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CONTENTS.

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By J. L. Hogg.

Presented by John Trowbridge June 29, 1909; received June 29, 1909.

Under the title "Friction and Force due to Transpiration as dependent on Pressure in Gases," there was published [1] some time ago an account of some experiments made to determine the relation between the friction of a gas and the pressure in it, and also the relation between the force exerted by a lamp on a mica vane, blackened on the face which is turned towards the lamp, and the mean pressure in the gas in which the vane is placed. The three-fold purpose of the investigation was pointed out there, viz.:

First, to investigate the relation between friction and pressure where the pressures were so small that "slip" is appreciable; second, to determine the relation of transpiration force in the special form of apparatus described there; [2] and, third, to make use, if possible, of these two relations to test the validity of the McLeod gauge measurements of pressure, and, if these measurements should prove unreliable, to make use of one of the relations named above to measure gas pressure.

There has been much delay in carrying out the investigation with the apparatus improved in the manner indicated in the closing paragraphs of that paper, but now some results have been obtained in so far as the friction problem is concerned.

As was pointed out in the paper mentioned, the investigation was defective in two respects. It was found that, in spite of the care which was taken to exclude mercury vapor from the apparatus, some of this vapor was undoubtedly present. This no doubt was due to the fact that the whole apparatus had to be maintained at a high temperature for long periods to insure drying, and thus the presence of the least speck of liquid mercury would cause, when evaporation took place, the diffusion of comparatively large quantities of the vapor through the

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[1] Proc. Am. Acad., **42**, 6 (1906).

[2] See p. 129 of that paper.

apparatus. Again, the logarithmic decrement due to the friction in the suspending fibre was not determined directly by experiment, and in the discussion of the results obtained its value was calculated.

The details of the method since used to exclude the mercury vapor and to determine the decrement due to friction in the fibre will appear later. Meanwhile a summary of what has been accomplished is given here.

First, the decrement due to the friction in the suspending fibre of the viscosity apparatus has been determined experimentally.

Second, mercury vapor has been excluded to such a degree that, even when the whole apparatus, in which the presence of the vapor would be objectionable, was kept at a temperature of 150° C., the mercury lines were absent from the spectrum of the gas enclosed.

Third, the value of the decrement has been obtained for hydrogen over a range of pressures extending from atmospheric pressure to 0.000016 mm. as indicated by the McLeod gauge.

Fourth, an equation relating pressure to decrement has been obtained which applies well at all pressures below 0.1 mm. as far as pressures have been measured. The equation, above mentioned, is of the form of Sutherland's equation given tentatively in the former paper. It is

$$\left(\frac{k}{l-\mu}-1\right)p=c.$$

In this equation k and c are constants to be determined from the observations; l is the decrement due to whatever friction there is in the gas under examination and to the friction in the fibre; p is pressure; μ is the decrement due to the friction in the fibre. Its value has been measured directly. The significance of the two slightly differing values of μ, namely, $\mu = 0.000020$ and $\mu = 0.000022$, which are found in the following table, will appear later when the measurement is discussed in detail. The first column of the table contains a series of values of the decrement for hydrogen, each of which corresponds to a definite pressure in the gas. The various values of the pressure are given in the second column. The first three of them were obtained from a manometer. Those which are marked thus,*, were obtained from measurements made with the McLeod gauge, while the others were obtained from a curve plotted from the directly observed values of the decrement and pressure. From two values of p, the corresponding values of l, and the value of μ, there are obtained two equations for the determination of the constants k and c in the above equation. These determined, it is clear that from any value of l, within the range

TABLE I.

HYDROGEN.

Log. Dec.	p (observed).	p (calculated) $\mu = 0.000020$.	p (calculated) $\mu = 0.000022$.
0.07942	760.0	......	......
0.07937	435.0	......	......
0.07927	103.0	......	......
0.07768*	8.89*	......	......
0.06902*	1.24*	......	......
0.05423*	0.410*	......	......
0.02861*	0.105*	0.109	0.109
0.01140	0.0300	0.0302	0.0302
0.01056	0.0275	0.0276	0.0275
0.00936*	0.0239*	......	......
0.00887	0.0225	0.0225	0.0225
0.00710	0.0175	0.0174	0.0174
0.00620	0.0150	......	......
0.00525	0.0125	0.0125	0.0125
0.00434*	0.0102*	0.0102	0.0102
0.00426	0.0100	0.00998	0.00998
0.00306*	0.00704*	0.00702	0.00702
0.00220	0.00500	0.00497	0.00496
0.00112	0.00250	0.00247	0.00246
0.000459*	0.00098*	0.00097	0.00097
0.000215*	0.00042*	0.00043	0.00043
0.000029*	0.000016*	0.000020	0.000015

indicated above, the corresponding value of p can be obtained from the formula by a simple calculation. The numbers thus obtained for the various values of l are given in the third and fourth columns.

It would, therefore, seem highly probable that so far as hydrogen is concerned the McLeod gauge can be relied upon for pressures as low as the lowest used, and which are recorded in Table I; and that, in the case of hydrogen, the measurement of friction can be used as a convenient and accurate method of measuring pressure, provided care is taken to exclude mercury vapor. This matter will be discussed at length later.

The details of the methods used to overcome the difficulties named above follow:

Measurement of Decrement due to the Friction in the Fibre.

Referring to Figure 1 it will be seen that the tube C is inserted in such a position that nothing can pass to the viscosity apparatus from the McLeod gauge, B, or from the pump, which is connected to D, without passing through it. This tube, C, therefore, replaces the tubes of sulphur and silver whose purpose was explained in the earlier paper. C is filled with granular charcoal, and is so arranged that either a cylindrical electrical heater or a long Dewar vessel can enclose it. When C had been placed in position and sealed in place, the whole apparatus was exhausted through D by means of the mechanical pump, and then dry air was allowed to pass in through an opening placed near the pump. The exhaustion was again performed and the admission of dry air repeated. This exhaustion and admission of air were carried out alternately many times for the purpose of removing the comparatively large quantities of moisture which had been formed in the vessel during the process of making the various joints in the construction of the apparatus. When it was certain that the whole apparatus had been made fairly dry, the cylindrical electric heater was placed about the tube C, and while the exhaustion proceeded the tube was raised to a temperature of about 150° C., to hasten the removal of the gas present in large quantities in the pores of the charcoal at atmospheric pressure, and which separates from the charcoal rather slowly under reduced pressure if the temperature is kept low. When the mercury pump had been used to secure a fairly high vacuum the other parts of the apparatus, viz., the McLeod gauge, the viscosity apparatus, and the connecting tubes were heated to about 150° C., for the purpose of removing from the glass the occluded gases. After the pumping had proceeded for some time under these conditions, the heater was re-

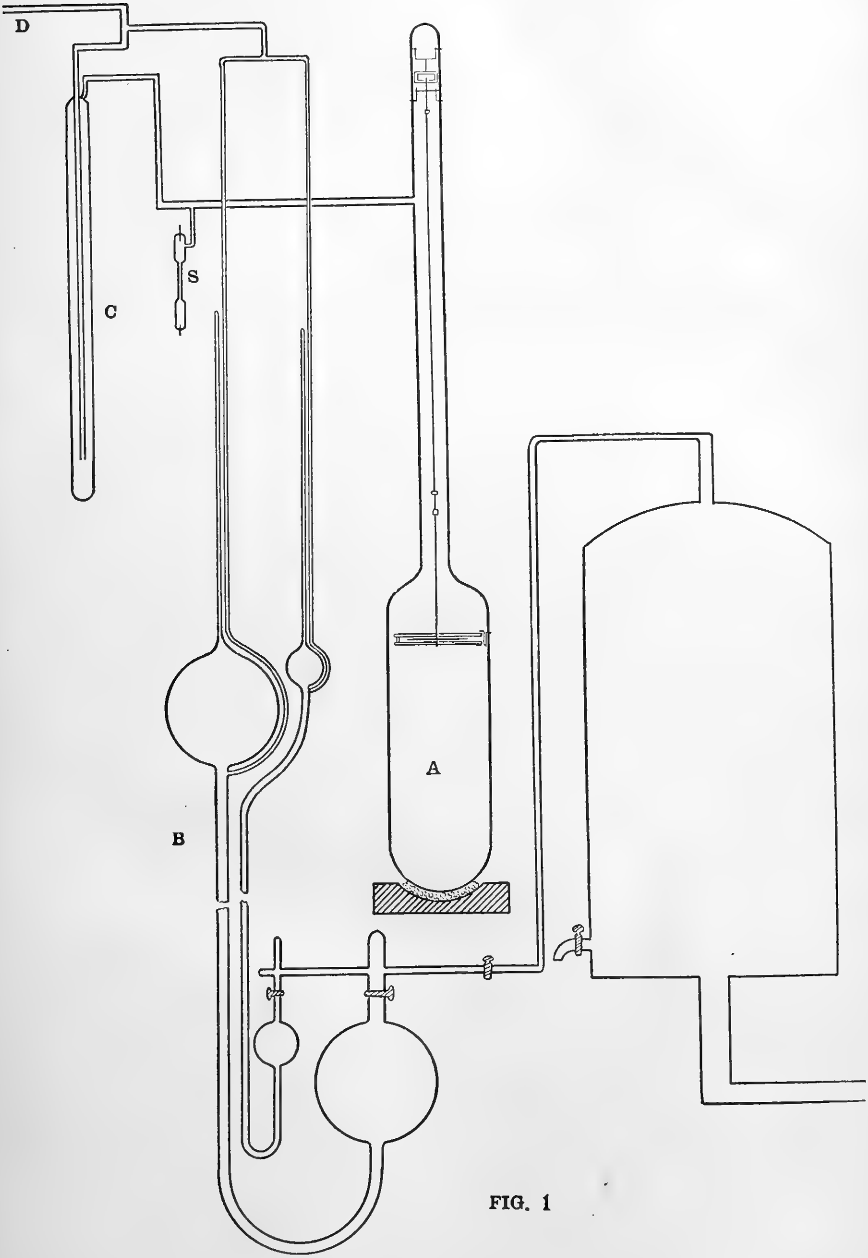

FIG. 1

moved from C and the Dewar vessel containing liquid air[3] was substituted for it, and the other part of the apparatus was allowed to cool down. The charcoal was allowed to absorb what it would at the temperature of the liquid air. Altogether the liquid air was kept surrounding the charcoal for about eighty hours, and from time to time during this interval a measurement of the decrement, l, was made. At first the diminution in the value of the decrement was fairly rapid, but after the first day the change was very slow. This, no doubt, was due in part to the slow passage of the gas towards the charcoal through the somewhat extended form of the apparatus. It was, also, probably due to the fact, which was noted later in the investigation, that at a given stage of exhaustion the raising of the free surface of the liquid air in the Dewar vessel surrounding C invariably produced a very appreciable diminution in the gas pressure in the apparatus, and the lowering of the free surface as the evaporation of the liquid air proceeded resulted in a distinct rise in the gas pressure. It is to be understood that the free surface was never allowed to fall as low as the top of the tube C, so that all of the charcoal was always below the free surface of the liquid air.

The following results show how the decrement changed with the time in the final forty-eight hours :

May 29,	12 M. to 2:53 A. M.	Decrement	0.000051
	7:15 P. M. to 8:58		0.000037
	10:53 P. M. to 1:36 A. M. (May 30)		0.000031
May 30,	1:36 A. M. to 4:48		0.000028
	11:11 A. M. to 2:06 P. M.		0.000037*
	2:06 P. M. to 5:27		0.000024*
	5:27 P. M. to 8:21		0.000028*
	8:21 P. M. to 11:49		0.000022

The smallest value of the decrement obtained was 0.000022, and this could be measured moderately well. Its error cannot, I think, be as much as ten per cent. Of course, it is clear that the true value of the decrement due to the friction in the fibre is somewhat less than this, for there is still, doubtless, some gas left to offer resistance to the moving disk, so that the number to be used for μ in the above equation should be somewhat smaller than 0.000022. I have ventured to

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[3] The liquid air used in this investigation was obtained at the Chemical Laboratory, Harvard University.

* These were taken in the afternoon when there is considerable jarring of the apparatus and are probably not so accurate as the others.

make use of the value 0.000020 as the true value to which the decrement will approach as the exhaustion is pushed higher and higher. It will be seen from Table I that the calculations are carried through not only with this value, but also with the actually measured value 0.000022. This is done simply to show what effect such a change in the value of μ has on the series of results obtained.

It may be of interest to state that, at the stage of exhaustion when $\mu = 0.000022$ was obtained, the McLeod gauge indicated a pressure certainly less than 0.000001mm. It is, to be sure, of little value to give the measurement of a pressure by the gauge where a column of mercury a fraction of a millimeter high requires to be measured, and especially is this true where the tube containing the mercury has been heated and cooled repeatedly. The mercury has a habit of sticking to the glass to such an extent that pressure measurements under the conditions mentioned are surely not reliable. The value of the pressure given above, then, only indicates the order of magnitude of the pressure. Though the factor of the gauge used was 95813, yet it was quite inadequate to measure the pressure of the gas in the vessel.

Removal of Water Vapor and Mercury Vapor from the Hydrogen in the Viscosity Apparatus.

For this purpose it was necessary to make arrangements by which no vapor should be carried into the apparatus with the entering gas, and also all the vapor which was already in the apparatus might be taken out. The following arrangement was finally adopted, Figure 2. E is a U-tube of small bore, and bent so that it may enter the long Dewar vessel already mentioned. For reasons which will appear later it was found necessary for the remainder of the investigation to replace the tube C, Figure 1, by this tube E. F is a tube leading from the gas generator. It enters G, which is similar to C of Figure 1. It can be surrounded by a heater or a Dewar vessel as circumstances may require. A connecting tube leads from G to a point on the tube H, which connects E to the viscosity apparatus A, Figure 1. I leads to the pump and McLeod gauge. Anything which proceeds from the pump or McLeod gauge towards the viscosity apparatus must pass through E. Moreover, the gas entering from the generator will, with the given arrangement, retard the diffusion of mercury vapor from the pump and gauge towards the viscosity apparatus. If there is no vapor entering with the gas, there can be none entering the viscosity apparatus without passing through E, and, since throughout the experiment this tube was kept surrounded by the liquid air, the pressure of the vapor due to diffusion from the mercury in the pump or gauge could never exceed the vapor pressure of mercury at the temperature of the liquid air. The tube G, when surrounded with the liquid air, was sufficient safeguard against the entrance of water vapor with the gas.

The method of removing all water vapor and mercury vapor already in the apparatus beyond the tube E was that of repeated exhaustion and filling with the gas to be experimented with, the whole apparatus meanwhile being kept at a high temperature.[4]

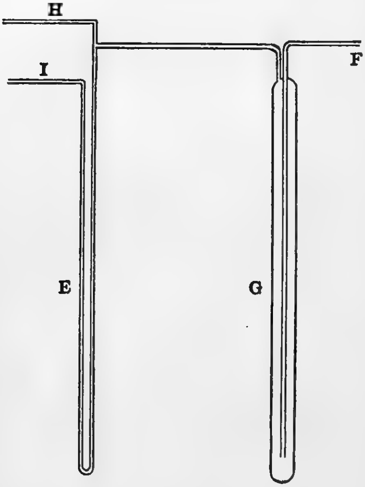

FIG. 2

At the first exhaustion, when the pressure had been reduced to a few centimeters of mercury, the tube G was surrounded by the electric heater, and the heat was applied to the oven in which the viscosity apparatus is placed. Practically the whole apparatus, except the gas generator, was kept hot while the pumping proceeded. After a fair vacuum was reached the pump was stopped and the hydrogen from the generator was allowed to enter very slowly, passing first over phosphoric pentoxide, and then over spongy platinum, heated in a combustion tube, before entering the tube G. This filling process was followed by another exhaustion under the same conditions. After the apparatus had been exhausted and filled a number of times in this way, when it seemed certain that the apparatus and the pores of the charcoal were filled with fairly pure hydrogen, the heater was removed from G, and the vessel containing the liquid air substituted for it. The same process of alternately exhausting and filling was continued, great care being taken in filling to allow the hydrogen to pass very slowly so that the drying process might be complete. Keeping the apparatus at a temperature of about 150° C. served to promote the evaporation of the mercury, which in all probability adhered to the inner glass surfaces. Comparatively large quantities of pure dry hydrogen were allowed to pass into the vessel and were then taken out. Each exhaustion would assuredly sweep out some vapor if it was present.

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[4] The suspended disk was, of course, lowered before this operation began.

It would naturally collect at E. We shall have evidence as to this later. After some days of incessant work the expected result was attained, as the character of the spectrum, obtained from the spectrum tube, S, showed. Even when the temperature of the viscosity apparatus was 150° C. the mercury lines were absent. The apparatus was then very slowly filled with hydrogen. The glass tube connecting G and H was then sealed off so that there were left no stop-cock joints to give trouble by leaking.

The Dewar vessel was removed from G, but the one surrounding E still remained. After the apparatus had cooled down to room temperature the disk of the viscosity apparatus was raised and adjusted as described in the former paper.[5]

METHOD OF EXPERIMENT.

The investigation of the relation of friction to pressure consists in measuring, for a given density of the gas, the logarithmic decrement of the suspended disk which is made to oscillate as a torsion pendulum between the two fixed plates of the apparatus.[6] The method of procedure was to measure the gas pressure in the apparatus by means of a manometer when the pressure was large, and by the McLeod gauge when it was small, and then to set the disk of the apparatus swinging and measure the decrement. Since the latter can be shown to be proportional to the resistance experienced by the disk, one gets data for the determination of the relation between friction and pressure.

It may be of interest to state here that in the first arrangement of the apparatus for the determination of the above relation instead of the simple bent tube E, a tube containing charcoal, similar to the tube C, was used. With this arrangement the mercury vapor was removed, but when observations on the decrement at different pressures were undertaken a difficulty presented itself. Although all of the tube containing the charcoal was immersed in the liquid air, the surface of which was always several inches above the top of the charcoal, yet it was found impossible to obtain a steady condition. As the evaporation of the liquid air proceeded, sufficient gas was given off from the charcoal to produce a large increase in pressure; as much as thirty per cent was observed. When a fresh supply of the liquid air was added the pressure diminished again. The difficulty became more serious as the pressure at which the observations were made became smaller.

The phenomenon was probably due to the fact that the fresh supply of liquid air was richer in nitrogen than it was after the process of

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[5] See pp. 133, 134.

[6] See pp. 124, 125 of former paper.

boiling had proceeded for some hours. The nitrogen is the more volatile, and so the boiling will proceed more vigorously just after a fresh supply of air has been added than at any other time. Consequently the temperature of the boiling liquid will be lower at first than it is later, and the charcoal will thus absorb better at each addition of liquid air to the Dewar vessel. The charcoal is necessary for the phenomenon, for when the tube E was substituting for the tube containing the charcoal, the effect disappeared, or became inappreciable.

It was suggested earlier in the paper that there would be adduced evidence to show that the mercury driven out from the apparatus collected in the tube E. After the measurement of pressure and decrement had proceeded down to the least value given in the table, the supply of liquid air in the Dewar vessel in which E was placed was allowed to disappear gradually. As the evaporation proceeded, it was found that the decrement increased much more rapidly than the pressure as indicated by the McLeod gauge, showing that vapor was finding its way into the apparatus.

Results.

In the first and second columns of Table I are contained the corresponding values of the decrement and pressure. Not all the numbers given in these columns were obtained by actual measurement. Only those which are marked with an asterisk were obtained in this way. The others were obtained as follows: A curve was plotted using the values of the pressure which were measured as abscissas and the corresponding values of the decrement as ordinates. This curve was drawn on such a scale that the value of the decrement, corresponding to any arbitrarily chosen pressure, could be obtained from the curve as accurately as it could be measured by the apparatus. The unmarked numbers in the first two columns were obtained by choosing arbitrarily a pressure and reading off from the curve the corresponding value of the decrement. In no case has this procedure involved an extrapolation.

After failing to obtain an analytical expression for the relation between the logarithmic decrement and the pressure which would be applicable over the whole range extending from very small pressures right up to atmospheric pressure, it was decided to find, if possible, an expression which would be applicable up to a certain pressure within the range for which it is known that the McLeod gauge measurements are quite reliable. Rayleigh[7] has shown that Boyle's Law holds down to 0.01mm. of mercury, and Baly and Ramsay[8] found the McLeod

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[7] Phil. Trans., 196 (1901).

[8] Phil. Mgg., 38 (1894).

gauge measurements reliable for hydrogen. Recently Scheel and Heuse[9] have applied a membrane manometer, devised by them, to test Boyle's Law, and McLeod gauge measurements for air, and they state that they find both valid down to about 0.0001 mm., provided proper care is taken in drying the gas.

An examination of the results at pressures less than 0.1 mm. showed that the equation given above, viz.:

$$\left(\frac{k}{l-\mu}-1\right)p=c,$$

served the purpose exceedingly well, if the experimentally determined value of μ given above was used, and if the constants c and k were determined by means of values of p between 0.1 mm. and 0.01 mm. of mercury, and the corresponding values of l.

The pressures chosen for the determination of these constants were 0.0239 mm. and 0.0150 mm., the former of these being a pressure actually measured by the gauge, while the latter was chosen arbitrarily. The measurement of l corresponding to the former was 0.00936, while the value of l corresponding to the latter was 0.00620, and was obtained from the curve as described above. The values of the constants calculated from the above data are

$$c=0.1491$$
$$k=0.0676.$$

The equation now takes the form

$$\left(\frac{0.0676}{l-0.000020}-1\right)p=0.1491.$$

How well the equation gives the relation existing between p and l can be seen by a comparison of the numbers in the second and third columns of the table. A number in the third column is obtained by choosing a value of l from the first column, inserting it in the equation and deducing the value of p, which is then placed in the third column in the same horizontal row as the chosen value of l. It will be seen that the various numbers in this column agree very well with the corresponding numbers in the second column, except at the very lowest pressure, 0.000016 mm. used, when the difference is about twenty-five per cent.

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[9] Verhandl. d. Deutsch. Physikal. Gesellsch., **11**, 1 (1909).

If instead of using the value 0.000020 for μ, we make use of 0.000022, which was the smallest value of the decrement actually measured, the values of c and k are

$$c = 0.1494$$
$$k = 0.0677$$

and the fourth column gives the values of p calculated, in the way described, from the equation with these values for the constants instead of those used in the preceding case. This calculation is carried out to call attention to the magnitude of the change produced by a slight change in the value of the constant, μ, which is subject to some uncertainty, as has been shown. It will be seen that it is only where the decrement, l, is very small that the difference between the two results is appreciable. The smallest value of p in the fourth column is nearer to the corresponding value of p, measured by the McLeod gauge; but the measured value is subject to an inaccuracy about as great as the difference between the measured and calculated values of p.

The results given above make it highly probable that the measurements of pressure by the McLeod gauge are reliable in the case of pure, dry hydrogen for pressures as low as the smallest pressure recorded in the table.

It is to be observed that for pressures below, say, 0.01 mm. of mercury the friction with which we have to do is largely external friction, and this is proportional to the density of the gas and the mean molecular speed. The friction, and, therefore, the decrement, corresponding to a given pressure will be smaller for hydrogen than for, say, oxygen, or mercury vapor. In the case of mercury vapor the decrement at a given low pressure ought to be about ten times as great as it is for hydrogen at the same pressure, since the molecular weight of mercury is about one hundred times that of hydrogen, while the mean molecular speed is about one-tenth as great as it is for hydrogen.

To be sure it does not follow that the decrement of a mixture of hydrogen and mercury vapor, in such proportions that the partial pressures of the two are the same, is simply the sum of the two decrements obtained when the gas and vapor are separate. If one accepts the expression deduced by Meyer[10] for the external friction of a gas, and applies the same method in considering external friction of mixtures as he does in dealing with the internal friction of mixtures, he will be able better to understand how the external friction of a mixture of

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[10] Kinetic Theory of Gases, p. 210 (Eng. Trans.).

gases depends upon the proportion in which the gases are mixed. Meyer shows that the coefficient of external friction is given by

$$\tfrac{1}{4}\beta m N \Omega,$$

where m is the molecular weight of the gas; N is the number of molecules per unit volume; Ω is the mean molecular speed; and β is a constant depending upon the solid surface. He gives some experimental evidence to show that β is independent of the gas.

In the case of a mixture of gases where there are N_1 molecules of one kind and N_2 molecules of another, in each unit volume we have, if N is the total number of molecules in the unit volume,

$$N = N_1 + N_2,$$

and the mean molecular weight is given by

$$m = \frac{N_1 m_1 + N_2 m_2}{N}$$

where m_1 and m_2 are the molecular weights of the two gases mixed.

Since the temperatures of the two gases are the same,

$$m_1 \Omega_1^2 = m_2 \Omega_2^2 = m\Omega^2.$$

Therefore,

$$m\Omega = m_1 \Omega_1 \sqrt{\frac{N_1}{N} + \frac{m_2}{m_1} \cdot \frac{N_2}{N}}.$$

If Boyle's Law holds, which seems a fair inference from the results given above, then we may write

$$m\Omega = m_1 \Omega_1 \sqrt{\frac{p_1}{p} + \frac{m_2}{m_1} \cdot \frac{p_2}{p}},$$

where p_1 and p_2 are the partial pressures and p is the whole pressure under the given conditions. If β is independent of the nature of the gas it follows that the ratio of the external friction of the mixture to the external friction of the gas whose partial pressure is p_1, if it were in the vessel alone, is

$$\frac{N m \Omega}{N_1 m_1 \Omega_1} = \frac{N m_1 \Omega_1 \sqrt{\frac{p_1}{p} + \frac{m_2}{m_1} \cdot \frac{p_2}{p}}}{N_1 m_1 \Omega_1} = \frac{N}{N_1} \sqrt{\frac{p_1}{p} + \frac{m_2}{m_1} \cdot \frac{p_2}{p}}.$$

If the mixture is one of hydrogen and mercury vapor such that $p_1 = p_2$, the above ratio becomes about 14. This means that if the pressure is measured by the McLeod gauge, which takes no account of the mercury vapor, the friction of the mixture would be about fourteen times as much as it would be with the same hydrogen pressure as in this case, but with the mercury vapor absent. If $p_1 = 1000 p_2$, the ratio is about 1.05, and if $p_1 = 10,000 p_2$, it becomes about 1.005.

It might be urged with regard to the method described above for freeing the hydrogen from mercury vapor that the lowest pressure of vapor obtainable by the method used is the pressure of mercury vapor at the temperature of liquid air boiling at atmospheric pressure. This pressure at 0° C. is about 0.0005 mm., but what it is at the lower temperature mentioned can hardly even be conjectured. We have simply to fall back upon the spectroscopic test. The above discussion shows, however, that if this pressure is less than 0.001 of the pressure of the hydrogen it will not very seriously affect the results. If it is as low as 0.0001 of the hydrogen pressure, then the error in the observations will easily be greater than any error introduced in this way. Considering the lowest pressure reached, namely, 0.000016 mm., the vapor pressure of mercury at the temperature of liquid air, boiling under atmospheric pressure, would require to be as low as 0.000,000,016 in order that the ratio of the partial pressures should be 1:1000.

This case serves to show how important it may be to consider mercury vapor when we are dealing with these very low pressures. It indicates that, in all high vacua work where we are considering the properties of a particular gas, it is important that great care should be taken to exclude this vapor. The McLeod gauge, of course, takes no cognizance of it, and in fact serves to introduce the vapor where it is not wanted. In all cases where the vacuum is high, and it is desirable to know the pressure in the vessel, and yet keep the gas pure, it would be desirable to have a gauge which would not introduce any impurity.

If the inference made above as to the validity of the McLeod gauge measurements on gas pressure is allowed, then we can say that reliance may be placed upon the measurements of pressure from decrement measurements in the apparatus used in this investigation. This method need introduce no mercury vapor, but it takes account of all that is in the vessel. Moreover, a discussion similar to that used for mercury vapor will show that in the case of oxygen the decrement, corresponding to a certain gas pressure, will be about four times as great as it is in the case of hydrogen. In the case of oxygen, therefore, a pressure of 0.00001 mm. should be measured with an accuracy of from five to ten per cent. This would indicate an absolute error of less than 0.000,001 mm.

The investigation is now being extended to the case of oxygen and nitrogen. The data obtained by using these gases, besides showing whether their behavior is like that of hydrogen, should give some more information regarding the quantity, β, which enters the foregoing discussion.

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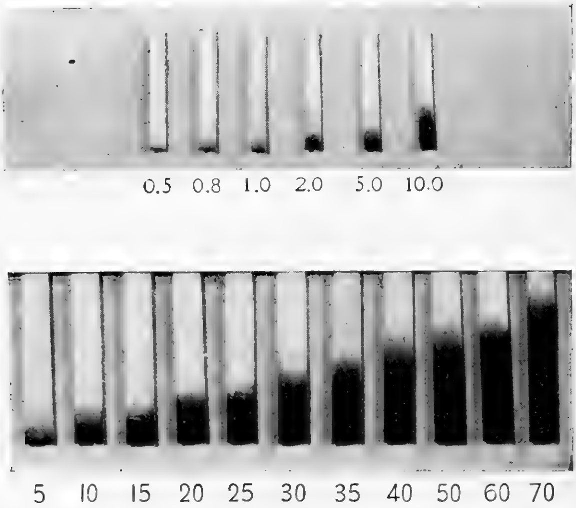

STANDARD ANTIMONY BANDS IN MICROMILLIGRAMS OF Sb_2O_3
AMMONIA DEVELOPMENT.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF HARVARD COLLEGE.

THE QUANTITATIVE DETERMINATION OF ANTIMONY BY THE GUTZEIT METHOD.

By Charles Robert Sanger and Emile Raymond Riegel.

With a Plate.

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OF HARVARD COLLEGE.

THE QUANTITATIVE DETERMINATION OF ANTIMONY BY THE GUTZEIT METHOD.

By Charles Robert Sanger and Emile Raymond Riegel.

Presented August 31, 1909. Received August 31, 1909.

The application of the so-called Gutzeit reactions to the quantitative determination of arsenic has been studied by Sanger and Black[1], who were able to use the general method of Gutzeit[2] for the convenient and reasonably accurate estimation of small amounts of arsenic. In studying the interference of the hydrides of sulphur, phosphorus, and antimony with the reaction of arsine on paper sensitized with mercuric chloride, the possibility of the quantitative determination of antimony by this method was apparent.

The action of stibine on mercuric chloride was first investigated by Franceschi[3], who obtained a white body, to which he gave the formula $SbHHg_2Cl_2$, analogous to the red compound formed by the action of arsine on mercuric chloride. This substance decomposes easily in moist air, turning dark, probably from the separation of mercury. When stibine is allowed to act upon sensitized mercuric chloride paper, as shown by Sanger and Black[1], no color is given to the strip from amounts of antimonious oxide up to about 70 micromilligrams (mmg.). Hydrochloric acid develops no color. But if the strip is treated with ammonia, a black band ensues, the length and intensity of which are proportional to the amount of antimonious oxide present. On this reaction we have based the following method for the determination of small amounts of antimony.

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[1] These Proceedings, **43**, 297 (1907); Jour. Soc. Chem. Ind., **26**, 1115 (1907); Zeitsch. f. anorg. Chem., **58**, 121 (1907); Suppl. ann. enciclop. chim., **24**, 372 (1907–08).

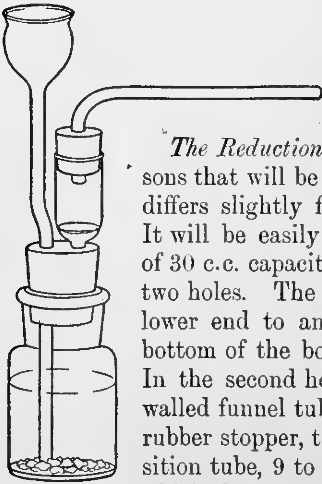
[2] Pharm. Zeitung, **24**, 263 (1879). In the original Gutzeit method, the evolved arsine was allowed to act upon paper containing argentic nitrate. From Flückiger in 1889 (Archiv d. Pharm., **227**, 1) came the suggestion of using mercuric chloride.

[3] L'Orosi, **13**, 397 (1890).

THE METHOD.

The procedure does not vary greatly from that used in the determination of arsenic as described by Sanger and Black[1]. Some details, therefore, of that method are necessarily repeated here.

Sensitized Mercuric Chloride Paper. A smooth filter paper of close texture, or a Whatman drawing paper of about 160 grams per square meter, is cut into strips of a uniform width of 4 millimeters. The strips are sensitized by drawing them repeatedly through a five per cent solution of recrystallized mercuric chloride until thoroughly soaked. They are then dried on a horizontal rack of glass tubing, and, when dry, are at once cut into lengths of six to seven centimeters. The small pieces are kept in the dark until needed, in a stoppered bottle over calcic chloride.



The Reduction Apparatus. (See Figure.) For reasons that will be explained later, the construction of this differs slightly from that used in the arsenic method. It will be easily seen from the figure. The bottle is of 30 c.c. capacity, closed by a pure rubber stopper with two holes. The thistle tube, which is constricted at its lower end to an opening of about 2 mm., passes to the bottom of the bottle and has a length of 17 to 18 cm. In the second hole of the stopper is inserted a straight-walled funnel tube of 17 to 20 mm. bore, carrying a pure rubber stopper, through which passes a right angle deposition tube, 9 to 10 cm. in length, the inner diameter of which should be as near 4 mm.as possible, but not less.

Reagents. These are exactly the same as in the arsenic method, and are entirely free from antimony. The zinc, Bertha spelter, is from the New Jersey Zinc Company of New York, and has been proved by repeated tests to be free from arsenic. The hydrochloric acid, from the Baker and Adamson Company of Easton, Pennsylvania, contains not over 0.02 milligram of arsenious oxide per liter. The quantity of diluted acid (one part to six of water) used in the analysis would not contain over 0.00004 milligram of arsenious oxide, an amount beyond the absolute delicacy of the method as applied to arsenic and hence of no influence in the determination of antimony.

Moisture Conditions in the Deposition Tube. As in the arsenic method, the moisture of the evolved hydrogen has an important bearing on the uniformity of the color bands. While excess of moisture must be avoided in the arsenic method by a cotton wool filter, it is necessary to have a much greater degree of saturation in order to obtain compact and uniform deposits on the strips from stibine. If the hydrogen is partially dried by cotton wool before impinging upon the sensitized paper, the bands are long, irregular and not comparable. By increasing the saturation and by making it as uniform as possible we have succeeded in determining the conditions under which the bands are short, regular, and perfectly comparable.

To effect this and at the same time to hold back any hydrogen sulphide which might be formed in the reduction, we use disks of lead acetate paper inserted in the straight-walled funnel tube and moistened with a definite amount of water. These disks are of filter paper of medium thickness, cut in quantity by means of a wad cutter or cork borer so as to fit loosely the bore of the funnel tube. They are saturated with normal lead acetate, dried, and kept in a well stoppered bottle.

Procedure. The deposition tube and funnel tube of the apparatus are cleaned and thoroughly dried. A lead acetate disk is then inserted in the funnel tube and moistened with one drop of water, delivered on the centre of the disk, so that the water spreads evenly to the circumference. Three grams of uniformly granulated zinc are placed in the bottle, a strip of sensitized paper is slipped wholly within the deposition tube to a definite distance, and the apparatus is put together. Five or ten cubic centimeters of diluted acid (1 to 6 ; normality, about 1.5) are then added through the thistle tube and allowed to act for about ten minutes. The acid is then poured off and fifteen cubic centimeters of fresh acid added. This procedure ensures a uniform degree of moisture saturation in the deposition tube, and the absence of arsenic in the reagents and apparatus is assured. The zinc is also rendered more sensitive, and a regular flow of hydrogen is quickly obtained on the second addition of acid.

In five minutes after this addition, the solution to be tested is introduced, either wholly or in aliquot part, which may be determined by weighing or measuring. In case it were necessary from the nature of the analysis to prove the absolute freedom of the apparatus and reagents from arsenic and antimony before adding the solution, the evolution of hydrogen would be continued for a longer time and the strip developed. The absence of contamination being thus assured, a fresh strip would be substituted before adding the solution to be tested. In ordinary work, however, this precaution is quite unnecessary.

After the solution is introduced, the reduction is continued for 30 to 40 minutes. No effect on the sensitized paper is observed unless the amount of antimony added is above 70 mmg., when a slight gray color may appear. Larger amounts would turn the paper still darker. If any color appears, it is an indication that the amount will be difficult to estimate, and hence another trial should be made with a smaller portion of the solution, or from less of the original substance.

The strip is now placed in a test tube and covered with normal ammonic hydroxide, which is allowed to act for five minutes. A black band is slowly developed, somewhat duller and considerably shorter than would be obtained from the same amount of arsenic, the latter difference being chiefly due to the moisture conditions in the deposition tube. The band is then compared with a set of standard bands. The amount of antimony in the entire solution follows from that determined in the aliquot part.

Standard Bands. A standard solution is made from pure, recrystallized tartar emetic, shown to be free from arsenic. 2.3060 grams are dissolved in water and made up to one liter. This solution (I) contains 1.0 mg. of antimonious oxide per cubic centimeter. From this, by dilution, are made two solutions containing respectively 0.01 mg. (II) and 0.001 mg. (III) per cubic centimeter. From definite portions of solutions II or III a series of bands is made by the above procedure, using a fresh charge of zinc and acid for each portion. The lower half of the Plate shows the actual size and shading of the set of bands, corresponding to the following amounts of antimonious oxide in micromilligrams: 5, 10, 15, 20, 25, 30, 35, 40, 50, 60, 70.

These bands have shown a fair degree of permanency, but fade slowly on exposure to moisture and light. They may be sealed in glass tubes with quicklime, if desired, as in the case of the corresponding ammonia-developed arsenic bands, but we have found it sufficient to mount them on a dry glass plate, which is covered by a dry plate of the same size. The two plates are then cemented together and bound with passepartout paper. The set thus mounted, if kept in a desiccator away from the light, will last for some time. In case a fresh set of standards is not available, a band may be approximately estimated from the accompanying Plate; the more accurate determination being made, if necessary, by comparison with freshly prepared bands from selected amounts.

Analytical Notes.

General Precautions. As in the arsenic method, the solution to be reduced should contain no interfering organic matter, except that any oxide of antimony obtained in the preparation for analysis may be eventually dissolved in tartaric acid. Sulphur in any form reducible to hydrogen sulphide should be removed as completely as possible, but small quantities of hydrogen sulphide will be completely retained by the lead acetate disk. There is little danger from phosphine, for phosphites and hypophosphites would be oxidized in any treatment of the substance to be analyzed which would convert the antimony to the oxide. Traces of phosphine would be readily recognized in presence of antimony[4], but are likely to interfere with its estimation. It is obvious that there must be a very thorough separation from arsenic.

The Evolution of Stibine in the Reduction Bottle. Sanger and Gibson[5] have shown that amounts of antimony under one milligram are practically all converted to hydride in the presence of zinc and hydrochloric acid, hence a retention of antimony by precipitation upon the zinc is not to be considered in the estimation of the small amounts provided for by this method.

Special Precautions. In order to be certain of uniformity in length and density of bands from the same concentration of solution, the following points must be observed :

1. The reduction bottles must be of equal capacity, and other parts of the apparatus of equal dimensions.
2. The amount of zinc must always be the same, similarly sensitized, and the granulation must be uniform.
3. The volume and concentration of the acid must be definite.
4. The moisture conditions in the deposition tube must be carefully regulated, as explained above.

In the "Analytical Notes" of the article by Sanger and Black[1], many suggestions will be found which will contribute to a clearer understanding of this method as well, but which are not included here for the sake of brevity.

ANALYTICAL DATA.

The method, as far as it concerns the determination of antimony in a solution properly prepared for reduction, was tested by the analysis of solutions containing varying amounts of antimony, which were unknown to the analyst. See Table, p. 26.

We do not claim for the method a greater accuracy than within ten per cent.

THE DELICACY OF THE METHOD.

Amounts of antimony as small as five micromilligrams are readily recognized by use of the 4 mm. strip. Less than this quantity may be

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[4] See Table II, Sanger and Black[1].

[5] These Proceedings, **42**, 719 (1907); Jour. Soc. Chem. Ind., **26**, 585 (1907); Zeitschr. f. anorg. Chem., **55**, 205 (1907).

TABLE.

No. of Analysis.	Sb_2O_3 taken. Tartar Emetic Solution.	Total Weight Diluted Solution.	Wt. Diluted Solution taken for Analysis.	Reading of Band.	Sb_2O_3 Found.	Sb_2O_3 Found. Mean.	Per Cent Sb_2O_3 Found.
	mg.	gm.	gm.	mmg.	mg.	mg.	
4	0.06	25.15	4.67	8	0.043		
			8.62	15	0.044		
			10.63	25	0.059	0.049	82
2	0.12	25.92	4.31	20	0.120		
			2.61	10	0.099		
			7.11	35	0.128	0.116	97
6	0.20	23.04	2.59	25	0.222		
			1.72	17	0.228		
			4.23	40	0.218	0.223	112
2, a	0.25	26.07	5.18	45	0.227		
			3.43	35	0.266		
			1.05	10	0.248	0.247	99
9	0.60	21.63	0.79	22	0.602		
			0.97	26	0.580		
			1.09	32	0.635	0.606	101
8	1.20	23.74	0.81	35	1.03		
			0.84	35	0.99	1.01	84
8, a	1.50	24.30	0.46	30	1.58		
			0.61	40	1.59		
			0.59	40	1.65	1.61	107
7	1.60	21.32	0.39	35	1.91		
			0.27	20	1.58		
			0.53	45	1.81	1.77	111
4, a	2.50	27.76	0.44	45	2.84		
			0.44	40	2.52		
			0.44	45	2.84	2.73	109
3, a	3.00	29.99	0.56	50	2.68		
			0.56	60	3.21		
			0.54	60	3.33	3.07	102
Average Percentage							100

indicated, but the estimation is difficult. By using smaller strips, however, a more accurate reading of the band may be obtained and the delicacy of the method increased. These small strips, as in the arsenic method, are made by cutting the large strip in two and again dividing these pieces lengthwise, giving a piece 2 mm. wide and 35 mm. long. This is inserted in a tube of 2 mm. diameter, affixed to the usual deposition tube by a rubber connector. A series of standards is then made of any amounts of the smaller quantities of which it may be desirable to get an approximate estimate. The upper part of the Plate shows the bands obtained from amounts of antimony equivalent to 0.5, 0.8, 1.0, 2.0, 5.0, and 10.0 mmg. of antimonious oxide.

The bands obtained from 0.5 and 0.8 mmg. are perfectly distinct, but not always differentiated with clearness. From amounts below 0.5 mmg. we have not been able to obtain any indication on the 2 mm. strip. It is safe, therefore, to set the practical limit of the delicacy of the method at 1 mmg. (0.001 mg.) of antimonious oxide (0.0008 mg. of antimony). The absolute delicacy, however, is very nearly half of this amount, — 0.0005 mg. of antimonious oxide, which is equivalent to 0.0004 mg., or one twenty-five-hundredth of a milligram of antimony.

Sanger and Gibson[5] were able to detect and identify by the Berzelius–Marsh method 0.005 mg. of antimonious oxide, but the deposit in the tube from 0.001 mg. was faint. It will thus be seen that the "band" method is much more delicate than the "mirror" method. It is also more convenient and accurate, for the bands are subject to no irregularity of formation comparable to the difficulty of obtaining a mirror of metallic antimony entirely free from oxide. The mirror method, however, is still of value as a confirmation of the other and a check upon its results. The two methods can be applied, if desired, to different portions of the solution which has been prepared for analysis.

The application of the method to the analysis of products containing antimony is under consideration in this laboratory, but we have contented ourselves for the present with showing that very small amounts of antimony may be estimated by it in a solution properly prepared for analysis. A study of its application should include the separation of small amounts of arsenic or antimony from relatively large amounts of the other, concerning which we have now no reliable information.

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THE EQUIVALENT CIRCUITS OF COMPOSITE LINES IN THE STEADY STATE.

By A. E. Kennelly.

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By A. E. Kennelly.

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Definitions and Purpose.

A *composite line* may be defined as an electrically conducting line formed of two or more successive sections, each section having its own length and its own particular uniformly distributed resistance, inductance, capacitance, and leakance. Each such section, considered separately, may be described as a *single line.* A composite line is, therefore, a successive connection of single lines which differ in linear constants.

It has been shown by the writer in a preceding paper[1] that any uniform single line, operated in the steady state, either by single-frequency alternating currents or by continuous currents, may be externally imitated by a symmetrical triple conductor. The triple conductor which can be substituted for a single line in a steady system of electric flow without disturbing the potentials, or currents, at or outside of the line terminals, may be defined as an *equivalent circuit* of the line. A star-connected equivalent circuit, with two equal line branches and a single leak, may be called an equivalent T; while a delta-connected equivalent circuit with two equal leaks, and a single line-resistance or impedance between them, may be called an equivalent Π. It is the object of this paper to extend the laws of equivalent circuits from single lines to composite lines, with or without loads, and also to present formulas for the distribution of current and potential over such composite lines.

Important Practical Application of the Problem.

An important application of this problem is found in telephony. With given sending and receiving apparatus, the commercial operativeness of a telephonic metallic circuit apparently depends only on the strength of alternating current, at a certain standard frequency, in

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[1] "Artificial Lines for Continuous Currents in the Steady State." See appended Bibliography.

the receiver. That is, it depends on the "receiving-end impedance" of the circuit, or the ratio of the impressed standard-frequency alternating emf. at the sending end, to the current-strength at the receiving end. If this receiving-end impedance of the circuit, including the impedance of the receiving apparatus, is not greater than 25,000 ohms (12,500 ohms per wire), at the angular velocity $\omega = 5000$ radians per second, commercial telephony will readily be possible with the standard Bell telephone apparatus used in the United States; unless the distortion of the speech-waves, due to unequal attenuation at different frequencies, is unusually great. If the circuit receiving-end impedance exceeds 200,000 ohms (100,000 ohms per wire) at $\omega = 5000$ radians per second, even expert telephonists will ordinarily be unable to converse with this apparatus over the line.

It is easy, with the aid of formulas given in the above-mentioned preceding paper, to find the equivalent $\sqcap$ of a simple single telephone line of given length, uniform linear constants, and assigned terminal conditions. But for most practical purposes this is not enough. Most long telephone lines in practical service are not single, but composite. Consider the case of a subscriber A, in Boston, talking to a subscriber B, in New York. First there is the terminal apparatus at A; then, say, a few kilometers of underground line in Boston. Next comes the long-distance overhead line from Boston to New York, perhaps consisting of more than one section and size of wire. Then come one or more sections of underground wire in New York, before we end the circuit in B's apparatus. At two or three intermediate exchanges in this circuit there may also be casual loads, formed by supervisory relays, or other instruments. The critical receiving-end impedance must not be exceeded in this composite circuit, if the talking is to be of satisfactory quality. Actual trial of the line by conversation will determine, with a fair degree of precision, whether the limiting permissible receiving-end impedance has been exceeded by the line. But the designing telephone engineer seeks to know, in advance, whether a certain projected composite line will, when constructed, fall within the permissible limit of receiving-end impedance. If working formulas can be developed, that are not too lengthy and complicated, for determining the receiving-end impedance of composite lines, they may help the designing engineer to decide questions of line construction.

In this paper the discussion will be principally confined to direct-current composite lines. The formulas thus derived are all easily presented, grasped, and checked by Ohm's law, since they involve only real numerical quantities. In the direct-current case the hyperbolic quantities used are all functions of simple real numerics, for which published tables are available. Identically the same formulas are, however, applicable to single-frequency alternating-current cases, by expanding their interpretation from real to complex numbers; or from one space-dimension into two, using impedances for resistances and plane-vectors for potentials and currents. Unfortunately, however, we have no tables of hyperbolic sines, cosines, and tangents available, as yet, for complex arguments except for the particular case of semi-imaginaries,[2] or plane-vectors of 45°; so that in working out the alternating-current cases, as, for example, in telephony, the engineer is delayed by having to assume the duties of a computer, and to work out his own hyperbolic sines, cosines, and tangents. However, even thus handicapped, it is claimed that the formulas here presented will not be too lengthy for the engineer to use in important cases. If hyperbolic tables of complex arguments were worked out and published, the formulas could, with their help, be applied almost as quickly and conveniently to alternating-current cases as they can be applied at present to direct-current cases. If, however, attempts are made to obtain alternating-current results of like precision without the use of hyperbolic functions, there seems to be no hope of helping the engineer. Only specially trained mathematicians could handle the long and complex resulting formulas.

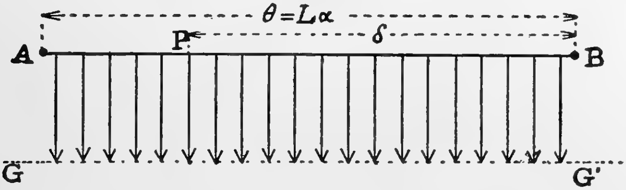

FIGURE 1. Uniform line with distributed resistance and leakance.

PRELIMINARY REVIEW OF SINGLE-LINE FORMULAS.

In order to pass to composite lines, we may first briefly review the laws of equivalent circuits for single lines. The fundamental formulas will be given for direct-current (D. C.) and for alternating-current (A. C.) circuits, in parallel columns.

Let AB, Figure 1, be a uniform single line operated to ground, or zero-potential, return circuit.

L = the length of the line in kilometers (or miles).

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[2] See Table appended to "The Alternating-Current Theory of Transmission-Speed over Submarine Cables," referred to in the Bibliography.

$r =$ the linear resistance of the line (ohms per wire km.).
$g =$ the linear leakance of the line (mhos per wire km.).
$l =$ the linear inductance of the line (henrys per wire km.).
$c =$ the linear capacitance of the line (farads per wire km.).
$n =$ the frequency of the impressed emf. at A (cycles per second).
$\omega = 2\pi n$, the angular velocity of the impressed emf. at A (radians per second).
$j = \sqrt{-1}$.

The attenuation constant of the line is

$$\text{D. C.} \quad \alpha = \sqrt{rg} \quad \frac{\text{hyps}}{\text{km.}}; \quad \text{A. C.} \quad \alpha = \sqrt{(r + jl\omega)(g + jc\omega)} \; \frac{\text{hyps}}{\text{km.}} \angle. \tag{1}$$

In the D. C. case α is a real numerical quantity which we may, for convenience of subsequent operation, define as a "linear hyperbolic angle," or "hyperbolic angle" per km. of length. Although it is a simple numeric per unit length of line, yet, since it forms the basis of argument in hyperbolic tables, we may call it a "hyperbolic angle" per unit length of line, and denote a hyperbolic unit angle as a "hyp." In the A. C. case α is a plane-vector "hyperbolic angle," or complex quantity, per unit length of line.

The hyperbolic angle subtended by the line AB is

$$\text{D. C.} \quad \theta = L\alpha \quad \text{hyps}; \qquad \text{A. C.} \quad \theta = L\alpha \quad \text{hyps} \angle. \tag{2}$$

θ is a real numeric for the D. C. case, and a plane-vector, or numeric at a definite angle in the reference plane, for the A. C. case. The surge-resistance, or surge-impedance, of the line is

$$\text{D. C.} \quad z = \sqrt{\frac{r}{g}} \quad \text{ohms}; \qquad \text{A. C.} \quad z = \sqrt{\frac{r + jl\omega}{g + jc\omega}} \quad \text{ohms} \angle. \tag{3}$$

The *surge-impedance* of an A. C. line is the impedance that the line offers at any point of its length to the propagation of waves of the frequency considered. It is a vector resistance, or impedance, often closely approximating numerically to $\sqrt{l/c}$. The *surge-admittance* of a line is the reciprocal of its surge-impedance.

In wave-propagation theory, and also in the steady-state theory here considered, θ and z, the hyperbolic angle and surge-impedance of a line, are its fundamental characteristics; while r, g, l, and c are its secondary or incidental characteristics.

Single Line Freed at Distant End.

If the line AB is freed at B, its resistance at A, measured to ground, is

$$R_{fA} = z \coth \theta \qquad \text{ohms.} \quad (4)$$

In the D. C. case the hyperbolic angle θ is a simple real quantity, z is a simple numerical resistance, and coth θ is the hyperbolic cotangent of θ, a real numeric, obtainable from tables of hyperbolic functions. Consequently, R_{fA} is a simple resistance in ohms. In the A. C. case, however, z is an impedance, or vector resistance, θ is also a vector quantity, and the hyperbolic cotangent of this vector is not ordinarily obtainable from any tables thus far published. It must be computed, say, with the aid of formula (142). The product of z and this cotangent is, therefore, a vector resistance, or impedance, R_{fA}. Similarly, all the remaining formulas of this paper may be regarded as applying either to D. C. or to A. C. cases; but the D. C. reasoning will be followed, for simplicity of numerical check.

At any point P (Figure 1) along the line, distant l' km. from B, its hyperbolic angular distance from B will be

$$\delta = l'a \qquad \text{hyps.} \quad (5)$$

The potential at P is

$$u_P = u_B \cosh \delta \qquad \text{volts,} \quad (6)$$

where u_B is the potential at the far free end B, defined by the condition

$$u_B = u_A / \cosh \theta \qquad \text{volts;} \quad (7)$$

whence

$$u_P = u_A \frac{\cosh \delta}{\cosh \theta} \qquad \text{volts.} \quad (8)$$

The curve of potential, or voltage to ground, plotted as ordinates along the line AB is, therefore, a curve of hyp. cosines, or a catenary. In the A. C. case the curve of vector lengths, or numerical values, of potential, plotted as ordinates along AB, is a sinusoid superposed upon a catenary.

The current-strength at the point P is

$$\dot{i}_P = \dot{i}_A \frac{\sinh \delta}{\sinh \theta} \qquad \text{amperes,} \quad (9)$$

where $\dot{i}_A$ is the current entering the line at A. The curve of current-strength plotted as ordinates along AB is, therefore, in the D. C. case, a curve of hyp. sines, or curve of catenary-slope.

The resistance of the line, at and beyond the point P, measured to ground is

$$R_{fP} = z \coth \delta \qquad \text{ohms,} \quad (10)$$

or

$$R_{fP} = R_{fA} \frac{\coth \delta}{\coth \theta} \qquad \text{ohms.} \quad (11)$$

SINGLE LINE GROUNDED AT DISTANT END.

If the line, instead of being freed at B (Figure 1), is grounded at B, its resistance at A is

$$R_{gA} = z \tanh \theta \qquad \text{ohms.} \quad (12)$$

At any point P, angularly distant δ hyps from B, the line resistance beyond P, measured to ground, is

$$R_{gP} = z \tanh \delta \qquad \text{ohms,} \quad (13)$$

or

$$R_{gP} = R_{gA} \frac{\tanh \delta}{\tanh \theta} \qquad \text{ohms.} \quad (14)$$

The potential at P, in terms of the potential u_A at A, is

$$u_P = u_A \frac{\sinh \delta}{\sinh \theta} \qquad \text{volts.} \quad (15)$$

The current-strength at P, in terms of the current-strength i_A entering the line at A, is

$$i_P = i_A \frac{\cosh \delta}{\cosh \theta} \qquad \text{amperes.} \quad (16)$$

For example, consider a line, AB, Figure 1, of $L = 100$ km., with a linear resistance r of 20 ohms per wire-km., and a linear leakance g of 20×10^{-6} mho per wire km. (20 micromhos per km.), corresponding to a linear insulation-resistance of 50,000 km-ohms. The attenuation-constant of this line is $\alpha = 2 \times 10^{-2}$ hyp. per km. by (1), and the hyperbolic angle subtended by the line is $\theta = 2$ hyps. by (2). The surge-resistance of the line is $z = 1000$ ohms by (3). Then the resistance offered by the line at A, when freed at B, is, by (4),

$$R_{fA} = 1000 \coth 2 = 1000 \times 1.037315 = 1{,}037.315 \text{ ohms,}$$

and when grounded at B, by (12),

$$R_{gA} = 1000 \tanh 2 = 1000 \times 0.964026 = 964.026 \text{ ohms.}$$

EQUIVALENT CIRCUITS OF SINGLE LINE.

The equivalent T of this line is a star-connection of three resistances AO, GO, BO (Figure 2), two of which — the line-branches AO, OB, — are equal; while OG is a leakage-resistance to ground. This equivalent T, when correctly proportioned, has the property of being able to replace the uniformly leaky line AB, without disturbing in any manner the system of potentials and currents outside the terminals ABG. Let $g' = 1/R'$ be the conductance of the leak OG′; then

$$g' = y \sinh \theta \quad \text{mhos,} \quad (17)$$

where $y = 1/z$ mhos, the reciprocal of the surge-resistance. We may call y the *surge-conductance* (A. C. *surge-admittance*).

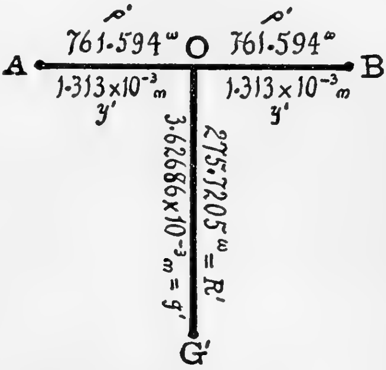

FIGURE 2. Equivalent T of uniform line.

Let ρ' be the resistance of each line-branch AO, OB; then

$$\rho' = z \tanh \frac{\theta}{2} = z \coth \theta - R' = R_f - R' \quad \text{mhos.} \quad (18)$$

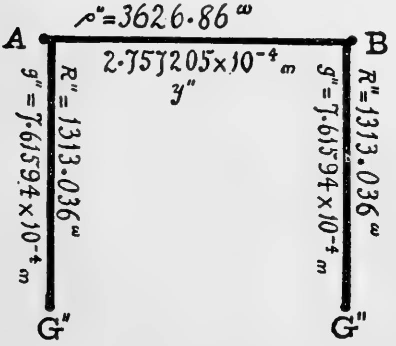

FIGURE 3. Equivalent ⊓ of uniform line.

Thus, for the line above considered, $g' = 0.001 \times \sinh 2 = 0.001 \times 3.62686 = 3.62686 \times 10^{-3}$ mho; while $R' = 1/g' = 275.7205$ ohms. $\rho' = 1000 \coth 2 - 275.7205 = 761.594$ ohms.

The equivalent ⊓ of the line is a delta-connection of three resistances A B, A G″, B G″ (Figure 3), the two "pillars" or leaks AG″, BG″, being equal conductances of g'' mhos each, and the "architrave" AB being the line-resistance ρ''.

$$\rho'' = z \sinh \theta \qquad \text{ohms} \quad (19)$$

and

$$g'' = 1/R'' = y \tanh \frac{\theta}{2} \qquad \text{mhos}$$

$$= y \coth \theta - y'' = G_g - y'' \qquad \text{mhos,} \quad (20)$$

where $y'' = 1/g''$ is the architrave conductance, and $G_g = 1/R_g$ is the conductance to ground of the line at one end, when grounded at the other end.

Thus, for the line considered, $\rho'' = 1000 \sinh 2 = 3626.86$ ohms, and $g'' = 0.001 \coth 2 - 2.757204 \times 10^{-4} = 7.6159 \times 10^{-4}$ mho.

Single Line Corresponding to a Symmetrical T or Π.

Reciprocally, any star connection of three resistances AO, GO, BO (Figure 2), having two equal line-branches AO and OB of ρ' ohms, with a leak to ground of $R' = 1/g'$ ohms, corresponds to some smooth uniform line of angle,

$$\theta = 2 \sinh^{-1} \sqrt{\frac{\rho'}{2R'}} \qquad \text{hyps,} \quad (21)$$

and of surge-resistance,

$$z = \sqrt{\rho'(\rho' + 2R')} \qquad \text{ohms.} \quad (22)$$

Likewise, any delta-connection ABG″G″ (Figure 3) with two equal grounded leaks of resistance $R'' = 1/g''$ ohms, connected by an architrave of ρ'' ohms, corresponds to a smooth uniform line of angle,

$$\theta = 2 \tanh^{-1} \sqrt{\frac{\rho''}{2R'' \times \rho''}} \qquad \text{hyps,} \quad (23)$$

and of surge-resistance,

$$z = R'' \tanh \frac{\theta}{2} \qquad \text{ohms.} \quad (24)$$

Equivalent Circuits of Single Line in Terms of Resistances of Line Free and Grounded.

If the line be first freed and then grounded at one end, say B (Figure 1), and the resistance of the line be measured correctly at the other end in each case (R_f and R_g respectively), we have for the equivalent T of the line,

$$\rho' = R_f \left(1 - \sqrt{1 - \frac{R_g}{R_f}}\right) \qquad \text{ohms,} \quad (25)$$

$$R' = R_f \sqrt{1 - \frac{R_g}{R_f}} \qquad \text{ohms.} \quad (26)$$

Similarly, we have for the equivalent Π of the line,

$$\rho'' = R_g \Big/ \sqrt{1 - \frac{R_g}{R_f}} \qquad \text{ohms,} \quad (27)$$

$$R'' = R_f\left(1 + \sqrt{1 - \frac{R_g}{R_f}}\right) \qquad \text{ohms.} \quad (28)$$

From which

$$\frac{\rho' + R''}{2} = R_f \qquad \text{ohms.} \quad (29)$$

$$r/g = R''\rho' = R'\rho'' = R_f\,R_g = z^2 \qquad (\text{ohms})^2 \quad (30)$$

$$\theta = L\alpha = \tanh^{-1}\sqrt{\frac{R_g}{R_f}} \qquad \text{hyps.} \quad (31)$$

The last two formulas serve to evaluate z and θ for any single line, when the sending-end impedances of that line (R_f and R_g) have been correctly measured.

Looped or Metallic-Return Single Circuits.

If we consider single metallic circuits, like those of wire-telephony, or of single-phase power-transmission,

Let $r_{||}$ = the linear resistance (ohms per loop km.).
$g_{||}$ = the linear leakance (mhos per loop km.).
$l_{||}$ = the linear inductance (henrys per loop km.).
$c_{||}$ = the linear capacitance (farads per loop km.).

Then

$$\left.\begin{array}{ll} r_{||} = 2\,r & \text{ohms per km.} \\ g_{||} = g/2 & \text{mhos per km.} \\ l_{||} = 2l & \text{henrys per km.} \\ c_{||} = c/2 & \text{farads per km.} \end{array}\right\} \quad (32)$$

where r, g, l, and c are the corresponding linear constants per wire km. Substituting in equations (1), (2), and (3), we have

$$\alpha_{||} = \alpha \qquad \text{hyps per loop km.,} \quad (33)$$

$$\theta_{||} = \theta \qquad \text{hyps,} \quad (34)$$

and

$$z_{||} = 2\,z \qquad \text{ohms.} \quad (35)$$

That is, the attenuation-constant, and the angle subtended by the looped line, are respectively identical with the attentuation-constant and angle subtended by one wire only operated to zero potential. The surge-impedance of the metallic circuit is double the surge-impedance of one wire to ground, or zero potential. The voltage impressed upon the loop is, however, double the voltage impressed on each wire singly worked to zero-potential plane, so that the current-strength in the circuit is the same with either method of computation.

The above conditions are illustrated in Figure 4, where ABB′A represents a simple metallic-return telephone circuit with a transmitter induction coil of impedance Z_s at A, and a receiver of impedance Z_r at B. One half of this circuit, with only one wire and ground return, is indicated at AB on the right hand. The length of the circuit has been taken as $L = 50$ km. (31.068 statute miles), and the following linear constants have been assumed:

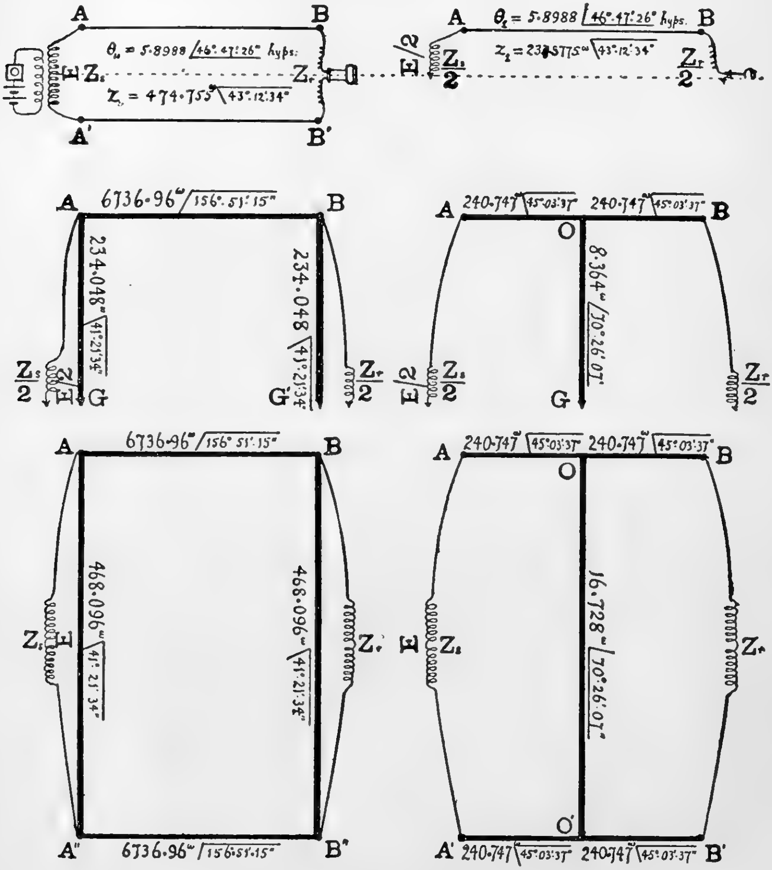

FIGURE 4. Equivalent circuits of lines with ground return and metallic return.

$r_{\shortparallel} = 55.92$ ohms per loop km. (90 ohms per loop-mile) ; $g_{\shortparallel} = 0$
$l_{\shortparallel} = 0.70 \times 10^{-3}$ henry per loop km. (1.126 millihenry per loop-mile)
$c_{\shortparallel} = 0.049{,}7 \times 10^{-6}$ farad per loop km. (0.08×10^{-6} farad per loop-mile); values which correspond to

$$r = 27.96 \text{ ohms per wire km.}$$
$$l = 0.35 \times 10^{-3} \text{ henry per wire km.}$$
$$c = 0.099{,}4 \times 10^{-6} \text{ farad per wire km.}$$

Substituting the above values in (1), (2), and (3), we obtain at $\omega =$ 5,000 radians per second :

$\alpha_{\shortparallel} = \alpha = 0.117{,}976{,}6\ \underline{/46^\circ\ 47'\ 26''}$ hyps per loop km., or per wire km.
$\theta_{\shortparallel} = \theta = 5.898{,}83\ \underline{/46^\circ\ 47'\ 26''}$ hyps for both the double line and the single line.
$z_{\shortparallel} =\ 474.755\ \overline{\backslash 43^\circ\ 12'\ 34''}$ ohms for the loop circuit.
$z = 237.377{,}5\ \overline{\backslash 43^\circ\ 12'\ 34''}$ ohms for the single line.

The equivalent ⊓ and ⊤ of one wire are indicated at ABGG′ and AOBG in Figure 4. The architrave impedance AB is $6{,}736.96\overline{/156^\circ\ 51'\ 15''}$ ohms, which is also the receiving-end impedance of each line, excluding the receiving instrument Z_r; because, if we ground the line at B, the current which will flow to ground at B will be the impressed potential at A divided by this architrave impedance.

The equivalent circuits of the loop line are indicated at ABB″A″ and AOBB′O′A′ (Figure 4). The former is a rectangle of impedances, and the latter an I of impedances. It will be seen that the rectangle ABB″A″ is merely a doublet of the single line ⊓, ABG′G; while the I, AOBB′O′A′ is merely a doublet of the single line ⊤, AOBG. The receiving-end-impedance of the loop-circuit is evidently $2 \times 6{,}736.96\overline{/156^\circ\ 51'\ 15''} = 13{,}473.92\overline{/156^\circ\ 51'\ 15''}$ ohms, excluding the receiving instrument Z_r.

Since, then, the equivalent circuits of metallic-circuit or loop-lines are mere doublets of those for their component single wires, and the latter are easier to think about and discuss, we will confine our attention to the latter.

COMPOSITE LINES.

First Case. *Sections of the same Attenuation-Constant and of the same Surge-Impedance.*

If a line AB (Figure 5) of L_1 km. is connected to a line CD of L_2 km., and each has the same attenuation constant α, and the same surge-resistance z ohms (conditions which imply the same linear constants), the line angles will be $\theta_1 = L_1\alpha$ and $\theta_2 = L_2\alpha$ hyps respectively. Then, if we free the composite line at D, the resistance at A is

$$R_f = z \coth(\theta_1 + \theta_2) \qquad \text{ohms,} \quad (36)$$

while, if the composite line be grounded at D, the resistance at A is

$$R_g = z \tanh(\theta_1 + \theta_2) \qquad \text{ohms.} \quad (37)$$

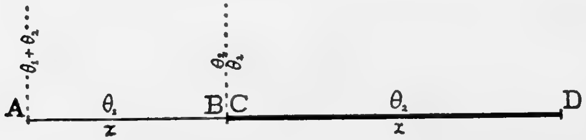

Figure 5. Composite line with sections of the same attenuation-constant and surge-resistance.

Reciprocally, freeing and grounding the composite line at A, we get resistances R_f and R_g at D, respectively the same as in (36) and (37).

It is evident, then, that the composite line differs in no way, except in length, from either of the component sections. The angle subtended by the whole line AD is the sum of the component section line-angles.

Second Case. *Sections of different Attenuation-Constant but of the same Surge-Impedance.*

If a section CD (Figure 5) of L_2 km. be connected to a section AB of L_1 km., and their respective linear constants r_2, g_2, and r_1, g_1 are such that their attenuation constants α_1, α_2 differ; while their surge-resistances z are the same, we assign the angles subtended by the sections $\theta_1 = L_1\alpha_1$ and $\theta_2 = L_2\alpha_2$ hyps. The angle subtended by the whole line will then be $\theta_1 + \theta_2$, as in the preceding case. That is, except for a disproportionality between the section-angles and their line-lengths, two sections of different attenuation-constant, but of the same surge-resistance, connect together like two sections of one and the same type of line. This is for the reason that in the unsteady state, or period of current building prior to the formation of the steady state here discussed, there is neither wave reflection nor discontinuity of wave propagation at the junction BC, when the surge resistance or impedance z is the same on each side thereof.

In order, however, to simplify the transition to complex cases later on, we may pause to consider the following case of two sections, with different a but the same z.

$$L_1 = 100 \text{ km.}, \; r_1 = 20 \text{ ohms/km.}, \; g_1 = 2 \times 10^{-5} \text{ mho/km.}$$
$$L_2 = 100 \text{ km.}, \; r_2 = 10 \text{ ohms/km.}, \; g_2 = 10^{-5} \text{ mho/km.}$$

Whence
$$a_1 = 0.02 \text{ hyp/km.}, \; z_1 = 1000 \text{ ohms};$$
$$a_2 = 0.01 \text{ hyp/km.}, \; z_2 = 1000 \text{ ohms.}$$

Merger Equivalent Circuits of Composite Lines.

Figure 6 shows the two lines at AB and CD respectively. It also shows the Π and T equivalent circuits of AB at A″B″G″G″ and A′OB′G′, likewise of CD at C″D″G″G″ and C′OD′G′. If we connect the sections together at BC, into a composite line AD, we virtually connect together some one pair of the combinations of equivalent circuits $\Pi_{AB}\Pi_{CD}$, $T_{AB}T_{CD}$, $\Pi_{AB}T_{CD}$, $T_{AB}\Pi_{CD}$. The first two combinations are shown at ABCDGGG and A′OBCOD′G′G′. If we merge together the two elements of any such pair by known formulas,[3] we arrive either at the equivalent Π, ADGG; or the equivalent T, AODG, of the composite line.

The equivalent Π or T of a composite line, computed by the merging of the Πs or Ts of the component sections, may be called the "merger Π" or "merger T" of the line, to distinguish them from the Π or T computed directly from the composite lines by the formulas to be presented later. The latter may be called, for distinction, the "hyperbolic Π" or T. For a given degree of precision, it will be found much easier to compute the hyperbolic Π or T of a composite line than to compute the merger Π or T. In all the examples given in this paper the equivalent Π and T of the various composite lines considered have both been derived hyperbolically, but have also been checked by the merging process.

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[3] "The Equivalence of Triangles and Three-Pointed Stars in Conducting Networks," A. E. Kennelly, *Electrical World and Engineer*, Vol. 34, No. 12, Sept. 16, 1899, pp. 413–414.

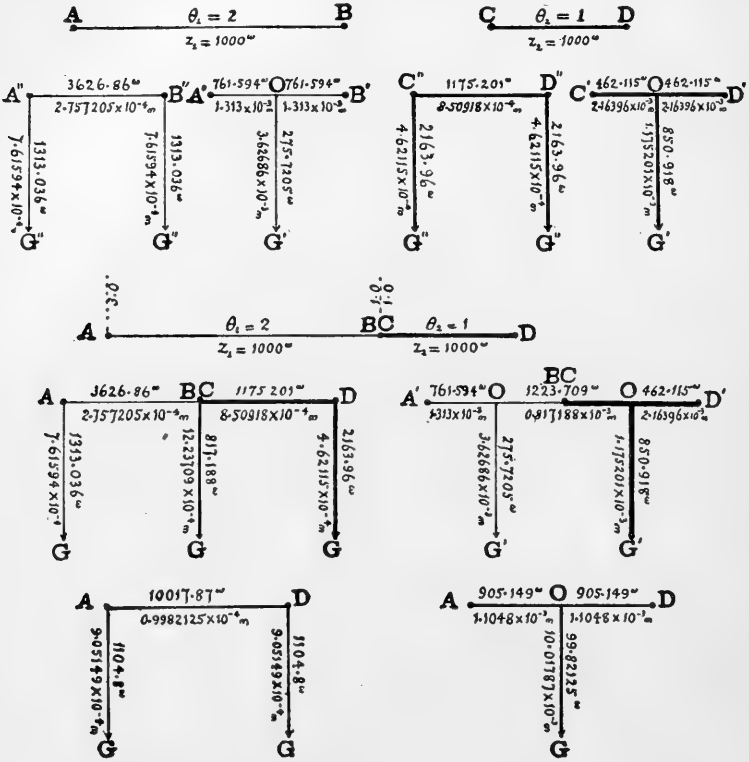

FIGURE 6. Composition of two sections with the same surge-resistance but with different attenuation-constants.

Equivalent ⊓.

In order to compute hyperbolically the equivalent ⊓ of the composite line AD (Figure 6) we proceed as follows:

Ground either end of the composite line AD, say the end D. Assign the junction-angle θ_2 at BC. Then the angle subtended by the composite line at A will be $\delta_A = \theta_1 + \theta_2$ hyps. The sending-end resistance of the composite line at A is, by (12) and (37),

$$R_{gA} = z_1 \tanh \delta_A \qquad \text{ohms} \quad (38)$$

$$= 1{,}000 \tanh 3 = 995.055 \text{ ohms.}$$

$$G_{gA} = 1/R_{gA} = y_1 \coth \delta_A \qquad \text{mhos} \quad (39)$$

$$= 0.001 \times \coth 3 = 10.049{,}7 \times 10^{-4} \text{ mho.}$$

Then the architrave resistance AD of the composite ⊓ will be:

$$\rho'' = z_1 \sinh \delta_A \qquad \text{ohms} \quad (40)$$
$$= 1{,}000 \sinh 3 = 10{,}017.87 \text{ ohms.}$$
$$y'' = 1/\rho'' = 0.998{,}212{,}5 \times 10^{-4} \text{ mho.}$$

The conductance g''_A of the leak at A is, by (20),

$$g''_A = y_1 \coth \delta_A - y'' \qquad \text{mho} \quad (41)$$
$$= 9.051{,}49 \times 10^{-4} \text{ mho.}$$

If we ground the composite line at A instead of at D, the angle subtended by the whole line at D will be $\delta_D = \theta_1 + \theta_2 = \delta_A$. The architrave resistance DA will be the same as that given in (40). The sending-end resistance R_{gD} and conductance G_{gD} will be identical with R_{gA} and G_{gA} respectively, by (38) and (39); so that the leak-conducttance g''_D at D will be identical with g''_A by (41). This completes the hyperbolic ⊓, ADGG of the composite line.

Equivalent T.

To find the hyperbolic equivalent T of the composite line AD (Figure 6), free the line at one end, say D. Then the angle subtended by the line at A will be, as before, $\delta_A = \theta_1 + \theta_2$ hyps.

The sending-end resistance of the line at A will be, by (4),

$$R_{fA} = z \coth \delta_A \qquad \text{ohms} \quad (42)$$
$$= 1{,}000 \coth 3 = 1{,}004.97 \text{ ohms.}$$

The conductance of the leak OG is, by (17),

$$g' = y \sinh \delta_A \qquad \text{mhos} \quad (43)$$
$$= 0.001 \sinh 3 = 10.017{,}87 \times 10^{-3} \text{ mhos.}$$

and its resistance is

$$R' = 1/g' = 99.821{,}25 \text{ ohms.}$$

The resistance of the AO branch is, then, by (18),

$$\rho' = R_{fA} - R' \qquad \text{ohms} \quad (44)$$
$$= 1{,}004.97 - 99.821 = 905.149 \text{ ohms.}$$

Similarly, if we free the composite line at A, instead of at D, the angle subtended by the line at D will be δ_D. As before, $\delta_D = \theta_2 + \theta_1 = \delta_A$

hyps. The sending-end resistance offered by the line at D will then be, by (4) and (42), identical with that found previously at A. The conductance of the leak will, by (17) and (43), be the same as that found from A. Finally, the resistance of the DO line-branch will, by (18) and (44), be identical with that of the AO branch (905.149^{ω}). This completes the ⊤ of the composite line.

We may infer from the above reasoning, and it may be readily demonstrated formally, that when a composite line is composed of sections differing in linear constants, but having the same surge-impedance, the angle subtended by the whole line is the same at either end, and whether the distant end be freed or grounded. Consequently the equivalent ⊓ and ⊤ of the composite line will be symmetrical. That is, the two leaks of the ⊓ are equal and the two line branches of the ⊤ are equal.

Conversely, it follows, from equations (21) to (24), that any composite line made up of sections differing in attenuation constant, but with the same surge-impedance, may be replaced by an equivalent single line of uniform attenuation and linear constants.

Third and General Case. Sections with Different Surge-Impedances.

Let a section AB of 100 km. (Figure 7) be connected to a section CD of 300 km., and let their respective linear constants be as follows:

$$r_1 = 20 \text{ ohms/km.}; \; g_1 = 20 \times 10^{-6} \text{ mho/km.}$$
$$r_2 = 10 \text{ ohms/km.}; \; g_2 = 2.5 \times 10^{-6} \text{ mho/km.}$$

from which

$$a_1 = 0.02 \text{ hyp/km.}; \; \theta_1 = 2 \text{ hyps}; \; z_1 = 1{,}000 \text{ ohms};$$
$$a_2 = 0.005 \text{ hyp/km.}; \; \theta_2 = 1.5 \text{ hyps}; \; z_2 = 2{,}000 \text{ ohms},$$

so that the surge-resistance of the two sections are unequal. It follows that the angle subtended by the composite line will differ at the two ends, and will also differ according to whether the distant end is freed or grounded.

Equivalent ⊓.

Let us ground the end A_2 of the composite line A_2D_2 (Figure 7). Then by formula (12), the sending-end resistance at B of the section BA grounded, will be

$$R_{gB} = z_1 \tanh \theta_1 \qquad \text{ohms} \quad (45)$$
$$= 1{,}000 \tanh 2.0 = 964.026{,}5 \text{ ohms.}$$

The angle of the section AB, at its end B, is $\delta_B = 2$ hyps. At the junction BC, however, the line-angle changes abruptly, owing to the change in surge-resistance, and at C, just across the junction it is

$$\delta_C = \tanh^{-1}\left(\frac{z_1}{z_2}\tanh\theta_1\right) = \tanh^{-1}\left(\frac{R_{gB}}{z_2}\right) \qquad \text{hyps. (46)}$$

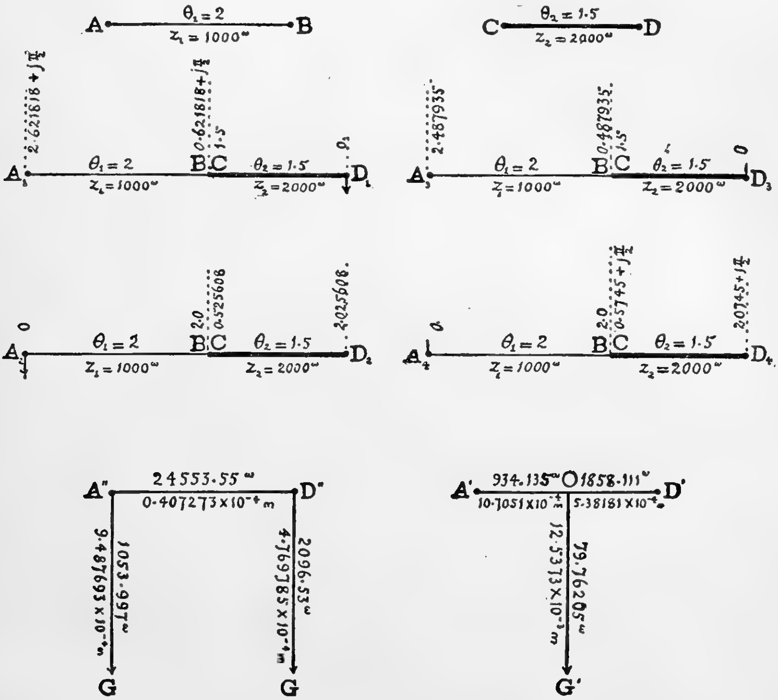

FIGURE 7. Composition of two sections of different surge-resistances and different attenuation-constants.

That is, the hyp-tangent of the new angle is the ratio of the sending-end resistance at B to the surge-resistance of the new section CD. In this case

$$\delta_C = \tanh^{-1}\left(\frac{964.026,5}{1000}\right) = \tanh^{-1} 0.964,026,5\,;$$

or, by tables of hyperbolic tangents, $\delta_c = 0.525,608$ hyp. We mark this angle opposite to C on the line A_2D_2 (Figure 7). The angle subtended at D_2 by the composite line is, therefore,

$$\delta_D = \theta_2 + \delta_C = 2.025{,}608 \text{ hyps.}$$

The sending-end resistance of the grounded composite line is then, at D_2, by (12), (37), (38), and (45),

$$\begin{aligned} R_{gD} &= z_2 \tanh \delta_D && \text{ohms (47)} \\ &= 2{,}000 \tanh 2.025{,}608 = 1931.58 \text{ ohms,} \end{aligned}$$

and the sending-end conductance,

$$\begin{aligned} G_{gD} &= y_2 \coth \delta_D = 1/R_{gD} && \text{mhos (48)} \\ &= 0.000{,}517{,}71 \text{ mho.} \end{aligned}$$

The formula for finding the architrave resistance of the equivalent ⊓ of the line AD is

$$\begin{aligned} \rho'' &= z_2 \sinh \delta_D \cdot \frac{\cosh \delta_B}{\cosh \delta_C} && \text{ohms (49)} \\ &= 2{,}000 \sinh 2.025{,}608 \times \frac{\cosh 2.0}{\cosh 0.525{,}608} \\ &= 24{,}553.55 \text{ ohms} \end{aligned}$$

and $\qquad y'' = 1/\rho'' = 0.407{,}273 \times 10^{-4}$ mho.

Formula (49) differs from the corresponding formula (40) of the preceding case by the application of the ratio $\frac{\cosh \delta_B}{\cosh \delta_C}$ or the ratio of the cosines of the line-angles across the junction BC.

The formula for finding the conductance of the leak at D is, as before (20) and (41),

$$\begin{aligned} g''_D &= G_{gD} - y'' = 1/R_{gD} - y'' && \text{mhos (50)} \\ &= 4.769{,}785 \times 10^{-4} \text{ mho.} \end{aligned}$$

In order to complete the equivalent ⊓ of the line AD hyperbolically, we must repeat the above process from the opposite end, by grounding the end D_1, as shown at A_1D_1 (Figure 7). The line angle at C is $\delta_C =$ 1.5 hyps. Across the junction BC this angle changes suddenly to

$$\begin{aligned} \delta_B &= \tanh^{-1}\left(\frac{z_2 \tanh \theta_2}{z_1}\right) && \text{hyps (51)} \\ &= \tanh^{-1} 1.810{,}296. \end{aligned}$$

This involves at first sight an impossible result; but in all cases of a hyperbolic tangent greater than unity, we may resort to the following formulas:

$$\left.\begin{aligned} \sinh\left(x \pm j\frac{\pi}{2}\right) &= \pm j \cosh x \\ \cosh\left(x \pm j\frac{\pi}{2}\right) &= \pm j \sinh x \\ \tanh\left(x \pm j\frac{\pi}{2}\right) &= \coth x \\ \coth\left(x \pm j\frac{\pi}{2}\right) &= \tanh x \end{aligned}\right\} \qquad \text{numeric.} \quad (52)$$

We thus obtain

$$\begin{aligned} \delta_B - j\frac{\pi}{2} &= \coth^{-1}\left(\frac{z_2 \tanh 1.5}{z_1}\right) \qquad \text{hyps} \quad (53) \\ &= \coth^{-1} 1.810,296 \\ &= 0.621,818 \ \text{hyp} \end{aligned}$$

and

$$\delta_B = 0.621,818 + j\frac{\pi}{2} \ \text{hyp.}$$

This difficulty with seemingly impossible antitangents or anticotangents is not encountered in the A. C. case.

We inscribe this value of δ_B opposite B on the line AD. The angle subtended by the whole line at A will then be

$$\theta_1 + \delta_B = \delta_A = 2.621,818 + j\frac{\pi}{2} \ \text{hyps.}$$

The sending-end resistance of the grounded composite line is then at A_1, by (12), (37), (38), and (47),

$$\begin{aligned} R_{gA} &= z_1 \tanh \delta_A \qquad \text{ohms} \quad (54) \\ &= 1,000 \tanh\left(2.621,818 + j\frac{\pi}{2}\right) \\ &= 1,000 \coth 2.621,618 = 1,010.64 \ \text{ohms,} \end{aligned}$$

and the sending-end conductance, as in (48),

$$\begin{aligned} G_{gA} &= y_1 \coth \delta_A \\ &= y_1 \coth\left(2.621,818 + j\frac{\pi}{2}\right) \\ &= 0.001 \tanh 2.621,818 = 9.894,966 \times 10^{-4} \ \text{mho.} \end{aligned}$$

The architrave resistance, as in (49), is

$$\rho'' = z_1 \sinh \delta_A \cdot \frac{\cosh \delta_C}{\cosh \delta_B} \qquad \text{ohms} \quad (55)$$

$$= 1{,}000 \cosh 2.621{,}818 \cdot \frac{\cosh 1.5}{\sinh 0.621{,}818}$$

$$= 24{,}553.55 \ \text{ohms}$$

and $$y'' = 1/\rho'' = 0.407{,}273 \times 10^{-4} \ \text{mho}.$$

The conductance of the ⊓ leak at A is, as in (50),

$$g''_A = G_{gA} - y''$$

$$= 9.487{,}693 \times 10^{-4} \ \text{mho}.$$

Equivalent T.

To compute the equivalent T of the composite line AD (Figure 7), free the line at one end, say D_3, and find the sending-end resistance at C in this condition. It is, by (4), (36), and (42),

$$R_{fc} = z_2 \coth \theta_2$$

$$= 2{,}000 \coth 1.5 = 2{,}209.59 \ \text{ohms}.$$

The line-angle changes abruptly at the junction BC from $\delta_C = 1.5$ to $\delta_B = 0.487{,}935$ hyp, by the condition

$$\delta_B = \coth^{-1}\left(\frac{z_2 \coth \theta_2}{z_1}\right) = \coth^{-1}\left(\frac{R_{fC}}{z_1}\right) \qquad \text{hyps} \quad (56)$$

$$= \coth^{-1} 2.209{,}59 = 0.487{,}935 \ \text{hyp}.$$

The line-angle at the end A_3 is thus $\theta_1 + \delta_B = 2.487{,}935$ hyps. The sending-end resistance at A_3 is finally, by (4),

$$R_{fA} = z_1 \coth \delta_A \qquad \text{ohms} \quad (57)$$

$$= 1{,}000 \coth 2.487{,}935 = 1{,}013.897 \ \text{ohms}.$$

The conductance of the leak OG′ is, by (43),

$$g' = y_1 \sinh \delta_A \cdot \frac{\cosh \delta_C}{\cosh \delta_B} \qquad \text{mhos} \quad (58)$$

$$= 0.001 \times \sinh 2.487{,}935 \times \frac{\cosh 1.5}{\cosh 0.487{,}935} = 12.537{,}3 \times 10^{-3} \ \text{mho}.$$

The resistance of the leak OG′ is, therefore, $R' = 1/g' = 79.762$ ohms. The resistance of the AO branch is then, by (18) and (44),

$$\rho' = R_{fA} - R' \qquad \text{ohms} \quad (59)$$
$$= 1{,}013.897 - 79.762 = 934.135 \text{ ohms.}$$

In order to complete the equivalent T of the line AD, we must repeat the above process from the opposite end, by freeing the end A, as shown at A_4D_4 (Figure 7). The line-angle at B is $\delta_B = 2.0$. Across the junction BC this angle changes suddenly to

$$\delta_C = \coth^{-1}\left(\frac{z_1 \coth \theta_1}{z_2}\right) \qquad \text{hyps} \quad (60)$$
$$= \coth^{-1}\left(\frac{1037.315}{2{,}000}\right) = \coth^{-1} 0.518{,}657{,}5.$$

In order to avoid an impossible operation, apply formula (52)

$$\delta_C - j\frac{\pi}{2} = \tanh^{-1} 0.518{,}657{,}5 = 0.574{,}50 \text{ hyp}$$
$$\delta_C = 0.574{,}5 + j\frac{\pi}{2} \text{ hyps.}$$

The line-angle at the end D_4 is thus $\theta_2 + \delta_C = 2.074{,}5 + j\frac{\pi}{2}$ hyps.

The sending-end resistance at D_4 is finally, by (4) and (57),

$$R_{fD} = z_2 \coth \delta_D \qquad \text{ohms} \quad (61)$$
$$= 2{,}000 \coth\left(2.074{,}5 + j\frac{\pi}{2}\right) = 2{,}000 \tanh 2.074{,}5 = 1{,}937.873 \text{ ohms.}$$

The conductance of the leak OG′ is, therefore, by (43) and (58),

$$g' = y_2 \sinh \delta_D \cdot \frac{\cosh \delta_B}{\cosh \delta_C} \qquad \text{mhos} \quad (62)$$
$$= 0.001 \sinh\left(2.074{,}5 + j\frac{\pi}{2}\right) \cdot \frac{\cosh 2.0}{\cosh\left(0.574{,}5 + j\frac{\pi}{2}\right)}$$
$$= 0.001 \cosh 2.074{,}5 \cdot \frac{\cosh 2.0}{\sinh 0.574{,}5} = 12.537{,}3 \times 10^{-3} \text{ mho.}$$

The resistance of the leak OG′ is, therefore, $R' = 1/g' = 79.762$ ohms. The resistance of the DO branch is then, by (18) and (59),

$$\rho' = R_{fD} - R' \qquad \text{ohms} \quad (63)$$
$$= 1{,}937.873 - 79.762 = 1{,}858.111 \text{ ohms.}$$

This completes the T of the composite line.

It may be inferred from the preceding reasoning that for the case of a composite line of two sections with different surge-impedances, the receiving-end impedance of the line in the absence of receiving instruments, which is the architrave of the line-⊓, has the same value from each end of the line. The leak of the composite line-T has also one and the same value, computed from either end. Both the ⊓ and the T are, however, dissymmetrical. Each requires two separate computations and line-angle distributions, one from each end.

Summary of Two-Section Formulas.

If we expand formulas (40) and (49), we obtain for the architrave of the composite line ⊓

$$\rho'' = z_1 \sinh \theta_1 \cosh \theta_2 + z_2 \cosh \theta_1 \sinh \theta_2 \qquad \text{ohms} \quad (64)$$

$$= \frac{z_1 + z_2}{2} \sinh (\theta_1 + \theta_2) + \frac{z_1 - z_2}{2} \sinh (\theta_1 - \theta_2) \quad \text{ohms [4]} \quad (65)$$

$$= z_1 \sinh \theta_1 \frac{\sinh \delta_D}{\sinh \delta_C} \qquad \text{ohms (line grounded at A)} \quad (66)$$

$$= z_2 \sinh \delta_D \frac{\cosh \delta_B}{\cosh \delta_C} \qquad \text{ohms (line grounded at A)} \quad (67)$$

$$= z_2 \sinh \theta_2 \frac{\sinh \delta_A}{\sinh \delta_B} \qquad \text{ohms (line grounded at D)} \quad (68)$$

$$= z_1 \sinh \delta_A \frac{\cosh \delta_C}{\cosh \delta_B} \qquad \text{ohms (line grounded at D).} \quad (69)$$

Similarly, if we expand formulas (58) and (62), we obtain

$$g' = y_1 \sinh \theta_1 \cosh \theta_2 + y_2 \cosh \theta_1 \sinh \theta_2 \qquad \text{mhos} \quad (70)$$

$$= \frac{y_1 + y_2}{2} \sinh (\theta_1 + \theta_2) + \frac{y_1 - y_2}{2} \sinh (\theta_1 - \theta_2) \quad \text{mhos} \quad (71)$$

$$= y_1 \sinh \theta_1 \frac{\sinh \delta_D}{\sinh \delta_C} \qquad \text{mhos (line freed at A)} \quad (72)$$

$$= y_2 \sinh \delta_D \frac{\cosh \delta_B}{\cosh \delta_C} \qquad \text{mhos (line freed at A)} \quad (73)$$

$$= y_2 \sinh \theta_2 \frac{\sinh \delta_A}{\sinh \delta_B} \qquad \text{mhos (line freed at D)} \quad (74)$$

$$= y_1 \sinh \delta_A \frac{\cosh \delta_C}{\cosh \delta_B} \qquad \text{mhos (line freed at D).} \quad (75)$$

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[4] Formulas (64) and (65) were first published as receiving-end impedances of a two-section composite line by Dr. G. di Pirro. See Bibliography.

Single Lines Equivalent to a Dissymmetrical ⊓ or T.

It is evident that formulas (21) to (24) apply only to a symmetrical ⊓ or T. Moreover, it may be seen that no single smooth and uniform line can correspond to a dissymmetrical ⊓ or T. This means that, in general, no single smooth and uniform line can be the counterpart of a composite line having sections of different surge-resistance. But if we reduce a dissymmetrical ⊓ to a symmetrical ⊓ and a terminal leak, we may apply equations (23) and (24) to transform the symmetrical ⊓ into an equivalent single line. It follows that any composite line may be resolved into one and only one uniform smooth line of the same length with a leak permanently applied to one end; or to an infinitude of such single uniform smooth lines having a leak at each end.

Similarly, the T of a composite line may be reduced to a symmetrical T plus a line-impedance at one end. By the use of equations (21) and (22), we may substitute a single smooth uniform line for the symmetrical T. Consequently, any composite line may be resolved into one and only one uniform smooth line of the same length with a line-impedance at one end; or, to an infinitude of such single uniform smooth lines having a line-impedance at each end.

COMPOSITE LINE WITH THREE SECTIONS OF DIFFERENT SURGE-IMPEDANCES.

A three-section composite line is indicated in Figure 8.

AB has a length L_1 of 100 km.
CD " " L_2 of 300 km.
EF " " L_3 of 50 km.

The respective linear constants are

$r_1 = 20$ ohms/km.; $r_2 = 10$ ohms/km.; $r_3 = 25$ ohms/km.
$g_1 = 20 \times 10^{-6}$ mho/km.; $g_2 = 2.5 \times 10^{-6}$ mho/km.; $g_3 = 4 \times 10^{-6}$ mho/km.
$\alpha_1 = 0.02$ hyp/km.; $\alpha_2 = 0.005$ hyp/km.; $\alpha_3 = 0.01$ hyp/km.
$\theta_1 = 2$ hyps; $\theta_2 = 1.5$ hyps; $\theta_3 = 0.5$ hyp.
$z_1 = 1000$ ohms; $z_2 = 2000$ ohms; $z_3 = 2500$ ohms.

Equivalent ⊓. First Method.

We proceed to compute the equivalent ⊓ of the composite line AF in the same manner as in connection with Figure 7. Ground the end F_1 and develop the line-angles towards A_1. As before,

$$\delta_D = \tanh^{-1}\left(\frac{R_{gE}}{z_2}\right) \qquad \text{and } \delta_B = \tanh^{-1}\left(\frac{R_{gC}}{z_1}\right) \qquad \text{hyps.} \quad (76)$$

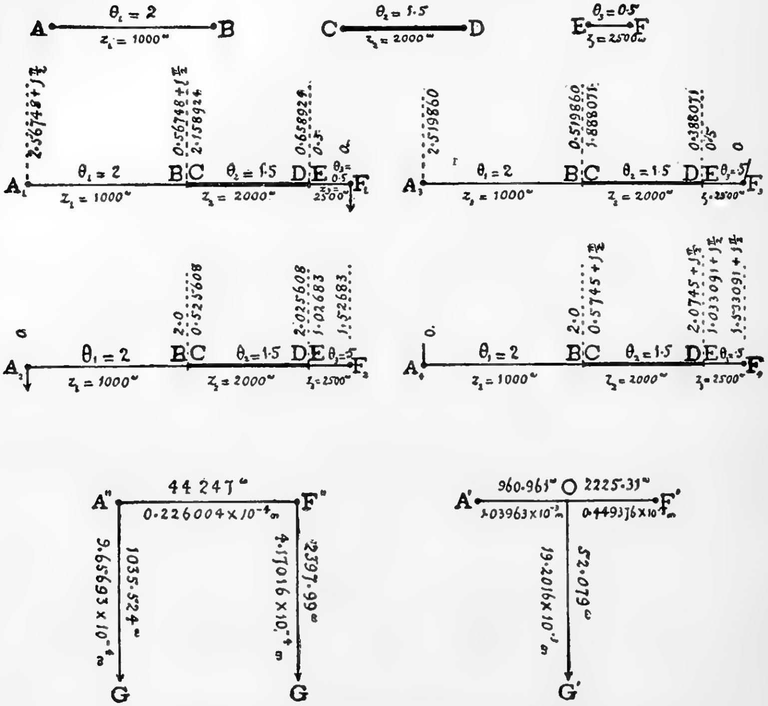

FIGURE 8. Composition of three sections of different surge-resistances.

The architrave resistance is then, following (49),

$$\rho'' = z_1 \sinh \delta_A \times \frac{\cosh \delta_C}{\cosh \delta_B} \times \frac{\cosh \delta_E}{\cosh \delta_D} \qquad \text{ohms} \quad (77)$$

$$= 1000 \cosh 2.567{,}48 \times \frac{\cosh 2.158{,}924}{\sinh 0.567{,}48} \times \frac{\cosh 0.5}{\cosh 0.658{,}923{,}6}$$

$$= 44{,}247 \ \text{ohms.}$$

The sending-end resistance at A is, as in (47),

$$R_{gA} = z_1 \ \tanh \delta_A \qquad \text{ohms} \quad (78)$$

$$= 1{,}000 \ \coth 2.567{,}48 = 1{,}011.84 \ \text{ohms}$$

The conductance of the ⊓ leak at A is, as in (50),

$$g''_A = 1/R_{gA} - 1/\rho'' \qquad \text{mhos.} \quad (79)$$

In order to complete the ⊓, we ground the line at A_2 (Figure 8), and develop the line-angles towards F_2. The architrave resistance is then

$$\rho'' = z_3 \sinh \delta_F \times \frac{\cosh \delta_D}{\cosh \delta_E} \times \frac{\cosh \delta_B}{\cosh \delta_C} \qquad \text{ohms} \quad (80)$$

$$= 44{,}247 \text{ ohms.}$$

The sending-end resistance at F is

$$R_{gF} = z_3 \tanh \delta_F \qquad \text{ohms} \quad (81)$$

$$= 2{,}500 \tanh 1.526{,}83 = 2{,}274.71 \text{ ohms.}$$

Again,

$$g''_F = 1/R_{gF} - 1/\rho'' \qquad \text{mhos.} \quad (82)$$

Equivalent ⊓. *Second Method.*

An alternative method of arriving at the architrave resistance, which we may call the second method, is by following (66) and (68). Grounding at A_2, we have

$$\rho'' = z_1 \sinh \theta_1 \cdot \frac{\sinh \delta_D}{\sinh \delta_C} \cdot \frac{\sinh \delta_F}{\sinh \delta_E} \qquad \text{ohms,} \quad (83)$$

and, grounding at F_1,

$$\rho'' = z_3 \sinh \theta_3 \cdot \frac{\sinh \delta_C}{\sinh \delta_D} \cdot \frac{\sinh \delta_A}{\sinh \delta_B} \qquad \text{ohms} \quad (84)$$

$$= 44{,}247 \text{ ohms.}$$

Equivalent T. *First Method.*

We proceed to compute the equivalent T of the composite line AF in the same manner as the T in Figure 7. Free the end F_3 and develop the line-angles towards A_3. As before,

$$\delta_D = \coth^{-1}\left(\frac{R_{fE}}{z_2}\right) \qquad \text{and } \delta_B = \coth^{-1}\left(\frac{R_{fC}}{z_1}\right) \qquad \text{hyps.} \quad (85)$$

The T leak conductance is then, following (58) and (75),

$$g' = y_1 \sinh \delta_A \cdot \frac{\cosh \delta_C}{\cosh \delta_B} \cdot \frac{\cosh \delta_E}{\cosh \delta_D} \qquad \text{mhos} \quad (86)$$

$$= 0.001 \sinh 2.519{,}86 \cdot \frac{\cosh 1.888{,}071}{\cosh 0.519{,}860} \cdot \frac{\cosh 0.5}{\cosh 0.388{,}071}$$

$$= 19.2016 \times 10^{-3} \text{ mho}$$

$$R' = 52.079 \text{ ohms.}$$

The sending-end resistance at A_3, as before, is

$$R_{fA} = z_1 \coth \delta_A \qquad \text{ohms} \quad (87)$$
$$= 1{,}013.04 \text{ ohms.}$$

The AO line branch is therefore $R_{fA} - R' = 960.961$ ohms.

Repeating the process from A_4 towards F_4, we have for the T leak conductance, as in (80),

$$g' = y_3 \sinh \delta_F \cdot \frac{\cosh \delta_D}{\cosh \delta_E} \cdot \frac{\cosh \delta_B}{\cosh \delta_C} \qquad \text{mhos.} (88)$$

The sending-end resistance at F is likewise

$$R_{fF} = z_3 \coth \delta_F \qquad \text{ohms} \quad (89)$$
$$= 2.500 \tanh 1.533{,}091 = 2{,}277.39 \text{ ohms,}$$

from which the resistance of the line branch FO follows.

Equivalent T. *Second Method.*

The second method of arriving at the T-leak conductance is by following (83) and (84). Freeing at A_4, we have

$$g' = y_1 \sinh \theta_1 \cdot \frac{\sinh \delta_D}{\sinh \delta_C} \cdot \frac{\sinh \delta_F}{\sinh \delta_E} \qquad \text{mhos,} \quad (90)$$

and freeing at F_3, after developing the line angles, we have

$$g' = y_3 \sinh \theta_3 \cdot \frac{\sinh \delta_C}{\sinh \delta_D} \cdot \frac{\sinh \delta_A}{\sinh \delta_B} \qquad \text{mhos.} \quad (91)$$

Composite Line of n Sections.

To compute the equivalent ⊓ of a composite line of *n* successive sections, ground the line at the A end and develop the line-angles towards the opposite end, following the process of (76). Find the architrave impedance according to formula (80) or (83). This may be regarded as formula (19) modified by the application of $(n - 1)$ ratios of cosines in (80), or of $(n - 1)$ ratios of sines in (83). The opposite end leak admittance will then be the sending-end admittance minus the architrave admittance. The process must be repeated after grounding the line at the distant end and developing line-angles towards A.

To compute the equivalent T, free the line at the A end and develop the line-angles towards the opposite end, following the process of (85).

Find the T-leak admittance by following formula (88) or (90). This may be regarded as formula (17) modified by the application of $n-1$ ratios of cosines in (88), or of $n-1$ ratios of sines in (90); that is, one such ratio for each junction. The opposite-end line-branch impedance will then be the sending-end impedance minus the leak impedance. The process must be repeated after freeing the line at the distant end and developing line-angles towards A.

One complete equivalent circuit, say the ⊓, of a composite line of n sections calls then for the determination $n-1$ line-angles first in one direction and then in the other. The formulas are well adapted to logarithmic computation. If, however, only the receiving-end impedance of the composite line is required, then we need only develop the line angles in one direction over the line so as to apply one of the architrave formulas, and neglect the pillars of the ⊓.

LOADED COMPOSITE LINES.

Definitions.

Loads in a line may be either *regular* or *casual.* Regular loads are such as are applied at regular intervals, in order to improve the current delivery on telephone lines. Casual loads are of an irregular or incidental character, such as might occur at section-junctions or at the ends of a composite line. In the former case they would be *intermediate* casual loads, and in the latter case, *terminal* casual loads. Only casual loads will be here discussed; because it is easy, with the aid of formulas already known, to substitute an equivalent smooth unloaded line for any uniformly loaded line.

Loads may also be divided into two classes; namely, (1) those applied in series with the line, or *impedance* loads, such as coils of impedance or resistance, and (2) those applied in derivation to the line, or *leak* loads.

INTERMEDIATE IMPEDANCE LOADS.

The case of an intermediate impedance load, of 100 ohms, inserted at the junction BC in the composite line last considered, is presented in Figure 9. The system differs from that of Figure 8 only in the addition of this load.

Equivalent ⊓. *First Method.*

To compute the equivalent ⊓, $A''F''GG$ (Figure 9), hyperbolically, ground the line at one end, say as at F_1, and develop the line-angles towards A_1. The only change in this process affected by the load is at the junction CB. The sending-end impedance at C is

$$R_{gC} = z_2 \tanh \delta_C \qquad \text{ohms} \quad (92)$$
$$= 2{,}000 \tanh 2.158{,}924 = 1{,}947.385 \text{ ohms.}$$

Consequently, if σ is the impedance of the load BC in ohms, the sending-end resistance at B is

$$R_{gB} = \sigma + z_2 \tanh \delta_C \qquad \text{ohms} \quad (93)$$
$$= 100 + 1{,}947.385 = 2{,}047.385 \text{ ohms,}$$

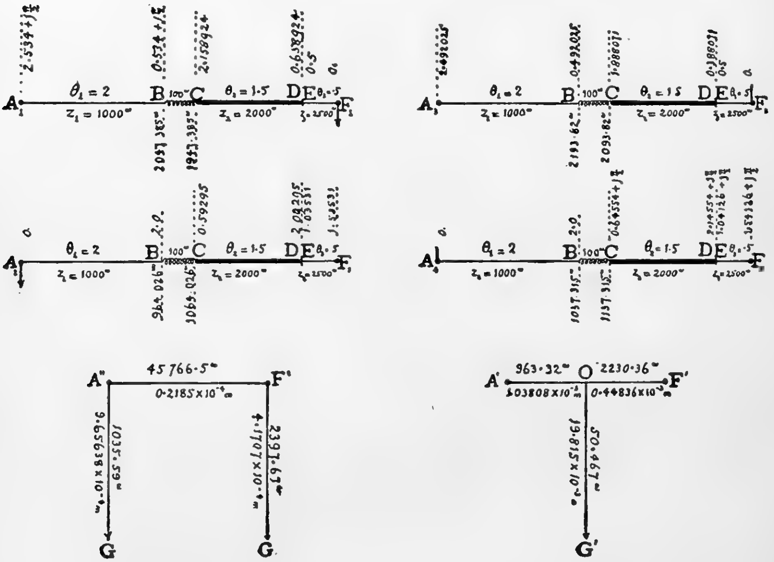

FIGURE 9. Three-section composite line with an intermediate impedance load.

and the new line-angle at B is

$$\delta_B = \tanh^{-1}\left(\frac{R_{gB}}{z_1}\right) \qquad \text{hyps} \quad (94)$$
$$= \tanh^{-1}\left(\frac{2{,}047.385}{1{,}000}\right) = 0.534 + j\frac{\pi}{2} \text{ hyp.}$$

Having established the angle of the whole line at A_1, the architrave impedance follows by formula (77) without further change. The A-leak is also obtained by formulas (78) and (79). In order to obtain the F-leak, and complete the ⊓, the line is grounded at the other end as at A_2 and the line-angles are developed towards F_2. At C, we have

$$\delta_C = \tanh^{-1}\left(\frac{100 + 964.026}{2{,}000}\right) = 0.592{,}95 \text{ hyp.}$$

Formulas (80), (81), and (82) then apply without change.

Equivalent ⊓. *Second Method.*

The alternative method for computing the architrave resistance of the line when grounded at A_2, and developed in angles, is

$$\rho'' = z_1 \sinh \theta_1 \cdot \frac{\sinh \delta_D}{\sinh \delta_C} \cdot \frac{\sinh \delta_F}{\sinh \delta_E} \cdot \frac{R_{gC}}{R_{gB}} \qquad \text{ohms, (95)}$$

and when grounded at F_1 it is

$$\rho'' = z_3 \sinh \theta_3 \cdot \frac{\sinh \delta_C}{\sinh \delta_D} \cdot \frac{\sinh \delta_A}{\sinh \delta_B} \cdot \frac{R_{gB}}{R_{gC}} \qquad \text{ohms. (96)}$$

That is, the effect of the load is to increase the architrave impedance in the ratio of the change of sending-end impedance across the load. In (95) this ratio is 1,064.026/964.026, and in (96) it is 2,047.385/1,947.385.

Equivalent T. *First Method.*

To compute the equivalent T of the loaded line in Figure 9, free the line at one end, as at F_3, and develop the line-angles towards A_3, as in (85). The only change effected by the load is in the angles at and beyond B. The sending-end impedance at C is

$$R_{fC} = z_2 \coth \delta_C \qquad \text{ohms (97)}$$
$$= 2{,}000 \coth 1.888{,}071 = 2{,}093.82 \text{ ohms.}$$

The sending-end impedance at B is, therefore,

$$R_{fB} = \sigma + z_2 \coth \delta_C \qquad \text{ohms (98)}$$
$$= 100 + 2{,}093.82 = 2{,}193.82 \qquad \text{ohms. (98)}$$

The new line-angle at B is then

$$\delta_B = \coth^{-1}\left(\frac{R_{fB}}{z_1}\right) \qquad \text{hyps (99)}$$
$$= \coth^{-1}\left(\frac{2{,}193.82}{1{,}000}\right) = 0.492{,}025 \text{ hyp.}$$

The T-leak admittance is now

$$g' = y_1 \sinh \delta_A \cdot \frac{\cosh \delta_C}{\cosh \delta_B} \cdot \frac{\cosh \delta_E}{\cosh \delta_D} \cdot \frac{G_{fC}}{G_{fB}} \qquad \text{mhos} \quad (100)$$

$$= 0.001 \cdot \sinh 2.492{,}025 \cdot \frac{\cosh 1.888{,}071}{\cosh 0.492{,}025} \cdot \frac{\cosh 0.5}{\cosh 0.388{,}071} \cdot \frac{2{,}193.82}{2{,}093.82}$$

$$= 19.815 \times 10^{-3} \text{ mho.}$$

Formula (87) then applies without change.

Repeating the process from the opposite end of the line, as at A_4F_4, we have

$$g' = y_3 \sinh \delta_F \cdot \frac{\cosh \delta_D}{\cosh \delta_E} \cdot \frac{\cosh \delta_B}{\cosh \delta_C} \cdot \frac{G_{fB}}{G_{fC}} \qquad \text{mhos} \quad (101)$$

$$= 19.815 \times 10^{-3} \text{ mho.}$$

Formula (89) then applies without change.

The effect of the load on the T-leak admittance formulas (86) and (88) is to alter them in the ratio of the impedances or admittances across the load, applying the said ratio in such a manner as to increase the result in the direct-current case.

Equivalent T. *Second Method.*

Formulas (90) and (91) of the alternative method are not altered by an intermediate impedance load, after the line-angles have been properly assigned.

Equivalence of Alternating-Current Transformers to Impedance Loads.

It may be observed that since the insertion of a transformer into a circuit, as, for example, the insertion of a "repeating-coil" into a telephone circuit, is theoretically equivalent to the insertion of impedance into the circuit without rupture of continuity, all cases of line transformers are capable of being dealt with by substituting for such transformers their equivalent intermediate impedance loads.[5]

TERMINAL IMPEDANCE LOADS.

A terminal impedance load is likely to present itself in a composite line, owing to the presence of terminal apparatus. The architrave impedance of a composite line ⊓, computed without any terminal load, can only represent the receiving-end impedance of the line when the

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[5] "On the Predetermination of the Regulation of Alternating-Current Transformers," A. E. Kennelly, *Electrical World and Engineer*, Sept. 2, 1899, Vol. 34, p. 343.

receiving apparatus is short-circuited. For example, in the case of Figure 4, if we short circuit the receiver Z_r the receiving-end impedance of each line is $6{,}736.96\overline{/156^\circ\ 51'\ 15''}$ ohms. With the receiver Z_r inserted, the receiving-end impedance is considerably changed, and this is the condition met with in practice. By applying half the impedance of the receiver as a terminal load to the line, the architrave of the new equivalent ⊓ gives the receiving-end impedance with the receiver included. If this is the result sought, it becomes unnecessary to compute the values of the leaks of this ⊓.

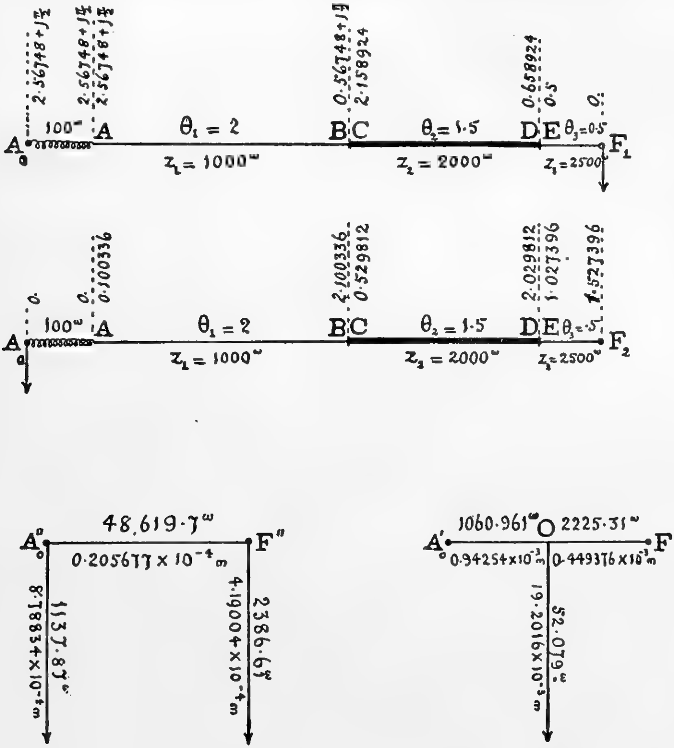

FIGURE 10. Three-section composite line with a terminal impedance load.

Equivalent ⊓. *First Method.*

Figure 10 represents the three-section composite line of Figure 8, with a terminal impedance of 100 ohms applied at A. To compute the equivalent ⊓ of the loaded line, ground F, as at F_1. Develop the line-angles towards A in the usual way. No change from the corresponding conditions of Figure 8 occurs until after we have reached δ_A. We then have

$$\begin{aligned} R_{gA} &= z_1 \tanh \delta_A \text{ ohms} \\ &= 1{,}000 \coth 2.567{,}48 = 1{,}011.607 \text{ ohms,} \end{aligned}$$

and if σ be the impedance of the terminal load at A_0,

$$\begin{aligned} R_{gA0} &= \sigma + z_1 \tanh \delta_A && \text{ohms} \quad (103) \\ &= z_0 \tanh \delta_A && \text{ohms} \quad (104) \\ &= 1{,}111.84 \text{ ohms,} \end{aligned}$$

where z_0 is the *apparent surge-impedance* of the line at A_0; or

$$\begin{aligned} z_0 &= z_1 + \sigma \coth \delta_A && \text{ohms} \quad (105) \\ &= R_{gA0}/\tanh \delta_A && \text{ohms} \quad (106) \\ &= 1{,}098.829 \text{ ohms.} \end{aligned}$$

The architrave resistance is then, following (77),

$$\begin{aligned} \rho'' &= z_0 \sinh \delta_A \cdot \frac{\cosh \delta_C}{\cosh \delta_B} \cdot \frac{\cosh \delta_E}{\cosh \delta_D} && \text{ohms} \quad (107) \\ &= 48{,}619.7 \text{ ohms.} \end{aligned}$$

The A-leak of the ⊓, as in the case of Figure (8), is

$$g''_A = 1/R_{gA0} - 1/\rho'' \qquad \text{mhos.} \quad (108)$$

To complete the ⊓, we ground the loaded line at A, as at A_0F_2, and develop the line-angles towards F, commencing with

$$\begin{aligned} \delta_A &= \tanh^{-1}\left(\frac{\sigma}{z_1}\right) && \text{hyps} \quad (109) \\ &= \tanh^{-1}\left(\frac{100}{1000}\right) = 0.100{,}336 \text{ hyp.} \end{aligned}$$

The architrave impedance is then

$$\begin{aligned} \rho'' &= z_3 \sinh \delta_F \cdot \frac{\cosh \delta_D}{\cosh \delta_E} \cdot \frac{\cosh \delta_B}{\cosh \delta_C} \cdot \frac{\cosh 0}{\cosh \delta_A} && \text{ohms} \quad (110) \\ &= 48{,}619.7 \text{ ohms.} \end{aligned}$$

The F-leak is then computed as in (82).

Equivalent Π. *Second Method.*

The alternative method gives

$$\rho'' = z_1 \cdot \frac{\sinh \delta_B}{\cosh \delta_A} \cdot \frac{\sinh \delta_D}{\sinh \delta_C} \cdot \frac{\sinh \delta_F}{\sinh \delta_E} \qquad \text{ohms,} \quad (111)$$

with the line grounded at A_0, and

$$\rho'' = z_3 \sinh \theta_3 \cdot \frac{\sinh \delta_C}{\sinh \delta_D} \cdot \frac{\sinh \delta_A}{\sinh \delta_B} \cdot \frac{R_{gA0}}{R_{gA}} \qquad \text{ohms,} \quad (112)$$

with the line grounded at F.

Equivalent T.

To arrive at the equivalent T of a composite line loaded with a terminal impedance, all that is necessary is to find the T of the same line unloaded, by preceding formulas, and then to add the terminal impedance to the proper line-branch of this T.

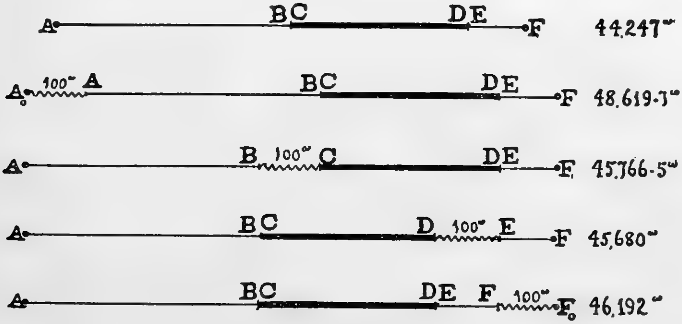

FIGURE 11. Diagram showing the influence of the location of an impedance load on the receiving-end resistance of a three-section composite line.

INFLUENCE OF LOCATION OF AN IMPEDANCE LOAD ON THE RECEIVING-END IMPEDANCE OF A COMPOSITE LINE.

It has been shown in a preceding paper that if a single smooth uniform line is terminally loaded with a given impedance, the change in the receiving-end impedance due to the load is the same, whichever end of the line the load may be applied to; *i. e.*, whether the load is applied at the sending or at the receiving end. In the case of a composite line, however, this proposition generally fails. The effect of a resistance coil of 100 ohms on the receiving-end resistance of the three-section composite line above discussed, is shown in Figure 11. Without the load, the receiving-end resistance of the line, or the architrave of its equivalent ⊓, is, by Figure 8, 44,247 ohms. If the load is added at the A end of the line, the receiving-end resistance becomes 48,619.7 ohms; but if added at the F end, it is only 46,192. When the same coil is inserted as an intermediate load, its influence on the receiving-end resistance is not so great. In A. C. composite lines, the opportunities for such variations are more marked. In all cases, however, the application of a terminal impedance σ to a line (single or composite), increases the receiving-end or architrave impedance of that line in the ratio $\frac{R_g + \sigma}{R_g}$; where R_g is the sending-end-impedance of the line at the loaded end before the load is applied. This is true whether the loaded end is made the sending or receiving end of the circuit. For single lines, R_g has the same value at either end, and therefore the ratio of increase in receiving-end impedance is the same at whichever end of a single line the load σ is applied; whereas, for composite lines, we have seen that R_g is different, in general, at the two ends.

INTERMEDIATE LEAK LOADS.

Equivalent ⊓. *First Method.*

Suppose a leak load to be applied at a junction between sections such as at DE (Figure 12). We proceed to compute the equivalent ⊓ of the loaded composite line by grounding one end, as at F_1. We develop the line-angles towards A_1. On arriving at E we have $R_{gE} = z_3 \tanh \theta_3 = 1{,}155.292$ ohms. Hence $G_{gE} = 1/R_{gE} = 8.655{,}82 \times 10^{-4}$ mho. To this sending-end admittance we add the admittance γ of the leak; so that the sending-end admittance at D, including the leak, is

$$G_{gD} = \gamma + G_{gE} \qquad \text{mhos} \quad (113)$$
$$= 13.655{,}82 \times 10^{-4} \text{ mho.}$$

Consequently the sending-end resistance at D, including the leak, is

$$R_{gD} = 1/G_{gD} \qquad \text{ohms,} \quad (114)$$
$$= 732.289 \text{ ohms.}$$

The line-angle at D is then

$$\delta_D = \tanh^{-1}\left(\frac{R_{gD}}{z_2}\right) \qquad \text{hyps} \quad (115)$$
$$= 0.383{,}964 \text{ hyp.}$$

The remaining line-angles are found in the regular way.

The architrave impedance is then

$$\rho'' = z_1 \sinh \delta_A \cdot \frac{\cosh \delta_C}{\cosh \delta_B} \cdot \frac{\cosh \delta_E}{\cosh \delta_D} \cdot \frac{R_{gE}}{R_{gD}} \qquad \text{ohms} \quad (116)$$

$$= 60{,}240 \text{ ohms.}$$

The A-leak is computed regularly from (78) and (79).

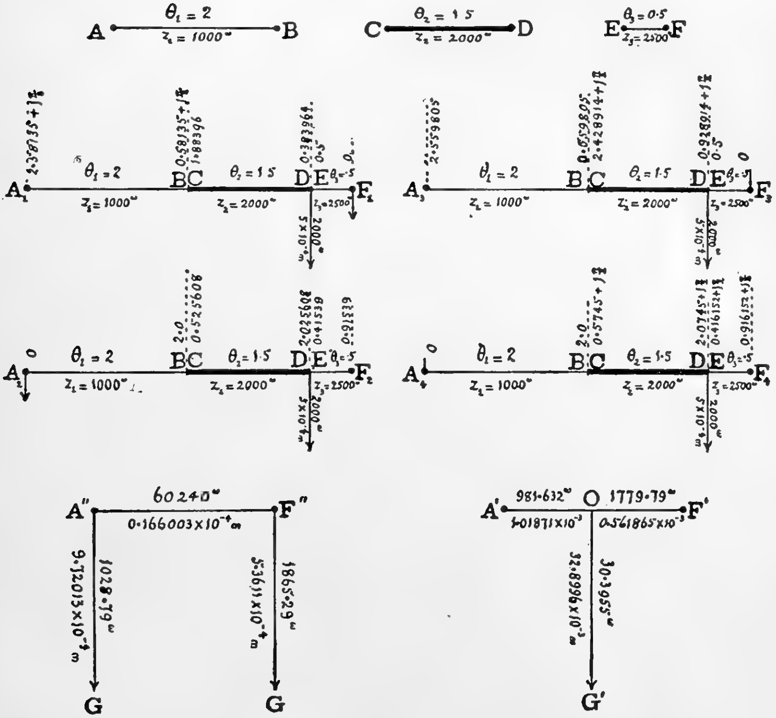

FIGURE 12. Composite line of three sections with intermediate leak load.

To complete the ⊓, ground the line at the opposite end, as at A_2, and develop the line-angles towards F_2, in the same manner as above. The architrave impedance is then

$$\rho'' = z_3 \sinh \delta_F \cdot \frac{\cosh \delta_D}{\cosh \delta_E} \cdot \frac{\cosh \delta_B}{\cosh \delta_C} \cdot \frac{R_{gD}}{R_{gE}} \qquad \text{ohms} \quad (117)$$

$$= 60{,}240 \text{ ohms.}$$

The F-leak is computed regularly from (81) and (82).

Equivalent ⊓. *Second Method.*

In the alternative method we have the regular formulas (83) and (84), unchanged by the intermediate leak load.

Equivalent T. *First Method.*

To complete the equivalent T, free one end of the line, say F, as at F_3 (Figure 12), and develop the line-angles towards A_3. At the loaded junction DE we have

$$G_{fD} = \gamma + G_{fE} \qquad \text{mhos, (118)}$$
$$= 6.848{,}47 \times 10^{-4} \text{ mho,}$$

and, following (114) and (115),

$$\delta_D = \coth^{-1}\left(\frac{R_{fD}}{z_2}\right) \qquad \text{hyps, (119)}$$
$$= 0.928{,}914 + j\frac{\pi}{2} \text{ hyp.}$$

The remaining line-angles follow regularly. The T-leak conductance also follows from (86) without change, and the line-branch AO is computed regularly by (18), (57), (59), and (87).

To complete the T, free the other end of the line as at A_4, and proceed, as above, to develop the line-angles towards F_4. The T-admittance must then conform to (88), and the line-branch impedance FO to (89).

Equivalent T. *Second Method.*

The alternative method of arriving at the T-leak admittance is by following (83) and (84). Freeing at A_4 (Figure 12), we have

$$g' = y_1 \sinh\theta_1 \cdot \frac{\sinh\delta_D}{\sinh\delta_C} \cdot \frac{\sinh\delta_F}{\sinh\delta_E} \cdot \frac{G_{fE}}{G_{fD}} \qquad \text{mhos, (120)}$$

and similarly, freeing at F_4, we have

$$g' = y_3 \sinh\theta_3 \cdot \frac{\sinh\delta_C}{\sinh\delta_D} \cdot \frac{\sinh\delta_A}{\sinh\delta_B} \cdot \frac{G_{fD}}{G_{fE}} \qquad \text{mhos. (121)}$$

TERMINAL LEAK LOADS.

Equivalent ⊓.

To arrive at the equivalent ⊓ of a composite line loaded with a terminal leak, such as that represented at AF in Figure 13, first compute the equivalent ⊓ of the same line unloaded, by preceding formulas, and then to the proper leak of the ⊓ add the terminal load leak, numerically in the D. C. case, vectorially in an A. C. case.

Equivalent T. *First Method.*

To compute the equivalent T, free one end of the line, say F, as at F_3 (Figure 13), and develop the line-angles towards A. We commence with

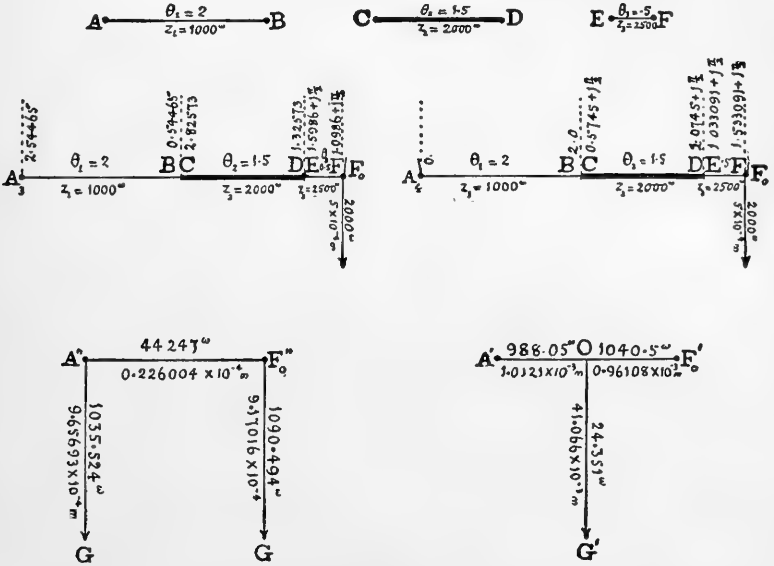

FIGURE 13. Composite line of three sections with terminal leak load.

$$\delta_F = \coth^{-1}\left(\frac{1/\gamma}{z_3}\right) \qquad \text{hyps} \quad (122)$$

$$= 1.098{,}6 + j\frac{\pi}{2} \text{ hyps},$$

where γ is the admittance of the load in an A. C. case or conductance of the load in the D. C. case (mhos).

The T-leak admittance is then

$$g' = y_1 \sinh \delta_A \cdot \frac{\cosh \delta_C}{\cosh \delta_B} \cdot \frac{\cosh \delta_B}{\cosh \delta_D} \cdot \frac{\cosh 0}{\cosh \delta_F} \qquad \text{mhos} \quad (123)$$

$$= 41.066 \times 10^{-3} \text{ mho},$$

and the line-branch impedance AO follows at once from (87).

To complete the T, the line is freed at A, as at A_4 (Figure 13), and the line-angles are developed toward F. We then have for the sending-end admittance at F,

$$G_{fF} = y_3 \ \tanh \delta_F \qquad \text{mhos.} \quad (124)$$

The sending-end conductance at F_0, including the leak admittance

$$G_{fF_0} = \gamma + y_3 \ \tanh \delta_F \qquad \text{mhos.} \quad (125)$$

The apparent surge-admittance y_0 at F_0 is defined by the condition,

$$G_{fF_0} = y_0 \ \tanh \delta_F \qquad \text{mhos,} \quad (126)$$

whence

$$y_0 = y_3 + \gamma \ \coth \delta_F \qquad \text{mhos.} \quad (127)$$

The T-leak admittance will then conform to

$$g' = y_0 \ \sinh \delta_F \cdot \frac{\cosh \delta_D}{\cosh \delta_E} \cdot \frac{\cosh \delta_B}{\cosh \delta_C} \qquad \text{mhos} \quad (128)$$

$$= y_3 \ \sinh \delta_F \cdot \frac{\cosh \delta_D}{\cosh \delta_E} \cdot \frac{\cosh \delta_B}{\cosh \delta_C} \cdot \frac{G_{fF_0}}{G_{fF}} \qquad \text{mhos,} \quad (129)$$

and the line-branch impedance FO follows at once from (89).

Equivalent T. *Second Method.*

By the alternative method, the T-leak admittance, when the line is freed at A, is

$$g' = y_1 \ \sinh \theta_1 \cdot \frac{\sinh \delta_D}{\sinh \delta_C} \cdot \frac{\sinh \delta_F}{\sinh \delta_E} \cdot \frac{G_{fF_0}}{G_{fF}} \qquad \text{mhos} \quad (130)$$

$$= 41.066 \times 10^{-8} \ \text{mho.}$$

Similarly, when the line is freed at F_0 (Figure 13), and the corresponding line-angles are set,

$$g' = y_3 \ \frac{\sinh \delta_E}{\cosh \delta_F} \cdot \frac{\sinh \delta_C}{\sinh \delta_D} \cdot \frac{\sinh \delta_A}{\sinh \delta_B} \qquad \text{mhos.} \quad (131)$$

The line-branch impedances are determined in the regular way.

Résumé of Rules applying to Casual Loads in Composite Lines.

In the accompanying Table the changes effected by loads in the formulas for ρ'' and g' are collected together as an aid to computation.

It will be seen that there is a certain symmetry in these changes that assists their application. Moreover, it is possible, after consulting the Table, to select in some particular case a method which avoids additional computation. Thus, in dealing with an intermediate leak, the first method calls for the application of the impedance ratio across the leak, to the formula for ρ''; whereas the second method calls for no change in its formula.

Table showing Changes made by Casual Loads in the Composite-Line Formulas for the Equivalent Π-Architrave and Equivalent T-Leak.

Nature of Load.	Change in the Formula for ρ''.		Change in the Formula for g'.	
	By First Method.	By Second Method.	By First Method.	By Second Method.
Intermediate impedance	None	R_{gM}/R_{gN}	G_{gM}/G_{gN}	None
Intermediate leak	R_{gM}/R_{gN}	None	None	G_{gM}/G_{gN}
Terminal impedance:				
At far end	$\cosh 0/\cosh \delta_N$	Subst.: $\frac{\sinh \delta_{N-1}}{\cosh \delta_{N0}}$ for $\sinh \theta_N$		
At near end	R_{gA0}/R_{gA} or subst. z_0 for z_1	R_{gA0}/R_{gA}		
Terminal leak:				
At far end			$\cosh 0/\cosh \delta_N$	Subst.: $\frac{\sinh \delta_{N-1}}{\cosh \delta_{N0}}$ for $\sinh \theta_N$
At near end			G_{fA0}/G_{fA} or subst. y_0 for y_1	G_{fA0}/G_{fA}

The ratios R_{gM}/R_{gN} and G_{gM}/G_{gN} denote respectively the ratios of sending-end impedance and sending-end admittance across the load, the ratio being taken in each case such that in the D. C. case it is greater than unity.

The far end is in all cases the end of the composite line which is to be considered as freed or grounded for the purposes of the computation, and the near end is the opposite end, or the end towards which the line-angles are developed.

It has been assumed for the purposes of the Table that the A end of the line happens to be the near end in all cases, and the N end the far end.

PLURALITY OF LOADS.

When several casual loads exist simultaneously in a composite line, each requires to be considered separately in the formulas for ρ'' and g', although no special treatment is involved thereby in computing g'' or ρ'. A particular case of this kind is shown in Figure 14, where the composite line of Figure 8 is loaded with an intermediate resistance of 100 ohms at the junction BC, a terminal resistance of 200 ohms at F and also with a terminal leak of 5000 ohms at F. The presence of the terminal resistance GH, however, converts the leak into an intermediate leak so far as concerns the process of computation.

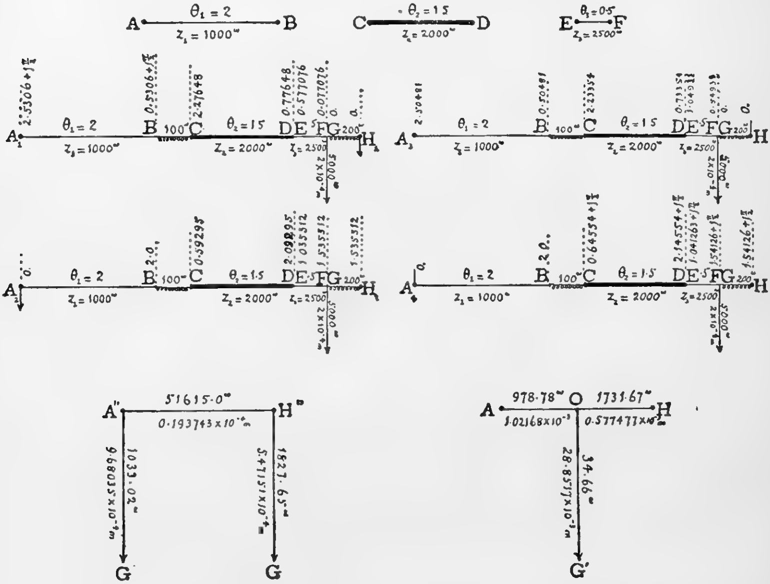

FIGURE 14. Composite line of three sections with two terminal and one intermediate load.

Equivalent ⊓. *First Method.*

In order to compute the equivalent ⊓, ground the line at one end, as at A_2 (Figure 14), and develop the line-angles towards H by preceding formulas. Referring to the Table, we have (a) one intermediate impedance at BC; (b) one intermediate leak at FG, and (c) one terminal impedance at the near end H, the distant end being grounded.

Consequently, so far as concerns the first method, we should make no change in the formula for ρ'' on account of (a), but introduce the ratio R_{gF}/R_{gG} for (b) and substitute z_0 for z_3 on account of (c). Consequently, following (77) with these changes,

$$\rho'' = z_0 \sinh \delta_H \cdot \frac{\cosh \delta_D}{\cosh \delta_E} \cdot \frac{\cosh \delta_B}{\cosh \delta_C} \cdot \frac{R_{gF}}{R_{gG}} \qquad \text{ohms} \quad (132)$$

$$= 1{,}936.87 \sinh 1.535{,}312 \cdot \frac{\cosh 2.092{,}95}{\cosh 1.035{,}31} \cdot \frac{\cosh 2.0}{\cosh 0.592{,}95} \cdot \frac{1809.74}{1565.14}$$

$$= 51{,}615 \text{ ohms.}$$

The H-leak is then found in the usual way.

Equivalent Π. *Second Method.*

Similarly, by reference to the Table, for changes in the ρ'' formula under the second method, we should introduce the ratio R_{gC}/R_{gB} for (a), make no change for (b), but introduce the ratio R_{gH}/R_{gG} for (c). Consequently, following (83) with these changes,

$$\rho'' = z_1 \sinh \theta_1 \cdot \frac{\sinh \delta_D}{\sinh \delta_C} \cdot \frac{\sinh \delta_F}{\sinh \delta_E} \cdot \frac{R_{gC}}{R_{gB}} \cdot \frac{R_{gH}}{R_{gG}} \qquad \text{ohms} \quad (133)$$

$$= 51{,}615 \text{ ohms.}$$

The A-leak is then computed in the regular manner.

If now we ground the line at the H-end, we obtain similarly, by the first method,

$$\rho'' = z_1 \sinh \delta_A \cdot \frac{\cosh \delta_C}{\cosh \delta_B} \cdot \frac{\cosh \delta_E}{\cosh \delta_D} \cdot \frac{\cosh 0}{\cosh \delta_F} \cdot \frac{R_{gG}}{R_{gF}} \qquad \text{ohms} \quad (134)$$

$$= 51{,}615 \text{ ohms,}$$

and by the second method,

$$\rho'' = z_3 \frac{\sinh \delta_E}{\sinh \delta_F} \cdot \frac{\sinh \delta_C}{\sinh \delta_D} \cdot \frac{\sinh \delta_A}{\sinh \delta_B} \cdot \frac{R_{gB}}{R_{gC}} \cdot \frac{G_{gF}}{G_{gG}} \qquad \text{ohms.} \quad (135)$$

Equivalent T. *First Method.*

Freeing the line at H, as at A_3H (Figure 14), we have

$$g' = y_1 \sinh \delta_A \cdot \frac{\cosh \delta_C}{\cosh \delta_B} \cdot \frac{\cosh \delta_E}{\cosh \delta_D} \cdot \frac{\cosh \delta_G}{\cosh \delta_F} \cdot \frac{G_{fC}}{G_{fB}} \qquad \text{mhos} \quad (136)$$

$$= 0.001 \cdot \sinh 2.504{,}81 \cdot \frac{\cosh 2.233{,}54}{\cosh 0.504{,}81} \cdot \frac{\cosh 1.049{,}31}{\cosh 0.733{,}54} \cdot \frac{1}{\cosh 0.549{,}31} \cdot \frac{2{,}146.46}{2{,}046.46}$$

$$= 28.851{,}7 \times 10^{-3} \text{ mho,}$$

and freeing the line at A, as at A$_4$, we have

$$g' = y_0 \sinh \delta_H \cdot \frac{\cosh \delta_D}{\cosh \delta_E} \cdot \frac{\cosh \delta_B}{\cosh \delta_C} \cdot \frac{G_{fB}}{G_{fC}} \cdot \frac{G_{fF}}{G_{fG}} \quad \text{mhos.} \quad (137)$$

Equivalent T. *Second Method.*

Freeing at H, we have

$$g' = y_3 \frac{\sinh \delta_E}{\cosh \delta_F} \cdot \frac{\sinh \delta_C}{\sinh \delta_D} \cdot \frac{\sinh \delta_A}{\sinh \delta_B} \quad \text{mhos,} \quad (138)$$

and freeing at A,

$$g' = y_1 \sinh \theta_1 \cdot \frac{\sinh \delta_D}{\sinh \delta_C} \cdot \frac{\sinh \delta_F}{\sinh \delta_E} \cdot \frac{G_{fG}}{G_{fF}} \quad \text{mhos.} \quad (139)$$

METHODS OF COMPUTATION ADAPTED TO ALTERNATING-CURRENT CASES.

There is especial need for brief methods of computation when A. C. cases are dealt with,[6] owing to the complexity of the vector arithmetic. In practice, the degree of precision desired will usually be much lower than that aimed at in the arithmetical examples of this paper, where the numerical values have been carried to six significant digits. Graphical methods may be frequently used with advantage, especially in the vector addition of complex hyperbolic angles. Traverse Tables as used by navigators may also be used with advantage for the resolution of vectors into complex quantities.

The following formulas are also useful:

$$\cosh (p \pm jq) = \sqrt{\cosh^2 p - \sin^2 q} \, \underline{/ \pm \tan^{-1}(\tanh p \cdot \tan q)} \quad (140)$$

$$\sinh (p \pm jq) = \sqrt{\sinh^2 p + \sin^2 q} \, \underline{/ \pm \tan^{-1}(\coth p \cdot \tan q)} \quad (141)$$

$$\tanh (p \pm jq) = \frac{\sinh 2p}{\cosh 2p + \cos 2q} \pm j \frac{\sin 2q}{\cosh 2p + \cos 2q} \quad (142)$$

$$\tanh^{-1}(p \pm jq) = \tfrac{1}{2} \log_e \sqrt{\frac{(1+p)^2+q^2}{(1-p)^2+q^2}} + j \left\{ \frac{\pi - \tan^{-1}\left(\dfrac{1+p}{\pm q}\right) - \tan^{-1}\left(\dfrac{1-p}{\pm q}\right)}{2} \right\} \quad (143)$$

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[6] A table of hyperbolic tangents of a vector variable or of tanh $r\underline{/\theta}$, is being prepared by the writer for values of r between 0 and 6, by steps of 0.1 or less; and for virtually all angles θ, by steps of one degree.

CONCLUSIONS.

Any composite line of any number of sections, with or without loads of any kind, operated in the steady state either by a direct current, or by an alternating current of one frequency, has the same receiving-end impedance from each end; so that, if one volt be applied to each end in turn, the current strength received at the other end will be the same.[7]

The equivalent circuits of such lines may always be computed either for the D. C. or A. C. case by the formulas given in this paper. That is, any such line may always be replaced by one delta connection or by one star connection of impedance, without disturbing the electrical conditions outside of the line.

Notation Employed

$a, a_{//}, a_1, a_2, a_3$	attenuation-constants of a single line, of a loop-line, and of different sections of a composite line (hyps. per km.).
$c, c_{//}, c_1, c_2, c_3$	linear capacitance of single line, loop-line, and sections (farads/km.).
$\delta, \delta_A, \delta_B$, ...	the hyp. angles of points on a line (hyps).
$G, G_g, G_{gA}, \ldots G_f, G_{fA}, \ldots$	the sending-end admittance (D. C. conductance) of a line, the admittance beyond a point on the same, when the far end is grounded, and when the far end is free (mhos).
$g, g_{//}, g_1, g_2, g_3$	linear conductance of single line, loop-line, and sections (mhos/km.).
$g' = 1/R'$	conductance of leak of a T (mhos).
$g'' = 1/R''$	conductance of leak of a ⊓ (mhos).
γ	conductance of a leak load (mhos).
i, i_A, i_P	current strength, at the sending-end, and at a point on the line (amperes).
j	$\sqrt{-1}$
$l, l_{//}, l_1, l_2, l_3$	linear inductance of single line, loop-line, and sections (henrys/km.).
L, L_1, L_2, L_3	length of a line and of sections (km.).

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[7] An exception should be noted in the case of any part of the composite line not obeying Ohm's law, as, for example, a fault in the insulation; so that the current through the fault is not proportional to the potential at the same.

l'	distance of a point on a line from its far end (km.).
n	frequency of single A. C. (cycles per second).
ω	angular velocity of A. C. (radians per second).
p, q	cartesian coördinates of a point in a plane.
$r, r_{//}, r_1, r_2, r_3$	linear resistance of a single line, loop-line, and sections (ohms/km.).
$R, R_g, R_{gA} \ldots, R_f, R_{fA} \ldots$	resistance of a line beyond a point on the same, the resistance when the far end is grounded, and when the far end is free, A. C. impedance (ohms).
$R' = 1/g'$	resistance (A. C. impedance) of leak of a ⊤ (ohms).
$R'' = 1/g''$	resistance (A. C. impedance) of leak of a ⊓ (ohms).
$\rho' = 1/y'$	resistance (A. C. impedance) of line-branch of ⊤ (ohms).
$\rho'' = 1/y''$	resistance (A. C. impedance) of architrave of ⊓ (ohms).
σ	resistance (A. C. impedance) of impedance load (ohms).
$\theta, \theta_{//}, \theta_1, \theta_2, \theta_3$	hyperbolic angle subtended by a single line, loop-line, and sections (hyps).
$y = 1/z$	surge-admittance (D. C. conductance) of a line (mhos).
$y' = 1/\rho'$	admittance (D. C. conductance) of a line-branch of a ⊤ (mhos).
$y'' = 1/\rho''$	admittance (D. C. conductance) of architrave of a ⊓ (mhos).
u, u_A, u_P	potential, at the sending-end, and at a point on the line (volts).
Z_r, Z_s	impedance of a terminal receiver, of terminal sending apparatus (ohms).
$z, z_{//}, z_1, z_2, z_3$	surge-impedance of a line, a loop-line, and sections (ohms).
z_0	apparent surge-impedance of a line to which an impedance load is prefixed (ohms).

BIBLIOGRAPHY.

O. Heaviside. Electrical Papers, ii, 248. London, Macmillan & Co., 1892.

M. I. Pupin. Propagation of Long Electrical Waves. Trans. Amer. Inst. Electr. Engrs., 1899, xvi, 93.

Wave Transmission over Non-Uniform Cables and Long-Distance Air-Lines. Trans. Amer. Inst. Electr. Engrs., 1900, xvii, 445.

Wave Propagation over Non-Uniform Conductors. Trans. Amer. Math. Soc., 1900, i, 259.

M. Leblanc. Formula for Calculating the Electromotive Force at any Point of a Transmission Line for Alternating Current. Trans. Amer. Inst. Electr. Engrs., 1902, xix, 759.

G. A. Campbell. Loaded Lines in Telephonic Transmissions. Phil. Mag., 1903, ser. 6, v, 313.

G. Roessler. Die Fernleitung von Wechselströmen. Berlin, Julius Springer, 1905.

G. Di Pirro. Sui Circuiti non uniformi. Atti dell' Assoc. Elettrotech., 1909, xii, No. 6; La Lumière Electrique, 1909, ser. 2, vii, 227.

A. E. Kennelly. A Contribution to the Theory of Telephony. Electr. World, 1894, xxiii, 208.

Resonance in Alternating Current Lines. Trans. Amer. Inst. Electr. Engrs., 1895, xii, 133.

Electric Conducting Lines of Uniform Conductor and Insulation Resistance in the Steady State. Harv. Eng. Journ., 1903, ii, 135.

The Alternating Current Theory of Transmission Speed over Submarine Cables. Trans. Internat. Electr. Cong. of St. Louis, 1904, i, 66.

High-Frequency Telephone Circuit Tests. Trans. Internat. Electr. Cong. St. Louis, 1904, iii, 414.

The Distribution of Pressure and Current over Alternating Current Circuits. Harv. Eng. Journ., 1905, iv, 149.

The Process of Building up the Voltage and Current in a Long Alternating Current Circuit. These Proceedings, 1907, xlii, 701.

Artificial Lines for Continuous Currents in the Steady State. These Proceedings, 1908, xliv, 97.

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Περὶ Φύσεως.

A STUDY OF THE CONCEPTION OF NATURE AMONG THE PRE-SOCRATICS.

By William Arthur Heidel,

Professor of Greek in Wesleyan University.

Περὶ Φύσεως.

A STUDY OF THE CONCEPTION OF NATURE AMONG THE PRE-SOCRATICS.[1]

By William Arthur Heidel.

Presented by M. H. Morgan, October 13, 1909; Received November 3, 1909.

Professor John Burnet says:[2] "So far as I know, no historian of Greek philosophy has clearly laid it down that the word used by the early cosmologists to express this idea of a permanent and primary substance was none other than φύσις;[3] and that the title Περὶ φύσεως, so commonly given to philosophical works of the sixth and fifth centuries B. C.,[4] means simply *Concerning the Primary Substance.* Both Plato and Aristotle use the term in this sense when they are discussing the

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[1] This paper was begun in the spring of 1908, and was read in substance before the Classical Club of Princeton University, Dec. 17, 1908.

[2] *Early Greek Philosophy*, 2d ed., 1908, p. 12 foll.

[3] Burnet, *ibid.*, p. 13 foll., p. 57, n. 1, rejects the traditional view that Anaximander so used ἀρχή, which, he says, "is in this sense purely Aristotelian." This statement, and the other that "To Anaximander ἀρχή could only have meant *beginning*," are open to question; cp. Hippocrates, Π. νούσων, 51 (7, 584 Littré) ὑπὸ τῶν ἀρχῶν διίσταται ὧν εἴρηκά οἱ πάντα, and *ibid.* (7, 590 Littré) ὅκως ἐργάζονται αἱ ἀρχαὶ τὴν θέρμην καὶ τὴν ταραχὴν τῷ ὑγρῷ ὑπάγουσαι ἐς νοῦσον. Cp. Philolaus, fr. 6 ἐπεὶ δὲ ταὶ ἀρχαὶ ὑπᾶρχον οὐχ ὁμοῖαι οὐδ' ὁμόφυλοι ἔσσαι, fr. 8 ἡμῖν μονὰς ὡς ἂν ἀρχὴ οὖσα πάντων, fr. 11 ἀρχὰ καὶ ἁγεμών, though I lay no stress on these, believing that all the so-called fragments of Philolaus, excepting fr. 16, which occurs in the *Eudemian Ethics*, are spurious. Cp. also note 166, below. This use of ἀρχή = causal principle may well have been old; cp. πηγή and ῥίζωμα = στοιχεῖον. The 'Aristotelian' sense of ἀρχή occurs in Plato, *Tim.* 48 B; cp. Diels, *Elementum*, p. 20. Burnet also says (p. 56) "That Anaximander called this something [i. e. his Ἄπειρον] by the name of φύσις, is clear from the doxographers." This statement likewise may fairly be challenged.

[4] Burnet here adds in a note: "I do not mean to imply that the philosophers used this title themselves; for early prose writings had no titles. The writer mentioned his name and the subject of his work in the first sentence, as Herodotus, for instance, does." As the titles were, in all probability, added later it is interesting to note the words of Galen, *de Elem. sec. Hippocr.* I. 9, p. 487 Kühn: τὰ γὰρ τῶν παλαιῶν ἅπαντα περὶ φύσεως ἐπιγέγραπται, τὰ Μελίσσου, τὰ Παρμενίδου, τὰ Ἐμπεδοκλέους, Ἀλκμαίωνός τε καὶ Γοργίου, καὶ Προδίκου, καὶ τῶν ἄλλων ἁπάντων. It was therefore, as we shall see, a sort of blanket-title.

earlier philosophy,[5] and its history shows clearly enough what its original meaning must have been. In Greek philosophical language, φύσις always means that which is primary, fundamental, and persistent, as opposed to what is secondary, derivative, and transient; what is 'given,' as opposed to that which is made or becomes. It is what is there to begin with."

"There is one important conclusion," says Professor Burnet,[6] "that follows at once from the account just given of the meaning of φύσις, and it is, that the search for the primary substance really was the thing that interested the Ionian philosophers. Had their main object been, as Teichmüller held it was, the explanation of celestial and meteorological phenomena, their researches would not have been called[7] Περὶ φύσεως ἱστορίη, but rather Περὶ οὐρανοῦ or Περὶ μετεώρων."

