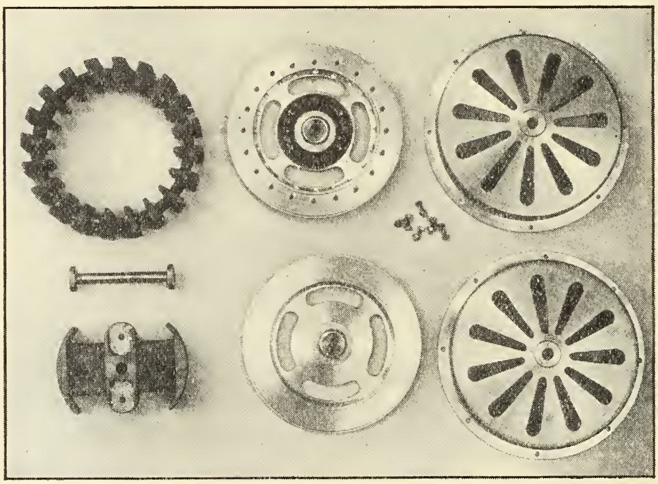


Fig. 2.—Parts of Motor-Gyrostat.

pierced with a number of radial slots; these are kept small so as to eliminate all danger of the fingers of an experimenter getting into contact with the rotating parts.

Fig. 2 is a photograph of the various parts of the motor-gyrostat, and will render clear the construction of the instrument.

It will be seen that this motor-gyrostat is designed to run on a continuous-current circuit. It can be connected up to run as a series motor, or as a shunt motor, and may be wound to suit all voltages. In the motors constructed up to the present the total mass of each instrument is $6\frac{1}{2}$ pounds; that of the rotating parts is 4 pounds. The diameter of the external case is $6\frac{1}{2}$ inches and the breadth is under 3 inches. It will thus be seen that the gyrostat is capable of being handled with ease and convenience. When going at full speed the rate of working in the motor is 100 volts.

The motor-gyrostat is provided with the following additional parts:—

(1) Attachment for showing ordinary precessional motion of gyrostat.

(2) Skate, for showing the "gyrostat on skate" experiment.

(3) Stilts and attachments, for showing the "gyrostat on stilts" experiment.

(4) Attachments for allowing of the motor-gyrostat being fitted up to show the crossed bifilar pendulum experiment due to Lord Kelvin.

(5) Mounting to enable the motor-gyrostat to demonstrate the principle of the gyrostatic compass.

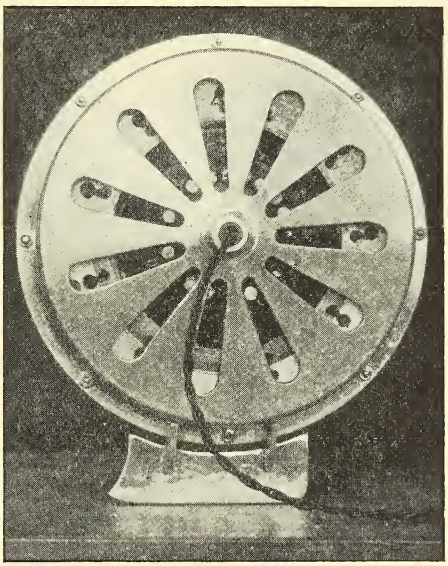


Fig. 3.—Motor-Gyrostat on skate.

(6) Apparatus to enable the motor-gyrostat to be fitted up as a stilt top of new design. This experiment is due to one of the authors.

(7) Apparatus to enable the motor-gyrostat to be mounted as a gyrostatic pendulum.

(8) Apparatus to enable the motor-gyrostat to be mounted to show the principle of the dirigible torpedo.

(9) Apparatus to enable the motor-gyrostat to be mounted so as to show the principle involved in the Schlick device for steadying ships at sea.

The accessories enumerated above are of entirely novel design, and for that reason it is proposed to describe them in some detail. In fig. 3 the motor-gyrostat is shown mounted on a skate. The skate attachment is readily slipped on the rim of the outer casing of the gyrostat, as shown in the figure. In this experiment the gyrostat is set down on the skate with its axis horizontal. If the axis becomes inclined to the horizontal, a couple tending to overbalance the gyrostat, that is, to produce angular momentum about a horizontal axis at right angles to the axis of spin, comes into play. The gyrostat in consequence precesses round on the skate. If the precessional motion is delayed, the inclination of the axis to the horizontal increases; if it is hurried, the gyrostat returns to the position in which its axis is horizontal and the centre of gravity is vertically above the support. With this motor-gyrostat such experiments are very striking. With the armature going at full speed—12,000 to 20,000 revolutions per minute—the balancing power is extraordinary; if the gyrostat is supported

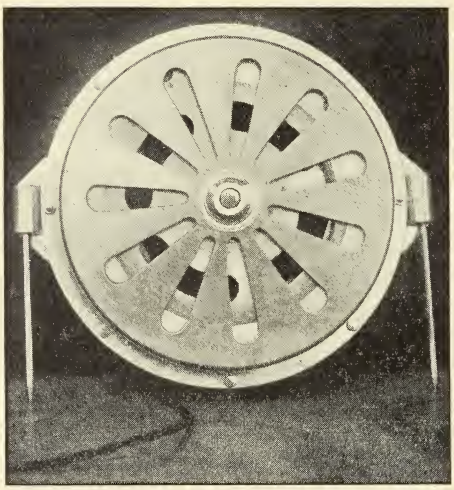


Fig. 4.—Motor-Gyrostat on stilts.

on the skate with its axis inclined to the horizontal it precesses round very slowly; a slight force tending to hurry up the precessional motion suffices to cause the instrument to stand up with its plane vertical. The spin can be readily altered by connecting up a rheostat in the circuit, and the dependence of the precessional motion upon the spin (that is, the angular momentum of the flywheel) demonstrated with the utmost ease.

In fig. 4 the motor-gyrostat is shown performing the two-stilt experiment invented by Lord Kelvin. To adapt the gyrostat for this experiment two pieces of magnalium, cast in the form shown in the figure, are screwed on the rim of the gyrostat at extremities of a diameter. The left-hand extremity is pierced with a cylindrical socket whose length is parallel to the plane containing the rim, and into this fits tightly one end of a rigid stilt; the other end of the stilt is pointed. The second magnalium casting carries a cylindrical cap, and in this rests loosely the upper end of a stilt pointed at both ends.

To perform the experiment the gyrostat is placed on the stilts as shown in the figure; it is adjusted so that the stilts are vertical and the axis of the gyrostat is horizontal, and it is then let go. The gyrostat balances with a to-and-fro motion on the stilts. This experiment, which is one of extreme difficulty with an ordinary gyrostat, is performed with the greatest ease with this motor-driven instrument.

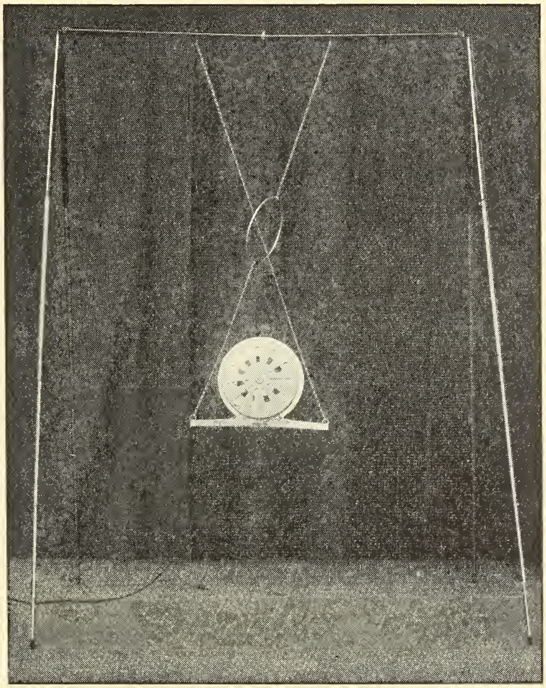


FIG. 5.—Motor-Gyrostat on bifilar support.

In this experiment it should be observed that the gyrostat, when without spin, is inclinationally and azimuthally unstable. The spin therefore removes two degrees of instability, and an illustration is obtained of the general theory according to which an even number of degrees of instability are removed by the spin. Thus a result which follows from a consideration of the roots of a determinantal equation which determines the periods of oscillation of the gyrostat about the equilibrium configuration, is confirmed experimentally.

In fig. 5 is shown the motor-gyrostat performing Lord Kelvin's bifilar experiment. The gyrostat is attached by means of bolts, provided with nuts, to a magnalium bar provided with two rings, one at each end. Fixed to the rings are two chains as shown. In the middle of one of the chains is a large ring as shown, and through this ring passes the second chain. The ends of the chains remote from the bar are fastened to two points on the same level whose distance apart is equal to the length of the magnalium bar. To perform the experiment the gyrostat is placed in the upright position with its rim and the chains in the same vertical plane, and is then left to itself. Instead of falling down, the gyrostat balances on the bar.

It is to be observed that here the gyrostat may fall over in two ways, either towards or away from the observer, and, further, in consequence of the crossed bifilar suspension the bar and gyrostat are also unstable azimuthally without spin. When the gyrostat is spinning rapidly the arrangement is one of stability, just as in the stilt experiment already described.

A general explanation of the experiment just described may be given as follows. Starting with the bar, gyrostat rim, and chains in one vertical plane, we may suppose the gyrostat to fall over slightly. In consequence of the tilting the gyrostat precesses so that its axis turns in a plane which is nearly horizontal. As a result the chains get out of the vertical, and as soon as this takes place a couple hurrying up the precessional motion is brought to bear on the gyrostat, which in consequence rears up into the vertical position. This holds for both positions in which it is possible for the gyrostat to fall over. Again, suppose, starting with the rim, bar, and chains in the same vertical plane, the chains get out of the vertical. A couple is brought to bear on the gyrostat tending to turn its axis in a horizontal plane. In consequence of gyrostatic action the gyrostat tilts over on the bar—in other words, it has a precessional motion about a horizontal axis in the plane of the flywheel. This results in a couple, due to gravity, being brought to bear on the gyrostat; this couple is such as to hurry up the precessional motion; the azimuthal motion is opposed and reversed, and with the reversal the gyrostat regains the upright position. This holds for both directions in which the bar tends to turn in consequence of the crossed bifilar. The result is complete stability. As performed by the aid of this motor-gyrostat the experiment is one of great beauty.

In fig. 6 is shown the motor-gyrostat fitted up to show the principle of the gyrostatic compass. The gyrostat is mounted in two bearings provided in a ring made of mahogany. The ring is held in the hands of the experimenter so that its plane is horizontal, and the arrangement is carried round in a horizontal circle. The gyrostat is stable, with its axis vertical, so long as the direction of the spin of the flywheel coincides with that in which the ring is being turned. If the direction in which the ring is being carried round is reversed, the gyrostat turns a somersault so as to

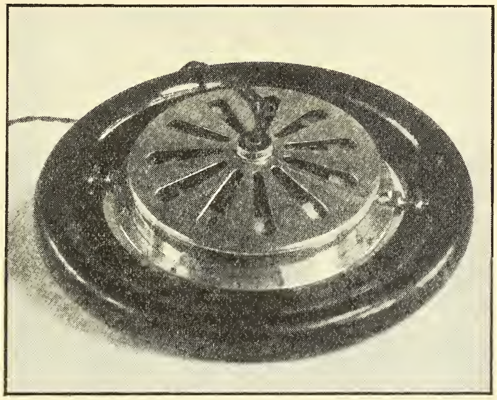


Fig. 6.—Motor-Gyrostat arranged to show the principle of the gyrostatic compass.

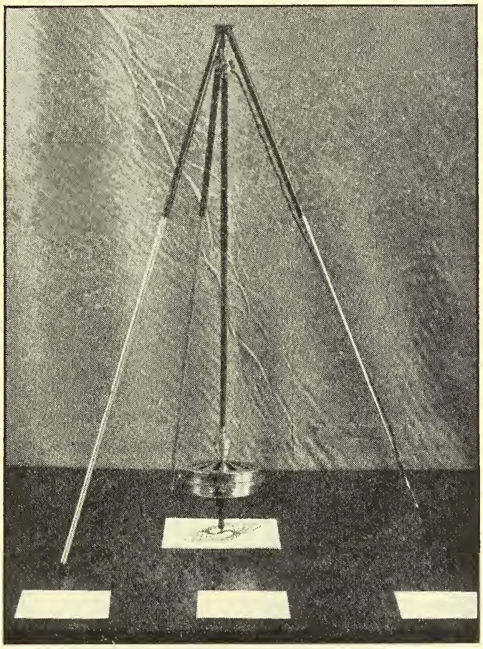


Fig. 7.—Motor-Gyrostat fitted up as gyrostatic pendulum.

render the direction in which the armature is rotating coincident with that in which the ring is being carried.

In the gyrostatic compass a heavy and rapidly rotating flywheel is mounted so that its axis is maintained horizontal by means of an elastic support. Under these circumstances the equilibrium position is that in which the axis of the flywheel is parallel to that about which the earth is spinning; that is, the equilibrium position of the flywheel is that in which its axis points true north and south. In the experiment described above the armature of the motor-gyrostat represents the flywheel, and the vertical axis about which the ring is carried represents the axis of the earth.

In fig. 7 the motor-gyrostat is shown fitted up as a gyrostatic pendulum. A support made of telescope tubing is attached to the apex of

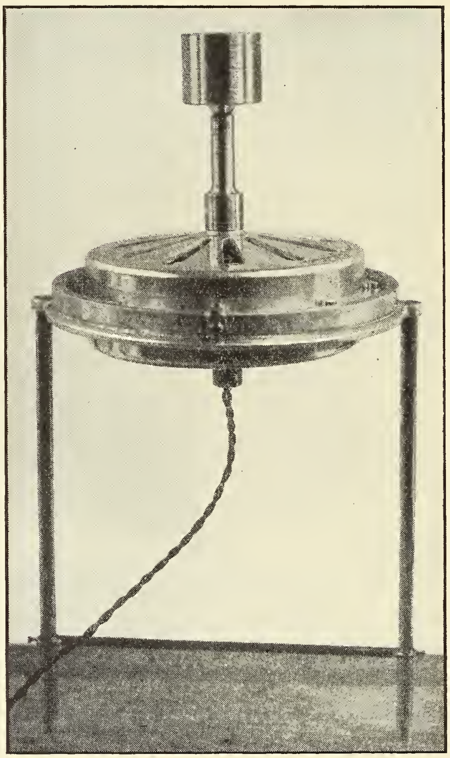


Fig. 8.—Motor-Gyrostat fitted up as "Gray" stilt top.

a triangular stand also made of telescope tubing, by means of a universal joint. The gyrostat is attached to the lower end of its supporting tube by means of a special cap provided with spring contact pieces to allow of the current being led into the the motor; the leading-in wires are carried down the inside tube. Screwed to the lower side of the gyrostat is a pen attachment.

By varying the speed of the motor and the initial inclination of the supporting tube to the vertical a large variety of designs may be obtained.

Fig. 8 shows the motor-gyrostat fitted up to show an experiment due to one of the authors. The motor-gyrostat is mounted, by means of two pivots screwed to the rim of the outer case, in two bearings carried by a magnalium casting in the form of a ring having a diameter slightly greater than that of the case of the gyrostat. Rigidly fastened to the ring at the extremities of a diameter, with their lengths at right angles to the plane of the ring, are two stiff legs terminating in points. To give rigidity to the legs a distance-piece is provided as shown. The line of the pivots is at right angles to the plane of the legs. Fastened to the top of the gyrostat, and in line with the axis of the flywheel armature, is a weight mounted as shown.

If the arrangement is placed on a table, with the legs and axis of the gyrostat vertical, and left to itself, it will balance on the legs. If the system tilts over in either direction the gyrostat precesses on its pivots so as to carry round the axis in a plane parallel to that containing the legs. But as soon as the axis of the gyrostat becomes inclined to the vertical, a couple, due to the weight fixed above the gyrostat, comes into play; and this couple is such as to hurry the precession. The gyrostat stands up towards the vertical, carrying with it the ring and legs. Again, tilting over of the gyrostat and weight in a vertical plane (from left to right or from right to left in the figure) is followed by precession of the gyrostat about the line joining the pointed feet of the stilts. But, as soon as the plane of the stilts get out of the vertical, a couple due to gravity, tending to hurry the precessional motion, comes into play, with the result that the gyrostat stands up vertically as before. Hence the arrangement is completely stable.

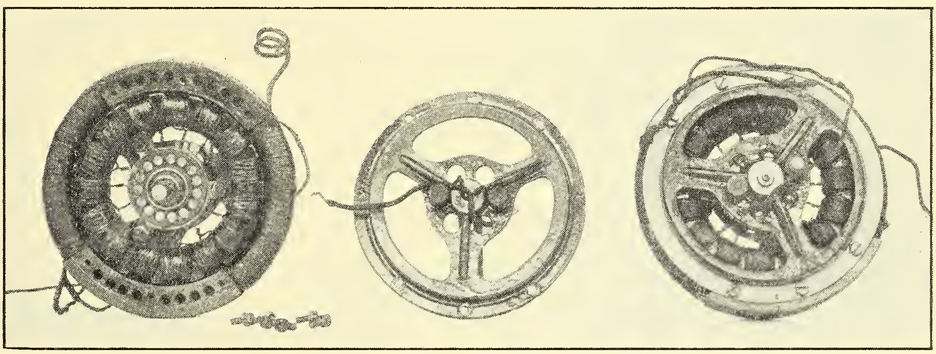


Fig. 9.—Alternative form of Motor-Gyrostat.

In fig. 9 an alternative design of motor-gyrostat is illustrated. For many purposes it is desirable that the gyrostat should be provided with an extended shaft which rotates with the armature, and it was to supply this want that the gyrostat under description was contrived. It differs from the gyrostat already described in having the field magnet system external to the armature ring. Here, as before, the non-rotating mass is reduced to a minimum. The shaft is of great strength to permit of the gyrostat being jerked to and fro without danger of damaging the shaft. With this type of gyrostat the authors have succeeded in obtaining speeds of from 25,000 to 30,000 revolutions per minute for considerable periods.

It may be pointed out that the accessories are so designed as to admit of the motor-gyrostat being rapidly adapted to any particular experiment. The design of the gyrostat itself is simple, and the instrument may readily be taken to pieces and the parts quickly assembled again. In all, five motor-gyrostats have been in use throughout the present winter in the Natural Philosophy Department of the University of Glasgow, and have been the means of greatly stimulating the interest of the students of physics and engineering in rotational motion. It is the opinion of the authors that the education of an engineer is incomplete unless he has handled gyrostatic apparatus; for experiments with such apparatus show, in a way that no other experiments can, the importance of carefully balancing rotating machinery, and of avoiding gyrostatic action of rotating parts where such action is a disadvantage. If the fact that we live on a top does not suffice to make top-spinning and gyrostatic action of universal interest, turbines and the rotating parts of other high-speed machinery make it an important subject for the engineer. For example, a turbine placed with its axis thwart-ship will, as the ship rolls, exert an alternating couple on the bearings in a plane parallel to that of the deck. If the axis of the turbine is fore and aft, such a couple has play as the ship pitches. In both cases a couple in a plane containing the axis and at right angles to the deck is called into play as the direction of the ship's head is altered.

In conclusion, the authors desire to express their deep thanks to Professor Gray for the lively and practical interest he has taken in the progress of the work described in the present paper. Rotational motion, spinning-tops, and gyrostatic action have always occupied a prominent position in the programme of dynamical subjects dealt with in the departments of Natural Philosophy presided over by Professor Gray, and it is to this fact that the authors owe what knowledge they possess of the properties of rotating bodies.

(*Issued separately August* 19, 1912.)

XXIV.—The Railway Transition Curve. By E. M. Horsburgh.

(MS. received November 21, 1911. Read December 4, 1911.)

If a railway circular curve spring directly from the tangent, the curvature at the point of contact is discontinuous, since it is zero for the straight line, and finite for the circle. This causes shock whenever rolling stock enters the curve, and at high speeds this is a matter of danger. Further, the outer rail is elevated on the curve, while the rails are at the same level on the straight. This indicates a further discontinuity. The transition curve is introduced to give both a gradual change of curvature and a gradual cant or super-elevation of the outer rail.

The curve to be found is one in which the curvature shall be a known function of some selected variable, and whose equation or equations contain a sufficient number of arbitrary constants to allow it to be fitted to the requirements of the case. Under the most general circumstances the equation should involve six arbitrary constants, since the curve should pass through two arbitrarily chosen points, and have at each of these points the required gradients and curvatures. This may be modified in practice, as the most general case is reducible to that of a transition curve springing from a straight line. In order to avoid difficulties of setting out, the equations obtained should be of the simplest nature.

If the curvature changes uniformly in passing along the transition curve its equation will be $\frac{d}{ds}\left(\frac{1}{\rho}\right)=k$, and as $1/\rho=y''/(1+y'^2)^{\frac{3}{2}}$, where ρ is the radius of curvature, the equation of the curve may be written $(1+y'^2)y'''-3y'y''^2-k(1+y'^2)^3=0$, where k is a known constant, and y', y'', y''' represent $\frac{dy}{dx}, \frac{d^2y}{dx^2}, \frac{d^3y}{dx^3}$. If the deflection angle be small, so that y'^2 is a negligible quantity, this simplifies to $y'''-3y'y''^2=k$.

The complete primitive, even if it could be obtained, would be of little practical use, and it would only contain three arbitrary constants. What is chiefly important is an approximation to the shape when the deflection angle is small, *i.e.* in the neighbourhood of the origin. Assuming that it may be obtained, and that it is expressible in the form of the convergent power series $y=\Sigma a_n x^n$, and taking the tangent and normal as axes of x and y respectively, and considering the curve as springing from a piece of straight track, the following conditions are satisfied:—$y_0=0$, $y'_0=0$, $y''_0=0$, so that a_0, a_1, and a_2 are all zero. Hence, a solution may be obtained in the form $y = \Sigma a_{3+4r} x^{3+4r}$. This shows that a very close approximation near the origin is given by the cubical parabola $y = lx^3$, the subsequent terms in the expansion being of the nature of correction terms. It is essential, however, to get exact compounding at the termination of the arc. It will be shown that it is possible to obtain this required exact compounding by using the cubical parabola in a particular way as a transition curve,

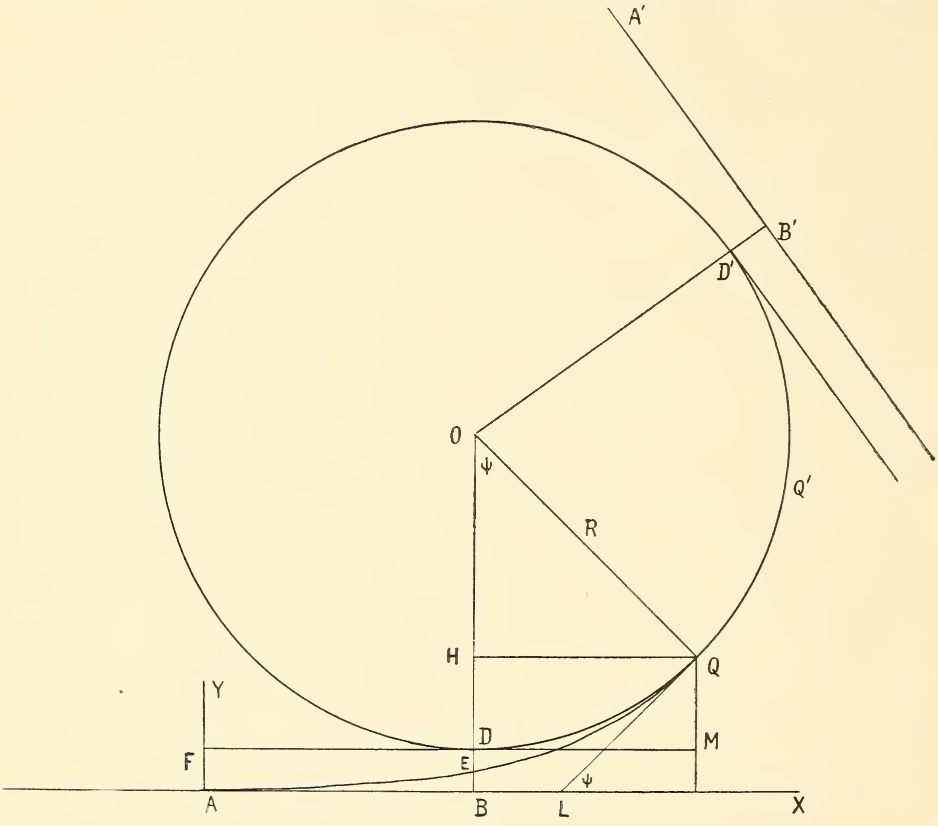


Fig. 1.

and yet under conditions which are, from the engineer's point of view, almost perfectly general. A further method of complete generality will be considered later.

Notation.

ABLX, original tangent or line of straight track, taken as x-axis or initial line.

FDM, parallel tangent, from which the actual circular curve is set out.

BD = AF = B′D′, the shift as calculated (β).

AEQ, transition curve (T.C.).

A, point of transition curve (P.T.C.) taken as origin of reference.

Q, point of compounding (P.C.C.).

D, point of circular curve (P.C.). The co-ordinates of D relatively to A are $AB = a$, $BD = \beta$.

R, radius of circular curve.

QM (η) is the perpendicular on DM (ξ), the tangent at D.

AY, a line perpendicular to AX, and taken as y-axis.

A′,B′,D′ denote corresponding points for the other tangent.

S and S′, points where the original and the parallel tangents intersect (points of tangent).

Let P be any point on the transition curve. Let the co-ordinates of P be (x, y) in cartesians and (r, θ) in polars. Let TP be the tangent at

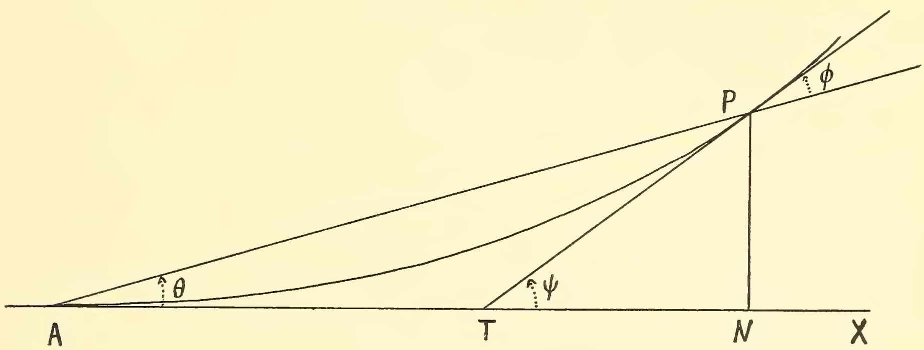


Fig. 2.

P, XTP is the deflection angle ψ, and $\tan\psi = \dfrac{dy}{dx} = y'$ is the gradient at P. When there is any danger of ambiguity the suffix $_1$ will be used to distinguish the elements at Q, the point of compounding. Though polar equations, freedom (or parametric) equations, or ordinary constraint equations may be used, the methods deduced must be of sufficient simplicity to be suitable for setting out in the field.

Although it is possible to fit a transition curve to a circle without shift or offset, yet a curve so calculated would be unsatisfactory. A simple method of procedure would be to select from practical considerations the approximate position of the P.C.C., by selecting an approximate value for ξ, and deducing an approximate value for the deflection angle ψ at the point of compounding. A convenient value of ψ is then chosen to locate accurately the point of compounding, Q. From this value of ψ, the radius of the circular curve or the degree of curve being known, the exact values of ξ and η are obtained, also AN, NQ (fig. 3), and l, and hence β (the shift), and a, and finally SB. The intermediate pegs on the circular and transition

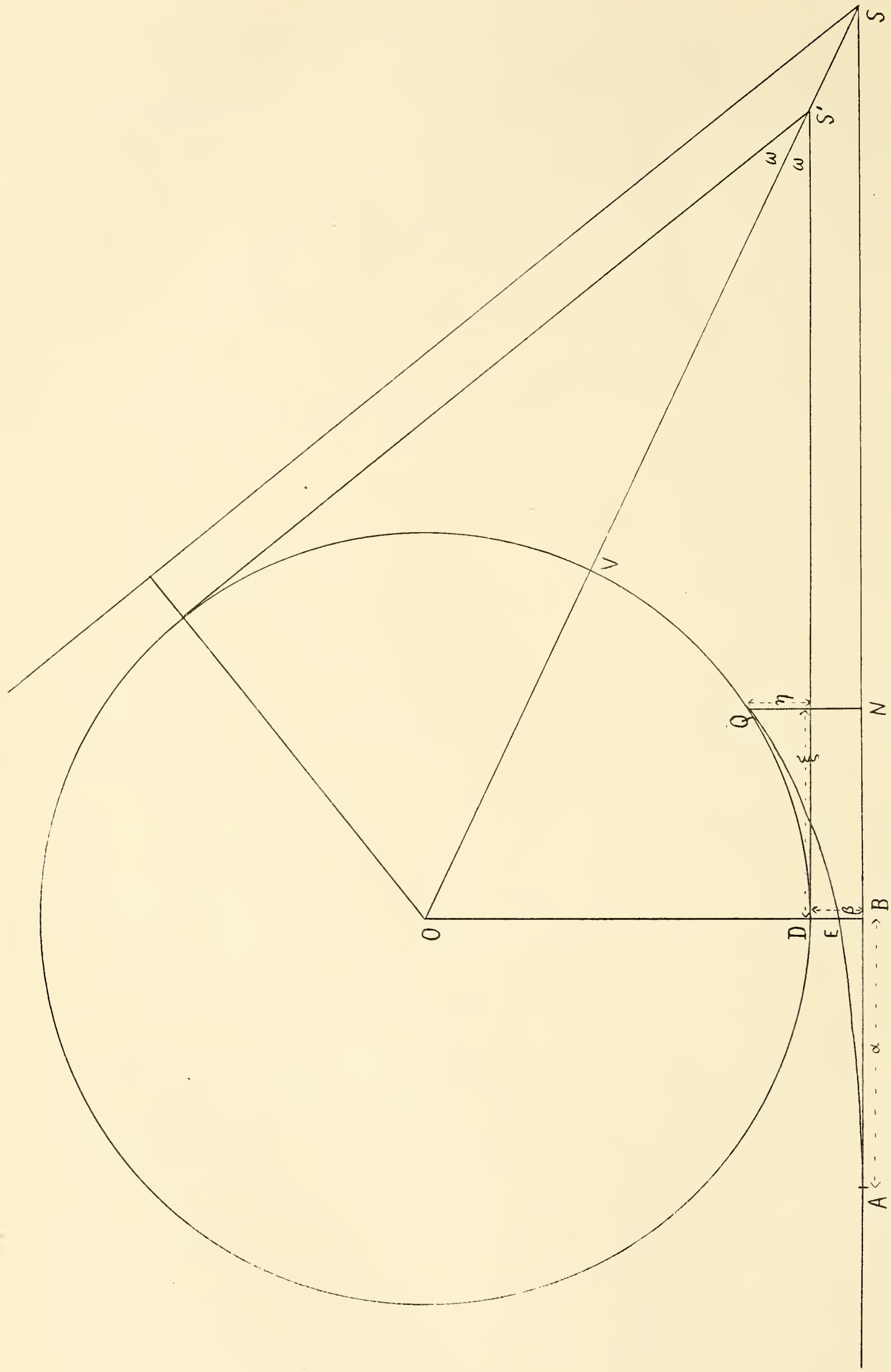


Fig. 3.

curves are then set out, and exact compounding is obtained by this use of the cubical parabola. The only restriction on the generality of this method is that when Q is given the position of A is fixed. The more general case will be examined later.

If neither the radius nor the degree of curve is given, but the point B is fixed, the calculations would be as follows:—BO $=(\mathrm{R}+\beta)$ is calculated, the approximate value of ξ selected as before, and the exact value of ψ at the point of compounding determined. Hence y_1 is calculated and the radius of the circular curve deduced. The shift is then known and the intermediate pegs may be set out as before, the compounding being exact.

$\mathrm{Y}=l\mathrm{X}^3$. Method of Exact Compounding, Radius or Degree of Curve being given.

If x and y, the co-ordinates of the P.C.C., be treated as unknowns, there are three unknown quantities, x, y, and l, to be determined at the point of compounding in terms of the known quantities ψ and R.

The equations are

$$y=lx^3.$$
$$y'=\tan\psi=3lx^2.$$
$$y''=\frac{(1+y'^2)^{\frac{3}{2}}}{\mathrm{R}}=\frac{\sec^3\psi}{\mathrm{R}}=6lx.$$

Hence x, y, and l may be found in terms of R and ψ:—

$$x=2\mathrm{R}\sin\psi\cos^2\psi.$$
$$l=1/(12\mathrm{R}^2\sin\psi\cos^5\psi).$$
$$y=\tfrac{2}{3}\mathrm{R}\sin^2\psi\cos\psi.$$

These formulæ are suitable for logarithmic computation, as are also the required values of ξ and η, viz. $\xi=\mathrm{R}\sin\psi$, $\eta=2\mathrm{R}\sin^2\frac{\psi}{2}$. Hence the values of α and β are both known, viz. $\alpha=x-\xi$, $\beta=y-\eta$. The point B is now known, since $\mathrm{SB}=(\mathrm{R}+\beta)\cot\omega$, where 2ω is the known angle between the tangents. The circular curve is then set out in the usual way from the tangent DS′, and the transition curve from the tangent AS, either by offsets, using the equation $\mathrm{Y}=l\mathrm{X}^3$, or by vectorial angles.

Before the exact position of the point of compounding is chosen, it is desirable to know the approximate values of the shift, and of the length of the transition curve. Since $\xi=\mathrm{R}\sin\psi$, and $x_1=2\mathrm{R}\sin\psi\cos^2\psi$, expanding in ascending powers of ψ, since the deflection angle is small, we have

$$\xi=\mathrm{R}\left(\psi-\frac{\psi^3}{6}+\cdots\right),\ x_1=2\mathrm{R}\left\{\psi-\frac{7}{6}\psi^3+\cdots\right\}.$$

Thus to a first approximation $x_1 = 2\xi$, or $AB = BN$ approximately, and this is nearly half the length of the transition curve. Thus when the length of the transition curve is given by practical considerations, the positions of the P.C.C. and the P.C. are both fixed within very narrow limits.

Also as

$$\eta = R(1 - \cos\psi)\ ;\ y = \tfrac{2}{3}R\sin^2\psi\cos\psi,$$

expanding in ascending powers of ψ,

$$\eta = R\left\{\frac{\psi^2}{2} - \frac{\psi^4}{24} + \ldots\right\},\ y = \frac{2}{3}R\left(\psi^2 - \frac{5}{6}\psi^4 + \ldots\right),$$

and hence to a first approximation $\beta = \frac{1}{6}R\psi^2$. Thus when two of these quantities are known, the third may be found, as when R and ψ are given the approximate value of the shift is obtained at once.

The simplicity of the preliminary trial calculations and the wide range of selection combine to render this method one of extreme flexibility.

An approximate value of the length of the curve to any degree of accuracy is easily obtained. Thus to a second approximation

$$s = x + \cdot 9y^2/x.$$

Example.—2° curve, angle between tangents 94° 38′. Compounding to occur near $\xi = 150'$.

Then

$$R = 100/\cdot 0349066 = 2864\cdot 787'.$$

$$\sin\psi = \frac{\xi}{R} = 150/2864\cdot 79\ ;\quad \therefore\quad \psi = 3^\circ\ 0'\ 5''.$$

As $\beta = \frac{1}{6}R\psi^2$ approximately, we have $\beta = 1\cdot 3'$ nearly.

For simplicity of calculation take $\psi = 3°$ as the value which determines Q, the point of compounding. Re-calculate ξ; hence

$$\xi = 149\cdot 932, \qquad \eta = 3\cdot 926.$$

For the co-ordinates of the point of compounding

$$x = 2R\sin\psi\cos^2\psi, \quad y = \tfrac{2}{3}R\sin^2\psi\cos\psi.$$

Hence

$$x = 299\cdot 041, \qquad y = 5\cdot 2240.$$

For the position of the origin,

$$a = x - \xi = 149\cdot 109,\ \beta = y - \eta = 1\cdot 298.$$

For the equation of the curve,

$$l = \frac{1}{12R^2\cos\psi\sin^5\psi}.$$

Hence

$$l = \cdot000{,}000{,}195{,}349,$$

and the equation of the transition curve is

$$Y = \cdot000{,}000{,}195{,}349X^3.$$

To determine the point B we have

$$BS = (R + \beta)\cot\theta\,;$$

hence,

$$BS = 2643 \cdot 21'.$$

The point B is now fixed, and also the points A, N, Q, D relatively to B. The circular curve is set out in the usual way from the point D in the tangent DS′, and the transition curve may be set out from the origin A by means of the equation

$$Y = \cdot000{,}000{,}195{,}349X^3.$$

The exactness of the compounding should be tested from this last equation. Thus at $X = 299\cdot041$ the recalculated values are

(i) $Y = 5\cdot2240$, (ii) $Y' = 3lX^2 = \tan 3°$, (iii) $Y'' = 6lX = \cdot000{,}350{,}505$,

which is the value given by the equation

$$y'' = \frac{(1 + y'^2)^{\frac{3}{2}}}{R}.$$

Thus the compounding is shown to be exact.

TABLE OF OFFSETS IN FEET.

X	30	60	90	120	150	180	210	240	270	300
Y	·0053	·0422	·1424	·3376	·6593	1·1393	1·8091	2·7005	3·8451	5·2744

If, instead of setting out the curve by offsets, it is to be laid out by theodolite and chain, the polar equation and a suitable selection of vectorial angles are required. Putting

$$X = r\cos\theta, \qquad Y = r\sin\theta,$$

the polar equation is

$$r^2 = \sin\theta/(l\cos^3\theta),$$

which is suitable for logarithmic computation. As

$$Y/X = \tan\theta = lX^2,$$

and

$$Y' = \tan\psi = 3lX^2, \quad \tan\psi = 3\tan\theta.$$

Thus, for small values of θ, $\psi = 3\theta$. This permits a suitable selection of values of θ to be made.

TABLE OF DISTANCES IN FEET FOR SELECTED VECTORIAL ANGLES.

θ	2′	7′	15′	30′	40′	50′	1°
r	54·572	102·10	149·45	211·37	244·08	272·90	298·97

Again, suppose that instead of ξ the approximate value of the shift is given, and instead of the degree of curve the value of R.

Example.—Given $R=1000$, $\omega=58°\ 50'$, and $\beta=1·35$ (approximately). Then

$$\psi=\left(\frac{6\beta}{R}\right)^{\frac{1}{2}},$$

and hence

$$\psi=5°\ 9'\ 24''.$$

Take

$$\psi=5°\ 10'$$

for ease of calculation, and redetermine β.

Then

$$\xi=R\sin\psi=90·0532.$$
$$\eta=R(1-\cos\psi)=4·0630.$$
$$x=2R\sin\psi\cos^2\psi=178·646.$$
$$y=\tfrac{2}{3}R\sin^2\psi=5·38442.$$
$$l=1/\{12R^2\sin\psi\cos^5\psi\}=·000,000,944,41.$$

Hence

$$\beta=y-\eta=1·3214,$$
$$a=x-\xi=88·5925,$$

and the equation of the curve is

$$Y=·000,000,944,41X^3.$$

As $SB=(R+\beta)\cot\omega=605·626'$, the P.C.C. is determined and the points A, N, Q, D are all fixed. Hence the circular and transition curves may be set out in the usual way.

TABLE OF OFFSETS.

X	20	40	60	80	100	120	140	150	160	170	180	190	200
Y	·008	·060	·204	·484	·944	1·63	2·59	3·19	3·87	4·64	5·51	6·48	7·56

EXACT COMPOUNDING. P.T.C. AND P.C.C. BOTH FIXED.

As the cubical parabola cannot be made to satisfy these conditions, an equation containing more arbitrary constants is required. Referring to the original differential equation, and considering terms of higher degree in x

than the third as correction terms, and remembering that the space rate of change of curvature might have been taken as a quadratic function of s or x, a suitable form of equation for the transition curve is seen to be $y = lx^3 + mx^4 + nx^5$, where l, m, and n are all unknown. To determine these values there are the three equations:—

$$\begin{aligned} lx^3 + mx^4 + nx^5 - y &= 0, \\ 3lx^2 + 4mx^3 + 5nx^4 - y' &= 0, \\ 6lx + 12mx^2 + 20nx^3 - y'' &= 0, \end{aligned}$$

where x, y, y', and y'' now denote known quantities at the point of compounding.

Solving this linear system, we have

$$l = \frac{y''}{2x} - \frac{4y'}{x^2} + \frac{10y}{x^3}.$$

$$m = -\frac{y''}{x^2} + \frac{7y'}{x^3} - \frac{15y}{x^4}.$$

$$n = \frac{y''}{2x^3} - \frac{3y'}{x^4} + \frac{6y}{x^5}.$$

These values of l, m, n require to be calculated and substituted in the equation $y = lx^3 + mx^4 + nx^5$, and a transition curve giving exact compounding at specified points is obtained.

As the calculations are more complicated than in the case of the cubical parabola they may be simplified by assuming $\xi = a$, and expressing x, y,′ y'', in terms of R and ψ, though it is probably best to express merely y' and y'' in terms of R and ψ. This gives for the values of l, m, and n:—

$$l = \frac{1}{R^2}\left[\left\{ \cdot 25 \sec^3\psi \operatorname{cosec}\psi \right\} - 2\operatorname{cosec} 2\psi\right] + \frac{10y}{x^3}$$

$$m = \frac{1}{R^3}\left[\left\{ -\cdot 25 \sec^3\psi \operatorname{cosec}^2\psi \right\} + \frac{7}{8}\left\{ \sec\psi \operatorname{cosec}^2\psi \right\}\right] - \frac{15y}{x^4}.$$

$$n = \frac{1}{R^4}\left[\left\{ \frac{1}{16}\sec^3\psi \operatorname{cosec}^3\psi \right\} - \left\{ \frac{3}{16}\sec\psi \operatorname{cosec}^3\psi \right\}\right] + \frac{6y}{x^5}.$$

Example.—3° curve, angle between the tangents 129° 8′.

$$\xi = a = 140', \qquad y = 6 \cdot 85'.$$

Then

$$R = \frac{s}{\theta} = 1909 \cdot 86'.$$

$$\sin\psi = \frac{\xi}{R} \quad \therefore \quad \psi = 4° \; 12' \; 14''$$

$$\eta = 5 \cdot 13847$$

$$\beta = y - \eta = 1 \cdot 71153$$

$$SB = (R + \beta)\cot\omega = 909 \cdot 044'.$$

Calculating, and substituting the values of l, m, and n, the equation of the transition curve becomes

$$y = \cdot 000{,}000{,}31294x^3 - \cdot 000{,}000{,}000{,}01145x^4 + \cdot 000{,}000{,}000{,}000{,}0292x^5.$$

The curve is then set out in the ordinary way.

A Method by use of Intrinsic and Freedom Equations to the Transition Curve.

If the curve chosen be such that the curvature changes uniformly in passing along the curve, then

$$\frac{d}{ds}\left(\frac{1}{\rho}\right) = c.$$

So with a properly chosen origin

$$\frac{d\psi}{ds} = cs, \quad \text{and} \quad \psi = \tfrac{1}{2}cs^2, \quad \text{or} \quad s = 2a\psi^{\frac{1}{2}},$$

the constants of integration being zero. This is the intrinsic equation of this transition curve.

Since $\rho = \dfrac{ds}{d\psi}$, at the point of compounding $a = \mathrm{R}\psi_1^{\frac{1}{2}}$, which determines the constant a. Also we have

$$dx = \cos\psi\, ds = \left(1 - \frac{\psi^2}{2\,!} + \frac{\psi^4}{4\,!} - \cdot \cdot \cdot\right) a\psi^{-\frac{1}{2}} d\psi$$

$$dy = \sin\psi\, ds = \left(\psi - \frac{\psi^3}{3\,!} + \cdot \cdot \cdot\right) a\psi^{-\frac{1}{2}} d\psi .$$

Integrating, and remembering that at the origin

$$x = 0,\ y = 0,\ \psi = 0,$$

the freedom equations of the curve are found to be

$$x = a\left(2\psi^{\frac{1}{2}} - \frac{1}{2\,!} \cdot \frac{2}{5}\psi^{\frac{5}{2}} + \frac{1}{4\,!} \cdot \frac{2}{9}\psi^{\frac{9}{2}} - \cdot \cdot \cdot\right)$$

$$y = a\left(\frac{2}{3}\psi^{\frac{3}{2}} - \frac{1}{3\,!} \cdot \frac{2}{7}\psi^{\frac{7}{2}} + \cdot \cdot \cdot\right).$$

This does not allow of exact compounding both at an arbitrarily chosen origin and at the P.C.C. When the P.C.C. is chosen, the origin and shift are thereby fixed. In other words, the curve represented by $\psi = \frac{1}{2}cs^2$ does not allow of compounding in the most general case.

It may be used, however, to give exact compounding when the P.C.C. is chosen arbitrarily and the origin deduced. This method possesses no advantage over the method of exact compounding by the cubical parabola, and the calculations required are more laborious.

In setting out the curve the origin is determined from the point of compounding by the co-ordinates

$$x_1 = \mathrm{R}(2\psi_1 - \tfrac{1}{5}\psi_1^3 + \tfrac{1}{108}\psi_1^5 - \cdot \cdot \cdot),$$
$$y_1 = \mathrm{R}(\tfrac{2}{3}\psi_1^2 - \tfrac{1}{21}\psi_1^4 + \cdot \cdot \cdot).$$

The curve is then set out by the freedom equations in the usual manner.

If a first approximation be taken $x = 2a\psi^{\frac{1}{2}}$, $y = \dfrac{2a}{3}\psi^{\frac{3}{2}}$, and ψ eliminated, the resulting curve is the cubical parabola $y = \dfrac{x^3}{12a^2}$, as is to be expected.

From the equation $s = 2a\psi^{\frac{1}{2}}$ the intermediate pegs on the curve could be set out by a process almost identical with Rankine's method for the circle by chain and theodolite. The methods by rectangular co-ordinates are, however, probably simpler.

Example.—

Given

$$\mathrm{R} = 1200', \quad \xi = 100', \quad 2\omega = 134° \ 16',$$

then

$$\beta = \frac{l^2}{24\mathrm{R}} \text{ (Froude)} = 1{\cdot}39' \text{ (approximate value)},$$

$$\sin\psi = \frac{\xi}{\mathrm{R}} = \frac{1}{12} \quad \therefore \quad \psi = 4° \ 46' \ 49''$$

$$\omega = 67° \ 8', \quad \mathrm{SB} = (\mathrm{R} + \beta)\cot\omega = 506{\cdot}077'$$

$$\xi = 100, \quad \eta = 4{\cdot}174.$$

Also

$$x_1 = \mathrm{R}(2\psi_1 - \tfrac{1}{5}\psi_1^3 +) = 200{\cdot}0964,$$
$$y_1 = \mathrm{R}(\tfrac{2}{3}\psi_1^2 - \tfrac{1}{21}\psi_1^4 +) = 5{\cdot}56589,$$

Hence

$$a = x_1 - \xi = 100{\cdot}096,$$
$$\beta = y_1 - \eta = 1{\cdot}392.$$

Table of Offsets.

ψ	·02	·03	·04	·05	·06	·07	·08	·09
x	98·037	120·06	138·62	154·97	169·74	183·33	195·94	207·80
y	·6536	1·2007	1·8486	2·5835	3·3961	4·2796	5·2287	6·2391

Inexact Compounding with the Cubical Parabola.

This occurs when the origin and the point of compounding are both specified, or even when the origin is fixed and the transition curve is made to touch the circular curve.

Example.—

$$R = 2000, \quad \xi = 80', \quad a = 220'.$$

Assuming

$$\beta = \frac{l^2}{24R} = \frac{90000}{48000} = 1\cdot 9 \text{ approximately. Take } \beta \text{ as } 1\cdot 9'.$$

$$\therefore \quad \sin\psi = \tfrac{1}{25}. \qquad \therefore \quad \psi = 2^\circ\ 17'\ 33''.$$

$$\eta = 1\cdot 6007$$

$$x_1 = 300$$

$$y_1 = \beta + \eta = 3\cdot 5007$$

$$\therefore \quad l_1 = y_1/x_1{}^3 = \cdot 000{,}000{,}12965.$$

Thus

$$Y = \cdot 000{,}000{,}12965X^3$$

is the equation of this transition curve. Hence the gradient of the T.C. at the point of compounding, $x = 300$, is tan 2° 0′ 18″, in which the angle differs by 17′ from the required angle. By calculating the value of y'' it may be shown that the radius of curvature differs by nearly 2290′ from the required value.

This illustrates the importance of the correct use of the transition curve, so as to obtain exact compounding.

The Sine Curve instead of Circular and Transition Curves.

The use of a sine curve would get rid of all trouble of fitting transition curves to the circular curve. This was suggested long ago by Mr Gravatt, but the method proposed was by offsets from the long chord. Such a method would usually be unsatisfactory, owing to the length of the offsets, and perhaps this accounts for the fact that the sine curve has been so little used in railway work. In the case of an excessively "flat" curve in open country such a method might, however, be employed.

The use of the sine curve, if it could be set out easily, would obviate all trouble with regard to calculation of shift and compounding, and it would only be necessary to see that the radius of curvature at the vertex was not less than a standard value. The curve might be set out easily, at least on flat ground, by offsets from three standard tangents—those at the two points of springing and at the vertex. A scheme for the determination of these offsets is shown, from which their values may be obtained rapidly by calculation or even by slide rule. This method might be useful also in the case of setting out tramway curves.

Let 2ω be the angle between the two pieces of straight track which are to be joined. AB = L, the long chord; V the vertex; VC = H; CAS the angle a. PR is drawn perpendicular to AS. Let A be the origin, AB the x-axis,

$P(x, y)$, $P'(x', y')$ two points on the curve, $AN = x$, $NP = y$, $AR = \xi$, $RP = \eta$, $VR' = u$, $R'P' = v$.

Then, by projection,

$$\xi = x \cos \alpha + y \sin \alpha,$$
$$\eta = x \sin \alpha - y \cos \alpha,$$

also

$$u = \frac{L}{2} - x',$$
$$v = H - y'.$$

The curve is set out by the co-ordinates ξ and η from the tangents AU and BU,′ and by the co-ordinates u and v from the tangent UVU′, the two arcs meeting near the point Z.

Where the ordinates are greatest, and where consequently there is most chance of error in setting out—viz., near the point Z, there is a safeguard in

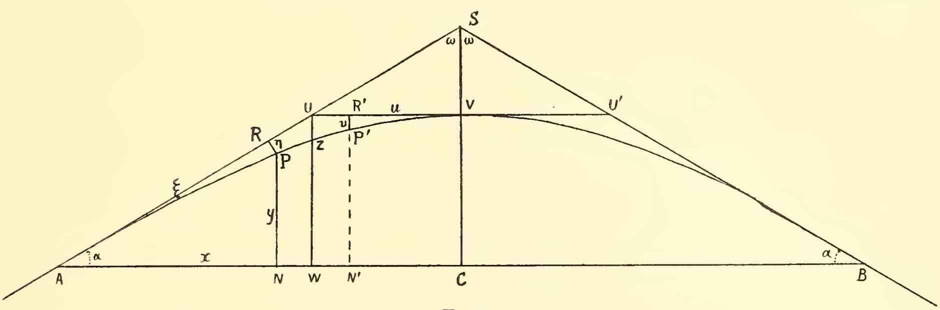


FIG. 4.

the fact that for correct junction of the two arcs in the field the offsets from the two tangents must determine the same points.

Owing to the symmetry about the axis SC the portion of curve VB is set out in the same manner as the portion AV. The offsets from the three tangents are all small, and so may be set out with ease and accuracy.

The equation of the curve referred to the origin A and the axis AB is

$$y = H \sin \frac{\pi x}{L},$$

and hence

$$\frac{dy}{dx} = H\frac{\pi}{L} \cos \frac{\pi x}{L}, \quad \text{or} \quad \tan \alpha = H\frac{\pi}{L}.$$

This gives L when H is known, and conversely; and hence we locate the points A, B, C, V, U, U′, W.

The radius of curvature at the vertex is $H \tan^2 \omega$, which from practical considerations must not be less than some given value.

A systematic method for obtaining the values of ξ, η, u, v, is indicated by the following calculation scheme.

log sin θ.	log cos θ.	$\theta°$.	N.	*d.*
$\bar{2}$·8946433	$\bar{1}$·9986591	$4\frac{1}{2}$	1	·05
$\bar{1}$·1943324	$\bar{1}$·9946199	9	2	·1
$\bar{1}$·3681853	$\bar{1}$·9878315	$13\frac{1}{2}$	3	·15
$\bar{1}$·4899824	$\bar{1}$·9782063	18	4	·2
$\bar{1}$·5828397	$\bar{1}$·9656153	$22\frac{1}{2}$	5	.25
$\bar{1}$·6570468	1·9498809	27	6	·3
$\bar{1}$·7180851	$\bar{1}$·9307658	$31\frac{1}{2}$	7	·35
$\bar{1}$·7692187	$\bar{1}$·9079576	36	8	·4
$\bar{1}$·8125444	$\bar{1}$·8810455	$40\frac{1}{2}$	9	·45
$\bar{1}$·8494850	$\bar{1}$·8494850	45	10	·5
$\bar{1}$·8810455	$\bar{1}$·8125444	$49\frac{1}{2}$	11	·55
$\bar{1}$·9079576	$\bar{1}$·7692187	54	12	·6
$\bar{1}$·9307658	1·7180851	$58\frac{1}{2}$	13	·65
$\bar{1}$·9498809	$\bar{1}$·6570468	63	14	·7
$\bar{1}$·9656153	$\bar{1}$·5828397	$67\frac{1}{2}$	15	·75
$\bar{1}$·9782063	$\bar{1}$·4899824	72	16	·8
$\bar{1}$·9878315	$\bar{1}$·3681853	$76\frac{1}{2}$	17	·85
$\bar{1}$·9946199	$\bar{1}$·1943324	81	18	·9
$\bar{1}$·9986591	$\bar{2}$·8946433	$85\frac{1}{2}$	19	·95

sin θ.	cos θ.	90	20	1
9969173	0784591	$85\frac{1}{2}$	19	·95
9876883	1564354	81	18	·9
9723699	2334454	$76\frac{1}{2}$	17	·85
9510565	3090170	72	16	·8
9238795	3826834	$67\frac{1}{2}$	15	·75
8910065	4539905	63	14	·7
8526402	5224986	$58\frac{1}{2}$	13	·65
8090170	5877853	54	12	·6
7604060	6494480	$49\frac{1}{2}$	11	·55
7071068	7071068	45	10	·5
6494480	7604060	$40\frac{1}{2}$	9	·45
5877853	8090170	36	8	·4
5224986	8526402	$31\frac{1}{2}$	7	·35
4539905	8910065	27	6	·3
3826834	9238795	$22\frac{1}{2}$	5	·25
3090170	9510565	18	4	·2
2334454	9722699	$13\frac{1}{2}$	3	·15
1564345	9876883	9	2	·1
0784591	9969173	$4\frac{1}{2}$	1	·05

Example.—Set out the transition sine curve for which $2\omega = 125° \ 34'$, and $H = 438'$.

Then

$$\omega = 62° \ 47', \qquad \alpha = 27° \ 13',$$

$$SA = \frac{\pi H}{2} \operatorname{cosec} \alpha = 1504·32 \qquad L = AB = \pi H \cot \alpha = 2675·53,$$

$$SV = H\left(\frac{\pi}{2} - 1\right) = 250·009, \qquad SC = \frac{\pi H}{2} = 688·009,$$

$$AF = H \cot \alpha = 851·646, \qquad \rho_v = 1655·94.$$

TABLE OF OFFSETS.

ξ.	η.
75·20	·03
150·30	·25
225·21	·85
299·83	2·00
374·07	3·90
447·84	6·72
521·05	10·63
593·61	15·79
665·44	22·36
736·48	30·50
806·64	40·36
875·86	51·99

TABLE OF OFFSETS.

u.	v.
66·89	1·35
133·78	5·39
200·66	12·10
267·55	21·44
334·44	33·34
401·33	47·74
468·22	64·54
535·11	83·65
601·99	104·94
668·88	128·29

The checks on the accuracy of this method are apparent on plotting or setting-out.

(*Issued separately August* 30, 1912.)

XXV.—The Elastic Strength of Flat Plates: An Experimental Research. By W. J. Crawford, B.Sc., Mechanical Engineering Department, Municipal Technical Institute, Belfast. *Communicated by* Dr C. G. Knott, Secretary.

(MS. received November 23, 1911. Read January 22, 1912.)

The question of the elastic strength of flat plates, supported or fixed at the edges, and subjected to uniform or concentrated loads upon their areas, is, from an engineering standpoint, one of the most unsatisfactory parts of mechanics; for exact solutions, rigorously based upon the laws of elasticity, have been obtained in only a few cases, chiefly for circular and elliptical forms.

Amongst practical engineers the confusion that exists on the subject is remarkable, for the author has, in the course of his inquiries, elicited the most contrary results. As a case in point, there seems no real knowledge whether rectangular and square plates should be ribbed along the diagonals or across the diameters. Again, as an instance of the doubt existing concerning the stress values in these plates, it may be mentioned that, in reply to an inquiry asking for help in this connection, a correspondent in the *American Machinist*, under date August 7, 1909, deplores the fact that he can obtain no assistance from English or American text-books and states that there is an entire absence of experimental data. Although the latter statement is not quite accurate, yet it can be confidently asserted that there have been very few experimental results recorded, and these few, for the most part, are not available in English.

Our ordinary scientific engineering books have very little to say about flat plates in general, and, in particular, with regard to square and rectangular plates practically nothing at all. The truth of the matter is, that the mathematics of the whole subject is beyond the domain of even the scientific engineer, and further, that even the most advanced mathematical research has been unsuccessful in completely solving all but the simplest cases.

The whole problem of the deformation of a thin isotropic elastic plate under given forces has for very long been of great interest to mathematicians. The general equations were first established by Navier, and the problem was afterwards attacked by Lamé, Sir William Thomson, de

St Venant, and others. It is still engaging the attention of mathematicians, and several papers of great analytical interest have recently been produced. Chief among these is Dr Dougall's treatise "An Analytical Theory of the Equilibrium of an Isotropic Elastic Plate." * The problem is there treated as a purely mathematical one, the object being to deduce the approximate differential equations for a thin plate from the exact equations of elasticity. It makes, however, no contribution to the solution of the equations for particular boundaries. Another recent paper, "Sur le problème d'analyse relatif à l'équilibre des plaques élastiques encastrées," by J. Hadamard, and crowned by the Paris Academy, is of much theoretical interest.

The results, however, of these analytical researches cannot be reduced to figures, except in a few special cases, as follows:—

1. The circular plate for free and fixed boundaries and for uniform and concentrated loadings.

2. The elliptical plate for free and fixed boundaries and for uniform loading. For concentrated loads the solution becomes very cumbersome, and, in fact, almost impracticable.

3. For certain forms of no practical interest, such as the limaçon of Pascal and in particular the cardioid.

The above refers to rigorous mathematical analysis only. But, as so often occurs in engineering theory, approximate solutions bearing a likelihood of success have been deduced for special cases. Thus, for square and rectangular plates, with fixed edges and uniform loading, Grashof gave, many years ago, an approximate solution, and this has been extensively used by boiler-makers and others in fixing working values. He assumed for the deflection an expression which (*a*) satisfies the conditions at the edge, (*b*) has the right sort of symmetry for a rectangle, (*c*) reduces to the known value for the deflection along a diameter in the limiting case in which the perpendicular diameter is very long. As the expression does not satisfy the fundamental differential equation, it cannot be correct, though it may not be very far wrong for points on the shorter diameter of a rectangular plate. Summarising, then:—

1. Rigorous solutions have been obtained for circular and elliptical plates, and for practically no others.

2. An approximate solution has been given for square and rectangular plates.

Very little experimental work seems to have been done on the subject of flat plates, and the results recorded are variable. It therefore seemed to the author that there was room for further work on this question. He

* See *Proceedings of the Royal Society of Edinburgh*, 1904.

decided to attack it as fully as possible, and for this he had to bear in mind, firstly, that the material chosen for the plates must be nearly homogeneous and isotropic; secondly, that the average results of many plates, of many thicknesses and many sizes, must be taken; and thirdly, that the apparatus used for experimental work must correctly interpret the conditions for edge clamping or supporting imposed by the theory.

The author decided to experiment with *fixed* plates for the reasons that it is easier to prevent leakage with fixed plates than with supported ones, that fixed plates are relatively more numerous and important in engineering than supported ones, and that the edge conditions can be better satisfied with fixed plates. It is, however, to be noticed that, should the equations for circular fixed plates with uniform load turn out to be true, it is certain that the equations for circular plates with load uniform or load concentrated at the centre and edge *supported*, would also be true, as the mathematical analysis is similar for them all, and one is derived from the other.

The objects of the present research may then be stated:—

1. To find how far the analytical equations for deflection, stress and strain, could be verified by experiment for the circular plate.

2. To find whether the equations given by Grashof for square and rectangular plates were near the truth.

3. To give a practical analysis of curvature for square and rectangular plates.

4. To throw some light on the question as to whether the elastic strength of the plate was determined by the maximum principal stress or the maximum principal strain.

In connection with 4, Grashof assumed that the elastic strength of the plate was proportional to the maximum principal strain, though in many works which quote his results this value, Ee_x (or Young's modulus multiplied by the maximum strain in the direction of x), is incorrectly given as the maximum principal stress.

In the following sets of experiments, close upon one hundred plates were used, and as these were distributed over circular, square, rectangular, and elliptical forms, with fixed edges only, the abundance of the results allows of an average being struck, and the author believes that the figures arrived at are as near the truth as experiments on this type of problem can be expected to be. In addition, the numerous results allow of many interesting points being noted, such as the effect of slightly loosening the clamped edges, the change of slope of the permanent set curves for circular plates, and the zero error and its causes at the origin.

In most experiments recorded, a hydraulic pump was employed to obtain the distributed pressure. With low pressures, such as a few pounds, this instrument is likely to introduce a considerable error. To avoid this, the author used a standard pressure-gauge tester with first glycerine and afterwards oil as the working fluid.

Preliminary Investigation.

A series of preliminary experiments on circular plates, conducted with comparatively simple apparatus, yielded results which were near enough to the theoretical values to warrant the undertaking of a more elaborate investigation.

The Rigorous Investigation.

The Dimensions, Form, and Type of Plates.

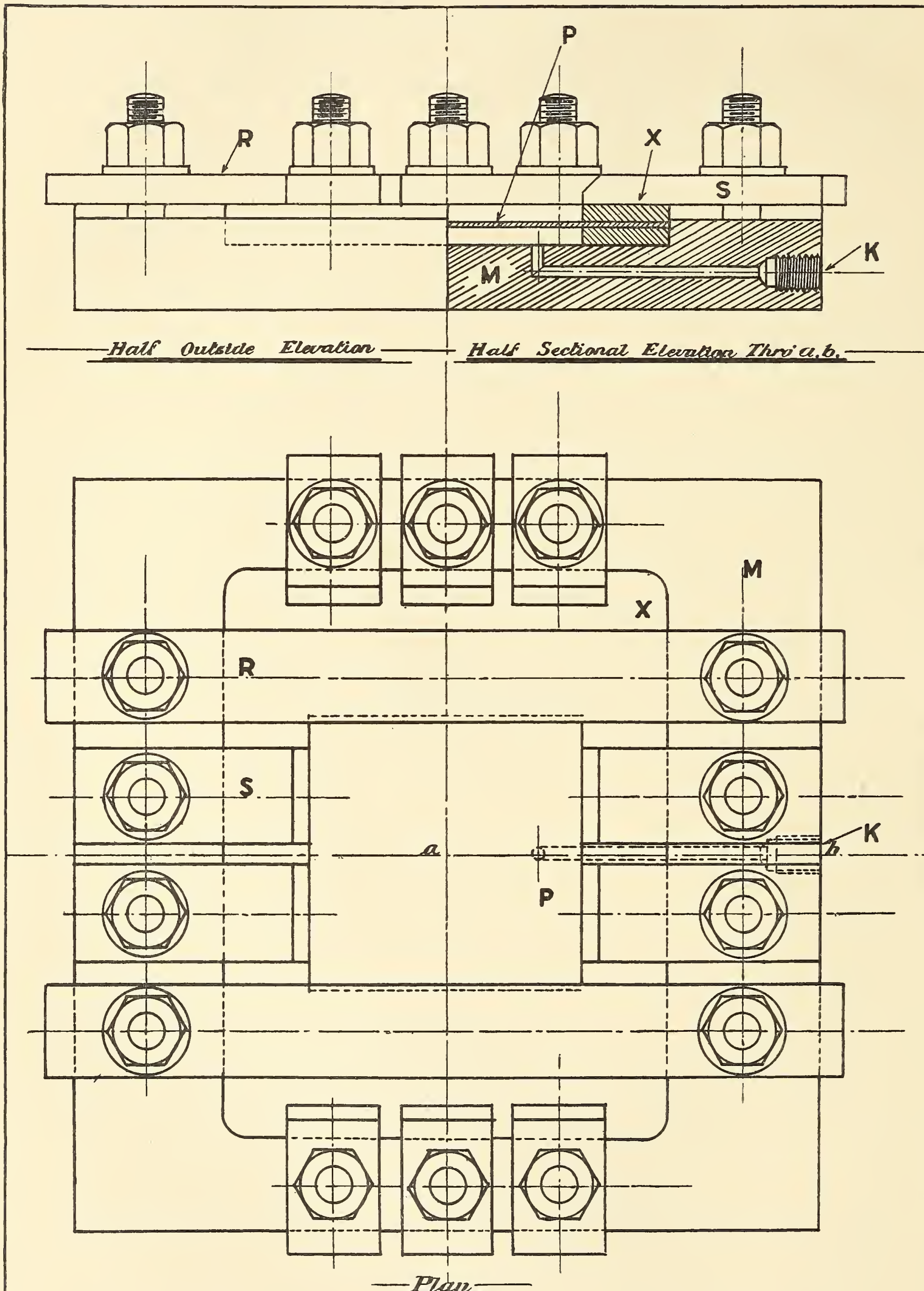
On account of the thinness of the plates (because of the formulæ only holding good for plates thin in comparison with their other dimensions, and owing to experimental reasons) it was found a difficult matter to turn them perfectly flat, even by experienced turners, and it became necessary to look around for a material that would obviate this difficulty. The material must satisfy the following conditions:—

1. It must be as nearly flat as possible in its natural state, and no part of it must be buckled or past its elastic limit.
2. Its thickness must be of great uniformity.
3. It must not be too expensive, because of the large number of plates required.

After much consideration it was determined to employ *planished steel* as fulfilling most of the above. This substance can be obtained in various thicknesses, ranging from ·050 in. to ·100 in. It receives such severe treatment in the process of rolling that it is perhaps more free from internal blemishes than ordinary mild steel sheet. It is very pure, an analysis showing over 99·5 per cent. iron. It is consequently very soft, the file tearing it with ease. It appears very homogeneous in structure, is supplied in flat sheets, and is of great uniformity of thickness. It is very ductile, has fairly well-defined elastic limit and yield points, and low breaking stress. It is used in practice for cylinder casings and the like.

The plates employed were of various gauges, and those were selected that showed almost perfect uniformity of thickness. The plates were mostly first drilled round with a series of small holes, cut out somewhat larger than required, and carefully filed down to size.

As it was necessary to experiment with plates of various forms and sizes, it became necessary to devise an apparatus that would do for all or



Apparatus for the Testing of Flat Plates of Various Shapes.

One Third Full Size.

Drawing No. 1.

for most of them. Accordingly, with this object in view, a modification of the original apparatus was constructed which is shown on the accompanying drawing No. 1, and which was found on trial to give satisfactory results.

M is a mild-steel block, over an inch thick. A rectangular recess, about 11 in. by 8 in., is machined into the centre of it. Two duplicate cast-iron pieces X X, externally fitting the recess and internally shaped to the form of the flat plates to be tested, are accurately machined and scraped flat. The flat plate rests between these duplicate blocks, and is clamped hard down by means of rigid clamps S, bars R, studs, and distance pieces. Underneath the bottom forming-piece X is placed a shaped piece of Jenkins K packing, and between the flat plate and the top of the lower piece X is placed another piece of the same packing, thus ensuring an even seating and preventing leakage. The oil is led to the under surface of the plate by the duct K. The drawing shows a square plate in position for testing.

Determination of the Elastic Constants.

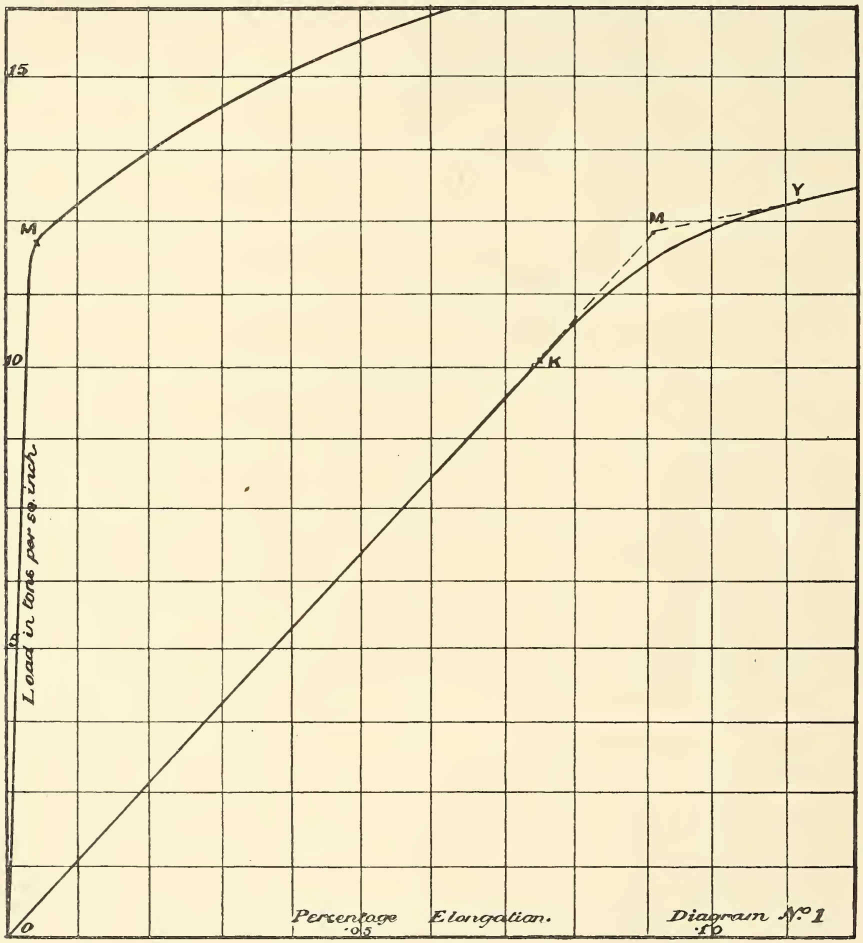
Young's Modulus.—The direct method was used. A strip of plate 8 in. long and $\frac{3}{4}$ in. wide, having enlarged ends for gripping in the chucks, was prepared, placed in the testing machine, and extensometer readings taken for small increments of load. In view of the importance of this curve, from the point of view not only of Young's modulus, but of the elastic limit and yield points, an example is given herewith (Diagram 1). The elastic limit occurs at K and the yield point at Y. Young's modulus is about 29,500,000 lbs. per sq. in. The elastic limit corresponds to 10 tons per sq. in. The yield point is at about 27,000 lbs. per sq. in. About one hundred readings were taken, to ensure accuracy, as the determination of an accurate yield point was of the first importance.

