PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

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PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

VOL. XXXVII. 1916–17.

I.—The Relation of Science to Industries and Education. Opening Address by John Horne, LL.D., F.R.S., President.

November 6, 1916.

At the opening of another session it seems to me not inappropriate to refer to the question of the relation of science to industries and to education. Not much that is new can be said on this theme. The neglect of science by the Government and the people and the persistent indifference to scientific training and methods have been exposed by prominent scientists for many years. Repeated appeals and warnings have had little or no effect. But this world war has, at last, aroused the Government and the practical Briton to realise that success in war and prosperity in peace under modern conditions depend upon the application of scientific methods, and the development of research in relation to the industries.

At the conference held in the rooms of the Linnæan Society, London, in May 1916, with Lord Rayleigh as chairman, various speeches bearing on this question were made with telling effect. It was clearly shown that science had not obtained its right place in the educational system of the country. It laboured under serious disadvantages in our public schools and in the competitive examinations for the Civil Service and the Army. The frank confession was made that, in one of our great public schools, the recognition of the value of science as part of a school education involved an unpleasant struggle. It was also revealed that some of our Government departments had made grave mistakes, resulting in serious losses to the country, through lack of scientific knowledge. In these cases it would be unjust to blame the individuals, for they are the products of a defective educational system, if it can be called a system, which has refused to grant equal privileges for the teaching of science.

The lack of appreciation of scientific methods and of the value of research is reflected in the development of some of our industries. A striking instance may be taken from the field of Applied Chemistry. Professor Meldola was an industrial chemist of a high order, with a great capacity for original research. He was a pioneer in the dyestuff industry after Perkin. He made many important discoveries, the full results of which were not realised in this country, but in Germany. Discouraged by the lack of enterprise on the part of the manufacturing firms that employed him, he accepted the Professorship of Chemistry at the Finsbury Technical College in 1885, a post which he held till his death in 1915. In a graceful tribute Lord Moulton says that Professor Meldola "represented fully the type of intellect and personality which must be developed amongst us if this war is not to be a mere incident in a long-drawn-out decline of our national greatness. We must have leaders and followers who, like him, are inspired with an equal devotion to knowledge and to the utilisation of knowledge."

In 1910 Meldola uttered this severe indictment of British methods: "The question of the cause of the decline of the British dyestuff industry resolves itself in reality into the question of the cause of Continental activity. The answer to this last question has been staring us broadly in the face for over thirty years. It is amazing that there should have ever been any doubt or any cause suggested than the true cause, which is Research, writ large. The foreign manufacturers knew what it meant and realised its importance, and they tapped the universities and technical high schools, and they added research departments and research chemists to their factories, while our manufacturers were taking no steps at all, or were calmly hugging themselves into a state of false security, based on the belief that the old order under which they had been prosperous was imperishable. It is true that when the effects of the new discoveries began to make themselves felt one or two factories did add a chemist to the staff, but the number and the means of work were totally inadequate. I happened to be one of them, and so I speak with some practical knowledge of the conditions. We were but a handful of light skirmishers against an army of trained legionaries. What could three or four—say half a dozen at a liberal estimate—research chemists, working under every disadvantage, do against scores, increasing to hundreds, of highly trained university chemists, equipped with all the facilities of research, encouraged and paid to devote their whole time to research, and backed up by technological skill of the highest order? The cause of the decline of our supremacy in this colour industry is no mystery—it is transparently and painfully obvious."

In his last address to the Institute of Chemistry in March 1915, he said, "I repeat it is not a business question, but a chemical question, and it is by chemical research alone that our colour industry can be saved in the long run. Consider the leeway that we have to make up. The German colour industry has been built up by the utilisation of the results of research carried on in the factories, universities, and technical schools for a period of over forty years. To suppose that we can retrieve our position after forty years of neglect by starting a company, the directorate of which is to consist solely of business people, is simply ludicrous." . . . "The group of industries which have arisen from the products of the tar still are not going to remain stagnant after the war, and it is scientific guidance and not mere assistance that will keep them alive. It is the expert, and the expert only, who can foresee the course of development, who can keep in touch with the progress of research, and who can direct with intelligence the campaign against our competitors. If such scientific direction is withheld, all schemes are sooner or later bound to end in failure. I deliver my second warning to a new generation after an interval of about thirty years; if it is again unheeded, so much the worse for the country."

The war has fortunately brought about a complete change from this state of apathy. On the initiation of the Chemical Society, an Association of British Chemical Manufacturers has been formed to promote closer co-operation between individual firms, to improve technical organisation, to promote industrial research, and to further co-operation between chemical manufacturers and our universities and technical schools. The Government has been instrumental in establishing a British Dyes Company with a capital of three million pounds to encourage the home industry and to render the nation more or less independent of foreign supplies. But our chief enemy is evidently determined, when the war is over, to make a great effort to retain his former position in foreign markets as regards this industry. A grant of one hundred thousand pounds for research was promised to this company by the Government. It is announced that several German firms producing dyes, fine chemicals, and drugs have combined with a capital of eleven million pounds to strengthen their position. It would seem that the competitive struggle in international trade after the war will be keener than ever.

Other industries which need not be enumerated have come to realise the importance of scientific research in relation to their future development. Of course there are great practical difficulties in carrying out this policy. These difficulties might be considerably diminished by the federation of firms in the separate industries.

Turning now to the mining industry, I may direct attention to the application of the results of scientific research to the development of a new field of iron-ore in Scotland. In the course of mapping the Mesozoic or Secondary rocks in the island of Raasay for the Geological Survey, my late friend and colleague, Horace B. Woodward, discovered a bed of oolitic iron-ore that had not previously been noticed. By means of the fossils he was able to show that the bedded ore occurred in the upper part of the Middle Lias. He called attention to the fact that the Raasay ironstone closely resembled that of Cleveland in Yorkshire, and occupied nearly the same horizon. He noted that the Raasay ore yielded 29 per cent. of metallic iron in the grey rock, and 37 per cent. in the brown rock. The results of this research were communicated to the Geological Section of the British Association at the Nottingham Meeting in 1893, and were published in the *Geological Magazine* the same year.

Here was a discovery of great potential importance, for if the bedded ore was of considerable horizontal extent, it implied a valuable addition to the iron-ore resources of Scotland. Its outcrop at the surface had been traced for about a mile. The comparison with the Cleveland deposit, whose yearly output varies from five million to six million tons, was a sufficient inducement to those engaged in the mining industry to develop the Raasay field. But, strange to say, sixteen years elapsed before any attempt was made to prove this field. In 1909, Mr Wallace Thorneycroft, the present President of the Institute of Mining Engineers, approached the Geological Survey, and obtained all available information regarding the disposition of the rocks. Having made arrangements with the owner, he systematically proved the field by a series of shallow bores, the thickness of the seam, its variation in quality, the horizontal extent that was workable. In these explorations he had the co-operation of one of the staff of the Geological Survey who had expert knowledge of the fossil zones of the Upper and Middle Lias, which was essential to the proper completion of the work. This exploitation was crowned with success, and the field is now being wrought by one of the leading firms in the mining industry in Scotland.

Still another example might be quoted of effective co-operation between the scientific staff of the Geological Survey and those engaged in the mining industry. One question of extreme economic importance at present is that of concealed coalfields, that is to say, coalfields of Carboniferous age which are covered by younger formations composed of Permian or Triassic strata.

In Scotland, part of the Ayrshire coalfield is overlain by volcanic rocks and red sandstones of Permian age. Special interest is centred in this area, because doubt had been cast on the existence of the Coal-measures beneath this Permian basin owing to a supposed unconformability near the top of the Carboniferous system. The problem has been solved by bores which have been put down through the Permian sandstones and lavas and through a succession of workable coals. As the work advanced the cores were examined from time to time by the officers of the Geological Survey in order to identify particular seams, and to determine points in the geological structure of the area.

Such co-operation is of the highest value, because it enables the Geological Survey to preserve permanent records of boring operations which have an important bearing on the future development of the mining industry. In the Scottish office of the Geological Survey this department is well organised, for the number of bore journals recorded is approximately about fourteen thousand, and the number of graphic vertical sections of the more important bores is about five thousand. Part of this material is confidential, and is not published without the written consent of the parties who gave it—an arrangement which is faithfully kept. In this manner the Geological Survey is accumulating a vast amount of economic detail of great value to the country.

Since the war began the Geological Survey has produced a series of reports on certain minerals of economic value occurring in Great Britain, which deserve special mention. Owing to the war, supplies of some minerals could not be obtained, and inquiries were made at the Department regarding the available home resources. Members of the staff were at once detailed to re-examine mines and quarries that had been abandoned, as well as those still in operation. In 1915 and 1916 five reports have been issued, relating to (1) Tungsten and Manganese Ores, (2) Barytes and Witherite, (3) Gypsum and Anhydrite, (4) Fluorspar, (5) Potash Felspar and other minerals. The method of treatment is excellent. In each report a description is given of the characters, sources, uses, and methods of preparation of the minerals, which is followed by an account of each of the mines or quarries from which the minerals have been obtained.

In his introduction to the series of reports, Dr Strahan, the Director, says that "one result of the work has been to bring into prominence the fact that what has been a waste-product has often become the most valuable asset of a mine under changed conditions as regards demand. Tin-mines closed on account of the prevalence of wolfram, and lead-mines closed through the prevalence of zinc-blende, have been re-opened to work those same ores; barytes and fluorspar have been left underground or thrown on the waste-heaps, but are now being recovered. It has become apparent also that some of our home products would be at least equal to material we have been importing, provided that they could receive equally careful preparation for the market, and that with improved treatment and greater facilities for transport they would be fit to compete with some of the foreign materials." From these observations it is clear that manufacturers have not made the most of our home resources when supplies could easily be obtained from abroad. Sir Robert Hadfield, in his opening statement as Chairman of the Ferrous Section of the Metallurgical Committee appointed by the Advisory Council, has called attention to the value of these reports, and has urged that similar information for the whole Empire should now be made available through one central source.

In this connection reference ought to be made to a scheme for the creation of a Government department of minerals and metals which has been submitted to Sir William M'Cormick as Chairman of the Advisory Council of Scientific and Industrial Research. The request is made by a powerful combination, including the Iron and Steel Institute, the Institute of Metals, the Institution of Mining Engineers, and the Institution of Mining and Metallurgy. The memorandum points out that at present various Government departments are concerned with the collection of information relating to the sources of supply of minerals and the production of metals which has led to considerable overlapping and duplication of effort. There is much force in this contention. The proposed department should include the geological surveys of the United Kingdom; it should collect and co-ordinate information bearing on the occurrence, uses, and economic value of minerals and their products; it should investigate all questions relating to the utilisation of the mineral or metallurgical resources of the Empire; it should circulate information on the development of mineral areas, output, capital employed, and markets; it should review from time to time the developed and undeveloped mineral resources to ensure that the mineral wealth of the Empire is being exploited with due regard to Imperial interests. This comprehensive scheme deserves careful consideration. If it were adopted, it would remove the overlapping in Government departments as regards these industries, it would bring about effective co-ordination of the institutions representing these industries, it would prevent the periodical scares as to coal supplies which have led to prolonged inquiries by Coal Commissions in the past.

Special acknowledgment ought to be made of the services rendered by the Royal Society to the Government and the nation at this critical time in the history of the country. On its initiative, a Board composed of representatives of twenty-seven scientific, including technical, societies has been formed to consider the relation of science to industry and education, to take such steps as may be necessary to promote the application of science to industries, and to discuss scientific questions in which international co-operation seems advisable. It is fortunate that our Society has two such competent representatives on this Board as Professor Gregory and Professor Walker, for difficult problems will have to be considered. The Board may be helpful in giving advice to the Government about matters affecting the reconstruction of our national life after the war.

The Government has at last recognised that the claims of science in relation to industries and education must be regarded from a fresh standpoint. Various committees and sub-committees have been appointed to deal with numerous complex problems. At present I will call attention only to two of these Committees.

The first of these, under the chairmanship of Sir J. J. Thomson, is to inquire into the position of science in our educational system, especially in universities and secondary schools. Attention is to be directed to the advancement of pure science, and to the interest of trades and industries dependent upon the application of science, to the provision of scholarships and bursaries for the encouragement of deserving students and scholars. The Lord President of the Council has asked that the Committee should receive the voluntary assistance of the universities, technical colleges, public schools and secondary schools generally, to facilitate the inquiries. He recognises that there is urgent need for prompt action on the part of the Government. We may reasonably hope for important developments from this Committee.

The Committee of the Privy Council for Scientific and Industrial Research, with the Advisory Council, under the chairmanship of Sir William M'Cormick, is of outstanding importance. A sum of forty thousand pounds has been placed on the estimates for the current financial year, to be administered by this Committee (1) "for instituting specific researches, (2) for establishing or developing special institutions or departments of existing institutions for the scientific study of problems affecting particular industries and trades, and (3) for the establishment and award of research studentships and fellowships." (See Note, p. 9.)

The National Physical Laboratory and the Imperial College of Science established by the Government have rendered valuable service to the State. But the scheme now realised of a permanent organisation for the promotion of industrial and scientific research is far-reaching. It applies to the

United Kingdom, and the most effective institutions and investigators are to be utilised irrespective of nationality. The Advisory Council is to co-operate with, and receive suggestions from, the Royal Society, scientific associations, the universities, technical colleges, and other institutions in which research can be carried on. It will surely be conceded that this organisation rests on a sound basis.

The first report of this Committee and of the Advisory Council, which has recently been issued, is of surpassing interest, and deserves careful consideration by everyone who is interested in the future welfare of the country. One must admire the thoroughness of their methods. In the numerous conferences that have taken place with the representatives of professional and other societies, it has transpired that the most highly organised industries have made most use of scientific research, while other trades have made little or no progress in this respect. At the same time it is admitted that the representatives of different industries have shown willingness to co-operate and to carry out research for the benefit of the trade as a whole. The Advisory Council recognise that research in pure science, which naturally finds its home in the universities, is essentially the basis of applied science. But in allocating grants they have been compelled to begin with industrial research, partly on account of the requirements of the manufacturers, and partly owing to the depletion of students and staffs in the universities and technical colleges through the war.

Two conditions are regarded by the Advisory Council as essential for the success of their work. The first is, a largely increased supply of men who are competent to undertake research; the second is, cordial co-operation among men of science, men of business, working men, professional and scientific societies, universities and technical colleges, local authorities and Government departments. It is obvious that in order to secure the required number of researchers there must be radical changes in the educational system of the country. A heavy responsibility rests with Boards of Education and all the great educational institutions to carry out these reforms with the aid of the legislature. Co-operation between all concerned is no less essential, for if the strife between capital and labour continues after the war, then our national welfare will be seriously imperilled.

The relation of science to industries and education is one of the absorbing questions of the present time. The Commonwealth of Australia has followed the lead of our own Government, and has produced a scheme for establishing an Institute of Science and Industry with an Advisory Council. Canada is acting on similar lines. The Government of the

United States has also realised the necessity of immediate action as regards this question. A National Research Council has been set up to encourage the increased use of scientific research in the development of American industries, to bring into co-operation the existing research organisations, and to employ scientific methods in strengthening the national defence. The scheme is cordially supported by cabinet ministers, scientific societies, the heads of the great universities and research foundations, and the leading investigators in the States.

The change in the mental outlook towards science in Great Britain has come in time. In this struggle for right and freedom the British nation has shown no decline. The people have displayed the same courage, fertility of resource, and tenacity of purpose as of yore. The time is ripe for changes in our educational system, and for the application of scientific research to our industries. If this policy is carried out to the full, it will certainly have a far-reaching effect on the future welfare of the country.

Note.—Since this address was written it has been officially announced that the Treasury has set apart a sum of one million pounds to be administered by the Committee of the Privy Council for Scientific and Industrial Research during the next five years.

(*Issued separately April* 2, 1917.)

II.—Notices of Fellows, Honorary and Ordinary, recently deceased. By The General Secretary.

(Read November 6, 1916.)

Emil Clement Jungfleisch was born in Paris in 1839. He devoted himself to chemistry and pharmacy, and in 1869 became Assistant to Berthelot, whom he succeeded in 1876. He has made numerous contributions to organic chemistry, and for his work on the different forms of tartaric acid was awarded the Jecker Prize of the Academy of Sciences. One of his latest contributions was a study of gutta-percha, in which he showed that the leaves of the plant could be used as a source of the material instead of the stem.

He was elected an Honorary Fellow of the Royal Society of Edinburgh in 1913, and died on April 24, 1916.

Elie Metchnikoff was born on May 15, 1845, at Ivanavka, in the Russian province of Kharkoff. He was early attracted to the study of nature, and after two years' study at Kharkoff University he proceeded to Germany, where he worked under Leuckart, first at Giessen, then at Göttingen. In 1870 he was appointed Professor of Zoology and Comparative Anatomy at Odessa, whence after twelve years he proceeded to Messina. Here he began his researches on the phagocytosis of the blood, and in course of time established his reputation as a pathologist. In 1888 he joined Pasteur in Paris, where to the end of his life he continued his studies in bacteriology and the close connection of micro-organisms with disease. His view on many of those questions are given in his book, *The Nature of Man*, published in 1903, and translated into English. Metchnikoff received many honours—was Copley Medallist of the Royal Society, a Member of the Institute of France, and in 1908 he was awarded the Nobel Medal and Prize for his researches on Immunity.

He was elected an Honorary Fellow of our Society in 1910, and died in Paris on July 15, 1916.

Sir William Ramsay, K.C.B., F.R.S., was born in Glasgow on October 2, 1852. He was educated at the Glasgow Academy and at the Glasgow University, and subsequently at the University of Tübingen. In 1880 he was elected Professor of Chemistry at University College, Bristol, where he remained seven years. From 1887 to 1913 he occupied the Chair of Chemistry in University College, London. He is most widely known for his investigations into the properties of liquids and vapours, and in 1894 was associated with Lord Rayleigh in the discovery of the gas Argon—a new element in the air—the presence of which, however, is clearly shown in Cavendish's *Experiments* of 1785. Ramsay also discovered in the air the element Helium, whose existence in the sun had been proved by spectroscopic evidence. In the course of further work on Argon other new atmospheric gases were discovered—Neon, Krypton, and Xenon. Sir William Ramsay was knighted in 1902, and received the Nobel Prize in 1904. He was the author of several books, including a work on *The Gases of the Air*.

He was elected an Honorary Fellow of our Society in 1905, and died on July 23, 1916.

Charles Rene Zeiller, Professor of Palæobotany in the National School of Mines, Paris, was one of the most distinguished workers in the field of fossil botany. In 1878 he published a volume on the plants of the French Coal Measures, and in 1903 two volumes on the Rhætic Flora of Tonkin. He also added considerably to our knowledge of the Permo-Carboniferous Floras of South Africa, Brazil, and India. His skill as a morphologist is shown by his researches into the structure of the Palæozoic fern Psaronius and the anatomy of Lepidostrobus. In 1909 he visited England for the first time to attend the Darwin Celebration at Cambridge, and received the University Honorary Degree.

He was elected an Honorary Fellow of our Society in 1913, and died in December 1915.

Sir Stair Agnew, K.C.B., a younger son of Sir Andrew Agnew, 7th Bt., was born in 1831. From Edinburgh University he proceeded to Trinity College, Cambridge, where he graduated as a Senior Optime in Mathematics in 1855. He rowed in the Oxford and Cambridge boat-race in 1854. For a short time he served in the army, but in 1860 was called to the Scottish Bar. He was appointed Queen's Remembrancer in 1870, and in 1881 was appointed Registrar-General and Keeper of Records for Scotland. He retired in 1909.

He was elected a Fellow of the Royal Society of Edinburgh in 1871, and on four different occasions served on the Council. His death took place at his residence in Edinburgh on July 12, 1916.

Alexander Russell Brown, M.A., B.Sc., Captain K.O.S.B., was born at Longriggend, Lanarkshire, in February 1889. He was educated at his native town and also at Airdrie Academy, where he was dux of his class. In 1905 he entered the University of Glasgow, where in 1910 he graduated with Honours both in Classics and in Mathematics. Two years later he graduated B.Sc. in Pure Science. As Carnegie Research Scholar in the University of Glasgow he investigated the Absorption of Light by Inorganic Salts, the results of which were published in our *Proceedings* in the year 1913. In 1912 he was appointed Science Master in Buckhaven Higher Grade School, but on the outbreak of war offered his services in the army. He obtained his lieutenancy in November 1914. He first went to France in August 1915, and was mentioned for gallant and distinguished conduct in Sir Douglas Haig's dispatch of April 30, 1916, and was gazetted Captain in the following May. He met his death on August 17, 1916.

He was elected a Fellow of the Royal Society of Edinburgh in 1913.

Robert Caird, LL.D., the head of the firm of Messrs Caird & Co., Shipbuilders, Greenock, was not only a leading authority on marine engineering and the construction of ships, but was a man of wide culture both in the ancient classics and also in modern languages. He joined the firm founded by his father in 1888. Nearly all the present vessels of the Peninsular and Oriental Company's fleet and many other great liners were built by him and his brothers. He was elected a Fellow of the Royal Society of Edinburgh in 1897, and from 1899 to 1901 was President of the Institution of Engineers and Shipbuilders in Scotland. He received the Honorary Degree of LL.D. from the University of Glasgow in 1900, and a few years previously had been created a Knight of the Crown of Italy. He also gave valuable assistance in the foundation and equipment of the new Natural Philosophy Institute in Glasgow University, and in other ways showed his keen interest in University affairs in Glasgow. He had a genuine interest not only in applied science, but also in the wider problems of the development of human civilisation. He was a man of rare generosity of spirit and warm affection. Latterly on account of failing health he was obliged to give up the many social and scientific gatherings in which he delighted.

He died at Greenock on December 1, 1915.

C. T. Clough, LL.D., was born in Yorkshire in 1853, and was educated at Rugby and St John's College, Cambridge. He early joined the staff of H.M. Geological Survey, then under the direction of Sir Andrew Ramsay, and after some years was transferred to Scotland. As a field geologist in intricate country he was second to none, and he had a wide reputation as a geologist of sound knowledge and judgment. He made special investigation of the Lothians coalfield and district, and of other important coalfields in the Lowlands of Scotland. In 1906 he was awarded the Murchison Medal of the Geological Society of London.

He was elected a Fellow of the Royal Society of Edinburgh in 1916, and received the same year the Honorary Degree of LL.D. from St Andrews. In the succeeding August, when crossing the railway line near Manuel, he misjudged the speed of a train that was being shunted, and was run over, sustaining serious injuries. He lingered for a few days in the Edinburgh Royal Infirmary, and died on August 27, 1916.

John Cook, M.A., LL.D., was a distinguished graduate of Aberdeen University, where for many years he acted as Assistant to the Professor of Natural Philosophy. After occupying for a few years the post of Mathematical Master in the High School of Arbroath, he proceeded to India in 1877 as Principal of the Doveton College, Madras. Five years later he was appointed Principal of the Mysore Central College at Bangalore, and in this position he devoted twenty-five years of his life to the development of the higher education in the Mysore provinces. He was author of a number of scientific text-books, which are widely used in the schools and colleges of South India, and he was also for a number of years in control of the Mysore Meteorological Department. After retiring from his Indian work he settled in Edinburgh, where he died December 30, 1915.

He was elected a Fellow of our Society in 1894.

Charles A. Cooper, LL.D., for nearly thirty years editor of the *Scotsman*, was elected a Fellow of the Royal Society of Edinburgh in 1891. His career as a journalist was steady and sure as he passed from Hull to London, and finally settled in Edinburgh in 1868. He died at Bournemouth on April 14, 1916, at the age of eighty-seven.

Arthur Dukinfield Darbishire, M.A., was born in 1879. Soon after graduating at Oxford in 1902 he started important experiments on the laws of heredity, the main results of which are embodied in his book, *Breeding and the Mendelian Discovery*, first edition 1911, third edition 1913. In 1911 he was appointed Lecturer in Genetics in the University of Edinburgh, and he began new work on Heredity in the University Experimental Farm at Fairslacks. When the war broke out in 1914 he was lecturing to the Graduate School of Agriculture at the University of Missouri and was offered a Research Fellowship there; but he resolved to offer himself, if possible, for military service, and finally enlisted as a private in the 14th Argyll and Sutherland Highlanders. He was just on the point of receiving a commission in the Royal Garrison Artillery when he died in a Military Hospital in the prime of life on December 26, 1915. On May 25, 1914, Mr Darbishire and Mr M. W. Gray communicated a paper "On the Inheritance of Certain Characters of the Wool of Sheep." Unfortunately the MS. was left by him in an incomplete form, and has never yet been published. Several chapters of an unfinished work have been edited by his sister, Miss Helen Darbishire, under the title *An Introduction to a Biology and other Papers* (Cassell, 1916).

He was elected a Fellow of our Society in 1912.

David Douglas was born at Stranraer in 1823, and was educated at Whithorn. In 1838 he came to Edinburgh, and shortly afterwards entered the employment of Messrs Blackwood, with whom he remained till 1847. He then formed the well-known firm of Edmonston & Douglas, publishers. He was an intimate friend of such well-known writers as Dr John Brown, the Rev. Dr Hanna, Cosmo Innes, and David Laing. His connection with the Philosophical Institution of Edinburgh brought him into close association with Thackeray and Dickens. In 1877 the firm of Edmonston & Douglas was dissolved, but Mr Douglas continued his labours as one of the firm of Douglas & Foulis.

He was elected a Fellow of the Royal Society of Edinburgh in 1866, and died in Edinburgh on April 4, 1916, at the advanced age of ninety-one.

Norman Hay Forbes, F.R.C.S.E., L.R.C.P. Lond., M.R.C.S. Eng., was born at Rawal Pindi, in the Punjab, on March 1, 1863, and was educated at the Bedford Grammar School and at Elizabeth College, Guernsey. He received his medical training at the Middlesex Hospital, and thereafter served for three years in the Royal Army Medical Corps. He finally settled at Church Stretton, Shropshire. He wrote a number of papers on medical subjects, which include "Observations on the Climate and Health Resorts of Scotland," and on the "Shropshire Highlands," and especially "Church Stretton as a Climatic Health Resort and After-Cure Station." He also contributed an article on "Food and Dietetics" to the *Practitioners' Encyclopædia of Medicine and Surgery*.

He was elected a Fellow of the Royal Society of Edinburgh in 1904, and died on June 27, 1916.

J. A. Harvie-Brown, LL.D., F.Z.S., of Dunipace House, near Larbert, was born in 1844. He was a keen zoologist, especially in the department of ornithology, and was the author of many papers as well as books on Natural History. Especially important are his series of volumes on the Vertebrate Fauna of different regions in the north of Scotland. He studied with great care the different problems relating to the migration of birds and of the dispersal and distribution of species. In pursuit of his favourite science he travelled extensively in various parts of Europe, and especially in the islands around the Scottish coasts.

He was elected a Fellow of the Royal Society of Edinburgh in 1881, and died on July 26, 1916.

Rev. George Laing was born in 1831, and studied at Edinburgh University and New College. He began his ministry at Penpont in 1857, and subsequently removed to Chapelshade Church, Dundee, where he resigned his charge about 1880, and after some years spent in travel on the Continent settled in Edinburgh. His publications include sermons in German.

He was elected a Fellow of the Royal Society of Edinburgh in 1886, and died on February 26, 1916.

Nicholas Henry Martin, Ph.C., F.C.S., was born at Trebarveth, Cornwall, on May 2, 1847, and served his apprenticeship with a Penryn chemist. Afterwards he proceeded to London, and passed through a thorough course of study at the Pharmaceutical School of Pharmacy. In 1875 he joined Mr Henry Brady, F.R.S., at Newcastle, and commenced the well-known business of Brady & Martin. He occupied various positions of importance, such as President of the British Pharmaceutical Conference, Chairman of the Newcastle Section of the Society of Chemical Industry, and generally took a leading part in the development of pharmacy.

He was elected a Fellow of the Royal Society of Edinburgh in 1903, and died at Ravenswood, Gateshead, on July 5, 1916.

Henry O'Connor, A.M.Inst.C.E.—an engineer well known in Edinburgh circles—was a lecturer at the Heriot-Watt College and the Royal Technical Institute in Glasgow. He served a term as President of the Institute of Gas Engineers, and contributed many valuable papers to the *Proceedings* of that body as well as to the Society of Arts. He had a strong interest in amateur dramatic art, and by his social qualities contributed much to the geniality of the meetings of the Pen and Pencil Club and of the Cap and Gown Club. He was elected a Fellow of the

Royal Society of Edinburgh in 1906. He was a keen Volunteer, and much interested in gunnery. When the war broke out he was mobilised with his corps and was Colonel of the 4th Royal Garrison Artillery. His health unfortunately broke down, and he died in Craigleith Military Hospital on December 1, 1915.

Thomas Parker, M.Inst.C.E., was born at Coalbrookdale on December 22, 1843. While still quite a young boy he entered the works of the Coalbrookdale Company, to which he returned after a few years spent in Manchester. While in charge of the electro-depositing department he made discoveries in connection with electric storage batteries which led to his starting business in Wolverhampton in partnership with Mr P. B. Elwell. This firm, although constituted originally to make accumulators, ere long applied itself to the construction of all kinds of electrical machines. Electric traction and electric locomotives especially engaged Mr Parker's attention. He was also interested in many other applications of science, such as the refining of metals, construction of slow-combustion stoves, the production of smokeless fuel, and the like.

He was elected a Fellow of the Royal Society of Edinburgh in 1892, and died December 5, 1915.

Sir Alexander Russell Simpson, M.D., was born at Bathgate on April 30, 1835. When he entered on his medical studies in the Edinburgh University he was at the same time apprenticed to Professor Goodsir. After graduating he studied at the Universities of Montpelier and Berlin, and on his return became Assistant to his famous uncle, Sir J. Y. Simpson, who then held the Chair of Midwifery at Edinburgh. After five years' private practice in Glasgow he was recalled to Edinburgh to succeed Sir J. Y. Simpson in the Chair of Midwifery; this was in 1870, and until his retirement in 1905 he carried on the duties of this Chair with high distinction. In addition to his University work Sir Alexander Simpson devoted much time to religious and philanthropic work, and was strongly interested in the Maternity Hospital in Lauriston Place, which was erected as a Memorial of his distinguished uncle. He was a man of wide culture, and conversed with ease in several foreign languages. After his retirement he made a tour round the world, from which he returned with many fresh interests. He was knighted in 1906 in recognition of his professional zeal and attainments. He retained his physical and mental powers to the end, his death on April 6, 1916, being the result of a motor accident.

He was elected a Fellow of our Society in 1871.

George Smith, F.C.S., was born at Woolwich in 1851. He went through the School of Mines, and in 1879 entered the service, as Works Manager of their Westquarter Factory in Stirlingshire, of Nobel's Explosives Company, and under his direction the factory grew in size and importance, and when Westquarter was no longer able to meet the demands placed on it a new factory was built at Linlithgow for powder fuse. He was a Fellow of the Institute of Chemistry, and was elected a Fellow of our Society in 1885. After his retirement he lived at Hastings and St Leonards-on-Sea, where he died November 19, 1915.

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III.—The Application of Operators to the Solution of the Algebraic Equation. By James Littlejohn, M.A. *Communicated by* The General Secretary.

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§ I. Foundation of the Calculus. Extended Significance of the Symbol D^{-1}.

The solution of the common quadratic

$$ax^2 - bx + c = 0,$$

which is usually written

$$\frac{b - \sqrt{b^2 - 4ac}}{2a},$$

is, when expanded,

$$x = \frac{c}{b} + \frac{ac^2}{b^3} + 2\frac{a^2c^3}{b^5} + 5\frac{a^3c^4}{b^7} + 14\frac{a^4c^5}{b^9} + 42\frac{a^5c^6}{b^{11}} + \text{etc.}$$

This expansion will be found to be the work of the operator $D_a^{-1} . D_c^{-1} . D_b^2$, each term being derived from its predecessor by this operator's agency, so that the whole solution may be written

$$(1 + \theta + \theta^2 + \theta^3 \text{ etc.}) \frac{c}{b} \quad \text{where} \quad \theta = D_a^{-1} . D_c^{-1} . D_b^2 .$$

In this operator the symbols have their usual signification, but with the following important exceptions:—

In the first place, $D_a^{-1} . \frac{1}{a}$ is not $\log a$, but is $\frac{a^0}{0}$; and again, D_a of q or any expression not containing a is to be written

$$D_a \text{ of } a^0 \times q, \quad \text{or} \quad \frac{0}{a}q,$$

and if, as always happens when we have one quantity (such as a) passing downwards, we have, sooner or later, another passing upwards, then there occurs the indeterminate form $\frac{0}{0}$, whose correct evaluation, we shall find, is -1.

In the solution written above, this form does not yet occur.

We have simply $ax^2 - bx + c = 0$, and one obvious way of obtaining an operand or free term is to write

$$x = \frac{c}{b} + \frac{a}{b}x^2.$$

From the initial value $x = \frac{c}{b}$ we derive a second, viz.

$$x = \frac{c}{b} + \frac{a}{b}\left(\frac{c}{b}\right)^2.$$

Now to convert the first term here into the second is the work of the operator $D_a^{-1} . D_c^{-1} . D_b^{2}$, and "solving the equation" therefore is simply carrying out *ad infinitum*, or so long as the process yields results tangibly different from zero, the operation here initiated and inculcated.

The other operand comes by writing

$$x = \frac{b}{a} - \frac{c}{a} \cdot \frac{1}{x}, \text{ leading to } x = \frac{b}{a} - \frac{c}{a} \times \frac{a}{b},$$

and again the first term is converted into the second by the operator $D_a^{-1} . D_c^{-1} . D_b^{2}$. Here now we have b passing downwards and a passing upwards.

The operations written full out are

$$\frac{c}{1} \cdot \frac{a^0}{0} \cdot \frac{1 \times 0}{b} \quad \text{or} \quad -\frac{c}{b}.$$

That $\frac{0}{0}$ should be evaluated as -1 will cause no difficulty if, instead of the zeros, we substitute two infinitesimally small quantities h, necessarily of opposite signs. Thus the two operators D_a^{-1} and D_b may be supposed either to overshoot their mark, or to fall short of it by an infinitesimal distance. In the former case we shall have $D_a^{-1} . D_b$ producing $\frac{a^h}{hb^h}$ from $\frac{b}{a}$, and then $D_c^{-1} . D_b$ on this will give

$$\frac{a^h}{h} \cdot \frac{-h}{b} \cdot \frac{c}{1} \quad \text{or} \quad -\frac{c}{b}.$$

In the latter case we have $\frac{h}{-h}$ instead of $\frac{-h}{h}$, in both cases therefore -1.

This step once taken, the way is clear. The operator acts now continuously and without further novel feature, producing

$$x = \frac{b}{a} - \frac{c}{b} - \frac{ac^2}{b^3} - 2\frac{a^2c^3}{b^5} - 5\frac{a^3c^4}{b^7} \text{ etc.,} \quad \text{or} \quad \frac{b + \sqrt{b^2 - 4ac}}{2a},$$

the second root, and if we confine ourselves to operands with integral indices, there is no third.

This style of solution, we shall see, can be extended to equations of every degree.

§ II. Harmonising of this Calculus with the Fluxional One.

To bring this calculus into harmony with that of Newton and Leibnitz, we begin by regarding the statement

$$\frac{d}{dx}x^n = nx^{n-1}$$

as universally true—true, that is, for all values of n, including zero or infinitesimally small quantities.

This will cause no difficulty, but the converse,

$$\int x^{n-1} = \frac{x^n}{n},$$

must also be regarded as holding without exception. In this view it is a mere corollary to the former.

We remark next that there exists no formal proof for the statement

$$\int \frac{1}{x} = \log x.$$

This is accepted in virtue of the converse

$$\frac{d}{dx}\log x = \frac{1}{x},$$

a statement which admits of proof.

Consider, however, the operation $D^{-1}\frac{1}{x}$ generally.

According to the rule just laid down, this must be evaluated as $\frac{x^0}{0}$ or $\frac{x^h_1}{h}$, where h is an infinitesimal, and the indeterminateness thus introduced is, in this new calculus, counteracted in the manner shown, and we reach the determinate value -1. In the Fluxional Calculus, again, the same indeterminateness is counteracted by the addition of a constant—after the manner of this calculus—the particular constant (if we can so call it) added being $-\infty$ in the form $-\frac{1}{h}$, so that we have $\frac{x^h - 1}{h}$, whose limit, for $h=0$, is $\log x$. And our justification for choosing this particular constant is, of course, that we thereby reach that evaluation which is in harmony with the established fact,

$$\frac{d}{dx}\log x = \frac{1}{x}.$$

In Physics, when a new theory or hypothesis is broached, it is considered sufficient if such theory be shown to involve no contradiction with other theories already established, and to be itself supported by incontrovertible facts. Further, the consideration of the latter, the facts underlying the hypothesis, must precede any attempt to explain the hypothesis or to fathom its significance. This is the position we now adopt here. Having squared the matter superficially with our formulæ, and shown that there is no antagonism between the new evaluation of $D^{-1}\frac{1}{x}$ and the former one, we shall have our hands quite full in considering some of the concomitants of this fact.

§ III. Solutions of the Cubic, Quartic, Quintic, etc.

We pass, therefore, to the application of the above mode of solution to the higher equations, beginning with the cubic $Ax^3+Bx^2+Cx+E=0$.

(*a*) The first operand is got by writing

$$x = -\frac{E}{C}-\frac{B}{C}x^2-\frac{A}{C}x^3,$$

and then, by substitution,

$$x = -\frac{E}{C}-\frac{B}{C}\left(\frac{E}{C}\right)^2+\frac{A}{C}\left(\frac{E}{C}\right)^3.$$

The conversion of $-\frac{E}{C}$ into $-\frac{B}{C}\left(\frac{E}{C}\right)^2$ and $\frac{A}{C}\left(\frac{E}{C}\right)^3$ is the work of the operators $D_B^{-1}.D_E^{-1}.D_C^{2}$ and $D_A^{-1}.D_E^{-2}.D_C^{3}$ respectively. By the aid of these operators we develop x as follows:—

$$x = -\frac{E}{C}$$

$$-\frac{B}{C}\left(\frac{E}{C}\right)^2+\frac{A}{C}\left(\frac{E}{C}\right)^3$$

$$-2\left(\frac{B}{C}\right)^2\left(\frac{E}{C}\right)^3+5\left(\frac{B}{C}\right)\left(\frac{A}{C}\right)\left(\frac{E}{C}\right)^4-3\left(\frac{A}{C}\right)\left(\frac{E}{C}\right)^5$$

$$-5\left(\frac{B}{C}\right)^3\left(\frac{E}{C}\right)^4+21\left(\frac{B}{C}\right)^2\left(\frac{A}{C}\right)\left(\frac{E}{C}\right)^5-28\left(\frac{B}{C}\right)\left(\frac{A}{C}\right)^2\left(\frac{E}{C}\right)^6+12\left(\frac{A}{C}\right)^3\left(\frac{E}{C}\right)^7$$

$$-14\left(\frac{B}{C}\right)^4\left(\frac{E}{C}\right)^5+84\left(\frac{B}{C}\right)^3\left(\frac{A}{C}\right)\left(\frac{E}{C}\right)^6-180\left(\frac{B}{C}\right)^2\left(\frac{A}{C}\right)^2\left(\frac{E}{C}\right)^7+165\left(\frac{B}{C}\right)\left(\frac{A}{C}\right)^3\left(\frac{E^8}{C}\right)-55\left(\frac{A}{C}\right)^4\left(\frac{E}{C}\right)^9,$$

etc., etc.

To get the third horizontal line here, pass the first operator over all the elements in the previous line. Then pass the second operator over the same elements to get those elements—*not already got*; for there must be no reduplication of elements. So for the other lines.

The triangular array thus created is seen to be the corner of an infinite rectangle. In other words, the solution of the quadratic being an expression singly infinite, the solution of the cubic is an expression doubly infinite, and we may so far anticipate by saying that the solution of the quartic, with all its terms present, is a triply infinite expression, that of the quintic quadruply infinite, and generally any solution of an equation of the n^{th} degree, in which no terms are wanting, is an expression $(n-1)^{\text{ly}}$ infinite.

Geometrically, the solution of the quadratic may be represented as a line, or an infinite series of elements ranged rectilinearly; the solution of the cubic may then be represented as a surface, with such elements disposed regularly all over it; the solution of the quartic is then an infinite rectangular parallelepiped or solid with such similar disposition of elements throughout it, but for the geometrical representation of solutions belonging to equations containing the fifth or higher powers of the unknown, and at the same time all lower powers, we require space of higher dimensions than 3.

Returning to our solution, we remark that the left-hand side is the solution of the quadratic $Bx^2+Cx+E=0$. The right-hand side is the solution of the cubic $Ax^3+Cx+E=0$.

All parallel lines in this development, whether vertical, horizontal, or slanting, are the work of the same operator. The vertical ray, *e.g.*,

$$-\frac{E}{C}, \quad \frac{5ABE^4}{C^6}, \quad -180\frac{A^2B^2E^7}{C^{11}}, \text{ etc.},$$

is, with all other verticals, the work of the operator $D_A^{-1}.D_B^{-1}.D_E^{-3}.D_C^{5}$.

The properties noticeable, in the indices and coefficients, which, of course, are constants, will not escape the reader.

(*b*) Meanwhile we write down another solution, the one belonging to

$$Ax^3 = -Bx^2-Cx-E \quad \text{or} \quad x = -\frac{B}{A}-\left(\frac{C}{A}\right)\frac{1}{x}-\left(\frac{E}{A}\right)\frac{1}{x^2}.$$

On writing

$$x = -\frac{B}{A}+\frac{C}{A}\left(\frac{A}{B}\right)-\frac{E}{A}\left(\frac{A}{B}\right)^2,$$

we see the operators to be $D_C^{-1}.D_A^{-1}.D_B^{2}$, and $D_E^{-1}.D_A^{-2}.D_B^{3}$. The form $\frac{0}{0}$ occurs at the outset with both.

Thus from

$$D_C^{-1} \,.\, D_A^{-1} \,.\, D_B^{2} \text{ on } -\frac{B}{A}$$

we have

$$-\frac{C}{1} \cdot \frac{A^0}{0} \cdot \frac{1 \times 0}{B} \quad \text{or} \quad \frac{C}{B},$$

and from

$$D_E^{-1} \,.\, D_A^{-2} \,.\, D_B^{3} \text{ on } -\frac{B}{A}$$

we have

$$-\frac{E}{1} \cdot \frac{A^1}{0 \times 1} \times \frac{1 \times 0 \times (-1)}{B^2} \quad \text{or} \quad -\frac{EA}{B^2}.$$

The development then proceeds as before, and we obtain

$$\begin{aligned} x = \; & -\frac{B}{A} \\ & \frac{C}{B} - \left(\frac{E}{B}\right)\left(\frac{A}{B}\right) \\ & \left(\frac{A}{B}\right)\left(\frac{C}{B}\right)^2 - 3\left(\frac{E}{B}\right)\left(\frac{C}{B}\right)\left(\frac{A}{B}\right)^2 + 2\left(\frac{E}{B}\right)^2\left(\frac{A}{B}\right)^3 \\ & 2\left(\frac{C}{B}\right)^3\left(\frac{A}{B}\right)^2 - 10\left(\frac{C}{B}\right)^2\left(\frac{E}{B}\right)\left(\frac{A}{B}\right)^3 + 15\left(\frac{C}{B}\right)\left(\frac{E}{B}\right)^2\left(\frac{A}{B}\right)^4 - 7\left(\frac{E}{B}\right)^3\left(\frac{A}{B}\right)^5 \\ & 5\left(\frac{C}{B}\right)^4\left(\frac{A}{B}\right)^3 - 35\left(\frac{C}{B}\right)^3\left(\frac{E}{B}\right)\left(\frac{A}{B}\right)^4 + 84\left(\frac{C}{B}\right)^2\left(\frac{E}{B}\right)^2\left(\frac{A}{B}\right)^5 - 84\left(\frac{C}{B}\right)\left(\frac{E}{B}\right)^3\left(\frac{A}{B}\right)^6 + 30\left(\frac{E}{B}\right)^4\left(\frac{A}{B}\right)^7, \\ & \text{etc., etc.} \end{aligned}$$

Here the same properties hold as in the former solution, the only exception to this being that the right-hand side, though the work of a cubic operator, is *not* the solution of a cubic trinomial form, as before.

(*c*) The third solution, in some respects the most difficult, comes by writing

$$Bx = -C - \frac{E}{x} - Ax^2,$$

from which we reach

$$x = -\frac{C}{B} + \frac{E}{B} \times \frac{B}{C} - \frac{A}{B} \cdot \left(\frac{C}{B}\right)^2;$$

the operators are thus $D_B^{-1} \,.\, D_E^{-1} \,.\, D_C^{2}$ and $D_A^{-1} \,.\, D_C^{-1} \,.\, D_B^{2}$, the former having in its first action the form $\frac{0}{0}$. These two operators, it will be noticed, are both quadratic ones, the two quadratic operators, viz., of the two previous solutions.

Another new feature in this solution is, that many of the elements have the value zero. This comes from a zero having appeared in the *numerator* of the coefficient before there is one in the denominator. Till the appearance of the counteracting zero in the denominator, then, the intervening elements are zero. But we shall first write down the solution itself.

$$
\begin{array}{rc}
x = & -\dfrac{C}{B} \\
& \dfrac{E}{C} - \left(\dfrac{A}{B}\right)\left(\dfrac{C}{B}\right)^2 \\
& \left(\dfrac{B}{C}\right)\left(\dfrac{E}{C}\right)^2 + \left(\dfrac{A}{B}\right)\left(\dfrac{E}{B}\right) - 2\left(\dfrac{A}{B}\right)^2\left(\dfrac{C}{B}\right)^3 \\
& 2\left(\dfrac{B}{C}\right)^2\left(\dfrac{E}{C}\right)^3 + 0 \quad + 3\left(\dfrac{A}{B}\right)^2\left(\dfrac{E}{B}\right)\left(\dfrac{C}{B}\right) - 5\left(\dfrac{A}{B}\right)^3\left(\dfrac{C}{B}\right)^4 \\
& 5\left(\dfrac{B}{C}\right)^3\left(\dfrac{E}{C}\right)^4 - \left(\dfrac{A}{C}\right)\left(\dfrac{E}{C}\right)^3 - 0 \quad + 10\left(\dfrac{A}{B}\right)^3\left(\dfrac{E}{B}\right)\left(\dfrac{C}{B}\right)^2 - 14\left(\dfrac{A}{B}\right)^4\left(\dfrac{C}{B}\right)^5 \\
& 14\left(\dfrac{B}{C}\right)^4\left(\dfrac{E}{C}\right)^5 - 5\left(\dfrac{A}{C}\right)\left(\dfrac{B}{C}\right)\left(\dfrac{E}{C}\right)^4 + 0 \quad - 2\left(\dfrac{A}{B}\right)^3\left(\dfrac{E}{B}\right)^2 + 35\left(\dfrac{A}{B}\right)^4\left(\dfrac{E}{B}\right)\left(\dfrac{C}{B}\right)^3 - 42\dfrac{A^5C^6}{B^{11}}, \\
& \text{etc., etc.}
\end{array}
$$

Here, when the operator $D_B^{-1} . D_E^{-1} . D_C^2$ acts on the element $\left(\dfrac{A}{B}\right)\left(\dfrac{E}{B}\right)$ in the second horizontal line, the result is zero. But we must keep record of the operations, which are as follows:—

$$\frac{(0)(-1)}{C^2} \times A \times \frac{E^2}{2} \times \left(\frac{1}{-1} \times \frac{1}{B}\right)$$

or zero. But with another action by the same operator there arises

$$\frac{(0)(-1)(-2)(-3)}{C^4} \times A \times \frac{E^3}{2 \cdot 3} \times \frac{B^0}{(-1)(0)} \quad \text{or} \quad -\frac{AE^3}{C^4}.$$

If the development be carried further, the same mode of calculating the coefficients must be observed. A little reflection will explain the appearance of these zero elements here. The two former roots had C and B for the denominators of their every element respectively. This root has *both* C and B.

In this root occurs, with changed sign, every element occurring in either of the preceding roots, with the solitary exception of the element $-\dfrac{B}{A}$, which belongs to the second root, and is the sum of all three. The function of the second operator here, therefore, is to take every element belonging to the first root and convert it gradually into an element of the second root. This it does by making the element pass through the value zero.

It ought to be added that a zero never appears in the denominator of a coefficient before there is one in the numerator, so that no elements become infinite.

(*d*) The above mode of representation soon ceasing to be practicable in the case of the quartic, quintic, etc., we adopt another notation.

For the first solution of the cubic given under paragraph (*a*) of this section, calling $D_B^{-1} . D_E^{-1} . D_C^2$ θ_1, and $D_A^{-1} . D_E^{-2} . D_C^3$ θ_2, we have

$$x = (1 + \theta_1 + \theta_1^2 + \text{etc.})(1 + \theta_2 + \theta_2^2 + \text{etc.}) - \frac{E}{C},$$

where the operational symbols $(1+\theta_1+\theta_1^2+\text{etc.})(1+\theta_2+\theta_2^2+\text{etc.})$ are evidently commutative. Similarly for the other two solutions.

(*e*) As the application of this process to the quartic, etc., is strictly similar, the only new feature being an additional operator or operators, we shall content ourselves by merely indicating these solutions. We shall also develop one of the solutions of the quartic to illustrate what is meant by saying that the solution of such an equation is a triply infinite expression.

In the quartic

$$Ax^4 + Bx^3 + Cx^2 + Ex + F = 0,$$

we have first

$$x = -\frac{F}{E} - \frac{C}{E}x^2 - \frac{B}{E}x^3 - \frac{A}{E}x^4 = -\frac{F}{E} - \frac{C}{E}\left(\frac{F}{E}\right)^2 + \frac{B}{E}\left(\frac{F}{E}\right)^3 - \frac{A}{E}\left(\frac{F}{E}\right)^4,$$

so that the operators are

$D_C^{-1} . D_F^{-1} . D_E^2$, $D_B^{-1} . D_F^{-2} . D_E^3$, and $D_A^{-1} . D_F^{-3} . D_E^4$, or θ_1, θ_2, and θ_3.

Therefore we have

$$x = (1 + \theta_1 + \theta_1^2 + \text{etc.})(1 + \theta_2 + \theta_2^2 + \text{etc.})(1 + \theta_3 + \theta_3^2 + \text{etc.}) \text{ of } -\frac{F}{E}.$$

This solution, corresponding to the first form for the cubic, we may distinguish as the direct solution, in contradistinction to the others, which are more or less of an inverse form. This direct solution, we shall see, always gives the least root.

For the second solution we write

$$x = -\frac{E}{C} - \frac{B}{C}x^2 - \frac{F}{C} \times \frac{1}{x} - \frac{A}{C}x^3 = -\frac{E}{C} - \frac{B}{C}\left(\frac{E}{C}\right)^2 + \frac{F}{C} \times \frac{C}{E} - \frac{A}{C}\left(\frac{E}{C}\right)^3,$$

and reach the operators $D_B^{-1} . D_E^{-1} . D_C^2$, $D_F^{-1} . D_C^{-1} . D_E^2$, and $D_A^{-1} . D_E^{-2} . D_C^3$, two quadratic ones and one cubic one. The form $\frac{0}{0}$ appears in the evaluation of the term $\frac{F}{C} \times \frac{C}{E}$. There are two different denominators for the elements, so that, as the solution develops, zero elements will appear as in the third form of the cubic given above.

From

$$x = -\frac{C}{B} - \frac{E}{B}\times\frac{1}{x} - \frac{F}{B}\times\frac{1}{x^2} - \frac{A}{B}x^2 = -\frac{C}{B} + \frac{E}{B}\times\frac{B}{C} - \frac{F}{B}\left(\frac{B}{C}\right)^2 - \frac{A}{B}\left(\frac{C}{B}\right)^2$$

we get the operators $D_B^{-1}\,.\,D_E^{-1}\,.\,D_C^{2}$, $D_F^{-1}\,.\,D_B^{-2}\,.\,D_C^{3}$, $D_A^{-1}\,.\,D_C^{-1}\,.\,D_B^{2}$, so that this solution will also exhibit different denominators and zero elements.

Finally, from

$$x = -\frac{B}{A} - \frac{C}{A}\frac{1}{x} - \frac{E}{A}\frac{1}{x^2} - \frac{F}{A}\frac{1}{x^3} = -\frac{B}{A} + \frac{C}{A}\times\frac{A}{B} - \frac{E}{A}\left(\frac{A}{B}\right)^2 + \frac{F}{A}\left(\frac{A}{B}\right)^3$$

we find as operators $D_C^{-1}\,.\,D_A^{-1}\,.\,D_B^{2}$, $D_E^{-1}\,.\,D_A^{-2}\,.\,D_B^{3}$, and $D_F^{-1}\,.\,D_A^{-3}\,.\,D_B^{4}$, a quartic, cubic, and quadratic operator, and the solution proceeds in descending powers of one coefficient only, viz. B.

The coefficients occurring in a development by a quadratic or cubic operator we have already learned, viz.

$$\frac{E}{C} + \frac{BE^2}{C^3} + 2\frac{B^2E^3}{C^5} + 5\frac{B^3E^4}{C^7} + 14\frac{B^4E^5}{C^9} + 42\frac{B^5E^6}{C^{11}} + 132\frac{B^6E^7}{C^{13}}, \text{ etc.,}$$

for the quadratic, and

$$\frac{E}{C} - \frac{AE^3}{C^4} + 3\frac{A^2E^5}{C^7} - 12\frac{A^3E^7}{C^{10}} + 55\frac{A^4E^9}{C^{13}} - 273\frac{A^5E^{11}}{C^{16}} + 1428\frac{A^6E^{13}}{C^{19}} - \text{etc.,}$$

for the cubic. (A cubic operator, with its three differentiations, is intrinsically − etc.) Those belonging to a quartic operator are

$$\frac{F}{E} + \frac{AF^4}{E^5} + 4\frac{A^2F^7}{E^9} + 22\frac{A^3F^{10}}{E^{13}} + 140\frac{A^4F^{13}}{E^{17}} + 969\frac{A^5F^{16}}{E^{21}} + 7084\frac{A^6F^{19}}{E^{25}}, \text{ etc.}$$

The direct solution of the quartic may be represented—on the flat at least—as follows:—

From the operators

$$D_C^{-1}\,.\,D_F^{-1}\,.\,D_E^{2},\quad D_B^{-1}\,.\,D_F^{-2}\,.\,D_E^{3},\quad D_A^{-1}\,.\,D_F^{-3}\,.\,D_E^{4} \text{ on } -\frac{F}{E}$$

we have

$$x = \qquad -\frac{F}{E}$$

$$-\frac{CF^2}{E^3} + \frac{BF^3}{E^4} \qquad\qquad -\frac{AF^4}{E^5}$$

$$-2\frac{C^2F^3}{E^5} + 5\frac{CBF^4}{E^6} - 3\frac{B^2F^5}{E^7} \qquad -6\frac{CAF^5}{E^7} + 7\frac{BAF^6}{E^8} \qquad -4\frac{A^2F^7}{E^9}$$

$$\text{etc., etc.} \qquad -28\frac{C^2AF^6}{E^9} + 72\frac{CBAF^7}{E^{10}} - 45\frac{B^2AF^8}{E^{11}} \qquad -45\frac{CA^2F^8}{E^{11}} + 55\frac{BA^2F^9}{E^{12}}$$

$$\text{etc., etc.} \qquad -275\frac{C^2A^2F^9}{E^{13}} + 858\frac{CBA^2F^{10}}{E^{14}}$$

etc., etc.

The elements $-\frac{F}{E}$, $-\frac{HF^4}{E^5}$, $-4\frac{A^2F^7}{E^9}$, etc., constituting the solution of the quartic trinomial $Ax^4+Ex+F=0$, we may suppose ranged along the z-axis, the other parallel lines being disposed parallel to the x and y axes respectively. The solution is thus triply infinite.

(f) In the case of an equation employing four or more operators, we simply develop one horizontal line after another.

Thus for the solution of the quintic $Ax^5+Bx^4-Cx^3+Ex^2+Fx+G=0$, belonging to the operand $\frac{E}{C}$, we have

$$x=\frac{E}{C}+\frac{F}{C}\cdot\frac{1}{x}+\frac{G}{C}\cdot\frac{1}{x^2}+\frac{B}{C}x^2+\frac{A}{C}x^3$$

leading to

$$\frac{E}{C}+\frac{F}{C}\cdot\frac{C}{E}+\frac{G}{C}\cdot\frac{C^2}{E^2}+\frac{B}{C}\cdot\frac{E^2}{C^2}+\frac{A}{C}\frac{E^3}{C^3},$$

and therefore to the operators

$$-D_F^{-1}\,.\,D_C^{-1}\,.\,D_E^{2},\quad D_G^{-1}\,.\,D_C^{-2}\,.\,D_E^{3},\quad D_B^{-1}\,.\,D_E^{-1}\,.\,D_C^{2},\quad -D_A^{-1}\,.\,D_E^{-2}\,.\,D_C^{3}.$$

These four, acting on the four elements of the second line, yield

$$-\frac{F^2C}{E^3}-3\frac{FGC^2}{E^4}+\frac{FB}{C^2}+2\frac{FAE}{C^3}-2\frac{G^2C^3}{E^5}+0+\frac{GA}{C^2}+2\frac{B^2E^3}{C^5}+5\frac{ABE^4}{C^6}+3\frac{A^2E^5}{C^7}.$$

E.g., let $A=B=F=G=1$, and $C=E=10$, then these elements give $x=1{\cdot}48$, which will be correct to two decimal places, though, for the closer approximation, the equation ought to be transformed.

The number of elements (including the zero) in this third line is 10, being the homogeneous products of four things two at a time; and generally the number of elements in the second, third, fourth, etc., horizontal lines representing a solution of an equation of the n^{th} degree (or an equation with $n+1$ terms) is ${}_{n-1}H_1$, ${}_{n-1}H_2$, ${}_{n-1}H_3$, etc.

(g) When the above series converge, then they converge to a root of the equation. When they first converge and ultimately diverge, then in spite of such divergence among the ultimately infinitesimal elements the root may be approximated to by following the series in its more important elements, the opening ones, though the calculation of the root may be facilitated by conducting the investigation into the proximity of the suggested root. We must, in fact, change the origin.

In the analogous case in Differential Equations, Poincaré has shown that the equations by which astronomers had, for over a century, been accustomed to calculate the motions of the heavenly bodies, do not, in their ultimate elements, converge, but diverge. Such a solution is accepted *pro tanto* and made the basis for a more accurate one. The quadratic $Ax^2 - Bx + C = 0$ will have a root belonging to the operand $\frac{C}{B}$ if the operator $D_A^{-1} . D_C^{-1} . D_B^2$ acts convergently, or if $B^2 > 4AC$. But let us suppose that we now introduce another root a, intermediate between the two roots of $Ax^2 - Bx + C = 0$, then this root a will make the quadratic operator of the smallest root act divergently, unless it be greater than twice this root, and if we introduce two roots a very close to each other, then they must be greater than four times the smallest root if the latter is to retain convergently-acting operators. None the less, this root belongs to the same operand of the new equation, and may be—though less rapidly—approximated to by using an operator with ultimately divergent action.

Divergence among elements infinitesimally small and differing in sign is an indeterminate case, to be settled only by conducting the investigation into the immediate neighbourhood of the suggested root and examining whether the new operators persist in their divergent action.

If, however, the operators—or any one of them—be steadily divergent in their action, the root obviously must be sought elsewhere.

§ IV. Proof that these are Solutions.

It may be considered advisable to give a more formal demonstration of the validity of these solutions than that based merely on continuity with the similar solutions of the quadratic, that is, solutions springing from operands with integral indices.

In the first place, then, these solutions may be established by Lagrange's Theorem. Lagrange himself, as is well known, applied his Theorem to such forms as $x = a + bx^n$, but the scope of the Theorem,

$$u = f(z) + \left\{ \phi(z) f'(z) \right\} \frac{y}{1} + \frac{d}{dz}\left[\phi(z) f'(z) \right]^2 \frac{y^2}{2!} + \text{etc.},$$

where $u = f(x)$ and $x = z + y\phi(x)$, is wide enough to embrace every solution —belonging to integral-indexed operands—of every algebraic equation by merely altering the opening term and the function ϕ.

To apply it, *e.g.*, to the solution of the quintic, see § III (f),

$$Ax^5 + Bx^4 - Cx^3 + Ex^2 + Fx + G = 0$$

belonging to the operand $\frac{E}{C}$, we put $y = 1$, $f(x) = x$, so that $f'(x) = 1$, so that we have

$$x = \frac{E}{C} + \phi(x),$$

where

$$\phi(x) = \frac{F}{C} \cdot \frac{1}{x} + \frac{G}{C} \cdot \frac{1}{x^2} + \frac{B}{C}x^2 + \frac{A}{C}x^3.$$

We have, therefore, for $\frac{1}{2!}\frac{d}{dx}\left\{\phi(x)\right\}^2$

$$\frac{1}{2!}\frac{d}{dx}\left[\frac{F^2}{C^2}\frac{1}{x^2} + \frac{G^2}{C^2} \cdot \frac{1}{x^4} + \frac{B^2}{C^2}x^4 + \frac{A^2}{C^2}x^6 + 2\frac{FG}{C^2} \cdot \frac{1}{x^3} + 2\frac{FB}{C^2}x + 2\frac{FA}{C^2}x^2 + 2\frac{GB}{C^2} + 2\frac{GA}{C^2}x + 2\frac{BA}{C^2}x^5\right],$$

or

$$-\frac{F^2}{C^2} \cdot \frac{1}{x^3} - 2\frac{G^2}{C^2} \cdot \frac{1}{x^5} + 2\frac{B^2}{C^2}x^3 + 3\frac{A^2}{C^2}x^5 - 3\frac{FG}{C^2}\frac{1}{x^4} + \frac{FB}{C^2} + 2\frac{FA}{C^2}x + 0 + \frac{GA}{C^2} + 5\frac{BA}{C^2}x^4,$$

which, with $x = \frac{E}{C}$, the operand or initial value, gives the third line in the development previously obtained from the operators, as

$$\frac{E}{C} \text{ and } \frac{F}{C} \cdot \frac{C}{E} + \frac{G}{C} \cdot \frac{C^2}{E^2} + \frac{BE^2}{C^3} + \frac{AE^3}{C^4}$$

are the first and second lines.

The reader can easily verify that the trend of the two expansions is the same, a term in Lagrange's development being a line, in the development from the operators.

If $\phi(z)$ contain $n-1$ terms consisting of positive or negative powers of z, the number of terms in its square, third power, etc., is also ${}_{n-1}H_2$, ${}_{n-1}H_3$, etc. See § III (f).

If we split Lagrange's $\phi(z)$ into $\psi(z) + f(z)$, his theorem still holds. We may now, starting from the case of the quadratic, build up a proof for the general case, by Induction, make $f(z)$ include z^{+3}, z^{+4}, etc., in succession.

Lagrange showed further that the equation $Ax^n + Bx + C = 0$ always yielded its least root from his application of his own Theorem. This was because he always took for operand $-\frac{C}{B}$ the operand for the direct solution which always leads to the numerically smallest root.

We may now extend this remark of Lagrange's by adding that if, in the equation

$$x^n + p_1 x^{n-1} + p_2 x^{n-2} \; . \; . \; . \; + p_{n-2} x^2 + p_{n-1} x + p_n = 0,$$

there be roots belonging to the operands

$$-\frac{p_n}{p_{n-1}}, \quad -\frac{p_{n-1}}{p_{n-2}} \; . \; . \; . \; -\frac{p_1}{1},$$

then these, as also the operands from which they spring, are, irrespective of sign, in ascending order of magnitude. For, take two consecutive operands,

$$-\frac{p_{r+1}}{p_r} \text{ and } -\frac{p_r}{p_{r-1}}.$$

If there be a root belonging to the operand $-\frac{p_{r+1}}{p_r}$, this implies, as we shall see, that its quadratic operator $D^{-1}_{p_{r-1}} \, . \, D^{-1}_{p_{r+1}} \, . \, D^{2}_{p_r}$ acts convergently, and this again means that $p_r^2 > 4p_{r+1} \, . \, p_{r-1} > p_{r+1} \, . \, p_{r-1}$.

Hence

$$\frac{p_r}{p_{r-1}} > \frac{p_{r+1}}{p_r}.$$

As it is not axiomatic that the roots follow in magnitude their operands, we shall show this to be true for $\phi(x) = Ax^3 + Bx^2 + Cx + E = 0$ by a proof that is easily seen to be general. Let the graph of this function be drawn, values of ϕ being indicated along the y-axis as usual. Let now two indicators, starting from the origin, move with equal velocities to the right and left along the x-axis, both pausing for a moment whenever either passes through a value of x that makes ϕ vanish. Let this occur first when $x = a_1$, then the origin being transferred to $(a_1, 0)$ and the function transformed, the absolute term vanishes, the coefficient of the original function which disappears is E, and the root which has disappeared —the least root thus—is the one which had E in the numerator of its every element, that is, the one whose operand was $-\frac{E}{C}$. The indicators now resuming their journey, let the next halt occur when x has a value ξ which causes $A\xi^2 + (3Aa_1 + B)\xi + 3Aa_1^2 + 2Ba_1 + C$ to vanish. The origin being again shifted, and the function transformed, the absolute term again vanishes, and now it is the original coefficient C that disappears. The root which has disappeared, then, is one having C or E (but not E alone) in the numerator of its every element, that is, the one whose operand was $-\frac{C}{B}$. The root ξ which has disappeared being $a_2 \backsim a_1$, the algebraic difference between the two least roots, there remains now a linear form $A\xi_1 + 2A(a_2 - a_1) + 3Aa_1 + B = 0$, giving

$$\xi_1 = -\frac{B}{A} - 2(a_2 - a_1) - 3a_1 = a_1 + a_2 + a_3 - 2(a_2 - a_1) - 3a_1 = a_3 - a_2 .$$

And $x = \xi_1 + a_2 = a_3$, a root shown retracted into a single term, its initial operand. On referring to the solutions (*a*), (*b*), and (*c*) in § III, it will be seen that (*a*) vanishes completely with E, and reduces the other two to single rays; the vanishing of C next causes (*c*) to vanish, and reduces (*b*) to a single term.

The proof here outlined is easily seen to be of general application, and may be extended to include equal and imaginary roots. A pair of equal roots will fall out in their turn when the indicator passes through their value, taking with them *two* coefficients, and when the real part of two imaginary roots $a \pm i\beta$ is reached, these will also disappear, along with two coefficients, if the origin be shifted so as to make the radius vector $\sqrt{(x-a)^2 + \beta^2}$ disappear.

This proposition is of importance practically in enabling us to see, often by mere inspection, that the roots cannot all belong to operands with integral indices. Such equations are in fact the exceptional ones, as will be seen from the following definition of an operand. As operand of an equation may, or must, be taken a value of x derived from any two terms in the equation, such as $Rx^r - R_1x^{r_1} = 0$, or $x = \left(\frac{R_1}{R}\right)^{\frac{1}{r - r_1}}$. If $r - r_1 = 1$, we have an integral-indexed operand, otherwise a fractional-indexed one.

To introduce the roots derived from fractional-indexed operands, we begin with the trinomial form $Ax^n - Bx^m + C = 0$. Lagrange's Theorem now failing, we employ common Reversion of Series.

Writing first

$$x^m = \frac{C}{B} + \frac{A}{B}x^n, \quad \text{or} \quad x = \left(\frac{C}{B} + \frac{A}{B}x^n\right)^{\frac{1}{m}}$$

we have first

$$x = \left(\frac{C}{B}\right)^{\frac{1}{m}}.$$

Next

$$x = \left[\frac{C}{B} + \frac{AC^{\frac{n}{m}}}{B^{\frac{n+m}{m}}}\right]^{\frac{1}{m}}$$

giving

$$x = \left(\frac{C}{B}\right)^{\frac{1}{m}} + \frac{1}{m} \cdot \frac{A}{1} \cdot \frac{C^{\frac{n-m+1}{m}}}{B^{\frac{n+1}{m}}}$$

The value of x^n derivable from this is

$$x^n = \left(\frac{C}{B}\right)^{\frac{n}{m}} + \frac{n}{m} \cdot \frac{A}{1} \cdot \frac{C^{\frac{2n-m}{m}}}{B^{\frac{2n}{m}}},$$

from which we have

$$x = \left[\frac{C}{B} + \frac{AC^{\frac{n}{m}}}{B^{\frac{n+m}{m}}} + \frac{n}{m}\frac{A^2}{1} \cdot \frac{C^{\frac{2n-m}{m}}}{B^{\frac{2n+m}{m}}} \right]^{\frac{1}{m}}$$

giving

(1) $$x = \left(\frac{C}{B}\right)^{\frac{1}{m}} + \frac{1}{m} \cdot \frac{A}{1} \cdot \frac{C^{\frac{n-m+1}{m}}}{B^{\frac{n+1}{m}}} + \frac{1}{m} \cdot \frac{2n-m+1}{m} \cdot \frac{A^2}{2\,!} \frac{C^{\frac{2n-2m+1}{m}}}{B^{\frac{2n+1}{m}}}$$

and the next two terms, got in the same way, are

$$\frac{1}{m} \cdot \frac{3n-m+1}{m} \cdot \frac{3n-2m+1}{m} \cdot \frac{A^3}{3\,!} \cdot \frac{C^{\frac{3n-3m+1}{m}}}{B^{\frac{3n+1}{m}}}$$

$$+ \frac{1}{m} \cdot \frac{4n-m+1}{m} \cdot \frac{4n-2m+1}{m} \cdot \frac{4n-3m+1}{m} \cdot \frac{A^4}{4\,!} \cdot \frac{C^{\frac{4n-4m+1}{m}}}{B^{\frac{4n+1}{m}}}.$$

I leave the reader to verify these last two terms, and also to verify the formula further, generally, by obtaining by direct inversion the root of $Ax^n - Bx + C = 0$.

(2) $$x = \frac{C}{B} + \frac{AC^n}{B^{n+1}} + \frac{2n}{1} \cdot \frac{A^2\,C^{2n-1}}{2\,!\;B^{2n+1}} + 3n\,(3n-1)\,.\frac{A^3\,C^{3n-2}}{3\,!\;B^{3n+1}} + 4n(4n-1)(4n-2)\,\frac{A^4\,C^{4n-3}}{4\,!\;B^{4n+1}}$$

etc., got from (1) by putting $m=1$. But the latter series is the development of the operator $D_A^{-1}.D_C^{-(n-1)}.D_B^{\,n}$, the general type of operator that acts on operands with integral indices. The reader may verify further that both formula (2) and operator hold when n is negative.

Turning to formula (1), we shall find that it also is the development of an operator $D_A^{-1}.D_C^{-\left(\frac{n}{m}-1\right)}.D_B^{\frac{n}{m}}$ similar to the operator $D_A^{-1}.D_C^{-(n-1)}.D_B^{\,n}$, but the symbols $D^{\frac{n}{m}}$ $D^{-\left(\frac{n}{m}-1\right)}$ can no longer be regarded as those of the Differential and Integral Calculus. To investigate the action of the symbol $D_B^{\frac{n}{m}}$ we recur to the formula

(3) $$D_B^{\,p} \,.\, B^{q} = q\,.\,(q-1)\,.\,.\,.\,.\,.\,(q-p+1)\,B^{q-p}\,.$$

In formula (1) $D_B^{\frac{n}{m}}$ has to act on indices of the form $\frac{1}{B^{\frac{rn+1}{m}}}$; but whether n be greater or less than m, so long as it is not a multiple of m, we do not take D a full time or times, and the left-hand terminal in (3) cannot be q or $\frac{rn+1}{m}$. This terminal has, in fact, become indeterminate. The right-hand terminal, on the other hand, remains fast, for the only thing we know about $D_B^{\frac{n}{m}}$ is that it diminishes the index of any power of B by $\frac{n}{m}$, and thus converts $\frac{1}{B^{\frac{rn+1}{m}}}$ into $\frac{1}{B^{\frac{(r+1)n+1}{m}}}$. The last multiplier, therefore, must have been this new index diminished by 1, or $\frac{(r+1)n+1}{m}-1$, retaining this feature of common differentiation if and as we may. We write, therefore, provisionally for $D^{\frac{n}{m}} \cdot \frac{1}{B^{\frac{rn+1}{m}}}$,

$$F\left\{\frac{(r+1)n+1}{m}-1\right\}\frac{1}{B^{\frac{(r+1)n+1}{m}}},$$

and for

$$(4) \qquad D_C^{-\left(\frac{n}{m}-1\right)} \cdot D_B^{\frac{n}{m}} \frac{C^{\frac{rn-rm+1}{m}}}{B^{\frac{rn+1}{m}}}, \quad \frac{1}{c} \cdot \frac{F\left\{\frac{(r+1)n+1}{m}-1\right\}}{F\left\{\frac{(r+1)(n-m)+1}{m}\right\}} \frac{C^{\frac{(r+1)(n-m)+1}{m}}}{B^{\frac{(r+1)n+1}{m}}},$$

where c is some possible factor independent of r.

Putting $r=0, 1, 2$, etc., in, we have from (1)

$$\frac{1}{c}\frac{F\left(\frac{n+1}{m}-1\right)}{F\left(\frac{n-m+1}{m}\right)}=\frac{1}{m}, \quad \frac{1}{c}\frac{F\left(\frac{2n+1}{m}-1\right)}{F\left(\frac{2n+1}{m}-2\right)}=\frac{1}{m}\cdot\frac{2n-m+1}{m}, \quad \frac{1}{c}\cdot\frac{F\left(\frac{3n+1}{m}-1\right)}{F\left(\frac{3n+1}{m}-3\right)}$$

$$=\frac{1}{m}\cdot\frac{3n-m+1}{m}\cdot\frac{3n-2m+1}{m}, \text{ etc., etc.}$$

From the first of these we see that $c=m$, and the others are then all satisfied by assigning to $F\left(\frac{rn+1}{m}-1\right)$ the form

$$(5) \qquad \left(\frac{rn+1}{m}-1\right)\left(\frac{rn+1}{m}-2\right)\left(\frac{rn+1}{m}-3\right) \ldots \textit{ad infinitum.}$$

I leave the reader to verify that, had we solved for x^m, we should have got

$$x^m=\frac{C}{B}+\frac{AC^{\frac{n}{m}}}{B^{\frac{n}{m}+1}}+\frac{2n}{m}\cdot\frac{A^2}{2!}\cdot\frac{C^{\frac{2n}{m}-1}}{B^{\frac{2n}{m}+1}}+\frac{3n}{m}\cdot\left(\frac{3n}{m}-1\right)\frac{A^3}{3!}\frac{C^{\frac{3n}{m}-2}}{B^{\frac{3n}{m}+1}}, \text{ etc.} \qquad (6)$$

and that this is the work of the same operator $D_A^{-1} \cdot D_C^{-\left(\frac{n}{m}-1\right)} \cdot D_B^{\frac{n}{m}}$ on $\frac{C}{B}$, as generally to get the p^{th} power (or extract the p^{th} root) of any root we have simply to raise the operand to the p^{th} power (or take its p^{th} root) and act on this with the same operator or operators unchanged.

Regarding (5) as an attempt on the part of the surviving terminal multiplier in (3) to reach its terminal partner which has disappeared, then in (6) we see that this quest will be successful when $\frac{n}{m}$ is an integer, and the operator $D^{\frac{n}{m}}$ coincides with the ordinary symbol of differentiation. If $\frac{n}{m}$ be not an integer in (6), however, and always in (1) where the root has been extracted, the terminal partner is never reached, the quest is perpetuated to infinity, and determinateness, tangibility, real quantity thus is only to be attained by the aid of a similar operator acting inversely. Thus unless $\frac{n}{m}$ be integral, the occurrence of $D^{\frac{n}{m}}$ necessitates the presence of $D^{-(\frac{n}{m}-1)}$, its "conjugate" operator, and in this way we avoid all reference to fractional differentiation (the occurrence of $D^{\frac{n}{m}}$ alone), speculation about which must be endless till we can control it by concrete results. (With this interpretation of $D^{\frac{n}{m}}$, *cf.* Gauss Π function.)

To illustrate this Method of Operators further, in possession of which, it will be seen, we have left both Lagrange's Theorem and the Differential Calculus behind, suppose we ascertain, by a test to be given presently, that the cubic $Ax^3+Bx^2+Cx-E=0$ has three roots belonging to the operand $\left(\frac{E}{A}\right)^{\frac{1}{3}}$. We write therefore

$$x^3=\frac{E}{A}-\frac{B}{A}x^2-\frac{C}{A}x,$$

or with $x=\left(\frac{E}{A}\right)^{\frac{1}{3}}$ as initial value,

$$x=\left\{\frac{E}{A}-\frac{BE^{\frac{2}{3}}}{A^{\frac{5}{3}}}-\frac{CE^{\frac{1}{3}}}{A^{\frac{4}{3}}}\right\}^{\frac{1}{3}}.$$

We now go through the formal process of extracting the root once. This gives

$$x=\left(\frac{E}{A}\right)^{\frac{1}{3}}-\frac{1}{3}\frac{B}{A}-\frac{1}{3}\cdot\frac{C}{1}\cdot\frac{E^{-\frac{1}{3}}}{A^{\frac{2}{3}}}.$$

The conversion of the operand $\left(\frac{E}{A}\right)^{\frac{1}{3}}$ into $-\frac{1}{3}\frac{B}{A}$ and $-\frac{1}{3}\frac{C}{1}\frac{E^{-\frac{1}{3}}}{A^{\frac{2}{3}}}$ is a determinate problem, with only one solution.* The necessary operators are $D_{(-B)}^{-1}.D_E^{-(\frac{2}{3}-1)}.D_A^{\frac{2}{3}}$ and $D_{(-C)}^{-1}.D_E^{-(\frac{1}{3}-1)}.D_A^{\frac{1}{3}}$, and these will give a rigorously

* Dropping the constant $\frac{1}{3}$, of course.

accurate account of the whole residue of the process to its most distant utterance.

The solutions so developed are

$$\left(\text{operators } D_{(-B)}^{-1} \cdot D_E^{-(\frac{2}{3}-1)} \cdot D_A^{\frac{2}{3}},\ D_{(-C)}^{-1} \cdot D_E^{-(\frac{1}{3}-1)} \cdot D_A^{\frac{1}{3}}\right).$$

$$\left(\frac{E}{A}\right)^{\frac{1}{3}}$$

$$-\frac{1}{3}\frac{B}{A} \qquad\qquad -\frac{1}{3}\frac{C}{1}\frac{E^{-\frac{1}{3}}}{A^{\frac{2}{3}}}$$

$$+\frac{1}{3}\cdot\frac{2}{3}\cdot\frac{B^2}{2!}\frac{E^{-\frac{1}{3}}}{A^{\frac{5}{3}}}+\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{B}{1}\cdot\frac{C}{1}\cdot\frac{E^{-\frac{2}{3}}}{A^{\frac{4}{3}}}+\frac{1}{3}\cdot 0\cdot\frac{C^2}{2!}\frac{E^{-1}}{A^1} \quad . \quad . \quad . \quad (7)$$

$$-\frac{1}{3}\cdot\frac{4}{3}\cdot\frac{1}{3}\cdot\frac{B^3}{3!}\frac{E^{-\frac{2}{3}}}{A^{\frac{7}{3}}}-\frac{1}{3}\cdot 1\cdot 0\cdot\frac{B^2}{2!}\cdot\frac{C}{1}\frac{E^{-1}}{A^2}+\frac{1}{3}\cdot\frac{2}{3}\cdot\frac{1}{3}\cdot\frac{B}{1}\cdot\frac{C^2}{2!}\frac{E^{-\frac{4}{3}}}{A^{\frac{5}{3}}}+\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{2}{3}\cdot\frac{C^3}{3!}\frac{E^{-\frac{5}{3}}}{A^{\frac{4}{3}}}$$

$$+\frac{1}{3}\cdot 2\cdot 1\cdot 0\cdot\frac{B^4}{4!}, \text{ etc.} \qquad +\frac{1}{3}\cdot\frac{5}{3}\cdot\frac{2}{3}\cdot\frac{1}{3}\cdot\frac{B^3}{3!}\frac{C}{1}\cdot\frac{E^{-\frac{4}{3}}}{A^{\frac{8}{3}}}+\frac{1}{3}\cdot\frac{4}{3}\cdot\frac{1}{3}\cdot\frac{2}{3}\frac{B^2}{2!}\cdot\frac{C^2}{2!}\frac{E^{-\frac{5}{3}}}{A^{\frac{7}{3}}}-\frac{1}{3}\cdot 1\cdot 0, \text{ etc.}$$

$$+\frac{1}{3}\cdot\frac{2}{3}\cdot\frac{1}{3}\cdot\frac{4}{3}\cdot\frac{C^4}{4!}\frac{E^{-\frac{7}{3}}}{A^{\frac{5}{3}}}, \text{ etc., etc.}$$

Before remarking on this development, I shall give an example. Rearranging, as we must always do, in powers of the surd, we have

$$x=-\frac{1}{3}\frac{B}{A}+\left(\frac{E}{A}\right)^{\frac{1}{3}}\left[1+\frac{1}{9}\frac{BC}{EA}-\frac{2}{81}\frac{B^3}{EA^2}+\frac{1}{81}\cdot\frac{C^3}{AE^2}+\frac{2}{81}\frac{B^2C^2}{A^2E^2}-\frac{7}{729}\frac{B^4C}{A^3E^2}, \text{ etc.}\right]$$
$$-\left(\frac{E}{A}\right)^{\frac{2}{3}}\left[\frac{1}{3}\frac{C}{E}-\frac{1}{9}\frac{B^2}{AE}-\frac{1}{27}\frac{BC^2}{AE^2}-\frac{5}{243}\frac{B^3C}{A^2E^2}+\frac{1}{243}\frac{C^4}{AE^3}-\frac{2}{729}\frac{B^5}{A^3E^2}, \text{ etc.}\right].$$

This gives for the equation $x^3+x^2+x-100=0$

$$x=-\frac{1}{3}+100^{\frac{1}{3}}\left[1+\frac{1}{900}-\frac{1}{4050}+\frac{1}{810000}, \text{ etc.}\right]-100^{\frac{2}{3}}\left[\frac{1}{300}-\frac{1}{900}-\frac{1}{270000}, \text{ etc.}\right]$$
$$=-\frac{1}{3}+(4{\cdot}6415)(1{\cdot}000865)-(21{\cdot}544)({\cdot}0022185) \text{ or } 4{\cdot}2644 \text{ (approx.)}.$$

And to get the other two roots we must affect 4·6415 and 21·544, first by ω and ω^2 respectively, and then by ω^2 and ω, where ω is a special cube root of 1.

It is advisable to give still another example before discussing the coefficients in (7). The reader will easily find the remaining $(n-m)$ roots of the equation $Ax^n-Bx^m+C=0$ given from the operand $\left(\frac{B}{A}\right)^{\frac{1}{n-m}}$ by the operator $D_{(-C)}^{-1} \cdot D_A^{-\frac{m}{n-m}} \cdot D_B^{\frac{n}{n-m}}$.

They are

$$\left(\frac{B}{A}\right)^{\frac{1}{n-m}}+\frac{1}{n-m}\cdot\frac{(-C)}{1}\cdot\frac{A^{\frac{m-1}{n-m}}}{B^{\frac{n-1}{n-m}}}-\frac{1}{n-m}\cdot\frac{n+m-1}{n-m}\cdot\frac{(-C)^2}{2!}\frac{A^{\frac{2m-1}{n-m}}}{B^{\frac{2n-1}{n-m}}}$$
$$+\frac{1}{n-m}\cdot\frac{2n+m-1}{n-m}\cdot\frac{n+2m-1}{n-m}\cdot\frac{(-C)^3}{3!}\frac{A^{\frac{3m-1}{n-m}}}{B^{\frac{3n-1}{n-m}}}, \text{ etc.}$$

The equation $x^4 - 12x + 7 = 0$ has three roots belonging to $x^3 = 12$. We have

$$x = \left\{\frac{12}{1} - \frac{7}{1} \cdot \frac{1}{x}\right\}^{\frac{1}{3}},$$

and then

$$x = \left\{\frac{12}{1} - \frac{7}{1}\left(\frac{1}{12}\right)^{\frac{1}{3}}\right\}^{\frac{1}{3}} = \left(\frac{12}{1}\right)^{\frac{1}{3}} - \frac{1}{3} \cdot \frac{7}{1}\left(\frac{1}{12}\right)^{\frac{1}{3}}\left(\frac{1}{12}\right)^{\frac{2}{3}}$$

The operator is therefore $D_{(-7)}^{-1} . D_1^{-\frac{1}{3}} . D_{12}^{\frac{4}{3}}$.

This gives

$$x = \left(\frac{12}{1}\right)^{\frac{1}{3}} + \frac{1}{3}\frac{(-7)}{1} \cdot \frac{1}{12} - \frac{1}{3} \cdot \frac{4}{3} \cdot \frac{(-7)^2}{2!} \cdot \frac{1^{\frac{1}{3}}}{12^{\frac{7}{3}}} + \frac{1}{3} \cdot \frac{8}{3} \cdot \frac{5}{3} \cdot \frac{(-7)^3}{3!} \cdot \frac{1^{\frac{2}{3}}}{12^{\frac{11}{3}}}$$
$$- \frac{1}{3} . 4 . 3 . 2 . \frac{(-7)^4}{4!} \cdot \frac{1^1}{12^{\frac{15}{3}}} + \frac{1}{3} \cdot \frac{16}{3} \cdot \frac{13}{3} \cdot \frac{10}{3} \cdot \frac{7}{3} \cdot \frac{(-7)^5}{5!}\frac{1^{\frac{4}{3}}}{12^{\frac{19}{3}}} - \text{etc.} \quad (8)$$

These operators no longer obey the Index Law (as did the operators with which we began), and each individual term must be derived directly from the operand. We bring the indices of 12 and 1 to the required value, and then the multiplying factor may be written down. The operator for (8) is $D_{(-7)}^{-1} . D_1^{-\frac{1}{3}} . D^{\frac{4}{3}}$, so that the integration with respect to 1 is a real integration.

The last coefficient $\left(\text{dropping the constant } \frac{1}{3}\right)$ is thus

$$\frac{\left(-\frac{19}{3}+1\right)\left(-\frac{19}{3}+2\right)\left(-\frac{19}{3}+3\right)\left(-\frac{19}{3}+4\right)\left(-\frac{19}{3}+5\right)\left(-\frac{19}{3}+6\right)}{\left(\frac{4}{3}\right)\left(\frac{4}{3}-1\right)}, \text{ etc.,}$$

and the practical rule is that the infinite factor to the right here, from and after the term $\dfrac{\left(-\frac{19}{3}+5\right)}{\frac{4}{5}}$, is to be evaluated as $+1$. (It may in reality be $+1$ or -1 as we please to include in it an odd or even number of terms.)* Thus the above multiplier is eventually +etc., being

$$\left(-\frac{16}{3}\right)\left(-\frac{13}{3}\right)\left(-\frac{10}{3}\right)\left(-\frac{7}{3}\right).$$

Turning now to (7) we find the operator here to be $D_{(-B)}^{-1} . D_E^{-(\frac{2}{3}-1)} . D_A^{\frac{2}{3}}$, so that the operation carried out with respect to E (as in every case when $\frac{n}{m} < 1$) is really a differentiation, or, as it is perhaps better to drop this word, it is an operation reducing E's index, and thus of essentially the same character as that carried out with respect to A. The elements in the numerator and denominator must therefore be represented as either

* But see later.

both increasing or both decreasing. We choose the latter, and introduce as many changes of sign from the indeterminate factor on the right as are needed to change, first, the sign of the element taken from the denominator and as many others as survive.

Thus to get the coefficient of

$$\frac{B^5}{5!} \cdot \frac{E^{-\frac{4}{3}}}{A^{\frac{11}{3}}} \quad \text{or} \quad \frac{B^5}{5!} \cdot \frac{1}{A^{\frac{11}{3}}} E^{\frac{4}{3}}$$

we change $-\frac{11}{3}$ to $+\frac{11}{3}$ and make it decrease till it meets $-\frac{4}{3}$, the index of E, stopping, of course, one short of this. Thus we have

$$\frac{\left(\frac{11}{3}-1\right)\cdot\left(\frac{11}{3}-2\right)\left(\frac{11}{3}-3\right)\left(\frac{11}{3}-4\right)\left(\frac{11}{3}-5\right)}{\frac{4}{3}}$$

or

$$+\left(\frac{8}{3}\right)\left(+\frac{5}{3}\right)\left(+\frac{2}{3}\right)\left(-\frac{1}{3}\right),$$

and the term is

$$+\frac{1}{3}\cdot\frac{8}{3}\cdot\frac{5}{3}\cdot\frac{2}{3}\cdot\frac{1}{3}\,\frac{B^5}{5!}\,\frac{1}{A^{\frac{11}{3}}}\,E^{\frac{4}{3}}.$$

$\left[\frac{(-B^5)}{5\,!}\right.$ introduces another − etc. sign here.$\left.\right]$

In the latter case, where $\frac{11}{3}-5$ and $-\frac{4}{3}$ have the same sign, there is no difficulty in seeing why the infinite factor to the right should be $+1$. But the same holds also for the above case, where the first factors that cancel, $\left(-\frac{19}{3}+5\right)$ and $\frac{4}{5}$, have unlike signs. In this case, both above and below, we pass through 0, and thus have the form $\frac{0}{0}$.

Having now ascertained the general form of operator for the solution of the trinomial form $Ax^n - Bx^m + C = 0$, we can complete our proof of the validity of such solutions for the general equation, left over from the beginning of the section. For clearly, if there be solutions of the trinomial form $Ax^n - Bx^m + C = 0$, got from the operator $D_A^{-1}.D_C^{-\left(\frac{n}{m}-1\right)}.D_B^{\frac{n}{m}}$ and the operand $-\left(\frac{C}{B}\right)^{\frac{1}{m}}$, and if the same operand remain available for the form $Ax^n + Rx^r - Bx^m + C = 0$, then the operator $D_R^{-1}.D_C^{-\left(\frac{r}{m}-1\right)}.D_B^{\frac{r}{m}}$ must take part in the solution of the latter, so as to recover the particular solution $(A=0)$ from the general, and so for the inclusion of $R_1x^{r_1}$, etc., in the form of the general equation.

The point about the non-reduplication of elements may be established by reversing for a few steps any tetranomial or pentenomial form. It is proper perhaps to point out further that, so far as the trinomial form $Ax^n - Bx + C = 0$ is concerned, there is another operator, $n \cdot \frac{A}{B} \cdot D_C^{-(n-1)} \cdot D_B^{n-1}$, equally effective in giving the solution. In the solution (a) of the cubic $Ax^3 + Bx^2 + Cx + E = 0$ in § III, *e.g.*, the operator $-2\frac{B}{C} \cdot D_E^{-1} \cdot D_C$ will give the left-hand ray, which is the solution of $B\xi^2 + C\xi + E = 0$. It will not, however, develop any ray parallel to this one, and therefore operators of this type must be rejected.

§ V. Convergency for a typical Operator.

We proceed now to deduce a test for the convergency of the development arising from the operators on any operand.

Let us examine the convergency of the n roots belonging to the operand $\left(\frac{C}{A}\right)^{\frac{1}{n}}$ in the trinomial $Ax^n \pm Bx^m \pm C = 0$ * $(n > m)$.

We have

$$x = \left\{\frac{C}{A} + \frac{B}{A} x^m\right\}^{\frac{1}{n}}$$

giving $\left(\frac{C}{A}\right)^{\frac{1}{n}}$ first, and then

$$\left\{\frac{C}{A} + \frac{BC^{\frac{m}{n}}}{A^{\frac{m}{n}+1}}\right\}^{\frac{1}{n}}.$$

This gives

$$x = \left(\frac{C}{A}\right)^{\frac{1}{n}} + \frac{1}{n} \frac{B}{1} \cdot \frac{C^{\frac{m-n+1}{n}}}{A^{\frac{m+1}{n}}},$$

so that the operator is $D_B^{-1} \cdot D_C^{-\left(\frac{m}{n}-1\right)} \cdot D_A^{\frac{m}{n}}$.

and the n roots, if they belong here, will be given by the formula

$$x = \left(\frac{C}{A}\right)^{\frac{1}{n}} + \frac{1}{n} \frac{B}{1} \cdot \frac{C^{\frac{m-n+1}{n}}}{A^{\frac{m+1}{n}}} + \frac{1}{n} \cdot \frac{2m-n+1}{n} \cdot \frac{B^2}{2!} \frac{C^{\frac{2m-2n+1}{n}}}{A^{\frac{2m+1}{n}}}$$
$$+ \frac{1}{n} \cdot \frac{3m-n+1}{n} \cdot \frac{3m-2n+1}{n} \cdot \frac{B^3}{3!} \frac{C^{\frac{3m-3n+1}{n}}}{A^{\frac{3m+1}{n}}} + \text{etc.}$$

* We take here such a combination of signs as excludes negative operands. We are considering merely arithmetical ratios, and for an imaginary root its modulus may be substituted.

The $(r+1)^{\text{th}}$ term here is

$$\frac{1}{n}\cdot\frac{rm-n+1}{n}\cdot\frac{rm-2n+1}{n}\cdot\frac{rm-3n+1}{n}\cdots\frac{rm-\overline{r-1}\,.\,n+1}{n}\cdot\frac{B^r}{r!}\cdot\frac{C^{\frac{rm-rn+1}{n}}}{A^{\frac{rm+1}{n}}}.$$

The $(r+2)^{\text{th}}$ term is

$$\frac{1}{n}\cdot\frac{(r+1)m-n+1}{n}\cdot\frac{(r+1)m-2n+1}{n}\cdots\frac{(r+1)m-(r-1)n+1}{n}\cdot\frac{(r+1)m-rn+1}{n}$$
$$\times\frac{B^{r+1}}{(r+1)\,!}\cdot\frac{C^{\frac{(r+1)(m-n)+1}{n}}}{A^{\frac{(r+1)m+1}{n}}}.$$

The ratio of the latter to the former is

$$\left[\frac{(r+1)m-n+1}{rm-n+1}\cdot\frac{(r+1)m-2n+1}{rm-2n+1}\cdot\frac{(r+1)m-3n+1}{rm-3n+1}\cdots\frac{(r+1)m-(r-1)n+1}{rm-(r-1)n+1}\cdot\right]$$
$$\frac{(r+1)m-rn+1}{(r+1)n}\cdot\frac{BC^{\frac{m-n}{n}}}{A^{\frac{m}{n}}}.$$

The expression within the square brackets, which we call L, is

$$\left(1+\frac{m}{rm-n+1}\right)\left(1+\frac{m}{rm-2n+1}\right)\ldots\text{ to }(r-1)\text{ terms},$$

and is seen to be akin to $\underset{p=\infty}{\mathrm{L}}\left(1+\frac{1}{p}\right)$.

To evaluate L when $r=\infty$, raise it to the power of $\frac{n}{m}$.

Thus

$$\left(1+\frac{m}{rm-n+1}\right)^{\frac{n}{m}}=1+\frac{n}{m}\cdot\frac{m}{rm-n+1}+\frac{n}{m}\left(\frac{n}{m}-1\right)\frac{1}{2!}\frac{m^2}{(rm-n+1)^2}=\frac{rm+1+\dfrac{\text{terms}}{(rm-n+1)}}{rm-n+1},$$

which, when r becomes infinite, tends to $\dfrac{rm+1}{rm-n+1}$.

Similarly, the second factor, $\dfrac{(r+1)m-2n+1}{rm-2n+1}$, by raising it to the $\left(\dfrac{n}{m}\right)^{\text{th}}$ power, becomes $\dfrac{rm-n+1}{rm-2n+1}$, and so for the others.

Hence

$$\mathrm{L}^{\frac{n}{m}}=\frac{(rm+1)}{1}\cdot\frac{(rm-n+1)}{rm-n+1}\cdot\frac{rm-2n+1}{rm-2n+1}\cdots\frac{rm-(r-2)n+1}{rm-(r-2)n+1}\cdot\frac{1}{rm-(r-1)n+1}$$
$$=\frac{rm+1}{rm-(r-1)n+1},\text{ which, when }r=\infty,\text{ becomes }\frac{m}{m-n}.$$

Hence when r becomes indefinitely great, $\mathrm{L}^{\frac{n}{m}}$ tends to the limit $\dfrac{m}{m-n}$, and L itself to $\left(\dfrac{m}{m-n}\right)^{\frac{m}{n}}$.

Hence the above ratio tends to the limit

$$(1) \quad . \quad . \quad . \quad . \quad . \quad \left(\frac{m}{m-n}\right)^{\frac{m}{n}} \cdot \frac{m-n}{n} \cdot \frac{BC^{\frac{m-n}{n}}}{A^{\frac{m}{n}}}$$

for indefinitely large values of r, which limiting value must therefore be <1. If this condition be fulfilled, then there are n roots of this trinomial form belonging to this operand $\left(\frac{C}{A}\right)^{\frac{1}{n}}$ and the operator $D_B^{-1} . D_C^{-(\frac{m}{n}-1)} . D_A^{\frac{m}{n}}$, and *vice versa*.

Similarly for the roots belonging to $\left(\frac{C}{B}\right)^{\frac{1}{m}}$ and the operator

$$D_A^{-1} . D_C^{-(\frac{n}{m}-1)} . D_B^{\frac{n}{m}}$$

in the same equation $Ax^n \pm Bx^m \pm C = 0$, and given by the formula

$$x = \left(\frac{C}{B}\right)^{\frac{1}{m}} + \frac{1}{m} \cdot \frac{A}{1} \cdot \frac{C^{\frac{n-m+1}{m}}}{B^{\frac{n+1}{m}}} + \frac{1}{m} \cdot \frac{2n-m+1}{m} \cdot \frac{A^2\, C^{\frac{2n-2m+1}{m}}}{2!\, A^{\frac{2n+1}{m}}}$$

$$+ \frac{1}{m} \cdot \frac{3n-m+1}{m} \cdot \frac{3n-2m+1}{m} \cdot \frac{A^3\, C^{\frac{3n-3m+1}{m}}}{3!\, A^{\frac{3n+1}{m}}}, \text{ etc.,}$$

we find the test for convergency to be

$$(2) \quad . \quad . \quad . \quad . \quad . \quad \left(\frac{n}{n-m}\right)^{\frac{n}{m}} \cdot \frac{n-m}{m} \cdot \frac{C^{\frac{n-m}{m}} . A}{B^{\frac{n}{m}}} < 1.$$

And finally for the roots belonging to $\left(\frac{B}{A}\right)^{\frac{1}{n-m}}$ and the operator

$$D_{(-C)}^{-1} \cdot D_A^{\frac{m}{n-m}} . D_B^{\frac{n}{n-m}} \cdot$$

given by the formula

$$x = \left(\frac{B}{A}\right)^{\frac{1}{n-m}} - \frac{1}{n-m} \cdot \frac{C}{1} \cdot \frac{A^{\frac{m-1}{n-m}}}{B^{\frac{n-1}{n-m}}} + \frac{1}{n-m} \cdot \frac{n+m-1}{n-m} \cdot \frac{C^2}{2!} \cdot \frac{A^{\frac{2m-1}{n-m}}}{B^{\frac{2n-1}{n-m}}}$$

$$- \frac{1}{n-m} \cdot \frac{2n+m-1}{n-m} \cdot \frac{n+2m-1}{n-m} \cdot \frac{C^3}{3!} \cdot \frac{A^{\frac{3m-1}{n-m}}}{B^{\frac{3n-1}{n-m}}}, \text{ etc.,}$$

we shall find the test to be

$$(3) \quad . \quad . \quad . \quad . \quad . \quad \left(\frac{m}{n}\right)^{\frac{n}{m}} \cdot \frac{m}{n-m} \cdot \frac{C . A^{\frac{m}{n-m}}}{B^{\frac{n}{n-m}}} < 1.$$

(Raise the factor corresponding to L here to the power of $-\frac{m}{n}$.)

From the second of these three tests,

$$\left(\frac{n}{n-m}\right)^{\frac{n}{m}} \cdot \frac{n-m}{m} \cdot \frac{C^{\frac{n-m}{m}} \cdot A}{B^{\frac{n}{m}}} < 1,$$

we deduce the test for the convergent action of the operator

$$D_A^{-1} \cdot D_C^{-(n-1)} \cdot D_B^{n}$$

which is the general type of operator for operands with integral indices.

This formula, with $m=1$, gives

$$\frac{n^n}{(n-1)^{n-1}} \cdot \frac{AC^{n-1}}{B^n}$$

as the test for the convergency of the root suggested by the operand $\frac{C}{B}$ in $Ax^n - Bx + C = 0$. By putting $n=2$, 3, 4, etc., we have the tests for quadratic, cubic, quartic, etc., operators, viz.

$$\frac{2^2}{1^1}\frac{AC}{B^2} < 1 \quad \frac{3^3}{2^2}\frac{AC^2}{B^3} < 1 \quad \frac{4^4}{3^3}\frac{AC^3}{B^4} < 1, \text{ etc., respectively.}$$

Suppose now that (2) above be true, then by raising

$$\left(\frac{n}{n-m}\right)^{\frac{n}{m}} \cdot \frac{n-m}{m} \cdot \frac{C^{\frac{n-m}{m}} \cdot A}{B^{\frac{n}{m}}}$$

to the power of $\frac{m}{n-m}$, we have

$$\left(\frac{n}{n-m}\right)^{\frac{n}{n-m}} \cdot \left(\frac{n-m}{m}\right)^{\frac{m}{n-m}} \cdot \frac{C \cdot A^{\frac{m}{n-m}}}{B^{\frac{n}{n-m}}} < 1 \qquad . \quad . \quad . \quad (4)$$

The numerical factor here is

$$\frac{n^{\frac{n}{n-m}}}{m^{\frac{m}{n-m}}} \frac{1}{(n-m)} \quad \text{or} \quad \left(\frac{n}{m}\right)^{\frac{n}{n-m}} \cdot \frac{m}{n-m} .$$

Comparing this now with the numerical factor in 3, we see that $\left(\frac{m}{n}\right)^{\frac{n}{m}}$ is a proper fraction raised to a power greater than 1, whereas $\left(\frac{n}{m}\right)^{\frac{n}{n-m}}$ is a mixed number or quantity greater than 1 raised to a power, and is therefore >1. The whole expression in (4) is greater than the similar expression in (3), and therefore if (2) hold, so does (3).

In other words, if there be m roots of the equation $Ax^n - Bx^m + C = 0$ belonging to the operand $\left(\frac{C}{B}\right)^{\frac{1}{m}}$, there are also $n-m$ roots belonging to the operand $\left(\frac{A}{B}\right)^{\frac{1}{n-m}}$.

Suppose now that (2) does not hold, or that

$$\left(\frac{n}{n-m}\right)^{\frac{n}{m}} \cdot \frac{n-m}{m} \cdot \frac{C^{\frac{n-m}{m}} \cdot A}{B^{\frac{n}{m}}} \nless 1.$$

Therefore

$$\left(\frac{n-m}{n}\right)^{\frac{n}{m}} \cdot \frac{m}{n-m} \cdot \frac{B^{\frac{n}{m}}}{C^{\frac{n-m}{m}} \cdot A} < 1.$$

(The case when this expression $=1$ we exclude for the present.)

Raising the expression last written to the power of $\frac{m}{n}$, we have

$$\frac{n-m}{n} \cdot \left(\frac{m}{n-m}\right)^{\frac{m}{n}} \cdot \frac{B \cdot C^{\frac{m-n}{n}}}{A^{\frac{m}{n}}} < 1,$$

and comparing this arithmetically with (1), we see that if (2) does not hold, (1) must hold, and there must be n roots of the trinomial $Ax^n - Bx^m + C = 0$ to be had from $\left(-\frac{C}{A}\right)^{\frac{1}{n}}$.

If the above expression,

$$\left(\frac{n-m}{n}\right)^{\frac{n}{m}} \cdot \frac{m}{n-m} \cdot \frac{B^{\frac{n}{m}}}{C^{\frac{n-m}{m}} \cdot A},$$

or its reciprocal $=1$, then either m and $n-m$ *both* must be submultiples of n, or some one of the coefficients A, B, and C must be incommensurable. Excluding the latter, the only possible case when m and $n-m$ both are submultiples of n is when $n=2m$, so that the equation is really a quadratic in x^m.

§ VI. Convergency in the General Case.

We have ascertained thus the existence in every case of n converging series belonging to the trinomial $Ax^n \pm Bx^m \pm C = 0$, the simplest form from which an operator can be formed. If another term $\pm Rx^r$ be included in this form, then either the new operator arising from this term will co-operate with the operator of the simpler form in the formation of a doubly infinite series which still converges, or this term $\pm Rx^r$ itself becomes the ridge of demarcation for solutions lying on either side of it, as the term $-Bx^m$ did in the trinomial form. In the general case, we start with the two coefficients on the right, testing for an integral-indexed solution, and if the different operators act convergently, we then turn to the second operand, $-\frac{p_{n-1}}{p_{n-2}}$, and so on. Whenever our way is blocked we pick out the term giving rise to the most divergently acting operator, and combine this in a binomial form with that coefficient farthest to the right which has not yet been made the numerator for a solution, and there will be roots belonging to the operand got from this binomial form by root extraction, unless this is frustrated by some term farther to the left, which is then taken as a new basis for roots by root extraction. Clearly, the farther we proceed to the left, or the greater r_2-r_1 is in the form $R_2x^{r_2}-R_1x^{r_1}$, giving the operand $\left(\frac{R_1}{R_2}\right)^{\frac{1}{r_2-r_1}}$, the greater number of roots we have from this one operand, and in this regard roots belonging to integral-indexed operands ($r_2-r_1=1$) are the least advantageous of all to calculate. This is to some extent counteracted by the fact that when we are evolving 3, 4, or more roots together the convergence is apt to be slow, and we shall often find it preferable to isolate any particular root we are following, as the smallest root in an equation reduced by Horner's process or a slight modification of this we shall give presently.

Examples:—

The equation x^3+x^2-2x-1 has one operand, $-\frac{1}{2}$ and 2 belonging to $x^2=2$.

The equation $7x^4+20x^3+3x^2-16x-8$ has three operands, $\left(\frac{8}{20}\right)^{\frac{1}{3}}$ and $1=-\frac{20}{7}$.

The equation $x^5+12x^4+59x^3+150x^2+201x-207$ has five operands, $\left(\frac{207}{1}\right)^{\frac{1}{5}}$. *

I go no further into the general theory at present, however, as the consideration of the exceptional cases would lead us too far. Another paper is necessary to deal with this point and the simplifications that may be introduced into these expressions for the roots as first thrown off by the operators.

A new basis is also afforded for the occurrence of algebraic solutions, this basis being the simplicity or complexity of the p^{th} roots of unity, p being the quantity (r_2-r_1) above, indicating the root of the operand taken. The sixth roots of unity being comparatively simple, so that they may be grouped in twos and threes, a sextic with six roots derived from $\left(\frac{p_6}{p_0}\right)^{\frac{1}{6}}$ can have these roots expressed algebraically. Not so the corre-

* See note at end.

sponding quintic however, which still remains algebraically intractable. This of course involves no contradiction with Abel's results, but is, on the contrary, in direct fulfilment of them, for these also are based on the private properties of the natural numbers.

Before giving two or three numerical examples, it may be formally stated that from the convergent action of all the operators at work follows the convergence to a finite magnitude of the root itself. The values we have investigated as the limiting values of the ratio of each element to its predecessor in the same ray are also the maximum values of that ratio. The ray as a whole, therefore, is less than the G.P. with the same initial element as first term and this maximum value for common ratio. In this way it may be shown that the whole solution (a), *e.g.* in § III, is

$$< \frac{1}{1+\frac{27}{4}\frac{AE^2}{C^3}} \times \frac{1}{1-\frac{4}{1}\frac{BE}{C^2}} \times \left(\frac{-E}{C}\right),$$

and that the first solution of the quartic in the same section is

$$< \frac{1}{1-\frac{256}{27}\frac{FE^3}{C^4}} \times \frac{1}{1+\frac{27}{4}\frac{AE^2}{C^3}} \times \frac{1}{1-\frac{4}{1}\frac{BE}{C^2}} \times \left(\frac{-E}{C}\right),$$

and so on.

It may be worth while perhaps to repeat that this condition, though sufficient, is not absolutely necessary; that is to say, a slight divergence, or even a more pronounced divergence among elements differing in sign, should not deter us following a development for some distance till it be thought expedient to decide the matter definitely, and at the same time accelerate the solution by shifting to a new origin. Finite roots, *e.g.*, are finite because the elements, after a certain point, all cancel each other out, or it is possible to re-cast the operators to make them do so. Only when the development is flagrantly and *ab initio* divergent must we abandon it as utterly worthless in giving information about the roots.

§ VII. Solution of Numerical Equations.

The form in which the operators first give off a solution is fairly well exemplified by the root of the equation, $x^3-49x^2+658x-1379=0$ belonging to $\frac{1379}{658}$. Both the operators $D_{49}^{-1}.D_{1379}^{-1}.D_{658}{}^{2}$ and $D_{1}^{-1}.D_{1379}^{-2}.D_{658}$

act convergently. These give

$$\frac{1379}{658}$$

$$+\left(\frac{49}{658}\right)^1\left(\frac{1379}{658}\right)^2 - \frac{1}{658}\cdot\left(\frac{1379}{658}\right)^3$$

$$+2\left(\frac{49}{658}\right)^2\left(\frac{1379}{658}\right)^3 - 5\left(\frac{49}{658}\right)\left(\frac{1}{658}\right)\left(\frac{1379}{658}\right)^4 + 3\left(\frac{1}{658}\right)^2\left(\frac{1379}{658}\right)^5$$

$$+5\left(\frac{49}{658}\right)^3\left(\frac{1379}{658}\right)^4 - 21\left(\frac{49}{658}\right)^2\left(\frac{1}{658}\right)\left(\frac{1379}{658}\right)^5 + \text{etc.}$$

$$+14\left(\frac{49}{658}\right)^4\left(\frac{1379}{658}\right)^5 - \text{etc.} \qquad\qquad + \text{etc.}$$

We develop, of course, in the direction of the important terms. The rate of convergency for these latter begins at ·15 and increases gradually to ·6, and by the aid of the logarithms of the three coefficients we may calculate the root with fair ease. The alternative method is to subtract the integral part of the root—this is seen from the opening term alone to be 2—as in Horner's process.

The reduced equation is $y^3 - 43y^2 + 474y - 251 = 0$.

The new operand $\frac{251}{474}$ suggests ·5 as the first figure of the new root, and now instead of subtracting ·5 we *divide* the root by ·5, division being an accelerated form of subtraction, and this latter being the kernel in Horner's unerring process.

With $y = \cdot 5(1+\theta)$ we have

$$\cdot 125(1+\theta)^3 - 10\cdot 75(1+\theta)^2 + 237(1+\theta) - 251 = 0,$$

or

$$\theta^3 - 83\theta^2 + 1727\theta - 197 = 0.$$

The operand here gives $\frac{197}{1727}$ or ·1 as the opening figure in θ. Put θ therefore $= \cdot 1(1+\theta_1)$, so that we have

$$\cdot 001(1+\theta_1)^3 - \cdot 83(1+\theta_1)^2 + 172\cdot 7(1+\theta_1) - 197 = 0,$$

or

$$\cdot 001\theta_1^3 - \cdot 827\theta_1^2 + 171\cdot 043\theta_1 - 25\cdot 129 = 0,$$

from which we have θ_1 in the form

$$\frac{25\cdot 129}{171\cdot 043} + \frac{\cdot 827}{171\cdot 043}\left(\frac{25\cdot 129}{171\cdot 043}\right)^2 = \cdot 147018.$$

And $x = 2 + \cdot 5[1 + \cdot 1(1\cdot 147018)]$, or 2·5573509 correct to five decimal places.

To get a still closer approximation, we must, of course, include more terms in the calculation of θ_1, or put $\theta_1 = \cdot 1(1+\theta_2)$. We are merely showing how best to accelerate the convergency. The series for θ above is already rapidly convergent.

When we turn to the other integral-indexed operands in this equation, $\frac{658}{49}$ and $\frac{49}{1}$, we find they both have a slightly divergent quadratic operator $D_{658}^{-1} . D_{1}^{-1} . D_{49}^{2}$, for 4×658 is slightly $>(49)^2$. But if we try the operands with fractional indices we shall find that these also have divergent operators. Now, whenever two or more operands are missing we may be sure of the presence of nearly equal roots. The series for the operands $\frac{658}{49}$ and $\frac{49}{1}$ themselves show this. The former, beginning at 13, increases rapidly, and the latter, beginning at $\frac{49}{1}$, decreases rapidly, and they have nearly become equal before there is any trace of divergency. Though the rate of convergency be rapid at first also, it later becomes very slow, approaching asymptotically its maximum value, which is slightly over 1. This slow convergence is another unmistakable symptom of proximity of roots. Once we are sure of nearly equal roots we may find a first approximation to their value in different ways, by following the series belonging to the operands which would give the roots in normal circumstances, or by finding the H.C.F. of the original function and its first derived. In this case we simply subtract the value of the root already found from 49, the sum of all three, and divide by 2. This gives 23·2 as the approximate value of the nearly equal roots, and the equation with its roots reduced by 23 is

$$y^3 + 20y^2 - 9y + 1 = 0.$$

There is now another point to be mentioned. In the above process of dividing the root by ·5, ·1, etc., the ultimate goal was to render negligible the coefficient of every power of the unknown above the first, leaving the solution finally in the form $a\theta = b$. But till a pair of equal roots are separated we must solve the *quadratic* in θ, the coefficient of θ^2 refusing to become negligible.

In the present case $20y^2 - 9y + 1 = 0$ gives $y = \cdot 2$ or $\cdot 25$.

Putting $y = \cdot 2\ (1 + \theta)$, therefore, we have

$$\cdot 008(1+\theta)^3 + \cdot 8(1+\theta)^2 - 1\cdot 8(1+\theta) + 1 = 0 \quad \text{or} \quad \theta^3 + 103\theta^2 - 22\theta + 1 = 0,$$

and the solution of this new quadratic $103\theta^2 - 22\theta + 1 = 0$ gives $\theta = \cdot 0656$ or $\cdot 1479$ and $y = \cdot 21312$ or $\cdot 2295$. The roots are now separated, and if one of them, say the first, be wanted more accurately, we must put $\theta = \cdot 06(1 + \theta_1)$ and solve for a single root θ_1.

The equation $7x^4+20x^3+3x^2-16x-8=0$ has, as already stated, three roots belonging to $\left(\frac{8}{20}\right)^{\frac{1}{3}}$ and a negative one beginning $-\frac{20}{7}$. The convergency is very slow, however, so that, though it be possible to locate the roots from the formula belonging to the equation as it stands, we may use it to exemplify the transformations that are often necessary in calculating roots belonging to operands with fractional indices. In the first place, it is always an improvement to have the opening coefficient 1. This coefficient becomes at least once, and perhaps twice, a denominator—a denominator, however, that soon becomes a numerator. In this case, then, multiply the roots by 7. Thus we have, with $y=7x$,

$$y^4+20y^3+21y^2-784y-2744=0.$$

The next transformation must aim at making the absolute term and one other coefficient in the centre predominate over the others, and, if it be possible, at removing a term, and with it an operator. In the present case, if we increase the roots by 5, we have, with $z=y+5$,

$$z^4-129z^2+6z-174=0,$$

and now we shall have two real roots from $\pm\left(\frac{129}{1}\right)^{\frac{1}{2}}$ as operand, and two imaginary ones from $\pm i\left(\frac{174}{129}\right)^{\frac{1}{2}}$ as operand.

For the development of the former we have

$$z^2=\frac{129}{1}-\frac{6}{1}\frac{1}{z}+\frac{174}{1}\cdot\frac{1}{z^2},$$

giving

$$z=\pm\left(\frac{129}{1}\right)^{\frac{1}{2}},$$

and taking first $+\left(\frac{129}{1}\right)^{\frac{1}{2}}$,

$$z=\left[\frac{129}{1}-\frac{6}{1}\left(\frac{1}{129}\right)^{\frac{1}{2}}+\frac{174}{1}\cdot\frac{1}{129}\right]^{\frac{1}{2}}.$$

or

$$\left(\frac{129}{1}\right)^{\frac{1}{2}}+\frac{1}{2}\frac{(-6)}{1}\cdot\frac{1^0}{129}+\frac{1}{2}\cdot\frac{174}{1}\cdot\frac{1^{\frac{1}{2}}}{129^{\frac{3}{2}}},$$

giving operators $D_{(-6)}^{-1}\,.\,D_1^{-\frac{1}{2}}\,.\,D_{129}^{\frac{3}{2}}$, $D_{174}^{-1}\,.\,D_1^{-\frac{3}{2}}\,.\,D_{129}^{\frac{5}{2}}$, and

$$z = \left(\frac{129}{1}\right)^{\frac{1}{2}}$$

$$+\frac{1}{2}\frac{(-6)}{1}\cdot\frac{1^0}{129} + \frac{1}{2}\cdot\frac{174}{1}\cdot\frac{1^{\frac{1}{2}}}{129^{\frac{3}{2}}}$$

$$+\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{(-6)^2}{2!}\frac{1^{\frac{1}{2}}}{129^{\frac{5}{2}}} + \frac{1}{2}\cdot 2\cdot\frac{(-6)}{1}\cdot\frac{(174)}{1}\cdot\frac{1^1}{129^3} + \frac{1}{2}\cdot 3\cdot\frac{(174)^2}{2!}\cdot\frac{1^2}{129^4}$$

$$+\frac{1}{2}\cdot 3\cdot 2\cdot\frac{(-6)^3}{3!}\cdot\frac{1^1}{129^4} + \frac{1}{2}\cdot\frac{7}{2}\cdot\frac{5}{2}\cdot\frac{(-6)^2}{2!}\frac{(174)}{1}\cdot\frac{1^{\frac{3}{2}}}{129^{\frac{9}{2}}}$$

$$+\frac{1}{2}\cdot\frac{9}{2}\cdot\frac{7}{2}\cdot\frac{(-6)}{1}\cdot\frac{(174)^2}{2!}\frac{1^{\frac{5}{2}}}{129^{\frac{11}{2}}} + \frac{1}{2}\cdot\frac{11}{2}\cdot\frac{9}{2}\cdot\frac{(174)^3}{3!}\frac{1^{\frac{7}{2}}}{(129)^{\frac{13}{2}}}, \text{ etc., etc.}$$

or

$$\left(\frac{129}{1}\right)^{\frac{1}{2}}\left[1 + \frac{174}{2\times 129^2} + \frac{3}{8}\cdot\frac{6^2}{129^3}, \text{ etc. (the other terms are very small)}\right]$$

$$-\frac{3}{129} - \frac{6\times 174}{129^3} + \text{very small terms,}$$

giving

$$z = -\cdot 02325 \pm \left(\frac{129}{1}\right)^{\frac{1}{2}}(1\cdot 00522).$$

For the imaginary roots we write

$$z^2 = -\frac{174}{129} + \frac{6}{129}z + \frac{1}{129}z^4, \text{ or with } z = i\left(\frac{174}{129}\right)^{\frac{1}{2}}$$

$$z = \left[-\frac{174}{129} + i\frac{6}{129}\left(\frac{174}{129}\right)^{\frac{1}{2}} + \frac{1}{129}\left(\frac{174}{129}\right)^2\right]^{\frac{1}{2}} = i\left(\frac{174}{129}\right)^{\frac{1}{2}} + \frac{1}{2}\cdot\frac{6}{1}\cdot\frac{174^0}{129} - i\frac{1}{1}\cdot\frac{174^{\frac{3}{2}}}{129^{\frac{5}{2}}}.$$

The operators are therefore $-iD^{-1}_{(+6)}\,.\,D^{-(\frac{1}{2}-1)}_{174}\,.\,D^{\frac{1}{2}}_{129}$ and $-D^{-1}_{1}\,.\,D^{-1}_{174}\,.\,D^{2}_{129}$,
and

$$z = i\left(\frac{174}{129}\right)^{\frac{1}{2}}$$

$$+\frac{1}{2}\cdot\frac{6}{1}\cdot\frac{174^0}{129} - i\frac{1}{2}\cdot\frac{1}{1}\cdot\frac{174^{\frac{3}{2}}}{129^{\frac{5}{2}}}$$

$$-i\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{6^2}{2!}\cdot\frac{174^{-\frac{1}{2}}}{129^{\frac{3}{2}}} - \frac{1}{2}\cdot 2\cdot\frac{6}{1}\cdot\frac{1}{1}\cdot\frac{174^{\frac{3}{2}}}{129^{\frac{6}{2}}} + i\frac{1}{2}\cdot\frac{1^2}{2!}\cdot\frac{174^{\frac{5}{2}}}{129^{\frac{8}{2}}}$$

$$-\frac{1}{2}\cdot 1\cdot 0\cdot\frac{6^3}{3!}\cdot\frac{174^{-1}}{129^{2}} + i\frac{1}{2}\cdot\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{6^2}{2!}\cdot\frac{1}{1}\cdot\frac{174^{\frac{1}{2}}}{129^{\frac{7}{2}}} + \frac{1}{2}\cdot 4\cdot 3\cdot\frac{6}{1}\cdot\frac{1^2}{2!}\cdot\frac{174^{\frac{4}{2}}}{129^{\frac{10}{2}}}$$

$$-i\frac{1}{2}\cdot\frac{11}{2}\cdot\frac{9}{2}\cdot\frac{1^3}{3!}\cdot\frac{174^{\frac{7}{2}}}{129^{\frac{13}{2}}}, \text{ etc., etc.}$$

$$= \pm i\left(\frac{174}{129}\right)^{\frac{1}{2}}\left[1 - \frac{1}{2}\cdot\frac{174}{129^2}, \text{ etc.}\right] + \frac{3}{129}, \text{ etc.} = \cdot 0232 \pm i\left(\frac{174}{129}\right)^{\frac{1}{2}} \times \cdot 99478.$$

The integral part of the root being soon apparent by this method, we can readily find a commensurable root, when there is one.