Considering its source, this declaration is of sufficient importance to justify an extended examination for its own sake, especially as it has not been adequately met by students of Greek thought;[8] but the purpose of this study is somewhat different. The words quoted from Professor Burnet serve, therefore, chiefly as a point of departure. It is proposed to consider three subjects, which are of importance in relation to the works entitled Περὶ φύσεως: (1) the historical relation of the studies so entitled to mythology and poetry; (2) the senses in which φύσις was employed before 400 B. C.; (3) the probable connotation of the title Περὶ φύσεως, judging by the direction of interest of the writers as indicated by the problems they raised.

Before proceeding to the consideration of these questions, however, it may be proper to touch briefly on several subjects suggested by the

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[5] Burnet here refers to Arist. *Phys.* 193 a 21 foll. and to Plato, *Legg.* 892 C φύσιν βούλονται λέγειν γένεσιν τὴν περὶ τὰ πρῶτα. Here he interprets γένεσιν with τὸ ἐξ οὗ γίγνεται. Though this use of γένεσις is as old as Homer (Ξ 201, 246), and though Plato could employ it in allusion to Homer (*Theaet.* 180 D), it would be ill-chosen to explain φύσις. Ast in his ed. (vol. III. 158) has, as it seems to me, correctly rendered the words: "Volunt illi naturam dici generationem eorum, quae primum orta sint," unless one prefers "quae prima sint." Cp. ὑπὲρ τῆς τῶν στοιχείων φύσεως, Diels, *Vorsokr.* II. 511, 15. Burnet might have referred with more propriety to Plato, *Legg.* 891 C, but it is to be noted that φύσις is singular.

[6] *Ibid.* p. 14.

[7] Burnet here refers to Plato, *Phaedo* 96 A and Eurip., fr. 910. We may add Theophrastus, *Ph. O.* fr. 5 (Diels, *Dox.* 480, 7) and fr. 9 (*ibid.* 485, 1). In the latter case ἡ π. φύσεως ἱστορία is opposed (speaking of Plato) to ἡ πραγματεία περὶ τῆς πρώτης φιλοσοφίας. Cp. n. 206, below. From Theophrastus the phrase was passed on to the doxographers. Thus Simplic. *in Phys.* (p. 23. 29 Diels) says: Θαλῆς δὲ πρῶτος παραδέδοται τὴν περὶ φύσεως ἱστορίαν τοῖς Ἕλλησιν ἐκφῆναι.

[8] Burnet's view has been briefly criticised by Professor Millerd, *On the Interpretation of Empedocles*, Chicago, 1908, pp. 18 foll.

words quoted from Professor Burnet. It is probably true that early prose writings had no formal titles; but our information on this point is really too scanty to admit of dogmatic statement.[9] It is reasonably certain that philosophical works were familiarly quoted as bearing the title Περὶ φύσεως some time before the close of the fifth century, as we may see from the works of Hippocrates;[10] and from the time of Xenophon, Plato, and Aristotle[11] onwards it must have been the accepted designation. In regard to the scope of the title Περὶ φύσεως and Professor Burnet's attempt to limit it narrowly to the meaning *Concerning the Primary Substance*, and to distinguish it, as if coördinate, from such titles as Περὶ οὐρανοῦ and Περὶ μετεώρων, we shall be in better position to decide at the conclusion of our inquiry. But, while it is clearly impossible, without writing a history of Greek philosophy, to refute his

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[9] Besides Herodotus, we have incorporated titles from Hecataeus (fr. 332 Müller), Antiochus of Syracuse (fr. 3 Müller), Alcmaeon (fr. 1), and Thucydides. It is possible that the Μικρὸς Διάκοσμος of Democritus had such a title; cp. Diog. Laert. IX. 41. We have, however, what are said to be the opening words of other works, but mention neither the name of the author nor the subject; e. g. Heraclitus, fr. 1; Archytas, fr. 1; Anaxagoras, fr. 1; Protagoras, fr. 1 and 4; Diogenes of Apollonia, fr. 1. For those who hold the fragments attributed to him to be genuine I may add, Philolaus, fr. 1. One may, of course, assume that the incorporated title was in these cases disregarded, either because a formal title had been substituted for it, or because it was considered negligible. The works of Hippocrates, however, do not have incorporated titles naming the author; but have in some cases an introductory sentence which announces the subject: e. g. Π. γυναικείης φύσιος (7, 312 Littré) περὶ δὲ τῆς γυναικείης φύσιος καὶ νοσημάτων τάδε λέγω; similarly Democritus, fr. 165 λέγω τάδε περὶ τῶν ξυμπάντων. Cp. also Hippocrates (Littré) 8, 10; 8, 408; 8, 466; 8, 556; 8, 512.

[10] Hippocr. Π. ἀρχ. ἰητρικῆς, 20 (1, 620 Littré) τείνει δὲ αὐτοῖς ὁ λόγος ἐς φιλοσοφίην, καθάπερ Ἐμπεδοκλῆς ἢ ἄλλοι οἳ περὶ φύσιος γεγράφασιν. ἐγὼ δὲ τοῦτο μέν, ὅσα τινὶ εἴρηται ἢ σοφιστῇ ἢ ἰητρῷ ἢ γέγραπται περὶ φύσιος, ἧσσον νομίζω τῇ ἰητρικῇ τέχνῃ προσήκειν ἢ τῇ γραφικῇ. Π. σαρκῶν, 15 (8,604 Littré) καὶ εἰσί τινες οἳ ἔλεξαν φύσιν ξυγγράφοντες ὅτι ὁ ἐγκέφαλός ἐστιν ὁ ἠχέων. In Hippocrates we find such titles as Π. φύσιος ὀστέων, Π. φύσιος παιδίου, Π. φύσιος ἀνθρώπου, Π. φύσιος γυναικείης. The meaning of these titles will be seen, I trust, in the sequel. It may excite comment that I quote Hippocrates indiscriminately. I do so because to do otherwise were to prejudge a question not yet settled — hardly even fairly put. I incline to the opinion that the works of the *Corpus Hippocrateum* (with possibly one or two exceptions) belong to the fifth century; at any rate, the conceptions and points of view they present show few traces of the influence of Socratic thought.

[11] Xen. *Mem.* I. 1, 14 τῶν τε περὶ τῆς τῶν πάντων φύσεως μεριμνώντων; Plato, *Legg.* 891 C; *Phaedo* 96 A (see above, note 7) ἐγὼ γάρ, ἔφη (sc. ὁ Σωκράτης), νέος ὢν θαυμαστῶς ὡς ἐπεθύμησα ταύτης τῆς σοφίας ἣν δὴ καλοῦσι περὶ φύσεως ἱστορίαν, which is of great importance since in this connexion Plato most clearly defines the relation of the Socratic-Platonic philosophy to that of the φυσικοί; for Aristotle it is hardly necessary to do more than refer to Bonitz's *Index* under the expressions οἱ φυσικοί, οἱ περὶ φύσεως, οἱ φυσιολόγοι, φυσιολογεῖν.

further statements that "the search for the primary substance really was the thing that interested the Ionian philosophers" and that "Greek philosophy began, as it ended, with the search for what was abiding in the flux of things;" it must be said that so to define the scope of Greek philosophy were to reduce it to terms which are well-nigh nugatory. Greek philosophy did, indeed, seek the permanent amid the flowing; but, as the first determined effort of the human mind to frame a science, it sought an explanation of the fleeting phenomena. This explanation it found ultimately in that which abides, and gave to it various names: but it was not the permanence, but the causality, of the *ὑποκείμενον* to which, as scientists, the Greek philosophers devoted their chief attention.[12] Aristotle was clearly right in refusing to regard the Eleatics, in so far as they adhered to their metaphysical principles which excluded causality and motion, as *φυσικοί.*

I.

"One may say that primitive man has only religious apperceptive masses." "No matter what historical phenomenon we may trace to a remote past, we come at last to religion. All human conceptions, so far as they fall within the intellectual horizon of a pre-scientific age, have developed out of mythical conceptions; but religious ideas constitute the content, or at least, the garb of myth." These words from the pen of the lamented Professor Usener[13] strike the key-note of this portion of our study.

As later Greek philosophy, so far as it was a philosophy of nature, grew out of the teachings of the pre-Socratics with only here and there a clearly marked infusion of metaphysics, ultimately derived from Socrates: so Greek philosophy as a whole was not a creation *e nihilo.* Long before the dawn of philosophy, properly so-called, the reflective thought of the Greeks had busied itself with many of the problems which later engaged the attention of the philosophers.[14] Even if we had no evidence to prove it, we should still have to assume it as a fact. We are not, of course, in position to trace even in the most general

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[12] In my study, *The Necessary and the Contingent in the Aristotelian System*, Chicago, 1896, pp. 7–10, I gave a brief analysis of the movement of pre-Socratic thought in logical terms. Somewhat more at length a similar study appeared in *The Logic of the Pre-Socratic Philosophy*, published as Chapter IX. of *Studies in Logical Theory*, by John Dewey, Chicago, 1903.

[13] *Vorträge und Aufsätze*, pp. 43 and 45.

[14] There is much philosophy held in solution in Greek mythology; but it is impossible to utilize it for historical purposes, because the early history of the myths is unknown. Unfortunately this is likely always to be the case.

outlines the stages in the process of organizing the confused mass of primitive human experience into a unified world of thought. We may be sure, however, that there never was a time when the human mind held even two wholly unrelated experiences; and there will never come a time when all human experiences shall constitute a perfect κόσμος. Somewhere between these limits history moves, the mind now energetically striving to achieve a synthesis, now supinely acquiescing in "the cult of odds and ends."

When the curtain of history rises on the Greeks, we find in Homer a strange condition. In the foreground there is a relatively well ordered society of gods and men; while in the shadows of the background lurk remnants of an ancient barbarism. Politically society is in unstable equilibrium, momentarily held together by a common cause: particularism clearly preceded, particularism follows. One can with difficulty banish the thought that the union of the Greeks under the suzerainty of Agamemnon was only a poet's dream, — an ideal never realized and perhaps never to be realized. Homeric religion is in much the same case: Zeus is king of all the gods, but even after his victory over the turbulent sons of Earth, his rule is precarious. The Titans fume; and the wife of his bosom nurses thoughts of treason.

As for the occurrences of daily life, they are the expression of divine powers [15] lurking everywhere and acting more or less capriciously. Nothing that occurs occasions much surprise, [16] and a ready explanation for even the most unexpected event is suggested by the inscrutable operations of the gods. This is not the atmosphere which surrounds and stimulates the birth of philosophy. But while Homer, on the whole, writes for entertainment and tells such tales as may fitly cheer a pleasant feast, there are not wanting in the *Iliad* passages which show that the Greeks of that age sometimes thought in a less light-hearted vein. Two portions in particular, the Διὸς Ἀπάτη [17] and the Θεομαχία, [18] contain unmistakable vestiges of earlier theogonic and cosmogonic poems. The tendency here appearing in Homer finds increasing favor with Hesiod and the cosmogonists of the eighth and seventh centuries B.C.

For reasons hardly intelligible to me it has become common to dis-

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[15] Cp. Adam, *The Religious Teachers of Greece*, p. 22. If Thales said πάντα πλήρη θεῶν, it was a survival of 'Homeric' thought out of harmony with the new philosophical movement. Such survivals, however, are common in all ages.

[16] Cp. Adam, *ibid.*, p. 24.

[17] *Il.* XIV.

[18] *Il.* XX, XXI. That this passage is cosmological was seen by Theagenes in the sixth century, B.C. (see *Schol. Il.* B on Υ, 67), and emphasized by Murray, *Rise of the Greek Epic*, p. 239 ff., and by Gilbert, *Die meteorologischen Theorien des griechischen Altertums*, p. 25, n. 2.

tinguish these interesting early thinkers from the illustrious company of the philosophers, headed by Thales, as if they belonged to different orders of existence. Certain it is that Aristotle was not aware of any such fundamental difference. "Even a lover of myth," he says,[19] "is in a sense a philosopher." Thales he calls the founder of the school of philosophy which inquires into the material cause of things; but he adds,[20] almost in the same breath, that "some think that the ancients who lived long before the present generation, and first framed accounts of the gods, had a similar view of nature." By late writers no distinction whatever is made between the two classes of thinkers; thus Hippolytus says,[21] "The poet Hesiod himself declares that he thus heard the Muses speak Περὶ φύσεως." Plato, on the other hand, says in a playful vein of the early philosophers,[22] "Each appears to me to recount a myth for our entertainment, as if we were children. One says that the things that are are three in number, and that certain of these somehow go to war with one another from time to time; then again they become reconciled, contract marriages, beget children, and rear their offspring. Another says there is a pair, — Moist and Dry, or Hot and Cold, — and gives away the bride and lets the pair cohabit. The Eleatic tribe out our way, however, going back to Xenophanes and even farther, recounts its tales as if all beings, so called, were one." However we may interpret the passage in detail, it is obvious that Plato notes and emphasizes the fundamental identity in point of view between the early cosmogonists and the golden tribe of philosophers. He shows how easy it is to state philosophical conceptions in mythological terms, and suggests by implication that the opposite procedure is equally easy.

Aristotle also clearly correlates *θεολόγοι* and *θεολογία* with *φυσιολόγοι* and *φυσιολογία* in such sort as to show that in his view words and concepts run alike parallel.[23] He likens the earliest philosophy to a lisping child,[24] and makes repeated attempts to restate in more acceptable form the opinions of his predecessors.[25] He would doubtless have

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[19] *Met.* 982^b 18.

[20] *Met.* 983^b 20 and 27 foll., transl. Ross. It is noteworthy that, though Aristotle does not expressly assent to the interpretation of the myth, he evidently has no thought of refuting it.

[21] *Philos.* 26 (Diels, *Dox.* 574, 14).

[22] Plato, *Soph.* 242 C. For this passage see Diels, *Vorsokr.*,^{2} 40, § 29.

[23] Cp., e.g., *Met.* 1071^b 26 foll., 1075^b 26 foll.

[24] *Met.* 993^a 15 foll. Cp. the interesting prelude to the myth, Plato, *Polit.* 268 E. This conception powerfully stimulated the tendency to allegorical interpretation, and accounts for Aristotle's freedom in reinterpreting his predecessors.

[25] I directed attention to several instances of somewhat violent reinterpretation

offered a like apology, only with larger charity, for the still earlier cosmogonists. Theophrastus[26] in the same spirit remarked upon the 'poetic' diction of Anaximander because he referred to the mutual encroachment of the elements as 'injustice.' Indeed, the mythical cast of much of the earlier philosophy is so marked as to constitute a serious problem to the historical student, who desires to interpret fairly the thought of the age. This fact, duly considered, throws light in both directions. It shows, on the one hand, that theogonists and cosmogonists employed the names of divinities to designate philosophical, or at any rate, quasi-philosophical concepts; but it also shows that the philosophers were not themselves conscious of a complete break with the past. Thus, while the theogonists pictured the origin and operations of the world in terms of the history and behavior of mythical characters, often so vaguely and imperfectly conceived[27] as at once to betray their factitious nature, the philosophers applied to their principles and elements names and epithets proper to the gods.[28] This course was, indeed, extraordinarily easy and natural to the Greeks, whose religion was in its higher phases essentially a worship of Nature.[29] But this very worship of Nature in her more significant aspects was in itself one of the chief influences which predisposed the Greeks to a philosophy of Nature.

There are certain picturesque effects of this intimate historical connexion of speculation on nature with theology (in the Greek sense), which are perhaps worth noting. Aristotle repeatedly uses the expression κόσμον γεννᾶν alongside κοσμοποιεῖν or κοσμοποιία in reference

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of his precursors in my study, *Qualitative Change in Pre-Socratic Philosophy* (Archiv. für Gesch. der Philos., 1906). There seem still to remain a few scholars who, even after the illustrations of this tendency noted by Natorp (e. g., Philos. Monatshefte, xxx. 345) and Burnet, are unaccountably blind to it.

[26] Apud Simpl. *In Phys.* I. 2, p. 24, 20 (Diels).

[27] See, e.g., Diels, *Parmenides Lehrgedicht*, p. 10; Rohde, *Psyche*, II. 114 and 115, n. 2; Ed. Meyer, *Gesch. des Altertums*, I, a (2d ed.), p. 100 foll.; Burnet, *Early Greek Philosophy*, (2d ed.) p. 74 foll.

[28] Cp. Otto Gilbert, *Ionier und Eleaten*, Rh. M., N. F., 64, p. 189. Empedocles deifies the Sphere, the elements, and the efficient causes, Love and Strife. The practice continues throughout Greek thought. The question is where religious belief ends and metaphor begins: see Millerd, *On the Interpretation of Empedocles*, p. 34. I do not doubt that Professor Millerd, as well as Gilbert (l. c. and *Meteorol. Theorien*, etc., p. 110, n. 1) and Adam, *The Religious Teachers of Greece*, pp. 184–190, 248, 250, go too far in accepting as sober belief what was in fact 'poetic' metaphor. See Burnet, p. 74 foll., p. 288 foll. Rohde says (*Psyche* II. 2) "Wer unter Griechen *unsterblich* sagt, sagt *Gott*: das sind Wechselbegriffe." This statement certainly requires qualification; but this is not the place to discuss the matter at length.

[29] Ed. Meyer, *Gesch. des Altertums*, I, a (2d ed.), pp. 97–100, distinguishes, — aside from purely magical beings, — two classes of gods: I. universal gods, con-

to philosophical accounts of creation;[30] and derivative forms of existence are called *ἔκγονοι* or *ἀπόγονοι* of the elements.[31] In other words, the philosophers were in effect giving the genealogy of the world.[32]

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ceived as presiding over certain spheres of the (physical or intellectual) world everywhere and for all men; II. particular gods, having locally or tribally circumscribed spheres. There is, of course, a certain overlapping. The gods of the first class exist as permanent beings by reason of the eternally identical activities proceeding from them; those of the second class attain permanence and personality by reason of the institution of a fixed cult. Many gods of the first class possess little or no cult, but stand as representatives of natural laws. "No one," says Professor Burnet, p. 75, n. 1, "worshipped Okeanos and Tethys, or even Ouranos." Since the superior gods of Greece are largely of this class, it is not difficult to see how religion proved a schoolmaster to lead the Greeks to philosophy.

[30] For examples see Bonitz's *Index*, 150^{a} 7 foll. Cp. such expressions as *γεννῶσι δὲ [παθητικαὶ δυνάμεις] τὸ θερμὸν καὶ ψυχρὸν κρατοῦντα τῆς ὕλης*, *Meteor.* 379^{a} 1; *μετὰ δὲ τούτους καὶ τὰς τοιαύτας ἀρχάς, ὡς οὐχ ἱκανῶν οὐσῶν γεννῆσαι τὴν τῶν ὄντων φύσιν*, *Met.* 984^{b} 8. Cp. Plato, *Theaet.* 153 A.

[31] Similar expressions abound, as, e.g. *τὰ δὲ ἄλλα ἐκ τούτων.* See my article, *Qualitative Change in Pre-Socratic Philosophy*, notes 36 and 41.

[32] In this connexion it is proper to refer to the beginnings of Greek historiography — both are *ἱστορίαι.* In each case it is the desire of the *ἵστωρ* to go back to first principles. Professor Millerd speaks of Empedocles' *Περὶ φύσεως* as a "world story;" such in truth it is. History appears to have grown up among the Greeks in connexion with Genealogy, dealing with *κτίσεις* and other similar events. In Xenophanes, according to tradition, the two interests of *ἱστορία* were naturally united. His physical derivation of the present world constituted his natural philosophy; on the historical side, he is reported to have composed poems on the founding of Colophon and the colonization of Elea. While this latter statement may be questioned (see Hiller, Rh. M., N. F. 38, 529) on external grounds, it is not *per se* improbable. The *Book of Genesis* similarly unites interest in creation and the derivation and early history of a people. It seems to be natural to the human mind to put explanation in the form of a *story*; even where it is a question of explaining how present phenomena occur, it is usual to cast the answer into the form of *origines.* This tendency has misled historians of Greek philosophy at many points into the vain endeavor to distinguish between the current cosmic processes and the story of creation. Another matter of much interest is the relation of creation-story and genealogy, which are thus united in *ἱστορίη περὶ φύσεως*, to the religious *ἱερὸς λόγος* or gospel. Of this I have spoken incidentally in another connexion; but it is obvious, even at a glance, that in *Genesis*, for example, they are virtually identical. In later schools of Greek philosophy the *naturae ratio* was clearly and consciously felt to be a gospel. It is therefore interesting to note that of the four Christian Gospels, three in various ways link the gospel story proper with the story of creation. Mark, the "human Gospel," omits this essential link. The later Gospels supply it: Matthew is content to trace the genealogy of Jesus to Abraham, from which point the story was familiar; Luke carries it back to Adam, "the son of God;" John goes back to the "beginning" and finds the *Λόγος*, or Gospel Incarnate, with God before, and preparatory to, creation. Hence he can dispense with a genealogy. One must bear in mind the supposed compelling force of genealogy in prayers. Among many peoples we find the practice of addressing

The intimate connexion of physical philosophy with theogony and cosmogony has thus been emphasized because it appears fundamental to any intelligent inquiry into the meaning and nature of the former; yet no one would deny that there is a distinction to be drawn between these cognate forms of speculation on the origin and operations of the world. The important point to determine is just wherein the essential difference consists.

In Plato there is a clear distinction drawn between *μῦθος* and *λόγος*; with him *μυθολογία* is associated with *ποίησις*, and, when contrasted with *λόγος* or *ἱστορία*, denotes that which is fictitious as opposed to sober truth. Herein Plato reflects the spirit of the sixth and fifth centuries, B. C., which brought science to the birth. Of that period Xenophanes is an interesting representative. We have seen that he combined the various interests of *ἱστορία*, and he naturally found himself in hostility to Homer[33] and all for which Homer stood. Homer stood for epic poetry, and epic poetry stood for *μῦθος*. To the mind of Xenophanes the myths of Titans, Giants, and Centaurs are *πλάσματα τῶν προτέρων . . . τοῖσ' οὐδὲν χρηστὸν ἔνεστι.* Indeed, what could such fictions profit an age that was busily engaged in sweeping the mists from the crest of Olympus to let in the dry light of reason? Hecataeus, another child of the sixth century and a *λογογράφος* or devotee of *ἱστορία*, in the introductory sentence of his *Genealogies*, says:[34] "I write the following as it seems to me in truth; for the tales (*λόγοι*) of the Greeks are many and, as I think, absurd." He employs the term *λόγοι* where a later writer would probably have said *μῦθοι*; for he refers to Greek mythical genealogies. Yet *λόγος* had even in his day come to mean prose[35] as opposed to epic composition, and Hecataeus proposed to use the new vehicle of artistic expression in the service of sober truth or *ἱστορία*.[36] It is noteworthy that he criticises the stories of "the

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the gods in prayer and enforcing the fulfilment of the request by giving the genealogy (or as Herodotus, I. 132 says, the *θεογονίη*) of the divinities. This is in turn connected with the magical procedure, which consists in "assigning the cause" and telling how that which, e. g., produced the wound (say, iron) originated, thus controlling the cause and effecting a cure. On this see Stewart, *The Myths of Plato*, p. 10 foll., who calls this the "aetiological myth."

[33] See Diels, *Parmenides Lehrgedicht*, p. 10.

[34] Fr. 332, Müller.

[35] What the substitution of prose for verse meant to philosophical thought can be best appreciated, perhaps, in connexion with Parmenides and Empedocles. Parmenides tried to write verse like a philosopher, and was ridiculed as a shabby poet; Empedocles tried to write philosophy like a poet, and is regarded as a fifth-rate thinker for his pains.

[36] For *ἱστορίη* see Stein on Hdt. I. 1; for *λόγος*, *ibid.*, I. 21. For the whole matter, see Bury, *Ancient Greek Historians*, p. 16.

Greeks,"[37] finding them utterly ridiculous. The new era of travel and research had brought to light many an evidence that things were not what they seemed, at least that much which passed for true and unquestionable among the Greeks was differently conceived or otherwise done in other lands.[38] The age of the Sophists merely made common property what had for a hundred years exercised the wits of the great leaders of the new thought.

We have seen that Greek religion in the Homeric age harbored two conceptions which contained the promise of disintegration, though they still dwelt peacefully side by side. According to the one conception every event was equally divine and so equally "natural," occasioning no surprise; according to the other, certain provinces of the world, physical and intellectual, were apportioned to the "wide-ruling gods" of Olympus. The former tended to dull the faculty of curiosity, the latter to stimulate it. For, in a sense, the Olympians were personified laws of Nature. With the increasing organization of experience came greater emphasis upon the "Götterstaat" and overlordship of Zeus, who assumed more and more the title of θεός *par excellence* and subordinated the lesser gods to himself, reducing them in the end to expressions of his sovereign pleasure. But back of Zeus, even in Homer, lurks the mysterious power of Μοῖρα, before whose might even the "pleasure" of Zeus avails little. As Zeus subdues the lesser gods, so Fate or Law subdues Zeus to her inexorable will. But the bright patterns woven into Greek mythology, based as they were upon personal caprice and

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[37] Bernays, Abh. der Berl. Akad., 1882, p. 70, refers to Anaxagoras (fr. 17 Diels: τὸ δὲ γίνεσθαι καὶ ἀπόλλυσθαι οὐκ ὀρθῶς νομίζουσιν οἱ Ἕλληνες), to Hecataeus (fr. 332), Philodemus (Π. εὐσεβείας p. 84, Gomp.: ὅσους φασὶν οἱ Πανέλληνες θεούς) and adds: "Es ist die vornehme Art der Philosophen von dem Volk zu reden." Compare also Empedocles, fr. 8 and 9 (Diels). The feeling is deeper than mere pride: it marks the exaltation of the philosophical λόγος, as the statement of φύσις, over the popular λόγος which stands for νόμος and μῦθος. Bury, *Ancient Greek Historians*, p. 51, n. 2, remarks that when Herodotus quotes and criticises οἱ Ἕλληνες he is contrasting the Greek tradition with that of Phœnicians, Persians, or Egyptians, and "is really quoting criticisms of Hecataeus on οἱ Ἕλληνες, that is, on the current mythology of epic tradition."

[38] It would be foolish to claim for any one cause the determining influence in giving direction and scope to the nascent rationalism of the sixth century. Travel and research could furnish the content and supply the materials for reflective thought; but both presuppose the divine curiosity which is the parent of philosophy. Many influences conspired to produce the revolution in thought; but travel may well have contributed most to convert curiosity into astonishment. The curious collections of strange and shocking customs, of which we find echoes in Herodotus, Hippocrates, the Διαλέξεις, etc., clearly originated in the sixth century, and supplied the arsenal of the militant Sophists.

anthropomorphic passions, ill comported with the growth of reason which demanded submission to universal law. Greek religion experienced the inevitable conflict between the imagination, the flowering of the capricious faculties of youth, and the reflective reason, in which the maturing powers assert their right to fixed habits of thought.

Now *φυσιολογία* is simply *λόγος* or *ἱστορία περὶ φύσεως*, — the child of the maturing age which set itself to discard or disregard childish things and to see things as they are. Thus *λόγος περὶ φύσεως* succeeds *μῦθος περὶ θεῶν*. The transition is natural; but it involves an element of opposition which could not help but be painful and even bitter as the extent and bearings of the inevitable conflict came to consciousness. The history of pre-Socratic philosophy is the history of this conflict; but the opposition was not final. The strain of conflicting ideals resulted in a new synthesis. Plato and Aristotle sought to effect such a synthesis, and the endeavor to perfect it is the characteristic of the main current of post-Aristotelian philosophy from the Stoics to Plotinus.

Gibbon's saying,[39] "Freedom is the first step to curiosity and knowledge," nowhere finds fuller application or illustration than in the history of Greek philosophical thought; and nowhere did the early Greek thinkers so much feel the need of asserting their freedom as in the sphere of opinion where there was an actual or possible clash with the received theology in the guise of *μῦθος*. From the first, philosophers had broken with it in intention, however much haunted they might have been individually or collectively by presuppositions formulated in their mythology. It should occasion no surprise to find inconsistencies and lapses from their principles; for such are common in all ages, because of the imperfect fluidity of the mental content, which refuses to be reshaped at a cast. Nor should we expect to find the principles operating to the regeneration of thought explicitly stated at the beginning: it is the rule that the clear enunciation of principles follows, often tardily, the tacit application of them. Plato speaks of the ancient feud between poetry and philosophy; and the point of contention concerns *μῦθος*.[40] Plato also well expresses the fundamental difference between the two. To him the poet is a *θεῖος ἀνήρ*,[41] a seer who works by inspiration;[42] the philosopher must follow the argument, even against

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[39] *Decline and Fall*, ch. 66.

[40] *Repub.* 607 B. See Adam's note *ad loc.* and *The Religious Teachers of Greece*, p. 2 foll., 401 foll.

[41] *Repub.* 368 A (with Adam's note).

[42] *Apol.* 22 A foll., etc.

his inclination: ὁ γὰρ λόγος ἡμᾶς ᾕρει, he says of himself[43] in apologizing for expelling Homer from the ideal state of the philosopher-king.

In the Epicurean *Epistle to Pythocles*[44] a distinction is drawn between such phenomena as admit of but one rational explanation and such as admit of several explanations equally consonant with the data of sense. In the former, the conclusion must be categorically affirmed; in regard to the latter, one must suspend judgment: "for one must conduct investigations into the operations of nature, not in accordance with vain dogmas and ex-cathedra pronouncements, but according as the phenomena demand. . . . But when one fails to state one possible explanation and rejects another that is equally consonant with the data of sense, it is evident that one falls wholly outside the breastworks of science and lapses into μῦθος."[45]

From the first φυσιολογία or ἱστορία περὶ φύσεως is characterized by the fact that it wholly disregards religious authority[46] (νομοθεσία of

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[43] *Repub.* 607 B. Following the lead of the argument is a commonplace in Plato: cp. *Euthyph.* 14 C, *Theaet.* 172 D, *Gorg.* 527 E, *Phaed.* 82 D, 115 B, *Repub.* 365 D, 394 D, 415 D, *Legg.* 667 A.

[44] Diog. Laert. x. 86–87.

[45] The fear of μῦθος was ever-present to Epicurus and his followers. See my *Epicurea* (American Journal of Philology, XXIII. p. 194) and compare Κύριαι Δόξαι, XI.–XIII. and Lucretius I. 68 foll., 102 foll., 151 foll., V. 1183 foll. See also Zeller, *Phil. der Griechen*, III. (a), 397, n. 2. Epicurus was, however, herein only following Democritus, fr. 297 (Diels): ἔνιοι θνητῆς φύσεως διάλυσιν οὐκ εἰδότες ἄνθρωποι, συνειδήσει δὲ τῆς ἐν τῷ βίῳ κακοπραγμοσύνης, τὸν τῆς βιοτῆς χρόνον ἐν ταραχαῖς καὶ φόβοις ταλαιπωρέουσι, ψεύδεα περὶ τοῦ μετὰ τὴν τελευτὴν μυθοπλαστέοντες χρόνου. Rohde, *Psyche*, II. 171, n. cast suspicion on the genuineness of this fragment; but it has been well discussed by Nestle, Philol. 67, 548. Epicurus required that one judge concerning what cannot be seen (τὰ ἄδηλα) on the analogy of that which is visible. In this also he followed the pre-Socratics. See Sext. Emp., VII. 140 Διότιμος δὲ τρία κατ᾽ αὐτὸν (i. e. Democritus) ἔλεγεν εἶναι κριτήρια· τῆς μὲν τῶν ἀδήλων καταλήψεως τὰ φαινόμενα· "ὄψις γὰρ τῶν ἀδήλων τὰ φαινόμενα," ὥς φησιν Ἀναξαγόρας (fr. 21 a, Diels), ὃν ἐπὶ τούτῳ Δημόκριτος ἐπαινεῖ. The same injunction was given to the physician; see Hippocrates, Π. διαίτης; I. 12 (6, 488 Littré). Epicurus was ridiculed for offering explanations which were foolish: cp. the delectable skit in Usener's *Epicurea*, p. 354, 27 foll., where he is taunted with believing a μυθαρίῳ γραώδει. But the charge was disingenuous, since the explanation in question was only one of several among which he allowed his followers to choose, since the matter was not one of which strict account was required of the faithful.

[46] It would be impossible to prove this without showing in detail — what is easy but requires more space than can be allotted to it here — how the conclusions of philosophers ran from the first counter to the fundamental assumptions of the received theology. The philosophers therefore came to be regarded as a godless crew: cp. Plato, *Apol.* 18 B C, 19 B, 23 D; Xen. *Mem.* I. 2, 31; Plut. *Pericles*, c. 32 (law of Diopithes, 432 B. C.).

Epicurus) and prejudice (*δεισιδαιμονία*),[47] and endeavors to explain natural phenomena on the basis of well considered facts and analogies,[48] assuming the constancy of nature and the universal reign of law.[49] Aristotle says that the early philosophers did not believe in chance,[50] and we find objection raised even to the conception of spontaneity,[51] which is made relative to human ignorance.

If one would catch the spirit of that age one must read the priceless repository of fifth century thought contained in the Hippocratean corpus and the fragments of the Sophists. So little remains to us of the

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[47] Rohde, *Psyche* II. p. 90 draws attention to the conscious opposition of philosophers to the magicians, etc. The same opposition developed among the philosophical and practical physicians, whence they also have been traditionally denounced as a godless crew. An interesting document in this regard is Hippocrates Π. *ἱερῆς νούσου*, quoted below, n. 133. See also Π. *παρθενίων* (8,468 Littré) : *τῇ Ἀρτέμιδι αἱ γυναῖκες ἄλλα τε πολλά, ἀλλὰ δὴ καὶ τὰ πουλυτελέστατα τῶν ἱματίων καθιεροῦσι τῶν γυναικείων, κελευόντων τῶν μάντεων, ἐξαπατεώμεναι.* Π. *εὐσχημοσύνης*, 5 (9, 234, Littré): The author says one must carry philosophy into medicine, and *vice versa*. The difference between the two disciplines is slight: among other things they have in common is *ἀδεισιδαιμονίη*; but medicine is not disposed to try to dethrone the gods — each in its own sphere !

[48] See Rohde, *Psyche*, II. 137. The pre-Socratic literature (including Hippocrates) is a remarkable repository of interesting observations and analogies, including a few carefully considered experiments.

[49] See Rohde, *Psyche*, II. 138 ; Milhaud, *Leçons sur les Origines de la Sciènce Grecque*, p. 11 foll. Aristotle says *Phys.* 261^{b} 25 : *φυσικὸν γὰρ τὸ ὁμοίως ἔχειν ἐν ἁπάσαις.* Hippocr. Π. *φύσιος ἀνθρώπου*, 5 (6, 42 Littré) in order to prove that something is *κατὰ φύσιν* says : *καὶ ταῦτα ποιήσει σοι πάντα πᾶσαν ἡμέρην καὶ νύκτα καὶ χειμῶνος καὶ θέρεος, μέχρις ἂν δυνατὸς ᾖ τὸ πνεῦμα ἕλκειν ἐς ἑωυτὸν καὶ πάλιν μεθιέναι, δυνατὸς δὲ ἔσται ἔστ' ἄν τινος τουτέων στερηθῇ τῶν ξυγγεγονότων.* Who could give a better statement of the constancy of natural law applied to a given case ? Π. *διαίτης* I. 10 (6, 486 Littré) *πῦρ, ὅπερ πάντων ἐπικρατέεται, διέπον ἅπαντα κατὰ φύσιν.* Leucippus (fr. 2 Diels) : *οὐδὲν χρῆμα μάτην γίνεται, ἀλλὰ πάντα ἐκ λόγου τε καὶ ὑπ' ἀνάγκης.* Hippocr. Π. *ἀέρων*, 22 (1, 66 Kühlewein) : *γίνεται δὲ κατὰ φύσιν ἕκαστα.* Epicurus and Lucretius (1, 150) regard the dictum "nullam rem e nilo gigni divinitus umquam" as the cornerstone of a rational view of the world : Aristotle repeatedly affirms that it was the common postulate of the early philosophers. Once (*de Gen. et Corr.* 317^{b} 29) he hints that the intervention of the gods was to be thereby excluded : *ὃ μάλιστα φοβούμενοι διετέλεσαν οἱ πρῶτοι φιλοσοφήσαντες, τὸ ἐκ μηδενὸς γίνεσθαι προϋπάρχοντος.*

[50] Arist., *Phys.* 196^{a} 5–11. This means, of course, that the philosophers believed their principles sufficient to account for things. When later writers charge the Atomists, for example, with having recourse to chance, this is said from the point of view of teleology : a purely physical cause was thought to be no cause at all. On the practical side, chance is luck. The physicians thought they could dispense with it ; see below, n. 152 and 153.

[51] Hippocr. Π. *τέχνης*, 6 (6, 10 Littré) *τὸ αὐτόματον οὐ φαίνεται οὐσίην ἔχον οὐδεμίην, ἀλλ' ἢ οὔνομα μοῦνον.* Cp. Π. *τροφῆς*, 14 (9, 102 Littré) *αὐτόματοι καὶ οὐκ αὐτόματοι, ἡμῖν μὲν αὐτόματοι, αἰτίῃ δ' οὐκ αὐτόματοι.* In the popular sense *τὸ αὐτόματον* is allowed, Π. *νούσων*, A, 7 (6, 152 Littré), Π. *χυμῶν*, 6 (5, 486 Littré).

authentic utterances of the philosophers of the sixth and fifth centuries B.C., that we should study with especial interest the body of literature emanating in great part from the pamphleteers who assimilated and disseminated the teachings of the great masters. The latter were, as is the wont of true men of science, more reserved than the motley crowd of pseudo-scientists who caught up their half-expressed conclusions and published them in the market places to eager laymen, for whom the scientists entertained only an ill-concealed contempt.[52] No opinion was so well established that they would not sap its roots; no question was too obscure to baffle explanation. A certain decorous respect was still shown for the gods; but they had in fact become supernumeraries so far as concerned the explanation of the world. Thus Hippocrates [53] says: "In matters human the divine is the chief cause; thereafter the constitutions and complexions of women"; but while the divine is then dismissed, the constitutions and complexions of women are considered at length and made to account for everything. In other cases, as, e. g., in the treatise Π. *ἱερῆς νούσου*, the gods are definitely ruled out as a particular cause, and only the elemental substances, which rule in the human frame, are recognized as divine.[54] Thus the divine working becomes another name for the operation of Nature.

A good illustration of this procedure is found in Hippocrates, Π. *ἀέρων ὑδάτων τόπων*. After remarking that the Scythians worship the eunuchs because they attribute their estate to a god and fear a like fate for themselves, the author says:[55] "I myself regard this as divine, as well as everything else. One is not more divine nor human [56] than another; but all are on the same level, and all are divine. Yet every one of these things has its natural cause, and none occurs without a natural cause. I will now explain how in my opinion this comes about." Whereupon the author proceeds to give a purely naturalistic explanation. You will note here the words *ἕκαστον . . . ἔχει φύσιν τὴν ἑαυτοῦ* [57]

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[52] See above, n. 37. For the physicians, see Hippocr. Π. *ἄρθρων*, 67 (4, 280 Littré), Π*ροορρητικόν*, 2 (9, 10 Littré), Π. *τέχνης*, 1 (6, 2 Littré).

[53] Π. *γυναικείης φύσιος*, 1 (7, 312 Littré). Similarly Π*ρογνωστικόν*, 1 (2, 112 Littré) it is required that the physician study the nature of the disease to see whether it is too powerful for the strength of the body, *ἅμα δὲ καὶ εἴ τι θεῖον ἔνεστι ἐν τῇσι νούσοισι, καὶ τουτέου τὴν πρόνοιαν ἐκμανθάνειν.* Yet, the main business of the physician is with the disease and its natural causes, which he must combat.

[54] Hippocr. Π. *ἱερῆς νούσου*, 18 (6, 394 Littré): *ταῦτα δ' ἐστὶ θεῖα, ὥστε μηδὲν διακρίνοντα τὸ νούσημα θειότερον τῶν λοιπῶν νουσημάτων νομίζειν, ἀλλὰ πάντα θεῖα καὶ ἀνθρώπινα πάντα· φύσιν δὲ ἔχει ἕκαστον καὶ δύναμιν ἐφ' ἑωυτοῦ.* For the last phrase see n. 57.

[55] Ch. 22, p. 64 Kühlewein.

[56] Cp. n. 54.

[57] Natorp, Philos. Monatshefte, 21, 581 detects in these words a protest against teleology. I think he is in error: it is rather a protest against the supposition of

καὶ οὐδὲν ἄνευ φύσιος γίνεται. — "Every thing has its natural cause and nothing occurs without a natural cause." Nature has usurped the power of deity. Lest any should fail to catch his meaning, the writer, after detailing his naturalistic explanation, repeats: "but as I said above, this is equally divine with other things; but everything occurs in accordance with natural law." Elsewhere[58] Hippocrates suggests that it is ignorance alone which inclines the vulgar to regard epilepsy as a divine visitation. It is in keeping with this view that teleology is excluded; even where a modern scientist would involuntarily slip into modes of expression which imply final causes, the pre-Socratics, though at a loss for a satisfactory explanation, offer no such suggestion.[59] To the Socratics it was a scandal that Anaxagoras made no teleological use of his Νοῦς.[60]

When nature was thus interpreted, it is clear that the gods must suffer. One recourse was to attribute the organization of the world to them, and then to have done with them. This is suggested by Hip-

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a direct intervention of the gods in the regular course of nature. The scientific assumption of proximate, special causes is perhaps an outgrowth of the suppositions of magic, for which see Ed. Meyer, *Gesch. des Altertums*, I. (a) p. 97. Heraclitus, fr. 1 (Diels) *διαιρέων ἕκαστον κατὰ φύσιν καὶ φράζων ὅκως ἔχει* appears to mean that the philosopher proposes to give in his philosophical *λόγος* both the general law or cause (for *φύσις* includes both; cp. Π. *ἱερῆς νούσου*, 1 (6, 352 Littré) *φύσιν μὲν ἔχει* (epilepsy) *ἣν καὶ τὰ λοιπὰ νουσήματα, ὅθεν γίνεται· φύσιν δὲ αὐτῇ καὶ πρόφασιν κτλ.*) and the proximate, particular cause. This latter promise he failed, of course, to keep; but that is true of every philosophy that has been, or ever will be, devised.

[58] Π. *ἱερῆς νούσου*, 1 (6, 352 Littré) *κατὰ μὲν τὴν ἀπορίην αὐτοῖσι τοῦ μὴ γινώσκειν τὸ θεῖον αὐτῇ διασώζεται.* The similarity of this case with that of *τύχη* and *τὸ αὐτόματον* (see above, n. 50 and 51) is at once apparent. Science can dispense with chance and God, in proportion as it apprehends the proximate causes of things. The religious bearings of this position need not be developed.

[59] Cp. Hippocrates, Π. *φύσιος παιδίου*, 19, 21 (7, 506 and 510 foll. Littré) in regard to the nails on fingers and toes, and in regard to the rising of milk to the breasts of the mother at parturition. Almost countless other examples might be cited. The significance of this fact is made clear when one thinks of the constant opposition of *τὸ οὗ ἕνεκα* to *τὸ ἀναγκαῖον* by Aristotle (*Hist. Animal.*, *Partt. Animal.*, etc.) and Galen (*De Usu Partt.*), the latter being the point of view of the pre-Socratics, the former that of the Socratic. Plato, *Tim.* 46 C foll. regards physical causes as mere *συναίτια, οἷς θεὸς ὑπηρετοῦσιν χρῆται τὴν τοῦ ἀρίστου κατὰ τὸ δυνατὸν ἰδέαν ἀποτελῶν.*

[60] See Diels, *Vorsokratiker*, Anaxagoras, § 47. Rohde, *Psyche* II. 192, n. 1 gives the impression that Anaxagoras employed teleology. Such a statement would be absurd. Our sources are explicit on this head. Proclus *ad Tim.* (ed. Diehl. I. 1) says: *Ἀναξ., ὃς δὴ δοκεῖ καθευδόντων τῶν ἄλλων τὸν νοῦν αἴτιον ὄντα τῶν γιγνομένων ἰδεῖν, οὐδὲν ἐν ταῖς ἀποδόσεσι προσχρῆται τῷ νῷ.* I add the passage because it is omitted by Diels. Cp. Gilbert, *Aristoteles und die Vorsokratiker*, Philol., 68, 392–395.

pocrates,[61] and was, apparently, the rôle assigned by Anaxagoras to his Νοῦς. Disguise it as he might, Aristotle could find no better solution of the problem. Plato[62] puts the question sharply as between God and Nature, and says that the majority favor the latter. Such, indeed, was for the moment the logical outcome of the pre-Socratic movement of thought. It might be allowed that the idea of God was innate;[63] but, like all other ideas, it was more likely to be regarded as having a history, and as requiring explanation along with the other immediate (*φύσις*) or mediate (*νόμος*) products of nature. Thus, among others, Critias[64] explained belief in the gods as a deliberate fiction concocted by a clever statesman to enforce morality beyond the reach of the law, supporting it with the natural fears inspired in man by *τὰ μετέωρα*. It is not necessary here to rehearse the familiar story of rationalism as applied to religion in the fifth century, B.C.;[65] but it is not too much to say that philosophy had deliberately enthroned Nature in the place of God.

But nature, thus completely depersonalized, could not so remain indefinitely. Conceived as the power that brings to pass all the events constituting the sum of experience, nature became in fact a Creator and Governor, only deprived of reason and purpose, and identified with the sum of existence.[66] The Greek mind, with its plastic imagination, was not likely, however, permanently to acquiesce in this impersonal view of nature, although Φύσις was extremely late in attaining personification as a deity.[67] Yet, as we shall see,[68] a good beginning was made in the pre-Socratic period. The transfer of the functions and attributes of the ancient gods to Φύσις by the philosophers of the

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[61] Π. *διαίτης*, I. 11 (6, 486 Littré) *φύσιν δὲ πάντων θεοὶ διεκόσμησαν.*

[62] *Soph.* 265 C *ζῷα δὴ πάντα θνητὰ καὶ δὴ καὶ φυτά . . . μῶν ἄλλου τινὸς ἢ θεοῦ δημιουργοῦντος φήσομεν ὕστερον γίγνεσθαι πρότερον οὐκ ὄντα; ἢ τῷ τῶν πολλῶν δόγματι καὶ ῥήματι χρώμενοι . . . τὴν φύσιν αὐτὰ γεννᾶν ἀπό τινος αἰτίας αὐτομάτης καὶ ἄνευ διανοίας φυούσης, ἢ μετὰ λόγου τε καὶ ἐπιστήμης θείας ἀπὸ θεοῦ γιγνομένης;*

[63] Hippocrates, Π. *εὐσχημοσύνης*, 6 (9, 234 Littré) *καὶ γὰρ μάλιστα ἡ περὶ θεῶν εἴδησις ἐν νόῳ αὐτὴ ἐμπλέκεται.*

[64] In the Satyr drama *Sisyphus*, fr. 25 (Diels).

[65] See Decharme, *La Critique des Traditions Religieuses chez les Grecs*, 1904.

[66] Cp. n. 62. With the necessary additions drawn from that passage the following definition of *φύσις* by Iamblichus (Stobaeus, I. 80, 9 Wachsmuth) well expresses the conception of the pre-Socratics: *φύσιν δὲ λέγω τὴν ἀχώριστον αἰτίαν τοῦ κόσμου καὶ ἀχωρίστως περιέχουσαν τὰς ὅλας αἰτίας τῆς γενέσεως.* Cp. also Hermes (Stobaeus I. 289, 26 Wachsmuth) *ἡ φύσις πάντων, φύουσα τὰ γιγνόμενα, φυὴν (= φύσιν) παρέχει τοῖς φυομένοις.*

[67] See K. Preisendanz, Philologus, XLVIII. (1908) pp. 474–5. Φύσις is worshipped in the tenth Orphic Hymn.

[68] See below, notes 106 foll.

sixth and fifth centuries eventually so charged Nature with personality that the Socratic teleology was a foregone conclusion. From Plato onwards, with few exceptions, philosophers proceed with the synthesis: the gods act according to the laws of nature, and Nature assumes the divinity of the gods.

II.

After thus sketching the setting of those works which by common consent bore the title Περὶ φύσεως, it is proposed in this section to consider the use of the term φύσις among the Greeks of the pre-Socratic period. Although this study is based upon a collection of passages nearly if not quite complete, it is not intended to treat the subject exhaustively, classifying each occurrence of the term. Such an exhibit, if carefully and intelligently made, would serve a valuable purpose ; its main uses would, however, be lexicographical rather than historical and philosophical. The purpose of this section is the more modest one of determining somewhat roughly the range of the term φύσις, in the period under discussion, as an index of the scope of the conception of Nature. While the chief emphasis will properly fall on works to be dated before 400 B.C., we shall have occasion to use, with proper precautions, also certain writings of later date, such as those of Plato and Aristotle. Indeed, the careful student is not likely to be greatly misled in this matter by any text of ancient Greek literature. The reason is already clear. The philosophy of the Greeks prior to 400 B.C., with the sole exception of that of Socrates, may all be properly described as concerning itself περὶ φύσεως. As such it is sharply contrasted with the later systems, the main interest of which, with few and relatively unimportant exceptions, lies elsewhere : to wit, in the spheres of logic, ethics, and metaphysics. This new interest did not date from Socrates, but had, like all conceptions, an interesting history. If we were here concerned with this history we should have to retrace our steps, beginning once more with Homer and the popular notions of the Greeks embodied in religion, mythology, and moral precepts. But all this would yield at most a Vorgeschichte ; for the *method*, which alone is of importance in philosophy proper, was created by Socrates.

There are, strictly speaking, only two periods in the history of occidental philosophy, the pre-Socratic and the Socratic. The first took external Nature as its point of departure, and fixed for all time the fundamental conceptions of physical processes. Even where it considered biological and intellectual processes, it started with mechanical notions and arrived in the end at materialistic conclusions. We may, if we choose, speak of the ethics or metaphysics of the pre-Socratics ;

but every careful student will be conscious of a fundamental difference. Socrates, by introducing the logical method of definition, based upon induction and employed in the interest of deduction, discovered a new order of existence, which was subject not to mechanical, but to teleological laws. Teleological facts were known from the beginning of time, and, as we have seen, Nature herself became, in the latter part of the pre-Socratic period, charged with personality in a measure which made a new interpretation of her operations a foregone conclusion; but teleology, considered *as a method of explanation*, was a discovery of the Socratics.

The significance of this fact can hardly be measured; certainly it has not been appreciated hitherto by historians of philosophy. Among the pre-Socratics conceptions have been found which were certainly alien to their range of thought; and the fundamental significance of the revolution wrought by Socrates still awaits the appreciation which is its due. Henceforth the world is definitively divided into two spheres, one subject to mechanical, the other subject to final, causes. The latter alone is really "intelligible"; of the other we may say ὅτι, not διότι. The later Greek systems owe their basic physical concepts ultimately, and almost exclusively, to the pre-Socratics: where these conceptions were in any way modified, the reasons for the change are commonly to be sought in obviously logical or metaphysical considerations traceable to the Socratics. Hence the two discrete streams of philosophical thought, though externally united, flow in the main peacefully side by side, clear and transparent everywhere save at the line of contact, where they become a trifle turbid. Plato and Aristotle constantly betray their dependence upon the predecessors of Socrates for their physical concepts; and where the post-Aristotelians departed from the specifically Platonic-Aristotelian doctrines, they harked back frankly to one or another of the pre-Socratics for their physical theories.

In the following synopsis the attempt has been made to classify the uses of the word φύσις in such sort as to suggest their relations one to another and to the root-meaning, which is assumed to be "growth." The scheme makes no claim to finality or completeness, being intended primarily as a means of displaying in a more or less logical order the chief connotations of the term. The inner history of the semasiology may be left to others whose interests incline them to such studies.[69]

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[69] I regret to say that I have not been able to obtain *Der Begriff der Physis in der griechischen Philosophie*, I Theil, von E. Hardy, Berlin, 1884. I know it only at second hand, chiefly through the reviews of Natorp (in Philosophische Monatshefte, 21 (1885), pp. 572–593) and of Lortzing (in Bursian's Jahresbericht, 96 (1899), pp. 223–225). There is a brief study of φύσις in Ch. Huit, *La Philosophie de la Nature chez les*

Synopsis of the Uses of φύσις.

φύσις: primary meaning, growth.

- I. φύσις as a process.
  - A. in the concrete: growth as a phenomenon or fact (φύσις = γένεσις)
  - B. in the abstract: growth as a law, principle or 'force' of nature.
- II. φύσις as the beginning of a process.
  - A. the starting point of the process considered impersonally as physical element, original condition, or place of origin. (Aristotle's "material cause.")
  - B. regarded as a person or originator. *Natura creatrix.* (Aristotle's "efficient cause.")
- III. φύσις as the end or result of a process. (Aristotle's "final cause," which, in the complete circle is identified with the "efficient cause.")
  - A. regarded from without, as the external frame or constitution.
    - 1. individual, = φυή, ἀκμή, (Aristotle's ἐντελέχεια).
    - 2. specific or generic, = ἰδέα, γέννα, γένος.
    - 3. universal, = κόσμος.
  - B. regarded from within, as character or constitution.
    - 1. physical: 'chemically' defined or analyzed into its constituent elements in pre-Socratic times, regarded with reference to its origin; (by the Socratics defined teleologically, with reference to its meaning or end).
    - 2. mental
      - a. regarded positively, as power, talent, instinct, native endowment.
      - b. regarded negatively, as natural limitations.

Let us now turn to the uses of φύσις, following the order of the synopsis and noting the implications involved in them. Etymologically φύσις means "growth:" as an abstract verbal its first suggestion (I.) is that of a process. The process of growth may be regarded concretely

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Anciens, Paris, 1901, pp. 65–69. Somewhat fuller is Woodbridge, *The Dominant Conception of the Earliest Greek Philosophy*, Philos. Rev., 1901, pp. 359–374, which was brought to my attention, after this article was in the hands of the printer, by Lovejoy, *The Meaning of φύσις in the Greek Physiologers*, Philos. Rev. (July), 1909, pp. 369–383. Professor W. A. Merrill's study of *The Signification and Use of the Word* NATURA *by Lucretius* (Proceedings of the American Philol. Ass'n, July, 1891, vol. 22, pp. xxxii–xxxiv) will serve as an interesting illustration of the influence of pre-Socratic usage. The same may be said of the articles *nature, kind,* and *kin*, in the Oxford English Dictionary. One cannot overlook the lexicographical studies of φύσις found in Aristotle's *Phys.* B, 1 (and in briefer form, *Met.* Δ, 4). Reference will be made to his distinctions at the proper points in the survey. There are several words of similar origin and meaning which should be studied in connexion with φύσις if a really exhaustive account of the word is to be given from a lexicographical point of view. Among them may be mentioned φυή and γέννα. Of course φύειν in all its uses is of the utmost importance; but, for our present purpose, these may be disregarded, except for occasional illustration.

(I. A) as a fact or phenomenon. This conception was to the Greeks so obvious [70] that the fact of natural growth lay at the foundation of their thought. Growth implies life, and life implies motion. This is true of Greek thought always. The growth denoted by *φύσις* refers to animal as well as to vegetable life; wherefore *φυτόν* appears originally to have applied to the former as well as to the latter. It is noteworthy that *φύσις*, as implying motion, seems always to denote a process or a phase of such process; that is to say, specifically the process itself, taken as a whole,[71] or its beginning, progress, or end. It does not lend itself, therefore, to use as an absolute *ἀρχή*: it is consequently always opposed, or subordinated to, creative force as such.[72] These ideas clearly hark back to the pre-Socratic period. In Empedocles we find *φύσις*, in the sense of absolute origination, denied;[73] in Aristophanes [74] we find *φύσις* in the sense of origin. It is difficult to classify certain uses of *φύσις*, where it may be rendered birth, descent, age, lineage, etc., but they may be set down here for convenience.[75]

But *φύσις*, as a process, may be viewed abstractly (I. B) as natural

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[70] Arist. *Phys.* 193^{a} 3 *ὡς δ' ἔστιν ἡ φύσις πειρᾶσθαι δεικνύναι γελοῖον.* These words apply to *φύσις* as a whole, which, according to Aristotle, is a process.

[71] There is an interesting passage in Plato's *Phaedo* 71 E foll., where he is applying to the soul the principles of the pre-Socratics: *οὐκ ἀνταποδώσομεν τὴν ἐναντίαν γένεσιν, ἀλλὰ ταύτῃ χωλὴ ἔσται ἡ φύσις; ἢ ἀνάγκη ἀποδοῦναι τῷ ἀποθνῄσκειν ἐναντίαν τινὰ γένεσιν; . . . τὸ ἀναβιώσκεσθαι.* Here *φύσις* is the circular process as a whole.

[72] Thus Arist. can say *ἡ δημιουργήσασα φύσις*, *De Partt. Anim.* 645^{a} 9, but that is said metaphorically; habitually *φύσις* is opposed to *δύναμις* and *τέχνη*, in that they operate from without, whereas *φύσις* resides within: *De Cael.* 301^{b} 17 *ἐπεὶ δὲ φύσις μέν ἐστιν ἡ ἐν αὐτῷ ὑπάρχουσα κινήσεως ἀρχή, δύναμις δ' ἡ ἐν ἄλλῳ ᾗ ἄλλο.* Cp. *Met.* 1049^{b} 8. *Met.* 1070^{a} 7 *ἡ μὲν οὖν τέχνη ἀρχὴ ἐν ἄλλῳ, ἡ δὲ φύσις ἀρχὴ ἐν αὐτῷ.* As the Stoics regarded God as immanent, they could speak of *Ζεὺς τεχνίτης*. In Plato, *Tim.* 41 C even the *θεοὶ θεῶν* are bidden: *τρέπεσθε κατὰ φύσιν ὑμεῖς ἐπὶ τὴν τῶν ζῴων δημιουργίαν.* Without discussing whether Plato's *δημιουργός* was regarded as a creator merely *κατ' ἐπίνοιαν* or not, it is clear that nature is supposed to proceed according to her own laws, and 'creation' is not *ἀπλῆ γένεσις*.

[73] Fr. 8 (Diels); *φύσις οὐδενός ἐστιν ἁπάντων | θνητῶν, οὐδέ τις οὐλομένου θανάτοιο τελευτή, | ἀλλὰ μόνον μίξις τε διάλλαξίς τε μιγέντων | ἐστί, φύσις δ' ἐπὶ τοῖς ὀνομάζεται ἀνθρώποισιν.* Aristotle, *Met.* 1014^{b} 35 curiously misinterprets *φύσις* here, equating it with *πρώτη σύνθεσις*, possibly because he misquoted *ἐόντων* for *ἁπάντων*, quoting (as usual) from memory. The slavish commentators do not correct him. Empedocles implies that laymen understand *φύσις* as *ἀπλῆ γένεσις*, which the philosophers one and all denied. Aristotle recognizes *φύσις = γένεσις*, *Phys.* 193^{b} 12 *ἔτι δ' ἡ φύσις ἡ λεγομένη ὡς γένεσις ὁδός ἐστιν εἰς φύσιν* (= *εἰς οὐσίαν*, cp. *Met.* 1003^{b} 7). *Met.* 1014^{b} 16 *φύσις λέγεται . . . ἡ τῶν φυομένων γένεσις.*

[74] *Av.* 691 *φύσιν οἰωνῶν γένεσίν τε θεῶν.* This occurs in the so-called 'Orphic cosmogony.'

[75] Cp. Soph. *Ant.* 726 *οἱ τηλικοίδε καὶ διδαξόμεσθα δὴ | φρονεῖν ὑπ' ἀνδρὸς τηλικοῦδε τὴν φύσιν*; *O. C.* 1295 *ὢν φύσει νεώτερος.* *Trach.* 379 *ἦ κάρτα λαμπρὰ καὶ κατ' ὄνομα*

law, principle, or force. As we have seen, *φύσις* and *φύειν* seemed to imply a growth from within, directed not by an external force or power, but obedient to its own laws. The importance of this conception cannot easily be measured. It expresses succinctly the opposition of *ἱστορία περὶ φύσεως* and *μῦθος περὶ θεῶν*. As Aristotle well puts it, *Phys.* 192^b 8 : *τὰ μέν ἐστι φύσει, τὰ δὲ δι' ἄλλας αἰτίας*. That which is *φύσει* is autonomous, or, as the Socratics would say, *αὐτόματον*. The pre-Socratics, when they use *τὸ αὐτόματον* strictly, deny its existence in nature, since every thing has its cause, though we may be ignorant of it. The law of nature is an inner constraint or *ἀνάγκη*.[76] Hence *φύσις*, besides being the embodiment of all natural laws, is also the *mode*[77] of operation, or *τρόπος*, and so comes to mean the customary.[78] Indeed habit becomes a "second-nature,"[79] and thus approaches *νόμος*.[80] It was apparently

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καὶ φύσιν. Probably the last (= lineage) should be classed under III. A, 2, but many cases present difficulties.

[76] Eurip. *Troad.* 886 *Ζεύς, εἴτ' ἀνάγκη φύσεος εἴτε νοῦς βροτῶν*. Here, as often, it is difficult to distinguish whether it is the *mode* or the *force* which predominates in the conception of law. The conception of *φύσις* as comparable to *ἀνάγκη* is neatly shown in Hippocr. Π. *διαίτης*, A, 28 (6, 502 Littré) *ψυχὴ μὲν οὖν αἰεὶ ὁμοίη καὶ ἐν μέζονι καὶ ἐν ἐλάσσονι· οὐ γὰρ ἀλλοιοῦται οὔτε διὰ φύσιν οὔτε δι' ἀνάγκην· σῶμα δὲ οὐδέκοτε τωὐτὸ οὔτε κατὰ φύσιν οὔθ' ὑπ' ἀνάγκης*. As has been already said, the Socratics did not really understand what the pre-Socratics meant by saying that a phenomenon occurs *ἀνάγκῃ* ; as it was opposed to what occurs according to design, it was rashly described almost indifferently as due to no cause at all, to *τύχῃ*, or to *τὸ αὐτόματον*. Cp. such popular phrases as *ἡ ἀναγκαία τύχη*, Soph., *Ai.* 485.

[77] Hippocrates, Π. *ὀστέων φύσιος*, 18 (9, 194 Littré) *ἡ δὲ ἐκ τῶν ἀριστερῶν φλέψ . . . τὴν αὐτὴν φύσιν ἐρρίζωται τῇ ἐν τοῖσι δεξιοῖσιν*. If one compares the analogous use of *δύναμιν*, e.g. Hippocrates, Π. *διαίτης*, A, 10 (6, 484 Littré) *θαλάσσης δύναμιν*, and the common adverbial use of *δίκην*, one is naturally struck by the circle of ideas from which the usage springs. The comparison shows the need of caution in inferring etymology from particular senses of a word. Cp. Soph., *Phil.*, 164 f. *βιοτῆς φύσιν* (= *τρόπον*).

[78] The association of *φύσις* with *τὸ εἰωθός* is common ; see, e.g. Hippocrates, Π. *ἱερῆς νούσου*, 14 (6, 388 Littré) *ἤ τι ἄλλο πεπόνθῃ πάθος παρὰ τὴν φύσιν ὃ μὴ ἐώθει*. *Προγνωστικόν*, 2 (2, 112 ff. Littré). It is the best sign in regard to the symptom, *εἰ ὅμοιόν ἐστι τοῖσι τῶν ὑγιαινόντων, μάλιστα δὲ εἰ αὐτὸ ἑωυτέῳ. οὕτω γὰρ ἂν εἴη ἄριστον, τὸ δὲ ἐναντιώτατον τοῦ ὁμοίου, δεινότατον*. (For *τὸ φύσει* in relation to likeness, see Proclus *in Platon. Crat.*, pp. 7, 18 ff., Pasquali.) *Ibid. passim* *τὸ ξύνηθες* is regarded as *κατὰ φύσιν*. [Arist.] *Probl.* 949^a 31 *τὸ πάλιν εἰς τὰ εἰωθότα ἐλθεῖν σωτηρία γίνεται αὐτοῖς ὥσπερ εἰς φύσεως κατάστασιν*. *Thucyd.* II. 45, 2 (advice to women) *τῆς τε γὰρ ὑπαρχούσης φύσεως μὴ χείροσι γενέσθαι ὑμῖν μεγάλη ἡ δόξα*.

[79] Democritus, fr. 33 *ἡ φύσις καὶ ἡ διδαχὴ παραπλήσιόν ἐστι. καὶ γὰρ ἡ διδαχὴ μεταρυσμοῖ τὸν ἄνθρωπον, μεταρυσμοῦσα δὲ φυσιοποιεῖ*. [Arist.] *Probl.* 949^a 27 *μέγα μέν τι καὶ τὸ ἔθος ἐστὶν ἑκάστοις· φύσις γὰρ ἤδη γίνεται*. Theophrastus, *C. P.* II. 5, 5 *τὸ γὰρ ἔθος* (referring to plant life) *ὥσπερ φύσις γέγονε*. Cp. Nauck, *Poet. Trag. Fr.* Adespota, 516 ; Xen. *Lacon.* 3, 4.

[80] The fact that the pre-Socratics contrasted *φύσις* and *νόμος* is instructive. They

on the analogy of such words as ἀνάγκη,[81] νόμος, αἰτία, δίκη, λόγος, etc., that the ubiquitous constructions κατὰ φύσιν, παρὰ φύσιν, φύσει, φύσιν ἔχειν,[82] were built. Though they often connote other notions, such as cause, their fundamental reference seems to be to what we call law. The frequency of such phrases is significant of the prevailing suggestion which φύσις had for the investigators περὶ φύσεως. There is here a marked contrast between the implicit and explicit signification of terms. Such phrases as παρὰ φύσιν have no proper sense except in relation to a teleological interpretation of nature;[83] but it is obvious that the pre-Socratics were not aware of this implication. They built up a structure of conceptions which of necessity led to teleology, but it was

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felt instinctively the parallelism of human and physical law, but the latter was consciously their point of departure. Yet in trying to interpret physical law, they necessarily imported conceptions derived from human law, as, e.g. the *δίκη* of Anaximander and Heraclitus. When Simonides said *ἀνάγκᾳ δ' οὐδὲ θεοὶ μάχονται* he meant much the same as the (intermittent) tyranny of Μοῖρα in Homer. I cannot but think that Pindar (Plato, *Gorg.* 483 C, 484 B) *νόμος ὁ πάντων βασιλεὺς θνατῶν τε καὶ ἀθανάτων — ἄγει δικαιῶν τὸ βιαιότατον ὑπερτάτᾳ χειρί* meant the same thing: cp. also the overruling God of Heraclitus, who is also Δίκη. So, at any rate, Plato interpreted the saying (*Gorg.* 483 C, *Legg.* 714 E), as did Hippocrates, Π. *γονῆς*, 1 (7, 470 Littré) *νόμος μὲν πάντα κρατύνει*, and the Anonymus Iamblichi (Diels, *Vorsokr.*^{2} 632, 31 foll.). Of course, in an age when *φύσις* and *νόμος* were contrasted, the opposite interpretation would also be found; cp. Plato, *Protag.* 337 C foll., Hdt., III. 38, VI. 104, Critias, fr. 25 (Diels). Cp. Galen, *De Usu Partium*, XI. 14 (III. 905 f. Kühn), and Nestle, Neue Jahrb. für d. klass. Altert., 1909, p. 10 foll. Zeller, *Ueber Begriff u. Begründung der sittlichen Gesetze*, Abh. d. Berl. Akad., 1882, cites some interesting phrases characteristic of the blending of *φύσις* and *νόμος*. Cp. Arist. *Cael.* 268^a 13, Arius Did. (Diels, *Dox.* 464, 24 ff.). The latter, speaking of the Stoics, says *κοινωνίαν δ' ὑπάρχειν πρὸς ἀλλήλους διὰ τὸ λόγου μετέχειν, ὅς ἐστι φύσει νόμος.* The common possession of reason is here the basis of law: conversely in Hippocrates, Π. *ἑπταμήνου*, 9 (7, 450 Littré) the possession of a common physical composition is the foundation of the inexorable law that all must die: *καί γε ὁ θάνατος διὰ τὴν μοίρην ἔλαχεν. ὥστε παράδειγμα τοῖς πᾶσιν εἶναι, ὅτι πάντα φύσιν ἔχει, ἐκ τῶν αὐτέων ἐόντα, μεταβολὰς ἔχειν διὰ χρόνων τῶν ἱκνουμένων.* Here *μοῖρα* has become expressly a physical law inhering in matter.

[81] Cp. Thucyd. v. 105 *ἡγούμεθα γὰρ τό τε θεῖον δόξῃ τὸ ἀνθρώπειόν τε σαφῶς διὰ παντὸς ὑπὸ φύσεως ἀναγκαίης, οὗ ἂν κρατῇ, ἄρχειν· καὶ ἡμεῖς οὔτε θέντες τὸν νόμον κτλ.* Cp. Plato, *Gorg.* 483 E; Eurip., *Troad.* 886; Hippocrates, Π. *σαρκῶν*, 19 (8, 614 Littré) *τῆς δὲ φύσιος τὴν ἀνάγκην, διότι ἐν ἑπτὰ τούτεων ἕκαστα διοικεῖται, ἐγὼ φράσω ἐν ἄλλοισιν.* Π. *διαίτης*, A, 5 (6, 476 foll. Littré) *πάντα γίνεται δι' ἀνάγκην θείην* is said from the point of view of Heraclitus.

[82] With *φύσιν ἔχειν* one should class such uses as *ἔφυ*, Soph. *Elect.* 860, where it states a natural law. One also meets *ἀνάγκην ἔχειν ὥστε* c. inf.

[83] Natorp, Philos. Monatsh. 21, p. 575 rightly refers to this fact; but he fails to observe that the pre-Socratics did not draw the obvious inference. In Aristotle, of course, the thought is clearly expressed, e.g. *Phys.* 193^a 32 *ὥσπερ τέχνη λέγεται τὸ κατὰ τέχνην, οὕτω καὶ φύσις τὸ κατὰ φύσιν λέγεται.*

the Socratics who seized the import of their labors, and, by introducing the teleological method, reconstituted philosophy. Even in the post-Socratic period teleology, because seen essentially from the pre-Socratic point of view, became, for example among the Stoics, an idle play-thing, being purely external.[84]

The step is short and easy from φύσις, regarded as a process eventuating in a result, to φύσις considered as the author or source of that which so results (II.). The distinction must lie in the degree of emphasis laid upon the beginning of the process as distinguished from its end, and, by consequence, in the degree of disruption visited upon the process as a whole. Such a separation is the result of analysis, and the relative prominence of the members into which the unitary process falls may reasonably be supposed to indicate the direction of interest of those who used the terms. This is, however, a point extraordinarily difficult to determine in a satisfactory way. It is safe to say that the layman is chiefly interested in φύσις, the result of the nature-process: he takes it for granted — his not to question why. It must, therefore, occasion no surprise that by far the most numerous uses of φύσις belong to this class (III.). The philosopher, also, must begin with the finished product and from it reason back to its source. In a peculiar way φύσις in this sense (II.) will occupy his attention; but it is obvious that the distinction between cause and law must be difficult to draw. Even in the philosophical and scientific literature of our day it is almost impossible to maintain a sharp distinction between them. We may be inclined to lay this to the charge of the Aristotelian usage; but this solution would fall short of historical truth. As we shall see, the fourfold causation of Aristotle, united in φύσις, is rooted in pre-Socratic usage, though Aristotle reinterpreted the pre-Socratic λόγος μίξεως, or chemical definition, converting it into a λόγος οὐσίας as the result of logical definition, and at the same time made explicit the unconscious teleology of the pre-Socratics by recognizing in the logical definition the final cause.

Touching the beginning of the process, the philosophers were chiefly interested in what Aristotle styled the "material cause" (II. A). There is no reason to doubt that the pre-Socratics used φύσις in this sense.[85] Aristotle speaks of Thales as the founder of the philosophy

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[84] From certain points of view modern philosophy, from Kant onwards, may be said to be the attempt to interpret the world in terms of teleology consciously conceived as the method of human thought. At bottom Pragmatism is hardly anything more than an effort to do this consistently, leaving no Absolute outside the teleological process.

[85] It is one of the many services of Burnet (see above, n. 3) that he directed

which deals with the material cause,[86] and says that the majority of the first philosophers regarded material causes as the sole causes of all things.[87] Empedocles[88] uses *φύσις* of the substance contributed by the parents to the birth of their offspring, and Hippocrates[89] does so likewise in the same connexion. In another passage Hippocrates well illustrates this force of *φύσις*. He is engaged in a polemic against the monists, who assert that all is one, and makes the point that a living being does not arise from even a multiplicity of substances unless they are mixed in the right proportions,[90] and hence *à fortiori*, could not arise from a single substance. He then proceeds:[91] "Such being the

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attention to this usage, though I cannot but differ from him in the interpretation of individual texts. It would serve no useful purpose to specify further instances. But it should be noted that *φύσις* in this sense means 'natural kind,' and hence is probably derived from III. A, 2. Cp. *ἰδέαι*, n. 89, and *εἴδεα*, n. 113.

[86] *Met.* 983^b 20, interpreted by 983^b 7 foll.

[87] *Met.* 983^b 7: *τῶν δὴ πρώτων φιλοσοφησάντων οἱ πλεῖστοι τὰς ἐν ὕλης εἴδει μόνας ᾠήθησαν ἀρχὰς εἶναι πάντων.* Proclus *in Tim.* (Diehl, I. p. 1) says to the same effect *οἱ μὲν πολλοὶ τῶν πρὸ τοῦ Πλάτωνος φυσικῶν περὶ τὴν ὕλην διέτριψαν.* Cp. Gilbert, *Aristoteles und die Vorsokratiker*, Philol. 68, 368 foll.

[88] Fr. 63 *ἀλλὰ διέσπασται μελέων φύσις· ἡ μὲν ἐν ἀνδρός.* Diels renders: "der Ursprung der Glieder liegt auseinander;" Burnet: "the substance of (the child's) limbs is divided between them, part of it in the man's and part in the woman's (body)." Here I agree in the main with Burnet. The phrase *μελέων φύσις* occurs also in Parm., fr. 16, 3, where Burnet gives it the same sense, whereas Diels renders: "die Beschaffenheit seiner Organe." In this case I agree with Diels.

[89] Π. *γονῆς*, 11 (7, 484 Littré) *ἐπὴν δέ τί οἱ νόσημα προσπέσῃ καὶ τοῦ ὑγροῦ αὐτοῦ, ἀφ' οὗ τὸ σπέρμα γίνεται, τέσσαρες ἰδέαι ἐοῦσαι, ὁκόσαι ἐν φύσει ὑπῆρξαν, τὴν γονὴν οὐχ ὅλην παρέχουσιν, κτλ.*

[90] Π. *φύσιος ἀνθρώπου*, 3 (6, 38 Littré). There is much in this discussion which applies the reasoning of Empedocles, for the interpretation of whose thought it is of extreme importance. It clearly presupposes and combats the theory of Diogenes of Apollonia (cp. espec. fr. 3, beginning). For the interpretation of Empedocles the statements regarding fit conditions of mixture for *γένεσις* are of especial interest, since they imply definite proportions and the admixture of all four elements. The intimate relation of Empedocles to the medical schools should be constantly borne in mind. Medicine, so far as it consisted in the ministration of medicaments, was essentially the art of interfering in the microcosmic *πόλεμος*, which reproduced in miniature the cosmic *πόλεμος*, and of preventing *ἐπικράτεια* of the several elements by combatting the overbearing and assisting those which were in danger of succumbing. One might be misled into supposing that Greek prescriptions were not precise, because few such are found in Hippocrates. The reason, I believe, is that Hippocrates insisted on a minute study of the individual case, for which precise prescriptions for general distribution would be unsuitable. That prescriptions were given by formula we know: cp. Hippocrates, Π. *εὐσχημοσύνης*, 10 (9, 238 Littré) *προκατασκευάσθω δέ σοι . . . ποτήματα τέμνειν δυνάμενα ἐξ ἀναγραφῆς ἐσκευασμένα πρὸς τὰ γένεα.* These are classified prescriptions.

[91] Π. *φύσιος ἀνθρώπου*, 3 (6, 38 Littré).

constitution (φύσις) of the universe and of man, it follows of necessity that man is not one substance, but each ingredient contributed to his birth keeps the self-same force (δύναμις) in the body that it had when contributed.[92] And each must return again to its natural kind (εἰς τὴν ἑωυτοῦ φύσιν), when man's body ceases to be, — the moist to the moist, the dry to the dry, the hot to the hot, and the cold to the cold. Such is the constitution (φύσις[93]) of animals and of all things else; all things originate in the same way, and all end in the same way; for their constitution is composed of the aforesaid substances and terminates in the same in the aforesaid manner, — whence it sprung into existence, thither also does it return."

Here we find peacefully side by side two uses of φύσις, (1) that of elemental constituent and (2) that of the resultant constitution. Among the strict monists there would be no real distinction, and thus there would be a show of reason for Professor Burnet's main contention if one limited its application to the Ionians and insisted on a strictly monistic interpretation of their thought;[94] but where a multiplicity of elemental constituents are recognized, the two uses must differ at least

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[92] This is interesting and important in view of its evident dependence upon Empedocles. Those who incline to regard Empedocles as a shifty and inaccurate pseudo-philosopher and decline to take seriously his doctrine of μίξις, as does Professor Millerd, *On the Interpretation of Empedocles*, p. 39 foll., should reckon with Hippocrates instead of relying entirely on scraps of his philosophical poem, especially when Aristotle agrees with Hippocrates. The fact that Aristotle found Empedocles' doctrine of the elements inconsistent with Aristotle's own misinterpretation of Empedocles' "union into one" (Millerd, p. 40) means absolutely nothing to those who know how prone the Stagirite was to find his own "indeterminate matter" in his predecessors. (See my essay *Qualitative Change*, etc., and Burnet, 2d ed. p. 57.) The fact is, and it ought to be emphasized, that the significance for the pre-Socratics of a knowledge of Hippocrates has been too much neglected even by scholars otherwise competent. The study of Qualitative Change which I published in 1906 would have gained immensely in value if I had then realized the evidential value of the Hippocratean corpus and of general Greek literature for these subjects and had incorporated the materials drawn from these sources which were then at my command. This is not, however, the proper occasion for a rehandling of that whole question, and it must therefore be postponed.

[93] This passage well illustrates the fact that, while the philosopher does speak of the elemental substance as φύσις, when he uses the term in a general way, as, e.g. the φύσις of a man or the φύσις of the universe, he means the "constitution" of things. This agrees well with the conclusion of Professor Millerd, *On the Interpretation of Empedocles*, p. 20.

[94] Such an interpretation I cannot accept for the Ionians (see my *Qualitative Change*, etc.), since strict monism implies the interpretation of τὸ ἕν as τὸ ὅμοιον, which appears distinctly first in the Eleatics. Even Diogenes is not to be regarded as a consistent monist, since he admitted distinctions in his One.

in this, that in the second sense *φύσις* is a collective comprising the individual *φύσεις*[95] of which it is the sum.[96]

It is probable that Democritus also spoke of the atoms as *φύσις* in the sense of elemental constituents of things, though this is not altogether certain.[97] Burnet likewise discovers this meaning in a fragment[98] of Diogenes of Apollonia, though as a would-be consistent monist Diogenes could ill distinguish. Closely allied to this force of *φύσις* is that in which *φύσις* appears as the natural or original place or condition of a thing. Thus Hippocrates[99] speaks of a joint, in dislocation, as leaving, and on being replaced, as returning to, its *φύσις*. It will be recalled that, according to Aristotle, each element has its *οἰκεῖος τόπος* to which it betakes itself as naturally as a cat returns home. Thus we find *ἡ ἀρχαία φύσις* denoting the original form or condition in Plato,[100] and *φύσις* coupled with *ἀρχαία κατάστασις*; but these turns lead naturally, if indeed they do not belong, to the use of *φύσις* as constitution.

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[95] The plural *φύσεις*, in this sense, is rare, cp. Arist., *Met.* 987^a 17; [Arist.], *De Mundo*, 396^b 14; Philodem., *De Morte* (Diels, *Vorsokr.*,^{2} 385, 17). [Plato], *Epin.* 981 D, uses the singular, not the plural, as one might gather from Diels, *Elementum*, p. 22.

[96] The recognition of this is common; e.g. Hippocrates, Π. *φύσιος ἀνθρώπου*, 4 (6, 38 foll. Littré) *τὸ δὲ σῶμα τοῦ ἀνθρώπου ἔχει ἐν ἑωυτῷ αἷμα καὶ φλέγμα καὶ χολὴν ξανθήν τε καὶ μέλαιναν, καὶ ταῦτ' ἐστὶν αὐτέῳ ἡ φύσις τοῦ σώματος, καὶ διὰ ταῦτα ἀλγέει καὶ ὑγιαίνει.* Cp. also Plato, *Phil.* 29 A.

[97] Democr. fr. 168. But the words of Simplicius are a comment on Arist., *Phys.* 265^b 24 *διὰ δὲ τὸ κενὸν κινεῖσθαί φασιν· καὶ γὰρ οὗτοι* (the Atomists) *τὴν κατὰ τόπον κίνησιν κινεῖσθαι τὴν φύσιν λέγουσι*, and may have no other warrant. But *τὴν φύσιν* in the Aristotelian passage means, almost certainly, "Nature," as Prantl renders it. On the other hand, Epicurus calls *τὸ κενόν* (which differs from *τὸ ναστόν*, according to Democritus, only as *μηδέν* from *δέν*) by the name of *ἀναφὴς φύσις*, though this may only be a periphrasis for *τὸ ἀναφές*. But see Arist., *Met.*, 985^b 4 foll.

[98] Fr. 2 *ἕτερον ὂν τῇ ἰδίᾳ φύσει*. This Burnet renders: "by having a substance peculiar to itself;" Diels says "anderes in seinem eigenen Wesen," which is probably the true meaning, implying constitution (composition ?).

[99] Π. *ἄρθρων*, 30 (4, 144 Littré); *ibid.* 61 (4, 262 Littré).

[100] *Symp.* 191 A. *ἡ φύσις δίχα ἐτμήθη*; 191 C *ἔστι . . . ὁ ἔρως ἔμφυτος ἀλλήλων τοῖς ἀνθρώποις καὶ τῆς ἀρχαίας φύσεως συναγωγεὺς καὶ ἐπιχειρῶν ποιῆσαι ἓν ἐκ δυοῖν καὶ ἰάσασθαι τὴν φύσιν τὴν ἀνθρωπίνην*; 192 E *ἡ ἀρχαία φύσις*; 193 C *εἰς τὴν ἀρχαίαν ἀπελθὼν φύσιν*. Cp. *Repub.* 547 B *ἐπὶ τὴν ἀρχαίαν κατάστασιν*. In Democritus, fr. 278 we find *ἀπὸ φύσιος καὶ καταστάσιος ἀρχαίης*. Protagoras (Diels, *Vorsokr.* II. 527, 1) is reported to have written a work Π. *τῆς ἐν ἀρχῇ καταστάσεως* (perhaps a sort of Π. *φύσεως ἀνθρώπου*) from which Nestle, Neue Jahrb. für klass. Altert., 1909, p. 8, thinks Plato freely transcribed the myth in the *Protag.* 320 C, foll. Hdt. VIII. 83 says *ἐν ἀνθρώπου φύσι καὶ κατάσασι*. Here belongs also Aristotle's *πρώτη σύνθεσις* (see n. 73) and Hippocrates' *ἡ ἐξ ἀρχῆς σύστασις*, Π. *διαίτης*, Α, 2 (6, 468 Littré).

We have seen that in the world of Homeric thought every event was regarded as due to the activity of the gods, and that, as the conception of Nature replaced that of the gods as a basis of explanation, φύσις was conceived as the source of the manifold activities of the world. The phenomena of life, cosmic and microcosmic, seeming to occur spontaneously and without external cause[101] and direction, naturally engrossed the attention of the philosopher and might well make it appear possible to dispense with a special cause of motion. Aristotle[102] complains that the first philosophers did not concern themselves with this question, confining themselves to the investigation of the material cause; and such anticipations of his efficient cause as he finds in the early cosmogonists and cosmologists bear the stamp of vital and psychic agencies, hardly distinguishable from the personifications of mythology. From these facts divergent conclusions have been drawn, some assuming that the mythical conceptions continued essentially unchanged, others finding a refined animism to which they give the name of hylozoism or hylopsychism. The first conclusion is shown to be false by the mechanical interpretation put upon the activities of the mythically named agencies;[103] the second presupposes distinctions which developed only at a later period.[104] In general the philosophers appear to have contented themselves with the recognition of the autonomy of nature, assigning no ground for her activity, since she seemed herself to be the sufficient explanation of events. The strict exclusion of divine agency not unnaturally suggests a conscious effort to eliminate such interference, though this inference might be wrong; on the other hand the habit of saying that certain phenomena occur "of themselves" or "of necessity" or "by chance" gave, as we have seen, great offense to the teleological Socratics. A modern philosopher, conscious of the difficulties presented by an attempt to define causality and necessity, would judge these early thinkers with less severity. But the constant criticism of pre-Socratic philosophers by their Socratic successors, due to the teleological prepossessions of

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[101] Spontaneous generation of animal life, for example, seems to have been generally accepted for lower forms. As philosophy advanced the higher forms of life were included, at least at the beginning of the world.

[102] Aristotle, *Met.* 984^a 18–985^b 22. Cp. Gilbert, *Aristoteles und die Vorsokratiker*, Philol., 68, 378 foll.

[103] In Empedocles this is obvious to all who regard him as a philosopher and consider the evidence; it is equally clear in regard to Parmenides. Cp. my *Qualitative Change*, n. 89, and see also *ibid.* nn. 55 and 65.

[104] For this see Burnet, ed. 2, p. 15 foll.

the latter,[105] is suggestive of the tardiness with which they came to consider the implications of causality and the laws of nature.

The use of φύσις, with more or less personification, as the author of a process (II. B), appears relatively late, as we should expect.[106] Hippocrates speaks of Nature as arranging the vitals in the inner parts;[107] says of the auricles of the heart that they are instruments by which she takes in the air, adding that they seem to be the handiwork of a good craftsman;[108] refers to the *vis medicatrix naturae*, Nature having discovered the methods without understanding and untaught;[109] she makes glands and hair;[110] she can prepare the way for and offer resistance to instruction;[111] she is all-sufficient;[112] she

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[105] It is perhaps unnecessary to cite passages, but the intrinsic interest of the following may justify one in quoting it. Arist. *De Partt. Animal.* 641^b 20: οἱ δὲ τῶν μὲν ζῴων ἕκαστον φύσει φασὶν εἶναι καὶ γενέσθαι, τὸν δὲ οὐρανὸν ἀπὸ τύχης καὶ τοῦ αὐτομάτου τοιοῦτον συστῆναι, ἐν ᾧ ἀπὸ τύχης καὶ ἀταξίας οὐδ' ὁτιοῦν φαίνεται. πανταχοῦ δὲ τόδε τοῦδε ἕνεκα, ὅπου ἂν φαίνηται τέλος τι πρὸς ὃ ἡ κίνησις περαίνει μηδενὸς ἐμποδίζοντος. ὥστε εἶναι φανερὸν ὅτι ἔστι τι τοιοῦτον, ὃ δὴ καὶ καλοῦμεν φύσιν. οὐ γὰρ δὴ ὅτι ἔτυχεν ἐξ ἑκάστου γίνεται σπέρματος, ἀλλὰ τόδε ἐκ τοῦδε, οὐδὲ σπέρμα τὸ τυχὸν ἐκ τοῦ τυχόντος σώματος. ἀρχὴ ἄρα καὶ ποιητικὸν τοῦ ἐξ αὐτοῦ τὸ σπέρμα. φύσει γὰρ ταῦτα· φύεται γοῦν ἐκ τούτου. ἀλλὰ μὴν ἔτι τούτου πρότερον τὸ οὗ τὸ σπέρμα· γένεσις μὲν γὰρ τὸ σπέρμα, οὐσία δὲ τὸ τέλος. Cp. Ed. Meyer, *Geschichte des Altert.* I. (a), p. 106: "Vielleicht noch verbreiteter (than the belief that divinities reside in inanimate objects, such as stocks and stones) ist der Glaube, dass die Götter in Tieren ihren Wohnsitz haben. Die Tiere sind lebendige Wesen, die eine willenstarke Seele haben wie der Mensch; nur sind sie nicht nur an Kraft dem Menschen vielfach überlegen, sondern vor allem viel geheimnisvoller, unberechenbarer und dabei zugleich durch ihren Instinkt viel sicherer und zielbewusster in ihrem Auftreten als der Mensch: sie wissen vieles, was der Mensch nicht weiss. Daher sind sie für die primitive Anschauung recht eigentlich der Sitz geheimnisvoller göttlicher Mächte." These same qualities of animals, as we shall see, shared in the development of the idea of φύσις which took the place of that of the gods for purposes of explanation.

[106] Not all the passages cited emphasize the agency of Nature, and the degrees of personification differ; but personification in any degree implies or suggests agency, and for convenience, if for no other reason, the uses should be considered together.

[107] Π. ἀνατομῆς, 1 (8, 538 Littré) τὰ μὲν ἓξ ἀνὰ μέσον ἐντὸς φύσις ἐκοσμήθη. Cp. Bonitz, *Index Arist.* 836^a 25.

[108] Π. καρδίης, 8 (9, 84 Littré) ἔστι δὲ ὄργανα τοῖσι ἡ φύσις ἁρπάζει τὸν ἠέρα. καίτοι δοκέω τὸ ποίημα χειρώνακτος ἀγαθοῦ.

[109] 'Επιδημ. VI. 5, 1 (5, 314 Littré) νούσων φύσιες ἰητροί. ἀνευρίσκει ἡ φύσις αὐτὴ ἑωυτῇ τὰς ἐφόδους, οἰκ ἐκ διανοίης, οἷον τὸ σκαρδαμύσσειν, καὶ ἡ γλῶσσα ὑπουργέει, καὶ ὅσα ἄλλα τοιαῦτα· ἀπαίδευτος ἡ φύσις ἐοῦσα καὶ οὐ μαθοῦσα τὰ δέοντα ποιέει. Π. τροφῆς, 39 (9, 112 Littré) φύσιες πάντων ἀδίδακτοι. Π. διαίτης, A, 15 (6, 490 Littré) ἡ φύσις αὐτομάτη ταῦτα ἐπίσταται. Cp. n. 117.

[110] Π. ἀδένων, 4 (8, 558 Littré) ἡ γὰρ φύσις ποιέει ἀδένας καὶ τρίχας.

[111] Νόμος, 2 (4, 638 Littré) πρῶτον μὲν οὖν πάντων δεῖ φύσιος (talent, natural aptitude)· φύσιος γὰρ ἀντιπρησσούσης, κενεὰ πάντα· φύσιος δὲ ἐς τὸ ἄριστον ὁδηγεούσης, διδασκαλίη τέχνης γίνεται.

[112] Π. τροφῆς, 15 (9, 102 Littré) φύσις ἐξαρκέει πάντα πᾶσιν.

produces natural species and legislates language;[113] in disease she may withhold signs, but may be constrained by art to yield them;[114] the means employed by her are likened to the means in use in the arts.[115] Such is the picture we find drawn of φύσις at the close of the pre-Socratic period. In the earlier writers such expressions are rare. Heraclitus[116] says that "nature loves to play at hide-and-seek," and Epicharmus[117] says "Eumaeus, wisdom is not confined to one place, but all living things have intelligence. The tribe of hens, if you will note sharply, does not bring forth living offspring but hatches eggs and causes them to acquire a living soul. This bit of wisdom — how this comes about — Nature alone doth know; she was self-taught."

Aside from such utterances as these[118] we are reduced to inferences from the general doctrines of philosophers, but it is not our plan to pursue this subject here. It may not be amiss, however, to remark that the type of pantheism found in Xenophanes,[119] vaguely anticipat-

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[113] Π. τέχνης, 2 (6, 4 Littré) οἶμαι δ' ἔγωγε καὶ τὰ ὀνόματα αὐτὰς (sc. τὰς τέχνας) διὰ τὰ εἴδεα λαβεῖν· ἄλογον γὰρ ἀπὸ τῶν ὀνομάτων τὰ εἴδεα ἡγεῖσθαι βλαστάνειν, καὶ ἀδύνατον· τὰ μὲν γὰρ ὀνόματα φύσιος νομοθετήματά ἐστι, τὰ δὲ εἴδεα οὐ νομοθετήματα, ἀλλὰ βλαστήματα. Cp. Plato's *Cratylus*. It is noteworthy that νόμος is here derived from φύσις, its products as only in a secondary degree accounted the result of Nature. Alongside this view ran the other which distinguished sharply between φύσις and νόμος, though here also νόμος is secondary. Hippocrates, Π. διαίτης, A, 11 (6, 486 Littré) says: νόμος γὰρ καὶ φύσις, οἷσι πάντα διαπρησσόμεθα, οὐχ ὁμολογέεται ὁμολογεόμενα· νόμον γὰρ ἔθεσαν ἄνθρωποι αὐτοὶ ἑωυτοῖσιν, οὐ γινώσκοντες περὶ ὧν ἔθεσαν· φύσιν δὲ πάντων (doubtless including man) θεοὶ διεκόσμησαν· ἃ μὲν οὖν ἄνθρωποι ἔθεσαν, οὐδέκοτε κατὰ τὠυτὸ ἔχει οὔτε ὀρθῶς οὔτε μὴ ὀρθῶς· ὁκόσα δὲ θεοὶ ἔθεσαν, ἀεὶ ὀρθῶς ἔχει.

[114] Π. τέχνης, 12 (6, 24 Littré) ὅταν δὲ ταῦτα μὴ μηνύωνται, μηδ' αὐτὴ ἡ φύσις ἑκοῦσα ἀφίῃ, ἀνάγκας εὕρηκεν (sc. ἡ τέχνη), ᾗσιν ἡ φύσις ἀζήμιος βιασθεῖσα μεθίησιν.

[115] Π. τέχνης, 8 (6, 14 Littré) ὧν γάρ ἐστιν ἡμῖν τοῖσί τε τῶν τεχνέων ὀργάνοις ἐπικρατέειν. Π. διαίτης is full of comparisons between the operations of nature and those of the arts.

[116] Fr. 123 φύσις κρύπτεσθαι φιλεῖ. I interpret this saying as referring to the game called κρυπτίνδα, and regard it as parallel to fr. 52 αἰὼν παῖς ἐστι παίζων, πεττεύων· παιδὸς ἡ βασιληίη. Bernays (Abh. der Akad. Berl., 1882, p. 43) said of the latter: "H. hatte seinen Zeus, insofern er unablässig Welten baut und Welten zerstört, ein 'spielendes Kind' genannt; der tiefsinnige Naturphilosoph wählte dieses Bild, um das Wirken der Naturkräfte allen menschlichen Fragen nach dem Zwecke zu entrücken." Heraclitus probably had little reason to fear teleological interpretation of nature. Perhaps the αἰών is playing a game of solitaire or playing against a dummy, now winning (κόρος), now losing (λιμός). Cp. Stein on Hdt. II. 122, 3. On similar lines one might explain the game of κρυπτίνδα.

[117] Fr. 4 (Diels). Cp. n. 109, above, and Ar., *Vesp.*, 1282. The genuineness of the fragment is not above suspicion.

[118] Cp. Eurip. fr. 920 ἡ φύσις ἐβούλεθ', ᾗ νόμων οὐδὲν μέλει.

[119] Cp. Burnet, 2d ed., p. 141 and Adam, *The Religious Teachers of Greece*, p. 209 foll. I incline to think that Adam somewhat overemphasized the degree of

ing that of the Stoics, inevitably contributed indirectly to the development of the conception of Nature as of a power more or less personally conceived but devoid of definite anthropomorphic attributes. This view of Nature was henceforth to prevail in ever-widening circles.

We now turn to consider *φύσις* regarded as the end of the process (III.). As has already been said the number and variety of cases which fall under this head are very great compared with the foregoing. In most respects there is little occasion for special remark in this connexion, since the usage of the pre-Socratic period coincides in the main with that of later times. Yet there are implications involved in this same usage which were drawn out and made explicit only in the Socratic age. Most interesting of all, perhaps, is the complete inversion of the conclusions of homely common sense and common usage introduced by the doctrine of Aristotle. Thus, e. g., he says :[120] "From what has been said, then, it is plain that *φύσις*, in the primary and strict sense, is the substantial entity (*οὐσία* = *φύσις* III.) of things which have in themselves, as such, a source of movement; for the matter is called *φύσις* (II. A) by reason of having a *capacity* to take this on, and the processes of becoming and growing (*φύσις* I.), by reason of being derived from it." In the circular process of the Socratic the end has become the beginning; that which the pre-Socratic called the reality has become a bare potentiality. Neither premise nor conclusion of this view would have been acceptable or even intelligible to the pre-Socratic, although, with one exception, the conceptions upon which the new view rests were common property. Yet that one exception is the corner-stone of Socratic philosophy.

When the pre-Socratic asked what a thing was, the answer he desired, if given with ideal completeness, would have presented its chemical formula. Now a formula is, I suppose, in origin and intention, a prescription. In the pre-Socratic schools, closely associated as they were with the schools of medicine, this procedure was natural : furthermore it was adequate, since the "things" they sought to define were material. But, as we have already seen, the Nature which the philosopher studied became at the end of the pre-Socratic period so charged with spiritual meaning, and in particular in the kingdom of *νόμος*, the son of *φύσις*, there was so much, non-material in character, which called for analysis, that a method of definition suited to the new objects of study became an urgent necessity. If the old method sought a defini-

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personality with which the *θεός* of Xenophanes is invested, especially as the negation of the popular view of the gods is so pronounced. What remains after the denials, while containing elements of personality, appears shadowy.

[120] *Met.* 1015^a 13 foll., transl. of Ross, modified.

tion of the material thing, yielding, as its final result, the formula of its production or origin with a view to its possible reproduction, the new method proposed to define the *idea* of the thing. Henceforth it mattered little whether the thing was material or not; nor did it matter whether it was actually or only "potentially" existent. These distinctions did not and could not arise until the new method supplanted that of the pre-Socratics.[121] The thing itself has a beginning, a source, and a history: it is transient. The idea of the thing (for the Socratic) had no relation to beginnings or history: it is eternal. The *idea* of a key, for example, is totally different from the key itself. The key is of brass or of iron: that is to say, it is *defined* with reference to its material source: the definition of the idea of a key, however, looks inevitably to its purpose, or end. Thus the limits of the process of φύσις, erected by this two-fold method of definition, are polar opposites. In either direction the quest was for the truly existent, and, the human mind being constituted as it is, the ultimate existence must be the first cause. To the Socratic the first cause must be the end or purpose; but, since historically this conception was a cadet and could not wholly supplant the first-born, the end must be in the beginning, even if it be only "potentially" present there. Like most Socratic ideas, the conception of the causality of φύσις, as the end of a process, was involved in many pre-Socratic expressions, though their significance was not realized. Attention was directed above to instances of personification (involving agency) of φύσις in the sense of constitution, talent, etc., falling under III. The same implication belongs to πέφυκε and φύσιν ἔχει with the infinitive. Nature thus becomes, as it is by Aristotle expressly regarded, a circular process, in which the end of one cycle is the beginning of another: ἄνθρωπος ἄνθρωπον γεννᾷ. The κύκλος γενέσεως thus established is, however, for the pre-Socratic a real process, with a clear history, comparable to the Orphic cycle, in which the immortal soul experiences the vicissitudes incident to sin. In Aristotle, where the process as a whole is all in all, the single moment tends to assume the guise of something having a reality only for the theorist, — a kind of psychologists' fallacy.

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[121] Hippocrates, Π. τέχνης, 2 (6, 2 foll. Littré) is an interesting discussion of the "existence" of arts, which could not have taken the form it actually takes if the Aristotelian distinctions had been current. "Potentiality" and "actuality" have no significance in relation to things which have a real history; the terms acquire meaning only in relation to an ideal construction, such as we find in the Aristotelian system, where the definition of the οὐσία of a thing has reference to its realization of an end as seen from without. Teichmüller, strangely enough, imported these conceptions into the pre-Socratics.

It has already been said that the practical man is concerned chiefly with the product, which he takes roughly for granted without too much curiosity as to its origin ; but he is intensely interested in its uses, whatever they may be. He does not reflect upon even this circumstance, however, proceeding in his pragmatic way to do the work in hand. When therefore he speaks of φύσις it is generally some aspect of nature as it is that he has in view. From this attitude springs the common usage of philosophical and quasi-philosophical circles, which regards chiefly things as things, without too much implication of further questionings. In so far as there is a suggestion of further questions, they concern the "constitution" of the thing — that is, "what it is" expressed in terms of "what it is made of." This is the regular sense of the phrase περὶ φύσεως as applied in titles of the works of Hippocrates,[122] and there is no reason to think that it bore a different sense when used as a title of distinctively philosophical writings.

If it were our purpose to treat fully of the uses of φύσις we should have to gather and discuss here the multitudinous meanings of the term which fall under the third head. This we could not do, however, without unduly and unprofitably increasing the bulk of this study ; for most developments of φύσις, regarded as the end of the process (III.), are of slight interest for the particular purposes of our inquiry. We may therefore here content ourselves with a summary glance at the ramifications of this main branch, adding such observations as may serve to throw light on philosophical and scientific conceptions.

We may then regard φύσις, as the end of the process, from without or from within. As seen from without it is the outward constitution or frame of a thing (III. A) ; viewed from within, it is its inner constitution or character. Under the former head we may distinguish (1) the individual frame,[123] (2) the specific or generic,[124] (3) the uni-

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[122] See above, n. 10 and n. 93. The titles of Hippocrates are probably not original, since in many instances they are in doubt, some works that bear specific titles being clearly parts of larger wholes. This is in keeping with the facts mentioned below, n. 204, relative to philosophical works. But in the case of Hippocrates the title in most cases merely reproduces in abbreviated form the subject as stated in the body of the work ; and the invariable meaning of φύσις, when used by Hippocrates in reference to the subject-matter of discourse, is "constitution."

[123] In the individual, φύσις denotes primarily the (perfect) stature attained, εἰς ἄνδρα τέλειον, εἰς μέτρον ἡλικίας, as Paul says, *Eph.* 4, 13. This is Aristotle's ἐντελέχεια, for which the whole creation groaneth. Aesch., *Pers.* 441 ἀκμαῖοι φύσιν shows that this association of ideas was popular.

[124] This head includes φύσις in the sense of 'birth,' 'lineage,' 'family,' and φύσις as sex ; for sex is a γένος. It also embraces θνητὴ φύσις, Democritus, fr. 297, Soph., *O. T.* 869, fr. 515, and Aesch., *Ag.* 633 χθονὸς φύσιν, 'earth's brood.' As (α) under this head should be classed φύσις denoting not the γένος itself but the

versal[125] frame of things. Difficult, and in some cases impossible, it is to distinguish clearly between the outward frame or constitution and the inner constitution or character of things (III. B). Each φύσις or frame has its inner constitution corresponding to it, which will of course vary according as the φύσις in question is individual, generic, or universal. Description or definition of the φύσις relates the individual or generic to the universal. Of course the crude methods of description and definition in use in the pre-Socratic period were not consciously generalized; but there was an evident desire, manifested most clearly in the parallel drawn between the microcosm and the cosmos, to find the universal in the particular. In accordance with the chemical mode of definition in vogue this desire assumed the form of the postulate that the constitution of individual things was the same as that of the world as a whole. We may, if we choose, denounce this procedure as crude logic, but it was instinctive logic, or logic in the making, for all that. The *differentiae specificae* were found chiefly in the proportions of the λόγος μίξεως, although this method was to a limited extent supplemented, though perhaps nowhere wholly supplanted, by the differentiation introduced in the universal through rarefaction and condensation, or — what practically amounts to the same thing — through heat and cold. As to the universal, the wide-spread conviction that each thing shares the attributes, or rather the constituents, of the world one and all in varying proportions, served as a bond of union, making things, on the physical side, capable of interaction, and, on the intellectual side, capable of being comprehended. The motive that inspired the postulation of a common principle for the explanation of the manifold data of sense is particularly evident in the case of the Pythagoreans, whose postulate that all is at bottom number or numerical relation has no meaning except that of rendering phenomena intelligible. This is clear even without accepting the so-called fragments of Philolaus, in which it is expressly stated. To Aristotle this principle descended in two forms. For physical theory, it provided a basis of interaction,

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specific differentiae, of which we have an early example in Hom. *Od.* 10, 303, the φύσις of the plant μῶλυ pointed out to Odysseus by Hermes; later we find, in the same class, φύσις denoting the characteristic differentiae of sex. Under (2) we might likewise include many uses in which φύσις = δύναμις, since the μέτρα of φύσις and δύναμις are specific differentiae. Cp. n. 85 above and **n.** 113, where natural kinds are called φύσιος βλαστήματα.

[125] In this sense φύσις practically = κόσμος. For the uses of κόσμος see Bernays, Abh. der Akad. Berlin, 1882, p. 6 foll. In this universal sense φύσις = τὰ φυόμενα, φύσις τῶν ὅλων, etc. For instances see Archytas, fr. 1; Eurip. fr. 910; Critias, fr. 19 (Diels); Δισσοὶ Λόγοι (*Dialexeis*), Diels, *Vorsokr.* II. 647, 15; Hippocrates, II. ἀρχαίης ἰητρικῆς, 20 (p. 24 foll., Kühlewein).

since, in order to interact, things must, according to his theory, be generically alike, though specifically they may be opposite or neutral in character. For logical theory, again, the universal is the foundation of the intelligible world.

It was said above that while the inquiry περὶ φύσεως regarded primarily the constitution of the world, viewed as a given fact, it did naturally imply a question as to its constituents and hence as to its origin. To this we have now added that this implied question involved for nearly all philosophers of early Greece the conception of φύσις as a λόγος μίξεως.[126] In effect we had already adverted to this fact in referring to the chemical definition of things as a congener of the medical prescription. In a curious passage[127] Aristotle dimly perceives that the λόγος μίξεως, which he appears to recognize only in Empedocles, is intimately related to logical definition, though he seems more fully aware of their differences than of their fundamental likeness. Chemical definition seeks to determine what matter entered into the making of the thing. Whether this matter is of one or more kinds makes little difference; since even the monist must somehow give variety to his unitary substance, and the Greek monists in particular appear to have conceived of concrete things as 'blends' of the derivative forms of matter. Logical definition, on the other hand, aims to discover what meanings or marks (teleologically interpreted) constitute the idea of the thing. Each method arrives at a λόγος: the first at a λόγος μίξεως; the second, at a λόγος οὐσίας.[128] In the Aristotelian scheme φύσις, as the λόγος οὐσίας, is the "formal cause." Among the pre-Socratics, the λόγος μίξεως of the cosmos was the object of scientific inquiry; and it was φύσις in this sense which, as we have seen, appears in the titular Περὶ φύσεως.

Thus far we have considered chiefly the physical φύσις or constitution (III. B, 1); but we must not overlook the fact that with the

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[126] Cp. n. 90 above. For φύσις involving λόγος μίξεως see Parmenides, fr. 16 and Epicharmus, fr. 2. The latter fragment, whether rightly or wrongly attributed to Epicharmus, clearly reflects the thought of Heraclitus, a supposed monist. On this subject see my study of *Qualitative Change.*

[127] *De Partt. Animal.* 642^a 2–31. The passage is too long to transcribe, but will well repay study.

[128] I cannot help feeling that the periphrastic use of φύσις is a by-product of logical definition and hence essentially peculiar to the Socratic period. The presence of such phrases as ἁ τῶ ἀριθμῶ φύσις, τᾶς τῶ ἀπείρω καὶ ἀνοήτω καὶ ἀλόγω φύσιος alongside ἀριθμὸς καὶ ἁ τούτω οὐσία and τᾳ̃ τῶ ἀριθμῶ γενεᾳ̃ (fr. 11), in Philolaus casts grave suspicion on the supposed fragments; for οὐσία in the pre-Socratics means not 'essence,' but 'reality.' Natorp, to be sure, in Philos. Monatshefte, 21, pp. 577, 582, finds a deep significance in these same phrases.

growth of interest in the microcosm φύσις as the mental constitution (III. B, 2) assumed considerable importance. Now φύσις (like its great rival, νόμος) ὁρίζει; and every delimitation implies a positive claim as well as a restrictive limitation. Thus φύσις positively regarded (III. B, 2 a), is as (native) endowment, talent, instinct, power, etc., opposed to (acquired) virtue, art, experience, wisdom;[129] negatively conceived (III. B, 2 b), φύσις marks the bounds set by nature to every creature, beyond which it may not pass.[130]

III.

A glance at the survey just given of the uses of φύσις will satisfy anyone that the conception of Nature in the pre-Socratic period was developed to a point at which little remained to be added. Certainly little was added in the course of subsequent Greek thought. Already our conclusion as to the connotation of φύσις when used as a comprehensive term has been stated; but it is desirable that this conclusion be confirmed by a consideration of the questions raised by those who wrote Περὶ φύσεως. Many a word having a wide range of meanings in the course of its development receives at different times an emphasis

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[129] Examples of native endowment, talent, or power, are exceedingly common; cp. Protagoras, fr. 3; Epicharmus, fr. 40; Critias, fr. 9; Democritus, fr. 21, 33, 176, 183, 242, etc. Of φύσις = instinct we have an instance in Democritus, fr. 278. In Democritus, fr. 267 φύσις means 'birthright.'

[130] "The metes and bounds of providence" furnish a favorite theme to singers and sages of all ages and peoples. Cp. for example, *Psalm* 104. Greek mythology found a text in the extravagance of the elemental water and fire respectively in the flood and in the conflagration of the world due to the escapade of Phaethon. Anaximander and Heraclitus called in the cosmic δίκη to curb such transgression. Xenophanes also recognized this principle in the periodicity of cosmic processes. With later philosophers it was a common theme. Democritus, fr. 3, couples δύναμις and φύσις; cp. also Archytas, fr. 1, and Herodotus, 8, 83. In Herodotus, 7, 16 α, it is said that the winds do not suffer the sea φύσι τῇ ἑωυτῆς χρᾶσθαι, which is explained afterwards by reference to ὕβρις. On this see my review of Hirzel, *Themis, Dike, und Verwandtes*, in A. J. P., XXIX, p. 216 foll. In Thucydides, 2, 35, 2 ὑπὲρ τὴν φύσιν is set definitely in relation to φθόνος, which opens up the kindred subject of the jealousy of the gods visited upon all who transgress their proper μέτρα, as we find it developed in the tragedians and Herodotus. In fact all things have their limitations, even God, according to the Greeks. There is an interesting passage in Hippocrates, Π. τέχνης, 8 (6, 12 Littré), where, after rebuking unreasonable critics of the art of medicine, the author says: εἰ γάρ τις ἢ τέχνην, ἐς ἃ μὴ τέχνη, ἢ φύσιν, ἐς ἃ μὴ φύσις πέφυκεν, ἀξιώσειε δύνασθαι, ἀγνοεῖ ἄγνοιαν ἁρμόζουσαν μανίῃ μᾶλλον ἢ ἀμαθίῃ. ὧν γάρ ἐστιν ἡμῖν τοῖσί τε τῶν φυσίων τοῖσί τε τῶν τεχνέων ὀργάνοις ἐπικρατέειν, τουτέων ἐστὶν ἡμῖν δημιουργοῖς εἶναι, ἄλλων δὲ οὔκ ἐστιν. As limitation and definition are the basis of intelligence and the guaranty of sanity, the Greeks had an antipathy to all extravagance. This appears most clearly in their aversion to the ἄπειρον in all forms.

falling now on one meaning, now on another, according to the direction of interest from time to time. We have had occasion to note this tendency in regard to *φύσις* and have seen, for example, that the personification of Nature has a clear history, arriving at the close of the pre-Socratic period at a stage that rendered the subsequent teleological interpretation of the world a foregone conclusion. It behooves us, therefore, to inquire what were the principal questions asked concerning Nature in the pre-Socratic period, in order, if possible, to determine the direction of interest upon which depends the selection of meanings attached to the term *φύσις*.

We may prosecute this inquiry in either of two ways. First, we may study the fragmentary remains of the literature of pre-Socratic philosophy and extract from its implicit logic the answer to our question. Or we may approach the matter indirectly, asking what were the ideals of science in that age as we find them reflected in the non-philosophical or only quasi-philosophical literature of the time and of the following period which received its inspiration from the pre-Socratics. Strictly both methods should be followed conjointly ; for only thus could we arrive at a conclusion that might be justly regarded as definitive. But a moment's thought will convince any reader that the limits of such a study as this could not possibly be made to yield to a detailed examination of the individual systems with a view to deducing from them the interests of their propounders. So comprehensive a review must be undertaken in connexion with a history of early Greek philosophy, which is not, and cannot be, the scope of this study. Our attention shall, therefore, be directed to the second means of approach, with only an occasional glance at the systems of the pre-Socratic philosophers themselves. We may pursue this course with the better conscience because it is self-evident that the scientific ideals of the age were, or soon became, common property, to the definition and development of which every man of science contributed what he had to offer. Nowhere does the unity of pre-Socratic thought more clearly appear than in this field, where philosophers and medical theorists coöperated in laying broad and sure foundations.

Hippocrates gives us the best glimpse of the scientific ideals of the age ; and it will prove worth our while to pause for a moment to learn what he has to teach us. The true physician is called the child of his art ;[131] he is disinterested in his devotion to it, since the love of one's art involves necessarily a love of mankind.[132] The charlatan was

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[131] Παραγγελίαι, 7 (9, 260 Littré) *ἰητρὸς ἀγαθὸς . . . ὁμότεχνος καλεόμενος.*

[132] Among the virtues which the physician is said to possess in common with the philosopher in Π. *εὐσχημοσύνης*, 5 (9, 232 Littré) is *ἀφιλαργυρίη*. Π. *ἰητροῦ*, 1 (9,

particularly despised, and his histrionic deportment decried.[133] The physician who desires to appear in public and address the people, should refrain from quoting the poets : such a procedure merely argues incapacity for honest work.[134] In public speech or writing, however, one must begin by laying down a proposition to which all may assent.[135]

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204 Littré) the physician is bidden τὸ δὲ ἦθος εἶναι καλὸν καὶ ἀγαθόν, τοιοῦτον δ' ὄντα πᾶσι καὶ σεμνὸν καὶ φιλάνθρωπον. Παραγγελίαι, 5 (9, 258 Littré) τίς γὰρ ὦ πρὸς Διὸς ἠδελφισμένος (called brother, because belonging to the fraternity: cp. Isocr. 19, 30) ἰητρὸς ἰητρεύειν πεισθείη ἀτεραμνίη ; The brotherhood of the fraternity leads to the fraternity of man ! *Ibid.* 6, ἢν δὲ καιρὸς εἴη χορηγίης ξένῳ τε ἐόντι καὶ ἀπορέοντι, μάλιστα ἐπαρκέειν τοῖσι τοιουτέοισιν. ἢν γὰρ παρῇ φιλανθρωπίη πάρεστι καὶ φιλοτεχνίη. Xen. *Mem.* I. 2, 60 refers to Socrates' refusal to receive remuneration for his informal instruction as evidence that he was φιλάνθρωπος and δημοτικός. In like manner Plato, *Euthyph.* 3 D, explains his lavish expenditure of wisdom as due to φιλανθρωπία, which would not only refuse to accept remuneration but would even display itself in paying the listener to boot. It seems evident that the exalted and even extravagant disinterestedness of Socrates reflects, though it doubtless carried beyond the common practice, the teaching of the medical schools, and possibly also of the early philosophical schools. In the medical Ὅρκος (4, 628 foll., Littré) the physician swears to regard his teacher as a father, sharing with him his substance, and his teacher's sons as his brothers ; if they desire to learn medicine, he swears διδάξειν τὴν τέχνην ταύτην . . . ἄνευ μισθοῦ καὶ ξυγγραφῆς. Socrates, like Paul, was a debtor to all men : he could receive pay from none ; for Socrates is the first great cosmopolitan. That the Sophists departed from this custom was one of Plato's severest charges against them. They were like the men of whom Xen. *Mem.* I. 2, 60 complains, who departed from the philanthropic and demotic way: οὐδένα πώποτε μισθὸν τῆς συνουσίας ἐπράξατο, ἀλλὰ πᾶσιν ἀφθόνως ἐπήρκει τῶν ἑαυτοῦ· ὧν τινες μικρὰ μέρη παρ' ἐκείνου (Socrates) προῖκα λαβόντες πολλοῦ τοῖς ἄλλοις ἐπώλουν, καὶ οὐκ ἦσαν ὥσπερ ἐκεῖνος δημοτικοί. Cp. Hippocrates, Π. εὐσχημοσύνης, 2 (9, 226 Littré) πᾶσαι γὰρ αἱ μὴ μετ' αἰσχροκερδείης καὶ ἀσχημοσύνης (sc. τέχναι) καλαί. These are the truly "liberal" arts.

[133] Π. ἰητροῦ, 4 (9, 210 Littré) ; Π. ἱερῆς νούσου, 1 (6, 354 Littré) ἐμοὶ δὲ δοκέουσιν οἱ πρῶτοι τοῦτο τὸ νόσημα ἀφιερώσαντες τοιοῦτοι εἶναι ἄνθρωποι οἷοι καὶ νῦν εἰσι μάγοι τε καὶ καθάρται καὶ ἀγύρται καὶ ἀλαζόνες, ὁκόσοι δὴ προσποιέονται σφόδρα θεοσεβέες εἶναι καὶ πλέον τι εἰδέναι· οὗτοι τοίνυν παραμπεχόμενοι καὶ προβαλλόμενοι τὸ θεῖον τῆς ἀμηχανίης τοῦ μὴ ἴσχειν ὅ τι προσενέγκαντες ὠφελήσουσιν, ὡς μὴ κατάδηλοι ἔωσιν οὐδὲν ἐπιστάμενοι, ἱερὸν ἐνόμισαν τοῦτο τὸ πάθος εἶναι, καὶ λόγους ἐπιλέξαντες ἐπιτηδείους τὴν ἴησιν κατεστήσαντο ἐς τὸ ἀσφαλὲς σφίσιν αὐτοῖσι, καθαρμοὺς προσφέροντες καὶ ἐπαοιδάς, κτλ. (With this passage cp. Plato, *Repub.* 364 B foll.). *Ibid.*, 18 (6, 396 Littré). Cp. also the portrait of the spurious philosopher, Π. εὐσχημοσύνης, 2 (9, 226 foll., Littré). Cp. n. 47, above.

[134] Παραγγελίαι, 12 (9, 266 foll., Littré). I read φιλοπονίης with the vulgate ; Littré reads φιλοπονίη.

[135] Π. σαρκῶν, 1 (8, 584 Littré) ἐγὼ τὰ μέχρι τοῦ λόγου τούτου κοινῇσι γνώμῃσι χρέομαι ἑτέρων τε τῶν ἔμπροσθεν, ἀτὰρ καὶ ἐμεωυτοῦ. (Littré misinterprets this: it means that he shares the common assumption of his predecessors !) ἀναγκαίως γὰρ ἔχει κοινὴν ἀρχὴν ὑποθέσθαι τῇσι γνώμῃσι βουλόμενον ξυνθεῖναι τὸν λόγον τόνδε περὶ τῆς τέχνης τῆς ἰητρικῆς, κτλ. Cp. Π. φύσιος ἀνθρώπου, 1 (6, 32 Littré) for the common assumption of the predecessors of whom he speaks at length in what follows. Π. τέχνης, 4 (6, 6 Littré) ἐστὶ μὲν οὖν μοι ἀρχὴ τοῦ λόγου, ἣ καὶ ὁμολογηθήσεται παρὰ πᾶσιν. Cp.

The physician will not indulge in useless dialectics,[136] but if he knows his art he will prefer to show it by deeds rather than words.[137] Life is fleeting, art is long,[138] and a cure may depend upon the moment.[139] Hence the physician must not restrict his attention to rational inference but must resort to the rule of rote together with reason;[140] he must therefore have a knowledge of practice as well as of theory.[141] The main object of medicine is to effect a cure;[142] above all the physician should avoid making much ado and accomplishing nothing.[143] The art of medicine is not, however, a mere routine; a good share of the ability of the physician is shown in his capacity to judge correctly touching what has been written;[144] for science is constituted by observations drawn from every quarter and brought into a unity.[145] An art or science attests its reality by what it accomplishes.[146] The art of medicine cannot always arrive at absolute certainty; but far from disputing the reality of medicine as an art or science because it does not attain strict accuracy in all things, one ought to praise it because of its desire to approximate it and to admire it because from extreme ignorance it has proceeded to great discoveries well and rightly made, and not by chance.[147]

Diog. of Apollonia, fr. 1: λόγου παντὸς ἀρχόμενον δοκεῖ μοι χρεὼν εἶναι τὴν ἀρχὴν ἀναμφισβήτητον παρέχεσθαι, τὴν δὲ ἑρμηνείαν ἁπλῆν καὶ σεμνήν. The latter ideal comports with the portrait of the true philosopher, Π. εὐσχημοσύνης, 3 (9, 228 Littré) εὐεπίῃ χρώμενοι, χάριτι διατιθέμενοι.

[136] Π. εὐσχημοσύνης, 1 (9, 226 Littré).

[137] Π. τέχνης, 13 (6, 26 Littré).

[138] Ἀφορισμοί, 1 (4, 458 Littré).

[139] Παραγγελίαι, 1 (9, 250 Littré).

[140] Παραγγελίαι, 1 (9, 250 Littré) δεῖ γε μὴν ταῦτα εἰδότα μὴ λογισμῷ πρότερον πιθανῷ προσέχοντα ἰητρεύειν, ἀλλὰ τριβῇ μετὰ λόγου. Plato and Aristotle oppose τριβή to τέχνη; but this τριβή is not ἄτεχνος (Plato, *Phaedr.* 260 E), but μετὰ λόγου.

[141] Π. ἄρθρων, 10 (4, 102 Littré) οὐκ ἀρκέει μοῦνον λόγῳ εἰδέναι τὴν τέχνην ταύτην, ἀλλὰ καὶ ὁμιλίῃ ὁμιλέειν.

[142] Π. ἄρθρων, 78 (4, 312 Littré).

[143] Π. ἄρθρων, 44 (4, 188 Littré) αἰσχρὸν μέντοι καὶ ἐν πάσῃ τέχνῃ καὶ οὐχ ἥκιστα ἐν ἰητρικῇ πουλὺν ὄχλον, καὶ πολλὴν ὄψιν, καὶ πουλὺν λόγον παρασχόντα, ἔπειτα μηδὲν ὠφελῆσαι.

[144] Π. κρισίμων, 1 (9, 298 Littré). Cp. Π. διαίτης, A, 1 (6, 466 Littré).

[145] Παραγγελίαι, 2 (9, 254 Littré) οὕτω γὰρ δοκέω τὴν ξύμπασαν τέχνην ἀναδειχθῆναι, διὰ τὸ ἐξ ἑκάστου τοῦ τέλους τηρηθῆναι καὶ εἰς ταὐτὸ ξυναλισθῆναι.

[146] Π. τέχνης, 5 and 6 (6, 8 foll. Littré). We even find a suggestion of definition in terms of the purpose of an art, Π. τέχνης, 3 (6, 4 Littré) καὶ πρῶτόν γε διοριεῦμαι ὃ νομίζω ἰητρικὴν εἶναι, τὸ δὴ πάμπαν ἀπαλλάσσειν τῶν νοσεόντων τοὺς καμάτους, κτλ. This and several other matters incline me to the opinion that Π. τέχνης belongs to the fourth century, though its general value for our purposes is not thereby appreciably affected.

[147] Π. ἀρχαίης ἰητρικῆς, 12 (1, 596 Littré) οὐ φημὶ δὴ διὰ τοῦτο δεῖν τὴν τέχνην ὡς

"There be," we read,[148] "who have reduced vilifying the sciences to a science, as those who engage in this pursuit opine. I think not so; but they are giving an exhibition of their own learning. To me it appears that to make a discovery, that were better made than left undiscovered, is the desire and function of understanding, and to advance to completion that which is half-finished, likewise; but to essay with ungentle words to shame the discoveries of others, oneself bettering nothing, but casting reproach upon the discoveries of those who know before those who do not know, this appears to me not the desire and function of understanding, but argues natural depravity even[149] more than want of science." Another interesting passage is the following:[150] "Medicine has long had an established principle and a method[151] of its own invention, in accordance with which the many excellent discoveries were made in the long lapse of time and in accordance with which also the rest will be made, if one, having proper capacity and a knowledge of past discoveries, shall take these as the point of departure for his quest. But whoso, casting these aside and rejecting all, shall essay to investigate after another method and in other fashion, and shall say that he has discovered aught, is deceived and deceives others; for that is impossible." Elsewhere we are assured[152] that the science of medicine has nothing left it to discover, since it now teaches everything, characters as well as proper seasons. He who has learned its teachings will succeed with or without the favor of fortune.[153]

From this it will be seen that the ancient art or science of medicine had not only developed the spirit of science and formulated in general its ideals, but that in some minds it had attained to a position of such independence that it might lay claim to finality. The fact that the claim

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οὐκ ἐοῦσαν οὐδὲ καλῶς ζητεομένην τὴν ἀρχαίην ἀποβαλέσθαι, εἰ μὴ ἔχει περὶ πάντα ἀκριβίην, ἀλλὰ πολὺ μᾶλλον, διὰ τὸ ἐγγύς, οἶμαι, τοῦ ἀτρεκεστάτου ὁμοῦ δύνασθαι ἥκειν λογισμῷ, προσίεσθαι, καὶ ἐκ πολλῆς ἀγνωσίης θαυμάζειν τὰ ἐξευρημένα, ὡς καλῶς καὶ ὀρθῶς ἐξεύρηται, καὶ οὐκ ἀπὸ τύχης.

[148] Π. *τέχνης*, 1 (6, 2 Littré).

[149] I read *ἔτι μᾶλλον*, and *ἀτεχνίης*.

[150] Π. *ἀρχαίης ἰητρικῆς*, 2 (1, 572 Littré).

[151] Cp. Π. *εὐσχημοσύνης*, 2 (9, 226 Littré), and above, n. 147, *καὶ οὐκ ἀπὸ τύχης*.

[152] Π. *τόπων τῶν κατὰ ἄνθρωπον*, 46 (6, 342 Littré) *ἰητρικὴ δή μοι δοκέει ἤδη ἀνευρῆσθαι ὅλη, ἥτις οὕτως ἔχει, ἥτις διδάσκει ἕκαστα καὶ τὰ ἔθεα καὶ τοὺς καιρούς. ὃς γὰρ οὕτως ἰητρικὴν ἐπίσταται, ἐλάχιστα τὴν τύχην ἐπιμένει, ἀλλὰ καὶ ἄνευ τύχης καὶ ξὺν τύχῃ εὐποιηθείη ἄν. βέβηκε γὰρ ἰητρικὴ πᾶσα, καὶ φαίνεται τῶν σοφισμάτων τὰ κάλλιστα ἐν αὐτῇ συγκείμενα ἐλάχιστα τύχης δεῖσθαι· ἡ γὰρ τύχη αὐτοκρατὴς καὶ οὐκ ἄρχεται, οὐδ' ἐπ' εὐχῇ ἐστιν αὐτὴν* (an *αὐτῆς*?) *ἐλθεῖν· ἡ δὲ ἐπιστήμη ἄρχεταί τε καὶ εὐτυχής ἐστιν, ὁπόταν βούληται ὁ ἐπιστάμενος χρῆσθαι, κτλ.*

[153] Cp. Π. *τέχνης*, 4 and 6 (6, 6 and 10 Littré). Π. *εὐσχημοσύνης*, 7 (9, 258 Littré) the charlatans are said to depend on luck.

was preposterous must not be allowed to obscure the significance of its being made; for at any time, past, present, or future, such assurance must be essentially subjective, based upon the sense of inner congruity or harmony of the world of thought organized and interpreted by the system. It was just this feeling of independence to which we attributed the growing sense of the autonomy of Nature that made it possible for philosophers to dispense with the intervention of the gods. The scientific movement in philosophy and medicine runs parallel courses with constant interaction. How constant and important this reaction of one upon the other really was we can never know. In the present state of our knowledge it would be foolish even to attempt to say; but that it is a fact, and a fact of large significance, none will deny. The physicians could not overlook the relation of the individual human organism to the world. They devoted themselves with keen intelligence to the study of atmospheric and climatic conditions[154] affecting the health of man, and in so doing could not avoid trenching on the domain of the physical philosopher. In countless other ways subjects of prime importance to the philosopher came within the purview of the writer on medicine. For all these questions the works of Hippocrates are for us an inexhaustible source of information, though they rarely enable us to refer an opinion to its responsible author. It is therefore a matter of interest to see the intimacy of the relation between these kindred disciplines recognized by the physicians.

The Hippocratean treatise *On Decorum*[155] sketches in ideal portraiture the man of science (especially the physician) and the philosopher and contrasts with them the charlatan, who appears in the colors familiar to all in the Platonic portraits of the Sophists. There the physician is called a god-like philosopher,[156] since he combines theory and practice of all that is true and beautiful. Philosopher and physician have the same virtues; their differences are slight.[157] Elsewhere, however, a distinction is drawn between the physician and the physical philosopher in respect to method. "There are those," we are told,[158] "who have essayed to speak or write concerning medicine, basing their argument on the hot or the cold, on the moist or the dry or any thing

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[154] Cp. especially the treatise Π. *ἀέρων, ὑδάτων, τόπων* (2, 12 foll. Littré; 1, 33 foll. Kühlewein).

[155] Π. *εὐσχημοσύνης* (9, 226 foll. Littré).

[156] *Ibid.* c. 5 (9, 232 Littré) *διὸ δεῖ . . . μετάγειν τὴν σοφίην ἐς τὴν ἰητρικὴν καὶ τὴν ἰητρικὴν ἐς τὴν σοφίην. ἰητρὸς γὰρ φιλόσοφος ἰσόθεος.*

[157] *Ibid. οὐ πολλὴ γὰρ διαφορὴ ἐπὶ τὰ ἕτερα· καὶ γὰρ ἔνι τὰ πρὸς σοφίην ἐν ἰητρικῇ πάντα, ἀφιλαργυρίη*, etc.

[158] Π. *ἀρχαίης ἰητρικῆς*, 1 (1, 570 foll. Littré; 1, 1 foll. Kühlewein).

else they choose, reducing the causes of human diseases and death to a minimum, one and the same for all, basing their argument on one or two; but in many of the novelties they utter they are clearly in the wrong. This is the more blameworthy, because they err touching an actual art which all men employ in the greatest emergencies and in which they honor most the skillful practitioners. Now there are practitioners, some bad, some excellent; which would not be true if medicine were not actually an art, and no observations or discoveries had been made in it. All would be equally unskilled and ignorant of it, and the cure of diseases would be wholly subject to chance. As a matter of fact, it is not so; but, as artisans in all other arts excel one the other in handicraft and knowledge, so also in medicine. Therefore I maintained that it had no need of vain hypotheses, as is the case in matters inaccessible to sense and open to doubt. Concerning these, if one essay to speak, one must resort to hypothesis. If, for example, one should speak and entertain an opinion touching things in the heavens or under the earth, it would be clear neither to the speaker nor to those who heard him whether his opinion was true or false; for there is no appeal to aught that can establish the truth." While the resort to hypothesis in medicine is here denounced there are instances of such use in the works of Hippocrates, notably in Περὶ φυσῶν.[159]

One more passage[160] relating to philosophy we may properly quote here. "Whoso is wont to hear men speak concerning the human constitution beyond the range of its bearing upon medicine, will find the following discourse unprofitable; for I do not say that man is wholly air, nor fire, nor water, nor earth, nor any thing else that is not clearly present in man. This I leave for whoso wills to say. Yet I think that those who say this are in error; for they agree in point of view, but not in statement. Nevertheless the argument in support of their point of view is the same; for they say that all that exists is one. This is the One and All; but they give it different names. One calls the One and All air; another, fire; a third, water; still another, earth. And each supports his argument with proof and evidence, which amounts to nothing. For, seeing that they are all of one mind, but say, one man this thing, another that, it is clear that they have no knowledge of the

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[159] Littré 6, 90 foll. The treatise is a Sophistic exercise, intended to prove that air, particularly the air in the body, is the cause of all diseases, and employs hypothesis avowedly. Cp. c. 15 (p. 114 Littré). The treatises Π. φύσιος ἀνθρώπου and Π. ἀρχαίης ἰητρικῆς aim their polemic at such exercises, as Littré justly observes, 6, 88.

[160] Π. φύσιος ἀνθρώπου, 1 (6, 32 foll. Littré). Littré, 6, 88, thinks the author of this treatise had definitely in mind, among others, the essay Π. φυσῶν.

matter. Of this one would be most thoroughly convinced if one attended their disputations; for when the self-same men dispute with one another in the presence of the self-same auditors, the same man never thrice in succession prevails in argument; but now one prevails, now another, and again he who has the most flowing speech before the mob. Surely it is fair to demand that he who claims to have the right opinion about things should cause his argument always to prevail, assuming that his opinion is true and that he properly sets it forth. As for me, I think that such men for want of understanding refute one another by the terms of their very argument and establish the contention of Melissus."

If, now, we recall to mind those ideals and conceptions anticipated above in the first section of this study, we shall have a fair notion of science as it was conceived among the Greeks of the fifth century B. C. But we have still to inquire just what questions the scientist addressed to nature; and to this quest we may now turn.

Science essays to determine the facts and to explain them. The one thing depends upon the other. If you find a rock and ask what it is, it becomes necessary to discover whether it is in position or not. It proves to be a boulder, and examination shows that it is metamorphic in character: finally it is identified as Laurentian, and its presence here is explained by reference to glacial action. The definition of the fact involves the explanation; but explanation is the motive of the scientific study of the fact, in contrast to the practical interest which leads merely to classification. The curious child, no less than the philosopher, asks the question, Why? But, while almost any answer, judiciously framed, will satisfy the child, the philosopher knows that the question may receive very different answers according to its specific intention. To ask why is to demand an explanation; and 'cause' is our generic name for explanation. Different as individual attempts at explanation may be, they are reducible to a few kinds. We are familiar with the four-fold causal principle of Aristotle, and with the fact that, while recognizing four kinds of causation and insisting that in explanation one should adduce all causes, he did not find it possible to reduce all to one, but was compelled to content himself in the ultimate analysis with two.[161]

This is, of course, not the place to discuss matters of metaphysics except so far as they pertain or contribute to our purpose; but there is here a point of some interest for us. We have noted that of Aristotle's causes, the material points to the past. It is that which is

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[161] Cp. Ritter-Preller, §§ 395–396.

there to begin with. "In the beginning," says the materialist, "was matter." "No," replies the theist, "in the beginning *God created* matter;" and thus a preface is placed before the beginning. The teleologist and the pragmatic explain all with reference to the end, which justifies the means. All alike endeavor to define the fact in the hope of explaining it; but it remained for a Socratic to detect the teleological import of logical definition and hence practically to identify it with the final cause. We have referred to the principal classes of philosophers with the exception of the positivist. If the materialist defines things with reference, so to speak, to the past, and the teleologist, with reference to the future, the positivist asks neither whence nor whither, but how. Definition for him becomes description, and description in universal, timeless terms. Such at least is the logic of his position. The reason why Aristotle did not find it possible to reconcile his ultimately two-fold causation in his 'formal' cause is that historically he was the heir of the pre-Socratic and the Socratic methods, of which the former deified the material, the latter the final, cause.[162] The degree of advancement in the formulation of the positivist attitude was not such as to compel a recognition in logic and metaphysics, although it would not be unfair to say that there was much of the positivist spirit in the scientific thought of the fifth century. Apparently it was the concreteness of Greek thinking, more than anything else, that obscured the significance of the scientific impulse as such. Every process, as we have seen, no matter how abstract, assumed in the thought of the Greeks the form of a series in time, or of a history with a proper beginning. How much of this was conscious device, how much instinctive procedure, we shall never know. Even the ideal construction of the world in Plato's *Timaeus* was, however, taken as an intended *vera historia* by the literal-minded Aristotle.

Accordingly we are not surprised to find that Aristotle sets down the pre-Socratics as mentioning only the material causes of things. This means, however, as we may now see, that they did not bring forward efficient causes — that is, chiefly, God — nor formal causes — that is, definitions or descriptions — nor final causes, as sufficient principles of explanation. It does not mean that they were not interested in the processes of nature as such or in their precise methods and laws. This no one would deny; but it is a point of prime importance, whose significance is frequently overlooked. What Hippocrates says of the monists is true of them all. "They agree in point of view, but not in statement." Why the difference in language? Because one kind of

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[162] The logical aspect of this situation I sought to set forth in my essay on *The Necessary and the Contingent in the Aristotelian System.*

primal matter seemed to lend itself better than another to the explanation of phenomena. The elements were interesting only as means to an end. It was the regularities of phenomena more than anything else that drew the attention of the philosopher; presumably it was this aspect of nature which counted most strongly in favor of a single primary substance. But the tendency to simplify was indulged too far and led ultimately to the opposite extreme.

Science, then, in attempting to explain things, assigns the cause and interprets the facts in accordance with analogies drawn from experience. In Hippocrates, Π. φυσῶν, c. 15 we read: "Airs, then, have been shown to be most mischievous in all diseases: other causes are only accessory and ancillary, but this has been shown to be the real cause of diseases. I promised to declare the cause of diseases, and I have shown that wind (πνεῦμα) lords it over other things and particularly over the bodies of living beings. I have applied the reasoning to known maladies, and in them the hypothesis has been shown to be true." "It is the function of the same intelligence to know the causes of diseases and to know how to treat them with all the resources of the art of healing."[163] What applies to the microcosm,[164] is equally true of the cosmos. The causes must be sought everywhere; for as Plato says,[165] citing Hippocrates as his authority, one cannot know the nature of man without knowing the nature of the whole. We are accustomed to think that strict science, based upon the knowledge of causes, dates from the age of Plato and Aristotle, but such is not the case.[166] In the *Republic*[167] Plato suggests that in the effort to read

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[163] Hippocrates, Π. τέχνης, 11 (6, 20 Littré).

[164] The comparison is old (cp. Anaximenes, fr. 2), though the expression only occurs later; cp. Democritus, fr. 34.

[165] *Phaedr.* 270 B foll.

[166] Cp. Arist. *De Partt. Animal.* 640^b 4 foll.; *De Sensu*, 436^a 15 καὶ ζωὴ καὶ θάνατος· περὶ ὧν θεωρητέον τί τε ἕκαστον αὐτῶν, καὶ διὰ τίνας αἰτίας συμβαίνει. φυσικοῦ δὲ καὶ περὶ ὑγιείας καὶ νόσου τὰς πρώτας ἰδεῖν ἀρχάς (cp. Hippocrates, Π. ἀρχαίης ἰητρικῆς, τὴν ἀρχὴν τῆς αἰτίης . . . νούσων τε καὶ θανάτου)· οὔτε γὰρ ὑγίειαν οὔτε νόσον οἷόν τε γίνεσθαι τοῖς ἐστερημένοις ζωῆς. διὸ σχεδὸν τῶν τε περὶ φύσεως οἱ πλεῖστοι καὶ τῶν ἰατρῶν οἱ φιλοσοφωτέρως τὴν τέχνην μετιόντες, οἱ μὲν τελευτῶσι εἰς τὰ περὶ ἰατρικῆς, οἱ δὲ ἐκ τῶν περὶ φύσεως ἄρχονται περὶ τῆς ἰατρικῆς. *De Gener. Animal.* 769^a 6 εἰρήκασι δέ τινες τῶν φυσιολόγων καὶ ἕτεροι (the medical writers) περὶ τούτων, διὰ τίν' αἰτίαν ὅμοια καὶ ἀνόμοια γίγνεται τοῖς γονεῦσι. Cp. *De Partt. Animal.* 641^a 7; *Met.* 1069^a 25 μαρτυροῦσι δὲ καὶ οἱ ἀρχαῖοι ἔργῳ· τῆς γὰρ οὐσίας ἐζήτουν ἀρχὰς καὶ στοιχεῖα καὶ αἴτια; *Ibid.* 988^b 22 ὅσοι μὲν οὖν ἕν τε τὸ πᾶν καὶ μίαν τινὰ φύσιν ὡς ὕλην τιθέασι, καὶ ταύτην σωματικὴν καὶ μέγεθος ἔχουσαν, δῆλον ὅτι πολλαχῶς ἁμαρτάνουσιν . . . καὶ περὶ γενέσεως καὶ φθορᾶς ἐπιχειροῦντες τὰς αἰτίας λέγειν κτλ. It is evident that Aristotle is here enlarging upon the criticism of the monists contained in Hippocrates, Π. φύσιος ἀνθρώπου, c. 1, quoted above, p. 119 foll.

[167] 368 D foll.

the character of justice one may perhaps gain some advantage from contemplating it as writ large in the history and constitution of the state and noting how it originated.[168] There were others who preferred to reverse the procedure, hoping to throw light on general nature by studying the nature of man. Of these we have an example in Hippocrates, Περὶ ἀρχαίης ἰητρικῆς. "Certain physicians and philosophers," he says,[169] "assert that one cannot know the science of medicine without knowing what man is, how he originally came into existence, and of what substances he was compounded in the beginning; and this he who would properly treat men must be thoroughly cognizant of. Now the contention of these men really looks to philosophy, as do Empedocles and others who have written Περὶ φύσεως. As for me, I consider that what a philosopher or physician has said or written Περὶ φύσεως has less relevancy to medicine than to painting; and I am of opinion that, so far as concerns knowledge Περὶ φύσεως, one can know nothing definite about it except from medicine; but this may be thoroughly learned when men go about it rightly. Hitherto, it seems to me, we are far from it: far, that is to say, from having a scientific knowledge of what man is (that is to say, what his constitution is), and to what causes he owes his origin and the rest, in any exact sense. Now so much at least it is indispensable that the physician should know Περὶ φύσεως and should greatly concern himself to know, if he is to do any part of his duty; to wit, what a man is (i. e. what his constitution is) relative to meat and drink, and what he is relative to the rest of his mode of life, and what results follow for the individual from particular things, and all this not merely in general terms, as e. g., 'cheese is unwholesome food, for it distresses one who eats plentifully of it'; but what particular distress it causes, and for what reason, and to what ingredient of the man's constitution it is unsuitable." The

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[168] Cp. also the myth in Plato's *Protagoras*, 320 C foll., where the virtues are illustrated by the story of their origin. An interesting contrast is presented by Aristotle, *De Gener. Animal.* 778ᵃ 16 foll., where he discusses the cases in which biological phenomena are to be interpreted teleologically or physically; γένεσις is for the sake of οὐσία, and οὐσία is the cause of γένεσις. The ancient physiologers thought otherwise; hence they recognized only material and efficient causes, not even discriminating between them. He states his own view thus: οὐ διὰ τὸ γίγνεσθαι ἕκαστον ποιόν τι, διὰ τοῦτο ποιόν τι ἐστίν, ὅσα τεταγμένα καὶ ὡρισμένα ἔργα τῆς φύσεώς ἐστιν, ἀλλὰ μᾶλλον διὰ τὸ εἶναι τοιαδὶ γίγνεται τοιαῦτα. The opposite argument is presented in Plato, *Euthyphro*, 10 A foll. The latter clearly represents the common logical procedure, based upon the common usage of the Greeks as established in the pre-Socratic period, though, strictly speaking, the former conforms perfectly to the teleological logic of the Socratics. This is another illustration of the inner contradiction of the Aristotelian logic.

[169] C. 20 (1, p. 24 Kühlewein).

writer then proceeds to say that the physician must study the particular food-stuff and its physiological action as well as the individual constitution, determining which of the humors is *πλείων ἐνεὼν καὶ μᾶλλον ἐνδυναστεύων ἐν τῷ σώματι*, and then knowing which humor is inimical[170] to the particular food-stuff and is roused to hostility by it, he can prescribe a suitable diet.

Here we find set up an ideal that science is still far from realizing. Only a year or two ago an eminent physician stated that the specific physiological action of drugs still remained undiscovered, with the possible exception of two or three. Even for foods a bare beginning has been made. We may recall that Hippocrates elsewhere[171] insists that each phenomenon has its own *φύσις* or natural cause (law?) and that Heraclitus likewise proposed to explain each thing according to its own law, thus aspiring to meet the two-fold requirement of science which aims to discover both the proximate causes of events and the ultimate statement of universal law. There is, moreover, a further interest attaching to the passage just quoted at length. It formulates three questions raised by philosophers and by physicians philosophically inclined: (1) what man is; (2) how he originated; and (3) of what he is composed. The first and third questions, as we have seen, practically coincide; the second agrees with its fellows, except that it regards the process rather than the result, which is, however, only an analysis read backward and cast into the time-form. Hippocrates does not object to the questions, as such; he merely regards them as too general and, therefore, as premature, considering the stage of advancement attained by positive science in his time. His attitude is instructive, however, since it is obviously that of a scientist of knowledge and discernment looking with critical eye upon the venturesome undertakings of less mature minds; for science naturally proceeds from the general to the particular.[172]

The same position is taken in the essay Περὶ διαίτης:[173] "I say that one

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[170] In the microcosm we thus have a picture in miniature of the cosmic *πόλεμος* of elemental forces, in which one element prevails (*ἐπικρατεῖ*) at one time, a second at another. It is the function of the physician to support (*βοηθεῖν*) the losing element and so to restore the harmony of a proper balance of powers. Cp., for example, II. *ἱερῆς νούσου*, 18 (6, 394 foll. Littré) *χρὴ δὲ καὶ ἐν ταύτῃ τῇ νούσῳ καὶ ἐν τῇσι ἄλλῃσιν ἁπάσῃσι μὴ αὔξειν τὰ νουσήματα, ἀλλὰ σπεύδειν τρύχειν προσφέροντα τῇ νούσῳ τὸ πολεμιώτατον ἑκάστῃ, καὶ μὴ τὸ φίλον καὶ σύνηθες.*

[171] See above, n. 57, and Plato, *Phaedr.* 270 B quoted below, n. 175.

[172] There is an interesting parallel to the procedure of Hippocrates in Aristotle's discussion of the winds, *Meteor.* 360^a 27 and the comments of Olympiodorus. See Gilbert, *Die meteorologischen Theorien des griechischen Altertums*, p. 524, n. 2.

[173] A,2 (6, 468 Littré).

who is to write a proper treatise on human dietetics must first of all know the constitution of man, — know and distinguish: he must know of what he was constituted in the beginning and distinguish (in the individual case) by what constituents he is ruled. Unless he knows his original composition, he will not be able to know the results that flow from it; unless he distinguish[174] the ruling constituent in the body, he will not be capable of administering what is beneficial to the man. This, then, the writer must know; but he must have learned, in addition, the action — whether due to nature or to human constraint and art — that each kind of meat and drink has which we employ by way of diet." To these, or similar, words of Hippocrates Plato refers in the *Phaedrus*[175] with cordial approval. It thus becomes a commonplace that distinction and, above all, analysis of a complex whole into its parts, are necessary to clear philosophical thought;[176] and that, in order to make clear the nature of anything, it is desirable by an act of imaginative synthesis to reconstitute the fact thus analyzed.

The boy who takes his watch to pieces and tries to put it together again, — usually with scant success, because synthesis lags far behind analysis, — indulges an ideal, rather than a practical, instinct. He has no thought of making watches, but wants to understand his time-piece. At the beginning of the *Politics*[177] Aristotle puts the matter clearly: "As in other departments of science, so in politics, the compound should always be resolved into the simple elements or least parts of the whole. We must therefore look at the elements of which the state is composed. . . . He who thus considers things in their first growth and origin, whether a state or anything else, will

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[174] I read *διαγνώσεται* for *γνώσεται*.

[175] 270 B *ἐν ἀμφοτέραις* (sc. medicine and rhetoric) *δεῖ διελέσθαι φύσιν, σώματος μὲν ἐν τῇ ἑτέρᾳ, ψυχῆς δὲ ἐν τῇ ἑτέρᾳ, εἰ μέλλεις, μὴ τριβῇ μόνον καὶ ἐμπειρίᾳ ἀλλὰ τέχνῃ, τῷ μὲν φάρμακα καὶ τροφὴν προσφέρων ὑγίειαν καὶ ῥώμην ἐμποιήσειν . . . ψυχῆς οὖν φύσιν ἀξίως λόγου κατανοῆσαι οἴει δυνατὸν εἶναι ἄνευ τῆς τοῦ ὅλου φύσεως;* Εἰ *μὲν* Ἱππο*κράτει γε τῷ τῶν* Ἀσκληπιαδῶν *δεῖ τι πιθέσθαι, οὐδὲ περὶ σώματος ἄνευ τῆς μεθόδου ταύτης . . .* Τὸ *τοίνυν περὶ φύσεως σκόπει τί ποτε λέγει* Ἱπποκράτης *τε καὶ ὁ ἀληθὴς λόγος· ἆρ᾽ οὐχ ὧδε δεῖ διανοεῖσθαι περὶ ὁτουοῦν φύσεως· πρῶτον μέν, ἁπλοῦν ἢ πολυειδές ἐστι οὗ πέρι βουλησόμεθα εἶναι αὐτοὶ τεχνικοὶ καὶ ἄλλον δυνατοὶ ποιεῖν, ἔπειτα δέ, ἂν μὲν ἁπλοῦν ᾖ, σκοπεῖν τὴν δύναμιν αὐτοῦ, τίνα πρὸς τί πέφυκε εἰς τὸ δρᾶν ἔχον ἢ τίνα εἰς τὸ παθεῖν ὑπὸ τοῦ, ἐὰν δὲ πλείω εἴδη ἔχῃ, ταῦτα ἀριθμησάμενον, ὅπερ ἐφ᾽ ἑνός, τοῦτ᾽ ἰδεῖν ἐφ᾽ ἑκάστου, τῷ τί ποιεῖν αὐτὸ πέφυκεν ἢ τῷ τί παθεῖν ὑπὸ τοῦ;* Κινδυνεύει.

[176] Cp. Plato, *Tim.* 57 D *διὸ δὴ συμμειγνύμενα αὐτά τε πρὸς αὐτὰ καὶ πρὸς ἄλληλα τὴν ποικιλίαν ἐστὶν ἄπειρα· ἧς δὴ δεῖ θεωροὺς γίγνεσθαι τοὺς μέλλοντας περὶ φύσεως εἰκότι λόγῳ χρήσεσθαι.* But to study the *ποικιλία* of things requires that the crazy-patchwork be set in order by analysis.

[177] 1252^a 24 foll., transl. Jowett. Aristophanes, *Thesmoph.* 11 foll. affords a good example of *φύσις* = 'constitution,' which at once suggests 'origin.'

obtain the clearest view of them." Quite apart from the obvious debt of Aristotle in this matter to Plato[178] and Hippocrates, it must be clear that this method of procedure has no relevancy to the distinctively Socratic doctrine of definition in terms of the end or purpose; it is a survival from the naturalistic or mechanical mode of thought, developed in the pre-Socratic age, which explains things in terms of their origin and physical constituents.

Socrates, the originator of the teleological method, could not understand this procedure. To his mind it belonged not to theory, but to the sphere of the practical arts. There is an extremely interesting passage touching this matter in Xenophon's *Memorabilia.*[179] "Nor did he (Socrates) converse," we are told, "about the constitution of the world (*περὶ τῆς τῶν πάντων φύσεως*), as the majority of the philosophers do, inquiring how that which the philosophers call the cosmos originated[180] and by what mechanical forces[181] (*ἀνάγκαις*) the phenomena of the heavens are brought about, but he even declared that they who worry their heads about such matters are fools." . . . "He inquired also concerning the philosophers, asking whether, in like manner as they who learn the human arts[182] think that they shall be able to make what they may learn either for themselves or for whomsoever they please, so also they who study things divine think that when they have learned by what mechanical forces they severally come about, they shall at their pleasure make winds and rains[183] and whatever of the

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[178] Especially *Repub.* 368 D foll., *Phaedr.* 270 C foll. Cp. Plato's summary of the *Republic* in *Tim.* 17 C *χθές που τῶν ὑπ' ἐμοῦ ῥηθέντων λόγων περὶ πολιτείας ἦν τὸ κεφάλαιον οἵα τε καὶ ἐξ οἵων ἀνδρῶν ἀρίστη κατεφαίνετ' ἄν μοι γενέσθαι.* For the thought that to understand a thing one should see it put together, cp. *Tim.* 27 C, 28 B, 90 E, etc.

[179] I. 1, 11 and 15.

[180] The MSS vary between *ἔφυ* and *ἔχει.* The former emphasizes the process of origination; the latter implies it in the question as to the truth about phenomena (*πῶς ἔχει*). Cp. Parmen. fr. 10. In Hippocrates *ὡς ἔχει* is often used in relation to *φύσις* = constitution.