In the experiments made, the elastic limit was fairly constant, averaging 10 tons per sq. in.; the breaking stress was low, averaging 20 tons. On the diagram the curve has been drawn to two scales. In determining values for the yield point, the point of intersection M has been taken. Although the elastic limit appears low at 10 tons, yet Dr J. H. Smith assures the author that in some steels he has obtained it down to 8 tons. Moreover, it is well known that the elastic limit is a fickle point, and is ready to jump about on the least provocation.

The average of three results gave 27,800 lbs. per sq. in. as the yield point.

The average of three results gave 29,530,000 lbs. per sq. in. as Young's modulus.

The Rigidity Modulus.—Although the rigidity modulus does not enter directly into the calculations for deflection, yet it was necessary to determine it for the purpose of finding Poisson's ratio. The multiplying factor in the deflection equation is $\frac{m^2-1}{m^2}$; if m is 4, this amounts to $\frac{15}{16}$, and if m is 3, it amounts to $\frac{8}{9}$, a ratio of 135 to 128, or 1·06 to 1. Although



this is not a very serious matter, as δ almost certainly would lie between $\frac{1}{3}$ and $\frac{1}{4}$, yet it was adjudged best to find m experimentally. Owing, however, to the greatest thickness of plate obtainable being $\frac{1}{10}$ in., this became a difficult matter.

A length of about 10 in., of square section $\frac{1}{10}$ in. side, was first prepared. A heavy disc whose moment of inertia was known was attached to one end, and the other end was gripped in a clamp. The method of oscillation was then used. The rigidity modulus thus obtained was 11,210,000 lbs. per sq. in. A piece of $\frac{1}{10}$ in. thick plate was then cut out and turned in the lathe to a diameter of about ·085 in., and made 6 in. long. The method of oscillation was used as above, but, to give variety, the applied moment of inertia was not a heavy disc, but a special apparatus used for such experiments in the laboratory. By this method, the modulus was 11,280,000 lbs. per sq. in. The average is thus 11,245,000. Several other experiments gave values differing but slightly from the above. Therefore Poisson's ratio

$$\delta = \frac{E}{2N} - 1$$

$$= \frac{1}{3 \cdot 19}$$

and

$$m = 3 \cdot 19.$$

For the fine work accomplished in preparing the specimens for the rigidity test, and indeed for the very accurate machine work throughout, that alone gives value to these experiments, the author has to thank Mr G. H. Longworth, workshop instructor at this Institute.

Readings for Deflection, Permanent Set, etc.

These were taken in three ways, as follows:—

1. The whole apparatus rested on a planed and scraped cast-iron base specially constructed for it, and on this base were erected two standards. A $\frac{5}{8}$-in. turned and polished steel bar, clamped to these standards, could be placed in any position and at any desired height over the plate. Readings were then taken between the face of the plate and the under surface of the bar by means of an inside micrometer gauge; and it is worthy of note in this connection that, by virtue of taking thousands of readings, the sense of touch became so fine that this direct method became perhaps the most accurate and convenient way of taking deflections. The micrometer used was marked to $\frac{1}{1000}$ in., but readings could be taken with some accuracy to $\frac{1}{10,000}$ in.

2. By means of a dial test indicator used in accurate machine tool work on the lathe for measuring eccentricity of the turning piece, and manufactured by Messrs Starrets. This was marked to $\frac{1}{2000}$ in., but readings to $\frac{1}{10,000}$ in. could be taken with almost dead accuracy. This little tester, no larger than a watch, was clamped to the side of the apparatus, and its pointer rested exactly on the centre of the flat plate. It was used chiefly on square and rectangular plates to obtain the pressures at which set first occurred, because it is a much more difficult matter to

catch this point in the case of square plates than of circular. In the latter, set begins at the same instant and equally for each fibre; but in the former, it begins only in a few fibres and gradually spreads to the rest.

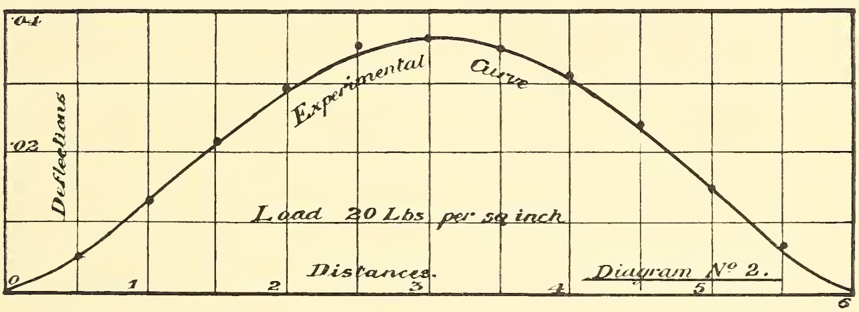
3. By means of a beam of light reflected from a mirror connected to the plate by a system of multiplying levers. This was chiefly used for the detection of zero errors and for obtaining permanent set curves. Before these permanent set curves could be drawn with accuracy, many months of experimenting was undertaken. It was found that, if as much as a drop of oil escaped from any of the connections during an investigation, the no-load reading was affected; should there occur an air-lock anywhere, there was a chance, during the working to and fro of the piston in the cylinder of the tester, that a little of this air could escape and thus affect the no-load result; when a certain quantity of oil had been admitted into the cylinder, that quantity had to be kept constant even though the flexure of the plate admitted more and more oil under it, thus using up the working fluid; the hand-wheel for moving the piston had always to be brought back to the same position, in this case against the stops; the weight-piston had to be turned to overcome static friction; in applying the weights to the weight-piston, the piston was allowed to rise slowly under small loads, and the final load reached gradually.

For the optical apparatus, two vertical standards were fixed in the mild-steel block into which the recess was cut for the plates, and a cross-bar was fitted to them, so that it lay right across the plate and above it. A flat was milled on this and a tiny groove made. A knife-edge to the top of which a lever was fixed rested in the groove made. From one end of the lever depended a round-ended bar which rested exactly at the centre of the plate, and on the other end was fixed a small circular mirror. A ray of light from an arc lamp was allowed to fall on the mirror and to be reflected to a screen a considerable distance away. In this way a very slight deflection of the plate gave a large movement of the spot, and the variation in position of the no-load deflection reading at the centre was ascertained. All the causes of irregularity were discovered with this apparatus, because the movement of the spot could be watched on the screen. The manipulation of the arc lamp, however, gave trouble, and, as absolute readings could be taken with sufficient accuracy with the micrometer, the apparatus was not largely used for direct measurement.

Results of Experiments on Circular Plates.

Results for Deflection for Plates fixed at the Circumference and having uniform load upon their areas. (All dimensions in inches and pounds.)

Plate	A, 6 in.	diameter,	thickness	·065 in.
„	B, 6 „	„	„	·065 „
„	C, 6 „	„	„	·055 „
„	D, 6 „	„	„	·066 „
„	E, 5 „	„	„	·061 „
„	F, 4 „	„	„	·067 „



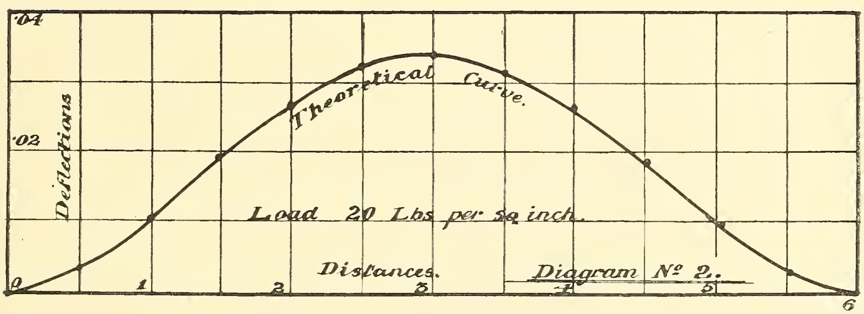
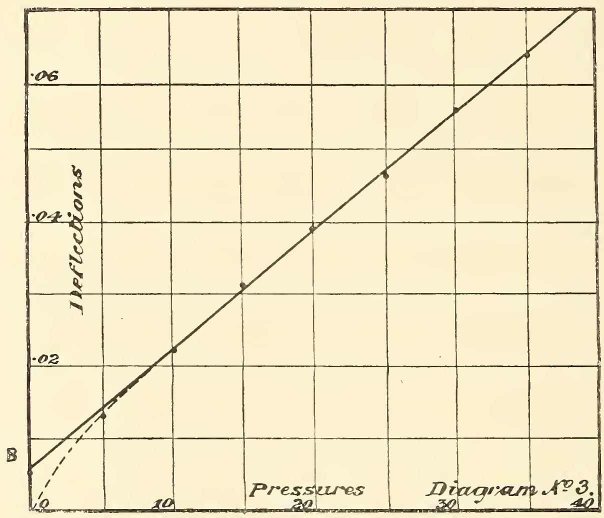


Diagram 2 is for plate A, and gives the theoretical and experimental deflection curves for a load of 20 lbs. per sq. in. It will be seen that the form of the two curves is almost identical, and as this was also conspicuous in the preliminary experiments, it will not be again referred to. The only point remaining for decision, with regard to deflection, is whether the deflection at the centre, as calculated from the equation, tallies with that found from experiment within small limits of experimental error. Diagram 3 shows, therefore, deflection at the centre plotted against load, and as the result is in the nature of a straight line, it is sufficient proof that the deflection varies directly with the pressure. In Diagram 3 there is a pronounced intercept OB, amounting to ·005 in., which, being interpreted, means that for no load the plate deflects this amount. It must be assumed that the plate is in a somewhat unstable state, similar to that of a wire which is being tested by direct loading for Young's modulus. The plate's initial irregular curvature corresponds to the slight and perhaps imperceptible kinks in the wire, which require a very small force for their removal, after which the wire follows the elastic law. The dotted portion of the line near the origin gives the hypothetical deflection there, and the straight line is made up by joining up the points that commence some slight distance away. It appears, then, that for little or no load the plate deflects at the centre an amount ·005 in. before it begins to follow the



elastic law, so that this quantity must be subtracted from the actual reading. When this is done at the centre and corresponding amounts at points along the diameter right up to the circumference, a curve is obtained for deflection that is almost indistinguishable from that obtained from the equation.

Plate B. Load 20 lbs. per sq. in.

At centre, theoretical value of deflection = ·0337 in.

„ experimental „ „ = ·039 − ·005 = ·034 in.

Plate C. Load 15 lbs. per sq. in.

At centre, theoretical value of deflection = ·0417 in.

„ experimental „ „ = ·054 − ·012 = ·042 in.

Plate D. Load 20 lbs. per sq. in.

At centre, theoretical value of deflection = ·0328 in.

„ experimental „ „ = ·040 − ·008 = ·032 in.

Plate E. Load 20 lbs. per sq. in.

At centre, theoretical value of deflection = ·0197 in.

" experimental " " = ·026 − ·006 = ·020 in.

Plate F. Load 20 lbs. per sq. in.

At centre, theoretical value of deflection = ·0061 in.

" experimental " " = ·008 "

Most of these results give a practical deflection almost identical with the theoretical. There were, however, a few cases in which there was a difficulty in determining accurately the intercept, owing to the shape of the curve near the origin. Just as to why some cases should show exceptions was at first very puzzling, for sometimes types of bad zero error were encountered.

Note on Zero Errors and the General Shape of the Deflection-Pressure Curve.

It was found that on every plate there was a zero error, and after much trial and experiment this was found to be due to two causes:—(*a*) A certain initial irregularity of the plate analogous to slight kinks in a wire. A small applied pressure took this out.

(*b*) A minute yielding of the fixed edges. At first sight it would seem impossible with the apparatus used that there could be the slightest *give*, especially when such pressure could be put upon the studs that the plate could be actually compressed at the edges. Yet it was often found that there was a minute slackness. With the larger size plates used this slackness was in general taken out by a few pounds pressure.

Diagram 4 has been drawn to indicate the nature of the zero error, and the general shape of the curve for the large-size plates. There are three main portions:—

1. From O to A. This is curved, and shows the effect of instability of the plate being removed.

2. From A to B. This is a perceptibly straight line and is the true elastic line. It usually commences a short distance from the origin and continues for a distance that depends upon the point at which permanent set begins for the plate. It does not cease at the permanent set point, but continues perceptibly straight up to a pressure from two to six times that which first produces set. This straight line is always present, is often very long, and is the line that must be taken to test the theory. When it is produced back on the vertical ordinate, it gives an intercept OS, and the value of this intercept must be subtracted from all readings on the straight-

line portion to obtain true values of deflection. The author has little doubt that an error due to slackness would occur even in riveted plates; for no matter how the plates were mechanically fixed to the retaining walls, there would probably be that minute *give* due to the limitations that are involved in making absolute and uniform contact between two surfaces, which, being magnified at the centre, especially with large plates, would produce a considerable zero error.

3. From B onwards. At a pressure varying from two to six times that which first produces permanent set, the elastic line AB begins to bend downwards, and there occurs the portion BC. The author believes this declension to be due solely to the arching of the plate; that above a certain

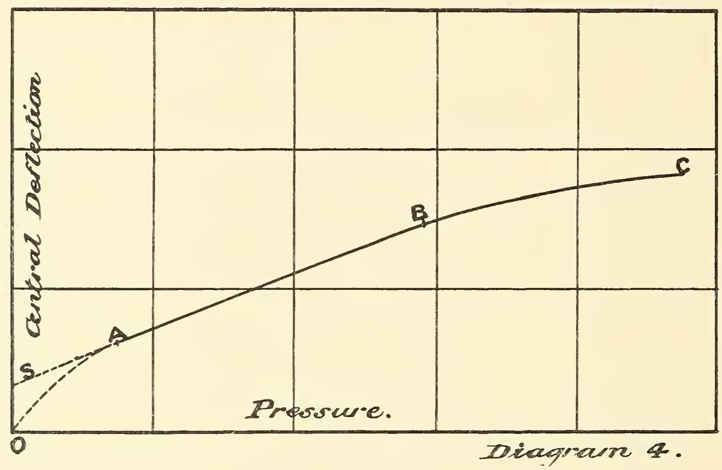


Diagram 4.

deflection the plate does not follow the flat-plate laws. In fact, from a flat it becomes an arched surface, and the resistance to bending increases.

For the smaller size plates the zero error is of a different nature. The initial want of flatness becomes negligible, and the error is almost solely due to give of the fixed edges. This initial slackness is not generally taken out by a few pounds pressure, but is only gradually removed. Hence readings for deflection for the first 20 or 30 lbs. pressure cannot be relied on, as they include not only the actual deflection reading, but the reading due to slackness as well. Therefore if values are taken from this portion of the curve they will be too high. If the graphs for plates E and F above be examined, it will be seen that the straight line has been drawn through the further points. These points are *not* on the falling portion of the curve, for with the smaller plates, especially the 4-in. size, it was found practically impossible to reach this region.

Turning once more to the results given, it will be seen that the experimental values for deflection are exceedingly close to the calculated ones. It is well known that such close results are obtained in ordinary experiments on the bending of beams, and are considered sufficient proof of the accuracy of the laws of bending. And there is no reason that with a sufficiently delicate apparatus the values for plates should be less accurate than those for simple beams.

As a final result for deflection, the following experiment is given. The plate was circular and 4 in. diameter. Its thickness was ·055 in., and certainly did not vary more than ·0002 in. on either side of this. The values on part of the straight line are as follows:—

Plate G.

Pressure in lbs. per sq. in.	Micrometer readings, in inches.	Difference in deflection, in inches.
65	·0655	0
70	·0635	·002
75	·0605	·003
80	·057	·0035
80 (new zero for oil adjustment).	·055	...
85	·053	·002
90	·050	·003
95	·047	·003
100	·045	·002
105	·042	·003
110	·040	·002
115	·037	·003
120	·034	·003
125	·031	·003
130	·028	·002

The sum of the differences gives ·0345 in. for a range of pressure of 65 lbs. This is equivalent to a deflection of ·01061 in. for a pressure of 20 lbs. per sq. in. The theoretical value is ·01101 in.

The 4 in. diameter circle was the most unfavourable for taking measurements, for, owing to its small size and the large amount of fixing area, the zero error was apt to be more troublesome than with the larger sizes. Bearing in mind this fact, the experimental and theoretical results are remarkably alike. And in general the author concludes that such a number of experimental results so closely focusing upon theoretical values could not have been obtained unless the theory was correct within half per cent. or less.

Since the deflection formula contains p, r^4, E, t^3, and constants, it might be sufficient to consider the matter proved. But it was determined to place it beyond all doubt by corroborative proof, if that was possible, and for this purpose it was necessary to fix within a few pounds at what pressure permanent set commenced in the plate. This was accomplished by measuring the pressure at which permanent set at the centre began.

Note on the Value of Permanent Set at the Centre as indicative of the Minutest Overstrain.

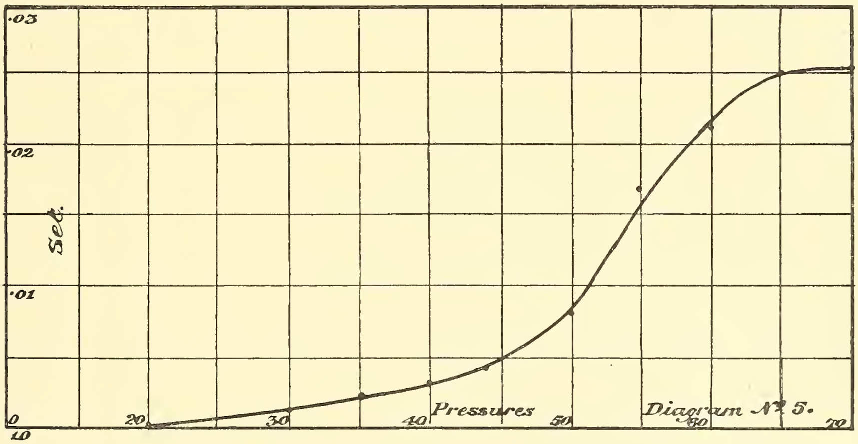
The question naturally arose as to whether, if set occurred at any point in the plate, it would be observed to the greatest degree by the alteration in the no-load deflection reading at the centre. Many experiments performed always gave a positive result, *i.e.* set was always first observed at the centre; not necessarily that set commenced at the centre, but that its results were observed there first. To see why this should be so requires some consideration, and for the purpose it is useful to think of a beam fixed at both ends and loaded uniformly. Suppose for the sake of illustration that the load is such that the outside fibres near the ends have passed the yield point and that the remainder of the beam is perfectly elastic. The bottom fibres are in tension and the top in compression at the end. Now, if the load is removed, the strained portion will not shorten to its original length, but will remain somewhat longer, with the result that the beam cannot become quite horizontal again, but will in its no-load state become slightly concave upwards. This concavity will be helped by the fact that the under side of the beam near the ends has passed its yield point on the compression side. The effects of concavity will be greatest at the centre. As more and more load is applied to the beam, the number of fibres that pass the yield point becomes greater and greater, and the no-load concavity increases proportionately. But when a certain pressure has been reached, the fibres at the *centre* begin to pass their yield points (because the maximum stress is not reached at the centre till later than at the ends), but in this case the top fibres are in tension, and the bottom in compression. When this point is reached there is then a kind of neutralising action, and the amount of concavity does not increase proportionately, and may even decrease a little. After a time, however, when most of the fibres have yielded, the concavity again increases. This reasoning is borne out by a consideration of the permanent set curves of circular plates. Diagram 5 gives such a curve. It will be noted that "set" increases gradually and proportionately for a time, then increases at a faster rate, then suddenly remains stationary. It is suggestive that this is owing to the neutralising action above referred to, and it is further suggestive that the method of taking the differences in the no-load deflection at the centre is a most delicate way of determining the point at which any part of the substance passes the yield point.

The two formulæ that give the maximum stress are:—

$$(1) \qquad f_1 = \frac{3}{4}\,\frac{r^2}{t^2} \cdot p.$$

$$(2) \qquad f_2 = \frac{2}{3}\,\frac{r^2}{t^2} \cdot p.$$

(The latter is only true if $m=3$; in the present case it requires to be multiplied by 1·01, but for simplicity of calculation the original form is retained and the slight correction made afterwards.)



No. (1) holds good if the elastic strength of the plate depends on the maximum principal stress, and No. (2) if it depends upon the maximum principal strain.

H is a plate 6 in. diameter, ·065 in. thick; when continued back, its permanent set curve cuts the horizontal ordinate at $p=20$.

I is a plate 6 in. diameter, ·066 in. thick; its permanent set curve cuts the horizontal ordinate at $p=20$.

J is a plate 6 in. diameter, ·063 in. thick; its permanent set curve cuts the horizontal ordinate at $p=19$.

Putting these values of p into the two formulæ, we have for

Plate H	(1)	$f_1=31{,}000$
	(2)	$f_2=27{,}500$
Plate I	(1)	$f_1=32{,}000$
	(2)	$f_2=28{,}400$
Plate J	(1)	$f_1=32{,}300$
	(2)	$f_2=28{,}700$

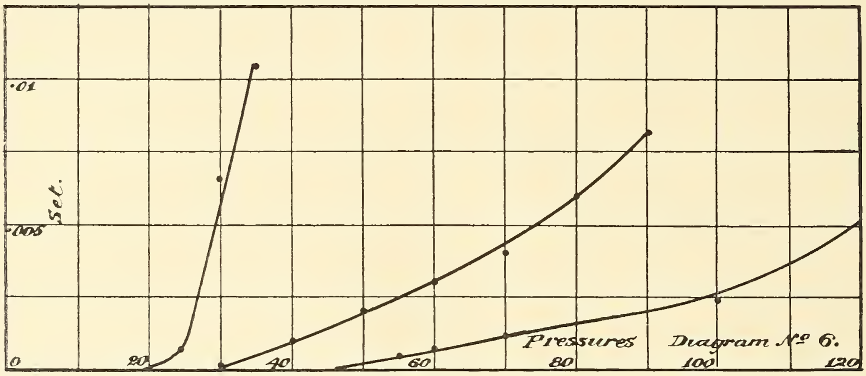
An average of the three gives for

(1) $f_1 = 31,800$
(2) $f_2 = 28,200$

and correcting (2) for the value of m by multiplying by 1·01, we have:

(1) $f_1 = 31,800$
(2) $f_2 = 28,500$

The theory, of course, only holds good up to the elastic limit of the material of the plate, but the author has found no perceptible breakdown in the law up to pressures much beyond those which first produce overstrain.



Now the natural yield point is 27,800 or thereabouts, and therefore the second of these formulæ, namely $f = \frac{2}{3}\frac{r^2}{t^2} \cdot p$, seems to be the correct one. It may be objected that it is impossible to tell where such curves as those given above cross the abscissa, owing to their tangential form when near it. This, of course, is true, and the only guide is to take as much of the curve as possible and endeavour by visual means to produce it back correctly. The probability is that the results are correct within two or three pounds per sq. in. pressure

Diagram 6 consists of the permanent set curves of three plates, J (6 in. diameter, ·063 in. thick), K (5 in. diameter, ·0645 in. thick), L (4 in. diameter, ·067 in. thick).

J. The curve cuts the horizontal ordinate at $p = 19$.

(1) $f_1 = 32,800$
(2) $f_2 = 28,700$

K. The curve cuts the horizontal ordinate at $p = 29$.

(1) $f_1 = 32,700$
(2) $f_2 = 29,000$

L. The curve cuts the horizontal ordinate at $p=46$.

$$(1)\quad f_1=30{,}700$$
$$(2)\quad f_2=27{,}300$$

The average of these results gives

$$(1)\quad f_1=31{,}900$$
$$(2)\quad f_2=28{,}600$$

No. (2) being corrected for m.

Again it appears that No. (2) is closer to the actual yield stress than No. (1), and that the formula $f=\frac{2}{3}\frac{r^2}{t^2}\cdot p$ is the more correct one.

The Question as to the Elastic Strength of the Plate.

The circular plate fixed at the edge and under uniform load on its area is an example of material under two principal stresses, circumferential and radial. At the circumference, where these are greatest, they are in the ratio of unity to Poisson's ratio. The maximum principal stress is therefore equal to the radial stress at the circumference.

The radial strain at the circumference is affected by the circumferential stress acting at right angles to the radial fibres. The strain thus induced by the combined action of the two stresses, if multiplied by Young's modulus, gives a measure of the elastic strength of the plate on the assumption that the maximum principal strain determines it.

The two theories then give for the plate

$$(1)\ \text{Maximum stress}\quad f_1=\frac{3}{4}\,\frac{r^2}{t^2}\cdot p,$$

$$(2)\ \text{Maximum strain}\quad f_2=\frac{2}{3}\,\frac{r^2}{t^2}\cdot p,$$

the latter being true if $m=3$. In the present case, with $m=3{\cdot}19$, the formula must be multiplied by 1·01.

The results stated above seem clearly to indicate, in the author's opinion, that the second of these is the more correct one, namely, that the elastic strength of the plate is a function of the maximum strain. He therefore arrives at the conclusion that, for a circular flat plate fixed at the circumference and uniformly loaded upon its area, the analysis as given in Thomson and Tait and elsewhere is correct and verified by experiment, and further, that of the two elastic strength theories, that of the principal strain more nearly accords with experimental results.

Square and Rectangular Plates fixed at the Edges and under Uniform Load on their Areas.

The experimental work that has been done on these plates seems to be very meagre, and there is perhaps nothing in the whole domain of mechanical engineering about which so little is known for certain, as the stress values in these common forms.

Recognising, then, that before there could be any certainty with regard to the matter, many plates would have to be tested and an average taken, the author decided to take up the matter as fully as possible. He had the more confidence in the work by reason of the results obtained for the circular plate.

The apparatus used for experimental work with these plates was exactly the same as for the circular, with the exception, of course, that the duplicate blocks between which the plates were held were of the required square or rectangular shape. The squares used were 6 in. by 6 in., $5\frac{1}{2}$ in. by $5\frac{1}{2}$ in., 5 in. by 5 in., $4\frac{1}{2}$ in. by $4\frac{1}{2}$ in., and 4 in. by 4 in.; the rectangles, 8 in. by 4 in., 7 in. by 4 in., 6 in. by 4 in., 5 in. by 4 in., and 8 in. by 6 in. There were thus five squares increasing regularly in length of side, and five rectangles having 4 in. as a basis for the short side, increasing regularly in length. These were thought enough to obtain sufficient points to determine the variation in stress with size and shape. The thickness of the plates also varied between limits which were small absolutely, but fairly large relatively; so that there were variations of size, thickness, and applied pressure.

The Square Plate.

As in the case of the circular plate, an attempt was first made to see how closely experimental central deflections were in agreement with theoretical values. The methods of detecting deflections were similar to those used for circular forms, and similar types of curves were obtained, the three main portions, the zero error, the elastic line, and the dip, being again conspicuous.

Grashof's formula for the deflection at the centre of a fixed rectangular plate is given by:

$$v=\frac{1}{32}\cdot\frac{p}{\mathrm{E}t^3}\cdot\frac{l^4b^4}{l^4+b^4},$$

where l is the length and b the breadth of the plate.

And for the square plate, where $l=b$, this reduces to:

$$v=\frac{1}{64}\cdot\frac{pb^4}{\mathrm{E}t^3}.$$

The following are the results obtained:—

Square plate.	Thickness.	Reference number.	Deflection corrected for zero error, for pressure 20 lbs per sq. in.	Theoretical deflection.
6×6	·065	1	·050	·04997
6×6	·069	2	·042	·04178
6×6	·067	3	·043	·04561
$5\frac{1}{2} \times 5\frac{1}{2}$	·064	4	·031	·03695
5×5	·065	5	·031	·02409
5×5	·055	6	·039	·039
5×5	·100	7	·008	·006616
5×5	·064	8	·026	·02523
5×5	·0635	9	·027	·02595
$4\frac{1}{2} \times 4\frac{1}{2}$	·061	10	·017	·01912
$4\frac{1}{2} \times 4\frac{1}{2}$	·070	11	·016	·01265
$4\frac{1}{2} \times 4\frac{1}{2}$	·066	12	·017	·01510
$4\frac{1}{2} \times 4\frac{1}{2}$	·065	13	·0165	·01581
4×4	·060	14	·015	·01254
4×4	·0635	15	·013	·01058

The results for the five plates may be stated concisely as follows:—

Square plate.	Average experimental deflection.	Average theoretical deflection.
6×6	·045	·04578
$5\frac{1}{2} \times 5\frac{1}{2}$	·031	·03695
5×5	·0262	·02417
$4\frac{1}{2} \times 4\frac{1}{2}$	·0166	·01567
4×4	·014	·01156
Grand average .	·0265	·0268

The average theoretical and experimental values are so close that there can be little doubt that the deflection formula for the square plate as given by Grashof is correct enough.

An attempt was made, however, to obtain the pressure at which set commenced for these plates. The maximum stress (or, what is equivalent to the maximum stress, the maximum strain by Young's modulus) is given by:

$$f = \frac{1}{4}\,\frac{pb^2}{t^2}.$$

It is to be noticed that, with the square plate, *one* fibre of the plate passes outside the elastic range first, and that there then follows a gradual elastic breakdown of the other fibres. In the circular plate, on the other hand, all the fibres at the circumference break down simultaneously, and set proceeds evenly on the application of further load. Hence, it is an extremely difficult matter to detect the pressure at which set *commences* in the square plate, and it is not possible to have accuracy of the same standard that can be obtained for deflections. Accordingly, the values of set given below can only be expected to *corroborate*, and not to verify, the results for deflection.

The following are the results obtained:—

Square plate.	Pressure.	Set at centre.	Thickness of plate.
6×6	15	·0003	·060
	20	·0007	
	25	·0012	
$5\frac{1}{2} \times 5\frac{1}{2}$	15	Just perceptible	·061
	20	·0005	
	25	·0007	
$4\frac{1}{2} \times 4\frac{1}{2}$	25	·0001	·066
	30	·0002	
	35	·0003	
	40	·0004	
	45	·0005	
	50	·00055	
	60	·0009	
	70	·0014	
	80	·0022	
4×4	20	·0002	·058
	25	·0005	
	30	·0007	
	35	·0008	
	40	·001	

These are summarised for convenience as follows:—

Square plate.	Pressure at which set commences (experimental).	Pressure at which set should commence (from formula).
6×6	15	11·13
$5\frac{1}{2} \times 5\frac{1}{2}$	15	13·68
$4\frac{1}{2} \times 4\frac{1}{2}$	25	23·92
4×4	20	23·37

These results are fairly close, and taken in conjunction with the deflection results (which were more susceptible of accurate treatment) seem, in the author's judgment, to verify the theoretical values as given by Grashof.

The Rectangular Plate.

The following are the results for deflection for various size rectangular plates, for a pressure of 20 lbs. per sq. in.:—

Rectangular plate.	Thickness.	Reference number.	Deflection corrected for zero error.	Theoretical deflection.
8 × 4	·0615	16	·026	·02192
8 × 4	·061	17	·027	·02247
8 × 4	·054	18	·033	·03238
7 × 4	·0615	19	·023	·02105
7 × 4	·061	20	·026	·02158
6 × 4	·0605	21	·018	·02043
6 × 4	·065	22	·020	·01648
5 × 4	·061	23	·019	·01693
5 × 4	·0615	24	·014	·01652
4 × 4	·060	14	·015	·01254
4 × 4	·0635	15	·013	·01058

The results for the five plates may be stated concisely as follows:—

Rectangular plate.	Average experimental deflection.	Average theoretical deflection.
8 × 4	·0287	·02559
7 × 4	·0245	·02131
6 × 4	·019	·01845
5 × 4	·0165	·01672
4 × 4	·014	·01156
Grand average .	·0205	·01872

It therefore appears that Grashof's theoretical deflection formula for the rectangular plate is nearly correct. However, it seems to the author that, as the length of the long side increases, the formula gives a value rather too low. The difference is, however, too slight to be of any practical importance.

Actual readings for set are given below. It is to be borne in mind that it is only the pressure at which set *commences* that is of any importance in the present investigation.

Rectangular plate.	Pressure.	Set at centre.	Thickness of plate.
8 × 4	20	·0002	·060
	25	·0004	
8 × 4	20	·0001	·061
5 × 4	25	·0001	·060
	30	·0002	
	35	·0005	

These results may be compared as follows:—

Rectangular plate.	Pressure at which set commences (practical).	Pressure at which set should commence (from formula).
8 × 4	20	13·29
8 × 4	20	13·73
5 × 4	25	17·62

The experimental results are too high. The parts of a rectangular plate that yield first are the extremities of the shorter diameter, and hence the accumulation of set must be very slow.

Graphic Analysis of Rectangular and Square Plates.

Being satisfied that the deflection at the centre of rectangular and square plates practically agrees with Grashof's formula, the author decided to make a practical analysis for curvature for a number of these plates. The method employed was the same for all. The curve of deflection, after

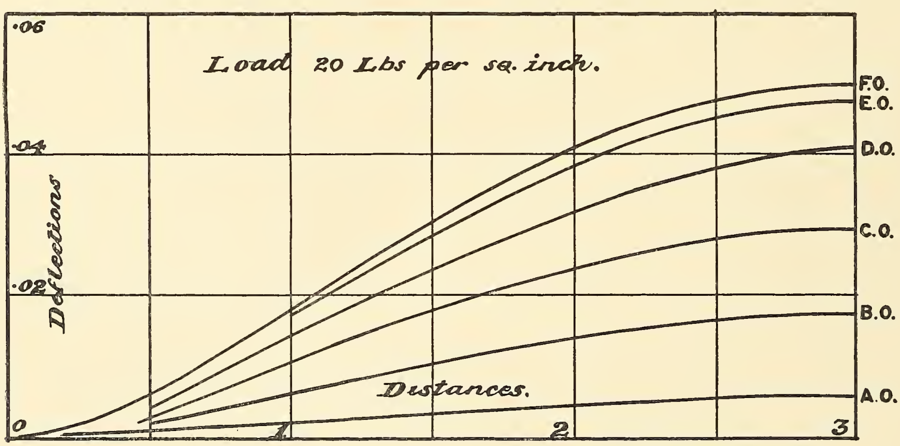
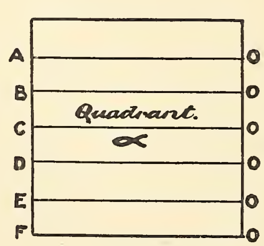


Diagram 7.



being corrected for error over its whole length, was set out on accurately squared paper. These deflection curves were taken not only along diameters, but also along various other directions in the plate. Tangents were then drawn to the deflection curves at numerous points, and thus readings for curves of slope obtained. The latter were then set out on squared paper, and, tangents being drawn as before, readings were obtained for curves of curvature. This work was done most carefully, and occupied several months. It was thought best to use such a practical method in preference to any method of finding equations to the curves by analysis, as any slight error in the continuity of the curves could be rectified by visual means, and, furthermore, accuracy of an engineering standard was all that was aimed at. The chief object, of course, was to obtain, in each case, the form of the curve of curvature, and thus to find out where the curvature was a maximum.

Diagram 7 gives the deflection contour lines for a square plate 6 in. by 6 in., thickness ·065 in., the curves being drawn for one quadrant. These curves are similar, of course, in both directions, the curvatures at the ends of the diameters being identical.

Diagram 8 shows the lines of deflection, slope, and curvature for a rectangular plate 8 in. by 6 in., thickness ·065 in., and loaded with 20 lbs. per sq. in.; the two last being derived from the former as explained above. The line OA is half the longer diameter.

Formulæ.

$E\frac{d^2y}{dx^2}$ is the extension in the direction of x (where E is Young's modulus), and $\frac{t}{2}E\frac{d^2y}{dx^2}$ (where t is the thickness of the plate) is a measure of the strength of the plate, assuming that this strength is proportional to the greatest strain. Calling this f_1, we have:

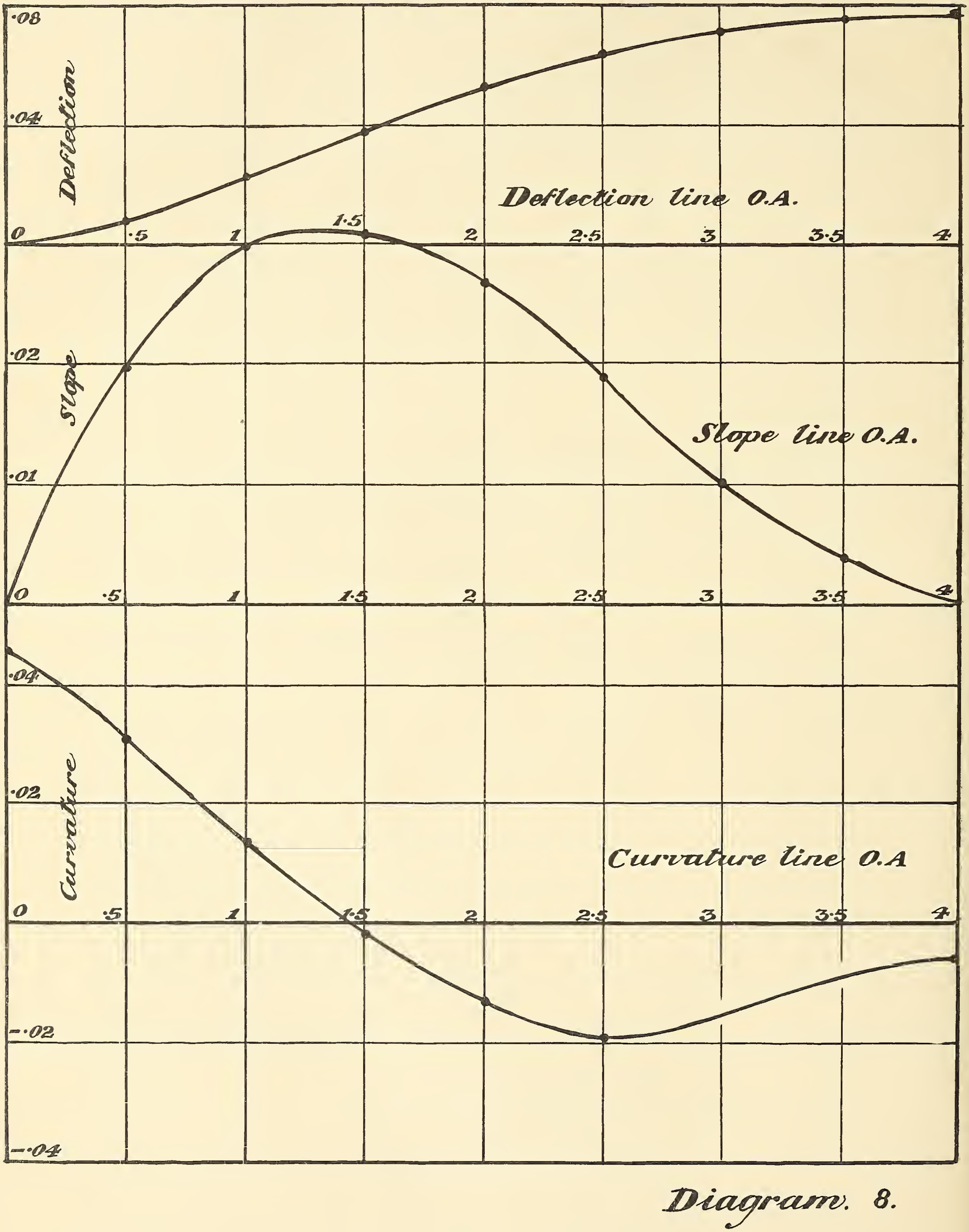
$$f_1 = \frac{t}{2}E\frac{d^2y}{dx^2},$$

and if $\frac{d^2y}{dx^2}$ be found from the lines of curvature at the ends of the diameters, f_1 can be calculated for these positions.

Now, Grashof's formula gives

$$f_2 = \frac{2b^4}{a^4+b^4} \cdot \frac{a^2}{t^2} \cdot p$$

as equivalent to the greatest stress at the ends of the long diameter, and



·08
·04
Deflection
0
·5
1
1·5
2
2·5
3
3·5
4
Deflection line O.A.
·02
·01
Slope
Slope line O.A.
0
·5
1
1·5
2
2·5
3
3·5
4
·04
·02
Curvature
0
·5
1
1·5
2
2·5
3
3·5
4
Curvature line O.A
−·02
−·04

Diagram. 8.

a symmetrical form for the greatest stress at the ends of the short diameter, where

a = half length of plate,
b = half breadth of plate,
t = thickness of plate,
p = applied pressure.

A comparison can thus be made between Grashof's theoretical results and the results obtained directly from the curves.

Tables for Graphical Analysis of Curvature for Rectangular and Square Plates.

General Explanation.—In all cases readings are for *one* quadrant of the plate. These readings are usually the average of those for all the quadrants. Diagram 9 shows generally along what lines measurements

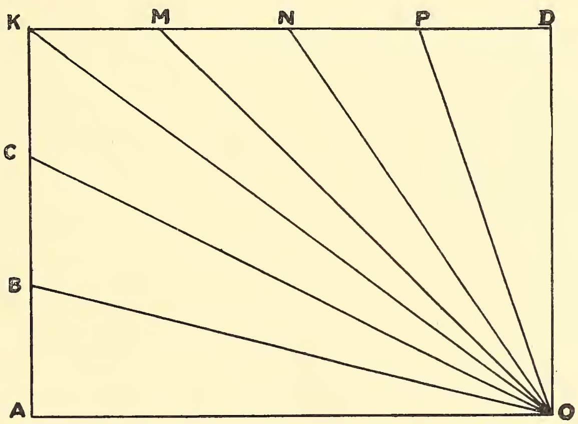


Diagram 9.

are taken. In the tables to follow, the points are given by means of polar co-ordinates, the origin being taken at the centre of the plate. The direction of the line is in each case given by the tangent of the angle; thus, in fig. 9, OA = $\tan^{-1} 0°$, OB = $\tan^{-1} ·25$, OC = $\tan^{-1} ·5$, OK = $\tan^{-1} ·75$, etc. In rectangular plates the axis of reference is along the long diameter.

[Table

Rectangular Plate, 8″ × 6″, Thickness ·065″. Load 20 lbs. per sq. inch.

Tan θ.	r.	Deflection.	Tan θ.	r.	Deflection.
0	0	·076	·25	0	·076
	½″	·074		·51	·074
	1″	·070		1·03	·070
	1½″	·062		1·54	·061
	2″	·052		2·06	·049
	2½″	·038		2·57	·036
	3″	·020		3·09	·019
	3½″	·008		3·60	·007
	4″	0		4·12	0
·5	0	·076	·75	0	·076
	·56	·074		·625	·073
	1·12	·068		1·25	·064
	1·68	·056		1·875	·048
	2·24	·041		2·5	·034
	2·79	·028		3·125	·016
	3·35	·014		3·75	·007
	3·91	·006		4·375	·001
	4·47	0		5	0
1·0	0	·076	1·5	0	·076
	·71	·073		·60	·073
	1·41	·057		1·20	·060
	2·12	·040		1·80	·044
	2·83	·021		2·40	·026
	3·53	·007		3·00	·010
	4·24	0		3·60	0
3·0	0	·076	∞ $\theta = 90°$	0	·076
	·53	·074		·5	·075
	1·05	·062		1·0	·063
	1·58	·046		1·5	·047
	2·11	·029		2·0	·030
	2·63	·0115		2·5	·012
	3·16	0		3·0	0

Slope and Curvature.

Tan θ.	r.	Slope.	Curvature.	Tan θ.	r.	Slope.	Curvature.
0	0	0	− ·006	·25	0	0	− ·006
	·5	·0045	− ·010		·62	·0042	− ·0145
	1·0	·010	− ·015		1·12	·015	− ·020
	1·5	·019	− ·020		1·62	·024	− ·014
	2·0	·027	− ·013		2·12	·027	− ·0083
	2·5	·031	− ·0019		2·62	·031	0
	3·0	·030	·011		3·12	·027	·013
	3·5	·020	·032		3·62	·019	·0275
	4·0	0	·046		4·12	0	·040

Slope and Curvature—contd.

Tan θ.	r.	Slope.	Curvature.	Tan θ.	r.	Slope.	Curvature.
	0	0	− ·009		0	0	− ·0155
	·47	·0045	− ·011		·5	·008	
	·97	·0115	− ·021		1·0	·015	− ·0146
	1·47	·025	− ·022		1·5	·022	
	1·97	·031	− ·001		2·0	·0265	− ·0086
·5	2·47	·0265	·010	·75	2·5	·029	
	2·97	·022	·013		3·0	·024	·018
	3·47	·0165	·013		3·5	·015	
	3·97	·010	·017		4·0	·0085	·011
	4·47	0	·021		4·5	·0036	
					5·0	0	·005
	0	0	− ·020		0	0	− ·024
	·24	·005			·6	·013	− ·022
	·74	·012	− ·018		1·1	·023	− ·017
	1·24	·022	− ·015		1·6	·029	− ·0015
1·0	1·74	·027	− ·0096	1·5	2·1	·032	0
	2·24	·029	0		2·6	·029	·020
	2·74	·0265	·0096		3·1	·018	·027
	3·24	·020	·018		3·6	0	·033
	3·74	·012	·023				
	4·24	0	·024				
	0	0	− ·031		0	0	− ·030
	·66	·019	− ·026		·5	·014	− ·026
	1·16	·030	− ·016		1·0	·025	− ·020
·0	1·66	·035	− ·008	∞	1·5	·034	− ·017
	2·16	·0375	·005	$\theta = 90°$	2·0	·040	− ·004
	2·66	·028	·040		2·5	·033	·038
	3·16	0	·072		3·0	0	·080

Rectangular Plate, 8″ × 6″, Thickness ·101″. Load 30 lbs. per sq. inch.

Tan θ.	r.	Deflection.	Tan θ.	r.	Deflection.
	0	·030		0	·030
	·5	·029		·5	·028
	1·0	·028		1·0	·0225
	1·5	·025		1·5	·018
0	2·0	·020	∞	2·0	·016
$\theta = 0°$	2·5	·015	$\theta = 90°$	2·5	·004
	3·0	·010		2·75	
	3·5	·003		3·0	0
	4·0	0			

Slope and Curvature.

Tan θ.	r.	Slope.	Curvature.	Tan θ.	r.	Slope.	Curvature.
	0	0	− ·0023		0	0	− ·010
	·5				·5	·005	− ·010
	1·0	·003	− ·005		1 0	·0105	− ·009
	1·5	·0077			1·5	·0142	− ·0073
0	2·0	·009	− ·006	∞	2·0	·015	·002
$\theta = 0°$	2·5	·012	− ·0026	$\theta = 90°$	2·5	·010	·015
	3·0	·012	·004		2·75	·006	
	3·5	·0075	·013		3·0	0	·030
	4·0	0	·018				

Rectangular Plate 8″ × 4″, Thickness ·069″. Load 35 lbs. per sq. inch.

Tan θ.	r.	Deflection.
0 $\theta = 0°$	0	·027
	·5	·0265
	1·0	·0260
	1·5	·0245
	2·0	020
	2·5	·015
	3·0	·011
	3·5	·004
	4·0	0
1·0	0	·027
	·71	·024
	1·41	·015
	2·12	·006
	2·83	0
·5	0	·027
	1·12	·0235
	2·24	·0120
	3·35	·0035
	4·47	0
·25	0	·027
	1·03	·024
	2·06	·0185
	3·09	·0065
	4·12	0
∞ $\theta = 90°$	0	·027
	·5	·024
	1·0	·015
	1·5	·007
	2·0	0

Slope and Curvature.

Tan θ.	r.	Slope.	Curvature.
0 $\theta = 0°$	0	0	−·002
	·5		
	1·0	·005	−·008
	1·5	·010	
	2·0	·019	−·014
	2·5	·021	
	3·0	·024	·0045
	3·5	·021	
	4·0	0	·038
·5	0	0	−·0062
	·47	·003	−·0064
	·97	·006	−·0062
	1·47	·0093	−·0043
	1·97	·0105	−·0011
	2·47	·01	·0025
	2·97	·007	·0062
	3·47	·0047	·0053
	3·97	·0023	·0046
	4·47	0	·004
∞ $\theta = 90°$	0	0	−·036
	·5	·015	−·020
	1·0	·020	−·008
	1·5	·0167	·038
	2	0	·060
·25	0	0	−·0040
	·62	·0033	−·0042
	1·12	·0047	−·0042
	1·62	·006	−·0042
	2·12	·0094	−·0047
	2·62	·011	−·001
	3·12	·010	·0046
	3·62	·0062	·0097
	4·12	0	·0145
1·0	0	0	−·015
	·33	·005	
	·83	·01	−·007
	1·33	·012	−·0026
	1·83	·012	·002
	2·33	·011	·00675
	2·58	·0063	
	2·83	0	·030

Rectangular Plate 8″ × 4″, Thickness ·065″. Load 20 lbs. per sq. inch.

Tan θ.	r.	Deflection.	Tan θ.	r.	Deflection.
	0	·0185		0	·0185
	·5	·0185		1·12	·015
0	1·0	·017	·5	2·24	·008
θ = 0°	1·5	·0163		3·35	·003
	2·0	·0145		4·47	0
	2·5	·011			
	3·0	·006			
	3·5	·003			
	4·0	0			
	0	·0185			
	·5	·015			
∞	1·0	·0095			
θ = 90°	1·5	·005			
	1·75				
	2·0	0			

Slope and Curvature.

Tan θ.	r.	Slope.	Curvature.	Tan θ.	r.	Slope.	Curvature.
	0	0	− ·0005		0	0	− ·0059
	1·5				·47	·003	− ·0047
	1·0		− ·00125		·97	·0045	
0	1·5	·0024	− ·0045		1·47	·0053	− ·0017
θ = 0°	2·0	·0056	− ·0077	·5	1·97	·0057	
	2·5	·0088	− ·0034		2·47	·0058	− ·0016
	3·0	·008	·004		2·97	·0042	
	3·5	·0062	·0084		3·47	·0028	·0025
	4·0	0	·013		3·97	·002	
					4·47	0	·004
	0	0	− ·027				
	·5	·010	− ·012				
∞	1·0	·0125	− ·003				
θ = 90°	1·5	·011	·012				
	1·75	·0077					
	2·0	0	·042				

Rectangular Plate 8″ × 4″, Thickness ·065″. Load 25 lbs. per sq. inch.

Tan θ.	r.	Deflection.	Tan θ.	r.	Deflection.
	0	·023		0	·023
	·5	·022		1·12	·018
	1·0	·0205		2·24	·0105
0	1·5	·018		3·35	·0015
θ = 0°	2·0	·0155	·5	4·47	0
	2·5	·012			
	3·0	·005			
	3·5	·0025			
	4·0	0			
	0	·023			
	·5	·0185			
∞	1·0	·0125			
θ = 90°	1·5	·0045			
	1·75				
	2·0	0			

Slope and Curvature.

Tan θ.	r.	Slope.	Curvature.	Tan θ.	r.	Slope.	Curvature.
	0	0	− ·002		0	0	− ·007
	·5	·00125			·47	·0031	− ·0053
	1·0	·0037	− ·0054		·97		
0	1·5	·0057			1·47	·0069	− 0026
$\theta=0°$	2·0	·0066	− ·0033	·5	1·97		
	2·5	·0087			2·47	·0077	·0015
	3·0	·0085	·002		2·97		
	3·5	·0062	·0072		3·47	·0042	·005
	4·0	0	·017		3·97	·0018	
					4·47	0	·0037
	0	0	− ·023				
	·5	·011	− ·019				
∞	1·0	·017	− ·009				
$\theta=90°$	1·5	·014	·021				
	1·75	·009					
	2·0	0	·046				

Rectangular Plate 8″ × 4″, Thickness ·055″. Load 15 lbs. per sq. inch.

Tan θ.	r.	Deflection.	Tan θ.	r.	Deflection.
	0	·023		0	·023
	·5	·0225		1·12	·016
	1·0	·023		2·24	·010
0	1·5	·020	·5	3·35	·0013
$\theta=0°$	2·0	·018		4·47	0
	2·5	·013			
	3·0	·008			
	3·5	·002			
	4·0	0			
	0	·023			
	·5	·019			
∞	1·0	·013			
$\theta=90°$	1·5	·005			
	1·75				
	2·0	0			

Slope and Curvature.

Tan θ.	r.	Slope.	Curvature.	Tan θ.	r.	Slope.	Curvature.
	0	0	− ·002		0	0	− ·0078
	·5	·0014			·47	·0035	
	1·0	·003	− ·0033		·97		
0	1·5	·004			1·47	·0077	− ·0026
$\theta=0°$	2·0	·0066	− ·006	·5	1·97		
	2·5	·009	− ·0046		2·47	·0083	·0023
	3·0	·011	0		2 97	·0057	
	3·5	·0075	·01		3·47	·0028	·0043
	4·0	0	·019		3·97		
					4·47	0	·0017
	0	0	− ·026				
	·5	·011	− ·017				
∞	1·0	·017	− ·005				
$\theta=90°$	1·5	·013	·016				
	1·75	·0085					
	2·0	0	·046				

Square Plate 6″ × 6″, Thickness ·065″. Load 20 lbs. per sq. inch.

Tan θ.	r.	Deflection.
0, $\theta = 0°$	0	·050
	·5	·0475
	1·0	·041
	1·5	·0295
	2·0	·0175
	2·5	·006
	2·75	
	3·0	0
1·0	0	·050
	·71	·0445
	1·41	·031
	2·12	·017
	2·83	·006
	3·53	·001
	4·24	0
·5	0	·050
	1·12	·037
	2·24	·012
	3·35	0

Slope and Curvature.

Tan θ.	r.	Slope.	Curvature.
0, $\theta = 0°$	0	0	− ·023
	·5	·0105	− ·020
	1·0	·020	− 019
	1·5	·026	− ·007
	2·0	·025	·007
	2·5	·020	·027
	2·75	·010	
	3·0	0	·047
1·0	0	0	− ·022
	·24	·005	
	·74	·016	− ·017
	1·24	·0205	− ·005
	1·74	·0205	·002
	2·24	·018	·00903
	2·74	·012	·011
	3·24	·008	·011
	3·74	·003	·008
	4·24	0	·004
·5	0	0	− ·020
	·35	·018	− ·026
	·85	·019	− ·014
	1·35	·021	− ·005
	1·85	·023	− ·004
	2·35	·018	·014
	2·85	·009	·022
	3·35	0	·019

Square Plate 5″ × 5″, Thickness ·065″. Load 20 lbs. per sq. inch.

Tan θ.	r.	Deflection.
0, $\theta = 0°$	0	·024
	·5	·023
	1·0	·018
	1·5	·0105
	2·0	·004
	2·5	0
1·0	0	·024
	·71	·021
	1·41	·013
	2·12	·0045
	2·83	·002
	3·54	0
·5	0	·024
	·56	·022
	1·12	·018
	1·68	·0095
	2·24	·0025
	2·80	0

Slope and Curvature.

Tan θ.	r.	Slope.	Curvature.	Tan θ.	r.	Slope.	Curvature.
0 θ=0°	0	0	−·014	·5	0	0	−·0125
	·5	·007	−·0135		·3	·004	
	1·0	·013	−·010		·8	·0085	−·010
	1·5	·016	0		1·3	·013	−·0055
	2·0	·012	·0158		1·8	·014	·0077
	2·5	0	·034		2·3	·007	·015
					2·8	0	·014
1·0	0	0	−·014				
	·54	·0077	−·0117				
	1·04	·012	−·006				
	1·54	·013	·0052				
	2·04	·0076	·0083				
	2·54	·004	·006				
	3·04	·0021					
	3·54	0	·003				

SQUARE PLATE 6″×6″, THICKNESS ·076″. LOAD 5 LBS. PER SQ. INCH.

Tan θ.	r.	Deflection.	Tan θ.	r.	Deflection.
0 θ=0°	0	·0078	·5	0	·0078
	·5	·007		·56	·0068
	1·0	·0062		1·12	·0052
	1·5	·0049		1·68	·0046
	2·0	·0029		2·24	·002
	2·5	·0013		2·79	·0011
	3·0	0		3·35	0

Slope and Curvature.

Tan θ.	r.	Slope.	Curvature.	Tan θ.	r.	Slope.	Curvature.
0 θ=0°	0	0	− 0034	·5	0	0	−·004
	·5	·0018	−·0028		·35	·0017	−·003
	1 0	·0025	−·0021		·85	·0021	−·0016
	1·5	·0028	−·001		1·35	·0031	0
	2·0	·0034	·0008		1·85	·003	0
	2·5	·003	·0036		2·35	·0026	·0015
	3·0	0	·008		2·85	·0017	·0033
					3·35	0	·0037

The following summary shows the results obtained for values of $\frac{t}{2}E\frac{d^2y}{dx^2}$ at the ends of the diameters of rectangular and square plates, with a comparison of the values obtained from Grashof's approximate formulæ. It is to be noted that these are the only points to which Grashof's formulæ apply.

Flat plates.			Grashof's maximum strain × E.		Experimental value of $\frac{t}{2}E\frac{d^2y}{dx^2}$.	
Size.	Thickness.	Pressure.	End short diameter.	End long diameter.	End short diameter.	End long diameter.
8 × 6	·065	20	64,710	36,180	76,780	44,150
8 × 6	·101	30	40,210	22,620	44,720	26,840
8 × 4	·069	35	56,650	13,840	61,120	38,710
8 × 4	·065	20	35,630	8,908	40,300	12,470
8 × 4	·065	25	44,540	11,140	44,150	16,310
8 × 4	·055	20	37,330	9,330	37,360	15,450
6 × 6	·065	20	42,600		45,100	
5 × 5	·065	20	29,580		32,630	
6 × 6	·075	5	7,791		8,997	

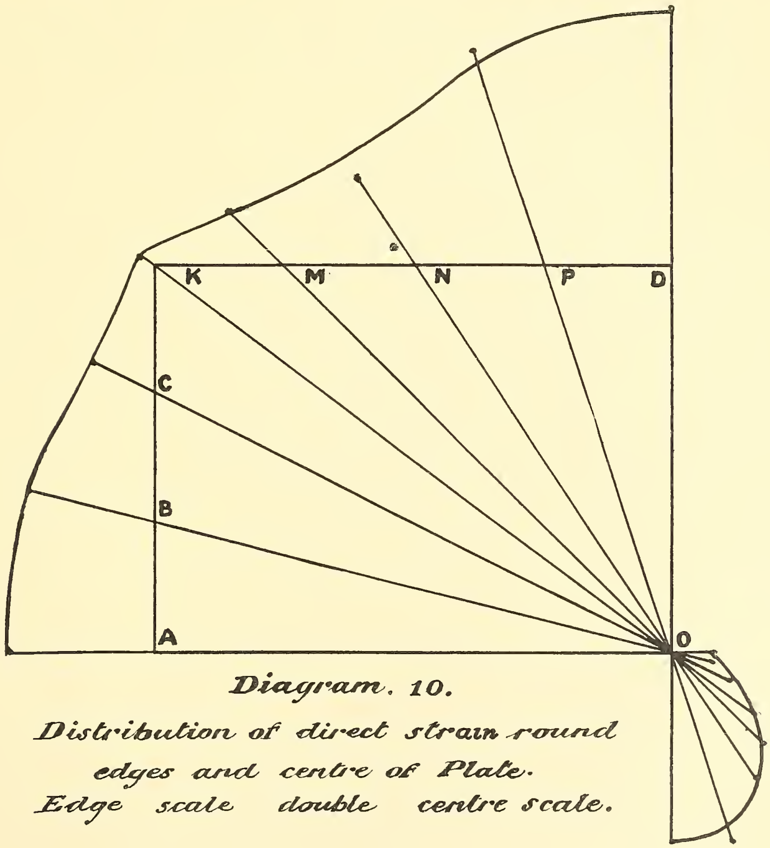


Diagram. 10.

Distribution of direct strain round edges and centre of Plate.

Edge scale double centre scale.

Diagram 10 shows the distribution of strain round the edge and at the centre of one quadrant of a rectangular plate.

From the above table it will be seen that in nearly every case the experimental value is greater than that given by Grashof, and the author believes engineers would be on the safe side if, when calculating stress values for square and rectangular plates, they multiplied Grashof's equations by 1·25.

The Position of the Ribs in Square and Rectangular Plates.

The analysis for curvature of these plates shows beyond doubt that the maximum stress is at the ends of the diameters. The natural position for ribbing would therefore be parallel to the sides, the primary ribs being at right angles across the centre, and *not* along the diagonals. Furthermore, the ribbing, if it altered at all, should be thickest at the edges and not at the centre. The function of ribbing, of course, is understood to prevent any portion of the plate from passing the elastic limit, and thus bringing the strength of flat plates into line with that of other engineering pressure members.

Some Effects observed during the Progress of the Investigation.

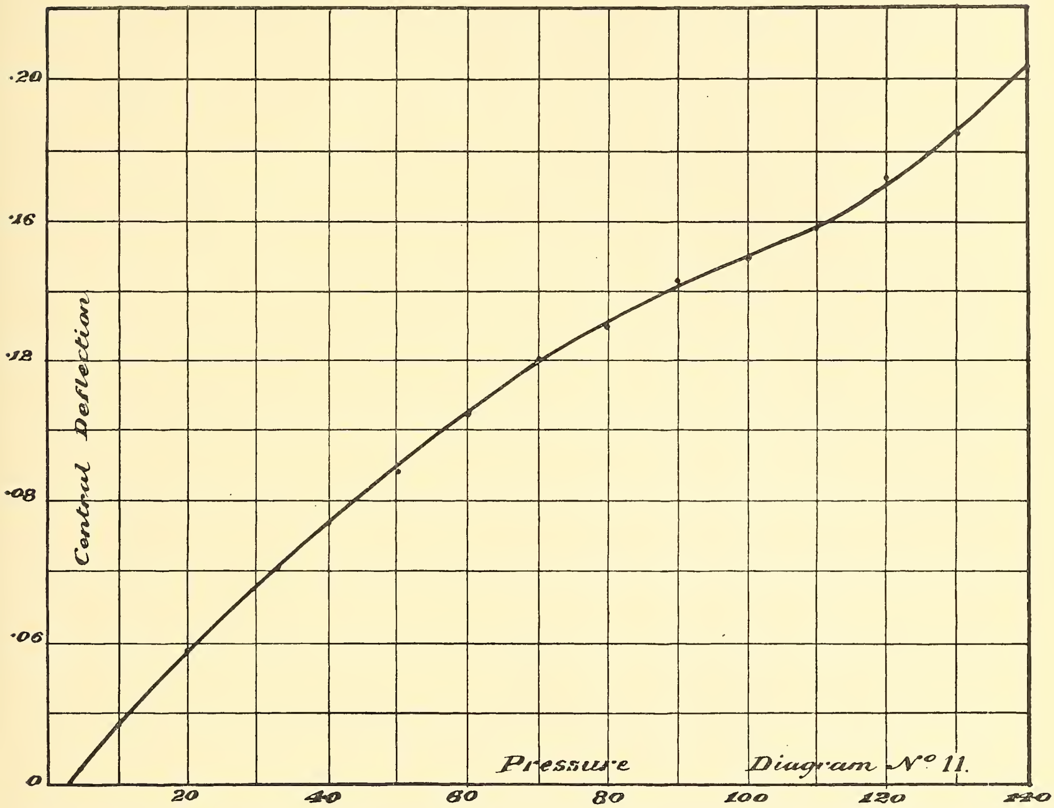
Reversing the Plate.—A circular plate 6 in. diameter and ·100 in. thick was tested for central deflection in the ordinary way, the maximum pressure used being insufficient to cause it to pass beyond the elastic range. The plate was then reversed and again tested, but no practical difference was noted in the magnitude of the deflections, except those produced by the low initial pressure.

Initial Slight Permanent Set.—A circular plate 6 in. diameter and ·066 in. thick was subjected to a pressure of 45 lbs. per sq. in., which produced a slight set. The pressure was then entirely removed and gradually reapplied. No practical difference was noted in the second series of deflection readings.

Slight Loosening of the Clamped Edges.—Several tests in which the fixing bars were not clamped hard down gave consistent results. In no case could a perfectly straight elastic line be obtained. The curve of pressure and central deflection bent slightly but persistently upwards.

Initial Heavy Permanent Set.—A plate 6 in. by 6 in. square and ·0653 in. thick was subjected to heavy set. The plate was then tested for deflection in the ordinary way. The result is given in Diagram 11. The general effect was to considerably lessen the deflections and make them below true values.

Dimensions such that Set begins almost from the first Application of Pressure.—Diagram 12 gives the deflection-pressure curve for a rectangular plate 8 in. by 6 in., ·064 in. thick. The elastic straight line cannot be discerned with accuracy. It is not only short, but is involved with the zero error.



SUMMARY OF RESULTS.

The results of this research, for flat plates fixed at the edges and under uniform load on their areas, are:—

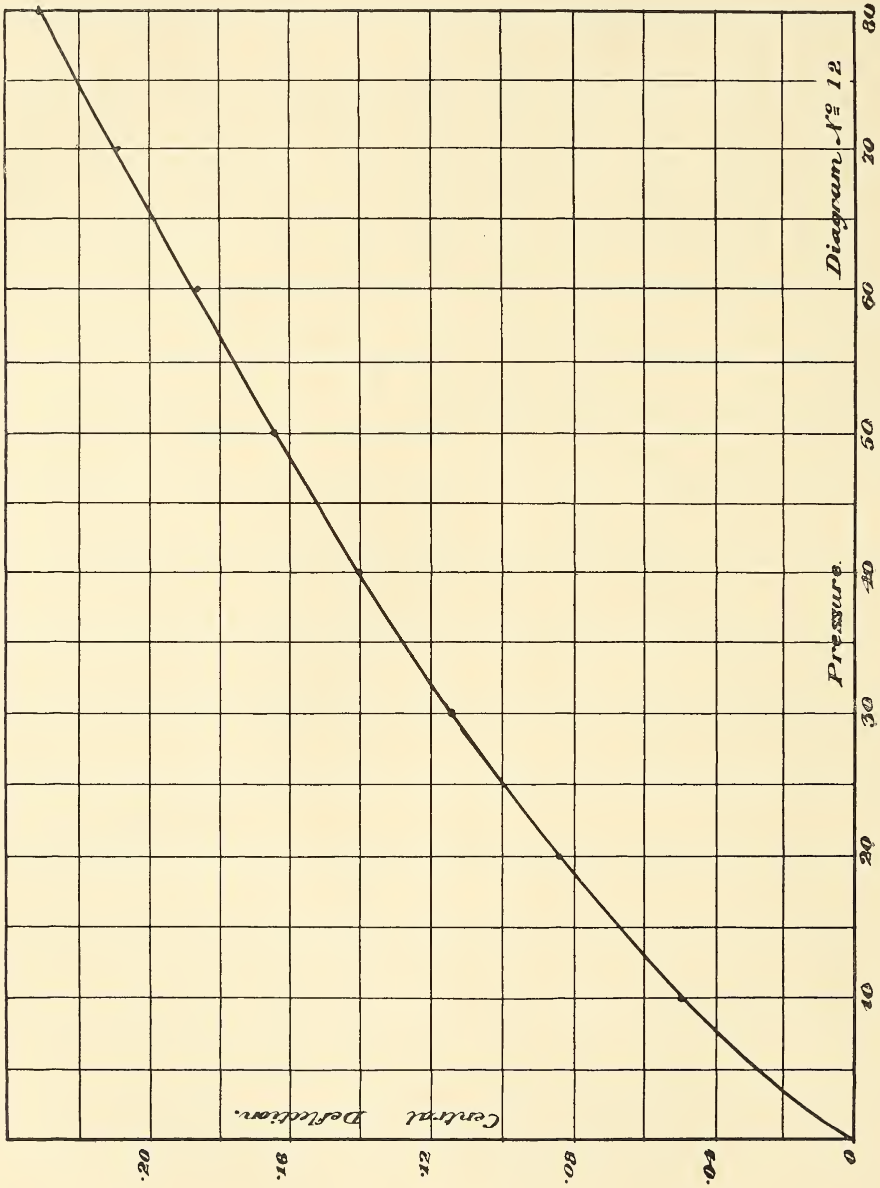
1. The analytical formulæ for stress, strain, and deflection in circular plates are correct.

2. The elastic strength of the circular plate is determined by the maximum principal strain rather than the maximum principal stress.

3. The formulæ for deflection given by Grashof for rectangular and square plates are practically correct.

4. In consideration of the practical results obtained for curvature, it would be well, in the case of rectangular and square plates, to multiply Grashof's formulæ for maximum stress by 1·25.

In conclusion, the author begs to state that this research was suggested to him by Dr J. H. Smith, Professor of Mechanical Engineering at the Belfast Municipal Technical Institute, who was of opinion that a thorough



experimental investigation of the subject would be of service to people who were chary of using the formulæ they could not prove, or for which no proof was obtainable.

The author desires also to express his thanks to Professor Love and to

Dr Dougall for their kindness in informing him of the latest phases of mathematical research on the subject; to Professor Morley for the loan of literature relating to the subject; and to Dr Bach of Stuttgart for the results of some of his tests on large riveted plates.

The author wishes also to record his thanks to Mr F. C. Forth, A.R.C.Sc.I., Principal of this Institute, for the kindly and sympathetic interest he has taken in what has proved to be a prolonged and laborious investigation.

Table I.—Circular Plates—Deflections.

The tabulated numbers are the pressures (lb./in.^{2}) and corresponding deflections at various points along a diameter of one brass and two mild steel plates of dimensions as shown, all lengths being in inches.

Brass Plate, Diameter 6″, Thickness $\frac{1}{8}$″.

Pressure.	Deflections at points measured from the centre						
	$2\frac{1}{4}$.	$1\frac{1}{2}$.	$\frac{3}{4}$.	0.	$\frac{3}{4}$.	$1\frac{1}{2}$.	$2\frac{1}{4}$.
10	·002	·004	·006	·007	·007	·004	·002
20	·002	·004	·009	·013	·011	·006	·003
25	·003	·010	·014	·017	·015	·011	·004
30	·005	·013	·017	·019	·017	·012	·005

	Deflections at points measured from circumference toward centre			
	$\frac{3}{16}$.	$\frac{3}{8}$.	$\frac{9}{16}$.	$\frac{3}{4}$.
30	·000	·001	·003	·004

Mild Steel Plate, Diameter 6″, Thickness $\frac{1}{8}$″.

Pressure.	Deflections at points distant from the centre					
	$2\frac{1}{2}$.	2.	$1\frac{1}{2}$.	1.	$\frac{1}{2}$.	0.
30	·0025	·004	·006	·0065	·007	·008
60	·0025	·0045	·008	·0115	·012	·012
80	·0035	·0095	·0155	·019	·023	·024
100	·0045	·0105	·018	·0235	·030	·030

Mild Steel Plate, Diameter 6″, Thickness $\frac{3}{16}$″.

Pressure.	Deflections at points distant from the centre					
	$2\frac{1}{2}$.	2.	$1\frac{1}{2}$.	1.	$\frac{1}{2}$.	0.
50	·000	·000	·002	·002	·003	·003
70	·000	·001	·003	·005	·005	·005
90	·001	·002	·004	·006	·006	·006
100	·001	·0025	·005	·006	·007	·008
130	·0015	·0035	·0065	·010	·0105	·0105
150	·0015	·005	·0095	·012	·0135	·0140

TABLE II.—PLANISHED STEEL PLATES (CIRCULAR)—DEFLECTIONS.