When the integral part is found, we may pause a moment to reconnoitre as to the presence of a commensurable root. If the root we are following be commensurable, it must be one of the series

$$\mathrm{I}+\frac{0}{\mathrm{A}},\ \mathrm{I}+\frac{1}{\mathrm{A}},\ \mathrm{I}+\frac{2}{\mathrm{A}},\ \text{etc.}\ \ldots\ \mathrm{I}+\frac{\mathrm{A}-1}{\mathrm{A}},$$

where I is the integral part of the root, and A the coefficient of the highest power of x. If among these mixed numbers, when expressed as improper fractions, there be one whose numerator divides the absolute term, then we should test for a finite root by factorisation.

From the operand $\frac{85}{2}$ in the equation $2x^3-85x^2-85x-87=0$ we have

$$x=\frac{85}{2}+\frac{85}{2}\times\frac{2}{85}-\left(\frac{85}{85}\right)^2\times\frac{2}{85},\ \text{etc.},$$

so that the integral part of the root is 43. Possible commensurable roots are therefore 43 and $43\frac{1}{2}$, and the latter leads to the factorised form $2(x-43{\cdot}5)(x^2+x+1)=0$.

The operand $\frac{121}{20}$ in the equation $20x^3-121x^2-121x-141=0$ gives $x=\frac{121}{20}+\frac{121}{121}+$a fraction, so that the integral part is 7.

The second of the series, 7, $7\frac{1}{20}$, $7\frac{1}{10}$ . . . $7\frac{19}{20}$ leads to the factorisation $(20x-141)(x^2+x+1)=0$.

It is, as will be seen from these two cases, possible to write the solution so as to exhibit line after line of zeros. But this would open the question of the simplification of operators, and in practice the above test is simply applied. Slightly divergent operators may also lead to a finite root.

The equation $(x-17)(x-8)(x-3)=x^3-28x^2+211x-408=0$ has two slightly diverging operators for the two larger roots. If we should pass a finite root over, the convergence begins to oscillate.

Addendum.

With regard to the quintic on p. 43, we say its roots belong to the operands $\left(\frac{207}{1}\right)^{\frac{1}{5}}$, because, though the operators belonging to these operands act divergently, the elements alternate in sign, and in view of the transformation $x=y-2$, which decides the matter. This transformation gives $y^5+2y^4+3y^3+4y^2+5y-321=0$, and leads to the solution

$$y=-\tfrac{2}{5}+1{\cdot}0013\left(\frac{321}{1}\right)^{\frac{1}{5}}-00153\left(\frac{321}{1}\right)^{\frac{2}{5}}-{\cdot}001047\left(\frac{321}{1}\right)^{\frac{2}{5}}-{\cdot}000872\left(\frac{321}{1}\right)^{\frac{4}{5}},$$

from which we have $y=2{\cdot}638$. See Burnside and Panton, chap. x, ex. 18. The other roots are then to be had from the 5th roots of unity.

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IV.—Experiments and Observations on Crustacea: Part I. Immersion Experiments on Ligia. By John Tait, M.D., D.Sc. (From the Laboratory of Physiology, Edinburgh University.) *Communicated by* Professor Sir E. A. Schäfer.

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Descended from marine ancestors, *Ligia oceanica*, like other members of the sub-order (Oniscoidea or "slaters") to which it belongs, has adopted a terrestrial mode of life. The other Oniscoidea have spread inland, invading regions far from the sea. Ligia, known as the sea-slater, is confined to the coastal fringe, and may possibly form a link between marine isopods and the more truly terrestrial forms.

The author has been led from some early observations on the blood of Ligia (5), (7) to study the animal as a whole. Thus, colour-change was found to occur in the creature in response to variation in its surroundings (6). Subsequent observations and experiments, the nature of which it is impossible to state briefly, have been carried out on Ligia and some of its relatives. It is proposed now to deal with these in a series of short communications.

Immersion Experiments.

One of the first points of interest in connection with the animal is the length of time it can survive immersion in salt and in fresh water respectively. The bearing of this question is at least threefold. To begin with, we know little regarding the ordinary habits of the animal; it is still a moot question whether it voluntarily enters the sea. Secondly, the distribution of Ligia along the sea-fringe being discontinuous, every suitable locality being nevertheless occupied, one wonders if the animal can be safely carried by way of the sea from one habitable zone to another across an uninhabitable area, such as a long stretch of sand. Lastly, one wishes to know for certain whether the animal has sprung directly from a sea-water stock, or whether its ancestors have passed through a fresh-water stage in attaining land adaptation.

Some experiments on immersion of Ligia have already been recorded by Hewitt (2), by Tait (5), and, in more detail, by Stewart (4). The last-named found that in sea-water one batch of four animals survived nine days, their death being ascribed "to insufficient food rather than to the external conditions." Another batch lived in sea-water for six days "without appearing to be in any way affected by the immersion." In "fresh" (presumably tap-) water Ligiæ did not survive more than two days. The internal gill-lamellæ of these last specimens were much swollen owing to absorption of water, the external lamellæ being little affected. It was claimed that immersion in sea-water produces microscopic changes in the internal gill-lamellæ; no change, however, could be detected on superficial examination.

Details concerning Technique, etc.—In the present investigation four immersion fluids were used, viz. sea-water, half sea-water, quarter sea-water, and distilled water. Each animal, after being washed in successive changes of the appropriate fluid, was immersed by itself in 150 c.c. of fluid in an Erlenmeyer flask, the water being aerated from a glass capillary that dipped below the surface out of reach of the gills (this last condition is specified because, when the aeration-tube dips deeply, bubbles of air may attach themselves to the gills). By having a large battery of flasks attached to the same aeration-pump—one of Dr J. F. Gemmill's many ingenious devices for this purpose—it was possible to conduct a great number of immersion experiments simultaneously. The water in the flasks was changed at suitable intervals. The temperature varied between 10·5 and 14·5 degrees Centigrade, the experiments being carried out from mid-January to mid-April. Permanent cessation of gill-movements was taken as the sign of death.

A word as to the manner in which the supply-stock of Ligia was kept in the laboratory. Previous experience (5) had shown that Ligia long survives if kept without food simply in a damp atmosphere. On this occasion an attempt was made to find a suitable food. When not provided with food they eat each other; they also eat Orchestia, if these are supplied. They refused to eat Porcellio; likewise pieces of cooked bacon or chicken. When blades of *Laminaria digitata* were provided they refrained from murder of their own kind and of Orchestia, and ate the vegetable matter freely. They showed distinct preference for Laminaria as compared with *Fucus nodosus* and *Fucus vesiculosus*. The animals lived in hundreds in a large tin box, which was kept closely covered. To maintain the atmosphere moist a piece of linen wrung out of half sea-water was laid inside. Twice a week the whole batch of animals was removed and bathed in three successive changes of clean sea-water; at the same time their box and the piece of linen stuff were thoroughly cleansed under the tap. Kept in this way the animals throve exceedingly. Practically the only deaths were those of moulting individuals, which are apt to get trampled upon.

Immersion in Sea-Water.—The stock of sea-water (obtained from Millport) had a chloride content equivalent to 3·25 grammes of NaCl per 100 c.c. Being perfectly clear to the eye, it was not filtered. Eight experiments were carried out (on four males and four females, all medium-sized). The individual survival periods were 35 (bis), 51, 52, 58, 65, over 83 (bis) days. Sex had no particular influence on the time of survival. There was no appreciable change in the external appearance of the dead animals.

These survival periods are greatly in excess of those recorded by Stewart. The experiments prove that Ligia can live for weeks and even months in sea-water; secondly, that it can endure fasting for an equally long period of time.

Immersion in Half Sea-Water.—Experiments were carried out on five medium-sized animals (sex undetermined) and on one large male. The periods of survival were 16, 18, 24 (large male), 34, 37, 42 days. Only in one case (that of the 37 days' survival) was there any appreciable change at death; this consisted in swelling of one solitary internal gill-lamella.

It is to be noted here that even the longest period of survival in half sea-water is much less than the average period of survival in full sea-water as determined in the preceding experiments. Reduction of the salinity of the sea-water to one-half distinctly diminishes the survival period.

Immersion in Quarter Sea-Water.—Six medium-sized animals were used (sex not determined). The periods of survival were 3, 5, between 7 and 9 (bis), 9, 15 days. Two of the animals showed marked œdema of the body (see below) before death. One of these, recorded above as surviving 9 days, when almost *in articulo mortis* was replaced in sea-water, where it slowly recovered power of movement; by next day the œdema had subsided: the animal continued to live in good health in sea-water for a week, after which it was removed to be fed for another experiment. Swelling of internal gill-lamellæ was noted in both these œdematous animals, but not in any of the others.

Immersion in Distilled Water.—The number of experiments reached beyond one hundred and fifty. This larger number was found necessary, because with a given batch of animals selected at random the survival-periods (short compared with those already recorded) were found to vary greatly. After the first 8 or 9 hours individual animals began to die, and this process might continue for at least 36 hours thereafter. The aim at first was to establish an average result; it was soon directed, however, towards the settlement of the condition or conditions responsible for the variability.

It will be convenient at this point to make a few general remarks regarding the effect of distilled water on the animals. Reference has already been made to the œdema produced in two of the animals immersed in one-quarter sea-water. A similar change is very common, but not invariable, after immersion in distilled water. Frequently the animal affected with œdema acquires a very un-Ligian contour: increasing considerably in length without any corresponding increase in breadth, it comes superficially to resemble one of the commoner Idoteæ. The elongation is due to separation of the calcified somites, the soft integument between these harder structures (normally hidden) being prominently visible on the dorsal aspect of the thorax. A flaccid Ligia that has died a natural death may be pulled into a similar form. The animal that takes on this elongated shape after immersion in distilled water is anything but flaccid, however. Its weight has considerably increased. With forced ventriflexion the integument between thorax and abdomen readily ruptures and some of the viscera escape along with a minimal amount of fluid. The fact that there is no gush of free fluid can indicate only that the condition is one of true œdema, in which the tissues as a whole have become infiltrated with water. When removed from the immersion-flask and placed on the table, animals in this condition are found to be still capable of feeble locomotion.

Other parts may participate in the swelling. Thus, one invariably finds that the lower lip protrudes, while the gills, instead of lying flat along the ventral aspect of the abdomen, project ventrally almost at right angles to the body axis. Some of the gill-lamellæ may be swollen into rounded bladders. There is much variation as regards the number and situation of the gills thus affected. As a rule the endopodites alone are swollen, but the swelling may affect exopodites; rarely the exopodites are affected to the exclusion of the endopodites. The gills may be swollen in cases where the body is not elongated, and *vice versa*.

It is plain that swelling of the gills is merely one part of a more general process. Nor does it seem necessary, as Miss Stewart has done, to refer the death of immersed Ligiæ to a primary change in the gills. From the fact that she observed swelling only in the internal gill-lamellæ this writer is inclined to argue that these lamellæ are more directly concerned with the respiratory function. Apart from the validity of the premises, one would demand more convincing proof than this.

While death may occur in distilled water without any sign of œdema, *i.e.* while water may fail to enter the animal in sensible quantity, in every case salts diffuse from the animal into the water. Repeated washing of a batch of Ligiæ in distilled water virtually frees them from such soluble salts as adhere to the surface of the body. Before immersion in the Erlenmeyer flasks the animals were so washed that a sojourn of two or three minutes in a final small quantity of distilled water permitted the escape of no chloride sufficient to be detected with silver nitrate. After a number of hours in the much larger quantity of distilled water contained in the Erlenmeyer flask, a marked white cloud was invariably produced on the addition of a drop of silver nitrate. It is quite probable that death from immersion in distilled water is primarily due to the loss of essential salts.

It has already been mentioned that an animal rendered œdematous by immersion in one-quarter sea-water may recover on prompt transference to full-strength sea-water. One had abundant opportunity of repeating this experiment—with similar findings—in the case of animals, whether œdematous or not, brought near the point of death by immersion in distilled water. With practice one came to estimate with considerable accuracy the chances of restoring the animal by this means. After high degree of inco-ordination of the gill-movements restoration is not possible, at any rate with immersion in simple sea-water; provided the inco-ordination is slight, recovery may be confidently expected.

We may now discuss the cause of variability in the period of survival of animals immersed in distilled water. As possible influencing conditions the following were experimentally taken into account: (1) sex; (2) colour, whether permanent (some animals, for example, are reddish, some blotched with white) or temporary (depending on retraction or expansion of chromatophores according to the prevailing background); (3) size; (4) the condition as regards moult. It was found that sex could be discounted; so also colour in so far as it did not depend simply on moult. Size proved to be of influence, a small animal having on the whole less chance than a large of surviving beyond a certain point. At the same time size is not the only factor, for a small animal may survive twice as long as another three or four times its size. A decisive influence is exercised by the state of the animal as regards moult.

As the question of moulting is to be discussed in the next communication, all that need be said at present is that a freshly moulted animal is easily distinguished from one that has moulted long since. Records were taken of the survival period of a large series of animals selected at random; after death each animal was examined to determine the state as regards moult. Without exception, those that long survived were found to have recently moulted; on the other hand, every animal that died unusually early was found to be approaching a new moult. Although no exact numerical statement can be made, I might express the result of these experiments in the following rough "rule": "Death within 13 hours —old cuticle; survival beyond 22 hours—fresh cuticle." It must be understood that the great proportion of deaths fell between these limits. The conclusion as to the importance of the moulting process is based on cases that fell beyond the limits.

The longest survival-period accurately measured was 44–45 hours. In two other records it is stated that the animal lived "more than 40 hours." Numerous survival-periods between 30 and 40 hours were recorded.

Another method was adopted to test the importance of moult in reference to the present question. Ligia moults in two stages. The cuticle of the posterior part of the body is first shed: about four days later the remaining cuticle is cast off. For some days before the first stage of moulting occurs the anterior part of the animal undergoes a conspicuous change of colour. In this way three stages in the immediate proximity of the actual moult are readily distinguished, viz. "just prior to moult," "between posterior and anterior moult," and "just subsequent to anterior moult." Successive sets of experiments were carried out on animals in these three stages.

Just Prior to Moult.—Twenty-nine animals were immersed. The exact survival-periods of twenty-four of these were as follows:—

Hours	13–14	14–15	15–16	16–17	17–18	18–19	19–20	20–21
No. of Animals	4	3	10	2	1	1	1	2

Three of the animals did not survive 13 hours. Two are recorded as having died "before 18 hours."

Between Posterior and Anterior Moult.—Sixteen animals were immersed. Exact records were obtained in the case of nine, thus:—

Hours	13–14	14–15	15–16	17–18	20–21
No. of Animals	1	1	5	1	1

Three of the remainder did not survive 13 hours; four are recorded as having died "between 12 and 23 hours."

Just Subsequent to Anterior Moult.—Eight animals were immersed. Exact records were obtained in the case of six, thus:—

Hours . . .	13–14	16–17	24–25	26–27	28–29	36–37
No. of Animals .	1	1	1	1	1	1

Two others are recorded as having died "between 22 and 35 hours." It will be noted that, whereas in the previous two sets of experiments no animals were known to survive beyond 21 hours, in the present set six out of eight, *i.e.* 75 per cent., lived over 22 hours.

These results with moulting animals not only confirm the previous experiments but they perhaps indicate that just subsequent to complete moult a sudden increase in resistance to the deleterious effect of distilled water is established.

General.

There can be no doubt that the immediate ancestors of Ligia were sea animals. If this terrestrial stock had been derived through a fresh-water line, it would not be so susceptible to damage from immersion in dilute sea-water. The physiological method of inquiry bears out the inference hitherto drawn simply from the distribution of the genus. All the known species frequent the sea-shore (see Stebbing (3)).

The question regarding its *invariable* occurrence in suitable localities,* conjoined with its complete absence from other localities (often miles in extent) along the shore, is no longer such a puzzle. It is quite open to suppose that the animal is carried past impassable areas by means of floating drift-material (the clinging habits of the creature must be kept in mind).

From the literature it is hard to extract a definite statement as to whether Ligia normally enters the sea. While collecting supplies for experiments the author paid particular attention to this point. Numerous pieces of evidence, all pointing in the same direction, were obtained. Large communities of Ligia were found completely immersed under stones that rested in supra-tidal saline pools. Heaps of stones, apparently too dry for permanent habitation, were discovered to be densely populated if in the immediate neighbourhood of such semi-permanent pools. Specimens on

* According to the observations of the writer, a "suitable locality" implies deeply fissured rock, or heaps and slabs of broken angular stone large enough to afford permanent dampness underneath. Ligia is not found among sand ; nor among decaying weed ; nor among stones, whether large or small, that are subject to frequent displacement by the waves.

rocks near the edge of the sea, attempting to escape pursuit, frequently ran into the water. More conclusive still, numerous examples were discovered half-way down the beach under stones just left bare by the receding tide. It is clear that immersion in salt water holds no terrors for Ligia.

The present set of experiments, however, equally brings into prominence the reverse side of the question. If Ligia can live in apparent comfort for nearly three months in sea-water, why does it prefer a land habitation? Some *immediate* cause must keep the animal to the landward rather than to the seaward side on the beach. In its general bearing this problem is common to all cases in which a terrestrial begins to spring from an aquatic stock; we should not forget that evolution has its physiological as well as its morphological side. In absence of experimental data the most plausible hypothesis for all such cases is the respiratory one. Atmospheric air with its 21 volumes per cent. of oxygen offers great advantages as a respiratory medium over water with its maximum of, say, 8 cm. oxygen per litre—in this connection see Winterstein (8). The most plausible general hypothesis is, however, but a feeble substitute for experiment.

An equally obscure problem, possibly cognate to the present work, is the determination of the obstacles, immediate and remote, that restrain Ligia from spreading inland. A vitally important agent (acting sometimes immediately, but as a rule remotely) for all sea-derived animals is rain, with its solvent or washing properties.

To be able to endure long periods of fasting is a common feature among Crustacea. Bethe (1), after commenting on the voracity of the common shore-crab, remarks: "It can also subsist for weeks without food." Lobsters are occasionally kept by fishermen and dealers for long periods without food. The writer has kept four specimens of the amphipod Orchestia in clean sea-water for 26 days, at the end of which period the animals were to all appearance as vigorous as before. The capacity of Ligia for enduring hunger should perhaps be kept in mind in pondering over the fact that its normal haunts often appear bare of any visible food material.

From the point of view of general physiology and of special crustacean physiology the most interesting facts elicited in the present inquiry are: firstly, the extraction of salts from Ligia and the œdema caused by distilled water (this line of work, however, is not entirely new); and, secondly, the increase in resistance to distilled water of recently moulted as compared with long since moulted animals. As these questions remain under investigation, discussion will be postponed to a later occasion.

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V.—Experiments and Observations on Crustacea: Part II. Moulting of Isopods. By John Tait, M.D., D.Sc. (From the Laboratory of Physiology, Edinburgh University.) *Communicated by* Professor Sir E. A. Schäfer.

(MS. received August 1, 1916. Read November 6, 1916.)

The moulting process in isopods has frequently attracted attention, the following notices on the subject having been collected:—

1880. Schöbl—*Porcellio.*
1881. Weber—*Trichoniscus*, *Haplophthalmus.*
1883. Friedrich—*Porcellio*, *Oniscus.*
1891. Leichmann—*Porcellio*, *Asellus*, *Sphæroma.*
1895. Němec—*Ligidium*, *Porcellio*, *Typhloniscus*, *Trichoniscus*, *Haplophthalmus*, *Platyarthrus.*
1898. Schönichen—*Oniscus*, *Porcellio*, *Armadillidium*, *Asellus.*
1901. Verhoeff—*Porcellio.*
1906. Webb and Sillem—*Armadillidium.*
1907. Pierce—*Armadillidium.*
Zuelzer—*Asellus.*
1911. Tait—*Ligia*, etc.
Wege—*Asellus.*
1912. Hankó—*Asellus.*
1913. Allee—*Asellus.*
Herold—*Ligidium*, *Trichoniscus*, *Oniscus*, *Philoscia*, *Platyarthrus*, *Porcellio*, *Cylisticus*, *Armadillidium*, *Asellus.*
1914. Aubin—*Porcellio.*

In practically all these papers—most contain mere incidental notices; that of Herold gives most detail—comment is made on the fact that the moult occurs in two stages (separated by an interval of days), which is certainly the most striking feature of the isopodan moult. The present communication comprises a brief descriptive account of Ligia in relation to its moult, an attempt to establish a morphological side to the subject of moulting, and, lastly, some observations bearing on the physiology of the question.

Descriptive.

When a recently moulted Ligia is examined under the dissecting microscope the external covering of the animal looks beautifully clean and fresh. The gills are silvery white. Certain parts of the ventral aspect and of the limbs of the animal are normally devoid of chromatophores; these in their shining clearness now stand out conspicuously against the dark chromatophore-bearing patches. A few parts are tinted light yellow, viz. the dactylopodites, the tips of the uropods, and the antennary flagella; otherwise the general hue is a clean grey.

As time goes on the cuticle becomes less transparent and dark in colour. The hidden parts of the gills gradually get coated with a dirty fur of black material. The extremities of the limbs, of the antennæ and of the uropods, change from yellow to red. By the depth of this redness one can best estimate the relative age of the cuticle.

These facts are here set down because, as we have seen in the first communication of the series, it is important in the case of immersion experiments to know whether a given animal has recently moulted or not. Moreover, blanching of the animal when placed on a white background—see Tait (1910)—is much more manifest when the cuticle is new.

Some days before moulting a chalky whiteness begins to develop on the ventral aspect of the anterior four (free) thoracic segments. At first limited to the narrow sternites, the whiteness invades the non-calcified parts, and eventually the whole anterior ventral aspect looks chalky.

This whiteness, found also in similar circumstances in the other Oniscoidea but apparently not in any marine or fresh-water isopod, is by Herold attributed to the development of definite plates, rich in calcium carbonate, just underneath the old cuticle. Herold's statement I am in a position neither to dispute nor to confirm. The idea that the chalky appearance might be due to inclusions of air was rendered improbable by the fact that it develops in Ligiæ completely immersed in sea-water. When the overlying cuticle is shed no sign of whiteness remains on the animal, nor does the exuviated skin appear white. One observation bearing on the subject is worth recording. A Ligia immersed for thirty-four days in half sea-water died in the act of moulting the posterior half of its cuticle. In this animal the usual chalky appearance had failed to develop, whereas all Ligiæ that underwent moult during immersion in full-strength sea-water exhibited the local white coloration.

As the ventral whiteness increases in area the dorsal aspect of the four anterior segments likewise alters in appearance. One obtains an impression as if the cuticle in this region were slightly separated from the underlying parts; the colour is lighter, and the deeper parts can no longer be clearly seen through the old cuticle. This part of the body seems also harder than usual to the touch. At the present stage the animal, with dark posterior and lighter anterior portion, may readily be mistaken for one that has just undergone posterior moult; indeed, it was some little time before I appreciated the distinction. By testing the rigidity of the posterior part with the fingers the possibility of error is avoided.

The change in appearance of the dorsal aspect is detectible at least five days before posterior moult. How long the chalky whiteness on the ventral aspect is present, I cannot say; Herold gives a period of two to three weeks for other Oniscoidea.

In captivity an animal about to moult apparently seeks to retire from the others, preferably to a damp spot. Moulting individuals were invariably to be found on the piece of wet cloth kept in the box that served for habitation, and this even at times when the other animals had clustered together in a different region. Dampness is certainly essential. On one occasion a Ligia that had just moulted its posterior part was placed in a small dry tin in a warm room. When the tin was opened an hour later the animal was apparently dead, all reaction to stimulation having ceased. It was now replaced in the tin on a strip of cloth wetted with tap-water. Half an hour afterwards it responded feebly to stimulation. When uncovered seven hours later it ran about actively. Moulting may occur equally well in sea-water and in an air-medium.

In posterior moulting (which, as well as anterior, I have observed on many occasions) a complete transverse separation takes place between the fourth and fifth (free) thoracic segments, while a longitudinal split occurs dorsally and laterally on each side of the fifth, sixth, and seventh segments; no longitudinal or other splitting occurs in any of the abdominal segments. The animal, remaining almost stationary, loosens itself from the cuticle by a series of writhing worm-like movements; finally, it walks away, leaving the old integument with dirty, black gill-lamellæ behind.

A period of four days having elapsed (temperature from 10·5° to 14·5° C.), the creature proceeds to moult the anterior portion. Two longitudinal cracks, this time involving the second, third, and fourth thoracic segments (no such split occurs in the first segment), appear dorsally and laterally. The limbs are early withdrawn, and the animal by active movements, usually in the forward direction but also slightly backwards (a rare movement in Ligia), gradually frees itself from the remaining covering. In this process, owing to the softness of the underlying cuticle, the head remains characteristically depressed, while the passively trailing integument of the half-moulted antennæ may be inadvertently stamped upon by the already free and rapidly hardening front limbs. As the antennary always adheres to the head-covering, by this last means the exuviation of the headpiece is incidentally aided.

Unlike the posterior covering of the animal, which is left behind in a fairly coherent or intact condition, the front integument may be torn apart into its constituent limbs, into a single portion representing the middle dorsal covering of segments 2, 3, and 4, and into a headpiece to which the antennæ and first thoracic somite with its pair of walking legs adhere.* As a rule, the last two larger portions (dorsal covering of segments 2, 3, and 4, and composite headpiece) come away united. Where separation occurs between them, the animal may, for a day or two after moult is otherwise complete, continue to wear on its back the three middle dorsal portions of somites 2, 3, and 4. A somewhat indistinctly recorded observation of Herold (p. 476, fourth paragraph) may refer to the occurrence of a similar phenomenon in Oniscus.

The limbs in any newly moulted half of the body being for a short period incapable of function, we find that, just subsequent to posterior moult, Ligia walks by means of its anterior four pairs of thoracic limbs, the hinder part of the body and uropods trailing limply behind; while, just subsequent to anterior moult, it walks on the hinder three pairs of limbs. In the latter case, should the fifth pair be thrown forwards simultaneously (as a rule these limbs move alternately), the centre of gravity of the animal now lies behind the anterior line of support; in consequence, the front of the body is tilted upwards to an angle of about 40 degrees with the horizontal plane. So soon as ever these limbs are drawn backwards again, the head-end of the animal drops to the ground. In progression by simultaneous forward and backward movement of the fifth pair of limbs, the fore-end of the body executes a series of upward-rising and downward-flopping movements extraordinary to witness. Should the animal come to rest with the fifth pair of limbs thrown forwards, its curiously fixed posture might remind one of Owen's restoration of Mylodon!

In this connection I shall quote Friedrich's account of Porcellio, which I lighted upon only after the above paragraph was written. "The movements of the animals at this time are very awkward. On moult of the hinder part of the body . . . they use the front four pairs for progression. On moult of the front half of the body, the three new pairs of the fifth, sixth, and seventh thoracic segments (of which the first pair is directed

* Macerated unmoulted Ligiæ may fall apart by disruption at precisely the same sites. Such animals are probably near the moult, and the fact is of physiological interest. It may also be of practical interest if moulting of Peracarida and of other malacostracan groups is to be systematically investigated (see next section).

forwards) are responsible for locomotion, which in the circumstances is highly insecure, the centre of gravity now lying beyond the point of support. The animal is forced to carry its head as high as possible, whereby its whole appearance assumes a comic aspect."

In popular descriptions of the moulting of Crustacea authors never fail to comment on the fact that the newly emerged animal is markedly larger than the cast-off shell. It is a curious fact that in Ligia, subsequent to posterior and prior to anterior moult, no increase in the girth of the posterior part of the body can be detected with the eye. In an Idotea under similar circumstances the disparity in thickness of the two portions is most striking, the anterior half of the animal then looking as if it had been tightly constricted with bands. Even when moult is complete Ligia is not obviously larger than before.

Morphological.

Under this heading two questions will be discussed. We have seen that in the process of exuviation as studied in Ligia, splitting is limited to the thoracic region; further, that the splitting is of two kinds, transverse and longitudinal. The transverse splitting is obviously inter-segmental. The longitudinal splitting appears to involve a break in the actual exoskeletal somitic ring—a process quite unique, if true.

Longitudinal Splitting.—As may be remembered, the longitudinal splitting involves the second to the seventh (free) thoracic segment, the first being unaffected.

A typical walking limb of a malacostracan may for convenience be said to consist of seven separate segments. Many isopods show only six segments in their walking legs. Hansen (1893) demonstrated that in such cases the proximal segment, or coxopodite, is fused with the corresponding body-ring. Among the Idoteæ he obtained illuminating cases of partial and of complete fusion, on the basis of which he established his conclusion—for an improved series, with illustrations, consult Calman (1909, p. 202).

As a succinct epitome of the matter I quote Calman's account. "The coxopodite has the form of a complete segment movably articulated with the body only in the Asellota. In all other Isopoda it is more or less completely fused with the body and is often expanded into a *coxal plate* overhanging the attachment of the limb and replacing the pleural expansion of the somite to which it belongs. . . . When the suture line disappears, as in most Oniscoidea, it is impossible to distinguish the coxal plate from a true pleuron. In all Isopoda, however, with the single exception of the genus *Plakarthrium* (Sphæromidæ), the coxopodites of the second thoracic somite (the first free somite) are completely coalesced with the body."

The trend of the present discussion will now be obvious. It is that the moult unmasks the line of junction between fused coxopodite and body-ring in the case of the second to the seventh (free) somites; in other words, that the longitudinal splitting is after all inter-segmental.

As proof I adduce the following facts. In *Idotea baltica* and in *Idotea emarginata* the line of junction between coxopodite and tergite (in the case of segments 2 to 7) is visible dorsally. During moult a longitudinal split occurs at these junctions, no corresponding split occurring in the first (free) segment. In *Asellus aquaticus*, in which the coxopodites do not form coxal plates, the body-rings are detached complete. Finally, in Amphipoda (*Gammarus, Orchestia, Talitrus*), in which the moult is not a double but a single event, splitting occurs along the line of the coxotergal junctions of all ambulatory limbs.

Examining the dorsal aspect of Ligia, one notes a faint line on each side running backwards from the animal's eye parallel to the lateral border. On the anterior four (free) thoracic somites this line is composed of two parallel furrows separated by a very narrow ridge; on the remaining three segments it is represented by a single furrow. When Ligia moults, the three posterior pairs of coxal plates separate from the tergites at the single furrow; the three preceding pairs separate at the more lateral of the two parallel furrows. The coxopodites are then found to have a convex upper border, while the tergites are hollowed out (as in some Idoteæ) to correspond. In those Oniscoidea in which the line of junction between coxopodite and tergite is not externally marked, e.g. *Oniscus, Porcellio, Philoscia*, a similar separation, disclosing similar moulding, occurs on moulting.

One or two points of general interest may be mentioned. In retaining traces of the original line of junction between coxopodite and tergite Ligia is more primitive than many Oniscoidea. The absence of splitting between the coxopodite of the first walking limb and its corresponding tergite gives interesting support to the conclusion (otherwise derived) that union of first coxopodite with body occurred early in the ancestors of Isopoda. The occurrence of coxotergal splitting in amphipods as well as isopods, and its absence in decapods, shows that this mode of moulting is an important arthrostracan, if not peracaridan, feature.

Transverse Splitting.—We should now be more inclined perhaps to attribute systematic importance to the site of the transverse split. According to Lockwood (1870) the cuticle of *Limulus* splits along a "thin narrow rim" which "runs round the under side of the anterior portion of the cephalic shield." In the moulting decapod transverse splitting occurs between thorax and abdomen. In the amphipod, as in the isopod, the transverse splitting is intrathoracic, not however between the fourth and fifth (free) segments, but between the first and second true segments, *i.e.* between the apparent head and thorax.

The functional separation of the isopod into two locomotor areas at the time of moult temporarily unmasks for us a primitive distinction between two groups of walking limbs. When discussing function of the limbs I shall return to this subject. In the meantime my only aim is to establish a case for the study of *moulting* in its relation to systems of classification.

Physiological.

In the case of amphipods and decapods moulting takes place in one single act. A Gammarus or a lobster, however, withdraws the anterior, not the posterior, part of its body first. In these animals the gills have a thoracic and not as in isopods an abdominal attachment. One might in consequence be tempted to suggest that the order of withdrawal of the body is determined by the position of the gills—an arrangement quite comprehensible from the physiological standpoint. The fact that the Brachyura (also with thoracic gills) are said to withdraw the abdomen first, may or may not upset this supposition—*cf.* also *Limulus.*

One or two interesting facts observed during experiment upon Ligia fall to be recorded under the heading of Physiology.

As we shall find later, the walking limbs of Ligia react in a very specific manner to sensory stimulation. It is remarkable that, immediately after either posterior or anterior moult, the freshly emerged limbs do not respond to stimulation of any part of the body. Whatever be the point at which the interruption occurs, the normal reflex mechanism is for the time being out of gear. One can scarcely refer the interruption to softness of the cuticle, for on such a supposition the muscles are discounted. The condition lasts only for some minutes.

Another curious fact observed in the case of the animal mentioned on page 61 as having been restored to life after virtual death by drying, is that the earliest motor reaction then elicited was one of the *posterior* and not of the *anterior* limbs, which failed to react. A hypothesis formed at the moment was that all the available internal fluid, essential for function of the reflex mechanism, was temporarily monopolised by the hinder half of the body. In a half-moult, such as occurs in isopods, we may assume that the newly moulted part swells by absorption of water; if this water cannot come from the outside, it may conceivably be abstracted from the other half of the body.

Many of the land isopods after moulting devour the cast-off cuticle. A similar observation has frequently been made in the case of the lobster. I have not observed anything similar in the case of Ligia, the exuviated cuticle, whether in air-medium or in sea-water, being apparently left untouched. Small pieces of the shell of a fowl's egg left in the tin beside the animals were similarly undisturbed.

To the author the most interesting physiological observation in respect of exuviation was the fact that several Ligiæ, after a long fast in clean sea-water, moulted normally, some of them surviving for quite long periods thereafter. Out of the set of eight animals recorded in the first communication of this series as having been immersed in sea-water, five moulted completely and one died during moult. The following table gives the number of days after immersion at which moult occurred:—

Posterior Moult.	Anterior Moult.	Total Period of Survival.
14 days	18 days	65 days
27 „	31 „	58 „
31 „	35 „	52 „
35 „	...	35 „ (died during moult)
46 „	?	>83 „
54 „	?	>83 „

From this it appears that after a fast of 54 days (during which there was constant expenditure of energy in movement) a Ligia can still moult normally. The fact is of special interest in view of ideas that still prevail with regard to the subject of moult. Since Réaumur's classical description (1712, 1718) of the moulting process in Astacus, it had been implicitly assumed that a crustacean moults because it has grown too large for its skin, or, to put it otherwise, that growth is the immediately determining cause of the moult. Having suggested the probability that the skin of crayfishes does not enlarge in the intervals between moults, Réaumur used these words: "Leur habit devient trop court et trop étroit, il les gêne, il faut qu'elles le quittent."*

Of late years the conceptions with regard to moulting have been under-

* In this connection the following sentence by a writer in the *Encyclopædia Britannica* is worth quoting: "As a rule Réaumur avoided theoretical questions, but when he took them up his manner of treatment was remarkably clear, chiefly on account of an ingenious use of metaphor, often expanding into allegory."

going change, chiefly perhaps owing to the work of the lamented G. Smith *
(1912, 1913)—*cf.* also Robson (1912). Smith established the existence of profound cyclic metabolic changes in the crab, which were not mere questions of calcium metabolism (Réaumur, strange to say, first proved the existence of the latter). His investigations included metabolism of glycogen, of fat and of blood pigments, in all of which cyclic change, related to the moulting periods, was detected. Arguing from the work of Potts (1906) and of Sexton and Matthews (1913), he declared that "growth and moulting are not necessarily connected processes," and that "moulting may take place without growth."

While the last conclusion was not absolutely warranted by the facts adduced in its support, it is certain that Smith's general contention, so far as it implies disavowal of the Réaumur conception, is correct. The results obtained in such a simple way with Ligia prove that moult may be independent of immediate intake of food. Whether the result applies to all Crustacea is another question. Smith singles out for special mention the fact that "starved or underfed crabs never moult in an aquarium, however near they may be to the moult when captured."

In a recent paper Paul and Sharpe (1916) thus express themselves: "Moulting is only the most noticeable part of a cyclic metabolic change in the crab. It comes at the end of a period when storage of reserve material has been the chief business of the animal." The Ligia experiments are in full accord with the first of these two statements, the significance of which they even extend; but hardly with the second.

For a further point of physiological interest in connection with moult the reader is referred to the immediately preceding communication of this series.

Summary.

(1) Ligia, like other iposods, moults in two stages. First the covering of the abdomen with that of the posterior three thoracic segments is exuviated. About four days later the anterior covering is thrown off. The external changes that occur in the cuticle as it ages, and the behaviour of the animal during moult, are described.

(2) At the moult the cuticle splits in two main directions: (i) transverse between the fourth and fifth (free) thoracic segments; (ii) longitudinal, at the coxotergal junctions of (free) thoracic segments 2 to 7, there being no coxotergal split in segment 1. Coxotergal splitting is an arthrostracan, if not a peracaridan, feature.

* Killed in action.

(3) Ligiæ that have fasted for many weeks in sea-water may still moult normally. It follows that onset of the moult is determined by an underlying cyclic change, not as Réaumur suggested by simple growth of the animal. A second moult during the period of fast was not observed.

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VI.—Experiments and Observations on Crustacea: Part III. Limb-Flexures and Limb-Taxis in the Peracarida. By John Tait, M.D., D.Sc. (From the Laboratory of Physiology, Edinburgh University.) *Communicated by* Professor Sir E. A. Schäfer.

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In order to obtain light on the functional heredity of the limbs of Ligia it was found necessary to consult the morphological papers of Boas (1883), of Hansen (1893), and of Calman (1904), on the classification of the Malacostraca, along with monographs and other records devoted to smaller groups. The result has been somewhat curious, for an investigation originally undertaken from a physiological standpoint has become semi-morphological. Finding that some of the accepted views with regard to the flexures of the limbs of Crustacea do not throw sufficient light on the case of Ligia, I have made an attempt, starting with Ligia, to restate the question of limb-flexion as met with in the Peracarida. From paucity of comparative knowledge such an essay cannot hope to have any finality. It is rather the belief that the morphologists have not said the last word on the subject that has induced me, tentatively and with second-hand resources, to enter upon a difficult task in an unfamiliar field.

For convenience I have arranged my remarks under sections, commencing with a description of the walking limbs of Ligia, which contains detail not given by Hewitt (1907) in his monograph on the animal. This first section will likewise serve as part introduction to a subsequent communication which will deal with gait.

The Walking Legs of Ligia.

Seven in number on each side, the walking limbs form a remarkably uniform series, as in all the Oniscoidea. Each limb consists of six movable segments (basipodite to dactylopodite). From before backwards they gradually increase in length, the seventh in order being nearly twice as long as the first.

Basipodites.—The largest and most powerful segment of each limb is the basipodite, which, arising laterally, is, in the position of flexion, directed medially. The successive basipodites increase slightly in length from before backwards. In flexion—the position of rest—they lie in grooves along the ventral soft tissue of the thorax, each pair posterior to its own sternite. These grooves have, generally speaking, a transverse direction, the anterior however running (latero-medially reckoned) somewhat backwards, the posterior somewhat forwards; consequently, when

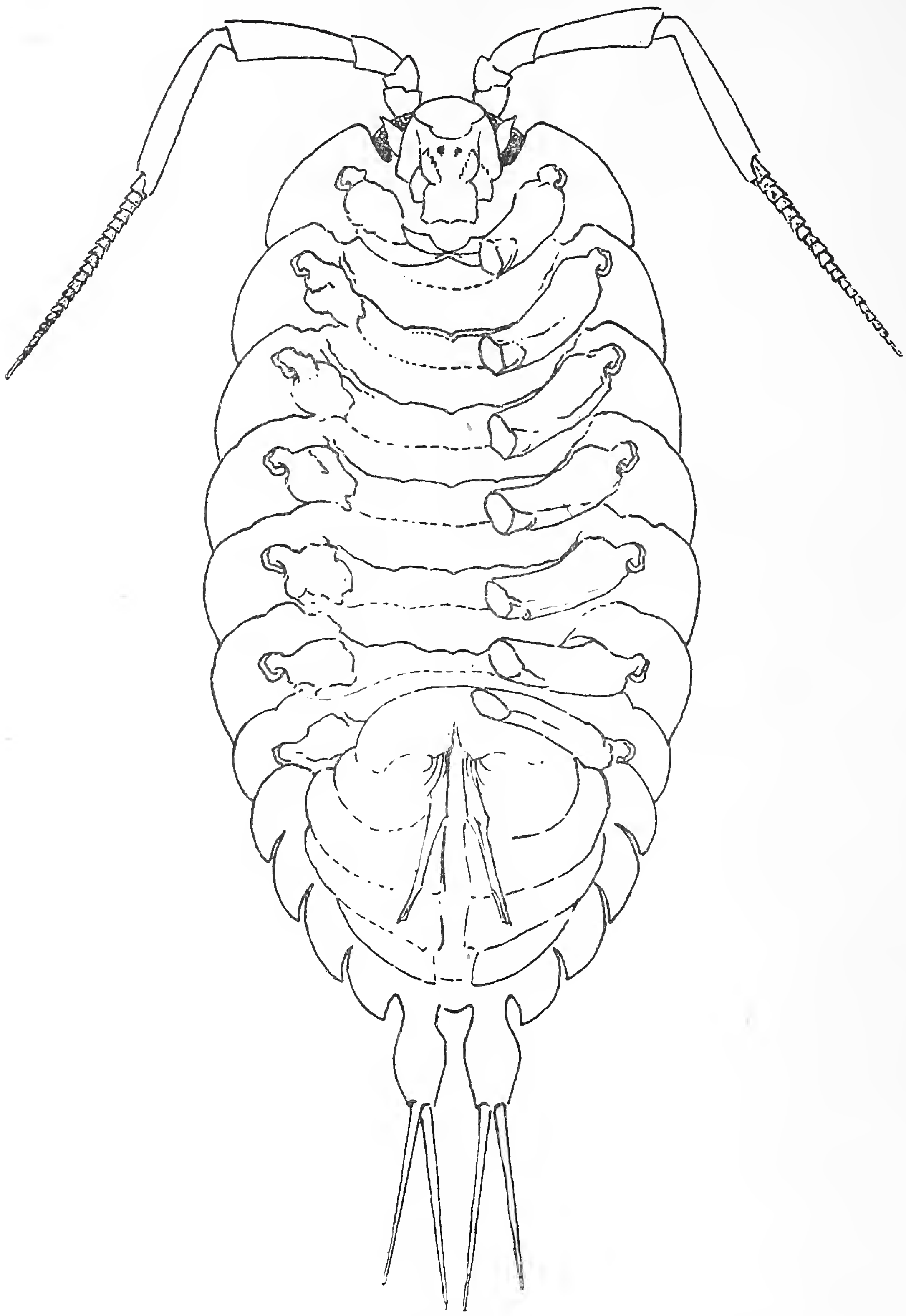


Fig. 1.—*Ligia oceanica* (male) from ventral aspect, to show the general alignment and the direction of flexion of the basipodites. On the right side of the animal (left side of illustration) the basipodites have been cut across at their proximal end. On the right side of the animal the basipodites have been left *in situ*.

the basipodites are flexed on the body, the antero-posterior distance between their distal ends is considerably less than that between their proximal ends (see fig. 1). This arrangement, to a certain extent but not altogether the result of the curved locus of origin of the basipodites, is connected with the function of the anterior limbs as pulling and of the posterior limbs as pushing agents. With lateral flexion of the thorax, say, to the right, the interspaces between the successive proximal ends of the right basipodites are obliterated, and they all lie in contact. Similarly, with ventriflexion of the thorax, the basipodites on both sides are brought into contact along their whole length.

Each basipodite is capable of movement through a right angle at the coxo-basal joint. In the extended position the anterior four are perpendicular to the surface of the ground, the posterior three sloping increasingly backwards.

Ischiopodites and the Ischio-Flexure.—In the ordinary squatting position of the animal the extreme four distal segments of the limb (meropodite to dactylopodite) form an axis which runs parallel to the basipodite, but in the reverse direction, viz. medio-laterally. Between these two oppositely directed parts intervenes an ischiopodite, the length of which in successive limbs increases from before backwards. The basi-ischial joint, almost straight when extended, flexes to a right angle in the same plane as, but with opposite sign to, the coxo-basal flexure. A statement almost identical applies to the ischio-meral joint. As a consequence, the intervention of the ischiopodite marks an important turning-point in the line of the limb, at which the sharp medial flexion of the basipodite on the body is neutralised, as it were. For convenience in description we shall refer to this seat of double flexion at the basi-ischial and ischio-meral joints as the "ischio-flexure" (see fig. 2, A).

In the death position of the limb and invariably after immersion in distilled water (see the first communication of this series) the flexion at the ischio-flexure is right-angled only, for then the ischio-meral joint is extended while the flexion at the more proximal joint remains unaltered. It would seem that flexion at the basi-ischial joint is, in Ligia as well as in many other isopods, more fundamental than flexion at the ischio-meral joint (see fig. 2, B).

The Post-ischial Limb and the Propodo-Flexure.—From one point of view the post-ischial limb may be looked upon as a single axis or lever, which in the squatting position of the animal forms a very mild curve concave ventralwards. It can also be considered as furnishing the basis of a compound flexural unit like the ischio-flexure—and like it flexing in the same vertical plane as that in which the basipodite flexes. The meropodite and carpopodite lie in one straight line; the joint between them permits of no bending in the vertical plane. Marked flexion, to more than a right angle, and in the opposite direction to that at the ischio-flexure, occurs at the carpo-propodal joint. In the same plane and in the same direction the dactylopodite flexes on the propodite. The combined angular movement of the last two joints, which we might call for shortness the "propodo-flexure," equals two right angles. Thus the propodo-flexure is complementary to the ischio-flexure; by means of it the laterally directed dactylopodite can be made to point medially like the basipodite.

After death there is usually some flexion of the carpo-propodal articulation, and Boas (1883) on comparative grounds singled out the high

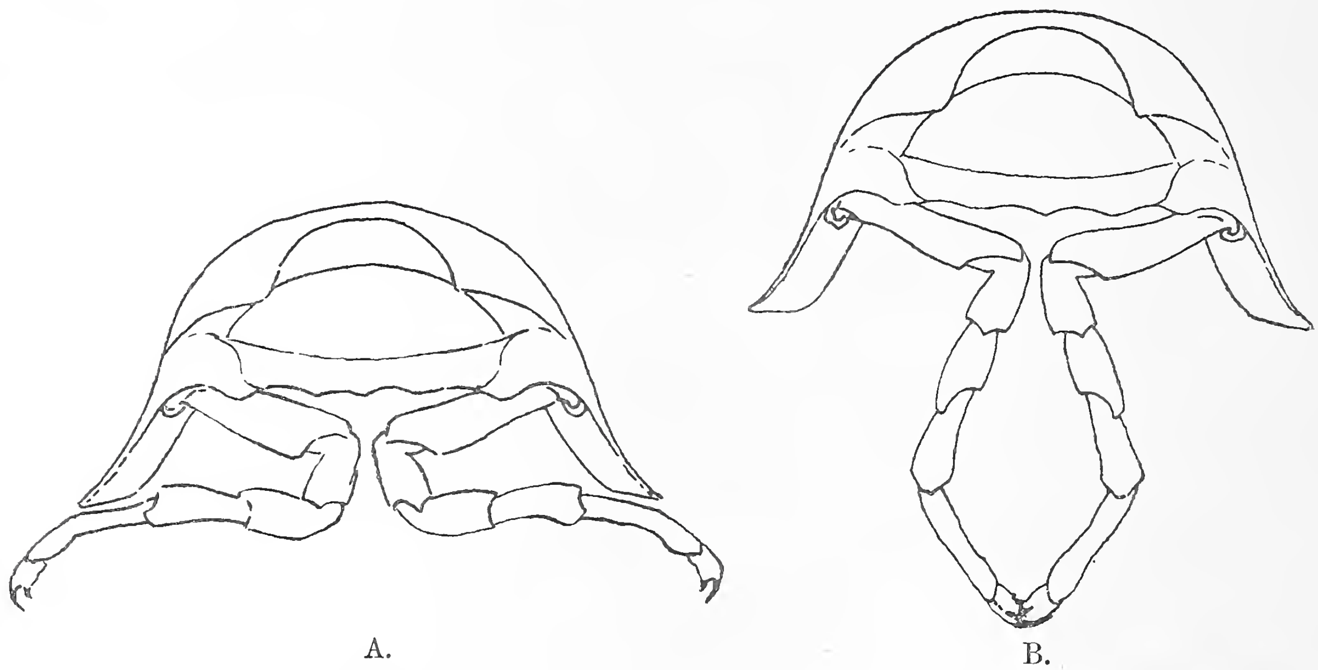


FIG. 2.—View of disarticulated fourth (free) thoracic segment of Ligia as seen from the front. In A (squatting position) there is acute flexion at the ischio-flexure. In B (position of limbs after death in distilled water) there is flexion at the basi-ischial articulation, the limb being extended at the ischio-meral articulation; at the carpo-propodal articulation there is also flexion.

degree of flexion possible at this joint as of special systematic value in grouping together the orders now included under Peracarida. As we shall soon see, however, to single out this characteristic alone as a peculiar feature of the peracaridan limb is trivial. At the same time it is of interest to note that in both the ischio-flexure and the propodo-flexure the more proximal of the two constituent joints is functionally the more important.

The Dactylopodites differ in many respects from the other limb segments. Like the dactylopodites of a shore-crab, they tend to be dark red in colour (see the last communication of this series), with a thicker and more heavily calcified wall and with less soft-tissue content than the more proximal segments. The latter are provided with chromatophores and with tactile hairs; the dactylopodites have neither. Each carries on its ventral aspect, arranged in the line of the post-ischial axis, two attenuated hooks, of which the more distal is the longer. These hooks are bent in the plane of the propodo-flexure, the inclination of the bend corresponding with that of the flexure.

The Flexion-Complex.—Let us now briefly pass in review the arrangement of one of these limbs, selecting, say, the limb of a land mammal for comparison. The basipodite evidently corresponds in function to an upper-arm or to a thigh, the ischio-flexure to an elbow or to a knee.* Beyond this point the correspondence is less exact. The post-ischial axis may indeed be likened to the forearm or to the leg of the mammal, and the dactylopodite to the (terminal phalanx with its) nail or claw. When we look, however, for a flexion system corresponding to that of the ankle, in its relation as regards sign of flexion to the knee, we have to select simply the propodo-flexure, leaving nothing to correspond to the phalangeal system of the mammal. In other words, the mammalian hind-limb exhibits an additional flexion system with new change of sign. In Ligia the "flexion-complex" is tri-alternate instead of quadri-alternate.†

While the general similarity to the arrangement of joints in a mammalian limb is obvious, it is, strange to say, less easy to trace an analogy between the limb of Ligia and the limb of a crab. The reason is that in the crab co-planar flexures do not occur in such regular sequence. Movement of the walking limb of a crab has been designed on quite a different plan from that observed in Ligia.

Limb-Taxis and Clinging Function.—Under this heading I shall discuss the general disposition of the seven pairs of limbs in relation to the body of the animal, commenting on one purpose thereby served.

As already emphasised, the flexion-complex is predominantly uniplanar, which means that the dactylopodite of each limb moves in approximately the same plane as its basipodite. The basipodite of the first walking limb moves in a vertical plane which is almost but not quite antero-posterior; the dactylopodite in consequence hooks into the ground in such a way as to resist a backward pull on the part of the limb. The basipodites of the third and fourth limbs are arranged transversely, and their dactylopodites hook into the ground in such a way as to prevent lateral displacement of the animal. The basipodites of the seventh limb, again, move in a more

* The use of the term "knee" to designate the carpo-propodal joint—Boas (1883), Hansen (1893)—is unhappy. On the other hand, Sars' (1869) description of the basi-ischial joint as a "geniculate bend" is appropriate.

† Properly speaking, the term "flexion-complex" should imply a complete specification of the movements of all the joints taken in order. With a series of six joints the possible complications are many. It is partly for simplicity, but also because this mode of description closely corresponds to the facts, that I describe the flexion-complex as uniplanar.

antero-posterior direction, and their dactylopodites catching into the ground resist a forward pull. Roughly, it may be said that the terminal claws, arranged along the border of the animal "from bow to stern," like so many boat-hooks held out from the side of a boat, give the animal an exceedingly firm grip; as may be seen by placing the creature on a sheet of sand-paper and subjecting it from any quarter to the draught of an electric fan.

The animal is thus excellently adapted for clinging. How the limbs are operated in progression need not concern us at present, and we shall avoid needless complications by postponing consideration of gait, which involves, in addition, a complex movement at the coxo-basal and a more simple movement at the mero-carpal articulation. The essential thing at this stage is to have a clear realisation of the general arrangement of this series of limbs, the long and powerful basipodites flexing medially under the body, the dactylopodites clutching the ground all along the lateral border.

Different Types of Limb-Taxis in the Peracarida.

Ligian Limb-Taxis in other Isopods.—While the Ligian variety of limb-taxis, associated with great clinging power, is a peculiarly isopodan feature, it is not characteristic of all the sub-orders. Perhaps in some Asellota, and certainly in the Phreatoicidea, and in the atypical Flabellifera, different arrangements prevail. From the point of view of walking limbs, the purely parasitic Epicaridea may in the meantime be neglected. In the more typical Flabellifera, in many Valvifera, and in the Oniscoidea the limb-taxis is Ligian. Some points of general interest here arise.

Members of the three last-named sub-orders resemble each other and differ from other Isopoda in the fact that the coxopodites of the walking limbs have become expanded into coxal plates more or less rigidly connected with the body. In the Asellota and in the Phreatoicidea the coxopodites are small and movable. One might be inclined to associate rigidity and lateral expansion of coxopodites with Ligian limb-taxis were it not that in some Asellota the taxis is like that in Ligia.

The typical Flabellifera and the Valvifera use the thoracic limbs chiefly for clinging, not so much for progression. While one naturally correlates the parasitic habits of Flabellifera with this peculiarity, a potential capacity for a similar life, so far as limbs is concerned, may be seen even in vegetable-feeding Idoteae. One has only to place an *Idotea baltica* in a vessel containing sea-water and a small fish, say, a slippery gunnel, to realise this. Whatever the immediate object, whether clinging to animal prey or to seaweed, the thoracic limbs of these animals serve as hold-fasts rather than as organs of locomotion. These facts possibly convey a hint that the well-developed locomotor power of the corresponding limbs of Oniscoidea represents reacquisition of a function temporarily in abeyance in their more immediate marine ancestors.

The general type of limb-taxis described as present in Ligia I shall henceforth call "isopodan." By this term is implied merely that the limbs with tri-alternate flexion-complex lie on the whole in planes transverse to the body, the basipodites being directed medially. Isopodan limb-taxis may be subdivided into varieties. For example, the "oniscoidean" variety is peculiar in that the various planes of flexion of the limbs vary uniformly in direction as one proceeds from front to back, not abruptly as in many Flabellifera and Valvifera.

Amphipodan Limb-Taxis.—In amphipods we meet with a slightly different type of limb-taxis. In each thoracic limb one can detect a uniplanar tri-alternate flexion-complex, composed of flexural units containing elements identical with those in the isopod limb. In most amphipods the first four limbs have an ischio-flexure, the apex of which points backwards and which is analogous to a mammalian elbow, while the ischio-flexure in the posterior three limbs points straight forwards like a knee. The front four dactylopodites might be considered as hooking into the ground so as to resist a backward pull on the part of the limb, the last three dactylopodites (at least in the conventional position often adopted in illustrations) then digging into the ground in the opposite direction. (Deviations from type and other complications will be discussed later.) To the amphipodan taxis the simile of a boat held fast by a series of boat-hooks projecting all round the gunwale does not apply; the taxis is exactly antero-posterior and oppositely directed, and might be roughly compared with that seen in the limbs of a horse. An identical limb-taxis occurs in the isopod *Phreatoicus*.

The basipodites in this case undergo flexion in a plane parallel to the sagittal. The coxopodites, virtually incapable of flexion at the coxo-somitic joint, in typical amphipods take the form of great, laterally compressed segments (we neglect the extra expansions on the first four), which, possibly for reasons connected with muscular efficiency,* lie in the same plane as that in which the basipodite flexes.

* On similar considerations one would expect a (mean) vertical transverse flattening of coxopodites in Ligia. It might be urged, however, that such a disposition would by resistance interfere with forward progression of the animal in the fluid-surrounding medium. Perhaps more important is the fact that in the typical isopodan limb with fused coxal plate the articulation between coxopodite and basipodite is analogous to a ball-and-socket joint, the coxo-basal articulation in the amphipod being a simple hinge.

At this juncture I shall quote from Calman (1909) a passage that caused me perplexity. "The lateral compression of the body in most Amphipoda has led to a separation of the thoracic legs into an anterior group of four (the two gnathopods and the first two peræopods) and a posterior of three, which are opposed to each other in the direction of the principal articulations." Then follows a sentence or two describing the structure of these limbs in more detail, to which is subjoined this footnote: "The correlation between the lateral compression of the body and this grouping of the legs is well illustrated by comparison with *Phreatoicus*, the only isopod where the body is laterally compressed and where the legs are divided into two groups exactly as in Amphipoda."

One would be bold perhaps to dispute the existence of any correlation. Independently of this, it is difficult to see how the lateral compression has led to a separation of the legs into groups. Here a more detailed specification seems to be necessary. We meet with separation of the legs into two groups in cases even of isopodan limb-taxis (Valvifera, Flabellifera), which involves *breadth* of body. Apart from either of these two forms of limb-taxis, we meet with separation of the legs into two groups showing contrast in the direction of the principal articulations, in other Amphipoda and Isopoda and even in Tanaidacea that show no particular lateral compression of the body; in all these the basipodites have a still different orientation, neither flexed under the body (isopodan type) nor in an antero-posterior direction (amphipodan type), but projecting laterally outwards. The real correlation, as Calman indicates in his footnote (as if still beset with qualms as to his logic), hinges upon the peculiarly antero-posterior plane of flexure of the limbs, not upon their separation into groups. Even in the end the "lateral compression of the body" is more apparent than real, being largely if not wholly due to the vertical depth of the coxal plates.* Chilton (1894), too, describing the genus *Phreatoicus*, remarks: "The lateral compression of the body is not so great and is chiefly seen in the pleon, where the pleura of the segments are produced downwards."

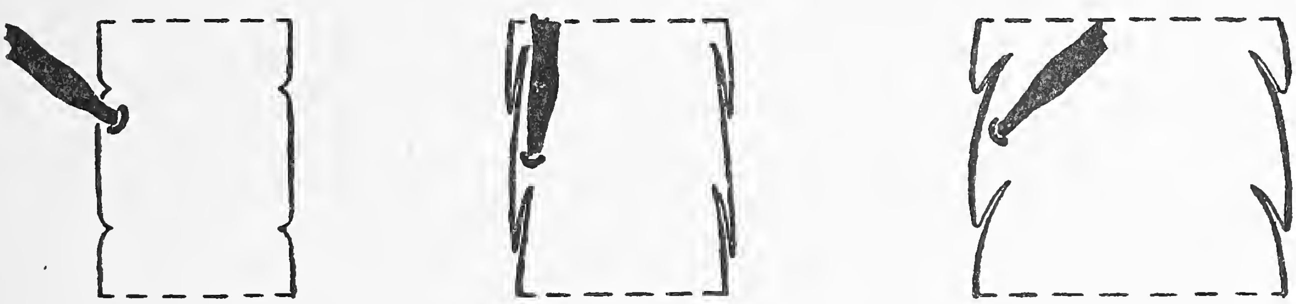
The matter therefore reduces itself to this: When the taxis is exactly antero-posterior, the coxal plates, if present, are antero-posterior. As we shall immediately see, the separation of the legs into groups is an independent modification.

Tanaidacean Limb-Taxis.—As tanaidacean we may for convenience designate the type of limb-taxis incidentally referred to under the last

* "The large coxal plates on the thoracic somites projecting downwards increase the depth of the body and add to the appearance of lateral compression"—Calman (1909).

sub-heading as associated with lateral projection of basipodites from the body. Just as in the amphipodan type, the limbs with uniplanar tri-alternate flexion-complex show a separation into two groups opposed in the direction of the chief articulations. The anterior basipodites project laterally and backwards, the posterior laterally and forwards. The consequence is that the dactylopodites still dig into the ground with a backward or forward inclination, as the case may be.

This type is present apparently in all the Tanaidacea; in the atypical Flabellifera (Gnathiidæ, Anthuridæ); perhaps in some Valvifera (Arcturidæ) and Asellota; and in the less typical Amphipoda (Caprellidæ, Cyamidæ). It is therefore widely prevalent, whereas isopodan taxis is wholly, and amphipodan almost wholly, confined to the respective orders indicated by the adjectival prefix.



A B C

Fig. 3.—Diagram to show the direction of the basipodites in different forms of limb-taxis. A, tanaidacean; B, amphipodan; C, isopodan.

In the diagram in Fig. 3 the three forms of taxis are schematically represented.

The Primitive Taxis.—We come now to an important question from a systematic point of view. Of the three types of limb-taxis, which is the most primitive?

The isopodan may probably be set aside at once as the least frequent and highly specialised. In fully developed form it demands great separation of the basal parts of the limbs and consequent breadth of body. It seems to be unique in the animal series.

The next in increasing order of frequency is the amphipodan, probably also specialised. A deep and narrow figure, at least in an aquatic animal, is mechanically ill-adapted for prone progression; without lateral grip on the walking surface it can readily be washed over on its side.

The mechanically simplest form, the earliest to occur (from Mysidacea onwards), the most widely prevalent, and for all these reasons the most *generalised,* is the tanaidacean, in which the limbs spread more or less directly outwards, affording a wide basis of support and attachment. Through the kindness of Dr J. H. Ashworth I had an opportunity of examining not only *Mysis* but fine spirit specimens of the syncaridan *Anaspides*. In this latter animal, too, the basal parts of the limbs flex outwards. The proof from comparative morphology seems to be complete.*

Conditions attaching to Reptant Limbs.

We shall now return to a discussion of the flexion-complex and of the grouping of the limbs into two opposed sets.

The Tri-alternate Flexion-Complex.—The quadri-alternate and tri-alternate flexion-complex in the limb of land mammals and of Ligia respectively is undoubtedly due to a long heredity of walking or crawling habits. Among the Vertebrata we meet with alternate flexion within the compass of the limb for the first time in Amphibia. In fishes there is hardly a hint of such an arrangement; in swimming mammals the complex shows all degrees of reversion to the flexural simplicity of a fin.

If we take the Peracarida as a group comparable to the Vertebrata, we find little or no alternate flexion in the limbs of swimming Mysidacea; in those of the Cumacea, especially in the posterior thoracic or fossorial limbs, we see indubitable evidence of a tri-alternate flexion-complex; finally, in the Tanaidacea, the Isopoda, and the Amphipoda the walking limbs may be said to show marked tri-alternate flexion.

The argument may be continued from the functional side. Imagine an unbending peg or projection (ventrally hinged and controlled by opposing antero-posterior muscles) to be used not for swimming but for progression over the ground. The limb pointing forwards, its distal end is fixed to the walking surface; the proximal end then describes an arc of a circle round this fixed point. The body of the animal, as it moves forward in correspondence, first rises and then falls again. This, in itself, like the galloping of a horse, is a strenuous and energy-wasting mode of progression. The limb has now to be moved forward again; it can do so only if there is a possibility of alternate flexion. (On analogous reasoning one might show an advantage in alternate flexion in a limb used for prehension.)

The general deduction appears unassailable. As an example of its application to specific cases, we may take the swimming function sub-

* One might perhaps associate the development of coxal plates (Isopoda and Amphipoda) with the more specialised limb-taxis.

served by the posterior three thoracic limbs of the Munnopsidæ. Apart from the mere morphological argument—*cf.* Hansen (1904) on the classification of the Asellota, and Beddard's (1886) figures of Munnopsidæ in the "*Challenger*" *Report*—this is *obviously* a new adaptation of previously reptant limbs. According to Sars (1889), *Desmosoma, Munnopsis, Ilyarachna,* and *Eurycope* all employ these limbs for backward propulsion. If it should be the case that the Mysidacea use the thoracic limbs only for forward swimming, and that backward swimming by means of these limbs is otherwise unknown among Peracarida, the phenomenon would, from a purely physiological point of view, be remarkable. Observations on the functional mechanism of the three posterior walking limbs of Ligia (as, for example, during anterior moult—see the immediately preceding paper of this series) would, however, indicate a possibility of deriving the backward progression of Munnopsidæ from a facility first acquired by reptant forms.*

As already indicated, the tri-alternate flexion-complex is no characteristic of the decapod walking limb. The limbs of reptant forms of the Eucarida have developed along another line from those of reptant forms of the Peracarida. At the same time the general principle holds, viz. that an elaborate flexion-complex found in any limb used for swimming means that the natant function has been secondarily derived from a more primitive reptant one—*cf.* the last pair of thoracic limbs in the Portunidæ.

Separation of Limbs into Two Groups.—As possible causes of the distinction between the two sets of limbs two contingencies suggest themselves. The phenomenon may be associated with the presence of the thoracic brood-pouch found throughout the group—one might suppose, for instance, that the basipodites converging from both ends of the thorax towards the middle might help to protect the brood-pouch. On the other hand, the separation might be connected simply with the function of locomotion.

The fact that there is no such separation in the Mysidacea, which possess a brood-pouch, and the further fact that the separation becomes apparent for the first time in Cumacea, which are reptant, would indicate that the division follows upon the acquisition of a crawling habit. The comparative evidence is not limited to the Peracarida. In Geoffrey Smith's (1908) figure of *Anaspides* in the natural position for walking (see also *Cambridge Natural History*, vol. iv, p. 111), the separation is

* Chilton (1894) mentions that the anthuridan *Cruregens* runs "backward and forward" with equal facility.

plainly shown, the taxis suggesting a simple inversion of the oniscoidean taxis.

Let us take the argument from analogy. Vertebrates have only two pairs of limbs between which to differentiate; the same kind of distinction occurs in them so soon as they are used for crawling.

Lastly, the functional argument. Suppose that the whole seven limbs of a reptant peracaridan were like the anterior three or four; the animal would be adapted for forward progression but not for halting dead or for holding on against a current of water directed from behind. Generally speaking, one should also bear in mind the important fact (explanatory of many things), that an animal creeping in water derives little attachment to the under-surface from gravity.

On all these counts it seems certain that the separation is an essential concomitant of a successful reptant life—at least in cases where the flexion-complex is uniplanar.

Several subsidiary questions remain for discussion. Is there any particular reason why the separation should commonly occur between the fourth and fifth walking limb? why should the line of division vary in position (in some isopods it occurs between the third and fourth pair of limbs)? and what leads to obliteration of the distinction in the Oniscoidea? While I intend to recur to these matters, I should here say that it is more easy to ask such questions than to provide answers. (The reader might perhaps refer to the immediately preceding paper of this series.) The most interesting question of all, and the greatest puzzle, is the presumable development of new neuro-muscular mechanisms in the reptant limb.

Land Adaptation.—The Mysidacea, the Cumacea, and the Tanaidacea are exclusively aquatic. Only among the Isopoda and the Amphipoda do we meet with land forms. In such cases the animals never have tanaidacean limb-taxis. The taxis which has proved suitable for progression on land is either isopodan or amphipodan.

In considering the design of any limb, reptant or natant, one might call attention to a mechanical principle known as "bending moment." In a branch of a tree breakage from wind or weight is most apt to occur at the junction with the trunk, and here the strength has to be greatest. The wood is hardest here; the branch also tapers as it is followed outwards, not simply in accordance with the provision of nutritional tubes for the leaves, but also for statical reasons. So in the limbs of an animal (even in a land animal they are never maintained all the time in a vertical position) the greatest bending moment is at the junction with the body.

The muscles at the joint in this region are strongest; the skeletal parts are also strongest.*

On change from water to land the question of gravity in its relation to the design of the limbs comes immediately to the forefront, for the bending moment at the proximal part of the limb is then greatly and permanently increased. In Peracarida with tanaidacean limb-taxis the transverse distance between the proximal part of any pair of limbs is slight, and on general grounds one might say that there is no available room for transversely arranged muscles to counteract the bending moment at the most proximal flexure. I shall not follow the problem into its details, for it is an involved one, and in any case cannot be settled simply on *a priori* reasoning. It is along these lines, however, that we may hope to explain the eventual success of isopods and amphipods in accommodating themselves to the conditions associated with a reptant life on land.

Rotation of the Plane of Flexure of the Limb.

In a developing mammal the limbs, originally straight outgrowths from the side of the body, by alternate flexions in one plane become bent into the position of the limbs of a tortoise. In this bending the "pre-axial border" remains anterior. Then the upper or humeral segment of the anterior extremity is rotated *horizontally* backwards from the shoulder through a right angle, while the corresponding segment of the posterior extremity is rotated to the same extent forwards from the hip. The consequence is that the pre-axial border of the fore-limb is lateral and the elbow points backwards, whereas the post-axial border of the hind-limb is lateral and the knee points forwards.

In the reptant Peracarida we are evidently confronted with a problem of analogous nature. As in the mammal, the site of rotation lies at the proximal part of the limb. In the bony limb the presence of a spheroidal joint may be essentially bound up with the possibility of this rotation (the retention of permanent "rotatibility" along with unimpared action of the proximo-distally arranged muscles suggests this). The working out of the corresponding problem as it affects the arthropod limb would not only have a systematic bearing but would form an interesting contribution to the study of analogy, a branch of inquiry instinctive to the

* This is independent of the question of *inertia*, which brings about reduction of mass, and consequently of muscle and supporting material, as one proceeds towards the distal extremity of the limbs of an animal designed for speed. The shifting of the muscles to the proximal part of the limb of an antelope and the provision of long tendons in the distal part come under this category.

physiologist but usually eschewed by the morphologist, and would provide material for a study of the more recondite principles underlying animal architecture.

As showing the suggestiveness of the problem, I may refer to a feature present in Amphipoda which distinctly gains in interest by comparison with the bony limb.

Backward rotation of the fore-limb of the land mammal has the disadvantage of making the digits point backwards, so that phalangeal flexion resists not a backward but a forward pull on the part of the limb. This defect is redressed by a subsequent secondary *torsion* of the forearm through two right angles, the dual bones here providing the

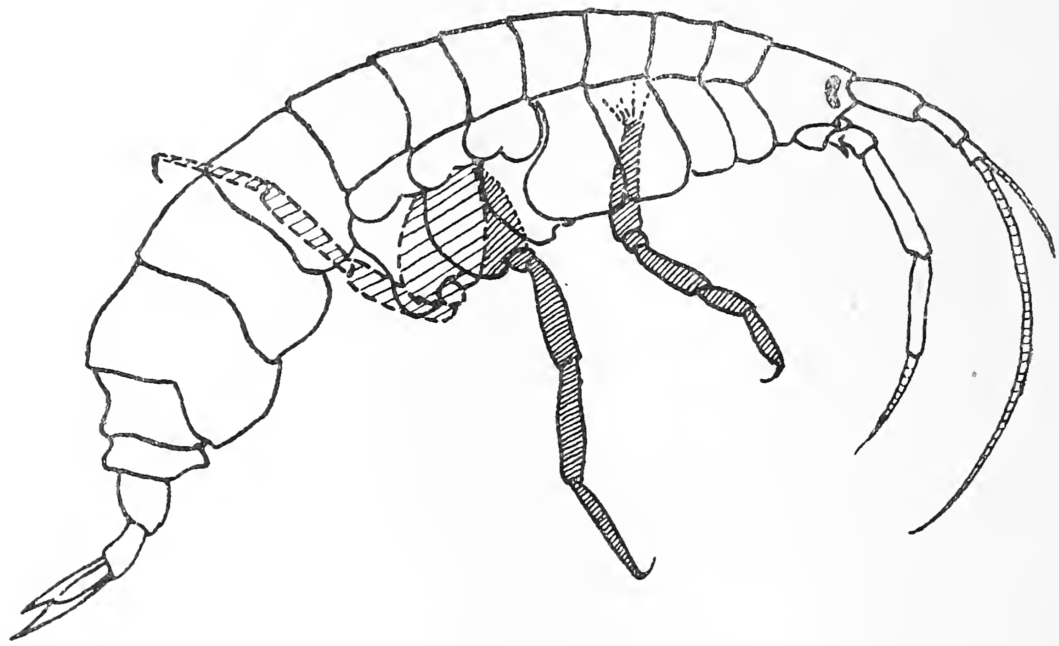


Fig. 4.—Diagram to show how one of the posterior thoracic limbs of Gammarus with anteriorly pointing dactylopodite is made to assist in forward progression. The limb is thrown over the back and the dactylopodite then points backwards.

structural basis for rotation in the axis of the limb. In the Gammaridea forward progression occurs by means of the posterior three limbs. Here the terminal segments, in their original position hooking into the ground so as to resist a forward pull on the part of the limb, have become adapted to resist a backward thrust by undoing of the first major flexure of the limb to the extent of a right angle and exaggeration of the second major flexure to the extent of another right angle, as in the diagram, fig. 4. Torsion of limb-segments is here avoided, the limb is simply thrown over the back, to which rearrangement the animal accommodates itself by adopting a new mode of progression, viz. with one or other lateral aspect addressed to the walking surface.

One naturally raises the question: Can extensive torsion of a segment of an arthropodan limb occur in the process of development or of evolution? In the literature I have found two papers, those of Herrick (1905) and of Emmel (1906), in which a total torsion of 90 degrees is proved to occur within the length of the regenerating cheliped of the lobster. From these papers, however, one cannot say whether the torsion actually involves the individual segments or not.

In this connection one might refer to the peculiar direction of dactylopodites in the posterior thoracic limbs of some of the Ampeliscidæ, Atylidæ, and Gammaridæ—see the illustrations in Sars (1895). In these animals the posterior dactylopodites hook in the same direction as the anterior. From a structural point of view it might seem a simple matter to reverse the direction of flexion of a dactylopodite (reversion of the direction of the carpo-propodal articulation can apparently occur—see the illustrations of *Arcturus anna* and *cornutus* in pl. xix of Beddard's (1886) "*Challenger*" *Report*). From the point of view of physiological co-ordination of movement, reversion of the direction of a dactylopodite is not so simple, for it must be presumed to involve far-reaching adjustments in the central nervous system. To assume a torsion of the mero-carpo-propodal axis avoids the central nervous system difficulty, but fresh *a priori* difficulties then seem to arise in regard to the peripheral muscles. In the absence of appropriate observations on the living animal one cannot of course profitably pursue the subject, but the interest of such cases is very great. (*Cf.* also the direction of the dactylopodite in the chela of Trischizostoma —Sars (1895), pl. xii.)

To recur, however, to the peculiar orientation of the posterior thoracic limbs in the Gammaridea, one might offer the suggestion that the isopod *Phreatoicus* progresses in a fashion similar to that common among the Amphipoda. In Chilton's (1894) account there is little direct reference to this point, but his discussion of the general build of the animal is suggestive.* I here set down in his own words his list of amphipodan similarities:—

"(1) The body, especially in the pleon, is more or less laterally compressed.

"(2) The pleura of the segments of the pleon are produced downwards, so as to protect the pleopoda on either side, just as in the Amphipoda.

"(3) The legs of the peraeon consist of an anterior series of *four* and a posterior series of *three*.

"(4) The general appearance of the legs and of the uropoda is not unlike that common among the Amphipoda.

* On page 208 Chilton says: "*Phreatoicus* walks erect or swims much in the same way as the Amphipoda."

"(5) The pleon is formed of six separate segments, and is better developed than in most Isopoda."

Even in the absence of any complete record as to the mode of use of the limbs the question is worth elaborating for a reason that will become more plain in the next succeeding section. We shall take the points under their appropriate numbers as Chilton has set them down.

(1) Greater narrowness of pleon than of peraeon is almost a mechanical necessity of the peculiar mode of gait present in Gammaridea. As regards lateral compression of the peraeon itself, Chilton in his discussion mentions amphipodan genera in which the body is more or less cylindrical, viz. *Caprella*, etc., *Corophium*, *Haplocheira*; and others in which the body is as much flattened as in most Isopoda, viz. *Icilius*, *Iphigenia*, and *Cyamus*. Many of these genera do not possess amphipodan taxis.

(2) There is no particular reason to suppose, as Chilton suggests, that downward prolongation of the pleura should arise "quite independently of the similar adaptation in the Amphipoda." Along with the gammaridean gait goes a peculiar use of the abdominal segments, which are rapidly flexed and extended. As he suggests, these pleural plates may have a function of protection for the pleopods; this would come into play particularly in *extension* of the abdomen. Moreover, beautifully fitted imbricating pleural plates may give mechanical support and guidance to the body segments in ventri- and dorsi-flexion (compare the arrangement in the peraeon of an amphipod with that in the peraeon of an isopod).

(3) Here I have no comment to make.

(4) Examination of Chilton's plates shows that the posterior three thoracic limbs have sufficient stretch to be thrown well dorsalwards. As he himself states, these limbs are "more amphipodan and flattened." The modification of the uropoda might well be correlated with what was said under (2) immediately above; in progression the uropods of Gammarus brush against the walking-surface.

(5) See also under (2) immediately above.

Should it be found on further investigation that *Phreatoicus* does employ the amphipodan mode of gait (if only to the extent of jerking itself forward by extension of the abdomen), we should of course be in a position to reverse the argument and to formulate clear correlations between structure and function. We should understand better the *raison d'être* of the so-called lateral compression of Amphipoda, or rather of vertical pleural plates, something of the principles governing build of uropods, and, lastly, the conditions that permit of concrescence of abdominal segments in Isopoda as compared with Amphipoda (in *Phreatoicus* almost alone among isopods are there six separate segments in the pleon). In this connection I might mention the comparison already drawn by Boas (1883, p. 565, footnote 2) between the "tail" region of natant and reptant decapods and that of natant and reptant vertebrates respectively.

General Discussion.

It will have been noted that the mode of treatment in this paper differs from that usually followed in a present-day morphological dissertation. The difference lies in the emphasis laid upon functional considerations (one is apt to forget that physiological evolution invariably accompanies morphological). The original object was to obtain light on the functional heredity of the limbs of Ligia. In seeking an answer to one question provisional answers to others have been obtained. It will help to focus matters if I first separate the different issues.

Morphological.—The ready method, due to Boas (1883), of discriminating between Peracarida and Eucarida by means of the flexures at the mero-carpal and carpo-propodal articulations proves to be merely part (perhaps an unessential one) of a wider principle which involves the limb as a whole. In the Peracarida the limb tends to flex alternately in one and the same (principal) plane. The arrangement is usually tri-alternate, but may become quadri-alternate, as in some of the Arcturidæ. In the reptant Eucarida the flexion-complex is of another kind, more difficult to describe in few words. Apparently correlated with the existence of a tri-alternate flexion-complex is a separation of the walking limbs into two opposed groups. According to the direction in which the proximal part of the limbs project from the body we distinguish three types of limb-taxis: one widely prevalent, the other two more restricted in occurrence.

These results are derived by following the existing classification, and to this extent merely give added colour to what is already known.* Can they be put to further use? Here I shall deal only with the Isopoda, and from want of the requisite knowledge in a very tentative way.

The Asellota form a homogeneous group, which has been revised of late years by Hansen (1905). The application of our principle to this sub-order might therefore be taken as a test case for the purpose of assessing practical utility. Hansen divides the Asellota into Asellidæ, Stenetriidæ, and Parasellidæ. Now, *Asellus* has isopodan limb-taxis;

* The fact that the question of limb arrangement has never been systematised partly accounts for the difficulty experienced in obtaining illustrations of inaccessible animals from aspects that make plain the taxis.

Stenetrium, or rather the species *antillense* figured in Hansen's paper, might have either tanaidacean or isopodan taxis; within the group of Parasellidæ, *Jæra* certainly has isopodan taxis, while some other forms are doubtful. Thus, figures of *Desmosoma* and of *Ischnosoma*, in which the basipodites occupy a position impossible in the case of Ligia, suggest tanaidacean taxis. Similarly, *Munnopsis typica* appears to have tanaidacean taxis, but Sars' (1899) figure, pl. lviii, in which, by exception, the ventral aspect appears, clearly shows the presence of isopodan taxis. It may be that all the Asellota have primitively an isopodan limb-taxis; on the other hand, some of them may not. Judging by this group there is no proof whatever that our principle is of any particular value for systematic purposes. In face of this unsatisfactory evidence, it would be useless to urge that the principle is based on an examination of wide groups of animal, including orders beyond the limits of the Isopoda.

Let us take the Valvifera, another homogeneous group. Though not recently revised it contains, according to the statement in Calman's (1909) invaluable book, six families — Idoteidæ, Chætiliidæ, Pseudidoteidæ, Holognathidæ, and Arcturidæ. The limb-taxis criterion makes a sharp line of division between the last family and the first five, which are all idotean in general aspect and have isopodan taxis. Thus the old separation of the Arcturidæ from the Idoteidæ—see Miers (1883)—is brought more into prominence than the other subdivisions. Here I am again unable to say whether our criterion is of any special value.

Sars' group Flabellifera is admittedly more heterogeneous than Valvifera. According to the test by limb-taxis, there is a clear line of division between the Gnathiidæ and Anthuridæ on the one hand and the remaining families of the sub-order on the other. Calman, not specially concerned with limb-taxis, remarks: "The Gnathiidæ . . . are an aberrant family whose relation to the more normal Flabellifera is not clear, and the same may perhaps be said of the Anthuridæ."

The taxis in the Epicaridea is isopodan. This may be compared with the statement, "the Epicaridea are closely related to some of the Flabellifera, the systematic value of the modifications due to parasitism having been here as elsewhere somewhat overestimated."

On the whole the amount of light we get by application of our new morphological criterion is problematical. Nor do we get much help in establishing a satisfactory phylogeny by considering that tanaidacean taxis is widespread and isopodan limited to its own order.

Physiological.—So soon as we consider the matter in its functional setting we obtain a new outlook. Isopodan taxis is specialised, an arrangement associated with increased clinging power. Here we see a possibility of establishing new phylogenetic relations between the various groups The Epicaridea may well be an offshoot from the more typical Flabellifera; the Valvifera, the Flabellifera, and perhaps the Oniscoidea may represent divergent branches from a common clinging stock with fused coxal plates (on this conception the tanaidacean taxis of the Arcturidæ would represent a secondary reversion, just as the tanaidacean (?) taxis in the Asellota is most probably a reversion). There is no need to pursue these speculations further; they are not by any means proved. My object is rather to show how, when the purely morphological method slows down or comes to a standstill, it may be reinforced by the physiological or functional. In other parts of this paper we have had further opportunity of proving the value of this latter method (which is merely a reversion to that so elaborately used by Cuvier) applied to a task nowadays considered to be the exclusive province of morphology.

Why physiology ceased to co-operate with morphology in this work is a question that belongs to the history of biology. Keith Lucas (1909), in an acute essay on functional evolution, has traced the origin of the anomaly to the period when a distinction first began to be drawn between homology and analogy. To his essay, perusal of which I can earnestly recommend to the reader, I should like to add a codicil.

Analogy and Homology. — According to Lankester (1888) it was Owen * who "gave precision and currency to the morphological doctrines which had taken their rise in the beginning of the century by the introduction of two terms 'homology' and 'analogy,' which were defined so as to express two different kinds of agreement in animal structure, which, owing to want of such 'counters of thought,' had been hitherto continually confused. *Analogous* structures in any two animals compared were by Owen defined as structures performing similar functions, but not necessarily derived from the modification of one and the same part in the 'plan' or 'architype,' according to which the two animals compared were supposed to be constructed. *Homologous* structures were such as, though greatly differing in appearance and detail from one another, and though performing widely different functions, yet were capable of being shown by adequate study of a series of intermediate forms to be derived from one and the same part or organ of the 'plan-form' or 'architype.' It is not easy to exaggerate the service

* Lankester appears here to have been in error. See Huxley (1894).

rendered by Owen to the study of zoology by the introduction of this apparently small piece of verbal mechanism; it takes place with the classificatory terms of Linnæus. And, though the conceptions of 'archi-typal morphology,' to which it had reference, are now abandoned in favour of a genetic morphology, yet we should remember, in estimating the value of this and of other speculations which have given place to new views in the history of science, the words of the great reformer himself: 'Erroneous observations are in the highest degree injurious to the progress of science, since they often persist for a long time. But erroneous theories, when they are supported by facts, do little harm, since everyone takes a healthy pleasure in proving their falsity' (Darwin)."

I have given the above quotation at length partly because of the concluding sub-quotation. Since reading Lucas's paper some years ago I have more than once endeavoured without success to come to a satisfactory decision as to the essential distinction between homology and analogy; the terminology presents such difficulties in the case of the "serial homologies" in the limbs. That there is no absolute distinction between the term "analogy" and the term "homology" (as frequently used) has finally been proved to me by a paper of Lankester (1870).

Lankester finds it necessary to distinguish between different kinds of homology, and his distinction is vital. I shall endeavour to condense his argument. He first points out that organs may be called *homogenetic* if the common possessors are derived from ancestors that possessed the same organ. But "homology" is often used in a wider sense. Thus, the four cavities of the bird's heart had been said to be homologous with the four cavities of the mammalian heart, in spite of the fact that the common ancestors of mammals and birds had in all probability but three heart cavities, and in spite of the further fact that the right ventricle of a bird's heart does not develop in the same way as the right ventricle of a mammalian heart. Again, certain muscles in the limbs of Sauropsida are still said to be homologous with other muscles in the limbs of Mammalia, although the presumption is that no such muscles were present in the limbs of the common amphibian ancestors. The crowning instance, however, is that of the "serial homologies," in which a correspondence is traced in detail between the structures composing, say, the fore-limb and those composing the hind-limb of one of the higher vertebrates. His conclusion is that something over and above simple *homogeny* is in such cases connoted by the term "homology," and he proceeds to state the question thus:—

"When identical or nearly similar forces, or environments, act on two or more parts of an organism which are exactly or nearly alike, the resulting modifications of the various parts will be exactly or nearly alike. Further, if, instead of similar parts in the same organism, we suppose the same forces to act on parts in two organisms, which parts are exactly or nearly alike and sometimes homogenetic, the resulting correspondences called forth in the several parts in the two organisms will be nearly or exactly alike. . . . I propose to call this kind of agreement *homoplasis* or *homoplasy.*"

Lankester's statement is exceedingly illuminating. If once we admit that in the term "homology" shall be included the idea of *homoplasy*, then the distinction between homology and analogy is purely relative; for in the end any two animals may be traced back to a common ancestor, and the distinction then depends entirely on the nearness or remoteness of the consanguinity.* If we restrict the connotation of homology to what is implied in Lankester's term *homogeny*, then the "serial homologies" are analogies. In view of this it would be much better to discard the two terms "homology" and "analogy" and to substitute therefor "homogeny" and "homoplasy."

Lucas has cogently remarked that the classical example of homology cited by Geoffroy St Hilaire, to whom the origin of the whole distinction is traceable, is no real example of a difference in function. "He (Geoffroy) pointed out that the fore-legs of mammals and the pectoral fins of fishes correspond in structure, though they perform such diverse functions as running, climbing, and swimming." To a modern physiologist with far deeper power of functional analysis, a fin and a walking leg may be essentially similar mechanisms. Indeed, it is difficult to cite any case of homogeny worth specifying as such, unless the function of the organs has undergone change, *i.e.* unless the case in question could not by any chance be taken as one of analogy; which assertion will commend itself to any histologist.

Lankester's statement is equally illuminating in its positive aspect. Outside the inherent unfolding impulse in organised structure, acting upon it and *moulding* it, are external forces or environments, and the interaction between the two is pictured as being direct. Here lies the essence of the question of *homoplasy*, or, if we like, of *analogy*, and to this very conclusion I had been led before reading Lankester's paper. Let us take the question in another way.

There is a limited number of animal phyla, representing different

* Lankester himself would still discriminate between homoplasy and analogy.

types of structure. In each case the structure of the individuals bears a relation of adaptation to certain physical conditions, also limited in number, which constitute their environment. In a general way this has been fully recognised by morphologists, but the essentially physiological element involved in adaptation is not always so clearly apprehended. It frequently happens that animals of different structural type are subjected to one and the same change of environment. Each has now the same physiological problem (or the same group of problems) to solve. The solution may or may not involve gross change in structure.

As an example I shall take a particular change in environment, which from its frequent occurrence has been and still is of first-class importance, viz. change from an aquatic to a land life. We shall restrict our attention to the effect upon the body-covering. The surface even of aquatic animals tends to become specialised into two regions—(1) respiratory and (2) general integumentary. If land adaptation, which involves *risk of evaporation* from the body-surface, is to occur, the general integument becomes absolutely or relatively impermeable to water (*cf.* Amphibia with Sauropsida and Mammalia). Even histologically such a change might be considered trivial. In order to permit the passage of gases the respiratory surface on the other hand has to remain moist.* We have only to consider the various groups of arthropods or of vertebrates to realise what profound structural changes may eventually be traced to the necessity of complying with this physiological principle.

While incidentally mentioning that this mode of viewing the respiratory organs might have set at rest Cuvier's difficulty with regard to them from the point of view of analogy (see Lucas's paper), I wish particularly to press home the fact that the whole process of structural or phylogenetic development is itself subordinate to the operation of external physical conditions, constituting the environment. The experimental analysis of these conditions, in so far as they affect the organism, has always been the business of physiology. It must also be the business of physiology (or at least of physiological technique) to investigate the relation between the unfolding structure and this controlling environment.

On every hand there are indications that the operation of the environment in modifying structure may be more direct than is assumed in the Darwinian principle of adaptation by elimination of unsuitable variations. The serial homoplasies, cases of reversion of all kinds (eyes,

* This assertion, partly a *post hoc* one, is worthy of treatment by the general physiologists.

limbs, etc.) are almost impossible to explain on the Darwinian hypothesis. Moreover, to the school of "developmental mechanics" we owe demonstration of the fact that in ontogenetic development there is an early stage "during which each cell is pursuing the proper course predetermined by its own inherent qualities; and that to this there succeeds a stage in which the interference of the general functional activity of the organism is necessary to the completion of the process."

In this connection consider the correlation in phylogenetic development of the fore- and hind-limb of higher vertebrates. Here we have to assume *either* that the moulding influence of the environment was very direct, *or then* that on the one hand there existed some co-ordinating communication, implying "interference of the general functional activity of the organism," between the fore- and hind-limb, or on the other that the unfolding impulse (with its material substratum) pursued a course predetermined by inherent qualities.

When, again, we compare the reptant peracaridan with the reptant vertebrate flexion-complex and limb-taxis, we see a correlation. It is far less precise in detail than that between the two vertebrate limbs, but nevertheless sufficient to excite wonder. Here there is no "co-ordinating communication" between the two respective unfolding impulses (with their material substratum); there is either predetermination or then the influence of the environment is direct.

These specific cases have been reintroduced to show that by deliberately selecting examples of homoplasy in non-homogenetic organs it may be possible to carry out investigations parallel to the ontogenetic experiments of the school of developmental mechanics—and with this advantage in the case at least of the "serial homologies," that one gets rid of any possible co-ordinating communication between the unfolding impulses (with their material substratum). One may thus study, as it were, the direct action of the environment. In this connection compare Henderson (1913).

I have here striven to make out a case for the systematic study of the homoplasies or analogies as opposed to that of the homogenies. Physiological study of the invertebrates suffers at present from a certain indefiniteness and scatteredness of aim. We are not without books on "comparative physiology." It is no discredit to the writers that they have difficulty in deducing principles and generalisations. It is the peculiar fortune of the physiologist to view the great achievements of morphology, to know that his science is directly and intimately involved, and yet to be unable to point to any great unifying physiological generalisation equivalent to those achieved by chemistry, by physics, and by morphology. Indeed, apart from the line taken by "general physiology," it is hard to see whither the study of physiology as a coherent body of science is tending.

To discourage study of invertebrate physiology in the interests of human physiology is certainly a mistake. From a union of physiology with comparative morphology there is much to be looked for; *prima facie* it is a great piece of fortune for the physiologist to have the ground so well charted by the morphologists. In the more recent books on comparative physiology the arrangement of subject-matter is strictly according to physiological activities. This change, while dictated by ideals of consistency, may be only a make-believe improvement in the interests of the whole subject. Just as in Wiedersheim's *Comparative Anatomy* (where the classification of matter is designed on strictly morphological lines), we find chapters with headings like these, "Organs of Nutrition," "Organs of Circulation," etc.; so in books on comparative physiology it might still be an advantage to drop the wider physiological and to employ an anatomico-functional arrangement.

Suppose we had a book that treated of the integument in all its physiological aspects, and similarly of other common organs (there are "τά τῶν πτερύγων τά τε τῶν σκελῶν"); without doubt such a book would bring a flood of light to bear upon the relation between structure and function; for, however much we may abstract structure from function and function from structure, when it comes to be a question of structural development, physiology and morphology are indivisible.

Simply to show how far we have already travelled I shall quote the following sentences from Wiedersheim (1886):—"The closely allied branches of science defined above are united together as *Morphology*, as opposed to *Physiology*, which concerns the functions of organs, apart from their morphological relations. Morphology alone leads us to a satisfactory explanation of the structural phenomena of the animal body, for it not only reveals to us the law of *heredity* and the consequent *relationship* of animals to one another, but, etc."

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Summary.

1. Each walking leg of Ligia shows three prominent flexures arranged alternately in one plane. Generally speaking, the planes of flexure of the whole series of seven limbs are transverse to the body, the basipodites flexing medially; this arrangement, here called isopodan limb-taxis, is associated with clinging power.

2. In Amphipoda and in Tanaidacea there are likewise three chief flexures in the limb, which are also arranged tri-alternately and in one plane. In the Tanaidacea the basipodites project laterally not medially. In the Amphipoda the basipodites flex antero-posteriorly. Tanaidacean limb-taxis seems to be the primitive form.

3. The tri-alternate flexion-complex in each limb of these Peracarida, as well as in certain limbs of Cumacea, is a reptant feature; so is the separation of the seven limbs into two opposing groups: an analogy with the limbs of reptant vertebrates is here traced. By simple inspection it is usually possible to say if a crustacean limb used for swimming has been secondarily modified from a reptant limb.

4. The flexion-complex in the limbs of reptant Eucarida is neither tri-alternate nor uniplanar as in the limbs of reptant Peracarida. This distinction is of more importance than that suggested by Boas, which relies upon the amount of flexion possible at the mero-carpal and at the carpo-propodal articulation respectively.

5. The Peracarida, like the Decapoda, may be divided into natant and reptant forms, with intermediate links. Of the reptant forms only those with isopodan or amphipodan limb-taxis have shown themselves capable of adopting a land life.

6. It is suggested that *Phreatoicus* may in progression employ its posterior peraeopods or its pleon in the same way as a Gammarus. Should it be found that the animal does so, one could formulate additional correlations between structure and function; these would involve vertical pleural plates, uropods, and the number of free segments in the pleon.

7. Considerations relating to limb flexures and limb-taxis have been tentatively applied to the classification of the Isopoda.

8. The different forms of limb-taxis in the Peracarida and certain features involving the flexion-complex, present problems analogous to the rotation and torsion that occur in the fore-limb of a developing mammal.

9. In a discussion on homology and analogy it is pointed out that homology in the sense of *homoplasy* (Lankester) cannot be effectively distinguished from analogy. The systematic study of analogies would appear to be more worthy of consideration than is generally acknowledged.

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VII.—On the Adelphic Integral of the Differential Equations of Dynamics. By Professor E. T. Whittaker, F.R.S.

(MS. received September 21, 1916. Read November 20, 1916.)

§ 1. *Ordinary and singular periodic solutions of a dynamical system.*—The present paper is concerned with the motion of dynamical systems which possess an integral of energy. To fix ideas, we shall suppose that the system has two degrees of freedom, so that the equations of motion in generalised co-ordinates may be written in Hamilton's form

$$\frac{dq_1}{dt}=\frac{\partial H}{\partial p_1}, \quad \frac{dq_2}{dt}=\frac{\partial H}{\partial p_2}, \quad \frac{dp_1}{dt}=-\frac{\partial H}{\partial q_1}, \quad \frac{dp_2}{dt}=-\frac{\partial H}{\partial q_2} \quad . \quad . \quad (1)$$

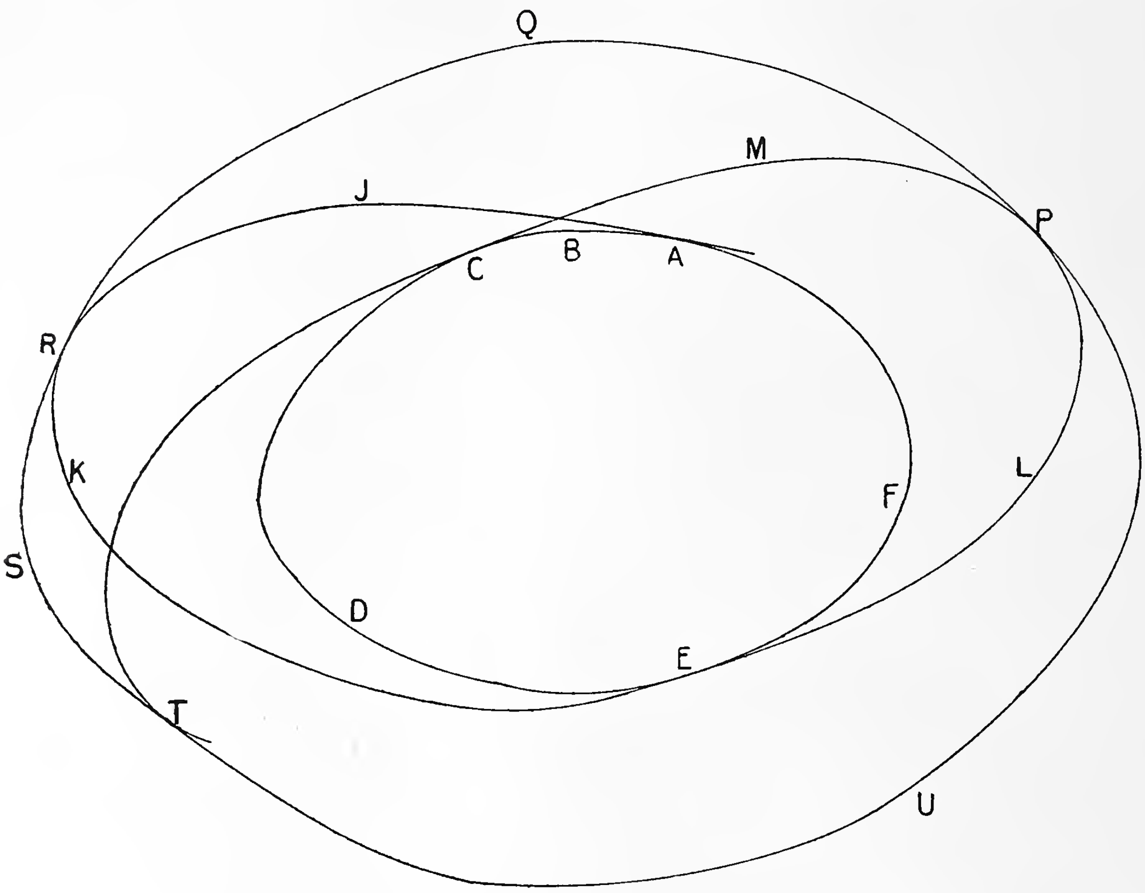
where (q_1, q_2) are the generalised co-ordinates, (p_1, p_2) are the generalised momenta, and where H is a function of (q_1, q_2, p_1, p_2) which represents the sum of the kinetic and potential energies.

The successive states of the system may be illustrated by the motion of a point whose co-ordinates referred to the axes are (q_1, q_2): the curve described by such a point is called a *trajectory*. Particular interest attaches to those trajectories which are *closed* curves: these are known as *periodic solutions*.

I wish to draw attention in the first place to a distinction which should be made in regard to these periodic solutions; the matter may perhaps be elucidated most readily by considering a particular problem, namely, that of the motion of a particle on the surface of an ellipsoid under no external forces. The particle describes a geodesic on the surface, so the periodic solutions are simply those geodesics which are closed curves. Now for a geodesic on an ellipsoid we have Joachimstal's equation

$$pd = \text{constant},$$

where p denotes the perpendicular from the centre of the ellipsoid on the tangent-plane at the point, and d is the diameter parallel to the tangent to the geodesic at the point. The same equation holds for the lines of curvature on the ellipsoid; so that every geodesic may be associated with a line of curvature, namely, that line of curvature for which pd has the same value as it has for the geodesic. We shall speak of the geodesic as "belonging to" the line of curvature. There is only one line of curvature having a prescribed value for pd, but there is an infinite number of geodesics having this value for pd, so that an infinite number of geodesics "belong to" each line of curvature. Now the line of curvature consists of two closed curves on the ellipsoid (being in fact the intersection of the ellipsoid with a confocal quadric): the region between these two portions of the line of curvature is a belt extending round the ellipsoid: and all the geodesics which belong to this line of curvature are comprised within this belt,* and touch the two portions of the line of curvature alternately. The matter is represented schematically in the diagram, where ABCDEF and PQRSTU are the two portions of the line of curvature, and AJRKELPMCT is an arc of one of the geodesics belonging to it, touching one of the



portions of the line of curvature at A, C, E, and touching the other portion at R, P, T.

In order that the geodesic may be closed, it is necessary (as in all poristic problems) that a certain parameter (depending in this case on the value of the constant *pd* of the line of curvature) should be a rational number: the geodesic is unclosed if this parameter is an irrational number. If it is closed, then there are ∞^1 other geodesics which belong to the same line of curvature and which are also closed; but if it is not closed, then no other geodesic belonging to this particular line of curvature can be a closed geodesic.†

* Ignoring the exceptional case of those geodesics which pass through an umbilicus.

† This is obvious in the case when the ellipsoid is of revolution: for then the two portions of the line of curvature are parallel circles on the surface, and the ∞^1 geodesics which belong to this line of curvature are obtained from each other by mere rotation about the axis of symmetry.

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[*Continued on page* iii *of Cover*.

Now consider the connection between the ∞^1 members of the family of geodesics which belong to the same line of curvature. It is known * that if

$$\phi\,(q_1, q_2, p_1, p_2) = \text{Constant}$$

is an integral of a dynamical system, then the infinitesimal contact-transformation which is defined by the equations

$$\delta q_1 = \epsilon \frac{\partial \phi}{\partial p_1}, \quad \delta q_2 = \epsilon \frac{\partial \phi}{\partial p_2}, \quad \delta p_1 = -\epsilon \frac{\partial \phi}{\partial q_1}, \quad \delta p_2 = -\epsilon \frac{\partial \phi}{\partial q_2}$$

(where ϵ is a small constant) transforms any trajectory into an adjacent curve which is also a trajectory. If we apply this theorem to the motion on the ellipsoid, we find without much difficulty † that the infinitesimal transformation which corresponds to the integral

$$pd = \text{Constant}$$

transforms any geodesic into another geodesic which belongs to the same line of curvature.

Summing up, we see that *the* ∞^2 *geodesics on an ellipsoid may be classified into* ∞^1 *families, each family consisting of* ∞^1 *geodesics: the members of any one family are either all closed or all unclosed: and a certain continuous group of transformations, which is closely associated with the integral* pd = *Constant, transforms any geodesic into all the geodesics which belong to the same family.*

Besides these geodesics which can be arranged in families, there are on the ellipsoid three other closed geodesics, namely, the three principal sections of the ellipsoid. These have quite a different character: they are solitary, instead of belonging to families: and the infinitesimal transformation which has just been mentioned tranforms them not into other geodesics but into themselves—that is, they are invariant under the transformation. This last property suggests a resemblance with the theory of "singular solutions" of ordinary differential equations of the first order: for if a differential equation of the first order admits a particular infinitesimal transformation, then this infinitesimal transformation changes the ordinary integral-curves into each other, but it leaves invariant the singular integral-curve. On account of this resemblance I propose to call a periodic solution (of a dynamical system with two degrees of freedom) *ordinary* if

* *Cf.*, *e.g.*, my *Analytical Dynamics*, § 144.

† As this problem of motion on an ellipsoid is only a special case of the general theory which is given later, I do not give the analysis relating to it in detail.

it belongs to a continuous family of ∞^1 periodic solutions for which the constant of energy has the same value, and which are transformed into each other by the infinitesimal transformation belonging to a certain integral (this is specified more closely later on); but a periodic solution is to be called *singular* if there is no periodic solution adjacent to it which corresponds to the same value of the constant of energy: the above-mentioned infinitesimal transformation leaves the singular periodic solutions invariant.

It should be noticed that we have inserted the condition "for which the constant of energy has the same value." If we suppose the constant of energy to vary, an "ordinary" periodic solution is in general a member of a continuous family of ∞^2 periodic solutions, whereas a "singular" periodic solution is a member of a family of ∞^1 periodic solutions.*

There are marked differences between the properties of "ordinary" and those of "singular" periodic solutions. For instance, the "asymptotic solutions" of Poincaré † can exist only in connection with *singular* periodic solutions, and not in connection with ordinary periodic solutions; an illustration of this is again afforded by the theory of geodesics on quadrics; for the only asymptotic solutions among the geodesics of quadrics are those geodesics which wind round and round the hyperboloid of one sheet, becoming ultimately asymptotic to the principal elliptic section of the hyperboloid: and this elliptic section is a singular periodic solution.

We must now examine into the existence of families of "ordinary" periodic solutions in the general dynamical system with two degrees of freedom. For this purpose we recall that in the solution of such systems by infinite trigonometric series,‡ the generalised co-ordinates (q_1, q_2) are ultimately expressed in the following way: each co-ordinate is a sum of terms like

$$a_{mn} \cos (m\beta_1 + n\beta_2)$$

where m and n are integers (positive, negative, or zero); the coefficients a_{mn} are functions of two of the constants of integration, α_1 and α_2 only; and the angles β_1 and β_2 are defined by equations

$$\beta_1 = \mu_1 t + \epsilon_1, \quad \beta_2 = \mu_2 t + \epsilon_2,$$

* The case of geodesic problems is exceptional, as in them the value of the constant of energy is immaterial.

† *Nouvelles Méth. de la Méc. Cel.*, i (1892), iii (1899).

‡ *Cf.*, *e.g.*, chap. xvi of my *Analytical Dynamics*.

where μ_1 and μ_2 are functions of a_1 and a_2 only, and ϵ_1 and ϵ_2 are the two remaining constants of integration.

Periodic solutions evidently arise when the constants a_1 and a_2 are such that μ_1 is commensurable with μ_2: the period of the solution is then $2\pi/\nu$, where ν is the largest quantity of which μ_1 and μ_2 are integer multiples.

Suppose then that a_1 and a_2 have such values. Then if the constant ϵ_1 be varied continuously, we obtain a family of periodic solutions, each having the same period (since this does not depend on ϵ_1). The constant of energy depends only on a_1 and a_2, and is therefore the same for each of these periodic solutions. The family is therefore a family of "ordinary" periodic solutions.

It might hastily be supposed that by varying ϵ_2 as well as ϵ_1 we should get a family of ∞^2 periodic solutions. But it is easily seen that the transformation which is obtained by varying ϵ_2 may be obtained by combining the transformation which consists in varying ϵ_1 with that which consists in adding a small constant to t. Now this latter transformation merely transforms every orbit into itself (each point being displaced in the direction of the tangent to the orbit), and so may be disregarded. The ϵ_1 and ϵ_2 transformations are therefore to be regarded as not distinct from each other.*

Singular periodic solutions are found chiefly in domains where the solution by purely trigonometric series is not possible.

§ 2. *Definition of the adelphic integral.*—Having now distinguished the "ordinary" and "singular" periodic solutions of a dynamical system, we shall consider those infinitesimal transformations which change each trajectory of the system into an adjacent trajectory, *in such a way that every ordinary periodic solution is changed into an adjacent periodic solution of the same family, i.e.* having the same period and the same constant of energy. In the notation we have just been using, this transformation corresponds to a small change in ϵ_1. This transformation will be called the *adelphic* transformation.† The adelphic transformation changes any solution of the dynamical system, whether periodic or not, into one of ∞^1 other solutions which stand in a particularly close relation to it, being in fact derived from it by a change of the constant ϵ_1 only.

* The only case of exception is when *all* the orbits of the system are periodic.

† From ἀδελφικός, *brotherly*, because these orbits stand in very close relation to each other, and also because the integral corresponding to the transformation stands in a much closer relation to the integral of energy than do the other integrals of the system.

To the adelphic transformation there corresponds an integral of the dynamical system: this integral we shall call the *adelphic integral* of the system.

As there is only one really distinct adelphic transformation of a given dynamical system with two degrees of freedom, so there is only one really distinct adelphic integral: all other adelphic integrals may be obtained from this by combining it in various ways with the integral of energy.*

In practically all the known soluble problems of dynamics with two degrees of freedom, the integral which enables us to effect the solution is an adelphic integral. Thus, when the trajectories are the geodesics on an ellipsoid, the adelphic integral is the equation $pd=$Constant. When the problem is that of two centres of gravitation, the adelphic integral is Euler's integral of that problem. When the solubility of the problem is due to the presence of an ignorable co-ordinate, say q_2, the corresponding integral (namely $p_2=$Constant) is adelphic.

In the present paper we shall find the adelphic integral for the general dynamical system with two degrees of freedom, and make this the basis from which to complete the integration of the system. It will appear that by this procedure we are enabled to overcome the difficulty formulated in Poincaré's celebrated theorem, that "the series of Celestial Mechanics, if they converge at all, cannot converge uniformly for all values of the time on the one hand, and on the other hand for all values of the constants comprised between certain limits." This unsatisfactory feature of the usual series springs from peculiarities which are deep-seated in the nature of the problem, and which are difficult to discern by the methods of solution employed in Celestial Mechanics. By fixing our attention in the first place on a single integral of the dynamical system, rather than attempting at once a complete solution, we shall find what these peculiarities are; for they manifest themselves very clearly in connection with the adelphic integral, and (as we shall see) may be so taken account of in its determination, that they no longer remain to trouble us in the final stages of the complete integration of the dynamical system.

§ 3. *The form of the Hamiltonian function.*—We now proceed to inquire how the adelphic integral of a dynamical system with two degrees of freedom may be determined.

* The integral of energy corresponds to that infinitesimal transformation which changes every orbit into itself, each point of an orbit being displaced in the direction of the tangent to the orbit.

The differential equations will be taken in the Hamiltonian form

$$\frac{dq_1}{dt}=\frac{\partial H}{\partial p_1},\quad \frac{dq_2}{dt}=\frac{\partial H}{\partial p_2},\quad \frac{dp_1}{dt}=-\frac{\partial H}{\partial q_1},\quad \frac{dp_2}{dt}=-\frac{\partial H}{\partial q_2} \quad . \quad . \quad . \quad (1)$$

where

$$H(q_1, q_2, p_1, p_2) = \text{Constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

is the integral of energy.

In general—at any rate in the problems of practical importance—it is possible * to choose the generalised co-ordinates in such a way that H can be expanded as an infinite series proceeding in powers of $\sqrt{q_1}$ and $\sqrt{q_2}$, and in trigonometric functions of multiples of p_1 and p_2: that is to say, in terms of the type

$$q_1^{\frac{1}{2}m} q_2^{\frac{1}{2}n} \cos(ip_1 + jp_2)$$

where m and n are integers (positive or zero) and i and j are integers (positive or negative or zero): moreover, if we call $(m+n)$ the "order" of a term, the terms of lowest order are linear in q_1 and q_2 and free from p_1 and p_2, so that they may be written $(s_1 q_1 + s_2 q_2)$, where s_1 and s_2 are constants. In most cases we find also the condition that $m-|i|$ is zero or an even integer, and $n-|j|$ is also zero or an even integer.

The Hamiltonian function H may therefore be expanded in the form

$$H = s_1 q_1 + s_2 q_2 + H_3 + H_4 + H_5 + \ldots \quad . \quad . \quad . \quad (3)$$

where H_r denotes the terms of order r, so we may write

$$\begin{aligned} H_3 = {} & q_1^{\frac{3}{2}}(U_1 \cos p_1 + U_2 \cos 3p_1) + q_1 q_2^{\frac{1}{2}}\{U_3 \cos p_2 + U_4 \cos(2p_1 + p_2) + U_5 \cos(2p_1 - p_2)\} \\ & + q_1^{\frac{1}{2}} q_2 \{U_6 \cos p_1 + U_7 \cos(2p_2 + p_1) + U_8 \cos(2p_2 - p_1)\} + q_2^{\frac{3}{2}}\{U_9 \cos p_2 + U_{10} \cos 3p_2\}, \end{aligned}$$

and

$$\begin{aligned} H_4 = {} & q_1^2(X_1 + X_2 \cos 2p_1 + X_3 \cos 4p_1) \\ & + q_1^{\frac{3}{2}} q_2^{\frac{1}{2}}\{X_4 \cos(p_1 + p_2) + X_5 \cos(p_1 - p_2) + X_6 \cos(3p_1 + p_2) + X_7 \cos(3p_1 - p_2)\} \\ & + q_1 q_2 \{X_8 + X_9 \cos 2p_1 + X_{10} \cos 2p_2 + X_{11} \cos(2p_1 + 2p_2) + X_{12} \cos(2p_1 - 2p_2)\} \\ & + q_1^{\frac{1}{2}} q_2^{\frac{3}{2}}\{X_{13} \cos(p_1 + p_2) + X_{14} \cos(p_1 - p_2) + X_{15} \cos(p_1 + 3p_2) + X_{16} \cos(p_1 - 3p_2)\} \\ & + q_2^2\{X_{17} + X_{18} \cos 2p_2 + X_{19} \cos 4p_2\}, \end{aligned}$$

the coefficients $U_1, U_2, \ldots U_{10}, X_1, X_2, \ldots X_{19}$ being constants.

It will appear that it is necessary to distinguish three cases: in each case an adelphic integral exists and will be determined, but the form of the adelphic integral is different in each of the three cases.

* *Cf., e.g., Analytical Dynamics*, §§ 184–6.

Case I. The ratio s_1/s_2 is an irrational number.

Case II. The ratio s_1/s_2 is a rational number, say equal to m/n (where m and n are integers and the fraction m/n is in its lowest terms) and terms in $\cos(np_1 - mp_2)$ are absent from H_3.

Case III. The ratio s_1/s_2 is a rational number, say equal to m/n, and terms in $\cos(np_1 - mp_2)$ are present in H_3.

We shall now determine the adelphic integral in each of these cases in turn.

§ 4. *Determination of the adelphic integral in Case I.*—Let us then first suppose that the Hamiltonian function is expanded as in § 3, and that the ratio s_1/s_2 is an irrational number. We shall now show how to set up formally a series which, if it converges, is an integral of the system.

If $\phi(q_1, q_2, p_1, p_2) = \text{Constant}$ is an integral, we must have (from the equations of motion)

$$\frac{\partial\phi}{\partial q_1}\frac{\partial H}{\partial p_1} + \frac{\partial\phi}{\partial q_2}\frac{\partial H}{\partial p_2} - \frac{\partial\phi}{\partial p_1}\frac{\partial H}{\partial q_1} - \frac{\partial\phi}{\partial p_2}\frac{\partial H}{\partial q_2} = 0 \quad . \quad . \quad . \quad . \quad (4)$$

an equation which we may write $(\phi, H) = 0$.

Let us see if this equation can be satisfied formally by a series proceeding in ascending powers of $\sqrt{q_1}$ and $\sqrt{q_2}$ and trigonometric functions of p_1 and p_2 (like the series for H), whose terms of lowest order are $(s_1q_1 - s_2q_2)$: so that we may write

$$\phi \equiv s_1q_1 - s_2q_2 + \phi_3 + \phi_4 + \phi_5 + \ldots$$

where ϕ_r denotes the terms which are of degree r in $\sqrt{q_1}$ and $\sqrt{q_2}$.