[181] Where the physical philosopher inquired *τίσιν (φυσικαῖς) ἀνάγκαις γίγνεται,* Socrates asked, if at all, *ᾗ ἕκαστα ὁ θεὸς μηχανᾶται,* Xen. *Mem.* IV. 7, 6. Cp. *ibid.* I. 4, 14 where *φύσει = θεοῦ προνοίᾳ*: *φύσις* has become the mechanism of God's providence.

[182] Cp. Aristoxenus, fr. 31 (Müller, *F. H. G.*, II. 281) *φησὶ δ' 'Α. ὁ μουσικὸς Ἰνδῶν εἶναι τὸν λόγον τόνδε· 'Αθήνησι γὰρ ἐντυχεῖν Σωκράτει τῶν ἀνδρῶν ἐκείνων ἕνα τινά, κἄπειτα αὐτοῦ πυνθάνεσθαι, τί ποιῶν φιλοσοφοίη· τοῦ δ' εἰπόντος, ὅτι ζητῶν περὶ τοῦ ἀνθρωπίνου βίου, καταγελάσαι τὸν 'Ινδόν, λέγοντα μὴ δύνασθαί τινα τὰ ἀνθρώπινα κατιδεῖν ἀγνοοῦντά γε τὰ θεῖα.* Compare the opinion of those who held that one cannot know the *φύσις* of man without knowing the *φύσις τοῦ ὅλου.*

[183] One is tempted to regard this as a hit at Empedocles; cp. fr. 111. Because of this expression Empedocles has been set down as a charlatan; but in the present

sort they may desire, or whether they do not even conceive such a hope, but are content merely to know how these phenomena occur." The difference between the physical and the teleological points of view is beautifully illustrated by the story told by Plutarch in his *Life of Pericles*: [184] "It is related that on a certain occasion the head of a goat with a single horn was brought from the country to Pericles, and that Lampon, the seer, when he saw the strong, solid horn growing out of the middle of the forehead, said that, there being in the city two rivals for power, Thucydides and Pericles, the power would come to the one to whom the sign was given. Anaxagoras, however, cutting open the skull, showed that the brain was not fully developed at the base, but shrunken from its integument and coming somewhat to a point, egg-like, at the spot where the horn sprouted. At the time Anaxagoras was applauded by those who were present; but Lampon's turn came shortly afterwards, when the power of Thucydides was broken and the affairs of the people came steadily under the direction of Pericles. There was nothing, however, so far as I can see, in the way of the physical philosopher and the seer [185] being equally in the right, the one

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state of his poem we are not in position to judge. The promise of fr. 2 is sufficiently modest (cp. Parmenides, fr. 10 and 11). I incline to think that fr. 111 belongs to the concluding passage of his philosophical poem, and voices the high hopes of the author that the secrets of nature will soon be laid bare. The age of Empedocles was intoxicated with the new wine of science and regarded nothing as too difficult to explain. Once the principles were fully understood, as in certain sciences (e.g. medicine, as we have seen) they were by some even then thought to be, it was not strange that men should hope to perform wonders of science equal to the most ambitious miracles of magic.

[184] C. 6.

[185] It is certain that the Socratic teleology, whether suggested by Socrates' reverence for *μαντική* or not, came to the rescue of divination at a time when it was in a bad way, as we may see from Thucydides. The identity of the two points of view is apparent: the question remains whether teleology is immanent in the process of nature or imposed on it from without. In a way *μαντική* differs from *ἱστορίη* chiefly in this that the latter attempts to know the present by reconstructing the past, while the former seeks to infer the future from the present. Hence the words of Pindar, *Pyth.* 9, 48 ff. are interesting: *κύριον ὃς πάντων τέλος* | *οἶσθα* (Apollo) *καὶ πάσας κελεύθους . . . χὤ τι μέλλει, χὠπόθεν ἔσσεται, εὖ καθορᾷς.* Knowledge of the end implies teleology: *ὅ τι μέλλει* is *ὅ τι ἔστι* thrown into the future, and *ὁπόθεν ἔσσεται* refers to the *κέλευθοι*, as Gildersleeve rightly says. Compare the praise of (Anaxagorean?) physical philosophy in Eurip. fr. 910 (the text of Diels, *Vorsokr.* 299, 23) *ὄλβιος ὅστις τῆς ἱστορίας* | *ἔσχε μάθησιν* | *μήτε πολιτῶν ἐπὶ πημοσύνην* | *μήτ' εἰς ἀδίκους πράξεις ὁρμῶν,* | *ἀλλ' ἀθανάτου καθορῶν φύσεως* | *κόσμον ἀγήρων, ᾗ τε συνέστη* | *χὤπῃ χὤπως.* *What* and *how* are the main questions; the latter includes the story, and hence the beginnings. Compare Plato, *Phaed.* 97 C *εἰ οὖν τις βούλοιτο τὴν αἰτίαν εὑρεῖν περὶ ἑκάστου ὅπῃ γίγνεται ἢ ἀπόλλυται ἢ ἔστι* with 96 A *ὑπερήφανος γάρ μοι ἐδόκει* (sc. *ἡ σοφία, ἣν δὴ καλοῦσι περὶ φύσεως ἱστορίαν*), *καὶ εἰδέναι τὰς αἰτίας ἑκάστου, διὰ τί γίγνεται ἕκαστον καὶ διὰ τί ἀπόλλυται καὶ διὰ τί ἔστι.*

well singling out the physical cause (*τὴν αἰτίαν*) the other the purpose (*τὸ τέλος*) ; for the former was, by hypothesis, inquiring from what physical conditions it sprung and how it came about in the course of nature (*ἐκ τίνων γέγονε καὶ πῶς πέφυκε*), whereas the latter was predicting to what purpose it came about and what it signified" (*πρὸς τί γέγονε καὶ τί σημαίνει*).

Democritus is reported to have said that he would rather make one contribution to the causal explanation of things than be made King of the Persians.[186] Surely this does not mean that he wanted to discover an atom; he was in search of the causal nexus in whatever form, and his atoms and void were only the last link in the chain. Men knew what it meant to explain : they did not confuse explanation with description, although they might content themselves with the latter, in default of the former. This was often the attitude of the physician, aware of his ignorance of the real cause. The words of Thucydides about the great plague well illustrate this point. "As to its probable origin," he says,[187] "or the causes which might or could have produced such a disturbance of nature, every man, whether a physician or not, may give his own opinion. But I shall describe its actual course, and the symptoms by which any one who knows them beforehand may recognize the disorder should it ever reappear."

It would be easy to multiply witnesses proving that the pre-Socratic philosophers aimed at nothing short of a complete understanding of the world in terms of its physical causes ; but enough has been said. There is, however, one passage in Plato to which reference should be made. In the *Phaedo*[188] Socrates sets forth, as only Plato could do it, the difference in point of view between the Socratic and the pre-Socratic philosophies. No contrast could be more clearly or sharply drawn : on the one hand we find an explanation of things beginning with matter and operating with mechanical causes, for which Socrates declares himself by nature unfitted ; on the other stands the teleological conception of the world for which Socrates is sponsor. Socrates tells how eagerly he took up the book of Anaxagoras in the hope of finding a real anticipation of his view, but only to meet with utter disappointment. Plato does not often touch directly upon the earlier philosophies, but here he has drawn a picture of their aims and methods which leaves nothing to be desired. Perhaps its full significance is hardly realized.

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[186] Fr. 118.

[187] II. 48, 3, transl. Jowett. In Hippocrates, especially in the works which may be classed as note-books, explanation commonly yields to description of the disease and its symptoms.

[188] 96 A foll.

It may be assumed, then, that in the conception of Nature developed by the pre-Socratics all the main senses of the term φύσις were combined; that is to say, Nature meant to them not only that out of which things grew or of which, in the last analysis, they are constituted; this was one of its meanings, but only one, and that not the most important. Certainly it would not be true to say even of the Ionians that they restricted themselves to the question as to the primary substance of the world. Nature (and φύσις) meant more than this: it included the law or process of growth exemplified in all things. Aristotle and Theophrastus suggest that Thales was led to the assumption that water was the primary substance by observations connected with evaporation and precipitation; be that as it may, it is certain that his successor Anaximander was more interested in the cosmic process of segregation than in his colorless Infinite, and thenceforward cosmic processes and laws occupy the attention of philosophers more and more. The main sense of Nature was, however, the sum of things as constituted by the elements and the cosmic laws and processes. This it was, the Natura Rerum, to the understanding of which the philosopher immediately addressed himself; and it was in this sense that the term φύσις occurs in the titular phrase Περὶ φύσεως. Yet, as we have seen, while the inquiry or ἱστορίη περὶ φύσεως concerned the question 'what is it' (ὅ τι ἐστί), the answer at once carried the inquirer to the further questions 'of what is it constituted' and 'how did it come about.' There is nothing startling in this conclusion. It is just what we might have expected, knowing the operations of the human mind. It is, however, not without a certain interest that we thus discover the ideals of present-day science informing and impelling the fathers of all science.

Science, however, merely formulates in the hierarchy of its ideals the interests of the plain man who goes about his daily business with no particular predilection for matters theoretical. The common mind is chiefly concerned with results, neither asking nor greatly caring how they were obtained. As for the underlying causes, material or efficient, which produced the results, they are relatively unimportant, except for the purpose of attaining the same object either actually or by way of ideal construction or verification. Thus every one has heard of the latest invention, say the aeroplane, and accepts it as a fact of interest. Many, though by no means all, know the names of the inventors; the human interest in personalities of distinction contributes not a little to the attitude of mind which fixes attention upon the author. Even smaller is the number of those who know of what materials the machine is constructed. That is a question of importance chiefly to the practical experimenter. Fewest of all are those who concern themselves about the natural laws involved in the attempt to navigate the air, of which the inventor must take advantage in the deft adjustment of his mechanical contrivance to the attainment of his cherished object. Many an experimenter even will be found to be lacking in a knowledge of these principles which absorb the attention of the theorist. The natural philosopher, however, will devote himself to the determination and formulation of the laws involved; from his point of view the inventor is of no consequence, and in his calculations the materials used in the contrivance will figure as a plus or minus quantity.

It remains for us to speak briefly of Professor Burnet's dictum [189] concerning the scope of the early Greek researches Περὶ φύσεως. Since he himself holds that the title is not original and finds it first mentioned in Euripides,[190] it is fair to judge it by the conceptions of the fifth century. But we may reasonably go farther and assert that the usage of the fifth and fourth centuries B.C. merely reflects the ideals of Greek science as they were gradually developed from the beginning. In the *Metaphysics* [191] Aristotle says: ‘ It is owing to their wonder that men both now begin and at first began to philosophize; they wondered originally at the obvious difficulties, then advanced little by little and stated difficulties about the greater matters, e. g. about the phenomena of the moon and those of the sun, and about the stars and about the genesis of the universe.” It is clear that the “obvious difficulties,” which are said to have originally excited the wonder of men, belong rather to the stages of preparation for technical philosophy, and that philosophy proper begins for Aristotle with the investigation of the phenomena of the heavens and of the origin of the universe. According to Plato [192] also it was the observed regularities of heavenly phenomena that begot the research into the nature of the universe. They were the θεῖα *par excellence*,[193] and wonder born of the observation of them was supposed to have produced the belief in the existence of gods.[194] It can hardly be doubted that in the early stages of philosophy the researches of investigators might have been almost indifferently characterized as περὶ μετεώρων or περὶ φύσεως ἱστορίη. Speaking of the distinction and elevation in oratory conferred upon Pericles by his familiarity with the lofty speculations of Anaxagoras, Plato says [195] πᾶσαι ὅσαι μεγάλαι τῶν τεχνῶν προσδέονται ἀδολεσχίας καὶ μετεωρολογίας φύσεως

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[189] Quoted above, p. 80.

[190] See above, n. 7.

[191] *Met.* 982b 12–17, transl. Ross.

[192] *Tim.* 47 A. Cp. *Epin.* 990 A and *Repub.* 530 A–531 A.

[193] Cp. n. 182 above.

[194] By Democritus, cp. Diels, *Vorsokr.* 365, 22 foll.

[195] *Phaedr.* 269 E.

πέρι; and even Aristotle comprehended in the term μετεωρολογία his philosophy of nature as a whole.[196] His *Physics* is rather the metaphysical consideration of the principles involved in the explanation of Nature. In the Hippocratean treatise Περὶ σαρκῶν occurs an instructive passage. "Concerning τὰ μετέωρα," we read,[197] "I do not want to speak except to show, in regard to man and the other animals, how they came about in the course of nature, and what the soul is, what is health and disease, what it is that produces health and disease in man, and from what cause he dies." The author, while professing to speak περὶ τῶν μετεώρων, proceeds to sketch the origin of things, giving in fact a miniature discourse Περὶ φύσεως after the manner of the philosophers, in the course of which he describes the segregation of the cosmic elements and then turns abruptly to tell of the origin of the various parts of the human organism. Each subject is introduced with the laconic but significant phrase, ὧδε ἐγένετο.[198]

We are thus brought face to face with the second sphere of interest included in the researches of early philosophy; for, however much the cosmos engaged the attention of the investigator, the microcosm soon, if not immediately, made good its claims. We have repeatedly remarked upon the intimate connexion of medicine, so far as it concerned physiology, with inquiries περὶ φύσεως. We need not now enlarge upon this theme. It is sufficient to call attention to the fact that it was recognized by Aristotle[199] as well as by the pre-Socratics.

But while the philosopher may have devoted the greater part of his attention to these two fields, nothing lay outside the sphere of his interest. Thus it is not improbable that the study of mathematics was associated with philosophy from the beginning and included in the scope of Περὶ φύσεως ἱστορίη. Aristotle, whose empirical method of determining what does and what does not belong to the subject matter of the several sciences is well known, says in the *Metaphysics*:[200]

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[196] See Gilbert, *Die meteorol. Theorien des griechischen Altertums*, p. 14.

[197] Π. σαρκῶν, 1 (8, 584 Littré) περὶ δὲ τῶν μετεώρων οὐδὲ (read οὐδὲν!) δέομαι λέγειν, ἢν μὴ τοσοῦτον ἐς ἄνθρωπον ἀποδείξω καὶ τὰ ἄλλα ζῷα, ὁκόσα (read ὅκως!) ἔφυ καὶ ἐγένετο, καὶ ὅ τι ψυχή ἐστιν, καὶ ὅτι τὸ ὑγιαίνειν, καὶ ὅτι τὸ κάμνειν, καὶ ὅτι τὸ ἐν ἀνθρώπῳ κακὸν καὶ ἀγαθόν, καὶ ὅθεν ἀποθνήσκει. This little treatise has been unduly neglected and deserves especial attention because of its intimate relation to pre-Socratic philosophy. Its date is hard to determine. Diels, *Elementum*, p. 17, n. 2, would assign it to the first half of the fourth century, B.C.

[198] Compare Arist., *De Partt. Animal.* 641^{a} 7 οὕτως γὰρ καὶ οἱ φυσιολόγοι τὰς γενέσεις καὶ τὰς αἰτίας τοῦ σχήματος λέγουσιν· ὑπὸ τίνων γὰρ ἐδημιουργήθησαν δυνάμεων. *Ibid.* 647^{a} 9 foll.; [Arist.] *Probl.* 892^{a} 23 foll.

[199] Cp. Arist., *De Longev.* 464^{b} 33 ff.; *De Partt. Animal.* 653^{a} 8 foll.; *De Sensu*, 436^{a} 17 foll.; *De Respir.*, 480^{b} 22 foll.

[200] 1005^{a} 19 foll., transl. Ross.

"We must state whether it belongs to one or to different sciences to inquire into the truths which are in mathematics called axioms, and into substance. Evidently the inquiry into these also belongs to one science, and that the science of the philosopher . . . And for this reason no one who is conducting a special inquiry tries to say anything about their truth or falsehood, — neither the geometer nor the arithmetician. *Some natural philosophers* (φυσικοί) *indeed have done so, and their procedure was intelligible enough ; for they thought that they alone were inquiring about the whole of nature and of being*" (περί τε τῆς ὅλης φύσεως καὶ περὶ τοῦ ὄντος). In like manner Plato[201] refers to the philosophers as those "who discourse and write about nature and the universe" (οἱ περὶ φύσεώς τε καὶ τοῦ ὅλου διαλεγόμενοι καὶ γράφοντες). Again[202] he pictures Hippias enthroned in the chair of philosophy at the home of Callias with a crowd of admiring students at his feet, who "appeared to be plying him with certain astronomical questions about nature and the phenomena of the heavens" (ἐφαίνοντο δὲ περὶ φύσεώς τε καὶ τῶν μετεώρων ἀστρονομικὰ ἄττα διερωτᾶν). Here περὶ φύσεως gives the general subject, which includes τὰ μετέωρα, and this in turn comprehends ἀστρονομικὰ ἄττα.[203] We may, therefore, safely say that Περὶ φύσεως was the general title[204] by which the comprehensive philosophical works of the early philosophers were called because they were devoted to the universal *Rerum Natura*.[205] For this reason also Περὶ

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[201] *Lysis*, 214 B.

[202] *Protag.*, 315 C.

[203] This seems also to be the interpretation put upon the passage by Gilbert, *Die meteorol. Theorien des griechischen Altertums*, p. 3, n. 3, although he emphasizes the (undoubted) fact that in many cases περὶ μετεώρων and περὶ φύσεως were used interchangeably.

[204] See Gilbert, *O. c.*, p. 6, n. 1 : "Es haben deshalb Anaximenes und Anaximander, Xenophanes und Parmenides, Empedokles und Anaxagoras jeder in *einem* Werke die Metaphysik, Physik, und Meteorologie gleichmässig behandelt. Auch des Diogenes von Apollonia angeführte Schriften μετεωρολογία und περὶ ἀνθρώπου φύσεως waren wohl nur Teile seines Werkes π. φύσεως. Erst Demokrit, der auch hierin epochemachend erscheint, hat — neben der Darstellung seines Gesamtsystems — in einer Menge von Specialschriften seine Forschungen niedergelegt." Diels, *Vorsokr.* p. 333, is of the same opinion regarding the titles attributed to Diogenes. It was the common tradition in after times that Π. φύσεως was the general title ; cp. D. L. IX. 5 (of Heraclitus) τὸ δὲ φερόμενον αὐτοῦ βιβλίον ἐστὶ μὲν ἀπὸ τοῦ συνέχοντος Περὶ φύσεως, διήρηται δὲ εἰς τρεῖς λόγους, εἴς τε τὸν περὶ τοῦ παντὸς καὶ πολιτικὸν καὶ θεολογικόν. Hippolytus, *Philos.* 2 (Diels, *Dox.* 555, 17) says of Pythagoras : καὶ οὗτος δὲ περὶ φυσικῶν (= περὶ φύσεως) ζητήσας ἔμιξεν ἀστρονομίαν καὶ γεωμετρίαν καὶ μουσικὴν καὶ ἀριθμητικήν. Cp. *ibid.* l. 24 : εἶτα ἐπειδὰν . . . περὶ ἄστρων καὶ φύσεως φιλοσοφήσωσι, κτλ. Philolaus, fr. 6, περὶ φύσειος καὶ ἁρμονίας ὧδε ἔχει. To the Pythagoreans, we are told, ἱστορία meant γεωμετρία ; cp. Nichomachus, apud Iamblichus, *Vita Pythag.* 89.

[205] It is therefore not surprising to find in Plato uses of φύσις corresponding to

φύσεως ἱστορία was set in sharp contrast[206] to the ethical and methodological studies of Socrates which resulted in the logic and metaphysics of Plato and Aristotle.

It is not surprising that science, sprung from the bosom of religion, and fostered by a spirit of reverence for truth in an age when the crumbling ruins of ancient beliefs testified to a loss of respect for the traditional gods, should have become in a measure itself a religion. Attention was called above to the fact that the philosophical system became in time invested with sanctity and was handed down as a *ἱερὸς λόγος*. In the Greek mysteries, even in the fifth century, and possibly in the sixth, *ἐποπτεία*, the final stage of initiation, included a vision of that most divine spectacle, the stellar universe. In Orphic and Pythagorean conventicles there was undoubtedly some consideration of its meaning, though one cannot say how much. Much nonsense is reported of the secrets of the Pythagoreans, but it probably had some basis in fact. The religion of the time tended more and more to become a matter of the individual, though the public forms were observed. Science, competing with religion and in educated circles to a considerable extent supplanting it, naturally appropriated its forms. The "Law" of Hippocrates[207] ends thus: "Things holy are revealed to holy men; to the profane it is forbidden, before they are initiated into the Mysteries of science." We are familiar with the beatitude pronounced by the poets upon those who were initiated in the Mysteries of Eleusis,[208] for they should see the gods and dwell with them, released from the distressing cycle of birth and death. Not unlike it is the inspired utterance of Euripides[209] in praise of the philosopher of nature: "Blessed is he who hath got knowledge of science, bent neither on harm to his neighbors nor on ways of injustice; but, contemplating the ageless order of undying nature, knoweth what it is and how. To such men there never cleaves desire for deeds of shame."

Wesleyan University,
Middletown, Conn., July 10, 1909.

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the Lucretian phrases *in rerum natura* and *in rebus*; thus, *Phaedo* 103 B *οὔτε τὸ ἐν ἡμῖν οὔτε τὸ ἐν τῇ φύσει*, and *Parm.* 132 D *τὰ μὲν εἴδη ταῦτα ὥσπερ παραδείγματα ἑστάναι ἐν τῇ φύσει.*

[206] Arist., *Met.* 987^b 1 foll. Cp. n. 7, above.

[207] Hippocrates, 4, 642 Littré. Cp. also the *Ὅρκος* (4, 628 foll. Littré).

[208] Cp. especially Pindar, fr. 114 (Bergk) *ὄλβιος ὅστις ἰδὼν | κεῖν' εἶσ' ὑπὸ χθόν'· οἶδε μὲν βίου τελευτάν, | οἶδεν δὲ διόσδοτον ἀρχάν.*

[209] Fr. 910. The text is quoted above, n. 185.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF HARVARD COLLEGE.

A REVISION OF THE ATOMIC WEIGHT OF PHOSPHORUS.

FIRST PAPER.—THE ANALYSIS OF SILVER PHOSPHATE.

By Gregory Paul Baxter and Grinnell Jones.

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By Gregory Paul Baxter and Grinnell Jones.

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Although phosphorus is one of the best known and most important elements, present knowledge concerning its atomic weight is somewhat inadequate. The early determinations of this constant by Dulong,[1] Pelouze,[2] Berzelius,[3] and Jacquelain[4] are widely discrepant and have no particular significance. Those by Schrötter, Dumas, van der Platts, and Berthelot, on the other hand, all give values not far from 31.0, and this value has been selected by the International Committee on Atomic Weights. Although these investigations have already been critically discussed by Clarke,[5] Brauner,[6] and others, a few of the more important sources of error are briefly pointed out here.

Schrötter,[7] the discoverer of red phosphorus, converted weighed quantities of this substance into phosphorus pentoxide by combustion in a stream of oxygen. As the mean of ten determinations which varied from 30.94 to 31.06, he obtained 31.03 for the atomic weight of phosphorus. The oxygen used was slightly moist, as Brauner has pointed out, since, although it was dried by phosphorus pentoxide, it was finally passed through a tube containing calcium chloride! The phosphorus pentoxide formed during the combustion must have retained this small amount of water, which would make the atomic weight of phosphorus appear too low. Schrötter admits that the combustion was incomplete, and since this error would tend to raise the atomic weight of phosphorus, he concludes that the true value is 31.00.

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[1] Ann. Chim. Phys. 1816, **2**, 149.

[2] C. R., 1845, **20**, 1053.

[3] Lehrbuch, 5th Ed., 1845, **3**, 1188.

[4] C. R., 1851, **33**, 693.

[5] A Recalculation of the Atomic Weights, Smith. Misc. Coll., 1897.

[6] Abegg, Handb. der anorg. Chem., 1907, vol. 3, part 3, p. 366.

[7] Ann. Chim. Phys., (3), 1853, **38**, 131.

Dumas[8] titrated the trichloride of phosphorus against silver after decomposing the trichloride with water. Since the sample used did not boil at constant temperature, but distilled between 76° and 78°, it must have been impure. If it contained oxychloride, as Clarke has suggested, the atomic weight of phosphorus would be found too high. Dumas overlooked the solubility of silver chloride and therefore used the wrong end-point in these titrations. Furthermore no precautions are mentioned either for preventing access of water to the material before weighing or for preventing the reduction of the silver salt by the phosphorous acid formed in the decomposition of the trichloride with water. Recalculated on the basis of the atomic weight of silver as 107.88, his five analyses give results which vary between 30.99 and 31.08. The average is 31.03.

Van der Platts[9] made two determinations by each of three different methods. He obtained the values 30.90 and 30.97 by the precipitation of silver from silver sulphate solution with phosphorus. His results from the analysis of silver phosphate were 31.08 and 30.95. He gives no details of the method of preparing and analyzing this substance, merely making the statement, "It is difficult to be sure of the purity of this salt." Finally, by the combustion of yellow phosphorus in oxygen he obtained the results 30.99 and 30.96. The very meagre descriptions of these experiments preclude criticism.

Using Leduc's data for the densities and compressibilities of phosphine and oxygen, Daniel Berthelot[10] has calculated, by the method of limiting densities, the molecular weight of phosphine to be 34.00 and the atomic weight of phosphorus to be 30.98.

Very recently Gazarian[11] has obtained a considerably lower value for the molecular weight of phosphine, 33.93. This value was calculated from the experimentally determined weight of the standard liter by the four methods of molecular volumes (Leduc), limiting densities (Berthelot), critical constants (Guye), and "indirect" limiting densities (Berthelot). The different methods give essentially identical results, except in the case of the direct method of limiting densities. By the latter method a value six-hundredths of a unit higher is obtained, but Gazarian rejects the result on the basis of inaccurate knowledge of the compressibility of phosphine. It is highly desirable to obtain more certain knowledge of the compressibility of phosphine, since the

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[8] Ann. Chem. Pharm., 1860, **113**, 28.
[9] C. R., 1885, **100**, 52.
[10] C. R., 1898, **126**, 1415.
[11] Jour. de Chim. Phys., 1909, **7**, 337.

method of limiting densities is the most reliable of all the methods for applying the correction to the densities made necessary by deviations from the laws of a perfect gas.

The other methods are burdened with arbitrary assumptions and empirical constants, and furthermore Baume[12] has shown that both the method of molecular volumes and the method of critical constants give correct results only with gases for which the ratio $\frac{T_c}{4\,P_c}$ is nearly 1, whereas for phosphine this ratio is 1.26.

If the molecular weight of phosphine be assumed to be 33.93, the atomic weight of phosphorus is 30.91. In the light of this low result it is unfortunate that Gazarian prepared phosphine by only one method, and that he did not determine the purity of the gas, i. e. by absorption. Gazarian used the method of Matignon and Trannoy[13] which consists in heating calcium phosphate and aluminum together until they react, and then treating the product of this reaction without further purification with water in a gas generator. Matignon and Trannoy show that the gas prepared in this way by them contained about three per cent of hydrogen, probably derived from calcium contained by the phosphide. In this case some calcium nitride would be formed, since the phosphide was made in air; and this would produce ammonia as an impurity in the phosphine. Although the gas was purified by fractional distillation, according to Gazarian's statements hydrogen is difficult to eliminate, and a proportion of only four-tenths of one per cent would be sufficient to lower the atomic weight of phosphorus one-tenth of a unit. Ammonia would be even more difficult to remove, since its boiling point is only 50° higher than that of phosphine. The effect of a given percentage of impurity is, however, much less with ammonia than with hydrogen, although in the same direction.

From the preceding brief summary it is evident that the uncertainty in the atomic weight of phosphorus is as great as one tenth of a unit, and that, as Brauner remarks at the conclusion of his review of the subject, "a revision of the atomic weight of phosphorus with modern means is urgently necessary."

The analysis of silver phosphate was selected as one of the most promising methods of attacking the problem, since the percent of silver can be determined exactly by a method which has been carefully studied, especially in this laboratory. The accuracy of the result will therefore depend primarily upon the success attained in preparing

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[12] Baume, J. Chim. Phys. 1908, **6**, 76 and 86.

[13] C. R., 1909, **148**, 167.

silver phosphate in a perfectly definite and pure state. The greater part of the following research was devoted to the solution of this problem which van der Platts found so difficult.

The analysis of the halogen compounds of phosphorus offers certain difficulties owing to the ease with which these substances are decomposed by water, and to the necessity for oxydizing the phosphorous acid resulting from the decomposition of the halogen compounds with water before the addition of silver nitrate. An investigation upon the tribromide of phosphorus is now in progress in this laboratory. Phosphonium compounds were found utterly unsuited for exact analysis on account of their instability.

Purification of Materials.

Water. All the water used in this research was made from the laboratory supply of distilled water by distillation, first from an alkaline permanganate solution, and then, after the addition of a trace of sulphuric acid, through a block tin condenser.

Ammonia. The best commercial ammonia was distilled into the purest water.

Nitric Acid. The best commercial concentrated acid was twice fractionally distilled through a platinum condenser, with the rejection of the first third of the distillate. Every sample was shown to be free from chloride by careful nephelometric tests.

Hydrochloric Acid. The best commercial C. P. acid, diluted with an equal volume of water, was distilled through a platinum condenser.

Hydrobromic Acid. This substance was prepared in conjunction with Mr. F. B. Coffin, who was engaged in a parallel research upon the atomic weight of arsenic.[14] Commercial bromine was converted into potassium bromide by addition to recrystallized potassium oxalate. In a concentrated solution of this bromide, in a distilling flask cooled with ice, bromine was dissolved, and distilled from the solution into a flask cooled with ice. A portion of the purified bromine was then converted into potassium bromide with pure potassium oxalate as before, and the remainder of the bromine was distilled from solution in this pure potassium bromide. The product obtained was thus twice distilled from a bromide, the bromide in the second distillation being essentially free from chlorine. This treatment has already been proved sufficient to free bromine from chlorine.[15]

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[14] Baxter and Coffin, These Proceedings, 1909, **44**, 179.

[15] Baxter, These Proceedings, 1906, **42**, 201.

Hydrobromic acid was synthesized from the pure bromine by bubbling hydrogen gas (made by the action of water on "hydrone") through the bromine warmed to 40°–44° and passing the mixed gases over hot platinized asbestos in a glass tube. The apparatus was constructed wholly of glass. The hydrogen was cleansed by being passed through two wash bottles containing dilute sulphuric acid, and through a tower filled with beads also moistened with dilute sulphuric acid. The hydrobromic acid gas was absorbed in pure water contained in a cooled flask. In order to remove iodine the solution of hydrobromic acid was diluted with water and twice boiled with a small quantity of free bromine. Then a small quantity of recrystallized potassium permanganate was added to the hydrobromic acid solution, and the bromine set free was expelled by boiling. Finally the acid was distilled with the use of a quartz condenser, the first third being rejected. It was preserved in a bottle of Nonsol glass provided with a ground-glass stopper.

The purity of the hydrobromic acid was tested by a quantitative synthesis of silver bromide. The silver used, which was kindly furnished by Mr. G. S. Tilley, had been prepared with all the necessary precautions for work on the atomic weights of silver and iodine.[16] The procedure used by Baxter[17] for the synthesis of silver bromide from a weighed amount of silver was followed in detail. In this experiment 6.02386 grams of silver yielded 10.48627 grams of silver bromide; hence, silver bromide contains 57.4452 per cent of silver, while Baxter found as the mean of 18 determinations 57.4453 per cent. The hydrobromic acid was evidently pure.

Silver Nitrate. Crude silver nitrate was reduced with ammonium formate, made by passing ammonia gas into redistilled formic acid. The reduced silver was washed with the purest water, until the wash waters no longer gave a test for ammonia with Nessler's reagent, and was fused on sugar charcoal. The buttons were then scrubbed with sea-sand and thoroughly cleansed with ammonia and nitric acid. They were then dissolved in redistilled nitric acid, in a platinum dish. After the silver nitrate solution had been evaporated on a steam bath until saturated, an equal volume of redistilled nitric acid was added and the solution was cooled. The precipitated silver nitrate was very completely drained in a centrifugal machine, provided with platinum Gooch crucibles to retain the salt.[18] A similar recrystallization fol-

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[16] Baxter and Tilley, Jour. Amer. Chem. Soc., 1909, **31**, 201.

[17] Baxter, These Proceedings, 1906, **42**, 208.

[18] Baxter, Jour. Amer. Chem. Soc., 1908, **30**, 286.

lowed. The final product was preserved in Jena glass vessels under a bell-jar.

Disodium Phosphate. One kilogram of Merck's best disodium phosphate was dissolved in hot water in a porcelain dish and hydrogen sulphide passed into the solution for several hours. After standing for twenty-four hours, the solution was again heated, saturated with hydrogen sulphide and filtered. The filtrate was slightly green, owing to the presence of iron. The solution was boiled to expel the hydrogen sulphide and a small amount of green precipitate filtered out. The filtrate was still distinctly green. The sodium phosphate was then crystallized fifteen times, five times in porcelain with centrifugal drainage of the crystals in a large porcelain centrifugal machine, ten times in platinum vessels with centrifugal drainage of the crystals in platinum Gooch crucibles. The green color concentrated in the first mother liquor.

When tested by means of the Marsh test, this material was found to contain only a mere trace of arsenic, which was estimated to be 0.01 mg. in ten grams of the salt. This small amount could have no effect on the analytical results, especially since the percentage of silver in silver arsenate is nearly the same as in silver phosphate. By means of the nephelometer it was proved that this material contained no chloride or other substances which could be precipitated by silver nitrate in the presence of dilute nitric acid.

Sodium Ammonium Hydrogen Phosphate. The best commercial microcosmic salt was recrystallized four times in platinum vessels. It was tested for arsenic by Marsh's method with negative results and gave no opalescence visible in the nephelometer when tested with silver nitrate and dilute nitric acid.

Preparation of Trisilver Phosphate.

Silver phosphate was prepared by mixing dilute solutions of silver nitrate with solutions of sodium and ammonium phosphates. Since it is not feasible to purify silver phosphate by recrystallization, the conditions of precipitation must be so chosen that a pure product will be obtained at once.

In order to avoid inclusion and occlusion of silver nitrate, sodium nitrate, sodium phosphate, or mono- or disilver phosphate, all of the solutions for precipitation were made about 0.03 N. All samples after precipitation were thoroughly washed and allowed to stand in water for at least twenty-four hours, in order to convert occluded acid phosphates into trisilver phosphate. Qualitative tests for nitrate with diphenylamine and for sodium by the spectroscope showed that all of the first three substances named could be completely washed out.

Joly[19] states that disilver phosphate is stable in the presence of phosphoric acid containing 40 per cent (11.8 N) of phosphoric anhydride, but is transformed into trisilver phosphate if the acid contains 38 per cent (11.0 N) or less of phosphoric anhydride. Since all the solutions used for the preparation of silver phosphate were nearly neutral, it is evident that the precipitation of disilver phosphate as a distinct phase in equilibrium with the solution is not to be feared.

It is, however, not such a simple matter to prove the absence of *occluded* disilver hydrogen phosphate or monosilver hydrogen phosphate. Much light is thrown on this point in a recent paper by Abbott and Bray[20] upon the dissociation constants of the three hydrogens of phosphoric acid, which were found to be 1.1×10^{-2}, 1.95×10^{-7} and 3.6×10^{-13} respectively. Since the phosphate ion ($PO_4^{\equiv}$) is almost completely hydrolyzed to the monohydrophosphate ion ($HPO_4^{=}$), even in slightly alkaline solutions, and since in slightly acid solutions the dihydrophosphate ion ($H_2PO_4^-$) acquires an appreciable concentration, the possibility of occlusion must be examined with especial care.

The concentrations in the following table are either taken directly from a table given by Abbott and Bray or calculated from these numbers with the help of the values of the dissociation constants of phosphoric acid. The values are expressed in formular weights per liter, the total concentration of the salt being in each case 0.05.

	$NaNH_4HPO_4$	$Na_2NH_4PO_4$
$H_2PO_4^-$	0.001184[21]	0.000002[22]
$HPO_4^{=}$	0.03265[21]	0.03219[21]
$PO_4^{\equiv}$	0.0000016[22]	0.001123[21]
OH^-	0.00000079[21]	0.000502[21]
H^+	0.0000000075[22]	0.000000000012[22]

It will be noted that the replacement of the remaining hydrogen in sodium ammonium hydrogen phosphate by sodium decreases the concen-

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[19] C. R., 1886, **103**, 1071.

[20] Jour. Amer. Chem. Soc., 1909, **31**, 755.

[21] These values are taken directly from the table of Abbott and Bray.

[22] These values are calculated from the others in the above table by the aid of the following equations:

$$(H^+)(OH^-) = 0.59 \times 10^{-14}$$

$$\frac{(H^+)(PO_4^{\equiv})}{(HPO_4^{=})} = 3{\cdot}6 \times 10^{-13} \qquad \frac{(H^+)(HPO_4^{=})}{(H_2PO_4^-)} = 1.95 \times 10^{-7}$$

tration of the hydrogen ion to 0.16 percent of its value in the microcosmic salt solution and decreases the concentration of the dihydrophosphate ion to 0.2 percent of its former value. The concentration of the monohydrophosphate ion remains essentially unchanged, while the concentration of the phosphate ion is increased seven hundred times. Disodium phosphate doubtless takes a position intermediate between the other two solutions in this regard, since it is more alkaline than microcosmic salt and less so than disodium ammonium phosphate. The numbers given above refer to solutions which are five times as strong as those used in this research, but the conditions in the more dilute solutions must be very similar. Furthermore, the exact values have no great importance, as the concentrations of the various ions change continuously during precipitation. It is evident from the figures given above and from the value of the dissociation constant of the second hydrogen of phosphoric acid that if the concentration of hydrogen ion increases above its value in a microcosmic salt solution, the concentration of the dihydrophosphate ion must increase greatly at the expense of the monohydrophosphate ion. If there is any tendency for the occlusion of disilver hydrogen phosphate or monosilver hydrogen phosphate, the amounts of these salts occluded would be expected to depend on the concentration of the undissociated molecules of these salts in the solution, and therefore on the concentration of the silver ion and on the concentration of the monohydrophosphate or dihydrophosphate ion respectively.

The exact concentrations of the ions during the precipitation cannot be calculated, since the solubility of silver phosphate in slightly acid solutions and the solubility-product of silver phosphate are not known. It is, however, easy to understand from a study of the conditions under which the various samples of silver phosphate were precipitated, that these concentrations must have varied greatly in the preparation of the different samples and therefore constancy of composition gives a strong presumption that there is very little or no tendency for the occlusion of the undesired acid salts.

Samples N and O. A 0.03 normal solution of silver nitrate was slowly poured into a 0.03 normal solution of disodium hydrogen phosphate with frequent shaking. This reaction may be roughly considered to take place in two stages represented by the equations

$$3\,AgNO_3 + 2\,Na_2HPO_4 = Ag_3PO_4 + NaH_2PO_4 + 3\,NaNO_3$$
$$3\,AgNO_3 + NaH_2PO_4 = Ag_3PO_4 + NaNO_3 + HNO_3$$

At the beginning of the precipitation the solution is very slightly alkaline and remains very nearly neutral during the addition of the first half of the silver nitrate. The concentration of the silver ion is kept very low by the excess of phosphate and, therefore, little occlusion of the acid salts is to be expected in spite of the fact that the solution contains appreciable concentrations of the monohydrophosphate and dihydrophosphate ions. The precipitate during this stage is very finely divided and does not settle well and, therefore, no attempt was made to collect it separately.

During the addition of the second half of the silver nitrate the solution becomes slightly acid and the solubility of the silver phosphate increases rapidly. The precipitate settles readily. During the second stage the conditions are more favorable for the occlusion of the acid phosphate, but only a small amount of silver phosphate is precipitated during this stage.

After standing a short time the mother liquor was decanted from the precipitate, and exactly the calculated amount of redistilled ammonia, diluted to one liter, was added to neutralize the excess of acid and complete the precipitation. Since this sample was evidently produced from a solution which was slightly acid at the beginning of the precipitation, although very nearly neutral at the end, and since it contained a considerable amount of silver, the conditions were favorable for the formation of acid salts.

Both precipitates were transferred to a large platinum dish and washed many times by decantation with the purest water. This washing was prolonged over more than twenty-four hours in order to give time for all soluble matter to be leached out. When the precipitates were tested for nitrate with diphenylamine, negative results were obtained. Sodium was found to be absent by spectroscopic tests. The precipitates were drained as far as possible in a platinum centrifugal machine, and the drying was completed by heating in platinum crucibles in an electric air bath for several hours, first at 90° and finally at about 130°. The dried lumps of silver phosphate were then gently ground in an agate mortar. The samples were preserved in platinum crucibles over sulphuric acid in the dark. All of the operations were performed in a dark room.

The sample prepared by pouring silver nitrate into disodium phosphate is designated Sample N, and the sample prepared by adding ammonia to the mother liquors is designated Sample O.

Sample P. A 0.03 normal solution of disodium ammonium phosphate was prepared by dissolving a weighed amount of disodium hydrogen phosphate and then adding the calculated amount of redistilled ammonia. The solution was then slowly poured into a 0.03 normal solution of silver nitrate. By this method of precipitation the solution is maintained as nearly neutral as is possible, because the excess of silver prevents the concentration of phosphate in solution from exceeding a very small value, so that neither can the solution become alkaline by hydrolysis nor can the concentration of hydrophosphate attain an appreciable value. The absence of the hydrophosphate ions would be expected to prevent the formation and occlusion of acid silver phosphate in this sample, whereas in Sample N the same result is probably brought about by the absence of the silver ion. Unfortunately both of these favorable conditions cannot be combined in one precipitation, as will be shown later. This precipitate settled readily. The washing, testing, and drying were carried out as already described for Samples N and O. This sample is designated Sample P.

Sample R. A 0.03 normal solution of sodium ammonium hydrogen phosphate was slowly poured into a similar solution of an equivalent amount of silver nitrate. Under these conditions the solution contains an excess of silver, which tends to produce occlusion of acid phosphates, since the solution becomes more and more acid as the precipitation proceeds, and as the precipitation is therefore far from complete, the concentrations of the two hydrophosphate ions gradually approach a very considerable value. At no stage could the solution become alkaline by hydrolysis. It should be noticed that the procedure differs from that used in preparing Sample N in that the precipitate is formed in the presence of an excess of silver nitrate instead of an excess of phosphate, and that this difference in the method of mixing greatly changes the conditions of precipitation.

The precipitate, which was designated Sample R, coagulated and settled quite readily. The washing and drying were completed as usual.

It will be shown that samples of silver phosphate prepared under these various conditions have nearly, if not exactly, the same composition. Further proof of the absence of acid phosphate in these samples is given by experiments to be described later which show that no water is given off when this material is fused.

An attempt to prepare a sample by pouring silver nitrate into disodium ammonium phosphate yielded unsatisfactory results. Since the disodium ammonium phosphate solution was alkaline, owing to hydrolysis, it contained free ammonia, which prevented the precipitation of silver phosphate at first. Nearly one-quarter of the silver nitrate was added before a permanent precipitate was produced. At the end of the precipitation the solution was of course essentially neutral. Even after standing for four days the precipitate had not appreciably settled. Since the coagulation of the precipitate seems to occur much more readily in the presence of excess of silver, a considerable amount of silver nitrate in solution was added. The precipitate coagulated and settled immediately. It was washed and dried as usual. This sample was somewhat darker in color than the other samples and gave a large amount of insoluble residue when treated with dilute nitric acid. The analysis showed that it contained about two hundredths per cent too much silver. This method of preparation is evidently unsatisfactory.

Three unsuccessful attempts were made to prepare silver phosphate from trisodium phosphate. The samples obtained in this way did not appear homogeneous after being dried and contained considerable sodium in spite of protracted washing. Two of these samples were found by analysis to contain, respectively, 4.4 and 4.1 per cent less silver than pure trisilver phosphate. The third of these samples was so unsatisfactory in appearance and in its behavior during its preparation that it was not analyzed. This method of preparing silver phosphate is evidently not suitable for our purpose. Time was lacking to investigate further this anomalous behavior.

Method of Analysis.

Unfortunately, owing to the high melting point of silver phosphate, it was not feasible to fuse the silver phosphate before its analysis in order completely to eliminate all water. Instead it was heated in a platinum boat, in a current of pure dry air, at a temperature of about 400° for seven hours, and then by means of bottling apparatus[23] it was inclosed in its weighing bottle without coming in contact with the moist air of the laboratory. During this heating the access of light to the sample was prevented. The continuous current of air which passed over the silver phosphate during the heating was driven by a water pump successively through an Emmerling tower containing beads moistened with silver nitrate solution, through a tower containing small pieces of fused caustic potash, then through three towers containing beads drenched with concentrated sulphuric acid, and finally through a long tube containing phosphorus pentoxide which had been resublimed in a current of air. The hard glass tube containing the platinum boat was surrounded by blocks of aluminum[24] which were jacketed with asbestos on the top and sides and heated directly from

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[23] Richards and Parker, These Proceedings, 1896, **32**, 59.

[24] Baxter and Coffin, These Proceedings, 1909, **44**, 184.

below by a large burner. The platinum boat was not attacked in the least, as was shown by the fact that its weight remained constant.

It was feared that in spite of this prolonged heating the silver phosphate still retained a trace of water, but by making the conditions in the different experiments as nearly uniform as possible it was hoped that the amount of water retained would be constant. Proof will be given later that the drying was highly efficient.

The salt thus prepared for analysis was allowed to stand over night in a desiccator covered with a black cloth in the balance room, and was then weighed in its glass-stoppered bottle by substitution, with the use of another weighing bottle of very similar surface and volume as a counterpoise.

The balance was a nearly new No. 10 Troemner balance. It was easily sensitive to 0.02 mg. The weights had already been used in an investigation of the atomic weight of sulphur,[25] and were restandardized with a very gratifying result. None of the corrections found differed by as much as 0.02 mg. from those found a year before, and only a few by 0.01 mg. The balance was provided with a few milligrams of radium bromide of radioactivity 10000 to dispel electrical charges generated during the handling of the weighing bottles with cork-tipped pincers.

The platinum boat containing the silver phosphate was transferred to an Erlenmeyer flask of "non-sol" glass of one liter capacity and treated with about 30 cubic centimeters of 5 normal nitric acid. Solution took place rapidly. The solution was not perfectly clear, however, owing to a very slight insoluble residue which sometimes settled out on standing. The solution was then heated on a steam bath until the residue dissolved completely. Upon the addition of about one liter of cold water a very slight opalescence was produced, which was visible only when the solution was carefully examined in a very favorable light. The solution was again warmed until it became perfectly clear. The water and nitric acid used in these processes did not give an opalescence visible in the nephelometer when treated with silver nitrate. The nature of this residue will be discussed more in detail after describing the remainder of the analytical process.

About eight hundred cubic centimeters of water was placed in a large glass-stoppered precipitating flask and a very slight excess of hydrobromic acid was added from a burette. The silver phosphate solution was then very carefully poured into the hydrobromic acid solution. This method of precipitation gives less opportunity for the occlusion

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[25] Richards and Jones, Pub. Car. Inst., 1907, No. **69**, 69.

of silver phosphate or nitrate than the reverse method. The occlusion of hydrobromic acid can do no harm. The flask was shaken for twenty minutes and was allowed to stand for several days until the precipitate had completely settled. Then the precipitate was collected upon a weighed Gooch crucible after many rinsings with pure water. In order to protect the mat of the Gooch crucible from disintegration, it was covered by a circular disk of thin platinum foil, perforated with many small holes. The precipitate was dried in an electrically heated air bath for several hours at 90°, then for some time at 130°, and finally for at least eight hours at 180°. After the crucible containing the precipitate had been weighed, the silver bromide was transferred to a porcelain crucible and the loss on fusion determined. The presence of the platinum disk covering the mat makes it possible to transfer very nearly all the silver bromide to the porcelain crucible without contamination with asbestos and therefore it is unnecessary to correct the loss on fusion for the small amount of silver bromide which is not fused. The loss on fusion, which represents water remaining in the silver bromide, was subtracted from the weight of the silver bromide. The asbestos shreds carried away by the wash waters and any silver bromide which may have escaped the Gooch crucible were collected by passing the filtrate through a very small filter paper. The paper was then burned and the residue, after treatment with a drop of nitric and hydrobromic acids to convert any reduced silver into silver bromide, was again gently heated and finally was weighed. The weight of the asbestos, corrected for the ash of the paper, was added to the weight of the silver bromide. In order to determine the soluble silver bromide, the filtrate was evaporated until most of the excess of nitric acid was driven off. The precipitating flask and all the flasks which had held the filtrate were rinsed with strong ammonia and the rinsings added to the evaporated wash water. Enough ammonia was added to make the solution alkaline and it was then diluted to one hundred cubic centimeters in a graduated flask. The amount of silver bromide present was determined by comparison in the nephelometer with a very similar solution containing a known amount of silver bromide. Both precipitates were dissolved in ammonia and reprecipitated at the same time and under precisely similar conditions[26] in the nephelometer tubes by a slight excess of nitric acid. The amount found in this way was added to the weight of the silver bromide.

In order to determine whether silver phosphate is occluded by silver

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[26] See Richards and Staehler, Pub. Carnegie Institute, No. 76, p. 20.

chloride, about six grams of silver phosphate were dissolved in nitric acid and the solution was diluted and poured into an excess of hydrochloric acid. After standing until the supernatant liquid was clear, the precipitate was washed very thoroughly with water and then dissolved in redistilled ammonia. The solution was diluted to one liter and the silver chloride was reprecipitated with nitric acid. The precipitate was filtered out and the filtrate evaporated in a platinum dish until concentrated. A little sodium carbonate was added and the dish was heated to expel all volatile ammonium salts. The residue was dissolved in about three cubic centimeters of water and treated with an excess of ammonium molybdate reagent with gentle warming. After standing for three days, not the slightest precipitate or yellow color had appeared, showing that no phosphate had been occluded by the silver chloride. Although not tested experimentally, it is reasonable to suppose that silver bromide also does not possess the property of occluding appreciable quantities of silver phosphate or phosphoric acid.

Insoluble Residue.

The presence of a slight residue or opalescence, after dissolving the dried silver phosphate in dilute nitric acid, proved the most perplexing difficulty which was encountered. The effort to discover the nature of this insoluble matter and eliminate it consumed a large part of the time devoted to this research. In an effort to make sure that it was not due to some unknown impurity, nineteen different samples of silver phosphate were prepared, the source of material, method of purification, and precipitation being varied. Disodium phosphate, trisodium phosphate, and sodium ammonium phosphate were carefully purified and converted into silver phosphate under varying conditions without appreciable effect upon the amount of the residue. Phosphorus oxychloride was twice fractionally distilled, converted into phosphoric acid, and then into disodium phosphate by means of sodium hydroxide made from sodium amalgam. The product was crystallized three times. Silver phosphate made from this material gave a slight residue, very similar to that obtained from the best samples made in other ways. Unfortunately, it was necessary to reject the analytical results obtained with this specimen because it was found to contain a small amount of metaphosphate. We did not succeed in preparing a sample of silver phosphate entirely free from the residue.

In the meantime attention had been devoted to the residue itself. The small amount of material available rendered this part of the investigation difficult. The silver phosphate, after its precipitation and washing, but undried, dissolves in dilute nitric acid, giving a solution which is perfectly clear to the naked eye, although some samples gave a barely visible opalescence in the nephelometer. The opalescence was much too small to have any effect on the analytical results. The dried samples invariably gave an opalescence.

Dry silver phosphate is very slowly darkened in color by the action of light. This effect is even more pronounced when silver phosphate is exposed to the light in the presence of water. These darkened samples gave a much greater residue than the undarkened material. The residue was insoluble in ammonia, slowly soluble in dilute nitric acid, especially when heated, and readily soluble in strong nitric acid. The addition of hydrochloric acid to these nitric acid solutions gave a precipitate of silver chloride, while ammonium molybdate indicated the presence of phosphate.

In order to determine whether or not a loss of weight occurs during the darkening by light, a sample of silver phosphate was dried and weighed as usual and found to weigh 3.01901 grams. It was then exposed to the direct action of bright sunlight for a day, while contained in a weighing bottle which was placed in a desiccator over sulphuric acid. It was found to have darkened slightly in color and to weigh 3.01903. The gain of 0.02 milligram is within the limit of error in the weighing. This sample, when treated with dilute nitric acid, gave a much larger residue than usual, which weighed 1.8 milligrams. This is much more residue than was usually found in samples containing from four to eight grams of silver phosphate. It is estimated that the samples which had been protected from the action of light as much as possible, except when unavoidably exposed to diffused daylight while being weighed or transferred to the furnace and solution flask, contained about one one-hundredth of a per cent of this residue.

Two analyses were made of the residue obtained by exposing silver phosphate *under water* to the action of light for several days, then dissolving the excess of silver phosphate in dilute nitric acid and thoroughly washing and drying the residue. 0.02674 gram of this residue yielded 0.03551 gram of silver chloride, which indicates that the residue contained 99.9 per cent of silver. In the case of another sample of the residue prepared and analyzed in the same way, 0.04320 gram of residue yielded 0.05747 gram of silver chloride, which indicates that the residue contained 100.1 per cent of silver. The mean of the two analyses is 100.0 per cent of silver. These analyses prove conclusively that when silver phosphate is acted on by light in the presence of water, it is so altered (perhaps by the formation of a subphosphate similar to subchloride), that when treated with very dilute nitric acid metallic silver remains.

It does not follow, however, that it would be a correct procedure to determine the per cent of this residue obtained from the samples used for analysis and apply a correction on the assumption that the material consisted of pure silver phosphate and a small amount of pure silver. This procedure would assume that the other product of decomposition is eliminated and not weighed. There are two facts which show that this assumption would be incorrect. In nearly every analysis, when the solution was diluted, after bringing the residue into solution by heating on the steam bath, a slight opalescence was produced. Careful tests of the water used showed that this opalescence was not due to impurity in the water. It seems probable that the substance which caused this opalescence was derived in part from the phosphate radical during the decomposition which produced the residue. The other fact is that dry silver phosphate does not lose weight when darkened by exposure to sunlight, although this treatment increases the amount of residue. The conclusion in regard to this residue may be summarized as follows : The washed moist silver phosphate was free from residue and contained silver and phosphoric acid combined in atomic proportions. During the drying and weighing a slight decomposition took place, undoubtedly owing in part at least to the action of light. It seems probable that during this decomposition no loss in weight took place, and therefore the sample contained the proper percentage of silver. When this slightly darkened silver phosphate is treated with cold dilute nitric acid, the unchanged silver phosphate and perhaps also a portion of the altered material dissolve, leaving a slight opalescence, which in some cases is deposited as a very slight residue on standing. This residue is estimated to be about 0.01 per cent of the weight of the silver phosphate. When the solution is warmed until perfectly clear, and then diluted, a very slight opalescence is usually produced which could be again cleared up by warming the solution. This opalescence is probably caused by the presence of the altered phosphate anion. If this explanation is correct, the presence of the residue cannot influence the result, and no correction need be applied. Until the exact nature of the decomposition products can be determined, there must remain some uncertainty in regard to whether or not any correction is necessary.

The uncertainty from this cause is, however, not very great. Even if all the phosphorus and oxygen corresponding to the residue of silver is removed before the weighing, the correction would be only twenty-three per cent of the weight of the residue. If the residue amounts to

0.01 per cent, as has been estimated, the maximum correction would be 0.002 per cent. If part of the oxygen is lost, but the phosphorus remains, the correction would of course be smaller. If there is no loss in weight by the action of light on the dry silver phosphate, no correction need be applied. From the evidence so far obtained the latter assumption seems rather more probable than any of the others, and therefore no correction has been applied.

THE DETERMINATION OF WATER IN THE DRIED SILVER PHOSPHATE.

In order to find out how efficient the drying of the silver phosphate had been, experiments were made to determine the amount of water retained by silver phosphate which had been dried for analysis as described above. (See page 147.) The water was determined by fusing the dried phosphate in a current of dry air and collecting the moisture set free in a weighed phosphorus pentoxide tube. Since the melting point of pure silver phosphate is considerably above the softening point of hard glass, it was found advantageous to lower the melting point of the phosphate by the use of silver chloride as a flux.

About fifteen grams of silver phosphate were placed in one end of a large silver boat and in the other end about twelve grams of previously fused silver chloride. The boat was then inserted in a hard glass tube and dried under the same conditions as prevailed in preparing the samples for the determination of the silver content. After the silver phosphate had been heated for seven hours in a current of purified air dried by phosphorus pentoxide, the air passing over the boat in the furnace was conducted through a weighed U-tube containing resublimed phosphorus pentoxide for one half hour. This was done to make sure that all the water which had been liberated from the silver phosphate without fusion had been swept out of the apparatus. In no case was there a gain in weight during this process of more than 0.05 mg., which is about the limit of error in weighing the phosphorus pentoxide tubes. The backward diffusion of moisture was prevented by a second tube containing pentoxide.

The carefully weighed phosphorus pentoxide tube was again attached to the tube containing the silver boat with its charge of silver phosphate and silver chloride. The latter tube was then heated hot enough to fuse the silver chloride, which flowed down to the silver phosphate and readily caused the entire charge to fuse completely. The liberated water was swept into the phosphorus pentoxide tube by a current of dry air for about thirty minutes. The tube was then reweighed to determine the water evolved by the fusion of silver phosphate. The pentoxide tube was weighed by substitution for a very similar counterpoise tube, one stop-cock of each tube being open during the weighing. Before being weighed both tubes were wiped with a damp cloth and allowed to stand near the balance for at least thirty minutes.

The following table gives the results of these experiments :

Sample.	Weight of Silver Phosphate.	Weight of Water.	Per Cent of Water.
P	13.50	0.00012	0.0009
P	15.64	0.00007	0.0004
O	15.66	0.00005	0.0003
O	16.62	0.00003	0.0002
		Average	0.0005

The amount of water evolved is hardly greater than the probable error in weighing the phosphorus pentoxide tubes, and is less than the probable error in determining the amount of silver in the salt. We are therefore justified in concluding that the material which was used for the determination of silver was essentially free from water and that no correction need be applied to the results for inefficient drying.

This result also furnishes evidence that the samples are free from acid phosphates, which, owing to conversion into pyro- or metaphosphate, would evolve water when fused, although it is possible that occluded acid phosphates might have been converted into pyro- or metaphosphates during the drying. Sample O, which was prepared under conditions most favorable for the formation of the acid silver phosphate, does not appear to contain more water than Sample P, which was prepared under conditions which were unfavorable to the formation of acid phosphate. Since these two samples, which differed most widely in their method of preparation, showed no difference in the amount of water retained, it seemed unnecessary to test the other samples also. Unfortunately this method of detecting acid phosphate is not very sensitive, owing to the unfavorable relation of the atomic weights involved, — one molecule of water corresponding to a deficiency of two atoms of silver.

THE SPECIFIC GRAVITY OF SILVER PHOSPHATE.

In order that the apparent weight of the silver phosphate might be corrected to the vacuum standard, the specific gravity of this salt was found by determining the weight of toluol displaced by a known quantity of salt. The specific gravity of the toluol at 25° referred to water at 4° was 0.8633. Great care was taken to remove air from the salt when covered with the toluol by warming the pycnometer, then placing it in a vacuum desiccator and boiling the toluol under reduced pressure. The salt and toluol were mechanically stirred to assist the escape of air bubbles. This process was repeated several times.

Weight of Silver Phosphate in Vacuum.	Weight of Displaced Toluol in Vacuum.	Volume of Silver Phosphate.	Density of Silver Phosphate.
grams	grams.	c. c.	25°/4°.
22.955	3.113	3.606	6.366
16.942	2.295	2.658	6.374
	Mean		6.37

Therefore the apparent weight of silver phosphate was corrected to the vacuum standard by adding 0.000044 gram per gram of salt. Similarly 0.000041 gram was added for every gram of silver bromide.

THE ADSORPTION OF AIR BY SILVER PHOSPHATE.

Since the silver phosphate was in a very finely divided condition and since many fine powders have the power of adsorbing appreciable quantities of air or other gases, the possibility of the adsorption of air by silver phosphate was investigated. The method of experimenting and the apparatus were very similar to that used by Baxter and Tilley for investigating the behavior of iodine pentoxide.

"Two weighing bottles were constructed with long, very well ground stoppers which terminated in stop-cocks through which the tubes could be exhausted. These tubes were very closely of the same weight and very nearly the same internal capacity. The tubes were first exhausted and compared in weight by substitution. Next they were filled with dry air and again weighed, the weighing being carried out with stop-cocks open. Both steps were then repeated with essentially the same results." [27]

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[27] Baxter and Tilley, Jour. Amer. Chem. Soc., 1909, **31**, 214.

In these two experiments, when air was admitted, the counterpoise gained 0.00028 and 0.00021 gram respectively (average 0.00025) more than the tube which was later to contain the silver phosphate. After 22.69 grams of pure dry silver phosphate had been placed in the tube, the tube and its counterpoise were exhausted and the difference in weight determined. When dry air at 25° C. and 766 mm. was admitted to both the tube containing the silver phosphate and the counterpoise, the counterpoise gained 0.00443 gram more than the tube. Therefore the air displaced by the silver phosphate was $0.00443 - 0.00025 = 0.00418$ gram. Since 22.69 grams of silver phosphate of density 6.37 have a volume of 3.56 c.c., the volume of pure air displaced at 25° C. and 766 mm. should weigh 0.00425 gram.[28]

The experiment was then repeated. After the air had been exhausted from the tube and its counterpoise, the tube containing the silver phosphate was heated gently. No gas was evolved. The tube and its counterpoise were then weighed by substitution. When dry air at 24.5° and 767 mm. was admitted to both, the counterpoise gained 0.00445 grams more than the tube containing the silver phosphate. Therefore the air displaced by the silver phosphate was $0.00445 - 0.00025 = 0.00420$ grams, whereas the weight of air displaced, calculated from the density of the salt, is 0.00426 gram.

The agreement between the experimental results and those calculated from the density of silver phosphate on the assumption that no adsorption takes place is close enough to show that no significant amount of adsorption occurs.

Discussion of the Results.

The following table contains all of the analyses not vitiated by a known impurity in the sample or by an accident during the analysis. One feature of this table requires further explanation. In Analysis 5 the silver was determined by precipitation as chloride instead of bromide. For every gram of silver phosphate there was obtained 1.02707 grams of silver chloride. Since Baxter found $AgBr : AgCl = 1.31017 : 1.00000$,[29] this analysis indicates that one gram of sample N is equivalent to $1.02704 \times 1.31017 = 1.34560$ grams of silver bromide. This result is placed in the table for comparison with the other analyses and is used in the computation of the mean.

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[28] Rayleigh's value for the density of air at 0° and 760 mm., 1.293 grams per liter, is used. Proc. Roy. Soc., **53**, 147.

[29] These Proceedings, 1906, **42**, 213.

SERIES I.

$3\,AgBr : Ag_3PO_4$

Number of Analysis.	Sample of Ag_3PO_4.	Weight of Ag_3PO_4 in Vacuum.	Weight of AgBr in Vacuum.	Weight of Asbestos.	Loss on Fusion.	Dissolved AgBr.	Corrected Weight of AgBr.	Ratio $\frac{3AgBr}{Ag_3PO_4}$.
		grams	grams	gram	gram	gram	grams	
1	O	6.20166	8.34427	0.00036	0.00034	0.00007	8.34490	1.34558
2	O	6.35722	8.55386	0.00041	0.00003	0.00011	8.55419	1.34559
3	N	5.80244	7.80792	0.00029	0.00005	0.00007	7.80819	1.34567
4	N	5.05845	6.80658	0.00019	0.00020	0.00012	6.80685	1.34564
5	N	3.34498	(AgCl) 3.43514	0.00029	0.00009	0.00008	(AgCl) 3.43544	1.34560
6	P	7.15386	9.62648	0.00046	0.00013	0.00013	9.62694	1.34570
7	P	7.20085	9.68929	0.00023	0.00005	0.00010	9.68947	1.34560
8	R	6.20182	8.34466	0.00041	0.00027	0.00012	8.34522	1.34561
9	R	5.20683	7.00543	0.00029	0.00040	0.00007	7.00605	1.34555

Average 1.34562

Per cent of Ag in Ag_3PO_4 . . . . . 77.300

A careful study of these results shows that the composition of silver phosphate is very nearly, if not quite, independent of the changes in the acidity of the solutions from which it is precipitated. Samples O and R were prepared under slightly more acid conditions than Samples N and P. The average amount of silver bromide obtained from one gram of Samples O and R is 1.34558 (77.297 per cent of silver), whereas the average from Samples N and P is 1.34564 (77.301 per cent of silver). This difference, if real and significant, is probably due to a very slight occlusion of disilver hydrogen phosphate. It does not seem probable that any basic salt was present in Samples N and P, because silver shows little tendency to form basic salts and the conditions of precipitation were not favorable for the formation of basic salts.

The difference between composition of the samples is so slight, both in absolute amount and by comparison with the differences between different analyses of the same sample, that in the present state of our knowledge it does not seem justifiable to reject the analyses of Samples N and O. This conclusion is supported by the fact that the water determinations failed to show a difference between these samples. The results, however, indicate that the average ratio 1.34562 (77.300 per cent of silver) may be very slightly too low, owing to the presence of disilver hydrogen phosphate. The ratio 1.34562, assuming the atomic weight of silver to be 107.88, and assuming that silver bromide contains 57.4453 per cent of silver, leads to an atomic weight of 31.043 for phosphorus, whereas the ratio 1.34564 derived from Samples N and P gives the value 31.037. The rounded-off value, 31.04, may be considered to be essentially free from error from this source.

We are greatly indebted to the Carnegie Institution of Washington for generous pecuniary assistance in pursuing this investigation; also to the Cyrus M. Warren Fund for Research in Harvard University for many pieces of platinum apparatus.

SUMMARY.

1. A careful study has been made of the conditions necessary for the preparation of pure trisilver phosphate.

2. It is found that silver phosphate can be almost completely dried without fusion by heating in a current of dry air.

3. The density of silver phosphate is found to be 6.37.

4. It is found that silver phosphate does not adsorb a significant amount of air.

5. Nine analyses, made with four different samples, show that one gram of silver phosphate yields 1.34562 grams of silver bromide, whence the per cent of silver in silver phosphate is 77.300.

Therefore,

If $Ag = 107.88$ $P = 31.04$
If $Ag = 107.87$ $P = 31.03$
If $Ag = 107.86$ $P = 31.02$

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THE REACTIONS OF AMPHIBIANS TO LIGHT.

By A. S. Pearse.

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Presented by E. L. Mark, December 8, 1909. Received November 24, 1909.

TABLE OF CONTENTS.

I. INTRODUCTION.

A. Historical.

Considerable interest has lately centred itself in the study of the behavior of animals under the influence of light, and the results of such studies have been largely used in formulating the various theories which attempt to account for the reactions of organisms after they have been subjected to external stimulation. Among vertebrates the amphibians offer particularly favorable material for such study, as the various species may be used for experimentation in or out of the water; they are, as a rule, very responsive to photic stimulation and are able to withstand severe operations without serious interference with their reactions. A large amount of work by a number of observers has already been done in the study of light responsiveness, and in the next few pages an attempt is made to summarize the results of those studies, so far as they apply to amphibians. For the sake of clearness this material will be considered from a comparative standpoint rather than in an historical order.

Amphibians react to light by giving motor responses. This motor reaction to illumination was first recorded by Configliachi and Rusconi ('19).[1] They observed that Porteus anguinus, the blind cave salamander of Europe, became restless when exposed to light, and this observation has been confirmed by later observers (Semper, '81; Dubois, '90; Beer, :01).[1] Since that time responsiveness to light has been noted in the following genera: Triturus, or Triton (Graber, '83, '84; Willem, '91), Necturus (Cope, '89, Reese, :06), Cryptobranchus (Reese, :06, B. G. Smith, :07), Diemyctylus (Jordan, '93), Spelerpes (Banta and McAtee, :06), Rana (Kühne, '78ᵃ; Loeb, '90; Parker, :03ᵇ; Torelle, :03, Yerkes, :03, :06; Dickerson, :06; Holmes, :06; Cole, :07), Acris

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[1] The numbers in parentheses indicate the year of publication of the article referred to, the title of which is given in full in the "Bibliography" at the end of the paper. An apostrophe indicates an omitted 18; a colon, an omitted 19.

(Cole, :07), Bufo (Graber, '84). As these are representative genera, it seems evident that photic stimulation exerts an influence of wide range among the amphibians.

Many amphibians show a marked tendency to orient the body and to move toward or away from the source of light. Configliachi and Rusconi ('19) observed that Proteus tended to go to the side of an enclosure farther from the light and remain there. Since then a number of observations have been made concerning the phototropism of amphibians. Thus, the following have been claimed to be positively phototropic: Rana sp. ? (Holmes, :06; Dickerson, :06), R. temporaria (Plateau, '89), R. clamata (Torelle, :03; Yerkes, :03, :06; Cole, :07), R. pipiens (Parker, :03^b; Torelle, :03), Acris gryllus (Cole, :07), Bufo clamita (Plateau, '89); and the five following negatively phototropic: Proteus anguinus (Configliachi and Rusconi, '19; Dubois, '90), Necturus (Cope, '89; Reese, :06; B. G. Smith, :07), Spelerpes maculicaudus (Banta and McAtee, :06), Rana (Loeb, '90). It will be seen from this list that the photic reactions of the Caudata are negative, while those of the Salientia are positive, with the exception of the observations by Loeb ('90), which do not agree with those of other writers.

Some amphibians show a tendency to come to rest in the shade. We would perhaps expect such a reaction in species which are normally negative in their phototropism, but Torelle (:03) has shown that the frog, which is strongly positive, will also go toward a shaded area and come to rest in it, though the animal then faces toward the light. Graber ('83, '84) had previously found that Triturus, Rana, and Bufo tended to come to rest in shadow.

The eyes are not essential for the light reactions, that is, such reactions may be brought about by stimulation through the skin. Configliachi and Rusconi ('19) ascribed the photic reactions of Proteus to the painful effect of light upon the skin, but Kohl ('95) showed that, while the eyes of this species are rudimentary, they might nevertheless be effective photoreceptors. It remained for Dubois ('90) to show that the reactions of Proteus might take place through the skin alone. He blackened the eyes and obtained a reaction from an individual in which only the tip of the tail was illuminated. Graber ('83, '84) observed reactions in Triturus, which were like those of normal individuals, after the eyes had been removed and the orbits filled with black wax. More recently Parker (:03^b) has shown that Rana is positively phototropic with and without the eyes; and Cole (:07), besides corroborating Parker's observations, has obtained like results from Acris. Korányi ('93) observed reflex leg movements in a frog, which had been rendered particularly sensitive by treating the brain with meat extract, when he threw a strong beam of light on its back. Reese (:06) found that when only the tip of the tail was illuminated in Cryptobranchus or Necturus, the individuals thus stimulated moved out of the lighted area.

The reactions brought about by stimulating the eye alone agree, in kind, with those brought about through the skin. Parker (:03[b]) found that frogs in which the skin was covered but the eyes were exposed, were positively phototropic, like individuals in which the eyes had been removed. Torelle (:03) made an observation which bears indirectly on the same point. She found that frogs which had one eye covered with black cambric went toward the light at an angle or made circus movements with the uncovered eye towards the centre.

The positive phototropism of amphibians is apparently a reaction toward a greater intensity of illumination; or, with the eyes, toward a greater illuminated area. Plateau ('89, p. 88) observed that Rana and Bufo, when placed in a box having two openings, went toward the larger aperture even though it was covered with a grating. Cole (:07) showed that when Acris was placed between two lights of the same quality and intensity but of different areas, it went toward the larger area, but when individuals in which the optic nerves had been cut were placed in the same situation, they went toward either light an approximately equal number of times. Rana also showed the same reaction toward the larger area when it was in normal condition. Torelle (:03) found that the direction of the illumination made no difference in photic responses, as frogs went toward the lighter end of a box when the illumination was from below, and Reese (:06) has made similar observations on Cryptobranchus and Necturus. Dickerson (:06, p. 32) says, "Frogs do not distinguish between a lighted space and a white solid. They will turn toward a white card or paper and try to jump through it, and they may struggle at the impossible task of working their way into the solid white surface made by the leaf edges of a closed book."

Torelle(:03) noted that frogs, when they were confined in a small space with an opening above, pointed the head upward toward the opening, and she supposed this to be evidence for the directive action of the rays. Objection may be made to this view on the ground that the opening offers the only opportunity for escape, and the animal, seeing the opening with its eyes, points its head toward it. If she had shown the same reaction with eyeless individuals, the evidence would have been more conclusive.

The rays toward the violet end of the spectrum are apparently most potent in producing photic reactions, and the rays toward the opposite *end approach in their effects the conditions brought about by dark.* Graber (**'83, '84**) found that Triturus did not come to rest in the colors toward the violet end of the spectrum when there was equal opportunity to remain in those nearer the opposite end. This was true of blinded as well as normal animals. He also (**'84**) found that Rana and Bufo reacted in much the same way. He states that his results could not have been due to the effect of temperature, as he performed experiments in which he used a heat screen for the blue and none for the red light, and the results were the same. Kühne (**'78**^{a}) had previously observed that normal frogs went from green toward blue light, while blinded individuals did not. Loeb (**'90**) states that the less refrangible rays do not affect light reactions to such an extent as those of greater refrangibility, and in this connection he remarks that a frog will jump towards a red cloth. (He found Rana to be negatively phototropic.) Torelle (**:03**) in speaking of the frog recorded in a stronger positive phototropism for blue light than for red, yellow, or green; and this was the same when the light was reflected, transmitted, or both. The individuals she used were indifferent to red light. Reese (**:06**) found blue to be most potent in causing reactions in Necturus and Cryptobranchus. Yerkes (**:03**, p. 586) suggested that the frog might be able to distinguish between red and white backgrounds, but, as he says (**:06**, p. 548), there is nothing to show that these reactions might not have been due to intensity differences. Holmes (**:06**, p. 350) in speaking of frogs sums up the whole matter by stating that "in general it may be said that where they are able to go toward one of two colors, of equal intensity, they move to the color lying nearest the violet end of the spectrum."

The phototropic reactions of amphibia are apparently not due to the direct stimulation of the central nervous system by light. Parker (**:03**^{b}) found that eyeless frogs responded positively when only the lower part of the body was illuminated from the side in such a manner that the central nervous organs were in shadow. The experiments of Dubois (**:90**) on blinded Proteus, and Reese (**:06**) on Cryptobranchus and Necturus offer additional evidence on this point. These animals reacted to a beam of light thrown on the tail, and hence beyond the limits of the central nervous organs.

Various internal and external factors may influence the responses of amphibians to light. It is probable that there are many factors which exert such a modifying influence. Those which are enumerated in the following paragraphs are known to alter the photic responses of certain amphibians by producing changes in their physiological states.

Breeding season. Jordan (**:93**, p. 271), in speaking of Diemyctylus, says they "usually conceal themselves under fallen leaves and among the tangle of water weeds. On warm, sunny days in early spring, however, they bask openly in the sunshine along the shore." Another instance is given by B. G. Smith, (:**07**, p. 6), who remarks that "Cryptobronchus comes forth but seldom in the daytime except during the breeding season," and (p. 32) "with the close of the breeding season, becomes more shy, avoids the light and is seldom seen in the open."

Temperature. Torelle (:**03**, p. 475) stated that the positive phototropism of the frog increased as the temperature was raised. If, however, the temperature rose above 30° C., these animals were indifferent to light, and if it fell below 8° C., they became negative. Cole (:**07**, p. 401) has shown conclusively that conditions of temperature influence the photic responses in Rana. As has been stated, his method was to place the animals between two lights of equal intensities but different areas. When a frog has been cooled to from 6° to 10° C., it went toward the smaller illuminated area, but after it became warm its reactions were uniformly toward the larger area.

Previous photic stimulation. Configliachi and Rusconi (:**19**) noticed that after Proteus had been exposed to light for some time, its reactiveness to that stimulus decreased. Reese (:**06**, p. 94), in experimenting with Cryptobranchus and Necturus, found that "the responses to light were much more marked for the first ten or a dozen stimulations." Torelle (:**03**, p. 47), on the other hand, observed that, after five to eight hours' exposure to light, frogs exhibited the same positive phototropism as before.

Stereotropism. Eigenmann and Denny (:**00**, p. 34) in speaking of Typhlotriton, say that "it seems probable that sterotropism rather than negative heliotropism accounts for the presence of this species in caves. Torelle (:**03**, p. 477) found that Rana was strongly stereotropic below 8° C. This stereotropism was associated with a change from positive to negative phototropism, and, as Holmes (:**06**, p. 349) has pointed out, may have been responsible for such change.

Age. Banta and McAtee (:**06**, p. 71) in their experiments with the cave salamander found that "all larvæ are very much more responsive to light stimulus than the adults, the young larvæ more so than the older."

Surrounding medium. Torelle (:**03**, p. 473) has shown that frogs will go toward the light under water as well as in air. The change in surrounding medium, and from walking to swimming, apparently does not alter the reactions.

B. Methods.

The experiments described in the present paper have been devoted (1) to extending the range of our knowledge of photic reactions among the amphibians, (2) to ascertaining more fully the nature of the photoreceptors involved, and (3) to determining how great a part the central nervous system takes in these reactions. It gives me great satisfaction to express my indebtedness to Professor G. H. Parker, under whose direction the work was accomplished.

All the experiments which are described in the succeeding pages were carried on in a dark room, the temperature of which usually varied between 17° C. and 21° C. The source of the light was a six-glower Nernst lamp, and as the amount of light it gave out varied under different conditions, the intensity used is given under the descriptions of the various experiments. All the amphibians used were collected in the vicinity of Cambridge, Massachusetts, with the exception of Necturus, which came from Venice, Ohio; Cryptobranchus from Oil City, Pennsylvania, and, through the kindness of Professor A. M. Banta, from Marietta, Ohio; and Diemyctylus from Jaffrey, New Hampshire. The aquatic species were kept in a large aquarium tank, four meters long by one and a half wide, in a cool basement room. The terrestrial forms were kept in cages, the floors of which were covered with earth and dead leaves, and individuals upon which operations had been performed were placed on a bed of moist excelsior in glass jars. Little trouble was experienced in keeping the animals in good condition. The frogs and toads were fed with meal worms, which they ate readily throughout the winter. The other species were not fed, though Cryptobranchus may have eaten frogs, which were kept for other purposes in the aquarium with it; and as one of those animals lived for two years, it is not improbable that it obtained such food from time to time. The experiments were carried out in the autumn and winter months (October 1 to April 1) of two different years.

Of the aquatic species used, Cryptobranchus was the most reactive. For experimental purposes Bufo was the most satisfactory of the land forms, both on account of its extreme activity and its greater ability to withstand dryness. Both Bufo fowleri and B. americanus were used, but the experiments on the two species were not kept separate. Dr. L. J. Cole informs me that Acris is much better than Bufo for work of this nature, but I have not had an opportunity to try it. The term "amphibians" in this paper does not include caecilians, whose reactions to light are, so far as I know, unstudied.

II. OBSERVATIONS.

A. The Photic Reactions of Normal Amphibians compared with those from which the Eyes have been removed.

In order to compare the reactions of amphibians in which both the skin and eyes acted as photoreceptors with those in which only the skin was open to stimulation, individuals were tested both in normal condition and after the eyes had been excised. The eyes were usually removed by making a single transverse cut as near the anterior edge of the ear drums as possible. The whole front of the head, including the olfactory lobes and a part of the cerebral hemispheres, was removed by this method of procedure (Figure 1). In Necturus and Cryptobranchus, however, only the eyes were excised. All the species stood the operation well and subsequently gave typical reactions, except Plethodon and Diemyctylus, which were apparently much weakened by it and were indifferent to light after the eyes had been removed. As a rule individuals were not used for experimentation until the day after the operation.

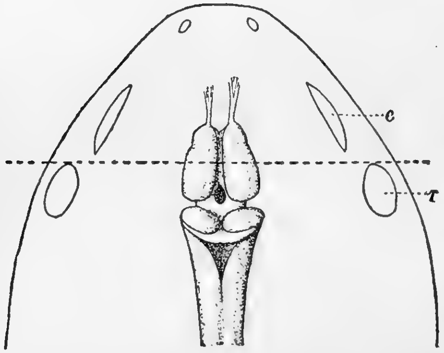

Figure 1. Dorsal view of toad's head showing the position of the brain. The dotted line indicates the plane of the cut used in removing the eyes. *e*, eye; *r*, ear.

The species studied fall naturally into two groups, aquatic and terrestrial. The former group included Necturus maculosus and Cryptobranchus allegheniensis, and the terrestrial species studied were Amblystoma punctatum, Plethodon cinereus, Diemyctylus viridescens, Rana clamata, R. sylvatica, Bufo fowleri, and B. americanus. The reactions of each species will be considered separately.

(*a*) *Necturus maculosus.*

The first experiments with this species were intended to show what influence light had upon its movements. Four individuals were placed successively in the centre of a large aquarium, which was illuminated from one end in such a way that the light had an intensity of about 220 candle-meters at its centre. Under these conditions an individual usually went at once to the end of the aquarium farther from the light. It then wandered about from one end to the other for some time, but finally came to rest as far as possible from the light. If the lamp was then changed to the opposite end of the aquarium, the animal again moved to the end which was farther from the light and came to rest.

In order to test the reactions of Necturus to light and shadow, the lamp was moved to the side of the aquarium and a movable screen interposed in such a way that one half of the aquarium was in shadow and the other half in light (220 candle-meters at the centre of the aquarium). Two animals were successively introduced. One of these, after wandering back and forth from one end to the other, came to rest in the shaded end of the aquarium. When the screen was changed to the opposite half, the animal moved again into the shaded area, and this action was repeated for five successive trials on two different occasions. The other individual remained at the side of the aquarium nearer the light, and in two experiments it kept going back and forth from light to shadow for more than one hour. It apparently did not avoid the light, but, by comparing the time it spent in the light with that spent in shadow during half an hour, it was found that three-fifths of that period had been passed in the shaded part of the aquarium. The first individual, then, invariably came to rest in the shadow, and the second one, while it continued to move actively, spent somewhat less time in the light than in the shadow.

The most decisive reactions shown by Necturus were brought about by illuminating a small area at its anterior or posterior end. The apparatus was in the same position as for the experiments just described, except that a screen was arranged in such a manner that a vertical band of light about five centimeters wide could be suddenly thrown on different regions of the body. Four individuals were used for these experiments and all of them behaved in essentially the same manner. After an animal had remained quiet in the dark for five minutes, it was suddenly illuminated, and a reaction usually took place within a few seconds. When the light fell on the tail, the animal moved forward, but when it was allowed to fall on the head, the movement was usually backward. Since the animals were never tested with the light until they had been quiet in the dark for five minutes, these reactions were without doubt due to the illumination, for they took place within a few seconds of the time when the light was thrown on the animals.

In order to discover whether the skin of Necturus was sensitive to light or not, the eyes were removed from two individuals and they were then tested by local stimulation as described in the last paragraph. Their reactions were similar to those of animals with eyes except in one particular. The average time which elapsed before the individuals with eyes moved out of the lighted area was shorter when the head was stimulated than when the light fell upon the tail, but the eyeless animals, on the contrary, reacted more quickly when the tail was stimulated. The results with normal animals agree with those of Reese (: **06**, p. 96) in his experiments on Necturus. He ascribed the shorter reaction time for the head to greater sensitiveness in that region, and he believed it to be due to stimulation received through the eyes. The present experiments with eyeless animals give support to his views, as the posterior end of the individuals tested was apparently more sensitive to photic stimulation after the eyes had been excised. The decreased sensitiveness of the head region may, however, have been due to the injury incident to the removal of the eyes, instead of the mere loss of the eyes themselves.

From the experiments described it is evident that Necturus is negatively phototropic and that it comes to rest in shaded areas. Both the skin and eyes act as photoreceptors, and the stimulation of either brings about negative reactions.

(b) *Cryptobranchus allegheniensis.*

The arrangement of the apparatus for the experiments with Cryptobranchus was the same as for those with Necturus. The reactiveness of this species to light was very marked. Seven individuals were placed successively in the middle of the aquarium, the illumination being from one end, whereupon they moved immediately to the end farther from the light. When the lamp was carried to the opposite end of the aquarium, they usually changed their position at once and again came to rest in the end farther from the light. In these reactions they were much more responsive than Necturus, though, as Reese (: **06**, p. 94) has observed, they often failed to respond readily after the first few reactions.

The reactions of Cryptobranchus to conditions of light and shadow were also pronounced. In testing these, half the aquarium was shaded by a screen which was changed from one end to the other at five minute intervals. An individual was placed in the aquarium and the screen changed ten times. It never failed to move at once to the shaded part of the aquarium, and furthermore it rested quietly in the shadow in the intervals between the changes.

The illumination of a small area at the anterior or posterior end of an individual produced the same reactions as in Necturus, but in Cryptobranchus they took place more quickly.

To test the sensitiveness of the skin to light, the eyes were removed from one individual and it was stimulated alternately on the head and tail by the same method as that used for Necturus. This animal usually responded within a few seconds to such illumination. In a series of fifty reactions it was found that the average time required for the animal to move out of the illuminated area was more than twice as great when the light fell upon the head as when the tail was illuminated in the same manner. The skin of Cryptobranchus is, then, a photoreceptor and the sensitiveness seems to be greater at the posterior than at the anterior end. Reese (:06, p. 94) has stated that, even with the eyes present, this species shows the greatest sensitiveness to light in the caudal region.

This eyeless individual was strongly photokinetic. It was placed in a flat porcelain dish about a meter below an ordinary gas burner, and after it had been allowed to remain in the dark for about an hour, the gas was suddenly lighted. There was an unfailing response to this illumination within a few seconds, the animal moving restlessly about in the dish. As the light was non-directive, and the animal often remained quiet for hours in the dark, this uniform response to sudden illumination showed this species to be strongly photokinetic. In this respect it was quite different from Necturus, which often did not respond to such stimulation for some time, even when the light intensity was 220 candle-meters.

In summarizing the results of the experiments upon Cryptobranchus, it may be said that it is negatively phototropic, that it comes to rest in shaded areas and is strongly photokinetic. These reactions apparently take place as readily when only the skin is stimulated by light as when the eyes are also affected.

The terrestrial amphibians were found to be much more satisfactory subjects for experimental work than the aquatic species. Not only was it easier to arrange the apparatus for the land forms, but more accurate results were obtained, as it was possible to orient the animals with a perfectly uniform relation to the light before each reaction. In all the experiments with terrestrial forms the apparatus shown in Figure 2 was used. After this apparatus had once been arranged, it was a simple matter to test one species after another, and to compare the reactions of normal animals with those of individuals without eyes. It will be seen from the figure that the two side screens (*s′*) were placed at the edge of the shadow made by the light that passed through the heat screen (*a*). Thus the greatest open space was away from the light, and, as far as the animal was able to see, the best chance for escape lay in that direction. An individual was not, then, subjected to the same conditions as one placed in a small box having a single opening. It does not seem improbable that any animal with eyes, after being handled and shut up in a small enclosure, would endeavor to escape by the most apparent opening; and the reactions could not in that case be interpreted as being due to the influence of light alone. The apparatus shown in Figure 2 is not open to such an objection.

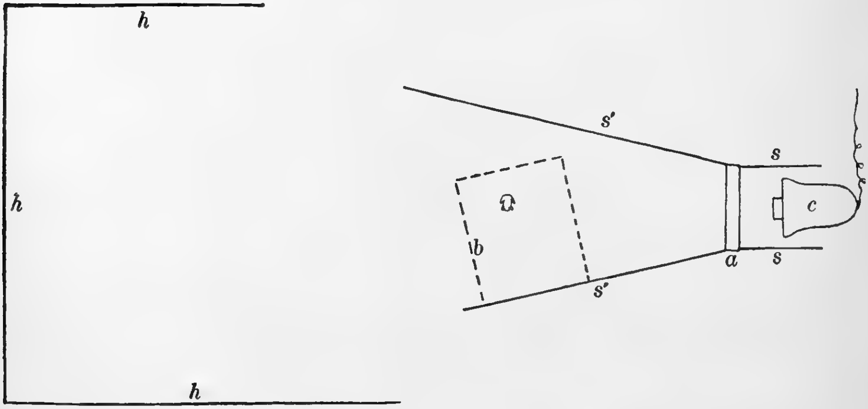

FIGURE 2. Plan of apparatus in which the reactions of terrestrial amphibians to light were tested. *a*, heat screen filled with water; *b*, screen for head of observer; *c*, lamp; *h*, screen extending to ceiling; *s*, *s′*, screen 25 cm. high.

The method of experimentation was to place an individual at a distance of seventy centimeters from the light (where the intensity was 225 candle-meters) and watch it through a small hole in the screen *b*, until a definite movement had taken place. After a reaction of this kind, the animal was held for a few seconds outside the screen *s*, where it could not see the light, in order to eliminate any directive effect produced by that stimulus, and it was then replaced ready for another reaction. To counteract the effects of compensatory movements, the animals were always turned in a clockwise direction between the reactions, and were placed with the right and left sides alternately toward the light, the long axis of the body being at right angles to the direction of the rays. To avoid effects due to fatigue, no more than twenty reactions were, as a rule, recorded from an individual on any one day. As the method of procedure was the same in all cases, and, as the only object in view was to compare the reactions of eyeless and normal animals, the discussion under each species will be limited mostly to the results obtained.

(*c*) *Amblystoma punctatum.*

Four individuals were used in the experiments upon this species. After the eyes had been excised, the two smaller animals, which measured about seven centimeters in length, did not survive more than a day or two. The two adult individuals, however, were apparently little affected by the operation, and one of them lived for forty-seven days after it. The results of the experiments are given in Table I. This species is shown to be negatively phototropic, both in the normal and eyeless condition. As might be expected, there were more movements without reference to the light after the eyes had been excised, but this may have been due to the effects of the operation. Whether this is true or not, the fact remains that the animals were able to respond negatively to light received through the skin.

TABLE I.[2]

PHOTIC REACTIONS OF AMBLYSTOMA PUNCTATUM, WITH AND WITHOUT EYES.

Condition of individuals		Normal			Eyeless		
Direction of movement		+	–	0	+	–	0
Reactions	Number	6	88	10	19	71	32
	Per cent	6	84	10	16	58	26

(*d*) *Plethodon cinereus erythronotus.*

This species manifested the same negative phototropism as the last, when in normal condition, but it did not stand the operations well.

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[2] In the tables which appear throughout this paper the following signs are used: "+" indicates a decided movement toward the light, "–" is used for a similar movement away from the light, and "0" signifies that the individual remained still for fifteen minutes or made a movement without apparent reference to the light.

This may have been due to the small size of the animal, which rendered it less able to withstand the unfavorable conditions in its environment after the eyes had been excised. The reactions summarized in Table II. show that the species was negatively phototropic when in normal condition. After the eyes had been excised, however, the movements were without apparent reference to the light. This indifference may, nevertheless, have been due to the effects of the operation rather than to lack of photic sensitiveness in the skin.

TABLE II.

PHOTIC REACTIONS OF PLETHODON CINEREUS ERYTHRONOTUS, WITH AND WITHOUT EYES.

Condition of individuals		Normal			Eyeless		
Direction of movement		+	–	0	+	–	0
Reactions	Number	4	30	0	12	9	20
	Per cent	12	88	0	29	22	49

(e) *Diemyctylus viridescens.*

Like Plethodon, this species did not stand the operation well and gave no reactions which were manifestly due to light after the eyes had been removed. Ten individuals were used, and the eyes were excised from eight of them. None of the latter lived more than twelve days after the operation. The results given in Table III. bring out the fact

TABLE III.

PHOTIC REACTIONS OF DIEMYCTYLUS VIRIDESCENS, WITH AND WITHOUT EYES.

Condition of individuals		Normal			Eyeless		
Direction of movement		+	–	0	+	–	0
Reactions	Number	242	88	10	30	29	57
	Per cent	71	26	3	26	25	49

that this species is positively phototropic; a condition which is not, so far as I know, found in any other caudate amphibian. All the individuals used were of the orange type of coloration, and it is possible that animals of this species having the green phase might give different results.

(*f*) *Rana clamata.*

Although the eyes were not excised from any individual of this species, the reactions observed are given in Table IV. for comparison with the next form. They agree essentially with those described by Parker (:03[b]) and Torelle (:03) for R. pipiens and R. viridescens. Five individuals were tested, and they all proved to be positively phototropic.

TABLE IV.

PHOTIC REACTIONS OF RANA CLAMATA.

Direction of movement		+	–	0
Reactions	Number	104	37	15
	Per cent	67	23	10

(*g*) *Rana sylvatica.*

This frog was more active than the last species, and some individuals gave more decided phototropic reactions than did any member of the

TABLE V.

PHOTIC REACTIONS OF RANA SYLVATICA, WITH AND WITHOUT EYES.

Condition of individuals		Normal			Eyeless		
Direction of movement		+	–	0	+	–	0
Individual No. 1		20	0	0	20	0	0
Individual No. 2		17	2	1	10	5	5
Individual No. 3		7	11	12	..	..	..
Individual No. 4		6	7	1	..	..	..
Total Reactions	Number	50	20	14	30	5	5
	Per cent	60	24	16	75	12	12

preceding species. There were, however, such differences in the reactions of the four animals used that they are tabulated separately. Individual No. 1 never failed to move straight toward the light. No. 2 was not as persistently positive after the eyes had been excised as before this operation, though it continued to give a majority of positive reactions. As individuals 3 and 4 were apparently indifferent to the light in their normal conditions, their eyes were not removed. The reactions of animals 1 and 2 were, however, strongly positive, and this condition remained even after the eyes had been excised; hence their skins served as photoreceptors as well as their eyes.

(*h*) *Bufo americanus and B. fowleri.*

Both these species were used for experimentation, but, as the records were not kept separate, their reactions cannot be distinguished and are given together in Table VI. The results include experiments with twenty normal animals and six in which the eyes had been excised. In removing the eyes from another individual, the head was cut diagonally so that the left ear was injured. This animal turned continually to the right, regardless of the direction of the light, and its reactions were therefore not included in the table. Although most of the individuals were adults, a few were immature, but none of them measured less than two centimeters in length. The results show the species to be positively phototropic in response to stimulation received through the skin as well as through the eyes.

TABLE VI.

Photic Reactions of Normal and Eyeless Toads.

Condition of individuals		Normal			Eyeless		
Direction of movement		+	–	0	+	–	0
Reactions	Number	802	265	11	126	47	17
	Per cent	74	25	1	66	25	9

It was also possible to show that the phototropic reactions of eyeless toads were not due to the effect of light upon the exposed ends of the optic nerves. On two occasions, after an individual had given ten successive positive responses, it was immediately oriented in such a manner that the anterior end of the body pointed away from the light. In both instances the animals turned at once and went directly toward the light, and this reaction was repeated on five successive trials. These reactions could not have been due to the direct stimulation of the optic nerves by light, as they were not exposed to such stimulation. The results are in agreement with those of Graber ('83), who filled the orbits of Triturus with black wax, and of Dubois ('90), who covered the eyes of Proteus with a mixture of gelatine and lampblack. Both these observers obtained phototropic reactions by stimulating the skin.

(*i*) *Conclusions.*

From the experiments described it may be said that photic sensitiveness is general in the skin of amphibians. While there is considerable variation in the phototropism of different species, and even of individuals of the same species, the reactions brought about by stimulation through the skin alone are like those produced when both the skin and eyes act as photoreceptors.

B. The Influence of Mechanical Stimulation on the Photic Reactions of the Toad.

In the experiments with terrestrial amphibians and light the observations were always made after the animals had been handled by the experimenter, and, though the response was decided in most cases and of such a nature as to attribute it to light, it is not impossible that mechanical stimulation through handling may have been responsible for more or less of it. In order to test this matter five toads which were known to be positively phototropic were placed successively in a box, the floor of which measured thirty-eight by ninety centimeters. The sides and floor of this box were of slate, and the ends were closed by glass heat-screens containing a layer of water 3.75 centimeters thick. The roof consisted of a coarsely woven black cloth stretched on a wooden frame, and the observations were made through the meshes of this cloth. A lamp giving a light intensity of 220 candle-meters was changed from one end to the other at five-minute intervals for a period of fifteen minutes. Four of the individuals when first placed in the apparatus went toward the light, and then wandered back and forth without evident reference to it, and apparently tried to escape from the enclosure. The fifth animal sat in the centre of the box, turning from one side to the other for three minutes, and then went away from the light. When the lamp was changed from one end of the apparatus to the other, only one of the individuals turned immediately and went toward it; the other four were apparently indifferent. In a later experiment, however, two toads were observed to be persistently positive, and they tried for as much as five minutes to move through a heat screen to the light.

Six toads were next placed together in a rectangular glass vessel (the floor of which measured twelve by twenty centimeters) and were subjected to approximately the same light conditions as in the last experiment. In jumping about they stimulated each other in a mechanical way. During fifteen minutes all the individuals remained mostly facing the light and making vain attempts to reach it, and only occasionally did one of them try to escape on the opposite side of the jar.

It is evident from these two experiments that mechanical stimulation exerts an influence on the phototropism of the toad by enforcing the effect of light, or, it could perhaps better be said, that the mechanical stimulation furnishes the *impulse* to locomotion, while the light is effective in *determining* the direction of the movement after locomotion has been established. For the purpose of the present paper, however, it makes no difference whether the responses obtained were due solely to the influence of light or whether they were reactions to light after mechanical stimulation. In either case the fact remains that both the skin and eyes of amphibians act as photoreceptors, and that definite reactions take place as a result of stimulation through either.

C. THE REACTIONS OF THE TOAD TO PHOTIC STIMULATION THROUGH THE EYES ALONE.

Experiments have been described in this paper which show that various amphibians react in the same way when either the skin alone is stimulated or when both the skin and eyes are affected. The next question which naturally arises is whether animals will react in the same way when the stimulation is received through the eyes alone. That such responses take place in Rana pipiens has been shown by Parker (:**03**^{b}, p. 33), who found this species to be positively phototropic when its entire surface was covered, with the exception of the eyes. In order to test the toad in a similar manner the apparatus shown in Figure 3 was used. Light was allowed to pass through a small opening (*e*) in a screen, which could be adjusted so that only a small area around the eye of the animal was illuminated. As an additional precaution against light reception through the skin, the individuals used were covered, except the eyes and feet, by a tight-fitting suit of soft leather. As might be expected, the movements of the two animals used in the experiments were slow. Each of these individuals was placed with its right and left side alternately toward the light, the long axis of the body being at right angles to the direction of the rays. The movements which resulted from this method of stimulation are summarized in Table VII. The results show that the toad gives the same sort of positive reactions when the eyes are stimulated as when the skin is illuminated.

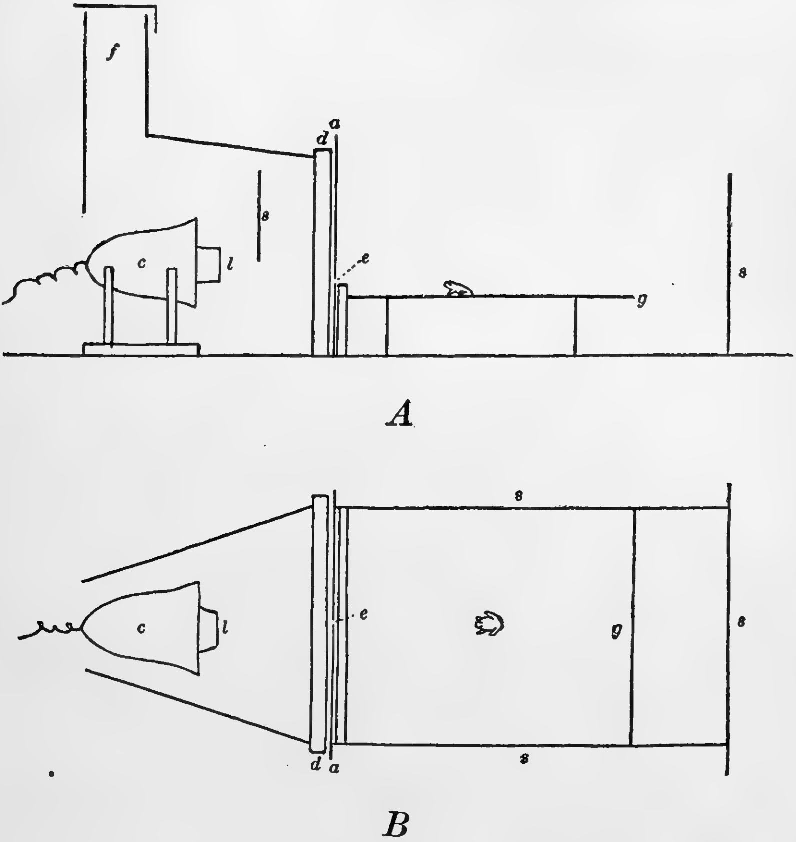

FIGURE 3. *A*, section of apparatus to test reactions of toads to stimulation through the eyes alone; *B*, ground plan. *a*, screen; *c*, lamp; *d*, heat screen; *e*, aperture for light; *f*, chimney for carrying away heat; *g*, slate upon which the animals were placed; *l*, source of light; *s*, screen.

If the reactions of the two individuals just described were due to unequal stimulation of the eyes, it ought to be possible to produce circus movements by stimulating only one eye. In order to obtain such unilateral stimulation, a flap was fastened in the leather suit used in previous experiments so that it could be made to cover either eye. The individuals were placed so that they faced the light with only the area about the uncovered eye illuminated. Under these circumstances seventy per cent of the movements (Table VIII.) were not toward the light but toward the side bearing the uncovered eye. These reactions are what might be expected from a positively phototropic species like the toad, as similar responses have been observed in many other animals. For example, circus movements have been noted in several arthropods after one eye had been blackened over or excised, by Holmes (:01, :05), Parker (:03[a]), and Rádl (:03). No observations

TABLE VII.

PHOTIC REACTIONS OF TOADS STIMULATED THROUGH THE EYES ALONE.

Direction of movement		+	−	0
Individual No. 1		73	14	13
Individual No. 2		72	22	6
Reactions	Number	145	36	16
	Per cent	72	18	10

TABLE VIII.

PHOTIC REACTIONS OF TWO TOADS FACING TOWARD THE LIGHT AND STIMULATED THROUGH ONLY ONE EYE.

Condition of individuals		Right eye covered			Left eye covered		
Direction of movement		Right	Left	+	Right	Left	+
Individual No. 3		33	45	22	63	21	16
Individual No. 4		0	97	3	66	5	29
Reactions	Number	33	142	25	139	26	35
	Per cent	17	71	12	69	13	18

of exactly this kind have been made on amphibians, although Torelle (:03, p. 474) found that a frog went toward the light with the long axis of the body oblique to the direction of the rays, or made circus movements, after one eye had been covered. She made no attempt, however, to stimulate the eye without also affecting the skin.

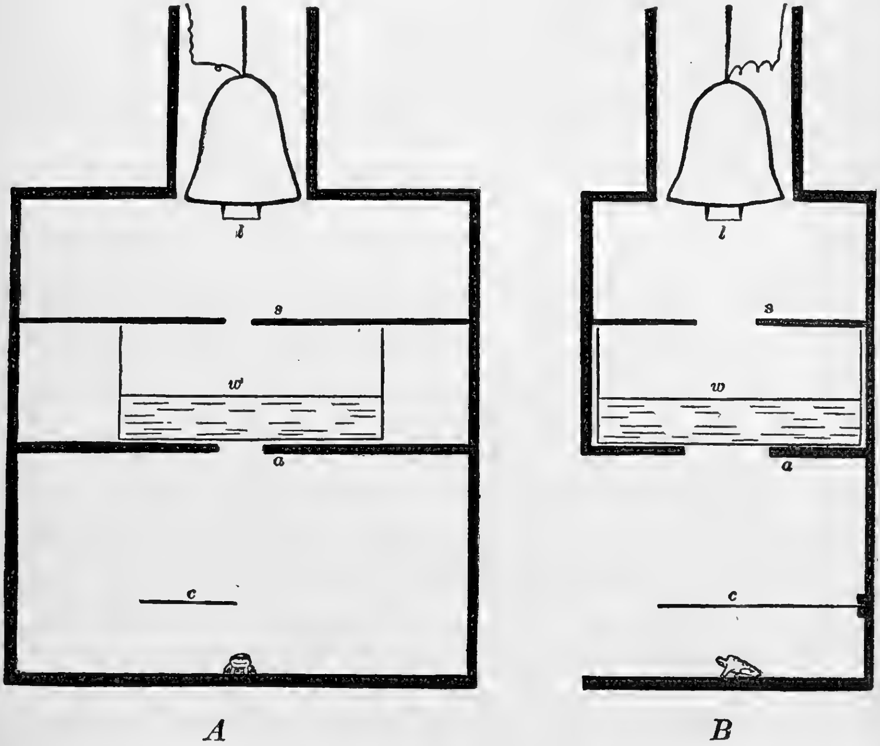

FIGURE 4. *A*, front sectional view through the middle of the apparatus for testing eyeless frogs under unilateral stimulation; *B*, sectional view from the side. *a*, wooden support for heat screen, which contained an oblong opening; *c*, adjustable screen of blackened sheet iron; *l*, source of light; *s*, black cardboard screen; *w*, glass dish containing water.

From these experiments it is apparent that the photic reactions of the toad, which are brought about by stimulation through the eyes, are due to intensity differences in the illumination of the two eyes, and the direction of the light rays is apparently of no significance.

D. The Reactions of Eyeless Toads to Unilateral Stimulation by Light from above.

The last experiments described showed that a toad would turn toward the illuminated side when only one eye was stimulated, even when such a movement did not take it into a region of greater light intensity. The next question which suggested itself was whether eyeless individuals would make similar movements when only one side was stimulated. In solving this problem, the apparatus shown in Figure 4 was used. It consisted of a wooden box (sixty centimeters high, forty-five wide, and twenty-eight deep) which was lined throughout with two layers of black cloth, except the floor, which was of slate. Light coming from above (*l*) passed through oblong openings in two screens (*a*, *s*) so that an area a little larger than a toad was illuminated on the floor of the apparatus, where the light intensity was 413 candle-meters. Each toad was so placed that the right and left sides were alternately illuminated, and an accurate unilateral division of light and shadow was secured by using a small movable screen (*c*) of blackened sheet iron.

In preparing individuals for these and subsequent experiments, a different method was used for excising the eyes from that followed heretofore. Instead of removing the whole upper jaw, a horizontal cut was made just above the nostrils, which met a vertical cut behind the eyes. The roof of the mouth was thus left intact, and there was consequently no interference with the respiratory movements. The plan followed in experimenting was to orient the individual facing the observer before each of the first ten reactions, while for the last ten it was faced in the opposite direction. Before and after the tests with light from above, each toad was tested ten times with light of the same intensity (413 candle-meters) from the side. The results of the reactions (Table IX.) with the light from above show a turning toward the side illuminated in seventy per cent of the cases, and, while the positive phototropism of the same individuals was slightly greater when they were illuminated from one side, the difference does not amount to enough to be significant. It may therefore be said that the positive phototropism of eyeless toads is due to intensity differences on the two sides of the body.

Payne (:07) has performed experiments of the same kind with the blind fish, Amblyopsis spelaeus, after the eyes had been excised, and obtained similar results. Apparently the direction of the light rays, as distinguished from intensity differences, has no influence on the reactions of either of these species.

TABLE IX.

Reactions of Six Eyeless Toads to Vertical and Horizontal Light.

Direction of light		Light from side			Light from above						Light from side		
Regions illuminated					Light on right side			Light on left side					
Direction of movement		+	–	0	+	–	0	+	–	0	+	–	0
Reactions	Number	54	1	5	42	10	8	48	5	7	38	9	13
	Per cent	90	2	8	70	17	13	80	8	12	62	15	23

E. The Effects of Illuminating Small Areas of Skin on Eyeless Toads.

In order to test the reactions of eyeless toads to local stimulation by light in various regions of the skin, individuals were placed two centimeters behind a screen containing a circular opening 3.2 millimeters in diameter, through which a horizontal beam of light passed. To render the rays of light as nearly parallel as possible a large condensing lens was interposed between the screen and the light. A small area of skin could thus be strongly stimulated by light; the light used had an intensity of 474 candle-meters. The three regions shown by the dotted areas in Figure 5 were stimulated, and they may be designated as the regions of the front leg, the hind leg, and the back. Before each of

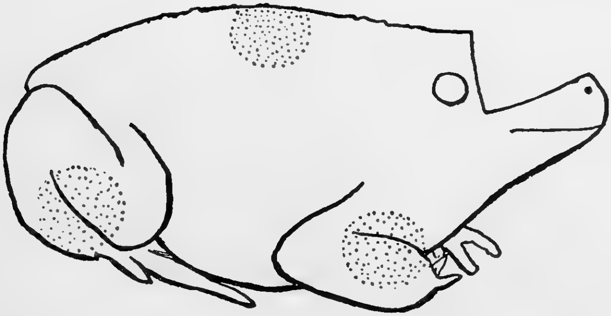

Figure 5. Toad, viewed from right side. The dotted areas indicate the regions illuminated.

the tests the individuals were tried in light of lesser intensity, but applied to the whole surface of the body, to see that they were positively phototropic.

TABLE X.

Local Skin Illumination of Eight Eyeless Toads.

Regions illuminated		Whole body			Front leg			Hind leg			Back		
Direction of movement		+	−	0	+	−	0	+	−	0	+	−	0
Reactions	Number	104	18	20	15	6	5	74	21	21	18	10	4
	Per cent	73	13	14	57	23	20	64	18	18	56	31	13

The experiments (Table X.) showed the toad to be positively phototropic in response to stimulation received through each of the regions tried, and there was no reason to assume that one region was more sensitive to such stimulation than another.

TABLE XI.

Summary of Daily Series of Twenty Reactions by Eleven Toads after Previous Exposure in the Light or in the Dark.

		Previously in dark			Previously in light		
Direction of reaction		+	−	0	+	−	0
First reaction	Number	29	20	6	33	10	3
	Per cent	53	36	11	59	17	23
First 5 reactions	Number	188	73	11	174	58	42
	Per cent	69	27	4	64	21	15
Last 15 reactions	Number	668	116	5	589	65	9
	Per cent	84	15	1	88	10	1
Total reactions	Number	851	179	16	763	121	51
	Per cent	81.3	17.1	1.5	81.6	12.9	5.5

Payne (:07) has shown a similar condition in Amblyopsis. He states (p. 323) that these fishes "seem to be equally sensitive on all parts of the body," after the eyes have been excised. Parker (:05[b], p. 419) and Reese (:06, p. 94) have, on the other hand, found the tail to be the most sensitive region in Ammocœtes and Cryptobranchus respectively. These few observations indicate that the comparative sensitiveness of the skin to photic stimulation varies in different species of vertebrates.

F. The Effect of Previous Conditions of Light Stimulation on Photic Reactions.

It had been noticed in a general way during the preceding experiments that when a toad was placed near a strong light the first reaction was more often away from the light than any of the subsequent responses were, and that the first reaction was usually slower than those which followed. G. Smith (:05) has shown that, when Gammarus is exposed to light, a pigment migration takes place toward the proximal ends of the retinula cells, and that as this migration progresses the animal changes its reactions from indifferent to strongly positive. As a pigment migration, as well as other changes, takes place when the eyes of amphibians are exposed to light, it was thought that there might be a similar influence on the reactions in this case, and experiments were accordingly carried out to test this question.

In these experiments toads were placed in the centre of a box which was ninety centimeters long and thirty-eight wide. The floor and sides were of slate, and both ends were closed by glass heat-screens which contained a layer of water 3.75 centimeters thick. Light, which had an intensity of 220 candle-meters at the spot where the toads were exposed to it, was admitted from one end, and before each reaction the individuals were placed with the right and left sides alternately toward the source of light. Eleven toads were kept first in the dark for five days and then in the light (three candle-meters) of a gas jet for an equal period of time. The eyes were thus exposed continuously to uniform light or dark, except when the animals were removed for the experiments, which occupied about half an hour daily. By taking twenty records from each individual each day, an attempt was made to get a series of a hundred reactions from each individual under the two conditions of previous exposure to light and to dark. In all but three cases these attempts were successful.

The results in Table XI. show that the first reaction in a series of twenty has the least tendency to be positively phototropic and that subsequent reactions are increasingly positive. There is, however, no great difference between the responses of individuals previously exposed to light and those previously in the dark. In Table XII. the reactions of each animal are shown, and it will be seen that the individuals often vary widely in their different reactions. For example, toad No. 13 was negatively phototropic after being in the dark, but strongly positive after exposure to light. Although the effect of previous stimulation is marked in some individuals, yet when we consider the total number of reactions, almost the same percentage of positive phototropism is shown after prolonged exposure to the light as after a similar period in the dark. These results agree with those of Torelle ('03), who found that eight hours of exposure to light did not change the positive phototropism of the frog.

TABLE XII.

Reactions of Individual Toads previously in the Light or in the Dark.

Condition	Previously in dark			Previously in light		
Direction of movement	+	–	0	+	–	0
Individual No. 11	94	6	0	100	0	0
Individual No. 12	89	11	0	76	6	3
Individual No. 13	26	74	0	95	5	0
Individual No. 15	89	10	1	74	9	1
Individual No. 22	94	5	1	82	14	4
Individual No. 23	88	9	3	64	25	7
Individual No. 24	72	8	2	83	14	4
Individual No. 25	97	9	0	64	34	2
Individual No. 26	76	22	2	10	1	18
Individual No. 27	59	8	7	26	3	12
Individual No. 28	73	27	0	89	10	1
Total reactions { Number	851	179	16	763	121	51
Per cent	81.3	17.1	1.5	81.6	12.9	5.5

Table XIII. shows the times which elapsed before the reactions recorded in Table XII. took place. No records were included which did not show twenty successive reactions on the day considered. Under (*a*) sixty such sets of daily records are included, and under (*b*), forty-three sets. The toads reacted more slowly after having been kept in the dark than after they had been exposed to light. The difference is not great and cannot be considered very significant in showing optic influence. The results may, however, be interpreted as indicating that prolonged exposure to light renders the toad more photokinetic.

G. The Reactions of Amphibians to Lights of Different Colors.

In testing the reactions of animals to lights of different wave lengths the apparatus shown in Figure 6 was used. Animals were placed in the position shown in the figure, and after each reaction they were rotated clockwise through 180°. The right and left sides were thus brought alternately toward the light, which had an intensity of 612 candle-meters (for white light) at the point where the animals were placed. The different colors were obtained by passing the white light of a Nernst lamp through colored screens. These screens were solutions of various substances held in rectangular glass jars which could be easily interchanged.[3] The colors used were red, yellow, green, and blue, and, though they were not perfectly monochromatic, they did not overlap significantly in the spectrum.

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[3] The substances used in making the solutions and the ranges of the colors obtained from them, as determined by an Engelmann spectroscope, were as follows:

Colors.	Substances.	Amount in grams.	c.c. of water.	Wave-length in μ.
Red	Fuchsin	0.10	750	0.605–0.608
Yellow	Potassium bichromate and Copper sulphate	63.00 15.00	750	0.540–0.605
Green	"Lichtgrün" and Copper sulphate	1.50 5.00	750	0.460–0.530
Blue	"Bleu de Lyon"	0.15	750	0.430–0.485

TABLE XIII.

AVERAGE REACTION TIMES IN MINUTES OF TOADS PREVIOUSLY IN THE LIGHT OR IN THE DARK.

Number of the reaction	1	2	3	4	5	6	7	8	9	10
(*a*) Previously in dark	8.3	3.5	2.2	2.0	1.8	1.4	1.3	1.1	1.1	1.1
(*b*) Previously in light	5.9	3.5	3.4	1.6	1.8	1.4	0.8	0.8	0.9	0.8
Number of the reaction	11	12	13	14	15	16	17	18	19	20
(*a*) Previously in dark	1.1	1.0	1.0	0.9	0.9	1.0	0.8	1.0	0.7	0.8
(*b*) Previously in light	0.7	0.7	0.6	0.7	0.6	0.7	0.7	0.6	0.5	0.5

(a) *Normal Individuals.*

For the experiments with animals in normal condition, Rana palustris was used. Six individuals were successively tested with the colors in the following order, blue, green, yellow, red, and then this

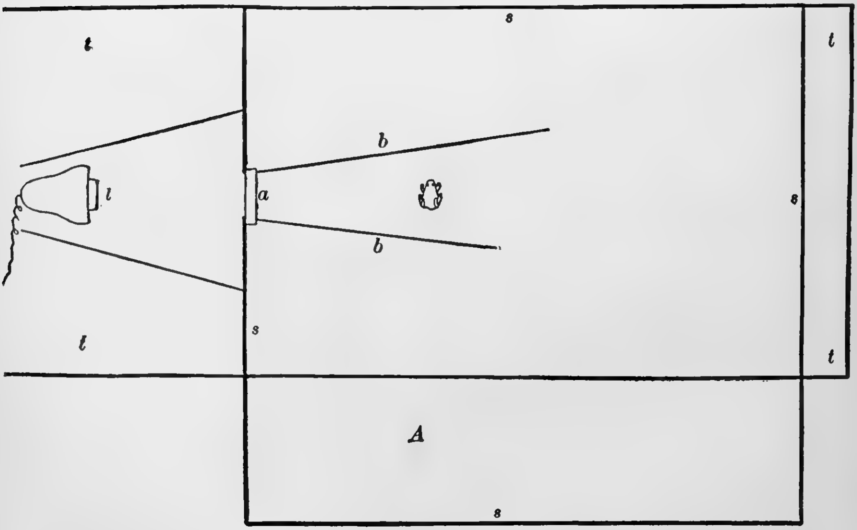

FIGURE 6. Plan of apparatus for testing the reactions of toads to colored lights. *A*, position of observer; *a*, heat and color screen; *b*, screen 25 cm. high; *l*, light; *s*, *s*, *s*, *s*, screen extending to ceiling; *t*, *t*, *t*, *t*, table.

order was reversed. The plan followed was to test all the individuals in one color and then to change the screen and test them again in the same order but with the next color; ten reactions being taken from each individual in every color. Each animal was thus actually subject to experiment for about one hour out of the six which were required to complete the series. A second half-dozen of frogs was tested in the same manner, except that the colors were used in the order red, yellow, green, blue, and then the order was reversed.

TABLE XIV.

Reactions of Rana palustris to Colored Lights.

Color of lights		Blue			Green			Yellow			Red		
Direction of movement		+	−	0	+	−	0	+	−	0	+	−	0
Reactions	First six individuals	96	13	11	80	14	26	69	20	33	66	16	39
	Second six individuals	81	25	14	68	38	14	55	39	26	45	29	46
	Total { Number	177	38	25	148	52	40	124	59	59	111	45	85
	Total { Per cent	74	16	10	62	22	16	52	24	24	46	19	35
Ave. reaction time in minutes		2.83			3.09			3.60			3.75		

The results (Table XIV.) show that blue is apparently the most effective in the production of positively phototropic reactions, and that there is a regular graduation from blue to red, both in the percentage of positive reactions and in the rapidity with which the movements took place. Other observers (p. 165) have obtained similar results in experiments with other species of amphibians. It is probable that these differences in the reactions are due to differences of the wave lengths, but they may be due to intensity differences.

(b) *Eyeless Individuals.*

The blue end of the spectrum is known to be more potent in affecting changes in the eyes of many animals, and in some species the sensitiveness to red is apparently lacking altogether. For example, Abelsdorff (:00, p. 562) observed that the pupil of the owl's eye enlarged in red light but contracted rapidly when it was exposed to blue light of low intensity. It therefore seemed not improbable that the differences in the frog's reactions to lights of different colors might have been due to stimulation received through the eyes; therefore another set of experiments was undertaken to ascertain if like results could be obtained through the stimulation of the skin alone.

As toads had been found to be more responsive than frogs after the eyes had been excised, they were used in testing the light reactions through the skin. The same apparatus (Figure 6) was used as in the experiments with normal animals, except that the light was passed through a square aperture, 2.7 centimeters on a side, and had an intensity of 874 candle-meters for white light at the point where the animals were placed. The method used for removing the eyes was the same as in previous experiments (p. 182). Three individuals were tested successively with white, red, yellow, green, and blue light in the order given. The next day two of the animals were tested again with the same colors but in the inverse order.

TABLE XV.

Reactions of Three Eyeless Toads to Colored Lights.

Color of lights		White			Red			Yellow			Green			Blue		
Direction of movement		+	–	0	+	–	0	+	–	0	+	–	0	+	–	0
Reactions	Number	48	1	1	38	2	10	37	2	11	38	5	7	37	3	10
	Per cent	96	2	2	76	4	20	74	4	22	76	10	14	74	6	20

It will be seen (Table XV.) that these toads gave about seventy-five per cent of positively phototropic reactions with every color. Apparently all the colors were equally effective in inducing photic responses. This fact is the more striking when we remember that the same color screens were used as in the experiments with normal amphibians (Table XIV.), in which case the blue was most potent. The reactions to white light, in the present instance, showed an almost perfect positive phototropism, and it seemed possible that the lesser degree of reactiveness shown in the responses to colored lights might have been due to differences in intensity, as the color-screens undoubtedly cut off much light. To ascertain if any difference would be manifest in the responses if the intensity were lowered, a diaphragm, having a circular aperture 2.8 millimeters in diameter, was interposed and experiments performed in which eyeless toads were placed at a distance of 275 centimeters from the lamp, where the intensity was 1.44 candle meters for white light. The colored screens cut the light down to what must have been considerably less than a candle-meter. The results obtained from seven toads not previously tested are shown in Table XVI.

TABLE XVI.

Reactions of Seven Eyeless Toads to Colored Lights of Low Intensity.

Color of lights		White			Red			Yellow			Green			Blue		
Direction of movement		+	–	0	+	–	0	+	–	0	+	–	0	+	–	0
Reactions	Number	84	6	10	56	26	28	76	18	16	65	21	24	59	28	25
	Per cent	84	6	10	50	23	27	68	17	15	58	19	23	53	25	22

Although the "positive percentages" in every color were lower than when light of greater intensity was used (Table XV.), the eyeless toads again showed positive phototropism in all the colors. There was also, in this case, a greater number of positive reactions when white light was used than when any of the colors were substituted for it. It is, then, apparent that in a decreased light intensity the number of positive reactions decreased, but no especial potency was shown by one color as compared with another as a means of inducing such reactions. The slight differences between the number of positive reactions produced by lights of different colors, as shown in the table, may be accounted for as being due to intensity differences. The colors, as judged by the human eye, could be arranged from more to less intense in the following order, yellow, green, red, blue; and it will be seen that the largest number of positive reactions was brought about by the most intense light, thus judged.

(c) *Summary.*

The results of the reactions of amphibians to colored lights may be briefly summarized as follows: normal animals were positively phototropic in all the colors tried, but there were more positive reactions toward the violet end of the spectrum than toward the red end; eyeless individuals were also positively phototropic in all the colors, but there was no difference in number between the positive reactions to the several colors. These results do not agree with those of most other observers.

In fact, Loeb ('**88**) has stated as a general law, that the more primitive the photoreceptor, the greater is its sensitiveness to the rays toward the violet end of the spectrum, as compared to those toward the opposite end. Graber ('**83**, p. 225) stated that in the phototropic responses of Triturus the rays became more and more like darkness in their effects as the red end of the spectrum was approached; and that this was true of eyeless individuals as well as those in normal condition. Dubois ('**90**, p. 358) observed that blue was more effective than red in producing responses from a blinded Proteus when only the tail was illuminated. Opposed to these observations are those of Kühne ('**78**^{b}, p. 119), who found that, while normal frogs rested in green when there was equal opportunity to rest in blue, blinded individuals showed no such reactions. The results described in the present paper agree with those of Kühne, and it seems to be evident that the photoreceptors in the skin of the frog and toad have little or no sensitiveness to color differences, as such.

H. Comparison of the Reactions of Eyeless Toads to Heat and to Light.

It has long been known that the skin of amphibians could be stimulated by heat, and the opinion has been expressed that there are receptors which are open to stimulation by either heat or light. Korányi ('**93**) showed that heat, as well as light, might produce motor reactions when it was applied to the skin of a frog. Parker (:**03**^{b}, p. 34) says: "It is conceivable that in the lower vertebrates, like the frog, the end organs of the skin are stimulated by radiant energy of a wide range, including what is for us both radiant heat and light, and that the descendants of these organs in the skins of higher vertebrates are more restricted in function and are ordinarily sensitive to radiant heat and its effects." Washburn (:**08**, p. 142) also says, "While, then, the nerve endings in the human skin are sensitive only to the slowest of these vibrations, the heat rays, those in the skin of the frog, may respond to the whole series."

During the experiments with eyeless toads the question arose as to whether the supposed photic reactions might not, after all, be due to the influence of heat. And, although a heat screen containing water was used in all experiments, there was a possibility that the light was converted into heat as it was absorbed by the skin, and that the sensitiveness was to heat rather than light. Furthermore, the part of the apparatus containing the lamp was warmed somewhat during a series of experiments and gave off a small amount of heat. A crude test as to the effect of this heat from the apparatus was made in the following way: On two occasions when a toad had gone successively ten times toward the light, an opaque screen was interposed in such a way that the light was cut off but the radiating heat from the apparatus was allowed to reach the toad. In both instances the individuals gave ten reactions without apparent reference to the heated apparatus, thus showing that the reactions had not been brought about by heat.

In order to test the sensitiveness of the toad to increased temperature, two eyeless individuals were suspended in such a way that the hind legs could be dipped into water. Neither of these animals made any movement under this method of treatment when the water was at room temperature (20° C.). The temperature of the water was then raised five degrees at a time, and there was no response until a temperature of 40° C. to 45° C. had been reached, when the animals quickly withdrew their legs from the hot water. It was evident, from these results, that the toad did not respond *readily* to increase in temperature. Reese (:06) found that Cryptobranchus also was comparatively insensitive to changes in the temperature of the surrounding medium, but, if the temperature was raised above 40° C., violent motor reactions occurred.

While these observations showed that amphibians might not be very sensitive to thermic stimulation, the possibility was not excluded that the assumed photic reactions might in reality be due to stimulation of the skin receptors by heat. If the positively phototropic reactions of blinded toads were due to the stimulation of such receptors, it ought to be possible to obtain similar reactions through the use of radiant heat instead of light. To ascertain if this were possible, an apparatus was arranged in which steam was passed through a vertical brass pipe which measured seven millimeters in diameter. The eyeless toads were placed near this pipe, and their reactions tested in the same manner as had previously been done with light. All these experiments were performed in the dark, but before and after the heat experiments each individual was tested with light (1.24 candle-meters) to ascertain whether it was positively phototropic or not. The method of experimenting in the dark was to orient the toad by using a mark at a known distance from the source of heat; then to listen until a movement was heard; after which the position of the animal was ascertained by feeling for it with the hand. In Table XVII. the signs +, —, and 0 are used to indicate movements in relation to the steam pipe as a source of heat, as they have previously been used for sources of light. As this table shows, toads placed near (10 to 20 cm.)

the heated pipe showed a slight tendency to move away from it, but beyond twenty centimeters they were apparently indifferent.

The amount of heat given off by the steam pipe as compared to that given off by the light apparatus was determined by means of a pair of thermometers. These thermometers were mounted in a wooden box (Figure 7), blackened inside and out and divided into two freely communicating compartments in each of which the blackened bulb of one of the thermometers (*A*, *B*) was enclosed. One of these compartments was permanently closed, while the other could be opened or closed at will by a slide (*d*). This apparatus was placed in such a position that the radiant heat to be measured fell directly upon the bulb of the thermometer *B* when the slide was out. After reading the thermometers at intervals and allowing the apparatus to become adjusted to the surroundings for two hours, the difference between the two thermometers was observed at one-minute intervals for twenty minutes while the compartment was open to receive the light or heat to be tested, and then for a like period of time with it closed. The

TABLE XVII.

REACTIONS OF FOUR EYELESS TOADS TO LIGHT AND TO RADIANT HEAT.

Nature of stimulation	Light			Distances from a hot pipe, in centimeters														
				10			20			30			40			50		
Direction of movement	+	−	0	+	−	0	+	−	0	+	−	0	+	−	0	+	−	0
Reactions No.	108	6	16	16	29	5	17	25	8	64	59	27	16	34	10	25	21	14
Reactions Per ct.	83	5	12	32	58	10	34	50	16	44	40	16	30	54	16	40	34	26

Nature of stimulation	Distances from a hot pipe, in centimeters															Light		
	60			70			80			90			100					
Direction of movement	+	−	0	+	−	0	+	−	0	+	−	0	+	−	0	+	−	0
Reactions No.	13	9	8	15	14	11	15	13	2	15	11	4	14	18	10	131	21	18
Reactions Per ct.	42	30	28	38	35	27	50	42	8	50	37	13	34	43	23	77	13	10

average difference between the two thermometers, when placed before the steam pipe was 0.064° C. while that for the light apparatus was 0.025° C. The amount of heat received by a thermometer at a distance of thirty centimeters from the heated pipe was therefore more than twice that received when the light apparatus was tested. As the toads were strongly positively phototropic to this light, and as the same individuals were indifferent when placed near the steam pipe, it is safe to conclude that thermo- and photo-reception are distinct processes in the toad's skin, and that, in this animal at least, heat does not give rise to tropic reactions unless there is very strong stimulation.

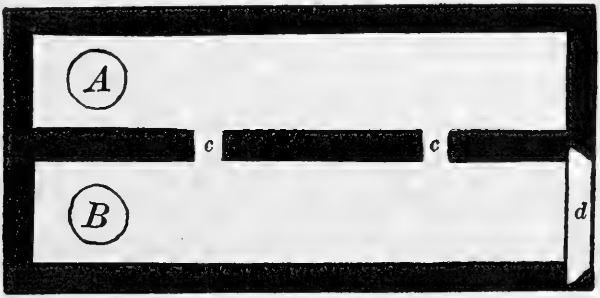

Figure 7. Plan of thermometer box. *A* and *B*, thermometers; *c*, *c*, positions of two of the ten circular openings between the two compartments; *d*, slide.

I. Experiments to determine the Influence of the Central Nervous Organs on the Photic Reactions of Amphibians.

Parker (:**05**ª) succeeded in obtaining photic responses from one of the lower fishes (Ammocœtes) after the entire brain had been removed, and he believed that such reactions were brought about by stimulation received through skin receptors and transmitted through the spinal nerves. To ascertain if similar reactions could be obtained from amphibians, experiments were undertaken with four species. The first to be tested was Rana pipiens. A sharp scalpel was inserted through the dorsal wall of the cranium and a transverse cut was made through the diencephalon ; this was followed by another cut behind the second vertebra which separated the cord from the myelencephalon. After such individuals had been tested, they were killed and hardened in alcohol. Subsequent dissection showed that the cuts had been successfully made in ten of the twelve individuals upon which operations had been performed. This method of procedure separated the cord from the brain, but did not interfere with the vital centres in the latter nor with the sympathetic system. These frogs were tested several times, for the two or three days during which they lived, by suspending them at the anterior end in such a way that the hind legs could be subjected to various stimuli. All of these individuals flexed the legs when they were touched with a brush which had been moistened in ten per cent acetic acid, and four of them reacted in the same manner when the light and heat from a Nernst lamp was thrown on the skin, a lens being used to bring the light to a focus; but not a single individual reacted to light from this lamp when the heat rays were cut off by interposing a flat-sided jar filled with water.

Ten toads were tested by the same methods as those used for the frogs, and, though they reacted to acid and the light with heat, no reactions were obtained when light alone was used.

As no photic reactions had been obtained from spinal frogs or toads, it was thought that such responses might be induced if the animals were rendered more sensitive; and experiments were accordingly undertaken in which the diencephalon and cord were transected in nine toads and 0.001 grain of strychnine inserted into the dorsal lymph space through a small slit in the skin. The individuals which had been treated in this manner were extremely sensitive to tactual stimuli, and the slightest jar of the table on which they were supported sufficed to throw their limbs into a state of spasmodic extension. When, however, a beam of light was focussed on the hind leg of such an individual, no indubitable responses were obtained.

Since the attempts to induce photic reactions in terrestrial amphibians had met with no success after the brain had been separated from the cord, I next turned my attention to the available aquatic species. The eyes were removed from a single Cryptobranchus, and its cord was cut behind the first vertebra. This individual was then placed in an aquarium, and light from a Nernst lamp was focussed upon its skin in various regions; and, although it had been found to be extremely responsive to light after the eyes had been removed, no such responses were obtained from it after the cord had been cut. It nevertheless continued to respond to tactual stimulation, and when the side was stroked gently with the finger, it jerked its legs and drew its tail away from the stimulated region. Chemical stimulation was also effective after the cord had been cut, for when a pellet of cotton moistened with ten per cent acetic acid was placed so that it touched the tail, the body was bent away from the stimulated area.

As the experiments with Cryptobranchus had given only negative results, it was determined to make cuts in various regions of the cord in different animals and determine whether the individuals thus treated would show differences in their behavior. The eyes were accordingly removed from four specimens of Necturus, and the cord was cut behind the fourth, ninth, eleventh, and twentieth vertebræ in the respective individuals. All these animals gave marked reactions to light when the illumination was anterior to the cut in the cord, but no responses were obtained from the region posterior to this cut, even when a strong beam of light was focussed on the skin. The regions posterior to the cut were, however, influenced by certain forms of stimulation, and responded by making withdrawing movements when they were stroked with a brush, or when cotton saturated with ten per cent acetic acid was placed in the water near them. All the individuals seemed to stand the operation well; the gill movements continued in a normal manner, and walking was carried on by the front legs, while the posterior part of the body dragged behind. All these animals lived more than five days, and one of them (with its cord cut behind the eleventh vertebra) lived thirty-six. This particular individual was extremely active, and when the front part of the body was in motion the hind legs also made walking movements, though they had a slower rate than that of the front legs. Furthermore, by gently pinching the tail the hind legs could be induced to walk when the front legs were quiet. In swimming, however, the trunk muscles of the whole body moved together. Loeb (:03) noted similar correlated swimming movements in Amblystoma larvæ after the cord had been transected. Notwithstanding such correlated movements, it may be said of the four specimens of Necturus that the parts of the body in front of and behind the cut in the cord carried on reactions more or less independently, and that the regions anterior to this cut responded to a greater range of stimuli.

As none of the spinal amphibians tested showed sensitiveness to light, even when reactions were easily induced by other forms of stimulation, it seems reasonable to conclude that their lack of sensitiveness to photic stimulation was not due to the absence of receptive or motor power, but to the fact that the ultimate control (centres or essential portions of reflex arcs) of these reactions lies in the brain and therefore anterior to the spinal cord.

In order to discover what parts of the brain were essential for the photic responses, experiments were carried out in which certain regions were excised and observations made of the deficiency phenomena thus brought about. The method followed was to excise all parts of the brain anterior to a certain region, and to carry the regions excised progressively backward in successive operations; the light reactions being tested at each step. On account of the large size of their brains, Necturus and Cryptobranchus were used for these experiments. The individuals were wrapped in a damp cloth, the head being allowed to protrude; and a T-shaped incision was then made in the skin on the dorsal side of the head, the stem of the T being toward the anterior end; after this the muscles were cut away and the bony roof of the cranial cavity carefully picked away with a pair of strong forceps. The brain was then cut across with a pair of scissors or a sharp scalpel and the parts anterior to the cut removed. The flaps of skin were drawn over the wound and stitched together with silk thread. The success of such operations was verified by subsequent dissection. The method used in testing photic reactions was to throw a vertical band of light (which had an intensity of about 220 candle-meters at the point where the animals were placed) upon the anterior or posterior end of an individual, and to observe the responses which took place. As such responses were like those previously described (p. 169), they need not be discussed in detail.

For a preliminary test as to the effect of such an operation as has just been described, aside from the actual cutting of the brain itself, the roof of the cranial cavities was removed from four individuals and the brain was left exposed to the water in which they were kept. These individuals seemed to be little affected by the operation, as they swam and walked in a normal manner; and when (twenty-four hours later) light was thrown on the anterior or posterior end of any one of them, it reacted in the same manner as an individual in which only the eyes had been excised. The exposure of the brain had, then, no obvious effect on the photic reactions of Necturus.

The eyes and telencephalon were next removed from six individuals, and five of them gave marked responses to light on the day after the operation. The other individual, which lived for fifteen days, gave no photic responses until the third day after the cerebral lobes had been excised, though it had apparently recovered from the operation before that time. These animals could doubtless have been kept alive for a long time if it had not been for the Saprolegnia which grew abundantly around the cut surfaces, and, even with this handicap, one of them lived for fifty days. The cerebral lobes are not, then, essential for the photic reactions of Necturus.

Owing to the scarcity of material, the number of operations had to be limited in the remaining experiments. The portions of the brain anterior to the mesencephalon were, therefore, excised in only one Necturus. This individual lived for twelve days and gave characteristic reactions when it was touched gently on the foot or tail, or when cotton which had been moistened in ten per cent acetic acid was placed in the water near it. When it was turned on its back, the righting reaction occurred, though this was accomplished with some difficulty. Light, however, called forth no response, even when a condensing lens was used to bring the rays to a focus on the skin. The investigations of Schrader ('87) and Loeser (:05) have demonstrated the fact that the mesencephalon exerts an inhibitory influence on those reflex actions that take place through the spinal nerves. These observers found that frogs were more responsive to external stimulation after the brain had been excised so as to leave only the myelencephalon than when such an operation did not include the mesencephalon. In other words, the midbrain had an inhibitory action on the reflexes controlled by the portions of the brain posterior to it, and when the more anterior brain regions (which originate the "spontaneous" reflexes) had been removed it rendered the frogs unusually sluggish. It is probable that the mesencephalon exerts a similar influence in other amphibians, and that the lack of responsiveness in Necturus was due to inhibition rather than lack of ability to respond to light. The following experiments support this view.

The portions of the brain anterior to the metencephalon were removed in two specimens of Cryptobranchus. Both these individuals were restless and usually continued to move about slowly for some time after locomotion had once been induced by any form of stimulation. When either of them was kept in dim light for an hour or two, however, it became quiet, and, if it was afterwards suddenly illuminated (with light having an intensity of about a thousand candle-meters), there was in most cases an active locomotor response and the movement continued for some time, even after the light had been shut off.

As the metencephalon is poorly developed in all amphibians, and as it has been shown to exert little, if any, influence on their ability to perform locomotor reactions, it is safe to conclude that the myelencephalon and the cord are the only portions of the central nervous system which are essential for the photic responses.

III. DISCUSSION AND CONCLUSIONS.

Photic responsiveness is a quality which is probably present in all amphibians, for the sixteen species which have been found to give reactions to light include representatives of most of the families of the class. Light has an orienting influence on all the species which have been studied; the Caudata are mostly negative in their phototropism, while the Salientia are positive. Such reactions are easily conceived to be of benefit to the different species under their ordinary conditions of environment, but whether the different types of reactions have arisen as the result of natural selection in the development of each species, or whether they are due to structural peculiarities which limit each species to certain stereotyped reactions and have hence caused it to frequent a particular habitat, or whether they have been brought about by other factors, are open questions. The negatively phototropic reactions of the nocturnal species would serve to bring them into places of concealment during the day. The positive reactions of the more diurnal forms would lead them toward the water (a large illuminated area) and thus facilitate their escape from pursuing enemies, or would take them into the bright sunlight, where insects were abundant and their hunger would be satisfied.

Under artificial conditions light has been shown to have a directive influence on the movements of all the amphibians which have been made the subject of experiment, but it does not follow that the presence of light will *induce* motor reactions in all these species, and there is, in fact, great variation between the different forms in this respect. For example, Cryptobranchus is strongly photokinetic and becomes restless when suddenly illuminated, while Necturus is comparatively indifferent to such stimulation. This photokinetic quality is apparently little developed in frogs and toads, though they are strongly phototropic. Generally speaking, there seems to be no correlation between the photokinesis and the phototropism of amphibians.

A given individual of any species is seldom consistently positive or negative in its phototropism, even when the conditions of light stimulation are uniform. This may be due to the influence of internal factors which bring about changes in the physiological state of the animal, or to external stimuli other than light which exert a modifying influence. Some of these modifying factors will be briefly considered, as far as they apply to the amphibians. Broadly speaking, the habits of the different forms are correlated with their phototropic responses and the species which are most truly terrestrial (Bufo americanus and Rana sylvatica) are most strongly positive, while the typical aquatic forms (Cryptobranchus allegheniensis and Necturus maculosus) are as decidedly negative. Therefore any variation from the conditions found in the normal habitat of a species might involve changes which would alter its ordinary phototropic responses. Previous exposure in light or dark does not usually exert a marked influence on the photic reactions of the toad, but some individuals were found to be positive after having been in the light, though they were negative after passing a similar period in the dark. Mechanical stimulation serves to initiate reactions which are directed by light, but it produces no marked changes in phototropism. Fatigue makes the photic responses more difficult to induce in some cases (e. g. Cryptobranchus), but does not alter their character. These few examples are typical and will serve to illustrate the influence of many factors on the photic reactions of amphibians. In general it may be said that, while various factors may give rise to changed phototropic responses in some individuals, the same factors may be without apparent influence in others. No stimulus, with the possible exception of decreased temperature (Torelle, :03) has been demonstrated to produce uniform changes in the light responses of amphibians. The internal causes which produce negative reactions in one species, or even in one individual of a species, while the same external conditions call forth positive reactions in other species or individuals, is practically an untouched field as far as the amphibians are concerned. The careful study of such a form as Diemyctylus, which undergoes marked changes in habitat during its life, ought to throw light on at least one aspect of this matter.

The next subject that deserves consideration is the nature of the photoreceptors upon which the sensitiveness of amphibians to light depends. There are at least two sets of nerve terminations which are open to photic stimulation, those of the retina and those of the skin. The investigation of the responses produced by light received through these two sets of endings is involved in considerable difficulty, for we are obliged to refer constantly to judgments formed through the human eye. We are able to form opinions as to the direction, intensity and color of light, and to judge the form, size, color, position, and movement of illuminated objects as they appear through our own eyes, but we have no conception of how these things appear when they are seen through the eyes of an amphibian, except as we can interpret its actions, and the problem becomes even more difficult when we attempt to consider the reception of light through the skin. There is some evidence that nervous connections exist in amphibians between these two kinds of photoreceptors and this complicates the matter still farther. Englemann ('85) observed that retinal changes were induced in the eyes of frogs by illuminating the skin. Furthermore, Fick ('90) found that the same changes took place after the optic nerves had been cut, and connections, if they exist, must therefore take some other course, in part at least, than that through the second nerve.

The eyes of amphibians are adapted for use in both air and water, and are hence not finely adjusted for visual discrimination in either medium. Binocular vision cannot be present, as the eyes are placed laterally, so that there is probably no overlapping in the fields. Nor is any definite image formed, as Beer ('**98**) has shown that the eye cannot be accommodated to any extent, and amphibians therefore depend upon motion rather than the form of objects to warn them of danger or to enable them to capture food. A frog or toad will allow a worm to lie in full view as long as it is quiet, but as soon as the worm moves it is devoured. The vision of amphibians is therefore limited to rather ill-defined outlines of the surrounding objects, and the comparative brightness or dulness, or possibly the colors, of objects will have considerable importance in determining the nature of the responses of an individual. The reactions brought about when the eyes alone are illuminated are similar to those which take place when such stimulation affects both the skin and eyes. When only one eye is stimulated by light coming from in front of a toad, the individual usually does not go toward the light but turns toward the stimulated side. These facts indicate that the eyes in their relations to objects in the field of vision serve more as direction eyes than as camera eyes. Cole has recently given additional support to this view by showing that amphibians placed between two lights of equal intensity but of different areas go toward the larger area ; thus demonstrating that the size of the area illuminated is of importance in the visual processes. Kühne ('**78**^{a}) has shown that the eye of the frog is sensitive to light rays from the whole range of the visible spectrum, and the results described in the present paper, as well as those of other observers (p. 165), indicate that the rays toward the violet end are most effective in producing photic responses. These apparent differences in sensitiveness to what appear to the human eye as colors may, however, be only differences in intensity when received by the frog's eye.

The skin is known to act as a photoreceptor in ten representative species of amphibians, and individuals show tropic reactions which are like those of animals in normal condition after their eyes have been excised. There is no great differentiation shown in the structure of the nerve endings in amphibians' skins, and Parker (:**03**^{b}, p. 34) has already been quoted as saying, "it is conceivable that in the lower vertebrates, like the frog, the end organs of the skin are stimulated by radiant energy of wide range, including what is for us both heat and light." There seems to be no doubt, however, that the amphibian skin is sensitive to light as such, and no tropic responses are induced by radiant heat having the same energy value as the light which does induce marked tropic reactions. Our knowledge of the comparative sensitiveness of the skin in different regions of the body is rather limited, but it shows that there is no uniformity among different amphibians in this respect. Cryptobranchus is most responsive when the tail region is illuminated, but the skin of the toad is equally sensitive on all parts of the body.

The fact that both the skin and eyes act as photoreceptors in fishes as well as amphibians has led to considerable speculation concerning the origin of the retina in higher vertebrates. Various theories have been put forward, but only two of them have direct relation to the field included in the present paper. Willem ('91) advanced the view that in its primitive condition light sensitiveness was distributed over the whole skin and that it had become gradually localized in the eyes of higher forms. Parker (:08) has pointed out an objection to this view in the fact that photic sensitiveness is lacking in the skin of the most primitive member of the vertebrate series (Amphioxus), though it possesses direction eyes which are closely connected with the central nervous organs. He believes that the development of photoreceptive power in the skins of vertebrates has been a separate process from that of the development of the retinas, which first arose in intimate connection with the central nervous system. This question cannot be regarded as definitely settled, and the results of the experiments described in the present paper throw little light upon it. The fact that photic sensitiveness is present in such a wide range of amphibians seems to support Willem's view, as the different forms have developed along extremely diverse lines.

Not only do the photoreceptive organs constitute important factors in a consideration of the photic reactions of amphibians, but variations in the light itself are important. Differences in intensity are significant in the reactions of the toad, for the percentage of positively phototropic responses decreases and the number of indifferent reactions increases when the light intensity is decreased. The direction of the incident rays of light which impinge on the photoreceptor is, however, of no apparent consequence. A toad in which only one eye is illuminated by light from in front turns toward the stimulated side instead of going toward the light, and an eyeless toad subjected to unilateral stimulation by light from above turns toward the illuminated side without regard to the direction of the rays. In general, then, the photic reactions of amphibians are brought about by intensity differences on the two sides of the body. Concerning the influence of the quality of the light, it may be said that both the skin and eyes of amphibians are open to stimulation by light rays which include the whole range of the visible spectrum. When the light is received through both the eye and skin receptors, the rays toward the violet end of the spectrum are most effective in producing tropic responses, but when the light is received through the skin alone, no such potency is shown by the more refrangible rays. The differences observed in the first case may therefore be interpreted as being due to stimulation received through the eyes, and we may conclude that the power of color perception, as distinct from light perception, is present in the eyes but absent in the skin. It is not certain, however, that these differences, which are supposedly due to differences in wave length, are not, after all, brought about by intensity differences.

Generally speaking, the parts of the central nervous system are segmentally arranged throughout the vertebrate series. Each neural segment is, however, capable of carrying on only the comparatively simple reflex actions which are concerned with the somatic segment which it controls. The complex reactions which involve correlated movements in different regions of the body depend upon correlation centres, and, the higher we go in the vertebrate scale, the more these centres become localized toward the anterior end of the nervous tube. A spinal eel is able to swim in a normal manner (Bickell, '97), but in the higher vertebrates spinal reactions show less correlative power, and there is a correspondingly greater importance attached to those reactions which are controlled through the brain. The fact that spinal fishes react to light (Parker, :03b), while spinal amphibians do not, is therefore perhaps to be expected and may be interpreted as new evidence of the progressive anterior localization of functions in the nervous system of vertebrates. However, Sherrington (:06, p. 9) has called attention to the fact that only stimuli of a particular kind will evoke certain reflexes. He was easily able to induce the croak reflex in a spinal frog by certain forms of stimulation, but he could not evoke it by others, and he also found that the scratch reflex could be called forth in spinal dogs by certain forms of tactual stimulation only. It is therefore possible that spinal amphibians may yet be induced to give photic reactions under some new method of stimulation. As far as the present evidence goes, however, the myelencephalon, as well as the cord, is essential for photic responses in which the skin is the receptor.

In the reactions of many organisms the ultimate direction of locomotion is determined by making many random movements and following such of them as lead away from conditions unfavorable to the organism or into conditions better adapted to its existence. Other organisms do not make great use of this method, but usually move directly toward or away from the source of stimulation, and Loeb ('90) has given the name of tropism to such responses. The light reactions of amphibians are characteristically tropic in nature, and, as has been stated, they are apparently brought about by unequal stimulation on the two sides of the body. This tropic character applies to the reactions whether they are induced by stimulation through the skin or eyes or through the simultaneous stimulation of both. In general, it may be said that the photic responses are of a typically reflex character and show little evidence of powers of association.

IV. SUMMARY.

(1) The following amphibians were found to be positively phototropic: Diemyctylus viridescens, Rana clamata, R. palustris, Bufo fowleri, B. americanus; and the negatively phototropic species studied were: Necturus maculosus, Cryptobranchus allegheniensis, Amblystoma punctatum, Plethodon cinereus erythronotus.

(2) Most of the species mentioned under (1), after the removal of their eyes, gave photic responses which were like those of normal individuals.

(3) The photic reactions of eyeless amphibians are not due to the direct stimulation of the central nervous system or the exposed ends of the optic nerves by light, but to the action of the skin as a photoreceptor.

(4) Mechanical stimulation (handling) does not change the character of the photic reactions, though it makes them more evident by inducing locomotion.

(5) Toads which are stimulated by light through the eyes alone react in the same manner as individuals stimulated through the skin or through both the skin and the eyes.

(6) The movements of eyeless toads stimulated unilaterally by light from above are toward the illuminated side; and toads stimulated through one eye only by light from in front do not go toward the light but turn toward the illuminated side. The photic reactions are therefore due to differences in light intensity on the two sides of the body and the direction of the rays is ineffective.

(7) After the eyes have been removed, Cryptobranchus and Necturus are most responsive when the tail is illuminated, but the skin of the toad is apparently of equal sensitiveness on all parts of the body.

(8) A prolonged period of time passed in light or dark had no effect on the nature of the phototropic responses of the toad.

(9) Cryptobranchus is strongly photokinetic, but in the other amphibians tested this quality was not strongly developed.

(10) When normal amphibians were used, blue light was the most effective in the production of tropic responses, but when eyeless indi-

viduals were tested with the same colored lights, the rays toward the blue end of the spectrum showed no such potency as compared with those nearer the opposite end. It may be said that, while both the skin and eyes are sensitive to the whole range of the visible spectrum, color sensitiveness is present only in the latter. It is possible, however, that the supposed color sensitiveness is due to the effects of what are intensity differences to the amphibian eye.

(11) A decrease in the intensity of the light brings about a correspondingly smaller number of positively phototropic responses and an increase in the number of indifferent reactions.

(12) The phototropic responses of eyeless toads are not due to the stimulation of heat-receiving organs in the skin. Thermo- and photo-reception are separate processes, and the former does not readily give rise to tropic reactions.

(13) Spinal amphibians gave no photic responses, but such reactions were induced in animals in which the brain anterior to the metencephalon had been excised.

V. BIBLIOGRAPHY.

Abelsdorff, G.
:00. Zur Erforschung des Helligkeits- und Farbensinnes bei Menschen und Thieren. Arch. f. Anat. u. Physiol., Physiol. Abth., Jahrg. 1900, pp. 561–562.

Banta, A. M., and McAtee, W. L.
:06. The Life History of the Cave Salamander, Spelerpes maculicaudus (Cope). Proc. U. S. Nat. Museum, Vol. 30, pp. 67–83, pls. 8–10.

Beer, T.
'98. Die Accommodation des Auges bei den Amphibien. Arch. f. ges. Physiol., Bd. 73, pp. 501–534.
:01. Ueber primitive Sehorgane. Wiener klin. Wochenschr., Jahrg. 1901, Nr. 11–13.

Bickel, A.
'97. Ueber den Einfluss der sensibelen Nerven und der Labyrinthe auf die Bewegungen der Thiere. Arch. f. ges. Physiol., Bd. 67, pp. 299–344.

Cole, L. J.
:07. An Experimental Study of the Image-forming Powers of Various Types of Eyes. Proc. Amer. Acad. Arts and Sci., Vol. 42, No. 16, pp. 335–417.

Cope, E. D.
'89. The Batrachia of North America. Bull. U. S. Nat. Museum, No. 34, 525 pp., 81 pls.