The tabulated numbers are the pressures (lb./in.^{2}) and corresponding deflections of the Plates A, B, C, D, E, F, the diameter d, the thickness t, and the deflections being in inches.

Plate.	Pressure.	Deflections at points distant from the circumference					
		$\frac{1}{2}$.	1.	$1\frac{1}{2}$.	2.	$2\frac{1}{2}$.	3.
A	10	·004	·007	·013	·016	·022	·021
$d=6$	15	·004	·010	·018	·023	·029	·029
$t=·065$	20	·0045	·013	·022	·028	·036	·036
	25	·008	·019	·029	·040	·053	·052

Plate.	Pressure.	Micrometer reading.	Central deflection.	Plate.	Pressure.	Micrometer reading.	Central deflection.
B	0	·273		C	0	·287	
$d=6$	5	·260	·013	$d=6$	5	·261	·026
$t=·065$	10	·251	·022	$t=·055$	10	·246	·041
	15	·242	·031		15	·232	·055
	20	·234	·039		20	·220	·067
	25	·226	·047				
	30	·217	·056				
	35	·209	·064	E	0	·206	
	40	·201	·072	$d=5$	5	·2015	·0045
	45	·189	·084	$t=·061$	10	·1945	·0115
	50	·182	·091		15	·188	·018
					20	·180	·026
					25	·175	·031
D	0	·315			30	·170	·036
$d=6$	5	·301	·014				
$t=·066$	10	·291	·024				
	15	·282	·033	F	0	·309	
	20	·274	·041	$d=4$	10	·304	·004
	25	·266	·049	$t=·067$	20	·301	·008
	30	·259	·056		30	·296	·013
	35	·253	·062		45	·288	·021
	40	·247	·068		55	·284	·023
	45	·240	·075		60	·282	·027
					70	·279	·030

Table III.—Planished Steel Plates (Circular)—Permanent Set.

The micrometer readings and permanent set measurements are taken after the load has been applied, and then reduced to the first (and lowest) value shown—load zero for Plates H, J, K; load 10 for Plate I.

Plate.	Load.	Micrometer reading.	Permanent set.
H	0	·273	0
$d=6$	10	·273	0
$t=·065$	20	·273	0
	30	·271	·002
	40	·266	·007
	50	·256	·017
J	0 to	·296	0
$d=6$	20	·296	0
$t=·063$	25	·295	·001
	30	·290	·006
	35	·286	·010
	40	·271	·025
	45	·257	·039
L	0 to	·3093	0
$d=4$	45	·3093	0
$t=·067$	55	·3088	·0005
	60	·3085	·0008
	70	·308	·0013
	100	·307	·0023
	120	·304	·0053

Plate.	Load.	Micrometer reading.	Permanent set.
I	10	·1525	0
$d=6$	20	·1525	0
$t=·066$	30	·1515	·001
	40	·149	·0035
	50	·143	·0095
	60	·130	·0225
	70	·126	·0265
	75	·1265	·0260
K	0	·078	0
$d=5$	10	·078	0
$t=·0645$	20	·078	0
	30	·0778	·0002
	40	·077	·001
	50	·076	·002
	60	·075	·003
	70	·0743	·0037
	80	·072	·006
	90	·070	·008

[Table IV.

Table IV.—Planished Steel Plates (Square).

The tabulated numbers are the pressures (lb./in.^{2}) and corresponding micrometer readings and central deflections of Plates Nos. 1 to 15, the length of side and thickness (t) being given in the first, fourth, and seventh columns, and all lengths being in inches.

Plate.	Pressure.	Deflection.
No 1	0	...
6×6	5	·024
$t=·065$	10	·042
	15	·054
	20	·067
No. 2	0	...
6×6	5	·014
$t=·069$	10	·027
	15	·038
	20	·048
	25	·058
	30	·069
No. 3	0	...
6×6	5	·0085
$t=·067$	10	·022
	15	·034
	20	·043
No. 4	0	...
$5\frac{1}{2}\times5\frac{1}{2}$	5	·0065
$t=·064$	10	·020
	15	·029
	20	·0385
	25	·0465
	30	·054
	35	·061
No. 5	0	...
5×5	5	·015
	10	·022
	15	·031
	20	·038
No. 6	0	...
5×5	5	·016
$t=·055$	10	·025
	15	·035
	20	·045
No. 7	0	...
5×5	5	·0035
$t=·100$	10	·0055
	15	·0075

Plate.	Pressure.	Deflection.
No. 8	0	...
5×5	20	·036
$t=·064$	25	·043
	30	·049
	35	·056
	40	·063
	45	·069
	50	·075
	55	·081
	60	·088
No. 9	0	...
5×5	5	·002
$t=·0635$	10	·010
	15	·016
	20	·0255
	25	·030
	30	·036
	35	·043
No. 10	0	...
$4\frac{1}{2}\times4\frac{1}{2}$	20	·0305
$t=·061$	30	·040
	35	·0455
	40	·0495
	45	·0535
	50	·0575
No. 11	0	...
$4\frac{1}{2}\times4\frac{1}{2}$	10	·012
$t=·070$	20	·020
	30	·028
	40	·036
	50	·044
	60	·053
	100	·088
No. 12	0	...
$4\frac{1}{2}\times4\frac{1}{2}$	10	·016
$t=·066$	20	·026
	30	·035
	40	·043
	50	·052
	65	·064
	100	·081

Plate.	Pressure.	Deflection.
No. 13	10	...
$4\frac{1}{2}\times4\frac{1}{2}$	20	·0105
$t=·065$	25	·0155
	30	·0205
	40	·0285
	50	·0375
	60	·0445
	70	·0530
	80	·0615
	90	·0715
	100	·0805
	120	·1005
No. 14	0	...
4×4	10	·012
$t=·060$	20	·022
	30	·029
	40	·038
	50	·044
	60	·053
	70	·059
	80	·065
	90	·071
	100	·078
	110	·086
	120	·093
	130	·101
	150	·117
No. 15	0	...
4×4	5	·0025
$t=·0635$	10	·007
	15	·011
	20	·014
	25	·017
	30	·020
	35	·024
	40	·027
	45	·029
	50	·0325
	55	·035
	60	·039
	65	·042

TABLE V.—PLANISHED STEEL PLATES (RECTANGULAR).

The tabulated numbers are the pressures (lb./in.^{2}) and corresponding central deflections of Plates Nos. 16 to 24, length, breadth, and thickness (t) being given in the first, fourth, and seventh columns, and all lengths being in inches, and pressures in lbs. per square inch.

Plate.	Pressure.	Deflection.
No. 16	0	...
8×4	10	·010
t=·0615	15	·016
	20	·023
	25	·029
	30	·036
	35	·042
	40	·049
	45	·0555
	50	·063
	55	·070
	60	·078
	70	·093
	80	·107
	90	·122
	100	·136
	110	·1505
	120	·164
No. 17	0	...
8×4	5	·006
t=·061	10	·012
	15	·021
	20	·026
	25	·034
No. 18	0	...
8×4	5	·005
t=·054	10	·016
	15	·0265
	20	·0365
	25	·046
	30	·053
	35	·061
	40	·069
	45	·078
	50	·086
No. 20	0	...
7×4	10	·009
t=·061	15	·018
	20	·023
	25	·031
	30	·037
No. 19	0	...
7×4	10	·007
t=·0615	15	·013
	20	·020
	25	·026
	30	·031
	35	·037
	40	·043
	45	·049
	50	·054
	55	·060
	60	·0655
	65	·072
	70	·078
	75	·084
	80	·090
No. 21	0	...
6×4	10	·002
t=·0605	15	·0085
	20	·015
	25	·0205
	30	·026
	35	·031
	40	·0365
	45	·042
	50	·047
	55	·052
	60	·056
	65	·061
	70	·0655
	75	·070
	80	·0745
	85	·079
	90	·083
	95	·088
	100	·093
	105	·096
	110	·101
	120	·1095
	130	·118
	140	·126
	150	·134
	155	·138
	160	·142
	170	·1495
	180	·1575
	190	·165
	200	·173
No. 22	0	...
6×4	10	·008
t=·065	20	·020
	25	·026
	30	·031
	35	·037
	40	·042
	45	·046
	50	·051
	55	·055
	60	·060
	65	·064
	70	·070
	80	·080
	90	·0895
	100	·099
No. 23	0	...
5×4	5	·002
t=·061	10	·007
	15	·012
	20	·0165
	25	·0225
No. 24	0	...
5×4	10	·005
	15	·010
	20	·015
	25	·019
	30	·024
	35	·030
	40	·034
	45	·0385
	50	·043
	55	·047
	60	·051
	65	·055
	70	·0585
	75	·062
	80	·066
	85	·070
	90	·073
	95	·077
	100	·0805
	105	·084

(*Issued separately September* 9, 1912.)

XXVI.—Comparison of Mr Crawford's Measurements of the Deflection of a Clamped Square Plate with Ritz's Solution. By Professor C. G. Knott (Secretary).

(MS. received July 24, 1912.)

When Mr Crawford's paper (see pp. 348-389) was passing through the press, Professor Love drew our attention to a paper by Walther Ritz, whose early death robbed the world of science of one whose brilliancy gave promise of a great future. This paper was published in *Crelle's Journal* in 1908 under the title, "Ueber eine neue Methode zur Lösung gewisser Variationsprobleme der mathematischen Physik." In illustration of this new method Ritz showed how to solve, by a series of successive approximations, the problem of the clamped rectangular plate.

I propose here to work out to the third approximation the numerical details of the deflections of a square plate subjected to a constant pressure on the one side.

Let z be the displacement of the point x, y, in the originally plane square plate of side a, referred to one corner as origin,—then Ritz's solution is, to the third approximation,

$$\frac{1}{l}z = 0\cdot674\xi_1\eta_1 + 0\cdot0308(\xi_1\eta_3 + \xi_3\eta_1) + 0\cdot0032\xi_3\eta_3$$
$$+ 0\cdot0040(\xi_1\eta_5 + \xi_5\eta_1) + 0\cdot0004(\xi_3\eta_5 + \xi_5\eta_3) + 0\cdot0000\xi_5\eta_5,$$

where $10^4 l/8a^4$ is proportional to the pressure, which is supposed to act uniformly on the one side of the square plate; where

$$\xi_n = \cos\frac{K_n x}{a} - \cosh\frac{K_n x}{a} - \left(\sin\frac{K_n x}{a} - \sinh\frac{K_n x}{a}\right)\frac{\cos K_n - \cosh K_n}{\sin K_n - \sinh K_n},$$
$$\eta_n = \text{similar function of } y,$$

and where K_n is the nth root of the equation:

$$\cos K \cosh K = 1.$$

Only the odd values of n enter into the expression for z; and for present purposes it will suffice to write down the values of K_1, K_3, K_5. These are:

$$K_1 = 3\pi/2 + 0\cdot01765 = 4\cdot7300$$
$$K_3 = 7\pi/2 + 0\cdot00003 = 10\cdot9956$$
$$K_5 = 11\pi/2 + \qquad = 17\cdot2787$$

With these values the following twelve values of ξ and y were calculated for x/a or $y/a = 1/8, 1/4, 3/8, 1/2$:—

$x/a=$	·125	·25	·375	·5	$=y/a.$
ξ_1	$-0·28234$	$-0·86278$	$-1·36283$	$-1·58425$	η_1
ξ_3	$-1·03858$	$-1·38050$	$+0·11758$	$+1·24600$	$_3$
ξ_5	$-1·50244$	$+0·52790$	$+0·78600$	$-1·41400$	$_5$

By combining these in various ways we can readily calculate the values of z/l corresponding to particular values of x and y. The following table gives these values for sixteen points in the quadrant of the square plate :—

$y/a.$	Values of z/l for various points x/a, y/a.				
·5	0·34140	0·95259	1·4043	1·5915	
·375	0·30950	0·84078	1·2335	1·4043	
·25	0·21357	0·57696	0·84078	0·95259	
·125	0·07868	0·21357	0·30950	0·34140	
	·125	·25	·375	·5	$=x/a$

For purposes of comparison with Mr Crawford's measured results, these numbers were divided by 1·5915, so that the maximum deflection at the centre of the plate was unity. It will suffice to calculate to three significant figures. The results are as follows :—

$y/a.$	Proportionate values of deflection.				
·5	·215	·599	·883	1·000	
·375	·195	·528	·776	·883	
·25	·134	·363	·528	·599	
·125	·050	·134	·195	·215	
	·125	·25	·375	·5	$=x/a$

It is useful to draw the graphs corresponding to these rows or columns of numbers, so as to interpolate for other values of x and y. Since the plate is clamped along each edge, each graph must begin tangential to the axis; and, since the central deflection is a maximum point, and the form of the bent plate is symmetrical about each diameter parallel to the sides, each graph must finish tangential to the axis. Guided by these considerations, we may draw the graphs with an accuracy as great as the numbers given.

Taking, then, Mr Crawford's measured deflections along a diameter in each of his three square plates, and reducing them all so that the central deflection is unity, we get the following comparison between experiment and Ritz's formula:—

Distance from centre.	Experiments.			Ritz's formula.
0·00	1	1	1	1
0·08	0·95		0·90	0·95
0·1		0·96		0·92
0·17	0·82		0·79	0·80
0·2		0·75		0·73
0·25	0·39		0·63	0·60
0 3		0·44		0·45
0·33	0·35		0 37	0·38
0·4		0·17		0·14
0·42	0·12		0·17	0·09
0·5	0	0	0	0

For two of the plates Mr Crawford gives a series of deflections along the diagonal. Taking these values and reducing as in the other cases, we find the following comparison:—

Distance from centre.	Deflections.		
	Experiments.		Ritz's formula.
0	1	1	1
$0·08 \times \sqrt{2}$		0·89	0·892
$0·1 \times \sqrt{2}$	0·875		0·85
$0·167 \times \sqrt{2}$		0·62	0·626
$0·2 \times \sqrt{2}$	0·542		0·52
$0·25 \times \sqrt{2}$		0·34	0·366
$0·3 \times \sqrt{2}$	0·187		0 22
$0·333 \times \sqrt{2}$		0·12	0·14
$0·4 \times \sqrt{2}$	0·083		0·02
$0·415 \times \sqrt{2}$		0·02	0·01
$0·5 \times \sqrt{2}$	0		0

The agreement throughout is, in the circumstances, quite satisfactory, and establishes the adequacy of Ritz's formula.

(*Issued separately September* 9, 1912.)

XXVII.—Experiments in Radioactivity; the Production of the Thorium Emanation and its Use in Therapeutics: Thorium X. By Dr Dawson Turner.

(Read June 3, 1912. MS. received June 14, 1912.)

The thorium emanation is given off more or less freely from the compounds of thorium, the best source, according to Professor Soddy, being a preparation of radiothorium in a moist condition. The important members of this group from the medical point of view are mesothorium, thorium X and its emanation.

The presence of the emanation can by shown by the ionising effect in discharging an electroscope, by the phosphorescence it imparts to zinc sulphide, and by its behaviour as a gas. The ionising effect can be produced at a considerable distance, and in consequence after a certain lapse of time. At a distance of ten feet there is a delay of about half a minute before the leaves begin to fall; when once the fall has begun, it continues steadily until the leaves are discharged. Experiments were performed to ascertain whether the presence of an electrified wire gauze screen placed between the source of the emanation and the electroscope would produce any recognisable effect. At first it appeared that when the screen was negatively electrified the rate of discharge was accelerated and *vice versa*, the rate of discharge being three seconds when the screen was negatively electrified, five seconds when unelectrified, and ten when positively electrified. On surrounding the electroscope with an earthed conductor this difference disappeared, so that the effects previously observed must have had an electrostatic origin. A powerful magnetic field interposed between the source of the emanation and the electroscope was also found to produce no recognisable effect.

There are various methods of preparing the active thorium compounds for therapeutic use.

1. By inhalation. In this case the medicament would enter the body by the lungs.

2. By ingestion.

3. By baths containing a solution of thorium X. It is found in bath treatment that considerable quantities of the active substance enter the body by inhalation as well as through the skin.

4. By local wet packs containing thorium salts.

5. By injection of radioactive water either into a tumour mass or into the veins. Czerny and Caan, in No. 14 of the *Münchener medizinische Wochenschrift*, narrate their experiences with mesothorium and thorium X. The latter was dissolved in physiological salt solution and injected either into the growth or into the veins of persons suffering from the growths.

Experiments carried out on animals showed that such experiments might easily be dangerous to life if too large doses were administered.

Thirty-six cases of tumours were treated, thirty-one carcinomas and five sarcomas; the strength of their solution of thorium X was such that 1 c.cm. equalled 1 to 3 Mache units.

The injections into the tumours were well borne. Twenty-four hours later a local swelling of the tumour occurred with pain and redness. This disappeared at the end of three days, and was followed by a diminution in the size due to the replacement of the cancer cells by dense connective tissue; a hæmorrhagic liquefaction sometimes took place.

When the intravenous injections were employed unexpected secondary or concomitant effects were sometimes observed, such as nausea, loss of appetite, dizziness, and weakness; no important organ suffered disturbance, nor was albumen found in the urine. The same swelling and subsequent shrinking of the tumour generally followed the intravenous injections. This seems to point to an elective action of the thorium X.

The effects produced by thorium X are due partly to the alpha rays it emits, and more particularly to the emanation of which it is the parent. The emanation, spreading by diffusion, conveys the action in every direction, and by disintegration coats the surrounding tissues with the active deposit which radiates alpha, beta, and gamma rays.

The author has only had the opportunity of trying, on 3rd May 1912, the thorium emanation in one case: a patient of Professor Caird suffering from an advanced excavating rodent ulcer. This case, which was inoperable, was recommended for radium treatment, but the area to be treated was too vast for the amount of solid radium in the author's possession. Under these circumstances the idea occurred of bringing the thorium emanation to the aid of the radium; the whole cavity was thoroughly sprayed with the thorium emanation some thirty times in the twenty-four hours, and 15 mgs. of pure radium were applied for twelve hours. Unfortunately the patient did not attend for further treatment, and has been lost sight of. The immediate effect of the application of the emanation seemed to be that of reducing the fœtor of the cavity.

Note.—Since the above paper was contributed the author has noticed that Dr Plesch is reported in the *Lancet* of 25th May 1912 to have treated two cases of suppuration of the accessory cavities of the nose by *inhalation* of the thorium emanation. The treatment was followed by a disappearance of the headache and a diminution in the secretion.

No previously recorded case of the therapeutic application *of a spray* of the thorium emanation is known to the author.

(*Issued separately September* 9, 1912.)

XXVIII.—Point Binomials and Multinomials in relation to Mendelian Distributions. By John Brownlee, M.D., D.Sc.

(Read November 20, 1911. MS. received February 16, 1912.)

THE subject of this paper is a consideration of the mathematics of some of the problems given by present-day developments of Mendelism. Breeding formulæ are becoming more and more complex, and it seems probable that many others are yet more complex. Without a suitable mathematical analysis it will be nearly impossible to analyse some of these by direct experiment, while a suitable analysis must always render the method of experimental attack much less obscure. Some of the formulæ in the present paper were calculated when I read my last paper * on this subject to this Society, but most of the developments seemed too remote from actual experimental work to render their publication useful. However, a paper published in the last number of the *Journal of Genetics*, by H. M. Leake, concerning the hybridisation of the cotton plant suggests that the time has come when they may prove of practical value.

Before approaching the mathematics, however, I think that some remarks on the notation of Mendelism may be profitably made. This notation seems to me specially cumbrous. When the elements are indicated by letters they are regularly written in sequence. Thus an organism containing three pairs of elements is denoted by (*aa bb cc*). If this mate with another of similar constitution but different properties, (AA BB CC), all the possible combinations of these appear in the second generation, and it becomes very fatiguing to the eye and brain to read the notation. I think that it would be preferable if the different elements which can combine were written in parallel rows. We have then the two arrangements above described denoted by

$$\left|\begin{array}{c} aa \\ bb \\ cc \end{array}\right| \qquad \left|\begin{array}{c} \mathrm{AA} \\ \mathrm{BB} \\ \mathrm{CC} \end{array}\right| ,$$

and when we come to consider such combinations as

$$\left|\begin{array}{c} a\mathrm{A} \\ \mathrm{BB} \\ c\mathrm{C} \end{array}\right| ,$$

* "The Inheritance of Complex Growth Forms on Mendel's Theory," *Proc. Roy. Soc. Edin.*, vol. xxxi. p. 251.

each acting part of the combination can be read at a glance. This will take up a little more space in printing, but the greater clearness will more than compensate.

So much for notation. There is, however, also a very easy use of symbolical multiplication which allows of the numbers of each special combination being immediately written down. As the F_2 generation is a complete picture of all possible combinations, this generation can always be at once written down directly.

If

$$\left|\begin{array}{c} aa \\ \\ bb \end{array}\right| \quad \text{and} \quad \left|\begin{array}{c} \mathrm{AA} \\ \\ \mathrm{BB} \end{array}\right|$$

are two races where a changes with A and b with B, we have the stable race, as far as (aa) and (AA) are concerned, represented by

$$(aa) + 2(a\mathrm{A}) + (\mathrm{AA})\,;$$

likewise that of (bb) and (BB), represented by

$$(bb) + 2(b\mathrm{B}) + (\mathrm{BB}).$$

If these be multiplied symbolically together, the numbers of any combination can be at once seen.

Thus those combinations containing the element (aa) are

$$\left|\begin{array}{c} aa \\ \\ bb \end{array}\right| \quad 2\left|\begin{array}{c} aa \\ \\ b\mathrm{B} \end{array}\right| \quad \left|\begin{array}{c} aa \\ \\ \mathrm{BB} \end{array}\right|,$$

and likewise the same holds for the other two terms.

In the same way symbolical multiplication may be applied to the mating of special cases. Thus, take the case given by Bateson on p. 193 of the current number of the *Journal of Genetics*, where the mating of (Ff Pp Ii ff Pp Ii) is discussed. Here, writing each gamete in columns, we have for the first eight possible gametes of the first parent

$$\left|\begin{array}{c} \mathrm{F} \\ \mathrm{P}, \\ \mathrm{I} \end{array}\right. \left|\begin{array}{c} \mathrm{F} \\ \mathrm{P}, \\ i \end{array}\right. \left|\begin{array}{c} \mathrm{F} \\ \mathrm{P}, \\ \mathrm{I} \end{array}\right. \left|\begin{array}{c} \mathrm{F} \\ \mathrm{P}, \\ i \end{array}\right. \left|\begin{array}{c} f \\ p, \\ \mathrm{I} \end{array}\right. \left|\begin{array}{c} f \\ p, \\ i \end{array}\right. \left|\begin{array}{c} f \\ p, \\ \mathrm{I} \end{array}\right. \left|\begin{array}{c} f \\ p, \\ i \end{array}\right.$$

and for the four possible gametes of the second parent, each duplicate,

$$\left|\begin{array}{c} f \\ p, \\ \mathrm{I} \end{array}\right. \left|\begin{array}{c} f \\ p, \\ \mathrm{I} \end{array}\right. \left|\begin{array}{c} f \\ p, \\ i \end{array}\right. \left|\begin{array}{c} f \\ p, \\ i \end{array}\right.$$

Multiplying symbolically, we have thirty-two different combinations. The first sixteen will be each unique, but the second half

$$\left\{\begin{vmatrix} f \\ p, \\ \mathrm{I} \end{vmatrix} + \begin{vmatrix} f \\ p, \\ \mathrm{I} \end{vmatrix} + \begin{vmatrix} f \\ p, \\ i \end{vmatrix} + \begin{vmatrix} f \\ p, \\ i \end{vmatrix}\right\}^2 .$$

With a little practice all the special forms existing may be easily worked out.

The general theorems on which this paper is based are given in the following lemmas. These in the form in which they are shown, so far as I know, are new, but they are so simple that it seems unlikely they have not been proved many times before. They depend on the well-known formulæ for the transference of the moments of a body round the centre of gravity to a point at a distance h therefrom, and on the values of the moments of a point binomial round the centre of gravity. If μ_2, μ_3, μ_4 be the moments of a curve round the centre of gravity, and μ'_2, μ'_3, μ'_4 be the moments round a point at a distance h, then

$$\begin{aligned} \mu'_2 &= \mu_2 + h^2 , \\ \mu'_3 &= \mu_3 + 3h\mu_2 + h^3 , \\ \mu'_4 &= \mu_4 + 4h\mu_3 + 6h^2\mu_2 + h^4 . \end{aligned} \qquad \text{(A)}$$

Also the moments of $(q+1)^n$ about the centre of gravity are given by

$$\mu_2 = \frac{c^2nq}{(q+1)^2} ,$$

$$\mu_3 = \frac{c^3nq(q-1)}{(q+1)^3} ,$$

$$\mu_4 = \frac{c^4nq}{(q+1)^2} \left\{ 1 + \frac{3(n-2)q}{(q+1)^2} \right\} .$$

Lemma I.—If an expression of distributed terms be multiplied by a second expression of distributed terms, the moments of the compound expression round the centroid vertical are given by

$$\begin{aligned} \mu_2 &= \zeta'_2 + \zeta''_2 , \\ \mu_3 &= \zeta'_3 + \zeta''_3 , \\ \mu_4 &= \zeta'_4 + \zeta''_4 + 2\zeta'^1\zeta''_2 , \end{aligned}$$

where ζ'_2, ζ''_2, etc., are the moments of the separate expressions, and μ_2, etc., those of the compound expression. Let the second expression be denoted by $a+b+c+ \ . \ . \ .$; then the centre of gravity of the compound expression is moved a distance equivalent to the distance of the middle point of the new expression from its centre of gravity. For the first can be conceived as concentrated at its centre of gravity, and if its mass be M, the distribution of the new expression will be equivalent to $M(a+b+c+...)$. We have therefore to obtain the moments of the compound expression round the new centre of gravity.

Let h_1, h_2, h_3, etc., be the distance of a, b, c, etc., from the centre of gravity then by the preceding formulæ (A)

$$M(a+b+c+\ ...)\mu_2 = M\zeta^2\Sigma a + M\Sigma ah^2\,,$$

or

$$\mu_2 = \zeta'_2 + \zeta''_2\,;$$

likewise

$$M(a+b+c+\ ...)\mu_3 = M\zeta_3\Sigma a + M\zeta_2\Sigma ah + M\Sigma ah^3 = \zeta'_3 + \zeta''_3\,,$$

since $\Sigma ah = 0$ by definition. So also

$$\mu_4 = \zeta'_4 + \zeta''_4 + 6\zeta'_2\zeta''_2\,.$$

And in general

$$\mu_2 = \Sigma\zeta'_2\,,$$
$$\mu_3 = \Sigma\zeta'_3\,,$$
$$\mu_4 = \Sigma\zeta_4 + 6\Sigma\zeta'_2\zeta''_2\,.$$

Lemma II.—The even moments μ_2 and μ_4 of $(1+q)^m\ (q+1)^n\ (1+1)^q\ (1+p)^k\ (p+1)^l$ etc., are constant if $m+n=$ const. and $k+l=$ const., and the odd moment μ_3 depends on the difference $m-n$, $k-l$, etc. This follows at once from the preceding since the second moment of $q+1$ is the same as those of $1+q$, while the third moment of these is equal but of opposite sign.

Lemma III.—If $(1+q)^m\ (q+1)^n\ (1+1)^p\ (1+p)^r\ (p+1)^s$ etc., be a distribution, then the first moments round an horizontal axis are the same if $m+n=c$, $n+s=c'$, etc. For, consider the distribution given by $a+b+c+...$, where a, b, and c are posited at equal distances m.

The first moment is

$$\frac{\dfrac{a^2}{m^2}+\dfrac{b^2}{m^2}+\dfrac{c^2}{m^2}+}{a\ +\ b\ +\ c\ +\ ...}.$$

Multiplying the distribution by $1+q$ and $q+1$ respectively, and limiting to three terms which is sufficient, we have the two expressions

$$a+(b+qa)+(c+qb)+qc,$$
$$qa+(qb+a)+(qc+b)+c.$$

The first moments are then

$$\frac{a^2+(b+qa)^2+(c+qb)^2+q^2c^2}{m^2(a+b+c)},$$

and

$$\frac{q^2a^2+(qb+a)+(qc+b)^2+c^2}{m^2(a+b+c)},$$

which are obviously identical.

The forms which the different point binomials tend to assume are interesting. As already shown by Professor Pearson, $(q+1)^n$ has the same slope as the curve given by his Type III., namely, by

$$y=\left(1+\frac{x}{a}\right)^{p} e^{-\gamma x}, \quad \text{where} \quad \gamma=\frac{p}{a}.$$

This curve has for the criterion

$$2\beta_2-3\beta_1-6=0,$$

where

$$\beta_1=\frac{\mu_3^2}{\mu_2^3} \quad \text{and} \quad \beta_2=\frac{\mu_4}{\mu_2^2}.$$

When the criterion is greater than zero, Type IV. arises. As just shown, the second and fourth moments of $(q+1)^p$ are the same as those of $(q+1)^m(1+q)^n$, when $m+n=p$; so that if the criterion be zero for the curve which represents the limit of $(q+1)^p$, it will be greater than zero for the curve representing $(q+1)^m(1+q)^n$, since the third moment of the latter is less than that of the former in the ratio $m-n$ to $m+n$, or, in other words, a curve having the same moment relationships as Type IV. will represent the result of mixed dominance.

I have not been able to prove that the limit of $(q+1)^m(1+q)^n$ is represented by

$$y=\frac{y_0}{\left(1+\frac{x^2}{a^2}\right)^m} e^{-\nu \tan^{-1}\frac{x}{a}},$$

but I think that it is highly probable that it is so, and, in addition, even if the limit is different, the curve must be nearly the same as that given by the Type IV. equation. This at once renders futile the class of criticism which, ignorant of mathematical principles, condemns Type IV. as inapplicable to practical problems on account of the imaginary roots in the denominator.

The form $(1+1)^n$ leads to the normal curve, as is well known.

The form $(1+n+1)^p$ has the moment relationships of Type IV., as shown in my former paper, and finally $(a+1+a)^p$ those of Type II.

The foregoing lemmas apply directly to the groupings which are possible on the Mendelian hypothesis. Four possibilities or combinations of possibilities have till now been experimentally ascertained or premised as the result of experiment:—

1. Blending;
2. Dominance;
3. Partial dominance;

and 4. Coupling or duplication of parts.

The first of these leads to groupings represented by the binomial or multinomial $(1+1)^p$ or $(1+0+1)^p$, etc. The second leads to groupings represented by $(3+1)^p$ or $(3+0+1)^p$, etc. The third to groupings represented by such multinomials as $(1+0+2+0+0+1)^p$, where the number of zeros is unequal on the two sides of the middle significant term. The fourth are represented by the groupings $1+2(n-1)+1$ and $\overline{n-1}+2+\overline{n-1}$, either alone or in combination, when n is a positive power of 2.

1. *Blending.*—Blending is known in many instances, as when a red cow mating with a white gives a roan; but of quantitative blending, to which the present analysis applies, I have not found many examples. One is given by H. M. Leake in the paper already referred to.* In this case he shows that when plants with a different leaf factor are mated the leaf factor of the cross is the arithmetical mean. When his diagram is examined it is found that he mated plants of a certain range of leaf factor with others of a like range of higher value, with a distance between the two approximately equal to the range of each type. For a first approximation each type can be represented by the distribution $1+2+1$, so that the F_2 generation will be given by

$$(1+2+1+0+0+1+2+1)^2,$$

or

$$1+4+6+4+1+2+8+12+8+2+1+4+6+4+1.$$

This is approximately what is obtained on experiment, but there is more compression, so that, although in fact the units of each type fell inside the limits chosen, many probably contained elements which, when pure, ranged beyond these limits. If we write, instead of the above,

$$(1+2+1+0+1+2+1)^2,$$

or

$$1+4+6+4+3+8+12+8+3+4+6+4+1,$$

the result will be nearly that shown by the diagram.

2. *Dominance.*—If dominance exist, the proportions in which the numbers will turn up is as 3 to 1, so that the general formula for a mixed dominance will be $(3+1)^m\ (1+3)^n$. This in general approximates to Type IV. Data on which to make analysis are very scarce, so scarce that I have not thought it worth while to carry out the arithmetical work necessary to test individual cases. There is, however, one case given by Mr Leake in the *Journal of Genetics* (*loc. cit.*), concerning the length of the vegetative period in hybrid cottons, which is sufficiently interesting to require an approximate solution. Here dominance obviously comes specially

* *Journal of Genetics*, vol. i. p. 227.

from one side. If the distribution given by Mr Leake be divided into six portions containing each four of the units and fitted to $(3+1)^3\,(1+3)^2$, we have the following comparison:—

Actual	+5	12	55	168	95	31	15	1·5
Theoretical	0	10	69	152	109	32	4	

This is a very good fit, considering the nature of the experiments, for the data do not admit of any very accurate analysis. The length of the vegetative period is dependent on the season, and the original crossings have obviously been selected, so that any better result could hardly be hoped for.

It is to be noted in this connection that the standard deviation of $(3+1)^m\,(1+3)^n$ is constant if $m+n=$ a constant, so that it affords a strict means of comparison between symmetrical and skew distributions if dominance holds. The degree of mixture of race cannot, however, be compared by this means, if blending holds in one case and dominance in another. The simplest case is that of the races blending with regard to four qualities, and is represented by $(1+1)^4$. The same when mixed dominance holds is $(3+1)\ (1+3)$; the extreme range is the same in both instances by hypothesis, yet in the former case $\mu_2=1$, and in the latter $\mu_2=1{\cdot}5$, so that between two equally mixed races a 50 per cent. difference might be observed.

Concerning cases 3 and 4 there are no data, so that further discussion is at present unnecessary.

Some remarks may be made in conclusion regarding the methods which should be observed in making experiments. Though no race is approximately pure, it is most probable that the extremes of each race are more pure than the members between. In making cross-fertilisation experiments, then, the extremes should be chosen, as in that way the mixtures will be much more easily analysed mathematically. When, as in Mr Leake's experiments, individuals are chosen from each race by special selection, it is hardly likely that results will be obtained which give much hope of satisfactory analysis.

[*Table*

Table of the Moments of the Principal Forms discussed.

(These, for any complex series, are sufficient by help of the preceding formulæ.)

Form.	μ^2.	μ^3.	μ^4.
$(1+1)^n$	$\frac{c^2 n}{4}$	0	$\frac{c^4 n}{4}\left\{1+\frac{3(n-2)}{4}\right\}$
$(q+1)^n$	$\frac{c^2 nq}{(q+1)^2}$	$\frac{c^3 nq(q-1)}{(q+1)^3}$	$\frac{c^4 n}{(q+1)^2}\left\{1+\frac{3n-2q}{(q+1)^2}\right\}$
$(1+p+1)^n$	$\frac{2nc^2}{p+2}$	0	$\frac{2pn+4n(3n-2)}{(p+2)^2}c^4$.

Appendix on Special Cases of Blending.

If a race with two elements denoted by (aa) be crossed with a race (bb) we have the stable population given by the proportions

$$(aa)+2(ab)+(bb).$$

This gives twice as many organisms having the mean quality as either extreme. In like manner take another race of greater stature, say $(cc)+2(cd)+(dd)$, and let them mate at random with the first population. The stable population will obviously consist of the proportions of

$$\begin{aligned}(aa)+(bb)+(cc)+(dd)&\\ +2(ab)+2(ac)+2(ad)&\\ +2(bc)+2(bd)&\\ +2(cd)&\,.\end{aligned}$$

There is a great number of special cases, all, however, agreeing in that the range is between (aa) and (dd), taking these as the extreme types, and that each mixed element has properties equivalent to the arithmetical mean of the elements.

(1) In the first case, let $(bb)=(cc)$ in the quality examined. The grouping will then be as follows:—

$$\begin{aligned}&(aa)+2(ab)+(bb)\\ &\qquad\qquad(cc)+2(cd)+(dd)\\ &\quad 2(ac)+2(ad)+2(bd)\\ &\qquad\quad 2(bc)\,,\end{aligned}$$

or

$$1\;+\;4\;+\;6\;+\;4\;+\;1.$$

In this case the permanent population is approximately normal, and only one mode appears. If the quality depend on two elements in each, if it is defined by

$$\left|\begin{matrix}aa\\ \\ bb\end{matrix}\right|\quad\left|\begin{matrix}\mathrm{AA}\\ \\ \mathrm{BB}\end{matrix}\right|,\ \text{etc.}\,,$$

the ordinary theorem of chance gives the distribution

$$(1+4+6+4+1)^2,$$

and so on.

(2) In this case, let $(ab)=(cc)$. The grouping is then as follows:—

$$\begin{array}{ccccccc}
(aa) & & +2(ab) & & +(bb) & & \\
 & & (cc) & & +2(cd) & & +(dd) \\
 & 2(ac) & & 2(bc) & & 2(bd) & \\
 & & & 2(ad) & & &
\end{array}$$

or

$$1 \;+\; 2+3 \;+\; 4 \;+\; 3+2 \;+\; 1,$$

or

$$(1+1+1+1)^2.$$

Again a unimodal curve shows itself. If the quality depends on n pairs of elements, then the general distribution is given by the multinomial,

$$(1+2+3+4+3+2+1)^n.$$

This quickly approximates to the normal curve.

(3) Let (cc) fall between (aa) and (ab); the grouping is then:—

$$\begin{array}{ccccccccccc}
(aa) & & & & +2(ab) & & & & +(bb) & & \\
 & & (cc) & & & & +2(cd) & & & & +(dd) \\
 & 2(ac) & & & & 2(cb) & & & & 2(bd) & \\
 & & & & & 2(ad) & & & & &
\end{array}$$

or

$$1 \;+\; 2+1+0 \;+\; 2+4 \;+\; 2+0 \;+\; 1 \;+\; 2 \;+\; 1,$$

or

$$(1+1+0+0+1+1)^2,$$

or we have in this case a multimodal curve.

(4) Let (cc) fall between (ab) and (bb); the grouping is then:—

$$\begin{array}{ccccccccccccccc}
(aa) & . & . & . & 2(ab) & . & . & . & (bb) & & & & & & \\
 & & & & & & & (cc) & . & . & . & 2(cd) & . & . & . \;\; (dd) \\
 & & & 2(ac) & & . & . & . & 2(bc) & . & . & . & 2(bd) & & \\
 & & & & & & & & 2(ad)\,, & & & & & &
\end{array}$$

$$1+0+0 \;+\; 2 \;+\; 2 \;+\; 0+1 \;+\; 4+1+0+2 \;+\; 2+0+0+1,$$

or

$$1+0+0+1+1+0+0+1)^2,$$

which again ultimately approaches normality. This curve, composed of two nearly equal races, has three modes.

(5) Let (cc) be greater than (bb), and we have the following, when a dot indicates a gap of one unit:—

Case (a).

$$(aa)+2(ab)+(bb)+\ \cdot\ +(cc)+2(cd)+(dd),$$

or

$$\begin{gathered}2(ac)+2(bc)+2(bd)\\ +2(ad),\end{gathered}$$

or

$$1 + 2 + 3 + 4 + 3 + 2 + 1,$$

or Case (2) from a different reason.

Case (b).

$$\begin{array}{cccccccc} (aa)+2(ab)+(bb)+ & \cdot & + & \cdot & + & \cdot & +(cc)+2(cd)+(dd) \\ & 2(ac) & & 2(ad) & & 2(bd) & \\ & & & 2(bc)\,, & & & \end{array}$$

or

$$1 + 2 + 1 + 2 + 4 + 2 + 1 + 2 + 1,$$

or

$$(1+1+0+1+1)^2 .$$

Case (c).

$$\begin{array}{cccccccccc} (aa)+2(ab)+(bb)+ & \cdot & + & \cdot & + & \cdot & + & \cdot & + & \cdot\ (cc)+2(cd)+(dd) \\ & & & 2(ac) & & 2(ad) & & 2(bd) & & \\ & & & & & 2(bc) & & & & \end{array}$$

or

$$1 + 2 + 1 + 0 + 2 + 4 + 2 + 0 + 1 + 2 + 1,$$

or Case (3) again.

(*Issued separately October* 31, 1912.)

XXIX.—On Novel Illustrations of Gyrostatic Action. By James G. Gray, D.Sc., Lecturer on Physics in the University of Glasgow.

(Read May 6, 1912. MS. received July 7, 1912.)

In this paper are described some new spinning-tops, gyrostatic apparatus, and illustrations of gyrostatic action devised by the author in the course of the last nine months. The apparatus has been built in the workshops of the Natural Philosophy Institute of the University of Glasgow, and has been employed in illustrating the lectures on rotational motion delivered in the Natural Philosophy classes in the course of the session just closed.

Fig. 1 shows a model aeroplane fitted with a small gyrostat in the position of the propeller. The gyrostat is therefore fixed on the front of the aeroplane with the plane of its flywheel at right angles to the length of the machine. The flywheel of the gyrostat, so fitted, represents exactly the rotor of the engine and the propeller of an actual aeroplane. The model, with its attached gyrostat, is suspended by means of a thin string so as to hang in a horizontal position. If the flywheel is spinning in either direction and the aeroplane is struck a small blow so that the whole model is made to turn quickly in a horizontal plane about the string, it will be found that for one direction of this turning the front end of the aeroplane rears up and the back end descends; for the other direction in which the aeroplane may be turned the front end of the aeroplane dips down and the back end rears up.

This experiment is of great practical value as illustrating a danger which persons not conversant with gyrostatic action would hardly foresee. The rotor of the engine and the propeller of an aeroplane combine to form a powerful gyrostat, the effect of which is to introduce a gyrostatic couple if the aeroplane is caused to turn in a horizontal plane. If G be the combined moment of inertia of the rotor and propeller, ω_1 their angular velocity, the magnitude of the gyrostatic couple tending to cause diving or rearing up of the aeroplane, at an instant at which the aeroplane is turning in azimuth with speed ω_2, is $G\omega_1\omega_2$. If the aeroplane is to be maintained in a nearly horizontal position use must be made of the tilting planes, and this necessitates stresses being set up in the framework of the machine. To avoid straining of the machine, ω_2 should be kept small—that is, the aeroplane should not be turned suddenly. These considerations show conclusively that aeroplane flights are accompanied by considerable danger from the gyrostatic action of the propeller, etc., if proper precautions are not taken.

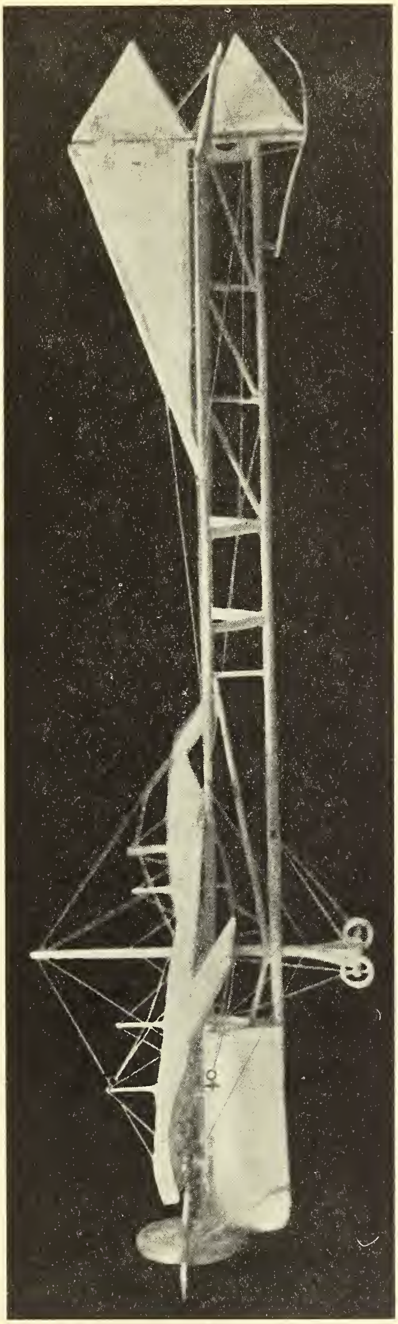


Fig. 1.—Model Aeroplane fitted with gyrostat to represent propeller and rotor of engine of actual aeroplane.

Where aeroplane flights are carried out for the amusement of spectators, manœuvres calling for sudden turning of the aeroplanes should be avoided.

In fig. 2 is shown a new stilt top due to the author. The construction of the instrument will be clear from the figure. A gyrostat is mounted as shown in two pivots, carried by a frame supported on two legs terminating in points. The pivots engage in sockets arranged in the frame of the gyrostat. The pivots are in the plane of the flywheel and lie in a plane at right angles to that containing the legs. Attached to a rod carried by

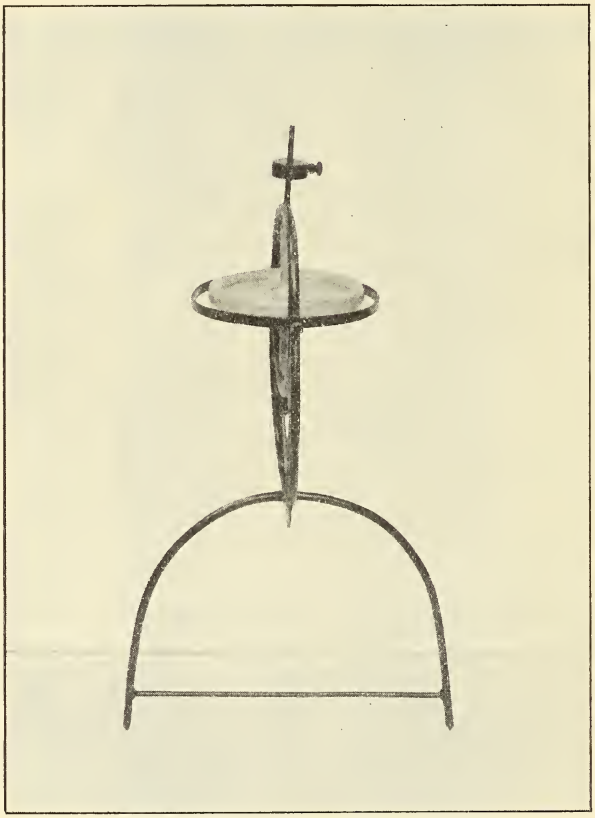


Fig. 2.—Gray Stilt Top.

the frame of the gyrostat, and in line with the axis of the gyrostat, is a weight. If the gyrostat is spun rapidly and the arrangement is set down on a table with the axis of the gyrostat, the rod carrying the weight, and the legs in one vertical plane, it will balance on the legs though doubly unstable without spin. The explanation of the action is given in a paper * by the author and Mr G. Burnside, and need not be repeated here.

In figs. 3, 4, and 5 are shown three gyrostatic bicycle-riders. The

* "On a Continuous-current Motor-Gyrostat with Accessories for demonstrating the Properties and Practical Applications of the Gyrostat," *Proc. Roy. Soc. Edin.*, 1912.

gyrostat shown in fig. 3 both steers and balances the bicycle. It depends for its action on the fact that the tube forming the upper part of the fork carrying the bearings of the back wheel is attached by means of a sleeved joint S to the frame carrying the fork of the front wheel and the gyrostat. The bicycle is similar, in fact, to the old-fashioned "high" bicycle.

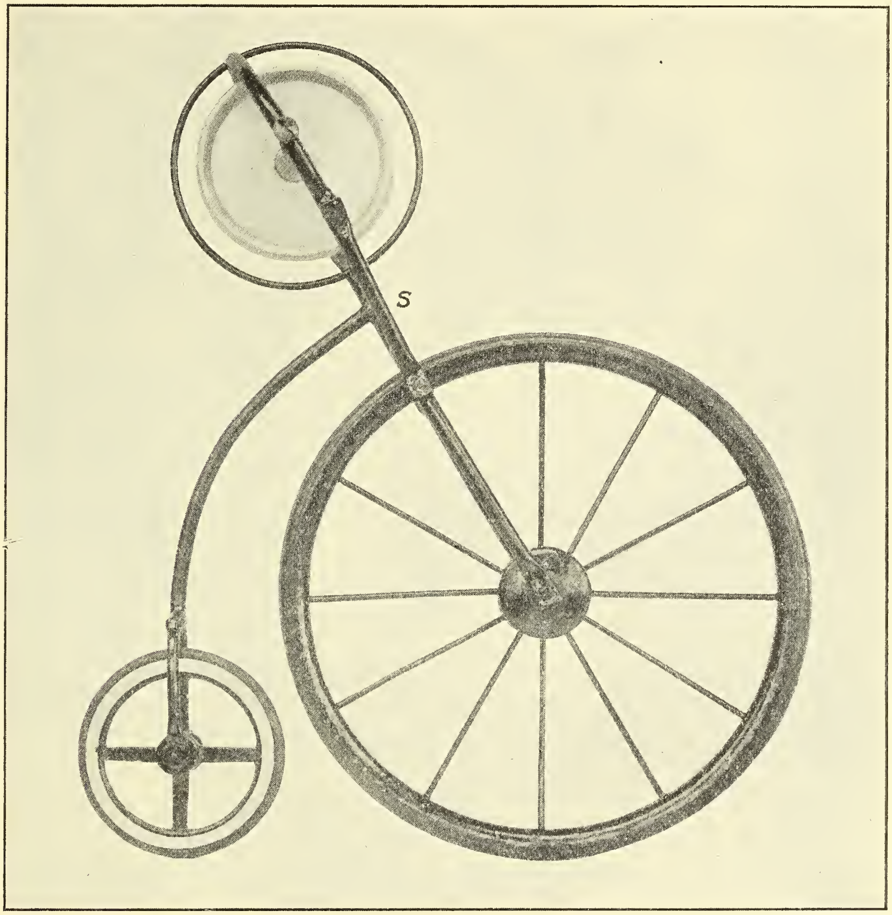


Fig. 3.—Gyrostatic Bicycle-rider.

Stability may be conferred in two ways upon the machine. In the first place, the gyrostat may be arranged to imitate exactly the action of a human rider. In balancing a bicycle the rider counteracts the tilting of the machine by turning the front handle-bar; if the machine tilts over to the left the bar is turned to the left, and the forward momentum of the machine and rider, aided by the gyrostatic action of the wheels (the latter factor is small in this case), results in the machine being brought into the vertical. To adapt the gyrostat to imitate the human rider the mass of the machine is made considerable, and the gyrostat is spun rapidly in a direction which coincides with that of the wheels of the bicycle when moving in the forward direction. When the machine tilts over to the left the gyrostat precesses round in azimuth, carrying with it the front wheel, and the precessional motion is such as to turn round the front wheel to the left; if the machine tilts over to the right the gyrostat turns the front wheel to the right. The result is to give the machine stability.

In a second mode of operating this form of bicycle the precessional motion of the gyrostat, brought about by the tilting of the machine, is automatically hurried. This is accomplished by giving considerable curve to the fork of the front wheel, with the result that the centre of gravity of that wheel lies considerably outside the line of the axis S. When so designed the front wheel is in unstable equilibrium when the bicycle is placed on the floor with the wheels in one vertical plane; the centre of gravity of the front wheel is then at the highest point of the circle in which it is constrained to move about the axis S, and if the wheel is turned slightly in either direction about that axis it will fall over, carrying with it the gyrostat. Thus if the gyrostat is spinning and precessional motion occurs, such motion is automatically hurried. If now the gyrostat is spun rapidly, and the bicycle is put down on the floor and pushed in the forward direction, it runs in a path which is practically a straight line, and the balancing power is very surprising. With this second method of mounting the gyrostat the bicycle will balance either when in motion or when at rest.

It is to be observed that, since the gyrostat is rigidly attached to the frame carrying the fork of the front wheel, the bicycle may be driven from the gyrostat.

In fig. 4 is shown a second bicycle fitted with a gyrostatic rider. In this case the gyrostat is mounted on the bicycle in two horizontal pivots, carried as shown by a fork attached to the bicycle. The line of the pivots is perpendicular to the plane containing the wheels of the bicycle, which are locked in one plane in this type of machine. Further, the construction is such that when the axis of the gyrostat is vertical the line of the pivots carrying the gyrostat lies considerably below the centre of gravity of the gyrostat and its frame. The device is stable when run on a stretched wire. If the machine tilts over in either direction the gyrostat precesses so that its axis turns in a plane which is nearly vertical; and since the centre of gravity of the gyrostat is above the line of the pivots, this precessional motion is hurried, with the result that the gyrostat regains the vertical position, carrying with it the bicycle. Again, if the gyrostat tilts over about the vertical on its supporting pivots, precession takes place about the line of contact of the wheels with the wire; the precession is again hurried, and the bicycle is brought into the vertical as before.

In fig. 5 is shown still another form of gyrostatic bicycle-rider. This arrangement is adapted to run on the floor as an ordinary bicycle, or on a tight or slack wire. The construction of the bicycle and gyrostat

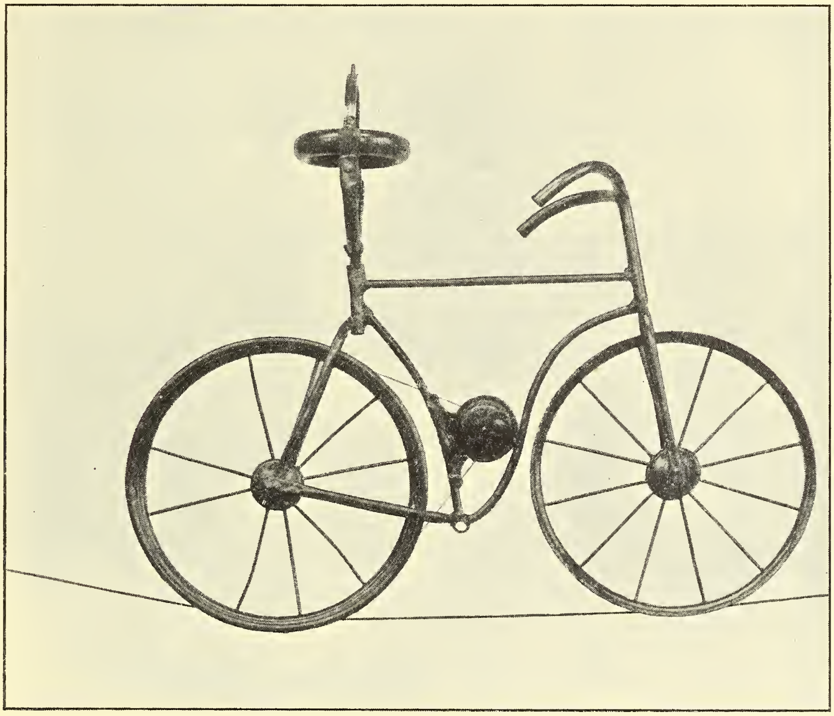


Fig. 4.—Gyrostatic Bicycle-rider.

mounting will be clear from the figure. The gyrostat frame, which is constructed from stiff wire, is attached to the bicycle by means of a sleeved joint *b*. To enable the bicycle to run on the floor the ends of the rod *r* are attached by means of strings to the extremities of the front handle-bar of the machine. If the gyrostat is spun rapidly very considerable stability is conferred upon the machine by the gyrostat. To run the device on a wire the strings are removed, the gyrostat is spun rapidly, and the bicycle placed on the wire and adjusted so that the wheels and the flywheel are in a vertical plane. The bicycle is then left to itself, when it balances perfectly. It will be evident from the figure and what has gone before how the stability is brought about.

A piece of apparatus in which a gyrostat is caused to imitate exactly the action of a human tight-rope balancer has been contrived by the author, and works perfectly. A framework to represent the body of the walker is mounted on two legs terminating in feet adapted to engage on a tight or slack wire. Mounted symmetrically on vertical pivots, carried by the framework, is a small gyrostat; the line of the pivots and that of the feet lie in one plane. The gyrostat is so mounted that its axis is horizontal

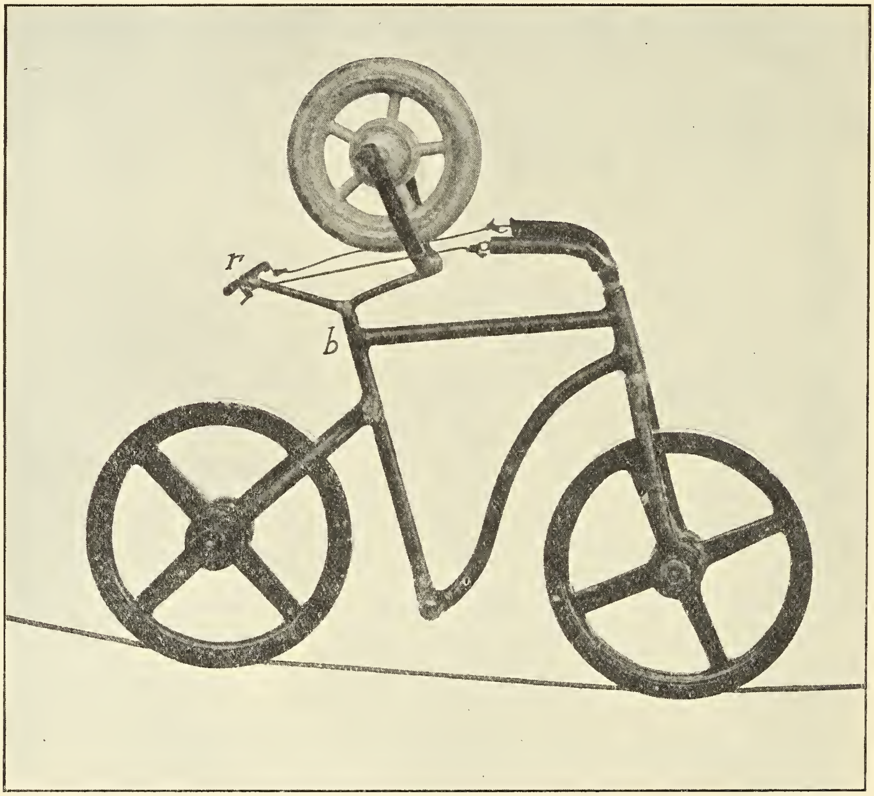


Fig. 5.—Gyrostatic Bicycle-rider.

when the line of the pivots is vertical. Attached to the frame of the gyrostat are two small arms terminating in hands, in which are placed a light balancing pole weighted at both ends. If the framework containing the spinning gyrostat is placed on the wire with the line of the pivots in a vertical plane containing the wire, and the gyrostat is properly spun, and adjusted so that its axis is in a line at right angles to the vertical plane of the wire, the contrivance will balance. To bring about this result the direction of spin is made such that tilting over of the apparatus causes the gyrostat to precess in the direction which results in the centre of gravity of the balancing rod being moved so as to counteract the tilting couple. The experiment is interesting inasmuch as the gyrostat detects the tendency of the apparatus to fall, and immediately corrects it.

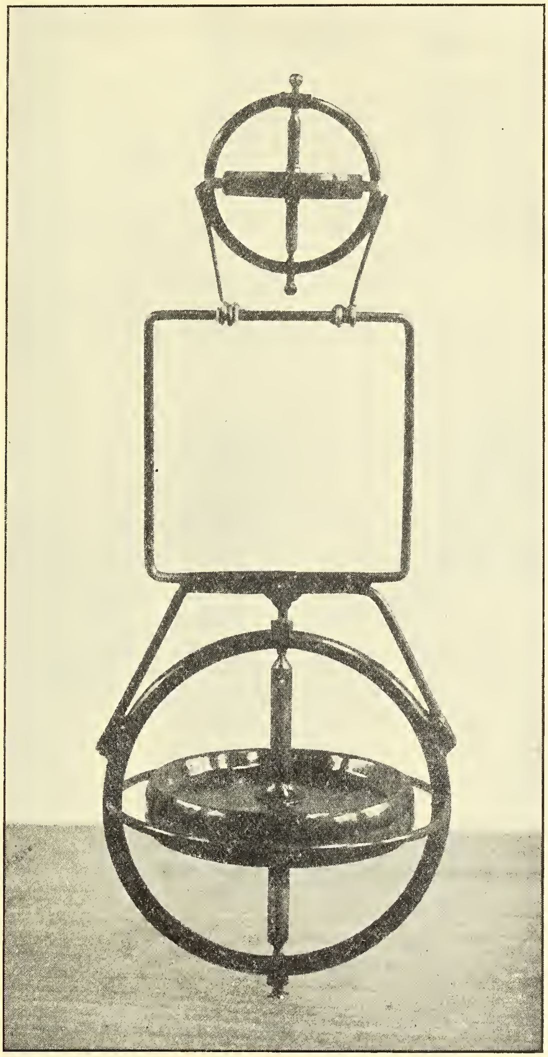


Fig. 6.—Acrobatic Top.

In figures 6 and 7 are shown two new tops, of which the construction will be clear from the diagrams. In the top illustrated in fig. 6 a large gyrostat carries a framework made of stiff wire. To this framework is attached, by means of two sleeved joints as shown, a small gyrostat. The two gyrostats are spun rapidly and the arrangement placed as shown on a table. The large gyrostat spins on the table as an ordinary top, and, in consequence of friction at the pivots of the flywheel, the frame and the small gyrostat are carried round with it. For one of the two directions in which the small gyrostat may be spun it will rise up and assume the position shown in the figure.

In figure 7 is shown a compound top made up of one large gyrostat and two small ones. A wire frame in which are carried, in sleeved joints as

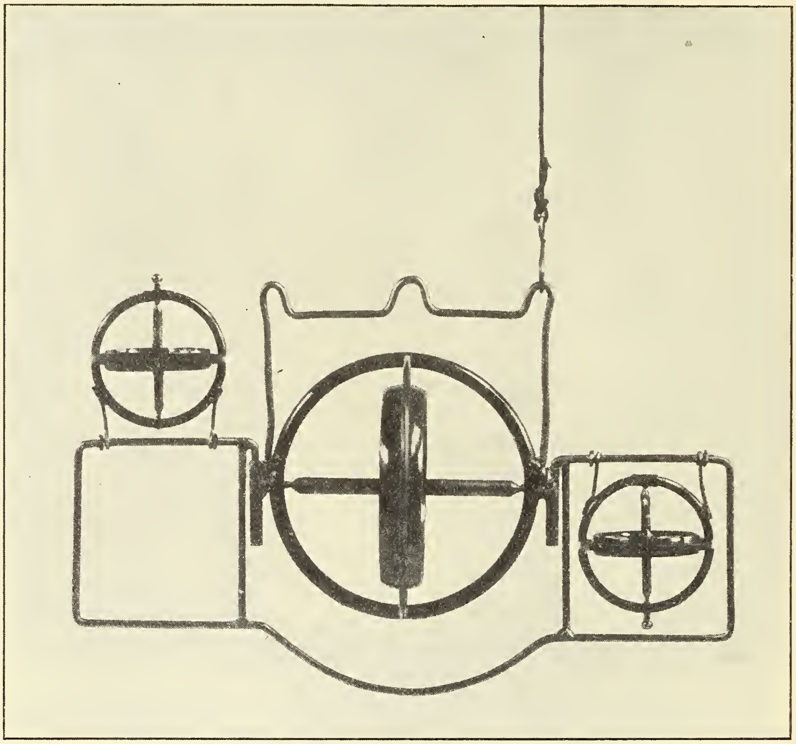


Fig. 7.—Acrobatic Top.

shown, the two small gyrostats, is mounted in two sockets carried by the frame of the large gyrostat. The whole arrangement is suspended from a stand by means of a string terminating in a hook as shown. Now, suppose the large gyrostat to be spinning rapidly, and the small gyrostats to be spinning in opposite directions. If the arrangement is hung up so that its centre of gravity is vertically under the hook the small gyrostats will both remain in the lower positions. If, however, the hook is placed to one side as shown, the large gyrostat will precess round so that its axis turns in a plane which is nearly horizontal, carrying with it the small gyrostats. One of these assumes the upright position as shown. If now the hook is transferred to the left-hand side of the frame the precessional motion is reversed; the small gyrostat, which previously assumed the upright position, now takes up a position in which its centre of gravity is vertically below its supporting joints, while the second small gyrostat assumes the upright position.

Experiments carried out with ordinary gyrostats are far from satisfactory if the old-fashioned method of spinning them by means of a string is adopted. By means of the following method angular velocities of about 12,000 revolutions per minute can be obtained. A leather disc is fitted on the spindle of an electric motor, and against this is pressed lightly the spindle of the gyrostat. After a little practice the operation of spinning becomes easy of accomplishment, and the experiments are rendered really beautiful.

The author desires to thank Professor Gray for the practical interest he has taken in the gyrostatic experiments here described.

(*Issued separately October* 31, 1912.)

XXX.—The Geometry of Twin Crystals. By Dr John W. Evans, of the Imperial Institute and Birkbeck College, London. *Communicated by* Professor J. W. Gregory, F.R.S.

(MS. received January 26, 1912. Read March 4, 1912.)

In a communication to the Mineralogical Society, published in September 1910 (*Min. Mag.*, vol. xv. p. 390), I suggested that the relation between the component structures of twin crystals was similar to that existing in combinations of crystals of different substances with definite relative orientation (Barker, *Min. Mag.*, vol. xiv., 1907, p. 235), and was determined by equality of molecular distances in the two structures in the plane of contact or composition.

It has been my purpose in the present paper to determine what are the possible geometrical relations between crystal structures in which such equality exists in all or some of the molecular rows in the plane of contact.

As a rule, rows with equal molecular distances are equivalent to one another. I have, accordingly, investigated in the first place the relations between crystal structures in which equivalent lines coincide in orientation. The treatment is much simplified by dealing with equivalent lines in general, not merely those which are parallel to possible edges and therefore to molecular rows; but special considerations applicable to the latter alone are added where necessary.

I next discuss the coincidence of lines which are not equivalent, but in which the molecular distances are either equal, or are related by a simple ratio so that equality exists between low multiples of them.

I am indebted to Mr H. Hilton (*Mathematical Crystallography*, Oxford, 1903; and *Min. Mag.*, vol. xiv., 1907, p. 261) for some of the methods employed. I have not thought it necessary to give proofs of well-known propositions, except where they assist in the development of the argument.

§ 1. Planes, Lines, and Directions; Operations.

i. It is convenient in describing the geometrical characters of a crystal to consider *planes* and *lines* (that is, straight lines) to possess orientation but not locality. In other words, parallel planes or lines are treated as identical; for they have the same physical and crystallographic characters, and parallel rows of molecules have equal molecular intervals.

Every line in a structure has two opposite *directions*, which may have either the same or different physical and crystallographic characters.

ii. It is often desirable to suppose that the disposition in space of the whole or part of a structure is changed subject to the condition that the distance between any two points is the same after the operation as it was before. Such a change is referred to as an *operation.*

§ 2. Coincidence and Equivalence.

i. If two structures are so related that not only every plane and line but every direction of one structure coincides with a plane, line, and direction of the other, possessing the same physical and crystallographic characters, they may be said to be *co-directional* (or parallel) to one another.

ii. If by an operation, as above defined, one structure can be brought into a co-directional relation to another, they are said to possess the *same form*; and a plane, line, or direction in one structure is said to be *equivalent* to any plane, line, or direction of another structure, if they can be brought into coincidence by an operation which brings the former structure into a co-directional relation to the latter.

iii. An operation which, when applied to a structure, leaves it in a co-directional relation to itself, as it existed before the operation, may be termed a *co-directional operation* (" symmetry operation " of Hilton), and every plane, line, or direction which can by means of a co-directional operation be brought into coincidence with another plane, line, or direction of the *same* structure, as it was before the operation, is likewise said to be *equivalent* to it. It is also often convenient to refer to a plane, line, or direction as equivalent to itself.

iv. It is clear that if two planes, lines, or directions are equivalent to the same plane, line, or direction, they are equivalent to one another; that if two planes are equivalent, the lines they contain are respectively equivalent; and that if two lines are equivalent, their directions are respectively equivalent; also that if two planes are equivalent, their normals are equivalent, and *vice versa.* Molecular rows parallel to equivalent lines must obviously have equal molecular distances.

v. If two structures are so related that every line in one coincides with an equivalent line in the other, they are said to be *co-linear*, and an operation that brings a structure into a co-linear relation with itself, as it existed before the operation, is said to be a *co-linear operation.*

vi. If the two directions of a line are equivalent, the line is said to be *biterminal.* If they are not equivalent, it is *uniterminal.*

If two structures are co-linear, the directions of every biterminal line in one must coincide with equivalent directions in the other, but the directions of a uniterminal line in one of two co-linear structures may either coincide with or be opposed to the equivalent directions of the corresponding line in the other. If the former be the case with all uniterminal lines, the structures are, as we have seen, co-directional; if the latter, they may be said to be *contra-directional.*

An operation which brings a structure into a contra-directional relation with itself, as it existed before, is said to be a *contra-directional operation.**

vii. All co-linear structures must either be co-directional or contra-directional, for if the directions of some uniterminal lines coincided with equivalent directions, and those of others were opposed to equivalent directions, other coincident lines which are intermediate in position to these could not be equivalent, for they would have different crystallographic relations in the two structures. Every co-linear operation must in like manner be either co-directional or contra-directional.

viii. The co-directional, contra-directional, and co-linear operations of a structure express the symmetry it possesses.

ix. If one structure can be brought into a co-directional relation with another, by an operation such as may be imitated by the movement of a rigid body, the two are said to be *congruent.* If, on the other hand, one structure can be brought by such an operation or movement into a contra-directional, but *not* into a co-directional, relation with another, they are said to be *enantiomorphic.* Two enantiomorphic structures are considered to possess the same form (§ 2, ii.) although their disposition in space prevents them from being brought into co-directional coincidence by such movements as can be carried out with a rigid body.

§ 3. Different Kinds of Operations.

i. *Reversal relatively to a Point.*—Every point in the structure is transferred to a new position such that the straight line joining the old and new positions is bisected by the same fixed point, the *point of reversal.*†

* Co-directional and contra-directional operations do *not* correspond to the operations of the first and second sort of Hilton.

† In describing these operations, the point, line, or plane of reversal or axis of rotation is supposed to have a definite position, but the result is independent of that position. The difficulty is avoided if it be supposed that *all* planes and lines of reversal and axes of rotation pass through the same point, and that this is also the point of reversal.

If a reversal relatively to a point be a co-directional operation of the structure, the two directions of every line in the structure must be equivalent to one another, or, in other words, every line is biterminal, and the structure is said to possess *point* (or central) *symmetry.* If the structure contain lines with non-equivalent directions, that is to say, uniterminal lines, the reversal will be a contra-directional operation of the structure.

ii. *Reversal relatively to a Line.*—Every point is transferred to a new position such that the straight line joining the old and new positions is perpendicularly bisected by the same fixed straight line, the *line of reversal.* If such a reversal is a co-directional operation of the structure to which it is applied, the line of reversal is said to be a *line of symmetry* of that structure.

iii. *Reversal relatively to a Plane.*—Every point is transferred to a new position such that the straight line joining the old and new positions is perpendicularly bisected by the same plane, the *plane of reversal.* If this is a co-directional operation, the plane is a *plane of symmetry* of the structure.

iv. *Rotation round a Line.*—Every point is rotated round a line, the axis of rotation, through the same angle and in the same cyclic direction. If the angle of rotation be an nth part of a complete turn, n is said to be the *cyclic number of the rotation.*

The cyclic number of the smallest rotation round a line, which is a co-directional operation, is said to be the *co-directional cyclic number of the line.*

On the other hand, the cyclic number of the smallest rotation (if any) round a line, which is a contra-directional operation, is the *contra-directional cyclic number of the line.*

In the same manner, the *co-linear cyclic number* of a line is that of the smallest rotation round it which is a co-linear operation of the structure.

v. The co-linear cyclic number forms the most convenient basis for the classification and nomenclature of the crystallographic systems and classes (*Min. Mag.*, vol. xv., 1910, p. 398).