Substituting in equation (4), and equating to zero the terms of lowest order, we have

$$s_1\frac{\partial\phi_3}{\partial p_1} + s_2\frac{\partial\phi_3}{\partial p_2} = s_1\frac{\partial H_3}{\partial p_1} - s_2\frac{\partial H_3}{\partial p_2}.$$

This evidently implies that to any term $A\cos(mp_1 + np_2)$ in H_3, there corresponds a term $\frac{s_1m - s_2n}{s_1m + s_2n} A\cos(mp_1 + np_2)$ in ϕ_3: so the value of ϕ_3 may be written down at once. Having thus determined ϕ_3, we equate to zero the terms in equation (4) which are of order 4 in $\sqrt{q_1}$ and $\sqrt{q_2}$: this gives the equation

$$s_1\frac{\partial\phi_4}{\partial p_1} + s_2\frac{\partial\phi_4}{\partial p_2} = s_1\frac{\partial H_4}{\partial p_1} - s_2\frac{\partial H_4}{\partial p_2} + (\phi_3, H_3).$$

As the quantities on the right-hand side are all known, we can solve this equation for ϕ_4 in the same way as the preceding equation was solved for ϕ_3: and thus combining our results we obtain for our integral-series ϕ

$$\text{Constant} = \phi \equiv s_1 q_1 - s_2 q_2 + q_1^{\frac{3}{2}}(\mathrm{U}_1 \cos p_1 + \mathrm{U}_2 \cos 3p_1)$$

$$+ q_1 q_2^{\frac{1}{2}}\left\{ - \mathrm{U}_3 \cos p_2 + \frac{2s_1 - s_2}{2s_1 + s_2}\mathrm{U}_4 \cos(2p_1 + p_2) + \frac{2s_1 + s_2}{2s_1 - s_2}\mathrm{U}_5 \cos(2p_1 - p_2)\right\}$$

$$+ q_1^{\frac{1}{2}} q_2 \left\{ \mathrm{U}_6 \cos p_1 + \frac{s_1 - 2s_2}{s_1 + 2s_2}\mathrm{U}_7 \cos(2p_2 + p_1) + \frac{s_1 + 2s_2}{s_1 - 2s_2}\mathrm{U}_8 \cos(2p_2 - p_1)\right\}$$

$$+ q_2^{\frac{3}{2}}\left\{ - \mathrm{U}_9 \cos p_2 - \mathrm{U}_{10} \cos 3p_2\right\}$$

$$+ q_1^2 \left[\left\{\frac{1}{2s_1 + s_2}\mathrm{U}_3\mathrm{U}_4 + \frac{1}{2s_1 - s_2}\mathrm{U}_3\mathrm{U}_5 + \mathrm{X}_2\right\}\cos 2p_1 + \left\{\frac{s_2}{(2s_1 + s_2)(2s_1 - s_2)}\mathrm{U}_4\mathrm{U}_5 + \mathrm{X}_3\right\}\cos 4p_1\right]$$

$$\begin{aligned}
+ q_1^{\frac{3}{2}} q_2^{\frac{1}{2}} \Bigg[& \frac{\cos(p_1 + p_2)}{s_1 + s_2}\left\{ - \frac{2s_1}{s_1 + 2s_2}\mathrm{U}_3\mathrm{U}_7 - \frac{6s_1 s_2}{(2s_1 - s_2)(s_1 - 2s_2)}\mathrm{U}_5\mathrm{U}_8 + \mathrm{U}_3\mathrm{U}_6 \right.\\
& \left. + \frac{s_2}{2s_1 + s_2}\mathrm{U}_4\mathrm{U}_6 - \mathrm{U}_1\mathrm{U}_3 + \frac{4s_2}{2s_1 + s_2}\mathrm{U}_1\mathrm{U}_4 + \frac{6s_2}{2s_1 - s_2}\mathrm{U}_2\mathrm{U}_5 + (s_1 - s_2)\mathrm{X}_4\right\}\\
& + \frac{\cos(p_1 - p_2)}{s_1 - s_2}\left\{ - \frac{6s_1 s_2}{(2s_1 + s_2)(s_1 + 2s_2)}\mathrm{U}_4\mathrm{U}_7 + \frac{2s_1}{s_1 - 2s_2}\mathrm{U}_3\mathrm{U}_8 - \mathrm{U}_3\mathrm{U}_6 \right.\\
& \left. + \frac{s_2}{2s_1 - s_2}\mathrm{U}_5\mathrm{U}_6 - \mathrm{U}_1\mathrm{U}_3 - \frac{4s_2}{2s_1 - s_2}\mathrm{U}_1\mathrm{U}_5 - \frac{6s_2}{2s_1 + s_2}\mathrm{U}_2\mathrm{U}_4 + (s_1 + s_2)\mathrm{X}_5\right\}\\
& + \frac{\cos(3p_1 + p_2)}{3s_1 + s_2}\left\{\frac{10s_1 s_2}{(2s_1 - s_2)(s_1 + 2s_2)}\mathrm{U}_5\mathrm{U}_7 + \frac{s_2}{2s_1 + s_2}\mathrm{U}_4\mathrm{U}_6 - 3\mathrm{U}_2\mathrm{U}_3 \right.\\
& \left. + \frac{2s_2}{2s_1 + s_2}\mathrm{U}_1\mathrm{U}_4 + (3s_1 - s_2)\mathrm{X}_6\right\}\\
& + \frac{\cos(3p_1 - p_2)}{3s_1 - s_2}\left\{\frac{10s_1 s_2}{(2s_1 + s_2)(s_1 - 2s_2)}\mathrm{U}_4\mathrm{U}_8 + \frac{s_2}{2s_1 - s_2}\mathrm{U}_5\mathrm{U}_6 - 3\mathrm{U}_2\mathrm{U}_3 \right.\\
& \left. - \frac{2s_2}{2s_1 - s_2}\mathrm{U}_1\mathrm{U}_5 + (3s_1 + s_2)\mathrm{X}_7\right\}\Bigg]
\end{aligned}$$

$$\begin{aligned}
+ q_1 q_2 \Bigg[& \cos 2p_1\left\{ - \frac{2}{2s_1 + s_2}\mathrm{U}_4\mathrm{U}_9 + \frac{2}{2s_1 - s_2}\mathrm{U}_5\mathrm{U}_9 + \frac{8s_2}{(s_1 + 2s_2)(s_1 - 2s_2)}\mathrm{U}_7\mathrm{U}_8 \right.\\
& \left. - \frac{2}{2s_1 + s_2}\mathrm{U}_3\mathrm{U}_4 - \frac{2}{2s_1 - s_2}\mathrm{U}_3\mathrm{U}_5 + \mathrm{X}_9\right\}\\
& + \cos 2p_2\left\{\frac{2}{s_1 + 2s_2}\mathrm{U}_6\mathrm{U}_7 - \frac{2}{s_1 - 2s_2}\mathrm{U}_6\mathrm{U}_8 + \frac{2}{s_1 + 2s_2}\mathrm{U}_1\mathrm{U}_7 + \frac{2}{s_1 - 2s_2}\mathrm{U}_1\mathrm{U}_8 \right.\\
& \left. + \frac{8s_1}{(2s_1 - s_2)(2s_1 + s_2)}\mathrm{U}_4\mathrm{U}_5 - \mathrm{X}_{10}\right\}\\
& + \frac{\cos(2p_1 + 2p_2)}{2s_1 + 2s_2}\left\{\frac{-2s_1}{2s_1 + s_2}\mathrm{U}_4\mathrm{U}_9 + \frac{6s_1}{2s_1 - s_2}\mathrm{U}_5\mathrm{U}_{10} + \frac{4s_2}{s_1 + 2s_2}\mathrm{U}_6\mathrm{U}_7 \right.\\
& \left. + \frac{2s_2}{s_1 + 2s_2}\mathrm{U}_1\mathrm{U}_7 + \frac{6s_2}{s_1 - 2s_2}\mathrm{U}_2\mathrm{U}_8 - \frac{4s_1}{2s_1 + s_2}\mathrm{U}_3\mathrm{U}_4 + (2s_1 - 2s_2)\mathrm{X}_{11}\right\}\\
& + \frac{\cos(2p_1 - 2p_2)}{2s_1 - 2s_2}\left\{\frac{2s_1}{2s_1 - s_2}\mathrm{U}_5\mathrm{U}_9 - \frac{6s_1}{2s_1 + s_2}\mathrm{U}_4\mathrm{U}_{10} + \frac{4s_2}{s_1 - 2s_2}\mathrm{U}_6\mathrm{U}_8 \right.\\
& \left. - \frac{2s_2}{s_1 - 2s_2}\mathrm{U}_1\mathrm{U}_8 - \frac{6s_2}{s_1 + 2s_2}\mathrm{U}_2\mathrm{U}_7 - \frac{4s_1}{2s_1 - s_2}\mathrm{U}_3\mathrm{U}_5 + (2s_1 + 2s_2)\mathrm{X}_{12}\right\}\Bigg]
\end{aligned}$$

$$+q_1^{\frac{1}{2}}q_2^{\frac{3}{2}}\Bigg[\frac{\cos(p_1+p_2)}{s_1+s_2}\Big\{U_6U_9-\frac{4s_1}{s_1+2s_2}U_7U_9+\frac{6s_1}{s_1-2s_2}U_8U_{10}-U_3U_6-\frac{s_1}{s_1+2s_2}U_3U_7$$

$$+\frac{2s_2}{2s_1+s_2}U_4U_6+\frac{6s_1s_2}{(2s_1-s_2)(s_1-2s_2)}U_5U_8+(s_1-s_2)X_{13}\Big\}$$

$$+\frac{\cos(p_1-p_2)}{s_1-s_2}\Big\{-U_6U_9-\frac{6s_1}{s_1+2s_2}U_7U_{10}+\frac{4s_1}{s_1-2s_2}U_8U_9-U_3U_6$$

$$-\frac{s_1}{s_1-2s_2}U_3U_8-\frac{6s_1s_2}{(s_1+2s_2)(2s_1+s_2)}U_4U_7-\frac{2s_2}{2s_1-s_2}U_5U_6+(s_1+s_2)X_{14}\Big\}$$

$$+\frac{\cos(p_1+3p_2)}{s_1+3s_2}\Big\{3U_6U_{10}-\frac{2s_1}{s_1+2s_2}U_7U_9-\frac{s_1}{s_1+2s_2}U_3U_7$$

$$+\frac{10s_1s_2}{(s_1-2s_2)(2s_1+s_2)}U_4U_8+(s_1-3s_2)X_{15}\Big\}$$

$$+\frac{\cos(p_1-3p_2)}{s_1-3s_2}\Big\{-3U_6U_{10}+\frac{2s_1}{s_1-2s_2}U_8U_9-\frac{s_1}{s_1-2s_2}U_3U_8$$

$$-\frac{10s_1s_2}{(s_1+2s_2)(2s_1-s_2)}U_5U_7+(s_1+3s_2)X_{16}\Big\}\Bigg]$$

$$+q_2^2\left[\left\{\frac{1}{s_1+2s_2}U_6U_7+\frac{1}{s_1-2s_2}U_6U_8-X_{18}\right\}\cos 2p_2+\left\{\frac{s_1}{(s_1-2s_2)(s_1+2s_2)}U_7U_8-X_{19}\right\}\cos 4p_2\right]$$

+ terms of the 5th and higher orders in $\sqrt{q_1}$ and $\sqrt{q_2}$ . . . . . (5)

The terms of higher order in the series may be determined in the same way as the terms in ϕ_3 and ϕ_4, and we thus obtain the complete expansion of ϕ.

We may note that instead of assuming $(s_1q_1-s_2q_2)$ as the lowest term of our integral, we might have assumed q_1, or q_2, or any linear function of q_1 and q_2; the integral then obtained would be merely a linear combination of our integral (5) with the integral of energy, whose lowest terms are $(s_1q_1+s_2q_2)$.

We may further note that in the above process, when finding ϕ_4, we may if we please add to ϕ_4 any terms of the form $\alpha q_1^2+\beta q_1q_2+\gamma q_2^2$, where α, β, γ are constants; for these terms are annulled by the operator $\left(s_1\frac{\partial}{\partial p_1}+s_2\frac{\partial}{\partial p_2}\right)$, and therefore ϕ_4 satisfies its differential equation just as well when these terms are present as when they are absent. The introduction of these terms into ϕ_4 will cause changes in the terms of higher order—in ϕ_5, ϕ_6, etc.: and the sum total of all the changes will merely amount to adding to our function ϕ a quadratic function of the two integrals which we know, namely, the integral of energy and the integral (5) itself.

Similarly we may add any terms of the form $(\alpha q_1^3+\beta q_1^2q_2+\gamma q_1q_2^2+\partial q_2^3)$ to ϕ_6: the ultimate effect is merely to add to our integral a cubic function of itself and the integral of energy. There is evidently nothing to be gained by doing this, and we may therefore omit these arbitrary terms in $\phi_4, \phi_6, \phi_8, \ldots$

§ 5. *An example of the integral found in § 4, with remarks on its convergence.*—As an example, consider the dynamical system which is specified by the Hamiltonian function

$$H = 2^{\frac{1}{2}} q_1 \sin^2 p_1 + q_2 \sin^2 p_2 - \frac{1 + 3.2^{\frac{1}{4}} q_1^{\frac{1}{2}} \cos p_1}{3(1 + 2^{\frac{5}{4}} q_1^{\frac{1}{2}} \cos p_1 + 2^{\frac{1}{2}} q_1 \cos^2 p_1 + 2 q_2 \cos^2 p_2)^{\frac{3}{2}}},$$

or expanding,

$$H = 2^{\frac{1}{2}} q_1 + q_2 + 2^{\frac{7}{4}} q_1^{\frac{3}{2}} (- \cos p_1 - \tfrac{1}{3} \cos 3p_1) \\ + 2^{-\frac{3}{4}} q_1^{\frac{1}{2}} q_2 \{ -2 \cos p_1 - \cos (p_1 + 2p_2) - \cos (p_1 - 2p_2) \} + \ldots \quad (6)$$

The corresponding integral, obtained by substituting in formula (5) is

$$\text{Constant} = \phi \equiv 2^{\frac{1}{2}} q_1 - q_2 + 2^{\frac{7}{4}} q_1^{\frac{3}{2}} (- \cos p_1 - \tfrac{1}{3} \cos 3p_1) \\ + 2^{-\frac{3}{4}} q_1^{\frac{1}{2}} q_2 \{ -2 \cos p_1 + (1 - \sqrt{2})^2 \cos (p_1 + 2p_2) + (1 + \sqrt{2})^2 \cos (p_1 - 2p_2) \} + \ldots \quad (7)$$

Now it may be verified readily by differentiation that this dynamical system possesses the integral

$$\text{Constant} = (q_2^{\frac{1}{2}} \sin p_2 + 2^{\frac{1}{4}} q_1^{\frac{1}{2}} q_2^{\frac{1}{2}} \sin p_2 \cos p_1 - 2^{\frac{3}{4}} q_1^{\frac{1}{2}} q_2^{\frac{1}{2}} \sin p_1 \cos p_2)^2 \\ - \frac{1 + 2^{\frac{1}{4}} q_1^{\frac{1}{2}} \cos p_1}{(1 + 2^{\frac{5}{4}} q_1^{\frac{1}{2}} \cos p_1 + 2^{\frac{1}{2}} q_1 \cos^2 p_1 + 2 q_2 \cos^2 p_2)^{\frac{1}{2}}},$$

which when expanded takes the form

$$\text{Constant} = q_2 + 2^{-\frac{1}{4}} (1 - \sqrt{2}) q_1^{\frac{1}{2}} q_2 \cos (p_1 + 2p_2) \\ - 2^{\frac{1}{4}} (1 + \sqrt{2}) q_1^{\frac{1}{2}} q_2 \cos (p_1 - 2p_2) + \ldots \quad (8)$$

It is evident, on comparing the series, that the series (7) is what would be obtained by subtracting twice the series (8) from the series (6), which represents the integral of energy. This shows that for the particular dynamical system we are considering, the ϕ-series (5) is identical with the expansion, formed by ordinary algebraic and trigonometric processes under conditions which ensure convergence, of a known integral: and the convergence of the series (5), for sufficiently small values of $\sqrt{q_1}$ and $\sqrt{q_2}$, is thereby established for this particular system.

It is by considering particular dynamical systems such as this, in which the convergence of the series can be proved, that I have formed the opinion that the series (5) is in general convergent, for sufficiently small values of q_1 and q_2, so long as the ratio s_1/s_2 is an irrational number. A general proof of its convergence would probably be very difficult, and I have not as yet succeeded in obtaining one. But the following considerations may be adduced in support of the opinion of convergence.

Since the ratio s_1/s_2 is an irrational number, none of the denominators (s_1+s_2), (s_1-s_2), $(2s_1+s_2)$, $(2s_1-s_2)$, (s_1+2s_2), $(3s_1+s_2)$, $\ldots$ can vanish, and therefore no term of the series can be infinite. The series is a power-series in $\sqrt{q_1}$ and $\sqrt{q_2}$, and it has been derived from another absolutely convergent power-series in $\sqrt{q_1}$ and $\sqrt{q_2}$, namely, the series for H, by operations which are of an ordinary algebraical and trigonometrical combinatory character, except as regards the operation of introducing the divisors of the type (ms_1+ns_2) (where m and n are positive or negative integers) in the integrations. We may therefore expect that the series will converge for sufficiently small values of $\sqrt{q_1}$ and $\sqrt{q_2}$, unless the smallness of some of these divisors causes the series to diverge for all values of $\sqrt{q_1}$ and $\sqrt{q_2}$, however small. Now the values of the integers m and n may indeed be so chosen that the divisor (ms_1+ns_2) may be as small as we please: but $|m|$ and $|n|$ are then large, and since $|m|$ and $|n|$ are not greater than the order of the term, this small divisor can occur only in a term of high order, where it will be more or less neutralised by the high powers of $\sqrt{q_1}$ and $\sqrt{q_2}$: and it was in fact shown many years ago by Bruns * that this state of things is consistent with the absolute convergence of a series. The example given by Bruns was the series

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{q_1^{\,m}q_2^{\,n}}{m-n\mathrm{A}}$$

where q_1 and q_2 are proper fractions and A is a positive irrational number which is an algebraic number, *i.e.* a root of an irreducible algebraic equation

$$\mathrm{A}^s+\mathrm{G}_1\mathrm{A}^{s-1}+\mathrm{G}_2\mathrm{A}^{s-2}+\ \ldots\ +\mathrm{G}_n=0$$

with integer coefficients G. If we multiply the numerator and denominator of any term in Bruns' series by

$$(m-n\mathrm{A}')\,(m-n\mathrm{A}'')\ \ldots$$

where A′, A″, . . . are the other roots of the algebraic equation, then the denominator becomes a polynomial in m and n with integer coefficients: and as it is never zero, it must be at least equal to unity: while in the numerator we now have a polynomial in m and n of degree $(s-1)$: whence it follows at once that Bruns' series converges.

The series (5) is much more complicated than Bruns' series: and although the analogy so far as it goes is favourable to the convergence of (5), yet our opinion must rest mainly on the undoubted convergence of (5) in the case of particular systems where a test is possible.

§ 6. *Use of the integral found in § 4 in order to complete the integration of the system.*—Still restricting ourselves to Case I, in which the ratio s_1/s_2 is an irrational number, we now know two integrals of the dynamical system, namely, the integral of energy (which is obtained by

* *Astr. Nach.* **109** (1884), p. 215.

equating the Hamiltonian function to a constant) and the integral expressed by equation (5). But it is known * that if, in any conservative holonomic dynamical system with two degrees of freedom, we know one integral in addition to the integral of energy, the system can be completely integrated, *i.e.* we can find expressions for the co-ordinates and momenta (q_1, q_2, p_1, p_2) in terms of the time and 4 arbitrary constants of integration. We shall now perform this process, which incidentally will show that the integral (5) is the adelphic integral of the system.

If we add the integral of energy to the integral (5), and divide throughout by $2s_1$, we obtain

$$\begin{aligned} l_1 = q_1 &+ q_1^{\frac{3}{2}}\left\{\frac{1}{s_1}\mathrm{U}_1 \cos p_1 + \frac{1}{s_1}\mathrm{U}_2 \cos 3p_1\right\} \\ &+ q_1 q_2^{\frac{1}{2}}\left\{\frac{2}{2s_1+s_2}\mathrm{U}_4 \cos(2p_1+p_2) + \frac{2}{2s_1+s_2}\mathrm{U}_5 \cos(2p_1-p_2)\right\} \\ &+ q_1^{\frac{1}{2}} q_2\left\{\frac{1}{s_1}\mathrm{U}_6 \cos p_1 + \frac{1}{s_1+2s_2}\mathrm{U}_7 \cos(2p_2+p_1) + \frac{1}{s_1-2s_2}\mathrm{U}_8 \cos(2p_2-p_1)\right\} \\ &+ \text{terms of the 4th and higher orders,} \end{aligned}$$

where l_1 denotes an arbitrary constant.

Similarly by subtracting the integral (5) from the integral of energy, and dividing by s_2, we obtain

$$\begin{aligned} l_2 = q_2 &+ q_1 q_2^{\frac{1}{2}}\left\{\frac{1}{s_2}\mathrm{U}_3 \cos p_2 + \frac{1}{2s_1+s_2}\mathrm{U}_4 \cos(2p_1+p_2) - \frac{1}{2s_1-s_2}\mathrm{U}_5 \cos(2p_1-p_2)\right\} \\ &+ q_1^{\frac{1}{2}} q_2\left\{\frac{2}{s_1+2s_2}\mathrm{U}_7 \cos(2p_2+p_1) - \frac{2}{s_1-2s_2}\mathrm{U}_8 \cos(2p_2-p_1)\right\} \\ &+ q_2^{\frac{3}{2}}\left\{\frac{1}{s_2}\mathrm{U}_9 \cos p_2 + \frac{1}{s_2}\mathrm{U}_{10} \cos 3p_2\right\} + \text{terms of the 4th and higher orders,} \end{aligned}$$

where l_2 represents a second arbitrary constant.

It is an easy matter to obtain q_1 and q_2 from these equations in terms of (l_1, l_2, p_1, p_2) by successive approximation: in fact, the first approximation gives $q_1 = l_1$, $q_2 = l_2$, and the second approximation gives

$$\begin{aligned} q_1 = l_1 &- l_1^{\frac{3}{2}}\left\{\frac{1}{s_1}\mathrm{U}_1 \cos p_1 + \frac{1}{s_2}\mathrm{U}_2 \cos 3p_1\right\} \\ &- l_1 l_2^{\frac{1}{2}}\left\{\frac{2}{2s_1+s_2}\mathrm{U}_4 \cos(2p_1+p_2) + \frac{2}{2s_1-s_2}\mathrm{U}_5 \cos(2p_1-p_2)\right\} \\ &- l_1^{\frac{1}{2}} l_2\left\{\frac{1}{s_1}\mathrm{U}_6 \cos p_1 + \frac{1}{s_1+2s_2}\mathrm{U}_7 \cos(2p_2+p_1) + \frac{1}{s_1-2s_2}\mathrm{U}_8 \cos(2p_2-p_1)\right\} \\ &+ \text{terms of the 4th and higher order in } \sqrt{l_1} \text{ and } \sqrt{l_2}, \end{aligned}$$

* *Cf., e.g., Analytical Dynamics*, § 121.

$$q_2 = l_2 - l_1 l_2^{\frac{1}{2}}\left\{\frac{1}{s_2}\mathrm{U}_3 \cos p_2 + \frac{1}{2s_1 + s_2}\mathrm{U}_4 \cos(2p_1 + p_2) - \frac{1}{2s_1 - s_2}\mathrm{U}_5 \cos(2p_1 - p_2)\right\}$$
$$- l_1^{\frac{1}{2}} l_2\left\{\frac{2}{s_1 + 2s_2}\mathrm{U}_7 \cos(2p_2 + p_1) - \frac{2}{s_1 - 2s_2}\mathrm{U}_8 \cos(2p_2 - p_1)\right\}$$
$$- l_2^{\frac{3}{2}}\left\{\frac{1}{s_2}\mathrm{U}_9 \cos p_2 + \frac{1}{s_2}\mathrm{U}_{10} \cos 3p_2\right\}$$
$$+ \text{terms of the 4th and higher order in } \sqrt{l_1} \text{ and } \sqrt{l_2}.$$

We know from the general theory of Dynamics * that the expressions thus found for q_1 and q_2 must be the partial differential coefficients with respect to p_1 and p_2 of some function of (l_1, l_2, p_1, p_2): and, in fact, we have obviously

$$q_1 = \frac{\partial \mathrm{W}}{\partial p_1}, \quad q_2 = \frac{\partial \mathrm{W}}{\partial p_2},$$

where

$$\mathrm{W} = l_1 p_1 + l_2 p_2 - l_1^{\frac{3}{2}}\left(\frac{1}{s_1}\mathrm{U}_1 \sin p_1 + \frac{1}{3s_1}\mathrm{U}_2 \sin 3p_1\right)$$
$$- l_1 l_2^{\frac{1}{2}}\left\{\frac{1}{s_2}\mathrm{U}_3 \sin p_2 + \frac{1}{2s_1 + s_2}\mathrm{U}_4 \sin(2p_1 + p_2) + \frac{1}{2s_1 - s_2}\mathrm{U}_5 \sin(2p_1 - p_2)\right\}$$
$$- l_1^{\frac{1}{2}} l_2\left\{\frac{1}{s_1}\mathrm{U}_6 \sin p_1 + \frac{1}{2s_2 + s_1}\mathrm{U}_7 \sin(2p_2 + p_1) + \frac{1}{2s_2 - s_1}\mathrm{U}_8 \sin(2p_2 - p_1)\right\}$$
$$- l_2^{\frac{3}{2}}\left\{\frac{1}{s_2}\mathrm{U}_9 \sin p_2 + \frac{1}{3s_2}\mathrm{U}_{10} \sin 3p_2\right\}$$
$$+ \text{ terms of the 4th and higher orders in } \sqrt{l_1} \text{ and } \sqrt{l_2} \quad . \quad . \quad . \quad (9)$$

The terms in which p_1 and p_2 occur otherwise than in the arguments of trigonometric functions are

$$p_1(l_1 + \text{terms of the 4th and higher order in } \sqrt{l_1} \text{ and } \sqrt{l_2})$$
$$+ p_2(l_2 + \quad \text{,,} \quad \text{,,} \quad \text{,,} \quad \text{,,} \quad)$$

Denote the coefficients of p_1 and p_2 in this expression by a_1 and a_2 respectively: express l_1 and l_2 in terms of a_1 and a_2 by reversion of series, and replace l_1 and l_2 throughout in the series (9) by these values in terms of a_1 and a_2; so that we now have

$$\mathrm{W} = a_1 p_1 + a_2 p_2 - a_1^{\frac{3}{2}}\left(\frac{1}{s_1}\mathrm{U}_1 \sin p_1 + \frac{1}{3s_1}\mathrm{U}_2 \sin 3p_1\right)$$
$$- a_1 a_2^{\frac{1}{2}}\left\{\frac{1}{s_2}\mathrm{U}_3 \sin p_2 + \frac{1}{2s_1 + s_2}\mathrm{U}_4 \sin(2p_1 + p_2) + \frac{1}{2s_1 - s_2}\mathrm{U}_5 \sin(2p_1 - p_2)\right\}$$
$$- a_1^{\frac{1}{2}} a_2\left\{\frac{1}{s_1}\mathrm{U}_6 \sin p_1 + \frac{1}{2s_2 + s_1}\mathrm{U}_7 \sin(2p_2 + p_1) + \frac{1}{2s_2 - s_1}\mathrm{U}_8 \sin(2p_2 - p_1)\right\}$$
$$- a_2^{\frac{3}{2}}\left\{\frac{1}{s_2}\mathrm{U}_9 \sin p_2 + \frac{1}{3s_2}\mathrm{U}_{10} \sin 3p_2\right\}$$
$$+ \text{terms of the 4th and higher orders in } \sqrt{a_1} \text{ and } \sqrt{a_2} \quad . \quad . \quad . \quad (10)$$

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* *Cf., e.g., Analytical Dynamics*, § 121.

and now p_1 and p_2 do not occur except in the arguments of trigonometric functions and in the terms $(\alpha_1 p_1 + \alpha_2 p_2)$.

Now the equations

$$q_1 = \frac{\partial W}{\partial p_1}, \quad q_2 = \frac{\partial W}{\partial p_2}, \quad \beta_1 = \frac{\partial W}{\partial \alpha_1}, \quad \beta_2 = \frac{\partial W}{\partial \alpha_2}$$

define a contact-transformation from the variables (q_1, q_2, p_1, p_2) to the variables $(\alpha_1, \alpha_2, \beta_1, \beta_2)$: so in terms of these new variables the differential equations take the form

$$\frac{d\alpha_1}{dt} = \frac{\partial H}{\partial \beta_1}, \quad \frac{d\alpha_2}{dt} = \frac{\partial H}{\partial \beta_2}, \quad \frac{d\beta_1}{dt} = -\frac{\partial H}{\partial \alpha_1}, \quad \frac{d\beta_2}{dt} = -\frac{\partial H}{\partial \alpha_2} \qquad . \quad . \quad (11)$$

But we know that α_1 = Constant and α_2 = Constant are two of the integrals of the system, since l_1 and l_2 are constant: and therefore

$$\frac{\partial H}{\partial \beta_1} = 0, \quad \frac{\partial H}{\partial \beta_2} = 0,$$

so when H is expressed in terms of $(\alpha_1, \alpha_2, \beta_1, \beta_2)$, it will be found to involve α_1 and α_2 only: and then the second pair of equations (11) give

$$\left.\begin{aligned} \beta_1 &= -\frac{\partial H(\alpha_1, \alpha_2)}{\partial \alpha_1} t + \epsilon_1 \\ \beta_2 &= -\frac{\partial H(\alpha_1, \alpha_2)}{\partial \alpha_2} t + \epsilon_2 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where ϵ_1 and ϵ_2 are arbitrary constants.

Thus we have the complete solution of the dynamical system expressed by the equations

$$\frac{\partial W}{\partial p_1} = q_1, \quad \frac{\partial W}{\partial p_2} = q_2,$$

$$\frac{\partial W}{\partial \alpha_1} = -\frac{\partial H(\alpha_1, \alpha_2)}{\partial \alpha_1} t + \epsilon_1, \quad \frac{\partial W}{\partial \alpha_2} = -\frac{\partial H(\alpha_1, \alpha_2)}{\partial \alpha_2} t + \epsilon_2,$$

where W is given by equation (10), and the four arbitrary constants of integration are $(\alpha_1, \alpha_2, \epsilon_1, \epsilon_2)$. On referring to the form of W, we see that these equations enable us to express q_1 and q_2 as purely trigonometric series, the arguments of the trigonometric functions being of the form

$$m\beta_1 + n\beta_2,$$

where m and n are integers (positive, negative, or zero) and where β_1 and β_2 are linear functions of the time, given by equations (12). *We have thus obtained expressions for the co-ordinates in terms of the time, by means of series in which the time occurs only in the arguments of trigonometric functions.*

It is moreover evident that a change in ϵ_1, in which the other constants of integration $(\epsilon_2, \alpha_1, \alpha_2)$ are left unaltered, does not affect either of the constants l_1 and l_2 (since these depend only on a_1 and a_2) and therefore does not affect the constant of the integral (5) or the constant of energy: this shows that all the orbits, which differ from each other only in having different values of the constant ϵ_1, have the same values for the constant of the integral (5) and the constant of energy: and hence that the infinitesimal transformation which corresponds to the integral (5) transforms these orbits into each other: that is to say, *the integral* (5) *is the adelphic integral of the dynamical system.*

§ 7. *Determination of the adelphic integral in Case II.*—We now proceed to the discussion of "Case II," in which the ratio s_1/s_2 is a rational number (say equal to m/n), but no term in cos (np_1-mp_2) is present among the third-order terms in the Hamiltonian function H. Certain terms of the series (5) now contain in their denominators the factor (ns_1-ms_2), which vanishes since $s_1/s_2=m/n$: and therefore the series (5) as it stands cannot converge in Case II, unless the terms which have zero denominators have numerators which also vanish. We have here come upon the real root of the principal difficulty of Celestial Mechanics: by removing it here, so as to obtain a valid adelphic integral in Cases II and III, we shall be enabled to remove it from the whole subject.

To fix ideas, we shall suppose that $s_1=2$, $s_2=1$, so that s_1/s_2 has the rational value 2, and the denominator (s_1-2s_2), which occurs frequently in the series (5), is zero.

In this case the equation for ϕ_3 becomes

$$2\frac{\partial\phi_3}{\partial p_1}+\frac{\partial\phi_3}{\partial p_2}=2\frac{\partial \mathrm{H}_3}{\partial p_1}-\frac{\partial \mathrm{H}_3}{\partial p_2},$$

and indeed the equation for any one of the functions ϕ_3, ϕ_4, ϕ_5, . . . takes the form

$$2\frac{\partial\phi_r}{\partial p_1}+\frac{\partial\phi_r}{\partial p_2}=\text{ a known sum of terms of the type } q_1^{\frac{1}{2}m}q_2^{\frac{1}{2}n}\sin(kp_1+lp_2).$$

Now in integrating the differential equations for ϕ_3, ϕ_4, . . . in § 4, we used only the "particular integral," which corresponds term-by-term to the known function on the right-hand side of the equation: so that, *e.g.*, the integral of the equation

$$s_1\frac{\partial\phi_3}{\partial p_1}+s_2\frac{\partial\phi_3}{\partial p_2}=q_1^{\frac{3}{2}}\sin p_1$$

would be taken to be

$$\phi_3=-\frac{q_1^{\frac{3}{2}}}{s_1}\cos p_1.$$

The reason for this was that the "complementary function," or arbitrary part of the solution of the differential equation, is a function of $(s_2p_1-s_1p_2)$, and so does not contain terms of the type appropriate to ϕ_3. But when $s_1=2$, $s_2=1$, the arbitrary part of the solution of the differential equation *does* contain terms of the type proper to ϕ_3, and these must be taken account of; so that we must take the integral of the equation

$$2\frac{\partial\phi_3}{\partial p_1}+\frac{\partial\phi_3}{\partial p_2}=q_1^{\frac{3}{2}}\sin p_1$$

to be

$$\phi_3=-\tfrac{1}{2}q_1^{\frac{3}{2}}\cos p_1+aq_1^{\frac{1}{2}}q_2\cos(p_1-2p_2),$$

where a is an arbitrary constant. *In this way we obtain terms with arbitrary coefficients in* ϕ_3, ϕ_4, ϕ_5, . . . : *and these arbitrary coefficients must be chosen in such a way as to remove terms with vanishing denominators from the subsequently determined part of* ϕ. This principle enables us to obtain, in Case II, an adelphic integral free from vanishing denominators.

§ 8. *Study of a particular dynamical system, as an illustration of the method of* § 7.—We shall now illustrate the working of this principle by an example. Consider the dynamical system which is specified by the Hamiltonian function

$$\left.\begin{aligned}\mathrm{H}=2q_1\sin^2 p_1+q_2\sin^2 p_2+&\frac{1}{2(1+2q_1^{\frac{1}{2}}\cos p_1+q_1\cos^2 p_1+2q_2\cos^2 p_2)^2}\\ -&\frac{1+q_1^{\frac{1}{2}}\cos p_1}{(1+2q_1^{\frac{1}{2}}\cos p_1+q_1\cos^2 p_1+2q_2\cos^2 p_2)^{\frac{3}{2}}}\end{aligned}\right\}\qquad(13)$$

If this be expanded in ascending powers of $\sqrt{q_1}$ and $\sqrt{q_2}$, we obtain

$$\begin{aligned}\mathrm{H}=2q_1+q_2&+q_1^{\frac{3}{2}}(-\tfrac{9}{2}\cos p_1-\tfrac{3}{2}\cos 3p_1)+q_1^2(\tfrac{75}{16}+\tfrac{25}{4}\cos 2p_1+\tfrac{25}{16}\cos 4p_1)\\ &+q_1q_2\{-3-3\cos 2p_1-3\cos 2p_2-\tfrac{3}{2}\cos(2p_1+2p_2)-\tfrac{3}{2}\cos(2p_1-2p_2)\}\\ &+q_2^2\{-\tfrac{9}{16}-\tfrac{3}{4}\cos 2p_2-\tfrac{3}{16}\cos 4p_2\}+\text{terms of the 5th and higher order}\\ &\qquad\text{in }\sqrt{q_1}\text{ and }\sqrt{q_2},\end{aligned}$$

so that in this case $s_1=2$, $s_2=1$.

As explained at the end of § 4, we may assume that the lowest term of the adelphic integral is simply q_2. Then if we write

$$\phi=q_2+\phi_3+\phi_4+\phi_5+\ .\ .\ .$$

the equation to determine ϕ_3 is

$$2\frac{\partial\phi_3}{\partial p_1}+\frac{\partial\phi_3}{\partial p_2}=0,$$

so by § 7,

$$\phi_3=aq_1^{\frac{1}{2}}q_2\cos(p_1-2p_2),$$

where a is an arbitrary constant.

The equation for ϕ_4 now becomes

$$\begin{aligned}2\frac{\partial\phi_4}{\partial p_1}+\frac{\partial\phi_4}{\partial p_2}=q_1q_2\Big\{\Big(6+\frac{9a}{2}\Big)\sin 2p_2+\Big(3+\frac{9a}{4}\Big)\sin(2p_1+2p_2)-\Big(3+\frac{9a}{4}\Big)\sin(2p_1-2p_2)\Big\}\\ +q_2^2(\tfrac{3}{2}\sin 2p_2+\tfrac{3}{2}\sin 4p_2)\end{aligned}$$

of which the integral is

$$\phi_4 = q_1 q_2 \left\{ -\left(3+\frac{9a}{4}\right)\cos 2p_2 - \left(\tfrac{1}{2}+\frac{3a}{8}\right)\cos(2p_1+2p_2) + \left(\tfrac{3}{2}+\frac{9a}{8}\right)\cos(2p_1-2p_2)\right\}$$
$$+ q_2^2\left(-\tfrac{3}{4}\cos 2p_2 - \tfrac{3}{16}\cos 4p_2\right).$$

The equation to determine ϕ_5 is now

$$2\frac{\partial\phi_5}{\partial p_1} + \frac{\partial\phi_5}{\partial p_2} = \frac{\partial H_5}{\partial p_2} + (\phi_3, H_4) + (\phi_4, H_3),$$

and we have to choose a so as to annul the terms in $\sin(p_1 - 2p_2)$ on the right-hand side of this equation. On calculating these terms, we find

$\left(\text{from } \frac{\partial H_5}{\partial p_2}\right)$ $\quad \frac{39}{2} q_1^{\frac{1}{2}} q_2 \sin(p_1 - 2p_2)$

$(\text{from } (\phi_4, H_3))$ $\quad -\frac{45}{2}\left(1+\frac{3a}{4}\right) q_1^{\frac{1}{2}} q_2 \sin(p_1 - 2p_2)$

$(\text{from } (\phi_3, H_4))$ $\quad +\frac{123}{8} a q_1^{\frac{1}{2}} q_2 \sin(p_1 - 2p_2).$

The quantity a must therefore satisfy the equation

$$\frac{39}{2} - \frac{45}{2}\left(1+\frac{3a}{4}\right) + \frac{123}{8}a = 0,$$

which gives

$$a = -2.$$

Substituting this value of a in ϕ_3 and ϕ_4 our integral becomes

$$\text{Constant} = q_2 - 2q_1^{\frac{1}{2}} q_2 \cos(p_1 - 2p_2)$$
$$+ q_1 q_2\{\tfrac{3}{2}\cos 2p_2 + \tfrac{1}{4}\cos(2p_1+2p_2) - \tfrac{3}{4}\cos(2p_1 - 2p_2)\} + q_2^2(-\tfrac{3}{4}\cos 2p_2 - \tfrac{3}{16}\cos 4p_2)$$
$$+ \text{terms of the 5th and higher orders in } \sqrt{q_1} \text{ and } \sqrt{q_2} \qquad (14)$$

Now it may be verified by differentiation that the dynamical system specified by equation (13) possesses the integral

$$\text{Constant} = \tfrac{1}{2}\{\sqrt{2q_2}\sin p_2 + q_1^{\frac{1}{2}}\sqrt{2q_2}\cos p_1 \sin p_2 - 2\sqrt{2q_1q_2}\sin p_1 \cos p_2\}^2$$
$$- \frac{1 + q_1^{\frac{1}{2}}\cos p_1}{(1 + 2q_1^{\frac{1}{2}}\cos p_1 + q_1\cos^2 p_1 + 2q_2\cos^2 p_2)^{\frac{1}{2}}} \qquad (15)$$

and this integral is adelphic, as may be shown by completing the solution, or more simply by remarking that the integral (15) is a function of the variables $(\sqrt{q_1}, \sqrt{q_2}, p_1, p_2)$ which is one-valued and free from singularities for a certain range of values, and therefore the infinitesimal transformation corresponding to it will also be one-valued and free from singularities, and so must transform closed orbits into closed orbits.

But *on expanding this integral* (15) *in ascending powers of* $\sqrt{q_1}$ *and* $\sqrt{q_2}$ *by the multinomial theorem, we arrive at the series* (14). This shows that, *for the dynamical system we are considering, the series obtained by the process of* § 7 *converges for all real values of* p_1 *and* p_2 *so long as* $|q_1|$

and $|q_2|$ *are inferior to certain fixed quantities, and that the series represents the adelphic integral of the dynamical system.*

§ 9. *Determination of the adelphic integral in Case III.*—The principle for the removal of vanishing divisors from the adelphic integral, which was explained in § 7 and illustrated in § 8, is not sufficient for the purpose if the Hamiltonian function contains, among its third-order terms, a term in $\cos(s_2 p_1 - s_1 p_2)$: for this term gives rise to a vanishing divisor in ϕ_3, and the arbitrary terms which are used in order to annul terms with vanishing divisors do not come into operation early enough to remove vanishing divisors from ϕ_3.

In this "Case III" we must make use of another principle (concurrently with the principle of § 7) which may be explained thus: Suppose that an integral of a system of differential equations in variables (q_1, q_2, p_1, p_2) is of the form

$$f(q_1, q_2, p_1, p_2) + \frac{g(q_1, q_2, p_1, p_2)}{\mu} = \gamma$$

where γ is the *arbitrary* constant and μ is a *definite* constant formed of quantities occurring in the differential equations. The integral in this form ceases to have a meaning when μ tends to zero. But we may derive from it an integral which has a meaning when $\mu \to 0$, by merely supposing first that μ is different from zero, and multiplying the equation throughout by μ, so that it becomes

$$\mu f(q_1, q_2, p_1, p_2) + g(q_1, q_2, p_1, p_2) = \mu\gamma$$

and then making $\mu \to 0$; the equation becomes

$$g(q_1, q_2, p_1, p_2) = c,$$

where c denotes $\text{Lt}_{\mu \to 0}(\mu\gamma)$. This is the desired form of the integral when μ is zero.

Our case is not so simple as this, since the vanishing divisor occurs not only in the inverse first power, but in an infinite series containing all the inverse powers. The method we follow, which will be illustrated in the next article, is really equivalent to using the principle of § 7 in order to remove all inverse powers of the small divisor except the first, and then using the principle of this article in order to remove this inverse first power.

§ 10. *Example of the principle of* § 9.—We shall now show by considering a particular dynamical system how the principle just mentioned is applied in order to obtain an adelphic integral free from vanishing divisors in "Case III."

Consider the dynamical system whose Hamiltonian function is

$$H = q_1 - 2q_2 + q_1^{\frac{3}{2}} U_1 \cos p_1 + q_1 q_2^{\frac{1}{2}} U_4 \cos(2p_1 + p_2) \quad . \quad . \quad (16)$$

Now if the Hamiltonian function is

$$\mathrm{H}=s_1q_1+s_2q_2+q_1^{\frac{3}{2}}\mathrm{U}_1\cos p_1+q_1q_2^{\frac{1}{2}}\mathrm{U}_4\cos(2p_1+p_2),$$

where s_1 and s_2 are arbitrary, the adelphic-integral series to which we are led by the method of § 4 is

$$\begin{aligned}\text{Constant}=\,&s_1q_1-s_2q_2+q_1^{\frac{3}{2}}\mathrm{U}_1\cos p_1+\frac{2s_1-s_2}{2s_1+s_2}q_1q_2^{\frac{1}{2}}\mathrm{U}_4\cos(2p_1+p_2)\\&+\frac{s_2}{2s_1+s_2}\mathrm{U}_1\mathrm{U}_4q_1^{\frac{3}{2}}q_2^{\frac{1}{2}}\left\{\frac{4\cos(p_1+p_2)}{s_1+s_2}+\frac{2\cos(3p_1+p_2)}{3s_1+s_2}\right\}\\&+\mathrm{U}_1^2\mathrm{U}_4q_1^2q_2^{\frac{1}{2}}\frac{s_2}{2s_1+s_2}\left\{-\frac{3}{3s_1+s_2}\,\frac{\cos(4p_1+p_2)}{4s_1+s_2}-\frac{6}{3s_1+s_2}\,\frac{\cos(2p_1+p_2)}{2s_1+s_2}\right.\\&\qquad\qquad\left.-\frac{6}{s_1+s_2}\,\frac{\cos p_2}{s_2}\right\}\\&+\mathrm{U}_1\mathrm{U}_4^2q_1^{\frac{3}{2}}q_2\frac{s_2}{2s_1+s_2}\left\{\frac{2(9s_1+s_2)}{(s_1+s_2)(3s_1+s_2)}\,\frac{\cos p_1}{s_1}+\frac{4}{s_1+s_2}\,\frac{\cos(3p_1+2p_2)}{3s_1+2s_2}\right\}\\&+\mathrm{U}_1\mathrm{U}_4^2q_1^{\frac{5}{2}}\frac{s_2}{2s_1+s_2}\cdot\frac{5s_1+s_2}{(3s_1+s_2)(s_1+s_2)}\cdot\frac{\cos p_1}{s_1}\\&+\text{terms of the 6th and higher orders in }\sqrt{q_1}\text{ and }\sqrt{q_2}\qquad . \quad . \quad (17)\end{aligned}$$

In our problem $s_1=1$, $s_2=-2$, so $2s_1+s_2$ is a vanishing denominator. This denominator makes its appearance in the fourth term of the above expression, and occurs in every subsequent term, being squared in the coefficient of the fifth-order term $q_1^2\,q_2^{\frac{1}{2}}\cos(2p_1+p_2)$. We must now modify this series (17) so as to obtain an integral which has no vanishing denominators.

In the first place, we apply the principle of § 9: the lowest term which is affected by the vanishing denominator is the term

$$\frac{2s_1-s_2}{2s_1+s_2}q_1q_2^{\frac{1}{2}}\mathrm{U}_4\cos(2p_1+p_2):$$

we therefore try to form an integral whose lowest term (discarding the non-essential factors $(2s_1-s_2)$ and U_4) shall be

$$q_1q_2^{\frac{1}{2}}\cos(2p_1+p_2).$$

If then we suppose this integral to be

$$\text{Constant}=\phi\equiv q_1q_2^{\frac{1}{2}}\cos(2p_1+p_2)+\phi_4+\phi_5+\phi_6+\ .\ .\ .$$

where ϕ_r denotes the terms of degree r in $\sqrt{q_1}$ and $\sqrt{q_2}$, and substitute in the equation $(\phi,\mathrm{H})=0$, we find on equating to zero the terms of order 4 that ϕ_4 is to be determined from the equation

$$\frac{\partial\phi_4}{\partial p_1}-2\frac{\partial\phi_4}{\partial p_2}=q_1^{\frac{3}{2}}q_2^{\frac{1}{2}}\mathrm{U}_1\{2\sin(p_1+p_2)+\sin(3p_1+p_2)\}$$

The integral of this is

$$\phi_4=q_1^{\frac{3}{2}}q_2^{\frac{1}{2}}\mathrm{U}_1\{2\cos(p_1+p_2)-\cos(3p_1+p_2)\}$$

to which, however, we may add terms of the type

$$\alpha q_1^2+\beta q_1q_2+\gamma q_2^2\qquad .\quad .\quad .\quad .\quad .\quad .\quad (18)$$

where α, β, γ are arbitrary constants, since these terms satisfy the differential equation and are of the type proper to ϕ_4. It should be noticed that these terms are not now superfluous, as they were in the general case studied in § 4; for in the general case the addition of such terms to ϕ_4 would merely be equivalent to adding on an arbitrary quadratic function of the integral of energy and the adelphic integral: but in our present case the adelphic integral does not begin with terms linear in q_1 and q_2, and therefore a quadratic function of it does not account for terms like those in (18). The arbitrary constants in (18) are to be determined in such a way as to make terms with vanishing denominators disappear from the higher-order terms of ϕ. Thus, writing now

$$\phi_4 = q_1^{\frac{3}{2}} q_2^{\frac{1}{2}} U_1 \{2 \cos (p_1 + p_2) - \cos (3p_1 + p_2)\} + \alpha q_1^2$$

and substituting in the differential equation satisfied by ϕ_5 which is

$$\frac{\partial \phi_5}{\partial p_1} - 2\frac{\partial \phi_5}{\partial p_2} = (\phi_4, H_3) \quad . \quad . \quad . \quad . \quad . \quad (19)$$

we find that on the right-hand side of (19) the terms involving $\sin (2p_1 + p_2)$ (which would lead to vanishing denominators on integration) are

$$-3q_1^2 q_2^{\frac{1}{2}} U_1^2 \sin (2p_1 + p_2) - 4\alpha q_1^2 q_2^{\frac{1}{2}} U_4 \sin (2p_1 + p_2)$$

and these will collectively vanish provided

$$\alpha = -\frac{3}{4}\frac{U_1^2}{U_4}$$

In this way, by repeated application of the principle of § 7, we are able to remove all terms with vanishing denominators and obtain an adelphic integral free from them.

§ 11. *Completion of the integration of the dynamical system in Cases II and III.*—Having now in §§ 7–10 overcome the difficulty caused by the presence of terms with vanishing divisors in the adelphic integral in Cases II and III, we can use this integral in order to integrate the dynamical system completely, just as was done for Case I in § 6. We thus obtain expansions for the co-ordinates in terms of the time in all cases: but these expansions are completely different in form, according as the dynamical system falls under Case I, II, or III. This result supplies the underlying explanation of Poincaré's theorem that the series of Celestial Mechanics cannot converge uniformly over any continuous range of values of the constants: for the series to which he was referring were of the kind which we have classified under Case I, and we have seen that when the constants s_1, s_2 are continuously varied, these series must be replaced by the series appropriate to Case II or Case III, whenever the ratio s_1/s_2 passes from an irrational to a rational value. The advantage of solving by means of the adelphic integral is that the forms of the adelphic integral corresponding to the three cases can be readily determined: and thus the difficulty is removed before the adelphic integral is used in order to obtain the complete expressions for the co-ordinates in terms of the time.

(*Issued separately April* 30, 1917.)

VIII.—The Family Budgets and Dietaries of Forty Labouring Class Families in Glasgow in War Time.* By Margaret Ferguson, from the Department of Physiology, University of Glasgow. *Communicated* by Professor D. Noël Paton.

(MS. received December 1, 1916. Read December 18, 1916.)

CONTENTS.

I. Introduction.

The following study of the diet of labouring class families in Glasgow was made as part of an investigation upon the etiology of rickets at present being carried out in the Physiological Department of the University of Glasgow. But the information gathered by Miss Lindsay in her study of the diet of the same class in 1911–1912 † makes possible a very interesting comparison between the conditions of living then and under the present war conditions, and the fact that these studies extended over three periods—(1) Summer of 1915, (2) Winter of 1915, (3) Spring of 1916 ‡—enables some idea of the progressive effects of war conditions to be obtained.

Forty families have been studied.

From the nature of the investigation, families have been chosen where there are young children. In *fourteen* of the families the children looked healthy and well nourished. In the other *twenty-six*, one child at least, and in most cases more than one child, suffered from rickets.

* The expenses of the investigation were defrayed by the Medical Research Committee under the National Health Insurance Act.

† Report on the Study of the Diet of Labouring Class Families in Glasgow.

‡ In February 1917 a second study was made of the dietaries of ten of the families. The results are given in a note at the end of this paper.

To compare the food requirements of the various families, it is convenient to express them in a common term. The requirements of an average man for a single day is taken as unity, and the relative requirements of the other members of the household are taken as follows:—

Woman,	equivalent to	0·8	of a man at	moderate labour.
Boy, 14–16	„ „	0·8	„	„
Girl, 14–16	„ „	0·7	„	„
Child, 10–13	„ „	0·6	„	„
Child, 6–9	„ „	0·5	„	„
Child, 2–5	„ „	0·4	„	„
Child, under 2	„ „	0·3	„	„

These allowances were made by Atwater in his diet studies in New York, and have been used in subsequent investigations. Expressed in this common term, the average of the families studied in 1915–1916 is equivalent to 3·51 men per day, as compared with 4·67 in the 1911–1912 families, that is, in the present studies the families were smaller as a whole or contained younger children.

In every case the food consumed by the family was weighed and noted for a period of seven days. Further particulars of the manner of investigating dietaries, the treatment of waste, etc., and the method by which the energy value is calculated, will be found in Professor Noël Paton's introduction to Miss Lindsay's report.*

The forty diets studied are very representative of the manner in which this class of the community lives. In connection with the same investigation, over 600 labouring class families in Glasgow were visited, and the conditions of living were very much the same as those of the forty families here described. Many complaints were voiced about the price of food; but as employment was easy to find, there was little actual privation, and where the children were suffering, it was the result of the indolence or drinking habits of one or both parents. The question of whether wages have risen sufficiently to compensate for the rise in prices is considered later.

Habits of living have not changed since Miss Lindsay remarked upon them. Food and coal are got in small quantities, the inadequate accommodation in the majority of the houses making this necessary. The rent is seldom saved from the income of each week, but is usually paid out of

* The schedules of the individual dietaries studies are not published as they were in Miss Lindsay's report, but they are preserved in the Physiological Laboratory, where they may be consulted.

the wages of every fourth week. The women say that during "rent week" the diet is poor. Part of the money goes to insurance, which is kept regularly paid. Clothing is got how and when possible, and is rarely made by the house-mother. As a rule everything is paid for as bought, for it is difficult to get credit. This was very strikingly shown at the beginning of the war, when for a week or two, till the Government Separation Allowances arrived, numbers of families were absolutely dependent upon the relief granted by the Soldiers' and Sailors' Families Association.

It is but fair to note here that there are wide differences in the character of the families visited, some having a high standard of home comfort, housing, etc., but unfortunately these have not necessarily the most satisfactory diet, as often so high a percentage of the income is spent upon rent and clothing that the food purchased is not sufficient.

II. Income.

The average income of the families studied is 30s. 0½d.,* as compared with 28s. 3¼d. in the 1911–1912 families, or 8s. 6d. compared with 6s. per man per week, an increase of 42 per cent.

Arranging in groups according to income, as was done by Miss Lindsay, we can compare as follows:—

Regular Income.	1911–12.		1915–16.	
	No. in Group.	Percentage of Total Families.	No. in Group.	Percentage of Total Families.
A. Children earning, income about 40s. .	7	11·7	5	12·5
B. Lodgers kept, " " 43s. .	8	13·3	4	10
C. Father only working, income over 31s.	...	...	5	12·5
D. " " " " 20s. to 31s.	13	23·3	10	25
S. Soldiers' wives, income average 26s. 2d.	...	...	7	17·5
E. Father only working, income under 20s.	5	8·3	...	...
Irregular Income.				
F. Over 20s.	7	11·7	6	15
G. Under 20s.	8	13·3	3	7·5

In addition, Miss Lindsay studied some Jewish and Italian families which are here omitted. Two groups of regular incomes are added, namely:

* Income is based on information supplied by the house-mothers. In no case was the employer appealed to for verification.

(S) soldiers' wives, who have been grouped by themselves as the slightly smaller number of children and the absence of the father reduces the average family equivalent by 1·1 men per day; and (C) householders earning over 31s. weekly.

Although the average income of the 1915–1916 series is only slightly in advance of that of 1911–1912, the number of families earning over 25s. is much greater. Leaving out those families where the children are earning or where lodgers are kept (income averaging 43s.), and dividing the remainder into two groups, those with an income above and those with an income below 25s., the following table shows a marked difference between the two periods:—

Comparison of Income.

	1911–12.			1915–16.		
	No. of Families.		Percentage.	No. of Families.		Percentage.
Below 25s. .	30	=	91·2	13	=	42
Above 25s. .	3	=	8·8	18	=	58

III. Expenditure.

In trying to estimate whether the standard of comfort is higher at the present time than in 1911–12 the expenditure must be considered, *i.e.* the amount and percentage of income spent on rent and food, and the surplus remaining for fuel, light, clothing, amusements, etc., and also the efficiency of the diets. It must be remembered that coal, light, and clothing have all increased in price.

Comparison of Expenditure.

	Average 1911–1912.	Average 1915–1916.
	s. d.	s. d.
A. Food . .	20 7 = 73 per cent. of Income	*18 8 = $62\frac{2}{9}$ per cent. of Income
B. Rent . .	3 9 = $13\frac{1}{3}$ " "	4 $0\frac{1}{4}$ = $13\frac{1}{3}$ " "
C. Remainder.	3 11 = $13\frac{2}{3}$ " "	7 4 = $24\frac{4}{9}$ " "

* On account of the smaller families the expenditure on food appears to be less than in 1911–12, but it is actually greater, being equivalent to an expenditure of 24s. 10d. for families having the same number of "men per day" as the earlier studies.

A. *FOOD.*

1. *Efficiency of the Diets.*

Comparison of Average Efficiency of the Diets.

	Energy in Calories per Man per Day.	Protein in Grams per Man per Day.	Fat in Grams per Man per Day.
1911–12	3163·0	110·0	83·0
1915–16	3297·6	102·0	90·4

There is very little difference in the energy value of the two groups. What difference there is is accounted for by the higher amount of fat consumed at the present time. The carbohydrates do not differ much. In 1915–16, owing to the high cost of butter, margarine was used instead of butter by 28—*i.e.* 70 per cent.—of the families. In 1911–12 few of the families studied used it; but when purchased at 6d. a lb. it forms an economical source of energy to the housekeeper, and this largely accounts for the increase in the consumption of fat. The average weight of butter or margarine consumed per man per day by 36 of the families studied in 1911–12 was 25 grams, and in 1915–16 this had risen to nearly 37 grams. The table on page 126, giving the food values obtainable for 1d., shows that in June 1916 margarine was, by as much as 35 per cent., a more economical source of energy than sugar.

The average protein content has fallen by 8 grams; and while in 1911–12 only 18·7 per cent. of the families had less than 90 grams, at the present time no less than 37·5 per cent. have this small amount.

A slightly higher percentage of protein used now comes from vegetable sources—63·6 per cent. compared with 61·1 per cent. in 1911–12. This indicates the distinct diminution which has taken place in the amount of meat consumed. In the 1911–12 dietaries the average consumption of meat per man per day was 120 grams, from which 10 per cent. of the total energy was supplied. In 1915–16 this average was only 86·4 grams, which was equivalent to only 7·4 per cent. of the energy in the food. The explanation of this is not that meat has risen in price out of proportion to the general increase, for such is not the case, but that meat is at all times one of the most expensive foodstuffs, and consequently is used in lesser quantities where the need for economy is felt.

Although the great majority of the diets approximate to the average, those of certain families in each group show marked divergences in energy value and in protein content.

The 1911–12 studies show variations from 1979 to 4255 Calories per man per day, and from 64 to 147 grams protein per man per day.

The 1915–16 studies show variations from 2222·8 to 4702 Calories per man per day, and from 61·4 to 148·4 grams protein per man per day.

The diet with the lower limit of the present dietary series was that of a family which was continually in straits owing to the lazy and drunken habits of the father. The house-mother was a very careful woman, and did the best she could with the money at her disposal. The food cost only 4·7 pence per man per day, and consisted largely of oatmeal, which, however, was insufficient in quantity. Both mother and children looked undernourished, and the children were very rachitic. Four other diets are under 2500 Calories. Three of those belong to group D, and are dealt with on p. 123. The other is that of the family mentioned on p. 125.

The 1915–16 diet having the highest energy value (4702 Calories) is that of a family where the father was doing very hard work in a shipyard, and the mother worked as a cleaner—also very arduous work. Money, however, was plentiful in this house, and the house-mother was very careless, so there is a possibility that some of the food was wasted without the knowledge of the visitor. Only two other diets have an energy value over 4000 Calories per man per day, and both of these are very little above that figure.

Comparison of—

A. Average Energy Value.

	1911–12.	1915–16.
	No. of Families.	No. of Families.
Below 3000 Calories . .	21=44 per cent.	12=30 per cent.
,, 3000–3500 ,, . .	17=35 ,,	14=35 ,,
Over 3500 ,, . .	10=21 ,,	14=35 ,,

B. Average Protein Content.

	1911–12.	1915–16.
	No. of Families.	No. of Families.
Below 100 grams . . .	15=31 per cent.	20=50 per cent.
Over ,, . . .	33=69 ,,	20=50 ,,

The majority of the later series of studies are higher in energy value than those done in 1911–12.

The protein content is not so high. Half the families instead of less than a third, as in 1911–12, have under 100 grams of protein per man per day.

Comparison of Energy Value and Protein in the Different Groups.

	1911–12.			1915–16.		
	No. of Families.	Energy Value.	Protein.	No. of Families.	Energy Value.	Protein.
A. Children earning, average 40s. . . .	7	3184	113·8	5	3568·5	109·1
B. Lodgers kept, average 43s.	8	3316	111·7	4	3202·8	105·7
C. Wage regular, over 31s. .	...	...	...	5	3505	108·8
D. " 20s. to 31s. . .	13	3458	117·8	10	3002	95·1
S. Soldiers' wives, average 26s.	...	...	...	7	3271·3	94·5
F. Income irregular, over 20s.	7	2994	108	6	3712·5	114
G. " " under 20s.	8	2797	96·6	3	2823·9	78·6

From these figures it will be seen that the energy and protein contents of the diets decrease progressively with the increase of poverty.

In both groups with irregular incomes in the 1911–12 studies the energy value is under 3000 Calories per man per day. In the present studies only group G (irregular income, under 20s.) does not reach this figure.

Three of the 1915–16 groups do not have 100 grams of protein per man per day. All the 1911–12 groups are above 100 grams except Class G (irregular income, under 20s.).

In the earlier studies, group F (families with irregular income, over 20s.), the energy value is low. Group F in the present studies has the highest average. The reason for this is that at present the average income is good—up to £2 in two of the families, since labourers, who constitute the great number of the heads of families in this group, are much in request just now, and the irregularity in some cases is simply the result of a varying amount of overtime, and not, as in the previous studies, of under-employment.

Group D of the recent studies, which has an average energy value of just over 3000 Calories and 95 grams of protein, is interesting. Of the ten families composing it, two were very superior, living in homes which were much above the average in comfort, 20 per cent. of the income being spent on rent, as against the average 13 per cent. One of these householders was a motor driver in the service of the G.P.O., earning 30s. weekly, and living in a house of three apartments; the other was a painter, only partly employed owing to seasonal slackness. One diet is low owing to bad marketing, *i.e.* buying the more expensive food, such as eggs and cooked meats. Three other families have insufficient diets, probably on account of the large number of young children in each, and the consequent stretching of income to cover all their needs.

It is in these families with regular incomes under 31s. that the most marked difference in favour of the 1911–12 studies is to be observed. At the present time their diets are about 500 Calories and 22·7 grams of protein per man per day lower than in 1911. It is here, where the father has a regular but small wage, with no overtime work, that the rise of prices is felt most.

2. *Nature of Foodstuffs used.*

There is very little change in the commodities used since Miss Lindsay made her studies. Bread is still the "staff of life." An average of about 40 per cent. of the total energy value in the studies comes from bread alone, and considerably more—51 per cent. in 1911–12, and 48 per cent. in the war-time dietaries—was got from cereals. Oatmeal is used by twenty-nine of the forty families, but only in one case, that of the family almost at starvation level already remarked upon, does it supply a large proportion of nourishment, and this because of the deficiency of the other foodstuffs. Since meat became dearer there has been a tendency to use the cheaper cuts, and this leads to a slightly greater variety in the food.

Here and there a house-mother puts into practice what she has learnt at "war economy" and other cookery classes, and utilises legumes, rice, etc.; but this is not widespread, and these are still used in relatively small quantities.

3. *Cost.*

The cost of food per man per day in 1911–12 averaged *7·07* pence over all, and *6·07* pence for the poorer families.

The average cost of the 1915–16 diets is *9·64* pence per man per day, 36·4 per cent. above that of 1911. The rise in the cost of living may be estimated by the smaller value which has been purchased for 1d. in the latter series of studies. The average is what might be expected considering the rise of prices (see p. 128).

In 1911–12 the yield was 453 Calories per 1d., compared with 350·5 in 1915–16. This is equivalent to a 29·4 per cent. rise in the cost of food.

Variations in Cost of Food Materials during 1915–1916.

The forty dietary studies extend over a period of a year, but they fall into three periods as follows:—

	No. of Families.
(*a*) May–June 1915	18
(*b*) November–December 1915	17
(*c*) Spring 1916	5

(*a*) Of the eighteen dietaries studied in June 1915, the average energy value purchased per 1d. is *335 Calories*. The value received by the different purchasers does not vary very much. Three have a value of over 400 Calories per 1d., viz. 459, 415, and 413 Calories. These get 75 per cent. of their protein from vegetable sources. Four get less than 300 Calories per 1d.—viz. 245, 280, 287, and 297 Calories per 1d.,—and this is due to the use of eggs, a great deal of milk, and other more expensive animal foods.

(*b*) Of the seventeen studies made in November-December 1915, the average value for 1d. is *380 Calories.* Here again the value received is wonderfully constant. Five are above 400—viz. 474, 436, 425·5, 419, and 418·5 Calories—and two under 350. Three of the families which were buying over 400 Calories per 1d. were nevertheless spending too little on food, as the energy value per man per day was under 3000 Calories in two of the diets, and the protein and fat, the more expensive items, each are under 80 grams in all three. The high energy value purchased per 1d. by these three women raises the average of this group somewhat, but there is no doubt that the cost of living fell somewhat during the Autumn of 1915, only to rise again in the Spring of 1916.

(*c*) Only five diets have been studied in the Spring of 1916. The average value purchased is *305 Calories per 1d.*

Although the number of families in this group is too small to eliminate the factor of individual marketing in determining the cost of living, those five diets are interesting. In two of them the energy value was 350 Calories per 1d., and in the other three about 270 Calories. One of the two cheaper diets is that obtained by a family of six—father, mother, and four children—on an income of 16s. for the week of the study. The father was a discharged, time-expired soldier, who had just obtained work on a railway and had not yet had a full pay. This diet consisted chiefly of bread, sugar, tea, and the cheapest of margarine. The last two days there was not even margarine, and the last three no meat. Potatoes, being 2s. 2d. a stone, were beyond their means. Oatmeal would have given a better energy value, but apart from that a good value was got. The other diet where an energy value of over 300 Calories was obtained is that of a very well cared-for family. The mother was in the habit of going long distances to secure bargains, buying, for example, the "end loaves" from the bakery at a farthing per pound below the usual price.

Thus we see that it is not possible at the present prices to get a varied diet with a higher value than 300 Calories per 1d. without the expenditure of a great deal of time and thought.

The value received in this group (305 Calories per 1d.) represents a rise of price of 48·5 per cent. since 1911–12. The following tables of prices bear out the results of the foregoing paragraph:—

Food Values obtainable for 1d., according to Retail Prices in Glasgow at various Periods (calculated from Cooper & Co.'s Glasgow Price List).

	Immediately pre-War.			June 1915.			November 1915.			June 1916.		
	Price per lb. in Pence.	Protein in Grams.	Calories.	Price per lb. in Pence.	Protein in Grams.	Calories.	Price per lb. in Pence.	Protein in Grams.	Calories.	Price per lb. in Pence.	Protein in Grams.	Calories.
Beef (stewing)	9	9·6	132	12	7·2	98·75	12	7·2	98·75	13	6·6	91
„ suet	10·5	1·9	337	10·5	1·9	337	10·5	1·9	337	11·5	1·7	308
„ corned	12	10	107	17	7	76	17	7	76	18	6·6	71
Mutton (cheaper parts)	8	6·8	132	12	4·5	88	12	4·5	88	12	4·5	88
Bacon	10·5	3·9	256	13·25	3·1	203	14·25	2·7	189	15	2·7	179
Cheese (Canadian)	8·5	15·5	241	12	11	171	10·5	12	195	13·5	9·8	152
Margarine	5·5	1	659	6	·9	587	6	·9	587	7	·8	504
Herring (fresh)	4	12·7	92	4	12·7	92	4	12·7	92	6	8·5	62
„ (smoked)	4	22·7	187	4	22·7	187	4	22·7	187	...	...	...
Sugar	2	...	930	3·5	...	...	3·75	...	...	5	...	372
Potatoes	·57	14·3	542	·57	14·3	542	·43	19·1	723	1·17	7·1	271
Oatmeal	1·23	59	1512	2·47	25·4	753	2·14	34	869	2·29	21·7	814
Wheat flour	1·43	36·26	1155	2·14	24·2	770	2·07	25	798	2·29	22·7	722
Bread	1·5	27·9	810	2	20·9	607	2	20·9	607	2·125	19·7	572
Lentils	2·5	47·2	648	3·5	33·7	463	7·5	16	216	3·5	33·7	463
Peas	2	56·8	827	3	37·9	552	4	28·4	414	5	22·7	331
Beans	2·5	41·8	640	2·5	41·8	640	3	34·8	533	3·5	24·1	457
Barley	2	19·3	825	2·5	15·4	660	3	12·9	550	3·5	11	471
Rice (Rangoon)	2	18·1	815	2	18·1	815	2	18·1	815	2·5	14·5	652

The analyses used are those given in Miss Lindsay's Report. See Appendix VI.

This table is very interesting as showing the fluctuation of the values of the main articles of diet. The tendency of prices is upward, with a slight fall in one or two of the commonest articles in Autumn 1915.

At pre-war prices oatmeal was the most economical source of both energy and protein; peas following for protein, and wheat flour for energy.

A year later (June 1915) beans were our cheapest source of protein, and rice of energy. Next came peas for protein, and flour as a source of energy.

In November 1915 oatmeal had almost recovered its position, being the most economical source of energy, and taking the second place as a source of protein. Beans were still the cheapest protein obtainable, and rice followed oatmeal as the cheapest source of energy.

In June 1916 (1) lentils and (2) oatmeal were the cheapest sources of protein, and (1) oatmeal and (2) flour the cheapest sources of energy.

Sugar and potatoes have undergone the greatest changes in value. At pre-war prices sugar was the most economical source of energy, following oatmeal and flour. Being restricted in import, sugar has been more subject to the causes which brought about the rise in prices than many other foodstuffs. Potatoes being mostly home grown did not rise immediately on the outbreak of war; indeed for a short time in the Autumn of 1915 they were selling at 5d. a stone, and formed the cheapest food at the time. An early frost in the Autumn of 1915, however, wasted part of the crop, and potatoes have become very dear. For a short period in June 1916 they were selling at 2s. 6d. a stone, a prohibitive price for the working-class housekeeper.

Animal foods are expensive at all times. At pre-war prices protein could be obtained from flour, lentils, peas, and oatmeal at about one-sixth of its cost when got from beef and mutton.

In June 1916, although meat had not risen in price to the same extent as the vegetable foodstuffs, it was still five times as dear as a source of protein, and over seven times as expensive as a source of energy.

[Table

Percentage Rise in the Cost of Food. A Comparison between Wholesale and Retail Prices.

	Wholesale Price.*			Retail Price per lb.		
	June 1914.	June 1916.	% Rise.	June 1914.	June 1915.	% Rise.
Beef per lb. (av. of all cuts)	6·25d.	10d.	60	(Av. of cheaper cuts) 9d.	13d.	44·4
Suet per lb.	...	...	...	10½	11½	9·5
Corned Beef per 72 lb.	41/6	63/9	54·8	12	18	50
Mutton per lb. (av. of all cuts)	8·5d.	10·75	26·5	(Av. of cheaper cuts) 8	12	50
American Bacon per 112 lbs.	73/6	95/6	30	10½	15	43
American Cheese per 112 lbs.	72/6	99/6	37·4	8½	13½	58·8
Margarine per 112 lbs.	78/	76/6	2 decrease	5½	7	27·2
Sugar	15/8	28/7	82·2	2	5	150
Potatoes per 168 lbs.	4/	8/7	114·5	·57	1·17	100
Oatmeal " 280 "	29/11	49/	63·3	1·23	2·29	86
Flour " 280 "	37/8	51/3	36	1·43	2·29	60·1
Bread	...	...	...	1·5	2·125	41·7
Lentils per 100 lbs.	13/3	26/6	100	2·5	3·5	40
Peas " 280 "	60/5	98/	62	2	5	150
Beans " 112 "	23/	34/6	50	2·5	3·5	40
Barley " 280 "	26/9	38/8	44·5	2	3·5	75
Rice " 112 "	10/10	16/3	50	2	2·5	25

Fish is omitted owing to the difficulty of obtaining reliable figures. Upon the whole it has advanced in price more than most other commodities.

The rise in the cost of food from June 1912 to June 1916, as shown by Miss Lindsay's dietary studies, is 48 per cent. The average rise in prices since the outbreak of war, as shown by the retail price list, is 67 per cent., but one or two of the commodities most used have not risen in price to that extent. Bread rose only 42 per cent., beef 44 per cent., mutton 50 per cent., rice 25 per cent., beans and lentils 40 per cent., and margarine 27 per cent.

The average rise of wholesale prices is about 14 per cent. less than the retail, being 53 per cent. compared with 67 per cent. on retail prices.

Certain commodities have risen in retail out of all proportion to the wholesale rise. The most marked differences exist in the price of peas, flour, oatmeal, sugar, margarine, and American cheese.

* From price list kindly supplied by the Scottish Wholesale Co-operative Society, Ltd. These are the prices at which the central society sells to the branches.

Sugar, the importation of which has been controlled, has been subject to a much greater rise of price than most other commodities.

The reason for the excessive rise in the cost of potatoes has already been noted.

The Board of Trade returns show that comparing June 1914 and June 1915 there was a rise of 68 per cent. on an average for all the cereals imported. Prices fell in Autumn 1915. In December the average price of imported cereals was only 47 per cent. above the pre-war price. Since then the corresponding figures have risen steadily, in May being 76 per cent. above what they were in June 1914. The over-all rise of price of dead meat imported was 67·5 per cent. from June 1914 to May 1916. With these the retail prices compare favourably. There was a fall in the price of wheat after June 1st, but so far the price of bread was not affected in Scotland.

B. *HOUSING AND RENT.*

The housing of the forty families studied does not show much variation, and is similar to what it was in 1911–12. In each case the house was of the poorer type of tenement dwelling. *Eleven* of the families were living in one-roomed houses, *twenty-six* lived in houses of two apartments, and *three* had three apartments. In every case the cubic space per person had been calculated approximately. Putting children upon the same footing as adults, the average air space per person is 376½ cubic feet, against a theoretical 1000 cubic feet. In this connection it is interesting to note that in a number of the families which have a well-proportioned and sufficient diet the children are nevertheless weak and deformed by rickets. In Port Sunlight rickets is very uncommon, and the infant death-rate low. The average income of labouring class families there was till recently no higher than that of the families visited in Glasgow, but the housing was much superior, each family having a cottage of its own, with an average for the fifty families visited of 1568 cubic feet per person and free access to the fresh air.

The percentage of income spent on rent is identical in the 1911–12 and the 1915–16 studies, namely 13·3. The housing factor may then be regarded as having remained constant.

C. *REMAINDER OF INCOME.*

The average amount of income remaining after deducting the cost of rent and food is higher in the 1915–16 than in the 1911–12 studies, 7s. 4d.—24·4 per cent. compared with 3s. 11d.—13·7 per cent. in 1911–12.

This remainder has to cover coal, light, clothing, insurance, amusements and holidays, also the cost of tea, which was not included in the food. Coal has risen from 1s. or 1s. 1d. to 1s. 6d. or 1s. 7d. a cwt.

The duty on tea has caused a rise of 33 per cent. in price. Insurance, light, and clothing are practically unaltered. A typical weekly expenditure of this remainder of income is as follows:—

Coal, 1 cwt.	1s. 6d.
Tea, ½ lb.	1s. 4d.
Gas	1s.
Life Insurance (whole family)	8d.
Remainder for clothing, etc.	2s. 10d.

The purchasing power of this portion of income, averaging 25 per cent. of the whole, has fallen by about 25 per cent., and this is equivalent to an increase of 6 per cent. in the cost of living.

D. *CHANGE IN COST OF LIVING.*

These investigations enable some answer to be given to the question of whether real wages have risen or fallen since the outbreak of war.

The percentage of unemployment has fallen from 3 per cent. in 1911 to 0·5 per cent. in June 1916. Thus under pre-war conditions unemployment represented a loss of 2·5 per cent. more of the weekly wage than in June 1916.

The weighted cost of food, calculated from the relative values purchased per penny in the family budgets studied at the two periods, has risen about 50 per cent. since 1911–12. In the families studied about two-thirds of income was spent on food, so that a rise of $\frac{2}{3} \times 50$ per cent. $= 33\frac{1}{3}$ per cent. of income would be necessary to compensate for the advance in the price of food. Adding to this the 6 per cent. rise from other expenditure, and subtracting 2·5 per cent., the difference in the loss of wages due to unemployment at the two periods, a rise in wages of 36·8 per cent. would be required to keep the standard of well-being constant.

In the recently published interim report of the Committee appointed by the Board of Trade to investigate the principal causes which have led to the increase of prices of commodities since the beginning of the war, the rise in the weighted cost of food is estimated at 65 per cent., which is equivalent to a rise of 45 per cent. in the cost of living among the working classes. These figures are calculated from June 1914 to September 1916.

According to figures supplied by the Board of Trade Department of Labour Statistics to the above Committee, the weighted cost of food has risen by 6 per cent. since June, when the last group of the present dietary studies was carried out. This is equivalent to a further advance of 4 per cent. in the cost of living, or an increase of 41 per cent. since the outbreak of war, according to the present calculations. The difference between this figure and that estimated by the Board of Trade Committee may be due to the following causes:—

1. That the character and quantity of the commodities purchased by the housekeeper has altered somewhat in the direction of economy. The use of less meat in some families, the substitution of cheap for the dear cuts of meat in others, and the increased consumption of margarine are instances of this.

2. That the "weighting" from which the Board of Trade figures are calculated does not exactly correspond with the proportionate use made of the various commodities by the labouring classes in Glasgow. As above stated, an average of 40 per cent. of the total energy in the food of the forty families studied came from bread, which had only advanced from 3d. to 4¼d. for the 2 lb. loaf (or 42 per cent.) by June 1916.

Conclusions.

If the results of these studies can be applied to the labouring classes in industrial centres generally, they show to June 1916—

1. That on an average the food supply was not less adequate than in pre-war times, although there was a tendency to a decreased consumption of protein in meat and an increased consumption of fat.

2. That the cost of energy in food had risen about 50 per cent.

3. That the total cost of living had probably increased by 37 per cent.

4. That the increase in the cost of living, resulting in a diminished supply of the necessaries of life, is being chiefly felt by the families of labouring men with a fixed wage, say from 20s. to 30s. weekly. Among the men who were irregularly employed before the war, or are now doing Government work, or are otherwise having a good deal of overtime work, the surplus of income over the necessary expenditure has materially increased.

In conclusion, I should like to express my indebtedness to the house-mothers for their kindly consent and co-operation, without which the studies could not have been made.

My thanks are also due to Sister Elinor, Miss Rutherfurd, and Mrs Scott for supplying the names of families for visitation; and to the workers at Queen Margaret Settlement, to Miss Walker and other ladies connected with Bridgeton District Office of the Charity Organisation Society, to the students of the Physiology class and to Dr Madge Robertson, all of whom very materially helped the investigation by visiting the homes daily and weighing the food.

Finally, I desire to acknowledge the extent to which I am indebted to Professor D. Noël Paton, who suggested and planned the work, and was always ready with kindly criticism and advice.

NOTE ADDED 15*TH MARCH* 1917.

The delay in publication has afforded an opportunity of comparing some of the foregoing diets with those of the same families since the Food Controller on February 2nd issued an appeal to the nation to adopt voluntarily a system of rationing, calling upon heads of families to try to limit their weekly purchases of the three staple articles of food for each person in the household to the following quantities:—

Bread	4 lb., *or* Flour 3 lb.
Meat	$2\frac{1}{2}$ lb.
Sugar	$\frac{3}{4}$ lb.

During the last week of February ten families, in all of which the father was the only wage-earner, were studied in the same manner as before. Only two of the house-mothers had heard of the ration, and neither of these had made any effort to confine herself in her purchases to the suggested quantities.

Income.

At the time of the first study two of the families belonged to group C, four to group D, one to group F, and three to group S (see p. 123). When the second study was made, five of the ten householders were absent on military service, and one, who was with the army during the first study, had been discharged and was making 30s. a week, thus bringing the family into group D.

Although the average income of these ten familes had materially increased—from 28s. 9d. to 36s. 10d.—this increase was chiefly confined to two, the first (M. 112) being that of a dock labourer whose earnings varied week by week, and amounted to 55s. 6d. during the second study instead of 30s. as in the former. The most marked change in circumstances was in M. 65, the family of a man who at the first study was earning 30s. weekly and at the second was absent in the motor transport. This family, from separation allowances, additional allowance from the father and a weekly proportion of the father's former wage, had 71s. weekly. Family H. 38, although the income was unchanged, owing to the father's habits was much better off in consequence of his enlistment. S. 84, the family of the soldier who was discharged, found it more difficult to live at the time of the second study on 30s. which the father earned than during the former study on a separation allowance of 27s. As the increase of the separation allowances had only been paid in one monthly instalment when the studies were made, and so far had not affected the weekly expenditure, it has been omitted in calculating the incomes. Six of the ten families had practically the same income as in 1915, and four were in a better position.

Omitting the family whose income had so markedly increased (M. 65), the average income was 33s. 6d. weekly.

Expenditure.

The average expenditure of the nine families (omitting M. 65) on food was 23s. 1½d., or 69 per cent. of income. As these families were equal, on an average, to 3·46 men per day, this is equivalent to a daily expenditure of 11d. per man per day on food. The amount spent on rent was practically unaltered.

An average of 6s. 1½d., or 18·3 per cent. of income, remained after rent and food were provided. Thus, in spite of the considerable advance in income, the proportion remaining for other expenses had decreased since the first study owing to the rise in food prices.

This "remainder of income" was unevenly distributed, being under 2s. in the three families with the smallest incomes. Three had a large surplus.

Owing to a further rise in the price of coal and tea, the purchasing power of this portion of income has fallen since the first study was made.

Efficiency of the Diets.

The dietary results are recorded in the following table (compare with table on p. 123):—

	First Study. Per man per day.				Second Study. Per man per day.			
	Protein in Grams.	Fat in Grams.	Energy in Calories.	Family Income.	Protein in Grams.	Fat in Grams.	Energy in Calories.	Family Income.
Income unchanged :—								
				s. d.				s. d.
M. 63 .	96·2	94·1	3378·6	23 0	98·5	77·1	3116	23 0
M. 62 .	83·1	86·2	2674	25 0	86·6	79·3	3087·6	25 0
S. 84 .	86	93	2836	27 0	77·5	63·8	2530	30 0
N. 31 .	128·9	128·2	4174	36 0	103	67·8	3112	39 0
H. 47 .	88·9	67·2	3003	22 0	85	62·3	2714	23 5
N. 151 .	92·8	107·5	3120·7	30 0	108·6	84	3136	29 9
	96·0	96·0	3197·7	27 2	93·2	72·4	2949·3	28 4
Income increased :—								
				s. d.				s. d.
M. 112 .	88	97·5	3317·7	30 0	105·9	98·1	3476	55 6
H. 38 .	80·7	80	2691	40 0	114·2	77·7	3314	40 0
M. 65 .	75·5	70·9	2491	30 0	91·1	68·2	2844	71 0
N. 150 .	148·4	105·3	3568	25 0	138·6	112·8	3690	35 6
	98·1	88·4	3017	31 3	112·4	89·2	3331	50 6

It will be seen that the families whose incomes had appreciably increased had a more generous diet. H. 38, as above stated, comes under this category.

As might be expected from the rise in the cost of food, those families whose incomes were practically unchanged had less generous diets than when first studied.

The average energy value had fallen very slightly, but the average protein content had increased a little; only five, instead of eight of these families as at the first study, having had less than 100g. per man per day.

Food Purchased per Penny.

All but two of these families were previously studied in 1915. The average value received by the ten families was 365 Calories per 1d. at the first study, and 273 in February 1917. This represents a fall of

33·7 per cent. in the food purchased per 1d. between the two sets of dietary studies, and of 66 per cent. compared with the values received in 1911–12.

Retail prices have risen somewhat beyond this, many commodities having doubled in price during the war; consequently these values purchased per 1d. indicate that the labouring man's wife, because of monetary considerations, or because of the scarcity of certain commodities, has altered the proportionate use made of the various food-stuffs, and thereby effected an economy. This is clearly shown from the following table. Bread, which is the cheapest food-stuff obtainable, was more largely used in the later studies than in the earlier.

AMOUNTS CONSUMED "PER PERSON"* IN LBS. PER WEEK.

	First Study.				Second Study.			
	Flour.	Potatoes.	Meat.	Sugar.	Flour.	Potatoes.	Meat.	Sugar.
Income unaltered:—								
M. 63	4·1	...	·31	1·87	4·73	1·23	·77	·87
M. 62	4·31	·43	1·06	·72	4·94	·11	·34	·62
S. 84	3·83	3·1	·77	·66	4·34	·6	·71	·65
N. 31	5·78	1·31	1·30	1·60	5·84	·54	1·23	·52
H. 47	3·07	3·07	·70	1·17	5·92	...	·48	·44
N. 151	3·11	3·11	1·17	1·04	4·19	2·66	1·02	·97
Averages	4·02	1·84	·88	1·18	4·99	·86	·76	·68
Income increased:—								
M. 112	3·5	3·48	1·32	1·1	6·08	1·62	1·38	·67
H. 38	3·54	3·69	1·03	·63	6·4	3·21	1·19	·6
M. 65	3·47	1·28	1·18	·8	6·72	...	·49	·36
N. 150	5·42	6·28	2·84	·43	5·45	4·27	2·53	·95
Averages	3·98	3·68	1·59	·74	6·16	2·28	1·40	·64

This seems to indicate that as a result of the non-availability of potatoes and sugar there has been an increased use of cereals even in those diets where the general intake of energy has decreased.

These figures bear out the conclusions published in the *British Medical Journal* of February 24, 1917, that it is not possible, on the bread, meat,

* The dietary requirements "per person," taking into account the proportionate number of men, women, and children in the community, has been estimated by the Food Committee of the Royal Society at ·77 of that of a man at moderate labour ("The Food Supply of the United Kingdom," Cd. 8421). This figure was used in making these calculations.

and sugar ration as the basis of a diet, to secure an adequate supply of energy, especially as the cost of meat has made its use to any extent impossible for the labouring classes.

As shown there, the flour, meat, and sugar purchased by the families studied in 1915–16, even at present prices, cost 6¼d. less per person than the ration, and yielded 2160 additional Calories per week; but the protein derived from these sources was only 342·9g. instead of 367·4g. as contained in the Food Controller's ration.

(*Issued separately April* 30, 1917.)

IX.—On some Causes of the Formation of Anticyclonic Stratus as observed from Aeroplanes. By Lieut. C. K. M. Douglas. *Communicated by* M. M'Callum Fairgrieve, M.A.

(MS. received December 9, 1916. Read January 22, 1917.)

Typical Form of Temperature Gradient in Anticyclones.

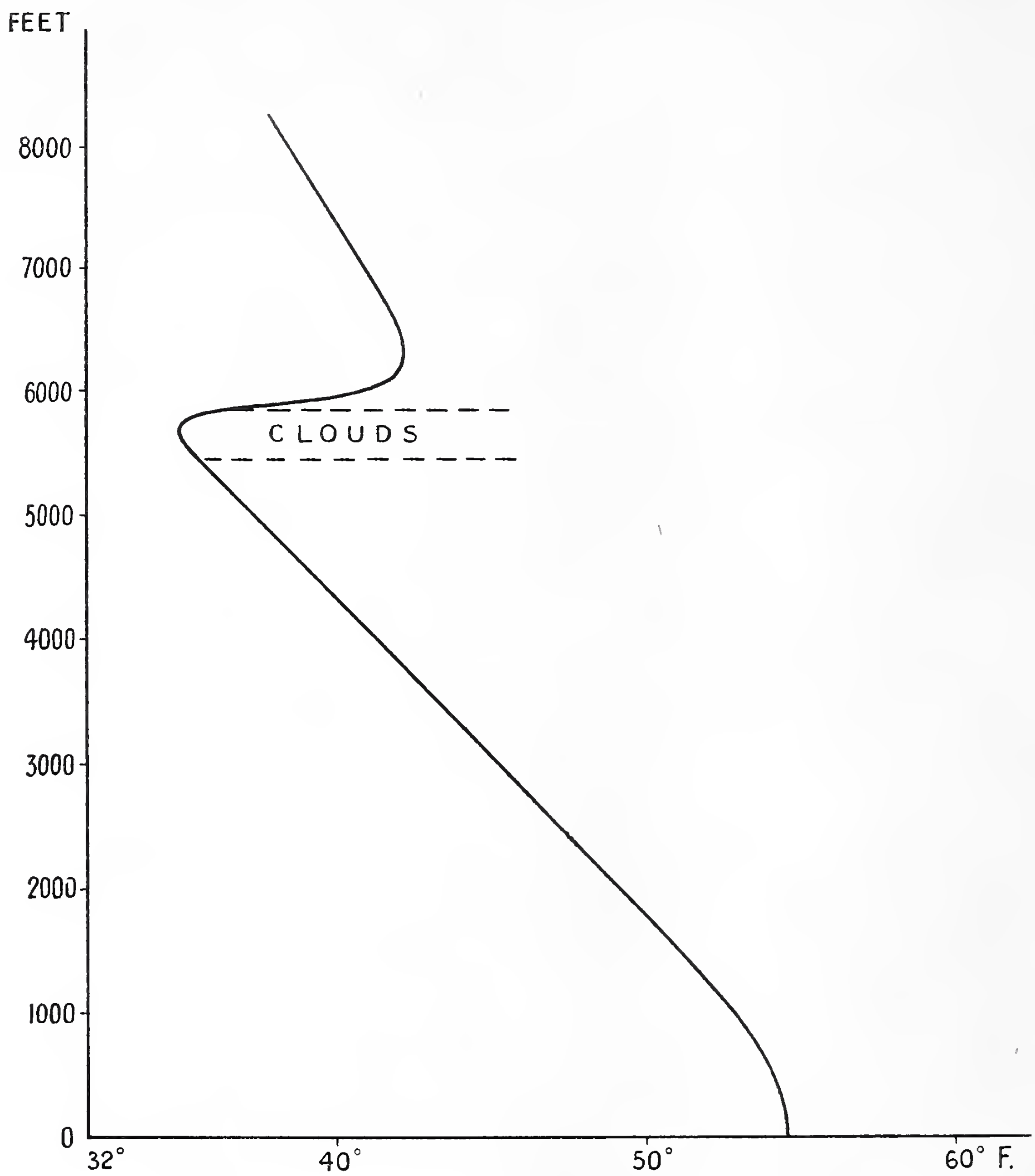
In a paper published elsewhere * I have summarised various observations of anticyclonic stratus. These observations were made from an aeroplane in Northern France during the year 1916, and showed that when stratus clouds are present and cover any considerable portion of the sky, there is a rise of temperature above the clouds, and further that the rise sometimes exists when there are no clouds, but only haze up to the level of minimum temperature. In both papers I include under "stratus" all clouds with approximately level tops, and into this class fall some clouds that are usually called strato-cumulus.

Since writing that paper I have made further numerous observations, which show clearly that round the northern and eastern sides of anti-cyclones the temperature gradient for a few thousand feet is usually 4° or 5° F. per 1000 feet (8° or 9° C. per km.); above this adiabatic region the temperature gradient is low, there being usually a well-marked rise of temperature for 500 or 1000 feet, followed by a gradual return to a gradient of about $3\frac{1}{2}$° F. per 1000 feet. The top of the lower adiabatic region is easily recognised by the top of the haze, and usually some cloud is present. The height of this level above the surface varies between 3000 and 6000 feet, and averages about 4500 feet; it is usually greater with northerly than with westerly winds, and greater near the boundary of the anticyclone than near the centre. In my earlier paper I gave the average total rise of temperature above layers of haze with patches of cloud as 4° F.; this was founded on numerous observations from April to July 1916, mainly with northerly winds. Observations from August to October showed the same average rise, and the rise was usually rather greater with west winds than with north winds. On the only occasion on which I obtained temperature observations above an unbroken layer of stratus there was a total rise of 9° F. above the clouds; this was at 6 a.m. on June 21, 1916, and the temperature curve is given on p. 138.

* "Weather Observations from an Aeroplane," *Journal of the Scottish Met. Soc.*, vol. xvii, No. 33, pp. 65–73.

This curve is typical in form for the northern or eastern sides of anticyclones, though the reversed gradient is less pronounced when the clouds are not continuous. The general form of the curve is probably similar in winter also, as the appearance of the cloud and haze layers is the same at all seasons.

It therefore seems probable that two layers of air of different origin are



responsible for the formation of anticyclonic stratus; the upper layer is very clear, and undoubtedly of a lower relative humidity than the lower layer, though I made no actual observations of this.

Descent of Air in Anticyclones.

On a few occasions in Northern France I have observed clear evidence of descent in an anticyclone of warm air to a lower level, though not to the surface of the earth. A good example occurred during the passage of a small anticyclone from the North Sea to Southern Russia from 22nd to 25th September 1916. On the 22nd there was a light surface wind from the E., which changed to S.S.W. about 9 a.m. on the 23rd; on the 24th it was still S.S.W., but on the 25th it was from the S.E. The temperatures for the period were as follows:—

Height in Feet (above Surface).	Temperatures in Degrees F.			
	September 22. 8.30 a.m.	September 22. 12 noon.	September 23. 6 a.m.	September 25. 1.30 p.m.
Surface (300 feet above sea-level)	° ...	° ...	° 44	° 72
500	52	...	48	...
1000	...	...	52	67·5
2000	46	...	50	63
3000	42	45·5	46	61
3800	...	...	43	...
4000	38	41·5	44	...
4500	...	39	54	54
5000	45	44	55·5	...
6000	43	...	53	47
6500	44	...	51·5	...

On both 22nd and 23rd September there was haze to the level of minimum temperature, but only a few small clouds. It is seen that between noon on the 22nd and 6 a.m. on the 23rd, while the warm air descended from about 5000 to 4500 feet and increased considerably in temperature, the temperature up to 3000 feet had hardly risen; possibly radiation from the hazy air during the night compensated for any increase of temperature due to descent. A further descent appears to have taken place between the 23rd and 25th; but the whole set of observations is complicated by the fact that the air was in motion throughout the period. The reversed temperature gradient was probably originally developed above the stratus clouds at 5000 or 6000 feet on the 20th and 21st, with a north wind.

Again, between 10 a.m. on 25th April and 12 noon on the 26th, 1916, the level of the top of the haze descended from about the 5000-feet level to the 3000-feet level, and the temperatures were as follows:—

Height in Feet (above Surface).	Temperatures in Degrees F.	
	April 25. 10 a.m.	April 26. 12 noon.
Surface	° 62	° 69
3,000	...	53
5,000	43	56
7,000	47	...
10,000	39	...
11,000	...	36

Thus a general increase of temperature took place except at about 10,000 feet, but the effect of convection from the ground within the lower adiabatic region was probably considerable. The wind was very light from the N.E. on the 25th and from the E. on the 26th, Northern France being under the influence of a shoulder of a large anticyclone centred over Russia.

During the period July 29 to August 1, 1916, there appeared to be general warming due to descent, but the effect of convection with the surface layer maintained the level of the top of the haze at about 4000 to 5000 feet; it reached the former height in the mornings, the latter in the evenings. The wind was very light from the N. or N.E. throughout the period.

In Scotland, when anticyclonic ridges arrive from the west I have often observed an effect apparently due to descent; the clouds which are usually present on the north-eastern sides of anticyclones dissolve away when the wind falls to a calm or light air.

Sir Napier Shaw * has shown that a current from south to north constantly tends to thicken, and can only be steadily maintained if there is an outflow for the excess air; this usually takes place close to the surface towards the west, and helps to explain the descent which appeared to take place behind the anticyclone of September 22–25 already referred to. In the same paper it is suggested that when a southerly current lies to the west of a northerly current, part of the excess air of the former may cross the high-pressure area and supply the reinforcement which is constantly required by the latter. If this takes place it would greatly help to explain the vertical temperature distribution of the northerly currents of anticyclones.

Suggested Explanations of Reversed Gradient.

In considering the origin of anticyclonic stratus and the accompanying vertical distribution of temperature, the difficulty lies not so much in accounting for the relatively high temperatures found at high levels, as in explaining the discontinuity involved in the well-marked reversed gradients.

We will first consider the effect of increasing pressure on cloudy air. The increase of pressure will raise the temperature except where clouds are present; the amount of cloud will be diminished, but those regions where there was originally most cloud will become regions of minimum temperature, with or without cloud. It is true that stratus could be formed in this way only if there was already some degree of stratification in the

* "Principia Atmospherica: a Study of the Circulation of the Atmosphere," *Proc. Roy. Soc. Edin.*, vol. xxxiv, 1914, pp. 77–112.

atmosphere, as regards the quantity of condensed moisture. This, however, does not seem an unreasonable assumption; I have often seen broken and shapeless clouds dissolve in the evening, and the last portions to dissolve are often flat patches all on one level, with a distinctly lower temperature gradient above them than below them. A layer of stratus formed in this way might have a somewhat undulating surface, but any protusions of cloud would be colder than the surrounding air, and would tend to be lowered by convection to the level of the other clouds; so that finally there would be the level surface which is characteristic of all stratus clouds unless they are disturbed from below. This process might result in the formation of more than one layer of stratus; these are in fact often found near the boundary of anticyclones, but the higher layers usually dissolve under the influence of a further increase of pressure, though a layer of haze with relatively low temperature may persist for some time afterwards.

Though layers of cloud or haze with a reversed gradient above them may be produced as described above, further investigation is required to account for such pronounced reversed gradients as that shown in fig. 1, and for the formation of stratus in layers where there was originally no cloud. The effect of the motion of the air falls next to be considered. Mr. C. J. P. Cave has shown that in clear weather the wind round the northern and north-eastern sides of anticyclones normally increases with height.* The pilot balloon ascents with the British Expeditionary Force in France during the summer of 1916 showed on several occasions that the increase in velocity was greater for an interval containing a region of reversed gradient than for an interval in the adiabatic region underneath; on some other occasions the increase in velocity in the region of reversed gradient was limited to the westerly component. Above the unbroken stratus of June 21, 1916, there was a very marked increase of the westerly wind.

The formula for the increase of wind velocity, U, with height, h, is given by Sir Napier Shaw as

$$\frac{1}{\mathrm{U}}\frac{d\mathrm{U}}{dh}=\frac{1}{\theta}\left(\frac{d\theta}{dh}+3{\cdot}42\times10^{-4}\frac{\Delta\theta}{\theta}\Big/\frac{\Delta p}{p}\right),$$

where p is the pressure, θ the absolute temperature, and $\Delta\theta$ and Δp the differences of temperature and pressure between two points on the same level, such that the line joining them is at right angles to the isobars.†

* *The Structure of the Atmosphere in Clear Weather*, by C. J. P. Cave, Cambridge, The University Press, 1912.

† "Upper Air Calculus and the British Soundings during the International Week (May 5–10) 1913," *Journal of the Scottish Met. Soc.*, vol. xvi, No. 30, pp. 167–178.

The first term of the right-hand side is usually negative, but in a region of reversed gradient becomes positive. The second term is nearly always positive as far as the west to east component is concerned, and on the north side of anticyclones would be large and positive above the level of the stratus clouds, as the air at that level is much warmer towards the centre of the anticyclone. It may therefore be expected that on the north side of anticyclones the wind velocity will increase more rapidly at levels where the temperature gradient is low than where it is high.

When warmer air is spreading from the west, as often happens in the northern half of anticyclones, the temperature will rise most where the

wind velocity is greatest. Thus reversed gradients already existing are intensified, and regions where the temperature gradient was originally relatively low may become regions of reversed gradient. This process is perhaps assisted by the action of convection * within those layers where the temperature gradient was originally relatively high; since the wind velocity increases with height and warm air is arriving, the temperature gradient will tend to fall in these layers also; but convection may maintain an adiabatic gradient, and thus retard the increase of temperature at the top of the layer, and so help to produce a reversed gradient above it. Convection within the adiabatic layers would also reduce the increase of wind velocity with height in those layers, and thus the increase within the regions of reversed gradient would become more marked than ever. Whether this explanation is true or not, the fact remains that temperature curves of the type shown on the right-hand side of the figure above are

* Mechanical mixing due to turbulent motion. See Note at end.

commoner than the type which would be obtained from the curve on the left-hand side by adding to the temperature an amount which increased continuously with the height. The figure illustrates how reversed gradients may be produced in this way from the inequalities in the temperature gradient which may always be expected to exist.

Layers of stratus appear often to be produced in this way when there is a rising temperature, especially in winter; near the northern boundary of the anticyclone there may be more than one layer, at different heights. During the spring and summer of 1916 good instances of stratus formed in this way occurred in Northern France on March 31, May 17, and June 21, with a west wind, and on July 21 with a north wind, which had spread round from the west; the figures for some of these instances were given in my earlier paper.

On the southern and western sides of anticyclones in Northern France in summer, solar radiation affected the temperatures to such an extent that no definite conclusions could be reached. There was occasionally an adiabatic gradient up to 10,000 feet. During the winter, in Northern France and Britain, stratus is common with easterly winds. In some cases this may have formed originally with a west wind, but probably it is more often formed as the result of cold air flowing under warm air. Mr Cave shows that very often the east wind increases to its gradient velocity at the height of about a kilometre, and then decreases. Probably the layer of maximum wind velocity is a layer of minimum temperature, and usually contains haze and some cloud.*

On the eastern sides of anticyclones cold air from the Arctic Sea sometimes flows under warm air. An instance of this occurred on June 15th–18th, 1916; on the 16th at 2 p.m. the temperature was 58° F. at the surface, 40° at 3500 feet, 43° at 4000 feet, 38° at 7500 feet, with haze up to 3500 feet and some stratus patches from 3000 to 3500 feet. On that occasion the wind did not decrease in velocity above the clouds, but there appeared to be an increase of the W. to E. component; this was probably due mainly to the horizontal temperature distribution, but perhaps partly also to the fact that the air constantly required to reinforce a northerly current was flowing from the west across the anticyclones, as suggested by Sir Napier Shaw in "Principia Atmospherica." The cirrus motion on the day in question was from the west. If the stratus clouds represented the dividing layer between two bodies of air of widely different origin,

* Probably the turbulence of the lower layers is of importance in these cases; otherwise there would probably be an adiabatic gradient up to the level of greatest wind velocity, and above that a constant low gradient. See Note at end.

the discontinuity in temperature can be explained. It is clear that the method of the formation of stratus described above represents a dividing line between the two methods already described,—*i.e.* the arrival of warmer air at the higher level in mild westerly conditions, and the intrusion of a cold surface current in easterly conditions—and that every intermediate method of formation may take place. In fact, all three methods of formation may be considered as due to the increase of the westerly component of the wind velocity above the level where the clouds form. A cold north-easterly type not infrequently gives way to a milder north-westerly type, with an increase in the quantity of stratus.

Other Factors in the Problem.

The reversed temperature gradient, which may be produced by one of the methods described above, is certainly the primary cause of the formation of anticyclonic stratus, but other factors have also to be considered. Among these are humidity, the difference in wind velocity above and below the clouds, and radiation. In damp weather the stratus cloud may have a thickness of 1500 feet or more, and low cloud masses underneath may be joined to it. Occasionally these may even cause showers of rain under the stratus; in France this sometimes happens both with north and with west winds, both of which are from the sea.

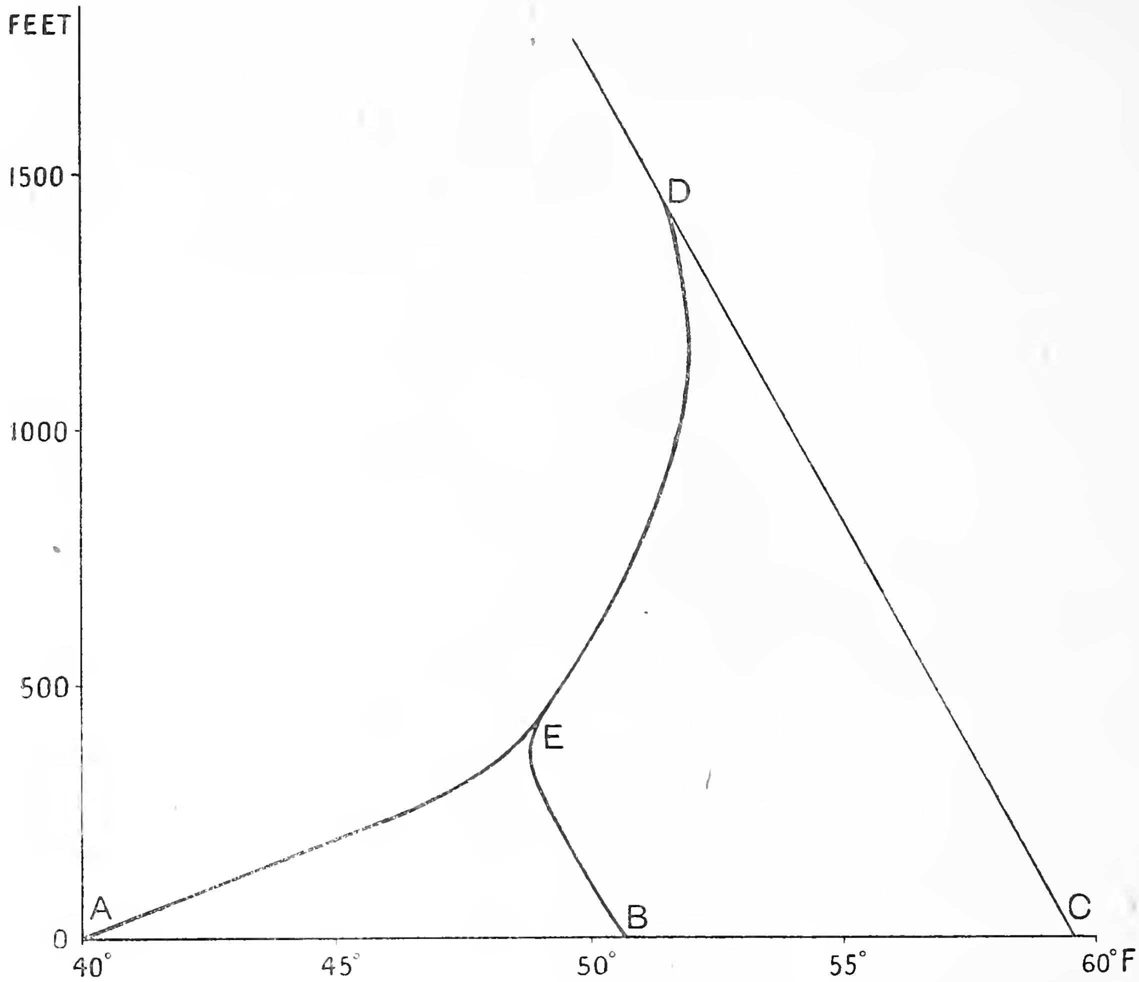
It is undoubtedly the rapid increase in the wind velocity just above the stratus in westerly conditions which produces the wavy effect so commonly seen. Difference in wind velocity may also sometimes cause clouds to form, when otherwise the cold air would lie undisturbed underneath the warm air. In winter, if a frost prevailed, clouds forming in this way would usually soon cause a thaw, as they would prevent radiation from the ground, but allow some of the sun's heat to pass through them. The frost at the beginning of February 1911 appeared to end in this way; on the night of February 2, stratus appeared simultaneously over the greater part of Britain, with a light northerly wind, and a thaw followed next day. Judging from autumn observations, I should say that in winter anticyclonic frosts there are probably often two quite distinct reversed gradients, the lower one within 1500 feet of the ground. If stratus clouds formed below the upper reversed gradient, the lower one would to a large extent die out.

In summer the most important effect of radiation is the absorption of the sun's heat by the clouds, which results in their gradual dissolution. The clouds do not always disappear altogether, as the effect of "cooling by warming" may maintain patches of the clouds; cumuli may also rise from below and spread out on reaching the reversed gradient. Ultimately the heating of the surface layers may result in the disappearance of the reversed gradient, as described in my earlier paper. It is not easy to distinguish the rise of temperature of hazy air due to absorption of heat from that due to convection from the surface; at night in summer the hazy air only seems to fall about 1° F. in temperature, except within 1500 feet of the ground. In the early morning in summer the top of the haze is much better defined than later in the day, and the tops of stratus clouds are usually more level, with the temperature minimum coinciding exactly with that level; these effects are probably partly due to radiation, partly to convection. In winter it is probable that the accumulated effect of radiation in cooling cloudy and hazy layers is of great importance.

Ground fogs may be included under the title of anticyclonic stratus, as they have a great resemblance to ordinary layers of stratus, and every intermediate type of cloud may be found. I have only made a few observations of ground fogs, but these indicate that they persist only so long as the temperature gradient above them is below the adiabatic rate for saturated air. There may be a reversed gradient within the fog, and I have also observed this on a few occasions with stratus clouds. Normally the adiabatic gradient persists to the top of a layer of stratus, or of haze.

On the average I have found that in fine and fairly calm weather in France, about sunrise in September, a reversed surface temperature gradient exists up to about 1000 or 1200 feet, and above 1500 feet the gradient is similar to that of the previous evening; the average rise up to 1000 feet appears to be about 10° F. On the morning of September 15, however, there was a rise of 8° F. for the first 500 feet, and above that an adiabatic gradient; on that occasion the air was drier than usual, and part of the previous night had been cloudy. The diagram on p. 146 shows a typical reversed gradient, C D being the temperature curve of the previous afternoon, A D the curve about sunrise, and B E D the curve two or three hours afterwards. About this time there are often small clouds due to convection at the level of E, and sometimes also flat patches which are probably due to the adiabatic expansion of the layer between E and the surface.

Over the North Sea, particularly in early summer, stratus often forms at the height of about 1000 or 1500 feet above the surface, and occasionally the same development takes place when mild air spreads over the land after a severe frost; possibly a curve of type B E D may be produced as a result of contact with the cold surface, an adiabatic gradient being maintained close to the surface by the turbulent motion of the wind in that region. In the case of a north wind, possibly a curve of type A E D is changed to one of type B E D when the air reaches warmer seas than those over which it has passed. This process may even cause stratus



clouds at rather higher levels, such as those of June 16, already referred to; but until more knowledge is available of the temperatures over the Arctic Sea, such an explanation must be considered as improbable for stratus layers at a greater height than 2000 feet above the surface.*

Summary.

The following is a summary of the more important conclusions which have been put forward in this paper:—

1. *The Nature and Distribution of Stratus in Anticyclones.*

(1) Stratus clouds have an adiabatic temperature gradient below them, and a reversed gradient above them; within the cloud the gradient is usually adiabatic. The same relations hold for well-defined layers of haze.

* See Note at end. The turbulent motion of the lower strata may reduce the temperature of layers even above 2000 feet, and the reversed gradient above the clouds be caused in this way.

(2) On the northern and eastern sides of anticyclones there is nearly always a layer of stratus, or of haze with cloud patches; the height of this layer varies between 3000 and 6000 feet above the surface, but the level is usually the same over a very large area.

(3) Stratus is common in winter on the southern sides of anticyclones.

2. *Descent of Air in Anticyclones.*

In shoulders of high pressure, and on the western sides of anticyclones, the level of a reversed temperature gradient is sometimes observed to descend somewhat.

3. *Some Causes of the Formation of Stratus in Anticyclones.*

(1) Stratus may be formed by the action of adiabatic compression on cloudy air, with the assistance of convection.

(2) Stratus may be produced as the result of the increase of the westerly component of the wind velocity above the layer in which it is formed. This method of formation may be subdivided as follows:—

(*a*) In westerly conditions with an increasing temperature, the increase is more pronounced where the wind velocity is high; layers whose temperature gradient is low may thus be changed to layers of reversed gradient, while convection maintains an adiabatic gradient in other layers.

(*b*) In cold easterly conditions the wind usually decreases above a certain level, and so may cause a reversed temperature gradient immediately above that level.

(*c*) In northerly conditions cold surface air may flow under warmer air from a more westerly point.

In general the increased westerly component is the result of horizontal temperature distribution, but in case (*c*) it may be partly due to the air required to reinforce a northerly current being supplied from the west.

(3) Stratus near the surface may be caused by contact with a cold land or sea.

Nov. 30, 1916.

Note on Turbulent Motion.

Since the above paper was written, some researches by Major G. I. Taylor on the subject of the conduction of heat by means of eddies have come to my notice.* Most of the clouds I have called "stratus" are

* "Eddy Motion in the Atmosphere," *Phil. Trans.*, A series, vol. 215 (1915), pp. 1–26. See also *Report on the Work carried out by the S.S.* "*Scotia*," 1913, London, 1914 (H.M. Stationery Office).

turbulent layers full of eddies, though the tops are approximately at one level. The eddies probably carry heat downwards from the level of the tops of the clouds to some distance below them. This would cause the adiabatic temperature gradient within the clouds, and the reversed gradient above them, which are characteristic of these clouds; and in some cases this may be the only cause of these phenomena. In the case of the more marked reversed gradients other factors are probably of importance, and the two layers of air may often be of widely different origin, as suggested in my paper. The arguments concerning the formation of reversed temperature gradients with west winds remain unaltered if the words "mixing due to turbulent motion" be substituted for "convection." In passing from a turbulent region to an undisturbed stratum, it is to be expected that there should be an increase of the west to east component of the wind velocity, and this is consistent with what I have observed. In some cases the difference of wind velocity may cause the turbulence, and in these cases the clouds might be said to be due to warm air flowing over cold air.

(*Issued separately April* 30, 1917.)

X.—Darwinism and Human Civilisation, with special reference to the Origin of German Military "Kultur." By Robert Munro, M.A., M.D., LL.D.

(An Address delivered before the Royal Society of Edinburgh on March 5, 1917.)

Natural Selection as applied to Man.

In 1859 the scientific world was startled by the publication of Charles Darwin's book on the *Origin of Species*, a work in which he advocated the doctrine that the various species of animals and plants, now inhabiting the globe, have been evolved by means of secondary causes from pre-existing and less differentiated forms of life—a process which he designated by the name of "Natural Selection." This theory is founded mainly on the struggle for existence which all living organisms have to undergo, not only against their natural enemies, but the overcrowding of their own species. The intensity of this contest becomes apparent to any careful observer who takes the trouble to look beneath the surface of his environment at the marvellous activity of the agencies at work in producing the countless living organisms which now tenant the earth. The most striking feature of these genetic operations, besides the astonishing number of seeds, fruits, eggs, young animals, etc., which come so profusely into existence, but of which so few come to maturity, is that those who survive have apparently no higher purpose in life than the propagation of their own species. Notwithstanding the activity of the never-ending agencies by which life is thus kept up as a going concern, the stock of wild animals, *i.e.* animals not in a state of domestication, seldom vary from year to year. The consequence of so profusely overcrowding the environment with the offspring of plants and animals is the premature death of the vast majority. The outcome of a contest under conditions where, of the millions born, only a small percentage can find the means of subsistence, is "the survival of the fittest," and the early decay and death of the weakest—most of which fall a prey to a number of cunning animals who feed on them in all stages of growth. This waste of creative energy can only be defended ethically on the ground that the life-struggle improves the future status of the successful competitors.

According to the doctrine of cosmic evolution, all the varied forms of organic life on the globe take their origin from the lowest known organisms, merely simple cells, occupying the borderland between the animal and vegetable kingdoms. From the cellular starting-point the development of mankind may be traced through a series of intermediate forms, gradually rising in complexity of structure with a corresponding specialisation of function, without the intervention of special acts of creation—as held by the earlier geologists. And here it is of importance to emphasise the fact that these intermediate links, or fossil ancestors, were all more or less different from each other, the later ones coming nearer to the existing types of mankind. The modifications thus effected were chiefly due to climatal alterations, the struggle for the ever-fluctuating means of subsistence, and some obscure variations in embryonic development, the result being the evolution of successors with more highly differentiated organs. On the other hand, concurrent with the natural causes which improved the species there are others which have an opposite effect. The process of degeneration which takes place in course of the life-history of certain animals is thus graphically described by Sir E. Ray Lankester: "Any new set of conditions coming to an animal which render its food and safety very easily obtained, seems to lead, as a rule, to degeneration: just as an active, healthy man sometimes degenerates when he becomes suddenly possessed of a fortune; or as Rome degenerated when possessed of the riches of the ancient world. The habit of parasitism clearly acts on animal organisation in this way. Let the parasitic life once be secured, and away go legs, jaws, eyes, and ears. The active, highly-gifted crab, insect, or annelid may become a mere sac, absorbing nourishment and laying eggs" (*Degeneration*, p. 33).

Among the more specialised of the lower animals the killing of one or more of their own species excites some alarm and fear, but little or no sense of compassion for the victims, nor do they manifest towards each other any altruistic attentions in cases of injury or sickness, except such as take their origin in duties emanating from parentage in the breeding season. Sudden cataclysms in the material world take place regardless of consequences to animal life. The death of thousands of innocent lives from the effects of earthquakes, volcanic eruptions, shipwrecks, explosions, abnormal floodings, etc., is of frequent occurrence, with as little manifestation of remorse as that of a man-eating tiger in the act of devouring his victim. The destruction of life due to the working of natural laws, the torturing of a mouse by a cat to make the poor timid beast a more palatable morsel, and the cruelties inflicted on helpless non-combatants in time of war, may be regarded as parallel acts, so far as the working of the laws of nature is concerned. The question of morality as regards physical suffering is an innovation by man himself, and forms one of the pillars of human civilisation.

The attainment of the erect attitude, which entirely relieved the fore-limbs of their function of locomotion, afforded man the opportunity of entering on a new phase of existence in which intelligence and mechanical skill became his governing factors. With the completion of the morphological changes involved in the attainment of this attitude, the evolution of the present human form, with the exception of some remarkable modifications in the skull and facial bones, was practically complete. Hence, as soon as bipedal locomotion became habitual and firmly secured on an anatomical basis, it does not appear that the osseous characters of the lower limbs would be sensibly affected by any subsequent increase in the quantity or quality of the brain-matter. The important and novel element introduced on the field of human life by the permanent assumption of the erect posture was the use to which the eliminated fore-limbs were put. By substituting for nature's means of defence and self-preservation a variety of implements, weapons, and tools made with their own hands, the subsequent well-being of these novel bipeds became dependent on their ability to interpret and utilise the laws and forces of nature. As time went on they began to recognise the value of the faculty of reasoning as the true source of inventive skill; and hence a premium was put on this commodity. In this way a stimulus to the production of new ideas and new inventions was constantly coming within the scope of their daily avocations, the result of which was a steady increment to human intelligence, and consequently an increase of brain-matter.

The far-reaching consequence of securing food supplies by means of agriculture and the domestication of animals led to more sedentary and social habits. The rise of towns and villages, concurrent with the development of various trades and industries, was but a matter of time—the outcome of which is now a vast system of international commerce. Already the greater portion of the earth capable of being cultivated is converted into gardens and fields whose choice productions are readily conveyed to all the large cities on the globe. The flesh of animals is abundant as human food, but it is no longer necessary to hunt the animals in their primeval haunts. Skin coats, dug-out canoes, and stone weapons are now lineally represented by woven fabrics, Atlantic liners, submarines, aeroplanes, Long Toms and bombs.

The Darwinian theory of Organic Evolution, together with a vast amount of criticism on its social effects on human institutions, has now been before the intellectual world for more than half a century, with the result that it has been accepted, if not even approved, by the leaders of thought in all departments of science and philosophy. At the outset it was strenuously opposed, not only by theologians, but by men eminent in natural science, such as Goodsir and Cuvier. But all these early controversies have now been relegated to the lumber-room of forgetfulness, without, apparently, leaving any bad effects on religion or ethics. The special code of moral and social laws, which became necessary for the guidance and protection of increasing generations, has not hitherto been grossly violated by any of our supposed civilised nations until the outbreak of the present European war, which has disclosed such a recrudescence of barbarous methods on the part of the Germans. That this relapse into primitive barbarism is partly founded on the facts of organic evolution is evident from the works of several German authors, notably those of Treitschke, Nietzsche, and Von Bernhardi. Three of the leading dogmas of the new German Kultur may be traced to that source, viz. (1) the little regard paid to the sacredness of human life; (2) that war, like natural selection, improves the efficiency of a nation's manhood; (3) that social inactivity, as disclosed by parasitic life, leads to the decay of races and nations.

Before, however, further discussing the causes which produced this retrograde phase of ethical culture among the German people, I wish to draw attention to remarks I made in the Friday evening lecture at the British Association held at Southport in 1903, on the earliest phase of human civilisation known to us, viz. that of the Palæolithic races of Europe as a contrast to that of the civilised world of to-day. My chief object in reproducing the following extract, being the concluding remarks of that lecture, is to emphasise the gloomy view I then took of the suicidal tendencies of the later discoveries and innovations into military and social life. It is somewhat remarkable that I should have then correlated the failure of food with the break-up of modern civilisation, as cause and effect, and that to-day scarcity of food is the most probable element which will decide the fate of the moral world now in a death struggle with destructive barbarism:—

"Were it possible for one of our Palæolithic ancestors to sit in judgment on the comparative merits of the two civilisations, his verdict would probably be something like this:—You have utilised the forces of nature to a marvellous extent, and thereby secured the means of greatly increasing the number of your fellow-creatures; but, at the same time, you have multiplied the sources of disease and misery. The invention of money has facilitated the accumulation and transmission of riches among the few; but it has impoverished the many and supplied incentives to fraud, theft, and all manner of crime. Patriarchal establishments have given way to social organisations governed by laws founded on moral sentiment and ethics; but their by-products are extreme luxury and extreme poverty. Hence, to support the weak and the unfortunate is no longer a matter of charity, but a moral obligation. Notwithstanding the size of your asylums, hospitals, and almshouses, they are always full and always on the increase. You have formulated various systems of religion, but whether founded on the principles of fetichism, polytheism, or monotheism, they are all more or less permeated with contradictory or controverted creeds and dogmas. National sport, as practised with weapons of modern precision, can only be characterised as legalised killing of helpless creatures. To shoot pigeons suddenly liberated from a box at a measured distance, or to slaughter overfed pheasants, or even to stalk semi-domesticated deer driven to the muzzle of a rifle—all, of course, within sight of a luncheon-basket,—is a poor substitute for the excitement and field incidents of the chase in Palæolithic times. With no better weapons than a wooden spear, or lance, tipped with a pointed flint, and a small dagger of bone or horn, we had, not infrequently, to encounter in mortal combat the mammoth, rhinoceros, cave-bear, or some other fierce and hungry animal which, like ourselves, was prowling in quest of a morning meal. Such scenes had many of the elements of true sport, and, being essential to our existence, were of daily occurrence. Moreover, from the standpoint of modern ethics, our method of sport put the combatants on something like a footing of equality, and gave our opponents a fair chance of escape. Nor did we in the least infringe the principles of modern societies against cruelty to animals. We cultivated physical qualities by the natural exercise of the senses; and so personal prowess was the distinguishing prerogative of our heroes. Thus we acquired the experience, skill, strength, and courage of practised athletes—qualities which left no room for fear or cowardice. With us brain-power passed almost directly from the generating organ to the muscles of the administrator; with you it has to pass through a complicated system of accumulators liable to various degrees of leakage; and it is this leakage which often sucks dry the blood-life of your civilisation. Finally, the permanence of your civilisation remains to be tested by the touchstone of time. For civilisations, like the genera and species of the organic world, have their life-histories determined by as fixed laws as those that govern the parallelogram of forces. To cosmic evolution, under which, to a large extent, our race flourished, you have superadded altruism, which means the survival of the weak as well as the strong. But altruism will continue to be a living force among civilised communities, only so long as your present and prospective food supplies hold out. For, after all, the essential problem of your social existence is how to provide food for an ever-increasing population. Whenever these necessaries of life become inadequate to meet the demands of the inhabitants of this globe, then your boasted civilisation comes to the end of its tether, and the only solution of the crisis will be a recurrence to the cosmic law of the survival of the fittest." (*The Times*, September 12, 1903. See also *Proceedings of the Royal Society of Edinburgh*, vol. xxv, p. 125.)

This outburst of prophetic pessimism was at the time regarded by current critics as an imaginative and far-fetched improbability. Who would have then thought that, in little more than a decade, one of the most scientific, cultured, and domesticated people in Europe would so unexpectedly adopt some of the worst phases of primitive barbarism in their ruling conduct towards their unoffending and helpless fellow-creatures? Nor was this action the result of a sudden impulse of revenge for oppressive conduct on the part of any of their neighbours. On the contrary, it seems to have been a deliberate policy, founded by the rulers of Germany with far-reaching foresight and secretly worked out for more than a decade.

Human Civilisation and German Military "Kultur."

In the code of savagery, under the name of Kultur, which has guided the conduct of the German soldiers in the present European war, with the approval of the Kaiser and his military authorities, it is maintained that in war the end justifies the means. Officers and men are taught the doctrine of "frightfulness" with all its associated horrors. Passenger and even hospital ships, with their living freight of helpless men, women, and children, are suddenly and without warning sent to watery graves by skilled sailors concealed in submarines in the wide ocean. Innocent non-combatants are bullied, beaten, and put to death; houses robbed and demolished; villages burnt to the ground; churches desecrated; and historic works of art destroyed — all by way of reprisals on the plea that their country is blockaded, or that some obscure civilian had fired on their soldiers, or given intelligence to the enemy, as if such acts, even if they were true, justified wholesale murders of innocent people.