Configliachi, P., e Rusconi, M.
'19. Del Proteo Anguino di Laurenti Monographia. Pavia. (Known to the writer only through the review by D. Ellis, '21.)

Dickerson, M. C.
:06. The Frog Book. New York, xvii + 253 pp., 112 pls.

DUBOIS, R.
'90. Sur la perception des radiations lumineuses par la peau, chez les Protées aveugles des grottes de la Carniole. Compt. Rend. Acad. Sci., Paris, tom. 110, pp. 358–361.

EIGENMANN, C. H., AND DENNY, W. A.
:00. The Eyes of the Blind Vertebrates of North America. III. The Structure and Ontogenetic Degeneration of the Eyes of the Missouri Cave Salamander, etc. Biol. Bull., Vol. 2, No. 1, pp. 33–41.

ELLIS, D.
'21. Observations on the Natural History and Structure of the Proteus Anguinus by Sig. Configliachi and Dr. Rusconi. Edinburgh Philosoph. Jour., Vol. 4, pp. 398–406, Vol. 5, pp. 84–112.

ENGLEMANN, T. W.
'85. Ueber Bewegungen der Zapfen und Pigmentzellen der Netzhaut unter dem Einfluss des Lichtes und des Nervensystems. Arch. f. ges. Physiol., Bd. 35, pp. 498–508, Taf. 2.

FICK, A. E.
'90. Ueber die Ursachen der Pigmentwanderung in der Netzhaut. Vierteljahrschr. d. naturf. Gesell. in Zürich, Jahrg. 35, pp. 83–86.

GRABER, V.
'83. Fundamentalversuche über die Helligkeits- und Farbenempfindlichkeit augenloser und geblendeter Thiere. Sitzb. Akad. Wissensch. Wien, Math.-naturw. Cl., Bd. 87, Abth. 1, pp. 201–236.
'84. Grundlinien zur Erforschung des Helligkeits- und Farbensinnes der Thiere. Prag, Leipzig, viii + 322 pp.

HOLMES, S. J.
:01. Phototaxis in Amphipoda. Amer. Jour. Physiol., Vol. 5, pp. 211–234.
:05. The Reactions of Ranatra to Light. Jour. Comp. Neurol. and Psychol., Vol. 15, No. 4, pp. 305–349.
:06. The Biology of the Frog. New York, x + 370 pp.

JORDAN, E. O.
'93. The Habits and Development of the Newt (Diemyctylus viridescens). Jour. Morph., Vol. 8, pp. 269–366, pls. 14–18.

KOHL, C.
'95. Rudimentäre Wirbelthieraugen. Dritter Theil. Bibliotheca Zoologica, Heft 14, pp. 181–274.

KORÁNYI, A. v.
'93. Ueber die Reizbarkeit der Froschhaut gegen Licht und Wärme. Centralbl. f. Physiol., Bd. 6, pp. 6–8.

KÜHNE, W.
'78ᵃ. Ueber den Sehpurpur. Untersuch. physiol. Inst., Heidelberg, Bd. 1, Heft 2, pp. 15–103, Taf. 1.
'78ᵇ. Das Sehen ohne Sehpurpur. Untersuch. physiol. Inst., Heidelberg, Bd. 1, Heft 2, pp. 119–138.

LOEB, J.
'88. Die Orientirung der Thiere gegen das Licht. Sitzb. physik.-med. Gesellsch. z. Würzburg, Jahrg. 1888, Nr. 1, pp. 1–5.
'90. Der Heliotropismus der Thiere und seine Übereinstimmung mit dem Heliotropismus der Pflanzen. Würzburg, 118 pp.
:03. Comparative Physiology of the Brain and Comparative Psychology. New York, xii + 309 pp.

LOESER, W.
:05. A Study of the Functions of Different Parts of the Frog's Brain. Jour. Comp. Neurol. and Psychol., Vol. 15, No. 5, pp. 355–373.

PARKER, G. H.
:03ᵃ. The Phototropism of the Mourning-cloak Butterfly, Vanessa antiopa Linn. Mark Anniversary Volume, No. 23, pp. 453–469, pl. 33.
:03ᵇ. The Skin and the Eyes as Receptive Organs in the Reactions of Frogs to Light. Amer. Jour. Physiol., Vol. 10, No. 1, pp. 28–36.
:05ᵃ. The Functions of the Lateral-line Organs in Fishes. Bull. U. S. Bureau of Fisheries, 1904, Vol. 24, pp. 183–207.
:05ᵇ. The Stimulation of the Integumentary Nerves of Fishes by Light. Amer. Jour. Physiol., Vol. 14, No. 5, pp. 413–420.
:08. The Sensory Reactions of Amphioxus. Amer. Acad. Arts and Sci., Vol. 43, No. 16, pp. 415–455.

PAYNE, F.
:07. The Reactions of the Blind Fish, Amblyopsis spelaeus, to Light. Biol. Bull., Vol. 13, No. 6, pp. 317–323.

PLATEAU, F.
'89. Recherches expérimentales sur la vision chez les arthropodes. Mém. cour. Acad. sci., lettres et beau-arts Belgique, tom. 43, pp. 1–93.

RÁDL, E.
:03. Untersuchungen über den Phototropismus der Thiere. Leipzig, viii + 188 pp.

REESE, A. M.
:06. Observations on the Reactions of Cryptobranchus and Necturus to Light and Heat. Biol. Bull., Vol. 11, No. 2, pp. 93–99.

SCHRADER, M. E. G.
'87. Zur Physiologie des Froschgehirns. Arch. f. ges. Physiol., Bd. 41, pp. 75–90.

SEMPER, C.
'81. Animal Life as Affected by the Natural Conditions of Existence. New York, xvi + 472 pp.

SHERRINGTON, C. S.
:06. The Integrative Action of the Nervous System. New York, xvi + 411 pp.

SMITH, B. G.
:07. The Life History and Habits of Cryptobranchus allegheniensis. Biol. Bull., Vol. 13, No. 1, pp. 5–39.

SMITH, G.
:05. The Effect of Pigment-migration on the Phototropism of Gammarus annulatus S. I. Smith. Amer. Jour. Physiol., Vol. 13, pp. 205–216.

TORELLE, E.
:03. The Response of the Frog to Light. Amer. Jour. Physiol., Vol. 9, No. 6, pp. 466–488.

WASHBURN, M. F.
:08. The Animal Mind, a Text-book of Comparative Psychology. New York, x + 333 pp.

WILLEM, V.
'91. Sur les perceptions dermatoptiques. Bull. Sci. France et Belgique, tom. 23, pp. 329–346.

YERKES, R. M.
:03. The Instincts, Habits, and Reactions of the Frog. Harvard Psychol. Studies, Vol. 1, pp. 579–638.
:06. The Mutual Relations of Stimuli in the Frog, Rana clamata Daudin. Harvard Psychol. Studies, Vol. 2, pp. 545–574.

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AVERAGE CHEMICAL COMPOSITIONS OF IGNEOUS-ROCK TYPES.

By Reginald Aldworth Daly.

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Contents.

Introduction: Purpose of the Paper.

The study of the igneous rocks has hitherto largely consisted in an analysis of their mineralogical and chemical composition, with the special intent to produce a satisfactory nomenclature and classification of the rocks as they occur throughout the world. This systematic petrography, though still pursued by a great number of workers, is now rivaled in interest and excelled in importance by its own offshoot, petrogeny. The science of the origin and history of the igneous rocks is reacting on the more purely descriptive subject, and at present petrologists are feeling their way toward a genetic classification of this great series of rock-types. Meantime, the much more numerous class of workers engaged on the problems of economic and general geology, of geochemistry and cosmogony, are raising highly important questions which belong to the field of petrogenesis. The problems thus raised are as fundamental as they are complex and difficult. For many of their solutions recourse must be had to the more modern geological reports and maps. With ever increasing skill and accuracy the distribution and relations of the rocks composing the earth's crust are being recorded by government officers and by geologists working in private capacity. For some thirty years past, as at present, the great body of geologists have mapped and described the igneous rocks in terms of what may be called the German system of nomenclature and definition. In particular, Rosenbusch's monumental treatises on the eruptive rocks have been, for a generation, the usual guide to the many authors who have described their findings among the igneous terranes of the world.

In view of these facts it is clear that a student in petrology who wishes to use the maps and memoirs should have a good conception of the rock-types recognized by Rosenbusch and by his hundreds of disciples among the field-geologists. It is true that in some details the usages of master and followers as regards names and classification have varied, but in a broad way Rosenbusch's definitions of the principal families and species of massive rocks have been used for maps and reports in all regions where modern work on igneous geology has been done. Just as the general sequence of the stratified rocks as first described in England, France, and Germany has been found to be closely paralleled in the rest of Europe and in the other continents, so the system of igneous rocks as at first developed from material largely collected in Europe has been nearly sufficient for the mapping of those rocks elsewhere. In the field as in the library the geologist soon learns that there is a persistent recurrence of types in the larger divisions of the earth's surface. The usefulness and objective character of Rosenbusch's classification are, therefore, proved by its adaptability in all the continents and islands.

Rosenbusch and his followers recognize some latitude of variation in the composition of each rock-type. The variation is both mineralogical and chemical, two rock specimens referred to a type showing differences in the proportions of the chemical elements found by analysis of the two rocks. In fact, no two analyses of granite, andesite, or any other one type have ever given precisely the same proportions of the dozen or more oxides which regularly make up an igneous rock. It is obvious that the student of map and memoir should, for many problems, have at hand the actual figures showing the most typical chemical composition of the rock-types to which his study is directed. In numerous cases an analysis of a single specimen is not so useful as that which could be made from a thorough mixture of specimens of the same rock-variety from all places on the globe where that variety occurs.

For obvious reasons such ideal analyses have never been made. In their stead the writer believes that the investigator of petrogenic and other world-problems may well use the averages calculated from the many excellent chemical analyses of rocks made since Rosenbusch's system of naming and classification has been in general use. It may, indeed, be argued that such averages would more nearly represent the chemistry of Rosenbusch's types than any of the respective single analysis which he has published in his treatise. These averages would be chemical "center-points" in his system of classification as *actually applied* to the terranes of the world.

So far as the writer is aware, the preparation of these averages has not hitherto been attempted to such an extent as to cover the chief families and species of igneous rocks. An approximation to the desired results is offered in the following tables.

The work of computing the averages has been lessened very greatly by the publication of Osann's "Beiträge zur chemischen Petrographie" (2nd part, Stuttgart, 1905). This remarkable book contains, in convenient arrangement, the statement of most of the eruptive-rock analyses (over 2400 in number) published in the interval between 1883 and 1901. The period of seventeen years lies within that during which systematic petrography has been dominated by Rosenbusch's names and definitions. In general, the number of analyses for each rock-species is so large that their average would be but slightly modified by the inclusion of the analyses made since 1900. In many cases, therefore, the extended labor required to search out from the literature the additional analyses, has not been considered necessary for the preparation of useful averages. For other averages it was necessary to include analyses published since 1900. The sources of such information are indicated below. Fortunately for the purpose, nearly the entire period since 1884 has seen the application of more or less refined methods of analysis; so that errors of observation for the leading oxides are relatively small.

METHOD OF CALCULATION.

The method of computation used is essentially like that employed by Washington and Clarke in their respective calculations of the "average composition" of all igneous rocks. In general, only the twelve more important oxides (including MnO) are recognized in the following tables. Distinctly "inferior" analyses were not considered. In each case the average was computed according to the actual numbers of determinations made by the analysts. Table I. shows these numbers for the respective rock-types, each column being headed by a key-number which corresponds with the named types of Table II. For some of the rocks BaO and SrO were computed. Their sum appears in the averages for CaO, as indicated in the tables. Similarly CO_2 and Cr_2O_3 were sometimes averaged and entered with H_2O and Fe_2O_3 respectively. As expected from the method employed, the average totals nearly always ran well over one hundred per cent. All averages were reduced to 100.00 per cent and entered in Table II. Each average analysis was then recalculated to 100.00 per cent after H_2O (and CO_2) had been subtracted. The results are also given in Table II., in which plutonics and corresponding effusives are grouped together. Magmatic relationships are often less obscured if these volatile oxides, which may be wholly or in part of exotic nature, are excluded. Finally, in order to facilitate reference to the tables, an index to the different rock-types was prepared and may be found below Table II.

It will be observed that certain rock-types have been omitted from the tables. The large class of "aschistic" dike-rocks is not represented because of their chemical similarity to the corresponding plutonic species. Other named varieties are omitted since their analyses are too few to give useful averages. In a few cases the mineralogical and chemical variations within each variety are so great that it has not seemed advisable to regard their averages as worthy of entry. Many other subordinate varieties of rock, though given special names, are chemically almost identical with the more important types entered in the tables and therefore have been excluded.

Sources of Information.

The immediate sources of the analytical statements used in the computations are as follows: —

1. Beiträge zur chemischen Petrographie, zweiter Teil, by A. Osann. Stuttgart, 1905.
2. Chemical Analyses of Igneous Rocks published from 1884 to 1900, by H. S. Washington. Prof. Paper, No. 14, U. S. Geological Survey, 1903.
3. Elemente der Gesteinslehre, 2nd edition, by H. Rosenbusch. Stuttgart, 1901.
4. Lehrbuch der Petrographie, 2nd edition, by F. Zirkel. Leipzig, 1893.
5. Studien über die Granite von Schweden, by P. J. Holmquist. Bull. Geol. Institution, University of Upsala, Vol. 7, 1906, p. 76.
6. Some Lava Flows of the Western Slope of the Sierra Nevada, California, by F. L. Ransome. Amer. Jour. Science, Vol. 5, 1898, p. 355.
7. Matériaux pour la Minéralogie de Madagascar. Nouv. Archives du Muséum, (4), Vol. 5, Paris, 1903.
8. Geology of the Yellowstone National Park, by A. Hague and others. Petrography by J. P. Iddings. Monograph No. 32, Part 2, U. S. Geological Survey, 1899.

9. Analyses of Rocks from the Laboratory of the United States Geological Survey, 1880 to 1903, by F. W. Clarke. Bulletin 228 of the Survey, 1904.
10. Geological and Petrographical Studies of the Sudbury Nickel District, by T. L. Walker, Quart. Jour. Geol. Soc., Vol. 53, 1897, p. 40.
11. Petrography and Geology of the Igneous Rocks of the Highwood Mountains, Montana, by L. V. Pirsson. Bull. 237, U. S. Geological Survey, 1905.
12. Geology of the North American Cordillera at the Forty-ninth Parallel, by R. A. Daly (forthcoming; analyses by M. F. Connor and M. Dittrich used in calculating some averages).

The sources of the analyses used in each average are indicated by the authors' names at the head of the corresponding columns in Table II.

TABLE I.

Showing the Number of Separate Determinations used in computing the Average Quantity of each Oxide in each Rock-type.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
SiO_2	47	114	184	236	64	24	40	50	7	5	8	23	50	48	7	13
TiO_2	22	74	60	87	40	10	30	20	5	4	5	14	35	26	6	5
Al_2O_3	47	114	180	232	63	23	40	49	7	5	8	23	49	48	7	13
Fe_2O_3	35	101	118	158	61	22	39	32	4	5	3	15	43	38	7	10
FeO	35	101	118	158	42	6	36	32	4	5	3	14	43	38	6	10
MnO	24	86	64	93	32	4	28	20	4	5	5	14	38	34	1	6
MgO	47	114	184	236	63	24	39	49	6	5	8	22	50	48	7	13
CaO	47	114	184	236	64	24	40	49	6	5	8	22	50	48	6	13
Na_2O	47	108	182	234	63	24	39	49	6	5	8	22	50	48	7	13
K_2O	47	108	182	234	63	24	39	49	6	5	8	21	50	48	7	13
H_2O	38	40	41	41	17	15	17	39	7	3	8	21	41	44	7	10
P_2O_5	15	34	73	81	27	4	23	17	3	3	..	7	34	25	4	4
BaO	8	..	35	35	..	..	..	..	..	..	..	..	..	..	..	..
SrO	5	..	21	21	..	..	..	..	..	..	..	..	..	..	..	..

	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
SiO_2	3	7	12	10	10	3	3	43	25	4	8	12	30	20	89	70
TiO_2	..	..	10	10	4	..	3	30	16	..	3	12	15	16	71	57
Al_2O_3	3	7	12	10	10	3	3	43	25	4	8	12	30	20	89	70
Fe_2O_3	3	7	12	10	6	3	3	30	18	2	2	12	24	18	86	69
FeO	2		12	10	6	3	2	30	18	2	2	12	24	18	86	69
MnO	..	..	10	10	4	3	2	30	15	1	3	12	14	11	66	53
MgO	3	7	12	10	10	3	3	41	25	4	8	12	30	20	89	70
CaO	3	7	12	10	10	3	3	43	25	4	8	12	30	20	89	70
Na_2O	3	7	12	10	10	3	3	43	25	4	8	12	30	20	85	67
K_2O	3	7	12	10	10	3	3	43	25	4	8	12	30	20	85	67
H_2O	3	7	9	10	10	3	1	26	23	4	6	12	30	17	47	36
P_2O_5	1	..	12	10	4	..	3	14	15	..	1	12	15	15	71	57

TABLE I.— *Continued.*

	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
SiO_2	87	33	20	24	10	7	41	198	161	20	17	11	9	24	17	5
TiO_2	51	16	13	13	9	6	26	132	113	13	6	5	8	16	10	4
Al_2O_3	87	33	20	24	10	6	41	197	160	20	17	11	9	24	17	4
Fe_2O_3	71	25	18	18	10	7	36	174	146	18	14	5	9	21	15	5
FeO	71	25	18	18	10	7	36	173	146	18	14	5	8	21	15	5
MnO	44	16	14	8	6	6	28	108	96	13	6	2	4	15	13	4
MgO	87	33	20	24	10	7	40	197	160	20	17	11	9	24	16	5
CaO	87	33	20	24	10	7	41	198	161	20	17	11	9	24	17	5
Na_2O	84	32	20	22	10	7	40	190	154	20	16	11	9	24	16	5
K_2O	84	32	20	22	10	7	39	190	154	20	16	11	9	23	16	5
H_2O	57	5	18	24	10	6	17	55	27	16	1	5	2	12	5	4
P_2O_5	47	14	13	11	9	6	27	135	116	14	6	4	9	16	11	4

	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64
SiO_2	2	12	4	4	3	4	3	4	49	3	11	4	7	6	6	6
TiO_2	2	4	1	4	3	..	..	2	27	..	11	2	3	3	2	6
Al_2O_3	2	12	4	4	3	4	2	4	49	3	11	4	7	6	6	6
Fe_2O_3	2	10	4	4	3	1	2	4	40	3	10	2	2	5	6	6
FeO	2	10	4	4	3	1	3	4	40	3	10	4	2	5	6	6
MnO	2	3	4	4	3	1	1	3	32	..	6	..	..	..	4	4
MgO	2	12	4	4	3	4	3	4	47	3	11	4	7	6	6	6
CaO	2	12	4	4	3	4	1	4	49	3	11	4	7	6	6	6
Na_2O	2	12	3	2	3	..	1	4	31	3	11	4	7	6	6	6
K_2O	2	12	2	1	3	..	..	4	31	3	11	4	7	6	6	6
H_2O	2	7	4	4	3	4	3	3	47	3	10	2	7	4	6	6
P_2O_5	2	1	1	4	3	..	1	2	25	3	11	2	2	2	2	6

TABLE I.— *Continued.*

	65	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**	**81**
SiO_2	24	20	4	20	16	4	1	2	7	7	5	9	26	3	10	4	10
TiO_2	8	14	4	4	11	3	1	2	2	6	5	3	23	3	10	4	9
Al_2O_3	24	20	4	20	16	4	1	2	7	7	5	9	26	3	10	4	10
Fe_2O_3	22	19	3	19	15	4	1	2	4	5	5	3	25	3	10	4	10
FeO	22	19	3	19	15	4	1	2	4	5	5	3	25	3	10	4	10
MnO	14	7	1	13	5	2	1	2	1	4	5	4	15	3	10	1	10
MgO	24	20	4	20	16	4	1	2	7	7	5	9	26	3	10	4	10
CaO	24	20	4	20	16	4	1	2	7	7	5	9	26	3	10	4	10
Na_2O	24	20	4	20	16	4	1	2	7	7	5	9	26	3	10	4	10
K_2O	24	20	4	20	16	4	1	2	7	7	5	9	26	3	10	4	10
H_2O	9	18	3	6	15	3	1	2	3	4	4	5	25	3	10	4	10
P_2O_5	19	9	3	16	8	1	1	2	5	2	4	6	23	3	10	4	9

	82	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**
SiO_2	11	4	8	5	2	6	15	5	5	8	15	10	20	4	15	16	6
TiO_2	11	4	8	5	2	6	9	2	3	7	9	8	16	4	10	12	5
Al_2O_3	11	4	8	5	2	6	15	5	5	8	15	10	19	4	15	16	6
Fe_2O_3	11	4	8	5	2	6	11	2	4	8	10	8	16	4	10	14	5
FeO	11	4	8	5	2	6	10	2	3	8	9	8	16	4	10	14	5
MnO	11	4	8	5	2	..	8	1	4	8	9	7	17	1	8	9	2
MgO	11	4	8	5	2	6	15	4	5	8	15	10	20	4	15	16	6
CaO	11	4	8	5	2	6	15	5	5	8	15	10	20	4	15	16	6
Na_2O	11	4	8	5	2	6	15	5	5	8	15	10	20	4	15	15	5
K_2O	11	4	8	5	2	6	15	5	5	8	15	10	20	4	15	15	5
H_2O	6	4	8	5	2	6	11	2	3	7	10	10	19	4	10	16	3
P_2O_5	8	4	8	5	2	5	7	..	..	7	5	7	18	4	6	11	3

TABLE II.

SHOWING THE AVERAGE COMPOSITIONS CALCULATED FOR THE PRINCIPAL IGNEOUS-ROCK TYPES.

GROUP I.

	PLUTONICS.				EFFUSIVES.			
	1	2	3	4	5	6	7	8
	Pre-Cambrian Granites, including 16 Analyses of Swedish Types (Osann).	Pre-Cambrian Granites of Sweden (Holmquist).	Granites younger than the Pre-Cambrian (Osann and Clarke).	Granite of all Periods (Osann and Clarke).	Liparite, including 40 Rhyolites (Osann).	Liparites, as named by authors (Osann).	Rhyolites, as named by authors (Osann).	Quartz Porphyry (Osann).
No. of Analyses.	47	114	184	236	64	24	40	50
SiO_2	71.06	69.81	69.73	69.92	72.60	72.90	72.62	72.36
TiO_2	.48	.54	.34	.39	.30	.48	.25	.33
Al_2O_3	14.10	13.76	14.98	14.78	13.88	14.18	13.77	14.17
Fe_2O_3	1.46	2.17	1.62	1.62	1.43	1.65	1.29	1.55
FeO	1.63	1.87	1.66	1.67	.82	.31	.90	1.01
MnO	.18	.26	.11	.13	.12	.13	.12	.09
MgO	.59	.84	1.08	.97	.38	.40	.38	.52
CaO	1.97 [1]	2.20	2.20 [2]	2.15 [3]	1.32	1.13	1.43	1.38
Na_2O	3.24	3.17	3.28	3.28	3.54	3.54	3.55	2.85
K_2O	4.50	4.38	3.95	4.07	4.03	3.94	4.09	4.56
H_2O	.69	.74	.78	.78	1.52	1.33	1.53	1.09
P_2O_5	.10	.26	.27	.24	.06	.01	.07	.09
Calculated as Water-free.								
SiO_2	71.56	70.33	70.28	70.47	73.72	73.89	73.75	73.16
TiO_2	.48	.54	.34	.39	.30	.49	.25	.33
Al_2O_3	14.20	13.86	15.10	14.90	14.10	14.37	13.99	14.33
Fe_2O_3	1.47	2.19	1.63	1.63	1.45	1.67	1.31	1.57
FeO	1.65	1.89	1.67	1.68	.83	.31	.91	1.02
MnO	.18	.26	.11	.13	.12	.13	.12	.09
MgO	.59	.85	1.09	.98	.40	.41	.39	.53
CaO	1.98 [1]	2.22	2.22 [2]	2.17 [3]	1.34	1.14	1.45	1.39
Na_2O	3.26	3.19	3.31	3.31	3.59	3.59	3.60	2.88
K_2O	4.53	4.41	3.98	4.10	4.09	3.99	4.16	4.61
P_2O_5	.10	.26	.27	.24	.06	.01	.07	.09
Each sum = 100.00.								

[1] Includes .08% BaO and .01% SrO.

[2] Includes .06% BaO and .02% SrO.

[3] Includes .06% BaO and .02% SrO.

GROUP II.

	Plutonics.					Effusives.		
	9	10	11	12	13	14	15	16
	Nordmarkite (Osann and Washington).	Pulaskite (Osann and Washington).	Akerite (Osann and Washington).	Average Alkaline Syenite, including 7 Nordmarkites, 5 Pulaskites, 9 Akerites, and 3 Laurvikites.	All Syenite, including five Types of "Alkaline" Syenite.	Trachyte (Osann and Rosenbusch).	Keratophyre (Osann and Washington).	Quartz Keratophyre (Rosenbusch).
No. of Analyses.	7	5	8	23	50	48	7	13
SiO_2	64.36	61.86	61.96	61.99	60.19	60.68	61.51	75.45
TiO_2	.45	.15	.99	.56	.67	.38	.45	.17
Al_2O_3	16.81	19.07	17.07	17.93	16.28	17.74	17.37	13.11
Fe_2O_3	1.08	2.65	2.35	2.22	2.74	2.64	1.92	1.14
FeO	2.71	1.49	3.37	2.29	3.28	2.62	3.35	.66
MnO	.15	.01	.09	.08	.14	.06	.01	.29
MgO	.72	.55	1.38	.96	2.49	1.12	1.26	34
CaO	1.55	1.47	3.41	2.55	4.30	3.09	1.08	.83
Na_2O	5.76	6.45	4.65	5.54	3.98	4.43	5.23	5.88
K_2O	5.62	5.75	3.80	4.98	4.49	5.74	5.29	1.26
H_2O	.70	.47	.93	.76	1.16	1.26	2.45	.69
P_2O_5	09	.08	....	.14	.28	.24	.08	.18
Calculated as Water-free.								
SiO_2	64.81	62.15	62.55	62.46	60.90	61.46	63.06	75.98
TiO_2	.45	.15	1.00	.56	.68	.38	.46	.17
Al_2O_3	16.93	19.16	17.23	18.07	16.47	17.97	17.81	13.20
Fe_2O_3	1.09	2.66	2.37	2.24	2.77	2.67	1.97	1.15
FeO	2.73	1.50	3.40	2.31	3.32	2.66	3.43	.66
MnO	.15	.01	.09	.08	.14	.06	.01	.29
MgO	.73	.55	1.39	.97	2.52	1.13	1.29	.34
CaO	1.56	1.48	3.44	2.57	4.35	3.13	1.11	.84
Na_2O	5.80	6.48	4.69	5.58	4.03	4.49	5.36	5.92
K_2O	5.66	5.78	3.84	5.02	4.54	5.81	5.42	1.27
P_2O_5	.09	.08	....	.14	.28	.24	.08	.18
Each sum = 100.00.								

GROUP III.

	Plutonic.	Effusive.	Plutonic.	Effusive.
	17	18	19	20
	Laurvikite (Osann).	Rhomb-porphyry (Washington).	Monzonite (Osann and Washington).	Latite (Ransome and Daly).
No. of Analyses.	3	7	12	10
SiO_2	57.45	57.45	55.25	57.65
TiO_2	....	....	.60	1.00
Al_2O_3	21.11	19.53	16.53	16.68
Fe_2O_3	2.89	6.47	3.03	2.29
FeO	2.39		4.37	4.07
MnO	....	....	.15	.10
MgO	1.06	1.28	4.20	3.22
CaO	4.10	3.11	7.19	5.74 [1]
Na_2O	5.89	6.35	3.48	3.59
K_2O	3.87	4.46	4.11	4.39
H_2O	.70	1.35	.66	.91 [2]
P_2O_5	.54	....	.43	.36
Calculated as Water-free.				
SiO_2	57.85	58.24	55.62	58.18
TiO_2	....	....	.60	1.01
Al_2O_3	21.26	19.79	16.64	16.84
Fe_2O_3	2.91	6.56	3.05	2.31
FeO	2.41		4.40	4.11
MnO	....	....	.15	.10
MgO	1.07	1.30	4.23	3.25
CaO	4.13	3.15	7.24	5.79 [1]
Na_2O	5.93	6.44	3.50	3.62
K_2O	3.90	4.52	4.14	4.43
P_2O_5	.54	....	.43	.36
Each sum = 100.00.				

[1] Includes .16% BaO and .07% SrO.

[2] Includes .14% CO_2.

GROUP IV.

	Plutonics.				Effusives.		
	21	22	23	24	25	26	27
	Foyaite (Osann and Rosenbusch).	Urtite (Osann).	Laurdalite (Osann).	Nephelite syenite (Osann).	Phonolite (Osann, Clarke, and Lacroix).	Leucite Phonolite (Osann and Washington).	Leucitophyre (Washington and Rosenbusch).
No. of Analyses.	10	3	3	43	25	4	8
SiO_2	56.11	45.61	54.36	54.63	57.45	54.89	49.83
TiO_2	.45	....	1.30	.86	.41	....	.71
Al_2O_3	21.33	27.76	19.99	19.89	20.60	21.28	19.00
Fe_2O_3	1.87	3.67	2.79	3.37	2.35	3.04	3.17
FeO	1.47	.50	2.58	2.20	1.03	1.49	3.59
MnO	.05	.15	.18	.35	.13	.01	.17
MgO	.55	.19	1.72	.87	.30	.66	1.79
CaO	1.72	1.73	2.96	2.51	1.50	2.31	5.69
Na_2O	8.48	16.25	8.28	8.26	8.84	5.62	7.19
K_2O	6.46	3.72	4.98	5.46	5.23	8.39	6.15
H_2O	1.50	.42	.22	1.35	2.04	2.31	1.93
P_2O_5	.01	....	.64	.25	.12	....	.78
Calculated as Water-free.							
SiO_2	56.96	45.80	54.48	55.38	58.65	56.19	50.82
TiO_2	.46	....	1.30	.87	.42	....	.72
Al_2O_3	21.65	27.88	20.03	20.16	21.03	21.78	19.38
Fe_2O_3	1.90	3.68	2.80	3.42	2.40	3.11	3.23
FeO	1.49	.50	2.59	2.23	1.05	1.53	3.66
MnO	.05	.15	.18	.35	.13	.01	.17
MgO	.56	.19	1.72	.88	.31	.68	1.83
CaO	1.75	1.74	2.97	2.54	1.53	2.36	5.80
Na_2O	8.61	16.32	8.30	8.38	9.02	5.75	7.33
K_2O	6.56	3.74	4.99	5.54	5.34	8.59	6.27
P_2O_5	.01	....	.64	.25	.12	....	.79
Each sum = 100.00.							

GROUP V.

	Pl.	Ef.	Plutonics			Effusives.				
	28	29	30	31	32	33	34	35	36	37
	Granodiorite (Osann and Clarke).	Dacite (Osann and Rosenbusch).	Quartz Diorite (Osann and Washington).	Diorite, includ'g Quartz Diorite (Osann).	Diorite, exclud'g Quartz Diorite (Osann).	All Andesite (Osann).	Augite Andesite (Osann).	Hypersthene Andesite (Osann).	Hornblende (Amphibole) Andesite (Osann).	Mica Andesite (Osann).
No. of Analyses.	12	30	20	89	70	87	33	20	24	10
SiO_2	65.10	66.91	59.47	58.38	56.77	59.59	57.50	59.48	61.12	62.25
TiO_2	.54	.33	.64	.80	.84	.77	.79	.48	.42	1.65
Al_2O_3	15.82	16.62	16.52	16.28	16.67	17.31	17.33	17.38	17.65	16.10
Fe_2O_3	1.64	2.44	2.63	2.98	3.16	3.33	3.78	2.96	2.89	3.62
FeO	2.66	1.33	4.11	4.11	4.40	3.13	3.62	3.67	2.40	2.20
MnO	.05	.04	.08	.13	.13	.18	.22	.15	.15	.21
MgO	2.17	1.22	3.75	3.88	4.17	2.75	2.86	3.28	2.44	2.03
CaO	4.66	3.27	6.24	6.38	6.74	5.80	5.83	6.61	5.80	4.05
Na_2O	3.82	4.13	2.98	3.34	3.39	3.58	3.53	3.41	3.83	3.55
K_2O	2.29	2.50	1.93	2.09	2.12	2.04	2.36	1.64	1.72	2.44
H_2O	1.09	1.13	1.39	1.37	1.36	1.26	1.88	.74	1.43	1.50
P_2O_5	.16	.08	.26	.26	.25	.26	.30	.20	.15	.40
Calculated as Water-free.										
SiO_2	65.82	67.67	60.31	59.19	57.56	60.35	58.65	59.92	62.01	63.20
TiO_2	.55	.33	.65	.81	.85	.78	.80	.48	.43	1.67
Al_2O_3	15.99	16.81	16.75	16.51	16.90	17.54	17.67	17.51	17.91	16.35
Fe_2O_3	1.66	2.47	2.67	3.02	3.20	3.37	3.85	2.98	2.93	3.67
FeO	2.69	1.35	4.17	4.17	4.46	3.17	3.69	3.70	2.44	2.23
MnO	.05	.04	.08	.13	.13	.18	.22	.15	.15	.21
MgO	2.19	1.23	3.80	3.93	4.23	2.78	2.90	3.31	2.48	2.06
CaO	4.71	3.31	6.33	6.47	6.83	5.87	5.92	6.66	5.88	4.11
Na_2O	3.86	4.18	3.02	3.39	3.44	3.63	3.60	3.44	3.88	3.61
K_2O	2.32	2.53	1.96	2.12	2.15	2.07	2.40	1.65	1.74	2.48
P_2O_5	.16	.08	.26	.26	.25	.26	.30	.20	.15	.41

Each sum = 100.00.

GROUP VI.

	Plutonics.		Effusives.					
	38	39	40	41	42	43	44	45
	All Norite (Osann and Walker).	All Gabbro (Osann).	All Basalt, including 161 Basalts, 17 Olivine Diabases, 11 Melaphyres, and 9 Dolerites (Osann).	Basalt, as named by Authors (including also Anamesite, Tachylite, etc.) (Osann).	Diabase (Osann).	Olivine Diabase (Osann).	Melaphyre (Osann).	Dolerite (Osann).
No. of Analyses.	7	41	198	161	20	17	11	9
SiO_2	50.16	48.24	49.06	48.78	50.12	50.10	50.60	49.50
TiO_2	1.64	.97	1.36	1.39	1.41	1.25	.68	1.42
Al_2O_3	18.51	17.88	15.70	15.85	15.68	14.43	17.40	14.37
Fe_2O_3	1.88	3.16	5.38	5.37	4.55	5.06	4.57	6.55
FeO	9.29	5.95	6.37	6.34	6.73	6.31	6.29	5.84
MnO	.14	.13	.31	.29	.23	.25	.46	.17
MgO	5.97	7.51	6.17	6.03	5.85	7.32	4.89	7.75
CaO	7.90	10.99	8.95	8.91	8.80	9.53	8.09	9.96
Na_2O	2.72	2.55	3.11	3.18	2.95	2.75	3.23	2.50
K_2O	.80	.89	1.52	1.63	1.38	.73	1.76	.84
H_2O	.76	1.45	1.62	1.76	1.93	2.00	1.83	.66
P_2O_5	.23	.28	.45	.47	.37	.27	.20	.44
Calculated as Water-free.								
SiO_2	50.54	48.95	49.87	49.65	51.11	51.12	51.54	49.83
TiO_2	1.65	.98	1.38	1.41	1.44	1.27	.69	1.43
Al_2O_3	18.65	18.15	15.96	16.13	15.99	14.73	17.73	14.47
Fe_2O_3	1.90	3.21	5.47	5.47	4.64	5.16	4.66	6.59
FeO	9.36	6.04	6.47	6.45	6.86	6.44	6.41	5.88
MnO	.14	.13	.32	.30	.23	.25	.47	.17
MgO	6.02	7.62	6.27	6.14	5.96	7.47	4.99	7.80
CaO	7.96	11.15	9.09	9.07	8.97	9.73	8.24	10.02
Na_2O	2.74	2.59	3.16	3.24	3.01	2.81	3.29	2.52
K_2O	.81	.90	1.55	1.66	1.41	.74	1.78	.85
P_2O_5	.23	.28	.46	.48	.38	.28	.20	.44
Each sum = 100.00.								

GROUP VII.

	Plutonics.				
	46	47	48	49	50
	Gabbro, excluding Olivine Gabbro (Osann).	Olivine Gabbro (Osann).	Norite, excluding Olivine Norite (Osann and Walker).	Olivine Norite (Osann).	Anorthosite (Osann and Washington).
No. of Analyses.	24	17	5	2	12
SiO_2	49.50	46.49	50.08	50.38	50.40
TiO_2	.84	1.17	1.44	2.04	.15
Al_2O_3	18.00	17.73	18.62	18.27	28.30
Fe_2O_3	2.80	3.66	2.35	.73	1.06
FeO	5.80	6.17	8.87	10.35	1.12
MnO	.12	.17	.11	.20	.05
MgO	6.62	8.86	6.22	5.32	1.25
CaO	10.64	11.48	7.89	7.91	12.46
Na_2O	2.82	2.16	2.53	3.18	3.67
K_2O	.98	.78	.71	1.02	.74
H_2O	1.60	1.04	1.01	.26	.75
P_2O_5	.28	.29	.17	.34	.05
Calculated as Water-free.					
SiO_2	50.31	46.97	50.60	50.51	50.78
TiO_2	.85	1.18	1.45	2.05	.15
Al_2O_3	18.30	17.92	18.81	18.32	28.51
Fe_2O_3	2.85	3.70	2.37	.73	1.07
FeO	5.89	6.24	8.96	10.38	1.13
MnO	.12	.17	.11	.20	.05
MgO	6.73	8.96	6.28	5.33	1.26
CaO	10.81	11.60	7.97	7.93	12.55
Na_2O	2.86	2.18	2.56	3.19	3.70
K_2O	1.00	.79	.72	1.02	.75
P_2O_5	.28	.29	.17	.34	.05
Each sum = 100.00.					

GROUP VIII.

	Plutonics.							Effusive.
	51	52	53	54	55	56	57	58
	Lherzolite (Osann).	Websterite (Osann).	Wehrlite (Osann).	Harzburgite, including Saxonite (Osann and Washington).	Dunite (Washington).	Pyroxenite (Osann).	All Peridotite (Osann).	Picrite (Osann).
No of Analyses.	4	4	3	4	3	4	49	3
SiO_2	42.09	53.65	48.13	43.85	40.06	49.82	44.39	43.24
TiO_2	.12	.14	.87	....	....	1.46	.88	....
Al_2O_3	4.83	1.66	6.50	5.00	.57	5.12	5.14	15.19
Fe_2O_3	4.98	1.90	2.01	2.54	2.29	1.83	3.88	8.62
FeO	4.58	5.35	11.73	6.30	7.32	7.44	6.70	7.89
MnO	.06	.17	.08	.12	.24	.09	.19	....
MgO	31.80	22.57	21.01	36.96	46.62	19.55	29.17	8.56
CaO	6.37	13.37	6.17	2.70	.35	13.00	6.31	13.78
Na_2O	1.02	.20	1.15	....	.01	.37	.64	.54
K_2O	.29	.07	.58	....	....	.21	.76	.48
H_2O	3.85	.85	1.62	2.53[1]	2.53	1.06	1.80	1.21
P_2O_5	.01	.07	.15	....	.01	.05	.14	.49
Calculated as Water-free.								
SiO_2	43.78	54.11	48.93	44.99	41.10	50.36	45.20	43.77
TiO_2	.12	.14	.88	....	....	1.48	.90	....
Al_2O_3	5.02	1.67	6.61	5.13	.58	5.17	5.25	15.37
Fe_2O_3	5.18	1.92	2.04	2.61	2.35	1.85	3.95	8.72
FeO	4.77	5.40	11.92	6.46	7.51	7.52	6.82	7.99
MnO	.06	.17	.08	.12	.25	.09	.19	....
MgO	33.08	22.76	21.36	37.92	47.83	19.76	29.70	8.66
CaO	6.62	13.49	6.27	2.77	.36	13.14	6.43	13.95
Na_2O	1.06	.20	1.17	....	.01	.37	.65	.55
K_2O	.30	.07	.59	....	....	.21	.77	.49
P_2O_5	.01	.07	.15	....	.01	.05	.14	.50
Each sum = 100.00.								

[1] Loss on ignition.

GROUP IX.

	PLUTONIC.	EFFUSIVES.		
	59	**60**	**61**	**62**
	Essexite (Osann and Rosenbusch).	Trachydolerite (Rosenbusch).	Limburgite (Zirkel).	Augitite (Osann, Washington, and Rosenbusch).
No. of Analyses.	11	4	7	6
SiO_2	48.40	54.81	41.69	42.25
TiO_2	1.71	.42	.67	2.52
Al_2O_3	16.67	20.01	14.80	16.26
Fe_2O_3	5.31	3.98	15.04	8.43
FeO	6.03	1.93		5.46
MnO	.15	....	....	....
MgO	4.48	2.32	8.64	5.49
CaO	9.05	5.60	11.98	9.75
Na_2O	4.45	5.86	3.52	4.45
K_2O	2.13	3.13	1.17	1.92
H_2O	.95	1.46	2.36	2.43
P_2O_5	.67	.48	.13	1.04
Calculated as Water-free.				
SiO_2	48.86	55.62	42.69	43.30
TiO_2	1.73	.43	.68	2.58
Al_2O_3	16.83	20.31	15.18	16.67
Fe_2O_3	5.36	4.04	15.43	8.64
FeO	6.09	1.96		5.59
MnO	.15	....	....	....
MgO	4.52	2.35	8.85	5.63
CaO	9.14	5.68	12.27	9.99
Na_2O	4.49	5.94	3.58	4.56
K_2O	2.15	3.18	1.19	1.97
P_2O_5	.68	.49	.13	1.07
Each sum = 100.00.				

GROUP X.

	Plutonics.		Effusives.					
	63	64	65	66	67	68	69	70
	Theralite (Osann).	Shonkinite (Pirsson).	All Tephrite.	All Basanite.	Nephelite Tephrite (Osann).	Leucite Tephrite (Osann and Washington).	Nephelite Basanite (Osann).	Leucite Basanite (Osann and Washington).
No. of Analyses.	6	6	24	20	4	20	16	4
SiO_2	45.61	48.66	49.14	44.41	46.91	49.90	44.20	45.34
TiO_2	1.96	.97	1.00	1.56	1.81	.16	1.64	1.30
Al_2O_3	14.35	12.36	16.57	15.81	15.25	16.94	15.64	16.59
Fe_2O_3	6.17	3.08	3.65	4.66	7.70	3.02	4.35	5.83
FeO	4.03	5.86	6.68	5.85	4.06	7.15	6.14	4.76
MnO	.19	.13	.30	.14	1.43	.23	.19	.01
MgO	6.05	8.09	3.98	8.20	2.95	4.22	8.89	5.43
CaO	9.49	10.46[1]	9.88	10.12	9.36	10.04	9.74	11.64
Na_2O	5.12	2.71	2.57	3.81	4.25	2.24	4.03	2.93
K_2O	3.69	5.15	3.39	2.37	2.63	3.57	1.83	4.55
H_2O	2.60	1.46	2.00	2.42	2.51	1.74	2.67	1.12
P_2O_5	.74	1.07	.84	.65	1.14	.79	.68	.50
Calculated as Water-free.								
SiO_2	46.83	49.38	50.15	45.51	48.12	50.79	45.41	45.86
TiO_2	1.98	.98	1.02	1.60	1.86	.16	1.68	1.31
Al_2O_3	14.73	12.55	16.90	16.20	15.65	17.24	16.07	16.78
Fe_2O_3	6.34	3.12	3.72	4.78	7.89	3.07	4.47	5.90
FeO	4.14	5.95	6.82	5.99	4.16	7.28	6.31	4.81
MnO	.19	.13	.31	.14	1.47	.23	.20	.01
MgO	6.22	8.21	4.06	8.41	3.02	4.30	9.13	5.49
CaO	9.75	10.62[2]	10.08	10.37	9.60	10.22	10.01	11.77
Na_2O	5.27	2.75	2.62	3.90	4.36	2.28	4.14	2.96
K_2O	3.79	5.23	3.46	2.43	2.70	3.63	1.88	4.60
P_2O_5	.76	1.08	.86	.67	1.17	.80	.70	.51
Each sum = 100.00.								

[1] Includes .40% BaO and .09% SrO.

[2] Includes .41% BaO and .09% SrO.

GROUP XI.

	PLUTONICS.		EFFUSIVES.		PLUTONIC.	EFFUSIVES.	
	71	**72**	**73**	**74**	**75**	**76**	**77**
	Fergusite (Pirsson).	Missourite (Pirsson and Daly).	Leucite Basalt (Osann and Rosenbusch).	Leucitite (Osann and Rosenbusch).	Ijolite (Osann).	Nephelinite (Rosenbusch).	Nephelite Basalt (Osann).
No. of Analyses.	1	2	7	7	5	9	26
SiO_2	51.70	44.27	46.47	47.72	43.51	41.17	39.87
TiO_2	.23	1.37	1.33	.52	1.07	1.35	1.50
Al_2O_3	14.50	10.73	15.97	18.19	19.54	16.83	13.58
Fe_2O_3	5.07	3.63	5.97	4.74	3.77	7.61	6.71
FeO	3.58	5.87	4.27	3.90	3.88	6.64	6.43
MnO	.01	.06	.01	.06	.16	.16	.21
MgO	4.55	13.05	5.87	3.45	2.94	3.72	10.46
CaO	7.40 [1]	11.46 [2]	10.54	7.27	9.89	10.12	12.36
Na_2O	2.93	1.07	1.69	4.51	10.58	6.45	3.85
K_2O	7.60	4.43	4.83	7.66	2.26	2.49	1.87
H_2O	2.25	3.23	2.32	1.51	.86	2.42	2.22 [3]
P_2O_5	.18	.83	.73	.47	1.54	1.04	94.
Calculated as Water-free.							
SiO_2	52.89	45.75	47.58	48.45	43.89	42.19	40.77
TiO_2	.24	1.41	1.36	.53	1.08	1.38	1.53
Al_2O_3	14.83	11.09	16.35	18.47	19.71	17.25	13.88
Fe_2O_3	5.18	3.75	6.11	4.81	3.80	7.79	6.86
FeO	3.66	6.07	4.37	3.96	3.91	6.81	6.57
MnO	.01	.06	.01	.06	.16	.17	.21
MgO	4.65	13.49	6.01	3.50	2.97	3.81	10.73
CaO	7.57 [4]	11.85 [5]	10.79	7.38	9.98	10.37	12.65
Na_2O	3.00	1.10	1.73	4.58	10.67	6.61	3.94
K_2O	7.79	4.57	4.94	7.78	2.28	2.55	1.90
P_2O_5	.18	.86	.75	.48	1.55	1.07	.96

Each sum = 100.00.

[1] Includes .30% BaO and .07% SrO.

[2] Includes .48% BaO and .18% SrO.

[3] Includes .29% CO_2.

[4] Includes .31 % BaO and .07 % SrO.

[5] Includes .50% BaO and .19% SrO.

GROUP XII.

	Plutonics.		
	78	79	80
	Alaskite (Osann).	Diorite of Electric Peak (Rosenbusch).	Malignite (Osann and Daly).
No. of Analyses.	3	10	4
SiO_2	76.47	62.21	50.34
TiO_2	.07	.60	.34
Al_2O_3	13.03	16.45	14.75
Fe_2O_3	1.04	2.53	4.18
FeO		2.89	2.75
MnO	.01	.02	.11
MgO	.06	3.32	4.23
CaO	.45	4.96	10.43
Na_2O	3.53	3.88 [1]	5.27
K_2O	4.81	2.21	5.21
H_2O	.52	.80 [2]	1.20
P_2O_5	.01	.13	1.19
Calculated as Water-free.			
SiO_2	76.87	62.71	50.95
TiO_2	.07	.60	.35
Al_2O_3	13.10	16.58	14.93
Fe_2O_3	1.05	2.55	4.23
FeO		2.92	2.78
MnO	.01	.02	.11
MgO	.06	3.35	4.28
CaO	.45	5.00	10.56
Na_2O	3.55	3.91 [1]	5.33
K_2O	4.83	2.23	5.27
P_2O_5	.01	.13	1.21
Each sum = 100.00.			

[1] Includes .07% Li_2O.

[2] Includes .05% Cl and .05% SO_3.

GROUP XIII.

	Effusives.						
	81	82	83	84	85	86	87
	Rhyolite of Yellowstone Park (Iddings).	Basalt of Hawaii (Osann).	Banakite (Osann).	Shoshonite (Osann).	Absarokite (Osann).	Leucite Absarokite (Osann).	Melilite Basalt (Osann).
No. of Analyses.	10	11	4	8	5	2	6
SiO_2	74.04	48.36	52.04	53.56	50.11	47.45	36.19
TiO_2	.18	.66	.76	.82	.96	.81	7.11
Al_2O_3	13.19	15.40	17.65	17.88	13.04	11.43	10.52
Fe_2O_3	1.35	6.48	4.66	4.51	4.58	3.22	8.48 [1]
FeO	1.01	10.07	2.75	3.05	3.94	5.78	5.97
MnO	.04	.80	.13	.07	.11	.12	....
MgO	.32	4.19	3.33	3.62	9.27	14.60	14.59
CaO	1.19	8.69	5.11	6.45	7.63	8.18	9.88
Na_2O	3.88	3.34	4.10	3.41	1.94	2.32	3.28
K_2O	3.75	1.30	5.03	3.76	4.15	2.99	2.03
H_2O	1.02 [2]	.43	3.74	2.32	3.58	2.50	1.94 [3]
P_2O_5	.03	.28	.70	.55	.69	.60	.01
Calculated as Water-free.							
SiO_2	74.80	48.57	54.06	54.84	51.97	48.67	36.90
TiO_2	.18	.66	.79	.84	1.00	.83	7.25
Al_2O_3	13.33	15.47	18.34	18.31	13.52	11.73	10.73
Fe_2O_3	1.37	6.51	4.84	4.62	4.74	3.30	8.65 [4]
FeO	1.02	10.11	2.85	3.12	4.08	5.93	6.09
MnO	.04	.80	.14	.07	.12	.12	....
MgO	.32	4.21	3.46	3.70	9.62	14.97	14.88
CaO	1.20	8.73	5.31	6.60	7.91	8.39	10.08
Na_2O	3.92	3.35	4.26	3.49	2.01	2.38	3.34
K_2O	3.79	1.31	5.22	3.85	4.31	3.06	2.07
P_2O_5	.03	.28	.73	.56	.72	.62	.01
Each sum = 100.00							

[1] Includes 2.85 % Cr_2O_3.

[2] Includes .02 % Li_2O and .23 % SO_3.

[3] Loss on ignition.

[4] Includes 2.47 % Cr_2O_3.

GROUP XIV.

Dike-rocks.

	88	89	90	91	92
	Granite-aplite (Osann and Washington).	Bostonite (Rosenbusch and Washington).	Grorudite (Osann and Washington).	Sölvsbergite (Osann and Washington).	Tinguaite (Osann and Washington).
No. of Analyses.	15	5	5	8	15
SiO_2	75.00	61.32	70.91	62.16	55.02
TiO_2	.30	.89	.48	.31	.36
Al_2O_3	13.14	18.43	11.50	17.58	20.42
Fe_2O_3	.58	3.84	4.58	3.05	3.06
FeO	.40	1.60	1.88	1.80	1.82
MnO	.07	.01	.39	.18	.22
MgO	.30	.46	.11	.48	.59
CaO	1.13	1.45	.39	1.11	1.67
Na_2O	3.54	5.75	5.43	7.30	8.63
K_2O	4.80	4.94	4.08	4.95	5.38
H_2O	.71	1.31	.25	1.04	2.77
P_2O_5	.03	. . . .	. . . .	.04	.06
Calculated as Water-free.					
SiO_2	75.54	62.14	71.09	62.82	56.59
TiO_2	.30	.90	.48	.31	.37
Al_2O_3	13.23	18.67	11.53	17.77	21.00
Fe_2O_3	.58	3.89	4.59	3.08	3.15
FeO	.40	1.62	1.89	1.82	1.87
MnO	.07	.01	.39	.18	.23
MgO	.30	.47	.11	.49	.61
CaO	1.14	1.47	.39	1.12	1.72
Na_2O	3.57	5.82	5.44	7.37	8.87
K_2O	4.84	5.01	4.09	5.00	5.53
P_2O_5	.03	. . . .	. . . .	.04	.06

Each sum = 100.00.

GROUP XV.

	Dike-rocks.					
	93	**94**	**95**	**96**	**97**	**98**
	Minette (Osann and Clarke).	Kersantite (Osann and Rosenbusch).	Vogesite (Osann).	Camptonite (Osann).	Monchiquite (Osann).	Alnöite (Osann and Washington).
No. of Analyses.	10	20	4	15	16	6
SiO_2	49.45	50.79	52.62	40.70	45.17	32.31
TiO_2	1.23	1.02	.54	3.86	1.90	1.41
Al_2O_3	14.41	15.26	14.86	16.02	14.78	9.50
Fe_2O_3	3.39	3.29	3.60	5.43	5.10	5.42
FeO	5.01	5.54	4.18	7.84	5.05	6.34
MnO	.13	.07	.84	.16	.35	.01
MgO	8.26	6.33	8.55	5.43	6.26	17.43
CaO	6.73	5.73	5.86	9.36	11.06	13.58
Na_2O	2.54	3.12	3.21	3.23	3.69	1.42
K_2O	4.69	2.79	2.83	1.76	2.73	2.70
H_2O	3.04 [1]	5.71 [2]	2.70	5.59 [3]	3.40	7.50 [4]
P_2O_5	1.12	.35	.21	.62	.51	2.38
Calculated as Water-free.						
SiO_2	50.99	53.87	54.08	43.10	46.76	34.93
TiO_2	1.27	1.08	.56	4.09	1.96	1.52
Al_2O_3	14.86	16.18	15.28	16.97	15.30	10.27
Fe_2O_3	3.50	3.48	3.70	5.76	5.28	5.86
FeO	5.17	5.88	4.29	8.30	5.23	6.85
MnO	.13	.07	.86	.16	.36	.01
MgO	8.53	6.71	8.79	5.76	6.48	18.84
CaO	6.95	6.09	6.02	9.92	11.45	14.68
Na_2O	2.62	3.31	3.30	3.42	3.82	1.53
K_2O	4.84	2.96	2.90	1.86	2.83	2.92
P_2O_5	1.14	.37	.22	.66	.53	2.59
Each sum = 100.00.						

[1] Includes .61% CO_2.

[2] Includes 2.61% CO_2.

[3] Includes 2.97% CO_2.

[4] Includes 4.35% CO_2.

Index to Table II.

Tinguaite	92	Vogesite	95
Trachydolerite	60	Websterite	52
Trachyte	14	Wehrlite	53
Urtite	22		

AVERAGE SPECIFIC GRAVITIES OF CERTAIN TYPES.

The average specific gravities of holocrystalline types have been calculated, with result shown in the following accessory table. Most of the determinations were taken from Osann's book.

	Number of Specimens averaged.	Average Specific Gravity.
Granite	58	2.660
Granodiorite	5	2.740
Syenite	11	2.773
Monzonite	2	2.805
Nephelite syenite	13	2.600
Diorite	17	2.861
Gabbro	19	2.933
Olivine gabbro	4	2.948
Anorthosite	6	2.715
Peridotite	21	3.176
Essexite	2	2.862
Theralite	3	2.917
Malignite	4	2.884

SOME APPLICATIONS.

The uses to which the averages may be put are diverse and, in certain instances, direct and important. A brief note in this place will indicate something of the range of the considerations affected.

1. The writer has found from personal experience that the averages have been of decided benefit in showing the chemical individuality and true nature of the igneous-rock types as actually mapped. To student and investigator alike such averages are, for many purposes, more valuable than single analyses. They help to show that eruptive rocks

do not form an infinite series, but that the varieties cluster about "center-points." Osann's great compilation proves that Rosenbusch's classification is an objective and "natural" one to a highly useful degree.

2. The obvious error involved in computing "the average composition of the primitive crust of the earth," or "the average igneous rock," or "the mean composition of the accessible parts of the earth's crust," by averaging a large number of analyses compiled at random, has not deterred a goodly number of authors from using such results as those deduced by Clarke, Washington, and Harker. These averages are bound to breed further errors when used as a basis for quantitative studies in geology or oceanography. The discovery of "the average igneous rock" is of the highest importance for many problems such as the chemical denudation of the lands and the chemical evolution of the ocean. The mean composition of the accessible crystalline rocks of the globe must ultimately be obtained by taking account of the relative volumes of the different rock-types. In computing the mean the average analyses for the principal individual species must be employed. Since the only approach to success is through the quantitative study of geological maps and memoirs, it is clear that for many years to come the averages for the types recognized in Rosenbusch's system are to be basal to the calculation.

A glance at Table II. shows, however, that this new world-average will differ little from the earlier world-averages with respect to one oxide, namely, soda. For each of the areally and volumetrically important rock-types the average soda never departs far from a mean of about three and one half per cent. The soda in the averages of Clarke, Washington, and Harker (calculated as water-free) is, respectively, 3.63 per cent, 3.34 per cent, and 3.90 per cent.[1] The agreement is fortunate, since, for example, the quantitative problem relative to the sodium in the ocean can be pursued without waiting for the close determination of "the average igneous rock." Incidentally, it may be remarked that the estimates of Joly[2] and Sollas[3] regarding the age of the ocean, as determined by the sodium content, need revision, since neither author has allowed for the great variations in the area of the lands during geological time.

3. The recurrence of the main types of igneous rock in every continent shows that general processes of differentiation have been at work

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[1] F. W. Clarke, Bull. 228, U. S. Geol. Survey, 1904, p. 16.

[2] J. Joly, Sci. Trans. Roy. Dublin Society, **7,** 23 (1899).

[3] W. J. Sollas, Quart. Jour. Geol. Soc., Presidential Address, **65,** p. lxxix (1909).

from the earliest recorded time. There is no reason to doubt that the diorite or the nephelite syenite of the pre-Cambrian periods have generally owed their origin to the same physico-chemical reactions as those responsible for the Mesozoic or Tertiary diorite or nephelite syenite. If this be true, the world-averages for the different principal types should be so many tests of theoretical conclusions as to the causes of the differentiation of those types. The question as to the derivation of augite andesite from basalt through fractional crystallization has been thus tested, with, so far as this test goes, an affirmative answer.[4]

Sometimes the averages themselves suggest lines of thought. For example, the average granite analysis (calculated water-free; 236 analyses) is close to the average of four analyses of the glassy base of augite andesite (calculated as water-free). The comparison may be made from the following table:

	Granite of all Periods.	Ground-mass (base) of Augite andesite.
No. of Analyses	**236**	**4**
	per cent.	per cent.
SiO_2	70.47	69.31
TiO_2	.39	....
Al_2O_3	14.90	17.11
Fe_2O_3	1.63	2.15
FeO	1.68	.60
MnO	.13	....
MgO	.98	.70
CaO (BaO and SrO)	2.17	2.63
Na_2O	3.31	3.20
K_2O	4.10	4.30
P_2O_5	.24	....
	100.00	100.00

The exact meaning of the correspondence between the two averages may not be discussed here; but it does suggest an explanation of the

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[4] Journal of Geology, **16**, 401 (1908).

common association of granites (and liparites) with andesites (and diorites) in nature. The question is open as to whether the primitive granite-liparite magma was not a polar differentiate of an andesitic magma, preferably by a settling-out of the phenocrystic constituents (in solid or liquid phases) from the andesitic magma.

Other related questions are raised by the comparison of the mean of average granite and average basalt with average diorite (including quartz diorite).

	1. Average Granite.	2. Average Basalt.	3. Mean of 1 and 2.	4. Average Diorite.
No. of Analyses	236	161	—	89
	per cent.	per cent.	per cent.	per cent.
SiO_2	70.47	49.65	60.06	59.19
TiO_2	.39	1.41	.90	.81
Al_2O_3	14.90	16.13	15.52	16.51
Fe_2O_3	1.63	5.47	3.55	3.02
FeO	1.68	6.45	4.06	4.17
MnO	.13	.30	.21	.13
MgO	.98	6.14	3.56	3.93
CaO	2.17[1]	9.07	5.62	6.47
Na_2O	3.31	3.24	3.28	3.39
K_2O	4.10	1.66	2.88	2.12
P_2O_5	.24	.48	.36	.26
	100.00	100.00	100.00	100.00

[1] Includes .06% BaO and .02% SrO.

Is basalt the basic pole, granite the acid pole, of a primitive differentiation of diorite magma? Is diorite the product of mixture of primitive, granitic crust and primary basalt still molten beneath? Though the averages give no answer, they tend to keep these fundamental queries before the eye of the petrologist.

4. The averages have been arranged so as generally to place

together those of plutonics and the corresponding effusive rocks. The comparisons show the truth of Rosenbusch's statement that the effusives are, on the whole, somewhat higher in silica and alkalies and lower in iron oxides, lime, magnesia, etc., than the respective plutonics.

The importance of this rule is at least two-fold. It proves the value of Rosenbusch's primary division into the deep-seated types and the surface lavas. It shows therewith one of the reasons why the Norm [5] Classification of igneous rocks is largely a failure so far as either the field-geologist or the student of petrogeny is concerned.

Secondly, the rule suggests clearly that at volcanic vents there is a general cause for the removal of iron, magnesium, and calcium oxides from the magmatic columns and that the cause is more effective in volcanic vents than in the average plutonic body. The cause is most probably to be found in the gravitative settlement of part of the ferromagnesian and other constituents of early crystallization. These constituents may settle out either as solid crystals or as liquid fractions immiscible near the consolidation point of the magma. Since, on the average, the column of fluid magma is taller in an active volcanic vent than in a plutonic mass, the overlying phase of the splitting magma should be, in general, slightly more acid and alkaline than the corresponding pole of differentiation in a deep-seated mass. In the nature of the case the more acid-alkaline pole is the one most liable to flow out at the surface. Though volcanic vents are much narrower than plutonic chambers and therefore subject to quicker chilling, with a resulting check to differentiation, this tendency is largely counterbalanced by the passage of very hot gases through vents. The mere agitation in the vents facilitates the separation. Whatever additional considerations are necessary to complete the comparison, it must here suffice to note that, as a rule, the laws of solution as applied to magmas seem to demand a differentiation with slow cooling, whereby a surface lava is less basic and ferromagnesian than the plutonic body feeding the vent of that lava. The corroboration of Rosenbusch's above-mentioned rule through the world-averages appears, therefore, to be of use in illustrating one of the world-wide influences controlling the origin of igneous rocks.

Some special conclusions regarding classification may be noted. From the averages it is evident that dacite is the effusive correspondent of granodiorite and not of quartz diorite. The contention of

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[5] Quantitative Classification of Igneous Rocks, by W. Cross, J. P. Iddings, L. V. Pirsson, and H. S. Washington, Chicago and London, 1903.

American geologists that the vast development of granodiorite in the Cordilleras of North and South America should alone give the name a primary place in rock classification, is again justified. The many occurrences of dacite throughout the world represent just so many additional masses of cooled magma which were chemically identical with, or closely related to granodiorite. In volumetric importance, as in mineralogical and chemical individuality, the granodiorite type should rank as of the same order as granite itself.

Quartz porphyry, liparite, and rhyolite show that essential identity of composition which has long been apparent from more qualitative comparison.

5. There is little noteworthy chemical difference between the average pre-Cambrian granite and the average granite of later periods. How far the differences in alumina and potash (columns **1**, **2**, and **3**) are due to the relative fewness of analyses of pre-Cambrian types cannot be stated. In spite of any such uncertainties the stability of the chemical type represented by granite throughout geological time is manifest. The explanation of the fact may well be found in Vogt's idea that granite is an "anchi-eutectic," a crystallized mother-liquor, a nearly extreme product of magmatic differentiation. It is possible that some of the older pre-Cambrian granite represents the differentiation of primeval magna. For many reasons it seems probable that most, if not all, post-Cambrian granites are differentiates from syntectic magma, chiefly composed of primary basaltic magma which has locally redissolved the ancient, acid shell overlying. In such case the splitting of the syntectic would ultimately give an acid differentiate similar to that formed in the primitive time. In general, differentiation in batholiths, when well advanced, restores the condition temporarily disturbed by magmatic assimilation. On this (confessedly hypothetical) view one may feel no surprise in noting a fairly steady composition in the granites from the average oldest type to the average youngest.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

ON THE APPLICABILITY OF THE LAW OF CORRESPONDING STATES TO THE JOULE-THOMSON EFFECT IN WATER AND CARBON DIOXIDE.

By Harvey N. Davis.

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Presented by John Trowbridge, December 8, 1909; Received December 30, 1909.

In the classical plug experiments of Joule and Kelvin certain gases were forced by pressure through a porous plug under circumstances which permitted the accurate measurement of any small resulting change in their temperature. It can easily be shown that a perfect gas would show no such change. As a matter of fact, hydrogen was found to be slightly warmer on the low pressure side of such a plug than on the high pressure side, while air, oxygen, nitrogen and carbon dioxide were slightly cooler. The ratio of the observed drop in temperature to the drop in pressure in such a plug has ever since been called the Joule-Thomson coefficient.

The results of such experiments afford the best known means of computing corrections for reducing the temperature scale of a gas thermometer to Kelvin's absolute thermodynamic scale. For this purpose one must know the Joule-Thomson coefficient of the gas in the thermometer at all temperatures between 0° C. and the t° C. at which the correction is desired. Unfortunately, none of the experiments either of Joule and Kelvin or of any of their successors are at temperatures other than between 0° C. and 100° C., except for certain inversion points of Olschewsky obtained under circumstances not yet fully understood. These are not enough to give a direct determination of the absolute thermodynamic scale above 100°. In order to get one indirectly, it has been customary to assume that, at least in the five gases, hydrogen, oxygen, nitrogen, carbon dioxide and air, the Joule-Thomson effect obeys the law of corresponding states. That is, it is assumed that if the coefficient for each gas is expressed in terms of the critical pressure and temperature of that gas as units, and if the results are plotted against the temperature expressed in the same "reduced" units, the resulting curves will be identical for all five gases. The observations at ordinary temperatures on hydrogen, whose critical temperature is very low, will then correspond to observations at very high temperatures on other gases, and will afford a useful though precarious extrapolation of their curves to above 1000° C.

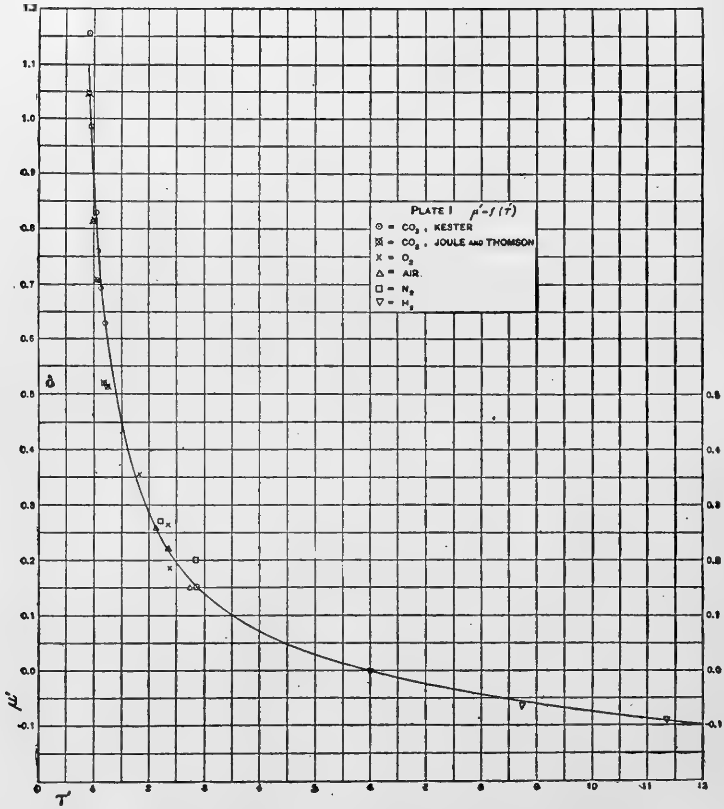

FIGURE 1. Reduced Joule-Thomson coefficient, μ', plotted against reduced temperature. From Buckingham's paper in the Bulletin of the Bureau of Standards, May, 1908. (See the note at the end of this paper.)

The experimental justification of this use of the law of corresponding states is, as yet, meager. Figure 1, which is taken from a recent paper by Buckingham, represents the available data. It will be seen that neither the hydrogen nor the carbon dioxide observations overlap those on the other three gases, and that the points for each of these three gases show such discrepancies among themselves as to make uncertain any judgment as to their agreement with each other. What evidence there is, is in favor of the validity of the law of corresponding states; but an accurate verification of it, especially for two substances with very different critical temperatures, would put the whole subject on a much more satisfactory basis.

In this paper it will be shown that this law is verified for carbon dioxide and water within the limit of error of the available observations on water. This limit of error is unfortunately quite as great as that of the oxygen, nitrogen and air observations plotted in Figure 1. Nevertheless, a multiplication of evidence, even of an inferior sort, is often valuable, and in this case there is an added interest because, if water, which is known to be anomalous in many ways through association, is found to obey the law of corresponding states as to its Joule-Thomson effect, it is probable that the permanent gases will also obey that law.

There are four sets of experiments on water which can be used. They were all undertaken for the purpose of determining the variation of the specific heat of superheated steam with pressure and temperature, an investigation which has since been more satisfactorily accomplished in other ways. Of the four observers, Griessmann[1] used a porous plug very much like that of Joule and Thomson, while the other three, Grindley,[2] Peake[3] and Dodge,[4] used what engineers call a throttling or wiredrawing calorimeter. The essential part of this instrument is a small orifice through which the steam flows tumultuously from one chamber into another, the high velocity of the steam being subsequently destroyed by friction at the surfaces of the walls of the second chamber and within the steam itself. During this process the kinetic energy of the steam is transformed into heat, all of which, if the thermal insulation is perfect, goes back into the steam. If this transformation is complete, the throttling calorimeter is exactly equivalent to a porous plug. To ensure this completeness, one of the three observers (Peake) put a quantity of wire gauze in the path of the steam from the orifice, and another (Dodge) used at times four small orifices instead of one larger one without noticeable change in the results. Grindley took no especial precautions of this sort, but the

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[1] Zeitsch. Ver. d. Ing., 1903, **47**, 1852 and 1880; also Forschungsarb., Ver. d. Ing., 1904, **13**, 1.

[2] Phil. Trans., 1900–1, **194A**, 1.

[3] Proc. Roy. Soc., 1905, A, **76**, 185.

[4] Jour. Am. Soc. Mech. Engs., 1907, **28**, 1265; and 1908, **30**, 1227.

fact that his results agree with those of Peake and of Griessmann shows that none were necessary in his apparatus.

This agreement is in many other ways a significant one, for it is inconceivable in view of the great differences in almost every respect between the details of the three sets of apparatus, that any serious systematic errors should have been present in any one of the sets of results without completely destroying the agreement between them. This is particularly true in the matter of heat insulation, where the precautions taken by the three observers had almost nothing in common except effectiveness. In Dodge's work also this point was carefully considered but the results are not so satisfactory. They will be discussed and a correction computed on page 262.

In all four cases the thermometry is the weakest part of the work. It is especially unfortunate for the present purpose that the original aim of the experiments did not require or suggest that the difference between the temperatures before and after the expansion be measured as such, as by a thermocouple or a differential resistance thermometer. The subtraction which must now be made of one reading on a mercury thermometer from another reading on another thermometer, to give a small difference, is not a particularly accurate method of getting that difference. The same is true of the determination of the pressure drop. The individual measurements were comparatively good, being made in three of the cases with carefully calibrated Bourdon or spring gauges, and in the fourth case by an extra measurement of the temperature of resaturation of the low side steam, but the differences needed in this paper must inevitably be subject to comparatively large errors. The reader must therefore be prepared for much lack of self-consistency in the results. It is hoped that the errors are largely incidental errors such as can be eliminated by averaging.

Grindley's experiments were performed in England during the winter of 1897–8. His data are given in full in his paper and are plotted in his Diagram 5 reproduced here as Figure 2. It will be observed that in every case his steam drops several pounds in pressure before it leaves the saturation line. This he explained by means of a curious and now discredited "heat of gasification." A better explanation is that his steam was initially slightly wet. Since this source of error affects the high side data of every one of his experiments, it might seem that all of his work must be rejected. It will be noticed, however, that his experiments are grouped into runs; that is, if in a certain experiment steam in a certain initial condition has been throttled to a certain low side pressure and temperature, then in later experiments of the same group, steam *in the same initial condition* is more and more throttled to lower low side pressures and temperatures, which when plotted together form the throttling curves of Figure 2. Since it is characteristic of throttling that the total heat, H, of the steam is the same on the high and low sides, it follows that H is constant along the whole of any throttling curve, and that any two low side points of a run may be taken, one as describing the high side conditions and the other as describing the low side conditions of a possible throttling experiment.

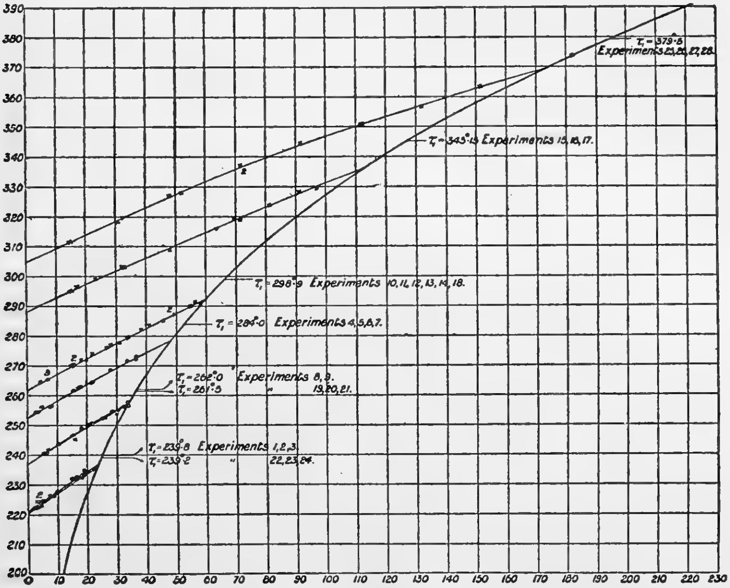

FIGURE 2. Grindley's throttling curves. Abscissae are pressures in lbs. per sq. in. Ordinates are Fahrenheit temperatures. From his paper in the Philosophical Transactions.

In other words, the slope of a throttling curve at any point is a value of the Joule-Thomson coefficient under corresponding conditions. It is therefore possible, even while rejecting all of Grindley's high side points together with that one of the low side points which is obviously affected by the same error, to use the remaining low side points in pairs. There were 101 of them in all, lying on seven throttling curves. They were first grouped so as to give 29 average points, the averaging being justified by the fact that for a range of not more than 5°, a throttling curve can be considered straight. These means were then taken two by two consecutively to give 22 values of the Joule-Thomson coefficient, each of which is assumed to correspond to the mean of the high and low side temperatures from which it was obtained. The values of the coefficient have been "reduced" by multiplying by 2.56, a factor which is the ratio of the critical pressure of water expressed in pounds per square inch (2947 lbs. per sq. in. or 200 atmospheres [5]) to its critical temperature in Fahrenheit degrees absolute (1149° F. abs. or 365° C. ord.[5]). The results are summarized in Table I, which gives

TABLE I.

SUMMARY OF GRINDLEY'S THROTTLING EXPERIMENTS.

Curve.	No. of Points.	Average Pressure		Average Temperature		Reduced Joule-Thomson Coefficient.
		lbs. per sq. in.	Reduced.	Fahr.	Reduced.	
A	1–1	141.6	0.0480	360.2	0.714	0.82
	1–2	121.7	0.0412	353.9	0.708	0.79
	2–1	101.6	0.0344	347.4	0.703	0.86
	1–2	81.7	0.0277	340.5	0.697	0.92
	2–2	61.0	0.0207	332.3	0.690	1.12
	2–1	40.3	0.0137	322.9	0.682	1.20
	1–1	22.7	0.0077	315.0	0.675	1.11
B	2–1	87.5	0.0296	326.3	0.685	1.00
	1–3	74.7	0.0253	321.0	0.680	1.13
	3–1	58.2	0.0197	313.6	0.673	1.15
	1–2	40.3	0.0137	306.1	0.667	1.00
	2–3	25.5	0.0086	299.9	0.662	1.14
C	3–5	50.7	0.0172	288.0	0.651	1.27
	5–4	37.5	0.0127	281.4	0.645	1.29
	4–8	24.3	0.0082	274.7	0.640	1.31
	8–4	12.4	0.0042	268.4	0.634	1.39
D	4–5	26.3	0.0089	267.2	0.633	1.40
	5–4	12.4	0.0042	259.5	0.626	1.47
E	8–5	24.2	0.0082	251.7	0.620	1.39
	5–4	12.8	0.0043	245.1	0.613	1.57
F *a*	6–7	12.2	0.0041	229.3	0.600	1.74
F *b*	6–5	11.5	0.0039	228.7	0.600	1.69

Column 2 indicates the number of observations involved in each of the two means used in each case. Thus 6–7 indicates that the mean used as the high side point of the pair included 6 of the points plotted in figure 2, while that used as the low side point involved 7.

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[5] Cailletet and Colardeau, Jour. de Phys., 1891, **10**, 333.

also the corresponding "reduced" pressures and temperatures. These values of the coefficient are plotted as open circles in Figure 6.

The experiments of Griessmann were performed in the mechanical engineering laboratory of the "Technische Hochschule" in Dresden, and were published in 1903. They were primarily undertaken to test the heat of gasification hypothesis already mentioned, and are a critical repetition of Grindley's work. The data are given in full in the paper in the Forschungsarbeiten, and are plotted in his Figure 7, which is reproduced here as Figure 3. He records 13 runs with 87 sets of low side observations, which with the 13 high side observations give 100 points on his diagram. Of these, three points on curve 2, one point on curve 7, three points on curve 8, and three points on curve 9 lie so far off the smooth curves determined by the neighboring points that they have arbitrarily been omitted from these calculations. The remaining 90 points, lying on 11 curves, have been grouped in 44 means

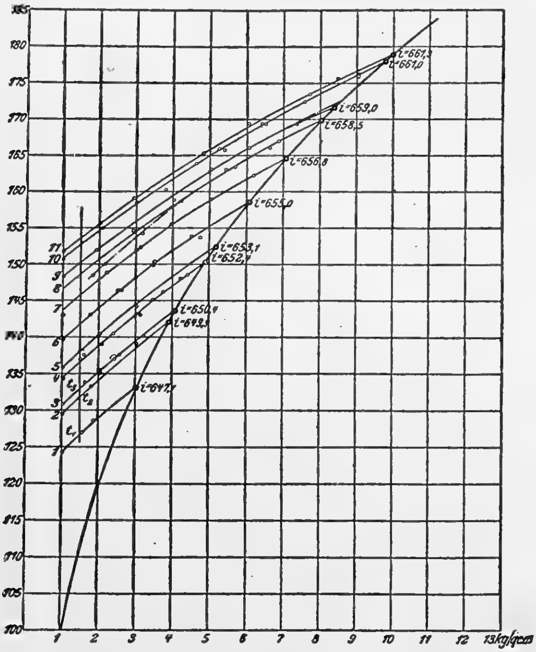

FIGURE 3. Griessmann's throttling curves. Abscissae are pressures in kg. per sq. cm. Ordinates are Centigrade temperatures. From his paper in the Forschungsarbeiten.

TABLE II.

Summary of Griessmann's Observations.

Curve.	No. of Points.	Average Pressure		Average Temperature		Reduced Joule-Thomson Coefficient.
		kgs./sq. cm.	Reduced.	Cent.	Reduced.	
1	2–2	2.19	0.0106	129.8	0.632	1.33
	2–1	1.37	0.0066	126.1	0.625	1.64
2	1–1	2.86	0.0138	137.6	0.644	1.34
	1–1	1.42	0.0069	131.4	0.634	1.56
3	2–2	2.97	0.0144	139.2	0.646	1.26
	2–2	1.81	0.0088	134.4	0.639	1.44
4	2–5	4.35	0.0210	148.2	0.660	1.19
	5–4	2.85	0.0138	142.3	0.651	1.30
	4–2	1.46	0.0071	136.5	0.642	1.47
5	1–1	4.11	0.0199	149.2	0.662	1.27
	1–1	2.53	0.0122	142.3	0.651	1.23
	1–1	1.52	0.0074	138.2	0.644	1.44
6	2–2	5.36	0.0260	156.1	0.673	1.07
	2–3	4.18	0.0202	152.2	0.667	1.06
	3–4	2.98	0.0144	147.9	0.660	1.22
	4–2	1.64	0.0079	142.5	0.651	1.42
7	2–2	5.56	0.0219	160.3	0.679	0.93
	2–3	3.64	0.0176	154.1	0.670	1.18
8	3–2	6.37	0.0308	165.4	0.687	0.88
	2–1	4.90	0.0237	161.0	0.680	1.08
9	1–2	6.46	0.0313	166.9	0.690	0.84
	2–1	3.73	0.0180	158.2	0.676	1.13
	1–1	1.46	0.0071	150.2	0.663	1.25
10	2–2	8.22	0.0398	174.1	0.701	0.88
	2–1	5.02	0.0243	164.7	0.686	0.99
	1–1	2.55	0.0123	156.8	0.673	1.23
	1–1	1.55	0.0075	152.8	0.668	1.29
11	1–2	9.05	0.0438	176.6	0.705	0.75
	2–2	7.17	0.0347	171.9	0.698	0.88
	2–1	5.53	0.0268	167.3	0.690	0.96
	1–1	3.89	0.0188	162.2	0.682	1.01
	1–1	2.48	0.0120	157.4	0.675	1.19
	1–1	1.52	0.0074	153.8	0.669	1.25

which have been used as above to give the 33 values of the Joule-Thomson coefficient which are presented in the following table. They are plotted as circles with diagonal crossbars in Figure 6. The reduction factor in this case is 0.324, Griessmann's pressures being in kilograms per square centimeter and his temperatures in Centigrade degrees.

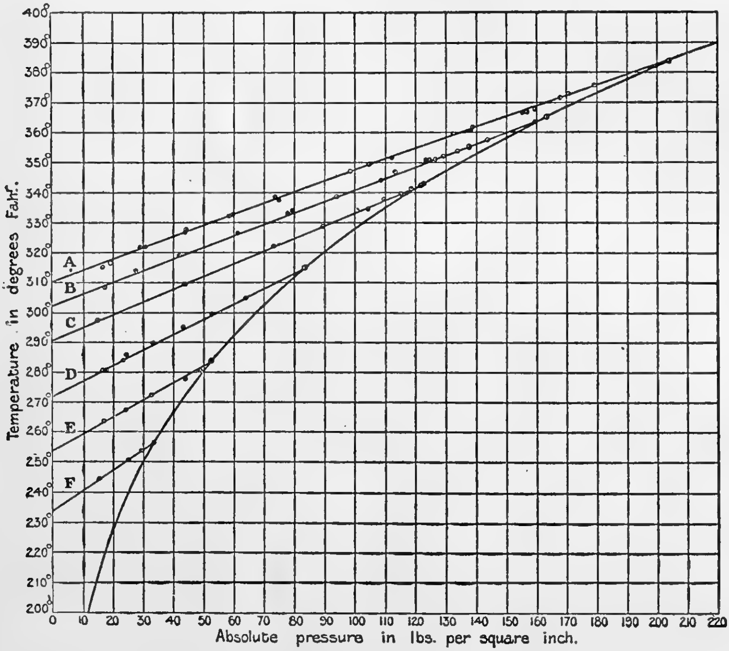

FIGURE 4. Peake's throttling curves. From his paper in the Proceedings of the Royal Society.

Peake's experiments were carried out in the engineering laboratory of Cambridge University in England and were begun in the fall of 1898. The appearance in 1900 of Grindley's work along almost identical lines at first inclined Peake to discontinue his investigation, but a careful examination of Grindley's data as compared with his own, led him to the discovery in both of the heat of gasification error already mentioned and to its true explanation, and his experiments were continued with this particular point in view. His apparatus was therefore redesigned so as to bring the steam as quickly as possible from the boiler to the orifice to avoid condensation on the way, and he, like

TABLE III.

Summary of Peake's Throttling Experiments.

Curve.	No. of Points.	Average Pressure		Average Temperature		Reduced Joule-Thomson Coefficient.
		mm. of Hg.[1]	Reduced.	Cent.	Reduced.	
A	3–3	9724	0.0639	192.3	0.730	0.85
	3–3	8503	0.0569	187.8	0.722	0.94
	3–2	7632	0.0502	184.4	0.716	0.90
	2–3	6282	0.0413	179.4	0.709	0.87
	3–2	4617	0.0304	173.1	0.699	0.94
	2–4	3239	0.0213	167.8	0.690	0.87
	4–4	1925	0.0127	162.5	0.683	1.05
B	4–9	7594	0.0499	181.5	0.712	0.97
	9–3	6128	0.0403	175.6	0.703	0.97
	3–2	4735	0.0312	170.0	0.695	0.95
	2–2	3359	0.0221	164.3	0.686	1.01
	2–2	1914	0.0126	158.2	0.675	1.00
C	2–2	6162	0.0405	171.8	0.697	1.09
	2–2	5762	0.0379	170.0	0.694	1.10
	2–1	5017	0.0337	166.6	0.689	1.05
	1–1	4163	0.0274	162.8	0.683	1.13
	1–1	3030	0.0199	157.4	0.675	1.11
	1–1	1513	0.0100	150.6	0.665	1.06
D	1–1	4212	0.0277	156.6	0.674	1.05
	1–1	3960	0.0260	155.3	0.671	1.14
	1–1	3547	0.0233	153.4	0.668	1.11
	1–1	3035	0.0200	151.0	0.665	1.17
	1–1	2502	0.0165	148.3	0.661	1.17
	1–2	1984	0.0131	145.6	0.656	1.36
	2–1	1460	0.0096	142.7	0.652	1.29
	1–1	986	0.0065	140.0	0.648	1.53
E	2–1	2045	0.0135	135.4	0.640	1.41
	1–1	1468	0.0096	132.1	0.635	1.35
	1–1	1050	0.0069	129.6	0.631	1.44
F	1–1	1404	0.0092	122.2	0.619	1.66
	1–1	1044	0.0069	119.7	0.615	1.63

[1] All of Peake's pressures were computed from suitable temperature measurements by means of Regnault's steam table. As a special precaution they have been recomputed with the new table of Holborn and Henning, and are therefore left in the metric units in which they were thus found. The "reduction factor" to give μ' is 238.

Griessmann, practically eliminated the effect which Grindley had found. His results are plotted as his Figure 4 which is reproduced as Figure 4 of this paper. He records 10 runs with 68 low side observations, making 78 points in all. Two of the high side points and two of the low side points still show traces of the wet steam effect and have therefore been rejected. The other low side points are much more self-consistent than Griessmann's. The ten runs correspond to only six throttling curves. The 74 satisfactory points were grouped into 33 means, giving the 27 values of the Joule-Thomson coefficient which are presented in Table III and are plotted as circles with horizontal crossbars in Figure 6.

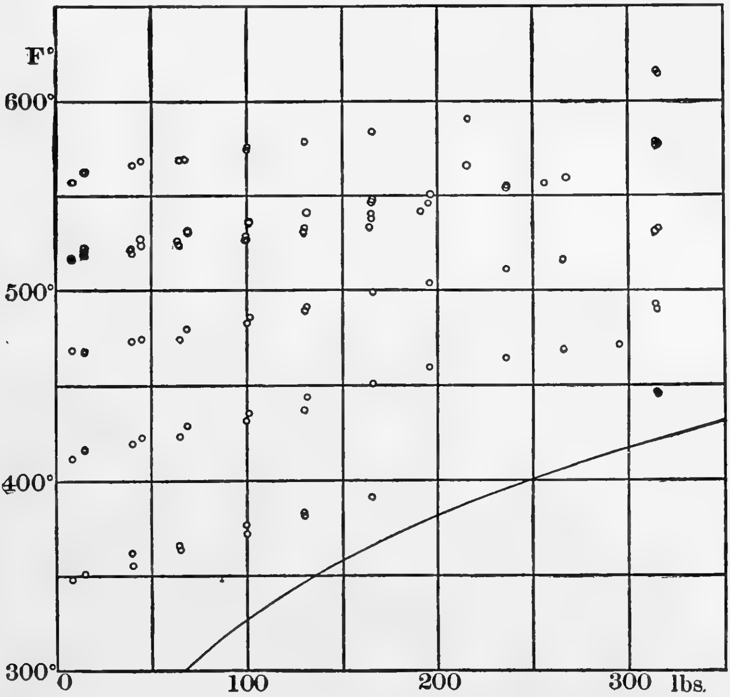

FIGURE 5. Dodge's throttling curves. Plotted from the original data sheets.

Dodge worked in the laboratories of the General Electric Company at Schenectady, N. Y., from 1901 to 1906. His data were not given at all in his first paper and were published only in part in his second paper. What follows is based on a study of the original records, the generous loan of which for this purpose is very gratefully acknowledged. On his advice, the first 26 of his 92 runs were disregarded as preliminary, and 9 other runs were rejected, either because of experimental mishaps, or because the log did not show satisfactorily steady conditions. The data selected were corrected for probable radiation and conduction losses in the way explained in the appendix of this paper (page 262).

Of the 47 selected tests, 14 were like those already discussed, except that the temperatures were much higher, the high side steam being superheated instead of saturated. The results of these 14 tests are plotted in Figure 5. It will be noticed that in every case a smooth curve through the low side points runs considerably below the corresponding high side point, just as did Grindley's curves. In Grindley's case this was because the entering steam carried water in suspension, the presence of which made the true total heat of the incoming mixture less than its apparent total heat regarded as homogeneous saturated steam, and dropped all the low side points onto throttling curves lower than those on which they apparently belonged. A similar phenomenon may be in evidence in Dodge's case, for although the incoming steam was superheated, it may still have been carrying in suspension a part of the water which had been sprayed into it for temperature regulation just before it reached the high side chamber.[6] It must, however, be admitted that if this explanation is to account for the whole of the discrepancy in Dodge's results, an extraordinarily large amount of water in suspension must have reached the high side chamber — from one to one and a half per cent of the whole weight present. It is therefore probable that there is another source of error not yet discovered. Nevertheless, if the high side points are disregarded and the low side points are taken together in pairs as in Grindley's case, it is probable that the resulting values of the Joule-Thomson coefficient will be trustworthy.

Each of the 14 runs was handled separately. It did not seem best to take consecutive points together as in the other cases, because, at the very high temperatures here dealt with, the temperature difference between consecutive points is much smaller than at lower temperatures, and so an error in either observation would make much more difference in the coefficient. Furthermore, the throttling curves are more nearly straight in this range than at lower temperatures. The lowest point of a run has therefore been taken with the point just beyond the middle

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[6] See the work of Knoblauch and Jakob, Forschungsarb., 1906, **34**, 109.

TABLE IV.

SUMMARY OF DODGE'S THROTTLING CURVE TESTS.

Test.	Average Pressure		Average Temperature		Reduced Joule-Thomson Coefficient.
	lbs. per sq. in.	Reduced.	Fahr.	Reduced.	
70 *a*	36.5	0.0124	563	0.892	0.52
	57.6	0.0196	569	0.895	0.39
	85.2	0.0289	572	0.899	0.36
70 *b*	54.2	0.0184	476	0.816	0.38
	73.0	0.0248	479	0.818	0.46
71	36.5	0.0124	356	0.711	0.72
	57.5	0.0195	362	0.716	0.62
	84.9	0.0288	369	0.722	0.75
72	36.5	0.0124	521	0.855	0.30
	57.4	0.0195	523	0.857	0.20
73	36.7	0.0125	418	0.765	0.55
	52.4	0.0178	424	0.770	0.48
	84.8	0.0288	248	0.774	0.44
74	54.0	0.0183	522	0.856	0.32
	72.6	0.0246	527	0.860	0.27
	102.2	0.0347	530	0.863	0.32
75	84.3	0.0286	373	0.726	0.58
	114.6	0.0389	381	0.733	0.52
76	127.3	0.0432	534	0.866	0.32
	101.0	0.0343	527	0.860	0.24
77	200.6	0.0681	547	0.877	0.50
	225.6	0.0765	551	0.881	0.44
78	57.5	0.0195	568	0.895	0.35
	105.0	0.0356	576	0.902	0.32
	142.0	0.0482	580	0.906	0.36
79	57.9	0.0196	527	0.860	0.50
	105.0	0.0356	535	0.867	0.49
	142.0	0.0482	548	0.878	0.57
80	90.7	0.0308	535	0.867	0.43
	120.4	0.0409	539	0.870	0.39
	152.0	0.0516	543	0.874	0.34
	184.4	0.0626	548	0.878	0.35
81	90.3	0.0306	484	0.822	0.54
	120.3	0.0409	489	0.826	0.49
	152.0	0.0516	495	0.832	0.48
	184.0	0.0625	501	0.837	0.47
82	90.3	0.0306	434	0.779	0.57
	120.3	0.0409	441	0.785	0.62
	152.0	0.0516	447	0.790	0.54
	184.0	0.0625	452	0.795	0.52
	213.5	0.0724	458	0.800	0.42

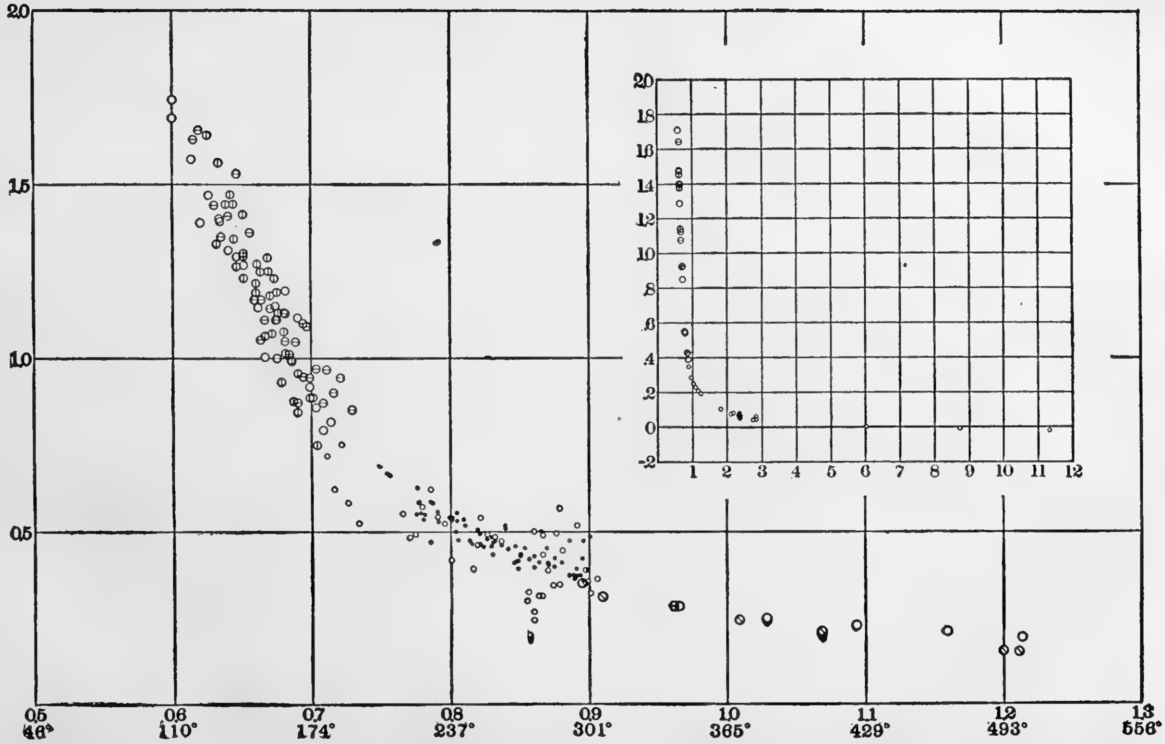

Figure 6. Reduced Joule-Thomson coefficient for steam and carbon dioxide plotted against reduced temperature. At the bottom are given the corresponding temperatures for steam on the Centigrade scale.

of that run, and so on, no point being used more than once. The 41 values of the coefficient obtained in this way are summarized in Table IV, and are plotted as small open circles in Figure 6. They lie in the range between 0.8 and 0.9 reduced temperature, filling a gap of considerable importance in that figure.

The remaining 33 of the selected runs cannot be handled in the same simple way, because the experiments which make up each of these runs are not so related as to give throttling curves, but are related in another way much better suited to the original purpose of the work, but much less suited to the present purpose. Nevertheless the gap between 0.7 and 0.9 in Figure 6 is so important that it is desirable to use every bit of information about it that can be obtained. These 33 additional runs have therefore been discussed at some length in the appendix of this paper, and, suitable corrections for the high side temperatures having been applied, the more favorable of them have been used to get the 77 values of the Joule-Thomson coefficient which are presented in Table IV. These values are plotted in Figure 6 as small black dots. They are more self-consistent than the values in Table IV above, but their trustworthiness is more uncertain as each involves two uncertain corrections of the original data instead of one. They are nevertheless valuable corroborative evidence.

Figure 6 is now complete. The 82 values of the coefficient which are summarized in Tables I., II., and III., lie in the range between 0.6 and 0.7 units of reduced temperature, and form a broad but reasonably well defined band, within which there is no evident tendency for either of the three sets of points to separate themselves from the others. The 118 values of the coefficient which were computed from Dodge's data, and which are presented in Tables IV. and V., lie between 0.7 and 0.9 and form a satisfactory continuation of the band. Above 0.9 are five large circles with diagonal crossbars representing on the same scale the original observations of Joule and Thomson on carbon dioxide, six large circles without crossbars representing Kester's[7] experiments, and one large circle with a horizontal crossbar representing Natanson's[8] result. These circles form a surprisingly good continuation of the curve suggested by the band of steam points. The law of corresponding states is therefore verified for carbon dioxide and water within the limits of error of the observations on the two substances.

The various values in Tables I to V have been grouped according to temperature and averaged. For this purpose a number was assigned

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[7] Phys. Zeitsch., 1905, **6**, 44; repeated and revised in Phys. Rev., 1905, **21**, 260.

[8] Wied. Ann., 1887, **31**, 502.

TABLE V.

Summary of Dodge's Main Series of Tests (Corrected as Described).

Test.	Average Pressure		Average Temperature		Reduced Joule-Thomson Coefficient.
	lbs. per sq. in.	Reduced.	Fahr.	Reduced.	
28	328	0.112	558	0.886	0.48
	“	“	534	0.865	0.50
	“	“	503	0.839	0.52
	“	“	430	0.775	0.55
	“	“	463	0.804	0.56
	“	“	574	0.901	0.49
29	380	0.129	568	0.895	0.42
	“	“	544	0.875	0.42
	“	“	520	0.853	0.45
	“	“	483	0.821	0.49
	“	“	460	0.803	0.54
	“	“	504	0.839	0.51
	“	“	538	0.869	0.45
31	330	0.112	511	0.846	0.41
	“	“	474	0.813	0.48
	“	“	441	0.785	0.47
32	379	0.129	558	0.886	0.38
	“	“	540	0.871	0.40
	“	“	524	0.857	0.42
	“	“	507	0.842	0.45
	“	“	493	0.830	0.48
	“	“	469	0.809	0.54
	“	“	444	0.787	0.58
36	385	0.131	563	0.891	0.39
	“	“	539	0.870	0.41
	“	“	516	0.850	0.44
	“	“	491	0.829	0.48
	“	“	460	0.801	0.54
37	338	0.115	571	0.898	0.39
	“	“	545	0.875	0.40
	“	“	516	0.850	0.43
	“	“	480	0.819	0.50
	“	“	456	0.798	0.54
41	205	0.070	562	0.890	0.36
	“	“	527	0.860	0.40
	“	“	462	0.803	0.50
	“	“	432	0.777	0.59
42	255	0.087	565	0.893	0.37
	“	“	540	0.871	0.40
	“	“	515	0.850	0.43
	“	“	492	0.829	0.46
	“	“	459	0.800	0.53
	“	“	437	0.781	0.55

TABLE V — (*continued*).

Test.	Average Pressure		Average Temperature		Reduced Joule-Thomson Coefficient.
	lbs. per sq. in.	Reduced.	Fahr.	Reduced.	
43	302	0.103	550	0.880	0.41
	“	“	533	0.865	0.43
	“	“	514	0.847	0.46
	“	“	495	0.832	0.47
	“	“	468	0.808	0.54
	“	“	444	0.786	0.56
56	256	0.087	513	0.848	0.38
	“	“	475	0.815	0.46
	“	“	435	0.780	0.55
59	252	0.086	560	0.888	0.38
	“	“	531	0.863	0.41
	“	“	488	0.826	0.48
	“	“	441	0.785	0.59
	“	“	463	0.804	0.53
60	204	0.069	500	0.837	0.46
	“	“	470	0.810	0.52
	“	“	431	0.776	0.62
61	168	0.057	493	0.830	0.43
	“	“	464	0.805	0.48
	“	“	432	0.777	0.58
	“	“	400	0.749	0.69
62	200	0.068	569	0.896	0.36
	“	“	513	0.848	0.42
64	302	0.103	550	0.880	0.41
	“	“	526	0.859	0.44
	“	“	493	0.830	0.49
	“	“	467	0.807	0.54
	“	“	436	0.780	0.54
68	166	0.056	485	0.823	0.46
	“	“	449	0.791	0.53
	“	“	409	0.756	0.66
69	162	0.055	483	0.821	0.47
	“	“	446	0.790	0.56
	“	“	406	0.754	0.67

to each of the values in Tables I, II, and III equal to the product of the total number of observations involved at both ends of the determination of the coefficient and the corresponding temperature drop measured in Centigrade degrees; proportional integral weights from 1 to 6 were then used in forming the weighted means in Table VI. The relative weights of the means themselves which are given in the last column of Table VI are proportional to the square roots of the sums of the above products which entered into each mean; they are given merely as a rough guide for anyone who may wish to use these means for other purposes. If weights had been assigned to Dodge's means on the same basis, they would have been misleadingly large because all the temperature differences retained were large (see the Appendix). The numbers in parentheses in the last column of Table VI are the number of separate coefficients involved in each of the means.

TABLE VI.

Summary of Weighted Means from Tables I to V.

Observer.	Temperature		Reduced μ'.	Weight.
	Cent.	Reduced.		
Grindley	109.4	0.600	1.719	4
	122.3	0.620	1.475	4
	131.0	0.633	1.399	3
	138.6	0.645	1.290	3
	158.4	0.676	1.121	4
	176.2	0.705	0.850	2
Griessmann	128.3	0.628	1.454	2
	134.7	0.646	1.377	5
	152.7	0.667	1.140	5
	167.0	0.690	0.923	4
Peake	120.6	0.616	1.641	1
	137.9	0.644	1.400	2
	160.1	0.679	1.075	4
	179.0	0.708	0.927	6
Dodge				
Table IV.	....	0.770	0.549	(16) [1]
	....	0.870	0.389	(25) [1]
Table V.	....	0.794	0.543	(34) [1]
	....	0.861	0.432	(43) [1]

[1] These are not weights comparable with those above. They give simply the number of observations involved in the corresponding means.

The small figure in the upper corner of Figure 6 is Buckingham's figure (Figure 1 of this paper) replotted on a different scale with the

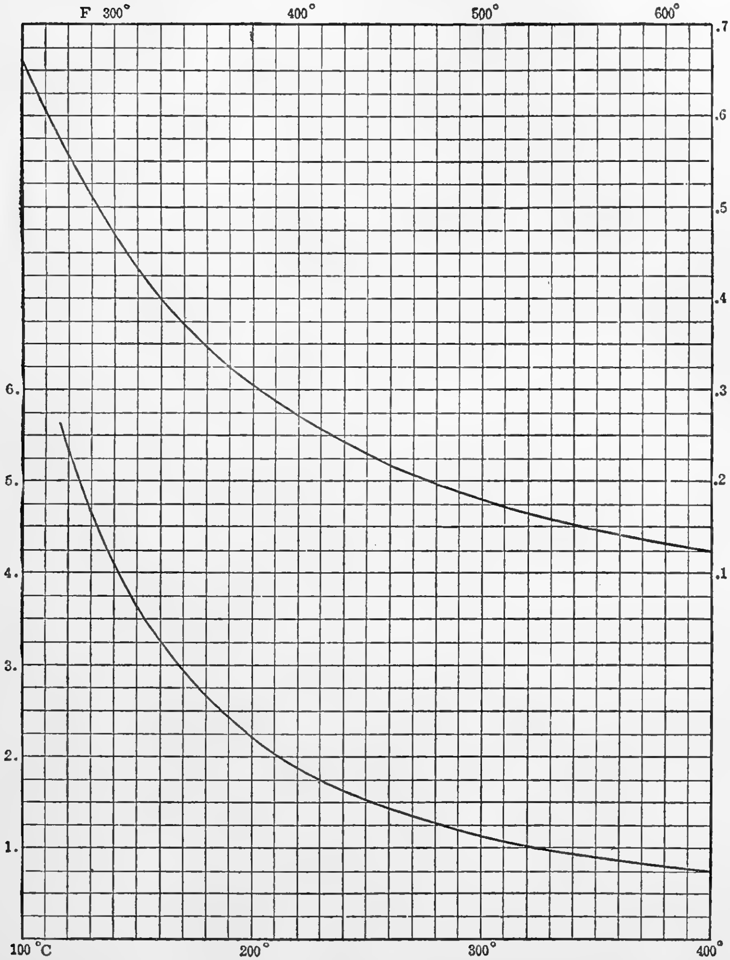

FIGURE 7. Joule-Thomson coefficient in ordinary units. In the lower part of the figure these are Centigrade degrees for a pressure drop of 1 kg. per sq. cm. (scale at left). In the upper part they are Fahrenheit degrees for a pressure drop of 1 lb. per sq. in. (scale at right).

18 means of Table VI added as large circles. The six small circles near $t=1$ are Kester's carbon dioxide points, the other carbon dioxide points being omitted for clearness. The other points in the figure are easily recognizable on comparison with Figure 1.

Figure 7 shows the smooth curve that best represents the band of Figure 6, translated back from "reduced" to ordinary units, both Centigrade and Fahrenheit. This curve has proved useful in several unexpected ways. For example, it will be made the basis of a discussion of the specific heat of very highly superheated steam in a later paper (see page 292 of these proceedings). It has also made certain cumbersome and uncertain computations in continuous flow calorimetry unnecessary (see "Power," June 2, 1908, page 871). It is hoped that the various scales of Figure 7 are open enough to make the curve useful to others.

All of the observations discussed in this paper have been examined with considerable care, both arithmetically and graphically, for traces of a systematic variation of the Joule-Thomson coefficient with pressure at constant temperature, without success. If such a variation exists even close to the saturation line, it is within the limit of error of the data.

APPENDICES.

Discussion of Dodge's Data.

In Dodge's apparatus the low side chamber was protected against loss of heat to its surroundings chiefly (although not wholly) by an independently heated steam jacket made in one piece with the wall of the chamber, and kept as nearly as possible at the same temperature as the low side steam. Thermometers were placed in this jacket and their temperatures recorded with the other routine data of each run. As a matter of fact, the jacket temperatures usually ran somewhat lower than the low side steam temperatures, so that some loss of heat by conduction through the chamber wall was to be expected. The high temperatures employed would also tend to make probable some loss of heat by radiation. The possibilities were tested in six special runs numbered 83 to 88, in which the partition between the high and the low side chambers, with its orifice, was completely removed. It was found that the low side thermometers in these tests did read somewhat lower than the high side thermometers although there was no throttling. The 27 observed differences can be fairly well represented by the empirical equation

$$\Delta t = \frac{12\,(\text{low side temp.} - \text{jacket temp.}) + \frac{1}{3}\,(\text{high side temp.})}{\text{flow in lbs. per hour}}.$$

The forms of the two terms in the numerator were intended to correspond to the two sorts of heat loss mentioned above. Corrections corresponding to this formula were accordingly applied to the main tests. The corrections in the tests summarized in Table IV averaged 2.4° F., and only occasionally amounted to 4°. Those in the tests summarized in Table V averaged 2.9° F. and only occasionally amounted to 5°.

The second set of corrections which are involved in Table V but not in Table IV are much more uncertain. As has been stated, the experiments of the runs of Table V could not be grouped into throttling curves whose various low side points could be combined with each other, all the high side observations being ignored except as indicating constancy of initial conditions, as was done in preparing Table IV. If the data were to be used at all, each low side point had to be taken with its own high side point. When this was done with only the radiation and conduction corrections made, the resulting values of the Joule-Thomson coefficient were not at all self-consistent, the values in each run which corresponded to small temperature drops and therefore to high mean temperatures being abnormally high. This tendency of the points near 0.9 in Figure 6 to swoop upward was unmistakable, and indicated clearly the presence in the tests of Table V of the same "wet steam" error shown in Figure 5 for the tests of Table IV.

The necessary corrections were obtained from the tests of Table IV. It seemed that they alone gave enough of a verification of the law of corresponding states to justify the drawing of a tentative curve like those of Figure 7, and this curve was then used to compute what correction would have to be applied to each of the high side temperatures of the tests of Table IV to make them self-consistent. These corrections were surprisingly constant. They were examined for systematic variations with mean pressure, with pressure drop and with quantity of steam discharged, without success. There seemed, however, to be a slight variation with the mean temperature and the following scheme was adopted :

If the mean reduced temperature is	decrease the high side temperature by
0.9	14°
0.85	13°
0.8	12°
0.75	11°

It should be noticed that these corrections were deduced wholly from the 14 throttling curve tests of Table IV. When they were applied to the tests of Table V, the resulting values of the coefficient showed none of their previous tendency to run high near 0.9, and were in general much more self-consistent. Further, they now agreed very definitely with the tests of Table IV in verifying the law of corresponding states and lay close along the tentative curve previously drawn. These facts, particularly the disappearance of the tendency to swoop near 0.9, seem to show that this reasoning is not a "circular fallacy," and that the values in Table V are a real corroboration of those in Table IV.

As a precaution against using these corrections too freely in cases where they might, perhaps, not apply, it seemed best to include in Table V only such of the 33 selected tests of the type in question as resembled the tests from which the corrections were determined in having comparatively large steam flow (more than 80 lbs. per hour). Furthermore, all tests or parts of tests were rejected for which the observed temperature drop was not as great as five times the correction, as the application of any correction amounting to more than 20 per cent of the quantity involved seemed unsafe. The 33 tests were thus reduced to 19, and these, corrected as above, gave the 77 values of the coefficient in Table V.

Note on the Vertical Scale of Figure 1.

The numerical values of the ordinates in Figure 1 are not the "reduced" Joule-Thomson effect in the ordinary sense, because Buckingham, in computing them, used 100 in. of mercury as his unit of pressure, but nevertheless expressed his critical pressures in atmospheres. The true reduced values of μ' are those indicated in the upper corner of Figure 6.

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NOTES ON CERTAIN THERMAL PROPERTIES OF STEAM.

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By Harvey N. Davis.

Presented by John Trowbridge, December 8, 1909; Received December 30, 1909.

1. Introduction.

It is the purpose of this paper to collect and correlate certain material on the thermal properties of steam. A part of this material was published in a technical journal a year ago.[1] Other parts of it have been contributed as discussion of papers by others in that journal and elsewhere. Still other parts of it have been used in a recent book.[2] The rest appears here for the first time. It all centers around a new determination of the total heat of saturated steam.

The previous determinations of the total heat of steam (H), and of the closely related latent heat of evaporation (L), will first be summarized. The most famous of them was published by Regnault in 1847. His experiments were so numerous, covered such a wide temperature range, and were characterized by such perfection of detail as to be accepted as the foundation of the engineering practise of the world,

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[1] Jour. Am. Soc. of Mech. Engs., 1908, **30**, 1419.

[2] Marks and Davis, Steam Tables and Diagrams.

and to remain standard for sixty years. He himself deduced from them the well-known linear formula

$$H = 606.5 + 0.305\, t \text{ calories.}$$

Others have represented them by second degree formulæ with negative second degree terms.

The more modern experimental work began in 1889 with a measurement of L at 0° C. by Dieterici.[3] He was followed by Griffiths,[4] Joly[5] and Smith,[6] working at various temperatures between 0° and 100° C., and finally in 1906 by Henning,[7] of the Reichsanstalt, who published an excellent series of values covering the range from 30° to 100° C. The results of all these observers are in excellent agreement and show that Regnault's formula for H gives values which are much too high near 0° and somewhat too low near 100°.

In 1908 the formula which is the basis of this paper was presented to the American Physical Society[8] and to the American Society of Mechanical Engineers.[9] It was based on the results of certain throttling experiments by Grindley,[10] Griessmann[11] and Peake.[12] These experiments were originally undertaken for the purpose of computing, with the help of Regnault's total heats, the variation with pressure and temperature of the specific heat, C_p, of superheated steam. This attempt was unsuccessful, because the total heats entered into the computations in such a way as to cause the errors in them to be tremendously magnified in the results. The desired information about C_p has since been obtained in other more direct ways, and the throttling experiments have been ignored. It is, however, possible, by reversing the computation processes of Grindley, Griessmann and Peake, to proceed from the recently determined values of C_p which were to have been their goal, back to a new determination of the values of H which were their starting point. The very sensitiveness of their procedure to errors in H ensures the insensitiveness of the

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[3] Wied. Ann., 1889, **37**, 494.

[4] Phil. Trans., 1895, **186** A, 261.

[5] In an appendix to Griffiths' paper, page 322.

[6] Phys. Rev., 1907, **25**, 145.

[7] Wied. Ann., 1906, **21**, 849.

[8] Phys. Rev., 1908, **26**, 407.

[9] Journal, loc. cit.

[10] Phil. Trans., 1900, **194** A, 1.

[11] Zeit. Ver. d. Ing., 1903, **47**, 1852, and 1880; also Forschungsarb., 1904, **13**, 1.

[12] Proc. Roy. Soc., 1905, **76** A, 185.

present procedure to errors in C_p. The result of such a reversal of their reasoning is the formula which was suggested two years ago, namely,

$$H = H_{100} + 0.3745\,(t - 100) - 0.000990\,(t - 100)^2.$$

This formula belongs only to the range between 100° and 190° C. Within this range its accuracy is believed to be of the order of one tenth of one per cent.

When the new formula was announced, there were no direct experimental determinations of H or L above 100° by which it could be checked except Regnault's, but more recently Henning [13] has published a continuation of his admirable research to 180°. The extent of the agreement of this with the formula will be discussed later.

As has been indicated, the computations leading to the new formula involve two different sorts of experimental data. The first of these, namely, the throttling experiments of Grindley, Griessmann and Peake, have been sufficiently discussed in a previous paper.[14] The second, the direct determinations of C_p mentioned above, will be discussed in the next section.

2. On the C_p Values Available for the Present Purpose.

There are three direct calorimetric determinations of the variation of C_p with pressure and temperature, namely, those of Lorenz,[15] of Knoblauch and Jakob [16] and of Thomas.[17] That of Lorenz was the earliest of the three and was, as he himself says, a preliminary survey for the sake of those engineers who could not afford to wait for more accurate work. It is not ordinarily considered comparable with Knoblauch's.

Both Knoblauch's and Thomas' results were obtained by determining the electrical energy necessary to increase by a known amount the temperature of previously superheated steam. In Knoblauch's apparatus the original superheating took place in an electrical preheater. The steam was then still further heated in a separate calorimeter, the energy added being the object of a direct measurement. In Thomas' case the separate preheating and calorimetric coils of Knoblauch's apparatus were replaced by a single coil, by means of which initially wet steam was brought, first just to dryness, and in a later experiment

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[13] Wied. Ann., 1909, **29**, 441.

[14] These Proceedings, page 241.

[15] Zeitsch. Ver. d. Ing., 1904, **48**, 698; Phys. Zeitsch., 1904, **5**, 383; and Forschungsarb., 1905, **21**, 93.

[16] Zeitsch. Ver. d. Ing., 1907, **51**, 81 and 124; Forschungsarb., 1906, **35**, 109.

[17] Proc. Am. Soc. Mech. Engs., 1907, **29**, 633.

to a high superheat. The amount of energy necessary for the superheating was then found by a subtraction. It is, therefore, liable to a percentage error much greater than that in either of the observed components. Knoblauch's method is obviously preferable to Thomas' in this respect.

His experimental arrangements also seem superior to Thomas'. In his separate calorimeter there were only small temperature differences between the inlet and outlet pipes; in Thomas' combination calorimeter there were very large differences. In Knoblauch's case the heat losses through these pipes were determined; in Thomas' case they were ignored. Furthermore, although both calorimeters were very carefully lagged, Knoblauch determined his radiation losses in each experiment, while Thomas, in the final form of his apparatus, relied on eliminating them, a difficult thing to be sure of. Finally, Knoblauch's thermometry is apparently more refined than Thomas'. It is therefore probable that wherever the two sets of results disagree, Knoblauch's are to be preferred.

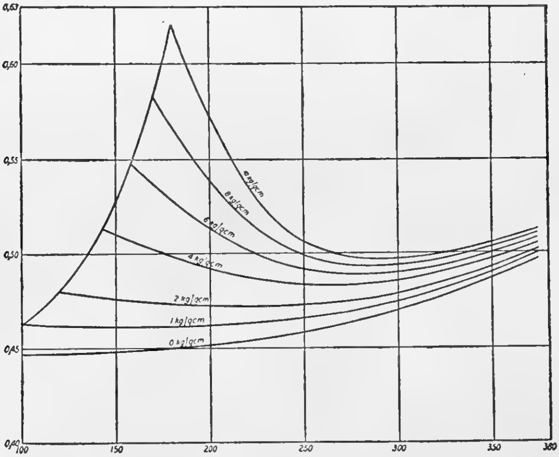

FIGURE 1. Knoblauch's Cp. diagram.

As a matter of fact, the two sets of results agree fairly well in the region of moderate superheats, as will be seen in Figures 1 and 2, but disagree fundamentally in exactly that part of the diagram which will be most used in what follows, namely, the region of moderate pressures and very low superheats (the lower left-hand corner of Figure 1). The sudden rise in Thomas' curves near saturation indicates, according to his interpretation, that a comparatively large amount of heat is required to change dry steam into slightly superheated steam. But it may also indicate that what he believed to be dry steam really carried a small amount of water floating as a mist. This would have to be evaporated at the expense of some extra heat in addition to that required for the actual superheating, and C_p would come out too large.

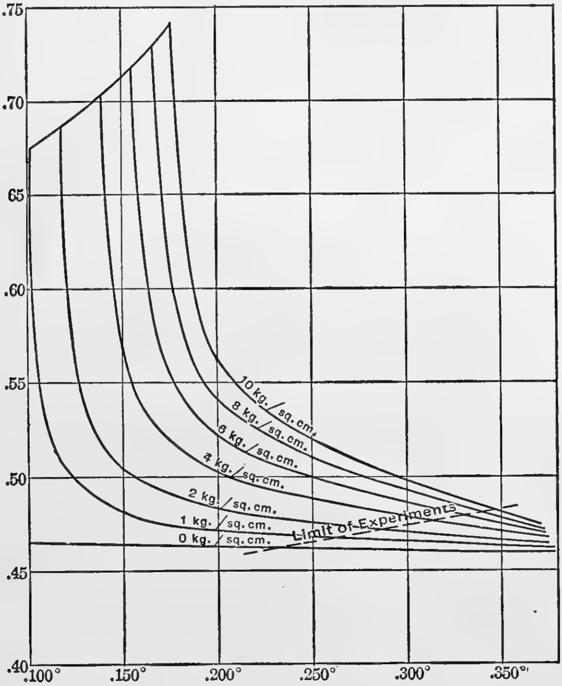

FIGURE 2.

That this explanation is a reasonable one is shown by a comparison of his apparatus with Knoblauch's. The latter's preheater, mentioned above, was a pipe made up of 15 sections each 20 cms. in diameter and 20 cms. long, each filled with a dense grid of constantan ribbons which ensured thorough mixing of the passing steam. All of the heat necessary for the desired superheating was ordinarily put in in the first one or two sections, and the sole purpose of the rest of the preheater was to bring the resulting mixture of highly superheated steam and floating mist into a homogeneous state. Knoblauch and Jakob say that traces of moisture were observable through several of the mixing sections, and it is easy to show that even if "several" means as few as two, and even if the steam in these sections had always had the greatest specific volume which it ever had, the floating mist must have persisted for a time which was never less than a second and averaged more than two seconds, and this after *all* of the heat necessary for the high superheat had been put in. In Thomas' apparatus, on the other hand, the evaporation and superheating had to take place in 24 quarter-inch holes in a soapstone block something like 5 inches long, and in a small chamber just above it, and a similar computation shows that even if the specific volume of the steam had never been greater than that of the original saturated steam, it must have passed the thermocouple, always within nine tenths of a second, sometimes within a thirtieth of a second, and on the average within less than half a second of the time when the *first* of the superheating heat was put in. It is, therefore, very probable that Thomas' "saturated steam" was slightly wet, and that the percentage of moisture passing the thermocouple decreased from experiment to experiment as the final superheat was increased, giving too high values of C_p near saturation. Knoblauch's values have therefore been used in preference to Thomas' in this work. Confirmations of this decision will be found on pages 287, 298 and 302.

3. The Total Heat of Saturated Steam.

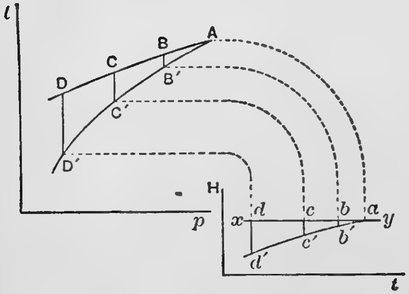

Fig. 3. Fig. 4.

Showing how the Total Heat Curve $ab'c'd'$ is obtained from a Throttling Curve ABCD.

A. *The determination of* $H - H_{100}$. — A part of the following account of the method by which the total heat of saturated steam has been computed is reprinted with minor changes from the Proceedings of the American Society of Mechanical Engineers.

Let Figure 3 represent a throttling curve of the sort published by Grindley, Griessman or Peake. Supposedly dry and saturated steam at the pressure and temperature corresponding to the point A is first throttled to lower pressure and temperature corresponding to the point B; then in a later experiment in the same run, it is throttled from exactly the same initial condition A to the condition C; then to D and so on. The well-known law of throttling is that the total heat in the condition B, or C, or D, is equal to that in the initial condition A.

The point B represents superheated steam at the pressure p_B; the point B′ represents saturated steam at the same pressure; and the amount of superheat at B is the measured temperature there minus the temperature at B′, which can be taken from a steam table. Also, by definition, the total heat at B equals that of saturated steam at the same pressure (point B′) plus the amount of heat required to superheat it at constant pressure from B′ to B. This is the integral of C_p from B′ to B, or simply the mean C_p from saturation multiplied by the known superheat. If C_p is known, this integral, or increment in the total heat between B′ and B, is easily evaluated.

This integral is not only the difference between the total heat of saturated steam at B′ and that of superheated steam at B; it is also the difference between the total heat of saturated steam at B′ and that of saturated steam at A; that is, between the two corresponding ordinates of the curve that gives the total heat of saturated steam as a function of the temperature, the curve sought in this paper. To draw a piece of this curve, one chooses arbitrarily some horizontal line such as xy in Figure 4, and lays off below it, at the proper temperatures, the distances bb', cc', dd', etc., which represent on the desired H-scale the integrals or total heat differences between B′ and B, C′ and C, D′ and D, etc. The curve $ab'c'd'$ is an isolated piece of the true curve of total heat against temperature. The *relative* height of its points, that is, its shape, is accurately determined; the *absolute* height above the usual zero of total heats, namely, that of water at 0°C., is as yet wholly unknown. The experiments of Grindley gave seven independent sample pieces of this sort, one for each throttling curve, their temperature ranges being known and greatly overlapping; similarly Griessmann's data gave eleven such sample pieces, and Peake's six.

As was explained in the preceding paper on the Joule-Thomson effect, Grindley's incoming steam (point A), and occasionally Peake's, was not quite dry, so that its total heat was not determined by its pressure and temperature. Whenever this seemed to be the case, the points A and a of Figures 3 and 4 were left out of consideration altogether. BCD would still be a curve of constant total heat, provided only that the quality of the incoming steam at A remained constant during a run, and $b'c'd'$ would still be a useful piece of the desired total heat curve.

All sample pieces of any one observer were then plotted carefully on very thin transparent rice paper, with vertical guide-lines at certain standard temperatures, which enabled these plots to be accurately oriented as far as rotation and horizontal displacement were concerned, but left them free to slide up and down over each other. The sheets were then piled on top of one another on a transparent table lighted from below, each one placed so as to make its piece of curve coincide most satisfactorily with the overlapping pieces already laid down. The exact relative displacements of the sheets were then carefully measured. This process was repeated for each of the three observers' sets of sheets independently, four different times for each set, in two very different orders and in those orders reversed, on different days, all with the object of avoiding as far as possible any routinizing effects of memory or habit which might disturb the real independence of the four determinations. The means of the measured displacements were then used to reduce each of the pieces of curve in any one of the sets to a zero common to all the curves of that set. The results are marked *Gy*, *Gs*, and *P* in Figure 5. They are plotted separately for clearness, but they are simply different experimental determinations of exactly the same real curve. The vertical scale of each is that indicated at the side of the diagram, but the height of each above its true zero is still unknown. Each of the circles represents at least one independent throttling observation, and some of them two or three independent observations that happened to coincide. It will be noticed that no one of the curves is more than a fifth of a scale division, or four tenths of a calorie, wide between centers. Each is, therefore, a self-consistent determination of the true curve within two tenths of a calorie, or about three hundredths of one per cent.

The next step was to establish a comparison between the three curves. The points of each were first divided into groups, each including some 20° of temperature range, and the mean point of each group was used to represent the group. This procedure is justified by the fact that so short a section of the total heat curve can be considered straight without serious error. There were eighteen such means, seven representing Grindley's points, five Griessmann's and six Peake's. These means were then plotted on three more sheets of rice paper, the resulting curves were superposed in the way already described, and a determination was made of the corrections necessary to reduce all three sets of means to a common but still arbitrary zero.

In the meantime successive means from each of the three curves taken separately were used to compute the values of the derivative dH/dt which are plotted with large circles in Figure 6. It is evident that the results from the three sources agree with each other in deter-

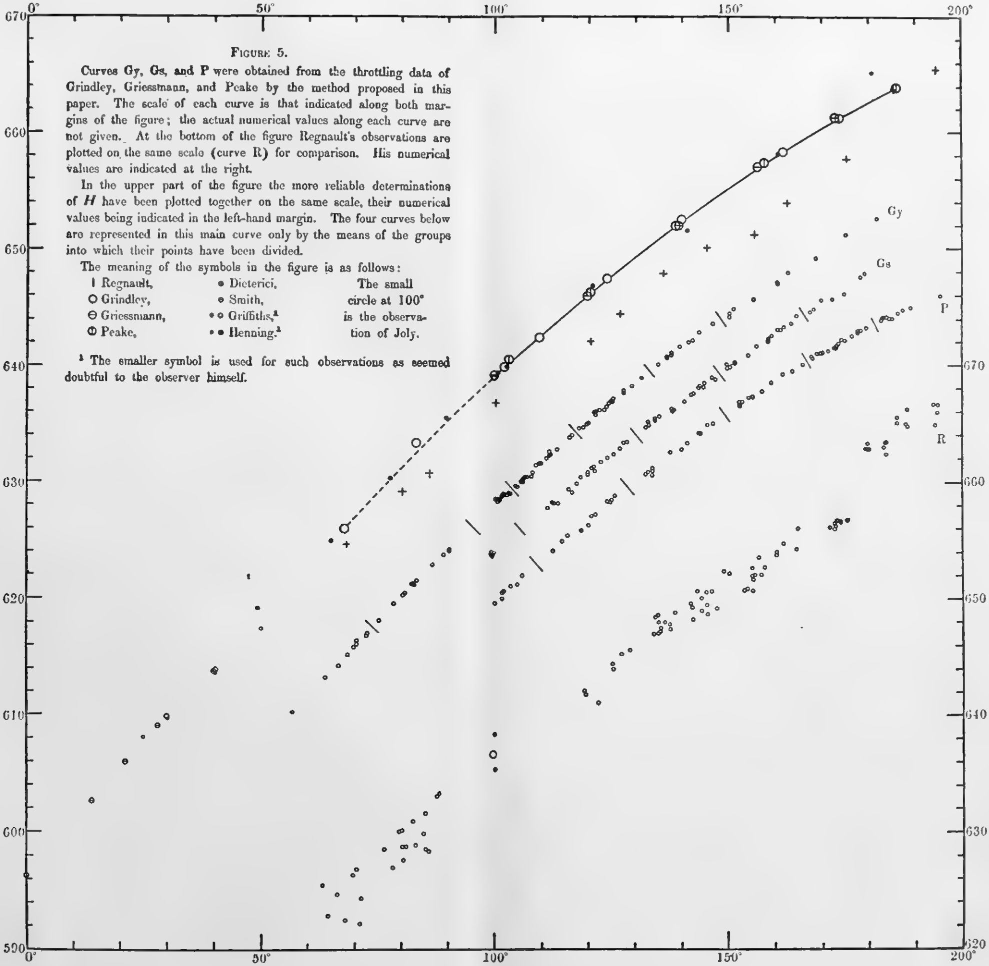

FIGURE 5.

Curves **Gy, Gs, and P** were obtained from the throttling data of Grindley, Griessmann, and Peake by the method proposed in this paper. The scale of each curve is that indicated along both margins of the figure; the actual numerical values along each curve are not given. At the bottom of the figure Regnault's observations are plotted on the same scale (curve R) for comparison. His numerical values are indicated at the right.

In the upper part of the figure the more reliable determinations of *H* have been plotted together on the same scale, their numerical values being indicated in the left-hand margin. The four curves below are represented in this main curve only by the means of the groups into which their points have been divided.

The meaning of the symbols in the figure is as follows:

I Regnault,	● Dieterici,	The small
○ Grindley,	◉ Smith,	circle at 100°
⊖ Griessmann,	• ○ Griffiths,[1]	is the observa-
⦶ Peake,	• ● Henning.[1]	tion of Joly.

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[1] The smaller symbol is used for such observations as seemed doubtful to the observer himself.

mining a straight line as the graph of dH/dt against t. The total heat curve itself can therefore be represented in the range between 100° and 190° by an equation of the second degree in t, within the limit of error of the available data. The form selected is

$$H = H_{100} + a\,(t - 100) - b\,(t - 100)^2.$$

The eighteen means, reduced to a common but still arbitrary zero, were

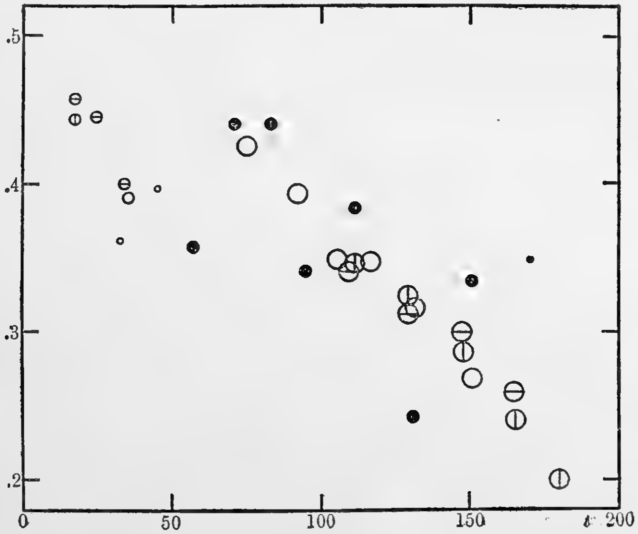

FIGURE 6. dH/dt plotted against t. The symbols refer to the same authorities as in Figure 5.

used to give a least squares determination of the constants H_{100}, a and b and of their probable errors, with the following results;

$$\begin{aligned} H_{100} &= \text{arbitrary} \pm 0.03, \\ a &= 0.3745 \pm 0.0014, \\ b &= 0.000990 \pm 0.000020. \end{aligned}$$

The agreement of the eighteen individual means with this formula is shown in the upper part of Figure 5, the curve being drawn to represent the formula as accurately as possible. It is also shown by the smallness of the three probable errors. Even if these errors are combined in the most unfavorable way, the change in the computed value of H at 200°C. is only 0.37 calories, or about one eighteenth of one per cent of H itself; at lower temperatures the change would be still less.

The value of H_{100} which was obtained from the least squares process is entered in the above table as "arbitrary" because it is measured from an arbitrary zero. This value of H_{100} was next subtracted from each of the eighteen means, giving new values for these means on a new scale whose zero is H_{100}. In other words, these means now represent the true values of $H - H_{100}$. The resulting values are given in Table I, and are represented within their limit of error by the formula

$$H - H_{100} = 0.3745\ (t - 100) - 0.000990\ (t - 100)^2.$$

TABLE I.

VALUES OF $H_t - H_{100}$ AND OF H_t.

	t	$H_t - H_{100}$	H_t*
Grindley	67.56	−13.23	625.88
	82.70	− 6.80	632.31
	101.80	+ 0.71	639.82
	109.27	+ 3.31	642.42
	123.82	+ 8.36	647.47
	139.92	+13.46	652.57
	161.55	+19.26	658.37
Griessmann . . .	99.61	+ 0.03	639.14
	119.80	+ 6.89	646.00
	139.18	+12.94	652.05
	156.01	+17.98	657.09
	172.60	+22.28	661.39
Peake	102.88	+ 1.15	640.26
	120.22	+ 7.16	646.27
	138.41	+12.87	651.98
	157.56	+18.36	657.47
	173.60	+22.22	661.33
	186.33	+24.77	663.88

* These values of H_t are computed from $H_t - H_{100}$ on the assumption that $H_{100} = 639.11$ mean calories (see page 281). They are inserted here for the convenience of the reader, but the values of $H_t - H_{100}$ are the significant part of this table and indeed of the whole paper.

This formula gives, in mean calories, the total heat of saturated steam at any temperature between 100° and 190° C. *in terms of that at* 100° C. A value for the fundamental constant H_{100} will presently be chosen from those available in the literature of the subject, but it should be remembered that even if this choice is wrong, or if new and different data near 100° are hereafter published, whatever merit the above equation may have will be wholly unaffected by the necessary change in H_{100}.

It is interesting to compare the self-consistency of this work, as represented by the narrowness of the bands of plotted points, with that of Regnault's observations, which are plotted at the bottom of Figure 5.[18] His band is at least eight or ten times as wide as any of those above it. It should also be noticed that something evidently happened to his apparatus at 178° C., and that allowing for this, his band shows unmistakably the same curvature as those above it. The observations above 178° C. were, as a matter of fact, the last he made, and he speaks definitely of serious trouble with his apparatus at the

TABLE II.

HENNING'S MEASUREMENTS OF H.

Temp.	Value of L.		Heat of the liquid.	H.	$H_{100}-H$.	H_{100}.	Variation from	
	As reported.	Reduced.					First mean.	Second mean.
30.12	(579.0)	(579.5)	30.1	(609.6)	....	....	....	....
49.14	569.55	570.07	49.09	619.16	....	....	....	....
64.85	559.47	559.98	64.77	624.75	+14.39	639.14	−0.19	−0.12
77.34	552.47	552.97	77.27	630.24	+ 8.99	639.23	−0.10	−0.03
89.29	545.76	546.26	89.24	635.50	+ 4.13	639.63	+0.30	+0.37
100.59	538.25	538.74	100.59	639.33	− 0.22	639.11	−0.22	−0.15
102.34	536.93	537.42	102.35	639.77	− 0.87	638.90	−0.43	−0.36
120.78	525.32	525.90	121.02	646.92	− 7.36	639.56	+0.23	+0.30
140.97	509.60	510.06	141.62	651.68	−13.79	637.89	−1.44	....
160.56	495.95	496.40	161.80	658.20	−19.06	639.14	−0.19	....
180.72	481.99	482.43	182.78	665.21	−23.79	641.42	+2.09	....
	Mean of all					639.33	±0.49	....
	Mean of first six					639.26	....	±0.19

The values of L in the second column are in terms of Henning's "15° Calorie" of 4.188 international Joules; those in the third column are reduced to mean calories of 4.1842 Joules. The heat of the liquid in the fourth column is from the steam tables of Marks and Davis. The probable error of each mean is 0.845 times the corresponding average error.

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[18] The large circle at the boiling point, 100° C., represents the mean of 38 points, of which only the highest and lowest are plotted.

very point at which the jump occurs; in fact, he had to renew many of its parts, and to watch it continually thereafter, so that his conditions may well have been somewhat changed. This discontinuity in his curve has been noticed by many writers, one of whom attributes it to a leak in his distributing valve, remedied at this point; but this is not definitely mentioned in the memoir.

The recent publication by Henning of his measurements of L between 100° and 180° gives a valuable test of the new formula. All his values in both papers are collected in the second column of Table II. They are expressed in terms of a calorie of 4.188[19] international Wattseconds. It is probable that the mean calorie (0° to 100°) is about 4.184(2) international Wattseconds, for the fine work of Reynolds and Moorby[20] by a mechanical method, leads, according to Smith,[21] to the value 4.1836, while the equally good work of Barnes[22] by an electrical method must now[23] be regarded as leading to the value 4.1849. Each of Henning's numbers should therefore be multiplied by $4.188 / 4.1842 = 1.00091$. The results are given in the third column of Table II. They lead to the values of H in the fifth column. In the sixth and seventh columns are given the values at the corresponding temperatures of

$$H - H_{100} = 0.3745\ (t - 100) - 0.000990\ (t - 100)^2.$$

and of H_{100} itself. The latter is practically constant as it should be if the new formula is true. It will be noticed that the probable error of the mean value of H is only one thirteenth of one per cent of that mean, and that this agreement is within the one tenth of one per cent which Henning claims for his observations. It will further be noticed that practically all of the discrepancy is in two of the last three values. If all three of these values are omitted, so that the range of the test is cut down to that between 65° and 121°, the probable error of the new mean is only ± 0.19 calories, or one thirty-fourth of one per cent.

In estimating the significance of the comparatively great disagreement between Henning's value at 180° and the new formula it should be remembered that Henning himself says, "Bei der höchsten Temperatur von 180° konnten nur an zwei tagen Versuchen angestellt werden" (instead of on four days as at most of the other temperatures).

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[19] This is Jäger and von Steinwehr's value for the 15° calorie. The justification for it has not yet been published.

[20] Phil. Trans., 1897, **190** A, 301.

[21] Monthly Weather Review, 1907, **35**, 458.

[22] Phil. Trans., 1902, **199** A, 149.

[23] Proc. Roy. Soc., 1909, **82** A, 390.

"Zu Beginn des dritten Tages versagte der Apparat seinen Dienst und es war infolge der durch die starke Hitze eintretenden allmählichen Veränderungen des Materials und inbesondere infolge der Abnutzung des Hahnes H nicht wieder der erforderliche Grad der Dichtheit zu erreichen." If a small leak of the same sort had been present without being noticed on the two days on which observations were made, its effect would have been to make the observed L too large, just as it seems to be. At any rate, the point at 180° is not entitled to nearly as much weight as the others. The point at 140° was, however, as far as Henning could judge, as good as any of the rest.

One other aspect of Henning's paper tends to minimize the significance of the disagreement at the two high temperatures. He is led by his points at 140° and 180° to the conclusion that the curve $L = f(t)$ is a straight line between 120° and 180°. Now, of course, it is possible that he and Regnault are both right in finding unexpectedly high values near 180°, and that, because of changing polymerization or some other disturbing condition, the character of the curve $L = f(t)$ between 120° and 180° is very different from that which it is known to have below 120° and from that which it must begin again to have somewhere above 180°, if it is to come vertically to zero at the critical temperature as is commonly supposed. This is, however, not probable, and until Henning's 180° point is definitely verified by observations with unquestionable apparatus, the writer will still believe that the formula proposed in this paper is nearer the truth than is Henning's straight line. The excellence of the confirmation between 65° and 121° and also at 160° seems more significant than the disagreements at 140° and 180°.

Another check of the new H formula can be obtained by computing from it the specific volume of saturated steam by means of Clapyron's equation

$$v = v' + J\frac{L}{T}\frac{1}{(dp/dv)_{\text{sat.}}}.$$

This check has been carried through independently by Peabody[24] and by the writer.[25] In both cases the necessary values of dp/dt were taken from the recent paper of Holborn and Henning on the saturation pressures of steam,[26] and the values of L were based on the formula proposed in this paper. The choice of a suitable value for H_{100} and of suitable values for the heat of the liquid which has to be subtracted

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[24] Proc. Am. Soc. Mech. Engs., 1909, **31,** 595.

[25] Marks and Davis, Steam Tables.

[26] Wied. Ann., 1908, **26,** 833.

from H to give L, was in each case accomplished independently of, and to a minor extent in disagreement with, the judgment of the other, but in each case the greatest difference between the computed values and those actually observed by Knoblauch, Linde, and Klebe[27] was under two tenths of one per cent, and in each case the average of the deviations was about one tenth of one per cent and they were nearly equally divided between plus and minus. It is probable that some of these deviations may properly be attributed to errors in the observed values.

The accuracy of the new H formula can now be estimated. It has been pointed out that the self-consistency of the computed points indicates a precision of the order of a twentieth of one per cent. The actual error is probably larger than this because of systematic errors in Knoblauch's specific heats. It is possible that these will ultimately be raised enough to make H_{190} a tenth or even a sixth of one per cent larger. Inasmuch, however, as the other two tests which have been applied, based on Henning's direct measurements of H and on Knoblauch, Linde, and Klebe's volume measurements, have both led to an estimated accuracy of a tenth of one per cent or better, a part of the outstanding disagreement in each case being furthermore reasonably attributable to possible errors on the observed as well as on the computed side of the comparison, it would seem that a claim of a tenth of one per cent for the accuracy of the new H formula between 100° and 190° is justified.

B. *The value of* H_{100}. — In what is to follow a suitable value for H_{100} will be necessary. Henning's work has already been shown to lead to the value

$$H_{100} = 639.26 \text{ mean calories (Henning).}$$

Another available value is that of Joly[28] who compared the latent heat of steam at 99.96° with the mean specific heat of water between 11.89° and 99.96°. The latter number is 0.99949 according to the curve used in the steam tables already mentioned. The resulting value of H_{100} is

$$H_{100} = 638.82 \text{ mean calories (Joly).}$$

In this determination of H_{100} Regnault's measurements will not be considered at all. They show unmistakable evidence of running lower than they should, probably for the same reason that makes

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[27] Forschungsarb., 1905, **21**, 33.

[28] *Loc. cit.*, on page 268.

Thomas' values of C_p at saturation correspondingly too high. Only recently has it become evident how difficult it is to remove the last traces of moisture from apparently dry steam, and if any remained in Regnault's steam, it would have made his results too low, just as they seem to be.

The mean of Henning's and Joly's values of H_{100} is 639.04 if both are weighted alike, or 639.11 if Henning's has (as it seems to deserve) twice the weight of Joly's. The final formula for H is, therefore,

$$H = 639.11 + 0.3745\,(t - 100) - 0.000990\,(t - 100)^2 \text{ mean calories.}$$

The steam table of Marks and Davis, which was computed before the appearance either of Henning's second paper or of Barnes' revision of his value of J, was based on $H_{100} = 639.08$, which, as it happens, is between the two means just found, and nearer to either of them than the limit of error of the work demands. The values of H, L and L/T in that table will be used in the rest of this paper as representing the best available data.

C. *Extrapolation formulæ for H and L.* — The range within which the new H formula holds has been set as from 100° to 190°. Above the latter temperature no observations are available. It is often important, however, both in scientific and in technical work, to have at least reasonably accurate steam tables at considerably higher temperatures. It is, therefore, desirable to develop as safe an extrapolation formula as possible for either H or L.

For this purpose the second degree H formula proposed above is wholly unsuited. Within the range for which it is proposed, it happens to be an unusually good three term Taylor's series development of the true function but it cannot be extrapolated safely either up or down.

That it cannot be used near 0°, is seen from Figure 6, where the small circles, not previously mentioned, represent values of the derivative of H with respect to t, obtained from the five sets of experimental values mentioned on page 268. It is evident that the graph of dH/dt against t is not a straight line over the whole range from 0° to 200°. No second degree formula that fitted the observations above 100° could be expected to reproduce those near 0° also.

That a second degree formula is no less unsatisfactory for a extrapolation to high temperatures can be shown as follows. Let it be assumed that the top of the steam dome on either the $p\ v$ or the T N (temperature-entropy) plane is round like Figure 7*a* and not pointed like Figure 7*b*.[29] This is the usual assumption, and it is corroborated

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[29] It follows from the Clapyron equation that if the dome is round on either plane, it will be on both.

by the work of a number of observers.[30] Now according to a familiar equation of thermodynamics

$$dH = T\,dN + v\,dp$$

for any transformation, and in particular for one along the saturation line. Dividing by dt and passing along that line to the critical temperature as a limit, gives

$$\lim_{T \doteq T_c}\left(\frac{dH}{dt}\right) = T_c \lim_{T \doteq T_c}\left(\frac{dN}{dt}\right) + v_c\left(\frac{dp}{dt}\right)_{\text{sat. crit.}}$$
$$= -\infty. \qquad\qquad + \text{constant.}$$

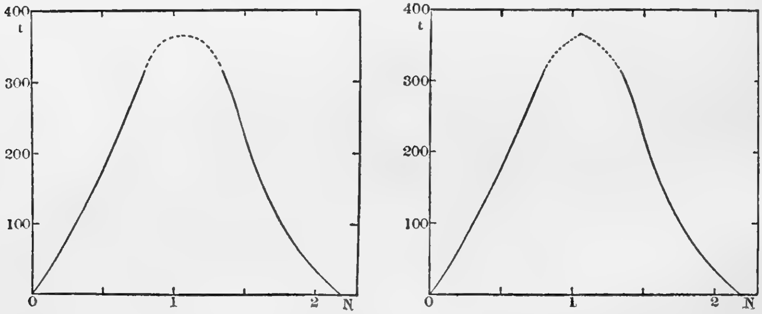

FIGURE 7*a*. FIGURE 7*b*.

The steam dome on the temperature-entropy plane. The full lines are drawn to scale; the dotted lines show two possible shapes near the critical point, of which the first is almost certainly right.

That is, H must not only pass a maximum below the critical temperature, but must approach that temperature with so sharp a turn downward as to reach it with a vertical tangent. The H curve is throughout a curve not only of constantly changing slope but also of constantly increasing curvature as is shown in the upper part of Figure 8, and it is only in very limited regions that the first three terms of a Taylor's development can be expected to represent it with sufficient exactness. It might be possible to invent a function having the general properties indicated by Figure 8, if one knew the value of H at the critical

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[30] See for example papers by Cailletet and Mathias, C. R., 1886, **102,** 1202, and 1887, **104,** 1563; by Amagat, C. R., 1892, **114,** 1093; and by Young, Phil. Mag., 1900, **50,** 291. See also the diagrams for normal pentane on pages 166 and 167 of Young's book on Stoichiometry, Longman's (1908).

temperature. Inasmuch as nothing is known about that final value of $H_{\text{sat.}}$, such an empirical treatment gives no promise of significance.

In the case of L, on the other hand, one learns from an inspection of figure 8, not only that $dL/dt = -\infty$ at the critical point, but also that $L = 0$ there. This led Thiesen in 1897,[31] to the fortunate suggestion that if the known values of L at ordinary temperatures can be represented by a formula of the form

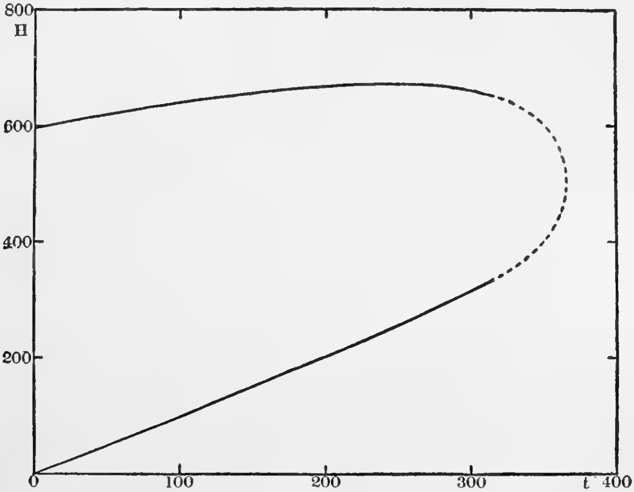

FIGURE 8. The steam dome on the *Ht* plane, showing the relationship between the graphs representing the "total heat of saturated steam" and the "heat of the liquid." The former (the upper boundary of the steam dome) is the curve that Regnault believed to be a straight line. It obviously passes a maximum and reaches the critical point with a negatively infinite derivative.

$$L = A\ (t_c - t)^n, \qquad n < 1,$$

one could also be sure that it gave correct values both for L and for dL/dt at the critical point, so that the use of the formula for other high temperatures would be, in a sense, an interpolation rather than an extrapolation. The constants can be determined and the formula tested in the range of the known L's by writting it in the logarithmic form

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[31] Verh. Phys. Gesch., Berlin, 1897, **16**, 80.

$$\log L = n \log (t_c - t) + \log A.$$

That is, if $\log L$ is plotted against $\log (t_c - t)$ one should get a straight line. This turns out to be remarkably near the truth. Thiesen originally suggested $n = 1/3$; Henning[32] showed that his observations below 100° could be represented by putting $n = 0.31249$ and $A = 94.21$; a careful plot, a year ago, of the values available before the appearance of Henning's work above 100°, but including the values in Table I. in this paper, led to $n = 0.3150$ and $A = 92.93$. The work has been carefully repeated this fall. Including Henning's new work and the values in this paper, 37 values of L are available. They were plotted logarithmically on a large scale, and the slope of the line that best represented them was determined graphically by stretching a thread among the points. This was done several times by each of two different people, their results being closely accordant. The average of their values of n was then used to compute A arithmetically. The result is exactly the same as that of a year ago, namely,

TABLE III.

Range.	No. of deviations.		Algebraic average of deviations in fractions of one per cent.
	+	−	
0° − 70°	2	9	−0.027%
70° − 130°	14	0	+0.023%
130° − 190°	4	8	−0.003% *
	37 = 20 + 17		

* This includes Henning's point at 181° with a deviation of +0.167% (see page 278); if this one point is omitted, the last value in the above table would be −0.018%.

$$L = 92.93\ (365 - t)^{0.3150}$$

The average of the numerical values of the differences between the 37 observed values of L and the numbers computed by means of the above formula is one fourteenth of one per cent, which is less than the probable accuracy of the measurements. It is true that there is some evidence of regularity among the deviations as the above table shows.

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[32] Wied. Ann., 1906, **21**, 870.

These average deviations are, of course, very small, but the larger deviations in each group tend to cluster and to approach the limit of accuracy of the measurements, so that the systematic variation may be real. In any case, its amplitude is so small that it deserves but little consideration at this time.

4. Discussion of the Specific Heat of Superheated Steam.

It is the purpose of the rest of this paper to collect and revise such useful computations of other thermal properties of steam as are affected by a change in the accepted values of the total heat of saturated steam, together with such other results as are valuable for comparison with them. Section 4 will be concerned with the specific heat of superheated steam. Many papers on this subject have been published during the last ten years, especially in the technical press. They can be roughly classified under the following heads.

A. Direct experimental determinations.
B. Indirect experimental determinations and computations from other data.
  a. Throttling experiments.
  b. Computations based on characteristic equations or on volume measurements.
  c. Computations based on the Joule-Thomson effect.
  d. Computations along the saturation line based on Planck's equation.
  e. Other computations.
C. Resumes and discussions.

Each of these possible sources of information will be discussed in turn, with the object, not so much of reviewing previous papers, as of getting by each method the best information that the new material in this and in the preceding paper makes possible.

A. *Direct experimental determinations.* — Three of the papers that belong in this subsection have already been discussed in Section 2. The conclusion there reached was that of the three, that of Knoblauch, Linde and Klebe was the most reliable. Their results will therefore be used as the point of departure of this section, it being the object of each subsection, either to test the justice of the decision that their work is preferable to Thomas', or to determine what changes should be made in their curves to bring them nearer to the truth.