Thus, in the triclinic system the co-linear cyclic number of every line is 1; in the monoclinic and orthorhombic systems the highest co-linear cyclic number of any line is 2; in the rhombohedral system it is 3; in the tetragonal, 4; in the hexagonal, 6; while in the cubic system there are always four lines with co-linear cyclic number 3.

If the tetragonal system is taken as an example, we find that in the IV Bk (chalcopyrite) and IV Uk classes (see § 4, xiv.) the co-directional cyclic number of the principal axis is 2, and the contra-directional and co-linear cyclic numbers 4, while in the remaining five classes the co-directional and co-linear cyclic numbers are 4: similar relations hold in the other systems.

§ 4. Combination, Equality and Equivalence of Operations.

i. The successive application of the operations A and B constitutes a third operation which may be expressed in the form of a product $A.B$, and the repeated application of the same operation A is, on the same principle, denoted by the corresponding power of A.

ii. If two operations A and B result in the same final change, they are said to be *equal*, and we may write $A=B$. For instance, a rotation with cyclic number 1 is equal to the absence of change, and one with the cyclic number 2 to a reversal relatively to the line which forms the axis.

iii. If, when applied to a structure S (that is to say, to a particular structure with a particular disposition in space), the results of two operations A and B are indistinguishable on account of the distribution of equivalent directions, the operations are said to be *equivalent* to one another, and we may write $[S]A\,(=)\,[S]B$; where $[S]$ indicates that the operation which follows is applied to the structure S, and $(=)$ expresses equivalence.

iv. The equality of two operations is independent of the structure to which they are applied, but it is convenient to consider equality as a special case of equivalence.

v. If an operation A is equal to an operation B followed by an operation C, and B is a co-directional operation of a structure S, the operation A, when applied to S, will be equivalent to the operation C, for $[S]A=[S]B.C\,(=)\,[S]C$.

vi. It is obvious that all co-directional operations of the same structure are equivalent to one another and to the absence of change, which may be denoted by 1. Thus, if A and B are co-directional operations of S, we may write $[S]A\,(=)\,[S]B\,(=)\,[S]1$. In the same manner, all contra-directional operations of the same structure are equivalent, so that, if A and B are contra-directional operations of S, $[S]A\,(=)\,[S]B\,(=)\,[S]R_i$, where R_i is a reversal relatively to a point; for the existence of a contra-directional operation implies the presence of uniterminal lines, so that a reversal relatively to a point will necessarily be contra-directional (§ 3, i.).

vii. It is easily seen that the combination of any number of co-directional operations is itself a co-directional operation, and so is a combination of an even number of contra-directional operations; whilst the combination of an odd number of contra-directional operations, or of a co-directional operation and a contra-directional operation, is a contra-directional operation.

As special cases it may be noted that (1) the combination of any contra-directional operation with a reversal relatively to a point is a co-directional operation, and that (2) every contra-directional operation is equivalent to any co-directional operation combined with reversal relatively to a point.

viii. As every co-linear operation is either co-directional or contra-directional, and a co-directional or contra-directional operation is always co-linear, all combinations of co-linear operations are themselves co-linear.

ix. The inverse of an operation is an operation in which the action of the original or direct operation is inverted. It is denoted by the symbol A^{-1}, where A is the direct operation. Then $A.A^{-1}=1$. If an operation A be a co-directional operation of a structure S, the inverse operation A^{-1} will also be a co-directional operation of S, for, since $[S]A.A^{-1}=[S]1$, and $[S]A\,(=)\,[S]1$, $[S]A^{-1}\,(=)\,[S]1$. In the same way the inverse of a contra-directional operation B is also a contra-directional operation of the original structure; for $[S]R_i.B\,(=)\,[S]1$, where R_i is a reversal relatively to a point (§ 4, vii.). Apply the inverse operation B^{-1}; then $[S]R_i$, $B.B^{-1}\,(=)\,[S]B^{-1}$, but $B.B^{-1}=1$; accordingly, $[S]R_i\,(=)\,[S]B^{-1}$. It follows from the above that the inverse of a co-linear operation is always a co-linear operation.

x. Let C_a, a rotation round the line c through an angle a, be the smallest rotation round c which is a co-linear operation of a structure S, then all rotations of the form C_{na} $(=C_a^n)$, where n is any integer, positive or negative, are also co-linear operations of S, which is not the case with any other rotation round the same line; for if a rotation $C_{(n+q)a}$, where n is an integer and q is a fraction less than 1, were a co-linear operation of S, $C_{(n+q)a}.C_{na}^{-1}=C_{(n+q-n)a}=C_{qa}$ would also be co-linear, which is, by hypothesis, impossible, for qa is less than a.

In the same way it can be shown that, if C_β be a co-directional operation of S, rotations of the form $C_{n\beta}$, where n is an integer, and no others round the same line are co-directional operations of S.

As a complete turn is always co-directional and co-linear, the co-directional and co-linear cyclic numbers must always be integers.

xi. The smallest rotation round a particular line which is a co-linear operation of a structure must be either the smallest co-directional rotation or the smallest contra-directional rotation round the same line. In the former case all co-linear rotations must be co-directional, and there are no contra-directional rotations. In the latter, if a rotation through an angle α be the smallest co-linear and contra-directional rotation, a rotation through an angle 2α will be the smallest co-directional rotation. The co-linear and contra-directional cyclic numbers will in this case be equal to one another and double the co-directional cyclic number. The contra-directional cyclic number will therefore always be an even integer.

The co-directional and co-linear cyclic numbers may be 1, 2, 3, 4, or 6; the contra-directional cyclic numbers 2, 4, or 6. Any others are inconsistent with the law of rational indices and the molecular constitution of matter.

A line with a co-directional, contra-directional, or co-linear cyclic number of 2 or more is said to be an axis of co-directional, contra-directional, or co-linear symmetry, as the case may be, with the corresponding cyclic number.

xii. If the co-directional cyclic number of a line be even, say $2m$, the line will be a line of symmetry of the structure; for since a rotation through a $2m$th part of a turn is co-directional, one through a half turn which is m times this is also co-directional, and it is equal to a reversal relatively to the axis.

xiii. The relation of a line to the symmetry of a crystal structure may be expressed by a symbol consisting of (1) the co-linear cyclic number of the line; (2) a capital letter U (unilateral) or B (bilateral), indicating the absence or presence of one or more planes of symmetry passing through to the line; (3) one of the following letters:—k, indicating that the line possesses contra-directional symmetry; u, that it is uniterminal; c, the presence of central or point symmetry; and h (helical), the presence of one or more lines of symmetry at right angles to the original line, and at the same time the absence of point symmetry, so that the terminations of the line are related in the same manner as the ends of a screw. It is necessarily unilateral.

Thus the symmetry of the principal axis of chalcopyrite is expressed by the symbol $4Bk$; that of tourmaline by $3Bu$; of beryl by $6Bc$; of dioptase by $3Uc$; of quartz (that is to say, ordinary or α quartz) by $3Uh$; of benitoite by $6Bk$.

This symbol may be referred to as the type symbol of the line.

xiv. The *crystallographic class* of a crystal structure may be expressed by the symbol of the line with highest co-linear cyclic number, written for this purpose as a Roman instead of an Arabic numeral. In the case of the classes of the cubic system, however, the symbol expressing the symmetry of the four lines with co-linear cyclic number 3 is chosen, but with the capital letter C (cubic) substituted for the numeral (*Min. Mag.*, vol. xv., 1910, p. 398).

A list of the symbols of the crystallographic classes is given in the accompanying table, with the usual names of the systems and classes, the name of a substance (where known) crystallising in each class, and the type symbols of the lines that may occur in it. It will be found convenient to refer to the classes by their symbols.

Class Symbol.	System.	Class.	Example.	Types of Line in each Class.
I Uu	triclinic	asymmetric	calcium thiosulphate	1Uu.
I Uc	,,	pinakoidal	plagioclase	1Uc.
II Uk	monoclinic	domatic	clinohedrite	1Uu, 1Bu, 2Uk.
II Uu	,,	sphenoidal	cane sugar	1Uu, 1Uh, 2Uu.
II Uc	,,	prismatic	augite	1Uc, 1Bc, 2Uc.
II Bu	orthorhombic	pyramidal	hemimorphite	1Uu, 1Bu, 1Uh, 2Bk, 2Bu.
II Uh	,,	bisphenoidal	epsomite	1Uu, 1Uh, 2Uh.
II Bc	,,	bipyramidal	staurolite	1Uc, 1Bc, 2Bc.
IV Uk	tetragonal	bisphenoidal	unknown	1Uu, 1Uh, 4Uk.
IV Bk	,,	scalenohedral	chalcopyrite	1Uu, 1Bu, 1Uh, 2Bk, 2Uh, 4Bk.
IV Uu	,,	pyramidal	wulfenite	1Uu, 1Uh, 4Uu.
IV Bu	,,	ditetragonal pyramidal	succin-iodimide	1Uu, 1Bu, 1Uh, 2Bk. 4Bu.
IV Uh	,,	trapezohedral	strychnine sulphate	1Uu, 1Uh, 2Uh, 4Uh.
IV Uc	,,	bipyramidal	scheelite	1Uc, 1Bc, 4Uc.
IV Bc	,,	ditetragonal bipyramidal	rutile	1Uc, 1Bc, 2Bc, 4Bc.
III Uu	rhombohedral or trigonal	trigonal pyramidal	sodium periodate	1Uu, 3Uu.
III Bu	,,	ditrigonal pyramidal	pyrargyrite	1Uu, 1Bu, 3Bu, 2Uk.
III Uh	,,	trapezohedral	α quartz	1Uu, 1Uh, 3Uh, 2Uu.
III Uc	,,	rhombohedral	phenakite	1Uc, 3Uc.
III Bc	,,	scalenohedral	calcite	1Uc, 1Bc, 3Bc, 2Uc.
VI Uk	hexagonal	trigonal bipyramidal	unknown	1Uu, 1Bu, 6Uk.
VI Bk	,,	ditrigonal bipyramidal	benitoite	1Uu, 1Bu, 1Uh, 2Bk, 6Bk, 2Bu.
VI Uu	,,	pyramidal	nepheline	1Uu, 1Uh, 6Uu.
VI Bu	,,	dihexagonal pyramidal	iodyrite	1Uu, 1Bu, 1Uh, 2Bk, 6Bu.
VI Uh	,,	trapezohedral	β quartz	1Uu, 1Uh, 2Uh, 6Uh.
VI Uc	,,	bipyramidal	apatite	1Uc, 1Bc, 6Uc.
VI Bc	,,	dihexagonal bipyramidal	beryl	1Uc, 1Bc, 2Bc 6Bc.
C Uu	cubic	tetrahedral pentagono-dodekahedral	sodium chlorate	1Uu, 3Uu, 1Uh, 2Uh.
C Bu	,,	hexakis tetrahedral	diamond	1Uu, 1Bu, 3Bu, 1Uh, 2Bk, 4Bk.
C Uh	,,	pentagono-icositetra-hedral	cuprite	1Uu, 1Uh, 3Uh, 2Uh, 4Uh.
C Uc	,,	dyakis do-dekahedral	pyrite	1Uc, 3Uc, 1Bc, 2Bc.
C Bc	,,	hexakis octahedral	galena	1Uc, 1Bc, 3Bc, 2Bc, 4Bc.

§ 5. Combinations of Reversals.

i. A second application of the same reversal (whether relatively to a point, line, or plane) restores a structure to its original disposition in space. In other words, a reversal is the inverse of itself. Thus, if R be a reversal, $R^2 = 1$ and $R = R^{-1}$.

ii. The combination of two equivalent reversals (or of any two operations equivalent to the same reversal) will be a co-directional operation; for if $[S]R\,(=)\,[S]R'$, where R and R' are reversals, $[S]R\,.\,R'\,(=)\,[S]R'^2 = [S]1$.

Inversely, if the combination of two reversals be a co-directional operation, one will be equivalent to the other, for, if $[S]R\,.\,R'\,(=)\,[S]1$, $[S]R\,(=)\,[S]R'^{-1} = [S]R'$.

iii. There are three groups, each of three reversals, such that the combination of any two reversals in the same group is equal to the third. These groups are (a) reversals relatively to a plane, its normal and a point, (b) reversals relatively to three lines at right angles, (c) reversals relatively to two planes at right angles and their line of intersection.

These relations are easily proved. For instance, in group (a), let any point in the structure have an angular distance ϕ from one direction of the normal considered as pole, the point of intersection of plane and normal being taken as centre and point of reversal,* and azimuth ψ from any meridian. Then, after a reversal relatively to the plane, the co-ordinates will be $\pi - \phi$ and ψ, and after a subsequent reversal relatively to its normal, $\pi - \phi$ and $\pi + \psi$, which are those that would result from a simple reversal relatively to the centre. The distance from the centre obviously remains unchanged throughout.

In the case of group (b), a direction of one line may be taken as pole, and one of a second line to mark the meridian. Then after a reversal relatively to the latter the co-ordinates will be $\pi - \phi$ and $-\psi$, and, after a subsequent reversal relatively to the third line, ϕ and $\pi + \psi$, which would also result from a simple reversal relatively to the first line.

The results of these combinations are independent of the order of application of the reversals, for let $R_1\,.\,R_2 = R_3$, where R_1, R_2, and R_3 are reversals belonging to the same group, then $R_1\,.\,R_2 = R_3 = R_3^{-1} = R_2^{-1}R_1^{-1} = R_2\,.\,R_1$.

iv. It follows from group (a) that a rotation with cyclic number 2 round an axis (§ 4, ii.), followed by a reversal relatively to a point,* is equal to a reversal relatively to the plane at right angles to the axis.

v. If the cyclic number of an axis of contra-directional symmetry be

* See note to § 3, i.

$2m$, where m is odd, there will be a plane of symmetry at right angles to the axis.

For, let C_a be the smallest rotation round the axis which is contra-directional; then C_{ma} $(=C^{m}{}_{a})$ is also contra-directional, since m is odd. Let R_l be a reversal relatively to the axis, R_p a reversal relatively to a plane at right angles to it, and R_i a reversal relatively to a point. Then $[S]R_p=[S]R_l \,.\, R_i=[S]C_\pi \,.\, R_i=[S]C_{ma} \,.\, R_i \; (=) \; [S]1$ (§ 4, vii.).

vi. A combination of two reversals R and R' relatively to lines l and l' making an angle θ with each other is equal to a rotation round an axis at right angles to both through an angle 2θ in the cyclic direction from l to l', or l' to l, according as R or R' is applied first. Thus $R \,.\, R'=C_{2\theta}$, and $R' \,.\, R=C_{-2\theta}$.

For, let any point on the structure have polar distance ϕ from one direction of the axis considered as pole, with the point of intersection of the two lines and axis as centre; assume its azimuth about the axis to be ψ, when the azimuth of l is 0, and that of l', θ. Then after the reversal relatively to l the polar distance and azimuth of the point will be $\pi-\phi$ and $-\psi$ respectively, and after the reversal relatively to l', ϕ and $\theta+\theta+\psi=2\theta+\psi$.

vii. In like manner a combination of two reversals R and R' relatively to planes making an angle θ with each other is equal to a rotation round the line of intersection through an angle 2θ; or, what comes to the same thing, a combination of reversals relatively to two planes whose normals n, n' make an angle θ with each other is equal to a rotation round an axis at right angles to both normals through an angle 2θ from n to n' or from n' to n, according as R or R' is first applied.

viii. The only combination of two reversals still to be considered is that in which a reversal relatively to a line is combined with reversal relatively to a plane whose normal makes an angle θ with the line. If the reversal relatively to a plane be resolved into a reversal relatively to its normal and a reversal relatively to a point, it follows that a combination of a reversal relatively to a line with a reversal relatively to a plane whose normal makes an angle θ with the line is equal to a rotation round an axis at right angles to the line and normal through an angle 2θ in a cyclic direction corresponding with the order of application of the reversals, combined with a reversal relatively to a point.

§ 6. The Symmetry of a Plane.

i. In the present communication the expression, *the symmetry of a plane,* is restricted to its symmetry in two dimensions. This is determined by the distribution of equivalent directions in the plane, and is independent of the remaining symmetry of the structure. It will include so much of the symmetry of the structure as a whole as is consistent with the fact that a plane is without thickness; that is to say, it may include a plane, line, or axis of symmetry at right angles to the plane, a line of symmetry lying in it, and point-symmetry; but not a plane, line, or axis of symmetry oblique to the plane, or an axis of symmetry lying in it with cyclic number greater than 2.

As there is, *on the assumption which has been made,* no difference between the opposite faces of a plane, a reversal of a plane relatively to the plane itself—resulting as it does merely in an interchange of the faces—will always be a co-directional operation of the plane, whether it be a co-directional operation of the structure as a whole or not.

If a reversal relatively to a plane be combined with an operation which is a co-directional operation of *both* the plane and the structure of which it forms part, the combination will be likewise a co-directional operation of the plane, but will or will not be a co-directional operation of the structure according as the reversal relatively to the plane is or is not a co-directional operation of the structure.

For instance, if a structure possess point symmetry, the normal to a plane will be a line of symmetry of the plane (§ 5, iii., group (*a*)), but will only be a line of symmetry of the structure if the plane be a plane of symmetry of the structure.

ii. Accordingly, the only operations which can be co-directional operations of a plane and not of the structure as a whole are (1) a reversal relatively to the plane, and (2) combinations of such a reversal with an operation which is a co-directional operation common to the plane and to the structure of which the plane forms part.

iii. A plane which is a plane of symmetry of the structure can possess no symmetry which is not shared by the latter.

iv. A rotation about the normal to a plane through a half turn is always a co-linear operation of the plane. The co-linear cyclic number of the normal in respect of the plane must therefore always be even, and if its co-linear cyclic number in respect of the structure as a whole be odd, the former must be twice the latter.

§ 7. Common Lines and Planes.

i. The coincidence of lines and of planes may be described in terms similar to those employed for the coincidence of structures (§ 2).

If equivalent lines of two structures coincide, the two lines constitute a *common line,* whether their equivalent directions also coincide or not. If each direction of one line coincide with an equivalent direction of the other, the lines are *co-directional.* If the component lines are biterminal or, in other words, if the two directions of each are equivalent to one another, the two lines will necessarily be co-directional. If, on the other hand, they are uniterminal, so that each has two non-equivalent directions, they will either be co-directional or they will have their equivalent directions opposed to each other, and will be *contra-directional.* The terms co-directional and contra-directional may also be applied to the common line formed by two co-directional or contra-directional lines.

ii. If two equivalent planes forming part of different structures coincide, they form a *common plane,* whether equivalent lines in the two planes coincide or not. If every line in one component plane coincide with an equivalent line in the other, so that every line in the common plane is a common line, the component planes and also the common plane are said to be *co-linear* (*cf.* § 2, v.). If in a co-linear common plane every direction in one component plane coincide with an equivalent direction in the other, so that every common line in the common plane is co-directional, the planes are also *co-directional* (§ 2, iii.). If every line in the component planes of a co-linear common plane is biterminal, the planes are necessarily co-directional. This will be the case if the structure possesses point symmetry or if the normal is a line of symmetry of the structure. If, however, there are lines with non-equivalent directions in the component planes of a co-linear common plane, the corresponding common lines must be either co-directional or contra-directional, and the planes will be co-directional or *contra-directional* accordingly. A contra-directional common plane will, however, contain co-directional common lines if the component planes contain biterminal lines (*cf.* § 2, vi.).

iii. If two equivalent planes coincide to form a common plane, the normals will coincide to form a common line. Inversely, a plane at right angles to a common line will be a common plane (§ 2, iv.).

§ 8. Twin Planes and Axes.

i. If in a compound crystal made up of two structures of the same form there is a co-linear common plane but the structures are not co-directional, the common plane is termed a twin plane, and the common line which is its normal a twin axis; and the two structures together constitute a twin crystal.

ii. A twin plane forms the most frequent plane of contact or "composition" between the component structures of a twin crystal, and is then always a possible face in each component structure, such faces being, of course, equivalent to each other. In such a plane there is a molecular net belonging to each structure, identical in shape and dimensions, and so placed that the interspaces of all parallel rows of molecules in the two nets are the same, in spite of the fact that the lattices, as a whole, of the two structures have in other respects a different disposition in space.

iii. Where a twin plane or other common plane forms a plane of contact, it may be termed a contact twin plane or contact common plane. Where, on the other hand, the two component planes of a twin plane or common plane coincide mathematically but not physically, it may be referred to as an abstract twin plane or common plane. In the same manner a common line parallel to a plane of contact may be termed a contact common line, and one which is not parallel to the plane of contact an abstract common line. Abstract common planes and lines have usually no direct structural significance.

iv. A twin plane like other common planes, and a twin axis like other common lines, may be co-directional or contra-directional.

§ 9. Twinning Operations.

i. We now proceed to ascertain the geometric relations of the component structures of a twin crystal as determined by the definition of a twin plane given in § 8. We may conceive the two structures as forming in the first place portions of one homogeneous crystalline structure and therefore co-directional with one another, and suppose that an operation is applied to one of them which results in such a change in its disposition in space that a contact or abstract twin plane results. Such an operation may be termed a twinning operation.* A twinning operation must accordingly bring a plane in the portion to which it is applied into coincidence with an equivalent plane in the other portion, in such a manner that the two planes are co-linear but the structures are no longer co-directional.

ii. We may restrict ourselves, however, to the case in which the two component planes of the twin plane were identical before the application of the twinning operation. For if by a twinning operation U a plane q' in the portion S' to which U is applied is brought into a co-linear relation

* It is scarcely necessary to explain that twin crystals are not formed in nature in this way, except in the case of twins formed by movements along gliding planes, when a new disposition in space is given to a portion of a crystal. Here it is the individual molecules that move, and not the structure as a whole, but the change satisfies the definition of an operation given in § 1, ii.

with a different but equivalent plane p in the undisturbed portion S, so as to form a twin plane, it is possible to obtain an indistinguishable result by first applying to S' a co-directional operation V, which will bring p', the portion of the plane p which lies in S', into the former position of q', and then applying the operation U. It will therefore be sufficient to consider the twinning operation $T = V.U$ by which the twinning plane is formed of two portions of what was originally the same plane (§ 4, v.).

It follows that the relation which exists between the component structures of any twin crystal may be brought about by applying to a portion of the untwinned structure an operation which is a co-linear operation of a plane but not a co-directional operation of the structure. It is also clear that every operation that satisfies these conditions is a twinning operation. Such a co-linear operation of a plane must be either a co-directional or a contra-directional operation of the plane.

iii. A reversal relatively to a plane is, as we have seen, always a co-directional operation of the plane itself, and will therefore, if applied to one portion of a uniform structure, convert the plane into a twin plane, provided of course that it is not a plane of symmetry of the structure, for if it were, the two portions of the structure would continue to have a co-directional relation with one another and to form a simple crystal.

iv. Any other operation U which is a co-directional operation of the same plane but not of the structure must be equal to the combination of an operation V which is a co-directional operation both of the plane and structure and a reversal R, relatively to the plane (§ 6, ii.). If, now, the operation U be applied to the structure, it will, since V is a co-directional operation of the structure, be equivalent to R (§ 4, v). Accordingly, every twinning operation which is a *co-directional* operation of a plane, is equivalent to a reversal relatively to the plane.

v. A contra-directional operation of a plane cannot be a co-directional operation of the structure, as a whole; it will therefore, if applied to a portion of the structure, always give rise to a twin plane. It may be resolved into a co-directional operation of the plane and a reversal relatively to a point (§ 4, vii.). But a co-directional operation of a plane must be either a co-directional operation of the structure, a reversal relatively to the plane, or an operation which when applied to the structure is equivalent to the latter (§ 6, ii.). The successive application of a co-directional operation of a plane and a reversal relatively to a point must therefore be equivalent to either a reversal relatively to a point or a reversal relatively to the normal to the plane (§ 5, iii., group (a)), and no other contra-directional twinning operations need be considered. But, in order that they may give rise to a *contra-directional* twin plane, the original plane must contain uniterminal lines; in other words, the normal must not be a line of symmetry of the structure, and the structure must not possess point symmetry.

vi. If these conditions are not fulfilled, the operations under consideration will be co-directional operations of the plane and give rise to co-directional twin planes, provided that in the case of a reversal relatively to a point the structure does not possess point symmetry, and in that of a reversal relatively to the normal this is not a line of symmetry of the structure; and they must, like all other co-directional twinning operations, be equivalent to reversals relatively to the plane (*see* § 9, iv., and § 12, i. and ii.).

§ 10. Modes of Twinning and Types of Twin Axes.

i. It has now been shown that every twin crystal may be obtained by a reversal of a portion of a structure relatively to a plane, a line, or a point, and that no other twinning operations need be considered. It has also been demonstrated that every such operation will result in the formation of a twin plane, provided that it is not a co-directional operation of the structure.

ii. Each of these three reversals may be termed a mode of twinning. If the twinning is by reversal relatively to a plane, which may be termed *plane* (or reflexion) *twinning,* the plane is the twin plane, and its normal the twin axis. If it is by reversal relative to a line, *line* (or rotation) *twinning,* the line is the twin axis, and the plane to which it is normal the twin plane. Finally, in twinning by reversal relatively to a point, *point* (or inversion) *twinning,* every plane has the properties of a twin plane, and every line may be regarded as a twin axis.

iii. In the case of plane twinning the corresponding twin plane is always co-directional, but the twin axis may be either co-directional or contra-directional; in line twinning the twin plane may be either co-directional or contra-directional, but the axis is always co-directional; and in point twinning both twin planes and axes may be either co-directional or contra-directional.

iv. As a twinning operation is not a co-directional operation of the structure (§ 9, ii.), a plane of symmetry cannot be a twin plane with plane twinning; a line of symmetry cannot be a twin axis with line twinning, and point symmetry is inconsistent with point twinning.

v. The different types of twin axes will now be enumerated. In each case the type symbol of the line which forms the twin axis is first given, and with it, in parentheses, the crystallographic classes (usually indicated by their symbols) in which it may occur. These are followed by the type symbol or symbols of the twin axis or axes which may be formed by the line, with examples in each case where such are known. The type symbol of a twin axis is formed by adding to the type symbol of the line the capital letter P, L, or I, according as it is an axis of plane, line, or point twinning. If it is an axis of two modes of twinning, both the corresponding letters are employed.*

Some of the more important types of twin axis are illustrated by diagrams which are stereographic projections on the twin plane of the faces of a general form. Two opposite equivalent parallel faces are represented by a single small disc *in the position of the projection of the upper face.* Opposite faces which are not equivalent do not occur in the same form. An upper face without an equivalent parallel lower face is distinguished by a plus sign with a disc as centre ; a lower face without an equivalent parallel upper face by a minus sign and disc *in the position which a parallel upper face would have occupied.*†

A short radial line drawn inwards from the circumference of the diagram indicates the position of a line of symmetry in the plane of projection. Other symmetry relations are obvious at once from the projection. For instance, if there is point symmetry there are no positive or negative symbols, for every line has two equivalent directions. Small marks placed on the circumference divide the diagrams into 2, 4, or 6 parts, in order to indicate more clearly the results of the application of the twinning operations to the structures.

The first of each group of diagrams shows the structure before the application of a twinning operation, and the other, or others, the same structure after one has been applied.

The position of an axis of line twinning lying in the plane of the paper is shown by a radiating line drawn outwards from the circumference of the diagram, and that of an axis of plane twinning by a shorter radiating line terminated by another longer line at right angles to it, while in the case of a twin axis with both line and plane twinning the radial line is continued outwards beyond the second line. Point twinning is indicated by a small circle in the centre of the diagram.

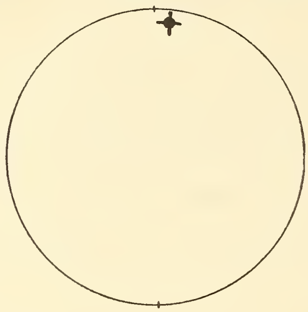
vi. *Uniterminal* lines with *odd* co-linear and co-directional cyclic number yield different twin crystals with the three different modes of twinning. With plane twinning the twinning plane is co-directional, while with line and point twinning it is contra-directional. With line twinning the twin axis is co-directional, while with plane and point twinning it is contra-directional.

1*Uu* (in all crystallographic classes without a centre of symmetry):

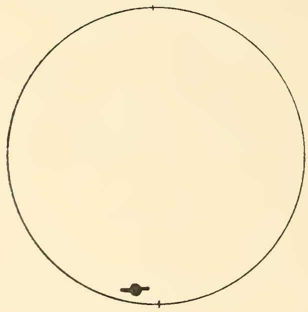
* A line cannot be an axis of all three modes of twinning at the same time (§ 12, i.).

† The presence of two non-equivalent faces is denoted by (see *Min. Mag.*, vol. xv. p. 401).

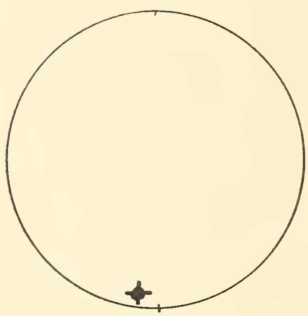
1*UuP,* twin axis of (1) twin of chalcopyrite on 101 (Fletcher, *Phil. Mag.*, vol. xiv., 1882, p. 276), (2) twins of right and left quartz symmetrical to a twin plane oblique to the principal axis, *e.g.* 125, or the plane at right angles to [210]; 1*UuL,* twin axis of twins of right (or left) quartz with oblique twin plane; 1*UuI,* all lines in the "Brazilian" twin of quartz except lines of symmetry and lines at right angles to them (§ 10, ii.).



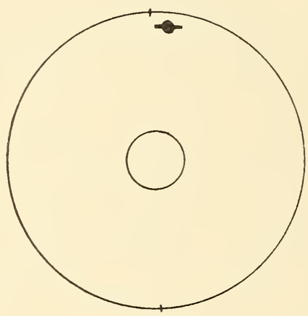
1Uu



1UuP



1UuL



1UuI

3*Uu* (in *III Uu, C Uu*): 3*UuP*; 3*UuL*; 3*UuI,* normals to the tetrahedron faces in the interpenetrant twins of sodium chlorate (*C Uu*).

1*Bu* (in *II Uk, II Bu, IV Bk, IV Bu, III Bu, VI Uk, VI Bk, VI Bu, C Bu*): 1*BuP,* the normal to a face of a trigonal prism of the first order in the twin of proustite described under 3*BuL* (1) (see § 16, ii.); 1*BuL,* twin axes of (1) twin of chalcopyrite on 111; (2) twin of pyrargyrite on rhombohedron face; (3) twin of pyrargyrite on a face of a trigonal prism of the first order, described by Haidinger (*Edinb. Journ. Sci.*, vol. i., 1824,

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E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

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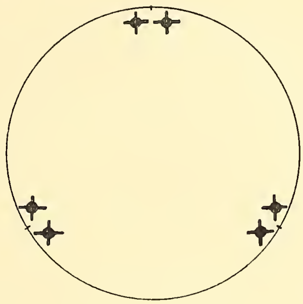
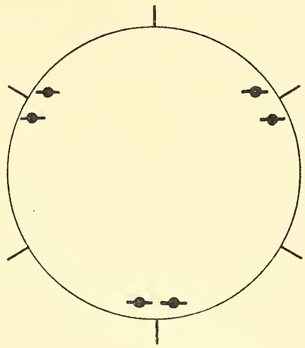
p. 326; but see Miers, *Min. Mag.*, vol. viii., 1888, p. 82), see 3*BuP*; 1*BuI*, any line in a plane of symmetry, but without other symmetry relations in a twin crystal where there is point twinning.

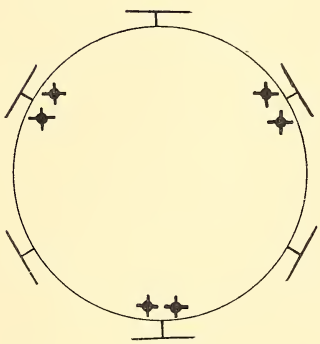
3*Bu* (in *III Bu*, *C Bu*): 3*BuP*, (1) principal axis of pyrargyrite in the twin 1*BuL* (3); 3*BuL*, (1) principal axis of proustite in twin on basal plane (Miers, *Min. Mag.*, vol. viii., 1888, p. 81); (2) normal to 111 in



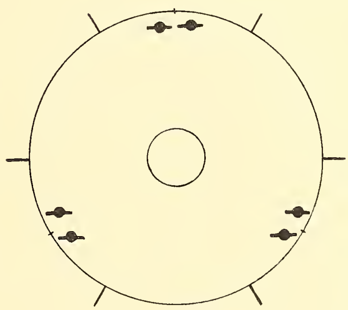
3Bu



3BuP



3BuL



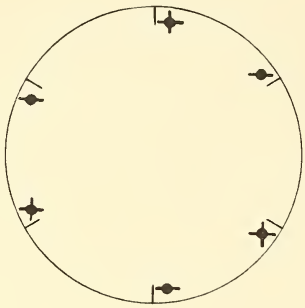
3BuI

twin of sphalerite on that face (Philipp Hochschild, *Neu. Jahrb. Min. Beilage*, vol. xxvi., 1908, pp. 197–205); 3*BuI*, (1) normals to tetrahedron faces of interpenetrant twins of eulytine and diamond (*III Bu* class), (2) principal axis of pyrargyrite in line twinning with the normals to the planes of symmetry as twin axes (Miers, *Min. Mag.*, vol. viii., 1888, p. 78); see 2*UkLI*.

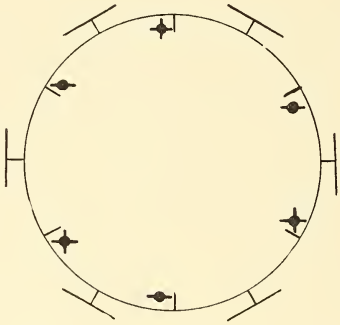
vii. Lines with *odd* co-linear and co-directional cyclic number, *at right angles to a line or lines of symmetry*, but *without point symmetry*

(§ 4, xiii. (3)), also yield different twin crystals with different modes of twinning. With plane twinning the twin plane is co-directional, while with line and point twinning it is contra-directional; the twin axis is always co-directional.

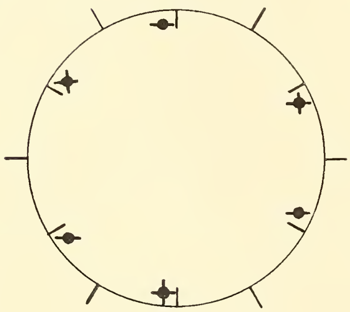
1*Uh* (in *II Uu*, *II Bu*, *II Uh*, *IV Uk*, *IV Bk*, *IV Uu*, *IV Bu*, *IV Uh*, *III Uh*, *VI Bk*, *VI Uu*, *VI Bu*, *VI Uh*, *C Uu*, *C Bu*, *C Uh*): 1*UhP*, 1*UhL*,



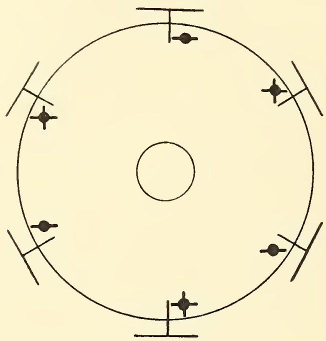
3Uh



3UhP



3UhL



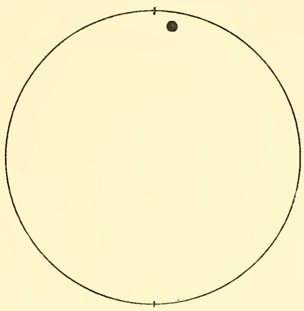
3UhI

and 1*UhI*. Examples occur in the twins of quartz enumerated under 3*Uh* and those of nepheline described on p. 445.

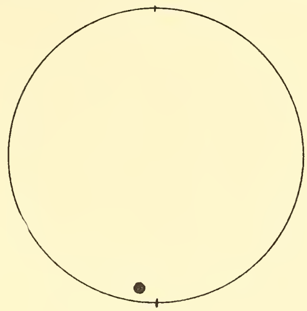
3*Uh* (in *III Uh*, *C Uh*): 3*UhP*, the principal axis of quartz in the lævo-dextrogyral β twin of quartz; 3*UhL*, the principal axis of quartz in the dextrogyral (or lævogyral) twin of quartz; 3*UhI*, the principal axis in the "Brazilian" twin of quartz (for a discussion of these twins of quartz, see Lewis, *Crystallography*, 1899, pp. 519–523).

viii. Lines with *odd* co-linear and co-directional cyclic number and *point* (central) symmetry have identical plane and line twinning, and there is no point twinning. The twin planes and axes are co-directional.

1*Uc* (in all classes with point symmetry): 1*UcPL*, twin axes of (1) all



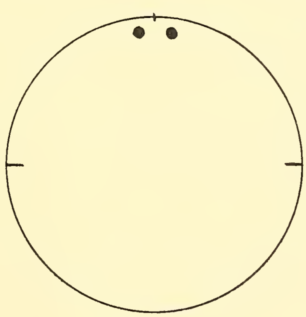
1Uc



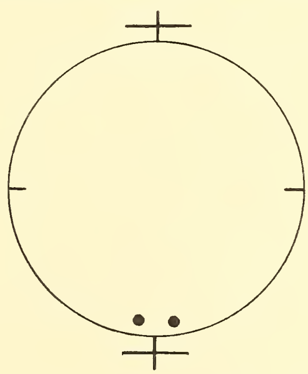
1UcPL

twins of plagioclase and other crystals of the *I Uc* class; (2) those oblique to a plane of symmetry in the *II Uc* class, as in the case of the Baveno twins of orthoclase; (3) those normal to a pyramid face in the *II Bc* class, as in the twins on 112 in redruthite and on 232 in staurolite.

3*Uc* (in *III Uc*, *C Uc*): 3*UcPL*, principal axis of phenakite in twin from Framont in the Vosges (E. Beyrich, *Pogg. Ann.*, vol. xxxiv., 1835, pp.



1Bc

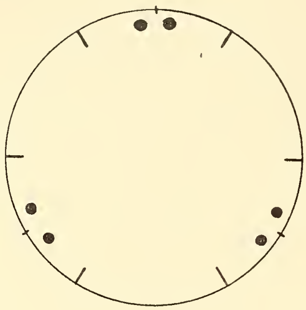


1BcPL

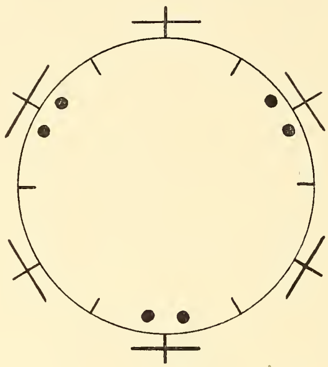
521–2; vol. xli., 1837, pp. 328–332; La Croix, *Minéralogie de la France*, Paris, vol. i., 1893–5, pp. 204–6).

1*Bc* (in *II Uc*, *II Bc*, *IV Uc*, *IV Bc*, *III Bc*, *VI Uc*, *VI Bc*, *C Uc*, *C Bc*): 1*BcPL*, twin axes of (1) the Karlsbad and Manebach twins of orthoclase;

(2) the twins of monoclinic amphiboles and pyroxenes on 100; (3) the twins of staurolite on 032 and 130, and of aragonite on 110; (4) twin of scheelite (*IV Uc*) on the faces of the prisms of the first and second order; (5) the supplementary twin of pyrite on faces of the rhombic dodecahedron.



3Bc



3BcPL

3*Bc* (in *III Bc, C Bc*): 3*BcPL*, twin axes of (*a*) twins of calcite and chabazite, on the basal plane, and (*b*) twins of spinel and fluor on the octahedron.

ix. Lines with co-linear and contra-directional cyclic number 2 or 6. As the plane of twinning is here a plane of symmetry (§ 5, v.), there cannot be plane twinning, and line and point twinning are equivalent (§ 12, iii.). The twin plane is contra-directional, and the twin axis co-directional.

2*Uk* (in *II Uk, III Bu*): 2*UkLI*, twin axes at right angles to the planes of symmetry in the twins of pyrargyrite referred to under 3*BuI*.

6*Uk* (in *VI Uk*): 6*UkLI*.

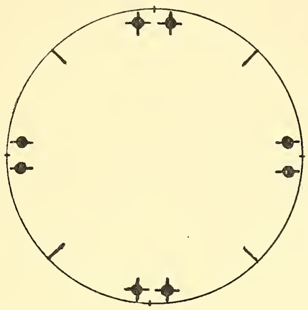
2*Bk* (in *II Bu, IV Bk, IV Bu, VI Bk, VI Bu, CBu*): 2*BkLI*, the normals to the rhombic dodecahedron in twins of eulytine with point twinning.

6*Bk* (in *VI Bk*): 6*BkLI*.

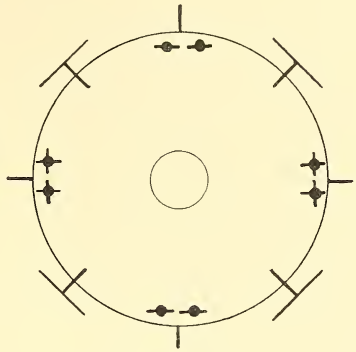
x. Other twin axes with *even* co-linear cyclic number are lines of symmetry of the untwinned structure and cannot be axes of line twinning. Plane and point twinning are equivalent (§ 12, ii.). The twin plane is co-directional, and this is also the case with the twin axis, except where its directions are unlike in the untwinned structure.

4*Uk* (in *IV Uk*): 4*UkPI*.

4*Bk* (in *IV Bk, C Bu*): 4B*kPI*, (*a*) the principal axis of chalcopyrite in twins on the basal plane; (*b*) the crystallographic axes in the interpenetrant twins of eulytine and the diamond.



4Bk



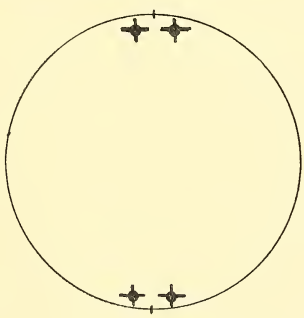
4BkPI

2*Uu* (in *II Uu, III Uh*): 2*UuPI*, the lines of symmetry in the "Brazilian" twins of quartz.

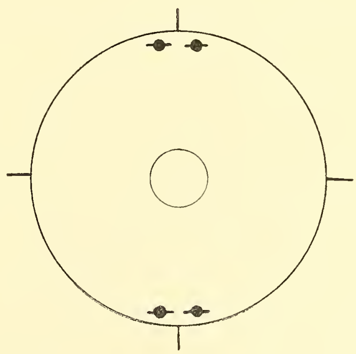
4*Uu* (in *IV Uu*): 4*UuPI*.

6*Uu* (in *VI Uu*): 6*UuPI*. The principal axis of the twins of nepheline referred to in § 19, vi. (see figures 6*Uu*, 6*UuPI*, p. 444).

2*Bu* (in *II Bu, VI Bk*): 2*BuPI*, vertical axis in the "symmetric" twins of hemimorphite on the basal plane.



2Bu



2 BuPI

4*Bu* (in *IV Bu*): 4*BuPI*.

6*Bu* (in *VI Bu*): 6*BuPI*.

2*Uh* (in *II Uh, IV Bk, IV Uh, VI Uh, C Uu, C Uh*): 2*UhPI*, the crystallographic axes of the interpenetrant twins of sodium chlorate.

4*Uh* (in *IV Uh, C Uh*): 4*UhPI*.

6*Uh* (in *VI Uh*): 6*UhPI*.

xi. The following lines cannot be twin axes:—

$2Uc$ (in $II\,Uc$, $III\,Bc$), $4Uc$ (in $IV\,Uc$), $6Uc$ (in $VI\,Uc$), $2Bc$ (in $II\,Bc$, $IV\,Bc$, $VI\,Bc$, $C\,Uc$, $C\,Bc$), $4Bc$ (in $IV\,Bc$, $C\,Bc$), $6Bc$ (in $VI\,Bc$), because reversals relatively to the line, to the plane to which it is normal, and to a point are all co-directional operations of the structure. For instance, the normal to the clinopinakoid in orthoclase and the normal to the rhombic dodecahedron faces in spinel cannot be twin axes.

§ 11. General Relation between Equivalent Twinning Operations.

i. If the same relation between the component structures of a twin crystal results from different twinning operations, these are said to be equivalent to one another.

ii. As all twinning operations are reversals (or equivalent to reversals), the combination of any two equivalent twinning operations must be a co-directional operation of the structure (§ 5, ii.); and if a co-directional operation be resolved into two reversals, either of these acting alone on the structure will be equivalent to the other, so that if one be a twinning operation, the other will be an equivalent twinning operation, and in like manner, if one be a co-directional operation, the other will be so likewise. If therefore, every combination of two reversals (see § 5) be examined to discover if it be a co-directional operation, every case in which two reversals are equivalent twinning operations will be ascertained.

§ 12. Equivalent Twinning Operations deduced from Group (*a*) of Three Reversals (§ 5, iii.).

i. The operation of reversal relatively to a point is equal to reversal relatively to a plane combined with reversal relatively to its normal. If, therefore, a twin crystal result from the reversal relatively to a plane of a portion of a structure possessing *point symmetry*, an indistinguishable result may be obtained by substituting a reversal relatively to the normal to the plane, and *vice versa*. The same twin plane and axis will correspond to both operations (§ 10, ii.). Inversely, if there be both plane and line twinning with the same twin plane and twin axis, the untwinned structure must possess point symmetry (see, for example, figures $1Uc$ and $1UcPL$, p. 435). It follows from this that point twinning cannot coexist with plane and line twinning which have a common twin axis (§ 10, iv.).

ii. A reversal relatively to a line is equal to a reversal relatively to the plane to which it is normal combined with reversal relatively to a point.

If, therefore, a *line of symmetry* be a twin axis of plane twinning; an indistinguishable twin crystal may be obtained by point twinning; and *vice versa*, if a twin crystal may be explained by point twinning, every line of symmetry is a twin axis of plane twinning (figures 2*Bu*, 2*BuPI*, p. 437, and figures 3*Uh* and 3*UhI*, p. 434).

iii. The operation of reversal relatively to a plane is equivalent to a reversal relatively to its normal combined with reversal relatively to a point. Accordingly, if a *plane of symmetry* be a twin plane of line twinning, the twin crystal may also be explained by point twinning. Again, if point twinning exist, every plane of symmetry is a twin plane of line twinning (figures 4*Bk* and 4*BkPI*, p. 437; also figures 3*Bu* and 3*BuI*, p. 433).

§ 13. Equivalent Twinning Operations connected by Group (*b*)

A reversal relatively to a line is equal to a combination of reversals relatively to two other lines at right angles to the first and to each other. Accordingly, if there be a twin axis of line twinning at right angles to a *line of symmetry*, there will be another twin axis of line twinning at right angles to the first and to the line of symmetry (figures 3*Uh* and 3*UhL*, p. 434).

§ 14. Equivalent Twinning Operations connected by Group (*c*).

i. A reversal relatively to a line is equal to a combination of reversals relatively to two planes meeting at right angles in that line. It follows that if a *line of symmetry* lie in a twin plane of plane twinning, there is another twin plane of plane twinning at right angles to the first and intersecting it in the line of symmetry; or, what comes to the same thing, if there is one twin axis of plane twinning at right angles to a line of symmetry, there must be another at right angles both to the first and to the line of symmetry (figures 3*Uh* and 3*UhP*, p. 434).

ii. Combining this result with that of § 13, we see that, if there be a twin axis either of line or plane twinning, or both, at right angles to a line of symmetry, there will be another twin axis with the same mode or modes of twinning at right angles to the first twin axis and to the line of symmetry (figures 1*Bc* and 1*BcPL*, p. 435). This is a special case of § 15 and § 16, i.

iii. A reversal relatively to a plane is equal to a combination of a reversal relatively to any other plane at right angles to the first and a reversal relatively to the line of intersection of the two planes; so that, if a reversal relatively to a plane at right angles to a *plane of symmetry* results in a twin crystal, this may also be obtained by a reversal relatively to the line of intersection of the two planes, and *vice versa*. Here the twin axes both of the plane twinning and the line twinning lie in the plane of symmetry and are at right angles to each other. Accordingly, if a twin axis of plane twinning lie in a *plane of symmetry*, there is a twin axis of line twinning in the same plane of symmetry and at right angles to the twin axis of plane twinning (figures $3Bu$ and $3BuP$, p. 433); and if a twin axis of line twinning be in a *plane of symmetry*, there is a twin axis of plane twinning at right angles to it in the same plane of symmetry (figures $3Bu$ and $3BuL$, p. 433).* These are special cases of § 16, ii., and § 17.

§ 15. Equivalent Twinning Operations connected by an Axis of Co-directional Symmetry.

i. It has been shown that a rotation through an angle 2θ is equal, in the first place, to a combination of reversals relatively to any two lines at right angles to the axis, making an angle θ with each other, and, secondly, to a combination of reversals relatively to two planes at right angles to two such lines (§ 5, vi., vii.). If the rotation be a co-directional operation of the structure, each of these pairs of reversals will represent either two twinning operations with the same mode of twinning, or two co-directional operations.

ii. It follows that, if a structure possess an axis of co-directional symmetry with cyclic number n, so that a rotation through an nth part of a complete turn is a co-directional operation of the structure, and if a line at right angles to the axis be a twin axis of plane or line twinning or both, a second line at right angles to the axis of co-directional rotatory symmetry and making an angle with the first line equal to the $2n$th part of a whole turn will also be a twin axis with the same mode or modes of twinning. There will accordingly be a succession of lines making angles of a $2n$th part of a complete turn with each other, which will all be twin axes with the same mode or modes of twinning. The $\overline{n+1}$th line will, however, coincide with the 1st, for n $2n$th parts of a complete turn are equal to a half turn. In the same way the $\overline{n+2}$th line will coincide with the 2nd, and so on, and the number of distinct lines and twin axes will be only n.

iii. If, therefore, a twin axis be at right angles to an axis of co-directional symmetry with cyclic number n, it will be one of n equivalent twin axes with the same mode or modes of twinning, at right angles to the same axis and making equal angles with each other.

* If a line in the plane of symmetry be an axis of both plane and line twinning, there must be a centre of symmetry, and the normal to the plane of symmetry will be a line of symmetry (§ 5, iii. (*a*) and § 11, ii.), so that the case falls within § 14, ii.

iv. If n be odd, all these n lines are equivalent, and so are the plane at right angles to them, and as the mode of twinning is the same in each case they may be described as *equivalent twin axes* and *twin planes* (figures $3Uh$, $3UhP$, and $3UhL$, p. 434).

v. If n be even, there cannot be more than $n/2$ *distinct* equivalent lines at right angles to the axis, unless the axis of co-directional symmetry also possess co-linear and contra-directional symmetry with cyclic number $2n$, in which case there will be n equivalent lines constituting n equivalent twin axes (figures $4Bk$, $4BkPI$, p. 437, where $n=2$; see also § 17).

vi. If, on the other hand, n be even and the axis does not possess co-linear and contra-directional symmetry with cyclic number $2n$, the n lines, which are twin axes with the same mode of twinning, must consist of two series such that those of one series bisect the angles between those of the other, and that the $n/2$ twin axes of each series are equivalent to one another, but not to those of the other series (see figures $6Uu$, $6UuP'$, and $6UuL'$, p. 444).

vii. The same reasoning may be employed in respect of *co-directional* reversals relatively to lines at right angles to the axis, or planes to which they are normal (in other words, planes passing through the axis); with the result that, if there be one such line or plane of symmetry, there will be a number of them equal to the co-directional cyclic number of the axis.

§ 16. Axes of Co-directional Symmetry which are Twin Axes.

i. Combining the results of § 14, ii., and § 15, we find that if an axis of co-directional symmetry with cyclic number n, and with n lines of symmetry at right angles to it, be a twin axis with line or plane twinning, or both, there will be n twin axes with the same mode or modes of twinning each at right angles to the axis and line of symmetry (see figures $3Uh$, $3UhP$, and $3UhL$, p. 434, and $3Bc$ and $3BcPL$, p. 436).

ii. Again, combining the results of § 14, iii., and § 15, if an axis of co-directional symmetry with cyclic number n, and with n planes of symmetry passing through it, be a twin axis of line or plane twinning, there will be n twin axes with plane or line twinning, as the case may be, at right angles to the axis, and each will lie in a plane of symmetry (see figures $3Bu$, $3BuP$, and $3BuL$, p. 433).

§ 17. Equivalent Twin Axes connected by an Axis of Contra-directional Symmetry.

i. A combination of a rotation through an angle 2θ and a reversal relatively to a point was shown in § 5, viii., to be equal to a combination of a reversal relatively to a line at right angles to the axis of rotation and a reversal relatively to a plane whose normal is at right angles to the axis and makes an angle θ with the line of reversal.

ii. If a structure has an axis of contra-directional symmetry with cyclic number n, a rotation through an nth part of a complete turn is a contra-directional operation of the structure, and a combination of the same rotation with a reversal relatively to a point will be a co-directional operation of the structure (§ 4, vii.); consequently, if a line at right angles to an axis of contra-directional symmetry with cyclic number n be a twin axis of plane twinning or line twinning, another line at right angles to the same axis and making an angle equal to a $2n$th part of a whole turn with the first will be a twin axis of line twinning or plane twinning.

iii. There will therefore, as in § 15, be n distinct twin axes, and, as n is always even (§ 4, xi.) in the case of an axis with contra-directional symmetry, there will be $n/2$ twin axes of line twinning and $n/2$ twin axes of plane twinning alternating with each other (figures $4Bk$ and $4BkPI$, p. 437).*

§ 18. Combinations of Twinning Operations.

i. As all twinning operations are reversals, or equivalent to reversals, a repetition of the same twinning operation will result in identity (§ 5, i.), so that a portion of a structure to which it has been applied twice or any even number of times will have the same disposition in space as a portion to which it has not been applied at all; and all portions to which it has been applied an odd number of times will have the same disposition in space as those to which it has been applied once only. These relations are well illustrated in the lamellar twinning of plagioclase.

ii. A combination of any number of equivalent twinning operations will obviously have the same result as the application of any one of them the same number of times.

iii. If, however, two twinning operations, which are independent, that is to say, which are not equivalent to one another, are applied to a portion of a structure, its disposition in space will differ from that of a portion to which neither has been applied and may depend on the order of application. We now proceed to consider the results of such combinations of twinning operations.

* The same lines cannot in this case be axes both of plane and of line twinning, for if they were, the structure would possess point symmetry (§ 12, i.), which is inconsistent with contra-directional symmetry.

§ 19. Combinations of Independent Twinning Operations with Parallel or Perpendicular Twin Axes.

i. If a twin axis of one twinning operation coincide with, or be at right angles to, a twin axis of an independent twinning operation, the result of a combination of these two twinning operations will not depend on the order in which they are applied; for a twinning operation does not alter the position of a line (though it may reverse its directions) which is parallel or at right angles to a twin axis produced by that twinning operation, and every combination of twinning operations the twin axes of which are parallel or at right angles to each other falls within one of the groups of three reversals (§ 5, iii.) in which the order of application of the operations makes no difference. If one of the twinning operations involve point twinning, it must necessarily have a twin axis parallel to a twin axis of the other twinning operation (§ 10, ii.).

ii. Let T_1 and T_2 be two independent twinning operations which satisfy these conditions, so that the result of their combination is independent of the order in which they are taken. These may be applied to a structure in such a manner that one portion is affected by neither, a second by T_1, a third by T_2, and the fourth by a combination of both, the nature of which we now proceed to determine in particular cases.

iii. If one of the twinning operations consist of point twinning, and the other of plane or line twinning, the combination will be a twinning operation with the same twin axis and twin plane as the plane or line twinning, but with the mode changed from plane to line twinning or *vice versa* (§ 5, iii., group (a)). There cannot be a combination of point twinning with a twinning operation resulting in plane and line twinning with the same twin axis, for such a twin axis implies symmetry relatively to a point, so that a reversal relatively to a point would be a co-directional operation.

iv. A combination of plane and line twinning having the same twin axis will obviously result in point twinning.

v. If twin axes of the two different twinning operations be at right angles, three cases may be distinguished. If both be twin axes with line twinning or both twin axes with plane line, the combination will consist of line twinning with a twin axis at right angles to both (groups (b) and (c)). If one be a twin axis with plane twinning and the other a twin axis with line twinning, the combination will be a twin axis at right angles to both with plane twinning (group (c)).

vi. In all these cases the combination of two twinning operations is itself a twinning operation, and as the three twinning operations form one of the groups of three reversals, each may be looked upon as a combination of the other two.

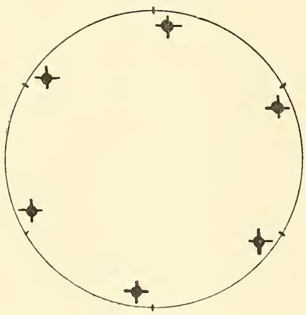
Let S be the portion to which neither twinning operation has been applied, S_1 that to which T_1, and S_2 that to which T_2 has been applied, and S_3 that subjected to a combination of both, T_3.

Then $[S]T_1=[S_1]1$; $[S]T_2=[S_2]1$; $[S]T_3=[S_3]1$; also $T_1.T_2=T_2.T_1=T_3$; $T_3.T_1=T_1.T_3=T_2$ and $T_2.T_3=T_3.T_2=T_1$.

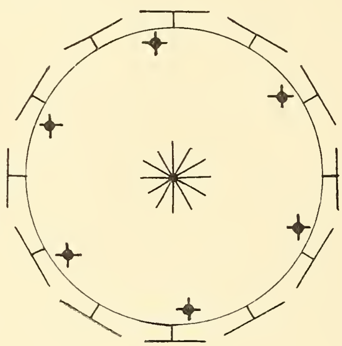
Then $[S_2]T_1=[S]T_2.T_1=[S]T_3=[S_3]1$, and so on.

The relations between all the different portions can in this manner be shown to be those indicated in the following table :—

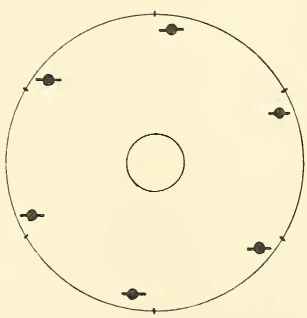
Disposition of the structure to which the operations are applied		S	S_1	S_2	S_3	
Resulting disposition of the structure	S	1	T_1	T_2	T_3	Operations applied (where $T_3=T_1.T_2$).
	S_1	T_1	1	T_3	T_2	
	S_2	T_2	T_3	1	T_1	
	S_3	T_3	T_2	T_1	1	



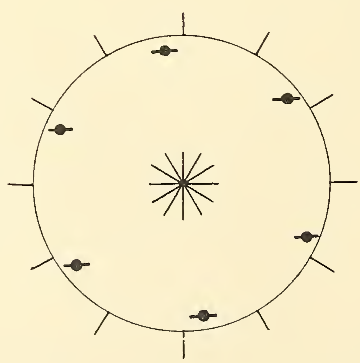
bUu



bUuP'



bUuPI



bUuL'

See next page.

These relations are illustrated by the twin crystals of nepheline described by H. Baumhauer (*Zeit. Kryst. Min.*, 1882, vol. vi. pp. 210–6, and 1891, vol. xviii. pp. 611–8), where T_1 is an operation of plane twinning with the faces of the hexagonal prisms of the first and second order as twin planes and their normals as twin axes; T_2 is an operation of point twinning equivalent to plane twinning with the basal plane as twin plane and the principal axis as twin axis; and T_3 is an operation of line twinning with the same twin planes and axes as T_1 (see p. 444, figures $6\,Uu$, $6\,UuP' = 1\,UhP = T_1$, $6\,UuPI = T_2$, $6\,UuL' = 1\,UhL = T_3$; the symbols P' and L' indicate the presence of axes of plane and line twinning respectively lying in the plane of projection. The short lines intersecting in the centre are common lines; see § 21).

vii. The well-known law (see § 8, ii.; and § 24, ii.) that in every twin crystal there is either a twin plane parallel to a possible face or a twin axis parallel to a possible edge, or both, is frequently inapplicable to the twinning relations between the two portions of a twin crystal which are connected by a twinning operation which is a combination of two independent twinning operations.

For instance, in crystals of plagioclase showing both albite twinning, and karlsbad twinning (with the vertical axis as twin axis), the twin axes of the two twinning operations are at right angles to each other. Their combination is therefore a twinning operation with a twin axis lying in the brachy-pinakoid at right angles to the vertical axis and neither normal to a possible face nor parallel to a possible edge.

§ 20. Combinations of Independent Twinning Operations with Twin Axes Oblique to one Another.

i. We will now consider the case of two independent twinning operations such that no twin axis resulting from one is parallel or at right angles to a twin axis resulting from the other.

Let T_1 and T_2 be the twinning operations, and a_1 and a_2 their respective twin axes making an angle θ with each other. Each operation will be a reversal relatively to a plane or a line, and not a reversal relatively to a point or equivalent to one (§ 19, i.). If a_1 and a_2 both remained fixed, and the operation T_1 were first applied and then T_2, the result would be a rotation through an angle 2θ round a line at right angles to a_1 and a_2 in the direction a_1, a_2, combined, if the modes of twinning were different with a reversal relatively to a point (§ 5, viii.). Thus $T_1 . T_2 = C_{2\theta}$ (or $C_{2\theta} . R_i$), where $C_{2\theta}$ is a rotation in the direction a_1, a_2, which is taken as positive. The first twinning operation T_1 will, however, change the position of the twin axis a_2 of the second operation T_2 to a'_2 on the other side of a_1, but so that the plane containing the twin axes, the line at right angles to them, and the angle between them remain the same. The successive application of the two twinning operations therefore results in a combination of T_1 and T'_2, where T'_2 is a twinning operation with the same mode of twinning as T_2, but with a'_2 as twin axis. This combination will be equal to a rotation in the negative direction a_2, a_1, round the same line, combined as before, if the modes of twinning be unlike, with a reversal relatively to a point. Then $T_1.T'_2 = C_{-2\theta}$ (or $C_{-2\theta}.R_i) = T_2.T_1$. If T_2 be applied first, a_1 will be moved in like manner to a'_1 on the other side of a_2, and the rotation will be in the positive direction a_1, a_2. Therefore $T_2.T'_1 = C_{2\theta}$ (or $C_{2\theta}.R_i) = T_1.T_2$, where T'_1 is a twinning operation similar to T_1 but with a'_1 as twin axis.

ii. A twin crystal subjected to two independent twinning operations, the result of the successive application of which depends on the order of their application, may consist of five differently oriented portions—one, S, to which neither has been applied, two, S_1, S_2, to which one, and two, S_3, S_4, to which both have been applied. The relations between these are shown in the following table:—

Disposition of the structure to which the operations are applied		S	S_1	S_2	S_3	S_4	
Resulting disposition of the structure	S	1	T_1	T_2	$C_{2\theta}$	$C_{-2\theta}$	Operations applied.
	S_1	T_1	1	$C_{-2\theta}$	T'_2	T_2	
	S_2	T_2	$C_{2\theta}$	1	T_1	T'_1	
	S_3	$C_{-2\theta}$	T'_2	T_1	1	$C_{-4\theta}$	
	S_4	$C_{2\theta}$	T_2	T'_1	$C_{4\theta}$	1	

If the two modes of twinning be different, the rotations 2θ and -2θ will be combined with a reversal relatively to a point. The above relations are illustrated by the twinning of bournonite and marcasite on dome faces aragonite on prism faces; and combinations of albite and pericline twinning in plagioclase.

§ 21. Cross Planes in Twin Crystals.

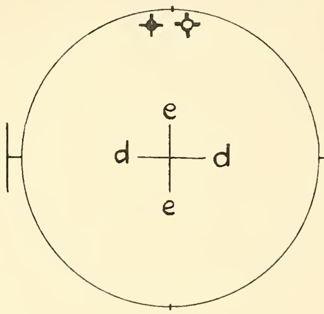
i. The plane of contact in twin crystals is not always a twin plane. It may be a plane in which some, but not all, of the lines are common lines along which the molecular intervals are the same in the two structures and determine their regular intergrowth. Even where the plane of contact is a twin plane, it is probably only the more important common lines (that is to say, those with the smallest molecular intervals) that determine the twinning.

ii. Let S', S'' be two structures of the same shape which form a compound crystal containing a common plane de made up of two equivalent planes $d'e'$ and $d''e''$ of S' and S'' respectively, and let d be a common line in de composed of the two equivalent lines d' and d'' (see figures, p. 448). Then the cyclic succession of the lines in $d'e'$ on either side of the line d' will be either the same as that of the lines in $d''e''$ on either side of d'', or opposite. In the former case, since d' coincides with d'', every other line in $d'e'$ must coincide with an equivalent line in $d''e''$. Every line in the common plane de is therefore a common line and the common plane is co-linear, so that, if the two structures are not co-directional, it will be a twin plane.

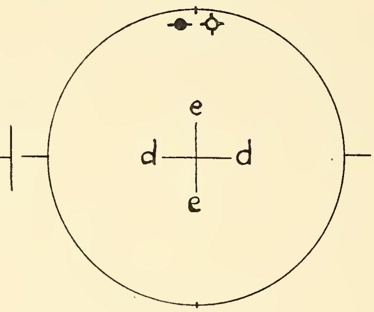
iii. If, on the other hand, the cyclic succession of the lines of the plane $d'e'$ on either side of d' be opposed to that of the lines in the plane $d''e''$ on either side of d'', the lines of $d'e'$ will not all coincide with equivalent lines of $d''e''$ and the common plane de will not be co-linear. If, however, the operation of reversal relatively to the line d' be applied to the structure S', the cyclic succession of the lines in the plane $d'e'$ will be reversed, so that the cyclic succession will become the same on either side of d' in $d'e'$ and d'' in $d''e''$, and as d still remains a common line, every line in $d'e'$ will coincide with an equivalent line in $d''e''$ and form a common line, and the common plane de will be co-linear. Every common plane which is not co-linear but in which there is at least one common line may therefore be converted into a co-linear common plane by the reversal of one of the component structures relatively to the common line.

iv. If the structure S' be again reversed relatively to the same line, the original common plane will be obtained. In this second reversal not only the line d but the line e at right angles to it in de will remain a common line. If, therefore, there be one common line in a common plane, there must be another at right angles to it. A common plane which is not co-linear but contains one or more pairs of common lines may be termed a *cross plane*. In figures $1UuP'_d$, $1UuL'_d$, $1UuP'_e$, and $1UuL'_e$, the short lines d and e intersecting at the centre are common lines and the plane of projection is in each case a cross plane. The discs with hollow centres indicate the original disposition of the structure as shown in figure $1Uu$, p. 432, while those with solid centres show the disposition of a portion of the structure to which the operation of plane or line twinning with twin axis d or e, as the case may be, has been applied; see § 22.

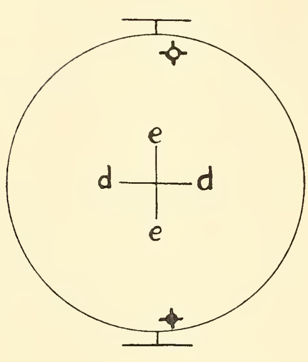
v. Every plane containing one or more pairs of common lines at right angles to each other will be a common plane, and if it is not co-linear will be a cross plane. For let *d* and *e* be two common lines at right angles to each other, *de* the plane containing them, and *n* its normal; then since *d* is a common line *ne* to which it is normal will be a common plane (§ 7, iii.). But since *e* is a common line in the common plane *ne*, *n* which is at right angles to *e* will also be a common line, and the plane *de* to which it is normal will be a common plane.



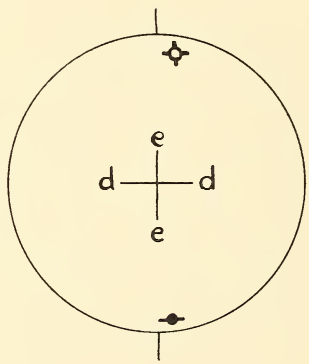
$1UuP'_d$



$1UuL'_d$



$1UuP'_e$

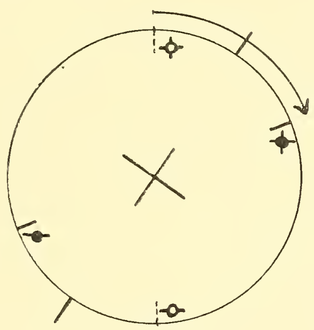


$1UuL'_e$

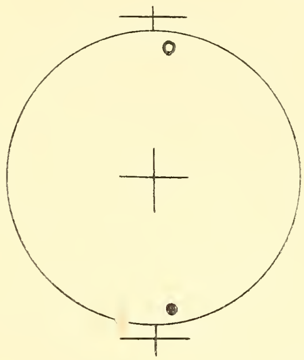
vi. A cross plane may be either a contact, or an abstract common plane (§ 8, iii.), the latter having usually no structural significance.

vii. It follows from paragraphs iii. and iv. that if two structures possess a co-linear common plane, a reversal of one relatively to a line in this co-linear plane will convert it into a cross plane, provided that the reversal is not a co-linear operation of the component planes. This is also the case with a reversal relatively to a plane at right angles to the co-linear plane.

viii. If l_1 be a line of symmetry in a co-linear common plane K, a rotation of one component structure S' round the normal n of K through any angle 2θ will convert K into a cross plane, provided that the rotation is not a co-linear operation of K (figure $1UhL'$). Let $C_{2\theta}$ be a rotation through an angle 2θ round the normal n, R_1 the operation of reversal relatively to l_1, and R_2 that of reversal relatively to l_2, making an angle θ with l_1, then $[S]C_{2\theta}=[S]R_1.R_2\,(=)\,[S]R_2$, for R_1 is a co-directional operation of the structure. Then, since the operation R_2 is equivalent to the rotation $C_{2\theta}$, it cannot, by hypothesis, be a co-linear operation. It must therefore result in a cross plane, and $C_{2\theta}$ must do so likewise.



1UhL'



1UcP'L'

(In these figures hollow discs and interrupted lines correspond to the original disposition of the structure.)

Again, if p_1 be a plane of symmetry at right angles to a co-linear common plane K, a rotation through an angle 2θ round the normal n of K, if it be not a co-linear operation of K, will in the same manner convert it into a cross plane; for if R_1 be a reversal relatively to p_1, and R_2 be one relatively to a plane p_2 also at right angles to K and making an angle θ with p_1, we have as before $[S]C_{2\theta}=[S]R_1R_2=[S]R_2$.

§ 22. Relations between Structures possessing a Cross Plane in Common.

i. We have seen that every cross plane can be obtained by the reversal of one of two structures which possess a co-linear common plane, de, relatively to a line d in that plane. This co-linear plane must be either (1) a plane common to two co-directional structures or, what comes to the same thing, two portions of the same structure, or (2) a twin plane.

ii. In the former case the operation, R_d, of reversal relatively to d, will be an operation of line twinning* with the line d as twin axis and the plane ne at right angles to the cross plane de, and cutting it in the line e at right angles to d, as twin plane (see figure $1UuL'_d$, p. 448, where L'_d denotes line twinning with d as twin axis).

iii. If, on the other hand, de, the co-linear common plane, be a twin plane, it may be obtained by applying a twinning operation, T, to one of two portions of the same structure. The cross plane will therefore result from the application of the operation $U=T.R_d$.