This revolting violation of the recognised rules of civilised warfare among the great nations of the world was, in the first place, deliberately devised and concocted by the Kaiser and his military advisers, for the purpose of carrying out their project of world domination; but how it came to receive the sanction of the entire German population remains an enigma to most people. We are now well on into the third year of this unparalleled war, and as it progresses the Germans seem to be adding to the enormity of their crimes by deporting civil populations from their homes to work as slaves in mines and other industries, and even to compel them to fight in the trenches against their own countrymen. In endeavouring to trace the causes of this novel and deplorable phase of German civilisation, we have to take into account the overweening conceit and vanity of the German people, in so readily adopting the unctuous idea that they are the chosen people of God to uphold and spread the principles of a beneficent civilisation throughout the world. The arrogant assumption that they are superior to all other nations in science, literature, arts, and social culture is unblushingly taught, not only by leading statesmen, but by learned professors in all departments of knowledge. The idea of world power, which has obsessed the German people since 1870, has so saturated their intellectual and moral faculties with a haze of what has been called "neurotic insanity," that they are incapable of seeing things, not even the current events of the present war, in their proper perspective. No other explanation can be given of the document issued, shortly after the outbreak of the war, to the civilised world (especially Americans) by ninety-three distinguished professors from all parts of the German empire, justifying the extirpation of the enemy and the destruction of his property for the sole purpose of creating terrorism.

But it is the doctrines taught in the works of German writers, especially Treitschke, Nietzsche, and Von Bernhardi, that are mainly responsible for so effectually educating the mass of the German people, that the entire population now seem to be strong defenders of the new Kultur, even of the latest phase of the submarine warfare, whose mission is to sink without warning any vessel that dares to come in sight of the British Isles. The following notes will give some idea of the dangerous teaching that has helped to effect this astounding transformation in the social culture of a hitherto peace-loving and civilised nation:—

Heinrich von Treitschke, a professor of history at Berlin from 1874 till his death in 1890, and a member of the Reichstag from 1871, became latterly notorious as an advocate of Prussian militarism, a policy which gave him great influence in Chauvinistic circles at Berlin. In all his writings he expresses a bitter hatred of the British people, characterising them as over-rich, and "living in the lucky aloofness of a wealthy island."

England is depicted as a worn-out, old-fashioned, degenerate power, and the shameless champion of barbarism and international tyranny. He is said to have been largely responsible for the anti-British feeling which became so pronounced in Germany during the late years of the nineteenth century. The dominant tone of Treitschke's writings, especially since the war of 1870, is the insolent arrogance with which he gave utterance to questionable statements without any reference to authorities, and the self-satisfied confidence he had in German Chauvinism. According to him, the State was the ultimate authority in all things, and the final appeal was not to a court of justice, but to the sword. Little notice was taken of his works in this country till the outbreak of the present war.

Friedrich Wilhelm Nietzsche (1844–1900) was from 1869 to 1879 professor of classical philology in the University of Bâle. For the next ten years he became a wandering invalid, dashing off these brilliant essays which have recently brought his name so prominently before the British public. During the last eleven years of his life he was insane, and died in an asylum. As an atheistic freethinker, a disbeliever in the cardinal virtues of Christianity, and a strong opponent of Prussian militarism, his philosophical teaching lacks both consistency and consecutiveness. Its keynote is the suppression of the weak and the poor—a doctrine which he seems to have imbibed from an imperfect knowledge of the Darwinian theory. Treitschke and Nietzsche differed widely in their political opinions. The former strongly upheld State supremacy, the latter denounced it. But yet they agreed in making the "will to power" their dominant watchword. Both loved war, and both hated England on account of her wealth and colonial possessions.

The now notorious book of General Friedrich von Bernhardi—*Germany and the Next War*—was published in the spring of 1912, and in the following year a supplementary volume appeared, which its translator entitles *Britain as Germany's Vassal*. The views expressed in these volumes may be briefly summarised as follows:—

(1) War was the great civilising influence in the world, and consequently it was the duty of every State to be ready for it.

(2) Germany, having risen superior to all other countries in science, literature, arts, commerce, and warlike achievements, was destined to be the world-Power of the future. To attain this object the German nation was to prepare secretly for a great European war; and when the necessary military preparations were ready, they were to advance a plausible *casus belli* and break up the Triple Entente by crushing, in the first place, France,

then Russia, and then Britain, before their enemies had time to bring their combined forces into action.

(3) Britain was a non-progressive nation in the world's history, and hence, in the interests of civilisation, she ought to be deprived of her vast colonial possessions, so as to give place for Germany's expansion as the coming world-Power.

(4) To rectify the current policy of the Triple Entente, which kept the Germans hedged in and circumscribed on all sides—a policy inspired by jealousy of the rapid strides they were making in wealth, commerce, and social culture—a European war was a necessity on political grounds. It was therefore the imperative duty of all the Germanic races and peoples to utilise every available means to counteract the hypocritical machinations of her enemies, and protect her legitimate interests. "Might was right," and war was to decide the destiny of mankind under the beneficial sway of Pan-Germanism.

(5) England is described as Germany's principal enemy; but as a decrepit, decadent Power she could be easily overpowered after France and Russia had been disposed of. Meantime, the highest aim of diplomacy was to keep the British people neutral in the coming Armageddon.

Notwithstanding the vast importance and significance of these astounding revelations by a highly placed general in the German army, they were utterly ignored by the rulers of this country till the actual outbreak of hostilities opened their eyes to the reality of Bernhardi's lucubrations. The striking parallelism between his recommendations as to how the war was to be carried out and the actual methods by which it has been conducted, together with the cynical frankness with which the downfall of Britain is prophesied, leaves no doubt as to the sources from which the Germans derived their present war policy. No argumentative evidence is required to show that the real responsibility for this deplorable war lies at the door of the Kaiser and his military advisers.

The plea of the War Lord and his outwitted diplomatists, now so frequently reiterated since the failure of their original plan of campaign, is that they are the innocent victims of a long-hatched conspiracy on the part of the Entente—a plea so manifestly false that it hardly requires any contradictory comments. In submitting their grievances—whatever these may have been—to the arbitrament of war so gigantic as that now raging throughout Europe, it is evident the authors are conscious of having stupidly blundered. Now their underhand diplomacy and lack of real statesmanship are unfavourably criticised by the neutral Powers all over the world. The imprudence of Bernhardi in forewarning Europe so openly of Germany's warlike intentions is not more astonishing than the unconcern displayed by our British statesmen. Although the influence of the civilian populations of the various countries engaged in this brutal war can avail but little in either stopping or mitigating its atrocities, it becomes the bounden duty of all to ponder seriously over the many object-lessons it has brought before us, because a considerable recasting of the international laws by which civilised nations have hitherto been governed has to be faced as soon as the abnormal fungus of Prussian militarism gets its *coup de grâce*. Our present code of international ethics is founded, not alone on experience of cosmic laws, but on deeply ingrained moral and altruistic sympathies of which mankind hold a monopoly.

The phenomena and agencies of the organic world, on which Treitschke, Bernhardi, and others justify their brutal disregard of human life, belong to the domain of primeval savagery from which modern civilised people have sprung, and of which condition they still retain traces. But what has the *modus vivendi* of our lower antecedents to do with the ethics of present-day civilisation, which has been laboriously constructed by successive increments of social and moral improvements during the long ages which have rolled past since man started on his human career? He has learned, however, that cosmic forces must be studied and obeyed by all mankind as well as by all other animals, because the more we know of their operations the greater our skill in controlling and utilising their results for the benefit of humanity. To trace moral laws to their primary rootlets, and to purge our beliefs of superstitions generated in days when scientific methods were too feeble to detect the errors on which they were founded, cannot but further the highest interests of human civilisation; for we are reminded in a thousand ways that success in life depends on strict obedience to the forces on which the universe is governed. Bernhardi, in founding his constantly reiterated statement that war is essential for the well-being of a nation on the principles of natural selection, forgets that both the methods of human civilisation and of German Kultur are at variance with the operations of the organic world. Modern civilisation is founded on the altruistic and moral inventions of mankind, and are directed not so much to the survival of the fittest as to the fitting of as many as possible to survive. Nature cuts off the weak, deformed, and stragglers in life's battle: war selects its victims from the physically strong and healthy of the nation's manhood, and leaves a feeble remnant to perpetuate the race. That racial degeneration is a consequence of social inactivity and an easily acquired supply of the necessaries of life is quite true, but war is not the only means by which such defects can be remedied. The keynote of German military Kultur is that the end justifies the means, and that these means are determined by the dogma that "Might is right." On the other hand, the dominant factors of cultured humanity are truth, justice, toleration, self-restraint, altruism, and "Honour bright." The former code of ethics is destructive, and leads to primitive barbarism; the latter is constructive, and aims at improving social life by adding further ekes to the hoary structure of human civilisation.

Opinions condemnatory of the bloodthirsty methods of German military Kultur since the commencement of the present war are not the monopoly of a few philosophical cranks, but the deliberate judgment of mankind. How this misanthropic virus should have so thoroughly permeated the minds of a whole nation as to overshadow and obliterate all moral and altruistic doctrines, is still a profound mystery. On this phase of the subject Lord Curzon, in presiding over a meeting at the Royal Hospital, Chelsea, spoke as follows:—

"If the war has taught them one thing more clearly convincing than another, it was that the literature and art of peoples were the true expressions of their character and psychology. The nation that produced a corrupt and debased literature was itself debased and corrupt. The Germans had for years been preaching the doctrine and cult of militarism, and infusing the virus of organised brutality into the minds of the German people. Books which preached the doctrine of brute force, material arrogance, and world-wide dominion were circulated and read by hundreds of thousands of people in Germany. All this had gone on for years, and then suddenly war broke out, and we found the revelation of this spirit in the acts and conduct of the people. In a moment, what the sculpture, the literature, and the art of the people had been, so they were on the battle-field. We now know there was no crime so black, no atrocity so revolting, no abomination so foul that it was not perpetrated by the German people, and perpetrated with pride and self-glorification." *

Since this earth became a suitable habitat for organic life, countless millions of living organisms have come to a premature end in consequence of sudden cataclysmic changes in the physical world; but no catastrophe under this category up to Anno Domini 1914 has taken place comparable to that produced by the hand of man, during the present European war, on account of the enormous destruction of life and property it has entailed. Let us hope that this attempt of the Germans to establish world-power on the basis that might is right will be defeated, and that freedom, justice, and

* From *The Times*, November 30, 1916.

the rights of small nationalities will triumph. But whatever the issue of the present world struggle may be, one thing is certain, that the reconstruction of the moral legislative system of the civilised world becomes an urgent necessity. And, even in the event of reaching a satisfactory agreement on that point, there remains to be solved the still more delicate problem: By what agency are its behests to be enforced? Is the *vis a tergo* to be moral suasion or physical force?

(*Issued separately April* 30, 1917.)

XI.—The Adsorption of Sulphur Dioxide by Charcoal at −10° C.

By **A. M. Williams**, M.A., B.Sc., 1851 Exhibition Scholar of the University of Edinburgh, 1911–14. *Communicated by* Professor James Walker, F.R.S.

(MS. received December 2, 1916. Read February 5, 1917.)

The object of this research was to find how the heat evolved on the adsorption of a vapour varied with the amount adsorbed. Work in this direction had already been done by Chappuis,* and more recently by Titoff.† Neither of these experimenters carried out the adsorption till the adsorbent was even approximately "saturated," with simultaneous measurement of the heat effect. To do this, then, was the aim of the author.

The adsorbent selected was blood charcoal (puriss. Merck). Similar charcoal had been employed before by the author and no further purification by means of acids, etc., was attempted. Its relative density had been found to be 1·628. The adsorbate selected was a vapour whose liquid boiled not far from room temperature, namely, sulphur dioxide with boiling-point at −10·1° C. The sulphur dioxide was prepared by redistillation of the liquid from a siphon. The gas was passed through sodium sulphite solution to free it from traces of sulphur trioxide, and through bulbs containing concentrated sulphuric acid to dry it thoroughly. It then passed into a spiral glass worm surrounded by a freezing mixture. The exit from the condensing flask led to another sulphuric acid bubbler and finally to a caustic alkali solution which prevented the gas escaping into the air.

Instead of a Bunsen ice calorimeter such as was employed by Chappuis and by Titoff, the author used a gas calorimeter after the manner of Dewar‡ and of Estreicher.§ The dimensions of the calorimeter were determined with reference to a large vacuum vessel possessed by the University College, London, Chemical Laboratory. This vessel was used to contain the freezing mixture in which the calorimeter was immersed. The calorimeter is shown in fig. 1. It consists of a Dewar's vessel, silvered save for a narrow

* *Wied. Ann.*, xix (1883), p. 21.

† *Zeits. f. physik. Chem.*, lxxiv (1910), p. 641. See also Joulin, *Ann. Chim. phys.* (5), xxii (1881), p. 398; and Favre, *Ann. Chim. phys.* (5), i (1874), p. 209.

‡ *Proc. Roy. Soc.*, lxxiv, A (1904), p. 122, and lxxvi, A (1905), p. 325.

§ *Zeits. f. physik. Chem.*, xlix (1904), p. 602, and *Anz. Akad. Wiss. Krakau*, 1910 A, p. 345.

strip up the side. The outer wall is extended into a ground-glass collar with a side tube at the base. A ground-glass stopper fits into the collar and the combination is sealed with mercury. Through the stopper which is also evacuated passes a capillary ending inside the calorimeter in a bulb which holds the charcoal. The gas to be adsorbed is led in through the capillary and the heat of adsorption causes evaporation of the calorimetric liquid — sulphur dioxide — and the gas evolved is collected through the side tube.*

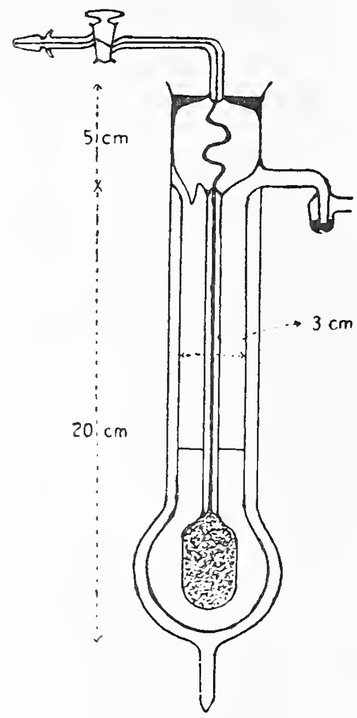


Fig. 1.

In Estreicher's experiments the rate of leak of sulphur dioxide from his vessel due to heat passing inwards was about 20 c.cm. of gas per minute, but only 5 per cent. of the total gas collected—2000 c.cm. In the author's case the leak was never more than 4 c.cm., and generally about 2 c.cm., per minute, but in some measurements it was as much as 80 per cent. of the total gas collected. Estreicher collected the gas evolved in a large aspirator containing water with a layer of white oil in which the dioxide is not so readily soluble as in water. The author was compelled to substitute mercury, as the gas was sufficiently soluble in any oil at his disposal to interfere seriously with the collection. An oil gauge was then used to indicate the pressure. It was now found that the leak appeared steadily to diminish. This was traced to cooling of the liquid sulphur

* The calorimeter was made to the author's design by Baumbach of Manchester. The first calorimeter which subsequently broke was resilvered and evacuated by the author, the second by the maker. The rate of leak was less in the first calorimeter. The author is indebted to Dr Whytelaw-Gray for many hints as to manipulation here.

dioxide owing to rapid evaporation into the atmosphere, and was obviated by joining on a mercury bubbler to the delivery tube from the calorimeter. When the mouth of the calorimeter was closed the gas bubbled through this and the atmosphere above the liquid soon became entirely gaseous sulphur dioxide, and the temperature remained steady at that, corresponding to (say) 0·5 mm. more than the barometric pressure.*

The greatest difficulty was the adjustment of the temperature of the freezing mixture round the vacuum vessel. The observations were taken during a spell of hot weather, which no doubt contributed to the steady rise in temperature of the mixture from $-10°$ C. The variation of the leak due to radiation inwards showed that the liquid sulphur dioxide was very sensitive to changes in temperature of the surrounding bath. The leak varied also with the level of the dioxide in the calorimeter. In itself the leak was small, but when the measurement was spread over two or three hours it soon amounted to a considerable percentage of the volume of the gas evolved during the experiment. As a result, observations of the leak had to be taken from (say) two hours before the gas was let in to be adsorbed till some time after the adsorption was adjudged complete. The author is convinced that the substitution of a freezing mixture for a bath of the calorimetric liquid itself—as used by Dewar—was a grave mistake, and impaired the ease and accuracy of the observations.

The apparatus made and devised by the author for the study of adsorption is shown in fig. 2, and is essentially a constant-volume apparatus. It consisted of a large bulb whose one end was attached to the ground-glass join to which fitted the charcoal bulb part of the calorimeter. The other end joined on to a capillary tube passing into ordinary quill tubing containing a fine tip of blue glass. From below this ran a side tube with a tap connecting with the mercury pump and the gas reservoir.†

The main tube continued downwards, then upwards parallel to the blue-point tube, and constituted with its connections a manometer, pressure readings being taken in the mirror scale behind. The volume from the blue point (marked b in the figure) to the tap of the charcoal bulb was found from the weight of mercury of known temperature required to fill it; and the volume from the tap to the bulb (including the hole in the tap) was similarly found before joining on the filled bulb to the capillary.‡

* It would have been possible by means of a tap to adjust the mercury so that the pressure was always (say) 78·0 cms., and so keep the dioxide at a fixed temperature.

† A two-way tap above the charcoal bulb would have been simpler.

‡ A change in volume on joining of 0·1 c.cm. would affect all the calculated values of the amount adsorbed 0·03 per cent., and the change must have been less.

The volume of the charcoal was known from its density and weight after complete evacuation.

The complete evacuation of the charcoal bulb took six days. The initial evacuation was accomplished by the water pump, the final by the mercury pump when the bulb was immersed in a bath of the vapour of boiling sulphur. At the end the last traces of adsorbed gas were removed by means of the second evacuated charcoal bulb immersed in liquid air. The evacuation was a very tedious process. The condition of the charcoal in

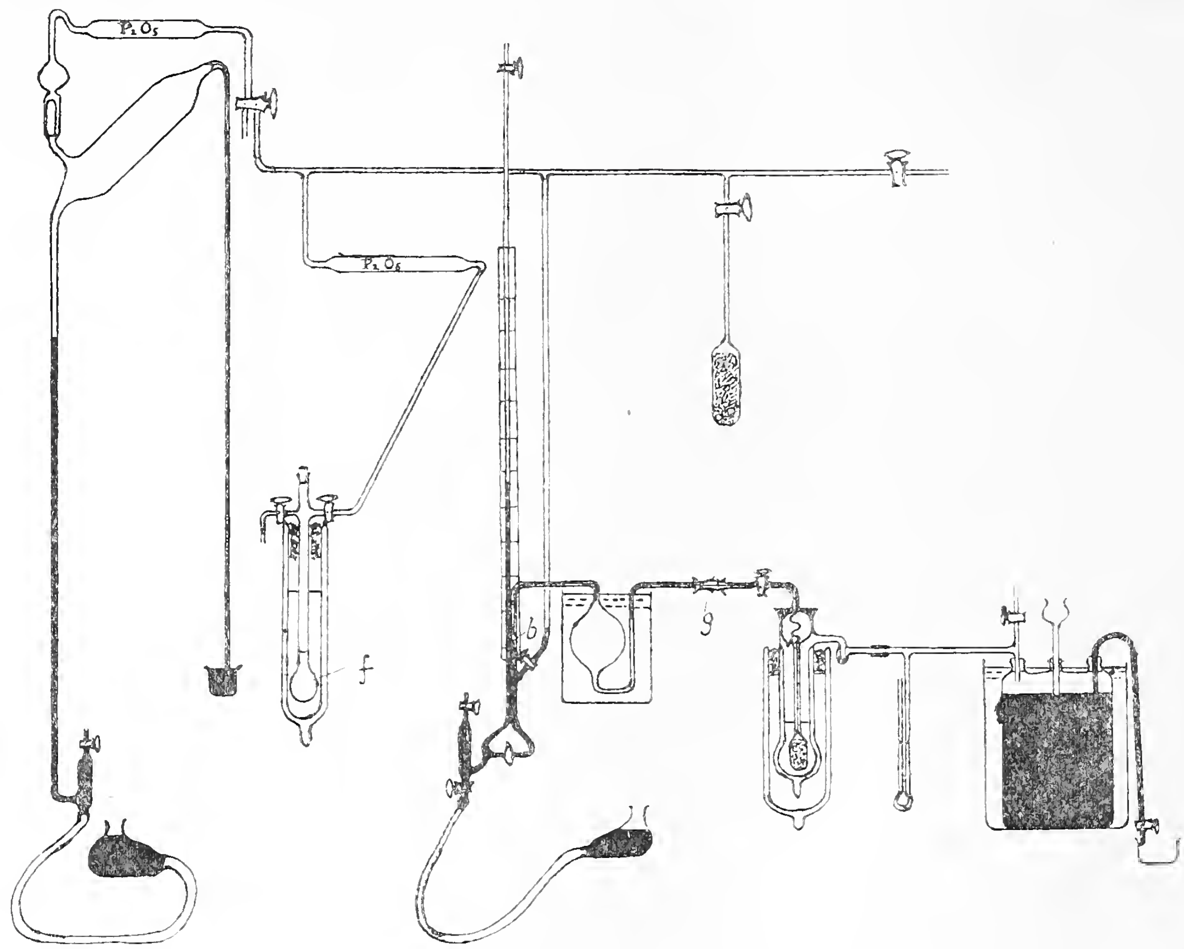


Fig. 2

a state of fine powder added to the difficulty, as it was borne upwards by the air leaving it. To prevent it going through all the tubes and connections a tiny plug of cotton-wool was placed in the head of the ground-glass join. The fine powder occasionally stuck in the capillary tube, and had to be brought back to the bulb by repeated tapping, or even, after disconnection at the join, by vigorous knocking on the bench. Though the volume of the bulb was 17 c.cm., it was "filled" by 4·5 gm. charcoal (density 1·63).

Sulphur dioxide to be adsorbed was kept in the bulb *f*, immersed in a freezing mixture of alcohol and carbon dioxide snow. When freshly made up this mixture froze the sulphur dioxide. The air was removed by freezing the dioxide in liquid air and evacuating the bulb, the solubility of the gas evolved being then 99·99 per cent. When the gas was required the mercury in the manometer was lowered, the requisite taps opened, and the freezing mixture removed until the manometer indicated the approximate pressure required. The taps were then closed, the freezing mixture replaced, and the mercury adjusted to touch the blue tip again. Mercury was also run through the tap behind the manometer before it was shut. The tap between the manometer tubes was now closed and that above the charcoal bulb opened, and the adsorption proceeded at *constant volume*. At the close the mercury was first roughly adjusted, and then on opening the closed manometer tap the pressure was accurately found with the mercury touching the fine blue point.

The heat of the adsorption vaporised some of the liquid dioxide around the charcoal bulb. This gas passed through the mercury bubbler and caused an increase in pressure, indicated on the oil gauge. Mercury was run out of the reservoir to keep the pressure constant. The volume thus collected every ten minutes was found by weighing to the nearest gram. A small correction was made for the volume of liquid dioxide and the thistle funnel tube; and another when necessary was applied for change in temperature of the initial volume of gas in the aspirator. The security of the joins, corks, etc., in the collection apparatus was regularly tested by pouring in mercury through the thistle funnel and finding if the amount recovered on adjusting the pressure was the same plus the natural leak during time of adjustment. When the gas was not being collected it passed through the tap shown above the reservoir into absorption vessels.

A short initial run was made, but owing to the fact that the liquid in the calorimeter had fallen below the shoulder of the charcoal bulb the heat effect registered was much less than it should have been. Two other complete runs were made, taking the pressure of adsorbed gas up to atmospheric. The comparison of the two sets of readings gives some idea of the accuracy of the calorimeter and method employed.

It is now necessary to indicate what calorimetric quantity is measured—that is, what is meant in this case by the "heat of adsorption." There may be defined, following Donnan,* three *isothermal* heats of adsorption, viz.: (1) *equilibrium*—the adsorption proceeds so that the vapour phase is constantly in equilibrium with the adsorbed phase; (2) *at constant pressure*—the gas at constant pressure, but not necessarily at equilibrium pressure is picked up by the adsorbent; (3) *at constant volume*—the total volume of the system is constant, and the gas is adsorbed with a fall of pressure. The heat effect registered by the author's arrangement was the *isothermal*

* The treatment by Freundlich, *Kapillarchemie*, pp. 107–11, is confused and erroneous.

heat of adsorption at constant volume plus the heat introduced by the gas entering the adsorption chamber at a temperature higher than that of the chamber.

Sulphur dioxide does not accurately obey the law $pV=RT$. In correcting, the author made use of the data obtained by various observers.

(1) In correcting for temperature he used the data given by Leduc * in the deduced form

$$V_t = V_0(1 + \cdot 00396t),$$

when t is near 20° C. and the pressure is constant and not more than two atmospheres.

(2) The pressure correction was calculated from the data of Jacquerod and Pintza,† and of Baume.‡ From their measurements of density at 0° C. a table was drawn up giving the values p' to be read for p the observed pressure, so as to make the gas conform to the equation $p'V = \text{constant} = RT/1 \cdot 023$. Specimen values thus found are given below.

p	76·00	60·00	40·00	20·00	10·00	0·00
p'	76·00	59·70	39·55	19·65	9·79	0·00

The application of the above corrections refers all volumes of gas to actual c.cms. at N.T.P., where the density of the gas is 1·023 times that calculated from the simple gas law $pV=RT$.

The observations are now given in the tables below, where

p = final pressure in cm. mercury;
a = c.cm. adsorbed measured at N.T.P.;
Δa = " " " " during an experiment;
β = c.cm. evolved from the calorimeter and measured at N.T.P.;
Δt = excess temperature of gas in reservoir over gas in adsorption bulb;
P = vapour pressure in cm. mercury.

As after a certain amount was adsorbed the rate of adsorption fell considerably, the stopcock was closed after forty minutes or so, and such non-equilibrium readings are given in brackets. In the course of work, however, opportunity was allowed for the adsorption to proceed for twenty-four hours or longer, and such readings are not enclosed in brackets. After the stopcock was closed a very small additional amount of gas would be adsorbed, but too small to affect appreciably the calculations.

* *Comptes Rendus*, cxlviii, p. 1173 (1907).
† *Ibid.*, cxxxix, p. 129 (1904).
‡ *Journal de Chim. phys.*, vi, p. 1 (1908).

This was shown by the fact that the rate of leak with stopcock open and shut was the same within the limits of concordance of readings taken. The quantity of heat measured never exceeded 95 gram calories, and in general varied between 55 and 30 calories. 80 per cent. of the effect was recorded in the first half-hour and 95 per cent. in the first hour.

Table I.—Mass of Charcoal = 4·428 gm. in Vacuo.

p.	α.	$\Delta\alpha$.	β.	Δt.	P.
0·10	94·6	94·6	165·5	28·6	76·1
(0·48) 0·36	240·0	145·1	219·9	28·6	”
0·92	387·7	147·7	215·3	29·1	”
(1·67) 1·64	547	159·0	213·1	28·6	”
(3·11)	705	157·7	204·7	28·9	76·3
(5·09) 4·50	858	151·3	186·4	29·3	”
(7·18)	1003	144·7	194·7	27·9	”
(10·32)	1137	134·2	178·5	27·5	”
(14·14) 13·64	1254	115·4	152·6	27·7	”
(24·63)	1432	178·6	226·4	28·9	76·5
(39·73)	1559	126·6	156·5	29·9	”
(54·02) 53·30	1637	77·3	91·9	30·4	76·4
(65·32)	1711	74·3	76·1	30·9	”
(72·76) 72·05	1815	104·1	106·6	31·2	”
(76·96) 75·50	1948	126·9	120·9	29·9	77·0
(77·73) 76·40	2065	114·8	110·6	30·5	”

Table II.—Mass of Charcoal = 4·428 gm.

p.	α.	$\Delta\alpha$.	β.	Δt.	P.
0·14	119·2	119·2	197·4	27·7	75·9
0·45	269·8	140·2	208·5	29·7	76·5
(1·06) 1·05	421·1	161·3	223·8	28·6	76·2
(2·67)	676	255·3	333·6	28·5	”
(5·29)	923	246·3	317·5	28·4	76·8
10·80	1176	253·5	...	...	”
(22·10)	1402	226·0	287·5	29·6	76·7
(43·82) 43·60	1584	181·8	226·2	30·6	76·6
(66·32)	1724	139·8	151·4	29·1	76·5
(73·20)	1837	113·1	113·8	30·5	”
(75·79) 75·41	1961	124·6	118·1	31·1	76·6
77·12	2104	143·0	...	...	77·1

In Table III are given the equilibrium values of p and α, taken from Tables I and II. The values of p are referred to $-10·1°$ C., when the pressure of the calorimetric dioxide would be 76·0 cm. mercury, using the fact that for a given α, p/P is nearly constant. α is referred to one gram charcoal.

Table III.—Mass of Charcoal = 1·000 gm.

p.	a.	p.	a.
0·10	21·4	0·14	26·9
0·36	54·2	0·45	61·0
0·92	87·6	1·05	95·1
1·64	123·5	10·68	266
4·48	193·8	43·2	358
13·59	283	74·9	443
53·0	370	76·0	475
71·7	410		
74·6	440		
75·4	466		

In Table IV are found the calorimetric results. a is a mean value of a during the adsorption, and is usually calculated from a logarithmic interpolation formula. Save at the beginning, this value does not greatly differ from the arithmetic mean of the initial and final amounts adsorbed in any interval. Instead of expressing the heat of adsorption in calories per c.cm. adsorbed, the ratio λ_v/λ_i is given of the isothermal heat of adsorption at constant volume to the "internal" heat of vaporisation of liquid sulphur dioxide. Thus with a usual notation

$$\lambda_i = (V_G - V_L)T\frac{dP}{dT} - P(V_G - V_L).$$

Again, since

$$\beta\{\lambda_i + P(V_G - V_L)\} = \lambda_v \Delta a + C_v \,.\, \Delta t \,.\, (\Delta a + \Delta a'),$$

where C_v is the specific heat at constant volume of the gas and $\Delta a'$ is the gas entering the adsorption chamber and not adsorbed, we have

$$\frac{\lambda_v}{\lambda_i} = \frac{\beta}{\Delta a}(1{\cdot}0934) - \Delta t\left(1 + \frac{\Delta a'}{\Delta a}\right)({\cdot}001436).$$

To obtain λ_v in terms of calories per c.cm. absorbed, the ratio must be multiplied by 0·250. Estreicher from the vapour pressure measurements of Matthias and Chappuis calculated that the heat of vaporisation of 1 gm. sulphur dioxide at $-10°$ C. was 95·7 cal. Favre had found 88·2 cal. Estreicher as the mean of three very concordant measurements of volumes evolved on heating gives 95·9 cal. as the observed heat. Later, by *weighing* the gas after absorption by a liquid he gives 95·3 cal. at $-11°$ C. The author from Regnault's vapour pressure figures calculated 95·5 cal. at $-10°$ C., assuming with Estreicher the density of the dioxide to be normal, and using the equation

$$\lambda = RT^2\frac{d \log P}{dT}.$$

Correcting for the departure from the gas law, we get 93·2 cal. at −10° C.* Taking 93·2 cal. as the heat of vaporisation of one gram of the dioxide at −10°, the factor 0·250 is easily calculated.

TABLE IV.

a.	λ_v/λ_i.	a.	λ_v/λ_i.
8·0	1·870	9·9	1·770
33·9	1·616	40·7	1·576
68·9	1·554	75·7	1·469
103·5	1·426	120·4	1·395
137·8	1·381	177·5	1·370
176	1·306	291	1·350
210	1·443	337	1·318
242	1·417	374	1·142
270	1·407	402	1·049
303	1·349	409	0·989
338	1·310		
361	1·257		
378	1·075		
398	1·026		
424	0·993		
454	1·012		

The results of Table III are presented in graphical form in fig. 3. The curve is of the same type as that obtained by Trouton † for the adsorption of water vapour by flannel. In the present case the upward bend occurs much later than $p/\mathrm{P} = \cdot 2$, the inflexion occurring about $p/\mathrm{P} = \cdot 63$ and the bend being pronounced only after $p/\mathrm{P} = \cdot 9$. The gradient does not appear to be infinite near $p/\mathrm{P} = 1$.

The calorimetric observations are shown in fig. 4. It will be seen that a *minimum* heat of adsorption is indicated. Such a minimum heat of adsorption is shown in Titoff's observations on the adsorption of ammonia by charcoal, and in Chappuis' results for the adsorption of sulphur dioxide by charcoal, etc. The author's curve then passes through a maximum and drops to run parallel to the horizontal axis, with the heat of adsorption equal to the heat of condensation, as might well be expected when $p = \mathrm{P}$. The inflexion in the last portion of the curve occurs near $a = 370$, while in fig. 3 the inflexion is near $a = 365$, so there appears to be some close connection between the final portion of the two graphs.

We may regard the heat of adsorption as the sum of at least two effects, namely, loss of potential energy of the adsorbate in yielding to the attrac-

* Mills, *Journ. of Phys. Chem.*, x, 1 (1906), calculates $\lambda = 94 \cdot 7$ cal. at −10° C.

† *Proc. Roy. Soc.*, lxxviii, A, p. 412.

tion of (1) the adsorbent and (2) the gas already adsorbed. The first effect will probably be represented by a function which diminishes to zero as more and more is adsorbed, if only because the sphere of molecular action of the adsorbent is attained. The second effect, on the other hand, may be represented by some function which will increase with the amount adsorbed, since the attracting "layers" will thicken. In the case of a vapour these "layers" will be able to exist "outside" the adsorbing

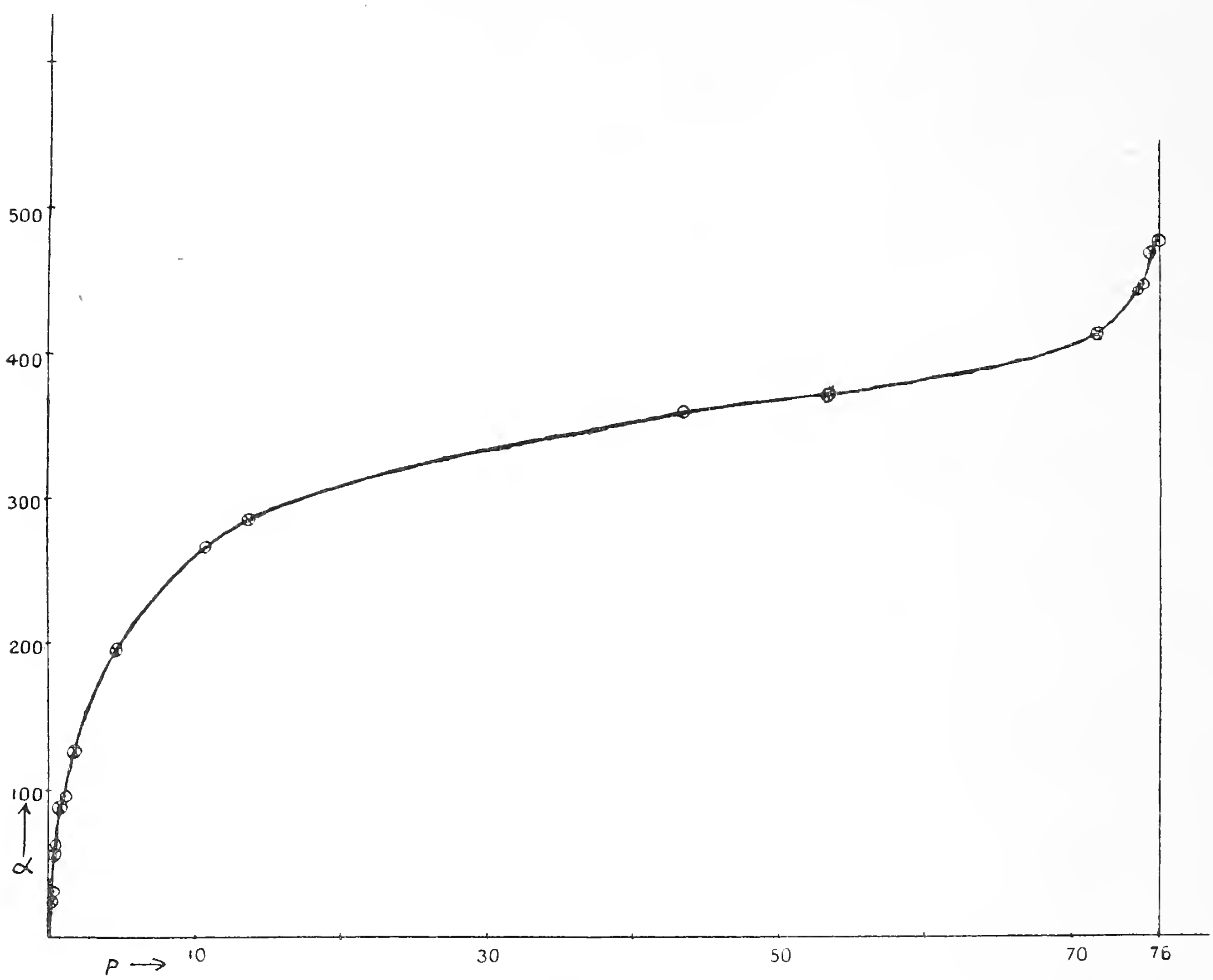


Fig. 3.

particles, so as to give the same final heat effect as condensation of vapour into liquid. (This effect, represented by the last portion of the curves above, will be absent in the case of a gas above its critical temperature, since the "outside layers" will never attain sufficient density to attract much of themselves after the sphere of molecular action of the adsorbent is passed.)

The sum of a steadily diminishing function and a steadily increasing function may quite well present a minimum, as found in the observations above.

It seems to the author that there should be a third term in the expression for the heat effect, namely, one representing the change in energy of the *adsorbent.* The Pouillet effect,* which is simply the heat given out on adsorption of a liquid by a powder, has been attributed to the compression of the "films" of liquid in contact with the powder. But in view of recent work on compression, the heat effect appears too large to be attributable solely to this cause. Thus the greatest loss of energy in compressing ether was found by Bridgman † to be 14 per cent. of the heat of condensation; and moreover, after a certain compression the total heat effect diminished. In the adsorption studied above (*cf.* also Titoff and Chappuis)

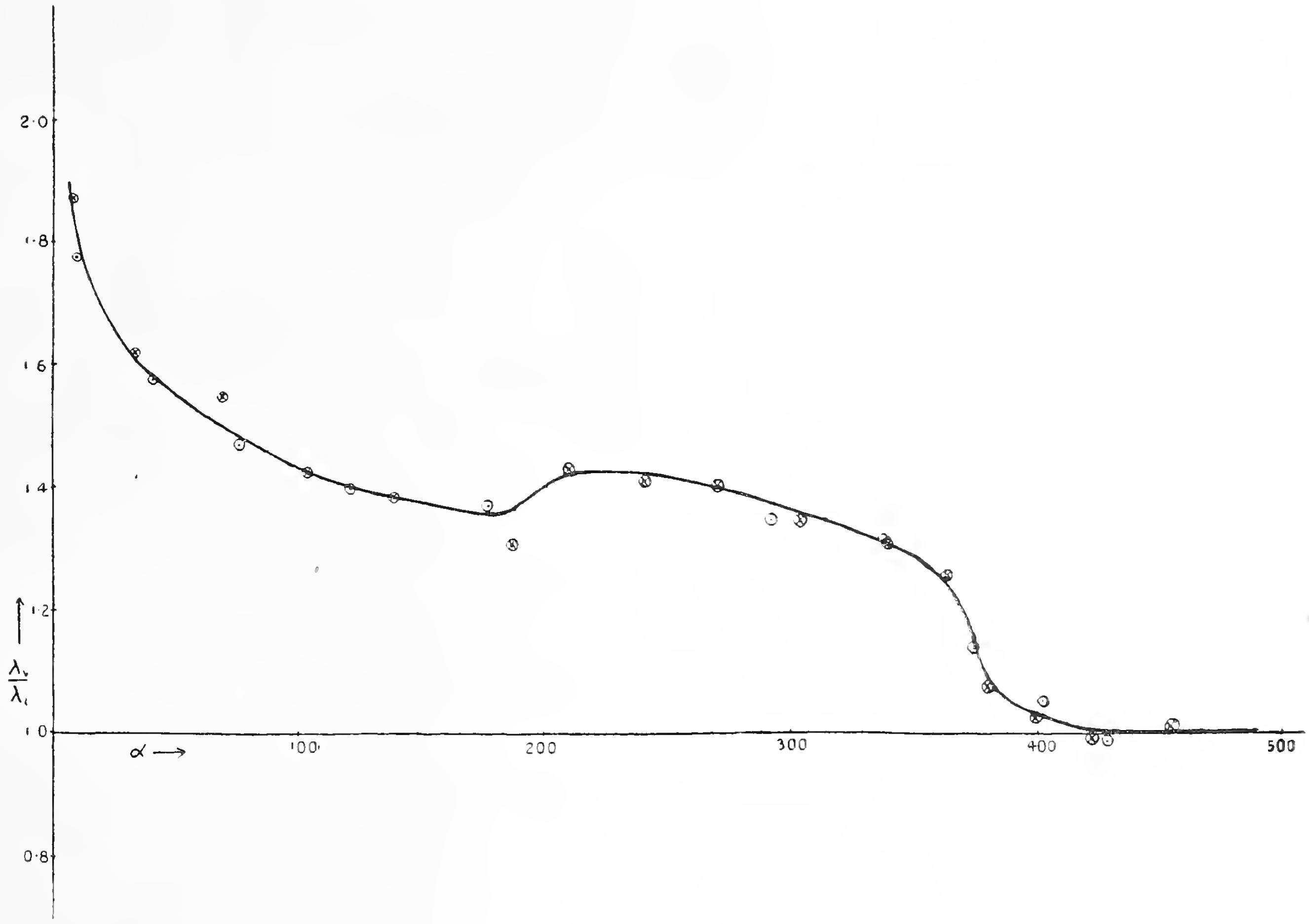


Fig. 4.

the initial heat effect exceeds by more than 80 per cent. the heat of condensation, and only after a great amount is adsorbed does it fall much below an excess of 40 per cent. It seems, therefore, unlikely that it is only the adsorbate which loses a considerable amount of energy on adsorption, and it looks very probable that initially at any rate the adsorbent loses energy on what may be not *contraction* but *expansion of surface* in embracing the adsorbed particles. ‡

* See Williams, *Trans. Far. Soc.*, x, p. 167 (1914).

† *Proc. Amer. Acad.*, xlix, 1 (1913).

‡ *Cf.* Donnan's negative surface tension of colloids.

Summary.

(1) The adsorption of sulphur dioxide by blood charcoal at $-10°$ C. was studied, and measurements were taken of the amount adsorbed, the pressure, and the isothermal heat of adsorption at constant volume.

(2) The adsorption isotherm is a typical vapour adsorption curve, and runs the same course as that found by Trouton for the adsorption of water vapour.

(3) The heat of adsorption curve passes through a minimum and a maximum, and finally runs parallel to the adsorption axis. A tentative explanation of this is offered.

In conclusion, the author would seek to express his thanks to Professor F. G. Donnan, F.R.S., for his advice given in the course of these experiments which were performed in 1913–14 in the Chemical Laboratories, University College, London.

(*Issued separately June* 7, 1917.)

XII.—The Hurlet Sequence in the East of Scotland and the Abden Fauna as an Index to the Position of the Hurlet Limestone. By Peter Macnair, F.G.S. (With One Plate.)

(Read December 18, 1916. MS. received February 2, 1917.)

CONTENTS.

I. Introduction.

In this communication an attempt is made to correlate the different members of the Lower Carboniferous Limestone Series of the East of Scotland with what has been chosen as the type section at Hurlet, near Paisley. For close on fifty years Scottish geologists have been endeavouring to correlate the Lower Carboniferous Limestones of Fife and the Lothians, and in general the Carboniferous Rocks of Scotland, with the Hurlet Section, but up till the present time the various attempts have only met with a partial success, the reason for this being, that our knowledge of the Hurlet section has by no means been either sufficiently detailed or accurate enough to admit of comparison with other widely separated Scottish sections. The important question also arises as to what extent the Hurlet Section can be considered as typical of the group of deposits as a whole. The recent revision of the Glasgow district by the officers of the Geological Survey and the publication of their work gave a renewed impetus to the study of these limestones, and within the last few years we have arrived at a much more accurate conception of the stratigraphical and palæontological characteristics of the Lower Limestone Series of the West of Scotland, and we are therefore in a much better position to institute a comparison with the East of Scotland sections than we were before. One of the most remarkable and important of all the fossiliferous horizons in this series is that found in the Hurlet Alum Shale just below the Hurlet Limestone. This faunal association is characterised by a brachiopod bed, a lamellibranch bed, and a bone bed. The fauna has been discovered on the same stratigraphical position over a large area in the West of Scotland, and a similar fauna has been found below the Hurlet Limestone at Charlestown and Abden in Fife, and on the same horizon in the Bilston Burn and at Aberlady Bay and Dunbar in the Lothians.

II. Classification of the Carboniferous Limestone Series of Scotland.

The Carboniferous Formation in the Midland Valley of Scotland has been divided into the following four main subdivisions:—1, The Coal Measure Series; 2, The Millstone Grit Series; 3, The Carboniferous Limestone Series; and 4, The Calciferous Sandstone Series. The Carboniferous Limestone Series has been further subdivided into the following three groups:—

Carboniferous Limestone Series.
- (3) Upper Group of three or more limestones, with thick beds of sandstone and coal.
- (2) Middle Group, containing several workable seams of coal, with Clayband and Blackband Ironstones associated with sandstones and shales, but with no limestones.
- (1) Lower Group, comprising several beds of limestone with sandstones, shales, some coals, and ironstones.

The group of limestones and associated strata to be discussed in this paper are practically all included in the lowest of the above three subdivisions, though some of the lower members are classed by the Geological Survey with the Calciferous Sandstone Series. We regard this line of demarcation as of a purely conventional character, and do not consider that it marks any important stratigraphical or palæontological break in the Carboniferous System.

III. Historical Review.

The Geological Survey began work in Scotland in 1854, and the map of the Hurlet district was published in 1878, but, unfortunately, the Explanatory Memoir for this sheet was never issued. An examination of the map, however, shows that the Hurlet Limestone position was correctly identified over the whole area included in the map from Howood in the south-west to Campsie in the north-east. When this district was being mapped the Hurlet Limestone and underlying coal were being worked all over the area, notably at Hurlet and Campsie, for the Alum Shale, and the characteristic features of the limestone and underlying sediments were such that their identity over the whole area included in the map could not possibly escape notice.

It may here be pointed out that the officers of the Survey have placed on this map the outcrops of the limestones lying below the Hurlet or Main Limestone, but at that time these do not appear to have been very accurately differentiated. One of the main points which this paper seeks to show is that almost immediately beneath the Hurlet Limestone, though generally separated from it by a variable thickness of strata, there occurs a limestone known as the Blackbyre Limestone, which, because of certain faunal and lithological characters, affords a much surer datum line than the Hurlet Limestone for correlation purposes, as it can be traced over wide areas where the Hurlet Limestone is not conspicuous. It appears to have been often confounded with the Hurlet Limestone.

The Survey Memoir in explanation of Sheet 32 appeared in 1861,* and describes the Lower Limestone Series as developed in the neighbourhood of Edinburgh, including the districts to the north and south of Bathgate and to the south-west of Edinburgh. In the former district Sir A. Geikie adopted the West Kirkton Limestone as the boundary line between the Carboniferous Limestone and the Calciferous Sandstone Series, and maintained that the Petershill Limestone is the continuation of the 8-foot limestone exposed in the Almond Section about 300 feet above the latter. In later publications he abandoned † this view, and correlated the Petershill Limestone with the Hurlet Limestone. In the course of this paper we hope to be able to show that the original correlation was the correct one.

The explanation to Sheet 33 appeared in 1886, ‡ and includes the two important areas of the Lower Limestone Series exposed on the Haddington coast at Aberlady and Dunbar. In these two areas the boundary line between the Carboniferous Limestone and Calciferous Sandstone Series was drawn at the lowest limestone exposed in the coast sections, which has

* *Memoirs of the Geological Survey, Scotland: Explanation of Sheet* 32, *Edinburghshire and Linlithgowshire.*

† *Memoirs of the Geological Survey, Scotland: Explanation of Sheet* 31 (1879), p. 20.

‡ *Ibid., Explanation of Sheet* 33, *Haddingtonshire.*

been named the Lower Longcraig Limestone. We believe the third limestone from the bottom, known as the Upper Longcraig Limestone, to be the equivalent of the Hurlet Limestone.

The map of West Fife, including the Lower Limestones of the coast section between Kinghorn and Kirkcaldy, was published in 1867, but was not then accompanied by an Explanatory Memoir. In this region the Seafield Tower Limestone exposed on the shore section south of Kirkcaldy is taken as the Hurlet Limestone, and is regarded as being upon the same horizon as the Charlestown Limestone on the north shore of the Firth of Forth.

The Survey Memoir in Explanation of Sheet 22 appeared in 1872,* and deals with the Lower Limestone Series in North Ayrshire in the district around Beith and Dalry, but in this Memoir no attempt has been made at a detailed correlation of the Ayrshire Limestones nor of their relationship to the Hurlet Series.

The Explanation to Sheet 23 appeared in 1873,† five years previous to the issue of the map of the Hurlet district, and it deals with the members of the Lower Limestone Series so well developed in the East Kilbride, Strathavon, Lesmahagow, and Carluke districts. A partial attempt is made to correlate the various limestones included within the sheet with one another, such as the Main Limestone, the Birkfield or first Calmy Limestone, the Kingshaw Limestones, and the Calderwood Cement throughout the district shown in the map, but no detailed attempt is made to correlate them with the limestones of the Hurlet district. It is important, however, to note that the Main or Hawthorn Limestone is taken as the equivalent of the Carluke Main, the Hurlet Main, Main Limestone of Wilsontown and Strathavon districts, Hawthorn Limestone of Muirkirk, and No. 2 Limestone of Lesmahagow district.

The Explanation to Sheet 31 ‡ was published in the year 1879. This sheet shows the Lower Limestone Series stretching along the southern part of the Campsie and Kilsyth Hills as far as Stirling, and also from the neighbourhood of Addiewell through the Bathgate Hills to Linlithgow on the east. In the first of these areas, as far east as Corrieburn, near Kilsyth, the Hurlet Limestone was correctly identified. In the Explanation to Sheet 32 the Tartraven and West Kirkton Limestone was taken as the boundary line between the Carboniferous Limestone and Calciferous

* *Memoirs of the Geological Survey, Scotland: Explanation of Sheet* 22, *Ayrshire, Northern District, and Parts of Renfrewshire and Lanarkshire.*

† *Ibid., Explanation of Sheet* 23, *Lanarkshire, Central Districts.*

‡ *Ibid., Explanation of Sheet* 31, *Lanarkshire N., Stirlingshire S., Linlithgowshire W.*

Sandstone Series. But in Sheet 31 and the accompanying Memoir that limestone is placed in the Calciferous Sandstone Series, and the thick limestone of Petershill is regarded as the equivalent of the Hurlet Limestone.

In *The Geology of Central and Western Fife and Kinross*, published in 1900,* the Charlestown Limestone is taken as the equivalent of the Hurlet Limestone in the upper reaches of the Firth of Forth, and the Seafield Tower Limestone as its easterly extension in the neighbourhood of Kirkcaldy. This is the same view as that already expressed in the map published in 1867.

In *The Geology of Eastern Fife*, published in 1902,† the White Coral Limestone seen on the shore near Coal Farm, St Monans, is taken as the equivalent of the Hurlet Limestone. We consider this to be the Blackbyre Limestone of the Hurlet district, the limestone lying immediately above this position exposed on the shore at Partan Crags, St Monans, being in the Hurlet Limestone position.

Such is a brief summary of the conclusions arrived at by the officers of the Geological Survey in their original survey of the Lower Carboniferous Limestone Series of Midland Scotland. A detailed examination of these maps and memoirs will show that they were unable to advance any very definite stratigraphical or palæontological evidence in support of the reasons which they had for adopting their Hurlet Limestone datum line in these widely separated localities. It will also be seen that after having fixed upon their Hurlet Limestone horizon everything above it in the Lower Limestone Series was grouped as the Hosie Limestones, and no attempt was made either to differentiate them out or to correlate them in detail.

Within the last few years a revision of the Carboniferous Limestone Series has been undertaken by the Geological Survey, and in this revision most of the original determinations of the Hurlet datum line have been maintained, with the notable exception that the West Kirkton-Tartraven Limestone has again been taken as the equivalent of the Hurlet Limestone.

The Geology of the Glasgow District was published in 1911,‡ and in this Memoir we have the first official account of the Hurlet Section, as it has already been pointed out that the Explanatory Memoir to Sheet 30 had never been published. This Memoir embodies the results of the

* *Memoirs of the Geological Survey of Scotland, District Memoirs: The Geology of Central and Western Fife and Kinross.*

† *Ibid., The Geology of Eastern Fife.*

‡ *Ibid., The Geology of the Glasgow District*, 1911.

recent revision, and we find that it adopts the Hurlet datum line as given in the map of 1878, which in Sheet 30 we regard as being everywhere correct. A valuable advance, however, is made in this Memoir, as the two important limestone horizons lying below the Hurlet Limestone, known respectively as the Blackbyre Limestone and Hollybush Limestone, are differentiated out and correlated with other exposures in the district. The overlying limestones all formerly grouped together as the Hosie Limestone are also differentiated out and correlated throughout the area under consideration.

In a series of papers published in the *Transactions of the Geological Society of Glasgow** the present writer, following up the work of the Survey, has been enabled to show that the various members of the Lower Limestone Series can be traced over large parts of Ayrshire, Renfrewshire, Lanarkshire, and Stirlingshire, not only by lithological characters and stratigraphical relationships, but also by the fact that the various fossiliferous horizons are characterised by certain faunal associations or groupings indicative of similar widespread physical conditions of deposition. These we believe to obtain over a large part of the West of Scotland, and it is the purpose of this present paper to attempt to trace them into the East of Scotland. Perhaps the most important of these faunal associations is that which occurs on the horizon of the Hurlet Alum Shale between the Blackbyre Limestone and the Hurlet Limestone. In one of the papers just referred to, the value of this fossil group in the establishment of the Hurlet datum line has already been discussed in some detail.†

To Mr A. Macconochie‡ belongs the merit of first correlating the Bilston Burn Section south-east of Edinburgh with that exposed between Kinghorn and Kirkcaldy on the other side of the Firth. One of the strong points in his correlation is the identification of the faunal association found below one of the lower limestones of the Bilston Burn with that occurring below the first Abden Limestone of Fife, a correlation

* "On the Distribution of *Posidonomya corrugata*, Eth. jun., in the Carboniferous Limestones of the Glasgow District," by P. Macnair and H. Conacher, *Trans. Geol. Soc. Glasgow*, vol. xiv, p. 309, 1912 "The Stratigraphy of the Limestones lying immediately above the Calciferous Lavas in the Glasgow District," by P. Macnair and H. Conacher, *ibid.*, vol. xv, p. 37, 1913. "The Hurlet Sequence in North Ayrshire," by Peter Macnair, *ibid.*, vol. xv, p. 200, 1914. "The Hurlet Sequence in North Lanarkshire," by Peter Macnair, *ibid*, vol. xv, p. 387, 1916.

† "The Hurlet Sequence in North Ayrshire," *Trans. Geol. Soc. Glasgow*, vol. xvi, p. 200, 1914.

‡ *Memoirs of the Geological Survey of Scotland: The Geology of the Neighbourhood of Edinburgh*, p. 611.

which we consider to be a very great advance in accurately determining the Hurlet datum line in the East of Scotland, and which we cordially homologate, and further correlate it with the Hurlet Alum Shale horizon of the West of Scotland. Another important recent advance was that made by Dr Crampton* when he correlated certain of the limestones exposed at Dunbar with those seen at Aberlady Bay, and the limestones in these two sections with certain limestones seen on the shore at St Monans on the north side of the Firth of Forth.

IV. The Hurlet Type Section and other Renfrewshire Sections.

We now pass to describe a series of sections beginning at Hurlet and passing by the way of the Campsie Hills, East Kilbride and Carluke to the East of Scotland at Cobbinshaw. Hereafter the sections in the Lothians and Fife will be considered and compared with those in the West of Scotland.

As it is only within the last few years that anything like a detailed knowledge of the character and sequence of the limestones in the Hurlet district has been arrived at, it is necessary that a summary of the most recent results should be given here so as to facilitate comparison with the other sections which it is the purpose of this paper to describe :—

			Ft.	In.
H.	27.	Top Marine Band, *Lingula, Nucula,* and small *Producti*	3	
	26.	Shales	10	
	25.	Sandstones	20	
	24.	Shales, with Johnstone Clayband Ironstone near top	100	
G.	23.	Top Hosie Limestone (Calderwood Cement), *Posidonomya corrugata*	1	
	22.	Lillies Coal and Oil Shale	2	
	21.	Shale	10	
	20.	Limestone, encrinital	1	6
	19.	Shales	12	
F.	18.	Main Hosie Limestone	4	
	17.	Hosie Sandstone	50	
	16.	Shales	100	
E.	15.	Blackhall Limestone, crinoidal on top, entomostracan below	6	
	14.	Shales, with Househill Clayband Ironstones at top	150	
D.	13.	Hurlet Limestone	3	
	12.	Alum Shale	6	
	11.	Coal impure with pyrites	5	
C.	10.	Baldernock Limestone (entomostracan lime with fish remains)	2	
	9.	Dark Blaes		6
	8.	Impure Limestone with rootlets	10	

* "The Limestones of Aberlady, Dunbar, and St Monans," by C. B. Crampton, *Trans. Edin. Geol. Soc.*, vol. viii, p. 374, 1905.

			Ft.	In.
B.	7.	Blackbyre Limestone, Brachiopods and Corals	10	
	6.	Coal	1	
	5.	Hollybush Sandstone, with Lady Ann coal in the middle	150	
	4.	Shale, with limey band full of *Rhynchonella pleurodon*	8	
A.	3.	Hollybush Limestone, *Productus latissimus* and *Lithostrotion*	4	
	2.	Coal	1	2
	1.	Sandstones, marls, and fireclays	...	

In the above table is shown the principal limestone horizons exposed in the neighbourhood of Hurlet. At the bottom of the section is the Hollybush Limestone, formerly wrought near the farm of that name. This limestone has a thickness of 4 feet, and is the lowest well-marked calcareous horizon to be met with in the Hurlet district. As we have elsewhere pointed out, it is characterised by the presence of bands of *Lithostrotion* and solitary corals with large *Producti,* including *P. giganteus, P. latissimus, P. punctatus,* and *P. semireticulatus.* Other brachiopods present are *Spirifer trigonalis* and *Seminula ambigua.* In the calcareous shale overlying the limestone there is a bed which is simply a mass of crushed specimens of *Rhynchonella pleurodon.**

At a distance varying from 10 to 20 fathoms above the Hollybush Limestone lies the Blackbyre Limestone, which can be seen in the railway cutting near the farm of that name a mile to the south-west of Hurlet. At this locality the upper part is composed of small encrinite fragments, while the lower is slabby and almost entirely composed of small brachiopods, principally *Productus longispinus* Sow. and *Spirifer duplicicosta* Phill. It is also important to note that plant remains are numerous, especially in association with the brachiopod bed in the limestone.† The Blackbyre Limestone was also formerly worked at Blackhall, where the evidence goes to show that it presents three distinct palæontological types, the lower part being characterised by the presence of large brachiopods with bands of *Lithostrotion* ; the middle part being slabby, with immense numbers of small brachiopods; while the upper part is fine grained, approaching the estuarine type, which was worked at Gallowhill, near Arkleston, about the beginning of the last century, and which yielded to Dr Scouler the type specimens of *Dithyrocaris testudinea* and *D. tricornis.* ‡

The Baldernock Limestone can be seen only in the Arkleston railway cutting, in which it occurs in three seams full of entomostraca

* See *Trans. Geol. Soc. Glasgow*, vol. xv, p. 206.

† *Ibid.*, p. 207.

‡ *Ibid.*, vol. xvi, p. 46.

and fish remains. Stratigraphically and palæontologically, it is clearly the equivalent of the Baldernock Limestone of the type locality near Campsie.

The next limestone horizon in ascending order is perhaps the most important in the whole group, namely, the Hurlet Limestone. It cannot now be seen in the neighbourhood of Hurlet, the best exposure in the district being that seen in the railway cutting at Arkleston. At this locality the limestone has a maximum thickness of 3½ feet, and is underlain by about 6 inches of alum shale much altered by contact with an intrusive sill. Below the sill comes some 18 feet of shale and fireclay, which rests upon the Baldernock Limestone. Unfortunately the Alum Shale fauna has not yet been identified in the Hurlet district owing to the paucity of exposures. In the Arkleston Section the shale is exceedingly thin and much altered by contact with the sill, and though I have searched it, I have not yet been successful in finding any fossils.

The next limestone in ascending order is that known as the Blackhall Limestone. It can be examined at Jenny's Well, near Blackhall, Paisley, where it is seen to be in several beds. The upper part, which is about 24 inches thick, is crinoidal, and contains marine shells; while the lower part, which consists of alternations of bituminous shale and limestone over 4 feet in thickness, is entirely composed of ostracods and fish remains, the ordinary marine fossils being absent. These characteristic features of the Blackhall Limestone have been found to hold good over a large area in the Glasgow district.

Immediately on the top of the Blackhall Limestone there occurs a bed of shale, which in the Glasgow district and at Thorntonhall, East Kilbride, is exceedingly fossiliferous, and has yielded a somewhat striking faunal assemblage. It was first described from the South Hill of Campsie by the late Dr John Young, where it lies immediately above the Blackhall Limestone, and it is fairly well exposed near the top of Baldow Glen, where all the characteristic fossils may be gathered. It has been found occupying a similar stratigraphical position over a large part of Renfrewshire and North Ayrshire, and it has been identified to the north of Glasgow, from Campsie to Kilsyth, and over a large part of North Lanarkshire, including the East Kilbride, Lesmahagow, and Carluke districts. We shall also show that it can be traced on the same stratigraphical horizon in the East of Scotland.

The next limestone horizon in ascending order is that known as the Main Hosie Limestone. The strata which lie between the Blackhall Limestone and the Main Hosie Limestone consist for the most part of shales in the lower division, and sandstones in the upper. The outcrop of massive encrinital limestone which is seen in the Levern Water just below the Cross Arthurlie Street Bridge, Barrhead, is regarded as the Main Hosie Limestone.* We shall see that this forms a well-marked fossiliferous horizon all over Central Scotland.

The next calcareous horizon in ascending order is the Top Hosie Limestone, sometimes known as the Calderwood Cement, because of its typical development in that district. Between it and the underlying Main Hosie comes the Lillies Coal and Oil Shale. The Top Hosie is nowhere exposed in the Hurlet district at the surface, but it has been found in the ironstone pits sunk by Messrs Baird at Corkerhill. It is a dark calmy limestone with strings of small gasteropods and brachiopods well preserved. The distinctive feature of this horizon is the presence above and below the limestone of shales crowded with *Posidonomya corrugata*, with which is associated a very characteristic faunal group.

The Top Marine horizon of the Lower Limestone Series is represented in the West of Scotland by a highly fossiliferous blaes, the upper part full of *Lingula*, the lower rich in small *Producti*. It is nowhere exposed at the surface in the Hurlet district, but was found in the pit at Corkerhill just referred to. It has also been long known to occur in the pits at Inkerman, where the calcareous shale passes into a limestone. To the north of Glasgow, in the Campsie district, this horizon is represented by a bed of limestone containing small brachiopods associated with *Ceriopora interporosa*. The limestones on this horizon are abundantly charged with the marine alga *Spirophyton cauda-galli*, which also occurs in great quantities in the associated sandstones.

Such, then, is a summary of the Hurlet type section, giving the principal characteristics of the limestone horizons from the Hollybush Limestone at the base to the Top Marine Band. It has been shown that there is unfortunately a great paucity of exposures in the district, and but few continuous sections of any length. For the purpose of strengthening and widening our conception of the Hurlet Sequence in Renfrewshire we briefly describe certain other sections.

An important section showing an almost continuous sequence from the Hollybush Limestone up to the Hurlet Limestone is exposed at Nethercraigs, $3\frac{1}{2}$ miles to the west of Hurlet. At this locality the strata have been thrown into a small basin, which is faulted against the volcanic rocks. The outcrops of the Hollybush, Blackbyre, and Hurlet Limestones appear in three more or less continuous rings, with the last-mentioned limestone

* *Trans. Geol. Soc. Glasgow*, vol. xv, p. 210.

in the centre. There can be no doubt as to the identity of the respective limestones, and the section throws much additional light upon the nature of the sequence between the Hollybush and Hurlet Limestones. Unfortunately the strata immediately underlying the Hurlet Limestone is not exposed in this section, and the existence of the Alum Shale fauna cannot be determined.

Another important section is that exposed in the burn at Meikle Corseford, Howood, about 8 miles to the west of Hurlet. Immediately below the Hurlet Limestone in this section comes the Alum Shale position, which has yielded a somewhat remarkable faunal association, and which seems to be of considerable significance. The fossils, which are exceedingly numerous, though somewhat crushed, are principally brachiopods and lamellibranchs. Of the former, the most common are *Schizophoria resupinata* and *Productus semireticulatus.* The most abundant lamellibranchs are *Sanguinolites abdenensis*, *Streblopteria ornata*, *Actinopteria persulcata*, and *Myalina verneuili*, the brachiopod band when clearly defined lying above the lamellibranch band. Now we shall show that these four lamellibranchs occur in association on exactly the same stratigraphical horizon in the Campsie district and at Corrieburn, also that they can be traced from Cobbinshaw through the Bathgate Hills to Charlestown on the Firth of Forth, and thence into the east of Fife and over the Lothians as far as Dunbar. The stratigraphical significance of this fauna in the Alum Shale will be discussed in more detail in a future part of this paper.

Another important section confirming the sequence in the Lower Limestone Series of this district is that seen in the Gryffe Water, between the Bridge of Weir and Crosslee. These sections have been described by me in detail in a series of papers* published in the *Transactions of the Geological Society of Glasgow*, to which the reader is referred. The Alum Shale is now covered up, but fragments containing the fauna can be picked up on the bings.

V. Sections between Campsie and Kilsyth.

In tracing the Hurlet Sequence into the Fife district and the East of Scotland we proceed in the first instance by the way of those sections which lie between Campsie and Kilsyth, and, as we shall see, they afford a remarkable amount of evidence bearing upon the correlation of the Lower Limestone Series in the East of Scotland with that of

* *Trans. Geol. Soc. Glasgow*, vol. xv, p. 215.

the Hurlet district. The first section that we have to notice is that exposed on the South Hill of Campsie, as seen in Craigenglen and Glen Wynd.

The South Hill of Campsie lies between Lennoxtown on the north and the Torrance of Campsie on the south, and reaches an altitude of close on

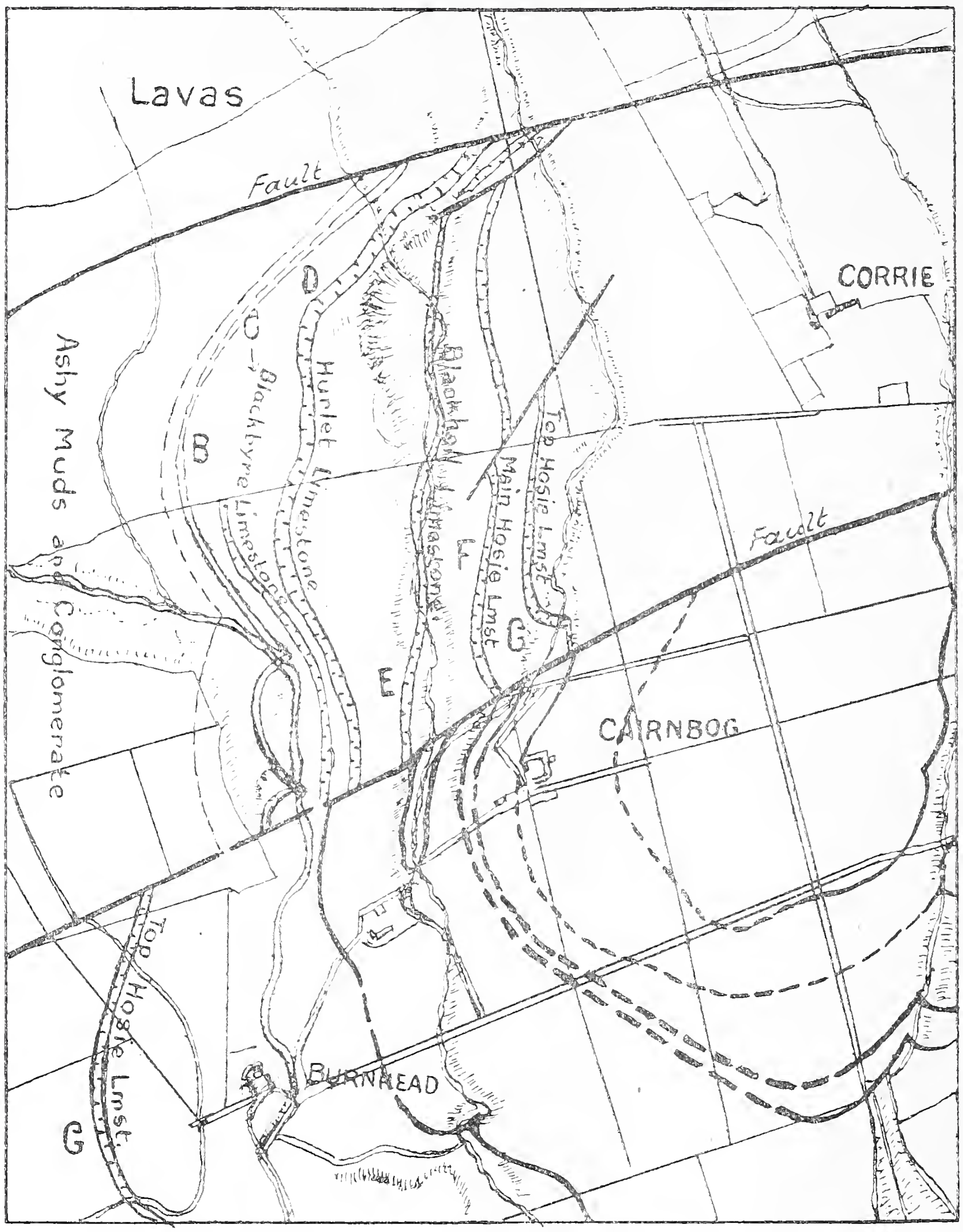


Fig. 1.—Sketch Map showing Lower Limestone Series at Corrieburn, Kilsyth. Based on H.M. Geological Survey Map.

700 feet above sea-level. On the south side of the hill the best sections are exposed in Craigenglen, Glen Whapple, and Glen Wynd, and on the north side of the hill in Baldow Glen. The strata range from the Hollybush Limestone up to the Hosie, which caps the hill. They are arranged in a flat syncline, and appear almost symmetrically on each side of the

hill. The Blackhall Limestone was formerly wrought at the foot of Craigenglen, where it is seen to be faulted against the Hollybush Limestone, from which a fairly continuous section can be traced to the shales above the Blackhall Limestone. In Glen Wynd, immediately to the west, the beds lying below the Hurlet Limestone are well shown, including the Alum Shale with its characteristic fauna, the Bone Bed, the Lamellibranch Bed, and the Brachiopod Bed in ascending order.

Proceeding to the north side of the hill we find a similar exposure in Baldow Glen. The second section given in the Plate at the end has been constructed partly from this section and from that exposed on the North Hill of Campsie, above the Milton of Campsie.

We now pass to a consideration of the sections exposed at Corrieburn, which we consider to be the most typical of all the sections of the Lower Limestone Series to be seen in Central Scotland; that is to say, it contains in the most generalised form all the features of the Lower Limestone Series as developed in the East and West of Scotland. If we had our choice we would be inclined to supplant the place of the Hurlet Section by that seen at Corrieburn.

At Corrieburn there are four stream sections which, proceeding from west to east, may be numbered 1, 2, 3, and 4 respectively. The following group of strata can be seen in these streams. It gives a complete section of the Lower Limestone Series from the Blackbyre Limestone to the Top Marine Band. The table is extracted with slight alterations from a paper on the stratigraphy of the limestone lying immediately above the Calciferous Lavas in the Glasgow district by Mr H. Conacher and myself:—*

			Ft.	In.
H.	27.	Sandstone with *Spirophyton cauda-galli* passing upwards into Top Marine Band	...	
	26.	Shales with *Posidonomya corrugata, Lingula, Orthoceras*, etc.	...	
G.	25.	Limestone (Top Hosie) (Calderwood Cement)	1	6
	24.	Black shales with rich marine fauna	6	
	23.	Limey shales	12	
F.	22.	Black limestone with *Bellerophon* and *Orthoceras* (Main Hosie)	2	
	21.	Shale	3	6
	20.	Sandstone	30	
	19.	Shale with nodules, Blackhall Shale fauna at the bottom	70	
E.	18.	Encrinital limestone with small solitary corals (upper part of Blackhall Limestone)	3	
E.	17.	Brown crusted entomostracan limestone (lower part of Blackhall Limestone)	3	

* *Trans. Geol. Soc. Glasgow*, vol. xvi, p. 49.

			Ft.	In.
	16.	Sandstone flaggy and ripple marked, limey and oolitic in part	150	
	15.	Limey shales with large encrinites and bryozoa	2	
D.	14.	Encrinital limestone (Hurlet Limestone)	15	
	13.	Black and grey shales, with characteristic brachiopod, lamellibranch, and fish fauna	6	
	12.	Coal		6
C.	11.	Ashy fireclay with limey nodules (position of Baldernock Limestone)	2	
B.	10.	Nodular green limestone with abundant brachiopods and corals (upper part of Blackbyre Limestone)	5	
B.	9.	Coral limestone represents a coral reef (lower part of Blackbyre Limestone)	4	
	8.	Limey shales with *Rhynchonella* and nodules containing corals and brachiopods	10	
	7.	Shale with ironstone bands	4	
	6.	*Naiadites crassa* bed	6	6
	5.	Shales with *Telangium affine* and marine fossils	12	
	4.	Beds containing *Naiadites crassa*	8	
	3.	Shales graduating down into volcanic mud with oolitic ironstone nodules	20	
	2.	Sandstone	3	8
	1.	Ashy muds and conglomerate	...	

In stream section No. 1 the beds 26 and 27 in the above table are exposed; they are brought down by a fault against the basal members of the section. In the remaining three streams the strata dip east and south-east at a low angle, strike north and south, and present their principal escarpments on the east banks of the streams. In stream No. 2 the strata from 1 to 15 are exposed, in stream No. 3 the strata from 16 to 19, and in stream No. 4 the strata from 20 to 26.

Beginning at the base of the section, we note in the first place the ashy muds and conglomerates well shown in the bed of the stream. The sandstone, No. 2 in the table, is on the same horizon as the Hollybush Sandstone. The *Naiadites crassa* bed, No. 6, is the same as that which occurs over a great part of North Ayrshire immediately below the Dockra Limestone. The Blackbyre Limestone, Nos. 9 and 10, presents all the characteristic features of that limestone as seen both in the East and West of Scotland. The Hurlet Coal is succeeded by the Alum Shale, carrying the Bone Bed, Lamellibranch band, and Brachiopod band in ascending order. The Hurlet Limestone, No. 14, by the accession of calcareous shales has attained a thickness of 15 feet, compared with 4 feet at Campsie. The next calcareous horizon is the Blackhall Limestone, Nos. 17 and 18 in the table. It presents exactly the same lithological and palæontological features as we have seen it possess at Blackhall in the Hurlet district, and it is overlaid by a highly fossiliferous blaes, No. 19 in the table, yielding the same fauna as was found by Dr Young in the pit on the South Hill, Campsie, and which has now been proved to form a well-marked horizon in the Lower Limestone Series of the West of Scotland.

After passing over 150 feet of shales and sandstones we reach the Main Hosie Limestone, No. 12. This limestone is 2 feet in thickness, and passes upwards into the limey shales, carrying a rich marine fauna.

The next calcareous horizon in ascending order is the Top Hosie or Calderwood Cement, No. 25 in table, which is well seen in relation to the underlying Main Hosie in stream No. 4, but is much better developed in stream No. 1, where it exhibits a striking similarity in appearance to the exposure in the type locality at Calderwood Glen.

In stream section No. 1 the sandstones with *Spirophyton caudi-galli* can be seen, but there is no exposure of the Top Marine Limestone, which is best seen about a mile and a half to the west in the Spouthead Burn, immediately to the south of the great fault.

VI. Sections between Hurlet and Cobbinshaw.

Having traced the Hurlet succession north-eastwards along the Campsie Hills, and, as we think, satisfactorily established the identity of the various fossiliferous horizons, we now proceed to consider the sections that lie between Hurlet and Cobbinshaw. There are numerous exposures along this line from west to east, but we shall only refer to those seen at Thorntonhall, Calderwood Glen, and that seen in the neighbourhood of Carluke.

At Thorntonhall and the Kittoch Water the Blackbyre and Hurlet Limestones with the intervening strata are well shown, though at this point the Alum Shale is thin, and the fauna has not yet been found. The Blackhall Limestone with its overlying fauna is well shown in the Kittoch Water a little above Arrotshole. Entering Calderwood Glen at Crossbasket Bridge, 2 miles east of East Kilbride, a series of sandstones are exposed in the bed of the stream, which further down are seen to rest upon volcanic detritus and lavas. Further upstream the sandstones are succeeded by beds of fireclay, coal, and limey shale, discovered some years ago by Mr R. G. Carruthers, and which yielded to him specimens of *Posidonomya becheri* and *Pterinopecten papyraceus.* This bed appears to form a well-marked horizon in the district.

Proceeding upstream we see outcropping on the right bank the Blackbyre Limestone, 2 feet thick. It is a cement in the upper part, and crinoidal in the lower, and passes down into a calcareous shale almost entirely made up of the crushed valves of *Productus semireticulatus* associated with the branches of *Lepidodendron*, which at once reminds us of the Blackbyre Limestone of the type locality, and, like it, is underlaid by a seam of coal. The limestone is followed by 6 feet of massive sandstone, on which rests the characteristic nodular limey fireclay found on the same position in the Hurlet district. Above the nodular fireclay comes 2 feet of entomostracan shale, followed by 4 feet of encrinital limestone, which we take as the equivalent of the Hurlet Limestone. A thin band of shale occurs between the entomostracan limestone and the Hurlet Limestone, but the Alum Shale fauna has not as yet been found at this locality.

About 350 yards further upstream the same section is repeated on the left bank, but the strata above the Hurlet Limestone are better shown than in the last exposure, and show the Blackhall Limestone in its characteristic development, with the highly fossiliferous blaes above it.

From this point to the waterfall above Calderwood Castle the sides of the glen are formed by a group of shales with bands of ironstone, which are succeeded by the thin bedded sandstones on which the castle is built.

About half a mile above Calderwood Castle the Calderwood Limestone Series is well shown in a steep cliff on the left bank of the stream. In the bed of the stream the Main Hosie Limestone appears in two posts, separated from each other by 2 feet 6 inches of highly fossiliferous shale, amongst which the following species occur:—*Pisocrinus globularis, Hydreionocrinus scoticus, Beyrichia bituberculata, Kirkbya urei, Seminula ambigua, Chonetes laguessiana, Crania quadrata, Discina nitida, Lingula mytiloides, Productus longispinus, P. semireticulatus, Rhynchonella pleurodon, Spirifer lineata, S. trigonalis, Spiriferina octoplicata, Dielasma hastata, Edmondia rudis, Axinus axiniformis, Myacites sulcata, Sanguinolites variabilis, Aviculopecten fimbriatus, Pecten sowerbii, Pleurotomaria contraria, Murchisonia urii, Orthoceras læve, Petalodus hastingsiæ,* and *Tomodus convexus*. This shale may be considered to be very rich in the remains of crinoids, small corals, polyzoa, entomostraca, and foraminifera. The mollusca and brachiopods are not nearly so abundant as in the limestone below the Hurlet Limestone, but Discina and Crania occur in abundance. The Top Hosie or Calderwood Cement occurs a little higher up in the same cliff section as the Main Hosie. The following is a list of the principal species that occur in the Calderwood Cement or associated shales of the neighbourhood:—*Serpulites carbonarius, S. mem-*

branaceus, Spirorbis caperatus, Dithyrocaris glabra, D. granulata, D. ovalis, D. testudinea, D. tricornis, Discina nitida, Lingula squamiformis, Productus semireticulatus, P. longispinus, Rhynchonella pleurodon, Streptorhynchus crenistria, Aviculopecten knockonniensis, Posidonomya corrugata, Nuculana attenuata, Protoschizodus æquilateralis, Nucula gibbosa, Sanguinolites plicatus, Orthoceras attenuatum, Nautilus sp.

Still further upstream, and on its left bank, is the escarpment known as the Black Craig, which shows a thick series of alternating bands of ironstone and shale. In the shale near the foot of the cliff is a bed rich in *Lingula squamiformis* and *Productus*, which marks the position of the Top Marine Band.

The next section we describe is that shown in the neighbourhood of Carluke. When the original survey of the Carluke district was made in 1873 the whole area was being opened up by mines and opencast workings, and at that time it was much more easy to study the group of strata under consideration than it is now, as many of the sections are not now exposed. The records that have been left are, however, particularly full and clear, especially those of Dr Rankine and Dr Hunter, and there can, we think, be little dubiety in correlating the limestones of the Carluke district with those of the Hurlet district and the West of Scotland generally.* It is not considered necessary to here discuss in detail the stratigraphical and palæontological evidence upon which the correlation is based, as that has already been done in a paper published in the *Transactions of the Geological Society of Glasgow*. The following table shows the principal horizons in the Hurlet district and their equivalents in the Carluke district:—

Hurlet District.	*Carluke District.*
Lingula Shale and Top Marine Band	Slingstone Limestone.
Johnstone C.B. Ironstone.	
Calderwood Cement	Lingula Limestone.
Lillies Coal and Ironstone.	
Main Hosie Limestone	Birkfield Limestone.
Hosie Sandstone	Raesgill Ironstones.
Blackhall Limestone	Foul Hosie Limestone.
Househill Ironstones	Selkirk Ironstones.
Hurlet Limestone	Second Calmy Limestone.
Alum Shale	Fossiliferous Blaes.†

* *Trans. Geol. Soc. Glasgow*, vol. xv, p. 404.

† At Thorn, $1\frac{1}{4}$ miles north-east of Carluke, the second Calmy Limestone is underlain by a fossiliferous blaes containing a varied fauna, with numerous shells of *Sanguinolites costellatus*. This we regard as the equivalent of the Hurlet Alum Shale fauna.

Hurlet District.	*Carluke District.*
Baldernock Limestone.	
Blackbyre Limestone	Main Limestone.
Hollybush Sandstone.	
Hollybush Limestone	*Productus giganteus* Limestone.

VII. Sections between Cobbinshaw and the Firth of Forth.

The Lower Limestone Series having been traced *pari passu* from the Hurlet Section eastwards through East Kilbride and Carluke to the neighbourhood of Wilsontown in the Clyde drainage area, we cross the watershed and enter the Forth basin at Cobbinshaw Reservoir, from which point the Lower Limestone Series may be traced by Baads Mill, Addiewell, Blackburn, over the Bathgate Hills to the shores of the Firth of Forth at Carriden. Between Bathgate and the Firth of Forth the sequence is considerably obscured by the intercalation of thick beds of lava, amongst which, however, certain well-marked limestone horizons can be more or less continuously followed.

Near Cobbinshaw Reservoir a pit is at present being worked by the Pumpherston Oil Company, in which there is a 2-feet limestone underlain by dark blaes carrying the Hurlet Alum Shale fauna. The state of preservation of the fossils in the Alum Shale at this locality has a striking resemblance to that exposed at Glen Wynd, near Campsie. They occur in loose blocks at the mouth of the pit, and the order of the superposition of the various bands has not yet been determined. Between the pit and the south end of Cobbinshaw Reservoir the coal and limestone were formerly wrought in a mine, and a small section showing the coal, limestone, and intervening strata can still be seen at the surface. The coal has also been worked at Viewfield Pit further to the south, and the shale which lies between the coal and limestone at these three localities has yielded the characteristic Hurlet Alum Shale fauna.

The crop of the Hurlet Limestone can be traced by workings across to Baads Mill on the Harwood Burn, and thence to Addiewell Bridge on the Breich Water. As the section seen in the Breich Water and its tributary, the Skolie Burn, is the most continuous in this part of the country, and as it can be linked up both stratigraphically and palæontologically with those of the West of Scotland, we now describe it in some detail. The section exposed is as follows:—

			Ft.	In.
G.	20.	Cement Limestone with *Aviculopecten* and *Posidonomya corrugata*	3	
	19.	Fossiliferous Shales	9	
	18.	Crinoidal Limestone	1	
	17.	Fossiliferous Shales	20	
F.	16.	Dark Crinoidal Limestone	1	
F.	15.	Fossiliferous Shales	4	
F.	14.	Dark Crinoidal Limestone	5	
	13.	Dark Shales with Spirifers, etc.	6	
	12.	Yellow Calcareous Shelly Sandstone	5	
	11.	Green Sandy Shales and Shaley Sandstone with Lingulæ	2	
	10.	Picrite Sill	...	
E	9.	Limestone	...	
	8.	Strata	...	
	7.	Coal	1	6
	6.	Strata	130	
D.	5.	Limestone encrinital	4	
	4.	Alum Shale	1	6
	3.	Coal	5	6
	2.	Strata	30	
C.	1.	Freshwater Limestone	1	3

The lowest stratum in the above table is a bed of freshwater limestone which lies beneath the Hurlet Limestone and coal, and which we consider to be upon the same horizon as the Baldernock Limestone of the Campsie district. The Alum Shale of this locality has yielded the typical faunal association of that horizon. The other limestones given in the table are clearly the equivalents as lettered of those seen in the West of Scotland.

It is not considered necessary in this paper to follow those limestones in detail to the Firth of Forth. Sufficient be it to say that all the stratigraphical and palæontological evidence available goes to show that the various horizons of the Hurlet Sequence can be more or less easily recognised. Thus the freshwater limestone of East Kirkton is the equivalent of the Baldernock Limestone of the west, while the West Kirkton or Tartraven Limestone is the same as the Cobbinshaw Limestone, that is, the Hurlet Limestone, of the West of Scotland. Between the Cobbinshaw Limestone and the Petershill Limestone there occurs a limestone with which is a shale carrying an identical fauna with that seen above the Blackhall Limestone in the West of Scotland. The Petershill Limestone is clearly the equivalent of the Main Hosie Limestone, and the Calderwood Cement or Top Hosie occurs above it in the Skolie Burn. The limestones seen on the shore at Carriden abundantly charged with *Spirophyton cauda-galli* are the representatives of the Top Marine Band in the west.

VIII. Sections between Charlestown and Pittenweem.

Crossing to the north of the Firth of Forth the next section that we describe is that seen in the neighbourhood of Charlestown, and we have no difficulty in correlating the different members of this section with the general sequence that has so far been established. Especially striking is the similarity between it and that seen at Corrieburn.

The Hurlet Limestone is exposed between high and low water mark on the shore opposite Charlestown railway station. It occurs in a small faulted down patch with quaquaversal dip. Underneath it lies the Hurlet Alum Shale and coal, the former carrying the fish, lamellibranch, and brachiopod fauna of this horizon. On the shore, in front of the breakwater, just west of the harbour, there occur a number of loose blocks of highly fossiliferous shale containing a fauna which bears a very strong resemblance to that found above the Blackhall Limestone of the West of Scotland, the blocks being crowded with *Spirifer urei*. Unfortunately we have not been able to give the exact stratigraphical position of this bed.

The next calcareous horizon in ascending order is the thick limestone of the Charlestown Quarries, which is estimated to lie about 150 feet above the Hurlet Limestone. The rich coralline, echinoderm, polyzoan, brachiopod, and molluscan fauna which it contains is identical with that seen in the Petershill Quarries of the Bathgate Hills, and we have no hesitation in considering this fauna to be the eastern representative of that characteristic of the Main Hosie Limestone of the West of Scotland.

The highest calcareous horizon seen in this neighbourhood is that exposed in the railway cutting three-quarters of a mile west of Charlestown railway station. It is calculated to lie some 180 feet above the Charlestown Limestone, and is separated from it by thick sandstones. This limestone, which is rich in *Spirophyton cauda-galli*, is on the same horizon as the lowest of the Carriden Limestones on the other side of the Firth, and is the same as the Top Marine Band of the West of Scotland.

The next section that we describe lies between Kinghorn and Kirkcaldy, and is not only one of the most continuous but also one of the most interesting in the East of Scotland. We hope to be able to show that the various members represented in this section can be correlated with those sections that have already been described.

The section begins with the first Abden Limestone and the strata lying immediately below, which outcrop on the shore about a quarter of a mile to the east of Kinghorn. At this point the underlying lavas begin to be less frequent, and we find resting upon the scoriaceous surface of a lava flow a thick bed of fine-grained, greenish-coloured mudstone full of the remains of plants, including stigmariæ and rootlets. Above this comes

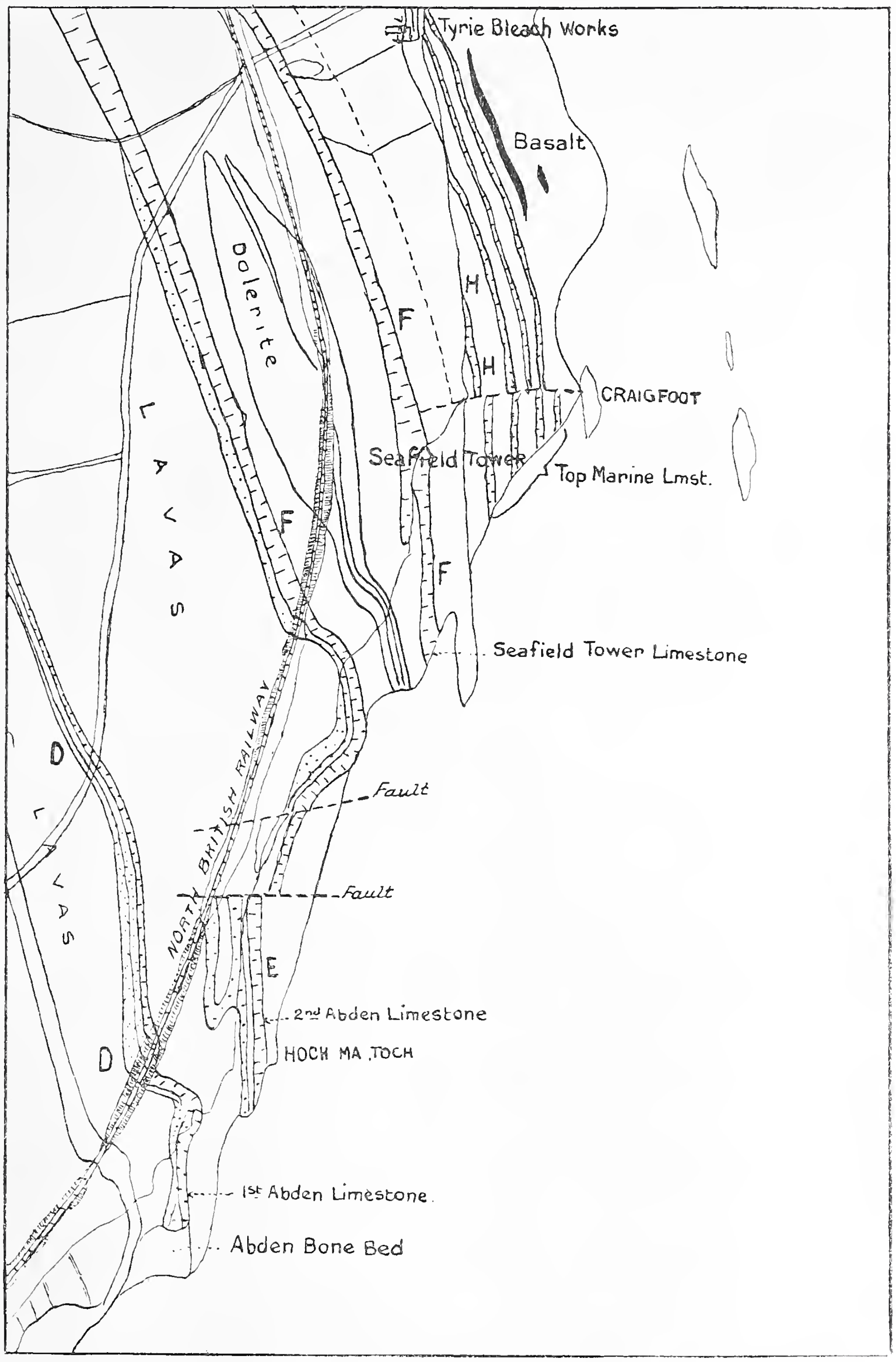


Fig. 2.—Sketch Map showing the Lower Limestone Series between Kinghorn and Kirkcaldy. Based on H.M. Geological Survey Map.

about 4 feet of black shale, at the bottom of which there is a layer literally crammed with the remains of fish scales, teeth, spines, and bones. This is known as the Bone Bed, and was first described by the Rev. Thomas Brown in the year 1861. The shales above it contain a Lamellibranch and

Brachiopod fauna, identical with that found in the Hurlet Alum Shale of the West of Scotland. This is succeeded by the first Abden Limestone, 9 or 10 feet thick, which is encrinital in its lower part, while its upper part is rich in marine fossils. The Brachiopod, Lammelibranch, and fish fauna of the shale lying below the first Abden Limestone, as well as the succession and general appearance of the group as a whole, is so like that seen on the shore at Charlestown railway station, that we do not hesitate for a moment in stating our belief that they are identical, and that the first Abden Limestone is the equivalent of the Cobbinshaw-Hurlet Limestone of the preceding part of this paper.

Over the first Abden Limestone comes a few thin beds of shale, which are succeeded by the lava forming part of the small headland known as Hoch-Ma-Toch. On the top of the lava rest more shales, which pass upwards into another marine limestone locally known as the second Abden Limestone. A group of dark shales occur below the second Abden Limestone, which carry a fauna similar to that found above the Blackhall Limestone in the West of Scotland. But the existence of a fossiliferous shale immediately above the limestone has not yet been determined. Stratigraphically this limestone should lie somewhere on the position of the Blackhall Limestone of the West of Scotland—that is, between the Hurlet Limestone below and the Main Hosie Limestone above—and this is clearly its position in the section under consideration, because it has below it the first Abden Limestone, which we have shown to be the equivalent of the Hurlet Limestone, and above it the Seafield Tower Limestone, which we believe to be the same as the thick Charlestown and Petershill Limestone, or the Main Hosie Limestone of the West of Scotland.

Passing over something like 100 feet of shale and reddened sandstone we reach the Seafield Limestone. This limestone is about 50 feet thick, consisting of bands separated from each other by intercalations of shale. It can be followed inland to the quarries at Inverteil, where it is practically identical, so far as its physical characters are concerned, with the section just described. The faunal assemblage shows it to be identical with the Charlestown and Petershill Limestone, as it contains similar species of corals, crinoids, polyzoa, brachiopods, lamellibranchs, gasteropods, cephalopods, and fishes, and is therefore upon the horizon of the Main Hosie Limestone of the West of Scotland.

The Seafield Tower or Main Hosie Limestone is surmounted by a bed of shale, which is succeeded by a thick mass of false-bedded pink sandstone, on which Seafield Tower stands. Above this come calcareous shales, with limestone bands and nodules, which stratigraphically should lie on the position of the Calderwood Cement Series of the West of Scotland.

On the top of the last-mentioned group come a bed of fireclay full of rootlets, and a thin seam of coal covered by more fireclay, and a bed of sandstone with worm burrows. In its upper part it becomes calcareous, and is crowded with fragmentary crinoid stems and *Spirophyton cauda-galli*. This limestone marks the beginning of the conditions that we have shown to be characteristic of the Top Marine Band in the West of Scotland, and at once reminds us of the exactly similar beds found at Corrieburn, Carriden, and Charlestown on this horizon.

We now come to the most north-easterly of the Fife sections in the Lower Carboniferous Limestone Series, namely, that exposed along the shore between St Monans and Pittenweem. This section was first examined in detail by Mr Kirkby, and his table has been published on pp. 149–150 of the *Geological Survey Memoir on Eastern Fife*. The section is altogether about 365 feet in thickness, and we believe that its various members can be correlated not only with the preceding Kinghorn-Kirkcaldy Section, but also with all the others that have been described. Especially striking is the similarity between it and the Corrieburn section. The following table is abridged from that by Mr Kirkby:—

			Ft.	In.
	19.	Shale grey, marine fossils	15	
H.	18.	Limestone, *Tabulipora scotica*, *Zaphrentis constricta*, *Spirophyton cauda-galli*	2	4
	17.	Shale, marine fossils	2	
	16.	Strata	77	
G.	15.	Shale and Cementstone, *Ulocrinus*, *Protoschizodus curtus*, *Schizodus wheeleri*, *Limatulina attenuata*	25	
	14.	Strata	47	
	13.	Cementstone, Crinoids, Productus	0	6
	12.	Strata	48	
	11.	Shale calcareous and fossiliferous in lower portion, *Platycrinus*, *Graphiocrinus*, *Bursacrinus*, *Nucula scotica*	50	
F.	10.	Limestone crinoidal	5	
	9.	Strata	38	
	8.	Shale, marine fossils	..	
E.	7.	Limestone, *Productus giganteus*, *Lithostrotion*	2	
	6.	Strata	10	
D.	5.	Limestone pseudo-brecciated and crinoidal	7	
	4.	Strata	12	
B.	3.	Limestone, white coral, *Lithostrotion junceum*, *L. irregulare*	18	
	2.	Strata	100	
A.	1.	Limestone and shale, *Lithostrotion*, solitary corals, and Brachiopods	2	

The above strata have been folded into a synclinal trough, whose western lip appears on the shore at the point where the St Monans Burn enters the sea, while its eastern lip can be seen on the shore opposite Coal Farm, half way between St Monans and Pittenweem. The centre of the

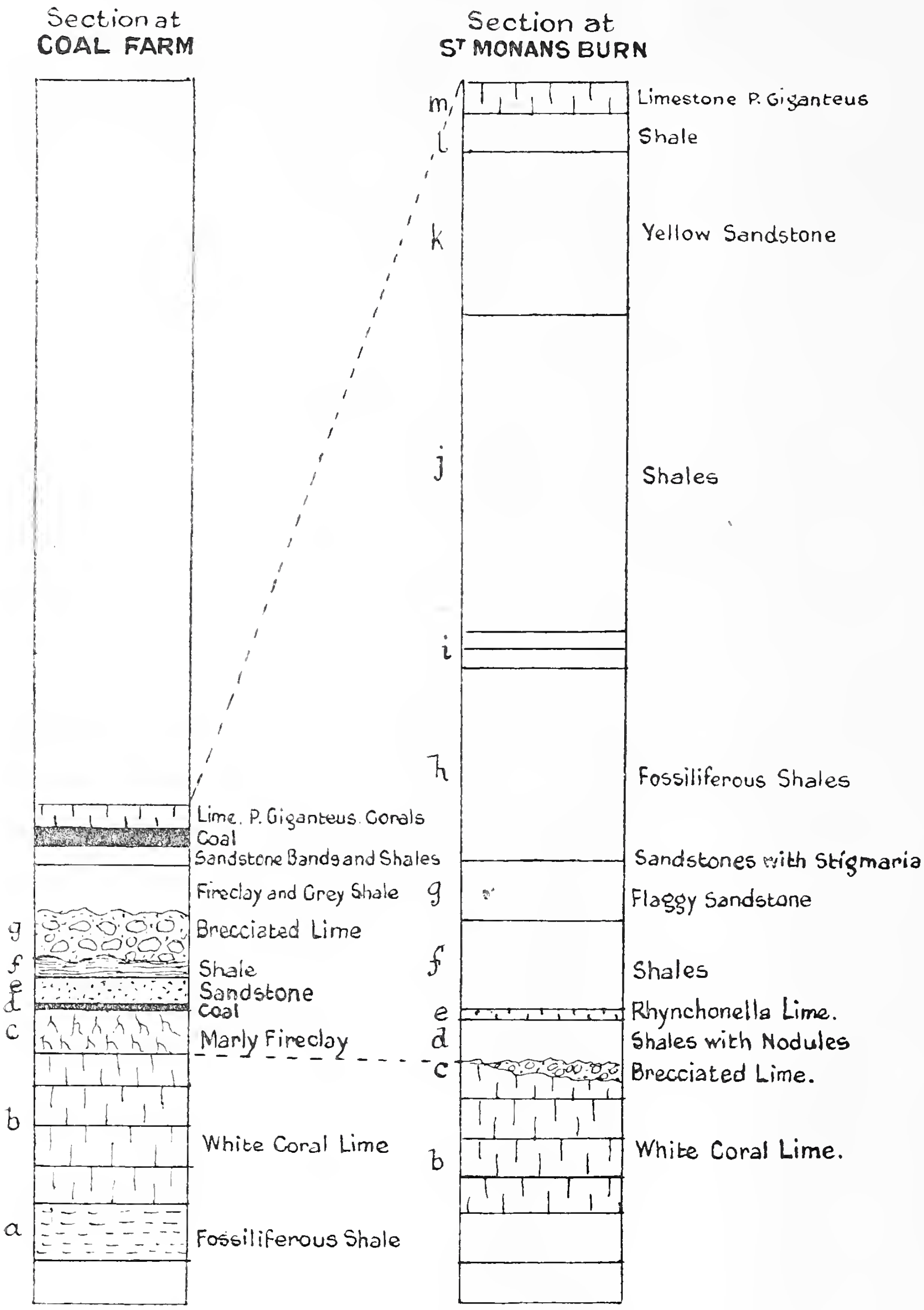


Fig. 3.—Comparative Vertical Sections of the Lower Limestone Series at the east and west ends of the St Monans syncline.

trough, which is not very symmetrical, appears immediately to the east of the harbour.

The White Coral Limestone, No. 3 in the above table, appears on the shore near Coal Farm. It presents all the lithological, stratigraphical, and palæontological features of the Blackbyre Limestone of the West of Scotland, especially as it is developed at Corrieburn, so that we have no hesitation in regarding it as the equivalent in the east of the Blackbyre Limestone of the west.

A comparison of that part of the section which lies between the White Coral Limestone, No. 3, and the *Productus giganteus* Limestone, No. 7, shows a most remarkable and interesting difference both in the thickness and general succession of the strata as developed at the opposite extremities of the syncline, though the distance between the two sections is less than a mile. This is shown in fig. 3.

The shale *a* in the section near Coal Farm lies immediately below the Coral Limestone, and has a striking resemblance to the fossiliferous shale lying below the Blackbyre Limestone of the West of Scotland. The similarity of the White Coral Limestone to the Blackbyre Limestone has already been noted, as it presents exactly the same stratigraphical, physical, and palæontological features as that limestone in the West of Scotland, the shale *f* in the section having yielded an exactly similar fauna to that which characterises the Hurlet Alum Shale horizon of the West of Scotland. The lamellibranch and the brachiopod bands are well marked, and the Bone Bed is represented by scales and fragments of fish remains, though they are not so abundant as at some of the other localities. The section at the west side of the syncline at St Monans Burn presents a number of striking differences from that seen at the Coal Farm end, showing that at the latter locality, land and estuarine conditions must have prevailed, whereas in the former more or less continuous sedimentation took place. The evidence afforded by these two sections, and the light which they throw upon the physical conditions which supervened upon the deposition of the Blackbyre Limestone all over the Midland Valley, is of exceeding great interest, but these details cannot be discussed here. The succeeding limestones in this section have been correlated with those of the West of Scotland, as indicated by the lettering in the table and in the series of comparative vertical sections given at the end. The palæontological evidence in support of these correlations will be given in the succeeding part of this paper.

IX. The Bilston Burn Section.

We recross to the south side of the Firth for the purpose of examining the sections exposed in the Bilston Burn and on the shore at Aberlady Bay and Dunbar. We shall see that the section at the first of these localities presents a strong resemblance to that seen between Kinghorn and Kirkcaldy, while those seen at the second and third localities approach more closely to that just described at St Monans.

The section in the Bilston Burn is situated about 7 miles to the south of Edinburgh, and begins just where the stream crosses the road at the Bilston Inn. The details of this section need not be repeated here. The following table shows our correlation of the main calcareous horizons with those of the West of Scotland:—

Hurlet District.	*Bilston Burn.*
Top Marine Band	Bilston Burn Limestone.
Top Hosie	Vexim Limestones.
Main Hosie	North Greens Limestone.
Blackhall Limestone	Gilmerton Limestone.
Hurlet Limestone	Limestone with Hurlet Alum Shale fauna in shale below.

X. Section at Aberlady Bay.

The Lower Carboniferous Limestone Group at Aberlady Bay is exposed between high and low water marks in a continuous section north of Aberlady and west of Kilspindie as far as Craigielaw Point. The beds are gently undulating, forming a syncline in the east and an anticline in the west. The dominant dip of the strata is southerly and parallel to the pitch of the folds. The section exposed is as follows:—

			Ft.	In.
	14.	Dark Shale with thin Limestone of Craigielaw Point	...	
F.	13.	Hard grey Limestone with encrinites and *Productus giganteus*	12	
	12.	Thin Coal and Fireclay	...	
	11.	Shale	7	
E.	10.	Hard grey Limestone with *Productus giganteus*	2	
	9.	Thin Coal in places		3
	8.	Shale	8	
	7.	False Bedded Sandstones	4	
	6.	Sandy black Shale	8	
D.	5.	Nodular dolomitic encrinital Limestone	12	
	4.	Coal		10
	3.	Grey Shale and Fireclay full of twisted rootlets	6	
B.	2.	White Coral Limestone	8	
	1.	Grey and greenish Shales, and yellow Sandstones	20	

The White Coral Limestone, No. 2 in the above table, is identical with that seen at Coal Farm, St Monans, as has been pointed out by Dr Crampton.* But not only is it identical with the coral limestone on the other side of the Firth, but it also presents a similar faunal assemblage to that found

* "The Limestones of Aberlady, Dunbar, and St Monans," *Trans. Edin. Geol. Soc.*, vol. viii, p. 374.

in the Blackbyre Limestone of the West of Scotland, while in the shale overlying it we have the Hurlet Alum Shale fauna. The shale is somewhat awkward to get at in this section, but both the brachiopod and the lamellibranch bed appear to be present in their relative positions. Fish remains occur, but the existence of a bone bed has not yet been determined. The nodular and dolomitic encrinital limestone, No. 5, consists chiefly of encrinite fragments, the upper part being shaley and more evenly bedded. This is the equivalent of the brecciated limestone of the St Monans Section. The hard grey limestone with *Productus giganteus*, No. 10, is the same as

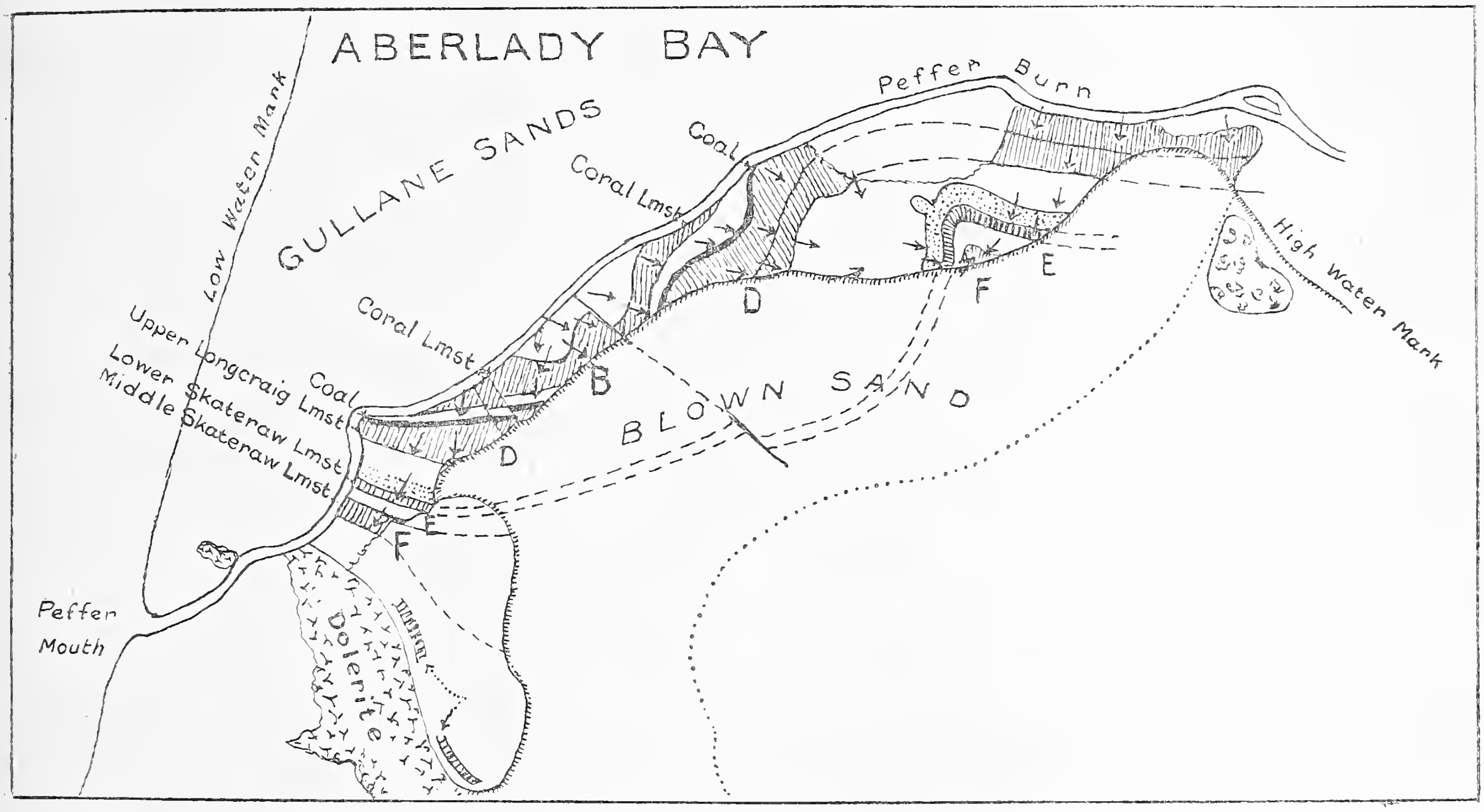


Fig. 4.—Sketch Map showing Lower Limestone Series at Aberlady Bay. Based on fig. 7 in the Geological Survey Memoir on *The Geology of East Lothian* (1910).

limestone No. 7 in the St Monans Section; and, lastly, the hard grey thick bedded limestone with encrinites and *Productus giganteus*, seen near Craigielaw Point, and the base of which is also seen in the Aberlady syncline, where it forms its highest bed, is the equivalent of limestone No. 10 in the St Monans Section, which we have shown to be the same as the Seafield Tower Limestone and the Charlestown and Petershill Limestone, that is, the Main Hosie Limestone of the West of Scotland.

XI. Section at Dunbar.

The last section which we describe is that exposed along the shore to the south-east of Dunbar, where the Lower Carboniferous Limestone Series is seen to be thrown into a synclinal trough, the basin being truncated on the north-east by the sea. It extends for a distance of 4 miles from the mouth of Broxburn to Longcraig. At Longcraig the Limestone Series has been faulted against the underlying Cement Stone Series. For a distance of about a mile towards the north-west the dip of the limestones is generally north at low angles, so that as we go west we successively ascend to higher and still higher platforms in the Limestone Series till we reach the topmost member, the Barness East Limestone, which can be seen at low-water mark near the foot of Dryburn. At this point the dip changes from north to north-east, and as we proceed westwards the succession is a descending one till we reach the horizon of the Longcraig Middle Limestone. Here a fault and broad dolerite dyke break the continuity of the section. The fault has a downthrow to the north-west of about 100 feet, and its effect is to bring the base of the Chapel Point Limestone against a lower limestone, the Middle Skateraw. The general stratigraphical succession and faunal sequence, as given in the following table,* is so similar to that already described at Aberlady Bay, St Monans, and Corrieburn, as not to call for any further description here:—

			Ft.	In.
	23.	Barness East Limestone, *Spirophyton cauda-galli*	6	
	22.	Strata, chiefly Sandstones	50	
	21.	Dryburn Foot impure Limestone, *Spirophyton cauda-galli*	2	
	20.	Sandstone and Shale	10	
H.	19.	Chapel Point Limestone, *Spirophyton cauda-galli*	10	
	18.	Shale and thin Coal smut	3	
	17.	Fireclay, Ganister and Sandstones	20	
	16.	Shale and thin flaggy Sandstone	80	
	15.	Skateraw Upper Limestone	2	
	14.	Black Shale	5	
F.	13.	Skateraw Middle Limestone	18	
	12.	Coal		6
	11.	Fireclay	1	6
	10.	Shale and Fireclay	6	6
E.	9.	Skateraw Lower Limestone	4	
	8.	Thin Coal in places	...	
	7.	Shale and Sandstone	23	
D.	6.	Longcraig Upper Limestone	18	
	5.	Thin Coal in places	...	
	4.	Shale partly sandy with the Hurlet Alum Shale fauna	5	
B.	3.	Longcraig Middle Limestone, *Lithostrotion* in reef-like masses	6	
	2.	Shale and Sandstone	25	
A.	1.	Longcraig Lower Limestone mixed with bands of calcareous Shale	...	

* This table has been copied from the Geological Survey Memoir on *The Geology of East Lothian* (1916), p. 137.

XII. Stratigraphical Comparisons and Considerations.

We now pass to certain stratigraphical comparisons and considerations which will be best understood by an examination of the series of comparative vertical sections given in the Plate. It will be noticed that in the first five of these—that is, the Hurlet, Campsie, Corrieburn, Calderwood Glen, and Carluke Sections—the datum line has been drawn at the Blackbyre Limestone. A similar position has been taken for the datum line in the St Monans, Aberlady Bay, and Dunbar Sections in the East of Scotland. In the remaining sections, which include those at Skolie Burn, Charlestown, Kinghorn, and Bilston Burn, the Hurlet Limestone with its underlying Alum Shale fauna has been taken as the datum line. The reason for this will be shown presently.

The outstanding feature of the first five sections, which we must constantly keep in mind, is the existence of an important unconformity and overlap at the junction of the Lower Limestone Series with the underlying contemporaneous volcanic rocks. In the Hurlet district the sediments lying below the Blackbyre Limestone attain a thickness of close on 1000 feet, and though they consist largely of barren sandstones and ashy muds and shales, a marine horizon is occasionally to be met with. If we turn to the section at Campsie, it will be found that in the Craigmaddie district there is a great thickness of sediments, estimated to be upwards of 600 feet, lying between the lavas and the Blackbyre Limestone. At Corrieburn, only some 4 miles further to the east, the sediments between the Blackbyre Limestone and the lavas is reduced to less than 70 feet. In the Calderwood Glen and Carluke districts similar variations indicative of the existence of a considerable unconformity and overlap have been established.

In addition to the above direct stratigraphical evidence of unconformity and overlap there is a pronounced colouration of the sediments up to the horizon of the Hurlet Limestone, due to the presence of decomposed volcanic material washed off the old land surface. After we reach the top of the Hurlet Limestone we find that the character of the limestones undergo a sudden change. The succeeding limestones are generally of a darker colour and of a more calmy nature than the underlying ones, and the terrigenous sediments are no longer tinged with the green or purple ashy material from the old volcanic platform, but consist for the most part of dark shales charged with fossils, with which are associated Clayband Ironstones.

When we cross the watershed to the basin of the Forth we find that below the Hurlet Limestone, in the Skolie Burn, Charlestown, Kinghorn, and Bilston Burn Sections, the thick Calciferous Sandstone Series has been divided into an upper or Oil Shale Group over 3000 feet in thickness, and a lower or Cement Stone Group also of considerable thickness. Over this area the Blackbyre Limestone or Coral Reef Limestone appears to be absent, at least in its typical form, our explanation of its absence being that the estuarine conditions which obtained over this part of the area were inimical to the existence of such true marine conditions as existed when the Blackbyre Limestone was formed. When we reach the St Monans Section in East Fife, we find that the Calciferous Sandstone Series presents quite a different development from that seen either in the West of Scotland or in the Lothians or Western Fife. In place of the widely separated marine platforms with comparatively few fossils seen in these districts we find a great succession of marine bands rich in organic remains, and having a total thickness amounting to about 4500 feet.

Though the Blackbyre Limestone appears to be absent in the Edinburgh district, we are fortunately not left in any dubiety regarding the approximate position of this important datum line, for the Hurlet Alum Shale with its characteristic faunal association, which has been shown to overlie immediately the Coral Reef or Blackbyre Limestone in the West of Scotland, forms a well-marked horizon in the Edinburgh district, enabling us to determine with sufficient exactitude the position of this important datum line.

Another important stratigraphical feature which we have to note is the local unconformity or break which is generally found to exist at the top of the Blackbyre or Coral Reef Limestone. It can be seen over a large part of North Ayrshire in the irregular hummocky top of the limestone itself, which also sometimes shows a brecciated structure, though on a much smaller scale than that seen at St Monans. That the Blackbyre Limestone had been upheaved into a land surface over a large part of North Ayrshire and the West of Scotland generally is also shown by the fact that it often has above it traces of coal or a fireclay containing large branching stigmariæ, which in many instances appear to have grown on the broken-up top of the coral reef.

Another important stratigraphical feature is the intercalation on various horizons in the Lower Limestone Series of thick beds of contemporaneous lavas. If the correlation advanced in this paper be accepted, it would throw a considerable amount of light upon these volcanic horizons in widely separated localities.

An examination of the comparative vertical sections given on the Plate shows that though the different calcareous horizons can be traced over the whole of Central Scotland, yet they vary considerably both in thickness and in character when traced from district to district. This feature is especially noticeable in the case of the Hurlet and Main Hosie Limestones, the former in the Renfrew and North Ayrshire districts, and the latter in the Bathgate Hills. The terrigenous sediments, though less constant in character than the limestones, can nevertheless be followed for considerable distances in which they retain well-marked lithological features, so that groups of sediments occurring between two known limestone horizons can often be identified over wide areas.

XIII. Palæontological Comparisons and Considerations.

It would be impossible within the limits of this paper to give a detailed analyses of the fossil contents of each of the limestone horizons under consideration, so that only the broader palæontological features will be indicated.

The Blackbyre or Coral Reef Limestone is characterised by the great abundance of colonial and solitary corals and brachiopods which it contains. The colonial corals belong mostly to the genus *Lithostrotion*, and include *L. irregulare*, *L. junceum*, and *L. portlocki*. They occur in reef-like masses over a large part of North Ayrshire, at Corrieburn, St Monans, Aberlady Bay, and Dunbar. The White Coral Limestone at St Monans has a thickness of about 18 feet. The corals evidently occupy the position in which they originally grew, though they are now bent and distorted by the pressure of the superincumbent strata, a not uncommon feature in existing coral reefs.

The solitary corals may be distributed throughout the limestone in single individuals, or they may occur in regular bands from 1 to 2 feet in thickness. The bands are partly calcareous and partly argillaceous, the corals being most abundant in the latter, from which at certain localities they can be gathered in great numbers. The genera represented include *Aulophyllum*, *Cyathophyllum*, *Cladochonus*, *Clisiophyllum*, *Dibunophyllum*, *Koninckophyllum*, *Zaphrentis*, and others.

The Blackbyre Limestone is also characterised by its extreme richness, both in species and individuals of brachiopods, thick bands being made up of the crushed shells of a single species such as *Productus semireticulatus* or *P. longispinus*. Amongst the genera represented are *Dielasma*, *Spirifera*, *Martinia*, *Reticularia*, *Spiriferina*, *Athyris*, *Retzia*, *Rhynchonella*, *Strophomena*, *Streptorhynchus*, *Schizophoria*, *Chonetes*, and *Productus*.

The characteristic features of the fauna of the strata lying between the Hurlet Coal and the Hurlet Limestone, occurring on the horizon known as the Hurlet Alum Shale, supply the key to the position of the Hurlet datum line over the East and West of Scotland. It has already been shown that in the West of Scotland the shale lying immediately below the Hurlet Limestone contains a band principally made up of the two brachiopods *Schizophoria resupinata* and *Productus semireticulatus.* Below this comes another band of shale, consisting almost entirely of the remains of the four lamellibranchs *Sanguinolites abdenensis, Streblopteria ornata, Actinopteria persulcata,* and *Myalina verneuili.* But an additional, and most remarkable, fact is that underlying the lamellibranch band in the Campsie district occurs a bone bed which is identical in every respect with the Bone Bed underlying the first Abden Limestone on the Fife coast north of Kinghorn. Like it, this bone bed varies from 1 to 2 inches in thickness, and is almost wholly composed of the scales, teeth, plates, spines, and bones of fishes, of which no two are in juxtaposition. The following table gives the characteristic species of this important horizon as it has been traced in the various sections that have been described:—

	Howood.	Glen Wynd.	M'Culloch's Slap.	Baldow Glen.	Boyd's Burn.	Burn Rannie.	Glorat Mine.	Corrieburn.	Thorn., Carluke.	Addiewell Pit 16.	Cobbinshaw.	Tartraven.	Rosyth Quarry.	Charlestown Rly. Stn.	First Abden Limestone.	Coal Farm, St Monans.	Bilston Burn.	Catcraig, Dunbar.
Telangium affine L. & H.				×		×		×						×	×	×		×
Archæocidaris urei Flem.	×			×					×	×				×				
Serpulites carbonarius M'Coy	×								×	×	×							
Productus longispinus Sow.	×	×	×	×						×								
,, *semireticulatus* Martin				×		×			×	×	×	×		×				×
Lingula squamiformis Phill.	×		×	×		×				×	×		×	×	×	×	×	×
Schizophoria resupinata Martin	×							×							×		×	×
Discina nitida Phill.	×					×					×				×			
Streblopteria ornata R. Eth., Jr.	×	×	×	×		×	×	×		×	×	×	×	×	×	×	×	×
Sanguinolites abdenensis R. Eth., Jr.	×					×	×	×			×		×	×	×	×	×	×
,, *costellatus* M'Coy	×	×	×					×	×	×								
Actinopteria persulcata M'Coy	×			×		×	×	×	×	×	×	×	×	×	×	×	×	×
Myalina verneuili M'Coy	×		×	×		×	×	×	×							×	×	
Posidonomya corrugata R. Eth., Jr.		×										×		×		×	×	
Bellerophon urei Flem.	×			×				×	×									
Orthoceras sp.	×							×	×		×							
Goniatite sp.																		