The most famous of all contributions to this subject is Regnault's direct experimental determination of C_p in 1862.[33] It seems not to be

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[33] Mem. Inst. de France, 1862, **26**, 167.

generally known that his computations involve one step which modern work has shown to be erroneous. He made four sets of experiments, all at atmospheric pressure, and all covering about the same range of superheat. In each experiment, first slightly superheated steam, and later highly superheated steam at the same pressure, was condensed in a water calorimeter. The heat released per gram of steam in the first process was then subtracted from that released per gram of steam in the second process and the difference divided by the difference in superheat to give C_p. The results which he deduced from his experiments will be found in the third column of Table IV. below.

The error which he made was in the determination of the quantity of water in his calorimeter. This he accomplished, not by weighing, but by a volumetric measurement in a sheet iron tank filled each time to a scratch on the glass tube that formed its neck. Regnault knew that the coefficients of expansion of the water and of the tank were such that the tank would hold fewer grams of water at the room temperatures at which he worked than at 0°, the temperature at which he had calibrated the tank. But he supposed that he also knew the specific heat of water to be an increasing function of the temperature at room temperatures as well as above 100° where he had carefully studied it. He therefore neglected both temperature changes, thinking that they neutralized each other, and used at all room temperatures the weight that would have filled the tank at 0°, and the specific heat 1.

TABLE IV.

A RECOMPUTATION OF REGNAULT'S VALUES OF C_p.

	Temp. range ($C°$).	R's value of C_p.	New value of C_p.	Kn's value of C_p.
Series 1	127.7–231.1	(0.46881) *	(0.4655) *	0.462
Series 2	137.7–225.9	0.48111	0.4769	0.462
Series 3	124.3–210.4	0.48080	0.4736	0.462
Series 4	122.8–216.0	0.47963	0.4780	0.462
Mean of last three . . .		0.48051	0.4762	

* ". . . les résultats de la première série, qui m'inspirent moins de confiance que les autres. . . ." Regnault, p. 178.

We now know that the specific heat of water decreases with increasing temperature from 0° to above 25°. There is some difference of opinion between Barnes and Dieterici, the two leading investigators of the subject, as to the exact shape of the curve of variation, but it is near enough to the truth to take, as in the steam tables already mentioned, a mean curve between that of Barnes and that of Dieterici, giving the former twice the weight of the latter.

Regnault's values of C_p have been recomputed from the data in his memoir, using his own value for the coefficient of expansion of sheet iron, modern data for the density of water, and the mean curve just mentioned for the specific heat of water. The new results are given in the fourth column of Table IV. They are somewhat lower than his original values and are thereby brought nearer to the corresponding values obtained by Knoblauch and Jakob, which are given in the fifth column of the table.

In the present unsettled state of our knowledge of C_p, Regnault's work should have considerable weight.

The only other important direct experimental determination of C_p is that of Holborn and Henning.[34] Their work, like Regnault's, was only at atmospheric pressure, but, unlike his, it covered a very wide temperature range, reaching 1400° C. It is certainly to be regarded as standard in the region of high superheats. It shows that in that region C_p increases with increasing temperature, but not as rapidly as Knoblauch's curves would indicate.

In a "Memorandum by the Chief Engineer for the year 1906 to the Executive Committee of the Manchester Steam Users Association,"[35] the National Physical Laboratory at Teddington, England, is said to have found $C_p = 0.532$ at saturation at 4.3 atmospheres (147° C.). This value lies remarkably close to Knoblauch's saturation curve.

Ba. *Throttling experiments.* — The failure of even the best throttling experiments to give satisfactory values of C_p by the ordinary methods has already been mentioned. A new method elaborated by Dodge[36] is much more promising, but no thoroughly reliable results have yet been obtained by it.

Bb. *Characteristic equations:* — If a sufficiently accurate characteristic equation, $f(p, v, t) = 0$, were known for superheated steam, much useful information about C_p could be obtained from Clausius's equation

$$\left(\frac{\partial c_p}{\partial p}\right)_T = - T\left(\frac{\partial^2 v}{\partial t^2}\right)_p.$$

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[34] Ann., 1907, **23**, 809.

[35] Manchester, June 4, 1907.

[36] Proc. Am. Soc. Mech. Engs., 1907, **28**, 1265 and 1908, **30**, 1227.

At the present time this is not a good way to get information about C_p for two reasons. In the first place, all of the most reliable set of volume measurements yet made (Knoblauch, Linde and Klebe) lie close to the saturation line, not one of them reaching either 50° of superheat or 190° of temperature. No characteristic equation based on them can be depended on at points far out in the superheated region. And in the second place, Clausius' equation involves a second derivative of the observed quantity v, and even the first derivative of an empirically determined function is liable to relative errors much larger than any in the observed quantity itself, while a second derivative is still more uncertain. This is illustrated by the fact that a characteristic equation of Tumlirz's form, which was shown by Linde to represent Knoblauch's volume measurements within four fifths of one per cent throughout their range, leads through Clausius' equation to the startling result that C_p does not vary at all with pressure at constant temperature, whereas it is known to vary within that same range by something like 60 per cent of its initial value.

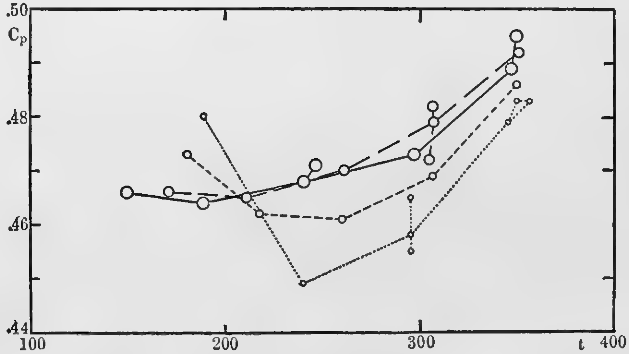

FIGURE 9. Knoblauch and Jakob's measurements of C_p, reduced to a pressure of 1 kg. per sq. cm. by means of Clausius' equation, using the characteristic equation developed by Linde to represent the volume measurements of Knoblauch, Linde, and Klebe. The smallest circles correspond to the highest original pressure (8 kg.), the next smallest to 6 kg., and so on. The progressive departure from a single curve with increasing pressure is marked.

The contention that even the best possible representation of Knoblauch's volume measurements is still too inaccurate to give reliable values of C_p, through Clausius' equation, can be further substantiated by an examination of the experimental data already described. Knoblauch and Jakob made observations on C_p at four pressures, all greater than one atmosphere. If these are all "reduced" to one atmosphere by means of Clausius' equation, using Linde's best characteristic equation to represent the volume measurements, the results, plotted in Figure 9, show deviations from a common curve that increase with the pressure. The same is more strikingly true in Thomas' case. If his results, so recomputed[37] as to partly eliminate the wet steam error already mentioned (see page 271), are similarly reduced to one atmosphere by means of Clausius' thermodynamic equation and Linde's best characteristic equation, the progressive departure with increasing pressure from the probable curve for one atmosphere is very marked, the 500 lb. and 600 lb. values disappearing beyond the bottom of the diagram altogether. That is, although Linde's second best equation gave no variation of C_p with pressure at all, his best one gives altogether too much. The experimental evidence is thus wholly against the reliability of any C_p values obtained by means of Clausius' equation from any volume measurements as yet available.

Bc. *The Joule-Thomson effect.* — There are three ways in which C_p can be connected with the Joule-Thomson coefficient μ. The first of these was suggested almost simultaneously by Linde and by Planck.[38] It is thermodynamically rigorous, except for the assumption of the form of an analytical expression for μ as a function of t. The one they used, namely,

$$\mu = \frac{Const.}{T^2},$$

was proposed by Joule and Thomson in their original memoir on air, and is not at all accurate, especially for steam. If it is replaced by a more complicated expression, the integration of the partial differential equation, to which the reasoning of Linde and Planck leads, is impossible.

A second equation connecting C_p with μ is used by Griessmann[39] in the discussion of his throttling experiments. It is not a thermodynamic equation in the true sense because it does not involve either of the two laws of thermodynamics; it is merely a manipulative identity that can be proved by the laws of partial differentiation — that is a truism. It says that at any point in any thermodynamic plane

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[37] Davis Proc. Am. Soc. Mech. Engs., 1908, **30**, 1433.

[38] Linde, Sitzungsber, bays. Akad., Math. Kl., 1897; Planck, "Vorlesungen über Thermodynamik," 1897, 117; Eng. ed., 1903, 124.

[39] Forschungsarb., 1904, **13**, 7 and 46.

$$C_p = \frac{(\partial H/\partial t)}{1 - \mu(\partial p/\partial t)}$$

provided only that both derivatives are taken in the same direction from the point. Griessmann uses the equation over the whole plane, but makes certain experimentally deduced assumptions which do not now seem to be justified.

The equation is likely to be most useful along the saturation line where dH/dt and dp/dt are both well known. Unfortunately μ is not as yet well known at such low temperatures, and it will be interesting to see whether, in the development of the subject, Griessmann's truism turns out to be more useful for the computation of C_p at saturation from μ or of μ from C_p.

The only use that will be made of the equation in this paper is to deduce from it the well-known theorem, usually attributed to Rankine, that at ordinary temperatures C_p at saturation must be numerically greater than dH_{sat}/dt.[40] At most temperatures this condition is so overwhelmingly fulfilled as to be of no value. At 0°C. it requires that C_p at saturation be as great as 0.44. Now if Knoblauch's saturation curve is continued to temperatures below 100° C., this condition will be found to require, either that the curve passes a minimum between 100° and 0°, or that it must lie somewhat higher between 100° and 150° so as to approach smoothly the right value at 0°. The existence of such a minimum has several times been suspected as a result of other indirect computations, and its experimental verification would be a matter of some interest; in the mean time the other alternative seems more probable, especially as it brings Knoblauch's values of C_p at atmospheric pressure into better agreement with Regnault's. Additional confirmation of this decision will be found on pages 293 and 300.

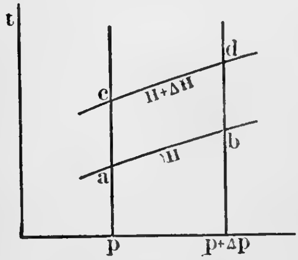

FIGURE 10.

The third of the methods referred to above for connecting C_p with μ is apparently new. It involves an equation which, like Griessmann's, is merely a manipulative identity or truism. It can be developed as follows. In Figure 10, let ab and cd be parts of two throttling curves on the usual tp diagram, the corresponding values of the total heat being H and $H + \Delta H$. Then at the pressures p and $p + \Delta p$ we have

---

[40] This follows at once from the fact that both μ and C_p are known to be positive.

$$C_p = \lim_{\Delta H \doteq 0} \frac{\Delta H}{t_c - t_a} \quad \text{and} \quad C_{p+\Delta p} = \lim_{\Delta H \doteq 0} \frac{\Delta H}{t_d - t_b},$$

from which

$$\frac{C_{p+\Delta p} - C_p}{C_p} = \lim_{\Delta H \doteq 0} \left[\frac{t_c - t_a}{t_d - t_b} - 1 \right].$$

Now, except for terms of higher order than ΔH and Δp,

$$t_b = t_a + \mu \Delta p,$$

$$t_d = t_c + \left[\mu + \left(\frac{\partial \mu}{\partial t} \right)_p (t_c - t_a) \right] \Delta p,$$

$$(t_d - t_b) = (t_c - t_a) \left[1 + \left(\frac{\partial \mu}{\partial t} \right)_p \Delta p \right].$$

Substituting this above gives

$$\frac{C_{p+\Delta p} - C_p}{C_p} = \lim_{\Delta H \doteq 0} \left[\frac{- \left(\frac{\partial \mu}{\partial t} \right)_p \Delta p}{1 + \left(\frac{\partial \mu}{\partial t} \right)_p \Delta p} \right]$$

and the limit sign is no longer necessary. Dropping it, dividing by Δp, and then letting Δp approach zero, gives

$$\frac{1}{C_p} \left(\frac{\partial C_p}{\partial p} \right)_H = - \left(\frac{\partial \mu}{\partial t} \right)_p.$$

Integrating this at constant H gives as the final equation[41]

$$C_p = C_{p_0} \epsilon^{-\int_{p_0}^{p} \left(\frac{\partial \mu}{\partial t} \right)_p dp}.$$

---

[41] The differential form of this equation can also be proved analytically as follows: For any three related quantities p, t, and H, one has the identity

$$\left(\frac{\partial p}{\partial t} \right)_H \left(\frac{\partial t}{\partial H} \right)_p \left(\frac{\partial H}{\partial p} \right)_t = -1.$$

But

$$\left(\frac{\partial t}{\partial p} \right)_H = \mu \text{ and } \left(\frac{\partial H}{\partial t} \right)_p = C_p,$$

and therefore

$$C_p = - \frac{1}{\mu} \left(\frac{\partial H}{\partial p} \right)_t. \qquad (1)$$

But for any function such as C_p which can be expressed in terms of any two of the variables p, t, and H, one has a second identity

$$\left(\frac{\partial C_p}{\partial p} \right)_H = \left(\frac{\partial C_p}{\partial p} \right)_t \left(\frac{\partial p}{\partial p} \right)_H + \left(\frac{\partial C_p}{\partial t} \right)_p \left(\frac{\partial t}{\partial p} \right)_H,$$

This formula has the disadvantage, as compared with Griessmann's, of involving the derivative of the inaccurately known function μ. This prohibits its use at the low temperatures close to saturation where μ is scarcely known at all, but makes much less difference at very high temperatures where the CO_2 points of Figure 5 of the preceding paper help to place the $\mu = f(t)$ curve with great definiteness. This method of computation is therefore at its best where many others fail completely.

The use of the new equation at ordinary temperatures is a matter requiring patience and much labor. First one computes and plots against t the derivative of the $\mu = f(t)$ curve of the preceding paper. Next one computes from the curve of μ itself the progress of some curve of constant H across the $p\ t$ plane; this is necessary so as to be able to express $\partial\mu/\partial t$ as a function of p in the integral. Then the integral has to be evaluated, either by replotting $\partial\mu/\partial t$ against p for the particular H curve in question and using an integraph, or by a step by step numerical process. The results are the Naperian logarithms of the desired ratios.

This process has been carried through for four curves in the region of moderate superheats. The results, which are presented in the first part of Table V., are in general in substantial agreement with the corresponding ratios computed from Knoblauch's curves, which are given in

---

Or $$\left(\frac{\partial C_p}{\partial p}\right)_H = \left(\frac{\partial C_p}{\partial p}\right)_t + \mu\left(\frac{\partial C_p}{\partial t}\right)_p. \tag{2}$$

Now from the definition of C_p

$$\left(\frac{\partial C_p}{\partial p}\right)_t = \left(\frac{\partial}{\partial p}\right)_t \left(\frac{\partial H}{\partial t}\right)_p = \frac{\partial^2 H}{\partial p \partial t}, \tag{3}$$

and from (1)

$$\left(\frac{\partial C_p}{\partial t}\right)_p = -\frac{1}{\mu}\frac{\partial^2 H}{\partial t \partial p} + \left(\frac{\partial H}{\partial p}\right)_t \frac{1}{\mu^2}\left(\frac{\partial \mu}{\partial t}\right)_p. \tag{4}$$

Substituting (3) and (4) in (2) and using (1) gives the desired equation. Neither of the laws of thermodynamics has been used.

The differential form of the equation can also be deduced immediately from the equation

$$\left(\frac{\partial C_p}{\partial p}\right)_t = -\left(\frac{\partial(\mu C_p)}{\partial t}\right)_p$$

which Grindley proves on pages 31 and 32 of his paper in the Philosophical Transactions. His proof depends twice over on each of the two laws of thermodynamics, but it need not have, as the above derivations show. The use which he makes of his form of the equation is quite different from that here proposed.

the last of each set of columns. The chief disagreement is along curve 1 where Knoblauch's curves are too condensed. This means either that his curve at atmospheric pressure should be lower, or that the lower part of his saturation curve, with the constant pressure curves near it, should be higher. The first of these possibilities would mean even less agreement between Knoblauch and Regnault than at present, and may therefore be rejected. The remaining possibility has already been suggested by the result obtained from Griessmann's truism (see page 290). Furthermore, it will be corroborated again in the next section (see page 300). It may therefore be accepted the more readily here.

At very high superheats, where the method is most valuable, the

TABLE V.

C_p Ratios obtained from the Joule-Thomson Effect.

Press. kg./sq. cm.	Curve 1.			Curve 2.			Curve 3.		
	Temp.	C_p/C_{p_0}.	Kn.	Temp.	C_p/C_{p_0}.	Kn.	Temp.	C_p/C_{p_0}.	Kn.
0.1	121.3	0.946	0.97	149.0	0.960	0.98	204.5	0.984	0.98
0.5	123.3	0.970	0.98	150.5	0.979	0.99	205.4	0.991	0.99
1.0	125.8	1.000	1.00	152.3	1.000	1.00	206.4	1.000	1.00
1.5	128.2	1.030	1.02	154.0	1.020	1.02	207.5	1.009	1.01
2.0	130.6	1.060	1.03	155.8	1.041	1.03	208.5	1.018	1.02
2.5	132.9	1.090	...	157.5	1.062	...	209.5	1.026	...
3.0	135.1	1.120	1.07	159.1	1.082	1.06	210.5	1.034	1.04
3.5	137.2	1.150	...	160.7	1.102	...	211.5	1.043	...
4.0	...	...	...	162.3	1.122	1.10	212.5	1.051	1.06
5.0	...	...	...	165.5	1.161	...	214.5	1.067	...
6.0	...	...	...	168.5	1.200	1.17	216.5	1.084	1.09
7.0	...	...	...	171.4	1.237	...	218.4	1.101	...
8.0	...	...	...	174.3	1.264	1.25	220.2	1.118	1.12
9.0	...	...	...	177.0	1.309	...	222.1	1.135	...
10.0	...	...	...	179.7	1.344	1.34	223.9	1.151	1.15
12.0	...	...	...	...	...	...	227.5	1.183	...
14.0	...	...	...	...	...	...	230.9	1.215	...
16.0	...	...	...	...	...	...	234.3	1.246	...
18.0	...	...	...	...	...	...	237.6	1.277	...
20.0	...	...	...	...	...	...	240.9	1.307	...
22.0	...	...	...	...	...	...	244.1	1.337	...
24.0	...	...	...	...	...	...	247.2	1.367	...
26.0	...	...	...	...	...	...	250.3	1.396	...
28.0	...	...	...	...	...	...	253.4	1.424	...

TABLE V — *continued.*

Press, kg./sq. cm.	Curve 4.			Curve 5, near 485° C. C_p/C_{p_0}.	Curve 6, near 600° C. C_p/C_{p_0}.	Curve 7, near 925° C. C_p/C_{p_0}.	Curve 8, near 1480° C. C_p/C_{p_0}.
	Temp.	C_p/C_{p_0}.	Kn.				
0.1	287.8	0.994	0.99	...	...	...	...
1.0	288.9	1.000	1.00	1.0000	1.0000	1.0000	1.0000
2.0	290.1	1.007	1.01	1.0025	1.0016	1.0009	1.0004
4.0	292.4	1.020	1.025	1.0053	1.0036	1.0020	1.0010
6.0	294.7	1.032	1.035	1.0082	1.0055	1.0031	1.0015
8.0	297.0	1.044	1.045	1.0106	1.0072	1.0041	1.0019
10.0	299.2	1.057	1.055	1.014	1.009	1.005	1.002
12.0	301.4	1.069	...	1.016	1.011	1.006	1.003
14.0	303.6	1.081	...	1.019	1.013	1.007	1.003
16.0	305.8	1.093	...	1.022	1.015	1.008	1.004
18.0	308.0	1.105	...	1.025	1.017	1.009	1.004
20.0	310.1	1.117	...	1.028	1.019	1.011	1.005
22.0	313.3	1.128	...	1.030	1.020	1.012	1.005
24.0	314.4	1.140	...	1.033	1.022	1.013	1.006
26.0	316.4	1.151	...	1.036	1.024	1.014	1.006
28.0	318.5	1.163	...	1.039	1.026	1.015	1.007

computation is simpler for two reasons. In the first place, μ and $\partial\mu/\partial t$ are both so small that the temperature can be assumed constant along a curve of constant H. $\partial\mu/\partial t$ is then constant in the integration. And in the second place $\partial\mu/\partial t$ can be computed from Buckingham's[42] equation for μ' against t', both in reduced units, namely,

$$\mu' = \frac{0.209}{t' - 0.32} - 0.0368.$$

This is corrected for the fact that although, in his paper, 100 inches of mercury is taken as the unit of pressure, his critical pressures are expressed in atmospheres. It was shown in the preceding paper that this equation can safely be assumed to hold for steam at very high superheats, since it is known to hold for the other gases which Buckingham discusses, and they are known to be connected with steam by a law of corresponding states.

This simpler process has been carried through for the four very high temperatures mentioned in the last part of Table V, with the results there presented. These results are the basis of the high superheat part of the Steam Tables of Marks and Davis.

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[42] Bul. Bureau of Standards, 1907, **3**, 263.

Bd. *Planck's equation:* — There remains the most interesting of all the indirect attacks on C_p at ordinary temperatures. In 1897 Planck published in his thermodynamics the equation

$$c_p = \frac{dH}{dt} - \frac{L}{T} + \frac{L}{u}\left[\left(\frac{\partial v}{\partial t}\right)_{p(\text{steam})} - \left(\frac{\partial v}{\partial t}\right)_{p(\text{water})}\right].$$

This equation holds only along the saturation curve. For its derivation the reader is referred to the English translation of Planck's book[43] or to Griessmann's paper.[44] The two partial derivatives must be such as to describe the behavior of superheated steam and of water, both close to the steam dome, not of steam within the steam dome. In practise, the second of these derivatives is always negligible in comparison with the first.

Two sorts of experimental material are necessary for computations with this equation, a set of total heat values (those proposed in this paper will be used), and a set of values of $(\partial v/\partial t)_p$ for superheated steam close to saturation. The latter can be based on the volume measurements of Knoblauch, Linde and Klebe,[45] or on those on Ramsay and Young,[46] or on those of Battelli.[47] These three sources will be considered in turn.

In the experiments of Knoblauch, Linde, and Klebe, the volume was held constant while the pressure and temperature were varied. Their results, when plotted on the $p\ t$ plane, gave isochors or lines of constant volume. These turned out to be straight lines within the limit of error of the measurements. Their slopes are entered with other data in the main table of the original paper. These slopes are values of $(\partial p/\partial t)_v$ and some manipulation is necessary to get from them the desired values of $(\partial v/\partial t)_p$. Let Figure 11 represent a portion of the $p\ t$ plane drawn, like an analytical geometry figure, with the same unit of length along each axis. Then

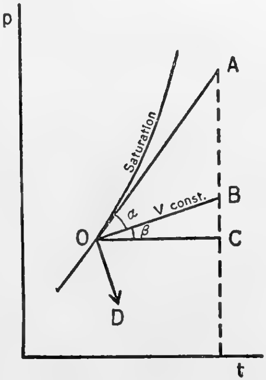

FIGURE 11

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[43] Treatise on Thermodynamics, 1903, 147.

[44] Forschungsarb., 1904, **13**, 8.

[45] Forschungsarb., 1905, **21**, 33.

[46] Phil. Trans., 1893, **183** A, 107.

[47] Mem. di Torino, 1893, **43**, 63; condensed in Ann. Chim. et Phys., 1894, **3**, 408.

$$\tan(\alpha+\beta)=\left(\frac{\partial p}{\partial t}\right)_{\text{sat.}}.$$

This derivative is well known from the work of Holborn and Henning.[48] Also

$$\tan\beta=(\partial p/\partial t)_v.$$

This is the derivative given by Knoblauch, Linde, and Klebe, as just explained. Along OB, v is constant; along OD, which is perpendicular to OB, v increases most rapidly. The following equations can then be verified, "Grad. v" being the space rate of v's increase along OD.

$$\text{Grad. } v=-\frac{1}{\sin\alpha}\frac{v_A-v_0}{\overline{OA}}.$$

$$\left(\frac{\partial v}{\partial t}\right)_p=\lim_{\Delta t\doteq 0}\frac{v_c-v_0}{\Delta t}=\lim_{\Delta t\doteq 0}\frac{\overline{OC}\cdot\text{Grad. } v\cdot\sin\beta}{\Delta t}$$

$$=-\frac{\sin\beta}{\sin\alpha}\frac{\overline{OC}}{\overline{OA}}\lim_{\Delta t\doteq 0}\frac{v_A-v_0}{\Delta t}$$

$$=-\frac{\sin\beta}{\sin\alpha}\cos(\alpha+\beta)\left(\frac{\partial v}{\partial t}\right)_{\text{sat.}}$$

The last term of Planck's equation can then be written in the form

$$\frac{L}{u}\left(\frac{\partial v}{\partial t}\right)_{p(\text{steam})}=AT\left(\frac{dp}{dt}\right)_{\text{sat.}}\left(\frac{\partial v}{\partial t}\right)_p=-AT\left(\frac{\partial v}{\partial t}\right)_{\text{sat.}}\frac{\sin(\alpha+\beta)\sin\beta}{\sin\alpha}.$$

In this transformation use has been made of the familiar Clapyron equation and of the definition of $\tan(\alpha+\beta)$. The computations are carried through by determining $(\alpha+\beta)$ and β from their tangents and getting α by subtraction. The necessary values of the differential coefficient $(\partial v/\partial t)_{\text{.sat}}$ were formed from the values of v in the Steam Tables of Marks and Davis by the usual finite difference formula

$$dv=\Delta v-\tfrac{1}{2}\Delta_2 v+\ldots.$$

The results of the computation are summarized in the first part of Table VI., and are plotted as black dots in Figure 12.

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[48] Wied. Ann., 1908, **26**, 835.

The necessary values of $(\partial v/\partial t)_p$ can also be obtained by differentiating the complicated characteristic equation which Linde has developed

TABLE VI.

VALUES OF C_p AT SATURATION FROM PLANCK'S EQUATION, USING:
a. KNOBLAUCH, LINDE, AND KLEBE'S EXPERIMENTAL DATA.
b. LINDE'S CHARACTERISTIC EQUATION.

Temp.	C_p.	Temp.	C_p.	Temp.	C_p.	
					Knoblauch.*	Thomas.*
101.4	0.46	140.9	0.54	100.6	0.484	0.519
102.4	0.55	143.0	0.65	126.3	0.506	0.533
108.1	0.44	143.2	0.52	153.1	0.560	0.585
110.7	0.56	144.1	0.52	170.1	0.615	0.634
112.4	0.49	149.8	0.58	193.2	0.722	0.737
114.8	0.40	150.2	0.54	205.1	0.794	0.808
115.3	0.36	153.7	0.56	215.8	0.875	0.885
119.1	0.50	154.2	0.60	225.1	0.956	0.966
119.3	0.52	157.6	0.56	235.8	1.067	1.075
122.1	0.49	159.6	0.58	245.2	1.179	1.184
122.6	0.53	163.7	0.64	253.5	1.293	1.298
126.3	0.49	166.0	0.59			
131.5	0.53	170.0	0.58			
131.9	0.51	174.6	0.62			
133.0	0.47	180.8	0.64			
139.1	0.53	183.0	0.66			

* The column headed "Knoblauch" is based on the *H* formula of this paper. That headed "Thomas" is based on a modified H formula derived from his values of C_p. It is inserted only for comparison with the preceding one (see page 299). In both columns values above 200° involve a doubtful extrapolation of Linde's equation beyond its proper range. All temperatures are on the centigrade scale.

TABLE VI — *continued.*

VALUES OF C_p AT SATURATION FROM PLANCK'S EQUATION, USING:

c. RAMSAY AND YOUNG'S EXPERIMENTAL DATA.		*d.* BATTELLI'S "TABLE M."	
Temp.	C_p.	Temp.	C_p.
130.3	0.69	99.5	0.46
133.2	0.69	129.9	0.42
136.9	0.60	133.9	0.48
144.3	0.72	140.6	0.49
154.1	0.74	143.8	0.50
168.2	0.64	149.2	0.50
180.4	1.04	160.9	0.49
191.3	0.80	178.5	0.48
		180.9	0.54
		192.0	0.55
		199.0	0.59

to represent the same data. This alternative computation seems worth while, partly because of the automatic smoothing effect which the use of an equation based on all the observations necessarily has, but more because it means a redistribution of the dependence of the computed values of C_p on the volume measurements on the one hand and the new H formula on the other. The results of eleven computations of this kind are summarized in the second part of Table VI., and five of them are plotted as circles in Figure 12.

Two conclusions can be drawn from Figure 12. In the first place, both sets of points agree in confirming the conclusion reached on page 272, that Knoblauch's saturation curve is nearer the truth than Thomas'. It will probably be argued this confirmation is simply a circular fallacy, inasmuch as the H formula of this paper was based on Knoblauch's values of C_p and might therefore be expected to lead back to them in the end. This is true only in a very small measure. The dependence of H on C_p is such that comparatively large changes in the C_p curves used at the beginning of this paper would have made only small changes in the H formula, C_p being a factor, not of H itself, but only of ΔH. And in the second part of the computation, the redependence of C_p on H is again insensitive to errors in the assumed function, which this time is H. All this can be strikingly illustrated as follows. It is easy to compute approximately by the method of Section 3 of this paper a value of ΔH near 140° and one near 180° using Thomas' values of C_p instead of Knoblauch's. These, with

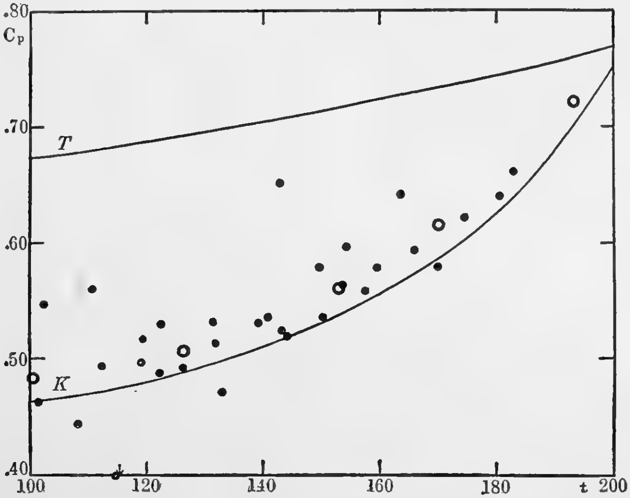

Figure 12. Values of C_p computed by Planck's method. The dots are based on the original volume measurements of Knoblauch, Linde and Klebe; the circles are based on Linde's characteristic equation. The lower curve is Knoblauch's saturation line; the upper one is Thomas'.

$\Delta H = 0$ at 100°, give a new second degree equation for $H = f(t)$ based wholly on Thomas' values. Finally this new H equation can be used with Linde's characteristic equation to compute, by means of Planck's equation, a set of values of C_p at saturation which are exactly comparable with those in the second part of Table VI., except that Knoblauch's C_p work is wholly replaced by Thomas'. If there is a circular fallacy in the confirmation mentioned at the beginning of this paragraph, the new results ought to confirm Thomas' C_p values at saturation just as definitely as the old ones did Knoblauch's. As a matter of fact, this is not at all the case. The new results are compared with the old in Figure 13, and agree strikingly in confirming Knoblauch's saturation curve. In other words, no matter which set of C_p values one starts with, one is led by this method of successive approximations to something much like Knoblauch's curve in the end.

The second conclusion that can be drawn from Figure 12 is that the true saturation curve, although close to Knoblauch's curve, probably runs somewhat higher in the range covered by these computations.

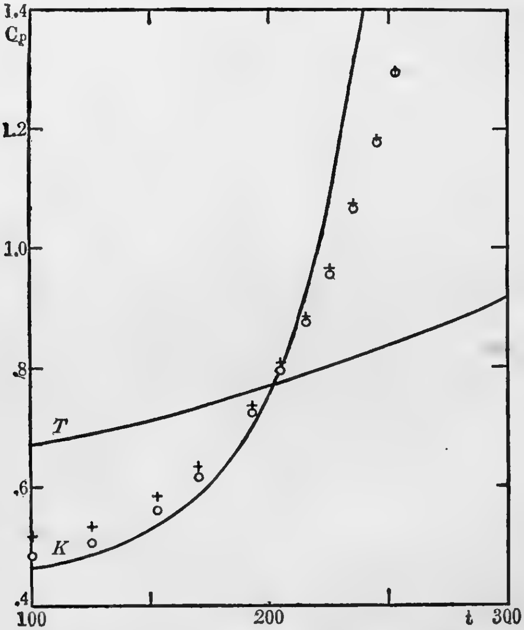

FIGURE 13. Values of C_p computed by Planck's method from Linde's characteristic equation. The circles come from an H formula based wholly on Knoblauch's C_p measurements, the crosses from a similar H formula based wholly on Thomas' C_p measurements. Both confirm Knoblauch's saturation curve (K) rather than Thomas' (T).

It will be remembered that the same conclusion was reached in two different ways in the last subsection (pages 290 and 293), and that it is further confirmed by the fact that Regnault's values near saturation at atmospheric pressure are higher than Knoblauch's.

The volume measurements of Ramsay and Young and of Battelli are not so conveniently arranged for the purposes of this particular computation. In both cases the temperature was held constant while the pressure and volume were varied. In the case of Ramsay and Young it is possible to rearrange the data so as to give approximate isochors which can then be reduced to suitable absolutely constant volumes by interpolation. The curves thus obtained are, however, irregular, and furthermore they show unmistakable evidences of a phenomenon exactly analogous to the wet steam error into which Thomas is believed to have fallen. The presence of this error, which took the form of surface condensation in the experimental bulb as saturation was approached, is specifically mentioned by the authors, but no attempt was made to eliminate it on the ground that, "however interesting from a theoretical point of view the absolute expansion of water-gas may be, in practise it is always in contact with a surface; and an indication of the behavior of steam in contact with glass cannot fail to be of use in considering the practical case of steam in contact with iron." It is therefore interesting to find that the values of C_p which have been computed from the data of Ramsay and Young and which are plotted as circles in Figure 14, run close to Thomas' saturation curve. This agreement is an indication that both are subject to the same error.

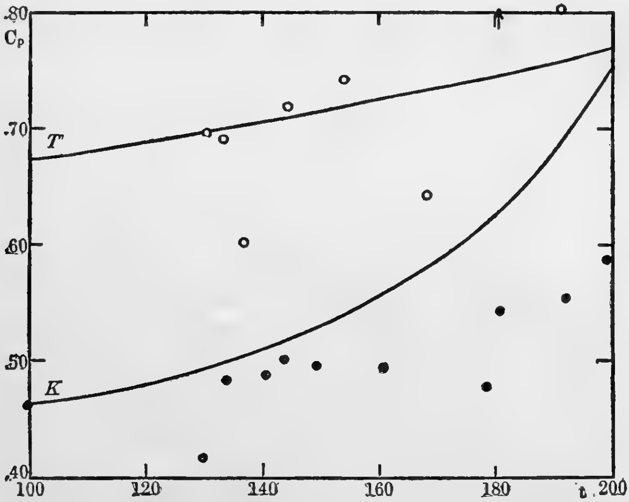

FIGURE 14. Values of C_p computed by Planck's method from the volume measurements of Ramsay and Young (circles) and of Battelli (dots).

Battelli was also troubled by surface condensation, but was at great pains, in discussing his results, to eliminate its effects. It has therefore seemed best to work not from his data, but from a table near the end of his memoir ("Table M"), in which are given certain graphically determined values of the coefficients in the formula

$$p = b\,t + a,$$

which he, like Knoblauch, Linde, and Klebe, uses to represent his isochors. The coefficient b in this formula is the same as the $(\partial p/\partial t)_v$ in the main table of Knoblauch's paper, and can be used in the same way. The values of C_p computed from Battelli's table M with condensation effects eliminated, run even lower than Knoblauch's saturation curve throughout the range of Figure 14. This indicates that Battelli rather more than eliminated the condensation errors in his discussion of his data.

The contrast between the values obtained from Ramsay and Young's work, where the wet steam error is known to exist, and those obtained from Battelli's work, where it is known to have been consciously eliminated, is so much like the contrast between Thomas' saturation curve and Knoblauch's as to be a striking verification of the conclusion reached on page 272.

It is not probable that either of the three sets of volume measurements are reliable enough to make the results computed in this section worthy of much consideration as new determinations of C_p. Their value is chiefly as corroborative evidence on one side or the other of the various doubtful points that have been mentioned.

Be. *Other indirect computations.* —
C. *Resumes and discussions.* — None of the papers which might be listed under Be or C are such as to be improvable by the use of the new material in this and in the preceding paper, or to be of importance in the present connection. They will not be discussed in detail.

Summary of this C_p discussion: —

1. Knoblauch's curves in general, and his saturation curve in particular, are much nearer the truth than Thomas'. The evidence for this is to be found on pages 287 and 298 to 302.

2. Knoblauch's saturation curve runs somewhat too low at low temperatures (see pages 290, 293 and 300).

3. The low temperature end of Knoblauch's 1 kg. curve should be somewhat raised, not only because of conclusion 2 above, but also so as to agree better with Regnault's recomputed results (see page 286).

4. Knoblauch's 1 kg. curve should be relocated at high superheats so as to agree with that of Holborn and Henning.

5. The spacing at high superheats of the curves corresponding to pressures higher than 1 kg. is best determined by a new method involving the Joule-Thomson coefficient (see pages 290 to 294).

6. The reconciliation, through Clausius' thermodynamic relation, of the accepted volume and specific heat measurements in the superheated region is impossible. This is probably the most important of the outstanding problems in this field.

All these conclusions have been embodied in the C_p diagram which is the basis of the Steam Tables already mentioned,[49] and it is partly for the purpose of gathering in one place all of the underlying evidence that justifies those tables, much of it unsuitable for presentation there, that this section of the present paper has been written. The C_p curves which were used were as faithful a translation and extrapolation of Knoblauch's curves as possible, except for the differences stated above. In particular they reproduced the tremendous rise of his saturation curve at even moderately high pressures and temperatures. It is probable that this feature of Knoblauch's curves, although near enough to the truth to satisfy the present needs of engineering practise, will have to be revised later. It is, however, the only rational guess yet published, and it is not worth while to cumber the literature with any more "harmonized" sets of C_p values at high pressures until there is something definite to build on. The problem of determining the true course of the high pressure end of the saturation curve on the C_p diagram is second in importance only to that mentioned at the end of the summary just above.

5. Clausius' "Specific Heat of Saturated Steam."

This section will be devoted to a revision of a computation first made by Clausius, which, although no longer of especial importance, is usually of considerable interest to students of thermodynamics. In the sixth chapter of the first volume of his "Mechanical Theory of Heat" he defines the specific heat of saturated steam as the quantity of heat that must be added to saturated steam at any temperature to turn it into saturated steam one degree hotter, account being taken of the fact that it will have to be compressed to keep it saturated. For steam and for most other substances it is negative at ordinary temperatures, because the work of compression is more than enough to provide the corresponding increase in the internal energy. But in the case of most substances including steam it is at ordinary temperatures an increasing function of the temperature and may, therefore, pass through zero and become positive if the temperature is sufficiently raised. This Clausius found to be actually the case for ether at ordinary temperatures and for chloroform above 130°, and the ex-

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[49] See page 97 of the Tables.

periments of Cazin and of Hirn confirmed this result. For such substances, the top of the temperature-entropy diagram must have the curious shape shown in Figures 15*a* and 15*b*.

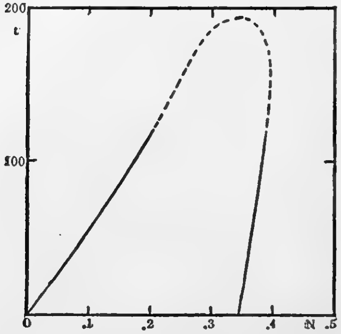

FIGURE 15*a*. Temperature-entropy diagram for ether.

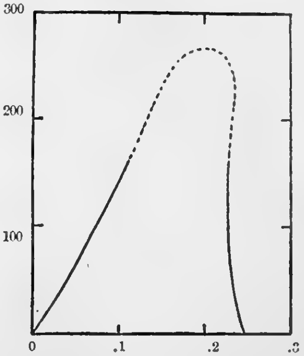

FIGURE 15*b*. Temperature-entropy diagram for chloroform.

The extrapolation formula for L in sub-section 3C enables one to compute this specific heat of saturated steam from Clausius' equation.

$$C_{\text{sat.}} = \frac{dL}{dt} + c - \frac{L}{T}$$

with the following results.

TABLE VII.

THE SPECIFIC HEAT OF SATURATED STEAM.

Temp.	$C_{\text{sat.}}$	Same acc. to Clausius.
0°	−1.69	−1.916
50°	−1.33	−1.465
100°	−1.08	−1.133
150°	−0.90	−0.879
200°	−0.80	−0.676
250°	−0.82	....
300°	−1.13	....

The necessary values of the specific heat of water, c, are taken from Marks and Davis' Steam Tables. Above 100° they are based on the experiments of Dieterici which run to 303°. The Table shows that $C_{\text{sat.}}$ passes its maximum below 250° without becoming zero or positive, and that at 300° it is already well on its way toward the value minus infinity which it has at the critical point. The temperature-entropy diagram for steam (see Figure 7a) is, therefore, fundamentally different in shape from that of ether or chloroform.

6. The Critical Volume of Water.

The extrapolation formula for L also makes possible a computation of the critical volume of water by the method of Cailletet and Mathias. These investigators announced in 1886[50] their well-known "law of the straight diameter," according to which, if the densities of a liquid and its saturated vapor are plotted against the corresponding temperatures to form a steam dome, the mid-points of the horizontal chords of the dome lie in a straight line. This law has been tested by a number of observers,[51] but particularly by Young, who proved that the diameter is accurately straight only in the case of a few "normal" substances of which normal pentane is the best known example, but that it is always nearly straight and can almost always be represented within the limit of error of the observations by a second degree equation in t. In certain cases, notably acetic acid and the alcohols, a third degree equation is necessary. All departures of the diameter from perfect straightness are commonly attributed to association in the liquid.

If the equation of the diameter is known, the substitution in it of the critical temperature gives the critical density with an accuracy far surpassing that of any known method of direct measurement. This accuracy is greatly increased by the fact that the diameter is always so nearly parallel to the t axis that even a considerable error in the critical temperature makes very little difference in the critical volume.

In applying this method to the determination of the critical density of water, one finds available in the third (1905) edition of Landolt and Bornstein's "Physikalische Tabellen" a satisfactory set of values

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[50] C. R., 1886, **102,** 1202.

[51] Mathias, Ann. de la Fac. des Sci. Toulouse, 1892, **6,** M1; C. R., 1892, **115,** 35; Mém. Soc. Roy. des Sci., Liege, 1899, **2**; Journ. de Phys., 1899, **8,** 407; and 1905, **4,** 77; Young, Journ. Chem. Soc., trans., 1893, **63,** 1237; Phil. Mag., 1892, **34,** 503; and 1900, **50,** 291; Guye, Archives des Sci. Phys. et Nat., 1894, **31,** 43; Tsuruta, Phys. Rev., 1900, **10,** 116. See also Young's "Stoichiometry," 1908, 165.

of the density of water up to 320°C. Furthermore, the pressure of saturated steam has been observed up to the critical point itself by a number of observers, of whom Cailletet and Colardeau[52] seem the most trustworthy. From their values and the extrapolation formula for L, one can compute the change of volume during vaporization up to 320° and indeed up to the critical point itself. The sum of these values and the volumes of the liquid mentioned above are the volumes of saturated steam up to 320°. The results are tabulated below and are plotted in Figure 16. The diameter is seen to be, as usual, nearly but not quite straight. It is not possible to represent the whole of it even by a third degree formula in t, because of the peculiar behaviour of the density of water at low temperatures. The 20 points above

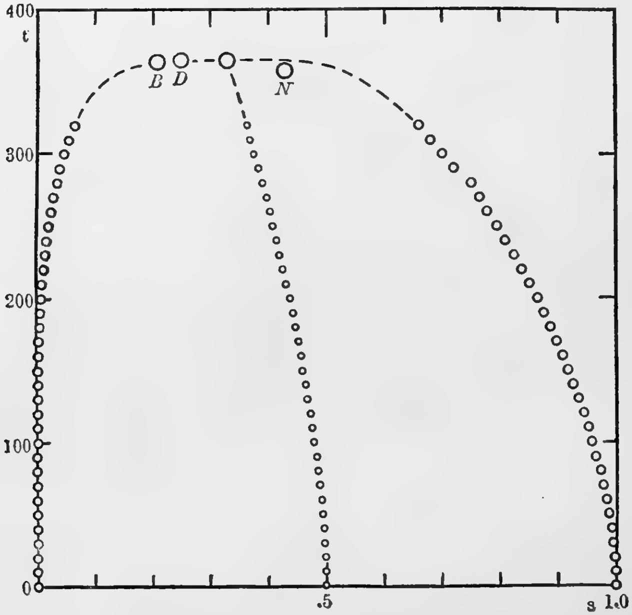

FIGURE 16. The steam dome on the temperature-density plane, with the "straight diameter" of Cailletet and Mathias, and the critical point according to Nadejdine (N), Battelli (B), Dieterici (D), and the present writer.

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[52] Journ. de Phys., 1891, **10,** 333; also Ann. Chem. et Phys., 1892, **25,** 519; also Physik. Rev., 1892, **1,** 14; also a short note in C. R., 1891, **112,** 563; see also Risteen, The Locomotive, 1907, **26,** 219.

TABLE VIII.

The Law of the Straight Diameter for Steam.

Temp. C°.	Density of Water.	Density of Steam.	Mean Density.	Formula.	Diff.
0°	0.9999	0.000005	0.5000	...	...
10°	0.9997	...	0.4999	...	...
20°	0.9982	...	0.4991	...	...
30°	0.9957	...	0.4978	...	...
40°	0.9922	0.00005	0.4961	...	...
50°	0.9881	0.00008	0.4941	...	...
60°	0.9832	0.00013	0.4917	...	...
70°	0.9778	0.0002	0.4890	...	...
80°	0.9718	0.0003	0.4860	...	...
90°	0.9653	0.0004	0.4828	...	...
100°	0.9584	0.0006	0.4795	...	...
110°	0.9510	0.0008	0.4759	...	...
120°	0.9434	0.0011	0.4722	...	...
130°	0.9352	0.0015	0.4684	0.4688	+0.0004
140°	0.9264	0.0020	0.4642	0.4644	+0.0002
150°	0.9173	0.0026	0.4599	0.4599	0.0000
160°	0.9075	0.0033	0.4554	0.4552	−0.0002
170°	0.8973	0.0041	0.4507	0.4504	−0.0003
180°	0.8866	0.0052	0.4459	0.4454	−0.0005
190°	0.8750	0.0064	0.4407	0.4403	−0.0004
200°	0.8628	0.0079	0.4354	0.4351	−0.0003
210°	0.850	0.010	0.430	0.431	+0.001
220°	0.837	0.012	0.424	0.424	0.000
230°	0.823	0.014	0.419	0.419	0.000
240°	0.809	0.017	0.413	0.413	0.000
250°	0.794	0.020	0.407	0.407	0.000
260°	0.779	0.024	0.402	0.401	−0.001
270°	0.765	0.029	0.397	0.395	−0.002
280°	0.75	0.03(4)	0.39(2)	0.388	−0.00(4)
290°	0.72	0.03(9)	0.38(0)	0.382	+0.00(2)
300°	0.70	0.04(6)	0.37(3)	0.375	+0.00(2)
310°	0.68	0.05(5)	0.36(8)	0.368	0.00(0)
320°	0.66	0.06(5)	0.36(2)	0.362	0.00(0)

120° can, however, be represented with an average deviation of about one ninth of one per cent by the formula

$$s = 0.4552 - 0.0004757\,(t-160) - 0.000000685\,(t-160)^2 \text{ gr./cm.}^3$$

It should be noticed, in judging of the reliability of the formula, that comparatively large relative errors in the density of steam make only very small relative errors in the mean of the densities. Thus in the most unfavorable case, at 320°, if an error in either dp/dt or in the extrapolated value of L made the computed change of volume wrong by five per cent, the resulting error in the mean of the densities would be less than half of one per cent.

The substitution of Cailletet and Colardeau's value for the critical temperature of water, $t_c = 365°\text{C.}$, in the equation of the diameter gives

$$s_c = 1/v_c = 0.329 \text{ gr./cm.}^3,$$

from which it follows that the critical volume is

$$v_c = 3.04 \text{ cm.}^3/\text{gr.}$$

There are three previous determinations with which this can be compared, two of which are direct measurements. These are

$$v_c = 2.33 \text{ cm}^3/\text{gr. (Nad.)},$$

found by Nadejdine[53] in 1885, and

$$v_c = 4.812 \text{ cm.}^3/\text{gr. (Batt.)},$$

found by Battelli[54] in 1890. In both cases a known weight of water was enclosed in a steel tube and heated at constant volume until the contents became homogeneous. If there was too little liquid, this occurred when it was all evaporated; if too much, when it had so expanded as to fill the tube; if just enough, at the critical point. The last case they hoped to recognize because of its corresponding to a higher temperature than either of the others. Such a method gives an excellent determination of the critical temperature, but it can hardly be expected to give an accurate determination of the critical volume. It amounts to trying to find the highest point of the steam dome by selecting experimentally its longest ordinate on the $v\ t$ plane. The extremely flat top of the steam dome makes this almost impossible, and it is interesting to notice that both Nadejdine and Battelli fell within the nearly flat region, one at one end and one at the other. The present determination lies between theirs and should be much more accurate than either.

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[53] Universitatäkija Investia Kiew., 1885, **6**, 32; Mel. Phys. et Chim. tirés du Bull. de l'Ac. de St. Pétersb., 1885, **12**, 299; Chem. CBl., 1885, **17**, 401.

[54] Mem. dell. Ac. di Torino, 1891, **41**, 76; Physikal. Rev., 1892, **2**, 1.

The third published value of the critical volume,

$$v_c = 4.025 \text{ cm.}^3/\text{gr. (Diet.)},$$

was computed by Dieterici[55] in 1904, from the empirical law of Young[56] that, for "normal" substances, the ratio of the actual to the gas-law density at the critical pressure and density is 3.8. Dieterici's belief that water becomes a "normal" substance at high temperatures, even though it is known to be very abnormal at ordinary temperatures, is based on the fact that the ratio of the change of internal energy during evaporation to the whole heat of evaporation, L, seemed to approach a value which he had predicted for "normal" substances. The present determination of v_c shows that water is, as one would have expected, still abnormal at the critical point. If interpreted in the usual way, it would indicate a polymerization factor of 1.3. Figure 17 shows how well this number fits a smooth curve through Ramsay's earlier large values of the polymerization factor at ordinary temperatures;[57]

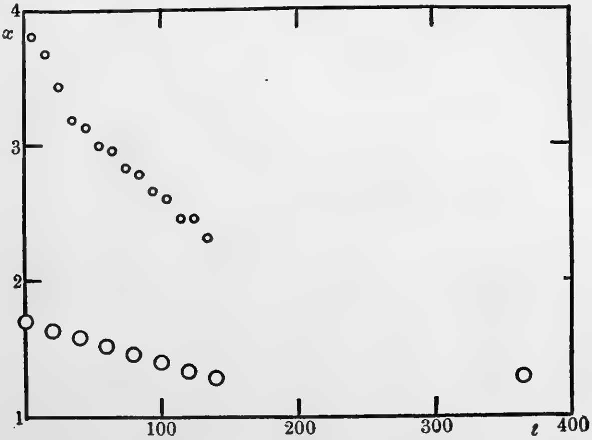

Figure 17. The polymerization factor for liquid water as a function of the temperature. The small circles below 150° are Ramsay's earlier values; the large circles below 150° are his revised values; the circle at 365° is the value indicated by the critical volume.

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[55] Wied. Ann., 1904, **15**, 864.

[56] Phil. Mag., 1892, 34, 507, and Jour. Chem. Soc., Papers, 1893, **63**, 1251.

[57] Phil. Trans., 1893, **184A**, 647; translated in Zeitsch. Phys. Chem., 1893, **12**, 433; second paper in Jour. Chem. Soc., 1893, **63**, 1089; translated in Zeitsch. Phys. Chem., 1893, **12**, 458.

there seems, however, to be little chance of reconciling it with his later "corrected" values.[58] This is an example of the uncertainty that seems to characterize the whole subject of polymerization in liquids, especially on its quantitative side.

The equation of the mean diameter which has just been obtained can also be used for the computation of a rough but useful extension of certain columns of the ordinary steam tables up to the critical point. As was mentioned on page 306, the extrapolation formula for L of Section 3 determines the change of volume during evaporation at all temperatures between 320° and the critical temperature. From these and the mean densities given by the equation of the diameter, it is easy to compute each of the densities separately, and to fill in the rest of the steam dome on the *t s* (temperature-density) planes. The results are shown by the dotted lines in Figure 16, and are given in detail at the end of Table I in the Steam Tables already mentioned. Any values obtained in this way are, of course, only rough approximations to the truth and should not be too much relied on.

Summary of the Results in this Paper.

1. It presents a new set of values for the difference between the total heat of saturated steam at certain temperatures between 65° and 190°C. and its value at 100°.

2. It shows that these differences can be represented within their limit of error by the first three terms of a Taylor's series, but that such a development should not be extrapolated far in either direction. The best direct measurements of H indicate that its value at 100° is 639.11 mean calories. If this be accepted, the proposed formula for H is

$$H = 639.11 + 0.3745\,(t - 100) - 0.000990\,(t - 100)^2 \text{ mean calories.}$$

The last two terms of the formula are the real contribution of this paper, and may still be valid, even if the first term is later found to be wrong.

3. Thiesen's formula for L with recomputed constants is shown to represent satisfactorily all of the reliable values of L, including those in this paper. It is believed to be the safest known means of extrapolating to high temperatures.

4. The literature on the specific heat of superheated steam is systematically discussed and revised in the light of the new values of H

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[58] Third paper; Proc. Roy. Soc., 1894, **56,** 171; translated in Zeitsch. Phys. Chem., 1894, **15,** 106.

and of the Joule-Thomson coefficient presented in an earlier paper. In particular the choice of Knoblauch's values of C_p as the foundation of the determination of H in this paper is justified.

5. It is shown that Clausius' specific heat of saturated steam passes its maximum without becoming zero or positive, so that the temperature-entropy diagram for steam must be essentially simpler than that for either ether or chloroform.

6. The extrapolation formula for L mentioned in 3 above is made the basis of a determination of the critical density of water by the method of Cailletet and Mathias. The result is

$$v_c = 3.04 \text{ cm}^3/\text{gr}.$$

The specific volumes of water and of saturated steam at other high temperatures have also been computed and embodied in a steam table running up to the critical temperature.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

THE SPECTRUM OF A CARBON COMPOUND IN THE REGION OF EXTREMELY SHORT WAVE-LENGTHS.

By Theodore Lyman.

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THE SPECTRUM OF A CARBON COMPOUND IN THE REGION OF EXTREMELY SHORT WAVE-LENGTHS.

By Theodore Lyman.

Presented December 8, 1909. Received January 3, 1910.

In the region of extremely short wave-lengths discovered by Schumann the spectra of two gases only are easily obtained; the one is due to hydrogen, the other to some compound of carbon.[1] The hydrogen spectrum consists of a great number of fine lines extending from λ 1675 to λ 1030; the wave-lengths of the most prominent of these lines have been determined.[2] The carbon spectrum consists of a considerable number of bands extending from the less refrangible end of the Schumann region to the neighborhood of λ 1300. The purpose of the present investigation was to measure the position of these bands.

The results are chiefly valuable because the bands in question fill the gap between λ 1854 and λ 1675 and form convenient standards of wave-length in a region which up to this time has lacked points of reference.

The appearance of the spectrum is shown in Plate VIII, Volume 13, of the Memoirs of this Academy. It is marked "Air." The bands are most intense in the less refrangible region, but they are all of the same general type with heads directed toward the region of shorter wave-length. The strongest bands are evidently double. The system, at least throughout its less refrangible part, forms a continuation of the "Fourth group" as described by Deslandres in his paper, "Spectre de bandes ultra-violet des composés hydrogénés et oxygénés du carbone."[3] The spectrum under investigation is thus related to the series of bright bands in the visible and ultra-violet attributed to carbon monoxide and often observed in ill-prepared vacuum tubes.

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[1] Smithsonian Contributions, 1903, **29,** No. 1413.

[2] Lyman, Memoirs of this Academy, 1906, **13,** 125.

[3] Comptes Rendus, 1888, **106,** 842.

It is only too easy to obtain the bands in the region of short wavelengths, for, to quote Schumann himself,[4] they are "the unwelcome attendants of all my spectra." In order to determine the cause of the phenomenon, however, experiments were made with both carbon monoxide and carbon dioxide and with a variety of conditions in the discharge tube. The results of these experiments may be stated as follows: Exactly the same bands are obtained when carbon monoxide is used as when carbon dioxide is employed, but in the former case the strength of the whole spectrum is considerably greater than in the latter. With increased current strength from a transformer, between five and twenty milliamperes the intensity of the bands increases in a uniform manner throughout the extent of the spectrum. When a spark gap is placed in series with the tube and a condenser is introduced in such a way as to produce a disruptive discharge, the spectrum at first weakens and then vanishes altogether. The effect is accompanied by a very marked decrease in pressure in the tube and by the formation of a dark deposit on the walls of the capillary. When precautions are taken to exclude the introduction of carbon monoxide or prevent its formation, the spectrum is greatly weakened if it does not vanish altogether.

These data go to confirm the results of Schumann, as they show that the spectrum is due to carbon monoxide. The occurrence of the bands when carbon dioxide is present may be explained by the fact that this gas is known to be transformed into carbon monoxide under the influence of light and the electric discharge.[5] The disappearance of the spectrum with the disruptive discharge is due to the destruction of the carbon monoxide. The oxygen set free by the reaction seems to combine with the electrodes, while the carbon is deposited. This property of a condenser discharge is useful, since it permits the spectroscopist to free his apparatus of an annoying impurity. The decrease in pressure which accompanies this reaction is often a striking and important phenomenon.

In making measurements in the region between λ 1880 and λ 2080 a concave grating of six foot radius with 15028 lines to the inch was employed. Schumann plates were used throughout the work. For the experiments in the region on the more refrangible side of λ 1880 the writer's vacuum spectroscope was employed[6] in the same manner as when the hydrogen spectrum was under investigation. An improvement in the discharge tube, however, has been introduced. The nature

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[4] Loc. cit., p. 16.

[5] Herchefinkel, Comptes Rendus, 1909, **149**, 395.

[6] See note 2.

of the change will be understood by consulting the illustration on page 90 of volume 27 of The Astrophysical Journal. The brass collar A is no longer provided with a screw thread as shown in the illustration, but it is now made to fit into the cup B air tight by means of a cone joint 2.8 cm. long. The discharge tube itself is no longer cast into the collar A with Khotinski cement, but is blown on a platinum tube 3.5 cm. long by 1.5 cm. in diameter. This tube is soldered into the collar A. By this arrangement the gas does not come in contact with grease in the joints, and the danger of leak is considerably reduced.

Measurements in the region between λ 1850 and λ 1675 where no fiducial lines exist were made by the two slit method.[7] In the region from λ 1675 to λ 1300 direct comparison was made with the spectrum of hydrogen.

The values of λ refer to the heads of bands, and they are accurate to 0.3 of an Angström unit. In the class of the double bands marked "*d*" the wave-length given is for the stronger component. The intensities are represented on a scale of ten. The absorption of fluorite, which begins to make itself felt near the end of the spectrum, renders the relative intensities of the most refrangible bands rather uncertain. As usual, the wave-lengths and frequencies are in vacuum.

In addition to their value as standards of wave-length, the results are of some theoretical importance. Deslandres in the paper just quoted[8] has used his measurements of the carbon spectrum to test his Laws. As the spectrum under discussion seems to form a continuation of that described by Deslandres, it is interesting to see if its bands also show the numerical relations described by the earlier investigator. In making the comparison, however, it will be necessary to confine the attention to those relations which deal with the heads of the bands, for the dispersion employed does not permit of the study of the lines of which each band is composed. It must also be remembered that the region of high frequencies is not perfectly adapted to such a test, since a small error in the wave-length is magnified in relations which deal with frequencies.

The laws under discussion are two in number: first, that a group of bands may be broken up into sets of series such that the differences in frequency of the heads of the bands in any one series form an arithmetical progression; second, that all the series are similarly constructed. The first rule may obviously be stated in another way, — the second differences of the frequencies of the heads of the bands in any one series are constant.

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[7] See note 2.

[8] Loc. cit.

TABLE I.

Fourth Group.

Series	Deslandres.				Lyman.											
	IV		V		VI		VII		VIII		IX		X		XI	
	N.	2nd Diffs.	N.	2nd Diffs.	N.	2nd Diffs.	N.	2nd Diffs.	N.	2nd Diffs.	N.	2nd Diffs.	N.	2nd Diffs.	N.	2nd Diffs.
	46981	. .	49821[1]													
	46499	—2	49349[1]	5	52247											
	46019	10	48872	28	51773	27										
	45529	22	48367	15	51272	12										
	45017	9	47847	10	50759	20	54309									
	44496	2	47317	—3	50226	6	53775	7								
	43973	13	46790	19	49687	10	53234	6								
	43437	. .	46244	11	49138	. .	52687	8	56341							
	. .	. .	45687	. .	. .	. .	52132	. .	55785	11						
	. .	. .	. .	. .	. .	. .	. .	. .	55218	9	58404					
	. .	. .	. .	. .	. .	. .	. .	. .	54642	. .	57820	5				
	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	57231	16				
	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	[56626]	2				
	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	56019	7				
	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	55405	17	58641			
	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	54774	. .	58008	19		
	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	57356	. .	61338	
	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	60672	
	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	59999	
	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	(57337)	
Av. 2nd Diff.	. .	**9**	. .	**13**	. .	**15**	. .	**7**	. .	**10**	. .	**9**	. .	**19**	. .	**7**

[1] From the writer's measurements.

Deslandres has analysed his Fourth Group into five series, characterized by small and not very regular second differences. The writer has been able to follow the arrangement into the region between λ 2000 and λ 1600 and has added seven new series of the same type. Table I.

TABLE II.

FOURTH GROUP.

Deslandres.			Lyman.					
First Differences.			First Differences.					
Series	IV	V	VI	VII	VIII	IX	X	XI
	482	472						
	480	477	474					
	490	505	501					
	512	520	513					
	521	530	533	534				
	523	527	539	541				
	536	546	549	547				
	. .	557	. .	555	556			
	. .	. .	. .	. .	567			
	. .	. .	. .	. .	576	584		
	. .	. .	. .	. .	. .	589		
	. .	. .	. .	. .	. .	605		
	. .	. .	. .	. .	. .	607		
	. .	. .	. .	. .	. .	614		
	. .	. .	. .	. .	. .	631	633	
	. .	. .	. .	. .	. .	. .	652	
	. .	. .	. .	. .	. .	. .	. .	666
	. .	. .	. .	. .	. .	. .	. .	673

shows these new members. They are numbered from VI. to XI.; series IV. and V. of Deslandres' are included in the table for the sake of comparison. The first two bands in the fifth series were measured by the writer. When it is remembered that the errors of observation make the fifth place in the frequencies very doubtful, it will be seen that the law of constant second differences is fairly well obeyed.

TABLE III.

Fifth Group.

Series	1		2		3		4		5		6	
	N.	2nd Diffs.	N.	2nd Diffs.	N.	2nd Diffs.	N.	2nd Diffs.	N.	2nd Diffs.	N.	2nd Diffs.
	65772	. .	66366	. .	67527	. .	69266	. .	70852	. .	71649	
	65531	109	65976	124	67204	116	68852	118	70721	118	71372	123
	65181	110	65462	106	66765	132	68320	129	70472	102	70972	100
	64721	130	64842	. .	66194	. .	67659	. .	70121	113	70472	
	64131	109	. .	. .	. .	. .	. .	. .	69657	84		
	63432	131	. .	. .	. .	. .	. .	. .	69109			
	62602											
Av. 2d Diffs.	. .	118	. .	115	. .	124	. .	123	. .	104	. .	112

Table II., which gives the first differences for each series, is arranged to show the similarity which exists among the members. It will be observed that the second rule is obeyed, for the series resemble each other. An exact similarity is not demanded by the rule as has been recently pointed out by Deslandres himself.[9] The arrangement of the series, however, does not permit of the "second progression"[10] mentioned by Olmsted and others.

In addition to the series VI. to XI. there appear to exist two others, in the region near λ 1800. These show larger second differences than the first type. They have not been included in the tables.

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[9] Comptes Rendus, 1904, **138**, 317.

[10] Comptes Rendus, 1902, **134**, 748; Zeits. f. Wiss. Photographie, 1906; **4**, 255.

TABLE IV.

λ.	I.	N.	λ.	I.	N.
1335.0	1	74,906	1615.1	2	61,916
1339.0	1	74,683	1623.4	1	61,599
1343.0	1	74,460	1629.6	3	61,365
1353.6	1	73,877	1630.3	6	61,338
1356.1	2	73,741	1648.2	5	60,672
1361.3	2	73,459	1653.3	4	60,485
1368.0	1	73,099	1666.7	1	59,999
1371.8	2	72,897	1669.9	6	59,884
1374.1	2	72,775	1685.3	1	59,337
1378.1	2	72,564	1688.5	1	59,224
1384.4	1	72,233	1698.8	1	58,865
1386.4	1	72,129	1705.3	6	58,641
1392.2	1	71,829	1712.2	7	58,404
1395.7	2	71,649	1723.9	6	58,008
1401.1	2	71,372	1729.5	8 *d*	57,820
1404.0	1	71,225	1743.5	3	57,356
1405.5	1	71,149	1747.3	7	57,231
1409.0	2	70,972	1774.9	8 *d*	56,341
1411.4	1	70,852	1785.1	6	56,019
1414.0	1	70,721	1792.6	10 *d*	55,785
1419.0	2	70,472	1801.9	2	55,497
1426.1	3	70,121	1804.9	8	55,405
1435.6	2	69,657	1811.0	10 *d*	55,218
1438.7	1	69,507	1825.7	7	54,774
1443.7	1	69,266	1830.1	9	54,642
1447.0	1	69,109	1837.2	1	54,431
1452.4	3	68,852	1841.3	8	54,309
1463.7	3	68,320	1846.7	2	54,151
1473.0	1	67,889	1849.4	4	54,072
1475.4	1	67,778	1859.6	10 *d*	53,775
1478.0	2	67,659	1870.3	3	53,467
1480.9	2	67,527	1878.5	10 *d*	53,234
1488.0	2	67,204	1891.2	6	52,876
1493.8	3	66,943	1898.0	10	52,687
1497.8	3	66,765	1914.0	1	52,247
1506.8	2	66,366	1918.2	7	52,132
1510.7	2	66,194	1931.5	6	51,773
1515.7	3	65,976	1933.6	2	51,717
1520.4	1	65,772	1950.4	4	51,272
1526.0	2	65,531	1951.7	5	51,237
1527.6	3	65,462	1953.0	5	51,203
1534.2	2	65,181	1970.1	8	50,759
1542.2	5	64,842	1991.0	1	50,226
1545.1	3	64,721	2007.2	5	49,821
1559.3	5	64,131	2012.6	8	49,687
1576.5	4	63,432	2026.4	7	49,349
1596.1	1	62,653	2031.7	1	49,220
1597.4	3	62,602	2035.1	4	49,138
1603.3	1	62,371	2047.0	8	48,852
1611.7	3	62,046	2068.4	8	48,347

On the more refrangible side of λ 1600 matters are not very satisfactory. The bands must be arranged into series showing very large second differences which are only approximately constant. These series, which are numbered from 1 to 7 go to make up the Fifth Group. Their frequencies together with the second differences are given in Table III. No attempt has been made to adopt an arrangement which would show the similarity between the members of the group. In fact these series fall in with the second rule only to a limited degree; 1 and 5 resemble each other, as do 2 and 6, and 3 and 7, but the relations are not exact.

The writer makes no claim that the arrangements given in this Fifth Group are the best possible, they are only the most obvious.

The spectrum contains a great many bands which are either too feeble to measure or whose positions are made uncertain by the tails of stronger bands; if these could be included in the series a better system would probably result.

It is to be remembered that although relations similar to Deslandres' laws have been proved to hold within the limit of error of observation for the distribution of lines within a band,[11] no such accuracy of agreement has been found when the laws of the distribution of the heads of the bands themselves have been tested. In fact, the rule of constant second differences as applied to the heads of bands must be looked upon as a first approximation only. The work which has just been described indicates that the approximation holds even in the region of extremely short wave-lengths.

In conclusion the writer wishes to point out that the important results of the investigation are the values of the wave-lengths contained in Table IV.

Jefferson Physical Laboratory,
Cambridge, Mass.,
December, 1909.

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[11] v. Carlheim-Gyllensköld, K. Svensk. Vetenskaps-Akad., Handl., 1907, **42**, No. 8.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL
LABORATORY, HARVARD UNIVERSITY.

EXPERIMENTS ON THE ELECTRICAL OSCILLATIONS OF A HERTZ RECTILINEAR OSCILLATOR.

By George W. Pierce.

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EXPERIMENTS ON THE ELECTRICAL OSCILLATIONS OF A HERTZ RECTILINEAR OSCILLATOR.

By George W. Pierce.

Presented December 8, 1909. Received January 3, 1910.

While engaged in calibrating a wavemeter for electric waves, I have made a series of measurements of the wave-length produced by a long Hertz rectilinear oscillator, consisting of two oppositely extending horizontal wires with a spark-gap between. By varying the length of the oscillator, wave-lengths from 16 to 63 meters were obtained. The experiments were conducted in a long room in the third story of the laboratory, so that the oscillator was at a height of 10 meters above the surface of the earth, and represents approximately the conditions that exist when the oscillator is alone in free space.

The experimental results, which give a relation of the wave-length to the length of the oscillator, may be not without interest; because of the existence of numerous very thorough mathematical discussions of the problem.

Apparatus and Plan of the Experiment. — A general idea of the experiment may be had by a reference to Fig. 1, which shows in ground plan the arrangement of the apparatus.

The wavemeter, shown at the left of the figure, consists of a variable condenser C in series with a loop of heavy wire L and a high-frequency electrodynamometer I. The loop of wire L is in the form of a square 30 cm. on a side. The condenser consists of two sets of semicircular plates — one set fixed and the other set movable by rotation about a vertical axis so as to permit variation of capacity by bringing a greater or less area of the two sets of plates into an interlapping position. A scale carried by the top movable plate passes under a fixed pointer, so that the position of the movable plates with respect to the fixed plates can be read after any adjustment of the apparatus.

The high-frequency dynamometer I is of the form previously employed by me in a series of experiments on resonance in wireless telegraph circuits,[1] and consists of a disc of silver, suspended by a quartz fibre, so as to hang near a small coil of a few turns of wire, with the axis of which the plane of the disc makes an angle of 45°, as is shown in Fig. 2. The disc is at M; and the coil, which in this experiment consisted of five turns of wire wound on a vulcanite tube, is shown at C, Fig. 2. The terminals from the coil are connected to binding posts by which the coil is put into the wavemeter circuit. The front of the disc M carries a small mirror, enabling the deflections of the disc to be measured by means of a telescope and scale.

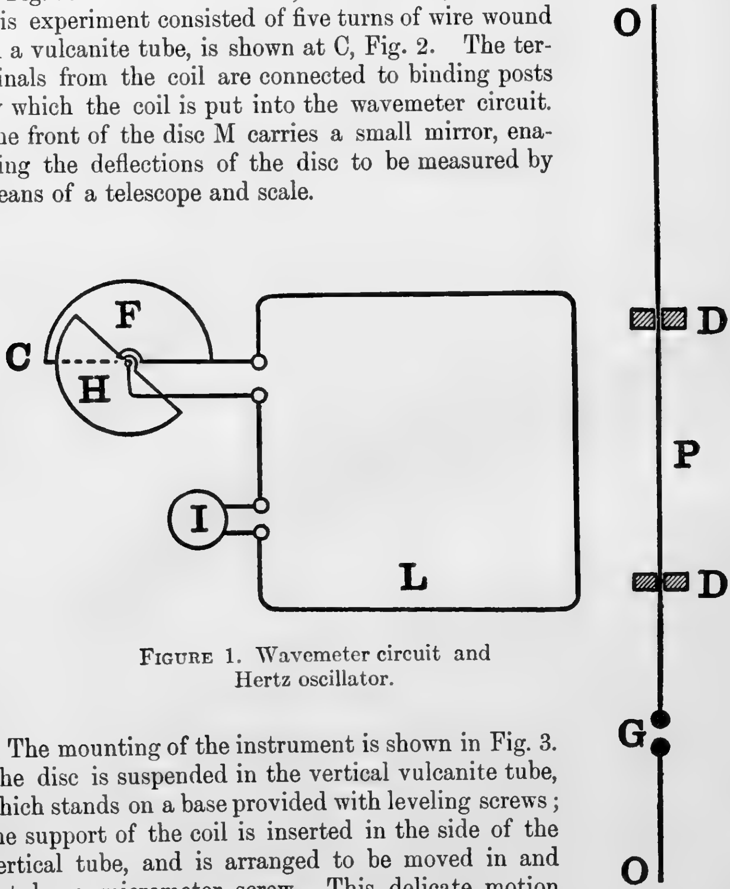

Figure 1. Wavemeter circuit and Hertz oscillator.

The mounting of the instrument is shown in Fig. 3. The disc is suspended in the vertical vulcanite tube, which stands on a base provided with leveling screws; the support of the coil is inserted in the side of the vertical tube, and is arranged to be moved in and out by a micrometer screw. This delicate motion of the coil in or out brings the coil nearer to or farther from the suspended silver disc so as to vary the sensitiveness of the instrument to make it suitable for measuring small or large oscillating currents.

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[1] Phys. Review, 1904, **19,** 196; 1905, **20,** 220; 1905, **21,** 367; 1906, **22,** 159; 1907, **24,** 152.

The action of the instrument is as follows : Oscillations in the coil induce oscillations in the disc, and between these two sets of oscillations there is a force which causes the disc to tend to set itself at right angles to the plane of the coil. A mathematical theory of the instrument, together with some experiments showing that the deflections of the disc are proportional to the square of the current in the coil, is given by me in volume 20, page 226, of the Physical Review for 1905.

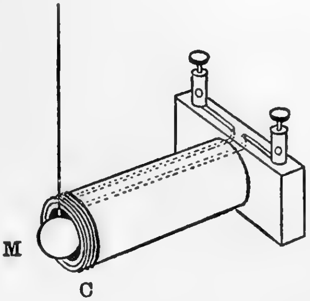

FIGURE 2. Coil and suspended disc of the high-frequency dynamometer.

In place of the dynamometer, a Geissler tube, connected to the two sides of the condenser, was used in some of the experiments.

The Calibration of the Wavemeter. — For wave-lengths greater than 350 meters, I have a set of standard oscillators whose periods have been determined by spark-photographs taken with the revolving mirror.[2] These could, however, not be employed in the present experiments, where the greatest length of oscillator that could be set up in the room had a wave-length of only 63 meters. It was, therefore, necessary to use another method of calibrating the wavemeter of Fig. 1; namely, by tuning it to resonance with an oscillator consisting of various lengths (4 to 17 meters) of two parallel wires, 1 mm. in diameter and 8 cm. apart. It was assumed that the wave-length of such a parallel-wire oscillator is four times the length of one of the wires. This assumption is on the supposition that there is a loop of potential at the free end of the oscillator, and that the velocity of the waves on parallel wires is equal to the velocity of light.

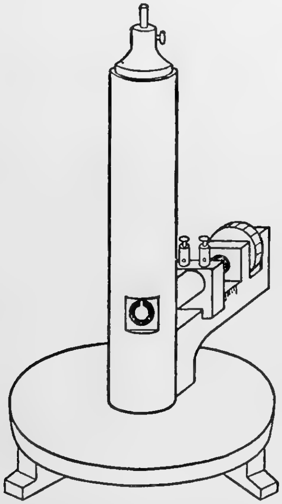

FIGURE 3. Mounting of dynamometer with variable sensitiveness.

In regard to the loop at the free end, Bumpstead[3] has shown that this loop of potential for a parallel-wire oscillator is really beyond the

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[2] Phys. Review, 1907, **24**, 152.

[3] Am. Jour. Sci., 1902, **14**, 359.

free end by an amount a little less than half the distance apart of the wires. This correction, applied to my experiments, amounts to less than one per cent in the case even of the shortest parallel-wire oscillator used in the calibration, and has been taken into account.

That the velocity of the waves on the wires is equal to the velocity of light has its theoretical basis in the fact that for rapid oscillations guided by parallel wires, the self-induction per unit of length multiplied by the capacity per unit of length is the reciprocal of the square of the velocity of light. That the velocity of propagation on the parallel wires is the velocity of light has been shown experimentally by Trowbridge and Duane[4] and by Saunders.[5] Recently also Diesselhorst[6] of the Reichsanstalt has made some experiments which indicate that the wave-length on the parallel wires differs from the wave-length in air by less than one-third of one per cent when the parallel wires are not more than 100 meters long.

Wave-length of the Wave Produced by the Hertz Oscillator. — If now we take the two parallel wires, separate them, and extend them out oppositely so as to form a Hertz oscillator, the capacity per unit of length diminishes, while the inductance per unit of length increases. Does the wave-length remain the same; namely, four times the length of the half-oscillator, or $\lambda = 2\,l$, where l is the length of the whole oscillator? Some theoretical writers (Abraham,[7] Rayleigh[8]) say that it does remain very approximately the same (if the diameter of the wire is a small fraction of the length); while, on the other hand, Macdonald[9] has concluded that λ is equal to $2.53\,l$, and he is supported in this conclusion by Pollock and Close.[10]

Experimental tests of the question have heretofore usually been made with very short vibrating systems, to which the theoretical deductions are not directly applicable. A. D. Cole[11] finds $\lambda = 2.52\,l$, for a Klemencic receiver 7 to 8 cm. long and 3.1 mm. in diameter. This is in good agreement with Macdonald's theoretical relation. It is doubtful, however, if Macdonald's equation, which was derived by considering the oscillator or receiver to be indefinitely thin in comparison with its length, was intended to apply to the relatively thick receivers of Cole's experiment.

Another very admirable set of measurements with short oscillators has recently been published by Webb and Woodman.[12] With an un-

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[4] Am. Jour. Sci., 1895, **49**, 297.

[5] Phys. Review, 1896, **24**, 152.

[6] Elektrotech. Zeits., 1908, **29**, 703.

[7] Wied. Ann. 1898, **66**, 435.

[8] Phil. Mag., 1904, **8**, 105.

[9] Electric Waves, 111.

[10] Phil. Mag. 1904, **7**, 635.

[11] Phys. Review, 1905, **20**, 268.

[12] Phys. Review, 1909, **29**, 89.

tuned receiver they have made measurements of the wave-length produced by rod oscillators of various lengths between 2 and 10 cm., and various diameters between 0.2 and 1.3 cm., and have obtained the wave-length a linear function of the length when the ratio of diameter to length is kept constant, and also the wave-length is a linear function of the ratio of diameter to length when the length is kept constant. By extrapolation from their measured values they find the limiting value of the ratio of the wave-length to the vibrator length, as the diameter approaches zero, to be 2.24.

Coming now to the experiments that have been made with the longer oscillators, I find two measurements mentioned by Drude[13] in which he obtains for a wire 1 mm. in diameter and 4 meters long the wave-length 8.42 meters, and for a wire 2.5 meters long the wave-length 5.24. *These two experiments give* $\lambda = 2.10\, l$.

Also there is a series of measurements by F. Conrat[14] for rectilinear oscillators 2 to 6 meters long (1 mm. diameter). These measurements are presented in Table I., and *show the average relation* $\lambda = 2.12\, l$.

TABLE I.

Conrat's Values for Relation of λ to l.

l. Length of Oscillator in Meters.	λ. Wave-length in Meters.	λ/l.
2.00	4.20	2.10
3.84	8.00	2.09
4.00	8.40	2.10
5.50	12.00	2.18
6.30	13.40	2.12
Average		2.12

My measurements, extending the experimental records in the direction of the longer waves, are given in Table II. The diameter of the wire employed was 1 mm. The result obtained is that *the wave-length of the oscillator is* 2.094 *times its length.* This is in good agreement with the results obtained by Drude and in fair agreement with those of Conrat.

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[13] Ann. d. Physik, 1903, **11**, 965.

[14] Ibid., 1907, **22**, 670.

Taking the present observation together with those of Drude and of Conrat it appears that the wave-length of a Hertz rectilinear is very close to 2.10 times the length of the oscillator, provided the oscillator is not less than two meters long and is of comparatively small diameter. The influence of the diameter in determining the wave-length was not tested further than by a single observation, in which it was found that an oscillator made of two brass tubes, each 6 meters long and 22 mm. in diameter, had a wave-length 2.14 times its length.

TABLE II.

Results obtained in Present Experiment.

l. Length of Oscillator in Meters.	Wave-length in Meters.	λ/l.
8.0	16.9	2.11
9.0	18.9	2.10
10.0	21.2	2.11
11.0	23.2	2.11
12.0	24.9	2.08
14.0	29.5	2.10
16.0	33.6	2.10
18.0	38.1	2.11
20.0	41.6	2.08
22.0	46.1	2.11
24.0	49.5	2.07
26.0	53.9	2.07
28.0	57.5	2.06
30.0	63.0	2.10
Average		2.094

Comparison of the Result with Abraham's Theoretical Relation. — The value obtained theoretically by Abraham, as a second approximation for the wave-length of a thin rod in terms of its length and diameter, is

$$\lambda = 2\,l(1 + 5.6\,\epsilon^2),$$

where

$$\epsilon = \frac{1}{4\log_e \frac{2\,l}{d}},$$

in which l is the length of the whole oscillator, and d its diameter. The formula was derived by applying Maxwell's equations to a long, perfectly conductive ellipsoid of revolution, and taking the limit approached by λ when the square of the minor axis of the ellipsoid vanishes in comparison with the square of the major axis. Under these conditions the major axis becomes the length of the rod-oscillator and the minor axis its diameter.

To show the size of the $5.6\,\epsilon^2$ term of Abraham's formula, the following table (Table III.) has been computed for various values of l/d, covering the range of the experiments by Webb and Woodman and those by Conrat and by me.

TABLE III.

COMPUTATION OF THE $5.6\epsilon^2$ TERM OF ABRAHAM'S FORMULA.

l/d.	$5.6\epsilon^2$	Range.
4	.081	Webb and Woodman.
5	.065	
7	.050	
10	.039	
160	.011	
2000	.005	Conrat.
4000	.0043	
6000	.0039	
8000	.0037	Writer.
12000	.0035	
20000	.0031	
30000	.0029	

It is seen that in the range of my experiments, the $5.6\,\epsilon^2$ term raises the theoretical value of the wave-length to $2.006\,l$, and in Conrat's range to $2.01\,l$. This term is, therefore, entirely inadequate to account for the 5 per cent excess of the experimental values over the theoretical values of Abraham.

Also the presence of the spark-gap in the oscillator seems to be without influence, as the values of Conrat were obtained for rods without a gap.

In discussing the question, raised by Pollock and Close,[15] as to whether a result obtained for an infinitely thin ellipsoid can be applied to an infinitely thin rod of uniform section, Lord Rayleigh[16] says: "It appears therefore that the wave-length of the electrical vibration associated with a straight terminated rod of infinitesimal section is equal to twice the length of the rod, whether the shape be cylindrical so that the radius is constant, or ellipsoidal so that the radius varies in a finite ratio at different points of the length, and that this conclusion remains undisturbed, even though the shape be not one of revolution." Lord Rayleigh, however, raises the question whether a sufficient reduction of the diameter of the rod to comply with Abraham's approximation is experimentally possible without too greatly diminishing the conductivity, which is assumed perfect in the theoretical discussion.

In reply to this note by Lord Rayleigh, Macdonald[17] expresses the view that the rate of damping of the free vibration associated with the terminated straight wire is very large, and in fact not far removed from the order of magnitude of the known result for a spherical vibrator. This large damping, if it exists, and especially if it is due to a large radiation from the wire near the ends, would account for a distortion of the current distribution in the conductor so as to give a wave-length larger than twice the length of the conductor.

Since the question of the conductivity of the wire and the damping of the oscillations has a bearing on the question of its period, it is proposed to give the results of a measurement made on the damping of one of the oscillators used in the present experiments.

Damping. — The damping factor of a rectilinear oscillator 14 meters long, consisting of two oppositely-extending horizontal wires 7 meters long and 1 mm. in diameter, was determined by a method recently given by K. E. F. Schmidt.[18] The spark-gap was 3 mm. long. Schmidt's method consists in determining the average square current in a low resistance wavemeter circuit for various adjustments of the wavemeter in the neighborhood of resonance. To get the mean square current in the wavemeter circuit the dynamometer shown in Figs. 2 and 3 was employed. The deflections of this instrument have been shown to be proportional to the square of the current. The values obtained are recorded in Table IV, which gives the wave-length adjust-

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[15] Loc. cit.

[16] Loc. cit.

[17] Phil. Mag., 1904, **8**, 276.

[18] Phys. Zeits., 1908, **9**, 13.

ment of the wavemeter and the corresponding relative deflections of the dynamometer.

TABLE IV.

FOR DETERMINING DAMPING.

Adjustment of Wavemeter. λ in Meters.	λ/λ_m.	D/D_m. Deflection relative to Maximum.
30.3	.992	.99
29.2	.960	.58
27.6	.908	.160
26.0	.859	.054
32.2	1.062	.39
33.9	1.111	.150
31.5	1.032	.71
30.5	1.000	1.00

These results are plotted in the curve of Fig. 4, in which the abscissas are λ/λ_m, and the ordinates D/D_m.

Schmidt's method of getting the damping from this curve consists in determining the width between the two branches of the curve at ordinates .55, .70, and .85, and then making use of a decrement diagram which he has computed and plotted in his original paper, to which the reader is referred. This method applied to the present case gives the values in Table V.

TABLE V.

DECREMENT BY SCHMIDT'S METHOD.

Ordinate.	Width of Res. Curve, reduced to Proper Scale.	δ
.55	.88	.32
.70	.64	.33
.85	.40	.32

The last column of this table gives the logarithmic decrement per complete oscillation. The value .32, including the Joulean decrement as well as the radiation decrement, is 40 per cent higher than the logarithmic decrement due to radiation alone, as computed by Abraham's formula for the decrement, which is

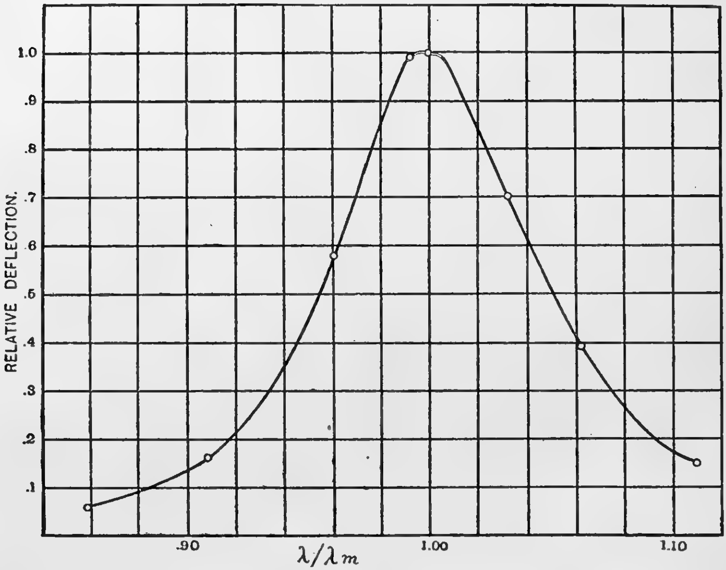

FIGURE 4. Resonance curve used in obtaining logarithmic decrement.

$$\delta = \frac{9.74}{4 \log_e \frac{2\,l}{d}}$$

$$= .23$$

The value of the decrement is, however, too small to produce a change in the measured value of the wave-length by more than a small fraction of one per cent.

Summary of Results. — Assuming that the wave-length produced by the parallel-wire oscillator is four times the length of one of the wires, the wave-length produced by the fundamental electrical vibration of a long, thin, rectilinear Hertz oscillator was found to be 2.094 times the total length of the oscillator, for oscillators of length between 8 and 30 meters.

This result is 4.5 per cent higher than Abraham's theoretical value computed by the formulas

$$\lambda = 2\,l(1 + 5.6\,\epsilon^2)$$
$$\epsilon = \frac{1}{4\log_e \frac{2\,l}{d}}.$$

The results obtained in the present experiments are in approximate agreement with two measurements given by Drude and with a series of measurements obtained by Conrat, both using oscillators of length between 2 and 6 meters.

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THE CONCEPTION OF THE DERIVATIVE OF A SCALAR POINT FUNCTION WITH RESPECT TO ANOTHER SIMILAR FUNCTION.

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In modern treatises on Mathematical Physics it is customary to define the derivative of a scalar function, taken at a given point in space in a given direction, in a manner which emphasizes the fact that this derivative is an invariant of a transformation of coördinates. According to this definition,[1] if through the point P a straight line be drawn in a fixed direction (s), if on this line a point P' be taken near P so that PP' has the direction s, and if u_P, $u_{P'}$ be used to represent the values at these points of the scalar point function u, then if the ratio

$$\frac{u_{P'} - u_P}{PP'} \tag{1}$$

approaches a limit as P' approaches P, this limit is called the derivative of u, at P, in the direction s. If u happens to be defined in terms of a system of orthogonal Cartesian coördinates, x, y, z, and has continuous derivatives with respect to these coördinates everywhere within a certain region, the limit just mentioned exists in this region and its value is

$$\frac{\partial u}{\partial x}\cdot\cos(x, s) + \frac{\partial u}{\partial y}\cdot\cos(y, s) + \frac{\partial u}{\partial z}\cdot\cos(z, s). \tag{2}$$

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[1] Hamilton, Elements of the Theory of Quaternions; Tait, Elementary Treatise on Quaternions; Gibbs, Vector Analysis; Maxwell, Treatise on Electricity and Magnetism; Webster, Dynamics of Particles and of Rigid, Elastic, and Fluid Bodies; Jeans, Mathematical Theory of Electricity and Magnetism; Lamé, Leçons sur les Coordonnées Curvilignes; Peirce, Theory of the Newtonian Potential Function; Generalized Space Differentiation of the Second Order; Czuber, Wienerberichte, **101**$_A$, 1417 (1892); Boussinesq, Cours d'Analyse Infinitésimale; H. Weber, Die Partiellen Differential-Gleichungen der Mathematischen Physik.

Of all the numerical values which the derivative of u can have at a given point, the greatest is to be found by making s normal to the level surface of u which passes through the point. This maximum value,

$$\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2\right]^{\frac{1}{2}} \tag{3}$$

is usually regarded as the value at the point of a vector point function called the gradient vector of u, the lines of which cut orthogonally the level surfaces of u, and the components of which parallel to the coördinate axes are

$$\frac{\partial u}{\partial x},\ \frac{\partial u}{\partial y},\ \frac{\partial u}{\partial z}. \tag{4}$$

This vector is, of course, lamellar.

The value of the tensor of the gradient vector is often called simply the "gradient" of u and is denoted by h_u. If at any point a straight line be drawn in the direction (n) normal to the level surface of u, in the sense in which u increases, and if a length h_u be laid off on this line, the projection,

$$h_u \cdot \cos(n,\ s), \tag{5}$$

of this length on any other direction (s) is numerically equal to the derivative of u in the direction s.

Most physical quantities — such as temperature, barometric pressure, density, inductivity — present themselves to the investigator as single valued point functions, which, except perhaps at one or more given surfaces of discontinuity, are differentiable in the sense just considered.

It is often desirable to differentiate a scalar function, u, at a point, in the direction in which another scalar function, v, increases fastest, and if (u, v) represents the angle between the gradient vectors of u and v at the point, the derivative is evidently equal to

$$h_u \cdot \cos(u,\ v). \tag{6}$$

It frequently happens that in a question of maxima and minima, one wishes to determine the greatest (or the smallest) value which a quantity U may have, subject to the condition that another quantity V shall have a given value (V_o). If these quantities can be represented by point functions, the problem geometrically considered requires one to find the parameter of a surface of the constant U family, which is tangent to the surface of the V family upon which V is everywhere equal to V_o; but at the point of tangency, the derivative of the function U in any direction in the tangent plane of the V surface is zero, that is, the normals to the U and V surfaces coincide, so that

$$\frac{\dfrac{\partial u}{\partial x}}{\dfrac{\partial v}{\partial x}} = \frac{\dfrac{\partial u}{\partial y}}{\dfrac{\partial v}{\partial y}} = \frac{\dfrac{\partial u}{\partial z}}{\dfrac{\partial v}{\partial z}}, \tag{7}$$

and these familiar equations usually furnish some general information about the problem independent of the value of V_o. As an extremely

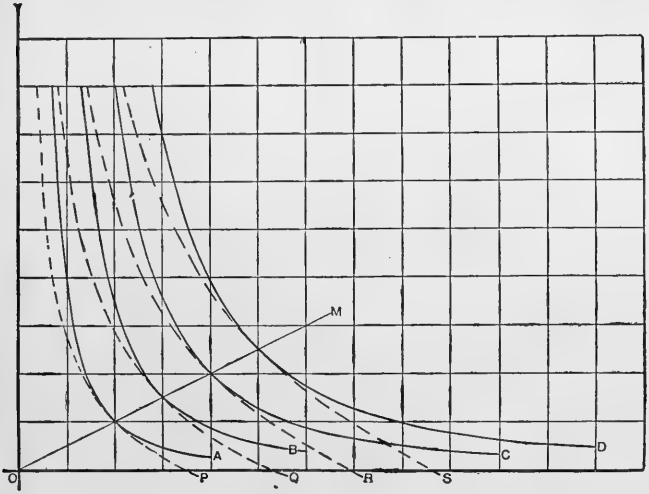

FIGURE 1.

simple example we may take the familiar problem concerning the relative dimensions of an open tank of square base ($x \times x$) and height y, which shall hold a given quantity ($V = x^2.y$) of water and have the smallest wet surface ($U = x^2 + 4xy$). Here we have the curve D of the V family, which has the given parameter, V_o, and are required to find that member of the P, Q, R, S family which touches D. The equation (7) becomes in this case $2y = x$, and it appears (Figure 1) that the curves of the two families which pass through any point of the line OM are at that point tangent to each other.

It is sometimes necessary to differentiate a point function, u, at a point P, in the direction of the line through the point, along which two other point functions, v, w, are constant; that is, along the line $v = v_P$, $w = w_P$. If

$$L = \begin{vmatrix} \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}, \quad M = \begin{vmatrix} \frac{\partial v}{\partial z} & \frac{\partial v}{\partial x} \\ \frac{\partial w}{\partial z} & \frac{\partial w}{\partial x} \end{vmatrix}, \quad N = \begin{vmatrix} \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} \end{vmatrix} \tag{8}$$

and if $R^2 = L^2 + M^2 + N^2$ — which is equal to $h_v^2 \cdot h_w^2$, if v and w are orthogonal — this direction is defined by the cosines L/R, M/R, N/R, and the derivative required is

$$\frac{1}{R}\left(L \cdot \frac{\partial u}{\partial x} + M \cdot \frac{\partial u}{\partial y} + N \cdot \frac{\partial u}{\partial z}\right). \tag{9}$$

If the maxima and minima of the function $u = f(x, y, z)$ are to be found under the condition that the functions v, w shall have given numerical values, the derivative of u taken in the direction in which v and w are constant must be made to vanish. Thus, if

$$u = x^2 + y^2 + z^2,$$

and if the conditions are

$$xyz = c^3 \text{ and } x + y = d,$$

equation (9) yields immediately the required relation

$$(xy + z^2)(y - x) = 0.$$

When $f'(u)$ is positive, the direction of the gradient vector of $f(u)$ coincides with that of the gradient vector of u itself: these directions are opposed when $f'(u)$ is negative. The tensors of both vectors are always positive. If

$$w = f(u), \quad h_w^2 = [f'(u)]^2 \cdot h_u^2, \text{ and } \cos(w, s) = \cos(u, s):$$

in particular, when

$$w = 1/u, \quad h_w = h_u/u^2 \text{ and } \cos(w, s) = -\cos(u, s),$$

so that

$$\frac{\partial}{\partial s}\left(\frac{1}{u}\right) = -\frac{\partial u}{\partial s} \cdot \frac{1}{u^2}$$

If u is the distance (r) to a point on a curve (s) from a fixed point outside the curve,

$$\frac{\partial r}{\partial s} = + \cos(s, r), \frac{\partial}{\partial s}\left(\frac{1}{r}\right) = - \frac{\cos(s, r)}{r^2}.$$

Any function of the complex variable $(ax + by + iz\sqrt{a^2 + b^2})$ has a gradient identically equal to zero, but every differentiable real point function has a gradient in general different from zero. The gradient of a function may be constant throughout a region of space: if the gradient of u is constant, the surfaces upon each of which u is constant form a parallel system. If the gradient of a function, u, is either constant or expressible in terms of u, any differentiable function of u has a gradient either constant or expressible in terms of u. If the gradient of u is expressible in terms of u alone $[h_u = f(u)]$, it is possible to form a function, $a\int\frac{du}{f(u)}$, of u the gradient of which shall be constant. If h_u is neither constant nor expressible in terms of u, no function of u exists the gradient of which is expressible in terms of u. The functions $u = \sin(x + y + z)$, $v = \sin(x + 2y - 3z)$, $w = \sin(5x - 4y - z)$ illustrate the fact that the gradient of each of three orthogonal point functions may be expressible in terms of the function itself.

If the gradient of each of two orthogonal point functions, u, v, were expressible as the product of a function of u and a function of v, so that $h_u = U_1 \cdot V_1$, and $h_v = U_2 \cdot V_2$, it would be possible to form two functions $\left[\int\frac{du}{U_1}, \int\frac{dv}{V_2}\right]$ of u alone and of v alone, respectively, the gradient of each of which would be expressible in terms of the other. If the gradient vectors of two functions have the same direction at every point of space, one of these functions is expressible in terms of the other. If the gradients of two real functions, u, v, are everywhere equal while the directions of their gradient vectors are different,

$$\frac{\partial(u+v)}{\partial x}\cdot\frac{\partial(u-v)}{\partial x} + \frac{\partial(u+v)}{\partial y}\cdot\frac{\partial(u-v)}{\partial y} + \frac{\partial(u+v)}{\partial z}\cdot\frac{\partial(u-v)}{\partial z} = 0, \quad (10)$$

and the functions $[u + v]$, $[u - v]$ are orthogonal, as are $F(u + v)$, $f(u - v)$, where F and f are any differentiable functions. If u and v are orthogonal functions, the functions $[F(u) + f(v)]$, $[F(u) - f(v)]$ have gradients numerically equal to each other at every point.

Two scalar point functions, the level surfaces of which are neither coincident nor orthogonal, may have gradients each of which is expressible in terms of the other: the gradient of $v = \frac{4}{3}x^3 - 4xy^2$ is equal at every point of the xy plane to the square of the gradient of $u = x^2 - y^2$. If u and v are orthogonal functions of x and y, the product of their gradients is equal to the Jacobian,

$$\frac{\partial u}{\partial x}\cdot\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\cdot\frac{\partial v}{\partial x}.$$

The differential equation

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = k^2,$$

which leads to systems of parallel surfaces, is of standard form. Its complete integral is

$$u = ax + by + z\sqrt{k^2 - a^2 - b^2} + d,$$

where a, b, d are arbitrary constants, and from this the general integral may be obtained in the usual manner.

If a direction s be determined at every point of a given region, T, by some law, the derivative of the function u becomes itself a scalar point function in T, and if this is differentiable, it may be differentiated at any point in any direction, say s. It is usually convenient to define s by means of three scalar point functions, l, m, n, the sum of the squares of which is identically equal to unity, and which represent the direction cosines of s. In this connection it is well to notice that if s has the direction at P of the tangent of a continuous curve which passes through the point, if P' be a point near P on the tangent and P'' a point near P on the curve, and if U is any differentiable scalar point function,

$$\frac{U_{P''} - U_P}{PP''}, \qquad \frac{U_{P'} - U_P}{PP'}$$

have the same limit, as P' and P'' approach P, that which has been defined as the derivative of U at P in the direction s. If, then, $\partial u/\partial s$ is differentiable

$$\begin{aligned}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial s}\right) &= \frac{\partial}{\partial x}\left(l\cdot\frac{\partial u}{\partial x} + m\cdot\frac{\partial u}{\partial y} + n\cdot\frac{\partial u}{\partial z}\right)\\ &= l\cdot\frac{\partial^2 u}{\partial x^2} + m\cdot\frac{\partial^2 u}{\partial x\cdot\partial y} + n\cdot\frac{\partial^2 u}{\partial x\cdot\partial z} + \frac{\partial u}{\partial x}\cdot\frac{\partial l}{\partial x} + \frac{\partial u}{\partial y}\cdot\frac{\partial m}{\partial x} + \frac{\partial u}{\partial z}\cdot\frac{\partial n}{\partial x},\end{aligned} \tag{11}$$

and

$$\frac{\partial^2 u}{\partial s^2} = l^2 \cdot \frac{\partial^2 u}{\partial x^2} + m^2 \cdot \frac{\partial^2 u}{\partial y^2} + n^2 \cdot \frac{\partial^2 u}{\partial z^2} + 2lm \cdot \frac{\partial^2 u}{\partial x \cdot \partial y} + 2mn \cdot \frac{\partial^2 u}{\partial y \cdot \partial z} + 2ln \cdot \frac{\partial^2 u}{\partial x \cdot \partial z}$$
$$+ \frac{\partial u}{\partial x}\left(l \cdot \frac{\partial l}{\partial x} + m \cdot \frac{\partial l}{\partial y} + n \cdot \frac{\partial l}{\partial z} \right) + \frac{\partial u}{\partial y}\left(l \cdot \frac{\partial m}{\partial x} + m \cdot \frac{\partial m}{\partial y} + n \cdot \frac{\partial m}{\partial z} \right)$$
$$+ \frac{\partial u}{\partial z}\left(l \cdot \frac{\partial n}{\partial x} + m \cdot \frac{\partial n}{\partial y} + n \cdot \frac{\partial n}{\partial z} \right). \tag{12}$$

If s' is a direction defined by the cosines l', m', n',

$$\frac{\partial^2 u}{\partial s' \cdot \partial s} = ll' \cdot \frac{\partial^2 u}{\partial x^2} + mm' \cdot \frac{\partial^2 u}{\partial y^2} + nn' \cdot \frac{\partial^2 u}{\partial z^2}$$
$$+ (lm' + l'm) \frac{\partial^2 u}{\partial x \cdot \partial y} + (mn' + m'n) \frac{\partial^2 u}{\partial y \cdot \partial z} + (nl' + n'l) \frac{\partial^2 u}{\partial z \cdot \partial x}$$
$$+ \frac{\partial u}{\partial x}\left(l' \cdot \frac{\partial l}{\partial x} + m' \cdot \frac{\partial l}{\partial y} + n' \cdot \frac{\partial l}{\partial z} \right) + \frac{\partial u}{\partial y}\left(l' \cdot \frac{\partial m}{\partial x} + m' \cdot \frac{\partial m}{\partial y} + n' \cdot \frac{\partial m}{\partial z} \right)$$
$$+ \frac{\partial u}{\partial z}\left(l' \cdot \frac{\partial n}{\partial x} + m' \cdot \frac{\partial n}{\partial y} + n' \cdot \frac{\partial n}{\partial z} \right), \tag{13}$$

and it is clear that the order of differentiation is usually not commutative. Derivatives of this kind are often found in differential equations of orders higher than the first which define functions in terms of simple curvilinear coördinates.

If for instance spherical coördinates are to be used, the second derivative of u taken in the direction in which θ increases fastest is

$$\frac{\partial^2 u}{\partial x^2} \cdot \cos^2\theta \cos^2\phi + \frac{\partial^2 u}{\partial y^2} \cdot \cos^2\theta \sin^2 y + \frac{\partial^2 u}{\partial z^2} \cdot \sin^2\theta + \frac{2\,\partial^2 u}{\partial x \cdot \partial y} \cdot \cos^2\theta \sin\phi \cos\phi$$
$$- \frac{2\,\partial^2 u}{\partial x \cdot \partial z} \cdot \sin\theta \cos\theta \cos\phi - \frac{2\,\partial^2 u}{\partial y \cdot \partial z} \cdot \sin\theta \cos\theta \sin\phi - \frac{\partial u}{r \cdot \partial x} \cdot \sin\theta \cos\phi$$
$$- \frac{\partial u}{r \cdot \partial y} \cdot \sin\theta \sin\phi - \frac{\partial u}{r \cdot \partial z} \cdot \cos\theta \tag{14}$$

and this, which contains derivatives of the first order, is in sharp contrast to the second derivative of u taken in the direction r, which is,

$$\frac{\partial^2 u}{\partial x^2} \cdot \sin^2\theta \cos^2\phi + \frac{\partial^2 u}{\partial y^2} \cdot \sin^2\theta \sin^2\phi + \frac{\partial^2 u}{\partial z^2} \cdot \cos^2\theta + \frac{2\,\partial^2 u}{\partial x \cdot \partial y} \cdot \sin^2\theta \sin\phi \cos\phi$$
$$+ \frac{2\,\partial^2 u}{\partial y \cdot \partial z} \cdot \sin\theta \cos\theta \sin\phi + \frac{2\,\partial^2 u}{\partial z \cdot \partial x} \cdot \sin\theta \cos\theta \cos\phi. \tag{15}$$

Sometimes s and s' are fixed directions so that l, m, n, l', m', n', are constants throughout T, and in this case the coefficients of $\partial u/\partial x$, $\partial u/\partial y$, $\partial u/\partial z$ in (12) and (13) vanish. The mutual potential energy W, of two magnetic elements, M, M', of moments, m, m', can be written in the form

$$m \cdot m' \frac{\partial^2}{\partial s \cdot \partial s'}\left(\frac{1}{r}\right), \tag{16}$$

where r is the distance MM' and s, s' are the directions of the axes of the elements. The force (due to the second magnet) which tends to move the first magnet in the direction of its own axis is then

$$- m \cdot m' \frac{\partial^3}{\partial s \cdot \partial s \cdot \partial s'}\left(\frac{1}{r}\right) \tag{17}$$

and these differentiations assume that the direction cosines of s and s' are constants.

In general, if s is the direction perpendicular to the level surface of u, and if h is the scalar point function which gives the value of $\partial u/\partial s$,

$$\frac{\partial^2 u}{\partial s^2} = \left(\frac{\partial h}{\partial x} \cdot \frac{\partial u}{\partial x} + \frac{\partial h}{\partial y} \cdot \frac{\partial u}{\partial y} + \frac{\partial h}{\partial z} \cdot \frac{\partial u}{\partial z}\right) \Big/ h. \tag{18}$$

In the case of oblique Cartesian coördinates in a plane, x increases fastest in a direction which is not perpendicular to the line along which it is constant. If the angle between the coördinate axes is ω,

$$\frac{\partial u}{\partial x} = h_u \cdot \cos\,(x, h_u), \quad \frac{\partial u}{\partial y} = h_u \cdot \cos\,(y, h_u), \quad \frac{\partial u}{\partial s} = h_u \cdot \cos\,(s, h_u),$$

$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x} \cdot \frac{\sin\,(y, s)}{\sin \omega} + \frac{\partial u}{\partial y} \cdot \frac{\sin\,(x, s)}{\sin \omega} \tag{19}$$

It is frequently necessary to differentiate one point function, U, with respect to another, u, and the process usually appears in the form of a kind of partial differentiation. If, for instance, U is to satisfy a differential equation in terms of a set of orthogonal curvilinear coördinates of which u is one, the derivatives of U with respect to u are to be taken on the assumption that the other coördinates remain constant. This large subject has been treated exhaustively in the many works on orthogonal coördinates which have been published since Lamé's classical treatise[2] appeared.

Given a function, u, it is, however, not generally possible to find a system of orthogonal functions of which u shall be one, and it is often convenient for a physicist to differentiate a physical function, U, with respect to another, u, without considering the existence of any other related functions. A physical point function has a value at every point in space which is not altered by changing the system of coördinates which fix the position of the point, and it is well to define the derivative of U with regard to u in a manner which shall emphasize the fact that the derivative is an invariant of a change of coördinates and which shall not assume that two functions (v, w) can be found orthogonal to each other and to u. When U and u are considered by themselves and not regarded as coördinated of necessity with other similar quantities, it is usually, if not always, the case that a "normal" derivative[3] is required.

The *normal* derivative, at any point, P, of the differentiable scalar point function U, with respect to the differentiable scalar point function u, may be defined as the limit, when PP' approaches zero, of the ratio

$$\frac{U_{P'} - U_P}{u_{P'} - u_P}, \tag{20}$$

where P' is a point so chosen on the normal at P of the surface of constant u which passes through P, that $u_{P'} - u_P$ shall be positive. If (U, u) denotes the angle between the directions in which U and u increase most rapidly, the normal derivatives of U with respect to u and of u with respect to U may be written

$$\frac{h_U}{h_u} \cdot \cos (U, u) \quad \text{and} \quad \frac{h_u}{h_U} \cdot \cos (U, u). \tag{21}$$

If $h_U = h_u$ these derivatives are equal. An example of this is the equality of $\partial n / \partial r$ and $\partial r / \partial n$ in a familiar application of Green's Theorem, where n and r represent the normal distance from a given surface and the distance from a given fixed point respectively. If U and u happen to be expressed in terms of a set (x, y, z) of orthogonal

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[2] Lamé, Leçons sur les Coordonnées Curvilignes et leur Diverses Applications; Salvert, Mémoire sur l'Emploi des Coordonnées Curvilignes; Darboux, Leçons sur les Systèmes Orthogonaux et les Coordonnées Curvilignes; Goursat, Cours d'Analyse Mathématique.

[3] Peirce, Short Table of Integrals, Theory of the Newtonian Potential Function; Generalized Space Differentiation of the Second Order.

Cartesian coördinates, the normal derivative of U with respect to u can be written

$$D_u U = \frac{\frac{\partial U}{\partial x}\cdot\frac{\partial u}{\partial x} + \frac{\partial U}{\partial y}\cdot\frac{\partial u}{\partial y} + \frac{\partial U}{\partial z}\cdot\frac{\partial u}{\partial z}}{{h_u}^2}, \tag{22}$$

and it is easy to see that this is equal to the ratio of the derivatives of U and u taken in the direction in which u increases most rapidly.

It is occasionally instructive to use the conception of normal differentiation in studying some of the general equations of Physics: thus in the uncharged dielectric about an electric distribution, the potential function, V, is connected with the inductivity of the medium, μ, by the familiar equation

$$\frac{\partial}{\partial x}\left(\mu\cdot\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\cdot\frac{\partial V}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu\cdot\frac{\partial V}{\partial z}\right) = 0, \tag{23}$$

in which μ is to be regarded as a point function discontinuous in general at each of a given set of surfaces at every point of which an equation of the form

$$\mu_1\cdot\frac{\partial V}{\partial n_1} + \mu_2\cdot\frac{\partial V}{\partial n_2} = 0 \tag{24}$$

is satisfied. Now (23) may be put into the form

$$\frac{\partial \log \mu}{\partial V} + \frac{\nabla^2 V}{h_V^2} = 0, \tag{25}$$

and according to Lamé's condition, the second term is a function of V only, if the level surfaces of V are possible level surfaces of a harmonic function.

It is easy to make from (25), by inspection, such simple deductions as those which follow in this paragraph. If V is harmonic, either the dielectric is made up of homogeneous portions separated from one another by equipotential surfaces, or the level surfaces of μ and of V are everywhere perpendicular to each other. If V, though not harmonic, satisfies Lamé's condition $[\nabla^2(V)/h_v^2 = F(V)]$ the level surfaces of the inductivity are equipotential; and if the level surfaces of V and μ are identical, V satisfies Lamé's condition. If when the plates of a condenser are kept at given potentials, the level surfaces of the inductivity of the dielectric are equipotential, the value of the potential function in the dielectric would be unchanged if μ were changed to $\Omega.\mu$, where Ω is any scalar point function orthogonal to V. If the continuous dielectric of a condenser in which the level surfaces of the inductivity, μ, are equipotential be changed so as to make the new potential function between the plates a function $[V' = f(V)]$ of the old, the new inductivity must satisfy an equation of the form $\mu' = \Omega.\mu / f'(V)$. If the V and the μ surfaces are neither coincident nor orthogonal, V cannot be harmonic, and if V is given and one value of the inductivity found, no other value of the inductivity with the same level surfaces as this can be found except by altering the old value at every point in a constant ratio. If V does not satisfy Lamé's condition, a new value of the inductivity found by multiplying the old value by any point function orthogonal to V, will yield the same value of V, but the level surfaces of the inductivity will be altered. If the V and the μ surfaces are not coincident, no change of the inductivity which leaves its surfaces unchanged can make these surfaces equipotential.

If a mass of fluid, the characteristic equation of which is of the form $p = f(\rho, T)$, is at rest under the action of a conservative field of force the components of which are X, Y, Z,

$$\frac{\partial p}{\partial x} = \rho \cdot X, \quad \frac{\partial p}{\partial y} = \rho \cdot Y, \quad \frac{\partial p}{\partial z} = \rho \cdot Z. \tag{26}$$

It follows immediately from these equations that p and V must be colevel, and the normal derivative of p with respect to V shows that equilibrium is impossible unless the distribution of temperature is such that the equipotential surfaces are also isothermal.

If the scalar point function, W, is expressed in terms of the three orthogonal point functions, u, v, w, the square of the gradient of W is well known to be equal to

$$h_u^2 \cdot \left(\frac{\partial W}{\partial u}\right)^2 + h_v \cdot \left(\frac{\partial W}{\partial v}\right)^2 + h_w^2 \cdot \left(\frac{\partial W}{\partial w}\right)^2.$$

If the vector point function Q is expressed in terms of u, v, w, the divergence of Q is equal to

$$h_u \cdot h_v \cdot h_w \left[\frac{\partial}{\partial u}\left(\frac{Q_u}{h_v \cdot h_w}\right) + \frac{\partial}{\partial v}\left(\frac{Q_v}{h_u \cdot h_w}\right) + \frac{\partial}{\partial w}\left(\frac{Q_w}{h_u \cdot h_v}\right)\right].$$

If the normal derivatives of u and v with respect to w be denoted by $D_w u$ and $D_w v$, it follows from the definition that

$$D_w(u+v) = D_w u + D_w v, \qquad D_w u^n = n \cdot u^{n-1} \cdot D_w u,$$

$$D_w(u \cdot v) = v \cdot D_w u + u \cdot D_w v, \quad D_w\left(\frac{u}{v}\right) = \frac{v \cdot D_w u - u \cdot D_w v}{v^2},$$

$$D_w f(u) = f'(u) \cdot D_w(u).$$

The normal derivative of u with respect to v is a scalar function which, if differentiable, has a normal derivative with respect to v, and since by definition

$$D_v x = \frac{D_x v}{h_v{}^2}, \tag{27}$$

$$D_v h_v = \frac{1}{h_v{}^2}\left\{ \frac{\partial h_v}{\partial x}\cdot\frac{\partial v}{\partial x} + \frac{\partial h_v}{\partial y}\cdot\frac{\partial v}{\partial y} + \frac{\partial h_v}{\partial z}\cdot\frac{\partial v}{\partial z} \right\}, \tag{28}$$

we may write

$$\begin{aligned}
D_v{}^2 u = {} & \frac{1}{h_v{}^4}\left\{ \frac{\partial^2 u}{\partial x^2}\cdot\left(\frac{\partial v}{\partial x}\right)^2 + \frac{\partial^2 u}{\partial y^2}\cdot\left(\frac{\partial v}{\partial y}\right)^2 + \frac{\partial^2 u}{\partial z^2}\cdot\left(\frac{\partial v}{\partial z}\right)^2 \right\} \\
& + \frac{2}{h_v{}^4}\left\{ \frac{\partial^2 u}{\partial x \cdot \partial y}\cdot\frac{\partial v}{\partial x}\cdot\frac{\partial v}{\partial y} + \frac{\partial^2 u}{\partial y \cdot \partial z}\cdot\frac{\partial v}{\partial y}\cdot\frac{\partial v}{\partial z} + \frac{\partial^2 u}{\partial z \cdot \partial x}\cdot\frac{\partial v}{\partial z}\cdot\frac{\partial v}{\partial x} \right\} \\
& + \frac{1}{h_v{}^3}\left\{ \frac{\partial u}{\partial x}\left(\frac{\partial h_v}{\partial x} - 2\,\frac{\partial v}{\partial x}\cdot\frac{\partial h_v}{\partial v}\right) + \frac{\partial u}{\partial y}\left(\frac{\partial h_v}{\partial y} - 2\frac{\partial v}{\partial y}\cdot\frac{\partial h_v}{\partial v}\right)\right. \\
& \left. + \frac{\partial u}{\partial z}\left(\frac{\partial h_v}{\partial z} - 2\frac{\partial v}{\partial z}\cdot\frac{\partial h_v}{\partial v}\right) \right\}
\end{aligned} \tag{29}$$

$$\begin{aligned}
D_w D_v u = {} & \frac{1}{h_v{}^2 h_w{}^2}\left\{ \frac{\partial^2 u}{\partial x^2}\cdot\frac{\partial w}{\partial x}\cdot\frac{\partial v}{\partial x} + \frac{\partial^2 u}{\partial y^2}\cdot\frac{\partial w}{\partial y}\cdot\frac{\partial v}{\partial y} + \frac{\partial^2 u}{\partial z^2}\cdot\frac{\partial w}{\partial z}\cdot\frac{\partial v}{\partial z} \right\} \\
& + \frac{1}{h_v{}^2 \cdot h_w{}^2}\left\{ \frac{\partial^2 u}{\partial x \cdot \partial y}\left(\frac{\partial w}{\partial y}\cdot\frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\cdot\frac{\partial v}{\partial y}\right) + \frac{\partial^2 u}{\partial y \cdot \partial z}\left(\frac{\partial w}{\partial z}\cdot\frac{\partial v}{\partial y} + \frac{\partial w}{\partial y}\cdot\frac{\partial v}{\partial z}\right)\right. \\
& \left. \qquad + \frac{\partial^2 u}{\partial x \cdot \partial z}\left(\frac{\partial w}{\partial z}\cdot\frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\cdot\frac{\partial v}{\partial z}\right)\right. \\
& + \frac{1}{h_v{}^2 \cdot h_w{}^2}\left\{ \frac{\partial u}{\partial x}\left(\frac{\partial^2 v}{\partial x^2}\cdot\frac{\partial w}{\partial x} + \frac{\partial^2 v}{\partial y \cdot \partial x}\cdot\frac{\partial w}{\partial y} + \frac{\partial v}{\partial x \cdot \partial z}\cdot\frac{\partial w}{\partial z}\right)\right. \\
& \left. \qquad - \frac{2}{h_v}\cdot\frac{\partial u}{\partial x}\cdot\frac{\partial v}{\partial x}\left(\frac{\partial h_v}{\partial x}\cdot\frac{\partial w}{\partial x} + \frac{\partial h_v}{\partial y}\cdot\frac{\partial w}{\partial y} + \frac{\partial h_v}{\partial z}\cdot\frac{\partial w}{\partial z}\right) \right\} \\
& \frac{1}{h_v{}^2 \cdot h_w{}^2}\left\{ \frac{\partial u}{\partial y}\left(\frac{\partial^2 v}{\partial y^2}\cdot\frac{\partial w}{\partial y} + \frac{\partial^2 v}{\partial z \cdot \partial y}\cdot\frac{\partial w}{\partial z} + \frac{\partial v}{\partial y \cdot \partial x}\cdot\frac{\partial w}{\partial y}\right)\right. \\
& \left. \qquad - \frac{2}{h_v}\cdot\frac{\partial u}{\partial y}\cdot\frac{\partial v}{\partial y}\left(\frac{\partial h_v}{\partial y}\cdot\frac{\partial w}{\partial y} + \frac{\partial h_v}{\partial z}\cdot\frac{\partial w}{\partial z} + \frac{\partial h_v}{\partial x}\cdot\frac{\partial w}{\partial x}\right) \right\}
\end{aligned}$$

$$\frac{1}{h_v^2 \cdot h_w^2}\left\{\frac{\partial u}{\partial z}\left(\frac{\partial^2 v}{\partial z^2}\cdot\frac{\partial w}{\partial z} + \frac{\partial^2 v}{\partial x \cdot \partial z}\cdot\frac{\partial w}{\partial x} + \frac{\partial v}{\partial z \cdot \partial y}\cdot\frac{\partial w}{\partial y}\right)\right.$$
$$\left. - \frac{2}{h_v}\cdot\frac{\partial u}{\partial z}\cdot\frac{\partial v}{\partial z}\left(\frac{\partial h_v}{\partial z}\cdot\frac{\partial w}{\partial z} + \frac{\partial h_v}{\partial x}\cdot\frac{\partial w}{\partial x} + \frac{\partial h_v}{\partial y}\cdot\frac{\partial w}{\partial y}\right)\right\} \qquad (30)$$

It is evident that $D_v D_w u$ is usually quite different from $D_w D_v u$.

In the transformation of a partial differential equation from one set of independent variables to another set which does not form an orthogonal system, derivatives occur which are not normal in the sense of the last paragraphs. If a mass of fluid is in motion under the action of given forces, it is usually convenient either to express the orthogonal coördinates of a particle which at the time t has the position (x, y, z) in terms of t and the coördinates x_0, y_0, z_0, which the same particle had at the origin of time, or to express x_0, y_0, z_0, as functions of x, y, z, t.

$$x_0 = f_1(x, y, z, t), \quad y_0 = f_2(x, y, z, t), \quad z_0 = f_3(x, y, z, t). \qquad (31)$$

In this case, it frequently happens that the level surfaces of f_1, f_2, f_3, are not orthogonal. According as we use the "historical" or the "statistical" method of studying the motion, we shall express the pressure and the density in terms of x_0, y_0, z_0, t, or in terms of x, y, z, t. Suppose the second method to have been chosen, and $\partial p / \partial x$ to have been found by the aid of Euler's Equations of Motion and the Equation of Continuity, and suppose that $\partial p / \partial x_0$ is needed. We shall then have

$$\frac{\partial p}{\partial x_0} = \frac{\partial p}{\partial x}\cdot\frac{\partial x}{\partial x_0} + \frac{\partial p}{\partial y}\cdot\frac{\partial y}{\partial x_0} + \frac{\partial p}{\partial z}\cdot\frac{\partial z}{\partial x_0}. \qquad (32)$$

If with the help of (31) we find the values of the determinants

$$L = \begin{vmatrix} \frac{\partial y_0}{\partial y} & \frac{\partial y_0}{\partial z} \\ \frac{\partial z_0}{\partial y} & \frac{\partial z_0}{\partial z} \end{vmatrix}, \quad M = \begin{vmatrix} \frac{\partial y_0}{\partial z} & \frac{\partial y_0}{\partial x} \\ \frac{\partial z_0}{\partial z} & \frac{\partial z_0}{\partial x} \end{vmatrix}, \quad N = \begin{vmatrix} \frac{\partial y_0}{\partial x} & \frac{\partial y_0}{\partial y} \\ \frac{\partial z_0}{\partial x} & \frac{\partial z_0}{\partial y} \end{vmatrix}, \qquad (33)$$

and put

$$Q = L\cdot\frac{\partial x_0}{\partial x} + M\cdot\frac{\partial x_0}{\partial y} + N\cdot\frac{\partial x_0}{\partial z},$$

$$R^2 = L^2 + M^2 + N^2,$$

we may write the results of differentiating all the equations of (31) with respect to x_0, y_0, z_0, in the form

$$\frac{\partial x}{\partial x_0} = \frac{L}{Q}, \frac{\partial y}{\partial x_0} = \frac{M}{Q}, \frac{\partial z}{\partial x_0} = \frac{N}{Q}, \tag{34}$$

so that

$$\frac{\partial p}{\partial x_0} = \frac{\frac{L}{R}\cdot\frac{\partial p}{\partial x} + \frac{M}{R}\cdot\frac{\partial p}{\partial y} + \frac{N}{R}\cdot\frac{\partial p}{\partial z}}{\frac{L}{R}\cdot\frac{\partial x_0}{\partial x} + \frac{M}{R}\cdot\frac{\partial x_0}{\partial y} + \frac{N}{R}\cdot\frac{\partial x_0}{\partial z}}, \tag{35}$$

and this is evidently equal to (9), the ratio of the directional derivatives of p and x_0 taken in the direction (s) in the (x, y, z) space in which both y_0 and z_0 are constant. If (s, p), (s, x) represent the angles between s and the directions of the gradient vectors of p and x respectively,

$$\frac{\partial p}{\partial x_0} = \frac{h_p \cdot \cos(s, p)}{h_{x_0} \cdot \cos(s, x_0)}. \tag{36}$$

It is convenient, therefore, to define the derivative of a scalar point function, u, with respect to another scalar point function, v, at any given point in any direction (s), as the ratio of the directional derivatives of u and v taken at the point in the direction s.

Derivatives of this kind which frequently appear in two dimensional problems in Thermodynamics and in Hydrokinematics, usually involve, as has been said, a transformation from one set of coördinates to another which is not orthogonal.

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THE EFFECT OF LEAKAGE AT THE EDGES UPON THE TEMPERATURES WITHIN A HOMOGENEOUS LAMINA THROUGH WHICH HEAT IS BEING CONDUCTED.

By B. Osgood Peirce.

CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL
LABORATORY, HARVARD UNIVERSITY.

THE EFFECT OF LEAKAGE AT THE EDGES UPON THE TEMPERATURES WITHIN A HOMOGENEOUS LAMINA THROUGH WHICH HEAT IS BEING CONDUCTED.

By B. Osgood Peirce.

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In many of the determinations of thermal conductivity which have been made during the last few years, the so called "wall method" has been employed. That is, one face of a plate or wall of the material to be experimented upon has been kept at one constant temperature for a long time while the opposite face has been maintained at another constant temperature, and the quantity of heat per square centimeter of either face, which under these circumstances has passed per second from one face to the other, has been measured in some convenient way.

In practice such a plate is of limited dimensions, and although it is easy to insure that the temperatures of the faces shall be nearly uniform, it is comparatively difficult to maintain a steady gradient from face to face at the edges so that the heat flow within the slab shall be the same as if the faces were infinite in extent. If, however, the faces of the specimen to be used are small enough, it is possible to prevent almost entirely the escape of heat at the edges by surrounding the periphery by an arrangement like a Dewar flask. This is impracticable when for any reason the plate has to be large, and in this case it is necessary to make the thickness of the wall so small compared with the dimensions of the faces that the lines of flow of heat from face to face in the central portion of the slab shall not be appreciably distorted by loss of heat through the edges of the wall.

Some time ago, in an attempt to obtain an accurate average value of the conductivity of a given stratum in a certain deep mine, I had occasion to apply the wall method to some blocks of stone which were not perfectly homogeneous, and in order to represent the material fairly it seemed best to use a slab eight centimeters thick for each determination. The slabs were square and the edges were covered with lagging to make the loss of heat through them as small as possible. Under these circumstances there was a very rough approximation to a uniform temperature gradient from the warm face to the cold one, at each edge, but it was difficult to measure the edge temperatures accurately and the areas of the faces were therefore made so large that the temperatures of points on the axis of the slab (that is, the line which joins the centres of the faces) would surely be the same within one one hundredth of a degree of the centigrade scale, in the final state, whether the whole of each edge was kept at the temperature of the warmer face or at the temperature of the colder face.

In anticipation of some further work of the same kind, I have been led to compute the final axial temperatures in a square slab ($a \times a \times c$) of thickness c, when one face is kept at temperature T_0 while the other face and all the edges are kept at the lower temperature T_1. The work is straightforward enough, but the computation when the slab is relatively broad is very laborious, and in view of the practical importance of the wall method in determinations of the conductivities of poor conductors of heat, it seems well to record some of the results.

The problem just stated is solved ($T_1 - WT_1 + WT_0$) when one has found[1] a solution (W) of the equation

$$\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 W}{\partial z^2} = 0 \tag{1}$$

which is equal to unity when $z = 0$, and to zero when $z = c$ for all positive values of x and y not greater than a; and which vanishes when $x = 0$, or $y = 0$, or $x = a$, or $y = a$, for all positive values of z not greater than c.

A convenient normal solution of (1) is

$$A\left(e^{\frac{k\pi z}{a}} - e^{\frac{k\pi(2c-z)}{a}}\right) \sin\frac{m\pi x}{a} \cdot \sin\frac{n\pi y}{a}, \tag{2}$$

where $k^2 = m^2 + n^2$, and it is evident that

$$W(x, y, z) =$$

$$\sum_{m=1}^{m=\infty}\sum_{n=1}^{n=\infty}\left(\frac{16}{\pi^2 mn \sinh\frac{\pi kc}{a}} \cdot \sinh\frac{\pi k(c-z)}{a} \cdot \sin\frac{m\pi x}{a} \cdot \sin\frac{n\pi y}{a}\right) \tag{3}$$

where m and n are odd integers.

---

[1] Byerly, Fourier's series, etc., p. 127.

The function

$$V = 1 - W(x, y, c - z), \tag{4}$$

which satisfies (1), is equal to unity when $z = 0$, and also for all positive values of z not greater than c, when $x = 0$, or $y = 0$, or $x = a$, or $y = a$. It vanishes when $z = c$, and the function

$$U = T_1 - W(T' - T_0) - V(T_1 - T') \tag{5}$$

or

$$T' - W(x, y, z) \cdot (T' - T_0) + W(x, y, c - z) \cdot (T_1 - T') \tag{6}$$

gives the temperatures in the slab if one face is kept at the temperature T_0, the other face at T_1, and the edges at T'. In an infinite slab of thickness c, the faces of which are kept at T_0 and T_1, the temperatures are given by the expression

TABLE I.

a	W
$\frac{1}{4}$ c	0.014
$\frac{1}{2}$ c	1.176
$\frac{2}{3}$ c	5.720
$\frac{4}{5}$ c	9.833
c	16.666
$\frac{3}{2}$ c	31.570
2 c	40.708
3 c	47.556
5 c	49.905

$$U_\infty = (T_1 - T_0)\frac{z}{c} + T_0 \tag{7}$$

so that the difference between the values of the temperature at any point in the slab in the ideal case and the real case is

$$(T_1 - T_0)\left[\frac{z}{c} - W(x, y, c - z)\right] + (T' - T_0)\left[W(x, y, z) + W(x, y, c - z) - 1\right]. \tag{8}$$

The last factor of this expression has its maximum value at the middle point of the axis where $z = \frac{1}{2}c$.

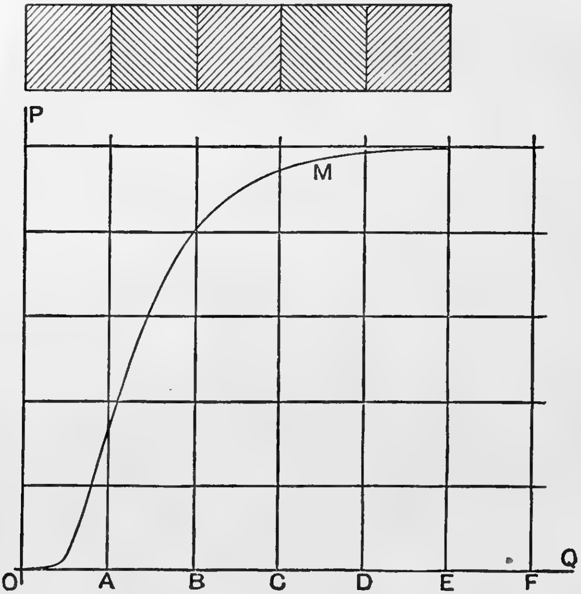

FIGURE 1. The ordinates of the curve show the temperatures, for different values of a, of a point Q in the centre of the axis (OS) of a square slab ($a \times a \times c$) of given thickness c, when one face ($a \times a$) is kept at the temperature 100° while the other face and the edges are kept at 0°. The horizontal unit is c, and it appears that when $a = 5\,c$, the temperature (49.9° +) of Q differs only slightly from the temperature (50°) which it would have if a were infinite. The shaded area above indicates the section of the slab for different values of a.

The value of W for the centre of the axis of the slab is given for several different values of a in Table I. When the ratio of a to c is large, the double series which defines W converges very slowly. Thus to obtain the last number in the table more than one hundred and fifty terms of the series were needed.

Figure 1 represents the numbers of Table I. graphically.

It is interesting to compare these results with similar ones for circular disks which Professor R. W. Willson and I obtained[2] several years ago.

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[2] These Proceedings, 1898, **34**, 1.

TABLE II.

Final Axial Temperatures in a Homogeneous Disk of Diameter d and Thickness c, when one Face ($z = 0$) is kept at 100° C., the other Face ($z = c$) at 0° C., and the Edge at the Uniform Temperature $\bar{\theta}$.

d/c	$z\ c$	$\bar{\theta}=0°$	$\bar{\theta}=100°$	$\bar{\theta}=50°$
$\frac{1}{2}$	$\frac{1}{4}$	14.05	99.88	56.95
$\frac{1}{2}$	$\frac{1}{2}$	1.30	98.70	50.00
$\frac{1}{2}$	$\frac{3}{4}$	0.12	85.95	43.03
1	$\frac{1}{4}$	42.32	96.07	69.20
1	$\frac{1}{2}$	13.93	86.07	50.00
1	$\frac{3}{4}$	3.95	57.68	30.80
$\frac{3}{2}$	$\frac{1}{4}$	58.15	88.83	73.49
$\frac{3}{2}$	$\frac{1}{2}$	28.54	71.46	50.00
$\frac{3}{2}$	$\frac{3}{4}$	11.17	41.85	26.51
2	$\frac{1}{4}$	66.41	82.86	74.63
2	$\frac{1}{2}$	38.39	61.61	50.00
2	$\frac{3}{4}$	17.14	33.59	25.36
3	$\frac{1}{4}$	72.84	77.12	74.98
3	$\frac{1}{2}$	46.98	53.02	50.00
3	$\frac{3}{4}$	22.88	27.16	25.02
4	$\frac{1}{4}$	74.48	75.51	74.99
4	$\frac{1}{2}$	49.27	50.73	50.00
4	$\frac{3}{4}$	24.49	25.52	25.01
6	$\frac{1}{4}$	74.97	75.03	75.00
6	$\frac{1}{2}$	49.96	50.04	50.00
6	$\frac{3}{4}$	24.97	25.03	25.00
10	$\frac{1}{4}$	75.00	75.00	75.00
10	$\frac{1}{2}$	50.00	50.00	50.00
10	$\frac{3}{4}$	25.00	25.00	25.00

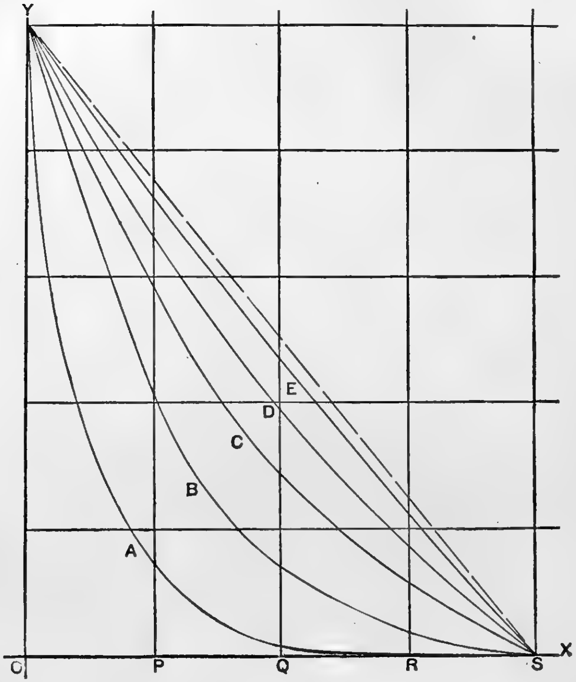

Figure 2. The curves show the final temperatures on the axis (OS) of a circular disk of given thickness (c) and of diameter d, when one face is kept at the temperature 100° and the other face and the rim at 0°. In A, B, C, D, and E, the diameter has the values $\frac{1}{2}c$, c, $\frac{3}{2}c$, $2c$, $3c$, respectively.

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ON EVAPORATION FROM THE SURFACE OF A SOLID SPHERE.

By Harry W. Morse.

CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

ON EVAPORATION FROM THE SURFACE OF A SOLID SPHERE.

PRELIMINARY NOTE.

By Harry W. Morse.

Presented by John Trowbridge, February 9, 1910. Received January 3, 1910.

The micro-balance of Salvioni and Nernst permits of following small changes in weight with considerable accuracy, provided the body under investigation has a mass not greater than a few milligrams. This balance consists merely of a fibre of quartz or glass, firmly held in a nearly horizontal position by being secured at one end, and provided at the other end with some means of attaching the object to be weighed. The weight is then, within quite wide limits of deflection, proportional to the deflection, and the balance is easily calibrated by means of small riders of known weight. Deflections are followed by means of a cathetometer or a microscope with micrometer eyepiece. Differences of 0.01 millimeter or even less are easily determined, and if the fibre be so chosen that a weight of 1 milligram gives a deflection of about a centimeter, there is no difficulty in detecting and measuring changes of weight of 0.001 milligram or less.

With such a balance the change of weight of small spheres of iodine has been followed at approximately constant temperature. Evaporation was allowed to go on in a large box with glass sides, and the two side doors of the case were left open before each series of readings to allow free circulation of air. It may therefore be assumed that the partial vapor pressure of iodine in the atmosphere about the evaporating spheres was constant. The temperature was constant within about 0.3° during each run.

After many attempts to obtain definite geometrical form by casting, fairly accurate spheres were made by pouring molten iodine into water. There is no difficulty in obtaining in this way approximately spherical pieces with radii varying from 1 millimeter down to 0.2 millimeter.

It was thought possible that there might be a change in the character of the surface as evaporation proceeded. The spheres were hard on the surface, and quite smooth as they came from the water, but they undoubtedly consist of a mass of very small irregular crystals and any roughening that might appear during the course of the experiment would lead to a considerable increase in surface. That such changes do not occur in disturbing amount is shown by the fact that the determinations made with small spheres fresh from formation fall accurately on the curve of measurements on spheres which have been evaporating for some hours. Microscopic examination corroborates this and shows also that the spherical shape is maintained practically unchanged until the sphere finally disappears completely.

In these experiments the spheres were supported on a nearly flat scale-pan of thin glass. This may introduce a variation in the surface exposed to the air, due to difference in the surface of contact between sphere and glass, and especially to be expected if the particles are not closely spherical. This factor is also shown to be negligible by the closeness with which the spherical form is kept during evaporation and also by the fact that turning the particle over has no measurable effect on the rate of evaporation.

Measurements on three spheres of different radii are given below.

These observations are plotted in the curve of Figure 1.

There is plenty of evidence that in any system made up of smaller and larger particles of the same substance, whether solid or liquid, the smaller particles are relatively unstable. So far, however, all of our knowledge about solids is of a purely qualitative nature, and no definite relation has ever been obtained based on vapor pressure or surface tension, and expressing quantitatively the change of vapor pressure or surface tension with change of radius. It has been many times noticed that, in a sealed tube containing iodine crystals of various sizes, the larger crystals grow at the expense of the smaller ones, which gradually disappear. In a few days this can be clearly proved, and the same effect has been noticed for water drops and for camphor and other rather volatile substances.

In the case of liquids it is possible to set up a definite relation between vapor pressure and curvature of drop. This has been done for water and a few other liquids, and the theory has been tested with some accuracy by experiments on the formation of fog by the expansion of saturated water vapor. For water the difference in vapor pressure between a drop of radius 0.001 millimeter and a flat surface is of the order of 0.001 mm. of mercury, so that the effect becomes almost insensible for drops of any size.

It was therefore expected that any influence of the size of the particle of iodine on the rate of evaporation would only appear for very small spheres indeed and that for all particles of sensible dimensions the rate would be proportional to the surface, so that

Sphere 1.		Sphere 2.		Sphere 3.	
Time.	Weight.	Time.	Weight.	Time.	Weight.
min.	mgms.	min.	mgms.	min.	mgms.
0	1.880				
20	1.770				
63	1.600				
103	1.420				
131	1.310				
140	1.260				
150	1.210				
169	1.140				
178	1.100				
189	1.050	187	1.066		
198	1.000	214	0.955		
295	0.638	228	0.907		
308	0.590	247	0.845		
319	0.557	263	0.759		
335	0.512	283	0.684	287	0.668
358	0.482	300	0.617	297	0.635
381	0.376	318	0.558	307	0.603
438	0.233	328	0.522	319	0.558
456	0.192	338	0.491	330	0.525
468	0.157	355	0.438	340	0.503
484	0.135	375	0.373		
498	0.104	390	0.337		
		456	0.160		
		466	0.147		
		476	0.126		
		486	0.105		
		496	0.087		
		506	0.070		
		521	0.048		
		531	0.036		
		536	0.028		
		542	0.022		
		548	0.017		
		554	0.011		
		560	0.006		
		576	0.000		

$$-\frac{dm}{dt} = ks;$$

or since the change in mass is being followed

$$-\frac{dm}{dt} = k_1 m^{\frac{2}{3}}.$$

The measurements show that this relation does not hold, even for spheres of radius 0.5 millimeter or more. The observed values do, however, agree accurately with the assumption that the rate of evaporation is proportional to the surface and at the same time inversely as the radius, so that

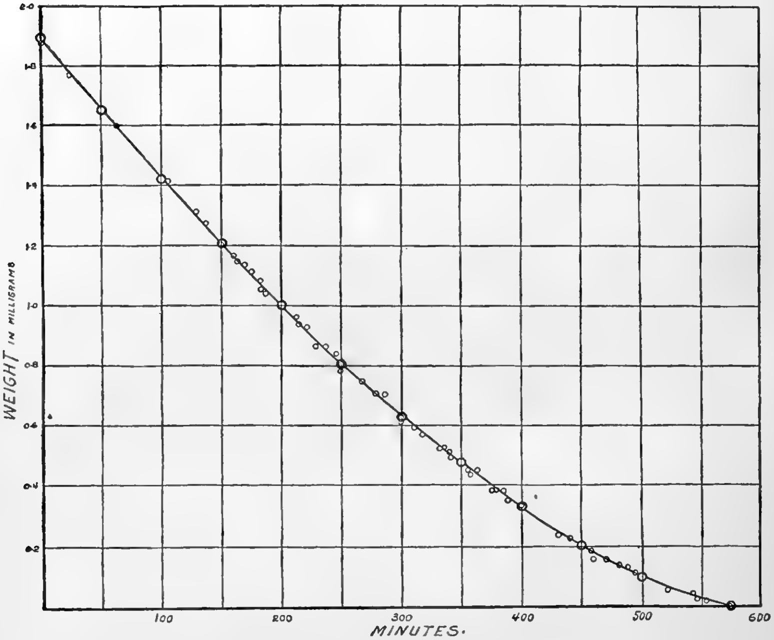

FIGURE 1. Evaporation from small spheres of Iodine. Small circles, observed values. Large circles, calculated values.

$$-\frac{dm}{dt} = k\frac{s}{r} \quad \text{or} \quad -\frac{dm}{dt} = k_2 m^{\frac{1}{3}}.$$

In the figure the large circles have been placed according to the formula

$$\frac{m_1^{\frac{2}{3}} - m_2^{\frac{2}{3}}}{t_2 - t_1} = K,$$

and the curve has been drawn through the points thus determined. The constant was calculated from the mean of all the observations and shows a probable error of a little less than 0.5 per cent. The results of the observations are given as smaller circles. In putting in the results for the smaller spheres or for those in which a full run down to zero of weight was not carried out, the original value of the mass of the sphere was placed on the curve and the times of the other observations on the same sphere were taken from this point. It is very probable that this method of choosing the highest weight has somewhat decreased the accuracy of the calculated constant, for it has been invariably observed that a measurable time elapses before a sphere falls into its regular rate of evaporation. It begins slowly, sometimes at not more than half its full rate, and several minutes elapse before it reaches its maximum value. It is probable that better agreement would have been obtained if a point farther along in the observations had been chosen and calculations made in both directions from this.

It seems clear that for spheres of iodine of mass ranging from 2 milligrams to very small values, the rate of evaporation is quite accurately proportional to the *radius.*

Before taking up any theory of this surprising result it will be best to have data on evaporation from masses having other geometrical shapes, and especially for a flat surface. It is expected that data on these points will be presented to the Academy in the near future.

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SOME MINUTE PHENOMENA OF ELECTROLYSIS.

By Harry W. Morse.

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SOME MINUTE PHENOMENA OF ELECTROLYSIS.

By Harry W. Morse.

Presented by John Trowbridge, December 8, 1909. Received January 6, 1910.

As the process of electrolysis is usually carried out there is very little opportunity to get any insight into its more minute mechanism. We are accustomed to think of each metal by having its own solution pressure, and by this we mean that it tends to go into solution under an impetus which varies with its position in the electro-motive force series. It is possible to calculate an osmotic pressure which would be just sufficient to balance this solution pressure and which would, if applied, cause equilibrium at the electrode. Under ordinary conditions electrochemical reactions are quite perfectly coupled. Equivalent amounts are dissolved at the anode and precipitated at the kathode, and it is not infrequent to state Faraday's Law in terms of the amounts thus dissolved and precipitated. But cases are well known where much more care must be taken in the statement of this law, as for example, where the air enters into reaction with one or both of the electrodes, or where the electrolyte itself attacks them. Very frequently a reaction of the form

$$M^{++} + M_{\text{metal}} \leftrightarrows 2\,M^{+}$$

causes a loss or gain not proportional to the amount of current which has passed through the electrolytic cell.

In the case of silver electrodes in a solution of silver nitrate it is usual to sum up the process as follows: —

During any unit of time after the circuit is closed

(1) An equivalent amount of silver dissolves at the anode.

(2) Silver migrates (as silver ion) toward the kathode and nitrate ion migrates toward the anode, each carrying its share of the current in proportion to its migration velocity.

(3) An equivalent amount of silver separates as metal at the kathode.

In the case of silver electrodes in pure water we might expect during each unit of time:

(1) At the anode, the formation of oxygen, or an oxide of silver, or the solution of silver, the sum total making one equivalent.

(2) The transfer of hydrogen ion (and later of silver ion if this is formed) toward the kathode, and of either or both of the ions O^{++} and OH^{-} toward the anode.

(3) At the kathode, evolution of hydrogen, and later precipitation of metallic silver, the two together making up one equivalent.

A case has recently come to my attention in which some of the more minute phenomena which accompany electrolysis are evident and in which lack of equivalence at the electrodes is especially evident. So far only qualitative observations have been made, but the data secured seem worthy of consideration.

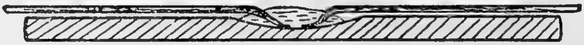

FIGURE 1. Electrolysis on microscopic slide between silver electrodes.

If pure water be electrolysed between small silver electrodes at voltages ranging from 1.40 to about 3.8 volts, and the space between and about the electrodes be observed under the microscope with powers of 50 or so, the following series of minute phenomena are visible: —

(1) A very short time after the circuit is closed a cloud of brownish particles, very small and in violent Brownian movement, is formed in the neighborhood of the anode. If silver foil is used as anode it can be seen to dissolve rapidly and a dark film of silver oxide remains. The particles first make their appearance at a slight distance from the anode, and appear to be due to the formation of a silver compound produced from the silver which has dissolved and one of the constituents of the water.

(2) This cloud consists of approximately spherical particles of diameter 0.3 to 1.0 mikron. It is readily soluble in very dilute acetic acid and slightly soluble in water, forming an alkaline solution. The particles appear to be silver oxide.

(3) If a cell of form similar to that shown in Figure 1 is used for the electrolysis, the particles move along the floor of the cell toward the kathode. During their migration toward the kathode they follow the current lines, and Figure 2 shows drawings made about half a minute apart, indicating the general appearance under a low magnifying power. The masses which move in this way are not the single

particles, which would not be visible at this magnification, but are clumps each containing a great many individual grains.

(4) While the above is occurring in the neighborhood of the anode a thin cloud of totally different appearance *may* appear about the kathode. The particles of this cloud are metallic in appearance, and they later disappear suddenly and completely when the growth of metallic silver begins at the front of the kathode. The kathode cloud seems to be effected by external conditions in greater degree than that from the anode. It is a function of the separation of the electrodes and the character of the kathode surface.

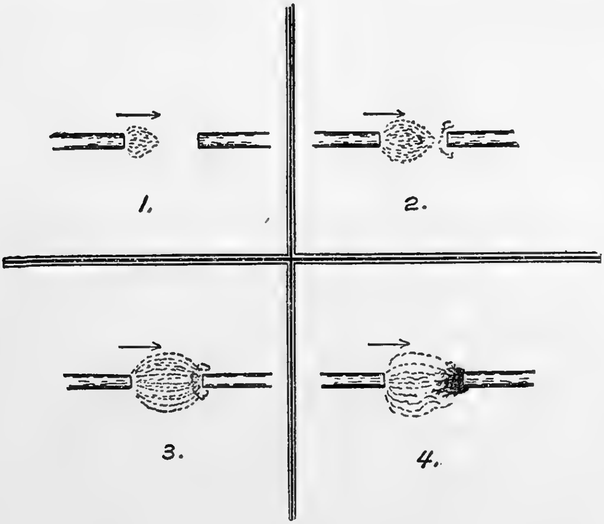

FIGURE 2. Minute phenomena of electrolysis between silver electrodes.

(5) The above described effects appear in the purest obtainable water and they are most evident in the best conductivity water, which has been recently prepared in quartz vessels and kept carefully from contact with air.

Electrolytes in very small concentration prevent the effect completely and cause the appearance of the usual gas bubbles at the anode and kathode. The following brief table shows how a few electrolytes behave in this respect.

Sodium Hydroxide	0.015	*N*	Cloud.
	.020	*N*	very slight cloud and bubbles at anode.
	above .020	*N*	Only bubbles at anode.
Sodium Chloride	0.0005	*N*	brown cloud, soluble in drop of acetic acid.
	.001	*N*	$\left.\begin{array}{l}\text{brown}\\ \text{white}\end{array}\right\}$ clouds $\begin{array}{l}\text{brown soluble}\\ \text{white insoluble}\end{array}$
	above .001	*N*	white cloud only.

(6) While the above effects are making their appearance in the electrolyte at slight distances from the electrodes nothing whatever happens at the kathode itself. The space between the electrodes may be active for several minutes without the appearance of either a bubble of gas or a crystal of silver. If very thin silver foil is used for electrodes solvent action on the anode is very evident and it is rapidly dissolved. A thin silver foil kathode shows signs of dissolving at the edges during the first minute or so of the passage of the current, but the action ceases immediately.

(7) There seems to be a limiting voltage below which these phenomena do not make their appearance. This is very close to 1.41 volts for electrodes 1 mm. apart. The upper limit of voltage, above which gas appears at the electrodes, is about 3.8 volts.

(8) Even in purest distilled water the phenomena are much more complicated than those so far described. The anode and kathode clouds are quite different in their behavior. That from the kathode appears to be composed of particles shot off at random, and these particles do not take any definite path after leaving the neighborhood of their parent electrode. The anode cloud, on the contrary, sticks closely together, and if the electrodes are at the mouth of a deep test-tube filled with water the anode cloud travels to the very bottom of the tube in such close coherence that it looks like a thin brown thread.

(9) The effect of a magnetic field on the behavior of these particles has been tried without definite result. They are relatively so large, and they move so slowly that an effect is hardly to be anticipated.

Attempt has been made to follow the changes in weight at each electrode during the electrolysis. The micro-balance was adapted for this purpose as shown in Figure 3. It is of course quite impossible to use any arrangement in which a fibre passes through the liquid surface. The effect of surface tension is far too great. But by placing both fibres and conducting wires under the surface of the electrolyte the difficulty is easily overcome. The balance loses but a small percentage of its sensitiveness when used with a heavy metal like silver or copper.

The fibres used were of quartz and about 8 cm. long. The conducting wires were of platinum about 0.04 mm. in diameter, and these were welded to small pieces of silver wire and held fast in hooks at the end of the fibres, so that the silver electrodes were presented to each other at a distance of about 1.5 mm. The sensitiveness was such that a 0.1 mg. rider at the end of either fibre caused a deflection of more than a centimeter. One of the (large) divisions of the micrometer eyepiece of the observing microscope corresponds to a change in weight of about 0.0001 mg., and a fraction of a division is easily read.