The twinning operation T must be (1) a reversal relatively to the plane de (figure $1UuP$, p. 432), (2) a reversal relatively to the normal n (figure $1UuL$), or (3) a reversal relatively to a point (figure $1UuI$). The operation $U=T.R_d$ will be, in the first case, a reversal relatively to the plane nd at right angles to de and passing through the lines n and d (§ 5, iii., group (c)). It will accordingly be an operation of plane twinning with e as twin axis and nd as twin plane (see figure $1UuP'_e$, where P'_e denotes plane twinning with e as twin axis). In the second case it will be a reversal relatively to the line e (group (b)), and therefore an operation of line twinning with the same axis and plane of twinning (see figure $1UuL'_e$) as in the first case. In the third it will be a reversal relatively to the plane ne, which is at right angles to d and to the plane de, and passes through the lines n and e (group (a)). It is an operation of plane twinning with d as twin axis, and ne as twin plane (see figure $1Uu\mathrm{P}'_d$).

In every cross plane, therefore, one common line at least is a twin axis.

§ 23. Number of Common Lines in Cross Planes.

i. We shall next determine how many common lines there may be in a cross plane. Let d_1 and d_2 be two common lines in a cross plane K; then a reversal R_1 of one structure relatively to d_1, and a reversal R_2 of the same structure relatively to d_2, will each result in a co-linear common plane which may be either co-directional or contra-directional. If these resulting common planes are both co-directional or both contra-directional, R_1 must, as applied to K, be equivalent to R_2, or, in symbols, $[K]R_1\ (=)\ [K]R_2$. If one be co-directional and the other contra-directional, R_1 must be equivalent to R_2 combined with a reversal relatively to a point, that is, $[K]R_1\ (=)\ [K]R_2.R_i$, where R_i is a reversal relatively to a point. Therefore, either $R_1.R_2$ or $R_1.R_2.R_i$ must be a co-directional operation of K (§ 5, ii.).

* The existence of the cross plane precludes, of course, its being a co-directional operation.

ii. Now if R_1 and R_2 make an angle θ with each other, $R_1.R_2$ will be equal to a rotation through 2θ round the normal to K, and $R_1.R_2.R_i$ to such a rotation combined with a reversal relatively to a point. But a reversal relatively to a point as applied to a plane is equivalent to a reversal relatively to its normal (for their combination is equal to a reversal relatively to the plane, which is a co-directional operation of the plane; see § 5, ii. and iii., group (a), and § 6, i.), and this is equal to a rotation round the normal through a half turn, π. The operation of rotation round the normal through either 2θ *or* $2\theta \pm \pi$ must accordingly be a co-directional and therefore, also, a co-linear operation of the plane. But a rotation round the normal through a half turn, π, is always a co-linear operation of a plane. It follows that, if two common lines in a cross plane meet at any angle θ, rotations round the normal through both the angle 2θ *and* the angle $2\theta \pm \pi$ will be co-linear operations of the plane.

Let such an angle of rotation be denoted by ϕ. Then, inversely, if ϕ be any co-linear operation of the component planes K' and K'', and if d_1 be a common line, there will be other common lines making angles $\frac{\phi}{2}$ and $\frac{\phi}{2} \mp \frac{\pi}{2}$ with d_1.

The following table may now be constructed:—

Co-linear cyclic number of normal, *in structure* .	1, 2	4	3, 6
Co-linear cyclic number of normal, *in plane* (§ 6, iv.)	2	4	6
ϕ = any co-linear rotation round normal in plane .	π	$\frac{\pi}{2}$, π	$\frac{\pi}{3}$, $\frac{2\pi}{3}$, π
θ = angle between any common line and a given common line $= \frac{\phi}{2} \mp \frac{\pi}{2}$.	0, $\frac{\pi}{2}$	0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$	0, $\frac{\pi}{6}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$, $\frac{2\pi}{3}$, $\frac{5\pi}{6}$
Total number of common lines	2	4	6

(See figures 6 *Uu*, 6 *UuP'*, 6 *UuL'*, p. 444; 1 UuP'_d, 1 UuL'_d, 1 UuP'_e, 1 UuL'_e, p. 448; 1 *UhL'* and 1 *UcP'L'*, p. 449.)

§ 24. Planes at Right Angles to Twin Planes and Cross Planes.

i. As the normal to a twin plane is a common line, a plane at right angles to a twin plane contains two common lines, for every line in a twin plane is a common line. It must consequently be either a cross plane or a co-linear plane. In the latter case it must be a twin plane, as the two structures are not co-directional.

ii. When the plane of contact of the two structures of a twin crystal is not a twin plane, it is found by experience to be at right angles to a twin plane, and must therefore be a cross plane. The structural reasons why this should be the case have already (§ 8, ii., and § 21, i.) been indicated.

iii. Similarly, a plane at right angles to a cross plane and intersecting it in one of its common lines will be a twin plane or a cross plane.

iv. Where, as in most pericline twins of plagioclase, the plane of contact is not a possible face, it appears to be always a cross plane and not a twin plane; one common line (the macro-axis in pericline twins) is a possible crystal edge, and the other the line of intersection of the plane of contact and a possible crystal face usually a plane of cleavage (the brachy-pinakoid in pericline twins) (*Min. Mag.*, vol. xv., 1910, pp. 392–3). This is illustrated by figure $1UcP'L'$ (p. 449), which shows the relation between the two component structures of a pericline twin of plagioclase, when they are projected on the cross plane which forms the plane of composition known as the rhombic section.

§ 25. Co-spatial Structures and Operations.

i. In the preceding pages the coincidence of equivalent lines in the constituent structures of compound crystals has been studied because it is believed that the determining factor in the joint growth of such structures is in most cases the coincidence of molecular rows in which the molecules are at an equal distance from one another, and that this equality mainly exists where the molecular rows are equivalent to one another. There are, however, lines in crystal structures which are not equivalent, but correspond to molecular rows in which the molecular intervals are either equal or stand in a simple ratio to one another. Such lines occur in a crystal which does not possess all the symmetry which could be associated with a system of axes and parameters to which it might be referred consistently with the rationality of the indices of its faces and edges.

ii. If two structures of the same form, which are not co-linear, are so related that every line having rational indices (and therefore parallel to a possible edge) in the one coincides with a line with rational indices in the other, the two structures may be said to be *co-spatial.*

iii. An operation which brings a structure into a co-spatial relation to itself as it existed before the operation may be described as a *co-spatial operation.*

iv. A co-spatial operation, though not a co-linear operation of the structure to which it is applied, will be a co-linear operation of a structure with higher symmetry, which may be referred to the same crystallographic axes, and is therefore a reversal or a rotation.

§ 26. Equispatial and Co-spatial Lines and Planes.

i. A line with rational indices which is brought by a co-spatial operation into the former position of another, non-equivalent, line may be said to be *equispatial* with it; for the molecular interspaces of such lines, or low integral multiples of such interspaces, will be equal to one another.

This follows from the fact that the structure in its new position can obviously be referred to the old axes in their original position with the same parameters, and that at least some lines will occupy the former place of equivalent lines.

ii. Similarly, if a plane with rational indices, and therefore parallel to a possible face, is brought by a co-spatial operation into coincidence with the former position of another, non-equivalent, plane of the same crystal structure, the two planes may be described as equispatial, for every line with rational indices in one will be equispatial with or equivalent to a corresponding line in the other.

iii. The term equispatial may be extended to include non-equivalent lines and planes with rational indices belonging to *different* structures, but capable of being brought into coincidence by an operation which brings those structures into a co-spatial relation with each other.

iv. If a line in one structure coincide with an equispatial line in another, the two lines may be described as co-spatial and as forming together a *co-spatial line.*

v. If a plane with rational indices of one structure coincide with an equispatial or equivalent plane of another structure in such a manner that every line with rational indices of one plane coincides with an equispatial or equivalent line of the other, but the two planes are not co-linear, the planes may be said to be co-spatial and to form a *co-spatial plane.*

A co-spatial plane may or may not be a common plane or a cross plane.

vi. If two equispatial planes coincide to form a plane that contains two, four, or six co-spatial lines, then that plane may be termed a *cross-spatial plane.*

vii. Compound crystals may be expected to occur in which the plane of contact is a co-spatial plane or cross-spatial plane; for these planes contain co-spatial lines, in which the molecular intervals, or low integral multiples of them, are equal to one another in the two structures.

viii. A co-spatial plane may be formed by applying to a part of a uniform structure either a co-spatial operation alone, or such an operation followed by an operation which, if the co-spatial operation had not been previously applied, would have been a twinning operation.

§ 27. Classification of Co-spatial Operations.

i. As we have seen, a co-spatial operation is either a reversal or a rotation. It cannot be a reversal relatively to a point, because that is always a co-linear operation. Nor can it be a reversal relatively to a plane at right angles to a line of symmetry, or a reversal relatively to a line at right angles to a plane of symmetry, because these operations are equivalent to a reversal relatively to a point (compare § 12, ii. and iii.).

ii. We shall first consider co-spatial operations which would be co-linear operations of crystals belonging to classes with higher symmetry in the same system. These must be reversals relatively to a line or plane, and therefore twinning operations, or a rotation with cyclic number four; for a rotation with cyclic number two is equal to a reversal relatively to the axis of rotation; and if a line is an axis of symmetry with co-linear cyclic number three or six in one class of a system, it will be so in all, so that if a rotation through a third or a sixth of a complete turn be co-linear in one class, it will be co-linear in all the classes of the same system (see § 3, v., and table, § 4, xiv.).

iii. Twinning operations which are also co-spatial operations give rise to supplementary twins, that is to say, twins of crystals in which the faces of the two component crystals together supply the faces required by the higher symmetry of another class of the same system.

In the supplementary twins formed by such a co-spatial operation there are only a limited number of twin planes, but all other planes with rational indices are co-spatial planes. For instance, in the supplementary twins of pyrites the planes parallel to the rhombic dodecahedron faces are twin planes, and those parallel to other possible faces are co-spatial planes; while the cubic faces are also cross planes.

Other analogous supplementary twins are formed by co-linear (here contra-directional) operations equivalent to a reversal relatively to a point, and in them every plane may be regarded as a twin plane (§ 10, ii.).

iv. A rotation with cyclic number 4 (that is, a rotation through a quarter of a complete turn) round a crystallographic axis in the cubic system is a co-linear and co-directional operation in the galena (*CBc*) and cuprite (*CUh*) classes; a co-linear and contra-directional operation in the diamond (*CBc*) class; and a co-spatial operation in the pyrite (*CUc*) and ulmannite (*CUu*) classes. In the case of the last-mentioned class there is no twin plane, but every plane with rational indices is a co-spatial plane. The compound crystal thus formed will be strictly analogous to a supplementary twin crystal. There is no other system in which a rotation with cyclic number 4 can be co-linear in one class and co-spatial in another.

v. A co-spatial operation may also correspond to a co-directional operation belonging to the symmetry of a class of a system other than that to which the crystal belongs. The only important case is that of a reversal relatively to an axis with co-linear and co-directional cyclic number 3. Such a reversal, R_l, is equivalent to a rotation with co-linear and co-directional cyclic number 6; for:—$[S]R_l=[S]C_{\frac{1}{2}}=[S]C_{\frac{1}{3}}.C_{\frac{1}{6}}$ (=) $[S]C_{\frac{1}{6}}$ (where $C_{\frac{1}{n}}$ is a rotation with cyclic number n). It is of course a twinning operation.

Thus a reversal of a rhombohedral crystal relatively to the principal axis, or of a cubic crystal relatively to the normal to an octahedron face, will be a co-spatial operation. For both these crystal structures can be referred to hexagonal axes.

In the first case the corresponding positive and negative rhombohedra coincide, and the same in the case with the scalenohedra (or such faces of the latter as the symmetry retains). The co-spatial planes formed by the coincidence of the positive and negative rhombohedral faces with each other contain two common lines—the edges between the rhombohedra of different sign, but as these are not at right angles, and the planes which coincide are not equivalent, the co-spatial planes which they form by their coincidence are not cross planes (§ 21, iv. and v.). In the second case the cube faces will coincide with certain of the faces of the trigonal triakis octahedron {122}, and other faces will coincide at the same time. The co-spatial planes formed by the coincidence of the faces of the cube and trigonal triakis octahedron will each contain two common lines corresponding to the intersections of the cube and trigonal triakis octahedron, but they will not be cross planes, for the reason already given.

Similar co-spatial operations may be conceived as occurring in other systems; thus prolectite, chondrodite, and clinohumite have the symmetry of the monoclinic system, but the three axes are at right angles to one another; a reversal, therefore, relatively to the vertical axis would be a co-spatial operation. Every such co-spatial operation will be a reversal and therefore a twinning operation; except in one possible case in which it would be a rotation with cyclic number 4.

§ 28. Combinations of Co-spatial and Twinning Operations.

We may now consider the case of co-spatial planes formed by a co-spatial operation followed by an operation which would have converted a plane into a twin plane if the co-spatial operation had not preceded it. If these two operations are reversals related to one another in the manner described in § 19, their combination will be equal to another reversal, and the compound crystal containing the resulting co-spatial plane will be a twin crystal. If, on the other hand, they are reversals which are not so related, the operation giving rise to the co-spatial plane will be such a rotation as is described in § 20.

Example.—If a portion of a quartz crystal be reversed relatively to the vertical axis, this will be a co-spatial operation and also a twinning operation (see figures 3*Uh* and 3*UhL*, p. 434). If the same portion be then reversed relatively to the normal to a rhombohedron face, these two operations will together be equivalent to a rotation round a line at right angles to the vertical axis and to the normal to the rhombohedron face, that is to say, round the normal to a face of a trigonal prism of the second order, through an angle twice that between the normal to the rhombohedron face and the vertical axis. The rhombohedral face in question will be a co-spatial plane.

Such a combination is believed by Professor v. Goldschmidt to occur in quartz twins with oblique twin axes (*Tsch. Min. Pet. Mitt.*, vol. xxiv., 1905, pp. 157–182), but might be explained as the result of the combination of two independent twinning operations.

§ 29. Cross-spatial Planes.

A cross-spatial plane may be formed as the result of a co-spatial operation followed by an operation which would have given rise to a cross plane if the co-spatial operation had not been already applied. As a co-spatial operation is usually a twinning operation, and an operation resulting in a cross plane is always one, the combined operation will be as a rule a twinning operation or rotation according as the combination falls under § 19 or § 20.

§ 30. Less Regular Combinations.

There are other compound crystals which do not contain contact or abstract twin planes or cross planes. Such crystals will rarely contain a co-spatial or cross-spatial plane, for these, as we have seen, are usually accompanied by a twin plane. There may be a single common line and a common plane at right angles to it (*Min. Mag.*, vol. xv., 1910, p. 395), or a co-spatial line with or without a co-spatial plane, or only a line or lines in which the molecular distances in the two components are approximately equal. The consideration of such combinations does not fall within the scope of the present paper.

(*Issued separately December* 2, 1912.)

XXXI.—On Inheritance of Hair and Eye Colour.

By John Brownlee, M.D., D.Sc.

(MS. received June 17, 1912. Read same date.)

Some time ago, in a paper published by the Royal Anthropological Institute, I applied a Mendelian analysis to that part of the observations made by the late Dr Beddoe (1) which refers to the colour of the hair. In that paper (2) I showed that these observations obeyed in a highly remarkable degree the law referred to, and that this result held from the north of Scotland, through the whole of England, Ireland, France, and Germany, to the south of Italy. At that time I was unable to make any application to the observations on eye colour also published in the same work, but I have now succeeded in completing the analysis.

The whole depends on a theorem of population stability which may be easily proved.

Let the population consist of a mixture of two races having two characters such as hair colour and eye colour inherited according to the Mendelian law of segregation. Let these qualities be denoted by (BB), (bb) for the hair, and (DD), (dd) for the eyes. Then the population may be considered given by

$$a^2\left|\begin{matrix}\mathrm{DD}\\ \mathrm{BB}\end{matrix}\right| + 2ab\left|\begin{matrix}\mathrm{DD}\\ \mathrm{B}b\end{matrix}\right| + b^2\left|\begin{matrix}\mathrm{DD}\\ bb\end{matrix}\right| + 2ac\left|\begin{matrix}\mathrm{D}d\\ \mathrm{BB}\end{matrix}\right| + (2ad+2bc)\left|\begin{matrix}\mathrm{D}d\\ \mathrm{B}b\end{matrix}\right|$$

$$+ 2bd\left|\begin{matrix}\mathrm{D}d\\ bb\end{matrix}\right| + c^2\left|\begin{matrix}dd\\ \mathrm{BB}\end{matrix}\right| + 2cd\left|\begin{matrix}dd\\ \mathrm{B}b\end{matrix}\right| + d^2\left|\begin{matrix}dd\\ bb\end{matrix}\right|^{*} \quad . \qquad (1)$$

If this population mate freely, and if all matings possess equal fertility, the relationship of the constants required for a stable population depends on whether coupling exists or not.

The meaning of the term "coupling" may be easily seen from a consideration of the different units in the above expression. It will be noticed that every term of the expression except that in the middle has either two eye units or two hair units the same. It is thus impossible when division takes

* The factors outside the brackets are the proportional numbers of each variety. The simple case is: if $x(\mathrm{A}, \mathrm{A})$ mate at random with itself and with $y(a, a)$ and all subsequent matings are equally probable, the stable population is given

$$x^2(\mathrm{A}, \mathrm{A}) + 2xy(\mathrm{A}, a) + y^2(a, a).$$

Cf. p. 462.

place for anything else to occur than that two constantly linked pairs are given off in equal numbers. Thus $\begin{vmatrix} d\mathrm{D} \\ bb \end{vmatrix}$ can only divide into $\begin{vmatrix} d \\ b \end{vmatrix}$ and $\begin{vmatrix} \mathrm{D} \\ b \end{vmatrix}$. But when we consider the case of $\begin{vmatrix} \mathrm{D}d \\ \mathrm{B}b \end{vmatrix}$ other things may easily happen. If D have a greater affinity for B than for b, then we may have more of the element $\begin{vmatrix} \mathrm{D} \\ \mathrm{B} \end{vmatrix}$ given off than of the element $\begin{vmatrix} \mathrm{D} \\ b \end{vmatrix}$.* But here also there is a necessary arithmetical relationship between the different elements resulting, and if n $\begin{vmatrix} \mathrm{D} \\ \mathrm{B} \end{vmatrix}$ elements occur for one $\begin{vmatrix} \mathrm{D} \\ b \end{vmatrix}$ it follows that there will also be n $\begin{vmatrix} d \\ b \end{vmatrix}$ for one $\begin{vmatrix} d \\ \mathrm{B} \end{vmatrix}$ even although the attraction of D for B might be different from that of d for b.

If the population (1) mate freely and if m $\begin{vmatrix} \mathrm{D} \\ \mathrm{B} \end{vmatrix}$ occur with n $\begin{vmatrix} \mathrm{D} \\ b \end{vmatrix}$ (where $m+n=\cdot 5$ and $2(ad+bc)$ is denoted by h), the next generation will be given by

$$\left.\begin{aligned} &(a^2+ab+ac+mh)^2 \begin{vmatrix} \mathrm{DD} \\ \mathrm{BB} \end{vmatrix} + 2(a^2+ab+ac+mh)(b^2+ab+bd+nh) \begin{vmatrix} \mathrm{DD} \\ \mathrm{B}b \end{vmatrix} \\ &\quad + (b^2+ab+bd+nh)^2 \begin{vmatrix} \mathrm{DD} \\ bb \end{vmatrix} + 2(a^2+ab+ac+mh)(c^2+ac+dc+nh) \begin{vmatrix} \mathrm{D}d \\ \mathrm{BB} \end{vmatrix} \\ &\quad + \left\{ \begin{aligned} &2(a^2+ab+ac+mh)(d^2+db+cd+mh) \\ +&2(b^2+ab+bd+nh)(c^2+ac+cd+nh) \end{aligned} \right\} \begin{vmatrix} \mathrm{D}d \\ \mathrm{B}b \end{vmatrix} \\ &\quad + 2(b^2+ab+bd+nh)(d^2+bd+dc+mh) \begin{vmatrix} \mathrm{D}d \\ bb \end{vmatrix} + (c^2+ac+cd+nh)^2 \begin{vmatrix} dd \\ \mathrm{BB} \end{vmatrix} \\ &\quad + 2(c^2+ac+cd+nh)(d^2+db+cd+mh) \begin{vmatrix} dd \\ \mathrm{B}b \end{vmatrix} + (a^2+db+cd+mh)^2 \begin{vmatrix} dd \\ bb \end{vmatrix} \end{aligned}\right\} \quad (2)$$

This has exactly the same form as that from which it is derived, but the relative proportions of the different classes may be different. If the population is stable we have as the sufficient conditions,

$$\frac{(a^2+ab+ac+mh)^2}{a^2} = \frac{(b^2+ba+bd+nh)^2}{b^2} = \frac{(c^2+cd+ca+nh)^2}{c^2} = \frac{(d^2+db+dc+mh)^2}{d^2},$$

as all the similar relationships hold if these are true.

Taking the first equation

$$\frac{(a^2+ab+ac+mh)^2}{a^2} = \frac{(b^2+ba+bd+nh)^2}{b^2},$$

* The assumption made here is that there is no special mortality or instability among the pairs which are actually formed.

we have, since the positive root must be taken,

$$a+b+c+\frac{mh}{a}=b+a+d+\frac{nh}{b},$$

or

$$c+\frac{mh}{a}=d+\frac{nh}{b},$$

or

$$abc+2mabd+2mb^2c=abd+2na^2d+2nabc,$$

$$\text{since } h=2(ad+bc);$$

$$=abd+(1-2m)a^2d+(1-2m)abc,$$

$$\text{since } 2(m+n)=1;$$

or

$$2m(abd+b^2c+a^2d+abc)=abd+a^2d,$$

or

$$2m(a+b)(ad+bc)=(a+b)ad$$

$$2m(ad+bc)=ad$$

$$2mbc=(1-2m)ad$$

$$=2nad.$$

The other equations also reduce to this, so that

$$\frac{ad}{bc}=\frac{m}{n}$$

is the criterion of stability if coupling exists. If there is no coupling,

$$m=n \quad \text{and} \quad ad=bc.$$

Some remarks may be made in this place concerning the meaning of coupling. It has two forms: either each unit has a special attraction for the corresponding unit originally associated with it, or on the other hand for the one with which it has come in contact when hybridisation occurs. The theory at present advanced by Mendelian biologists makes in my notation $\frac{m}{n}=2^p-1$ when p is a positive integer. I confess that I cannot follow the arguments on which this is based. The facts seem to me much more in line with the conditions of stability in chemical solutions. If there be a solution, say, of Na_2SO_4 and HCl, the relative proportions of the four possible substances depend on the rate at which the reactions between Na_2SO_4 and HCl and between NaCl and H_2SO_4 take place. Denoting these respectively by n and m, if the amount of these four substances be respectively a, d, b, c, equilibrium will exist if $nad=mbc$. Or, in other words, the equation of chemical equilibrium is the same as that of the stability of the population considered. The advantage of this method of looking at the matter is that it implies no special values of m and n. Short, therefore, of some fundamental reason for the value $\frac{m}{n}=2^p-1$, it is better to consider that other values may be possible and that facts on one side or the other are at present of more importance than theories. The only difference in this case is that either $\left|\begin{matrix}\mathrm{D}\\\mathrm{B}\end{matrix}\right. + \left|\begin{matrix}d\\b\end{matrix}\right.$ or $\left|\begin{matrix}\mathrm{D}\\b\end{matrix}\right. + \left|\begin{matrix}\mathrm{B}\\d\end{matrix}\right.$ must exist; thus four different compounds cannot all appear together, but if an average of a large number of examples is taken the result must be the same.

Referring back to the expression for a freely mating population, we see that the fact that it forms a perfect square is not a sufficient criterion of stability. All that is stable is the relation of the eyes alone or of the hair alone. Thus, taking formula (2) and summing each line as regards number, we have for the total of the first line, or the terms containing (DD),

$$(a^2+ab+ac+mh)^2+2(a^2+ab+ac+mh)(b^2+ab+bd+nh)+(b^2+ab+bd+nh)^2,$$

or

$$(a^2+ab+ac+mh+b^2+ab+bd+nh)^2,$$

or

$$(a^2+ab+ac+ad+b^2+ab+bc+bd)^2,$$

since $m+n=\cdot 5$
and $h=2ad+2bc$;

or

$$(a+b)^2(a+b+c+d)^2;$$

the second line, *i.e.* the terms containing (Dd), is equal to

$$2(a+b)(c+d)(a+b+c+d)^2,$$

and the third to

$$(c+d)^2(a+b+c+d)^2,$$

and the proportions of the original population (1) are exactly maintained.

Shortly written as before shown, the general formula may be denoted by

$$\left\{a\left|\begin{matrix}\mathrm{D}\\\mathrm{B}\end{matrix}\right.+b\left|\begin{matrix}\mathrm{D}\\b\end{matrix}\right.+c\left|\begin{matrix}d\\\mathrm{B}\end{matrix}\right.+d\left|\begin{matrix}d\\b\end{matrix}\right.\right\}^2.$$

This is the typical stable Mendelian population without coupling if $ad=bc$; if coupling exists, $\dfrac{ad}{bc}=\dfrac{m}{n}$ is the criterion, and stability in the population is only established after many generations.

Suppose equal numbers of two populations mix and mating is free: suppose also that the coupling ratio is 7, one actually found by Bateson and Punnett (3). Then if mating is free the first generation will be given by the ratio

$$\left|\begin{matrix}\mathrm{DD}\\\mathrm{BB}\end{matrix}\right|+2\left|\begin{matrix}\mathrm{D}d\\\mathrm{D}b\end{matrix}\right|+\left|\begin{matrix}dd\\bb\end{matrix}\right|.$$

With a ratio of 7 the next generation will be represented by

$$\left\{8\left|\begin{matrix}\mathrm{D}\\\mathrm{B}\end{matrix}\right.+7\left|\begin{matrix}\mathrm{D}\\\mathrm{B}\end{matrix}\right.+\left|\begin{matrix}\mathrm{D}\\b\end{matrix}\right.+\left|\begin{matrix}d\\\mathrm{B}\end{matrix}\right.+7\left|\begin{matrix}d\\b\end{matrix}\right.+8\left|\begin{matrix}d\\b\end{matrix}\right.\right\}^2,$$

or

$$\left\{15\left|\begin{matrix}D\\B\end{matrix}\right. + \left|\begin{matrix}D\\b\end{matrix}\right. + \left|\begin{matrix}d\\B\end{matrix}\right. + 15\left|\begin{matrix}d\\b\end{matrix}\right.\right\}^2,$$

which when expanded gives

							Total
225	$\begin{matrix}DD\\BB\end{matrix}$	+30	$\begin{matrix}DD\\Bb\end{matrix}$	+1	$\begin{matrix}DD\\bb\end{matrix}$		256
30	$\begin{matrix}Dd\\BB\end{matrix}$	+452	$\begin{matrix}Dd\\Bb\end{matrix}$	+30	$\begin{matrix}Dd\\bb\end{matrix}$		512
1	$\begin{matrix}dd\\BB\end{matrix}$	+30	$\begin{matrix}dd\\Bb\end{matrix}$	+225	$\begin{matrix}dd\\bb\end{matrix}$		256
Total	256		512		256		

Hence $\frac{m}{n}=225$ in place of 7.

The subsequent matings can be easily calculated by the application of the form in expression (2). The first term is $(225+15+15+\frac{7}{16}\cdot452)^2$, and the rest are found likewise.

Applying the process seriatim with suitable approximations we have the successive values of $\frac{ad}{bc}$ given in Table I.

Table I.

	Value of $\frac{ad}{bc}$ or $\frac{m}{n}$.
After first generation . . .	225
,, second ,, . . .	56
,, third ,, . . .	27
,, fourth ,, . . .	19
,, fifth ,, . . .	14
,, sixth ,, . . .	11
,, seventh ,, . . .	9·6
,, eighth ,, . . .	8·6
,, ninth ,, . . .	8·3

It is thus seen that stability is attained only after a considerable number of generations in a free-mating population if coupling exists.

It is possible to introduce a shortened notation. In all circumstances these populations after one generation consist of numbers which are those of a perfect square. If we write this in the following way we can at once proceed to the full expression with little trouble.

Thus

$$\left|\begin{array}{cc} a \left|\begin{array}{c} \mathrm{D} \\ \mathrm{B} \end{array}\right| & b \left|\begin{array}{c} \mathrm{D} \\ b \end{array}\right| \\ c \left|\begin{array}{c} d \\ \mathrm{B} \end{array}\right| & d \left|\begin{array}{c} d \\ b \end{array}\right| \end{array}\right|^2 \quad \text{or more shortly} \quad \left|\begin{array}{cc} a & b \\ c & d \end{array}\right|$$

denotes

$$\left|\begin{array}{ccc} a^2 \left|\begin{array}{c} \mathrm{DD} \\ \mathrm{BB} \end{array}\right| & 2ab \left|\begin{array}{c} \mathrm{DD} \\ \mathrm{B}b \end{array}\right| & b^2 \left|\begin{array}{c} \mathrm{DD} \\ bb \end{array}\right| \\ 2ac \left|\begin{array}{c} \mathrm{D}d \\ \mathrm{BB} \end{array}\right| & \begin{array}{c} 2ad \\ +2bc \end{array} \left|\begin{array}{c} \mathrm{D}d \\ \mathrm{B}b \end{array}\right| & 2bd \left|\begin{array}{c} \mathrm{D}d \\ bb \end{array}\right| \\ c^2 \left|\begin{array}{c} dd \\ \mathrm{BB} \end{array}\right| & 2cd \left|\begin{array}{c} dd \\ \mathrm{B}b \end{array}\right| & d^2 \left|\begin{array}{c} dd \\ bb \end{array}\right| \end{array}\right|$$

Each of the four sides of the complete expression is the square of the corresponding terms of the contracted expression, and the term in the middle the sum of twice the product of the diagonal elements.

One or two other examples of the rate at which stability is approached in one generation are shown in the following table:—

Table II., showing Rate of Approximation to a Stable Population.

Form of Population.	Value of $\frac{m}{n}$.	Value of $\frac{ad}{bc}$ at Commencement.	Value of $\frac{ad}{bc}$ after One Generation.
$\left\|\begin{array}{cc} 9 & \cdot 5 \\ 1 & 1 \end{array}\right\|^2$	9	18	15·6
$\left\|\begin{array}{cc} 11 & 1 \\ 1 & 1 \end{array}\right\|^2$	9	11	10·5
$\left\|\begin{array}{cc} 10 & 1 \\ 1 & 1 \end{array}\right\|^2$	9	10	9·76
$\left\|\begin{array}{cc} \sqrt{10} & 1 \\ 1 & \sqrt{10} \end{array}\right\|^2$	9	10	9·67
$\left\|\begin{array}{cc} 9\cdot 5 & 1 \\ 1 & 1 \end{array}\right\|^2$	9	9·5	9·46
$\left\|\begin{array}{cc} 5 & 1 \\ 1 & 1 \end{array}\right\|^2$	3	5	4·1
$\left\|\begin{array}{cc} 4 & 1 \\ 1 & 1 \end{array}\right\|^2$	3	4	3·7

Before quoting any examples of Dr Beddoe's figures it will be well to state clearly his hair and eye categories. He recognises five types of hair colour. The meanings of these types seem to me as follows:—

(1) *Jet black.*—This is a true single hue, and persons possessing this colour of hair are with few exceptions those who possess two distinct pure-black elements in the gametes. The exceptions, so far as I have seen, are a few persons who have one red and one jet-black element. In manhood this may resemble jet black very closely, but the colour of the hair on the body usually shows some trace of the ruddy pigment. These are, however, so few in number that they do not disturb the calculation.

(2) *Dark hair.*—This is really a mixture consisting of one jet-black element and one element of either medium hair or fair hair. Black is thus imperfectly dominant.

(3) *Brown hair.*—This consists of those who are true brown or medium and of those who possess one brown element and one fair element.

(4) *Fair hair.*—This again is a pure pigment, the person possessing it having two fair elements.

(5) *Red hair.*—In this group are included the pure reds, the mixtures of red and fair hair, and the mixtures of red and brown hair.

For purposes of analysis it is necessary to combine the last three classes.

Eyes are more difficult.* Dr Beddoe recognises three classes:—

(1) *Light eyes.*—This includes, in my opinion, the pure blue, the grey or pale yellow, and the mixture of these. All are distinct varieties, and can be distinguished with fair accuracy after a certain amount of practice.

(2) *Mixed eyes.*—This class contains a certain proportion of those eyes which are a mixture of the shades of eye just mentioned and of the chocolate and dark-yellow eyes.

(3) *Dark eyes.*—This class contains all the pure-dark eyes and I think the pure-yellow eyes, as, on account of the manner in which the dark pigment of the back of the iris frequently shows both internally and externally, these may look dark except on careful inspection. It also contains many eyes which a moment's careful inspection would show to be either mixed dark and grey or dark and blue eyes. The latter types of eye are much more common than the true dark or chocolate eye. That they have not been more definitely distinguished is somewhat surprising.

It is obvious from what has been said that the last two classes must at least in the first instance be placed together.

We thus have six equations to determine four unknown quantities. The success of this fitting must be the test of the truth of these statements.

* See Appendix.

As an example, the figures Dr Beddoe obtained by observation in the town of Caen are given. The numbers are as follows:—

	Light, Medium and Red Hair.	Dark Hair.	Jet-black Hair.	Total.
Light eyes	149·5* (149·5)	27 (27·15)	1·5* (1·11)	178
Mixed and dark eyes . .	51·5*	93·5*	16	161
Total . . .	201 (201)	120·5* (120·0)	17·5* (17·9)	339

In this case $a^2=149{\cdot}5$ and $(a+b)^2=178$. This gives on solution $a=12{\cdot}23$ and $b=1{\cdot}11$, so that we obtain $2ab=27{\cdot}15$ as against 27 found and $b^2=1{\cdot}23$ as against 1·5 found. Whether we regard the 1·5 as really one individual or two, the fit is exceedingly good. The same process applied to the total gives $(a+c)^2=201$ and $(a+b+c+d)^2=339$, so that $(a+c)=14{\cdot}18$ and $(b+d)=4{\cdot}23$, which give $2(a+c)\,(b+d)=120$ as against 120·5 and $(b+d)^2=17{\cdot}9$ as against 17·5 found.

This example illustrates how the race mixture can be analysed and the closeness with which the numbers accord with such distribution of the population as is given by the Mendelian theory. Such complete correspondence is of course rare. Another example almost equally good is that of Bradford. Here the numbers are even larger, the sample of the population observed numbering 1400 persons. In this case the theoretical numbers are printed in brackets above the actual:—

	Light, Medium and Red Hair.	Dark Hair.	Jet-black Hair.	Total.
Light eyes	(663) 663	(117·8) 117	(5·24) 6	786
Total (all eyes) . .	(968) 968	(392·4) 387	(39·6) 45	1400

The method of testing the suitability of such fitting is that given by Professor Pearson (4). The differences are taken between each theoretical and actual number; these are squared, divided by the corresponding theoretical number, and summed. In the case of the totals this is equal to $\frac{(5{\cdot}4)^2}{39{\cdot}6}+\frac{(5{\cdot}4)^2}{392{\cdot}4}$ or ·81.

* Where ·5 occurs, the indications were so nearly equal that the individual was recorded half in one class and half in another.

This sum is denoted by the symbol χ^2; the value of P or the probability that the fit might be worse is then obtained from the published tables (5). For the above figures $P = \cdot 67$; that is, if 1400 persons were observed by random sampling 100 times, in 67 of these a worse fit might be expected than that found. In the case of the upper line the fit is practically perfect.

In what follows, the figures relating to Scotland are chiefly used. Concerning the suitability of these it may be remarked that (excluding Glasgow and Edinburgh, where the recent Irish immigrations have introduced a large element unassimilable on account of the difference in religion, and which therefore fulfil none of the conditions necessary to the application of the present theory), Dr Beddoe made observations in 43 localities in which the characteristics of the hair and eyes were noted in more than 150 persons.

If 43 cases are noted at random, the number of good fits and bad fits may easily be calculated from the probability table already referred to. We find χ^2 should be less than unity in ·393 of the cases; greater than unity and less than two in ·239; greater than two and less than three in ·125; and greater than three in the remainder, namely, ·223. The following table is divided into two classes—the towns with the larger districts, and the country districts. It is seen that the number expected not only is realised but largely exceeded; in other words, except for the fact that the number of towns and large districts in which χ^2 is greater than three is twice that expected, the number of small values of χ^2 is much in excess of that required. The exception is to be expected as into these towns specially the immigration has been much the greatest in recent years.

Table III., showing the Distribution of the Forty-three Districts in Scotland according to the Actual Findings and the Theoretical Proportions expected by the Theory of Chance.

Values of χ^2.		0–1.	1–2.	2–3.	3–.
Towns and large districts	Actual	6	2	...	5
	Theoretical	5·1	3·2	1·9	2·9
Small districts	Actual	22	3	2	3
	Theoretical	11·7	7·1	4·4	6·7
Total	Actual	28	5	2	8
	Theoretical	11·7	10·3	6·3	9·7

For comparison of hair and eyes a further selection has been made. Only those towns and districts in which χ^2 is less than unity have been analysed, as it is only in those we can expect sufficient freedom of mating to allow of the degree of coupling being determined.

These number twenty-seven in all. An analysis has been made in the manner already indicated. The values of a and b have been determined from the numbers showing the combinations of hair and light eyes, and the values of $a+c$ and $b+d$ similarly from the totals of each colour of hair when all eyes are grouped together. The four elements of the population are thus found, and a, b, c, d being thus known, the ratio of ad to bc may be calculated and the degree of coupling of the eyes and hair known.

In the adjoining table these values are given. The numbers of persons observed and the probable proportions in which the present population is derived from the three great races of Europe are given for comparison:—

Table IV., showing the Constitution of the Population in Different Districts in Scotland, with Dr Beddoe's Reference Numbers.

	No. of Persons.	Teutonic Race.	Alpine Race.	Mediterranean Race.	$\frac{D}{B}$	$\frac{D}{b}$	$\frac{d}{B}$	$\frac{d}{b}$	$R=\frac{m}{n}$	$X^2 < 1$
15. Beauly, etc.	170	47	32	21	7·3	1·3	·7	·7	5·6	<1
16. Inverness town	200	32	42	26	6·5	1·4	·9	1·2	6·2	<1
18. ,, district	500	38	39	23	7·0	1·3	·7	1·0	7·7	<1
19. Keith, etc.	200	36	40	24	6·7	1·18	·62	1·13	9·8	<1
20B. Forres	210	37	46	17	7·4	·9	·9	·8	7·3	<1
30. Kirkcaldy, etc.	300	44	39	17	7·4	·9	·9	·8	7·3	<1
34. Perth	665	42	36	22	7	1	·83	1·17	9·8	<1
37. Auchterarder	180	43	33	24	6·8	1·1	·8	1·28	9·9	2·5
38. Forteviot	300	42	35	23	6·4	·8	1·37	1·43	8·3	1·2
40. Callander	150	37	37	26	7·0	1·6	·4	1·0	10·9	<1
47. Breadalbane	199	41	30	29	6·5	1·6	·69	1·3	8·8	<1
51. Athol	290	39	39	22	7·1	1·1	·73	1·07	9·5	<1
57. Great Glen	200	44	38	18	7·5	1·2	·66	·64	6·0	<1
72. Ayr	500	42	36	22	7·2	1·2	·69	·99	9·1	2·3
73. Maybole	250	39	41	20	7·4	1	·67	·93	10·3	<1
74. Sanquhar	200	36	41	23	9·7	1·7	1·25	1·76	7·8	<1
76. Upper Galloway	250	38	41	21	6·99	1·17	·93	·91	5·8	<
78. Dumfries	200	39	42	19	6·9	·9	1·25	·95	4·6	<1
86. Leith, etc.	200	46	37	17	7·8	·7	·48	1·02	23·3	<1
88. Dunbar	150	42	44	14	9·6	·9	·89	·85	10·2	1·2
89. Midlothian	300	54	32	14	7·8	·8	·76	·64	8·2	<1
90. Newhaven	176	52	33	15	9·7	·8	1·48	1·32	10·2	7
100. Duns	230	48	42	10	7·4	5	1·65	·45	4·1	<1
109. Jedburgh	150	44	39	17	8·8	·6	1·27	1·57	18·1	<1
115. Rulewater, etc.	180	47	38	15	10·1	·8	1·32	1·2	11·4	<1
116. Teviotdale	272	44	40	16	7·45	·7	1	·83	8·8	<1
117. Langholm	200	44	42	14	7·33	·61	1·24	·82	7·8	<1

It is seen on inspection that in these twenty-seven cases the degree of successful fitting when the persons with light eyes are considered is exceedingly good. In twenty-two cases χ^2 is less than unity as against 10·6 expected; but as there must be some correlation between the two sets of results, the great excess is not unexpected. In two cases χ^2 is between 1 and 2, and in two between 2 and 3. In two of the latter cases the presence of a single individual would make the fit good, and only one individual could be expected considering the small numbers observed. It may be taken, then, that in these twenty-seven districts at the present moment the conditions for the applications of the theory may be held to exist.

In the table just given the value of the ratio $ad:bc$ is stated in each case. For convenience it will in future be noted by the letter R. It has a wide range of variation in value. The lowest value is 4·1 and the highest 23·3; but of the twenty-one different values eighteen lie between 7 and 11. The mean is 9·14, and the probable error of this $\pm$·48. A number, however, such as the ratio at present considered has for each individual observation a very high probable error. I have been unable to evaluate the expression for the probable error of R in terms of the frequencies, and it is difficult to make a reliable estimate of this; but by an application of the formula given by Mr Udny Yule (6) for the probable error of the same ratio in the fourfold division, it must be large. The average number of observations in each case does not much exceed two hundred, and, taking this value and making a rough estimate, it would seem that the probable error when $R=9$ is 2. That is to say, that in half the cases R should lie between 7 and 11. As we have just seen, two-thirds lie in this interval. When these ratios are considered from the point of view of the median it is found that the latter lies almost exactly in the same place. As small values of the ratio are just as likely to arise from emigration as large values from immigration, it therefore seems probable that the number 9 approximately represents the value of the ratio. The only value which is possible on the current theory of Mendelism is 7, namely 2^3-1. The observations do not favour this value, so that the latter cannot be taken with reasonable probability.

Leaving Scotland for further verification, it seems best to take only large numbers. Dr Beddoe gives eight instances in which the criteria demanded in Scotland approximately hold, and in which the numbers observed are upwards of four hundred. These are collected in Table V., p. 469.

The mean value of R in the case of these towns is 9·4, with a probable error of $\pm$·67, so that they show no certain difference from the result obtained. If anything, they render the value 7 obtained by the Mendelians less probable. In the absence of other evidence we may take it that R ≒ 9, and that if it differs much from that, it is in excess rather than in defect.

As the result of these calculations it is seen that if we take collectively all those with light eyes and distribute them according to the colour of the hair, the number of those with dark hair is always equal to twice the product of the square roots of the numbers of those possessing light hair

Table V., showing the Values of R in several Large Towns and Districts.

Reference to *Races of Britain.*	Place.	Number of Observations.	R.
Page 162	Manchester	475	9
„ 179	St Austell	850	8·6
„ 180	Truro	500	10·3
„ 183	Gloucester	500	10
„ 177	Chippenham	650	6·8
„ 163	Bradford	1400	8·4
„ 199	Bourges	420	10·8
„ 212	Vienna	1700	10·8

and black hair. The proportions in which the eyes are divided among the different types of hair show also that something mathematically equivalent to coupling takes place with apparent uniformity. This is the Mendelian law, and the evidence seems to me sufficient to prove that something at least analogous to segregation takes place. Whether the actual mechanism is Mendelian or not, it is evident that any other theory which seeks support must lead to the same numerical relationship.

We now come to the discussion of mixed and dark eyes. Light eyes have been shown to fulfil the necessary conditions for Mendelian inheritance, but the other groups evidently have some different significance. This is best understood by referring again to Expression (1), or

$$\left|\begin{array}{rl|rl|rl|} a^2 & \begin{array}{c} bb \\ \mathrm{BB} \end{array} & 2ab & \begin{array}{c} bb \\ \mathrm{BD} \end{array} & b^2 & \begin{array}{c} bb \\ \mathrm{DD} \end{array} \\ 2ac & \begin{array}{c} bd \\ \mathrm{BB} \end{array} & (2ad+2bc) & \begin{array}{c} bd \\ \mathrm{BD} \end{array} & 2bd & \begin{array}{c} bd \\ \mathrm{DD} \end{array} \\ c^2 & \begin{array}{c} dd \\ \mathrm{BB} \end{array} & 2cd & \begin{array}{c} dd \\ \mathrm{BD} \end{array} & d & \begin{array}{c} dd \\ \mathrm{DD} \end{array} \end{array}\right.$$

which is stable if $\mathrm{R} = \dfrac{ad}{bc}$.

The ratios of mixed eyes to dark eyes in each class of hair are therefore

$$\frac{2ac}{c^2}, \quad \frac{2ad+2bc}{2cd}, \quad \frac{2bd}{d^2},$$

or

$$2, \quad 1+\frac{bc}{ad}, \quad \frac{2bc}{ad},$$

cancelling and multiplying by $\frac{c}{a}$,

or

$$2R, \quad R+1, \quad 2.$$

This ratio is evidently independent of the relative proportions of the different elements of the population. If $R=9$, which is the value it approximates to in the majority of cases, this ratio becomes 9 : 5 : 1. Nine districts in Scotland have values of R approximating to 9; they range from 8·2 to 9·9. The relative proportions of mixed and of dark eyes are given in the following table:—

Table VI.—Percentage Mixed and Dark Eyes associated with each Class of Hair.

Eyes.	Light Hair.		Dark Hair.		Black Hair.	
	Mixed.	Dark.	Mixed.	Dark.	Mixed.	Dark.
Selected districts . . .	6·7	6·7	7·3	11·5	·74	3·14
All districts	6·9	6·3	6·3	12·6	·82	2·56

It is a matter of observation that many mixed eyes are classed as dark, and it seems reasonable to suppose that a fixed proportion are so classed; but the figures given by the selected districts cannot be adapted to the ratios given above by transferring the same proportion from each group of mixed eyes to the corresponding group of dark eyes which we have shown takes place. The numbers, however, in the last group, that of black hair, are small, and the error of the ratio, which is approximately 1 in 4, may be large.

The second group of ratios—*i.e.* that for the whole twenty-seven groups—is more nearly in accord with the supposition that a fixed proportion of the mixed eyes are called dark; but it would seem probable that with each change of the constitution of the gamete as regards hair colour a mixed eye tends to assume a darker hue to the casual observer, though it may well be that this is due as much to the colour of the eyelashes as of the eye itself. In fact, the difference to be explained is not so great but that it might be accounted for on this supposition.

One other point requires to be considered. In this paper fair-haired and medium-haired persons have been classed together, and the question arises as to the effect this may have on the relative proportions of mixed and dark eyes, as it might well be that a mixed grey and chocolate eye and a mixed blue and chocolate eye would impress an observer differently. Personal observation renders it probable that the latter is more often classed as dark, and the figures bear out this observation. The proportions are shown in the accompanying table

Table VII.

Ratio of Fair to Medium Hair.	Light Hair.		Ratio.	Dark Hair.		Ratio.	Black Hair.		Ratio.
	Mixed Eyes.	Dark Eyes.		Mixed Eyes.	Dark Eyes.		Mixed Eyes.	Dark Eyes.	
>1·2	5·8	5·1	1·13	5·4	11·5	·47	·95	2·60	·37
>1<1·2	6·9	6·5	1·06	5·6	11·2	·50	·84	2·24	·37
<1	6·2	6·8	·91	5·7	11·3	·50	·74	2·91	·25

It is to be noted that the ratio of mixed to dark eyes tends in the groups of light hair and black hair to decrease with the decrease of light hair and to remain constant in the group of dark hair. From such facts no certain inferences can be drawn, but the suggestion is that a mixed blue and chocolate eye is somewhat darker on the average than a mixed grey and chocolate eye.

Conclusions.

(1) Many of Dr Beddoe's populations are stable in a Mendelian sense. Though this does not necessarily imply that the theory as stated by Mendel is the only explanation of the arithmetical proportions found, any other theory claiming to explain the facts of heredity must also explain these relative proportions.

(2) That linkage between hair colour and eye colour exists. The coupling factor is more likely to be 9 than 7, and therefore does not agree with the present Mendelian theory. It is quite possibly to be explained on the analogy of chemical equilibrium.

(3) That it is possible that the colour of the hair has, in addition to this, some other effect in altering the colour of the eyes; but the evidence is not sufficient to prove this, and it may be only due to the fact that dark eyelashes tend to lend a darker appearance to eyes than would be found justified on a more careful examination.

(4) A further result of the analysis made in this paper is that Dr Beddoe's figures give no suggestion of the presence of any race in this country which had different hair and eye relationships from those pertaining to the three races generally considered to form the basis of the European population. This, of course, does not exclude the possibility of an older race surviving in sufficient numbers to form a considerable part of the British population; but, so far as the survey is valid, this race must have had a hair and eye complex closely allied to one or other of the hair and eye complexes considered in this paper.

APPENDIX.

On the Categories of Eye Colour, with a Record of One Observation.

Eye colour is the subject of much controversy. I am personally of the opinion that all categories that have been described are very imperfect. In the first place, apart from actual colour, the pigment of the posterior layer of the iris may be seen at times with more or less prominence along the inner and outer edges of the iris, often causing the eye to appear darker than the colour alone would permit.

Again, mixed eyes are of two kinds—those in which the pigment is (1) diffuse and (2) discrete, that is, in spots; but as far as my observations go, I have never seen pigment in the eyes of children which was not present in the eyes of one or other of the parents. In mixed eyes the pigment tends to collect more markedly near the inner edge of the iris, so that in a mixed chocolate and grey eye we may have both the chocolate and the grey pigment in the inner part, and the outer edge simulating a blue eye.

Of actual types of pure as distinct from mixed eyes I recognise four:—

(1) The pure blue eye, in which there is no pigment in the iris, such grey as appears being due to strands of connective tissue.

(2) The grey or pale yellow, in which there is always visible pigment present in little masses, quite distinct from definite strands of connective tissue.

(3) The deep yellow eye, a more or less rare form, not much exceeding 1 per cent. of the adult population as seen in Glasgow.

(4) The dark-brown or chocolate eye, of which the shades vary, but in all of which the iris is sensibly the same colour from the inner margin to the outer.

All these types of eyes may be found mixed, and as regards eyes the population may be taken as given by

$$m^2(a, a) + n^2(b, b) + p^2(c, c) + q^2(d, d) + 2mn(a, b) + 2mp(a, c)$$
$$+ 2mq(a, d)2np(b, c) + 2nq(b, d) + 2pq(c, d).$$

Now, some of these types are very difficult to distinguish, especially in children. Of the varieties which are very difficult to distinguish are: (1) the mixture of yellow and grey from the mixture of chocolate and grey, a small amount of chocolate pigment being not unlike yellow; and (2) the mixture of chocolate and grey from the mixture of chocolate and blue, the connective tissue of the latter simulating grey pigment when masked by a veil of chocolate pigment.

Last summer I examined a school of nearly one hundred children in Skye, a school where the population may be considered free-mating and uncontaminated by immigration. As each child was shown to me I stated to an amanuensis my decision concerning the eye colour, and the numbers are as follows:—

Class 1.	Pure blue	12
,, 2.	Pure grey	9
,, 3.	Dark yellow	1
,, 4.	Chocolate	4
,, 5.	Mixed blue and grey	23
,, 6.	Blue and yellow	2
,, 7.	Blue and chocolate	1
,, 8.	Grey and yellow	18
,, 9.	Grey and chocolate	18
,, 10.	Yellow and chocolate	3

The difficulties above mentioned show themselves at once; but if classes 3, 4 and 10 be combined, and if classes 6, 7, 8, and 9 be also combined, we have the following figures:—

	Actual Figures.	Theoretical* Proportions.
Pure blue	12	12·39
Mixed grey and blue	23	21·54
Pure grey	9	9·36
Mixed blue or grey and chocolate or yellow	39	38·95
Chocolate and yellow	8	8·76
Pure and mixed	91	91·00

These results are too close to be wholly chance, but as it is a solitary instance they are advanced with diffidence. They are, however, in complete accordance with those given in the preceding notes on "Inheritance of Hair and Eye Colour."

* Fitted by the method of least squares.

REFERENCES.

(1) Beddoe, *Races of Britain*, 1885.

(2) Brownlee, "A Note on the Possibility of Analysing Race Mixtures," *Jour. Roy. Anth. Inst. Great Britain and Ireland*, July 1911.

(3) Bateson and Punnett, "The Interrelations of Genetic Factors," *Proc. Roy. Soc.*, March 1911.

(4) Pearson, "On the Criterion that a given System of Deviations from the Probable in the case of a Correlated System of Variables is such that it can be reasonably supposed to have arisen from Random Sampling," *Phil. Mag.*, July 1900.

(5) Elderton, "Tables for Testing the Goodness of Fit of Theory to Observation," *Biometrika*, vol. i. p. 155.

(6) Yule, "On the Association of Attributes in Statistics," *Phil. Trans.*, vol. cxciv., A, 1900, pp. 284–5.

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OBITUARY NOTICES.

David Harris, F.S.S. By Professor David Fraser Harris.

(Read January 6, 1913.)

David Harris, F.S.S., was born at Dunster, Somerset, in June 1842. He came to Edinburgh in 1863 as manager in Scotland of the British Medical and General Life Association and the Britannia Fire Association, which companies flourished greatly under his management. In 1869 Mr Harris, then a Fellow of the Statistical Society, published an address on the "Principles and Bonus Appropriations of Life Assurance," a paper received with much interest.

In 1872 Mr Harris was invited to join the late Mr A. B. Fleming of Hillwood, Corstorphine, in the business undertaking which in a few years became Messrs A. B. Fleming & Co., Ltd., Caroline Park, Granton. As managing director of this company he notably increased the volume of its business. Although not a chemist, he was deeply interested in the chemical problems bound up with the purification of lubricating oils of mineral, vegetable, and animal origin, as also in the manufacture of black and of coloured printing inks, for which his firm was famous. He studied carefully the principles of trichromatic printing, and was closely associated with those who in this country developed the technique of this beautiful process. His services to the printing trade were acknowledged by the printers and lithographers of Edinburgh themselves presenting him with an illuminated address.

Mr Harris was a Fellow of the Society of Arts and a member of the Society of Chemical Industry, and was for many years a member of the Parish Council of the Parish of Cramond.

As one of the directors of the Edinburgh Chamber of Commerce, Mr Harris published in 1885 a pamphlet entitled, "Depression of Trade and National Progress and Prosperity, viewed in the Light of Statistics," which drew forth a good deal of favourable comment at the time. As a member of the Chamber of Commerce, he served for a number of years on the Board of Managers of the Edinburgh Royal Infirmary, and while holding this position drew up a statistical analysis of the income and expenditure of that noble charity.

No account would be complete without some allusion to Mr Harris's great activities in the philanthropic and evangelical religious circles of the city. In 1867 he founded the well-known and still flourishing institution, the Edinburgh Industrial Brigade Home for receiving destitute lads, teaching them a trade, and enabling them to become useful citizens. In this undertaking he was assisted by Lord Polwarth, the late Rev. Dr Thomas Guthrie, and the late Mr E. Erskine Scott. He was also closely associated with the "Mars" training-ship in the Tay, the Discharged Prisoners' Aid Society, the Edinburgh Night Refuge, and the Grove Laundry—all charities that have done valuable work. Mr Harris was elected a Fellow of this Society in 1896: he died at Bath on June 17, 1912.

Professor George Chrystal, M.A., LL.D. By Dr J. Sutherland Black and Professor C. G. Knott.

(Read January 6, 1913.)

Part I.—Life and Career. By Dr J. S. Black.

George Chrystal was born on the 8th of March 1851, at Mill of Kingoodie, in the parish of Bourtie, some thirteen miles to the north-west of the city of Aberdeen. His father, William Chrystal of Gateside, who achieved some success both in agriculture and in commerce, is described as having been a man who made his way, without any initial advantages, by sheer force of character and the exercise of great natural ability and originality. His son, the subject of this memoir, received his early education at the parish school of Old Meldrum, some two miles distant from his home. From an early age he gave marked promise of intellectual distinction, though physically he was far from strong, and was hampered by a lameness which he afterwards outgrew, but which precluded him from joining in some of the more boisterous activities of boyhood. Early in the 'sixties the family removed to Aberdeen, where in 1863 he entered the Grammar School. Of this period few memorials survive, beyond a number of medals which show that he maintained the early promise of his childhood. In 1866 he gained the Williamson Scholarship, and in 1867 he passed, in his seventeenth year, into the University. Among his teachers here, he was accustomed to refer to Bain as having perhaps had the greatest influence on his whole intellectual development. But he also acknowledged his deep indebtedness to Geddes, Fuller, Nicol, Thomson, and others, of whom some account, inspired by himself, will be found in the recently published *Life* of his elder contemporary, William Robertson Smith, who had graduated with the highest distinction in 1866, and whose brilliant career is known to have been a pattern and an incentive to so many of the ablest men of the younger generation. Chrystal, who was the friend and contemporary at Aberdeen of Sir William M. Ramsay, seems at first to have been much attracted to classical scholarship, and his interest in the Literæ Humaniores continued strong and keen to the end of his life; but ultimately the study of the mathematical sciences became the absorbing pursuit of his academic years. In this he undoubtedly was following the natural bent of his genius; and a happy determining circumstance was his connection with Dr David Rennet, the talented and stimulating extra-mural teacher to whose instruction he resorted in his summer vacations, and to whom many years afterwards he dedicated his *Introduction to Algebra*, "in memory of happy hours spent in his classroom in days of old." That he did not specialise thus early to the exclusion of all other intellectual pursuits may be judged from the fact that, when he graduated, in 1871, he received the Town's Gold Medal, "awarded annually to the most distinguished scholar at the termination of the Arts curriculum." By this time, however, he had found a clear vocation in the sphere of the mathematical sciences, as is strikingly shown by the fact that within less than a year he gained all the honours accessible to students in this department: the Simpson Prize in Mathematics, the Arnott Prize in Natural Philosophy, the Fullerton Scholarship in Mathematics and Physics, the Ferguson Mathematical Scholarship open to recent graduates from any of the four Universities of Scotland, and, finally, an open scholarship at Peterhouse, Cambridge.

At Cambridge he commenced residence in 1872, and in 1875 he graduated as second wrangler and second Smith's prizeman. Of his teachers during these three years the most influential and formative undoubtedly was Clerk Maxwell, who had delivered his inaugural lecture as Cavendish Professor of Experimental Physics in October 1871, had published his *Treatise on Electricity and Magnetism* in 1873, and had opened the Cavendish Laboratory in June 1874. Clerk Maxwell was not one of those whose instruction came within the ordinary scope of work for the Mathematical Tripos, and Chrystal, who from an early date threw himself with great ardour into the work of the laboratory, seems to have been regarded by some of his friends as having "wasted" a good deal of time there. But this was not his own opinion, nor was it borne out by the decision of the examiners. He found himself so well abreast of his proper tripos work that he had ample leisure not only for this but for other parerga, such as that of writing an essay on "Wit and Humour in English Poetry," which won the Members' Prize in 1873, and also for full enjoyment of undergraduate companionship and the characteristic recreations of the place. Writing long afterwards to the late Dr Adam of Emmanuel, who was engaged on a memoir of that eminent scholar Mr R. A. Neil of Pembroke, he said: "The happiest days of my life were my undergraduate days at Peterhouse, and the chief joy of that time was my friendship with Neil." He rowed a little, and was for a time an energetic volunteer. Some sentences from his own retrospect of the three undergraduate years at Cambridge as compared with the four undergraduate years he passed in

Aberdeen are worth quoting here. Speaking of Aberdeen, he says: "The work in all the ordinary classes was very elementary. The course was the same for everyone, viz. the old seven subjects, plus a course of natural history, which included both zoology and a little geology. Yet there was a great variety, and if we did not get much of any one thing, all that we got was highly digestible, and men who went conscientiously through that course carried with them in after-life, for the most part, an intellectual mark that was unmistakable." "When I went to the University of Cambridge . . . I found that the course there for the ordinary degree in Arts was greatly inferior in educational quality to the Scottish one. On the other hand, the courses in Honours were on a very much higher standard, although they suffered greatly from the chaotic organisation of the English universities." He goes on to say: "I have frequently been tempted to think that the three years I spent as an undergraduate at Cambridge were wasted years of my life: if they were to be valued merely by the amount of new knowledge acquired, they were largely wasted; but, on the other hand, they were of great advantage to me in other respects. I made the acquaintance of a large number of the ablest young men of my generation; and it was no small matter to come even within view of such men as Cayley, Adams, Stokes, and Maxwell, and to have lived for a time within the College walls which had sheltered Tait and Kelvin. Cambridge at that time presented strange contrasts. Although almost decadent as an educational institution, it numbered among its members, as the names I have just quoted prove, perhaps the greatest galaxy of intellectual stars that ever illustrated any period of the history of a university."

Shortly after his graduation in 1875, Chrystal was elected Fellow and Lecturer of Corpus Christi College, with which society he retained a lifelong connection, having been subsequently chosen an Honorary Fellow. For some two years he lectured in mathematics and physics to the students of a group of colleges which included both Corpus and Peterhouse; in this period of his life, besides his activity and success as a teacher, a noteworthy feature was the part he took in promoting certain measures of University and College reform. It was during these years, too, that he carried out his important work in connection with the experimental verification of Ohm's law, and began the article "Electricity" for the ninth edition of the *Encyclopædia Britannica*, which will be dealt with more fully in another part of this notice.

In the summer of 1877 a vacancy occurred in the chair of Mathematics in the University of St Andrews, and Chrystal made application for the post, fortified by testimonials of exceptional strength and cordiality from all the most eminent of the Cambridge men of science with whom he had been brought into contact: Clerk Maxwell, Sir William Thomson, Professor Tait, Sir George Stokes, and Mr Routh (pre-eminent in the annals of Cambridge coaches) united in praising what he had already accomplished and in forming the happiest auguries for his future. But, in view of his youth—it will be remembered that he was little over twenty-six,—he had not much expectation of success, and he used to tell afterwards with glee how much he was surprised on a certain Saturday morning to receive from the Home Secretary a telegram, followed by a letter, informing him that he had received the appointment and was expected to enter upon its duties at the earliest possible date, which proved to be the following Monday. At St Andrews, apart from his professorial work, he found strenuous employment in the completion of the "Electricity" article already referred to.

In June 1879 he married Miss Margaret Ann Balfour, whom he had known from his childhood, and in the following month he was elected by the Curators to the chair of Mathematics in the University of Edinburgh in succession to Professor Kelland. His inaugural lecture, delivered on October 30th, opened a career of uninterrupted professorial activity, which extended over thirty-two years, and which was destined to be memorable in the history of Scottish education.

If Professor Chrystal was far from agreeing with Poisson in the saying which he used to quote, that "La vie n'est bonne qu'à deux choses—à faire les mathématiques et à les professer," he none the less found great happiness in being able all through life to surrender himself with love and devotion to the twofold task of a discoverer and of a teacher in the subject he professed.

Professor Chrystal was in fact not merely the brilliant Professor of Mathematics during his thirty-two years' occupancy of the Edinburgh chair: he created an epoch in the study of that subject not only within the University, but throughout Scotland at large. Nay, more, of the great and far-reaching changes made in the whole educational system of the country during the last forty years he might well have said without the slightest shadow of boasting: *quorum magna pars fui.* We may here recapitulate, in a few sentences, the principal stages in that great movement.

In 1878, while he was still in St Andrews, the Royal Commission appointed in 1876, with Lord President Inglis as chairman, to inquire into the Universities of Scotland, had issued their Report; and, though the consequent executive changes did not take place until eleven years later, after the passage of the Universities (Scotland) Act, 1889, the problems raised by the Report were from the first eagerly and anxiously discussed in academic circles and had a prominent place in the thoughts of Professor Chrystal. The Universities, however, were only a part, though no doubt a highly important part, of the problems of national education to the solution of which so many of the best years of his life were devoted. In 1882 the Educational Endowments (Scotland) Act was passed, establishing a Commission with compulsory powers, Lord Balfour of Burleigh being chairman, and Mr Alexander Gibson, advocate, secretary. Professor Chrystal was not a Commissioner, but he was on terms of intimate friendship with the secretary, and there can now be no impropriety in saying that the many questions the Commission had to deal with were frequently discussed by Mr Gibson with Professor Chrystal and their common friend Professor Robertson Smith, who about that time was much in Edinburgh, and that Mr Gibson found their opinion always helpful and generally such as might profitably be suggested for the consideration of his Commission. These discussions served to deepen in Chrystal's mind the interest he had long felt in educational reform as it ought to be regarded by a statesman. In 1872 Lord Young's Act had revolutionised primary education in Scotland, but on his return to his native country in 1877 Chrystal remarked with concern that secondary education had not only not kept pace with primary education, but had, on the whole, retrograded. Secondary schools were dying, or, even if apparently prosperous, far from efficient. The Universities, he said, were "unwholesomely prosperous"; their standard, like that of the secondary schools, was "below the level of the cultivated nations of Europe." His reflections soon led him to become more and more the advocate of extending the policy of state aid to secondary schools.

In 1886 he was appointed to represent the University on the newly constituted governing body of the Heriot Trust, and he continued to hold office until 1902. The task was a congenial one. He took the greatest interest in the affairs of the Trust, and in his capacity of Governor he was delighted to be able to play an influential part in laying the foundations of the new organisation both of George Heriot's School and of the Heriot-Watt Technical College. Meanwhile, another important development of his educational activities had taken place. In 1885 the Scotch Education Department had been reconstituted, the Secretary for Scotland being made Vice-President of the Committee of Council, and one result of this important administrative change was that for several years it fell to Professor Chrystal, along with some other Scottish professors, to take part in the inspection of secondary schools. It was in the course of his labours in this connection that the idea occurred to him of a simultaneous written examination so arranged that he might be able to report on all the schools he visited in a uniform manner; and when he came to write his report on the schools he had visited, he sketched a complete scheme for the Leaving Certificate examination which was immediately taken up, and in most of its essential features ultimately adopted by the Department. The importance of this step can be more fully appreciated now, after the lapse of a quarter of a century, than was possible at the time.

The institution of the Leaving Certificate examination was almost immediately followed by the Universities (Scotland) Act, 1889, by which another Commission, this time armed with executive powers, was brought into being. The result, as is well known, was a fundamental change in the Scottish University system. A series of ordinances were issued, which reorganised the finance and internal management of the Universities, greatly widened the curriculum, by introducing into it an elaborate system of options, set up a new system of Honours degrees, and admitted women to lectures and graduation.

The administrative work involved in bringing all these changes into operation required much time, labour, and executive capacity; and when Professor Chrystal was appointed Dean of the Faculty of Arts on the resignation of Professor Campbell Fraser in 1891, he had a formidable duty to face. It is the simple truth to say that, so far as Edinburgh was concerned, the task of carrying out the reforms embodied in the new ordinances fell very largely on the new Dean, and, as the Minute of Senatus drawn up at the close of his twenty years' term of office records: "To his knowledge of public opinion, to his mastery of the educational problems of the day, and to his unwearying zeal and administrative capacity, it was mainly due that these changes were successfully accomplished."

It was not long before some even of the fundamental alterations made by the Commissioners called urgently for revision, in view of the rapid developments that were taking place. In 1907, as the result of much consideration and many years' toilsome experience, the Edinburgh University Court formulated a very important new ordinance giving power to establish a three-term session and to overhaul completely the scheme for graduation in Arts. The reforms foreshadowed in that ordinance did not become effective until the beginning of the session 1909–10, the details having had to be worked out in the interval through the Senatus. On Chrystal, as Dean of the Faculty of Arts, rested the main burden of devising the new regulations and of piloting them into harbour after the leading principles had been decided on. How much strenuous effort this cost, only those who were in intimate touch with him at the time can ever know. In his Promoter's Address of 1908 he thus referred to the impending changes in words in which those nearest to him were concerned to detect a premonitory note of weariness:—

"I am keenly interested in the developments that lie before us; but I must confess that I shrink from the labour that they will involve. Yet the whole of my career has been a turmoil of University reform, beginning at Cambridge; and it may as well end as it began, if it be decreed that it is to continue any longer."

It was natural, and indeed inevitable, that Professor Chrystal, who had so great a share in the remodelling of the curricula of the Universities of Scotland, should be called upon to take part in the inception and execution of the further reforms in the primary and secondary schools of Scotland which had been rendered necessary by this radical change. In obedience to this call he took a leading part in the business of framing a new system for the training of teachers. When the first Edinburgh Provincial Committee charged with the administration of this system entered upon its duties, he joined it as a representative of the University and was elected chairman. Dr George Macdonald, the Assistant Secretary to the Scotch Education Department, has allowed us to transcribe the following appreciation of Professor Chrystal's public services in this connection:—

"The immediate tasks that confronted the Committee were three in number. In the first place, the Department's new Regulations for the Training of Teachers—a code that involved a veritable revolution in the educational system of the country—were submitted to them in draft for consideration and discussion. Many of the problems were new to Chrystal, who had never taken any active part in primary school administration. But he mastered the whole subject in an astonishingly brief space of time, and made himself the guiding spirit in the Committee's deliberations—deliberations which resulted in certain important modifications being made.

"In the second place, negotiations of an exceedingly delicate character had almost at once to be entered upon with representatives of the two great Presbyterian Churches as well as with certain prominent members of the Episcopalian community, the object in view being to arrange for the transference of the existing Training Colleges to the new Committee. These negotiations were of a very protracted and difficult kind, and it was largely due to Chrystal's tact and fairmindedness, and to his success in securing the complete confidence of everyone concerned, that matters were brought to a satisfactory issue. It is true that in the case of the Episcopalian Training College no transfer took place. But even there the Committee and the Church parted company on the friendliest terms. As far as the two Presbyterian bodies were concerned, much of the work which had to be done had to be done in common with the Provincial Committees of Glasgow and Aberdeen. Chrystal was chairman of a Joint Committee representing the three, so that his labours in this matter secured appreciation from a circle much wider than his position as chairman of the Edinburgh Committee alone might have implied.

"In the third place, the whole of the business arrangements for the new body had to be organised. How complex an undertaking this was may be gathered from the fact that an expenditure of somewhere about £30,000 a year of public funds was involved. Innumerable individual susceptibilities had to be taken account of and considered. All sorts of contingencies had to be provided for. Yet the whole machinery was brought into working order without friction of any kind in a comparatively brief period. If mistakes were made in this matter of detail or in that, Chrystal himself made none. And he was, as I know from the personal testimony of those who worked under him, most unselfish in taking upon his own shoulders, ungrudgingly and uncomplainingly, the burden of rectifying any error for which anyone else was responsible.

"His work in these and in many other ways met with comparatively scant recognition from the public. I daresay the average man might have thought more of him if he had accepted the knighthood which the Government is understood to have offered him. But, as you know, he cared for none of these things, and was content with the consciousness of having done his duty. If, however, the circle of those who learned, through his connection with the Provincial Committee, to appreciate his worth and to care for his personality was small, the measure of that appreciation, and of the liking that was engendered, was large indeed. I do not know any instance of a public man whose labours and whose personality have been spoken of with greater or more uniform cordiality by all of those who were privileged to be in touch with what he was doing."