The Hurlet Limestone does not present any outstanding palæontological features by which it can be readily recognised. It is a crinoidal limestone, with occasional bands of *Lithostrotion*, and with an occasional solitary coral. According to Dr Young, the principal fossils of the Campsie Main Limestone are *Productus*, *Spirifer*, and *Athyris*, but it cannot be pronounced a rich fossiliferous deposit. He also considers the following shells to be peculiar to this division of the Campsie strata, namely, *Productus mesolobus*, *P. aculeatus*, and *P. fimbriatus*.

In the West of Scotland the Blackhall Limestone usually presents a lower part, which is a brown-crusted entomostracan limestone, and an upper part, which is an encrinital limestone with solitary corals. In the east the estuarine lower part is absent, and only the upper or marine part is developed. Over the limestone comes a dark shale, which carries a faunal assemblage which has been traced all over the West of Scotland and into Fife and the Lothians. The following are some of the more characteristic forms:—*Chonetes hardrensis*, *Leda brevirostris*, *Nucula luciniformis*, *Dentalium priscum*, *D. inornatum*, *Euomphalus carbonarius*, *Pleurotomaria conica* var. *decussata*, *Bellerophon oldhami*, *Goniatites gilbertsoni*, *G. mucronatus*, *G. striatus*, *G. vesica*, *Nautilus subsulcatus*, *N. biangulatus*, *Orthoceras cinctum*, *O. goldfusianum*, and *O. pygmæum*. *Spirifer urei*, a shell comparatively rare on other horizons, is abundant on this, and its large numbers are usually sufficient to identify the horizon at once. Other abundant forms are *Loxonema curvilinea* and *Cyrtoceras rugosum*.

The Main Hosie Limestone as developed in the Glasgow district and at Bathgate, Charlestown, Seafield, and Skateraw in the East of Scotland, presents a rich faunal assemblage. Amongst other forms we have *Lithostrotion junceum*, *Zaphrentis patula*, *Lonsdaleia floriformis*, *Cyathophyllum regium*, *Aulophyllum fungites*, *Alveolites depressa*, and *A. septosa*. The last, besides being found growing in large irregular masses in the limestone, frequently occurs on shells such as *Productus giganteus*, of which fine examples are occasionally found. In certain of the localities crinoid remains are abundant, belonging to the genera *Hydreionocrinus*, *Platycrinus*, and *Poteriocrinus*. The polyzoa are also exceedingly abundant upon this horizon, and are represented by the genera *Ceriopora*, *Diastopora*, *Fenestella*, *Glauconome*, *Polypora*, and *Rhabdomeson*. Some of the other common forms are *Phillipsia eichwaldi* var. *mucronata*, *Productus giganteus*, *P. youngianus*, *P. aculeatus*, *Terebratula hastata*, the three varieties; several species of *Aviculopecten*, *Pinna flabelliformis*, *P spatula*, *Euomphalus dionysii*, large examples of *Dentalium ingens*, *Tomodus convexus*, *Psammodus porosus*, and several other palatal teeth.

In ascending order the next recognisable faunal association is that associated with the Calderwood Cement Limestone. Here, again, we have an estuarine and marine type in association, the former represented by the shales containing *Posidonomya corrugata*, scales, plates, and bones of fishes of an estuarine character, and land plants, the marine type by the cement limestone full of brachiopods and other purely marine forms. A list of the more characteristic species of this horizon is given on pp. 188–189.

In the West of Scotland the principal features of the Top Marine Band are a bed of shale which is simply a mass of *Lingula squamiformis* and limey shales, sometimes passing into limestone largely made up of single ossicles of a small crinoid. At certain localities the calmy limestone or calcareous shales are largely made up of the crushed valves of *Schizophoria resupinata* and other small brachiopods, with which is often associated the polyzoan *Ceriopora interporosa*, which at certain localities occurs in great abundance. Another outstanding feature of this horizon is the abundant occurrence in the sandstones and limestones of the marine alga *Spirophyton cauda-galli*.

XIV. Position in the Avonian Sequence.

If the foregoing correlation of the Dunbar Section with the Hurlet Sequence prove to be the correct one—and it seems to us to rest upon a fairly strong foundation of facts—it enables us to form some idea of the relationships of the Lower Carboniferous Limestone Series of the Midland Valley of Scotland to the Carboniferous Limestones of the North of England. In the year 1898* the late Mr William Gunn of the Geological Survey showed that the group of the lower Scottish limestones about Dunbar and round the Midlothian Coalfield does not represent any part of the mountain limestone of Yorkshire, but is the equivalent of the upper part of the Yoredale Series of Phillips, and which were included by him in the Millstone Grit. He identified one of the Longcraig Limestones, or perhaps the set of them (it does not seem quite clear which), as the equivalent of the Eelwell Limestone of North Northumberland and the Five Yards Limestone of Weardale and Teesdale. The Skateraw Limestone (probably the Middle Skateraw) was considered by him to be the equivalent of the Acre Limestone of Lowick, and a still higher member of the Dunbar Group (probably the Chapel Limestone) he regarded as the equivalent of the Dryburn Limestone of Lowick and the Great Limestone of Teesdale and Weardale.

Professor E. J. Garwood in his paper on the Lower Carboniferous

* "Correlation of the Carboniferous Rocks of England and Scotland," *Geol. Mag.*, p. 342, 1898; also *Trans. Edin. Geol. Soc.*, vol. vii, p. 361, 1899.

Succession in the North-west of England * has correlated the limestones of Northumberland and North Cumberland from the Oxford Limestone to the Acre Limestone with his Upper Sub-Zone of the *Dibunophyllum* Zone in the North-western Province, which he has further correlated with the D_2 Zone of the South-western Province and Midlands. Though we have not had the privilege of examining Professor Garwood's ground, it seems to us that his various bands, sub-zones, and zones rest upon a thoroughly secure stratigraphical and palæontological basis, presenting phenomena similar to that which we have met with in the region just described.

In the present state of our knowledge of Carboniferous stratigraphy and palæontology we consider it particularly undesirable that too much reliance should be placed upon the appearance and disappearance of certain forms over limited areas, or that too much importance should be attributed to varieties or mutations supposed to represent an evolutionary series, and assumed to be of value as time indices. The late Dr Vaughan and his co-workers have as a rule based their correlations upon the general character of certain faunal assemblages, and have been enabled thereby to trace certain horizons from place to place, and it is upon this principle that the marine strata of the Scottish Carboniferous System from the Hollybush Limestone up to the Upper Limestone Series have been correlated with the Upper Sub-Zone D_2 of Vaughan's *Dibunophyllum* Zone. That there is a close resemblance between the faunal assemblages of the two areas cannot be disputed, but that these indicate anything more than a similarity of physical conditions we consider has yet to be proved.

XV. Physical Conditions of Deposition.

Judging from the evidence that has been adduced in the foregoing part of this paper, it will be seen that the physical conditions which prevailed during the deposition of that part of the Carboniferous Formation which lies below the Blackbyre Limestone presented three well-marked and diverse types, namely, that seen in the West of Scotland, that seen in the Lothians, and that seen in East Fife. But when we reach the horizon of the Hurlet Limestone, and from that up to the Top Marine Band, a fairly uniform condition of deposit seems to have prevailed over Central Scotland.

The Hollybush and Blackbyre Limestones where typically developed are full of such clear water marine forms as foraminifera, corals, crinoids, echinoids, polyzoa, lamellibranchs, gasteropods, cephalopods, brachiopods, and

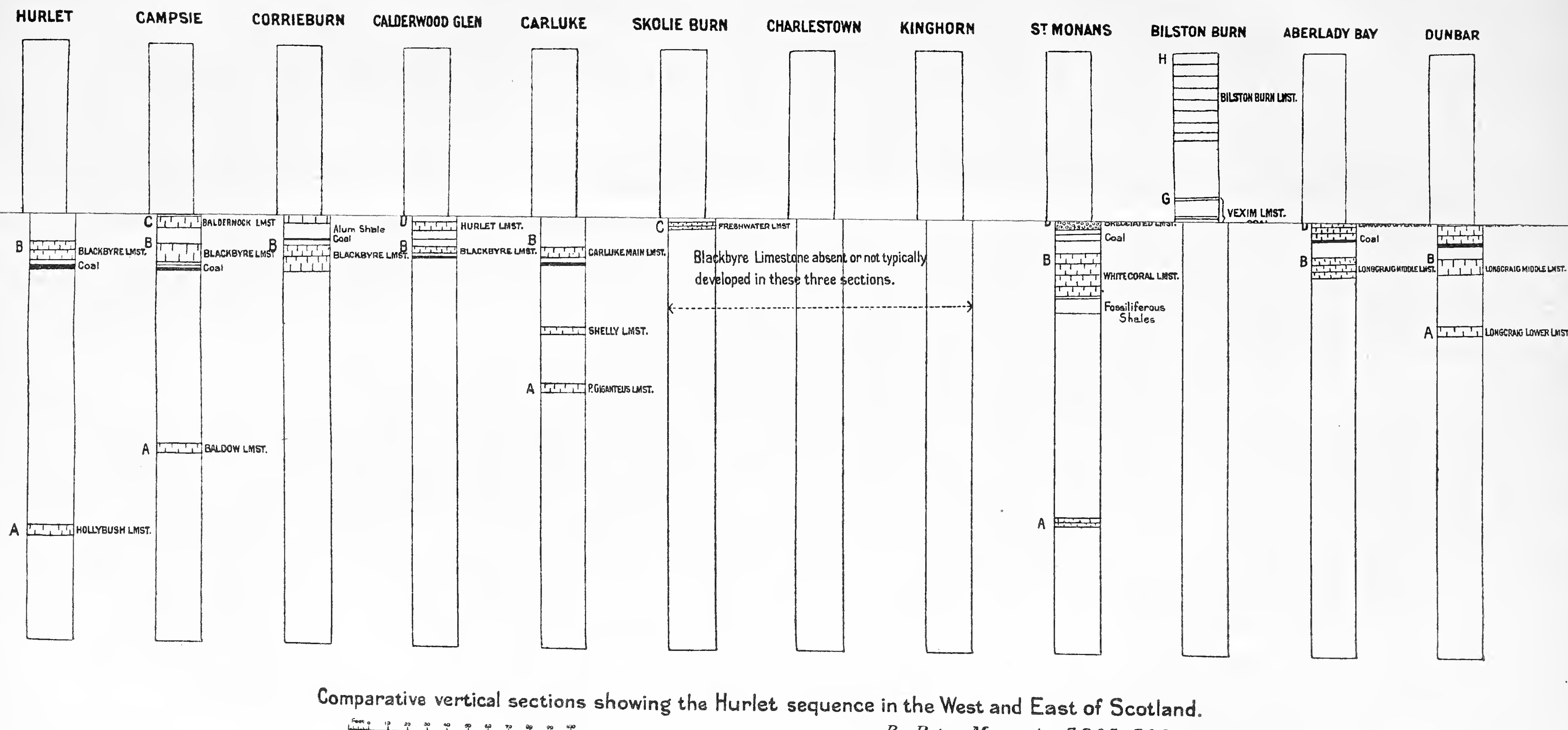
* "The Lower Carboniferous Succession in the North-west of England," *Quart. Jour. Geol. Soc.*, vol. lxviii, p. 449.

the crushing teeth of fishes. Such compound corals as *Lithostrotion* and *Lonsdaleia* occur in regular bands in the position in which they grew, or in nests associated with brachiopods, gasteropods, polyzoa, etc. These limestones appear to have been formed of the detritus of calcareous organisms laid down beyond the reach of terrigenous sediments.

When we reach the top of the Blackbyre Limestone we find that the character of the limestones undergoes a sudden change, showing that the clear water coastal conditions under which the lower limestones were deposited had been replaced by an upheaval of the sea bottom, so that when the land again began to sink below the level of the sea, estuarine and lagoon conditions prevailed, in which river-borne mud derived from the land was being deposited. The succeeding limestones from the Hurlet Limestone up to the Top Marine Limestone indicate that the physical conditions prevailing over Central Scotland during their deposition were of a fairly uniform character compared with those which existed before the deposition of the Hurlet Limestone.

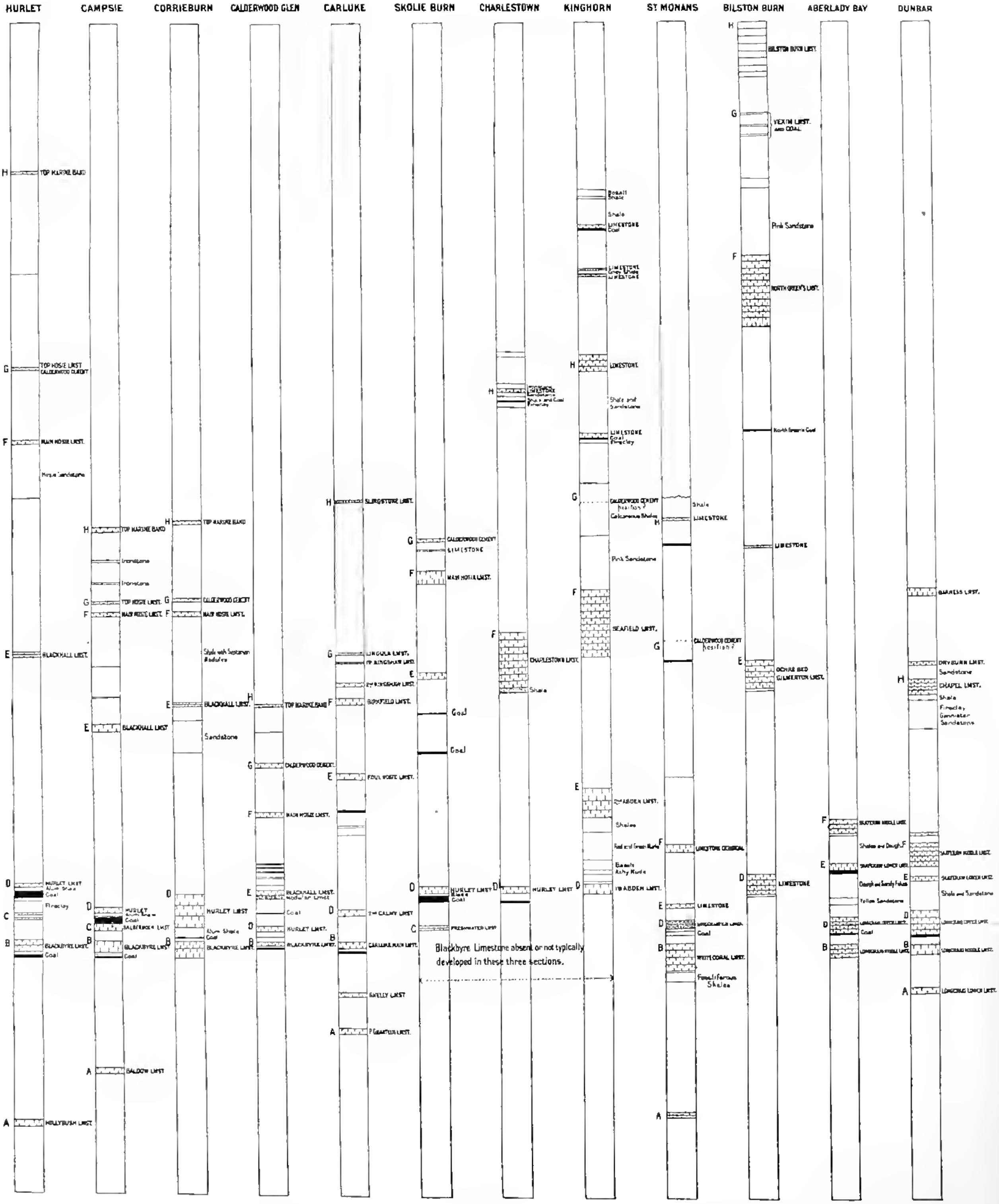
XVI. Summary of Conclusions.

The main object, then, of this paper has been to show that the different calcareous horizons in the Hurlet Sequence from the Hollybush Limestone up to the Top Marine Band can be traced right across Central Scotland to the East Coast. In the neighbourhood of Edinburgh the Hollybush and Blackbyre Limestones appear to be absent or at least are not typically developed owing to the existence over that area of estuarine conditions under which the oil shales were accumulated. But the occurrence of the Baldernock Limestone in its typical freshwater or estuarine aspect, the Hurlet Coal, and above all the Hurlet Alum Shale with its characteristic bone bed and lamellibranch and brachiopod faunas, present such a strong combination of stratigraphical and palæontological evidence as to leave no doubt in our mind that we are here on the position of the Hurlet Limestone and its associated strata, and that the Hurlet datum line can now be definitely fixed on clear stratigraphical and palæontological evidence. It has also been shown that the succeeding limestones, namely, the Blackhall, the Main Hosie, the Top Hosie or Calderwood Cement, and the Top Marine Limestone, can be identified over the whole of the area under consideration, not only on evidence of a purely lithological and stratigraphical character, but also by the fact that these different horizons present certain faunal assemblages which are sufficiently characteristic of the beds in which they occur to enable us to trace them over wide districts with a considerable degree of certainty.



Comparative vertical sections showing the Hurlet sequence in the West and East of Scotland.

By Peter Macnair, F.R.S.E., F.G.S.



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By Peter Macnair, F.R.S.E., F.G.S.

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MODEL INDEX.

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Cells, Liver,—Intra-cellular Canaliculi in.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

Liver,—Injection within Cells of.
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CONTENTS.

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[*Continued on page* iii *of Cover.*

XVII. Literature.

The literature bearing more or less directly upon the stratigraphy and palæontology of the Lower Carboniferous Limestone Series in Central Scotland is such an extensive one, that no adequate list of it can be given here. But underneath is a list of the principal publications in which it may be obtained:—

Transactions of the Royal Society of Edinburgh.
Maps and Memoirs of the Geological Survey of Scotland.
Prize Essays and Transactions of the Highland and Agricultural Society of Scotland.
Quarterly Journal of the Geological Society.
Monographs of the Palæontographical Society.
Geological Magazine.
Transactions of the Edinburgh Geological Society.
Transactions of the Geological Society of Glasgow.

(*Issued separately June* 25, 1917.)

XIII.—The Arithmetical Mean and the "Middle" Value of Certain Meteorological Observations. By L. Becker, Ph.D., Regius Professor of Astronomy in the University of Glasgow.

(MS. received April 30, 1917. Read May 7, 1917.)

The arithmetical mean of similar meteorological observations is usually regarded as their representative value. The object of this paper is to show that in a certain case this assumption is not borne out by observations.

The investigation refers to the maximum temperature in the shade as observed at Glasgow Observatory in the forty-eight years 1868 to 1916. For each of the 73 periods of 5 days the arithmetical mean of the (240) maximum temperatures observed in the forty-eight years was calculated, and the average maximum temperature, t_0, was interpolated from these means for each day of the year. Let $M(\tau)$ designate the number of days of same date on which the maximum temperature lies within the limits $t_0+\tau\pm\frac{1}{2}$ degree. For each day of the year $M(\tau)$ was counted, τ being successively all the positive and negative integers. These numbers change from day to day in a regular way for the same value of τ, and no appreciable error is committed by combining the numbers counted on all the days of the month and ascribing the result to the middle of the month. Table I contains the $M(\tau)$ per 1000 days. The figures for January are based on 1488 ($=48\times 31$) observations, and similarly for the other months.

We may assume that when the same cause is acting on different occasions, and is disturbed in a haphazard way, the effects will not be identical, but they will be grouped round an average effect, which effect is the most probable to occur. In the event of the effect having been measured and expressed in figures, the most probable value of the effect is the arithmetical mean of all these figures. It then follows that the individual values will be arranged round the arithmetical mean according to the Law of Errors. Hence the probability $\phi(\tau)$ of a departure $\pm\tau$ from the arithmetical mean is expressed as follows:

$$\phi(\tau)=h\pi^{-\frac{1}{2}}e^{-h^2\tau^2}.$$

Thus if there is only one constant cause to an effect, the arithmetical mean coincides with the "middle" value, *i.e.*, that value which occupies a middle position when the figures expressing the effect are arranged in order of magnitude, and the positive and negative departures from the arithmetical mean are equally probable. None of these properties are shown by the actual countings given in Table I, and it follows that the individual values are not grouped about only one value. It is noteworthy that the figures in a column can approximately be represented by two superposed theoretical error curves whose zeros do not coincide. This might have been expected in view of the fact that the temperature on a certain day is not only due to the heat radiated by the sun, but also to the direction of the wind, etc. Under these circumstances the best representative value of such or similar data is the middle value t'_0, for there are even chances of an observation lying above or below this value.

Let $t'_0 = t_0 + m$. The zero point from which τ is counted must be changed by m so as to give equal sums on both sides of the zero. m is interpolated from the figures in Table I, and its values are given in the bottom line of the table. According to these values of m the middle value lies in winter up to 0°·5 F. above the arithmetical mean, and in summer up to 0°·7 F. below the arithmetical mean.

As to the grouping of the individual temperatures about the middle value, let $N(\tau)$ designate the number of days of same date in the forty-eight years on which the maximum temperature lies within the limit $t_0 + \tau - \frac{1}{2}$ and $t'_0 + \tau + \frac{1}{2}$, t'_0 being the middle value of the maximum temperature on that date. Hence $N(\tau) = M(\tau + m)$, and $N(\tau)$ can be obtained from Table I by interpolation. The result is compiled in Table II. In the winter months the temperatures are scattered about 8° F. further to the negative side of the middle value than towards the positive side, and from March to October the extremes of the high temperatures lie much further from the middle value than the extreme low temperatures do, the scattering in September extending 10° F. further to the positive side than to the negative side. Hence the curves representing $N(\tau)$ with abscissæ τ are steep for positive τ in winter and for negative τ in summer.

When all the months are taken together the $N(\tau)$ curve becomes symmetrical. And this might have been expected. For the heat of the sun is the one cause which operates the whole year in the same direction, while the other causes, wind direction, etc., act in a haphazard way during a year, and produce accidental departures. The average scattering of the temperatures in a year about their respective middle values well agrees with that calculated from the Theory of Errors. In applying this theory we might adapt its terminology to our case, and call the "probable error," $r = 0{\cdot}4769/h$, the "probable scattering." The frequency numbers, $N(\tau)$, are given by the function [*Continued on page* 214.

TABLE I.—M(τ).

τ.	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
+22°	...	...	...	...	...	...	...	...	1	...	...	...
21	...	...	...	...	...	...	...	...	1	...	...	...
20	...	...	...	...	...	1	1	1	...	...	...	...
19	...	...	...	1	...	...	1	...	...	...	...	...
18	...	...	...	1	2	3	1	1	...	...	...	...
17	...	...	1	2	1	...	2	1	1	...	...	...
16	...	...	1	2	2	7	3	1	1	1	...	...
15	...	...	1	7	2	6	...	3	1	1	...	...
14	...	...	3	4	7	8	5	1	3	...	...	...
13	1	1	6	8	9	11	4	5	3	1	1	1
12	1	1	4	4	14	12	6	9	1	2	3	6
11	4	1	4	11	7	10	10	5	5	2	4	7
10	8	9	8	8	16	12	7	17	4	3	8	18
9	23	15	7	9	21	19	21	16	9	7	20	24
8	37	24	19	18	25	19	23	18	12	16	35	41
7	48	39	33	26	24	24	22	20	19	22	44	47
6	59	58	41	31	37	30	28	26	23	40	37	52
5	73	67	63	45	36	45	37	27	38	44	51	52
4	59	84	60	49	39	41	43	35	49	57	55	58
3	63	69	80	54	52	51	50	47	72	73	62	57
2	60	64	66	81	63	62	64	58	72	83	64	54
+ 1	58	68	80	71	62	57	67	85	86	95	78	60
0	56	61	60	85	78	67	93	91	114	108	79	64
− 1	52	57	77	85	85	67	94	116	115	91	64	50
2	59	55	60	88	79	89	83	113	105	89	89	57
3	60	55	64	68	69	81	85	103	83	77	63	57
4	53	60	55	66	57	52	79	77	67	47	53	59
5	54	57	46	51	61	70	73	50	51	38	53	57
6	48	45	46	31	45	68	36	28	32	30	35	38
7	37	33	38	34	42	32	26	21	15	27	21	44
8	26	33	29	29	26	19	17	12	10	24	31	26
9	11	16	17	16	15	13	9	5	4	10	18	14
10	20	8	15	7	7	10	6	3	3	5	11	11
11	8	7	9	5	9	8	2	3	...	4	4	13
12	6	3	5	1	4	4	1	1	...	1	6	11
13	4	4	1	...	1	...	1	...	...	1	5	7
14	5	3	1	1	1	...	...	1	...	1	2	3
15	3	...	...	1	1	...	...	...	...	...	1	5
16	1	...	...	...	...	1	...	...	...	...	1	3
17	...	1	...	...	1	1	...	...	...	...	...	1
18	2	...	...	...	...	...	...	...	...	...	...	1
19	...	1	...	...	...	...	...	...	...	...	...	1
20	1	1	...	...	...	...	...	...	...	...	1	...
− 21	...	...	...	...	...	...	...	...	...	...	1	1
Sums.												
0 to + 22	522	530	507	474	458	451	442	422	458	501	501	509
0 to − 22	478	470	493	526	542	549	558	578	542	499	499	491
m.	+0°·38	+0°·47	+0°·10	−0°·30	−0°·52	−0°·73	−0°·63	−0°·75	−0°·37	...	...	+0°·15

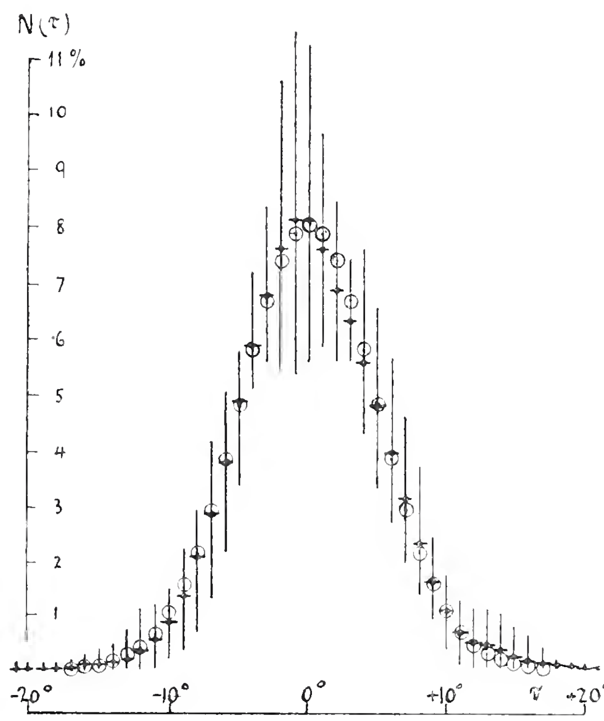
TABLE II —N(τ).

τ.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	Year.	Theory.
+21	...	...	...	...	...	...	...	...	1	...	...	...	...	...
20	...	...	...	...	...	1	1	1	1	...	...	...	...	...
19	...	...	...	1	1	1	1	1	...	...	...	...	...	...
18	...	...	...	1	1	2	2	1	...	...	...	...	1	...
17	...	...	...	2	1	4	2	1	1	...	...	...	1	...
16	...	...	1	4	2	7	2	2	1	1	...	...	2	1
15	...	...	1	5	5	8	3	2	2	1	...	...	2	1
14	...	...	3	6	9	10	4	4	2	...	...	...	3	1
13	...	...	5	7	11	11	6	7	2	1	1	1	4	3
12	1	...	4	7	11	11	7	8	3	1	2	4	5	4
11	2	1	5	8	12	12	10	13	4	2	4	8	7	7
10	7	5	6	9	17	16	16	16	6	4	9	15	11	11
9	18	11	9	12	23	19	21	18	10	8	21	25	16	15
8	31	20	17	19	26	23	22	20	14	15	34	37	23	22
7	44	32	30	28	30	30	26	24	20	24	41	46	31	30
6	56	49	43	36	35	39	33	27	29	36	42	51	40	39
5	66	65	57	45	38	43	40	33	42	46	49	54	48	48
4	65	76	65	52	45	49	48	43	57	57	56	56	56	58
3	62	74	73	63	56	57	58	57	71	71	61	56	63	67
2	61	69	73	74	64	59	68	76	80	85	67	56	69	74
+ 1	59	66	73	79	72	63	84	92	97	97	75	60	77	79
0	56	64	69	84	81	71	92	109	113	102	76	61	82	80
− 1	54	59	70	86	82	82	89	115	112	96	74	56	82	79
2	57	56	66	81	75	79	85	106	98	88	78	54	77	74
3	58	56	62	70	64	66	82	83	78	73	67	57	68	67
4	57	58	56	59	58	66	72	56	61	52	56	58	59	58
5	54	58	49	46	53	63	50	34	43	37	48	54	49	48
6	50	50	45	34	43	44	31	22	26	31	36	46	38	39
7	41	40	39	31	33	23	19	14	13	27	26	40	29	30
8	29	33	29	25	21	14	12	7	7	21	25	29	21	22
9	19	22	20	14	12	11	7	4	4	12	19	17	13	15
10	15	13	14	7	7	8	3	2	2	5	11	12	8	11
11	11	7	9	3	6	5	2	1	...	3	6	12	5	7
12	7	5	5	1	3	1	1	1	...	2	5	11	4	4
13	5	4	1	...	1	...	1	...	...	1	4	8	2	3
14	5	3	1	1	1	...	...	...	...	1	3	4	2	1
15	4	1	...	...	1	1	...	...	...	...	1	4	1	1
16	2	...	...	...	...	1	...	...	...	...	1	3	1	1
17	1	1	...	...	...	...	...	...	...	...	...	2	...	...
18	1	...	...	...	...	...	...	...	...	...	...	1	...	...
19	1	1	...	...	...	...	...	...	...	...	...	1	...	...
20	1	1	...	...	...	...	...	...	...	...	1	1	...	...
−21	...	...	...	...	...	...	...	...	...	...	1	...	...	...
Sums.														
0 to + 21	500	500	500	500	500	500	500	500	500	500	500	500	500	500
0 to − 21	500	500	500	500	500	500	500	500	500	500	500	500	500	500

$$N(\tau) = 1000\int_{\tau-\frac{1}{2}}^{\tau+\frac{1}{2}} \phi(\Delta)d\Delta = 1000\int_{h(\tau-\frac{1}{2})}^{h(\tau+\frac{1}{2})} \pi^{-\frac{1}{2}}e^{-tt}dt.$$

This function is tabulated in treatises on the Method of Least Squares.* The observed frequency numbers are best represented by the $N(\tau)$ calculated with the probable scattering of $\pm 3{\cdot}3°$ F. These values appear in the last column of Table II.

It appears to me that the probable scattering of temperature is an important representative figure in the definition of a climate.



The points of both yearly curves are also drawn in the diagram, in which the vertical lines show the range of $N(\tau)$ during the year, the horizontal lines mark the observed points, and the circles indicate the calculated points.

I am indebted to Mr J. Connell for effecting the countings utilised in this paper.

* See Chauvenet, *Spherical Astronomy*, vol. ii, table ix, A.

(*Issued separately June* 25, 1917.)

XIV.—On some Nuclei of Cloudy Condensation. Part III.

By Dr John Aitken, F.R.S.

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In communications to this Society in 1893 * and in 1912,† and in other papers, I have shown that some of the nuclei in the atmosphere have a power of condensing water vapour even when the air is unsaturated, owing to the material of which they are composed having an affinity for water. All kinds of nuclei have an affinity for water vapour, and by their surface action condense some water at all degrees of humidity, so causing increase in their size and hazing effect; but the particles having an affinity for water condense very much more and cause fogs in unsaturated air. In the present communication I intend to deal with the more common form of nuclei—that is, those which have little or no chemical affinity for water—and with those nuclei which require supersaturation to cause condensation; the reason for this investigation being some statements which have appeared in scientific journals in this country and abroad regarding the nuclei counted by the condensation method. It has been definitely stated that the greater number of the nuclei counted are not dust particles but only ions, or aggregations of them. It therefore seemed desirable that further investigation should be made into the subject to see if there was anything to support this ionic theory of the condensation nuclei in the air, and at the same time to inquire into their origin.

The best way of attacking this question seemed to be to make an investigation into the sizes of the nuclei in the atmosphere; because if any of them are formed by the combination of ions, then we would expect to find in the air some nuclei of all sizes—from the size of ions which require a very high degree of supersaturation to make them centres of condensation, up to a size requiring only a very slight supersaturation. For this purpose some new apparatus was necessary to enable us to sift and grade the particles into their different sizes, and to give some idea of their relative numbers. The method adopted of sorting out the different-sized nuclei was to make them centres of condensation at different degrees of supersaturation, all the larger ones being showered down with slight supersaturation, and the smaller ones with increasing degrees of it.

* "On some Observations made without a Dust-Counter," *Proc. Roy. Soc. Edin.*, 1892–93.

† "The Sun as a Fog-Producer," *Proc. Roy. Soc. Edin.*, vol. xxxii, part ii, No. 16.

It was therefore necessary that we should have some form of apparatus that could be easily, accurately, and quickly manipulated, and that would give definite degrees of expansion to any desired amount. Fig. 1 shows the general arrangement of the different parts of the apparatus used, though not in their relative positions. T is the test-flask in which the cloudy condensation takes place, and which is illuminated by the incandescent gaslight L in the dark lantern D, the light being concentrated in T by means of a globe of water C. Three tubes pass through the stopper S of the test-

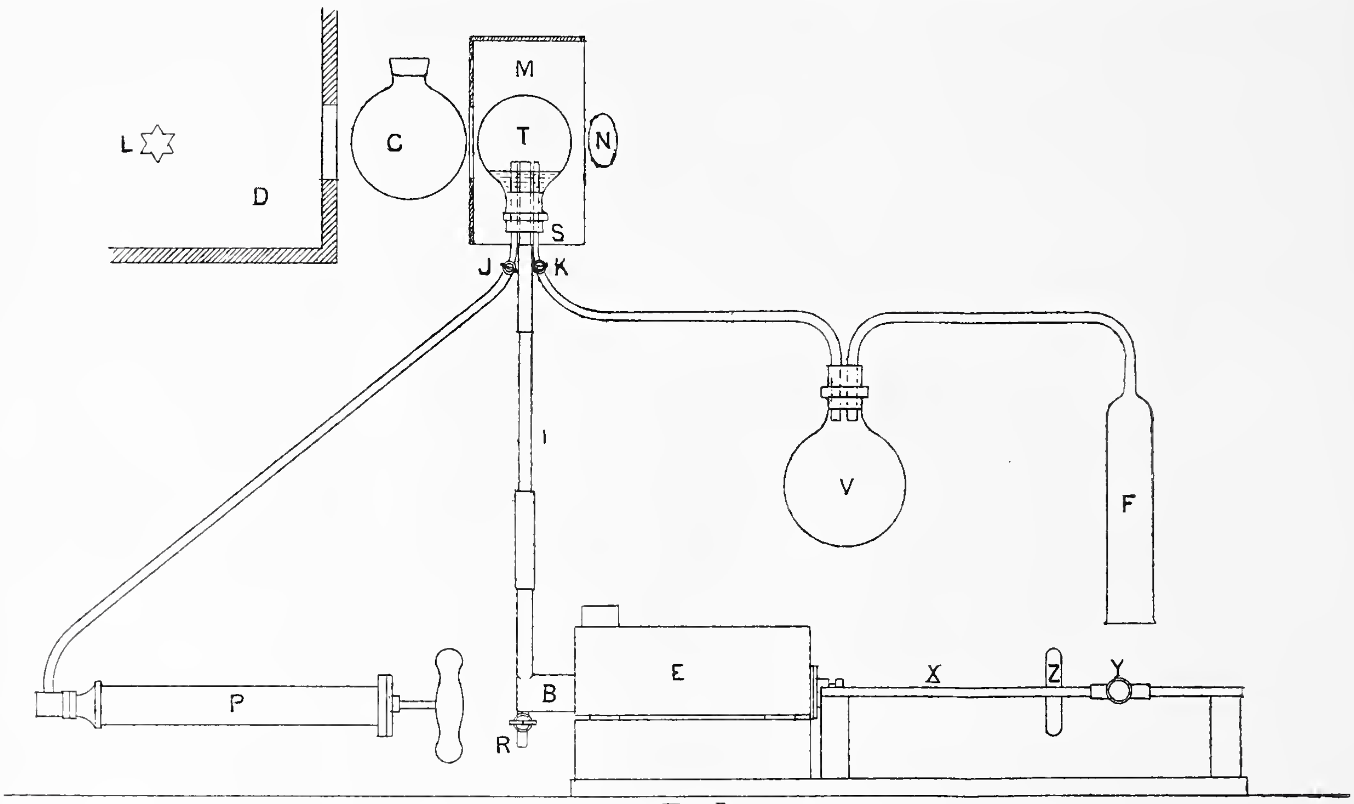


Fig. I

flask T. One of these tubes connects the flask with the pump P, and is provided with a stopcock J. The pump has a capacity of 152 c.c. The middle tube connects with what we will call the expander E, while the third tube, which is provided with a stopcock K, goes to the source from which we wish to draw, by means of the pump P, the air to be tested in T. F is a cotton-wool filter which is introduced when required; N is a lens for observing the condensation in T; M is a box, blackened inside, surrounding the test-flask to cut off all light that might interfere with the observation of what is taking place in the flask.

The expander, which is the new part of the apparatus, is shown in horizontal section in fig. 2. It is practically a pump working in water. The barrel B of the pump is fitted with a piston H of two cupped leathers, and is enclosed in the small tank G filled with water, the top end of the pump opening into the tank. By this arrangement no air can leak past the piston and interfere with the tests. In order to get definite amounts of expansion, the bar X carries a slide Y under which there is an ordinary boxwood centimetre scale U. On the slide Y is a projecting part which stops the pump-handle Z at any desired point. When Z is pushed full in and the slide is pushed up to it, the edge of Y is at zero on the scale; so that when the slide is moved to any figure on the scale and the expander-handle Z drawn out till it is stopped by it, we know exactly what length of stroke we have made, and, knowing the diameter of the pump and the capacity of the test-flask and tubes, we can calculate the expansion.

The expander E is connected with the test-flask T by means of the tube I. This tube is made up of a short length of glass tube 6 mm. inside diameter, and two short lengths of rubber tube which have spiral wires inside to prevent them from yielding to outside pressure. The glass part

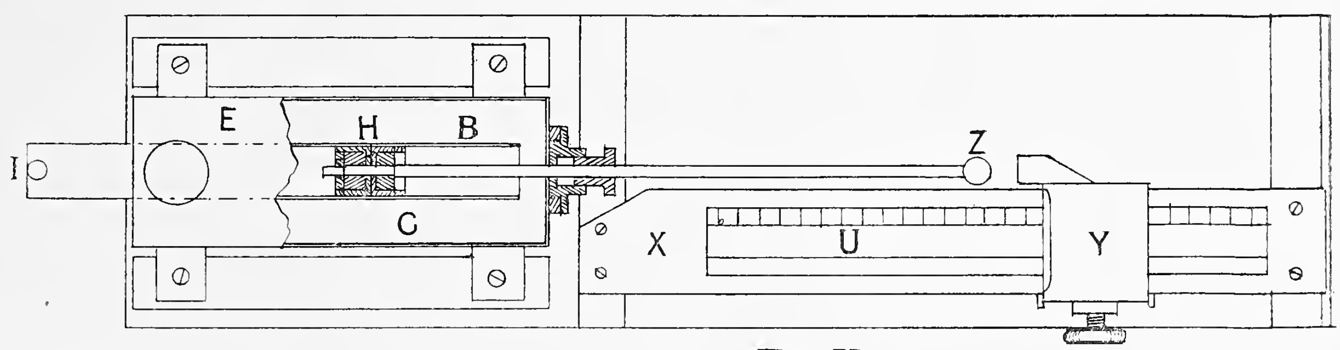


Fig. II

of this tube serves the useful purpose of showing if there is any leakage in the piston H, the water showing in the tube. If leakage occurs, it can be removed by means of the tap R. The substance we wish to test is put in the flask V, which is supplied with dustless air by the filter F. When testing the effects of heat, the flask V is removed and a tube put in its place between the filter and the test-flask; the tube is heated by a bunsen burner under it, and the substance to be tested put in the tube.

It may be thought that the tube I connecting the expander E with the test-flask is unnecessarily long. There are reasons, however, for the length. First, the flask T should be at a convenient height for observation; second, the handle Z of the expander should be at a convenient height for quick manipulation; and, third, the pipes connecting the test-flask must have a considerable degree of freedom, as during the tests it requires frequently to be turned upside down and from side to side, to wet the inside walls. If this is not done we cannot be sure the air inside is saturated, unless we allow long intervals of time between each expansion.

As stated, if we know the length of stroke and the diameter of the expander pump, also the capacity of the test-flask and its tubes, we can calculate the expansion in the test-flask. To simplify matters and save calculations, an easier method, however, was adopted. The capacity of the expander barrel was carefully measured with water, and found to contain 4·6 c.c. per cm. of stroke. If the capacity of the test-flask and tubes was 460 c.c., then 1 cm. of stroke would give 1 per cent. of expansion of the air in the flask. In these tests a smaller flask was generally used, as it enabled the tests to be made more quickly and only half the quantity of air was used. It held about 400 c.c. of water. The flask was first filled with water, then 216 c.c. were measured out; this with 14 c.c. in the tubes makes 230 c.c. to be operated on, so that when the slide is put at 1 cm. on the scale the expansion is 2 per cent. In writing the above degree of expansion we may either use the ordinary mathematical expression of $v_2/v_1 = 1{\cdot}02$, or we may write $\frac{2}{100}$ or $\frac{1}{50}$, or simply 2 per cent. The latter seems to be the simplest with our new apparatus, and will be adopted in what follows.

Though we can easily get definite degrees of expansion, we cannot by any means get such definite degrees of supersaturation. First, because we can only get very slight degrees of saturation while there is any dust in the air, however great the expansion may be, since the amount of condensing surface presented by the cloud particles prevents it. Second, the size of the vessel has a great effect even when there are no nuclei present. If the vessel be only 2 or 3 centimetres in diameter, the air is rapidly heated by its walls, and also rapidly unburdens itself of the vapour to the nearest surfaces. This diffusion of vapour, I have previously shown (*loc. cit.*), takes place with very great rapidity. Third, the rate of expansion is an important factor, and in small vessels is of very great importance; but even in large vessels it has a considerable effect. The exchanges of heat between the air and the walls of the vessel and the diffusion of the vapour take place so quickly that it is only when the expansion is instantaneous and the air dustless that the calculated supersaturation can be attained. All decrease in the rate of expansion means lower supersaturation, even in dustless air, and this is particularly the case where high supersaturations are required. For instance, if we use an ordinary air-pump to expand the air, we find that the length of stroke required to produce condensation on ions is never the same in two tests, because we cannot make the successive expansions at the same rate; and the wire-drawing effect of the valve further complicates matters. For this reason there is no valve in the expander used in these tests, and the air, instead of having to pass through a valve of about 1 or 2 mm. diameter, has an opening of 6 mm. The expander has the disadvantage, however, that, though the stroke may be made almost instantaneously, expansion is slowest at the end, while it is better that it should be quickest at that stage. Still, in spite of that defect, condensation can be steadily produced on ions with about a 25 per cent. expansion. Professor Wilson's latest figure is slightly lower than this, but his apparatus is better arranged for giving expansions at great rapidity and at nearly constant speeds.

In what follows it will be frequently necessary to speak of the size of the particles or nuclei. So before proceeding further, and to save constant qualification, it will be well to say what is meant here by the term "size." As already stated, all particles have an affinity for water, in the same way as all large solid surfaces have; but we do not know that this surface attraction is the same for all kinds of particles: we have therefore to assume that it is. When the particles are large they all become centres of condensation with the slightest supersaturation; but when they are very small this does not happen unless the air is to some degree supersaturated, and the smaller they are the higher the degree of supersaturation required to make them active. When, then, we use the word "size," we mean their condensing power, which does not necessarily mean their relative dimensions, though probably not far from it.

The first use to which the new apparatus was put was an inquiry into the size of dust particles in the atmosphere. In my early communications it is stated that almost all the particles in the air become active and are carried down in raindrops with a supersaturation of 2 per cent.; and other observers have found the same thing. We were wrong, however, though correct for the air we experimented with — that is, the air of a room in which gas is burning. On testing air drawn directly from the outside, or air of a room in which no gas was burning, a different result was obtained, and particles requiring a higher degree of expansion to make them active were always found.

Before going further it will be as well to explain in greater detail the method of working when making a test. The tube K is connected with a pipe leading through the frame of the window to the outer air. The stopcocks J and K are opened, and a number of strokes made with the pump P to clear out the air in the test-flask T and replace it with the air to be tested. Stopcocks J and K are then closed, and T is taken out of its support and moved so as to wet its inside walls and then replaced. The flask being now full of the air to be tested, we can proceed to clear out the larger particles so as to ascertain if there are any small ones. The method of clearing out the larger ones is to make them centres of condensation, so causing them to fall out of the air as raindrops. A few particles, owing to their affinity for water, will do this of their own accord if time be given. Many will fall out with a 1 per cent. expansion; but it takes many expansions and a long time to remove them, as they are very small and fall slowly, and, further, many of them evaporate before reaching the bottom owing to the rise in the temperature of the air in the flask. A 2 per cent. expansion was therefore selected as a convenient and more practical lower limit to be used in the removal of the larger particles.

We may now proceed in either of two ways to clear out the larger particles. We may set the expander to give a 2 per cent. expansion and make a number of quick expansions causing successive showers of very small drops, and continue this till no drops appear. But even with a 2 per cent. expansion this is a very slow process, and involves great waste of time. Much time can be saved, however, if we set the expander to give, say, an 8 per cent. expansion; and if, instead of making the expansion quickly, it be done slowly, the clearing-out process can be greatly accelerated. In this method of working, though we are making expansions of 8 per cent., practically no supersaturation is produced, because so long as there are plenty of large nuclei present there is no possibility of causing supersaturation; the large amount of surface presented by the dense cloud prevents it. The advantage of the process is that we grow the cloud particles to such a size that they fall, and continue falling during the whole time the expansion is being made. To obtain the best result the expansion should begin with extreme slowness, and should then be quickened; the very slow expansion picks out a small number of the largest particles, and these, becoming larger by the water deposited on them, continue to grow as the expansion is quickened. By this means fewer nuclei become active and grow larger, so ensuring their falling out. If a quick expansion is made at first, too many particles become active, and so grow to a less size; fewer therefore fall out before the air recovers its temperature. These slow, large expansions of 8 per cent. can be made till the number of nuclei becomes small, and may even be made at a certain speed till all are down; but it is better to stop the large expansions when the particles are becoming few, and slide the stop on the expander down to 2 per cent. When this is done, a few quick expansions of 2 per cent. clear out all the remaining large particles, leaving only the smaller particles, which require higher supersaturations. During this process the test-flask must be frequently turned over to wet the interior, particularly when finishing clearing at each degree of expansion.

While the clearing process is going on we get a rough indication of the number of large particles in the air tested. In these tests a record was kept of the number of expansions at 8 per cent. and the number at 2 per cent. required to clear them out. It is, of course, only a rough indication, as the numbers brought down in the different showers will probably vary a good deal.

The sample of air being now free of all the large particles—that is, those capable of becoming nuclei with a 2 per cent. expansion,—we next proceed to test for any smaller particles that may be in the air. For this purpose the slide on the expander is now moved to, say, 4 per cent., when the handle of the expander is quickly drawn to the stop and the test-flask examined to see if any cloud particles are formed. If a shower falls, the treatment is repeated till condensation ceases, the number of showers observed being entered in the notebook. When the showers stop, the slide on the expander is moved up to, say, 6 per cent., and the process is repeated till the showers stop, the result being entered in the notebook. Higher and higher expansions are continued till all condensation ceases, showing that all nuclei, large and small, have been brought down.

Table I.—Nuclei from Different Sources.

No. of Test.	Source of Nuclei.	Density of First Condensation at 2 p.c.	Number of Showers at Higher Expansions.								Remarks.
			4 p.c.	6 p.c.	8 p.c.	10 p.c.	12 p.c.	14 p.c.	16 p.c.	18 p.c.	
1	Outside air	Dense	9	3	1						
2	"	"	3	2							
3	Room air	Very dense	0								
4	Electricity	0	$\frac{1}{5}$	...	...	1	...	...	1	$\frac{1}{4}$	Electrophorus, two electrodes.
5	"	0	0	...	...	$\frac{1}{4}$	...	1	...	1	One electrode.
6	"	Slight	10	7	4	$\frac{1}{2}$	$\frac{1}{10}$	...	...	...	Small spark.
7	"	Very dense	1	...	...	...	...	...	...	...	With condenser and spark.
8	Sun and SO_2	...	5	2	$\frac{1}{4}$	...	...	...	...	...	Short exposure.
9	"	Dense fog	0	...	...	...	...	...	...	...	Longer exposure.
10	El and SO_2	...	3	6	1	...	...	...	...	...	Very little SO_2.
11	"	Dense fog	0	...	...	...	...	...	...	...	More SO_2 added.

The results of the first tests with outside air and with the air in the room where gas was burning are entered in Table I. The outside air was always found to contain some small nuclei requiring more than a 2 per cent. expansion to make them active. Their number and size were variable. The table contains the results of two tests of outside air. In the first column is entered the number of the test; in the second, the source from which the air was drawn; in the third, the density of the first condensation at 2 per cent. expansion. In the next series of columns are entered the numbers of showers given at the different expansions above

2 per cent. When the tests given in this table were made, no record was kept of the number of showers required to clear at 2 per cent. expansion. Its density was simply entered, and when there was no condensation this was noted.

Consider the first test recorded, namely, outside air. The first condensation at 2 per cent. was dense—that is, above the usual density given by air from that source. After all those nuclei were cleared away with 2 per cent. expansions, the expansion was increased to 4 per cent., when a succession of expansions gave nine showers before condensation ceased. The expansion was then increased to 6 per cent., when three more showers were obtained; and one shower was brought down with an 8 per cent. expansion. It will be noticed that in the second test the number of very small particles was much less, and that none were found quite so small as in the first sample. The third entry in the table is the result of a test of the air in a room in which gas was burning. In this case there were no nuclei requiring more than a 2 per cent. expansion to bring them down. It may be mentioned that before the gas was lit there were plenty of very small particles and the air was similar to the outside air at the time, but shortly after the gas was lit the small particles could not be detected. Two explanations may be offered of their disappearance. One is, that they may get entangled and brought down in the first dense condensations. The other is, that when particles become densely crowded the small ones tend to disappear, probably by aggregating to form larger particles. Tests were made to see if the former supposition was correct. A quantity of room air was diluted with filtered air so as to reduce the density of the first condensation and allow of the larger particles being cleared with fewer expansions, but this seldom showed the presence of very small particles. It seems possible that the great number of ions produced by the burning gas may attach themselves to the nuclei, and these in turn, owing to their electric charges, may attach themselves to each other. That is, the ions by charging the particles may cause them to aggregate to form large nuclei.

Electricity and Nuclei.

The nuclei produced by the electric discharge were now tested. The flask V between the filter and the test-flask had two wires led into it—one through the stopper, the other through a hole in the side—and cemented airtight. Using a steel point and an electrophorus as a source of electricity, there were formed a few nuclei almost all requiring very high degrees of expansion to make them active (see test 4 in the table). The nuclei formed by the point discharge are very few, owing to their being deposited by the electricity, and one discharge of the electrophorus gave as great an effect as many discharges, owing to each succeeding discharge undoing the effect of the previous one. Little difference was found if there was no outlet for the electricity, the only wire entering the flask being the one with the discharging point (see test 5). The glass seems to act as the other electrode.

Spark Discharge.

The discharging point was now placed near the other electrode so that a very small spark could pass. The result is shown in test No. 6. The number of nuclei was now greatly increased, and many of them were large enough to be active with a 2 per cent. expansion. When blunt electrodes were used, with a greater distance between them, and a more powerful spark from a small induction machine with two small Leyden jars, the nuclei gave an extremely dense fog with the slightest supersaturation (see test No. 7). After remaining eighteen hours the nuclei were tested. When all the large nuclei had been disposed of, only one shower remained which required more than 2 per cent. expansion. These tests show that the number and the size of the nuclei can be changed by altering the length and density of the discharge.

Sulphurous Acid and Light.

The nuclei produced by the action of sunlight on sulphurous acid was tested by placing a little of the diluted acid in the flask V and drawing the air into the test-flask. With short exposure to light there were a number of small nuclei requiring more than a 2 per cent. expansion, but none large (see test No. 8). A longer exposure to light gave a dense fog with the slightest expansion.

Sulphurous Acid and Electric Discharge.

If a very weak solution of sulphurous acid be put in the flask V and an electric discharge be made in it, a great number of small nuclei are formed (see test No. 10). With stronger sulphurous acid, the condensation was very dense with the slightest expansion. This greater effect of the electric discharge produced by the presence of sulphurous acid over that given by air is probably due to the peroxide of hydrogen and nitrogen compounds formed by electric discharge.

Tests in Pure Air.

It was thought that some information might be obtained by an examination of the nuclei in purer air than can be obtained in this district. Loch Awe, where on a previous occasion some investigations were made in very pure air, was again selected. Before going there, however, the method of testing by means of the expander was systematised and developed, so as to make it give more definite information than had been previously got from it. Further, the method of illuminating the test-flask had to be changed; in place of gaslight, sky light had to be used. It was therefore necessary to practise here with the new arrangements before going north, so that the readings at the two places might be taken under the same conditions. Tests were also made on my return home, with the same arrangement as to lighting. These were made in the end of September and beginning of October last.

Table II.—Nuclei in the Air.

Date.	Hour.	Number of Showers to clear at		Number of Showers to clear at Higher Expansions.			Number of Dust Particles in the Air.	Wind.	Remarks.
		8 p.c.	2 p.c.	4 p.c.	6 p.c.	8 p.c.			
Sept. 22	11	6	4	2	1	...	...	S.W. ·5	Falkirk air.
23	11.30	6	5	2	$\frac{1}{4}$	...	...	S.W. 1	,,
27	12	1	8	5	3	2	...	E.N.E. 1	Loch Awe.
,,	3.30	1	8	7	4	2	6,000	,,	Local pollution.
,,	4.30	0	4	2	2	...	1,100	,,	Free from local pollution.
28	3	2	3	3	2	...	10,000	E.	Local pollution.
29	11.30	4	8	9	3	1	14,000	E.N.E. 2	,,
,,	12	1	2	1	1	...	1,250	,,	Free from local pollution.
,,	3	2	5	6	2	...	10,000	,,	Local pollution.
,,	3.30	0	4	2	$\frac{1}{4}$	...	1,125	,,	Free from local pollution.
30	10.30	3	5	6	1	...	14,000	,,	Local pollution.
,,	11	0	2	1	$\frac{1}{4}$	...	500	,,	Free from local pollution.
,,	3.40	3	3	3	...	...	3,500	...	Air in room.
,,	3.45	16	4	2	...	...	...	...	Air in room after burning wax vesta.
Oct. 1	11.45	2	2	1	...	...	2,500	S.W. ·2	Free from local pollution.
,,	...	2	3	2	...	...	2,500	...	Air in room.
,,	...	14	4	2	...	...	...	...	After burning wood match.
4	12	4	3	1	...	...	20,000	Calm	Falkirk air.
,,	4	6	4	3	...	...	25,000	,,	,,
5	1	5	2	2	$\frac{1}{4}$	...	7,000	W.S.W. 2	,,
,,	4	4	2	1	...	...	7,400	W.	,,
6	11	3	3	1	...	...	6,000	W.S.W. 1	,,
,,	4	6	4	2	...	...	22,000	,,	,,

The method of making these tests will be understood from an examination of Table II. In the first two columns are entered the date and hour of the observation. Then after the test-flask has been filled with the air to be tested, and its inside walls wetted, the two stopcocks are closed, the expander stop having been previously set at 0. The stop is now moved back to give an 8 per cent. expansion, and the handle Z slowly drawn up to the stop, when a fairly dense condensation appears in the flask. Time is given for the cloud to settle, but if it is dense it will not all settle, much of it evaporating. When it has ceased falling, the handle of the expander is put back to 0 and time allowed for the air to get saturated, when another slow expansion is made and another shower falls. This process of slow, large expansions is continued till the condensation gets thin. The number of expansions required to secure this is noted and entered in the third column of the table. When the condensation becomes thin there is a risk of the high expansion giving too high a supersaturation. The stop should therefore be put back to a 2 per cent. expansion, and quick expansions at this rate continued till the showers cease, the number of these 2 per cent. expansions being entered in the fourth column.

By the above process all the nuclei in the sample of air which were active with a slight expansion have been cleared out. We then proceed to test if there are any nuclei too small to respond to a 2 per cent. expansion. The stop is now moved back to give 1 or 2 per cent. greater expansion. In these tests an advance of 2 per cent. was adopted. Setting the slide to give a 4 per cent. expansion, a quick motion is given to the handle, and if there are any small nuclei a shower will fall. The process is repeated till the showers stop, after which the expansion is increased by 2 per cent. and the treatment continued till all condensation ceases, and the number of showers at each expansion are entered in columns 5, 6, and 7. Of course, the expansion must always be kept under 25 per cent., as ions become active with that degree of expansion.

Again referring to Table II, it will be seen that the air at Falkirk generally required a number of large, slow expansions at 8 per cent. and a number at 2 per cent. to clear the air of the larger particles. After the large particles were cleared out there were always sufficient smaller particles to give showers at 4 per cent., and in some cases at 6 per cent. The number of these small particles varied with the weather and the direction of the wind. From a great number of tests, not here recorded, it was found that there were always fewer particles with southerly than with easterly winds, and that the easterly air contained smaller particles than the southerly. We will refer to this point later, but it may be stated here that the greater number and smaller size of the nuclei in the easterly air has no connection with the direction of the wind, but is due to the position of the place of observation relatively to the local pollution.

In one respect I was unfortunate in not getting any pure air at Loch Awe during this visit. The wind never blew from the N.W. quadrant, the only direction which brings air with 100 to 200 particles per c.c. On this occasion the wind was generally E.N.E., and the lowest number observed was 500 per c.c. Though unfortunate in this respect, I was fortunate in another way. The E.N.E. wind brought to the window of the room where the tests were made the polluted air from two houses distant some few hundred yards. At first it looked as if the persistent E.N.E. wind was going to stop the work by bringing polluted air to the place of observation. However, on hunting about cross-wind-ways with the dust-counter I found a room where there was no local pollution and the numbers were low and constant. The test-flask was therefore taken to this window, filled with pure air, and returned to the operating-room and tested. The constant presence of polluted air at the window, instead of being a disadvantage, turned out to be an advantage, as it gave me an opportunity of comparing newly polluted air with air free from pollution. It will be seen from Table II that not only did the locally polluted air contain a greater number of particles as counted by the dust-counter, and require a greater number of showers to clear at a 2 per cent. expansion, but it had more very small particles. While the locally polluted air had particles requiring 8 per cent. quick expansion to make them active, the purer air had few requiring a 6 per cent. expansion.

From these tests it would seem that time plays an important part in the disappearance of the very small particles, greater numbers being present in newly polluted air than in ordinary air. This disappearance of the small nuclei has been observed in laboratory tests. If we confine a quantity of air for a considerable time, the number of particles gets reduced, but the very small ones tend to disappear more quickly, and in many cases nothing but particles requiring only a 2 per cent. expansion remain. The very small particles seem either to get deposited on the sides of the vessel or become attached to other particles. It will be noticed that the number of particles counted by the dust-counter and entered in column 8 of Table II does not quite agree with the number of showers given by the new apparatus. One reason for this is that the tests were not made at the same time, one being made some time after the other, and so not made with the same air; and as the number varies quickly in polluted air, the differences are only what might be expected.

One naturally asks what is the origin of these very small nuclei in the polluted air at Loch Awe. They came with the products of combustion, but to what are they due? A partial answer to this question suggested itself while working at Loch Awe. On two occasions when the air in the room was fairly pure, and contained no particles requiring more than an expansion of 4 per cent. to make them active, a match was burned and the products thoroughly mixed with the air in the room. This of course gave a great increase in the total number of particles, which required a great number of expansions to clear; but it will be seen from Table II that there was no increase in the number of very small particles. From this it would appear that neither flames nor solids in combustion are the source of the very small particles. It may be pointed out that these tests are open to the objection that the very small particles may have been brought down by the great number of showers. I have already shown reason for supposing that they did not exist at the beginning of the test. That they can escape being brought down by the showers is evident from the results of test No. 3, Table III. In that experiment the nuclei were so numerous that a large amount of filtered air had to be added before beginning the sifting process, and a very great number of showers was required to clear out the larger particles, yet there remained nuclei enough for many showers up to very high expansions. In that case there was probably no electric charge, as the temperature at which they were produced was not high enough to cause its escape from the iron.

Nuclei produced by Heat.

As the very small particles found in polluted air do not seem to be produced by combustion, an investigation of the effects of heat on different substances was made. In a paper, "On Dust, Fogs, and Clouds,"* read before this Society in 1881, I pointed out that when bodies are cleansed by heat enormous numbers of nuclei are produced, and that it was possible easily to detect the impurity driven off a small piece of iron wire only $\frac{1}{2000}$ of a grain in weight. This suggested that some of the very small nuclei in the air might have their origin in the something driven off by the heat of the fire from the coal and other bodies exposed to it. As this early experiment gave no indication of the size of these nuclei, the subject was investigated with our new methods.

In order to test the effect of heat in producing nuclei from different kinds of matter, it was necessary to have a vessel which would stand heating without giving rise to nuclei. Tubes of a number of different substances were therefore tested. To do this the flask V was removed and a tube introduced in its place, the air being drawn from the filter through the tube into the test-flask. The tubes tested were of iron, brass, copper, glass, porcelain, alundum, and two kinds of silica tubes. One of these latter is transparent and looks like glass, and the other opaque and looks as if composed of white silk threads stretched lengthwise. Two

* *Trans. Roy. Soc. Edin.*, vol. xxx, part i.

bunsen burners were used in these tests—one an old form, which does not give a very high temperature; the other the recent form, which gives much higher temperatures.

The first thing to be done was to heat the tube very highly to cleanse it, filtered air being drawn through it to carry away the impurities. This air always gave very dense condensation; the test-flask looked as if it were packed with cotton-wool. After the high-temperature flame had been applied for some time, moving it over some length of the tube, the low-temperature flame was used to see if it gave any nuclei at the lower temperature, as at the higher temperature all the tubes gave great quantities of nuclei. At a low red heat some of the tubes were inactive, but all of them became active after the temperature was raised a little higher. It soon became evident that, if we simply observe the effect of the flame on the tube, we can get the information wanted without testing the air for nuclei. So long as the flame is unaltered by the presence of the tube, no nuclei are produced; but when the temperature is such that the flame above the tube is different in appearance from what it is below, it is found that some action has taken place inside the tube which produces nuclei. Take, for instance, the glass tube: it is inactive till the temperature is high enough to show the sodium colouring in the flame; whenever that appears, dense fogs are formed in the test-flask. In most cases the flame above the tube, when examined with a pocket spectroscope, showed an increase in the brightness of the D lines.

There is, however, an interesting exception to this rule. The transparent silica tube produced very little change in the flame, and yet at a certain temperature it was an active nucleus-producer. Watching this tube as the temperature was raised, it was noticed that it had the appearance of cracking into pieces. Bright, shining facets appeared, which looked as if they were reflecting light, which, however, could not be the case, as there was none to reflect, the room being dark except for the pale light of the bunsen flame. These shining facets were self-luminous, and shone with the brightness of any small piece of opaque substance which happened to be in the tube for testing. It was noticed that so long as these bright cracks were absent the tube did not produce nuclei, but whenever they appeared nuclei also appeared in the test-flask, and the density of the condensation was roughly proportional to the extent of the facets. It may be mentioned that these crack-like facets, or at least some of them, appeared at the same places on successive heatings. One irregular crack seemed to pass nearly round the tube, which looked as if it would fall in pieces, but no change was noticed when the tube was cold. These bright cracks seem to be the source from which come the nuclei produced by this tube. The cause of the luminosity of these cracks may be worth investigating.

A great number of tests were made with the tubes and with substances placed in them and heated to a temperature below that at which the tube was active. For instance, about 10 cm. of fine iron wire was coiled up into a length of 2 cm., and after the tube had been cleansed by heat was drawn by a magnet from the cold part of the tube into the cleansed part and heated to a temperature much under red heat. As was found in the early experiments, this gave a very great number of nuclei; so dense was the fog

Table III.—Nuclei produced by Heat and Chemical Action.

No. of Test.	Substance used.	Strokes of Pump to Dilute.	Number of Showers to clear at		Number of Showers to clear at Higher Expansions.									Remarks.
			8 p.c.	2 p.c.	4 p.c.	6 p.c.	8 p.c.	10 p.c.	12 p.c.	14 p.c.	16 p.c.	18 p.c.	20 p.c.	
1	Iron wire (Hot)	12	28	20	22	9	7	3	2	½	...	...	...	Much under red.
2	,, (Hot)	8	9	3	4	3	2	1	½	...	...	...	...	Effect of successive heatings all under red.
3	,, (Hot)	6	3	3	6	5	4	3	3	1	...	...	...	
4	,, (Hot)	0	1	2	4	10	10	7	6	6	4	2	1½	
5	,, (Hot)	0	0	1	4	7	6	10	8	6	5	4	3	
6	,, (Hot)	7	13	2	½	...	...	...	...	...	...	...	...	Dull red.
7	Platinum wire . .	6	13	7	3	½	...	...	...	...	...	...	...	Red.
8	Phosphorus (Cold)	0	0	½	3	6	2	1	...	...	...	...	...	Air drawn quickly through.
9	,, (Cold)	Very dense fog			...	...	...	...	...	...	...	...	...	3 minutes.
10	Magnesium (Cold)	0	0	...	...	½	...	4	...	2	...	½	...	Dry air 30 minutes.
11	,, (Cold)	0	12	9	5	3	2	1	...	...	...	...	...	Moist air 30 minutes.
12	,, (Cold)	0	...	...	...	...	...	...	1	4	2	1	...	,, 2 ,,
13	Zinc (Cold)	0	...	...	...	...	1	2	4	4	3	2	1	,, 3 ,,
14	,, amalgamated (Cold)	0	0	10	18	21	7	2	1	...	...	...	...	,, 30 ,,

in this case that it would have been difficult to test it for the size of the nuclei. A considerable quantity of filtered air was therefore added, till the condensation was thin enough for testing; the result is given in test No. 1, Table III, where it will be seen that the air was first diluted with 12 pumpfuls of filtered air, after which 28 showers at slow expansions of 8 per cent. and 20 showers at 2 per cent. were required to clear out the larger nuclei. When all these were disposed of there still remained large numbers of smaller nuclei, as the air gave 22 showers at 4 per cent., 9 at 6 per cent., and others at higher expansions up to ½ shower at 14 per cent.

In tests Nos. 2, 3, 4, 5, and 6, Table III, are given the results of heating another piece of very fine iron wire 6 cm. long in the clear silica tube. It will be seen from test No. 2 that the effect of a temperature much under red was similar to that in test No. 1, the condensation being so dense at first that much filtered air had to be added before clearing out the larger nuclei, while after these were all down there were many small nuclei requiring higher expansions to make them active. The result of a second heating is shown in test No. 3. The condensation is still very dense, and there are still many very small nuclei. Test No. 4 shows the effect of a slightly higher temperature. There is now a great decrease in the number of the larger particles, but a great increase in the number of the very small ones, some requiring an expansion of 20 per cent. to make them active. Test No. 5 gives the result of the next heating, when, as will be seen, there were almost no large nuclei, but many very small ones—some so extremely small that they required the highest possible expansions. In this test there were still smaller nuclei than shown in the table; 2 showers were obtained with an expansion of 24 per cent., showing the presence of nuclei requiring a supersaturation almost as great as that required by ions, so indicating almost molecular dimensions. It will be noticed that in these tests the number of large nuclei was greatest at first and decreased with each successive heating. The probable explanation is that these large nuclei are due to the effect of heat on the impurities on the surface of the iron, and that as these are driven off the large nuclei disappear; there now being less impurity, the nuclei become smaller, and some of these small nuclei may be due to the oxidation of the iron as the temperature rises. In test No. 6 is given the effect of heating the iron to a red heat. The nuclei are again very abundant and large, and the air had to be greatly diluted before it could be tested for small nuclei; but scarcely any of these were found, less than a shower requiring more than 2 per cent. expansion. These large nuclei are probably the result of chemical action combined with the escape of electricity at the high temperature.

A piece of thick copper wire was also tested in the tube. Like the iron, it gave very dense condensations when heated much below red heat. When the temperature was raised the number and size of the nuclei decreased, but at red heat it was not so active as iron, though it continued to give small nuclei. On afterwards examining the copper it was found to be blackened by the chemical action.

Platinum Tube.

Some tests were made with a small platinum tube. It was found that a good deal of heating did not stop its producing nuclei at a low red heat. Catalysis was suspected as the cause of these nuclei, as oxidation is not likely to take place. Some experiments were therefore made to see if its action could be checked by purifying the air. The air was passed through a cotton-wool filter wetted with caustic potash, a method previously found effective in clearing out impurities; it had no effect, however, in reducing the number of nuclei produced by the red-hot platinum. The presence of water vapour was suspected of playing a part in the action. As it was impossible to use chemical methods of drying the air for experiments of this kind, since the cure is worse than the disease, a physical one was adopted. The air after passing the caustic and cotton-wool filters was drawn through a coil of pipe placed in a freezing mixture; but again no decided decrease could be observed in the condensation. These experiments are somewhat unsatisfactory, because the density of the condensation depends greatly on the temperature of the tube, and it is not easy without elaborate apparatus to keep this constant; again, the rate at which the air is drawn through the tube affects its temperature, so that any effect of the purification of the air might easily be lost in the differences of temperature.

As already stated, all the tubes became active producers of nuclei whenever their temperature was raised high enough to cause discoloration of the bunsen flame. The platinum tube also complied with this rule. The flame above the platinum tube was always brighter than below it, and showed the D lines more brilliantly. Other samples of platinum gave varying amounts of discoloration, but all gave some. After the platinum tube had been highly heated to thoroughly cleanse it, the nuclei produced at a low red heat were all small, requiring high expansions to make them active, similar to what was found with iron. They are so unstable that they all disappeared in ten minutes, probably owing to their having an electric charge. This quick disappearance takes place whether the nuclei are kept in the damp test-flask or in an intermediate dry one. If, however, we heat the tube to a high temperature, the nuclei are almost all large, just as was found with iron, and are as stable as the ordinary dust in the atmosphere, some remaining in suspension for more than a day.

A few tests were made with platinum wire placed in the flask V and heated red by electricity. These gave the same result as the tube. The results are entered in test No. 7, Table III, from which it will be seen that there were very few small particles, though a great quantity of nuclei.

Porcelain Tube.

The effect on this tube was interesting. At a good red heat it gave great quantities of very small nuclei, which were very unstable. All disappeared in three minutes, but by keeping the highest temperature up for about ten minutes the particles grew to be large enough to condense with the slightest expansion.

Alundum Tube.

This tube proved quite unsuitable for tests of this kind. It is so porous that it allows air and dust particles to pass through its walls. The result is that the test-flask cannot be cleared, but always gives a considerable amount of condensation. When, however, the tube was heated to a low red, the condensation changed to the woolly type, thus showing a very great increase in the number of nuclei. The flame at the same time became tinged with orange colour. The increased density of the condensation was probably not due to the passage through the walls of the densely packed nuclei in the flame, as the increase did not take place till the tube was heated to a dull red. The electric charge on the nuclei in the flame probably causes them to adhere to the walls of the pores in the tube. Difference in temperature would also produce a like effect.*

Nuclei produced at Ordinary Temperatures.

In all the above cases where nuclei are produced by heating different substances, there are signs of chemical action, or in some cases of physical action, causing disintegration at their surfaces and the liberation of small particles. This wearing away of the different kinds of matter under the influence of hot gases is well known in many industries. Iron bars of grates slowly disappear, and the linings of furnaces grow thinner, even where there is no friction to account for the loss, and the above experiments with earthenware tubes seem to indicate that their surfaces are carried away in minute particles. Though this latter form of breaking down may not take place to any great extent at ordinary temperatures, yet, one naturally asks, may not this production of nuclei occur even at ordinary temperatures when there is any chemical action?

Phosphorus.

Phosphorus when exposed to air naturally suggested itself as an example of this, and some experiments were made with it. A very small piece, the size of 1 mm. cut off the point of a pin, was attached to the glass tube by which the filtered air entered the flask V. It was kept in its place simply by the capillarity of the water attached to it. If the air was quickly pumped through the flask, the condensation was not too dense for testing. The nuclei under these conditions were all small, very few being large enough to respond to a 2 per cent. expansion (see test No. 8, Table III); but there were a number of showers at higher expansions.

* "On the Formation of Small Clear Spaces in Dusty Air," *Trans. Roy. Soc. Edin.*, vol. xxxii, part ii.

If, however, two or three minutes were allowed for the nuclei to collect, there was a dense fog with the slightest supersaturation, requiring filtered air to be added before testing, when it was found that almost all the nuclei were large. It looks as if the particles coagulate and grow in size when they are in sufficient numbers and time is given for the action; the aggregation of the nuclei to form large ones is probably assisted by the great number of ions produced by the oxidising phosphorus. The very small particles first formed are very short-lived, their electric charges possibly causing them to adhere to each other or to the walls of the flask; but the large ones remain many hours, possibly because of the ions having neutralised each other.

Aluminium, Magnesium, Zinc, and Lead.

Inquiry was now made to see if any metals produced nuclei while oxidising at ordinary temperatures, in the same way as they do at high temperatures, the ordinary metals which have the power of displacing hydrogen being selected. Aluminium, which is generally supposed to oxidise rapidly at first though slowly afterwards, was first tested. 50 cm. of aluminium wire 1·6 mm. thick was polished with emery and wound into a spiral form, care being taken to touch it as little as possible. The wire was then introduced into the flask V and the apparatus filled with filtered air. After resting a time the air was drawn into the test-flask, but extremely few nuclei were found, and all required very high expansions.

Magnesium was next tested in dry air, and found to be much more nucleus-producing than aluminium. 15 cm. of magnesium ribbon was polished, wound into a spiral, and hung in the flask V. After being pumped clear it was left for half an hour. On testing, a number of showers were obtained (see test No. 10, Table III). As will be seen, there were no large particles; the greater number of them responded to an expansion at 10 per cent. In some other tests the numbers were greater and the particles larger. The conditions were now changed, as it was thought that the moisture in the air might probably be the oxidising agent. A few drops of water were put in the flask along with the magnesium to moisten the air. On testing after thirty minutes' action, a great increase both in the size and the number was found (see test No. 11). By comparing the figures in this test with the result of the tests of the air at Falkirk (Table II), it will be seen that about twice as many particles were produced by it as were observed in the polluted air of the place named.

A test was also made of the effect of the action of magnesium for only two minutes, and the result is given in test No. 12, Table III, from which it will be seen that the few particles produced were extremely small. The numbers given probably do not give the correct rate of production, as there was not time for the water vapour to get diffused through all the flask. The similarity of these tests to those made by chemical action at high temperatures is evident. When the action begins, and is slight, the nuclei in both cases are very small, and as the action increases in activity they become larger. Time also has an important effect. It was also noticed that the magnesium was most active if not highly polished but only drawn two or three times through the emery cloth.

The magnesium was removed from the flask V and a piece of ordinary commercial zinc 5 cm. $\times$ 2 cm. put in its place. The zinc was polished immediately before testing, but it proved much less active than magnesium. In dry air only a few nuclei were observed, and in damp air all the nuclei were small, none being active with less than 8 per cent. expansion (see test No. 13). The time given for the zinc to act was only three minutes, because it was found that if left for an hour all the particles had disappeared.

In the next tests amalgamated zinc was used. The metal in this condition gave far more and larger particles than when simply polished. As will be seen from test No. 14, Table III, it gave many showers requiring only an expansion of 2 per cent. and many more showers at 4 per cent., but none requiring more than 12 per cent. expansion. As it is possible that others may wish to repeat this experiment with amalgamated zinc, it will save them trouble if I give an account of my experiences with it. Some time after I had completed the above results, I wished to investigate a point which had been observed but not understood at the time. It was noticed that the zinc after a time had become inactive, and if left overnight there were no nuclei remaining, all being deposited and no new ones made. Occasionally the action could be started by supplying a fresh supply of air; at other times this had no effect. The total cessation of all activity while in the same air looked as if this depended, not simply on water vapour, but on the presence in the air of some impurity which got used up; just as was found when investigating the action of radio-activity on sulphurous acid in producing large nuclei, when it was found to depend on some impurity in the air.*

On beginning to investigate this point, the amalgamated zinc previously used was tried, but its action was now weak and uncertain. A new piece of zinc was therefore amalgamated, but on testing was found to be quite inactive. As this piece was cut from a different sheet from the first one, it was thought the failure might be due to some difference in the

* "The Sun as a Fog-Producer," *Proc. Roy. Soc. Edin.*, vol. xxxii, part ii, No. 16.

samples. Another piece cut from the same sheet as the first was therefore prepared, but on trial it also failed to give any nuclei. Much time was spent in trying to discover the cause of the difference: flasks, tubes, etc., were all changed, but without result, and at one time it looked as if the first results would have to be scrapped and put down to some unknown cause—not an infrequent occurrence in experiments of this kind, where very minute quantities of impurities produce very observable results. On thinking over all the conditions of the experiments, I remembered that the last two pieces of zinc were very carefully amalgamated, and that they looked more brilliant than the first piece. Reflecting on what that meant, it appeared that in the two last samples I had been testing not zinc but mercury. Taking one of the inactive pieces, I rubbed it with a piece of emery cloth, to expose the zinc. On now testing, it was found to be as active as the first, and in some tests gave more nuclei than shown in Table III.

Having obtained the conditions for making the zinc active and dependable, it was possible to proceed with the inquiry as to whether or not the activity was due to impurities in the air. To purify the air the caustic filter was used. We cannot, however, draw the purified air directly into the flask containing the zinc, because the filter alters the vapour contents of the air, and vapour has an effect on the activity; so if we wish to compare the condensation given by pure air with that given by impure air, we must have the same amount of vapour in both cases. To accomplish this a coil of pipe was introduced between the filter and the flask. The coil was wetted inside and placed in cold water, some 15° F. below the temperature of the room. By this means the damp air from the purifying filter had some of its water taken out, and the dry air from the other filter was brought to the same humidity. By attaching the purifying filter and the ordinary one alternately to the coil, we obtained the supplies of ordinary or pure air at the same humidity. After a number of trials there was no evidence that the impurities played any perceptible part in the action. The purified air, like the other, gave great quantities of nuclei. The explanation of the stoppage of the action seems to be that the zinc gets coated with a film of water, which stops the action, since its activity can be restored by heating and drying its surface. The tests of lead gave negative results.

Calcium, Potassium, and Sodium.

Attention was now directed to the most active of all the metals, to see if they also produced nuclei while oxidising. Calcium was first tested.

A piece of the metal was polished and put in a glass cup suspended in the flask V, but it was quite inactive. It was thought this might be due to the rapid rate at which it got covered with oxide stopping the action before it could be tested. Arrangements were therefore made for polishing the metal while it was in the filtered air. A glass tube was passed through the stopper of the flask. At the lower end of this tube was an arrangement for holding a piece of the metal. Inside the tube was a rat-tail file fixed to a piece of rubber tube on the top of the glass one, by means of which the file could be made to polish the calcium without letting in dusty air. Further, as there was water in the flask, a deep cup had to be fixed under the metal to prevent the filings from falling into the water, so generating hydrogen and making nuclei by bubbling. The conclusion come to was that if calcium is a nucleus-producer it is an extremely feeble one, as nothing but odd drops were obtained after filing. It must of course be remembered that the extent of surface cleaned was only 2 or 3 sq. mm.

Sodium and potassium were also tested, being placed in the glass cup in the flask V, but neither of these metals gave any nuclei.

We see from these experiments on metals that phosphorus is not the only substance which gives off nuclei while cold. It is well known that phosphorus when oxidising gives off not only nuclei but also a great number of ions; but whether the liberation of the nuclei by the metals was accompanied by ions was not determined, as the insulation of the electroscope used was not good enough to show it.

There is an interesting point connected with the behaviour of all very small nuclei, whether they be ions or very fine particles, and that is that after they have become centres of condensation they do not on drying return to their original condition. If we make a high enough expansion to cause condensation on the ions, or if we use a much higher expansion and get the fog-like condensation without ion nuclei, and if before the particles settle we return the pressure to its original condition, the cloud particles do not entirely evaporate, but leave larger nuclei behind. They will now be found to be active with a less degree of supersaturation. Some are active with a 2 per cent. expansion, and others require higher expansions, but none require very high. These particles soon disappear, probably owing to their electric charge. It is impossible to say that none of the nuclei return to their original state, but the density of the cloud which is given by a second expansion at 8 per cent. roughly corresponds to the density of the first cloud. The same thing happens when very small nuclei have once been made active with a high expansion: they afterwards answer to a much lower one. They act just like "penny dips": after each coating they come out larger. It is thus possible to bring all the very smallest nuclei down with only a 2 per cent. expansion. First give a quick expansion of 8 or 10 per cent., and immediately return the pressure; after that dip many will be found to respond to a 2 per cent. expansion. When these are all down, another high expansion will grow more of the remainder to the size that will be active at 2 per cent. expansion, and so on. It should be noted that the air in this experiment is never very dry, but it will not be saturated, owing to the sudden increase of 25 or 30 per cent. in the pressure due to the return of the piston, which provides more than enough heat to dissolve the particles which have not fallen out.

The activity of the different metals after polishing declines at different rates. Zinc is soon inactive. Magnesium falls off somewhat rapidly at first, but keeps up its action, though feebly, for a long time, as even an old unpolished piece gives some nuclei; these, however, may be due to the broken ends, or to clean surfaces produced in the handling. Amalgamated zinc remains fairly active for days.

It may be as well to state that the figures in Table III do not represent anything definite; they only indicate the nature of the results. A slight change in the temperature in the first seven tests would alter all the figures, higher temperatures tending to increase the size and the number of the nuclei. Similarly with the last seven tests: anything that increased or diminished the activity of the action would change all the figures. It should also be noted that all these tests were made in darkness, so that the effects of ultra-violet light on the zinc and other metals are excluded.

After the above was written I found that Professor C. T. R. Wilson * had made a number of experiments with his apparatus on the nuclei produced by metals. He tested zinc, amalgamated zinc, lead, copper, and tin. His conclusion was: "In no case were the metals found to produce nuclei requiring only slight expansions to catch them." He did find, however, that most of them have an effect. When the expansion was great enough to cause condensation on the ions—that is, when v_2/v_1 was between 1·25 and 1·38—the condensation in the presence of the metals was denser than without them. The great difference in the results of our tests seems to receive its explanation in the conditions of the two methods of experimenting. In Professor Wilson's apparatus the metals were in the test-chamber, and therefore exposed to saturated air, and their surfaces would

* *Phil. Trans.*, A, vol. 192, pp. 403-453.

soon be covered with a film of water, which we have found stops all discharge of nuclei; or the absence of the large nuclei when amalgamated zinc was used might have been due to a perfect coating of mercury.

Dust in an Electric Field.

A few studies were made of the effect of an electric field on dust particles, to see how the dust moved under the strain. For these experiments a large glass beaker 12 cm. diameter and 25 cm. deep was used. The top was covered with a piece of wood with a hole in the centre through which projected an insulated metal rod 6 mm. diameter. This rod was connected with a gold-leaf electroscope and kept charged by a very small friction electrical machine, which supplied either + or − electricity. The machine had a cylinder of only 1·5 cm. diameter. The limit of the charge was regulated by the electroscope, which caused a discharge when the potential was high enough to cause the leaves to diverge to an angle of about 45°, when they came in contact with an earthed conductor. For dust, the products of combustion of a small piece of magnesium ribbon were used. The ribbon was burned under the inverted beaker, and when cold the wooden cover was slipped underneath and turned with the cover upwards, and the insulated metal rod passed through the hole into the interior among the dust. Magnesia was used because the size of the particles enables us to follow their movements more readily than can be done with very small particles.

On first charging the rod there is very little perceptible effect, but after a minute or so flocculation is seen to be rapidly taking place, and on examining the rod and the inside surface of the beaker these are found to be covered with projecting threads composed of dust particles. Some of the threads may be a centimetre or more in length. Between the threads on the rod and those on the glass will be seen a number of threads in rapid movement going between the projecting threads on the rod and the sides and falling at the same time. So far as could be observed, there was little action at first, but very soon dust particles were seen to be attracted, and adhered to the rod. These particles, owing to their projecting from the rod, acquire a higher electric density than the rod: this causes other particles to be attracted and attached to them, causing an increase in their length, so increasing the charge at their points, and thus quickening up the activity. These threads all point radially and horizontally from both surfaces, and they seem to grow till their charge causes them to break away and dart to a projecting thread on the other surface, where they get discharged and repelled back again, and fall when passing from rod to side, the whole action resulting in a rapid flocculation of the dust, some of which falls and some adheres to the surfaces. These flocculated threads of dust projecting radially all round give us a new illustration of the so-called "tubes or lines of force."

The amount of electricity involved in these movements is very small. If in place of using an electrified rod in the centre of the enclosure we insert a glass tube 5 mm. in diameter and run a metal wire down the inside and electrify the wire, the induced charge on the glass is sufficient to show all the phenomena. And for the same reason it makes little difference whether we use the induced charge on the inside of the beaker to do the work, or provide a good conducting and earthed surface all round, or use metal plates, one electrified and one earthed.

That this process of flocculation depends on the high electric density accumulating on the projecting dust particles on the surface seems to be proved by the fact that, though we can get similar flocculation from smoke, from burning naphtha, and from other sources, we do not get it if the particles form a wet surface. Further, if we cover the electrified rod with glycerine there is no flocculation in the magnesia dust till enough dust is deposited on the rod to neutralise the wetting and levelling power of the glycerine; when this is effected, little specks of dry dust appear and the process goes on as before. Or if we coat the inside of the beaker we get little flocculation, though the rod gets a much denser coating of the dust under these conditions. Fumes produced by hydrochloric acid and ammonia, also smoke from brown paper, give no flocculation either with + or − electrification, owing to the attracted particles forming a wet deposit.

Ion Nuclei.

We shall now return to the point raised in the first paragraph of this paper, namely: Are the condensation nuclei in the atmosphere dust particles, or, as is now generally asserted, only aggregations of ions? On what foundation the latter theory has been built it is difficult to see, unless it be the fact that there are a great number of particles in the air which move in the same way as ions in an electric field, though only at about $\frac{1}{1000}$ of the velocity. To this point reference will be made later. In the meantime we will try to find if there is any evidence of ions ever combining to produce nuclei which are active with slight degrees of supersaturation such as we find in the atmosphere.

To test this point some experiments were made with the new apparatus. First some radium salt was put on the top of the test-flask, where it produced plenty of ions, as was evidenced by the density of the showers produced by an expansion of 26 per cent. The nuclei produced by these cloud particles were then cleared away, and the air left free of nuclei of any size larger than ions. It was found that the radium might be left for an hour, or a day, without the ions combining to produce particles requiring an expansion less than that required for ions. The ions evidently did not grow in damp air, so the conditions were altered. The flask V was put in its place, and the small bottle of the radium with the cork removed was put in it and the dusty air cleared out. After hours of exposure in the dry air to the action of the α, β, and γ rays, there was no condensation till the expansion was high enough to act on the ions. A bottle with hydroxide of thorium with the cork out was put in V in place of the radium. The emanations from this body are very powerful ionisers, yet it gave no nuclei larger than ions. So that neither in dry nor in moist air do the ions show any signs of combining to produce large particles. I wish now to refer to an experiment on this subject described in Part I.* In that experiment pure hydrogen was burned in filtered air. Now, the products from that flame gave no nuclei even with fairly high expansions, and yet there must have been large quantities of ions in it. It is true the time for combination was short; but as the ions would be very numerous, one would have expected some combinations to have taken place.

It is not contended that because the ions under the above conditions did not combine to form larger nuclei, therefore they never do so. But it fairly rests with those who uphold that theory to show under what conditions they do combine. Further, it is not contended that ions may not under certain conditions cause the formation of large nuclei. My own experiments described in a previous paper † show that they do. If, for instance, there be sulphurous acid and other impurities in the air, then enormous numbers of large particles are produced by ions. These, however, are not aggregations of ions, but are due to the ions bringing about a chemical action which results in the formation of large particles.

From these artificial conditions let us now turn to nature. An examination of the figures in Tables I and II shows that there are no very small particles in the atmosphere. Now, if the nuclei of cloudy condensation were really made up of aggregations of ions, there ought to be particles of all sizes in the air, from ions upwards. This, however, is not the case; seldom are there any particles requiring more than a 6 per cent. expansion to

* "On some Nuclei of Cloudy Condensation," *Trans. Roy. Soc. Edin.*, vol. xxxix, part i, No. 3.

† "The Sun as a Fog-Producer," *Proc. Roy. Soc. Edin.*, vol. xxxii, part ii, No. 16.

make them active. Further, Table II shows that in the pure air of Loch Awe there were few requiring more than a 4 per cent. expansion, and that in a locality where the ions in the atmosphere were free to act, and where also the particles were few, thus reducing the possibility of any very small particles being lost in the testing. It will also be seen from the table that the smallest particles were found in the newly polluted air of inhabited districts.

A great amount of research has been directed to the observation of the electricity discharged by heated bodies. The discharge of the electricity is supposed by some to cause the formation of the nuclei, due to heat. Now, in the experiments here described it has been shown that enormous quantities of nuclei are given off from all kinds of matter when heated to temperatures far below that at which either + or − electricity escapes—that is, at temperatures a long way below a visible red in a dark room. Further, it has been shown that, when the temperature is raised high enough to cause the escape of electricity, some chemical or disintegrating action is also at the same time taking place, as is shown by the spectroscopic examination of the flame after it has passed over the heated body. This seems to point to these two influences being the cause of the nuclei given off by bodies when heated.

All particles produced by heat, and also those produced by other causes, are very small when the temperature is just high enough to cause some action, and as the temperature is increased the particles become larger; but it is difficult to say whether the greater size is due to the higher temperature or to the greater crowding of the particles, since all particles have a tendency to grow by aggregating, while at the same time, of course, diminishing in numbers. There is also an important condition which greatly affects the aggregation and also the disappearance of the particles, and that is their electric charge. It was noticed that the nuclei produced by the electric discharge and those produced by heat while electricity is escaping are all short-lived. This will be seen from Table III. In tests 1 to 5, while the temperature was under red there were always plenty of very small particles produced by the cleansing process which were capable of an independent existence, as many did not aggregate though densely crowded. These particles were also very stable, and remained in the air for a long time. But when the temperature was high enough to allow of the escape of electricity, all the very small particles disappeared (see test No. 6). As already stated, it is probably the charge given by the ions in burning gas that causes the aggregation and disappearance of all the very small nuclei in room air.

Concluding Remarks.

Let us now see if these experiments help us to understand what these small particles are which in my early papers are called dust particles, which form the nuclei of cloudy condensation in the atmosphere, and which are now called large ions and have been investigated by means of the electric field. In many of the papers defending the latter view there seems to be considerable misunderstanding, as some of the writers associate these very fine particles with the dust raised by winds, etc. Now, though one particle of that kind of dust may weigh more than thousands of the finer particles, their number is quite negligible. It is certainly very difficult to understand how there can be such enormous numbers of solid and liquid particles in the air. That there are such numbers is evident, however, and we must do our best to understand the conditions and not simply say it is impossible. The whole difficulty seems to lie in the size of the particles. If they are extremely small the whole difficulty vanishes.

For illustration of the minuteness of the particles let us take the experiment with the $\frac{1}{2000}$ of a grain of iron wire heated to a temperature much below red. From its contaminated surface the heat drove off many thousands of particles capable of causing condensation with but slight supersaturation, though it would have required a very fine balance to detect the loss. And yet it had lost something, as it ceased to be active after a short time unless it was heated higher or touched with something not purified by heat, after which it would be again active. Or take another example, in the experiment made with pure hydrogen burned in dustless air, previously referred to, where it was found that hydrogen burning under these conditions produced no nuclei that caused condensation with slight expansion. This, so far as I know, is the only example of chemical action in gases not accompanied by the formation of particles. While the burning hydrogen gave no particles, yet if a minute speck of cotton—so small as to be carried by the gentle current in the tubes—passed through the hydrogen flame there were always produced thousands of particles which caused condensation with slight expansion. There was no dubiety in this observation, since it could be repeated by simply tapping the filter, and after a short time the passage of the speck of cotton was indicated by a bright flash in the almost invisible hydrogen flame, followed by the presence of condensation in the test-flask. The extremely small amount of matter in condensation nuclei is evident from these and other experiments.

If one might venture an opinion as to the difference in this case where the combustion of the hydrogen gave no particles, while the burning of the speck of cotton gave many, it is that while the burning hydrogen added nothing to the air that was not there before, it only increased the humidity and the number of ions; while the cotton introduced new elements, and the possibility of chemical action taking place in the gases. Or the particles might be due to impurities on the cotton, forming nuclei as in the case of the heated iron wire.

There is a difficulty which I have always felt with regard to the calculation of the size of nuclei. The radius has been calculated from the supersaturation required to make them centres of condensation—in other words, the higher vapour pressure due to their convex curvature over that at a flat surface. But when one comes to ask, What are we to measure to get the size of the nucleus—suppose it is a small solid particle: is it the radius of the solid? This seems doubtful. It is generally admitted that on the surface of all solids there is condensed a film of any gas or vapour that may be present. Now, though this film may be extremely thin, yet two thicknesses of an extremely thin film may add very materially to the diameter of the very small particles. Whatever its action may be, it introduces a disturbing element into the calculation of their size. For instance, the wonderful nucleus-producing power of phosphorus seems to depend greatly on its power of condensing water vapour. This condensing power can easily be seen when it is oxidising in the presence of wet surfaces, as it forms a heavy fog; but it is only when the particles are closely packed and probably grown to some size that this fog takes place. In the tests with this substance, when the air was quickly drawn away from it there was no such action, yet its affinity for water enabled the very small particles to become active with but slight expansion; but if a short time was given for the particles to increase in numbers and to grow, they gave a fog without expansion which became very dense with the slightest expansion. The particles in this case must have been extremely minute, as the piece of phosphorus from which the thousands of nuclei came was no larger than a pin point; and their activity would seem to depend on their affinity for water vapour making them very much larger. The above speculations are advanced with considerable hesitation, as I have not the knowledge necessary for forming an opinion on the points, and I leave them as a suggestion.

In this investigation it has been shown that there is no proof that ions alone ever form nuclei large enough to cause condensation with but little supersaturation, and there is no evidence that large particles are ever formed by the ions in the atmosphere, as in pure air there are no nuclei smaller than can be brought down with a slight expansion. The small particles are found in the air of polluted districts, the results of chemical or heat action; and the air containing the greatest number of particles or the so-called large ions is also found in polluted districts, in the centres of manufacturing industries where great amounts of coal are burned. In these areas the air has many thousands of particles per c.c., while the air of uncontaminated areas has only hundreds, which represent the pollution not got rid of, added to the natural supplies from volcanoes, meteoric matter, dried water spray, and other sources.

Some of those who have investigated the large ions by means of the electric field seem to admit the necessity of a nucleus to which an ion is attached and which gives it its charge; and, so far as can be judged from the results of this investigation, this seems to be correct, because in the experiments described, when nuclei are formed by chemical or heat action there is always something thrown off which forms a nucleus to which the ions naturally attach themselves. If, then, this be the correct explanation, why call charged particles *ions*? In suspensoid colloidal solution there are just such similar small particles of matter; in both cases they are in brownian movement, they have electric charges, and they move in an electric field. But the small particles in liquids are not called ions; then why call similar particles in gases ions? The movement of the particles in air corresponds to cataphoresis in liquids, with slight differences. In liquids the particles have a repulsive action on each other, and do not tend to coagulate unless their charge be destroyed; while in gases there seems to be no repulsive action, and the particles tend to come into contact and adhere to each other and to the sides of the vessel. In cataphoresis the particles adhere to the electrodes, while in air they may adhere to the plates or be charged by them and repelled. In this manner it seems possible they may become carriers of electricity from one plate to the other, and so come to be counted more than once. This action, however, can be checked, as we have shown, by the field plates being covered with glycerine or a similar substance.

No doubt the electricity which escapes at high temperatures plays some part in the behaviour of the nuclei, as it was found that when both were given off at the same time the life of the particles was short, owing to their charges causing combination and deposition on the sides of the vessel. But all particles, whether electrified or not, tend to disappear. So far as these tests go, there has not been found anything in gases corresponding to suspensoid colloidal solution in liquids. The very smallest particles all disappeared in a short time, owing probably to there being no repulsive zone surrounding the small particles in air corresponding to that in liquids. In liquids these fine particles remain for a long time in suspension, though Graham, who first studied them, regarded them as unstable.

Small particles in liquids keep separate from each other, while in gases they tend to coalesce—a difference in behaviour which is very important. By this action the very small particles in air tend to diminish in numbers but to grow in size. Nature seems to have no use for these extremely small particles till they are aggregated and grown large enough to become cloud particles with the slightest supersaturation. These experiments seem to indicate that what are now called large ions are nothing more than very small solid or liquid particles with an ion or ions attached.

(*Issued separately June* 28, 1917.)

XV.—Experiments and Observations on Crustacea: Part IV. Some Structural Features Pertaining to Glyptonotus.* By John Tait, M.D., D.Sc. (From the Scottish Oceanographical Laboratory and from the Physiological Laboratory of Edinburgh University.) (With twenty-two figures in the text.)

(MS. received March 8, 1917. Read May 7, 1917.)

In the immediately preceding paper of this series each walking limb of *Ligia* was described as comprising a uniplanar system of alternate flexures. This mode of description, adopted for the special purpose then in hand, was stated to be incomplete; for, when the limbs are used not for clinging but for progression, a complex movement occurs at the coxo-basal and a more simple movement at the mero-carpal articulation, neither of these being in the common plane of flexure of the other articulations. As the investigation of the first of these movements had involved minute examination of the largest male individuals, it was with unusual interest that I learned of the existence in the Scottish Oceanographical Laboratory of spirit specimens of gigantic isopods whose limb segments could be taken between the thumb and forefinger and moved about almost like those of a full-grown shore-crab. This isopod is *Glyptonotus antarcticus*, Eights, one of the Idoteidæ.

Dr W. S. Bruce having kindly placed at my disposal a number of the laboratory specimens as well as some examples of the related *Glyptonotus acutus*, Richardson, I was able to confirm the result of examination of the various articulations in *Ligia*. Examination of the animals proved to be interesting, not only because of the limb flexures but because of other structural features. As the existing descriptions of *Glyptonotus* have been written entirely from a systematic point of view, and besides have dealt only with the exterior of the animal, I have ventured here to set down some additional points relating to its anatomy, many of them suggested by considerations pertaining to function. It needs no saying that function cannot be profitably studied in a dead animal; at best one can only hazard guesses from comparison with the behaviour of allied living forms. All functional questions here discussed can be readily settled,

* The references to preceding papers of this series will be found in the bibliography at the end.

however, at the proper time on the live creature. In this way the paper, while chiefly structural, may serve to heighten the interest of some subsequent antarctic biologist in the physiology of *Glyptonotus*.

The physiological interest is not to be considered as a thing by itself. In the last paper of this series I attempted to show that functional considerations have an intimate bearing on structural problems. Since that paper was handed in for publication and when the present communication had been almost completed, E. S. Russell published a book (1916) dealing with the history of biology, the main argument of which is that one cannot afford to study form apart from function. In the present and more especially in the succeeding few papers, I will give examples of a method by which it seems to me possible, apart from the line taken by the school of "Entwicklungsmechanik," to attack the problem of organised structure. When a reasonable number of examples has been brought forward it is my intention to discuss the matter in its wider bearings, for there are sufficient indications that morphology has reached a stage when the formulation of new method is desirable. The issue also affects the present outlook of physiologists upon their particular branch of study.

Previous Accounts.

Glyptonotus antarcticus was first described and figured as a new genus and species of Crustacea by the American naturalist Eights (1833).* Short notices of his detailed account, which had been contributed to a local publication, subsequently appeared in two larger journals—see bibliography—and in 1856 the original account with plates was reproduced with a prefatory note by Dana. Dana groups the animal as an isopod under the family Idoteidæ.

It is only of recent years that *Glyptonotus* has been taken in any quantity. In his revision of the Idoteidæ, Miers (1883) mentions that he had seen no specimens. Gerstaecker (1881–1895) in *Bronn's Tierreich* contents himself with reproducing one of Eights' original illustrations. A few specimens brought to Europe from the German station in South Georgia, 1882–1883, supplied Pfeffer (1887) with material for additional illustration and systematic description. A new species, *G. acutus*, obtained by the first French expedition under Charcot, 1903–1905, was described and figured by Richardson (1906); the same species was further described and figured by Hodgson (1910) from specimens taken in Scott's first Antarctic expedition, 1901–1904. Both species were obtained in the *Scotia* expedition

* For a biographical notice of Eights, see Clarke (1916).

under Dr Bruce, 1902–1904,* and likewise in the second French expedition under Charcot, 1908–1910—see Richardson (1913).

Biological Details.—It would appear from the localities in which the animals were collected, *G. antarcticus* from South Georgia, South Orkneys, South Shetlands, and *G. acutus* from Coats Land, Belgica Strait, McMurdo Sound, that the former is a more northern, the latter a more southern form. At the end of his account of *G. antarcticus* Eights remarks: "I procured them from the southern shores of the New South Shetland Islands. They inhabit the bottom of the sea, and are only to be obtained when thrown far upon the shores by the immense surges that prevail when the detached glaciers from the land precipitate themselves into the ocean." The German specimens were washed ashore in a storm—see Pfeffer (1887, 1890). The *Français*, the *Pourquoi Pas*, the *Scotia*, and the *Discovery* specimens were got chiefly by dredging, but also in traps (*Scotia*) and with the "senne" (*Pourquoi Pas*).

The animals do not appear to have been much studied in the live condition. It cannot be said for certain whether they ever leave the bottom and swim freely in the water. As they were got in traps baited with penguin and seal meat, Dr Bruce expressed the opinion that they are carnivorous. According to Hodgson (1910), who accompanied the *Discovery* expedition, *G. acutus* is "of a dull brown colour and of sluggish habits." To Pfeffer (1887, 1890) it was reported that *G. antarcticus* during life is "lobster-red." Dr Bruce informs me that the latter animal varies greatly in colour, a statement that is supported by the appearance of the spirit specimens. The temperature of the water from which they were taken was in the neighbourhood of 0° C.

A suggestive fact is that *G. acutus* has been recorded from much greater depth than *G. antarcticus.* The only determinations of depth for the latter animal are to be found in the *Scotia* records, which are sufficiently significant. Specimens were taken with almost unfailing regularity in Scotia Bay at 10 to 12 fathoms (about 20 metres). In the same bay traps were set on a number of occasions at 50 fathoms (90 metres), but no *G. antarcticus* entered these. On the other hand, Richardson (1913) reports *G. acutus* as occurring at various depths from low water on the beach to 70 metres. The *Scotia* obtained this species off Coats Land in 161 fathoms (300 metres).

* *G. antarcticus* was taken in record quantity by the *Scotia*. I had access to more than eighty specimens. For a photograph of *G. antarcticus* by Dr Bruce, see *Scot. Nat. Antarct. Exped.*, vol. iv, pl. xi.

The Legs.

Within the family Idoteidæ, Miers (1883) distinguishes two subfamilies, Glyptonoteinæ and Idoteinæ. A distinguishing mark of the first is: "The three anterior pairs of legs with the penultimate joint or propus dilated and forming, with the reflexible dactylus, a prehensile hand." Under subfamily Idoteinæ we read: "Legs all ambulatory; the three anterior pairs with the penultimate joint not dilated." We may conveniently call the three anterior pairs of legs in *Glyptonotus* the *gnathopods*, and the four posterior pairs the *peræopods*. Before considering the special features of these two groups of limbs, one might comment upon certain features common to both and to the limbs of *Ligia*.

Common Features.

The taxis is isopodan, the flexion-complex uniplanar and tri-alternate. To cover a complete specification in serial order of the flexures of these limbs we may use the term "full flexion-complex," as distinct from "uniplanar flexion-complex," in which the limb is considered as moving in one "principal" plane.

The Full Flexion-complex.—Two new movements are involved in the full flexion-complex. Assuming the principal plane to be vertical and transverse, the axis about which movement occurs at the mero-carpal articulation is vertical, not antero-posterior as in the case of most of the joints. The carpus articulates with the merum somewhat in the same way as the human wrist articulates with the forearm, forming in the rest position one straight line with the merum. The articulation being a simple hinge, from this position the carpus may be bent backwards or forwards like a patent door.

The coxo-basal articulation is not a simple hinge. When the basipodite is fully flexed on the ventral surface of the body it can still be made to execute an angular movement, of 30 to 70 degrees according to the particular limb in question, about its long axis. When fully extended (by convention extension occurs about an antero-posterior axis) it can be made to bend forwards about a transverse axis, and from here again in the medial direction it can be made to describe a cone about a vertical axis. In the extended position rotation about the long axis of the basipodite is minimal, and probably does not occur in the living animal; contrariwise, in the fully flexed position there is no possibility of bending out of the principal plane. In either position the additional movements allow of a backward and forward swing of the (flexed) limb as a whole, as in progression. The movement at the mero-carpal articulation supplements the amount of this antero-posterior swing so far as the distal portion of the limb is concerned.

The existence of the extra movements at the coxo-basal and mero-carpal articulations serves to establish an increased similarity between the flexural arrangements of the isopodan and those of the mammalian limb. In the human arm, for example, not only may rotation of the humerus about its own axis occur, but also flexion of the bone in three separate planes at right angles to each other. The mero-carpal flexion of the isopod limb corresponds in relative direction, and in serial order in the full flexion-complex, to the latero-medial flexion between wrist and forearm. When one compares in detail the full flexion-complex of the isopod limb with that of the limb of a shore-crab, the very contrast in design between the two crustacean types serves if anything to heighten the similarity between the flexural arrangements of the isopodan and of the mammalian limb.* To save space the comparison with the crab may be presented in the form of a table, the limb in each case being supposed to lie in the transverse plane and to be arranged in the ordinary position for walking.

Name of Articulation.	Direction of Axis of Joint.		Variety of Joint.	
	Crab.	Isopod.	Crab.	Isopod.
Coxo-somitic . .	Vertical	...	Simple hinge (patent door)	Segmental fusion
Coxo-basal . .	Antero-posterior	Movement about 3 axes: rotation about long axis of basip.	Simple hinge (patent door)	Analogous to "spheroidal"
Basi-ischial . .	...	Antero-posterior	Segmental fusion	Simple hinge
Ischio-meral . .	Vertical	"	Simple hinge	"
Mero-carpal . .	Antero-posterior	Vertical	"	Simple hinge (patent door)
Carpo-propodal .	Transverse	Antero-posterior	Simple hinge (patent door)	Simple hinge
Propodo-dactylic .	Antero-posterior	"	Simple hinge	"

To summarise:—All the articulations *in the brachyuran limb* are simple hinges, and have only one degree of freedom of movement; the flexion-complex in the principal plane (the uniplanar flexion-complex) is

* It should be noted that the isopodan limb during progression has a different alignment from the mammalian. One can demonstrate the mode of movement of the isopodan limb by moving sideways with one's lower limbs flexed.

bi-alternate only; occurring alternately with each joint that permits of movement in the principal plane is an articulation whose axis lies in the principal plane. *In the isopod limb* five articulations out of six permit of movement in the principal plane, forming a tri-alternate flexion-complex; at the most proximal articulation there are three degrees of freedom of movement, while rotation about the long axis of the moving segment may also occur; immediately distal to the elbow- or knee-bend occurs an articulation whose axis lies in the principal plane.

The matter may be still more compactly embodied in a diagram—see fig. 1. Imagine the six movable segments of the limb in each case to be pulled outwards in a straight line from the body. If *a* and *b* represent simple hinges moving respectively about a vertical axis situated in the plane of the paper and about an axis normal to the plane of the paper, and if B represents a universal hinge capable among other things of the same

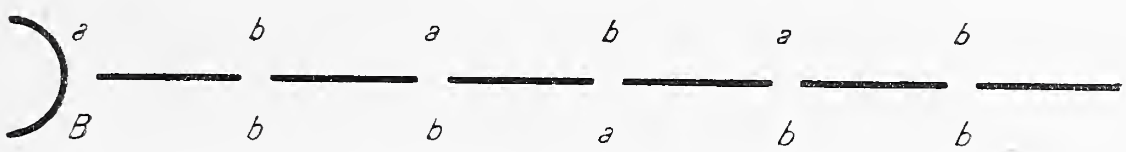


Fig. 1.—Diagram to show the flexion-complex of a brachyuran compared with that of an isopodan limb.

a, simple hinge joint whose axis is vertical and in the plane of the paper; *b*, simple hinge joint whose axis is normal to the plane of the paper; B, universal hinge whose chief movement is like that of *b*.

The upper row of letters represents the brachyuran, the lower row the isopodan flexion-complex.

movement as at *b*, then the upper row of letters in the figure will represent the full flexion-complex in the brachyuran, the lower row that in the isopodan limb.

In both types of limb the linkage of segments confers on the dactylopodite, within a restricted range, complete freedom of movement in space. In the isopod limb this end is attained in essentially the same way as in a reptant vertebrate (or "tetrapodan") limb. The fact that there exists an alternate form of linkage, the brachyuran, which leads to the same result, might remind one that the problem of conferring, with the help of different varieties of hinges, upon the terminal link of a series complete freedom of movement in space, theoretically admits of an enormous number of solutions. This ideal problem however is, in the animal, subject to many conditions of restraint: the question of bending moment is involved; the question of inertia; the provision of a specially complex set of muscles at a universal joint; restriction of complete freedom of movement in correlation with the build of the body and with the position of the limb in the series of limbs; and so on. The point to insist upon—

cf. Tait (1917, III, p. 81)—is that the problem of limb-design in relation to the environment is worthy of study in and for itself. By comparison with the limbs, say, of other reptant Crustacea and of insects, one might in this way arrive at wider conceptions.

The Coxo-basal Articulation.—The most interesting articulation in the isopod limb is (of course) the coxo-basal. The arrangements pertaining to this joint may be described under four headings: (1) the articular part of the coxa, (2) the articular part of the basipodite, (3) the articular membrane, and (4) the muscles.

It may be as well to offer first of all a few general remarks upon crustacean (or arthropodan) joints. The rigid links or segments between which the joints lie are not solid like bones; they are open cylinders; the articulating extremities are similarly portions of hollow cylinders. In movable crustacean joints the bearing surfaces are not oiled as are the diarthrodial joints of vertebrates; they are not even inclosed within the articular membrane, though this is always continuous like the capsule of a diarthrodial bony joint; they lie as it were outside the "capsule," if one can make such a statement. These bearing surfaces, frequently two in number in each of the two segments contributing to the formation of a simple hinge joint, are stronger and more heavily built than the other parts of the extremities of the segments; at the bearing surfaces alone do the articulating segments come into direct contact. The articular membrane, like the capsule of a diarthrodial joint, becomes thickened at parts (especially near the bearing surfaces) to form definite ligaments, these being invariably short. Generally speaking, in any two articulating segments of an appendage the proximal end of the distal segment is received into the dilated distal extremity of the proximal segment. One other difference between the arthropodan and the vertebrate joint may be mentioned, though this does not concern us just at present. The diarthrosis, or most movable joint, of the vertebrate is, so to speak, a terminal stage; originating as a synarthrosis, it rarely reverts again to a synarthrodial condition. The crustacean joints, primitively movable, tend in many cases to develop into immovable joints—compare the union between basipodite and ischiopodite of the crab or between coxopodite and tergite of isopods, and also the very common concrescence of body segments in Crustacea.

The *articular part of the coxa* is best described under two headings—the articular foramen and the bearing surface. In *Glyptonotus* the lateral projections of the coxæ do not form ventrally hanging plates; the ventral surface of all the coxopodites is situated in the same horizontal plane as the sternites. The articular foramen, considerably larger than the inserted articular extremity of the basipodite, has a border which therefore lies in the horizontal plane. Its outline may be compared with that of a Cupid's heart—see fig. 2. The apex of the heart points on the whole

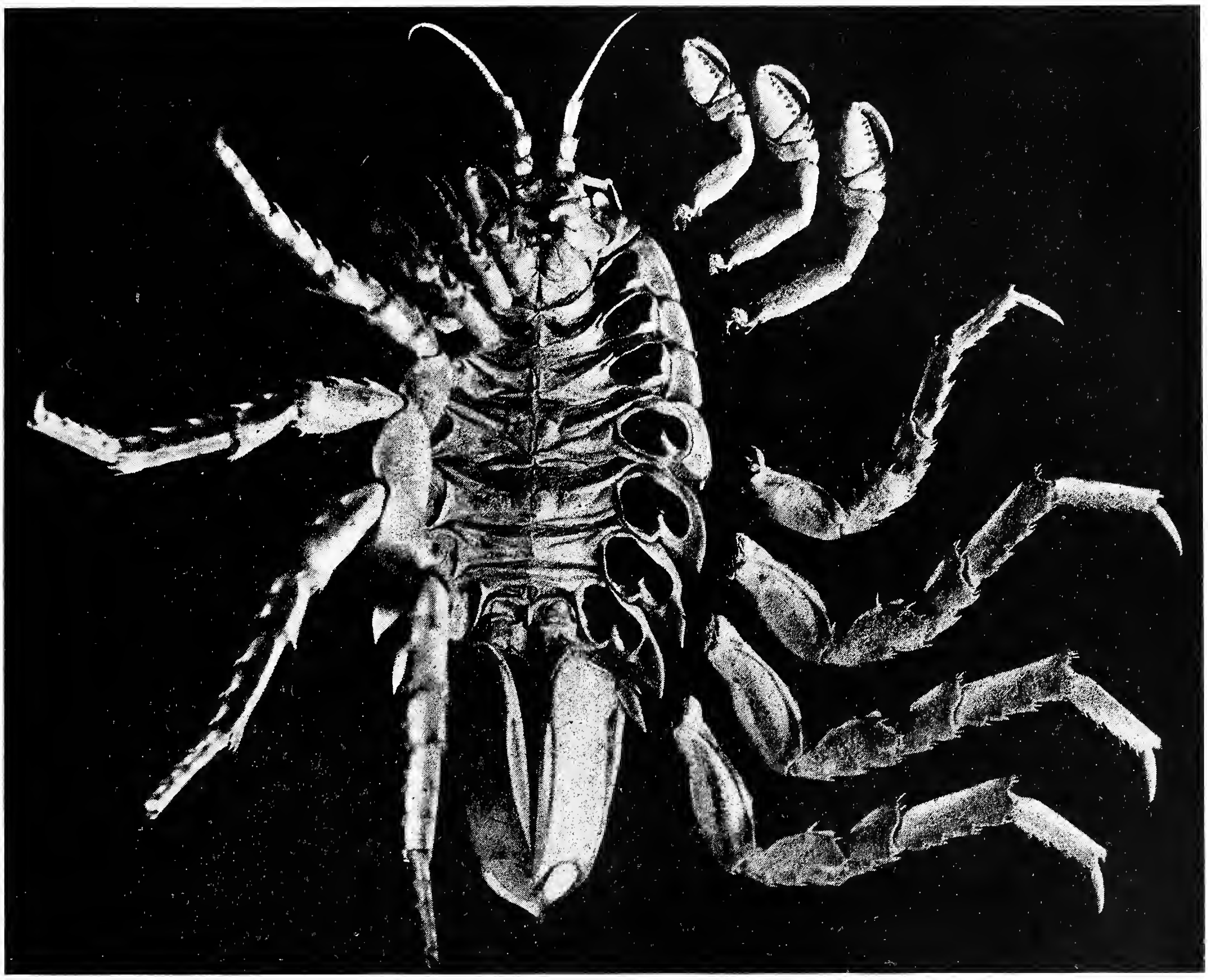


FIG. 2.—*Glyptonotus antarcticus* from the ventral aspect. Photograph. Natural size. The thoracic limbs on the left side of the animal have been disarticulated at the coxo-basal joint.

Note (i) the flat ventral surface and the absence of ventrally hanging coxal plates; (ii) the division of the thoracic limbs into two groups and the system of alternate flexures in each limb; (iii) the presence of an articular spur in each coxal foramen (those in the anterior three foramina are not distinct in the photograph—*cf.*, however, fig. 15, p. 280); (iv) the presence of a medial split in the majority of the thoracic sternites.

medially; the base is asymmetrically cleft by a medially pointing process, which will be referred to as the *articular spur* of the coxa. Owing to the direction of this spur, which does not point towards the apex but rather towards the anterior border of the Cupid's heart, the anterior of the two *bays* into which the base is thus divided is deeper and has a wider sweep than the posterior.

The bearing surface is wholly confined to the lateral (or in the case of the posterior coxæ to the posterior) aspect of the articular foramen. In this region alone does the foramen have any vertical depth. A smooth wall, arising dorsally from the border of each bay, sweeps from about the middle of each border of the Cupid's heart round the bay to meet at the articular spur, the whole ventral surface of which is also smooth. These two *bay-walls* along with the ventral surface of the articular spur form the bearing surface. The articular spur, broad-based and heavily calcified, tapers to a point at the free end; projecting ventrally near its base is a rounded boss—the *coronoid boss*; the apical part of the spur forms quite a pronounced *hook*, with its concavity, and therefore also its point, directed

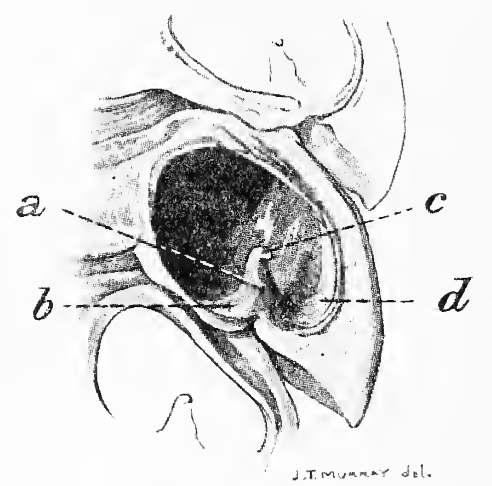


Fig. 3.—The left coxal foramen of the sixth true thoracic somite of *Glyptonotus*. × 2.

a, coronoid boss; *b*, posterior bay-wall; *c*, terminal hook on articular spur; *d*, anterior bay-wall.

ventralwards. By means of this spur the weight of the body segment is transferred to the basipodite, which has a special knife-edge that rides across the concavity of the hook. This is therefore the most important part of the bearing surface, the smooth walls arising dorsally from the border of the two bays being merely guiding surfaces for corresponding rounded processes on the basipodite—see fig. 3.

The whole lateral part of the coxa, which carries the bearing surface, and thus comes into direct contact with the basipodite, is evidently designed for strength. The dorsal wall, which fuses with the tergite, and the anterior and posterior (intersegmental) walls are likewise strong and rigid, offering in this respect a marked contrast to the ventral wall in its medial part, for the latter becomes progressively thinner and less rigid as we trace it towards the sternite.