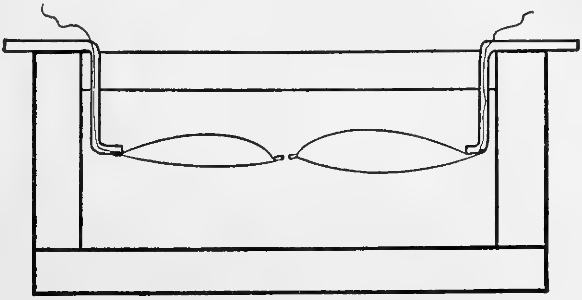

FIGURE 3. Microcoulometer.

With this instrument the following qualitative changes were noticed.

(1) Immediately on closing the circuit a very slight *decrease* in the weight of each electrode. This change was observed in four of six experiments and must therefore be classed as doubtful until further proof is obtained of its correctness.

(2) Thereafter for several minutes an *increase* in the weight of each electrode, the anode gaining much faster than the kathode. This effect is quite certain and considerable. It is accompanied by a change in color at the anode, which turns dark, and probably represents the formation of silver oxide or peroxide. The increase in weight at the kathode is seen to be due to the deposition of silver.

(3) From then on decrease in weight at the anode, and increase at the kathode, finally approaching proportionality.

The most important point which has been brought out in this preliminary exploration seems to be that of the complete lack of equivalence at the two electrodes. As observed under a high power, the entire anode may be eaten away, and the electrolyte space filled with masses of silver oxide, in some cases without a visible change at the kathode. Not even a bubble of gas makes its appearance. If platinum is used as kathode in place of silver, not the smallest amount of current can be sent through the cell without the appearance of streams of minute bubbles.

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Cambridge, Mass.,
December, 1909.

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Vol. XLV. No. 16. — May, 1910.

CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

AIR RESISTANCE TO FALLING INCH SPHERES.

By Edwin H. Hall.

CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

AIR RESISTANCE TO FALLING INCH SPHERES.

By Edwin H. Hall.

Presented January 12, 1910; Received January 12, 1910.

In 1903[1] I published an account of experiments which I had made with falling bronze spheres, one inch in diameter, in the tower of the Jefferson Physical Laboratory. The especial object of these experiments was to look for a southerly deviation, from the plumb line vertical, of the course of the falling balls, several observers, from the time of Hooke, 1680, to Rundell, 1848, having reported finding such a deviation, though Gauss and Laplace, both of whom discussed the matter theoretically about 1803, could find no cause for the phenomenon.

The general mean of the deviations observed by myself in the north and south plane in the experiments referred to, experiments much more careful and extensive than those which any one else had made in this matter, was a southerly movement of about 0.005 cm. in a fall of about 23 m. The probable error was about 0.004 cm., and I should have regarded the case as practically closed in favor of the negative if my predecessors had not, almost without exception, reported a considerable southerly excursion. On the whole I was disposed to try the question further, and accordingly applied in 1904 for permission to make experiments for this purpose in the great monument at Washington, D. C., where a sheer fall of about 165 m. is possible. The monument is in the care of the War Department, and at first the authorities applied to acted favorably upon my petition. A few months later, and before I had made any overt preparations for the work proposed, some change of management or of mind occurred in the Department, and the permission previously granted me was courteously but firmly withdrawn, "for the reason that the monument was designed as a memorial to General Washington." I have long since come to

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[1] Physical Review, 1903, **17**, 179 and 245; These Proceedings, 1904, **39**, 339.

the conclusion that this action was a fortunate one for me, as the investigation would certainly have been tedious and expensive and would probably have been inconclusive.

But the easterly deviation also was, incidentally, measured in my experiments at the Jefferson Laboratory, and the general mean value found for it was 0.149 cm., whereas the value given by the theoretical formula,

$$y = \tfrac{1}{3}\, gu \cos \lambda \times t^3,$$

where u is the angular velocity of the earth's rotation, λ is the latitude, and t is the time of fall in seconds, is 0.177 [2] cm. for the case in hand. The probable error of the observed general mean is perhaps greater than that for the southerly deviation, but is not great enough to account for the difference between the observed and the theoretical easterly value. I did not give in any of my previous papers on this subject the formula of Gauss or that of Laplace for the easterly deviation of a body falling in air, though I had given considerable attention to their treatment of the effect of air resistance, but closed my discussion of the matter thus: "The mean easterly deviation actually found in these experiments, 0.149 cm., differs 0.03 cm. from this theoretical value, — a quantity too large to be accounted for by the resistance of the air. I attach but little significance to this discrepancy, as the conditions for determining the easterly deviation in my work were plainly not so good as those for determining the southerly deviation."

Thus the matter stood till last April, when I received from Professor Hagen of the Vaticana Specola Astronomica the suggestion that I should make some experiments to find out how much the resistance of the air really amounted to, in order to see whether it might not after all go some distance toward explaining the discrepancy between the observed and the calculated easterly deviation. Father Hagen puts the statement of Gauss concerning the effect of air resistance so clearly, that I shall copy his words, changing, however, the nomenclature slightly. He writes:

"Gauss puts the height of the fall, determined by *linear* measure, $= f$, and $\frac{1}{2} gt^2 = f + \delta$, determined from the observed *time* of the fall. The difference δ is owing to the resistance of the air. Then

$$\text{Deviation } y = \tfrac{2}{3} \cos \lambda u t \left(f - \tfrac{1}{2}\delta\right).\text{"}$$

It was easy to carry out the suggestion thus given, and accordingly in October I reëstablished the releasing part of my apparatus at the top

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[2] I have given this previously as 0.179, but 0.177 is more nearly correct.

of the Laboratory tower and had a new cloth tube suspended for the balls to drop through. This tube, like the old one, which had wasted away, was about 35 cm. in diameter, and the balls fell along its axis.

At the bottom of the tower the receiving apparatus was now a horizontal plate of brass, fastened at one end but free at the other, so as to be capable of up and down motion. Near the free end of this plate a square hole, about 5 cm. on each side, was cut. Over this hole was placed in some cases a sheet of lead somewhat narrower than the hole but long enough to be clamped fast to the brass plate at each end. Later a thin sheet of wood was placed over the hole before each fall. In either case the ball, after falling from the top of the tower, would strike the cover of the hole and break through it, the first shock of its impact pulling the brass plate down far enough to break the contact which made part of an electrical circuit including a chronograph. At the top of the tower the release of the ball broke the same electrical circuit, which was, however, closed a fraction of a second later. It is hardly necessary to give further details of the apparatus except this, that the chronograph, which was driven by an electric motor at the rate of about 3 cm. per second, was not under the best of control, and it was accordingly necessary to make a greater number of trials than would otherwise have been required in order to determine the time of fall with sufficient accuracy. It should be added that the rate of the clock giving the second signals at the chronograph was not very accurately known, as it varied somewhat from day to day, probably because of changes of temperature. Its error may have been as much as half a minute per day, but was probably less than this. An error of this magnitude is not serious for our present purpose, and the clock was in my calculations assumed to be correct.

On the 16th of October 17 balls were dropped with such success as to give usable records. The mean time of fall was 2.176 seconds, with a probable error about 0.002 second.

On the 25th of October I made another series of trials, dispensing with the protecting cloth tube. In this series records were obtained from 15 balls, the mean time of fall being 2.174 seconds, with a probable error about 0.004 second. It appears, then, that the presence of the tube has little if any influence on the time of fall.

The latitude of Cambridge being 42° 22′, very nearly, and the elevation above sea level very slight, we find that, according to the general formula for g as a function of λ, its value here is, to the first decimal place, 980.4. Accordingly we have as Gauss's $f + \delta$, the distance a body would fall in vacuum in 2.176 seconds,

$$f + \delta = \tfrac{1}{2} \times 980.4 \times 2.176^2 = 2321 \text{ cm.}$$

The distance f, the actual length of the fall, as measured by a steel tape which was tested by a Brown and Sharpe steel meter rod, was 2285 cm. Accordingly $\delta = 36$ cm., and the easterly deviation should be, according to Gauss,

$$y = \tfrac{2}{3} \cos 42^\circ\, 22' \times \frac{6.28}{86400} \times 2.176\,(2285 - 18) = 0.177 \text{ cm.},$$

that is, to the third place of decimals the value of the easterly deviation is not in our case affected by the resistance of the air, if I have correctly understood and used the formulas of Gauss.

Coefficient of Air Resistance.

It is perhaps worth while, since observations on the air resistance offered to the motion of spherical bodies are not over numerous, to work out from the data here at hand the coefficient of this resistance for the spheres here used, — bronze spheres, one inch in diameter, ground to a smooth surface, but left in a slightly greasy condition by their experience of being dropped into beds of tallow in their use six years ago.

The mere buoyant effect of air on bronze may properly be neglected in this discussion, as it is very small.

If we assume that the resistance of the air is proportional to the square of the velocity of the falling sphere, within the moderate range of velocity here considered, we have, as the net accelerating force on a ball of m grams, $(mg - kv^2)$ dynes, where k is the constant coefficient of resistance. Accordingly, writing c for $m \div k$, we find as the increment of velocity

$$dv = \left(g - \frac{v^2}{c}\right) dt, \tag{1}$$

whence

$$\frac{dv}{g - \frac{v^2}{c}} = \frac{c\,dv}{(gc - v^2)} = dt. \tag{2}$$

This equation, integrated for v between the limits 0 and v, and for t between the limits 0 and 2.176 (the observed value), gives

$$\frac{c}{2\sqrt{gc}} \left[\log \frac{\sqrt{gc} + v}{\sqrt{gc} - v}\right]_0^v = \tfrac{1}{2} \sqrt{\frac{c}{g}} \log \frac{\sqrt{gc} + v}{\sqrt{gc} - v} = 2.176. \tag{3}$$

We have further, if s is the distance fallen, from (2)

$$ds = vdt = \frac{vdv}{g - \frac{v^2}{c}}. \qquad (4)$$

Integrating this equation for s between the limits 0 and 2285 (the observed value) and for v between 0 and v, we get

$$s = 2285 = -\frac{c}{2}\Big[\log(v^2 - gc)\Big]_0^v = -\frac{c}{2}\log\left(1 - \frac{v^2}{gc}\right). \qquad (5)$$

Writing now (3) in the form

$$\frac{\sqrt{gc} + v}{\sqrt{gc} - v} = \epsilon^{4.352\sqrt{\frac{g}{c}}} \qquad (6)$$

and (5) in the form

$$v = \sqrt{gc\left(1 - \epsilon^{-\frac{4570}{c}}\right)}, \qquad (7)$$

and substituting for v in (6), we get

$$\frac{\sqrt{gc}\left(1 + \sqrt{1 - \epsilon^{-\frac{4570}{c}}}\right)}{\sqrt{gc}\left(1 - \sqrt{1 - \epsilon^{-\frac{4570}{c}}}\right)} = \epsilon^{4.352\sqrt{\frac{g}{c}}},$$

or

$$\frac{1 + \sqrt{1 - \epsilon^{-\frac{4570}{c}}}}{1 - \sqrt{1 - \epsilon^{-\frac{4570}{c}}}} = \epsilon^{4.352\sqrt{\frac{g}{c}}},$$

or

$$\frac{1 + \sqrt{1 - 10^{-\frac{1984.726}{c}}}}{1 - \sqrt{1 - 10^{-\frac{1984.726}{c}}}} = 10^{1.89005\sqrt{\frac{980.4}{c}}}. \qquad (8)$$

The value of c which satisfies this equation I find to be about 48000. The value of k, the coefficient in question, is m, the mass of the ball, which is about 73.8 gm., divided by c.

$$k = 73.8 \div 48000 = 0.00154.$$

In Alger's "Exterior Ballistics" I find the following passage:

"Expressing the retardation caused by the resistance of the air in the form $A \frac{d^2}{w} v^n$, in which d is the diameter of the projectile in inches, w its weight in pounds and v its velocity in f. s., Mayevski's equations are as follows:"

"v between 790 f. s. and 0 f. s.,

$$\text{"}\frac{dv}{dt} = - A_7 \frac{d^2}{w} v^2 \qquad \log A_7 = 5.669 \ldots (-10).$$

"The coefficient A depends on the shape of the projectile. In Mayevski's calculations the 'ogival' form [the shape of an ordinary artillery 'shell'] is assumed, the 'ogival' heads having two calibers radius. A would be greater with hemispherical heads."

Mayevski's formula is equivalent to

$$\text{Resistance (poundals)} = - w \frac{dv}{dt} = A d^2 \times v^2.$$

Taking this formula for the case in which d is one inch, the diameter of the bronze balls, and the velocity is 1 cm. per second, we get for the "ogival" form,

$$\text{Resistance (poundals)} = A \div \overline{30.5}^2,$$

$$\text{" \quad (dynes)} = A \div \overline{30.5}^2 \times (453 \times 30.5) = 0.00069.$$

This is about 45 per cent of the value, 0.00154, found above for k in the case of spherical one inch balls.

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Proceedings of the American Academy of Arts and Sciences.

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CONTRIBUTIONS FROM THE GRAY HERBARIUM OF HARVARD UNIVERSITY.

New Series. — No. XXXVIII.

I. A preliminary Synopsis of the Genus *Echeandia.* By C. A. Weatherby.

II. Spermatophytes, new or reclassified, chiefly *Rubiaceae* and *Gentianaceae.* By B. L. Robinson.

III. American Forms of *Lycopodium complanatum.* By C. A. Weatherby.

IV. New and little known Mexican Plants, chiefly *Labiatae.* By M. L. Fernald.

V. Mexican Phanerogams — Notes and new Species. By C. A. Weatherby.

CONTRIBUTIONS FROM THE GRAY HERBARIUM OF HARVARD UNIVERSITY.—NEW SERIES, NO. XXXVIII.

Presented by B. L. Robinson, January 12, 1910. Received February 15, 1910.

I. A PRELIMINARY SYNOPSIS OF THE GENUS ECHEANDIA.

By C. A. Weatherby.

The genus *Echeandia*, founded on *Anthericum reflexum* Cav., was proposed by Ortega in his Novarum Plantarum Decades in 1798, and has been generally maintained by botanists since. Kunth, in 1843, recognized three species under it. Baker, monographing the *Anthericeae* in 1877, could find no clear lines of demarcation between these species and referred all the material known to him to the original species. Hemsley, though suspecting that more than one species was concerned, retained Baker's treatment because of insufficient material for a satisfactory revision. Since the date of his work, the increasingly thorough floristic exploration of Mexico has revealed a number of obviously distinct forms, several of which have been singly described by various botanists. The genus can hardly yet be considered as thoroughly understood; but a brief synopsis, which shall contrast the characters of the different species and bring together the existing information concerning them, may be of service, even though it can lay no claim to finality. The following is an attempt at such a synopsis.

Echeandia is, so far as known, a strictly American genus and chiefly confined to Mexico and Central America. The material at hand shows one species collected in Venezuela. The genus is very closely related to *Anthericum* L., from which, indeed, it is separated by only one constant character — its connate anthers. Although the American species of *Anthericum* are more numerous than those of *Echeandia*, the two groups show a distinctly parallel development, both containing species with smooth and with roughened filaments, smooth and scabrous stems and ovoid and oblong capsules. In particular, *E. macrocarpa* and *A. stenocarpum*, and *E. Pringlei* and *A. tenue* are nearly indistinguishable except by the characters of their anthers.

I have preferred, at least for the present, to regard plants which differ only in comparatively superficial foliar and habital characters as varieties of a single species, rather than specifically distinct. I have, however, made an exception in the group of forms closely related to *E. reflexa.* Here, because of imperfect material of *E. reflexa* and *E. paniculata* and of certain puzzling specimens from Yucatan, I have not been able to arrive at a wholly clear conception of the relationships of the different forms; and I have allowed described species to stand as such, rather than make new combinations which later might have to be withdrawn.

For the loan of specimens, and for other kindly assistance in the preparation of this paper, I am indebted to Captain John Donnell Smith, to Mr. Brandegee of the University of California, Dr. Rose of the National Herbarium, and Dr. Greenman of the Field Museum. All specimens cited are in the Gray Herbarium, unless otherwise specified.

ECHEANDIA Ort. Perianth rotate, spreading or reflexed in flower, after anthesis the withered segments cohering above the ovary and persistent until pushed off by the expanding capsule; segments 6, distinct, three-nerved, about equal in length, the inner often broader. Stamens 6, hypogynous, shorter than the perianth; filaments filiform or clavate, smooth or more or less papillose- or crispate-roughened; anthers linear, hastate at base, the filament attached in the sinus, usually equalling or longer than the filaments, connate in a cylindrical tube which surrounds the style, introrse. Ovary sessile, three-lobed; style filiform, a little longer than the tube of anthers; stigma small, capitate. Capsule ovoid or oblong, triangular, loculicidal. Seeds numerous, angulate-compressed, black, minutely papillose. — Roots fibrous, clustered, often thickened or fusiform. Leaves basal or rarely the lower part of the stem leafy. Stem scapiform, bracted, simple or branched above, the branches virgate. Flowers yellow or white, on usually slender jointed pedicels in clusters of 1–4 on the stem and its branches, in the axils of chartaceous bracts, each pedicel subtended by a similar smaller bractlet; the clusters in virgate racemes.

a. Filaments smooth; leaves strictly basal, not sheathing the stem, *b.*
  b. Stem scabrous, 1–4-bracted . . . . 1. *E. parviflora.*
  b. Stem smooth, 6–9-bracted, *c.*
    c. Leaves spreading, falcate, 15 cm. or less long . . 2. *E. brevifolia.*
    c. Leaves erect, narrowed at base, more than 15 cm. long, *d.*
      d. Leaves broad, 2 cm. or more . . . . 3. *E. nodosa.*
      d. Leaves narrow, not over 1 cm. wide . 3. *E. nodosa, var. lanceolata.*

a. Filaments more or less crispate- or papillose-roughened, *e*.
  e. Leaves broad, 0.8–3.5 cm. wide, membranous in drying, soft, the principal nerves usually connected by conspicuous cross-veinlets, *f*.
    f. Stem smooth; flowers chiefly yellow, as far as known, *g*.
      g. Capsule ovoid or short-oblong, 6–9 mm. long, 5–7 mm. broad; inner perianth-segments oblong-lanceolate, little broader than the outer, *h*.
        h. Leaves lanceolate or even ovate-lanceolate, 20–25 cm. long, 2.8–5 cm. wide, not more than 8 times as long as wide. 4. *E. macrophylla.*
        h. Leaves linear or narrowly lanceolate, 24–42 cm. long, 1.2–2.3 cm. wide, at least 12 times as long as wide. 4. *E. macrophylla, var. longifolia.*
      g. Capsule oblong, 1–1.8 cm. long, 4–6 mm. wide; inner perianth-segments ovate or ovate-lanceolate, often much broader than the outer, *i*.
        i. Leaves for the most part sheathing the stem but confined to its base; stem about 2-bracted, *j*.
          j. Leaves narrow, 8–13 mm. wide, *k*.
            k. Leaves usually several (6–10), suberect . 5. *E. macrocarpa.*
            k. Leaves few (2–4), spreading, short in proportion to the stem. 5. *E. macrocarpa, var. formosa.*
          j. Leaves broader, 1.5–2 cm. wide . . . . 6. *E. reflexa.*
        i. Stem leafy for about a third of its height, the leaves passing gradually into 3–6 reduced bracts . . . . . 7. *E. paniculata.*
    f. Stem scabrous, at least below; flowers white . . . . 8. *E. albiflora.*
  e. Leaves narrow, 2–5 mm. wide or less, firm, closely and prominently veined, mostly without visible cross-veinlets, *l*.
    l. Leaves 2–5 mm. wide, minutely scabrous beneath; stem 2-bracted; inflorescence mostly branched . . . . 9. *E. flexuosa.*
    l. Leaves 2 (–2.5) mm. wide or less, scabrous-ciliate on the margins, elsewhere smooth; stem 3–6-bracted; inflorescence mostly simple. 10. *E. Pringlei.*

1. E. PARVIFLORA Baker. Leaves membranous, linear, not very prominently nerved, 4–8 mm. wide, 6–22 cm. long, suberect or somewhat spreading and falcate; stem scabrous or hirtellous at least below, simple or sometimes with as many as 5 branches; pedicels rather short and stout, in fruit 6–8 mm. long, jointed below the middle or toward the base; filaments smooth; capsule (seen on the Pringle specimen only) broadly oblong, 3.5–5 mm. wide, 6–9 mm. long. — Engl. Bot. Jahrb. viii. 209 (1887). — GUATEMALA: Santa Rosa, alt. 900 m., May, 1892, *John Donnell Smith*, Pl. Guat., no. 3528. MEXICO: Mt. Orizaba, Cordoba, 830 m., Aug. 20, 1891, *Henry E. Seaton*, no. 485, in part. State of Guerrero, dry hillsides, near Iguala, alt. 915 m., July 29, 1907, *Pringle*, no. 10,388.

2. E. BREVIFOLIA Watson. Leaves membranous, with cross-veinlets,

short, 12–15 cm. long, 6 mm. wide, acuminate, spreading and somewhat falcate, not sheathing the stem; stem about 6 dm. tall, smooth, 6-bracted, with few (3–4) branches; pedicels slender, in fruit 11–14 mm. long, jointed below the middle; filaments smooth; capsule short-oblong, 4–4.5 mm. wide, 7–8 mm. long. — Proc. Am. Acad. xxi. 441 (1886). — Mexico: State of Chihuahua, Hacienda San Miguel near Batopilas, Sept., 1885, *Palmer*, no. 229.

3. E. nodosa Watson. Leaves membranous, with cross-veinlets, linear-lanceolate, narrowed at base, not sheathing the stem, 18–40 cm. long, 2–2.7 cm. wide; stem smooth, 6–9-bracted, with 6–7 branches, which rarely branch again; pedicels slender, jointed below the middle, in fruit 11–14 mm. long; filaments smooth, shorter than the anthers; capsule oblong, 3.5–4 mm. wide, 8–9 mm. long. — Proc. Am. Acad. xxvi. 156 (1891). ?*Phalangium ramosissimum* Presl, Rel. Haenk. i. 127 (1825). ?*Anthericum ramosissimum* R. & S. Syst. vii. 469 (1829). ?*Echeandia Haenkeana* Kunth, Enum. iv. 629 (1843). — Mexico: State of Jalisco, near Guadalajara, 12 Nov., 1888, *Pringle*, no. 2151. Dry rocky bluffs of barranca near Guadalajara, 23 Sept., 1891, *Pringle*, no. 3870. — Flowers apparently small as in *E. macrophylla*; the perianth-segments narrow, whitish in drying. From Presl's description it seems highly probable that this plant is the same as his *Phalangium ramosissimum*. In the absence of authentic material, however, I hesitate to make the new combination required by the transfer of Presl's species to *Echeandia*.

Var. **lanceolata**, n. var., a forma typica recedit habitu graciliore, foliis angustioribus 6–10 mm. latis, pedicellis 1 cm. longis, capsulis minoribus 3.5 mm. latis 5–6 mm. longis. — Mexico: State of Sinaloa, Copradia, Oct. 20, 1904, *Brandegee*, type (in Herb. Univ. Cal., sheet no. 119,863). Ymala, Sept. 28 to Oct. 8, 1891, *Palmer*, no. 1677. Culiacan, Sept. 17, 1904, *Brandegee* (in Herb. Univ. Cal., sheet no. 119,856). — The name *lanceolata* was applied to this plant, on herbarium labels, by Mr. Brandegee, who at that time was inclined to regard it as a good species. It seems, however, hardly specifically distinct from *E. nodosa*. The specimen on sheet no. 119,856 of the University of California Herbarium has broader leaves than the other two plants cited and may be regarded as a transitional form between the extreme development of the variety and typical *E. nodosa*.

4. **E. macrophylla** Rose, in hb., foliis omnino radicalibus caulis basin vaginantibus lanceolatis 20–25 cm. longis 2.8–5 cm. latis in apicem acuminatum angustatis, caule 7 dm. alto glabro 2-bracteato, ramis 5–6 saepe 2 ex axilla unica, pedicellis infra medium vel prope basin articulatis, floribus parvis, perianthii segmentis 1–1.3 cm. longis

lineari- vel oblongo-lanceolatis latitudine subaequalibus, interioribus paulum latioribus acutis, exterioribus obtusiusculis, filamentis clavatis modice crispatis in floribus (novellis) visis quam antherae duplo brevioribus, capsulis ovoideis 7 mm. longis 5 mm. latis. — MEXICO: State of San Luis Potosi, grassy slopes, Las Canoas, 16 June, 1890, *Pringle*, no. 3183.

Var. **longifolia,** n. var., foliis late linearibus 24–42 cm. longis 1.2–2.3 cm. latis saepius solum radicalibus, caule 6.2–9 cm. alto, ramis paucis (1–3), pedicellis 1–2 cm. longis, filamentis antheras aequantibus vel eis brevioribus, capsulis ovoideis vel breviter oblongis 7–9 mm. longis 5–6 mm. latis, ceteris praecedentis. — ? *E. terniflora* Lindley, Bot. Reg. xxv. Misc. no. 144 (1839), not Ort. *E. terniflora* Baker, Journ. Linn. Soc. xv. 288 (1877), in part, not Ort.; Hemsl. Biol. Cent.-Am. Bot. iii. 376, in part, not Ort. — MEXICO: State of Oaxaca, vicinity of Choapam, alt. 1150–1400 m., July 28 & 29, 1894, *Nelson*, no. 910, type (in U. S. Nat. Herb.). State of Vera Cruz, Zacuapan, dry sunny fields, Nov., 1908, *Purpus*, no. 3761. Orizaba, *Botteri*, no. 1185. Ibid., Cordoba, 830 m., Aug. 20, 1891, *H. E. Seaton*, no. 485, in part. Vallée de Cordova, 23 Avril, 1865–66, *Bourgeau*, no. 2307. VENEZUELA: prope coloniam Tovar, 1854–55, *Fendler*, no. 1549. The Bourgeau plant has entirely the habit and the fruit of this species, but the filaments are nearly smooth. It seems somewhat transitional between this and the preceding group. — Flowers yellow according to Lindley's description; white with yellow anthers according to a note on Fendler's label. The plant seen by Lindley was possibly *E. reflexa*, but from his description, seems rather to belong here.

5. E. MACROCARPA Greenman. Leaves chiefly basal, suberect, rather narrowly linear, (6) 8–15 mm. broad, membranous, the cross-veinlets usually prominent, long in proportion to the stem, usually 6–10 in number; stem 1–2-bracted, glabrous, simple or few-branched; pedicels jointed below the middle, rather stout, in fruit 1–1.7 cm. long; flowers apparently rather large, the perianth-segments 1.5–1.7 cm. long, the inner ovate-lanceolate; filaments moderately roughened, equalling or slightly longer than the anthers; capsules oblong, 1–1.8 cm. long, 4–6 mm. wide. — Proc. Am. Acad. xxxix. 73 (1903). *E. terniflora* Hemsl. Biol. Cent.-Am. Bot. iii. 376, in part, not Ort. — MEXICO: State of San Luis Potosi, near Tancanhuitz, May 2, 1898, *Nelson*, no. 4393, type; region of San Luis Potosi, alt. 1850–2450 m., *Parry & Palmer*, no. 890. "Mexico," no locality, *Ehrenberg*, no. 31. "Chiapas, etc.," *Ghiesbreght*, no. 875. Vallée de Mexico, Santa Fé, 6 Juillet, 1865–66, *Bourgeau*, no. 413. Guanajato, 1880, *A. Dugès*. State of Oaxaca, vicinity of Cerro San Felipe, alt. 3000–3400 m., 1894, *Nelson*, no. 1056

(in U. S. Nat. Herb.). — A specimen from Mt. Orizaba, 3000 m., Aug. 5, 1891, *H. E. Seaton*, no. 180, is probably a reduced form of this species. — Flowers yellow according to Ghiesbreght's label. Difficult to separate from *E. reflexa*, except by purely habital characters.

Var. **formosa,** n. var., foliis paucis (circa 4) caulis basin extremam vaginantibus patulis caule duplo brevioribus late linearibus circa 1 cm. latis summum 2 dm. longis, caule simplice, pedicellis gracilibus, floribus magnis aureis, ceteris formae typicae. — MEXICO: State of Chiapas, near San Christobal, alt. 2100–2500 m., Sept. 18, 1895, *Nelson*, no. 3143 (in U. S. Nat. Herb. Sheet no. 233,087). — Flowers "rich yellow" according to Nelson's note.

6. E. REFLEXA (Cav.) Rose. Leaves rather closely sheathing the base of the stem, broadly linear, 27–40 cm. long, 1.5–2.2 cm. wide, acuminate, membranous, the cross-veinlets prominent; stem about 7 dm. tall, smooth, rather slender, bearing 2–3 foliaceous bracts, in the single specimen seen with two branches; pedicels jointed below the middle, in fruit 1.4–1.7 cm. long; perianth-segments broad, 1.5 cm. in length; filaments strongly roughened, at least in the young flower shorter than the anthers; capsule (immature) oblong, 1 cm. long, 4 mm. wide. — Contr. U. S. Nat. Herb. x. 93 (1906). *Anthericum reflexum* Cav. Ic. Pl. iii. 21, t. 241 (1795); Willd. Sp. Pl. ii. 140 (1799). *Echeandia terniflora* Ort. Nov. Pl. Dec. 90, 135, & 136, t. 18 (1798); Redouté, Lil. vi. t. 313 (1812); Kunth, Enum. iv. 627 (1843); Baker, Journ. Linn. Soc. xv. 288 (1877), in part; Hemsl. Biol. Cent.-Am. Bot. iii. 376 (1885), in part. *Phalangium reflexum* Poir. Encycl. Meth. Bot. v. 249 (1804). *Conanthera Echeandia* Pers. Syn. i. 370 (1805); Link & Otto, Ic. Pl. Rar. 5, t. 3 (1828). — MEXICO: State of Morelos, ledges, Sierra de Tepoxtlan, near Cuernavaca, alt. 2300 m., August 22, 1906, *Pringle*, no. 10,289. — Although the form represented by Mr. Pringle's plant here cited was the first of the genus to be collected, it seems not to be common. His specimen is the only one I have seen which, in its combination of broad leaves, few-branched stem, yellow, rather broad perianth-segments, strongly roughened filaments and oblong capsules, agrees well with Cavanilles's and Ortega's plates.

7. E. PANICULATA Rose. Stem tall, with 6–7 panicled branches, leafy above the base for about a third of its height, the leaves passing gradually into 3–6 reduced bracts; leaves membranous, with cross-veinlets, linear, long-attenuate at apex, up to 5 dm. long, 1.5–3 cm. wide; flowers rather large, yellow; perianth-segments 1.5 cm. long, the outer oblong-linear, the inner ovate, 6 mm. wide; filaments clavate, strongly roughened, about equalling the anthers; capsule not seen. — Contr. U. S. Nat. Herb. x. 93 (1906). — MEXICO: State of Morelos,

near El Parque, Sept. 21, 1903, *Rose & Painter*, no. 844 (in U. S. Nat. Herb., sheets nos. 454,954 & 454,955). — No fruit of this species has been preserved, but its floral characters place it clearly very near *E. reflexa.* So far as the material at hand shows, it differs from that species only in its more leafy stem and more branched inflorescence and may very probably prove to be no more than a variety of it. — Here are doubtfully placed the specimens from two collections of *C. F. Gaumer* namely from Yucatan, Izamal, Sept., 1895, no. 843 and Chicankanab, no. 1995 (the latter in Herb. Field Mus. Nat. Hist., sheet no. 58,793). These specimens have neither fruit nor good flowers and in their absence can hardly be placed definitely. They have mostly a much-branched inflorescence, several(7–8)-bracted stem and the leaves pass abruptly into the much reduced bracts. In this respect they differ from *E. paniculata;* and the branches of the inflorescence are more slender and the flower-buds smaller than in either that species or *E. reflexa*, although the plants are quite as robust.

8. E. ALBIFLORA (Schlecht. & Cham.) Mart. & Gal. Leaves basal, several, lanceolate-linear, narrowed to an acute apex, the principal nerves united by transverse veinlets, membranous, glabrous, about 36 cm. long, 1.8–2 cm. wide ; stem scabrous or hirtellous below ; inflorescence paniculate ; pedicels slender, 10 mm. long, jointed below the middle ; flowers white ; perianth-segments lanceolate ; filaments retrorsely papillose-crispate, equalling the anthers ; capsule ? — Bull. Acad. Brux. ix. 386 (1842) ; Kunth, Enum. iv. 628 (1843). *Conanthera albiflora* Schlecht. & Cham. Linnaea, vi. 50 (1831). *Echeandia leucantha* Klotzsch, fide Kunth, l. c. — I have seen no material referable to this species. The above description is taken chiefly from that of Kunth.

9. E. FLEXUOSA Greenman. Leaves firm, closely and prominently veined, suberect, minutely scabrous beneath, 2–5 mm. wide, variable in length (reaching 8 dm.), long-acuminate ; stem 9 dm. high or less, smooth, 2–3-bracted, the lower bract sometimes elongated and setaceous, reaching 15 cm. in length ; pedicels jointed near or below the middle, rather stout, in fruit 12–16 mm. long ; flowers rather large with lanceolate perianth-segments ; filaments moderately roughened, shorter than or nearly equalling the anthers ; capsule oblong, 6–9 mm. long, 3–4 mm. wide. — Proc. Am. Acad. xxxix. 73 (1903). — MEXICO : State of Oaxaca, Mts. of Jayacatlan, alt. 1400 m., 10 Sept., 1894, *Lucius C. Smith*, no. 188. State of Jalisco, Rio Blanco, July, 1886, *Palmer*, no. 185 ; bluffs of the barranca of Guadalajara, 1400 m., 19 July, 1902, *Pringle*, no. 11,197.

10. E. PRINGLEI Greenman. Leaves firm, closely and prominently

veined, scabrous-ciliate on the margins, elsewhere smooth, 1.5–2 (2.5) mm. wide, 1–3 dm. long; stem 2.7–6 dm. high, slender, glabrous, simple, bearing 3–6 bracts; pedicels jointed near the base, in fruit 10–14 mm. long; filaments moderately roughened, shorter than the anthers; capsule oblong, 3–3.5 mm. wide, 7 mm. long. — Proc. Am. Acad. xl. 28 (1904). — MEXICO: State of Jalisco, dry calcareous hills above Etzatlan, 2000 m., 24 Oct., 1904, *Pringle*, no. 8812; grassy plains near Guadalajara, 1500 m., 4 Oct., 1903, *Pringle*, no. 11,715; hillsides of Zapotlan, alt. about 1500 m., Aug. 8, 1905, *P. Goldsmith*, no. 122; near Etzatlan, Oct. 2, 1903, *Rose & Painter*, no. 7544 (in U. S. Nat. Herb.).

EXCLUDED SPECIES.

E. eleutherandra K. Koch, Ind. Sem. Hort. Berol. App. 4 (1861) = *Anthericum echeandioides*, acc. to Baker.

E. graminea Mart. & Gal. Bull. Acad. Brux. ix. 387 (1842) = *Anthericum leptophyllum.*

E. leptophylla Benth. Pl. Hartw. 25 (1840) = *Anthericum leptophyllum.*

E. scabrella Walp. Ann. iii. 1010 (1853) = *Anthericum scabrellum.*

E. pusilla Brandegee, Univ. Cal. Pub. Bot. iii. 377 (1909) = form of *Anthericum leptophyllum.*

II. SPERMATOPHYTES, NEW OR RECLASSIFIED, CHIEFLY RUBIACEAE AND GENTIANACEAE.

BY B. L. ROBINSON.

Ranunculus trisectus Eastwood, n. sp.,[1] glaber vel paulo pilosus 1–2 dm. altus simplex vel 2–3-ramosus, ramis ascendentibus; foliis radicalibus orbicularibus trisectis, diametro 2–3 cm., basi reniformibus cum sinu saepissime angusto; segmentis approximatis, medio late cuneato, lateralibus inaequaliter bipartitis, superiore parte trilobata majore; omnibus lobulis similibus oblongis 2–3 mm. latis duplo longioribus, apice et basi callosis, sinubus obtusis; petiolis striatis basi membranaceis dilatatis et persistentibus; foliis caulinis 1–3 sessilibus vel breviter petiolatis 3–5-sectis, segmentis integris vel lobatis, ultimis lobulis oblongo-linearibus ad apicem et sinum callosis, basi petiolorum vel foliorum membranaceo amplexicauli; pedunculis altis, fructiferis

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[1] This species, elaborated by Miss Alice Eastwood from material in the Gray Herbarium, is here published at her request.

saepe 5–6 mm. longis, floriferis multo brevioribus; sepalis purpurascentibus orbiculatis 6–7 mm. latis et longis, concavis, cum pilis canis et sericeis parce investis; petalis aurantiacis cuneatis 5–15 mm. latis, sepala multo superantibus, apice undulatis rotundatis, basi cum squamula hemicycla supra brevem unguem; staminibus numerosis, loculis antherarum separatis, dorso filamentis planis; acheniis spicatis, receptaculo subulato albo membranaceo pilosello; stylis purpureis vel flavis rectis vel curvatis et divaricatis, apice saepe deciduis. — Alpine Wallowa mountains, eastern Oregon, altitude 2745 m.. growing at base of cliffs, *William C. Cusick*, 16 August, 1907, no. 3200 (type, in Gray Herb.). Under the same species are included with some doubt the following, all collected by Mr. Cusick at the same locality: — no. 3188, strong growing plants, some with smooth, others with hairy akenes but otherwise identical; 3325 d, with akenes all hairy; 3326 with both hairy and smooth akenes. Among the older specimens in the Gray Herbarium are 3219 a collected in 1907 with heads of akenes more globular and hairy, styles purplish, 1513 of 1888 and 2006 of 1898. These all show great variability in size of flowers and height of stems but the plants have an individuality which makes them appear quite distinct from *R. Suksdorfii* with which they have been confused. In general this species differs from *R. Suksdorfii* in having more orbicular leaves with more deeply cut divisions, narrower basal sinus, the ultimate lobules obtuse and narrowing slightly to the base thus making the dividing space rounded rather than acute. The akenes are not angled, hairy instead of smooth, and the style curves outward more noticeably and is less strongly subulate.

Tococa Peckiana, n. sp., fruticosa 3–6 m. alta; ramis valde compressis brunneis fistulosis parce praesertim nodos versus glanduloso-hispidulis; foliis late ovatis modice disparibus membranaceis 5-nerviis supra appresse setulosis rugosis siccitate nigrescentibus subtus tomentellis flavidi-viridibus margine integriusculis hispidulis apice angustissime caudato-attenuatis, majoribus 1.4–2.2 dm. longis 7–12 cm. latis, petiolo crasso hispidulo 2–2.5 cm. longo prope apicem vesciculifero, vesciculis ovoideis subcoriaceis 1–1.2 cm. longis; foliis minoribus 1.2–1.5 dm. longis ab vesciculis destitutis; panicula terminali pedunculata ca. 8 cm. longa, ramis patentibus dichotomo-cymiferis; floribus sessilibus; calycis tubo subgloboso 4–5 mm. diametro parce glanduloso-hispidulo, limbo brevissimo membranaceo obscure 5-lobato; petalis ovatis subcoriaceis minute papillosis. — BRITISH HONDURAS, in thickets, near Manatee Lagoon, 16 July, 1905, *Prof. Morton E. Peck*, no. 68 (type, in Gray Herb.). A species of the § *Hypophysca* and related apparently to *T. guyanensis* Aubl., from which, however, it may be readily distinguished by its less unequal, more nearly entire leaves, smaller, thicker-walled vescicles, and especially by its sessile flowers.

Cynoctonum oldenlandioides (Wall.), n. comb. *Mitreola oldenlandioides* Wall. Cat. no. 4350 (1828), without description; G. Don, Syst. iv. 172 (1837), where distinctions are slightly indicated; A.DC. Prod. ix. 9 (1845), where described and distinguished chiefly by the widely divergent lobes of the fruit; Hook. Ic. t. 827 (1852), where admirably figured. The change from *Mitreola* to *Cynoctonum* becomes necessary under the Vienna Rules, though it is certainly to be regretted that the well established *Mitreola* was not included in the list of nomina conservanda.

Cynoctonum paniculatum (Wall.), n. comb. *Mitreola paniculata* Wall. Cat. no. 4349 (1828), without description; G. Don, Syst. iv. 171 (1837); A.DC. Prod. ix. 9 (1845); Progel in Mart. Fl. Bras. vi. pt. 1, 266, t. 71 (1868).

Cynoctonum pedicellatum (Benth.), n. comb. *Mitreola pedicellata* Benth. Jour. Linn. Soc. i. 91 (1857).

Centaurium Beyrichii (Torr. & Gray), n. comb. *Erythraea trichantha β angustifolia* Griseb. in DC. Prod. ix. 60 (1845). *E. Beyrichii* Torr. & Gray ex Torr. in Marcy, Expl. Red Riv. 291 (1853).

Centaurium cachanlahuen (Molina), n. comb. *Gentiana Cachanlahuen* Molina, Sagg. Chil. 147 (1782); also in the German edition by Brandis, 310 (1786). *G. peruviana* Lam. Encycl. ii. 642 (1786). *Chironia chilensis* Willd. Sp. Pl. i. 1067 (1798). *Erythraea chilensis* Pers. Syn. i. 283 (1805). *E. Cachanlahuan* Roem. & Schultes, Syst. iv. 167 (1819).

CENTAURIUM CALYCOSUM (Buckl.) Fernald, var **nana** (Gray), n. comb. *Erythraea calycosa*, var. *nana* Gray, Syn. Fl. ii. pt. 1, 113 (1878).

Centaurium floribundum (Benth.), n. comb. *Erythraea floribunda* Benth. Pl. Hartw. 322 (1849).

Centaurium macranthum (Hook. & Arn.), n. comb. *Erythraea macrantha* Hook. & Arn. Bot. Beech. 438 (1841). *E. mexicana* Griseb. ex Hook. & Arn. l. c. 302, 438. *Gyrandra chironioides* Griseb. in DC. Prod. ix. 44 (1845). *Erythraea chironioides* Torr. Bot. Mex. Bound. 156 (1859), in part.

Centaurium madrense (Hemsl.), n. comb. *Erythraea madrensis* Hemsl. Biol. Cent.-Am. Bot. ii. 346 (1882). *Gyrandra chironioides* Griseb. in Seem. Bot. Herald. 318 (1856), not Griseb. in DC. Prod. ix. 44 (1845).

Centaurium micranthum (Greenm.), n. comb. *Erythraea micrantha* Greenm. Proc. Am. Acad. xxxix. 83 (1903).

Centaurium multicaule, n. sp., verisimiliter bienne multicaule caespitosum 5–10 cm. altum basi densissime foliatum ; radice simplice 2–6 cm. longa ; caulibus 4–22 subsimplicibus 4-angulatis gracilibus apice 1–2(rarius 3)-floris, ramis 1–2 erectis ; foliis radicalibus rosulatis obovato-spatulatis 1–2 cm. longis 4–8 mm. latis apice rotundatis basi in petiolum attenuatis ; foliis caulinis 3–4-jugis lineari-oblongis vel linearibus 8–10 mm. longis 1–2.7 mm. latis 1-nerviis crassiusculis ; pedunculis 1.5–4 cm. longis erectis nudis unifloris ; floribus pentameris ; calycis lobis linearibus attenuatis 6 mm. longis margine scariosis quam tubus corollae paulo brevioribus ; corolla 1.5 cm. longa tubo constricto flavido, limbi lobis ellipticis 6 mm. longis 2 mm. latis apice rotundatis ; filamentis antheras subaequantibus gracilibus ; stigmate capitato-subbilobo. — MEXICO : most meadow, Hacienda of St. Diego, Chihuahua, 2 June, 1891, *C. V. Hartman*, no. 717 (type, in Gray Herb.). This plant of somewhat striking tufted habit was distributed as *Erythraea calycosa*, but differs from that species rather markedly in its lower stature, much smaller flowers, and clustered chiefly 1-flowered stems.

Centaurium nudicaule (Engelm.), n. comb. *Erythraea nudicaulis* Engelm. Proc. Am. Acad. xvii. 222 (1882).

Centaurium pauciflorum (Mart. & Gal.), n. comb. *Erythraea pauciflora* Mart. & Gal. Bull. Acad. Brux. xi. 373 (1844).

Centaurium Pringleanum (Wittr.), n. comb. *Erythraea Pringleana* Wittr. Bot. Gaz. xvi. 85 (1891).

Centaurium quitense (HBK.), n. comb. *Erythraea quitensis* HBK. Nov. Gen. et Spec. iii. 178 (1818). *Cicendia quitensis* Griseb. Linnaea, xxii. 33 (1849). *Erythraea divaricata* Schaffner ex Schlecht. Bot. Zeit. xiii. 920 (1855). *Erythraea chilensis* Benth. Pl. Hartw. 89 (1842), non Pers. *Centaurium divaricatum* Millsp. & Greenm., Field Columb. Mus. Bot. Ser. ii. 309 (1909).

Centaurium retusum (Rob. & Greenm.), n. comb. *Erythraea retusa* Rob. & Greenm. Proc. Am. Acad. xxxii. 38 (1896).

Centaurium setaceum (Benth.), n. comb. *Erythraea setacea* Benth. Bot. Sulph. 128 (1845).

Centaurium tenuifolium (Mart. & Gal.), n. comb. *Erythraea macrantha β major* Hook. & Arn. Bot. Beech. 438 (1841). *E. tenuifolia* Mart. & Gal. Bull. Acad. Brux. xi. 372 (1844). *Gyrandra speciosa* Benth. Bot. Sulph. 127, t. 45 (1845).

Centaurium trichanthum (Griseb.), n. comb. *Erythraea tricantha* Griseb. Gen. et Spec. Gent. 146 (1839).

Centaurium venustum (Gray), n. comb. *Erythraea chironioides* Torr. Bot. Mex. Bound. 156, t. 42 (1859), not *Gyrandra chironioides* Griseb. *Erythraea venusta* Gray, Bot. Calif. i. 479 (1876).

LISIANTHUS CUSPIDATUS Bertoloni, Nov. Comm. Bonon. iv. 408, t. 38 (1840). *Leianthus cuspidatus* Griseb. in DC. Prod. ix. 82 (1845). This species is reduced to a synonym of *Leianthus nigrescens* (Cham. & Schlecht.) Griseb. by Hemsley, Biol. Cent.-Am. Bot. ii. 345 (1882) and of *Lisianthus nigrescens* Cham. & Schlecht. by Miss Perkins in Engl. Jahrb. xxxi. 493 (1902). An examination of Bertoloni's excellent plate of his *Lisianthus cuspidatus* leads to the belief that it represents a species markedly distinct from *L. nigrescens.* Conspicuous differences are to be found in the following features. In *L. cuspidatus* the leaves are narrowed to a subcuneate base, the corolla is much more deeply lobed, the lobes distinctly surpassing the pistil, while in *L. nigrescens* the leaves are rounded to a somewhat amplexicaul base and the corolla-lobes are only 4–11 mm. long being somewhat overtopped by the stigma. A specimen, collected in the Sapoti Barranca near the City of Guatemala by Sutton Hayes, July, 1860, and now in the Gray Herbarium, corresponds in all respects to the plate of Bertoloni, and fully justifies the separation of the species. The lobes of its corolla are 1.7 cm. in length. *Lisianthus nigrescens* Hook., in Curt. Bot. Mag. t. 4043, would appear to be *L. cuspidatus* Bert.

Lisianthus oreopolus, n. sp., suberectus 7 dm. vel ultra altus perennis; caule tereti (juventate solum plus minusve tretragono) levissime basi lignescenti; foliis sessilibus lanceolato-oblongis acuminatis membranaceis 8–11 cm. longis 1.5–2.4 cm. latis basi amplexicaulibus biauriculatis subtus pallidioribus internodia multo superantibus; panicula laxa 3 dm. longa 2 dm. diametro; ramis ramulisque ascendenti-patentibus saepius alternis; pedicellis propriis (supra bracteolas) brevibus 1–2 mm. longis saepe curvatis; calyce graciliter ovoideo acutiuscule angulato 1 cm. longo fere a basi 5-lobo, lobis tenuibus attenuatis corollae appressis; corolla infundibuliformi 4 cm. longo glaberrima flava, tubo proprio gracili, faucibus longiusculis gradatim ampliatis, lobis 1.4–1.6 cm. longis lanceolatis acutissimis late patentibus; et staminibus et stylo exsertis; stigmate peltato margine revoluto. — MEXICO: Temperate region, mountain of Chiapas, flowering in June, *Ghiesbreght,* no. 702bis (type, in Gray Herb.). A species in habit similar to *L. nigrescens* Cham. & Schlecht., but differing in its yellow corolla with considerably longer and much more widely spreading lobes.

Lisianthus viscidiflorus, n. sp., erectus 1–1.2 m. altus floribus exceptis glaberrimus; caule subtereti levissimo angulis parvis prominulis 2 e costis mediis foliorum decurrentibus paululo ancipitali; internodiis inferioribus brevissimis 8–12 mm. longis, intermediis 2–6 cm. longis, superioribus ad 19 cm. longis; foliis lanceolato-oblongis sessilibus amplexicaulibus 7–12 cm. longis 1–2.2 cm. latis acutis crassiusculis basi biauriculatis; panicula laxissima 3 dm. longa 2–3 dm. diametro, ramis patenti-ascendentibus infra nudis apice saepissime trichotomis 3–5-floris, ramulis lateralibus saepius 2–3.5 cm. longis 1-floris apicem versus saepissime arcuatis bibracteolatis; floribus viscosis; calyce herbaceo breviter subcylindrico basi turbinato, lobis juventate acutis mox apice erosis maturitate obtusissimis viscidis; corolla 3–3.5 cm. longa, tubo rectiusculo verisimiliter atrorubenti, limbo ca. 1 cm. diametro viscidissimo, dentibus deltoideis 3 mm. longis viridescentibus; staminibus inclusis; stigmate modice exserto peltato. —GUATEMALA: Coban, Dept. Alta Verapaz, alt. 1350 m., August, 1907, *H. von Tuerckheim*, no. II. 1308 (type, in Gray Herb.). Distributed as *Leianthus brevidentatus* Hemsl., a species described as having dense inflorescence, short pedicels, shorter corolla with lobes scarcely 2 mm. long, very acute calyx-lobes appressed to the corolla, etc., differences which would certainly appear to be of specific value. It is, furthermore, scarcely likely that the viscidity which is such a conspicuous feature of the present species could have been present in *L. brevidentatus* in like degree and have escaped mention.

Schultesia Hayesii, n. sp., annua erecta gracilis 3–4 dm. alta glaberrima supra ramosa; radice fibrosa; caule subtereti leviter 6-angulato foliato; foliis linearibus, inferioribus brevibus, superioribus 4–5 cm. longis 2–3 mm. latis angustissime attenuatis basi paulo angustatis sessilibus 3-nerviis subtus pallidioribus; ramis patenti-ascendentibus simplicibus saepissime alternis apice 2-bracteolatis et 1-floris; bracteolis anguste linearibus 3 cm. longis; floribus supra bracteolas sessilibus 4-meris; calyce anguste ovoideo 3–3.6 cm. longo, tubo castaneo levissimo evenio; alis semilanceolatis 3 mm. latis viridibus venosis sursum in dentes calycis subsetaceos gradatim attenuatis; corolla 4 cm. longa verisimiliter purpurea, lobis late ovatis breviter acuminatis 1 cm. longis; ovario 4 angulari 1.4 cm. longo 4 mm. lato. — PANAMA: Rio Grande Station, Panama railway, 13 December, 1859, *Sutton Hayes*, no. 160 (type, in Gray Herb.). This species is closely related to *S. heterophylla* Miq. but differs in several points. The stems are perceptibly 6-angled; the leaves are decidedly longer and relatively narrower than in *S. heterophylla* and the middle ones equal or often exceed the internodes, while in *S. heterophylla* they are much exceeded by the internodes. Finally the lobes of the corolla are only 1 cm. long, i. e. one third the length of the tube, those of *S. heterophylla* on the other hand being 1.6 cm. long, i. e. more than half the length of the tube.

Schultesia Peckiana, n. sp., decumbens, verisimiliter annua, habitu *S. lisianthoidi* similis 6–7 dm. alta laxe ramosa glaberrima; caule tereti laevissimo; foliis lanceolati-ovatis tenuibus sessilibus acutissimis basi rotundatis; cymis laxe etiam atque etiam dichotomis; floribus in dichotomis solitariis 1.5 cm. longis erectis; pedicellis 8–30 mm. longis rectis nudis; calycis lobis 4 anguste lanceolatis acutissimis in media parte herbaceis margine scariosis vix carinatis ex alatis; corolla rubescenti vel purpurascenti fere ad mediam partem 4-secta; lobis ovatis acutis; filamentis gracilibus, basi exappendiculatis; antheris mucronatis. — British Honduras: about plantations and in the openings of the forests, near Manatee Lagoon, 27 January, 1906, *Prof. Morton E. Peck*, no. 318 (type, in Gray Herb.). A species considerably resembling *S. lisianthoides* (Griseb.) Benth. & Hook. f., but readily distinguished by its pedicelled flowers.

Evolvulus sericeus Sw., var. **glaberrimus**, n. var., ubique glaberrimus gracillimus, caulibus a basi patenti-ramosis suberectis 2.5–3 dm. altis; calyce etiam glaberrimo, aliter formae typicae simillimus. — British Honduras: low pine ridge near Manatee Lagoon, 28 March, 1906, *Prof. Morton E. Peck*, no. 372 (type, in Gray Herb.). A form remarkable for the complete absence of the silky pubescence, which is to some extent present in all other specimens examined, even those of the form *glabratus* Chod. & Hassl., which has decidedly silky-villous calyces.

Schwenkia oxycarpa, n. sp., perennis erecta suffrutescens scoparia 5–6 dm. alta; radice fibrosa; caulibus teretibus cortice fusco-griseo obtectis; ramis gracillimis ascendentibus vel erectis viridibus teretibus; foliis linearibus acutis sessilibus crassiusculis subglabris 5–7 mm. longis vix 1 mm. latis saepissime curvatis vel tortis 1-nerviis; inflorescentia ca. 1 dm. longa gracillima spiciformi; floribus fasciculatis sessilibus parvis; calyce turbinato ca. 1.3 mm. longo obscure strigilloso, dentibus lanceolatis acutis tubum subaequantibus; corolla 4 mm. longa atrocyanea rectiuscula, limbi dentibus 5 clavellatis quam sinuum lobi obovati crassiusculi subbipartiti vix longioribus; staminibus fertilibus 4 didynamis tubo corollae inclusis; capsula lanceolato-ovoidea acuta 2 mm. longa firmiuscula minute papillosa. — British Honduras: open damp ground, near Sibune River, 4 May, 1906, *Prof. Morton E. Peck*, no. 417a (type, in Gray Herb.). This noteworthy species, through some accident associated with no. 417 (an *Angelonia*), is clearly of § *Brachyhelus* and most nearly approaches the east Brazilian *S. fasciculata* Benth. It differs, however, in its essentially glabrous stem and rhachises, its never fascicled leaves neither perceptibly cuneate at the base nor revolute on the margin, and finally in its lance-ovoid capsule.

Angelonia ciliaris, n. sp., caulibus gracilibus inaequaliter 4-angulatis in angulis conspicue ciliatis; foliis sessilibus oblongo-lanceolatis acutis basi vix angustatis rotundatis 2–2.5 cm. longis ca. 5 mm. latis serratis supra laxe villosis margine ciliatis subtus in costa media solum longiuscule ciliatis aliter glabris; foliis floralibus late ovatis acutis subcordatis conspicue longeque ciliatis, inferioribus ca. 1 cm. longis pedicellum subaequantibus, superioribus ca. 3 mm. longis pedicello triplo brevioribus; ramis inflorescentiae ca. 1. dm. longis racemiformibus, pedicellis oppositis ascendenti-patentibus filiformibus ca. 1 cm. longis apice nutantibus; calycis segmentis lanceolatis acuminatis 3.5 mm. longis; corolla ca. 1 cm. diametro, sacco lato, appendice interiori ca. 0.7 mm. longa; capsula depresse globosa 5 mm. diametro. — BRITISH HONDURAS: on open damp ground, near Sibune River, 4 May, 1906, *Prof. Morton E. Peck*, no. 417 (type, in Gray Herb.). This species differs from *A. angustifolia* Benth. in its conspicuously ciliated stem and leaves, broader-based bracts, and smaller flowers; from *A. salicariaefolia* H. & B. it may be readily distinguished by its smaller flowers and much more sparing pubescence of much longer non-glandular hairs.

Isidorea pungens (Lam.), n. comb. *Ernodea pungens* Lam. Ill. i. 276 (1791). *E. pedunculata* Poir. Encyc. Suppl. ii. 581 (1811). *Isidorea amoena* A. Rich. Mém. sur les Rubiacées, 204, t. 15, f. 1 (1829), and Mém. Soc. Hist. Nat. Par. v. 284, t. 25 (1834).

Bikkia campanulata (Brong.), n. comb. *Grisia campanulata* Brong. Bull. Soc. Bot. Fr. xii. 406 (1865).

Bikkia Pancheri (Brong.), n. comb. *Bikkiopsis Pancheri* Brong. l. c. 405.

Bikkia retusiflora (Brong.), n. comb. *Grisia retusiflora* Brong. l. c. 407.

Houstonia mucronata (Benth.), n. comb. *Hedyotis mucronata* Benth. Bot. Sulph. 19 (1844). *Houstonia fruticosa* Rose, Contrib. U. S. Nat. Herb. i. 132 (1890), 239 (1893); Greenman, Proc. Am. Acad. xxxii. 292 (1897).

Houstonia umbratilis, n. sp., herbacea repens multicaulis ramosa obscure strigillosa; caulibus gracillimis interplexis subquadrangularibus foliosis, nodis radicantibus, internodiis 2–9 mm. longis; foliis parvis ovatis membranaceis acutiusculis brevissime petiolatis utrinque strigillosis subtus paululo pallidioribus uninerviis obscure reticulato-venosis 2.5–4 mm. longis 1.8–3 mm. latis, stipulis brevissimis; pedunculis filiformibus 1.5 cm. longis terminalibus 1-floris; calyce basi turbinato, tubo lobos ovato-lanceolatos acutiusculos anthesi aequante; corolla infundibuliformi in siccitate nigrescenti, tubo 5 mm. longo, lobis ovatis patentibus; staminibus 4 (eis speciminis observati exsertis, antheris lineari-oblongis filamenta aequantibus); fructu seminibusque ignotis.

— MEXICO: shaded cliffs of mountains, near Monterey, Nuevo Leon, 25 April, 1906, *C. G. Pringle*, no. 13,877 (type, in Gray Herb.). An attractive little matted plant with the habit of *H. serpyllifolia* Michx. and *H. serpyllacea* (Schlecht.) C. L. Sm. but differing from the former in its more shortly petioled, more acute leaves, and much smaller flowers, and from the latter in its membranaceous strigillose but unciliated leaves, more filiform stems, etc. The absence of fruit and seeds naturally throws a slight doubt upon the generic position, but the general habit, as well as such technical traits as are manifest, are those of *Houstonia*.

Neurocalyx calycinus (R. Br.), n. comb. *Argostemma calycinum* R. Br. in Bennett, Pl. Jav. Rar. 97 (1838). *Neurocalyx Wightii* Arn. Ann. Nat. Hist. iii. 22 (1839). *N. Hookeriana* Wight, Ic. i. t. 52 (1840).

Rondeletia leptodictya, n. sp., fruticosa 2 m. alta; ramis gracilibus rubro-brunneis flexuosis teretibus mox glabratis; foliis oppositis obovato-oblongis acuminatis basi modice angustatis tenuibus supra viridibus tenuiter (sub lente) reticulatis glabris vel subglabris subtus juventate griseo-tomentosis 6–11 cm. longis 2.5–5 cm. latis; petiolis gracilibus 5–12 mm. longis pubescentibus; stipulis ovato-lanceolatis acutis brunneis 4 mm. longis erectis; pedunculis terminalibus 4–5.5 cm. longis gracilibus arachnoideis; floribus sessilibus dense capitatis; calycis tubo albo-lanato subgloboso 1.8 mm. diametro, lobis limbi 4 vix inaequalibus oblanceolatis viridibus vix 2 mm. longis; corolla sanguinea, tubo gracili sursum vix ampliato 1.4 cm. longo griseo-arachnoideo, lobis limbi 4 patentibus 2–3 mm. longis, ore nudo; stylo exserto. — MEXICO: banks of the Rio Petatlan near the boundary between Michoacan and Guerrero, alt. 500 m., 24 November, 1898, *E. Langlassé*, no. 666 (type, in Gray Herb.). Near *R. elongata* Bartl., but with calyx-lobes much shorter (scarcely a fifth the length of the corolla-tube), the limb of the corolla smaller, and the stipules much shorter than the petioles.

Rondeletia rufescens, n. sp., fruticosa; ramis teretibus tarde glabratis cortice griseo tectis, ramulis et pedunculis et petiolis dense rufo-tomentosis; foliis lanceolato-oblongis 9–15 cm. longis 3.2–5 cm. latis apice basique acuminatis tenuibus supra obscure reticulatis et molliter puberulis subtus albido-tomentosis, nerviis lateralibus ca. 10-jugis; inflorescentiis terminalibus thyrsoideis flexuosis ca. 1.5 dm. longis rufo-tomentosis; cymulis superioribus subsessilibus inferioribus 2–12 mm. longe pedicellatis bracteis lineari-subulatis ca. 3 mm. longis suffultis multifloris; floribus brevissime pedicellatis aut sessilibus; calycis tubo subgloboso minute hirsuto, lobis 4 linearibus inaequalibus intus glabris; corollae tubo gracillimo in fauces distincte ampliato appresse griseo-puberulo vel arachnoideo 1 cm. longo; limbi lobis 4 suborbicularibus 1 mm. longis extus rufo-hispidulis intus et ore nudis; stylo paulo exserto, apice bifido nigro. — *Rondeletia* J. D. Sm. Enum. Pl. Guat. i. 16 (1889). *R. villosa* J. D. Sm. l. c. ii. 94 (1891), not Hemsl. — GUATEMALA: Coban, Depart. Alta Verapaz, alt. 1475 m., March, 1881, *H. von Tuerckheim*, no. 582 of Mr. J. Donnell Smith's distribution (type, in Gray Herb.). This plant is clearly distinct from *R. villosa* Hemsl., which has considerably broader (ovate) stipules and a very different closely matted white pubescence on the lower surface of the leaves, a more slender and denser inflorescence, etc.

Var. **ovata**, n. var., minus rufescens; foliis ovatis brevioribus 7–9 cm. longis basi rotundatis, aliter formae typicae similis. — *R. villosa*, forma *strigosissima* J. D. Sm. Enum. Pl. Guat. vii. 15 (1905), nomen. — GUATEMALA: Tactic, Depart. Alta Verapaz, alt. 550 m., March, 1903, *H. von Tuerckheim*, no. 8401 of Mr. J. Donnell Smith's distribution.

Rondeletia secundiflora, n. sp., arborescens; ramulis gracilibus teretibus dense griseo-strigillosis; foliis ovato-lanceolatis apice basique acuminatis tenuissimis 7–9 cm. longis 2–3.5 cm. latis utrinque appresse pilosiusculis subtus paulo pallidioribus, nerviis ca. 8-jugis; petiolo gracili 4–6 mm. longo griseo-piloso; stipulis a basi deltoidea subulatis 2 mm. longis; inflorescentiis 6–8 cm. longis spiciformibus plus minusve recurvis valde secundis, rhachi hirsutulo, cymulis parvis subsessilibus paucifloris numerosis; floribus deflexis; calycis tubo subgloboso dense patentimque sordido-hirsuto, lobis 4 modice inaequalibus minus dense indutis 1.4–2 mm. longis erectis spatulato-linearibus vel anguste lanceolatis; corolla 9 mm. longa extus strigillosa, tubo gracili cylindrica, limbo 4-lobo, lobis suborbicularibus patulis 1.3 mm. diametro, ore nudo. — GUATEMALA: in woods, along the road from Patin to Esquintla, 21 July, 1860, *Dr. Sutton Hayes* (type, in Gray Herb.). This species is obviously related to *R. capitellata* Hemsl. but may be readily distinguished by the shaggy-hirsute tube and lance-linear or spatulate lobes of the calyx.

Rondeletia septicidalis, n. sp., fruticosa; ramis teretibus plus minusve flexuosis griseo-brunneis; foliis oppositis ovatis vel lanceolato-ovatis apice basique acuminatis firmiusculis 11–16 cm. longis 2–7 cm. latis utrinque viridibus subtus pallidioribus supra glaberrimis subtus basin versus obscure pilosulis, nerviis lateralibus ca. 8-jugis, petiolo 1–2.3 cm. longo glabro vel glabriusculo; stipulis anguste lanceolatis glabris 5 mm. longis acutis; inflorescentiis in axillis superioribus spiciformibus 1–1.5 dm. longis, pedunculo 1.5–3.5 cm. longo gracili tereti, rhachi simillimo obscure arachnoideo;

cymulis vulgatim 2–3-floris breviter pedicellatis bracteolis linearibus suffultis; calyce anguste campanulato basi turbinato, tubo griseo arachnoideo, lobis 4 lanceolato-linearibus deflexis modice inaequalibus tubum subaequantibus glabriusculis; corolla coccinea, tubo gracili subcylindrico sursum paulo ampliato basin versus glabriusculo supra cum limbo patente plus minusve arachnoideo ca. 17 cm. longo, lobis 4 orbicularibus ca. 3 mm. diametro tenuiter margine crispulis; ore nudo; staminibus 4 in ore affixis paulo exsertis, antheris lineari-oblongis; capsula subglobosa ca. 4 mm. diametro septicidali, valvis bifidis. — Mexico: Chicharras, Chiapas, alt. 920–1840 m., *E. W. Nelson*, no. 3755 (type material in U. S. Nat. Mus. and Gray Herb.). This plant possesses so precisely the habit and most of the technical features of a *Rondeletia* that it seems best to refer it to this genus, though it will form an exception among the known species in the fact that its fruit is septicidal.

Hymenodictyon floribundum (Hochst. & Steud.), n. comb. *Kurria floribunda* Hochst. & Steud. Flora, xxiv. pt. 1, Intell. 28 (1841), name only; ibid. xxv. 234 (1842), with description. *Hymenodictyon Kurria* Hochst. Flora, xxvi. 71 (1843).

Bouvardia gracilipes, n. sp., fruticosa; ramis gracilibus teretibus cortice griseo tectis glabris, ramulis valde compressis, internodiis longiusculis glabris, nodis stipulisque puberulis; foliis oppositis breviter petiolatis tenuibus ovatis acuminatis basi rotundatis 5–7 cm. longis 2–3.5 cm. latis supra laete viridibus glabris subtus pallidioribus in costa venisque obscure puberulis; petiolo 2 mm. longo sordide tomentello; ocreis pallidis ca. 1 mm. longis marginem versus tomentellis cum appendicibus filiformibus breviter pubescentibus ca. 2 mm. longis munitis; inflorescentiis terminalibus laxis 8–12-floris glabris; pedunculis 2–4 cm. longis trichotomis, bracteis linearibus 1–3 mm. longis, ramulis lateralibus 3–4 cm. longis vicissim trichotomis; pedicellis filiformibus 1.5–2 cm. longis apice denique uncinatis; calycis dentibus 4 linearibus 1 mm. longis erectis in fructu inflexis persistentibus; corolla non visa; fructu 6 mm. lato 4.5 mm. alto pallide viridi sub lente albido-lineato quasi strigilloso. — Mexico: Tepic, 5 January to 6 February, 1892, *Dr. E. Palmer*, no. 1971 (type, in Gray Herb.). Although this species is described from fruiting material and without knowledge of the corolla, it is believed that the unusually loose inflorescence with filiform at length hooked pedicels yields characters sufficiently distinctive for ready recognition.

Bouvardia longiflora (Cav.) HBK., var. **induta**, n. var., foliis ovato-rhomboideis acutis supra scabriusculo-puberulis subtus tomentosis; corolla extus tomentella. — Mexico: "Chiapas, etc." *Dr.*

Ghiesbreght, the specimen associated in the Gray Herbarium with Ghiesbreght's nos. 108 and 692 which, however, represent the more typical form of the species, being nearly glabrous. Forms to some extent intermediate in their pubescence and somewhat peculiar in their thinnish mostly obtusish leaves are shown by Langlassé's no. 1049 from near the boundary of Michoacan and Guerrero, as well as by Purpus's no. 1249 from Tehuacan, Puebla.

BOUVARDIA TERNIFOLIA (Cav.) Schlecht., var. **angustifolia** (HBK.), n. comb. *B. angustifolia* HBK. Nov. Gen. et Spec. iii. 384 (1818). *B. triphylla*, var. *angustifolia* Gray, Syn. Fl. i. pt. 2, 24 (1884). Although *B. angustifolia* HBK. has been treated as an independent species in various works of recent date, an increasingly complete series of intergrading specimens leaves no doubt that Dr. Gray was right in regarding this plant as merely a variety. Priority of the specific name of Cavanilles requires the new combination.

Lygistum ignitum (Vell.) Ktze., var. **micans** (K. Schum.), n. comb. *Manettia ignita*, var. *micans* K. Schum. in Mart. Fl. Bras. vi. pt. 6, 171 (1889).

Lygistum Rojasianum (Chod. & Hass.), n. comb. *Manettia Rojasiana* Chod. & Hass. Bull. Herb. Boiss. ser. 2, iv. 91 (1904).

Lygistum Smithii (Sprague), n. comb. *Manettia Smithii* Sprague, Bull. Herb. Boiss. ser. 2, v. 267 (1905).

Gonzalagunia bracteosa (J. D. Sm.), n. comb. *Gonzalea bracteosa* J. D. Sm. Bot. Gaz. xxxiii. 252 (1902).

Gonzalagunia leptantha (A. Rich.), n. comb. *Gonzalea leptantha* A. Rich. Fl. Cub. Fanerog. ii. 16 (1853).

Gonzalagunia ovatifolia (J. D. Sm.), n. comb. *Gonzalea ovatifolia* J. D. Sm. Bot. Gaz. xxvii. 336 (1899).

Gonzalagunia Petesia (Griseb.), n. comb. *Gonzalea Petesia* Griseb. Mem. Amer. Acad. new ser. viii. 504 (1863). *Gonzalagunia hirsuta* γ *Petesia* Ktze. Rev. Gen. i. 284 (1891).

Gonzalagunia thyrsoidea (J. D. Sm.), n. comb. *Gonzalea thyrsoidea* J. D. Sm. Bot. Gaz. xiii. 188 (1888).

Tarenna mollis (Wall.), n. comb. *Rondeletia ? mollis* Wall. Cat. no. 8454 (1847). *Webera mollis* Hook. f., Fl. Brit. Ind. iii. 104 (1882).

Tarenna mollissima (Hook. & Arn.), n. comb. *Cupia mollissima* Hook. & Arn. Bot. Beech. 192 (1833). *Stylocorine mollissima* Walp. Rep. ii. 517 (1843). *Webera mollissima* Benth. ex Hance, Jour. Linn. Soc. xiii. 105 (1873).

Tarenna odorata (Roxb.), n. comb. *Webera odorata* Roxb. Hort. Bengal. 15 (1814), and Fl. Ind. i. 699 (1832). *Cupia odorata* DC.

Prod. iv. 394 (1830). *Webera macrophylla* Roxb. Hort. Bengal. 85 (1814), and Fl. Ind. i. 697 (1832). *Cupia macrophylla* DC. l. c.

Casasia nigrescens Wright in herb. *Randia nigrescens* Griseb. Cat. Pl. Cub. 123 (1866), where the combination *Casasia nigrescens* Wright is implied though not definitely made. *Randia nigrescens* Wright & Sauvalle, Fl. Cub. 60 (1873). *Randia nigricans* K. Schum. in Engl. & Prantl, Nat. Pflanzenf. iv. Abt. 4, 77 (1891), by obvious clerical error.

Hamelia hypomalaca, n. sp., fruticosa ramosa; ramis curvatis teretibus cortice brunneo-griseo lenticellifero tectis; ramulis dense tomentellis; foliis ternis ovalibus obtuse acuminatis basi brevissime acuminatis saepe inaequilateralibus 6.5–9 cm. longis 4–5.5 cm. latis membranaceis supra laete viridibus obscure puberulis subtus multo pallidioribus molliter griseo-tomentellis vel denique glabrescentibus; petiolo gracili ca. 2 cm. longo tomentello; cymis terminalibus ca. 9-floris modice laxis tomentellis, ramis recurvis, pedicellis 2–9 mm. longis; floribus pro genere majusculis; calyce tomentello, dentibus brevibus subulatis; corolla flava 4 cm. longa, tubo proprio brevi, faucibus longis ampliatis, limbi lobis 5 late ovatis acuminati-mucronatis; fructu immaturo ca. 8 mm. longo. — MEXICO: State of Durango, 15 August, 1897, *Dr. J. N. Rose*, no. 2304 (type, in U. S. Nat. Mus. and Gray Herb.). Closely related to *H. ventricosa* Sw., but readily distinguished by its tomentulose leaves, loose inflorescence, and somewhat smaller flowers.

Hoffmannia Conzattii, n. sp., fruticosa glabra; ramis subteretibus obsolete solum et obtuse subtetragonis apicem versus foliatis deorsum longe floriferis; foliis obovato- vel oblanceolato-oblongis breviter caudato-acuminatis basi longe attenuatis tenuiter membranaceis utrinque glaberrimis supra in siccitate nigrescentibus subtus pallidioribus viridibus 11–16 cm. longis 3.5–6 cm. latis; costa media supra impressa, nerviis lateralibus ca. 8-jugis oppositis vel alternis; petiolo 1.8–2.5 cm. longo glabro; stipulis ovatis caducis; cymis subsessilibus oppositis lateralibus numerosis subapproximatis ca. 6-floris; pedicellis calycem subaequantibus; tubo calycis subgloboso 2.5 mm. longo, limbo breviter patentimque 4-dentato; corolla ca. 6 mm. longa ad mediam partem 4-fida, lobis anguste oblongis saepissime patentibus; antheris anguste oblongis exsertis; fructu ignoto. — MEXICO: Colonia Melchor Ocampo, Canton de Córdoba, Vera Cruz, alt. 1200 m., *Prof. C. Conzatti*, 19 June, 1896, no. 168 (type, in Gray Herb.). This species in foliage closely resembles *H. calycosa* J. D. Sm., but is readily distinguished by its exceedingly short calyx-lobes. From *H. Ghiesbreghtii* (Lem.) Hemsl. it differs in its subterete wingless branches. *H. longepetiolata* Polak.

appears by its description to have longer petioles and considerably larger flowers.

Hoffmannia cuneatissima, n. sp., fruticosa; ramis teretibus griseis etiam in lignescentia cum pilis brevibus crispis rufescentibus denique sparsis inconspicuisque tectis; foliis oppositis vel ternis deflexis tenuibus acuminatis oblanceolatis 1–1.6 dm. longis 3–4.5 cm. latis basi longissime cuneatis supra glabriusculis subtus paulo pallidioribus praesertim in nerviis venisque crispe puberulis; cymis axillaribus pedunculatis 4–8-floris; pedunculis ad ca. 1 cm. longis ascendentibus gracilibus rufo-pubescentibus; pedicellis 1–2 mm. longis; calyce turbinato-subtereti 2 mm. longo crispe pubescenti, limbi dentibus 4 lanceolati-deltoideis primo suberectis denique patentibus ca. 1.2 mm. longis cum denticulis 4 minimis glandulosis alternantibus; corolla flavida extus puberula ca. 1 cm. longa ad mediam partem 4-fida; lobis oblongis obtusiusculis in media parte crassiusculis dorso carinatis carina crispe puberula; bacca nigrescenti 5 mm. diametro; seminibus numerosis brunneis compressiusculis foveolatis. — Mexico: mountain cañon near Cuernavaca, alt. 200 m., 29 May, 1898, *C. G. Pringle*, no. 7662 (type, in Gray Herb.); and previously in the same locality, 20 Nov., 1895, *C. G. Pringle*, no. 7075 (Gray Herb.) and 31 July, 1896, *C. G. Pringle*, no. 7248 (Gray Herb.). This species belongs clearly to the same group as *H. affinis* Hemsl. and *H. lenticellata* Hemsl., but with its thin, thoroughly membranaceous leaves and rufous-pubescent branches cannot well be placed in either of these species.

Hoffmannia Rosei, n. sp., fruticosa 3 m. alta; ramis flexuosis dense pulverulo-puberulis et obscure strigillosis, internodiis brevibus 5–12 mm. solum longis; foliis oppositis oblanceolatis membranaceis acuminatis basi longe attenuatis 6–12 cm. longis 3.4–5 cm. latis utrinque obscure strigilloso-puberulis vel supra glabriusculis subtus in costa et nerviis lateralibus dense minuteque pulverulo-puberulis; cymis axillaribus oppositis graciliter pedunculatis 5–9-floris subcircinatis; pedunculis 1–1.3 cm. longis pulverulis rubescentibus; pedicellis similibus ca. 2 mm. longis; calyce ovoideo strigilloso, dentibus 4 brevibus anguste deltoideis cum glandulis 4 parvis alternantibus; corolla alba 7 mm. longa pulverula ad partem paulo infra mediam 4-fida, lobis limbi oblongis acutis tenuibus nec carinatis nec pubescentibus. — Mexico: along a brook near Pedro Paulo, Tepic, 3 August, 1897, *Dr. J. N. Rose*, no. 1968 (type, in U. S. Nat. Mus. and Gray Herb.). Very near *H. cuneatissima*, described above, but with opposite leaves, mere puberulence instead of pubescence, and unkeeled corolla-lobes.

Antirrhoea chinensis (Champ.), n. comb. *Guettardella chinensis* Champ. in Hook. Kew. Journ. Bot. iv. 197 (1852).

Timonius polygamus (Forst.), n. comb. *Erithalis polygama* Forst. Prod. 17 (1786). *E. obouata* Forst. l. c. 93, mere mention in index. *Timonius Forsteri* DC. Prod. iv. 461 (1830); Drake del Castillo, Ill. Fl. Ins. Pacif. 193 (1890), which see for further synonymy.

Stylocorine alpestris (Wight), n. comb. *Pavetta ? lucens* R. Br. in Wall. Cat. no. 6168 (1828), name only. *Coffea alpestris* Wight, Ic. t. 1040 (1848–1856). *Webera lucens* Hook. f. Fl. Brit. Ind. iii. 106 (1882), as to var. 1. *Stylocorine breviflora* Schlecht. ex Hook. f., l. c. — Foliis oblanceolatis. Var. **grumelioides** (Wight), n. comb. *Coffea grumelioides* Wight, Ic. t. 1041 (1848–1856). *Webera lucens* Hook. f., l. c. as to var. 2. — Foliis obovatis.

Stylocorine longifolia (G. Don), n. comb. *Ixora macrophylla* R. Br. in Wall. Cat. no. 6165 (1828), name only, not Bartl. *Ixora longifolia* G. Don Syst. iii. 573 (1834). *Pavetta longifolia* Miq. Fl. Ind. Bot. iii. 275 (1856–1859). *Webera longifolia* Hook. f. Fl. Brit. Ind. iii. 105 (1882).

Rudgea crassiloba (Benth.), n. comb. *Coffea crassiloba* Benth. in Hook. Jour. Bot. iii. 233 (1841). *Rudgea Schomburgkiana* Benth. Linnaea, xxiii. 459 (1850).

CEPHAELIS ELATA Sw. Prod. 45 (1788). Here apparently belongs *Cephaleis punicea* Vahl., Eclog. i. 19 (1796) and consequently *Uragoga punicea* K. Schum. in Engl. & Prantl, Nat. Pflanzenf. iv. Abt. 4, 120 (1891), a name which, through apparent clerical error, has been cited by Durand & Jackson, Ind. Kew. Suppl. 1, 445 (1906), as "*Uragoga phoenicea* K. Schum," a combination said by them to equal "*Palicourea punicea* R. & P." However, Ruiz & Pavon do not appear to have created any such binomial, though DeCandolle's *Palicourea punicea* (Prod. iv. 526, 1830) was based upon *Psychotria punicea* R. & P. Fl. Per. ii. 62, t. 212 fig. a (1799), a species obviously not of *Cephaelis.* Schumann's "*Uragoga phoenicea,*" which seems never to have been published by its supposed author, appears to have given rise to *Cephaelis phoenicea* J. D. Sm. Pl. Guat. v. 39 (1899), which as to plants cited is clearly *C. elata* Sw.

Cephaelis sphaerocephala (Muell. Arg.), n. comb. *Psychotria sphaerocephala* Muell. Arg. Flora, lix. 550, 553 (1876).

Nertera Arnottianiana (Walp.), n. comb. *Leptostigma Arnottianum* Walp. Rep. ii. 463 (1843). *Hedyotis repens* Clos in Gay, Fl. Chil. iii. 208 (1847). *Coprosma calycina* Gray, Proc. Am. Acad. iv. 306 (1860).

Coprosma australis (A. Rich.), n. comb. *Ronabea ? australis* A. Rich. Voy. Astrolabe Bot. i. 265 (1832). *Coprosma grandifolia* Hook.

f. Fl. N. Z. i. 104 (1853). *Pelaphia grandifolia* Banks & Soland. ex Hook. f., l. c.

Coprosma quadrifida (Labill.), n. comb. *Canthium quadrifidum* Labill. Nov. Holl. Pl. i. 69, t. 94 (1804). *Marquisia Billardierii* A. Rich. Mém. sur les Rubiacées, 112 (1829), & Mém. Soc. Hist. Nat. Par. v. 192 (1829). *Coprosma Billardieri* Hook. f. in Hook. Lond. Jour. Bot. vi. 465 [bis] (1847). *Coprosma microphylla* A. Cunn. ex Hook. f., l. c.

Richardia muricata (Griseb.), n. comb. *Richardsonia muricata* Griseb. Cat. Pl. Cub. 143 (1866). *Spermacoce* (*Borreria*) *richardsonioides* Wright in Sauv. Fl. Cub. 73 (1873).

Crusea hispida (Mill.), n. comb. *Crucianella hispida* Mill. Dict. ed. 8, no. 4 (1768). *Spermacoce rubra* Jacq. Hort. Schönb. iii. 3, t. 256 (1798). *Crusea rubra* Schlecht. & Cham. Linnaea, v. 165 (1830).

Borreria asperifolia (Mart. & Gal.), n. comb. *Diphragmus scaber* Presl, Bot. Bemerk. 81 (1844), not *Borreria scabra* (Schum. & Thonn.) K. Schum. *Spermacoce asperifolia* Mart. & Gal. Bull. Acad. Brux. xi. pt. 1, 132 (1844).

Borreria nesiotica n. sp., suffrutescens glaberrima 4 dm. vel ultra alta ramosa; ramis ascendentibus subteretibus parte superiori 4-angulatis basim versus foliosissimis saepe purpurascentibus; foliis oppositis anguste lanceolatis basi apiceque attenuatis laevissimis etiam ad marginem paulo revolutum 2–4.5 cm. longis 3–12 mm. latis modice venosis subtus paululo pallidioribus axillis saepe proliferis; verticillis plerisque 4 distantibus 9–12 mm. diametro hemisphaericis a bracteis 2 majoribus oppositis 1–2 cm. longis ovato-lanceolatis obtusiusculis basi ampliato setoso-dentatis et ca. 4 minoribus ovatis obtusis 5 mm. longis suffultis; calyce glabro breviter et subaequaliter 4-lobato cum dentibus intermediis brevissimis; corolla glabra; staminibus exsertis; stigmate brevissime bilobato; seminibus papillosis nigris non transverse sulcatis. — *Spermacoce* (*Boneria*), sp. Vasey & Rose, Proc. U. S. Nat. Mus. xiii. 148 (1890). *Spermacoce* sp. Brandegee, Zoe, v. 27 (1900). — SOCORRO ISLAND (of the Revillagigedo Group), *A. W. Anthony*, 1897 (type, in Gray Herb.); previously collected by *C. H. Townsend*, March, 1889; and later by *F. E. Barkelew*, 27 May to 3 July, 1903, no. 208. In habit somewhat resembling *B. verticillata* (L.) G. F. W. Mey., but readily distinguished by its 4-lobed calyx. Also somewhat like forms of the highly variable *B. tenella* (HBK.) Cham. & Schlecht., but having much shorter calyx-lobes (about one third the length of the tube), glabrous foliage, etc.

Borreria rhadinophylla, n. sp., gracillima ramosa prostrata, caulibus elongatis valde flexuosis obsolete quadrangularibus foliosis tenuiter patenteque pubescentibus plus minusve rubescentibus fere filiformibus sed basim versus induratis et lignescentibus, nodis hirsutulis; foliis anguste linearibus subfiliformibus 1-nerviis glabris margine revolutis apice acutissimis 1–2 cm. longis; vaginis brevissimis pauci- (saepius 2-) setis; verticillis remotis plerumque 2 subglobosis ca. 1 cm. diametro; calyce longe 2-lobato, lobis lanceolato-linearibus acutissimis herbaceis sursum fimbriato-ciliatis, dentibus intermediis multo brevioribus scariosis; corolla alba hypocraterimorpha 4-loba 2.5 mm. longa, lobis ovato-oblongis apicem versus hispidis, tubo intus basim versus pubescente; staminibus 4 in summa parte tubi affixis, leviter exsertis; fructu et seminibus non visis. — BRITISH HONDURAS, on dry sandy pine ridges, 23 October, 1905, *Prof. Morton E. Peck*, no. 180 (type, in Gray Herb.). From its 2-lobed calyx this species would seem to stand near the polymorphous *B. verticillata* (HBK.) Cham. & Schlecht. but with all due recognition of the extraordinary variability of that species, it does not seem possible that this delicate filiform plant should be included among its forms. Among the distinctions noted is the form of the stigma, which in *B. verticillata* is barely lobed, but in *B. Peckiana* distinctly bifid with short but actually filiform lobes.

BORRERIA VERTICILLATA (L.) G. F. W. Mey., var. **thymiformis**, n. var., pumila 6–8 cm. alta subglabra; caulibus multis gracilibus laxis flexuosis a caudice crassa nigrescente oriuntibus; foliis ovato-ellipticis 7–11 mm. longis 2–5 mm. latis; capitibus parvis ca. 8 mm. diametro terminalibus. — MEXICO: about 29 km. southwest of the city of Oaxaca, alt. 2300–2900 m., 10–20 September, 1894, *E. W. Nelson*, no. 1410 (type, in Gray Herb. and Herb. U. S. Nat. Mus.). This plant, although maintaining all the floral traits of the species, is so strikingly different from the usual forms as to be well worthy of varietal distinction. Were it not connected with the more typical forms by such intermediates as L. C. Smith's no. 40 from the Cuilapan Mountains, it could certainly pass as a distinct species.

Erigeron Deamii, n. sp., suffruticulus gracillimus pumilus 1 dm. altus irregulariter a basi ramosus, ramis teretibus strigosis foliosissimis ascendentibus saepius 1-capitatis; foliis linearibus (infimis anguste oblanceolatis) ca. 1 cm. longis ca. 1 mm. latis utrinque strigilloso-hispidulis 1-nerviis saepe in axillis proliferis; pedunculis filiformibus ca. 3 cm. longis rectis vel apicem versus plus minusve nutantibus 1-capitatis subappresse pubescentibus; capitibus hemisphaericis ca. 8 mm. diametro; involucri squamis argute linearibus attenuatis subaequalibus media parte viridibus hirsutulis margine pallidis scariosis ca. 4 mm. longis; flosculis disci numerosis, corollis 2.3 mm. longis apicem versus flavidulis, achaeniis compressis sparse hirsutulis 1.3 mm.

longis, pappi setis ca. 12 tenuibus albis 2.4 mm. longis; flosculis liguliferis ca. 40, ligulis angustis albis vel purpureo-tinctis tubo subaequilongis apice saepissime bidentatis, achaeniis et pappi setis eis flosculorum disci similibus. — GUATEMALA: growing on rocks in bottom of cañon, Fiscal, Guatemala, alt. 1130 m., 3 June, 1909, *Charles C. Deam*, no. 6159 (type, in Gray Herb.). This species is obviously of the affinity of *E. mucronatus* DC., *E. exilis* Gray, and *E. Karwinskianus* DC. From the first of these it differs in having narrower (linear rather than lanceolate) leaves, smaller heads, and relatively as well as absolutely shorter rays (exceeding the disk scarcely by one third). *E. exilis* Gray has the involucral bracts and peduncles very much more closely and finely puberulent, and *E. Karwinskianus* DC. is described as having the leaves glabrous on both surfaces.

Verbesina medullosa, n. sp., frutescens 1.2–1.8 m. alta; caulibus crassiusculis teretibus foliosis medullosis omnino exalatis juventate tomentellis serius subglabratis; foliis alternis ovatis majusculis 1.2–1.5 dm. longis 4–6 cm. latis crenato-serratis penninerviis supra scabris puberulis viridibus subtus griseo-tomentellis apice attenuatis caudato-acuminatis basi in petiolum alatum biauriculatum sensim angustatis, alis petioli transverse valde rugosis margine integriuscula revoluta; capitulis numerosis parvis 9 mm. altis in corymbis compositis planiusculis bracteatis dispositis; involucri subturbinati squamis villoso-tomentellis pallide viridibus apicem versus purpurascentibus; flosculis disci ca. 20, corollis albidis 4 mm. longis tubo extus puberulo dentibus limbi suberectis brevibus deltoideis, flosculis liguliferis ca. 3 fertilibus, ligulis ovalibus parvis albis tubo vix longioribus; achaeniis valde immaturis obovatis valde compressis margine sursum ciliolatis apice biaristatis. — GUATEMALA: along railway, Fiscal, alt. 1130 m., 9 June, 1909, *Charles C. Deam*, no. 6250 (type, in Gray Herb.). This species differs in its wingless stem and branches from such forms of *V. turbacensis* HBK. as have unlobed leaves. From *V. sublobata* Benth., it may be distinguished by its more bluntly toothed (crenate-serrate) unlobed leaves which are more gradually narrowed to the winged petiole.

Trixis Deamii, n. sp., fruticosa 1.5 m. alta laxe ramosa; ramis exalatis teretibus gracilibus griseis glabratis; ramulis striatulis viridibus tomentellis foliosis; foliis rhomboideo-obovatis acute acuminatis basi subabrupte angustatis subintegris tenuibus supra atroviridibus pilosiusculis planis subtus griseo-sericeis 3.5–7 cm. longis 1.5–3 cm. latis nullo modo decurrentibus; petiolo ca. 4 mm. longo gracili villosulo subtus carinato; capitulis prope apicem ramulorum aggregatis ca. 2 cm. longis 12-floris a foliis longioribus plus minusve excessis et obscuratis;

bracteis involucri exterioris ca. 4 elliptico-lanceolatis alternis acuminatis ca. 12 mm. longis tenuibus foliis similibus; squamis involucri proprii 8 lanceolati-linearibus attenuatis ca. 14 mm. longis dorso glanduloso-puberulis medio herbaceis margine subscareosis demum stellato-patentibus divaricatis apice falcatis; corollis ca. 1 cm. longis laete flavis; achaeniis 5 mm. longis columnaribus papilloso-setulosis; pappi setis albo-fulvescentibus ca. 9 mm. longis. — GUATEMALA: along river, alt. 230 m., Zacapa, 19 June, 1909, *Charles C. Deam*, no. 6359 (type, in Gray Herb.). This shrub differs from such related species as *T. megalophylla* Greenman, *T. silvatica* Robinson & Greenman, *T. Nelsonii* Greenman, and *T. rugulosa* Robinson & Greenman, in its much thinner, flatter, softer, and essentially entire leaves of rhombic-obovate form. From *T. frutescens* P. Browne and its relatives the present plant is readily distinguished by its larger outer involucre, the silky under surface of its leaves, etc.

Chaptalia semifloscularis (Walt.), n. comb. *Perdicium semiflosculare* Walt., Fl. Car. 204 (1788). *Chaptalia tomentosa* Vent. Desc. Jard. Cels, t. 61 (1800). *Tussilago integrifolia* Willd. Sp. Pl. iii. 1964 (1804). *Gerbera Walteri*, Sch. Bip. in Seem. Voy. Herald. 313 (1856). *Thyrsanthema semifloscülare* (Walt.) Ktze. Rev. Gen. i. 369 (1891).

III. AMERICAN FORMS OF LYCOPODIUM COMPLANATUM.

By C. A. Weatherby.

Lycopodium complanatum L. occurs in the western hemisphere in two distinct and geographically isolated areas. In the north, it ranges from Newfoundland to Alaska, and southward to northern Idaho and (in its variety *flabelliforme*) to the mountains of North Carolina. It is apparently entirely absent from the United States south of these points; but it reappears in south-central Mexico and extends thence through Central America to Bolivia and southern Brazil. It has also been reported from the West Indies. Specimens from these areas show, on examination, four more or less well-marked variant tendencies — two (one with a subsidiary variation) in the north, and in the south, two others, separable from each other and from both of the northern forms.

The northern forms have been clearly distinguished by Prof. Fernald.[1] The two southern (one chiefly Mexican, the other chiefly

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[1] Rhodora, iii. 280 (1901).

South American) are connected by various intermediates, but, in their extreme development, are sufficiently diverse to warrant varietal distinction. Indeed, since Humboldt and Bonpland described their *Lycopodium thyoides* in 1810, it has been recognized by most botanists that some, at least, of the tropical material differed from typical *L. complanatum* of northern Europe and North America; and *L. thyoides* has been rather generally maintained as a variety, differently defined by different authors. Neither its relation to the northern forms, however, nor its exact identity in regard to the other tropical form seems to have worked out with entire clearness. Lloyd and Underwood, in their Review of the North American Species of Lycopodium,[2] called attention to the habital difference between Mexican and Central American, and northern specimens; but, partly owing, no doubt, to their reluctance to describe varieties, carried their studies no further. Dr. Christ,[3] in a brief but clear note, has pointed out the distinctions between the two southern forms; but he seems to be in error in referring the prevailing South American form to typical *L. complanatum.* The plant of northern Europe and America which, as Prof. Fernald has shown, should be regarded as the type of the Linnaean species, is low, and habitally as well as in the characters of its branchlets and their leaves, quite different from the taller South American plant. Dr. Christ seems also to have been in error in identifying the other tropical extreme, which has broad branchlets and long leaves with conspicuously spreading tips, with *L. thyoides* H. & B. The original description of this species in Willd. Sp. Pl. v. 18, emphasizes rather strongly the appressed leaves.[4] In view of the facts that the type specimens were from Venezuela, and that the appressed-leaved form is apparently much the more common throughout South America, it seems best to follow the first diagnosis, and to restrict *L. thyoides* to that form.

In spite of their complete geographic separation, there is nothing to warrant the segregation of the tropical forms as separate species. The characters which distinguish them are of too little importance in themselves and too inconstant. They are rather to be considered as extreme developments of tendencies which are traceable also in occasional specimens of the northern plant, but are there not so strongly developed. The earliest varietal designation of the South American plant and that which, under the Vienna Rules, it should bear, is *L. complanatum, β tropicum* Spring, based on *L. thyoides* H. & B. The other, prevailingly Mexican, extreme seems to be without an available name.

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[2] Bull. Torr. Bot. Club, xxvii. 165 (1900).

[3] Bull. Herb. Boiss., sér. 2, ii. 707 (1902).

[4] "foliis semper adpressis."

The following synopsis will serve to define these American tendencies of *L. complanatum*, as understood by the writer. The specimens cited are all in the Gray Herbarium.

* Branchlets ascending, or, if spreading, lax and irregular; ultimate branchlets often more or less elongated.

+— Ultimate branchlets comparatively broad, 2–5 mm. wide, conspicuously flattened, usually ascending and only moderately elongated; their leaves 3–5 mm. long.

Lycopodium complanatum L. Branches mostly not over 3 dm. long; peduncles bearing 1–2(–4) spikes; tips of the lateral leaves usually appressed or incurved. — Sp. Pl. 1104 (1753), excl. citation of Dill. Musc. t. 59 f. 3. — North America: Newfoundland to Alaska, south to Maine and northern Idaho. Also in Eurasia.

Var. **validum,** nom. nov. More robust; branches usually 3–4.5 dm. long; peduncles bearing 4–6(–9) spikes; tips of the lateral leaves conspicuously spreading. — *L. complanatum* Fourn. Enum. Pl. Mex. i. 146, at least in part, not L.; Hemsl. Biol. Cent.-Am. Bot. iii. 701, at least in part, not L. *L. complanatum*, var. *thujoides* Christ, Bull. Herb. Boiss. sér. 2, ii. 707 (1902), not *L. thyoides* H. & B. — Mexico: Chiapas; Bergwald zwischen San Cristobal Las Casas und Huitztan, *C. & E. Seler*, no. 2273; Chiapas "etc.," *Ghiesbreght*, no. 600; Oaxaca, Cerro San Felipe, alt. 2000 m., *Gonzalez & Conzatti*, no. 889; region d'Orizaba, *Bourgeau*, no. 3159, in part; Hidalgo, Trinidad, *C. G. Pringle*, no. 11,856 (a form with the ultimate branchlets lax, elongated, and somewhat attenuate at tip). No. 3196 in John Donnell Smith's Plants of Guatemala shows a form intermediate between this and the following variety.

+— +— Ultimate branchlets narrow, not more than 2 mm. wide, less conspicuously flattened, somewhat convex above, sometimes much elongated (to 12 cm.) and loosely spreading; their leaves 2–3 mm. long, the tips usually closely appressed.

Var. tropicum Spring in Mart. Fl. Bras. i. pt. 2, 116 (1840). *L. thyoides* H. & B. in Willd. Sp. Pl. v. 18 (1810); ? HBK. Nov. Gen. et Sp. i. 38 (1815); Presl, Rel. Haenk. 77 (1825); Raddi, Fil. Bras. 80 (1825), at least in part. *L. complanatum β adpressifolium* Spring, Monog. Lycopod. i. 102 (1842), excl. syn. *L. anceps* Wallr. *L. complanatum*, "var. *L. thuyoides* HBK." Baker, Handb. of the Fern Allies, 28 (1887). *L. complanatum*, var. *thyoides* Hieron. Engl. Bot. Jahrb. xxxiv. 576 (1905). — Colombia: *Moritz;* Santa Marta, *Purdie.* Ecuador: in Andibus quitensibus, *Jameson;* Andibus, *Spruce*, no. 5412 (a doubtful plant which seems to have suffered some injury to its leaves). Peru:

Andes, *Jameson.* BOLIVIA: Yungas, *Bang*, no. 395. BRAZIL: *Riedel; Claussen;* Herb. U. S. So. Pac. Expl. Exp., no. 27; Prov. Minas Geraes, *Widgren*, no. 984½. Burchell's no. 2223, from Brazil, of which the specimen in the Gray Herb. shows only the tip of a stem, is perhaps referable to var. *validum.*

** Branchlets spreading or recurved, forming a regular flabelliform spray; ultimate branchlets usually short, 0.5 to 4 cm. long, broad as in *L. complanatum* but with shorter leaves.

Var. FLABELLIFORME Fernald. Peduncles usually bearing 4 spikes. — Rhodora, iii. 280 (1901). *L. complanatum* Amer. auth. in part. — NORTH AMERICA: Nova Scotia to the mountains of North Carolina, Kentucky, Iowa, and Minnesota.

Var WIBBEI Haberer. Peduncles 1-spiked. — Rhodora, vi. 102 (1904). NORTH AMERICA: northern Vermont and central New York.

IV. NEW AND LITTLE KNOWN MEXICAN PLANTS, CHIEFLY LABIATAE.

BY M. L. FERNALD.

Juncus albicans, n. sp., caespitosus; caulibus 5–7 dm. altis tenuibus striatis albido-viridibus; vaginis basilaribus laxis albicantibus demum fuscis, auriculis cartilagineis, laminis subteretibus anguste canaliculatis; inflorescentiis decompositis 2–6 cm. longis, ramis suberectis, floribus subremotis vel aggregatis; bractea infima frondosa inflorescentiam plerumque superante; floribus 4–5 mm. longis albido-stramineis; bracteolis tenuibus albicantibus; sepalis petalisque subaequilongis patentibus lanceolatis apice subulatis anguste membranaceo-marginatis; staminibus 6 sepalis circa dimidio brevioribus, antheris filamentisque aequantibus; fructibus trigono-ellipsoideis truncatis breve mucronatis 3–4 mm. longis pallide stramineis nitidis; seminibus 0.5 mm. longis oblique ellipsoideis brevissime albo-caudatis. — CHIHUAHUA: vicinity of Chihuahua, altitude about 1300 m., May 1–21, 1908, *Edward Palmer*, no. 161 (type, in Gray Herb.). [It should be noted that two plants have been distributed under no. 161, but, as the other belongs in the *Cruciferae*, little confusion is likely to result.] Nearly related to *J. dichotomus* Ell. of the southern and eastern United States. Differing in its very pale color, the softer texture of the prophylla, perianth, and capsule, and the distinctly white-caudate longer seeds.

Palmer's no. 253, collected May 28–31, 1906, at Tobar, Durango, is provisionally placed with *Juncus albicans*, though it may eventually prove to be distinct. It has less cartilaginous auricles, smaller flowers, and more ascending sepals, but the material at hand is over-mature and has lost all its seeds.

Juncus Pringlei, n. sp., dense caespitosus; caulibus erectis gracilibus rigidis 1.5–2.5 dm. altis sulcatis; cataphyllis basilaribus mucroniferis stramineis, supremis laminigeris lamina 4–10 cm. longa; inflorescentia densa 3–7-flora a bractea infima vix superata; floribus 4.5–5 mm. longis; sepalis lanceolatis petala subaequantibus apice subulatis dorso crassis viridibus lateribus castaneis marginibus membranaceis pallidis; staminibus 6, antheris linearibus flavidis quam filamentum longioribus; fructibus trigono-ellipsoideis mucronatis nitidis pallide castaneis vel olivaceis 5–6 mm. longis; seminibus 0.4 mm. longis ellipsoideis mucronatis. — OAXACA: Cuesta de San Juan del Estado, altitude 2125 meters, August 31, 1894, *C. G. Pringle*, no. 5818 (type, in Gray Herb.). An interesting addition to the little group of species, *J. Drummondii* E. Meyer, *J. Parryi* Engelm., and *J. Hallii* Engelm., all of which are confined to the cordillera of western North America. *J. Pringlei* closely simulates *J. Hallii* of Colorado and Utah, but differs in its blunt-pointed, not retuse, capsule; and, unlike any of its three allies, it has mucronate instead of caudate-appendaged seeds.

Scutellaria spinescens, n. sp., fruticosa 1–2 dm. alta; caule crasso tortuoso cortice cinereo, ramis implicatis rigidis spinescentibus cinereohirtellis, pilis minutis; foliis ellipticis vel oblongis integris breve petiolatis rugosis cinereo-hispidulis, majoribus 1 cm. longis; floribus axillaribus; pedicellis 5 mm. longis; calyce 2.5–3 mm. longo glanduloso-hispido; corolla curvata pilosa 2 cm. longa flava vel rubella, tubo anguste cylindrico. — COAHUILA: by a brook in San Lorenzo Cañon, near Saltillo, September 21–23, 1904, *Edward Palmer*, nos. 392 (type, in Gray Herb.) and 394. A characteristic dwarf shrub closely simulating *S. suffrutescens* Watson, which, however, has very minutely pulverulent glandless branches, leaves, and calyx. The corolla of *S. spinescens*, as shown by Dr. Palmer's material, is very variable in color (as is that of *S. suffrutescens*); the material under no. 392 having the corolla canary-yellow passing to salmon, with the galea reddish; while no. 394 has the corolla of various shades of red, with yellow only on the sides of the galea.

SALVIA SANCTAE-LUCIAE Seem. Bot. Herald, 327 (1856). In the writer's synopsis of Mexican Salvias (Proc. Am. Acad. xxxv. 514), this plant was placed in the *Vulgares* and was taken to be the same as a plant of that section collected by Dr. Edward Palmer in Tepic. Subsequently the writer has studied Seemann's original material at Kew and it proves to be, not a plant of the *Vulgares* as stated by Seemann in the original description, but a characteristic member of the *Membranaceae.* It is identical with the Tepic plant which the writer has described as *S. cladodes* (Proc. Am. Acad. xxxv. 497).

Salvia (Membranaceae) Langlassei, n. sp., suffruticosa; caule gracile duro flexuoso obtuse quadrangulato, ramis sordido-villosis; foliis ramorum membranaceis lanceolatis vel anguste ovatis basi rotundatis apice acuminatis 3–4.7 cm. longis 1.3–1.8 cm. latis acute serratis supra strigosis venis subtus pilosis, petiolis 5–10 mm. longis; racemo elongato; verticillis 9–14-floris demum 2–2.5 cm. distantibus; bracteis reniformibus acuminatis 6–9 mm. longis glabris lucidis purpurascentibus; pedicellis 4 mm. longis glanduloso-hispidis; calyce campanulato purpurascente glanduloso-hispido fructifero 8 mm. longo, labiis subaequalibus, superiore late ovato 1.5 mm. longo, inferiore cum lobis ovatis mucronatis; corolla violacea. — MICHOACAN or GUERRERO: in argillaceous soil of the Sierra Madre at 1700 meters altitude, January 27, 1899, *Langlassé*, no. 805 (type, in Gray Herb.). Closely related to *S. Sanctae-Luciae* Seem., but with slender stems said by M. Langlassé to be "volubile," thinner leaves with very different pubescence, and with shorter, broader calyx-lobes.

Salvia (Angustifoliae) urolepis, n. sp., herbacea circa 1 m. alta; caulibus gracilibus retrorse pubescentibus, pilis brevibus cinereis; foliis late lanceolatis vel anguste ovatis basi subcuneatis apice acutis 3.5–5 (–9) cm. longis crenato-serratis supra viridibus puberulis subtus albo-pannosis, petiolis gracilibus 1–2 cm. longis pilosis; racemis gracilibus, primariis 1.2 demum 3 dm. longis; bracteis lanceolato-attenuatis 9–13 mm. longis deciduis; verticillis 12-floris demum 3–3.5 cm. distantibus; calyce tubuloso-campanulato fructifero 6–7 mm. longo caerulescente albido-piloso, labiis subaequalibus, superiore late ovato mucronato, inferiore cum lobis deltoideo-ovatis subaristatis; corolla azurea 12–16 mm. longa, tubo exserto, galea oblonga 4–6 mm. longa pilosa, labio inferiore 6–9 mm. longo cum lobo medio valde majore; stylo piloso. — NUEVO LEON, by brooks of the Sierra Madre above Monterey, August 25, 1903, September 4, 1904, and September 19, 1907, *C. G. Pringle*, nos. 11,906, 13,281, and 13,978 — all collected from the same colony (type, in Gray Herb.). Apparently most nearly related to *S. oblongifolia* Mart. & Gal., which differs in its narrower glabrous leaves, shorter and broader bracts, and the greener somewhat viscid puberulence of the calyx.

SALVIA LAVANDULOIDES HBK., var. LATIFOLIA Benth. Pl. Hartw. 21 (1839), and in DC. Prodr. xii. 303 (1848) as *nomen nudum;* Fernald, Proc. Am. Acad. xxxv. 506 (1900). A fine collection of this plant, made by Mr. E. W. Nelson at an altitude of 2125–3040 m. on Mt.

Patamban, MICHOACAN, January 28–31, 1903 (no. 6575), exactly matches Hartweg's no. 171 which is the type of the variety. In studying the variety in the light of this more adequate material an important character is noted in the glabrous or glabrate lower surface of the leaves, those of typical *S. lavanduloides* being canescent-tomentose beneath.

Salvia (Angustifoliae) moniliformis, n. sp., caulibus altis minute pilosis; ramis elongatis valde ascendentibus; foliis ramorum lanceolatis utrinque acutis 3–4 cm. longis crenato-serratis supra viridibus trigosis subtus pallidis pilosis; racemis spiciformibus demum 3–4 dm. longis; verticillis 10–40-floris demum 8–9 cm. distantibus; bracteis lanceolato-ovatis attenuatis caeruleis albido-pilosis deciduis; pedicellis 1–2 mm. longis; calyce cylindrico albido-caeruleo piloso costato fructifero 8 mm. longo, labiis subaequalibus lanceolato-attenuatis 3 mm. longis; corolla caerulea circa 8 mm. longa, tubo paulo exserto, galea puberula, labio inferiore multo longiore. — MEXICO: open woods on hillside at 2735 meters altitude, Iztaccihuatl, January, 1906, *C. A. Purpus*, no. 1720 (type, in Gray Herb.). Distributed as *S. lavanduloides* HBK., but more nearly related to *S. remota* Benth., which, however, has much smaller calyces (in maturity 4 mm. long) which are less prominently bilabiate.

Salvia (Vulgares) lilacina, n. sp., herbacea 1–1.5 m. alta; caulibus minute puberulis valde sulcatis purpurascentibus; foliis ovatis acuminatis basi rotundatis 4–6 cm. longis serratis supra minute strigosis venis subtus strigosis, petiolis 5–10 mm. longis; racemis gracilibus permultis 6.5–12.5 cm. longis; verticillis 10–20-floris approximatis demum 1 cm. distantibus; bracteis lanceolato-aristatis 1.5 mm. longis caducis; pedicellis 2–3 mm. longis; calyce purpurascente tubuloso-campanulato 3–3.5 mm. longo strigoso, labio superiore ovato acuminato 1 mm. longo, labio inferiore cum lobis subaristatis; corolla lilacina 12 mm. longa pilosa, tubo ventricoso exserto, galea labiam inferiorem subaequante; stylo piloso. — MICHOACAN: near Uruapan, October 15, 1904, *C. G. Pringle*, no. 13,279 (type, in Gray Herb.). Closely related *S. Ghiesbreghtii* Fernald, which has the midrib of the leaf densely lanate beneath, the puberulence of the branches coarser, and the few racemes more elongate.

Salvia (Vulgares) uruapana, n. sp., herbacea annua, 7 dm. alta; caule gracile minute piloso, pilis retrorsis appressis, internodiis 3.5–10 cm. longis; foliis ovatis subcordatis acuminatis 4–5 cm. longis 2.6–3.5 cm. latis crenato-serratis supra pallide viridibus minute puberulis vel glabratis subtus cinereis minute pilosis vel glabratis, margine piloso-ciliato; racemis elongatis, primariis 3 dm. longis; verticillis 3–10-floris demum 3 cm. distantibus; bracteis lanceolato-caudatis demum 7–10 mm. longis; pedicellis demum 6–7 mm. longis tenuibus albido-pilosis; calyce tubuloso-campanulato fructifero 9 mm. longo 3 mm. diametro cinereo-piloso valde bilabiato, labio superiore oblongo acuminato 2.5 mm. longo, inferiore rectiusculo 4 mm. longo cum lobis lanceolato-aristatis; corolla azurea 12 mm. longa, tubo vix exserto, galea brevissima pilosa, labio inferiore multo longiore; stylo glabro. — MICHOACAN: lava fields, Uruapan, October 16, 1904, *C. G. Pringle*, no. 13,280 (type, in Gray Herb.). Strongly simulating *S. leptostachys* Benth., from which it differs in its much longer, more slender, and unequally cleft greener calyx, the longer, more pubescent pedicels, and the more copiously pilose leaf-margin.

Salvia (Vulgares) lenta, n. sp., caulibus lentis gracilibus 5 dm. altis pilosis, pilis cinereis nodulosis; foliis ovatis acuminatis basi subcuneatis 6.5–9 cm. longis 3.5–4 cm. latis argute serratis utrinque pilosis; petiolis 1–1.5 cm. longis; racemo elongato 2 dm. longo; verticillis 8–12-floris demum 1.5–2 cm. distantibus; bracteis lanceolato-ovatis acuminatis pilosis deciduis; pedicellis demum 2–3 mm. longis pilosis; calyce tubuloso-campanulato circa 4 mm. longo dense piloso, pilis albidis nodulosis, labio superiore ovato obtuso 1 mm. longo, inferiore breviore cum lobis deltoideis acutis; corolla caerulea minute pilosa 1 cm. longa, tubo exserto, labiis subaequalibus; stylo piloso. — MICHOACAN OR GUERRERO: in granitic soil, at 1100 meters altitude, Real de Guadelupe, September 10, 1898, *Langlassé*, no. 343 (type, in Gray Herb.). Nearly related, apparently, to *S. Warszewicziana* Regel, which has broad cordate acuminate bracts, a secund inflorescence, and the lips of the corolla very unequal, the upper glandular.

Salvia (Vulgares) fallax, n. sp., fruticosa; ramis gracilibus elongatis lignosis brunnescentibus juventate dense sordido-villosis, pilis nodulosis; foliis ovatis acuminatis basi subcuneatis 6–11 cm. longis 3.5–6 cm. latis argute serratis utrinque pilosis, pilis albidis nodulosis; petiolis gracilibus villosis 2–5 cm. longis; racemis gracilibus 1–1.5 dm. longis; verticillis 3–6-floris demum 1 cm. distantibus; bracteis atropurpureis anguste ovato-caudatis deciduis; pedicellis demum 2 mm. longis; calyce atro-purpureo tubuloso-campanulato hirsuto fructifero 5–6 mm. longo, labio superiore ascendente ovato acuminato, labio inferiore rectiusculo 1.5 mm. longo cum lobis deltoideo-aristatis; corolla azurea 9 mm. longa, tubo vix exserto, galea villosa, labio inferiore paulo breviore; stylo piloso. — *S. Sanctae-Luciae* Fernald, Proc. Am. Acad. xxxv. 514 (1900), not Seemann. — TEPIC: near the town of Tepic, January and February, 1892, *Edward Palmer*, no. 1964 (type, in Gray Herb.). Closely related to *S. lenta* Fernald and apparently also to *S. Warczewicziana* Regel. In the writer's synopsis of *Salvia* published in 1900 he mistook this plant, from the description alone, for *S. Sanctae-Luciae* Seem.; but he has since examined Seemann's type and finds that it is not this plant but a species of the *Membranaceae* (see above).

Salvia (Scorodoniae) rupicola, n. sp., fruticosa; ramis gracilibus subteretibus lignosis albescentibus cortice fibrilloso, juventate brunnescentibus glanduloso-pilosis; foliis oblongis vel anguste ovatis crenatis utrinque obtusis 1–2 cm. longis supra rugosissimis viridibus hispidis glandulosisque subtus pallidis glanduloso-pilosis, petiolo 2–3 mm. longo; racemis gracilibus 4.5–9 cm. longis; rhachi purpurascente glanduloso-hispidulo; verticillis circa 8-floris remotis demum 1.5–2 cm. distantibus; bracteis ovatis 2 mm. longis; pedicellis 2 mm. longis; calyce tubuloso-campanulato livido fructifero 6 mm. longo glanduloso-hispido, labio superiore obtuso 1.5 mm. longo, labio inferiore obtuso vix 1 mm. longo; corolla circa 1 cm. longa, tubo ventricoso exserto; galea pilosa, labio inferiore paulo breviore; stylo piloso. — HIDALGO: on rocks, Ixmiquilpan, 1903, *C. A. Purpus*, no. 431 (type, in Gray Herb.). In habit similar to *S. fruticulosa* Benth., which has the branchlets, lower leaf-surfaces, calyces, etc., stellate-pannose; nearer related, apparently, to *S. Gonzalezii* Fernald, which is less fruticose, with darker branches, glandless softer pubescence, broad-ovate leaves, and larger calyx.

Salvia (Scorodoniae) tepicensis, n. sp., caulibus gracilibus obtuse angulatis dense piloso-hispidis, pilis viscidis; foliis oblongo-ovatis obtusis supra viridibus rugosis setosis subtus albo-villosis 3–3.5 cm. longis basi subcordatis, petiolo brevi gracili viscido-hispido; racemis simplicis elongatis 1.5 dm. longis; verticillis 6–10-floris remotis demum 2.5–3 cm. distantibus; bracteis lanceolato-ovatis acuminatis dentatis 4 mm. longis; calyce azureo anguste campanulato fructifero 7–8 mm. longo valde costato, costis glanduloso-setulosis, labio superiore obtuso 3 mm. longo, inferiore obtuso 2 mm. longo; corolla azurea 1.5 cm. longa, tubo paulo ventricoso exserto, galea pilosa, labio inferiore multo longiore; stylo villosissimo. — TEPIC: near the town of Tepic, January 5–February 6, 1892, *Edward Palmer*, no. 1984 (type, in Gray Herb.). Related to *S. Gonzalezii* Fernald and *S. rupicola* Fernald. From the former distinguished by its characteristic glandular spreading pubescence, the long lip of the corolla, and the villous style; from the latter by its more herbaceous character, its much longer pubescence (of branches, leaves, and calyx), its larger prominently costate calyx, and the longer corolla with a comparatively long lip.

Salvia (Scorodoniae) dasycalyx, n. sp., fruticosa 1.5 m. alta; ramis gracilibus valde quadrangulatis superne decussatim bifariam pilosis; foliis ramorum lanceolatis acuminatis basi subcuneatis 3.5–5.5 cm. longis paulo rugosis utrinque glabris vel venis supra pilosis venis subtus albidis, petiolis 2–5 mm. longis pilosis; paniculis densis thyrsoideis, secundariis 3.5–5 cm. longis; bracteis lanceolato-attenuatis 3–4 mm. longis; calyce turbinato circa 3 mm. longo purpurascente dense villoso, pilis albidis planis, lobis brevissimis latis; corolla violacea 7–8 mm. longa, tubo incluso, galea pilosa labiam inferiorem subaequante. — MICHOACAN or GUERRERO: in argillaceous soil at 1800 meters altitude, Sierra Madre, January 23, 1899, *Langlassé*, no. 779 (type, in Gray Herb.). Closely simulating *S. thyrsiflora* Benth., from which it differs in its glabrous leaves and smaller shaggy-villous calyces.

Salvia (Cyaneae) umbratilis, n. sp., fruticosa 1 m. alta; ramis gracilibus puberulis; foliis membranaceis glabris rhomboideo-ovatis acuminatis basi cuneatis 8 cm. longis crenato-serratis, dentibus mucronatis; petiolis gracilibus 1.5–3.5 cm. longis; racemo 1.5 dm. longo; verticillis 2–6-floris demum 2 cm. distantibus; bracteis ovato-acuminatis 2 mm. longis subpersistentibus; pedicellis filiformibus 5–6 mm. longis divergentibus minute hispidis; calyce campanulato demum 11 mm. longo valde 9-costato costis setulosis, labio superiore ascendente late deltoideo mucronato, labio inferiore 4 mm. longo cum lobis porrectis anguste deltoideis aristatis; corolla cyanea 2.5–3 cm. longa pilosa rectiuscula, tubo paulo ventricoso, galea 7 mm. longa, labio inferiore paulo breviore; stylo glabro. — MICHOACAN or GUERRERO: in argillaceous soil of damp forests, at 1200 meters altitude, Sierra Madre, February 19, 1899, *Langlassé*, no. 904 (type, in Gray Herb.). Nearest related to *S. phaenostemma* Donnell Smith, which has the leaves more rounded at base, the calyx longer and purberulent (with subequal lobes), and the pedicels ascending.

Salvia (Tubiflorae) arbuscula, n. sp., arborea vel fruticosa circa 2.5 m. alta; ramis lanatis, pilis brunneis; foliis ovatis oblique subcordatis acuminatis circa 1 dm. longis crenato-serratis supra viridescentibus tomentosis cum pilis stellatis subtus albido-pannosis cum pilis stellatis; petiolis 1–1.5 cm. longis stellato-tomentosis; racemis densis primario 2.5 dm. longo; verticillis 20–30-floris demum 3 cm. distantibus; bracteis minutis deciduis; calyce tubuloso-campanulato valde costato 5 mm. longo albido-lanato, labio superiore late deltoideo cuspidato 1 mm. longo, inferiore cum lobis anguste deltoideis mucronatis; corolla purpurea curvata 2.5–3 cm. longa vix ventricosa villosa, galea rectiuscula 7 mm. longa, labio inferiore 4 mm. longo; stylo glabro. —

MICHOACAN or GUERRERO: at 1500 metres altitude in the Sierra Madre, January 20, 1899, *Langlassé*, no. 767 (type, in Gray Herb.). A handsome species nearest related to *S. Rosei* Fernald, but abundantly distinct in the pubescence of its branches, calyx and corolla, as well as the small calyx and the glabrous style.

Hyptis (Hypenia § Longiflorae) Langlassei, n. sp., fruticosa circa 2 m. alta; ramis glabris rufescentibus; foliis crassis coriaceis glabris lanceolatis acuminatis basi subcuneatis, superioribus 1–1.7 dm. longis 2–3.5 cm. latis acute dentatis; panicula trichotoma ramis 1.5–2.7 dm. longis cymulas item semel vel bis trichotomas 2–7 cm. longas laxe patentes gerentibus, rhachi glanduloso-puberulo; bracteis ovato-lanceolatis acuminatis integris puberulis, inferioribus 2.5 cm. longis, superioribus 1 cm. longis; pedicellis demum 4–11 mm. longis; calyce campanulato anthesi 4–5 mm. fructifero 8–9 mm. longo glanduloso-puberulo et glanduloso-hispido, pilis brevibus albidis squamosis; labiis patentibus lanceolato-aristatis; corolla sanguinea puberula 2 cm. longa, tubo infundibuliforme, galea 2–3 mm. longa lobis rotundis labiam inferiorem subaequante; staminibus stiloque exsertis glabris. — MICHOACAN or GUERRERO: in granitic soil at 1800 m. altitude, Sierra Madre, February 10, 1899, *Langlassé*, no. 854 (type, in Gray Herb.). Closely related to *H. Nelsoni* Fernald, of the mountains of Jalisco, which has the leaves broad and clasping at base, the pubescence much finer (that of the calyx merely a fine puberulence), and the hardly aristate calyx-lobes much shorter.

V. MEXICAN PHANEROGAMS — NOTES AND NEW SPECIES.

BY C. A. WEATHERBY.

Anthericum tenue, n. sp., gracillimum scaposum, radicibus fasciculatis nonnullis apice nonnullis basin versus tuberoso-incrassatis, foliis marcidis in collo laxe fibroso 3 cm. longo supra radicem persistentibus foliis suberectis pluribus radicalibus subulatis duris glabris marginibus minute ciliolatis exceptis 1.5–2.8 dm. longis circa 1 mm. latis caule paulum brevioribus in apicem longum acicularem productis, caulibus gracilibus glabris 6–9-bracteatis ex speciminibus visis simplicibus 2.8–3.6 dm. altis, floribus in bractearum axillis 2–3-fasciculatis, pedicellis 7–10 mm. longis infra medium articulatis, perianthii segmentis 1 cm. longis albis (fide Nelsonii), staminibus quam perianthium tertiam partem brevioribus, antheris 3 mm. longis liberis, filamentis 4 mm. longis muricatis, capsulis immaturis ovoideis quam perianthium marcescens duplo brevioribus. — GUERRERO: between Ayusinapa and Petatlan altitude 1500–2000 m., Dec. 14, 1894, *Nelson*, no. 2120 (in hb. U. S. Nat. Mus.). Near *A. leptophyllum* Baker, from which it differs in its even more slender habit, narrower and longer leaves, and several-bracted stem. Very similar also to *Echeandia Pringlei* Greenman, but with free anthers.

Anthericum uncinatum, n. sp., scaposum, radicibus medio incrassatis, collo radicis dense fibroso, foliis (6–7) 8–12 cm. longis 6–10 mm. latis pallide viridibus saepius patentibus valdeque falcatis in siccis conduplicatis membranaceis marginibus manifestis albis cartilagineis ciliolatis lente nervatis, caulibus circa 3 dm. altis simplicibus scabris vel hirtellis 1–2-bracteatis bracteis setaceo-acuminatis chartaceis, pedicellis floriferis 5–7 mm. longis infra medium articulatis, perianthii flavi (?) segmentis 8–12 mm. longis, filamentis papilloso-crispatis circa 5 mm. longis antheris longioribus, capsulis immaturis brevibus ovoideis. — DURANGO: Otinapa, July 25–Aug. 5, 1906, *Palmer*, no. 437. Near *A. scabrellum* Baker, from which it differs in its cartilaginous-margined and strongly falcate leaves; similar to those of *A. drepanoides* Greenman. From the latter species it differs in its scabrous stem, smaller size, and fewer, chartaceous bracts. In *A. drepanoides* the bracts are about 5, and the lower are foliaceous and falcate, like the root-leaves.

Nemastylis (§ **Chlamydostylus**) **latifolia**, n. sp., bulbo ovoideo tunicis brunneis friabilibus, caule simplici subflexuoso in speciminibus visis circa 4.5 dm. alto folium unicum erectum bracteamque vaginantem gerente, folio radicali uno lineari-lanceolato longe acuminato apice setaceo 3 dm. longo 1–1.5 cm. lato plicato valde nervato, folio caulino simili inflorescentia breviore vel eam aequante ejus vagina 3–3.5 cm. longa scariosi-marginata, bractea acuminata scariosi-marginata 7.5–8.5 cm. longa, spatha 5.3 cm. longa valvis acuminatis aequilongis vel exteriore paulum longiore, floribus in spatha 4, pedicellis filiformibus spatham aequantibus vel exsertis, perianthiis albis marcescentibus paulum caerulescentibus 3 cm. (?) latis, filamentis brevissimis minus quam 1 mm. longis, antheris 1 cm. longis connectivis angustis, styli ramis filiformibus antheras subaequantibus parte indivisa circa 1 mm. longa, fructu non viso. — GUERRERO: hills, near Iguala, alt. 915 m., July 29, 1907, *Pringle*, no. 10,391. Distinguished from all the other Mexican species hitherto described by its very short, almost obsolete filaments. In this respect it resembles some of the South American species, but is not satisfactorily referable to any of them.

Quercus (§ **Erythrobalanus**) **rysophylla**, n. sp., arborea magna, cortice nigricante aspera vel profunde sulcata, foliis integris ovato-lanceolatis 14–21 cm. longis 4.5–8 cm. latis basi cordatis vel rarius truncatis in apicem acutum sensim angustatis apice (in foliis immaturis) arista gracili 3–4 mm. longa munitis coriaceis glabris vel subtus in axillis nervorum barbatis pallide viridibus subnitidis valde reticulato-rugosis nervis supra impressis subtus prominentibus marginibus leviter incrassatis durisque sicut nervis marginalibus, petiolis 5–7 mm. longis crassis supra planis tomentosis vel glabratis, stipulis persistentibus linearibus 1.2–1.5 cm. longis, floribus femineis 2–4 folii in axilla singula sessilibus, cupulae immaturae squamis late ovatis obtusis glabris vel minute furfuraceis, glandibus non visis. — NUEVO LEON: Sierra Madre, Monterey, *Pringle*, nos. 10,225, 10,226, 10,379. A well-marked species, nearest *Q. nectandraefolia* Liebmann.

Mirabilis Pringlei, n. sp., caulibus herbaceis circa 1 m. altis ramosis, ramis dense glanduloso-puberulentibus, foliis late ovatis vel suborbiculatis 7–10 cm. longis 5–9 cm. latis integris cordatis acutis vel breviter acuminatis ciliolatis praeter nervos glanduloso-puberulentibus subtus sparse et minute pubescentibus pilis brevibus adpressis, inflorescentiae foliis parvis subsessilibus, inflorescentia divaricato-cymosa non congesta, cymis breviter pedunculatis, involucris unifloris campanulatis glandulosis ejus laciniis ovatis obtusis in anthesi tubam subaequantibus, perianthiis pallide roseis 2.5–3 cm. longis cylindraceis basi paulum dilatatis et quam ovarium latioribus limbo angusto, staminibus 5 longe exsertis perianthii tubo duplo longioribus, anthocarpiis glabris tuberculatis circa 7 mm. altis 5 mm. latis pentagonis in angulis costatis basi late truncatis. — GUERRERO: under limestone cliffs, Iguala Cañon, alt. 915 m., July 23, 1907, *Pringle*, no. 10,384. Near *M. exserta* Brandegee, from which it differs in its tuberculate, five-ribbed anthocarp and in the shape of its perianth which, at base, is broader than the ovary. From *M. Jalapa* and its immediate allies it differs, as does *M. exserta*, in its long-exserted stamens and style and in its more open inflorescence.

OXYBAPHUS GLABER Watson. The type material of this species consisted only of a portion of the panicle. The following amplified description, drawn up largely from the specimen of Mr. Pringle's cited below, may, therefore, be of service.

Perennial; stem stout, glabrous, 8 dm. high, simple below, branching above, the lower internodes numerous and short (2 cm. long); leaves linear, 4–8 cm. long, 3–6 mm. wide, thick, glabrous; panicle large and open, its branches opposite and strictly glabrous; involucres somewhat campanulate, 4–8 mm. high, about 1 cm. across when mature, glabrous or minutely strigillose with short yellow hairs, on slender glabrous pedicels 4–8 mm. long; flowers cleistogamous (?), the perianth inconspicuous, equalling or shorter than the involucre; fruit lance-ovate in outline, acute at the apex, narrowed at the base, with five narrow but prominent smooth ribs, the space between more or less strongly tuberculate, glabrous or minutely strigillose between the ribs.— Am. Nat. vii. 302 (1873). — Kanab, South Utah, *Mrs. A. P. Thompson.* CHIHUAHUA: sand hills near Paso del Norte, Sept. 20, 1886, *Pringle*, no. 1126. A specimen from Kansas, sand hills, Kearny Co., Aug. 29, 1897, *A. S. Hitchcock*, no. 421b perhaps belongs here also.

There is in the Gray Herbarium a plant clearly referable to this species, but differing from the typical form in its pubescent pedicels and involucres. It seems worthy of recognition as: var. **recedens**, n. var., a forma typica differt pedicellis involucrisque pubescentibus. — CHIHUAHUA: between Casas Grandes and Sabinal, altitude 1550–1700 m., Sept. 4–5, 1899, *Nelson*, no. 6351.

In the course of a recent attempt to rearrange, with the aid of Mr. Standley's excellent monograph, the Mexican specimens of *Nyctaginaceae* in the Gray Herbarium, it became apparent that, under the Vienna Rules, several new combinations in the genus *Oxybaphus* were required. They are accordingly proposed here, as follows:

Oxybaphus texensis (Coult.), n. comb. *Allionia corymbosa*, var. *texensis* Coult. Contr. U. S. Nat. Herb. ii. 351 (1894). *Allionia texensis* Small, Fl. Southeast. U. S. 406 (1903). — *Coulter's* no. 912, from Mexico, but without more definite locality, should apparently be referred here.

Oxybaphus coahuilensis (Standley), n. comb. *Allionia coahuilensis* Standley, Contr. U. S. Nat. Herb. xii. 347 (1909).

Oxybaphus melanotrichus (Standley), n. comb. *Allionia melanotricha* Standley, l. c. 351. The following, not cited by Mr. Standley, belongs here: CHIHUAHUA: mountains near Pilares, 23 Sept., 1891, *C. V. Hartman*, no. 743.

Oxybaphus pseudaggregatus (Heimerl), n. comb. *Mirabilis pseudaggregata* Heimerl, Ann. Cons. et Jard. Genèv. v. 183 (1901). *Allionia pseudaggregata* Standley, l. c. 356. — The following specimens belong here: SAN LUIS POTOSI: alt. 1850–2500 m., 1878, *Parry & Palmer*, no. 768; in montibus San Miguelito, 1876, *Schaffner*, no. 177. Vallée de Mexico, Guadelupe, 1er Août, 1865, *Bourgeau*, no. 651.

Urvillea biternata, n. sp., fruticosa 1–2 m. alta glabra vel ramulis minute pulverulentibus, ramis 3–5-costatis costis obtusis interdum rubris inter costas planiusculis vel leviter sulcatis, foliis biternatis, foliolis membranaceis glabris vel subtus praeter nervos sparse pubescentibus punctis lineisque pellucidis minute punctatis ovatis subtus pallidioribus, terminalibus 11–15 cm. longis 4.5–5.5 cm. latis obtuse acuminatis mucronulatis supra medium paucis dentibus crenatis basi abrupte angustatis sicut in petiolulam alatam 1–2 cm. longam, lateralibus similibus minoribus interdum obliquis acumine breviore, inflorescentiae paniculis angustis axillaribus longe (ad 8 cm.) pedunculatis 2-cirrhosis, sepalis 5, 3 mm. longis concavis obtusis late ovatis minute pubescentibus duobus exterioribus paulum minoribus, petalis 4, 3 mm. longis obovatis vel suborbiculatis unguiculatis rotundatis, duobus superioribus squamas gerentibus latas cucullatas apice in appendicem longam deflexam productas appendice et marginibus barbatas summo dorso crista dilatata subflabelliforme instructas, duorum inferiorum squamis minoribus margine barbatis summo dorso cuspidatis, disci glandis duobus oblongis basi latioribus et callosis inter callos concavis, staminibus 8, filamentis crassis extra sparse villosis, antheris introrsis, fructu trialato subobovato 1.8 cm. longo 1.3 cm. lato apice leviter emarginato vel rotundato basi subacuto. — Guerrero: Iguala Cañon, alt. 915 m., July 24, 1907, *Pringle*, no. 10,380. An anomalous species, distinguished from all the other species of *Urvillea* by its biternate leaves. In habit it resembles some species of *Serjania*, but has the fruit of *Urvillea.*

Euphorbia (§ Anisophyllum) chalicophila, n. sp., erecta annua (?) basi ramosa, caulibus teretibus gracilibus 3.5–4 dm. altis dichotome ramosis pilis albis crispatis dense vestitis, foliis oppositis lanceolatis basi valde obliquis subcordatis falcatis acutis vel obtusiusculis brevissime petiolatis ab apice fere ad basin serrulatis pilosis, caulinis 15–19 mm. longis 3–5 mm. latis, involucris brevissime pedicellatis in cymosulas paucifloras bracteatas ad apices ramulorum congestis turbinatis 0.6 mm. altis extus glabris intus hirtellis non fissis, lobis ovato-lanceolatis pectinatis, glandulis transverse ellipticis 0.5 mm. longis subconcavis appendice rubra vel rubella 0.5 mm. lata integra vel emarginata, capsulis 1.5 mm. altis brevipedunculatis glabris vel sparse pilosis, seminibus laevibus griseis ovatis haud angulatis 1 mm. longis. — Jalisco: gravelly banks of gullies near Guadalajara, alt. 1525 m., October 12, 1903, *Pringle*, no. 11,846. In habit and in the characters of the involucre very like narrow-leaved forms of *E. brasiliensis* Lam., but differing in being pilose throughout and in its smooth seeds.

Euphorbia (§ Anisophyllum) chamaecaula, n. sp., perennis rubescens, caulibus ex apice radicis pluribus prostratis ramosis compressis infra nodos paulum dilatatis glabris, foliis oppositis brevissime petiolatis late ovatis basi subcordatis obliquis apice obtusis integris glabris vel facie superiore sparse pilosis, caulinis 6–8 mm. longis 4.5–6 mm. latis, ramulinis minoribus, involucris in axillis foliorum solitariis vel apicibus ramulorum in cymosulas paucifloras aggregatis pedicellatis campanulatis extus intusque glabris, lobis parvis ovatis fimbriatis, glandulis ellipticis 0.6 mm. longis, appendice conspicua alba flabelliforme integra vel crenulata 0.5 mm. lata, pedicellis 2.5 mm. longis vel brevioribus, capsulis 2 mm. longis 1.5 mm. latis subacute carinatis omnino glabris, seminibus pallidis oblongis apice apiculatis quadrangularibus inter angulos subtransverse vel irregulariter rugosis. — JALISCO: gravelly plain near Guadalajara, Oct. 14, 1903, *Pringle*, no. 11,848. Near *E. prostrata*, from which it differs as follows: *E. prostrata*, plant green, leaves strictly oblong, abruptly rounded at apex, capsules hairy on the angles, glands with very short or no appendages. *E. chamaecaula*, leaves mostly ovate, tapering somewhat to the obtuse apex, plant reddish, capsule entirely glabrous, glands with conspicuous white fan-shaped appendages.

Manihot intermedia, n. sp., fruticosa erecta 1–2 m. alta omnino glabra, foliis orbiculatis palmatis non peltatis fere ad petiolam profunde 7–8-lobatis, supra viridibus subtus pallidis venis albis reticulatis, lobis medianis foliorum inferiorum lanceolatis sinuata-lobatis infra apicem late et abrupte rhombeo-dilatatis apice setaceo-mucronatis, duobus lobis lateralibus parvis lanceolatis integris, lobis medianis foliorum superiorum leviter sinuatis nec lobatis nec rhombeo-dilatatis, petiolis limbo brevioribus vel eum subaequantibus glaucis, racemis brevibus 3–4 cm. longis 3–4 ad apicem ramulorum fasciculatis patulis, bracteis pedicellas aequantibus vel paulum superantibus lineari-setaceis, pedicellis 5–10 mm. longis saepe bracteas duas oppositas parvas infra medium gerentibus, florum masculorum perianthiis gamophyllis 5-lobatis campanulatis circa 15 mm. altis basi rotundatis extus glauco-caerulescentibus intus flavescentibus venosis extus intusque glabris, laciniis deltoideis tubo triplo brevioribus, staminibus longioribus perianthium aequantibus, capsulis glabris globosis in siccitate rugosis, seminibus laevibus ellipticis latere interiore planis vel obtusissime angulatis exteriore convexis. — GUERRERO; limestone cliffs of Iguala Cañon, alt. 915 m., July 29, 1907, *Pringle*, no. 13,938. Intermediate between *M. carthaginensis* and *M. acutiloba*, having nearly the foliage of the former but the flowers of the latter; and apparently differing from both in its bracted pedicels.

Ipomœa (§ Pharbitis) igualensis, n. sp., volubilis tota papilloso-hirsuta pilis plus minusve flavescentibus 2–3 mm. longis vel caulibus glabrescentibus, marginibus foliorum bractearum sepalorumque pilis similibus dense papilloso-ciliatis, foliis longe petiolatis (ad 2 dm.) ovato-orbiculatis cordatis breviter acuminatis 7.5–12 cm. longis 7–13 cm. latis, pedunculis petiolos subaequantibus vel superantibus 3-floris, inflorescentia capitata congesta, ejus bracteis duabus late ovatis cuspidatis venosis membranaceis 17 mm. longis pedicellas brevissimas floriferas sicut involucrum includentibus et occultantibus, sepalis circa 13 mm. longis acutis, duobus exterioribus latioribus ovatis 5 mm. latis intus circa 10-nervatis, tribus interioribus lanceolatis 2–2.5 mm. latis, corolla 5 cm. longa pallide purpurea tubo angusto infundibuliforme, tubo et plicis dense pilosis, limbo glabro, capsulis non visis. — GUERRERO; Iguala Cañon, alt. 760 m., September 21, 1905, *Pringle*, no. 10,054. Apparently near *I. hirtiflora* Mart. & Gal., from which it differs in its almost setose pubescence.

JUSTICIA PACIFICA (Oerst.) Hemsl. Mr. Pringle's no. 10,145, from Balsas in the state of Guerrero, agrees excellently with Oersted's description. The original specimens were in fruit only and the species was doubtfully referred to *Justicia* by Hemsley. Mr. Pringle's plant shows a glabrous corolla 2.5 cm. long with the short tube and broad limb characteristic of *Justicia*. The species would seem, then, to be certainly a *Justicia* and allied to *J. furcata*, but differing from all forms of that species in its grayish-puberulent stem, spicate inflorescence, ciliate bracts and in the very broad white margins of its calyx-lobes.

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CONTRIBUTIONS FROM THE ROGERS LABORATORY
OF PHYSICS, MASSACHUSETTS INSTITUTE
OF TECHNOLOGY.

LIII. — *ON THE EQUILIBRIUM OF THE SYSTEM CONSISTING OF LIME, CARBON, CALCIUM CARBIDE AND CARBON MONOXIDE.*

By M. DeKay Thompson.

Investigations on Light and Heat made and published, wholly or in part, with Appropriation
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LIII. — ON THE EQUILIBRIUM OF THE SYSTEM CONSISTING OF LIME, CARBON, CALCIUM CARBIDE AND CARBON MONOXIDE.

By M. deKay Thompson.

Presented by H. M. Goodwin, February 9, 1910. Received February 20, 1910.

1. Introduction.

While the author of the following paper was working on the subject indicated in the title above, an article dealing with the same matter appeared in the Electrochemical and Metallurgical Industry.[1] The present writer's results did not agree with those in the article referred to, and it was therefore thought best to publish a preliminary paper on the subject, which was accordingly presented at the October meeting of the American Electrochemical Society in New York. As the work has now been brought to a close, the following article will be made complete, including all of the preliminary publication that is necessary for clearness.

According to the Phase Rule[2] the substances taking part in the reaction $CaO + 3C \rightleftarrows CaC_2 + CO$ form a monovariant system, that is to say, for any given temperature there is a definite pressure of carbon monoxide which will preserve equilibrium. In order that equilibrium can exist the reaction must be reversible. The fact that this reaction is reversible has been shown by Rothmund[3] and others.[4] Rothmund also attempted to measure the temperature of formation of carbide by heating to different temperatures lime and carbon, and testing the charge immediately afterwards to see if it reacted with water, giving off acetylene. The furnace used consisted of a carbon tube through

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[1] C. A. Hansen, Electrochem. Met. Ind. 1909, **7**, 427.

[2] See Findlay, "The Phase Rule," p. 16.

[3] Zeitschr. f. anorg. Chem. 1902, **31**, 136.

[4] A. Frank, Zeitschr. f. angew. Chem. 1905, **44**, 1733.

which an electrical current was passed. The assumption that must be made with regard to the partial pressure of the carbon monoxide is that it is constant and is due to oxygen of the air acting on the carbon tube, giving one-third of an atmosphere.[5] Unless the temperature is raised above that corresponding to one-third of an atmosphere, no carbide would be found. By repeated trials this temperature could be located within certain limits, if the above assumption is true. In this way Rothmund found 1620° C. as the temperature of formation. Similar experiments were repeated later by Rudolphi,[6] who found the temperature of formation to lie between 1800 and 1819° C., that is, about 200° higher than Rothmund's value. The temperature measurements were made by an optical method, as were also Rothmund's. Finally, Lampen,[7] by a method similar to the above, using a Wanner pyrometer for temperature measurements, found 1725° C. for the temperature of formation. It seemed evident, from the poor agreement of these results, all obtained by the same method, that some other method would have to be used in which the pressure of the carbon monoxide could also be measured, as these differences might be due simply to different values of this quantity. It was the object of the following investigation to make these measurements.

2. Method and Results.

The method decided on was to heat the charge in a vacuum furnace connected with a mercury manometer and to measure the temperature of the charge and pressure of the carbon monoxide when equilibrium is reached. A small Arsem [8] vacuum furnace, made by the General Electric Company, was the apparatus used. It consists of a cylindrical bronze casting 24 centimeters in inside diameter and 39 centimeters in length. Parallel to the axis in the center of the casting and fastened to the lid, is a graphite helix, 27 centimeters in length, 5.1 in outside diameter and 0.5 in thickness of wall. The helix is clamped at each end by water-cooled electrodes. The lid is fastened to the casting with a number of cap-screws and a lead washer. The whole furnace is immersed in water with the exception of a tower projecting from the center

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[5] Rothmund erroneously assumes the pressure of the carbon monoxide to be 1/5 atmosphere, probably because this is the partial pressure of oxygen in the atmosphere. Taking into consideration that every mole of oxygen produces two of carbon monoxide, 1/3 atmosphere is the result obtained.

[6] Zeitschr. f. anorg. Chem. 1907, **54**, 170.

[7] Jour. Am. Chem. Soc., 1906, **28**, 864.

[8] Trans. Am. Electrochem. Soc., 1906, **9**, 163.

of the lid containing a mica window, making it possible to see the hot material held in the center of the spiral. The support for the crucible is a graphite rod held by the lower electrode, but insulated by lava rings. The lid also contains a pipe by which the furnace may be exhausted. A Geryk oil pump was used for obtaining the vacuum. The pressure could be read by a wooden scale divided in millimeters on a mercury gauge completely evacuated and sealed off at one end, thus making a siphon barometer. The temperature of the gas contained in the furnace is not constant, but all that determines the equilibrium besides the pressure is the temperature of the solid substances and of the gas in contact with it. Of course the pressure must be constant throughout the furnace.

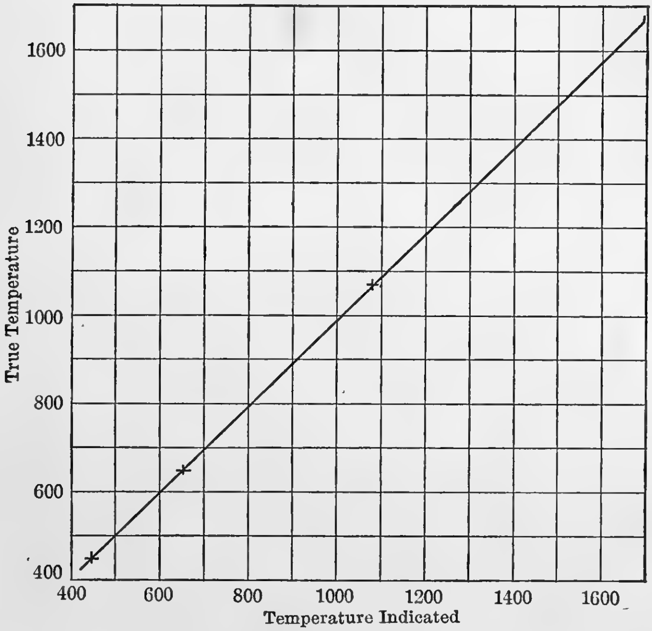

Figure 1. Calibration of Thermoelectric Junction.

In the first experiments the temperature was measured by a Wanner pyrometer, which rendered it necessary to replace the mica window by one of glass clamped between rubber and sealed up with paraffin. In calibrating the pyrometer a similar piece of glass was placed between the amylacetate standard and the instrument. The Wanner was found

TABLE I.

Calibration of Thermoelectric Junction.

True Temperature.		Temperature read directly from scale.
Melting point of Gold . .	1065° C.	1075° C.
Melting point of Aluminum	655° C.	650° C.
Boiling Sulphur	445° C.	440° C.

to be unreliable, however, apparently due to inconstancy in the amyl-acetate standard.[9] The furnace was therefore calibrated by means of a platinum platinum-rhodium junction, that is, the temperature of the crucible was measured while the power was held constant. The temperature was then subsequently determined by measuring the power applied. Heating was furnished by an alternating current with a frequency of sixty cycles per second. This was taken from a transformer-switchboard so arranged that the voltage could be varied in steps of about twelve volts. For the finer regulation a carbon plate rheostat, in which current regulation could be obtained by varying the compres-

TABLE II.

Calibration of Furnace.

Kilowatts.	Temperature by thermo-electric junction.	Remarks.
3.60	968°	1st spiral
7.03	1185°	
8.98	1325°	
9.71	1368°	
3.12	925°	2d spiral
4.99	1062°	
7.10	1180°	
8.03	1225°	
9.00	1262°	
9.81	1325°	2d spiral repeated on following day
7.68	1180°	
6.15	1100°	

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[9] The temperatures measured in the former article on this subject are accordingly from 100° to 150° too low, but the general conclusions there reached are not affected.

sion on the plates was found satisfactory. The terminals were copper boxes filled with water.

Figure 1 gives the calibration of the junction. The galvanometer was a Siemens and Halske instrument made for this special purpose, but which did not read as high as the melting point of platinum.

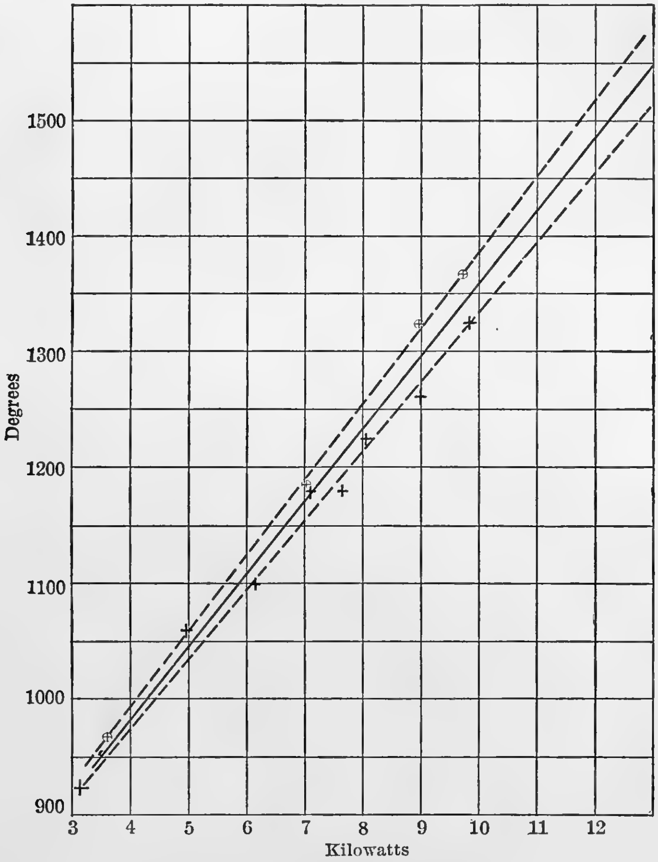

FIGURE 2. Calibration of Furnace.

In Table II and Figure 2 the calibration of the furnace is given. The power was obtained from voltmeter and ammeter readings. The ammeter scale read to five amperes and was connected to a current transformer with a ratio of 60 to 1. This instrument was not calibrated.

Two voltmeters with scales from 0–65 and 40 to 160 were used. These were calibrated so as to make them comparable with each other. The alternating current instruments were of the Thomson type made by the General Electric Company.

In calibrating the furnace the wires of the pyrometer were protected by fused silica tubes which extended up into the tower in the lid of the furnace. The tubes were covered at the junction by a short graphite tube. This projected through a hole in the cap of the crucible containing the charge and rested in the charge. The bare wires were brought out of the furnace at the top of the tower between rubber washers; the furnace was then evacuated and the calibration taken.

TABLE III.

Variation of Temperature in Crucible.

Distance of junction from bottom of crucible.	Temperature.
0.0 cm.	1220°
0.6	1220°
1.8	1225°
3.3	1225°
4.0	1220°
4.4	1215°
4.8	1205°

The power was 8.36 kilowatts.

The carbon shield surrounding the spiral was not used in these experiments on account of the fact that carbon absorbs a large amount of gas which is not easily removed. It will be evident from the method of experimenting described below that its use would not be permissible.

In the figure a circle is put around those points taken with the first spiral. It is evident from this figure that this method of obtaining the temperature is not as accurate as the Wanner pyrometer would be were it in good condition.

It will be seen that there is no regular difference in the calibration of the two spirals, except that all the points of the first coil lie on the upper dotted line, while some of the points for the second coil lie on the upper as well as the lower. This is probably due to the fact that the second spiral was calibrated more than once. It was thought best under the circumstances to draw the solid line midway between the two extremes and take this for estimating the temperature.

A further test was made to see how constant the temperature was throughout the length of the crucible. For this purpose the junction, protected by silica tubes, was lowered through the window in the tower into the crucible and the furnace heated without pumping out the air. There was no lid on the crucible in this experiment. The results are given in Table III.

It is seen that without the lid and with no charge in the crucible the temperature is quite constant, which would be improved, if anything, when the charge is in the crucible and the lid in position.

The carbide used in the following experiments was made from Merk's lime and Acheson graphite powder in the form of turnings from graphite electrodes. Carbide was made by heating a mixture of the two in an arc furnace consisting of a graphite electrode and graphite crucible. By the loss in weight method[10] it analyzed 78 per cent pure. The impurities must have been carbon and lime which were not harmful for these experiments.

The first experiments were carried out at from 1700° to 2000°, but no consistent results could be obtained. After a run at these temperatures it was found that the walls of the furnace were always lined with a white powder, whether lime and carbon were heated alone or when carbide was in an atmosphere of carbon monoxide. It was found when carbide was heated in carbon monoxide to about 1800° only graphite was left in the crucible and the white powder was formed on the walls. When carbide was heated alone in a vacuum the walls of the furnace were lined with a thin sheet of calcium, which easily peeled off and took fire when brought in contact with moisture. Graphite was left behind in the crucible. These two facts taken together show that calcium reduces carbon monoxide according to the equation:

$$\mathrm{Ca} + \mathrm{CO} = \mathrm{CaO} + \mathrm{C}.$$

Therefore, if carbide is to be produced, it must either be below the temperature where it breaks up into its elements, or the velocity of the reaction

$$\mathrm{CaO} + 3\ \mathrm{C} = \mathrm{CaC_2} + \mathrm{CO}$$

must be greater than the velocity of the preceding reaction. The latter is evidently the state of affairs in the manufacture of carbide, but equilibrium measurements could hardly be made under this condition.