Another important activity of the last ten years of his life was his membership of the Committee which, as one of the results of the South African War, had been appointed by the War Office to advise in regard to the education of officers. This gave him yet another opportunity of placing at the disposal of his country his administrative gifts and his ripe experience as a teacher and a master of educational methods. The deliberations of the Committee may not perhaps have yet borne fruit in the establishment of an ideal system of military education; but Chrystal's services were recognised and appreciated by successive Secretaries of State.

The foregoing paragraphs will serve to indicate in some degree how strenuous was Professor Chrystal's life as investigator, as author and teacher, and as administrator, from his twenty-sixth to his fifty-ninth year. But in his holiday employments and recreations also he showed the same characteristic zeal and thoroughness. At Cambridge, as we have said, he was for some time a zealous volunteer and as good a shot as his myopic blue eyes permitted; all his life he was a keen angler and for many years an energetic cyclist. He was deeply interested in both the science and the art of photography, in the pursuit of which he was indefatigable and most successful. In travel also he found great pleasure and refreshment. His first visit to the Continent was in 1874, when he studied for some months in Tübingen; in the summer of 1876 he took a Cambridge reading party (which, as it happened, included Sir Martin Conway and Prof. F. O. Bower, then undergraduates of Trinity) to Sterzing, Tyrol, and found a new recreation in mountaineering; in later life he frequently visited France, Germany, Norway, and Italy, and in 1892 he spent some weeks in the Western States of America. His "literature," to use the term in the eighteenth-century sense, was remarkably extensive, and was founded on a wide knowledge of the classics, for which, as we have said, he retained an early taste. He did not lose hold even of Greek, which in the case of men engrossed in other pursuits is apt to become rather a faint memory in middle life. German was the first modern foreign language which he mastered, and German books, especially poetry books, were long one of his favourite relaxations. He spoke German fluently, and had much of Heine by heart. In French he had read very widely after his predilections; but in his maturity it was perhaps to Norse and Italian that he turned with the greatest enthusiasm. He professed to be a desultory reader and to have forgotten much, if not most, of what he had read, but, in Bacon's phrase, he was both "a full man" and "a ready man"; and, as might be expected in one of such wide and varied knowledge and experience, his talk was always interesting and informing; and this combined with his genial kindliness to make him a delightful host, guest, travelling companion, comrade, and friend. Amid all his many interests his home life was the greatest. Mrs Chrystal died in 1903, leaving a family of six sons and daughters to the care of a devoted father, who was also the most enthusiastic and the most intimate of their friends.

It was towards the autumn of 1909 that Professor Chrystal's friends began to notice in him symptoms of impaired health, and some loss of the indefatigable vigour that had hitherto characterised him. Indeed, he began, to their dismay, to speak sometimes of withdrawing from his many activities. The first actual step in this direction was taken in October 1909, when, at the end of the first four years' term of office as chairman of the Edinburgh Provincial Committee, he intimated his inability to accede to a request which had been urgently addressed to him that he should consent to accept nomination as a member of the new Committee. "I have been medically advised," he wrote, "that for some time to come I must diminish the amount of business for which I am responsible, if I am not to court final unfitness for all business whatsoever. As the work of your Committee was the last faggot added to the bundle, it must be the first removed." After touching upon the arduous work of the Committee during the preceding four years, he went on to allude to his personal relations with his colleagues. "These relations, I am happy to say, are not clouded by a single unpleasant recollection. The name of the local administrator is 'writ in water.' He must look for his reward in the approbation of his own conscience, and in the keen sense of friendly comradeship which is generated by sharing a common enterprise for what is believed to be the public good. Such reward is enough, in my opinion, for any man; certainly enough for me. During the four years that I have worked with you I have learned to know and like many men with whom I should otherwise never have become intimate. I hope the friends I have thus made will remember me as long as I shall remember them, and with equal pleasure. . . . I thank you for the uniform kindness and courtesy with which you have treated me during my term of office. The best I can wish for my successor is that his Committee may show him the same consideration as you have always given to me."

The Committee, in recording their grateful sense of his eminent services, spoke of his public work in connection with education in Scotland, in what had really been a fresh chapter of its record, as having merited and as having received "the warmest appreciation of all who are conversant with the subject"; and added: "He has been an ideal chairman, and his resignation has been received by all with a profound sense of personal loss. He has justly earned their sincere and affectionate regard. They have admired his great business capacity and the singular thoroughness and devotion with which he carried out the duties of his responsible office. They appreciate no less his courtesy, his fairness of mind, his personal kindness, and his unselfish readiness to credit to others the success of work which he had himself inspired."

The incipient breakdown in health, hinted at in the letter just quoted, was not destined to be arrested, and the obscure illness gradually became more marked in its character as the session advanced. At its close he was still able, however, to undertake a tour in Italy; and the energy and cheerfulness he displayed throughout those weeks will never be forgotten by those who were privileged to be his companions. Milan, Perugia, Assisi, and (chiefly) Rome were the cities visited; and the brief diary he kept indicates unflagging delight in the glories of nature and the miracles of art. Neither this visit to Italy, however, nor treatment at Harrogate and a visit to Northumberland in the following summer sufficed to effect any lasting improvement in his health. He continued to fight bravely on, and proved equal to the discharge of his professorial duties to the end of the winter session 1910–11. When the spring had far advanced there remained no room for doubt in the minds of those who were nearest him that his trouble was incurable, and that all the highest professional skill could now do was to mitigate the inevitable suffering incident to a distressing and mortal illness. Yet he continued to find pleasure and refreshment in his work; and though at the beginning of the winter session of 1911–12 the University Court had granted him extended leave of absence, his enthusiasm and strength of purpose enabled him to attend at the University and award the bursaries as late as 21st October. The end came on the morning of Friday, 3rd November, in the eighth month of his sixty-first year. He was laid to rest on 8th November in the churchyard of Foveran, Aberdeenshire, where his parents are buried, and at the same hour an impressive service, attended by a large congregation, which included many students as well as representatives of the various public bodies with which he had been associated, was held in St Giles' Cathedral, Edinburgh.

A letter from the Secretary of the Royal Society of London, announcing that the King had been pleased to approve of the recommendation of his name for the award of a Royal medal "on account of his contributions to mathematical and physical science, especially, of late years, on the seiches of lakes," arrived in Edinburgh only two hours after his death. It was felt by the Council of the Royal Society that the award thus made should not be cancelled, but that the medal should be transmitted to his family as a visible token of the admiration with which the Society regarded his work. In giving his sanction to this proposal, the Royal donor caused also the following message to be sent: "The King trusts that you will be so good as to convey to the family the assurance of His Majesty's sincere sympathy in the terrible loss that they have sustained, through which so distinguished a career has been brought to a close."

Professor Chrystal's close connection with this Society began with his Edinburgh career and continued without intermission to the end. He joined it early in 1880, and in November of that year was elected to the Council. He served three terms of office as Councillor (1880–3, 1884–7, 1893–5), and two terms as Vice-President (1887–93, 1895–1901), finally succeeding Professor Tait as General Secretary in 1901. Of Professor Tait as General Secretary it is recorded in the minutes that "the Council always felt that in his hands the affairs of the Society were safe, that nothing would be forgotten, and that everything that ought to be done would be brought before it at the right time and in the right way." The ideal thus set up was certainly taken up and realised in a very notable manner by his successor. In the Council of the Society as the chairman's right-hand man he impressed one afresh at every meeting with a sense of extraordinary alertness and resourcefulness, tactfulness and courtesy, good sense and sagacity, so that when the meeting came to an end, however anxious the deliberations had been, one felt that it had been a privilege, a pleasure, and an education to be present.

Part II.—Scientific Work. By Professor C. G. Knott.

Chrystal's cast of mind was fundamentally physical. This was clearly shown during his student days at Aberdeen; but it was not till he went to Cambridge that he found opportunity for real scientific work. In the year in which he took his degree the British Association had appointed a small committee consisting of Clerk Maxwell, Everett, and Schuster to test experimentally the validity of Ohm's law in electricity. Clerk Maxwell undertook to have the test made in the Cavendish Laboratory, and to Chrystal was entrusted the task of making the experiments. Two forms of experiment were devised by Maxwell, the second proving to be the more satisfactory. The general idea of this experiment was to balance against each other in a Wheatstone bridge a thin and thick wire of the same material, and then pass through the system in alternation a strong and weak current. When the galvanometer showed no deflection under those conditions, the weak current was reversed in direction. Since this reversal of the weaker currents did not affect the equilibrium, it followed that Ohm's law was true within the limits of error of the experiment. The experiments, which involved considerable difficulties of manipulation, were carried out with great success by Chrystal, who wrote the report which was presented to the Glasgow meeting of the British Association in 1876.

A third form of experiment was devised by Chrystal himself, being a modification of one already tried by Schuster. It was based upon the fact that in an induction coil the induced current at break of the primary has a higher maximum intensity than the induced current at make. If, then, the induction currents from the secondary circuit of an induction coil, whose primary is made and broken by a tuning-fork, are passed through a galvanometer, the induced currents will not balance in their effects if resistance depends on strength of current. Certain effects, which at first (as in Schuster's experiments) seemed to indicate a departure from Ohm's law, were traced by Chrystal to the galvanometer. The explanation of these peculiar effects was given by Chrystal in a paper on "Bi- and Uni-lateral Galvanometer Deflections," which was published in the *Philosophical Magazine* for December 1876. Maxwell, writing to Tait on 5th February 1876, put the results obtained by Chrystal in these words: "Ohm's law has now been tested with currents that make the wire swag and swelter, and it is now at least 10^5 to 1 that if Schuster observed anything it was not an error of Ohm's law." As indicating the impression which Chrystal's personality had made on Maxwell, the following quotation from a later letter to Tait is of interest. In the summer of 1878 Tait had evidently asked Maxwell for some help in conduction of heat calculations, and Maxwell replied: "If you mean that I am, by the aid of Fourier, to get up the theory of a square box, and let you have it before the Edinburgh University Library opens, then in that case also you will not bother me, for I will not do it. Nevertheless, I have heard Chrystal say that the variable state of a parallelepiped was more tolerable than that of a cylinder, and he therefore cut his paraffin into a square prism. He also said that in this matter Poisson was of more use than Fourier." The most direct expression we have of Maxwell's opinion of Chrystal's capacity as an experimentalist is contained in the testimonial with which, on 10th July 1877, he supported Chrystal's application for the chair of Mathematics in St Andrews. "Of Mr Chrystal's papers," he wrote, "the most important is that on the 'Testing and Verification of Ohm's Law.' . . . The difficulties which he encountered and overcame in the course of this work can be appreciated only by one who, like myself, has had opportunity of watching his progress through all its stages." The testimonial ends with a reference to his "extensive and thorough culture, his original and penetrating intellect, and his untiring energy."

No doubt it was on Maxwell's recommendation that Chrystal was asked to contribute the electrical articles to the ninth edition of the *Encyclopædia Britannica*; and before he left Cambridge to take up his new duties in

St Andrews the manuscript of the article "Electricity" must have been in the printer's hands. The volume containing this article appeared in 1878. When in 1879 Chrystal became a candidate for the chair of Mathematics in the University of Edinburgh, Maxwell strengthened his former testimonial by adding these words: "I think it is of the greatest importance that, in a university in which the time that the majority of students can give to mathematics is so limited, their attention should be specially directed to those branches which will be most useful to them in their subsequent study of natural philosophy. This has always been kept in view in the University of Edinburgh. . . . That Professor Chrystal is well qualified to maintain the old reputation of the University is amply shown by the article 'Electricity.' . . . I have reason to know something of the amount of matter which must be gone through in order to write such an article, and of the difficulty of co-ordinating it, and I can confidently assert that the manner in which Professor Chrystal has made use of this mass of matter shows that he has the power, so invaluable in a professor, of giving such an account of what has been done in any subject as will give his students the greatest advantage in dealing with it themselves."

The article "Electricity" was followed in due course by the supplementary article "Magnetism," and the two are best considered together.

In gathering material for these and other contributions to the *Encyclopædia Britannica,* Chrystal spared no pains in getting access to original sources. With unerring discrimination he sifted out from the mass of accumulated and rapidly accumulating experimental results those which were essential in the progress of our knowledge of electrical science. Not only are the articles compact history, but the varied experience of all types of scientific investigator is woven into a unity under the formative influence of Faraday's conceptions and Maxwell's fruitful methods. Theory and experiment go hand in hand. Where necessary, mathematics of a high order are introduced; but the student not familiar with higher mathematics has no difficulty following the general argument and appropriating for his own purposes both the methods and the best results of experiment. In short, as an exposition of the development of the sciences of electricity and magnetism down to the date of publication, these two articles, concise and clear cut in their literary form, have never been surpassed for thoroughness of treatment, clearness of vision, unity of plan, and lucidity of expression. They at once became the English text-book for all real students of electricity and magnetism.

In addition to these two great articles, Chrystal also wrote for the *Encyclopædia Britannica* the articles "Electrometer," "Galvanometer," "Goniometer," "Mathematics," "Parallels," "Perpetual Motion"; and the biographies of J. von Lamont, Mascheroni, Michell, Montucla, R. Murphy, Musschenbroek, Oughtred, Pascal (part), G. Peacock, Pell, Pfaff, Playfair, Plücker, Poggendorf, Poisson, Recorde, Rheticus, Riemann, Robins, Sturm. Of these, "Pascal" and "Poisson" are of particular interest.

"Mathematics" is a brief exposition of the historical development of the fundamental ideas of the various branches of the science, and in "Parallels" there is a clear account of the rise of non-Euclidian geometry. An address on this subject was given by Chrystal before the Royal Society of Edinburgh during his first year here—indeed, before he was formally elected a Fellow. In the same year he contributed along with Professor Tait an obituary notice of Professor Kelland, his predecessor in the Edinburgh chair. A peculiar interest attaches to this obituary notice, inasmuch as Chrystal gives in it a remarkably appreciative account of Kelland's investigations in wave motion—a subject which, towards the close of his own life, Chrystal was destined still further to elucidate in his masterly papers on seiches.

An important part of the scientific labours of a Scottish University Professor of Mathematics is the practical work of teaching. When Chrystal came to Edinburgh every Arts student had to study mathematics as one of the seven compulsory subjects. There were no options. These conditions were not conducive to a high standard in mathematical study; but even in these early days many a man of classical or philosophical attainments trembled as he entered the examination hall and sat down to tackle the algebra or the Euclidean geometry paper. The first year of Chrystal's professoriate struck terror to the hearts of those unfortunates to whom the *pons asinorum* was a bridge of sighs. Keen, rapid, logical, full of suggestions as to wider realms of mathematical delights, Chrystal transformed the whole atmosphere of the class-room. "Principles," "symmetry," "form"—not an endless wrestling with examples—were his watchwords; yet his exercises were splendid training. Eagerly the mathematical minds followed his fascinating lead; despondingly and despairingly those not so gifted fell hopelessly behind, perceiving faintly, if at all, the finely knit sequence of ideas which formed the thread of his discussions. Nevertheless, when the time of testing came, the really intelligent, hard-working student got full credit for his limited mathematical powers; for, with all his strenuous and successful labours to raise the standard of mathematical teaching, Chrystal was essentially just. With the close of the winter session of 1879–80 the University and Academic world of Edinburgh knew that a fresh force had come into their midst.

In these days in the Scottish Universities, there were during summer no regularly constituted classes in the Arts Faculty. Consequently, freed from the trammels of class work by the beginning of April, Chrystal found opportunity to resume with eagerness his experimental investigations, from which his St Andrews career had completely divorced him. Tait invited him to work in the Physical Laboratory and to utilise to the full all its appliances. It was my first year as Tait's assistant, and the incursion of this young professor of twenty-eight years into our midst gave all our minds a new orientation. His constant presence in the laboratory during the summer months and his ready accessibility at all times gave a great impetus to the experimental study of electricity and magnetism. Tait himself was at the time fully occupied with the corrections to be applied to the *Challenger* thermometers and with the related work on high pressures. This work was being done in the basement by a few of the senior students working directly under Tait's supervision; and Tait was rarely seen in the upper rooms where most of the other laboratory work was going on. Summer after summer Chrystal flitted through these laboratories, busy with his own researches, but not too busy to take a keen interest in all that was being done. Many a helpful suggestion he gave for new lines of work, and many an eager student did he encourage by inviting his co-operation in some special bit of investigation. The advanced students of these years came into more direct contact with him than with Tait, and owed much of their scientific progress to his sympathetic help. My own research work in magnetism, which has continued over many years, had its origin in a conversation over a passage in the article "Magnetism."

The first work to which Chrystal devoted himself was the comparison of inductances and capacities according to the methods soon to be expounded in his important paper on the differential telephone, for which he was awarded the Keith prize. This paper, indeed, contains for the first time the complete theory of the Wheatstone bridge through which a periodic current is passing, when coefficients of mutual and self-induction are given their full significance. In the course of his investigations he constructed many forms of apparatus, which were the pioneers of the more elaborate and refined methods of the present day. Side issues of enticing interest often led him away for a time from the main trend of his researches. On one of these, the wire telephone, he made two communications to our Society, and showed some of the experiments. The accounts of these in our *Proceedings* are mere abstracts; a much fuller description will be found in *Nature* of 29th July 1880.

During this first strenuous year at Edinburgh, Chrystal also presented a paper on Minding's theorem, which Tait had shortly before discussed by quaternion methods. Chrystal developed the subject in illustration of Plücker's complexes and congruences.

After a few years Chrystal found himself compelled to give up experimental work, in large measure, no doubt, on account of increasing demands on his time. The tercentenary celebrations of 1884 were followed by an awakened interest in University reform: the mathematics department was being rapidly developed, and much of Chrystal's leisure must have been spent in preparing his text-book on *Algebra.* Also, as he himself once said, he found that he was monopolising all the best pieces of apparatus in the Physical Laboratory, so that the senior students were seriously handicapped.

The publication of the first volume of Chrystal's *Algebra* marked an epoch in the teaching of mathematics in our schools and colleges. Already in 1885, as President of Section A of the British Association at its meeting in Aberdeen, Chrystal had pointed out in clear, unequivocal terms the need of a revolution in the presentment of algebraic theory. The following quotations are not altogether inapplicable even in these days:—

"In the higher teaching, which interests me most, I have to complain of the utter neglect of the all-important notion of algebraic form. I found, when I first tried to teach University students co-ordinate geometry, that I had to go back and teach them algebra over again. . . . I found that their notion of higher algebra was the solution of harder and harder equations. . . . Many examination candidates, who show great facility in reducing exceptional equations to quadratics, appear not to have the remotest idea before hand of the number of solutions to be expected. . . . The whole training consists in example grinding. What should be merely the help to attain the end has become the end itself. The result is that algebra, as we teach it, is neither an art nor a science, but an ill-digested farrago of rules whose object is the solution of examination problems. . . . The end of all education nowadays is to fit the student to be examined; the end of every examination not to be an educational instrument, but to be an *examination* which a creditable number of men, however badly taught, shall pass. We reap, but we omit to sow. Consequently our examinations, to be what is called fair—that is, beyond criticism in the newspapers—must contain nothing that is not to be found in the most miserable text-book that anyone can cite bearing on the subject. One of my students, for example, who was plucked in his M.A. examination—and justly so, if ever man was—by the unanimous verdict of three examiners, wrote me an indignant letter because he believed, or was assured, that the paper set could not have been answered out of Todhunter's *Elementary Algebra*. . . . The problem for the writer of a text-book has come now, in fact, to be this—to write a booklet so neatly trimmed and compacted that no coach, on looking through it, can mark a single passage which the candidate for a minimum pass can safely omit. Some of these text-books I have seen, where the scientific matter has been, like the lady's waist in the nursery song, compressed so 'gent and sma',' that the thickness of it barely, if at all, surpasses what is devoted to the publisher's advertisements.

"The cure for all this evil is simply to give effect to a higher ideal of education in general, and of scientific education in particular. . . . Science cannot live among the people . . . unless we have living contact with the working minds of living men. It takes the hand of God to make a great mind, but contact with a great mind will make a little mind greater. . . . In the future we must look to men and to ideas, and trust less to systems. Systems of examination have been tested and found wanting in nearly every civilised country on the face of the earth. . . . The University of London . . . has for many years pursued its career as a mere examining body. It has done so with rare advantages in the way of Government aid, efficient organisation, and an unsurpassed staff of examiners. Yet it has been a failure as an instrument for promoting the higher education—foredoomed to be so, because, as I have said, you must sow before you can reap."

Within a year of uttering these stirring words, Chrystal presented to the mathematical world of teachers his solution of part of the difficulty in the form of the first volume of his well-known *Algebra*. Its merits were immediately recognised. The first object he set before him was "to develop algebra as a science, and thereby to increase its usefulness as an educational discipline." The introduction into an elementary text-book of such subjects as the complex variable, the equivalence of systems of equations, the use of graphical methods, and the discussion of problems of maxima and minima, was a new feature; but subsequent developments have fully vindicated the innovations. Graphical methods of representing simple functions, and the transition to the solution of equations, are now the stock-in-trade of every modern *Algebra*. From these methods, the principles of co-ordinate geometry flow naturally and simply; and Chrystal himself, somewhat later, made a further valuable contribution to the nomenclature of the subject by his introduction of the expressive terms, "the constraint equation" and "the freedom equations of a curve." The second volume followed in 1889, and was marked by the same lucid and logical treatment of the more advanced parts. In 1898 Chrystal further enriched the literature of the subject by bringing out his *Introduction to Algebra* for the use of secondary schools and colleges.

Although he did not for many years take up any experimental work, he was always ready when occasion offered to advise and help others engaged in such work. For example, he took a keen interest in the Ben Nevis Observatory, and devised special forms of hygrometer and anemometer for use in these altitudes, especially during the winter season. He also turned his attention to the invention of an electrical method for reversing deep-sea thermometers.

In 1883, through the initiative of the late Mr Yule Fraser, at that time mathematical master in George Watson's College, the Edinburgh Mathematical Society was founded. As its first secretary I was brought into still closer touch with Chrystal, who took a warm interest in its welfare from the first. In these early years he contributed a number of notes on mathematical subjects, and greatly encouraged the members by his presence at the meetings. His paper on certain inverse roulette problems contains elegant solutions of particular cases of the general problem: Given the body centrode and the roulette for one point of a plane figure moving in its plane, to find the space centrode. The most elaborate paper which he communicated to the Mathematical Society was entitled "On the Theory of Refraction of approximately Axial Pencils of Light through a Series of Lenses, more especially with regard to Photographic Doublets and Triplets." He was led to take up the subject in connection with his own photographic work. The theory is worked out in a very simple form, and instructions are given the student of photography how most easily to determine the constants of his system of lenses.

Between 1891 and 1896 Chrystal communicated three fairly elaborate papers on differential equations. There was a rumour at one time that he purposed writing a book on this subject; whether this was so or not, the character of these papers shows that he had given careful consideration to the logical foundations of the theory of certain parts. For example, in the first paper he proves the defective nature of Lagrange's demonstration of the rule for the solution of the partial differential equation of the first order, and supplies a demonstration free from the fallacy which had been generally current since Lagrange's days. Similarly, in the second paper he aims at establishing rigorously a fundamental theorem regarding the equivalence of systems of ordinary linear differential equations. This leads to a systematic way for solving systems of this kind without the introduction of superfluous arbitrary constants; and the paper ends with illustrations of the practical use of the method. In the third paper, that on the p-discriminant of a differential equation of the first order, Chrystal begins by pointing out that Cayley, in his early discussions of singular solutions, was led to a proposition which was erroneous, or at least very misleading. Cayley stated that when the differential equation had no singular solution it did not admit of an algebraic primitive. Chrystal, on the other hand, showed that although it is true that, when a singular solution exists, the primitive is algebraic, yet it is the exception and not the rule that there is a singular solution when the primitive is algebraic. It is interesting to note that there has been a steady demand for copies of this particular paper on the p-discriminant and the connected theory of envelopes.

The outstanding characteristic of these mathematical papers is the endeavour after rigorous demonstration. They are clearly written, and are put together in a way which shows a keen perception of the essential nature of the problem contemplated and the inborn power of the teacher in presenting his demonstrations so as to be easily followed by the intelligent student. Chrystal, indeed, possessed the intuitive art of the born teacher. Wide and deeply read in all that was best in mathematical literature, possessing at the same time an artistic appreciation of the beauty of form and logical sequence, he never failed to impart to his presentation of a mathematical argument the distinctive personal touch which appeals to the real student. The following sentences drafted by his eldest son, Mr George Chrystal of the Home Office, give what may be regarded as Chrystal's own estimate of himself in the ranks of mathematicians:—

"My father in familiar conversation with me always declined the title of a great original mathematician. How far this was justified, I have no means of judging; but his real bent seemed to be towards physical science—towards the concrete rather than the abstract. With this, however, he had a keen appreciation, a great knowledge, and a thorough understanding of what had been achieved by the giants of the mathematical world—the Cayleys and the Riemanns, whose results, as he used to tell me, were sometimes reached by stages and processes which even these great men themselves could not always thoroughly explain or account for.

"What he regarded (I believe, though he never told me so in terms) as his special service to mathematics was that, by study and diligence and the exercise of the intellectual power which he possessed, he had been able to consolidate some of the conquests made by the great mathematicians, his predecessors and contemporaries, and had evolved and excogitated a method by which the diligent student of average ability could retread the path which had conducted the man of genius to his discoveries.

"This method required two things: *in the first place*, the abandonment of the traditional practice of occupying, as it were, isolated points in the terrain to be conquered by science, from which isolated forays or raids were conducted under the guise of problem-solving and other virtuosities. Henceforth the pupil was to be conducted by an orderly series of reasonings up a sort of inclined plane from one well-defined conception to another, to the higher levels of the science—morphology, in the words of Sylvester, was to be introduced into algebra and mathematical analysis in general.

"*Secondly*: even in its elementary stages the science of algebra required setting in order, and the morphological method required a new, a precise, and to some minds a 'forbidding' terminology. This was the 'raising of the standard' playfully and ruefully described by Mr J. M. Barrie in *An Edinburgh Eleven*."

All who had any acquaintance with Chrystal's work will endorse these impressions, which spring from a sympathetic intercourse between a father and a son who had many intellectual interests in common, save perhaps that of mathematics. It is interesting in this connection to establish a correspondence between Chrystal and his great colleague, Peter Guthrie Tait. Tait was one of the examiners when Chrystal took his degree at Cambridge. The similarity of their mental gifts was brought out by the fact, often referred to by Tait, that Chrystal was distinctly the best candidate from Tait's standpoint. Tait used to say that Chrystal easily outdistanced the other Wranglers on the problems which demanded real thought! Another link between Tait and Chrystal was the preparation of the second edition of Thomson and Tait's *Natural Philosophy*, vol. i., the proofs of Part I. having been read by Chrystal and Burnside. Chrystal's appointment to the Edinburgh chair was hailed with intense delight by Tait, who knew he had gained a colleague who could think in physical lines. The full facilities he offered Chrystal to work in the laboratory have been already referred to. Each had for the other the greatest admiration, and much mutual help was given before their communications took final form in the printed page of the Royal Society *Proceedings*. In mathematics, and especially in geometry, Tait's was, no doubt, the more original mind; but Chrystal's knowledge of recent developments was wider, and probably he had a juster view of the perspective of things. Tait revelled in the direct and powerful methods of quaternions; but Chrystal used to say that it was Tait the man, and not quaternions the method, which was the real factor in any investigation which Tait made. Chrystal took a passing interest in Hamilton's calculus during his early years at Edinburgh, and gave a course of lectures on vector methods. But they never gripped his mind—chiefly, as he expressed it, because they did not fall in line with the general symmetrical methods of modern analysis which specially appealed to him. Chrystal and Tait were at one in their love for physical investigation, and both were able to utilise to the full their mastery of mathematical symbolism.

I have heard Tait express the hope that when he retired from the chair of Natural Philosophy Chrystal would be his successor; but when the time of retirement came the situation had altered. Had it been Chrystal's fortune early in his professorial career to have had official control of a physical laboratory, he would certainly have founded a strong experimental school.

On Tait's death in 1901, our Society, looking around for a General Secretary, naturally turned their eyes to Chrystal, whose administrative powers and driving force had become widely recognised. It is not necessary again to speak at length of the exceptional ability shown by Chrystal in carrying on the work of the Royal Society. It seems important, however, to bring to light the valuable work he did in connection with the change of abode from the Royal Institution in Princes Street to the present location in George Street.

It had been apparent for many years that the west wing of the Royal Institution, which had been assigned to the Royal Society from the beginning, was becoming far too limited to accommodate the growing activities of the Society. Accordingly in 1903, largely through the initiative of Sir John Murray, a scheme was set on foot for securing for science the whole of the building. The Royal Society would have thus been able to find accommodation for its invaluable library, and other scientific societies might have found shelter under the same roof. A representative committee was formed, and the Secretary for Scotland approached on the subject. Mr Graham Murray, now Lord Dunedin, met the deputation, and spoke very sympathetically in favour of the movement. The difficulty was how to provide accommodation for the schools of art which occupied other parts of the building.

Meanwhile the Liberal Government came into power; and one of its earliest acts was to introduce a Bill handing over to a specially appointed board the whole of the building for the purposes of art. There was at first no provision in this Bill for securing in any way the vested rights of the Royal Society of Edinburgh. The situation demanded supreme vigilance; and fortunately in the person of the General Secretary we possessed the very man for the emergency. As early as 1885, in his British Association address already quoted from, Chrystal, who at that time had had no direct experience of the wonderful way in which our legislators acquire knowledge and draft Bills, wrote these words:—

"We all have a great respect for the integrity of our British legislators, whatever doubts may haunt us occasionally as to their capacity in practical affairs. The ignorance of many of them regarding some of the most elementary facts that bear on everyday life is very surprising. Scientifically speaking, uneducated themselves, they seem to think that they will catch the echo of a fact or the solution of an arithmetical problem by putting their ears to the sounding-shell of uneducated public opinion."

His experience during the early stages of the fight which our Society had to make for recognition must have brought back to his mind more than once the memory of these sentences. In his reply to the first deputation which was received in Edinburgh, the Secretary for Scotland, while sympathising with the object, was of opinion that it was not supported by a body of public opinion. Before the second deputation was received in London, Chrystal had collected a vast array of facts and had fostered in the body of the Scottish representatives a public opinion which there was no gainsaying. By the sympathy and support of the Scottish Members of Parliament, as well as of the Fellows of the Royal Society of London and other eminent scientific men, more was achieved than was at first hoped for. Only a man of Chrystal's alertness of mind, clearness of vision, knowledge of affairs, fairmindedness, and yet determination to have the Society's rights recognised, could have successfully manœuvred the Society through this time of strain and stress when its status and efficiency were threatened. Through all the cross currents of opinion, while many acted valiantly and worked effectively, it was Chrystal who was the man at the helm. He was the prompter, supplying needed information at every stage to those who came forward in the interests of the Society. In such work success is its own reward, and Chrystal never grudged the time and energy he was called upon to devote to the cause of science in securing for all time from the Government of the day a generous recognition of its claims.

In this, as in other similar cases, Chrystal's labours were wholly disinterested. There was never anything personal or selfish in his aims. It was this detachment from self-interest that added to the strength of his appeals and secured the recognition of the principles for which he fought.

Meanwhile, before the introduction of the Government's Galleries Bill (Scotland), the progress of which through its various stages will be found discussed by Sir William Turner in his Presidential Address on the occasion of the opening of the new rooms (8th November 1909), Chrystal's advice had been sought by Sir John Murray in connection with the observations of seiches on the Scottish lakes. This Sir John proposed to make part of his general survey of the fresh-water lochs of Scotland, and to this end he set up forms of limnometer which had been designed by Forel and others in Switzerland. The problem was just the kind to awaken Chrystal's keenest interest. It involved hydrodynamical problems of great difficulty, which could be surmounted only by use of mathematics of a high order. It demanded experimentation of an enticing nature, full of difficulties, and ever presenting new conditions to be considered. Into this work Chrystal accordingly threw himself wholeheartedly. Not only so, but he drew round him a devoted band of helpers, who accumulated new data by patient observation along the shores of several Scottish lochs, and who also arranged under his supervision ingenious experimental models of seiche phenomena in lakes. His papers on this subject, which will be found enumerated below, are recognised as among the most important bearing upon the subject. For this work he was awarded the Gunning Victoria Prize by our Society and a Royal Medal by the Royal Society of London.

His seiche investigations brought him into close touch with F. A. Forel, the veteran naturalist of Lake Geneva, whom he advised the Council to invite to deliver an address to the Society. This Professor Forel did in the summer of 1911. Unfortunately Chrystal was at the time too ill to receive the genial Swiss or to listen to his interesting lecture on the Fata Morgana.

To our Society Professor Chrystal's family have gifted his unique collection of books and papers bearing on the subject of seiches; these, with copies of his own valuable contributions, are now arranged, partly chronologically, partly according to size and country, in several volumes, which cannot fail to be for the future student a most important compendium of literature.

Chrystal's last published paper, "On the Theory of the Leaking Microbarograph," is, in a sense, a continuation of his investigations into the causes of seiches. It has, however, a much wider application, and is an important contribution to the theory and method of observation of small and rapid barometric fluctuations.

The list of papers appended is for the most part arranged in chronological order.

List of Contributions by George Chrystal to Scientific Literature.

1. Report of the Committee for testing experimentally Ohm's Law. *B.A. Reports, Glasgow Meeting*, 1876.
2. On Bi- and Uni-lateral Galvanometer Deflections. *Phil. Mag.*, December 1876.
3. Articles "Electricity" and "Electrometer." *Ency. Brit.*, 1878.
4. Articles "Galvanometer" and "Goniometer." *Ency. Brit.*, 1879.
5. Obituary Notice of Professor Kelland (in conjunction with Professor Tait). 1879. *Proc. R.S.E.*, vol. x.
6. On Minding's System of Forces. 1880. *Trans. R.S.E.*, vol. xxix. Abstract in the *Proc.*, vol x.
7. Address on Non-Euclidean Geometry. 1880. *Proc. R.S.E.*, vol. x.
8. On a New Telephone Receiver. 1880. *Proc. R.S.E.*, vol. x.; also in *Nature*, vol. xxii., 24th June 1880.
9. On the Differential Telephone. 1880. *Trans. R.S.E.*, vol. xxix. Abstract in *Proc.*, vol. x.
10. On the Wire Telephone and its Application to the Study of the Properties of strongly Magnetic Metals. *Proc. R.S.E.*, vol. x. See also *Nature*, vol. xxii., 29th July 1880, for a fuller account.
11. Note on Thomas Muir's Transformation of a Determinant into a Continuant. 1881. *Trans. R.S.E.*, vol. xxx.
12. On a Special Class of Sturmians. 1881. *Trans. R.S.E.*, vol. xxx.
13. Article "J. von Lamont" in *Ency. Brit.*, 1882.
14. Remarks on Dielectric Strength. 1882. *Proc. R.S.E.*, vol. xi.
15. Articles "Magnetism," "Mascheroni, "Mathematics," "Michell," "Montucla," in *Ency. Brit.*, 1883.
16. Present Fields of Mathematical Research. 1883. Opening Address to Edinburgh Mathematical Society. Never published.
17. Articles "R. Murphy," "Musschenbroek," in *Ency. Brit.*, 1884.
18. Mathematical Models chiefly of the Surfaces of the Second Degree. 1884. *Edin. Math. Soc.*, vol. ii.
19. Application of the Multiplication of Matrices to prove a Theorem in Spherical Trigonometry. 1884. *Edin. Math. Soc.*, vol. ii.
20. On the Discrimination of Conics enveloped by the Rays joining the Corresponding Points of Two Projective Ranges. 1884. *Edin. Math. Soc.*, vol. ii.
21. On a Problem in the Partition of Numbers. 1884. *Edin. Math. Soc.*, vol. ii.

22. Articles "Oughtred," "Parallels," "Pascal" (part), "G. Peacock," "Pell," "Perpetual Motion," "Pfaff," "Playfair," "Plücker," "Poggendorf," "Poisson," in *Ency. Brit.*, 1885.
23. On the Problem to construct the Minimum Circle enclosing Given Points in a Plane. 1885. *Edin. Math. Soc.*, vol. iii.
24. On Certain Formulæ for Repeated Differentiation. 1885. *Edin. Math. Soc.*, vol. iii.
25. On a Method for obtaining the Differential Equation to an Algebraic Curve. 1885. *Edin. Math. Soc.*, vol. iii.
26. On the Hessian. 1885. *Trans. R.S.E.*, vol. xxxii.
27. Address as President of Section A of the British Association at the Aberdeen Meeting of 1885. *B.A. Reports.*
28. Articles "Recorde," "Rheticus," "Riemann," "Robins," in the *Ency. Brit.*, 1886.
29. Article "Sturm" in the *Ency. Brit.*, 1887.
30. On certain Inverse Roulette Problems. 1887. *Edin. Math. Soc.*, vol. v.
31. On the Inequality $mx^{m-1}(x-1) \gtrless x^m - 1 \gtrless m(x-1)$ and its Consequences. 1888. *Edin. Math. Soc.*, vol. vi.
32. An Electrical Method of reversing Deep-Sea Thermometers. 1888. *Proc. R.S.E.*, vol. xv. Title only.
33. An Elementary Discussion of the Closeness of the Approximation in Stirling's Theorem. 1889. *Edin. Math. Soc.*, vol. vii. Title only.
34. A Demonstration of Lagrange's Rule for the Solution of a Linear Differential Equation, with some Historical Remarks on Defective Demonstrations hitherto current. 1891. *Trans. R.S.E.*, vol. xxxvi.
35. A Fundamental Theorem regarding the Equivalence of Systems of Ordinary Linear Differential Equations and its Applications to the Determination of the Order and the Systematic Solution of a Determinate System of such Equations. 1895. *Trans. R.S.E.*, vol. xxxviii.
36. A Summary of the Theory of the Refraction of Thin approximately Axial Pencils through a Series of Media bounded by Spherical Surfaces, with Application to a Photographic Triplet, etc. 1896. *Edin. Math. Soc.*, vol. xiv.
37. On the p-Discriminant of a Differential Equation of the First Order, and on Certain Points in the General Theory of Envelopes connected therewith. 1896. *Trans. R.S.E.*, vol. xxxviii.
38. Some Elementary Theorems regarding Surds. 1901. *Edin. Math. Soc.*, vol. xix.

39. Note on the Mathematical Theory of Miller's Trisector, and its Relation to other Solutions of the Problem of Trisection. 1901. *Proc. R.S.E.*, vol. xxiv.

40. On the Relation of Miller's Trisector to the Quartic Trisectrix, with a Description of a Seven-bar Limaçonograph. 1901. *Proc. R.S.E.*, vol. xxiv.

41. Obituary Notice of Professor Tait, *Nature*, 25th July 1901.

42. On the Theory of Seiches. With experimental illustrations by E. M. Wedderburn. 1903. *Proc. R.S.E.* Title only.

43. Obituary Notice of Luigi Cremona. 1904. *Proc. R.S.E.*, vol. xxiv.

44. Some Results in the Mathematical Theory of Seiches. 1904. *Proc. R.S.E.*, vol. xxv.

45. Some Further Results in the Mathematical Theory of Seiches. 1905. *Proc. R.S.E.*, vol. xxv.

46. On the Hydrodynamical Theory of Seiches. 1905. *Trans. R.S.E.*, vol. xli.

47. Calculations of the Periods and Nodes of Lochs Earn and Treig from the Bathymetric Data of the Scottish Lake Survey. In conjunction with E. M. Wedderburn. 1905. *Trans. R.S.E.*, vol. xli.

48. An Investigation of the Seiches of Loch Earn by the Scottish Lake Survey. Part I.—Limnographic Instruments and Methods of Observation. Part II. (by James Murray)—Limnographic Observations. 1906. *Trans. R.S.E.*, vol. xlv.

49. An Investigation of the Seiches of Loch Earn by the Scottish Lake Survey. Part III.—Observations to determine the Periods and Nodes. Part IV.—Effect of Meteorological Conditions upon the Denivellation of Lakes. Part V.—Mathematical Appendix on the Effect of Pressure Disturbances upon the Seiches in a Symmetric Parabolic Lake. 1908. *Trans. R.S.E.*, vol. xlvi.

50. On the Theory of the Leaking Microbarograph, and on some Observations made with a Triad of Dines-Shaw Instruments. 1908. *Proc. R.S.E.*, vol. xxviii.

Books.

1. Algebra, an Elementary Text-Book for the Higher Classes of Secondary Schools and for Colleges. A. & C. Black, Edinburgh. Part I., 1886; Part II., 1889.

2. Introduction to Algebra for the use of Secondary Schools and Technical Colleges. A. & C. Black, London. 1902.

APPENDIX.

CONTENTS.

PROCEEDINGS OF THE STATUTORY GENERAL MEETING.

The 129th Session, 1911–1912.

At the Annual Statutory Meeting of the Royal Society of Edinburgh, held in the Society's Lecture Hall, 24 George Street, on Monday, 23rd October 1911, at 3 p.m.,

Professor J. C. Ewart, F.R.S., Vice-President, in the Chair,

the Minutes of last Statutory Meeting, 24th October 1910, were read, approved, and signed.

On the motion of Professor J. C. Ewart, seconded by Dr Horne, Mr E. M. Horsburgh and Mr A. Hewat were appointed Scrutineers, and the ballot for the New Council commenced.

The Treasurer's Accounts for the past year, 1910–1911, were submitted. These, with the Auditors' Report, were read and approved.

The Scrutineers reported that the following New Council had been duly elected:—

Principal Sir Wm. Turner, K.C.B., D.C.L., F.R.S., President.

Professor J. C. Ewart, M.D., F.R.S.,
John Horne, LL.D., F.R.S., F.G.S.,
James Burgess, C.I.E., LL.D., M.R.A.S.,
Professor T. Hudson Beare, M.Inst.C.E.,
Professor F. O. Bower, M.A., D.Sc., F.R.S.,
Professor Sir Thomas R. Fraser, M.D., LL.D., F.R.C.P.E., F.R.S.,
} Vice-Presidents.

Professor George Chrystal, M.A., LL.D., General Secretary.

Cargill G. Knott, D.Sc.,
Robert Kidston, LL.D., F.R.S., F.G.S.,
} Secretaries to Ordinary Meetings.

James Currie, M.A., Treasurer.

John S. Black, M.A., LL.D., Curator of Library and Museum.

ORDINARY MEMBERS OF COUNCIL.

Professor D. Noël Paton, M.D., B.Sc., F.R.C.P.E.
William S. Bruce, LL.D.
Professor F. G. Baily, M.A.
J. G. Bartholomew, LL.D., F.R.G.S.
Ramsay H. Traquair, M.D., LL.D., F.R.S.
Professor James Walker, D.Sc., Ph.D., LL.D., F.R.S.
Professor Arthur Robinson, M.D , M.R.C.S.
Sir W. S. M'Cormick, M.A., LL.D.
Professor Crum Brown, M.D., LL.D., F.R.S.
Professor T. H. Bryce, M.A., M.D.
Benjamin N. Peach, LL.D., F.R.S., F.G.S.
William Allan Carter, M.Inst.C.E.

On the motion of Dr Knott, thanks were voted to the Scrutineers.

On the motion of Mr Currie, seconded by Dr Horne, the Auditors were thanked and reappointed.

On the motion of Dr Kidston, thanks were voted to the Treasurer.

PROCEEDINGS OF THE ORDINARY MEETINGS, Session 1911-1912.

FIRST ORDINARY MEETING.

Monday, 13th November 1911.

Professor J. C. Ewart, F.R.S., Vice-President, in the Chair.

The following communications were read :—

1. Obituary Notices of the Rev. Professor R. Flint, Dr Alexander Bruce, and the Very Rev. Dr James MacGregor. *Proc.*, vol. xxxi. pp. 687–693.
2. Experiments to show how Failure under Stress occurs in Timber, its Cause, and Comparative Values of the Maximum Stresses induced when Timber is fractured in Various Ways. By Angus R. Fulton, B.Sc., A.M.Inst.C.E. Communicated by Professor Peddie. *Trans.*, vol. xlviii. pp. 417–440.
3. On a New Method of Measuring Mental Processes in Normal and Insane People, with special reference to Maniac Depressive Insanity. By George Rutherford Jeffrey, M.D. (Glasg.), M.R.C.P. (Edin.). Communicated by Sir James Crichton-Browne. *Proc.*, vol. xxxii. pp. 73-109.
4. The Freezing-points of Rhombic Sulphur and of *Soufre nacré.* By Professor Alex. Smith and C. M. Carson. *Proc.*, vol. xxxii. pp. 1–3.

The Chairman read the following excerpt from the Minutes of Council Meeting, held on 13th November 1911 :—

"The President and Council wish to put on record their deep sense of the great loss which the Society has sustained by the death of Professor Chrystal.

"Professor Chrystal became a Fellow of the Society in 1880, a few months after his appointment to the Chair of Mathematics in the University of Edinburgh. In November of the same year he took his seat at the Council board. He subsequently served as Vice-President for two terms of office; and on completion of his second term of office in 1901, he was elected General Secretary in succession to the late Professor Tait.

"During his thirty-one years of Fellowship he contributed a number of papers of marked originality and importance on mathematical and physical subjects. For two of these he was awarded the Keith and Gunning Prizes respectively. It is pathetic to note that, two hours after his death, his family received official information that the King had approved of the award of a Royal Medal by the President and Council of the Royal Society of London to Professor Chrystal.

"As General Secretary he guided the affairs of the Society with untiring zeal and never-failing success. His great services during the critical time which preceded the Society's removal to its present habitation cannot be over-estimated. Keen, determined, resourceful, and tactful, he was ready for all emergencies. The Society's new Rooms are indeed a lasting memorial to his devotion, perseverance, and foresight.

"Wise in counsel, sound in judgment, prompt in action, Professor Chrystal was an ideal administrator. His character was strong in its simplicity. In intercourse with his colleagues he never failed in reasonableness and truest courtesy. Above all, he has left behind him the memory of a trusted and faithful friend."

Mr J. A. S. Watson signed the Roll, and was duly admitted a Fellow of the Society.

SECOND ORDINARY MEETING.

Monday, 20th November 1911.

Professor T. Hudson Beare, B.Sc., Memb.Inst.C.E. Vice-President, in the Chair.

The following communications were read :—

1. Point Binomials and Multinomials in Relation to Mendelian Distributions. By Dr John Brownlee. *Proc.*, vol. xxxii. pp. 396-405.
2. The Influence of the Ratio of Width to Thickness upon the Apparent Strength and Ductility of Flat Test-bars of Mild Steel. By W. Gordon, B.Sc., A.M.I.Mech.E., and G. H. Gulliver, B.Sc., A.M.I.Mech.E. *Trans.*, vol. xlviii. pp. 195–214.

3. Observations on the Temperature of some *Diving* and *Swimming* Birds. By Professor Sutherland Simpson, M.D., D.Sc. *Proc.*, vol. xxxii. pp. 19–35.
4. The Theory of Circulants from 1861–1880. By Dr Thomas Muir. (*Held as read.*) *Proc.*, vol. xxxii. pp. 136–149.

Robert Alexander Houstoun, M.A., Ph.D., D.Sc., was balloted for, and duly elected a Fellow of the Society.

William Allan Carter, C.E., was appointed the Society's Representative on George Heriot's Trust in room of Dr R. M. Ferguson, resigned.

THIRD ORDINARY MEETING.

Monday, 4th December 1911.

Professor F. O. Bower, F.R.S., Vice-President, in the Chair.

The following communications were read :—

1. Obituary Notice of the Hon. Lord M'Laren. By Dr C. G. Knott. *Proc.*, vol. xxxi. pp. 694–696.
2. On *Branchiura Sowerbyi*, Beddard, and on a New Species of Limnodrilus with Distinctive Characters. By Dr J. Stephenson. Communicated by Professor J. C. Ewart, F.R.S. *Trans.*, vol. xlviii. pp. 285–303.
3. The Railway Transition Curve. By E. M. Horsburgh, M.A., B.Sc. *Proc.*, vol. xxxii. pp. 333–347.
4. (*a*) The Preparation and Properties of Basic Copper Nitrate, and the Hydrates of Copper Nitrate. By Messrs A. C. Cumming and Alex. Gemmell. *Proc.*, vol. xxxii. pp. 4–11.
(*b*) On the Reduction of Ferric Iron (1) by Sulphurous Acid, and (2) by Zinc Dust. By Messrs A. C. Cumming and E. W. Hamilton Smith. *Proc.*, vol. xxxii. pp. 12–16.
(*c*) Note on a Perforated Silica Plate for excluding Flame Gases from a Crucible during Ignition. By Mr A. C. Cumming. *Proc.*, vol. xxxii. pp. 17–18.
Above three Papers communicated by Professor James Walker.

The following Candidates for Fellowship were balloted for, and declared duly elected :— Adolphus Edward Bridger, M.D. (Edin.), F.R.C.P. (Edin.), B.Sc. (Paris), B.L. (Paris); Ralph Allen Sampson, M.A., D.Sc., F.R.S. ; William Joseph Maloney, M.D. (Edin.).

FOURTH ORDINARY MEETING.

Thursday, 14th December 1911, *at* 8 *p.m.*

Principal Sir William Turner, K.C.B., President, in the Chair.

At the request of the Council an Address was delivered :—

On Photography in Natural Colours. (*With Lantern Illustrations.*) By Dr Inglis Clark.

FIFTH ORDINARY MEETING.

Monday, 8th January 1912.

Dr Horne, F.R.S., Vice-President, in the Chair.

The following Communications were read :—

1. On the Structure of *Stenomyelon Tuedianum*, Kidst. By Dr R. Kidston, F.R.S., and Professor D. T. Gwynne-Vaughan. *Trans.*, vol. xlviii. pp. 263–271.
2. A Monograph on the General Morphology of the Myxinoid Fishes, based on a Study of Myxine. Part IV.—On Some Peculiarities of the Afferent and Efferent Branchial Arteries of Myxine. By Dr F. J. Cole. Communicated by Dr R. H. Traquair, F.R.S. *Trans.*, vol. xlviii. pp. 215–230.
3. The Effect of changing the Daily Routine on the Diurnal Rhythm in Body Temperature. By Professor Sutherland Simpson, M.D., D.Sc. *Trans.*, vol. xlviii. pp. 231–261.
4. Scottish National Antarctic Expedition : Observations on the Anatomy of the Weddell Seal (*Leptonychotes Weddelli*). Part II.—Genito-urinary Organs. By Professor David Hepburn, M.D., etc. *Trans.*, vol. xlviii. pp. 191–194.

Sir Thomas Carlaw Martin, LL.D., J.P., was balloted for, and declared duly elected a Fellow of the Society.

SIXTH ORDINARY MEETING.

Monday, 22nd January 1912.

James Burgess, C.I.E., LL.D., Vice-President, in the Chair.

The following Communications were read :—

1. Note upon the Structure of Ternary Alloys. By G. H. GULLIVER, B.Sc., A.M.I.Mech.E. *Proc.*, vol. xxxii. pp. 36–39.
2. The Absorption of Light by Inorganic Salts :
   No. V.—Copper and the Alkali Metals. By Dr R. A. HOUSTOUN. *Proc.*, vol. xxxii. pp. 40–49.
   No. VI.—The Cobalt Chloride Colour Change. By ALEX. R. BROWN, M.A. Communicated by Dr R. A. HOUSTOUN. *Proc.*, vol. xxxii. pp. 50–61.
3. Observations on the Structure and Affinities of *Branchiomaldane vincenti*, Langerhans. By Dr J. H. ASHWORTH. *Proc.*, vol. xxxii. pp. 62–72.
4. The Elastic Strength of Flat Plates. By W. J. CRAWFORD, B.Sc. Communicated by Dr C. G. KNOTT. *Proc.*, vol. xxxii. pp. 348–392 (including Dr KNOTT's note).

The following Gentlemen signed the Roll, and were duly admitted Fellows of the Society :—Dr R. A. HOUSTOUN, Dr A. E. BRIDGER, Prof. R. A. SAMPSON.

SEVENTH ORDINARY MEETING.

Monday, 5th February 1912.

Sir Thomas R. Fraser, M.D., LL.D., F.R.S., Vice-President, in the Chair.

The following Communications were read :—

1. On the Bionomics of *Nematus Ericksonii* (Hartig), the Large Larch Sawfly. By Dr R. STEWART MACDOUGALL.
2. The Molecular Theory of Magnetism in Solids. By Professor W. PEDDIE. *Proc.*, vol. xxxii. pp. 216–246.
3. Note on the Torsional Oscillations of Magnesium Wires. By Mr G. P. SEAMAN. Communicated by Professor W. PEDDIE. *Proc.*, vol. xxxii. pp. 247–250.

EIGHTH ORDINARY MEETING.

Monday, 19th February 1912.

Professor J. C. Ewart, F.R.S., Vice-President, in the Chair.

The following Communications were read :—

1. Entomostraca of the Scottish National Antarctic Expedition. By Dr THOMAS SCOTT. Communicated by Dr J. H. ASHWORTH. *Trans.*, vol. xlviii. pp. 521–599.
2. Cephalopoda of the Scottish National Antarctic Expedition. By Dr W. E. HOYLE. *Trans.*, vol. xlviii. pp. 273–283.
3. The Tunicata of the Scottish National Antarctic Expedition. By Professor W. A. HERDMAN. *Trans.*, vol. xlviii. pp. 305–320.
4. Notes on Hamiltonian Dynamics. By Professor ANDREW GRAY, F.R.S. *Proc.*, vol. xxxii. pp. 164–174.
5. An Investigation into the Effects of Seasonal Changes on Body Temperature. By Professor SUTHERLAND SIMPSON. *Proc.*, vol. xxxii. pp. 110–135.

Sir THOMAS CARLAW MARTIN signed the Roll, and was duly admitted a Fellow of the Society.

The following Candidates for Fellowship were Balloted for and declared duly elected :—JOHN STEPHENSON, M.B., D.Sc., ARNOLD HARTLEY GIBSON, D.Sc., JOHN GEORGE LINDSAY, M.A., B.Sc.

NINTH ORDINARY MEETING.

Monday, 4th March 1912.

Professor T. Hudson Beare, B.Sc., M.Inst.C.E., Vice-President, in the Chair.

The following Communications were read :—

1. The Geometry of Twin Crystals. By Dr JOHN W. EVANS. Communicated by Professor J. W. GREGORY, F.R.S. *Proc.*, vol. xxxii. pp. 416–457.

2. Temperature Observations in Loch Earn, with a Further Contribution to the Hydrodynamical Theory of Temperature Oscillations in Lakes. By E. M. WEDDERBURN, W.S. *Trans.*, vol. xlviii. pp. 629–695.
3. Transverse Induction Changes in Demagnetised and Partially Demagnetised Iron in relation to the Molecular Theory of Magnetism. By JAMES RUSSELL. *Proc.*, vol. xxxii. pp. 292–314.

TENTH ORDINARY MEETING.

Monday, 18th March 1912.

James Burgess, C.I.E., LL.D., Vice-President, in the Chair.

The following Communications were read :—

1. On *Rhetinangium Arberi*, a new Type of Fossil Stem from Pettycur. By Dr W. T. GORDON. Communicated by Professor JAMES GEIKIE. *Trans.*, vol. xlviii. pp. 813–825.
2. The Sun as a Fog Producer. By Dr JOHN AITKEN, F.R.S. *Proc.*, vol. xxxii. pp. 183–215.

The following Candidates for Fellowship were balloted for, and declared duly elected:—ANDERSON GRAY M'KENDRICK, M.B.; WILLIAM SMITH SYME, M.D.

ELEVENTH ORDINARY MEETING.

Monday, 6th May 1912.

Principal Sir William Turner, K.C.B., President, in the Chair.

The following Communications were read :—

1. (*a*) On Walking and Climbing Gyrostats ; (*b*) On Novel Illustrations of Gyrostatic Action. (*b*), *Proc.*, vol. xxxii. pp. 406–415. By JAMES G. GRAY, D.Sc.
2. On a Continuous-current Motor Gyrostat with Accessories for demonstrating the Properties and Practical Applications of the Gyrostat. By JAMES G. GRAY, D.Sc., and GEORGE BURNSIDE. *Proc.*, vol. xxxii. pp. 321–332.
3. The Effect of Vibration upon the Structure of Alloys. By G. H. GULLIVER, B.Sc., A.M.I.Mech.E. *Proc.*, vol. xxxii. pp. 315–320.
4. On the Singular Solutions of Partial Differential Equations of the First Order. By H. LEVY, M.A., B.Sc. Communicated by D. GIBB, M.A., B.Sc. *Proc.*, vol. xxxii. pp. 150–163.

The Rev. LAUCHLAN MACLEAN WATT, B.D., and J. T. EWEN, B.Sc., signed the Roll, and were duly admitted Fellows of the Society.

The following Candidates for Fellowship were balloted for, and declared duly elected:—GEORGE RUTHERFORD JEFFREY, M.D., F.R.C.P.; BANAWARI LAL CHAUDHURI, B.A. (Cal.), B.Sc. (Edin.).

FIRST SPECIAL MEETING.

Monday, 13th May 1912.

Dr Horne, F.R.S., Vice-President, in the Chair.

The following communications were read :—

1. Report on Rock Specimens dredged by the *Michael Sars* in 1910, by H.M.S. *Triton* in 1882, and by H.M.S. *Knight Errant* in 1880. By Dr B. N. PEACH, F.R.S. *Proc.*, vol. xxxii. pp. 262–291.
2. A Study in Chromosome Reduction. By Dr A. ANSTRUTHER LAWSON. *Trans.*, vol. xlviii. pp. 601–627.
3. Caradocian Cystidea from Girvan. By Dr F. A. BATHER, F.R.S. Communicated by Dr HORNE, F.R.S. *Trans.*, vol. xlix.

TWELFTH ORDINARY MEETING.

Monday, 20th May 1912.

Professor F. O. Bower, F.R.S., Vice-President, in the Chair.

The following communication was read :—

The Cancer Statistics of Scotland for the Years 1861–1900, and their Indications as to the Prevalence, Age Incidence, and Distribution of the Disease in Scotland. By Sir GEORGE THOMAS BEATSON, M.D., K.C.B. Communicated by Dr BYROM BRAMWELL.

Dr G. R. JEFFREY signed the Roll, and was duly admitted a Fellow of the Society.

THIRTEENTH ORDINARY MEETING.

Monday, 3rd June 1912.

Principal Sir William Turner, K.C.B, President, in the Chair.

The following Communications were read :—

1. Experiments in Radio-activity ; the Production of the Thorium Emanation, and its Use in Therapeutics : Thorium X. By Dr DAWSON TURNER, B.A., F.R.C.P.E. *Proc.*, vol. xxxii. pp. 393–395.
2. On the Use of Antiseptics in Autolysis of Animal and Vegetable Matter. By DOROTHY COURT, B.Sc. Communicated by Dr E. WESTERGAARD. *Proc.*, vol. xxxii. pp. 251–261.
3. The Equilibrium of the Circular-arc Bow-Girder. By Professor A. H. GIBSON, D.Sc. *Trans.*, vol. xlviii. pp. 391–416.

The following Communications were held as read :—

4. The Marine Mollusca of the Scottish National Antarctic Expedition :—Part II., being a Supplementary Catalogue. By JAMES COSMO MELVILL, M.A., D.Sc., and ROBERT STANDEN, Assistant Keeper, Manchester Museum. Communicated by Dr W. S. BRUCE. *Trans.*, vol. xlviii. pp. 333–366.
5. Scottish National Antarctic Expedition :—Observations on the Anatomy of the Weddell Seal (*Leptonychotes Weddelli*). Part III., The Respiratory System and the Mechanism of Respiration. By Professor DAVID HEPBURN, M.D. *Trans.*, vol. xlviii. pp. 321–332.

Professor A. H. GIBSON signed the Roll, and was duly admitted a Fellow of the Society.

The following Candidates for Fellowship were balloted for, and declared duly elected :— ROBERT JOHN MATHIESON INGLIS, Assoc.Memb.Inst.C.E., ALEXANDER NINIAN BRUCE, D.Sc., M.D., ROBERT FOSTER KENNEDY, M.D., M.B., B.Ch.

FOURTEENTH ORDINARY MEETING.

Monday, 17th June 1912.

Professor Cossar Ewart, F.R.S., Vice-President, in the Chair.

The following Communications were read :—

1. On Inheritance of Hair and Eye Colour. By JOHN BROWNLEE, M.A., M.D. *Proc.*, vol. xxxii. pp. 458–474.
2. (*a*) The Upper Cambrian Rocks at Craigeven Bay, Stonehaven :
   (*b*) The Downtonian and Old Red Sandstone Rocks of Kincardineshire. By ROBERT CAMPBELL, M.A., D.Sc. Communicated by Professor GEIKIE, F.R.S. *Trans.*, vol. xlviii. Pt. IV.
3. The Amphipoda of the Scottish National Antarctic Expedition. By Professor CHARLES CHILTON, M.A., D.Sc. (N.Z.). Communicated by Dr W. S. BRUCE. *Trans.*, vol. xlviii. pp. 455–520.
4. The Cestoda of the Scottish National Antarctic Expedition. By JOHN RENNIE, D.Sc., and ALEXANDER REID. *Trans.*, vol. xlviii. pp. 441–453.
5. The Brachiopoda of the Scottish National Antarctic Expedition. By J. W. JACKSON, F.G.S. Communicated by Dr W. S. BRUCE. *Trans.*, vol. xlviii. pp. 367–390.

FIFTEENTH ORDINARY MEETING.

Monday, 1st July 1912.

Dr Horne, F.R.S., Vice-President, in the Chair.

The following communications were read :—

1. Multiple Neuroma of the Central Nervous System ; their Structure and Histogenesis. By the late Dr ALEXANDER BRUCE and Dr JAMES W. DAWSON. Communicated by Dr A. NINIAN BRUCE. *Trans.*, vol. xlviii. pp. 697–798.
2. On Magnetic Induction, at High and Intermediate Temperatures, in Ferric Oxide heated in Air and in Hydrogen, and on the Possible Chemical Changes indicated by the Observations. By G. E. ALLAN, D.Sc., and JOHN BROWN, M.A., B.Sc. Communicated by Professor A. GRAY, F.R.S. *Proc.*, vol. xxxiii.

Dr A. NINIAN BRUCE signed the Roll, and was duly admitted a Fellow of the Society.

SIXTEENTH AND LAST ORDINARY MEETING.

Monday, 15th July 1912.

Principal Sir William Turner, K.C.B., President, in the Chair.

PRIZES.

The Council having awarded :—

1. The Neill Prize for the biennial period 1909–10, 1910–11, to James Murray, F.R.S.E., for his paper on "Scottish Rotifers collected by the Lake Survey (Supplement)," and other papers on the "Rotifera" and "Tardigrada," which appeared in the *Transactions* of the Society—(this Prize was awarded after consideration of the papers received within the five years prior to the time of award : see Neill Prize Regulations) ;—

2. The Keith Prize for the biennial period 1909–10, 1910–11, to Professor Alexander Smith, B.Sc., Ph.D., of New York, for his researches upon "Sulphur" and upon "Vapour Pressure," appearing in the *Proceedings* of the Society,—these Prizes were presented.

On Presenting the Neill Prize the Chairman read the following statement :—

The Neill Prize for the biennial period 1909–10, 1910–11, to James Murray, F.R.S.E., for his papers in the Society's *Transactions* on "Scottish Rotifers collected by the Lake Survey," and on Scottish and Arctic Tardigrada.

Mr Murray took part in the systematic Bathymetrical Survey of the Scottish Fresh Water Lochs, under the superintendence of Sir John Murray and Mr Laurence Pullar, from 1902–1907.

During that time, in addition to the systematic sounding work and other systematic physical observations, Mr Murray found time to make a critical study of the Rotifera and Tardigrada, and devoted most of his leisure time to the microscopical examination and description of the forms found in the many lochs and ponds investigated by him.

His principal results as regards these two classes of lowly organised animals are given in six papers, which have been published in the *Transactions* of the Royal Society during the years 1905–1908. In these papers he describes twenty-six new species of *Rotifera*, including a representative of a new genus named *Microdina*, and of the Tardigrada he describes twenty new species. But probably of more interest are the distribution lists which he gives of these groups which are of the greatest interest to all students of these organisms.

On presenting the Keith Prize the Chairman read the following statement :—

The work on Sulphur was published in seven papers. At the time these investigations were begun, the published observations upon the behaviour of melted sulphur were full of apparent inconsistencies, and could not be formulated in harmony with physico-chemical theory.

The first step was to settle the disputed question as to the relations of amorphous and soluble sulphur in the melt. Measurements of freezing-points and of the corresponding proportions of amorphous sulphur in the freezing liquid showed that Raoult's law held rigorously. This established the existence of liquid amorphous sulphur dissolved, but distinct from the melted soluble sulphur.

The fact that melted sulphur, when kept at a given temperature, gives, on chilling, very inconstant proportions of amorphous sulphur was next investigated. It was discovered that the introduction of sulphur dioxide and other foreign substances greatly influenced the proportions. These foreign bodies were proved to act catalytically, and retard or hasten the change from amorphous to soluble sulphur. The establishment of this conclusion at once afforded a basis for explaining a large proportion of the apparent inconsistencies in the older as well as the more recent observations. In connection with this work, the proportions of amorphous sulphur present in equilibrium at various temperatures were measured.

In the fifth paper, studies of some other peculiarities in the behaviour of melted sulphur were described, and all the results were shown to harmonise with a theory of the relation of the two liquid forms as dynamic isomers.

Precipitated sulphur was the subject of the sixth paper, and it was shown that, when first liberated, the sulphur consists of droplets of liquid amorphous sulphur. In presence of weak acids, or in neutral or alkaline solutions, this changes wholly to crystalline, soluble sulphur. In presence of active acids, the amount of amorphous sulphur surviving in the final product is proportioned to the concentration of the acid.

In the seventh paper, the generally accepted melting-points (or freezing-points) of the various forms of sulphur, determined before the complex nature of the problem which such measurements involved was in the least suspected, were subjected to revision, and the correct values, in harmony with the theory, were given.

The work on Vapour Pressures is described in seven papers. The first two deal with a simple device, named the "submerged bulblet," by which boiling-points and vapour pressures of liquids and of non-fusing solids may be determined with the use of only minute amounts of material.

In the third and fifth papers, forms of apparatus for the exact study of vapour pressures, and named respectively the static and dynamic "isoteniscope," are described. To ascertain the possibilities of the methods, values for water, which agree with the best previous determination, were obtained by the static method, and values for benzene and for ammonium chloride by the dynamic method.

The fourth paper describes a determination of the vapour pressures of mercury. These were made because exact values were required for the subject of the sixth paper, and the existing results (*e g.*, those of Regnault, Ramsay and Young, and others) were highly inconsistent with one another, and the methods used were open to serious criticism.

The sixth paper deals with the constitution of calomel vapour, a matter long but inconclusively discussed by chemists. By making measurements of the vapour pressures of mercury, of calomel, and of a mixture of the two, and applying the laws of chemical equilibrium to the resulting data, it was shown inclusively that the vapour is wholly composed of mercury and corrosive sublimate. The close quantitative correspondence showed that in these measurements the order of accuracy was much higher than in any previous measurements of vapour pressures at elevated temperatures.

The seventh paper shows that, as the laws of chemical equilibrium applied to the result of the preceding paper predict, calomel, *when dried in the most rigorous manner*, exercises, even at high temperatures, no measurable pressure whatever. This is the only successful experimental confirmation of a familiar and important application of the theory.

At the request of the Council an Address was delivered:—

On Pigments Old and New, and their Identification in Pictures. (*With Lantern Illustrations.*) By Principal A. P. Laurie, M.A., D.Sc., Heriot-Watt College, Edinburgh, Professor of Chemistry, Royal Academy.

The following Candidates for Fellowship were balloted for, and declared duly elected:— Arthur Dukinfield Darbishire, M.A.; Edmund Taylor Whittaker, Sc.D., F.R.S.; The Most Hon. The Marquis of Linlithgow; Robert Black Thomson, M.B. (Edin.).

LAWS OF THE SOCIETY,

As revised 26th October 1908.

[By the Charter of the Society (printed in the *Transactions*, vol. vi. p. 5), the Laws cannot be altered, except at a Meeting held one month after that at which the Motion for alteration shall have been proposed.]

I.

THE ROYAL SOCIETY OF EDINBURGH shall consist of Ordinary and Honorary Fellows. **Title.**

II.

Every Ordinary Fellow, within three months after his election, shall pay Two Guineas as the fee of admission, and Three Guineas as his contribution for the Session in which he has been elected; and annually at the commencement of every Session, Three Guineas into the hands of the Treasurer. This annual contribution shall continue for ten years after his admission, and it shall be limited to Two Guineas for fifteen years thereafter.* Fellows may compound for these contributions on such terms as the Council may from time to time fix. **The fees of Ordinary Fellows residing in Scotland.**

III.

All Fellows who shall have paid Twenty-five years' annual contribution shall be exempted from further payment. **Payment to cease after 25 years.**

IV.

The fees of admission of an Ordinary Non-Resident Fellow shall be £26, 5s., payable on his admission; and in case of any Non-Resident Fellow coming to reside at any time in Scotland, he shall, during each year of his residence, pay the usual annual contribution of £3, 3s., payable by each Resident Fellow; but after payment of such annual contribution for eight years, he shall be exempt from any further payment. **Fees of Non-Resident Ordinary Fellows.** In the case of any Resident Fellow ceasing to reside in Scotland, and wishing to continue a Fellow of the Society, it shall be in the power of the Council to determine on what terms, in the circumstances of each case, the privilege of remaining a Fellow of the Society shall be continued to such Fellow while out of Scotland. **Case of Fellows becoming Non-Resident.**

* A modification of this rule, in certain cases, was agreed to at a Meeting of the Society held on the 3rd January 1831.

At the Meeting of the Society, on the 5th January 1857, when the reduction of the Contributions from £3, 3s. to £2, 2s., from the 11th to the 25th year of membership, was adopted, it was resolved that the existing Members shall share in this reduction, so far as regards their future annual Contributions.

V.

Defaulters.

Members failing to pay their contributions for three successive years (due application having been made to them by the Treasurer) shall be reported to the Council, and, if they see fit, shall be declared from that period to be no longer Fellows, and the legal means for recovering such arrears shall be employed.

VI.

Privileges of Ordinary Fellows.

None but Ordinary Fellows shall bear any office in the Society, or vote in the choice of Fellows or Office-Bearers, or interfere in the patrimonial interests of the Society.

VII.

Numbers unlimited.

The number of Ordinary Fellows shall be unlimited.

VIII.

Fellows entitled to Transactions and Proceedings.

All Ordinary Fellows of the Society who are not in arrear of their Annual Contributions shall be entitled to receive, gratis, copies of the parts of the Transactions of the Society which shall be published subsequent to their admission, upon application, either personally or by an authorised agent, to the Librarian, provided they apply for them within five years of the date of publication of such parts.

Copies of the parts of the Proceedings shall be distributed to all Fellows of the Society, by post or otherwise, as soon as may be convenient after publication.

IX.

Mode of Recommending Ordinary Fellows.

Candidates for admission as Ordinary Fellows shall make an application in writing, and shall produce along with it a certificate of recommendation to the purport below,* signed by at least *four* Ordinary Fellows, two of whom shall certify their recommendation from personal knowledge. This recommendation shall be delivered to the Secretary, and by him laid before the Council, and shall be exhibited publicly in the Society's rooms for one month, after which it shall be considered by the Council. If the Candidate be approved by the Council, notice of the day fixed for the election shall be given in the circulars of at least two Ordinary Meetings of the Society.