The *articular part of the basipodite* is in cross section much smaller than the articular foramen of the coxa. The bearing surface is limited to the lateral aspect, the medial part of the proximal end of the segment ending in a thin edge which becomes continuous with the articular membrane. The bearing surface, prolonged beyond the rest of the segment, shows three smooth areas, all continuous, which fit against the bay-walls and the articular spur of the coxa. Viewed from the lateral (or postero-lateral) aspect (I here describe the arrangement in one of the peræopods; the gnathopods show unessential variations from type) this portion might remind one of the posterior aspect of the distal end of the human femur—see fig. 4. It has two rounded *condyles*, the anterior more prominent than

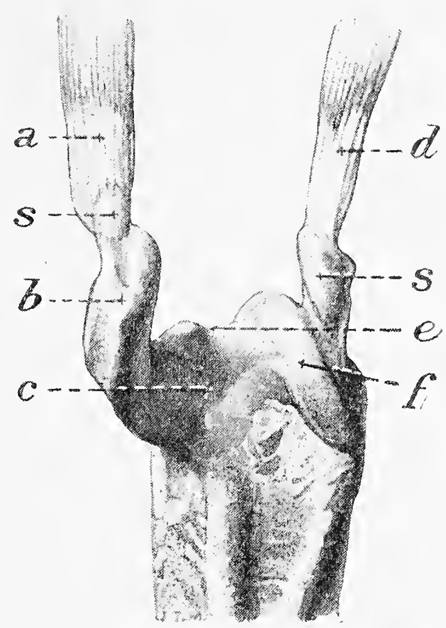


FIG. 4.—Proximal end of basipodite of second left peræopod of *Glyptonotus*, seen from postero-lateral aspect. ×4. This basipodite corresponds to the coxal foramen shown in the previous figure; note, however, the difference in magnification.

a, tendon of anterior extensor muscle; *b*, anterior condyle; *c*, coronoid fossa; *d*, tendon of posterior extensor muscle; *e*, intercondylar ridge; *f*, posterior condyle; *s*, sesamoid calcification.

the posterior, separated by a depression. There are great and essential differences, however, from the distal articular extremity of the femur. Stretching across the hollow between the condyles, and at a lower level than their upper ends, is the previously mentioned knife-edge or *intercondylar ridge*, a slight groove in which, the *intercondylar groove*, bears against the concavity of the hook on the articular spur of the coxa. The posterior aspect of the anterior condyle and the anterior aspect of the posterior condyle are also smooth, as indeed is the whole interior of the intercondylar hollow. Over a less area the anterior aspect of the anterior and the posterior aspect of the posterior condyle are also smooth. Thus the whole lateral aspect of the proximal extremity of the basipodite is modelled to correspond with the lateral aspect of the articular foramen of the coxa. A fossa, the *coronoid fossa*, just distal to the intercondylar ridge receives the coronoid boss of the articular spur in extreme extension of the basipodite. In extreme flexion the basipodite rests against the medial part of the coxal foramen, viz. in the apex of the Cupid's heart.

In this way, but by an utterly different, and it may be added less efficient, arrangement, great freedom of movement is obtained as in a spheroidal bony joint. The mode of union at the essential point, by means of a scarcely perceptible groove fitting into the concavity of a hook, is mechanically unstable; thus we explain the existence of supplementary arrangements, such as the intercondylar ridge on either side of the intercondylar tubercle, and the great guiding surfaces of the condyles themselves, which play against the bay-walls of the coxal articular foramen (the condyles also serve for muscular attachment).

The *articular membrane*, covered with soft cuticle, has a proximal and a distal line of attachment. On the medial aspect of the articulation, where there are no bearing surfaces, it is fixed to the free edge of the segments; at the bearing surfaces it is attached just internal to the smooth parts, where the edge again becomes free. In some regions the membrane is long and loose, in others short and thickened. In the subjoined table corresponding regions of attachment are set side by side, the general nature of the membrane between these regions being also specified.

Coxal Attachment.	Basipodal Attachment.	Nature of Membrane.
Border of coxal foramen on either side of apex of Cupid's heart	Proximal edge of basipodite on medial aspect	Large and loose; stretched only in extreme extension of basipodite
Summit of anterior bay-wall	Anterior condyle just internal to smooth surface	Loose
Dorsal aspect of articular spur	Medial free edge of intercondylar ridge	Short
Apex of hook on articular spur	Free edge of intercondylar groove	Strong stout ligament, which undergoes torsion in rotation of basipodite
Summit of posterior bay-wall	Posterior condyle just internal to bearing surface	Loose

The muscles are four in number, two flexors and two extensors. When the basipodite is forcibly pulled out of the coxal foramen the point of insertion of each muscle is rendered plain by the adhering tendon. The *anterior extensor* is inserted into the upper tip of the anterior condyle, the *posterior extensor* having a corresponding attachment to the posterior condyle—see fig. 4. The *anterior flexor* is inserted at the base of the anterior condyle on its medial aspect; the posterior flexor is inserted, not into the posterior condyle, but into the free edge of the basipodite in its medial part. Reference to fig. 5, in which the point of insertion of each muscle is shown relatively to the intercondylar groove, will make plain the way in which these four muscles combine to produce any particular movement at the joint (forward bending, backward bending, extension, flexion, rotation of the basipodite about its long axis, etc.), for the fixed point, the intercondylar groove, lies in the centre of the four outlying points of muscular attachment.

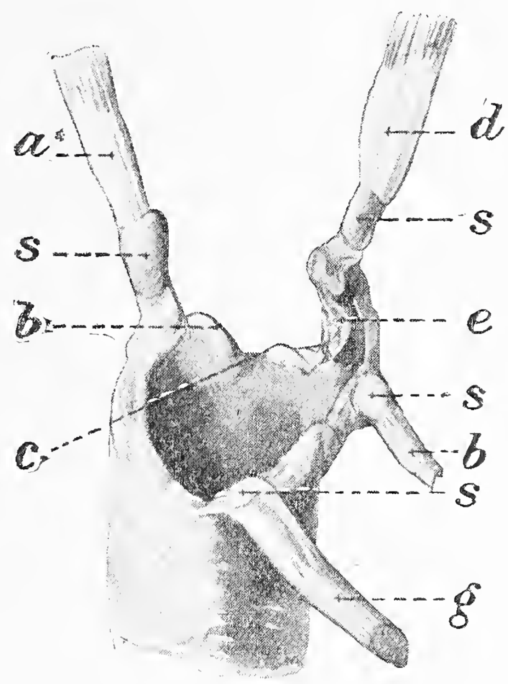


Fig. 5.—Proximal end of basipodite of second left peræopod of *Glyptonotus*, seen from antero-medial aspect. × 4. Same basipodite as in the previous figure.

a, tendon of posterior extensor muscle; *b*, posterior condyle; *c*, intercondylar groove; *d*, tendon of anterior extensor muscle; *e*, anterior condyle; *f*, tendon of anterior flexor muscle; *g*, tendon of posterior flexor muscle; *s*, sesamoid calcification.

Within each of the tendons is developed a small "sesamoid" calcification, each being laid down in relation to the articular membrane, which in these regions (see preceding table) is loose and lies in contact with the tendons. These structures are of no little interest; for, if we could determine the conditions that give rise to calcification here, we should be in a better position to understand the principles that underlie formation of the crustacean skeleton as a whole.* Apart from this wider question, their

* A full discussion of the various possibilities that arise in examination of the sesamoid calcifications from this point of view would occupy undue space, and I will state simply that repeated compression, as opposed to tension, in the long axis of the tendon appears to be one of the most probable out of many possible factors involved in producing the calcification. Quite apart from the conditions that *produce* and *maintain* the special structure, is one result achieved by the presence of the calcifications, viz. protection of the loose

existence might warn one against designating any and every newly discovered calcification in a crustacean limb as a remnant of a primitive segment—see later under Pleopods, p. 273.

Owing to the ease with which crustacean muscles become detached from their origin, and also owing to the brittle condition of the muscular fibres in the preserved specimens, considerable difficulty was experienced in determining the site of origin of the individual muscles. The following details, however, may be taken as reliable—see fig. 6. The posterior flexor arises near the mid-dorsal ridge from a narrow longitudinal area just lateral to the line of the dorsal longitudinal muscles of the trunk (see p. 265).

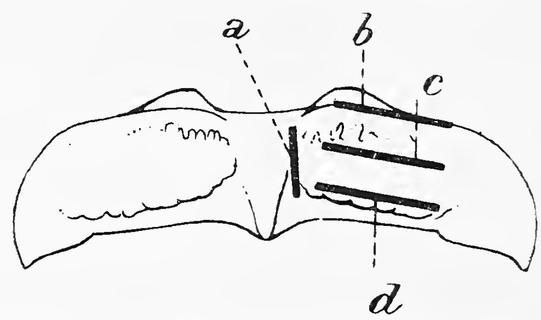


Fig. 6.—Outline drawing of a thoracic tergite of *Glyptonotus*; from dorsal aspect. To show the origins of the limb muscles; *cf.* fig. 8. Natural size.

a, origin of posterior flexor; *b*, origin of anterior flexor; *c*, origin of anterior extensor; *d*, origin of posterior extensor.

The posterior extensor arises by a transverse origin from the posterior edge of each sculptured triangle on its own side of the tergite. The anterior extensor arises from an extensive transverse area in front of this. The anterior flexor arises from another transverse area still farther forward, the greater part of its fibres being derived from a ventral inturning of the anterior border of the somite. It is worthy of mention that no muscular fibres are derived from the interior of the coxopodite. All the muscles operating a given basipodite spring exclusively from the tergite corresponding to that basipodite.

The Peræopods.

Most figures of *Glyptonotus* (that by Dr Bruce is an exception) show the peræopods directed posteriorly in the line of the body axis, the dactylopodites even of the first pair being situated behind the fifth (free) thoracic segment. It can be readily shown that the centre of gravity of the extended body lies within the limits of this segment. Consequently, unless

articular membrane against nipping between the bearing surfaces, with consequent greater freedom of movement at the joint. Only the articular membrane in the neighbourhood of the condyles incurs risk of nipping; that on the medial aspect of the joint, in relation to the tendon of the posterior flexor, is not liable to be nipped.

there is a marked difference in specific gravity between the fore and the hind part of the animal (a thing in itself unlikely), the position taken up by the limbs after death can hardly correspond with that adopted in walking.

The dactylopodites of the first pair of peræopods can reach forwards to a considerable distance in front of the head; those of the second pair can reach forward level with the front of the head; and it is probable that at any phase of gait on a horizontal surface, at least one limb of the four is in contact with the ground at a point in front of the centre of gravity.

While it is not possible to reconstruct the gait from an examination of the dead animal, one can formulate certain conditions with which the gait probably complies. General considerations may come first. In the forward progression of any reptant animal the distal extremity of each limb in contact with the ground may be considered as moving backward relatively to some point on the body of the animal; there being no slip between the distal extremity and the walking surface, the path of this relative backward movement, where as in isopods there is no swaying of the body from side to side, is rectilinear and parallel to the antero-posterior axis of the animal. The two dactylopodites of a pair of limbs may thus be considered as describing parallels of equal length equidistant from the axis of the body.

If the pair of limbs is moved simultaneously, like pleopods or like swimming limbs, these parallels may be looked upon as forming opposite sides of a rectangle. If they move in alternate rhythm, the backward paths of the dactylopodites relatively to some fixed point in the mid line of the body also form opposite sides of a rectangle, with this difference, however, that the dactylopodite on one side is in a different phase of (periodic) movement from that on the other—see fig. 7. The mode of operation of any pair of limbs is to a first approximation determined if we can specify the length of stride in relation to the length of the animal, and the relation in phase of the movement on one side to that on the other.

We now come to data pertaining specially to *Glyptonotus*. The first two pairs of peræopods have an antero-posterior reach *exactly* equal to the length of the body. The antero-posterior reach of the third pair is about three-quarters, that of the fourth pair one-half the length of the body. In the structural arrangements of the limbs there is nothing to indicate that any two successive limbs on one side ever intercross; it may be taken for granted that the dactylopodite of a given limb is invariably in front of that of its successor. These considerations make it probable that, if all the limbs have the same period, the front three pairs are not employed in their full antero-posterior reach during ordinary progression, but with a reach perhaps of half * the body length, which arrangement in turn would imply that, if the animal remains horizontal, the anterior two pairs are employed well forwards.†

The second pair has the greatest lateral stretch, and probably extends farthest on either side during walking, while the last pair moves in lines nearest to the central axis. If the animal walks horizontally, it probably carries its body at some considerable height above the walking-surface; this is suggested by the direction and length of the basipodites, by the

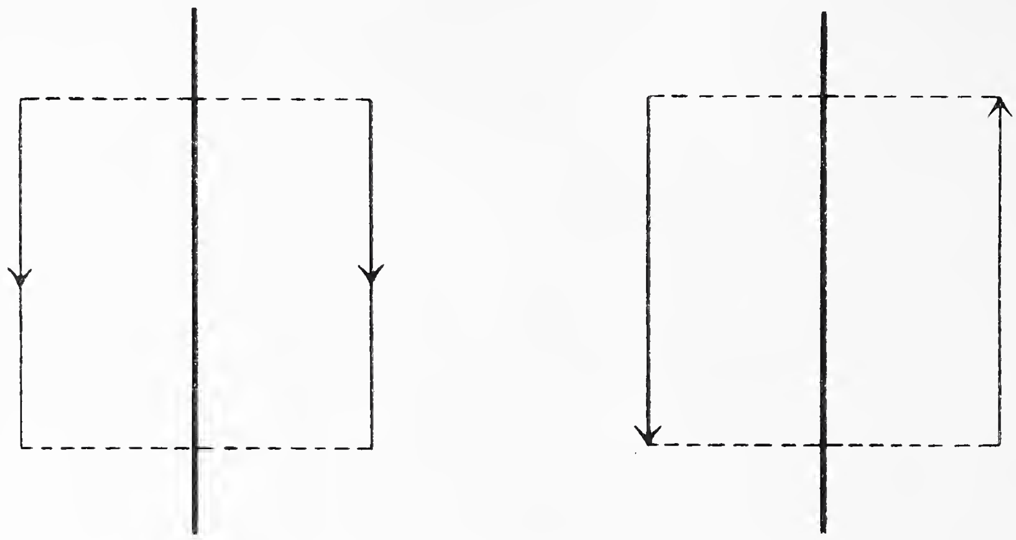


Fig. 7.—Two diagrams to illustrate possible varieties of gait in a pair of limbs.

The central vertical line in each of the diagrams represents the axis of the body; the two thinner lines on each side represent the excursion of the dactylopodites relative to some fixed point in the body axis. The animal is supposed to be moving towards the top of the page, and the dactylopodites, represented by arrowheads, are in contact with the ground. A, simultaneous movement of limbs; B, alternate movement.

length of the dactylopodites, and by the fact that space must be allowed for the opening of the uropodal valves. On the other hand, it may move over the bottom with head depressed and pleon pointing upwards.

It may be mentioned that the mode of operation of the limbs might well be of high comparative interest. We have here an animal whose walking limbs have been reduced from the normal complement of seven to four. Does the central nervous mechanism function in the same way as the corresponding mechanism of the last four walking limbs of *Ligia*, or has it developed different functional peculiarities? Have all the limbs the same period? Are they assisted by the pleopods, and is their action in any way centrally correlated with the motor mechanism of the pleon?

* This statement is based on examination of the gait of *Ligia*, whose limbs are employed with a reach of just half the body length.

† Since this was written, Mr R. S. Clarke, who accompanied Shackleton's recent expedition, has informed me that *Glyptonotus* carries its anterior peræopods in front of the centre of gravity of the body, and that pairs of limbs are used alternately.

The forward direction and mode of articulation of the basipodites, as well as the absence of hooks or special curvature on the dactylopodites, suggest that in the evolution of the animal clinging power has been sacrificed. Dr Bruce informs me that in spite of its size the living *Glyptonotus* can be handled with impunity, which is more than can be said of some smaller isopods—see Stebbing (1893, p. 343). The power of clinging or of clasping implies the existence ("to every action there is an equal and opposite reaction") of some oppositely directed (paired) mechanism, transverse or antero-posterior as the case may be.* The limbs of *Glyptonotus* show such obvious arrangement neither in the transverse nor in the antero-posterior direction; indeed the predominantly unidirectional orientation of the resting peræopods might suggest that in the natural economy the animal is given either to hanging head downwards from a support or, on a level surface, to depressing its head and tilting its pleon upwards.

When the functional employment of the walking limbs of *Ligia* comes to be described it will be shown that the creature's "internal world," as von Uexküll (1909) puts it, is in large degree pieced together of impressions of *surfaces* touched by the limbs; the content of its psyche largely centres around the ventral part of the body with its cluster of limbs. Without something to cling to, *Ligia* is deprived of its main source of sensory communication with the external world; its power of orientation (and probably its perception of orientation) is related not so much to gravity as to a fixed surface of some kind. The animal is just as much at home on the under surface or on the vertical face of a rock as on the upper surface thereof ("he clasps the crag with crooked hands"), which peculiarity in turn is undoubtedly connected with the large number of limbs it possesses. In *Glyptonotus* the walking limbs are reduced in number and the dactylopodites straightened out. It would be of much interest to determine just in what proportion this animal's power of progression and of body orientation is dependent on gravity on the one hand and on contact with an adequate surface on the other. Can it walk upside-down adhering to under-surfaces like *Ligia* or like the water-encompassed Idoteinæ with their numerous limbs and curved dactylopodites?

Doflein (1910) has drawn attention to a probable function of the hairs or setæ which clothe the appendages and other movable parts of Crustacea. Species of *Leander* resting upon rough bottom seek to obtain contact with surrounding objects through a maximal number of these hairs; on the limbs the setæ are especially numerous in the vicinity of the articulations, and Doflein's view is that a sense equivalent to the muscular sense of

* In this connection compare the feet of different birds.

vertebrates is subserved by these organs. In connection with *Glyptonotus* one might draw special attention to the elaborate and beautifully regular arrangement of setæ on the ventral aspect of the segments of the walking limbs other than the basipodites and dactylopodites. The animal would seem to afford exceptional opportunity for experiment on the function of these sense organs.

The Gnathopods.

Striking features of these limbs are: (1) the adjustment to move in parallel planes close to each other and the abrupt change in direction of the principal plane of movement compared with the mean direction of the principal planes of the peræopods; (2) slenderness of the basipodites; (3) shortness of the post-ischial axis, owing especially to reduction in length of the merum and carpus, which diminishes the lateral or antero-lateral reach of the limb; (4) vertical depth of the propus and recurvature of the dactylus to form a grasping finger; and (5) dense aggregation of setæ along the ventral border of the mero-carpo-propus. In addition to this a ventrally projecting rim on the lateral aspect of each coxal foramen (see fig. 2) prevents such extreme extension of the basipodite as occurs in the case of the peræopods; any two gnathopods of a pair can be opposed in the middle line, the peræopods being incapable of medial apposition. We may take up these peculiarities, not severally, but conjointly.

Prehensile limbs in all animals tend to develop in close association with the head, which carries the mouth and certain specialised sense organs. So true is this that when we find exceptions we look for the operation of special conditions. Thus, grasping power on the part of the hind limbs of vertebrates is found in the climbing Primates and among birds. *Phronima sedentaria* among amphipods, which has the fifth pair of thoracic limbs chelate, is exceptional in its place of abode. The occurrence of a group of three anterior pairs of grasping limbs, all similar, appears to be limited to the Glyptonotinæ, and is a very special feature of their organisation.

It may be that the natural food of these animals is at times difficult to handle, demanding the conjoint manipulation of many limbs. In the gut of two dissected specimens I found numbers of amphipods almost intact. It is not impossible that the brush of setæ on the propus is in part an adaptation for securing these smooth-bodied creatures. *Glyptonotus*, however, eats dead meat as well as amphipods, as is shown by the presence of mammalian or avian muscle and connective tissue among the gut contents —*cf.* p. 248. It also eats ophiuroids.

All three gnathopods have a forward reach to a transverse line a little in front of the head, along which line the "hands" can be arranged in regular series, those of the posterior gnathopods being outermost. The lateral stretch of this last pair of limbs just corresponds with that of the antennary flagella, and is considerably less than that of the anterior pair of peræopods, which have also a greater forward reach.

It is possible that the gnathopods work in association with tactile impressions derived from the antennary flagella. Owing to the backward current caused by the pleopods, "olfactory" stimuli will tend to reach the antennules from in front, not as in a crab from behind—*cf.* Bethe (1897)—so that the animal will approach its food or other desired object head on. The flexural arrangements of the gnathopods, the provision of setæ on a limited (ventral) region of the limb, and the fact that the thorax is capable of ventriflexion chiefly in this region, suggest that food is gathered or secured under the body, the animal bestriding or settling down upon its prey. In any case, it is plain that these prehensile limbs are used not so much for reaching outwards as for manipulating portions of food already in the vicinity of the mouth region.

The slenderness of the basipodites and the shortness of the merum and carpus are correlated with a feeble development of the basipodal condyles. The coxo-basal articulation of the long walking legs with their well-developed basipodal condyles is designed to resist a much greater bending moment. This slighter build of the gnathopods would in turn indicate that any living prey taken by the animal is small and incapable of powerful defence.

The Peræon.

Most of the spirit specimens show a certain amount of ventriflexion of the body, especially in the fore and mid part of the peræon. The animal can be extended until each lateral border forms a straight line, and all the females with full brood-pouch are thus fully extended. Dorsiflexion is prevented by abutting of the posterior border of each thoracic tergite against the pair of sculptured triangles on the succeeding tergite. The possibility of lateral flexion is all but absent.

A slight amount of flexion occurs between the more anterior segments of the pleon. Between pleon and peræon a movement of ventriflexion of some 10 degrees is permitted; so also at each of the joints between the first and second, the fifth and sixth, and the sixth and seventh ("free") thoracic segments. Between the second and third, *above all between the third and fourth,* where an abrupt change in the direction of the limbs occurs, and between the fourth and fifth segments, the amount of movement is greater.

Taken altogether, these various articulations allow the head to be bent to an angle of 80 degrees with the lateral border of the pleon. At the articulation between the cephalosome and the second true thoracic segment the soft intersegmental cuticle has disappeared.

On the whole, the body is very compact and rigid; in particular, it allows of no elongation or longitudinal separation of segments, as does that of *Ligia*—see Tait (1917, I). It is the less surprising, then, to find that the thoracic sternites are thin and flexible, and that, with the exception of the last segment, their calcareous skeleton is divided in the middle line—see fig. 2. This peculiarity, indicated in one of Eights' original figures, seems to have attracted slight attention, Pfeffer (1887) alone referring to it. A similar arrangement, in this case affecting segments 2 to 7, is present in

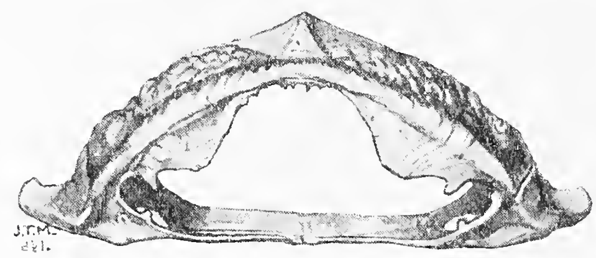


FIG. 8.—Seventh true thoracic somite of *Glyptonotus* from the front. Natural size. To show the ventral inturnings of the anterior border of the tergite.

Chiridotea, Harger; see plate iv, fig. 2 in Gerstaecker, or fig. 374 in Richardson (1905). It may with considerable reason be interpreted as a device for allowing of distension of the body after a meal; in females with ripening ovaries the ventral wall does in reality protrude, and in this connection it may be mentioned that the length of a specimen of *Ligia* at the time of capture is frequently greater than after a sojourn of some days without food.

We have already had occasion to refer to the ventral inturning of the anterior border of each thoracic tergite. This does not involve the border in its medial but only in its more lateral part—see fig. 8. Owing to the presence of these inturnings the lateral portion resists any bending in the main plane of curvature of the tergite, as when the ring as a whole is compressed from side to side. The middle portion of each tergite is flexible, and in consequence of the additional medial splitting of the sternite each segmental ring *when detached* (with the exception of the very last, which possesses a rigid sternite) can be readily compressed from side to side

Owing to the absence of ventral inturning on the middle part of each tergite, space is provided for the dorsal longitudinal muscles, which lie a short distance on each side of the middle line. As in the thoracic segments of *Ligia*—see Hewitt (1907)—each bundle of muscle fibres of this series arises immediately behind the anterior border of one segment and is inserted into the anterior border of the succeeding segment. The ventral longitudinal muscles, with attachments similar to those of the dorsal series, lie more laterally, where the sternites have greatest rigidity. From the anterior surface of each ventral inturning arise a number of small muscular bundles, which spread out radially to be inserted into the overlying intersegmental cuticle; these presumably contract during the act of extension of the body.

The epimera are not flattened and imbricated like those of many Crustacea. In the immediately preceding paper of this series (p. 84), it was suggested that flattening and imbrication of pleural plates may give mechanical support and guidance during ventri- and dorsi-flexion. While the prevailing view is that they are designed for protection of underlying parts, one cannot believe that the whole story is therein told; for whenever we find flattening and imbrication of pleural plates in Crustacea, whether in peræon or in pleon, we discover the corresponding region to be movable. It is even possible that imbricating pleural plates subserve during movement a sensory as well as a mechanical function. In any case the hypothesis that specifically connects them with movement is more open to experiment than that which vaguely attributes to them a "protective" function.

In the case of *Glyptonotus*, which is peculiar in that many of the basipodites in extension are visible from the dorsal aspect, it might be said that the existence of such free limb movement is incompatible with the presence of ventrally directed lateral plates, in other words, that protective structures have to be sacrificed in the interests of mobility; but against this line of argument is the fact that they are equally absent in the three anterior segments, the basipodites of which remain hidden on dorsal view. Noting that the body of the animal as a whole is characterised by immobility rather than by mobility, one might say that the phenomena presented are at least not opposed to our hypothesis.

As has been pointed out in all accounts of *Glyptonotus*, the line of junction between coxopodites and tergites is visible only in the case of segments 5, 6, and 7. Indeed, the distinction at present drawn between the antarctic genus *Glyptonotus* and the more widely distributed northern genus *Chiridotea* * lies in the number of epimera thus obviously separated from the thoracic tergites, *Chiridotea* showing a series of six separate epimera.

* As a rule in this paper I shall use Harger's term *Chiridotea* to include Miss Richardson's two genera *Chiridotea* and *Mesidotea*.

To appreciate the relation between these two types of structural conformation it is necessary to take a wider survey of the Idoteidæ. Considering for a moment Richardson's (1905) classification of the large number of forms occurring in the seas round the continent of North America, let us arrange her genera according to this single criterion, thus:—

(1) All seven epimera separated—*Erichsonella* (3 species);

(2) The last six epimera separated, the first completely fused—*Mesidotea* (2 species), *Chiridotea* (2 species), *Idotea* (8 species), *Pentidotea* (4 species), *Cleantis* (3 species);

(3) The last three separated, the preceding four fused — *Colidotea* (1 species);

(4) All seven epimera fused—*Synidotea* (14 species), *Edotea* (3 species). Premising, by way of parenthesis, that one of her generic types—*Eusymmerus*—based upon a single specimen, and therefore unreliable, has been omitted from the above list, one remarks that the serial order here exhibited is known on purely morphological grounds to represent the order of evolution of fusion.

Type (1), represented by *Erichsonella* among the Idoteidæ, is rare among isopods, so rare that Calman (1909, p. 203) states: "In all Isopoda, with the single exception of the genus *Plakarthrium* (Sphæromidæ), the coxopodites of the second thoracic somite (the first free somite) are completely coalesced with the body." Type (3), which would include *Glyptonotus* and *Symmius*, Richardson (1904), is less uncommon. Types (2) and (4) are the most prevalent.

One peculiar feature about the series is that progression from one type to another is not continuous, but step-like. Between (2) and (3) there is a sudden jump, and an equally sudden jump between (3) and (4). Arguing too from the number of representatives in each group, we should say that types (2) and (4) appear in each case to represent a more stable set of conditions than either type (1) or type (3).

In looking for some explanation of the anomaly we inevitably think of the moulting process in isopods—the reader is here referred to a previous communication, Tait (1917, II). So soon as we correlate the step-like progression with the stages and phenomena observed in the moult, we begin to realise how important it is, in attempting to frame proper conceptions with regard to criteria of classification, to study not the dead but the living animals, to compare not exclusively structure with structure but structure with questions pertaining to function. From this point of view too the occurrence of a specimen like "*Eusymmerus*," which had the last *two* epimera separated and the preceding *five* fused, acquires a new interest.

Merely to state a problem is not to solve it; but already by implication a shrewd blow has been dealt at the presumed degree of consanguinity underlying one of the systematists' generic criteria; a number of further problems has also been suggested.

Glyptonotus evidently moults after the fashion of other isopods. Among the specimens in the Oceanographical Laboratory two were found to have just completed full moult, the half of the body anterior to the usual transverse line of split being still quite soft. A considerable number had obviously undergone complete moult at no long date before capture, the body being rigid but fragile. No specimens were at the stage just subsequent to posterior and prior to anterior moult.

The last-mentioned fact is not surprising when one considers that the locomotor apparatus is almost limited to the posterior half of the body. In the state of half-moult a *Glyptonotus* would presumably be in an unusually helpless condition, having for a time at least only the fourth pair of peræopods by which to row itself along. Seeing that there exists a special relation between moulting and locomotion in isopods, and seeing that three pairs of walking limbs in *Glyptonotus* have been transformed into prehensile organs, observations on the behaviour of the animal during moult would be of exceptional interest. It may be that the moult sets a limit to the number of isopodan limbs that can thus be modified, and also that it determines the general balance between the half of the body behind and that in front of the fifth thoracic segment.

As was probably inevitable, no evidence bearing on the meaning of the present "generic" separation between *Glyptonotus* and *Chiridotea* was obtained from the dead specimens. Whether these two branches of Miers' "Subfamily" owe their present external similarity to direct heredity or to convergence is a point that cannot be settled offhand, even though *Chiridotea* should be found off Florida and also in the Caspian Sea. Some biologists might perhaps be willing to assume, and even to defend, a purely hereditary connection in such a case; this makes it all the more necessary to keep an open mind and to refuse to admit unproved assumptions. In this connection, *cf.* also Racovitza and Sevastos (1910).

Intersomitic Articulations.—In flexion and extension of the thoracic segments upon each other the movement at each articulation occurs about a transverse axis, which in each case is situated more ventrally than dorsally. In correspondence with this arrangement, the intersegmental cuticle on the dorsal aspect is long and loose, being folded inwards between the segments in extension of the body (*cf.* p. 265); the intersegmental cuticle on the ventral aspect is much shorter.

The axis of each hinge is not actually on the ventral surface, but cuts across just at the line of junction of the epimera with the tergites, *i.e.* some little distance above the floor of the peræon cavity. Consequently in the movement of body flexion the two tergites of an adjoining pair are increasingly separated from each other, while the corresponding sternites are approximated; the relative movement of the parts being like those of the gripping ends and handles respectively of a spring clothes-pin, the (more elongated) handles of the clothes-pin corresponding to the dorsal, the (shorter) gripping ends to the ventral parts of the somites—see fig. 9.

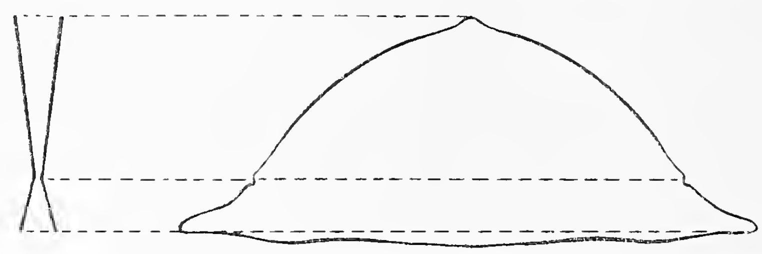


Fig. 9.—Diagram to illustrate the sectional form of a thoracic somite of *Glyptonotus* and the nature of the articulation between the somites.

On the right-hand side is shown the outline of a somite in end section; on the left-hand side is represented the "rocker" articulation between the somites as it appears when viewed from the side. The bearing point in this rocker articulation lies at the level of the coxo-tergal junction. A line drawn across the somite at this level divides it into two approximately equal areas.

It is obvious that by such an arrangement the internal volume of the peræon is not much altered by change from the flexed to the extended position. If we disarticulate a segment and, looking at the open end of the ring, imagine a line drawn across at the level of the axis of movement, we see that the area above this line is approximately equal to that below the line. Not only so, but the transverse diameter of the segment successively diminishes from below upwards, so that a large antero-posterior movement of any small slice taken across the dorsal part will cause a much less change of volume than an equal antero-posterior movement of a slice of similar depth taken across the ventral part.

In many isopods, including *Glyptonotus*, the pleon is rigid (in this connection we may for the moment discount the cephalosome; see, however, p. 286), and it seems probable on general grounds that this principle of constant internal volume might be applied to explain certain features in the conformation of Crustacea generally. In seeking to apply it, each animal or group of similar animals should, however, be considered on its own merits, for possible change of internal volume might turn out to be a complicating factor, involving a study in itself. Even in *Glyptonotus* one cannot be certain whether distension or retraction of the articular membranes in the limbs does not accompany body flexion. This could be settled by taking plethysmographic records, perhaps even of a single limb, during flexion or extension of the peræon; similarly a manometer would record any pressure changes within the animal. What I wish to bring out is that design and correlation of parts are susceptible of experimental treatment, and until the subject comes to be systematically handled in this way we can hope to have but the vaguest ideas on the matter.

The position of the centres of rotation in the hinge between two somites is determined not so much by the conformation of the hard structures as by the presence of very short stout ligaments at the essential points, viz. at the level of the junction of the epimera with the tergites. It would serve no immediate purpose to describe the structure of the bearing surfaces between the somites, more especially as the epimeral bearing surfaces vary in form along the series.

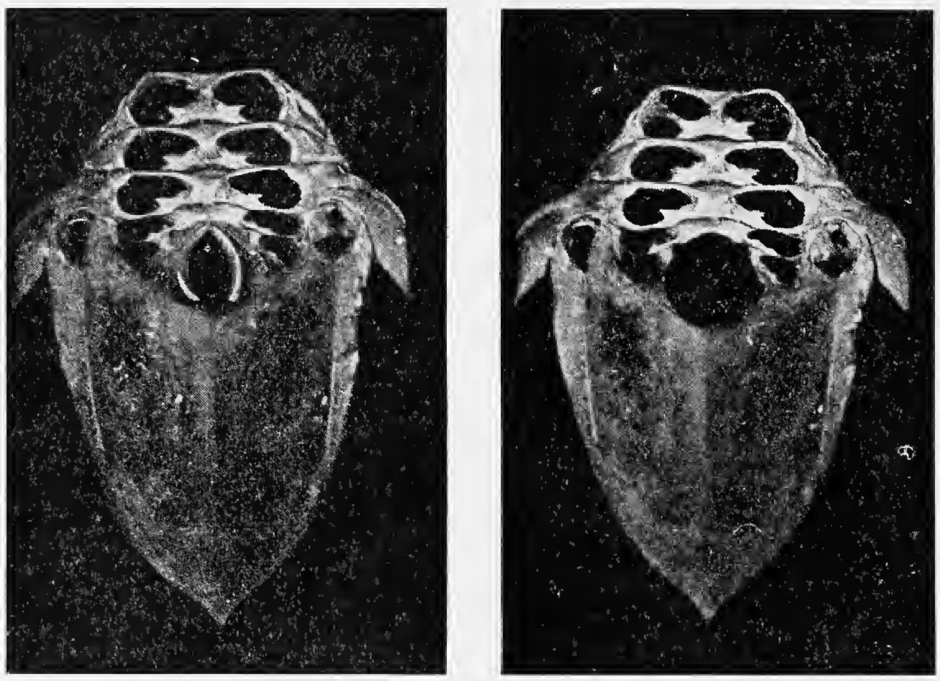
The Pleon and its Appendages.

It is only in the rarest cases among Isopoda that one finds the telson distinct from the sixth segment of the pleon. In *Glyptonotus* the seventh, sixth, fifth, and fourth primitive segments are all fused together, the separate fusions having obviously occurred in succession from the caudal end forwards. Only three of the six interpleonic articulations permit of movement.

The fusion between segments 4 and 5, obviously the most recent, and overlooked by previous authors, is present in both species of *Glyptonotus*. The line of junction between segments 5 and 6 is sharply marked on the medial dorsal ridge, where the anterior of the two overhangs the posterior; less sharply, yet in parts with sufficient distinctness, between the lateral border and the medial ridge. Caudal to the fifth segment the medial ridge, lying at a new and lower level, is not interrupted by any cleft, yet the original line of junction between segments 6 and 7 has not been entirely obliterated. On its dorsal aspect the telson proper is homogeneously pitted with fine depressions, whose distribution corresponds with the area ventrally covered by soft cuticle; this pitted region is likewise, and as a consequence, the chromatophore-bearing region. In tracing the anterior limit of these pits, in some specimens mapped out with quite a sharp anterior boundary, one traces the original line of separation between the sixth segment and the telson, the general situation of the line being further rendered plain by a pair of short transverse depressions, one immediately on each side of the medial ridge. The sixth segment, as wide as the telson, is by much the shortest.

The simplest way to appreciate the skeletal structure of the pleon in its ventral aspect is to remove the internal contents and to disarticulate the pleopods.

The under surface of the pleon, or more properly the floor of the pleon cavity—see fig. 10—presents features complementary to those just described as visible from the dorsal aspect. Segments 1, 2, 3, and 4, between which



A B

Fig. 10.—Two photographs of the ventral aspect of the pleon of *Glyptonotus* after removal of the appendages. Natural size.

In A the circumanal cuticle has been left *in situ*. Owing to drying of the preparation the anal valves, which naturally lie in apposition in the middle line, are open, and the anus appears larger than usual.

In B the circumanal cuticle, including the anal valves, has been removed to display the exact extent of the calcareous skeleton.

lie the sole movable articulations, are the only ones provided with complete sternites. The fifth sternite has no middle part, its lateral calcified portions reaching forward in fusion with the corresponding parts of sternite 4. Thus the fifth sternite gains attachment to a medial portion only by proxy, much as the eighth costal cartilage in man gains attachment to the sternum. No calcified part in this region * can be identified as a remnant of the sixth primitive sternite. The ventral surface of the telson proper, as in all Crustacea, is devoid of calcification.

The under skeleton of the pleon cavity curiously reminds one of the skeleton of the human chest as seen from the front, for the anterior

* As we shall see later, the calcareous sternite of the sixth segment has not wholly disappeared, though it takes no part in formation of the floor of the pleon cavity.

sternites, like the upper costal cartilages, are arranged transversely, while, owing to the forward direction of the lateral parts of sternites 4 and 5, corresponding to the upward trend of the inferior costal cartilages, a large "angle" is formed not unlike the subcostal angle of the human being. This *poststernal angle* is bridged across by soft cuticle—the *circumanal cuticle*—which is raised into two longitudinal folds, the *anal valves*, one on each side of the anus. The ventral edges of the anal valves are rigid, being formed of two chitinified half-hoops, which, hinged at each end, open like the jaws of an ordinary metal-mounted leather purse, and in the closed position are brought into accurate opposition in the middle line—*cf.* the description by Milne Edwards and Bouvier (1902) of the anus of *Bathynomus*.

There are at least three structural modifications with which one might seek to correlate the increasing tendency, as one proceeds caudalwards, to obliteration of the middle parts of the posterior pleonic sternites. These are the successive fusions of the corresponding body-rings, the forward displacement of structures pertaining to the anus and the development of valved uropods. By examination of *Glyptonotus* alone one cannot determine whether all three are real correlations, still less can one decide the relation as regards cause and effect between any two of the four modifications involved; at the same time such problems are of high structural interest.

The anus is situated opposite the dorsal parts of segments 5 and 6, and the internal cavity of the pleon may be said to end at the level of the sixth segment as in *Bathynomus*, the narrow space between roof and floor of the telson proper being wholly filled with vascular tissue. In sagittal section the internal cavity of the pleon is roughly wedge-shaped, or the cavity as a whole might be described as slipper-shaped. The roof slopes at first gently, and at the fifth segment steeply downwards, while the floor inclines uniformly upwards to meet the roof at the narrow sixth segment. In this way vertical depth for play of pleopodal muscles is retained until just near the termination of the cavity. These muscles in solid mass occupy the internal parts of the cavity, leaving in the middle a long, narrow, vertical-walled tunnel to be occupied in its dorsal fourth by the heart, in its ventral three-fourths by the gut. Only the first pleonic tergite presents an inturning of the anterior border, the other tergites being flattened calcareous hoops without prominent ventral projections. At the same time analogous, vertically arranged, membranous partitions subdivide the two lateral parts of the cavity into separate muscular compartments. These are especially marked in the fourth and fifth segments.

Like the floor of the peræon cavity, that of the pleon lies wholly in one plane, and for this reason alone, apart from other peculiarities, it stands out distinct from the adjoining parts. It is narrower from side to side than the roof, the lateral walls in their ventral part sloping not only ventrally but medially. In consequence the bases of the pleopods are separated in the middle line only by a slight gap, which however widens as one proceeds caudalwards. Such medial approximation of the appendages, still retained in the pleon though departed from in the peræon of *Glyptonotus*, is a primitive feature in Crustacea.

Projecting ventrally from the middle of each of the three anterior sternites is a process, the *medial sternal process*, which on each lateral aspect carries a bearing surface for contact with the basal part of a pleopod. The least distinct of these three processes is the third, the most marked being the first; the latter somewhat resembles the crista galli of the human ethmoid bone, and is arranged with its sharp edge in the sagittal plane.

The pleopods are not only brought close to the middle line, but are also arranged in very close succession antero-posteriorly. The articular foramina in the sternites being relatively large, it follows that the surrounding calcareous skeleton is more reticular than solid in appearance. Although the tergites overlap, the sternites, where separate, are placed almost edge to edge, or more strictly the anterior border of the posterior segment slightly overlaps on the ventral aspect the posterior border of the anterior. The rigid bars that intervene between the successive articular foramina are exceedingly slim and narrow; in the interests of strength they are deepened in the vertical plane, *i.e.* in the direction of muscular tension, and project prominently into the interior of the pleon cavity. Another and more marked internal vertical projection is formed above each calcareous side of the poststernal angle.

As in the peræon, dorsal and ventral longitudinal bundles of muscle fibres move the somites on each other. The dorsal bundles lie just lateral to the median ridge on each side of the heart. Owing to lateral compression of the pleonic floor the ventral bundles are much nearer the middle line than those in the peræon: when the pleopods are removed these muscular bundles can be seen crossing the medial portions of the articular foramina in the sternites; traced caudalwards they deviate laterally in accordance with the more lateral situation of the posterior foramina. Curiously enough, these muscles persist posteriorly, although the corresponding somites have fused. They may have acquired new relations and uses, *e.g.* in connection with the anal apparatus. *Cf.* here Flower's (1891) account of muscles in the limbs of the horse.

The Pleopods.

The pleopods have been carefully described by Pfeffer (1887), who pointed out that the anterior three in virtue of their rigidity probably function as natatory organs, and the posterior two, which are much softer, as simple branchiæ. The long axis of each of the anterior three is in its more proximal part concave posteriorly; this conformation serves equally well for beating the water and for packing of the branchiæ into small compass in the fully flexed position. These pleopods can be extended ventrally and forwards through much more than a right angle; the two posterior branchiæ are hardly capable of half this amount of ventral extension. In some of the spirit specimens the anterior three pairs had become fixed in extension. In both endopodite and exopodite of each pleopod a large blood-vessel runs along each lateral border; the two vessels are connected by a great number of transverse vessels.

The protopodite or sympodite consists of two chief segments with a doubtful trace of a third. The last takes the form of a single transverse calcification on the anterior aspect of the articular membrane joining each of the three anterior pleopods to the pleon. Milne Edwards and Bouvier (1902) have described a similar calcification at the base of the anterior aspect of the pleopods of *Bathynomus,* and Bouvier, following Hansen (1893)—see also Hansen (1903)—evidently regards the structure in question as the remains of a true segment, a view that appears to have met with general assent—*cf.* Richardson (1905, p. 133) and Calman (1909, p. 204).

The inference is by no means an inevitable one. The calcification is embedded rather in the articular membrane than directly united to the sympodite; nor does it follow the movement of the sympodite in extension or flexion of the pleopod. An alternative suggestion is that the structure has arisen in response to functional need of some kind (*cf.* the development of muscles in relation to the intertergal articular membranes of the peræon), whether *de novo* or by splitting of the next succeeding calcification. The occurrence of sesamoid bones in a mammal or bird and of analogous structures in the walking limbs of *Glyptonotus* (see p. 257), interesting and fundamentally important as the phenomenon is from what might be called a "physiomorphic" point of view, is of little significance to higher morphology. More than this, subdivision of segments occurs with too great frequency in Crustacea (witness the multiarticulate subdivision of primitive limb segments in the "Polycarpinea," the double sternites of *Glyptonotus* and of *Chiridotea,* the split tergites of *Gnathia,* and so on) for us to hail any newly discovered calcification as *prima facie* evidence of a primitive appendicular segment.*

The first indubitable segment of the sympodite is very short, yet it forms a complete ring. Posteriorly and proximally it bears a groove, which bears against an articular spur (derived from sternite) in the sternal articular foramen (this relationship may be real evidence against the segmental nature of the first-mentioned calcification). The most distal segment, which carries the exopodite and endopodite, is longer than the preceding and has a greater transverse width, overhanging its predecessor on the medial aspect, *i.e.* in the region of the medial sternal process. The form of this element resembles that of the corresponding segment in the pleopods of *Bathynomus*—see Milne Edwards and Bouvier (1902, pl. vi, fig. 1). The sympodite of each of the last two branchiæ is short and but slightly calcified; no third calcification can be made out in it.

Reference to fig. 10 will show that the articular foramina in the sternites are transversely elongated, that each has an articular spur, that the anterior foramina are large and the posterior small, and, finally, that the long axis of the posterior two foramina tends to be increasingly twisted out of the transverse plane. Some of these points are of importance in determining the precise line of modification that has occurred in the uropods.

The muscles that move each sympodite on the body are two in number, an extensor and a more powerful flexor. The extensor muscle, very slightly broader at its origin from the tergite than at its insertion, passes vertically downwards to be attached to the anterior aspect of the first complete ring of the sympodite in its middle and lateral part. The flexor muscle, posterior to the extensor, thicker, shorter, and more fan-shaped, takes origin from the whole lateral part of the tergite and converges to a stout tendon inserted into the posterior part of the above segment, only more laterally than the extensor. The more medial insertion of the extensor muscle as compared with that of the flexor might almost be inferred from the appearance of the sternal foramina—see fig. 10. The muscles that operate the last two pairs of pleopods are very feeble.

The Uropods.

The serial correspondences of the parts of the uropodal appendages of Valvifera are apparently still unsettled. In Calman's (1909) book we

* Since this was written I have found Lloyd's (1908) account of *five* separate plates in the pleopodal sympodite of Indian Ocean forms of *Bathynomus giganteus*. It would be manifestly absurd to apply Hansen's conception to this case.

read: "Each consists of a large plate formed by the expanded protopodite with the small endopodite at its tip while the exopodite is vestigial or absent," to which succeed the following sentences as a footnote, "What is here called the exopodite is usually regarded as the endopodite, and *vice versa*. The interpretation given above depends on the assumption that the uropods have reached their present position by a movement of *rotation*, not of simple translation." As to the constitution of the uropodal sympodite I take the following specific statement from Pfeffer (1887): "The first segment cannot be made out, having disappeared completely in the tail-shield. The second segments are large plates which completely cover the plates of the pleopods."

In failing to specify the exact nature of the rotation assumed, Calman's footnote is not clear. Pfeffer's statement as to the fate of the original uropodal sympodite is apparently at variance with that implied in Calman's phrase "formed by the expanded protopodite." I will here attempt to show (1) that the large plate of the uropod is a compound piece, and (2) that what is usually called the exopodite of the uropod is a real exopodite.

Noting the oblique position of the long axis of the sternal foramina corresponding to the last two pleopods, one concludes that the anterior border of the large uropodal plate is its original medial border. The anterior part of this plate is prolonged in front of its articulation with the sixth pleonic segment; so too is each pleopodal sympodite (or rather the most distal segment thereof) prolonged medially beyond the articulation with the corresponding sternite: this probably means that the greater part of the uropodal plate corresponds to the distal segment of the pleopodal sympodites.

More conclusive evidence as to the orientation of the uropods is, however, obtained from the form of the articular foramen in the sixth pleonic segment, which shows an articular spur projecting backwards and laterally from the *anterior border* of the foramen—see fig. 11. It is evident that this articular spur serially corresponds with those in the other sternites; its direction shows that the sixth sternite as a whole has undergone a rotation more complete than that of the fourth and fifth sternites, but with the same angular sign. This means that the lateral surface of the opened uropod corresponds to the anterior surface of the pleopods, the medial or gill-directed surface of the uropod corresponding to the posterior surface of the pleopods. The mode of articulation of the uropodal plate with the sixth sternite by means of an articular spur is evidence that the plate is compound.

Having determined by what kind of rotation the uropodal sympodite has acquired its present position, it is easy to settle the relationships of the two terminal pieces. The larger and more rigid of the two, which is also the more lateral and the more ventral in the fully flexed position of the sympodite, is obviously the exopodite. The more medial and more dorsal segment, less well developed and almost hidden by the exopodite when the uropod is closed, is the endopodite.

The gill-directed surface of the uropodal sympodite is covered with soft cuticle and is obviously branchial in function, for between the main blood-vessels, which run antero-posteriorly at each border, is arranged a great number of transversely coursing vessels exactly like those in the exopodites

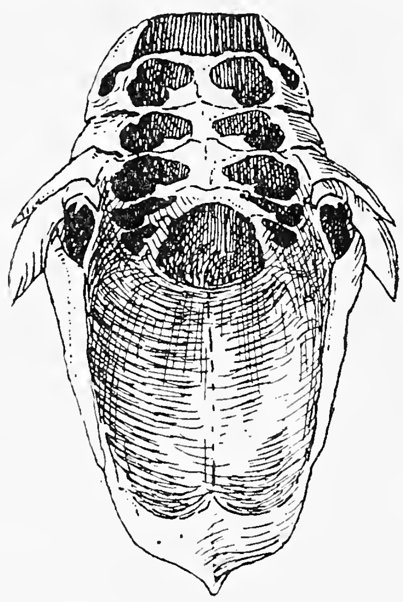


Fig. 11.—Ventral aspect of the pleon of *Glyptonotus* after removal of the appendages. Natural size. Somewhat schematised in order to bring the uropodal and the pleopodal articular foramina into view at the same time.

An articular spur is present on the posterior-medial border of each pleopodal articular foramen; a similar spur occurs on the anterior border of the two uropodal articular foramina. As can also be seen in the previous figure, the posterior articular foramina are seen to be increasingly twisted forwards.

and endopodites of the pleopods. On similar grounds one infers that the ventral aspect of the telson is a respiratory surface.

The flexor and extensor muscles which move the uropod are more feeble than those which move the anterior three pleopods. They lie in a special compartment of the pleon, lateral and slightly posterior to those occupied by the muscles of the fourth and fifth pleopod. On the medio-ventral aspect these muscles are covered simply by soft cuticle, through which they are easily visible; the medial border of the articular foramen in the sixth segment is similarly non-calcified.

The Cephalosome.

When the body is transversely divided along the line of the first movable thoracic articulation the internal hard structures of the anterior portion, which we may in this particular instance call *the cephalosome*, are readily brought into view (to remove the friable spirit-fixed muscles it suffices to hold the preparation under the water-tap). Internally the cephalosome contains three distinct compartments, the head proper and the first and second true thoracic segments. Of these compartments the first and the last are large, the interposed somite being very narrow. The separation between them is effected by means of calcified septa, which may be considered as inturnings of the original free border of some of the somites involved.

The First Intracephalosomic Septum, viz. that between the head proper and the peræon, is exceptionally well marked, and therefore of more than

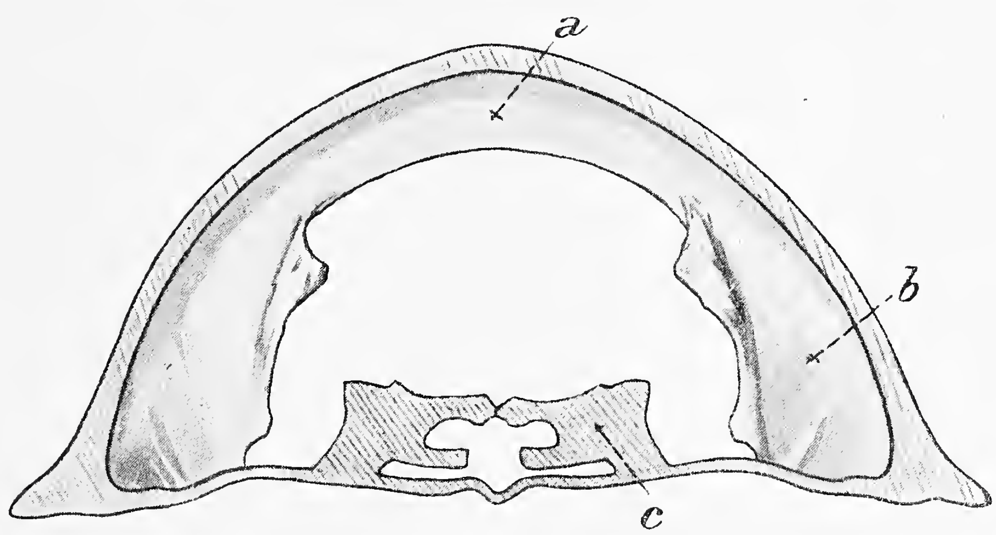


FIG. 12.—The first intracephalosomic septum of *Glyptonotus*. ×2. Semi-diagrammatic. *a*, medial plate-like girder ; *b*, triangular expansion on which medial girder rests ; *c*, alar plate.

usual interest. It consists medially of a crescent-shaped vertical plate which, resting upon its two ends, appears to support the roof of the cavity at this part like a bow-string girder whose upper and lower booms are both curved—see fig. 12. Laterally each end of this plate broadens out into, or is supported upon, a laminar expansion of triangular shape, which, gaining attachment to the floor of the cavity, fills in the angular space at the join of tergite and sternite. These triangular expansions are not arranged in the coronal plane, the lateral angle of each triangle running forwards as well as outwards, the free edge or unattached *side* of the triangle being correspondingly directed posteriorly as well as medially. The free edge lies approximately in the same coronal plane as the medial girder-like plate, which means that the main body of each triangular expansion lies in front of this plane.

On external examination of the cephalosome the position of this septum is distinctly but not obtrusively marked. Previous authors, dealing solely with the external conformation of the animal, have commented upon the beautifully regular curved line by which the second true thoracic segment is evidently marked off from the parts in front. The curved line in question consists in its two lateral parts, *i.e.* where the carapace is smooth, of a groove; in its medial part, where the carapace is sculptured and where complete intersegmental fusion has occurred, of a ridge—see fig. 13. The triangular expansions of the first intracephalosomic septum are attached along the course of each lateral groove. From about the region where each of the two lateral smooth areas on the head proper comes to a pointed

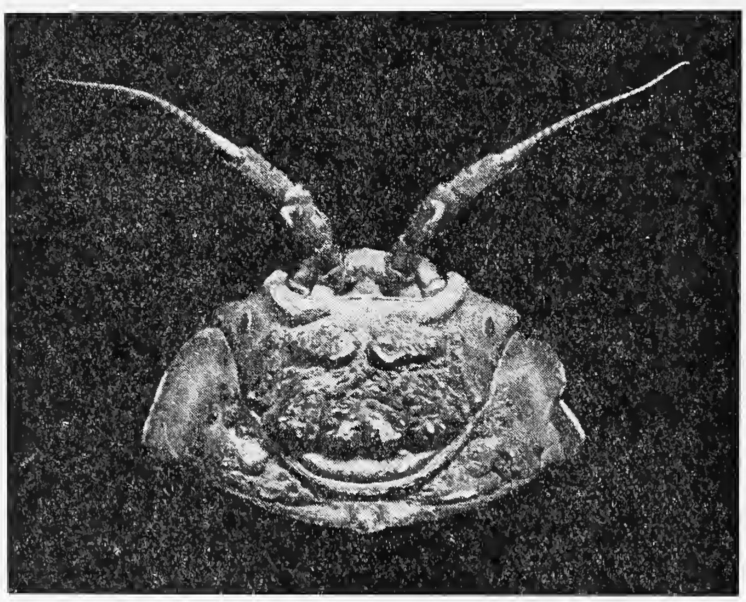


Fig. 13.—Dorsal aspect of the "cephalosome" of *Glyptonotus*. Photograph. Natural size.

The cephalosome proper is sunk into the second true thoracic somite, the separation between them consisting of a pair of lateral grooves and of a medial ridge. Just in front of the medial ridge and behind the sculptured part of the cephalon is a crescent-shaped area, which is the first thoracic somite. The deep transverse groove in front of this somite corresponds to the medial girder of the first intracephalosomic septum.

end medially and posteriorly, the first intracephalosomic septum is continued coronally across as the medial crescent-shaped plate, its position being now externally marked by a deep transverse depression behind the highly sculptured region of the head proper.

For visceral, vascular, and nervous communication between head and thorax a wide medial foramen, the *cephalo-thoracic foramen*, is left internally—see fig. 12. The inferior border or boundary of this foramen is not formed by the sternite, the opening being partially blocked in this region by a pair of structures, partly calcified, partly chitinous, which project upwards from a region just in front of the articular foramina for the maxillipeds and behind the corresponding foramina for articulation of the second, and especially of the first, maxillæ. These structures, to which

Lloyd (1908) has given the name of "sternal alæ," will be more minutely described under the heading of Ventral Endophragmal Skeleton.

The Second Intracephalosomic Septum.—The shape of this septum, in end elevation, is shown in fig. 14. More feebly developed than the first, with which it fuses laterally, it exactly follows the curved line previously referred to as marking the separation between cephalosome proper and second true thoracic somite, which means that its lateral portions lie considerably anterior to its middle part. The free edge of each lateral triangular expansion is the chief feature that renders the identity of this part of the septum distinct in the interior, for the main body of each expansion appears to fuse with the corresponding, much larger, triangular

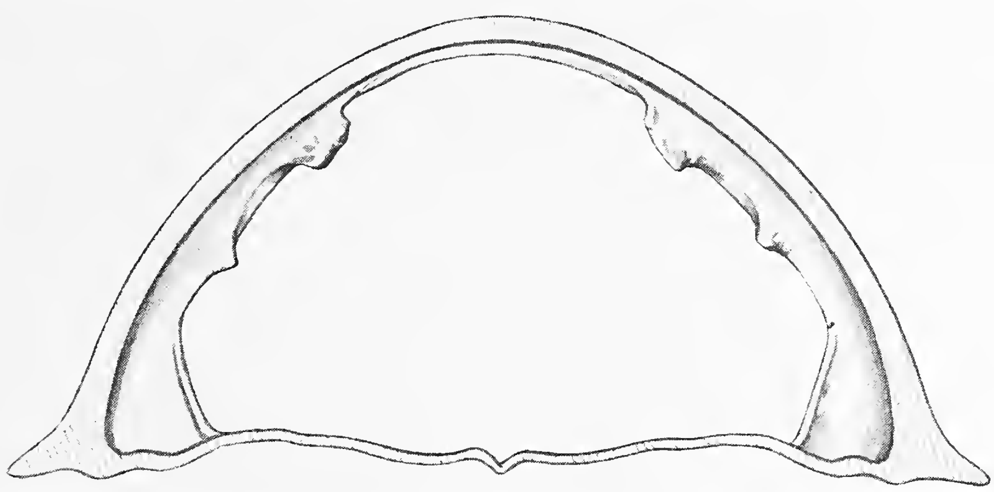


FIG. 14.—The second intracephalosomic septum of *Glyptonotus*. ×2. Semi-diagrammatic.

This septum intervenes between the first and second true thoracic somites—*cf.* fig. 16.

expansion of the first septum. As we shall see later, there is almost complete separation between the two by means of a narrow cleft which leads in from the exterior on the lateral part of the cephalosome.

A Function of the Intracephalosomic Septa.—Whatever the conditions that led to the formation of the complex internal skeleton described above, one result certainly achieved is that the head region of the animal is strongly braced against compression applied from without. In seas where glacial boulders are dropped from above, and where even deep-water animals are tossed about by great surges, liability to accident by crushing is by no means remote. Especially vital parts of an isopod are the head and the abdomen. In *Glyptonotus* the rigidity of the latter largely depends on the strong mid-dorsal ridge, that of the former on the system of internal struts. In other isopods there are rudimentary homologues of the highly developed first intracephalosomic septum found in *Glyptonotus*. These take the form of two inturnings of the dorso-lateral portion of the posterior border of the cephalon, and are to be seen in *Ligia*; also in

Chiridotea entomon, for access to a dried disarticulated specimen of which in the Royal Scottish Museum I have to thank Dr James Ritchie: the "tergal alæ" described by Lloyd (1908) in the cephalosome of *Bathynomus* likewise come under the category. In all these cases, however, a medial girder is absent, while the inturnings are not solidly planted upon the cephalosomic floor as in *Glyptonotus*.

The Ventral Skeleton.—Curiously enough, when the oral appendages are removed from the cephalosome, the articular foramina for the mandibles appear to be directly continuous with the row of articular foramina for the peræopods and gnathopods—see fig. 15. As if they did not belong to the

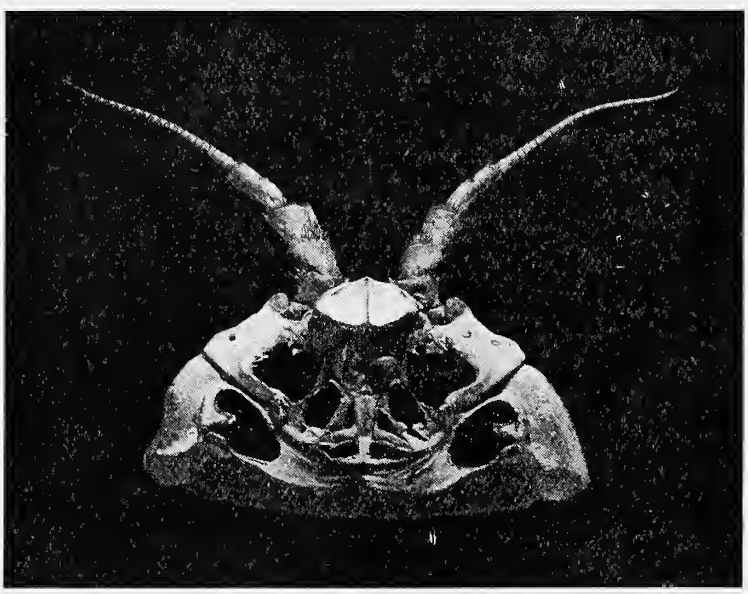


Fig. 15.—Ventral aspect of the "cephalosome" of *Glyptonotus* after removal of the appendages. Photograph. Natural size.

The articular foramina for the pair of mandibles and also for the first pair of gnathopods are large and widely spaced, those for the first maxillæ, the second maxillæ, and the maxillipeds being increasingly approximated towards the middle line.

series, the articular foramina for the maxillipeds and for the first and second maxillæ are medially clustered together, and are borne by a special skeletal framework—the *maxillo-sternal framework*—which is movably connected with the buccal frame formed by the main or heavy skeleton of the cephalosomic floor. From a purely mechanical point of view this last feature of the construction of parts might suggest an analogy with the oral region of one of the higher vertebrates, in which the hyoid skeleton, light, delicate, and of secondary mechanical importance, has become inclosed within the angle formed by the mandibula and tends to relinquish its bony union with the cranium (according to this comparison the maxillæ and maxillipeds of the isopod would correspond to the tongue of the vertebrate).

The articular foramina for the first pair of gnathopods are widely separated; those for the maxillipeds are closely approximated in the middle line; so too are the articular foramina for the second maxillæ; from this point forwards the foramina diverge like the limbs of a V, those for the mandibles being once again widely separated. In other words, of the primitive cephalic somites only the one corresponding to the mandibles shows indubitable evidence of participation in the general lateral expansion of the body. Consequently we might suppose that by the strong lateral development of this particular primitive somite the more posterior cephalic somites have been crowded out from the lateral region, and thus their sternites left, as it were, floating. A similar movable maxillo-sternal framework is obviously present in *Bathynomus*—see Milne Edwards and Bouvier (1902, pl. iv, fig. 9).

Attention may be drawn to a kind of antero-posterior symmetry in the arrangement of the sternal articular foramina for the appendages of *Glyptonotus.* We have seen that the appendages of the mesosome are all widely separated. As we pass from mesosome to metasome the double longitudinal row of sternal articular foramina suddenly becomes drawn together medially, gradually to diverge farther back, attaining greatest divergence at the uropods. Similarly, as we pass from mesosome to cephalosome proper, the series of sternal articular foramina are suddenly approximated towards the middle line, to diverge anteriorly, attaining greatest divergence at the mandibles. One might also note that the cephalon is, as it were, pushed backwards into an excavation in the front of the thorax, the pleon being similarly received into an excavation in the hinder part of the thorax. A transverse cut carried across between the two anterior points of the farthest forward thoracic epimera would pass through the mouth; a similar cut at the level of the posterior points of the farthest back thoracic epimera would pass through the anus. The oral and the pleopodal are very closely packed as compared with the thoracic appendages.

Having obtained a general idea of the skeleton of the cephalosome, we pass to a more detailed consideration of some of the parts. We may designate the lateral triangular expansions of the first and of the second intracephalosomic septum as the "first (triangular) lamina" and the "second (triangular) lamina" respectively; the thoracic somites will be numbered according to their position in the primitive series of eight; and some new descriptive terms will be introduced.

The First Thoracic Somite.—Examination of the interior of the cephalosome having made plain the position of this somite, it is easy to trace its boundaries on the external dorsal aspect. Its greatest (antero-posterior) length was found to be in the mid-dorsal region; from this point lateralwards its length was seen to decrease continuously, until the somite appeared to fade away where the second triangular lamina joins the first. In other words, it is wedge-shaped, and its wedge-like insertion between the cephalon and the second thoracic somite may be readily recognised in fig. 13.

To external view the somite appears to end laterally at the groove between the two lateral smooth areas, cephalic and second thoracic respectively, on the dorsal aspect of the cephalosome. Inspection of this groove shows that it is merely the dorsal termination of a deep cleft—the (*lateral*) *cephalo-thoracic cleft*—which cuts in medially from the lateral border of the cephalosome. Passing a bristle into this cleft and looking at the two triangular laminæ on that side, one notes, thanks to the transparent thinness of the second lamina, that the point of the bristle moves freely in front of the latter, being capable of covering any point in its whole area right up to the medial free edge. In other words, this second lamina, forming the posterior wall of the cleft, is an inturning of the original free border of the lateral part of the second thoracic somite, or more properly is a portion of the coxa of the second thoracic appendage.

Partly by the same method, partly by examination of the ventral part of the cephalosome and the interior of the mandibular articular foramen, one sees that the anterior wall of the cleft, co-extensive with the posterior wall, though of much less extent than the whole first triangular lamina, is formed by an inturning of the posterior free edge of the cephalosome proper. Is this a portion of the first thoracic somite, or can the latter be elsewhere identified?

Careful examination of the region in proximity to the free edge of the second triangular lamina shows that between this free edge and the surface of the first triangular lamina is another calcareous fold or reduplication exactly similar to the free edge of the second lamina and running all the way up and down parallel and closely contiguous to it—see fig. 16. The arrangement may be made more plain by means of a diagrammatic horizontal section through the cephalosome—see fig. 17. This reduplication, hidden at the bottom of the cephalo-thoracic cleft, and therefore far removed from the extreme lateral border of the animal, is evidently a remnant of the first thoracic tergite, which, therefore, cannot have participated in the general lateral expansion of the cephalosome. Comparison with *Chiridotea entomon*, Harger, in which the position of the first thoracic somite is better marked externally, confirms this view.

Let us now turn to the ventral aspect. The posterior (or postero-lateral) border of the mandibular articular foramen is obviously formed of two separate elements, separated by a deep fissure—see fig. 18. One half, the more lateral, is simply a strongly calcified portion of the main latero-ventral skeleton of the cephalosome proper. As this part carries the posterior of the two bearing surfaces at each end of the axis of the mandibular hinge, we can be reasonably certain that it is a portion of the

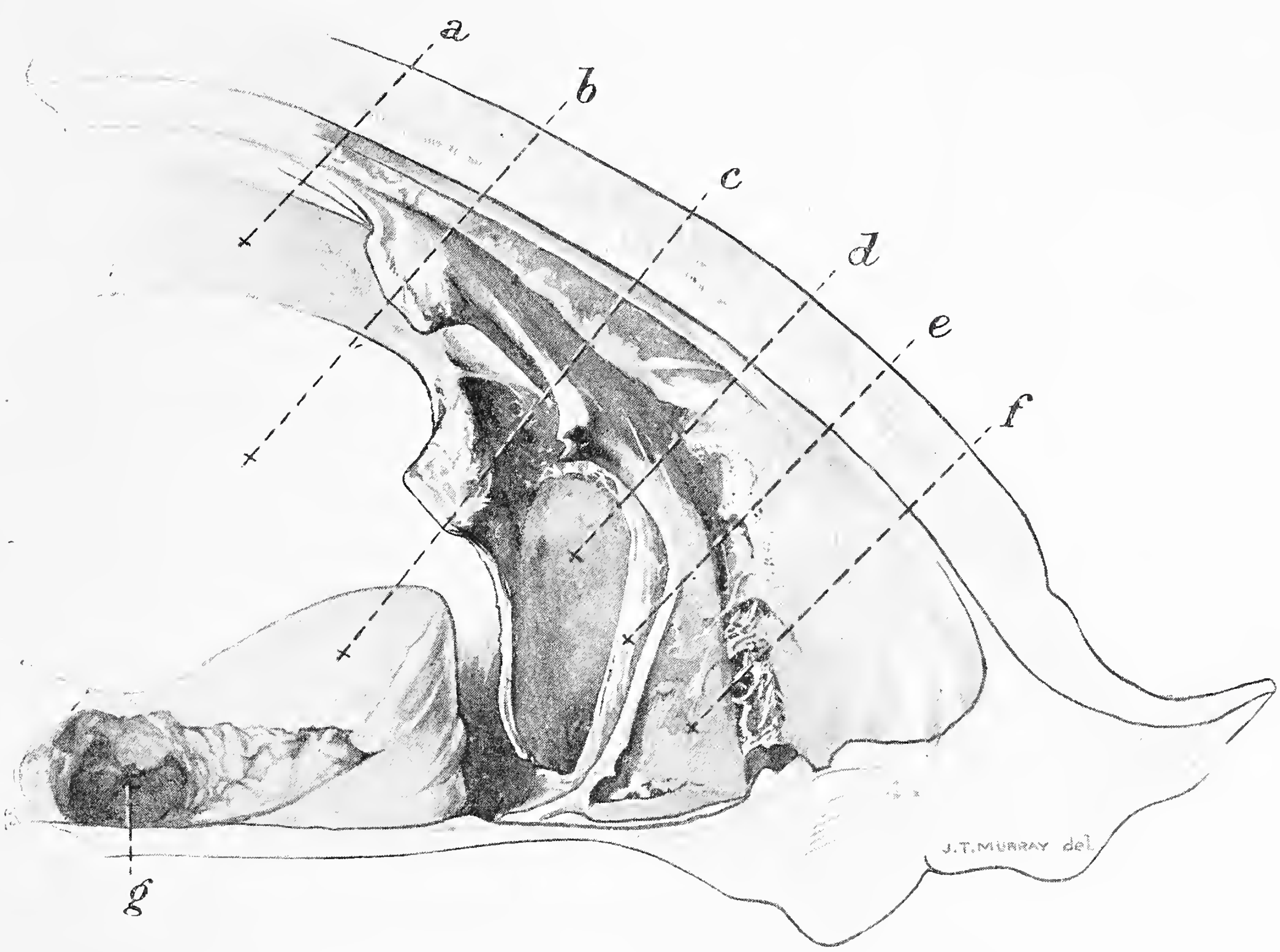


FIG. 16.—End view of the interior of the "cephalosome" of *Glyptonotus*. ×6. To show the alar piriform bodies, and the relation of the first to the second triangular lamina.

a, medial plate-like girder of first septum; *b*, cephalo-thoracic foramen; *c*, capsule of alar piriform body partially lifted behind (this structure is wholly rounded, not concave behind as is suggested by the figure); *d*, first triangular lamina (probably belonging to mandibular somite); *e*, first thoracic somite; *f*, second triangular lamina (belonging to second thoracic somite); *g*, foramen for nerve-chain.

primitive mandibular somite; which in turn means that the anterior wall of the cephalo-thoracic cleft, if not the whole of the first triangular lamina, has been formed by the mandibular somite.

The more medial half of the border of the articular foramen consists of a curiously shaped calcification, the *styloid calcification,* which projects ventrally as a styloid process. Though it lies pronouncedly in front of the anterior border of the second thoracic sternite, it is welded firmly to this sternite, as well as to the mandibular somite. When we trace to its ventral destination the fold or reduplication, which in the last paragraph but one was shown to represent the lateral part of the first thoracic tergite, we discover that, after curving round the medial free edge of the first triangular lamina near the floor of the cephalosome (see fig. 16), it ends in the styloid calcification.

The position of the styloid calcification therefore corresponds with the ventral termination of the first thoracic tergite. Whether it is exclusively formed by the first thoracic somite cannot be said: it is possible (though

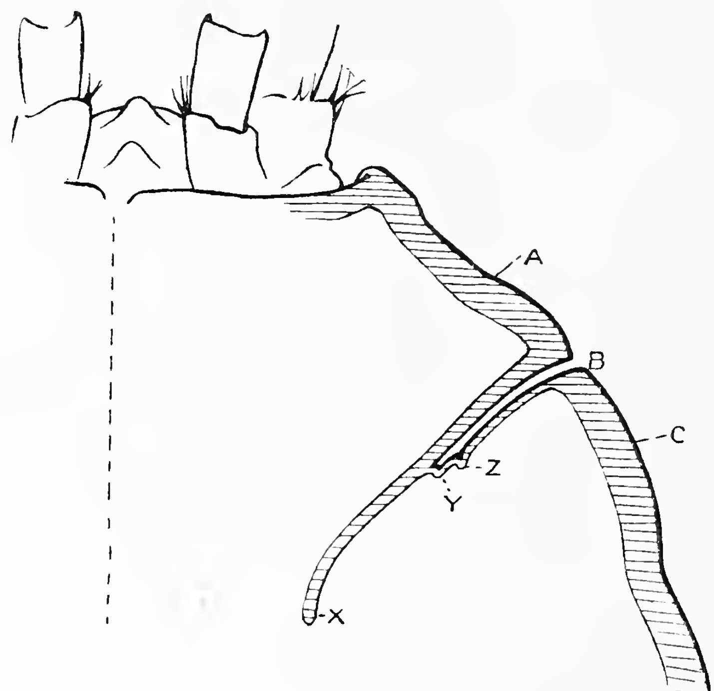


Fig. 17.—Diagrammatic horizontal section through the "cephalosome" of *Glyptonotus*. To show the relation of parts near the lateral cephalo-thoracic cleft.

A, mandibular somite; B, cephalo-thoracic cleft; C, second thoracic somite; X, first triangular lamina; Y, first thoracic somite, rudimentary and confined to the bottom of the cephalo-thoracic cleft; Z, second triangular lamina, belonging to second thoracic somite.

on the whole improbable) that maxillary somitic elements likewise enter into its composition.

While the posterior limits of the mandibular somite of the cephalon can be successfully traced in parts, it is a striking fact that no element in the roof or side-wall of the cephalosome can be identified as belonging to the maxillary somites. In the interior of the cephalon there are calcified structures of undetermined origin (see under Ventral Endophragmal Skeleton) which are directly connected to maxillary sternites, but these are far removed from the roof and lie ventral to the gut.

One might also call attention to the fact that whereas in *Glyptonotus* the first and second thoracic somites have fused together, in *Chiridotea entomon*, Harger, the articulation between these two somites is movable.

The Maxillo-sternal Framework has a longitudinal axis composed of a medial calcareous bar or *keel*, which, commencing between the two articular foramina for the maxillipeds, runs forwards towards the mouth, widening as it goes—see fig. 18. Ventrally the keel projects as a ridge (the dorsal aspect is correspondingly grooved like a gutter), with transverse indentations corresponding to the joins between successive sternites; the articulations at these joins are rigid, not movable as in *Bathynomus*—see Milne

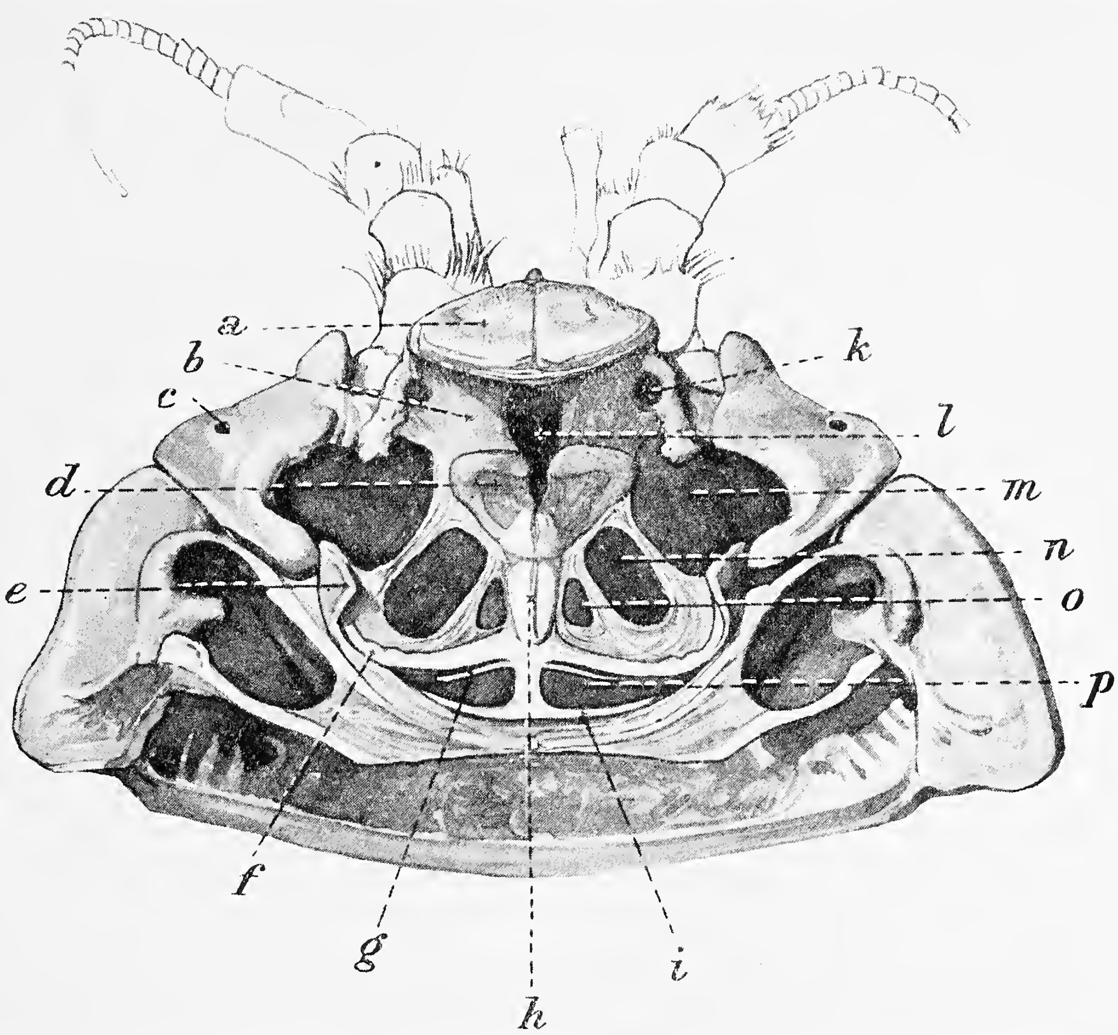


FIG. 18.—Ventral aspect of "cephalosome" of *Glyptonotus*. 2½. Somewhat schematised. The calcified parts of the maxillo-sternal framework (with the exception of the borders of the paragnaths) are rendered in a lighter shade than the membranous parts.

a, labrum; *b*, lateral border of mouth; *c*, ventral eye; *d*, paragnath, with sickle-shaped calcification extending round lateral and ventral border; *e*, styloid process; *f*, transverse alar bar (the reference line is carried to a point which marks the lateral limit of attachment of the alar plate); *g*, anterior bar of first thoracic sternite; *h*, keel of maxillo-sternal framework; *i*, posterior bar of first thoracic sternite; *k*, socket for anterior pivot of mandible; *l*, mouth; *m*, articular foramen for mandible; *n*, articular foramen for first maxilla; *o*, articular foramen for second maxilla; *p*, articular foramen for maxilliped.

Edwards and Bouvier (1902). Anteriorly the keel sends out two lateral, sickle-shaped, calcareous branches, which form the skeletal framework of the paragnaths. It ends just behind the posterior border of the mouth, being connected to the anterior heavy skeleton of the head by means of two broad bands of soft tissue, the lateral borders of the mouth.

Laterally attached on either side of the posterior part of the keel are two slender, calcareous, transverse bars, one behind and one in front of the articular foramen for the maxillipeds. These bars, which end blindly, form the remains of the calcareous sternite of the first thoracic somite which has evidently lost its rigid connection with the dorsal arch or tergite, including the styloid calcification. The pair of posterior bars is movably articulated with the sternite of the second thoracic somite.

Immediately in front of the anterior of these two bars and rigidly connected to the keel is a stronger and more important calcification, the *alar bar*, which runs for some distance transversely, to end by curving forward just medial to the styloid calcification, with which it forms a movable articulation. Neglecting the pair of slender bars belonging to the first thoracic somite, one might say that the calcareous maxillo-sternal framework is cruciform, the keel and the pair of alar bars forming the four limbs of a cross. The two membranous bands which intervene between the three foramina for the mandibles and maxillæ may be described as stretching across the two anterior quadrants of this cross.

We have now seen that the maxillo-sternal framework, with which are connected the paragnaths, the maxillæ, and the maxillipeds, is at every point movably united to the surrounding parts. Near the mouth especially, where it is joined to the surrounding skeleton by long membranous bands, the framework is capable of a considerable range of upward and downward movement, which will no doubt come into play in the process of feeding. We may safely assume that a similar mechanism is present in all isopods. I have already had occasion to point out—Tait (1917, I)—that the œdema produced by immersion of *Ligia* in distilled water causes the paragnaths to protrude, an effect which could occur only if the skeleton carrying the paragnaths is movable.

Reference has already been made to certain ventral endophragmal structures within the cavity of the cephalon. Being rigidly connected to the maxillo-sternal framework, these structures participate in all its movements.

The Ventral Endophragmal Skeleton, which might also be considered as part of the maxillo-sternal framework, has been but little studied in isopods, Lloyd's account of it in *Bathynomus* being the most complete. It consists in the main of the structures named "sternal alæ" by Lloyd, but also of two paired rods which spring dorsally from the keel of the maxillo-sternal framework.

When exposed by dissection from behind the "sternal alæ" of *Glyptonotus* appear as two smooth, pear-shaped objects, which, lying transversely on the floor of the cephalosome, meet medially by means of their pointed extremities (the pear stalks) at some little distance above the floor—*cf.* fig. 16. In this way a medial ventral channel is left for the nerve-chain, the foregut being supported above these structures—see also fig. 21.

The anterior cheek or surface of each of these *alar piriform bodies* differs from its dorsal and posterior surface (which is wholly chitinous) in being rigid and calcareous. The anterior of each pear is occupied with soft tissue, which seems to be a cylindrical or tubular structure folded two or three times upon itself. This internal tissue, which is not fatty, as Lloyd reports in the case of *Bathynomus,* is most probably a part of the maxillary excretory gland.

The calcareous anterior surface, which we may call the *alar plate,* is arranged in a vertical transverse plane; it springs from the alar bar of the maxillo-sternal framework. On the posterior aspect each alar plate is slightly concave to accommodate the internal soft tissue of the pear. The chitinous surface, or *alar capsule,* convex externally, concave internally, is like a skull-cap; attached along the upper border of the alar plate, it is folded backwards and downwards from this line of attachment so as completely to cover the underlying tissue; at its inferior edge it seems to be free, and it is readily lifted from behind. With a little care it may be removed almost complete. It is thin, translucent, and flexible, and appears to act simply as a covering to the underlying soft tissue.

In contrast with the attached capsule the alar plate is a complicated structure. Dissected free of the softer parts it presents an appearance that Lloyd has compared to the wings of a butterfly—see fig. 12. From the central body of the plate spring four *pterygoid processes,* one at each corner, *i.e.* a superior pair and an inferior pair. For descriptive purposes each alar plate might be considered as held in position by four attachments to surrounding parts. Two of these attachments occur at the extremity of pterygoid processes; the other two attachments are brought about by means of special calcareous bars situated in front of the plate, one of these bars being a member of the pair of rods already mentioned as springing from the dorsal aspect of the keel of the buccal frame.

The superior lateral and the inferior medial pterygoid process end freely. The superior medial process is tied to its neighbour of the opposite side by a band of soft tissue. The inferior lateral process is fused to the transverse bar in the buccal frame which separates the articular foramen for the maxilliped from the articular foramina for the two maxillæ, whence it follows that the alar plate can move only with movements of the buccal frame.

When viewed from behind the alar plate seems to be rigidly fixed only at its infero-lateral corner, while its more medial half is to all appearance ill-supported. On its anterior aspect, however, this latter part abuts against the rod-like projection already mentioned as arising from the internal aspect of the keel of the buccal frame. Springing from a point that corresponds to the anterior angle of the articular foramen for the second maxilla, *i.e.* some little distance in front of the alar plate, and rigidly connected to the keel, this calcareous rod runs backwards and upwards at an angle of about 30 degrees with the cephalosomic floor to unite, apparently by a chitinous joint, with the alar plate just at the indentation between the superior and the inferior medial pterygoid processes. Its (mathematical) projection upon the cephalosomic floor would coincide with the chitinous band separating the articular foramina for the two maxillæ. This rod, which we must reckon as a constituent part of the ventral endophragmal skeleton, besides acting as a very efficient strut to the alar plate, no doubt gives origin to some of the muscles which move the maxillæ.

The fourth attachment of the alar plate to surrounding parts takes place by means of a very long process which springs from the superior medial pterygoid process—see fig. 19. Lloyd has described it under the name of the "anterior portion of the sternal ala"; we shall call it the *pharyngeal process.* It has a broad base, which is firmly fused to the pterygoid process along the whole width of the latter, and is situated at a higher level than the attachment of the rod-like strut described in last paragraph, which it consequently hides from view. The basal portion is a triangular lamina which lies in a horizontal plane, and whose apex is produced into a long process like a sabre with back and edge arranged vertically. At its commencement the sabre is straight, horizontal, and calcareous; its distal two-thirds, chitinous and flexible, sweeps upwards in a curve to gain attachment by a clubbed extremity to the internal surface of the cephalon a little above the fossa for the antennule. The pharyngeal process does not confer any special rigidity upon the alar plate; on the contrary, it derives its own fixity from its firm attachment to the plate. According to Lloyd, it gives origin to the muscles which move the foregut. The triangular basal portions of the two pharyngeal processes, diverging like a V, form a platform upon which the foregut rests. The two sabre-like extremities intervene as thin slips between the pharynx and foregut on the one hand and the adductor calcification of the mandible (see p. 291) on the other.

It may here be mentioned that the alar plate gives direct attachment to the first or basal segment of the first maxilla. Previous workers must have noted that this appendage in *Glyptonotus,* if not in all similar isopods, is peculiarly difficult to disarticulate, in spite of the fact that the basal segment in question is very narrow and the corresponding articular foramen in the sternal region exceptionally wide—*cf.* Hansen (1903, p. 22). The point of articulation is situated on the lower border of the alar plate between the two inferior pterygoid processes.

It remains to comment upon the morphological significance of the ventral endophragmal skeleton. Lloyd has shown that in *Bathynomus*

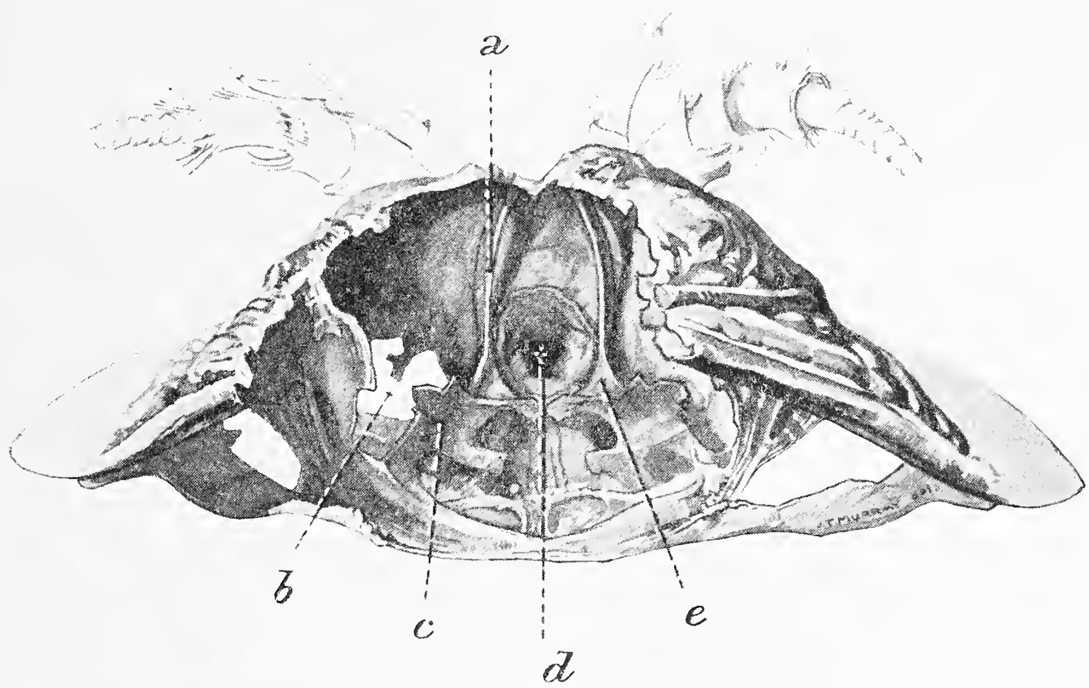


Fig. 19.—Dissection of interior of "cephalosome" of *Glyptonotus.* To show alar plates and pharyngeal processes. × 2½.

The cephalosome has been opened from the dorsum, more freely on the left side than on the right. The ventral appendages on the left side had been removed, those on the right side being left *in situ.*

a, chitinous anterior part of pharyngeal process; *b*, articular foramen for left mandible; *c*, alar plate; *d*, pharynx; *e*, triangular calcareous part of pharyngeal process.

It will be observed that the left alar plate is attached (to the transverse alar bar) only by the infero-lateral pterygoid process. On the right side this arrangement is not so plain because the basal segment of the first maxilla (left *in situ*) blocks the gap below the infero-medial pterygoid process.

it gives origin to muscles for the foregut and for the post-mandibular oral appendages. It seems to be a rule in the anatomy of *Glyptonotus* that the muscles which move the appendages on the body never arise from the sternite, but always from the tergite of the corresponding somite (this we observe throughout the mesosome and metasome and also in the mandibular somite). We have seen that no tergite corresponding to the maxillary somites can be recognised on the dorsal aspect of the cephalon, while the lateral part of the first thoracic tergite is so greatly reduced as to be quite unsuited for any effective muscular attachment. In so far as it gives attachment to the muscles of the post-mandibular oral appendages the ventral endophragmal skeleton fulfils the function of tergites. Consequently one would correlate the presence of this skeleton with the absence of tergites: which suggestion in turn raises its own problems.

The Oral Appendages.

The oral appendages of Crustacea possess a special importance for the systematist, and in the case of *Glyptonotus* they have been carefully described. Our knowledge of the function of these appendages is, however, very meagre, so that I have ventured to set down some additional details as a guide to any worker with access to living material—in which connection it may be pointed out that few Crustacea seem better adapted for experimental work along this line than *Glyptonotus*, with its large size and readily accessible mouth parts.

Hansen (1893) has sought to advance our knowledge of the morphology of these appendages by attempting to establish detailed homologies between them. Thus he says: "In order to understand the structure of the maxillæ in the Malacostraca we must commence with the maxillipeds." It is unnecessary to expound the assumption that underlies such a statement. In a recondite matter of this kind the quest of homologies is apt to resolve itself, as indeed it usually does, into mere reasoning in a circle.

The Mandibles articulate with the cephalosome by a simple hinge (an anterior and a posterior articulatory *pivot* projecting from each mandible fit into corresponding *sockets* in the heavy skeleton of the cephalosome). The posterior of the two sockets is situated within the mandibular articular foramen; the anterior lies in front of the foramen at the base of the labrum. Each mandible may be said to consist of a proximal *body*, semicylindrical in shape, and of a distal *biting process*, which is carried in a smooth curve (its neck) from the body forwards, medialwards, and ventralwards to end in the *biting surface*, which includes a molar and an incisor process.

On the body we recognise the following important parts: (1) the two articulatory pivots; (2) a large medially placed, cylindroid *maxillary polished surface*, which rubs against the antero-lateral aspect of the hard lateral lobe of the first maxilla; (3) a much smaller, posteriorly placed, flattened, *styloid polished surface*, which rubs against the styloid calcification; (4) a *polished groove* medially situated at the join of the body with the neck and burnished by friction against the lateral border of the paragnath; (5) a postero-lateral *abductor process* for attachment of the abductor muscle. Attached by a band of connective tissue to the free edge of the appendage, just dorsal to the maxillary polished surface, is a separate *adductor calcification,* into which is inserted the powerful adductor muscle. These parts are shown in fig. 20.

The line joining the two articulatory pivots does not correspond with the main axis of the semi-cylindrical body. The latter axis lies medial and ventral to that of the mandibular hinge, so that in adduction of the two mandibles not only are the biting processes brought together, but each mandibular body as a whole is carried nearer the middle line and also

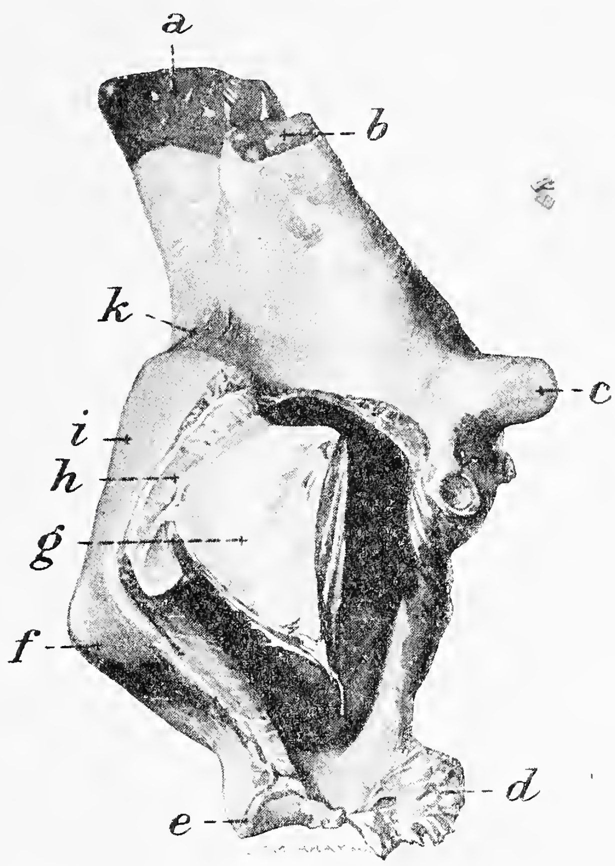


Fig. 20.—Right mandible of *Glyptonotus* from dorsal aspect. × 6.

a, incisor process; *b*, molar process; *c*, anterior articulatory pivot; *d*, abductor process; *e*, posterior articulatory pivot; *f*, styloid polished surface; *g*, adductor calcification; *h*, membranous band between adductor calcification and free edge of mandibular body; *i*, maxillary polished surface; *k*, polished groove, for contact with lateral border of paragnath.

nearer the plane of the buccal frame, thus restricting the space available for movement of the first maxillæ. The axis of the mandibular hinge, while mainly antero-posterior, also runs from behind forwards and medialwards, so that in adduction the mandible is carried to some extent backwards as well as medially. Especially during adduction do the maxillary polished surface and the polished groove come in contact with their appropriate neighbours, so that when molar compression is applied to the food no adjacent crevices are left into which portions might be squeezed.

The connective tissue band between the adductor calcification and the mandible is broad and flexible and allows of a considerable range of simple hinge movement, the necessity for which we can appreciate when we consider how short are the attached muscular fibres and how greatly excentric is the axis of the mandibular body as compared with that of the mandibular joint. On the medial aspect, where it comes in contact with the pharyngeal process of the alar plate, the adductor calcification is smooth and flattened. The two adductor muscles take up a large part of the total space within the cephalon. Each is shaped like an inverted pyramid, and takes origin from almost the whole sculptured region of its own half of the head. At their origin the right and the left muscle meet in the middle line, and between them the foregut rises to a sharp ridge like the roof of a house —see fig. 21. Each abductor muscle, taking origin lateral to the corresponding adductor, is slender, and acts with slight mechanical advantage, the abductor process extending to a less distance from the axis of the joint than does the attachment of the adductor calcification. The mandible is capable of an angular movement of nearly 40 degrees.

Within each mandible is left a cavity of considerable size, which is occupied with glandular tissue. Neither palp nor lacinia mobilis are present on the external aspect. In complete adduction the curved incisor edge of the right mandible is received within the concavity of the correspondingly curved incisor edge of the left, while to prevent slip an additional isolated incisor process on the left mandible is received within the concavity of the curved incisor edge of the right.

The Maxillæ.—The only figures showing the complete skeleton of the maxillæ in any of the Valvifera are to be found in a paper by Hansen (1886).* The skeleton of the distal parts of the maxillæ of *Glyptonotus* has already been figured; that of the more proximal parts is less clearly defined than would appear to be the case in *Chiridotea.*

The basal segment of the first maxilla consists of a solid chitinous or but slightly calcified rod (it is flexible), which articulates with the inferior border of the alar plate, a rounded cavity in the border being prepared to receive the head of the segment. Up to the point of attachment of the medial lobe this proximal rod is stout; beyond this point it thins down, no second element being applied to it as in the case of *Chiridotea.*

The two lobes spring from the proximal rod at a right angle, the skeleton of the medial lobe being in its proximal half a mere localised thickening of the surrounding articular membrane, the corresponding part of the lateral lobe being a calcified gutter open on the medial aspect. While the freely projecting distal half of each lobe is hollow, the proximal

* These figures, which refer to the maxillæ of *Chiridotea*, have been copied in Calman's book (1909).

half of both is contained within one and the same sheet of articular membrane. Consequently from mere inspection of this one type it is impossible to say whether the two were primitively separate and have since partially united, or whether they have arisen by splitting of a single primitive part. Strong muscles are attached to the gutter in the proximal half of the lateral lobe. These arise medially from some part of the medial endophragmal skeleton. There is little independent movement of the one lobe relatively to the other.

The structure of the two terminal parts of the first maxilla make it plain that their function is different, that of the lateral lobe with its stout, darkly pigmented prongs being mechanical, that of the medial lobe with its long, delicate terminal filaments set with innumerable fine hairs being sensory (possibly for special sense, *e.g.* taste).

To the existing descriptions of the second maxillæ and maxillipeds of *Glyptonotus* I have nothing to add.

The Alimentary Canal.

Considered simply as a motor mechanism for intake, onward propulsion and expulsion of food, the alimentary canal of many Crustacea, like that of vertebrates, is more complicated at either end than in the intervening parts, the complexity of the anterior end, as also in vertebrates, being greater than that of the posterior. In *Glyptonotus* the alimentary tube consists of a small and elaborately designed foregut (associated with which are the oral appendages and the ventral endophragmal skeleton of the cephalon), of a large dilated midgut, and of a hindgut of internal volume less than half that of the midgut. The separation between the three parts is very distinct. The whole tube, with the exception of one part, to be mentioned below, runs in a straight line from mouth to anus—see fig. 21.

The Foregut.

The foregut is practically confined to the cephalon, and may be said to terminate at or just beyond the cephalo-thoracic foramen. It begins as a short and narrow tube, the *pharynx*, which runs from the mouth dorsalwards to expand very soon into a dilated chamber, the foregut proper or *vestibule*, to which the terms "stomach" and "gizzard" have also been applied. The use of the term "gizzard" (chosen apparently as an improvement on the older and admittedly unsuitable term "stomach") is itself misleading, for the name suggests that the function of the organ is to triturate the food. This idea is disposed of by the condition of the ingesta discovered in the midgut of the dissected specimens. When the food had consisted of amphipods these were found, according to size, almost intact or cut into longitudinal blocks of about $\frac{1}{4}$ inch length. Similarly meat food was found in strings of similar blocks attached to each other by tags

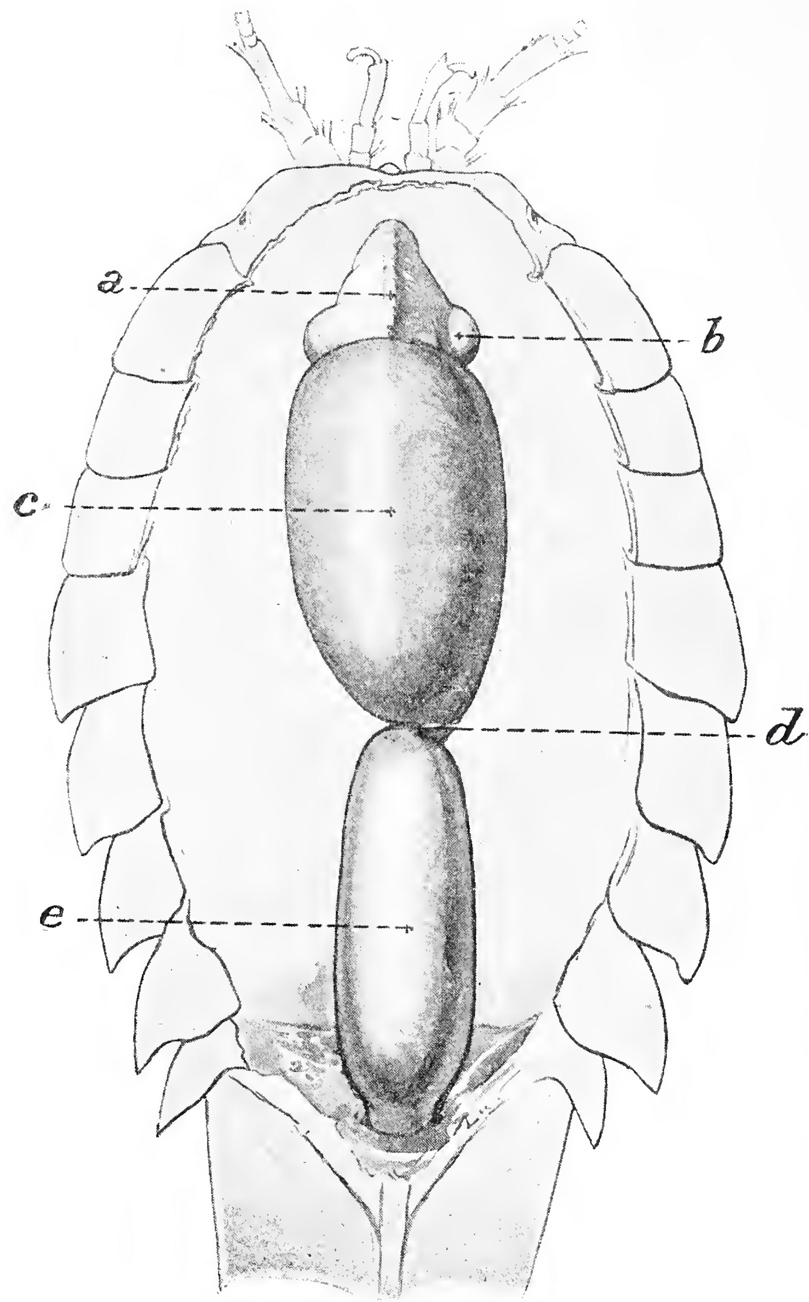


FIG. 21.—Alimentary canal of *Glyptonotus*. × 1½.
a, vestibule; *b*, alar piriform body, upon which vestibule rests; *c*, midgut; *d*, sphincter region between midgut and hindgut; *e*, hindgut.

of connective tissue. The cutting had evidently been done by the incisor processes of the mandibles, the length of the blocks corresponded roughly to the reach of these processes from the position of abduction to that of adduction, and the food had evidently been "bolted" without the occurrence of any further process of comminution in the vestibule.

So far as its motor function is concerned, the whole foregut is merely a propelling mechanism; in other words, the process of *swallowing* is incomplete until the midgut is reached. This view is confirmed by the feeble rigidity of every part of the wall of the vestibule. In Lloyd's account, too, of the foregut of *Bathynomus*, in which the discovery of some form of gastric mill was apparently expected, there is no clear evidence of the existence of such.

In construction the foregut of *Glyptonotus* is fundamentally similar to that of *Bathynomus*, varying however in details, more especially in the posterior part. The involutions of the vestibular wall, to which Lloyd has given distinctive names, are all present, though their position is not so easily recognised from the external aspect. Thus the pair of "anterior ampullæ" and the pair of "posterior ampullæ" are readily seen in the interior. The two "upper valvular processes" take the form of a single medial chitinous plate, which may or may not be medially cleft at the posterior end. This plate is not an obvious duplicature of the wall like the other involutions, but a thin, stiffish sheet of chitin, which reminds one of the capsule attached to the alar plate. Slightly concave inferiorly, it is attached by its anterior edge to the roof of the vestibule, whence it projects freely backwards. The pair of "lower valvular processes" take the form of two long parallel tongue-shaped elevations of the vestibular floor, which extend to a much greater distance posteriorly than the free extremity of the overlying chitinous plate.

The upper and the lower valvular processes are best seen from the posterior end, as when the cephalosome has been detached from the thorax —see fig. 22. The posterior free edge of the vestibule, which projects slightly into the midgut, does not lie exactly in the (vertical) plane of the cephalo-thoracic foramen, but slopes obliquely from above downwards and backwards. Surrounding this free edge on every side is a sinus formed by a forward pouching of the midgut.

We may now turn for a moment to consider the process of manducation and of swallowing. In manducation the gnathopodal hands (each one of which can be brought to lie under the mouth), the first maxillæ and the mandibles would appear to be chiefly concerned. The gnathopods probably act as packers, holding the food towards the mandibles and pushing it forwards between each bite. The lateral lobes of the first maxillæ, working like many-pronged forks in the cleft between the paragnaths, probably help in keeping the food in the middle line. Repeated mandibular adductions alone are insufficient to cause any forward movement, as one can readily prove by trial on the dead animal with a wad of softened paper to represent food.

The food having entered the pharynx, it would seem almost necessary to assume that it is here propelled not simply by the *vis a tergo* of the oral appendages, but by some mechanism peculiar to the canal itself—and Pearson (1908) has shown that special constrictor and dilator muscles are connected with the pharynx of *Cancer*. The infoldings of the wall of the vestibule, which all have a caudal trend, seem to be structural adaptations connected with the further propulsion of the food.

We might perhaps correlate the mobility of the buccal frame with the existence of the large chamber of the vestibule. Were the cephalic wall rigid on every side, distension of the vestibule with food would necessitate

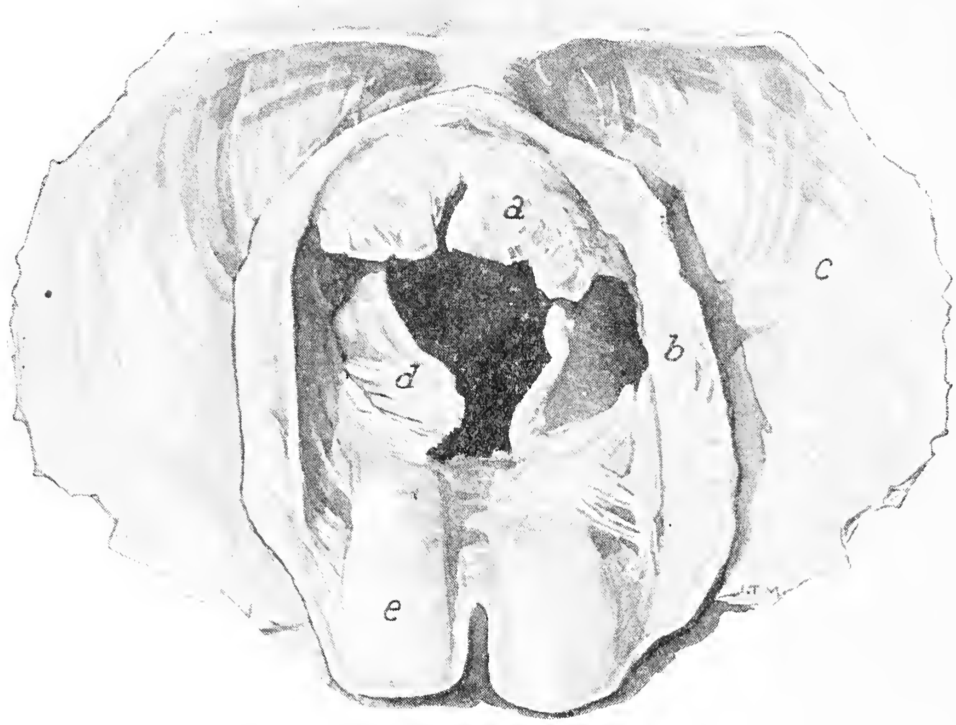


Fig. 22.—*Glyptonotus*. Opening of foregut into midgut as seen from behind. Magnified about 5 diameters.

a, "upper valvular process" of Lloyd; *b*, posterior free edge of vestibule; *c*, wall of midgut; *d*, "posterior ampulla" of Lloyd; *e*, "lower valvular process" of Lloyd.

on each occasion a shifting of an equivalent volume of cephalic contents into the thorax—and at this juncture it might be mentioned that in the dead animal extreme abduction of the mandibles of itself causes a downward movement of the buccal frame. Supposing that the ventral endophragmal skeleton, which is rigidly connected to the buccal frame, were ultimately shown to be derived from tergal elements, one might on the above correlation hypothesis explain why it has come to lie beneath the gut.

In classifying the Malacostraca the number of oral appendages is of fundamental importance. Until we know, however, what maxillipeds are for, the statement that decapods have three pairs, members of the Peracarida one pair, members of the Euphausiacea, etc., none at all, is of systematic interest, but stands for nothing more. Again, we have only the vaguest idea as to the function of the two pairs of maxillæ present in all Crustacea. In order to understand the relation between different types of oral appendicular apparatus it would seem advisable to experiment upon some large form which is at the same time sufficiently simple—and Crustacea thus doubly qualified are rare. Consequently an experimental investigation of the mouth parts of *Glyptonotus* might greatly help to clear up the matter.

The Midgut.

It is remarkable that this part of the gut in the spirit-preserved specimens is fixed in a much dilated condition, its cavity being considerably greater than the volume of the contained food, which can be shaken about as a compacted cylindrical roll within it. In some examples the inner lining shows longitudinal rugæ on the ventral aspect, in other cases the whole internal wall seems to be stretched smooth. It is hard to believe that during life this part of the gut is incapable of contracting on the food; if it does so contract, it is not plain how the dilatation has occurred, unless it be by shrinkage of the muscles and other structures occupying the lateral compartment of the thorax. In one example, whose extreme breadth was 49 mm., the width of the midgut was 20 mm., *i.e.* two-fifths of the breadth of the body.

The midgut, which is fusiform, is drawn to a point at the posterior end, which is situated, according to circumstances, opposite the sternite of the sixth or seventh of the (true) thoracic somite. Here the alimentary tube is greatly narrowed down, without being involuted into the hindgut, as in the specimen of *Bathynomus* examined by Lloyd. In some individuals the roll of food material was found to be continuous along this constriction, in others the gut at this part was tightly shut down for a distance of some millimetres. It is plain that this intermediate short portion between midgut and hindgut acts as a sphincter.

In every case there was a slight want of alignment between the posterior end of the midgut and the anterior end of the hindgut, the former deviating towards the right, so that the narrow connecting tube formed a sigmoid bend. A sigmoid bend with similar direction has been described by Collinge (1916) as an abnormality in *Idotea linearis*.

The hepatic cæca were too much macerated to permit of examination. I failed not only to discover the number of cæca, but even to determine their length.

The Hindgut.

Even when it contains no food the hindgut remains widely open in the preserved specimens. In its thoracic part it is shaped like the end of a spindle; in its pleonic part its cross-section is quadrilateral. The interior of the thoracic portion is perfectly smooth; that of the pleonic portion is thrown into innumerable slight ridges, which confer upon it a reticulate appearance. These ridges, which tend on the whole to run transversely, probably correspond to strands of muscular tissue under the lining, for Miller (1910) has shown that such bundles of muscular fibres occur in the corresponding ridges in the posterior part of the hindgut of the crayfish (*Cambarus*), in which animal the anterior part of the hindgut is likewise smooth. Round the anus the ridges have a radial arrangement—*cf.* Miller's account of radiating muscular bundles round the anus of the crayfish.

The anus is smaller than one might expect from the size of the anal valves. The arrangement of parts in this region may be described as follows. The circumanal cuticle forms a strong diaphragm, which closes the wide posterior opening in the ventral skeleton of the pleon. Though elevated at one portion into two deep parallel folds (the anal valves), the cuticle still extends across the floor of the space between these valves. At the very posterior part of this floor the cuticle is perforated by the anus, and to the edge of the perforation, but to no other part of the diaphragm, the hindgut is attached.

It is possible that the pleopodal muscles may play some part in the onward propulsion of food material along the pleonic portion of the hindgut. When working with *Gammarus*—see Tait (1908, 1910)—I had frequent opportunity of observing that this animal, replaced in sea-water after a longish sojourn in moist air, invariably begins to defæcate. The observation was of interest as pointing to the existence of a natural provision for maintaining the hygiene of the beach, waste being eliminated only when the tide is up. The process may be an independent reflex, in which contact of some part (*e.g.* the anus) with sea-water acts as the exciting stimulus, but it may also, in part at least, be a secondary result of the renewed activity of the pleopods consequent upon immersion. The same phenomenon is observable in the case of *Ligia*, whose pleopods also become active on immersion. It is likewise to be seen in *Carcinus*, whose gut is not sandwiched between pleopodal muscles; which fact rather suggests that the process is an independent reflex. It is worth while mentioning, however, that *Carcinus* helps to extract fæcal masses from the anus by means of its chelæ, whence one might infer that the local mechanism concerned in defæcation is more feeble in this animal than in *Gammarus* or in *Ligia*. In *Idotea*, and presumably in *Glyptonotus*, fæcal pellets are shot backwards to a distance by the current caused by the pleopods.

The Eyes and the Chromatophore-System.

The eyes of *Glyptonotus* are of interest. On first examination they appear to be entirely dorsal in position. In both species, however, each eye is divided into two quite separate parts, one lying on the dorsal the other on the ventral surface.

Eights (1833) described their situation thus: "Eyes . . . placed near the lateral and anterior margin of the head, so deeply impressed in the margin of the shell as to be easily distinguished from beneath." Pfeffer (1887) detected the true ocular nature of the ventral pigmented spot thus indicated by Eights; his account is as follows: "The eyes are situated, as the systematic diagnosis states, on the surface of the head, while in the genus *Idotea* they are situated on the border. The morphological relation between these two conditions may be conceived as follows: In all isopods a narrow border tends to run round the whole periphery of the animal; so also in *Idotea*, in which the border is continued midway across the eye without interfering with the power of vision in this region; for the border is here transparent and participates in formation of the cornea. In this way an *Idotea* can see in the upward direction, horizontally and downwards. In *Glyptonotus* the transparency of the border has been lost; the border, in this case stout and strongly pigmented, courses right across the eye as in *Idotea*. Thus the animal is deprived of vision in the horizontal direction; above and below, the skin over the eye has remained transparent. Consequently *Glyptonotus* has one eye on the dorsal aspect of the head and another on the ventral aspect; the latter has a true cornea, even if less regular and distinct than that of the upper one." Though Pfeffer's description is somewhat roundabout, I have quoted his observations for the sake of his comparison with the eye of *Idotea*.

In *Chiridotea* the eye is entirely dorsal, a notch in the cephalic carapace denoting where the eye once extended over the lateral margin. The extinct *Proidotea* had likewise a notched cephalic carapace, the dorsal eye in this case being apparently situated somewhat nearer the lateral border than in *Chiridotea*—see Racovitza and Sevastos (1910). Whether *Proidotea* had in addition a ventral eye is not known.

Physiological experiment sheds a suggestive light upon these structural peculiarities, for V. Bauer (1905) in experiments on colour change in *Idotea* came to the conclusion that the apparently single eye of this animal acts as a double structure, the upper half functioning separately from the lower. His discovery in brief was this. When one half of the eye—upper or lower, it did not matter—was illuminated, the other half being kept dark, the colour of the animal as a whole became dark. When the whole eye was simultaneously either darkened or illuminated, the change to a dark colour did not occur. His results, of course, incidentally explain how *Idotea* changes colour in response to a change of background; on a dark background the eye is unequally illuminated, on a white background it is wholly illuminated.

Colour change of isopods is due to retraction or expansion of chromatophores in the hypodermis. *Ligia* and *Sphæroma* also undergo colour change in response to change of background, though some other isopods do not—see Tait (1910, 1911). A recent worker, Pieron (1914), whose paper I have failed to obtain, has apparently dealt further with colour change in isopods. The presumption is that the phenomenon is widely distributed throughout the order.

Examination of the spirit specimens of *Glyptonotus* shows not only that black chromatophores are present under the cuticle, but that these are retracted in the lighter coloured animals and expanded in the darker. In other words, the animal is capable of changing colour—and this it probably does according to the mode of illumination of the eyes. From the literature I have been unable to discover whether *Chiridotea* possesses chromatophores. It need hardly be said, however, that the whole question of the structure of the eye in relation to colour change is worthy of study.

During part of the year at least *Glyptonotus* lives under ice, and there are no records to show in what proportion light can penetrate through the frozen surface of the sea. Hellen-Hansen (1912) has found that the extreme limit at which light can affect a photographic plate exposed under open sea-water lies between 1000 and 1700 metres. Lest one should imagine that colour change of isopods can occur only under the incidence of intense light, I may mention that *Ligia* exposed on appropriate light or dark backgrounds under feeble red illumination in a photographic dark-room completely changes colour within the space of an hour or two.

Summary.

1. Advantage has been taken of the large size of *Glyptonotus* to study certain structural features, especially skeletal, which cannot be so readily investigated in smaller isopods. Where possible, an attempt has been made to correlate structural peculiarities with functional use.

2. *The Legs.*—The peculiar articulation, analogous to a spheroidal bony joint, between the coxopodites and basipodites of the thoracic limbs, is described in detail. A comparison is instituted between the full flexion-complex of the isopodan and of the brachyuran walking limb. The peræopods are contrasted with the gnathopods of *Glyptonotus*, and functional peculiarities pertaining to each group of limbs are discussed.

3. *The Perœon.*—The medial split in the thoracic sternites is interpreted as a device for allowing of distension of the body, say, after a meal. The arched thoracic somites articulate with each other in a special way, not by a crossed articulation, like that in a pair of scissors, but by a rocker articulation, like that in a spring clothes-pin; this combination of arch and rocker articulation is interpreted as a means of preventing change of internal volume in body flexion. The phenomena pertaining to the moulting process throw light upon the numerical grouping of the successive fusions of coxæ with somites which have occurred in isopods.

4. *The Pleon.*—The pleon consists of four movable portions, the last four of the seven constituent pieces being welded together. Of the four fused segments, only the first has a complete sternite, like those of somites 1, 2, and 3. In the skeleton of the pleonic floor a wide opening is thus left posteriorly, which is closed by a diaphragm of soft cuticle; part of this diaphragm is elevated into two long parallel folds or valves, one on each side of the anus.

5. *The Pleonic Appendages.*—The protopodite of each of the anterior three pleopods is composed of two complete pieces; a third piece more proximally situated is interpreted, not as evidence of an additional primitive segment, but as a secondary development in the articular membrane. The uropods have acquired their present position by a forward rotation of the sternite of the sixth primitive somite, and the gill-directed surface of the uropod corresponds to the posterior surface of the pleopods. It follows that what has commonly been called the exopodite of the uropod is a real exopodite. In addition to the pleopods, the internal walls of the whole uropodal chamber subserve a respiratory function.

6. *The Cephalosome.*—Two thoracic somites are fused with the head to form a cephalosome. By welded inturnings of the anterior border of these two and of the posterior border of the mandibular cephalic somite a strong internal bracing is formed. The tergites of the maxillary somites have apparently disappeared from the dorsal aspect of the cephalon; the endo-skeletal structures described by Lloyd in *Bathynomus*, and by him named "sternal alæ," functionally correspond to these tergites in so far as they serve for attachment of muscles for the maxillæ. These "sternal alæ"

(which also appear to form a covering for the maxillary excretory gland) with other ventral endoskeletal structures are described in detail. All are rigidly fixed to the maxillo-sternal framework, which is capable of independent movement.

7. *The Alimentary Canal.*—The foregut of *Glyptonotus* is not a gastric mill; the muscle-provided involutions of its wall (exactly analogous to those in *Bathynomus*) are concerned simply with onward propulsion of the food, *i.e.* with swallowing. Between the midgut and the hindgut, the two opposed ends of which are not in exact alignment, is a strongly contractile part of the gut, which acts as a sphincter. The hepatic cæca could not be examined. *Glyptonotus* is carnivorous.

8. *The Eyes and the Chromatophore-System.*—By experiment on colour change of *Idotea* it has been shown that the eye of this animal acts as a double mechanism, the ventral half being functionally separable from the dorsal. *Glyptonotus* has retractile chromatophores, and probably undergoes colour change in accordance with different modes of eye illumination. Its eye is divided into two entirely separate parts, one lying on the dorsal the other on the ventral aspect of the cephalon.

The cost of providing the illustrations which accompany this paper was defrayed by a grant from the Earl of Moray Endowment for the promotion of research in the University of Edinburgh; and the expense of preparing the blocks was defrayed by a grant from the Carnegie Trust.

I have to record my thanks to Dr W. S. Bruce, not only for the material provided, but for information relating to the quarters inhabited by *Glyptonotus*.