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[10] Lunge, Chemische–technische Untersuchungs Methoden, 5te Auflage, Band II, 711. The drying tube contained a layer of P_2O_5 besides one of Ca Cl_2, which the escaping gas had to pass first.

From a number of experiments, which it is not necessary to reproduce here, it seemed that 1500° C. was about the highest temperature at which equilibrium could be measured. This conclusion was based on the quantity of white powder found on the walls of the furnace after runs at different temperatures. Some further experiments at about this temperature showed that it would be impossible to differentiate between the pressure of carbon monoxide and occluded gases that came out of the carbon spiral and the charge on heating in a vacuum. It was therefore decided to heat the charge in some indifferent gas, which could be drawn off and analyzed for the amount of carbon monoxide present. Hydrogen was of course the only gas available. Nitrogen could not be used on account of the fact that it is absorbed by calcium carbide forming calcium-cyanamide. Hydrogen would have no action on carbide,[11] but it does enter into an equilibrium with carbon monoxide according to the reaction

$$H_2O + C \rightleftarrows H_2 + CO$$

which is the reaction of water-gas formation. If an appreciable quantity of water were produced from hydrogen and carbon monoxide this would react with the carbide and form acetylene and in analyzing for carbon monoxide by absorption in cuprous chloride solution, acetylene would be mistaken for the former. It can be shown, however, that the quantity of water vapor formed is too small to have any effect. The free energy of this reaction is given by the equation[12]

$$\Delta F = -27950 + 31.76\,T + 4.58\,T \log \frac{p_{H_2O}}{p_{CO}\,p_{H_2}}$$

where T is the absolute temperature and the p's are partial pressures. At equilibrium $\Delta F = 0$, therefore placing the right-hand side of the equation equal to zero, and substituting for T its value 1773° absolute, we find that for 1500° C.

$$\frac{p_{H_2O}}{p_{CO}\,p_{H_2}} = 0.000324.$$

If p_{H_2} equals about 90 centimeters of mercury as it does in the following experiments,

$$\frac{p_{H_2O}}{p_{CO}} = 0.0029$$

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[11] Moisson, "The Electric Furnace," p. 211.

[12] Bodländer, Zeitschr. f. Elektrochem. 1902, **8**, 833.

or $p_{H_2O} = 0.003\ p_{CO}$, which is a negligible quantity. The temperature of the gas, however, is not all at 1500°, but falls off to the temperature of the water cooled walls of the furnace. At 1000° C. $p_{H_2O} = .063\ p_{CO}$ which is still a relatively small amount. What actually happens is that at the higher temperatures where the velocity of the reaction is great, the equilibrium varies uniformly with the temperature, but as the gas reaches the cooler portions of the furnace, due to convection currents, it suddenly becomes chilled to a point where the reaction practically stops, leaving the concentrations at values corresponding to the higher temperatures.

Experiment 1.

The charge consisted of lime, carbon, and calcium carbide mixed together. A loosely fitting lid with a quarter-inch hole in the center covered the crucible. The mixture was placed in the furnace, the furnace was evacuated, and the charge heated to 1000° for an hour to drive off gases that invariably come off on the first heating, and particularly to get rid of any water contained as hydrate of calcium. If this were not done water would come off during the run and react with the carbide present. The furnace was then evacuated to a pressure of 0.05 centimeters of mercury and carbon monoxide let in to 1.25 centimeters. This was generated from strong sulphuric acid and potassium ferrocyanide and was washed with two drying towers of soda lime and a phosphorous pentoxide tube. Hydrogen was then admitted to a final pressure of 63.6 centimeters. This was generated from hydrochloric acid and zinc and was purified by two bottles of permanganate, a hot copper gauze, two towers of soda lime, and a phosphorus pentoxide tube. The furnace was filled with hydrogen in three quarters of an hour. The volume of the furnace, after allowing for the solids present during a run was 19.9 liters. The run began at 9.45 A. M. and lasted till 4.00 P. M. The power was held constant at 12.0 kilowatts corresponding to 1485° C. The following table gives the analysis for carbon monoxide, made by drawing off 100 cubic centimeters into a Hempel burette and absorbing with acid cuprous chloride solution.

Time.		Per cent Carbon Monoxide.
9.45 A. M.	Sample taken as furnace warmed up.	1.05
1.42 P. M.		Less than 0.1

It was evident from this result that the quantity of gas corresponding to equilibrium at this temperature could not be analyzed by a Hempel apparatus. The experiment was continued till 4.00 P. M. to make sure equilibrium had been reached. The method used to determine the small quantity of carbon monoxide present in this and all the following experiments was to draw about half the gas in the furnace through two Liebig bulbs sealed together and filled with cuprous chloride solution. These were tilted at an angle so the gas bubbled through the liquid on leaving each of the five spheres of which a Liebig bulb is composed. The gas then passed a column seven centimeters long of soda lime and another similar one of phosphorous pentoxide. This whole apparatus was made entirely of glass closed by two glass stopcocks. The bulbs, in which the air was displaced by hydrogen, were hung in the balance case by a platinum wire the day before the final weight was taken. The air in the balance case was dried by two beakers of sulphuric acid and the temperature was read from a thermometer in the case. The volume of the bulbs was determined by the bottle method for specific gravity, in which a large desiccator took the place of the bottle. This was necessary in order to be able to reduce the weighings to vacuo. From the total weight in grams of carbon monoxide absorbed the number of moles is formed by dividing by 28, the molecular weight of the gas. This, however, gives only a fraction of the total amount in the furnace. The total amount is calculated as follows. If $n_1 =$ the total number of moles in the furnace before any gas is removed, n_2 the number after a certain amount had been drawn off through the absorption bulbs, $p_1 =$ the pressure in the furnace when the absorption began and p_2 the pressure at the end, then

$$p_1 v = n_1 R T_1$$
$$p_2 v = n_2 R T_2$$

where v equals the volume of the furnace. The temperatures were equal to those of the water surrounding the furnace and were made equal to each other at the start and finish.

Therefore $$\frac{n_1}{n_2} = \frac{p_1}{p_2}$$

also $$n_1 - n_2 = m$$

if $m =$ the number of moles absorbed.

Solving $$n_1 = \frac{m}{1 - \frac{p_2}{p_1}}$$

If p_3 = the total pressure during the run, which is greater than p on account of the higher temperature, the pressure in millimeters of carbon monoxide is computed by the formula

$$p = \frac{m \times .0821 \times T \times 760 \times p_3}{19.9 \times p_1}$$

in which T is the absolute temperature of the gas in the furnace at the beginning and at the end of the absorption.

At the end of the absorption the pressure of hydrogen in the absorption bulbs was only about half an atmosphere, consequently enough hydrogen had to be let in to bring the pressure to one atmosphere, after which the bulbs were again hung in the balance case and weighed the following day. The variation due to temperature and pressure change in the weight of hydrogen filling the bulbs was negligible. All weighings given in the following are reduced to vacuo. The data thus obtained after the above run were the following :

Initial weight bulbs	175.3392 grams
Final " "	175.3482 "
Gain in weight	0.0090 "

The time taken for absorbing the gas was 6 hrs.

p_1 = 68.5 cm. of mercury
p_2 = 38.6 " " "
p_3 = 89.0 " " "

$$\text{whence } p_{CO} = \frac{.00074 \times .0821 \times 285 \times 760 \times 89}{19.9 \times 68.5}$$

$$= 0.86 \text{ mm. of mercury.}$$

On opening the furnace white powder was found on the lid.

Experiment 2.

The same charge as used in Experiment 1 was ground up and replaced in the crucible. Part was tested with water and gave off acetylene vigorously. It was heated for an hour to 1000° and evacuated to a pressure of 0.05 centimeter of mercury. No carbon monoxide was admitted. Hydrogen was let in to 6.72 centimeters in 1 hr. 40 min.

Duration of run : 6 hrs.
Power : 11.7 K. W.
Temperature : 1465° C.

Initial weight bulbs	182.5989 grams
Final " "	182.6061 "
Gain	0.0072 "

Time taken for absorption 4½ hours.

$p_1 = 67.2$ cm. of mercury
$p_2 = 38.1$ " " "
$p_3 = 91.8$ " " "

$$\therefore p_{CO} = \frac{.000589 \times .0821 \times 288 \times 760 \times 91.8}{19.9 \times 67.2} = 0.73 \text{ mm.}$$

On opening the furnace somewhat more white powder found on the walls than in Experiment 1.

This experiment was carried out with the idea of approaching the equilibrium from the side which generates carbon monoxide. To decide whether this had been done in the above experiment it was necessary to see whether the bulbs would gain no weight if the furnace were filled with hydrogen and part then drawn through the bulbs. The following blank experiment was therefore carried out. The furnace was evacuated to a pressure of 0.15 centimeter of mercury, hydrogen was let in to 1 centimeter and again evacuated to 0.15. This operation was repeated and hydrogen then let in to 67.6 centimeters. The final filling took 1 hr. 40 min.

The gas then drawn through the weighed bulbs for 3 hrs. 45 min.

$p_1 = 67.1$ cm. of mercury
$p_2 = 38.1$ " " "

Initial weight reduced	182.606 grams
Final " "	182.627 "
Gain	0.021 "

If the whole amount of gas could have been drawn through the gain in weight would have been 0.049 gram. This gain in weight must have been due to oxygen, which might not have been removed or which might have gotten in while filling the furnace. This would have been converted to carbon monoxide by the hot carbon spiral giving too high a pressure for equilibrium. Equilibrium in Experiment 2 was therefore approached from the same side as in Experiment 1. This remark holds good for all the following experiments.

Experiment 3.

As the previous experiments agreed fairly well, it was thought desirable to try a lower temperature, to make sure that the gain in weight of the absorption bulbs was really due to carbon monoxide and not to some impurity in the hydrogen.

The charge consisted of fresh carbide, lime and carbon. The furnace was evacuated to 0.15 centimeter and was heated to 1000° till the occluded gases coming off gave a pressure of 6 centimeters, which required about ten minutes. It was then evacuated to 0.15 centimeter with the furnace still at 1000°. Hydrogen was let in to 2.4 centimeters and evacuated to 0.15. The furnace was then cooled and filled with hydrogen to a pressure of 67.0 in 1 hr. 40 min.

Power : 8.24 K. W.
Temperature : 1250°
Duration of run : 6 hrs.

The solution of cuprous chloride had been used in a previous experiment.

Initial weight of absorption bulbs	183.6340	grams
Final " " " "	183.6372	"
Gain	0.0032	"

$p_1 = 67.2$ cm. of mercury
$p_2 = 34.8$ " " "
$p_3 = 90.4$ " " "

$$\therefore p_{CO} = \frac{0.000248 \times .0821 \times 286 \times 760 \times 90.4}{19.9 \times 67.2} = 0.30 \text{ mm.}$$

Experiment 4.

The charge was the same material as in the previous experiment with some lime and carbon added and mixed up with the rest.

The furnace was evacuated to a pressure of 0.2 centimeter and heated to 900° for two hours. It was then evacuated to 0.15 centimeter, hydrogen was admitted to 2.4 and again evacuated to 0.15. It was finally filled with hydrogen to 67.3 centimeters in 1 hr. 40 min.

Power : 8.8 K. W.
Temperature : 1270°
Duration of run : 7 hrs. 10 min.

The absorption bulbs were refilled.

Initial weight	182.9303 grams
Final "	182.9316 "
Gain	0.0013 "

$p_1 = 66.8$ cm. of mercury
$p_2 = 37.2$ " " "
$p_3 = 90.3$ " " "

$$\therefore p_{CO} = \frac{0.000103 \times .0821 \times 285 \times 760 \times 90.3}{19.9 \times 66.8} = 0.13 \text{ mm.}$$

Experiment 5.

The object of the following experiment was to see if measurements might not be carried out at a somewhat higher temperature where the pressure would be greater and the determination therefore more accurate.

The charge was the same carbide used in experiment 4 to which about one half as much lime and carbon, previously heated to redness, was added. The furnace was then evacuated to 0.2 centimeter and heated to 900° for 1¼ hours. It was then evacuated to 0.2 centimeter and hydrogen let in to 2.3; again evacuated to 0.12 centimeter and filled with hydrogen to 67.7.

The charge was then heated 7 hours with 10.0 kilowatts, corresponding to 1370°. This must have established equilibrium at this temperature. The power was then raised to 12.6 kilowatts corresponding to 1525° for 4¼ hours.

The cuprous chloride was the same used in Experiment 4.

Initial weight	182.9243 grams
Final "	182.9277 "
Gain	0.0034 "

Time taken for absorption 3 hrs.

$p_1 = 66.3$ cm. of mercury
$p_2 = 37.7$ " " "
$p_3 = 90.0$ " " "

$$\therefore p_{CO} = \frac{.000280 \times .0821 \times 285 \times 760 \times 90.0}{19.9 \times 663} = 0.34 \text{ mm.}$$

On opening the furnace a larger amount of powder than any other experiments here given was found on the walls. This, taken in connection with the small pressure found and the experiments referred to in the Introduction seem to indicate that at this temperature the carbon monoxide was removed by calcium coming from the decomposition of carbide.

It is true that this equilibrium was really approached from the side of too little carbon monoxide, but as the velocity of the reaction is the same in both directions at equilibrium, this cannot account for the low pressure of carbon monoxide.

Experiment 6.

The hydrogen used in the following experiments was generated electrolytically on platinum electrodes dipping into sulphuric acid of 1.2 specific gravity. The cathodes were contained in a porous cup closed at the top by a cork stopper covered with paraffin, through which projected glass tubes, into which the electrodes were sealed. There was also a tube through which hydrogen could escape. The porous cup stood in a small battery jar. The hydrogen tube was connected to a mercury manometer so that the pressure in the cathode compartment could be kept from 0.1 to 1.0 centimeter above the atmosphere, thereby preventing air from leaking in. In Experiment 6 only one such electrolytic cell was used, but for the last two experiments another cell was connected in series with the first, thus requiring only half the time for filling the furnace. The hydrogen first passed through a soda lime tube, then the hot copper gauze used in the previous experiments, then two soda lime towers and phosphorous pentoxide tube. Hydrogen was passed over the hot copper for at least half an hour before any was let into the furnace, in order to sweep out the air in the tube. The object in using electrolytic hydrogen was to show that the above gains in weight were not due to impurities in the hydrogen generated from zinc and hydrochloric acid.

The carbon monoxide used in the following experiments was generated by allowing formic acid to drop from a separatory funnel into concentrated sulphuric acid.

In order to see if all the carbon monoxide was absorbed by the two Liebig bulbs containing cuprous chloride in the following experiments a second absorbing apparatus similar to the above was used with one Liebig bulb in place of two. This was filled with a 3 per cent solution of neutral gold chloride. This has been found to oxidize carbon monoxide to dioxide without affecting hydrogen.[13] Gold chloride in an excess of potassium hydrate is even more sensitive to carbon monoxide, but it was found that hydrogen reduced the gold in the alkaline solution to a black powder if left in contact with the solution over night.

The charge consisted of about equal portions of powdered carbide and a mixture of lime and carbon. It had been used in a previous run.

The cuprous chloride in the Liebig bulbs had been used in the three previous experiments, but as a little was tested with water and gave a heavy white precipitate it was not thought necessary to change the solution.

The furnace was evacuated to a pressure of 0.28 centimeter and hydrogen was let in to 1.0 centimeter; then evacuated to 0.1 and carbon monoxide let in to 0.3 centimeter. Hydrogen was then admitted to 67.3 centimeters requiring three hours with a current of about 14 amperes.

Duration of run: 6¼ hours.
Power: 11.8 K. W.
Temperature: 1475°

Initial weight cuprous chloride bulbs	173.1312 grams
Final " " " "	173.1383 "
Gain	0.0073 "
Initial weight gold chloride bulb	116.8119 grams
Final " " " "	116.8158 "
Gain	0.0036 "
Total gain	0.0110 "

The gain in the gold chloride bulbs was relatively large, probably on account of the cuprous chloride having taken so much carbon monoxide into solution that it was not so good an absorber as when fresh.

$p_1 = 65.7$ cm. of mercury
$p_2 = 34.5$ " "
$p_3 = 92.0$ " "

$$\therefore p_{\text{CO}} = \frac{0.000703 \times 0.0821 \times 287 \times 760 \times 92}{19.9 \times 65.7} = 0.88 \text{ mm.}$$

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[13] Phillips, Am. Chem. Journ. 1894, **16**, 273.

Experiment 7.

The charge consisted of powdered carbide with some coarser pieces on top. It was heated in the furnace at 1050° for an hour and a quarter and evacuated to 0.1 centimeter. Hydrogen was then let in to a pressure of 1.6 centimeters, the furnace was evacuated to 0.10 and carbon monoxide let in to 0.25 centimeter. Finally hydrogen was let in to 68.3 centimeters requiring one hour and a half with 14 amperes.

Duration of run: 6 hours
Power: 11.8 K. W.
Temperature: 1475°

The cuprous chloride bulbs were refilled, but not the gold chloride.

Initial weight cuprous chloride bulbs	173.7646	grams
Final “ “ “ “	173.7695	“
Gain	0.0049	“

Initial weight gold chloride bulb	116.8121	grams
Final “ “ “ “	116.8126	“
Gain	0.0005	“
Total gain	0.0054	“

Time required for absorption, 3 hours.

$$\therefore p_{\text{CO}} = \frac{0.00065 \times 0.0821 \times 291 \times 760 \times 95}{19.9 \times 69.3} = 0.81 \text{ mm.}$$

Experiment 8.

The charge was the same material as used in Experiment 7.

The furnace was evacuated and heated for an hour and fifty minutes at 1050°. It was then evacuated to a pressure of 0.12 centimeter and hydrogen let in to 2.0, again evacuated to 0.15 and carbon monoxide let in to 0.28 centimeter. Hydrogen was then admitted to 77.6 centimeters requiring an hour and a quarter.

Duration of run: 6 hours, 10 minutes.
Power: 11.4 K. W.
Temperature: 1445° C.
The cuprous chloride bulbs were refilled.

Initial weight of cuprous chloride bulbs	167.4274	grams
Final “ “ “ “ “	167.4312	“
	0.0038	“

The tube previously used for gold chloride was filled with cuprous chloride.

Initial weight of second cuprous chloride tube	119.3358	grams
Final " " " " " "	119.3363	"
Gain	0.0005	"
Total gain	0.0043	

$p_1 = 67.1$ cm of mercury
$p_2 = 37.7$ " "
$p_3 = 92.7$ " "

The time taken for absorption was 3¼ hours.

$$\therefore p_{\mathrm{CO}} = \frac{.000350 \times 0.0821 \times 287 \times 760 \times 92.7}{19.9 \times 67.1} = 0.44 \text{ mm.}$$

3. DISCUSSION OF RESULTS.

For convenience the results obtained above are collected in the following table.

TABLE IV.

No. of Exp.	Duration in Hours.	Kilowatts.	Temp. Centigrade.	Gain in Weight of 1st Bulb.	Gain in Weight of 2d Bulb.	Time taken for Absorption in Hours.	Initial Pressure CO in mm. Hg.	Final Pressure CO in mm. Hg.
1	6½	12.0	1485	0.0090	. . .	6	12.5	0.86
2	6	11.7	1465	0.0072	. . .	4½	0.0	0.73
3	6	8.2	1250	0.0032	. . .	3½	0.0	0.30
4	7	8.8	1270	0.0013	. . .	2⅔	0.0	0.13
5	4½	12.6	1525	0.0034	. . .	3	0.0	0.34
6	6¼	11.8	1475	0.0073	0.0036	4	2.0	0.88
7	6	11.8	1475	0.0049	0.0005	3	1.5	0.81
8	6	11.4	1445	0.0038	0.0005	3¼	1.3	0.44

In all of these experiments, even at 1250°, there was some white powder on the walls of the furnace. Whether a slight decomposition of carbide into its elements takes place at this temperature could not be decided by this means, as the white powder may have been due to two causes, both the decomposition of carbide and to volatilization of some impurity in the lime or carbon. The best evidence that the carbide does not break up at 1475° and does break up at 1525° is that equilibrium could be measured at the former but not at the latter temperature. Attention was called to the possibility of lime itself being somewhat volatile at 1500°, since a piece of Merk's lime heated at the melting point of platinum for an hour also produced a layer of white powder on the walls of the furnace.

As Experiments 1 and 2 were carried out at temperatures equally above and below the temperature in experiments 6 and 7, the average of these four may be taken, with the result

$$p_{\mathrm{CO}} \text{ at } 1475°\mathrm{C.} = 0.82 \pm .02 \text{ mm.}$$

Through these results were obtained from the same side of the equilibrium, different amounts of carbon monoxide were present at the beginning in each case, which makes the evidence that equilibrium had been reached conclusive.

From this result, the pressure obtained in Experiment 8 at a temperature 30° lower may be checked by the integrated van't Hoff equation :

$$4.57 \log_{10} \frac{p_2}{p_1} = Q \left(\frac{1}{T_1} - \frac{1}{T_2} \right)$$

where p_2 and p_1 and the pressures of carbon monoxide corresponding to the absolute temperatures T_1 and T_2 and Q is the heat absorbed by the reaction, when it proceeds from left to right. Q has been calculated[14] to be 121000 calories at room temperature, with a negative temperature coefficient of 3.3 calories per degree.

Therefore $$Q = 121000 - 3.3\, t,$$

where t equals centigrade degrees above room temperature, which for high temperatures may be considered as degrees above zero. For 1460° C. Q therefore equals 116000 calories. Substituting in the above equation the absolute temperatures corresponding to 1475° and 1445°, the value of $\frac{p_2}{p_1}$ comes out 1.79. The ratio between the pressures found by experiment is 1.86, which is very satisfactory agreement.

If the pressure at 1270° is calculated from that at 1475°, using the value of Q corresponding to the mean temperature 1370°, the result is

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[14] Trans. Am. Electrochem. Soc., 1909, **15**, 197.

0.0093 millimeter, that is, it is below a measurable quantity. The fact that in one case 0.13 and in another 0.3 millimeters were found is due to the insufficient time allowed to absorb this very small amount of carbon monoxide.

From the value of the equilibrium pressure obtained at 1475° it is possible by the above formula to calculate the pressure at higher temperatures and see approximately what is the shape of the pressure temperature curve. The value of Q corresponding to the mean of each set of temperatures is used. 1475° is always taken as the lower temperature. The results of this computation are given in Table IV and Figure 3.

TABLE V.

Pressures of Carbon Monoxide Computed from the Value Determined at 1475°.

Temperature Degrees Centigrade.	Equilibrium Pressure of Carbon Monoxide in Centimeters.		
	I Lower Limit.	II Mean.	III Upper Limit.
1475	0.05	0.08	0.13
1575	0.31	0.50	0.79
1675	1.54	2.53	4.00
1775	6.6	10.7	17.
1875	25.0	40.5	64.
1975	81.	133.0	210.

It is evident that the error in this curve is due practically entirely to the error in the temperature measurements, for while the value of p_1 is accurate to 2.5 per cent, the temperature is uncertain by 25°, and the value 0.82 millimeters might correspond to 1500° or 1450° as the two extremes. This would mean the true value at 1475° might be 1.3 or 0.5 millimeters as the two extremes. If now the curve be computed first with the value 1.3 in place of 0.82, and again with 0.5, the values under I and III in Table V are obtained. The values are plotted in Figure 3 in broken curves. From these curves it is seen the temperature corresponding to 1/3 of an atmosphere lies between 1800° and

1875°, with which Rudolphi's values agrees the best of all the three referred to in the Introduction.

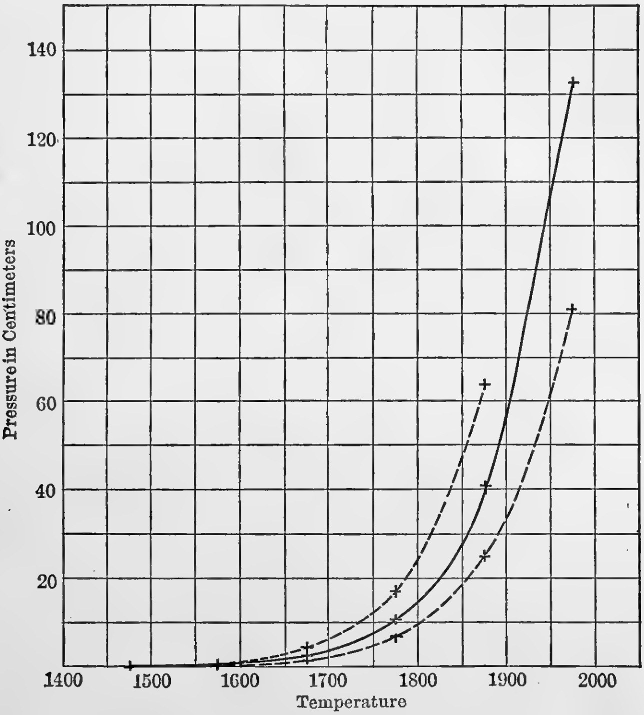

Figure 3. Pressure of Carbon Monoxide Computed from the Value Determined at 1475° C.

The free energy increase of the reaction taken from left to right at 1475° C. is

$$\begin{aligned}\Delta F &= RT \log \frac{760}{0.82} \\ &= 4.57 \times 1748 \log_{10} 927 \\ &= + 23700 \text{ calories}\end{aligned}$$

As the temperature rises ΔF decreases till at 1920°, where the equilibrium pressure equals an atmosphere, $\Delta F = 0$. Above 1920° ΔF becomes negative.

SUMMARY OF RESULTS.

1. The equilibrium pressure of carbon monoxide in the reaction

$$CaO + 3\ C \rightleftarrows CaC_2 \times CO$$

was measured at 1475° and 1445°. The results were in good thermodynamic agreement.

2. A little below 1445° C. the pressure becomes too small to measure; a little above 1475° decomposition of calcium carbide into its elements prevents measurement of equilibrium.

3. With the aid of the heat of the reaction the vapor pressure curve at higher temperatures was computed which cannot be realized experimentally on account of the decomposition of calcium carbide.

4. The free energy increases of the reaction

$$CaO + 3\ C = CaC_2 + CO$$

at 1475° is +23700 calories.

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By John Trowbridge.

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1. Reflection of Cathode Rays.

In the course of this paper I shall refer to certain hydrodynamical analogies which the discharges of electricity through gases present; not with the conviction that in these discharges we have to deal with questions of flow alone. The complicated phenomena give large scope both to theories of flow and molecular theories: the hydrodynamical analogies are more striking in discharges through gases at comparatively high pressures; while molecular theories apply best in highly rarefied gases. There seems to be a certain continuity here similar to that between motions of matter in the liquid state and in the gaseous state, when such matter is subjected to forces which can produce movement or flow of the particles.

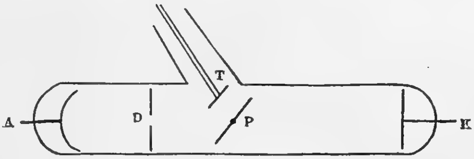

Figure 1.

The conditions of electrical discharges in a tube represented in Figure 1 remind one of the flow of a fluid interrupted by a plane lamina. A is a cathode, K an anode, D a diaphragm, P a plane lamina which can be moved about an axis perpendicular to the plane of the paper, Figure 1 being a plan of the discharge tube. P can also serve as an anode.

At the striæ stage the electrical conditions in the tube are very little modified by turning the lamina through small inclinations to the line of discharge. The striæ remain practically unaffected in shape and position until the angle between the normal to the lamina and the axis of flow reaches 50°. This phenomenon is analogous to the case of a lamina subjected to the flow of a liquid (Lamb's Hydrodynamics, pages 94 and 111). It is also analogous to the conditions presented by the impact of wind on vanes.

By means of a side adjunct a thermo pile, T, was introduced in order to measure the heat excited by the reflection of the cathode rays passing through the diaphragm D and reflected from the lamina, when the latter was inclined to the axis of the cathode rays at varying angles. Here also there was an action similar to the reflection of a stream of liquid from the lamina, proceeding in the direction of the cathode rays. The angle between the normal to the lamina and the axis of flow or discharge could vary largely without affecting the amount of heat from the reflected cathode beam shown by the thermopile.

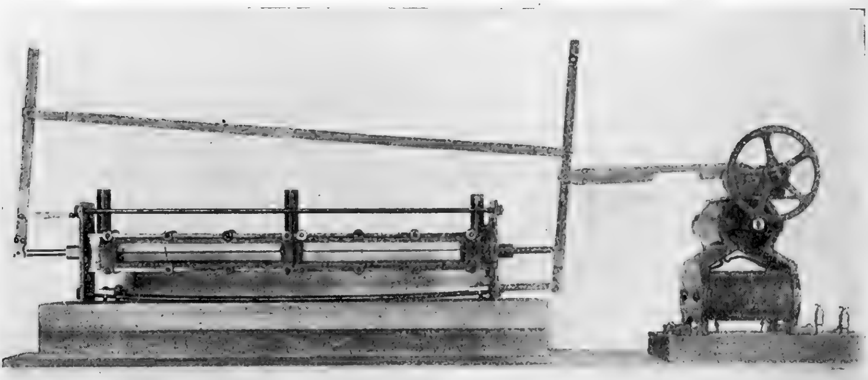

FIGURE 2.

2. STRIÆ.

The striæ, or stratifications, in Geissler tubes constitute a very beautiful and mysterious phenomenon of the discharge of electricity through gases, and if one could follow the mechanism involved perfectly one could feel sure of having penetrated far into questions of the method of propagation of electricity. There seems no reason to doubt that the striæ are phenomena of ionization; but the regularity of the striæ leads one to ask if this regularity could arise from some pulsation or rhythmical action, — the ionization being, so to speak, on top of such a rhythmical action. When the striæ are excited by a storage battery, they are perfectly steady, and when one is sure that there are no breaks in the circuit, a telephone introduced into the circuit is silent; moreover, self-induction included in the circuit does not affect the striæ.

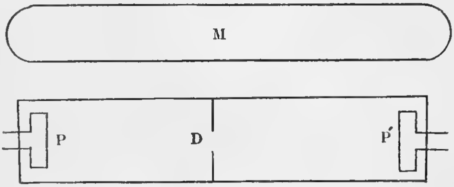

FIGURE 3.

Under certain conditions the current from a storage battery oscillates or pulsates, but such oscillations or pulsations do not seem to modify the appearance of the stratifications. If, on the other hand, there is a flow from the cathode which pulsates at a different rate from a supposititious flow from the anode, one might expect striæ, or accumulation of ionic disturbances at regular intervals. An hydrodynamical analogy is afforded by the motion of two pistons moving against each other at different rates in a channel filled with water.

Figure 2 represents an apparatus by means of which two pistons driven in opposite directions by a motor cause waves in a trough filled with water.

FIGURE 4.

Figure 3 shows the arrangement, in plan, by means of which the ripples are studied. M is a mercury lamp of the Cooper Hewitt form. This is placed directly behind the trough containing the pistons. The surface of the water, totally reflecting the light, forms a dark line which under the motion of the pistons undulates in waves, which can be studied by instantaneous photography. P and P′ are the pistons, and D is a diaphragm with a rectangular orifice. Figure 4 represents a case in which P moves twice as fast as P′. The waves are formed nearer the slower-moving piston.

All who have worked in the field of discharge of electricity through gases must recognize the suggestiveness of the theory of ionization by collision, especially in reference to striæ; but one who was ignorant of this theory, in seeing the action of the cathode rays in apparently driving the striæ into the anode, might attribute this action to an actual repelling force arising from the cathode. When this supposititious force is diverted by a magnet, the striæ reappear and more current flows. One ignorant, too, of the many facts of ionization by collision might further suppose that heavier particles of slower motion might be held back by swifter particles issuing from the cathode. These views of a mind not biased by ionization theories would appear to be supported by the phenomena presented by the tube represented in Figure 5.

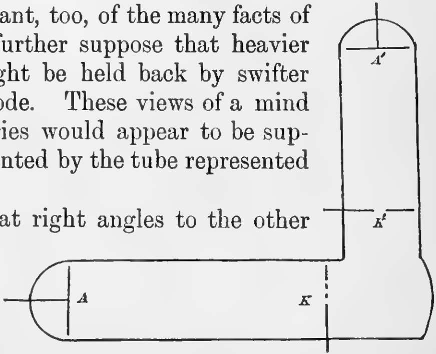

FIGURE 5.

One branch of this tube is at right angles to the other branch. There are two anodes, A and A′, and two perforated cathodes, K and K′. When a multiple circuit is formed by leading in the current to the two anodes and out by one cathode, K, striæ form in the branch A′K′ after they disappear in the branch AK; and they persist in the branch A′K′ when the branch AK appears to be nearly at the X-Ray stage. One looking at the branch A′K′ would suppose that the rarefication of the entire tube was low, and gazing at the branch AK would think it very high. The bend in the tube acts like a magnet in allowing the striæ to emerge from the anode A′; and it does this by enfeebling by reflection the effect of the cathode rays in the branch A′K′.

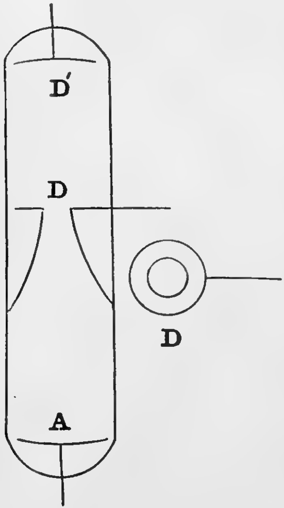

FIGURE 6.

The function of the cathode beam seems to be twofold: it forces back the striæ, and at higher exhaustions it ionizes the gas; for the current ceases to flow at high exhaustions when the cathode beam is strongly diverted by a magnet. These functions are illustrated by the phenomena in a tube represented in Figure 6.

Between the anode A and a cathode D the glass tube is constricted. The cathode D is a circular disc with an orifice a little larger than the glass orifice. The cathode rests upon the ground walls of this orifice, presenting no metallic surface toward the anode A. The cathode beam produces an orange fluorescence toward

D′, and is marked in the direction toward A by a white beam which produces hardly a perceptible fluorescence. The latter beam does not come from the metallic surface of the cathode, but seems to come from the gas in the region DD′. At comparatively high exhaustions this latter portion of the cathode beam ceases to ionize the gas and the current ceases; the potential between A and D rises to the full potential of the battery — indicating an open circuit. When, however, D′ is made the cathode, the current is immediately re-established and the cathode beam from D′ ionizes the gas between D′ and A. The tube acts as a rectifier; for when D is made the anode and A the cathode, a current passes; on reversal of the current, when at the same exhaustion, no current passes in the opposite direction.

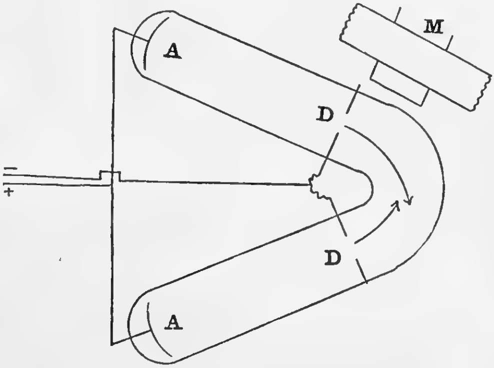

FIGURE 7.

It is interesting to observe the effect of a transverse magnetic field on the discharge in this tube when A is made a cathode and D an anode, and striæ appear in the portion DD′. The magnetic field placed near A diverts the cathode beam and striæ advance in the portion DD′. While this field is still on, another transverse magnetic field placed near D′ diverts the striæ independently of the action of the field at A. This indicates the well known fall of potential from striæ to striæ.

The rectification observed under proper conditions in the tube (Figure 6) suggests other forms of tubes by which rectification can be produced. Even with a straight cylindrical tube the current can be stopped at high exhaustions by touching the outside of the tube with the finger, thus diverting the cathode beam by electrostatic action; while it readily passes when the current is reversed. The phenomenon of rectification is shown in a practical way in the U-shaped tube represented in Figure 7. It is provided with two anodes, A and A′, and two cathodes, D and D.′ The cathodes have orifices at their centres. The two anodes are connected together, and the two cathodes — the tube forming a multiple circuit. A transverse magnetic field can be so placed near one cathode that no current will pass in the branch of the tube of which it is a part, while the current passes freely in the other branch of the U tube. This form of tube rectifies an alternating current.

The apparent repelling or driving back action of the cathode beam on striæ is shown in a suggestive manner in a straight cylindrical tube when a diaphragm is inserted between the anode and the cathode. We will take for illustration one branch of the U-shaped tube (Figure 7), and suppose that the current is led into the tube at A and out at D. A metallic diaphragm with a small hole at its centre is inserted in the tube about one third of the distance beween A and D, measured from the anode A — the latter also having an orifice at its centre. The striæ are slowly driven back by the cathode rays as the exhaustion proceeds. At a definite stage of this exhaustion a stria takes refuge behind the diaphragm nearer the anode, where it is protected from the driving back action of the cathode rays; finally at higher exhaustions this stria is driven through the orifice in the anode and shelters itself behind the anode.

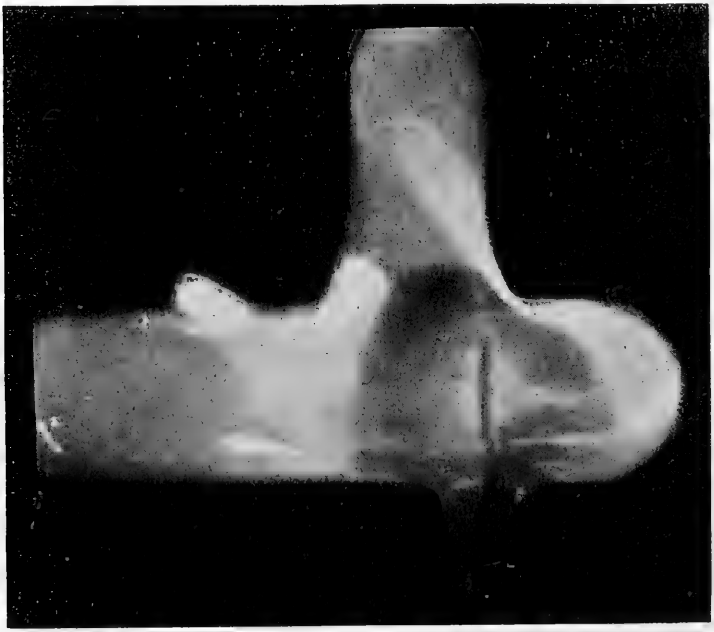

Figure 8.

At a still higher state of rarefaction a stria issues from the orifice in the anode, and this also shelters itself behind the diaphragm on the side toward the anode. There are, thus, three definite stages of stratification in this form of tube. At a pressure of four centimetres fine striæ appear on the side of the orifice in the diaphragm opposite to the anode. These soon disappear with increasing rarefaction. At a pressure of approximately 3 mm. a large stria shelters itself behind the diaphragm. This fades into the orifice in the anode with diminishing pressure; and at a pressure of approximately .15 mm. a large stria wells up out of the orifice in the anode and takes a similiar place near the diaphragm. When the state of canalstrahlen is reached, all striæ have been driven into the anode. Can we regard these strahlen as a stratification which cannot be driven back by the cathode rays? In this form of tube we find evidence of successive states of stratification which may depend upon positive rays of different velocity.

When we turn from our observation of stratification in the neighborhood of the cathode instead of in the neighborhood of the anode, we find that a stratification always takes place on the glass wall close to the entrance of the cathode, or to its sealing in place. It can be produced equally well by causing the cathode to approach the wall of the tube opposite to this sealing in place. Figure 8 represents the phenomenon in a tube with a dome-shaped chamber near the electrode. We seem to have two dissected striæ: one on the wall of the tube nearest to the cathode, which provides a beautiful light blue cathode beam thrown into the dome; and another stria on the opposite wall of the dome. The original cathode beam excites both positive and negative rays in these striæ. In considering these detached striæ it seems that the cathode rays in striking the glass walls can excite both positive and cathode rays.

When a spark gap is inserted in a circuit containing a discharge tube which is properly exhausted to the striæ stage, the latter apparently disappear — the light of the tube becomes more brilliant and fluorescence is generally manifested. This is also the case when a condenser is discharged through the tube. The eye cannot perceive any evidence of stratifications; for the brightness of the pilot spark, together with the fluorescence both of the gas and of the glass walls effectually shield any striæ of lesser radiance which might be present. It is not possible to employ a revolving mirror. The only method which seemed to promise any results in detection of possible stratifications was the employment of a portrait lens of large aperture — four inches —in photographing single discharges. Accordingly a discharge tube was filled with hydrogen and exhausted to the striæ stage. A condenser of .02 m f capacity was charged to a difference of potential of 100.000 volts and discharged through the rarefied tube by flat copper bands of inappreciable self-induction. The photographs showed unmistakable striæ, superposed upon the general illumination of the tube. It is difficult to reproduce the photographs by half tones.

With an anode consisting of a ring of wire placed in a cylindrical tube .5 mm. internal diameter, a striation is formed at a short distance from the anode by condenser discharges, and there are traces of similar striations at greater distances along the tube. If these striations are formed by ionization by collision, the time of ionization is that of the duration of the pilot spark, a time which at present is beyond our power of measurement.

3. Doppler Effect.

When two anodes and two cathodes are employed in the form of tube represented in Figure 7, there are two canalstrahlen which emanate from orifices in the cathodes in opposite directions. One might suppose that the Doppler effect would be modified by collision of the particles in these rays and that the effect would certainly be less than when only one anode and one cathode were employed — the current thus passing through but one branch of the U tube. It is true that the difference of potential is less between A and D when the tube is coupled in multiple circuit than when only one branch of the tube is connected to the battery; but this difference in the case I studied was comparatively small. With both branches of the tube constituting a multiple circuit there were two strong canalstrahlen passing through the orifices in D which were undistorted and which gave the same Doppler effect which was obtained when only one branch of tube was excited; it seems difficult to reconcile this result with any theory of collision.

4. Conclusions.

1. The striæ in Geissler tubes are analogous to waves set up in narrow channels by opposing pulsations of different periods.
2. Striæ are greatly influenced by the direction of cathode rays. Certain forms of tubes, described in this article, can imitate the action of a transverse magnetic field in apparently increasing the conductibility of the rarefied gas and restoring the condition of stratification.
3. Striæ can be formed by condenser discharges; and such striæ lead one to suppose a time of ionization beyond our power of measure-

ment. By means of a suitably placed diaphragm successive stages in stratification can be produced.

4. By modification of the form of discharge tubes rectification of alternating discharges is possible.

5. The Doppler effect in hydrogen is not modified by causing two canalstrahlen to oppose each other.

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BUDDHAGHOSA'S DHAMMAPADA COMMENTARY,

and the Titles of its three hundred and ten Stories, together with an Index thereto and an Analysis of Vaggas I–IV.

By Eugene Watson Burlingame,

Harrison Fellow, University of Pennsylvania.

BUDDHAGHOSA'S DHAMMAPADA COMMENTARY.

By Eugene Watson Burlingame.

Presented by Charles R. Lanman, December 8, 1909. Received February 5, 1910.

Prefatory Remarks. — My interest in Hindu Folk-tales was first aroused by Professor Morris Jastrow, Jr., of the University of Pennsylvania, who introduced me to the famous Arabian classic Kalīla wa Dimna, giving me generously of his time, and granted me the privilege of collaborating in the preparation of an English translation of the recently published Cheikho recension of the text. Professor Morton W. Easton, of the same University, to whom I am no less indebted for valuable assistance in my work, then induced me to make a serious study of the corresponding Sanskrit collections, Pañcatantra and Hitopadeça, and encouraged me to prosecute researches in the closely related Pāli collections. When, therefore, Provost Harrison of the University of Pennsylvania, the giver of the Harrison Foundation, granted me leave of absence from the University for this purpose, I placed myself under the direction of Professor Charles R. Lanman, of Harvard University. It was at his suggestion that I undertook the task upon which, under his most wise and kindly guidance, I am at present engaged, that of translating into English the important Buddhist work entitled Buddhaghosa's Commentary on the Dhammapada.[1]

Divisions of the Buddhist Texts. — In order to give the reader a clear idea of the relation in which Buddhaghosa's Dhammapada Commentary stands to the Buddhist Canon, it will be necessary to describe briefly the principal divisions of the Buddhist Scriptures. They fall into three principal divisions called Piṭakas (Baskets); first, the Sutta Piṭaka; secondly, the Vinaya Piṭaka; thirdly, the Abhi-

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[1] Several years ago my attention was first attracted to this fascinating collection of stories by reading a brief description of it in Professor Rhys Davids's American Lectures on Buddhism. The passage that caught my eye occurs on page 69, and closes as follows: "Cannot some one undertake a translation for us into English of these strange and interesting old-world stories about a collection of verses so widely popular among Buddhists, and now attracting so much attention in the West?" Nevertheless, it is due wholly and entirely to Professor Lanman that I am able to answer "Yes."

dhamma Piṭaka. Speaking broadly, the first relates to Doctrine; the second, to Discipline; the third, to what we may call Psychology. The first two Piṭakas alone concern us. Each of the Piṭakas falls into several subdivisions. The Sutta Piṭaka consists of five groups, called Nikāyas; namely, Four Nikāyas the Greater, and One Nikāya the Less. The first four Nikāyas are called the Āgamas, and are as follows: (1) Dīgha; (2) Majjhima; (3) Saŋyutta; (4) Aṅguttara. The Dīgha and Majjhima consist of Dialogues of the Buddha, arranged somewhat after the manner of the Dialogues of Plato; the Saŋyutta and Aṅguttara contain sayings of the Buddha, arranged according to subject and length respectively. These four Nikāyas are the oldest parts of the Canon, and are the source of most of our knowledge of the tenets and history of primitive Buddhism. The Lesser Nikāya, called the Khuddaka, consists of fifteen books, grouped in three pentads. Of these fifteen books, perhaps the most famous are the Thera- and Therīgāthā (or Hymns of the Monks and Nuns), the Sutta Nipāta (a very old collection of poetical dialogues and epic pieces), the Udāna (or Solemn Utterances of the Buddha), the Jātakas, and the Dhammapada. As the above-given titles indicate, the Lesser Nikāya is a miscellaneous, but none the less exceedingly important, collection. It is not relevant to our purpose to consider the subdivisions of the Vinaya. Suffice it to say that it contains a number of highly interesting stories, designed to explain the circumstances under which various rules and ceremonies were established.

The Dhammapada and its Commentary. — The Dhammapada, then, is one of fifteen books belonging to the Khuddaka Nikāya, which latter is the fifth division of the Sutta Piṭaka; and the Sutta Piṭaka is one of the three major divisions of the Sacred Scriptures of the Buddhists. The Dhammapada is an anthology of about 423 stanzas uttered by the Buddha on a great variety of religious subjects. Many such anthologies were current in the early ages of Buddhism, and so great was the popularity they acquired that in addition to the anthology included in the Buddhist Canon other similar collections have come down to us. For example, in 1878, Samuel Beal published a translation of a Chinese Dhammapada; in 1898, Émile Senart deciphered and published part of a Kharoṣṭhī Ms. of the Dhammapada, the fruit of the mission of Dutreuil de Rhins; and Richard Pischel, shortly before his death, brought out specimens of a Central Asiatic Dhammapada. The precise relation between the Dhammapada of the Buddhist Canon and the other collections has not yet been determined; nor is it important for our immediate purpose. It is sufficient to say that by a fortunate circumstance one of these anthologies was included in the Buddhist

Canon. This Anthology consists of twenty-six parts, or books (vaggas), the arrangement of the stanzas being by subjects, such as Heedfulness, The Fool, The Wise Man, The Buddha, Pleasure, Anger, and so on. The relation between the Anthology and its Commentary will at once become clear from an example. Suppose we had a collection of detached sayings of Christ; such as, for example, "Labor not for the meat which perisheth;" or, "He that is without sin among you, let him first cast a stone at her." The Commentary bears much the same relation to the Sacred Stanzas as the Gospel narrative to the Sacred Sentences. The parallel is not a perfect one, for the Commentary does not rank as canonical; besides which, there are certain other important differences.

The Commentary consists of upwards of three hundred stories (vatthus), distributed in twenty-six books (vaggas), corresponding to the parts of the Dhammapada described above. Ordinarily each story consists of eight subdivisions, as follows: (1) quotation of the stanza (gāthā) to illustrate which the Buddha told the story; (2) a brief statement of the occasion and the person or persons about whom the story was told; (3) the story proper; or, more strictly, the Story of the Present (paccuppanna-vatthu), closing with the utterance of (4) the stanza or stanzas; (5) word-for-word commentary or gloss on the stanza; (6) a brief statement of the spiritual benefits which accrued to the hearer or hearers; (7) the Story of the Past; or, more accurately, the Story of Previous Existences (atīta-vatthu); (8) identification of the personages of the Story of the Past with those of the Story of the Present. Sometimes the Story of the Past is omitted, together with the accompanying Identification; but it is so much expected as a matter of course, that at the end of the story of Nanda the Herdsman (iii. 8), where none occurs, the author is at some pains to say that, as no one asked the Teacher about Nanda's deed in a previous existence, the Teacher said nothing about it. It will readily be seen that the Dhammapada Commentary closely resembles, both in form and content, the commentary on the famous Jātaka collection; indeed, so close is the connection between the two that it would not be inappropriate to call the Commentary a supplement to the Jātaka. The Commentary constantly refers to the Jātaka, every now and then borrows a story from it, sometimes showing interesting variants, and as often gives a different version of some familiar Jātaka story. The stories of the Dhammapada Commentary stand in precisely the same relation to the stanzas of the Dhammapada as the Jātaka stories do to the Jātaka stanzas. The Dhammapada Commentary has sometimes been referred to as a sort of Buddhist Acta Sanctorum; it would perhaps be more appropriate to speak of it as a Collection of Stories about Buddhist

Saints and Sinners, designed to illustrate the maxim, " Whatsoever a man soweth, that shall he also reap."

Editions of the Dhammapada Commentary. — In 1855, extracts from the Commentary were published by Fausböll in his edition of the Dhammapada. The second edition of this work, published in 1900, contains only the text and translation of the Dhammapada. In 1906–9 appeared the first two instalments of the Pāli Text Society edition of the Commentary, edited by Professor H. C. Norman of Benares. These two parts together make up Volume I, and contain the first four vaggas. Since the publication of Fausböll's first edition of the Dhammapada, editions of the Commentary, in whole or in part, printed in Burmese or Cingalese letters, have appeared; and at present H. R. H. Prince Vajira-ñāṇa is engaged in publishing an edition of the work at Bangkok. The editions which form the basis of my work are as follows: (1) Pāli Text Society, Vol. I, Parts 1–2, London, 1906–9; (2) Burmese, edited by Ū Yan, Rangoon, 1903; (3) Cingalese, edited by W. Dhammānanda Mahā Thera and M. Ñāṇissara Thera, Colombo, 1898–1908.

Translations of parts of the Commentary. — Only a few of the stories have ever been translated into any European language. Such of the Jātaka stories as are identical with stories contained in the Commentary, or similar to them, will be found in the Cambridge translation of the Jātaka. An English version of three of the stories will be found in Warren's Buddhism in Translations: Patipūjikā (iv. 4), pp. 264–7; Visākhā (iv. 8), pp. 451–481; Godhika (iv. 11), pp. 380–3. Four more stories were translated into French by Godefroy de Blonay and Louis de la Vallée Poussin under the title Contes Bouddhiques, and were published in the Revue de l' Histoire des Religions. Volume xxvi (1892) contains two of these stories: Cakkhupāla (i. 1), pp. 180–193; and Maṭṭhakuṇḍalī (i. 2), pp. 193–200; Volume xxix (1894), the two others: Kosambikā bhikkhū (i. 5), pp. 329–337; Viḍūḍabha (iv. 3), pp. 195–211. In 1870, Captain T. Rogers published, under the title Buddhaghosha's Parables, an English translation of a late Burmese version of a few of the stories. References to the Jātakas and to Rogers's Parables are given in the Analysis.

Purpose of this paper. — The purpose of this paper is two-fold. First, it is hoped, by means of a Table giving the titles of the stories, and by an Alphabetic Index to those titles, to render the work in its entirety more accessible to scholars. In particular, it is hoped that the proper names of eminent Buddhists and the information about them may prove of special value as material for the Buddhist onomasticon of Professor Rhys Davids. That the contents of the last two thirds of the Commentary are virtually almost inaccessible to Occidental students is a fact that deserves especial emphasis as an ample justification of the present paper. Norman's edition of the first third is of course easily had; but it may well be doubted whether there are more than two or three copies of the Cingalese edition in the western hemisphere, or more than one copy of the Burmese. And there is probably not one bookseller in the United States who would even attempt to procure directly such rare exotics. And even if a considerable number of copies were to be found in the great libraries of America, it is still true that the Burmese and Cingalese letters are so troublesome that very few Occidentals, even among the students of Pāli, have learned to read these native editions with facility. Secondly, it is hoped, by means of an Analysis of the first third of the work, to afford some idea of its structure, contents, and style, not only to professed students of Sanskrit and Pāli, but also to students of Comparative Literature, and to the general reader as well.

In case the paper shall subserve, to however small a degree, the purpose for which it is intended, a large share of the credit belongs, not to me, but to my friend and teacher, Professor Lanman, who, in the midst of pressing duties, has given me unreservedly of his time and labor, and has assisted me in countless ways. I wish to thank him most heartily for his many kindnesses to me during the progress of my work.

Note on the Table of Contents and Alphabetical Index. — Unfortunately, Fausböll has numbered the stanzas of the Dhammapada from the beginning continuously; and this bad example has been followed by the Burmese edition; and, to make a bad matter worse, its numeration (from 163 to 208, and from 416 to 424) disagrees with that of Fausböll. The Cingalese edition does not number the gāthās. In the following table, the numbers of the gāthās are given in **heavy type** and in square brackets immediately after the title of the story: first, the number of the gāthā as counted from the beginning of its vagga[2]; second, the number as counted continuously from the beginning. If, for the latter numeration, on account of the disagreement just mentioned, more than one number has to be given, or if, on account of variation in the titles, more than one title has to be given, they are distinguished by a prefixed F (meaning Fausböll), or B (meaning Burmese), or C (meaning Cingalese). The stories are numbered from the beginning of each book. The number as counted

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[2] This is the only proper method. To ignore such important and historically significant native divisions is extremely reprehensible and unpractical.

continuously from the beginning to the end of the work is ignored of a purpose and upon principle. In the columns at the right are given the numbers of the pages on which the stories begin (not end). PTS means Pāli Text Society, B Burmese, C Cingalese. In the Alphabetical Index, the stories are cited by book (vagga : in Roman numerals) and story (vatthu : in Arabic). Thus, xiv. 3 means the third story of the fourteenth book. Exponential numbers indicate imbedded stories. Thus, in ii. 1 are imbedded ii. 1^a, 1^b, 1^c, 1^d, 1^e, 1^f.

TITLES OF STORIES OF THE DHAMMAPADA COMMENTARY.

Yamaka-vagga = Book I.

Story		PTS	B	C
1.	Cakkhupāla thera [**1** = **1**]	3	44	1
2.	Maṭṭhakuṇḍalī [**2** = **2**]	25	58	12
3.	Tissa thera (B) = Thulla Tissa thera (PTS and C) [**3–4** = **3–4**]	37	67	18
4.	Kāḷi yakkhinī [**5** = **5**]	45	72	22
5.	Kosambikā bhikkhū [**6** = **6**]	53	77	26
6.	Cūla Kāḷa and Mahā Kāḷa [**7–8** = **7–8**]	66	84	33
7.	Devadatta [**9–10** = **9–10**]	77	91	38
8.	Aggasāvakā (PTS and C) = Sāriputta thera (B) [**11–12** = **11–12**]	83	95	41
9.	Nanda thera [**13–14** = **13–14**]	115	116	58
10.	Cunda sūkarika [**15** = **15**]	125	123	64
11.	Dhammika upāsaka [**16** = **16**]	129	125	66
12.	Devadatta [**17** = **17**]	133	128	68
13.	Sumanā devī [**18** = **18**]	151	139	77
14.	Dve sahāyakā bhikkhū [**19–20** = **19–20**]	154	141	78

Appamāda-vagga = Book II.

Story		PTS	B	C
1.	Udena (PTS and C) Sāmāvatī (B) [**1–3** = **21–23**]	161	145	81
	1^a Udena-uppatti	161	145	81
	1^b Ghosaka-seṭṭhi-uppatti	169	150	85
	1^c Sāmāvatī-uppatti	187	162	95
	1^d Vāsuladattā	191	166	97
	1^e Māgandiyā	199	170	101
	1^f Maraṇa-paridīpaka	203	173	103
2.	Kumbhaghosaka seṭṭhi [**4** = **24**]	231	190	116
3.	Cūla Panthaka thera [**5** = **25**]	239	195	120
4.	Bāla-nakkhatta-ghuṭṭha [**6–7** = **26–27**]	256	205	128
5.	Mahā Kassapa thera [**8** = **28**]	258	207	130
6.	Dve sahāyakā bhikkhū (PTS) = Pamatt-appamattā dve sahāyakā bhikkhū (B and C) [**9** = **29**]	260	208	131
7.	Mahāli-pañha (PTS and C) = Magha (B) [**10** = **30**]	263	210	132
8.	Aññatara bhikkhu [**11** = **31**]	281	221	140
9.	Nigamavāsī Tissa thera [**12** = **32**]	283	222	141

Citta-vagga = Book III.

Story		PTS	B	C
1.	Meghiya thera [**1–2 = 33–34**]	287	224	143
2.	Aññatara bhikkhu [**3 = 35**]	290	226	145
3.	Ukkaṇṭhit-aññatara-bhikkhu (PTS and C) = Aññatara ukkaṇṭhita bhikkhu (B) [**4 = 36**]	297	232	149
4.	Bhāgineyya-saṅgharakkhita thera (PTS and C) = Saṅgharakkhita-bhāgineyya thera (B) [**5 = 37**]	300	243	151[3]
5.	Cittahattha thera [**6–7 = 38–39**]	305	236	154
6.	Pañcasata vipassakā bhikkhū (PTS and C) = Pañcasata bhikkhū (B) [**8 = 40**]	313	241	158
7.	Pūtigatta Tissa thera [**9 = 41**]	319	245	160
8.	Nanda gopāla [**10 = 42**]	322	248	162
9.	Soreyya thera [**11 = 43**]	325	249	164

Puppha-vagga = Book IV.

Story		PTS	B	C
1.	Paṭhavi-kathā-pasutā pañcasata bhikkhū [**1–2 = 44–5**]	333	254	167
2.	Marīci-kammaṭṭhānika thera [**3 = 46**]	335	256	168
3.	Viḍūḍabha (PTS and C) = Viṭaṭūbha (B) [**4 = 47**]	337	257	169
4.	Patipūjikā (PTS and C) = Pātipūjikā kumārikā (B) [**5 = 48**]	362	272	181
5.	Macchariya Kosiya seṭṭhi [**6 = 49**]	366	274	183
6.	Pāṭhikājīvaka (PTS and C) = Pāveyyakājīvaka (B) [**7 = 50**]	376	280	187
7.	Chattapāṇi upāsaka [**8–9 = 51–52**]	380	283	189
8.	Visākhā [**10 = 53**]	384	286	191
9.	Ānanda-thera-pañha [**11–12 = 54–55**]	420	308	209
10.	Mahā-Kassapa-thera-piṇḍapāta-dinna [**13 = 56**]	423	310	210
11.	Godhika-thera-parinibbāṇa [**14=57**]	431	315	214
12.	Garahadinna [**15–16 = 58–59**]	434	317	216

Bāla-vagga = Book V.

Story		B	C
1.	Kumuduppalātīta-duggata-sevaka (C) = Aññatara purisa (B) [**1 = 60**]	324	221
2.	Mahā-Kassapa-thera-saddhivihārika [**2 = 61**]	335	230
3.	Ānanda seṭṭhi [**3 = 62**]	339	233
4.	Ganṭhi-bhedakā corā [**4 = 63**]	342	235
5.	Udāyi thera [**5 = 64**]	343	235
6.	Bhadda-vaggiyā(C) = Tiŋsa-matta-pāveyyakā bhikkhū (B) [**6 = 65**]	344	236
7.	Suppabuddha kuṭṭhī [**7 = 66**]	345	237
8.	Kassaka [**8 = 67**]	347	238
9.	Sumana mālā-kāra [**9 = 68**]	349	240
10.	Uppalavaṇṇā therī [**10 = 69**]	353	243
11.	Jambukājīvaka (C) = Jambuka thera (B) [**11 = 70**]	355	245
12.	Ahipeta [**12 = 71**]	363	251
13.	Saṭṭhikūṭa peta [**13 = 72**]	366	253
14.	Sudhamma thera (C) = Citta gahapati (B) [**14–15 = 73–4**]	369	256
15.	Vanavāsī Tissa thera (C) V. T. sāmaṇera (B) [**16 = 75**]	376	261

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[3] The Colombo edition has no page 153.

Paṇḍita-vagga = Book VI.

Story		B	C
1.	Rādha thera [**1 = 76**]	389	271
2.	Assaji-punabbasukā [**2 = 77**]	392	273
3.	Channa thera [**3 = 78**]	393	274
4.	Mahā Kappina thera [**4 = 79**]	394	275
5.	Paṇḍita sāmaṇera [**5 = 80**]	403	281
6.	Lakuṇṭaka-bhaddiya thera [**6 = 81**]	415	291
7.	Kāṇa-mātā [**7 = 82**]	416	292
8.	Vighāsādā dosa-vuttā pañcasata bhikkhū (C) = Pañcasata bhikkhū (B) [**8 = 83**]	418	294
9.	Dhammika thera [**9 = 84**]	420	295
10.	Dhamma-savaṇa [**10–11 = 85–86**]	422	296
11.	Āgantukā pañcasata bhikkhū (C) = Pañcasata āgantukā bhikkhū (B) [**12–14 = 87–89**]	423	297

Arahanta-vagga = Book VII.

Story		B	C
1.	Jīvaka-pañha [**1 = 90**]	424	298
2.	Mahā Kassapa thera [**2 = 91**]	426	299
3.	Belaṭṭha-sīsa thera [**3 = 92**]	428	301
4.	Anuruddha thera [**4 = 93**]	429	302
5.	Mahā Kaccāyana thera [**5 = 94**]	431	303
6.	Sāriputta thera [**6 = 95**]	432	304
7.	Kosambivāsī-Tissa-thera-sāmaṇera [**7 = 96**]	435	306
8.	Sāriputta-thera-pañha-vissajjana [**8 = 97**]	437	308
9.	Khadiravaniya Revata thera [**9 = 98**]	438	309
10.	Aññatarā itthi [**10 = 99**]	445	314

Sahassa-vagga = Book VIII.

Story		B	C
1.	Tamba-dāṭhika-cora-ghātaka [**1 = 100**]	446	315
2.	Dāru-ciriya thera (C) = Bāhiya-dāru-ciriya thera (B) [**2 = 101**]	450	318
3.	Kuṇḍala-kesi-therī [**3–4 = 102–3**]	454	322
4.	Anattha-pucchaka brāhmaṇa [**5–6 = 104–5**]	459	326
5.	Sāriputta-therassa mātula-brāhmaṇa [**7 = 106**]	461	327
6.	Sāriputta-therassa bhāgineyya [**8 = 107**]	462	328
7.	Sāriputta-therassa sahāyaka brāhmaṇa [**9 = 108**]	463	328
8.	Dīghāyu kumāra (C) = Āyuvaḍḍhana kumāra (B) [**10 = 109**]	464	329
9.	Saṅkicca-sāmaṇera [**11 = 110**]	466	331
10.	Khāṇu-koṇḍañña thera [**12 = 111**]	474	337
11.	Sappa-dāsa thera [**13 = 112**]	475	338
12.	Paṭācārā therī [**14 = 113**]	478	340
13.	Kisā Gotamī [**15 = 114**]	484	344
14.	Bahu-puttikā therī [**16 = 115**]	487	347

Pāpa-vagga = Book IX.

Story		B	C
1.	Cūḷeka-sāṭaka brāhmaṇa [**1 = 116**]	488	348
2.	Seyyasaka thera [**2 = 117**]	491	350
3.	Lājā devadhītā [**3 = 118**]	492	351

Story		B	C
4.	Anāthapiṇḍika seṭṭhi [**4–5** = **119–120**]	494	353
5.	Asaññata-parikkhāra bhikkhu [**6** = **121**]	497	355
6.	Biḷāla-padaka seṭṭhi [**7** = **122**]	498	356
7.	Mahādhana vāṇija [**8** = **123**]	501	358
8.	Kukkuṭa-mitta-nesāda [**9** = **124**]	502	359
9.	Koka-sunakha-luddaka [**10** = **125**]	507	362
10.	Maṇikāra kulūpaga Tissa thera [**11** = **126**]	509	364
11.	Tayo bhikkhū (C) = Tayo janā (B) [**12** = **127**]	511	366
12.	Su-ppabuddha Sakya [**13** = **128**]	515	369

Daṇḍa-vagga = Book X.

Story		B	C
1.	Chab-baggiyā [**1** = **129**]	517	370
2.	Chab-baggiyā [**2** = **130**]	518	371
3.	Sambahulā kumārakā [**3–4** = **131–2**]	519	371
4.	Kuṇḍadhāna thera [**5–6** = **133–4**]	519	372
5.	Visākhādīnaŋ upāsikānaŋ uposatha-kamma [**7** = **135**]	523	375
6.	Ajagara peta [**8** = **136**]	524	376
7.	Mahā Moggallāna thera [**9–12, 137–140**]	527	378
8.	Bahubhaṇḍika thera (C) = B. bhikkhu (B) [**13** = **141**]	531	381
9.	Santati mahāmatta [**14** = **142**]	535	384
10.	Pilotika-thera (C) = -Tissa-thera (B) [**15–16** = **143–4**]	538	387
11.	Sukha sāmaṇera [**17** = **145**]	540	388

Jarā-vagga = Book XI.

Story		B	C
1.	Visākhāya sahāyikā [**1** = **146**]	548	394
2.	Sirimā [**2** = **147**]	550	396
3.	Uttarā therī [**3** = **148**]	554	398
4.	Adhimānikā bhikkhū (C) = Sambahulā adhimānikā bhikkhū (B) [**4** = **149**]	555	399
5.	Janapada-kalyāṇī-rūpa-nandā therī [**5** = **150**]	556	400
6.	Mallikā devī [**6** = **151**]	559	403
7.	Lāḷudāyi thera [**7** = **152**]	561	404
8.	Ānanda-therassa udāna-gāthā [**8–9** = **153-4**]	564	406
9.	Mahādhana seṭṭhi-putta [**10–11** = **155–6**]	565	407

Atta-vagga = Book XII.

Story		B	C
1.	Bodhi rājakumāra [**1** = **157**]	568	409
2.	Upananda Sakyaputta [**2** = **158**]	571	412
3.	Padhānika Tissa thera [**3** = **159**]	573	414
4.	Kumāra Kassapa thera (C) = Kumāra-Kassapa-mātu-therī (B) [**4** = **160**]	574	415
5.	Mahā Kāla upāsaka [**5** = **161**]	578	417
6.	Devadatta [**6** = **162**]	579	419
7.	Saṅgha-bheda-parisakkana[4] [**F7, BC7–8** = **F163, B163-4**]	580	419

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[4] This story is told in connection with the stanzas beginning "Sukaraŋ sādhunā sādhuŋ" and "Sukarāni asādhūni." B and C give both stanzas, but Fausböll omits the first. Cp. Dh. (1900), p. 38.

Story		B	C
8.	Kāla thera [**F8, BC9 = F164, B165**]	581	420
9.	Cūla Kāla upāsaka [**F9, BC10 = F165, B166**]	583	421
10.	Atta-d-attha thera [**F10, BC11 = F166, B167**]	584	422

Loka-vagga = Book XIII.

Story		B	C
1.	Aññatara dahara bhikkhu [**1 = F167, B168**]	585	423
2.	Suddhodana-rāja (B) = Suddhodana (C) [**2–3 = F168–9, B169–170**]	587	424
3.	Pañcasata vipassakā bhikkhū [**4 = F170, B171**]	588	425
4.	Abhaya rājakumāra [**5 = F171, B172**]	589	426
5.	Sammuñjani thera(C) = Sammajjana thera (B) [**6 = F172, B173**]	590	427
6.	Aṅgulimāla thera [**7 = F173, B174**]	591	428
7.	Pesakāra-dhītā [**8 = F174, B175**]	592	428
8.	Tiŋsa bhikkhū [**9 = F175, B176**]	595	431
9.	Ciñcā māṇavikā [**10 = F176, B177**]	596	432
10.	A-sadisa-dāna [**11 = F177, B178**]	600	434
11.	Kāla nāma Anāthapiṇḍika-putta (C) = Anāthapiṇḍika-putta Kāla (B) [**12 = F178, B179**]	603	437

Buddha-vagga = Book XIV.

Story		B	C
1.	Māra-dhītaro (C) = Māgaṇḍiyā (B) [**1–2 = F179–180, B180–181**]	606	439
2.	Yamaka-pāṭihāriya (C) = Dev-orohaṇa (B) [**3 = F181, B182**]	609	442
3.	Erakapatta nāgarājā [**4 = F182, B183**]	628	456
4.	Ananda-thera-uposatha-pañha [**F5–7, C5–6**[5] **= F183–5, B184–6**]	632	459
5.	Anabhiratibhikkhu [**F8–9, C7–8 = F186–7, B187–8**]	633	460
6.	Kosala-rañño purohita Aggidatta-brāhmaṇa (C) = Aggidatta-brāhmaṇa (B) [**F10–14, C9–13 = F188–192, B189–193**]	635	462
7.	Ānanda-thera-pucchita-pañha [**F15, C14 = F193, B194**]	639	465
8.	Sambahulā bhikkhū [**F16, C15 = F194, B195**]	640	466
9.	Kassapa-dasabalassa suvaṇṇa-cetiya [**F17–18, C16–17 = F195–6**]	*	466

Sukha-vagga = Book XV.

Story		B	C
1.	Ñāti-kalaha-vūpasamana [**1–3 = F197–9, B196–8**]	641	468
2.	Māra [**4 = F200, B199**]	643	470
3.	Kosala-rañño parājaya [**5 = F201, B200**]	644	470
4.	Aññatarā kuladārikā [**6 = F202, B201**]	645	471
5.	Goṇaṭṭha upāsaka (B) = Aññatara upāsaka (C) [**7 = F203, B202**]	646	472

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[5] C omits the stanza beginning "Khantī paramaŋ tapo titikkhā" (F184).

* B omits the stanzas beginning "Pūjārahe pūjayato" and "Te tādise pūjayato," and the story connected therewith.

Story		B	C
6.	Pasenadī-Kosala rājā [**8** = **F204**, **B203**]	648	473
7.	Tissa thera (B) = Aññatara bhikkhu (C) [**9** = **F205**, **B204**]	650	474
8.	Sakka devarājā (C) = Sakkupaṭṭhāna (B) [**10–12**, **B10–13** = **F206–208**, **B205–208**[6]]	651	475

Piya-vagga = Book XVI.

Story		B	C
1.	Tayo janā pabbajitā (B) = Tayo bhikkhū (C) [**1–3** = **209–211**]	653	477
2.	Aññatara kuṭumbika [**4** = **212**]	655	479
3.	Visākhā [**5** = **213**]	656	480
4.	Licchavī [**6** = **214**]	657	480
5.	Anitthigandha-kumāra [**7** = **215**]	658	481
6.	Aññatara brāhmaṇa [**8** = **216**]	660	482
7.	Pañcasata dārakā [**9** = **217**]	662	484
8.	Anāgāmī thera [**10** = **218**]	663	485
9.	Nandiya [**11–12** = **219–220**]	664	486

Kodha-vagga = Book XVII.

Story		B	C
1.	Rohinī khattiya-kaññā [**1** = **221**]	666	488
2.	Aññatara bhikkhu [**2** = **222**]	669	489
3.	Uttarā upasikā [**3** = **223**]	670	491
4.	Mahā-Moggallāna-thera-pañha-pucchita [**4** = **224**]	677	496
5.	Sāketaka-brāhmaṇa (C) = Buddha-pitu-brāhmaṇa (B) [**5** = **225**]	678	497
6.	Puṇṇā nāma Rājagaha-seṭṭhi-dāsī [**6** = **226**]	681	498
7.	Atula upāsaka [**7–10** = **227–230**]	683	500
8.	Chab-baggiya-bhikkhū [**11–14** = **231–234**]	685	502

Mala-vagga = Book XVIII.

Story		B	C
1.	Go-ghātaka-putta [**1–4** = **235–8**]	686	503
2.	Aññatara brāhmaṇa [**5** = **239**]	690	506
3.	Tissa thera [**6** = **240**]	691	507
4.	Lāḷudāyi thera [**7** = **241**]	693	508
5.	Aññatara kulaputta [**8–9** = **242–3**]	695	510
6.	Sāriputta-therassa saddhi-vihārika (C) = Cūḷa Sāri (B) [**10–11** = **244–5**]	697	511
7.	Pañcasata upāsakā [**12–14** = **246–8**]	699	512
8.	Tissa dahara [**15–16** = **249–250**]	700	513
9.	Pañca upāsakā [**17** = **251**]	702	515
10.	Meṇḍaka seṭṭhi [**18** = **252**]	704	516
11.	Ujjhāna-saññī thera [**19** = **253**]	711	522[7]
12.	Subhadda paribbājaka [**20–21** = **254–5**]	712	522

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[6] B divides F207–8 into three stanzas, thus:

B206 Bālasaŋgatacārī hi dīgham addhāna socati
Dukkho bālehi saŋvāso amitteneva sabbadā

B207 Dhīro ca sukhasaŋvāso ñātīnaŋ va samāgamo
Tasmā hi: Dhīraŋ paññañ ca bahussutañ ca dhorayha

B208 Sīlaŋ vatavantam āriyaŋ taŋ tādisaŋ sappurisaŋ
Sumedhaŋ bhajetha nakkhattapathaŋ va candimā.

[7] Pages 522–529 of the Colombo edition are numbered (by a printer's error) 122–129.

Dhammaṭṭha-vagga = Book XIX.

Story		B	C
1.	Vinicchaya-mahāmaccā [**1–2 = 256–7**]	713	523
2.	Chab-baggiya-bhikkhū [**3 = 258**]	714	524
3.	Ekūdāna-thera-khīṇāsava (B) = Ekuddāna-khīṇāsava-thera (C) [**4 = 259**]	715	525
4.	Lakuṇṭaka-bhaddiya-thera [**5–6 = 260–261**]	716	526
5.	Sambahulā bhikkhū [**7–8 = 262–3**]	717	526
6.	Hatthaka [**9–10 = 264–5**]	718	527
7.	Aññatara brāhmaṇa [**11–12 = 266–7**]	719	528
8.	Titthiya [**13–14 = 268–9**]	720	529
9.	Bālisika (C) = Ariya-bālisika (B) [**15 = 270**]	722	530
10.	Sambahulā bhikkhū (C) = Sambahulā sīlādi-sampannā bhikkhū (B) [**16–17 = 271–2**]	722	530

Magga-vagga = Book XX.

Story		B	C
1.	Pañcasata bhikkhū [**1–4 = 273–6**]	724	531
2.	Pañcasata bhikkhū (C) = Anicca-lakkhaṇa (B) [**5 = 277**]	725	533
3.	Pañcasata bhikkhū (C) = Dukkha-lakkhaṇa (B) [**B6**, [**C6–7**[8] = **B278**]	726	533
B4.	Anatta-lakkhaṇa [**B7 = B279**]	726	[8]
B5 C4.	Padhāna-kammika Tissa thera [**8 = 280**]	727	534
B6 C5.	Sūkara-peta [**9 = 281**]	728	535
B7 C6.	Poṭṭhila thera [**10 = 282**]	732	538
B8 C7.	Pañca mahallakā therā [**11–12 = 283–4**]	734	539
B9 C8.	Suvaṇṇakāra thera [**13 = 285**]	736	541
B10 C9.	Mahādhana vāṇija [**14 = 286**]	738	543
B11 C10.	Kisā Gotamī [**15 = 287**]	740	544
B12 C11.	Paṭācārā [**16–17 = 288–9**]	741	545

Pakiṇṇaka-vagga = Book XXI.

Story		B	C
1.	Gaṅgārohana (C) = Attano pubba-kamma (B) [**1 = 290**]	742	546
2.	Kukkuṭaṇḍa-khādikā [**2 = 291**]	749	551
3.	Bhaddiyā bhikkhū [**3–4 = 292–3**]	751	552
4.	Lakuṇṭaka-bhaddiya thera [**5–6 = 294–5**]	752	553
5.	Dāru-sākaṭika-putta [**7–12 = 296–301**]	753	555
6.	Vajji-puttaka bhikkhu [**13 = 302**]	756	557
7.	Citta gahapati [**14 = 303**]	758	558
8.	Cūḷā Subhaddā [**15 = 304**]	759	559
9.	Eka-vihārī thera [**16 = 305**]	762	562

Niraya-vagga = Book XXII.

Story		B	C
1.	Sundarī paribbājikā [**1 = 306**]	763	563
2.	Duccarita-phalānubhavana-sattā [**2 = 307**]	766	565
3.	Vaggu-mudā-tiriya-bhikkhū [**3 = 308**]	767	565

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[8] In the Colombo edition the story entitled "Dukkha-lakkhaṇa" is told in connection with stanzas 6-7, and the story entitled "Anatta-lakkhaṇa" is omitted.

Story		B	C
4.	Anāthapiṇḍika-bhāgineyya-Khemaka-seṭṭhi-putta (B) = Khema (C) [**4–5** = **309–310**]	767	566
5.	Dubbaca bhikkhu [**6–8** = **311–313**]	769	567
6.	Issā-pakatā itthi [**9** = **314**]	770	568
7.	Sambahulā āgantukā bhikkhū [**10** = **315**]	771	569
8.	Niganthā [**11–12** = **316–317**]	772	570
9.	Titthiyā sāvakā [**13–14** = **318–319**]	773	571

Nāga-vagga = Book XXIII.

Story		B	C
1.	Attānaŋ ārabbha kathita [**1–3** = **320–322**]	775	572
2.	Hatthācariya-pubbaka bhikkhu [**4** = **323**]	777	573
3.	Parijiṇṇa-brāhmaṇa-putta (B) = Aññatara-brāhmaṇa-putta (C) [**5** = **324**]	777	574
4.	Pasenadī-Kosala [**6** = **325**]	782	577
5.	Sāṇu sāmaṇera [**7** = **326**]	783	578
6.	Pāveyyaka hatthi (B) = Baddheraka hatthi (C) [**8** = **327**]	786	581
7.	Sambahulā bhikkhū (B) = Pañcasata disā-vāsī bhikkhū (C) [**9–11** = **328–330**]	787	581
8.	Māra [**12–14** = **331–333**]	790	583

Taṇhā-vagga = Book XXIV.

Story		B	C
1.	Kapila-maccha [**1–4** = **334–7**]	792	585
2.	Sūkara-potikā [**5–10** = **338–343**]	797	589
3.	Vibbhanta bhikkhu [**11** = **344**]	800	592
4.	Bandhanāgāra [**12–13** = **345–6**]	802	593
5.	Khemā therī [**14** = **347**]	804	594
6.	Uggasena-seṭṭhi-putta [**15** = **348**]	805	595
7.	Cūḷa Dhanuggaha paṇḍita (B) = Daharaka bhikkhu (C) [**16–17** = **349–350**]	809	599
8.	Māra [**18–19** = **351–2**]	812	601
9.	Upakājīvaka [**20** = **353**]	814	602
10.	Sakka-pañha (B) = Sakkadevarājā (C) [**21** = **354**]	814	603
11.	Aputtaka seṭṭhi [**22** = **355**]	817	605
12.	Añkura [**23–26** = **356–9**]	819	606

Bhikkhu-vagga = Book XXV.

Story		B	C
1.	Pañca bhikkhū [**1–2** = **360–361**]	821	607
2.	Haŋsa-ghātaka bhikkhu [**3** = **362**]	823	609
3.	Kokālika [**4** = **363**]	825	611
4.	Dhammārāma thera [**5** = **364**]	827	613
5.	Vipakkha-sevaka bhikkhu [**6–7** = **365–6**]	828	613
6.	Pañc-aggadāyakā brāhmaṇā [**8** = **367**]	830	615
7.	Sambahulā bhikkhū [**9–17** = **368–376**]	832	616
8.	Pañcasata bhikkhū [**18** = **377**]	838	621
9.	Santakāya thera [**19** = **378**]	839	622
10.	Nañgala-kula thera [**20–21** = **379–380**]	840	623
11.	Vakkali thera [**22** = **381**]	842	624
12.	Sumana sāmaṇera [**23** = **382**]	843	625

Brāhmaṇa-vagga = Book XXVI.

Story		B	C
1.	Pasāda-bahula-brāhmaṇa [**1 = 383**]	854	633
2.	Sambahulā bhikkhū [**2 = 384**]	855	633
3.	Māra [**3 = 385**]	855	634
4.	Aññatara brāhmaṇa [**4 = 386**]	856	634
5.	Ānanda thera [**5 = 387**]	857	635
6.	Aññatara brāhmaṇa pabbajita [**6 = 388**]	858	636
7.	Sāriputta thera [**7–8 = 389–390**]	858	636
8.	Mahā Pajāpati Gotamī [**9 = 391**]	860	638
9.	Sāriputta thera [**10 = 392**]	861	638
10.	Jaṭila brāhmaṇa [**11 = 393**]	862	639
11.	Kuhaka brāhmaṇa [**12 = 394**]	863	639
12.	Kisā Gotamī [**13 = 395**]	865	641
13.	Eka brāhmaṇa [**14 = 396**]	865	641
14.	Uggasena-seṭṭhi-putta [**15 = 397**]	866	642[9]
15.	Dve brāhmaṇā [**16 = 398**]	867	642
16.	Akkosalabhāradvāja [**17 = 399**]	867	643
17.	Sāriputta thera [**18 = 400**]	869	644
18.	Uppalavaṇṇā therī [**19 = 401**]	870	645
19.	Aññatara brāhmaṇa [**20 = 402**]	871	645
20.	Khemā bhikkhunī [**21 = 403**]	871	646
21.	Pabbhāravāsī Tissa thera [**22 = 404**]	872	646
22.	Aññatara bhikkhu [**23 = 405**]	874	648
23.	Sāmaṇerā (B) = Cattāro sāmaṇerā (C) [**24 = 406**]	876	649
24.	Mahā Panthaka thera [**25 = 407**]	878	651
25.	Pilindavaccha thera [**26 = 408**]	879	651
26.	Aññatara thera [**27 = 409**]	880	652
27.	Sāriputta thera [**28 = 410**]	881	653
28.	Mahā Moggallāna thera [**29 = 411**]	881	653
29.	Revata thera [**30 = 412**]	882	654
30.	Candābha thera [**31 = 413**]	883	654
31.	Sīvali thera [**32 = 414**]	885	656
32.	Sundara-samudda-thera [**33 = 415**]	887	657
33.	Jaṭila thera [**34 = 416**]	890	660
	33ª. Jotikassa uppatti	890	660
	33ᵇ. Jaṭila thera	899	667
34.	Jotika thera [**34 = F416, B417**[10]]	905	671
35.	Naṭa-puttaka thera (B) = Naṭa-pubbaka (C) [**35 = F417, B418**]	906	672
36.	Naṭa-puttaka thera [**36 = F418, B419**]	907	672
37.	Vaṅgīsa thera [**37–38 = F419–420, B420–421**]	907	673
38.	Dhammadinnā therī [**39 = F421, B422**]	909	674
39.	Aṅgulimāla thera [**40 = F422, B423**]	911	675
40.	Devahita brāhmaṇa (B) = Devaṅgika brāhmaṇa (C) [**41 = F423, B424**]	911	676

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[9] Pages 642–677 of the Colombo edition are numbered (by a printer's error) 624–659.

[10] Story 34 repeats the stanza of Story 33.

ALPHABETIC INDEX TO THE TITLES OF THE STORIES OF THE DHAMMAPADA COMMENTARY.

Kāla nāma Anāthapiṇḍika-putta (C) = A.-p.-K. (B), xiii. 11.
Kāḷi yakkhinī, i. 4.
Kisā Gotamī,[11] viii. 13; xx. 11; xxvi. 12.
Kukkuṭaṇḍa-khādikā, see Ekā k.-kh.
Kukkuṭa-mitta-nesāda, ix. 8.
Kuṭumbika, see Aññatara k.
Kuṇḍadhāna thera, x. 4.
Kundala-kesi-therī, viii. 3.
Kumāra Kassapa thera (C) = K.-K.-mātu-therī (B), xii. 4.
Kumuduppalātīta-duggata-sevaka (C) = Aññatara purisa (B), v. 1.
Kumbhaghosaka seṭṭhi, ii. 2.
Kuladārikā, Kulaputta, see Aññatarā, -a.
Kuhaka brāhmaṇa, xxvi. 11.
Kūṭa peta, see Saṭṭhi-k. p.
Koka-sunakha-luddaka, ix. 9.
Kokālika, xxv. 3.
Koṇḍadhāna thera, see Kuṇḍadhāna thera.
Kosambikā bhikkhū, i. 5.
Kosambivāsī-Tissa-thera-sāmaṇera, vii. 7.
Kosala-rañño parājaya, xv. 3.
Kosala-rañño purohita Aggidatta-brāhmaṇa (C) = A.-b. (B), xiv. 6.
Khadiravaniya Revata thera, vii. 9.
Khāṇu-koṇḍañña thera, viii. 10.
Khema (C) = Anāthapiṇḍika-bhāgineyya-Khemaka-seṭṭhi-putta (B), xxii. 4.
Khemā therī, xxiv. 5.
Khemā bhikkhunī, xxvi. 20.
Gaṅgārohana (C) = Attano pubbakamma (B), xxi. 1.
Ganṭhi-bhedakā corā, v. 4.
Garahadinna, iv. 12.
Goghātaka putta, xviii. 1.
Gotamī, see Kisā G. and Mahā Pajāpati G.
Godhika-thera-parinibbāṇa, iv. 11.
Goṇaṭṭha upāsaka (B) = Aññatara upāsaka (C), xv. 5.
Ghosaka-seṭṭhi-uppatti, ii. 1^b.
Cakkhupāla thera, i. 1.
Cattāro sāmaṇerā (C) = Sāmaṇerā (B), xxvi. 23.
Candābha thera, xxvi. 30.
Ciñcā māṇavikā, xiii. 9.
Citta gahapati (B) = Sudhamma thera (C), v. 14.
Citta gahapati, xxi. 7.
Cittahattha thera, iii. 5.
Cunda sūkarika, i. 10.
Cūla Kāla upāsaka, xii. 9.
Cūla Kāla and Mahā Kāla, i. 6.
Cūla Dhanuggaha paṇḍita (B) = Daharaka bhikkhu (C), xxiv. 7.
Cūla Panthaka thera, ii. 3.
Cūla Sāri (B) = Sāriputta-therassa saddhivihārika (C), xviii. 6.
Cūlā Subhaddā, xxi. 8.
Cūlaka-sāṭaka-brāhmaṇa, ix. 1.
Chab-baggiyā, x. 1; x. 2; xvii. 8; xix. 2.
Chattapāṇi upāsaka, iv. 7.
Channa thera, vi. 3.
Janapada-kalyāṇī-rūpa-nandā therī, xi. 5.
Jaṭila thera, xxvi. 33 and 33^b.
Jaṭila brāhmaṇa, xxvi. 10.
Janā, see Tayo janā.
Jambuka thera (B) = Jambukājīvaka (C), v. 11.
Jīvaka-pañha, vii. 1.
Jotika thera, xxvi. 34.
Jotika-uppatti, xxvi. 33^a.
Ñāti-kalaha-vūpasamana, xv. 1.
Tamba-dāṭhika-cora-ghātaka, viii. 1.
Tayo janā (B) = Tayo bhikkhū (C), ix. 11.
Tayo janā pabbajitā (B) = Tayo bhikkhū (C), xvi. 1.
Tayo bhikkhū, ix. 11; (C) = Tayo janā pabbajitā (B), xvi. 1.
Tiŋsa bhikkhū, xiii. 8.
Tiŋsa-matta-pāveyyakā bhikkhū, v. 6.
Titthiya, xix. 8.
Titthiyā sāvakā, xxii. 9.

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[11] Müller, in his Glossary of Pāli Proper Names (JPTS. 1888), gives only one Kisā Gotamī, as does also Kern in his Manual of Indian Buddhism (page 16, note 3). But are not the virgin of the Warrior caste who greeted the Buddha from the roof of her palace (Jā. i. 60^{26}–61^{14}), and the frail widow, daughter of a poverty-stricken house, described in these passages as sorrowing over the loss of her first-born son, two entirely different persons?

ANALYSIS OF THE STORIES OF THE DHAMMAPADA COMMENTARY, BOOKS I–IV.

Ayaŋ pan' ettha saṅkhepo.

Book I. Story 1. Cakkhupāla, Elder.[12]

ILLUSTRATING STANZA 1 = 1.

Mahāsuvaṇṇa, a rich householder of Sāvatthi, made a vow to a tree-spirit, whereby he became the father of two sons. Since the tree had been protected (pālitaŋ) by him, he named them Mahā Pāla[13] and Culla Pāla. When they reached manhood, their parents set them up in households of their own.[14] **(3-4)**

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[12] Cf. Rogers, pp. 1–11.

[13] Called Cakkhupāla after he wins Arahatship by sacrificing his eyes. *Cakkhu* is the Pāli word for "eye."

[14] The numbers printed in heavy type and in parentheses at the end of each paragraph indicate the pages of Norman's text which are summarized in the paragraph concerned.

At this time the Teacher was in residence at Jetavana monastery. (He spent one rainy season at Banyan-tree monastery, erected by his relatives; nineteen at Jetavana, erected by Anāthapiṇḍika; six at Eastern-grove, erected by Visākhā.) Anāthapiṇḍika and Visākhā went to the monastery twice each day with the usual offerings. One day the former refrained from asking questions for fear of wearying the Teacher. Knowing this, Buddha preached with such vehemence that fifty of the seventy million inhabitants of Sāvatthi became noble disciples. The noble disciples performed two duties daily: before breakfast, they dispensed alms; after breakfast, bearing the usual offerings, they went to hear the Law. (**4–5**)

Mahāpāla followed them one day and was so affected by the discourse that he asked Buddha to make him a monk. Taking leave of his brother, who did his utmost to dissuade him, he was admitted and professed. After five years had passed, he came to Buddha and asked him how many were the Burdens of the Religious Life. On being told that there were two, namely, the Burden of memorizing and preaching the Scriptures, and the Burden of the development of Spiritual Insight by ascetic practices and meditation, he chose the latter as being better suited to his advanced years. The Teacher instructed him in the ascetic practices leading to Arahatship, and he set out with sixty disciples. (**5–8**)

The inhabitants of a village 120 leagues distant received them hospitably, obtained the privilege of entertaining them during the rainy season, and built them a monastery. A physician also offered his services. Mahāpāla, on learning that the monks purposed to avail themselves of the four postures (walking, standing, sitting, and reclining), announced that he should content himself with the first three, and vowed not to stretch his back in repose. After encouraging each other to be vigilant, they entered upon the observance of the rainy season. (**8–9**)

At the end of the first month Mahāpāla's eyes began to trouble him. The physician treated him, but as he never lay down to rest, the treatment did him no good. However, he resolutely kept his vow, until finally, one night at the end of the middle watch, he lost simultaneously his eyesight and the Depravities, and became an Arahat. The monks and villagers, learning that he had lost his eyesight, expressed their sympathy, and assured him that they would take care of him. At the end of the rainy season, the monks also attained Arahatship. (**9–13**)

When the monks expressed a desire to see the Teacher, Mahāpāla, knowing that there was a forest on the way haunted by evil spirits, and fearing that he would be a hindrance to the monks, sent them on ahead, directing them to ask his brother to send some one to lead him, and to greet Buddha and the eighty abbots in his name. After taking leave of the villagers, who were reluctant to part with them, they went and did their master's bidding. Cullapāla sent his nephew Pālita, first admitting him as a monk, that he might escape the dangers of the journey. (**13-15**)

Pālita, after waiting upon Mahāpāla for a fortnight, led him to the village. In spite of the protests of the inhabitants, they continued on their journey until they reached the forest, where the youth, hearing the voice of a woman, left his uncle and broke the vow of chastity. Returning, he confessed his sin, removed his yellow robes, and assumed the garb of a householder. But Mahāpāla would have nothing more to do with him, and he departed in tears. (**15-17**)

So intense was Mahāpāla's morality that Sakka's throne showed signs of heat. Looking about, he beheld the Elder. Fearing that if he failed to go to his assistance, his head would split into seven pieces, he disguised himself as a wayfarer, went to him, and offered to lead him to Sāvatthi. Shortening the distance by his magic power, Sakka brought him to his destination that very evening. Cullapāla cared for him tenderly and gave him two novices to wait on him. (**17-19**)

One night after a heavy rain Cakkhupāla took a walk in the cloister and trampled many insects to death. Some visiting monks reported the matter to the Teacher, who replied that as Cakkhupāla did not see the insects, he was innocent of offense. The monks then asked how it was that the Elder, though destined to attain Arahatship, became blind. Buddha replied that it was because of a sin he committed in a previous existence. The monks asked the Teacher to tell them about it, and he did so. (**19-20**)

Story of Cakkhupāla's sin in a previous existence. A woman of Benares promised a physician that she and her children would become his slaves in case he succeeded in curing her of an affection of the eyes. He did so; but she, repenting of her bargain, attempted to deceive him by telling him that her eyes were worse than ever. He discovered that she was deceiving him, and got revenge by giving her an ointment that made her blind. That physician was Cakkhupāla. (**20-21**)

The Teacher, warning his hearers to take the lesson to heart, pronounced Stanza 1, at the conclusion of which, thirty thousand monks attained Arahatship. (**21-4**)

Book I. Story 2. Maṭṭhakuṇḍalī.[15]

ILLUSTRATING STANZA 2 = 2.

At Sāvatthi lived a Brahman of a disposition so niggardly that people called him Adinnapubbaka (Never-gave-a-farthing). He had an only son, whom he dearly loved. Desiring to give the boy a pair of earrings, but at the same time to avoid unnecessary expense, he beat out the gold himself, made him a pair, and gave them to him; wherefore people called the boy Maṭṭhakuṇḍalī (The-boy-with-the-burnished-ear-rings). When the boy was sixteen years old he had an attack of jaundice. The mother wished to have a physician called, but the father demurred at the thought of paying him his fee, inquired of various physicians what remedies they were accustomed to prescribe for such and such an ailment, and treated him himself. The boy grew steadily worse and was soon at the point of death. Realizing this, and fearing that those who came to see his son would also see the wealth the house contained, the Brahman carried his son outside and laid him down on the terrace. **(25–6)**

That very morning the Exalted One, arising from a Trance of Great Compassion, and surveying the world with the eye of a Buddha, beheld Maṭṭhakuṇḍalī lying on the terrace at the point of death. Foreseeing that Maṭṭhakuṇḍalī, and through him many others, would attain the Fruit of Conversion, Buddha visited him on the following day. The youth made an Act of Faith in Buddha, died, and was reborn in the world of the Thirty-three. **(26–8)**

Adinnapubbaka, after having the body of his son cremated, went daily to the cemetery and bewailed his loss. Maṭṭhakuṇḍalī, desiring to convert his father, assumed the form he had borne upon earth, and went and wept also. The Brahman asked the youth why he was weeping. The latter replied: "I need a pair of wheels for my chariot. The sun and moon are just what I want, and I weep because I cannot get them." The Brahman told him he was a fool. "But which of us is the bigger fool," said the youth, "I, who weep for what exists, or you, who weep for what does not exist?" The youth then told him that he was his son, and that he had attained his present glory by making an Act of Faith in the Buddha. Thereupon the father sought refuge in the Buddha, the Law, and the Order, and took upon himself the Five Precepts. The son, after urging his father to visit the Buddha, disappeared. **(28–33)**

The Brahman invited Buddha and his monks to dine with him.

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[15] Cf. Rogers, pp. 12–17.

Buddha accepted the invitation. The Brahman asked him whether it was possible to attain rebirth in heaven by a simple Act of Faith. Buddha instanced the case of Maṭṭhakuṇḍalī, and then said: "It is not one hundred, or two hundred, — there is no counting the number of those who have attained rebirth in heaven by making an Act of Faith in me." To convince the bystanders, he summoned Maṭṭhakuṇḍalī, who appeared in all his glory and confirmed the Buddha's words. Buddha then dwelt upon the importance of a right attitude of the thoughts and of a believing heart, and pronounced Stanza 2. (33-5)

At the conclusion of the stanza eighty-four thousand persons obtained Comprehension of the Law. The god Maṭṭhakuṇḍalī was established in the Fruit of Conversion; likewise Adinnapubbaka, who devoted his great wealth to the religion of Buddha. (37)

Book I. Story 3. Tissa the Fat, Elder.[16]

ILLUSTRATING STANZAS 3–4 = 3–4.

Tissa, a son of the sister of Buddha's father, became a monk late in life. He lived well on the Buddha's alms, and spent most of his time sitting in smoothed garments in the Buddha's own room. He grew to be fat and well-liking. One day he so far presumed on his kinship with the Buddha as to snub some monks who came to pay their respects. The monks resented this; whereupon the Elder, informing them who he was, threatened to extirpate their whole race, and went and complained to the Buddha. The latter, after asking him a few questions about his behavior, told him that he was in the wrong, and directed him to apologize to the monks. This he refused to do. The monks remarked that he was strangely obstinate and intractable; whereupon the Buddha, informing them that it was not the first time he had so conducted himself, related the following story of the past: (37-9)

Devala and Nārada. Once upon a time, when Brahmadatta reigned at Benares, two ascetics, Devala and Nārada, obtained lodging for the night in Potter's Hall. After Nārada had lain down, Devala, in order to start a quarrel, lay down in the door-way. Nārada, having occasion to go out during the night, trod on Devala's matted locks. Devala then changed his posture, putting his head where his feet had been. When Nārada returned, he trod on his neck. In spite of Nārada's protests that it was all an accident, Devala cursed him, saying, "When

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[16] Cf. Rogers, pp. 18–24.

the sun rises to-morrow may your head split into seven pieces." Nārada then pronounced the curse, "When the sun rises to-morrow may the head of the guilty person split into seven pieces;" but foreseeing that the curse would fall upon Devala, he took pity on him and by his supernatural power prevented the sun from rising. (**39–41**)

The people, who were unable, by reason of the darkness, to pursue their wonted occupations, went to the king and begged him to make the sun rise for them. The king, after surveying his own actions and perceiving that he had been guilty of no sin, concluded that the darkness must have been caused by a quarrel of the monks. He learned the circumstances of the quarrel from Nārada, who told him that Devala might escape the consequences of the curse by begging his pardon. The king pleaded with Devala to do this; but the latter obstinately refused until finally the king, losing his patience, forcibly compelled him to do so. Nārada forgave him, but said to the king, "Since this man did not beg my pardon of his own free will, take him to the pond near the city, place a lump of clay on his head, and make him stand in the water up to his neck. He then said to Devala, "I will send forth my magical power and cause the sun to rise; at that moment duck in the water, rise, and go your way." As soon as the sun's rays touched Devala, the lump of clay split into seven pieces; whereupon he ducked in the water, rose, and made his escape. (**41–3**)

"At that time," said the Teacher, "Ānanda was the king, Tissa was Devala, and I was Nārada. At that time too he was just as obstinate." And admonishing Tissa, he spoke Stanzas 3–4. At the conclusion of the discourse, a hundred thousand monks obtained the Fruits. The multitude derived profit from the instruction given, and the obstinate Elder became amenable to discipline. (**43–5**)

Book I. Story 4. Kāḷi, the Ogress.

ILLUSTRATING STANZA 5=5.

The only son of a widow did all the farm and household work, and cared for his mother to boot. One day the mother proposed to procure him a wife. The son protested that he was able to care for his mother himself, but finally told her of a young woman that suited him and allowed her to bring her home and install her in the house. She turned out to be barren. Thereupon the mother proposed to procure him another wife. The son objected. The barren wife overheard the discussion, and fearing that she might be supplanted by a wife of their selection, procured him another wife herself. (**45–6**)

It then occurred to the barren wife that if her rival bore a child she would become sole mistress of the household. Accordingly she said to the new wife, "As soon as you 've conceived, let me know." To this the other agreed, and when she conceived, told the barren wife. The latter mixed a drug in her rival's food, and caused an abortion. After this had happened twice, the new wife, on the advice of the women of the neighborhood, held no further communication with her fellow. The latter, suddenly discovering that her rival was great with child, employed the same tactics as before, with the result that she killed both child and mother. (**46-7**)

Just before the mother died, she uttered the prayer that she might be reborn as an Ogress, able to devour the children of her persecutor. Thereafter, in three successive existences, the fruitful and the barren wife returned hatred for hatred. (**47**)

The Fruitful Wife was first reborn as a Cat. The Barren Wife was reborn as a Hen. The Cat ate the eggs of the Hen, who prayed that in her next existence she might be able to devour the offspring of her enemy. (**48**)

The Barren Wife, at the end of her existence as a Hen, was reborn as a Leopardess. The Fruitful Wife, at the end of her existence as a Cat, was reborn as a Doe. Thrice the Doe brought forth young, and thrice the Leopardess went and devoured the Doe's offspring. The Doe prayed that in her next existence she might be able to devour the offspring of her enemy. (**48**)

The Fruitful Wife, at the end of her existence as a Doe, was reborn as an Ogress. The Barren Wife, at the end of her existence as a Leopardess, was reborn at Sāvatthi as an Heiress. The Ogress devoured the first and the second child of the Heiress; but when the latter was about to be delivered of her third child, she eluded her enemy by retiring to the house of her father. Here she was safely delivered of a son. A few days later, while the mother was sitting in the grounds of the Monastery, suckling the child, she saw the Ogress approaching. The terrified mother, seizing the child in her arms, fled, closely pursued by the Ogress, into the very presence of the Teacher. (**48-50**)

When the Teacher learned the circumstances of the quarrel, he said to the Ogress: "Why do you return hatred for hatred? Love your enemies;" and he pronounced Stanza 5, at the conclusion of which the Ogress was established in the Fruit of Conversion. (**50-51**)

The Teacher said to the mother, "Give your child to this Ogress." "I am afraid to, Venerable sir." "Be not afraid; you have nothing to fear from her." The mother obeyed. The Ogress kissed the child, caressed him, returned him to the arms of his mother, and began to weep. The Teacher, learning that she had suffered greatly in the past, comforted her, and directed the Heiress to take her home with her and care for her tenderly. Thenceforth they befriended each other in every possible way. It was Kāḷi who established the Eight Ticket-Foods, which are kept up even to this day. **(51–3)**

Book I. Story 5. The Monks of Kosambi.[17]

ILLUSTRATING STANZA 6=6.

Two monks with a retinue of five hundred monks each resided at Kosambi; one a student of the Vinaya, the other, of the Suttas. One day the latter committed the sin of leaving water standing in the bathroom, for which he was reproved by his brother, who, however, on being informed that the offense was unintentional, assured him that he was guiltless. The Vinaya scholar then proceeded to tell his pupils that the Sutta scholar had committed sin and had no conscience about it. The latter, hearing of this, declared the former to be a liar, and was shortly thereafter excommunicated. Then ensued a quarrel in which monks, nuns, unconverted persons, and deities from the lowest heaven to the highest were involved. **(53–4)**

The circumstances of the quarrel among the monks were reported to Buddha, who sent word to them to patch up their differences. Twice he did this, and twice the answer came back that they would not. Then he went in person, pointed out to both factions the wrong involved in their actions, and laid down rules of conduct for their observance. Hearing that they were quarrelling again, he went to them the second time, urged them to be united, and spoke to them long and earnestly on the unprofitableness of discord, illustrating his remarks by telling the Laṭukika Jātaka, the Sammodamāna Jātaka,[18] and the story of Brahmadatta, Dīghati, and Dīghāvu.[19] But in spite of his best efforts, he was unable to restore harmony. **(54–6)**

Disheartened by his failure to reconcile their differences, he left them, went quite alone to the village of Bālaka the salt-maker, where he discoursed to the Elder Bhagu on the solitary life; thence to Eastern Bamboo Deer-park where he discoursed to the three noble youths on the bliss to be found in the sweets of concord; and from there to Protected Forest, where he spent the rainy season pleasantly, attended by the elephant Pārileyyaka. **(56–7)**

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[17] Cf. Jā. iii. 486–490.

[18] The text says simply "Vaṭṭaka-jātaka," i. e., "Quail Jātaka," of which there are several.

[19] Vinaya i. 342–349 (translated SBE. xvii. 293–305).

The lay brethren of Kosambi, learning the reason of the Teacher's departure, snubbed the monks until they came around to a proper view of things and asked to be pardoned. This the laymen declined to do until the monks apologized to the Teacher. But as the rainy season was then at its height, they were unable to go to the Teacher, and had a very unpleasant time as a result. The Teacher, however, spent the time pleasantly, attended by an elephant. (57)

Buddha, the Elephant, and the Monkey. A noble elephant named Pārileyyaka, who had left his herd on account of the excessive annoyances to which he had been subjected, came to Protected Forest, paid obeisance to the Teacher, swept the ground with the branch of a tree, gave the Teacher water to drink, heated water for his bath, and brought him wild fruits. When the Teacher went to the village to collect alms, the elephant took his bowl and robe, put them on the top of his head, and accompanied him as far as the village. Then he gave him his bowl and robe, and waited right there until he returned; whereupon he advanced to meet him, took his bowl and robe as before, deposited them in his place of abode, performed the usual courtesies, and fanned him with the branch of a tree. During the night he paced back and forth in the interstices of the forest with a big club in his trunk, protecting the Teacher from attacks of beasts of prey. (Thus the forest came to be called Protected Forest.) At sunrise he gave him water to rinse his mouth with, and in the same manner performed all the other duties. (57–9)

The elephant's courteous attentions to the Teacher excited in a monkey the desire to do likewise. One day he found some honey and presented it to the Teacher. The latter accepted it, but refrained from eating it. It turned out that there were some insects' eggs in it. These the monkey carefully removed; the Teacher then ate the honey. The monkey was so delighted that he leaped from one branch to another and danced about in great glee. A branch broke, down he fell on the stump of a tree, and a splinter pierced his body. So he died. But because of his faith in the Teacher he was reborn in the world of the Thirty-three. (59–60)

When it became known that the Teacher was living there, Anāthapiṇḍika and others requested Ānanda to procure for them the privilege of hearing the Teacher. Ānanda, accompanied by five hundred monks, went to the forest. Not knowing how Buddha would feel about receiving so many visitors, he left the monks outside, and approached the Teacher alone. Pārileyyaka assumed a threatening attitude, but abandoned it at the command of his master. Learning that Ānanda had come with five hundred monks, Buddha instructed him to ask them to come in. He then spoke to them in praise of the solitary life, pronouncing Stanzas 328–330, at the conclusion of which all were established in Arahatship. Ānanda announced the request of Anāthapiṇḍika and the others, and Buddha bade the monks take bowl and robe and set out. **(60–62)**

Pārileyyaka went and stood cross-wise on the road. The Teacher, knowing that he wished to give alms to the monks, ordered them to wait. The elephant went into the forest, gathered a great quantity of fruit, and presented it to the monks. When they had finished eating, Buddha took bowl and robe and set out. The elephant again went and stood cross-wise on the road. Buddha, knowing that he wished to hinder his departure, reproved him. The elephant thrust his trunk in his mouth and retreated weeping. When they reached the village, Buddha ordered the elephant to go no farther. As Buddha passed out of sight the elephant's heart broke, and he died; but because of his faith in the Teacher he was reborn in the world of the Thirty-three. **(62–3)**

When the Teacher arrived at Sāvatthi, the monks of Kosambi went thither to beg his pardon. The king of Kosala and Anāthapiṇḍika threatened to keep them out, but were dissuaded from so doing. Buddha humiliated the quarrelsome monks by assigning them places separate from the others; and when they threw themselves at his feet and begged for pardon, he reproved them for their sinful conduct, related the story of Brahmadatta, Dīghati, and Dīghāvu[20] once more, and pronounced Stanza 6, at the conclusion of which the assembled monks were established in the Fruits. **(63–5)**

Book I. Story 6. Cūla Kāḷa and Mahā Kāḷa.[21]

ILLUSTRATING STANZAS 7–8 = 7–8.

Cūla Kāḷa, Majjhima Kāḷa, and Mahā Kāḷa, were three brothers who lived in Setavya. Cūla Kāḷa and Mahā Kāḷa, the youngest and oldest respectively, drove a caravan, and Majjhima Kāḷa sold the wares. One day the caravan halted between Sāvatthi and Jetavana, and Mahā Kāḷa, leaving the wagons in charge of Cūla Kāḷa, went and listened to the Teacher. He was so affected by the discourse that he resolved to become a monk, turned over his property to Cūla Kāḷa, and in spite of the latter's protests carried out his resolution. Cūla

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[20] See note 19, p. 492. The text calls this story "Devakosambika-jātaka;" another instance of the loose use of titles.

[21] Cf. Rogers, pp. 25–31.

Kāḷa also became a monk, but with the intention of leaving the Order and taking his brother with him. **(66–8)**

After Mahā Kāḷa had been professed, he inquired of the Teacher how many were the Burdens of the Religious Life, and upon being told that there were two: namely, the Burden of Study and the Burden of Insight, he chose the latter as being better suited to his advanced years. He had the Teacher instruct him in the ascetic practices that one performs in a cemetery, and at the end of the first watch, while the others were asleep, he went to the cemetery and spent the night in meditation, returning to the monastery before the others had risen. **(68)**

For some time Mahā Kāḷa followed the routine laid down for him by the cemetery-attendant without success. Meanwhile Cūla Kāḷa wondered at his brother's perseverance and pined for son and wife. Finally Mahā Kāḷa attained Arahatship by contemplating the destruction by fire of the corpse of a beautiful girl. **(68–71)**

At this time the Teacher, accompanied by the Congregation of Monks, visited Setavya. Mahā Kāḷa sent Cūla Kāḷa to attend to the seating arrangements. Cūla Kāḷa's wives subjected him to such ridicule that he then and there left the Order. Mahā Kāḷa's wives then laid plans to recover their husband. Now Cūla Kāḷa had only two wives, while Mahā Kāḷa had eight. The monks therefore openly expressed the opinion that Mahā Kāḷa would succumb to their wiles. The Teacher, however, told them that they were wrong; and comparing Cūla Kāḷa to a feeble tree standing on the edge of a precipice, and Mahā Kāḷa to a rocky mountain, pronounced Stanzas 7–8. Mahā Kāḷa escaped from the clutches of his wives by soaring through the air. At the conclusion of the stanzas, the assembled monks were established in the Fruits. **(71–7)**

Book I. Story 7. Devadatta.

ILLUSTRATING STANZAS 9–10 = 9–10.

One day the Venerable Sāriputta preached a sermon on the two-fold duty of giving alms and urging others to do likewise. Thereupon a lay brother invited him to bring his retinue of a thousand monks and take a meal with him. Sāriputta accepted the invitation; and the lay brother, with the assistance of the inhabitants of Rājagaha, each of whom responded to his request to give alms according to his ability, entertained the monks handsomely. Now a certain householder had given the lay brother a costly robe, with the understanding that if the supply of food proved insufficient, he was to sell it and buy more food with the proceeds; otherwise he might give it to whomsoever he wished. It turned out that there was an ample supply of food, and the question arose what to do with the robe. The lay brother submitted the question to popular vote, with the result that as between Sāriputta and Devadatta there was a majority of four in favor of the latter. But as soon as Devadatta put on the robe everybody remarked that it was not at all becoming to him, and would have suited Sāriputta much better. This incident was reported to the Teacher, who replied that it was not the first time Devadatta had worn unbecoming robes, and then told the following story of the past: **(77-80)**

The Elephant Hunter and the Noble Elephant. Once upon a time, when Brahmadatta reigned at Benares, there lived an elephant hunter who made his living by killing elephants and selling their tusks. One day he saw thousands of elephants go into a forest and fall on their knees before some Private Buddhas. Concluding that it was the yellow robe that inspired their reverence, he went to a pond where a Private Buddha was bathing, stole his robes, and went and sat down on the elephant path with spear in hand and upper robe drawn over the head. The elephants, supposing that he was a Private Buddha, made obeisance to him and went on their way. The last elephant to come he killed with a thrust of his spear; then, removing the tusks, he buried the rest of the carcass, and departed. **(80-81)**

A little while later, the Future Buddha was born as a young elephant, and in the course of time he became the leader of the herd. The hunter was still engaged in his nefarious business. The noble creature, observing the diminution of his herd, and suspecting who was at the bottom of it, sent the other elephants on ahead and brought up the rear himself, walking with a long, slow stride. The hunter threw his spear at him and darted behind a tree. The elephant resisted the temptation to encircle man and tree with his trunk and crush the offender, and contented himself with saying, "Why did you commit so grievous a sin? You have put on robes suited to those that are free from the Depravities, but unbecoming to you." **(81-2)**

"At that time," said the Teacher, "Devadatta was the elephant hunter, and I was the noble elephant. This is not the first time he has worn unbecoming robes." Then he pronounced Stanzas 9–10, at the conclusion of which many of his hearers were established in the Fruits. **(82-3)**

Book I. Story 8. The Chief Disciples.

ILLUSTRATING STANZAS 11–12 = 11–12.

The Future Buddha, after receiving recognition at the hands of twenty-four Buddhas beginning with Dīpañkara, and after fulfilling the Perfections, was reborn in the Tusita heaven. Urged by the deities to save the world, he made the Five Great Observations, was born of Queen Māyā, passed his youth in the enjoyment of great magnificence in three mansions suited to the three seasons, beheld the Four Ominous Sights, resolved to become a monk, renounced son and wife, was greeted by Kisā Gotamī, made the Great Retirement and the Great Struggle, defeated the hosts of Māra, and attained omniscience under the Bo-tree. At the request of Brahma he proclaimed the Law and converted the Five Monks, Yasa and Fifty-four Companions, the Thirty Young Nobles, and the Three Brothers; after which he visited King Bimbisāra and accepted from him the grant of Bamboo Grove monastery, where he took up his abode and Sāriputta and Moggallāna came to him. **(83–8)**

Upatissa (Sāriputta) and Kolita (Moggallāna) were born on the same day and brought up in great luxury. They acquired a sense of the impermanence of things while witnessing Mountain-top festivities, and were for a time disciples of Sañjaya. Desiring something more than he could give them, they travelled about India listening to various teachers, and were converted to the religion of Buddha by Assaji. After making an unsuccessful attempt to persuade Sañjaya to accompany them, they went to the feet of Buddha, who admitted and professed them as members of the Order and made them his chief disciples. **(88–96)**

The other disciples accuse Buddha of showing favoritism in bestowing the highest dignity on new-comers and passing over what they allege to be the prior claims of the Five Monks, Yasa and his Fifty-four Companions, the Thirty Young Nobles, and the Three Brothers. Buddha denies the charge and declares that it is his wont to bestow on every man that for which he has made his wish. By way of illustration he relates the following stories of the past: **(96–7)**

Mahā Kāḷa and Cūla Kāḷa. Aññākoṇḍañña in his existence as Cūla Kāḷa bestowed the gift of first-fruits nine times on the Buddha Vipassī and for seven days bestowed great largess on the Buddha Padumuttara, making the wish that he might be the first to comprehend the Law. The fact of his attaining this distinction was no proof of favoritism, but rather the fruit of that earnest wish. **(97–9)**

Yasa and his Fifty-four Companions performed many meritorious deeds in the dispensation of a previous Buddha, making the wish that they might thereby attain Arahatship. In a later dispensation they banded themselves together for the performance of good works, and went about caring for the dead bodies of paupers. One day they came upon the dead body of a pregnant woman. They carried the body to the cemetery, Yasa and four others undertook the duty of cremating it, and the rest returned to the village. While Yasa was engaged in turning the body over and over, he acquired a sense of the impurity of the body. This he communicated to the four others, who in turn communicated it to the rest. Yasa also went and communicated it to his mother, his father, and his wife. It was due entirely to this that Yasa obtained in the women's apartments, the disposition of mind requisite to Conversion and that he and the others developed Specific Attainment. **(99-100)**

The Thirty Young Nobles made their wish to attain Arahatship under previous Buddhas and performed works of merit. In a later dispensation they gave themselves up to the pleasures of sense, but on hearing the admonition addressed to Tuṇḍila they kept the Five Precepts for seventy thousand years. **(100)**

The Three Brothers, Uruvela Kassapa, Nadī Kassapa, and Gayā Kassapa, entertained the Buddha Phussa, their oldest brother, and made the wish to attain Arahatship thereby. After undergoing rebirth as gods during ninety-two cycles of time, they obtained the fulfilment of their wish. (At that time Bimbisāra was their superintendent, the lay brother Visākha their steward, and the three ascetics with matted locks were the three royal princes.) Their serving men had a very different experience. The latter diverted to their own use the food they had been ordered to bestow in alms. After undergoing rebirth as ghosts during four Buddha-intervals, they came and begged food and drink of the Buddha Kakusandha, who referred them to the Buddha Koṇāgamana, who referred them to the Buddha Kassapa, who comforted them with the assurance that, in the dispensation of his successor Gotama, their kinsman Bimbisāra would be king, and would obtain relief for them by transferring to them the merit he would earn by giving alms to the Teacher. Thus at last they obtained celestial food, drink, and robes, and became gods. **(100-104)**

Sarada and Sirivaḍḍha. Sāriputta and Moggallāna were born as Sarada and Sirivaḍḍha respectively at the time when the Buddha Anomadassī appeared in the world. Sarada retired from the world with seventy-four thousand followers, entertained Anomadassī, and held the flower parasol over him for seven days, making the wish that he might thereby become the chief disciple of a Buddha. Upon receiving assurance that his wish would be fulfilled, he sent word to Sirivaḍḍha to make his wish for the place of second disciple. Thereupon Sirivaḍḍha entertained Anomadassī and made his wish. So what Sāriputta and Moggallāna obtained was only that for which they had made their wish under Anomadassī. **(104–112)**

Sāriputta and Moggallāna then related their experiences from the Mountain-top festivities to their final interview with Sañjaya. Buddha then contrasted the attitude of Sañjaya with that of his own faithful followers, and pronounced Stanzas 11–12, at the conclusion of which many of his hearers were established in the Fruits. **(113–114)**

Book I. Story 9. Nanda, Elder.[22]

ILLUSTRATING STANZAS 13–14 = 13–14.

After the events related in the last story, Buddha visited his father Suddhodana and established him in the Fruits of the First Two Paths by pronouncing Stanzas 168–169. On the following day, while the festivities connected with Nanda's marriage were going on, Buddha went into the house to collect alms, placed his bowl in Nanda's hands, wished him happiness, and then went out without taking the bowl. So profound was Nanda's reverence for the Teacher that he did not dare ask him to take the bowl; but, expecting that the Teacher would ask for it sooner or later, he followed him first to the head of the stairs, then to the foot of the stairs, then to the court-yard. Here Nanda wished to turn back. But the Teacher went straight ahead, and Nanda, much against his will, followed. When Nanda's bride, Country Beauty, learned what had happened, she ran after him as fast as she could, with tears streaming down her face and hair half combed, and begged him to return. This caused a quaver in Nanda's heart, but the Teacher still gave no indication that he wished to have the bowl returned, and Nanda kept right on. When they reached the Monastery, the Teacher said: "Nanda, would you like to become a monk?" That was the last thing in the world Nanda wanted to do just then; but his reverence for the Teacher was so profound that he promptly said "Yes." Thereupon the Teacher admitted him to the Order. **(115–116)**

After receiving his son Rāhula into the Order, and establishing his father in the Fruit of the Third Path, the Teacher, accompanied by the Congregation of Monks, went into residence at Jetavana. By this time

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[22] Cf. Jā. ii. 92–4.

Nanda had become thoroughly dissatisfied with the Religious Life, and one day he told his brethren that he was going to return to the World. When this was reported to the Teacher, he asked Nanda what was the matter. Nanda told him that he was so deeply in love with Country Beauty that he could not keep his mind on his religious duties. The Teacher, taking him by the arm, led him to a burnt field, and showed him a singed monkey that had lost ears, nose, and tail, sitting on a charred stump; then, by his supernatural power, conducting him to the world of the Thirty-three, he showed him five hundred pink-footed celestial nymphs. Then said the Teacher: "Nanda, which do you regard as being the more beautiful, Country Beauty or these five hundred pink-footed celestial nymphs?" Nanda replied: "Venerable sir, Country Beauty is as far inferior to these nymphs as she is superior to that singed monkey." "Cheer up, Nanda; I guarantee that you will win these nymphs if you only persevere in the Religious Life." The Teacher allowed it to become generally known that he had made this promise to Nanda; whereupon the latter was subjected to such intense ridicule by his brethren that he returned to his religious duties with redoubled energy. In a short time he attained Arahatship; whereupon he went to the Teacher and said, "Venerable sir, I release the Exalted One from his promise." "But," said the Teacher, "when you attained Arahatship, at that moment I was released from my promise." **(116–121)**

One day Nanda told the other monks that he no longer had any desire to go back to the life of a householder. The monks reported this statement to the Teacher, who compared Nanda's former state to that of an ill-thatched house, and his latter state to that of a well-thatched house, and pronounced Stanzas 13–14, at the conclusion of which many of his hearers were established in Fruits. **(121–2)**

The monks were amazed at the Teacher's complete success in winning Nanda's obedience by employing the nymphs as a lure. But the Teacher said: "This is not the first time Nanda has been won to obedience by the lure of the opposite sex. The same thing happened once before." And he told the following tale of the past: **(122–3)**

Kappaṭa and the Donkey. Once upon a time, when Brahmadatta reigned at Benares, there lived in that city a merchant named Kappaṭa; and he had a donkey. Every day the merchant loaded the donkey down with pottery and made him go at least seven leagues. One day he made a trip to Takkasilā; and while he was engaged in disposing of his wares, he let the donkey run loose. The donkey, seeing a female of his species, went up to her. She greeted him in a friendly way and said, "Where have you come from?" "From Benares." "On what errand?" "On business." "How big a load do you carry?" "A big load of pottery." "How many leagues do you go, carrying a big load like that?" "Seven leagues." "In the various places you go to, do you have anybody to rub your feet and back?" "No." "If that's the case, you must have a mighty hard time." (Of course animals don't have anybody to rub their feet and back; she talked the way she did simply to strengthen the bonds of love between them.) As the result of her talk, he became dissatisfied with his job. After the merchant had disposed of his wares, he returned to the donkey, and said, "Come, Jack, let's be off." "You go yourself; I'm not going." The merchant tried without success to persuade him, and then said, "I will beat you till I break every bone in your body; think that over." Said the donkey, "If you beat me, I will plant my fore feet, and let fly with my hind feet, and knock out your teeth; think that over." The merchant was at a loss to account for the donkey's conduct, until he saw the female. Then he changed his tactics and said, "If you will go with me, I will bring you home a mate like that." "In that case," said the donkey, "I'll go home with you and travel fourteen leagues a day hereafter." And off he went. **(123–5)**

"At that time," said the Teacher, "Country Beauty was the female donkey, Nanda was the donkey, and I was the merchant. In former times, too, Nanda was won to obedience by the lure of the female sex." **(125)**

Book I. Story 10. Cunda, the Pork-butcher.

ILLUSTRATING STANZA 15 = 15.

Cunda, the pork-butcher, was a selfish, brutal, irreligious man. After a course of evil conduct lasting fifty-five years, he was attacked by a frightful disease, and while he yet lived, the Avīci hell yawned before him. He went stark mad, and began to crawl about the house on his hands and knees, squealing and grunting like a pig. His kinsmen ran out of the house, barricaded the doors, and mounted guard. After he had raved for seven days he died, and was reborn in the Avīci hell. **(125–7)**

Some monks who passed the house during his madness thought that preparations for a big entertainment were in progress, and so reported the matter to the Teacher. The latter told them the real facts of the case, remarked that the irreligious man sorrows both here and hereafter, and pronounced Stanza 15, at the end of which many were established in the Fruits. **(127–9)**

Book I. Story 11. The Faithful Lay Brother.

ILLUSTRATING STANZA 16 = 16.

A certain lay brother distinguished for his benefactions and religious zeal, was attacked by mortal illness, and desiring to hear the Law, requested the Teacher to send him some monks. Just as the monks were beginning the recitation, a host of deities drove up in their chariots and said, "We would take you with us." The layman, wishing to hear the Law, said to the deities "Hold;" whereupon the monks, mistaking his meaning, arose and departed. The layman's children, to whom the deities were invisible, began to weep; whereupon the layman, to confirm their faith, performed a miracle, urged them to follow the example he had set in performing good works, and then, stepping into a celestial chariot, was reborn as a deity. **(129-131)**

When the monks told the Teacher that the layman had refused to hear the Law, he informed them of the real facts of the case, assured them that the religious man rejoices both here and hereafter, and pronounced Stanza 16, establishing many in the Fruits. **(131-2)**

Book I. Story 12. Devadatta.

ILLUSTRATING STANZA 17 = 17.

The story of Devadatta from the time he retired from the world to the time he was swallowed up by the earth is related in detail in the Jātakas;[23] the following is an abridgment of it: **(133)**

When the Future Buddha lived at Anupiya Mango-grove, eighty thousand kinsmen observed on his person the marks and characteristics of a Tathāgata, and each dedicated a son to his service. In the course of time, all of these young men became monks, with the exception of Bhaddiya, Anuruddha, Ānanda, Bhagu, Kimbila, and Devadatta. One day Anuruddha's brother Mahānāma went to Anuruddha and said, "There is n't one of our family that has become a monk; you become a monk, and I'll follow your example." **(133)**

(Now Anuruddha had been brought up in softness and luxury, and had never heard the word *is n't.* Once the six princes engaged in a game of ball, wagering a cake on the result. Anuruddha lost and sent word to his mother to send him a cake, which she did. This happened three times. The fourth time his mother sent word: "There is n't cake to send." The son replied, "Send me some *is n't* cake." The mother, in order to teach her son a lesson, sent him an empty bowl

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[23] Jā. vi. 129–131; v. 333–7; iv. 158–9.

covered with another empty bowl. The tutelary deities of the city filled the bowl with celestial cakes. The mother found out what had happened, and thereafter, whenever her son sent for cakes, sent him an empty bowl, which the deities filled with celestial cakes. How could a youth who was ignorant of the meaning of the word *is n't*, be expected to know the meaning of *monk*?) **(133-5)**

Anuruddha replied to Mahānāma: "What does this word *monk* mean?" Mahānāma told him. Anuruddha replied that he was too delicate to become a monk. "Well then," said Mahānāma, "learn farming, and adopt the life of a householder." Anuruddha replied, "What does this word *farming* mean?" **(135-6)**

(How could you expect a youth to know the meaning of *farming* who did n't know where rice comes from? Once a discussion arose among the three princes Kimbila, Bhaddiya, and Anuruddha, as to where rice comes from. Kimbila thought it came from the granary; Bhaddiya, from the kettle; Anuruddha, from the golden bowl.) **(136)**

Mahānāma explained to Anuruddha what was implied by the term *farming;* whereupon Anuruddha, aghast at the endless routine of manual labor, said, "Well then, live the householder's life yourself; I have no use for it." He went to his mother and said, "Mother, give me your permission to become a monk." "All right, if your friend King Bhaddiya will do the same." Anuruddha had no little difficulty in persuading Bhaddiya to do this; but finally the latter agreed to retire from the world in seven days. Then Bhaddiya, Anuruddha, Ānanda, Bhagu, Kimbila, and Devadatta, together with the barber Upāli, set out with four-fold array, and crossed over into foreign territory. Here the six princes sent back the army, took off their ornaments, made a bundle of them, gave them to Upāli, and ordered him to return. When Upāli had gone a little way, he was overcome with fear that the fierce Sakyans, thinking that he had put their princes to death, would retaliate by killing him; accordingly he untied the bundle, hung the ornaments up on a tree, and returned to his masters. Then the six princes, taking Upāli with them, went to the Teacher, and said: "We, Venerable sir, are proud Sakyans; this man has been a servitor of ours for a long time; admit him to the Order first; to him first we will offer respectful salutations; so will our pride be humbled." Thereupon the Teacher first admitted Upāli to the Order, and after him the others. Bhaddiya attained Threefold Knowledge, Anuruddha Supernatural Vision, afterwards Arahatship, Ānanda was established in the Fruit of Conversion, and Bhagu and Kimbila by the development of Insight attained Arahatship. Devadatta attained the lower grade of Magic Power. **(136-8)**

When the Teacher and the monks went into residence at Kosambi great numbers of people flocked thither and said, "Where is the Teacher? Where is Sāriputta? Moggallāna? Kassapa? Bhaddiya? Anuruddha? Ānanda? Bhagu? Kimbila?" But nobody said, "Where is Devadatta?" Thereupon Devadatta said to himself, "I retired from the world with these monks; I, like them, belong to the Warrior caste; but unlike them, I am the object of nobody's solicitude. With whom can I make common cause, that I may obtain gain and honor for myself? Bimbisāra? He will have nothing to do with me. The king of Kosala? Neither will he. What about Bimbisāra's son Ajātasattu? He does n't know anybody's virtues or vices. He 's the very man!" **(138-9)**

Accordingly Devadatta assumed the form of a child girded about with snakes, and descending from the sky, sat in Ajātasattu's lap. Perceiving that he was frightened, Devadatta told him who he was, and resumed his proper form. Ajātasattu bestowed all manner of attentions upon Devadatta, until there arose in the latter's mind, overmastered by gain and honor, the evil thought, "It is I who ought to run the Congregation of Monks." Thereupon he went to the Teacher and said: "Venerable sir, the Exalted One is stricken in years; let him live a life of ease in this world; I will run the Congregation of Monks; make over the Order to me." But the Teacher repulsed Devadatta, called him a "lick-spittle," and caused proclamation to be made concerning him at Rājagaha.[24] Thereupon Devadatta cherished resentment against the Teacher, and resolved to make trouble for him. **(139-140)**

So Devadatta went to Ajātasattu and said: "Youth, aforetime men were long-lived, but nowadays they don't live long; this makes it probable that you won't live long. You kill your father and become king, and I 'll kill the Buddha and become Buddha." When Ajātasattu was established in the kingdom,[25] Devadatta made three attempts on the life of the Buddha. First he hired some men to kill him, but they deserted their posts and obtained the Fruit of Conver-

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[24] Oldenberg, relying on Fausböll's faulty text, says regarding this proclamation (SBE. xx. p. 239, note 2): "It is not referred to by the Dhammapada commentator." Norman, however, gives the same reading as the Vinaya.

[25] It is interesting to note that this account does not say that Ajātasattu killed his father. The Vinaya says (ii. 190–191) that Ajātasattu's designs were discovered and that Bimbisāra abdicated in favor of his son. The Jātaka (vi. 129, lines 20–22) refers to the section of the Vinaya quoted above, and then goes on to say that Ajātasattu killed his father! In the Dīgha (i. 85 $^{15-18}$) Ajātasattu confesses to the Buddha that he killed his father.

sion. Then he climbed to the top of Vulture Peak and hurled down a rock, but succeeded only in wounding the Teacher. Last of all he despatched the elephant Nālāgiri against the Teacher, but Ānanda stood in the breach and the Teacher subdued the elephant. Buddha informed the monks that this was not the first time Ānanda had risked his life for him, and related the Cūḷahaŋsa, Mahāhaŋsa, and Kakkaṭa Jātakas. **(140–141)**

After that neither the people nor the king would have anything more to do with Devadatta. Then the latter went to Buddha and made the Five Demands, but was again repulsed. Finally Devadatta caused a schism in the Order by persuading five hundred monks to make common cause with him, but Sāriputta and Moggallāna convinced them of the error of their ways by preaching and performing miracles before them, and returned with them through the air. When the Teacher saw Sāriputta returning with this splendid retinue, he remarked that this was not the first time he had done so, and related the Lakkhaṇa Jātaka.[26] **(141–4)**

During the Teacher's residence at Rājagaha, he related many Jātakas about Devadatta's evil deeds in previous existences. For example when the monks told him that Devadatta was imitating him, he related the Vīraka, Kandagalaka, and Virocana Jātakas; with reference to his ungratefulness, he related the Javasakuṇa Jātaka; commenting on his wickedness, he told the Kuruṅga Jātaka; hearing the remark that Devadatta had renounced the joys of the householder's life only to fall away from the estate of a monk, he told the Ubhatobhaṭṭha Jātaka. The Teacher then retired from Rājagaha to Sāvatthi and took up his residence at Jetavana Monastery. **(144–6)**

Devadatta suffered from sickness for nine months, at the end of which, realizing that his end was near, he was overwhelmed with remorse, and resolved to make his peace with the Teacher. So he caused himself to be carried on a litter to Jetavana. The Teacher refused to see him. When Devadatta raised himself from the litter and assumed a sitting posture with both feet resting on the ground, the earth gave way under his feet, and slowly swallowed him up. As his jaws touched the earth, he cried out, "I seek refuge in Buddha;" whereupon the Teacher made him a monk, prophesying that at the end of a hundred thousand cycles of time he would be reborn as a Private Buddha named Aṭṭhissara. After the earth had swallowed up Devadatta, he was re-

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[26] Jā. i. 142. Chalmers remarks: "Unlike this Jātaka, the Vinaya . . . gives a share of the credit to Moggallāna." But elsewhere (Jā. iv. 158, lines 3–4) the Jātaka distinctly says that it was Sāriputta *and* Moggallāna.

born in the Avīci hell, where he suffered excruciating tortures, being encased in an iron shell and impaled on iron stakes. **(146–8)**

When the monks commented on what had happened to Devadatta, the Teacher told them that Devadatta had suffered similar experiences in previous existences, and related the Sīlavanāga, Khantivādi, and Culla Dhammapāla Jātakas. When the multitude rejoiced at his death, the Teacher told them that the same thing had happened before, and related the Mahāpiṅgala Jātaka. Finally the monks inquired where he had been reborn. The Teacher replied, "In the Avīci hell;" and reminding them that irreligious men suffer both here and hereafter, he pronounced Stanza 17, at the end of which many were established in the Fruits. **(148–150)**

Book I. Story 13. Sumanā.

ILLUSTRATING STANZA 18 = 18.

Anāthapiṇḍika and Visākhā were so intimately acquainted with the needs of the monks that they were much sought after to accompany those who desired to carry alms to the monks. When Visākhā left her house, she appointed a granddaughter to dispense alms in her place. Anāthapiṇḍika assigned a similar duty to his oldest daughter. The latter attained the Fruit of Conversion, married, and was succeeded by a younger sister. She also attained the Fruit of Conversion, married, and was succeeded by the youngest daughter Sumanā. **(151)**

Sumanā attained the Fruit of the Second Path, but remained unmarried. Thereat she sickened, would eat nothing, and sent for her father. When the latter asked her what was the matter, she addressed him as "youngest brother," and died. Anāthapiṇḍika, unable to quiet his grief, went to the Teacher and told him what had happened. "Why do you grieve?" said the Teacher. "Know you not that death is certain for all?" "I know that, Venerable sir; but my daughter talked incoherently when she died, addressing me as 'youngest brother.'" "She spoke quite correctly," replied the Teacher, "for she had attained the Fruit of the Second Path, while you have attained only the Fruit of Conversion."[27] Thereupon the Teacher informed Anāthapiṇḍika that Sumanā had been reborn in the Tusita heaven, and pronounced Stanza 18, at the conclusion of which many were established in the Fruits. **(151–4)**

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[27] Compare the story of Kavi in Manu, ii. 150 (Lanman's Reader, 61 ^{15}).

Book I. Story 14. The Two Brethren.

ILLUSTRATING STANZAS 19–20 = 19–20.

Two noble youths who had been friends retired from the world together. The older of these assumed the Burden of Insight and attained Arahatship; the younger assumed the Burden of Study, acquired the Tipiṭaka, and became renowned as a master of the Law. One day the younger monk learned from some pupils of his older brother that the latter knew only one Nikāya and one Piṭaka, and that of the four-lined Stanzas he knew none at all. Becoming greatly puffed up at the thought of his own superior learning, he resolved to seize the first opportunity to ask his older brother some embarrassing questions. **(154–5)**

Somewhat later the older monk came to pay his respects to the Teacher. The latter, knowing what was in the mind of the younger monk, anticipated his designs, and asked both monks several questions. The younger monk answered all the questions about the Trances and the Eight Attainments, but failed to answer a single question the Teacher asked him about the Paths. The older monk, however, answered all the the questions correctly. The Teacher praised the older monk highly, and pronounced Stanzas 19–20, at the end of which many were established in the Fruits. **(155–9)**

Book II. Story 1. Udena.[28]

ILLUSTRATING STANZAS 1–3 = 21–23.

1a. Rise and Career of Udena.

Once upon a time two kings named Allakappa and Veṭhadīpaka, who had been friends since boyhood, retired from the world and became forest hermits. One day Veṭhadīpaka died and was reborn as a powerful spirit. Desiring to see his brother, he disguised himself as a wayfarer and paid him a visit. Allakappa told him that the elephants were giving him a lot of trouble; whereupon Veṭhadīpaka gave him a lute to charm elephants with, and taught him the proper spells. "Twang this string and utter this spell," said he, "and the elephants will run away without so much as taking a look behind them; twang this string and utter this spell, and they will retreat, eyeing you as they go; twang this string and utter this spell and the leader of the herd will come up and offer you his back." Veṭhadīpaka then departed,

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[28] Cf. Rogers, pp. 32–60.

and after that Allakappa got along famously with the big beasts. (161-4)

At this time Parantapa was king at Kosambi. One day the king and the queen were sitting out in the open air sunning themselves. The queen, who was great with child, was wearing the king's scarlet blanket; and as they chatted together the queen removed the king's signet ring from his finger and slipped it on her own. Just then a monster vulture, mistaking the queen for a piece of meat, swooped down, caught up the queen in his talons, carried her off to the forest, and deposited her in the fork of a banyan tree. The following morning she gave birth to a son, whom she called Udena. (164-5)

Now the banyan tree was not far from the hermitage of Allakappa. The latter, discovering mother and child, escorted them to the hermitage and cared for them tenderly. After a time, the mother, fearing that if the hermit went away she and her child would be left alone in the forest to die, tempted the hermit to break his vow of chastity. The latter yielded to the temptation, and thereafter the two lived together as man and wife. (165-6)

One day Allakappa read it in the stars that the king of Kosambi was dead. He told the queen, and the latter burst into tears. Then said the hermit, "Why do you weep?" "Because he was my husband." "Weep not; death is certain for all." "I know, sir." "But why do you continue to weep?" "Because of my son; if he could only be there, he would be crowned king." "Cease weeping; I will arrange all that." Thereupon the hermit gave the boy the lute to charm elephants with and taught him the proper spells. The hermit then said to the mother, "Give your son the necessary instructions, that he may go hence and become king." The mother told the boy that he was the son of Parantapa, king of Kosambi; that a monster bird had carried her off just before he was born; that he was to go forth and claim his kingdom; and that in case the ministers refused to believe him, he was to show them his father's scarlet mantle and signet ring. Then the prince bade farewell to his father and mother, mounted the back of the oldest elephant of the herd, and whispered in his ear, "My lord, I am the son of Parantapa, king of Kosambi; obtain for me the kingdom of my father." The elephant trumpeted, saying, "Let all the hosts of the elephants assemble;" and immediately all the hosts of the elephants assembled. Then the elephant trumpeted again, saying, "Let the old elephants retire, and the young elephants withdraw;" and immediately the old elephants retired, and the young elephants withdrew. So Udena set out with a prodigious host of warrior elephants, and going to the gates of Kosambi, cried out with a loud voice, "Give me battle or my kingdom." Then he cried out again, "I am the king's son;" and held up the mantle and the ring, that all might see them; whereupon the citizens opened the gates, and hailed him as their king. **(166-9)**

1b. Rise and Career of the Treasurer Ghosaka.

Once upon a time there was a famine in the kingdom of Ajita, and a certain man named Kotūhalaka took his wife and infant son and set out for Kosambi in search of food. When the provisions for the journey failed, the father proposed to the mother to cast the child away, but the mother protested vigorously, and suggested that they carry the child by turns. While the father was carrying the child in his arms, the child fell asleep; whereupon the father, allowing the mother to precede him, laid the child on a couch of leaves under a bush, and went on his way. When the mother discovered what had happened, she begged her husband to restore the child to her, and he did so. (In consequence of having cast his child away on this occasion, Kotūhalaka was himself cast away seven times in a later existence. Let no one regard a sinful deed as a small matter.) **(169-170)**

Continuing on their journey, they arrived at the house of a herdsman. One of the herdsman's cows had just calved, and a festival was being held in honor of the event. The herdsman received the visitors hospitably, set abundant food before them, and then sat down to eat his own meal. Kotūhalaka watched the herdsman feed a bitch that lay under his stool, and thought to himself: "How fortunate is that bitch to get food like that to eat!" During the night Kotūhalaka died of indigestion, and was conceived in the womb of the bitch whose lot he envied. **(170-171)**

Now a Private Buddha was accustomed to take his meals in the house of the herdsman; and Kotūhalaka's widow, realizing what an opportunity she had to store up merit for the future, bestowed alms on him faithfully every day. By and by the bitch gave birth to a single pup. The herdsman reserved the milk of one cow for the pup, and in a short time the latter grew to be a fine big dog. The Private Buddha fed him every day with his own hand, and the dog became so fond of the Private Buddha that he performed all manner of services for him. Some time later the Private Buddha took leave of the herdsman, and setting his face towards Gandhamādana, soared into the air. Thereupon the dog set up a howl of grief, and when the Private Buddha passed out of sight, his heart broke, and he died. (Dogs, they say, are straightforward; men think one thing with their heart, but say another with their lips.) The dog was reborn in the world of the Thirty-three with a retinue of a thousand celestial nymphs. (If you ask, "Of what was this the consequence?" it was because he barked so affectionately at the Private Buddha.) **(171–3)**

In consequence of having devoted himself to sensual pleasures, he fell from the world of the Thirty-three, and was conceived in the womb of a harlot of Kosambi. When the child was born, and the harlot learned that it was a boy, she had him cast away on a dust-heap. A man who happened to pass by took a fancy to the child, and saying to himself, "I have gained a son," took him home with him. **(173–4)**

That day there was a conjunction of the moon with a certain lunar mansion; and a treasurer of Kosambi, meeting an astrologer, asked him what the sign betokened. The astrologer said, "This day is born in Kosambi a child who will become the principal treasurer of the city." It so happened that the treasurer's wife was at that very time great with child; and he immediately sent word to find out whether she had been delivered or no. When the messenger brought back word that she had not, the treasurer summoned a female slave and said to her, "Here are a thousand pieces of money; scour the city and find a boy that was born to-day and bring him hither to me." The slave returned with the foundling. The treasurer thought to himself: "If a daughter is born to me I will marry her to this boy and make him treasurer; but if a son is born to me, I will kill this boy." A few days later his wife gave birth to a son. The treasurer then set about to carry out his plan. **(174–5)**

[The reader will bear in mind that the adopted son of the treasurer was none other than the harlot's son who had been cast away on the dust-heap, and that he must needs be cast away six times more in consequence of the evil deed he committed when, in his existence as Kotūhalaka, he cast away his own son; that he must needs be rescued through the effect of the merit he earned in his existence as a dog by barking so affectionately at the Private Buddha; and that, inasmuch as all the hosts of heaven and earth cannot interfere with the operation of the law of cause and effect, the astrologer's prophecy concerning him was at last to be fulfilled. The boy's name was Ghosaka.]

First the treasurer had Ghosaka laid at the door of the cattle-pen, hoping that he would be trampled to death. But the bull stood over him, allowing the cows to pass out on either side of him, and the herdsman took him home. **(175)**

The treasurer recovered Ghosaka, and then had him placed on the caravan trail, expecting that he would either be trampled by the oxen, or crushed by the wheels of the carts. But when the oxen saw the boy, they stopped with one accord, and the whole caravan stood stock still until its leader discovered what was the matter and rescued the boy. **(176)**

Ghosaka was recovered by the treasurer, who then had him cast away under a bush in the cemetery. Along came a goatherd with his goats. The goatherd's suspicions were aroused by the peculiar actions of a she-goat; whereupon he made an investigation, discovered the boy, and rescued him. **(176–7)**

Ghosaka was again recovered by the treasurer and thrown down a precipice. He fell into a clump of bamboo, and a basket-maker rescued him. **(177)**

In spite of the treasurer's attempts on his life, Ghosaka lived and thrived and grew to manhood. He was a thorn in the flesh of the treasurer, who could not look him straight in the face. Finally the treasurer resorted to desperate measures. He went to a potter, gave him a thousand pieces of money, and said to him, "I have a job for you." "What is it?" "I have a base-born son; I 'll send him to you to-morrow; get him into a room, take a sharp razor, cut him into bits, and throw them into the chatty." "All right." The next day the treasurer said to Ghosaka, "Go and tell the potter to finish up the job I gave him yesterday." "Very well," said Ghosaka; and started out. When he had gone a little way, the treasurer's own son, who was playing ball with some other boys, stopped him and said to him, "Where are you going?" Ghosaka told him. "Let 's change places," said the treasurer's son; "these boys have won a lot of money from me, and you 're such a good ball-player that you can easily win it back for me." So Ghosaka took his foster-brother's place in the game, and the treasurer's own son carried his father's message to the potter. That night the despised Ghosaka returned home; the treasurer's son did not. The treasurer cried out, "Woe is me!" and rushed to the potter, who said to him, "Master, make no noise; I have done the job." The wicked treasurer was overwhelmed with sorrow and grief at the thought that he had shed innocent blood, even as Buddha says in Stanzas 137–140. **(177–9)**

The treasurer made one more attempt on Ghosaka's life. He wrote a letter to the superintendent of his estate, saying, "This is my base-born son; kill him, and I will do what is right for you;" pinned it to the hem of Ghosaka's clothing, and ordered Ghosaka to carry it to the superintendent. (The treasurer had never taught Ghosaka to read, for he expected sooner or later to kill him.) When Ghosaka remarked that he needed provisions for the journey, the treasurer said, "Not at all; in such and such a village lives a friend of mine who is a treasurer;

he will give you something to eat." When Ghosaka stopped at the village treasurer's house, the treasurer's wife took a fancy to him, and the daughter of the household fell madly in love with him. (It was she that had been his wife in his former existence as Kotūhalaka, and it was through the merit she acquired by bestowing alms on the Private Buddha that she was reborn as the treasurer's daughter. No wonder that her old passion for him returned!) When the treasurer's daughter discovered that Ghosaka was carrying his death-warrant, she secretly removed it and substituted another letter of her own composition, which read as follows: "This is my son Ghosaka. Bestow treasure upon him; prepare for the festival of his marriage to the daughter of the village treasurer; build him a splendid palace; and provide him with a strong guard of soldiers. When you have so done, send me word, saying, 'I have done this and that.'" When the superintendent read the letter he immediately did as he was told. **(180-182)**

When the treasurer learned how miserably his last attempt had failed, he cried out, "What I would do, that I do not; what I would not do, that I do," sickened, and was soon at the point of death. Ghosaka and his bride visited him in his last moments. Just as the treasurer was about to die, he lifted up his voice, intending to say, "These my treasures shall *never* be Ghosaka's;" but by a slip of the tongue said instead, "These my treasures shall *ever* be Ghosaka's." King Udena confirmed Ghosaka in his inheritance and made him the principal treasurer of the city. When the treasurer Ghosaka learned from his wife how narrow had been his escape from death, he resolved to forsake the life of Heedlessness, and to live the life of Heedfulness, and thereafter he dispensed a thousand pieces of money daily in alms to the poor. **(182-7)**

1c. Rise and Career of Sāmāvatī.

At this time the treasurer Ghosaka learned from some merchants who had lately returned from Bhaddavatī that there lived in that city a merchant of great wealth and high standing, named Bhaddavatiya; and desiring to be friends with him, Ghosaka sent him a present. Bhaddavatiya returned the compliment; and thus, though they had never seen each other, they became fast friends. A little later a pestilence broke out in Bhaddavatiya's city; and the treasurer, taking his wife and daughter, set out for Kosambi, intending to ask Ghosaka to help them. After a hard journey they reached Kosambi, and secured lodgings in a hall near the city gate. Bhaddavatiya told his wife that Ghosaka was accustomed to dispense a thousand pieces of money daily in alms to the poor, and suggested that they send their daughter to him to procure food until they recovered sufficient strength to pay him a visit. **(187–8)**

So it happened that the daughter of a wealthy house accompanied poor folk to Ghosaka's hall for alms. "How many portions will you have?" "Three." That night her father died. "How many portions will you have?" "Two." That night her mother died. "How many portions will you have?" "One." A householder named Mitta, who remembered that she had taken more on the two previous days, said to her, "I suppose that is all you can hold to-day." This cruel remark cut her to the quick, and she said, "Sir, don't think I took more for myself; before we were three, yesterday two, to-day I am left alone." She then told him the whole story, whereupon he took pity on her and adopted her as his oldest daughter. She rendered such valuable assistance in the administration of the hall where Ghosaka's alms were distributed as to attract the attention of Ghosaka himself, who, upon learning that she was the daughter of Bhaddavatiya, gave her a retinue of five hundred women and made her as his own oldest daughter. One day King Udena saw her, fell in love with her, and married her. She became one of his queen-consorts, and the women of her retinue ladies-in-waiting. **(188–191)**

1d. Vāsuladattā.

Another of Udena's queen-consorts was Vāsuladattā, daughter of Caṇḍapajjota, king of Ujjeni. Udena gained possession of her in the following way: **(191–2)**

One day King Caṇḍapajjota said to his ministers, "Is there any other monarch so powerful as I am?" "Of course not," said they; "but yet King Udena of Kosambi is pretty powerful." "Well then, let's take him prisoner." "It can't be done; he understands how to charm elephants, and has more elephants at his disposal than any other king." "I suppose it can't be done." "Well, if your heart is set on doing it, you might try this stratagem: Have a wooden elephant made, and send it out somewhere near him; he will go a long way after a good mount, and you can take him prisoner as he approaches." "That *is* a stratagem!" **(192)**

Thereupon Caṇḍapajjota had a mechanical elephant made of wood, and turned it loose where Udena would be sure to see it. It looked exactly like a real elephant; moreover, it was fitted with mechanical appliances worked from the inside, so that it moved hither and thither just like a real elephant; its belly held sixty men, who worked the mechanism, and every now and then dumped out a quantity of elephant dung. Udena immediately mounted his elephant and started out in pursuit. Caṇḍapajjota posted an ambuscade. Udena tried to charm the wooden elephant by twanging his lute and uttering spells, but the wooden elephant paid no attention to him, and only made off faster than ever. Udena, unable to keep up with the wooden elephant, mounted his horse, left his army behind, and started out alone. Thereupon he was drawn into the ambuscade and captured. **(192–3)**

Caṇḍapajjota kept Udena in prison for three days, and then offered to release him if Udena would divulge the charm. "I will do so," said Udena, "provided you will pay me homage." "That I will not do," replied Caṇḍapajjota; "but will you divulge it to another, if the other will pay you homage?" "Yes." "Well then, there is a hunchbacked woman in this house; I will have her sit inside a curtain; you remain outside and teach her the charm." "Very well." Caṇḍapajjota then went to his daughter, the beautiful Princess Vāsuladattā, and said to her, "There is a leper who knows a priceless charm; you sit inside a curtain; he will remain outside, and teach you the charm; then tell me what it is." (Caṇḍapajjota employed this stratagem to protect his daughter's chastity.) **(193–4)**

One day Udena repeated the charm over and over again to Vāsuladattā, but the latter was unable to reproduce it correctly. Thereupon Udena lost his patience, and cried out, "What's the matter with you, you thick-lipped hunchback?" Vāsuladattā retorted angrily, "How dare you speak thus? do I look like a hunchback?" Udena raised the curtain, and immediately they both knew why Caṇḍapajjota had deceived them. Vāsuladattā yielded her chastity to Udena; and after that there were no more lessons. The king frequently asked his daughter, "How are you getting along with your lessons?" and always received the answer, "Very well." **(194–5)**

One day Udena said to Vāsuladattā, "If you will save my life, I will make you queen-consort and provide you with five hundred ladies-in-waiting." "Very well," replied Vāsuladattā; and she went and said to her father, "Father, in order that I may perfect myself in this charm, it will be necessary for me to dig a certain medicinal root in the dead of night at a time indicated by the stars; therefore please have one door left open, and put an elephant at my disposal." **(195–6)**

(Now King Caṇḍapajjota, in consequence of having bestowed alms on a Private Buddha in a previous existence as a slave, was possessed of the five conveyances: a female elephant, which could travel 50 leagues a day; a slave, who could travel 60 leagues; two horses, 100 leagues; and an elephant named Nālāgiri, 120 leagues.) **(196–8)**

One day, when Caṇḍapajjota was absent, Udena filled several big leather sacks with gold and silver, put them on the back of the female elephant, assisted Vāsuladattā to mount, and away they went. As soon as Caṇḍapajjota learned what had happened, he sent out a force in pursuit. Udena opened the sacks and scattered coins along the route; Caṇḍapajjota's men delayed pursuit to pick them up; so Udena easily escaped. It was thus that Vāsuladattā came to be one of King Udena's queen-consorts. **(198–9)**

1e. Māgandiyā.