X.

Honorary Fellows, British and Foreign.

Honorary Fellows shall not be subject to any contribution. This class shall consist of persons eminently distinguished for science or literature. Its number shall not exceed Fifty-six, of whom Twenty may be British subjects, and Thirty-six may be subjects of foreign states.

* "A. B., a gentleman well versed in science (*or Polite Literature, as the case may be*), being "to our knowledge desirous of becoming a Fellow of the Royal Society of Edinburgh, we hereby "recommend him as deserving of that honour, and as likely to prove a useful and valuable "Member."

XI.

Personages of Royal Blood may be elected Honorary Fellows, without regard to the limitation of numbers specified in Law X. Royal Personages.

XII.

Honorary Fellows may be proposed by the Council, or by a recommendation (in the form given below*) subscribed by three Ordinary Fellows; and in case the Council shall decline to bring this recommendation before the Society, it shall be competent for the proposers to bring the same before a General Meeting. The election shall be by ballot, after the proposal has been communicated *viva voce* from the Chair at one Meeting, and printed in the circulars for Two Ordinary Meetings of the Society, previous to the day of election. Recommendation of Honorary Fellows. Mode of election.

XIII.

The election of Ordinary Fellows shall take place only at one Afternoon Ordinary Meeting of each month during the Session. The election shall be by ballot, and shall be determined by a majority of at least two-thirds of the votes, provided Twenty-four Fellows be present and vote. Election of Ordinary Fellows.

XIV.

The Ordinary Meetings shall be held on the first and third Mondays of each month from November to March, and from May to July, inclusive; with the exception that when there are five Mondays in January, the Meetings for that month shall be held on its second and fourth Mondays. Regular Minutes shall be kept of the proceedings, and the Secretaries shall do the duty alternately, or according to such agreement as they may find it convenient to make. Ordinary Meetings.

XV.

The Society shall from time to time publish its Transactions and Proceedings. For this purpose the Council shall select and arrange the papers which they shall deem it expedient to publish in the Transactions of the Society, and shall superintend the printing of the same. The Transactions.

XVI.

The Transactions shall be published in parts or *Fasciculi* at the close of each Session, and the expense shall be defrayed by the Society. How Published.

* We hereby recommend____
for the distinction of being made an Honorary Fellow of this Society, declaring that each of us from our own knowledge of his services to (*Literature or Science, as the case may be*) believe him to be worthy of that honour.

(To be signed by three Ordinary Fellows.)

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To the President and Council of the Royal Society of Edinburgh.

XVII.

The Council.

That there shall be formed a Council, consisting—First, of such gentlemen as may have filled the office of President; and Secondly, of the following to be annually elected, viz.:—a President, Six Vice-Presidents (two at least of whom shall be Resident), Twelve Ordinary Fellows as Councillors, a General Secretary, Two Secretaries to the Ordinary Meetings, a Treasurer, and a Curator of the Museum and Library.

The Council shall have power to regulate the private business of the Society. At any Meeting of the Council the Chairman shall have a casting as well as a deliberative vote.

XVIII.

Retiring Councillors.

Four Councillors shall go out annually, to be taken according to the order in which they stand on the list of the Council.

XIX.

Election of Office-Bearers.

An Extraordinary Meeting for the election of Office-Bearers shall be held annually on the fourth Monday of October, or on such other lawful day in October as the Council may fix, and each Session of the Society shall be held to begin at the date of the said Extraordinary Meeting.

XX.

Special Meetings; how called.

Special Meetings of the Society may be called by the Secretary, by direction of the Council; or on a requisition signed by six or more Ordinary Fellows. Notice of not less than two days must be given of such Meetings.

XXI.

Treasurer's Duties.

The Treasurer shall receive and disburse the money belonging to the Society, granting the necessary receipts, and collecting the money when due.

He shall keep regular accounts of all the cash received and expended, which shall be made up and balanced annually; and at the Extraordinary Meeting in October, he shall present the accounts for the preceding year, duly audited. At this Meeting, the Treasurer shall also lay before the Council a list of all arrears due above two years, and the Council shall thereupon give such directions as they may deem necessary for recovery thereof.

XXII.

Auditor.

At the Extraordinary Meeting in October, a professional accountant shall be chosen to audit the Treasurer's accounts for that year, and to give the necessary discharge of his intromissions.

XXIII.

General Secretary's Duties.

The General Secretary shall keep Minutes of the Extraordinary Meetings of the Society, and of the Meetings of the Council, in two distinct books. He shall, under the direction of the Council, conduct the correspondence of the Society, and superintend its publications. For these purposes he shall, when necessary, employ a clerk, to be paid by the Society.

XXIV.

The Secretaries to the Ordinary Meetings shall keep a regular Minute-book, in which a full account of the proceedings of these Meetings shall be entered; they shall specify all the Donations received, and furnish a list of them, and of the Donors' names, to the Curator of the Library and Museum; they shall likewise furnish the Treasurer with notes of all admissions of Ordinary Fellows. They shall assist the General Secretary in superintending the publications, and in his absence shall take his duty. **Secretaries to Ordinary Meetings.**

XXV.

The Curator of the Museum and Library shall have the custody and charge of all the Books, Manuscripts, objects of Natural History, Scientific Productions, and other articles of a similar description belonging to the Society; he shall take an account of these when received, and keep a regular catalogue of the whole, which shall lie in the hall, for the inspection of the Fellows. **Curator of Museum and Library.**

XXVI.

All articles of the above description shall be open to the inspection of the Fellows at the Hall of the Society, at such times and under such regulations as the Council from time to time shall appoint. **Use of Museum and Library.**

XXVII.

A Register shall be kept, in which the names of the Fellows shall be enrolled at their admission, with the date. **Register Book.**

XXVIII.

If, in the opinion of the Council of the Society, the conduct of any Fellow is unbecoming the position of a Member of a learned Society, or is injurious to the character and interests of this Society, the Council may request such Fellow to resign; and, if he fail to do so within one month of such request being addressed to him, the Council shall call a General Meeting of the Fellows of the Society to consider the matter; and, if a majority of the Fellows present at such Meeting agree to the expulsion of such Member, he shall be then and there expelled by the declaration of the Chairman of the said Meeting to that effect; and he shall thereafter cease to be a Fellow of the Society, and his name shall be erased from the Roll of Fellows, and he shall forfeit all right or claim in or to the property of the Society. **Power of Expulsion.**

THE KEITH, MAKDOUGALL-BRISBANE, NEILL, AND GUNNING VICTORIA JUBILEE PRIZES.

The above Prizes will be awarded by the Council in the following manner:—

I. KEITH PRIZE.

The Keith Prize, consisting of a Gold Medal and from £40 to £50 in Money, will be awarded in the Session 1913–1914 for the "best communication on a scientific subject, communicated,* in the first instance, to the Royal Society during the Sessions 1911–1912 and 1912–1913." Preference will be given to a paper containing a discovery.

II. MAKDOUGALL-BRISBANE PRIZE.

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded at the commencement of the Session 1914–1915, for an Essay or Paper having reference to any branch of scientific inquiry, whether Material or Mental.

2. Competing Essays to be addressed to the Secretary of the Society, and transmitted not later than 8th July 1914.

3. The Competition is open to all men of science.

4. The Essays may be either anonymous or otherwise. In the former case, they must be distinguished by mottoes, with corresponding sealed billets, superscribed with the same motto, and containing the name of the Author.

5. The Council impose no restriction as to the length of the Essays, which may be, at the discretion of the Council, read at the Ordinary Meetings of the Society.

* For the purposes of this award the word "communicated" shall be understood to mean the date on which the manuscript of a paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

They wish also to leave the property and free disposal of the manuscripts to the Authors; a copy, however, being deposited in the Archives of the Society, unless the paper shall be published in the Transactions.

6. In awarding the Prize, the Council will also take into consideration any scientific papers presented * to the Society during the Sessions 1912–13, 1913–14, whether they may have been given in with a view to the prize or not.

III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr Patrick Neill of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate:

1. The Neill Prize, consisting of a Gold Medal and a sum of Money, will be awarded during the Session 1913–1914.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented * to the Society during the two years preceding the fourth Monday in October 1913,—or failing presentation of a paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award.

IV. GUNNING VICTORIA JUBILEE PRIZE.

This Prize, founded in the year 1887 by Dr R. H. Gunning, is to be awarded quadrennially by the Council of the Royal Society of Edinburgh, in recognition of original work in Physics, Chemistry, or Pure or Applied Mathematics.

Evidence of such work may be afforded either by a Paper presented to the Society, or by a Paper on one of the above subjects, or some discovery in them elsewhere communicated or made, which the Council may consider to be deserving of the Prize.

The Prize consists of a sum of money, and is open to men of science resident in or connected with Scotland. The first award was made in the year 1887.

In accordance with the wish of the Donor, the Council of the Society may on fit occasions award the Prize for work of a definite kind to be undertaken during the three succeeding years by a scientific man of recognised ability.

* For the purposes of this award the word "presented" shall be understood to mean the date on which the manuscript of a paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

AWARDS OF THE KEITH, MAKDOUGALL-BRISBANE, NEILL, AND GUNNING VICTORIA JUBILEE PRIZES.

I. KEITH PRIZE.

1st Biennial Period, 1827–29.—Dr Brewster, for his papers "on his Discovery of Two New Immiscible Fluids in the Cavities of certain Minerals," published in the Transactions of the Society.

2nd Biennial Period, 1829–31.—Dr Brewster, for his paper "on a New Analysis of Solar Light," published in the Transactions of the Society.

3rd Biennial Period, 1831–33.—Thomas Graham, Esq., for his paper "on the Law of the Diffusion of Gases," published in the Transactions of the Society.

4th Biennial Period, 1833–35.—Professor J. D. Forbes, for his paper "on the Refraction and Polarization of Heat," published in the Transactions of the Society.

5th Biennial Period, 1835–37.—John Scott Russell, Esq., for his researches "on Hydrodynamics," published in the Transactions of the Society.

6th Biennial Period, 1837–39.—Mr John Shaw, for his experiments "on the Development and Growth of the Salmon," published in the Transactions of the Society.

7th Biennial Period, 1839–41.—Not awarded.

8th Biennial Period, 1841–1843.—Professor James David Forbes, for his papers "on Glaciers," published in the Proceedings of the Society.

9th Biennial Period, 1843–45.—Not awarded.

10th Biennial Period, 1845–47.—General Sir Thomas Brisbane, Bart., for the Makerstoun Observations on Magnetic Phenomena, made at his expense, and published in the Transactions of the Society.

11th Biennial Period, 1847–49.—Not awarded.

12th Biennial Period, 1849–51.—Professor Kelland, for his papers "on General Differentiation, including his more recent Communication on a process of the Differential Calculus, and its application to the solution of certain Differential Equations," published in the Transactions of the Society.

13th Biennial Period, 1851–53.—W. J. Macquorn Rankine, Esq., for his series of papers "on the Mechanical Action of Heat," published in the Transactions of the Society.

14th Biennial Period, 1853–55.—Dr Thomas Anderson, for his papers "on the Crystalline Constituents of Opium, and on the Products of the Destructive Distillation of Animal Substances," published in the Transactions of the Society.

15th Biennial Period, 1855–57.—Professor Boole, for his Memoir "on the Application of the Theory of Probabilities to Questions of the Combination of Testimonies and Judgments," published in the Transactions of the Society.

16th Biennial Period, 1857–59.—Not awarded.

17th Biennial Period, 1859–61.—John Allan Broun, Esq., F.R.S., Director of the Trevandrum Observatory, for his papers "on the Horizontal Force of the Earth's Magnetism, on the Correction of the Bifilar Magnetometer, and on Terrestrial Magnetism generally," published in the Transactions of the Society.

18th Biennial Period, 1861–63.—Professor William Thomson, of the University of Glasgow, for his Communication "on some Kinematical and Dynamical Theorems."

19th Biennial Period, 1863–65.—Principal Forbes, St Andrews, for his "Experimental Inquiry into the Laws of Conduction of Heat in Iron Bars," published in the Transactions of the Society.

20th Biennial Period, 1865–67.—Professor C. Piazzi Smyth, for his paper "on Recent Measures at the Great Pyramid," published in the Transactions of the Society.

21st Biennial Period, 1867–69.—Professor P. G. Tait, for his paper "on the Rotation of a Rigid Body about a Fixed Point" published in the Transactions of the Society.

22nd Biennial Period, 1869–71.—Professor Clerk Maxwell, for his paper "on Figures, Frames, and Diagrams of Forces," published in the Transactions of the Society.

23RD BIENNIAL PERIOD, 1871–73.—Professor P. G. TAIT, for his paper entitled "First Approximation to a Thermo-electric Diagram," published in the Transactions of the Society.

24TH BIENNIAL PERIOD, 1873–75.—Professor CRUM BROWN, for his Researches "on the Sense of Rotation, and on the Anatomical Relations of the Semicircular Canals of the Internal Ear."

25TH BIENNIAL PERIOD, 1875–77.—Professor M. FORSTER HEDDLE, for his papers "on the Rhombohedral Carbonates," and "on the Felspars of Scotland," published in the Transactions of the Society.

26TH BIENNIAL PERIOD, 1877–79.—Professor H. C. FLEEMING JENKIN, for his paper "on the Application of Graphic Methods to the Determination of the Efficiency of Machinery," published in the Transactions of the Society; Part II. having appeared in the volume for 1877–78.

27TH BIENNIAL PERIOD, 1879–81.—Professor GEORGE CHRYSTAL, for his paper "on the Differential Telephone," published in the Transactions of the Society.

28TH BIENNIAL PERIOD, 1881–83.—THOMAS MUIR, Esq., LL.D., for his "Researches into the Theory of Determinants and Continued Fractions," published in the Proceedings of the Society.

29TH BIENNIAL PERIOD, 1883–85.—JOHN AITKEN, Esq., for his paper "on the Formation of Small Clear Spaces in Dusty Air," and for previous papers on Atmospheric Phenomena, published in the Transactions of the Society.

30TH BIENNIAL PERIOD, 1885–87.—JOHN YOUNG BUCHANAN, Esq., for a series of communications, extending over several years, on subjects connected with Ocean Circulation, Compressibility of Glass, etc.; two of which, viz., "On Ice and Brines," and "On the Distribution of Temperature in the Antarctic Ocean," have been published in the Proceedings of the Society.

31ST BIENNIAL PERIOD, 1887–89.—Professor E. A. LETTS, for his papers on the Organic Compounds of Phosphorus, published in the Transactions of the Society.

32ND BIENNIAL PERIOD, 1889–91.—R. T. OMOND, Esq., for his contributions to Meteorological Science, many of which are contained in vol. xxxiv. of the Society's Transactions.

33RD BIENNIAL PERIOD, 1891–93.—Professor THOMAS R. FRASER, F.R.S., for his papers on *Strophanthus hispidus*, Strophanthin, and Strophanthidin, read to the Society in February and June 1889 and in December 1891, and printed in vols. xxxv., xxxvi., and xxxvii. of the Society's Transactions.

34TH BIENNIAL PERIOD, 1893–95.—Dr CARGILL G. KNOTT, for his papers on the Strains produced by Magnetism in Iron and in Nickel, which have appeared in the Transactions and Proceedings of the Society.

35TH BIENNIAL PERIOD, 1895–97.—Dr THOMAS MUIR, for his continued communications on Determinants and Allied Questions.

36TH BIENNIAL PERIOD, 1897–99.—Dr JAMES BURGESS, for his paper "on the Definite Integral $\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$, with extended Tables of Values," printed in vol. xxxix. of the Transactions of the Society.

37TH BIENNIAL PERIOD, 1899–1901.—Dr HUGH MARSHALL, for his discovery of the Persulphates, and for his Communications on the Properties and Reactions of these Salts, published in the Proceedings of the Society.

38TH BIENNIAL PERIOD, 1901–03.—Sir WILLIAM TURNER, K.C.B., LL.D., F.R.S., &c., for his memoirs entitled "A Contribution to the Craniology of the People of Scotland," published in the Transactions of the Society, and for his "Contributions to the Craniology of the People of the Empire of India," Parts I., II., likewise published in the Transactions of the Society.

39TH BIENNIAL PERIOD, 1903–05.—THOMAS H. BRYCE, M.A., M.D., for his two papers on "The Histology of the Blood of the Larva of *Lepidosiren paradoxa*," published in the Transactions of the Society within the period.

40TH BIENNIAL PERIOD, 1905–07.—ALEXANDER BRUCE, M.A., M.D., F.R.C.P.E., for his paper entitled "Distribution of the Cells in the Intermedio-Lateral Tract of the Spinal Cord," published in the Transactions of the Society within the period.

41ST BIENNIAL PERIOD, 1907–09.—WHEELTON HIND, M.D., B.S., F.R.C.S., F.G.S., for a paper published in the Transactions of the Society, "On the Lamellibranch and Gasteropod Fauna found in the Millstone Grit of Scotland."

42ND BIENNIAL PERIOD, 1909–11.—Professor ALEXANDER SMITH, B.Sc., Ph.D., of New York, for his researches upon "Sulphur" and upon "Vapour Pressure," appearing in the Proceedings of the Society.

II. MAKDOUGALL-BRISBANE PRIZE.

1st Biennial Period, 1859.—Sir Roderick Impey Murchison, on account of his Contributions to the Geology of Scotland.

2nd Biennial Period, 1860–62.—William Seller, M.D., F.R.C.P.E., for his "Memoir of the Life and Writings of Dr Robert Whytt," published in the Transactions of the Society.

3rd Biennial Period, 1862–64.—John Denis Macdonald, Esq., R.N., F.R.S., Surgeon of H.M.S. "Icarus," for his paper "on the Representative Relationships of the Fixed and Free Tunicata, regarded as Two Sub-classes of equivalent value; with some General Remarks on their Morphology," published in the Transactions of the Society.

4th Biennial Period, 1864–66.—Not awarded.

5th Biennial Period, 1866–68.—Dr Alexander Crum Brown and Dr Thomas Richard Fraser, for their conjoint paper "on the Connection between Chemical Constitution and Physiological Action," published in the Transactions of the Society.

6th Biennial Period, 1868–70.—Not awarded.

7th Biennial Period, 1870–72.—George James Allman, M.D., F.R.S., Emeritus Professor of Natural History, for his paper "on the Homological Relations of the Cœlenterata," published in the Transactions, which forms a leading chapter of his Monograph of Gymnoblastic or Tubularian Hydroids—since published.

8th Biennial Period, 1872–74.—Professor Lister, for his paper "on the Germ Theory of Putrefaction and the Fermentive Changes," communicated to the Society, 7th April 1873.

9th Biennial Period, 1874–76.—Alexander Buchan, A.M., for his paper "on the Diurnal Oscillation of the Barometer," published in the Transactions of the Society.

10th Biennial Period, 1876–78.—Professor Archibald Geikie, for his paper "on the Old Red Sandstone of Western Europe," published in the Transactions of the Society.

11th Biennial Period, 1878–80.—Professor Piazzi Smyth, Astronomer-Royal for Scotland, for his paper "on the Solar Spectrum in 1877–78, with some Practical Idea of its probable Temperature of Origination," published in the Transactions of the Society.

12th Biennial Period, 1880–82.—Professor James Geikie, for his "Contributions to the Geology of the North-West of Europe," including his paper "on the Geology of the Faroes," published in the Transactions of the Society.

13th Biennial Period, 1882–84.—Edward Sang, Esq., LL.D., for his paper "on the Need of Decimal Subdivisions in Astronomy and Navigation, and on Tables requisite therefor," and generally for his Recalculation of Logarithms both of Numbers and Trigonometrical Ratios,—the former communication being published in the Proceedings of the Society.

14th Biennial Period, 1884–86.—John Murray, Esq., LL.D., for his papers "On the Drainage Areas of Continents, and Ocean Deposits," "The Rainfall of the Globe, and Discharge of Rivers," "The Height of the Land and Depth of the Ocean," and "The Distribution of Temperature in the Scottish Lochs as affected by the Wind."

15th Biennial Period, 1886–88.—Archibald Geikie, Esq., LL.D., for numerous Communications, especially that entitled "History of Volcanic Action during the Tertiary Period in the British Isles," published in the Transactions of the Society.

16th Biennial Period, 1889–90.—Dr Ludwig Becker, for his paper on "The Solar Spectrum at Medium and Low Altitudes," printed in vol. xxxvi. Part I. of the Society's Transactions.

17th Biennial Period, 1890–92.—Hugh Robert Mill, Esq., D.Sc., for his papers on "The Physical Conditions of the Clyde Sea Area," Part I. being already published in vol. xxxvi. of the Society's Transactions.

18th Biennial Period, 1892–94.—Professor James Walker, D.Sc., Ph.D., for his work on Physical Chemistry, part of which has been published in the Proceedings of the Society, vol. xx. pp. 255–263. In making this award, the Council took into consideration the work done by Professor Walker along with Professor Crum Brown on the Electrolytic Synthesis of Dibasic Acids, published in the Transactions of the Society.

19th Biennial Period, 1894–96.—Professor John G. M'Kendrick, for numerous Physiological papers, especially in connection with Sound, many of which have appeared in the Society's publications.

20th Biennial Period, 1896–98.—Dr William Peddie, for his papers on the Torsional Rigidity of Wires.

21st Biennial Period, 1898–1900.—Dr Ramsay H. Traquair, for his paper entitled "Report on Fossil Fishes collected by the Geological Survey in the Upper Silurian Rocks of Scotland," printed in vol. xxxix. of the Transactions of the Society.

22nd Biennial Period, 1900–02.—Dr Arthur T. Masterman, for his paper entitled "The Early Development of *Cribrella oculata* (Forbes), with remarks on Echinoderm Development," printed in vol. xl. of the Transactions of the Society.

23rd Biennial Period, 1902–04.—Mr John Dougall, M.A., for his paper on "An Analytical Theory of the Equilibrium of an Isotropic Elastic Plate," published in vol. xli. of the Transactions of the Society.

24th Biennial Period, 1904–06.—Jacob E. Halm, Ph.D., for his two papers entitled "Spectroscopic Observations of the Rotation of the Sun," and "Some Further Results obtained with the Spectroheliometer," and for other astronomical and mathematical papers published in the Transactions and Proceedings of the Society within the period.

25th Biennial Period, 1906–08.—D. T. Gwynne-Vaughan, M.A., F.L.S., for his papers, 1st, "On the Fossil Osmundaceæ," and 2nd, "On the Origin of the Adaxially-curved Leaf-trace in the Filicales," communicated by him conjointly with Dr R. Kidston.

26th Biennial Period, 1908–10.—Ernest MacLagan Wedderburn, M.A., LL.B., for his series of papers bearing upon "The Temperature Distribution in Fresh-water Lochs," and especially upon "The Temperature Seiche."

III. THE NEILL PRIZE.

1st Triennial Period, 1856–59.—Dr W. Lauder Lindsay, for his paper "on the Spermogones and Pycnides of Filamentous, Fruticulose, and Foliaceous Lichens," published in the Transactions of the Society.

2nd Triennial Period, 1859–61.—Robert Kaye Greville, LL.D., for his Contributions to Scottish Natural History, more especially in the department of Cryptogamic Botany, including his recent papers on Diatomaceæ.

3rd Triennial Period, 1862–65.—Andrew Crombie Ramsay, F.R.S., Professor of Geology in the Government School of Mines, and Local Director of the Geological Survey of Great Britain, for his various works and memoirs published during the last five years, in which he has applied the large experience acquired by him in the Direction of the arduous work of the Geological Survey of Great Britain to the elucidation of important questions bearing on Geological Science.

4th Triennial Period, 1865–68.—Dr William Carmichael M'Intosh, for his paper "on the Structure of the British Nemerteans, and on some New British Annelids," published in the Transactions of the Society.

5th Triennial Period, 1868–71.—Professor William Turner, for his papers "on the Great Finner Whale; and on the Gravid Uterus, and the Arrangement of the Fœtal Membranes in the Cetacea," published in the Transactions of the Society.

6th Triennial Period, 1871–74.—Charles William Peach, Esq., for his Contributions to Scottish Zoology and Geology, and for his recent contributions to Fossil Botany.

7th Triennial Period, 1874–77.—Dr Ramsay H. Traquair, for his paper "on the Structure and Affinities of *Tristichopterus alatus* (Egerton)," published in the Transactions of the Society, and also for his contributions to the Knowledge of the Structure of Recent and Fossil Fishes.

8th Triennial Period, 1877–80.—John Murray, Esq., for his paper "on the Structure and Origin of Coral Reefs and Islands," published (in abstract) in the Proceedings of the Society.

9th Triennial Period, 1880–83.—Professor Herdman, for his papers "on the Tunicata," published in the Proceedings and Transactions of the Society.

10th Triennial Period, 1883–86.—B. N. Peach, Esq., for his Contributions to the Geology and Palæontology of Scotland, published in the Transactions of the Society.

11th Triennial Period, 1886–89.—Robert Kidston, Esq., for his Researches in Fossil Botany, published in the Transactions of the Society.

12th Triennial Period, 1889–92.—John Horne, Esq., F.G.S., for his Investigations into the Geological Structure and Petrology of the North-West Highlands.

13th Triennial Period, 1892–95.—Robert Irvine, Esq., for his papers on the Action of Organisms in the Secretion of Carbonate of Lime and Silica, and on the solution of these substances in Organic Juices. These are printed in the Society's Transactions and Proceedings.

14th Triennial Period, 1895–98.—Professor Cossar Ewart, for his recent Investigations connected with Telegony.

15TH TRIENNIAL PERIOD, 1898–1901.—Dr JOHN S. FLETT, for his papers entitled "The Old Red Sandstone of the Orkneys" and "The Trap Dykes of the Orkneys," printed in vol. xxxix. of the Transactions of the Society.

16TH TRIENNIAL PERIOD, 1901–04.—Professor J. GRAHAM KERR, M.A., for his Researches on *Lepidosiren paradoxa*, published in the Philosophical Transactions of the Royal Society, London.

17TH TRIENNIAL PERIOD, 1904–07.—FRANK J. COLE, B.Sc., for his paper entitled "A Monograph on the General Morphology of the Myxinoid Fishes, based on a study of Myxine," published in the Transactions of the Society, regard being also paid to Mr Cole's other valuable contributions to the Anatomy and Morphology of Fishes.

1ST BIENNIAL PERIOD, 1907–09.—FRANCIS J. LEWIS, M.Sc., F.L.S., for his papers in the Society's Transactions "On the Plant Remains of the Scottish Peat Mosses."

2ND BIENNIAL PERIOD, 1909–11.—JAMES MURRAY, Esq., for his paper on "Scottish Rotifers collected by the Lake Survey (Supplement)," and other papers on the "Rotifera" and "Tardigrada," which appeared in the Transactions of the Society—(this Prize was awarded after consideration of the papers received within the five years prior to the time of award: see Neill Prize Regulations).

IV. GUNNING VICTORIA JUBILEE PRIZE.

1ST TRIENNIAL PERIOD, 1884–87.—Sir WILLIAM THOMSON, Pres. R.S.E., F.R.S., for a remarkable series of papers "on Hydrokinetics," especially on Waves and Vortices, which have been communicated to the Society.

2ND TRIENNIAL PERIOD, 1887–90.—Professor P. G. TAIT, Sec. R.S.E., for his work in connection with the "Challenger" Expedition, and his other Researches in Physical Science.

3RD TRIENNIAL PERIOD, 1890–93.—ALEXANDER BUCHAN, Esq., LL.D., for his varied, extensive, and extremely important Contributions to Meteorology, many of which have appeared in the Society's Publications.

4TH TRIENNIAL PERIOD, 1893–96.—JOHN AITKEN, Esq., for his brilliant Investigations in Physics, especially in connection with the Formation and Condensation of Aqueous Vapour.

1ST QUADRENNIAL PERIOD, 1896–1900.—Dr T. D. ANDERSON, for his discoveries of New and Variable Stars.

2ND QUADRENNIAL PERIOD, 1900–04.—Sir JAMES DEWAR, LL.D., D.C.L., F.R.S., etc., for his researches on the Liquefaction of Gases, extending over the last quarter of a century, and on the Chemical and Physical Properties of Substances at Low Temperatures: his earliest papers being published in the Transactions and Proceedings of the Society.

3RD QUADRENNIAL PERIOD, 1904–08.—Professor GEORGE CHRYSTAL, M.A., LL.D., for a series of papers on "Seiches," including "The Hydrodynamical Theory and Experimental Investigations of the Seiche Phenomena of Certain Scottish Lakes."

THE COUNCIL OF THE SOCIETY,

October 1912.

President.

SIR WILLIAM TURNER, K.C.B., M.B., F.R.C.S.L. and E., LL.D., D.C.L., D.Sc. (Camb. and Dub.), F.R.S., Knight of the Royal Prussian Order *Pour le Mérite*, Principal and Vice-Chancellor of the University of Edinburgh.

Vice-Presidents.

JOHN HORNE, LL.D., F.R.S., F.G.S., formerly Director of the Geological Survey of Scotland.
JAMES BURGESS, C.I.E., LL.D., M.R.A.S.
T. HUDSON BEARE, M.Inst.C.E., Professor of Engineering in the University of Edinburgh.
FREDERICK O. BOWER, M.A., D.Sc., F.R.S., F.L.S., Regius Professor of Botany in the University of Glasgow.
SIR THOMAS R. FRASER, M.D., LL.D., Sc.D., F.R.C.P.E., F.R.S., Professor of Materia Medica in the University of Edinburgh.
BENJAMIN N. PEACH, LL.D., F.R.S., F.G.S., formerly District Superintendent and Acting Palæontologist of the Geological Survey of Scotland.

General Secretary.

CARGILL G. KNOTT, D.Sc., Lecturer on Applied Mathematics in the University of Edinburgh.

Secretaries to Ordinary Meetings.

ROBERT KIDSTON, LL.D., F.R.S., F.G.S.
ARTHUR ROBINSON, M.D., M.R.C.S., Professor of Anatomy in the University of Edinburgh.

Treasurer.

JAMES CURRIE, M.A.

Curator of Library and Museum.

JOHN SUTHERLAND BLACK, M.A., LL.D.

Councillors.

* RAMSAY H. TRAQUAIR, M.D., LL.D., F.R.S.
JAMES WALKER, D.Sc., Ph.D., LL.D., F.R.S., Professor of Chemistry in the University of Edinburgh.
SIR W. S. M'CORMICK, M.A., LL.D.
ALEXANDER CRUM BROWN, M.D., D.Sc., F.R.C.P.E., LL.D., F.R.S., Emeritus Professor of Chemistry in the University of Edinburgh.
THOMAS H. BRYCE, M.A., M.D., Professor of Anatomy in the University of Glasgow.
WILLIAM ALLAN CARTER, Memb.Inst.C.E.
ANDREW WATT, M.A., Secretary to Scottish Meteorological Society.
JAMES H. ASHWORTH, D.Sc., Lecturer on Invertebrate Zoology in the University of Edinburgh.
GEORGE A. GIBSON, M.A., LL.D., Professor of Mathematics in the University of Glasgow.
RALPH A. SAMPSON, M.A., D.Sc., D.C.L., F.R.S., Astronomer Royal for Scotland, and Professor of Astronomy in the University of Edinburgh.
D'ARCY W. THOMPSON, C.B., B.A., F.L.S., Professor of Natural History in the University College, Dundee.
EDMUND T. WHITTAKER, Sc.D., F.R.S., Professor of Mathematics in the University of Edinburgh.

Society's Representative on George Heriot's Trust.

WILLIAM ALLAN CARTER, Memb.Inst.C.E.

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* Dr Traquair died November 18, 1912.

ALPHABETICAL LIST OF THE ORDINARY FELLOWS OF THE SOCIETY,

Corrected to 1st January 1913.

N.B.—*Those marked * are Annual Contributors.*

B. prefixed to a name indicates that the Fellow has received a Makdougall-Brisbane Medal.
K. „ „ „ Keith Medal.
N. „ „ „ Neill Medal.
V. J. „ „ „ the Gunning Victoria Jubilee Prize.
C. „ „ „ contributed one or more Communications to the Society's TRANSACTIONS or PROCEEDINGS.

Date of Election.			Service on Council, etc.
1898	C.	* Abercromby, the Hon. John, LL.D., 62 Palmerston Place, Edinburgh	
1898		Adami, Prof. J. G., M.A., M.D. Cantab., F.R.S., Professor of Pathology in M'Gill University, Montreal	
1911		* Adams, Archibald Campbell, A.M.I.Mech.E., A.M.I.E.E., Consulting Engineer, 1 Old Smithhills, Paisley	
1896		* Affleck, Sir Jas. Ormiston, M.D., LL.D., F.R.C.P.E., 38 Heriot Row, Edinburgh	
1871		Agnew, Sir Stair, K.C.B., M.A., formerly Registrar-General for Scotland, 22 Buckingham Terrace, Edinburgh 5	1882–85, 1886–89, 1891–93, 1895–98.
1875	C. K. V. J.	Aitken, John, LL.D., F.R.S., Ardenlea, Falkirk	
1895		* Alford, Robert Gervase, Memb. Inst. C.E., 1 Windmill Hill, Hampstead, London, N.W.	
1889		* Alison, John, M.A., Head Master, George Watson's College, Edinburgh	
1894		Allan, Francis John, M.D., C.M. Edin., M.O.H. City of Westminster, Westminster City Hall, Charing Cross Road, London	
1888	C.	Allardice, R. E., M.A., Professor of Mathematics in Stanford University, Palo Alto, Santa Clara Co., California 10	
1906		Anderson, Daniel E., M.D., B.A., B.Sc., Green Bank, Merton Lane, Highgate, London, N.	
1893		Anderson, J. Macvicar, Architect, 6 Stratton Street, London	
1883		Anderson, Sir Robert Rowand, LL.D., 16 Rutland Square, Edinburgh	
1905		Anderson, William, F.G.S., Box 4303, Johannesburg, Transvaal, South Africa	
1905		* Anderson, William, M.A., Head Science Master, George Watson's College, Edinburgh, 29 Lutton Place, Edinburgh 15	
1903		Anderson-Berry, David, M.D., LL.D., F.R.S.L., M.R.A.S., F.S.A. (Scot.), Versailles, Highgate, London, N.	
1905		* Andrew, George, M.A., B.A., H.M.I.S., Balwherrie, Strathearn Road, Broughty Ferry	
1881	C.	Anglin, A. H., M.A., LL.D., M.R.I.A., Professor of Mathematics, Queen's College, Cork	
1906		Appleton, Arthur Frederick, F.R.C.V.S., Lieut.-Col., Army Headquarters, Pretoria, South Africa	
1899		Appleyard, James R., Royal Technical Institute, Salford, Manchester 20	
1893		* Archer, Walter E., 17 Sloane Court, London, S.W.	
1910	C.	Archibald, E. H., B.Sc., Professor of Chemistry, Syracuse University, Syracuse, N.Y., U.S.A.	
1907		* Archibald, James, M.A., Head Master, St Bernard's School, 1 Leamington Terrace, Edinburgh	

Date of Election.			Service on Council, etc.
1911	C.	* Ashworth, James Hartley, D.Sc., Lecturer on Invertebrate Zoology, University of Edinburgh, 4 Cluny Terrace, Edinburgh	1912–
1907		* Badre, Muhammad, Ph.D., Almuneerah, Cairo, Egypt 25	
1896	C.	* Baily, Francis Gibson, M.A., Memb. Inst. E.E., Professor of Electrical Engineering, Heriot-Watt College, Edinburgh, Newbury, Colinton, Midlothian	1909–12.
1877	C.	Balfour, I. Bayley, M.A., Sc.D., M.D., LL.D., F.R.S., F.L.S., King's Botanist in Scotland, Professor of Botany in the University of Edinburgh and Keeper of the Royal Botanic Gardens, Inverleith House, Edinburgh	1888–91.
1905	C.	Balfour-Browne, William Alexander Francis, M.A., Barrister-at-Law, Claremont, Holywood, County Down, Ireland	
1892		* Ballantyne, J. W., M.D., F.R.C.P.E., 19 Rothesay Terrace, Edinburgh	
1902	C.	Bannerman, W. B., C.S.I., I.M.S., M.D., D.Sc., Surgeon General, Indian Medical Service, Madras, India 30	
1889		* Barbour, A. H. F., M.A., M.D., LL.D., F.R.C.P.E., 4 Charlotte Square, Edinburgh	
1886		Barclay, A. J. Gunion, M.A., 729 Great Western Road, Glasgow	
1883	C.	Barclay, G. W. W., M.A., Raeden House, Aberdeen	
1910		* Barclay, Lewis Bennett, C.E., 13 Cargill Terrace, Edinburgh	
1903		Bardswell, Noël Dean, M.D., M.R.C.P. Ed. and Lond., King Edward VII. Sanatorium, Midhurst 35	
1882	C.	Barnes, Henry, M.D., LL.D., 6 Portland Square, Carlisle	
1904		Barr, Sir James, M.D., LL.D., F.R.C.P. Lond., 72 Rodney Street, Liverpool	
1874		Barrett, Sir William F., F.R.S., M.R.I.A., formerly Professor of Physics, Royal College of Science, Dublin, 6 De Vesci Terrace, Kingstown, County Dublin	
1887		Bartholomew, J. G., LL.D., F.R.G.S., The Geographical Institute, Duncan Street, Edinburgh	1909–12.
1895	C.	Barton, Edwin H., D.Sc., A.M.I.E.E., Fellow Physical Society of London, Professor of Experimental Physics, University College, Nottingham 40	
1904		* Baxter, William Muirhead, St Colms, 21B Strathearn Road, Edinburgh	
1888		Beare, Thomas Hudson, B.Sc., Memb. Inst. C.E., Professor of Engineering in the University of Edinburgh (VICE-PRESIDENT)	1907–1909. V-P 1909–
1897	C.	* Beattie, John Carruthers, D.Sc., Professor of Physics, South African College, Cape Town	
1892		Beck, Sir J. H. Meiting, Kt., M.D., M.R.C.P.E., Drostdy, Tulbagh, Cape Province, South Africa	
1893	B. C.	* Becker, Ludwig, Ph.D., Regius Professor of Astronomy in the University of Glasgow, The Observatory, Glasgow 45	
1882	C.	Beddard, Frank E., M.A. Oxon., F.R.S., Prosector to the Zoological Society of London, Zoological Society's Gardens, Regent's Park, London	
1887		Begg, Ferdinand Faithfull, Bartholomew House, London	
1886		Bell, A. Beatson, 17 Lansdowne Crescent, Edinburgh	1889–92, 1893–96, 1897–1900.
1906		Bell, John Patrick Fair, F.Z.S., Fulforth, Witton Gilbert, Durham	
1900		* Bennett, James Bower, C.E., 5 Hill Street, Edinburgh 50	
1887		Bernard, J. Mackay, of Dunsinnan, B.Sc., Dunsinnan, Perth	
1893	C.	* Berry, George A., M.D., C.M., F.R.C.S., 31 Drumsheugh Gardens, Edinburgh	
1897	C.	Berry, Richard J. A., M.D., F.R.C.S.E., Professor of Anatomy in the University of Melbourne, Victoria, Australia	
1904		* Beveridge, Erskine, LL.D., St Leonards Hill, Dunfermline	
1880	C.	Birch, De Burgh, C.B., M.D., Professor of Physiology in the University of Leeds, Lyddon Hall, Leeds 55	
1907		* Black, Frederick Alexander, Solicitor, 59 Academy Street, Inverness	
1884		Black, John S., M.A., LL.D. (CURATOR OF LIBRARY AND MUSEUM), 6 Oxford Terrace, Edinburgh	1891–94. Cur. 1906–
1897		* Blaikie, Walter Biggar, The Loan, Colinton	
1904	C.	* Bles, Edward J., M.A., D.Sc., Elterholm, Cambridge	
1898	C.	* Blyth, Benjamin Hall, M.A., Memb. Inst. C.E., 17 Palmerston Place, Edinburgh 60	
1894		* Bolton, Herbert, M.Sc., F.G.S., F.Z.S., Director of the Bristol Museum and Art Gallery, Bristol	
1872	C.	Bottomley, J. Thomson, M.A., D.Sc., LL.D., F.R.S., F.C.S., 13 University Gardens, Glasgow	

Date of Election.			Service on Council, etc.
1886		Bower, Frederick O., M.A., D.Sc., F.R.S., F.L.S., Regius Professor of Botany in the University of Glasgow, 1 St John's Terrace, Hillhead, Glasgow (VICE-PRESIDENT)	1887–90, 1893–96, 1907–09. V-P 1910–
1884	C.	Bowman, Frederick Hungerford, D.Sc., F.C.S. (Lond. and Berl.), F.I.C., Assoc. Inst. C.E., Assoc. Inst. M.E., M.I.E.E., etc., 4 Albert Square, Manchester	
1901		Bradbury, J. B., M.D., Downing Professor of Medicine, University of Cambridge 65	
1903	C.	* Bradley, O. Charnock, M.D., D.Sc., Principal, Royal Veterinary College, Edinburgh	1907–10.
1886		Bramwell, Byrom, M.D., F.R.C.P.E., 23 Drumsheugh Gardens, Edinburgh	1890–93.
1907		* Bramwell, Edwin, M.B., F.R.C.P.E., F.R.C.P. Lond., 24 Walker Street, Edinburgh	
1912		Bridger, Adolphus Edward, M.D. (Edin.), F.R.C.P. (Edin.), B.Sc. (Paris), B.L. (Paris), Foley Lodge, Langham Street, London, W.	
1895		Bright, Charles, Memb. Inst. C.E., Memb. Inst. E.E., F.R.A.S., F.G.S., Consulting Engineer to the Commonwealth of Australia, The Grange, Leigh, Kent, and Members' Mansions, Victoria Street, London, S.W. 70	
1893		Brock, G. Sandison, M.D., 6 Corso d'Italia, Rome, Italy	
1901	C.	* Brodie, W. Brodie, M.B., Thaxted, Dunmow, Essex	
1907		Brown, Alexander, M.A., B.Sc., Professor of Applied Mathematics, South African College, Cape Town	
1864	C. K. B.	Brown, Alex. Crum, M.A., M.D., D.Sc., F.R.C.P.E., LL.D., F.R.S., Emeritus Professor of Chemistry in the University of Edinburgh, 8 Belgrave Crescent, Edinburgh	1865–68, 1869–72, 1873–75, 1876–78, 1911– Sec. 1879–1905. V-P 1905–11.
1898		* Brown, David, F.C.S., F.I.C., Willowbrae House, Willowbrae Road, Edinburgh 75	
1911		* Brown, David Rainy, Chemical Manufacturer (J. F. Macfarlan & Co.), 1 Glenorchy Terrace, Edinburgh	
1883	C.	Brown, J. J. Graham, M.D., F.R.C.P.E., 3 Chester Street, Edinburgh	
1885	C.	Brown, J. Macdonald, M.D., F.R.C.S., 64 Upper Berkeley Street, Portman Square, London, W.	
1909	C.	* Brownlee, John, M.A., M.D., D.Sc., Ruchill Hospital, Bilsland Drive, Glasgow	
1912		* Bruce, Alexander Ninian, D.Sc., M.D., 8 Ainslie Place, Edinburgh 80	
1906		* Bruce, William Speirs, LL.D., Antarctica, Joppa, Midlothian	1909–12.
1898	K. C.	* Bryce, T. H., M.A., M.D. (Edin.), Professor of Anatomy in the University of Glasgow, 2 The University, Glasgow	1911–
1870	C. K.	Buchanan, John Young, M.A., F.R.S., 26 Norfolk Street, Park Lane, London, W.	1878–81, 1884–86.
1887	C.	Buist, J. B., M.D., F.R.C.P.E., 1 Clifton Terrace, Edinburgh	
1905		Bunting, Thomas Lowe, M.D., 27 Denton Road, Scotswood, Newcastle-on-Tyne 85	
1902		* Burgess, A. G., M.A., Mathematical Master, Edinburgh Ladies' College, 64 Strathearn Road, Edinburgh	
1894	C. K.	* Burgess, James, C.I.E., LL.D., Hon. A.R.I.B.A., F.R.G.S., Hon. M. Imp. Russ. Archæol. Soc., and Amer. Or. Soc., M. Soc. Asiat. de Paris, M.R.A.S., *H.* Corr. M. Batavian Soc. of Arts and Sciences, and Berlin Soc. Anthrop., *H.* Assoc. Finno-Ugrian Soc. (VICE-PRESIDENT), 22 Seton Place, Edinburgh	1895–98, 1899–1902. V-P 1908.
1902		* Burn, Rev. John Henry, B.D., The Parsonage, Ballater	
1887		Burnet, John James, Architect, 18 University Avenue, Hillhead, Glasgow	
1888		Burns, Rev. T., D.D., F.S.A. Scot., Minister of Lady Glenorchy's Parish Church, Croston Lodge, Chalmers Crescent, Edinburgh 90	
1896		* Butters, J. W., M.A., B.Sc., Rector of Ardrossan Academy	
1887	C.	Cadell, Henry Moubray, of Grange, B.Sc., Bo'ness	
1897		* Caird, Robert, LL.D., Shipbuilder, Greenock	
1910		* Calderwood, Rev. Robert Sibbald, Minister of Cambuslang, The Manse, Cambuslang, Lanarkshire	
1893	C.	Calderwood, W. L., Inspector of Salmon Fisheries of Scotland, South Bank, Canaan Lane, Edinburgh 95	
1894		* Cameron, James Angus, M.D., Medical Officer of Health, Firhall, Nairn	

Date of Election.			Service on Council, etc.
1905	C.	Cameron, John, M.D., D.Sc., M.R.C.S. Eng., Anatomy Department, Middlesex Hospital Medical School, London, W.	
1904		* Campbell, Charles Duff, 21 Montague Terrace, Inverleith Row, Edinburgh	
1899	C.	* Carlier, Edmund W. W., M.D., M.Sc., F.E.S., Professor of Physiology, University, Birmingham	
1910		Carnegie, David, Memb. Inst. C.E., Memb. Inst. Mech. E., Memb. I. S. Inst., 33–35 Charterhouse Square, London, E.C. 100	
1905	C.	* Carse, George Alexander, M.A., D.Sc., Lecturer on Natural Philosophy, University of Edinburgh, 3 Middleby Street, Edinburgh	
1901		Carslaw, H. S., M.A., D.Sc., Professor of Mathematics in the University of Sydney, New South Wales	
1905		Carter, Joseph Henry, F.R.C.V.S., Rowley Hall, Burnley, Lancashire	
1898		* Carter, Wm. Allan, Memb. Inst. C.E., 32 Great King Street, Edinburgh (Society's Representative on George Heriot's Trust)	1911–
1898		Carus-Wilson, Cecil, F.R.G.S., F.G.S., 16 Waldegrave Park, Strawberry Hill, Middlesex 105	
1908		Cavanagh, Thomas Francis, M.D., 396 Eccleshall Road, Sheffield	
1882		Cay, W. Dyce, Memb. Inst. C.E., 39 Victoria Street, Westminster, London	
1899		Chatham, James, Actuary, 7 Belgrave Crescent, Edinburgh	
1912		Chaudhuri, Banawari Lal, B.A.(Cal.), B.Sc. (Edin.), Assistant Superintendent, Natural History Section, Indian Museum, 120 Lower Circular Road, Calcutta, India	
1874		Chiene, John, C.B., M.D., LL.D., F.R.C.S.E., Emeritus Professor of Surgery in the University of Edinburgh, Barnton Avenue, Davidson's Mains 110	1884–86, 1904–06.
1891		* Clark, John B., M.A., Head Master of Heriot's Hospital School, Lauriston, Garleffin, Craiglea Drive, Edinburgh	
1911		* Clark, William Inglis, D.Sc., 29 Lauder Road, Edinburgh	
1903		* Clarke, William Eagle, F.L.S., Keeper of the Natural History Collections in the Royal Scottish Museum, Edinburgh, 35 Braid Road, Edinburgh	
1909		Clayton, Thomas Morrison, M.D., D.Hy., B.Sc., D.P.H., Medical Officer of Health, Gateshead, 13 The Crescent, Gateshead-on-Tyne	
1875		Clouston, Sir T. S., M.D., LL.D., F.R.C.P.E., 26 Heriot Row, Edinburgh 115	
1904	C.	Coker, Ernest George, M.A., D.Sc., Professor of Mechanical Engineering and Applied Mechanics, City and Guilds Technical College, Finsbury, Leonard Street, City Road, London, E.C.	
1904		Coles, Alfred Charles, M.D., D.Sc., York House, Poole Road, Bournemouth, W.	
1888	C.	Collie, John Norman, Ph.D., F.R.S., F.C.S., Professor of Organic Chemistry in the University College, Gower Street, London	
1904	C.	* Colquhoun, Walter, M.A., M.B., 18 Walmer Crescent, Ibrox, Glasgow	
1909		* Comrie, Peter, M.A., B.Sc., Head Mathematical Master, Boroughmuir Junior Student Centre, 19 Craighouse Terrace, Edinburgh 120	
1886		Connan, Daniel M., M.A.	
1872		Constable, Archibald, LL.D., 11 Thistle Street, Edinburgh	
1894		Cook, John, M.A., 30 Hermitage Gardens, formerly Principal, Central College, Bangalore, Director of Meteorology in Mysore, and Fellow, University of Madras, India	
1891		* Cooper, Charles A., LL.D., 41 Drumsheugh Gardens, Edinburgh	
1905		* Corrie, David, F.C.S., Nobel's Explosives Company, Polmont Station, Midlothian 125	
1911		* Cowan, Alexander C., Papermaker, Valleyfield House, Penicuik, Midlothian	
1908		Craig, James Ireland, M.A., B.A., Director of the Computation Office, Survey Department, Giza (Branch Office), The White House, Giza, Egypt	
1875		Craig, William, M.D., F.R.C.S.E., Lecturer on Materia Medica to the College of Surgeons, 71 Bruntsfield Place, Edinburgh	
1907		* Cramer, William, Ph.D., Lecturer in Physiological Chemistry in the University of Edinburgh, Physiological Department, The University, Edinburgh	
1903		Crawford, Lawrence, M.A., D.Sc., Professor of Mathematics in the South African College, Cape Town 130	
1887		Crawford, William Caldwell, 1 Lockharton Gardens, Colinton Road, Edinburgh	
1870		Crichton-Browne, Sir Jas., M.D., LL.D., D.Sc., F.R.S., Lord Chancellor's Visitor and Vice-President of the Royal Institution of Great Britain, 72 Queen's Gate, and Royal Courts of Justice, Strand, London	
1886		Croom, Sir John Halliday, M.D., F.R.C.P.E., Professor of Midwifery in the University of Edinburgh, late President, Royal College of Surgeons, Edinburgh, 25 Charlotte Square, Edinburgh	
1898		* Currie, James, M.A. Cantab. (Treasurer), Larkfield, Goldenacre, Edinburgh	Treas. 1906–

Date of Election.			Service on Council, etc.
1904		* Cuthbertson, John, Secretary, West of Scotland Agricultural College, 6 Charles Street, Kilmarnock 135	
1885		Daniell, Alfred, M.A., LL.B., D.Sc., Advocate, The Athenæum Club, Pall Mall, London	
1912		* Darbishire, Arthur Dukinfield, M.A., Lecturer in Genetics at the University of Edinburgh, 63 Frederick Street, Edinburgh	
1884		Davy, R., F.R.C.S. Eng., Consulting Surgeon to Westminster Hospital, Burstone House, Bow, North Devon	
1894		* Denny, Archibald, Cardross Park, Cardross, Dumbartonshire	
1869	C. V. J.	Dewar, Sir James, Kt., M.A., LL.D., D.C.L., D.Sc., F.R.S., V.P.C.S., Jacksonian Professor of Natural and Experimental Philosophy in the University of Cambridge, and Fullerian Professor of Chemistry at the Royal Institution of Great Britain, London 140	1872–74.
1905		* Dewar, James Campbell, C.A., 27 Douglas Crescent, Edinburgh	
1906		* Dewar, Thomas William, M.D., F.R.C.P., Kincairn, Dunblane	
1904		Dickinson, Walter George Burnett, F.R.C.V.S., Boston, Lincolnshire	
1884		Dickson, the Right Hon. Charles Scott, K.C., LL.D., 22 Moray Place, Edinburgh	
1888	C.	Dickson, Henry Newton, M.A., D.Sc., The Lawn, Upper Redlands Road, Reading 145	
1876	C.	Dickson, J. D. Hamilton, M.A., Fellow and Tutor, St Peter's College, Cambridge	
1885	C.	Dixon, James Main, M.A., Litt. Hum. Doctor, Professor of English, University of Southern California, Wesley Avenue, Los Angeles, California, U.S.A.	
1897		* Dobbie, James Bell, F.Z.S., 12 South Inverleith Avenue, Edinburgh	
1904		* Dobbie, James Johnston, M.A., D.Sc., LL.D., F.R.S., Principal of the Government Laboratories, London, 4 Vicarage Gate, Kensington, London, W.	1905–08.
1881	C.	Dobbin, Leonard, Ph.D., Lecturer on Chemistry in the University of Edinburgh, 6 Wilton Road, Edinburgh 150	1904–07.
1867	C.	Donaldson, Sir James, M.A., LL.D., Principal of the University of St Andrews	1870–73.
1896		* Donaldson, William, M.A., Viewpark House, Spylaw Road, Edinburgh	
1905		* Donaldson, Rev. Wm. Galloway, F.R.G.S., F.E.I.S., The Manse, Forfar	
1882		Dott, David B., F.I.C., Memb. Pharm. Soc., Ravenslea, Musselburgh	
1901		* Douglas, Carstairs Cumming, M.D., D.Sc., Professor of Medical Jurisprudence and Hygiene, Anderson's College, Glasgow, 2 Royal Crescent, Glasgow 155	
1866		Douglas, David, 22 Drummond Place, Edinburgh	
1910		* Douglas, Loudon MacQueen, Author and Lecturer, 3 Lauder Road, Edinburgh	
1908	C.	Drinkwater, Harry, M.D., M.R.C.S. (Eng.), F.L.S., Grosvenor Lodge, Wrexham, North Wales	
1901		* Drinkwater, Thomas W., L.R.C.P.E., L.R.C.S.E., Chemical Laboratory, Surgeons' Hall, Edinburgh	
1878		Duncanson, J. J. Kirk, M.D., F.R.C.P.E., 22 Drumsheugh Gardens, Edinburgh 160	
1904		* Dunlop, William Brown, M.A., 4A St Andrew Square, Edinburgh	
1903		* Dunstan, John, M.R.C.V.S., 1 Dean Terrace, Liskeard, Cornwall	
1892	C.	Dunstan, M. J. R., M.A., F.I.C., F.C.S., Principal, South-Eastern Agricultural College, Wye, Kent	
1899		* Duthie, George, M.A., Inspector-General of Education, Salisbury, Rhodesia	
1906	C.	* Dyson, Frank Watson, M.A., F.R.S., Astronomer Royal, Royal Observatory, Greenwich 165	1907–10.
1893		Edington, Alexander, M.D., Howick, Natal	
1904		* Edwards, John, 4 Great Western Terrace, Kelvinside, Glasgow	
1904		* Elder, William, M.D., F.R.C.P.E., 4 John's Place, Leith	
1875		Elliot, Daniel G., American Museum of Natural History, Central Park West, New York, N.Y., U.S.A.	
1906	C.	* Ellis, David, D.Sc., Ph.D., Lecturer in Botany and Bacteriology, Glasgow and West of Scotland Technical College, Glasgow 170	
1897	C.	* Erskine-Murray, James Robert, D.Sc., 77 Kingsfield Road, Watford, Herts	
1884		Evans, William, F.F.A., 38 Morningside Park, Edinburgh	
1879	C. N.	Ewart, James Cossar, M.D., F.R.C.S.E., F.R.S., F.Z.S., Regius Professor of Natural History, University of Edinburgh, Craigybield, Penicuik, Midlothian	1882–85, 1904–07. V-P 1907–12.

Date of Election.			Service on Council, etc.
1902		* Ewen, J. T., B.Sc., Memb. Inst. Mech. E., H.M.I.S., 104 King's Gate, Aberdeen	
1878	C.	Ewing, Sir James Alfred, K.C.B., M.A., B.Sc., LL.D., Memb. Inst. C.E., F.R.S., Director of Naval Education, Admiralty, Froghole, Edenbridge, Kent 175	1888–91.
1900	C.	Eyre, John W. H., M.D., M.S. (Dunelm), D.P.H. (Camb.), Guy's Hospital (Bacteriological Department), London	
1910		* Fairgrieve, Mungo M'Callum, M.A. (Glasg.), M.A. (Cambridge), Master at the Edinburgh Academy, 37 Queen's Crescent, Edinburgh	
1875		Fairley, Thomas, Lecturer on Chemistry, 8 Newton Grove, Leeds	
1907	C.	Falconer, John Downie, M.A., D.Sc., F.G.S., Lecturer on Geography, The University, Glasgow.	
1888	C.	Fawsitt, Charles A., 9 Foremount Terrace, Dowanhill, Glasgow 180	
1883	C.	Felkin, Robert W., M.D., F.R.G.S., Fellow of the Anthropological Society of Berlin, 47 Bassett Road, North Kensington, London, W.	
1899		* Fergus, Andrew Freeland, M.D., 22 Blythswood Square, Glasgow	
1907		* Fergus, Edward Oswald, 12 Clairmont Gardens, Glasgow	
1904		* Ferguson, James Haig, M.D., F.R.C.P.E., F.R.C.S.E., 7 Coates Crescent, Edinburgh	
1888		Ferguson, John, M.A., LL.D., Professor of Chemistry in the University of Glasgow 185	
1898		* Findlay, John R., M.A. Oxon., 27 Drumsheugh Gardens, Edinburgh	
1899		* Finlay, David W., B.A., M.D., LL.D., F.R.C.P., D.P.H., Emeritus Professor of Medicine in the University of Aberdeen, Honorary Physician to His Majesty in Scotland, 23 Dundonald Road, Glasgow, W.	
1911		Fleming, John Arnold, F.C.S., etc., Pottery Manufacturer, Woodburn, Rutherglen, Glasgow	
1906		* Fleming, Robert Alexander, M.A., M.D., F.R.C.P.E., Assistant Physician, Royal Infirmary, 10 Chester Street, Edinburgh	
1900	C. N.	* Flett, John S., M.A., D.Sc., LL.D., Director of the Geological Survey of Scotland, 33 George Square, Edinburgh 190	
1872	C.	Forbes, Professor George, M.A., Memb. Inst. C.E., Memb. Inst. E.E., F.R.S., F.R.A.S., 11 Little College Street, Westminster, S.W.	
1904		Forbes, Norman Hay, F.R.C.S.E., L.R.C.P. Lond., M.R.C.S. Eng., Corres. Memb. Soc. d'Hydrologie médicale de Paris, Druminnor, Church Stretton, Salop	
1892		* Ford, John Simpson, F.C.S., 4 Nile Grove, Edinburgh	
1910		* Fraser, Alexander, Actuary, 17 Eildon Street, Edinburgh	
1858		Fraser, A. Campbell, Fellow of the British Academy, Hon. D.C.L. Oxford, LL.D., Litt.D., Emeritus Professor of Logic and Metaphysics in the University of Edinburgh, 34 Melville Street, Edinburgh 195	1879–82.
1896		* Fraser, John, M.B., F.R.C.P.E., formerly one of H.M. Commissioners in Lunacy for Scotland, 54 Great King Street, Edinburgh	
1867	C. K. B.	Fraser, Sir Thomas R., Kt., M.D., LL.D., Sc.D., F.R.C.P.E., F.R.S., Professor of Materia Medica in the University of Edinburgh, Honorary Physician to the King in Scotland, 13 Drumsheugh Gardens, Edinburgh. (Vice-President)	1870–73, 1877–79, 1883–86, 1894–97. V-P 1911–
1891		* Fullarton, J. H., M.A., D.Sc., 23 Porchester Gardens, London, W.	
1891		* Fulton, T. Wemyss, M.D., Scientific Superintendent, Scottish Fishery Board, 41 Queen's Road, Aberdeen	
1907		* Galbraith, Alexander, Superintendent Engineer, Cunard Line, Liverpool, 93 Trinity Road, Bootle, Liverpool 200	
1888	C.	Galt, Alexander, D.Sc., Keeper of the Technological Department, Royal Scottish Museum, Edinburgh	
1901		Ganguli, Sanjiban, M.A., Principal, Maharaja's College, and Director of Public Instruction, Jaipur State, Jaipur, India	
1899		Gatehouse, T. E., Assoc. Memb. Inst. C.E., Memb. Inst. M.E., Memb. Inst. E.E., Fairfield, 128 Tulse Hill, London, S.W.	
1867		Gayner, Charles, M.D., F.L.S.	
1900		Gayton, William, M.D., M.R.C.P.E., Ravensworth, Regent's Park Road, Finchley, London, N. 205	
1909	C.	* Geddes, Auckland C., M.D., Professor of Anatomy, Royal College of Surgeons in Ireland, Dublin	
1880	C.	Geddes, Patrick, Professor of Botany in University College, Dundee, and Lecturer on Zoology, Ramsay Garden, University Hall, Edinburgh	

Date of Election.			Service on Council, etc.
1861	C. B.	Geikie, Sir Archibald, K.C.B., D.C.L. Oxf., D.Sc., LL.D., Ph.D., Pres. R.S., Foreign Member of the Reale Accad. Lincei, Rome, of the National Acad. of the United States, of the Academies of Stockholm, Christiania, Göttingen, Corresponding Member of the Institute of France and of the Academies of Berlin, Vienna, Munich, Turin, Belgium, Philadelphia, New York, etc., Shepherd's Down, Haslemere, Surrey	1869–72, 1874–76, 1879–82.
1871	C. B.	Geikie, James, LL.D., D.C.L., F.R.S., F.G.S., Professor of Geology in the University of Edinburgh, Kilmorie, Colinton Road, Edinburgh	1882–85, 1888–91, 1897–99. V-P 1892–97. 1900–05.
1909		* Gentle, William, B.Sc., 12 Mayfield Road, Edinburgh 210	
1910	C.	* Gibb, David, M.A., B.Sc., Lecturer in Mathematics, Edinburgh University, 15 South Lauder Road, Edinburgh	
1912	C.	* Gibson, Arnold Hartley, D.Sc., Professor of Engineering, University College, Dundee	
1910		* Gibson, Charles Robert, Lynton, Mansewood, by Pollokshaws	
1881	C.	Gibson, George Alexander, D.Sc., M.D., LL.D., F.R.C.P.E., 3 Drumsheugh Gardens, Edinburgh	1901–04.
1890		* Gibson, George A., M.A., LL.D., Professor of Mathematics in the University of Glasgow, 10 The University, Glasgow 215	1905–08. 1912–
1877	C.	Gibson, John, Ph.D., Professor of Chemistry in the Heriot-Watt College, 16 Woodhall Terrace, Juniper Green	1892–94, 1897–1900.
1911		Gidney, Henry A. J., L.M. and S. Socts. Ap. (Lond.), F.R.C.S. (Edin.), D.P.H. (Camb.), D.O. (Oxford), Army Specialist Public Health, c/o Thomas Cook & Sons, Ludgate Circus, London	
1900		Gilchrist, Douglas A., B.Sc., Professor of Agriculture and Rural Economy, Armstrong College, Newcastle-upon-Tyne	
1880		Gilruth, George Ritchie, Surgeon, 53 Northumberland Street, Edinburgh	
1907		Gilruth, John Anderson, M.R.C.V.S., Professor, University, Melbourne, Australia 220	
1909		* Gladstone, Hugh Steuart, M.A., M.B.O.U., F.Z.S., Capenoch, Thornhill, Dumfriesshire	
1911		Gladstone, Reginald John, M.D., F.R.C.S. (Eng.), Lecturer on Embryology and Senior Demonstrator of Anatomy, Middlesex Hospital, London, 1 Gloucester Gate, Regent Park, London, N.W.	
1898		* Glaister, John, M.D., F.R.F.P.S. Glasgow, D.P.H. Camb., Professor of Forensic Medicine in the University of Glasgow, 3 Newton Place, Glasgow	
1910		Goodall, Joseph Strickland, M.B. (Lond.), M.S.A. (Eng.), Lecturer on Physiology, Middlesex Hospital, London, Annandale Lodge, Vanbrugh Park, Blackheath, London, S.E.	
1901		Goodwillie, James, M.A., B.Sc., Liberton, Edinburgh 225	
1899		* Goodwin, Thomas S., M.B., C.M., F.C.S., 25 Worple Road, Isleworth, and Derwent Lodge, London Road, Spring-grove, Isleworth, Middlesex	
1897		Gordon-Munn, John Gordon, M.D., Heigham Hall, Norwich	
1891		* Graham, Richard D., 11 Strathearn Road, Edinburgh	
1898	C.	* Gray, Albert A., M.D., 4 Clairmont Gardens, Glasgow	
1883	C.	Gray, Andrew, M.A., LL.D., F.R.S., Professor of Natural Philosophy in the University of Glasgow 230	1903–06. V-P 1906–09.
1910		Gray, Bruce M'Gregor, C.E., Assoc. Memb. Inst. C.E., Westbourne Grove, Selby, Yorkshire	
1909	C.	* Gray, James Gordon, D.Sc., Lecturer in Physics in the University of Glasgow, 11 The University, Glasgow	
1910		* Green, Charles Edward, Publisher, Gracemount House, Liberton	
1886		Greenfield, W. S., M.D., F.R.C.P.E., Emeritus Professor of General Pathology in the University of Edinburgh, Kirkbrae, Elie, Fife	
1897		Greenlees, Thomas Duncan, M.D. Edin., Amana, Tulse Hill, London 235	
1905	C.	* Gregory, John Walter, D.Sc., F.R.S., Professor of Geology in the University of Glasgow, 4 Park Quadrant, Glasgow	1908–11.
1906		Greig, Edward David Wilson, M.D., B.Sc., Captain, H.M.'s Indian Medical Service, Byculla Club, Bombay, India	
1905		Greig, Robert Blyth, F.Z.S., Whangara, Wallington, Surrey	
1910		* Grimshaw, Percy Hall, Assistant Keeper, Natural History Department, The Royal Scottish Museum, 49 Comiston Drive, Edinburgh	
1899		* Guest, Edward Graham, M.A., B.Sc., 5 Newbattle Terrace, Edinburgh 240	

Date of Election.			Service on Council, etc.
1907	C.	* Gulliver, Gilbert Henry, B.Sc., A.M.I. Mech. E., Lecturer in Experimental Engineering in the University of Edinburgh, 10 Stanley Street, Portobello	
1911	C.	* Gunn, James Andrew, M.A., M.D., D.Sc., Department of Pharmacology, University Museum, Oxford	
1888	C.	Guppy, Henry Brougham, M.B., Rosario, Salcombe, Devon	
1911		* Guy, William, F.R.C.S., L.R.C.P, L.D.S. Ed., Consulting Dental Surgeon, Edinburgh Royal Infirmary ; Dean, Edinburgh Dental Hospital and School ; Lecturer on Human and Comparative Dental Anatomy and Physiology, 11 Wemyss Place, Edinburgh	
1910	B. C.	Gwynne-Vaughan, D. T., F.L.S., Professor of Botany, Queen's University, Belfast, The Cottage, Balmoral, Belfast 245	
1911		Hall-Edwards, John Francis, L.R.C.P. (Edin.), Hon. F.R.P.S., Senior Medical Officer in charge of X-ray Department, General Hospital, Birmingham, 141A and 141B Great Charles Street (Newhall Street), Birmingham	
1905	B. C.	* Halm, Jacob E., Ph.D., Chief Assistant Astronomer, Royal Observatory, Cape Town, Cape of Good Hope	
1899		Hamilton, Allan M'Lane, M.D., LL.D., 36 East 40th Street, New York	
1876	C.	Hannay, J. Ballantyne, Cove Castle, Loch Long	
1896	C.	* Harris, David Fraser, B.Sc. (Lond.), D.Sc. (Birm.), M.D., F.S.A. Scot., Professor of Physiology in the Dalhousie University, Halifax, Nova Scotia 250	
1888	C.	Hart, D. Berry, M.D., F.R.C.P.E., 5 Randolph Cliff, Edinburgh	
1869		Hartley, Sir Charles A., K.C.M.G., Memb. Inst. C.E., 26 Pall Mall, London	
1877	C.	Hartley, Sir Walter Noel, Kt., D.Sc., F.R.S., F.I.C., Fellow of King's College, London, Savile Club, Piccadilly, W.	
1881		Harvie-Brown, J. A., of Quarter, LL.D., F.Z.S., Dunipace House, Larbert, Stirlingshire	
1880	C.	Haycraft, J. Berry, M.D., D.Sc., Professor of Physiology in the University College of South Wales and Monmouthshire, Cardiff 255	
1892	C.	* Heath, Thomas, B.A., formerly Assistant Astronomer, Royal Observatory, Edinburgh, 11 Cluny Drive, Edinburgh	
1893		Hehir, Patrick, M.D., F.R.C.S.E., M.R.C.S.L., L.R.C.P.E., Surgeon-Captain, Indian Medical Service, Principal Medical Officer, H.H. the Nizam's Army, Hyderabad, Deccan, India	
1890	C.	Helme, T. Arthur, M.D., M.R.C.P.L., M.R.C.S., 3 St Peter's Square, Manchester	
1900		Henderson, John, D.Sc., Assoc. Inst. E.E., Kinnoul, Warwick's Bench Road, Guildford, Surrey	
1908		* Henderson, William Dawson, M.A., B.Sc., Ph.D., Lecturer, Zoological Laboratories, University, Bristol 260	
1890	C.	Hepburn, David, M.D., Professor of Anatomy in the University College of South Wales and Monmouthshire, Cardiff	
1881	C. N.	Herdman, W. A., D.Sc., F.R.S., Past Pres. L.S., Professor of Natural History in the University of Liverpool, Croxteth Lodge, Ullet Road, Liverpool	
1908		* Hewat, Archibald, F.F.A., F.I.A., 13 Eton Terrace, Edinburgh	
1894		Hill, Alfred, M.D., M.R.C.S., F.I.C., Valentine Mount, Freshwater Bay, Isle of Wight	
1902		* Hinxman, Lionel W., B.A., Geological Survey Office, 33 George Square, Edinburgh 265	
1904		Hobday, Frederick T. G., F.R.C.V.S., 6 Berkely Gardens, Kensington, London, W.	
1885		Hodgkinson, W. R., Ph.D., F.I.C., F.C.S., Professor of Chemistry and Physics at the Royal Military Academy and Royal Artillery College, Woolwich, 89 Shooter's Hill Road, Blackheath, Kent	
1911		Holland, William Jacob, LL.D. St Andrews, etc., Director Carnegie Institute, Pittsburg, Pa., 5545 Forbes Street, Pittsburg, Pa.	
1881	C. N.	Horne, John, LL.D., F.R.S., F.G.S., formerly Director of the Geological Survey of Scotland (VICE-PRESIDENT), 12 Keith Crescent, Blackhall, Midlothian	1902–05, 1906–07. V-P 1907–
1896		Horne, J. Fletcher, M.D., F.R.C.S.E., The Poplars, Barnsley 270	
1904	C.	* Horsburgh, Ellice Martin, M.A., B.Sc., Lecturer in Technical Mathematics, University of Edinburgh, 11 Granville Terrace, Edinburgh	
1897		Houston, Alex. Cruikshanks, M.B., C.M., D.Sc., 19 Fairhazel Gardens, South Hampstead, London, N.W.	
1912	C.	* Houstoun, Robert Alexander, M.A., Ph.D., D.Sc., Lecturer in Physical Optics, and Assistant to the Professor of Natural Philosophy, University, Glasgow, 11 Cambridge Drive, Glasgow	