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(*Issued separately July 5, 1917.*)

XVI.—Experiments and Observations on Crustacea: Part V. A Functional Interpretation of certain Structural Features in the Pleon of Macrurous Decapods. By John Tait, M.D., D.Sc. (From the Marine Laboratory, Aberdeen, and the Department of Physiology, Edinburgh University.)

(MS. received March 31, 1917. Read May 7, 1917.)

In the last paper of the present series * it was pointed out that the articulations between the thoracic somites of *Glyptonotus* are so designed as to minimise change of internal volume during flexion and extension of the body. The pleon of the long-tailed Decapoda forms a system comparable to the series of thoracic somites of *Glyptonotus*; and as the macrurous decapods execute very rapid strokes of the pleon, one might expect that change of volume during movement is slight, for otherwise there would be waste of energy owing to inertia. It was decided to examine the matter experimentally.

The animals used for experiment were *Homarus*, *Astacus*, and *Nephrops*, and the first observations were made on formalin-fixed specimens. After disarticulation of the pleon from the thorax the internal contents were scooped out, the cavity was filled with water, and the pleon subjected to passive movement. The internal volume was found to diminish with flexion, the water flowing over the edge of the first somite. The experiments, however, could hardly be taken to imitate the natural conditions, for, owing to hardening, the ventral soft cuticle was seen to fold unequally and irregularly.

When the same experiment is carried out on a recently exuviated pleon, or even on an exuviated pleon, which has lain for some time in preservative, passive flexion causes no change of internal volume. In the exuviated pleon the ventral soft cuticle is seen to fold exactly in the middle, the internal face of one half being accurately applied to that of the other half. To make the experiment still more delicate, the first pleonic somite was sealed with a waxed cork, while a narrow glass tube passing through the cork served to narrow down the opening at the upper end and thus to act as a gauge. In this way it was proved that change from full extension to full flexion occurs without any change of volume whatsoever.

* *Ante*, p. 268.

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[*Continued on next page.*

In *Nephrops* the pleonic somites are ring-shaped, the anterior ones being circular, the posterior elliptical; in this case the axis of the intersomitic joint goes right across the middle of the somite. In *Astacus* the pleonic somites are bow-shaped, the greatest transverse diameter being on the ventral aspect; here the axis of the intersomitic joint lies nearer the ventral than the dorsal aspect, the arrangement resembling that in *Glyptonotus.* The pleonic somites of *Homarus*, while not accurately circular, resemble those of *Nephrops* rather than those of *Astacus*, and the joint axis lies near the middle of the ring. Seeing that movement of these different types of pleon occurs in each case without change of internal volume, it is plain that the position of the various hinges is intimately correlated with the form in cross-section of the somites, any variation in the one involving a corresponding variation in the other; that is to say, the principle of constant internal volume, a physiological principle, can be invoked to account for a structural correlation observed in the pleon.

I have to thank Dr H. C. Williamson and Mr J. Mackenzie for material, and Dr T. W. Fulton for granting facilities in the Marine Laboratory at Aberdeen.

(*Issued separately July* 9, 1917.)

XVII.—Observations on the Blood in Gas Poisoning. By James Miller, M.D., Captain R.A.M.C., and Harry Rainy, M.D., F.R.C.P.E.

(MS. received June 30, 1917. Read May 21, 1917.)

The subject of the following paper was suggested by the study of a case (No. 9 of our series) which was admitted to the Second Scottish General Hospital, Craigleith, suffering from general weakness and a certain degree of breathlessness for which no very obvious cause was apparent. Investigation revealed a somewhat unusual condition of the blood, and on seeking for a cause for this it occurred to one of us (H. R.) that it might be due to the effects of a comparatively slight gas poisoning which the patient had sustained, but to which he had not attached much importance. Investigations were therefore instituted in other cases under our own care and that of our colleagues, with the result that the surmise was confirmed, and we are now able to submit the results of the examination of fifty cases and to reach our conclusions from a sufficiently extended series of observations.

The great bulk of the work of this joint paper has fallen upon Captain Miller, who, as pathologist to the hospital, took supervision of the laboratory and blood counts; Dr Rainy's share being mainly the initiation of the investigation and the providing of certain of the clinical cases, especially in the earlier stages of the research. The authors, however, hold themselves jointly responsible for the views which the communication expresses.

The effects produced by the inhalation of chlorine gas may be divided into immediate and remote.

As regards the immediate effects upon the body tissue generally, Schäfer (*British Medical Journal*, 1915, ii, 245) has shown experimentally that considerable quantities of Ringer's solution, saturated with the gas, may be introduced directly into the circulation without producing any very marked effect beyond a quite temporary diminution of blood-pressure and a slight increase in the depth of respiration.

With inhalation, on the other hand, the results are always serious. Even with air containing only 1 per cent. of chlorine a profound and sudden change occurs, the animal's respirations become very deep, then convulsive, and ultimately cease altogether. The blood-pressure rises slightly at first, then falls rapidly, whilst at the same time there is marked slowing of the pulse. On substituting air for the mixture of air and chlorine, respiration returns immediately to the normal, while the blood-pressure rises temporarily to a much greater height than normal.

A second period of inhalation produces the same results at the end of a longer period, but usually leads to the death of the animal.

Administration of a 2 per cent. chlorine mixture produces similar effects, with a cessation of respiration, which are not recovered from. Artificial respiration has no effect. With a mixture of 5 per cent. or greater concentration a fatal result is rapidly and inevitably produced.

Schäfer considers that these effects must be due to a local effect on the lungs, since the chlorine which is inhaled cannot be carried to the tissues in a free state. Moreover, there is evidence, in animals killed in the above manner, that there is no poisoning of the tissues, because their muscles contract briskly and the heart responds to stimulation. The only visible change is in the lungs. These, even after the shortest exposure to a fatal dose, are intensely red, and distended; they possess a solid feel, and are not crepitant, although small pieces still float. In Schäfer's opinion the fatal result is due to obstruction in the pulmonary capillaries, which makes it impossible for the blood to pass freely to the left side of the heart. There is apparently not only no constriction of the bronchioles, but experimental methods indicate that they are actually more permeable.

Microscopic sections show the pulmonary capillaries engorged with blood; there is œdema of the interstitial tissue and of the air vesicles. Probably the œdema is due to the vascular obstruction. It is significant that the epithelium of the bronchial tubes is well preserved.

Leonard Hill (*British Medical Journal,* 1915, ii, 801) confirms Schäfer's results in the main, but does not consider that death is due to stasis in the pulmonary vessels. He believes, on the contrary, that obstruction in the air passages, through contraction of the bronchial muscle, bears an important part in the symptoms, and further suggests that Schäfer's results may have been due to the relatively high concentration of the gas which he employed in his experiments. He interprets the sequence of events in the lung as follows:—Just as lymph is poured out after a superficial burn of the skin, or the application of a blistering fluid, or in a septic wound under the influence of bacterial toxins or antiseptics, so an exudation of lymph in the lungs is excited by the action of chlorine. The epithelial lining, both that of the mucous membrane and of the capillary wall, is damaged by the poison, and fluid is withdrawn, by osmotic forces, from the damaged vessels. In short, the classical phenomena of inflammation are exhibited, ending in capillary stasis as a result of concentration of their contents through the exudation of plasma. Death, however, is regarded by Hill as due, not to capillary stasis, but to the presence of fluid in the air cells and passages, the man being actually drowned in his own secretions.

In man the immediate effects of chlorine gas poisoning since the Germans introduced gas into warfare in the spring of 1915 has been observed in a large number of cases. Notes on a series of 685 cases were published in the *British Medical Journal*, 1915, ii, 165, by Black, Glenny, and McNee. Cases have also been described by Broadbent (*British Medical Journal*, 1915, ii, 247), and an excellent summary will be found in Hurst's *Medical Diseases of the War*, 1917.

The most marked immediate effects are burning pain in the throat and eyes, associated with a feeling of suffocation. Pain is also felt in the chest, especially behind the sternum. Respiration is painful, rapid, and difficult. Retching and vomiting very commonly occur, the latter sometimes giving temporary relief. Severe headache soon comes on, and unconsciousness may occur almost immediately; at other times it is delayed for some hours. In very severe cases the face may assume a pale, greenish-yellow colour; in less severe cases it is violet-red, and the ears and finger-nails blue. The skin is cold and the temperature subnormal. The pulse-wave is large, unless collapse is present, and it rarely rises to over 100 per minute. Respirations are jerky, shallow, and rapid, often 40 or even 80 to the minute. The auxiliary muscles of respiration are in active motion. Coughing may be frequent and painful, and much frothy sputum is brought up.

On examination the percussion note is impaired, and auscultation reveals the presence of moist sounds of varying qualities over the whole chest.

The dyspnœic and asphyxial stage lasts some thirty-six hours, after which the patient may fall asleep and waken much better. After a few hours of comparative quiet, symptoms of bronchial irritation begin to show themselves. Sometimes broncho-pneumonia supervenes. The sputum becomes viscid, yellowish, or greenish, with occasional streaks of blood. Respirations are rapid and shallow, 70–80 to the minute. The pulse is small and rapid and the temperature rises, sometimes to 104° F. Other complications which may supervene are pleurisy, empyema, and gangrene of the lung.

The post-mortem examination of acute cases shows thin, light yellow, frothy secretion filling the trachea and bronchi. The mucous membrane of the respiratory passages is swollen and œdematous. A slight degree of œdema of the glottis is observed in some cases. The lungs are voluminous, and subpleural hæmorrhage occurs. On section the lungs are of a deep maroon-red colour, with abundant secretion flowing from the cut surface. Light grey patches of acute emphysema are observed along the borders. Microscopically, the portions of the lung not affected by emphysema show marked congestion of the vessels. The alveoli are filled with a homogeneous amorphous exudate taking on eosin stain. Occasionally fibrin threads, red cells, and leucocytes are met with.

The heart, more especially the right side, is dilated.

The stomach shows evidence of acute catarrh. The mucosa is covered with thick yellowish mucus and shows submucous hæmorrhages, which are sometimes extensive.

The more remote effects of gassing are various. An analysis of 50 cases observed by us results as follows:—

Conjunctivitis	2 cases.
Dyspnœa and bronchitis	33 ,,
Vomiting and dyspepsia	6 ,,
Headache, tremors, nervous symptoms, and loss of sleep	10 ,,

Broadbent (*British Medical Journal,* August 14, 1915) records five cases of nephritis following upon gas poisoning. The albumen was not present when the men were first admitted to hospital, but appeared in the course of a few days. Epithelial and granular casts were present in abundance. Broadbent says that "it looks as if in some cases the chlorine or bromine damages the lung epithelium so severely that it does not allow absorption into the general circulation, while in others the gas passes through the lungs without affecting them permanently, but then sets up an acute nephritis." The observations of Schäfer are against any such damage of tissue other than that of the lung. Leonard Hill combats this view of Broadbent, stating that the nephritis should be ascribed to the intense and prolonged dyspnœa and the struggles for breath. He says that albuminuria is a common result of the very temporary dyspnœa which athletes suffer in a race. It results from want of oxygen in the kidneys. We have not observed evidence of kidney damage in our cases.

The striking thing about these cases of gassing is the prolonged disability of the men. The symptoms persist for months and years in many cases. We have seen a number of such cases; the following may be taken as instances:—

Cases 3 and 6.—Twelve months after gassing, marked conjunctivitis and tremors.

Case 9.—Eleven months after gassing, still suffering from breathlessness.

Case 15.—Twelve months after gassing, still suffering from nausea and vomiting.

Case 49.—Eighteen months after gassing, still suffering from dyspepsia, breathlessness, vomiting, sleeplessness.

Case 45.—Twenty-two months after gassing, still symptoms of bronchitis.

Thus the condition is one of very great importance from the point of view of the health of the army, the return of men to service, and the determination of pensions.

Gases used for offensive purposes have been, and are, many and various. They may be divided into two groups: (*a*) cloud gas discharged from cylinders and carried by the wind; (*b*) gas shells and gas bombs in which the poisonous substance is contained in the liquid form in the missile and is converted into a cloud of vapour on the explosion. The gas first used by the Germans in the spring and early summer, 1915, was almost certainly chlorine. It is certain that subsequently other gases were added. The above description of symptoms applies mainly to the 1915 gas.

It is, of course, difficult to obtain exact information from a man who has been gassed as to the smell of the gas and his first symptoms, but we have been able to classify our cases as follows:—

17 cases occurred in 1915.
31 cases occurred in 1916 or 1917.
18 were cases of cloud gassing.
19 were cases of shell gassing.
9 were poisoned with British gas.
27 were poisoned with gas which was either probably or certainly German.

As regards the blood change which we are describing, there is no marked difference. If the gassing is severe, the change is marked whether the gas be cloud or shell. Of the 15 cases in which the lymphocyte count was over 50 per cent., 5 were definitely cases of cloud gassing and 10 cases of shell gassing.

Our attention was first drawn to the blood changes in gas poisoning by the following case:—An officer was gassed (by British gas) October 1915. He was exposed for about twenty minutes, but only complained at the time of feeling dazed. He suffered from cough for three months, and in September 1916, when in Craigleith Hospital, he complained of lassitude, and of some breathlessness and fatigue on exertion. He was anæmiclooking, but on examination nothing was found but a few rhonchi in the lungs. The case being a puzzling one, a blood examination was resorted to, and on making a differential count 39 per cent. of polymorphs and 51 per cent. of lymphocytes were found. The total number of leucocytes was 6562 per c.m. The conjunction of a lymphocytosis with gassing was regarded as significant, and on further investigation the sign was found to be characteristic of such cases. We published a short note in the *Lancet*, January 6, 1917, describing 14 cases with blood counts. Our observations have now been extended to 50 cases, the main points in which will be found in Table I.

In our original communication we made the statement that the change takes some time to develop, probably three to four months. Further observation has shown that this is not necessarily so, for in Cases 33 and 39, which were observed respectively one month and six weeks after gassing, the change was already well marked.

Recently we had the opportunity of examining the blood of an officer who was accidentally gassed in this country with chlorine. The accident happened on January 24, 1917, and the blood was examined on January 27, 1917. The red cells numbered 5,500,000, the hæmoglobin was 98 per cent., and the white cells were 5900 per c.m. Beyond a slight leucopenia, there was therefore no change. The differential count was normal, polymorphs 68·5 per cent., and lymphocytes 25·5 per cent.

Little can be gathered from the evidence of a single case, but what evidence there is goes to show that the gassing results in a destruction of leucocytes, but that at such an early stage there is no stimulation of any one type of leucocyte. The case was a very slight one, the patient returning to duty within a week of the gassing.

In order to ascertain whether the relative lymphocytosis is due to an actual increase in the number of lymphocytes or to a diminution in the polymorpho-nuclear leucocytes, we have constructed Table II, giving a series of cases taken at random in which a leucocyte count was made in addition to the differential count. From the two data—number of leucocytes per cubic centimetre, and percentage of the two main types found—the actual number of lymphocytes and polymorpho-nuclear leucocytes has been given in figures. The first lines of the table give the average in normal cases at what may be taken as the physiological limits and the mean (Gulland and Goodall, *The Blood*, 2nd edition, p. 78). This table shows that in all cases the sign is marked (the only exceptions in the table being Nos. 10, 11, and 16, which were comparatively slight), and there is an absolute increase in the number of lymphocytes. In certain cases

TABLE I.

Case.	Poly-morphs.	Lympho-cytes.	Mono-nuclears.	Eosino-phils.	Mast.	Period between Gassing and Examination.	Immediate Symptoms.	Symptoms at Time of Examination.
	Per cent.	Per cent.	Per cent.	Per cent.	Per cent.			
1	50·2	40·8	4·6	3·4	·8	3 months.	Nausea.	No symptoms at present.
2	53·19	40·0	5·5	·4	·8	3 „	Marked symptoms.	Discharged from army as result of gassing, September 1916.
3	47·6	44·6	2·6	1·8	1·8	12 „	„ „	Conjunctivitis and tremors still present.
4	33·5	60·0	3·7	1·0	·7	5 „	Fairly severe gassing.	Still unable to march any distance; tremors.
5	58·2	33·4	2·8	4·6	·9	12 „	Slight case.	No symptoms.
6	51·1	45·2	1·4	2·2	...	12 „	Fairly severe symptoms.	Eye symptoms, conjunctivitis, tremors.
7	58·5	35·0	3·8	2·5	...	2 „	No symptoms.	No symptoms.
8	37·5	56·3	3·2	2·6	...	12 „	Breathless, unconscious for 4 hours.	
9	39·9	51·7	4·3	3·2	·8	11 „	Dazed, cough 3 months.	Breathlessness.
10	62·3	30·2	4·3	1·2	1·9	12 „	Slight case.	No symptoms.
11	72·1	23·4	2·5	2 0	...	9 „	„ „	„ „
12	35·3	58·2	2·7	3·2	·2	17 „	Unconscious, dyspnœa, nausea.	
13	47·9	47·6	2·4	·3	1·5	12 „	Unconscious, 4 days' intense nausea and vomiting.	Persistent pain in chest.
14	50·0	40·0	6·4	3·4	·2	2½ „	Half unconscious, dyspnœa.	Huskiness of voice.
15	36·2	55·4	2·3	5·08	·4	1 month.	Twice gassed, unconscious second time.	Vomiting, dyspepsia, pain in head.
16	75·7	21·7	1·2	1·2	...	2 months.	Unconscious.	Cough, tightness in chest.
17	49·3	45·6	3·0	2·0	...			
18	54·5	36 3	4·0	4·5	·6			
19	56·2	37·0	1·9	3·9	·9	1 month.	Choking sensation.	No symptoms.
20	53·8	38·6	4·3	2·1	1·2	6 months.	Slight case.	„ „
21	40·3	54·8	1·8	2·4	·6	13 „	„ „	Gastric pain, vomiting, losing weight.
22	58·5	37·4	2·5	1·1	·2	12 „	Lost power of legs but did not go sick.	Shortness of breath.
23	57·5	35·2	6·7	·3	...	17 „	Slight case.	Chronic otitis media, no present symptoms.
24	59·9	37·9	1·1	·5	·3	1 month.	Could not eat food for several days; stretcher case.	No symptoms, suffering from scabies, discharged fit.

25	47·2	48·3	1·6	2·4	·2	3 months.	Became unconscious 4–5 hours after attack.	Languid, breathless, has sick spells.
26	32·1	63·7	2·8	·9	·3	3 ,,	Became unconscious in $1\frac{1}{2}$ hours after attack.	Headache, cough, breathlessness.
27	50·5	43·2	2·5	3·5	·5	12 months.	Unconscious 4 days.	Has frequent attacks like pneumonia, which last 3–4 days and subside.
28	52·4	43·7	1·5	1·8	·3	3 ,,	Cough and breathlessness.	Cough persists but is much improved.
29	70·3	26·9	1·1	1·4	...	3 ,,	Cough, vomiting, and unconscious.	Dyspnœa and pain in chest.
30	53·2	41·9	·9	3·8	...	2 ,,	,, ,, ,,	,, ,, ,,
31	49·4	49·4	·9	·3	...	3 ,,	Slight cough, copious catarrh.	Cough and spit.
32	56·4	41·7	·4	1·3	...	3 ,,	Vomiting, severe headache.	Severe headache.
33	35·5	56·2	5·0	5·0	·5	1 month.	Vomited; stretcher case.	Dyspnœa and pain in chest.
34	47·0	49·0	1·08	1·9	·2	3 months.	,, ,, ,,	,, ,, ,,
35	50·4	44·6	1·7	3·2	...	12 ,,	No symptoms.	No symptoms.
36	68·0	30·0	2·0	...	...	2 ,,	Vomited.	Tight feeling in chest.
37	37·8	54·8	2·6	3·5	1·1	12 ,,	Cough and vomiting.	Wheezing and cough.
38	56·3	39·3	1·7	2·7	...	4 ,,	Not unconscious, vomiting not severe.	Bronchitis.
39	44	54·3	·3	1·3	...	6 weeks.	Unconscious 4 hours, very severe case.	Breathlessness, tightness in chest, giddiness.
40	66·7	35·3	2·0	2·0	...	11 months.	Unconscious 2 hours, back to duty in 6 days.	Complains of shortness of breath.
41	50·0	45·0	3·0	2·0	...	6 ,,	Did not feel much at time. A few days after, cough and breathlessness.	Bronchitis.
42	59·4	34·9	2·9	1·8	1·0	20 years.	In bed with bronchitis for 5 weeks. Indigestion and bronchitis 6 months.	None, but liable to bronchitis.
43	56·6	35·2	6·7	1·5	...	15 ,,	Pain, choking sensation. Bronchitis lasting 3 weeks.	Almost complete loss sense of smell.
44	47·7	47·5	4·0	·4	·4	2 months.	Slight.	Breathlessness and cough.
45	44·3	52·2	1·8	1·7	...	22 ,,	Unconscious 14 hours. Severe acute bronchitis.	Chronic bronchitis.
46	29·5	66·5	2·0	1·9	...	3 ,,	Choking and vomiting.	Cough, breathlessness, pain in chest, sleeplessness.
47	43·1	52·4	2·2	2·3	...	3 ,,	,, ,, ,,	Cough, pain in chest, tremors, dyspnœa, sleeplessness.
48	40·0	57·5	1·8	1·7	...	$2\frac{1}{2}$,,	Unconscious, breathless, vomiting.	Pain over heart, breathlessness, sleeplessness.
49	40·4	56·6	3·0	1	...	18 ,,	Breathless.	Dyspepsia, vomiting, breathlessness, sleeplessness.
50	63·6	35·0	·8	·6	...	$2\frac{1}{2}$,,	Dyspnœa, vomiting.	Dyspnœa on exertion.

TABLE I.

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3	47·6	44·6	2·6	1·8	1·8	12 "	" "	Conjunctivitis and tremors still present.
4	33·5	60·0	3·7	1·0	·7	5 "	Fairly severe gassing.	Still unable to march any distance; tremors.
5	58·2	33·4	2·8	4·6	·1	12 "	Slight case.	No symptoms.
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7	58·5	35·0	3·8	2·5	...	2 "	No symptoms.	No symptoms.
8	37·5	56·3	3·2	2·6	...	12 "	Breathless, unconscious for 4 hours.	
9	39·9	51·7	4·3	3·2	·8	11 "	Dazed, cough 3 months.	Breathlessness.
10	62·3	30·2	4·3	1·2	1·9	12 "	Slight case.	No symptoms.
11	72·1	23·4	2·5	2·0	...	9 "	" "	" "
12	35·3	58·2	2·7	3·2	·2	17 "	Unconscious, dyspnœa, nausea.	
13	47·9	47·6	2·4	·3	1·5	12 "	Unconscious, 4 days' intense nausea and vomiting.	Persistent pain in chest.
14	50·0	40·0	6·4	3·4	·2	2½ "	Half unconscious, dyspnœa.	Huskiness of voice.
15	36·2	55·4	2·3	5·08	·4	1 month.	Twice gassed, unconscious second time.	Vomiting, dyspepsia, pain in head.
16	75·7	21·7	1·2	1·2	...	2 months.	Unconscious.	Cough, tightness in chest
17	49·3	45·6	3·0	2·0	...			
18	54·5	36·3	7·0	1·5	·6			
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20	53·8	38·6	4·3	2·1	1·2	6 months.	Slight case.	" "
21	40·3	54·8	1·8	2·4	·6	13 "	" "	Gastric pain, vomiting, losing weight.
22	58·5	37·4	2·5	1·1	·2	12 "	Lost power of legs but did not go sick.	Shortness of breath.
23	57·5	35·2	6·7	·3	...	17 "	Slight case.	Chronic otitis media, no present symptoms.
24	59·9	37·9	1·1	·5	·3	1 month.	Could not eat food for several days; stretcher case.	No symptoms, suffering from scabies, discharged fit.
25	47·2	48·3	1·6	2·1	·2	3 months.	Became unconscious 4–5 hours after attack.	Languid, breathless, has sick spells.
26	32·1	63·7	2·8	·9	·3	3 "	Became unconscious in 1½ hours after attack.	Headache, cough, breathlessness.
27	50·5	43·2	2·5	3·5	·5	12 months.	Unconscious 4 days.	Has frequent attacks like pneumonia, which last 3–4 days and subside.
28	52·4	43·7	1·5	1·8	·3	3 "	Cough and breathlessness.	Cough persists but is much improved.
29	70·3	26·9	1·4	1·4	...	3 "	Cough, vomiting, and unconscious.	Dyspnœa and pain in chest.
30	53·2	41·9	·9	3·8	...	2 "	" " "	" " "
31	49·1	49·4	·9	·3	...	3 "	Slight cough, copious catarrh.	Cough and spit.
32	56·4	41·7	·4	1·3	...	3 "	Vomiting, severe headache.	Severe headache.
33	35·5	56·2	5·0	5·0	·5	1 month.	Vomited; stretcher case.	Dyspnœa and pain in chest.
34	47·0	49·0	1·08	1·9	·2	3 months.	" " "	" " "
35	50·4	44·6	1·7	3·2	...	12 "	No symptoms.	No symptoms.
36	68·0	30·0	2·0	...	...	2 "	Vomited.	Tight feeling in chest.
37	37·8	54·8	2·6	3·5	1·1	12 "	Cough and vomiting.	Wheezing and cough.
38	56·3	39·3	1·7	2·7	...	4 "	Not unconscious, vomiting not severe.	Bronchitis.
39	44	54·3	·3	1·3	...	6 weeks.	Unconscious 4 hours, very severe case.	Breathlessness, tightness in chest, giddiness.
40	66·7	35·3	2·0	2·0	...	11 months.	Unconscious 2 hours, back to duty in 6 days.	Complains of shortness of breath.
41	50·0	45·0	3·0	2·0	...	6 "	Did not feel much at time. A few days after, cough and breathlessness.	Bronchitis.
42	59·4	34·9	2·9	1·8	1·0	20 years.	In bed with bronchitis for 5 weeks. Indigestion and bronchitis 6 months.	None, but liable to bronchitis.
43	56·6	35·2	6·7	1·5	...	15 "	Pain, choking sensation. Bronchitis lasting 3 weeks.	Almost complete loss sense of smell.
44	47·7	47·5	4·0	·4	·4	2 months.	Slight.	Breathlessness and cough.
45	44·3	52·2	1·8	1·7	...	22 "	Unconscious 11 hours. Severe acute bronchitis.	Chronic bronchitis.
46	29·5	66·5	2·0	1·9	...	3 "	Choking and vomiting.	Cough, breathlessness, pain in chest, sleeplessness.
47	43·1	52·1	2·2	2·3	...	3 "	" " "	Cough, pain in chest, tremors, dyspnœa, sleeplessness.
48	40·0	57·5	1·8	1·7	...	2½ "	Unconscious, breathless, vomiting.	Pain over heart, breathlessness, sleeplessness.
49	40·4	56·6	3·0	1	...	18 "	Breathless.	Dyspepsia, vomiting, breathlessness, sleeplessness.
50	63·6	35·0	·8	·6	...	2½ "	Dyspnœa, vomiting.	Dyspnœa on exertion.

there is some diminution in the number of polymorpho-nuclear leucocytes, which tends to accentuate the sign, but the lymphocytosis is nevertheless an absolute one. Moreover, in Cases 13 and 15, which show a fairly well-marked leucocytosis, there is still a lymphocytosis. These cases are given in detail below.

TABLE II.

Case.		Leucocytes. Per cubic millimetre.	Polymorphs. Per cent.	Lymphocytes. Per cent.	Total.	
					Polymorphs.	Lymphocytes.
	Normal { minimum	5,500	...	...	3850	1,100
	Normal { average	7,000	70	20	4900	1,400
	Normal { maximum	9,000	...	...	6300	1,800
1	Severe case	5,000	50	40	3500	2,930
3	Persistent symptoms	5,968	47	44	2820	2,676
4	,, ,,	6,570	33	60	2168	3,942
7	Slight case	7,812	58	35	4530	2,734
9	Severe case	6,562	39	51	2593	3,360
10	Very slight case	8,124	62	30	5036	2,437
11	,, ,,	7,184	72	23	5047	1,638
16	Severe case	9,686	75	21	7264	2,034
12	,,	5,000	35	58	1750	2,900
13	Moderately severe	8,122	47	47	3816	3,816
39	Severe case	11,562	50	46	5781	5,300
15	,, (2nd exam.)	17,000	37	58	6401	10,034
13	,, ,,	15,000	44	46	6600	6,900

Table III classifies the cases according to the degree of lymphocytosis. It will be seen that all with a count of 50 per cent. or more of lymphocytes are severe cases, with the exception of 21, which was a slight case of gassing, but the symptoms have persisted to the present time, *i.e.* sixteen months after the gassing. Of those with a lymphocyte count of 40 per cent. or over, all, with the exception of three, 14, 31, and 35, are severe cases or cases with persistent symptoms. To these one other, 44, should perhaps be added, but it is of too recent occurrence to be sure of the course it will follow. Too much stress cannot, of course, be laid on these percentages, but we have found that they are wonderfully constant on repeated examination, and in blood counts made by different observers.

A classification of the cases according to the period which has elapsed since the gassing (Table IV) does not yield any very definite information beyond the fact that the sign is a very persistent one, and has not disappeared in any instance in our experience.

In this relation we were fortunate in being able to examine the blood of two schoolmasters (Cases 42 and 43) who had been gassed with chlorine, the one fifteen and the other twenty years previously. Both suffered severely at the time, and symptoms persisted, in the one for some weeks, and in the other for many months. The counts in each instance give a

TABLE III.

Case No.	Lymphocytes. Per cent.	Interval.	Symptoms.
Cases with Lymphocyte Count of over 50 per cent.			
46	66·5	3 months	Severe case, breathlessness, cough.
26	63·7	3 "	" " unconscious.
4	60·0	5 "	" " symptoms still.
12	58·2	17 "	" " unconscious.
48	57·5	2½ "	" " breathlessness.
49	56·6	18 "	" " " vomiting.
8	56·3	12 "	" " unconscious 4 hours.
33	56·2	1 "	" " vomiting.
15	55·4	1 "	" " persistent symptoms.
21	54·8	13 "	Slight, persistent gastric symptoms.
37	54·8	12 "	Fairly severe, cough and wheezing still.
39	54·3	6 weeks	Severe case, unconscious.
47	52·4	3 months	" " breathlessness, cough.
45	52·2	22 "	Chronic bronchitis.
9	51·7	11 "	Fairly severe, dazed, cough still.
Cases with Lymphocyte Count of over 40 per cent.			
31	49·4	3 months	Slight case.
34	49·0	3 "	Vomited, stretcher case.
25	48·3	3 "	Unconscious.
13	47·6	12 "	Severe case, unconscious.
44	47·5	2 "	Slight case, persistent symptoms.
6	45·2	12 "	Fairly severe, eye symptoms still.
41	45·0	6 "	" " case, symptoms still.
3	44·6	12 "	Marked case, eye symptoms still.
35	44·6	12 "	Slight case, no present symptoms.
28	43·7	3 "	" "
27	43·2	12 "	Severe case, still symptoms.
30	41·9	2 "	Unconscious.
32	41·7	3 "	Vomited, headaches.
1	40·8	3 "	Severe case.
2	40·0	3 "	Marked symptoms.
14	40·0	(2) "	Partly unconscious, symptoms still.

slight relative increase of lymphocytes, but they are probably not outside the limits of error.

On investigating the cases more closely, they may be classified into groups. In the first place, a group the counts of which approximate more or less closely to the normal, which are, in other words, either normal or

Table IV.

Months.	Case No.	Poly-morphs. Per cent.	Lympho-cytes. Per cent.	Symptoms.
Over Six Months.				
22	45	44·3	52·2	Severe case, unconscious 14 hours, still bronchitis.
18	49	40·4	56·6	Fairly severe case, symptoms still present.
17	12	35·3	58·2	Severe case, unconscious.
17	23	57·5	35·2	Slight case, no present symptoms.
13	21	40·3	54·8	" "
12	8	37·5	56·3	Severe case, unconscious for 4 hours.
12	37	37·8	54·8	Fairly severe, cough and wheezing still.
12	13	47·9	47·6	Severe case, unconscious, but no symptoms now.
12	6	51·1	45·2	Fairly severe, with eye symptoms.
12	35	50·4	44·6	Slight case, no present symptoms.
12	3	47·6	44·6	Marked case, with eye symptoms.
12	27	50·5	43·2	Severe case, still complains.
12	22	58·5	37·4	Did not go sick, still breathless.
12	5	58·2	33·4	Slight case.
12	10	62·3	30·2	" " no present symptoms.
11	40	66·7	35·3	Fairly severe, symptoms still present.
11	9	39·9	51·7	" " dazed, cough still.
9	11	72·1	23·4	Slight case, no present symptoms.
Six Months or under.				
6	41	50·0	45·0	Fairly severe, persistent bronchitis.
6	20	53·8	38·6	Slight case.
5	4	33·5	60·0	Severe, symptoms still.
4	38	56·3	39·3	Vomiting, persistent bronchitis.
3	1	50·2	40·8	Nausea, severe gassing.
3	2	53·1	40·0	Marked symptoms, symptoms still.
3	25	47·2	48·3	Unconscious.
3	26	32·1	63·7	"
3	28	52·4	43·7	Slight cough, breathlessness.
3	29	70·3	26·9	Unconscious.
3	31	49·4	49·4	Slight case.
3	32	56·4	41·7	Vomiting and headache.
3	34	47·0	49·0	Vomited, stretcher case.
3	46	29·5	66·5	Fairly severe, persistent dyspnœa.
3	47	43·1	52·4	" " " "
2½	48	40·0	57·5	" " " "
2	7	58·5	35·0	Very slight case.
2	14	50·0	40·0	Half unconscious, slight symptoms still.
2	16	75·7	21·7	Unconscious.
2	30	53·2	41·9	"
2	36	68·0	30·0	Slight case.
2	44	47·7	47·5	" " persistent bronchitis.
1½	39	44·0	54·3	Severe case.
1	15	36·2	55·4	Twice gassed, unconscious, symptoms still.
1	19	56·2	37·0	Choking sensation.
1	24	59·9	37·9	Stretcher case.
1	33	35·5	56·2	Vomiting, stretcher case.

give a lymphocyte count of less than 40 per cent. Of these eight—viz. 5, 7, 10, 11, 19, 20, 22, and 23—are definitely slight cases in which the gassing was a secondary consideration.

Taking Case 22 as an example:—

This is the case of a man who was gassed on September 25, 1915, through lifting his helmet for a couple of seconds only. He lost the power of his legs for some ten minutes, but did not vomit. He has since childhood suffered from bronchitis and asthma, but has been worse since the gassing. He did not go sick until some time afterwards, but he has since been discharged from the army on account of his asthma. This is a case in which a condition previously present was aggravated by gassing, but clearly the actual gassing was not severe.

There is another series of cases in which counts approximately normal were obtained, but in which the gassing was moderately severe. These require a rather more careful scrutiny. Such are 16, 24, 29, 36, 38, 40, and 50.

Case 16 was gassed in July 1916. His statement is that he was rendered unconscious, and he complained when admitted to hospital of cough and tightness in the chest. He was sent to a convalescent home soon after, where he was punished for breaking bounds, and he has since rejoined his unit and has received promotion. It is clear that he cannot have been a severe case of gas poisoning.

Case 24 was gassed with cloud gas in August 1916, through not having his helmet on. He was not rendered unconscious, but had to be carried back on a stretcher. He could not eat food for several days. He suffered from cough. His symptoms had practically disappeared before admission to hospital. A note from the medical officer of the convalescent home to which he was sent states that he had neither gastric nor pulmonary symptoms when discharged. He has since rejoined his unit.

Case 29 was gassed with shell gas at the end of July 1916. He had no helmet on, and suffered immediately from vomiting and breathlessness. These symptoms continued. Fourteen hours later he became unconscious, and remained so all day. He continued to suffer from cough with expectoration, and when admitted to hospital complained of breathlessness on exertion and pain in the chest. No heart lesion was found on auscultation. Although showing an almost normal blood count on October 25, 1916, a month later the count was as follows:—polymorphs 58·7 per cent., lymphocytes 39·3 per cent. The man was suffering from psoriasis. This man volunteered the statement that the gas smelt like ammonia.

Case 36 was gassed with shell gas on August 21, 1916. He vomited, but did not become unconscious. He complains of having a large amount of expectoration every morning, in addition to a tight feeling in the chest and a pain in the left side. He looks very well, and nothing can be made out on examination. The film which was sent from a distance was bad. No great reliance can therefore be placed on the result, but, as the man has returned to duty, the case is obviously a slight one. The medical officer of the convalescent home regarded the man as making the most of his condition.

Case 38 was gassed at the same time as 29. He states that the shell gas had a sickly sweet odour. He vomited immediately, but had no cough or breathlessness, and did not become unconscious. He was able to put on his helmet. He continued in action all night, and felt better in the morning. His age is 41, and he has suffered from cough for many years. His present attack of bronchitis began two months after the gassing. He is evidently a case of chronic bronchitis of long duration.

Case 40 was gassed in December 1915. He is uncertain whether it was shell or cloud gas, but it had a "pleasant smell." It made his eyes water and caused severe coughing. He felt "intoxicated," and became unconscious, remaining in this condition for about two hours. He vomited freely in the dressing station. He returned to duty after six days. He has always been subject to asthma and bronchitis, but since being gassed he has been very short-winded. His medical officer has no doubt that he exaggerated the story of the gassing, as he makes the most of all his ailments. His present attack of bronchitis does not appear to have been connected with the gassing. A second examination of his blood, a month after the first, resulted as follows:—polymorphs 57·5 per cent., lymphocytes 33·5 per cent.

It will be seen that, of the above-mentioned cases, three are much less marked cases of gas poisoning than the story of the patient would indicate, all having returned to duty within three months of the gassing. Other two are cases of chronic bronchitis which happened to be gassed, but a considerable interval elapsed between the gassing and "going sick." No. 29, the only remaining one of the series, is apparently a clear case of pretty severe gas poisoning without marked blood change. A complication in this case, however, is a skin condition—psoriasis.

Case 50 is of recent origin, but there is every reason to anticipate, from the progress of the patient, that his recovery will be reasonably rapid.

We may place in relation to this group the two cases of gas poisoning in schoolmasters many years ago, Nos. 42 and 43. Both of these, after a fairly sharp illness, recovered, and were able to resume their usual avocations.

The cases with marked and persistent respiratory symptoms are most numerous. The common complaints are pain in the chest, cough, and breathlessness. Such cases are 2, 8, 9, 12, 25, 26, 27, 28, 30, 33, 34, 37, 39, 41, and 45.

Case 9 is the case already noted, which first drew our attention to the condition.

Case 31 was gassed July 25, 1916 (shell). He was partly buried by the explosion and stunned. He had breathlessness from the first, also vomiting. He was conscious at the time of a sweet, rather pleasant, odour. At the time of examination, October 20, 1916, he had a slight cough, and somewhat copious catarrhal expectoration. He had a scorbutic skin condition. The percentage of polymorphs and lymphocytes was the same, viz. 49·4 per cent. of each. This case is classified as one of the four slight cases we have observed with a high lymphocyte percentage.

Case 37 was gassed September 25, 1915, by British gas from a burst cylinder. He was rendered unconscious by the shell, which burst the cylinder, so that he had a good dose of the gas. When he recovered consciousness he started coughing and vomiting. He complained (October 10, 1916) of pain in the right side of the chest, and was troubled with wheezing and coughing. His medical man, not being able to make out much in the way of physical signs, regarded the man as a humbug. His blood count proved to be:—polymorphs 37·8 per cent., lymphocytes 54·8 per cent., showing that he was a genuine case of gas poisoning.

Case 39 was gassed October 15, 1916 (by shell gas). He noticed a sweetish odour which caught his breath. He was unconscious for four hours. Oxygen was administered for four days. His most prominent symptoms, when seen by us, were breathlessness, pain and tightness in the chest, and giddiness. He complained also of impaired digestion. His blood count was:—polymorphs 44 per cent., lymphocytes 54·3 per cent. His leucocytes numbered 11,562.

Case 41 was gassed May 1916 (shell). He did not feel much at the time. A few days afterwards he developed cough and breathlessness. He is still suffering from bronchitis. His blood count is:—polymorphs 50 per cent., lymphocytes 45 per cent.

The remaining cases may be classified into groups according to the nature of the chief symptoms. The larger proportion show at some period more or less marked symptoms associated with the respiratory passages. In another series of cases the symptoms are mainly gastric. Not infrequently both types are to be met with together. Still another series contains those cases in which nervous symptoms predominate.

In a small group of two cases, 3 and 6, gassed at the same time (September 1915), by British gas, the chief symptom was obstinate conjunctivitis. This was associated with tremors. It is a striking evidence of the profound effect which gassing has upon the tissues of the body that a year after the gassing the conjunctiva should exhibit such marked inflammatory change.

Nervous phenomena are exhibited by a large number of cases, sometimes in the form of what may be called neurasthenia, as in Cases 15 and 27. Headache is severe and persistent in some, *e.g.* 15 and 32. Tremors were present in Cases 3, 4, and 6.

Sleeplessness is a nervous phenomenon observed in a series of cases which have come under our observation quite recently, 46, 47, 48, and 49. These men stated that they got scarcely a wink of sleep. The sister in charge confirmed this, and noticed also that when they did sleep they were restless, shouted, sat up, and exhibited symptoms of terror. With the exception of one who was gassed in September 1915, they were all gassed in January 1917, about the same time.

Gastric symptoms are among the most obstinate in some cases. Two of these, 13 and 15, are worth recording, as they are associated with a high leucocyte count.

Case 13 was gassed by British gas in September 1915. He lost consciousness some hours afterwards, and remained unconscious for four days. For several days afterwards he suffered from intense nausea and vomiting, and these persist, in some degree, up to the present time. He was sent home, and after a month returned to the Front, and remained there from November 1915 till July 1916. He remained fairly well until May, when he began to suffer from pains, particularly in the shoulder and chest, like a knife going through him. He was again sent home, and has been twice in Craigleith Hospital, with intervals at convalescent homes. He has constant nausea and disinclination for food, and occasionally vomits. The pains in the chest and limbs also persist to the present time. His blood shows a persistent moderate leucocytosis (15,000). His differential count on last examination was:—polymorphs 44 per cent., lymphocytes 46 per cent. There is thus an absolute increase in the polymorphs as well as in the lymphocytes, although it is more marked in the case of the lymphocytes (see Table II).

Case 15 is a very similar case who was gassed on two separate occasions. First, July 30, 1916, slightly, owing to sleeping through a gas attack, in his dugout; and, on a second occasion, August 3, 1916, while trying to extricate an officer from his dugout after the explosion of a shell. On the latter occasion he was rendered unconscious. His case is complicated by neurasthenia following shell shock. He suffers from persistent pains in the head and dyspepsia, with frequent vomiting. His blood shows:—polymorphs 36·2 per cent., lymphocytes 55·4 per cent. The number of leucocytes per cubic millimetre on the last examination was 17,000. An analysis of the cases as arranged in the above groups is given in Table V.

Table V.—Analysis of Cases in regard to Relation between Percentage of Lymphocytosis and Severity of Symptoms.

Group I. Lymphocyte percentage less than 40 per cent.
1. Slight cases. (Total eight.) Nos. 5, 7, 10, 11, 19, 20, 22, 23.
2. Cases originally moderately severe. (Total seven.) Nos. 16, 24, 29, 36, 38, 40, 50.
3. Old cases occurring many years ago, and associated at the time with severe symptoms. (Total two.) Nos. 42, 43.
4. Case insufficiently recorded to determine classification. (Total one.) No. 18.
Group II. Lymphocyte percentage of 40 per cent. or over.
1. Slight cases. (Total four.) 14, 31, 44, with slight respiratory symptoms; 35, with no symptoms.
2. Intermediate cases. (Total three.) 1, 21, with gastric symptoms; 28, with respiratory symptoms.
3. Severe cases. (Total twenty-four.)
(*a*) Symptoms respiratory. (Eighteen cases.) 2, 8, 9, 12, 25*, 26, 27*, 30, 33, 34, 37, 39, 41, 45, 46, 47, 48, 49*.
(*b*) Symptoms gastric. (Four cases.) 13, 15*, 25*, 49*.
(*c*) Nervous symptoms. (Eleven cases.)
i. Neurasthenia, Nos. 15*, 27*.
ii. Headache, Nos. 15*, 26*, 32.
iii. Tremor, Nos. 3 †, 4, 6 †.
iv. Insomnia, Nos. 46*, 47*, 48*, 49*.
4. Case insufficiently recorded to determine classification. (Total one.) No. 19.

The cases marked with an asterisk appear in the list in more than one place, as several prominent symptoms were present. The two cases marked with a dagger are of interest because, in addition to the other symptoms, they exhibit a long-continued conjunctivitis which has persisted for months after the tissue cells originally injured must have been replaced by new ones.

The type of leucocyte which is increased is the small lymphocyte with relatively large, deeply staining nucleus, and comparatively little protoplasm. In a few cases, *e.g.* 49, there was a fair sprinkling of larger lymphocytes, with a broader rim of protoplasm and larger nucleus. No other type of blood cell appeared to be influenced in any constant fashion.

The last point which arises is the question of the cause of the blood change. A relative or absolute lymphocytosis is met with as an accompaniment of a large number of pathological conditions. Of the diseases associated with *relative* lymphocytosis mentioned in Gulland and Goodall (*loc. cit.*, p. 79), only one, chronic catarrh of the small intestine, seems to have any relation to the condition met with in gas poisoning. Undoubtedly gastric catarrh is a prominent symptom in a large proportion of these cases.

Another pathological condition, in which the lymphocytosis is not merely relative but *absolute*, is whooping-cough. This is, of course, a subacute condition of the larger bronchial tubes, believed to be due to a specific organism, the bacillus of Bordet and Gengou. This disease presents a very close analogy to the cases of gassing, in which the respiratory tract is mainly affected.

The blood change would appear to have no relation to the nature of the gas employed. Apparently, therefore, it is due to some change set up which is common to all types of gas poisoning. It may be that it is the chronic catarrh of the respiratory or alimentary mucous membrane which explains it, but we are still in the early stages of our knowledge of the effects of gas poisoning on the body. They are undoubtedly very profound and persistent. So far as we have been able to ascertain, no mention is made of a blood change, such as we have observed in any official study of cases of gas poisoning, or in experimental records published by physiological investigators. We know, however, that catarrh of the respiratory and gastric mucous membranes is present in the early stages, and tends to persist. Lymphocytosis is met with in tuberculosis, syphilis, whooping-cough, and gastro-intestinal catarrh, and other subacute and chronic infections. From analogy, therefore, one would judge that chronic catarrh is the main factor in the production of the change. We are at present extending our observations to other chronic inflammatory diseases of the lungs and mucous surfaces, and we hope on a subsequent occasion to be able to present further statistics with reference to the occurrence of lymphocytosis in these diseases.

Summary.

1. In cases of gas poisoning in which symptoms persist there is an increase in the number of lymphocytes, relative and absolute, in the circulating blood. In slight cases this may not be beyond the normal limits, or in excess of what may be met with from other causes. In any marked case, however, the change is sufficiently striking to be of some importance in cases where the medical officer is in doubt as to the reliance to be placed upon the statements of men complaining of having been gassed.

2. The blood change is elicited by a differential count of the leucocytes,

and it may be taken that a count in which the percentage of lymphocytes approaches that of the polymorpho-nuclear leucocytes indicates that the patient is still suffering from the effects of gassing, provided always that there is no other complicating disease present which might produce a similar change. A slight relative lymphocytosis is not an uncommon finding, and particularly in men from overseas, so that no great reliance can be placed upon the sign unless it is marked, *i.e.* unless the percentage of lymphocytes approaches closely that of the polymorpho-nuclear cells.

3. The cell which is increased is the ordinary small lymphocyte of the blood. There may be, in some cases, a diminution in the number of polymorpho-nuclear leucocytes which will, of course, accentuate the sign, but the increase of lymphocytes is an absolute one. Moreover, it appears in cases with a high leucocyte count.

4. The change is one which develops early, probably within a month of the gassing, and continues for a long time, in cases with persistent symptoms for at least eighteen months.

5. The change appears to be independent of the kind of gas, and it is shown by patients exhibiting many varieties of symptoms.

6. It is not clear what the change is due to, but from analogy with other conditions exhibiting a lymphocytosis it is probable that chronic inflammatory change in respiratory and gastric mucous membranes is at least a factor.

In conclusion, we should like to thank our colleagues in No. 2 Scottish General Hospital for their co-operation, notably Captain Glen, who has taken a great deal of trouble in looking out cases; also Miss MacLean, M.D., and Mr Malcolm Smith, who carried out many of the differential counts. Dr Graham Brown and Miss MacNeil, M.B., supplied us with a number of cases from Seafield War Hospital, Leith; and Major Wallace, C.M.G., Red Cross Commissioner, gave us access to a number of others.

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XVIII.—Vanishing Aggregates. By Professor William H. Metzler.

(MS. received June 1, 1917. Read July 9, 1917.)

In 1888 Dr Thomas Muir gave the following theorem:—*

Theorem A.—If any two determinants A and B of the nth order be taken, and from these two sets of determinants be formed, namely, first, a set of nCr determinants, each of which is in r rows identical with A and in the remaining rows with B; and, secondly, a set of the same number of determinants each of which is in r columns identical with A and in the remaining columns with B, then the sum of the first set of determinants is equal to the sum of the second set.

Let the αth and βth be two complementary selections of a and b respectively of the n numbers 1, 2, . . . n (where $a+b=n$), and let $\Delta\begin{pmatrix} a & b \\ \alpha & \beta \end{pmatrix}$ be the determinant formed by taking for its αth selection of a rows the αth selection of a rows from A, and for its βth (*i.e.* the complementary) selection of b rows the βth selection of b rows of B; *i.e.* the rows from A occupy the same positions in $\Delta\begin{pmatrix} a & b \\ \alpha & \beta \end{pmatrix}$ that they do in A, and similarly for those from B.

Let $\Delta\begin{pmatrix} \alpha & \beta \\ a & b \end{pmatrix}$ represent the determinant formed similarly from the columns, then the theorem may be stated symbolically thus:

Theorem A.

$$\Sigma\Delta\begin{pmatrix} a & b \\ \alpha & \beta \end{pmatrix} = \Sigma\Delta\begin{pmatrix} \alpha & \beta \\ a & b \end{pmatrix},$$

where there are $\dfrac{|\underline{n}}{|\underline{a}\,|\underline{b}}$ determinants on each side of the equation.

The object of this paper is to extend this theorem so as to involve k instead of two determinants.

Let $a+b+c+\ . \ . \ . +k=n$, and let the αth, βth, γth, . . . κth selections of a, b, c, . . . k respectively of the n numbers 1, 2, 3, . . . n be a set of complementary selections. Let $\Delta\begin{pmatrix} a & b & c & \ldots & k \\ \alpha & \beta & \gamma & \ldots & \kappa \end{pmatrix}$ be the determinant formed by taking for its αth selection of a rows the αth selection of a rows from the determinant A, for its βth selection of b rows the βth selection of b rows from the determinant B, and so on, so that the rows taken from the determinants A, B, C, . . . K occupy the same positions in

* *Proc. R.S.E.*, vol. xv, p. 103.

$\Delta\binom{a\ b\ c\ \ldots\ k}{\alpha\ \beta\ \gamma\ \ldots\ \kappa}$ that they did in the determinants from which they were taken. Let $\Delta\binom{\alpha\ \beta\ \gamma\ \ldots\ \kappa}{a\ b\ c\ \ldots\ k}$ be the determinant formed similarly from the columns.

Theorem B.—Then

$$\Sigma\Delta\binom{a\ b\ c\ \ldots\ k}{\alpha\ \beta\ \gamma\ \ldots\ \kappa}=\Sigma\Delta\binom{\alpha\ \beta\ \gamma\ \ldots\ \kappa}{a\ b\ c\ \ldots\ k},$$

where the number of determinants on each side is

$$\frac{\underline{|n}}{\underline{|a}\ \underline{|b}\ \ldots\ \underline{|k}}.$$

Before considering the proof of this theorem, let us consider an example where $n=4$, $a=2$, $b=c=1$, and therefore the number of determinants formed in each set is

$$\frac{\underline{|4}}{\underline{|2}\ \underline{|1}\ \underline{|1}}=12.$$

We have

$$\begin{aligned}
&R(a_{11}a_{22}b_{33}c_{44})+R(a_{11}a_{22}c_{33}b_{44})+R(a_{11}b_{22}a_{33}c_{44})\\
&+R(a_{11}c_{22}a_{33}b_{44})+R(a_{11}b_{22}c_{33}a_{44})+R(a_{11}c_{22}b_{33}a_{44})\\
&+R(b_{11}a_{22}a_{33}c_{44})+R(c_{11}a_{22}a_{33}b_{44})+R(b_{11}a_{22}c_{33}a_{44})\\
&+R(c_{11}a_{22}b_{33}a_{41})+R(b_{11}c_{22}a_{33}a_{44})+R(c_{11}b_{22}a_{33}a_{44})\\
=\ &C(a_{11}a_{22}b_{33}c_{44})+C(a_{11}a_{22}c_{33}b_{44})+C(a_{11}b_{22}a_{33}c_{44})+\ldots+C(c_{11}b_{22}a_{33}a_{44})
\end{aligned}$$

Where for convenience R $(a_{11}a_{22}b_{33}c_{44})$ stands for

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14}\\ a_{21} & a_{22} & a_{23} & a_{24}\\ b_{31} & b_{32} & b_{33} & b_{34}\\ c_{41} & c_{42} & c_{43} & c_{44}\end{vmatrix},$$

and $C(a_{11}a_{22}b_{33}c_{44})$ for

$$\begin{vmatrix} a_{11} & a_{12} & b_{13} & c_{14}\\ a_{21} & a_{22} & b_{23} & c_{24}\\ a_{21} & a_{32} & b_{33} & c_{34}\\ a_{41} & a_{42} & b_{43} & c_{44}\end{vmatrix}.$$

The truth of this relation may be seen by expanding each determinant involved, by Laplace's theorem, in terms of minors of the second order containing the a's and their complementaries. Then it will be seen that the coefficient of any minor of the second order in the a's on one side is equal to the coefficient of the same minor on the other side of the equation.

Thus taking the coefficient of

$$\begin{vmatrix} a_{11} & a_{12}\\ a_{21} & a_{22}\end{vmatrix}$$

on each side gives the relation

$$\begin{vmatrix} b_{33} & c_{34} \\ b_{43} & c_{44} \end{vmatrix} + \begin{vmatrix} c_{33} & b_{34} \\ c_{43} & b_{44} \end{vmatrix} = \begin{vmatrix} b_{33} & b_{34} \\ c_{43} & c_{44} \end{vmatrix} + \begin{vmatrix} c_{33} & c_{34} \\ b_{43} & b_{44} \end{vmatrix},$$

which is an example of Theorem A for the two determinants

$$\begin{vmatrix} b_{33} & b_{34} \\ b_{43} & b_{44} \end{vmatrix} \text{ and } \begin{vmatrix} c_{33} & c_{34} \\ c_{43} & c_{44} \end{vmatrix}.$$

If we had expanded in terms of the b's and their complementaries and taken the coefficient of any b such as b_{11}, we would have

$$\begin{vmatrix} a_{22} & a_{23} & c_{24} \\ a_{32} & a_{33} & c_{34} \\ a_{42} & a_{43} & c_{44} \end{vmatrix} + \begin{vmatrix} a_{22} & c_{23} & a_{24} \\ a_{32} & c_{33} & a_{34} \\ a_{42} & c_{43} & a_{44} \end{vmatrix} + \begin{vmatrix} c_{22} & a_{23} & a_{24} \\ c_{32} & a_{33} & a_{34} \\ c_{42} & a_{43} & a_{44} \end{vmatrix}$$

$$= \begin{vmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ c_{42} & c_{43} & c_{44} \end{vmatrix} + \begin{vmatrix} a_{22} & a_{23} & a_{24} \\ c_{32} & c_{33} & c_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix} + \begin{vmatrix} c_{22} & c_{23} & c_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{vmatrix},$$

which is another example of Theorem A for the determinants

$$(a_{22}a_{33}a_{44}) \text{ and } (c_{22}c_{33}c_{44}).$$

For the proof of Theorem B it is sufficient to observe that the truth of the case for three determinants is seen by expanding by Laplace's theorem in terms of minors formed from the a's and their complementaries; then the coefficient of any minor of the a's on the one side is equal to the coefficient of the same minor on the other side by Theorem A. Having thus established the theorem for three determinants, it is extended in a similar manner to four, five, and so on up to any number k.

We might use for our k determinants the mutually exclusive minors of order n formed from any n rows of a determinant, of order nk. That is, these minors would have no two columns alike. If we select the minors so that some of them will have columns alike, it is apparent that some of the terms on the right in Theorem B will have columns alike, and therefore disappear.

Syracuse University,
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XIX.—The Bone-Cave in the Valley of Allt nan Uamh (Burn of the Caves), near Inchnadamff, Assynt, Sutherlandshire.

By **B. N. Peach**, LL.D., F.R.S., and **J. Horne**, LL.D., F.R.S. With Notes on the Bones found in the Cave, by **E. T. Newton**, F.R.S. (With Four Plates and Six Text-Figures.)

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CONTENTS.

A characteristic feature of the plateau of Cambrian Limestone in the neighbourhood of Inchnadamff is the occurrence in it of swallow-holes, caves, and subterranean channels which are intimately associated with the geological history of the region. The valley of Allt nan Uamh (Burn of the Caves), locally known as the Coldstream Burn, furnishes striking examples of these phenomena. One of the caves in this valley yielded an interesting succession of deposits, from which were collected abundant remains of mammals and birds. The discovery of bones of the Northern Lynx, the Arctic Lemming, and the Northern Vole among these relics, and the collateral evidence of the materials forming some of these layers, seem to link the early history of this bone-cave with late glacial time, or at least with a period before the final disappearance of local glaciers in that region.

Several caves in the limestone cliff on the southern slope of the valley of Allt nan Uamh were noted by us in the course of the Geological Survey of the district in 1885, but as our attention was directed mainly to the complicated tectonics of the region, no time was spent on their exploration. Fortunately, during a visit to Inchnadamff in 1889, an opportunity was afforded of a systematic examination of the cave now under description, when we were assisted in the work by the Rev. Mr Short, who had considerable experience in cave exploration in England, and by Mr Clarence Fry.

The collection of bones was submitted to Mr E. T. Newton, F.R.S., for determination, and a brief preliminary report on the deposits and the fauna was communicated to the Geological Section of the British Association in 1892.*

Before describing the sequence of deposits in this bone-cave, a brief account will be given of the physical and geological features of the district, the glaciation, and the underground drainage. For these phenomena have an important bearing on the initiation of the cave, and on the interpretation of the evidence furnished by the earlier deposits.

I. Physical and Geological Features of the District.

The accompanying map (fig. 1) shows the prominent topographical features of the region. In the central tract a marked depression runs from Allt Sgiathaig (Skiag Burn), which flows into Loch Assynt from the north, southwards by Inchnadamff, and along the valley of the Loanan to Ledmore. It is bounded on the east by an undulating plateau which is prominently developed between the Traligill river and Allt nan Uamh. Beyond this plateau there is a conspicuous range of high ground, extending from Glas Bheinn (2541 feet) in a south-easterly direction to Conamheall and Ben More (3273 feet). It is continued southwards in the Breabag range, whose highest points rise above the level of 2000 feet. Allt nan Uamh drains the western slope of Breabag, and joins the river Loanan about a mile and a quarter south from Inchnadamff.

On the west side of the central depression there is a group of isolated hills, Spidean Coinich (2508 feet), Beinn Gharbh (1769 feet), and Canisp (2779 feet), with lofty escarpments facing the west and gentle slopes towards the east. These physical features bear a close relation to the geological structure of the region.

Detailed descriptions of the complicated tectonics of the district surrounding Inchnadamff have been given in the memoir on "The Geological Structure of the North-West Highlands of Scotland." † For our present purpose it will be sufficient to indicate the distribution of the rock formations in relation to the glaciation and the underground drainage.

The depression running along the Skiag Burn and the valley of the Loanan to Loch Awe coincides in a general way with the boundary between the belt of country lying to the east that has been affected by the great series of post-Cambrian movements and the undisturbed area to the west.

* *Brit. Assoc. Rep.* for 1892, p. 720. See also *Trans. Inv. Sci. Soc.*, vol. iv, p. 118.

† *Mem. Geol. Surv.* (1907), pp. 508–525. See also Geol. Surv. 1-inch sheets 107 and 101.

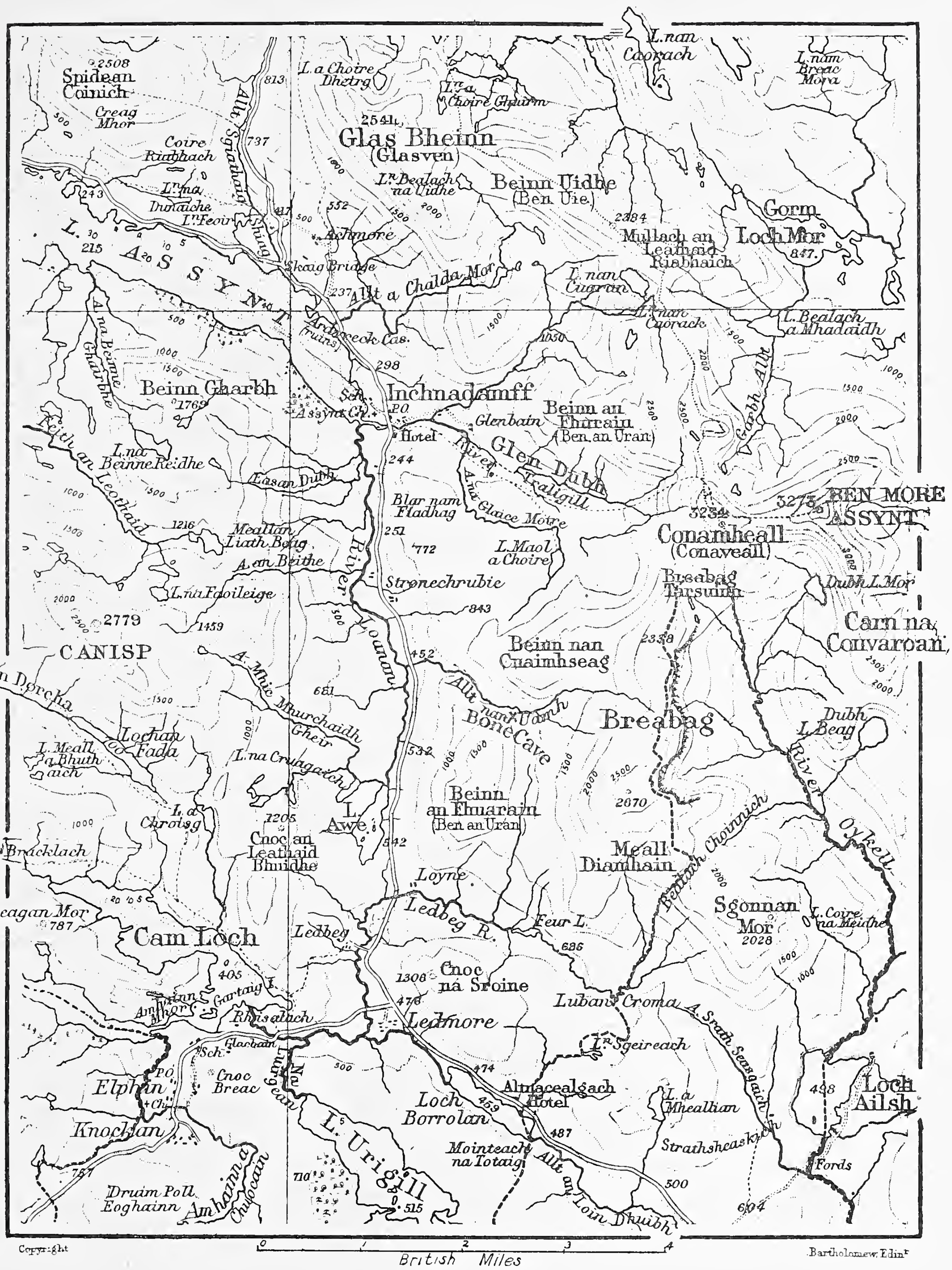


Fig. 1.—Map of the district surrounding Inchnadamff and Allt nan Uamh (Burn of the Caves).

The group of isolated hills from Spidean Coinich to Canisp are composed of Cambrian quartzites with intrusive sheets of igneous material, resting partly on Torridon Sandstone, and partly on the ancient platform of Lewisian Gneiss, all in normal sequence.

The range of mountainous ground extending from Glas Bheinn by Conamheall to Ben More is composed of thrust masses of Cambrian quartzite, Lewisian Gneiss, and, to a limited extent, on Ben More of Torridon Sandstone, which have been driven westwards by the Glencoul and Ben More thrusts. The Breabag range consists mainly of displaced Cambrian quartzites (fig. 4, p. 336) with intrusive sheets of igneous material. Further east on Sgonnan Mòr, a core of Lewisian Gneiss is laid bare in association with Torridon Sandstone and Cambrian quartzite, the whole succession on that mountain overlying the Ben More thrust-plane.

The undulating plateau lying between the central depression and the eastern range of mountains consists mainly of Cambrian dolomite and limestone, repeated by numerous thrusts and folds. It stretches from Achumore to Inchnadamff, thence up the Traligill river for a distance of 2 miles, and southwards to Allt nan Uamh and the Ledbeg river. Beyond the granitic intrusion of Cnoc na Sròine it has an extensive development, for it spreads over the peaty flat, about 4 miles in width, south-east of the hamlets of Elphin and Knockan. In the plateau between Inchnadamff and Allt nan Uamh the three lowest groups of the Durness sequence of Cambrian dolomites and limestones (1, Ghrudaidh, 2, Eilean Dubh; 3, Sail Mhòr) are represented; in the peaty moorland south-east of Elphin, only the two lowest have been recorded.

The accompanying section (fig. 2) shows the complicated arrangement of the Cambrian dolomites and limestones between the Traligill river and Allt nan Uamh, where the zones have been heaped up by major and minor thrusts. These piled-up calcareous masses are resting on a sole or thrust-plane that truncates the underlying strata. Of special interest are the two outliers of displaced materials above the Ben More thrust-plane which have been left on the limestone plateau. They form Beinn an Fhuarain and Beinn nan Cnaimhseag to the south and north of Allt nan Uamh. With the exception of a core of Lewisian Gneiss on the west face of Beinn an Fhuarain, these outliers are composed of Torridon Sandstone with a small development of basal quartzite and pipe-rock. They clearly point to the original westward extension of the materials overlying the Ben More thrust-plane, and to their isolation by prolonged denudation from the main mass to the east of Breabag (fig. 2).

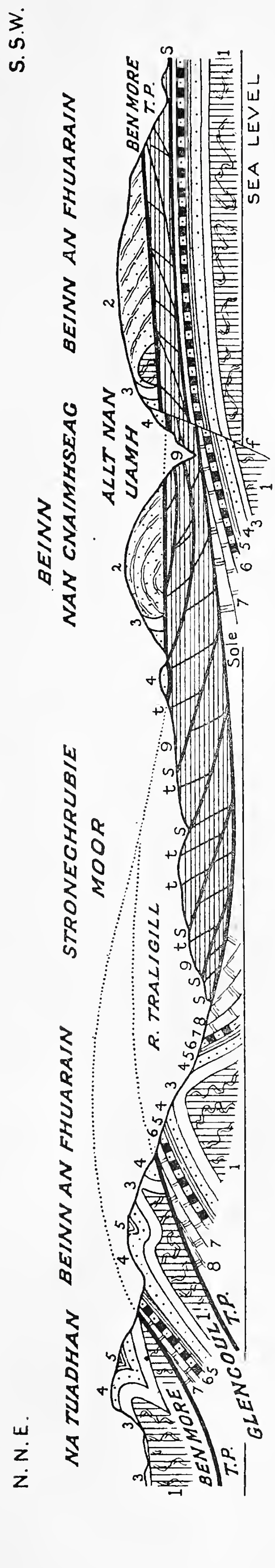


FIG. 2.—Section from Na Tuadhan near Ben More across the River Traligill and Allt nan Uamh to Beinn an Fhuarain.

1, Lewisian Gneiss; 2, Torridon Sandstone; 3, basal quartzite (Cambrian); 4, pipe-rock; 5, fucoid-beds; 6, serpulite grit; 7, basal limestone (Ghrudaidh); 8, Eilean Dubh Limestone; 9, piled-up masses of limestone (Imbricate structure), chiefly of Eilean Dubh group; T.P., thrust-planes (Glencoul, Ben More); S, soles or major thrusts; *t*, minor thrusts; *f*, subsequent faults.

II. Glaciation of the District surrounding Inchnadamff and Allt nan Uamh.

In the mountainous district around Inchnadamff there is clear evidence of intense glaciation. The most striking feature of the glacial phenomena is the evidence pointing to the conclusion that during the maximum glaciation the ice-shed did not coincide with the existing watershed.* The striæ at great elevations and the distribution of the boulders prove that the ice-parting lay to the east of the present watershed during the climax of glacial conditions. The ice must have accumulated to a great thickness on the less elevated plateau occupied by the Moine schists east of the Ben More and Breabag range.

During the maximum glaciation the general movement of the ice-sheet at great elevations in this district was in a westerly direction. On Glas Bheinn (see map, fig. 1), at a height above 2000 feet, the striæ point W. 5° N.; on Bealach an Uidhe, between Glas Bheinn and Beinn Uidhe, the direction is W.S.W. at an elevation of about 2000 feet. On the quartzite of Beinn an Fhuarain, east from Inchnadamff, between the 2000- and 2250-feet contour-lines, the trend is north of west. In the high pass north of Conamheall that leads into Coire Mhadaidh, at a level of 2750 feet, the direction is W. 10° S. or W.S.W. Farther south, on the long ridge of Breabag, finely striated surfaces of quartzite have been recorded which point to an ice-movement in a westerly direction.

Confirmatory evidence of this westerly movement is obtained on the mountains north and south of Loch Assynt. On Quinag, at an elevation of 1750 feet, the striæ point W. 5° N., and on Beinn Gharbh, about the 1500-feet contour-line, the direction varies from W. 10° S. to W.S.W. On the eastern slope of Canisp, the striæ point north of west, indicating an ice-movement up the slope in the direction of the plateau of Lewisian Gneiss.

The evidence furnished by the dispersal of the boulders also points to a westerly ice-movement across the mountainous district around Inchnadamff during the maximum glaciation. About 2 miles east from Inchnadamff, on the crest of Beinn an Fhuarain,—a hill composed of Cambrian rocks, we found boulders of thrust Lewisian Gneiss that have been carried westwards from the deep corries north of Ben More Assynt. Farther north, on Mullach an Leathaid Riabhaich, similar blocks of Lewisian Gneiss rest on the quartzite at a height of 2250 feet. On Breabag, on the

* Peach and Horne, "The Ice-shed in the North-West Highlands during the Maximum Glaciation," *Brit. Assoc. Rep.* for 1892, p. 720.

quartzite ridge that runs south from Breabag Tarsuinn, about the 2000-feet level, blocks of thrust gneiss and Moine schist have been recorded. Farther south, along the same ridge, north of Meall Diamhain (see map, fig. 1), small boulders of thrust gneiss and granulitic quartz-schist occur on outcrops of fucoid-beds and quartzites. The boulders of thrust gneiss have been derived from the slice of this material lying to the east above the Ben More thrust-plane. The blocks of granulitic schists have been carried westwards from the Moine schist area, the average height of which is lower than that of the Breabag ridge. It follows that during this westerly movement the Moine schist erratics must have been transported to levels several hundred feet higher than the sources from which they were derived.

Boulder clay is sparsely distributed in the mountainous district around Inchnadamff. It appears in the catchment basins of the Cassley and the Oykell, also in the upper part of the valley of the Ledbeg river. The drift deposits consist chiefly of moraines which have a wide distribution. An examination of the morainic material and of the boulders on the mounds points to a period of confluent glaciers when the Assynt mountains became independent centres of dispersion of the ice. In the valley of the Cassley river, which drains the great corries east of Ben More Assynt and Carn na Conbhairean, boulders of Cambrian quartzite have been traced for about 15 miles down to Invercassley, across the area occupied by the Moine schists. Again, on the Moine schist plateau south-east from Sgonnan Mòr, moraines occur containing blocks of Cambrian quartzite and thrust Lewisian Gneiss which have been borne from that mass of high ground.

When we pass westwards to the central region around Inchnadamff there is evidence to prove that local ice streamed off the eastern slopes of Canisp and Beinn Gharbh, which coalesced with that radiating from the Breabag range. Moraines of retreat are to be found near Stronechrubie and along the valley of the Loanan. Again, part of the confluent glacier ice in the neighbourhood of Inchnadamff moved northwards up the Skiag valley, carrying boulders of the intrusive porphyrite of Beinn Gharbh in its train.

The ground around Inchnadamff is above the level of the raised beaches on the seaboard of the West Highlands. The surface of Loch Assynt is 215 feet above Ordnance Datum line. We are thus deprived of evidence which might enable us to determine the stage when local glaciers ceased to exist in the mountainous region of Assynt.

III. Drainage of Underground Water and Formation of Caves in the Plateau of Cambrian Limestone.

The limestone plateau lying between the most westerly thrust-plane or "sole" and the great lines of displacement to the east (Glencoul, Ben More) is admirably adapted for the circulation of underground water, as the limestone is soluble and is traversed by innumerable minor thrusts. These piled-up calcareous masses, as already indicated, lie between impervious strata (Cambrian quartzites and fucoid-beds) to the west and the displaced Lewisian Gneiss, Torridon Sandstone, and Cambrian quartzites forming the eastern range of high ground. The streams descending from the high ground to the west that feed the Skiag Burn and the Loanan river, flow over impervious rocks and suffer no diminution before reaching Loch Assynt. On the other hand, the tributaries draining the western slopes of the Glas Bheinn and Breabag range, on reaching the limestone plateau, either suffer diminution or disappear to issue again at lower levels before sinking to the underlying floor of impervious strata (see map, fig. 1).

Near the northern limit of the limestone plateau north of Achumore, no streams descend from the western slope of Glas Bheinn between Loch Gainmhich and Allt a Chalda Mòr. The plateau, which is here comparatively narrow, is dotted with swallow-holes. The water that falls on its surface disappears below ground and probably supplies Allt a Chalda Beag, which issues near the outcrop of the Glencoul thrust-plane, and, flowing across the limestone belt, enters Loch Assynt near Castle Bay.

The Big Chalda stream (Allt a Chalda Mòr) draining Glas Bheinn and Beinn Uidhe, on reaching the outcrop of the Glencoul thrust-plane at the eastern margin of the limestone belt, loses part of its waters along this plane. Between the Big Chalda and Allt Poll an Droighinn, a tributary of the Traligill river, swallow-holes and open chasms occur along the outcrop of the Glencoul thrust-plane where the water descending the quartzite slopes to the east disappears.

The phenomena connected with the drainage of underground water in the limestone plateau are best displayed in the area drained by the Traligill river and its tributaries and Allt nan Uamh. About a mile and a half up the Traligill from Inchnadamff Hotel the stream suddenly plunges into a cavern along the outcrop of an important thrust-plane or "sole." This line of disruption forms a prominent feature in the landscape (Pl. I), the surface of the plane of movement giving rise to a well-marked slope on the northern side of the channel. The accompanying section (fig. 3) shows the relations of the strata where this "sole" appears at the surface in the Traligill.

On the north side of the Traligill between the river and the outcrop of the Glencoul thrust-plane the Cambrian strata form an arch, on whose southern limb there is a normal ascending sequence from the basal quartzites (3 in fig. 3) to the limestones of the Eilean Dubh group (8 in fig. 3) exposed on the northern bank of the river channel. Here they are truncated by a thrust which has caused the basal limestones (7 in fig. 3) to override the members of the younger group (8 in fig. 3).

After flowing underground for about a quarter of a mile, the Traligill reappears but with diminished volume (Pl. I). About two hundred yards

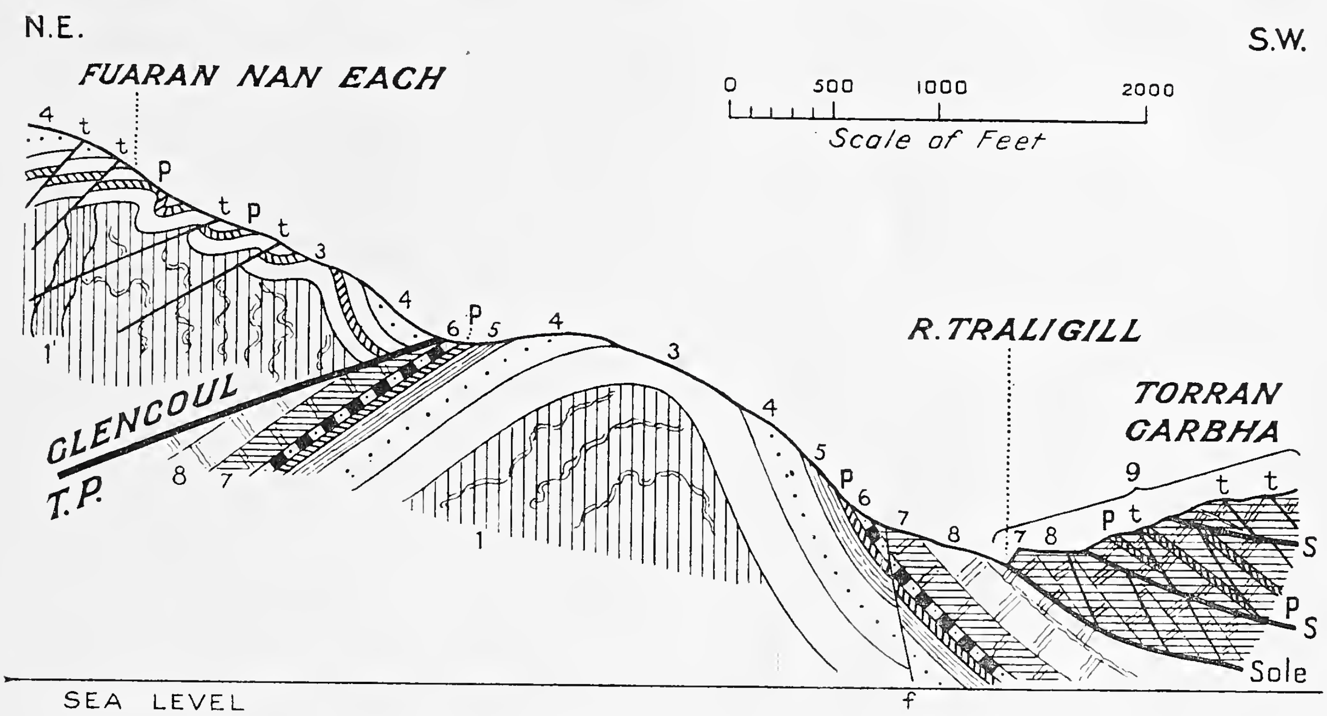


Fig. 3.—Section from Fuaran nan Each across the Traligill River to Torran Garbha.

1, Lewisian Gneiss; 3, basal quartzite (Cambrian); 4, pipe-rock; 5, fucoid-beds; 6, serpulite grit; 7, basal limestone (Ghrudaidh); 8, Eilean Dubh Limestone; 9, piled-up masses of limestone (Imbricate structure), chiefly of Eilean Dubh group; P, porphyrite sills; T.P., Glencoul thrust-plane; S, soles or major thrusts; *t*, minor thrusts.

farther down the stream a sudden increase in the volume of water is perceptible, the probable cause of which will now be indicated.

About half a mile up the valley from the point where the Traligill enters its subterranean channel, and about two hundred yards south from the river, caves occur which are locally named Uamh an Tartair (Cave of Roaring, literally Great Noise) and Uamh an Uisge (Cave of Water). They lie along the outcrop of the Traligill thrust-plane just described, which is prolonged south-eastwards in the direction of Cnoc nan Uamh. Descending one of these caves, the observer encounters an underground river which is seen to leap over a waterfall and pass down a steep slope into a wide cavern. It is probably fed by several streams, which, during heavy rains, flow down the western slope of Breabag and plunge into large swallow-holes on reaching the limestone plateau (fig. 4). It is highly probable also

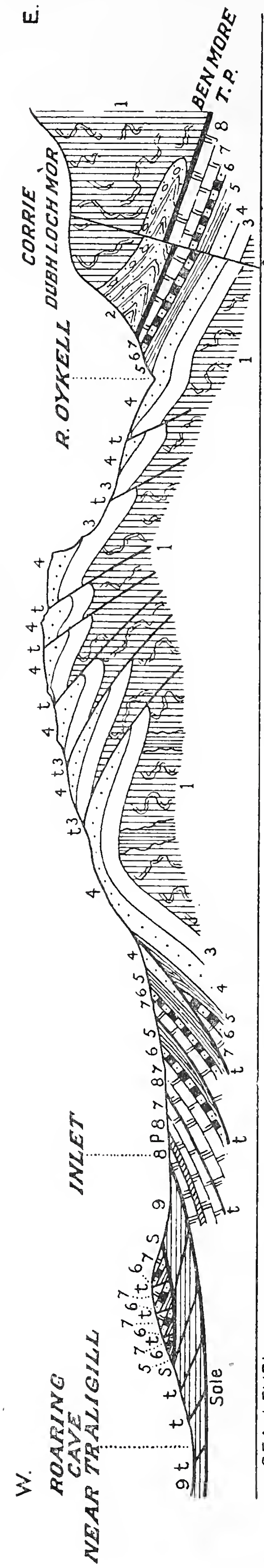


FIG. 4.—Section from the Roaring Cave near the Traligill River across Breabag and the River Oykell to Corrie Dubh Loch Mòr.

1, Lewisian Gneiss; 2, Torridon Sandstone; 3, basal quartzite (Cambrian); 4, pipe-rock; 5, fucoid-beds; 6, serpulite grit; 7, basal limestone (Ghrudaidh) 8, Eilean Dubh Limestone; 9, piled-up masses of limestone; P, porphyrite sill; T.P., Ben More thrust-plane; S, major thrusts; *t*, minor thrusts.

that this underground river, which is not visibly connected with any surface stream, may be the source of the accession to the Traligill river below its point of issue from the subterranean channel.

Loch Maol a' Choire, a small sheet of water, situated on the limestone plateau about three-quarters of a mile south from the Traligill river (see map, fig. 1, p. 329), is now almost a closed basin. The rocks are concealed by the surrounding peat, but from the available evidence it would appear that the loch is probably floored with fucoid-beds and serpulite-grit, and perhaps partly with glacial material. A dry channel connecting this loch with the Traligill is traceable on the surface of the ground, evidently representing an old stream course carved by the water issuing from the lake. In the lower part of this channel occurs a cave (Uamh Cailliche Pearag) which forms a tunnel with an aperture up stream. It is clear that the water at one time entered this chimney, flowed out of the tunnel, and pursued its course above ground till it joined the Traligill.

Throughout the limestone plateau the most striking example of the disappearance and reappearance of a stream is furnished by the Allt nan Uamh or Coldstream Burn. It rises near the crest of the Breabag range above the 2000-feet contour-line (fig. 1, p. 329), descends the quartzite slopes, and, soon after reaching the limestone belt, plunges beneath the surface, and runs underground for a distance of about a mile. It reappears as a powerful spring (Fuaran Allt nan Uamh, spring of the Burn of the Caves), about half a mile above the junction of the Coldstream Burn with the Loanan river. In periods of excessive rainfall or rapid melting of snow in the upper part of the catchment basin, when the volume of water is too large for the subterranean channel, the swollen stream reverts to its old water-course. But under normal conditions the channel remains dry for about a mile above the Fuaran Allt nan Uamh.

IV. Relation of the Creag nan Uamh Bone-Cave to the Glacial Deposits in the Valley of Allt nan Uamh.

About a mile and a quarter up Allt nan Uamh from its point of junction with the Loanan river, a prominent crag of Eilean Dubh limestone (named on the 6-inch map Creag nan Uamh, Crag of the Caves) appears on the south side of the valley at a height of about 150 to 200 feet above the dry channel of the stream. The bone-cave is the most easterly of three caves at the base of this crag (Pl. II, fig. 1). The steep slope between the caves and the stream course is composed of highly denuded glacial drift, partly covered with scree material (Pl. II, fig. 1). On both sides of the valley at this point there is a considerable development of drift (Pl. III, figs. 1 and 2). On the northern slope it forms a well-marked terrace (fig. 5), whose surface is about the level of the bone-cave. A corresponding terrace is observable about 1200 yards farther down the valley. No satisfactory sections of this deposit are exposed, but from the absence of morainic contour we infer that it represents the ground-moraine produced during the maximum glaciation, or at a later stage during the period of confluent glaciers.

It seems reasonable to conclude that the valley of Allt nan Uamh was originally filled with this impervious glacial drift up to the level of the caves (see dotted line *a*, fig. 5), so that the water entering the limestone was obliged to escape at the edge of the terrace where it bounded the limestone crag.

A later stage in the history of the valley is indicated by the dotted line *b* (fig. 5), when the stream had removed part of the drift terrace, so that the water issued from the Otters' Cave. Here the limestone appears at the surface through a thin covering of drift, about half-way down the slope (Pl. II, fig. 1, O.C.). Since that period the stream has excavated its channel to its present level (*c*, fig. 5), without reaching the solid rock for some distance above and below the line of section. About half a mile farther down, however, the rocky floor is exposed where the stream issues from its subterranean channel (Fuaran Allt nan Uamh). Between the caves and the point where the Coldstream Burn reappears the bottom of the valley displays a succession of alluvial terraces marking stages in the removal of the glacial drift.

V. Sequence of Deposits in the Creag nan Uamh Bone-Cave.

The exploration of the most easterly cave in Creag nan Uamh yielded a definite succession of deposits, given below in descending order:—

1. Peaty matter from a few inches to 1 foot in thickness.
2. Lenticular layer of calcareous marl about 1 foot thick.
3. Red clay or cave-earth from 1 to 3 feet in thickness, which furnished abundant remains of mammals and birds with indications of occupation by man.
4. Fine grey clay, about 6 inches thick, containing quartzite blocks.
5. A bed composed of limestone fragments yielding bones of mammals and birds.
6. A layer of gravel consisting of stones mostly foreign to the cave.

A careful consideration of the evidence has led us to the following conclusions regarding the significance of these deposits, which are discussed in sequence, beginning with the oldest (fig. 6).

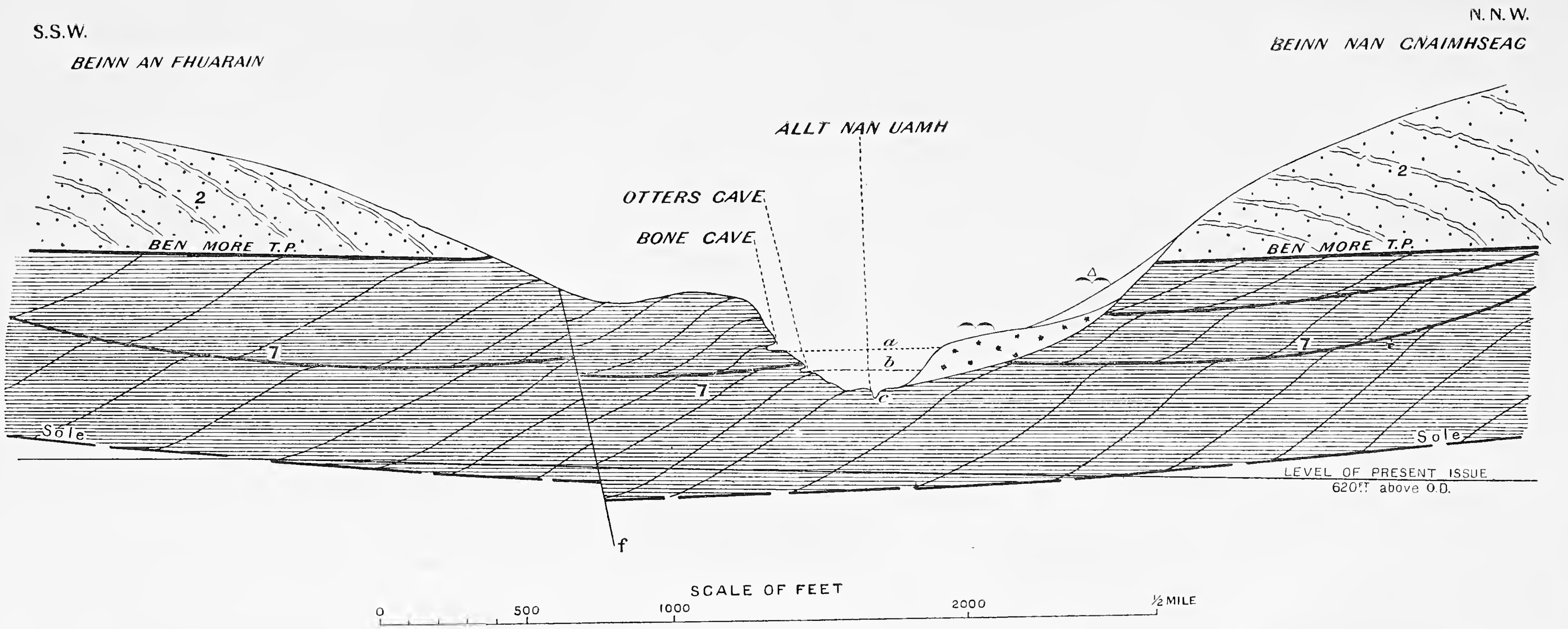


Fig. 5.—Section across the valley of Allt nan Uamh from Beinn an Fhuarain (Hill of the Spring) to Beinn nan Cnaimhseag (Hill of the Bearberry), showing the position of the Bone-Cave and the Otters' Cave.

2, Thrust Torridon Sandstone; 7, piled-up masses of Eilean Dubh Limestone; T.P., Ben More thrust-plane; ⌒, drift terrace; △, scree debris; *a*, dotted line indicates probable extension of drift when water issued from Bone-Cave; *b*, probable extension of drift when water issued from Otters' Cave; *c*, present dry channel of Allt nan Uamh.

6. The gravel on the floor of the cave is composed of well-rounded stones, comprising quartzites, porphyrites, serpulite-grit, fucoid-beds, and limestones. Most of the materials are foreign to the cave. They might have been obtained by streams traversing the quartzite slopes of Breabag and the drift-covered limestone plateau. The arrangement of the layers and the interlocking of the pebbles indicate that the gravel was deposited by a stream issuing from the cave. It is evident, therefore, that the stream transporting these stones from the upper part of the catchment basin of Allt nan Uamh must have flowed underground on reaching the limestone plateau, and that part of the current at least must have found a channel through the bone-cave. It is quite possible that this gravelly bed may

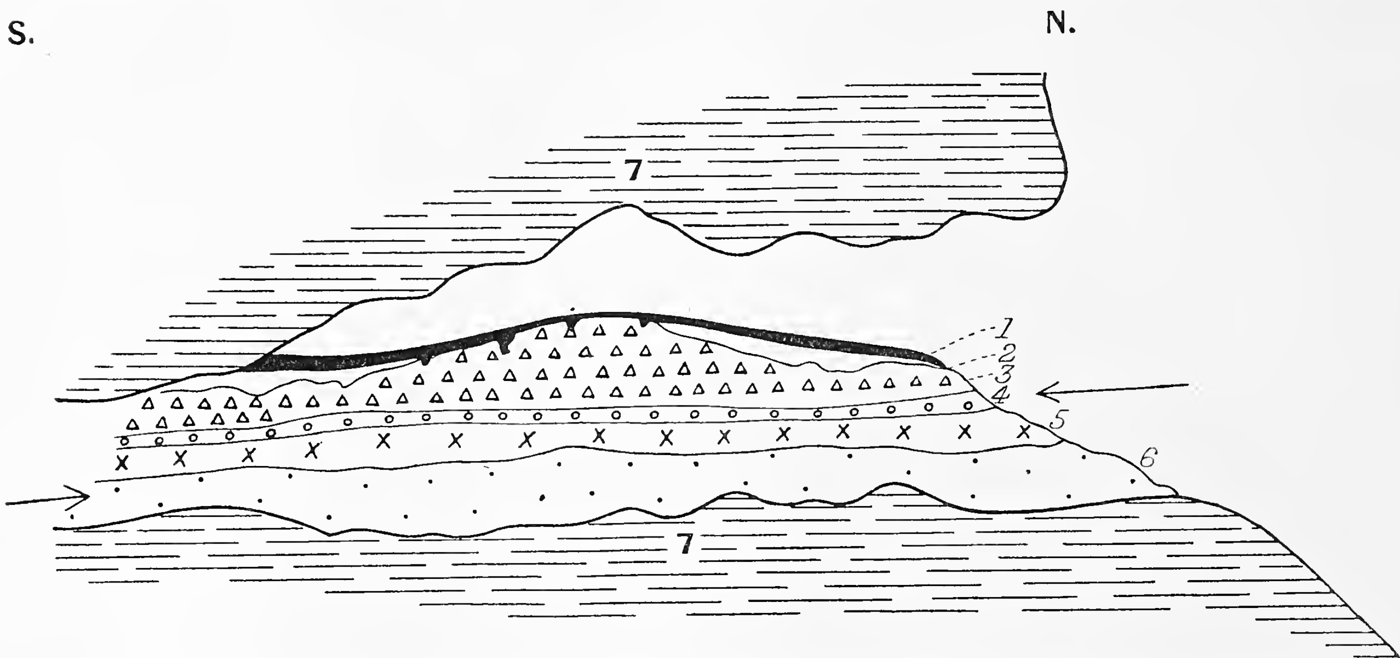


Fig. 6.—Diagrammatic section showing sequence of deposits in Creag nan Uamh Bone-Cave. 1, peaty layer; 2, marl; 3, cave-earth; 4, grey clay; 5, bed of limestone fragments; 6, gravel.

have accumulated on the floor of the cave while glaciers still existed in the high ground to the east.

5. The bed of fine splinters of limestone overlying the gravel indicates that the cave had already become a dry one, and that the water had begun to circulate at a lower level. The limestone fragments appear to be largely due to frost action on the roof and sides of the cavern. The almost entire absence of red clay or *terra rossa* would seem to indicate that there was very little drip from the roof during the accumulation of this bed.

From this layer were exhumed the remains of the Arctic Lemming, Field Vole (*Microtus agrestis*), Rat Vole (*Microtus ratticeps*), Bear, and birds. The great number of pinion bones and anchylosed dorsal vertebral bones of birds shows that the deposit must have been formed very slowly. The pinion bones of ptarmigan occur in lenticular layers containing the remains of hundreds of individuals.

4. The compact grey clay with small quartzite boulders, forming bed 4, resembles to some extent the morainic material in the adjoining valley. No fragment of limestone from the roof or walls of the cavern was observed in this deposit. These facts suggest that the layer was due to morainic material, derived from the quartzite slopes of Breabag, which had been carried on the surface of a glacier and shot into the cave from the lobe of ice that passed down the valley of the Coldstream Burn. If the material had been derived from the high terrace of drift left in the valley (see fig. 5), a greater variety of stones would have been found in the clay. No organic remains were noted in this deposit.

3. Bed 3 is perhaps the most interesting member of the succession. It consists of red clay or *terra rossa*—a true cave-earth—with occasional splinters of limestone, and, at the east end of the cave, some stalagmite. Its thickness is variable and its surface uneven (see fig. 6), showing that the falls from the roof were very irregular. The mammals obtained from this deposit include Northern Lynx, Reindeer, Red-deer, and Otter, as well as a very large number of Frog bones; and there is just a possibility that the Arctic Lemming and Rat Vole occur also in this bed, but definite evidence is wanting. It is worthy of note that the antlers of Reindeer represent very young individuals, while those of the Red-deer are very massive.

No less interesting is the evidence pointing to the conclusion that during the accumulation of this bed the cave was tenanted by man. In various layers, fireplaces and firestones, split and burnt bones were observed but no artefacts were detected. It was noted that some of the Reindeer antlers had been sawn off. The Fox, Otter, and Badger, whose remains are associated with those of other mammals in the cave-earth, need not have been contemporary with the period of occupation by man, for these animals may have burrowed into the deposit in historic times.

The surface of the bare portions of bed 3 was pitted with numerous small conical holes formed by drips from the roof of the cave. These yielded abundant remains of the common Frog and Toad, and the Natter-jack Toad. Some of the larger pits were filled with the long bones of these creatures, packed closely together, thus indicating a protracted period for their accumulation. Near the eastern wall of the cave the *terra rossa* and the limestone splinters were locally cemented by a small amount of stalagmite not only at the surface but at different depths farther down.

The cave-earth probably marks a change to wet and milder conditions than those which prevailed during the deposition of bed 5, composed, as already indicated, of limestone fragments and yielding bones of mammals and birds.

2. The layer of whitish marl occurred chiefly on the western side of the floor of the cave (see fig. 6). The remains obtained from it consist almost wholly of the shells of small Pupa-like land snails, pointing to a long period when, with the exception of a few splinters, only the limestone-loving land shells dropped from the walls and roof of the cave.

1. The thin peaty layer at the top is composed almost wholly of excrement of sheep. In recent years it has been the custom during severe snowstorms to drive the sheep into this part of the valley to enable them to find shelter in the caves of Creag nan Uamh.

Among the mammals obtained from this bone-cave the Arctic Lemming is of special interest because the remains of this animal were found by the late Mr James Bennie in the arctic plant bed of the ancient lake at Corstorphine, near Edinburgh. At the latter locality the palæontological evidence indicates that the deposit is probably of late glacial age.

In his published description of this old lake deposit Mr Bennie did not record his discovery of the jaw-bone of the Lemming which had been determined by Mr E. T. Newton, F.R.S. Special thanks are due to Mr William Evans, F.R.S.E.,* for calling attention to this discovery and obtaining permission from the Geological Survey to have the jaw-bone re-examined by Mr Newton. He reported: "I do not think there can be any doubt as to the larger jaw belonging to a Lemming, and it is closely allied to the Arctic Lemming, which is now called *Dicrostonyx torquatus*; but being imperfect one cannot speak with certainty as to the species."

The arctic plant bed at Corstorphine occurs in the lower part of a succession of lacustrine deposits filling a silted-up lake. The seeds and leaves collected by Mr Bennie were determined by the late Mr Clement Reid, F.R.S., who stated that the vegetation consists mainly of dwarf willow and birch with a few herbaceous plants belonging to species still living within the Arctic Circle.† Amongst the arctic species are dwarf birch (*Betula nana*), willows (*Salix polaris, S. herbacea, S. reticulata*), the white dryas (*Dryas octopetala*), and *Oxyria digyna*. From the evidence of the plants Mr Reid inferred that the deposit is probably of late glacial age.‡ This conclusion is confirmed by the occurrence in the plant bed of remains of *Lepidurus* (*Apus*) *glacialis*, a phyllopod now found living only in the freshwater lakes of Greenland and Spitsbergen. The discovery of the remains of the Arctic Lemming in the same deposit is another link in the chain of evidence indicating the climatic conditions which then prevailed in Scotland.

* *Proc. Roy. Phys. Soc.* (1906), vol. xvi, part 8; also *The Scot. Naturalist*, No. 17, May 1913, p. 97.

† *Brit. Assoc. Rep.*, 1892, p. 716.

‡ *Origin of the British Flora*, 1899, p. 62.

The Corstorphine lake occupied a hollow in the boulder clay. The ice had finally retreated from the lowlands of Scotland, but glaciers still lingered in some of the Highland valleys. The deposits of the 100-feet beach, which have yielded at certain localities shells of an arctic type and bones of the small arctic seal, were then being laid down. Indeed, all the available evidence seems to point to the conclusion that the Corstorphine Lemming is of late glacial age.

The deposits of the Creag nan Uamh bone-cave are certainly later than the accumulation of the ground-moraine in the valley of Allt nam Uamh. They seem to us of special interest and importance from the light which they throw on the mammalian and avian life that flourished in the North-West Highlands since the climax of the Ice Age and during Neolithic time.

Our special thanks are due to Mr Spencer L. Arnot for the photographs of the bone-cave and the drift deposits in the valley of Allt nan Uamh which are reproduced in Pls. II and III, to Dr W. Inglis Clark for the photograph of the thrust-plane in the Traligill river from which Pl. I has been made, and to the Carnegie Trustees for a grant in aid of the illustrations. Mr John Mathieson, H.M. Ordnance Survey, has kindly revised the spelling of the Gaelic place-names appearing in the text.

VI. Notes on Bones found in the Creag nan Uamh Cave, Inchnadamff, Assynt, Sutherland. By E. T. Newton, F.R.S.

A small collection of bones from Creag nan Uamh cave, Inchnadamff, was submitted to me for examination by my colleagues Drs Peach and Horne in the year 1890; but only a brief reference to the fauna of the cave was made by them in their short preliminary report [*Brit. Assoc. Rep.* for 1892, p. 720]. During the last few months this series of remains has been re-examined and some 36 distinct vertebrate forms recognised: 15 mammals, 17 birds, 3 amphibians, and 1 fish.

The presence among these remains of Reindeer, Bear, Northern Lynx, Arctic Lemming, and *Microtus ratticeps* show that the deposits of this cave are not of very recent origin, but that a considerable lapse of time must have taken place since they were accumulated; indeed, it seems highly probable that some of these deposits (Bed 5) are of late Pleistocene age, although no characteristic extinct species has been observed. All the above-named mammals are living forms. The Reindeer and Bear are known to have been living in this country in historic times, and it is possible that the other three, although giving a boreal aspect to the fauna of the cave, may have continued to live in Sutherland until a much more recent date than is usually supposed. On the other hand, neither of these three species has hitherto been found in modern deposits in this country, and, in so far as the southern parts of Britain are concerned, may be taken as characteristic Pleistocene forms; but the case is not quite the same for the north of Scotland.

The Lynx is represented among these remains by a femur and a metatarsal bone, both of which agree with corresponding bones of a Northern Lynx in the British Museum except in being a little smaller, but in this respect they are like the bones of this species from Teesdale described by W. Davis (*Geol. Mag.*, 1880, p. 346). We have no evidence that the Lynx was living in Britain in historic times, although it may have lingered in the wilder parts of northern England and Scotland without being recorded. All the remains of Lynx yet discovered in Britain are from caves or rock fissures, and in no case have they been associated with extinct Pleistocene species.

A single small canine tooth of a Bear from Bed 5 is scarcely sufficient for the definite determination of the species, yet it seems most probable that it represents the Brown Bear (*Ursus arctos*) which was living in Britain in early historic times (A.D. 500–1000).

The Otter is represented by a few bones from Bed 3, and the Badger by a skull and several bones from the same horizon, but there is always the possibility of the latter animal having burrowed into the deposit at a later date. The Stoat and Weasel have been found in Bed 5, parts of skulls and limb-bones of both species having been met with. A single caudal vertebra of a Fox alone represents that animal.

Portions of Red-deer and Reindeer antlers occur in this cave, and similar remains have been recorded from many deposits of prehistoric and modern date in various parts of Scotland, where Reindeer were still living in the twelfth century.

There are a few limb-bones of a large Hare; and as no such fossil remains appear hitherto to have been recorded from Scotland, it is of peculiar interest to ascertain, if possible, to what species and variety these belong. The specimens available for comparison are a femur and a humerus, both nearly perfect, and an upper incisor tooth. There are also portions of a tibia, an ulna and some foot-bones, but these do not give much help. The femur and humerus both present characters of their proximal ends which at once show their affinity to the Variable Hare (*L. variabilis*) rather than to the Common or Brown Hare (*Lepus europæus*); and the incisor tooth has cement in its anterior groove, thus pointing to a similar affinity. It is not easy, however, to decide to which variety of *L. variabilis* these remains should be referred, and this determination rests chiefly upon measurements. One naturally, in the first place, compared them with the modern Scottish Blue-hare (now called *Lepus variabilis scoticus*), but the bones of my own example of this species were so much smaller and more slender than the Assynt bones that it seemed unlikely they could be the same. Length of Assynt femur, 132 mm.; Scotch Hare, 115 mm. Length of Assynt humerus, 111 mm.; Scotch Hare, 92 mm. On further comparison with measurements of specimens in the College of Surgeons Museum, given by Mr Martin A. C. Hinton in his masterly paper on Fossil Hares (*Sci. Proc. R. Dublin Soc.*, vol. xii, N.S., p. 225, 1909) and with specimens of Russian Variable Hares in my own collection, I find that there is a considerable overlap in the sizes of these varieties. The largest Scotch Hare is very nearly as large as the Assynt fossil; but on the other hand some of the Russian *L. variabilis* are smaller. As a matter of fact, the Assynt humerus is longer and the femur more robust than are these bones in the largest of my Russian specimens, and approach the dimensions of the Kentish fossils given by Mr Hinton. There remains, therefore, a doubt regarding the variety represented by these Assynt remains, and this will, I think, be best shown by recording them as *Lepus variabilis scoticus?*

Dicrostonyx, the Arctic Lemming, is undoubtedly represented by a few jaws and teeth found in Bed 5 and perhaps also in Bed 3; there are likewise some limb-bones which in all probability belong to the same form. Until the last few years British fossil Lemmings, of the *D. torquatus* type, have generally been referred to that species; but more recently Mr Martin A. C. Hinton and others have shown that there are dental and cranial differences among these fossils which necessitate a further subdivision, and Mr Hinton has described one of these under the name of *D. henseli*, and, for another, revives the name of *D. gulielmi* of Sanford. Of these species the former is allied to the recent *D. hudsonius*, while the latter more nearly resembles the recent *D. torquatus*.

One could have wished for better material to indicate the species inhabiting the Assynt district in Pleistocene times; but fortunately there are two characteristic upper molars preserved (m^1 and m^2), and these show the greater development of their posterior angles as in *D. torquatus* (see figure by Barrett Hamilton, *Hist. Brit. Mams.*, part xiv, p. 388, 1913) and not the reduced condition seen in the recent *D. hudsonius* and in the fossil *D. henseli*. Although the characters of the Assynt fossils may not be so marked as in the figure just referred to, they agree very closely with recent specimens in the British Museum. There are four more or less imperfect lower jaws, and only one of these retains the last molar tooth (m_3), which is said to be characteristic but less distinctly so than the upper molars. This hindermost lower molar in our fossil has its anterior angles as fully developed as in some of the recent *D. torquatus*, and the same may be said of the second lower molar (m_2). It is clear, therefore, that this Assynt Lemming belongs to the *D. torquatus* type as distinguished from *D. hudsonius*. With regard to the size of these Assynt specimens, the series of molar teeth in two of them measures 7·5 mm. (alveolar length), and in a third, probably young jaw, 6·5 mm. Among the specimens of *D. torquatus* type now in the British Museum there is much difference of size, the alveolar length of the lower molar series varying from 6·0 mm. to 7·7 mm. These differences may be due to age, and perhaps to local varieties not yet distinguished, but they fall into two groups—those with the molar series measuring from 6·0 mm. to 6·6 mm. in length, and a larger form, two specimens, measuring respectively 7·3 mm. and 7·7 mm. There is a remarkable agreement, therefore, between the measurements of these recent and our fossil specimens. The first of the larger recent forms is from N.W. Siberia and the second from Wellington Channel (75° N. 93° W.), that is, far to the N.E. of North America. Most of the smaller recent forms are from Discovery Bay (82° N. 65° W.). From this it will be seen that both large and small forms occur in the extreme N.E. of North America, although their habitat may be 500 miles apart; and the smaller form lives on the most northerly land yet discovered. From what has been said above, there appears to be no valid reason for separating the Assynt Lemmings specifically from the recent forms, which at present are included under the name of *D. torquatus*. But what about fossil forms?

The only fossil Lemming of *D. torquatus* type which, so far as I know, has been recognised and named, is *D. gulielmi*, and this agrees with *D. torquatus* in the pattern of its teeth, but is said to be distinguished "by its considerably larger size, shorter and broader incisive foramina, broader nasals, and heavier teeth." As we have no skulls from Assynt for comparison, and as the patterns of the teeth are alike, the larger size and heavier teeth are the only characters left for comparison, and in these particulars the Assynt specimens do not agree with *D. gulielmi*, the alveolar length of the three molars being 8·3 mm. Until better specimens are forthcoming, the Assynt Lemming will be referred to *Dicrostonyx torquatus*.

An incomplete skull of a small Water Vole is most probably referable to the black Scottish variety now called *Arvicola amphibius reta*, Miller; but its horizon and that of a lower front tooth of a Bank Vole (*Evotomys glareolus*) are uncertain. Jaws of the Field Vole (*Microtus agrestis*) and also of *Microtus ratticeps* have been found in Bed 5.

Avian remains are numerous and represent several genera and species. The bones of Ptarmigan (*Lagopus mutus*) are by far the most numerous, and the greater number of these are metacarpal and metatarsal bones; among which were a few of larger size which seem to belong to Red-grouse (*Lagopus scoticus*); and it thus appears that at the time these cave deposits were accumulating, Ptarmigan were much more abundant than Red-grouse in the north of Scotland. Several genera of Ducks have been identified, while the Little Auk (*Mergulus alle*) and the Puffin (*Fratercula arctica*) are each represented by a single bone.

Remains of the Common Frog (*Rana temporaria*) were found in large numbers in "pockets" in Bed 3, and with them a few bones of the Common Toad (*Bufo vulgaris*). A small humerus and one or two other bones agree so closely with corresponding parts of the Natter-jack Toad (*Bufo calamita*) that I have, although with hesitation, included that species in the list.

Fish vertebræ and portions of skulls occurred in some numbers in Bed 5, and belong to either salmon or trout.

*List of Vertebrata from Creag nan Uamh Cave.**

	Bed 5.	Bed 3.	Bed 3 or 5.
Mammals.			
Lynx, *Felis lynx*, Linn.	...	S	S
Stoat, *Mustela erminea*, Linn.	S	...	S
Weasel, „ *vulgaris*, Linn.	S	...	S
Fox, *Vulpes alopex*, Linn.	...	...	S
Otter, *Lutra vulgaris*, Erxleb.	...	S	SL
Badger, *Meles taxus*, Schreber	...	...	SL
Bear, *Ursus arctos*, Linn.	S	...	...
Red-deer, *Cervus elaphus*, Linn.	...	S	S
Reindeer, „ *tarandus*, Linn.	...	S	S
Hare, *Lepus variabilis scoticus ?*	S	...	SL
Lemming, *Dicrostonyx torquatus*, Pallas	S	...	S
Water Vole, *Arvicola amphibius reta ?*, Miller	...	...	L
Field Vole, *Microtus agrestis*, Linn.	S	...	SL
Rat Vole, „ *ratticeps*, Key and Bl.	S	...	SL
Bank Vole, *Evotomys glareolus*, Schreber	...	...	L
Birds.			
Chaffinch ?, *Fringilla*	S	...	...
Barnacle Goose, *Bernicla leucopsis*, Bechst.	...	...	L
Swan, *Cygnus olor ?*, Gmel.	...	...	L
Mallard ?, *Anas boscas ?*, Linn.	...	...	L
Teal, *Querquedula crecca*, Linn.	S	...	...
Wigeon, *Mareca penelope*, Linn.	S	...	...
Tufted Duck, *Fuligula cristata*, Leach	S	...	...
Long-tailed Duck, *Harelda glacialis*, Linn.	S	...	SL
Eider Duck, *Somateria mollissima*, Linn.	L	...	L
Common Scoter, *Oedemia nigra*, Linn.	S	...	S
Velvet Scoter, *Oedemia fusca*, Linn.	...	...	L
Ptarmigan, *Lagopus mutus*, Montin.	SL	...	SL
Red-grouse, *Lagopus scoticus*, Lath.	SL	...	...
Golden Plover, *Charadrius pluvialis*, Linn.	...	...	L
Grey Plover, *Squatarola helvetica*, Linn.	S	...	...
Little Auk, *Mergulus alle*, Linn.	S	...	...
Puffin, *Fratercula arctica*, Linn.	...	...	L
Amphibia.			
Frog, *Rana temporaria*, Linn.	S	S	S
Toad, *Bufo vulgaris*, Laur.	...	S	...
Natter-jack ?, *Bufo calamita ?*, Laur.	...	S	...
Fish.			
Salmon or Trout	S	...	...

* The letter S indicates that the specimen is located in the Geological Survey Collection Edinburgh ; the letter L, in the Geological Survey Collection, London.

View of Traligill River issuing from underground channel along bared outcrop of thrust-plane. Limestone plateau in middle distance. Quartzite mountain of Breabag in background.

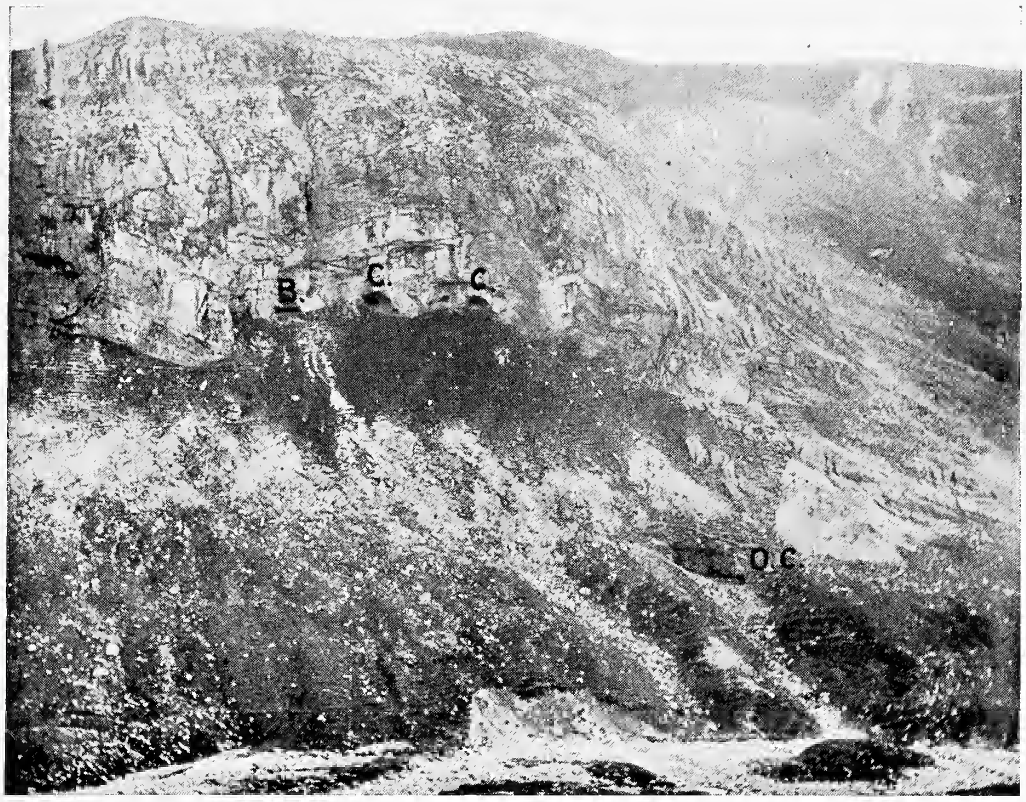


Fig. 1.—Creag nan Uamh, limestone escarpment with three caves on same level (B. Bone-cave; C. C. Caves). Glacial drift and scree material on lower slope (O. C. Otters' Cave). Dry channel of Allt nan Uamh in foreground.

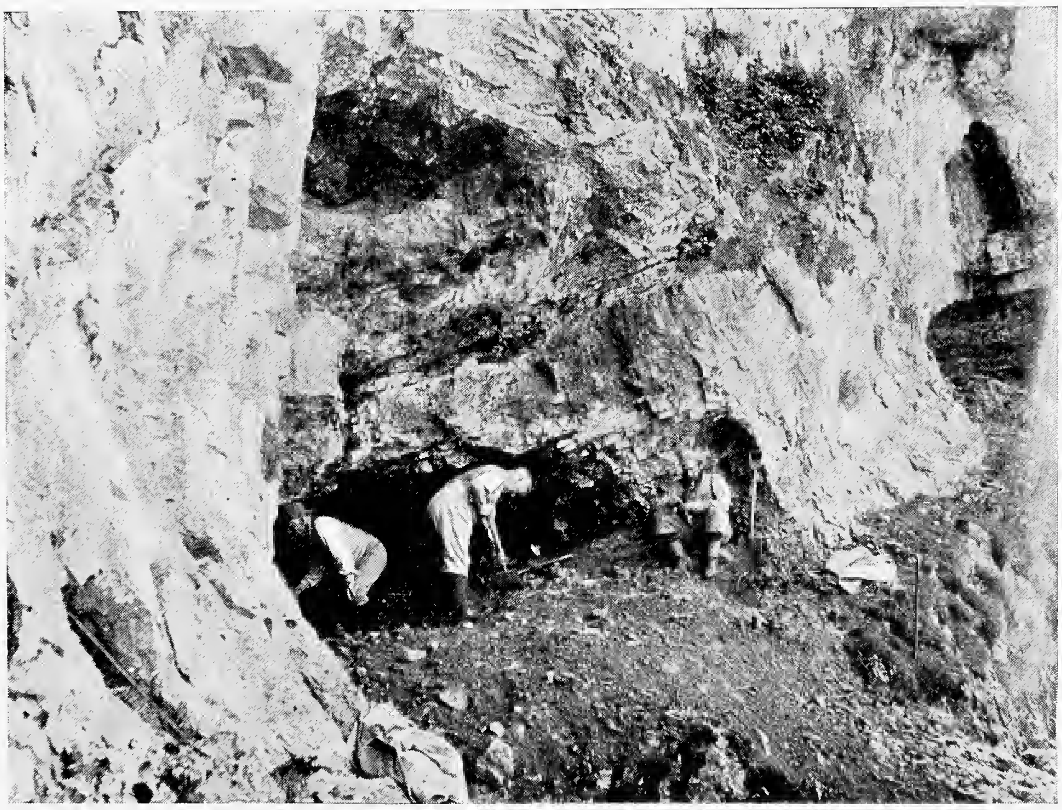
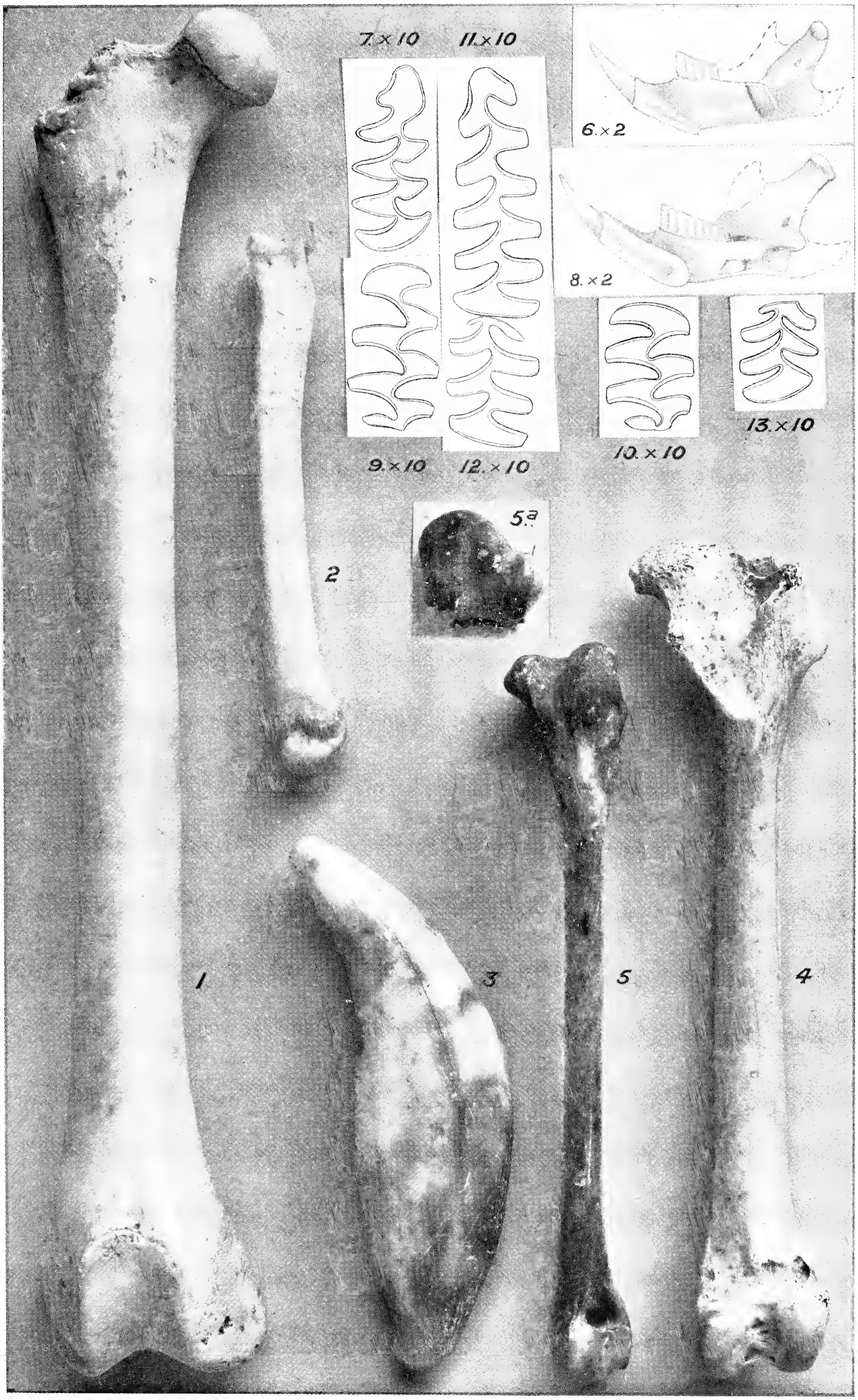


Fig. 2.—View of Bone-cave after excavation.