Māgandiyā was another of Udena's queen-consorts. She was the daughter of a Brahman named Māgandiya, who lived in the Kuru country. Māgandiyā was the name of her mother, and she had an uncle named Māgandiya. She was as beautiful as a celestial nymph. One after another the sons of the most prominent families presented themselves as suitors for her hand; but the Brahman refused them all, telling them that they were not worthy of her. **(199)**

One day the Teacher, knowing that the Brahman and his wife were capable of attaining the Fruit of the Third Path, went to the place where the Brahman was tending the sacred fire. The Brahman was so impressed with the majestic appearance of his visitor that he then and there offered him his daughter in marriage. The Teacher said nothing. The Brahman went home in great haste, told his wife that he had found a husband for their daughter, caused the latter to be dressed in gala attire, and then all three went to the Teacher. **(199–200)**

By this time the Teacher had moved away from the place of his interview with the Brahman, leaving a foot-print. "Where can he have gone?" said the Brahman; and then, seeing the foot-print, he said to his wife, "There is his foot-print." Now the Brahman's wife was well versed in the Three Vedas; and after considering the foot-print, and turning over in her mind the texts relating to foot-prints, she said, "Husband, that is not the foot-print of one who follows the Five Lusts." "Hush, wife, you 're always seeing alligators in the water-vessel and thieves hiding in the house." Then the Brahman saw the Teacher and said, "There is the man." The Brahman immediately went to him and said, "I bestow my daughter upon you; cherish her tenderly." The Teacher replied, "Brahman, I have something to say to you;" and then told him that from the time of the Great Retirement to the time of the Session under the Banyan-tree Māra had pursued him relentlessly, only to be defeated at every point, that Māra's daughters had then tempted him in various forms without exciting in him the lust of the flesh, and that nothing would induce him to touch the maiden who stood before him with so much as the sole of his foot.

Thereupon the Brahman and his wife were established in the Fruit of the Third Path. Māgandiyā, however, cherished the most bitter hatred of the Teacher ever after. **(200–202)**

The Brahman and his wife entrusted Māgandiyā to the care of her uncle, who adorned her with all the adornments and presented her to King Udena. The king immediately fell in love with her and married her, making her queen-consort and giving her a retinue of five hundred ladies-in-waiting. **(202–203)**

1f. Death of Sāmāvatī.

At this time there were living in Kosambi three treasurers, Ghosita, Kukkuṭa, and Pāvāriya. At the beginning of the rainy season five hundred monks returned from the Himālaya country and went about the city collecting alms. The three treasurers saw them and provided them with food during the four rainy months. When the rains were at an end the monks took leave of their hosts and retired to Himālaya, promising to return the following year. And this became an established custom. Several years later, the monks on their return from Himālaya took up their abode in the forest under a gigantic banyan tree. The oldest monk thought to himself: "This tree must be tenanted by a very powerful tree-spirit; I wish he would give us some water to drink;" and immediately the spirit gave them water to drink. Then the monk thought, "I wish he would give us some water to bathe in;" and immediately the spirit gave them water to bathe in. Then the monk thought of food, and there it was! "Well!" said the monk, "this spirit gives us everything we think of; let's have a look at him." Immediately the tree split open, and out came the spirit. Said the monks, "Spirit, you have great power; what did you do to get it?" But it was a very modest spirit; and so said, "Don't ask me." "Please tell." After considerable urging, the spirit told his story. **(203–204)**

It seems that the spirit had once been a servant of Anāthapiṇḍika. One fast-day Anāthapiṇḍika, on learning that his servant had not been told the significance of the day, ordered a meal to be prepared for him. The servant observed that no one else was eating, learned why, and followed suit. He then went out and did his day's work, was taken sick, and died that very night. "My master," said the spirit, "was devoted to Buddha, the Law, and the Order; and it was through him and in consequence of the fast I observed that I was reborn as a tree-spirit." **(204–206)**

Thereupon the monks sought refuge in Buddha, the Law, and the Order, and on the following day, after conferring with the three treasurers, visited the Teacher, attained Arahatship, and were admitted to the Order. A little later Ghosita, Kukkuṭa, and Pāvāriya came to the Teacher, bearing rich offerings, and were established in the Fruit of Conversion. For two weeks the treasurers remained with the Teacher, giving generously of their store, and then, after obtaining the Teacher's promise to visit them, returned to Kosambi. Here they erected Ghosita, Kukkuṭa, and Pāvāriya monasteries, and here the Teacher visited them, dividing his time equally among the three. After the treasurers had entertained the Teacher for some time, their gardener Sumana asked and received permission to entertain him for a single day. **(206–208)**

Now at this time King Udena was in the habit of giving Queen Sāmāvatī eight pieces of money every day to buy flowers with. This money the queen turned over to a female slave, Khujjuttarā, who went regularly to the gardener Sumana's and bought flowers. On the day appointed for the Teacher's visit Sumana said to her: "To-day I expect to entertain the Teacher, and shall have use for my flowers; wait and listen to the Law, and then, if there are any flowers left, you may have them." Khujjuttarā harkened to the Law, and was established in the Fruit of Conversion. Now hitherto it had been Khujjuttarā's practice to spend only four pieces of money on flowers, and to pocket the rest. That day, however, she spent the entire amount on flowers, and returned with so many that the queen's curiosity was aroused, and the whole story came out. From that time on Khujjuttarā stole no more; but becoming as it were a mother to Sāmāvatī, went regularly every day to hear the Teacher, and returned and preached the Law to the queen and her retinue exactly as she had heard it. She soon knew the Tipiṭaka so well as to win from the Teacher the title of "Pre-eminent." Queen Sāmāvatī and her retinue were established in the Fruit of Conversion. **(208–210)**

One day Sāmāvatī expressed to Khujjuttarā a desire to see the Teacher. Khujjuttarā said, "It 's a serious matter to live in a king's palace; once in, you can't get out." The queen begged her to arrange it in some way. Khujjuttarā then told her to make holes in the walls of the palace and to render homage to the Teacher from within. Māgandiyā came to know of this. **(210–211)**

Now Māgandiyā had cherished the most bitter hatred of the Teacher and his followers ever since the Teacher refused to marry her; and as soon as she learned that Sāmāvatī and her attendants were making a practice of rendering homage to the Teacher through holes in the walls of the palace, she said to herself, "I know what 's to be done to him; I know what 's to be done to them." Thereupon Māgandiyā went to

King Udena and told him that Sāmāvatī was planning to kill him, and had made holes in the walls of the palace for that purpose. The king, however, refused to believe her; and when he learned what the real facts were, had the holes sealed up and windows made in the upper storey. (Upper-storey windows came in at this time, we are told.) **(211)**

Māgandiyā then determined to drive the Teacher out of the city, and to this end employed ruffians to follow him about and heap abuse upon him. Ānanda proposed to the Teacher that they should go elsewhere; but this the Teacher declined to do, and comparing himself to an elephant engaged in the fray, pronounced Stanzas 320–322. After seven days the uproar ceased; and Māgandiyā, perceiving that she could do nothing against the Teacher, renewed her determination to destroy the women who were his supporters. **(211–213)**

Māgandiyā then procured from her uncle eight live cocks and eight dead cocks, and presented the live cocks to Udena, suggesting that he ask Sāmāvatī to cook them for him. Udena did so, and Sāmāvatī replied, "I and my followers do not take life." "Now," said Māgandiyā, "see whether she will cook them for the hermit Gotama." Māgandiyā then substituted the dead cocks for the live cocks, and Sāmāvatī immediately obeyed directions. "See," said Māgandiyā, "they won't do it for the like of you, but they 'll do it readily enough for outsiders." The king, however, still refused to believe her. **(213–215)**

Now the king was accustomed to divide his time equally among his three consorts, spending a week at a time in the apartment of each. Māgandiyā, knowing that the king would go to Sāmāvatī's apartment on the following day, carrying with him, as was his custom, the lute Allakappa had given him, procured a snake from her uncle and placed it in the cavity of the lute, stopping the end of the lute with a bunch of flowers. Then she said to him, "Whose apartment do you visit to-day?" The king told her. "Don't do it," said she; "last night I had a bad dream, and I fear that something will happen to you." But the king went, just the same, and Māgandiyā, much against his wishes, followed after. The king placed the lute beside his pillow and lay down on the bed. Māgandiyā secretly removed the bunch of flowers from the lute, and out came the snake. Māgandiyā screamed as if in terror, and after reproaching the king for disregarding her warning, turned to Sāmāvatī and her attendants and reviled them, saying, "You wretched scoundrels, what do you hope to gain by killing your most gracious sovereign?" The king was consumed with anger, and now believed all that Māgandiyā had said. **(215–216)**

Sāmāvatī urged her attendants to remain true to the principles of their religion, and to cherish no bitter feelings toward the king or Māgandiyā. The king took his bow, which required a thousand soldiers to string, and shot a poisoned arrow at Sāmāvatī's breast. But so great was the power of Sāmāvatī's love that the arrow turned back and, as it were, penetrated the king's heart. Thereupon the king threw himself at Sāmāvatī's feet and cried out, "Be thou my refuge." Sāmāvatī replied, "In whom I have myself sought refuge, in him do thou also seek refuge." Then the king sought refuge in Buddha and thereafter was a most generous benefactor of the Order. **(216–220)**

Māgandiyā thought to herself: "Everything I do turns out badly; what *shall* I do next?" Finally she resorted to the desperate expedient of directing her uncle to fire Sāmāvatī's palace. Her uncle wrapped the palace in cloths saturated with oil, barred the doors, set fire to the building in several places at once, and Sāmāvatī and her five hundred attendants perished in the flames. By devoting themselves to earnest meditation on the element of pain, some of the victims obtained the Fruit of Conversion, others the Fruit of the Second Path, still others the Fruit of the Third Path. (According to a passage in the Udāna, the monks reported to the Teacher what had happened and questioned him regarding the future state of the victims. The Teacher assured them that none failed to obtain a suitable reward, and warned them that all beings are constantly experiencing both happiness and misery.) **(220–222)**

When the king learned what had happened, he was overwhelmed with grief, and at once perceived that Māgandiyā was at the bottom of it. But knowing that he could not intimidate the latter, he resorted to artifice and said to his ministers, "Now that Sāmāvatī is dead, I can sleep in peace; whoever did this deed must have loved me greatly." Māgandiyā overheard this remark and said triumphantly, "It was I" "Well," said the king, "I am delighted. Send for your relatives, and I will reward you properly." The king bestowed handsome presents on Māgandiyā and her relatives; whereupon many persons who were in no way related to her came forward and claimed relationship. When the king had caught them all, he had them subjected to excruciating tortures and put to death. **(222–4)**

One day the Teacher overheard the monks remark that the cruel death of Sāmāvatī and her attendants was undeserved. "Quite right," said the Teacher, "if you regard only this existence; but their sad end was the result of an evil deed committed in a previous existence;" and he went on to tell them that in a previous existence Sāmāvatī and her attendants had once attempted to burn a Private Buddha to death. **(224–5)**

Then the monks asked the Teacher: "How did Khujjuttarā come to be a hunchback? how did she become so wise? how did she obtain the Fruit of Conversion? how did she come to be an errand-girl?" The Teacher told them that she became a hunchback through mocking a Private Buddha, that she acquired wisdom by waiting on some Private Buddhas, and the Fruit of Conversion by giving them her bracelets, and that she became an errand-girl because she once asked a nun to do a menial service for her. **(225–7)**

Again the monks asked the Teacher: "Sāmāvatī and her attendants perished by fire and Māgandiyā and her kinsfolk by torture; which of these live and which of these are dead?" The Teacher replied: "They that are heedless, though they live a hundred years, yet are they dead; they that are heedful, be they dead or alive, yet are they alive. Māgandiyā, while she yet lived, was dead already; Sāmāvatī and her attendants, though they be dead, yet are they alive; the heedful never die." Then he pronounced Stanzas 21–23, at the conclusion of which many were established in the Fruits. **(227–231)**

Book II. Story 2. Kumbhaghosaka.

ILLUSTRATING STANZA 4 = 24.

A pestilence once broke out in Rājagaha and a certain treasurer and his wife were attacked by the disease. Realizing that they were about to die, they bade farewell to their son Kumbhaghosaka, directing him to bury their treasure in the earth, flee for his life, and return later and dig it up again. Kumbhaghosaka buried the treasure, fled to a jungle, and after twelve years returned. No one recognized him; and this made him fear that if he dug up the treasure, he might be subjected to annoyance; therefore he decided to make his own living, and obtained a position as a cart-driver. **(231–2)**

One day King Bimbisāra heard the sound of Kumbhaghosaka's voice, and immediately exclaimed, "That is the voice of some rich man." A female slave heard the remark, made an investigation, and reported to the king that it was only a cart-driver. The king refused to believe her; whereupon she said, "Give me a thousand pieces of money, and I will make you master of his wealth." The king complied with her request. **(232–3)**

Now the female slave had a daughter whom she resolved to employ in the accomplishment of her design. Accordingly she obtained lodging for herself and her daughter in Kumbhaghosaka's house, and contrived to seduce Kumbhaghosaka to violate her daughter. When she had so far succeeded in her purpose, she contracted a marriage between

Kumbhaghosaka and her daughter, and Kumbhaghosaka was obliged to dig up some of his money to defray the expenses of the wedding festivities. In this way the whole story came out; but the king, instead of confiscating Kumbhaghosaka's wealth, praised him for his industry, confirmed him in his inheritance, and gave him his daughter in marriage. **(233-8)**

When the Teacher heard this story, he commented on it and pronounced Stanza 24, establishing many in the Fruits. **(238-9)**

Book II. Story 3. Little Roadling.[29]

ILLUSTRATING STANZA 5 = 25.

The daughter of a rich treasurer of Rājagaha yielded her chastity to a slave, and fearing that she would be discovered, fled with her lover to a distant place. When the time of her delivery was near at hand she expressed a desire to return home; but her lover, fearing to accompany her, put her off from one day to another, until finally she took matters into her own hands and started out alone. The pains of travail came upon her by the way, and she was delivered of a son. Just then her lover, who had learned her destination from the neighbors, arrived on the scene, and found her quite willing to go back with him. As the child had been born by the road, they agreed to call him Roadling. After a time the same thing happened again, and again they called the second child Roadling, distinguishing between the two by calling the older "Big Roadling," and the younger "Little Roadling." **(239-241)**

One day Big Roadling heard some other boys talking about their uncles and grandfathers, and said to his mother, "Have n't we any?" "Oh, yes!" said she; "you have a grandfather who is a rich treasurer, living at Rājagaha, and many other relatives there besides." "Why don't we go and see them?" The mother evaded the question, and spoke of the matter to her husband. "Why won't you take the children to their grandfather's? You don't suppose my parents are going to eat you alive, do you?" "I should never dare to face them, but I am willing to take them as far as the city." "That will do; all I want is to have them see their grandparents." So all four started out for Rājagaha, and when they reached the city, the mother sent word to her parents that she had returned. Her parents refused to see her, but sent her a sufficient sum of money for her support, and told her that she might go with her husband and live wherever she desired. The children, however, they consented to receive into their house; and

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[29] Cf. Jā. i. 114–120. Rogers, pp. 61–71.

that is how Big Roadling and Little Roadling came to be brought up in their grandfather's house. **(241–2)**

Big Roadling used to accompany his grandfather to hear the Teacher preach the Law, and one day told his grandfather that he would like to become a monk. His grandfather was greatly delighted, and took him to the Teacher, who received him as a monk, and somewhat later professed him. After a time Big Roadling attained Arahatship, and desiring to have his brother attain what he had attained, went to his grandfather and asked permission to receive Little Roadling into the Order. The grandfather readily gave his consent, and so Little Roadling also became a monk. **(242–4)**

Now in a previous existence under the Buddha Kassapa, Little Roadling had once made fun of a dullard monk; and in consequence of this act, he was now unable to master a single stanza in the course of four whole months. Big Roadling was so disgusted that he expelled him from the monastery. Little Roadling, however, was greatly attached to the religion of Buddha, and did not give up the monastic life. **(244)**

One day Jīvaka Komārabhacca went to Big Roadling and asked him, "How many monks are there under the Teacher?" "Five hundred." "I invite them all to take a meal with me to-morrow." "The layman Little Roadling is a dullard; I accept the invitation for everybody but him." When Little Roadling heard his brother speak thus, he decided to give up the monastic life on the morrow. The Teacher became aware of his intention, led him into his own perfumed chamber, gave him a piece of cloth, and said to him, "Little Roadling, face towards the East, rub this cloth, and say as you do so, 'Removal of Impurity, Removal of Impurity.'" The Teacher then went, accompanied by the monks, to Jīvaka's house. **(244–6)**

After Little Roadling had rubbed the cloth for a time, he perceived that it had become soiled, and a sense of the impermanence of things came to him. At that moment an apparition of the Teacher appeared before him and pronounced the Stanzas beginning with the words, "Impurity is Lust . . . Impurity is Hatred . . . Impurity is Infatuation." At the conclusion of the Stanzas Little Roadling attained Arahatship, acquired Four-fold Knowledge, and became a master of the Three Piṭakas. (This was because, in a former existence as a king, he gained a sense of impermanence by contemplating a cloth which had become soiled with the sweat of his brow.) **(246–7)**

When Jīvaka offered the Water of Donation to the Teacher, the latter placed his hand over the vessel, and said, "Are there no monks in the monastery?" Big Roadling replied, "No, indeed."

The Teacher said, "Yes, there are." Jīvaka sent a servant to find out. At that moment Little Roadling, aware of what his brother had said, exercised his supernatural power and filled the Mango-grove with a thousand monks. Jīvaka's servant returned and said, "The whole Mango-grove is full of monks." The Teacher said to him, "Go and tell Little Roadling to come hither." The servant went to the grove and called out, "Little Roadling, come hither." Thereupon the cry went up from a thousand throats, "Here I am! Here I am!" The servant went back to the Teacher and said, "They all say they 're Little Roadling." "Well, then," said the Teacher, "go back and take by the hand the first one who says he 's Little Roadling, and the rest will vanish." The servant did as he was told and soon returned with his man. **(247-8)**

After the meal Little Roadling returned thanks, and the Teacher, accompanied by the monks, withdrew. When the monks assembled in the evening, they discussed Little Roadling's expulsion from the monastery and subsequent attainment of Arahatship, and were loud in their praises of the Buddha. All of a sudden the Buddha appeared in their midst and said to them, "This is not the first time Little Roadling has shown himself a dullard; aforetime, too, he was a dullard. Nor is it the first time I have assisted him; aforetime, too, I assisted him, and by my assistance he attained no less success in the things of this world than he has just attained in higher things." "Tell us all about it," said the monks; whereupon the Teacher began the following story of the past: **(248-250)**

The World-renowned Teacher, the Young Man, and the King of Benares. A young man of Benares once went to Takkasilā and became a pupil of a World-renowned Teacher. He was most faithful in the performance of his duties as a pupil, but such a dullard was he that after a long term of residence he was unable to repeat a single Stanza. Finally he became discouraged, and went to his Teacher and told him that he was going to give it up as a bad job and go back home. The Teacher had by this time become much attached to his pupil by reason of the latter's dutifulness to him; so he took him to the forest and taught him a charm, telling him that it would insure him a living, and impressing it upon him that he must recite it over and over again to avoid the possibility of forgetting it. And this is the way the charm went: "You 're at it, you 're at it; why are you at it? *I* know what you 're at." When the young man had mastered the charm he returned to Benares. **(250-251)**

It so happened just at this time that the King of Benares made a careful examination of his thoughts, words, and deeds, for the purpose of discovering in what particulars he might have failed. So far as he could see, his conduct had been quite correct; but then he reflected, "A person never sees his own faults; it takes another person to see them." Accordingly, he decided to find out just what was the candid opinion of his subjects; and after nightfall he put on a disguise, and went about the streets eavesdropping. **(251–2)**

The first house the king came to was that of the young man who had just returned from Takkasilā. The king observed that some robbers were in the act of breaking into the house; so he took his stand in the shadow of the house and awaited developments. The robbers made such a noise effecting an entrance that they woke up the young man; whereupon the latter began to recite his charm: "You 're at it, you 're at it; why are you at it? *I* know what you 're at." The robbers exclaimed, "We 're discovered; run for your lives!" dropped their spoils, and fled. The next day the king sent for the young man, got him to teach him the spell, and presented him with a thousand pieces of money. **(252–3)**

That very day the Prime Minister went to the royal barber, presented him with a thousand pieces of money, and said, "The next time you go to shave the king, cut his throat with a razor; then you shall be Prime Minister, and I shall become king." "Agreed," said the barber. A day or two later the barber went in to shave the king; and as he sharpened his razor, he said to himself, "One stroke, and it 's all done." Just at that moment the king began to recite the charm: "You 're at it, you 're at it; why are you at it? *I* know what you 're at." Beads of sweat stood out on the forehead of the barber; he threw his razor away in terror, and flung himself at the feet of the king. Now kings know a thing or two; and the King of Benares immediately exclaimed, "Villain, you thought I did n't know." "Sire, spare my life." "Have no fear; only tell me the truth." "It was the Prime Minister that put me up to this." Thereupon the king banished the Prime Minister, and appointed in his place the young man who taught him the spell. **(253–4)**

"At that time," said the Teacher, "Little Roadling was the young man, and I was the World-renowned Teacher. Aforetime, too, Little Roadling was a dullard, and I helped him." The Teacher closed his discourse by telling the Cūḷakaseṭṭhi Jātaka and identifying the births. On a later occasion the monks commented on Little Roadling's determination never to give up; whereupon the Teacher assured them that the highest rewards are within reach of the persevering disciple, and pronounced Stanza 25, establishing many in the Fruits. **(254–5)**

Book II. Story 4. When Foolish Folk Made Holiday.

ILLUSTRATING STANZAS 6–7 = 26–27.

On a certain occasion the foolish, ignorant people of Sāvatthi used to smear themselves with cow-dung, and give themselves up to license for a period of seven days. They went about the city insulting everybody they met, even their own kinsmen, and persons devoted to the religious life; and would desist only on the payment of a forfeit. During this period of disorder the Teacher and the monks remained within the walls of the monastery. When the noble disciples told him of the insults to which they had been subjected, he expressed his disapproval of the misconduct of the foolish folk, and pronounced Stanzas 26–27, at the conclusion of which many were established in the Fruits. **(256-8)**

Book II. Story 5. Kassapa the Great, Elder.

ILLUSTRATING STANZA 8 = 28.

On a certain occasion, during the time when the Elder Kassapa was living in Pipphali Cave, he went to Rājagaha to collect alms; and after he had eaten his meal, he sat down and endeavored to obtain by Supernatural Vision a comprehension of Birth and Rebirth. The Teacher, seated at Jetavana, exercised Supernatural Vision, and at once perceived what Kassapa was about. "That is beyond your range, Kassapa," said he; "only a Buddha is able to comprehend the Totality of Existences." Then the Teacher sent forth an apparition of himself, which went to Kassapa and pronounced Stanza 28. At the conclusion of the Stanza, many were established in the Fruits. **(258-260)**

Book II. Story 6. The Two Brethren.

ILLUSTRATING STANZA 9 = 29.

Two brethren obtained a subject of meditation from the Teacher, and retired to the forest. One of them was heedful and zealous, and in a short time attained Arahatship. The other was heedless and lazy. When the two brethren returned to the Teacher, and the latter learned how they had spent their time, he compared the zealous monk to a race-horse and the lazy monk to a hack, and pronounced Stanza 29, establishing many in the Fruits. **(260-263)**

Book II. Story 7. Mahāli's Question.

ILLUSTRATING STANZA 10 = 30.

One day a Licchavi prince named Mahāli, who had heard the Suttanta entitled Sakka's Question recited by the Teacher, went to the latter and asked him, "Did you ever see Sakka?" "Oh, yes," replied the Teacher. "It must have been a counterfeit of Sakka," returned Mahāli, "for it is a difficult matter to get a look at Sakka." "Nevertheless," said the Teacher, "I am well acquainted with Sakka; and what is more, I know all about the meritorious deeds by means of which he rose to the lordship of the gods." Then the Teacher enumerated Sakka's meritorious deeds in his human existence as Magha. "Tell me all about Magha," said Mahāli. "Well, then, listen," replied the Teacher, and then told the following story of the past: **(263-5)**

Magha.[30]

Once upon a time a youth named Magha went about his native village in the kingdom of Magadha doing all manner of good works; and in the course of time gathered others about him, until finally there were thirty-three persons in the village keeping the Five Precepts and doing works of merit. The village headman observed their actions, and said to himself, "If these men would only drink strong drink and do as other men do, I should get something out of it." Accordingly he said to them, "What 's this you 're doing?" "Treading the Heavenly Path." "That 's no occupation for householders; why don't you eat fish and flesh, drink strong drink, and have a good time?" Magha and his companions rejected his suggestion; whereupon he determined to destroy them. **(265-7)**

The village headman went to the king and told him that there was a band of robbers in the village. The king immediately ordered them to be trampled to death by elephants. But the elephants refused to go near them. When this was reported to the king, he concluded that there must be a reason for it; accordingly he had the thirty-three youths summoned before him, told them the charge the village headman had brought against them, and listened to their story. The result was that he begged their pardon for having so misunderstood them, made the village headman their slave, gave them an elephant to ride on, and placed the entire resources of the village at their disposal. **(267-8)**

At this the youths rejoiced greatly and resolved to abound yet more

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[30] Cf. Jā. i. 199-206.

in good works. So they summoned a carpenter, and had him erect a rest-house for the multitude at the junction of four highways. As they had lost all desire for women, they would not allow women to share in the work. (268-9)

Now there were four women living in Magha's house, Joy, Thoughtful, Goodness, and Wellborn. One day Goodness bribed the carpenter to give her the chief share in the erection of the hall. The carpenter made a pinnacle, cut this inscription on it, "This Hall is named for Goodness," wrapped the pinnacle in a cloth, and laid it aside. When the hall was nearly completed, the carpenter said to his masters, "We have forgotten something." "What is it?" said they. "A pinnacle." "Let's get one." "But it's too late to season the wood." "Well, what *is* to be done?" "Perhaps we might find one ready-made." The carpenter immediately procured the pinnacle he had made for Goodness; and thus Goodness obtained the chief share in the erection of the hall. (269-270)

The thirty-three youths prepared thirty-three wooden seats, and entertained visitors handsomely, the elephant going out to meet each arrival and performing the usual courtesies. Magha planted an Ebony-tree near the hall, and under the tree set up a stone seat. Joy provided a lotus tank, and Thoughtful a flower garden. Wellborn, thinking that it was a sufficient distinction to be a cousin of Magha, did nothing but adorn herself. Magha, having fulfilled the Seven Injunctions, was at the end of his allotted term of life reborn in the world of the Thirty-three as Sakka, king of the gods; Magha's companions were also reborn there, as was also the carpenter, who became Vissakamma. (270-272)

Now at this time there were Asuras dwelling in the world of the Thirty-three; and when they became aware that some entirely new gods had been born in their midst, they prepared strong drink to welcome them. Sakka forbade his companions to touch it, and they obeyed him; but the Asuras got very drunk. Then Sakka gave the signal, and his companions picked up the Asuras by the heels, and flung them down into the abyss. Thereupon there sprang up at the foot of Mount Sineru the Palace of the Asuras and the Tree that is called Pied Trumpet-flower. And when the conflict between the Gods and the Asuras was over, and the Asuras had been defeated, there sprang into existence the City of the Thirty-three, crowned with a magnificent palace called the Palace of Victory. A Coral-tree sprang up to correspond with the Ebony-tree Magha had planted, and at the foot thereof, to correspond with the stone seat he had set up, stood Sakka's Yellowstone Throne. The elephant was reborn as the god

Erāvaṇa; since there are no animals in the world of the Thirty-three, whenever Erāvaṇa wished to go into the garden to play, he would lay aside his godhead and become an elephant for the time being. Erāvaṇa created gigantic water-pots for each member of Sakka's retinue. Each vessel held seven tusks; each tusk, seven tanks; each tank, seven lotus plants; each plant, seven flowers; each flower, seven leaves; each leaf, seven celestial nymphs, who danced unceasingly. For Sakka he created a water-pot much larger than the others. Above hung a canopy with a fringe of bells whose sound was as the music of the celestial choir. Beneath it was a jewelled couch, where Sakka reclined in state. Such was the splendor in the enjoyment of which Sakka lived. **(272–4)**

When Goodness, Joy, and Thoughtful died, they were reborn in the world of the Thirty-three; and through the effect of their respective benefactions there arose a mansion named Goodness, a lotus tank named Joy, and a creeper-grove named Thoughtful. When Wellborn died, she was reborn as a crane in a mountain cave. **(274–5)**

Sakka surveyed his handmaidens, and desiring that Wellborn should be reborn as one of them, went to her in disguise, conducted her to the world of the Thirty-three, let her see her friends, and assured her that she could attain equal happiness by keeping the Five Precepts. This she promised to do. After a few days Sakka, desiring to test her sincerity, lay down on the sand in the form of a fish. The crane, thinking that it was dead, seized it in her beak. Just as she was about to swallow it, it wiggled its tail, whereupon the crane dropped it. Three times Sakka tried this stratagem, and three times the crane, discovering that the fish was alive, refused to eat it. Then Sakka resumed his proper form, praised the crane, and departed. **(275–7)**

At the end of her existence as a crane, Wellborn was reborn at Benares as the daughter of a potter. Sakka disguised himself as a peddler, filled a cart with precious jewels disguised as cucumbers, went to the city, and cried out, "Cucumbers in exchange for Five Precepts." The inhabitants of the city brought kidney-beans, and when the peddler refused them, they said, "What are these 'precepts' like? are they black or brown?" "Neither," said the peddler. "Oh," said they, "we have heard a potter's daughter say, 'I keep the precepts;' you might try her." So Sakka went to the potter's daughter, revealed himself to her, gave her the jewels, praised her, and departed. **(277–8)**

At the end of her existence as a potter's daughter, Wellborn was reborn in the world of the Asuras as the daughter of Vepacitti, king of the Asuras, a bitter enemy of Sakka. One day Vepacitti assembled all the hosts of the Asuras, and giving his daughter a wreath of flowers, directed her to choose a husband. At that moment Sakka, disguised as an aged Asura, sat down in the outer fringe of the assembly. The maiden immediately threw the wreath of flowers over his head and chose him for her husband. He took her by the hand, shouted out, "I am Sakka," and flew up into the air. The Asuras cried out, "We have been fooled by old Sakka," and started up in pursuit. **(278-9)**

Sakka's charioteer, Mātali, brought up the chariot Victory, and Sakka, after assisting Wellborn to mount, set out for the city of the gods. When they reached the Forest of the Silk-cotton Trees, the fledglings of the Garuḷa birds, fearing that they were going to be crushed to death, shrieked aloud; whereupon Sakka said to his charioteer, "Let not these creatures perish on my account; turn back the chariot." At this the Asuras concluded that reinforcements must have come up, and abandoned the pursuit. Sakka bore Wellborn to the city of the gods and made her chief among twenty-five millions of celestial nymphs. Thereafter, when the Asuras made preparations to attack Sakka, the latter placed at the gates of his city images of Indra bearing the thunderbolt. When the Asuras saw the images, they invariably concluded that Sakka was no longer there, and departed. **(279-280)**

The Teacher extolled Magha's earnestness, and pronounced Stanza 30, at the conclusion of which Mahāli was established in the Fruit of Conversion, and many others were established in the Three Fruits. **(280-281)**

Book II. Story 8. A Certain Monk.

ILLUSTRATING STANZA 11=31.

A certain monk had the Teacher instruct him in the ascetic practices which lead to Arahatship, and retired to the forest to meditate. In spite of his best efforts, he was unable to attain Arahatship; therefore he decided to return to the Teacher and ask him to assign him a specific subject of meditation. On the way he caught sight of a forest fire; whereupon he hastily climbed a bare mountain, and as he watched the fire, concentrated his mind on the following thought: "As this fire goes its way consuming all obstacles both great and small, so also ought I to go, consuming all obstacles both great and small with the fire of knowledge of the Noble Path." **(281-2)**

As the Teacher sat in his Perfumed Chamber, he became aware of what the monk was doing, and sent forth an apparition of himself, which went to the monk and pronounced Stanza 31. At the conclusion of the Stanza, the monk attained Arahatship. **(282-3)**

Book II. Story 9. Tissa of the Market Town, Elder.

ILLUSTRATING STANZA 12=32.

A certain noble youth who was born and brought up in a market town not far from Sāvatthi, was received into the Order by the Teacher, and was thereafter known as Tissa of the Market Town, Elder. He wanted little, was satisfied with what he had, and lived an active, blameless life. All his life long he remained within the borders of his native village, in spite of the fact that in near-by Sāvatthi, Pasenadī Kosala, Anāthapiṇḍika, and others were bestowing alms, the like of which had never been seen before. One day the Teacher sent for him, and said to him, "Monk, it is no wonder that you, who have such a one as I am for your Master, should want little." When the other monks asked the Teacher to explain himself, the latter told them the following story of the past: **(283–4)**

Sakka and the Parrot.[31] Once upon a time a great many parrots lived in a grove of fig-trees in the Himālaya country. The king-parrot, when the fruit of the tree in which he lived had come to an end, ate whatever he could find, drank the water of the Ganges, and being very happy and contented, stayed where he was. In fact, he was so happy and contented that the abode of Sakka began to shake. Thereupon Sakka decided to put him to the test, and by his supernatural power withered up the tree. When Sakka perceived that this made no difference at all to the parrot, he decided to give the parrot his choice of a boon; whereupon, taking the form of a royal goose, and preceded by Well-born in the form of an Asura nymph, he went to the parrot and asked him why his heart delighted in a tree that was withered and rotten. (This story is identical with the Mahāsuka Jātaka, which will be found in the Tenth Nipāta;[32] only the setting is different. The Jātaka goes on to say that the parrot replied, "This tree has been good to me in the past; why should I forsake it now?" Thereupon Sakka caused the tree to bloom anew, and to bear ambrosial fruit.) **(284–5)**

"At that time," said the Teacher, "Ānanda was Sakka, and I was the parrot. It is no wonder that Tissa wants little, having found a Teacher like me." Then he pronounced Stanza 32, at the end of which Tissa attained Arahatship, and many others were established in the Fruits. **(285–6)**

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[31] Cf. Jā. iii. 491–4.

[32] Norman calls attention to the fact that it actually occurs in the Ninth Nipāta.

Book III. Story 1. Meghiya, Elder.

ILLUSTRATING STANZAS 1–2 = 33–34.

According to a story, the details of which will be found in the Suttanta entitled Meghiya, the Elder Meghiya made little progress in wrestling with the flesh until the Teacher impressed upon him the importance of bringing the thoughts into subjection, by pronouncing Stanzas 33–34; whereupon Meghiya was established in the Fruit of Conversion, and many others in the Three Fruits. **(287–9)**

Book III. Story 2. A Certain Monk.

ILLUSTRATING STANZA 3 = 35.

Seventy monks once had the Teacher instruct them in the ascetic practices which led to Arahatship, and went to a certain village named Mātika in the kingdom of Kosala to collect alms. A lay sister, mother of the owner of the village, offered them hospitality, and provided them with food and lodging during the three rainy months. At her request the monks instructed her in the ascetic practices, which she performed with such diligence that in advance of her instructors she attained the Three Paths and Fruits and the Supernatural Faculties. As she was thus enabled to know the precise needs of the monks, thereafter she ministered to them so successfully that in a short time they too attained Arahatship. At the close of the rainy season they took leave of their hostess and returned to the Teacher. When the latter remarked, "You look as if you had fared well," the monks replied, "We did, indeed; our hostess knew the secret desires of our hearts, insomuch that no sooner did we think of our needs than she immediately supplied them." **(290–293)**

A certain monk heard this and was immediately seized with a desire to enjoy so pleasant an experience. Accordingly he had the Teacher instruct him in the ascetic practices, went to the house of the lay sister, and accepted her offer of food and lodging. He found everything exactly as the monks had represented it. But then the thought occurred to him, "If I should entertain a sinful thought, she would doubtless seize me by the top-knot, and treat me as people treat thieves; I had best get away from here." So he returned to the Teacher and told the latter what had made him change his plans. The Teacher admonished him to control his thoughts, pronounced Stanza 35, thereby establishing many in the Fruits, and sent the monk back to the house of the lay sister. The latter ministered to the needs of the monk so successfully that in a short time he attained Arahatship. One day, while the monk was experiencing the bliss of the Path and the Fruit, he was filled with gratitude towards the lay sister, and became curious to know whether she had befriended him in previous existences. So he called up before his mind ninety-nine previous existences, and to his horror perceived that in each of these existences she had murdered him. "Oh, what a sinner she has been!" thought he. At the same moment the lay sister, sitting in her own chamber, became aware of what was passing through his mind. "Call up one more existence," said she. By the power of Supernatural Audition the monk immediately heard what she said; whereupon he called up before his mind the hundredth existence, and perceived that in that existence she had spared his life. Then he rejoiced greatly, and straightway passed into Nibbāna. **(293-7)**

Book III. Story 3. A Certain Discontented Monk.

ILLUSTRATING STANZA 4 = 36.

The son of a certain treasurer of Sāvatthi performed the duties of a layman so faithfully as to win the appellation "Faithful." But after he had become a monk he grew discontented over the multitudinous duties imposed upon him, and said so to the Teacher. The latter replied, "You have only one duty to perform; and that is to guard your thoughts; if you do that, you have done all." The Teacher then pronounced Stanza 36, at the conclusion of which the discontented monk was established in the Fruit of Conversion, and many others were established in the Three Fruits. **(297-300)**

Book III. Story 4. Saṅgharakkhita's Nephew, Elder.

ILLUSTRATING STANZA 5 = 37.

A certain noble youth of Sāvatthi retired from the world, was admitted to the Order, and in a short time attained Arahatship. His name was Saṅgharakkhita. About this time a son was born to his youngest sister and named after him. When Saṅgharakkhita's nephew reached the age of manhood, he followed his uncle's example and entered the Order. At the beginning of the rainy season the younger monk procured two sets of monastic robes, intending to present one of them to his uncle, and for this purpose set out for his uncle's quarters. When he arrived at his destination, he discovered that the older monk had not yet returned; so he swept the place carefully, procured water for washing the feet, prepared a seat, and sat down, awaiting his uncle's return. When he saw his uncle coming he went out to meet him, took his bowl and robe, seated him, fanned him with a palm-leaf fan, gave him water to drink, washed his feet, brought him the set of robes he had procured for him, formally presented them to him, and then, taking the palm-leaf fan into his hands, resumed fanning him. Said the older monk, "Nephew, I have a complete set of robes; use these yourself." The younger monk pleaded with his uncle to reconsider his answer, but the older monk remained obdurate. The younger monk was so bitterly disappointed that he then and there decided to give up the monastic life and return to the life of a householder. So as he stood there beside the older monk, swinging the palm-leaf fan to and fro, he pondered in his mind ways and means of earning a living. Finally the following thought occurred to him: **(300–302)**

"I will sell this set of robes, and buy me a ewe; ewes are very prolific; every lambkin the ewe drops I will sell; in this way I shall be able to accumulate a lot of money. When I have done that, I will procure me a wife. She will bear me a son, whom I will name after my uncle. I will put my son in a go-cart, and taking son and wife along, go and pay my respects to my uncle. As I journey by the way I will say to my wife, 'Just hand me my son; I wish to carry him.' She will reply, 'What 's the need of your carrying the boy? go ahead and push this go-cart;' then she will take the boy into her arms and say, 'I 'll carry him myself;' whereupon, finding the child too heavy for her, she will let him fall. Then I will say to her, 'You would n't let me carry the child, in spite of the fact that you could n't carry him yourself;' and having thus said, I will bring down my stick on her back" . . . At that moment the younger monk swung his fan with great force, and brought it down on the head of his uncle.[33] **(302–303)**

The older monk considered within himself, "Why did my nephew strike me on the head?" and immediately became aware of what was passing through his nephew's mind. So he said, "Nephew, you did n't succeed in hitting the woman; but why should an aged Elder suffer for it?" The younger monk was so ashamed of himself that he immediately threw his fan away and started to run off. But the novices and young monks ran after him, caught him, and brought him before the Teacher, who said to him, "Be not disturbed; only guard your thoughts hereafter," and pronounced Stanza 37, establishing the young monk in the Fruit of Conversion, and many others in the Three Fruits. **(303–305)**

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[33] Compare the story of the Brahman and his Jar, in the Pañcatantra, Hertel's ed., v. 7.

Book III. Story 5. Cittahattha, Elder.[34]

ILLUSTRATING STANZAS 6–7 = 38–39.

A noble youth of Sāvatthi once became a monk for no other reason than to obtain an easy livelihood. After a few days he tired of the monastic life and returned to the world. Six times he became a monk, and as many times returned to the life of a householder; wherefore his brethren called him Cittahattha (Thought-controlled). In the meantime his wife became great with child. Once more he decided to become a monk, and entered the inner chamber of his house to procure his yellow robe. There on the bed lay his wife asleep. Her garments were in disarray, saliva was flowing from her mouth, she was snoring, her mouth was wide open. Her appearance reminded him of a bloated corpse. At that moment he obtained a sense of impermanence, and taking the yellow robe, left the house and went to the monastery. A short time after this, his seventh reception into the Order, he attained Arahatship. The Teacher, contrasting Cittahattha's former and latter states, pronounced Stanzas 38–39. The monks said, "How could a youth destined to Arahatship abandon the monastic life six times?" "Easily enough," said the Teacher; "I did the same thing myself." Then he told the following story of the past: **(305-311)**.

Kuddāla and his Spade. Once upon a time, when Brahmadatta reigned at Benares, a Pandit named Kuddāla was admitted to a certain heretical Order, but after a few months renounced the monastic life, all because of his attachment for a blunt spade with which he used to till the ground. This happened six times. Finally Kuddāla made up his mind to put temptation out of his way; so he took the spade to the bank of the Ganges, closed his eyes, and threw it into the water. As he did so he shouted as loud as he could, "I have conquered!" At that moment along came the King of Benares, returning from a successful expedition. When the King heard Kuddāla's exclamation of victory, he went up to him and asked him what he meant by it. Kuddāla replied, "Those whom you have conquered will have to be conquered again; but I have conquered myself for good and all." At that moment Kuddāla attained Specific Attainment by gazing on the water; whereupon he sat cross-legged in the air and instructed the king in the Law. The King of Benares then and there retired from the world with all his followers, and shortly afterwards his royal enemy followed his example. **(311-313)**

"At that time," said the Teacher, "I was the Pandit Kuddāla." **(313)**

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[34] Cf. Jā. i. 311–313.

Book III. Story 6. How Five Hundred Monks Attained Insight.

ILLUSTRATING STANZA 8=40.

Five hundred monks once had the Teacher instruct them in the ascetic practices that lead to Arahatship, and retired to a certain forest. In this forest lived a great many powerful tree-spirits, who took a dislike to the monks and determined to get rid of them. Accordingly the spirits caused the monks to see bodiless heads and headless trunks, to hear the voices of demons, and to catch all manner of diseases. After a time the monks returned to the Teacher and related their experiences. "I will provide you with a weapon," said he; whereupon he rehearsed the Sutta entitled Metta, and told them to return to the forest and do the same. When they did so, the hearts of the spirits were suffused with love, and the monks quickly attained Insight. The Buddha, seated in his Perfumed Chamber, became aware of what had happened in the forest, and sent forth an apparition of himself, which went to the monks and pronounced Stanza 40. At the conclusion of the Stanza the five hundred monks attained Arahatship, and returned, praising the golden body of the Teacher. **(313-318)**

Book III. Story 7. Tissa of the Diseased Body, Elder.

ILLUSTRATING STANZA 9=41.

A noble youth of Sāvatthi once became a monk and was thereafter known as Tissa. As time went on, he was attacked by boils, and his condition grew steadily worse until finally his brethren, unable to do anything for him, abandoned him and left him to his fate. Now the Buddhas are wont, twice a day, to survey the world; at early dawn, from the Rim of the World to the Perfumed Chamber; and in the evening, from the Perfumed Chamber to the outer world. One evening, accordingly, as the Tathāgata surveyed the world, Tissa of the Diseased Body appeared within the net of his knowledge. He immediately went to him, and, assisted by the monks, bathed him with warm water, alleviating his sufferings. Then the Teacher pronounced Stanza 41, at the conclusion of which Tissa attained Arahatship and passed into Nibbāna, and many of the bystanders were established in the Three Fruits. When the monks expressed surprise that a noble youth destined to attain Arahatship should have been visited with such an affliction, the Teacher told them that it was no more than he deserved, and related the following story of the past: **(319-321)**

The Cruel Fowler. In the dispensation of the Buddha Kassapa, Tissa was a fowler. In order that the birds he caught might not be able to escape, he was in the habit of breaking their legs and wing-bones and throwing them all together in a heap. This was the cause of his suffering in a later existence. One day, however, he bestowed alms on a monk, saying, "May I obtain the highest Fruits of the religion you profess." In consequence of this meritorious deed he was enabled to attain Arahatship in a later existence. **(322)**

Book III. Story 8 Nanda the Herdsman.

ILLUSTRATING STANZA 10 = 42.

Nanda was a herdsman of Anāthapiṇḍika. One day he went to his master's house to listen to the Teacher and was established in the Fruit of Conversion. He entertained the Teacher for seven days, and when the latter departed, accompanied him on his way for a considerable distance, and finally bidding him farewell, turned back. He had not gone far when he was shot and killed by the stray arrow of a hunter. The monks reported the incident to the Teacher, and remarked that if the latter had not gone to visit Nanda, Nanda would not have died. "You are greatly mistaken," said the Teacher; "there is no such thing as escape from death." Then the Teacher solemnly warned them that ill-regulated thoughts do a man much more harm than external enemies, and pronounced Stanza 42, at the conclusion of which many were established in the Fruits. (No one asked the Teacher about Nanda's deed in a previous existence; therefore the Teacher said nothing about it.) **(322–5)**

Book III. Story 9. Soreyya, Elder.

ILLUSTRATING STANZA 11 = 43.

When the Teacher was in residence at Sāvatthi, there was a treasurer's son named Soreyya living in the city of Soreyya. One day, accompanied by a friend, he entered a splendid carriage, and, surrounded by a considerable retinue, drove out of the city for a dip in the swimming-pool. As they passed out of the city gate Soreyya caught sight of the Elder Mahā Kaccāyana in the act of putting on his monastic robes. The golden hue of the Elder's body attracted the attention of Soreyya, who immediately exclaimed, "Would that this Elder were my wife; or else that the hue of my wife's body were like the hue of his body." In consequence of this wicked wish Soreyya was instantly transformed into a woman. Soreyya, much embarrassed, immediately left the carriage, joined a caravan-train bound for Takkasilā and was eventually married to the son of a treasurer of that city, becoming the mother of two sons. **(325–7)**

(There are no men who have not been women at some time or other; and no women who have not, at some time or other, been men. For example, men who commit adultery endure punishment in hell for a hundred thousand years, and on returning to human estate at the end of that period, have to spend a hundred existences as women. Even the Elder Ānanda, who fulfilled the Perfections for a space of a hundred thousand cycles of time, once committed adultery in an existence as a blacksmith, and as a result was obliged to spend fourteen existences as a woman, and seven existences more before the effect of his evil deed was completely exhausted. Women may obtain rebirth as men by such works of merit as almsgiving, ready obedience to their husbands, and so on.) **(327)**

So Soreyya, who, as a treasurer of Soreyya, was already the father of two sons, became, as the wife of a treasurer of Takkasilā, the mother of two more, making four children in all. Now just at this time, Soreyya's carriage-companion paid a visit to Takkasilā; and Soreyya, who happened to see him from the window, invited him to the house and entertained him handsomely. "Madam," said the guest, "I never saw you before; why is it that you have been so kind to me? do you know me?" Soreyya then told him the whole story. "Oh," said the guest, "it is easy enough to remedy all this; the Elder Mahā Kaccāyana lives near by; just beg his pardon, and everything will be all right again." Soreyya did so, and immediately became a man again. Mahā Kaccāyana admitted him to the Order, and Soreyya, after committing his two youngest sons to the care of the treasurer of Takkasilā, went back to Sāvatthi with Mahā Kaccāyana. **(327–330)**

When the natives learned what had happened, they were much excited, and went to Soreyya and said, "This is a strange state of affairs; you are the mother of two sons, and the father of two more; which pair of children have you the stronger affection for?" Soreyya replied, "For the pair of which I am the mother." After a time Soreyya attained Arahatship. The next time he was asked this question he replied, "My affection is set nowhere." When Soreyya's latest reply was reported to the Teacher, the latter remarked that Soreyya, having now obtained mastery over his thoughts, was accomplishing for others what neither father nor mother had power to accomplish. The Teacher then pronounced Stanza 43, at the conclusion of which many were established in the Fruits. **(330–332)**

Book IV. Story 1. The Monks who talked about tilling the Soil.

ILLUSTRATING STANZAS 1–2 = 44–45.

One evening five hundred monks who had accompanied the Teacher on his rounds began to talk about the varieties of soil they had seen. The Teacher told them that they might better be occupied with tilling the soil of their hearts, and pronounced Stanzas 44–45, at the end of which all five hundred monks attained Arahatship. (333–5)

Book IV. Story 2. The Elder who contemplated a Mirage.

ILLUSTRATING STANZA 3 = 46.

A certain monk who had made little progress in the practice of meditation once saw a mirage. He immediately concentrated his mind upon the following thought: "Just as this mirage appears substantial to those that are far off, but vanishes on nearer approach, so also is this existence." Then, seeing a waterfall, he thought, "Just as this spray is dissipated and no more seen, so also is this existence." The Teacher, sitting in his Perfumed Chamber, became aware of the monk's Attainment, and pronounced Stanza 46; whereupon the monk attained Arahatship and returned, praising the golden body of the Teacher. (335–7)

Book IV. Story 3. Viḍūḍabha.

ILLUSTRATING STANZA 4 = 47.

At Sāvatthi, lived Prince Pasenadi, son of the King of the Kosalans; at Vesālī, a prince of the Licchavi line, named Mahāli; at Kusinārā, Prince Bandhula, son of the King of the Mallas. These three princes resorted to a world-renowned teacher at Takkasilā for instruction, and, chancing to meet in a hall outside of the city, became warm friends. After acquiring the various branches of learning, they took leave of their teacher, departed together, and went to their several homes. Pasenadi's father was so pleased with his son's attainments that he made him king. Mahāli devoted himself to the task of educating the Licchavi princes, but over-exerting himself, lost the sight of his eyes; whereupon the princes erected a gate for him, and ever afterwards remained his most devoted and loyal pupils. Bandhula received a slight at the hands of the Malla princes, which made him so angry that he determined to kill them and seize the throne. When he informed his mother and father of his plan, they told him that it was bound to fail, inasmuch as the kingdom of the Mallas was an hereditary kingdom. Thereupon he decided to go to Sāvatthi and live with his friend Pasenadi. King Pasenadi received him with distinguished honors, and made him Commander-in-chief of his army. Bandhula sent word to his mother and father to come and live with him, and they did so. **(337-9)**

One day King Pasenadi saw from his terrace a great company of monks passing along the street. "Where are they going?" said he. One of his retinue replied, "Sire, every day two thousand monks go to the house of Anāthapiṇḍika to obtain food, medicine, and the other requisites; five hundred, to Cūla Anāthapiṇḍika's; a like number to Visākhā's and to Suppavāsā's." "I, too, will serve the Congregation of Monks," thought the king; and immediately went to the Teacher and asked to be allowed the privilege. For seven days the king entertained Buddha and the monks, and when he bade farewell to the Teacher, he invited the latter to come regularly to his house thereafter. The Teacher declined the invitation, however, on the ground that many other persons desired his presence, and sent Ānanda in his place. For seven days the king served Ānanda and the monks in person; during the three following days he was so remiss in the performance of his duty to the monks that the latter dropped off, one by one, until finally Ānanda was the only one left. The king was so provoked at the conduct of the monks that he went to the Teacher and complained. The Teacher exonerated the monks from blame, and told the king that the monks lacked confidence in him. **(339-341)**

"A family must possess nine distinctive marks," said the Teacher, "to be entitled to the privilege of entertaining monks. They must rise courteously to meet them; greet them pleasantly; seat them comfortably; conceal not what they possess; possessing much, give much; possessing good things, give good things; present their offerings with deference; sit to hear the Law; speak in an agreeable tone of voice. It was doubtless because you failed in your duty to the monks that they left your house. Just so the wise men of old time went to a place where they felt secure." The Teacher then told the following story of the past: **(341-2)**

Kesava, Kappa, Nārada, and the King of Benares.[34] Once upon a time, when Brahmadatta reigned at Benares, a hermit named Kesava, accompanied by his following, accepted the offer of the King to entertain them during the rainy season. The monks were so annoyed by the cries of elephants, however, that they dropped off, one by one, until finally Kesava was left alone with his faithful pupil Kappa. After

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[34] Cf Jā. iii. 142-145.

a time even Kappa was unable to stand the noise any longer, and left his master. Thereupon Kesava fell sick, and begged the King to send him back to his followers. The King immediately did so, sending Nārada and three other ministers with him. As soon as Kesava was restored to his companions he recovered his health, and was soon well and happy. When Nārada asked him how he liked a hermit's fare after enjoying the hospitality of a king, Kesava replied that he was now completely happy since, after all, a sense of security and confidence was the main thing. **(342–4)**

"At that time," said the Teacher, "the King was Moggallāna; Nārada was Sāriputta; the pupil Kappa was Ānanda; while the hermit Kesava was I myself." **(344–5)**

Thereupon King Pasenadi bethought himself how he might regain the confidence of the monks, and concluded that the best way would be to take to himself as wife the daughter of some kinsman of the Buddha. Accordingly he sent ambassadors to the Sakyans, requesting one of their daughters in marriage. The King of the Sakyans, fearing that he would incur the enmity of King Pasenadi by refusing his request, put the matter before his nobles. Mahānāma said, "I have a daughter by one of my slave-women, and she is very beautiful; why not send her?" Accordingly the King of the Sakyans sent Mahānāma's daughter to King Pasenadi, and the latter married her. Her name was Vāsabhakhattiyā. **(345–6)**

In due time Vāsabhakhattiyā became the mother of a son. Pasenadi sent to his grandmother, asking her to give the child a name. She selected the name Vallabha (Beloved); but the messenger, being a little deaf, understood her to say Viḍūḍabha, and so reported to the King of Kosala. Accordingly the child was named Viḍūḍabha. When Viḍūḍabha was seven years old, he said to his mother, "Mother, the other boys get presents from their maternal grandfathers; why is it that I don't get any? haven't you any mother or father?" "Oh, yes!" said she; "your grandparents are Sakyan kings; but they live a long way off, and that's the reason why you don't get any presents." When Viḍūḍabha was sixteen years old, he expressed one day a desire to visit his grandparents. At first Vāsabhakhattiyā demurred at his request; but afterwards she consented to let him go, taking the precaution, however, to send the following letter ahead of him: "I am happy where I am; for the sake of my husband, say nothing to him." So Viḍūḍabha took leave of his father and, accompanied by a large retinue, set out. **(346–7)**

When the Sakyan princes learned of Viḍūḍabha's approaching visit, they decided not to render homage to him, and therefore sent away all the princes who were younger than he. Viḍūḍabha rendered homage to his grandfather and the other princes, but noticed that no one rendered homage to him. When he spoke of this it was explained to him that all those about him were his seniors; and this explanation satisfied him. One day, however, a female slave, while engaged in scrubbing the seat on which Viḍūḍabha was wont to sit, remarked, "Here 's where the son of the slave-woman Vāsabhakhattiyā sits!" A soldier happened to overhear what she said, and in a short time the remark became common gossip. When it came to the ears of Viḍūḍabha, he swore the following oath, "Just as these Sakyans now wash my bench with water, so also, when I am king, will I wash my bench with their blood." **(347–8)**

When Viḍūḍabha returned to Sāvatthi, and the King of Kosala learned that Vāsabhakhattiyā was really the daughter of a slave-woman, he was filled with rage at the King of the Sakyans, and degraded Viḍūḍabha and his mother to the position of slaves. About that time the Teacher went to visit the King of Kosala; and upon learning that the truth had leaked out, said to the king, "What does the family of the mother matter? the family of the father is the only thing worthy of consideration." Thereupon King Pasenadi restored Viḍūḍabha and Vāsabhakhattiyā to their former rank. **(348–9)**

Just at this time Bandhula, the Commander-in-chief of King Pasenadi's army, dismissed his wife Mallikā on the ground of barrenness. The Teacher bade her return to her husband, and Bandhula took her back; whereupon she conceived a child in her womb. One day the longing of pregnancy came upon her, and she said to her husband, "I long to bathe in the lotus tank of Vesālī, and to drink the water thereof." "Very well," said Bandhula. So he took his bow, which required a thousand men to string, assisted Mallikā to mount the chariot, and drove to Vesālī, entering the city by the gate erected in honor of Mahāli. Now Mahāli lived near this gate; and when he heard the rumble of Bandhula's chariot, he said to himself, "There is trouble brewing for the Licchavi princes." Now the lotus tank was guarded within and without by strong guards, and fenced in with an iron grating the meshes of which were so fine that not even birds could get through. Bandhula alighted from his chariot, drove the guards away, tore down the grating, and admitted his wife to the tank. So Mallikā bathed in the lotus tank of Vesālī, and drank the water thereof. Then Bandhula assisted her to mount the chariot, and drove back by the way he came. **(349–351)**

The guards reported Bandhula's insolence to the Licchavi princes, who were exceedingly angry, and immediately mounted their chariots, five hundred strong, and set out to capture Bandhula. Mahāli warned them that Bandhula would slay them all, but the princes paid no attention to his warning. Bandhula waited until the file of chariots was so straight that but one chariot-front appeared to view; and then, stringing his mighty bow, he let an arrow fly. The arrow passed through the body of every one of the five hundred men. Not realizing what had happened, they continued the pursuit; but Bandhula immediately stopped his chariot and cried out, "You are all dead men; I will not fight with the dead." "Do we look like dead men?" "Loosen your girdles." They did so, and the instant they did so, five hundred dead men lay on the ground. **(351-3)**

Bandhula returned to Sāvatthi with Mallikā. Sixteen times Mallikā bore twin sons to Bandhula, and all of them became mighty men. Bandhula by his upright conduct incurred the hostility of the unjust judges, who went to the king and falsely accused him of designs on the throne. Thereupon the king ordered Bandhula and his sons to proceed to the frontier and put down an insurrection, and at the same time suborned men to lie in wait for them on their return, kill them, and bring back their heads. Bandhula and his sons quickly put the marauders to flight, and were murdered on their return. News of the murder was brought to Mallikā on the morning of the day on which she had invited the Chief Disciples to be her guests. As she was entertaining the monks, one of the servants dropped a dish and broke it. Sāriputta said to her, "Heed it not." Mallikā drew from the folds of her dress the letter she had received that morning, and replied, "If I heed not the murder of my husband and two and thirty sons, I am not likely to heed the breaking of a mere dish." After the departure of the monks Mallikā addressed her sons' wives, assuring them that their husbands, having lived blameless lives, had obtained only the fruit of deeds in previous existences, and urged them to cherish no bitter feelings against the king. The king soon learned that the charges brought against Bandhula were false; whereupon he made amends to Mallikā, and at her request permitted her to return to her family, and to send back her sons' wives to theirs. **(353-5**

King Pasenadi appointed to the post of Commander-in-chief a nephew of Bandhula, Dīghakārāyaṇa by name. Dīghakārāyaṇa did not forget that Pasenadi had caused his uncle to be murdered, and waited for a chance to get even. Now at that time the Teacher was residing in a village near-by; and Pasenadi, being greatly troubled in spirit, set out with a small body-guard to pay him a visit. As Pasenadi was about to enter the Perfumed Chamber, he handed the royal insignia to Dīghakārāyaṇa, who immediately hurried back to Sāvatthi and proclaimed Viḍūḍabha king. That night Pasenadi died, and when the news was brought to Viḍūḍabha, the latter ordered the funeral rites to be performed. (355–6)

Viḍūḍabha remembered the oath he had sworn against the Sakyans, and set out with a large force, intending to kill them all. The Teacher, aware of the impending destruction of his kinsmen, seated himself under a small tree near Kapilavatthu. Viḍūḍabha was surprised to see him there, and said to him, "Why do you sit here rather than under the great banyan tree that grows in my kingdom?" "The shade of my kinsmen refreshes me," replied the Teacher. Then Viḍūḍabha knew that the Teacher had gone there to protect his kinsmen, and immediately returned to Sāvatthi. The Teacher rose and returned to Jetavana. Three times this happened. Then the Teacher, realizing that his kinsmen must needs be slain through the effect of the evil deed they committed in a previous existence when they threw poison into the water, went no more to the tree. So Viḍūḍabha went forth to slay his enemies. The Sakyans, as kinsmen of the Buddha, were unwilling to kill any of their enemies, and therefore made only a show of resistence, with the result that Viḍūḍabha destroyed them utterly, and washed his bench with their blood. (357–9)

Mahānāma, rather than eat with Viḍūḍabha, attempted suicide; but such was the effect of the merit he had accumulated, that he was translated to the palace of the Nāgas, where he remained for twelve years. Viḍūḍabha searched for him in vain, and then set out on his return journey. At nightfall Viḍūḍabha pitched his camp in the bed of the river Aciravatī; during the night a violent storm arose, the river bed was filled with a raging torrent, and Viḍūḍabha and his retinue perished in the waters. (359–360)

When the monks referred to the destruction of the Sakyans, the Teacher told them that it was the effect of their throwing poison into the river in a previous existence. When they commented on the fact that Viḍūḍabha was swept away in the height of his glory the Teacher pronounced Stanza 47, establishing many in the Fruits. (360–362)

Book IV. Story 4. Patipūjikā.

ILLUSTRATING STANZA 5 = 48.

Once upon a time, while the god Mālabhārī was amusing himself in the company of a thousand celestial nymphs in the Garden of the Thirty-three, one of the nymphs fell from that existence, and was reborn in a noble family of Sāvatthi. Remembering her former estate, she made the wish that she might be reborn as Mālabhārī's wife, and her life abounded in good works. When she married, her devotion to her husband was so conspicuous that she became known as Patipūjikā (Husband-honorer). On her death she was reborn, according to the wish she had made, as Mālabhārī's wife. It was now evening in the world of the Thirty-three. When she told the other nymphs that men lived only a thousand years, they were greatly surprised, but when she added that in spite of the shortness of human life, men were heedless and sluggish, they hardly credited her words. The Teacher, drawing a lesson from Patipūjikā's history, warned the monks of the shortness of human life, and pronounced Stanza 48, at the conclusion of which many were established in the Fruits. (362-6)

Book IV. Story 5. Kosiya, the Niggardly Treasurer.[35]

ILLUSTRATING STANZA 6 = 49.

There once lived not far from Rājagaha a treasurer named Kosiya, who was as niggardly as he was wealthy; and that was saying a great deal. So niggardly was he, in fact, that on a certain occasion he compelled his wife to carry her cooking implements up to the seventh storey of the house to prepare a cake for him, for fear that otherwise he might have to share his treat with the neighbors. The Teacher, aware of what was going on, bade Moggallāna transport the treasurer, his wife, and the cake to Jetavana. Suddenly the treasurer saw Moggallāna, poised in the air, looking in through the window. Moggallāna indicated that he wished to have something to eat. After a good deal of hesitation, the treasurer said to his wife, "Cook him just one tiny little cake, and let's get rid of him." One after another, the cakes they baked grew to an enormous size, until finally, out of sheer desperation, the treasurer presented them all to Moggallāna. The latter then preached the Law to the treasurer and his wife, dwelling on the importance of almsgiving, after which he transported them, together with the cakes, to Jetavana. The cakes provided an ample meal for the whole Congregation of Monks. After the meal the Teacher delivered his customary discourse, at the end of which the treasurer and his wife were established in the Fruit of Conversion. The treasurer then devoted his entire wealth to the religion of Buddha. The latter, referring to the subject in the course of a conversation with the monks, gave high praise to Moggallāna for his share in the conversion of the niggardly treasurer, and pronounced Stanza 49, establishing many in the Fruits.

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[35] Cf. Jā. i. 345–349.

Continuing his discourse, the Teacher informed the monks that this was not the first time Moggallāna had converted the treasurer, and then related the Illīsa Jātaka. **(366–376)**

Book IV. Story 6. Pāṭhika, the Naked Ascetic.

ILLUSTRATING STANZA 7 = 50.

The wife of a certain householder of Sāvatthi was accustomed to give food to a naked ascetic named Pāṭhika. One day she expressed a desire to go and hear the Teacher; but the ascetic, desiring to retain his place, urged her not to do so. Accordingly she decided to invite the Teacher to be her guest, and sent her young son to deliver the message. Pāṭhika found out where the boy was going, and told him to give the Teacher wrong directions, saying that in case the latter failed to come, he and the boy would have all the more to eat. The boy did as the ascetic told him; but the Teacher, knowing the way himself, came at the appointed time. The ascetic was greatly provoked, reviled his benefactor, and left the house. The Teacher, observing that the mind of his hostess was agitated, and learning the reason why, urged her to pay no attention to the sins of others, but rather to heed her own shortcomings; and pronounced Stanza 50, at the conclusion of which she was established in the Fruit of Conversion. **(376–380)**

Book IV. Story 7. Chattapāṇi, Lay Disciple.

ILLUSTRATING STANZAS 8–9 = 51–52.

Chattapāṇi was a lay disciple of Sāvatthi who had entered upon the Third Path. When King Pasenadi Kosala came to pay his respects to the Teacher, Chattapāṇi, out of respect for the Teacher, withheld homage. This irritated the king, but the Teacher justified Chattapāṇi's conduct, and the king said no more about it. One day the king saw Chattapāṇi pass through the courtyard with a parasol in his hand and sandals on his feet. He caused Chattapāṇi to be summoned; whereupon Chattapāṇi laid aside his parasol and sandals, and came into the king's presence without them. The king said, "Why did you lay aside parasol and sandals?" Chattapāṇi replied, "Because I was summoned into the presence of a king." "Oh," said the king, "so at last you know that I am a king." "I always did," replied Chattapāṇi. "Why, then, did you withhold homage from me on the day I went to see the Teacher?" "Out of respect for the Teacher." "Very well; we 'll let the past rest." The king then requested Chattapāṇi to preach the Law in the palace, but Chattapāṇi, not being a monk, declined. Then King Pasenadi sent word to the Teacher, saying, "Mallikā and Vāsabhakhattiyā of the Royal Household desire to hear the Law." The Teacher deputed Ānanda to preach the Law in the palace. Somewhat later Ānanda reported to the Teacher that Vāsabhakhattiyā, unlike Mallikā, had made little progress; whereupon the Teacher, contrasting their attitudes, pronounced Stanzas 51–52, establishing many in the Fruits. **(380–384)**

Book IV. Story 8. Visākhā.

ILLUSTRATING STANZA 10 = 53.

Visākhā was the daughter of Dhanañjaya, a treasurer of the city of Bhaddiya in the kingdom of Bengal. Dhanañjaya's father, Meṇḍaka, was one of five persons of limitless wealth living in Bimbisāra's territory. Now King Bimbisāra was a connection by marriage of King Pasenadi Kosala, and one day received a request from the latter to move one of the families of limitless wealth to the kingdom of Kosala. Since this was too great an undertaking, Bimbisāra did the next best thing, and sent Dhanañjaya. So Dhanañjaya, accompanied by his family and following, removed to the kingdom of Kosala, and settled in a place called Sāketa, seven leagues from Sāvatthi. By this time Visākhā, who was established in the Fruit of Conversion at the early age of seven, had grown to womanhood. **(384–7)**

At this time there was living in the neighboring city of Sāvatthi a young man named Puṇṇavaddhana, son of the treasurer Migāra, who had agreed to marry a girl possessed of the Five Beauties, if such could be found. Eight Brahmans devoted themselves to the task of finding him a wife, and one day noticing Visākhā, and discovering that she was possessed of the Five Beauties, they went to her father, Dhanañjaya, and asked him to give her in marriage to their master, Puṇṇavaddhana. Dhanañjaya consented, and the Brahmans hastened to inform Migāra. Thereupon Migāra the treasurer and King Pasenadi Kosala, accompanied by their retinues, paid a visit to the treasurer Dhanañjaya. In the meantime Dhanañjaya caused a magnificent trousseau to be made for his daughter, and provided her with a splendid dowry. **(387–397)**

When it was time for Visākhā to go, her father enjoined upon her the observance of Ten Injunctions, which were as follows: The in-door fire is not to be carried outside; the out-door fire is not to be carried inside; give only to him that gives; give not to him that gives not; give both to him that gives, and to him that gives not; sit happily; eat happily; sleep happily; tend the fire; honor the household divinities. Migāra happened to be sitting in the next room, and overheard all that Dhanañjaya said. Dhanañjaya then appointed eight sponsors for Visākhā, and directed them to try her in case any charges were brought against her. He then entrusted his daughter to the care of King Pasenadi and the treasurer, who returned with her to Sāvatthi. So Visākhā, arrayed in a magnificent parure, and accompanied by a splendid retinue, entered Sāvatthi in the train of the King, and immediately won the hearts of all the inhabitants. **(397–9)**

That night Visākhā's thoroughbred mare gave birth to a foal; whereupon Visākhā arose, went to the stable, and bathed the mare. When her father-in-law learned that she had left the house at night, he was much displeased, but refrained from making further inquiries. Now Migāra was much attached to a certain sect of naked ascetics, who, when they learned that a disciple of Gotama had become the wife of his son, urged Migāra to put her out of the house. Somewhat later, at the close of a day on which Migāra had entertained the naked ascetics, he overheard Visākhā remark that he was eating "stale fare." Migāra then and there ordered her out of the house. Visākhā, however, claimed the right of being tried before her eight sponsors; accordingly Migāra had the sponsors summoned, and brought three charges against his daughter-in-law: first, that she had accused him of eating what was unclean; secondly, that she had left the house at night; thirdly, that she performed the work of menials. Visākhā cleared herself of guilt on the first count by explaining that all she meant to say was that her father-in-law was living on stale merit instead of acquiring fresh merit; then she explained that she had left the house at night for no other purpose than to care for her mare; the third charge was withdrawn. **(399–402)**

Migāra then asked Visākhā to explain the hidden meaning of the Ten Injunctions. "The first," said Visākhā, "means that I must not speak of the faults of my mother-in-law, or father-in-law, or husband, to others; the second, that if I hear others speak of their faults, I must not tell them what I have heard; the third, that I should give to those only who return borrowed articles; the fourth, that I should not give to those who fail to return borrowed articles; the fifth, that I should give to anyone in needy circumstances, whether or not he is able to repay me; the next three mean that I must not sit or eat or sleep until I have first attended to the needs of my mother-in-law, father-in-law, and husband; the ninth means that I must look upon them as upon a flame of fire; the tenth, that I must look upon them as my divinities." **(402–404)**

Thereupon Migāra, finding no fault in Visākhā, asked her to pardon him. She did so, but told him that now she should leave the house of her own accord. She consented to stay, however, on the condition that she should be allowed to entertain the Buddha. On the occasion of the Teacher's first visit, Migāra and his wife were established in the Fruit of Conversion. Visākhā's life abounded in good works; and she lived to be an hundred and twenty years old. She endeavored to sell her magnificent trousseau, intending to devote the proceeds to the work of the Order; but finding that no one else was rich enough to buy it, made up the price herself, and erected a splendid monastery. The Teacher informed the monks that Visākhā's noble life was the fruit of good works performed in the dispensations of Padumuttara and Kassapa, and then pronounced Stanza 53, establishing many in the Fruits. (**404–420**)

Book IV. Story 9. The Elder Ānanda's Question.

ILLUSTRATING STANZAS 11–12 = 54–55.

Once upon a time the Elder Ānanda pondered the following thought in his mind: "The Exalted One possesses three kinds of perfumes; but each of these goes with the wind. Is there, perhaps, a kind of perfume that goes against the wind?" So he went to the Teacher and put the question to him. The Teacher replied, "Certainly there is a kind of perfume that goes against the wind." "Which kind is it?" "The perfume of good works." Then the Teacher pronounced Stanzas 54–55, at the conclusion of which many were established in the Fruits. (**420–423**)

Book IV. Story 10. Sakka bestows Alms on Mahā Kassapa.

ILLUSTRATING STANZA 13 = 56.

Sakka's five hundred wives once endeavored to obtain the privilege of bestowing alms on Mahā Kassapa, but the latter refused them the privilege, on the ground that he preferred to allow the poor to accumulate merit by so doing. When Sakka learned of this, he disguised himself as an old, broken-down weaver, transformed Wellborn into an old woman, and had no difficulty at all in persuading Kassapa to accept his alms. When Kassapa discovered that it was Sakka from whom he had accepted alms, he reproached him for deceiving him and defrauding the poor. But Sakka explained that he hoped by the performance of this and similar works of merit to make his own lustre equal to that of three other deities who had hitherto outshone him. The Teacher, becoming aware of what had happened, pronounced Stanza 56, at the conclusion of which many were established in the Fruits. (**423–430**)

Book IV. Story 11. How the Elder Godhika attained Nibbāna.

ILLUSTRATING STANZA 14 = 57.

The Elder Godhika found himself so impeded in the practice of ecstatic meditation by a disease which had attacked him that he drew a razor and cut his throat, passing at once to Nibbāna. Māra searched everywhere in hope of discovering where he had been reborn; but the Teacher informed him that he was engaged in a futile task, and pronounced Stanza 57, establishing many in the Fruits. **(431–4)**

Book IV. Story 12. Garahadinna.

ILLUSTRATING STANZAS 15–16 = 58–59.

At Sāvatthi once lived two friends, Sirigutta and Garahadinna; the former, a lay disciple of the Buddha; the latter, an adherent of the Naked Ascetics. These heretics used to say to their disciple Garahadinna, "Go and ask your friend Sirigutta why he visits the hermit Gotama, and what he expects to get out of him, and see if you can't persuade him to transfer his allegiance to us." So Garahadinna used to ask his friend Sirigutta why he visited the hermit Gotama, and what he expected to get out of him, and tried with all his might to persuade him to transfer his allegiance to the Naked Ascetics. After a time Sirigutta became very weary of hearing this sort of talk, and one day said to Garahadinna, "What do your masters know, anyway?" "Oh, sir, don't talk that way; there is nothing my masters don't know. They know all about the past, the present, and the future. They know everybody's thoughts, words, and actions. They know just what is going to happen, and just what is not going to happen." "You don't say." "Indeed I do." "Well, if that 's the case, pray convey my compliments to your masters, and tell them that I should like to have the privilege of entertaining them." The heretics at once accepted. **(434–6)**

Sirigutta had a long ditch dug, and had it filled with dung and slime. Then he had cords stretched across, rugs laid on the cords, and the seats so placed with one edge resting on the ground and the other on the cords, that the instant the heretics sat down, they would be tipped over backwards and precipitated into the mass of filth at the bottom of the ditch. In order that the rugs might not be smeared with filth, Sirigutta stationed men all along the line with orders to pull the rugs out from under when the heretics sat down. He did n't take the trouble to provide any food or drink for his guests. Thought he, "If

Garahadinna's masters really know just what is going to happen, they 'll stay away from here." (**436–7**)

But Garahadinna's masters came, just as Sirigutta expected they would. Sirigutta told them to sit down all at once, and when they did so, they were immediately tipped over backwards, and precipitated into the mass of filth at the bottom of the ditch. As they crawled out, Sirigutta's men belabored them with clubs until they were glad enough to escape with their lives. Garahadinna had Sirigutta haled before the king and asked the king to give him the full extent of the law; but when the king investigated the matter, he decided that it was Garahadinna, rather than Sirigutta, who deserved to be punished, and therefore had Garahadinna beaten soundly. (**437–9**)

Garahadinna cherished deep resentment against Sirigutta for a long time, and finally determined to serve Buddha and his monks somewhat as Sirigutta had served the Naked Ascetics. He employed much the same stratagem, except that instead of filling the ditch with filth, he had it filled with glowing coals. But the Buddha caused an enormous lotus-flower to spring up from the bed of coals, whereon he sat, surrounded by his five hundred monks. By a second miracle he created an abundant supply of food, whereof all partook. Then he pronounced Stanzas 58–59, at the end of which the multitude obtained clear comprehension of the law, and Garahadinna and Sirigutta attained the Fruit of Conversion. In the evening, referring to a similar experience he had in a previous existence, he related the Khadiraṅgāra Jātaka. (**439–447**)

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RECORDS OF MEETINGS, 1909–1910.

OFFICERS AND COMMITTEES FOR 1910–1911.

LIST OF THE FELLOWS AND FOREIGN HONORARY MEMBERS.

STATUTES AND STANDING VOTES.

RUMFORD PREMIUM.

INDEX.

(Title Page and Table of Contents.)

RECORDS OF MEETINGS.

Nine hundred ninety-first Meeting.

OCTOBER 13, 1909. — STATED MEETING.

The PRESIDENT in the chair.

There were thirty-four Fellows present.

The Corresponding Secretary announced that letters had been received from F. J. Furnivall and Hermann Jacobi, accepting Foreign Honorary Membership; from F. G. Benedict, Arthur W. Ewell, J. H. Ropes, W. W. Fenn and G. M. Lane, accepting Resident Fellowship; from W. J. Spillman, American Secretary of the Universal Scientific Association, suggesting the establishment of technical vocabularies in the international language, Esperanto, for the various sciences; from Mrs. Simon Newcomb and family, announcing the death on July 11th, 1909, of Simon Newcomb; from Harvard University, requesting the presence of a delegate at the inauguration of Abbott Lawrence Lowell, as its President; from the Nobel Prize Committees, inviting competition for the Nobel prizes of 1910; from Dr. J. Zavodny, enclosing a pamphlet of the Export-verein für Böhmen, Mähren und Schlesien, in Prag, and requesting admission into the Academy as Corresponding member; from Anaboli Pavlov, a theory of numbers (in Russian); from H. G. Wadlin, E. A. Filene and C. Bertrand Thompson, suggesting an exhibit at the "1915" Boston Exposition to be held Nov. 1–27, 1909; from Carlos A. Hesse, suggesting changes in the Calendar; from the Aero Club of America, inviting the Academy to take part in the proceedings at the presentation of medals to Messrs. Wilbur and Orville Wright, as discoverers of the art of flying; from the American Philosophical Society, requesting the Academy to co-operate with other scientific societies in urging the government of the

United States to send a vessel to explore and survey the coast of Wilkes Land and other parts of Antarctica.

The following deaths were announced by the Chair: —

John M. Ordway, Associate Fellow in Class I, Section 3; Simon Newcomb, Associate Fellow in Class I, Section 1.

On motion of E. L. Mark, it was

Voted, that a committee be appointed to investigate the question of co-operation with other scientific societies in urging the Government to send a vessel to explore the coast of Wilkes Land.

The question of an exhibit at the Boston "1915" Exposition was referred to the Librarian, with full power.

President Trowbridge gave a paper entitled "The Future of Aeroplanes."

The following papers were presented by title: —

"The Principle of Relativity and Non-Newtonian Mechanics." By Gilbert N. Lewis and Richard C. Tolman. Presented by C. R. Sanger.

"Friction in Gases at Low Pressures." By J. L. Hogg. Presented by John Trowbridge.

"The Quantitative Determination of Antimony by the Gutzeit Method." By Charles Robert Sanger and Emile Raymond Riegel.

"The Preparation and Properties of Pyrosulphuryl Chloride and Chlorsulphonic Acid." By Charles Robert Sanger, Emile Raymond Riegel and Lawrence Haines Whitney.

"A Revision of the Atomic Weight of Phosphorus." By Gregory P. Baxter and Grinnell Jones.

"The Equivalent Circuits of Composite Lines in the Steady State." By A. E. Kennelly.

"*Περὶ Φύσεως*. A Study of the Conception of Nature among the Pre-Socratics." By William A. Heidel. Presented by Morris H. Morgan.

Nine hundred ninety-second Meeting.

NOVEMBER 10, 1909.

The PRESIDENT in the chair.

There were thirty-seven Fellows present.

The Corresponding Secretary read the following letters: an invitation from the Museum of Fine Arts, to the opening of its new building; from the Secretary of the International Congress of Americanists, a notification of the 17th Congress.

The Committee on the proposed action regarding Antarctic exploration reported as follows: —

"We believe that it is fitting for governments to take part in exploration. Our government has already done it in moderate measure; other governments have done more.

We believe, also, that it is fitting for learned societies to take part in promoting government exploration by making recommendations to this end.

We find that the particular plan under consideration deserves our support, because the work proposed is well worthy of investigation; it touches a region in which our previous national exploration gave good results but left much to be done. There is abundant room for coöperative exploration in the Antarctic regions by various countries.

We therefore recommend that favorable action be taken by the Academy on the communication from the American Philosophical Society."

W. M. Davis.
A. Lawrence Rotch.

On motion of the Corresponding Secretary it was

Voted, That the Academy take favorable action on the communication from the American Philosophical Society.

The following letter from Alexander Agassiz regarding his presentation of a new building to the Academy was read by the President: —

October 16 [1909].

My dear Mr. Trowbridge,

I have at last bought the house adjoining the Academy's building on Newbury Street, — No. 26, so that on my return from the West I shall be ready to make my proposition to the Academy for their consideration and decision. The house is let for 2 years but I fancy we could obtain possession earlier. In meanwhile the architects can perfect the plans. It will be necessary while building, for the Academy to get shelter from the Historical Society or Natural History Society and to hire a room for their office for say 18 months. As the Academy will have ample room, I think we could increase the number of members by 150 or 200, which would pay for increased expense of running the building when completed. With the great increase in number of Professors at Tufts, Boston University, Institute of Technology, and Harvard, there ought to be no difficulty in filling our number. I leave for the West the 22d, not to return till Nov. 12th. In meantime, you will perhaps get one of our lawyer members to look over our Statutes, By-laws and Charter, and make out a plan for us to submit to the Members at a properly called meeting to decide on my suggestions or such modifications of them as are advisable. I propose to deliver the building complete to the Academy and hope that increase of new members will pay running expenses. The building will have

1 large Meeting Room 42 × 46 I
1 " Reading " " " " II
Janitor's quarters and bed room 3. III
Basement Hall 1, 2 rooms for Committee meetings.

The stack or shelf room of I, II and basement will give room for 10 M. additional books without a new stack.

As I go off for the winter the 13th of December, I hope we can have the meeting of the Academy before that time and appoint a committee to examine the plans and report to the Academy what action they think best for the Academy.

The location is excellent — near all electric cars, near the Natural History Society, the Institute, Tufts Medical School and Boston University, and I hope the building may become a scientific and literary club while remaining the domicile of the Academy.

Yours very truly,

A. Agassiz.

After discussion, on motion of Professor Wolff, it was

Voted, That a committee of three be appointed by the President to consider the general plan suggested by Professor Agassiz.

On motion of Professor Webster it was unanimously

Voted, That the Academy expresses its hearty thanks to Professor Agassiz for his very generous proposition.

The following communication was given by Professor Kittredge, " Moot Points about Chaucer."

Nine hundred ninety-third Meeting.

DECEMBER 8, 1909.

The PRESIDENT in the chair.

There were fifty-six Fellows present.

The Corresponding Secretary read the following letters:—from the President of the 8th International Zoological Congress, inviting delegates to the congress; from the family of Henry Charles Lea, announcing his death; from the Comité Géologique de la Russie, announcing the death of M. Serge Nikitin; from the Königlich Böhmische Gesellschaft der Wissenschaften, announcing the death of Phil. Dr. Karl Domalép.

The death of Henry Charles Lea, an Associate Fellow in Class III., Section 3, was announced by the Chair.

On motion of E. C. Pickering the following Resolution was passed: *Resolved*, That the American Academy of Arts and Sciences desires to express its entire approval of the recommendations of the President of the United States, in his annual message to Congress, regarding the administration of the Naval Observatory. The Academy believes that the scientific work of the Observatory should be under the direction of a scientific man, and that in this way its efficiency will be greatly increased.

Resolved, That a copy of these resolutions be transmitted to the President of the United States, by the Secretary.

On motion of Mr. Webster it was

Voted, To give the above Resolution to the public press.

It was suggested by the Corresponding Secretary, that Standing Vote No. 10 precluded giving the above Resolution to the public press.

On motion of Mr. Bowditch, it was

Voted, That in the opinion of the Academy, Standing Vote No. 10 does not apply to making public the vote just passed.

The President read the names of the Committee appointed at the last meeting to consider the general plan suggested by Professor Agassiz, viz.: Dr. H. P. Walcott, Professor John C. Gray, and President A. Lawrence Lowell.

The President re-read the letter of Professor Agassiz, read at the last meeting of the Academy, and after considerable discussion the following votes were passed:—

On motion of A. G. Webster, it was

Voted, That the Academy accepts with profound gratitude the very generous gift of Professor Agassiz.

Voted, That a committee on Policy be selected to consider all questions relating to the enlarged functions of the Academy.

On motion of C. P. Bowditch it was

Voted, That in the opinion of the Academy an increase of membership is desirable.

Voted, That the committee on the general plan suggested by Professor Agassiz be authorized to apply to the Legislature for an amendment to the charter which will permit such increase.

Professor Derr exhibited some lantern photographs taken in the Yellowstone National Park.

The following papers were presented by title: —

"Buddha-ghosa's Commentary on the Dhammapada, an Analysis of the First Four Books of the Buddhist *Acta Sanctorum* in Pali, with an Index to the 304 Stories of the Burmese Edition." By Eugene Watson Burlingame. Presented by C. R. Lanman.

"The Effect of Leakage at the Edges upon the Conduction of Heat in a Homogeneous Lamina." By B. O. Peirce.

"The Resistance of the Air to a Swinging Magnet." By B. O. Peirce.

"The Differentiation of Scalar Point Functions with Respect to Other Similar Functions." By B. O. Peirce.

"The Spectrum of a Compound of Carbon in the Region of Extremely Short Wave-Lengths." By Theodore Lyman.

"Average Chemical Compositions of Igneous-Rock Types." By Reginald A. Daly.

"Experiments on the Electrical Oscillations in a Hertz Rectilinear Oscillator." By George W. Pierce.

"On the Applicability of the Law of Corresponding States to the Joule-Thomson Effect in H_2O and CO_2." By Harvey N. Davis. Presented by John Trowbridge.

"Notes on Certain Thermal Properties of Steam." By Harvey N. Davis. Presented by John Trowbridge.

"Discharge of Electricity through Gases." By John Trowbridge.

"Measurement of Pressure and Density in Gases with the Micro Balance." By H. W. Morse. Presented by John Trowbridge.

"Some Minute Phenomena of Electrolysis." By H. W. Morse. Presented by John Trowbridge.

"The Reactions of Amphibians to Light." By Arthur Sperry Pearse. Presented by E. L. Mark.

Nine hundred ninety-fourth Meeting.

JANUARY 12, 1910. — STATED MEETING.

The PRESIDENT in the chair.

There were twenty-nine Fellows present.

In the absence of the Corresponding Secretary, the President read the following: — a letter from B. Beernaert, Minister of State of Belgium, sending three hundred and seventy-five copies of a manifesto against criticism of Belgium concerning its African possessions; circulars from the committee of the Third International Congress of Botany to be held at Brussels, May 14–22, 1910; a circular from the Committee of the First International Congress of Entomology, to be held at Brussels, August 1–6, 1910; from the Museo Nacional, Mexico, sending the felicitations of the new year; an announcement from the Société d'Emulation d'Abbeville, of the death of M. P.-C.-E. Prarond; an announcement from the Société Royale Norvégienne des Sciences of Trondhjem of the death of M. M. H. Foslie.

The following deaths were announced: — James Barr Ames, Resident Fellow in Class III., Section 1; F. W. Maitland, Foreign Honorary Member in Class III., Section 1.

The following gentlemen were elected members of the Academy: —

Arthur Fairbanks, of Boston, as Resident Fellow in Class III., Section 4.

William Arthur Heidel, of Middletown, as Associate Fellow in Class III., Section 2.

On motion of B. L. Robinson, it was

Voted, That Professor W. G. Farlow be appointed delegate to the International Botanical Congress to be held at Brussels, May 14 to 22, 1910.

On motion of C. R. Cross, it was

Voted, To appropriate the sum of five hundred dollars ($500) from the unexpended balance of the income of the Rumford Fund, to be applied at the discretion of the Committee.

The President announced that, in pursuance of the vote at the last meeting of the Academy, he appointed the following gentlemen a Committee on Policy, to consider all questions relating to the enlarged functions of the Academy: Messrs. Webster, Rotch, Ernst, Lyman, Walcott, W. M. Davis and Trowbridge.

The following communication was given by Dr. D. G. Lyon: "Harvard Explorations in Samaria."

The following papers were presented by title: —

"Air Resistance to Falling Inch Spheres." By Edwin H. Hall.

"Contributions from the Gray Herbarium of Harvard University. New Series No. XXXVIII." I. A preliminary synopsis of the Genus Echeandia. By C. A. Weatherby. II. Spermatophytes, new or reclassified, chiefly Rubiaceae and Gentianaceae. By B. L. Robinson. III. American Forms of Lycopodium complanatum. By C. A. Weatherby. IV. New and little known Mexican Plants, chiefly Labiatae. By M. L. Fernald. V. Mexican Phanerogams — Notes and New Species. By C. A. Weatherby. Presented by B. L. Robinson.

Nine hundred ninety-fifth Meeting.

FEBRUARY 9, 1910.

The PRESIDENT in the chair.

There were present thirty-four Fellows.

The Corresponding Secretary read the following: — a circular from the American Philosophical Society, with Resolutions adopted, urging upon Congress the establishment of a National Bureau of Seismology; a letter from Arthur Fairbanks, accepting Resident Fellowship; a letter from William A. Heidel, accepting Associate Fellowship; a circular from the Königlich Böhmische Gesellschaft, announcing the death of Dr. Ottokar Hostinsky; two letters from the "Boston 1915 Committee"; a letter from A. Biddlecombe, showing "proof of the truth of his theory that electricity is material motion in a special condition, etc."; a circular from the President and Fellows of Harvard University, announcing the inauguration of Abbott Lawrence

Lowell as President; a letter from President Taft in answer to Resolutions forwarded to him by the Academy relative to the Naval Observatory, requesting that copies of the Resolutions be sent to the President of the Senate and the Speaker of the House of Representatives.

On motion of Professor Webster it was

Voted, To send copies of the Resolutions regarding the Naval Observatory to the President of the Senate and the Speaker of the House of Representatives.

Voted, That the Librarian be appointed a delegate to the Boston 1915 Directorate conference to be held March 3, 1910.

Professor W. M. Davis gave a paper entitled: —

"The Italian Riviera Levante: a study in Geographical Description."

Dr. Percival Lowell exhibited and described transparencies of photographs of Mars and Saturn, taken at the Lowell Observatory.

The following papers were presented by title: —

"Evaporation from the Surface of Small Solid Spheres." By H. W. Morse. Presented by John Trowbridge.

"On the Equilibrium of the System Consisting of Lime, Carbon, Calcium Carbide, and Carbon Monoxide." By M. de Kay Thompson. Presented by H. M. Goodwin.

"A Study of the Greek Epigram before 300 B. C." By Florence Alden Gragg. Presented by H. W. Smyth.

Nine hundred ninety-sixth Meeting.

MARCH 9, 1910. — STATED MEETING.

The PRESIDENT in the chair.

There were thirty-nine Fellows present.

The Corresponding Secretary read the following: — a letter from William H. Niles, resigning Fellowship; a letter from H. G. Chase, announcing the death of Professor A. E. Dolbear; a letter from Dr. Edward Kohlrausch, announcing the death of W. F. Kohlrausch; a circular from Senator Augusto Righi, President of the Royal Academy of Science, Bologna, announcing the competition for the Elia De Cyon prize in 1911; a circular from G. Spiller, Secretary, announcing the Universal Race Congress to be held in London in July, 1911; a circular from Signor Vito Volterra, announcing the publication of the mathematical works of Count Julius Charles of Fagnano.

The following deaths were announced by the Chair: —

William Sellers, Associate Fellow in Class I., Section 4; Samuel W. Johnson, Associate Fellow in Class I., Section 3; William Frederick Kohlrausch, Foreign Honorary Member in Class I., Section 2.

The President announced that the Massachusetts Legislature had complied with the request of the Academy and had passed the following amendment to the Charter of the Academy: —

[Chapter 129.]

COMMONWEALTH OF MASSACHUSETTS.

In the year One Thousand Nine Hundred and Ten.

An Act relative to the American Academy of Arts and Sciences.

Be it enacted by the Senate and House of Representatives in General Court assembled and by the authority of the same, as follows: —

SECTION 1. Section four of chapter forty-six of the acts of the year seventeen hundred and seventy-nine, passed May fourth, seventeen hundred and eighty, which incorporated the American Academy of Arts and Sciences, is hereby amended by striking out in the proviso at the end of said section, the word "two" before the word "hundred," and inserting in place thereof the word: — three, — so as to read as follows: — SECTION 4. That the fellows of the said academy, may from time to time, elect such persons to be fellows thereof, as they shall judge proper; and that they shall have full power and authority from time to time to suspend, expel, or disfranchise, any fellow of the said academy, who shall by his conduct render himself unworthy of a place in that body, in the judgment of the academy; and also to settle and establish the rules, forms, and conditions of election, suspension, expulsion, and disfranchisement: provided, that the number of the said academy who are inhabitants of this state, shall not at any one time, be more than three hundred, nor less than forty.

SECTION 2. Said chapter forty-six is hereby further amended by striking out Section six and inserting in place thereof the following: — SECTION 6. That the fellows of the said academy may, and shall, forever, hereafter, be deemed capable, in the law, of having, holding, and taking, in fee simple or any less estate, by gift, grant, devise, or otherwise, any lands, tenements, or other estate, real and personal: provided, that the said real estate shall not exceed in value the sum of one hundred thousand dollars, and the said personal estate shall not exceed in value the sum of three hundred thousand dollars; all the sums mentioned in the preceding section of this act to be valued in silver, at the rate of six shillings and eight pence by the ounce; and the annual interest and income of the said real and personal estate, together with the fines and penalties aforesaid, shall be appropriated for premiums, to encourage improvements and discoveries in agriculture, arts, and manufactures, or for other purposes consistent with the end and design of the institution of the said academy, as the fellows thereof shall determine.

SECTION 3. This act shall take effect upon its passage.

HOUSE OF REPRESENTATIVES, February 25, 1910.

Passed to be enacted. JOSEPH WALKER, *Speaker.*
In Senate, February 28, 1910.
Passed to be enacted. ALLEN T. TREADWAY, *President.*
February 28, 1910.
Approved. EBEN S. DRAPER.

Office of the Secretary,
BOSTON, March 1, 1910.

A true copy.

Witness the Great Seal of the Commonwealth.
ISAAC H. EDGETT,
[Seal.] *Deputy and Acting Secretary of the Commonwealth.*

The following gentlemen were elected Members of the Academy: —

Clifford Herschel Moore, of Cambridge, as Resident Fellow in Class III., Section 2 (Philology and Archaeology).

Charles Pomeroy Parker, of Cambridge, as Resident Fellow in Class III., Section 2 (Philology and Archaeology).

Voted, That the sum of four hundred dollars from the unexpended balance of the appropriation for publication from the income of the Rumford fund be transferred to the amount available for use at the discretion of the Committee.

The Chair appointed the following Councillors to serve as Nominating Committee: —

John E. Wolff, of Class II.

Henry P. Talbot, of Class I.

George L. Kittredge of Class III.

The following gentlemen were appointed a Committee to revise the Statutes: —

Charles R. Lanman.

Charles R. Cross.

Frederic J. Stimson.

The following communication was given: —

"Some New Factors in Determining the Location of Wineland the Good." By M. L. Fernald.

Mr. Henry H. Edes gave an account of "Some Lacunae in the Archives of the Academy." These Lacunae were letters written to the Academy by the following: —

George Washington, March 22, 1781; Count Rumford, February 15, 1797; Marquis de Chastellux, five letters in 1781 and '82; Chevalier de la Luzerne, March 20, 1781; Peter Wargentin, March 20, 1781; Marquis de Marbois, May 20, 1781; Richard Price, three letters in 1781 and '83; J. J. L. Delalande, November 30, 1781; J. L. D'Alembert, December 11, 1781; Leonardus Euler, March 11, 1782; Count de Gébelin, June 24, 1782; E. S. Jeaurat, three letters in 1782 and '83; Thomas Brand Hollis, two letters in 1783; Joseph Priestley, June 23, 1785; John C. Lettsom, February 1, 1793; J. F. Blumenbach, November 29, 1795; Nathaniel Bowditch, August 28, 1797, and were procured from the descendants of Joseph Willard, former Secretary and Vice President of the Academy.

On motion of Professor Webster, it was

Voted, That in view of the unusual nature of the Communication, the Academy depart from its usual custom of not expressing an opinion on communications presented to it, and give a hearty vote of thanks to Mr. Edes for his success in restoring to the Academy these valuable documents.

It was then

Voted, That the thanks of the Academy be given to the descendants of Dr. Joseph Willard, its first Corresponding Secretary, for restoring to its files a collection of papers, mostly letters accepting Fellowship in the Academy.

Nine hundred ninety-seventh Meeting.

APRIL 13, 1910.

Vice-President THOMSON in the chair.

There were thirty-six Fellows and three guests present.

The Corresponding Secretary read the following: — letters from Charles P. Parker and Clifford H. Moore, accepting Resident Fellowship; a letter from V. M. Slipher, accepting Associate Fellowship; a letter from John Ritchie, Jr., resigning Fellowship; a card from the Historical Society of Pennsylvania, requesting the presence of the President at the opening of the New Hall of the Society; a letter and circulars from the Argentine Scientific Society, concerning the International American Scientific Congress to be held in Buenos Aires in July, 1910, commemorating the Centenary of the Revolution of May, 1810; circulars of the World's Congress of International Associations to be held under the patronage of the Belgian government, in May, 1910; circulars of the eleventh International Geological Congress and the second International Agrogeological Conference to be held in Stockholm in 1910; a circular from the Boston-1915 Director, announcing the publication of "The Chronicle of Boston-1915."

The Chair announced the death of Alexander Agassiz, Resident Fellow in Class II., Section 3, and President of the Academy from 1895 to 1903; of Morris Hicky Morgan, of Class III., Section 2; and of William Graham Sumner, Associate Fellow in Class III., Section 3.

On motion of the Corresponding Secretary, it was

Voted, To refer the appointment of delegates to the three International Congresses, to the President.

Vice-President Thomson announced that the Rumford Premium had been awarded to Professor Robert Williams Wood for his discoveries in light, and particularly for his researches on the optical properties of sodium and other metallic vapors.

The two medals were then presented to Professor Wood, who expressed his appreciation of the honor conferred upon him. He then gave an address on "Photography with Invisible Rays."

Nine hundred ninety-eighth Meeting.

May 11, 1910. — Annual Meeting.

The President in the chair.

Thirty-six Fellows and one guest present.

The Corresponding Secretary read the following: — a notice of the death of Alexander Agassiz from The Faculty of the Museum of Comparative Zoölogy; a notice from the clerk of the Probate Court of the City of Newport, that the Academy is named as a beneficiary under the will of Alexander Agassiz; a circular from the Association des Ingénieurs Electriciens sortis de l'Institut électrotechnique Montefiore, giving the conditions of a triennial prize; a circular announcing papers to be given at the 17th Congress of Americanists at Buenos Aires; a circular from the Secretary of the International Hygiene exhibition to be held in Dresden, 1911.

The following report of the Council was read: —

Since the last report of the Council the deaths of eleven members have been noted: three Resident Fellows, — James Barr Ames, Morris Hicky Morgan, Alexander Agassiz; six Associate Fellows, — John Morse Ordway, Simon Newcomb, Henry Charles Lea, William Sellars, Samuel William Johnson, William Graham Sumner; two Foreign Honorary Members, — Frederic William Maitland, Wilhelm Friedrich Kohlrausch.

Two Resident Fellows have resigned

New members elected are: Resident Fellows, 8; Associate Fellows, 2; Foreign Honorary Members, 2.

The roll of the Academy therefore now includes 191 Resident Fellows, 84 Associate Fellows, and 61 Foreign Honorary Members.

The annual report of the Treasurer was read, of which the following is an abstract: —

General Fund.

Receipts.

Balance, April 30, 1909	$507.57	
Investments	2,043.63	
Assessments	1,870.00	
Admission fees	80.00	
Rent of offices	756.58	
Sale of book plates	265.00	$5,522.98

Expenditures.

Expense of House	$1,358.10	
Expense of Library	2,691.59	
Expense of Meetings	141.54	
Treasurer	131.75	
Income transferred to principal	224.38	$4,547.35
Balance, April 30, 1910		975.83
		$5,522.98

Rumford Fund.

Receipts.

Balance, April 30, 1909	$2,155.19	
Investments	2,956.43	
Sale of publications	19.00	
Unexpended balance returned	55.95	$5,186.57

Expenditures.

Research	$2,375.00	
Periodicals and binding	232.75	
Books and binding	50.69	
Publication	388.48	
Medals	350.00	
Sundries	287.80	
Income transferred to principal	142.54	$3,827.26
Balance April 30, 1910		1,359.31
		$5,186.57

C. M. Warren Fund.

Receipts.

Balance, April 30, 1909	$544.95	
Investments	406.41	$951.36

Expenditures.

Research	$100.00	
Vault rent (part)	4.00	
Income transferred to principal	13.88	
Charged to reduce premium on bonds	50.00	$167.88
Balance, April 30, 1910		783.48
		$951.36

PUBLICATION FUND.

Receipts.

Balance, April 30, 1909	$692.99	
Appleton Fund investments	618.82	
Centennial Fund investments	2,312.17	
Sale of publications	402.43	$4,026.41

Expenditures.

Publication	$2,545.02	
Vault rent (part)	12.50	
Income transferred to principal	146.94	$2,704.46
Balance, April 30, 1910		1,321.95
		$4,026.41

The following reports were also presented: —

REPORT OF THE LIBRARIAN.

The work of cataloguing the library has progressed during the past year, and is now almost completed. Two alcoves of Society publications, half the dictionaries and the bibliography, only, remain uncatalogued.

There is now no more room for books in the stack-building, and if we remain in this house, shelving must be put up in the house — which is not fire-proof — or another story must be added to the stack-building.

The number of bound volumes in the library at the last report was 29,911. 1105 volumes have been added during the past year, making the number of bound volumes now on the shelves 31,016. The number of volumes added includes 990 gifts and exchanges, 70 purchased by the General Fund, and 48 by the Rumford Fund.

86 volumes have been borrowed from the library by 30 persons, including 19 Fellows.

All books borrowed during the year have been returned, except 11, 5 of which were borrowed within two weeks; and of the 8 remaining out at the last report, all have been returned except 3.

The expenses charged to the library are as follows: Miscellaneous, $506.70 (which includes $153.13 for cataloguing); Binding, $738.25 General, and $84.55 Rumford, Funds; Periodical subscriptions, $446.64 General, and $164.68 Rumford, Funds; making a total of $1184.89 for the General, and $249.23 for the Rumford, Funds, as the cost of subscriptions and binding.

Of the appropriation of $50 from the Rumford Fund, plus $68.86, the unexpended balance from last year, $50.69 has been paid for Books and binding.

A. Lawrence Rotch, *Librarian.*

May 11, 1910.

Report of the Rumford Committee.

The following grants in aid of researches on light and heat have been made by the Rumford Committee during the year 1909–10: —

June 9, 1909. Professor W. W. Campbell of the Lick Observatory, for the purchase of certain parts of a quartz spectrograph	$300
Professor M. A. Rosanoff, of Clark University, in further aid of his research on the fractional distillation of binary mixtures	200
October 13, 1909. Professor L. R. Ingersoll, of the University of Wisconsin, for the continuation of his work on the optical constants of metals, additional	300
December 8, 1909. Professor Joel Stebbins, of the University of Illinois, in further aid of his researches with the selenium photometer	300
Professor W. W. Campbell, of the Lick Observatory, in furtherance of his researches on the polariscope study of the solar corona by means of a Hartmann photometer, additional	125
February 9, 1910. Professors C. E. Mendenhall, of the University of Wisconsin, and Augustus Trowbridge, of Princeton University, in aid of their research on ether drift upon the intensity of radiation	250
Professor C. E. Mendenhall, in furtherance of a research on free expansion of gases, additional	250
Mr. Frank W. Very, for the purchase of photographic glass plates of the spectrum from George Higgs, London, a sum not to exceed	50
Professor M. De K. Thompson, of the Massachusetts Institute of Technology, in aid of his research on the high temperature equilibrium of the system of materials employed industrially in the carbide process for the fixation of atmospheric nitrogen	100

It was voted on February 9, 1910, that the sum of $250 be granted to Professor Gilbert N. Lewis, in aid of the preparation of abstracts of publications on light and heat for the forthcoming International Physico-chemical Tables.

On March 9, 1910, it was voted to appropriate the sum of $100 for the purchase and binding of periodicals for the library, a considerable number of back volumes of several periodicals including a complete set of the Physikalische Zeitschrift having been secured : this sum to be paid from the amount available for use at the discretion of the Committee.

The following papers have been published in the Proceedings of the Academy during the present year at the expense of the Rumford Fund.

Vol. 45, No. 8. "On the Applicability of the Law of Corresponding States to the Joule-Thomson Effect in Water and Carbon Dioxide." By Harvey N. Davis.

Vol. 45, No. 9. "Notes on Certain Thermal Properties of Steam." By Harvey N. Davis.

Vol. 45, No. 10. "The Spectrum of a Carbon Compound in the Region of Extremely Short Wave-Lengths." By Theodore Lyman.

Vol. 45, No. 18. "On the Equilibrium of the System consisting of Lime, Carbon, Calcium Carbide and Carbon Monoxide." By Maurice De K. Thompson.

Reports of the progress of researches which have been aided by grants from the Rumford Fund have been received from Messrs P. W. Bridgman, W. W. Campbell, A. L. Clark, W. J. Fisher, E. B. Frost, L. R. Ingersoll, N. A. Kent, F. E. Kester, C. E. Mendenhall, R. S. Minor, J. A. Parkhurst, M. A. Rosanoff, F. A. Saunders, J. Stebbins, F. A. Very.

At a meeting of the Committee held on February 9th, it was unanimously voted for the first time, and at a meeting held on March 9th, for the second time, to recommend to the Academy, the award of the Rumford Premium to Charles Gordon Curtis for his improvements in the utilization of heat as work in the steam-turbine.

Charles R. Cross, *Chairman.*

May 11, 1910.

Report of the C. M. Warren Committee.

The C. M. Warren Committee beg leave to report that grants have been made during the past year to the following persons, in aid of the researches specified : —

Dr. J. Elliott Gilpin, Johns Hopkins University, for the study of the nature and source of petroleum	$100
Dr. E. W. Washburn, University of Illinois, for the construction of an adiabatic calorimeter for the measurement of heats of dilution and of solution	150

The research by Professor A. W. Foote, of Yale University, on the "Nature of Precipitated Colloids," in aid of which a grant of $300 was made by the Warren Committee in 1909, has been published.

Reports of progress have been received from Dr. Frederic Bonnet, Jr., and from Dr. J. Elliott Gilpin in regard to researches for which money has been contributed from the Warren Fund, and the results of both these investigations it is hoped will be published during the coming year.

LEONARD P. KINNICUTT, *Chairman.*

May 11, 1910.

REPORT OF THE PUBLICATION COMMITTEE.

Between May 1, 1909, and May 1, 1910, there were published nine numbers of Volume XLIV. (Nos. 18–26) and fifteen numbers of Volume XLV. of the Proceedings. In Volume XLIV. there were included two biographical notices. The total publication amounted to 714 + v pages, with four plates, of which three numbers (Nos. 8, 9, 10 of Volume XLV.) have been paid for by the income of the Rumford Fund.

Five numbers of the Proceedings are in press, of which one number (No. 18) has been authorized by the Rumford Committee to be published at the expense of the Rumford Fund.

There was available for the use of the Committee on Publication an unexpended balance from last year of $110.96, an appropriation of $2500, and an amount of $378.55 from the sale of publications up to March 4, 1910, — in all $2989.51 from the Publication Fund. Bills against this fund to the amount of $2545.02 have been approved by the Chairman of the Committee, and have been submitted to the Treasurer. This leaves an unexpended balance of $444.49.

Bills aggregating $388.48, incurred in publishing Rumford papers, have been forwarded to the Rumford Committee.

G. W. PIERCE, *Acting Chairman.*

May 11, 1910.

REPORT OF THE HOUSE COMMITTEE.

During the year 1909–10 the House has been occupied as heretofore with the exception of the first floor, which has been vacant since

November 17th. It has not been let because tenants could not be given a lease of any length of time.

On the first of May, 1909, there was a balance of $109.45 to the credit of the House Expenses appropriation, and at the annual meeting of May 12, 1900, $1450 was appropriated, making an amount of $1559.45 for use during the year.

Of this amount, $1358.10 has been expended for current expenses, leaving a balance of $201.36 toward the expenses of the coming year.

The woodwork on the outside of the house should be painted, and the windows re-puttied and painted, if the building is to be occupied another winter. In anticipation of the gift of Mr. Agassiz, this was not done last autumn as it should have been.

WILLIAM R. WARE, *Chairman.*

May 11, 1910.

FINANCIAL REPORT OF THE COUNCIL.

The income for the year 1910–11, as estimated by the Treasurer, is as follows: —

GENERAL FUND	Investments	$1,660.95	
	Assessments	1,800.00	$3,460.95
PUBLICATION FUND	Appleton Fund	$614.82	
	Centennial Fund	2,312.17	$2,926.99
RUMFORD FUND	Investments		$2,888.18
WARREN FUND	Investments		$329.78

The above estimates, less 5 per cent to be added to the capital, leave an income available for appropriation as follows: —

General Fund	Income	$3,287.90	
	Unappropriated, 1909–10	440.68	
	Unexpended approp'tion, 1909–10	534.95	$4,263.53
Publication Fund			2,780.64
Rumford Fund			2,743.77
Warren Fund			313.29

The following appropriations are recommended: —

GENERAL FUND.

House expenses	$1,200	
Library expenses	1,400	
Books, periodicals, and binding	900	
Expenses of meetings	150	
Treasurer's office	150	$3,800

PUBLICATION FUND.

Publication		$2,500

RUMFORD FUND.

Research	$1,000	
Periodicals and binding	150	
Books and binding	50	
Publication	700	
To be used at discretion of Committee	800	
To be used at discretion of Committee, the unexpended balance of 1909–10	350	$3,050

WARREN FUND.

Research		$300

In accordance with the recommendation in the foregoing report it was

Voted, To appropriate for the purposes named the following sums: —

From the income of the General Fund, $3800.
From the income of the Publication Fund, $2500.
From the income of the Rumford Fund, $3050.
From the income of the Warren Fund, $300.

On motion of the Treasurer, it was

Voted, That the assessment for the ensuing year be ten dollars ($10).

On the recommendation of the Rumford Committee, it was

Voted, To award the Rumford Premium to Charles Gordon Curtis for his improvements in the utilization of heat as work in the steam-turbine.

The annual election resulted in the choice of the following officers and committees: —

JOHN TROWBRIDGE, *President.*
ELIHU THOMSON, *Vice-President for Class I.*
HENRY P. WALCOTT, *Vice-President for Class II.*
JOHN C. GRAY, *Vice-President for Class III.*
EDWIN H. HALL, *Corresponding Secretary.*
WILLIAM WATSON, *Recording Secretary.*
CHARLES P. BOWDITCH, *Treasurer.*
A. LAWRENCE ROTCH, *Librarian.*

Councillors for Three Years.

Hammond V. Hayes, of Class I.
Merritt L. Fernald, of Class II.
Henry H. Edes, of Class III.

Finance Committee.

John Trowbridge,
Eliot C. Clarke,
Francis Bartlett.

Rumford Committee.

Charles R. Cross,
Edward C. Pickering,
Erasmus D Leavitt,
Arthur G. Webster,
Elihu Thomson,
Theodore W. Richards,
Louis Bell.

C. M. Warren Committee.

Leonard P. Kinnicutt,
Henry P. Talbot,
Charles R. Sanger,
Theodore W. Richards,
Arthur A. Noyes,
George D. Moore,
James F. Norris.

The following Standing Committees were chosen: —

Publication Committee.

George W. Pierce, of Class I.
Walter B. Cannon, of Class II.
Albert A. Howard, of Class III.

Library Committee.

Harry M. Goodwin, of Class I.
Samuel Henshaw, of Class II.
Henry W. Haynes, of Class III.

Auditing Committee.

Henry H. Edes,
Frederic J. Stimson.

House Committee.

Arthur G. Webster, A. Lawrence Rotch,
Louis Derr.

The following gentlemen were elected Fellows of the Academy: —

Roland Burrage Dixon, of Cambridge, as Resident Fellow in Class III., Section 2 (Philology and Archæology).

Archibald Cary Coolidge, of Boston, as Resident Fellow in Class III., Section 3 (Political Economy and History).

Worthington Chauncey Ford, of Boston, as Resident Fellow in Class III., Section 3 (Political Economy and History).

Edward Caldwell Moore, of Cambridge, as Resident Fellow in Class III., Section 4 (Literature and the Fine Arts).

Sir David Gill, of London, as Foreign Honorary Member in Class I., Section 1 (Mathematics and Astronomy).

At his request, Robert Wheeler Willson, of Cambridge, Resident Fellow in Class I., Section 2, was transferred to Class I., Section 1.

On motion of E. C. Pickering the nominations for Associate Fellowship were referred back to the Council.

Professor Robert W. Willson gave a communication on Halley's Comet.

The following papers were presented by title: —

"On the Magnitude of an Error which usually Affects the Results of Magnetic Tests upon Iron and Steel Rings." By B. O. Peirce.

"The Effects of Sudden Changes in the Inductances of Certain Forms of Electric Circuits and their Mechanical Analogies." By B. O. Peirce.

"The Influence of the Magnetic Characteristics of the Iron Core of an Induction Coil upon the Manner of Establishment of a Steady Current in the Primary Circuit." By B. O. Peirce.

"The Effect of the Damping due to the Surrounding Medium upon the Form of the Oscillations of a Swinging Body." By B. O. Peirce.

"Some Illustrations of the Effects of Sudden Changes in the Resistances of Inductive Circuits." By B. O. Peirce.

"The Forms of the Magnetic Diagrams for Low Fields of Certain very Pure Kinds of Soft Iron which at very High Excitations show extraordinarily Large Values of I." By B. O. Peirce.

"The Reactions of Earthworms to Acids." By S. H. Hurwitz. Presented by E. L. Mark.

"On the Electromagnetic and the Thermomagnetic Effects in Soft Iron." By Edwin H. Hall and L. L. Campbell.

American Academy of Arts and Sciences

OFFICERS AND COMMITTEES FOR 1910–11.

PRESIDENT.

JOHN TROWBRIDGE.

VICE-PRESIDENTS.

Class I.	Class II.	Class III.
ELIHU THOMSON,	HENRY P. WALCOTT,	JOHN C. GRAY.

CORRESPONDING SECRETARY.

EDWIN H. HALL.

RECORDING SECRETARY.

WILLIAM WATSON.

TREASURER.

CHARLES P. BOWDITCH.

LIBRARIAN.

A. LAWRENCE ROTCH.

COUNCILLORS.

Class I.	Class II.	Class III.
WILLIAM L. HOOPER,	HAROLD C. ERNST, *Terms expire* 1911.	FREDERIC J. STIMSON.
WILLIAM R. LIVERMORE,	THEOBALD SMITH, *Terms expire* 1912.	CHARLES R. LANMAN.
HAMMOND V. HAYS,	MERRITT L. FERNALD, *Terms expire* 1913.	HENRY H. EDES.

COMMITTEE OF FINANCE.

JOHN TROWBRIDGE, ELIOT C. CLARKE, FRANCIS BARTLETT.

RUMFORD COMMITTEE.

CHARLES R. CROSS, *Chairman*,

ERASMUS D. LEAVITT, EDWARD C. PICKERING, ELIHU THOMSON,
ARTHUR G. WEBSTER, THEODORE W. RICHARDS, LOUIS BELL.

C. M. WARREN COMMITTEE.

LEONARD P. KINNICUTT, *Chairman*,

HENRY P. TALBOT, THEODORE W. RICHARDS, GEORGE D. MOORE,
CHARLES R. SANGER, ARTHUR A. NOYES, JAMES F. NORRIS.

COMMITTEE OF PUBLICATION.

GEORGE W. PIERCE, of Class I, *Chairman*,

WALTER B. CANNON, of Class II, ALBERT A. HOWARD, of Class III.

COMMITTEE ON THE LIBRARY.

A. LAWRENCE ROTCH, *Chairman*,

HARRY M. GOODWIN, of Class I, SAMUEL HENSHAW, of Class II,
HENRY W. HAYNES, of Class III.

AUDITING COMMITTEE.

HENRY H. EDES, FREDERIC J. STIMSON.

HOUSE COMMITTEE.

ARTHUR G. WEBSTER, *Chairman*.

A. LAWRENCE ROTCH, LOUIS DERR.

LIST

OF THE

FELLOWS AND FOREIGN HONORARY MEMBERS.

(Corrected to July 20, 1910.)

RESIDENT FELLOWS. — 195.

(Number limited to two hundred.)

CLASS I. — *Mathematical and Physical Sciences.* — 79.

SECTION I. — *Mathematics and Astronomy.* — 13.

Solon Irving Bailey	Cambridge
William Elwood Byerly	Cambridge
Seth Carlo Chandler	Wellesley Hills
Percival Lowell	Boston
Edward Charles Pickering	Cambridge
William Henry Pickering	Cambridge
Arthur Searle	Cambridge
William Edward Story	Worcester
Henry Taber	Worcester
Harry Walter Tyler	Boston
Oliver Clinton Wendell	Cambridge
Robert Wheeler Willson	Cambridge
Paul Sebastian Yendell	Dorchester

SECTION II. — *Physics.* — 27.

Alexander Graham Bell	Washington
Louis Bell	Boston
Clarence John Blake	Boston
Francis Blake	Weston
George Ashley Campbell	New York
Harry Ellsworth Clifford	Newton
Charles Robert Cross	Brookline

Louis Derr	Brookline
Alexander Wilmer Duff	Worcester
Arthur Woolsey Ewell	Worcester
Harry Manley Goodwin	Roxbury
Edwin Herbert Hall	Cambridge
Hammond Vinton Hayes	Cambridge
William Leslie Hooper	Somerville
William White Jacques	Newton
Frank Arthur Laws	Boston
Henry Lefavour	Boston
Theodore Lyman	Brookline
Charles Ladd Norton	Boston
Benjamin Osgood Peirce	Cambridge
George Washington Pierce	Cambridge
Abbott Lawrence Rotch	Boston
Wallace Clement Sabine	Boston
John Stone Stone	Boston
Elihu Thomson	Swampscott
John Trowbridge	Cambridge
Arthur Gordon Webster	Worcester

SECTION III. — *Chemistry.* — 21.

Gregory Paul Baxter	Cambridge
Arthur Messinger Comey	Chester, Pa.
James Mason Crafts	Boston
Charles William Eliot	Cambridge
Henry Fay	Boston
Charles Loring Jackson	Cambridge
Walter Louis Jennings	Worcester
Leonard Parker Kinnicutt	Worcester
Gilbert Newton Lewis	Boston
Charles Frederic Mabery	Cleveland
George Dunning Moore	Worcester
James Flack Norris	Boston
Arthur Amos Noyes	Boston
Robert Hallowell Richards	Jamaica Plain
Theodore William Richards	Cambridge
Charles Robert Sanger	Cambridge
Stephen Paschall Sharples	Cambridge
Francis Humphreys Storer	Boston
Henry Paul Talbot	Newton
William Hultz Walker	Newton
Charles Hallet Wing	Boston

SECTION IV. — *Technology and Engineering.* — 18.

Comfort Avery Adams	Cambridge
Alfred Edgar Burton	Boston
Eliot Channing Clarke	Boston
Heinrich Oscar Hofman	Jamaica Plain
Ira Nelson Hollis	Cambridge
Lewis Jerome Johnson	Cambridge
Arthur Edwin Kennelly	Cambridge
Gaetano Lanza	Boston
Erasmus Darwin Leavitt	Cambridge
William Roscoe Livermore	New York
Hiram Francis Mills	Lowell
Cecil Hobart Peabody	Brookline
Andrew Howland Russell	Paris
Albert Sauveur	Cambridge
Peter Schwamb	Arlington
Henry Lloyd Smyth	Cambridge
George Fillmore Swain	Boston
William Watson	Boston

CLASS II. — *Natural and Physiological Sciences.* — 60

SECTION I. — *Geology, Mineralogy, and Physics of the Globe.* — 16.

Henry Helm Clayton	Milton
Algernon Coolidge	Boston
William Otis Crosby	Jamaica Plain
Reginald Aldworth Daly	Cambridge
William Morris Davis	Cambridge
Benjamin Kendall Emerson	Amherst
Oliver Whipple Huntington	Newport
Robert Tracy Jackson	Cambridge
Thomas Augustus Jaggar, Jr.	Brookline
Douglas Wilson Johnson	Cambridge
Charles Palache	Cambridge
John Elliott Pillsbury	Washington
Robert DeCourcy Ward	Cambridge
Charles Hyde Warren	Auburndale
John Eliot Wolff	Cambridge
Jay Backus Woodworth	Cambridge

Section II. — *Botany.* — 11.

Frank Shipley Collins	Malden
William Gilson Farlow	Cambridge
Charles Edward Faxon	Jamaica Plain
Merritt Lyndon Fernald	Cambridge
George Lincoln Goodale	Cambridge
John George Jack	Jamaica Plain
Edward Charles Jeffrey	Cambridge
Benjamin Lincoln Robinson	Cambridge
Charles Sprague Sargent	Brookline
Arthur Bliss Seymour	Cambridge
Roland Thaxter	Cambridge

Section III. — *Zoölogy and Physiology.* — 23.

Robert Amory	Boston
Francis Gano Benedict	Boston
Henry Pickering Bowditch	Jamaica Plain
William Brewster	Cambridge
Louis Cabot	Brookline
Walter Bradford Cannon	Cambridge
William Ernest Castle	Cambridge
Samuel Fessenden Clarke	Williamstown
William Thomas Councilman	Boston
Harold Clarence Ernst	Jamaica Plain
Samuel Henshaw	Cambridge
Edward Laurens Mark	Cambridge
Charles Sedgwick Minot	Milton
Edward Sylvester Morse	Salem
George Howard Parker	Cambridge
James Jackson Putnam	Boston
Herbert Wilbur Rand	Cambridge
Samuel Hubbard Scudder	Cambridge
William Thompson Sedgwick	Boston
William Morton Wheeler	Boston
James Clarke White	Boston
Harris Hawthorne Wilder	Northampton
William McMichael Woodworth	Cambridge

Section IV. — *Medicine and Surgery.*—10.

Edward Hickling Bradford	Boston
Arthur Tracy Cabot	Boston
Reginald Heber Fitz	Boston

Samuel Jason Mixter	Boston
William Lambert Richardson	Boston
Theobald Smith	Jamaica Plain
Oliver Fairfield Wadsworth	Boston
Henry Pickering Walcott	Cambridge
John Collins Warren	Boston
Francis Henry Williams	Boston

CLASS III. — *Moral and Political Sciences.* — 56.

SECTION I. — *Philosophy and Jurisprudence.* — 7.

Joseph Henry Beale	Cambridge
John Chipman Gray	Boston
Francis Cabot Lowell	Boston
Hugo Münsterberg	Cambridge
Josiah Royce	Cambridge
Frederic Jesup Stimson	Dedham
Samuel Williston	Belmont

SECTION II. — *Philology and Archæology.* — 19.

Charles Pickering Bowditch	Jamaica Plain
Lucien Carr	Cambridge
Franklin Carter	New Haven
Roland Burrage Dixon	Cambridge
Jesse Walter Fewkes	Washington
William Watson Goodwin	Cambridge
Henry Williamson Haynes	Boston
Albert Andrew Howard	Cambridge
Charles Rockwell Lanman	Cambridge
David Gordon Lyon	Cambridge
Clifford Herschel Moore	Cambridge
George Foot Moore	Cambridge
Charles Pomeroy Parker	Cambridge
Frederick Ward Putnam	Cambridge
Edward Robinson	New York
Edward Stevens Sheldon	Cambridge
Herbert Weir Smyth	Cambridge
Franklin Bache Stephenson	Boston
John Williams White	Cambridge

SECTION III. — *Political Economy and History.* — 12.

Charles Francis Adams	Lincoln
Thomas Nixon Carver	Cambridge
Archibald Cary Coolidge	Boston
Andrew McFarland Davis	Cambridge
Ephraim Emerton	Cambridge
Worthington Chauncey Ford	Boston
Abner Cheney Goodell	Salem
Henry Cabot Lodge	Nahant
Abbott Lawrence Lowell	Cambridge
James Ford Rhodes	Boston
Charles Card Smith	Boston
Frank William Taussig	Cambridge

SECTION IV. — *Literature and the Fine Arts.* — 18.

Francis Bartlett	Boston
Arlo Bates	Boston
Le Baron Russell Briggs	Cambridge
Henry Herbert Edes	Cambridge
Arthur Fairbanks	Boston
William Wallace Fenn	Cambridge
Kuno Francke	Cambridge
Edward Henry Hall	Cambridge
Thomas Wentworth Higginson	Cambridge
George Lyman Kittredge	Cambridge
Gardiner Martin Lane	Boston
William Coolidge Lane	Cambridge
Edward Caldwell Moore	Cambridge
James Hardy Ropes	Cambridge
Denman Waldo Ross	Cambridge
William Robert Ware	Milton
Herbert Langford Warren	Cambridge
Barrett Wendell	Boston

ASSOCIATE FELLOWS. — 80.

(Number limited to one hundred.)

Class I. — *Mathematical and Physical Sciences.* — 31.

Section I. — *Mathematics and Astronomy.* — 12.

Edward Emerson Barnard	Williams Bay, Wis.
Sherburne Wesley Burnham	Williams Bay, Wis.
George Davidson	San Francisco
Fabian Franklin	Baltimore
George William Hill	West Nyack, N. Y.
Edward Singleton Holden	West Point
Emory McClintock	Morristown, N. J.
Eliakim Hastings Moore	Chicago
Charles Lane Poor	New York
George Mary Searle	Washington
Vesto Melvin Slipher	Flagstaff, Ariz.
John Nelson Stockwell	Cleveland

Section II. — *Physics.* — 6.

Carl Barus	Providence
George Ellery Hale	Pasadena, Cal.
Thomas Corwin Mendenhall	Worcester
Albert Abraham Michelson	Chicago
Edward Leamington Nichols	Ithaca
Michael Idvorsky Pupin	New York

Section III. — *Chemistry.* — 7.

Frank Austin Gooch	New Haven
Eugene Waldemar Hilgard	Berkeley
John William Mallet	Charlottesville, Va.
Edward Williams Morley	West Hartford, Conn.
Charles Edward Munroe	Washington
John Ulric Nef	Chicago
Ira Remsen	Baltimore

SECTION IV. — *Technology and Engineering.* — 6.

Henry Larcom Abbot	Cambridge
Cyrus Ballou Comstock	New York
William Price Craighill	Charlestown, W. Va.
John Fritz	Bethlehem, Pa.
Frederick Remsen Hutton	New York
Robert Simpson Woodward	New York

CLASS II. — *Natural and Physiological Sciences.* — 31.

SECTION I. — *Geology, Mineralogy, and Physics of the Globe.* — 9.

Cleveland Abbe	Washington
George Jarvis Brush	New Haven
Thomas Chrowder Chamberlin	Chicago
Edward Salisbury Dana	New Haven
Walter Gould Davis	Cordova, Arg.
Samuel Franklin Emmons	Washington
Grove Karl Gilbert	Washington
Raphael Pumpelly	Newport
Charles Doolittle Walcott	Washington

SECTION II. — *Botany.* — 6.

Liberty Hyde Bailey	Ithaca
Douglas Houghton Campbell	Palo Alto
John Merle Coulter	Chicago
Cyrus Guernsey Pringle	Charlotte, Vt.
John Donnell Smith	Baltimore
William Trelease	St. Louis

SECTION III. — *Zoölogy and Physiology.* — 8.

Joel Asaph Allen	New York
Charles Benedict Davenport	Cold Spring Harbor, N. Y.
Franklin Paine Mall	Baltimore
Silas Weir Mitchell	Philadelphia
Henry Fairfield Osborn	New York
Addison Emory Verrill	New Haven
Charles Otis Whitman	Chicago
Edmund Beecher Wilson	New York

SECTION IV. — *Medicine and Surgery.* — 8.

John Shaw Billings	New York
William Stewart Halsted	Baltimore
Abraham Jacobi	New York
William Williams Keen	Philadelphia
William Osler	Oxford
Theophil Mitchell Prudden	New York
William Henry Welch	Baltimore
Horatio Curtis Wood	Philadelphia

CLASS III. — *Moral and Political Sciences.* — 18.

SECTION I. — *Philosophy and Jurisprudence.* — 4.

Joseph Hodges Choate	New York
William Wirt Howe	New Orleans
Charles Sanders Peirce	Milford, Pa.
George Wharton Pepper	Philadelphia

SECTION II. — *Philology and Archæology.* — 6.

Timothy Dwight	New Haven
Basil Lanneau Gildersleeve	Baltimore
William Arthur Heidel	Middletown
Thomas Raynesford Lounsbury	New Haven
Rufus Byam Richardson	New York
Andrew Dickson White	Ithaca

SECTION III. — *Political Economy and History.* — 4.

Henry Adams	Washington
Arthur Twining Hadley	New Haven
Alfred Thayer Mahan	New York
Henry Morse Stephens	Berkeley

SECTION IV. — *Literature and the Fine Arts.* — 4.

James Burrill Angell	Ann Arbor
Horace Howard Furness	Wallingford, Pa.
Herbert Putnam	Washington
John Singer Sargent	London

FOREIGN HONORARY MEMBERS.—61.

(Number limited to seventy-five.)

Class I. — *Mathematical and Physical Sciences.* — 18.

Section I. — *Mathematics and Astronomy.* — 7.

Arthur Auwers	Berlin
Sir George Howard Darwin	Cambridge
Sir David Gill	London
Sir William Huggins	London
Felix Klein	Göttingen
Émile Picard	Paris
Jules Henri Poincaré	Paris

Section II. — *Physics.* — 4.

Oliver Heaviside	Torquay
Joseph Larmor	Cambridge
John William Strutt, Baron Rayleigh	Witham
Sir Joseph John Thomson	Cambridge

Section III. — *Chemistry.* — 5.

Adolf, Ritter von Baeyer	Munich
Emil Fischer	Berlin
Jacobus Henricus van't Hoff	Berlin
Wilhelm Ostwald	Leipsic
Sir Henry Enfield Roscoe	London

Section IV. — *Technology and Engineering.* — 3.

Maurice Lévy	Paris
Heinrich Müller-Breslau	Berlin
William Cawthorne Unwin	London

Class II.— *Natural and Physiological Sciences.* — 22.

Section I. — *Geology, Mineralogy, and Physics of the Globe.* — 4.

Sir Archibald Geikie	London
Julius Hann	Vienna
Albert Heim	Zurich
Sir John Murray	Edinburgh

SECTION II. — *Botany*. — 6.

Jean Baptiste Edouard Bornet	Paris
Adolf Engler	Berlin
Sir Joseph Dalton Hooker	Sunningdale
Wilhelm Pfeffer	Leipsic
Hermann, Graf zu Solms-Laubach	Strassburg
Eduard Strasburger	Bonn

SECTION III. — *Zoölogy and Physiology*. — 5.

Ludimar Hermann	Königsberg
Hugo Kronecker	Bern
Sir Edwin Ray Lankester	London
Elias Metschnikoff	Paris
Magnus Gustav Retzius	Stockholm

SECTION IV. — *Medicine and Surgery*. — 7.

Emil von Behring	Marburg
Sir Thomas Lauder Brunton, Bart.	London
Angelo Celli	Rome
Sir Victor Alexander Haden Horsley	London
Robert Koch	Berlin
Joseph Lister, Baron Lister	London
Friedrich von Recklinghausen	Strassburg

CLASS III. — *Moral and Political Sciences*. — 20.

SECTION I. — *Philosophy and Jurisprudence*. — 4.

Arthur James Balfour	Prestonkirk
Heinrich Brunner	Berlin
Albert Venn Dicey	Oxford
Sir Frederick Pollock, Bart.	London

SECTION II. — *Philology and Archæology*. — 7.

Ingram Bywater	London
Friedrich Delitzsch	Berlin
Hermann Diels	Berlin
Wilhelm Dörpfeld	Athens
Henry Jackson	Cambridge
Hermann Georg Jacobi	Bonn
Gaston Camille Charles Maspero	Paris

SECTION III. — *Political Economy and History.* — 5.

James Bryce	London
Adolf Harnack	Berlin
John Morley, Viscount Morley of Blackburn	London
Sir George Otto Trevelyan, Bart.	London
Pasquale Villari	Florence

SECTION IV. — *Literature and the Fine Arts.* — 4.

Georg Brandes	Copenhagen
Samuel Henry Butcher	London
Jean Léon Gérôme	Paris
Rudyard Kipling	Burwash

STATUTES AND STANDING VOTES.

STATUTES.

Adopted May 30, 1854: *amended September* 8, 1857, *November* 12, 1862, *May* 24, 1864, *November* 9, 1870, *May* 27, 1873, *January* 26, 1876, *June* 16, 1886, *October* 8, 1890, *January* 11, *and May* 10, 1893, *May* 9, *and October* 10, 1894, *March* 13, *April* 10, *and May* 8, 1895, *May* 8, 1901, *January* 8, 1902, *May* 10, 1905, *February* 14 *and March* 14, 1906, *January* 13, 1909.

CHAPTER I.

Of Fellows and Foreign Honorary Members.

1. The Academy consists of Resident Fellows, Associate Fellows, and Foreign Honorary Members. They are arranged in three Classes, according to the Arts and Sciences in which they are severally proficient, viz.: Class I. The Mathematical and Physical Sciences; — Class II. The Natural and Physiological Sciences; — Class III. The Moral and Political Sciences. Each Class is divided into four Sections, viz.: Class I., Section 1. Mathematics and Astronomy; — Section 2. Physics; — Section 3. Chemistry; — Section 4. Technology and Engineering. Class II., Section 1. Geology, Mineralogy, and Physics of the Globe; — Section 2. Botany; Section 3. Zoölogy and Physiology; — Section 4. Medicine and Surgery. Class III., Section 1. Theology, Philosophy, and Jurisprudence; — Section 2. Philology and Archæology; — Section 3. Political Economy and History; — Section 4. Literature and the Fine Arts.

2. The number of Resident Fellows residing in the Commonwealth of Massachusetts shall not exceed two hundred, of whom there shall not be more than eighty in any one of the three classes. Only residents in the Commonwealth of Massachusetts shall be eligible to election as Resident Fellows, but resident fellowship may be retained after removal from

the Commonwealth. Each Resident Fellow shall pay an admission fee of ten dollars and such annual assessment, not exceeding ten dollars, as shall be voted by the Academy at each annual meeting. Resident Fellows only may vote at the meetings of the Academy.

3. The number of Associate Fellows shall not exceed one hundred, of whom there shall not be more than forty in either of the three classes of the Academy. Associate Fellows shall be chosen from persons residing outside of the Commonwealth of Massachusetts. They shall not be liable to the payment of any fees or annual dues, but on removing within the Commonwealth they may be transferred by the Council to resident fellowship as vacancies there occur.

4. The number of Foreign Honorary Members shall not exceed seventy-five; and they shall be chosen from among persons most eminent in foreign countries for their discoveries and attainments in either of the three departments of knowledge above enumerated. There shall not be more than thirty Foreign Members in either of these departments.

CHAPTER II.

Of Officers.

1. There shall be a President, three Vice-Presidents, one for each Class, a Corresponding Secretary, a Recording Secretary, a Treasurer, and a Librarian, which officers shall be annually elected, by ballot, at the annual meeting, on the second Wednesday in May.

2. There shall be nine Councillors, chosen from the Resident Fellows. At each annual meeting, three Councillors shall be chosen, by ballot, one from each Class, to serve for three years; but the same Fellow shall not be eligible for two successive terms. The nine Councillors, with the President, the three Vice-Presidents, the two Secretaries, the Treasurer, and the Librarian, shall constitute the Council. Five members shall constitute a quorum. It shall be the duty of this Council to exercise a discreet supervision over all nominations and elections. With the consent of the Fellow interested, they shall have power to make transfers between the several sections of the same Class, reporting their action to the Academy.

3. The Council shall at its March Meeting receive reports from the Rumford Committee, the C. M. Warren Committee, the Committee on Publication, the Committee on the Library, the President and Record-

ing Secretary, and the Treasurer, proposing the appropriations for their work during the year beginning the following May. The Treasurer at the same meeting shall report on the income which will probably be received on account of the various Funds during the same year.

At the Annual Meeting, the Council shall submit to the Academy, for its action, a report recommending the appropriations which in the opinion of the Council should be made for the various purposes of the Academy.

4. If any office shall become vacant during the year, the vacancy shall be filled by a new election, at the next stated meeting, or at a meeting called for this purpose.

CHAPTER III.

OF NOMINATIONS OF OFFICERS.

1. At the stated meeting in March, the President shall appoint a Nominating Committee of three Resident Fellows, one for each Class.

2. It shall be the duty of this Nominating Committee to prepare a list of candidates for the offices of President, Vice-Presidents, Corresponding Secretary, Recording Secretary, Treasurer, Librarian, Councillors, and the Standing Committees which are chosen by ballot; and to cause this list to be sent by mail to all the Resident Fellows of the Academy not later than four weeks before the Annual Meeting.

3. Independent nominations for any office, signed by at least five Resident Fellows, and received by the Recording Secretary not less than ten days before the Annual Meeting, shall be inserted in the call for the Annual Meeting, which shall then be issued not later than one week before that meeting.

4. The Recording Secretary shall prepare for use, in voting at the Annual Meeting, a ballot containing the names of all persons nominated for office under the conditions given above.

5. When an office is to be filled at any other time than at the Annual Meeting, the President shall appoint a Nominating Committee in accordance with the provisions of Section 1, which shall announce its nomination in the manner prescribed in Section 2 at least two weeks before the time of election. Independent nominations, signed by at least five Resident Fellows and received by the Recording Secretary not later than one week before the meeting for election, shall be inserted in the call for that meeting.

CHAPTER IV.

OF THE PRESIDENT.

1. It shall be the duty of the President, and, in his absence, of the senior Vice-President present, or next officer in order as above enumerated, to preside at the meetings of the Academy; to direct the Recording Secretary to call special meetings; and to execute or to see to the execution of the Statutes of the Academy. Length of continuous membership in the Academy shall determine the seniority of the Vice-Presidents.

2. The President, or, in his absence, the next officer as above enumerated, shall nominate members to serve on the different committees of the Academy which are not chosen by ballot.

3. Any deed or writing to which the common seal is to be affixed shall be signed and sealed by the President, when thereto authorized by the Academy.

CHAPTER V.

OF STANDING COMMITTEES.

1. At the Annual Meeting there shall be chosen the following Standing Committees, to serve for the year ensuing, viz.: —

2. The Committee on Finance to consist of three Fellows to be chosen by ballot, who shall have, through the Treasurer, full control and management of the funds and trusts of the Academy, with the power of investing and of changing the investment of the same at their discretion.

3. The Rumford Committee, to consist of seven Fellows to be chosen by ballot, who shall consider and report to the Academy on all applications and claims for the Rumford premium. They shall also report to the Council in March of each year on all appropriations of the income of the Rumford Fund needed for the coming year, and shall generally see to the due and proper execution of the trust. All bills incurred on account of the Rumford Fund, within the limits of the appropriation made by the Academy, shall be approved by the Chairman of the Rumford Committee.

4. The C. M. Warren Committee, to consist of seven Fellows to be chosen by ballot, who shall consider and report to the Council in March of each year on all applications for appropriations from the income of the C. M. Warren Fund for the coming year, and shall generally see to the due

and proper execution of the trust. All bills incurred on account of the C. M. Warren Fund, within the limits of the appropriations made by the Academy, shall be approved by the Chairman of the C. M. Warren Committee.

5. The Committee on Publication, to consist of three Fellows, one from each class, to whom all communications submitted to the Academy for publication shall be referred, and to whom the printing of the Proceedings and Memoirs shall be entrusted. This Committee shall report to the Council in March of each year on the appropriations needed for the coming year. All bills incurred on account of publications, within the limits of the appropriations made by the Academy, shall be approved by the Chairman of the Committee on Publication.

6. The Committee on the Library, to consist of the Librarian *ex officio,* and three other Fellows, one from each class, who shall examine the Library and make an annual report on its condition and management. This Committee, through the Librarian, shall report to the Council in March of each year, on the appropriations needed for the Library for the coming year. All bills incurred on account of the Library, within the limits of the appropriations made by the Academy, shall be approved by the Librarian.

7. The House Committee to consist of three Fellows. This Committee shall have charge of all expenses connected with the House, including the general expenses of the Academy not specifically assigned to other Committees. This Committee shall report to the Council in March in each year on the appropriations needed for their expenses for the coming year. All bills incurred by this Committee within the limits of the appropriations made by the Academy shall be approved by the Chairman of the House Committee.

8. An auditing Committee, to consist of two Fellows, for auditing the accounts of the Treasurer, with power to employ an expert and to approve his bill.

9. In the absence of the Chairman of any Committee, bills may be approved by a member of the Committee designated by the Chairman for the purpose.

CHAPTER VI.

OF THE SECRETARIES.

1. The Corresponding Secretary shall conduct the correspondence of the Academy, recording or making an entry of all letters written in its name, and preserving on file all letters which are received; and at each

meeting he shall present the letters which have been addressed to the Academy since the last meeting. Under the direction of the Council, he shall keep a list of the Resident Fellows, Associate Fellows, and Foreign Honorary Members, arranged in their Classes and in Sections in respect to the special sciences in which they are severally proficient; and he shall act as secretary to the Council.

2. The Recording Secretary shall have charge of the Charter and Statute-book, journals, and all literary papers belonging to the Academy. He shall record the proceedings of the Academy at its meetings; and after each meeting is duly opened, he shall read the record of the preceding meeting. He shall notify the meetings of the Academy, apprise officers and committees of their election or appointment, and inform the Treasurer of appropriations of money voted by the Academy. He shall post up in the Hall a list of the persons nominated for election into the Academy; and when any individual is chosen, he shall insert in the record the names of the Fellows by whom he was nominated.

3. The two Secretaries, with the Chairman of the Committee of Publication, shall have authority to publish such of the records of the meetings of the Academy as may seem to them calculated to promote its interests.

4. Every person taking any books, papers, or documents belonging to the Academy and in the custody of the Recording Secretary, shall give a receipt for the same to the Recording Secretary.

CHAPTER VII.

Of the Treasurer.

1. The Treasurer shall give such security for the trust reposed in him as the Academy shall require.

2. He shall receive all moneys due or payable to the Academy and all bequests and donations made to the Academy. He shall pay all bills due by the Academy, when approved by the proper officers (except those of the Treasurer's office, which may be paid without such approval). He shall sign all leases of real estate in the name of the Academy. All transfers of stocks, bonds, and other securities belonging to the Academy shall be made by the Treasurer with the written consent of one member of the Committee of Finance. He shall keep an account of all receipts and expenditures, shall submit his accounts annually to the Auditing

Committee, and shall report the same at the expiration of his term of office or whenever called on so to do by the Academy or Council.

3. The Treasurer shall keep separate accounts of the income and appropriation of the Rumford Fund and of other special funds, and report the same annually.

4. The Treasurer may appoint an Assistant Treasurer to perform his duties, for whose acts, as such assistant, the Treasurer shall be responsible; or the Treasurer may employ any Trust Company, doing business in Boston, as agent to perform his duties, the compensation of such Assistant Treasurer or agent to be paid from the funds of the Academy.

CHAPTER VIII.

Of the Librarian and Library.

1. It shall be the duty of the Librarian to take charge of the books, to keep a correct catalogue of them, to provide for the delivery of books from the Library, and to appoint such agents for these purposes as he may think necessary. He shall make an annual report on the condition of the Library.

2. The Librarian, in conjunction with the Committee on the Library, shall have authority to expend such sums as may be appropriated, either from the General, Rumford, or other special Funds of the Academy, for the purchase of books, periodicals, etc., and for defraying other necessary expenses connected with the Library.

3. To all books in the Library procured from the income of the Rumford Fund, or other special funds, the Librarian shall cause a stamp or label to be affixed, expressing the fact that they were so procured.

4. Every person who takes a book from the Library shall give a receipt for the same to the Librarian or his assistant.

5. Every book shall be returned in good order, regard being had to the necessary wear of the book with good usage. If any book shall be lost or injured, the person to whom it stands charged shall replace it by a new volume or set, if it belongs to a set, or pay the current price of the volume or set to the Librarian; and thereupon the remainder of the set, if the volume belonged to a set, shall be delivered to the person so paying for the same.

6. All books shall be returned to the Library for examination at least one week before the Annual Meeting.

7. The Librarian shall have custody of the Publications of the Academy. With the advice and consent of the President, he may effect exchanges with other associations.

CHAPTER IX.

Of Meetings.

1. There shall be annually four stated meetings of the Academy; namely, on the second Wednesday in May (the Annual Meeting), on the second Wednesday in October, on the second Wednesday in January, and on the second Wednesday in March. At these meetings, only, or at meetings adjourned from these and regularly notified, or at special meetings called for the purpose, shall appropriations of money be made, or alterations of the statutes or standing votes of the Academy be effected.

Special meetings shall be called by the Recording Secretary at the request of the President or of a Vice-President or of five Fellows. Notifications of the special meetings shall contain a statement of the purpose for which the meeting is called.

2. Fifteen Resident Fellows shall constitute a quorum for the transaction of business at a stated or special meeting. Seven Fellows shall be sufficient to constitute a meeting for scientific communications and discussions.

3. The Recording Secretary shall notify the meetings of the Academy to each Resident Fellow; and he may cause the meetings to be advertised, whenever he deems such further notice to be needful.

CHAPTER X.

Of the Election of Fellows and Honorary Members.

1. Elections shall be made by ballot, and only at stated meetings.

2. Candidates for election as Resident Fellows must be proposed by two Resident Fellows of the section to which the proposal is made, in a recommendation signed by them; and this recommendation shall be transmitted to the Corresponding Secretary, and by him referred to the Council. No person recommended shall be reported by the Council as a

candidate for election, unless he shall have received the approval of at least five members of the Council present at a meeting. All nominations thus approved shall be read to the Academy at any meeting, and shall then stand on the nomination list until the next stated meeting, and until the balloting. No person shall be elected a Resident Fellow, unless he shall have been resident in this Commonwealth one year next preceding his election. If any person elected a Resident Fellow shall neglect for one year to pay his admission fee, his election shall be void; and if any Resident Fellow shall neglect to pay his annual assessments for two years, provided that his attention shall have been called to this article, he shall be deemed to have abandoned his Fellowship; but it shall be in the power of the Treasurer, with the consent of the Council, to dispense (*sub silentio*) with the payment both of the admission fee and of the assessments, whenever in any special instance he shall think it advisable so to do. In the case of officers of the Army or Navy who are out of the state on duty, payment of the annual assessment may be waived during such absence if continued during the whole official year and if notification of such absence be sent to the Treasurer.

3. The nomination and election of Associate Fellows shall take place in the manner prescribed in reference to Resident Fellows.

4. The nomination and election of Foreign Honorary Members shall take place in the manner prescribed for Resident Fellows, except that the nomination papers shall be signed by at least seven members of the Council before being presented to the Academy.

5. Three-fourths of the ballots cast must be affirmative, and the number of affirmative ballots must amount to eleven to effect an election of Fellows or Foreign Honorary Members.

6. If, in the opinion of a majority of the entire Council, any Fellow — Resident or Associate — shall have rendered himself unworthy of a place in the Academy, the Council shall recommend to the Academy the termination of his Fellowship; and provided that a majority of two-thirds of the Fellows at a stated meeting, consisting of not less than fifty Fellows, shall adopt this recommendation, his name shall be stricken off the roll of Fellows.

CHAPTER XI.

OF AMENDMENTS OF THE STATUTES.

1. All proposed alterations of the Statutes, or additions to them, shall be referred to a committee, and, on their report at a subsequent stated meeting or a special meeting called for the purpose, shall require for

enactment a majority of two-thirds of the members present, and at least eighteen affirmative votes.

2. Standing votes may be passed, amended, or rescinded at a stated meeting, or a special meeting called for the purpose by a majority of two-thirds of the members present. They may be suspended by a unanimous vote.

CHAPTER XII.

Of Literary Performances.

1. The Academy will not express its judgment on literary or scientific memoirs or performances submitted to it, or included in its publications.

STANDING VOTES.

1. Communications of which notice has been given to the Secretary shall take precedence of those not so notified.

2. Associate Fellows, Foreign Honorary Members, and Resident Fellows, who have paid all fees and dues chargeable to them, are entitled to receive one copy of each volume or article printed by the Academy on application to the Librarian personally or by written order within two years of the date of publication. Exceptions to this rule may be made in special cases by vote of the Academy.

3. The Committee of Publication shall fix from time to time the price at which the publications of the Academy may be sold. But members may be supplied at half this price with volumes which they are not entitled to receive free, and which are needed to complete their sets.

4. Two hundred extra copies of each paper accepted for publication in the Memoirs or Proceedings of the Academy shall be placed at the disposal of the author, free of charge.

5. Resident Fellows may borrow and have out from the Library six volumes at any one time, and may retain the same for three months, and no longer.

6. Upon special application, and for adequate reasons assigned, the Librarian may permit a larger number of volumes, not exceeding twelve, to be drawn from the Library for a limited period.

7. Works published in numbers, when unbound, shall not be taken from the Hall of the Academy, except by special leave of the Librarian.

8. Books, publications, or apparatus shall be procured from the income of the Rumford Fund only on the certificate of the Rumford Committee that they, in their opinion, will best facilitate and encourage the making of discoveries and improvements which may merit the Rumford Premium; and the approval of a bill incurred for such purposes by the Chairman shall be accepted by the Treasurer as proof that such certificate has been given.

9. A meeting for receiving and discussing scientific communications may be held on the second Wednesday of each month not appointed for stated meetings, excepting July, August, and September.

10. No report of any paper presented at a meeting of the Academy shall be published by any member without the consent of the author, and no report shall in any case be published by any member in a newspaper as an account of the proceedings of the Academy.

RUMFORD PREMIUM.

In conformity with the terms of the gift of Benjamin, Count Rumford, granting a certain fund to the American Academy of Arts and Sciences, and with a decree of the Supreme Judicial Court for carrying into effect the general charitable intent and purpose of Count Rumford, as expressed in his letter of gift, the Academy is empowered to make from the income of said fund, as it now exists, at any Annual Meeting, an award of a gold and a silver medal, being together of the intrinsic value of three hundred dollars, as a premium to the author of any important discovery or useful improvement in light or in heat, which shall have been made and published by printing, or in any way made known to the public, in any part of the continent of America, or any of the American islands; preference being always given to such discoveries as shall, in the opinion of the Academy, tend most to promote the good of mankind; and to add to such medals, as a further premium for such discovery and improvement, if the Academy see fit so to do, a sum of money not exceeding three hundred dollars.

INDEX.

2583