Date of Election.				Service on Council, etc.
1893		Howden, Robert, M.A., M.B., C.M., D.Sc., Professor of Anatomy in the University of Durham, 14 Burdon Terrace, Newcastle-on-Tyne		
1899		Howie, W. Lamond, F.C.S., 26 Neville Court, Abbey Road, Regent's Park, London, N.W.	275	
1883	C.	Hoyle, William Evans, M.A., D.Sc., M.R.C.S., Crowland, Llandaff, Wales		
1910		Hume, William Fraser, D.Sc. (Lond.), Director, Geological Survey of Egypt, Helwân, Egypt		
1886		Hunt, Rev. H. G. Bonavia, Mus.D. Dub., Mus.B. Oxon., The Vicarage, Burgess Hill, Sussex		
1911		Hunter, Gilbert Macintyre, M. Inst. C.E., M.I.E.S, Resident Engineer Nitrate Railways, Iquique, Chile, and Maybole, Ayrshire		
1887	C.	Hunter, James, F.R.C.S.E., F.R.A.S., Rosetta, Liberton, Midlothian	280	
1887	C.	Hunter, William, M.D., M.R.C.P.L. and E., M.R.C.S., 54 Harley Street, London		
1908		Hyslop, Theophilus Bulkeley, M.D., M.R.C.P.E., 5 Portland Place, London, W.		
1882	C.	Inglis, J. W., Memb. Inst. C.E., 26 Pitt Street, Edinburgh		
1912		* Inglis, Robert John Mathieson, Assoc.M.Inst.C.E., Engineer, Northern Division, North British Railway, Tantah, Peebles		
1904	C.	Innes, R. T. A., Director, Government Observatory, Johannesburg, Transvaal	285	
1904		* Ireland, Alexander Scott, S.S.C., 2 Buckingham Terrace, Edinburgh		
1875		Jack, William, M.A., LL.D., Emeritus Professor of Mathematics in the University of Glasgow		1888–91.
1894		Jackson, Sir John, LL.D., 48 Belgrave Square, London		
1889		* James, Alexander, M.D., F.R.C.P.E., 14 Randolph Crescent, Edinburgh		
1901		* Jardine, Robert, M.D., M.R.C.S. Eng., F.R.F.P.S. Glas., 20 Royal Crescent, Glasgow	290	
1912	C.	* Jeffrey, George Rutherford, M.D. (Glasg.), F.R.C.P. (Edin.), etc., Bootham Park Private Mental Hospital, York		
1906		* Jehu, Thomas James, M.A., M.D., F.G.S., Lecturer in Geology, University of St Andrews, St Ronan's, St Andrews		
1900		* Jerdan, David Smiles, M.A., D.Sc., Ph.D., Temora, Colinton, Midlothian		
1895		Johnston, Col. Henry Halcro, C.B., R.A.M.S., D.Sc., M.D., F.L.S., Orphir House, Kirkwall, Orkney		
1903	C.	* Johnston, Thomas Nicol, M.B., C.M., Pogbie, Humbie, East Lothian	295	
1902		Johnstone, George, Lieut. R.N.R., formerly Marine Superintendent, British India Steam Navigation Co., 26 Comiston Drive, Edinburgh		
1874		Jones, Francis, M.Sc., Lecturer on Chemistry, 17 Whalley Road, Whalley Range, Manchester		
1905		Jones, George William, M.A., B.Sc., LL.B., Scottish Tutorial Institute, Edinburgh and Glasgow, 25 North Bridge: Coraldene, Kirk Brae, Liberton, Edinburgh		
1888		Jones, John Alfred, Memb. Inst. C.E., Fellow of the University of Madras, Sanitary Engineer to the Government of Madras, c/o Messrs Parry & Co., 70 Gracechurch Street, London		
1907		* Kemp, John, M.A., Sea Bank School, North Berwick	300	
1912		Kennedy, Robert Foster, M.D. (Queen's Univ., Belfast), M.B., B.Ch. (R.U.I.), Instructor in Neurology, and Chief of the Neurological Clinic, Cornell University, New York, 52 West 53rd Street, New York.		
1909		Kenwood, Henry Richard, M.B., Chadwick Professor of Hygiene in the University of London, 126 Queen's Road, Finsbury Park, London, N.		
1908		* Kerr, Andrew William, F.S.A. Scot., Royal Bank House, St Andrew Square, Edinburgh		
1903	C. N.	* Kerr, John Graham, M.A., F.R.S., Professor of Zoology in the University of Glasgow		1904–07.
1891		Kerr, Joshua Law, M.D., The Chequers, Mittagong, Sydney, Australia	305	
1908		Kidd, Walter Aubrey, M.D., 12 Montpelier Row, Blackheath, London		
1886	C. N.	Kidston, Robert, LL.D., F.R.S., F.G.S. (SECRETARY), 12 Clarendon Place, Stirling		1891–94, 1903–06. Sec. 1909–
1907		* King, Archibald, M.A., B.Sc., formerly Rector of the Academy, Castle Douglas; Junior Inspector of Schools, La Maisonnette, Clarkston, Glasgow		
1880		King, W. F., Lonend, Russell Place, Trinity		
1883		Kinnear, the Right Hon. Lord, P.C., one of the Senators of the College of Justice, 2 Moray Place, Edinburgh	310	
1878		Kintore, the Right Hon. the Earl of, M.A. Cantab., LL.D. Cambridge, Aberdeen and Adelaide, Keith Hall, Inverurie, Aberdeenshire		
1901		* Knight, Rev. G. A. Frank, M.A., St Leonard's United Free Church, Perth		

Date of Election.			Service on Council, etc.
1907		* Knight, James, M.A., D.Sc., F.C.S., F.G.S., Head Master, St James' School, Glasgow, The Shieling, Uddingston, by Glasgow	
1880	C. K.	Knott, C. G., D.Sc., Lecturer on Applied Mathematics in the University of Edinburgh (formerly Professor of Physics, Imperial University, Japan) (Gen. Secretary), 42 Upper Gray Street, Edinburgh	1894–97, 1898–01, 1902–05. Sec. 1905–1912. Gen. Sec. 1912–
1886		Laing, Rev. George P., 17 Buckingham Terrace, Edinburgh 315	
1878	C.	Lang, P. R. Scott, M.A., B.Sc., Professor of Mathematics, University of St Andrews	
1910		* Lauder, Alexander, D.Sc., F.I.C., Lecturer in Agricultural Chemistry, Edinburgh and East of Scotland College of Agriculture, 13 George Square, Edinburgh	
1885	C.	Laurie, A. P., M.A., D.Sc., Principal of the Heriot-Watt College, Edinburgh	1908–11.
1894	C.	* Laurie, Malcolm, B.A., D.Sc., F.L.S., 19 Merchiston Park, Edinburgh	
1910	C.	* Lawson, A. Anstruther, B.Sc., Ph.D., D.Sc., F.L.S., Professor of Botany, University of Sydney, New South Wales, Australia 320	
1905		* Lawson, David, M.A., M.D., L.R.C.P. and S.E., Druimdarroch, Banchory, Kincardineshire	
1910	C.	* Lee, Gabriel W., D.Sc., Palæontologist, Geological Survey of Scotland, 33 George Square, Edinburgh	
1903		* Leighton, Gerald Rowley, M.D., Professor of Pathology and Bacteriology, Royal Veterinary College, Edinburgh, Sunnyside, Russell Place, Edinburgh	
1874	C. K.	Letts, E. A., Ph.D., F.I.C., F.C.S., Professor of Chemistry, Queen's College, Belfast	
1910		Levie, Alexander, F.R.C.V.S., D.V.S.M., Veterinary Surgeon, Lecturer on Veterinary Science, Veterinary Infirmary, 12 Derwent Street, Derby 325	
1905		* Lightbody, Forrest Hay, 56 Queen Street, Edinburgh	
1889		* Lindsay, Rev. James, M.A., D.D., B.Sc., F.R.S.L., F.G.S., M.R.A.S., Corresponding Member of the Royal Academy of Sciences, Letters and Arts, of Padua, Associate of the Philosophical Society of Louvain, Annick Lodge, Irvine	
1912		* Lindsay, John George, M.A., B.Sc. (Edin.), Science Master, Royal High School, 33 Lauriston Gardens, Edinburgh	
1912		* Linlithgow, The Most Honourable the Marquis of, Hopetoun House, South Queensferry	
1903		Liston, William Glen, M.D., Captain, Indian Medical Service, c/o Grindlay, Groom & Co., Bombay, India 330	
1903		* Littlejohn, Henry Harvey, M.A., M.B., B.Sc., F.R.C.S.E., Professor of Forensic Medicine, Dean of the Faculty of Medicine in the University of Edinburgh, 11 Rutland Street, Edinburgh	
1898		* Lothian, Alexander Veitch, M.A., B.Sc., Training College, Cowcaddens, Glasgow	
1884		Low, George M., Actuary, 11 Moray Place, Edinburgh	
1888		Lowe, D. F., M.A., LL.D., formerly Head Master of Heriot's Hospital School, Lauriston, 19 George Square, Edinburgh	
1900		Lusk, Graham, Ph.D., M.A., Professor of Physiology, Cornell University Medical College, New York, N.Y. 335	
1894		* Mabbott, Walter John, M.A., Rector of County High School, Duns, Berwickshire	
1887		M'Aldowie, Alexander M., M.D., Glengarriff, Leckhampton, Cheltenham	
1907		MacAlister, Donald Alexander, A.R.S.M., F.G.S., 26 Thurloe Square, South Kensington, London, S.W.	
1891		Macallan, John, F.I.C., 3 Rutland Terrace, Clontarf, Dublin	
1888	C.	M'Arthur, John, F.C.S., Woodfield, Maplehurst, Horsham, Sussex 340	
1883		M'Bride, P., M.D., F.R.C.P.E., 10 Park Avenue, Harrogate, and Hill House, Withypool, Dunster, Somerset	
1903		* M'Cormick, Sir W. S., M.A., LL.D., Secretary to the Carnegie Trust for the Universities of Scotland, 13 Douglas Crescent, Edinburgh	1910–
1899		* M'Cubbin, James, B.A., Rector of the Burgh Academy, Kilsyth	
1905		* Macdonald, Hector Munro, M.A., F.R.S., Professor of Mathematics, University of Aberdeen, 52 College Bounds, Aberdeen	1908–11.
1894		* Macdonald, James, 2 Garscube Terrace, Edinburgh 345	
1897	C.	* Macdonald, James A., M.A., B.Sc., H.M. Inspector of Schools, Stewarton, Kilmacolm	
1904		* Macdonald, John A., M.A., B.Sc., Johannesburg College, Johannesburg, Transvaal	
1886		Macdonald, the Right Hon. Sir J. H. A. (Lord Kingsburgh) P.C., K.C.B., K.C., LL.D., F.R.S., M.I.E.E., Lord Justice-Clerk, and Lord President of the Second Division of the Court of Session, 15 Abercromby Place, Edinburgh	1889–92.

Date of Election.			Service on Council, etc.
1904		Macdonald, William, B.Sc., M.Sc., Agriculturist, Editor *Transvaal Agricultural Journal*, Department of Agriculture, Pretoria Club, Pretoria, Transvaal	
1886		Macdonald, William J., M.A., 15 Comiston Drive, Edinburgh 350	
1901	C.	* MacDougall, R. Stewart, M.A., D.Sc., Professor of Biology, Royal Veterinary College, Edinburgh, 9 Dryden Place, Edinburgh	
1910		Macewen, Hugh Allan, M.B., Ch.B., D.P.H. (Lond. and Camb.), Local Government Board, Whitehall, London, S.W.	
1888	C.	M'Fadyean, Sir John, M.B., B.Sc., LL.D., Principal, and Professor of Comparative Pathology in the Royal Veterinary College, Camden Town, London	
1878	C.	Macfarlane, Alexander, M.A., D.Sc., LL.D., 317 Victoria Avenue, Chatham, Ontario, Canada	
1885	C.	Macfarlane, J. M., D.Sc., Professor of Botany and Director of the Botanic Garden, University of Pennsylvania, Philadelphia, Pennsylvania, U.S.A. 355	
1897		* MacGillivray, Angus, C.M., M.D., D.Sc., 23 South Tay Street, Dundee	
1878		M'Gowan, George, F.I.C., Ph.D., 21 Montpelier Road, Ealing, Middlesex	
1880	C.	MacGregor, James Gordon, M.A., D.Sc., LL.D., F.R.S., Professor of Natural Philosophy in the University of Edinburgh, 24 Dalrymple Crescent, Edinburgh	1902–04.
1903		* M'Intosh, Donald C., M.A., D.Sc., 3 Glenisla Gardens, Edinburgh	
1911		M'Intosh, John William, A.R.C.V.S., Grasmead, 88 Underhill Road, E. Dulwich, London, S.E. 360	
1869	C. N.	M'Intosh, William Carmichael, M.D., LL.D., F.R.S., F.L.S., Professor of Natural History in the University of St Andrews, 2 Abbotsford Crescent, St Andrews	1885–88.
1895	C.	* Macintyre, John, M.D., 179 Bath Street, Glasgow	
1882		Mackay, John Sturgeon, M.A., LL.D., formerly Mathematical Master in the Edinburgh Academy, 69 Northumberland Street, Edinburgh	1895–98, 1900–03.
1873	C. B.	M'Kendrick, John G., M.D., F.R.C.P.E., LL.D., F.R.S., Emeritus Professor of Physiology in the University of Glasgow, Maxieburn, Stonehaven	1875–78, 1885–88, 1893–94, 1900–02. V-P 1894–1900.
1912	C.	M'Kendrick, Anderson Gray, M.B., Captain Indian Medical Service, Officiating Statistical Officer to the Government of India, Sanitary and Medical Departments, Chillingham, Simla, Punjab, India 365	
1900	C.	* M'Kendrick, John Souttar, M.D., F.R.F.P.S.G., 2 Buckingham Terrace, Glasgow	
1910	C.	* Mackenzie, Alister Thomas, M.A., M.D., D.P.H., Principal, College of Hygiene and Physical Training, Dunfermline	
1911		* M'Kenzie, Kenneth John, M.A., Master of Method to Leith School Board, 17 East Trinity Road, Leith	
1894		* Mackenzie, Robert, M.D., Napier, Nairn	
1904		* Mackenzie, W. Leslie, M.A., M.D., D.P.H., LL.D., Medical Member of the Local Government Board for Scotland, 1 Stirling Road, Trinity 370	
1905		Mackenzie, William Colin, M.D., F.R.C.S., Demonstrator of Anatomy in the University of Melbourne, Elizabeth Street North, Melbourne, Victoria	
1910		* MacKinnon, James, M.A., Ph.D., Professor of Ecclesiastical History, Edinburgh University, 12 Lygon Road, Edinburgh	
1904		* Mackintosh, Donald James, M.V.O., M.B., C.M., LL.D., Supt. Western Infirmary, Glasgow	
1869	C.	Maclagan, R. C., M.D., F.R.C.P.E., 5 Coates Crescent, Edinburgh	
1899		Maclean, Ewan John, M.D., M.R.C.P. Lond., 12 Park Place, Cardiff 375	
1888	C.	Maclean, Magnus, M.A., D.Sc., Memb. Inst. E.E., Professor of Electrical Engineering in the Royal Technical College, 51 Kerrsland Terrace, Hillhead, Glasgow	
1876		Macmillan, John, M.A., D.Sc., M.B., C.M., F.R.C.P.E., F.R.C.S.E., 22 George Square, Edinburgh	
1907		* Macnair, Peter, Curator of the Natural History Collections in the Glasgow Museums, Kelvingrove Museum, Glasgow	
1898	C.	Mahâlanobis, S. C., B.Sc., Professor of Physiology, Presidency College, Calcutta, India	
1908		Mallik, Devendranath, B.A., B.Sc., Professor of Physics and Mathematics, Patna College, Bankipur, Bengal, India 380	
1912		Maloney, William Joseph, M.D.(Edin.), Professor of Neurology at Fordham University, New York City, N.Y., U.S.A.	
1880	C.	Marsden, R. Sydney, M.D., C.M., D.Sc., D.P.H. Hon., L.A.H. Dub., M.R.I.A., F.I.C., F.C.S., Rowallan House, Cearns Road, and Town Hall, Birkenhead	

Date of Election.			Service on Council, etc.
1909	C.	* Marshall, C. R., M.D., M.A., Professor of Materia Medica and Therapeutics, Medical School, Dundee, Arnsheen, Westfield Terrace, West Newport, Fife	
1882	C.	Marshall, D. H., M.A., Professor, Union and Alwington Avenue, Kingston, Ontario, Canada	
1901	C.	* Marshall, F. H. A., M.A., D.Sc., Lecturer on Agricultural Physiology in the University of Cambridge, Christ's College, Cambridge 385	
1888	C. K.	Marshall, Hugh, D.Sc., F.R.S., Professor of Chemistry in University College, Dundee	
1903		Martin, Nicholas Henry, F.L.S., F.C.S., Ravenswood, Low Fell, Gateshead	
1912		* Martin, Sir Thomas Carlaw, LL.D., J.P., Director, Royal Scottish Museum, 4 Gordon Terrace, Edinburgh	
1885	C.	Masson, Orme, D.Sc., F.R.S., Professor of Chemistry in the University of Melbourne	
1898	C. B.	* Masterman, Arthur Thomas, M.A., D.Sc., Inspector of Fisheries, Board of Agriculture, Whitehall, London 390	1902–04.
1911		Mathews, Gregory Macalister, F.L.S., F.Z.S., Langley Mount, Watford, Herts	
1906		* Mathieson, Robert, F.C.S., Rillbank, Innerleithen	
1902		Matthews, Ernest Romney, Assoc. Memb. Inst. C.E., F.G.S., Bessemer Prizeman, Soc. Engineers, Bridlington, Yorkshire	
1901	C.	* Menzies, Alan W. C., M.A., B.Sc., Ph.D., F.C.S., Professor of Chemistry, Oberlin College, Oberlin, Ohio, U.S.A.	
1888		Methven, Cathcart W., Memb. Inst. C.E., F.R.I.B.A., Durban, Natal, S. Africa 395	
1902	C.	Metzler, William H., A.B., Ph.D., Corresponding Fellow of the Royal Society of Canada, Professor of Mathematics, Syracuse University, Syracuse, N.Y.	
1885	C. B.	Mill, Hugh Robert, D.Sc., LL.D., 62 Camden Square, London	
1908		* Miller, Alexander Cameron, M.D., F.S.A. Scot., Craig Linnhe, Fort-William, Inverness-shire	
1910		* Miller, John, M.A., D.Sc., Professor of Mathematics, Royal Technical College, 2 Northbank Terrace, North Kelvinside, Glasgow	
1905		* Miller-Milne, C. H., M.A., Headmaster, Daniel Stewart's College, 4 Campbell Road, Murrayfield, Edinburgh 400	
1909		Mills, Bernard Langley, M.D., F.R.C.S.E., M.R.C.S.L., D.P.H., Lt.-Col. R.A.M.C., formerly Army Specialist in Hygiene, 84 Grange Crescent, Sharrow, Sheffield	
1905		* Milne, Archibald, M.A., B.Sc., Lecturer on Mathematics and Science, Edinburgh Provincial Training College, 108 Comiston Drive, Edinburgh	
1904	C.	* Milne, James Robert, D.Sc., Lecturer on Natural Philosophy, 11 Melville Crescent, Edinburgh	
1886		Milne, William, M.A., B.Sc., 70 Beechgrove Terrace, Aberdeen	
1899		* Milroy, T. H., M.D., B.Sc., Professor of Physiology in Queen's College, Belfast, Thornlee, Malone Park, Belfast 405	
1889	C.	Mitchell, A. Crichton, D.Sc., Formerly Director of Public Instruction in Travancore, India, 103 Trinity Road, Edinburgh	
1897		* Mitchell, George Arthur, M.A., 9 Lowther Terrace, Kelvinside, Glasgow	
1900		* Mitchell, James, M.A., B.Sc., 4 Manse Street, Kilmarnock	
1899		* Mitchell-Thomson, Sir Mitchell, Bart., 6 Charlotte Square, Edinburgh	
1911		Modi, Edalji Manekji, D.Sc., LL.D., Litt.D., F.C.S., etc., Proprietor and Director of Arthur Road Chemical Works, Meher Buildings, Tardeo, Bombay, India 410	
1906	C.	Moffat, Rev. Alexander, M.A., B.Sc., Professor of Physical Science, Christian College, Madras, India	
1890	C.	Mond, R. L., M.A. Cantab., F.C.S., Combe Bank, near Sevenoaks, Kent	
1887	C.	Moos, N. A. F., L.C.E., B.Sc., Professor of Physics, Elphinstone College, and Director of the Government Observatory, Colaba, Bombay	
1896		* Morgan, Alexander, M.A., D.Sc., Principal, Edinburgh Provincial Training College, 1 Midmar Gardens, Edinburgh	
1892	C.	Morrison, J. T., M.A., B.Sc., Professor of Physics and Chemistry, Victoria College, Stellenbosch, Cape Colony 415	
1901		Moses, O. St John, I.M.S., M.D., D.Sc., F.R.C.S., Captain, Professor of Medical Jurisprudence, 26 Park Street, Wellesley, Calcutta, India	
1892	C.	Mossman, Robert C., Superintendent of Publications, Argentine Meteorological Office, Sarmiento 947, Buenos Ayres	
1874	C. K.	Muir, Thomas, C.M.G., M.A., LL.D., F.R.S., Superintendent-General of Education for Cape Colony, Education Office, Cape Town, and Mowbray Hall, Rosebank, Cape Colony	1885–88. V-P 1888–91.
1888	C.	Muirhead, George, Commissioner to His Grace the Duke of Richmond and Gordon, K.G., Speybank, Fochabers	

Date of Election.			Service on Council, etc.
1907		Muirhead, James M. P., J.P., F.R.S.L., F.S.S., Markham's Buildings, Cape Town. 420	
1887		Mukhopâdhyay, Asûtosh, M.A., LL.D., F.R.A.S., M.R.I.A., Professor of Mathematics at the Indian Association for the Cultivation of Science, 77 Russa Road North, Bhowanipore, Calcutta	
1891	C.	* Munro, Robert, M.A., M.D., LL.D., Hon. Memb. R.I.A., Hon. Memb. Royal Society of Antiquaries of Ireland, Elmbank Largs, Ayrshire	1894–97, 1900–03. V-P 1903–08.
1896		* Murray, Alfred A., M.A., LL.B., 20 Warriston Crescent, Edinburgh	
1907	C. N.	* Murray, James, Hill Farm Bungalow, Froxfield, Hants, England	
1877	C. B. N.	Murray, Sir John, K.C.B., LL.D., Ph.D., D.Sc., F.R.S., Knight of the Royal Prussian Order *Pour le Mérite*, Norwegian Order of St Olav, Director of the Challenger Expedition Publications. Office, Villa Medusa, Boswell Road. House, Challenger Lodge, Wardie, and United Service Club 425	1881–84, 1889–91, 1901–03. Sec. 1891–1900. V-P 1884–89, 1903–06.
1907		* Musgrove, James, M.D., F.R.C.S. Edin. and Eng., Bute Professor of Anatomy, University of St Andrews, The Swallowgate, St Andrews	
1902		Mylne, Rev. R. S., M.A., B.C.L. Oxford, F.S.A. Lond., Great Amwell, Herts	
1888		Napier, A. D. Leith, M.D., C.M., M.R.C.P.L., 28 Angas Street, Adelaide, S. Australia	
1897		Nash, Alfred George, B.Sc., F.R.G.S., C.E., Belretiro, Mandeville, Jamaica, W.I.	
1906		* Newington, Frank A., Memb. Inst. C.E., Memb. Inst. E.E., 7 Wester Coates Road, Edinburgh 430	
1898		Newman, Sir George, M.D., D.P.H. Cambridge, Lecturer on Preventive Medicine, St Bartholomew's Hospital, University of London: Dene, Hatch End, Middlesex	
1884		Nicholson, J. Shield, M.A., D.Sc., Professor of Political Economy in the University of Edinburgh, 3 Belford Park, Edinburgh	1885–87, 1892–95, 1897–1900.
1880	C.	Nicol, W. W. J., M.A., D.Sc., 15 Blacket Place, Edinburgh	
1878		Norris, Richard, M.D., M.R.C.S. Eng., 3 Walsall Road, Birchfield, Birmingham	
1906		* O'Connor, Henry, C.E., Assoc. Memb. Inst. C.E., 1 Drummond Place, Edinburgh 435	
1888		Ogilvie, F. Grant, C.B., M.A., B.Sc., LL.D., Director of the Science Museum and the Geological Survey, 15 Evelyn Gardens, London, S.W.	1901–03.
1888		Oliphant, James, M.A., 11 Heathfield Park, Willesden Green, London	
1886		Oliver, James, M.D., F.L.S., Physician to the London Hospital for Women, 18 Gordon Square, London	
1895	C.	Oliver, Sir Thomas, M.D., LL.D., F.R.C.P., Professor of Physiology in the University of Durham, 7 Ellison Place, Newcastle-upon-Tyne	
1884	C. K.	Omond, R. Traill, 3 Church Hill, Edinburgh 440	1901–04.
1908		Page, William Davidge, F.C.S., F.G.S., M. Inst. M.E., 10 Clifton Dale, York	
1905		Pallin, William Alfred, F.R.C.V.S., Major in the Army Veterinary Corps, c/o Messrs Holt & Co., 3 Whitehall Place, London	
1892		Parker, Thomas, Memb. Inst. C.E., Severn House, Iron Bridge, Salop	
1901		* Paterson, David, F.C.S., Lea Bank, Rosslyn, Midlothian	
1886	C.	Paton, D. Noël, M.D., B.Sc., F.R.C.P.E., Professor of Physiology in the University of Glasgow, University, Glasgow 445	1894–97, 1904–06, 1909–12.
1889		* Patrick, David, M.A., LL.D., c/o W. & R. Chambers, 339 High Street, Edinburgh	
1892		* Paulin, Sir David, Actuary, 6 Forres Street, Edinburgh	
1881	C. N.	Peach, Benjamin N., LL.D., F.R.S., F.G.S. (Vice-President), formerly District Superintendent and Acting Palæontologist of the Geological Survey of Scotland, 72 Grange Loan, Edinburgh	1905–08, 1911–1912. V-P 1912–
1907		* Pearce, John Thomson, B.A., B.Sc., School House, Tranent	
1904		* Peck, James Wallace, M.A., Chief Inspector, National Health Insurance, Scotland, 83 Princes Street, Edinburgh 450	
1889		* Peck, William, F.R.A.S., Town's Astronomer, City Observatory, Calton Hill, Edinburgh	
1887	C. B.	Peddie, Wm., D.Sc., Professor of Natural Philosophy in University College, Dundee, Rosemount, Forthill Road, Broughty Ferry	1904–07, 1908–11.

Date of Election.			Service on Council, etc.
1900		Penny, John, M.B., C.M., D.Sc., Great Broughton, near Cockermouth, Cumberland	
1893		Perkin, Arthur George, F.R.S., 8 Montpellier Terrace, Hyde Park, Leeds	
1889		* Philip, Sir R. W., M.A., M.D., F.R.C.P.E., 45 Charlotte Square, Edinburgh 455	
1907	C.	Phillips, Charles E. S., Castle House, Shooter's Hill, Kent	
1905		* Pinkerton, Peter, M.A., D.Sc., Head Mathematical Master, George Watson's College, Edinburgh, 36 Morningside Grove, Edinburgh	
1908	C.	* Pirie, James Hunter Harvey, B.Sc., M.D., F.R.C.P.E., 6 St James Terrace, Hillhead, Glasgow	
1911		* Pirie, James Simpson, Civil Engineer, 28 Scotland Street, Edinburgh	
1906		Pitchford, Herbert Watkins, F.R.C.V.S., Bacteriologist and Analyst, Natal Government, The Laboratory, Pietermaritzburg, Natal 460	
1886		Pollock, Charles Frederick, M.D., F.R.C.S.E., 1 Buckingham Terrace, Hillhead, Glasgow	
1888		Prain, Sir David, Lt.-Col., Indian Medical Service (Retired), C.M.G., C.I.E., M.A., M.B., LL.D., F.L.S., F.R.S., For. Memb. K. Svensk. Vetensk. Akad.; Hon. Memb. Soc. Lett. ed Arti d. Zelanti, Acireale; Pharm. Soc. Gt. Britain; Corr. Memb. K. Bayer Akad. Wiss., etc.; Director, Royal Botanic Gardens, Kew, Surrey	
1902		* Preller, Charles Du Riche, M.A., Ph.D., Assoc. Memb. Inst. C.E., 61 Melville Street, Edinburgh	
1892		* Pressland, Arthur J., M.A. Camb., Edinburgh Academy	
1875	C.	Prevost, E. W., Ph.D., Weston, Ross, Herefordshire 465	
1908		* Pringle, George Cossar, M.A., Rector of Peebles Burgh and County High School, Bloomfield, Peebles	
1903		* Pullar, Laurence, The Lea, Bridge of Allan	
1911		Purdy, John Smith, M.D., C.M. (Aberd.), D.P.H. (Camb.), F.R.G.S., Chief Health Officer for Tasmania, Islington, Hobart, Tasmania	
1898		* Purves, John Archibald, D.Sc., 13 Albany Street, Edinburgh	
1897		* Rainy, Harry, M.A., M.B., C.M., F.R.C.P. Ed., 16 Great Stuart Street, Edinburgh 470	
1899		* Ramage, Alexander G., 8 Western Terrace, Murrayfield, Edinburgh	
1884		Ramsay, E. Peirson, M.R.I.A., F.L.S., C.M.Z.S., F.R.G.S., F.G.S., Fellow of the Imperial and Royal Zoological and Botanical Society of Vienna, Curator of Australian Museum, Sydney, N.S.W.	
1911		* Rankin, Adam A., Vice-President, British Astronomical Association, West of Scotland Branch, 324 Crow Road, Broomhill, Glasgow, W.	
1891		* Rankine, John, K.C., M.A., LL.D., Professor of the Law of Scotland in the University of Edinburgh, 23 Ainslie Place, Edinburgh	
1904		Ratcliffe, Joseph Riley, M.B., C.M., c/o The Librarian, The University, Birmingham 475	
1900		Raw, Nathan, M.D., M.R.C.P. (London), B.S., F.R.C.S., D.P.H., 66 Rodney Street, Liverpool	
1883	C	Readman, J. B., D.Sc., F.C.S., Belmont, Hereford	
1889		Redwood, Sir Boverton, Bt., D.Sc. (Hon.), F.I.C., F.C.S., Assoc. Inst. C.E., Wadham Lodge, Wadham Gardens, London	
1902		Rees-Roberts, John Vernon, M.D., D.Sc., D.P.H., Barrister-at-Law, National Liberal Club, Whitehall Place, London	
1902		Reid, George Archdall O'Brien, M.B., C.M., 9 Victoria Road South, Southsea, Hants 480	
1908	C.	* Rennie, John, D.Sc., Lecturer on Parasitology, and Assistant to the Professor of Natural History, University of Aberdeen, 60 Desswood Place, Aberdeen	
1908		Richardson, Linsdall, F.L.S., F.G.S., Organising Inspector of Technical Education for the Gloucestershire Education Committee, 10 Oxford Parade, Cheltenham	
1875		Richardson, Ralph, W.S., 10 Magdala Place, Edinburgh	
1906	C.	* Ritchie, William Thomas, M.D., F.R.C.P.E., 9 Atholl Place, Edinburgh	
1898	C.	Roberts, Alexander William, D.Sc., F.R.A.S., Lovedale, South Africa 485	
1880		Roberts, D. Lloyd, M.D., F.R.C.P.L., 23 St John Street, Manchester	
1900		* Robertson, Joseph M'Gregor, M.B., C.M., 26 Buckingham Terrace, Glasgow	
1896		* Robertson, Robert, M.A., 25 Mansionhouse Road, Edinburgh	
1902	C.	* Robertson, Robert A., M.A., B.Sc., Lecturer on Botany in the University of St Andrews	
1896	C	* Robertson, W. G. Aitchison, D.Sc., M.D., F.R.C.P.E., 2 Mayfield Gardens, Edinburgh 490	
1910		* Robinson, Arthur, M.D., M.R.C.S., Professor of Anatomy, University of Edinburgh, 35 Coates Gardens, Edinburgh (SECRETARY)	1910–1912. Sec. 1912–

Date of Election.			Service on Council, etc.
1881		Rosebery, the Right Hon. the Earl of, K.G., K.T., LL.D., D.C.L., F.R.S., Dalmeny Park, Edinburgh	
1909	C.	* Ross, Alex. David, M.A., D.Sc., F.R.A.S., Lecturer in Natural Philosophy in the University of Glasgow, 7 Queen's Terrace, Glasgow	
1906		* Russell, Alexander Durie, B.Sc., Mathematical Master, Falkirk High School, Dunaura, Heugh Street, Falkirk	
1902	C.	* Russell, James, 22 Glenorchy Terrace, Edinburgh 495	
1880		Russell, Sir James A., M.A., B.Sc., M.B., F.R.C.P.E., LL.D., Woodville, Canaan Lane, Edinburgh	
1904		Sachs, Edwin O., Architect, Chairman of the British Fire Prevention Committee, Vice-President of the International Fire Service Council, 8 Waterloo Place, Pall Mall, London, S.W.	
1906		Saleeby, Caleb William, M.D., 13 Greville Place, London	
1912		* Sampson, Ralph Allen, M.A., D.Sc., D.C.L., F.R.S., Astronomer Royal for Scotland, Professor of Astronomy, University, Edinburgh, Royal Observatory, Edinburgh	
1903		* Samuel, John S., 8 Park Avenue, Glasgow 500	
1903		* Sarolea, Charles, Ph.D., D.Litt., Lecturer on French Language, Literature, and Romance Philology, University of Edinburgh, 21 Royal Terrace, Edinburgh	
1891		Sawyer, Sir James, Kt., M.D., F.R.C.P., F.S.A., J.P., Consulting Physician to the Queen's Hospital, 31 Temple Row, Birmingham	
1900	C.	* Schäfer, Edward Albert, M.R.C.S., LL.D., F.R.S., Professor of Physiology in the University of Edinburgh	1900–03, 1906–09.
1885	C.	Scott, Alexander, M.A., D.Sc., F.R.S., 34 Upper Hamilton Terrace, London, N.W.	
1880		Scott, J. H., M.B., C.M., M.R.C.S., Professor of Anatomy in the University of Otago, New Zealand 505	
1905		Scougal, A. E., M.A., LL.D., formerly H.M. Senior Chief Inspector of Schools and Inspector of Training Colleges, 1 Wester Coates Avenue, Edinburgh	
1902		Senn, Nicholas, M.D., LL.D., Professor of Surgery, Rush Medical College, Chicago, U.S.A.	
1871		Simpson, Sir A. R., M.D., Emeritus Professor of Midwifery in the University of Edinburgh, 52 Queen Street, Edinburgh	
1908		* Simpson, George Freeland Barbour, M.D., F.R.C.P.E., F.R.C.S.E., 43 Manor Place, Edinburgh	
1900	C.	* Simpson, James Young, M.A., D.Sc., Professor of Natural Science in the New College, Edinburgh, 25 Chester Street, Edinburgh 510	
1911	C.	Simpson, Sutherland, M.D., D.Sc. (Edin.), Professor of Physiology, Medical College, Cornell University, Ithaca, N.Y., U.S.A., 118 Eddy Street, Ithaca, N.Y., U.S.A.	
1900		Sinhjee, Sir Bhagvat, G.C.I.E., M.D., LL.D. Edin., H.H. the Thakur Sahib of Gondal, Gondal, Kathiawar, Bombay, India	
1903		* Skinner, Robert Taylor, M.A., Governor and Head Master, Donaldson's Hospital, Edinburgh	
1901		* Smart, Edward, B.A., B.Sc., Tillyloss, Tullylumb Terrace, Perth	
1891	C. K.	* Smith, Alexander, B.Sc., Ph.D., Department of Chemistry, Columbia University, New York, N.Y., U.S.A. 515	
1882	C.	Smith, C. Michie, B.Sc., F.R.A.S., formerly Director of the Kodaikânal and Madras Observatories, Winsford, Kodaikânal, South India	
1885		Smith, George, F.C.S., 5 Rosehall Terrace, Falkirk	
1911		* Smith, Stephen, B.Sc., Goldsmith, 12 Murrayfield Avenue, Edinburgh	
1907	C.	Smith, William Ramsay, D.Sc., M.B., C.M., Permanent Head of the Health Department, South Australia, Belair, South Australia	
1880		Smith, William Robert, M.D., D.Sc., LL.D., Professor of Forensic Medicine and Toxicology in King's College, University of London, Principal of the Royal Institute of Public Health, London 520	
1899		Snell, Ernest Hugh, M.D., B.Sc., D.P.H. Camb., Coventry	
1880		Sollas, W. J., M.A., D.Sc., LL.D., F.R.S., Fellow of University College, Oxford, and Professor of Geology and Palæontology in the University of Oxford	
1910		* Somerville, Robert, B.Sc., Science Master, High School, Dunfermline, 38 Cameron Street, Dunfermline	
1889	C.	Somerville, Wm., M.A., D.Sc., D.Oec., Sibthorpian Professor of Rural Economy and Fellow of St John's College in the University of Oxford, 121 Banbury Road, Oxford	
1911	C.	* Sommerville, Duncan M'Laren Young, M.A., D.Sc., Lecturer in Mathematics and in Applied Mathematics, University of St Andrews 525	
1882		Sorley, James, 82 Onslow Gardens, London	
1896		* Spence, Frank, M.A., B.Sc., 25 Craiglea Drive, Edinburgh	
1874	C.	Sprague, T. B., M.A., LL.D., Actuary, 29 Buckingham Terrace, Edinburgh	1885–87.

Date of Election.			Service on Council, etc.
1906		Squance, Thomas Coke, M.D., Physician and Pathologist in the Sunderland Infirmary, President Sunderland Antiquarian Society, Sunderland Naturalists' Association, 15 Grange Crescent, Sunderland	
1891		* Stanfield, Richard, Professor of Mechanics and Engineering in the Heriot-Watt College, Edinburgh 530	
1912	C.	Stephenson, John, M.B., D.Sc. (Lond.), Indian Medical Service, Professor of Biology, Government College, Lahore, India.	
1910		* Stephenson, Thomas, F.C.S., Editor of the *Prescriber*, Examiner to the Pharmaceutical Society, 9 Woodburn Terrace, Edinburgh	
1886	C.	Stevenson, Charles A., B.Sc., Memb. Inst. C.E., 28 Douglas Crescent, Edinburgh	
1884		Stevenson, David Alan, B.Sc., Memb. Inst. C.E., 84 George Street, Edinburgh	
1888	C.	Stewart, Charles Hunter, D.Sc., M.B., C.M., Professor of Public Health in the University of Edinburgh, Usher Institute of Public Health, Warrender Park Road, Edinburgh 535	
1902		* Stockdale, Herbert Fitton, Director of the Royal Technical College, Glasgow, Clairinch, Upper Helensburgh, Dumbartonshire	
1889		* Stockman, Ralph, M.D., F.R.C.P.E., Professor of Materia Medica and Therapeutics in the University of Glasgow	1903–05.
1906		Story, Fraser, Lecturer in Forestry, University College, Bangor, North Wales	
1907		* Strong, John, M.A., Rector of Montrose Academy, Linksgate, Montrose	
1903		Sutherland, David W., M.D., M.R.C.P. Lond., Captain, Indian Medical Service, Professor of Pathology and Materia Medica, Medical College, Lahore, India 540	
1905		Swithinbank, Harold William, Denham Court, Denham, Bucks	
1912		* Syme, William Smith, M.D. (Edin.), 10 India Street, Glasgow	
1885	C.	Symington, Johnson, M.D., F.R.C.S.E., F.R.S., Professor of Anatomy in Queen's College, Belfast	1892
1904		* Tait, John W., B.Sc., Rector of Leith Academy, 18 Netherby Road, Leith	
1898	C.	Tait, William Archer, D.Sc., Memb. Inst. C.E., 38 George Square, Edinburgh 545	
1895		Talmage, James Edward, D.Sc., Ph.D., F.R.M.S., F.G.S., Professor of Geology, University of Utah, Salt Lake City, Utah, U.S.A.	
1890	C.	Tanakadate, Aikitu, Professor of Natural Philosophy in the Imperial University of Japan, Tokyo, Japan	
1870		Tatlock, Robert R., F.C.S., City Analyst's Office, 156 Bath Street, Glasgow	
1899		* Taylor, James, M.A., Mathematical Master in the Edinburgh Academy	
1892		Thackwell, J. B., M.B., C.M., 423A Battersea Park Road, London, S.W. 550	
1885	C.	Thompson, D'Arcy W., C.B., B.A., F.L.S., Professor of Natural History in University College, Dundee	1892–95, 1896–99, 1907–10, 1912–
1907		* Thompson, John Hannay, M.Sc. (Durh.), M. Inst. C.E., M. Inst. Mech. E., Engineer to the Dundee Harbour Trust, Earlville, Broughty Ferry	
1905		* Thoms, Alexander, 7 Playfair Terrace, St Andrews	
1887		Thomson, Andrew, M.A., D.Sc., F.I.C., Rector, Perth Academy, Ardenlea, Pitcullen, Perth	
1911		* Thomson, Frank Wyville, M.A., M.B., C.M., D.P.H., D.T.M., Lt.-Col. I.M.S. (Retired), Bonsyde, Linlithgow 555	
1896		* Thomson, George Ritchie, M.B., C.M., General Hospital, Johannesburg, Transvaal	
1903		Thomson, George S., F.C.S., Ferma Albion, Marculesci, Roumania	
1906		* Thomson, Gilbert, M.Inst.C.E., 164 Bath Street, Glasgow	
1887	C.	Thomson, J. Arthur, M.A., Regius Professor of Natural History in the University of Aberdeen	1906–08.
1906	C.	Thomson, James Stuart, F.L.S., Zoological Department, University, Manchester 560	
1880		Thomson, John Millar, LL.D., F.R.S., Professor of Chemistry in King's College, London, 18 Lansdowne Road, London, W.	
1899		* Thomson, R. Tatlock, F.C.S., 156 Bath Street, Glasgow	
1912	C.	Thomson, Robert Black, M.B., Edin., Professor of Anatomy, South African College, Cape Town	
1870		Thomson, Spencer C., Actuary, 10 Eglinton Crescent, Edinburgh	
1882		Thomson, Wm., M.A., B.Sc., LL.D., Registrar, University of the Cape of Good Hope, University Buildings, Cape Town 565	
1876	C.	Thomson, William, Royal Institution, Manchester	
1911		* Tosh, James Ramsay, M.A., D.Sc. (St Ands.), Thursday Island, Queensland, Australia	

Date of Election.			Service on Council, etc.
1874		Tuke, Sir J. Batty, M.D., D.Sc., LL.D., F.R.C.P.E., formerly M.P. for the Universities of Edinburgh and St Andrews, 20 Charlotte Square, Edinburgh	1887–90, 1893–95, 1897-1900.
1888		Turnbull, Andrew H., Actuary, The Elms, Whitehouse Loan	
1905		* Turner, Arthur Logan, M.D., F.R.C.S.E., 27 Walker Street, Edinburgh 570	
1906	C.	* Turner, Dawson F. D., B.A., M.D., F.R.C.P.E., M.R.C.P. Lond., Lecturer on Physics, Surgeons' Hall, and formerly Physician in charge of Electrical Department, Royal Infirmary, Edinburgh, 37 George Square, Edinburgh	
1861	K. N. C.	Turner, Sir William, K.C.B., M.B., F.R.C.S.L. and E., LL.D., D.C.L., D.Sc., F.R.S., Knight of the Royal Prussian Order *Pour le Mérite*, Principal and Vice-Chancellor of the University of Edinburgh (PRESIDENT), 6 Eton Terrace, Edinburgh	1866–68, 1895–97. Sec. 1869–91. V-P 1891–95, 1897–1903. P. 1908–
1895		Turton, Albert H., M.I.M.M., 18 Harrow Road, Bowenbrook, Birmingham	
1898	C.	* Tweedie, Charles, M.A., B.Sc., Lecturer on Mathematics in the University of Edinburgh, Duns, Berwickshire	1907–10.
1889		Underhill, T. Edgar, M.D., F.R.C.S.E., Dunedin, Barnt Green, Worcestershire 575	
1910		Vincent, Swale, M.D. Lond, D.Sc. Edin., etc., Professor of Physiology, University of Manitoba. Winnipeg, Canada	
1911	C.	* Walker, Henry, M.A., D.Sc., Teacher, 18 Station Road, Dalbeattie	
1891	C. B.	* Walker, James, D.Sc., Ph.D., LL.D., F.R.S., Professor of Chemistry in the University of Edinburgh, 5 Wester Coates Road, Edinburgh	1903–05, 1910–
1873	C.	Walker, Robert, M.A., LL.D., University, Aberdeen	
1902		* Wallace, Alexander G., M.A., 56 Fonthill Road, Aberdeen 580	
1886	C.	Wallace, R., F.L.S., Professor of Agriculture and Rural Economy in the University of Edinburgh	
1898		Wallace, Wm., M.A., Belvedere, Alberta, Canada	
1891		* Walmsley, R. Mullineux, D.Sc., Principal of the Northampton Institute, Clerkenwell, London	
1907		Waters, E. Wynston, Medical Officer, H.B.M. Administration, E. Africa, Malindi, British East Africa Protectorate, *via* Mombasa	
1901	C.	* Waterston, David, M.A., M.D., F.R.C.S.E., Professor of Anatomy, King's College, London 585	
1904		* Watson, Charles B. Boog, Huntly Lodge, 1 Napier Road, Edinburgh	
1911		* Watson, James A. S., B.Sc., etc., Assistant in Agriculture, University of Edinburgh, Downieken, Dundee	
1900		* Watson, Thomas P., M.A., B.Sc., Principal, Keighley Institute, Keighley	
1910		* Watson, William John, M.A., LL.D. Aberdeen, B.A. Oxon., Rector of the Royal High School, Edinburgh, 17 Merchiston Avenue, Edinburgh	
1907		* Watt, Andrew, M.A., Secretary to the Scottish Meteorological Society, 6 Woodburn Terrace, Edinburgh 590	1912–
1911		Watt, James, W.S., F.F.A., 24 Rothesay Terrace, Edinburgh	
1911		* Watt, Rev. Lauchlan Maclean, B.D., Minister of St. Stephen's Parish, 7 Royal Circus, Edinburgh	
1896		Webster, John Clarence, B.A., M.D., F.R.C.P.E., Professor of Obstetrics and Gynæcology, Rush Medical College, Chicago, 706 Reliance Buildings, 100 State Street, Chicago	
1907	B. C.	* Wedderburn, Ernest Maclagan, M.A., LL.B., 7 Dean Park Crescent, Edinburgh	
1903	C.	* Wedderburn, J. H. Maclagan, M.A., D.Sc., 95 Mercer Street, Princeton, N.J., U.S.A. 595	
1904		Wedderspoon, William Gibson, M.A., LL.D., Indian Educational Service, Senior Inspector of Schools, Burma, The Education Office, Rangoon, Burma	
1896		Wenley, Robert Mark, M.A., D.Sc., D.Phil., Litt.D., LL.D., Professor of Philosophy in the University of Michigan, Ann Arbor, Michigan, U.S.A.	
1909	C.	* Westergaard, Reginald Ludovic Andreas Emil, Professor of Technical Mycology, Heriot-Watt College, Ashestiel, Lasswade Road, Liberton, Edinburgh	
1896	C.	White, Philip J., M.B., Professor of Zoology in University College, Bangor, North Wales	
1890		White, Sir William Henry, K.C.B., Sc.D., D.Sc., D.Eng. "John Fritz" Medallist, LL.D., F.R.S., formerly Assistant Controller of the Navy, and Director of Naval Construction, Cedarcroft, Putney Heath, London 600	
1881		Whitehead, Walter, F.R.C.S.E., formerly Professor of Clinical Surgery, Owens College and Victoria University, Birchfield, 235 Wilmslow Road, Manchester	

Date of Election.			Service on Council, etc.
1911		* Whittaker, Charles Richard, F.R.C.S. (Edin.), F.S.A. (Scot.), Lynwood, Hatton Place, Edinburgh	
1912		* Whittaker, Edmund Taylor, Sc.D., F.R.S., Professor of Mathematics in the University of Edinburgh	1912–
1879		Will, John Charles Ogilvie, of Newton of Pitfodels, M.D., 17 Bon-Accord Square, Aberdeen	
1908		* Williamson, Henry Charles, M.A., D.Sc., Naturalist to the Fishery Board for Scotland, Marine Laboratory, Aberdeen 605	
1910	C.	* Williamson, William, 9 Plewlands Terrace, Edinburgh	
1900		Wilson, Alfred C., F.C.S., Voewood Croft, Stockton-on-Tees	
1911		* Wilson, Andrew, Assoc. M. Inst. C.E., 51 Queen Street, Edinburgh	
1902		* Wilson, Charles T. R., M.A., F.R.S., 108 Huntingdon Road, Cambridge, Sidney Sussex College, Cambridge	
1895		Wilson-Barker, David, R.N.R., F.R.G.S., Captain-Superintendent Thames Nautical Training College, H.M.S. "Worcester," off Greenhithe, Kent 610	
1882		Wilson, George, M.A., M.D., LL.D.	
1891		* Wilson, John Hardie, D.Sc., University of St Andrews, 39 South Street, St Andrews	
1902		Wilson, William Wright, F.R.C.S.E., M.R.C.S. Eng., Cottesbrook House, Acock's Green, Birmingham	
1908		* Wood, Thomas, M.D., Eastwood, 182 Ferry Road, Bonnington, Leith	
1886	C.	Woodhead, German Sims, M.D., F.R.C.P.E., Professor of Pathology in the University of Cambridge 615	1887–90.
1884		Woods, G. A., M.R.C.S., 1 Hammelton Road, Bromley, Kent	
1911		* Wrigley, Ruric Whitehead, B.A. (Cantab.), Assistant Astronomer, Royal Observatory, Edinburgh	
1890		* Wright, Johnstone Christie, Conservative Club, Edinburgh.	
1896		* Wright, Sir Robert Patrick, Chairman of the Board of Agriculture for Scotland, Corsewall, Colinton, Midlothian	
1882		Young, Frank W., F.C.S., H.M. Inspector of Science and Art Schools, 32 Buckingham Terrace, Botanic Gardens, Glasgow 620	
1892		Young, George, Ph.D., "Bradda," Church Crescent, Church End, Finchley, London, N.	
1896	C.	* Young, James Buchanan, M.B., D.Sc., Dalveen, Braeside, Liberton	
1904		Young, R. B., M.A., D.Sc., F.G.S., Professor of Geology and Mineralogy in the South African School of Mines and Technology, Johannesburg, Transvaal 623	

LIST OF HONORARY FELLOWS OF THE SOCIETY

At 1st January 1913.

HIS MOST GRACIOUS MAJESTY THE KING.

Foreigners (limited to thirty-six by Law X.).

Elected.
1897 Émile Hilaire Amagat, Membre de l'Institut, St Satur, Cher, France.
1900 Georg F. J. A. Auwers, Grosslichserfelde u. Bellevue-Strasse 55, Berlin, Germany
1900 Adolf Ritter von Baeyer, Universität, München, Germany.
1905 Waldemar Christofer Brögger, K. Frederiks Universitet, Christiania, Norway.
1905 Moritz Cantor, Gaisbergstrasse 15, Heidelberg, Germany.
1902 Jean Gaston Darboux, Secrétariat de l'Institut, Paris, France.
1910 Hugo de Vries, Universiteit, Amsterdam, Holland.
1905 Paul Ehrlich, K. Institut für Experimentelle Therapie, Sandhofstrasse 44, Frankfurt-a.-M. Germany.
1908 Emil Fischer, Universität, Berlin, Germany.
1910 Karl F. von Goebel, Universität, München, Germany.
1905 Paul Heinrich Groth, Universität, München, Germany.
1888 Ernst Haeckel, Universität, Jena, Germany.
1883 Julius Hann, Universität, Wien, Austria.
1908 George William Hill, West Nyack, New York, U.S.A.
1910 Jacobus Cornelius Kapteyn, Universiteit, Groningen, Holland.
1897 Gabriel Lippmann, Université, Paris, France.
1895 Carl Menger, Universität, Wien, Austria.
1910 Élie Metchnikoff, Institut Pasteur, Paris, France.
1910 Albert Abraham Michelson, University, Chicago, U.S.A.

Elected
1897 Fridtjof Nansen, K. Frederiks Universitet, Christiania, Norway.
1908 Henry Fairfield Osborn, Columbia University, New York, N.Y., U.S.A.
1910 Wilhelm Ostwald, Universität, Leipzig, Germany.
1908 Ivan Petrovitch Pawlov, Wedenskaja Strasse 4, St Petersburg, Russia.
1910 Frederick Ward Putnam, Peabody Museum of Harvard University, Cambridge, Mass., U.S.A.
1889 Georg Hermann Quincke, Bergstrasse 41, Heidelberg, Germany.
1908 Magnus Gustaf Retzius, Högskolan, Stockholm, Sweden.
1908 Augusto Righi, Regia Università, Bologna, Italy.
1905 Eduard Suess, Afrikanergasse 9, Wien 11/2, Austria.
1908 Louis Joseph Troost, Université, Paris, France.
1905 Wilhelm Waldeyer, Universität, Berlin, Germany.
1910 August F. L. Weismann, Universität, Freiburg-im-Breisgau, Germany.
1905 Wilhelm Wundt, Universität, Leipzig, Germany.

Total, 32.

British Subjects (limited to twenty by Law X.).

1889 Sir Robert Stawell Ball, Kt., Hon., M.A. (Cantab.), LL.D., F.R.S., M.R.I.A., Lowndean Professor of Astronomy in the University of Cambridge, Observatory, Cambridge.
1892 Colonel Alexander Ross Clarke, C.B., R.E., F.R.S., Strathmore, Reigate, Surrey.
1900 Sir David Ferrier, Kt., M.A, M.D.., LL.D., F.R.S., Emer. Professor of Neuro-Pathology, King's College, London, 34 Cavendish Square, London, W.
1900 Andrew Russell Forsyth, M.A., D.Sc., F.R.S., Formerly Sadlerian Professor of Pure Mathematics in the University of Cambridge, Trinity College, Cambridge.
1910 James George Frazer, D.C.L., LL.D., Litt.D., F.B.A., Fellow of Trinity College, Cambridge, Professor of Social Anthropology in the University of Liverpool, Trinity College, Cambridge.
1892 Sir David Gill, K.C.B., LL.D., F.R.S., formerly His Majesty's Astronomer at the Cape of Good Hope, 34 De Vere Gardens, Kensington, London, W.
1895 Albert C. L. G. Günther, M.A., M.D., Ph.D., F.R.S., 2 Lichfield Road, Kew Gardens, Surrey.
1908 Sir Alexander B. W. Kennedy, Kt., LL.D., F.R.S., Past Pres. Inst. C.E., 1 Queen Anne Street, Cavendish Square, London, W.
1908 Sir Edwin Ray Lankester, K.C.B., LL.D., F.R.S., 29 Thurloe Place, S. Kensington, London, S.W.
1910 Sir Joseph Larmor, Kt., M.A., D.Sc., LL.D., D.C.L., F.R.S., M.P. University of Cambridge since 1911, Lucasian Professor of Mathematics in the University of Cambridge, Secretary of the Royal Society, St John's College, Cambridge.
1900 Archibald Liversidge, LL.D., F.R.S., Em.-Professor of Chemistry in the University of Sydney, Hornton Cottage, Hornton Street, Kensington, London, W.
1908 Sir James A. H. Murray, LL.D., D.C.L., Editor of the Oxford English Dictionary, Oxford.
1905 Sir William Ramsay, K.C.B., LL.D., F.R.S., formerly Professor of Chemistry in the University College, London, 19 Chester Terrace, Regent's Park, London, N.W.
1886 The Rt. Hon. Lord Rayleigh, O.M., P.C., J.P., D.C.L., LL.D., D.Sc. Dub., F.R.S., Corresp. Mem. Inst. of France, Terling Place, Witham, Essex.
1908 Charles Scott Sherrington, M.A., M.D., LL.D., F.R.S., Holt Professor of Physiology in the University of Liverpool, 16 Grove Park, Liverpool.
1905 Sir Joseph John Thomson, D.Sc., LL.D., F.R.S., Cavendish Professor of Experimental Physics, University of Cambridge, Trinity College, Cambridge.
1900 Sir Thomas Edward Thorpe, Kt., C.B., D.Sc., LL.D., F.R.S., formerly Principal of the Government Laboratories, Imperial College of Science and Technology, South Kensington, London, S.W., Whinfield, Salcombe, South Devon.
1910 Alfred Russel Wallace, O.M., LL.D., D.C.L., F.R.S., Old Orchard, Broadstone, Wimborne, Dorset.

Total, 18.

ORDINARY FELLOWS OF THE SOCIETY ELECTED

During Session 1911–12.

(Arranged according to the date of their election.)

20*th November* 1911.

Robert Alexander Houstoun, M.A., Ph.D., D.Sc.

4th December 1911.

Adolphus Edward Bridger, M.D., F.R.C.P., etc.
Ralph Allen Sampson, M.A., D.Sc., F.R.S. William Joseph Maloney, M.D.

8th January 1912.

Sir Thomas Carlaw Martin, LL.D., J.P.

19th February 1912.

John Stephenson, M.B., D.Sc. John George Lindsay, M.A., B.Sc.
Arnold Hartley Gibson, D.Sc.

18th March 1912.

Anderson Gray M'Kendrick, M.B. William Smith Syme, M.D.

6th May 1912.

George Rutherford Jeffrey, M.D., F.R.C.P.
Banawari Lal Chaudhuri, B.A. (Cal.), B.Sc.

3rd June 1912.

Robert John Mathieson Inglis, Assoc. M.Inst. C.E.
Alexander Ninian Bruce, D.Sc., M.D. Robert Foster Kennedy, M.D., M.B., B.Ch.

18th July 1912.

Arthur Dukinfield Darbishire, M.A.
Edmund Taylor Whittaker, Sc.D., F.R.S.
The Most Hon. The Marquis of Linlithgow.
Robert Black Thomson, M.B.

ORDINARY FELLOWS DECEASED AND RESIGNED

During Session 1911–12.

DECEASED.

George Chrystal, M.A., LL.D.
Francis J. Martin, W.S.
Very Rev. Norman Macleod, D.D.
John Francis Sutherland, M.D.
William J. Vandenbergh, S.S.C., etc.
Alexander Taylor Innes, M.A., LL.D.
Francis T. Bond, B.A., M.D.
The Rt. Hon. Lord Lister, O.M.
Sir William H. Allchin, M.D.
Very Rev. J. M'Murtrie, D.D.
David Harris.
Andrew Wilson, Ph.D.
Sir Robert Pullar, LL.D.
James F. Pullar.
J. M'Lauchlan Young, F.R.C.V.S.
John Muter, M.A., F.C.S.
John J. Charles, M.A., M.D.

RESIGNED.

Duncan Scott Macnair, Ph.D., B.Sc. Charles Stewart Lowson, M.B., C.M.

FOREIGN HONORARY FELLOWS DECEASED.

Ferdinand Zirkel. Jules Henri Poincaré. F. A. Forel.

BRITISH HONORARY FELLOW DECEASED.

Joseph Dalton Hooker, O.M.

ABSTRACT

OF

THE ACCOUNTS OF JAMES CURRIE, ESQ.

As Treasurer of the Royal Society of Edinburgh.

SESSION 1911–1912.

I. ACCOUNT OF THE GENERAL FUND.

CHARGE.

1. Arrears of Contributions at 2nd October 1911		£101 17 0
2. Contributions for present Session:—		
1. 143 Fellows at £2, 2s. each	£300 6 0	
145 Fellows at £3, 3s. each	456 15 0	
	£757 1 0	
Less Subscriptions for present Session included in 1911 Accounts	7 7 0	
	£749 14 0	
2. Fees of Admission and Contributions of twelve new Resident Fellows at £5, 5s. each	63 0 0	
3. Fees of Admission of eight new Non-Resident Fellows at £26, 5s. each	210 0 0	
		1022 14 0
3. Arrears of Contributions written off in previous Accounts recovered		6 6 0
4. Interest received—		
Interest, less Tax £22, 17s. 9d.	£369 8 8	
Annuity from Edinburgh and District Water Trust, less Tax £3, 1s. 2d.	49 8 10	
		418 17 6
5. Transactions and Proceedings sold		157 12 5
6. Annual Grant from Government		600 0 0
7. Income Tax repaid for year to 5th April 1912		25 18 11
Amount of the Charge		**£2333 5 10**

DISCHARGE.

1. Taxes, Insurance, Coal and Lighting:—		
Inhabited House Duty	£0 6 3	
Insurance	10 10 6	
Coal to 25th April 1912	18 8 0	
Gas to 13th May 1912	0 2 9	
Electric Light to 3rd May 1912	7 10 1	
Water 1911–12	7 10 8	
		£44 8 3
2. Salaries:—		
General Secretary for Session 1910–11 and for period from 30th September to 3rd November 1911	£109 6 3	
Librarian	96 1 0	
Assistant Librarian	28 19 4	
Office Keeper	86 10 0	
Treasurer's Clerk	25 0 0	
		345 16 7
Carry forward		£390 4 10

Brought forward . . .		£390 4 10
3. Expenses of Transactions :—		
Neill & Co., Ltd., Printers	£188 7 7	
Do. for illustrations	0 6 6	
Do. (Ben Nevis)	35 12 8	
M'Farlane & Erskine, Lithographers	68 17 6	
Hislop & Day, Engravers	21 13 9	
Orrock & Son, Bookbinders	35 13 0	
Anglo Engraving Co.	2 5 0	
A. H. Searle, Lithographer	8 10 0	
		361 6 0
4. Expenses of Proceedings :—		
Neill & Co., Ltd., Printers	£401 6 0	
Do. (for illustrations)	12 18 0	
M'Farlane & Erskine, Lithographers	4 17 6	
Hislop & Day, Engravers	24 5 11	
R. & R. Clark, Ltd., Printers	0 18 6	
		444 5 11
5. Books, Periodicals, Newspapers, etc. :—		
Otto Schulze & Co., Booksellers	£146 15 6	
James Thin, do.	60 18 0	
R. Grant & Son, do.	7 6 2	
Wm. Green & Sons, do.	0 15 6	
International Catalogue of Scientific Literature	17 0 0	
Robertson & Scott, News Agents	1 8 0	
Egypt Exploration Funds Subscription	3 3 0	
Ray Society do.	1 1 0	
Palæontographical Society do.	1 1 0	
Journal de Conchyliologie	3 2 10	
Orrock & Son, Bookbinders	77 1 8	
R. Friedländer & Sohn, Berlin	10 13 8	
The *Times* to 2nd February 1913	2 12 0	
Wm. Blackwood & Sons, Printers	1 17 6	
T. & A. Constable, Printers	3 1 6	
		337 17 4
6. Other Payments :—		
Neill & Co., Ltd., Printers	£64 19 0	
R. Blair & Son, Confectioners	38 9 1	
S. Duncan, Tailor (uniforms)	4 14 0	
Lantern Exhibitions, etc., at Lectures	10 10 0	
Lindsay, Jamieson & Haldane, C.A., Auditors	6 6 0	
National Telephone Co.	10 2 6	
A. Cowan & Sons, Ltd.	7 11 0	
G. Waterston & Sons	7 2 6	
J. & T. Scott.	2 0 4	
Robert Maule & Son	9 5 3	
Petty Expenses, Postages, Carriage, etc.	73 3 3	
Burn Bros., Plumbers	8 6 1	
T. & A. Constable, Printers	5 12 6	
James Gray & Son, Ironmongers	2 5 0	
		250 6 6
7. Interest Paid on Borrowed Money :—		
Makerstoun Magnetic Meteorological Observation Fund		4 10 0
8. Irrecoverable Arrears of Contributions written off		4 4 0
Arrears of Contributions outstanding at 1st October 1912 :—		
Present Session	£64 1 0	
Previous Sessions	38 17 0	
		102 18 0
Amount of the Discharge		**£1895 12 7**

Amount of the Charge		**£2333 5 10**
Amount of the Discharge		**1895 12 7**
Excess of Receipts over Payments for 1911-1912		**£437 13 3**
Floating Balance due by the Society at 2nd October 1911		£449 7 9
Deduct Excess of Receipts as above		437 13 3
Floating Balance due by the Society at 1st October 1912		**£11 14 6**
Being—		
Loan from the Makerstoun Magnetic Meteorological Observation Fund		£218 13 4
Less Balance due by Union Bank of Scotland, Ltd., on Account Current	£201 18 10	
Balance in hands of Librarian	5 0 0	
		206 18 10
		£11 14 6

II. ACCOUNT OF THE KEITH FUND

To 1st October 1912.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Account Current at 2nd October 1911		£63 1 2
2. Interest Received :—		
On £896, 19s. 1d. North British Railway Company 3 per cent. Debenture Stock for year to Whitsunday 1912, less Tax £1, 11s. 4d.	£25 6 10	
On £211, 4s. North British Railway Company 3 per cent. Lien Stock for year to Lammas 1912, less Tax 7s. 4d.	5 19 4	
		31 6 2
3. Income Tax repaid for year to 5th April 1912		1 18 8
		£96 6 0

DISCHARGE.

1. Professor Alexander Smith, B.Sc., Ph.D.—Money Portion of Prize for 1909–11	£50 9 8
2. Alexander Kirkwood & Son, Engravers, for Gold Medal	16 0 0
3. Balance due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1912	29 16 4
	£96 6 0

III. ACCOUNT OF THE NEILL FUND

To 1st October 1912.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Account Current at 2nd October 1911	£49 3 0
2. Interest Received :—	
On £355 London, Chatham and Dover Railway $4\frac{1}{2}$ per cent. Arbitration Debenture Stock for year to 30th June 1912, less Tax 18s. 8d.	15 0 10
3. Income Tax repaid for year to 5th April 1912	0 18 8
	£65 2 6

DISCHARGE.

1. James Murray—Money Portion of Prize for 1909-11	£15 19 0
2. Alexander Kirkwood & Son, Engravers, for Gold Medal	16 0 0
3. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1912	33 3 6
	£65 2 6

IV. ACCOUNT OF THE MAKDOUGALL-BRISBANE FUND

To 1st October 1912.

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1911	£171 1 3
2. INTEREST received:— On £365 Caledonian Railway Company 4 per cent. Consolidated Preference Stock No. 2 for year to 30th June 1912, less Tax 17s.	13 15 0
3. INCOME TAX repaid for year to 5th April 1912	0 17 0
	£185 13 3

DISCHARGE.

BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1912	£185 13 3

V. ACCOUNT OF THE MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND

To 1st October 1912.

CHARGE.

1. BALANCE due by General Fund at 2nd October 1911	£219 3 4
2. INTEREST received on Balances due by General Fund at Deposit Receipt Rates to 1st October 1912	4 10 0
	£223 13 4

DISCHARGE.

1. N. T. S. Wilsmore, Grant in aid of the publication of the Annual Tables of Constants and Numerical Data, Chemical, Physical and Technological	£5 0 0
2. BALANCE due by General Fund at 1st October 1912	218 13 4
	£223 13 4

VI. ACCOUNT OF THE GUNNING VICTORIA JUBILEE PRIZE FUND

To 1st October 1912.

(Instituted by Dr R. H. GUNNING of Edinburgh and Rio de Janeiro.)

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Account Current at 2nd October 1911	£89 5 4
2. INTEREST received on £1000 North British Railway Company Consolidated Lien Stock for year to Lammas 1912, less Tax £1, 15s.	28 5 0
3. INCOME TAX repaid for year to 5th April 1912	1 15 0
	£119 5 4

DISCHARGE.

Balance due by Union Bank of Scotland, Ltd., on Account Current at 1st October 1912	£119 5 4

STATE OF THE FUNDS BELONGING TO THE ROYAL SOCIETY OF EDINBURGH

As at 1*st October* 1912.

1. GENERAL FUND—

1. £2090, 9s. 4d. three per cent. Lien Stock of the North British Railway Company at 76⅝ per cent., the selling price at 1st October 1912	£1,596 12 0
2. £8519, 14s. 3d. three per cent. Debenture Stock of do. at 78¾ per cent., do.	6,709 5 5
3. £52, 10s. Annuity of the Edinburgh and District Water Trust, equivalent to £875 at 154 per cent., do.	1,347 10 0
4. £1811 four per cent. Debenture Stock of the Caledonian Railway Company at 104⅞ per cent., do.	1,899 5 9
5. £35 four and a half per cent. Arbitration Debenture Stock of the London, Chatham and Dover Railway Company at 111¼ per cent., do.	38 18 9
6. Arrears of Contributions, as per preceding Abstract of Accounts	102 18 0
	£11,694 9 11
Deduct Floating Balance due by the Society, as per preceding Abstract of Accounts	11 14 6
Amount	£11,682 15 5

Exclusive of Library, Museum, Pictures, etc., Furniture of the Society's Rooms at George Street, Edinburgh.

2. KEITH FUND—

1. £896, 19s. 1d. three per cent. Debenture Stock of the North British Railway Company at 78¾ per cent., the selling price at 1st October 1912	£706 7 0
2. £211, 4s. three per cent. Lien Stock of do. at 76⅝ per cent., do.	161 6 1
3. Balance due by Union Bank of Scotland, Ltd., on Account Current	29 16 4
Amount	£897 9 5

3. NEILL FUND—

1. £355 four and a half per cent. Arbitration Debenture Stock of the London, Chatham and Dover Railway Company at 111¼ per cent., the selling price at 1st October 1912	£394 18 9
2. Balance due by Union Bank of Scotland, Ltd., on Account Current	33 3 6
Amount	£428 2 3

4. MAKDOUGALL-BRISBANE FUND—

1. £365 four per cent. Consolidated Preference Stock No. 2 of the Caledonian Railway Company at 98½ per cent. x.d., the selling price at 1st October 1912	£359 10 6
2. Balance due by Union Bank of Scotland, Ltd., on Account Current	185 13 3
Amount	£545 3 9

5. MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND—

Balance due by General Fund at 1st October 1912	£218 13 4

6. **GUNNING VICTORIA JUBILEE PRIZE FUND**—Instituted by Dr Gunning of Edinburgh and Rio de Janeiro—

1. £1000 three per cent. Consolidated Lien Stock of the North British Railway Company at 76⅜ per cent., the selling price at 1st October 1912 . .	£763	15	0
2. Balance due by Union Bank of Scotland, Ltd., on Account Current . .	119	5	4
AMOUNT	£883	0	4

EDINBURGH, 14*th October* 1912.—We have examined the six preceding Accounts of the Treasurer of the Royal Society of Edinburgh for the Session 1911–1912, and have found them to be correct. The securities of the various Investments at 1st October 1912, as noted in the above Statement of Funds, have been exhibited to us.

LINDSAY, JAMIESON & HALDANE,
Auditors.

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MODEL INDEX.

Schäfer, E. A.—On the Existence within the Liver Cells of Channels which can be directly injected from the Blood-vessels. Proc. Roy. Soc. Edin. vol. , 1902, pp.

Cells, Liver,—Intra-cellular Canaliculi in.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp.

Liver,—Injection within Cells of.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. 1902, pp.

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