Fig. 1.—Valley of Allt nan Uamh, with denuded terrace of glacial drift in foreground. Limestone cliff with caves in middle distance.

Fig. 2.—Allt nan Uamh looking east, with denuded terrace of ground-moraine in foreground. Limestone cliff with caves in middle distance. Quartzite mountain of Breabag in background.



Description of this Plate is given on page 349.

EXPLANATION OF PLATE IV.

(Figs. 1–5*a* natural size.)

Fig. 1. Lynx, right femur, front view.
Fig. 2. ,, ,, metatarsal, front view.
Fig. 3. Ursus sp., canine tooth.
Fig. 4. *Lepus variabilis scoticus?* femur, back view.
Fig 5. ,, ,, ,, humerus, front view.
Fig. 5*a*. ,, ,, ,, humerus, proximal end.
Fig. 6. *Microtus ratticeps*, right ramus, lower jaw. × 2.
Fig. 7. ,, ,, right lower front molar tooth m_1, surface view. × 10.
Fig. 8. *Dicrostonyx torquatus* right ramus, lower jaw. × 2.
Fig. 9. ,, ,, right upper m^1. × 10.
Fig. 10. ,, ,, right upper m^2. × 10.
Fig. 11. ,, ,, left lower m_1. × 10.
Fig. 12. ,, ,, left lower m_2. × 10.
Fig. 13. ,, ,, right lower m_3. × 10.

(*Issued separately October* 15, 1917.)

XX.—The Square Roots of a Linear Vector Function. By Frank L. Hitchcock. *Communicated by* The General Secretary.

(MS. received February 26, 1917. Read May 19, 1917.)

The equation in linear vector functions

$$\phi^2 = \omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

was proposed by Tait,* and an elegant solution was obtained by him which does not require a determination of the axes of ω. He showed that upon this equation depends the separation of the pure and the rotational parts of a homogeneous strain. The problem appears to be interesting also from the point of view of algebraic analysis. The number and character of the solutions is more varied, given different types of the function ω, than we might at first suppose. In fact there are two forms which may be assigned to ω such that the equation does not permit of solution. Otherwise the number of solutions is 2, 4, 8, or infinity. Keeping Tait's notation for the cubic in ω, the two cases of failure arise when $m = 0$; hence the solution is always possible for a physically existent strain ω. But there are also cases where $m = 0$, and the solution exists. Furthermore, Tait's solution ceases to be determinate when ω has an infinite number of axes—for example, when ω is real and self-conjugate with a pair of equal roots.

In a former paper were given four normal or type expressions covering all cases in the sense that a given ω can always be thrown into one of the four forms.† In the same paper it was proved that if two linear vector functions are commutative, and if the first has an axis not an axis of the other, then the first is reducible, *i.e.* possesses an infinite number of axes. Now if $\phi^2 = \omega$, it is obvious that ω and ϕ are commutative, since any number of linear vector functions are associative. Hence we have the theorem—

Theorem I.—A linear vector function is reducible if it has an axis not possessed by its square root.

The root ϕ, on the other hand, can evidently not have an axis which is not an axis of ϕ^2. Then if ϕ^2 be assigned to have a finite number of axes, ϕ has those, and no others. Whence this second theorem—

* *Quaternions*, 3rd ed., chap. v, Ex. 17 (*a*), p. 144; solution, p. 298, note. Also in Kelland and Tait's *Introduction to Quaternions*, chap. x.

† *Proc. Roy. Soc. Edin.*, vol. xxxv, Part ii (No. 17), p. 170 (1915).

Theorem II.—A given linear vector function and its square root are of the same class except when the given function is reducible.

If ω is reducible, it will be shown that we have, in general, an infinite number of *irreducible* values for ϕ, as well as a finite number of reducible values.

Considering in order the possible forms of ω, if ω is of Class I, *i.e.* if ω has just three axes, the matter is simple, for ϕ is fully determined by possessing as axes the axes of ω, and as latent roots the square roots of those of ω, each taken with either sign. Hence there are eight solutions, except when $m=0$, *i.e.* when ω has a vanishing latent root, the number of distinct values of ϕ falling to four.

When ω is of Class II, *i.e.* has but two distinct axes, we may write

$$\omega\rho = g\rho + c\beta S\beta\beta_1\rho + c_1\beta_1 S\alpha\beta\rho \quad . \quad . \quad . \quad . \quad (2)$$

where c, c_1, and $S\alpha\beta\beta_1$ do not vanish. β is the double axis, g the double root, β_1 and $g+c_1S\alpha\beta\beta_1$ the single axis and root. The plane of α and β is precessive, *i.e.* repeated application of ω to any vector in that plane brings it more and more into the direction β. Now, as above noted, ϕ has β and β_1 for axes, and no others. The latent roots of ϕ corresponding to these axes must be the square roots of those of ω, taken with either sign, four possibilities. It only remains to find the effect of ϕ on α. Let the latent roots of ϕ be selected and called r and r_1. Assume

$$\phi\alpha = x\alpha + y\beta + z\beta_1,$$

where x, y, z are to be determined. Operating again with ϕ, we have

$$\begin{aligned}\phi^2\alpha &= x\phi\alpha + y\phi\beta + z\phi\beta_1 \\ &= x^2\alpha + xy\beta + xz\beta_1 \\ &\quad + ry\beta + r_1z\beta_1\end{aligned}$$

by the values of $\phi\alpha$, $\phi\beta$, and $\phi\beta_1$ just assumed. But by (2)

$$\phi^2\alpha = \omega\alpha = g\alpha + c\beta S\alpha\beta\beta_1,$$

whence equating values of $\phi_2\alpha$

$$x^2 = g, \quad y(x+r) = cS\alpha\beta\beta_1, \quad z(x+r_1) = 0.$$

By the first of these equations x is one of the square roots of g. By the second equation x cannot be the negative of that square root of g already selected to be r, for if so either c or $S\alpha\beta\beta_1$ would vanish, contrary to hypothesis. Hence $x=r$. Thus y is determined. Since two latent roots of ω are unequal, we cannot have $x+r_1=0$; hence $z=0$. Thus $\phi\alpha$ is uniquely determined when r and r_1 have been selected. We therefore have in general four distinct solutions of ϕ when ω is of Class II. If the single root of ω vanishes, the number falls to two, r_1 being zero.

If, however, the double root g vanishes, then ϕ ceases to exist for $x=r=0$, and either c or $S\alpha\beta\beta_1$ vanish, contrary to hypothesis. As a simple example of a function without square root, take $\omega\rho = jSi\rho + kSk\rho$, of which the solution requires that $\phi j=0$ and $\phi^2 i = -j$, incompatible conditions.

When ω is of Class III, *i.e.* has but one distinct axis, we may write

$$\omega\rho = g\rho + c\beta S\beta\gamma\rho + c_1\gamma S\gamma\alpha\rho \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where c, c_1 and $S\alpha\beta\gamma$ do not vanish. γ is the triple axis and g its latent root. γ must be an axis of ϕ, and we select r, the latent root of ϕ, to be one of the square roots of g. Assume

$$\phi\alpha = x'\alpha + y'\beta + z'\gamma, \quad \phi\beta = x\alpha + y\beta + z\gamma,$$

whence

$$\begin{aligned}
\phi^2\beta &= x\phi\alpha + y\phi\beta + z\phi\gamma \\
&= xx'\alpha + xy'\beta + xz'\gamma \\
&\quad + xy\alpha + y^2\beta + yz\gamma \\
&\quad + rz\gamma \\
&= \omega\beta \\
&= g\beta + c_1\gamma S\alpha\beta\gamma, \text{ by (3).}
\end{aligned}$$

Again

$$\begin{aligned}
\phi^2\alpha &= x'\phi\alpha + y'\phi\beta + z'\phi\gamma \\
&= x'^2\alpha + x'y'\beta + x'z'\gamma \\
&\quad + xy'\alpha + yy'\beta + y'z\gamma \\
&\quad + rz'\gamma \\
&= \omega\alpha \\
&= g\alpha + c\beta S\alpha\beta\gamma, \quad \text{by (3).}
\end{aligned}$$

Equating coefficients, we have these two sets of equations,

$$x(x'+y)=0, \quad xy'+y^2=g, \quad xz'+z(y+r)=c_1 S\alpha\beta\gamma,$$
$$x'^2+xy'=g, \quad y'(x'+y)=cS\alpha\beta\gamma, \quad y'z+z'(x'+r)=0.$$

By the second equation of the lower set we cannot have $x'+y=0$, since c and $S\alpha\beta\gamma$ are different from zero. Hence by the first equation of the upper set $x=0$. Then by the right hand of the upper set $y+r$ is not zero; and $y=r$ by the second equation. Thus x, y, and z are uniquely determined when r has been selected.

Writing $x=0$ and $y=r$ in the lower set, we have $x'=r$ by similar reasoning. y' and z' are then uniquely determined by the last two equations.

The reasoning fails and the solution is impossible if $g=0$. As an example, we cannot have ϕ such that $\phi^2=j\mathrm{S}i\rho+k\mathrm{S}j\rho$.

When ω is of Class IV, *i.e.* reducible, we may write

$$\omega\rho=g\rho+\gamma\mathrm{S}\lambda\rho \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where the axes of ω are γ and any vector perpendicular to λ. Let α be now selected *any* vector perpendicular to λ. Let us assign $\phi\alpha=\beta$, where β is a different vector perpendicular to λ. We now have

$$\phi\beta=\phi^2\alpha=\omega\alpha=g\alpha,$$

that is, ϕ is involutory as regards the directions of α and β. If γ is not perpendicular to λ it is an axis of ϕ (assuming that γ and λ are actual vectors, *i.e.* that ω is not a mere dilation). For if γ is not an axis of ϕ, both γ and $\phi\gamma$ are axes of ω having the same root, and ω has two different planes of axes, which is impossible. Hence $\phi\gamma$ is determined except for sign. Since $\phi\alpha$ is arbitrary within the plane $\mathrm{S}\lambda\rho=0$, we may construct an infinite number of different functions ϕ satisfying (1). When so constructed the latent roots of ϕ are, besides that corresponding to γ, the two square roots of g, opposite in sign. Or we may begin by choosing any two vectors perpendicular to λ to be axes of ϕ, with roots $\pm\sqrt{g}$, respectively. On the other hand, if we assign the same value of $\sqrt{g}$ to two axes of ϕ, ϕ becomes reducible and has only two forms, corresponding to choice of signs for $\phi\gamma$.

It remains to consider the case where $\mathrm{S}\gamma\lambda=0$, which includes the problem of finding the square root of a shear. In the plane of axes all is as before. ϕ is then fully determined when we know $\phi\lambda$. Assume r selected as $+\sqrt{g}$ with μ and ν axes of ϕ in the plane $\mathrm{S}\lambda\rho=0$, so that $\phi\mu=r\mu$ and $\phi\nu=-r\nu$. Also assume $\phi\lambda=x\lambda+y\mu+z\nu$. Then

$$\begin{aligned}\phi^2\lambda&=x\phi\lambda+y\phi\mu+z\phi\nu\\&=x^2\lambda+xy\mu+xz\nu\\&\qquad+ry\mu-rz\nu\\&=\omega\lambda\\&=g\lambda\;+\gamma\lambda^2.\end{aligned}$$

Expanding $\gamma\lambda^2$ in terms of μ and ν, we may write $\gamma\lambda^2=a\mu+b\nu$ where a and b are known. Equating coefficients of λ, μ, ν, we have

$$x^2=g,\quad y(x+r)=a,\quad z(x-r)=b.$$

Since $x=\pm r$, it follows that a or b must be zero. Hence if λ is real, μ or ν must coincide in direction with γ, the double axis of ω. We then have *either* y or z *arbitrary*. The solution again fails when $g=0$ (unless $\lambda^2=0$)

for we cannot have simultaneously $\phi\gamma=0$ and $\phi^2\lambda=\gamma\lambda^2$; this, however, may be regarded as that limiting case of (2) where both roots of ω vanish.

The imaginary case $\lambda^2=0$ is easily examined by the foregoing methods, and may be left to the curiosity of the reader.

Finally, if ω is a mere dilation, *i.e.* if $\omega=g$, we clearly have an infinite number of values of ϕ, by assigning $\phi\alpha=+\sqrt{g}\,\alpha$, $\phi\beta=-\sqrt{g}\,\beta$, $\phi\gamma=\pm\sqrt{g}\,\gamma$, with $S\alpha\beta\gamma$ not zero*; also when $g=0$ we have solutions of the form $\phi\rho=\beta S\beta\gamma\rho$.

The more important of the foregoing special cases are easily summed up. By inspection of the various cases we have

Theorem III.—Solutions of the equation $\phi^2=\omega$ fail to exist when, and only when, ω has a double, or a triple axis with vanishing latent root.†

Theorem IV.—If a linear vector function has an infinite number of axes, it has an infinite number of square roots; except as provided in Theorem III.

Theorem V.—A linear vector function having all its latent roots positive has always one, and only one, square root having its latent roots all positive.

The methods by which these results have been obtained can be applied to cube roots and higher roots of a linear vector function, and the corresponding theorems differ but little from those for square roots. The process consists essentially in writing ω in normal form, assuming a ϕ with undetermined coefficients, raising ϕ to the proper power by direct operation, and equating to the known form of ω.

More generally, if ϕ satisfies the equation with scalar coefficients

$$a_n\phi^n + a_{n-1}\phi^{n-1} \; . \; . \; . \; + a_o = \omega$$

ϕ and ω are commutative and the same method of attack might be used. But to solve a quadratic we should more naturally proceed thus: given $a\phi^2+b\phi+c=\omega$, we complete the square on the left just as in ordinary algebra, then extract the square root, by the rules of the present paper, or otherwise. The more general quadratic, $\phi^2+p\phi=q$, where p and q are

* Comparing our results with Tait's solution (note 1), with which, of course, they are fully in accord, it is interesting to note that when ϕ has an infinite number of values, the denominator $\omega+g_1$ in Tait's solution becomes $\omega-g$, hence $(\omega+g_1)^{-1}$ is indeterminate.

† General conditions for the existence of a solution of the equation in matrices $X^n=A$ are given by H. Kreis, *Vierteljahrschrift Nat. Ges. Zurich*, 1908, liii, p. 375. While his results hold for matrices of any order, their form renders them needlessly cumbrous for the present case. Cayley, by a method akin to Tait's in using the symbolic cubics of the known and unknown matrices, showed how the square root can be extracted, considering the general case (eight solutions) only. *Proc. Roy. Soc. Edin.*, 1872, vii, p. 675.

linear vector functions, arises in the solution of linear differential equations of the second order. If p and ϕ are commutative the equation depends for its solution on the extraction of the square root of a linear vector function.* The non-commutative case must be reserved for treatment in a separate paper.

* See Tait's fascinatingly suggestive paper, *Proc. Roy. Soc. Edin.*, vii, p. 311.

(*Issued separately December* 10, 1917.)

XXI.—Contributions to the Knowledge of the Family Chermesidæ. No. I.: The Biology of the Chermes of Spruce and Larch and their Relation to Forestry.

By H. M. Steven, B.Sc., Carnegie Research Scholar in Entomology, The University of Edinburgh. *Communicated* by Dr R. Stewart MacDougall.

(Read May 25, 1917. MS. received June 13, 1917.)

CONTENTS.

PART I.—INTRODUCTORY.

1. Introduction.

Extensive and intensive research on the Chermes group has been carried out on the Continent during the last ten years, but there has been no close study of this group in Britain. This paper is a record of introductory research in this country on this complex but interesting and important group.

These Aphids possess several well-marked features, namely:—

A. Only conifers serve as food-plants.

B. Two host-plants and a period of two years are normally required for the complete cycle of a species, and several parts of each host are utilised for food purposes. This has resulted in a well-marked polymorphism.

C. If two hosts are required, one is constantly a species of the genus Picea, on which a gall is produced. The other may be a species of Larix, Pinus, Abies, or Pseudotsuga, but no gall is produced on any of these Genera.

D. Sexual reproduction in this group is either limited to one of the five generations or is absent altogether. The eggs are always stalked.

Systematists are agreed that the group Chermes is closely allied to the well-known Phylloxera group. Cholodkovsky * (29), who has worked on the Chermes for over a quarter of a century, considers the classification to be as follows:—

Family APHIDÆ.
Sub-family PHYLLOXERIDÆ.
Genus *Chermes*.

In 1915 he admitted as sub-genera what Börner considers Genera. In 1907 Carl Börner (3) suggested the following classification:—

Family CHERMESIDÆ (Passerini, 1867).
Sub-families: A, PHYLLOXERINÆ.
B, CHERMESINÆ, with the following Genera: *Pineus* (Shimer), *Cnaphalodes* (Macquart), *Chermes* s. str. (Börner), *Dreyfusia* (Börner). In 1909 he (8) added the Genera *Cholodkovskya*, *Aphrastasia*, and *Gillettea*.

In this classification the Phylloxera-Chermes group is raised to the rank of a family. In 1913 Börner (10) raised the Chermes group to family rank as follows:—

Family CHERMESIDÆ.
Tribe PINEINI.
Genus *Pineus*.
Tribe CHERMESINI.
Genera *Chermes*, *Gillettea*, *Aphrastasia*, *Dreyfusia*, *Cnaphalodes*, *Cholodkovskya*. The species to be dealt with in this paper are contained in the two Genera *Chermes* s. str. (Börner), and *Cnaphalodes* (Macquart). The generic characters of these Genera are as follows:—

(1) *Chermes* s. str.

The dorsal plates of the first-stage larva of the Fundatrix and Colonici sistens bear groups of two to six double-contoured wax-pores surrounding a small spine. These plates are neither fused together nor polygonal in shape. The secondary vein of the hind wing of the winged forms leaves the longitudinal vein at a right angle.

(2) *Cnaphalodes*.

The head and prothoracic plates of the Colonici sistens first-stage larva are fused together into a homogeneous shield. The dorsal plates of the

* The numbers in brackets refer to "Literature," p. 379.

first-stage larva of the Fundatrix bear one central, double-contoured wax-pore. The secondary vein of the hind wing of the winged forms leaves the longitudinal vein at an acute angle.

2. Nomenclature of the Generations.

The main features of the various generations are as follows:—

I. The first-stage larva hatches in autumn, and is the hibernating stage on spruce. Development is completed in spring. This generation is apterous in all stages. Its feeding habits begin the hypertrophy which results in the formation of the gall. This generation is parthenogenetic, and each female lays a very large number of eggs.

II. Stages 1 to 4 of this generation are passed within the chambers of the gall. This gall is formed by the coalescence of the spruce needles. When the fourth or nymph stage has completed its development the gall opens, and the final moult gives the winged adult. This winged adult (*a*) may remain on spruce, giving rise to Generation I, or (*b*) may migrate to the non-Picea host. The females of this generation are not so prolific as those of Generation I.

III. The first-stage larva of this generation is the hibernating stage on the intermediate or non-Picea host, except in the genus Pineus, where the hibernating insect is a third- or fourth-stage larva. This generation resembles I, but the females generally lay fewer eggs. The parthenogenetic adult may give rise to two kinds of larvæ—(*a*) larvæ which remain undeveloped for a period of time, (*b*) larvæ which develop into apterous parthenogenetic adults. There may be a series of the latter group.

IV. This generation arises from Group III (*b*). It gives a generation which is winged in the adult stage. It resembles Generation II, but development does not take place within a gall. The adults migrate to spruce, and there lay a small number of eggs.

V. This is the sexual generation, which is apterous in all stages. The female lays but one egg, which begins the cycle again.

The following is a table showing the names given to these generations by the various authors:—

	Lichtenstein, 1878.	Blochmann, 1889.	Dreyfus, 1889.	Cholodkovsky, 1895.	Börner, 1907.	Börner, 1908.	Nüsslin, 1908.	Marchal, 1913.	Steven, 1917.
I.	Fundatrix.	...	...	...	...	...	...	...	...
II.	Emigrantes.	...	Migrantes. Emigrantes. Migrantes alatæ.	Migrans alata.	Cellaris (*a*) monœca, (*b*) diœca.	Fundatrigenia (*a*) monœca, (*b*) diœca.	Migrans cellaris (*a*) monœca, (*b*) diœca.	Gallicola (*a*) alata non-migrans, (*b*) migrans alata.	Gallicola (*a*) migrans, (*b*) non-migrans.
III.	Gemmantes.	Aliencolæ.	Virgines. Virgines secundæ.	Emigrantes. Fundatrix spuria. F. intermedia. Exsul.	Virgo (*a*) hiemalis, (*b*) æstivalis.	Virginogenia (*a*) hiemalis, (*b*) æstivalis.	Emigrans + Exsul. Emigrans + Exsul æstivalis.	Exsules (*a*) sistens, (*b*) progrediens.	Colonici (*a*) sistens, (*b*) progrediens.
IV.	Pupifera.	Remigrantes.	Sexupara.	...	...	...	...	...	...
V.	Sexuales.	...	...	...	...	...	...	...	...

The author has selected the above names for the following reasons:—

Generation II.—He considers that the name Gallicola describes the outstanding feature of this generation in the family Chermesidæ. The terms (*a*) migrans and (*b*) non-migrans are less clumsy than (*a*) alata non-migrans and (*b*) migrans alata, while the words monœcious and diœcious have special meanings in botany.

Generation III.—The name Colonici has been established by Burdon in British literature. So long as the non-Picea host is considered intermediate, this term may stand (see Part IV, Section I). Cholodkovsky has accepted the terms (*a*) sistens and (*b*) progrediens.

3. Previous Research.

The galls made by the Chermes species were observed before the insects themselves. In 1583 the Dutch botanist Clusius (30) referred to the galls, but it was not until the eighteenth century that it was discovered that insects lived within the galls. Linney (41, 42), Hartig (38, 39), Kaltenbach (40), and Ratzeburg (51) studied the insects. Blochmann (1) discovered the males in 1887, and between that time and 1889 Blochmann (2), Cholodkovsky (14, 15, and 16), and Dreyfus (32, 33) independently discovered the regular migrations. Further, Blochmann (2) was the first to regard spruce as the original host of the Chermes. Dreyfus enunciated what has been termed the "Parallel-row Theory." It has been defined by Börner as follows: "The Chermes not only pass through a heterogonism * in five separate generations, but one and the same species can live in related rows by the temporary suppression of the true heterogonous circles."

Cholodkovsky (14–29) has continued his researches to the present day. When Blochmann published that the intermediate host of *Chermes abietis*, L., was larch, Cholodkovsky sought to determine how this species lived in Northern Europe, where larch was absent. Finally, he discovered what he considered to be two separate species, one with a cycle of five generations on spruce and larch, which he named *Chermes viridis*, Ratz., and another closely resembling the above, but with a cycle of two generations on spruce only. He called this second species *Chermes abietis*, Kalt. He found the same for the species *Chermes strobilobius*, Kalt., there being a non-migrating species, *Chermes lapponicus*, with two varieties, "præcox" Cholod. and "tardus" Dreyfus. We thus have the establishment of the so-called parthenogenetic species, there being no sexual generation in *Chermes abietis*,

* Heterogonous: producing offspring dissimilar to the parent. Heterogonism: state of being heterogonous.—Murray's *Dictionary*.

Kalt., and *Ch. lapponicus*, Cholod. The following is a graphical description of the life-histories of these species according to Cholodkovsky (29):—

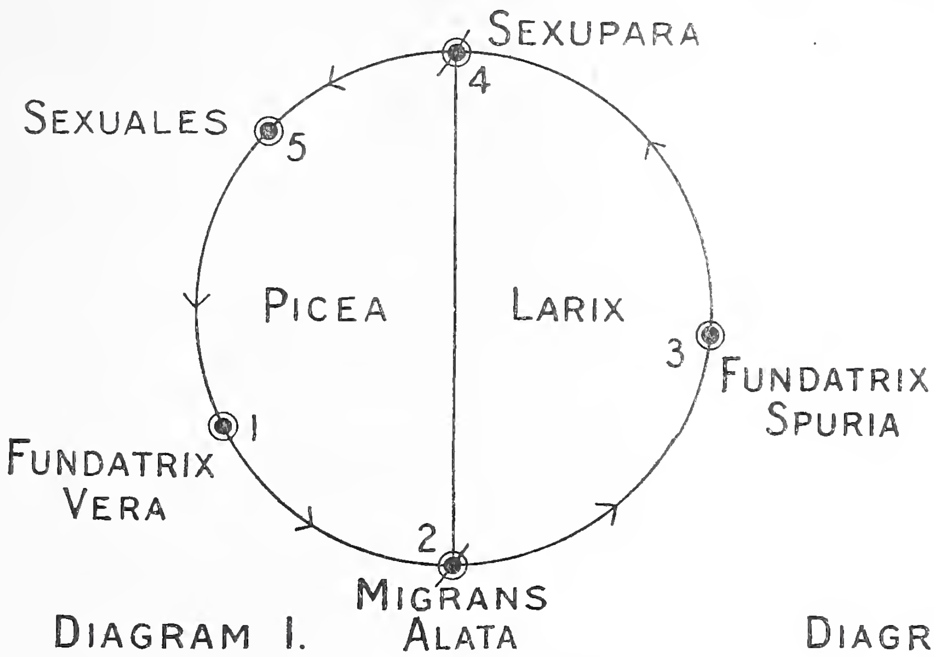


DIAGRAM 1.

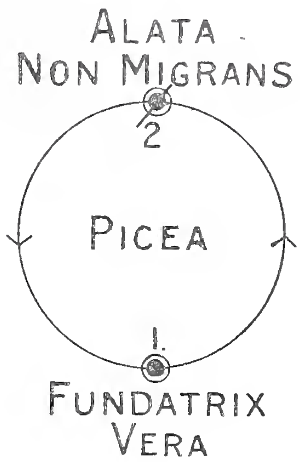


DIAGRAM 2.

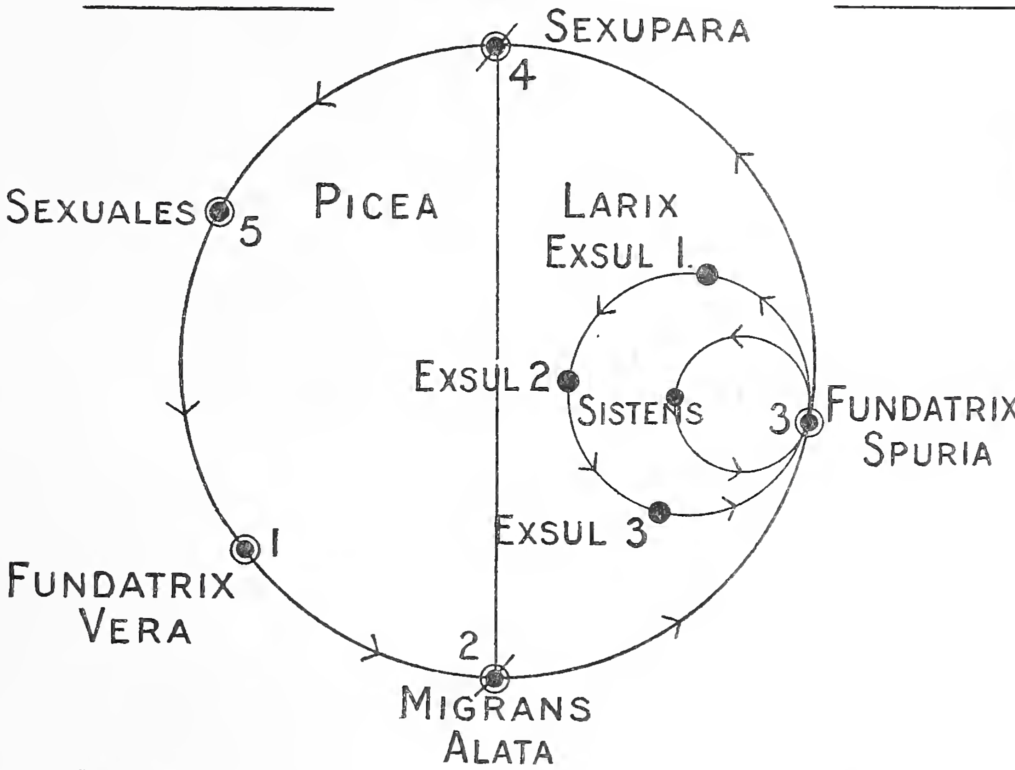


DIAGRAM 3.

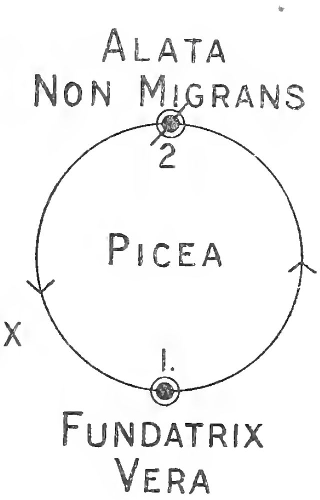
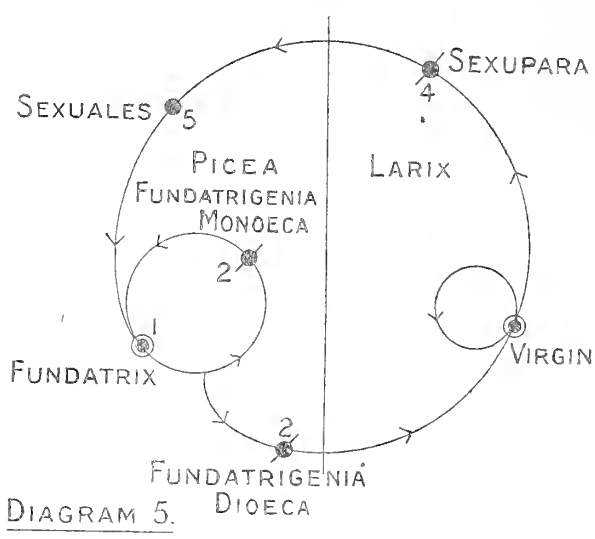


DIAGRAM 4.

Diagram 1.—Graphical representation of the life-history of *Chermes viridis* (Ratz.), according to Cholodkovsky.
Diagram 2.—Graphical representation of the life-history of *Chermes abietis*, Kalt., according to Cholodkovsky.
Diagram 3.—Graphical representation of the life-history of *Chermes* (*Cnaphalodes*) *strobilobius*, Kalt., according to Cholodkovsky.
Diagram 4.—Graphical representation of the life-history of *Chermes* (*Cnaphalodes*) *lapponicus*, Chol., according to Cholodkovsky.

Börner (3) made the "Parallel-row Theory" of Dreyfus the foundation for his work, and sought experimental proof. He (6) succeeded in obtaining gallicolæ migrantes of *Chermes abietis* from one gall produced by one fundatrix derived from a gallicola non-migrans. From these data and from morphological evidence he has deduced that the parthenogenetic species of Cholodkovsky have no existence, and he has united the two branches into one species, *Chermes abietis*, L.; similarly for *Cnaphalodes strobilobius*, Kalt. Further, he (3) stated that from the eggs laid by the colonici two types of larvæ hatched, the sistens and progrediens types (hiemalis and æstivalis of Börner).

The following are the cycles of the above species according to Börner (4–10):—



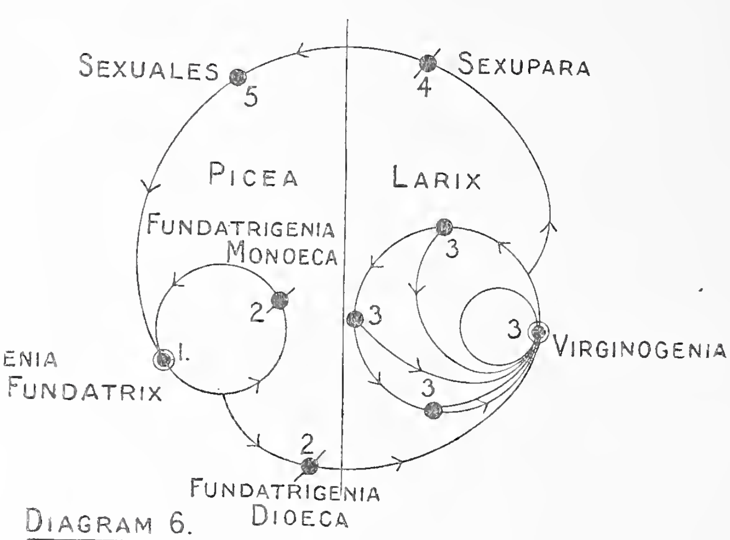


Diagram 5.—Graphical representation of the life-history of *Chermes abietis* (L.), Börner, according to Börner.

Diagram 6.—Graphical representation of the life-history of *Cnaphalodes strobilobius* (Kalt.), Börner, according to Börner.

In reply Cholodkovsky maintained the existence of the parthenogenetic species, and stated that he had not observed two different kinds of larvæ hatching from eggs laid by colonici. In 1909, while on holiday in Switzerland, he (27) observed gallicolæ of abietis type on spruce, which in themselves and their progeny differed from those of *Chermes abietis*, Kalt., observed by him in Estland, Russia. Cholodkovsky then concluded that there were three species of this type in Europe: a Western species, which he named *Chermes occidentalis*, probably possessing both gallicolæ migrantes and non-migrantes, with two species, *Chermes abietis*, Kalt., and *Chermes viridis*, Ratz., in North and East Europe, the former with non-migrantes and the latter with migrantes gallicolæ.

Nüsslin (45) has contributed to the knowledge of the genus Dreyfusia, and has published valuable and suggestive criticisms on the theories as to the origin of migration (46).

Marchal (43) in France has worked on the genera Pineus and Dreyfusia. He has shown that the species of these genera have a complex life-history on the intermediate hosts, pine and silver fir respectively.

Gillette (37), Patch (47), and Crystal (31) have contributed to the knowledge of the North American species, while Stebbing (49, 50) has traced the biology of a spruce (*Picea morinda*)—silver fir (*Abies webbiana*) species. Patch (47) observed gallicolæ of the abietis type settling entirely on spruce and there laying yellow eggs.

Buckton's (11) descriptions are untrustworthy.

Burdon (13) reviewed Cholodkovsky's first monograph, and has researched on methods of control for Chermes (12). Burdon's work was very helpful in spreading a more accurate knowledge of this group in this country.

From the above outline it will be seen that for the spruce-larch genera and species there are two main groups of questions yet to be finally settled:—

A. Is the existence of the parthenogenetic species real? If so, are there three species of the genus Chermes s. str. in Europe, namely, *Chermes occidentalis*, Chol., *Ch. abietis*, Kalt., and *Ch. viridis*, Ratz.? What are their life-histories and distribution in Europe?

B. What is the degree of complexity of the life-history of each species on the larch host?

4. Technique.

The various workers have used varying methods in conducting their biological experiments. The aim of the author was to begin by approximating the experiments as closely as possible to nature, and gradually evolving a more artificial and controlled system, when it was seen that such could be safely done. Thus in the introductory experiments here recorded interference was reduced to a minimum.

Fundatrices and colonici, and later their progeny, were observed from birth. In selected cases egg-laying was observed under a binocular microscope, and the rate of egg-laying determined. The adult gallicolæ were dealt with as follows:—

A. Part allowed to remain on galled twig of spruce.
B. Part transferred to clean spruce.
C. Part transferred to clean larch.

B showed the effect of disturbance, and eliminated the possible error due to that cause. The sexuparæ were transferred to clean spruce. In one half of the experiments the twig was sleeved before the winged stage was reached and after transference; in the other half the insects were allowed freedom. In later experiments muslin cages were used. In the laboratory egg-clusters of colonici were placed separately in petri-dishes, and allowed to hatch, in order to determine the proportion of sistens and progrediens in each egg-cluster. In every case the experiments were supplemented by numerous observations on the experimental area.

The experiments to determine the value of the fumigation of coniferous nursery stock as a method of control were carried out in the following way:—A strong wooden box was procured and placed on one of its narrow ends. Two small apertures were cut, one on the top to allow the gases to escape after the experiment, and another in one of the sides, near the base, to allow the chemicals to be introduced after the front lid had been screwed on. A piece of glass was fitted into the top aperture and a wooden door into the side one. The box was then lined with heavy packing paper. A tray to hold the plants was fitted into the box. The following conifers were fumigated:—

Spruce (*Picea excelsa*), 2 year 2 year old,* very badly infested with fundatrices of a Chermes s. str. species.

Larch (*Larix europœa*), 2 year 2 year old, very badly infested with colonici of *Cnaphalodes strobilobius*.

Pine (*Pinus sylvestris*), 2 year 1 year old, badly infested with colonici of *Pineus pini*.

Fifteen plants of each kind were placed on the tray in the box and the front lid firmly screwed on. The chemicals were then introduced under the tray through the small side aperture. After the experiment the top aperture was opened for ten minutes, then the front lid was unscrewed and the plants taken out and planted. Unfumigated conifers, infested in the same way as those above, were planted out on another area for comparison as to the effects of the fumigants on the trees.

The effect on the Chermesidæ was determined by the future development and egg-laying of the insects.

The following fumigants were used:—

Hydrocyanic Acid Gas.—This gas was generated by adding potassium cyanide to a mixture of sulphuric acid and water. 98 per cent. potassium cyanide and 66° Baumé sulphuric acid (about 93 per cent. purity) are desirable. The proportions used were 1 : 2 : 5, *i.e.* 1 part potassium cyanide to 2 parts sulphuric acid to 5 parts water. The acid was mixed with the water, then the potassium cyanide immediately added. This gas was also generated from sodium cyanide. 126 per cent. purity is desirable for this chemical, and, moreover, it must contain less than 1 per cent. of sodium chloride. The amount of sodium cyanide used was 30 per cent.

* *I.e.*, 2 years in a seed bed, then transplanted and left for 2 years.

less than that of potassium cyanide, as the former liberates about 30 per cent. more hydrocyanic acid gas than does the latter.

Nicotine.—The gas was produced by vaporising a liquid compound containing about 39 per cent. of nicotine in a vaporiser.

Carbon bisulphide.—This gas was produced by allowing the liquid to vaporise.

PART II.—BIOLOGICAL.

The biological experiments were carried out in a small wood at Drumshoreland, West Lothian. In this wood were mixed *Picea excelsa* (Lk.), *P. alba* (Lk.), *P. nigra* (Lk.), *Larix europœa* (D.C.), *L. leptolepis* (Gordon), *Pinus sylvestris* (L.), *Pseudotsuga douglasii* (Carr.), and hardwoods. The age of the trees varied from five to fifty years.

(1) Genus *Chermes* s. str.

The author has found the species of this genus on the following hosts in Scotland:—

Primary or Picea Host: *Picea excelsa* (Link.), *P. alba* (Link.), *P. orientalis* (Link. and Carr.), *P. morinda* (Link.), *P. sitkensis* (Trautr. and Meyer).

Intermediate or non-Picea Host: *Larix europœa* (D.C.), *L. leptolepis* (Gordon), *L. occidentalis* (Nutt.).

Generation I. Fundatrix.—Fundatrices were found hatching both from fertilised eggs and eggs laid by gallicolæ during August and September. These larvæ were active, and wandered over the branches before settling; this activity aided in the dispersion of the larvæ where such had hatched from eggs laid by gallicolæ. The larvæ from the different sources differed as follows:—

A. Those hatching from fertilised eggs were green.

B. Those hatching from eggs laid by gallicolæ were yellow.

In one or two days after hatching the larvæ settled down in a crevice at the bases of fairly strong buds, but seldom the buds of leading shoots. Each inserted its suctorial mouth-apparatus, secreted short curly "wool," and hibernated. Frequently many larvæ were present around a bud, and then they spread on to the bud and down the stem; only one or two such larvæ completed their development. Feeding began around the middle of March. The first moult took place during the first week in April. The suctorial apparatus seemed to be withdrawn before moulting, as the cast skin was seldom anchored to the bud. The newly moulted larva was soft and naked for about a day, then "wool" was again secreted. The second moult took place during the last week in April; the buds of spruce began to burst about this date. The larva moulted for the last time during the first week in May. The adult fundatrices differed as follows:—

A. Green laying green eggs.

B. Yellowish-green laying yellowish-green eggs.

The latter group were much the more numerous on the experimental area. By mid-May the effects of the feeding of the fundatrices became evident. The bases of the needles near the fundatrices became swollen, and such needles remained stunted. Five to ten days later, numerous reddish papillæ appeared on the swollen parts.

Generation II. Gallicola.—The first-stage larvæ began to hatch out during the last week in May. They immediately crowded amongst the deformed needles. As a result of the feeding of these larvæ further swelling of the needles took place, and a compact gall was formed by mid-June. Only the needles at the base of the shoot were affected; the shoot continued to grow, but was usually bent. Three moults took place within the gall at intervals of about two weeks. The galls began to open from 26th July. These opening first were derived from Group B fundatrices. The nymphs (fourth-stage larvæ) climbed out of the gall on to the surrounding needles and moulted, becoming winged gallicolæ. These were dealt with as described under "Technique," I, 4. The majority of the gallicolæ, transferred to or allowed to remain on spruce, settled and began egg-laying in twenty-four hours. The eggs were bright yellow, and numbered twenty to fifty. No "wool" was secreted by the gallicolæ. Those gallicolæ transferred to larch and, allowed freedom, climbed to the top of the needles and flew away. When the twigs were sleeved a few settled down and laid bright yellow eggs. In twenty to twenty-two days the eggs on spruce had hatched, giving yellow fundatrices; by that time all the yellow eggs laid on larch had shrivelled up without hatching, or the larvæ hatching from these eggs had died. Galls of this type continued to open until 10th September. The above experiments were repeated many times, and all gave the same results.

Only two galls, derived from green fundatrices (Group A), were obtained. These galls opened on 1st and 3rd August. The gallicolæ were slightly more green in colour than those of Group B. They were dealt with as before. No result was obtained from those remaining on or transferred to spruce. Those transferred to larch needles settled down, and laid twenty to thirty dark-green eggs under the protection of the wings of the mother.

Between 1st and 10th September, in the Royal Botanic Garden, Edinburgh, and in another wood on the Drumshoreland area, adult gallicolæ belonging to the genus Chermes s. str. were found on spruce needles. The colour of the eggs laid by them was bright green, markedly different both from that of the eggs laid by gallicolæ of Group A on larch, and from that of the eggs laid by gallicolæ of Group B on spruce. These gallicolæ appeared to correspond to those in the Börner experiment (6) establishing the unity of the non-migrans and migrans branches of the gallicolæ of *Chermes abietis*, L., C.B., and to those described by Cholodkovsky (27) as *Chermes occidentalis* in Switzerland.

Generation III. Colonici.—In nineteen to twenty-two days dark-green larvæ hatched from the eggs laid by the gallicolæ migrantes on larch. These larvæ crept from beneath the wings of their dead mother, and in one to two days migrated from the needle to the bark of the branch or trunk. Small cracks and irregularities of the bark were used for protection. Strongly lighted zones, such as tops of exposed trees and upper sides of branches, were avoided. The larvæ inserted their suctorial apparatus, secreted short curly "wool," and hibernated. From 1st March onwards the first-stage larvæ of the colonici woke up and began to feed. The time of awakening and moulting was very irregular, probably due to varying degrees of protection. By 1st April some colonici had moulted three times, become adult, and begun egg-laying. Egg-laying, however, was not general until the beginning of May. Fifteen to thirty green eggs were laid under the protection of copious "wool" secreted by the mother. The egg-laying continued about a month. The eggs hatched in twenty to twenty-two days. The first larvæ hatching from a clutch of eggs were of the progrediens type; they migrated to the needles (see Generation IV). Those hatching later behaved in a different manner. Most of them settled down around their dead mother, secreted short curly wool, and thus passed the summer and winter. The majority of the sistens larvæ did not hatch until the end of June. Fresh larch shoots were offered to some on hatching, and they quickly climbed on to the needles. These latter sistentes were constantly observed during summer; they remained undeveloped, and all such larvæ had died by mid-August. Numerous sistentes were observed in nature on shaded dwarf-shoot needles; they did not secrete wool nor feed, and all died off before the end of the summer. These observations seemed to indicate that there was still some impulse within these sistentes urging them to attempt to develop immediately.

Generation IV. Sexupara.—The sexupara generation developed on the larch needles. The feeding of the larvæ resulted in a decrease in the chlorophyll, and the needles became kneed. The larvæ were to be found on the larch needles from the end of May to the end of June. The four moults take place at intervals of seven to ten days. Most of the nymphs moulted, and became adult during the last week in June. The adult sexuparæ were yellow when newly moulted, and became dark green in a few hours. A little "wool" was secreted. The adult sexuparæ were transferred to spruce, and they settled principally on needles one to three years old. They laid five to ten yellow green eggs under cover of wings and "wool."

Generation V. Sexuales.—In twelve to fifteen days from laying, the eggs hatched out. The sexuales larvæ were very small and light yellow green in colour. The four moults took place at intervals of about seven days. The adults were found during the last week in July. Their colour was yellow green, and no colour difference between male and female was noticed. The female laid the single fertilised egg near a bud, under the protection of a little patch of wool. The egg was yellowish-green in colour.

(2) Genus *Cnaphalodes.*

The author has found the species of this genus on the following hosts in Scotland:—

Primary or Picea Host: *Picea excelsa, P. orientalis, P. alba, P. sitkensis.*

Intermediate or non-Picea Host: *Larix europæa, L. leptolepis, L. occidentalis.*

Generation I. Fundatrix.—Fundatrices were found hatching from fertilised eggs during August, and from eggs laid by gallicolæ during August, September, and October. In both cases the fundatrices were light brown, gradually turning to a very dark brown. At first the larvæ were active, then they settled down on a shaded, hence weak, bud. They secreted long single strands of coarse "wool," which gave the larvæ a plumose appearance, and hibernated. Frequently both the terminal and side buds were covered with larvæ. The fundatrices wakened later than do those of Chermes s. str. This was probably due to their choice of shaded buds, which were not touched by the sun's rays until a later date. Feeding began about the 1st April, and the first moult took place towards the end of the month. The second-stage larvæ secreted short curly "wool" from the numerous dorsal wax glands. The two succeeding moults took place at intervals of about seven days, and egg-laying was general by mid-May. The adult female, which was dark bronze green in colour, secreted copious "wool" for the protection of her fifty to one hundred eggs. The egg-colour varied with the age of the egg, yellow when laid, gradually becoming greener and darker until a brownish-green colour was reached. Before the end of May the effect of the feeding of the fundatrix was clearly visible. The needles of the opening bud remained stunted; chlorophyll production was impeded, and the tips of the needles became a bright rose colour. The growing point was usually killed.

Generation II. Gallicola.—The eggs hatched out from 1st June onwards. Short "wool" was secreted by the light green larvæ. These larvæ continued to hatch until the end of June, but most of the galls had closed up by mid-June, so that larvæ hatching later had to remain outside the gall. These larvæ did not develop, and finally died. Their presence on the outside of the gall was characteristic for this genus, and probably due to the greater rapidity in the formation of the gall as a result of the fundatrix being situated actually on the gall. The rate of development within the gall varied as follows:—

A. In a gall produced by a fundatrix hatching from a fertilised egg development was rapid; the three moults took place at intervals of five to seven days. Such galls turned brown and opened from 5th to 12th July. The nymphs climbed out and the final moult took place, generally in the morning. The wings of the red adults had straightened and firmed by early afternoon, when the principal migration of the day took place. The progeny of each experimental gall was dealt with as described in "Technique" (Section I, 4). Eggs were only laid on spruce when the twig was sleeved, but the gallicolæ settled readily on the larch needles and laid twenty to forty dark brownish-green eggs. Practically no "wool" was secreted by the gallicolæ. After fifteen days all the eggs laid on spruce had shrivelled up without hatching, while those on larch had hatched.

B. In galls produced by fundatrices hatching from eggs laid by gallicolæ development was slower and variable. At the time of the opening of those of Group A the larvæ in those of Group B were in the second stage. In some of the latter the next two moults took place at intervals of about seven days, and the galls opened around 25th July; in others the development was slower, and galls of this type continued to open during August until the 20th of September. The progeny of these galls were dealt with as before. The gallicolæ settled readily on spruce and laid twenty to thirty bright orange-coloured eggs under the protection of copious wool. In spite of numerous experiments only one gallicola was induced to settle and oviposit on a larch needle. A few larvæ hatched from these orange eggs, but they died. As the season advanced the size of the gallicolæ decreased. These later females laid fewer eggs, and they secreted more "wool."

Generation III. Colonici.—The larvæ hatching from the eggs laid by the gallicolæ migrantes on larch needles were dark brown in colour. They generally settled in the axils of needles on the branches of older trees, and were not noticeable until the needles fell off. When the trees were very young they settled on the bark of the shoot. No "wool" was secreted before or during hibernation. These larvæ began to feed from the 1st March onwards. Development was variable; some had reached the adult stage by the beginning of April, but in most cases not till the end of that month. The adult female was bronze green in colour and powdery wool was secreted from the last segments of the abdomen. Egg-laying was closely observed under a porro-prism. The eggs were, as a rule, laid in the axil of, or on, a dwarf shoot. The female used the end of the abdomen in selecting a place for oviposition. The ovipositor remained a few minutes on the bark in anchoring the stalk of the egg, then the stalk was passed out and remained coiled like a spring. The egg took a few minutes to pass out. The colour of the egg was yellow when laid, turning a bronze-green colour a few hours later. A pinkish tinge appeared a few days afterwards. Five to twelve eggs per day were laid until thirty-five to fifty had been laid. Unfavourable weather frequently stopped egg-laying for a number of days. Egg-laying continued until mid-May, the first eggs hatching before the last were laid.

(*a*) The first larvæ hatching from the eggs which were laid first were of the progrediens type.

(*b*) The larvæ hatching from the eggs laid last were of the sistens type. They settled on the twig, and remained undeveloped during the summer and winter. These were very few in number, and hatched out much later than those of the progrediens type. Some climbed on to needles and died.

The black active progrediens larvæ climbed on to the needles and, in nature, frequently fed beside the larvæ of Chermes s. str. The feeding caused the bending of the needle. The moults took place at intervals of about seven days. The third moult gave two forms—a nymph of the sexupara, and a fourth-stage progrediens. The apterous progredientes became adult during the first half of June and laid twenty to thirty brownish eggs under the protection of copious "wool." These eggs hatched in fifteen to twenty days. The proportion of sistens and progrediens larvæ in the progeny of this and succeeding generations was variable. The ancestry of these females, *i.e.* whether derived from gallicolæ migrantes or from sistentes of previous years, may be a factor causing this variability, but this can only be determined by control experiments extending over many years. One factor, however, seemed to be the climatic conditions. In the summer of 1916 the percentage of progrediens larvæ was much higher than in the summer of 1917, with the result that the individuals of Generations II and III of the progredientes were more numerous in the former than in the latter summer. The larvæ, hatching from the eggs laid by Generation I, were in most cases principally of the progrediens type. In some cases, however, the proportion was about half and half, while in one experiment the progeny consisted entirely of sistentes. The females of Generation II became adult during the second half of July. The progeny of Generation II was as I, but the average percentage of sistentes was higher. The females of Generation III became adult about the beginning of September. The percentage of sistentes in the progeny of this generation was still higher than the preceding generation. The inclement weather caused the death of the progrediens larvæ in the first or subsequent stages. In each generation the sistens larvæ remained undeveloped and hibernated, while the progrediens larvæ developed. Only three moults were observed in progredientes, Generations II and III, while four is the normal number in the progrediens type. Further research is necessary to clear up this point.

Generation IV. Sexupara.—The adult stage was reached during the second half of June. The sexuparæ were transferred to spruce; they settled principally on one- to three-year-old needles. Five to ten yellow brown eggs were laid under the protection of the wings and copious "wool."

Generation V. Sexuales.—The eggs hatched in ten to fifteen days. The larvæ were straw-coloured, and lived under the "wool" and wings. The four moults took place at intervals of five to seven days. The adult stage was reached during the second half of July. The female laid the single fertilised egg under a bark scale near a shaded bud. The egg was straw-coloured. This egg hatched in twenty to twenty-five days.

Part II. General Conclusions.

(1) *Chermes* s. str.

1. Two separate cycles have been proved to be present in Britain:

(*a*) A cycle of two generations, Fundatrix and Gallicola non-migrans. The Fundatrix and Gallicola non-migrans lay yellow eggs on spruce, and the galls open over an extended period from the end of July until mid-September. This is the species *Chermes abietis*, Kalt., of Cholodkovsky.

(*b*) A cycle of five generations, Fundatrix, Gallicola migrans, Colonici, Sexupara, and Sexuales. The Fundatrix lays green eggs on spruce, and the Gallicola migrans lays very dark green eggs on larch. The galls open during a limited period in the first half of August. This is the species *Chermes viridis*, Ratz., of Cholodkovsky. There is probably a cycle with both Gallicola migrans and non-migrans corresponding to *Chermes abietis*, L., Börner, or *Chermes occidentalis*, Chol.

2. Sistens and progrediens larvæ hatch from the eggs laid by the colonici. Many of the sistens larvæ do not settle down on the bark, but migrate to the needles and die.

(2) *Cnaphalodes*.

3. Two separate cycles have been shown to be present in Britain:

(*a*) A cycle of two generations, Fundatrix and Gallicola non-migrans. The latter lays bright orange-coloured eggs under the protection of copious "wool." The galls open over an extended period from the end of July until the end of September. This is the species *Chermes lapponicus*, Chol., var. *tardus*, Dreyfus, of Cholodkovsky.

(*b*) A cycle of five generations, Fundatrix, Gallicola migrans, Colonici, Sexupara, and Sexuales. The Gallicola migrans lays dark bronze-coloured eggs without any "wool" covering on larch. The galls open during a limited period in the first half of July. This is the species *Chermes strobilobius*, Kalt., of Cholodkovsky.

4. Sistens and progrediens larvæ hatch from the eggs laid by the colonici and progredientes of *Cnaphalodes strobilobius*, Kalt. The relative proportion of each type is variable.

It has thus been shown that the above cycles are not confined to North and East Europe, where larch is either absent or the European species is replaced by *Larix siberica*, but are present in Britain, where larch and spruce grow side by side. On his experimental area the author estimates that 90 per cent. of the species of Chermes s. str. and Cnaphalodes on spruce were the non-migrating, parthenogenetic species, although the branches of the spruce and the larch were often interlocked. This fact suggests that the non-migrating species have not arisen because the intermediate host larch was absent, but because they are the more successful species even when spruce and larch are both present.

The question whether the above cycles should be considered as those of separate species or of biological races of one species can only be determined by further research, both morphological and biological. Statistical research, such as that done by Philiptschenko (48), will be important in determining this question.

PART III.—RELATION TO FORESTRY.

1. Methods of Infection.

The methods of infection are the same in both the genera Chermes s. str. and Cnaphalodes. Conditions favouring infection are proximity of the hosts and favourable weather conditions. These explain the rapid spread of the pest in forest nurseries, which are usually sheltered, with the trees crowded together.

(*a*) *Spruce.*

Spruce is infected from two sources—(*α*) By sexuparæ. The parthenogenetic adaptations on larch are made at the expense of the sexupara generation. Thus on the area under observation, although colonici were very numerous on larch, the number of sexuparæ attaining maturity, and still more those reaching spruce and laying eggs, was small. Thus infection from that source was not serious. (*β*) By gallicolæ non-migrantes. The majority of such gallicolæ settled on the tree on which they were born. This, together with the high fertility of the two generations constituting that cycle, caused the rapid increase on the host. Thus infection from this source was serious.

(*b*) *Larch.*

Larch is infected by gallicolæ migrantes. This is the source of the first infection, but the species are thereafter principally carried on on this host by the parthenogenetic adaptations.

2. Damage to Spruce.

(*a*) Genus *Chermes* s. str.

The species of this genus attack strong growing spruce, hence the damage is primary. Under normal conditions such damage is unimportant, but, allied with unsuitable soil or atmospheric conditions, the work of this genus may play an important part in killing the host. On the area under observation the generations of the non-migrating species did the much greater damage.

(*b*) Genus *Cnaphalodes*.

The species of this genus only attack thin shaded branches, hence the damage, as regards spruce, is secondary. Shaded spruce, however, are quickly killed as the galls terminate the twigs. Thus the damage would be important where, in thinning a wood, the shaded spruce were left for soil protection. As before, the non-migrating species was found to be the more dangerous.

3. Damage to Larch.

The damage to larch by the species of the two genera Chermes s. str. and Cnaphalodes has undoubtedly been greatly increased by the planting of that conifer in localities and under conditions very different from those of its natural habitat, with a consequent weakening of the tree. The species of both the genera are usually present on the same tree.

(*a*) Genus *Chermes* s. str.

The species of this genus is principally a bark-feeder as regards larch, hence the damage is difficult to estimate. Frequently the bark is whitened by the "wool" and cast skins of the colonici; later the bark turns black. The feeding of these numerous colonici, at a time when growth should be at a maximum, must have a weakening effect on the host. The punctures are small, but they are made at a time when girth is increasing, so that they will be greatly increased in size and become a possible source of infection by wound parasitic fungi. The damage resulting from the feeding of the sexuparæ is unimportant, as its duration is short.

(*b*) Genus *Cnaphalodes*.

The species of this genus is a twig- and leaf-feeder on larch. The colonici do damage similar to that by colonici of Chermes s. str. In the author's opinion the principal damage is done by the progredientes. They were frequently so numerous as to whiten the larch needles. To repair the damage, the dwarf shoots and dormant buds began to grow. These, together with the elongating terminal shoots, provided new and succulent food for the succeeding generations of progredientes. Thus the struggles of the host to free itself only resulted in its more complete subjugation.

4. Control.

The high parthenogenetic developments of the species of the genera Chermes and Cnaphalodes on both spruce and larch make it impossible that any considerable benefit would result from attempting to eliminate either host from any particular plantation. As the species of the Chermesidæ quickly increase on any decline in the health of their hosts, great care should be taken that the planting area is clearly suitable to the conifer which it is desired to plant. The author is convinced that there is no practical method of controlling the pests after a plantation has been formed. Specimen trees may be sprayed as suggested by Burdon (12).

During his observations two facts impressed the author, namely:—

(*a*) The species of Chermesidæ are frequently widely present in forest nurseries.

(*b*) These pests often do serious damage to their hosts immediately after a plantation has been formed.

These facts suggested that it would be exceedingly advantageous if the conifers used in the formation of a plantation were free from Chermesidæ. The author considered that the most practical and thorough method to ensure this was the fumigation of the nursery stock immediately before dispatch to the planting area. The fumigation of certain classes of nursery stock is compulsory in some countries and is practised in many others, but not to any great extent in this country.

The methods employed in carrying out the fumigation experiments have been described under "Technique," I, 4. They were as simple as possible, and will have to be elaborated to ensure complete success. The plants were much more seriously infested than would be normally met with, hence this method of control was rigorously tested.

The details and results of the experiments have been embodied in the following table:—

[Table

Number of Experiment.	Date of Fumigation.	Trees fumigated.	Fumigant.	Amount used.	Temperature.	Duration of Fumigation.	Effect on Chermesidæ.	Effect on the Trees.
1.	March 21st.	Spruce and larch.	Hydrocyanic acid gas from potassium cyanide.	1 oz. KCN to 200 cub. ft. of space.	38° F.	1 hour.	Spruce : 100 p.c. of fundatrices killed. Larch : 90 p.c. of colonici estimated killed.	In Experiments 1–7 inclusive the trees did not appear to be in any way injured by the fumigants. In every case the trees which were fumigated and planted out were less retarded in growth than those which were only transplanted. Experiment 8 was delayed in order to obtain a higher temperature, and the transplantation at this latter date and not the fumigant appeared to injure the trees.
2.	,,	,,	,,	1 oz. KCN to 150 cub. ft. of space.	38° F.	,,	Spruce : 100 p.c. of fundatrices killed. Larch : 90 p.c. of colonici estimated killed.	
3.	March 22nd.	,,	,,	1 oz. KCN to 100 cub. ft. of space.	39° F.	,,	Spruce : 100 p.c. of fundatrices killed. Larch : 95 p.c. of colonici estimated killed.	
4.	,,	,,	,,	1 oz. KCN to 75 cub. ft. of space.	39° F.	,,	Spruce : 100 p.c. of fundatrices killed. Larch : 95 p.c. of colonici killed.	
5.	,,	,,	,,	1 oz. KCN to 50 cub. ft. of space.	39° F.	,,	Spruce : 100 p.c. of fundatrices killed. Larch : 99 p.c. of colonici killed.	
6.	March 26th	Scots pine.	Hydrocyanic acid gas from sodium cyanide.	1 oz. NaCN to 100 cub. ft. of space.	42° F.	,,	Pine : 100 p.c. of colonici killed.	
7.	,,	Spruce and larch.	Nicotine.	$\frac{1}{4}$ oz. to compound containing 39 per cent. nicotine to 100 cub. ft. of space.	42° F.	3 hours.	Spruce : 25 p.c. of fundatrices killed. Larch : 100 p.c. of colonici killed.	
8.	April 26th.	Spruce, larch, and Scots pine.	Carbon bisulphide.	8 oz. to 100 cub. ft. of space.	...	24 hours.	The plants did not live, hence effect of fumigant on Chermesidæ could not be determined.	

In addition to these experiments, a considerable number of spruce and larch, 2 year 1 year old, not so badly infested with Chermesidæ, hence healthier plants, were fumigated, as were the plants in Experiment 3. The fumigation was in this case completely successful.

In the opinion of the author, Experiments 1–5 were not completely successful as regards larch, because the plants of this conifer were farthest from the generator during fumigation, and the concentration of the fumigant around them was too weak owing to the fumigation box not being sufficiently gas-tight. No reason can be given why Experiment 7 was not successful as regards spruce.

The temperature at which the fumigation of these pests by hydrocyanic acid gas was successful is much lower than that considered the optimum. This is important, as the average air temperature, at the time when fumigation must be carried out, is low.

These experiments have shown that fumigation is a safe, efficient, and cheap method of killing Chermesidæ on nursery stock. The practising of this method of control will ensure that the further spread of these pests will be limited, and that the conifers will get a chance to establish themselves in their new environment. The methods will have to be improved before a definite set of instructions can be given, but the following may be stated now:—

(*a*) The fumigation should be carried out immediately before dispatch of the plants to the planting area.

(*b*) The fumigation should be carried out not later than the 1st April, as the insects begin to become adult and lay eggs after that date.

(*c*) Hydrocyanic acid gas, generated from potassium cyanide at the rate of 1 oz. to 100 cubic feet of space, or from sodium cyanide at the rate of 1 oz. to 130 cubic feet of space, appears to be the most useful fumigant.

This method of control should prove of especial value in this country at this time, as very extensive planting of conifers will have to be carried out in the years following the close of the war.

Part III. General Conclusions.

1. The non-migrating species of the genera Chermes s. str. and Cnaphalodes are more serious enemies to spruce than are the migrating species. The species of Chermesidæ, however, are only serious enemies of spruce when allied with unsuitable soil or atmospheric conditions.

2. The collective damage to larch by the colonici of the species of Chermes s. str. and Cnaphalodes and the progredientes of Cnaphalodes is serious in Britain.
3. The fumigation of coniferous nursery stock before dispatch to the planting area has proved a practical method of limiting the further distribution of these pests, and of ensuring that the plants get a reasonable chance of establishing themselves in their new environment.

PART IV.—GENERAL.

1. Theories as to the Origin of Migration.

All authors, from Blochmann to Nüsslin, have considered that spruce was the original host of the Chermesidæ; for instance, Cholodkovsky (29) thinks that the cycle was an annual one and on spruce only. Sexuales were produced towards the end of the summer. The winged forms were transported by wind to trees of other genera, and there adapted themselves to feeding and breeding. The migration back to spruce took place in a similar way to the first migration.

In 1907 Börner (3) introduced a new theory which reversed the above theory. He held that the Picea host was intermediate, and that Pine was the primary host. Nüsslin (46) pointed out the many difficulties which this new theory raised, and Börner (9) abandoned it for that of Mordwilko.

Mordwilko (44) considers that all aphids were originally polyphagous, and that the present migrations are remnants of that ancestral polyphagia. The relative suitability of the hosts as regards food and breeding is the impulse inducing any particular migration. He divides modern aphids into groups on these lines:—

A. A group in which there are no real migrations; two different host plants are not necessary, but the species are widely polyphagous. Here come numerous Aphidinæ, some Lachninæ and Schizoneurinæ.

B. A group in which there is facultative migration; two host plants may be utilised, while the polyphagia of the species is limited; *e.g. Siphocoryne xylostei*, Schr., according to Mordwilko (44) can complete uninterruptedly its life-cycle on honeysuckle. The part from the first winged parthenogenetic females to the Sexuparæ and the winged males can, however, be passed on an umbelliferous host.

C. A group in which migration is obligatory; two host plants are necessary, while the species are only slightly polyphagous. Here come a few Aphidinæ, some Schizoneurinæ and Pemphiginæ. Here also come the Chermes species.

Mordwilko's theory is based on wide data, but it assumes that the same phenomena, within this diverse group (Aphids in its widest sense), arose in the same way. All the species of the Chermesidæ, which possess a sexual generation, pass that generation on spruce. On the other hand, the parthenogenetic development has arisen on a number of other genera of conifers. In the author's opinion the Blochmann theory explains this phenomenon in the more satisfactory way.

In conclusion, I wish to express my indebtedness to Dr R. Stewart MacDougall for the help which he has given me in carrying out this research, and also to Professor I. Bayley Balfour for facilities for observation and experiment granted in the Royal Botanic Garden, Edinburgh. I am also indebted to the factors of the Right Hon. the Earl of Buchan for permission to carry out experiments at Drumshoreland, West Lothian.

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(*Issued separately January* 15, 1918.)

OBITUARY NOTICES.

James Burgess, C.I.E., LL.D. By **C. G. Knott,** D.Sc., LL.D.

(MS. received November 12, 1917.)

James Burgess was born on August 14, 1832, at Kirkmahoe, Dumfriesshire, Scotland. He received his education chiefly at Glasgow, and was trained as a teacher. In 1855 he was appointed Professor of Mathematics in the Doveton College, Calcutta, and after six years proceeded to Bombay, where he became head of the Sir Jamsetjee Jejeebhoy Parsee Benevolent Institution. Here he became greatly interested in archæological matters, and began to contribute to the *Bombay Gazette* a series of valuable geographical and architectural notes. Some of these took the form of guide-books, and his activity in this direction led to his devoting more and more of his time to antiquarian research. In 1868 Dr Burgess was appointed Secretary of the Bombay Geographical Society, and while holding that position he did his first great service to the scientific world by starting the *Indian Antiquary* in 1872. Among his main objects were the publication of all kinds of historical and archæological work carried out in India, and the encouragement of research in these lines. A feature was the reproduction of English translations and abstracts of work done by European scholars in other languages. His editorial work and two books which he published, viz. *Temples of Satruñjaya,* in 1869, and the *Rock-cut Temples of Elephanta,* in 1871, attracted the attention of the Government, and in 1874 Dr Burgess was appointed Archæological Surveyor and Reporter to Government for Western India. In 1881 his sphere of work extended, and he became Archæological Surveyor and Reporter to Government for Southern India. During this period he published a series of large, well-printed, and handsomely illustrated quarto Reports on the archæology and architecture of most of the famous sites of Western and Southern India. These are specially valuable on the architectural side, but also contain the most ancient inscriptions. They constitute the beginning of the New Imperial Series of the Archæological Survey.

On the retirement of General Sir Alexander Cunningham from the directorship of Northern India in 1886, Dr Burgess was appointed Director-

General of the Archæological Survey in India for all three districts. After three years he retired from this office, and with his retirement the office came to an end. The work, however, continued, one very important development being the institution by Dr Burgess of the *Epigraphia Indica*, an official periodical devoted to the publication of Sanskrit and other inscriptions, with facsimiles, translations, annotations, etc. This periodical is still the leading organ of this branch of Indian research.

Meanwhile he had settled in Edinburgh, and continued to fulfil his engagement with the Government, viz., to publish a number of volumes based on the drawings he had accumulated during his tenure of office. The last appeared in 1911. In 1910 he re-edited Fergusson's *Indian Architecture*, and in 1913 published his *Chronology of Modern India*, A.D. *1494–1894*. He also edited, with additions, translations of Grünwedel's *Buddhistic Art in India* and Bühler's *Indian Sects of Jaina*. It may be mentioned that at his instance the Orientalist Congress adopted the present accepted scheme of transliteration of Indian alphabets.

After 1913 increasing infirmities of age considerably curtailed his literary activities; but his mental faculties remained unimpaired, and he was dictating important correspondence only a few days before his death. He died on October 3, 1916, in the eighty-fifth year of his age.

His antiquarian work must be regarded as the most important of all he undertook; but it is well to remember that he was a man of keen interest in other lines of study. While engaged in education in India he published for the sake of his Indian students various English classics, with notes explanatory, philological, and critical. These are mines of information. He also published an *Introduction to Arithmetic, containing the Theory and Practice of Whole Numbers, with Tables of the Coins, Weights and Measures in use in British India and the United Kingdom.* This is in many respects, a very original work. He had indeed a strong bias toward arithmetical and mathematical calculations. He contributed interesting articles for a number of years to the *Times of India Calendar*, and drew up an important discussion on "Hypsometrical Measures by means of the Barometer and the Boiling-point Thermometer," published in the *Journal of the Asiatic Society of Bengal*. An article by him on the same subject appeared about the same time in the *Philosophical Magazine* for 1863 (vol. xxv, pp. 29–37). His most important and lasting contribution to mathematical literature was his paper in our *Transactions* (vol. xxxix, 1897) on the "Calculation and Tabulation of the Error-function Definite Integral." For this he was awarded the Keith Prize in 1898.

Dr Burgess received from the Edinburgh University the honorary degree of LL.D. in 1881, and in 1885 was created Companion of the Order of the Indian Empire. He was an Honorary Associate of the Royal Institute of British Architects; Honorary Member of the Imperial Russian Archæological Society, the American Oriental Society, and the Royal Philosophical Society of Glasgow; Fellow of the University of Bombay; Honorary Associate of the Finno-Ugrian Society; Corresponding Member of the Ethnological Society of Berlin and of the Batavian Society of Arts and Sciences; Fellow of the Royal Geographical Society; and Member of the Société Asiatique, Paris. He was also a member of the Royal Asiatic Society of Great Britain and Ireland, which he joined in 1886, and of which he was at the time of his death almost the oldest surviving member.

He was for many years one of the external members of the Library Committee of the University of Edinburgh, and devoted a great deal of his time to the preparation of a new catalogue.

He was elected a Fellow of the Royal Society of Edinburgh in 1894, served two terms of three years as a Member of Council, and one term of six years (1908–1914) as Vice-President.

His taste for mathematical calculation led him to give valuable help in the preparations for the Napier Tercentenary Celebration in 1914, although the state of his health prevented him attending the Congress.

Dr Burgess took a strong personal interest in the mission work of the Free Church of Scotland and (later) of the United Free Church, and served for many years on their Committees.

The following list of books and papers, although not exhaustive, contains his most important contributions to archæology:—

The Rock-cut Temples of Ajanta. Pamphlet, 1868.

The Temples of Satruñjaya, photographed by Sykes and Dwyer, with historical and descriptive introduction (one plate and forty-six photographs). Atlas fol. Bombay, 1869. Reprint of text. 8vo. Ahmadabad, 1878.

Notes of a Visit to Somanâth, Girnâr, etc., in Kâthiâwâd. 18mo. Bombay, 1869.

Notes of a Visit to Gujarât. 12mo. Bombay, 1870.

Photographs from Somanâth, Girnâr, Junâgadh, etc., in Kâthiâwâd, with descriptive text. Bombay, 1870.

The Rock-cut Temples of Elephanta or Ghârâpurî. Illustrated (with drawings and photographs). 8vo and oblong fol. Bombay, 1871.

Photographs of Architecture and Scenery in Gujarât and Rājputāna (by Bourne and Shepherd), with historical and descriptive letterpress. Fol. Calcutta, 1874.

Report of the Archæological Survey in Belgaum and Kaladgi Districts. Roy. 4to. London, 1874.

Memorandum on the Buddhist Caves at Junnar. In conjunction with J. F. Fleet. 1874.

Memorandum on the Remains of Gumli, Gop, etc. 1875.

Report of the Archæological Survey: Kâthiâwad and Kachh. Roy. 4to. London, 1876.

The Rock-cut Temples of Elura or Verul, with twelve photographs. Cr. 8vo. Bombay, 1877.

Report of the Archæological Survey: Bidar and Aurangâbâd. Roy. 4to. London, 1878.

Pali Sanskrit and old Canarese Inscriptions from the Bombay Presidency and Parts of the Madras Presidency and Maisur, arranged and explained. In conjunction with J. F. Fleet. London, 1878.

Notes on the Rock-temples of Ajanta and their Wall-paintings. Demy 4to. Bombay, 1879.

The Cave Temples of India. In conjunction with Jas. Fergusson. 8vo. London, 1880.

Inscriptions, etc., from the Cave-temples of Western India. Demy 4to. Bombay, 1881.

Notes on the Amarâvati Stûpa. 4to. Madras, 1882.

Buddhist Cave-temples and Brahmanical and Jaina Caves in Western India; and the companion volume, The Buddhist Cave Temples and their Inscriptions. Roy. 4to. London, 1883.

Notes and Inscriptions from Temples in the Madura District. 1886.

Tamil and Sanskrit Inscriptions. 1886.

List of Ancient Monuments for Conservation in the Madras Presidency. 1886–7.

List of Antiquarian Remains in the Bombay Presidency. Demy 4to. Bombay, 1885. Second edition, revised by H. Cousens, 1897.

Buddhist Stûpas of Amarâvati and Jaggayyapeta. Roy. 4to. London, 1887.

"Archæological Research in India," Actes du Congrès Int. des Orientalistes, 1889.

Epigraphia Indica. Vols. 1 and 2. 4to. Calcutta, 1891–94.

Muhammadan Architecture of Bharoch, Cambay, Dholka, etc., in Gujarât. Roy. 4to. London, 1896.

The Ancient Monuments, Temples, Sculptures, etc., of India. Pt. i, 170 plates. Fol. London, 1897.

Muhammadan Architecture of Ahmadabad. 2 vols. Roy. 4to. 1900–5.

Buddhist Art in India, translated from A. Grünwedel's *Handbuch*. Revised and enlarged: 154 illustrations. 8vo. London, 1901.

"Indian Architectural Details," Journ. of Indian Art, vol. 3, No. 32, fifteen plates. Fol. London, 1890.

The Gandhâra Sculptures (twenty-five plates and thirty-eight text-blocks). Ibid., vol. 8, pp. 23–40 and 73–92.

"The Great Stûpa at Sânchi-Kânâkheda," Journ. R. As. Soc., 1902, pp. 29–45.

"Sketch of Archæological Research in India during half a century," Journ. Bom. B. R. As. Soc. Centenary vol., 1905, pp. 131–148.

J. Burgess and H. Cousens.—The Antiquities of Dabhoi. Fol. Edinburgh, 1888.

The Architectural Antiquities of Northern Gujarât. Roy. 4to. London, 1903.

J. Burgess, E. W. Smith, and A. Führer.—Sharqi Architecture of Jaunpur. 4to. 1889.

In addition to what has been mentioned above, Dr Burgess was also the author of:—

Observations on the Tides, with reference to the Computation of the Times of High Water at Bombay, 1864. From the "Bombay Almanac," 1864.

A Guide-book to the Elura Caves.

Notes on Hindu Astronomy, and the History of our Knowledge of it. From the "Journal of the Royal Asiatic Society," October, 1893.

Note on Finding the Logarithmic Sines and Tangents of Small Arcs, Proc. Roy. Soc. Edin., vol. xxii, 1898.

Indian Architecture—an Outline Sketch. From the Imperial Gazetteer of India, "The Indian Empire," vol. 2, pp. 155–205, 1907.

Benjamin Hall Blyth, M.A., Past-Pres. Inst.C.E. By W. A. Tait, M.Inst.C.E.

(MS. received December 4, 1917.)

Benjamin Hall Blyth, secundus, was the eldest son of a well-known civil engineer of the same name, and was born in Edinburgh on May 25, 1849. He was educated at Merchiston Castle and the University of Edinburgh, where he graduated in Arts, half a century ago, at the early age of eighteen. He was then apprenticed to the firm of B. & E. Blyth, founded by his father and uncle, and, after serving his time, was admitted in 1871 to partnership in the firm, which had then become Blyth & Cunningham.

During his apprenticeship the firm was engaged on the construction, among other large works, of the Callander and Oban Railway, through the heart of the Scottish Highlands. Mr Blyth had thus the best possible opportunity of obtaining a thoroughly practical knowledge of every variety of field work, which stood him in good stead when he came personally to have the oversight of extensive works.

In 1892, on the retirement of the late Mr George Miller Cunningham, he became senior partner of the firm of Blyth & Westland, now Blyth & Blyth.

As a member of the successive firms above referred to, Mr Blyth was responsible for the design and construction of many large and important undertakings, representing a cost for works alone of ten millions sterling.

The first large work of which he personally took charge was the Citadel Station at Carlisle, involving the reconstruction of the lines of four English and three Scottish railways, in order to separate the passenger from the goods traffic and to remove several dangerous level crossings. At the same time his firm were constructing for the Caledonian Railway Company the original Central Station in Glasgow, with its connecting lines, including a large viaduct over the Clyde. Other stations which have been built or reconstructed by his firm include the present Waverley and Princes Street Stations in Edinburgh, the General Station at Perth, the Joint Station at Paisley, and the Central Station at Leith.

For a couple of years preceding Mr Cunningham's retirement the firm were joint engineers for the Glasgow Central (Underground) Railway—a work presenting many troublesome points, to which Mr Blyth gave very close attention.

Among important bridges designed and carried out by his firm were the new North Bridge, Edinburgh, the new Broomielaw Bridge, Glasgow, and others over the rivers Ayr, Dee, Gala, Spey, Tay, and Tweed.

One of the last works on which Mr Blyth was engaged was a large new dock at Methil intended mainly for the exporting of Fife coal. This work comprised the construction of a sea wall more than a mile in length, which had to be carried out regardless of the severe storms which are so frequent on the east coast.

These engineering works, in so far as they suggest physical strength and solidity, may be said to be peculiarly apt memorials of him who was responsible for their design and construction.

Although his professional career was after the time known as the "railway mania," he saw a portion at least of the good times for engineers which, thanks to peace agreements, etc., are now at an end. Then, as now, almost every large work required Parliamentary sanction, which could only be obtained by following the course prescribed in Standing Orders—namely, detailed advertisements in the Edinburgh or London *Gazette* and in the local newspapers in the middle of November, followed by the deposit of plans and sections at the end of that month. The greatest secrecy had to be maintained, especially in railway schemes, lest some opponent should come forward with a competitive proposal. At the last available moment, and of course under great pressure, a large staff of assistants would make the necessary surveys, etc., in the field during the day, and develop them indoors at night, with the result that men, regardless of wetting and other discomforts, did not have their clothes off for a fortnight or more at a time.

He was consulting engineer to the North British and the Great North of Scotland Railway Companies.

Like his father, Mr Blyth had a high reputation as a professional witness, and his services were in great demand in Parliamentary, court, and arbitration proceedings, where he was able both to express his points with great clearness and to take good care of himself in cross-examination.

Blyth was ever ready to maintain the best traditions of his profession. He took enormous trouble on numerous occasions to assist, gratuitously and whole-heartedly, brother engineers who, in his view, had been shabbily treated by the companies, authorities, or individuals who employed them.

It is, of course, common knowledge that Parliament has not yet solved the problem of housing. Blyth used to refer humorously to a passage in one of his firm's early specifications which read as follows: "Proper accommodation shall be provided for the workmen, and there shall be not more than two navvies in one bed."

He read several papers to the Institution of Civil Engineers, and frequently took part in the discussions upon others. He was elected a member of the Institution in 1877, and after serving for some time on the Council was chosen as President in 1914, being the first engineer practising in Scotland to hold that office.

While President he had the satisfaction of persuading the Council to refrain from practising one war economy which might have had the effect of interrupting the Institution's annual grant to the National Physical Laboratory.

While holding the office of President, Blyth was asked by the War Office to preside over a Commission, to be nominated by himself from leading members of the Institution, to advise as to the best designs, material to be used, and method of construction to be adopted in connection with the hutted camps throughout the country. Most of the then existing hutted camps were inspected, and a voluminous report was prepared and handed to the War Office for future guidance.

He was also the first Chairman, and was largely intrumental in the formation, of the Metropolitan Munitions Committee, but failing health ultimately compelled him to resign that position.

It was probably a professional brother who paraphrased the first Psalm thus:—

"That man hath railway business
Who walketh all the day
In converse with rough working men,
And keeps in Blyth's way."

Arising out of an arbitration in regard to the available rainfall at the head waters of the river Tweed, he was joint author of a paper published by the Royal Society of Edinburgh. In this and another nearly similar arbitration, where Parliament had decreed that a series of rain gaugings were to be taken for a limited number of years, he readily joined in a recommendation to the authorities concerned that these gaugings should be continued as a means of adding to the available data upon this very important subject.

In regard to University matters, Mr Blyth always regretted that science degrees in engineering were only created some years after he had finished his college course.

He was a member of a committee in raising a fund for duplicating the Natural Philosophy Chair in Edinburgh University, and he was a hearty supporter of the proposal to fix age limits for all future members of the Senatus.

Blyth was fond of many sports and pastimes—archery, bowls, curling, football, golf, shooting, etc.; but probably he was most in evidence in playing at and legislating upon golf.

He was the first Chairman of the Rules of Golf Committee, set up by the Royal and Ancient Club of St Andrews in preference to a Golfing Union. He took great pains in framing clear decisions upon the various knotty points which were submitted from all parts of the globe.

When there was a proposal to abolish golf on Bruntsfield Links, Blyth, who had long ceased to play there, as he had plenty of private courses to play on, threw his whole weight and Parliamentary skill into the protection of his less fortunate fellow-citizens, with the result that golfing on Bruntsfield Links was only stopped by the Town Council after they had provided a proper substitute course on the Braid Hills.

He married, in 1872, Millicent, youngest daughter of Thomas Edward Taylor, of Dodworth Hall, Yorkshire, who predeceased him in 1914; and he is survived by an only daughter, the wife of Mr John Charles Couper, W.S.

At the date of his death, Mr Hall Blyth held the following appointments:—Chairman of the Edinburgh and District Tramways Company; Chairman of the Scottish Canadian Mortgage Company; Director of the National Bank of Scotland; Director of the Edinburgh Life Insurance Company; Director of Merchiston Castle Schools; Director of the Royal Hospital for Sick Children.

Mr Hall Blyth had an impressive personality. Exceptionally tall and massive, he had a distinctive appearance on the platform. His campaigns in East Lothian as Unionist candidate in opposition to Lord Haldane and Mr J. D. Hope were carried through with characteristic vigour and energy. For some years he was Chairman of the Unionist Association for Haddingtonshire. In recognition of his services he was presented with his portrait, the presentation being made at Haddington by the Rt. Hon. A. J. Balfour.

APPENDIX.

CONTENTS.

PROCEEDINGS OF THE STATUTORY GENERAL MEETING

Beginning the 134th Session, 1916-1917.

At the Annual Statutory Meeting of the Royal Society of Edinburgh, held in the Society's Lecture Room, 24 George Street, on Monday, October 23, 1916, at 4.30 p.m.

Dr John Horne, F.R.S., President, in the Chair.

The Minutes of the last Statutory Meeting, October 25, 1915, were read, approved, and signed.

Dr Clark Trotter signed the Roll, and was duly admitted a Fellow of the Society.

The President nominated as Scrutineers of the Voting Paper, Lord Salvesen and Professor MacKinnon.

The ballot for the election of Office-Bearers and Members of Council was then taken.

The Treasurer submitted his Report for the preceding Session, drawing special attention to the depreciation in value of the Society's investments.

On the motion of Sir E. A. Schäfer the Treasurer's Report was adopted.

The Secretary moved that Messrs Lindsay, Jamieson & Haldane, C.A., be reappointed auditors for the ensuing session. This was agreed to.

The Scrutineers reported that the Balloting Papers had all been in order, and the following Council had been duly elected:—

John Horne, LL.D., F.R.S., F.G.S., President.

Benjamin N. Peach, LL.D., F.R.S., F.G.S., Professor Sir E. A. Schäfer, M.R.C.S., LL.D., F.R.S., The Right Hon. Sir J. H. A. Macdonald, P.C., G.C.B., K.C., LL.D., D.L., F.R.S., M.I.E.E., Professor R. A. Sampson, M.A., D.Sc., F.R.S., Professor D'Arcy Thompson, C.B., B.A., F.R.S., Professor James Walker, D.Sc., Ph.D., LL.D., F.R.S.,	Vice-Presidents.

Cargill G. Knott, D.Sc., LL.D., General Secretary.

Professor Arthur Robinson, M.D., M.R.C.S., Professor E. T. Whittaker, Sc.D., F.R.S.,	Secretaries to Ordinary Meetings.

James Currie, M.A., Treasurer.

A. Crichton Mitchell, D.Sc., Hon. D.Sc. (Geneva), Curator of Library and Museum.

ORDINARY MEMBERS OF COUNCIL.

W. B. Blaikie, LL.D.
Principal O. C. Bradley, M.D., D.Sc.
R. Stewart MacDougall, M.A., D.Sc.
W. A. Tait, D.Sc., M.Inst.C.E.
J. H. Ashworth, D.Sc.
Professor C. G. Barkla, D.Sc., F.R.S.
Professor C. R. Marshall, M.A., M.D.
John S. Black, M.A., LL.D.
Sir George A. Berry, M.D., C.M., F.R.C.S.
John S. Flett, M.A., D.Sc., LL.D., F.R.S.
Professor Magnus Maclean, M.A., D.Sc., M.Inst.C.E.
Professor David Waterston, M.A., M.D., F.R.C.S.E.

Society's Representative on George Heriot's Trust, William Allan Carter, M.Inst.C.E.

The President, in the name of the Society, thanked the Scrutineers for their Report.

The Secretary announced that Messrs. Stewart and Beaton, the Librarians, were still on active service, and the work of the Library was being efficiently carried on by Miss Le Harivel.

PROCEEDINGS OF THE ORDINARY MEETINGS,

Session 1916-1917.

FIRST ORDINARY MEETING.

Monday, November 6, 1916.

John Horne, Esq., LL.D., F.R.S., F.G.S., President, in the Chair.

The President opened the Session with a short Address on the Relation of Science to Industries and Education.

The following Communication was read:—

Experiments and Observations arising from a Consideration of *Ligia oceanica* (the common Slater). Part I. Immersion Experiments. Part II. Moulting of Isopods. By John Tait, M.D., D.Sc. Communicated by Sir E. A. Schäfer.

SECOND ORDINARY MEETING.

Monday, November 20, 1916.

John Horne, Esq., LL.D., F.R.S., F.G.S., President, in the Chair.

At the Meeting of December 18 the following alterations in the Rules will be moved and seconded:—

"In order to abolish the distinction between Resident and Non-Resident Fellows, the words 'Resident in Scotland' in Rule VI shall be deleted, and Rule VII shall be wholly deleted. As a consequence the present Rules Nos. VIII–XXIX will become Nos. VII–XXVIII."

Mr J. H. R. Kemnal signed the Roll and was duly admitted a Fellow of the Society.

The following Communications were read:—

1. On the Adelphic Integral of the Differential Equations of Dynamics. By Professor E. T. Whittaker, F.R.S.

2. A Special Table of Logarithms. By Frank Robbins. Communicated by the General Secretary.

3. Sketch of a projected new Branch of Biology: illustrated by Observations chiefly on Crustacea. (*a*) Limb Flexures and Limb Taxis in the Peracarida. (*b*) The Eyes of Glyptonotus. (*c*) Respiratory Arrangements in the Peracarida. (*d*) The Organ of Smell in Amphipods and Isopods. By John Tait, M.D., D.Sc. Communicated by Sir E. A. Schäfer.

THIRD ORDINARY MEETING.

Monday, December 4, 1916.

John Horne, Esq., LL.D., F.R.S., F.G.S., President, in the Chair.

At the Meeting of December 18 the following alterations in the Rules will be moved and seconded:—

"In order to abolish the distinction between Resident and Non-Resident Fellows, the words 'Resident in Scotland' in Rule VI shall be deleted, and Rule VII shall be wholly deleted. As a consequence the present Rules Nos. VIII–XXIX will become Nos. VII–XXVIII."

The following Communications were read:—

1. The Gametophyte of Psilotum. By G. P. Darnell-Smith, B.Sc., F.I.C. Communicated by Professor F. O. Bower, F.R.S.

2. Transverse and Codirectional Induction Changes in Demagnetised Iron and Nickel in relation to the Molecular Theory of Magnetism. Part II. By James Russell. (*With Lantern Illustrations.*)

3. The Magnetic Test of Molecular Arrangement in Crystals: Magnetite and the α, β, γ forms of Iron. By Professor W. Peddie, D.Sc.

FOURTH ORDINARY MEETING.

Monday, December 18, 1916.

John Horne, Esq., LL.D., F.R.S., F.G.S., President, in the Chair.

The following alterations in the Rules were, on the motion of Dr SUTHERLAND BLACK, seconded by Professor JAMES WALKER, F.R.S., agreed to by the Society:—

"In order to abolish the distinction between Resident and Non-Resident Fellows, the words 'Resident in Scotland' in Rule VI shall be deleted, and Rule VII shall be wholly deleted. As a consequence the present Rules No. VIII–XXIX will become Nos. VII–XXVIII."

The following Communications were read:—

1. The Family Budgets and Dietaries of Forty Labouring Class Families in War Time. By Miss MARGARET FERGUSON. Communicated by Professor NOËL PATON.
2. The Hurlet Sequence in the East of Scotland. By PETER MACNAIR, F.G.S. (*With Lantern Illustrations.*)

FIFTH ORDINARY MEETING.

Monday, January 22, 1917.

John Horne, Esq., LL.D., F.R.S., F.G.S., President, in the Chair.

The following Communications were read:—

1. Obituary Notice of Professor GWYNNE-VAUGHAN. By Professor F. O. BOWER, F.R.S.
2. Obituary Notices of Deceased Fellows during the Session 1915–16. By THE GENERAL SECRETARY.
3. On some Causes of the Formation of Anticyclonic Stratus, as observed from Aeroplanes. By Lieut. C. K. M. DOUGLAS. Communicated by M. M'CALLUM FAIRGRIEVE, M.A.
4. The Structure, Bionomics, and Forest Importance of *Myelophilus minor*. By WALTER RITCHIE, B.Sc. Communicated by Dr R. STEWART MACDOUGALL. (*With Lantern Illustrations.*)

SIXTH ORDINARY MEETING.

Monday, February 5, 1917.

John Horne, Esq., LL.D., F.R.S., F.G.S., President, in the Chair.

The following Communications were read:—

1. The Gametophyte Generation of the Psilotaceæ. By Professor A. A. LAWSON. (*With Lantern Illustrations.*)
2. The Anatomy and Affinity of *Stromatopteris moniliformis*, Mett. By J. M. THOMPSON, M.A., B.Sc. (*With Lantern Illustrations.*)
3. Preliminary Note on the Peculiarities of the Tides round Western Australia. By Professor and Mrs A. D. ROSS.

SEVENTH ORDINARY MEETING.

Monday, February 19, 1917.

Professor E. A. Schäfer, LL.D., F.R.S., Vice-President, in the Chair.

The following Communications were read:—

1. The Bone Cave in the Valley of Allt nan Uamh (Burn of the Caves), near Inchnadamff, Assynt, Sutherlandshire. By Dr PEACH, F.R.S., and Dr HORNE, F.R.S. With Notes on the Bones found in the Cave, by E. T. NEWTON, F.R.S. (*With Lantern Illustrations.*)
2. The Adsorption of Sulphur Dioxide by Charcoal at −10° C. By A. M. WILLIAMS, M.A., B.Sc. Communicated by Professor JAMES WALKER, F.R.S.

EIGHTH ORDINARY MEETING.

Monday, March 5, 1917.

Dr John Horne, Esq., LL.D., F.R.S., F.G.S., President, in the Chair.

The Annual Election of Fellows took place. The following were elected:—George Barnhill Burnside, Brysson Cunningham, T. Cuthbert Day, Robert W. Dron, Alexander Gibson, John Harrison, James Colquhoun Irvine, Andrew King, Sir Donald Macalister, Hector Copland Macpherson, Louis William Gunther Malcolm, A. Ernest Maylard George Fowlie Merson, Frederick Phillips, Henry Harold Scott, Sir George Adam Smith, John Tait, William White Taylor, John M'Lean Thompson, Wallace Thorneycroft, Donald Francis Tovey.

The following Communication was read:—

Darwinism and Human Civilisation, with special reference to German Military "Kultur." By Dr Robert Munro.

NINTH ORDINARY MEETING.

Monday, March 19, 1917.

Dr John Horne, Esq., LL.D., F.R.S., F.G.S., President, in the Chair.

The following Communications were read:—

1. On some Nuclei of Cloudy Condensation. Part III. By Dr John Aitken, F.R.S. (*With Lantern Illustrations.*)

2. Note on the Salmon of the River Lochy as shown by a Collection of Scales taken in 1916. By W. L. Calderwood, Esq.

Mr John Harrison, J.P., Dr John Tait, Professor Donald F. Tovey, and Professor C. G. Barkla signed the Roll, and were duly admitted Fellows of the Society.

TENTH ORDINARY MEETING.

Monday, May 7, 1917.

Dr John Horne, Esq., LL.D., F.R.S., F.G.S., President, in the Chair.

The following Communications were read:—

1. The Arithmetical Mean and the "Middle" Value of certain Meteorological Observations. By Professor L. Becker, Ph.D.

2. Phycomycetous Fungi from the Lower Coal Measures. By Dr David Ellis.

3. Experiments and Observations on Crustacea. Parts IV and V. By Dr J. Tait.

Sir Donald Macalister, G. F. Merson, and Dr W. W. Taylor signed the Roll, and were duly admitted Fellows of the Society.

Dr John Aitken's Meteorological Papers.

Owing to the steady demand for Dr Aitken's well-known Papers on Dust, Fogs, and Clouds, and on Dew, published in the Society's *Transactions*, Vols. XXX and XXXIII (1880 and 1887), the Parts containing these Papers have now been almost exhausted. Under these circumstances the Council felt it their duty to reprint the two Papers and issue them as one pamphlet. The Reprint may be obtained through the Society's Publishers, Robert Grant & Son, 107 Princes Street, price seven shillings and sixpence.

ELEVENTH ORDINARY MEETING.

Monday, May 21, 1917.

Dr John Horne, Esq., LL.D., F.R.S., F.G.S., President, in the Chair.

The following Communications were read:—

1. Observations on the Blood in Gas Poisoning. By James Miller, M.D., Capt. R.A.M.C.(T.), and Harry Rainy, M.D., F.R.C.P.E.

2. The Chermes of Spruce and Larch and their relation to Forestry. By H. M. Steven, B.Sc. Communicated by Dr R. Stewart MacDougall.

3. The Square Roots of a Linear Vector Function. By Frank L. Hitchcock. Communicated by the General Secretary.

TWELFTH ORDINARY MEETING.

Monday, June 4, 1917.

Dr John Horne, Esq., LL.D., F.R.S., F.G.S., President, in the Chair.

The Makdougall-Brisbane Prize award for the Biennial Period 1914–1916.

The Council of the Royal Society of Edinburgh having awarded the Makdougall-Brisbane Prize to Robert Alexander Houstoun, Ph.D., D.Sc., for his series of papers on "The Absorption of Light by Inorganic Salts," published in the *Proceedings* of the Society; and the Gunning Victoria Prize award for the Quadrennial Period 1912–1916, to Sir Thomas Muir, C.M.G., LL.D., F.R.S., for his series of memoirs upon "The Theory and History of Determinants and Allied Forms," published in the *Transactions* and *Proceedings* of the Society between the years 1872 and 1915; these Prizes will be presented at the July Meeting.

The following Communications were read:—

1. The Highland Border Rocks in the Aberfoyle District. By Professor T. J. Jehu and Dr Robert Campbell. (*With Lantern Illustrations.*)
2. On Knots, with a Census of the Amphicheirals with twelve Crossings. By Miss Mary G. Haseman. Communicated by the General Secretary.

THIRTEENTH ORDINARY MEETING.

Monday, June 18, 1917.

Dr John Horne, Esq., LL.D., F.R.S., F.G.S., President, in the Chair.

The Makdougall-Brisbane Prize award for the Biennial Period 1914–1916.

The Council of the Royal Society of Edinburgh having awarded the Makdougall-Brisbane Prize to Robert Alexander Houstoun, Ph.D., D.Sc., for his series of papers on "The Absorption of Light by Inorganic Salts," published in the *Proceedings* of the Society; and the Gunning Victoria Prize award for the Quadrennial Period 1912–1916, to Sir Thomas Muir, C.M.G., LL.D., F.R.S., for his series of memoirs upon "The Theory and History of Determinants and Allied Forms," published in the *Transactions* and *Proceedings* of the Society between the years 1872 and 1915; these Prizes will be presented at the July Meeting.

The following Communications were read:—

1. On the Improvement of the Gregorian Calendar. The Discussion was opened by Alexander Philip, LL.B. (*With Lantern Illustrations.*)

Mr Philip stated that his object was to simplify the calendar so as to maintain the date of the vernal equinox at or about the 21st of March, since any alteration involving a change in that date involved also a change in the tables for the calculation of Easter. He believed that many of the advantages of a real simplification could be obtained by taking one day from August and adding it to February of the *following* year. This proposal was superior to any other which had been formulated, and might easily receive the cordial support of the most conservative defenders of the Gregorian calendar. Moreover, the change could be effected without any trouble, disturbance, or expense. If such a change were carried out, further advantages could be obtained by making the almanac year run from the 1st of March. In this case only one Dominical letter would be required for each year, whether leap year or common year. By this change also the year would be divided into four three-monthly quarters of 91 days, and on the basis of such a calendar Mr Philip showed that a Perpetual Calendar could be subsequently established with a minimum of disturbance.

Dr W. B. Blaikie and Professor Sampson took part in the discussion and expressed the opinion that if any change was to be made they preferred Mr Philip's earlier suggestion, as described to the Society some years ago (see his *Proposal for a Simplified Calendar* (1907), and *The Reform of the Calendar* (1914)).

2. Note upon an Observation on Insects and Light. By Professor J. Graham Kerr, F.R.S.

FIRST SPECIAL MEETING.

Monday, 9th July 1917.

Dr John Horne, Esq., LL.D., F.R.S., F.G.S., President, in the Chair.

The Makdougall-Brisbane Prize for the Biennial Period 1914–1916 was presented to Robert Alexander Houstoun, Ph.D., D.Sc., for his series of papers on "The Absorption of Light by Inorganic Salts," published in the *Proceedings* of the Society.

The Gunning Victoria Prize for the Quadrennial Period 1912–1916 was presented to Sir Thomas Muir, C.M.G., LL.D., F.R.S., for his series of memoirs upon "The Theory and History of Determinants and Allied Forms," published in the *Transactions* and *Proceedings* of the Society between the years 1872 and 1915.

The following Communications were read :—

1. The Origin, Rupture, and Closure of Ovarian Follicles. By Professor A. Robinson, M.D.
2. Development of the Heart in Man. By Professor D. Waterston, M.D.
3. Compound Determinants. By Professor E. T. Whittaker, F.R.S.
4. Vanishing Aggregates. By Professor W. H. Metzler.
5. A Further Contribution to our Knowledge of *Platyzoma microphyllum*, R.Br. By Dr J. M'L. Thompson.

PROCEEDINGS OF THE STATUTORY GENERAL MEETING

Ending the 134th Session, 1916–1917.

At the Annual Statutory Meeting of the Royal Society of Edinburgh, held in the Society's Lecture Room, 24 George Street, on Monday, October 22, 1917, at 4.30 p.m.

Dr Ben. N. Peach, F.R.S., Vice-President, in the Chair,

the Minutes of the last Statutory Meeting of October 23, 1916, were read, approved, and signed.

The Chairman nominated as Scrutineers of the Voting Papers, Mr W. Hume Kerr and Mr C. H. Milne.

The Ballot for the Election of Office-Bearers and Members of Council was then taken.

The Secretary submitted the following Report:—

By the death of our Assistant Librarian, Mr Wm. J. Beaton, who was killed at the front on September 24, 1917, the Society has lost a devoted and efficient servant. Almost since the outbreak of the war both the Librarians—Mr George Stewart and the Assistant Librarian—have been in the service of their country. They have continued to hold office in this Society during these years.

Mr Beaton joined the 15th Royal Scots, and after being wounded in the "Big Push" in July 1916, he returned to this country to convalesce. He subsequently obtained a Commission in the Machine Gun Company, and it was in that capacity, when inspecting the position of the guns under his care, that he lost his way during a fog, and found himself when the fog lifted close to the German lines; there in full view of his own men he met his death at the hands of the snipers. He was alive when rescued from "No Man's Land," but expired almost immediately thereafter. Lieut. Beaton was an only son, and the Society desire to express their deep sympathy with his parents and sisters.

Mr George Stewart is a sergeant in the 4th Royal Scots, and has served continuously through the Dardanelles, Egyptian, and Palestine campaigns. He is expected home on his first leave before the end of the year. He received special mention in dispatches some months ago.

The activities of the Society have continued with but slight abatement during the past year in spite of war conditions. The number of papers read at our meetings during 1916–17 amounted to 34, of which 20 have been, or are being, printed in the *Proceedings*, and 7 in the *Transactions*. 4 of the papers which are not yet in the printers' hands will certainly be published in due course. Of the papers read 6 were in mathematics, 3 in physics, 4 in meteorology and tides, 6 in botany, 8 in zoology, 3 in geology, 2 in anatomy, and 2 in physiology. If we take 31 as the number of papers which have been or will be published in the *Proceedings* and *Transactions* of the last session, and compare this with the 41 papers of 1913–14, and the 41 papers of 1914–15, we see that the diminution in publication on account of war conditions amounts to about one-fourth. It is certain, however, that there will be a greater fall-off in publication during the coming session, partly on account of the energies of scientific workers being devoted to war purposes, and partly on account of the necessity of keeping down our expenses.

The disastrous fire which destroyed Messrs Neill & Co.'s printing works in May of 1916 disorganised the publication of both our *Proceedings* and *Transactions*. One whole Part of the *Transactions* had to be reset and reprinted, and a considerable portion of another Part. This also caused great delay, so that what ought to have been published in 1915–16 did not appear until this year. Only one Part of the *Transactions* was issued in 1916; and during this last session we have made up arrears by issuing five Parts instead of the customary three.

Our financial loss on account of the fire was considerable, and this, added to the increase in prices, has seriously affected our finances, regarding which the Treasurer will give a separate Report.

The new method for the election of Fellows has now been in existence for two years, and has proved in every respect a successful change of procedure. Last March the Society elected 21 new Fellows, and we have lost during the session by death and resignation 9 Ordinary Fellows and 1 Honorary Fellow. There are at this moment 627 Fellows, of whom 261 are annual subscribers.

Two prizes were awarded during the year—the Makdougall-Brisbane Prize to Dr R. A. Houstoun of Glasgow University, and the Gunning Victoria Prize to Sir Thos. Muir of South Africa.

Great progress has been made in the cataloguing of the Library, which has been carried out under the direct supervision of Dr Sutherland Black, who voluntarily retired from the Curatorship last year. Through the united efforts of our Assistant Librarian—Miss Le Harivel—and Miss Dorothy Charlton (specially engaged by Dr Black for the purpose of cataloguing) we not only know what journals and books we have in our possession, but are able, at a moment's notice, to lay our hands upon them. It is appropriate at this time that special thanks be given to Dr Black and these two ladies for their devoted work in this connection.

The Treasurer submitted his Report for the preceding Session, drawing special attention to the re-investment of the Society's funds in the War Loan, and to the causes of increased expenditure during the preceding Session.

On the motion of the Hon. Lord Guthrie, the Treasurer's Report was adopted, and Messrs Lindsay, Jamieson & Haldane, C.A., were reappointed auditors for the ensuing Session.

The Scrutineers reported that the Balloting Papers had all been in order, and that the following Council had been duly elected:—

John Horne, LL.D., F.R.S., F.G.S., President.

Vice-Presidents:
The Right Honourable Sir J. H. A. Macdonald, P.C., G.C.B., K.C., LL.D., D.L., F.R.S., M.I.E.E.,
Professor R. A. Sampson, M.A., D.Sc., F.R.S.,
Professor D'Arcy Thompson, C.B., B.A., F.R.S.,
Professor James Walker, D.Sc., Ph.D., LL.D., F.R.S.,
Professor George A. Gibson, M.A., LL.D.,
Robert Kidston, LL.D., F.R.S., F.G.S.,

Cargill G. Knott, D.Sc., LL.D., General Secretary.

Secretaries to Ordinary Meetings:
Professor Arthur Robinson, M.D., M.R.C.S.,
Professor E. T. Whittaker, Sc.D., F.R.S.,

James Currie, M.A., Treasurer.

A. Crichton Mitchell, D.Sc., Hon. D.Sc. (Geneva), Curator of Library and Museum.

ORDINARY MEMBERS OF COUNCIL.

J. H. Ashworth, D.Sc., F.R.S.
Professor C. G. Barkla, D.Sc., F.R.S.
Professor C. R. Marshall, M.A., M.D.
John S. Black, M.A., LL.D.
Sir George A. Berry, LL.D., M.B., F.R.C.S.E.
John S. Flett, M.A., D.Sc., LL.D., F.R.S.
Professor Magnus MacLean, M.A., D.Sc., M.Inst.C.E.
Professor David Waterston, M.A., M.D., F.R.C.S.E.
Professor F. O. Bower, M.A., D.Sc., F.R.S., F.L.S.
Professor P. T. Herring, M.D., F.R.C.P.E.
Professor T. J. Jehu, M.A., M.D., F.G.S.
Alexander Lauder, D.Sc., F.I.C.

Society's Representative on George Heriot's Trust — William Allan Carter, M.Inst. C.E.

The Chairman, in the name of the Society, thanked the Scrutineers for their Report

ABSTRACT

OF

THE ACCOUNTS OF JAMES CURRIE, ESQ.

As Treasurer of the Royal Society of Edinburgh.

SESSION 1916–1917.

I. ACCOUNT OF THE GENERAL FUND.

CHARGE.

1. Arrears of Contributions at 30th September 1916		£143 17 0
2. Contributions for present Session:—		
1. 156 Fellows at £2, 2s. each	£327 12 0	
109 Fellows at £3, 3s. each	343 7 0	
	£670 19 0	
Less—Contribution for present Session, included in 1915–1916 Accounts	3 3 0	
	£667 16 0	
2. Fees of Admission and Contributions of twenty-one new Resident Fellows at £4, 4s. each	88 4 0	
3. Commutation Fee in lieu of future contributions of one Fellow	5 5 0	
		761 5 0
3. Interest received—		
Interest on £7830 five per cent. War Stock, 1929–47, to 1st June 1917	£110 0 8	
Less—Interest paid to Union Bank of Scotland, Ltd., on special overdraft	40 10 6	
	£69 10 2	
Other Interest, less Tax, £80, 11s.	241 13 5	
Annuity from Edinburgh and District Water Trust, less Tax, £13, 2s. 6d.	39 7 6	
		350 11 1
4. Transactions and Proceedings		98 10 0
5. Annual Grant from Government		600 0 0
6. Income Tax repaid for year to 5th April 1917		128 2 10
7. Napier Tercentenary Memorial Volume:—		
Donations and Receipts from Sale of Volume		36 7 2
Amount of the Charge		**£2118 13 1**

DISCHARGE.

1. Taxes, Insurance, Coal and Lighting:—		
Inhabited House Duty	£0 6 3	
Insurance	22 16 11	
Coal, etc., to 10th May 1917	30 14 0	
Gas to 9th May 1917	2 8 4	
Electric Light to 19th September 1917	5 12 7	
Water, 1916–17	4 4 0	
		£66 2 1
2. Salaries:—		
General Secretary, 1916–17	£100 0 0	
Librarian	120 0 0	
Assistant Librarian	50 0 0	
Interim Assistant Librarians	88 10 0	
Office Keeper	94 10 0	
Treasurer's Clerk	25 0 0	
		478 0 0
Carry forward		£544 2 1

Brought forward . . .			£544 2 1
3. Expenses of Transactions :—			
Neill & Co., Ltd., Printers, Payments to account . . .		£450 0 0	
Do. Do. Balance due at 30th September 1917		430 13 4	
Hislop & Day, Ltd., Engravers		19 10 9	
Orrock & Son, Bookbinders		179 3 0	
André Sleigh & Anglo, Ltd., Printers		10 10 0	
Bemrose & Sons, Ltd., Printers		51 18 0	
Percy Highley		10 10 0	
A. Ritchie & Son, Lithographers		52 6 6	
C. Hodges & Son, Lithographers		13 13 9	
M'Farlane & Erskine, Lithographers		95 8 0	
		£1313 13 4	
Less—Donations, etc., received from—			
Mrs Eliz. Gray	£25 0 0		
Dr John Aitken	10 0 0		
Carnegie Trustees	243 13 3		
Do. towards Dr Kidston's Paper	32 13 0		
Do. do. Dr Collinge's Paper	36 0 0		
Do. do. Mr Smellie's Paper	3 13 9		
		351 0 0	
			962 13 4
4. Expenses of Proceedings :—			
Neill & Co., Ltd., Printers, Payments to account . . .		£650 0 0	
Do. Do. Balance due at 30th September 1917		75 14 6	
Hislop & Day, Ltd., Engravers		17 3 10	
Orrock & Son, Bookbinders		0 14 6	
			743 12 10
5. Books, Periodicals, Newspapers, etc. :—			
W. Green & Son, Ltd., Booksellers		£0 14 0	
Society of Chemical Industry		0 3 0	
A. F. Bird, Publisher		2 5 6	
Williams & Norgate, Publishers, Subscription . . .		1 4 0	
Robertson & Scott, News Agents		4 2 8	
Wilson Ross & Co., Ltd., Publishers		4 11 4	
Egypt Exploration Fund, Subscription		4 4 0	
Ray Society Do.		1 1 0	
Board of Scientific Societies, Donation		20 0 0	
James Thin, Bookseller		74 10 5	
R. Grant & Son, Booksellers		7 1 2	
			119 17 1
6. Other Payments :—			
Neill & Co., Ltd., Printers, Balance due at 30th September 1917		£121 10 5	
E. Sawers, Purveyor		30 15 0	
S. Duncan, Tailor (uniforms)		5 13 0	
Orrock & Son, Bookbinders		63 3 6	
Andrew H. Baird		2 5 0	
Lindsay, Jamieson & Haldane, C.A., Auditors . . .		6 6 0	
Post Office Telephone Rent		12 0 0	
A. Cowan & Sons, Ltd.		15 12 6	
Special Honorarium to General Secretary		50 0 0	
Miss Le Harivel		10 0 0	
James Gray & Son		14 13 6	
Gillies & Wright		16 19 11	
R. Graham, Slater		9 17 4	
Dundas & Wilson, C.S.		7 2 10	
Oliver Typewriter Coy., Ltd		6 1 3	
Burn Brothers		10 10 0	
Petty Expenses, Postages, Carriage, etc.		98 16 9	
			481 7 0
7. Investments Made		£7435 15 2	
Less—Realised		7415 16 5	
			19 18 9
8. Arrears of Contributions outstanding at 30th September 1917 :—			
Present Session		£66 3 0	
Previous Sessions		74 11 0	
			140 14 0
Amount of the Discharge . .			**£3012 5 1**

Amount of the Charge			**£2118 13 1**
Amount of the Discharge			**3012 5 1**
Excess of Payments over Receipts for 1916–1917 . .			**£893 12 0**
Floating Balance in favour of Society at 30th September 1916			172 19 5
Floating Balance due by the Society at 30th September 1917 .			**£720 12 7**
Being—			
Balance due to Neill & Co., Ltd., at 30th September 1917 . .		£627 18 3	
Due to General Secretary		100 0 0	
Due to Union Bank of Scotland, Ltd., on Account Current . .		1 16 10	
		£729 15 1	
Less—Due by Treasurer		9 2 6	
			720 12 7

II. ACCOUNT OF THE KEITH FUND

To 30th September 1917.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Account Current at 30th September 1916			£28 15 10
2. Interest Received :—			
On £896, 19s. 1d. North British Railway Company 3 per cent. Debenture Stock from 15th May 1916, to 23rd March 1917, £23, 3s. 9d., less Tax, £5, 15s. 11d.		£17 7 10	
On £211, 4s. North British Railway Company 3 per cent. Consolidated Lien Stock from 30th June 1916, to 26th January 1917, £3, 12s. 4d., less Tax, 18s. 1d.		2 14 3	
On £650 five per cent. War Loan, 1929–47, to 1st June 1917	£9 1 11		
Less—Interest paid to Union Bank of Scotland, Ltd., on special overdraft . .	2 15 11		
		6 6 0	
			26 8 1
3. Investments Realised			626 5 10
4. Income Tax repaid for year to 5th April 1917			9 12 9
			£691 2 6

DISCHARGE.

1. Investments Made :—	
Cost of £650 five per cent. War Loan, 1929–47	£617 4 8
2. Balance due by Union Bank of Scotland, Ltd., on Account Current at 30th September 1917	73 17 10
	£691 2 6

III. ACCOUNT OF THE NEILL FUND

To 30th September 1917.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Account Current at 30th September 1916		£18 1 8
2. Interest Received :—		
On £355 London, Chatham and Dover Railway $4\frac{1}{2}$ per cent. Arbitration Debenture Stock from 30th June 1916, to 26th January 1917, £9, 2s. 6d., less Tax, £2, 5s. 7d. . . .	£6 16 11	
On £15 four and a half per cent. War Loan, 1925–45, to 1st December 1916	0 6 9	
On £300 five per cent. War Loan, 1929–47, to 1st June 1917 .	4 8 7	
		11 12 3
3. Investment Realised		267 10 11
4. Income Tax repaid for year to 5th April 1917		3 19 2
		£301 4 0

DISCHARGE.

1. Investments Made :—			
Cost of £284, 4s. 3d. five per cent. War Loan, 1929–47	£269	19	5
2. Balance due by Union Bank of Scotland, Ltd., on Account Current at 30th September 1917	31	4	7
	£301	4	0

IV. ACCOUNT OF THE MAKDOUGALL-BRISBANE FUND

To 30th September 1917.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Account Current at 30th September 1916				£45	8	11
2. Interest Received :—						
On £365 Caledonian Railway Company 4 per cent. Consolidated Preference Stock No. 2 from 30th June 1916, to 26th January 1917, £8, 6s. 9½d., less Tax, £2, 1s. 8½d. . . .	£6	5	1			
On £150 four and a half per cent. War Loan, 1925–45, to 1st December 1916	3	7	6			
On £400 five per cent. War Loan, 1929–47, to 1st June 1917 .	7	3	2			
				16	15	9
3. Investment Realised				231	11	5
4. Income Tax repaid for year to 5th April 1917				4	7	4
				£298	3	5

DISCHARGE.

1. Dr Robert Alex. Houstoun—Money portion of Prize 1914–16	£14	0	0
2. Investments Made :—			
Cost of £242, 2s. 1d. five per cent. War Loan, 1929–47	229	19	9
3. Balance due by Union Bank of Scotland, Ltd., on Account Current at 30th September 1917	54	3	8
	£298	3	5

V. ACCOUNT OF THE MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND

To 30th September 1917.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Account Current at 30th September 1916				£14	5	4
2. Interest Received :—						
On £220 four and a half per cent. War Loan, 1925–45, to 1st December 1916	£4	19	0			
On £250 five per cent. War Loan, 1929–47, to 1st June 1917 .	5	11	4			
				10	10	4
3. Income Tax repaid to 5th April 1917				1	4	9
				£26	0	5

DISCHARGE.

1. W. C. M. Lewis.—In aid of publication of the Annual Tables of Constants, etc. .	£5	0	0
2. Investment Made :—			
Cost of £18, 8s. 5d. five per cent. War Loan, 1929–47	17	7	6
3. Balance due by Union Bank of Scotland, Ltd., on Account Current at 30th September 1917	3	12	11
	£26	0	5

VI. ACCOUNT OF THE GUNNING VICTORIA JUBILEE PRIZE FUND

To 30th September 1917.

(Instituted by Dr R. H. Gunning of Edinburgh and Rio de Janeiro.)

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Account Current at 30th September 1916		£115 7 2
2. Interest Received :—		
On £1000 North British Railway Company 3 per cent. Consolidated Lien Stock from 30th June 1916, to 26th January 1917, £17, 2s. 9d., less Tax, £4, 5s. 8d.	£12 17 1	
On £15 four and a half per cent. War Loan, 1925–45, to 1st December 1916	0 6 9	
On £570 five per cent. War Loan, 1929–47, to 1st June 1917	8 6 2	
		21 10 0
3. Investment Realised		528 2 7
4. Income Tax repaid to 5th April 1917		7 6 6
		£672 6 3

DISCHARGE.

1. Sir Thomas Muir—Money Portion of Prize, 1912-16	£105 0 0
2. Investments Made :—	
Cost of £554, 4s. 3d. five per cent. War Loan, 1929–47	526 9 5
3. Balance due by Union Bank of Scotland, Ltd., on Account Current at 30th September 1917	40 16 10
	£672 6 3

STATE OF THE FUNDS BELONGING TO THE ROYAL SOCIETY OF EDINBURGH

As at 30th *September* 1917.

1. GENERAL FUND—

1. £7830 five per cent. War Loan, 1929–47, at 94⅜ per cent.	£7389 11 3
2. £52, 10s. Annuity of the Edinburgh and District Water Trust, equivalent to £875 at 113½ per cent.	993 2 6
3. Arrears of Contributions, as per preceding Abstract of Accounts	140 14 0
	£8523 7 9
Deduct Floating Balance due by the Society, as per preceding Abstract of Accounts	720 12 7
Amount	£7802 15 2

Exclusive of Library, Museum, Pictures, Furniture, etc., at the Society's Rooms, George Street, Edinburgh.

2. KEITH FUND—

1. £650 five per cent. War Loan, 1929–47, at 94⅜ per cent.	£613 8 9
2. Balance due by Union Bank of Scotland, Ltd., on Account Current	73 17 10
Amount	£687 6 7

3. NEILL FUND—

1. £300 five per cent War Loan, 1929–47, at 94⅜ per cent.	£283 2 6
2. Balance due by Union Bank of Scotland, Ltd., on Account Current	31 4 7
Amount	£314 7 1

4. **MAKDOUGALL-BRISBANE FUND—**

	£	s.	d.
1. £400 five per cent. War Loan, 1929–47, at 94⅝ per cent.	£377	10	0
2. Balance due by Union Bank of Scotland, Ltd., on Account Current	54	3	8
AMOUNT	£431	13	8

5. **MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND—**

	£	s.	d.
1. £250 five per cent. War Loan, 1929–47, at 94⅝ per cent.	£235	18	9
2. Balance due by Union Bank of Scotland, Ltd., on Account Current	3	12	11
AMOUNT	£239	11	8

6. **GUNNING VICTORIA JUBILEE PRIZE FUND**—Instituted by Dr Gunning of Edinburgh and Rio de Janeiro—

	£	s.	d.
1. £570 five per cent. War Loan, 1929–47, at 94⅝ per cent.	£537	18	9
2. Balance due by Union Bank of Scotland, Ltd., on Account Current	40	16	10
AMOUNT	£578	15	7

EDINBURGH, 16*th October* 1917.—We have examined the six preceding Accounts of the Treasurer of the Royal Society of Edinburgh for the Session 1916–1917, and have found them to be correct. The securities of the various Investments at 30th September 1917, as noted in the above Statement of Funds, have been exhibited to us.

LINDSAY, JAMIESON & HALDANE, C.A.
Auditors.

THE COUNCIL OF THE SOCIETY.

October 1917.

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CHANGES IN FELLOWSHIP DURING SESSION 1916–17.

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T. CUTHBERT DAY.
ROBERT W. DRON, A.M.Inst.C.E.
ALEX. GIBSON, M.B., Ch.B., F.R.C.S. Eng.
JOHN HARRISON, J.P.
JAMES COLQUHOUN IRVINE, Ph.D., D.Sc.
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LOUIS WILLIAM GUNTHER MALCOLM.
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JOHN M'LEAN THOMPSON, M.A., D.Sc.
WALLACE THORNEYCROFT.
DONALD FRANCIS TOVEY, B.A.

ORDINARY FELLOWS DECEASED.

WALTER E. ARCHER.
B. HALL BLYTH, M.A., V.P.Inst.C.E.
JOHN FERGUSON, M.A., LL.D.
Rev. H. G. BONAVIA HUNT.
CHARLES FREDERICK POLLOCK, M.D., F.R.C.S.E.
ROBERT ROBERTSON, M.A.
A. E. SCOUGAL, M.A., LL.D.
NICHOLAS SENN, M.D., LL.D.
T. EDGAR UNDERHILL, M.D., F.R.C.S.E.

FOREIGN HONORARY FELLOWS DECEASED.

ADOLF RITTER VON BAEYER.
JEAN GASTON DARBOUX.

ORDINARY FELLOW RESIGNED.

GEORGE FRANCIS SCOTT ELLIOT.

INDEX.

Index of Papers published in the "Transactions" during Session 1916–17.

(*Arranged under the Authors' Names.*)

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MODEL INDEX.

Schäfer, E. A.—On the Existence within the Liver Cells of Channels which can be directly injected from the Blood-vessels. Proc. Roy. Soc. Edin., vol. 1902, pp.

Cells, Liver,—Intra-cellular Canaliculi in.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp.

Liver,—Injection within Cells of.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp.

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